

**MODELING FINANCIAL MARKETS WITH
HETEROGENEOUS INTERACTING AGENTS**

by

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ABSTRACT OF THE THESIS

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Financial market has been extensively recognized as a complex system, where large number of heterogeneous agents contribute to price formation of asset. Interactions and adaptations of these agents form the core foundation of market operations and its resultant characteristic properties. These market agents are highly diverse in their perception of the world around them and in the way they respond to it. Various studies of statistical properties of financial markets and price fluctuations have revealed a rich set of typical characteristics known as stylized facts. Agent-based models that can reproduce these stylized facts and explain the roots of complex dynamics of financial market have been subject of intense research in recent time. The Minority Game Model proposed by Challet and Zhang is one such model that presents a simplified paradigm of financial market. Another model proposed by Lux and Marchesi offers a different perspective to agent-based modeling, where parallels are drawn between the physical system with a large number of interacting units and financial markets. The Minority Game model succeeds to a certain extent in reproducing stylized facts and explaining behavioral foundation of it. However, in attempt to present a simplified picture of market scenario

both these models make certain assumptions that dilute the heterogeneity aspect of the real market. In real world markets, agents are truly diverse in their thinking, strategy, action and analyzing ability. Due to these unrealistic assumptions, the model can be validated only with a very limited spectrum of parameters. Also, it's difficult to point out precisely which aspects of the game contribute to some of the stylized facts producible with the model. To improve on these issues, we have developed a model and a simulator based on modified minority game, which we are referring to as "adapted minority game". The main focus of our research is on improving the heterogeneity aspect of agents, their interactions, and bringing fundamental value of asset into the Minority Game model. Our model introduces fundamentalist agents into the minority game model and also allows agents to have different historical memory and time horizons. Furthermore, agents are free to switch from one trading strategy group to another to improve their chances of performing better. Reproducing the stylized facts still remains the benchmark for validating our model. Our adapted minority game succeeds to an extent in expanding the spectrum of parameters that can be used for modeling the market. Agents' interactions and adaptations have been tracked down to the basis of stylized facts. An interesting property of periodic volatility is successfully demonstrated with our model.

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Chapter 1: Introduction

1.1 Background

Financial market has been extensively recognized as a complex system with large number of agents involved in the price formation. Heterogeneous interacting agents are considered to be the foundation of any financial market. These agents are highly diverse in their perception of the world around them and in the way they respond to them. The study of statistical properties of financial market and price fluctuations divulges a rich set of properties. Such characteristic properties in market behavior that can be generalized over different markets are known as *stylized facts*. Examples of stylized facts include distribution of price changes, autocorrelation of returns, volatility clustering etc. Agent-based models that capture these stylized facts and complex dynamics of financial market have generated considerable interest across many disciplines. Studies have revealed that the traditional approach of statistical analysis of financial market and price series are inadequate to explain the origin of stylized facts in market behavior. Furthermore, the advances made in field of mathematical modeling, computational power and simulation technologies over the past decade have propelled the development of such market models, which can be used as analytical tools for facilitating the understanding of market operations.

An important aspect of financial markets is the interplay between the agents and information. Agents in the market make their trading decision based on the piece of information they receive. Agent-based financial market models have been subject of intense research in recent time [8, 10, 11, 15, 20]. The Minority Game Model proposed

by Challet and Zhang is one such model that succeeds to a large extent in reproducing the stylized facts with highly simplified paradigm of financial market [5]. Due to its richness and simplicity MG has attracted a lot of further studies [1, 8, 10]. Minority Game Model is basically a mathematical formulation of El Farol Bar problem that was originally proposed by Brian Arthur in 1994 [2]. El Farol Bar problem is the study of how many individuals may reach a collective solution to a problem under adaptation of one's expectation about the future. MG is an extended model of El Farol Bar problem for collective behavior of agents in an idealized situation where they have to compete through adaptation for finite resources. It is a dynamical system of many interacting degrees of freedom. The MG simply involves an odd number of agents opting repeatedly between the options of buying (1) or selling (0) a quantity of asset. The resource level of asset is finite, which gives it the minority nature. The agents use inductive reasoning with strategies that map the series of recent price fluctuations into their action for next time step.

Stochastic multi-agent market model proposed by Lux and Marchesi offers a different perspective to the agent-based modeling [15]. Their work shows the resemblance between the physical system in which large number of units interact and the financial market with interacting agents. The interactions of large number of market participants is believed to be the core reason of scaling property observed in financial price series. However, it is in direct contradiction the prevalent 'Efficient Market Hypothesis'. The efficient market hypothesis states that the current price already contains all information about the market and past price can not help in predicting future prices.

Therefore the market is efficient in aggregating available information. On the other hand, the ‘Interacting Agents Hypothesis’ says that the price changes arise endogenously from the trading process and mutual interactions of market participants. The model manages to replicate some of the stylized facts at the same time showing conflict between efficient market hypotheses and interacting agents hypothesis.

1.2 Motivation

As pointed out in the previous section, both MG model and Lux model thrive to certain extent in reproducing the stylized facts and explaining the behavioral foundation of it. However, in attempt to present a simplified picture of market scenario both these models make certain assumptions. These assumptions though seemingly reasonable dilute the heterogeneity aspect of the real world market. In real world market, agents are truly heterogeneous in their thinking, strategy, action and analyzing ability. Because of these assumptions the model can be validated only with very limited spectrum of parameters.

For instance, in MG all the agents are assumed to have same historical memory. That means all agents have same amount of access to the historical information and they all make their decision based upon the same length of recent outcomes. This is definitely not the case in real world market where agents display high degree of heterogeneity. Furthermore, in MG it is assumed that all agents evaluate their strategies in the same time horizon. This is again not true in actual market. Thus agents’ diversity in the model is very limited. One more key aspect that seems to be absent in the MG model is the fundamental value of asset. The MG model doesn’t take into consideration the impact of

fundamental value on the evolution of actual market price. That means there is no fundamentalist in the market. **Fundamentalists** are the agents who rely heavily on the fundamental value of asset to determine their trading action at any given time. Fundamental value of asset emerges from fundamental sources of information such as intrinsic value of a company, its business, dividends, interest rates etc. Due to these certain unrealistic postulations, the set of parameters that can imitate the real world market has been tapered to a large extent.

To improvise on these issues, we have developed a model and simulator based on modified minority game. The model consists of three groups of agents. One group is of fundamentalists, who follow the ‘Efficient Market Hypothesis’ and form their decisions based on fundamental value of asset. Agents in other two groups play the minority game but with two groups having different historical memory and using different time horizons to evaluate their own performance. This allows us to study the impact of different memory and different time horizon on agent’s success rate, price volatility and evolution of market price. Furthermore, the agents are allowed to switch the group with a certain endogenous and time-varying probability based on the difference between the momentary profits earned by individuals in each group. Reproducing the stylized facts still remains the benchmark for validating this model. The effort is made to expand the spectrum of parameters validated by original MG model. Thus the central objective of our work is to present a more realistic market model with subtle changes and bringing in a few improvisations to original MG model.

1.3 Outline

In this chapter we have provided the overview of popular financial market models, their approach and how we expect to improve on the minority game model. The rest of the thesis is organized in 4 chapters. The second chapter provides more detailed description of financial market operations and stylized facts. It also discusses the essentials of El Farol Bar Problem and evolution of minority game from it. We briefly touch upon the limitations of original minority game as a market model. The chapter concludes with overview of Lux and Marchesi model. The third chapter focuses on analytical approach for our adaptive minority game model and precise details of this model. The fourth chapter presents our module design and flowchart for the simulator based on adaptive minority game. In fifth chapter the implementation detail and results of adaptive minority game models are presented. The comparisons are made between results achievable with original MG and our model. The final chapter presents the conclusion and suggested directions for future work in this field.

Chapter 2: Market Models and Stylized Facts

2.1 Financial Time Series & Stylized Facts

Present day financial markets generate a great amount of data and hold plenty of vital information throughout the day that is recorded on different time scales. Price changes in financial time series can be articulated in several ways. The change in asset's price over a period of time is known as *return*. The obvious way to represent return is simple price difference for specific time step.

$$R(t) = P(t + \Delta t) - P(t) \quad (2.1)$$

The net return can be defined as

$$R(t) = [P(t + \Delta t) - P(t)] / P(t) \quad (2.2)$$

The most useful form of return is logarithmic return (normalized return), which is defined as

$$R(t) = \ln P(t + \Delta t) - \ln P(t) \quad (2.3)$$

The advantage of using logarithmic return instead of absolute return or net return is the scale invariance of log changes with respect to the price scales. It facilitates more meaningful comparison of price changes.

During recent time, research in field of financial market has shifted to study of high frequency data, which reveals remarkably stable non-trivial empirical laws [18]. Such properties, common across a wide variety of assets, markets and time periods are called **stylized facts**. Ability to reproduce these properties is considered a prerequisite for any good market model. It is important here to note that the stylized facts are not laws but

they are common denominators among the properties widely observed in studies of real world scenarios. They are qualitative representation of typical characteristics of empirical data. Stylized facts have emerged from various independent studies in last 20 years [6, 14, 18, 24, 27].

Financial markets have been found to exhibit various properties such as fat tail distribution, absence of autocorrelation in return, aggregational gaussianity, Gain/loss asymmetry, intermittency, conditional heavy tails, leverage effect, Asymmetry in time scales, long term correlation in volatility and volatility clustering [6]. Out of these, we will concentrate mainly on three stylized facts - fat tail distribution, volatility clustering and absence of autocorrelation in return - as they are widely accepted as the standard gauge for market models. We briefly discuss these important properties in following section.

2.1.1 Fat Tail distribution of return

The statistical analysis of probability distributions of price changes reveals very high probability of large changes. Several studies have confirmed that distribution of returns is strongly non-Gaussian. For small time scales (daily or higher frequency) it tends to display a power-law or Pareto-like tail. For very large time scales (a few months) it exhibits Quasi-Gaussian distribution. Figure 2.1 shows the comparison of Gaussian distribution with other symmetric Levy distributions [3]. The PDF for price changes of financial assets have sharper peak around zero change when compared to the Gaussian distribution. Also, the curve remains well above the horizontal axis for large changes

whereas Gaussian distribution has almost attained zero [12]. This is widely known as **fat tailed distribution**. The fat tail distribution can be characterized by a power law of exponent $1 + \alpha$ [15]. This is in contrast to the normal distribution, which decay very quickly after first two standard deviations.

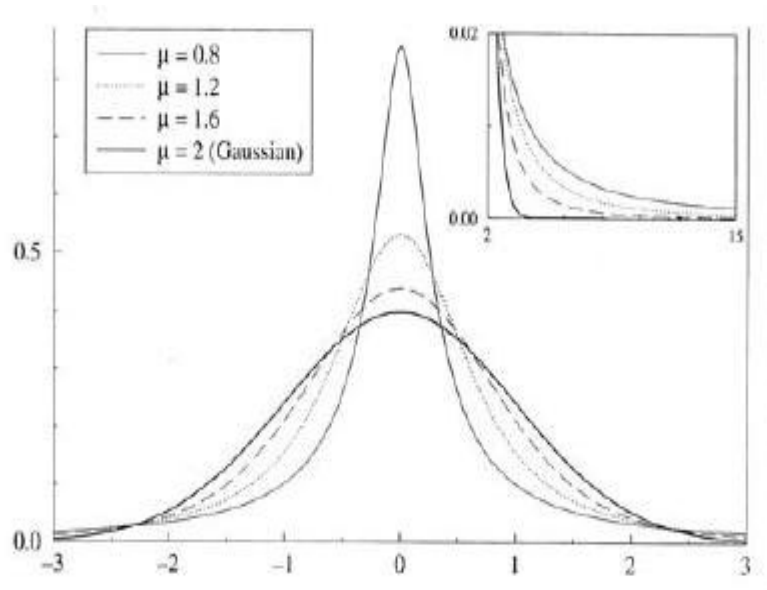


Figure 2.1 Comparison of Gaussian distribution (μ mean = 2) with other symmetric Levy probability distribution functions

2.1.2 Absence of auto-correlation in return

It has been observed in wide variety of financial markets that price changes do not exhibit any significant autocorrelation. Returns usually display very weak autocorrelation for initial few lags and then drops down to zero for subsequent lags. This indicates that returns have very 'short memory'. Absence of long-time autocorrelation in return is in good agreement to the 'Efficient Market Hypothesis'. Efficient market hypothesis states

that it is not possible to consistently outperform the market by using any information that the market already knows, except through luck. It assumes that the movements of financial prices are an immediate and unbiased reflection of incoming news about future earning prospects [15]. Thus if returns exhibit considerable correlation, it can be used to form a trading strategy to exploit the information and make significant profit. This will effectively tend to bring down the correlation in longer run. The autocorrelation function for return can be defined as:

$$C(\tau) = E[(R_t - \mu)(R_{t+\tau} - \mu)] / \sigma^2 \quad (2.4)$$

Here, τ = lag

R_t = Return at time t

μ = Mean of return

σ^2 = Variance of return

2.1.3 Volatility Clustering

Standard deviation of price changes over a period of time is known as volatility. In other words, volatility represents swings in supply and demand of asset, which according to efficient market hypothesis is unbiased reflection of incoming news about future earning prospects. Since volatility is a direct measure of amount of information coming in the market, it is a good indicator of amount of risk involved with any particular trading strategy. Time series of financial asset frequently shows property of volatility clustering. That means large changes are followed by large changes, of either sign, and small changes are followed by small changes [17]. Thus changes of similar nature tend to cluster together, resulting in persistence of the amplitude of price changes. The market

switches between periods of high and low activity, with long duration of periods. The main cause of volatility clustering is the interaction between various heterogeneous agents in the market and their transition from one pool to another as it forces the switching between high and low activity regimes. This concept is further explained in the next chapter.

Volatility is calculated as:

$$\text{Volatility (t)} = \sigma_{p'} * \sigma_p, \quad (2.5)$$

Where, $p' = p(t) - p(t-1)$

$$\sigma_{p'} = \sqrt{\frac{1}{t} \sum_{i=1}^t (p' - \overline{p'})^2}$$

t = time window of volatility

Figure 2.2 – 2.5 show historical data recorded on daily basis and stylized facts observed in Dow Jones Industrial Average from 1928 to 2007. The graphs have been generated by us using data available from Yahoo finance [28]. Figure 2.2 and 2.3 shows price trajectories for DJIA. The price series tend to exhibit different patterns across different markets and different stocks but eventually the properties extracted from these price series demonstrate striking resemblance. Thus price series itself is not one of a stylized property to model on, but is an important aspect that contributes to other characteristics. Figure 2.4 displays that in return price series, large variations are followed by large variations and small variations are followed by small variations. We can also see some big spikes and herding of higher returns. This feature substantiates clustering of volatility that we discussed earlier. In figure 2.5 we have plotted probability distribution function of normalized return (equation (2.3)), which demonstrates sharper

peak and heavier tail compared to normal distribution. Figure 2.6 confirms that returns are weekly correlated over time and results in mere noise after first few lags. Very little correlation that is observed in initial lags is due to the amount of time the market takes to absorb and react to the newly arrived information. Thus DJIA time series exhibits properties that are in good agreement to the stylized facts that we discussed.

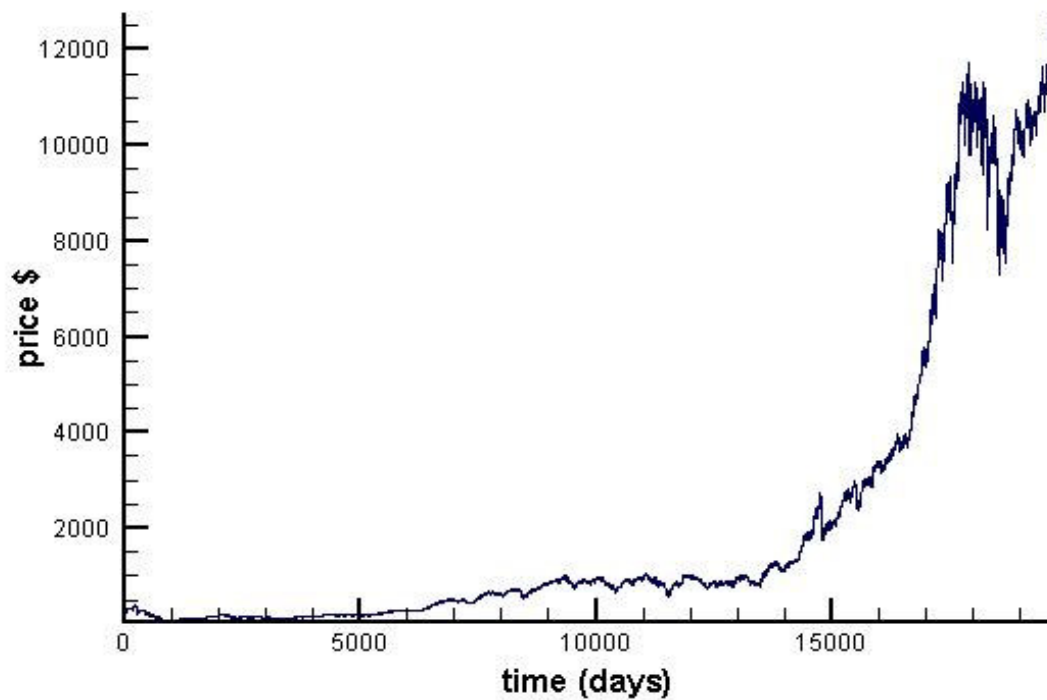


Figure 2.2 DJIA Price Series (1928 – 2007)

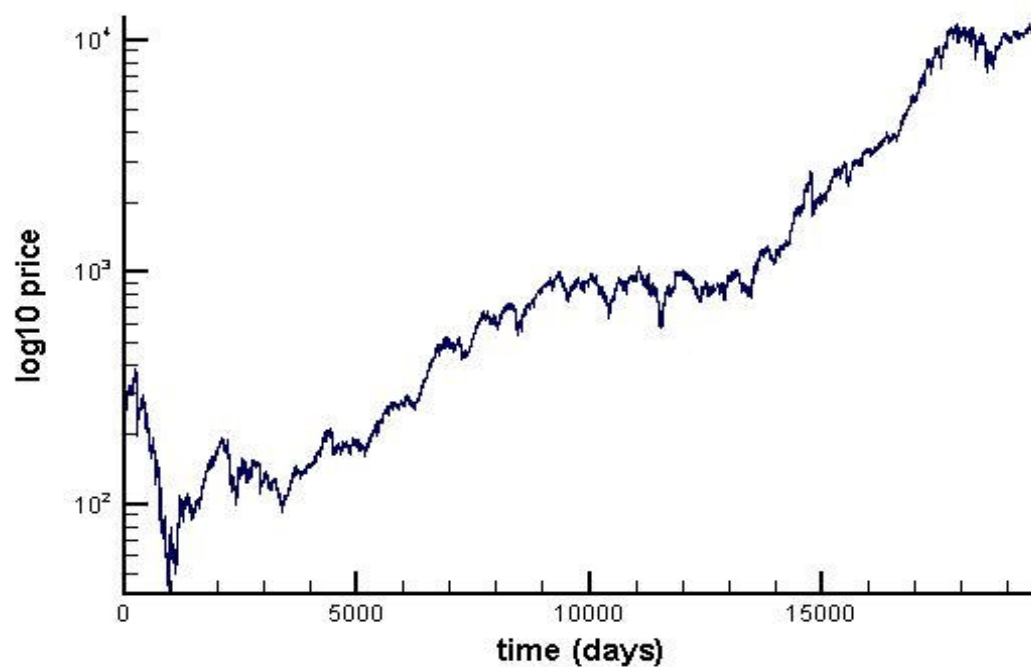


Figure 2.3 DJIA Logarithmic Price Series (1928 – 2007)

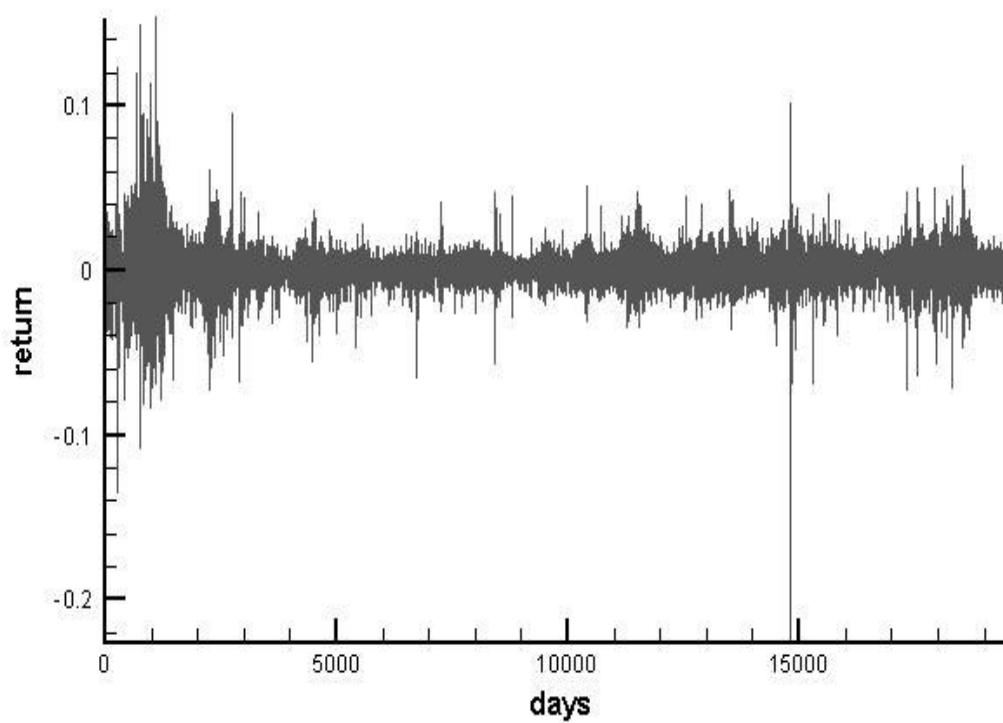


Figure 2.4 DJIA Return (1928 – 2007)

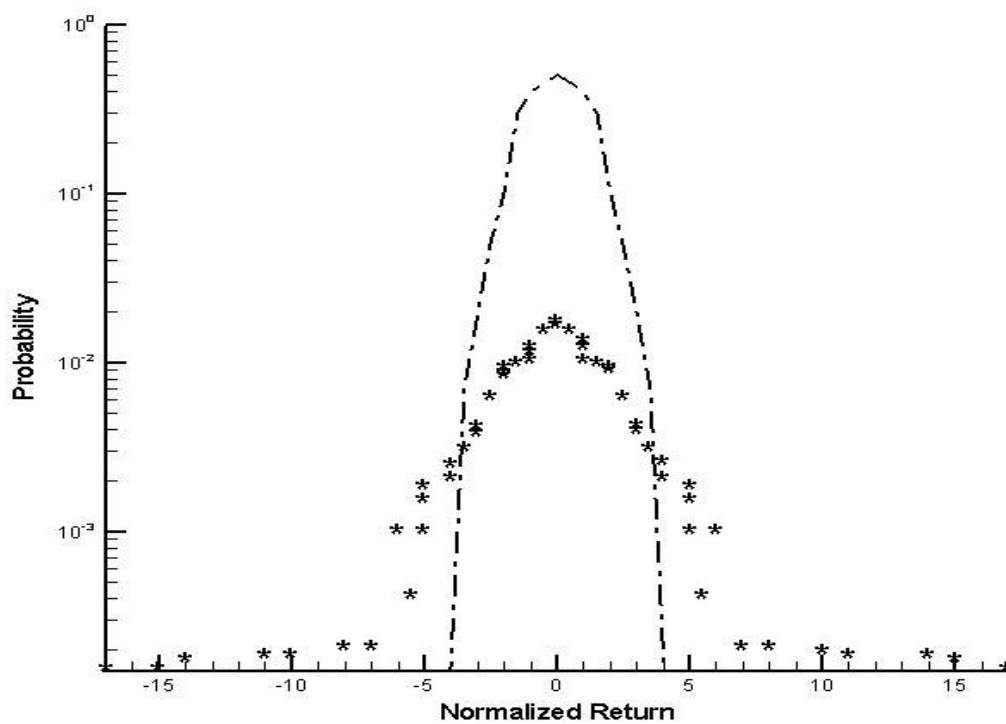


Figure 2.5 DJIA Probability Distribution Function Of Normalized Return

(dashed curve: normal distribution, *: DJIA return)

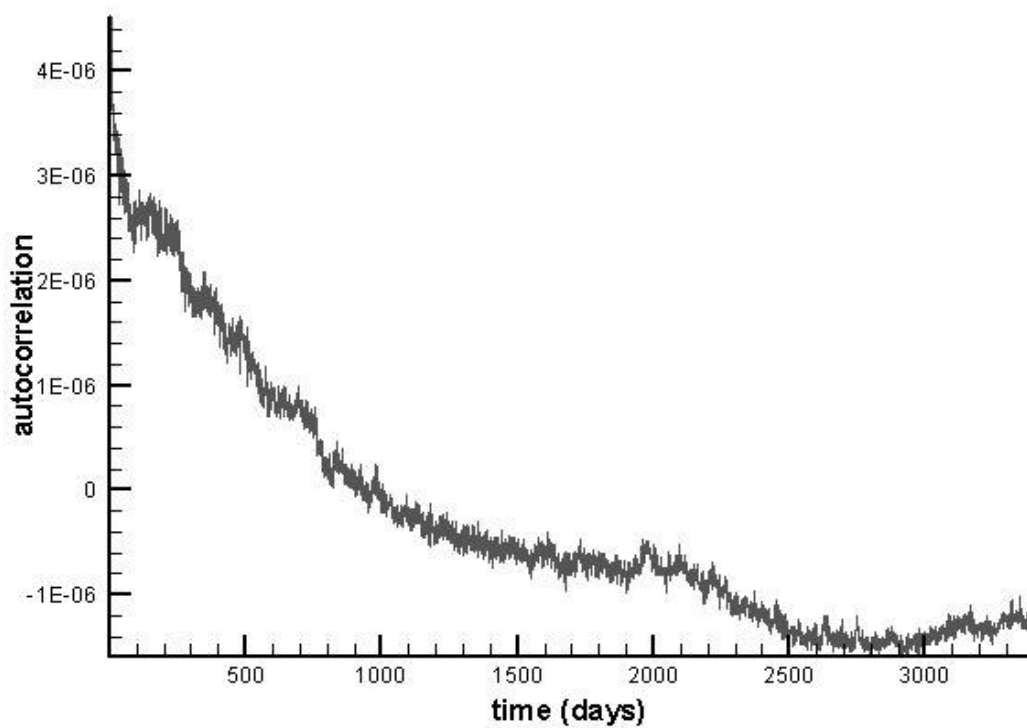


Figure 2.6 DJIA Autocorrelation in Absolute Return

2.2 El Farol Bar Problem

El Farol Bar problem was first proposed by Brian Arthur in 1994 [2]. It is an example of inductive reasoning in scenario of bounded rationality. Due to limited knowledge and analyzing capability of agents, inductive reasoning generates a feedback loop in where the agent commits an action based on his expectations of other agents' actions. These expectations are built based on what other agents have done in the past. Inductive reasoning assumes that with the help of feedback, agents could ultimately reach perfect knowledge about the game and arrive on steady state [21].

The problem is posed in the following way: N people have to decide independently each week whether to go to a bar that offers entertainment on a certain night. Space in bar is limited and the evening is enjoyable if it's not too crowded – specifically, if fewer than 60% of the possible 100 are present. There is no prior communication between the agents and the only information available is the number of people who came in past weeks. Thus there is no deductively rational solution to this problem, since given only the number attending in the recent past; a large number of expectation models might be reasonable. So, without the knowledge of which model other agents might choose, a reference agent can not choose his in a well defined way. If all believe most will go, nobody will go, invalidating that belief. Similarly, if all believe very few people will attend; all will end up in the bar. In order to advance the attendance next week each agent is given a fixed number of predictors which map the past week attendance figure into next week. Also, agents need not necessarily know how many total agents are participating in the game, but they do know how many agents attended

the bar in past weeks. For example, the total number of agents in system is 100 and attendances in recent weeks are (right most is the most recent):

63 42 72 53 49 36 70 39 51 40 44 84 35 19 47 54 41

Following are some of the possible predictors:

- same as 3 weeks ago: 47
- mirror image around 50 of last week's attendance: 59
- minimum of last 5 weeks: 19
- rounded average of last 3 weeks: 48

Each agent monitors his predictors by keeping an internal score of them which is updated every week by giving points or not to all of them depending on whether they correctly predicted the outcome or not. At each week agent chooses his predictor with the highest score to decide his action. Computer simulation demonstrated that attendance fluctuated around 60%. Figure 2.7 shows the bar attendance for first 100 weeks [2]. The reason for this somewhat surprising feature is that agents adapt to the hypothesis and belief models in the aggregate environment that they jointly create. Even though this problem deals with non-market context it offers a very good framework to build a simple market model.

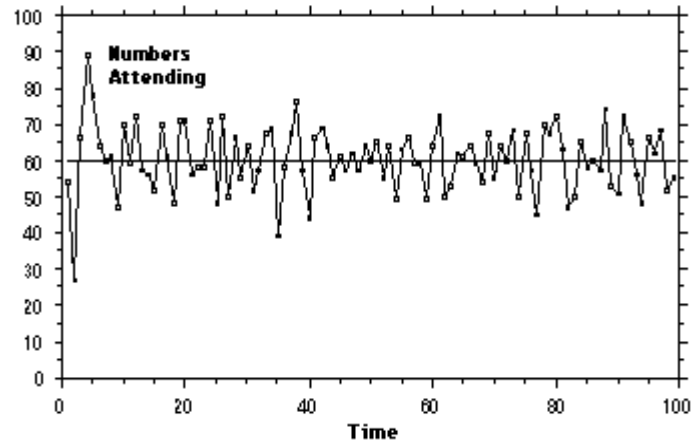


Figure 2.7 Bar Attendance

2.3 Minority Game As A Market Model

El Farol Bar problem can simply be extended to market scenario. At each time step agent can buy or sell an asset. After each time step, price of the asset is determined by a simple supply-demand rule. If there are more buyers than sellers, the market price is high and if there are more sellers than buyers, the market price is low. If the price is high, sellers do well, while if the price is low, buyers win the round. Thus minority group always wins.

Challet and Zhang gave a precise mathematical definition for the El Farol bar problem, which is known as Minority Game (MG) [5]. The underlying principle of MG is again inductive thinking of agents. That means agents rely on trial and error inductive approach rather than trying to find deductively rational solution. In its most basic form MG is a simple evolutionary game that has a population of N (odd) agents. At each time step of the game (trading round), each of the N agents take an action deciding either to

buy ($a_i(t)=1$) or to sell ($a_i(t)=-1$) one unit of stock. For simplicity purpose, only *one type of stock or asset* is taken into consideration here. The resource level is kept finite. The payoff of the game is to declare that the agents who take minority action win, whereas majority losses. Thus payoff function of agent i is given by:

$$g_i(t) = -a_i(t).A(t) \quad (2.6)$$

$$\text{where, } A(t) = \sum_N a_i(t)$$

The function $g_i(t)$ represents outcome of the current round of the game for agent i and ensures that agents with minority action are rewarded. That means if $g_i(t) > 0$, agent i won the round and if $g_i(t) < 0$, agent i lost the round. The absolute value of g_i represents the margin by which agent won or lost the round. Furthermore, it is assumed that agents are quite limited in their analyzing power and they can only retain last m bits of the system's signal (market outcome) and make their next decision based only on these m bits. Here, m is called **historical memory length** of the agent and is assigned at the start of the game.

signal	prediction
000	1
001	0
010	0
011	1
100	1
101	0
110	1
111	0

Table 2.1 A possible strategy for some agent with $m=3$

Each agent has some finite number of strategies S . A strategy is defined to be the next action (whether to buy or to sell) given a specific sequence of last m outcomes. Table 2.1 shows the example of one such strategy [5]. Since there are 2^m possible inputs for each strategy, the total number of possible strategies for a given m is 2^{2^m} . At the beginning of the game each agent is assigned randomly drawn S strategies from the pool of 2^{2^m} strategies. The assignment is different for each agent and thus, agents may or may not share the same set of strategies. From the simulation tests performed by Challet and Zhang, it has been observed that agents tend to perform poorly if the number of assigned strategies S is too big. It has been observed from their results that average performance of agents tend to degrade significantly if number of assigned strategies is more than 8. However, the overall operation of the market model is not greatly affected by the choice of S . The reason for this behavior is that agents are more likely to get confused if they are provided with bigger strategy bag since they would switch the strategy immediately if another strategy has one virtual point more than the one currently in use. Setting a higher threshold for switch could improve this result.

Initially at the start of the game, each agent draws randomly one out of his S strategies and uses it to predict next step. In an attempt to learn from the past mistakes, after each time step, each agent assigns one virtual point to all his strategies that might have correctly predicted the actual outcome, i.e., strategies that would have placed the agent into the minority group. Thus agent reviews not only the strategy he has just used but all the strategies in his bag that could have actually come up with the right prediction.

For example, **virtual points** of agent i 's strategy j is ζ_{ij} and assuming strategy j was used for the current round then,

$$\begin{aligned}\zeta_{ij}(t) &= \zeta_{ij}(t-1) && \text{if } g_i(t) < 0 \\ &= \zeta_{ij}(t-1) + 1 && \text{if } g_i(t) > 0\end{aligned}\tag{2.7}$$

The points are collected over a specific interval of time for each agent. The interval of time over which agents accumulate virtual points of their strategies and evaluate their own performance in the game is known as **time horizon** T . For next time step agent picks the strategy with the highest virtual points and makes his decision based on it. Since agent keeps track of how his strategies are performing, updates their points, and picks the strategy that is performing best, he is constantly *adapting*. This original MG model functions as infinite time horizon market where agents keep collecting the points through out the length of the game. However, various studies of financial markets and economy has pointed out that most market agents operate and evaluate their performance in limited time span. Subsequent work by Hart, Jefferies and Johnson in [9] have presented time horizon version of minority game. In MG, the memory of the agents is very essential as it is related to agents' ability to identify patterns in the available information and use it to their advantage. This is because of the fact that agents' strategies are mapping of recent past outcome patterns to the current time step prediction. That means agents with longer historical memory can recognize the recent trend more efficiently. However, the question that how bigger memory is advantageous for agent demands further research. We will address this issue later in chapter 5 of this thesis. Furthermore, the memory determines the *dimension of strategy space*. The minority

nature of the game makes it impossible to achieve a complete steady state in the community. This is a basic form of minority game as a market model [21]. With this simple artificial market scenario the resultant dynamics shows great richness.

2.4 Lux-Marchesi Model

The Lux-Marchesi model [15] draws attention to the scaling property observed in financial price series. Even though the scaling property is not considered as a stylized fact, it is an interesting feature of price series as it demonstrates resemblance between financial market and physical systems which consist of large number of interacting particles obeying universal scaling laws. Lux and Marchesi came up with a multi agent model of financial market, which supports the idea that scaling in financial prices arise from mutual interactions of market agents. The model consists of two types of agents, *fundamentalists* and *noise traders*. Noise traders are further classified as *optimistic* or *pessimistic* depending on the amount of risk an individual is willing to take in pursuit to succeed. **Optimists** buy additional units of asset expecting price to go further high in future, whereas **pessimists** sell part of their actual holdings of asset in order to avoid loss. Fundamental value of the asset dominates the trading strategy of **fundamentalists**, whereas **noise traders** look at price trends, patterns and consider behavior of other agents as source of information. The important feature of this model is movement of individuals from one group to another. That means *switching of trading strategy*. The agents switch trading strategy with some time varying probability so that their chances of making profit increase. Thus profits earned by individuals in each group acts as a driving force for such switches.

Switches between the optimists and pessimists are governed by the majority of opinion among noise traders and the actual price trend. Movements between fundamentalists and noise traders depend on the *profit difference* W between two groups. While calculating profit of fundamentalist, a discount factor (which is < 1) has to be taken into account because fundamentalist's gain is realized only in future when price reverts back to fundamental value. Since fundamentalists believe that digression of the market price from the fundamental value is just momentary and asset price will eventually approach the fundamental value, their gain is prolonged for that time interval. Given that, they can not immediately invest this earned profit, it needs to be discounted by a factor that is controlled by the time it takes for the market price to revert to its fundamental value. Thus gain of fundamentalist is given by:

$$\text{Gain} = [(p_f - p) / p] * d \quad (2.8)$$

Here, d is a discount factor. This model uses discount factor of 0.75. In actual market this factor can vary depending on how frequently the company that issued stocks publishes the information about its sales and profit which would affect the fundamental value. Profit of optimistic noise traders consists of short term capital gains due to increase of market price or losses in case of fall of market price. This gain is realized immediately. Since pessimistic noise traders rush out of the market in order to avoid losses, their gain is given by the difference between the average profit rate from alternative investments and the price change of the asset they sell. Thus gain of pessimistic noise traders can be defined as $R - p'$, where R is average return from other investments and is assumed to be constant. For the simplicity purpose, there is only one type of stock listed in this model market. Also, all agents trade only one unit of stock in every trading cycle.

For calculating the **transition probability** from one group to another, Lux and Marchesi have used mass statistical formalization approach inspired by statistical physics [26]. As a simple formalization of movements into and out of these groups, exponential functions are used. Also, the *frequency of revaluation of opinion or strategy* by agents is considered an important parameter for calculating this probability. This is the frequency at which agents evaluate their performance and tend to switch to more successful group. This frequency is symbolized with V_1 and V_2 in equations below. For example, if $V_1 = 5$ trading cycles, agents will reevaluate their strategy after every 5 trading cycles and decide whether to switch to other group with probability π_{+-} and π_{-+} . Thus, actual transition probability is combined effect of these two factors.

Transition probability from optimistic to pessimistic is:

$$\pi_{+-} = V_1 \exp(U_1) \quad (2.9)$$

Transition probability from pessimist to optimistic is:

$$\pi_{-+} = V_1 \exp(-U_1) \quad (2.10)$$

Here, $U_1 = \alpha_1 x + (\alpha_2 / V_1) p'(t)/p(t)$

x : majority opinion = $(n_+ - n_-) / (n_+ + n_-)$

n_+ : number of optimists

n_- : number of pessimists

$p(t)$: market price of one unit of stock at time t

$p'(t)$: price trend = $p(t+1) - p(t)$

V_1 : frequency of strategy revaluation (in number of trading cycles)

α_1 : factor of importance that individuals place on the majority opinion

α_2 : parameter for actual price trend in forming expectations about future price changes

In here, U_1 is an influential term covering those factors that are decisive for the pertinent changes of behavior. Parameters V_1 , α_1 and α_2 are same for all the agents in the market. Furthermore, they all are constant and setup right at the beginning of simulation. Both α_1 and α_2 are typically in range of 0 to 1.

Transition probability from noise trader to fundamentalist is:

$$\pi_{nf} = V_2 \exp(U_2) \quad (2.11)$$

Transition probability from fundamentalist to noise trader is:

$$\pi_{fn} = V_2 \exp(-U_2) \quad (2.12)$$

Here, $U_2 = \alpha_3 * \text{profit differential}$

α_3 : factor of pressure exerted by profit difference

V_2 : frequency of strategy revaluation (in number of trading cycles)

In here, V_2 and α_3 are constant and same for all the agents. α_3 is typically in the range of 0 to 1. Profit differential (W) is simply the difference of average gains of agents in different groups. That means agent compares his own profit with average gain of all other agents in groups other than his own.

Apart from agents switching group, other two important building blocks of this model are *price changes* and *changes in fundamental value*. The price changes are driven

by supply and demand in the market, which originate from decisions of agents. Excess demand or supply generated by noise traders can simply be calculated by number of total optimists and pessimists, assuming their trading volume to be constant.

Thus, excess demand by noise traders is:

$$(ED)_n = t_v (n_+ + n_-) \quad (2.13)$$

Here, t_v : trading volume (total number of stocks traded in one trading cycle, either sold or bought)

n_+ : number of optimists in the current trading cycle t

n_- : number of pessimists in the current trading cycle t

Fundamentalists' sensitivity (γ) to relative deviation of price from the fundamental value contributes to the excess demand or supply.

Excess demand by fundamentalists is:

$$(ED)_f = \gamma (p_f - p) n_f \quad (2.14)$$

Here, γ : sensitivity to deviation of price from fundamental value

n_f : number of fundamentalists in the current trading cycle t

$p_{f,t}$: fundamental value of one unit of stock in the current trading cycle t

In here, γ is a constant and is same for all the fundamentalists in the market. Its range is 0 to 1. The overall excess demand or supply is sum of both these components $(ED)_n$ and $(ED)_f$. Furthermore, the model assumes that changes of the log of fundamental value follows a normal distribution with mean zero and time invariant variance σ^2 . Thus,

$$\ln(p_{f,t}) = \ln(p_{f,t-1}) + \epsilon_t \Delta t \quad (2.15)$$

Where, $\epsilon_t \sim N(0, \sigma)$

Following is the conceptual construct of model's market operations:

1. The new information about company's sales and prospects arrives in the market, which has a normal distribution with mean zero. All incoming values of sales above 0 are transformed into 1 (sales expected to increase and stock price expected to go up) and all values below 0 are transformed into -1 (sales expected to decrease and stock price expected to go down)
2. The noise traders set themselves up as optimistic or pessimistic. This is done uniformly randomly by flipping a coin.
3. Noise traders decide their action of whether to buy or to sell depending on the actions of all other noise traders in previous cycles multiplied by their sensitivity to get influenced by others (α_1), the nature of the news (+1 or -1 from step 1) multiplied by the news sensitivity (α_2), and current trend of the fundamentalists multiplied by the propensity to imitation (κ_i - confidence factor of noise trader i , in range of 0 to 1).
4. Fundamentalists decide their action of buying or selling by comparing the market price to the fundamental value. That means, if $p > p_f$, sell a unit of asset and if $p < p_f$, buy a unit of asset.

5. After all trading is completed; price and returns are computed based on supply demand rule. Excess demand leads to increase of the prevailing price and excess supply leads to decrease of the prevailing price.

$$p(t+1) = p(t) + [\text{number of buyers} - \text{number of sellers}] \quad (2.16)$$

Returns are calculated using equation (2.3).

6. If the return of the asset moves in the direction suggested by incoming information, irrational agents (agents with high sensitivity to get influenced by other traders) among the noise traders become more confident on other noise traders and herding behavior of them increases. If the return doesn't follow the arrived information, the confidence decreases. The confidence factor κ_i of noise traders is initially set to 0.5 and it increases or decreases by the amount of return after each trading cycle.
7. After each cycle, agents can switch the group with certain time varying probability defined earlier in equations (2.9) - (2.12).

The simulation tests performed by Lux-Marchesi confirm that even though the fundamental price follows the market price evolution very closely, the time paths of returns extracted from price series do not reflect distributional characteristics of fundamental value. This result is in agreement to the return series observed in wide variety of real world markets and it suggests that distribution of returns is non-gaussian and statistical properties of increments differ fundamentally. For instance, DJIA return distribution in figure 2.5 confirms this behavior. Other stylized facts such as fat tail distribution, clustering of volatility, absence of autocorrelation in return and high

frequency of extreme events are also producible with Lux-Marchesi model. It also demonstrates that even though the scaling properties are not present in the external driving factors of their simulated market, they are generated by the interaction of agents with heterogeneous strategies.

2.5 Financial Market Models and Simulators

In recent year, various analytical approaches and simulation methods have been employed to explore complex economic dynamics of financial markets. Traditional analytical methods in finance have been found to be highly macroscopic with number of unrealistic assumptions [24]. Also, interactions between market players are overlooked to a large extent with these sort of analytical methods. Such macroscopic simulation techniques typically use top down approach where agents' heterogeneity and market situations are oversimplified. This approach fails to explain the grounds for the stylized facts observed in financial market. Also, because of the complexity and number of assumptions, it is hard to find out which aspects of the models are responsible for producing stylized facts [7]. In this thesis we have tried to come up with a model that has simple framework and minimal postulations. Also, modeling each individual agent and keeping track of their interactions have been paid ample attention in our model. We will describe this model in next chapter.

2.6 Limitations Of Original MG As Market Model

Ever since it's arrival, MG has been focus of intense study. Basic MG as realistic market model has quite a few limitations such as [10].

- L1** Agents' heterogeneity and wealth are limited.
- L2** There are no interactions between agents.
- L3** The payoff function of the game is too simple [equation (2.6)].
- L4** All agents trade at each time step.
- L5** All agents deal equal quantity of asset every time.
- L6** Unable to produce periodic volatility property observed in various markets.
- L7** Impact of asset's fundamental value on the market is overlooked.
- L8** Limited parameter sets that can produce stylized facts.
- L9** Only one type of stock is offered in the market model.

A few researchers have come up with certain modifications to original Minority Game model [5] to overcome some of these limitations. For example, The Grand-Canonical MG addresses the issue of agent's selection whether or not to trade at a given time step depending on his confidence level [10]. Thus not all the agents trade in each trading cycle. It also allows agents to trade multiple units of asset in one time step. One more variation of MG known as Colored MG has agents playing with different frequencies [19]. That means trading frequency of different agents can vary from several times during a day to once in several months. The $\$$ -game proposed by Anderon and Sornette offers a different payoff function where the gain at time t depends on the trading action of agents at time $t-1$ [1]. Main focus of our research is to improve on the heterogeneity aspect of agents, their interactions and introduce fundamental value of asset into MG market model.

Chapter 3: Adapted Minority Game

In this chapter we will describe our model, which we are calling as “adapted minority game”. In our model we are taking a bottom-up approach as it allows us to concentrate on interactions of agents with wide range of spectrum for parameters. This approach has shown its advantages and has become quite popular in recent time with various microscopic simulation models based on this approach evolving in the fields of finance, physical science, biology, social science etc. [3,12,24]. The **bottom-up approach** means, we first create the market environment and generate various elements in the system. These elements interact with each other and the market environment by well-defined analytical methods. Here each element is modeled individually and it’s possible to track the dynamics of each element over the time. For instance, market price, asset, fundamental price, returns, volatility etc. are modeled as market environment parameters. On the other hand, agents, agents’ trading strategies, agents’ adaptation, agents’ pool transitions etc. are modeled as independent elements, which evolve through a set of predefined rules. In contrast to this, traditional models of financial market analysis use the **top-down approach**, where statistical methods are applied to a chunk of market data and in conjunction with certain hypothesis, the relationship between various market parameters and agents are estimated. It often assumes that agents are completely rational and homogeneous in nature. With this approach it’s very difficult to point out which factors contribute to typical market properties or stylized facts. Following sections describe our adaptive minority game model.

3.1 Types of Agents

In adaptive minority game model we have divided market agents in 3 pools. The first pool of agents is of *fundamentalists*. Agents in this pool follow the efficient market hypothesis. That means they assume that upcoming price fluctuations will follow the movements suggested by incoming news about the future earning prospects. Fundamentalists believe that the price of the asset (p) may temporarily deviate from the fundamental value (p_f) of asset but eventually will revert to it. Thus market would be efficient in longer run. Fundamental value of asset is the discounted sum of expected future earnings. It is related to the current and prospective states of the company that has issued the asset. Fundamentalist's trading strategy is very straightforward. Fundamentalist buys asset when actual market price is believed to be below fundamental value and sells asset when market price goes above fundamental value [15]. This fundamental value is a perception of agent based on his knowledge about the asset, company's prospects and the market, and in general can be different for different agents. In our model we assume that the fundamental value of stock is the same for all agents in a trading cycle and its relative changes follow normal distribution from cycle to cycle as per equation (2.16).

Agents in other 2 pools play the minority game. However agents in these pools have different historical memory length m . That means different agents decide their trading action looking at different lengths of recent past outcomes of market. Here, the full strategy space for both pools is different. Similar to original MG model described in section 2.3, agents are assigned fixed number of strategies S randomly drawn from the

full strategy space. Furthermore, agents in different pools use different time horizons T to evaluate their individual performances. Thus, agents collect and maintain the virtual points of their strategies over different period of time lengths. After the specified window of time horizon, agent discards virtual points of all his strategies and starts afresh. This feature is in contrast to original MG model, where strategy points for all agents are kept right from the beginning till the end of the game. Thus agents operate on infinite time horizon basis. In real world market this is not true, where agents tend to exhibit limited time horizon in evaluating their strategies [1,9,10,18]. Also, it's a well researched observation that market price of the assets depends only on last few values of price and after a certain threshold, the older price series doesn't help much in predicting future trend. Absence of autocorrelation in longer run observed in variety of markets and assets supports our assumption that in real world market agents operate in finite time horizon.

3.2 Agents' Decision Making

At each time step of the game, agents have to decide whether to buy or sell a unit of asset. A fundamentalist will buy the asset if market price is less than fundamental value and will sell the asset if market price is more than fundamental value. Since we want to make sure that the typical characteristics of financial price series are not fashioned on the basis of exogenous factors that are unrealistic, we are assuming that relative log changes of the fundamental value follow normal distribution with mean zero and time invariant variance σ^2 as in equation (2.15). Original MG model doesn't have fundamentalists in the market, so we are using this from Lux-Marchesis model described in section 2.4. Here, change in fundamental value is an exogenous factor that affects

market through operation of fundamentalists. That means it is an external force which is not generated from within the system. These changes in fundamental value serve as new information coming into the market. By modeling fundamental value changes as normal distribution, we have ensured that the news arrival process in our model does not clutch fat tails [14].

Agents in other two pools who play minority game are assigned a set of S strategies. Agents in first pool have historical memory of length $m1$ and time horizon of $T1$, whereas agents in second pool have historical memory of length $m2$ and time horizon of $T2$. The full strategy space for these two pools will be 2^{m1} and 2^{m2} respectively. At the start of the game agent picks a strategy randomly to decide his action. After each time step, the agent assigns a virtual point to all the strategies in his bag that would have correctly predicted the right outcome. For the subsequent time steps agent chooses the strategy with the highest virtual points. If there are more than one strategies with the same highest virtual points, one strategy is chosen randomly. At the end of each time step, the total action of agents is computed and the market price is determined by supply-demand rule as per equation (2.16). Agents who end up on the minority side win the round. In here, agents have only two choices: “buy” or “sell”. There is no option to “hold”. This is a simplified model and future work can include extending this model with “hold” option.

3.3 Adaptation and Interaction of Agents

Agents evaluate their performance at the end of each time step. Agents are allowed to switch from one pool to another with certain endogenous time varying probability. That means agents from fundamentalists pool can switch to minority game pool and agents from minority game pool can switch to fundamentalist's pool. The main incentive for agents to switch pool is the profit earned by the respective groups' agents. Thus agents tend to switch to more successful pool and trading strategy. Since there is a possibility that one pool of minority game agents may perform consistently better than other minority game pool, it may result in all minority game agents tending to switch to that pool. Thus, the switch from one minority game pool to another minority game pool is *not allowed* in order to avoid market to be flooded with agents having same memory length and time horizon.

In order to calculate the transition probability from one pool to another, profit earned by agents in each pool needs to be speculated. For fundamentalists, the deviation between market price and fundamental value is considered as a source of arbitrage opportunities. However gain earned by fundamentalists is realized in future only depends on uncertain time interval for reversal of market price to fundamental value. This factor has to be taken into account when calculating profit earned by fundamentalists. Thus the gain for a fundamentalist is given by equation (2.8)

The gain of agents playing minority game consists of two components. First is the change in market price of stock. Second is the dividend paid by the company that issued

the stock. When the company makes a loss, the dividend is waived. Otherwise we are assuming dividend to be constant. This assumption is reasonable in this scenario as we are keeping the resource level (number of available stocks) in the market constant. Thus gain of minority game agents is given by:

$$\text{Gain} = (p' + D) / p \quad (3.1)$$

Here, $p' = dp/dt$, price change and D is the dividend of asset.

It's important here to compare the gain given by equation (3.1) with the payoff function of MG agents given by equation (2.6). The payoff function stands for the margin by which agent won or lost the current round, which effectively represents excess demand or supply in the market context. This excess demand or supply affects the market price as per equation (2.15). Thus gain is simply the profit made by agents from their respective investments. Here profit is combination of price change and dividend of the asset.

For calculating the transition probability, we are using the same approach as Lux-Marchesi model and modifying equations (2.11) and (2.12) as described below. The driving force for pool transition still remains the profit difference (W), which in turn depends on number of factors such as price change, fundamental value, discount factor, dividend and frequency of strategy revaluation. The transition probability from fundamentalist to minority game pool is defined as:

$$\text{Prob} (F \rightarrow \text{MG}) = v (N_f / N) * \exp (U) \quad (3.2)$$

Here, v = Frequency of revaluation of strategy

N_f = Number of fundamentalist agents

N = Total number of agents

$$U = \alpha_f ((D + (p' / v)) / p - ((p_f - p) / p) * d)$$

α_f = Factor of profit difference influence for fundamentalists

In here, α_f is a constant and typically in range of 0 to 1.

Similarly, transition probability from minority game pool to fundamentalist pool is defined as:

$$\text{Prob} (\text{MG} \rightarrow \text{F}) = v (N_{mg} / N) * \exp (-U) \quad (3.3)$$

Here, v = Frequency of revaluation of strategy

N_{mg} = Number of agents playing minority game

N = Total number of agents

$$U = \alpha_{mg} ((D + (p' / v)) / p - ((p_f - p) / p) * d)$$

α_{mg} = Factor of profit difference influence for minority game agents

In here, α_{mg} is a constant and typically in the range of 0 to 1. As we can see, the main difference between equations (2.11), (2.12) and (3.2), (3.3) is the way in which term U is defined. Since we don't have optimists and pessimists in our model and Lux-Marchesi model doesn't have minority game agents, the profit differential function is modified here appropriately.

Here, it's vital to note that the transition probability is bounded by $0 \leq P \leq 1$. The condition that ensures this bound is that agents tend to switch their pool only when profit earned by agents in other pool is more than their own profit. Here the exponential

function is driven by U , which has the profit difference as its main component. The exponential function itself ensures *postivity of all probabilities* and *symmetry of pool transitions*. As for the *upper bound* of the probability, frequency of strategy revaluation is the decisive factor. The most frequent revaluation should be every 2 trading cycles, which ensures that upper bound on the probability is met for all the feasible values that U can take up. We will further discuss choice of parameters in chapter 5.

Also, it's important to note that agents' **interaction** is in the sense that they compare their own performance with average performance of agents in every pool other than his own. The agent does not compare his own performance with other agents in the same pool since if average performance of other pools isn't better than his own performance, he is already in the best possible pool (strategically). If his own performance is worse than average performance of some other pool, he would tend to switch to that pool.

3.4 Generic Algorithm

Following is the generic algorithm that we have developed for simulation of our adapted minority game model. This algorithm is used to obtain the main stylized facts and analysis of price series. We made subtle modifications to this algorithm to perform other simulation tests discussed in chapter 5 such as impact of agent's memory on success rate, impact of time horizon on volatility etc., though the main framework remains the same. These modifications will be discussed in the following chapter.

Initial market set up

1. Initialize the time counter to 0
2. Setup fundamental price series that follows price change of normal distribution.
3. Initialize arrays to hold time series of market price, volatility and return
4. Set the resource level of asset (number of available stocks in the market) to $0.5 * N$, where N is total number of agents

Initial agents set up

1. Setup desired number of agents in all 3 pools
2. Generate full strategy space for agents playing minority game. For first pool of MG agents, it is 2^{m1} and for second pool of MG agents it is 2^{m2}
3. Initialize minority game agents with memory size and time horizon length
4. Initialize minority game agents with randomly drawn strategies from full strategy space
5. Generate a random history string for the chosen memory length to begin the game. This acts as an initial input for minority game agents
6. Set up virtual points counters for each strategy for each minority game agent. Initialize all of them to 0 ($\zeta_{ij}=0$)
7. Setup counters to keep track of each agent's success rate. Success rate is simply the ratio of number of times agent has ended up in the winning group to the number of times agent has traded. It is only used for the post-simulation analysis and is not used by agents during the game
8. Define frequency of strategy revaluation (v) for all the agents

9. Define agents' sensitivity to profit difference for pool transition. This is α_f for fundamentalist in equation (3.2) and α_{mg} for minority game agents in equation (3.3)

Agents' trading action & market operation

1. Based on their strategies, agents have to decide whether to buy (1) or sell (0) one unit of asset.
 - Agents playing minority game pick a strategy randomly for the first iteration. For the subsequent iterations strategy with the highest virtual points ζ_{ij} is picked. Based on the chosen strategy agent decides the action appropriate to the most recent history pattern for his memory length.
 - Fundamentalist compares the market price with fundamental value and decides the action based on it. If the market price is below the fundamental value ($p < p_f$), he will buy a unit asset. If the market price is above the fundamental value ($p > p_f$), he will sell a unit of asset.
2. Once all the trading decisions are made, total number of buyers and sellers are computed. If there are more buyers than sellers, sellers are declared as winner and if there are more sellers than buyers, buyers win the round.
3. The supply demand rule determines the market price as per equation (2.16). If there are more buyers then price goes up by the amount of difference between buyers and sellers. If there are more sellers then price goes down by the amount of difference between sellers and buyers.

4. Strategy points (ζ_{ij}) of all minority game agents are updated as per equation (2.7).
All the strategies that would have predicted the right outcome are assigned one virtual point.
5. Success rates of all agents are updated, including fundamentalists and all minority game agents.
6. Time horizon counter is incremented by one for all the minority game agents. If the counter is equal to the pre-defined time horizon of agent, it is reset to 0 and all the strategy points of that agent are wiped out and re-initialized to 0 as discussed in section 3.1.
7. Pool transition probabilities (from minority game pool to fundamentalist pool or vice versa) are calculated for all the agents based on equation (3.2) and (3.3). Agents switch pools based on this probability. Thus, actual number of agents switching pools is calculated by multiplying number of agents in that group with respective transition probabilities.
8. Various statistics such as volatility, price return, absolute return, pdf, and autocorrelation are computed for later analysis as will be seen in chapter 5.
9. Increment the time counter and if it has not reached the total number of iterations (total number of trading cycles, which can be set to anything), go to step 1.

Chapter 4: Simulator Design

In this chapter we will provide the overall design of the simulator that we have developed for adapted minority game model.

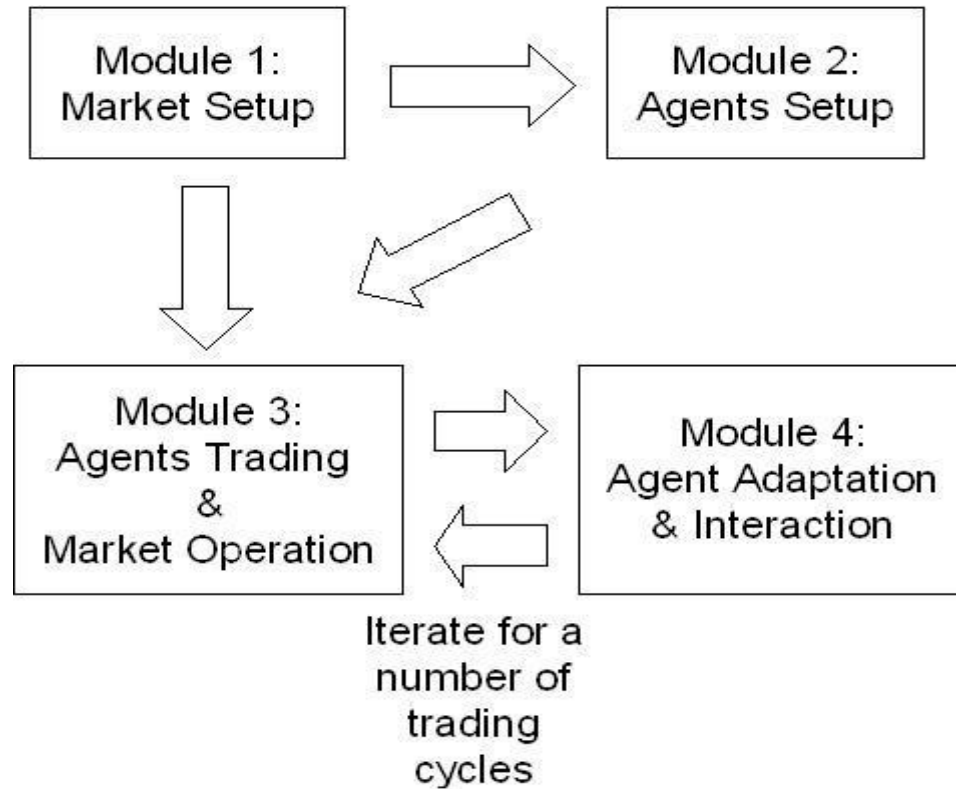


Figure 4.1 Overall Module Structure

Figure 4.1 shows the basic building blocks of our simulator. It consists of 4 core modules that interact with each other. Module 1 and 2 deal with market and agents' initialization. Actual trading and market operations take place in module 3, whereas module 4 encapsulates agents' adaptations and interactions. Here, module 3 and 4 form a closed loop.

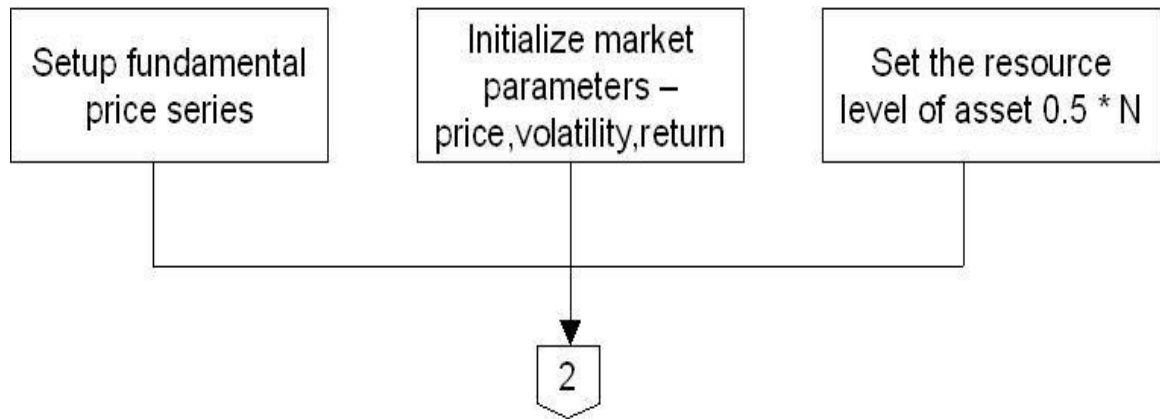


Figure 4.2 Module 1: Market Setup

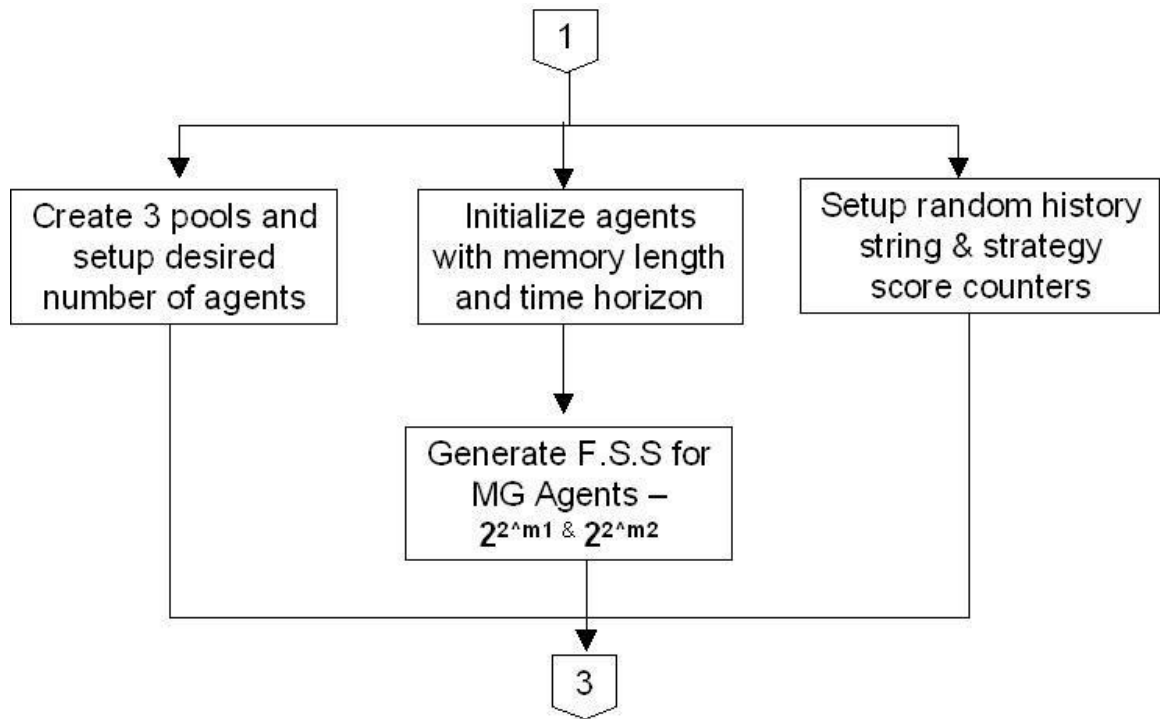


Figure 4.3 Module 2: Agents Setup

Market setup module initializes the fundamental price series, which serves as incoming information in the market. As we already discussed, it is simulated with logarithmic changes in fundamental value following normal distribution. The module

also initializes arrays to hold various market parameters such as price series, volatility, return etc. Further, the resource level in the market is setup. Module 2 operations comprise of agents setup and initialization. It also generates a random history string that is being used by MG agents at the start of the game. Furthermore the full strategy space is generated and agents are assigned memory length, time horizon and strategies. Counters for agents' performance monitoring and strategy evaluation are setup in this module.

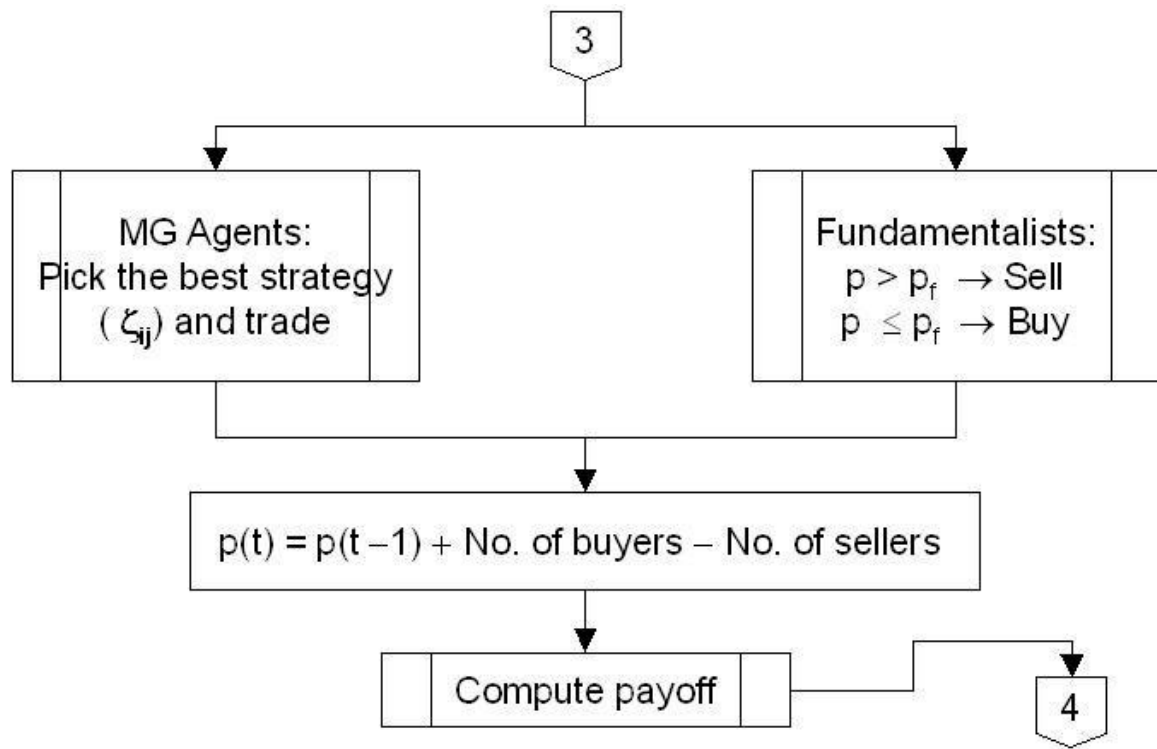


Figure 4.4 Module 3: Agents' Trading & Market Operation

Module 3 and 4 forms the core of our market model. All the information from module 1 and 2 is passed onto module 3, where agents make their trading decisions. After all agents have completed their action, the market price is updated and the payoff for the current trading cycle is computed. Module 3 interacts with module 4. Agents' adaptation

and performance evaluation take place in module 4. The information flow between module 3 and 4 is bi-directional. In module 4, agents compute their gain and calculate the pool transition probability. Based on this probability agents tend to switch their pool. Also, MG agents' strategy scores are updated in this module. Once the pool transition phase is over, the control is passed back to module 3. The loop formed by module 3 and 4 is iterated for the defined number of trading cycles.

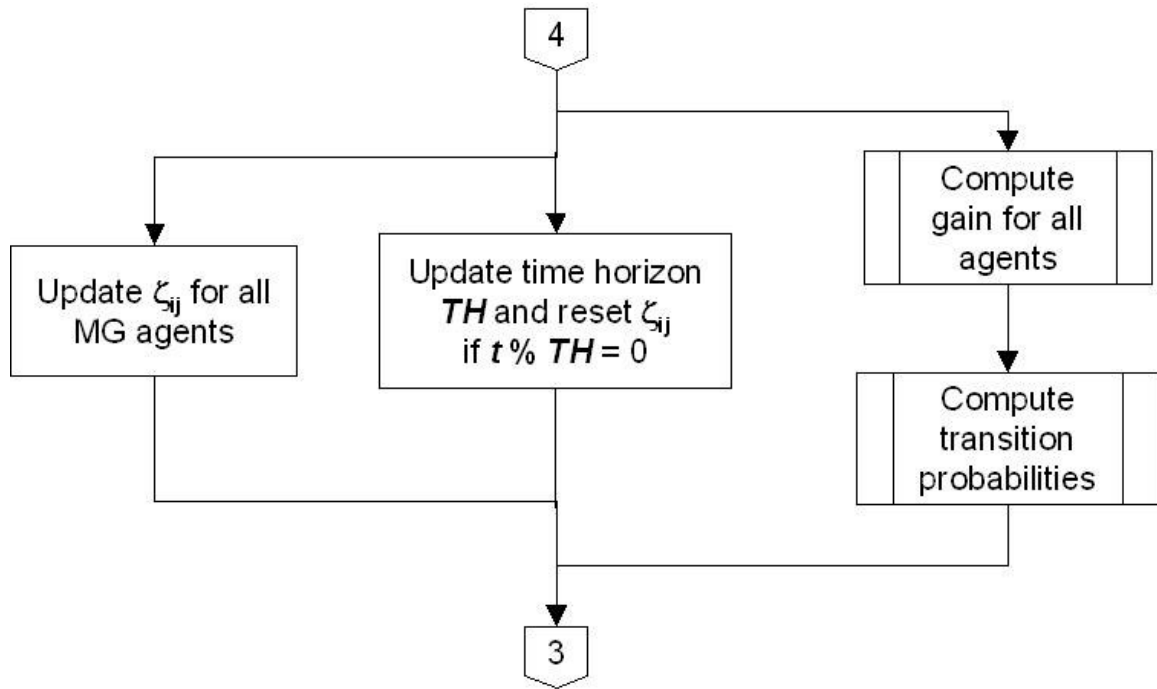
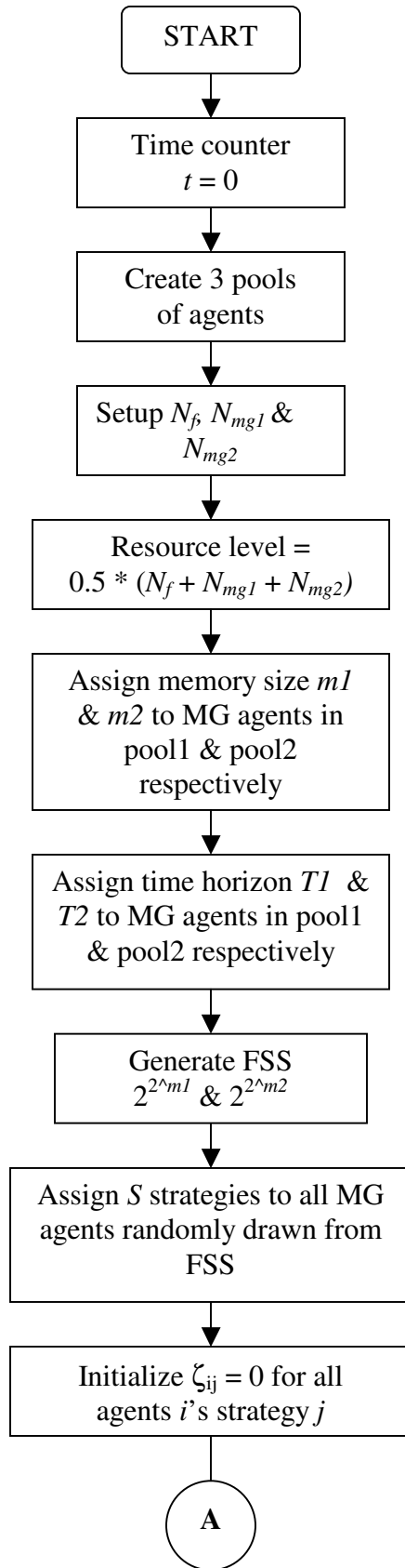
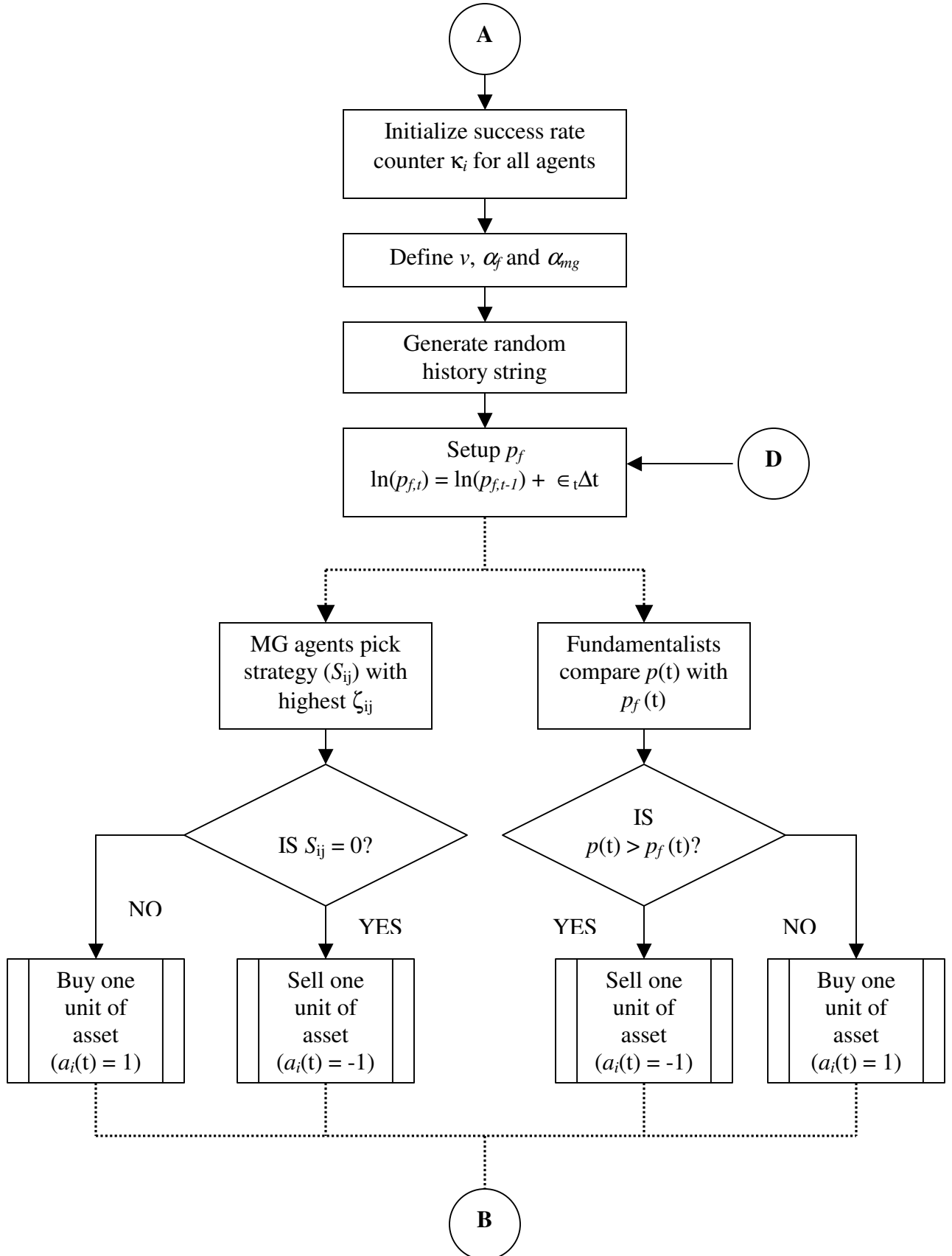
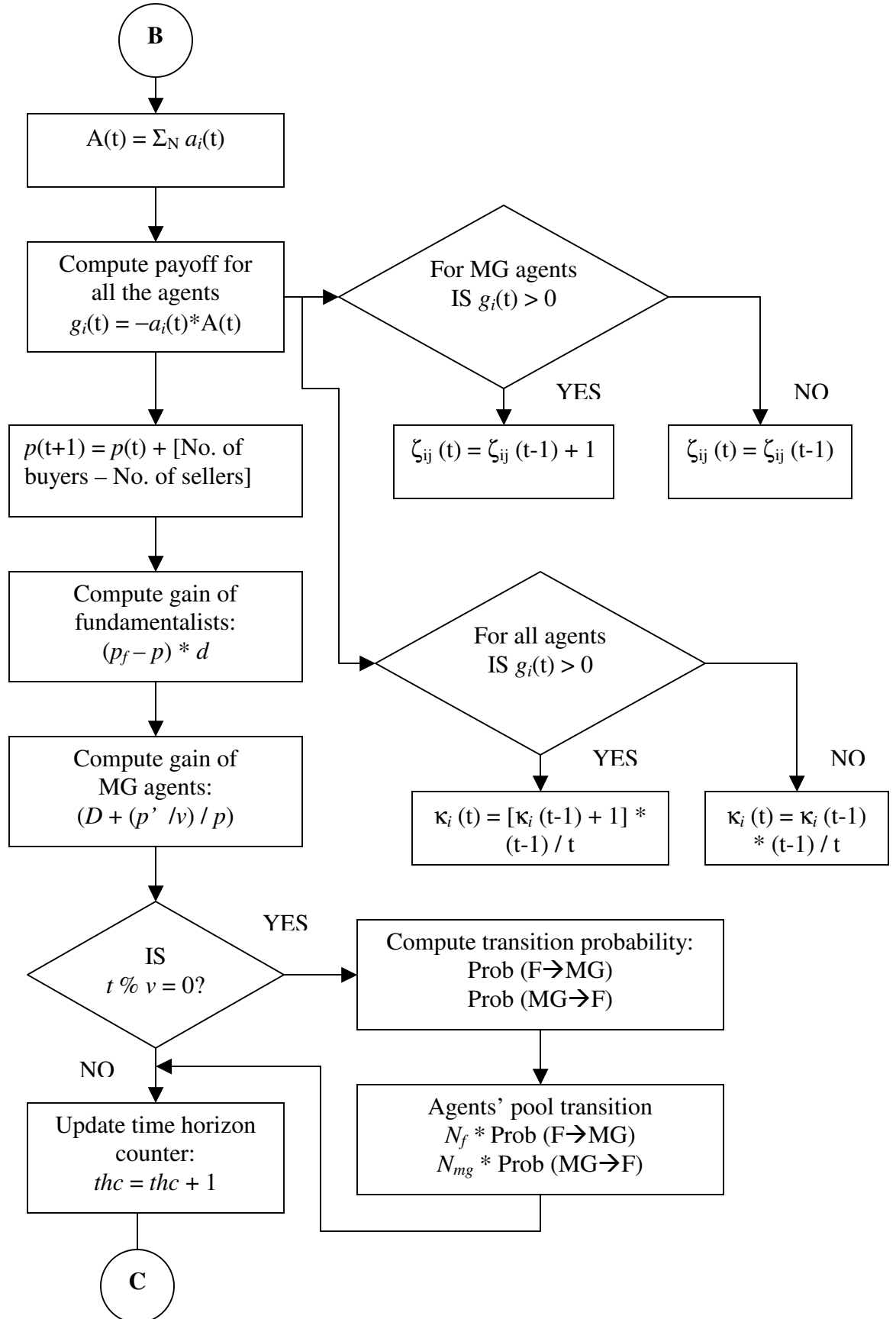


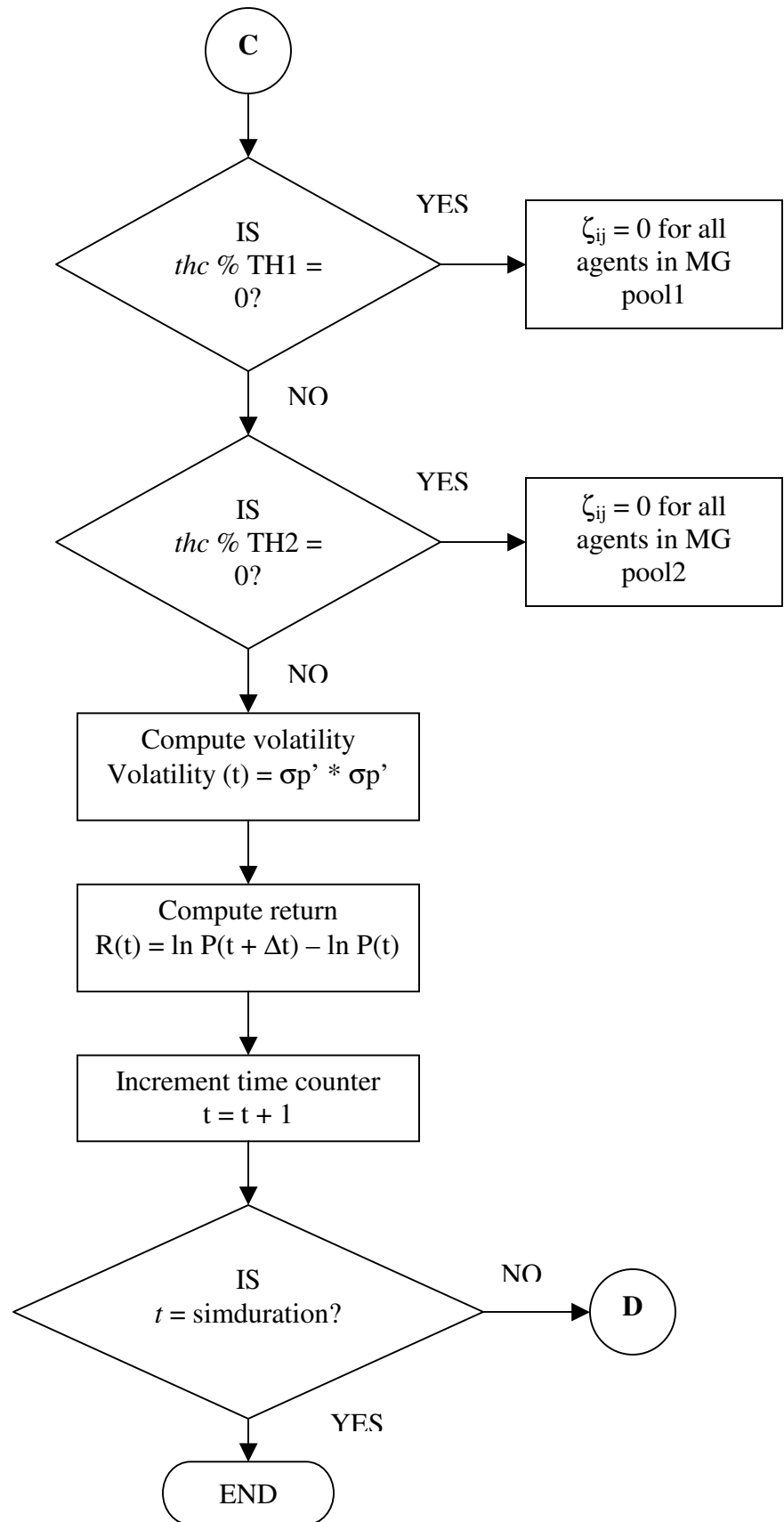
Figure 4.5 Module 4: Agents' Adaptation & Interaction

Next we have presented the flowchart of the simulator based on the explained module design.









Chapter 5: Implementation and Results

5.1 Implementation Overview

We have simulated the adapted minority game model in 2 phases. In the first phase we developed the simulator with the simplified approach where all the agents belonging to the same pool have same memory and time horizons. That means all the agents in pool 1 playing minority game have memory $m1$ and time horizon $T1$, and all the agents in pool 2 playing minority game have memory $m2$ and time horizon $T2$. This is the *baseline version* of our adaptive minority game model that we have described in the previous chapter. We will be referring this version as “*Divided MG Pool Adaptive Model*”.

In the second phase we have tried to explore the full spectrum of heterogeneity of agents. In this scenario, the agents are now divided in 2 pools, only based on their playing strategy: an agent is either a fundamentalist or a minority game player. We have got rid of two separate pools for minority game agents and now all the minority game agents belong to the one pool only. The agents playing minority game have the *full spectrum of memory and time horizon*. That means minority game players may or may not share same memory and time horizon. The range of this spectrum is discussed in section 5.3. All agents are randomly assigned certain memory length and time horizon at the start of the game. Except for this random assignment of memory and time horizon to agents, all the rules of the game remain same as described in chapter 3. This simulation scenario allows us to take a step further in the direction to make market agents highly diverse and study

their impact on financial price series. This version of our model will be referred to as “*Randomized MG Pool Adaptive Model*”.

5.2 Platform and Tools

The entire simulator has been developed using Perl. The foremost decisive factors while opting Perl over other languages were its modularization, arbitrary data structure, minimal overhead with random number generation and highly efficient array handling. Also, Perl is highly efficient in memory management, file-handling operations and demonstrates significantly less overhead associated with various data types as compared to most other programming languages. Furthermore, we have found it to be very robust and simple for this particular application, as we have focused to model individual agents of the market and follow their interactions. For the visualization and graphing purpose we have used TecPlot 360 [25]. All the simulation tests have been performed on a system running Linux CentOS 4.2.

5.3 Model Parameters and Validation Benchmarks

Table 5.1 and 5.2 show the parameters that we have opted for the simulation of “divided MG pool adaptive model”. From simulation tests performed over wide variety of parameters, we have observed that the memory length of 2 to 16 is a good range for modeling financial market and produces acceptable results, keeping stylized facts as a benchmark. If the memory of agents is very large, the number of agents using the same strategy will be very few. Thus there will be very few agents in the market who will be using the best strategy (the one with which the probability of winning is highest) at any

given time. Same thing can be said for the worst strategy as well. It is highly unlikely that any agents will be having same strategy in the game. The impact of this on the market is that they tend to cancel each other out and causes market volatility to drop significantly.

Number of agents in pool-1 – MG players	138
Number of agents in pool-2 – MG players	138
Number of agents in pool-3 – Fundamentalists	225
Number of assigned strategies for each agent playing MG	6
Memory of agents in pool-1 – $m1$	4
Memory of agents in pool-2 – $m2$	6
Time Horizon for agents in pool-1 – $T1$	42
Time Horizon for agents in pool-2 – $T2$	30
Resource level	250
Simulation duration	1000 days

**Table 5.1 Simulation Parameters for
Divided MG Pool Adaptive Model**

Thus with memory length of higher than 16, the model is unable to produce important property of volatility clustering discussed in section 2.1.3 and returns show significant drop compared to what we can see in figure 2.4 for DJIA. We are considering the results

returned by our simulation tests with choice of various parameters as *acceptable* if they meet *three basic conditions*:

1. The probability distribution function of return should demonstrate fat tail distribution with exponent in range of 3.8 to 4.2. Various studies of various real world markets have shown that this exponent is typically 4 on average [23].
2. The autocorrelation of return should not be greater than 0.01 for lag of more than 30 trading cycles and should continuously be decaying.
3. The volatility should never be less than average volatility for consecutive 15 trading cycles. Here, 15 trading cycle is the assumed time period for the newly arrived information to be absorbed by the agents and this assumption is in sync with results shown by study of actual markets [7]. Also, this ensures that market does not die for the lack of activity.

Resource level in table 5.1 is the total number of stocks or units of asset available in the market model. The resource level is constant through out the game and should always be less than $0.5 \times N$, where N is total number of agents in the game including minority game agents and fundamentalists. This gives the minority nature to the game ensuring that minority action always wins.

Furthermore, the total number of agents in pool 1 and pool 2 must be greater than the number of fundamentalists. It doesn't matter how greater this number is as long as it is greater. If the number of fundamentalists is greater than total number of agents playing minority game, the market becomes inactive after first 50-75 trading cycles. The graph of

price series goes flat and return drops down to zero. The main cause of this characteristic is herding tendency of fundamentalists as they dominate the market with same trading strategy. In this scenario, there is a large fraction of agents in the market who are opting for the same action and cling onto it for sufficiently long duration, which tends to bring down the market volatility to zero. Once the fundamental price drifts significantly from the market price, it is perceived as an arbitrage opportunity by all the fundamentalists and would switch the trading strategy all around the same time. This forces a turbulent period in the market for a short while and subsequently turns into tranquil phase once again. Thus with large number of fundamentalists in the model, it fails to imitate real world market. Apart from these rules, there aren't any constraints on choice of other parameters in table 5.1 such as total number of agents in the game or number of assigned strategies for each agent. That means, any other choice of parameters would produce similar results and would not affect model's ability to generate stylized facts in qualitative manner.

Table 5.2 shows the parameters used for calculating transition probabilities of agents defined in equations (3.2) and (3.3). These parameters are mostly adopted from Lux-Marchesi model. Except for frequency of strategy revaluation (ν), all other parameters are sensitive to change and modifying them can lead to poor performance, which would fail to produce stylized facts. Future work could explore more to make the model more robust to these parameters. However since all these parameters except for dividend of asset are not exactly the driving factors in the market context and are used exclusively for calculating the transition probability while modeling, there are no striking benefits, worth investing significant research efforts for this.

discount factor d	0.75
Frequency of strategy revaluation ν	1/3
factor of profit difference influence α_f & α_{mg}	0.5
variance σ^2	0.5
Fundamentalists's sensitivity γ	0.01
dividend of asset D	0.0004

**Table 5.2 Transition Probability Parameters for
Divided MG Pool Adaptive Model**

5.4 Results of Divided MG Pool Adaptive Model

5.4.1 Reproducing Stylized Facts with Divided MG Pool Adaptive Model

Results of our divided MG pool adaptive model simulation are shown in figures 5.1 – 5.8. Figure 5.1 shows the evolution of asset price and figure 5.2 represents logarithmic price series for the same. The price trajectories demonstrate good resemblance to real world market data, for instance Dow Jones Industrial Average price series that we discussed in chapter 2, figure 2.2. As we discussed in chapter 2, price series tends to exhibit different patterns across different markets and assets. However these price series are typical in the sense that over the long period the asset value is always rising. Also, markets would demonstrate occasional non-periodic crashes and recovery from it. Thus price series itself isn't a stylized fact but form the basis for other

characteristics that are derived from it. Thus, here the comparison is not in terms of absolute values of asset but rather overall characteristic of the series in its entirety.

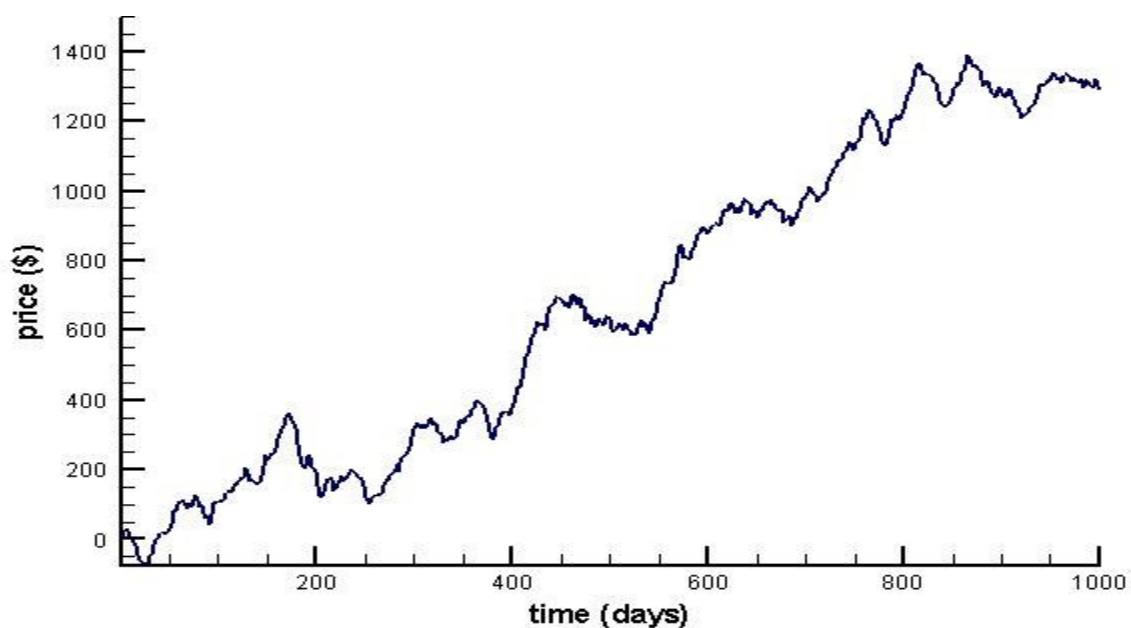


Figure 5.1 Evolution of Price Series for Divided MG Pool Adaptive Model

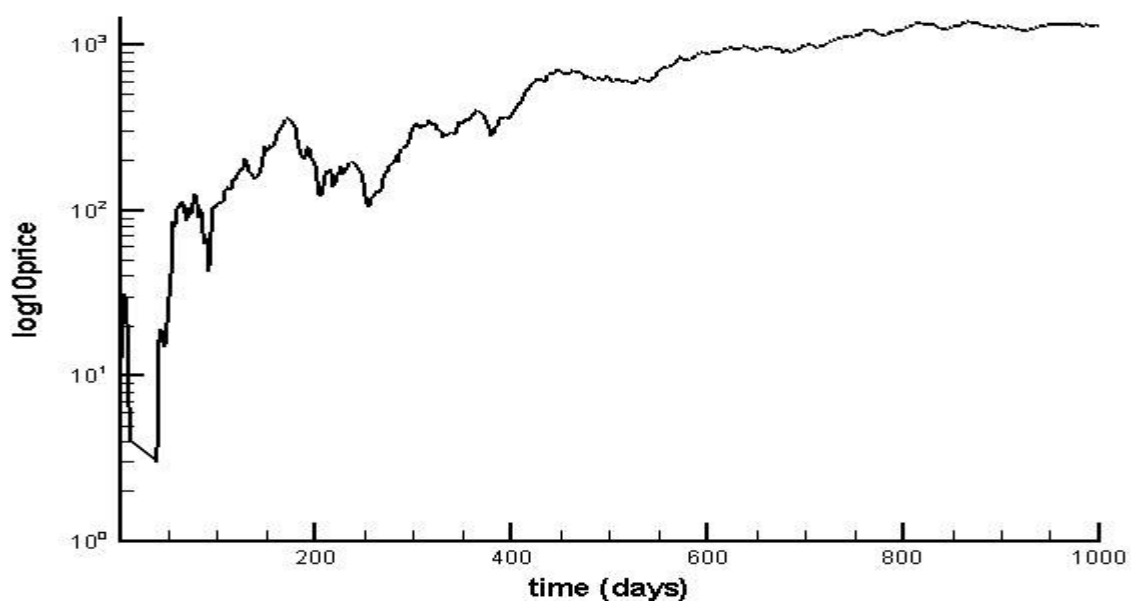


Figure 5.2 Logarithmic Price Series for Divided MG Pool Adaptive Model

Figure 5.3 shows the return price time series. Since we start the game with market price of asset set to zero, the return shows larger fluctuations in the beginning but with evolution of market price the net return subsides considerably and exhibits the range and characteristic that is very similar to actual market returns like the one shown in figure 2.4 for DJIA. The visible difference in the return price curves of figure 2.4 and 5.3 is because of the vertical axis scale. Figure 5.4 shows volatility clustering phenomenon (calculated using equation (2.5)) where extreme events are followed by extreme events. This reinforces the common understanding in financial market community that volatilities are not independent. Various studies of financial markets around the world have shown that volatility displays significant autocorrelation [6, 10, 18, 22]. In our model, since agents are constantly adapting and seeking to use the best strategy, there is a very high probability that at any given time there are large number of agents who are using similar strategies, which contributes to large fluctuations in supply-demand. This results in extreme events and consequently in high volatility. Thus adaptation of agents and their migrations from one group to another can be attributed as the foremost factors for clustering of volatility.

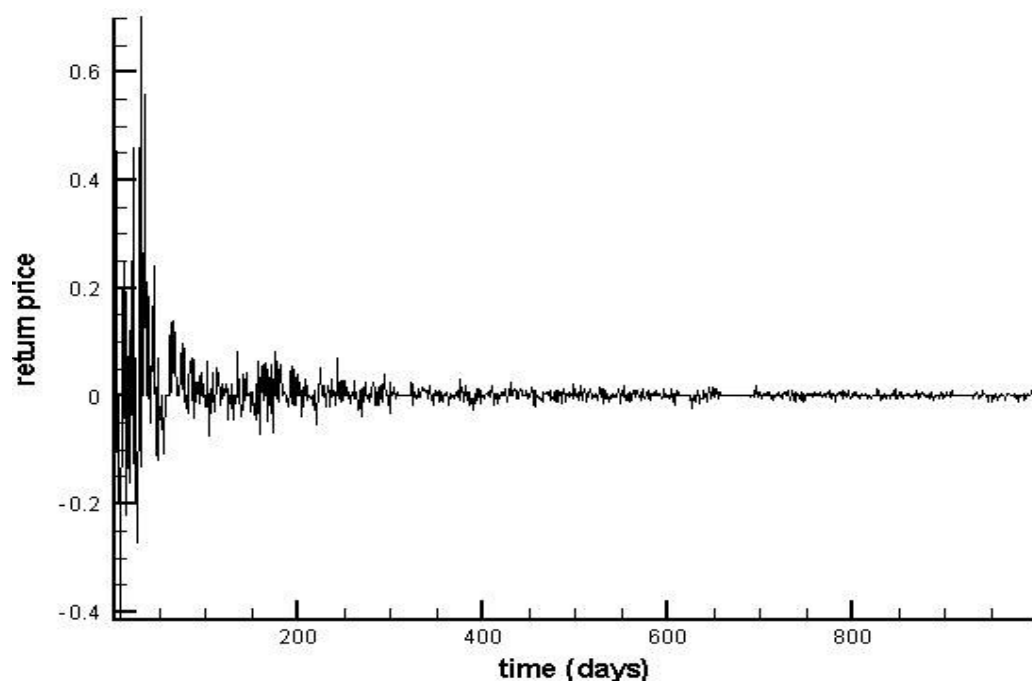


Figure 5.3 Return Price for Divided MG Pool Adaptive Model

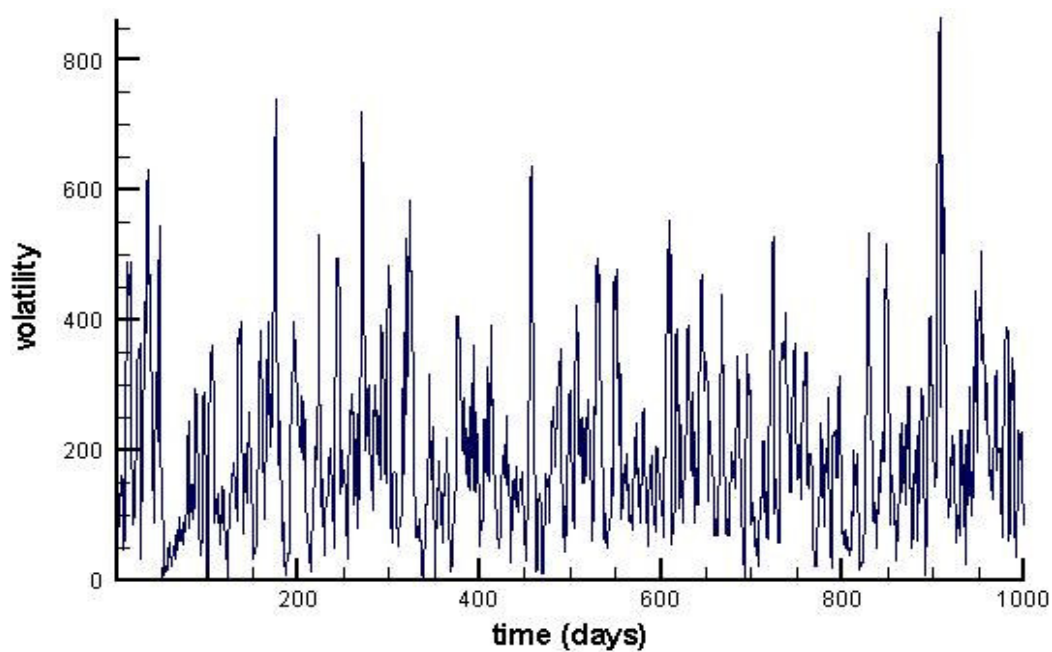


Figure 5.4 Volatility for Divided MG Pool Adaptive Model

Figure 5.5 illustrates the distribution of absolute return. For the purpose of comparison, the normal distribution has been plotted on the same graph. It is very obvious from this plot that *return distribution has much fatter tail compared to gaussian distribution*. This implies greater frequency of extreme events than what is expected with normal distribution. The return curve remains well above zero even for larger changes. Additionally, the decay evidently follows power-law distribution given by,

$$\mathcal{P}(x) = x^{-\alpha}$$

This is in contrast to normal distribution, which decays quite quickly after couple of initial standard deviations. Thus returns obtained by our model exhibit heavier tails and resemble the return properties observed in real markets. For instance, see the probability distribution of return in DJIA shown in figure 2.5.

Autocorrelation of returns is shown in figure 5.6. The autocorrelation fluctuates from -0.5 to 0.65 for first few lags but for later lags it becomes very weak. Comparing it with DJIA return properties (figure 2.6) confirm great similarities. The similarities are in the sense that the autocorrelation of return is continuously decaying and asymptotically approaching zero. As we pointed out earlier in section 2.1.2, absence of autocorrelation supports efficient market hypothesis. The diverse nature of agents contributes largely to this property and fortifies the existence of fundamentalists in the model. The presence of fundamentalists ensures that any agents playing minority game cannot take advantage of any statistical arbitrage opportunity associated with any particular strategy for sufficiently longer duration. This implies that price changes do not exhibit significant correlation except for very short duration. To be more precise, this duration is the amount of time it

takes for fundamentalists to absorb the new information coming in to the market and act according to it. Thus action of fundamentalists tends to nullify the correlation between successive price changes. In section 5.4.4, figure 5.12; we have shown the plot of autocorrelation for original MG where there are no fundamentalists in the market and it fails to produce similar result.

While we succeed in reproducing the stylized facts such as fat tail distribution, volatility clustering and weak autocorrelation of return, which are also producible with original MG model and Lux-Marchesi model, we also add new results of impact of memory length on agents' performance and periodic volatility. These results are discussed in next subsection.

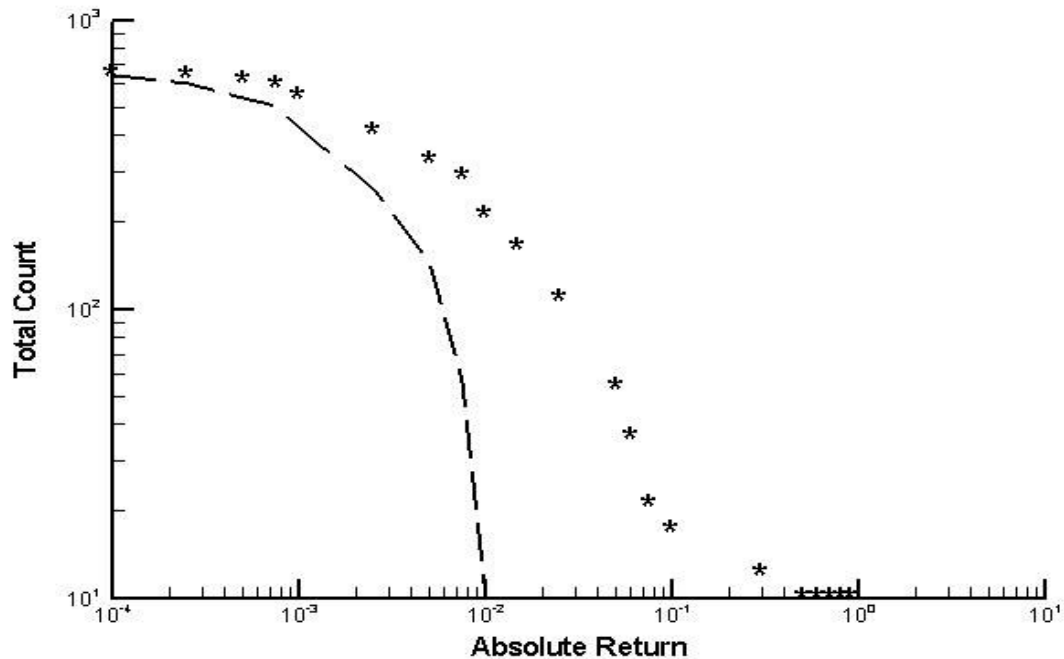


Figure 5.5 Distribution of Absolute Return for Divided MG Pool Adaptive Model

(dashed curve: gaussian distribution, *: absolute return)

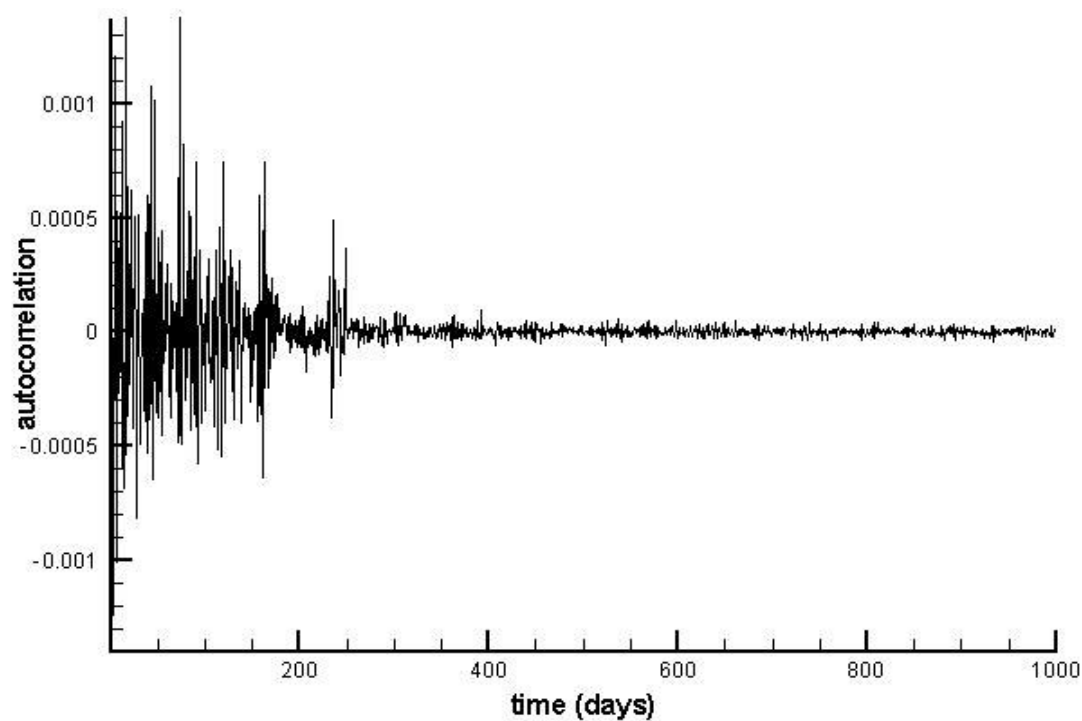


Figure 5.6 Autocorrelation in Return for Divided MG Pool Adaptive Model

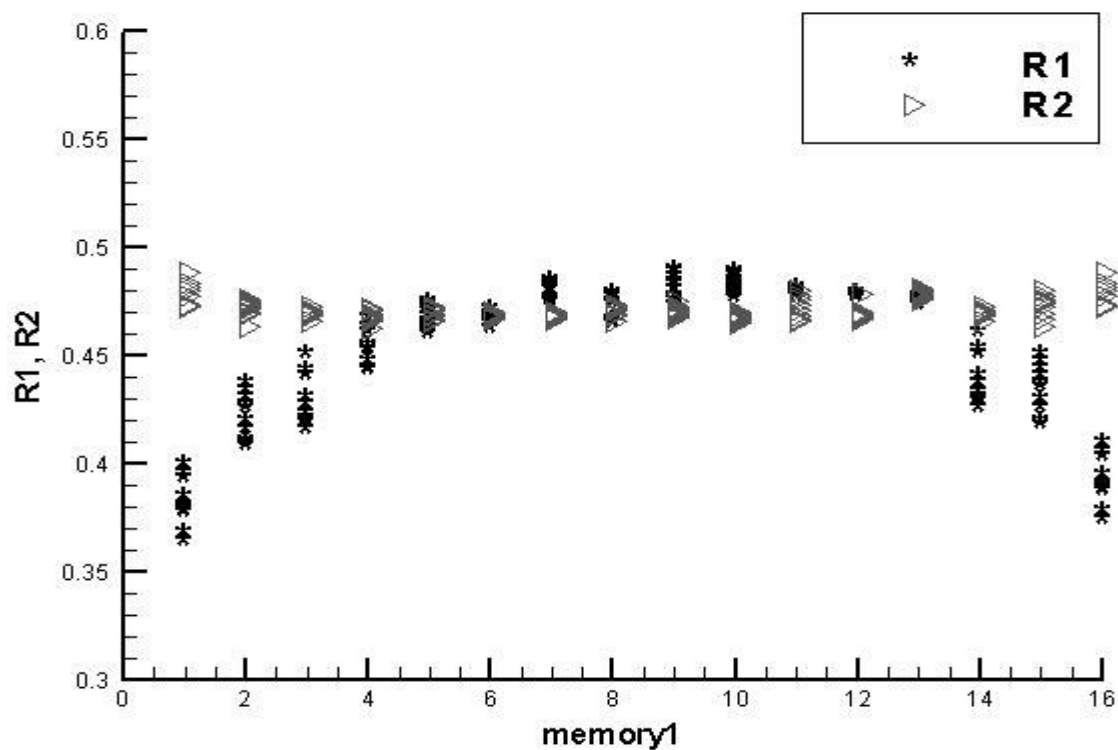


Figure 5.7 Impact of memory length on agent's success rate

5.4.2 Impact of Memory Length

Figure 5.7 demonstrates the **impact of memory length** on agent's success rate. For this part of simulation we have kept the memory length of agents in pool 2 (m_2) constant at 6. The simulation tests have been run for 100 times for each variation of memory length of agents in pool 1 and it has been averaged out in graph to represent mean and standard deviation. We can observe that initially when we increase the memory length of pool 1 agents, their success rates seem to be improving. There is also a slight drop in pool 2 agents' success rates with increasing m_1 . This is because the sequence of the winning groups contains information about the strategies of the agents and agents with more memory can exploit this information more efficiently. However, the gain in success rate due to increasing memory length lessens beyond a certain threshold and is completely wiped out with considerable larger memory length. As we can see from the plot, agents seem to be performing best when they have the memory length of 7. After that, the gain from the increasing memory appears to be constant and it shows a fall beyond memory length of 10. We have performed these simulation tests with wide range of combinations of parameters in table 5.1 and have found this result to be consistent. Thus, varying number of agents or their time horizons do not impact this outcome. Furthermore, it's important to note here that choice of m_2 as 6 has no impact on the results that we have achieved. That means opting for some other value of m_2 would not affect this behavior and best memory length is not merely $m_2 + 1$. We have carried out simulation runs with different values of m_2 ranging from 2 to 16, and have confirmed this behavior with agents in pool1 performing best when memory length is 7. This result

implies that changes in prices are related in shorter duration of time period but do not hold strong relation in longer run.

The result also supports the concept of short-term autocorrelation observed in returns (figure 5.6). Also, agents playing with shorter memory can outperform agents playing with longer memory. One reason for that could be that agents playing with longer memory have to switch more frequently between their strategies as their strategy space is significantly bigger and the number of possible combinations within each strategy is also lot more. Thus there is very high likelihood that multiple strategies are having identical virtual points, forcing agents to switch back and forth between those strategies. There is no obvious best or worst strategy for these agents. This tends to bring down their performance in long run.

5.4.3 Impact of Time Horizon

To study the **impact of time horizon** on average volatility in the market, we have performed another simulation test. This means that after every time horizon number of iterations the virtual points (ζ_{ij}) of the strategies of MG agents in a given pool are all reset to zero. In this case we have set the memory length of both pool 1 and pool 2 agents to 3. Simulation tests have been performed by varying time horizon $T1$ and $T2$ and keeping all other parameters fixed. Also, $T1$ and $T2$ are set equal and the entire simulation test has been run for 1000 cycles with different values for $T1$ and $T2$. For each value of $T1$ and $T2$ simulation has been run for 5 times and results are averaged out to show the median and two maximum and two minimum findings of average volatility. We

have allowed $T1$ and $T2$ to take on value in range of 1 to 60. Thus we have performed 300 independent simulation runs for this entire test. Figure 5.8 shows the simulation results. As we can see, the average volatility (calculated using equation (2.5)) increases with increasing time horizon.

Furthermore, it shows remarkable periodicity in time horizon of 2^{m+1} . Here, $m=m1=m2=3$. This is a very vital result obtained from our simulation test. Studies have shown that financial markets tend to exhibit seasonal periodicity in volatility [22]. For instance, prominent U.S. stock indices: Dow Jones Industrial Average, S&P 500, S&P MidCap 400, S&P SmallCap 600 all display periodicity in volatility. Expected stock returns exhibit strong seasonal pattern in the form of day of the week, month of the year and holiday effects [13]. Similarly, Italian stock market has been found to exhibit very strong periodic behavior in volatility with a one trading day period in the MIB30 index [22]. Though there has been significant research efforts invested to recognize and justify the factors involved in periodicity behavior, till now there hasn't been any concrete finding that has evolved and enjoyed universal reception [13].

In our model, it can be argued that periodic volatility is just a result of periodic resetting of agents' virtual points. However, that's not the only aspect causing this behavior since the periodicity is actually more closely related to the agents' memory length rather than their time horizon. We have carried out simulation tests with different memory lengths for agents and have observed similar periodic behavior of volatility with periodicity of 2^{m+1} . For example in figure 5.8, with memory length of $m = 3$, we can see

repetition of volatility pattern at periodicity of 16, which is 2^{m+1} . This result is independent of choice of value of m and is consistent across all full spectrums of $T1$ and $T2$.

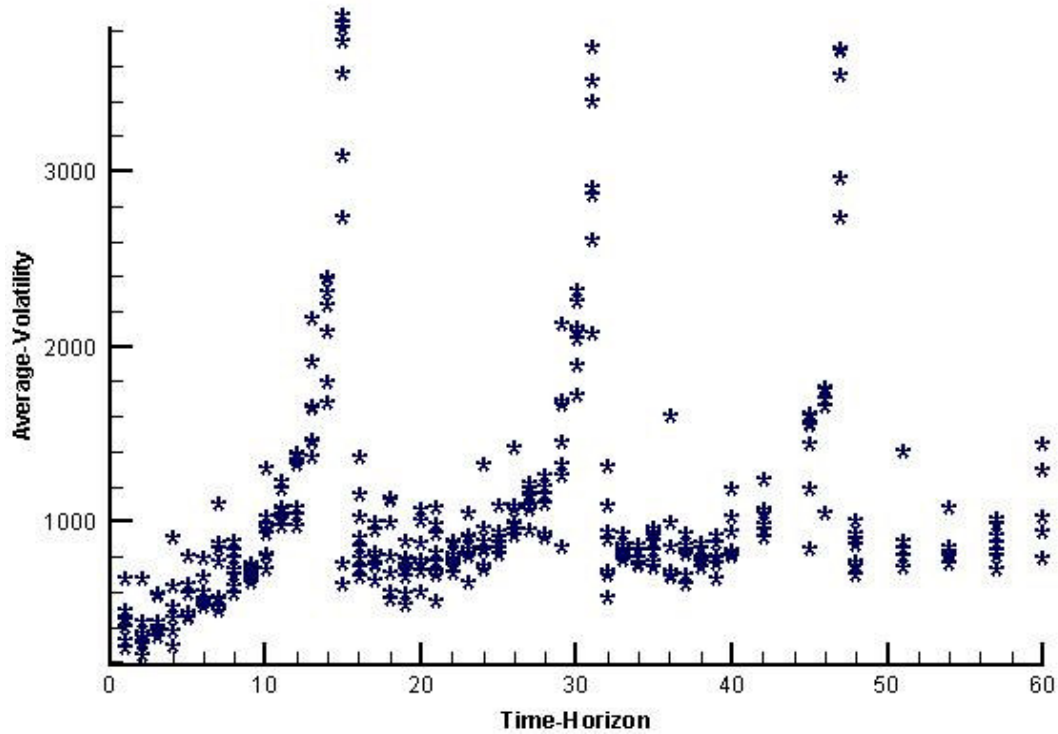


Figure 5.8 Impact of Time-Horizon on Average Volatility

Another argument that can be drawn here is that what's the need for resetting agents' virtual points and why don't agents just stop learning when they are performing optimally. The obvious reason is that there is no optimum strategy for sufficiently long duration of time since market is constantly evolving and all the agents continuously try adapting to it. The minority nature of the game plays an important part in ensuring that agents can't perform consistently well without adaptation and learning. However, with time horizon feature, agents tend to perform poorly for a short while when their virtual

points are reset and they start gathering knowledge about the market fresh again. One probable alteration to this could be to have agents give more weight to new data (virtual points assigned for the new time horizon) and lower weight to old data (virtual points from older time horizons). It's important to note that in original minority game model [5], there is no periodic resetting of agents' virtual points. Agents keep collecting points for their strategies through out the entire duration of the game. That means it acts as an infinite time horizon market model. In our model we speculate that time horizon and agents' memory act as a proxy to this factor and contribute to the periodicity in volatility. The original minority game model though can replicate stylized facts, fails to produce this interesting trait.

5.4.4 Results of Original MG with higher regime of memory & time-horizon

Figures 5.9 – 5.12 show the results for original MG [5], described in section 2.3 with higher memory and time horizon for agents.

Total number of agents	501
Memory of each agent	9
Time Horizon of each agent	72
Number of assigned strategies for each agent	6
Resource level	250
Simulation duration	1000 days

Table 5.3 Simulation Parameters for Original MG

Table 5.3 shows the parameters used for this simulation test. All other parameters except for memory and time horizon are same as in the original paper. The results, presented below, clearly show that the price series does not comply with real world market scenario and fails to generate stylized properties. For instance, price series in figure 5.9 shows some highly unrealistic values and fluctuations. The same holds true for return series as well, shown in figure 5.10. Figure 5.11 shows the volatility results. We can see some patches of volatility clustering in it but it's not consistent throughout the length of the simulation and has a few unreasonably long periods of low activity. Also, from figure 5.12 we can see that autocorrelation of return isn't following the stylized pattern. Decaying of autocorrelation isn't obvious and we can see some sudden spikes in it at certain places. Comparing these results with those of DJIA presented in figures (2.2) – (2.6) demonstrates great contradiction in terms of their inherent characteristics. These results support the claim that MG has limitation as financial market model in higher regime of memory and time horizon. The agents are, though, heterogeneous in terms of their strategies, effectively they all play with same parameters and tendency. That means all agents despite using different strategies have same memory, time horizon, number of strategies and common inclination to be in minority group. There are no agents in the market who are conceptually playing with different tendency. Thus their nature of heterogeneity is typically restricted, contributing to this limitation. In the next section, we will show that with our adaptive minority game, the model behaves much more efficiently even in higher regime of memory.

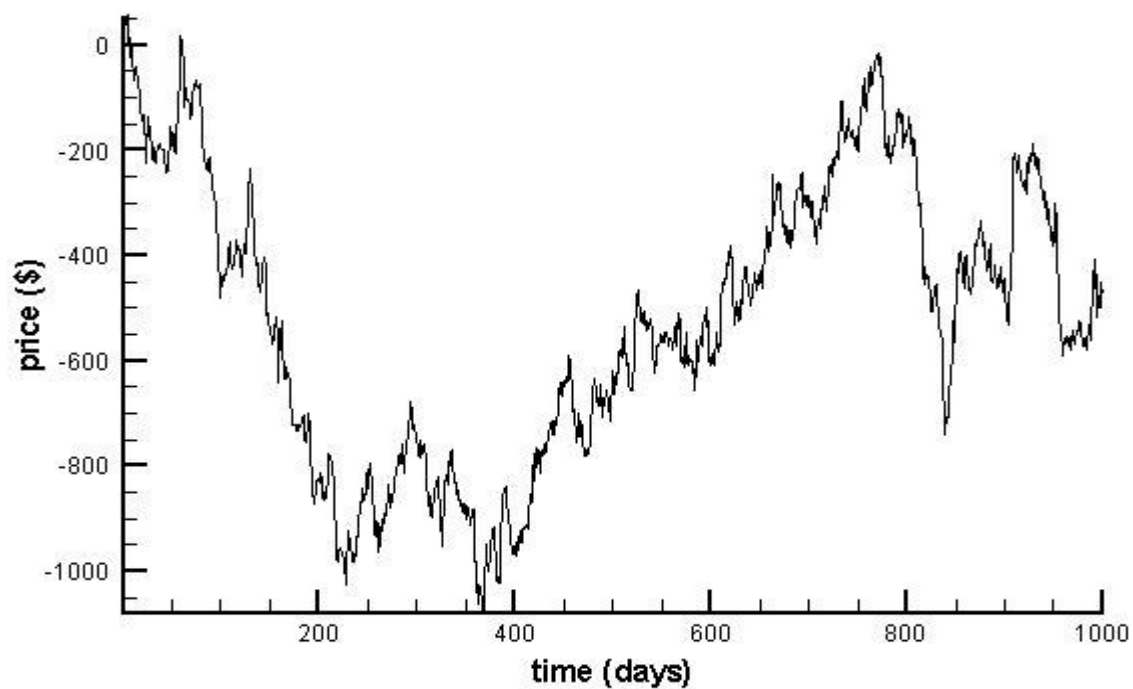


Figure 5.9 Price Series for Original MG with $m = 9$, $T = 72$

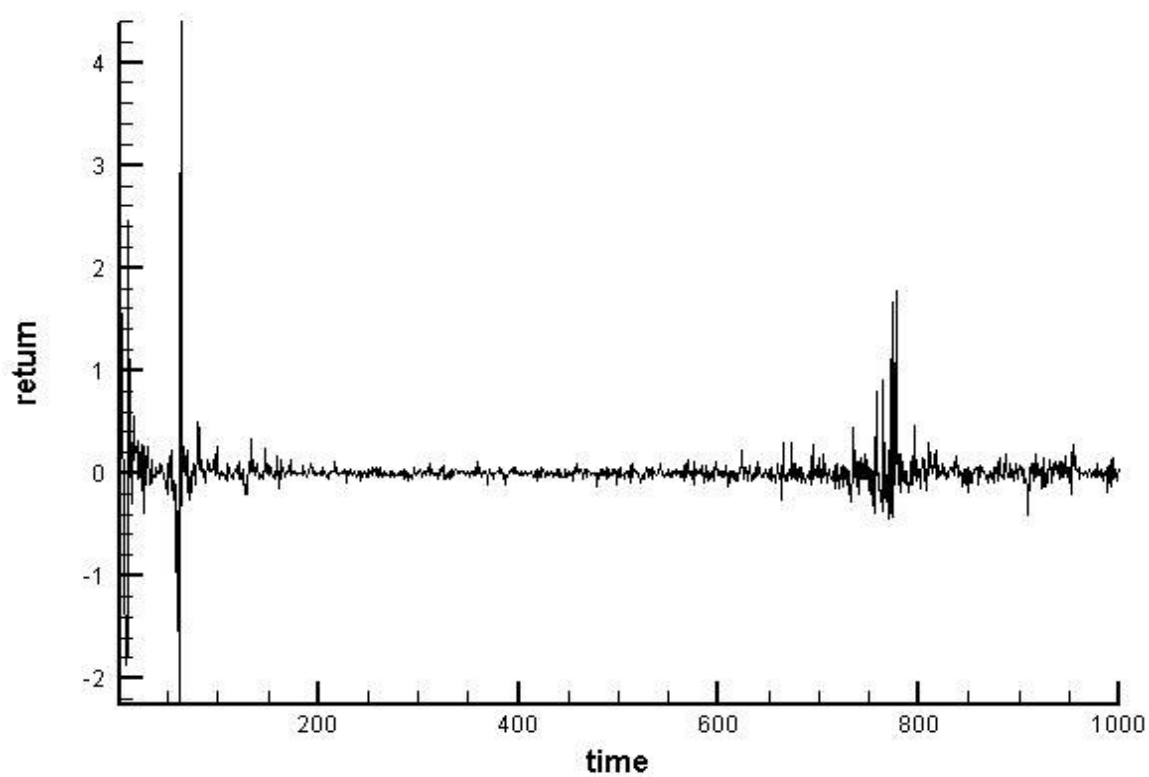


Figure 5.10 Return Price for Original MG with $m = 9$, $T = 72$

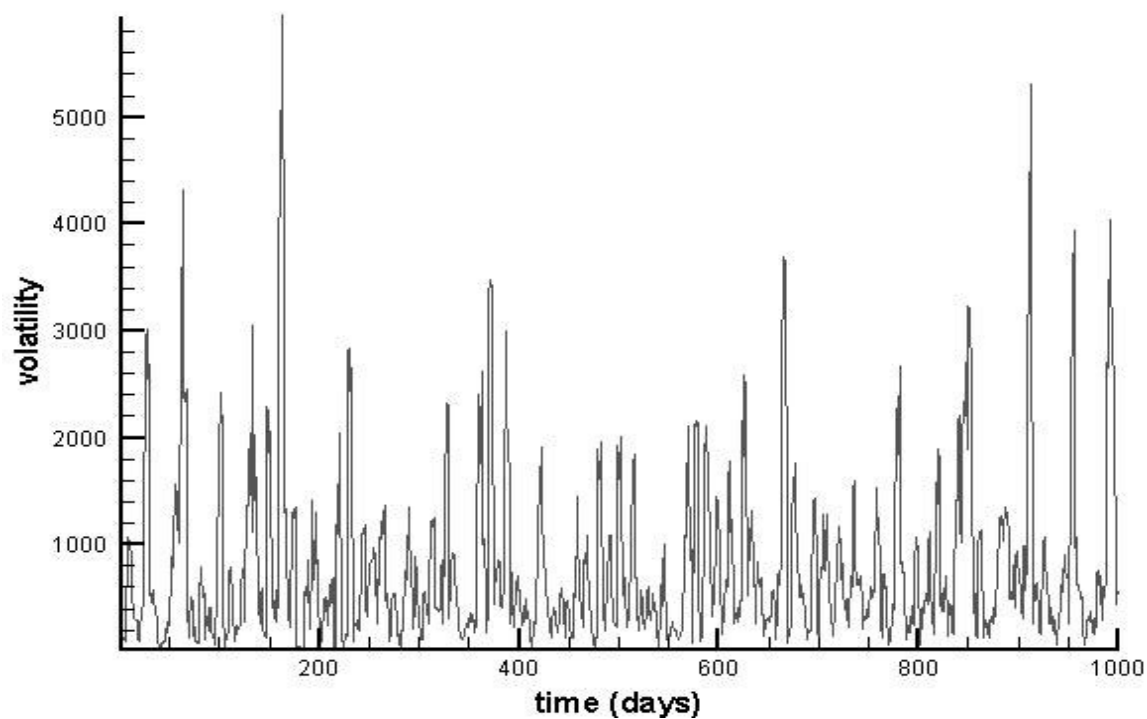
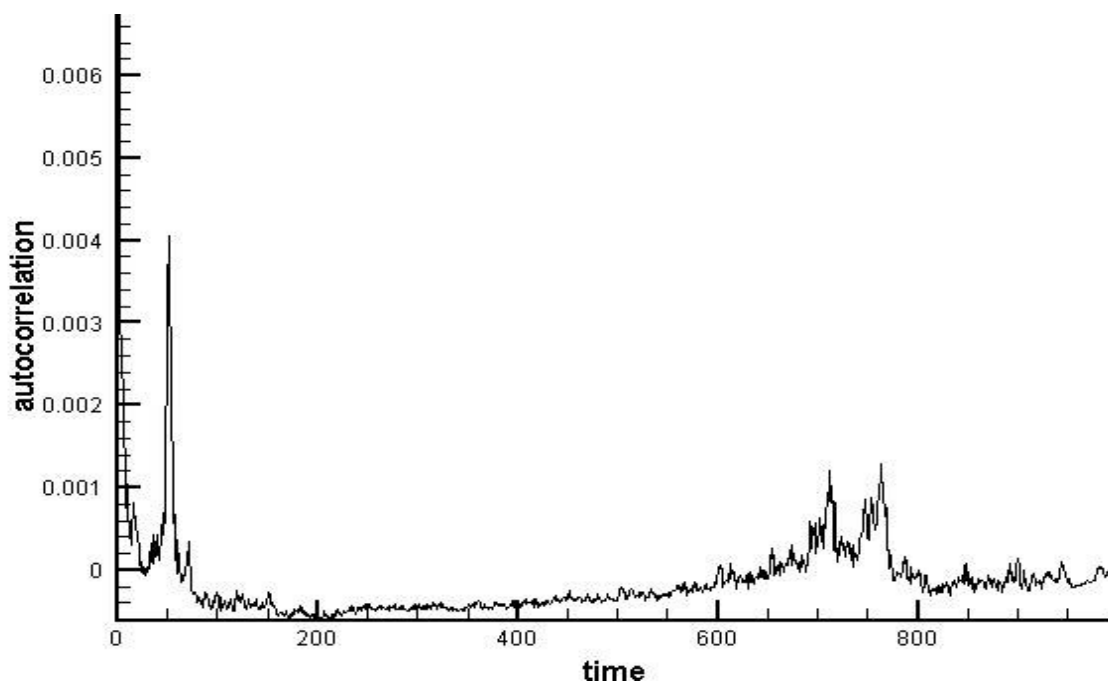


Figure 5.11 Volatility for Original MG with $m = 9$, $T = 72$



**Figure 5.12 Autocorrelation in Absolute Return for
Original MG with $m = 9$, $T = 72$**

5.4.5 Discussion Of Results

As we pointed out earlier, original MG can imitate financial market with very limited spectrum of parameters. Outside of this spectrum it fails to capture the vital characteristics of market considering the benchmarks that we discussed in section 5.3 for validating the result. There has been a small number of parameter sets for MG that can be validated to model financial markets [8]. We have attempted to expand this spectrum. For example, a number of studies have pointed out that with original MG model [5], the price series and returns are not in harmony with stylized facts for higher regime of memory length and time horizon [8, 10, 20]. From our tests of original MG with broad range of values for memory and time horizon, we have found that agent's memory typically should not be more than 7 and time horizon has to be lesser than 48. With higher range of memory and time horizon it fails to capture fat tail distribution and autocorrelation in return (see figure 5.12). Also, price series demonstrates highly unstable market as shown in figure 5.9.

In contrast to this, our adaptive minority game model can replicate the market's typical characteristics with relatively broad spectrum of parameters. Agent's memory can be as high as 16 whereas time horizon can vary from 8 to 172. We will show the results with these parameters in next section. Thus, our model does succeed to expand the spectrum, though still imposes a bound on it. We speculate the main reason for this limitation to be the payoff function of minority game (equation (2.6)). With significantly higher memory and time horizon, the payoff function in its original form inflicts barrier on agents' learning or adaptation rate. This can be considered as one of the intrinsic

limitation of MG. As we earlier discussed, agents with very high memory switch more often between their strategies, leading to a lot more random behavior from overall market perspective. Furthermore, the agents with very high time horizon are at high risk of being outperformed by other agents due to their slow learning rate.

5.5 Results of Randomized MG Pool Adaptive Model

Figures 5.13 – 5.18 show the results of our Randomized MG Pool Adaptive Model simulation, where agents are not restricted to groups in terms of their memory and time horizon. Agent can either be fundamentalist or minority game player. Agents are assigned memory and time horizon from the full spectrum possible for our model, which is 1 to 16 for memory and 8 to 172 for time horizon. All other parameters remain same as in table 5.1 and 5.2, except that the MG agents are not split in two pools. As the simulation results show, the model satisfactorily produces all the stylized facts with full spectrum as well. This is in sharp contrast to the results achievable with original MG model in higher regime of memory and time horizon shown in section 5.4.4.

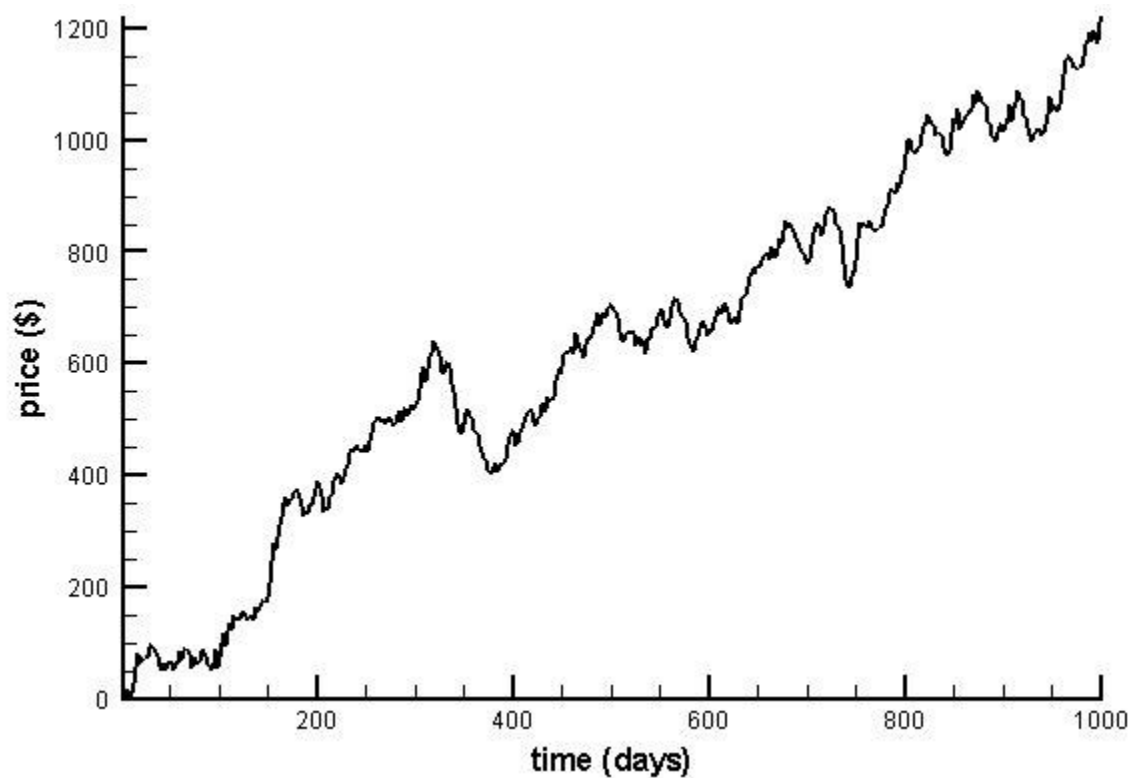


Figure 5.13 Price Series with Full Spectrum of Heterogeneity

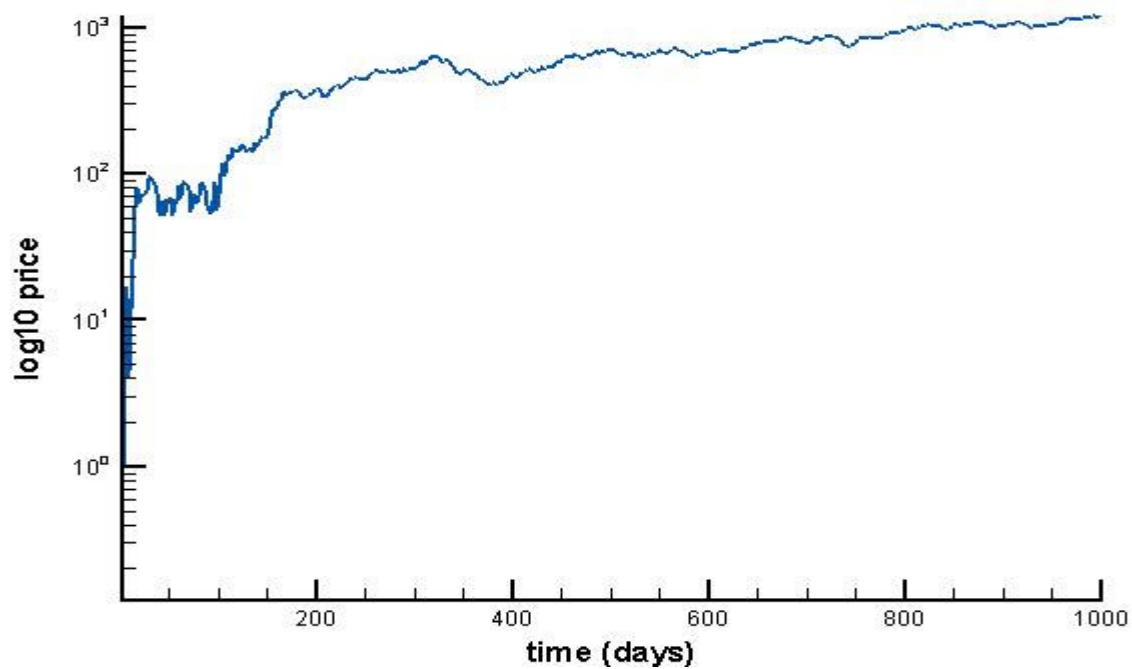


Figure 5.14 Logarithmic Price Series with Full Spectrum of Heterogeneity

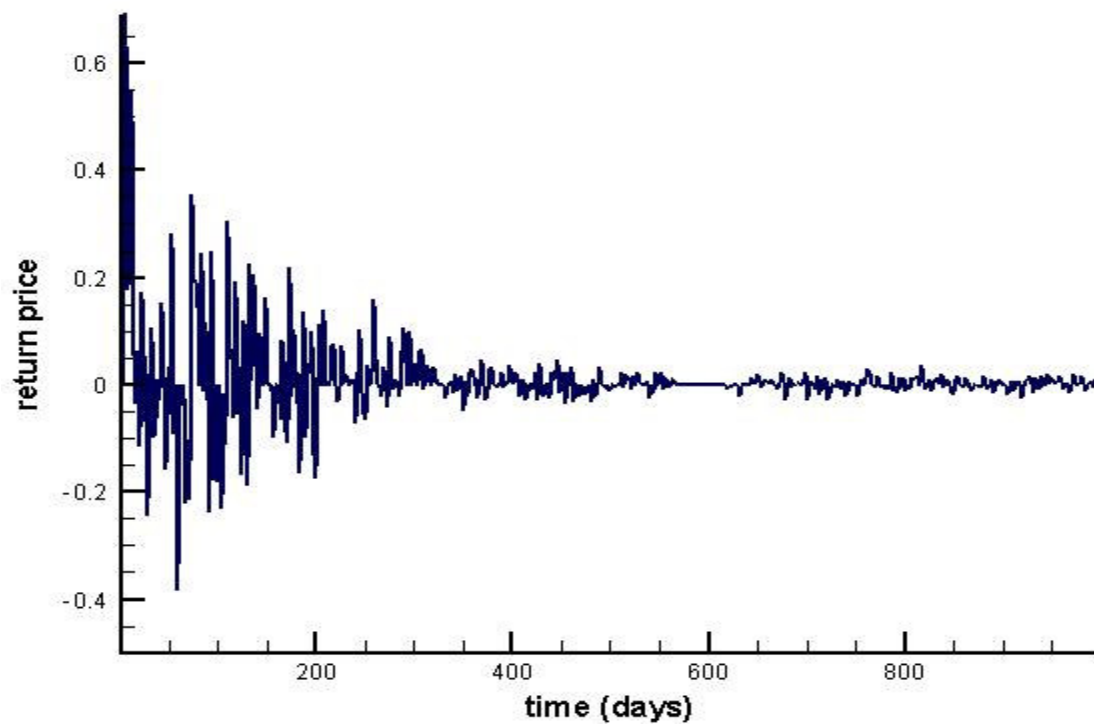


Figure 5.15 Return Price with Full Spectrum of Heterogeneity

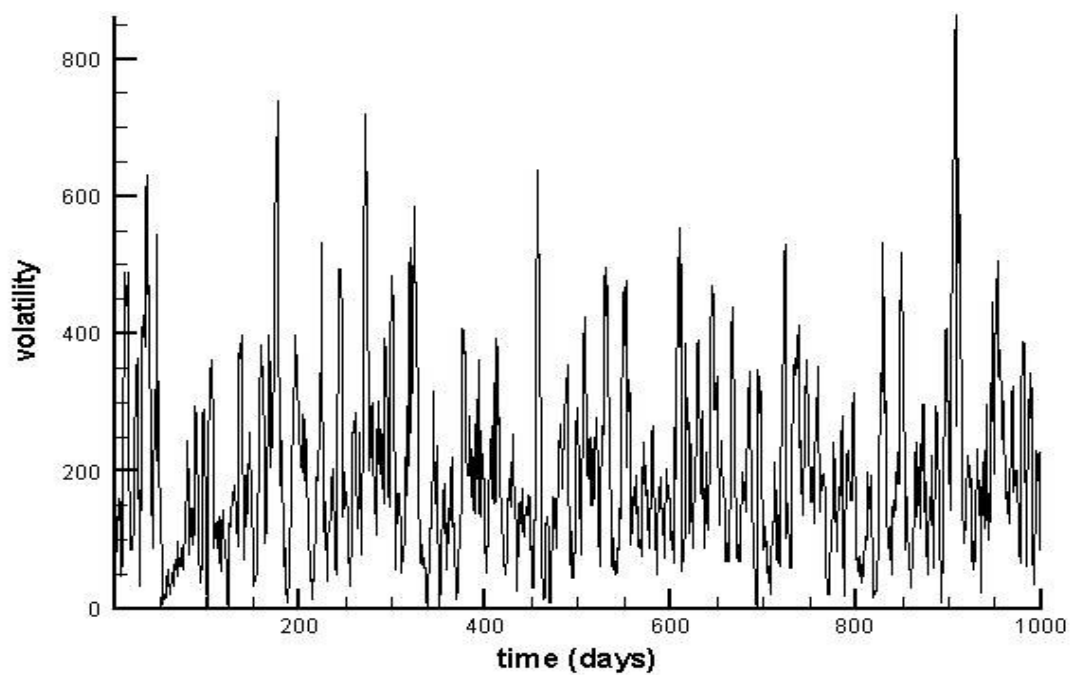


Figure 5.16 Volatility with Full Spectrum of Heterogeneity

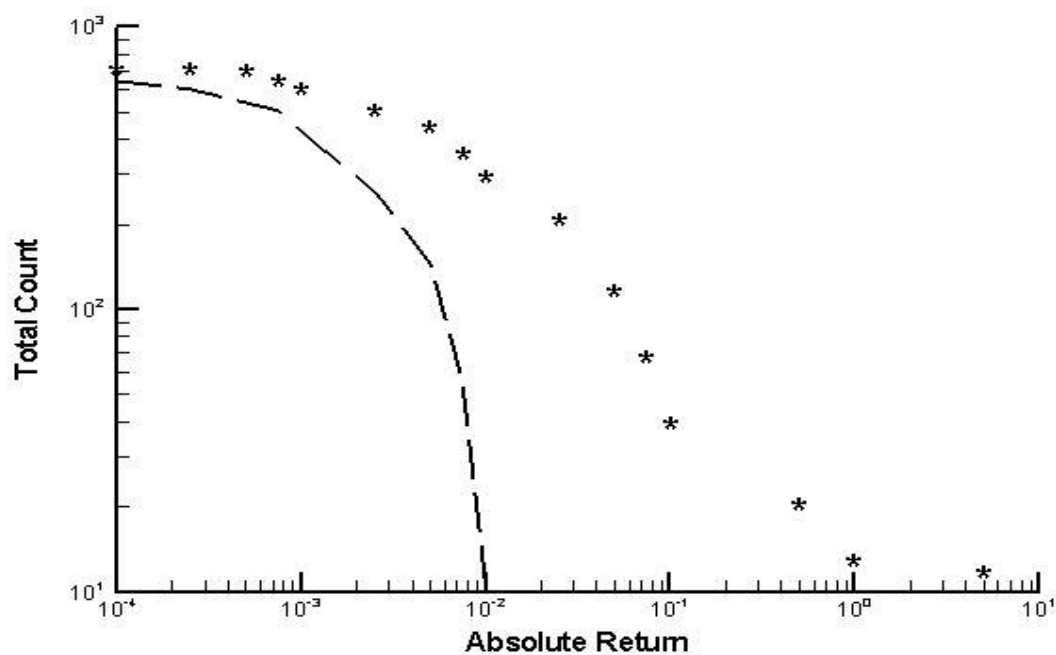


Figure 5.17 Distribution of Absolute Return

(dashed curve: gaussian distribution, *: absolute return)

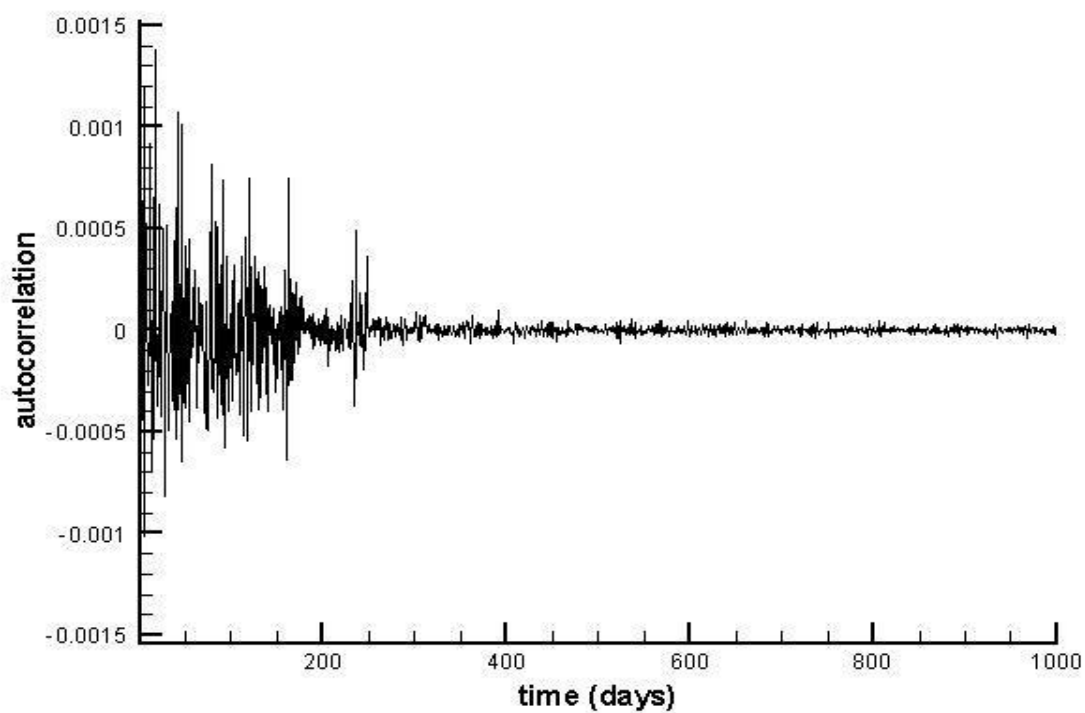


Figure 5.18 Autocorrelation in Return

5.6 Discussion of Results

Since now we have the simulation results of original MG model (section 5.4.4), Divided MG Pool Adaptive Model (section 5.4.1) and Randomized MG Pool Adaptive Model (section 5.5), we are better placed to have qualitative comparison of these models. From this comparison, we can draw that our model contributes to address the limitations L1, L2, L6, L7, L8 listed in section 2.6. If we compare figures 5.1 – 5.6 to figures 5.13 – 5.18, we can see that they are statistically very similar. This means that the new choice of parameters doesn't significantly affect the performance of our model and it exhibits much *more robustness* compared to original MG model over a wider range of memory and time horizon. This is an important improvement since there is often a need to model financial markets with variety of types of agents whose strategy, behavior, knowledge and information resources available at their disposal to analyze the market are highly diverse. The adaptive model presented here addresses these issues.

Introduction of *fundamentalists* in the market has added another dimension to the strategy aspect. Also, with agents having different memory lengths and time horizon expands the spectrum of their knowledge and analyzing ability. The agents' tendency to switch to more successful pools, and effectively strategy, makes the market scenario much more realistic with agents' interactions and adaptation. Periodic volatility feature discussed in section 5.4.3 is also a unique result producible with our model. Furthermore, our model provides us a better idea of how well agents perform with varying memory length, which is not very clear from the original MG model due to its limitation in higher memory regime.

To summarize, with *adaptive minority game* we can model the financial markets and reproduce the stylized facts with the following rules:

1. Total number of agents playing minority game should always be more than number of fundamentalists in the market. It doesn't matter how much more, as long as it's more, for the reason explained in section 5.3.
2. Memory length of minority game agents should typically be in the range of 2 to 16.
3. Time horizon should typically be in the range of 8 to 172. This range ensures a reasonable learning rate of agents. It's perfectly fine to choose time horizon outside this range as well, however very high value of it results in extremely slow learning rate for agents. These agents are easily outperformed by agents with time horizon in above-mentioned range.

Choice of other parameters in table 5.1 within bound of above mentioned rules will not significantly impact the model performance. However, parameters in table 5.2 are sensitive to change and would not necessarily produce similar results with modifications.

Chapter 6: Conclusion and Future Work

The core objective of this thesis was to develop a financial market model that can resemble to real world market and at the same time keep check on the assumptions that are made throughout the development of the model. Also we intended to ensure that the authenticity of these assumptions do not digress a great deal from actual market scenario. We had set up the stylized facts and certain other important characteristics observed in wide variety of markets as benchmark for validating results achieved by our model. We selected Minority Game toy model as a groundwork for our research and built upon that with certain alterations, which were subtle in nature but demonstrated rich characteristics. The main contribution of this research work is *improved heterogeneity* aspect of agents in MG market model while we have tried to *curtail some of the unreasonable assumptions*. The original MG model assumes that: (1) memory length of all the agents in market is same, (2) all agents operate in infinite time horizon and (3) fundamental value of asset has no impact on market operations. We have got rid of these assumptions in our model and agents were populated with different memory lengths and time horizons. Also we added some fundamentalists to the market and allowed agents to switch their trading strategies, which contributed in improving heterogeneity of agents.

The simulation tests that we performed with wide range of parameters successfully reproduced vital stylized facts such as fat tail distribution, volatility clustering and absence of autocorrelation in return. The model succeeds to explain the basic foundation of these stylized facts and parameters that contribute to each of them.

Furthermore we demonstrated *enhancement to the spectrum of parameters*, which can be used to model financial market. That means we can choose from much wider spectrum of memory length and time horizon than what's possible with original MG model. The simulation also captured a very important property of *periodic volatility with varying time horizon*. Finally we studied the impact of various memory lengths on agent's success rate and found that agents with relatively larger memory tend to perform better, but there is a certain threshold beyond which increase in memory length doesn't improve agents' success rate. On the contrary, the success rates of agents degrade with memory larger than this threshold and they perform poorly in longer run.

Future work on this line could include:

- Agents trading at different frequencies and different volumes. Both in original MG model and our model, all agents trade one unit of stock in each time step. The model can be extended with “hold” option for the agents apart from buy and sell. For this, some conditions or threshold needs to be decided that would have agents hold from participating in bid if that condition is not met. Thus agents' heterogeneity could further be improved and more randomness can be brought into the market.
- As we pointed out in section 5.4.5, more efficient payoff function for the game needs to be researched in order to further expand the spectrum of parameters with which the model can be validated.

- A different function can be tried for periodic resetting of agents' virtual points at the end of each time horizon. One probable solution is to assign more weight to the new data and lesser weight to the old data.
- The concept of various market clearing mechanisms could be brought into modeling and how it would impact agent's strategy and adaptation. Market clearing mechanism means that the fundamental value of the asset is changed until the supply and demand for the asset exactly match and the market can be cleared of any excess supply or demand. The Grand-Canonical MG [10] has already tried incorporating this concept in market model.

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