SEQUENCE DESIGN WITH APPLICATIONS TO ULTRA-WIDEBAND COMMUNICATION SYSTEMS

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ABSTRACT OF THE DISSERTATION

Sequence Design with Applications to Ultra-Wideband Communication Systems

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To enable high data rate target applications in a typical ultra-wideband (UWB) dense multipath channel, DS-UWB signaling suffers from a severe inter-symbol-interference (ISI) and multiple access interference (MAI). In an attempt to avoid complexity at the receiver, we tackle these problems from the transmitter side and consider ternary sequence based UWB (TS-UWB) signaling as a unifying descriptor of a number of impulse-based UWB schemes.

We propose a technique which employs ternary complementary sets to improve the performance of orthogonal pulse based multi-channel UWB systems. By assigning mutually orthogonal (MO) complementary sets to the users, both MAI and multipath interference can be significantly suppressed. One of the major impediments to deploying such systems is a high peak-to-average power ratio (PAPR). We show that PAPR can be upper bounded by an autocorrelation merit function of column sequences of the corresponding complementary set matrix. Hence, we propose a design of complementary sets satisfying a column correlation constraint. The design algorithm recursively builds a collection of MO complementary set matrices starting from a companion pair of sequences. We relate correlation properties of column sequences to that of the companion pair and illustrate how to select an appropriate companion pair to ensure that a given column correlation

constraint is satisfied.

To support high-speed and multirate data services, we construct ternary orthogonal variable spreading factor (OVSF) code with a zero-correlation zone (ZCZ). Compared to the conventional Walsh code based OVSF code, the proposed ZCZ-OVSF code maintains its code orthogonality even in a multipath channel or when synchronism is lacking and, thus, is suitable for quasi-synchronous (QS) TS-UWB systems. We further propose an adaptive data rate transmission scheme which maximizes the uplink aggregate throughput of the system by allocating OVSF codes to the active users based on their signal to interference and noise ratios (SINR).

Finally, we propose an adaptive transmission approach that suggests selecting a ternary beamforming sequence for a TS-UWB signal based on the signs of the reflection coefficients corresponding to a few strongest paths so that the signal energy at the output of the simple non-adaptive receiver is enhanced.

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List of Abbreviations

1D	One Dimensional
2D	Two Dimensional
3G	Third Generation
ACF	Autocorrelation Function
BCP	Binary Complementary Pair
BKP	Binary Knapsack Problem
CCF	Crosscorrelation Function
CDMA	Code Division Multiple Access
DS	Direct Sequence
FCC	Federal Communications Commission
FHSS	Frequency Hopping Spread Spectrum
IMT	International Mobile Telecommunication
ISI	Inter Symbol Interference
LOS	Line of Sight
MAI	Multiple Access Interference
MC	Multicarrier
MF	Matched Filter
MO	Mutually Orthogonal
NLOS	Nonline of Sight
OFDM	Orthogonal Frequency Division Multiplexing
OVSF	Orthogonal Variable Spreading Factor
PAPR	Peak to Average Power Ratio
PHY	Physical Layer
QS	Quasi-Synchronous
SINR	Signal to Interference and Noise Ratio
TCP	Ternary Complementary Pair
TS	Ternary Sequence
UMTS	Universal Mobile Telecommunications System
UWB	Ultra Wideband
WCDMA	Wideband Code Division Multiple Access
WPAN	Wireless Personal Area Network
ZCZ	Zero-Correlation Zone

Chapter 1

Introduction

1.1 Background and Motivation

Ultra-wideband (UWB) is defined as any radio technology having a spectrum that occupies a bandwidth greater than 20 percent of the center frequency, or a bandwidth of at least 500 MHz. By its rulemaking proposal in 2002, the Federal Communications Commission (FCC) has mandated that UWB radio transmissions can legally operate in the range from 3.1 GHz up to 10.6 GHz at a low transmit power. To support high data-rate short-range wireless personal area network (WPAN), UWB signaling is selected to provide a higher speed physical layer (PHY) enhancement to IEEE 802.15.3 standard for applications which involve image and multimedia transmission. The large bandwidth of UWB waveforms significantly improve the channel capacity, at the same time, bring the following main challenges and problems in signaling design for direct sequence based UWB (DS-UWB) communications.

- *Interference:* At distances between 4-10 m, the typical nonline-of-sight (NLOS) UWB channel environment has an rms delay spread of 14 ns, while the worst case channel environment has an rms delay spread of 25 ns [1]. The large bandwidth of UWB waveforms increase the ability of the receiver to resolve multipath reflections. Hence, in a UWB dense multipath channel, DS-CDMA UWB signaling suffers from severe multipath and multiple access interference (MAI).

- *High Speed:* Shorter length spreading sequences can be selected to support higher data rate. However, the inter-symbol interference (ISI) increases and degrades the system performance. Multicode or multicarrier scheme can be an alternate solution. One of the main challenges involved in the design of these types of signaling is the peak-to-average power ratio (PAPR) reduction.

- *Low Cost:* The performance of a DS-UWB system is often determined by the amount of multipath energy that can be collected at the receiver. A large number of RAKE fingers are needed in order to sufficiently capture multipath energy. To mitigate the effect of ISI caused by the time-dispersive nature of UWB channel, an equalizer can be used to improve system performance. However, all these complicate the receiver and raise the cost.

In an attempt to avoid introducing complexities at the receiver, we tackle the above problems from the transmitter side with an emphasis placed on sequence design for improved system performance.

A sequence is a discrete-time complex-valued signal in which the order of its elements is well defined and significant. The order of elements determines correlation functions of the sequence. Communication devices typically employ matched filter (MF) receivers whose output is characterized by the autocorrelation function (ACF) of the target sequence and crosscorrelation functions (CCF) of the target sequence with other sequences forming the input signal. Receiver performance requires that the magnitude of out-of-phase ACF and the magnitude of CCF are as small as possible. Sequences with low out-of-phase ACFs have widespread applications in parameter identification, synchronization, timing estimation, and pulse compression radar system. The direct sequence code-division multiple-access (DS-CDMA) systems and frequency-hopping spread-spectrum (FHSS) systems require sequence sets with small CCF [2, 3]. Signals with good correlation properties are of special importance for high rate and low power UWB communications for several reasons. In particular, synchronization efficiency, multipath resolution, ISI suppression, and multiple access interference suppression can be significantly degraded when the number of chips per symbol and the corresponding correlation sidelobe suppression are reduced.

Motivated by the fact that ternary signaling with epochs of zero (power off) amplitude is easy to implement with impulse radios, we study ternary sequences with elements in $\{+1, 0, -1\}$ including binary sequences as a special case. We suggest ternary sequence based UWB (TS-UWB) signaling [4–11] as a unifying descriptor of a number of impulse-based UWB signaling schemes. Allowing some of the chips to be zero enables significant improvement in the correlation properties of the employed signaling.

We study both ternary complementary sets of sequences [6, 8, 9] and ternary zero-correlation zone (ZCZ) sequences [7, 10, 11]. The former are two-dimensional (2D) sequences or sequence arrays and the latter are traditional one-dimensional (1D) sequences. Based on the well-known Welch bound [12], a non-trivial 1D sequence set may never satisfy the ideal correlation property [2], i.e., zero out-of-phase ACF and zero CCF values. However, a complementary 2D sequence set can achieve the ideal correlation property. Here, the correlation function is defined based on the sum of correlation functions of all 1D row sequences in the array. Complementary sets can be recursively constructed from a complementary pair [13]. It has been proved that the length of a binary complementary pair (BCP), i.e., a Golay pair [14], can only be $2^{\alpha}10^{\beta}26^{\gamma}$, where α, β, γ are non-negative integers [2]. In [15], Gavish and Lempel have shown that ternary complementary pairs (TCPs), which include BCPs as a special case, exist for any sequence length. Hence, ternary signaling may not only improve the correlation properties, but also provide more flexibility in choosing sequence lengths in the system design. Similar to multicarrier DS-CDMA signals [16], signaling employing 2D sequences typically suffers from a high PAPR which may limit its application or/and result in performance degradation. We propose the construction of complementary sets with optimum PAPR reduction.

Compared to 2D sequences, although 1D sequences can not achieve ideal correlation properties, their PAPR and hardware generation complexity are both smaller. We design ternary 1D sequence set whose correlation values equal to zero for a contiguous set of delays starting from a single delay. Thus, the proposed ZCZ sequences can suppress both multipath and multiuser interference.

Ternary complementary sets and ZCZ sequence sets provide sequence solutions for DS-UWB systems with no channel state information at the transmitter. Based on limited multipath channel information, we also propose a set of beamforming sequences which can coherently combine the energy at the output of a non-adaptive correlator receiver.

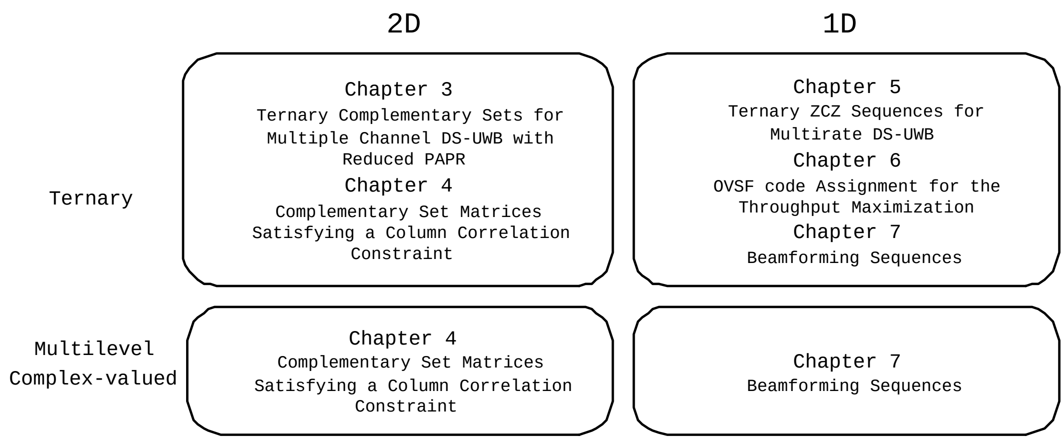

Figure 1.1: Classification of sequences

The sequences that we considered in this thesis are depicted in Figure 1.1. Ternary sequences are our main focus, nevertheless, we also extend our research to multilevel and complex-valued sequences.

1.2 Road Map

In this dissertation, we propose several sequence construction methods and attempt to solve sequence related issues for UWB communications.

In Chapter 2, we first define sequence correlations, merits and operations, followed by the introduction of several special sequences, i.e., perfect ternary sequences, ternary ZCZ sequences, OVSF codes and complementary sets. We also present three lemmas which can extend complementary sets from small to large sizes.

In Chapter 3, we propose a technique which employs ternary complementary sets for an orthogonal-pulse based multichannel DS-UWB system. By assigning MO complementary sets to different users, both MAI and multipath interference can be significantly suppressed. We derive an upper bound which shows that the column sequences of the complementary set with small out-off-phase aperiodic ACFs can lower PAPR of the system. Therefore, a construction method is proposed for the design of ternary complementary sets with reduced PAPR.

In Chapter 4, motivated by the problem of reducing PAPR of transmitted signals, we consider a design of complex-valued complementary set matrices whose column sequences satisfy a correlation constraint. The design algorithm recursively builds a collection of MO complementary set matrices starting from a companion pair of sequences. We relate correlation properties of column sequences to that of the companion pair and illustrate how to select an appropriate companion pair to ensure that a given column correlation constraint is satisfied. We also reveal a design of the companion pair which leads to complementary set matrices with Golay column sequences. By exploiting the well-known Welch bound, sufficient conditions for the existence of companion pairs which satisfy a set of column correlation constraints are also given.

In Chapter 5, we construct ternary ZCZ sequence sets with a both periodic and aperiodic ZCZ to suppress the multipath and multiuser interference. We further construct a tree-structure for ternary OVSF codes with a ZCZ of arbitrary lengths. ZCZ sequences have been studied in [17–22]. However, there are no previous studies of trees formed by codes of different spreading factors and a common ZCZ.

In Chapter 6, we focus on the uplink aggregate throughput maximization for a quasi-synchronous (QS) TS-UWB system. QS systems [23] may employ GPS receivers at the base station and mobiles to correct for propagation delays [24]. The relative time delay between signals of different users is random but can be maintained within a given interval [25]. We model the relative time delay Δ being upper bound as $|\Delta| \leq \tau$ (e.g., see [26]). Hence, the MAI is zero when the length of the constructed ZCZ is no less than the τ. The constructed ternary ZCZ-OVSF codes ensure that the multipath and MAI are zero [4, 7] and thus, improve the system throughput. We also propose an optimal code assignment which can maximize the aggregate throughput by allocating OVSF codes to the active users based on their SINRs.

In Chapter 7, in view of target high data rate applications and corresponding simple receiver design, our approach focuses on signal designs that enhance the performance of a non-adaptive receiver. We propose an adaptive transmission approach which suggests selecting a ternary sequence for a DS-UWB signal based on the signs of the reflection coefficients corresponding to a few strongest paths so that the signal energy at the output of the receiver is enhanced.

Chapter 2

Definitions and Preliminaries

Sequences are denoted by boldface lowercase letters (e.g., $\mathbf{x}$), their elements by corresponding lowercase letters with subscripts (x_0), boldface uppercase letters denote matrices ($\mathbf{X}$), and calligraphic letters denote either sets of numbers or sets of sequences ($\mathcal{X}$).

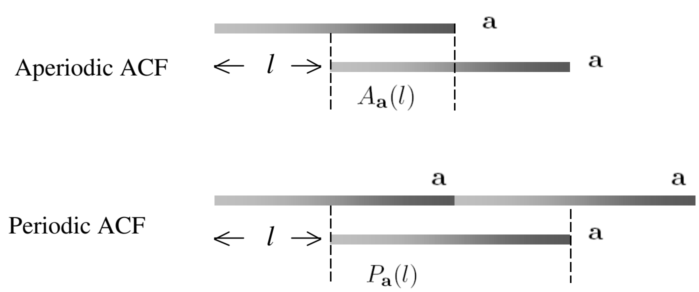

Figure 2.1: Aperiodic ACF and periodic ACF

2.1 Correlation Functions

Let $\mathbf{a} = (a_0, a_1, ..., a_{n-1})$ denote a sequence of length n with $a_i \in \mathcal{C}$, $0 \leq i \leq n-1$, where $\mathcal{C}$ is the set of complex numbers. As illustrated in Fig. 2.1, the aperiodic and periodic ACFs of $\mathbf{a}$ are

$$A_{\mathbf{a}}(l) = \sum_{i=0}^{n-1-l} a_i a_{i+l}^*, \quad 0 \leq l \leq n-1, \tag{2.1}$$

$$P_{\mathbf{a}}(l) = \sum_{i=0}^{n-1} a_i a_{i \oplus_n l}^*, \quad 0 \leq l \leq n-1, \tag{2.2}$$

where a_i^* denote the complex conjugate of a_i, and $\oplus_n$ denotes modulo-n addition. It follows that

$$P_{\mathbf{a}}(l) = A_{\mathbf{a}}(l) + A_{\mathbf{a}}(n-l), \quad 0 \leq l \leq n-1. \tag{2.3}$$

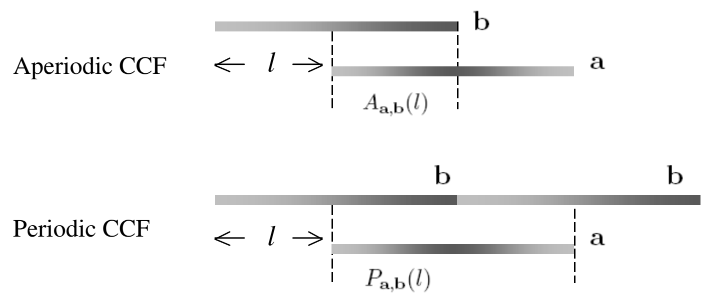

Figure 2.2: Aperiodic CCF and periodic CCF

Let $\mathbf{b} = (b_0, b_1, ..., b_{n-1})$, where $b_i \in \mathcal{C}$, $0 \leq i \leq n-1$. As shown in Fig. 2.2, the aperiodic and periodic crosscorrelation functions of $\mathbf{a}$ and $\mathbf{b}$ are defined, respectively, as,

$$A_{\mathbf{a},\mathbf{b}}(l) = \begin{cases} \sum_{i=0}^{n-1-l} a_i b_{i+l}^*, & 0 \leq l \leq n-1 \\ \sum_{i=0}^{n-1+l} a_{i-l} b_i^*, & 1-n \leq l < 0 \\ 0, & |l| \geq n \end{cases}, \tag{2.4}$$

$$P_{\mathbf{a},\mathbf{b}}(l) = \sum_{i=0}^{n-1} a_i b_{i \oplus_n l}^*, \quad 0 \leq l \leq n-1. \tag{2.5}$$

Table 2.1 lists the correlation function parameters which are commonly used to judge the merits of a sequence design, and are termed *correlation merits*. $\lambda_{\mathbf{a}}^A$, $\lambda_{\mathbf{a}}^P$, $S_{\mathbf{a}}^A$, and $S_{\mathbf{a}}^P$ are autocorrelation merits, and $\lambda_{\mathbf{a},\mathbf{b}}^A$, $\lambda_{\mathbf{a},\mathbf{b}}^P$, $S_{\mathbf{a},\mathbf{b}}^A$, and $S_{\mathbf{a},\mathbf{b}}^P$ are common crosscorrelation merits. For example, it is well known that binary m-sequences satisfy $\lambda_{\mathbf{a}}^P = 1$, and the sequences with $\lambda_{\mathbf{a}}^A \leq 1$ are called Barker sequences. Furthermore, a small value of $S_{\mathbf{a}}^A$ can significantly reduce the PAPR of OFDM signals, if the elements of $\mathbf{a}$ are assigned across all carriers [9, 27].

Table 2.1: Correlation Merits

$\lambda_{\mathbf{a}}^A$	$=$	$\max_l\{\lvert A_{\mathbf{a}}(l)\rvert, 1 \leq l \leq n-1\}$	$S_{\mathbf{a}}^A$	$=$	$\sum_{l=1}^{n-1} \lvert A_{\mathbf{a}}(l)\rvert$
$\lambda_{\mathbf{a}}^P$	$=$	$\max_l\{\lvert P_{\mathbf{a}}(l)\rvert, 1 \leq l \leq n-1\}$	$S_{\mathbf{a}}^P$	$=$	$\sum_{l=1}^{n-1} \lvert P_{\mathbf{a}}(l)\rvert$
$\lambda_{\mathbf{a},\mathbf{b}}^A$	$=$	$\max_l\{\lvert A_{\mathbf{a},\mathbf{b}}(l)\rvert, \ \lvert l \rvert \leq n-1\}$	$S_{\mathbf{a},\mathbf{b}}^A$	$=$	$\sum_{l=1-n}^{n-1} \lvert A_{\mathbf{a},\mathbf{b}}(l)\rvert$
$\lambda_{\mathbf{a},\mathbf{b}}^P$	$=$	$\max_l\{\lvert P_{\mathbf{a},\mathbf{b}}(l)\rvert, 0 \leq l \leq n-1\}$	$S_{\mathbf{a},\mathbf{b}}^P$	$=$	$\sum_{l=0}^{n-1} \lvert P_{\mathbf{a},\mathbf{b}}(l)\rvert$

Table 2.2: Sequence Operations

$\mathbf{a}^*$	$=$	$(a_0^*, a_1^*, ..., a_{n-1}^*)$
$-\mathbf{a}$	$=$	$(-a_0, -a_1, ..., -a_{n-1})$
$\overleftarrow{\mathbf{a}}$	$=$	$(a_{n-1}, a_{n-2}, ..., a_1, a_0)$
$\mathbf{ab}$	$=$	$(a_0, a_1, ..., a_{n-1}, b_0, b_1, ..., b_{n-1})$
$\mathbf{a} \otimes \mathbf{b}$	$=$	$(a_0, b_0, a_1, b_1, ..., a_{n-1}, b_{n-1})$
$\mathbf{a} \cdot \mathbf{b}$	$=$	$a_0 b_0 + a_1 b_1 + ... + a_{n-1} b_{n-1}$
$\mathbf{a} \odot \mathbf{b}$	$=$	$(a_0 b_0, a_0 b_1, ..., a_0 b_{n-1}, a_1 b_0, a_1 b_1, ..., a_1 b_{n-1}, ..., a_{n-1} b_{n-1})$
$f_i(\mathbf{a})$	$=$	$(a_1, -a_0, a_3, -a_2, ..., a_{n-1}, -a_{n-2})$
$f_c(\mathbf{a})$	$=$	$(a_{\frac{n}{2}}, a_{\frac{n}{2}+1}, ..., a_{n-1}, -a_0, -a_1, ..., -a_{\frac{n}{2}-1})$

2.2 Sequence and Matrix Operations

In Table 2.1, we list the following sequence operations: complex conjugation, negation, reversal, concatenation, interleaving, inner product and Kronecker product. Furthermore, we introduce two sequence reshaping functions $f_i(\cdot)$ and $f_c(\cdot)$ defined on sequences of even length.

2D sequence can be represented using a *matrix*

$$\mathbf{C} = \begin{bmatrix} \mathbf{r}_1 \\ \mathbf{r}_2 \\ \vdots \\ \mathbf{r}_m \end{bmatrix} = \begin{bmatrix} \mathbf{c}_0^T & \mathbf{c}_1^T & \cdots & \mathbf{c}_{n-1}^T \end{bmatrix} = \begin{bmatrix} c_{1,0} & c_{1,1} & \cdots & c_{1,n-1} \\ c_{2,0} & c_{2,1} & \cdots & c_{2,n-1} \\ \vdots & \vdots & \ddots & \vdots \\ c_{m,0} & c_{m,1} & \cdots & c_{m,n-1} \end{bmatrix}_{m \times n}, \qquad (2.6)$$

where *row sequences* $\mathbf{r}_i = (c_{i,0}, c_{i,1}, c_{i,2}, \cdots, c_{i,n-1})$, $1 \le i \le m$, and *column sequences*

$$\mathbf{c}_j^T = \begin{bmatrix} c_{1,j} \\ c_{2,j} \\ \vdots \\ c_{m,j} \end{bmatrix}, \quad 0 \le j \le n-1. \qquad (2.7)$$

Correlation functions and merits of $\mathbf{c}^T$ can be defined as the same as $\mathbf{c}$. Hence, for notation simplicity and without causing any confusion, column sequences are still denoted by a normal row vectors,

$$\mathbf{c}_j = \begin{bmatrix} c_{1,j} & c_{2,j} & \cdots & c_{m,j} \end{bmatrix}, \quad 0 \le j \le n-1. \qquad (2.8)$$

$\mathbf{c}_j^T$ only helps to represent a column vector in a matrix.

Let $\mathbf{C} = [c_{i,j}]$ and $\mathbf{D} = [d_{i,j}]$ be two matrices of equal dimensions, then $\mathbf{C}^* = [c_{i,j}^*]$, $-\mathbf{C} = [-c_{i,j}]$, and $\mathbf{C}^T = [c_{j,i}]$. $\mathbf{C} \otimes \mathbf{D}$ is the matrix whose ith row sequence is obtained by interleaving ith row sequences of $\mathbf{C}$ and $\mathbf{D}$. $\mathbf{CD}$ denotes the matrix whose ith row sequence is the concatenation of ith row sequences of $\mathbf{C}$ and $\mathbf{D}$.

2.3 Ternary Sequences

$\mathbf{a} = (a_0, a_1, ...a_{n-1})$ is a ternary sequence when $a_i \in \{1, 0, -1\},\ 0 \le i \le n-1$. Ternary sequences include binary sequences as a subset. Let $\lambda_{\mathbf{a}} = \delta_{\mathbf{a}}/n$ denote the deficiency ratio of a ternary sequence $\mathbf{a}$ of length n, where $\delta_{\mathbf{a}}$ denotes the number of zero elements in $\mathbf{a}$.

2.3.1 Perfect Ternary Sequences

A perfect ternary sequence satisfies

$$P_{\mathbf{a}}(l) = \begin{cases} (1-\lambda_{\mathbf{a}})n & l = 0 \\ 0 & 1 \le l \le n-1 \end{cases}. \tag{2.9}$$

2.3.2 Ternary ZCZ Sequence Set

A set of m ternary sequences $\{\mathbf{a}_1, \mathbf{a}_2, ..., \mathbf{a}_m\}$ each of length n is said to be a ZCZ sequence set for both periodic and aperiodic correlations, if

$$A_{\mathbf{a}_i}(l) = P_{\mathbf{a}_i}(l) = \begin{cases} (1-\lambda_{\mathbf{a}})n, & l = 0 \\ 0, & 1 \le |l| \le L_{zcz} \end{cases} \tag{2.10}$$

$$A_{\mathbf{a}_i,\mathbf{a}_j}(l) = P_{\mathbf{a}_i,\mathbf{a}_j}(l) = 0. \qquad |l| \le L_{zcz} \tag{2.11}$$

Ternary ZCZ sequence set can be denoted as $\mathcal{T}(n, m, L_{zcz})$, where n, m and L_{zcz} are , respectively, referred to as the length of the sequences, the family size, and the length of the ZCZ.

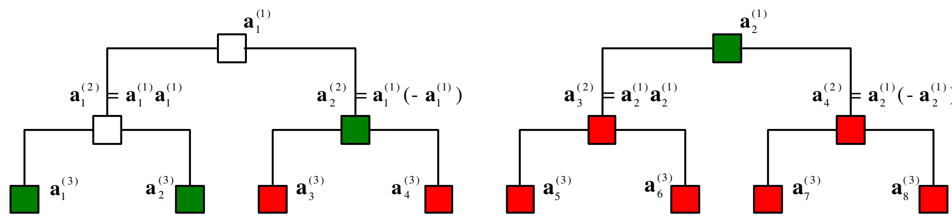

Figure 2.3: OVSF code tree: $\mathbf{a}_i^{(k)}$ denotes the ith code in kth layer

2.4 OVSF Codes

The tree structure of OVSF codes is shown in Fig. 2.3. Let $\mathbf{a}_i^{(k)}$ denote the ith code in kth layer, where $i \in [1, 2^k]$. The $(k+1)$th layer codes can be constructed as follows,

$$\begin{cases} \mathbf{a}_{2i-1}^{(k+1)} = \mathbf{a}_i^{(k)}\mathbf{a}_i^{(k)} \\ \mathbf{a}_{2i}^{(k+1)} = \mathbf{a}_i^{(k)}(-\mathbf{a}_i^{(k)}) \end{cases}. \tag{2.12}$$

The conventional Walsh code based binary OVSF codes are constructed by $\mathbf{a}_1^{(1)} = (++)$ and $\mathbf{a}_2^{(1)} = (+-)$. Let $n^{(1)}$ denote the length of the first layer code, then the kth layer code of length $n^{(k)} = 2^{k-1}n^{(1)}$, $k = 1, 2, 3....$

The orthogonality of the assigned codes is guaranteed by the fact that none of the parent-child code pairs is simultaneously assigned to the users. For instance, if $\mathbf{a}_2^{(2)}$ has been assigned, the codes $\mathbf{a}_3^{(3)}$ and $\mathbf{a}_4^{(3)}$ are blocked, since $\mathbf{a}_2^{(2)}$ is a prefix of $\mathbf{a}_3^{(3)}$ and $\mathbf{a}_4^{(3)}$. As a result, the assigned code set has a prefix-free property. Let us assume k_ith layer code is assigned to the ith user. Based on the Kraft inequality, we have

$$\sum_{i=1}^{M} 2^{-k_i} \leq 1, \tag{2.13}$$

where M is the total number of active users.

2.5 Complementary Sequences

2.5.1 Complementary Pairs

A pair of sequences $\{\mathbf{c}_1, \mathbf{c}_2\}$ is called a complementary pair, if

$$A_{\mathbf{c}_1}(l) + A_{\mathbf{c}_2}(l) = 0, \forall\, l \neq 0. \tag{2.14}$$

Complementary pairs were first introduced by Golay, where the pairs are over the binary alphabet $\{1, -1\}$, as known as Golay pairs. It has been proved that the length of a binary complementary pair (BCP) can only be $2^\alpha 10^\beta 26^\gamma$, where α, β, γ are non-negative integers [2]. In [15], Gavish and Lempel investigated ternary complementary pairs. The so-called ternary complementary pair (TCP) includes BCP as a special case, and exists for any sequence length.

2.5.2 Complementary Sets

A set of m sequences $\{\mathbf{c}_1, \mathbf{c}_2, ..., \mathbf{c}_m\}$, each having length n, is called a complementary set if

$$\sum_{i=1}^{m} A_{\mathbf{c}_i}(l) = 0, \qquad 1 \le l \le n-1. \tag{2.15}$$

A complementary set can be represented using a *complementary set matrix*

$$\mathbf{C} = \begin{bmatrix} \mathbf{r}_1 \\ \mathbf{r}_2 \\ \vdots \\ \mathbf{r}_m \end{bmatrix} = \begin{bmatrix} \mathbf{c}_0 \\ \mathbf{c}_1 \\ \vdots \\ \mathbf{c}_{n-1} \end{bmatrix}^T = \begin{bmatrix} c_{1,0} & c_{1,1} & \cdots & c_{1,n-1} \\ c_{2,0} & c_{2,1} & \cdots & c_{2,n-1} \\ \vdots & \vdots & \ddots & \vdots \\ c_{m,0} & c_{m,1} & \cdots & c_{m,n-1} \end{bmatrix}_{m \times n}, \tag{2.16}$$

where the *row sequences*, $\mathbf{r}_i = (c_{i,0}, c_{i,1}, c_{i,2}, \cdots, c_{i,n-1})$, $1 \le i \le m$, are complementary sequences, that is, $\sum_{i=1}^{m} A_{\mathbf{r}_i}(l) = 0$, $1 \le l \le n-1$. $\mathbf{c}_j = (c_{1,j}, c_{2,j}, c_{3,j}, \cdots, c_{m,j}), 0 \le j \le n-1$, are *column sequences*.

Complementary sets can be constructed from perfect ternary sequences [28, 29]. Let $\mathbf{c} = (c_0, c_1, ...c_{n-1})$ be a perfect ternary sequence of length n, then all its shift versions form a complementary set as follows,

$$\mathbf{C} = \begin{bmatrix} c_0 & c_1 & c_2 & \cdots & c_{n-2} & c_{n-1} \\ c_1 & c_2 & c_3 & \cdots & c_{n-1} & c_0 \\ c_2 & c_3 & c_4 & \cdots & c_0 & c_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ c_{n-1} & c_0 & c_1 & \cdots & c_{n-3} & c_{n-2} \end{bmatrix}. \tag{2.17}$$

2.5.3 Mutually Orthogonal Complementary Sets

Complementary sets $\{\mathbf{a}_1, \mathbf{a}_2, ..., \mathbf{a}_m\}$ and $\{\mathbf{b}_1, \mathbf{b}_2, ..., \mathbf{b}_m\}$ are mates if

$$\sum_{i=1}^{m} A_{\mathbf{a}_i,\mathbf{b}_i}(l) = 0, \quad 1-n \le l \le n-1. \tag{2.18}$$

The mutually orthogonal (MO) complementary set is a collection of complementary sets in which any two are mates. Let $\mathbf{M}_{m,n}^k$ denote the MO complementary set consisting of k complementary sets each having m complementary sequences of length n. For binary sequences, k cannot exceed m [2], that is, the maximum number of mutually orthogonal complementary sets is equal to the number of complementary sequences in a set. Hence, $\mathbf{M}_{m,n}^m$ is called a complete complementary code of order m [30].

A set of k mutually orthogonal complementary sets, $\{\mathbf{C}_1, \mathbf{C}_2, ...\mathbf{C}_k\}$, form a MO complementary set matrix

$$\mathbf{M}_{m,n}^k = \begin{bmatrix} \mathbf{C}_1 & \mathbf{C}_2 & \cdots & \mathbf{C}_k \end{bmatrix}_{m\times kn} \tag{2.19}$$

and its column sequences are denoted as $\mathbf{u}_i, 0 \le i \le kn-1$. An upper bound on an autocorrelation merit of column sequences is termed *the column correlation constraint.* If there exists at least one MO complementary set matrix satisfying a given column correlation constraint, then this constraint is called *an achievable column correlation constraint.* For example, let

$$\lambda_{\mathbf{u}}^A = \max\left\{\lambda_{\mathbf{u}_i}^A, 0 \le i \le kn-1\right\}, \tag{2.20}$$

if $\lambda_{\mathbf{u}}^A \le \lambda^A$, then $\mathbf{M}_{m,n}^k$ is called a MO complementary set matrix satisfying a column correlation constraint λ^A, and λ^A is an achievable column correlation constraint.

2.5.4 Extension Operations

We describe two operations for extending complementary set matrices, namely, *length-extension* and *size-extension.*

Lemma 2.1 [2]: Let $\{\mathbf{a}, \mathbf{b}\}$ be a complementary pair, then $\{\overleftarrow{\mathbf{b}^*}, -\overleftarrow{\mathbf{a}^*}\}$ is its mate, and both $\{\mathbf{a}\overleftarrow{\mathbf{b}^*}, \mathbf{b}(-\overleftarrow{\mathbf{a}^*})\}$ and $\{\mathbf{a} \otimes \overleftarrow{\mathbf{b}^*}, \mathbf{b} \otimes (-\overleftarrow{\mathbf{a}^*})\}$ are complementary pairs.

Proof. Let us first prove that $\{\overleftarrow{\mathbf{b}}^*, -\overleftarrow{\mathbf{a}}^*\}$ is a mate of $\{\mathbf{a}, \mathbf{b}\}$. A proof for binary sequences can be found in *Theorem 11* of [13]. For complex-valued sequences, the complementarity of $\{\overleftarrow{\mathbf{b}}^*, -\overleftarrow{\mathbf{a}}^*\}$ follows from

$$\begin{aligned} A_{\overleftarrow{\mathbf{b}}^*}(l) + A_{-\overleftarrow{\mathbf{a}}^*}(l) &= (A_{\overleftarrow{\mathbf{b}}}(l))^* + (A_{-\overleftarrow{\mathbf{a}}}(l))^* \\ &= A_{\mathbf{b}}(l) + A_{-\mathbf{a}}(l) \\ &= A_{\mathbf{b}}(l) + A_{\mathbf{a}}(l) \\ &= 0 \end{aligned}$$

for $1 \leq l \leq n-1$, where n denotes the sequence length. We further show that the pair $\{\overleftarrow{\mathbf{b}}^*, -\overleftarrow{\mathbf{a}}^*\}$ is orthogonal to $\{\mathbf{a}, \mathbf{b}\}$ in the complementary sense,

$$\begin{aligned} A_{\mathbf{a},\overleftarrow{\mathbf{b}}^*}(l) + A_{\mathbf{b},-\overleftarrow{\mathbf{a}}^*}(l) &= A_{\mathbf{a},\overleftarrow{\mathbf{b}}^*}(l) - A_{\mathbf{b},\overleftarrow{\mathbf{a}}^*}(l) \\ &= (A_{\mathbf{a}^*,\overleftarrow{\mathbf{b}}}(l))^* - A_{\mathbf{b},\overleftarrow{\mathbf{a}}^*}(l) \\ &= A_{\mathbf{b},\overleftarrow{\mathbf{a}}^*}(l) - A_{\mathbf{b},\overleftarrow{\mathbf{a}}^*}(l) \\ &= 0 \end{aligned}$$

for every l. Refer to the proofs of *Theorem 6* and *Theorem 13* from [13]. If $\{\mathbf{a}_1, \mathbf{b}_1\}$ is a complementary pair and $\{\mathbf{a}_2, \mathbf{b}_2\}$ is one of its mates, then both $\{\mathbf{a}_1\mathbf{a}_2, \mathbf{b}_1\mathbf{b}_2\}$ and $\{\mathbf{a}_1 \otimes \mathbf{a}_2, \mathbf{b}_1 \otimes \mathbf{b}_2\}$ are complementary pairs. □

Lemma 2.2: Let $\{\mathbf{a}_1, \mathbf{b}_1\}$, $\{\mathbf{a}_2, \mathbf{b}_2\}$, ..., $\{\mathbf{a}_m, \mathbf{b}_m\}$ be m complementary pairs of length n. Then, $\{\mathbf{a}_1, \mathbf{b}_1, \mathbf{a}_2, \mathbf{b}_2, ..., \mathbf{a}_m, \mathbf{b}_m\}$ is a complementary set of $2m$ complementary sequences.

Proof. $\sum_{i=1}^{m} (A_{\mathbf{a}_i}(l) + A_{\mathbf{b}_i}(l)) = 0, \;\; 1 \leq l \leq n-1.$ □

Lemmas 2.1-2.2 imply that, if a complementary set consists of $m/2$ complementary pairs, the sequence length can be recursively doubled as follows. Let

$$\mathbf{C}^{(p)} = \begin{bmatrix} \mathbf{r}_1^{(p)} \\ \mathbf{r}_2^{(p)} \\ \vdots \\ \mathbf{r}_{m-1}^{(p)} \\ \mathbf{r}_m^{(p)} \end{bmatrix}_{m\times n^{(p)}} \quad \text{and} \quad \mathbf{D}^{(p)} = \begin{bmatrix} \overleftarrow{\mathbf{r}}_2^{(p)} \\ -\overleftarrow{\mathbf{r}}_1^{(p)} \\ \vdots \\ \overleftarrow{\mathbf{r}}_m^{(p)} \\ -\overleftarrow{\mathbf{r}}_{m-1}^{(p)} \end{bmatrix}^*_{m\times n^{(p)}} \tag{2.21}$$

be, respectively, an m by $n^{(p)}$ complementary set matrix and its mate, where m is an even number, and $\{\mathbf{r}_1^{(p)}, \mathbf{r}_2^{(p)}\}, \{\mathbf{r}_3^{(p)}, \mathbf{r}_4^{(p)}\}, ...\{\mathbf{r}_{m-1}^{(p)}, \mathbf{r}_m^{(p)}\}$ are assumed to be complementary pairs. A complementary set matrix $\mathbf{C}^{(p+1)}$ of dimension m by $n^{(p+1)} = 2n^{(p)}$ can be constructed recursively as either,

$$\mathbf{C}^{(p+1)} = \mathbf{C}^{(p)} \, \mathbf{D}^{(p)} \tag{2.22}$$

or

$$\mathbf{C}^{(p+1)} = \mathbf{C}^{(p)} \otimes \mathbf{D}^{(p)}. \tag{2.23}$$

We term (2.22) and (2.23) *length-extension operations.*

Lemma 2.3 [13]: A MO complementary set matrix $\mathbf{M}_{2m,2n}^{2k}$ can be constructed recursively as either,

$$\mathbf{M}_{2m,2n}^{2k} = \begin{bmatrix} \mathbf{M}_{m,n}^{k}\mathbf{M}_{m,n}^{k}, (-\mathbf{M}_{m,n}^{k})\mathbf{M}_{m,n}^{k} \\ (-\mathbf{M}_{m,n}^{k})\mathbf{M}_{m,n}^{k}, \mathbf{M}_{m,n}^{k}\mathbf{M}_{m,n}^{k} \end{bmatrix} \tag{2.24}$$

or

$$\mathbf{M}_{2m,2n}^{2k} = \begin{bmatrix} \mathbf{M}_{m,n}^{k}\otimes\mathbf{M}_{m,n}^{k}, (-\mathbf{M}_{m,n}^{k})\otimes\mathbf{M}_{m,n}^{k} \\ (-\mathbf{M}_{m,n}^{k})\otimes\mathbf{M}_{m,n}^{k}, \mathbf{M}_{m,n}^{k}\otimes\mathbf{M}_{m,n}^{k} \end{bmatrix}. \tag{2.25}$$

Proof. Refer to the proof of *Theorem 12-13* in [13]. $\square$

We term (2.24) and (2.25) *size-extension operations.*

Chapter 3

Ternary Complementary Sets for Multiple Channel DS-UWB with Reduced PAPR

3.1 Introduction

We have proposed a multichannel TS-UWB signaling which transmits orthogonal pulses carrying the same information bit. We demonstrated how, by employing ternary mutually orthogonal (MO) complementary set of sequences as the spreading sequences, the multipath interference and multiple access interference are efficiently suppressed.

Akin to the multicarrier direct-sequence code-division multiple access (MC-DS-CDMA) signaling [16], the transmitted signal of the multichannel UWB system is the sum of the signals from all parallel channels, so its envelope and transmitted power may vary significantly. Hence, such signaling potentially suffers a high PAPR which may limit its application. Great efforts have been devoted to PAPR analysis and reduction in multicarrier modulation systems (see, e.g., [16, 27, 31]). However, to our best knowledge, there is no PAPR analysis for impulse based multichannel UWB signaling in the literature, this being a promising technique for high bit-rate transmission, in particular, for indoor environments where multipath can be significant.

We derive an upper bound on the PAPR for multichannel DS-UWB system using orthogonal pulses. The bound shows that sequence with small out-off-phase aperiodic ACF can lower the PAPR of the system. This result is similar to the bound derived by Tellambura [27] for orthogonal frequency-division multiplexing (OFDM) signaling. We construct the spreading sequence sets which can result in sequences with small aperiodic ACF. The spreading sequence sets preserve the complementarity and orthogonality to mitigate the multipath and multiuser interference.

3.2 Multiple Channel DS-UWB

3.2.1 Multipath and Multiuser Interference Cancellation

In this section, we introduce a technique for multichannel DS-UWB systems that employ complementary sets as spreading sequences. Fig. 3.1 demonstrates, in a simple single user multipath environment, how a complementary pair suppress the multipath interference with the aid of orthogonal channels.

At transmitter, the same information bit is spread by two complementary sequences which are respectively assigned to orthogonal channels. The transmitted signal is the summation of signals from two subchannels. Despreading in the receiver is accomplished on a channel-by-channel basis using a set of correlators. The multipath interference at the output of a correlator is the autocorrelation of the sequence assigned on this channel with a time shift l. By adding the output of two correlators, the autocorrelation sidelobes are cancelled by the complementarity of the complementary pair. The MAI can be suppressed in a similar way if mutually orthogonal complementary sets are assigned to different users.

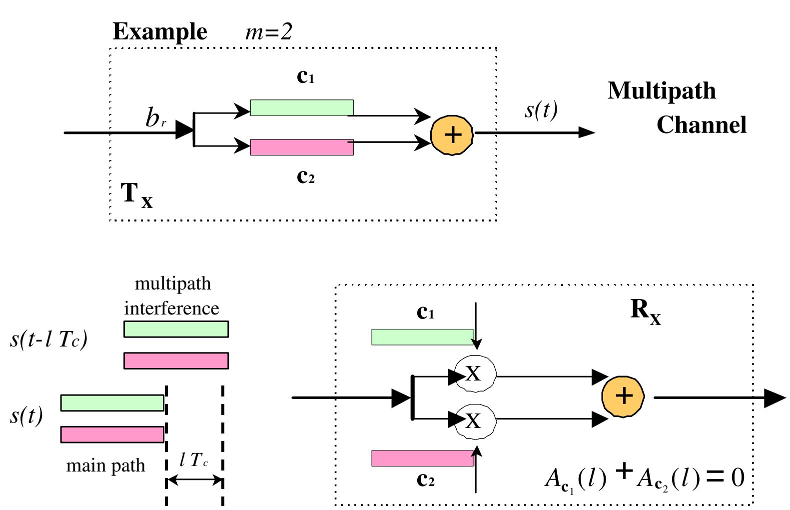

Figure 3.1: Example: multipath interference cancellation by complementary sets and orthogonal pulses

3.2.2 System Model

The same information bit is spread over a set of m parallel channels, each corresponding to one of a set of orthogonal pulses. The transmitted signal is given by

$$s(t) = \sum_r b_r \sum_{i=1}^{m} p_i(t - rT_b), \tag{3.1}$$

where the pulse train of the ith channel is

$$p_i(t) = \sum_{j=0}^{n-1} c_{i,j}\psi_i(t - jT_c). \tag{3.2}$$

The binary antipodal symbol b_r is transmitted over m parallel channels and r is its index. The m spreading sequences $(c_{i,0}, c_{i,1}, \cdots, c_{i,n-1})$, $i = 1, 2, ...m$, assigned respectively to m parallel channels are ternary sequences of length n. T_c is the chip duration time and $T_b = nT_c$ is the bit period. $\psi_i(t)$ is the signaling pulse chosen from the orthogonal set of pulses and assumed known to the receiver. The impulse response of the UWB channel with L resolvable paths is

$$h(t) = \sum_{l=0}^{L-1} \alpha_l \delta(t - \tau_l), \tag{3.3}$$

where α_l and τ_l denote the channel gain and the propagation delay of the l_{th} path, respectively. When sufficient multipath resolution is available, small changes in the propagation time only affect the path delay and path component distortion can be neglected. Under these assumptions, path coefficients α_l can be modelled as independent real valued random variables whose sign is a function of the material properties and, generally, depends on the wave polarization, angle of incidence, and the frequency of the propagating wave [32]. We quantize the multipath delay into bins, i.e. $\tau_l = lT_c$.

For an asynchronous UWB system with K users, the corresponding received signal model is:

$$r(t) = \sum_{k=1}^{K} \sum_{l=0}^{L-1} \alpha_l s^{(k)}(t - lT_c - \tau^{(k)}) + n(t), \tag{3.4}$$

where the superscript k indicates the parameter of kth user. $\tau^{(k)}$ accounts for propagation delay and lack of synchronism between transmitters, $n(t)$ is a white Gaussian noise process. For the system with short sequences, the delay $\tau^{(k)}$ is assumed to be uniformly distributed in the interval $[0, nT_c)$, and here, we quantize it into bins.

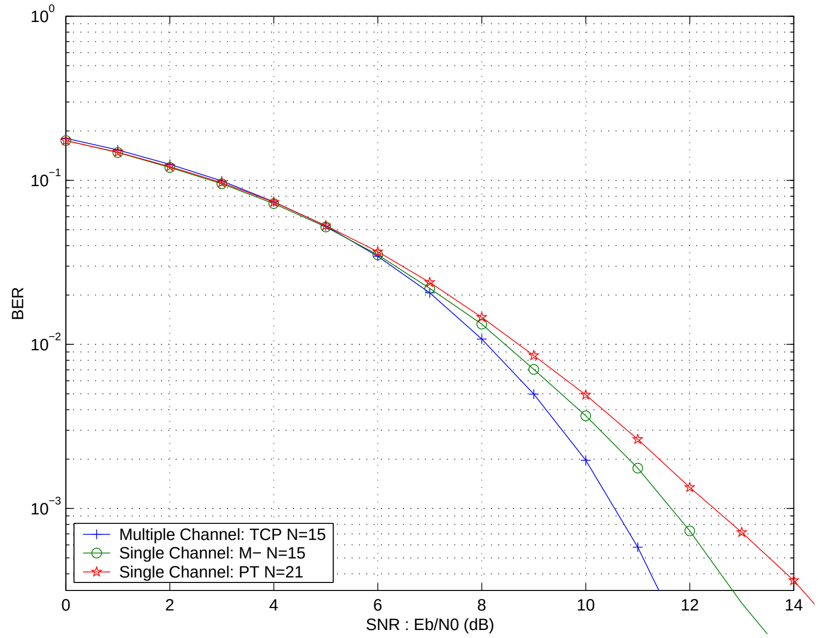

Figure 3.2: Single User: Parallel channel UWB system with TCP versus Single channel UWB system with m- and perfect ternary sequence

3.2.3 BER Performance

We study correlator receiver performances for TCP, m-sequences, and perfect ternary sequences in a single user system. For multiuser system, we compare ternary complementary sets, Walsh codes, Gold-like sequences [2] and preferred m-sequences [33]. Bit energy for all signaling schemes is normalized. The mean power of multipath components are chosen to be equal to the average value given in [34], which is based on the indoor line of sight (LOS) measurements performed in 23 homes. In [34], it is observed that the line of sight component and the first 10 paths account for 33% and 75% of the total power, respectively. The sign of the reflected path coefficient is modelled as a uniformly distributed random variable [35]. The path power is quantized into bins corresponding to a chip duration $T_c = 0.4$ns. We assume that each bin contains exactly one multipath component (emulating a dense multipath environment) and the delay spread is assumed to be 4 nanosecond. The effects of interchip interference has been assumed negligible.

In Fig. 3.2, a TCP with length 15 has been assigned to a single user for its two orthogonal channels, i.e. $[++-+++++-0-+-0+]$ and $[++-+++---0+-+0-]$. We compare it to a single user single channel UWB system using the same

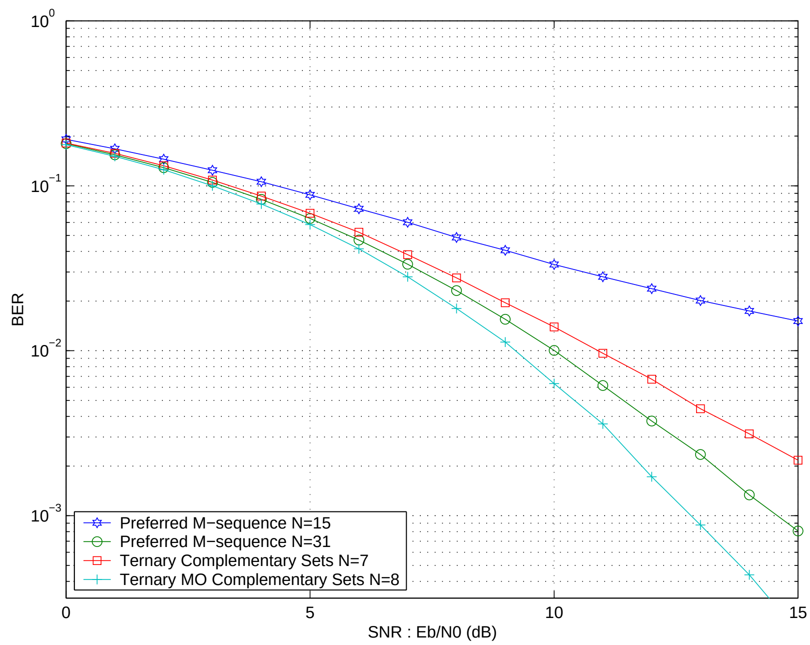

Figure 3.3: Two Users: Parallel channel UWB system with ternary complementary sets versus Single channel UWB system with preferred m-sequences

length m-sequence $[- - - + - - + + - + - + + + +]$ and perfect ternary sequence $[+ + + + + - +0 + 0 - + + -00 + -0 - -]$. The results show that UWB signaling employing TCP suffers less multipath interference than the signaling using m-sequence and perfect sequences. At $SNR = 10^{-3}$, 1 dB and 2 dB gain can be respectively achieved by employing TCP over m-sequence and perfect ternary sequences.

Figure 3.3 demonstrates BER performances of complementary sets and preferred m-sequence in a two user scenario. MO complementary sets constructed from binary complementary pair $\{(+-), (++)\}$ and $\{(--), (-+)\}$ by using extension methods (see Sec. 2.5.4) are assigned to two users. Based on the approach described in Eq. 2.17, we derive two complementary sets from perfect ternary sequence $[+ - -0 - 00]$ and $[+00 - 0 - -]$ and assign them to two users. Note that, by employing MO complementary sets, the multiple access interference is mitigated significantly.

In a dense multipath UWB environment, the spreading sequence with short length commonly suffers sever ISI, while with multiple orthogonal channels, MO ternary complementary set of sequence length 7 has better performance than the preferred m-sequences with length 31. We can observe 2 dB gain at $SNR = 10^{-3}$.

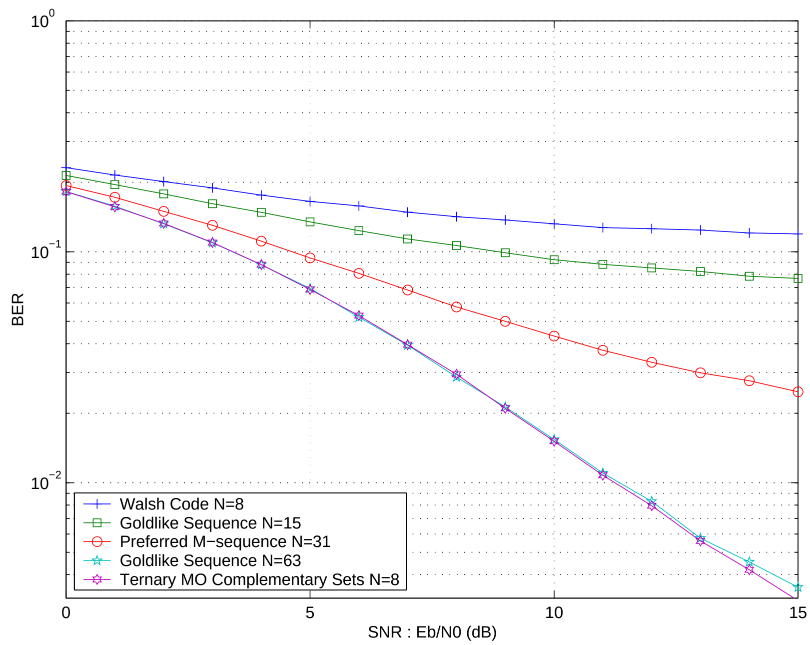

Figure 3.4: Four Users: BER performance for MO ternary complementary sets, Walsh-Hadamard codes and preferred m-sequences

In Figure 3.4, we respectively assign four ternary MO complementary sets to four users. Ternary complementary set of sequences of length 8 can achieve the same BER performance as Gold-like sequences of length 63. Hence, without any loss of the BER performance, in a multiple channel UWB system, we can increase the date rate more than 7 times than the single channel UWB signals employing Gold-like sequences.

We have demonstrated that, when the spreading sequence set $\{(c_{i,0}^{(k)}, c_{i,1}^{(k)}, \cdots, c_{i,n-1}^{(k)})\}$, $i = 1, 2, 3, ...m$, assigned to kth user is a ternary complementary set and ternary complementary sets between any two users are mutually orthogonal, the multipath interference as well as multiple access interference are efficiently suppressed by simply employing a correlator receiver. However, we need solve the potential PAPR problem caused by multiple channels while at the same time maintaining the complementarity and orthogonality of the spreading sequence sets.

3.2.4 PAPR Analysis

Complementary set with m complementary sequences of length n can be represented as a matrix,

$$\mathbf{C} = \begin{bmatrix} \mathbf{c}_0 \\ \mathbf{c}_1 \\ \vdots \\ \mathbf{c}_{n-1} \end{bmatrix}^T = \begin{bmatrix} c_{1,0} & c_{1,1} & c_{1,2} & \cdots & c_{1,n-1} \\ c_{2,0} & c_{2,1} & c_{2,2} & \cdots & c_{2,n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ c_{m,0} & c_{m,1} & c_{m,2} & \cdots & c_{m,n-1} \end{bmatrix}_{m \times n}, \tag{3.5}$$

where $\{\mathbf{c}_j, j = 0, 1, ...n-1\}$, are column sequences of the complementary set matrix. Note that the information bits won't affect the instantaneous power of the transmitted signal, since all the parallel channels carry the same information bit at the same time. Thus, in this section, we can analysis the PAPR and derive an upper bound on it by ignoring the information bits.

The corresponding multichannel UWB signal of a single user is given by

$$s_{\mathbf{c}_j}(t) = \sum_{i=1}^{m} c_{i,j}\psi_i(t - jT_c), \quad \text{for} \quad j = 0, 1, ...n-1. \tag{3.6}$$

where j is the chip index. The corresponding instantaneous envelope power is,

$$\begin{aligned} P_{\mathbf{c}_j}(t) &= s^2_{\mathbf{c}_j}(t) = \left(\sum_{i=1}^{m} c_{i,j}\psi_i(t - jT_c)\right)\left(\sum_{l=1}^{m} c_{l,j}\psi_l(t - jT_c)\right) \\ &= 2\sum_{l=1}^{m-1}\sum_{i=1}^{m-l} c_{i,j}c_{i+l,j}\psi_i(t - jT_c)\psi_{i+l}(t - jT_c) \\ &\quad + \sum_{i=1}^{m} c^2_{i,j}\psi^2_i(t - jT_c). \end{aligned} \tag{3.7}$$

We assume that orthogonal pulses have zero amplitude outside the chip interval. Hence, the instantaneous power over symbol period is given by,

$$P_{\mathbf{c}}(t) = \sum_{j=0}^{n-1} P_{\mathbf{c}_j}(t)U(t - jT_c), \tag{3.8}$$

where

$$U(t) = \begin{cases} 1 & t \in [\,0, T_c) \\ 0 & \text{otherwise} \end{cases}. \tag{3.9}$$

The peak amplitude of all the orthogonal pulses may or may not have the same value. We define

$$\lambda = \max_{i} \left(\sup_{t \in [0, T_c]} |\psi_i(t)| \right) \quad \text{for} \quad i = 1, 2, ...m, \tag{3.10}$$

where $\sup_{t \in [0,T_c]} |\psi_i(t)|$ denotes the peak value of pulse $|\psi_i(t)|$ over time period $[0, T_c]$. Hence, the peak envelope power (PEP) corresponding to the spreading sequence set $\mathbf{C}$ is given by,

$$\begin{aligned} \text{PEP}(\mathbf{C}) &= \sup_{t \in [0, T_b]} P_{\mathbf{c}}(t) = \max_{\mathbf{c}_j} \left(\sup_{t \in [jT_c, (j+1)T_c]} P_{\mathbf{c}_j}(t) \right) \\ &\leq \lambda^2 \max_{\mathbf{c}_j} \left(\left| \sum_{i=1}^{m} c_{i,j}^2 \right| + 2 \left| \sum_{i=1}^{m-1} \sum_{l=i+1}^{m} c_{i,j} c_{l,j} \right| \right) \\ &\leq \lambda^2 \max_{\mathbf{c}_j} \left(A_{\mathbf{c}_j}(0) + 2 \sum_{l \neq 0} \left| A_{\mathbf{c}_j}(l) \right| \right). \end{aligned} \tag{3.11}$$

We normalize the information symbol energy over m channels to be 1. Thus,

$$\int_0^{T_b} P_{\mathbf{c}}(t) dt = \sum_{i=1}^{m} \left(\int_0^{T_b} \sum_{j=0}^{n-1} c_{i,j}^2 \psi_i^2(t - jT_c) dt \right) = 1. \tag{3.12}$$

The average power corresponding to the spreading sequence set $\mathbf{C}$ over a symbol period T_b is,

$$P_{av}(\mathbf{C}) = \frac{1}{T_b} \int_0^{T_b} P_{\mathbf{c}}(t) dt = \frac{1}{T_b}. \tag{3.13}$$

From (3.11) and (3.13), the PAPR of the multichannel UWB system can be upper bounded as follows,

$$\text{PAPR}(\mathbf{C}) = \frac{PEP(\mathbf{C})}{P_{av}(\mathbf{C})} \leq \frac{\lambda^2}{T_b} \max_{\mathbf{c}_j} \left(A_{\mathbf{c}_j}(0) + 2 \sum_{l \neq 0} \left| A_{\mathbf{c}_j}(l) \right| \right). \tag{3.14}$$

This upper bound highlights the intimate relationship of PAPR with the aperiodic ACF of $\mathbf{C}$'s column sequences. In the next section, we construct the ternary complementary set $\mathbf{C}$ whose column sequences have reduced aperiodic ACFs, thus lowering the PAPR of a multichannel UWB system.

3.3 Ternary Complementary Sets with Reduced PAPR

Let $\mathcal{T}(m, \delta, S^A)$ denote a ternary sequence set, in which any sequence $\mathbf{c}$ of length m with δ zero elements satisfies

$$\sum_{l=1}^{m-1} |A_{\mathbf{c}}(l)| \leq S^A. \tag{3.15}$$

We introduce a stepwise algorithm for the design of the ternary complementary set matrix whose column sequences are in $\mathcal{T}(m, \delta, S^A)$, where m is an even number. Based on (3.14), the PAPR of the system is bounded by

$$PAPR(\mathbf{C}) \leq \frac{\lambda^2}{T_b}\left(m - \delta + 2S^A\right). \tag{3.16}$$

3.3.1 Design Algorithm

Step 1: We do the exhaustive search to find seed sequence $\mathbf{c}_0 = (c_{1,0}, c_{2,0}, ..., c_{m,0}) \in \mathcal{T}(m, \delta, S^A)$, such that,

$$\begin{aligned} \mathbf{c}_1 &= f_i(\mathbf{c}_0) \\ &= (c_{2,0}, -c_{1,0}, c_{4,0}, -c_{3,0}, ..., c_{m,0}, -c_{m-1,0}) \\ &\in \mathcal{T}(m, \delta, S^A). \end{aligned} \tag{3.17}$$

Step 2: By employing length-extension operations, we can recursively extend

$$\mathbf{C}^{(0)} = \begin{bmatrix} \mathbf{c}_0 \\ \mathbf{c}_1 \end{bmatrix}^T = \begin{bmatrix} c_{1,0} & c_{2,0} & c_{3,0} & c_{4,0} & \cdots & c_{m,0} \\ c_{2,0} & -c_{1,0} & c_{4,0} & -c_{3,0} & \cdots & -c_{m-1,0} \end{bmatrix}^T \tag{3.18}$$

to be the PAPR reduced ternary complementary set $\mathbf{C}^{(p)}$ of dimension m by 2^{p+1}.

Proposition 3.1: $\mathbf{C}^{(0)}$ is a ternary complementary set with PAPR reduction.

Proof. $\mathbf{c}_0$ and $\mathbf{c}_1$ satisfy that $\{(c_{2i-1,0}, c_{2i-1,1}), (c_{2i,0}, c_{2i,1})\}$, $i = 1, 2, ...m/2$, are respectively $m/2$ ternary complementary pairs. Based on *Lemma 2.1* and *Lemma 2.2*, $\mathbf{C}^{(0)}$ is a ternary complementary set. On the other hand, both $\mathbf{c}_0$ and $\mathbf{c}_1$ are in $\mathcal{T}(m, \delta, S^A)$, thus

$$PAPR(\mathbf{C}^{(0)}) \leq \frac{\lambda^2}{T_b}\left(m - \delta + 2S^A\right). \tag{3.19}$$

□

Proposition 3.2: The extended sequence sets $\mathbf{C}^{(p)}$, $p = 0, 1, 2, 3...$, are ternary complementary sets with PAPR reduction.

Proof. The complementarity of $\mathbf{C}^{(p)}$ can be verified by using *Lemmas 3.1-3.2*. The column sequences of $\mathbf{C}^{(p+1)}$, i.e., $\left\{\mathbf{c}_0^{(p+1)}, \mathbf{c}_1^{(p+1)}, ...\mathbf{c}_{n^{(p+1)}-1}^{(p+1)}\right\}$, can be expressed in terms of the column sequences of $\mathbf{C}^{(p)}$ as follows

$$\mathbf{c}_i^{(p+1)} = \begin{cases} \mathbf{c}_i^{(p)} & \text{for} \quad 0 \le i \le n^{(p)} - 1 \\ -\mathbf{c}_{i-n^{(p)}}^{(p)} & \text{for} \quad n^{(p)} \le i \le \frac{3}{2}n^{(p)} - 1 \\ \mathbf{c}_{i-n^{(p)}}^{(p)} & \text{for} \quad \frac{3}{2}n^{(p)} \le i \le 2n^{(p)} - 1 \end{cases},$$

where $n^{(p)} = 2^{p+1}$. Note that $\mathbf{c}_0^{(0)}$ and $\mathbf{c}_1^{(0)}$, the column sequences of $\mathbf{C}^{(0)}$, are in set $\mathcal{T}(m, \delta, S^A)$. Thus, all column sequences of $\mathbf{C}^{(p)}$, i.e. $\mathbf{c}_0^{(p)}$, $\mathbf{c}_1^{(p)}$,... $\mathbf{c}_{n^{(p)}-1}^{(p)}$, $p = 0, 1, 2, ...$, are in the same ternary sequence set $\mathcal{T}(m, \delta, S^A)$ as well. $\square$

3.3.2 Numerical Results

After exhaustive computer search, we can find $\mathbf{c}_0 = (+ - - + + + 0 +) \in \mathcal{T}(8, 1, 5)$. It can be verified that, $\mathbf{c}_1 = (- - + + + - + 0) \in \mathcal{T}(8, 1, 5)$. Thus, $\mathbf{C}^{(0)}$ is given by,

$$\mathbf{C}^{(0)} = \begin{bmatrix} c_{1,0} & c_{1,1} \\ c_{2,0} & c_{2,1} \\ \vdots & \vdots \\ c_{8,0} & c_{8,1} \end{bmatrix} = \begin{bmatrix} + & - & - & + & + & + & 0 & + \\ - & - & + & + & + & - & + & 0 \end{bmatrix}^T.$$

By employing length-extension operations, we construct ternary complementary set $\mathbf{C}^{(1)}$ and $\mathbf{C}^{(2)}$ from $\mathbf{C}^{(0)}$ as follows,

$$\mathbf{C}^{(1)} = \begin{bmatrix} + & - & - & + & + & + & 0 & + \\ - & - & + & + & + & - & + & 0 \\ - & + & + & - & - & - & 0 & - \\ - & - & + & + & + & - & + & 0 \end{bmatrix}^T, \tag{3.20}$$

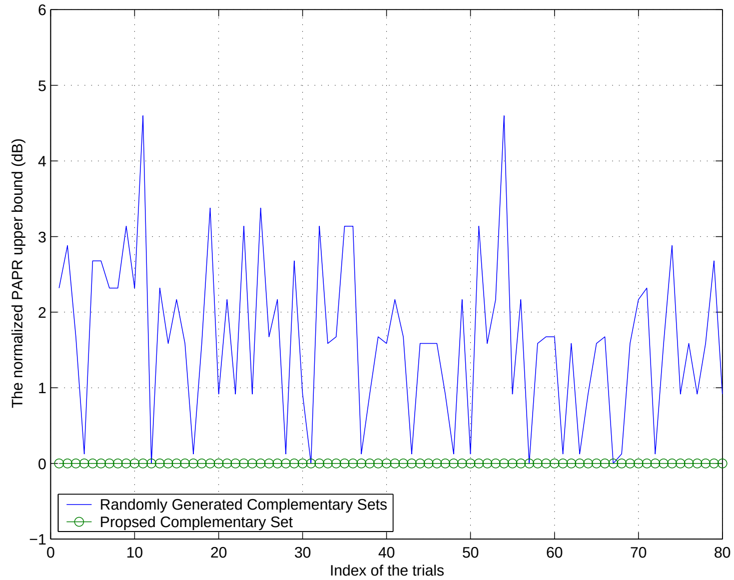

Figure 3.5: Normalized PAPR upper bound of randomly generated complementary sets versus that of proposed PAPR reduced ternary complementary set

and

$$\mathbf{C}^{(2)} = \begin{bmatrix} + & - & - & - & - & + & - & - \\ - & - & + & - & + & + & + & - \\ - & + & + & + & + & - & + & + \\ + & + & - & + & - & - & - & + \\ + & + & - & + & - & - & - & + \\ + & - & - & - & - & + & - & - \\ 0 & + & 0 & + & 0 & - & 0 & + \\ + & 0 & - & 0 & - & 0 & - & 0 \end{bmatrix}_{8\times 8} .$$

We normalize the PAPR upper bound of the 8 by 8 ternary complementary set $\mathbf{C}^{(2)}$ constructed in the above example. For comparison purpose, we randomly generate complementary sets with the same dimension. Fig. 3.5 demonstrates that the proposed ternary complementary set may lower the PAPR upper bound by 4.5 dB over that of randomly generated complementary sets.

Chapter 4

Complementary Set Matrices Satisfying a Column Correlation Constraint

4.1 Introduction

Complementary sequence sets have been introduced by Golay [14, 36], as a pair of binary sequences with the property that the sum of their aperiodic autocorrelation functions is zero everywhere except at zero shift. Tseng and Liu [13] generalized these ideas to sets of binary sequences of size larger than two. Sivaswamy [37] and Frank [38] investigated the multiphase (polyphase) complementary sequence sets with constant amplitude sequence elements. Gavish and Lemple considered ternary complementary pairs over the alphabet $\{1, 0, -1\}$ [15]. The synthesis of multilevel complementary sequences is described in [39]. These generalizations of a binary alphabet lead to new construction methods for complementary sets having a larger family of lengths and cardinalities. However, all these studies focus either on the set complementarity or on the design of orthogonal families of complementary sets. Correlation properties of column sequences of the complementary set matrix (i.e., the matrix whose row sequences form a complementary set) have not been considered.

In Chapter 3 (see also [6, 40, 41]), a technique for a multicarrier direct-sequence code-division multiple access (MC-DS-CDMA) system [42, 43] that employs complementary sets as spreading sequences has been investigated. Each user assigns different sequences from a complementary set to his subcarriers. By assigning MO complementary sets to different users, both multiple access interference and multipath interference can significantly be suppressed. Similar to conventional multicarrier systems, one of the major impediments to deploying such systems is a high PAPR. We have stressed in the previous chapter that the PAPR can be upper bounded by the maximum sum-of-out-of-phase aperiodic autocorrelation magnitude of column sequences of the employed complementary sets. Hence, in this work, we search for ways of constructing sequence sets which are characterized by both their complementarity and a desired column correlation constraint.

By generalizing earlier work of Boyd [44], Popović [45] has demonstrated that PAPR corresponding to any binary Golay sequence (i.e., a sequence having a Golay complementary pair) is at most two. This has motivated Davis and Jedwab to explicitly determine a large class of Golay sequences as a solution to the signal envelope problem [46]. Here, we also consider Golay column complementary sets.

We describe a construction algorithm for the design of 2^{t+1} MO complementary set matrices of size $2^t m$ by 2^{t+p+1}, where t and p can be any non-negative integer, and m is an even number. The construction process is based on a set of sequence/matrix operations, starting from a companion sequence pair which can be constructed based on the Quads introduced in [47]. These operations preserve the alphabet (up to the sign) of the companion pair. We illustrate how, by selecting an appropriate companion pair we can ensure that column sequences of the constructed complementary set matrix satisfy a correlation constraint. For $t = 0$, companion pair properties directly determine matrix column correlation properties. For $t \geq 1$, reducing correlation merits of the companion pair may lead to improved column correlation properties. However, further decrease of the maximum out-of-phase aperiodic autocorrelation of column sequences is not possible once the correlation of the companion pair is less than a threshold determined by t. We also present a method for constructing the companion pair which leads to the complementary set matrix with Golay column sequences.

We exhaustively search for optimum companion pairs leading to binary complementary sets with a minimum out-of-phase autocorrelation magnitude or a minimum sum-of-out-of-phase autocorrelation magnitude for column sequences of length at least up to 28, or leading to optimum complementary sets whose column sequences are ternary sequences with a single zero element of length at least up to 24. However, exhaustive search is infeasible for medium and long sequences. We instead suggest finding companion pairs with a small, if not minimum, column correlation constraint. By exploiting properties of the companion pair, we convert the problem into a search for two sequences of length $m/2$ with low autocorrelation and crosscorrelation merits, a long standing problem in literature (e.g. see [48–51]). We further derive an improved cost function and show how it leads to reduced achievable maximum out-of-phase column correlation constraint. Sufficient conditions for the existence of companion pairs which satisfy various column correlation constraints are also derived.

4.2 Definitions

4.2.1 Companion Pair

Let $\mathbf{a}$ be a sequence of length m, where m is an even number. We define a sequence $\mathbf{b}$ as a companion of $\mathbf{a}$ if

$$\mathbf{C} = \begin{bmatrix} \mathbf{a} \\ \mathbf{b} \end{bmatrix}^T \tag{4.1}$$

is a complementary set matrix which consists of $m/2$ complementary pairs. $\mathbf{C}$ is called a companion matrix and $(\mathbf{a}, \mathbf{b})$ is called a companion pair.

4.2.2 Expanded Sequences Set

Let us also define an expanded sequence set $\mathcal{R}_{\mathbf{c}}^{(v)}$, which is a collection of row sequences of matrix $\mathbf{R}_{\mathbf{c}}^{(v)}$ recursively constructed from a sequence $\mathbf{c}$, as follows,

$$\mathbf{R}_{\mathbf{c}}^{(v)} = \begin{bmatrix} \mathbf{R}_{\mathbf{c}}^{(v-1)}\mathbf{R}_{\mathbf{c}}^{(v-1)} \\ \mathbf{R}_{\mathbf{c}}^{(v-1)}(-\mathbf{R}_{\mathbf{c}}^{(v-1)}) \end{bmatrix}, \quad v = 1, 2, 3..., \tag{4.2}$$

where

$$\mathbf{R}_{\mathbf{c}}^{(0)} = \begin{bmatrix} \mathbf{c} \\ -\mathbf{c} \end{bmatrix}_{2\times m}. \tag{4.3}$$

Let $\mathcal{R}_{\mathbf{c},\mathbf{d}}^{(v)} = \mathcal{R}_{\mathbf{c}}^{(v)} \cup \mathcal{R}_{\mathbf{d}}^{(v)}$ which consists of 2^{v+2} sequences of length $2^v m$. For example, $\mathcal{R}_{\mathbf{c}}^{(0)} = \{\mathbf{c}, -\mathbf{c}\}$, $\mathcal{R}_{\mathbf{d}}^{(0)} = \{\mathbf{d}, -\mathbf{d}\}$, and $\mathcal{R}_{\mathbf{c},\mathbf{d}}^{(0)} = \{\mathbf{c}, \mathbf{d}, -\mathbf{c}, -\mathbf{d}\}$.

4.3 Construction of Complementary Set Matrices from a Companion Pair

4.3.1 Recursive Construction Algorithm

Let $\mathcal{X}(m)$ denote a sequence set which consists of all length m sequences whose elements are from the alphabet $\mathcal{X}$. We summarize a recursive construction of a MO complementary set matrix $\mathbf{M}^{2^{t+1}}_{2^t m, 2^t n^{(p)}}$ with elements from $\mathcal{X}$, where $n^{(p)} = 2^{p+1}$, m is an even number, and $t, p = 0, 1, 2, ...$

Step 1: The construction starts from a companion pair $\mathbf{c}_0$ and $\mathbf{c}_1$ which are in $\mathcal{X}(m)$. They form an m by 2 companion matrix

$$\mathbf{C}^{(0)} = \begin{bmatrix} \mathbf{c}_0 \\ \mathbf{c}_1 \end{bmatrix}^T . \tag{4.4}$$

Step 2: By employing the length-extension operation p times, we extend $\mathbf{C}^{(0)}$ to an m by $n^{(p)} = 2^{p+1}$ complementary set matrix $\mathbf{C}^{(p)}$. $\mathbf{C}^{(p)}$ and its mate $\mathbf{D}^{(p)}$ constructed from Eq. (2.21) form a MO complementary set matrix

$$\mathbf{M}^2_{m,n^{(p)}} = \begin{bmatrix} \mathbf{C}^{(p)} & \mathbf{D}^{(p)} \end{bmatrix}_{m \times 2^{p+2}} . \tag{4.5}$$

Step 3: Starting with $\mathbf{M}^2_{m,n^{(p)}}$, we can construct the MO complementary set matrix $\mathbf{M}^{2^{t+1}}_{2^t m, 2^t n^{(p)}}$ by repeating the size-extension operation t times, where $p, t = 0, 1, 2, ...$

In this chapter, we will alternately use either "the constructed MO complementary set matrix" or, simply, $\mathbf{M}^{2^{t+1}}_{2^t m, 2^t n^{(p)}}$ when referring to the above constructed MO complementary set matrix.

4.3.2 Companion Pair Design

Proposition 4.1: Let us arrange the elements of $\mathbf{c}_0 = (c_{1,0}, c_{2,0}, ...c_{m,0})$ into $m/2$ arbitrary pairs, e.g., $(c_{x,0}, c_{y,0})$. Then, its companion sequence $\mathbf{c}_1 = (c_{1,1}, c_{2,1}, ...c_{m,1})$ can be constructed as either

$$c_{x,1} = c^*_{y,0}, \quad c_{y,1} = -c^*_{x,0} \tag{4.6}$$

or

$$c_{x,1} = -c^*_{y,0}, \quad c_{y,1} = c^*_{x,0} . \tag{4.7}$$

Proof. Let us assume $\mathbf{c}_1$ is constructed using (4.6). In this case,

$$\mathbf{C}^{(0)} = \begin{bmatrix} \mathbf{c}_0 \\ \mathbf{c}_1 \end{bmatrix}^T = \begin{bmatrix} \cdots & c_{x,0} & \cdots & c_{y,0} & \cdots \\ \cdots & c^*_{y,0} & \cdots & -c^*_{x,0} & \cdots \end{bmatrix}^T . \tag{4.8}$$

Here, $\mathbf{r}_x^{(0)} = (c_{x,0}, c^*_{y,0})$ and $\mathbf{r}_y^{(0)} = (c_{y,0}, -c^*_{x,0})$, respectively, the xth and the yth row sequence of $\mathbf{C}^{(0)}$ form a complementary pair, since

$$A_{\mathbf{r}_x^{(0)}}(l) + A_{\mathbf{r}_y^{(0)}}(l) = 0,\ 1 \le l < 2. \tag{4.9}$$

Based on *Lemma 2.2*, $\mathbf{C}^{(0)}$ is a complementary set matrix consisting of $m/2$ complementary pairs. Hence, $\mathbf{c}_0$ and $\mathbf{c}_1$ form a companion pair. □

Example 4.1: $f_i^*(\mathbf{c}_0)$ is a companion of $\mathbf{c}_0$, since

$$\mathbf{C}^{(0)} = \begin{bmatrix} \mathbf{c}_0 \\ f_i^*(\mathbf{c}_0) \end{bmatrix}^T = \begin{bmatrix} c_{1,0} & c_{2,0} & c_{3,0} & c_{4,0} & \cdots & c_{m-1,0} & c_{m,0} \\ c^*_{2,0} & -c^*_{1,0} & c^*_{4,0} & -c^*_{3,0} & \cdots & c^*_{m,0} & -c^*_{m-1,0} \end{bmatrix}^T \tag{4.10}$$

is a companion matrix. It can be verified that $f_c^*(\mathbf{c}_0)$ is also a companion of $\mathbf{c}_0$.

Remark: A companion pair of length n consists of $n/2$ Quads of the following types

$$\begin{bmatrix} c & d \\ -d^* & c^* \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} c & d \\ d^* & -c^* \end{bmatrix}, \tag{4.11}$$

where $c, d \in \mathcal{C}$. (4.11) is a complex generalization of binary Quads proposed in [47]. Note that binary Golay pair of length n are formed by $n/2$ binary Quads nested in a special way (e.g., see *Proposition 2.1* in [52]). This "nested" condition is not required by companion pairs. Hence, binary Golay pair is a special case of the companion pair.

The companion pair has the following properties,

Property 4.1 (Commutation): If $\mathbf{c}_0$ is a companion of $\mathbf{c}_1$, then $\mathbf{c}_1$ is also a companion of $\mathbf{c}_0$.

Proof. In Eq. (4.1), $\mathbf{C}$ is still a companion matrix when the column sequences $\mathbf{a}$ and $\mathbf{b}$ are switched. □

Property 4.2 (Inner product): If $\mathbf{c}_0$ and $\mathbf{c}_1$ form a companion pair, then $\mathbf{c}_0 \cdot \mathbf{c}_1 = 0$.

Proof. Based on Eqs. (4.6) and (4.7), $\sum_{i=1}^{m} c_{i,0}c_{i,1} = 0$. □

Property 4.3 (Negation property): If $(\mathbf{c}_0, \mathbf{c}_1)$ forms a companion pair, then $(-\mathbf{c}_0, \mathbf{c}_1)$, $(\mathbf{c}_0, -\mathbf{c}_1)$ and $(-\mathbf{c}_0, -\mathbf{c}_1)$ are also companion pairs, and they are considered as the same companion pair.

Proof. Based on Eqs. (4.6) and (4.7), $\pm\sum_{i=1}^{m} c_{i,0}c_{i,1} = 0$. □

Corollary 4.1: Binary sequences $\mathbf{c}_0$ and $\mathbf{c}_1$ form a companion pair, if and only if $\mathbf{c}_0 \cdot \mathbf{c}_1 = 0$.

Proof. From Property 4.2, $\mathbf{c}_0 \cdot \mathbf{c}_1 = 0$ is for any companion pair. Furthermore, if $\mathbf{c}_0$ and $\mathbf{c}_1$ are binary sequences such that $\mathbf{c}_0 \cdot \mathbf{c}_1 = 0$, there must exist $m/2$ pairs of (x, y) satisfying $c_{x,0}c_{x,1} + c_{y,0}c_{y,1} = 0$, where $1 \leq x \leq m$ and $1 \leq y \leq m$. Hence, row sequences of $\mathbf{C}^{(\mathbf{0})}$ can be arranged into $m/2$ pairs, where each pair $\mathbf{r}_x^{(0)} = (c_{x,0}, c_{x,1})$ and $\mathbf{r}_y^{(0)} = (c_{y,0}, c_{y,1})$ satisfies $A_{\mathbf{r}_x^{(0)}}(l) + A_{\mathbf{r}_y^{(0)}}(l) = 0$, $1 \leq l < 2$. Consequently, $\mathbf{C}^{(0)}$ is a companion matrix. □

4.3.3 Column Sequence Properties

Lemma 4.1: All column sequences of the constructed complementary set matrix $\mathbf{M}^2_{m,n^{(p)}} = \left[\mathbf{C}^{(p)}\ \ \mathbf{D}^{(p)}\right]$ are in $\mathcal{R}^{(0)}_{\mathbf{c}_0,\mathbf{c}_1} = \{\pm\mathbf{c}_0, \pm\mathbf{c}_1\}$, where $(\mathbf{c}_0, \mathbf{c}_1)$ is the companion pair for the construction.

Proof. Let

$$\mathbf{C}^{(p)} = \begin{bmatrix} \mathbf{c}_0^{(p)} \\ \mathbf{c}_1^{(p)} \\ \vdots \\ \mathbf{c}_{n^{(p)}-1}^{(p)} \end{bmatrix}^T . \tag{4.12}$$

For $\mathbf{C}^{(p+1)}$ constructed by the length-extension (2.22), we have,

$$\mathbf{c}_i^{(p+1)} = \begin{cases} \mathbf{c}_i^{(p)} & 0 \leq i \leq n^{(p)} - 1 \\ -\mathbf{c}_{i-n^{(p)}}^{(p)} & n^{(p)} \leq i \leq \frac{3}{2}n^{(p)} - 1 \\ \mathbf{c}_{i-n^{(p)}}^{(p)} & \frac{3}{2}n^{(p)} \leq i \leq 2n^{(p)} - 1 \end{cases} . \tag{4.13}$$

It follows that column sequences of $\mathbf{C}^{(p+1)}$ are equal to, or are a negation of, column sequences of $\mathbf{C}^{(p)}$. Since column sequences of $\mathbf{C}^{(0)}$ are $\mathbf{c}_0$ and $\mathbf{c}_1$, all column sequences of $\mathbf{C}^{(p)}$, $p = 0, 1, 2, ...$, are in $\mathcal{R}^{(0)}_{\mathbf{c}_0,\mathbf{c}_1}$. In addition, for $\mathbf{C}^{(p+1)}$ constructed using the length-extension (2.23), the interleaving of the corresponding row sequences doesn't change the column sequences and, thus, its column sequences are also in $\mathcal{R}^{(0)}_{\mathbf{c}_0,\mathbf{c}_1}$. Any column sequence of $\mathbf{D}^{(p)}$ can be found in $\mathbf{C}^{(p+1)}$ and, consequently, it is in $\mathcal{R}^{(0)}_{\mathbf{c}_0,\mathbf{c}_1}$ as well. □

Lemma 4.2: All column sequences of the constructed MO complementary set matrix $\mathbf{M}^{2^{t+1}}_{2^t m, 2^t n^{(p)}}$ are in $\mathcal{R}^{(t)}_{\mathbf{c}_0,\mathbf{c}_1}$, where $(\mathbf{c}_0, \mathbf{c}_1)$ is the companion pair and $t, p = 0, 1, 2, ...$

Proof. Let $\{\mathbf{u}^{(t)}_i, 0 \leq i < r = 2^{2t+1} n^{(p)}\}$ denote column sequences of $\mathbf{M}^{2^{t+1}}_{2^t m, 2^t n^{(p)}}$. The size-extension (2.24) implies,

$$\begin{cases} \mathbf{u}^{(t+1)}_i & = -\mathbf{u}^{(t+1)}_{i+2r} & = \mathbf{u}^{(t)}_i(-\mathbf{u}^{(t)}_i) \\ \mathbf{u}^{(t+1)}_{i+r} & = \mathbf{u}^{(t+1)}_{i+3r} & = \mathbf{u}^{(t)}_i \mathbf{u}^{(t)}_i \end{cases}, \tag{4.14}$$

where $0 \leq i < r$. Based on *Lemma 4.1*, $\mathbf{u}^{(0)}_i$, i.e., column sequences of $\mathbf{M}^2_{m,n^{(p)}} = [\mathbf{C}^{(p)}\ \mathbf{D}^{(p)}]$, are in $\mathcal{R}^{(0)}_{\mathbf{c}_0,\mathbf{c}_1}$. From (4.2), (4.3), and (4.14), we have that

$$\mathbf{u}^{(t)}_i \in \mathcal{R}^{(t)}_{\mathbf{c}_0,\mathbf{c}_1}, \ \ t = 0, 1, 2, ... \tag{4.15}$$

When $\mathbf{M}^{2^{t+1}}_{2^t m, 2^t n^{(p)}}$ is constructed using size-extension (2.25), the proof is analogous. □

4.4 Properties of the Constructed Complementary Set Matrix

In this section, column correlation properties of the constructed MO complementary set matrix are related to ACFs of the companion pair. We illustrate how to satisfy a column correlation constraint by selecting an appropriate companion pair. We also construct the companion pair which leads to complementary set matrices with Golay column sequences. Since number of zeros in an energy-normalized sequence can affect its PAPR (see e.g. [6, 15]), we also discuss the number of zeros in column sequences at the end of this section.

4.4.1 Column Correlation Properties

Theorem 4.1: MO complementary set matrix $\mathbf{M}^{2^{t+1}}_{2^t m, 2^t n^{(p)}}$ satisfies a column correlation constraint, if and only if the companion pair $(\mathbf{c}_0, \mathbf{c}_1)$ is selected so that all sequences in

$\mathcal{R}^{(t)}_{\mathbf{c}_0,\mathbf{c}_1}$ satisfy the constraint.

Proof. The proof is a direct consequence of *Lemma 4.2.* □

Corollary 4.2: Complementary set matrices $\mathbf{C}^{(p)}$ and $\mathbf{D}^{(p)}$ constructed from a companion pair $(\mathbf{c}_0, \mathbf{c}_1)$, satisfy a column correlation constraint, if and only if $\mathbf{c}_0$ and $\mathbf{c}_1$ satisfy the constraint.

Proof. The proof follows by setting $t = 0$ in *Theorem 4.1.* □

The minimum achievable column correlation constraint for the constructed MO complementary set matrix $\mathbf{M}^{2^{t+1}}_{2^t m, 2^t n^{(p)}}$ is a function of its size and alphabet and can be expressed as follows,

$$\lambda_{min}(t,m) = \min\left\{\max\left\{\lambda_{\mathbf{c}} : \mathbf{c} \in \mathcal{R}^{(t)}_{\mathbf{c}_0,\mathbf{c}_1}\right\} : (\mathbf{c}_0, \mathbf{c}_1) \text{ is a companion pair}, \mathbf{c}_0, \mathbf{c}_1 \in \mathcal{X}(m)\right\},$$

where λ is any autocorrelation merit. The following *Lemma 4.3* is a key in relating column correlation constraints to the correlation of the companion pair. In particular, it leads to the minimum achievable column correlation constraint $\lambda^A_{min}(t, m)$.

Lemma 4.3: The ACF of any column sequence $\mathbf{u}^{(t)}_i$ of $\mathbf{M}^{2^{t+1}}_{2^t m, 2^t n^{(p)}}$ can be expressed in terms of the ACF of $\mathbf{c}_0$ or $\mathbf{c}_1$, recursively, as follows,

$$A_{\mathbf{u}^{(v+1)}_j}(l) = \begin{cases} 2A_{\mathbf{u}^{(v)}_i}(l) \pm A_{\mathbf{u}^{(v)}_i}(s-l), & 0 \le l < s; \\ \pm A_{\mathbf{u}^{(v)}_i}(l-s), & s \le l < 2s. \end{cases} \tag{4.16}$$

where '+' holds for $j = i + r$ or $j = i + 3r$, '−' holds for $j = i$ or $j = i + 2r$, when size-extension (2.24) is used; '+' holds for $j = 2i + 1$, '−' holds for $j = 2i$, when size-extension (2.25) is employed; $\mathbf{u}^{(0)}_i \in \{\mathbf{c}_0, \mathbf{c}_1, -\mathbf{c}_0, -\mathbf{c}_1\}$; $0 \le i < r = 2^{2v+1} n^{(p)}$, $s = 2^v m$, and $v = 0, 1, 2, ..., t-1$.

Proof. Eq. (4.16) can be derived based on (4.14). Note that, $\mathbf{u}^{(0)}_i \in \mathcal{R}^{(0)}_{\mathbf{c}_0,\mathbf{c}_1} = \{\mathbf{c}_0, \mathbf{c}_1, -\mathbf{c}_0, -\mathbf{c}_1\}$ and the negation of a sequence doesn't change its ACF. □

Similar recursive equations can be found for periodic ACFs based on Eq. (2.3).

Proposition 4.2 (*A sufficient condition for* S^A): Let $S^A_{\mathbf{c}_0} \leq S^A_0$, $S^A_{\mathbf{c}_1} \leq S^A_0$, and $A_{\mathbf{c}_0}(0) = A_{\mathbf{c}_1}(0) = E$, where $(\mathbf{c}_0, \mathbf{c}_1)$ is a companion pair. Then, a sufficient condition for $\mathbf{M}^{2^{t+1}}_{2^t m, 2^t n^{(p)}}$ to satisfy the column correlation constraint S^A_t is

$$S^A_t \geq 4^t S^A_0 + 2^{t-1}(2^t - 1)E \ . \tag{4.17}$$

Proof. Let $\{\mathbf{u}^{(t)}_i, 0 \leq i < 2^{2t+1}n^{(p)}\}$ be the column sequences of $\mathbf{M}^{2^{t+1}}_{2^t m, 2^t n^{(p)}}$. Clearly, $\mathbf{u}^{(0)}_i \in \{\mathbf{c}_0, \mathbf{c}_1, -\mathbf{c}_0, -\mathbf{c}_1\}$. Then, based on (4.16), we have that

$$\begin{aligned} S^A_{\mathbf{u}} &= \max_i \left\{ S^A_{\mathbf{u}^{(t)}_i}, 0 \leq i < 2^{2t+1}n^{(p)} \right\} \\ &= \max_i \left\{ \sum_{l=1}^{2^t m-1} |A_{\mathbf{u}^{(t)}_i}(l)|, 0 \leq i < 2^{2t+1}n^{(p)} \right\} \\ &\leq 4^t S^A_0 + 2^{t-1}(2^t - 1)E, \end{aligned} \tag{4.18}$$

for $t = 0, 1, 2, ...$ Hence, (4.17) is sufficient for $S^A_t \geq S^A_{\mathbf{u}}$ which proves the proposition. □

Proposition 4.3 (*A sufficient condition for* λ^A): Let $\lambda^A_{\mathbf{c}_0} \leq \lambda^A_0$, $\lambda^A_{\mathbf{c}_1} \leq \lambda^A_0$, and $A_{\mathbf{c}_0}(0) = A_{\mathbf{c}_1}(0) = E$, where $(\mathbf{c}_0, \mathbf{c}_1)$ is a companion pair. Then, a sufficient condition for $\mathbf{M}^{2^{t+1}}_{2^t m, 2^t n^{(p)}}$ to satisfy the column correlation constraint λ^A_t is

$$\lambda^A_t \geq \max\left\{(2^t - 1)E,\ (2^{t+1} - 1)\lambda^A_0\right\} . \tag{4.19}$$

Proof. Let $\{\mathbf{u}^{(t)}_i, 0 \leq i < 2^{2t+1}n^{(p)}\}$ be the column sequences of $\mathbf{M}^{2^{t+1}}_{2^t m, 2^t n^{(p)}}$. (4.16) implies

$$\begin{aligned} \lambda^A_{\mathbf{u}} &= \max_i \left\{ \lambda^A_{\mathbf{u}^{(t)}_i}, 0 \leq i < 2^{2t+1}n^{(p)} \right\} \\ &= \max_{l,i} \left\{ |A_{\mathbf{u}^{(t)}_i}(l)|, 1 \leq l < 2^t m, 0 \leq i < 2^{2t+1}n^{(p)} \right\} \\ &\leq \max\left\{(2^t - 1)E, (2^{t+1} - 1)\lambda^A_0\right\} \end{aligned} \tag{4.20}$$

where $t = 0, 1, 2, ...$ Hence, if (4.19) holds, we have $\lambda^A_t \geq \lambda^A_{\mathbf{u}}$. □

Proposition 4.4 (*A necessary condition for* λ^A): Let $A_{\mathbf{c}_0}(0) = A_{\mathbf{c}_1}(0) = E$, where $(\mathbf{c}_0, \mathbf{c}_1)$ is a companion pair. An achievable column correlation constraint λ^A_t of $\mathbf{M}^{2^{t+1}}_{2^t m, 2^t n^{(p)}}$ must satisfy

$$\lambda^A_t \geq (2^t - 1)E. \tag{4.21}$$

Proof. (4.16) implies that $|A_{\mathbf{u}_k^{(t)}}(m)| = (2^t - 1)E$ for $k = 2^{2t+1}n^{(p)} - 1$. Hence, $\lambda_t^A \geq \max_{l,i}\{|A_{\mathbf{u}_i^{(t)}}(l)|, 1 \leq l < 2^t m, 0 \leq i < 2^{2t+1}n^{(p)}\} \geq (2^t - 1)E$. $\square$

Corollary 4.3: Let $A_{\mathbf{c}_0}(0) = A_{\mathbf{c}_1}(0) = E$ and $\lambda_0^A = \max\{\lambda_{\mathbf{c}_0}^A, \lambda_{\mathbf{c}_1}^A\}$, where $(\mathbf{c}_0, \mathbf{c}_1)$ is a companion pair. For $\mathbf{M}^{2^{t+1}}_{2^t m, 2^t n^{(p)}}$, $t \geq 1$, when

$$\lambda_0^A \leq \frac{2^t - 1}{2^{t+1} - 1} E, \tag{4.22}$$

the minimum column correlation constraint

$$\lambda_{min}^A(t, m) = (2^t - 1)E \tag{4.23}$$

is achievable.

Proof. If (4.22) holds, based on *Proposition 4.3*, $\mathbf{M}^{2^{t+1}}_{2^t m, 2^t n^{(p)}}$ satisfies the column correlation constraint $\lambda_t^A = (2^t - 1)E$. On the other hand, *Proposition 4.4* states that $\lambda_t^A \geq (2^t - 1)E$ must hold. $\square$

Example 4.2: To construct the complex-valued complementary set matrices $\mathbf{C}^{(p)}$ and $\mathbf{D}^{(p)}$ with a column correlation constraint $\lambda^A = 1$, we can choose a companion pair $\mathbf{c}_0 = (+, j, -, j)$ and $\mathbf{c}_1 = f_i^*(\mathbf{c}_0) = (\bar{j}, -, \bar{j}, +)$, where $+$ denotes 1, $-$ denotes -1, $j = \sqrt{-1}$ denotes the imaginary unit and $\bar{j}$ denotes $-j$, which satisfy $\lambda_{\mathbf{c}_i}^A \leq 1$, $i = 0, 1$. Then, the companion matrix is

$$\mathbf{C}^{(0)} = \begin{bmatrix} + & j & - & j \\ \bar{j} & - & \bar{j} & + \end{bmatrix}^T. \tag{4.25}$$

$$\mathbf{M}^4_{8,8} = \left[\begin{array}{c|c|c|c}
+\bar{j}-\bar{j}-j-\bar{j} & +\bar{j}-\bar{j}-j-\bar{j} & -j+j+\bar{j}+j & +\bar{j}-\bar{j}-j-\bar{j} \\
j-\bar{j}-\bar{j}+\bar{j}- & j-\bar{j}-\bar{j}+\bar{j}- & \bar{j}+j+j-j+ & j-\bar{j}-\bar{j}+\bar{j}- \\
-\bar{j}+\bar{j}+j+\bar{j} & -\bar{j}+\bar{j}+j+\bar{j} & +j-j-\bar{j}-j & -\bar{j}+\bar{j}+j+\bar{j} \\
j+\bar{j}+\bar{j}-\bar{j}+ & j+\bar{j}+\bar{j}-\bar{j}+ & \bar{j}-j-j+j- & j+\bar{j}+\bar{j}-\bar{j}+ \\
-j+j+\bar{j}+j & +\bar{j}-\bar{j}-j-\bar{j} & +\bar{j}-\bar{j}-j-\bar{j} & +\bar{j}-\bar{j}-j-\bar{j} \\
\bar{j}+j+j-j+ & j-\bar{j}-\bar{j}+\bar{j}- & j-\bar{j}-\bar{j}+\bar{j}- & j-\bar{j}-\bar{j}+\bar{j}- \\
+j-j-\bar{j}-j & -\bar{j}+\bar{j}+j+\bar{j} & -\bar{j}+\bar{j}+j+\bar{j} & -\bar{j}+\bar{j}+j+\bar{j} \\
\bar{j}-j-j+j- & j+\bar{j}+\bar{j}-\bar{j}+ & j+\bar{j}+\bar{j}-\bar{j}+ & j+\bar{j}+\bar{j}-\bar{j}+
\end{array}\right]_{8\times 32} \tag{4.24}$$

By employing length-extension (2.22), the complementary set matrix $\mathbf{C}^{(1)}$ and its mate $\mathbf{D}^{(1)}$ can be obtained as

$$\mathbf{C}^{(1)} = \begin{bmatrix} + & \bar{j} & - & \bar{j} \\ j & - & \bar{j} & - \\ - & \bar{j} & + & \bar{j} \\ j & + & \bar{j} & + \end{bmatrix}_{4\times 4}, \ \mathbf{D}^{(1)} = \begin{bmatrix} - & j & - & \bar{j} \\ \bar{j} & + & \bar{j} & - \\ + & j & + & \bar{j} \\ \bar{j} & - & \bar{j} & + \end{bmatrix}_{4\times 4}. \tag{4.26}$$

Based on *Corollary 4.2*, $\mathbf{C}^{(1)}$ and $\mathbf{D}^{(1)}$ are complementary set matrices with a column correlation constraint $\lambda^A = 1$.

Example 4.3: Let again $\mathbf{c}_0 = (+, j, -, j)$ and $\mathbf{c}_1 = f_i^*(\mathbf{c}_0) = (\bar{j}, -, \bar{j}, +)$, then, any sequence $\mathbf{c}$ in $\mathcal{R}^{(1)}_{\mathbf{c}_0,\mathbf{c}_1}$ satisfies $\lambda^A_{\mathbf{c}} \leq \lambda^A = 4$. Starting with $\mathbf{M}^2_{4,4} = [\mathbf{C}^{(1)}\ \mathbf{D}^{(1)}]$ constructed in *Example 4.2* and applying the size-extension (2.24), we obtain $\mathbf{M}^4_{8,8}$ as shown in (4.24). Based on *Theorem 4.1*, $\mathbf{M}^4_{8,8}$ satisfies a column correlation constraint $\lambda^A = 4$. On the other hand, *Corollary 4.3* implies that $\mathbf{M}^4_{8,8}$ achieves its lower bound $\lambda^A_1 = E = 4$, since $\lambda^A_0 = 1 \leq \frac{E}{3}$.

Example 4.4: Let us consider how to construct $\mathbf{M}^4_{8,8}$ satisfying a column correlation constraint $S^A = 12$. Since $t = 1$ and $m = 4$, based on *Proposition 4.2*, a sufficient condition for $S^A_1 = 12$ is $S^A_0 \leq 2$, for $E = 4$. The companion pair $(\mathbf{c}_0, \mathbf{c}_1)$ in *Examples 4.2-4.3* satisfies $S^A_{\mathbf{c}_i} \leq S^A_0 = 2$, for $i = 0, 1$. Thus, $\mathbf{M}^4_{8,8}$ in (4.24) must also satisfy a column correlation constraint $S^A = 12$. Let $\{\mathbf{u}^{(1)}_i, 0 \leq i \leq 31\}$ denote column sequences of $\mathbf{M}^4_{8,8}$ in (4.24), we can verify that $\max\left\{S^A_{\mathbf{u}^{(1)}_i}, 0 \leq i \leq 31\right\} = 12$.

Remark: For the case $t = 0$, *Corollary 4.2* implies that the correlation constraint for the companion pair is also the column correlation constraint of $\mathbf{M}^2_{m,n^{(p)}} = [\mathbf{C}^{(p)}\ \mathbf{D}^{(p)}]$. For $t \geq 1$, based on *Proposition 4.2*, small S^A_0 may also help in reducing the column correlation constraint S^A_t. However, *Corollary 4.3* implies that it is not necessary to search for the companion pair with smaller λ^A_0, once the lower bound $\lambda^A_{min}(t, m) = (2^t - 1)E$ has been achieved.

4.4.2 Golay Column Sequences

Theorem 4.2: Column sequences of complementary set matrices $\mathbf{C}^{(p)}$ and $\mathbf{D}^{(p)}$ are Golay sequences, if and only if the companion sequences $\mathbf{c}_0$ and $\mathbf{c}_1$ are both Golay.

Proof. Lemma 4.1 states that column sequences of $\mathbf{C}^{(p)}$ and $\mathbf{D}^{(p)}$ are either $\pm\mathbf{c}_0$ or $\pm\mathbf{c}_1$. Note that, a negation of a Golay sequence is also a Golay sequence. $\square$

We present a constructive method to obtain the companion pair from which an m by n complementary set matrix with Golay column sequences can be constructed, where $m = 2^{q+1}$, $n = 2^{p+1}$, $p, q = 0, 1, 2....$

Theorem 4.3: Let

$$\mathbf{H}_i^{(q)} = \begin{bmatrix} \mathbf{h}_{i,0}^{(q)} \\ \mathbf{h}_{i,1}^{(q)} \end{bmatrix}_{2\times 2^q} = \begin{bmatrix} \mathbf{h}_{i,0}^{(q-1)} \overleftarrow{\mathbf{h}_{i,1}^{(q-1)}} \\ \mathbf{h}_{i,1}^{(q-1)} (-\overleftarrow{\mathbf{h}_{i,0}^{(q-1)}}) \end{bmatrix}, \tag{4.27}$$

where $\mathbf{h}_{i,0}^{(q)}$ and $\mathbf{h}_{i,1}^{(q)}$ are two row sequences of $\mathbf{H}_i^{(q)}$, $i = 0, 1$. The initial matrices are

$$\mathbf{H}_0^{(0)} = \begin{bmatrix} + & + \\ + & - \end{bmatrix}_{2\times 2}, \quad \mathbf{H}_1^{(0)} = \begin{bmatrix} + & - \\ + & + \end{bmatrix}_{2\times 2}. \tag{4.28}$$

Then, $\{\mathbf{h}_{0,0}^{(q)}, \mathbf{h}_{0,1}^{(q)}\}$ and $\{\mathbf{h}_{1,0}^{(q)}, \mathbf{h}_{1,1}^{(q)}\}$ are respectively Golay complementary pairs and, furthermore, $f_i(\mathbf{h}_{0,0}^{(q)}) = \mathbf{h}_{1,0}^{(q)}$ and $f_i(\mathbf{h}_{0,1}^{(q)}) = -\mathbf{h}_{1,1}^{(q)}$, for $q = 0, 1, 2....$

Proof. See Appendix. $\square$

Example 4.5: Let $q = 2$, then $\mathbf{c}_0 = \mathbf{h}_{0,0}^{(2)} = (++-+---+)$ and $\mathbf{c}_1 = \mathbf{h}_{1,0}^{(2)} = (+-++-+++)$. Based on *Theorem 4.3*, $\{\mathbf{c}_0, \mathbf{h}_{0,1}^{(2)}\}$ and $\{\mathbf{c}_1, \mathbf{h}_{1,1}^{(2)}\}$ are, respectively, Golay complementary pairs, where $\mathbf{h}_{0,1}^{(2)} = (+----+--)$ and $\mathbf{h}_{1,1}^{(2)} = (+++---+-)$. Thus, the companion sequences $\mathbf{c}_0$ and $\mathbf{c}_1$ are Golay sequences. The length-extension (2.23) for $p = 2$ allows for constructing the following complementary set matrix

$$\mathbf{C}^{(2)} = \begin{bmatrix} + & - & - & - & + & - & + & + \\ + & - & - & - & - & + & - & - \\ - & + & + & + & + & - & + & + \\ + & - & - & - & + & - & + & + \\ - & + & + & + & - & + & - & - \\ - & + & + & + & + & - & + & + \\ - & + & + & + & + & - & + & + \\ + & - & - & - & + & - & + & + \end{bmatrix}_{8\times 8} \tag{4.29}$$

whose column sequences are Golay. Hence, the PAPR of all column sequences of $\mathbf{C}^{(2)}$ is at most two [46].

4.4.3 Number of Zeros of Ternary Complementary Sets

Proposition 4.5: Let the companion sequence $\mathbf{c}_0$ be a length m sequence with z zeros, then, any column sequence of $\mathbf{M}^{2^{t+1}}_{2^t m, 2^t n^{(p)}}$ contains $2^t z$ zeros.

Proof. Based on Eqs. (4.6) and (4.7), $\mathbf{c}_0$ and its companion $\mathbf{c}_1$ have the same number of zeros. The number of zeros in each column sequence does not change after each length-extension operation. Each size-extension operation doubles the length of column sequences, as well as the number of zeros. □

Example 4.6: In this example, we consider a ternary complementary set matrix and its mate with a column correlation constraint $S^A = 5$. Let us set $m = 8$ and $z = 1$. We can find the companion pair $\mathbf{c}_0 = (+--+++0+)$ and $\mathbf{c}_1 = f_i(\mathbf{c}_0) = (--+++-+0)$ which satisfy $S^A_{\mathbf{c}_i} \leq 5$, $i = 0, 1$. The companion matrix is

$$\mathbf{C}^{(0)} = \begin{bmatrix} +--+++\ 0+ \\ --+++-+0 \end{bmatrix}^T. \tag{4.30}$$

Using length-extension (2.22), we extend $\mathbf{C}^{(0)}$ as

$$\mathbf{C}^{(2)} = \begin{bmatrix} + & - & - & - & - & + & - & - \\ - & - & + & - & + & + & + & - \\ - & + & + & + & + & - & + & + \\ + & + & - & + & - & - & - & + \\ + & + & - & + & - & - & - & + \\ + & - & - & - & - & + & - & - \\ 0 & + & 0 & + & 0 & - & 0 & + \\ + & 0 & - & 0 & - & 0 & - & 0 \end{bmatrix}_{8\times 8} \quad \mathbf{D}^{(2)} = \begin{bmatrix} - & + & + & + & - & + & - & - \\ + & + & - & + & + & + & + & - \\ + & - & - & - & + & - & + & + \\ - & - & + & - & - & - & - & + \\ - & - & + & - & - & - & - & + \\ - & + & + & + & - & + & - & - \\ 0 & - & 0 & - & 0 & - & 0 & + \\ - & 0 & + & 0 & - & 0 & - & 0 \end{bmatrix}_{8\times 8} \tag{4.31}$$

Based on *Theorem 4.1*, complementary set matrices $\mathbf{C}^{(2)}$ and $\mathbf{D}^{(2)}$ satisfy the column correlation constraint $S^A = 5$. Furthermore, by setting $t = 0$ in *Proposition 4.5*, we have that any column sequence of $\mathbf{C}^{(p)}$ and $\mathbf{D}^{(p)}$ has only one zero.

4.5 Search for Companion Pairs

4.5.1 Exhaustive Search Algorithm

Let $\mathcal{X}(n, \lambda)$ denote a subset of all sequences of $\mathcal{X}(n)$ which satisfy the correlation constraint λ. For example, let $\mathcal{B}(2^t m) = \{\mathbf{c} \,|\, c_i \in \mathcal{B}, 1 \leq i \leq 2^t m\}$, then $\mathcal{B}(2^t m, S^A) = \{\mathbf{c} \,|\, \mathbf{c} \in \mathcal{B}(2^t m), S_{\mathbf{c}}^A \leq S^A\}$, where $\mathcal{B} = \{+1, -1\}$. Clearly, all column sequences of binary MO complementary set matrix $\mathbf{M}_{2^t m, 2^t n^{(p)}}^{2^{t+1}}$ with a column correlation constraint S^A can be found in $\mathcal{B}(2^t m, S^A)$. Hence, to construct a MO complementary set matrix $\mathbf{M}_{2^t m, 2^t n^{(p)}}^{2^{t+1}}$ whose column sequences are in $\mathcal{X}(n, \lambda)$, we need to find a companion pair $(\mathbf{c}_0, \mathbf{c}_1)$ such that $\mathcal{R}_{\mathbf{c}_0, \mathbf{c}_1}^{(t)} \subseteq \mathcal{X}(n, \lambda)$ (see *Lemma 4.2*).

Let us index all $K = |\mathcal{X}(m)|$ sequences as $\mathbf{x}_i$, for $1 \leq i \leq K$. When a column correlation constraint λ is given, desired companion pairs can be obtained by exhaustive computer search over $\mathcal{X}(m)$, as described in Table 4.1.

Table 4.1: Exhaustive Search Algorithm

```
j=0;
for   i = 1, 2, 3, ..., K   loop
    if   R_{x_i}^{(t)} ⊆ X(2^t m, λ),
        j = j + 1; y_j = x_i;
        for   l = j − 1, j − 2, ...1,
            check if (y_j, y_l) is a companion pair;
        end
    end
end   loops
```

Note that the ACFs of sequences in $\mathcal{R}_{\mathbf{x}_i}^{(t)}$ can be computed recursively using (4.16). For binary sequences, we can simply check if $\mathbf{x} \cdot \mathbf{y} = 0$ to determine the companion pair.

4.5.2 Minimum Achievable Column Correlation Constraint

The exhaustive search algorithm can be easily modified to search for the companion pair with a minimum achievable column correlation constraint. However, the computing load is heavy, especially for large m and t. Let $t = 0$ in (4.16), the companion pair for the construction of $\mathbf{M}_{m, n^{(p)}}^2 = [\mathbf{C}^{(p)} \;\; \mathbf{D}^{(p)}]$ with a minimum achievable column correlation constraint is

$$
\begin{aligned}
(\mathbf{c}_0, \mathbf{c}_1) \quad = \quad & \arg \min_{(\mathbf{x},\mathbf{y})} \{\max\{\lambda_{\mathbf{x}}, \lambda_{\mathbf{y}}\} : \\
& (\mathbf{x}, \mathbf{y}) \text{ is a companion pair and } \mathbf{x}, \mathbf{y} \in \mathcal{X}(m)\} . \qquad (4.32)
\end{aligned}
$$

Based on *Propositions 4.2-4.4*, the above companion pair may also lead to MO complementary set matrix $\mathbf{M}^{2^{t+1}}_{2^t m, 2^t n^{(p)}}$ with a reduced column correlation constraint for $t \geq 1$. Hence, in the following, we consider companion pairs leading to an achievable or a minimum achievable column correlation constraint $\lambda_{min}(m) = \lambda_{min}(t = 0, m)$ for the case $t = 0$ only.

In Table 4.2, λ^A and S^A denote the minimum autocorrelation constraints for arbitrary binary sequences. λ^A_{min} and S^A_{min} denote the minimum achievable column correlation constraints for binary companion pairs. N_u, counted up to the commutative property 4.1 and the negation property 4.3, denotes the number of corresponding companion pairs. It can be observed that, for lengths up to 28, the companion pairs can achieve the minimum autocorrelation constraints of binary sequences. Hence, the proposed companion pair based construction method can build optimum binary complementary sets (and their mates) with a minimum λ^A or S^A for column sequences of length at least up to 28. In Table 4.3, a similar conclusion can be drawn for ternary sequences with a single zero element of length at least up to 24. We also can observe that most of these optimum companion pairs can achieve λ^A_{min} and S^A_{min} simultaneously.

When m is large, the exhaustive computer search is infeasible. Hence, the existence of a companion pair satisfying a correlation constraint is an important problem considered in the next section.

4.6 The Existence of Companion Pairs

The existence of a sequence of a desired correlation constraint has been studied in literature. For example, binary sequences with $\lambda^A = 1$ exist only for lengths $2, 3, 4, 5, 7, 11$ and 13, and are called binary Barker sequences; binary m-sequences [2] with $\lambda^P = 1$ exist for length $m = 2^l - 1$, $l = 2, 3, 4....$ In this section, we exploit the correlation properties of companion pairs and analyze their existence for correlation constraints λ^A and λ^P.

4.6.1 The Companion Pair Constructed Using Two Arbitrary Sequences

In the following Cases 1 and 2, we illustrate how a companion pair of length m can be formed using two arbitrary sequences $\mathbf{s}_0$ and $\mathbf{s}_1$ of length $m/2$. We study the correlation properties of the companion pair $(\mathbf{c}_0, \mathbf{c}_1)$ constructed from $\mathbf{s}_0$ and $\mathbf{s}_1$.

Case 1: Let $\mathbf{c}_0 = \mathbf{s}_0 \otimes \mathbf{s}_1$ and $\mathbf{c}_1 = \mathbf{s}_1^* \otimes (-\mathbf{s}_0^*)$. Then $\mathbf{c}_1$ is a companion of $\mathbf{c}_0$ since $\mathbf{c}_1 = f_i^*(\mathbf{c}_0)$. The ACFs of the companion pair can be expressed in terms of the ACFs of $\mathbf{s}_0$ and $\mathbf{s}_1$ and their crosscorrelation functions,

$$A_{\mathbf{c}_0}(l) = \begin{cases} A_{\mathbf{s}_0,\mathbf{s}_1}(\frac{l-1}{2}) + A_{\mathbf{s}_0,\mathbf{s}_1}(\frac{-l-1}{2}), & l \in \text{odd} \\ A_{\mathbf{s}_0}(\frac{l}{2}) + A_{\mathbf{s}_1}(\frac{l}{2}), & l \in \text{even} \end{cases} \tag{4.33}$$

$$A_{\mathbf{c}_1^*}(l) = \begin{cases} -A_{\mathbf{s}_0,\mathbf{s}_1}(\frac{l+1}{2}) - A_{\mathbf{s}_0,\mathbf{s}_1}(\frac{-l+1}{2}), & l \in \text{odd} \\ A_{\mathbf{s}_0}(\frac{l}{2}) + A_{\mathbf{s}_1}(\frac{l}{2}), & l \in \text{even} \end{cases} \tag{4.34}$$

$$P_{\mathbf{c}_0}(l) = \begin{cases} P_{\mathbf{s}_0,\mathbf{s}_1}(\frac{l-1}{2}) + P_{\mathbf{s}_0,\mathbf{s}_1}(\frac{m-l-1}{2}), & l \in \text{odd} \\ P_{\mathbf{s}_0}(\frac{l}{2}) + P_{\mathbf{s}_1}(\frac{l}{2}), & l \in \text{even} \end{cases} \tag{4.35}$$

$$P_{\mathbf{c}_1^*}(l) = \begin{cases} -P_{\mathbf{s}_0,\mathbf{s}_1}(\frac{l+1}{2}) - P_{\mathbf{s}_0,\mathbf{s}_1}(\frac{m-l+1}{2}), & l \in \text{odd} \\ P_{\mathbf{s}_0}(\frac{l}{2}) + P_{\mathbf{s}_1}(\frac{l}{2}), & l \in \text{even} \end{cases} \tag{4.36}$$

where $0 \le l \le m-1$.

Lemma 4.4: Let $\mathbf{c}_0 = \mathbf{s}_0 \otimes \mathbf{s}_1$ and $\mathbf{c}_1^* = \mathbf{s}_1 \otimes (-\mathbf{s}_0)$, then

$$\begin{cases} \lambda_{\mathbf{c}_i}^A \le \max\{\lambda_{\mathbf{s}_0}^A + \lambda_{\mathbf{s}_1}^A, 2\lambda_{\mathbf{s}_0,\mathbf{s}_1}^A\} \\ \lambda_{\mathbf{c}_i}^P \le \max\{\lambda_{\mathbf{s}_0}^P + \lambda_{\mathbf{s}_1}^P, 2\lambda_{\mathbf{s}_0,\mathbf{s}_1}^P\} \end{cases} \tag{4.37}$$

and

$$\begin{cases} S_{\mathbf{c}_i}^A \le S_{\mathbf{s}_0}^A + S_{\mathbf{s}_1}^A + S_{\mathbf{s}_0,\mathbf{s}_1}^A \\ S_{\mathbf{c}_i}^P \le S_{\mathbf{s}_0}^P + S_{\mathbf{s}_1}^P + 2S_{\mathbf{s}_0,\mathbf{s}_1}^P \end{cases}, \tag{4.38}$$

where $i = 0, 1$.

Proof. See Appendix. □

Case 2: Let $\mathbf{c}_0 = \mathbf{s}_0\mathbf{s}_1$ and $\mathbf{c}_1 = \mathbf{s}_1^*(-\mathbf{s}_0^*)$. Then $\mathbf{c}_1$ is a companion of $\mathbf{c}_0$ since

$\mathbf{c}_1 = f_c^*(\mathbf{c}_0)$. The aperiodic ACFs of the companion pair can be expressed as

$$A_{\mathbf{c}_0}(l) = \begin{cases} A_{\mathbf{s}_0}(l) + A_{\mathbf{s}_1}(l) + A_{\mathbf{s}_0,\mathbf{s}_1}(l - \frac{m}{2}), & 0 \leq l < \frac{m}{2} \\ A_{\mathbf{s}_0,\mathbf{s}_1}(l - \frac{m}{2}) & \frac{m}{2} \leq l < m \end{cases} \tag{4.39}$$

$$A_{\mathbf{c}_1}(l) = \begin{cases} A_{\mathbf{s}_0}(l) + A_{\mathbf{s}_1}(l) - A_{\mathbf{s}_0,\mathbf{s}_1}(\frac{m}{2} - l), & 0 \leq l < \frac{m}{2} \\ -A_{\mathbf{s}_0,\mathbf{s}_1}(\frac{m}{2} - l) & \frac{m}{2} \leq l < m \end{cases}. \tag{4.40}$$

Lemma 4.5: Let $\mathbf{c}_0 = \mathbf{s}_0\mathbf{s}_1$ and $\mathbf{c}_1^* = \mathbf{s}_1(-\mathbf{s}_0)$, then

$$\lambda_{\mathbf{c}_i}^A \leq \lambda_{\mathbf{s}_0}^A + \lambda_{\mathbf{s}_1}^A + \lambda_{\mathbf{s}_0,\mathbf{s}_1}^A, \quad i = 0, 1. \tag{4.41}$$

Proof. The proof is along the lines of the proof of *Lemma 4.4*. □

4.6.2 Existence

Without loss of generality, we assume that $\mathbf{s}_i$ are complex-valued sequences of length $m/2$ and $A_{\mathbf{s}_i}(0) = P_{\mathbf{s}_i}(0) = m/2$, $i = 0, 1$.

Lemma 4.6 (Welch bound [12]): Let $\{\mathbf{s}_i, i = 0, 1, ..., K-1\}$, denote a set of K complex-valued sequences of length N. If $A_{\mathbf{s}_i}(0) = P_{\mathbf{s}_i}(0) = N$ for all i, then,

$$P_{max} \geq N\sqrt{\frac{K-1}{NK-1}} \tag{4.42}$$

$$A_{max} \geq N\sqrt{\frac{K-1}{2NK-K-1}} \tag{4.43}$$

where

$$P_{max} = \max_{0 \leq i,j < K, i \neq j} \{\lambda_{\mathbf{s}_i}^P, \lambda_{\mathbf{s}_i,\mathbf{s}_j}^P\} \tag{4.44}$$

$$A_{max} = \max_{0 \leq i,j < K, i \neq j} \{\lambda_{\mathbf{s}_i}^A, \lambda_{\mathbf{s}_i,\mathbf{s}_j}^A\}. \tag{4.45}$$

Proof. The proof can be found in [12]. □

The following *Theorems 4.4-4.5* restate the companion pair existence conditions from *Theorems 4.1-4.2* in terms of the $\{\mathbf{s}_0, \mathbf{s}_1\}$ pair existence conditions from *Lemmas 4.4-4.5*.

Theorem 4.4: MO complementary set matrix $\mathbf{M}^2_{m,n^{(p)}}$ with a column correlation constraint λ^A exists if there exists a sequence pair $\{\mathbf{s}_0, \mathbf{s}_1\}$ with $A_{max} = \frac{1}{2}\lambda^A$.

Proof. *Theorem 4.1* states that MO complementary set matrix $\mathbf{M}^2_{m,n^{(p)}}$ satisfying the column correlation constraint λ^A exists, if and only if we can find a companion pair $(\mathbf{c}_0, \mathbf{c}_1)$, such that,

$$\lambda^A_{\mathbf{c}_i} \leq \lambda^A, \quad i = 0, 1. \tag{4.46}$$

Based on (4.37), a sufficient condition for (4.46) is,

$$\lambda^A_{\mathbf{s}_i} \leq \frac{\lambda^A}{2}, \ i = 0, 1 \ \text{ and } \ \lambda^A_{\mathbf{s}_0,\mathbf{s}_1} \leq \frac{\lambda^A}{2}. \tag{4.47}$$

Based on (4.41),

$$\lambda^A_{\mathbf{s}_i} \leq \frac{\lambda^A}{3}, \ i = 0, 1 \ \text{ and } \ \lambda^A_{\mathbf{s}_0,\mathbf{s}_1} \leq \frac{\lambda^A}{3}. \tag{4.48}$$

Hence, by comparing (4.47) and (4.48), we can set $A_{max} = \frac{1}{2}\lambda^A$. □

Proposition 4.6: Sequence pair $\{\mathbf{s}_0, \mathbf{s}_1\}$ of length $\frac{m}{2}$ with $A_{max} = \frac{1}{2}\lambda^A$ exists only if

$$\lambda^A \geq \frac{m}{\sqrt{2m-3}}. \tag{4.49}$$

Proof. Let $A_{max} = \frac{1}{2}\lambda^A$, $K = 2$, and $N = \frac{m}{2}$ in (4.43) of *Lemma 4.6*, then (4.49) follows. □

Corollary 4.4: Let $\mathbf{c}_0 = \mathbf{s}_0 \otimes \mathbf{s}_1$ and $\mathbf{c}_1 = \mathbf{s}_1 \otimes (-\mathbf{s}_0)$ be a binary companion pair of length m, and $\mathbf{u}_i$ denote column sequences of the constructed $\mathbf{M}^2_{m,n^{(p)}}$, $0 \leq i < 2n^{(p)}$. Then, $\lambda^A_W \leq \lambda^A_{\mathbf{u}} \leq \lambda^A_B$, where

$$\begin{aligned} \lambda^A_{\mathbf{u}} &= \max_i \left\{ \lambda^A_{\mathbf{u}_i^{(t)}}, 0 \leq i < 2n^{(p)} \right\}, && (4.50) \\ \lambda^A_B &= \max \left\{ \lambda^A_{\mathbf{s}_0} + \lambda^A_{\mathbf{s}_1}, 2\lambda^A_{\mathbf{s}_0,\mathbf{s}_1} \right\}, && (4.51) \\ \lambda^A_W &= \lceil \frac{m}{\sqrt{2m-3}} \rceil. && (4.52) \end{aligned}$$

Proof. λ^A_W is derived from *Theorem 4.4* and (4.49) by noting that $\lambda^A_{\mathbf{u}}$ is an integer for binary sequences. λ^A_B follows from *Lemma 4.4*. □

Theorem 4.5: MO complementary set matrix $\mathbf{M}^2_{m,n^{(p)}}$ with a column correlation constraint λ^P exists, if there exists $\{\mathbf{s}_0, \mathbf{s}_1\}$ of length $m/2$ with $P_{max} = \frac{1}{2}\lambda^P$.

Proof. The proof follows along the lines of the proof of *Theorem 4.4* and is omitted. □

Proposition 4.7: A length $m/2$ sequence pair $\{\mathbf{s}_0, \mathbf{s}_1\}$ with $P_{max} = \frac{1}{2}\lambda^P$ exists only if,

$$\lambda^P \geq \frac{m}{\sqrt{m-1}}. \tag{4.53}$$

Proof. Setting $P_{max} = \frac{1}{2}\lambda^P$, $K = 2$ and $N = \frac{m}{2}$ in (4.42) leads to (4.53). □

4.6.3 Achievable Column Correlation Constraints

Theorems 4.4-4.5 suggest searching for sequences $\mathbf{s}_0$ and $\mathbf{s}_1$ of length $m/2$ with good autocorrelation and crosscorrelation merits to form a companion pair with a small achievable column correlation constraint. Former is a long standing problem (e.g. see [48–51]). In [50], good binary sequence pairs with small $\lambda^A_{\mathbf{s}_0}$, $\lambda^A_{\mathbf{s}_1}$ and $\lambda^A_{\mathbf{s}_0,\mathbf{s}_1}$ were found by using simulated annealing search algorithm, and were listed in Tables I and II. Based on *Corollary 4.4*, we present λ^A_W and λ^A_B of their corresponding binary companion pairs in Table 4.4, where the reference [50] indicates that data is obtained by using sequences from this reference. However, the cost function for the simulated annealing in [50] is not optimal in our case. We instead minimize the cost function

$$f(\mathbf{s}_0, \mathbf{s}_1) = \max\left\{\lambda^A_{\mathbf{s}_0} + \lambda^A_{\mathbf{s}_1}, 2\lambda^A_{\mathbf{s}_0,\mathbf{s}_1}\right\} \tag{4.54}$$

to obtain an improved λ^A_B.

In Table 4.5, sequence pairs $\{\mathbf{s}_0, \mathbf{s}_1\}$ of length $m/2 = 63, 84$ and 100 obtained using simulated annealing based on (4.54) are presented. The corresponding ACF merit $\lambda^A_{\mathbf{u}}$ is calculated and compared to that of the sequence pairs from [50]. The proposed sequence pairs lead to companion pairs with an improved autocorrelation correlation merit.

4.7 Conclusion

We have considered a construction algorithm for MO complementary set matrices satisfying a column correlation constraint. The algorithm recursively constructs the MO complementary set matrix, starting from a companion pair. We relate correlation properties of column sequences to that of the companion pair and illustrate how to select an appropriate companion pair to satisfy a given column correlation constraint. We also reveal a method to construct the Golay companion pair which leads to the complementary set matrix with Golay column sequences. An exhaustive computer search algorithm is described which helps in searching for companion pairs. Based on exhaustive search results, the companion pair based construction algorithm leads to optimum binary complementary sets (and their mates) with a minimum column correlation constraint λ^A or S^A for column sequences of length at least up to 28, or leads to optimum ternary complementary sets whose column sequences are ternary sequences with a single zero element of length at least up to 24. Exhaustive search is infeasible for relatively long sequences. Hence, we instead suggest a strategy for finding companion pairs with a small, if not minimum, column correlation constraint. Based on properties of the companion pair, the strategy suggests a search for any two shorter sequences by minimizing a cost function in terms of their autocorrelation and crosscorrelation merits, from which the desired companion pair can be formed. An improved cost function is derived to further reduce the achievable column correlation constraint λ^A. By exploiting the well-known Welch bound, sufficient conditions for the existence of companion pairs are also derived for column correlation constraints λ^A and λ^P.

We have left the general problem of finding MO complementary set matrices with a minimum column correlation constraint as an open question. An important step towards solving the general problem is to find new construction approaches for MO complementary set matrices. A design algorithm based on N-shift cross-orthogonal sequences can be found in [30]. However, their column correlation properties can be evaluated only on a case by case basis.

4.8 Appendix

4.8.1 Proof of Theorem 4.3

$$\mathbf{H}_0^{(0)} = \begin{bmatrix} \mathbf{h}_{0,0}^{(0)} \\ \mathbf{h}_{0,1}^{(0)} \end{bmatrix} = \begin{bmatrix} + & + \\ + & - \end{bmatrix}_{2\times 2}$$

and

$$\mathbf{H}_1^{(0)} = \begin{bmatrix} \mathbf{h}_{1,0}^{(0)} \\ \mathbf{h}_{1,1}^{(0)} \end{bmatrix} = \begin{bmatrix} + & - \\ + & + \end{bmatrix}_{2\times 2}.$$

It can be verified that $\{\mathbf{h}_{0,0}^{(0)}, \mathbf{h}_{0,1}^{(0)}\}$ and $\{\mathbf{h}_{1,0}^{(0)}, \mathbf{h}_{1,1}^{(0)}\}$ are respectively Golay complementary pairs. Based on *Lemma 2.1*, $\{\mathbf{h}_{0,0}^{(q)}, \mathbf{h}_{0,1}^{(q)}\}$ and $\{\mathbf{h}_{1,0}^{(q)}, \mathbf{h}_{1,1}^{(q)}\}$, $q = 1, 2, 3...$, constructed from (4.27) are guaranteed to be Golay complementary pairs.

We observe that $f_i(\mathbf{h}_{0,0}^{(0)}) = \mathbf{h}_{1,0}^{(0)}$ and $f_i(\mathbf{h}_{0,1}^{(0)}) = -\mathbf{h}_{1,1}^{(0)}$. Let $f_i(\mathbf{h}_{0,0}^{(q)}) = \mathbf{h}_{1,0}^{(q)}$, $f_i(\mathbf{h}_{0,1}^{(q)}) = -\mathbf{h}_{1,1}^{(q)}$, then,

$$f_i(\mathbf{h}_{0,0}^{(q+1)}) \;=\; f_i(\mathbf{h}_{0,0}^{(q)}\ \overleftarrow{\mathbf{h}_{0,1}^{(q)}}) = f_i(\mathbf{h}_{0,0}^{(q)})\ \overleftarrow{(-f_i(\mathbf{h}_{0,1}^{(q)}))} = \mathbf{h}_{1,0}^{(q)}\ \overleftarrow{\mathbf{h}_{1,1}^{(q)}} = \mathbf{h}_{1,0}^{(q+1)}\,.$$

In a similar way, we have that $f_i(\mathbf{h}_{0,1}^{(q+1)}) = -\mathbf{h}_{1,1}^{(q+1)}$. This ends the proof.

4.8.2 Proof of Lemma 4.4

We give the proof for $\lambda_{\mathbf{c}_0}^P \le \max\left\{\lambda_{\mathbf{s}_0}^P + \lambda_{\mathbf{s}_1}^P, 2\lambda_{\mathbf{s}_0,\mathbf{s}_1}^P\right\}$ and $S_{\mathbf{c}_0}^A \le S_{\mathbf{s}_0}^A + S_{\mathbf{s}_1}^A + S_{\mathbf{s}_0,\mathbf{s}_1}^A$. Other proofs are similar.

$$\begin{aligned}
\lambda_{\mathbf{c}_0}^P &= \max_l \{|P_{\mathbf{c}_0}(l)|, 1 \le l \le m-1\} \\
&= \max_l \left\{|P_{\mathbf{s}_0}(l) + P_{\mathbf{s}_1}(l)|, 1 \le l \le \frac{m}{2} - 1; |P_{\mathbf{s}_0,\mathbf{s}_1}(l) + P_{\mathbf{s}_0,\mathbf{s}_1}(-l-1)|, 0 \le l \le \frac{m}{2} - 1\right\} \\
&\le \max_l \left\{|P_{\mathbf{s}_0}(l)| + |P_{\mathbf{s}_1}(l)|, 1 \le l \le \frac{m}{2} - 1; 2|P_{\mathbf{s}_0,\mathbf{s}_1}(l)|, 0 \le l \le \frac{m}{2} - 1\right\} \\
&= \max\left\{\lambda_{\mathbf{s}_0}^P + \lambda_{\mathbf{s}_1}^P, 2\lambda_{\mathbf{s}_0,\mathbf{s}_1}^P\right\}
\end{aligned}$$

and

$$\begin{aligned}
S_{\mathbf{c}_0}^A &= \sum_{l=1}^{m-1} |A_{\mathbf{c}_0}(l)| \\
&= \sum_{l=0}^{\frac{m}{2}-1} |A_{\mathbf{s}_0,\mathbf{s}_1}(l) + A_{\mathbf{s}_0,\mathbf{s}_1}(-l-1)| + \sum_{l=1}^{\frac{m}{2}-1} |A_{\mathbf{s}_0}(l) + A_{\mathbf{s}_1}(l)| \\
&\le \sum_{l=0}^{\frac{m}{2}-1} |A_{\mathbf{s}_0,\mathbf{s}_1}(l)| + \sum_{l=-\frac{m}{2}+1}^{-1} |A_{\mathbf{s}_0,\mathbf{s}_1}(l)| + \sum_{l=1}^{\frac{m}{2}-1} |A_{\mathbf{s}_0}(l)| + \sum_{l=1}^{\frac{m}{2}-1} |A_{\mathbf{s}_1}(l)| \\
&= S_{\mathbf{s}_0}^A + S_{\mathbf{s}_1}^A + S_{\mathbf{s}_0,\mathbf{s}_1}^A.
\end{aligned}$$

Table 4.2: Binary Companion Pairs

m	λ^A	S^A	λ^A_{min}	S^A_{min}	N_u	Example Companion Pair
2	1	1	1	1	1	$-+$ $++$
4	1	2	1	2	6	$---+$ $-+++$
6	2	5	2	5	37	$-+---+$ $--+-++$
8	2	6	2	6	20	$-+----++$ $+++--+-+$
10	2	9	2	9	44	$+-+++++--+$ $---++--+-+$
12	2	8	2	8	16	$++++-+-++--+$ $--++-----+-+$
14	2	13	2	13	180	$+-+------++--+$ $----++--++-+-+$
16	2	12	2	/	192	$-+++------+--+-+$ $+--++-+-+------+$
16	2	12	/	12	88	$++--+++++-+-+--+$ $-+++-+++-+--+-++$
18	2	17	2	17	16	$++++----+-+--++--+$ $-+-++-+-----++--++$
20	2	14	2	/	22	$+++++-+++---+-++-+--$ $++-++------++---+-+-$
20	2	14	/	14	6	$+++++-+---+-++---++-$ $+--+++--+-+++-+-----$
22	3	23	3	23	16	$++++++---++-+-+-++--$ $+-$ $+++--+++++++--+--+-+$ $-+$
24	3	20	3	20	6	$++--+++------+-+-+--$ $+--+$ $+--++-++-+-+-------+$ $++--$
26	3	27	3	27	28	$++++++---++---+-+-+-$ $-+--+-$ $+++---+++++++-++--+-$ $-+-+-+$
28	2	28	2	/	12	$+++--+++------+-+-+-$ $-+--+--+$ $++-++-+--+---+---+--$ $-++++---$
28	2	28	/	28	6	$++++++-+--+-+---+++-$ $--+--++-$ $++--+++-++-++-+++++-$ $---+-+-+$

Table 4.3: Ternary Companion Pairs

m	λ^A	S^A	λ^A_{min}	S^A_{min}	N_u	Example Companion Pair
2	0	0	0	0	1	0 + + 0
4	1	1	1	1	2	0 + + − + + − 0
6	1	2	1	2	16	+ + + 0 − + + + 0 − + −
8	1	3	1	3	2	0 + + + − − + − + + + − − + − 0
10	2	6	2	6	10	+ + 0 + − + − − + + + + + − 0 − + − − +
12	1	5	1	5	2	0 + + + − − − + − − + − + + + − − − + − − + − 0
14	1	6	1	6	2	0 + + + + + − − + + − + − + + + + + + − − + + − + − + 0
16	2	9	2	⁄	25232	+ + + + + + − − + + − + − + + 0 + + + 0 + + + − − − + − + + − +
16	2	9	⁄	9	4	0 + + + + + − − + + − − + − + − + + + + + − − + + − − + − + − 0
18	2	14	2	14	8	+ + + 0 − − − − + − + − − + + − − + + + − − + + − − − − − + − + 0 − + −
20	2	15	2	⁄	1274	+ 0 + + + + + − + − − + − + + + − − + + + + + + + − + − − + − 0 + + − − + + + −
20	2	15	⁄	15	58	+ + + + + − − − + + − − + − − + − + − 0 + + + − 0 − − + − + + − + + + − + + + −
22	2	14	2	⁄	1012	+ + + 0 + + + + − − − + + − + + − − + − + − + + + + + − − + + + − − + − − + − + 0 + − +
22	2	14	⁄	14	4	+ + − − − − − − − + + − − + − + − + − − + 0 0 + − − + − + − + − − + + − − − − − − − + +
24	2	19	2	⁄	94	+ + + + + + − + + − − − + − + − + + − − 0 − + + + + + + − + − − + − − + − − − + − − − + + 0 − −
24	2	19	⁄	19	52	+ + + + + − + − 0 − + − + + − + − − + + + − − + + + + − − − − + + + − + + + − + + − + − − + − 0

Table 4.4: λ_W^A and λ_B^A for Long Binary Seed Sequences

m	62	74	82	106	118	122	126	134
λ_W^A	6	7	7	8	8	8	8	9
λ_B^A [50]	16	18	16	18	20	22	22	22
λ_B^A	13	15	15	18	18	18	19	20
m	146	158	168	182	186	200	218	240
λ_W^A	9	9	10	10	10	11	11	11
λ_B^A [50]	24	24	28	24	24	28	30	28
λ_B^A	22	22	24	24	24	27	28	28

Table 4.5: Achievable λ^A for Long Binary Seed Sequences

m	merits	$\mathbf{s}_0$ and ($\mathbf{s}_1$)
126	$\lambda_B^A = 19$ $\lambda_\mathbf{u}^A = 17$ $\lambda_B^A = 22$ [34] $\lambda_\mathbf{u}^A = 17$ [34]	$+-+-+--+-+-+++++++--+$ $--+-+++--+---+++---++$ $--++-++----++-+---+++$ $(++-+++-++--++----+-++$ $-+++--++++--+++++-+-+$ $+++-+----++-+-+-+---+)$
168	$\lambda_B^A = 24$ $\lambda_\mathbf{u}^A = 20$ $\lambda_B^A = 28$ [34] $\lambda_\mathbf{u}^A = 21$ [34]	$+--+--+++++----+-++--$ $+-++--++--++-++----++$ $-+++-+-+---+++-++++--$ $+-++-+-++------+-----$ $(+---+------++---++++-$ $--+---++-+++---+-+-++$ $-+-+--+++++---++++++-$ $++-++-+++-+-++-+--+--)$
200	$\lambda_B^A = 27$ $\lambda_\mathbf{u}^A = 23$ $\lambda_B^A = 28$ [34] $\lambda_\mathbf{u}^A = 25$ [34]	$-+++++-+++++----+-++$ $++---++-+-+-++--+-++$ $++++---++++-+-+--+-+$ $-+++++-++---+-++--+-$ $-+--++++---+-+-+-+++$ $(--+++-++-+---+++--++$ $+-++--+-+--+--+-+---$ $-----++--+----+++--+$ $-+++++----++--+-+++-$ $-++-+-++-++-+-+++--+)$

Chapter 5

Ternary ZCZ Sequences for Multirate DS-UWB

5.1 Introduction

A zero correlation zone (ZCZ) sequence set has the periodic and/or aperiodic correlation values equal to zero for a contiguous set of delays starting with a single delay. Thus, it can significantly alleviate the multipath interference and multiple access interference. Recently, there has been a considerable interest in applying the ZCZ sequences in the quasi-synchronous CDMA type systems. ZCZ sequence set design was first studied by Suehiro in [53]. Fan [17, 20, 54] proposed binary, quadriphase and polyphase ZCZ sequence sets derived from complementary sets. Torii and Nakamura proposed ZCZ set construction based on perfect sequences and unitary matrices [55]. Cha *et al.* proposed a ternary ZCZ sequence set constructed by cyclically shifting preferred ternary pairs [19].

We propose ternary ZCZ sequence set construction based on MO complementary sets [13]. Compared with earlier work on ZCZ sets, constructed sets have both periodic and aperiodic zero correlation zone. For example, a binary periodic ZCZ sequence set with $(n, m, L_{zcz}) = (32, 4, 4)$ can be generated by Fan's method [20], where n is the length of the sequence, m is the family size (namely, the number of sequences in the set) and L_{zcz} is the length of the periodic ZCZ. Corresponding constructed sequences allow for an improved system performance relative to Fan's sets due to the fact that the ZCZ is both in the periodic and aperiodic sense.

In the next generation of wireless systems, to access a mixture of multimedia applications, the system must support variable transmission rates for different users. It is possible to support higher data rates in DS-CDMA systems by assigning a multiple of orthogonal constant-spreading-factor codes to a link. This mode of operation is called multicode CDMA (MC-CDMA) [56]. Multicode approach suggests splitting high data rate streams into several low rate data substreams. Each data substream is spread by a sequence and all the substreams are transmitted in parallel using synchronous multicode channels. Since a higher data rate is achieved by increasing the number of parallel code channels, the processing gain can be kept sufficiently large to alleviate the ISI for any particular sequence. Different data rates can be supported by changing the size of the sequence set assigned to a user. Instead of being limited by single sequence ISI, the data rate is limited by the sequence set correlation properties.

We study the BER performance of a multicode TS-UWB system employing the proposed ternary ZCZ set and experiencing a dense multipath [57]. A comparison is given with the performance of a single spreading sequence (with a reduced processing gain per sequence), and with comparable examples employing Fan's [20] and Cha's [19] ZCZ sets.

In an alternative scheme (also known as OVSF-CDMA), each user is assigned a single orthogonal variable-spreading-factor (OVSF) code [58]. A higher data rate access is possible by using a lower spreading factor. Both MC-CDMA and OVSF-CDMA have been proposed in UMTS/IMT-2000 [58] for supporting variable data rates. OVSF codes are commonly adopted in the forward link of synchronous DS-CDMA systems as channelization codes to accommodate multiple users with different transmission rates. In the precious work [59], OVSF codes with a tree structure are constructed based on Walsh codes. In this case, the orthogonality is easily lost when the synchronism is lacking or in a multipath scenario. Thus, the conventional OVSF codes are not suitable for UWB systems which usually suffer in a dense multipath environment [57].

In [6], we propose the two-dimensional (2D) OVSF sequence set (matrix) with ideal correlation properties (i.e. zero autocorrelation sidelobes and zero cross-correlation functions), which significantly improve the interference-rejection capability of the multicarrier DS-CDMA or multichannel DS-UWB systems. In such a system, each user applies a unique 2D OVSF sequence set (i.e. an m by n matrix, where m is the number of multichannels and n is the sequence length). The same information bit is spread by different sequences within the set and parallel transmitted over m multiple orthogonal channels. Despreading in the receiver is accomplished on a channel-by-channel basis using a set of m correlators matched to the spreading sequences for respective channels. However, the ideal correlation properties are easily lost when m channels undergo different fadings [6]. On the other hand, the same information bits transmit over m channels , resulting in higher complexities on transceiver design compared with that in a single channel scheme. The multichannel scheme also potentially suffers a high peak-to-average power ratio (PAPR) because the transmitted signal is the sum of the signals from parallel channels. As the power amplifier has a limited peak output power, an increased PAPR reflects that the average radiated power has to be reduced to avoid the nonlinear distortion of transmitted signal [60]. Hence, it is necessary to construct a class of one-dimensional (1D) OVSF codes which has better correlation properties than Walsh code based OVSF codes to support multirate users in DS-UWB systems.

It is known that 1D spreading sequences with ideal correlation properties don't exist [54]. However, we can construct zero-correlation zone (ZCZ) sequence sets whose periodic and aperiodic correlation values equal to zero for a contiguous set of delays starting with a single delay [7]. In this chapter, we also propose a stepwise algorithm for the construction of 1D OVSF codes with both periodic and aperiodic ZCZ. The proposed ternary OVSF codes have the same orthogonality as the conventional Walsh code based OVSF codes and can support the same number of multirate users. In addition, the proposed codes present a ZCZ which allows for significant alleviation of multipath and multiuser interference of particular interests for TS-UWB systems.

5.2 ZCZ Sequence Set Design

5.2.1 Construction Method 1: Based on Ternary Complementary Pair

Let $\{\mathbf{c}_1, \mathbf{c}_2\}$ be any ternary complementary pair (TCP) [15] of length n. Let $\mathbf{H}_{t\times t} = [h_{ij}]$ be a ternary orthogonal matrix, where $\sum_{j=1}^{t} h_{ij}h_{kj} = 0,\ \forall i \neq k$. By using Kronecker product $\odot$, we construct the $2t$ by $2nt$ matrix

$$\begin{bmatrix} \mathbf{H}\odot\mathbf{c}_1 & \mathbf{H}\odot\overleftarrow{\mathbf{c}_2} \\ \mathbf{H}\odot\mathbf{c}_2 & \mathbf{H}\odot(-\overleftarrow{\mathbf{c}_1}) \end{bmatrix} = \begin{bmatrix} h_{11}\mathbf{c}_1 & h_{12}\mathbf{c}_1 & \cdots & h_{1t}\overleftarrow{\mathbf{c}_2} \\ h_{21}\mathbf{c}_1 & h_{22}\mathbf{c}_1 & \cdots & h_{2t}\overleftarrow{\mathbf{c}_2} \\ \vdots & \vdots & \ddots & \vdots \\ h_{t1}\mathbf{c}_2 & h_{t2}\mathbf{c}_2 & \cdots & -h_{tt}\overleftarrow{\mathbf{c}_1} \end{bmatrix}. \tag{5.1}$$

The ternary ZCZ sequence set is given by padding n zeros between any two sequence elements in the same row of the above matrix. E.g. $((h_{11}\mathbf{c}_1)\mathbf{z}_n(h_{12}\mathbf{c}_1)\mathbf{z}_n\cdots\mathbf{z}_n(h_{1,t}\overleftarrow{\mathbf{c}_2})\mathbf{z}_n)$ is one ternary ZCZ sequence in the set $\mathcal{T}(4nt, 2t, n)$, where $\mathbf{z}_n$ denotes the sequence consisting n continuous zero elements.

Example: Let $\{\mathbf{c}_1, \mathbf{c}_2\} = \{(++-), (+0+)\}$ and

$$\mathbf{H} = \begin{bmatrix} + & + & 0 \\ + & - & 0 \\ 0 & 0 & + \end{bmatrix}.$$

Based on construction method 1, one sequence in ZCZ set $\mathcal{T}\{36, 6, 3\}$ is $(++-000++-000000000+0+000+0+000000000)$.

5.2.2 Construction Method 2: Based on MO complementary set

Let $\mathbf{M}^k_{m,n}$ be a MO complementary set matrix where $\mathbf{c}_{i,j}$ denotes the ith sequence in the complementary set $\mathbf{C}_j$,

$$\mathbf{M}^k_{m,n} = [\mathbf{C}_1\ \mathbf{C}_2\ ...\ \mathbf{C}_k] = \begin{bmatrix} \mathbf{c}_{1,1} & \mathbf{c}_{1,2} & \cdots & \mathbf{c}_{1,k} \\ \mathbf{c}_{2,1} & \mathbf{c}_{2,2} & \cdots & \mathbf{c}_{2,k} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{c}_{m,1} & \mathbf{c}_{m,2} & \cdots & \mathbf{c}_{m,k} \end{bmatrix}. \tag{5.2}$$

Ternary ZCZ sequence set $\mathcal{T}(2mn, k, n)$ can be constructed by padding n zeros between adjacent sequences in the same column as follows,

$$\mathcal{T}(2mn, k, n) = \{\mathbf{a}_i = \mathbf{c}_{1,i}\mathbf{z}_n\mathbf{c}_{2,i}\mathbf{z}_n\cdots\mathbf{c}_{m,i}\mathbf{z}_n,\ i = 1, 2...k\}. \tag{5.3}$$

Example: Let MO complementary set

$$\mathbf{M}^2_{2,3} = \begin{bmatrix} ++- & +0+ \\ +0+ & +-- \end{bmatrix}.$$

Based on construction method 2, one ZCZ sequence in $\mathcal{T}\{12, 2, 3\}$ is $(++-000+0+000)$.

5.2.3 ZCZ Sequences Comparison

The proposed ternary ZCZ sequence sets have the advantage over that constructed by Fan's [17, 54] and Cha's [19] approaches with regard to available sequence lengths. The length of Cha's ternary ZCZ sequences is of the form $4 \times 2^p, (p = 1, 2, 3...)$ which is a subset of lengths described in method 1. Fan's ZCZ set is constructed from binary MO complementary sets. If $k = 2$, binary $\mathbf{M}^2_{m,n}$ contains two Golay Pairs which lead to binary ZCZ sets $(2^{2p+1}n, 2^{p+1}, 2^{p-1}n)$, where Golay length $n = 2^\alpha 10^\beta 26^\gamma$, α, β, γ are nonnegative integers. However, based on the construction method 2, when $k = 2$, we can obtain the ternary ZCZ sequence set $\mathcal{T}(2^{2p+2}n, 2^{p+1}, 2^p n)$, where n is the length of the TCP which can be any positive integers [15]. Thus, the proposed methods have much more flexibilities in selecting ZCZ sequence length.

In Table 5.1, we compare the family size for various lengths of ZCZ in two cases, namely 128 and 256, which are possible for all the three methods. Note that, the ZCZ in Fan's and Cha's methods is only in a periodic sense, but the proposed method can construct sequences with both periodic and aperiodic ZCZ.

Table 5.1: Family Sizes of ZCZ Sequence Sets

Sequence Length $N = 128$

L_{zcz}	2	4	8	16	32
Fan's	32	16	8	4	2
Cha's	32	18	10	4	2
Proposed	32	16	8	4	2

Sequence Length $N = 256$

L_{zcz}	2	4	8	16	32	64
Fan's	64	32	16	8	4	2
Cha's	64	38	20	10	4	2
Proposed	64	32	16	8	4	2

5.3 ZCZ OVSF Codes Design

The construction algorithm is described as follows.

Step 1: Starting with a TCP $\{\mathbf{c}_1, \mathbf{c}_2\}$ of length n, the first layer code can be constructed as follows,

$$\begin{cases} \mathbf{a}_1^{(1)} & = & \mathbf{c}_1 \ \mathbf{z}_n \ \mathbf{c}_2 \ \mathbf{z}_n \\ \mathbf{a}_2^{(1)} & = & \overleftarrow{\mathbf{c}_2}\mathbf{z}_n(-\overleftarrow{\mathbf{c}_1})\mathbf{z}_n \end{cases}, \tag{5.4}$$

where $\mathbf{z}_n$ denotes the sequence consisting of n zero elements, and $\overleftarrow{\mathbf{c}_1}$ denotes the reverse of $\mathbf{c}_1$.

Step 2: Based on (2.12), we can recursively construct the kth layer codes.

Proposition 5.1: The constructed set $\{\mathbf{a}_i^{(k)}, i = 1, 2, ..., 2^k, k = 1, 2, ...\}$ is an OVSF code set with a periodic and aperiodic ZCZ of length n.

Proof. See Appendix. □

Compared with the conventional Walsh code based OVSF codes, the TCP based OVSF codes preserve the orthogonality and, in addition, present a ZCZ in both periodic and aperiodic correlation functions. The parameters of the proposed OVSF codes are listed in Table 5.2, where k and n are, respectively, the layer index and the length of the TCP. Note that, the codes in an OVSF code tree form a ZCZ sequence set of $L_{zcz} = n$, as long as any two of them is not a parent-child pair.

Table 5.2: OVSF Codes Constructed from TCP of Length n

Layer Index	k
Number of Codes	2^k
Code Length	$2^{k+1}n$
L_{zcz}	n

Property 5.1 *(arbitrary length of L_{zcz})*: The OVSF code set $\{\mathbf{a}_i^{(k)}, i = 1, 2, ..., 2^k, k = 1, 2, ...\}$ can be constructed with arbitrary length of L_{zcz}.

Proof. Proposition 5.1 states that the L_{zcz} of the constructed OVSF code set is determined by the length of the TCP. It is known that the TCP exists for arbitrary sequence length [15]. □

Property 5.2 *(constant deficiency ratio)*: Let $\lambda_{\mathbf{c}} = \delta_{\mathbf{c}}/n$ denote the deficiency ratio of a ternary sequence $\mathbf{c}$ of length n, where $\delta_{\mathbf{c}}$ is the number of zero elements of $\mathbf{c}$. The constructed ternary ZCZ sequences in the same OVSF code tree have a constant deficiency ratio

$$\lambda_{\mathbf{a}_i^{(k)}} = \frac{1}{2} + \frac{\delta_{\mathbf{c}_1} + \delta_{\mathbf{c}_2}}{4n}, \quad i = 1, 2, ..., 2^k, k = 1, 2, ... \tag{5.5}$$

Proof. The proof follows from Eqs. (5.4) and (2.12). □

Property 5.3 *(additional zero-correlation duration)*: Let $\mathbf{a}_i^{(u)}$ and $\mathbf{a}_j^{(v)}$ be any two codes from the constructed OVSF code set $\{\mathbf{a}_i^{(k)}, i = 1, 2, ..., 2^k, k = 1, 2, ...\}$. Let us assume $u < v$. $\mathbf{a}_i^{(u)}$ is the mother code of $\{\mathbf{a}_s^{(v)}, s = 2^{v-u}(i-1)+1, 2^{v-u}(i-1)+2, ..., 2^{v-u}i\}$. If $\mathbf{a}_i^{(u)}$ is not the mother code of $\mathbf{a}_j^{(v)}$, they have an additional zero-correlation duration as follows,

$$P_{\mathbf{a}_s^{(v)}, \mathbf{a}_j^{(v)}}(l) = 0, \quad 2^{v+1} - n \leq l \leq 2^{v+1} - 1; \tag{5.6}$$

$$A_{\mathbf{a}_s^{(v)}, \mathbf{a}_j^{(v)}}(l) = 0, \quad 2^{v+1} - n \leq |l| \leq 2^{v+1} - 1. \tag{5.7}$$

Proof. It can be proved by noticing the n ending zeros of all constructed codes. □

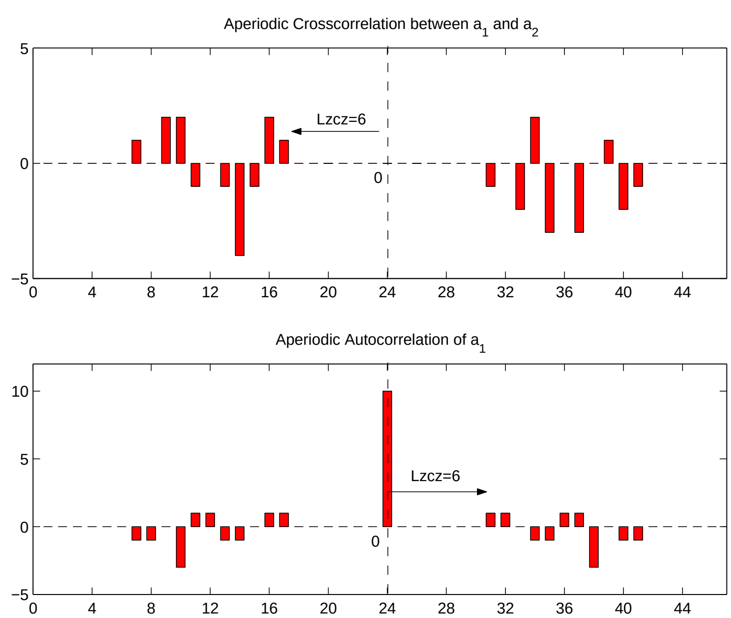

Figure 5.1: Upper plot: the aperiodic cross-correlation of code $\mathbf{a}_1^{(1)}$ and $\mathbf{a}_2^{(1)}$; Lower plot: the aperiodic autocorrelation values of code $\mathbf{a}_1^{(1)}$.

Example: Based on TCP $\{\mathbf{c}_1, \mathbf{c}_2\} = \{++-+0+, ++--0-\}$ of length $n = 6$, a ZCZ sequence pair with $L_{zcz} = 6$ can be constructed as,

$$\mathbf{a}_1^{(1)} = (++-+0+000000++--0-000000); \quad (5.8)$$

$$\mathbf{a}_2^{(1)} = (-0--++000000-0-+--000000). \quad (5.9)$$

The above ZCZ sequence pair forms the first layer OVSF codes. Based on Eq.(2.12), we can generate higher layer OVSF codes. For example,

$$\begin{aligned}\mathbf{a}_1^{(2)} &= \mathbf{a}_1^{(1)}\mathbf{a}_1^{(1)} = (++-+0+000000++--0-000000 \\ &\quad ++-+0+000000++--0-000000); \quad (5.10)\\ \mathbf{a}_2^{(2)} &= \mathbf{a}_1^{(1)}(-\mathbf{a}_1^{(1)}) = (++-+0+000000++--0-000000 \\ &\quad --+-0-000000--++0+000000). \quad (5.11)\end{aligned}$$

The L_{zcz} of the OVSF codes is $n = 6$. Fig. 5.1 shows the ZCZ of codes $\mathbf{a}_1^{(1)}$ and $\mathbf{a}_2^{(1)}$.

5.4 Ternary ZCZ Sequences for Multicode DS-UWB

5.4.1 System Model

A set of m spreading sequences $\{\mathbf{c}_1, \mathbf{c}_1, ..., \mathbf{c}_m,\}$ of length n is assigned to a single user. m consecutive information symbols $\{b_1, b_2, ...b_m\}$ are transmitted over m parallel code channels simultaneously. The symbol rate $R_s = m/T_s$, where T_s is the symbol period containing n chips of duration T_c. By increasing the number of code channels from 1 to m, we can adapt the system data rate from $1/T_s$ to m/T_s.

The transmitted baseband signal is given by

$$s(t) = \sum_r \sum_{i=1}^{m} b_{i,r} \sum_{j=0}^{n-1} c_{i,j}\psi(t - jT_c - rT_s), \quad (5.12)$$

where $b_{i,r}$ is the rth binary antipodal symbol transmitted using the ith multicode sequence; $\psi(t)$ is the unit energy chip pulse with duration T_c and assumed known to the receiver. The spreading sequence set $\{\mathbf{c}_1, \mathbf{c}_1, ..., \mathbf{c}_m,\}$ is suggested to be the ternary ZCZ sequence set introduced in the preceding section. The UWB channel with L resolvable paths is modeled as

$$h(t) = \sum_{l=0}^{L-1} \alpha_l \delta(t - \tau_l), \tag{5.13}$$

where α_l and τ_l denote the channel gain and the propagation delay of the lth path, respectively. When sufficient multipath resolution is available, small changes in the propagation time only affect the path delay and path component distortion can be neglected. Under these assumptions, path coefficients α_l can be modelled as independent real valued random variables whose sign is a function of the material properties and, generally, depends on the wave polarization, angle of incidence, and the frequency of the propagating wave [32]. For simplicity, we quantize the multipath delay into bins, i.e. $\tau_l = lT_c$. For a single user multicode UWB system, the corresponding received signal model is:

$$r(t) = \sum_{l=0}^{L-1} \alpha_l s(t - lT_c) + n(t), \tag{5.14}$$

where $n(t)$ is a white Gaussian noise process with power spectral density $N_0/2$.

5.4.2 Numerical Results

We first compare the single user correlator receiver BER performance for multiple and single code systems employing a ternary ZCZ set and an m-sequence, respectively. Two data rates, i.e., Rate A and Rate B are assumed. For Rate A, $n = 16$ for single code scheme employing an m-sequence of length 15 padded with a zero and $n = 128$ for the multicode scheme with $m = 8$. For Rate B, $n = 8$ for a single code scheme employing an m-sequence of length 7 padded with one zero and $n = 32$ for the multicode scheme with $m = 4$.

The mean power of the multipath component is selected equal to the average value given in [34], which is based on the indoor line of sight (LOS) measurements performed in 23 homes. In [34], it is observed that the line of sight component and the first 10 multipath bins account for 33% and 75% of the total power, respectively. The sign of the reflected path coefficient is modeled as a uniformly distributed random variable [35]. The path power is quantized into 0.4 nanosecond bins corresponding to a chip duration T_c. We assume that each bin contains exactly one multipath component (emulating a dense

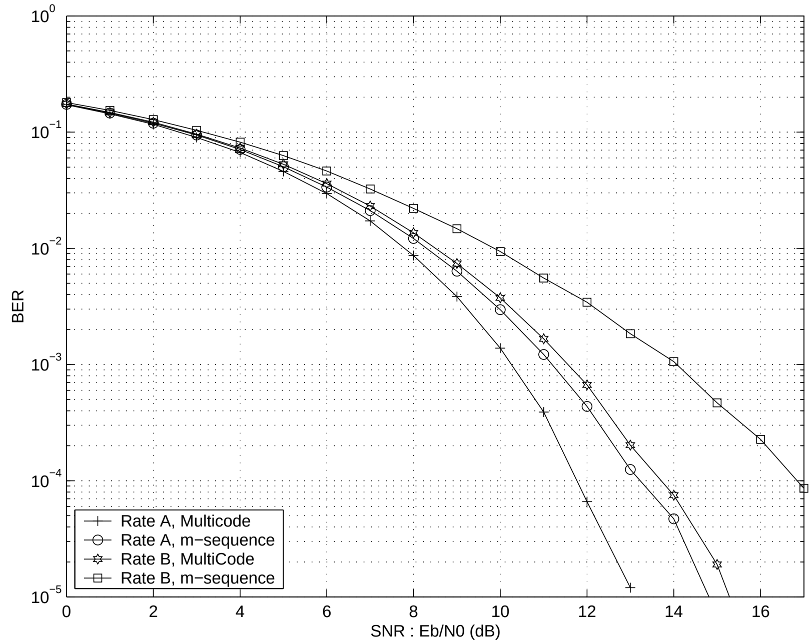

Figure 5.2: BER performance for multicode system using ternary ZCZ sequences versus that of single code system using m-sequence

multipath environment) and that the delay spread is restricted to be 4 nanoseconds. The effect of interchip interference has been assumed negligible.

In Figure 5.2, the average BER is plotted against the SNR per bit. For a multicode system, we employ the ternary ZCZ sequence set constructed from complementary pair $\{++, -+\}$ with parameter $(n, m, L_{zcz}) = (128, 8, 8)$ and $(n, m, L_{zcz}) = (32, 4, 4)$ for Rate A and B respectively. For single code scheme, we employ m-sequence with ending zero $(- - - + - - + + - + - + + + + 0)$ and $(+ + + - + - - 0)$ for Rate A and Rate B respectively. BER performance improvement can be observed for both Rate A and Rate B. E.g., larger than 2 dB and 1 dB gains can be achieved by using ternary ZCZ set based multicode scheme over single code scheme when the target BER is 10^{-3}.

Figure 5.3 demonstrates that when employing ternary ZCZ set, the BER performance does not change when the data rate is varied by increasing the number of code channels. The ternary ZCZ sequence set parameters are $(n, m, L_{zcz}) = (128, 8, 8)$, the number of code channels varies from 1 to 8. The three almost flat BER performance curves lie at $BER = 10^{-2}, 10^{-3}$ and 10^{-4} for $SNR = 8, 10$ and 12 dB respectively.

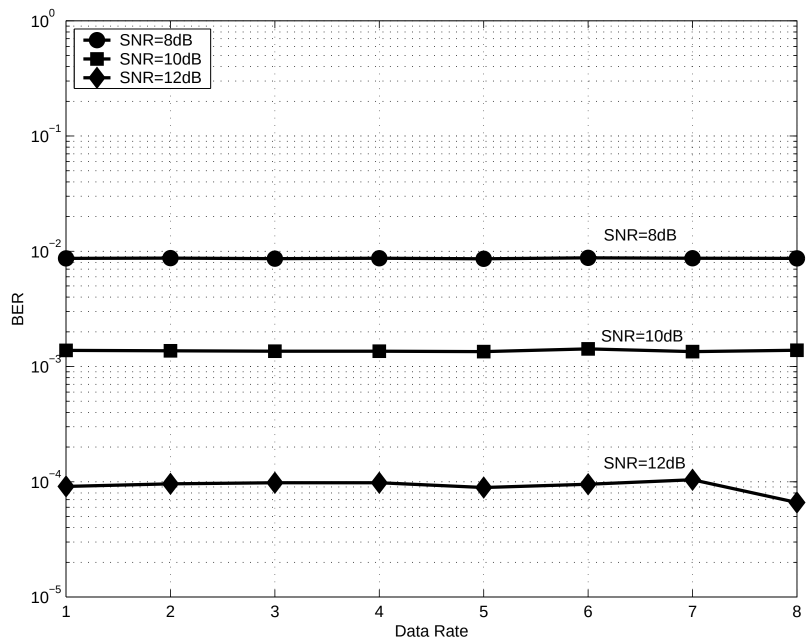

Figure 5.3: BER performance for multicode system using ternary ZCZ sequences vs number of code sequences

In Figure 5.4, we compare the BER performance of the multicode UWB system employing Cha's ternary ZCZ set $(n, m, L_{zcz}) = (32, 4, 5)$ [19], Fan's binary ZCZ set $(n, m, L_{zcz}) = (32, 4, 4)$ [20] and the constructed ternary ZCZ set with parameter $(n, m, L_{zcz}) = (32, 4, 4)$. At $BER = 10^{-4}$, the system employing proposed ternary ZCZ set can achieve 2 dB and 4 dB gains over the system using Fan's binary ZCZ set and Cha's ternary ZCZ set, respectively. This is due to the fact that the proposed ternary sets have a ZCZ in both periodic and aperiodic sense. Note that, the sequences from the proposed ternary set have the same PAPR as that ones proposed by Cha.

5.5 OVSF Codes with a ZCZ for Synchronous TS-UWB

5.5.1 System Model

The impulse response of the UWB channel with L resolvable paths is

$$h(t) = \sum_{l=0}^{L-1} \alpha_l \delta(t - \tau_l), \tag{5.15}$$

where α_l and τ_l denote the channel gain and the propagation delay of the l_{th} path, respectively. Path coefficients α_l are modelled as independent real valued random variables

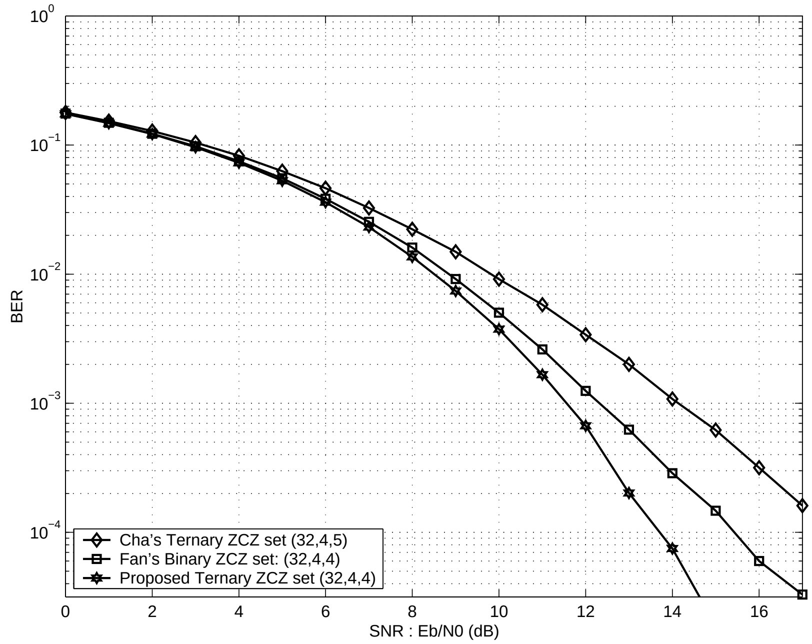

Figure 5.4: BER performance comparison for multicode UWB system for different ZCZ sets

whose sign is a uniformly distributed random variable [35]. The transmitted signal for user m is given by

$$s_m(t) = \sum_r b_m, r \sum_{i=0}^{n-1} c_{m,n} \psi(t - rT_s - nT_c), \tag{5.16}$$

where r is the index of the information symbols and n is the length of spreading sequences. The spreading sequence for mth user is the OVSF code chosen from the OVSF code tree. b_r are binary antipodal symbols, T_c is the chip duration time and $T_s = nT_c$ is the symbol period. $\psi(t)$ is the unit energy signaling pulse and assumed known to the receiver. We consider a synchronous CDMA UWB system in which M active users share the same physical channel. The corresponding received signal model is:

$$r(t) = \sum_{m=1}^{M} \sum_{l=0}^{L-1} \alpha_l s_m(t) + n(t), \tag{5.17}$$

where $n(t)$ is a white Gaussian noise with zero mean and two-sided power spectral density $N_0/2$.

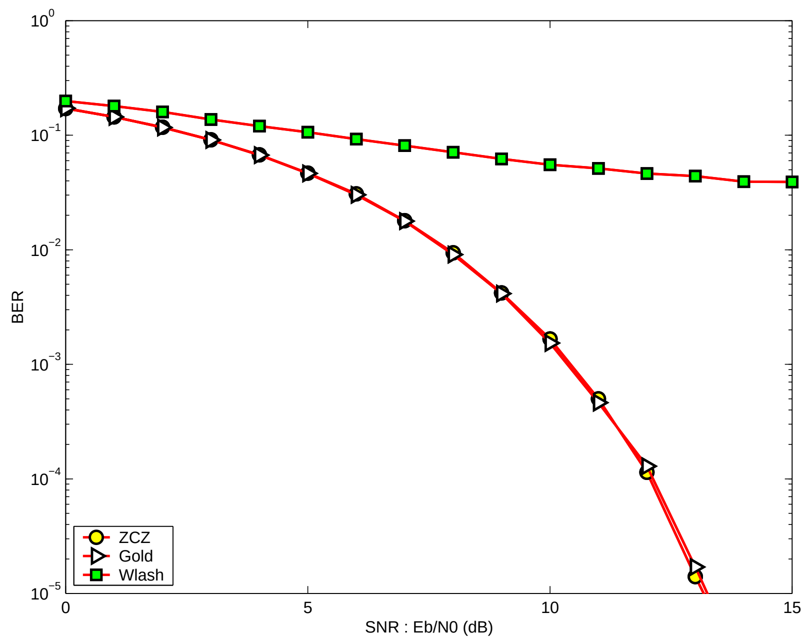

Figure 5.5: Comparison of the BER performances of ZCZ sequence, Gold sequence and Walsh code in a single user DS-UWB system

5.5.2 Numerical Results

We study correlator receiver performance of DS-UWB systems employing different types of OVSF codes. The mean power of multipath components are chosen to be equal to average value given in [34]. The path power is quantized into 0.4 nanosecond bins corresponding to a chip duration T_c. We assume that each bin contains exactly one multipath component (emulating a dense multipath environment) and the delay spread was restricted to be 4 nanosecond. The effects of interchip interference has been assumed negligible.

In a single user DS-UWB system, we compare the BER performances of the proposed ZCZ sequence to that of Walsh code $(++----++++----++++----++++----++)$ and Gold sequence $(----+--+-++--+++++---++-+++-+-+)$. For comparison purpose, since both the sequence lengths of Walsh code and Gold sequence are the power of 2. The ZCZ sequence that we employ here is $(++++0000++--0000-+-+0000-++-0000)$ which is constructed from binary complementary pair $\{++,--\}$. Fig. 5.5 shows that the OVSF code with $L_{zcz}=4$, which may help suppress the multipath interference from the first 4 indirect paths, has the same BER performance as the Gold sequence. However, the user employing a Walsh code has

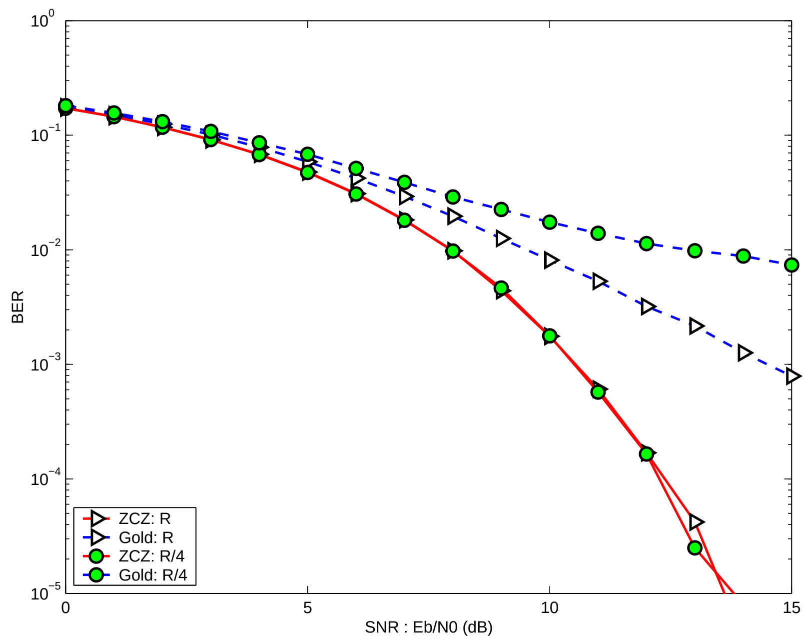

Figure 5.6: BER performance comparison for the proposed OVSF code with a ZCZ and OVSF codes based on Gold sequence for DS-UWB

the worst BER performance.

In Fig. 5.6, we compare the correlator receiver performance of DS-UWB systems employing different types of OVSF codes. For OVSF codes with a ZCZ, we use the tree structure constructed in Sec. 5.3. The four codes assigned to four users are marked in black so that none of the above codes is the mother code of the others. We also construct the OVSF codes based on Gold sequence using the method in (2.12) for comparison purpose. The first layer OVSF codes are chosen from the Gold sequence set with sequence length 31. R and $R/4$ denote the codes selected from the first and third layer respectively. We observe that, in multiuser scenario, proposed OVSF codes have much better BER performance than Gold sequence based OVSF codes.

5.6 Conclusion

We construct ternary ZCZ sequence sets with both periodic and aperiodic ZCZ and a tree structure of ZCZ OVSF codes. Both can be employed in DS-UWB systems for multiple rate purposes. A multicode UWB system using ternary ZCZ sequence set can adapt its data rate by changing the size of the code set to satisfy different data rate requirements without compromising its BER performance. DS-UWB system employing ZCZ OVSF codes can assign the users different length spreading codes and, thus, satisfy different data rate requirements. Multicode and OVSF systems employing the proposed ZCZ sequences have a notably improved BER performance over systems using comparable binary or ternary sequences. We argue that the improvement is due to the unique periodic and aperiodic ZCZ property of proposed sequence sets.

5.7 Appendix: Proof of Proposition 5.1

Lemma 5.1a: The same layer sequences $\{\mathbf{a}_i^{(k)}, i = 1, 2, ..., 2^k\}$ form a ZCZ sequence set with a periodic and aperiodic ZCZ of length $L_{zcz} = n$.

Proof. Based on *Theorem 11* in [13], for a complementary pair $\{\mathbf{c}_1, \mathbf{c}_2\}$ of length n, we have

$$A_{\mathbf{c}_1}(l) + A_{\mathbf{c}_2}(l) = 0, \quad 1 \leq l \leq n-1; \tag{5.18}$$

$$A_{\overleftarrow{\mathbf{c}_2}}(l) + A_{-\overleftarrow{\mathbf{c}_1}}(l) = 0, \quad 1 \leq l \leq n-1; \tag{5.19}$$

$$A_{\mathbf{c}_1,\overleftarrow{\mathbf{c}_2}}(l) + A_{\mathbf{c}_2,-\overleftarrow{\mathbf{c}_1}}(l) = 0, \quad 1-n \leq l \leq n-1; \tag{5.20}$$

Then, Eqs. (5.18), (5.19) and (5.20) imply that,

$$A_{\mathbf{a}_i^{(1)}}(l) = P_{\mathbf{a}_i^{(1)}}(l) = 0, \;\; 1 \leq l \leq n, \;\; i = 1, 2. \tag{5.21}$$

$$A_{\mathbf{a}_1^{(1)},\mathbf{a}_2^{(1)}}(l) = P_{\mathbf{a}_1^{(1)},\mathbf{a}_2^{(1)}}(l) = 0, \; -n \leq l \leq n. \tag{5.22}$$

Thus, $\{\mathbf{a}_1^{(1)}, \mathbf{a}_2^{(1)}\}$ is a ZCZ sequence set with $L_{zcz} = n$. If we assume $\{\mathbf{a}_i^{(k-1)}, i = 1, 2, ..., 2^{k-1}\}$ is a ZCZ sequence set of $L_{zcz} = n$, then for $\{\mathbf{a}_i^{(k)}, i = 1, 2, ..., 2^k\}$ we have

$$A_{\mathbf{a}_i^{(k)}}(l) = 2A_{\mathbf{a}_r^{(k-1)}}(l) \pm A_{\mathbf{a}_r^{(k-1)}}(2^k n - l) = 0, \;\; 1 \leq l \leq n, \tag{5.23}$$

$$P_{\mathbf{a}_i^{(k)}}(l) = 2A_{\mathbf{a}_r^{(k-1)}}(l) \pm 2A_{\mathbf{a}_r^{(k-1)}}(2^k n - l) = 0, \;\; 1 \leq l \leq n, \tag{5.24}$$

where $r = \lceil \frac{i}{2} \rceil$ and $i = 1, 2, ...2^k$. Let $s = \lceil \frac{j}{2} \rceil$, $j = 1, 2, ..., 2^k$, and $i \neq j$. When $r = s$, we have

$$A_{\mathbf{a}_i^{(k)},\mathbf{a}_j^{(k)}}(l) = A_{\mathbf{a}_r^{(k-1)}}(l) - A_{\mathbf{a}_r^{(k-1)}}(l) + A_{\mathbf{a}_r^{(k-1)}}(2^k n - l) = 0, \tag{5.25}$$

$$\begin{aligned} P_{\mathbf{a}_i^{(k)},\mathbf{a}_j^{(k)}}(l) &= A_{\mathbf{a}_r^{(k-1)}}(l) - A_{\mathbf{a}_r^{(k-1)}}(l) + A_{\mathbf{a}_r^{(k-1)}}(2^k n - l) - A_{\mathbf{a}_r^{(k-1)}}(2^k n - l) \\ &= 0. \end{aligned} \tag{5.26}$$

When $r \neq s$,

$$
\begin{aligned}
A_{\mathbf{a}_i^{(k)},\mathbf{a}_j^{(k)}}(l) &= A_{\mathbf{a}_r^{(k-1)},\mathbf{a}_s^{(k-1)}}(l) \pm A_{\mathbf{a}_r^{(k-1)},\mathbf{a}_s^{(k-1)}}(l) \pm A_{\mathbf{a}_r^{(k-1)},\mathbf{a}_s^{(k-1)}}(2^k n - l) \\
&= 0, \qquad (5.27) \\
P_{\mathbf{a}_i^{(k)},\mathbf{a}_j^{(k)}}(l) &= A_{\mathbf{a}_r^{(k-1)},\mathbf{a}_s^{(k-1)}}(l) \pm A_{\mathbf{a}_r^{(k-1)},\mathbf{a}_s^{(k-1)}}(l) \pm A_{\mathbf{a}_r^{(k-1)},\mathbf{a}_s^{(k-1)}}(2^k n - l) \\
&\quad \pm A_{\mathbf{a}_r^{(k-1)},\mathbf{a}_s^{(k-1)}}(2^k n - l) = 0, \qquad (5.28)
\end{aligned}
$$

where $-n \leq l \leq n$. $\square$

Lemma 5.1b: Any two sequences from different layers $\mathbf{a}_i^{(u)}$ and $\mathbf{a}_j^{(k)}$, where $u \neq k$, have a periodic and aperiodic ZCZ of length $L_{zcz} = n$, if no one is the mother code of the other.

Proof. Let us assume $u < k$. $\mathbf{a}_i^{(u)}$ is the mother code of $\{\mathbf{a}_v^{(k)}, v = 2^{k-u}(i-1)+1, 2^{k-u}(i-1)+2, ..., 2^{k-u}i\}$. Since $j \neq v$, based on Eqs. (5.25)-(5.28), $\mathbf{a}_i^{(u)}$ and $\mathbf{a}_j^{(k)}$ have a periodic and aperiodic ZCZ of length $L_{zcz} = n$. $\square$

Lemmas 5.1a-b imply that the constructed set $\{\mathbf{a}_i^{(k)}, i = 1, 2, ..., 2^k, k = 1, 2, ...\}$ is an OVSF code set with a periodic and aperiodic ZCZ of length n.

Chapter 6

OVSF Code Assignment for Throughput Maximization

6.1 Introduction

To support high-speed and multirate data services in third-generation (3G) mobile communication systems, orthogonal variable-spreading-factor (OVSF) codes are being used in Universal Mobile Telecommunications System (UMTS)/ International Mobile Telecommunication 2000 (IMT-2000) with wideband code-division multiple access (WCDMA) technology [58, 61–64]. OVSF codes can be represented as a binary code tree [59, 65], in which the codes at lower layer are of shorter spreading factor and can be used for a higher data rate access. The code orthogonality is preserved if none parent-and-child pair has been assigned to any two active users. This prefix-free condition imposes a constraint on the OVSF code assignment, which can be described by the well-known Kraft inequality [66].

Recent studies of CDMA systems employing OVSF codes focus on dynamic code assignment schemes and attempt to solve the code placement and code replacement problems (e.g., see [67–69]). The former addresses how to place a new call to avoid the code tree becoming too fragmented and, thus, it may have significant impact on the OVSF codes utilization. The latter addresses, when a new call arrives, how to relocate codes to reduce code blocking.

We study the adaptive code assignment scheme which allocates the OVSF codes to active users based on their SINRs such that the aggregate throughput of the system is maximized. Different layer codes on the OVSF code tree lead to different throughput. Hence, throughput is a function of the spreading sequence length and the SINR. The kth layer OVSF codes are of length $2^{k-1}n^{(1)}$, $k = 1, 2, 3, ...$, where $n^{(1)}$ denotes the length of the first layer codes. Hence, the aggregate throughput maximization becomes a discrete optimization problem. In a set of OVSF codes, each user chooses a single code with a corresponding throughput and a cost associated with the Kraft inequality constraint. The objective is to maximize the overall throughput. This is similar to the classical binary Knapsack problem (BKP) [70], in which a hitch-hiker has to fill his knapsack by selecting from among various possible objects with specific volume and comfort to maximize his total comfort. The BKP is NP-complete so that it can not be solved by simple algorithms. Our problem is more complex in that the code set has infinite number of items (the OVSF code tree potentially has infinite number of layers).

The throughput maximization problem associated with OVSF codes has been studied in [71]. However, throughput functions may vary for different system setups and, thus, the OVSF code assignment algorithm which is optimal for a system setup may not be optimal for other systems. We describe three types of throughput functions which include throughput functions in [71] as a special case. Based on the properties of throughput functions, an optimal OVSF code assignment algorithm is proposed.

The optimality of the code assignment is verified in a quasi-synchronous ternary sequence based UWB (QS TS-UWB) system [4–11]. QS [23] operation is obtained, for example, by using GPS receivers at the base station and mobiles to correct for propagation delays and obtain a universal clock [24]. However, due to synchronization errors, the relative time delay between the signals of different users is random, but can be maintained in a certain time range [25]. We model the relative time delay Δ being upper bound as $|\Delta| \leq \tau$ (e.g., see [26]).

We construct ternary OVSF codes with a ZCZ of arbitrary length L_{zcz}, and apply these to a QS TS-UWB system. ZCZ sequences have been intensively studied in the literature (e.g., see [17–19, 21, 22]). However, all these studies focus on ZCZ sequence sets in which all sequences are of a fixed length. The analysis in [26] demonstrates that the multiple access interference (MAI) of the QS-CDMA system is determined by the cross-correlation between spreading codes around the origin. As a result, the MAI is cancelled when $\tau \leq L_{zcz}$.

Based on the knowledge of relative time delay parameter τ, we demonstrate how to select an ZCZ-OVSF code tree to increase the aggregate throughput of the system. Once the OVSF code tree is selected, the optimal code assignment can further maximize the aggregate throughput by allocating OVSF codes to the active users based on their SINRs.

6.2 Throughput Analysis

6.2.1 System Model

The information bit stream is organized in packets, each containing J information bits. With channel coding, the total size of each packet is $L \geq J$ bits. The packet needs to be retransmitted when at least one error has been detected. The probability of a successful packet transmission $q(\gamma)$ depends on the SINR γ at the receiver of the desired user. If all transmissions are statistically independent, the number of necessary transmissions to receive a packet correctly is a geometric random variable with an expected value $1/q$. The throughput of a user can be interpreted as the number of information bits correctly received per second. Thus, when a kth layer OVSF code of length $n^{(k)}$ is assigned to a user, its throughput can be expressed as follows,

$$\Gamma_k(\gamma) = \frac{\alpha}{2^k} q\left(k, \gamma\right), \quad k = 1, 2, 3, ..., \tag{6.1}$$

where $\alpha = 2J/LT_c n^{(1)}$, and T_c is the chip duration.

Property 6.1 *(monotonic non-decreasing)*: If $\gamma_1 \geq \gamma_2$, then

$$\Gamma_k(\gamma_1) \geq \Gamma_k(\gamma_2), k = 1, 2, 3, ... \tag{6.2}$$

Proof. It follows from the fact that $q(\gamma)$ is a monotonic non-decreasing function. □

Property 6.2 *(throughput at high SINR)*:

$$\lim_{\gamma \to +\infty} \Gamma_k(\gamma) = \frac{\alpha}{2^k}, k = 1, 2, 3, ... \tag{6.3}$$

Proof. It can be proved by noticing that $\lim_{\gamma \to +\infty} q(\gamma) = 1$. □

Property 6.3 *(throughput at low SINR)*:

$$\lim_{\gamma \to 0} \Gamma_k(\gamma) = c \cdot \frac{\alpha}{2^k}, k = 1, 2, 3, ... \tag{6.4}$$

where c is a non-negative constant.

Proof. It can be proved by noticing that $\lim_{\gamma \to 0} q(\gamma) = c$, where c is a non-negative constant. □

Proposition 6.1 *(upper bound)*: Let the throughput functions be

$$\Gamma_k = \frac{\alpha}{2^k} q(k, \gamma) \quad k = 1, 2, 3, ... \tag{6.5}$$

The aggregate throughput of the system employing OVSF codes is bounded by

$$\sum_{i=1}^{M} \Gamma_{k_i}(\gamma_i) \leq \alpha. \tag{6.6}$$

Proof. Based on *Properties 6.1-6.2*,

$$\sum_{i=1}^{M} \Gamma_{k_i}(\gamma_i) \leq \sum_{i=1}^{M} \frac{\alpha}{2^{k_i}} = \alpha \sum_{i=1}^{M} 2^{-k_i} \leq \alpha. \tag{6.7}$$

□

6.2.2 Throughput Functions

Let us introduce three types of Γ_k which satisfy the *Properties 6.1-6.3*.

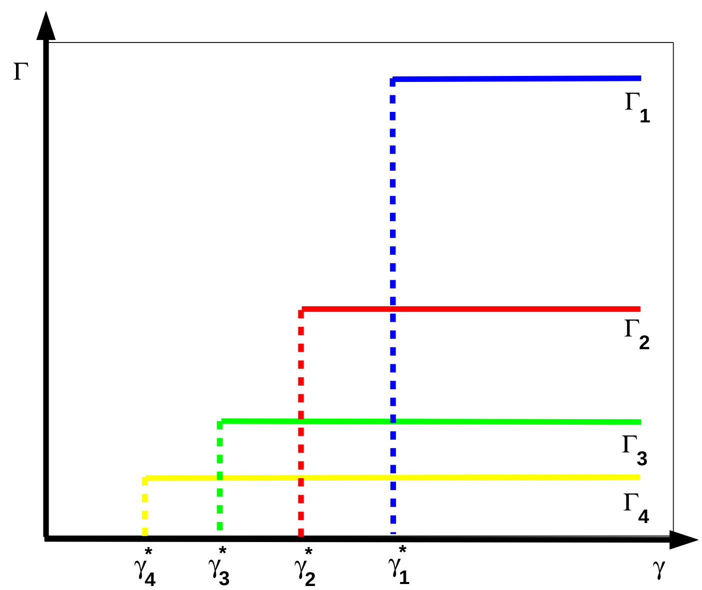

Figure 6.1: Type-I throughput function Γ_k

Type-I Γ_k as shown in Fig. 6.1

$$\Gamma_k(\gamma) = \begin{cases} \alpha/2^k, & \gamma \geq \gamma_k^* \\ 0, & \text{otherwise} \end{cases}, \tag{6.8}$$

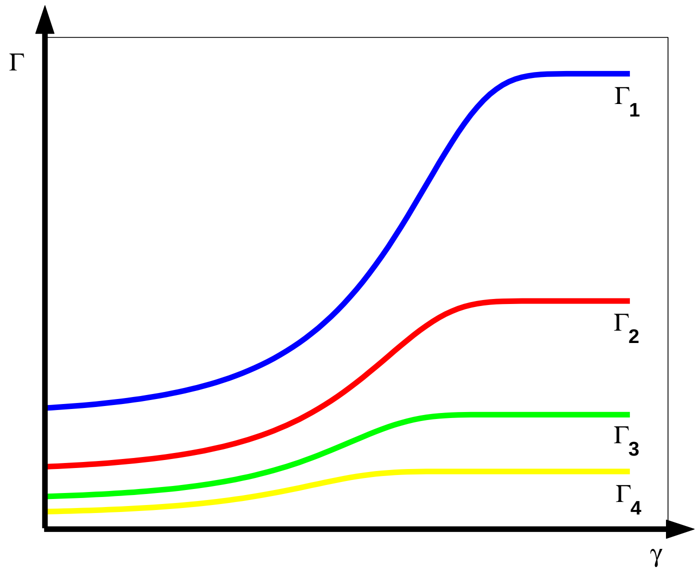

Figure 6.2: Type-II throughput function Γ_k

where $\gamma_k^* \geq \gamma_{k+1}^*, k = 1, 2, 3, ...$

Type-II Γ_k as shown in Fig. 6.2 A set of Type-II throughput functions is characterized by the fact that there is no intersection between any two of them.

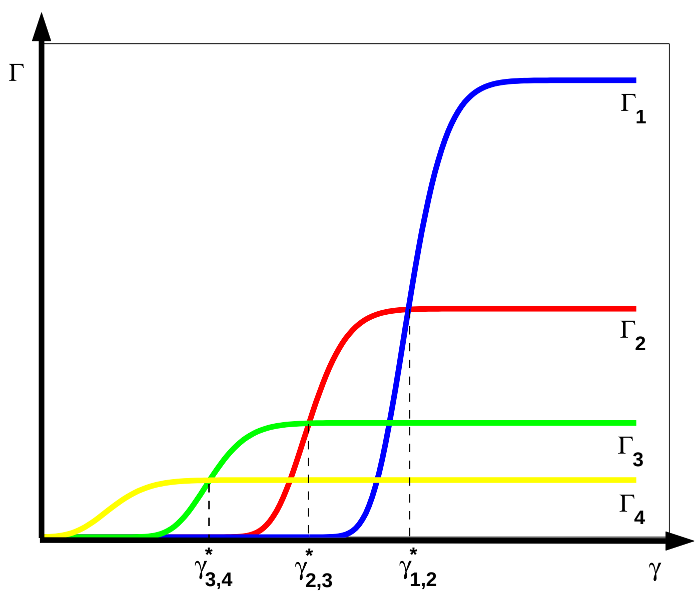

Figure 6.3: Type-III throughput function Γ_k

Type-III Γ_k as illustrated in Fig. 6.3 Let $k < j$,

$$(\Gamma_k - \Gamma_j)(\gamma) \begin{cases} > 0, & \gamma > \gamma^*_{k,j} \\ = 0, & \gamma = \gamma^*_{k,j} \\ < 0, & \gamma < \gamma^*_{k,j} \end{cases}, \tag{6.9}$$

where $(\Gamma_k - \Gamma_j)(\gamma) = \Gamma_k(\gamma) - \Gamma_j(\gamma)$ and $\gamma^*_{k,k+1} \geq \gamma^*_{k+1,k+2}$, $k, j = 1, 2, 3, ...$

Property 6.4 *(maximum Γ_k)*: Let $\gamma^*_{0,1} = +\infty$. When $\gamma^*_{k-1,k} \geq \gamma \geq \gamma^*_{k,k+1}, k = 1, 2, 3, ...$, then

$$(\Gamma_k - \Gamma_j)(\gamma) \geq 0, \qquad \forall j \neq k \quad \text{and} \quad j = 1, 2, 3, ... \tag{6.10}$$

Proof. If $j > k$, then $\gamma \geq \gamma^*_{k,k+1} \geq \gamma^*_{k+1,k+2} \geq ... \geq \gamma^*_{j-1,j}$. Thus,

$$\Gamma_k(\gamma) \geq \Gamma_{k+1}(\gamma) \geq ... \geq \Gamma_{j-1}(\gamma) \geq \Gamma_j(\gamma). \tag{6.11}$$

If $j < k$, then $\gamma \leq \gamma^*_{k-1,k} \leq \gamma^*_{k-2,k-1} \leq ... \leq \gamma^*_{j,j+1}$,

$$\Gamma_k(\gamma) \geq \Gamma_{k-1}(\gamma) \geq ... \geq \Gamma_{j+1}(\gamma) \geq \Gamma_j(\gamma). \tag{6.12}$$

□

6.2.3 Examples of Throughput Functions

For example, let us assume a single user uncoded BPSK system in an AWGN channel with two-sided spectral density $N_0/2$. J information bits are transmitted in a packet. The packet needs to be retransmitted if one or more than one errors have been detected. The transmitted power is P_t and the link gain is h. Let $\gamma = n^{(1)} T_c P_t h / 2N_0$, thus,

$$\Gamma_k(\gamma) = \frac{\alpha}{2^k}\left[1 - \frac{1}{2}\text{erfc}\left(\sqrt{2^k \gamma}\right)\right]^J, \quad k = 1, 2, 3, ..., \tag{6.13}$$

where erfc$(\cdot)$ is the complementary error function and $\alpha = 2/n^{(1)}T_c$. When J is relative small, the $\{\Gamma_k, k = 1, 2, 3, ...\}$ are Type-II throughput functions. Type-III throughput functions are those in (6.13) with relative large J. Type-I throughput models an ideal situation where $q(k, \gamma) = 1$ if the SNR $2^k \gamma \geq \gamma_0$ in (6.1). Thus, $\gamma^*_k = \gamma_0 2^{-k}$ in (6.8).

However, if we fix the transmit bit energy E_t instead of the constant transmit power P_t, the throughput function

$$\Gamma_k(\gamma) = \frac{\alpha}{2^k}\left[1 - \frac{1}{2}\text{erfc}\left(\sqrt{\gamma}\right)\right]^J, \quad k = 1, 2, 3, ..., \tag{6.14}$$

where $\gamma = E_t h/N_0$, is another example of a Type-II throughput function.

The examples for coded systems can be the ones from [72], where the throughput performance of binary IR HARQ (incremental redundancy Hybrid ARQ) schemes based on turbo codes over block-fading channels are studied. In general, the throughput function presents either a step or a continuous nondecreasing behavior. We consider Type-I, Type-II, and Type-III functions as shown in Figs. 6.1-6.3.

6.2.4 Aggregate Throughput

In a non-adaptive case, the OVSF codes can be assigned to the M active users as follows,

$$k_i = \begin{cases} p, & 1 \le i \le 2^{p+1} - M \\ p+1, & 2^{p+1} - M + 1 \le i \le M \end{cases}, \tag{6.15}$$

where $p = \lfloor \log_2 M \rfloor$. The codes on the pth or $(p+1)$th layer arbitrarily assigned to the M users as long as none of parent-child pairs is assigned. However, if the SINR γ of the users are periodically estimated, then we can accordingly assign the active users the OVSF codes other than on the pth or $(p+1)$th layer to maximize the overall throughput of the system, i.e.,

$$\max_{\{k_i\}} \left\{ \sum_{i=1}^{M} \Gamma_{k_i}(\gamma_i) = \sum_{i=1}^{M} \frac{\alpha}{2^{k_i}} q\left(k_i, \gamma_i\right) \right\} \tag{6.16.a}$$

$$\text{subject to:} \quad \sum_{i=1}^{M} 2^{-k_i} \le 1, \qquad k_i \in \{1, 2, 3, ...\}. \tag{6.16.b}$$

For simplicity, throughout this chapter, we assume that assigning a different layer code to the ith user may not change the SINR of other users. An example can be the downlink of a synchronous CDMA system in a frequency-flat channel. The orthogonality of the OVSF codes remains unchanged when a user select codes from a different layer. Another example is the uplink of a QS-CDMA system employing OVSF codes with a ZCZ which we will discuss in details in Section 6.3.

The problem stated in (6.16) can be considered as an extended binary Knapsack problem (BKP) [70]. BKP belongs to a discrete optimization problem and is NP-complete so that it can not be solved with polynomial complexity. In BKP, a hitch-hiker has to fill up his knapsack of capacity v by selecting from among various possible objects with volume v_i and comfort c_i to maximize his total comfort. Formally,

$$\max \sum_{i=1}^{m} c_i x_i \tag{6.17.a}$$

$$\text{subject to:} \quad \sum_{i=1}^{m} v_i x_i \le v \tag{6.17.b}$$

where

$$x_i = \begin{cases} 1, & \text{if object} \quad i \quad \text{is selected} \\ 0, & \text{otherwise} \end{cases}. \tag{6.18}$$

In the problem stated in (6.16), for each γ_i, we have a set of infinite codes each having a value $\Gamma_k(\gamma_i)$ and a cost 2^{-k}, $k = 1, 2, 3, ...$ Our goal is selecting one code in each set to maximize the total value under a total cost 1.

6.3 OVSF Codes Assignment

6.3.1 Self Maximization and Group Maximization

Definition 6.1 *(self maximization)*: The self maximization is defined as a code assignment which maximizes a single user throughput and satisfies a constraint k_a, that is,

$$\mathcal{S}(k_a, \gamma) = \arg\max_{k} \left\{ \Gamma_k(\gamma), k \ge k_a \right\}. \tag{6.19}$$

Definition 6.2 *(group maximization)*: Let $\overrightarrow{\gamma} = \{\gamma_1, \gamma_2, ..., \gamma_m\}$, where $\gamma_1 \ge \gamma_2 \ge ... \ge \gamma_m$, and $v \le 1$. Then, the group maximization is defined as a code assignment which maximizes the overall throughput of m users under the constraint v and k_a,

$$\mathcal{G}\left(v, k_a, \overrightarrow{\gamma}, m\right) = \arg\max_{\{k_i\}} \left\{ \sum_{i=1}^{m} \Gamma_{k_i}(\gamma_i) : \sum_{i=1}^{m} 2^{-k_i} \le v, \forall k_i \ge k_a \right\}, \tag{6.20.a}$$

$$\Gamma_{\max}(v, k_a, \overrightarrow{\gamma}, m) = \max_{\{k_i\}} \left\{ \sum_{i=1}^{m} \Gamma_{k_i}(\gamma_i) : \sum_{i=1}^{m} 2^{-k_i} \le v, \forall k_i \ge k_a \right\}. \tag{6.20.b}$$

Theorem 6.1 *(self maximization)*: For the ith user of γ_i, then

1) for Type-I Γ_k,

$$\mathcal{S}(k_a, \gamma_i) = \begin{cases} k_a, & \gamma_i \ge \gamma^*_{k_a} \\ k, & \gamma^*_k \le \gamma_i < \gamma^*_{k-1} \le \gamma^*_{k_a}. \end{cases} \tag{6.21}$$

2) for Type-II Γ_k,

$$\mathcal{S}(k_a, \gamma_i) = k_a. \tag{6.22}$$

3) for Type-III Γ_k,

$$\mathcal{S}(k_a, \gamma_i) = \begin{cases} k_a, & \gamma_i \geq \gamma^*_{k_a, k_a+1} \\ k, & \gamma^*_{k,k+1} \leq \gamma_i < \gamma^*_{k-1,k} \end{cases}. \tag{6.23}$$

Proof. If a k_ath layer code is available, then all its child codes with layer index $k > k_a$ are also available. The proofs for Type-I and Type-II Γ_k are straightforward. The result for Type-III Γ_k can be proved by using *Property 6.4*. □

Lemma 6.1 *(priority of users)*: If $\mathcal{S}(k_a, \gamma_i) = k_i$, $\mathcal{S}(k_a, \gamma_j) = k_j$, and $\gamma_i \geq \gamma_j$, then $k_i \leq k_j$ and $\Gamma_{k_i}(\gamma_i) \geq \Gamma_{k_j}(\gamma_j)$.

Proof. The proof follows the fact that throughput functions are monotonic non-decreasing functions. □

Lemma 6.2 *(priority of codes)*: If $\mathcal{S}(k_a, \gamma_i) = k_i$, $\mathcal{S}(k_b, \gamma_i) = k_i^{'}$, and $k_a \leq k_b$, then $k_i \leq k_i^{'}$ and $\Gamma_{k_i}(\gamma_i) \geq \Gamma_{k_i^{'}}(\gamma_i)$.

Proof. It can be proved based on (6.21)-(6.23). □

Lemma 6.3 *(pair maximization)*: Let $\gamma_i \geq \gamma_j$,

$$\mathcal{S}(k_a, \gamma_i) = k_i; \;\; \mathcal{S}(k_b, \gamma_j) = k_j \tag{6.24}$$

$$\mathcal{S}(k_b, \gamma_i) = k_i^{'}; \;\; \mathcal{S}(k_a, \gamma_j) = k_j^{'} \tag{6.25}$$

If $k_a \leq k_b$, then $\Gamma_{k_i}(\gamma_i) + \Gamma_{k_j}(\gamma_j) \geq \Gamma_{k_i^{'}}(\gamma_i) + \Gamma_{k_j^{'}}(\gamma_j)$, for

1) Type-I Γ_k;

2) Type-II Γ_k satisfying $\frac{d(\Gamma_k - \Gamma_j)(\gamma)}{d\gamma} \geq 0$, $\forall \gamma \geq 0, k \leq j$.

3) Type-III Γ_k satisfying $\frac{d(\Gamma_k - \Gamma_{k+1})(\gamma)}{d\gamma} \geq 0$, $\forall \gamma \geq \gamma^*_{k,k+1}, k = 1, 2, 3, ...$

Proof. See Appendix. □

In the following, we focus on the throughput functions with a constraint on q functions as described in *Lemma 6.3*.

Theorem 6.2 *(initial assignment)*: Let $\gamma_1 \geq \gamma_2 \geq ... \geq \gamma_M$, and $v \leq 1$. The OVSF codes can be assigned to M active users as follows,

$$\mathcal{S}\left(K_i, \gamma_i\right) = k_i, \qquad i = 1, 2, ...M, \tag{6.26.a}$$

$$\text{where} \qquad K_i = \begin{cases} 1 + \left\lfloor -\log_2\left(v\right)\right\rfloor, & i = 1 \\ 1 + \left\lfloor -\log_2\left(v - \sum_{j=1}^{i-1} 2^{-k_j}\right)\right\rfloor, & 1 < i < M \\ \left\lceil -\log_2\left(v - \sum_{j=1}^{M-1} 2^{-k_j}\right)\right\rceil, & i = M \end{cases} \tag{6.26.b}$$

Then, if $i \leq j$, the resulting code assignment $\{k_i, 1 \leq i \leq M\}$ satisfies

$$\sum_{i=1}^{M} 2^{-k_i} \leq v, \tag{6.27.a}$$

$$\Gamma_{k_i}(\gamma_i) \geq \Gamma_{k_j}(\gamma_j), \tag{6.27.b}$$

$$k_i \leq k_j, \tag{6.27.c}$$

$$\Gamma_{k_i}(\gamma_i) + \Gamma_{k_j}(\gamma_j) \geq \Gamma_{k_j}(\gamma_i) + \Gamma_{k_i}(\gamma_j). \tag{6.27.d}$$

Proof. See Appendix. □

Lemma 6.4 *(stopping criteria)*: Let $\gamma_1 \geq \gamma_2 \geq ... \geq \gamma_m$ and $\overrightarrow{\gamma}_{i+1} = \{\gamma_{i+1}, \gamma_{i+2}, ..., \gamma_m\}$. Let $\{k_i, 1 \leq i \leq m\}$ denote the initial code assignment and, thus, $v_i = v - \sum_{j=1}^{i-1} 2^{-k_j}$, then $\forall t \geq t_0$,

$$\begin{aligned} &\Gamma_{\max}\left(v_i - 2^{-k_i - t_0}, k_i + t_0, \overrightarrow{\gamma}_{i+1}, m - i\right) \geq \\ &\Gamma_{\max}\left(v_i - 2^{-k_i - t}, k_i + t, \overrightarrow{\gamma}_{i+1}, m - i\right), \end{aligned} \tag{6.28}$$

where

$$t_0 = \left\lceil \log_2\left(\frac{m - i + 1}{2^{k_i} v_i}\right)\right\rceil. \tag{6.29}$$

Proof. When the ith user increase its initial code from the k_ith layer to $(k_i + t)$th layer, then

$$v'_{i+1} = v - \sum_{j=1}^{i-1} 2^{-k_j} - 2^{-k_i - t} = v_i - 2^{-k_i - t}. \tag{6.30}$$

If $t = t_0$, we have

$$(m-i+1)2^{-(k_i+t_0)} \leq v_i. \tag{6.31}$$

Then, a $(k_i + t_0)$th layer code is available for each jth user, where $j > i$. Thus,

$$\begin{aligned} \mathcal{G}\left(v_i - 2^{-k_i-t_0}, k_i + t_0, \overrightarrow{\gamma}_{i+1}, m-i\right) &= \{\mathcal{S}(k_i + t_0, \gamma_{i+1}), \mathcal{S}(k_i + t_0, \gamma_{i+2}), \\ &\quad ..., \mathcal{S}(k_i + t_0, \gamma_m)\}. \end{aligned} \tag{6.32}$$

In a similar way, for $\forall t > t_0$,

$$\begin{aligned} \mathcal{G}\left(v_i - 2^{-k_i-t}, k_i + t, \overrightarrow{\gamma}_{i+1}, m-i\right) &= \{\mathcal{S}(k_i + t, \gamma_{i+1}), \mathcal{S}(k_i + t, \gamma_{i+2}), \\ &\quad ..., \mathcal{S}(k_i + t, \gamma_m)\}. \end{aligned} \tag{6.33}$$

Based on *Lemma 6.2*, (6.28) can be proved by (6.32) and (6.33). □

Theorem 6.3 *(group maximization)*: Let $\overrightarrow{\gamma_i} = \{\gamma_i, \gamma_{i+1}, ..., \gamma_m\}$, where $1 \leq i \leq m$ and $\gamma_1 \geq \gamma_2 \geq ... \geq \gamma_m$. Let $\{k_i, 1 \leq i \leq m\}$ denote the initial code assignment, and $v_i = v - \sum_{j=1}^{i-1} 2^{-k_j}$. If

$$\mathcal{G}\left(v_i - 2^{-k_i-t}, k_i + t, \overrightarrow{\gamma}_{i+1}, m-i\right), \quad 0 \leq t \leq t_0 = \left\lceil \log_2\left(\frac{m-i+1}{2^{k_i} v_i}\right)\right\rceil, \tag{6.34}$$

are known, and let

$$\delta = \arg\max_t \left\{\Gamma_{\max}\left(v_i - 2^{-k_i-t}, k_i + t, \overrightarrow{\gamma}_{i+1}, m-i\right) + \Gamma_{k_i+t}(\gamma_i), 0 \leq t \leq t_0\right\}. \tag{6.35}$$

Then the group maximization

$$\mathcal{G}\left(v_i, k_i, \overrightarrow{\gamma_i}, m-i+1\right) = (k_i + \delta) \cup \mathcal{G}\left(v_i - 2^{-k_i-\delta}, k_i + \delta, \overrightarrow{\gamma}_{i+1}, m-i\right). \tag{6.36}$$

Proof.

$$\begin{aligned} \Gamma_{\max}\left(v_i, k_i, \overrightarrow{\gamma}_i, m-i+1\right) = \max_t \Big\{&\Gamma_{\max}\left(v_i - 2^{-k_i-t}, k_i + t, \overrightarrow{\gamma}_{i+1}, m-i\right) \\ &+\Gamma_{k_i+t}(\gamma_i), t \geq 0\Big\}. \end{aligned} \tag{6.37}$$

Note that, k_i is the initial code assignment for the ith user and, thus, for $\forall t > 0$, $\Gamma_{k_i+t}(\gamma_i) \leq \Gamma_{k_i}(\gamma_i)$. Hence, (6.36) can be proved based on *Lemma 6.4*. □

Table 6.1: Initial OVSF Codes Assignment Algorithm

Input:	$\gamma_1 \geq \gamma_2 \geq \gamma_3 \geq ... \geq \gamma_M$
	FOR $\quad i = 1 : M,$
	$K_i = \begin{cases} 1, & i = 1 \\ 1 + \left\lfloor -\log_2 \left(1 - \sum_{j=1}^{i-1} 2^{-k_j}\right)\right\rfloor, & 1 < i < M \\ \left\lceil -\log_2 \left(1 - \sum_{j=1}^{M-1} 2^{-k_j}\right)\right\rceil, & i = M \end{cases}$
	$k_i = \mathcal{S}\left(K_i, \gamma_i\right)$
	END
Output:	$\{k_i, i = 1, 2, ...M\}$

6.3.2 OVSF Codes Assignment Algorithms

Table 6.1 presents an initial code assignment algorithm, which is an important initialization step for optimal code assignment demonstrated in Table 6.2.

Theorem 6.4: The initial code assignment algorithm maximizes the aggregate throughput for Type-I Γ_k.

Proof. For Type-I Γ_k, the initial assignment remains unchanged after group maximization.

□

Table 6.2: Optimum OVSF Codes Assignment Algorithm

Input:	$\gamma_1 \geq \gamma_2 \geq \gamma_3 \geq ... \geq \gamma_M$
Initial assignment output:	$\{k_1, k_2, ..., k_M\}$
Group maximization:	$\mathcal{G}\left(v_i, k_i, \overrightarrow{\gamma_i}, M - i + 1\right) =$
	$(k_i + \delta) \cup \mathcal{G}\left(v_i - 2^{-k_i-\delta}, k_i + \delta, \overrightarrow{\gamma}_{i+1}, m - i\right)$
	$\delta = \arg\max_t \left\{\Gamma_{\max}\left(v_i - 2^{-k_i-t}, k_i + t, \overrightarrow{\gamma}_{i+1}, M - i\right)\right.$
	$\left. +\Gamma_{k_i+t}(\gamma_i), 0 \leq t \leq \left\lceil \log_2\left(\frac{M-i+1}{2^{k_i} v_i}\right)\right\rceil \right\}$
	$\mathcal{G}\left(v_M, k_M, \overrightarrow{\gamma}_M, 1\right) = \mathcal{S}(k_M, \gamma_M)$
Optimum assignment Output:	$\mathcal{G}\left(1, 1, \overrightarrow{\gamma_1}, M\right)$

As shown in Table 6.2, the optimum OVSF code assignment maximizes the aggregate throughput of the system by recursively operating the group maximization. For each group maximization, an initial code assignment is operated.

Theorem 6.5: The optimum code assignment algorithm maximizes the aggregate throughput for

1) Type-I Γ_k;

2) Type-II Γ_k satisfying $\frac{d(\Gamma_k - \Gamma_j)(\gamma)}{d\gamma} \geq 0$, $\forall \gamma \geq 0, k \leq j$.

3) Type-III Γ_k satisfying $\frac{d(\Gamma_k - \Gamma_{k+1})(\gamma)}{d\gamma} \geq 0$, $\forall \gamma \geq \gamma^*_{k,k+1}, k = 1, 2, 3, ...$

Proof. For M users with $\gamma_1 \geq \gamma_2 \geq ... \geq \gamma_M$, the optimum code assignment $\mathcal{G}\left(1, 1, \overrightarrow{\gamma_1}, M\right)$ can be recursively achieved by the group maximization described in *Theorem 6.3.* For the Mth user, its group maximization is reduced to self maximization. □

6.4 Uplink Throughput Maximization for a QS-DS UWB Systems

6.4.1 System Model

We consider a QS-CDMA UWB system in which M active users share the same physical channel. Each user transmits data to a single base station. The received signal is

$$r(t) = \sum_{i=1}^{M} \sqrt{P_i} b_i(t - l_i T_c) \sum_{n=0}^{N-1} c_{i,n} \psi(t - nT_c - l_i T_c) + n(t), \tag{6.38}$$

where $\psi(t)$ is the unit energy signaling pulse assumed here to be equal to zero outside an interval which is equal to or smaller than the chip duration T_c, and assumed known to the receiver. The spreading sequence $\mathbf{c}_i = \{c_{i,0}, c_{i,1}, ...c_{i,N-1}\}$ is selected from the OVSF code tree. $b_i(t)$ is the data signal consisting of a sequence of rectangular pulses with positive or negative unit amplitude and the duration $T_b = NT_c$. P_i is the received signal power of user i determined by

$$P_i = \frac{(1-\lambda) E_c h_i}{T_c}, \tag{6.39}$$

where λ is the deficiency ratio of the employed OVSF codes, h_i is the link gain of the ith user, and E_c is the chip energy. For simplification, we have quantized the propagation delay into bins, and l_i is modelled as a discrete uniformly distributed random variable on $[-\tau, \tau]$, where τ is an integer and $0 \leq \tau \leq N/2$. We further assume the time delays of all users are i.i.d. $n(t)$ is the white Gaussian noise with two-sided spectral density $N_0/2$.

When the correlator receiver is tuned to the ith user, the output is,

$$Z_i = \sqrt{P_i T_b}\left(b_{i,0} + \sum_{j=1, j\neq i}^{M} I_{i,j} + \eta\right), \tag{6.40}$$

where η is a zero-mean Gaussian random variable with variance $\sigma_\eta^2 = (2P_iT_b/N_0)^{-1}$. $b_{i,0}$ is the desired signal in the decision interval $[0, T_b]$. The MAI from the jth user is denoted as $I_{i,j}$, which can be formulated in terms of the continuous two bits, i.e., $b_{j,-1}$ and $b_{j,0}$, of the jth user, and the aperiodic cross correlation functions of $\mathbf{c}_i$ and $\mathbf{c}_j$.

$$I_{i,j}(l_{i,j}) = \sqrt{\frac{P_j}{P_i}}\left[b_{j,-1}A_{\mathbf{c}_i,\mathbf{c}_j}(l_{i,j} - N) + b_{j,0}A_{\mathbf{c}_i,\mathbf{c}_j}(l_{i,j})\right], \tag{6.41}$$

where $l_{i,j}$ represents the time delay of the jth user relative to the ith user. Let $\xi = \sum_{j\neq i} I_{i,j}(l_{i,j})$. The data signal $b_j(t)$ and the time delay $l_{i,j}$ are independent from those of different users. As a result, the variance of ξ equals to the sum of the variance of $I_{i,j}(l_{i,j})$, that is

$$\begin{aligned}
\sigma_\xi^2 &= \sum_{j\neq i} E\left\{I_{i,j}^2(l_{i,j})\right\} \\
&= \sum_{j\neq i} \frac{P_j}{2P_i}\left\{(A_{\mathbf{c}_i,\mathbf{c}_j}(l_{i,j} - N) + A_{\mathbf{c}_i,\mathbf{c}_j}(l_{i,j}))^2 + (A_{\mathbf{c}_i,\mathbf{c}_j}(l_{i,j} - N) - A_{\mathbf{c}_i,\mathbf{c}_j}(l_{i,j}))^2\right\} \\
&= \sum_{j\neq i} \frac{P_j}{P_i}\left\{A^2_{\mathbf{c}_i,\mathbf{c}_j}(l_{i,j} - N) + A^2_{\mathbf{c}_i,\mathbf{c}_j}(l_{i,j})\right\}.
\end{aligned} \tag{6.42}$$

Let us assume, in the QS-CDMA system, the relative time delay $l_{i,j}$ can be maintained such that $|l_{i,j}| \leq L_{zcz}$, where L_{zcz} is the ZCZ of the employed OVSF codes. Based on the definition of ZCZ and *Property 5.3* of the proposed ZCZ sequences, the MAI can be reduced as $\sigma_\xi^2 = 0$. Thus, the signal to interference and noise ratio (SINR) for the ith user is,

$$\mathrm{SINR}_i = \frac{1}{\sigma_\xi^2 + \sigma_\eta^2} = \frac{(1-\lambda)Nh_iE_c}{N_0/2}, \quad \text{when} \quad |l_{i,j}| \leq L_{zcz}. \tag{6.43}$$

Let us further assume the QS-DS UWB system is an uncoded BPSK system in which J information bits are transmitted in a packet. The packet needs to be retransmitted if one or more than one errors have been detected. When the relative delays within the ZCZ and the kth layer code is assigned to the user, we have

$$\Gamma_k = \frac{1}{2^{k+1}nT_c}\left[1 - \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{2^{k+1}n(1-\lambda)h_iE_c}{N_0/2}}\right)\right]^J \quad k = 1, 2, 3, ... \tag{6.44}$$

Based on $\gamma_i = h_i E_c / N_0$ of the ith user, we attempt to maximize the overall throughput of the QS-DS-UWB system,

$$\max_{\{k_i\}} \sum_{i=1}^{M} \frac{\alpha}{2^{k_i}} \left[1 - \frac{1}{2}\text{erfc}\left(\sqrt{2^{k_i}\beta\gamma_i}\right)\right]^J \tag{6.45.a}$$

$$\text{subject to:} \quad \sum_{i=1}^{M} 2^{-k_i} \leq 1, \qquad k_i \in \{1, 2, 3, ...\}. \tag{6.45.b}$$

where $\beta = 4(1-\lambda)n$ and $\alpha = 1/2nT_c$.

6.4.2 OVSF Code Tree Selection

When the OVSF code tree with $L_{zcz} = \tau$ is selected, the multiple access interference is cancelled and, thus, the power control scheme is not necessary. All active users can simply maintain the same chip energy. The TCPs of the same length and different deficiency ratio may further affect the aggregate throughput of the system.

Proposition 6.2: Let $\lambda(n)$ and $\lambda'(n)$ be the deficiency ratios of two TCPs of length n. Let $\{k_i\}$ denote the code assignment for the ith users of SINR γ_i, $i = 1, 2...M$. If $\lambda(n) \geq \lambda'(n)$, then we have

$$\sum_{i=1}^{M} \Gamma_{k_i}(\gamma_i) \leq \sum_{i=1}^{M} \Gamma'_{k_i}(\gamma_i), \tag{6.46}$$

where $\Gamma_{k_i}(\gamma_i)$ and $\Gamma'_{k_i}(\gamma_i)$ denotes the throughput of the ith user employing OVSF code tree constructed by the TCP of deficiency ratio $\lambda(n)$ and $\lambda'(n)$ respectively.

Proof. $q(\gamma)$ is a monotonic non-decreasing function, thus,

$$\frac{\Gamma_{k_i}(\gamma_i)}{\Gamma'_{k_i}(\gamma_i)} = \frac{q\left(\frac{2^{k_i}n(1-\lambda(n))h_i E_c}{N_0/2}\right)}{q\left(\frac{2^{k_i}n(1-\lambda'(n))h_i E_c}{N_0/2}\right)} \leq 1. \tag{6.47}$$

□

Based on *Proposition 6.2*, we suggest selecting the TCP of length $n = \tau$, and with the minimum $\lambda(n)$.

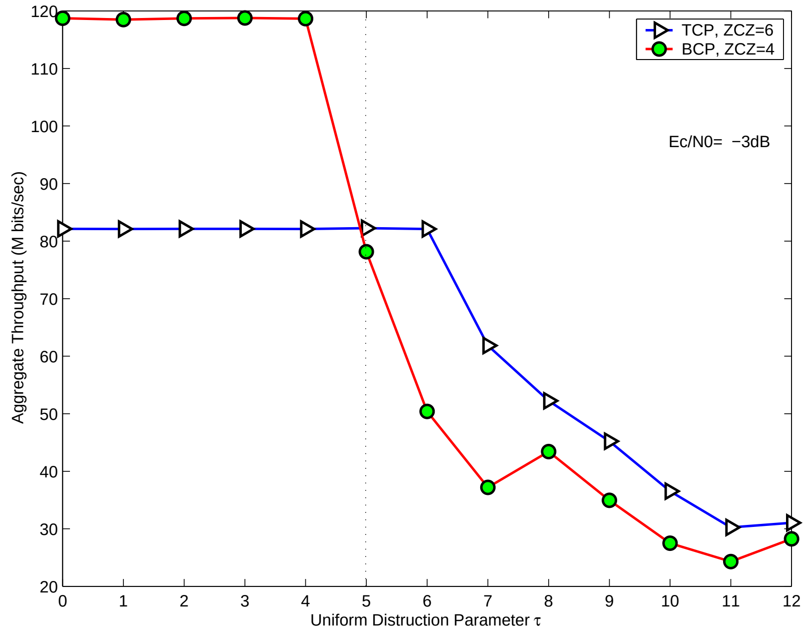

Figure 6.4: OVSF code tree selection: BCP-based OVSF code tree with $L_{zcz} = 4$ v.s. TCP-based OVSF code tree with $L_{zcz} = 6$, $E_c/N_0 = -3$ dB

6.4.3 Numerical Results

We assume that the multipath interference has been suppressed by the ZCZ sequences. In the QS-DS UWB system, M active links (M users and 1 base station) are in the system. The $\gamma_i = E_c d_0^2 / N_0 d_i^2$, where d_i, the distance between the ith user and the base station, is a uniformly distributed random variable in $[d_0, 5d_0]$ and may vary from time to time. Each transmitter maintains the same chip energy independent of its data rate. The chip rate $1/T_c = 1$ GHz, and the packet size $J = 1024$.

We first demonstrate how to select the OVSF code tree. One of the candidate OVSF trees is constructed from the TCP $\{++-+0+, ++--0-\}$, as shown in the example in Section 5.3. The tree has a $L_{zcz} = 6$. The other OVSF code tree is constructed from a BCP $\{++-+, ++--\}$ with $L_{zcz} = 4$. The aggregate throughput is calculated based on the average aggregate throughput over 10^4 sets of random positions of 4 users at $E_c/N_0 = -3$ dB. Fig. 6.4 illustrates how τ affects the aggregate throughput of the system. When $\tau < 5$, the OVSF code tree with less sequence length shows its advantage over the OVSF code tree with longer length due to the time delays tensely around the origin. When $\tau \geq 5$, the time delays may fall outside $L_{zcz} = 4$, hence if an active user suffers a sever near-far problem, its effective throughput may significantly reduced. In this case, the OVSF code tree with larger L_{zcz} shows its advantage.

Figure 6.5 illustrates the aggregate throughput gain of the proposed adaptive data rate transmission scheme over the non-adaptive transmission when the OVSF codes of $L_{zcz} = 6$ is selected. We assume $\tau \leq L_{zcz}$. In the non-adaptive transmission case, the OVSF codes are assigned to the users based on (6.15). We can observe the average aggregate throughput gain of initial and optimal code assignment over the non-adaptive scheme at relative low E_c/N_0 regions. For particular relative distances between the users and the base station, the optimal code assignment may have instantaneous aggregate throughput gain over the initial code assignment. If the average aggregate throughput is considered, as shown in Fig. 6.5, the performance of the initial code assignment approaches that of the optimal one due to the fact that the instantaneous aggregate throughput gain has been averaged out. Thus, the optimal code assignment trades the computation complexity only for the instantaneous aggregate throughput. Based on *Proposition 6.1*, both adaptive and non-adaptive schemes approach the aggregate throughput upper bound $\alpha = 1/2nT_c = 83.3$(Mbps) at high E_c/N_0 region.

6.5 Conclusion

We study the aggregate throughput maximization problem associated with OVSF codes for three types of throughput functions. We propose two code assignment scheme: initial code assignment and optimal code assignment. Both algorithms adapt the data rate of active users based on their SINRs and, thus, obtain the aggregate throughput gain. Optimal code assignment trades the long code allocation time to achieve the instantaneous maximum aggregate throughput.

We construct ternary OVSF codes with a ZCZ of arbitrary lengths. The OVSF codes can be employed in a QS-CDMA UWB system to suppress both the multipath and multiple access interference. The relative time delay of the QS CDMA system is modelled as a uniformly distributed random variable on $[-\tau, \tau]$. When τ is large, the time delays may frequently fall outside the ZCZ of the OVSF codes, thus causes the MAI to the active users,

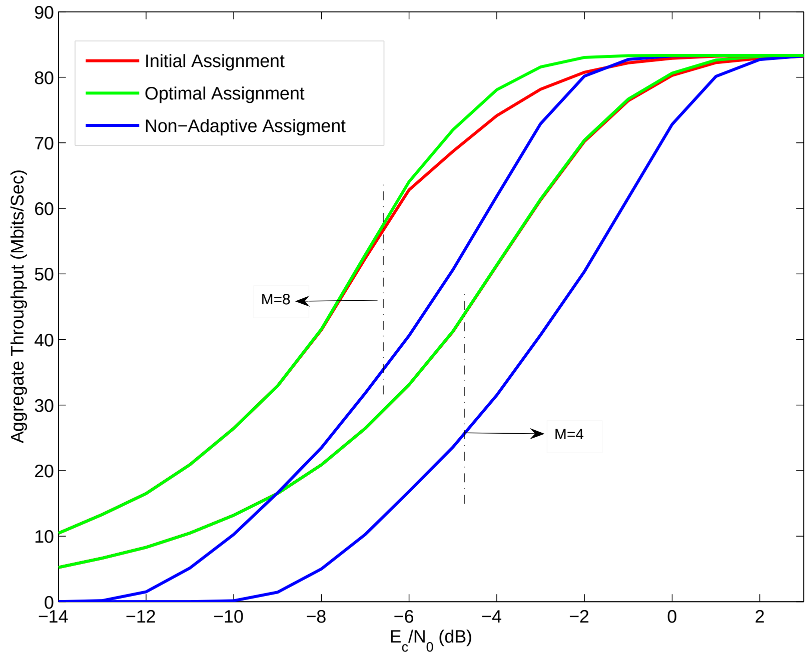

Figure 6.5: The aggregate throughput gain of the adaptive data rate transmission scheme for the DS-UWB system employing OVSF code tree with $L_{zcz} \geq \tau$

and further reduces the aggregate throughput of the system. Based on the knowledge of τ, we demonstrate how to select the OVSF code tree of different L_{zcz} to improve the aggregate throughput of the system. Once the OVSF code tree is selected, based on the knowledge of SINRs, we can further obtain the aggregate throughput gain by adaptively allocating OVSF codes to the active users.

6.6 Appendix

6.6.1 Proof of Lemma 6.3

Proof for Type-I Γ_k: Based on (6.21), if $\gamma^*_{k_a} \geq \gamma^*_{k_b}$, then

$$\Gamma_{k_i}(\gamma_i) + \Gamma_{k_j}(\gamma_j) \begin{cases} = \Gamma_{k_i'}(\gamma_i) + \Gamma_{k_j'}(\gamma_j), & \gamma_i \geq \gamma_j \geq \gamma^*_{k_a} \quad \text{or} \quad \gamma^*_{k_a} \geq \gamma_i \geq \gamma_j \geq \gamma^*_{k_b} \\ & \text{or} \quad \gamma^*_{k_b} \geq \gamma_i \geq \gamma_j \\ > \Gamma_{k_i'}(\gamma_i) + \Gamma_{k_j'}(\gamma_j), & \gamma_i \geq \gamma^*_{k_a} \geq \gamma_j \quad \text{or} \quad \gamma^*_{k_a} \geq \gamma_i \geq \gamma^*_{k_b} \geq \gamma_j \end{cases} \tag{6.48}$$

Proof for Type-II Γ_k: If $k_a \le k_b$, $(\Gamma_{k_a} - \Gamma_{k_b})(\gamma)$ is a monotonic non-decreasing function. Hence, if $\gamma_i \ge \gamma_j$,

$$\begin{aligned}
& \left(\Gamma_{k_i}(\gamma_i) + \Gamma_{k_j}(\gamma_j)\right) - \left(\Gamma_{k_i'}(\gamma_i) + \Gamma_{k_j'}(\gamma_j)\right) \\
= \; & \left(\Gamma_{k_a}(\gamma_i) + \Gamma_{k_b}(\gamma_j)\right) - \left(\Gamma_{k_b}(\gamma_i) + \Gamma_{k_a}(\gamma_j)\right) \\
= \; & \left(\Gamma_{k_a} - \Gamma_{k_b}\right)(\gamma_i) - \left(\Gamma_{k_a} - \Gamma_{k_b}\right)(\gamma_j) \ge 0. \qquad (6.49)
\end{aligned}$$

Proof for Type-III Γ_k: We have that $(\Gamma_k - \Gamma_{k+1})(\gamma)$ is a monotonic non-decreasing function for $\forall \gamma \ge \gamma^*_{k,k+1}, k = 1, 2, 3, ...$ Note that $\gamma^*_{k_a,k_a+1} \ge \gamma^*_{k_a+1,k_a+2} \ge ... \ge \gamma^*_{k_b-1,k_b}$, thus,

$$\begin{aligned}
& \left(\Gamma_{k_a} - \Gamma_{k_b}\right)(\gamma) \\
= \; & \left(\Gamma_{k_a} - \Gamma_{k_a+1}\right)(\gamma) + \left(\Gamma_{k_a+1} - \Gamma_{k_a+2}\right)(\gamma) + ... + \left(\Gamma_{k_b-1} - \Gamma_{k_b}\right)(\gamma) \qquad (6.50)
\end{aligned}$$

is a monotonic non-decreasing function for $\gamma \ge \gamma^*_{k_a,k_a+1}$. Then based on (6.23), we can prove it as follows.

For $\gamma_i \ge \gamma_j \ge \gamma^*_{k_a,k_a+1}$,

$$\begin{aligned}
& \left(\Gamma_{k_i}(\gamma_i) + \Gamma_{k_j}(\gamma_j)\right) - \left(\Gamma_{k_i'}(\gamma_i) + \Gamma_{k_j'}(\gamma_j)\right) \\
= \; & \left(\Gamma_{k_a} - \Gamma_{k_b}\right)(\gamma_i) - \left(\Gamma_{k_a} - \Gamma_{k_b}\right)(\gamma_j) \le 0. \qquad (6.51)
\end{aligned}$$

For $\gamma_i \ge \gamma^*_{k_a,k_a+1} \ge \gamma_j \ge \gamma^*_{k_b,k_b+1}$, without loss of any generality, let us assume $\gamma^*_{t,t+1} \ge \gamma_j \ge \gamma^*_{t+1,t+2}$, then

$$\begin{aligned}
& \left(\Gamma_{k_i} - \Gamma_{k_i'}\right)(\gamma_i) = \left(\Gamma_{k_a} - \Gamma_{k_b}\right)(\gamma_i) \ge \left(\Gamma_{k_a} - \Gamma_{k_b}\right)(\gamma^*_{k_a,k_a+1}) \\
= \; & \left(\Gamma_{k_a+1} - \Gamma_{k_b}\right)(\gamma^*_{k_a,k_a+1}) \ge \left(\Gamma_{k_a+1} - \Gamma_{k_b}\right)(\gamma^*_{k_a+1,k_a+2}) \\
= \; & \left(\Gamma_{k_a+2} - \Gamma_{k_b}\right)(\gamma^*_{k_a+1,k_a+2}) \ge ... \ge \left(\Gamma_{t} - \Gamma_{k_b}\right)(\gamma^*_{t,t+1}) \\
= \; & \left(\Gamma_{t+1} - \Gamma_{k_b}\right)(\gamma^*_{t,t+1}) \ge \left(\Gamma_{t+1} - \Gamma_{k_b}\right)(\gamma_j) = \left(\Gamma_{k_j'} - \Gamma_{k_j}\right)(\gamma_j) \qquad (6.52)
\end{aligned}$$

For $\gamma_i \ge \gamma^*_{k_a,k_a+1} \ge \gamma^*_{k_b,k_b+1} \ge \gamma_j$, we have $\left(\Gamma_{k_j'} - \Gamma_{k_j}\right)(\gamma_j) = 0$. For the cases $\gamma_i \le \gamma^*_{k_a,k_a+1}$, the *Lemma* can be proved in a similar way.

6.6.2 Proof of Theorem 6.2

Proof of (6.27.a): It can be proved by noticing that

$$k_M \geq K_M = \left\lceil -\log_2\left(v - \sum_{j=1}^{M-1} 2^{-k_j}\right)\right\rceil \geq -\log_2\left(v - \sum_{j=1}^{M-1} 2^{-k_j}\right). \tag{6.53}$$

Proof of (6.27.b) and (6.27.c): If $i \leq j$,

$$\begin{aligned} K_j &= 1 + \left\lfloor -\log_2\left(v - \sum_{m=1}^{j-1} 2^{-k_m}\right)\right\rfloor \\ &= 1 + \left\lfloor -\log_2\left(v - \sum_{m=1}^{i-1} 2^{-k_m} - \sum_{m=i}^{j-1} 2^{-k_m}\right)\right\rfloor \\ &\geq 1 + \left\lfloor -\log_2\left(v - \sum_{m=1}^{i-1} 2^{-k_m}\right)\right\rfloor = K_i. \end{aligned} \tag{6.54}$$

Let $k_i' = \mathcal{S}(K_j, \gamma_i)$. Since $k_i = \mathcal{S}(K_i, \gamma_i)$ and $k_j = \mathcal{S}(K_j, \gamma_j)$, based on *Lemmas 6.1-6.2*,

$$\Gamma_{k_i}(\gamma_i) \geq \Gamma_{k_i'}(\gamma_i) \geq \Gamma_{k_j}(\gamma_j), \tag{6.55}$$

$$k_i \leq k_i' \leq k_j. \tag{6.56}$$

Proof of (6.27.d): The proof is very similar to the proof of *Lemma 6.3* and, thus, is omitted.

Chapter 7

Beamforming Sequences

7.1 Introduction

We study an adaptive approach that exploits the increased phase coherence of ultra-wideband (UWB) channels [73] with a "discrete" number of reflectors relative to narrow-band wireless channels. The proposed multipath beamforming technique suggests selecting the transmitted signal waveform based on limited multipath channel state information so that the signal energy at the output of a non-adaptive correlator is enhanced. The approach falls into the general category of adaptive signal design where the transmitter has a full or partial knowledge of the channel. Narrowband literature on this topic is abundant; nevertheless, even though somewhat related UWB literature exists (see, e.g. [74]), an approach that explicitly exploits the distinct phase coherence characteristic of impulsive UWB signaling has not been found in the literature.

In view of target high data rate applications and corresponding simple receiver design, our approach focuses on signal designs that enhance the performance of the non-adaptive correlator.

The first approach exhaustively searches ternary beamforming sequences that maximize one of two criteria. The optimal ternary beamforming sequences minimize detector's error probability, the suboptimal one enhances the signal energy at the output of a matched filter based on the sign of the reflection coefficients of several strongest reflectors. Here we assume the signs of the reflection coefficients are known, as would be natural in a TDD system. It is obvious that the exhaustive computer search is only suitable for relatively short sequences.

We also provide a constructive approach for multilevel beamforming sequence design.

This method is based on the autocorrelation properties of perfect ternary sequence, the idea of pre-RAKE [75] and Lüke's approach to the construction of binary Alexis sequences [76]. In [75], the authors suggested concentrating all the processing required for the RAKE combination at the transmitter and keeping the receiver as simple as a non-combining single path receiver. In [76], Lüke suggested an approach to the construction of binary Alexis sequences [77] of various lengths. The aperiodic ACF of Alexis sequences vanishes in a broad window. We combined these ideas to achieve beamforming gain in a multipath environment by employing a multilevel beamforming sequence and a receiver correlation sequence at the transmitter and receiver sides respectively. Note that, the only knowledge we need to construct the beamforming sequence is the signs of several strongest multipath coefficients.

7.2 Signal Model

The received signal model is:

$$r(t) = \sum_{r} \sum_{l=0}^{L-1} b_r \alpha_l p(t - rT_s - \tau_l) + n(t), \tag{7.1}$$

where

$$p(t) = \sum_{i=0}^{n-1} c_i \psi(t - iT_c). \tag{7.2}$$

b_r are binary antipodal symbols of duration $T_s = nT_c$. n is the length of the ternary spreading sequence $\mathbf{c} = (c_0, c_1, c_2, ...c_{n-1})$ whose elements are in $\{-1, +1, 0\}$, $\psi(t)$ is the signaling pulse assumed here to be equal to zero outside an interval which is equal to or smaller to the chip interval $[0, T_c]$ and assumed known to the receiver. L is the number of multipath components. The delays τ_l are either fixed or chosen randomly. The α_l are real channel coefficients, each with a random sign. In slowly changing environments determining, tracking, and conveying the signs of reflection coefficients to the transmitter would require a non-significant overhead.

It is assumed that the transmitter knows the sign of the first L_c paths (e.g., based on receiver feedback) and can select the spreading sequence which attempts to maximize the energy at the output of the matched filter. The resulting effect is that the first L_c

paths are combined coherently in a beamforming manner. The crosscorrelation between any one of the L_c path components times the channel coefficient and the line of sight path component is positive and, thus, enhances the received signal energy.

7.3 Beamforming Sequence Design

7.3.1 Exhaustive Computer Search for Ternary Beamforming Sequence

We address exhaustive computer search maximization of two criteria only for relatively short sequences.

The energy-based criterion for beamforming sequence selection is:

$$\begin{aligned} &\textstyle\sum_{l=0}^{L_c-1} \operatorname{sign}(\alpha_l) E\{|\alpha_l|\} R(l) - \sum_{l=L_c}^{n-1} E\{|\alpha_l|\}|R(l)| \\ &- \textstyle\sum_{l=0}^{L_c-1} E\{|\alpha_l|\}|R(n-l)| - \sum_{l=L_c}^{n-1} E\{|\alpha_l|\}|R(n-l)| - B, \end{aligned} \tag{7.3}$$

where

$$\begin{aligned} B = &\textstyle\sum_{k=1}^{\lfloor (L-1)/n \rfloor} E\{|\alpha_{kn}|\}|R(0)| \\ &+ \textstyle\sum_{l=n+1, l_{\oplus n} \neq 0}^{L-1} E\{|\alpha_l|\}(|R(l_{\oplus n})| + |R(n - l_{\oplus n})|). \end{aligned} \tag{7.4}$$

$R(\tau)$ is the autocorrelation function of $p(t)$ with T_c being suppressed (assumed to be equal to 1) for notational simplicity, and $\oplus_n$ is modulo-n operation. Here we assume that the mean values of the channel coefficients are known. Simply setting these values to be 1, we obtain another criterion for the case when we only know the signs of first several strongest channel coefficients.

The optimal bit-error rate criterion for sequence selection is based on the probability of detection error.

$$p(\text{error}) = E_{\boldsymbol{\alpha}, \mathbf{b}}\{p(\text{error}|\boldsymbol{\alpha}, \mathbf{b})\}, \tag{7.5}$$

where the expectation is over $L-1$ random channel coefficients $\boldsymbol{\alpha} = \{\alpha_1, \dots, \alpha_{L-1}\}$ (note that the sign of the first L_c coefficients might be known) and $\lfloor L/n \rfloor$ interfering symbols $\mathbf{b} = \{b_1, \dots, b_{\lfloor L/n \rfloor}\}$. For a correlator detector, the conditional probability of error is:

$$p(\text{error}|\boldsymbol{\alpha}, \mathbf{b}) = \begin{cases} \frac{1}{2}\text{erfc}(\sqrt{E_b/N_0}) & A_b \geq 0 \\ 1 - \frac{1}{2}\text{erfc}(\sqrt{E_b/N_0}) & A_b < 0 \end{cases}, \tag{7.6}$$

where erfc(.) is the complementary error function. Here, $E_b = A_b^2$, where

$$\begin{aligned} A_b = \textstyle\sum_{l=0}^{n-1} \alpha_l R(l) + \sum_{l=1}^{n-1} b_1 \alpha_l R(n-l) \\ + \textstyle\sum_{l=n+1, l_{\oplus_n} \neq 0}^{L-1} b_{\lfloor l/n \rfloor} \alpha_l R(l_{\oplus_n}) \\ + b_{1+\lfloor l/n \rfloor} \alpha_l R(n - l_{\oplus_n}) + \textstyle\sum_{k=1}^{\lfloor (L-1)/n \rfloor} b_k \alpha_{kn} R(0). \end{aligned} \tag{7.7}$$

Note that the energy-based criteria, as defined above, is based on the mean values of channel coefficients, whereas, the probability of error criteria requires the knowledge of the coefficient probability distributions. A particularly important component of the sequence design is the PAPR of the signal. For ternary based signaling, arbitrarily padding the transmitted sequence with zeros is helpful to achieving good ACF properties. The tradeoff is an increase in PAPR. A reasonable approach is to find the best sequence under a constraint on the number of zeros (or, more generally, under a PAPR constraint). Even so, as will be demonstrated in the following section, the single pulse (maximum PAPR solution) sequence is not always optimum for signaling in multipath channels when beamforming is aided with limited channel state information at the transmitter.

7.3.2 Constructive Design of Multilevel Beamforming Sequence

The construction of multilevel beamforming sequence is based on the properties of perfect ternary sequences, the idea of pre-RAKE and Lüke's approach to the binary Alexis sequences design. A perfect ternary sequence $(c_0, c_1, ...c_{n-1})$, for which

$$P_{\mathbf{a}}(l) = \begin{cases} (1-\lambda_{\mathbf{a}})n & l = 0 \\ 0 & 1 \leq l \leq n-1 \end{cases} \tag{7.8}$$

is linearly combined with its first $(L_c - 1)$ left cyclic shifts where the combining coefficients are the signs of the first L_c paths. Thus, the multilevel beamforming sequence $(\tilde{c}_0, \tilde{c}_1, ..., \tilde{c}_{n+\tilde{L}-1})$ is obtained by appending $\tilde{L} < L-1$ zero guard chips to

$$\tilde{c_i} = \sum_{l=0}^{L_c-1} \text{sign}(\alpha_l) c_{(i+l)_{\oplus_n}}, i = 0, 1, ...n-1. \tag{7.9}$$

By periodically extending the mother perfect ternary sequence up to length $n + \tilde{L}$, we obtain the receiver correlation sequence. By employing this pair of sequences and a simple correlation receiver, the signal replicas corresponding to the first L_c paths are coherently combined at the output of the correlator.

Example: Let us assume that the multipath number is 11, the signs of the first three paths (including the direct path) are {+, -, +} respectively, and the mother sequence is the Hoholdt's perfect ternary sequence [28]

$$(+ + + + + - +0 + 0 - + + -00 + -0 - -).$$

Then, the transmitter beamforming sequence of length 31 is {1 1 1 -1 3 -2 2 -1 0 2 -1 -1 2 -1 1 -2 2 -2 0 1 -1 0 0 0 0 0 0 0 0 0 0 } and the received correlation sequence is {+ + + + + - + 0 + 0 - + + - 0 0 + - 0 - - + + + + + + - + 0 + 0}.

We note that the transmitter beamforming sequence is a multilevel signal with a number of levels no larger than $2L_c+1$, while the received correlation sequence is still ternary. In this example, the energy from the first, second, and the third path is coherently combined. The multipath interference from other paths will be suppressed due to the perfect ACF properties of the mother perfect ternary sequence. The noise penalty increases since we employ different beamforming and receiver correlation sequence. Thus, this approach trades multipath interference suppression and beamforming gain for SNR loss and an increase in the number of levels of the beamforming sequence. The simulation result in the next section shows that even with no zero guard chips, a multilevel beamforming sequence can still achieve lower BER than a single pulse sequence.

7.4 Numerical Results

Two sets of experiments have been conducted. Bit energy for all signaling schemes is normalized, so that, for example, a ternary sequence of length seven with three zero pulses have pulses with a peak 30% higher and single impulse sequence pulses 160% higher than the corresponding m-sequence pulses. The path power is quantized into 0.4 nanosecond bins corresponding to a chip duration T_c. It is assumed that each bin can contain only one path; the sign of the reflected path coefficient is modelled as a uniformly distributed random variable [35]. Effects of interchip interferences have been assumed negligible. Employed multipath beamforming was based on the sign of first L_c path coefficients. For

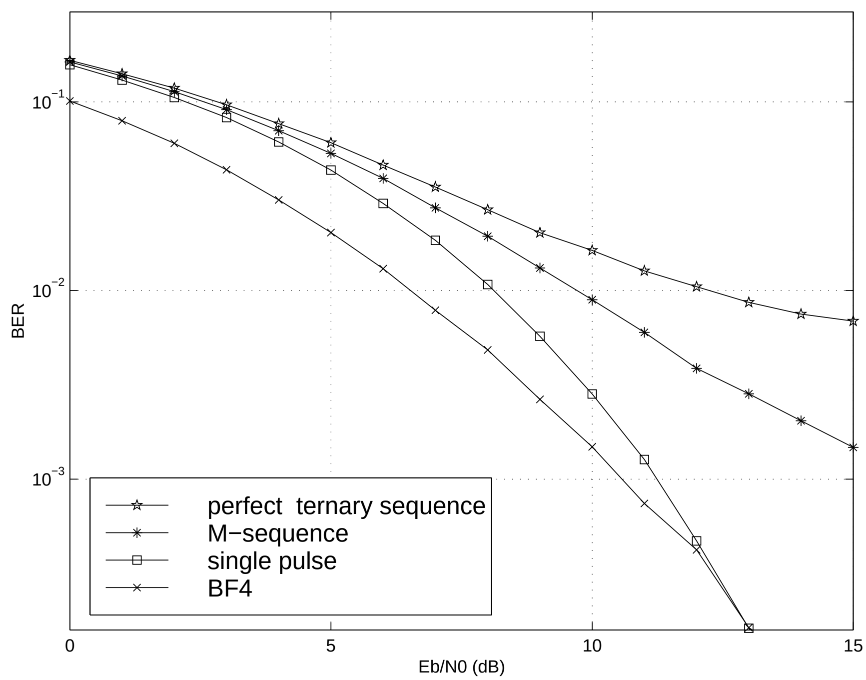

Figure 7.1: Short sequence BER performance for an exponential channel profile and supersymbol path delays.

each simulation block, one of 2^{L_c} optimum ternary sequences was selected based on the sign of the first L_c reflected path coefficients.

In the first set of experiments, we focus on high rate direct sequence ultra-wideband schemes with low spreading gain $N = 7$, which, for a chip time of $T_c < 1$ nanoseconds would correspond to data rates of over 100Mbits/Second. Note that such high rate systems employed in channels with significant multipath can experience significant inter-symbol interference which can dominate the performance degradation, in particular, at high SNRs. We have employed two sets of channel coefficients. One is an exponentially decaying profile, the other is a deterministic set of coefficients based on the indoor line of sight (LOS) measurements (performed in 23 homes) [34]. In the latter case, channel coefficients are chosen to be equal to average values given in [34] where it is observed that the line of sight component and the first 10 paths account for 33% and 75% of the total power, respectively. In both cases, the distribution of the arrival times of individual multipath components follows a modified Poisson model (namely, the $\Delta - K$ model) [78]. Here, we have used k=0.5 (see [1]). Two scenarios have been considered. In the first, the delay

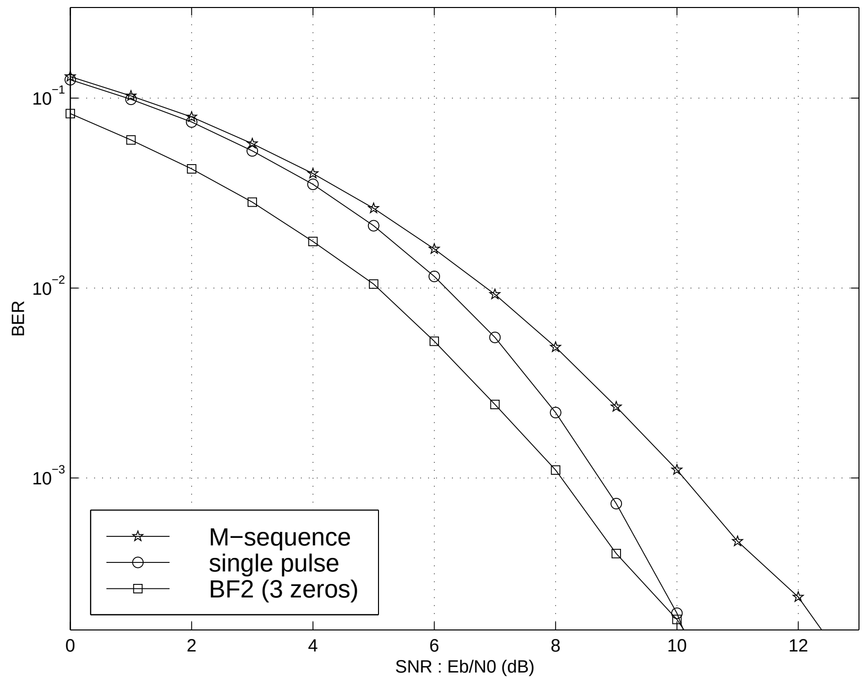

Figure 7.2: Short sequences BER performance for an exponential channel profile and subsymbol path delays: PAPR optimized sequences.

spread was restricted to be less than 8 bins, and in the second to be less than 11 bins We study correlator receiver performance for beamforming, m-, single impulse and ternary sequences.

Figures 7.1 and 7.2 depict the results based on the exponential multipath profile decreasing 10 dB (in power) over a 4 nanosecond excess delay and the decreasing 10 dB over a 2.8 nanosecond excess delay, respectively. Results in Figure 7.3 is based on the profile in [34] truncated to 2.4 nanoseconds. Figure 7.1 demonstrates the gain of the multipath beamforming approach which requires knowledge of the first four indirect path coefficient, means and signs over the non-adaptive single impulse, m-, and perfect ternary sequence. For a range of SNRs both single pulse and the beamforming approach can overcome the inter-symbol interference induced error floor observed when either perfect ternary or m-sequences are used. Figure 7.2 demonstrates that beamforming based on two path coefficients which constrains the number of zeros to be less or equal to three can still improve over the non-adaptive single pulse sequence. Figure 7.3 demonstrates that correlator receiver with multipath beamforming and ternary sequences can capture

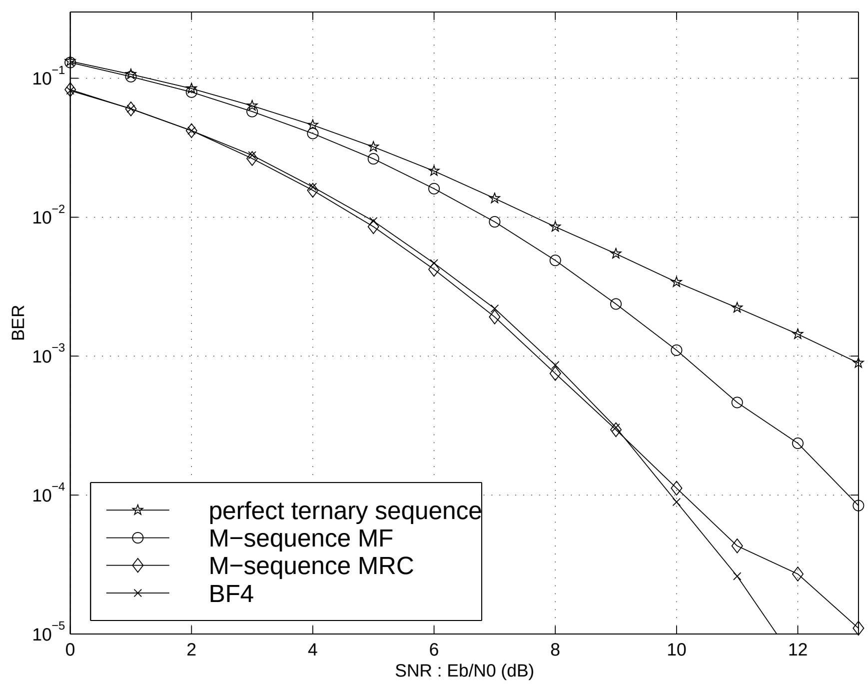

Figure 7.3: Short sequences BER performance.

as much energy as the maximum ratio combining (MRC) receiver when m-sequences are used.

The second set of experiments focuses on the long beamforming sequences and assumes that each bin contains exactly one multipath component (emulating a dense multipath environment) and an exponential profile decreasing 10 dB in power over a 4.4 nanosecond excess delay. Employed pair of transmitter multilevel beamforming and receiver correlation sequences is based on a Hoholdt's perfect ternary sequence and has been introduced in Section 7.3.2. Figure 7.4 depicts that a 2 dB beamforming gain can be achieved by employing beamforming sequences over m- and single pulse sequences. Figure 7.5 demonstrates that the beamforming approach benefits from the multipath interference suppression and beamforming gain exceed the noise penalty due to difference in transmitting and receiving sequences when the excess delay is larger than 1.6ns. In Figure 7.6, we shorten the transmitter multilevel beamforming sequence by reducing the number of zero guard chips. At the receiver, the correlation sequence will accordingly shrink. The more zeros we reduce, the higher the multipath and inter-symbol interference. Even in the case of zero guard chips, the SNR required for achieving the BER of 10^{-3} is smaller for the beamforming

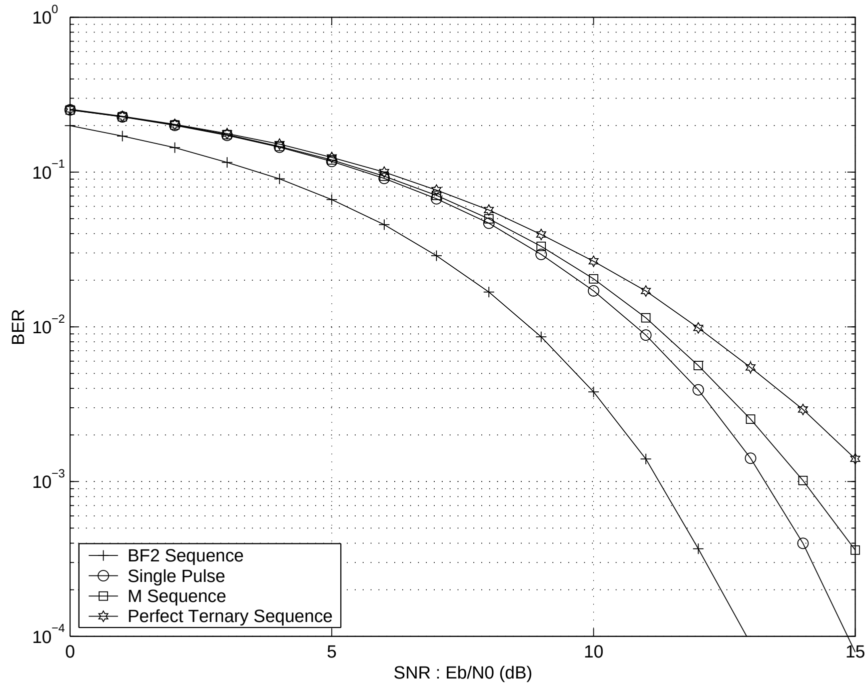

Figure 7.4: Long sequences BER performance for an exponential channel profile.

approach than that for the single pulse sequence.

7.5 Conclusions

We study an adaptive transmission approach that exploits the increased phase coherence of UWB channels with a “discrete” number of reflectors relative to narrow-band wireless channels. In a LOS environment, based on the signs (and/or mean) of a few strongest multipaths’ coefficients, the transmitter adaptively selects the beamforming sequence so that the signal energy of these paths coherently combines at the output of a receiver correlator. By employing both the ternary and multilevel beamforming sequences, better performance can be achieved than by using a single pulse sequence while still keeping a low PAR.

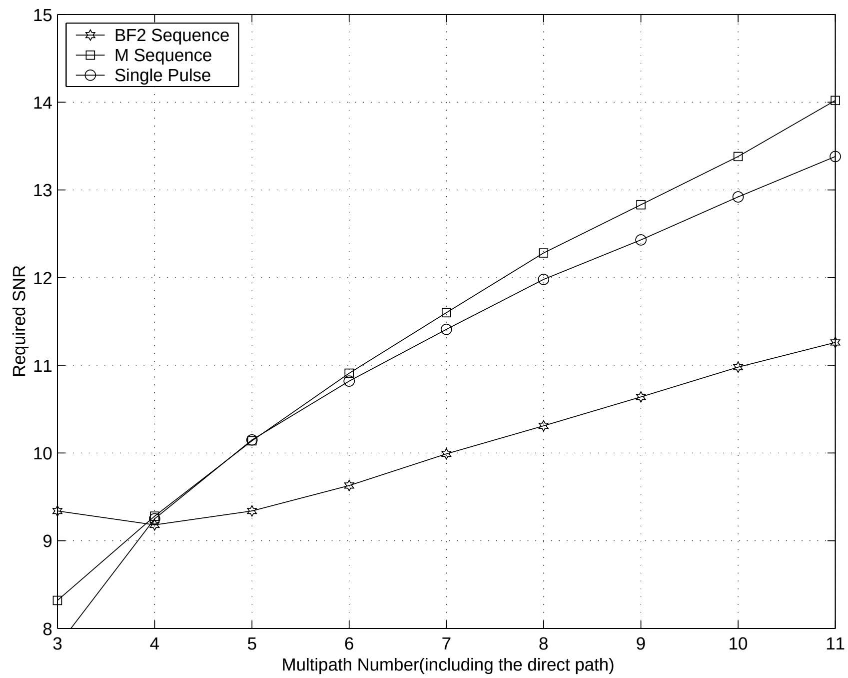

Figure 7.5: Required SNR vs number of multipath components for BER=10^{-3}

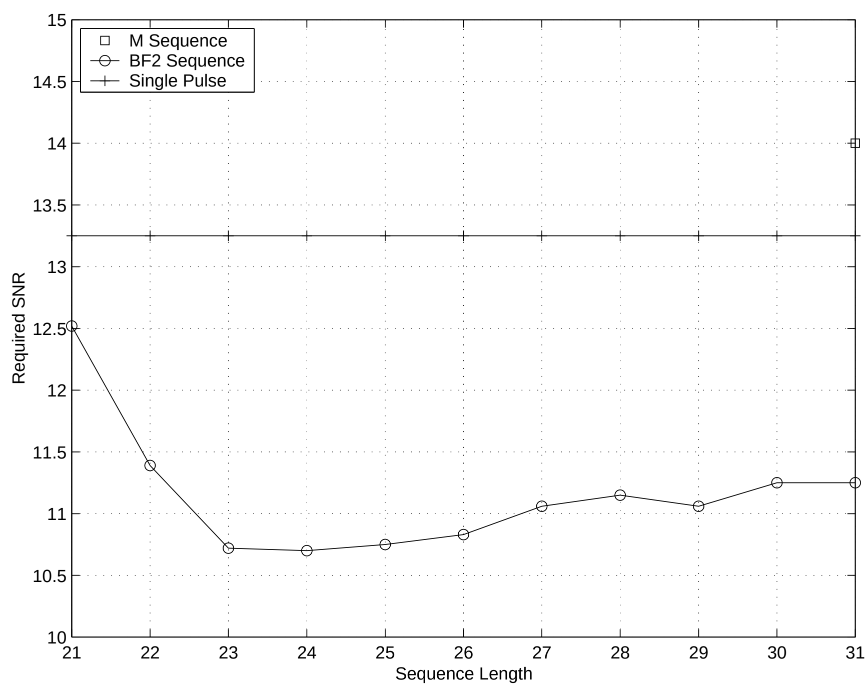

Figure 7.6: Required SNR vs sequence length for BER=10^{-3} (Increasing ISI experienced with a decrease in zero padding for beamforming sequences.)

Chapter 8

Conclusion and Future Work

8.1 Conclusion

We studied TS-UWB signaling which has epochs of zero signal amplitude including binary antipodal signaling as a special case. Allowing some of the chips to be zero enables significant improvement in the correlation properties of the employed signaling and provides new construction methods for spreading sequence sets of diverse sequence lengths and family sizes.

To achieve an ideal correlation property, i.e., all cross-correlations and all sidelobes of the ACFs are zeros, we proposed a technique employing complementary sets as spreading sequences for an orthogonal-pulse based multichannel UWB system. As a result, both multiple access and multipath interference can be significantly suppressed. Similar to conventional multicarrier systems, one of the major impediments to deploying such systems is high PAPR. We have stressed that correlation properties of column sequences in complementary set matrices play an important role in the reduction of PAPR. Hence, we generalized our construction procedure to design the complex-valued MO complementary set whose column sequences satisfy a correlation constraint.

We constructed OVSF codes with both a periodic and aperiodic ZCZ. A higher data rate access can be achieved by assigning a lower spreading factor code to the user. The proposed OVSF codes can be employed in a quasi-synchronous code-division multiple access (QS-CDMA) UWB system, for which the relative time delay Δ between the signals of different users is being upper bounded as $|\Delta| \leq \tau$. Based on the knowledge of τ, we demonstrated how to select the OVSF code tree with a different ZCZ to increase the aggregate throughput of the QS-TS-UWB system.

We also studied an adaptive transmission approach that exploits the increased phase coherence of UWB channels with a "discrete" number of reflectors relative to narrow-band wireless channels. The proposed multipath beamforming technique suggests selecting a ternary sequence for a DS-UWB signal based on the signs of the reflection coefficients corresponding to a few strongest paths so that the signal energy at the output of a non-adaptive correlator receiver is enhanced.

8.2 Future Work

In this thesis, we have left the general problem of finding MO complementary set matrices with a minimum column correlation constraint as an open question. An important step towards solving the general problem is to find new construction approaches for MO complementary set matrices. A design algorithm based on N-shift cross-orthogonal sequences can be found in [30]. However, their column correlation properties are intractable. The optimality of the constructed binary/ternary complementary sets have been verified by exhaustive computer search to a limited sequence length. We hope our research can stimulate the research in this thread and, thus, our results can be further evaluated.

We use simulated annealing search algorithm for finding companion pairs of relatively long length sequences. The simulated annealing is a statistical computational technique for obtaining optimal or near-optimal solutions for combinational optimization problem [79]. The search process may stop at the local minimum if the starting temperature, the decrement of temperature, the determination of equilibrium condition and the stopping criterion have not been well selected. Hence, the results in Table 4.4 and Table 4.5 may be further improved.

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