SYSTEM RELIABILITY ESTIMATION AND COMPONENT REPLACEMENT ANALYSIS FOR ELECTRICITY TRANSMISSION AND DISTRIBUTION SYSTEMS

by

JOSE FRANCISCO ESPIRITU NOLASCO

A Dissertation submitted to the

Graduate School-New Brunswick

Rutgers, The State University of New Jersey

in partial fulfillment of the requirements

for the degree of

Doctor of Philosophy

Graduate Program in Industrial and Systems Engineering

Written under the direction of

Dr. David W. Coit

and approved by

____

____

____

____

New Brunswick, New Jersey

October, 2007

ABSTRACT OF THE DISSERTATION

System Reliability Estimation and Component Replacement Analysis

for Electricity Transmission and Distribution Systems

by JOSE FRANCISCO ESPIRITU NOLASCO

Dissertation director: Dr. David W. Coit

This PhD dissertation focuses on the development of mathematical methods that can be readily applied to obtain the reliability of Electricity Transmission and Distribution Systems (ETDS) and to determine long-term component replacement strategies for aging ETDS components. This work has devoted research efforts to develop electric power reliability models that can be used to accurately approximate the system outage rate, average repair time, and expected system downtime of ETDS configurations used by the power industry for different types of outages. Additionally, new component criticality importance measures have been developed. Several existing popular reliability criticality importance measures (e.g., Birnbaum, Reliability Achievement Worth) cannot be directly applied to these power systems, because they have been developed mainly for components with specified finite mission times. Alternatively, for ETDS, the different components within the system exhibit outage rates and repair rates instead of probability of failure for a specified time interval.

Most of the U.S. power grid was built in the early 1960s. ETDS are often built with redundancy to minimize the number and duration of interruptions. They have been

operating reliably in the past, but as equipment ages, it fails more frequently and it becomes economically important to plan the expensive replacements and/or restorations of aging equipment.

Determining the planned retirement of aged equipment in the ETDS is an important research area, because the aged equipment is continuously used until it fails. It can take more than one year to complete the whole replacement process of some critical components. The component replacement analysis method proposed is based on an integrated iterative dynamic programming and integer programming approach. This method works under the consideration of heterogeneous assets with different ages subject to annually budget constraints.

The method developed can be applied to systems composed with sets of heterogeneous assets. This is a new solution methodology that offers distinct benefits to previous methods, which only pertained directly to a system composed of homogeneous assets. This research leads to many research contributions specific to ETDS. However, the replacement analysis model represents a novel approach that can be applied to many types of systems and problems.

Acknowledgements

Life is like a book filled with many chapters, stories and major characters, and now I am about to close one of the most important chapters in my life, one that has not been easy but I have really enjoyed. Like in many books, this is just a summary of some of the most important characters of this chapter. Please do not blame me if I do not mention your name, it is just that it is almost impossible for me to mention all the people that made my life more enjoyable during my PhD studies.

First, I would like to specially thank my advisor Dr. David W. Coit for his constant support and expert advise. Dr. Coit, you are a very bright person and also a very good friend. I am so glad I chose you as my advisor. I also would like to thank Dr. James T. Luxhøj for his invaluable advise and confidence. You are the most enthusiastic person that I met during my studies. I also extend my gratitude to Dr. Elsayed A. Elsayed and Dr. Frank Felder for their comments and opinions to improve the quality of my dissertation.

During these years at Rutgers, I had the opportunity to meet wonderful people, but I would like to specially mention my dear friends Erol Zeren, Frank Chen and Abdullah Karaman. Thank you for your support and priceless companion. I know that life is going to take us to different roads but I am sure we will meet in the same destination called "success." I wish you all the best.

Life is a constant learning process, one keeps learning from the best valuable people. I would like to mention a person that I deeply admire, Dr. Susan Albin, from whom I

have learned how to face adversity in life, with fearlessness and courage. I will continue to pray for you and ask God to renew your strength daily.

I would like to thank my beloved parents. If I were going to be born again, I would choose you again as my parents. Thank you for your constant love, encouragement and help throughout my whole career. Maribel, Miriam and Rosaura, I love you so much.

Juan and Efi, thanks for being such a nice family and considering me part of it, I will always be grateful to you. I hope you know that you can count on me for anything.

Undoubtedly, the most important character in my life's book is my wife Heidi, who has always been a constant source of support in the good and in the bad moments. I am sure we will keep growing together, professionally and personally. I hope you know that you were, are and will be my first real true love. I love you.

Jose F. Espiritu

July 19, 2007

Table of Contents

List of Tables

List of Illustrations

Chapter 1

Introduction

This thesis is concerned with the development of new mathematical models to estimate the reliability of electricity transmission and distribution systems. Moreover, new reliability importance measures are also developed in order to rank the importance of each component in the system, and finally, a new component replacement analysis model, based on combined integer programming and dynamic programming was developed and applied to some commonly used electricity transmission and distribution network configurations.

Our modern society is dependent on a cost-effective reliable electric power supply, since electricity is an essential service that contributes to prosperity and quality of life. Therefore, maintaining a highly reliable power supply is a very important factor for power systems design and operation. In order to provide high quality electricity supply at a reasonable cost, a well-designed, efficiently operated and maintained, reliable transmission and distribution network is required. Reliability is a key aspect of power system design and planning. The concept of power system reliability is extremely broad and covers all aspects of the ability of the system to satisfy the customer requirements.

Unreliable electric power supplies can be extremely costly to electric utilities and their customers. The economic and social effects of loss of electrical service can have significant impacts on both the utility supplying electric energy and the end-users of electrical service. According to SGI Federal (2003), the cost of a major power outage confined to one state can be on the order of tens of millions of dollars per day. If a major power outage affects multiple states, then the cost can exceed 100 million dollars. The power system is vulnerable to system abnormalities such as control failures, protection or communication system failures, and disturbances, such as lightning, and human operational errors. Therefore, maintaining a reliable power supply is a very important issue for power systems design and operation.

In the past years, several surveys related to the estimation of large-scale customer interruption costs have been developed by utilities in the USA and Canada. The results of these surveys have been applied in a number of areas of utility planning, including transmission line design (Dalton *et al*., 1996), distribution circuit design (Williams & Ochoa, 1995), and substation design (Vojdani *et al*., 1996).

A power system can be mainly divided in three parts which are (1) generation, (2) transmission, and (3) distribution. Generation refers to the production of electricity. Transmission networks are the link that connects generators of electricity with the distribution system, and the final retail customers of electricity are connected through distribution networks. The point of connection between a distribution network and a transmission network is often described as a bulk supply point.

For power systems, the number and duration of supply interruptions characterize the continuity of supply (Sand *et al*., 2004). It is neither technically nor economically feasible for a power system to ensure that electricity is continuously available on demand. Instead, the basic function of a power system is to supply power that satisfies the system load and energy requirement, as economically as possible, at acceptable levels of continuity and quality. Voltage quality is usually measured in terms of acceptable values of voltage (i.e., voltage level, harmonics, voltage dips, etc.), while continuity of supply refers to uninterrupted electricity supply service.

Power quality includes both voltage quality and continuity of supply. Reliability refers to the ability of a power system to provide an adequate and secure supply of electrical energy at any point in time. Within the power systems industry, the term "reliability" is often used to describe what is traditionally defined as "availability."

Part of this research is concerned with the development and extension of mathematical models that can be used to accurately approximate the reliability of actual electricity transmission/distribution network configurations used by the power industry. The results then can be used for assessment of proposed modifications to an existing industrial distribution system configuration to minimize the costs of interruptions to both the utility and its industrial customers. Later, new importance criticality measures are developed, that can be used to prioritize the most important components in the system to indicate where the main investments should be made in order to increase the availability of the system. Another extension of the present research covers the timely component replacement analysis of several commonly used electricity transmission and distribution configurations.

1.1 Motivation

System reliability (Elsayed 1996; Ayyub, 2003) is related to the probability that a product or service operates properly for a specified period of time under the design operating conditions without failure. In standard reliability theory, this probabilistic perspective has been generally used to model and analyze the reliability of a product or service. Reliability related measures such as availability, mean time to failure, criticality importance measures, etc., are also based on such probabilistic perspective.

In the power industry (Billinton & Allan, 1983; Billinton & Allan, 1984; Billinton & Li, 1994, etc.), the function of an electric power system is to satisfy the system load requirement with a reasonable assurance of continuity and quality. Thus, reliability is mainly related to the ability of the system to provide an adequate and uninterrupted supply of electrical energy to the final customers and is used for evaluation of system availability. However, reliability related measures in this industry are fundamentally different to those used in traditional reliability practice. When considering electric power system reliability, researchers and analysts are interested in how component outages and repair rates affect the associated overall system outage rates and downtimes. Therefore, the primary interest is devoted to quantify system failure impacts at the system-level. This quantification is then translated to system expected outage rates, mean outage duration (repair time) and overall downtime.

Due to the different perspectives that traditional and electric power systems maintain with regard to system reliability evaluation, the first phase of this work has devoted research efforts to extend existing electric power reliability models for transmission and distribution configurations as long as failure definition for the models can be specified.

Many of the electric utilities were built in the early 1960s and they are still using the original equipment since they were built. As components fail, they are replaced, but the system is still aging. There is an increased risk of suffering power outages due to the prolonged use of this aging equipment. Thus, an important aspect for decision-makers and managers is to know which components in the system are the most important such that the right investments can be made in order to increase the system reliability.

Due to economic constraints, investments to upgrade aging systems need to be made appropriately and intelligently. Therefore, a company or utility needs to know the reliability importance or importance ranking of the components within the system. Thus, there is a need for new quantitative criticality measures that can be directly applied to electricity transmission and distribution systems, such that managers and other decision-makers have useful metrics to evaluate where investments could be made in order to improve the functioning and reliability of the overall system.

A third extension of the existing research work proposes a capital replacement analysis method for planning for the timely replacement of the different components of several commonly used electricity transmission and distribution configurations. For this task, a heterogeneous group of components with vastly differing outage costs can be collectively and simultaneous analyzed to determine a planned multiple year-horizon replacement plan. The existing replacement analysis literature for "fleets" of assets only pertains to homogeneous assets and can not be applied for these systems. The developed model represents a new advancement.

1.2 Background

Prior to the 1960s, the reliability of power systems was often estimated by extrapolating the experience obtained from existing systems and using rule-of-thumb methods to forecast the reliability of new systems. During the 1960s, considerable work was performed in the field of power system reliability. The most significant publications were two company papers (Gaver *et al*., 1964; Montmeat *et al*., 1965) by a group of

Westinghouse Electric Corporation and Public Service and Gas Company. These papers introduced the concept of a fluctuation environment to describe the failure rate of transmission system components.

The techniques presented in these papers were approximations, which provided results within a few percent of those obtained using more theoretical techniques, such as Markov processes. The application of Markov chains in the power system reliability field was initially illustrated in Billinton & Bollinger (1968). The Markov approach is limited in application because it becomes more difficult to apply and computationally prohibitive as the number of components in the systems increases. For actual problems and actual systems, the use of Markov chains has been severely limited because of this computational complexity. Broadwater *et al*. (1994) presented the first application to apply linked lists and pointer concepts to reliability analysis. It took into account constraints associated with switching operations, but it was relatively slow due to running numerous power flow calculations. Montmeat *et al*. (1965) took into account constraints associated with switching operations, but it was relatively slow due to running numerous power flow calculations.

1.3 Research contributions

There are several research contributions that are developed in the present work. The first one is the development of mathematical recursive equations to estimate the system availability for electricity transmission and distribution systems. The approximations were developed for four different types of overlapping outages, (component sustained outages overlapping other component sustained outages, component sustained outages overlapping component maintenance outages, component sustained outages overlapping component transient outages). All of the approximations developed were implemented in an Excel-based software for the evaluation of electricity transmission and distribution systems, such as the breaker-and-a-half, the breaker-and-a-third, etc.

The second main research contribution is the development of new criticality importance measures, such that they can be directly applied to power systems. These new importance measures can be directly applied by managers of the power system industry to rank the importance of the different components in the system such that directions for improvement of the system can be identified.

The third, and major, contribution of this thesis is the development of a capital replacement analysis method based on combined dynamic programming and integer programming that can be applied for any application with heterogeneous assets and actual constraints on replacement costs. The recursive equations, criticality importance measures and the replacement analysis method are applied to some commonly used electricity transmission and distribution systems by the power industry.

1.4 Proposal organization

In **Chapter 2,** an overview of different types of power generation is presented. The characteristics of the electricity transmission/distribution and some common configurations used by the power industry are introduced, and a discussion of the different types of single component outages and overlapping outages that this type of systems can experience is presented.

Chapter 3 presents the Markov modeling of repairable electricity transmission and distribution systems and components, which is used to obtain exact solution for the different metrics of the different electric configurations. The continuous time Markov chain technique is applied to the evaluation of the reliability of electricity transmission systems. Although theoretically sound and valid, the method becomes cumbersome when the system is composed of a large number of components. Therefore, it is necessary to develop approximate equations that can accurately approximate the availability of the system for large systems without having to solve complex Markov chains.

In **Chapter 4,** equations to approximate the system outage rate, average repair time and expected downtime are developed. The main objective in this chapter is to develop electric power system reliability equations to accurately estimate the system outage rate, average repair time and expected downtime, for any electricity transmission/distribution system configuration. The new models are extensions of commonly applied reliability estimation models. The new models developed, can be used for any power utility to obtain an approximation of the system reliability.

The mathematical expressions developed consider: (*i*) component sustained outages overlapping component sustained outages, (*ii*) component sustained outages overlapping component maintenance outages, and (*iii*) component temporary outages overlapping component sustained outages.

In **Chapter 5**, different existing reliability importance measures are proposed. Previously developed criticality or importance measures (e.g., Birnbaum, Fussell-Vesely) are generally very useful but they cannot be directly applied in the ETDS area because these methodologies were developed on the assumption that there is a definite time period of interest (or mission time) for in the system. In the area of ETDS, the different components of the system have no definite time period and the system is expected to work endlessly without any failure. In addition to this, component and system "reliability" is expressed in terms of outage rates rather than probability of failure. In the present work, some of the pre-existing importance criticality measures have been transformed to be readily applied in the ETDS area. These new extended measures pertain to the outage rate of the component instead of the probability of failure. The proposed metrics are applied to different typical system configurations in the power industry such as breaker-and-a-half and the dual element spot network (DESN) design configurations.

In **Chapter 6**, a component replacement analysis methodology is developed and applied to radial electricity distribution systems. The proposed method is a novel technique which requires solving iteratively dynamic and integer programming programs to obtain the optimal replacement schedules for different types of electricity transmission and distribution configurations. The objective function is to minimize the net present value of unmet demand (unreliability), maintenance and purchase costs subject to annual budgetary constraints.

The method developed in Chapter 6 cannot be readily applied to complex ETDS because direct estimation of each component opportunity cost cannot be directly obtained given that opportunity costs are neither linear nor separable. Therefore, in **Chapter 7**, a Taylor series expansion model is developed to determine the opportunity costs associated with each component in the system. Then, the method is applied to more complex ETDS, such as the DESN.

Finally, in **Chapter 8**, future research is presented. New component cost-based criticality importance measures will be developed to consider reliability metrics commonly used by the power industry, such as the system average interruption frequency index (SAIFI) and the customer average interruption duration index (CAIDI). Also, the development and application of a multi-objective evolutionary algorithm to the power systems area will be performed. The objectives to be considered can potentially be the multi-state stationary availability, the expected multi-state capacity, the expected unsupplied demand, and the loss of load probability.

Chapter 2

Power systems overview

The main purpose of an electric power system is to provide energy to final customers, as economically as possible and with a reasonable assurance of continuity and quality. An electric power system can be mainly divided in three major components which are (1) the generation system, (2) a high voltage transmission grid, and (3) the distribution system. The high voltage transmission system links generators to substations, which supply power to the user through the distribution system. Interruptions in these connecting links can potentially disrupt the flow of power from generators to end-users. In the following sections, a brief description of the main functional parts of a power system is presented.

2.1. Power generation

According to the United States Department of Energy (DOE), the USA operates a fleet of approximately 10,000 power plants and the average thermal efficiency is around 33%. Power plants are generally long-lived investments, and the majority of the existing capacity is 30 or more years old.

A power station is a facility for the generation of electric power. At the center of nearly all power stations there is a generator, i.e., a rotating machine that converts mechanical energy into electrical energy by creating relative motion between a magnetic field and a conductor. The energy source harnessed to turn the generator varies widely. It depends on what fuels are available and the types of technology that the power company has access to. There are different ways to generate electric power including hydroelectric, thermal, nuclear, solar and wind.

2.2. Electricity transmission

The electric power transmission systems are among the most complex networks and the largest systems that exist in the world. Transmission systems often traverse a long distance to transport the energy over various networks to load points. A common design strategy for improving the reliability of power supply is by parallel redundancy of transmission lines. However, multiple transmission line outages can alter the transmission system operating configuration and, possibly, result in supply interruptions to a large number of customers.

The electric power transmission is one process in the delivery of electricity to consumers. It refers to the “bulk” transfer of electrical power from place to place. Typically, power transmission is between the power plant and a substation in the vicinity of a populated area. This is distinct from electricity distribution, which is concerned with the delivery from the substation to the consumers. Due to the large amount of power involved, transmission normally takes place at high voltage (110 kV or above). Electricity is usually sent over long distance through overhead power transmission lines.

A power transmission system is sometimes referred as a "grid" in which redundant paths and lines are commonly present. Power can be directed from any power plant to any load center, through a variety of routes, based on the economics of the transmission path and the cost of power.

Currently, transmission-level voltages are usually considered to be 110 kV and above. Lower voltages such as 66 kV and 33 kV are usually considered sub-transmission voltages but are occasionally used on long lines with light loads. Voltages less than 50 kV are usually used for distribution. According to the Department of Energy (DOE) the

USA operates about 157,000 miles of high voltage (>230kV) electric transmission lines and it is estimated that power outages and power quality disturbances cost the economy from $25 to $180 billion annually. These costs could increase if outages become more frequent or longer in duration as components age and fail more frequently.

2.3 Electricity distribution

The transition from electricity transmission to electricity distribution usually occurs at the substation. Substations take power from transmission-level voltages and distribute it to lower voltage distribution lines. The distribution system is generally considered to begin at the substation and end at the customer's meter.

The distribution system is an important part of the total electrical supply system, as it provides the final link between a utility’s bulk transmission system and its customers. It has been reported that 80% of all customer interruptions occur due to failures in the distribution systems (Chowdhury & Koval, 1998). Electricity distribution is generally considered to include medium-voltage (less than 50 kV) power lines, low-voltage electrical substations and pole-mounted transformers, and low-voltage (less than 1,000 V) distribution wiring.

2.4 Electricity transmission/distribution network configurations

There are many types of distribution/transmission networks that can be used to supply customers (Gonen, 1986), but electrical configurations can be broadly classified into two basic groups: radial or interconnected. Figure 2.1 shows a radial design. In this system, a single incoming power service is received and distributes power to the facility; there is no duplication of equipment and, failure of any one component in the series path between the source node and the load point results in a power interruption to all loads downstream of the failed component. This type of configuration is typical of long rural lines with isolated load areas.

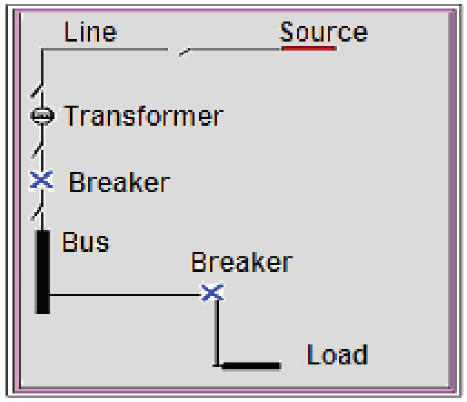

Figure 2.1 Radial configuration

An interconnected (meshed type) network is shown in Figure 2.2. This is generally found in urban areas and is used for facilities requiring a more reliable power supply. This configuration has multiple connections to other points of supply. These points of connection are normally open but allow various configurations by closing and opening switches. The breaker-and-a-half, breaker-and-a-third and breaker-and-a-fourth are some examples of these types of configurations (Billinton & Satish, 1995; Kasztenny *et al*., 2004; Satish & Billinton, 1995). The benefit of the interconnected model is that, in the event of a fault or required maintenance, a small area of network can be isolated and the remainder can continue providing a supply of power to the end-user.

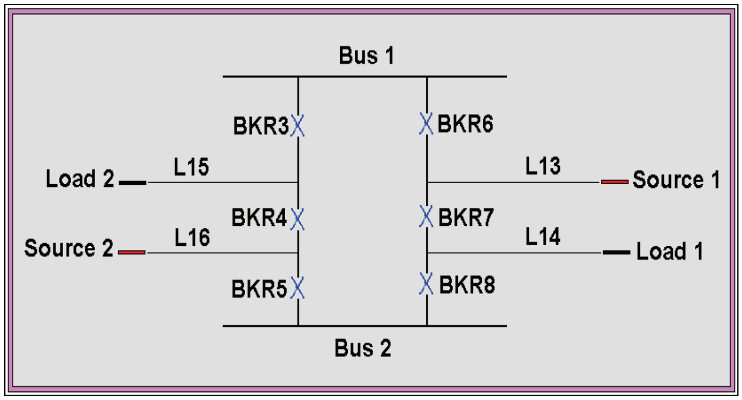

Figure 2.2 Breaker-and-a-Half configuration – Two Diameters

According to the OSHA, the main characteristics that distinguish transmission lines from distribution lines are that transmission lines are operated at relatively high voltages. Transmission lines transmit large quantities of power, and they transmit the power over large distances.

2.5. Substations

A substation is a high-voltage electric system facility. It is used to switch generators, equipment, and circuits or lines in and out of a system. It is also used to change AC voltages from one level to another, and/or change alternating current to direct current or direct current to alternating current. Some substations are small with little more than a transformer and associated switches. Others are very large with several transformers and dozens of switches and other equipment. There are four main types of substations:

• **Step-up transmission substations** receive electric power from a nearby generating facility and use a large power transformer to increase the voltage for transmission to distant locations.

• **Step-down transmission substations** are located at switching points in an electrical grid. They connect different parts of a grid and are a source for subtransmission lines or distribution lines.

• **Distribution substations** are located near the end-users. Distribution substation transformers change the transmission or subtransmission voltage to lower levels for use by end-users.

• **Underground distribution substations** are also located near the end-users. Distribution substation transformers change the subtransmission voltage to lower levels for use by end-users.

According to the USA DOE, there are approximately 10,287 transmission substations and 2,179 distribution substations. Transmission substations use transformers to convert or increase a generator's voltage for long distance transmission, while the distribution substation steps power down to distribution voltage levels and splits it into many directions. In short, substations are critical components of the distribution system, and a loss of only 4% of transmission substations would result in a 60% loss of connectivity.

2.6. Types of outages

Each component of the ETDS can fail or be associated with an outage in several different ways. It is beneficial to treat these different categories of outages differently. For instance, separation of events is important if they have different effects on the system or are associated with very different restoration processes and costs. Three particular outage events are sustained, temporary and scheduled (maintenance).

- **Permanent or sustained outages** are associated with damaged faults requiring the component to be repaired or replaced.
- **Transient or temporary outages** are associated with undamaged faults that are restored by manual/automatic switching or fuse replacement.

- **Scheduled or maintenance outages** are outages which are planned in advance in order to perform preventive maintenance, and in general, a scheduled outage is not performed (or considered in the reliability evaluation) if, by this action alone, the load point is disconnected.

Each of these types of outages can be included in the reliability assessment procedure. In the present research, four types of outages are considered. These are related with overlapping outages and are applied to typical electricity transmission and distribution configurations currently used by the power industry. An explanation of each of these types of failures is demonstrated by using a simple parallel system shown in Figure 2.3. In the example each of the outage types is occurring in accordance with a homogeneous Poisson process.

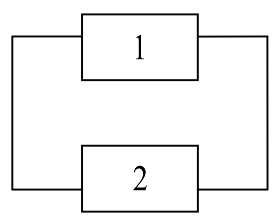

Figure 2.3 Two component parallel system

Notation:

λ_1, λ_2;	Sustained outage rates for components 1 and 2
r_1, r_2;	Average sustained repair time/outage for components 1 and 2
$\tilde{\lambda}_1, \tilde{\lambda}_2$;	Scheduled outage rates for components 1 and 2
$\tilde{r}_1, \tilde{r}_2$;	Average maintenance repair time/outage for components 1 and 2
λ_1^t, λ_2^t;	Transient outage rates for components 1 and 2
r_1^t, r_2^t;	Average transient repair time/outage for components 1 and 2

2.6.1 Component sustained outages overlapping component sustained outages

An overlapping sustained outage for the system in Figure 2.3 occurs when the first component experienced a sustained outage (at rate λ_1) and during the repair time of component 1 (r_1), component 2 has a sustained outage (at rate λ_2), or when component number 2 has a sustained outage (at rate λ_2) and during its repair time (r_2), component number 1 has a sustained outage (at rate λ_1).

2.6.2 Component sustained outages overlapping component maintenance outages

Considering a two component parallel system, if component 1 is down due to a scheduled outage (at rate $\tilde{\lambda}_1$) and during its maintenance time ($\tilde{r}_1$), component 2 has a sustained outage (at rate λ_2), then this called a sustained outage overlapping component maintenance outage. Alternatively, if component 2 is unavailable due to a scheduled outage (at rate $\tilde{\lambda}_2$) and during component 2 maintenance time ($\tilde{r}_2$) component 1 has a sustained outage (at rate λ_1), it is also an outage of this type.

In the operation of electricity transmission and distribution systems, it is important to note that when the system is first weakened by a sustained outage, the policy followed is that the healthy component is never taken out of service for a scheduled maintenance action because it will cause a loss of power. Similarly for multiple component (more than two) system structures, preventive maintenance is never performed (even if scheduled) if it leads to a power interruption.

2.6.3 Component transient outages overlapping component sustained outages

In a system consisting of two components arranged in parallel as in Figure 2.3, this type of overlapping outage occurs when a component in the system has a transient or sustained outage while another component is being repaired because it had either a sustained or transient outage, e.g., when the first component has a sustained outage (at rate λ_1) and during the repair time (r_1), the second component has a transient outage (at rate λ_2^t), or when the first component has a transient outage (at rate λ_1^t) and during its transient repair time (r_1^t) the second component has a sustained outage (at rate λ_2).

2.7 Conclusions

In the present chapter, an overview of power systems is presented. Definition of the different outages was made, and a simple two component parallel system was used to demonstrate the different types of overlapping outages. In practice, the reliability estimation of electricity transmission and distribution systems for any configuration, can be approximated based on minimal cut-sets. For these systems, exact solutions are preferable for fixed designs (also still difficult) but when optimizing or determining replacement policies, accurate approximations are required because of computational issues. Next, Chapter 3 introduces the Markov modeling of repairable components, which is used to obtain exact solution for the different metrics of the different electric configurations.

Chapter 3

Modeling of overlapping outages

Continuous time Markov chain techniques can be applied to evaluate the reliability of electricity transmission systems (Ross, 2002, 2003; Resnick, 2002). However, the method is generally not efficient when the system is composed of a large number of components. Therefore, it is also necessary to develop approximate equations that can very accurately approximate the availability of the system without having to solve complex continuous time Markov chains. This is particularly important when determining an optimal design architecture or when planning replacement policies over an extended planning period.

The failure probabilities and frequencies for the combinations of outages described in Section 2.5 can be computed for the different types of electric configurations in the reliability modeling of ETDS. By using continuous time Markov chains, failure event probability and frequency can be obtained using state transition matrices as in Medicherla *et al*. (1994). Another alternative is to develop a suitable set of expressions from the Markov models as in Vohra *et al*. (1987). These expressions can be stored and utilized for all first-order and second-order cut-sets. This is advantageous because it avoids the repetitive computation of the state transition matrix in solving for a number of events in a power system consisting of a number of station configurations.

Chan & Asgarpoor (2005) present a method to find the optimum maintenance policy for a component. Using Markov processes, the state probabilities are calculated and the optimal value of the mean time to preventive maintenance was obtained by maximizing the availability of a single component with respect to the mean time to minimal preventive maintenance.

3.1 Component sustained outages overlapping component sustained outages

3.1.1 Markov model for two repairable components

A Markov model is presented for sustained outages overlapping component sustained outages for two components; each component can be in either the up-state or the down-state. Let λ_1, λ_2, μ_1 and μ_2 be the sustained outage rates and repair rates for components 1 and 2. Outages and repairs occur as a homogeneous Poison process. The Markov chain model assumes constant outage and repair rates, and exponentially distributed time-between failures and repair times.

In this case, each of the components can be in one of two states, either working or failed. There are two components, and thus, there are 2^2 or 4 possible states in which the system can exist. These are enumerated in Table 3.1.

Table 3.1. Space diagram states

State	Component 1	Component 2
1	Up	Up
2	Down	Up
3	Up	Down
4	Down	Down

The corresponding state space diagram is shown in Figure 3.1. It is important to mention that in the model, a transfer from state 1 and 4 or between states 2 and 3, is not possible because such transfers require two simultaneous changes in the states of the components involved. The probabilities of such simultaneous occurrences are assumed to be negligibly small.

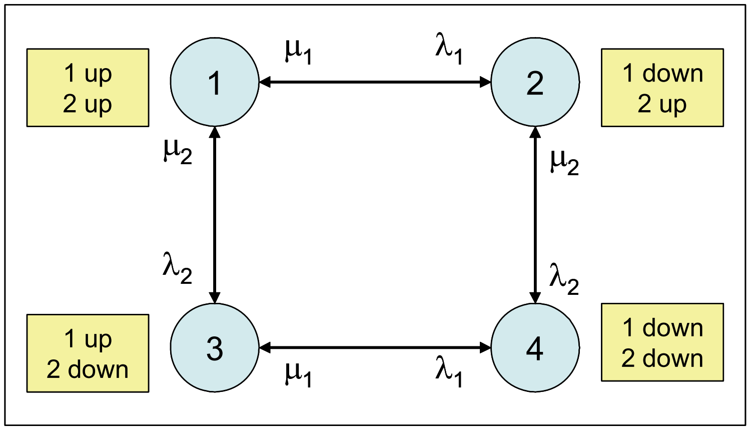

Figure 3.1. State-space diagram for two different repairable components

For this case, the ρ-matrix, stochastic transitional probability matrix (**P**) and the Markov differential equations, in vector-matrix notation, are as follows:

$$\rho = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \\ S_4 \end{array} \begin{array}{c} \begin{array}{cccc} S_1 & S_2 & S_3 & S_4 \end{array} \\ \begin{bmatrix} 0 & \lambda_1 & \lambda_2 & 0 \\ \mu_1 & 0 & 0 & \lambda_2 \\ \mu_2 & 0 & 0 & \lambda_1 \\ 0 & \mu_2 & \mu_1 & 0 \end{bmatrix} \end{array}$$

$$\mathrm{P} = \begin{bmatrix} 1-(\lambda_1+\lambda_2) & \lambda_1 & \lambda_2 & 0 \\ \mu_1 & 1-(\lambda_2+\mu_1) & 0 & \lambda_2 \\ \mu_2 & 0 & 1-(\lambda_1+\mu_2) & \lambda_1 \\ 0 & \mu_2 & \mu_1 & 1-(\mu_1+\mu_2) \end{bmatrix}$$

$$\begin{bmatrix} P_1'(t) \\ P_2'(t) \\ P_3'(t) \\ P_4'(t) \end{bmatrix} = \begin{bmatrix} -(\lambda_1+\lambda_2) & \mu_1 & \mu_2 & 0 \\ \lambda_1 & -(\lambda_2+\mu_1) & 0 & \mu_2 \\ \lambda_2 & 0 & -(\lambda_1+\mu_2) & \mu_1 \\ 0 & \lambda_2 & \lambda_1 & -(\mu_1+\mu_2) \end{bmatrix} \begin{bmatrix} P_1(t) \\ P_2(t) \\ P_3(t) \\ P_4(t) \end{bmatrix}$$

Then we have the following set of equations:

$$P_1'(t) = -(\lambda_1+\lambda_2)\ P_1(t) + \mu_1 P_2(t) + \mu_2 P_3(t) \tag{3.1}$$

$$P_2'(t) = \lambda_1\ P_1(t) - (\lambda_2+\mu_1) P_2(t) + \mu_2 P_4(t) \tag{3.2}$$

$$P_3'(t) = \lambda_2 P_1(t) - (\lambda_1 + \mu_2)P_3(t) + \mu_1 P_4(t) \tag{3.3}$$

$$P_4'(t) = \lambda_2 P_2(t) + \lambda_1 P_3(t) - (\mu_1 + \mu_2)P_4(t) \tag{3.4}$$

The steady-state probabilities can be computed by the simultaneous solution of

$$\alpha \mathrm{P} = \alpha \tag{3.5}$$

where, $\alpha = \begin{bmatrix} P_1 & P_2 & P_3 & P_4 \end{bmatrix}$, and $P_1 + P_2 + P_3 + P_4 = 1$

Assuming the system starts in state 1, $P_1(0) = 1, P_2(0) = P_3(0) = P_4(0) = 0$, the solutions are the following set of Equations (3.6 - 3.9):

$$P_1(t) = \frac{\mu_1\mu_2}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)} + \frac{\lambda_2\mu_1 e^{-((\mu_2+\lambda_2)t)}}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)} + \frac{\lambda_1\mu_2 e^{-((\mu_1+\lambda_1)t)}}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)} + \frac{\lambda_1\lambda_2 e^{-((\mu_1+\lambda_1+\lambda_2+\mu_2)t)}}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)} \tag{3.6}$$

$$P_2(t) = -\frac{\lambda_1\mu_2 e^{-((\mu_1+\lambda_1)t)}}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)} - \frac{\lambda_2\lambda_1 e^{-((\mu_2+\lambda_2+\mu_1+\lambda_1)t)}}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)} + \frac{\lambda_1\mu_2}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)} + \frac{\lambda_1\lambda_2 e^{-((\lambda_2+\mu_2)t)}}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)} \tag{3.7}$$

$$P_3(t) = \frac{\lambda_1\lambda_2 e^{-((\mu_1+\lambda_1)t)}}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)} - \frac{\lambda_2\mu_1 e^{-((\mu_2+\lambda_2)t)}}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)} + \frac{\lambda_2\mu_1}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)} - \frac{\lambda_1\lambda_2 e^{-((\mu_1+\lambda_1+\lambda_2+\mu_2)t)}}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)} \tag{3.8}$$

$$P_4(t) = \frac{\lambda_1\lambda_2 e^{-((\mu_1+\lambda_1+\lambda_2+\mu_2)t)}}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)} - \frac{\lambda_2\lambda_1 e^{-((\mu_2+\lambda_2)t)}}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)} + \frac{\lambda_1\lambda_2}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)} - \frac{\lambda_1\lambda_2 e^{-((\mu_1+\lambda_1)t)}}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)} \tag{3.9}$$

Taking the limit as t approaches ∞ in Equations 3.6 - 3.9, the steady-state availabilities in the system are:

$$P_1 = \frac{\mu_1\mu_2}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)} \tag{3.10}$$

$$P_2 = \frac{\lambda_1\mu_2}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)} \tag{3.11}$$

$$P_3 = \frac{\mu_1 \lambda_2}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)} \tag{3.12}$$

$$P_4 = \frac{\lambda_1 \lambda_2}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)} \tag{3.13}$$

Based on the steady state solutions for the system, equations to compute system outage rate, average repair duration, and total system downtime for components connected in series and in parallel can be obtained. The Markov chain analysis results are general and can be used for different system structures (series, parallel) depending on the mapping of systems states to outcomes.

3.1.1.1 Case 1: components connected in series

Considering the case when two repairable components are connected in series as in Billinton & Li (1994) and Billinton & Allan (1984). The steady-state probability of both components being in operating condition is given by Equation 3.10. To obtain the outage rates and repair rates for the system, it is necessary to first obtain the outage rates and repair rates of a single component that is equivalent to the two components connected in series in the diagram shown in Figure 3.2. Thus, the probability of the single component being in the up-state can be obtained.

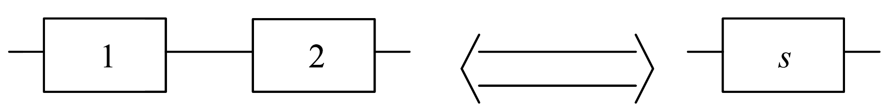

Figure 3.2. Components connected in series

For the equivalent component, the steady-state probability of being in the good state (up) is,

$$P_1 = \frac{\mu_s}{\lambda_s + \mu_s} \tag{3.14}$$

For the single component to be equivalent to the two series components, Equations 3.10 and 3.14 must be identical, thus,

$$\frac{\mu_s}{\lambda_s + \mu_s} = \frac{\mu_1 \mu_2}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)} \tag{3.15}$$

Rearranging and solving for μ_s yields,

$$\mu_s = \frac{\lambda_s \mu_1 \mu_2}{\lambda_1 \lambda_2 + \lambda_1 \mu_2 + \lambda_2 \mu_1} \tag{3.16}$$

Expressing Equation 3.16, in terms of mean repair times r_1, r_2, and r_s where

$$r_1 = \frac{1}{\mu_1}, \quad r_2 = \frac{1}{\mu_2} \quad \text{and} \quad r_s = \frac{1}{\mu_s} \tag{3.17}$$

And substituting equation 3.17 into 3.16 we obtain the average repair time for two components connected in series

$$r_s = \frac{\lambda_1 r_1 + \lambda_2 r_2 + \lambda_1 r_1 \lambda_2 r_2}{\lambda_1 + \lambda_2} \tag{3.18}$$

From the above equation, we can say that for component 1, the number of outages per unit time is λ_1, and every time the component is down, it takes on average, r_1 time units to repair. $\lambda_1 r_1$ is also an approximation of the fraction of the time the component 1 is down for $\lambda_1 r_1 << 1$. When $\lambda_1 r_1$ and $\lambda_2 r_2$ is small ($\lambda_1 r_1 << 1$ and $\lambda_2 r_2 << 1$), Equation 3.18 reduces to:

$$r_s = \frac{\lambda_1 r_1 + \lambda_2 r_2}{\lambda_1 + \lambda_2} \tag{3.19}$$

The system outage rate for two components connected in series is

$$\lambda_s = \lambda_1 + \lambda_2 \tag{3.20}$$

The expected system downtime can then be approximated, as in Billinton & Allan (1983) as,

$$U_s = \lambda_1 r_1 + \lambda_2 r_2 \tag{3.21}$$

3.1.1.2 Case 2: components connected in parallel

In the case where the components are connected in parallel, the system fails only if both components fail. From the equations derived from the Markov model, Equation 3.13 corresponds to the case when the system is down (state 4). The steady state probability P_4 can be set equal to the unavailability for the parallel two-component system as follows,

$$P_4 = \frac{\lambda_p}{\lambda_p + \mu_p} \tag{3.22}$$

Therefore,

$$\frac{\lambda_p}{\lambda_p + \mu_p} = \frac{\lambda_1 \lambda_2}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)} \tag{3.23}$$

$$\lambda_p = \frac{\mu_p \lambda_1 \lambda_2}{\lambda_1 \mu_2 + \lambda_2 \mu_1 + \mu_1 \mu_2} \tag{3.24}$$

Since repairing either component brings up the system to the working state, the equivalent repair rate is equal to the sum of the two individual repair rates (for exponentially distributed repair times). That is,

$$\mu_p = \mu_1 + \mu_2 \tag{3.25}$$

Combining equations 3.24 and 3.25;

$$\lambda_p = \frac{(\mu_1 + \mu_2)\lambda_1 \lambda_2}{\lambda_1 \mu_2 + \lambda_2 \mu_1 + \mu_1 \mu_2} = \frac{(r_1 + r_2)(\lambda_1 \lambda_2)}{1 + \lambda_1 r_1 + \lambda_2 r_2} = \frac{\lambda_1 \lambda_2 r_1 + \lambda_1 \lambda_2 r_2}{1 + \lambda_1 r_1 + \lambda_2 r_2} \tag{3.26}$$

In the case of two components connected in parallel, for component 1, the number of failures per unit time is λ_1, and every time the component is down, it takes an average, r_1 time units to repair. Therefore, $\lambda_1 r_1$ is a close approximation to the fraction of time the component is down. For highly reliable components, as in the case of electricity transmission systems, this number is very small. Similarly, $\lambda_2 r_2$ is also small ($\lambda_1 r_1 << 1$ and $\lambda_2 r_2 << 1$), Then we can express Equation 3.26 as the following approximation to obtain the system outage rate.

$$\lambda_p \approx \lambda_1 \lambda_2 r_1 + \lambda_1 \lambda_2 r_2 = \lambda_1 \lambda_2 (r_1 + r_2) \tag{3.27}$$

The average repair time and the system downtime (unavailability) can be computed as,

$$r_p = \frac{1}{\mu_p} = \frac{1}{\mu_1 + \mu_2} = \frac{r_1 r_2}{r_1 + r_2} \tag{3.28}$$

$$U_p = \lambda_p r_p \tag{3.29}$$

3.1.2 Markov model for three repairable components

The following is a Markov model for three different repairable components. Each component can be in either the up-state or the down-state. As in the previous example, let λ_1, λ_2, λ_3, μ_1, μ_2 and μ_3 be the outage rates and repair rates for the three components. Each of the components can be in one of two states, and since there are three components, there are 2^3 or 8 possible states. The total numbers of states in the system are shown in Table 3.2.

Table 3.2. Possible states for a three component repairable system

State	Component 1	Component 2	Component 3
1	Up	Up	Up
2	Down	Up	Up
3	Up	Down	Up
4	Up	Up	Down
5	Down	Down	Up
6	Down	Up	Down
7	Up	Down	Down
8	Down	Down	Down

Representation of the states is represented as a state transition diagram in Figure 3.3. The diagram illustrates and enumerates all the possible system states and also shows the transition modes from one state to another. Again, as in the previous example, transfers from states 2, 3 and 4 to state 8 or between states 5, 6 and 7 to state 1, are not allowed because such transfers require two simultaneous changes in the states of the components involved. The probabilities of such simultaneous occurrences are assumed to be negligibly small.

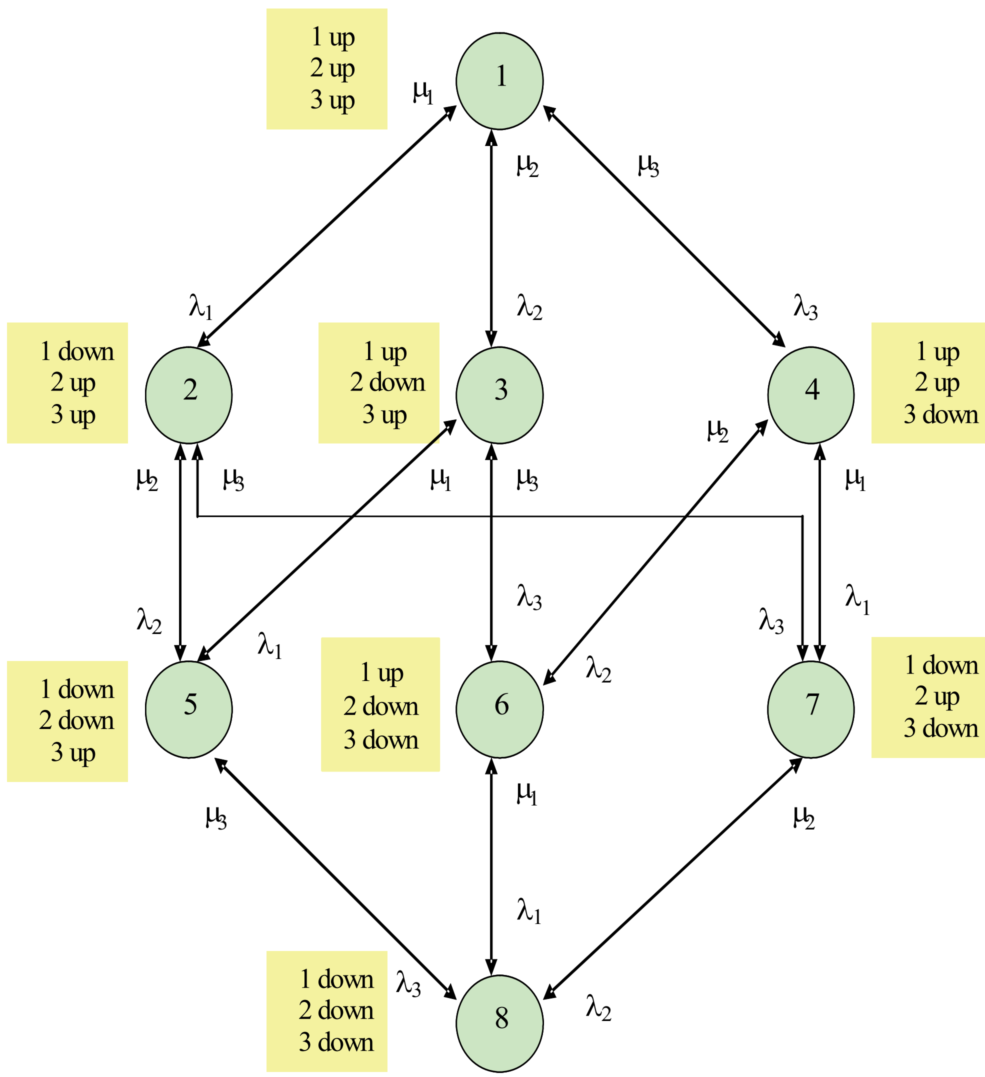

Figure 3.3. State-space diagram for three different repairable components

The ρ-matrix, the stochastic transitional probability matrix (**P**) and the Markov differential equations, in vector-matrix notation, are:

$$
\rho = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \\ S_7 \\ S_8 \end{array}
\begin{array}{c}
\begin{array}{cccccccc} S_1 & S_2 & S_3 & S_4 & S_5 & S_6 & S_7 & S_8 \end{array} \\
\begin{bmatrix}
0 & \lambda_1 & \lambda_2 & \lambda_3 & 0 & 0 & 0 & 0 \\
\mu_1 & 0 & 0 & 0 & \lambda_2 & 0 & \lambda_3 & 0 \\
\mu_2 & 0 & 0 & 0 & \lambda_1 & \lambda_3 & 0 & 0 \\
\mu_3 & 0 & 0 & 0 & 0 & \lambda_2 & \lambda_1 & 0 \\
0 & \mu_2 & \mu_1 & 0 & 0 & 0 & 0 & \lambda_3 \\
0 & 0 & \mu_3 & \mu_2 & 0 & 0 & 0 & \lambda_1 \\
0 & \mu_3 & 0 & \mu_1 & 0 & 0 & 0 & \lambda_2 \\
0 & 0 & 0 & 0 & \mu_3 & \mu_1 & \mu_2 & 0
\end{bmatrix}
\end{array}
$$

$$
P = \begin{bmatrix}
1-(\lambda_1+\lambda_2+\lambda_3) & \lambda_1 & \lambda_2 & \lambda_3 & 0 & 0 & 0 & 0 \\
\mu_1 & 1-(\lambda_2+\lambda_3+\mu_1) & 0 & 0 & \lambda_2 & 0 & \lambda_3 & 0 \\
\mu_2 & 0 & 1-(\lambda_1+\lambda_3+\mu_2) & 0 & \lambda_1 & \lambda_3 & 0 & 0 \\
\mu_3 & 0 & 0 & 1-(\lambda_2+\lambda_1+\mu_3) & 0 & \lambda_2 & \lambda_1 & 0 \\
0 & \mu_2 & \mu_1 & 0 & 1-(\lambda_3+\mu_2+\mu_1) & 0 & 0 & \lambda_3 \\
0 & 0 & \mu_3 & \mu_2 & 0 & 1-(\lambda_1+\mu_3+\mu_2) & 0 & \lambda_1 \\
0 & \mu_3 & 0 & \mu_1 & 0 & 0 & 1-(\lambda_2+\mu_1+\mu_3) & \lambda_2 \\
0 & 0 & 0 & 0 & \mu_3 & \mu_1 & \mu_2 & 1-(\mu_1+\mu_2+\mu_3)
\end{bmatrix}
$$

$$
\begin{bmatrix} \dot{P}_1(t) \\ \dot{P}_2(t) \\ \dot{P}_3(t) \\ \dot{P}_4(t) \\ \dot{P}_5(t) \\ \dot{P}_6(t) \\ \dot{P}_7(t) \\ \dot{P}_8(t) \end{bmatrix} =
\begin{bmatrix}
-(\lambda_1+\lambda_2+\lambda_3) & \lambda_1 & \lambda_2 & \lambda_3 & 0 & 0 & 0 & 0 \\
\mu_1 & -(\lambda_2+\lambda_3+\mu_1) & 0 & 0 & \lambda_2 & 0 & \lambda_3 & 0 \\
\mu_2 & 0 & -(\lambda_1+\lambda_3+\mu_2) & 0 & \lambda_1 & \lambda_3 & 0 & 0 \\
\mu_3 & 0 & 0 & -(\lambda_2+\lambda_1+\mu_3) & 0 & \lambda_2 & \lambda_1 & 0 \\
0 & \mu_2 & \mu_1 & 0 & -(\lambda_3+\mu_2+\mu_1) & 0 & 0 & \lambda_3 \\
0 & 0 & \mu_3 & \mu_2 & 0 & -(\lambda_1+\mu_3+\mu_2) & 0 & \lambda_1 \\
0 & \mu_3 & 0 & \mu_1 & 0 & 0 & -(\lambda_2+\mu_1+\mu_3) & \lambda_2 \\
0 & 0 & 0 & 0 & \mu_3 & \mu_1 & \mu_2 & -(\mu_1+\mu_2+\mu_3)
\end{bmatrix}
\begin{bmatrix} P_1(t) \\ P_2(t) \\ P_3(t) \\ P_4(t) \\ P_5(t) \\ P_6(t) \\ P_7(t) \\ P_8(t) \end{bmatrix}
$$

Then we have the following set of balance equations:

$$(\lambda_1 + \lambda_2 + \lambda_3)\ P_1(t) = \mu_1 P_2(t) + \mu_2 P_3(t) + \mu_3 P_4(t) \qquad (3.30)$$

$$(\mu_1 + \lambda_2 + \lambda_3)\ P_2(t) = \lambda_1 P_1(t) + \mu_2 P_5(t) + \mu_3 P_7(t) \qquad (3.31)$$

$$(\lambda_1 + \mu_2 + \lambda_3)\ P_3(t) = \lambda_2 P_1(t) + \mu_1 P_5(t) + \mu_3 P_6(t) \qquad (3.32)$$

$$(\lambda_1 + \lambda_2 + \mu_3)\ P_4(t) = \lambda_3 P_1(t) + \mu_2 P_6(t) + \mu_1 P_7(t) \tag{3.33}$$

$$(\mu_1 + \mu_2 + \lambda_3)\ P_5(t) = \lambda_2 P_2(t) + \lambda_1 P_3(t) + \mu_3 P_8(t) \tag{3.34}$$

$$(\lambda_1 + \mu_2 + \mu_3)\ P_6(t) = \lambda_3 P_3(t) + \lambda_2 P_4(t) + \mu_1 P_8(t) \tag{3.35}$$

$$(\mu_1 + \lambda_2 + \mu_3)\ P_7(t) = \lambda_1 P_4(t) + \lambda_3 P_2(t) + \mu_2 P_8(t) \tag{3.36}$$

$$(\mu_1 + \mu_2 + \mu_3)\ P_8(t) = \lambda_3 P_5(t) + \lambda_1 P_6(t) + \lambda_2 P_7(t) \tag{3.37}$$

and

$$P_1 + P_2 + P_3 + P_4 + P_5 + P_6 + P_7 + P_8 = 1 \tag{3.38}$$

Solving the balance equations and computing the steady-state availability for each state in the system we have

$$P_1 = \frac{\mu_1 \mu_2 \mu_3}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)(\lambda_3 + \mu_3)} \tag{3.39}$$

$$P_2 = \frac{\lambda_1 \mu_2 \mu_3}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)(\lambda_3 + \mu_3)} \tag{3.40}$$

$$P_3 = \frac{\lambda_2 \mu_1 \mu_3}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)(\lambda_3 + \mu_3)} \tag{3.41}$$

$$P_4 = \frac{\lambda_3 \mu_1 \mu_2}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)(\lambda_3 + \mu_3)} \tag{3.42}$$

$$P_5 = \frac{\lambda_1 \lambda_2 \mu_3}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)(\lambda_3 + \mu_3)} \tag{3.43}$$

$$P_6 = \frac{\lambda_2 \lambda_3 \mu_1}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)(\lambda_3 + \mu_3)} \tag{3.44}$$

$$P_7 = \frac{\lambda_3 \lambda_1 \mu_2}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)(\lambda_3 + \mu_3)} \tag{3.45}$$

$$P_8 = \frac{\lambda_1 \lambda_2 \lambda_3}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)(\lambda_3 + \mu_3)} \tag{3.46}$$

3.1.2.1 Case 1: components connected in series

To determine reliability metrics for a series system, the strategy again is to equate the system availability to the corresponding steady state probability from the Markov chain. For the single component to be equivalent to the three series components, Equations 3.14 and 3.39 must be equal, thus,

$$\frac{\mu_s}{\lambda_s + \mu_s} = \frac{\mu_1 \mu_2 \mu_3}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)(\lambda_3 + \mu_3)} \tag{3.47}$$

Solving for μ_s on the left side, we find,

$$\mu_s = \frac{\lambda_s \mu_1 \mu_2 \mu_3}{\lambda_1 \lambda_2 \lambda_3 + \lambda_1 \lambda_2 \mu_3 + \lambda_1 \lambda_3 \mu_2 + \lambda_1 \mu_2 \mu_3 + \lambda_2 \lambda_3 \mu_1 + \lambda_2 \mu_1 \mu_3 + \lambda_3 \mu_1 \mu_2} \tag{3.48}$$

Equation 3.48 can be expressed in terms of mean repair times r_1, r_2, and r_s where

$$r_1 = \frac{1}{\mu_1}, \quad r_2 = \frac{1}{\mu_2}, \; r_3 = \frac{1}{\mu_3} \quad \text{and} \quad r_s = \frac{1}{\mu_s} \tag{3.49}$$

Substituting Equation 3.49 into 3.48, we obtain the average repair time for three components connected in series

$$r_s = \frac{\lambda_1 r_1 + \lambda_2 r_2 + \lambda_3 r_3 + \lambda_1 \lambda_2 r_1 + \lambda_1 \lambda_3 r_1 + \lambda_2 \lambda_3 r_1 + \lambda_3 r_3 \lambda_1 r_1 \lambda_2 r_2}{\lambda_1 + \lambda_2 + \lambda_3} \tag{3.50}$$

The following approximation, as in Billinton & Allan (1983), can be made to obtain the average repair time:

$$r_s = \frac{\lambda_1 r_1 + \lambda_2 r_2 + \lambda_3 r_3}{\lambda_1 + \lambda_2 + \lambda_3} \tag{3.51}$$

The system outage rate and expected system downtime for three components connected in series are

$$\lambda_s = \lambda_1 + \lambda_2 + \lambda_3 \tag{3.52}$$

$$U_s = \lambda_1 r_1 + \lambda_2 r_2 + \lambda_3 r_3 \tag{3.53}$$

3.1.2.2 Case 2: components connected in parallel

Following a similar procedure as in the modeling of two components connected in parallel, the system outage rate, average repair time and expected downtime are:

$$\lambda_p = \frac{(r_1 + r_2 + r_3)(\lambda_1\lambda_2\lambda_3)}{1 + \lambda_1 r_1 + \lambda_2 r_2 + \lambda_3 r_3} \approx \lambda_1\lambda_2\lambda_3(r_1 r_2 + r_1 r_2 + r_2 r_3) \tag{3.54}$$

$$r_p = \frac{r_1 r_2 r_3}{r_1 r_2 + r_1 r_3 + r_2 r_3} \tag{3.55}$$

$$U_p = \lambda_p r_p \tag{3.56}$$

3.2 Component sustained outages overlapping component maintenance outages

The following is a Markov model for component maintenance outages overlapping component sustained outages for two components. Each component can be in either the up, down or maintenance state. Let λ_1, λ_2, μ_1 and μ_2 be the sustained outage rates and repair rates for components 1 and 2, and $\tilde{\lambda}_1$, $\tilde{\lambda}_2$, $\tilde{\mu}_1$ and $\tilde{\mu}_2$ be the maintenance outage rates and maintenance repair rates for components 1 and 2. Each of the components can be in one of three states, since there are 2 components, there are 3^2 or 9 possible states. However, in practice there are only 8 states because there will never be scheduled maintenance on both components. These are shown in Table 3.3.

Table 3.3. Diagram states

State	Component 1	Component 2
1	Up	Up
2	Up	Down
3	Down	Up
4	Up	Maintenance
5	Maintenance	Up
6	Down	Maintenance
7	Maintenance	Down
8	Down	Down

The state space diagram for two components considering sustained outages overlapping maintenance is shown in Figure 3.4. As in the previous models, transfers requiring two simultaneous changes in the states of the components involved are not considered because the probabilities of such simultaneous occurrences are assumed to be negligibly small.

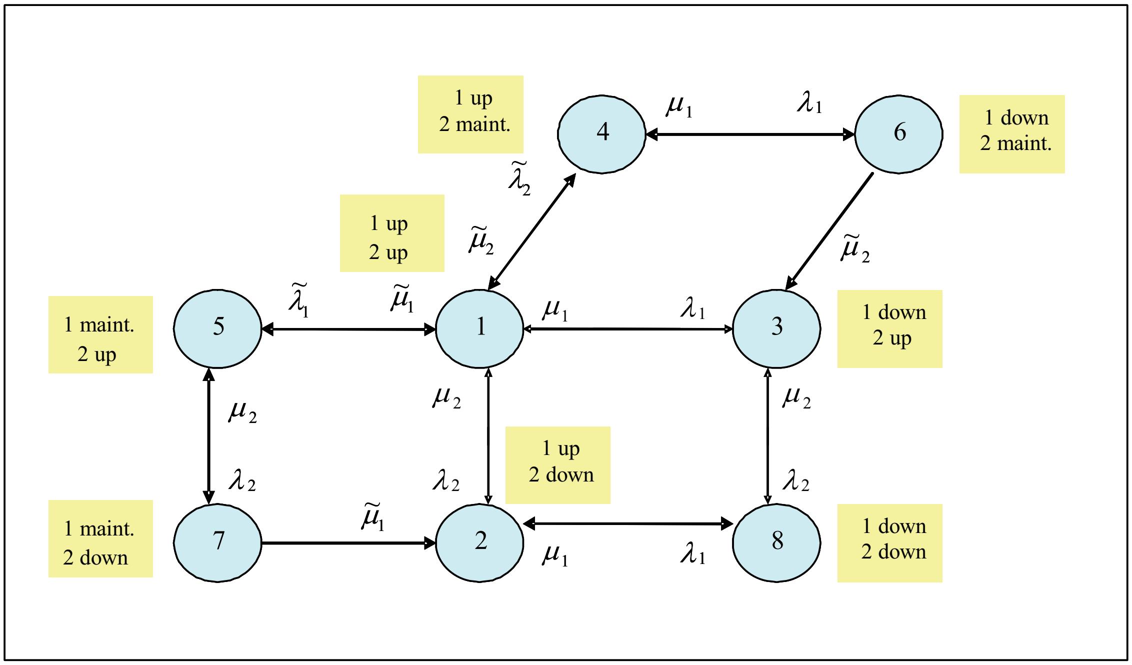

Figure 3.4. State space diagram for two different repairable components with maintenance

In the first example for overlapping component sustained outages, composed of two components arranged in parallel, the method used for solving the Markov chains, involved the solution of Laplace and inverse Laplace transforms. In the second case, for three components arranged in parallel for overlapping component sustained outages, the method used to solve the problem was the solution of the balance equations since only the steady-state availability (long term behavior of the system) was considered. In the present example, for two components in parallel, considering component sustained outages overlapping component maintenance outages, the solution technique employs a partitioning technique, which is based on the characteristics of the state space diagram.

Let Z_1 have states 1, 2, 3 and 8; Z_2 states 4 and 6; and Z_3 states 5 and 7. Balance equations for states contained in Z_1 are,

$$(\lambda_1 + \tilde{\lambda}_2 + \tilde{\lambda}_1 + \lambda_2)P_1 = \tilde{\mu}_2 P_4 + \mu_1 P_3 + \tilde{\mu}_1 P_5 + \mu_2 P_2 \tag{3.57}$$

$$(\lambda_1 + \mu_2)P_2 = \lambda_2 P_1 + \mu_1 P_8 + \tilde{\mu}_1 P_7 \tag{3.58}$$

$$(\mu_1 + \lambda_2)P_3 = \mu_2 P_8 + \lambda_1 P_1 + \tilde{\mu}_2 P_6 \tag{3.59}$$

$$(\mu_1 + \mu_2)P_8 = \lambda_2 P_3 + \lambda_1 P_2 \tag{3.60}$$

Balance equations for states contained in Z_2 are as follows,

$$(\lambda_1 + \tilde{\mu}_2)P_4 = \mu_1 P_6 + \tilde{\lambda}_2 P_1 \tag{3.61}$$

$$(\mu_1 + \tilde{\mu}_2)P_6 = \lambda_1 P_4 \tag{3.62}$$

Balance equations for states contained in Z_3 are as follows,

$$(\lambda_2 + \tilde{\mu}_1)P_5 = \mu_2 P_7 + \tilde{\lambda}_1 P_1 \tag{3.63}$$

$$(\mu_2 + \tilde{\mu}_1)P_7 = \lambda_2 P_5 \tag{3.64}$$

Finally,

$$A_{4x4}Z_1 = B_{4x2}Z_2 + C_{4x2}Z_3 \tag{3.65}$$

$$D_{2x2}Z_2 = E_{2x4}Z_1 \tag{3.66}$$

$$F_{2x2}Z_3 = E_{2x4}Z_1 \tag{3.67}$$

Defining everything in terms of Z_1, then we have the following set of equations:

$$Z_2 = D_{2x2}^{-1}E_{2x4}Z_1 \tag{3.68}$$

$$F_{2\times2}Z_3 = E_{2\times4}Z_1 \tag{3.69}$$

$$A_{4x4}Z_1 = B_{4x2}D_{2x2}^{-1}E_{2x4}Z_1 + C_{4x2}F_{2x2}^{-1}G_{2x4}Z_1 \tag{3.70}$$

Substituting Equations 3.68 and 3.69 into Equation 3.70,

$$A_{4x4}Z_1 = B_{4x2}D_{2x2}^{-1}E_{2x4}Z_1 + C_{4x2}F_{2x2}^{-1}G_{2x4}Z_1 \tag{3.71}$$

Obtaining matrices A through G we have:

$$A = \begin{bmatrix} \lambda_1 + \tilde{\lambda}_2 + \tilde{\lambda}_1 + \lambda_2 & -\mu_2 & -\mu_1 & 0 \\ -\lambda_2 & \lambda_1 + \mu_2 & 0 & -\mu_1 \\ -\lambda_1 & 0 & \lambda_2 + \mu_1 & -\mu_2 \\ 0 & -\lambda_1 & -\lambda_2 & \mu_1 + \mu_2 \end{bmatrix}$$

$$B = \begin{bmatrix} \tilde{\mu}_2 & 0 \\ 0 & 0 \\ 0 & \tilde{\mu}_2 \\ 0 & 0 \end{bmatrix} \qquad C = \begin{bmatrix} \tilde{\mu}_1 & 0 \\ 0 & 0 \\ 0 & \tilde{\mu}_1 \\ 0 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} \lambda_1 + \tilde{\mu}_2 & -\mu_1 \\ -\lambda_1 & \mu_1 + \tilde{\mu}_2 \end{bmatrix} \qquad E = \begin{bmatrix} \tilde{\lambda}_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$F = \begin{bmatrix} \lambda_2 + \tilde{\mu}_1 & -\mu_2 \\ -\lambda_2 & \mu_2 + \tilde{\mu}_1 \end{bmatrix} \qquad G = \begin{bmatrix} \tilde{\lambda}_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$P = A_{4x4} - B_{4x2}D_{2x2}^{-1}E_{2x4} - C_{4x2}F_{2x2}^{-1}G_{2x4} \tag{3.72}$$

$$PZ_1 = 0$$

Normalizing,

$$A_{4x4}Z_1 = B_{4x2}Z_2 + C_{4x2}Z_3$$

Then, we have,

$$(1,1,1,1)\begin{bmatrix} Z_1(1) \\ Z_1(2) \\ Z_1(3) \\ Z_1(4) \end{bmatrix} + (1,1)\begin{bmatrix} Z_2(1) \\ Z_2(2) \end{bmatrix} + (1,1)\begin{bmatrix} Z_3(1) \\ Z_3(2) \end{bmatrix} = 1$$

$$(1,1,1,1)[Z_1] + (1,1)D^{-1}E[Z_1] + (1,1)F^{-1}G[Z_1] = 1$$

$$\rho = (1,1,1,1) + (1,1)D^{-1}E + (1,1)F^{-1}G$$

and finally,

$$\hat{P} = \begin{bmatrix} Z \\ \rho \end{bmatrix}, \qquad \hat{P} Z_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \qquad \text{and} \qquad Z_1 = \hat{P}^{-1}\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

A Matlab code was developed to obtain the long run probabilities of the system modeled. As it can be observed, the determination of closed-form expressions to calculate the reliability of the system and the steady-state availability becomes complicated. As the number of states in the models increases, in the case of electricity transmission and distribution network configurations, it will be necessary to obtain solutions for systems composed of a large number of components. Therefore, it is necessary to develop approximations that can be easily applied without having to solve complex continuous time Markov chains every time a new configuration or replacement policy needs to be evaluated.

To fully evaluate system reliability using Markov chains, it will be necessary obtain solutions for all different types of overlapping outages such as overlapping component sustained outages, component transient outages overlapping component sustained outages and component sustained outages overlapping component maintenance outages. The

Markov chains become large and the approach is cumbersome as the number of components in the system increases, and alternatives are required to efficiently solve large problems and obtain solutions.

3.3 Conclusions

The complete Markov model can be applied without too much difficulty to relatively small systems, but it becomes rather complex when applied to larger systems. To overcome this difficulty, in the next chapter, equations are developed to approximate the system outage rates and repair durations. The aim of the set of equations developed is that they can be readily applied to different ETDS by managers of ETDS. The essence of these approximations is to develop a set of recursive equations which can be applied to large complex systems without the necessity of solving time consuming and complex equations.

Chapter 4

System reliability modeling and analysis

In the present chapter, new mathematical equations for different types of overlapping outages (overlapping component sustained outages, component sustained outages overlapping component maintenance outages, component transient outages overlapping component sustained outages) are developed (Coit *et al*., 2004; Espiritu *et al.,* 2005). They can be applied to obtain the system average repair time, expected system downtime and system outage rate for any electric power configuration as long as the minimal cut-sets can be determined.

The Markov technique is a widely used method for the reliability modeling of electricity transmission and distribution systems (Chan. & Asgarpoor, 2005; Billinton & Bollinger, 1968; Ramamoorty & Gopal, 1970). However, it becomes less amenable for calculations as the system becomes larger and more complex. In such cases, it is necessary to develop alternative methods which can be used in the evaluation of more complex network configurations.

4.1 Reliability evaluation in electric power systems

Literature in the area of electric power system reliability has focused on obtaining outage rates for series and parallel configurations when considering different outage cases. Literature in this area notes that it has become standard practice to use approximate evaluation techniques for system reliability evaluation.

Billinton & Allan (1984), Billinton & Li (1994) and Billinton & Zhang (2000) discuss the reliability evaluation of power systems. Particularly, they describe approximation techniques for the reliability analysis of series and parallel system configurations (with two and three components). Moreover, Billinton & Allan (1983) and Billinton & Li (1994), note that for complex systems, a series-parallel transformation based on the system minimal cut-sets can provide a good approximation to the actual system reliability metric. In the current project, the previous models have been extended by using recursive relationships to accommodate series-parallel systems of any size or any number of components within a cut set.

4.2 Objectives

The main objective in this chapter is to develop electric power system reliability equations to accurately estimate the system outage rate, average repair time and expected downtime, for any electricity transmission/distribution system configuration. The main objective of the expressions developed is that they can be readily applied by a power utility to obtain an efficient and accurate approximation to the system reliability for any electricity transmission/distribution configuration. The estimation of the true system reliability is based on minimal cut-sets, which provide a lower-bound approximation to the real system reliability.

The mathematical expressions developed, consider the following types of overlapping outages:

- Component sustained outages overlapping component sustained outages
- Component sustained outages overlapping component maintenance outages
- Component temporary outages overlapping component sustained outages

The following assumptions are made throughout these analyses:

1) Component failures are statistically independent. Failure at one component does not impact the outage rate of the other components.
2) Time between component outages and repair durations are distributed in accordance with known exponential distributions.
3) In the case of scheduled maintenance, no maintenance is performed in a component if this will cause a failure in the system.

A minimal cut-set approximation was used as a general approach to analyze more complex design configurations. If the system is a series-parallel system, then each cut-set represents each parallel structure, called a subsystem. If another configuration is appropriate, then cut-sets are initially determined. Each of these cut-sets are considered and referred to as "subsystems" because of the methodology employed.

Throughout this analysis, for subsystems and series-parallel systems, what is called "maintenance" is taking into account maintenance outages combined with sustained outages, and what is called "temporary" is considering temporary outages combined with sustained outages, the following notation is used.

λ_{ij} = Sustained outage rate for component j in subsystem i

$\lambda_{S_i}^{m_i}$ = Sustained outage rate for a parallel subsystem i with m_i components

$\lambda_{s\text{-}p}$ = Sustained outage rate for series-parallel system

$\tilde{\lambda}_{ij}$ = Maintenance outage rate for component j in subsystem i

$\tilde{\lambda}_{S_i}^{m_i}$ = Maintenance outage rate for subsystem i with m_i components

$\tilde{\lambda}_{s-p}$ = Maintenance outage rate for series-parallel system

λ_{ij}^{t} = Temporary outage rate for component j in subsystem i

$\lambda_{S_i}^{t\,m_i}$ = Temporary outage rate for subsystem i with m_i components

λ_{s-p}^{t} = Temporary outage rate for series-parallel system

r_{ij} = Average repair time for component j in subsystem i

$r_{S_i}^{m_i}$ = Average repair time for subsystem i with m_i components

$r_{s\text{-}p}$ = Average repair time series-parallel system

$\widetilde{r}_{ij}$ = Average maintenance time/outage for component j in subsystem i

$\widetilde{r}_{S_i}^{m_i}$ = Average maintenance time/outage for subsystem i with m_i components

$\widetilde{r}_{s-p}$ = Average maintenance time series-parallel system

r_{ij}^{t} = Average maintenance time/outage due to temporary outage for component j in subsystem i

$r_{S_i}^{t\,m_i}$ = Average maintenance time/outage due to temporary outage for subsystem i with m_i components

r_{s-p}^{t} = Series-parallel system average maintenance outage time (hr/outage) due to temporary outage

U_{s-p} = Expected downtime for series-parallel system

$U_{S_i}^{m_i}$ = Expected downtime for subsystem i due to sustained outage with m_i components

$\widetilde{U}_{S_i}^{m_i}$ = Expected downtime for subsystem i due to maintenance outage with m_i components

$\widetilde{U}_{s-p}$ = Expected downtime for series-parallel system due to maintenance outages

$U_{S_i}^{t\,m_i}$ = Expected downtime for subsystem i due to temporary outage with m_i components

U_{s-p}^{t} = Expected downtime for series-parallel system due to temporary outages.

4.3 Development of system reliability metrics

Figure 4.1 presents a series-parallel system with n subsystems connected in series and each of these subsystems has m_i components connected in parallel.

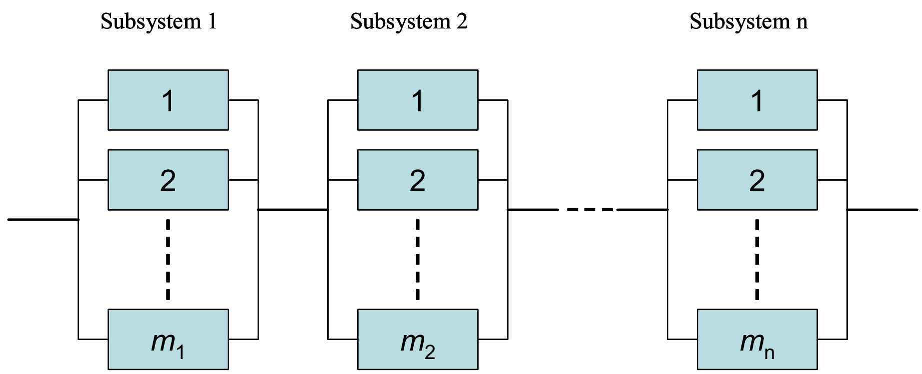

Figure 4.1 Series-Parallel System

A series-parallel approximation can be used to approximate more complex configurations based on minimum cut-sets, which is a lower bound approximation to system reliability. In this approach, each cut-set is considered as a "subsystem." The formulas given by Billinton & Allan (1983, 1984) and Billinton & Li (1994) have been generalized and extended to analyze this type of configuration.

4.4. Sustained outages overlapping component sustained outages

4.4.1 System outage rate

Billinton & Allan (1983, 1984) and Billinton & Li (1994) present an outage rate equation for systems with a series configuration:

$$\lambda_{s-p} = \sum_{i=1}^{n} \lambda_{S_i}^{m_i}$$

That is, the outage rate for a series-parallel design is the sum of the outage rates associated to each of the parallel subsystems. Based on the equations presented by Billinton & Allan (1983, 1984) and Billinton & Li (1994) a general formulation for subsystem *i* can be given as:

$$\lambda_{S_i}^{m_i} = \left[\prod_{j=1}^{m_i} \lambda_{ij}\right] \sum_{j_1<j_2<\ldots<j_{m_i}} r_{j_1} r_{j_2} \ldots r_{j_{m_i}} = \left[\prod_{j=1}^{m_i} \lambda_{ij}\right] \sum_{j=1}^{n} \prod_{k \neq j} r_{ik} \tag{4.1}$$

Equation 4.1 is a direct extension of the approach presented by Billinton & Allan (1983) for two and three component parallel subsystems. Note that for two components Equation 4.1 yields:

$$\lambda_{S_1}^{2} = \lambda_{11}\lambda_{12}\left(r_{11} + r_{12}\right) = \lambda_{11}(r_{11}\lambda_{12}) + \lambda_{12}(r_{12}\lambda_{11})$$

The system outage rate follows the following rationale as in Billinton & Allan (1983): "a two component parallel system fails if the first system component fails, at rate λ_{11}, and during the repair time of such component, r_{11}, the second system component fails, at rate λ_{12}, or if the second system component fails, at rate λ_{12}, and during the repair time of such component, r_{12}, the first system component fails, at rate λ_{11}."

A set of recursive equations has been developed to obtain the system outage rate (Coit *et al*., 2004; Espiritu *et al.,* 2005). This recursive approach is applied to most of the metrics proposed. A recursive formula for $\lambda_{S_i}^{m_i}$, a parallel subsystem, is given by:

$$\lambda_{S_i}^{m_i} = \lambda_{S_i}^{m_i - 1}\lambda_{im_i}\left(r_{S_i}^{m_i - 1} + r_{im_i}\right) \tag{4.2}$$

Equation 4.2 follows from the idea that a parallel system with m_i components can be regarded as a new parallel system with two components. The first "component" of this new system has an associated failure rate of $\lambda_{S_i}^{m_i - 1}$. That is, this first "component" includes the first $m_i - 1$ actual components connected in parallel. The second component of the transformed system is just the last component in the original system configuration, i.e., m_i. Figure 4.2 graphically describes the transformation discussed.

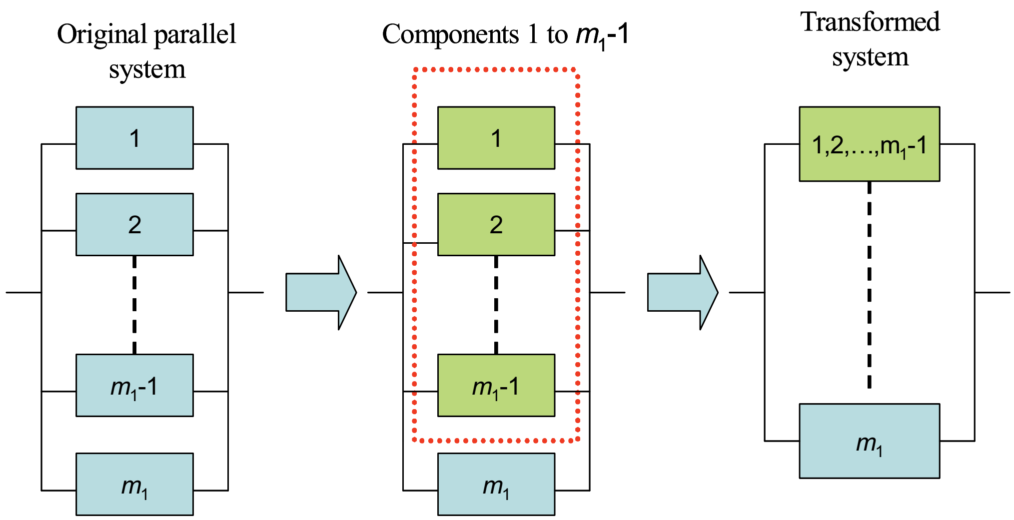

Figure 4.2: Parallel System Transformation

The recursion consists of exactly m_i computations or recursions. The first recursion considers only the first component and its outage rate is computed, i.e., $\lambda_{S_i}^1 = \lambda_{i1}$. The second recursion considers only the first two components its outage rate is computed, i.e., $\lambda_{S_i}^2$. Thus,

$$\lambda_{S_i}^2 = \lambda_{S_i}^{m_i-1}\lambda_{im_i}\left(r_{S_i}^{m_i-1} + r_{im_i}\right) = \lambda_{S_i}^1\lambda_{i2}\left(r_{S_i}^1 + r_{i2}\right) = \lambda_{i1}\lambda_{i2}\left(r_{i1} + r_{i2}\right).$$

In the same form, the remaining recursions can be used to obtain

$$\lambda_{S_i}^{m_i} = \lambda_{S_i}^{m_i-1}\lambda_{im_i}\left(r_{S_i}^{m_i-1} + r_{im_i}\right).$$

Mathematically the recursion is as follows:

1. $\lambda_{S_i}^1 = \lambda_{i1}$

2. $\lambda_{S_i}^2 = \lambda_{S_i}^{m_i-1}\lambda_{im_i}\left(r_{S_i}^{m_i-1} + r_{im_i}\right) = \lambda_{S_i}^1\lambda_{i2}\left(r_{S_i}^1 + r_{i2}\right) = \lambda_{i1}\lambda_{i2}\left(r_{i1} + r_{i2}\right)$

3. $\lambda_{S_i}^3 = \lambda_{S_i}^{m_i-1}\lambda_{im_i}\left(r_{S_i}^{m_i-1} + r_{im_i}\right) = \lambda_{S_i}^2\lambda_{i3}\left(r_{S_i}^2 + r_{i3}\right) = \lambda_{i1}\lambda_{i2}\lambda_{i3}\left(r_{i1}r_{i2} + r_{i1}r_{i3} + r_{i2}r_{i3}\right)$

4. $\lambda_{S_i}^4 = \lambda_{S_i}^{m_i-1}\lambda_{im_i}\left(r_{S_i}^{m_i-1} + r_{im_i}\right) = \lambda_{S_i}^3\lambda_{i4}\left(r_{S_i}^3 + r_{i4}\right) = \lambda_{i1}\lambda_{i2}\lambda_{i3}\lambda_{i4}\left(r_{i1}r_{i2}r_{i3} + r_{i1}r_{i2}r_{i4} + r_{i1}r_{i3}r_{i4} + r_{i2}r_{i3}r_{i4}\right)$

m_i. $\lambda_{S_i}^{m_i} = \lambda_{S_i}^{m_i-1}\lambda_{im_i}\left(r_{S_i}^{m_i-1} + r_{im_i}\right)$

It must be noted that Equation 4.2 depends on the a priori knowledge of $r_{S_i}^{m_i-1}$, the associated outage (repair) time associated to the first $m_i - 1$ parallel components. Thus, the same rationale has been applied to obtain metrics related to the average repair time and the total outage time.

4.4.2 Expected outage duration (average repair time):

Direct approximation equations for systems with a series configuration have been proposed by Billinton & Allan (1983, 1984) and Billinton & Li (1994) as:

$$r_{s-p} = \frac{\sum_{i=1}^{n} \lambda_{S_i}^{m_i} r_{S_i}^{m_i}}{\lambda_{s-p}}$$

A general formulation for subsystem *i* can be given as:

$$r_{S_i} = \frac{\prod_{i=1}^{n} r_{ij}}{\sum_{j=1}^{n} \prod_{k \neq j} r_{ik}}$$

A recursive formula for $r_{S_i}^{m_i}$ can be obtained by:

$$r_{S_i}^{m_i} = \frac{r_{S_i}^{m_i-1} r_{im_i}}{r_{S_i}^{m_i-1} + r_{im_i}} \qquad (4.3)$$

As in Equation 4.2, the recursive formula presented in Equation 4.3 consists of exactly m_i computations or recursions. The first recursion considers only the first component and its outage duration is computed, i.e., $r_{S_i}^{1} = r_{i1}$. The second recursion considers only the first two components and its outage duration is computed, i.e.,

$r_{S_i}^2 = \frac{r_{S_i}^1 r_{i2}}{r_{S_i}^1 + r_{i2}} = \frac{r_{i1} r_{i2}}{r_{i1} + r_{i2}}$. The same process can be applied to the remaining recursions to determine $r_{S_i}^{m_i} = \frac{r_{S_i}^{m_i - 1} r_{im_i}}{r_{S_i}^{m_i - 1} + r_{im_i}}$.

Mathematically, the recursion is as follows:

1. $r_{S_i}^1 = r_{i1}$

2. $r_{S_i}^2 = \frac{r_{S_i}^1 r_{i2}}{r_{S_i}^1 + r_{i2}} = \frac{r_{i1} r_{i2}}{r_{i1} + r_{i2}}$

3. $r_{S_i}^3 = \frac{r_{S_i}^2 r_{i3}}{r_{S_i}^2 + r_{i3}} = \frac{r_{i1} r_{i2} r_{i3}}{r_{i1} r_{i2} + r_{i1} r_{i3} + r_{i2} r_{i3}}$

4. $r_{S_i}^4 = \frac{r_{S_i}^3 r_{i4}}{r_{S_i}^3 + r_{i4}} = \frac{r_{i1} r_{i2} r_{i3} r_{i4}}{r_{i1} r_{i2} r_{i3} + r_{i1} r_{i2} r_{i4} + r_{i1} r_{i3} r_4 + r_{i2} r_{i3} r_{i4}}$

m_i. $r_{S_i}^{m_i} = \frac{r_{S_i}^{m_i - 1} r_{im_i}}{r_{S_i}^{m_i - 1} + r_{im_i}}$

4.4.3 System outage time

Finally, average system outage time for series-parallel systems is given by:

$$U_{s-p} = \lambda_{s-p} r_{s-p} = \sum_{i=1}^{n} U_{S_i}^{m_i}$$

Where:

$$U_{S_i}^1 = \lambda_{S_i}^1 r_{S_i}^1$$

$$U_{S_i}^2 = \lambda_{S_i}^2 r_{S_i}^2$$

$$U_{S_i}^3 = \lambda_{S_i}^3 r_{S_i}^3$$

$$U_{S_i}^4 = \lambda_{S_i}^4 r_{S_i}^4$$

$$U_{S_i}^{m_i} = \lambda_{S_i}^{m_i} r_{S_i}^{m_i}$$

4.5 Sustained outages overlapping component maintenance outages

The same rationale for developing the recursive formulas for obtaining reliability metrics related to overlapping component-sustained outages has been applied for this case (Coit *et al*., 2004; Espiritu *et al.,* 2005). Recursive formulas for each of the metrics of interest follow:

4.5.1 System maintenance outage rate

For a series-parallel configuration:

$$\tilde{\lambda}_{s-p} = \sum_{i=1}^{n} \tilde{\lambda}_{S_i}^{m_i}$$

Considering each parallel subsystem:

$$\tilde{\lambda}_{S_i}^{m_i} = \tilde{\lambda}_{S_i}^{m_i-1}\left(\lambda_{im_i}\tilde{r}_{S_i}^{m_i-1}\right) + \tilde{\lambda}_{im_i}\left(\lambda_{S_i}^{m_i-1}\tilde{r}_{im_i}\right) \quad .. \quad (4.4)$$

$\lambda_{S_i}^{m_i}$ can be obtained using Equation 4.2.

The recursive formula presented in Equation 4.4 will consist of exactly m_i computations or recursions assuming $\lambda_{S_i}^{m_i-1}$ has been computed. The same rationale discussed for Equations 4.2 and 4.3 can be applied to the recursions in Equation 4.4. For the remainder of this section only the mathematical illustration of the recursion technique is presented.

It is important to mention that this approximation is less accurate because all possible failure combinations may not be considered. However it is very accurate for smaller cut-sets and its accuracy decreases as the size of the cut-set increases.

For the current equation, mathematically the recursion is illustrated as follows:

1. $\tilde{\lambda}_{S_i}^{1} = \tilde{\lambda}_{i1}$

2. $\widetilde{\lambda}_{S_i}^{2} = \widetilde{\lambda}_{S_i}^{1}\left(\lambda_{i2}\widetilde{r}_{S_i}^{1}\right)+\widetilde{\lambda}_{i2}\left(\lambda_{S_i}^{1}\widetilde{r}_{i2}\right)=\widetilde{\lambda}_{i1}\left(\lambda_{i2}\widetilde{r}_{i1}\right)+\widetilde{\lambda}_{i2}\left(\lambda_{i1}\widetilde{r}_{i2}\right)$

3. $\widetilde{\lambda}_{S_i}^{3} = \widetilde{\lambda}_{S_i}^{2}\left(\lambda_{i3}\widetilde{r}_{S_i}^{2}\right)+\widetilde{\lambda}_{i3}\left(\lambda_{S_i}^{2}\widetilde{r}_{i3}\right)$

4. $\widetilde{\lambda}_{S_i}^{4} = \widetilde{\lambda}_{S_i}^{3}\left(\lambda_{i4}\widetilde{r}_{S_i}^{3}\right)+\widetilde{\lambda}_{i4}\left(\lambda_{S_i}^{3}\widetilde{r}_{i4}\right)$

m_i. $\widetilde{\lambda}_{S_i}^{m_i} = \widetilde{\lambda}_{S_i}^{m_i-1}\left(\lambda_{im_i}\widetilde{r}_{S_i}^{m_i-1}\right)+\widetilde{\lambda}_{im_i}\left(\lambda_{S_i}^{m_i-1}\widetilde{r}_{im_i}\right)$

It must be noted that the recursion equations depend on the a priori knowledge of $\widetilde{r}_{S_i}^{m_i-1}$, the associated maintenance outage time associated to the first $m_i - 1$ parallel components. Thus, the same rationale has been applied to obtain metrics related to the average repair time due to maintenance outage and the total outage time.

4.5.2 Expected outage duration (average maintenance time)

If system design follows a series-parallel configuration:

$$\widetilde{r}_{s-p} = \frac{\sum_{i=1}^{n}\widetilde{\lambda}_{S_i}^{m_i}\widetilde{r}_{S_i}^{m_i}}{\widetilde{\lambda}_{s-p}}$$

Considering each parallel subsystem:

$$\widetilde{r}_{S_i}^{m_i} = \frac{1}{\widetilde{\lambda}_{S_i}^{m_i}}\left[\widetilde{\lambda}_{S_i}^{m_i-1}\left(\lambda_{im_i}\widetilde{r}_{S_i}^{m_i-1}\right)\left(\frac{r_{im_i}\widetilde{r}_{S_i}^{m_i-1}}{r_{im_i}+\widetilde{r}_{S_i}^{m_i-1}}\right)+\widetilde{\lambda}_{im_i}\left(\lambda_{S_i}^{m_i-1}\widetilde{r}_{im_i}\right)\left(\frac{\widetilde{r}_{im_i}r_{S_i}^{m_i-1}}{\widetilde{r}_{im_i}+r_{S_i}^{m_i-1}}\right)\right] \tag{4.5}$$

$\lambda_{S_i}^{m_i}$ and $r_{S_i}^{m_i}$ are obtained using Equations 4.2 and 4.3 respectively. This expression appears complex but it can be readily explained. The average repair time is computed from two distinct failure types; (1) the first $m_i - 1$ components have an outage due to maintenance at rate $\lambda_{S_i}^{m_i-1}$, and the m_i^{th} component has a sustained outage during repair time, or (2) the m_i^{th} component has a maintenance outage and during its repair time, the "component" composed of the $m_i - 1$ components has a sustained outage.

Mathematically, Equation 4.5 leads to the following recursions:

1. $\tilde{r}_{S_i}^{1} = \tilde{r}_{i1}$

2. $$\tilde{r}_{S_i}^{2} = \frac{1}{\tilde{\lambda}_{S_i}^{2}}\left[\tilde{\lambda}_{S_i}^{1}\left(\lambda_{i2}\tilde{r}_{S_i}^{1}\right)\left(\frac{r_{i2}\tilde{r}_{S_i}^{1}}{r_{i2}+\tilde{r}_{S_i}^{1}}\right)+\tilde{\lambda}_{i2}\left(\lambda_{S_i}^{1}\tilde{r}_{i2}\right)\left(\frac{\tilde{r}_{i2}r_{S_i}^{1}}{\tilde{r}_{i2}+r_{S_i}^{1}}\right)\right]$$
$$= \frac{1}{\tilde{\lambda}_{S_i}^{2}}\left[\tilde{\lambda}_{i1}\left(\lambda_{i2}\tilde{r}_{i1}\right)\left(\frac{r_{i2}\tilde{r}_{i1}}{r_{i2}+\tilde{r}_{i1}}\right)+\tilde{\lambda}_{i2}\left(\lambda_{i1}\tilde{r}_{i2}\right)\left(\frac{\tilde{r}_{i2}r_{i1}}{\tilde{r}_{i2}+r_{i1}}\right)\right]$$

3. $$\tilde{r}_{S_i}^{3} = \frac{1}{\tilde{\lambda}_{S_i}^{3}}\left[\tilde{\lambda}_{S_i}^{2}\left(\lambda_{i3}\tilde{r}_{S_i}^{2}\right)\left(\frac{r_{i3}\tilde{r}_{S_i}^{2}}{r_{i3}+\tilde{r}_{S_i}^{2}}\right)+\tilde{\lambda}_{i3}\left(\lambda_{S_i}^{2}\tilde{r}_{i3}\right)\left(\frac{\tilde{r}_{i3}r_{S_i}^{2}}{\tilde{r}_{i3}+r_{S_i}^{2}}\right)\right]$$

4. $$\tilde{r}_{S_i}^{4} = \frac{1}{\tilde{\lambda}_{S_i}^{4}}\left[\tilde{\lambda}_{S_i}^{3}\left(\lambda_{i4}\tilde{r}_{S_i}^{3}\right)\left(\frac{r_{i4}\tilde{r}_{S_i}^{3}}{r_{i4}+\tilde{r}_{S_i}^{3}}\right)+\tilde{\lambda}_{i4}\left(\lambda_{S_i}^{3}\tilde{r}_{i4}\right)\left(\frac{\tilde{r}_{i4}r_{S_i}^{3}}{\tilde{r}_{i4}+r_{S_i}^{3}}\right)\right]$$

m_i. $$\tilde{r}_{S_i}^{m_i} = \frac{1}{\tilde{\lambda}_{S_i}^{m_i}}\left[\tilde{\lambda}_{S_i}^{m_i-1}\left(\lambda_{im_i}\tilde{r}_{S_i}^{m_i-1}\right)\left(\frac{r_{im_i}\tilde{r}_{S_i}^{m_i-1}}{r_{im_i}+\tilde{r}_{S_i}^{m_i-1}}\right)+\tilde{\lambda}_{im_i}\left(\lambda_{S_i}^{m_i-1}\tilde{r}_{im_i}\right)\left(\frac{\tilde{r}_{im_i}r_{S_i}^{m_i-1}}{\tilde{r}_{im_i}+r_{S_i}^{m_i-1}}\right)\right]$$

4.5.3 Outage time

The outage time approximations for sustained failures overlapping with maintenance failures are as follows,

$$\tilde{U}_{s-p} = \tilde{\lambda}_{s-p}\tilde{r}_{s-p} = \sum_{i=1}^{n}\tilde{U}_{S_i}^{m_i}$$

$$\tilde{U}_{S_i}^{m_i} = \tilde{\lambda}_{S_i}^{m_i}\tilde{r}_{S_i}^{m_i}$$

4.6 Component temporary outages overlapping component sustained outages

The recursive approach (Coit *et al*., 2004) is now applied for obtaining system forced outage rate, expected outage duration and total downtime due to temporary outages overlapping component-sustained outages.

4.6.1 System outage rate

Considering a series-parallel configuration:

$$\lambda_{s-p}^{t} = \sum_{i=1}^{n} \lambda_{S_i}^{t\,m_i}$$

For a parallel system with two components and considering temporary outages overlapping component sustained outages, we have the following:

$$\lambda_{S_i}^{t\,m_i} = \lambda_{11} r_{11} \lambda_{12}^{t} + \lambda_{12} r_{12} \lambda_{11}^{t} + \lambda_{11}^{t} r_{11}^{t} \lambda_{12} + \lambda_{12}^{t} r_{12}^{t} \lambda_{11} = \lambda_{11} \lambda_{12}^{t} (r_{11} + r_{12}^{t}) + \lambda_{12} \lambda_{11}^{t} (r_{12} + r_{11}^{t})$$

A two component parallel system fails (1) if during a sustained outage of the first component, at rate λ_{11}, and during its repair time, r_{11}, the second component has a temporary outage, at rate λ_{12}^{t}, or (2) if the second component goes down due to a sustained outage, λ_{12}, and during its repair time, r_{12}, the first component has a temporary outage, λ_{11}^{t}, or (3) if the first component has a temporary outage, λ_{11}^{t}, and during its temporary repair time, r_{11}^{t}, the second component has a sustained outage, λ_{12}, or (4) if the second component experiments a temporary outage, λ_{12}^{t}, and during its temporary repair time, r_{12}^{t}, the first component has a sustained outage, λ_{11}.

Based on this reasoning, a recursive formula for outages due to temporary outages overlapping component sustained outages for parallel subsystem is given as follows. This equation is based on the inequality that since $r_{11} >> r_{12}^{t}$ and $r_{12} >> r_{11}^{t}$, and therefore, we have:

$$\lambda_{S_i}^{t\,m_i} = \lambda_{11} \lambda_{12}^{t} r_{11} + \lambda_{12} \lambda_{11}^{t} r_{12}$$

For each parallel subsystem:

$$\lambda_{S_i}^{t\,m_i} = \lambda_{S_i}^{t\,m_i-1} \left(\lambda_{im_i} r_{im_i}\right) + \lambda_{im_i}^{t} \left(\lambda_{S_i}^{m_i-1} r_{S_i}^{m_i-1}\right) \tag{4.6}$$

$\lambda_{S_i}^{m_i-1}$ and $r_{S_i}^{m_i-1}$ can be obtained by applying Equations 4.2 and 4.3 respectively.

For Equation 4.6, the recursion technique can be illustrated as follows:

1. $\lambda_{S_i}^{t\,1} = \lambda_{i1}^{t}$

2. $\lambda_{S_i}^{t\,2} = \lambda_{S_i}^{t\,1}\left(\lambda_{i2} r_{i2}\right) + \lambda_{i2}^{t}\left(\lambda_{S_i}^{1} r_{S_i}^{1}\right) = \lambda_{i1}^{t}\left(\lambda_{i2} r_{i2}\right) + \lambda_{i2}^{t}\left(\lambda_{i1} r_{i1}\right)$

3. $\lambda_{S_i}^{t\,3} = \lambda_{S_i}^{t\,2}\left(\lambda_{i3} r_{i3}\right) + \lambda_{i3}^{t}\left(\lambda_{S_i}^{2} r_{S_i}^{2}\right)$

4. $\lambda_{S_i}^{t\,4} = \lambda_{S_i}^{t\,3}\left(\lambda_{i4} r_{i4}\right) + \lambda_{i4}^{t}\left(\lambda_{S_i}^{3} r_{S_i}^{3}\right)$

m_i. $\lambda_{S_i}^{t\,m_i} = \lambda_{S_i}^{t\,m_i-1}\left(\lambda_{im_i} r_{im_i}\right) + \lambda_{im_i}^{t}\left(\lambda_{S_i}^{m_i-1} r_{S_i}^{m_i-1}\right)$

4.6.2 Expected outage duration (average maintenance time)

For a series-parallel system configuration:

$$r_{s-p}^{t} = \frac{\sum_{i=1}^{n} \lambda_{S_i}^{t\,m_i} r_{S_i}^{t\,m_i}}{\lambda_{s-p}^{t}}$$

$\lambda_{S_i}^{t\,m_i}$ can be obtained by applying Equation 4.6. Thus, for each parallel subsystem, the expected repair time due to component temporary outages overlapping component maintenance outages can be obtained by applying Equation 4.7.

$$r_{S_i}^{t\,m_i} = \frac{1}{\lambda_{S_i}^{t\,m_i}}\left[\lambda_{S_i}^{t\,m_i-1}\left(\lambda_{im_i} r_{im_i}\right)\left(\frac{r_{im_i} r_{S_i}^{t\,m_i-1}}{r_{im_i} + r_{S_i}^{t\,m_i-1}}\right) + \lambda_{im_i}^{t}\left(\lambda_{S_i}^{m_i-1} r_{S_i}^{m_i-1}\right)\left(\frac{r_{im_i}^{t} r_{S_i}^{m_i-1}}{r_{im_i}^{t} + r_{S_i}^{m_i-1}}\right)\right] \qquad (4.7)$$

$\lambda_{S_i}^{m_i-1}$, $r_{S_i}^{m_i-1}$ and $\lambda_{S_i}^{t\,m_i-1}$ are given by Equations 4.2, 4.3 and 4.6 respectively. The recursion technique is illustrated as follows:

1. $r_{S_i}^{t\,1} = r_{i1}^{t}$

2. $$r_{S_i}^{t\,2} = \frac{1}{\lambda_{S_i}^{t\,2}}\left[\lambda_{S_i}^{t\,1}\left(\lambda_{i2} r_{i2}\right)\left(\frac{r_{i2} r_{S_i}^{t\,1}}{r_{i2} + r_{S_i}^{t\,1}}\right) + \lambda_{i2}^{t}\left(\lambda_{S_i}^{1} r_{S_i}^{1}\right)\left(\frac{r_{i2}^{t} r_{S_i}^{1}}{r_{i2}^{t} + r_{S_i}^{1}}\right)\right]$$
$$= \frac{1}{\lambda_{S_i}^{t\,2}}\left[\lambda_{i1}^{t}\left(\lambda_{i2} r_{i2}\right)\left(\frac{r_{i2} r_{i1}^{t}}{r_{i2} + r_{i1}^{t}}\right) + \lambda_{i2}^{t}\left(\lambda_{i1} r_{i1}\right)\left(\frac{r_{i2}^{t} r_{i1}}{r_{i2}^{t} + r_{i1}}\right)\right]$$

3. $$r_{S_i}^{t\,3} = \frac{1}{\lambda_{S_i}^{t\,3}}\left[\lambda_{S_i}^{t\,2}\left(\lambda_{i3} r_{i3}\right)\left(\frac{r_{i3} r_{S_i}^{t\,2}}{r_{i3} + r_{S_i}^{t\,2}}\right) + \lambda_{i3}^{t}\left(\lambda_{S_i}^{2} r_{S_i}^{2}\right)\left(\frac{r_{i3}^{t} r_{S_i}^{2}}{r_{i3}^{t} + r_{S_i}^{2}}\right)\right]$$

4. $$r_{S_i}^{t\,4} = \frac{1}{\lambda_{S_i}^{t\,3}}\left[\lambda_{S_i}^{t\,3}\left(\lambda_{i4} r_{i4}\right)\left(\frac{r_{i4} r_{S_i}^{t\,3}}{r_{i4} + r_{S_i}^{t\,3}}\right) + \lambda_{i4}^{t}\left(\lambda_{S_i}^{3} r_{S_i}^{3}\right)\left(\frac{r_{i4}^{t} r_{S_i}^{3}}{r_{i4}^{t} + r_{S_i}^{3}}\right)\right]$$

m_i. $$r_{S_i}^{t\,m_i} = \frac{1}{\lambda_{S_i}^{t\,m_i}}\left[\lambda_{S_i}^{t\,m_i-1}\left(\lambda_{im_i} r_{im_i}\right)\left(\frac{r_{im_i} r_{S_i}^{t\,m_i-1}}{r_{im_i} + r_{S_i}^{t\,m_i-1}}\right) + \lambda_{im_i}^{t}\left(\lambda_{S_i}^{m_i-1} r_{S_i}^{m_i-1}\right)\left(\frac{r_{im_i}^{t} r_{S_i}^{m_i-1}}{r_{im_i}^{t} + r_{S_i}^{m_i-1}}\right)\right]$$

4.6.3 Outage time

For a series-parallel system, the associated outage time can be approximated as,

$$U_{s-p}^{t} = \lambda_{s-p}^{t} r_{s-p}^{t} = \sum_{i=1}^{n} U_{S_i}^{t\,m_i}$$

where,

$$U_{S_i}^{t\,m_i} = \lambda_{S_i}^{t\,m_i} r_{S_i}^{t\,m_i}$$

4.7 Simulation based testing of developed metrics

The results obtained with the approximation techniques were evaluated to test the accuracy of the approximations obtained with the equations developed in the present section (Coit *et al*., 2004). A simulation analysis software (BlockSim manufactured by ReliaSoft) was used to simulate examples of parallel and series-parallel systems. BlockSim is a discrete-event simulation program. Discrete event simulation involves modeling a system as it progresses through time and is particularly useful for the analysis of engineering systems. For the systems under analysis, the component outage rates are in outages per year, the repair times are in hours per outage, and both time between outages and repair times are assumed to be exponentially distributed. After this data has been input, a one year time frame was selected for simulating the systems and 100,000 replications were run.

Simulation results were computed on two series-parallel systems. Figure 4.3 and Tables 4.1 through 4.4 present the equivalent series-parallel reliability block diagram and the component parameters for the first example. The components labeled L_1 and L_2 represent two lines, components B_1, B_2 and B_3 are three breakers while S_1 and S_2 represent buses. This configuration has nine different sets of components that guarantee system failure. Such sets commonly known as minimal cut sets are: $\{L_1, L_2\}$, $\{L_1, B_2, B_3\}$, $\{L_2, B_1, B_3\}$, $\{L_1, S_2, B_3\}$, $\{L_2, S_1, B_3\}$, $\{B_1, B_2\}$, $\{S_1, B_2\}$, $\{B_1, S_2\}$ and $\{S_1, S_2\}$. For the simulation, it was assumed that equivalent components have the same reliability metrics (share the same data). BlockSim was used to approximate reliability metrics of the actual configuration and compare them to the analytical results obtained with the formulas for series-parallel reliability analysis (minimal cut-sets) previously presented. Table 4.1-4.4 present different system reliability metrics considering various cases of component data.

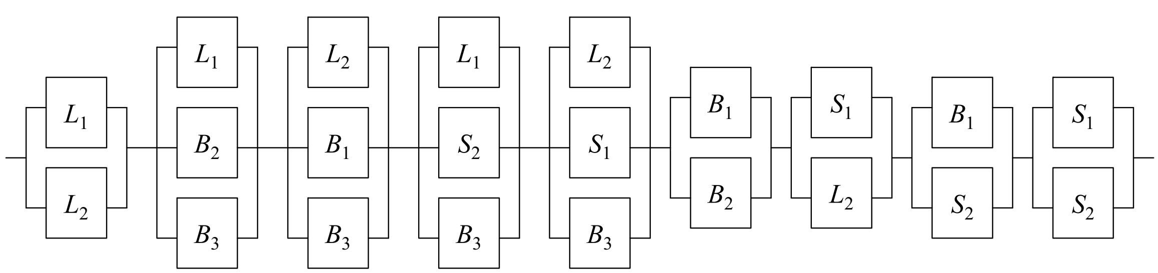

Figure 4.3: Series-Parallel Transformation - Example 1

Table 4.1: Series-Parallel Component Data and Results

Case 1					
Components	Outage rate	Average repair time	**System**	**Simulation**	**Analytical**
L1	0.519	24	Outage rate	0.0025	0.0027
L2	0.519	24	Repair time	14.36	14.45
B1	0.15	72	Downtime	0.0359	0.0390
B2	0.15	72	Variance	1E-8	
B3	0.15	72			
S1	0.25	12			
S2	0.25	12			
Case 2					
Components	Outage rate	Average repair time	**System**	**Simulation**	**Analytical**
L1	0.619	36	Outage rate	0.0058	0.0059
L2	0.619	36	Repair time	23.44	23.56
B1	0.2	96	Downtime	0.136	0.139
B2	0.2	96	Variance	1.6E-7	
B3	0.2	96			
S1	0.3	18			
S2	0.3	18			
Case 3					
Components	Outage rate	Average repair time	**System**	**Simulation**	**Analytical**
L1	0.719	96	Outage rate	0.0168	0.0170
L2	0.719	96	Repair time	43.15	43.53
B1	0.25	120	Downtime	0.725	0.740
B2	0.25	120	Variance	6.4E-7	
B3	0.25	120			
S1	0.35	36			
S2	0.35	36			
Case 4					
Components	Outage rate	Average repair time	**System**	**Simulation**	**Analytical**
L1	0.819	192	Outage rate	0.057	0.053
L2	0.819	192	Repair time	73.67	75.83
B1	0.35	156	Downtime	4.1993	4.0192
B2	0.35	156	Variance	9E-6	
B3	0.35	156			
S1	0.65	72			
S2	0.65	72			

Table 4.2. Low outage rates increasing repair times

Case 1					
Components	Outage rate	Average repair time	**System**	**Simulation**	**Analytical**
L1	0.076	5.4	Outage rate	0.00014	0.00011
L2	0.096	6.8	Repair time	2.9285	3.9091
B1	0.065	9.3	Downtime	0.00041	0.00043
B2	0.087	6.4	Variance	9E-6	
B3	0.034	5.8			
S1	0.187	9.1			
S2	0.109	8.9			
Case 2					
Components	Outage rate	Average repair time	**System**	**Simulation**	**Analytical**
L1	0.076	12.6	Outage rate	0.0002	0.00017
L2	0.096	13.8	Repair time	5	6.47
B1	0.065	10.2	Downtime	0.0010	0.0011
B2	0.087	12.1	Variance	8.5E-6	
B3	0.034	11.4			
S1	0.187	13.8			
S2	0.109	13.4			
Case 3					
Components	Outage rate	Average repair time	**System**	**Simulation**	**Analytical**
L1	0.076	32.5	Outage rate	0.0005	0.0004
L2	0.096	43.6	Repair time	32.060	41.525
B1	0.065	42.4	Downtime	0.01603	0.01661
B2	0.087	26.3	Variance	1E-8	
B3	0.034	26.8			
S1	0.187	31.7			
S2	0.109	29.8			
Case 4					
Components	Outage rate	Average repair time	**System**	**Simulation**	**Analytical**
L1	0.076	87.45	Outage rate	0.0011	0.0011
L2	0.096	98.32	Repair time	48.00	46.09
B1	0.065	65.78	Downtime	0.0528	0.0507
B2	0.087	76.54	Variance	4E-8	
B3	0.034	87.67			
S1	0.187	98.99			
S2	0.109	92.16			

Table 4.3. Medium outage rates increasing repair times

Case 1					
Components	Outage rate	Average repair time	**System**	**Simulation**	**Analytical**
L1	1.56	5.4	Outage rate	0.0522	0.0524
L2	1.32	6.8	Repair time	4.23	4.12
B1	2.56	9.3	Downtime	0.2211	0.21639
B2	2.47	6.4	Variance	4E-8	
B3	2.97	5.8			
S1	2.4	9.1			
S2	2.7	8.9			
Case 2					
Components	Outage rate	Average repair time	**System**	**Simulation**	**Analytical**
L1	1.56	12.6	Outage rate	0.0732	0.0788
L2	1.32	13.8	Repair time	6.318	6.201
B1	2.56	10.2	Downtime	0.4625	0.4886
B2	2.47	12.1	Variance	9E-6	
B3	2.97	11.4			
S1	2.4	13.8			
S2	2.7	13.4			
Case 3					
Components	Outage rate	Average repair time	**System**	**Simulation**	**Analytical**
L1	1.56	32.5	Outage rate	0.1939	0.2109
L2	1.32	43.6	Repair time	16.713	16.200
B1	2.56	42.4	Downtime	3.2406	3.4166
B2	2.47	26.3	Variance	1.44E-6	
B3	2.97	26.8			
S1	2.4	31.7			
S2	2.7	29.8			
Case 4					
Components	Outage rate	Average repair time	**System**	**Simulation**	**Analytical**
L1	1.56	87.45	Outage rate	0.4985	0.5442
L2	1.32	98.32	Repair time	43.487	41.703
B1	2.56	65.78	Downtime	21.6785	22.6950
B2	2.47	76.54	Variance	2.401E-5	
B3	2.97	87.67			
S1	2.4	98.99			
S2	2.7	92.16			

Table 4.4. High outage rates high repair times

Case 1					
Components	Outage rate	Average repair time	**System**	**Simulation**	**Analytical**
L1	1.78	12.6	Outage rate	0.0279	0.0239
L2	1.94	13.8	Repair time	6.537	7.284
B1	0.87	10.2	Downtime	0.1824	0.1741
B2	0.59	12.1	Variance	4E-8	
B3	0.76	11.4			
S1	1.54	13.8			
S2	1.32	13.4			
Case 2					
Components	Outage rate	Average repair time	**System**	**Simulation**	**Analytical**
L1	1.78	32.5	Outage rate	0.0701	0.0654
L2	1.94	43.6	Repair time	17.981	17.553
B1	0.87	42.4	Downtime	1.2605	1.1480
B2	0.59	26.3	Variance	6.4E-7	
B3	0.76	26.8			
S1	1.54	31.7			
S2	1.32	29.8			
Case 3					
Components	Outage rate	Average repair time	**System**	**Simulation**	**Analytical**
L1	1.78	87.45	Outage rate	0.1976	0.1667
L2	1.94	98.32	Repair time	39.929	44.626
B1	0.87	65.78	Downtime	7.8901	7.4393
B2	0.59	76.54	Variance	1.089E-5	
B3	0.76	87.67			
S1	1.54	98.99			
S2	1.32	92.16			
Case 4					
Components	Outage rate	Average repair time	**System**	**Simulation**	**Analytical**
L1	5.32	57.45	Outage rate	4.701	5.1644
L2	5.87	68.32	Repair time	17.5268	17.4176
B1	12.45	35.78	Downtime	82.3936	89.9519
B2	14.32	39.54	Variance	3.721E-5	
B3	13.43	47.67			
S1	11.89	28.99			
S2	10.12	22.16			

The results presented in Tables 4.1 through 4.4 show that the proposed approximation technique as a general recursive formula generates accurate approximations. The analytical results yield a relative absolute deviation below 10% for many of the cases analyzed for system downtime.

Figure 4.4 presents a second example series-parallel system. Components L_1, L_2 and L_3 represent lines, components B_1, B_2, B_3 and B_4 are breakers, and the buses are represented by S_1 and S_2. For this configuration, fourteen different sets of components guarantee system failure. These minimal cut sets are: $\{L_1, L_2, L_3\}$, $\{L_1, B_2, B_4\}$, $\{L_3, B_1, B_3\}$, $\{L_1, S_2, B_2\}$, $\{L_3, S_1, B_3\}$, $\{L_1, L_2, S_2, B_3\}$, $\{L_2, L_3, S_1, B_2\}$, $\{L_1, L_2, B_3, B_4\}$, $\{L_1, L_3, B_2, B_3\}$, $\{B_1, B_4\}$, $\{L_2, L_3, B_1, B_2\}$, $\{S_1, B_4\}$, $\{B_1, S_2\}$ and $\{S_1, S_2\}$. It has been assumed that equivalent components have the same component reliability metrics.

Simulation has again been used to approximate reliability metrics of the actual configuration and compare them to the analytical results obtained with the formulas for series-parallel reliability analysis previously presented. Table 4.5 presents system reliability metrics considering various cases of component data. Similar to the results for the other examples, the results in Table 4.5 indicate that the approximations are generally accurate for components in the ETDS area.

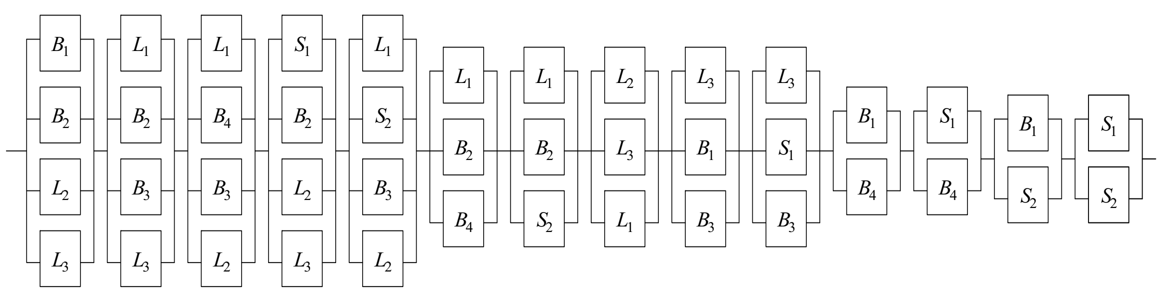

Figure 4.4: Series-Parallel Transformation – Example 2

Table 4.5: Series-Parallel Component Data and Results– Example 2

Case 1					
Components	Outage rate	Average repair time	**System**	**Simulation**	**Analytical**
L1	0.519	24	Outage rate	0.0011	0.0012
L2	0.519	24	Repair time	14.54	17.5
L3	0.519	24	Downtime	0.016	0.021
B1	0.15	72	Variance	1.75E-8	
B2	0.15	72			
B3	0.15	72			
B4	0.15	72			
S1	0.25	12			
S2	0.25	12			
Case 2					
Components	Outage rate	Average repair time	**System**	**Simulation**	**Analytical**
L1	0.619	36	Outage rate	0.0026	0.0028
L2	0.619	36	Repair time	24.62	24.64
L3	0.619	36	Downtime	0.064	0.069
B1	0.2	96	Variance	5.5E-7	
B2	0.2	96			
B3	0.2	96			
B4	0.2	96			
S1	0.3	36			
S2	0.3	36			
Case 3					
Components	Outage rate	Average repair time	**System**	**Simulation**	**Analytical**
L1	0.719	96	Outage rate	0.0056	0.0059
L2	0.719	96	Repair time	37.15	36.10
L3	0.719	96	Downtime	0.208	0.213
B1	0.25	120	Variance	7E-6	
B2	0.25	120			
B3	0.25	120			
B4	0.25	120			
S1	0.35	36			
S2	0.35	36			
Case 4					
Components	Outage rate	Average repair time	**System**	**Simulation**	**Analytical**
L1	0.819	192	Outage rate	0.023	0.023
L2	0.819	192	Repair time	53.04	50.87
L3	0.819	192	Downtime	1.22	1.17
B1	0.35	156	Variance	4.5E-5	
B2	0.35	156			
B3	0.335	156			
B4	0.35	156			
S1	0.65	72			
S2	0.65	72			

4.8 Minimal Cut-Sets for Other Configurations

The models developed in the present chapter can be applied to any configuration once minimal cut-sets have been determined. Then, each cut-set acts as a subsystem in an equivalent series-parallel configuration. The reason it is an approximation, instead of an exact calculation, is because the series-parallel reliability calculations requires independent subsystem failure times. However, if the same component is in more than one cut-set, then the "subsystems" are not actually independent.

Typical electricity distribution configurations used by power companies are the breaker-and-a-half, breaker-and-a-third and the dual element spot network (DESN) configurations. Next a description of them is presented.

4.8.1 Breaker-and-a-Half

Figures 4.5 and 4.6 present functional block diagrams for breaker-and-a-half configurations with one diameter and two diameters, respectively. The breaker-and-a-half configuration with two diameters is used to further demonstrate the methodology. In Figure 4.6, the components labeled 13, 14, 15 and 16 represent the four lines; components 3, 4, 5, 6, 7, 8 represent six breakers while components 1 and 2 represent the buses.

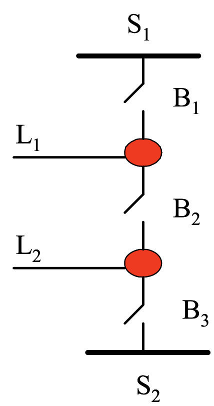

Figure 4.5: Breaker-and-a-Half Configuration – One Diameter

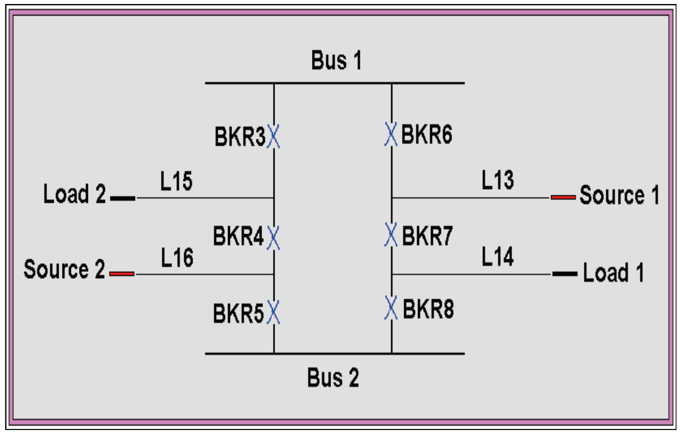

Figure 4.6: Breaker-and-a-Half Configuration – Two Diameters

The breaker-and-half configuration has two load points. Separate equivalent series-parallel models were developed for the three following cases:

- Failure at load 1
- Failure at load 2
- Failure at loads 1 and 2

Tables 4.6, 4.7 and 4.8 and Figures 4.7, 4.8 and 4.9 present the minimal cut-sets and equivalent series-parallel reliability block diagrams for these three cases. From these models, the reliability metrics can be determined.

Table 4.6: Minimal Cut Sets for Breaker-and-a-Half for Failure at Load 1

{LINE 14}	{LINE 13, BKR 4, BKR 5}	{LINE 13, BKR 3, BKR 8}
{BKR 7, BKR 8}	{LINE 13, BKR 4, BUS 2}	{LINE 13, BUS 1, BKR 5}
{BKR 7, BKR 5}	{LINE 13, BKR 4, BKR 8}	{LINE 13, BUS 1, BUS 2}
{BKR 7, BUS 2}	{LINE 13, BKR 3, BKR 5}	{LINE 13, BUS 1, BKR 8}
{LINE 13, LINE 16}	{LINE 13, BKR 3, BUS 2}	{LINE 13, BKR 5, BKR 6}
{LINE 16, BKR 6, BKR 7}	{LINE 16, BKR 3, BKR 7}	{LINE 13, BUS 2, BKR 6}
{LINE 16, BUS 1, BKR 7}	{LINE 16, BKR 4, BKR 7}	{LINE 13, BKR 8, BKR 6}

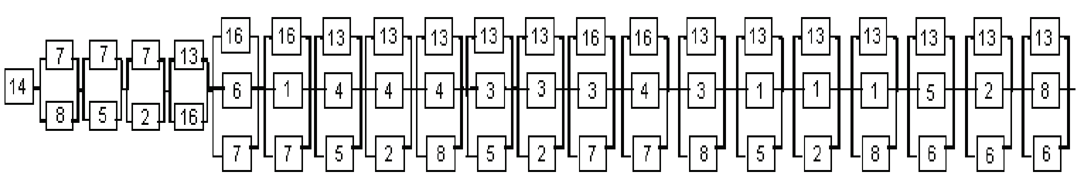

Figure 4.7: Breaker-and-a-Half Transformation at Load 1

Table 4.7: Minimal Cut Sets for Breaker-and-a-Half for Failure at Load 2

{LINE 15}	{LINE 16, BKR 6, BKR 7}	{LINE 16, BKR 6, BUS 2}
{BKR 4, BKR 6}	{LINE 16, BUS 1, BKR 7}	{LINE 16, BUS 1, BUS 2}
{BKR 4, BKR 3}	{LINE 16, BKR 3, BKR 7}	{LINE 16, BKR 3, BUS 2}
{BKR 4, BUS 1}	{LINE 16, BKR 6, BKR 8}	{LINE 16, BKR 6, BKR 5}
{LINE 13, LINE 16}	{LINE 16, BUS 1, BKR 8}	{LINE 16, BUS 1, BKR 5}
{LINE 13, BKR 4, BKR 5}	{LINE 16, BKR 3, BKR 8}	{LINE 16, BKR 3, BKR 5}
{LINE 13, BKR 4, BUS 2}	{LINE 13, BKR 4, BKR 8}	{LINE 13, BKR 4, BKR 7}

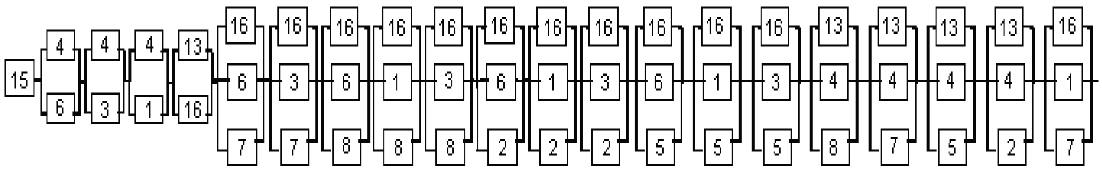

Figure 4.8: Breaker-and-a-Half Transformation at Load 2

Table 4.8: Minimal Cut Sets for breaker-and-a-half for failure at Load 1 & Load 2

{LINE 14,LINE 15}	{BKR 3, BKR 4, LINE 14}	{LINE 13, LINE 15, BKR 3, BKR 5}
{LINE 13,LINE 16}	{BUS 1, BKR 4, LINE 14}	{LINE 13, LINE 15, BKR 3, BUS 2}
{BKR 4, BKR 5, LINE 13}	{BKR 6, BKR 4, LINE 14}	{LINE 13, LINE 15, BKR 3, BKR 8}
{BKR 4, BUS 2, LINE 13}	{BKR 7, BKR 8, BKR 4, BKR 6}	{LINE 13, LINE 15, BKR 5, BUS 1}
{BKR 4, BKR 8, LINE 13}	{BKR 7, BKR 8, BKR 4, BKR 3}	{LINE 13, LINE 15, BUS 2, BUS 1}
{BKR 7, BKR 6, LINE 16}	{BKR 7, BKR 8, BKR 4, BUS 1}	{LINE 13, LINE 15, BKR 8, BUS 1}
{BKR 7, BUS 1, LINE 16}	{BKR 7, BKR 5, BKR 4, BKR 6}	{LINE 13, LINE 15, BKR 6, BKR 5}
{BKR 7, BKR 3, LINE 16}	{BKR 7, BKR 5, BKR 4, BKR 3}	{LINE 13, LINE 15, BKR 6, BUS 2}
{BKR 7, BKR 8, LINE 15}	{BKR 7, BKR 5, BKR 4, BUS 1}	{LINE 13, LINE 15, BKR 6, BKR 8}
{BKR 7, BUS 2, LINE 15}	{BKR 7, BUS 2, BKR 4, BKR 6}	{LINE 14, LINE 16, BKR 6, BKR 8}
{BKR 7, BKR 5, LINE 15}	{BKR 7, BUS 2, BKR 4, BKR 3}	{LINE 14, LINE 16, BUS 1, BKR 8}
{LINE 14,LINE 16,BKR 6,BUS 2}	{BKR 7, BUS 2, BKR 4, BUS 1}	{LINE 14, LINE 16, BKR 3, BKR 8}
{LINE 14,LINE 16,BUS 1,BUS 2}	{LINE 14,LINE 16,BKR 6,BKR 5}	{LINE 14, LINE 16, BKR 3, BKR 5}
{LINE 14,LINE 16,BKR 3,BUS 2}	{LINE 14,LINE 16,BUS 1,BKR 5}	

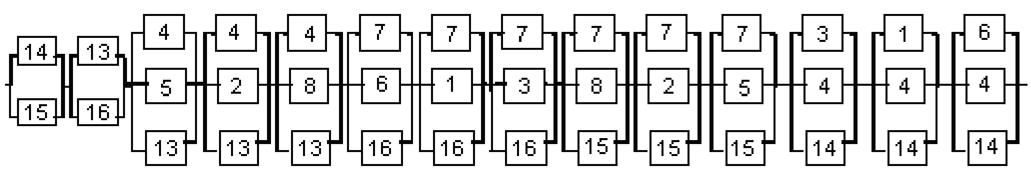

Figure 4.9: Breaker-and-a-Half Transformation at Loads 1 and 2 (up to third order sets)

4.8.2 Breaker-and-a-Third

Figures 4.10 and 4.11 present functional block diagrams for breaker-and-a-third configurations with one diameter and two diameters, respectively. The breaker-and-a-third configuration with two diameters is used to further demonstrate the methodology. In

Figure 4.11, components 13, 14, 15, 16, 27 and 28 represent the lines, components 3, 4, 5, 6, 7, 8, 23 and 24 the breakers, and finally components 1 and 2 represent the buses.

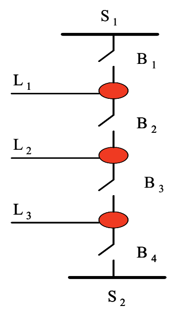

Figure 4.10: Breaker-and-a-Third Configuration – One Diameter

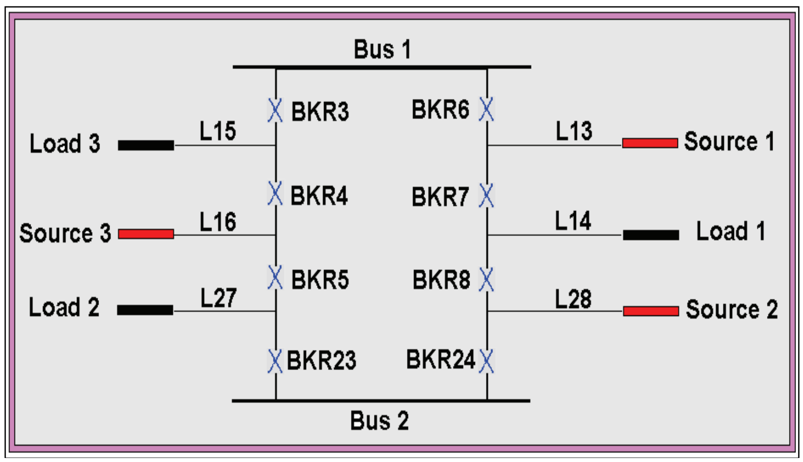

Figure 4.11: Breaker-and-a-Third Configuration – Two Diameters

The breaker-and-a-third configuration has three load points. Separate equivalent series-parallel models were developed for the following cases:

- Failure at load 1
- Failure at load 2
- Failure at load 3
- Failure at loads 1, 2 and 3

Tables 4.9, 4.10, 4.11 and 4.12 and Figures 4.12, 4.13 and 4.14 present the minimal cut-sets and equivalent series-parallel reliability block diagrams for these cases. From these models, the reliability metrics can be determined. The minimal cut sets and the series parallel transformation for breaker-and-a-third for simultaneous failures at loads 1 and 2, 1 and 3, 2 and 3 are shown in Appendix A.

Table 4.9: Minimal Cut Sets for Breaker-and-a-Third for Failure at Load 1

{LINE 14}	{BKR 7, BKR 8}	{LINE 13, LINE 16, LINE 28}
{LINE 28, BKR 7, BUS 2}	{LINE 28, BKR 7, BKR 24}	{LINE 28, BKR 7, BKR 23}
{LINE 28, BKR 7, BKR 5}	{LINE 13, BKR 8, BKR 6}	{LINE 13, BKR 8, BKR 3}
{LINE 13, BKR 8, BUS 1}	{LINE 13, BKR 8, BKR 4}	{LINE 13, BKR 8, LINE 16, BKR 23}
{LINE 13, BKR 8, LINE 16, BKR 5}	{LINE 13, BKR 8, LINE 16, BUS 2}	{LINE 13, BKR 8, LINE 16, BKR 24}
{LINE 13, LINE 28, BKR 6, BKR 24}	{LINE 13, LINE 28, BUS 1, BKR 24}	{LINE 13, LINE28, BKR 4, BKR 24}
{LINE 13, LINE 28, BKR 3, BKR 24}	{LINE 13, LINE 28, BUS 1, BUS 2}	{LINE 13, LINE 28, BKR 6, BUS 2}
{LINE 13, LINE 28, BKR 4, BUS 2}	{LINE 13, LINE 28, BKR 3, BUS 2}	{LINE 13, LINE 28, BUS 1, BKR 23}
{LINE 13, LINE 28, BKR 6, BKR 23}	{LINE 13, LINE 28, BKR 3, BKR 23}	{LINE13, LINE28, BKR4, BKR 23}
{LINE 13, LINE 28, BKR 6, BKR 5}	{LINE13, LINE 28, BUS 1, BKR 5}	{LINE 13, LINE 28, BKR 3, BKR 5}
{LINE 13, LINE28, BKR4, BKR 5}	{LINE 28, LINE 16, BKR 7, BKR 4}	{LINE28, LINE 16, BKR7, BKR 3}
{LINE 28, LINE 16, BKR 7, BUS 1}	{LINE28, LINE 16, BKR7, BKR 6}	

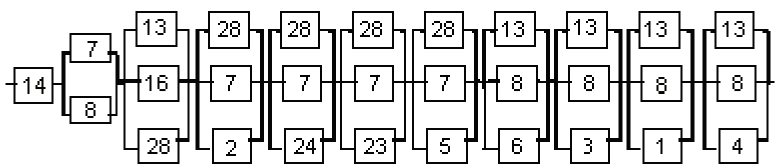

Figure 4.12: Breaker-and-a-third transformation – Load 1 (up to third order sets)

Table 4.10: Minimal Cut Sets for Breaker-and-a-Third for Failure at Load 2

{LINE 27}	{BKR 5, BKR 23}	{BKR 5,BUS 2}
{BKR 5,BKR 24}		
{LINE 28, BKR 5, BKR 8}	{LINE 28, BKR 5, BKR 7}	{LINE 16, BKR 24, BKR 6}
{LINE 16, BKR 23, BKR 4}	{LINE 16, BKR 23, BKR 3}	{LINE 16, BKR 23, BUS 1}
{LINE 16, BKR 23, BKR 6}	{LINE 16,BUS 2,BKR 3}	{LINE 16,BUS 2,BUS 1}
{LINE 16,BUS 2,BKR 4}	{LINE 16,BUS 2,BKR 6}	{LINE 13, LINE 16, LINE 28}
{LINE 16,BKR 24,BKR 4}	{LINE 16,BKR 24,BKR 3}	{LINE 16,BKR 24,BUS 1}
{LINE 13, KR 6,LINE 28,BKR 5}	{LINE 28, BKR 5, LINE 13, BUS 1}	{LINE 13, BKR 3, LINE 28, BKR 5}
{LINE 13,LINE 28,BKR 4,BKR 5}	{LINE 13, LINE 16, BKR 7, BKR 24}	{LINE 13, LINE 16, BKR 8, BKR 24}
{LINE 13,LINE 16,BKR 23,BKR 8}	{LINE 13,BKR 23,LINE 16,BKR 7}	{LINE 28,LINE 16,BKR4, BKR 8}
{LINE 16,BUS 2,BKR 7,BKR 3}	{LINE 16,BUS 2,BKR 8,BKR 3}	{LINE 28,LINE 16,BKR 6,BKR 7}
{LINE 28,LINE 16,BUS 1,BKR 7}	{LINE 28,LINE 16,BKR 3,BKR 7}	{LINE 28,LINE 16,BKR 4,BKR 7}
{LINE 28,LINE 16,BKR 6,BKR 8}	{LINE 28,LINE 16,BUS 1,BKR 8}	{LINE 28,LINE 16,BKR 3,BKR 8}

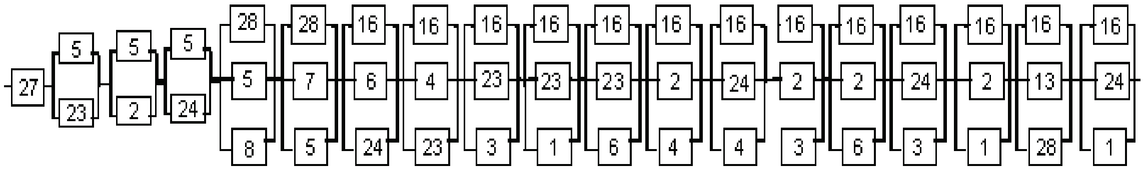

Figure 4.13: Breaker-and-a-third transformation – Load 2 (up to third order sets)

Table 4.11: Minimal Cut Sets for Breaker-and-a-Third for Failure at Load 3

{LINE 15}	{BKR 3, BKR 4}	{BKR 4, BUS 1}
{BKR 6, BKR 4}	{LINE 13, LINE 16, LINE 28}	{LINE 13, BKR 8, BKR 4}
{LINE 16, BKR 3, BKR 24}	{LINE 16, BKR 24, BKR 6}	{LINE 13, BKR 7, BKR 4}
{LINE 16, BKR 3, BKR 23}	{LINE 16, BUS 2, BKR 6}	{LINE 13, LINE 16, BKR 7, BKR 24}
{LINE 16, BKR 3, BKR 5}	{LINE 16, BKR 23, BKR 6}	{LINE 13, LINE 16, BKR 7, BKR 23}
{LINE 16, BKR 3, BUS 2}	{LINE 16, BKR 5, BKR 6}	{LINE 13, LINE 16, BKR 7, BKR 5}
{LINE 16, BKR 24, BUS 1}	{LINE 13, LINE 16, BKR 8, BKR 24}	{LINE 13, LINE 16, BKR 7, BUS 2}
{LINE 16, BKR 24, BUS 2}	{LINE 13, LINE 16, BKR 8, BKR 23}	{LINE 13, LINE 28, BKR 4, BKR 5}
{LINE 16, BKR 24, BKR 23}	{LINE 13, LINE 16, BKR 8, BKR 5}	{LINE 13, LINE 28, BKR 4, BKR 23}
{LINE 16, BKR 24, BKR 5}	{LINE 13, LINE 16, BKR 8, BUS 2}	{LINE 13, LINE 28, BKR 4, BKR 24}
{LINE 16, LINE 28, BKR 7, BKR 6}	{LINE 16, LINE 28, BKR 8, BKR 6}	{LINE 13, LINE 28, BKR 4, BUS 2}
{LINE 16, LINE 28, BKR 7, BUS 1}	{LINE 16, LINE 28, BKR 8, BKR 3}	
{LINE 16, LINE 28, BKR 7, BKR 3}	{LINE 16, LINE 28, BKR 8, BUS 1}	

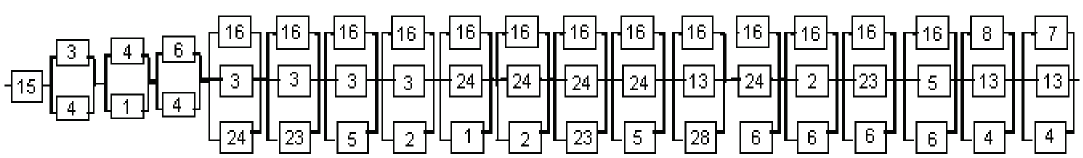

Figure 4.14: Breaker-and-a-third transformation – Load 3 (up to third order sets)

Table 4.12: Minimal Cut Sets for Breaker-and-a-Third for Failure at Loads 1, 2 and 3

{LINE 13, LINE 16, LINE 28}	{LINE 14, LINE 15, LINE 27}	{LINE 16, LINE 28, BKR 7, BKR 6}
{LINE 14, LINE 15, BKR 5, BKR 23}	{LINE 15, LINE 27, BKR 7, BKR 8}	{LINE 16, LINE 28, BKR 7, BKR 3}
{LINE 14, LINE 15, BKR 5, BKR 24}	{LINE 14, LINE 16, BKR 6, BKR 24}	{LINE 16, LINE 28, BKR 7, BUS 1}
{LINE 14, LINE 15, BKR 5, BUS 2}	{LINE 14, LINE 16, BKR 3, BKR 24}	{LINE 13, LINE 28, BKR 4, BKR 5}
{LINE 14, LINE 27, BKR 3, BKR 4}	{LINE 14, LINE 16, BUS 1, BKR 24}	{LINE 28, LINE 15, BKR 5, BKR 7}
{LINE 14, LINE 27, BKR 6, BKR 4}	{LINE 14, LINE 16, BUS 1, BUS 2}	{LINE 13, LINE 27, BKR 4, BKR 8}
{LINE 14, LINE 27, BUS 1, BKR 4}	{LINE 14, LINE 16, BKR 6, BUS 2}	{LINE 13, LINE 16, BKR 8, BKR 24}
{LINE 14, LINE 16, BKR 23, BUS 1}	{LINE 14, LINE 16, BKR 6, BKR 23}	{LINE 13, LINE 16, BKR 8, BUS 2}
{LINE 14, LINE 16, BKR 3, BUS 2}	{LINE 14, LINE 16, BKR 3, BKR 23}	{LINE 13, LINE 16, BKR 8, BKR 23}

{LINE 14, LINE 27, LINE 13, BKR 8, BKR 4}	{LINE 15, LINE 27, LINE 28, BKR 5, BKR 7}
{LINE 14, LINE 27, LINE 13, BKR 7, BKR 4}	{LINE 15, LINE 27, LINE 28, BKR 23, BKR 7}
{LINE 14, LINE 27, LINE 16, BKR 3, BKR 24}	{LINE 15, LINE 27, LINE 28, BUS 2, BKR 7}
{LINE 14, LINE 27, LINE 16, BKR 3, BKR 23}	{LINE 15, LINE 27, LINE 28, BKR 24, BKR 7}
{LINE 14, LINE 27, LINE 16, BKR 3, BKR 5}	{LINE 15, LINE 27, LINE 13, BKR 8, BKR 4}
{LINE 14, LINE 27, LINE 16, BKR 3, BUS 2}	{LINE 15, LINE 27, LINE 13, BKR 8, BKR 3}
{LINE 14, LINE 27, LINE 16, BUS 1, BKR 24}	{LINE 15, LINE 27, LINE 13, BKR 8, BUS 1}
{LINE 14, LINE 27, LINE 16, BKR 24, BKR 6}	{LINE 15, LINE 27, LINE 13, BKR 8, BKR 6}
{LINE 14, LINE 27, LINE 16, BUS 2, BKR 6}	{BKR 7, BKR 8, BKR 5, BKR 23, LINE 15}
{LINE 14, LINE 27, LINE 16, BKR 23, BKR 6}	{BKR 7, BKR 8, BKR 5, BUS 2, LINE 15}
{LINE 14, LINE 27, LINE 16, BKR 5, BKR 6}	{BKR 7, BKR 8, BKR 5, BKR 24, LINE 15}
{LINE 14, LINE 15, LINE 28, BKR 5, BKR 8}	{BKR 5, BKR 23, BKR 3, BKR 4, LINE 14}
{LINE 14, LINE 15, LINE 28, BKR 5, BKR 7}	{BKR 5, BUS 2, BKR 3, BKR 4, LINE 14}
{LINE 14, LINE 15, LINE 16, BKR 23, BKR 4}	{BKR 5, BKR 24, BKR 3, BKR 4, LINE 14}
{LINE 14, LINE 15, LINE 16, BKR 23, BKR 3}	{BKR 5, BKR 23, BUS 1, BKR 4, LINE 14}
{LINE 14, LINE 15, LINE 16, BKR 23, BUS 1}	{BKR 5, BUS 2, BUS 1, BKR 4, LINE 14}
{LINE 14, LINE 15, LINE 16, BKR 23, BKR 6}	{BKR 5, BKR 24, BUS 1, BKR 4, LINE 14}
{LINE 14, LINE 15, LINE 16, BUS 2, BKR 4}	{BKR 5, BKR 23, BKR 6, BKR 4, LINE 14}
{LINE 14, LINE 15, LINE 16, BKR 24, BKR 4}	{BKR 5, BUS 2, BKR 6, BKR 4, LINE 14}
{LINE 14, LINE 15, LINE 16, BKR 24, BKR 6}	{BKR 5, BKR 24, BKR 6, BKR 4, LINE 14}
{LINE 14, LINE 15, LINE 16, BKR 24, BUS 1}	{BKR 7, BKR 8, BKR 3, BKR 4, LINE 27}
{LINE 14, LINE 15, LINE 16, BKR 24, BKR 3}	{BKR 7, BKR 8, BUS 1, BKR 4, LINE 27}
{LINE 14, LINE 15, LINE 16, BUS 2, BKR 6}	{BKR 7, BKR 8, BKR 6, BKR 4, LINE 27}
{LINE 14, LINE 15, LINE 16, BUS 2, BUS 1}	{LINE 14, LINE 28, LINE 16, BKR 6, BKR 8}
{LINE 14, LINE 15, LINE 16, BUS 2, BKR 23}	{LINE 14, LINE 28, LINE 16, BUS 1, BKR 8}
{LINE 14, LINE 15, LINE 13,LINE 28,LINE 16}	{LINE 14, LINE 28, LINE 16, BKR 3, BKR 8}
{LINE 14, LINE 13, LINE 16, BKR 23, BKR 8}	{LINE 14, LINE 13, LINE 16, BKR 23, BKR 7}
{LINE 27, LINE 16, LINE 28, BKR 7, BKR 6}	{LINE 15, LINE 13, LINE 28, BKR 4, BKR 5}
{LINE 27, LINE 16, LINE 28, BKR 7, BUS 1}	{LINE 15, LINE 13, LINE 28, BKR 3, BKR 5}
{LINE 27, LINE 16, LINE 28, BKR 7, BKR 3}	{LINE 15, LINE 13, LINE 28, BUS 1, BKR 5}
{LINE 27, LINE 13, LINE 28, BKR 4, BKR 5}	{LINE 15, LINE 13, LINE 28, BKR 6, BKR 5}
{LINE 27, LINE 13, LINE 28, BKR 4, BKR 23}	{LINE 15, LINE 16, LINE 28, BKR 7, BKR 6}
{LINE 27, LINE 13, LINE 28, BKR 4, BUS 2}	{LINE 15, LINE 16, LINE 28, BKR 7, BUS 1}
{LINE 27, LINE 13, LINE 28, BKR 4, BKR 24}	{LINE 15, LINE 16, LINE 28, BKR 7, BKR 3}
{LINE 15, LINE 13, LINE 16, BKR 8, BKR 24}	{LINE 15, LINE 16, LINE 28, BKR 7, BKR 4}
{LINE 15, LINE 13, LINE 16, BKR 8, BKR 23}	

4.8.3 DESN Configuration

The DESN configuration has been adopted (by Hydro One) as a basic design for supplying electricity to significant load areas. Figure 4.15 presents a functional diagram of a DESN configuration. Components 12 and 13 representing supply lines, components 1 and 2 are the power transformers, buses are defined by components 6 and 7, and finally, breakers are represented with components 3, 4, 5, 10, 20 and 11.

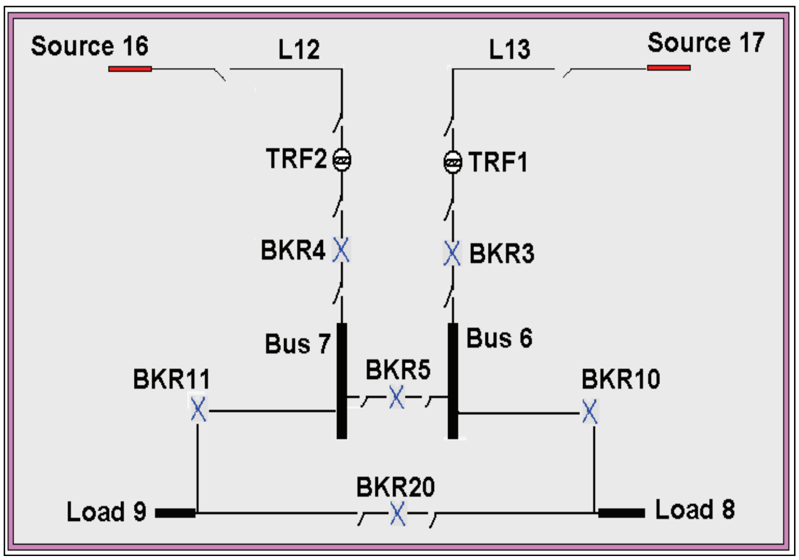

Figure 4.15: DESN Configuration

Tables 4.13, 4.14 and 4.15 and Figures 4.16, 4.17 and 4.18 present the minimal cut sets and the equivalent series-parallel model that is used as an approximation to compute reliability metrics.

Table 4.13: DESN Minimal Cut Sets for Load 9

{BUS 7}	{BKR 11}	{BKR 4, BKR 5}
{LINE 13, BKR 4}	{TRF 1, BKR 4}	{TRF 2, BKR 5}
{LINE 13, TRF 2}	{TRF 1, TRF 2}	{LINE 12, BKR 5}
{LINE 13, LINE 12}	{TRF 1, LINE 12}	{BKR 4, BKR 3}
{BKR 4, BUS 6}	{LINE 12, BKR 3}	{TRF 2, BKR 3}
{TRF 2, BUS 6}	{LINE 12, BUS 6}	

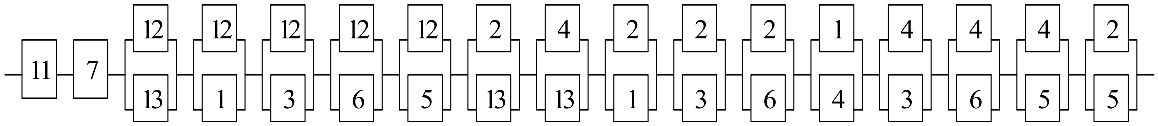

Figure 4.16: DESN Series-Parallel Transformation with Failure at Load 1

Table 4.14: DESN Minimal Cut Sets for Load 8

{LINE 12, LINE 13}	{BUS 6}	{BKR 10}
{ LINE 13,TRF 2}	{LINE 13, BKR 4}	{LINE 13, BUS 7}
{LINE 13, BKR 5}	{LINE 12, TRF 1}	{LINE 12, BKR 3}
{ TRF 1, TRF 2}	{ TRF 1, BKR 4}	{TRF 1, BUS 7}
{ TRF 1, BKR 5}	{BKR 3, TRF 2}	{BKR 3, BKR 4}
{BKR 3, BUS 7}	{BKR 3, BKR 5}	

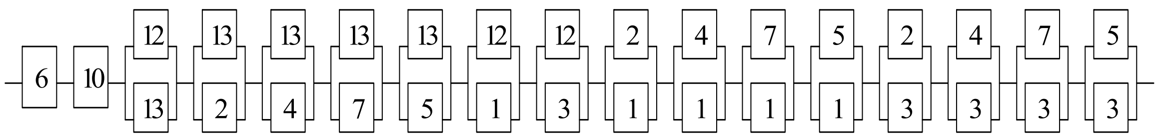

Figure 4.17: DESN Series-Parallel Transformation with Failure at Load 2

Table 4.15: DESN Minimal Cut Sets for Load 8 and Load 9

{BUS 7, BUS 6}	{TRF 2, BKR 3}	{BUS 6, BKR 4}
{BUS 7, BKR 10}	{TRF 2, TRF 1}	{BUS 6, TRF 2}
{BKR 11, BUS 6}	{TRF 2, LINE 13}	{BUS 6, LINE 12}
{BKR 11, BKR 10}	{LINE 12, BKR 3}	{BUS 7, BKR 3}
{BKR 4, BKR 3}	{LINE 12, TRF 1}	{BUS 7, TRF 1}
{BKR 4, TRF 1}	{LINE 12, LINE 13}	{BUS 7, LINE 13}
{BKR 4, LINE 13}		

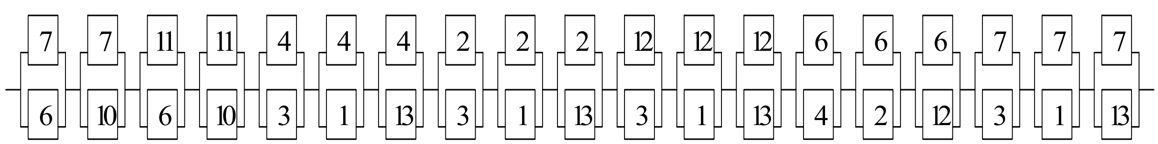

Figure 4.18: DESN Series-Parallel Transformation with Failure at Load 1 and Load 2

4.9 Conclusions

In this chapter commonly used approximation techniques for analyzing the reliability of series, parallel and series-parallel systems have been developed. They can be used to analyze any kind of electricity transmission/distribution configuration, as long as failure definition is specified and cut-sets can be determined.

In particular, a series-parallel transformation has been presented to obtain reliability metrics for the following cases:

(1) Component sustained outages overlapping component sustained outages.

(2) Component sustained outages overlapping component maintenance outages.

(3) Component temporary outages overlapping component sustained outages.

The series-parallel approximation can be applied to any system provided the sets of components that guarantee system failure, commonly known as minimal cut sets, are known. In order to test the effectiveness of the developed approximation models, extensive experimentation has been performed. For different data cases, two different configurations were simulated to obtain outage rates, average repair time and total system downtime. The simulation results provided satisfactory results about the adequacy of the proposed metrics.

Chapter 5

Component Criticality Importance Measures for the Power Industry

New component reliability importance measures have been developed to be used within the power industry (Espiritu *et al.*, 2007). These measures can be used by designers and managers to identify and rank the most important components within the system, and specifically, to identify where investments should be made to increase the overall system availability. Several existing popular reliability criticality measures cannot be directly applied to the power systems, because they have been developed mainly for components with specified finite mission times. Alternatively, for ETDS, the different components within the system exhibit outage rates and repair rates instead of probability of failure for a specified time interval.

5.1. Introduction

ETDS provide a power supply to the customers as economically and reliably as possible. Increasing the investment in the planning or operating phase can increase system availability but it can also lead to increased costs. Consequently, the economic considerations may become prohibitive. On the other hand, under-investment can lead to other problems, including excessive maintenance cost and loss of power to the consumer. Thus, the economic and reliability constraints become competitive, and it is critical to have timely and accurate indicators and metrics to characterize the reliability of the system. Therefore, there is a need for a formal methodology to calculate the importance of each component of the system and to rank them. In that way, managers and administrators can prioritize where investments should be made to upgrade old and aging equipment and to guarantee the maximum increase of reliability considering the whole distribution system.

Reliability analysts have proposed different analytical and empirical critical importance measures which rank the components regarding their importance in the system. Several importance measures such as Birnbaum (1969); Fussell-Vesely (1975); Reliability Achievement Worth, Reliability Reduction Worth (Gandini, 1990; Wendai *et al.,* 2004; Levitin *et al.,* 2003), have been proposed in the past. But none of these measures can be directly applied to ETDS because these methodologies were developed on the assumption that there is a definite time period (or mission time) for the system. In contrast to this, ETDS have no definite life time period and are expected to work endlessly without any service interruptions. In addition, for ETDS, component and system reliability are more meaningful in terms of outage rates rather than probability of failure.

In this chapter, importance measures such as Birnbaum, Criticality Importance, Reliability Achievement Worth (RAW), Reliability Reduction Worth (RRW) and Fussell-Vesely have been extended to make them compatible with ETDS. The new proposed metrics are applied to some commonly used electrical configurations, including breaker-and-a-half, breaker-and-a-third and the dual element spot network (DESN) for ETDS.

System reliability is generally defined as the probability that the system provides the service for which was intended for a specified mission time, under the condition that the components of the system can be in two possible states, either failed or perfect functioning. However some systems, like in the distribution systems area, have more complex behavior and the components and system exhibit failures as a stochastic process in terms of outage rates. The total system unavailability can be expressed in terms of total system downtime. Selection of an inappropriate metric, lead to erroneous conclusions about the system reliability and incorrect conjectures regarding reliability improvement efforts.

In the power systems area, Hamoud *et al*. (2004) presented a method that relates the reliability of the electricity distribution systems at a specific load point to the reliability of each individual component. The assessment of component criticality was performed by using a computer program and it was performed for each component in the system. Hilber & Bertling (2004) proposed an importance index method for defining the importance of individual components in an electrical network with respect to total interruption cost. In their approach, they considered several load points simultaneously and they ranked the components in terms of total system interruption cost.

Hamoud *et al.* (2003) proposed a method for quantifying the risk associated with the failure of the SCADA (Supervisory Control and Data Acquisition) systems used in power systems. The calculated risk is used to identify the importance of stations and to establish the reliability requirements for the SCADA system that has the lowest capital cost.

Due to economic constraints, investments to upgrade aging systems need to be made appropriately. Therefore, a company or utility needs to know the reliability importance or importance ranking of the components within the system. Thus, there is a need for new quantitative criticality measures that can be directly applied to the ETDS area, such that managers and other decision makers have useful metrics to evaluate where investments could be made in order to improve the functioning and reliability of the overall system.

5.2. Background: Overview of criticality measures

Importance measures are widely used in systems engineering to identify components within the system that more significantly influence the system behavior with respect to reliability, risk and/or safety. The information gathered by the use of importance measures provides management with useful insights for the safe and efficient operation of the system.

Importance measures (or reliability importance indices) are valuable in establishing direction and prioritization of actions related to an upgrading effort (reliability improvement) in system design, or suggesting the most effective way to operate and maintain system status. In general, importance measures (Meng, 1995; Hoyland & Rausand, 1994; Levitin *et al.,* 2003; Ramirez-Marquez & Coit, 2005; etc.) are used to quantify the contribution of individual elements of a system to the overall system performance (e.g., reliability, risk, availability).

Birnbaum (1969) first introduced the concept of importance in 1969 and he contributed one of the most widely used reliability importance measures. Analytically, it is defined by:

$$I_i^B(t) = \frac{\partial R_s(t)}{\partial R_i(t)} = R_s(t; R_i(t) = 1) - R_s(t; R_i(t) = 0) \tag{5.1}$$

where:

$I_i^B(t)$ = Birnbaum importance of component *i*

$R_s(t)$ = system reliability at time *t*

$R_i(t)$ = reliability of component *i* at time *t*

$R_s(t; R_i(t) = 0)$ = system reliability at time *t* given component *i* is failed

$R_s(t; R_i(t) = 1)$ = system reliability at time t given component i is perfectly working

The Birnbaum importance ranking represents the maximum loss in system reliability when component i switches from the condition of perfect functioning ($R_i(t) = 1$) to the condition of certain failure ($R_i(t) = 0$). A weakness of Birnbaum importance measure is that $I_i^B(t)$ does not depend on the specific numerical value of component reliability, $R_i(t)$, or unreliability, $F_i(t)$. Therefore, two components may have a similar metric $I_i^B(t)$ value, although these current levels of reliability could differ substantially. In practice, the less reliable component is generally a greater concern, i.e., more critical.

The Criticality Importance (CI) measure is another popular existing metric. The CI metric is a natural extension of the Birnbaum metric. The CI metric includes the component unreliability, $F_i(t)$, whereas the Birnbaum measure does not. In this way, a less reliable component is given more attention, i.e., is more critical. The CI ranking is mathematically expressed as:

$$I_i^{CR}(t) = I_i^B(t)\frac{F_i(t)}{F_s(t)} = \left[R_s(t; R_i(t) = 1) - R_s(t; R_i(t) = 0)\right]\frac{F_i(t)}{F_s(t)} \tag{5.2}$$

where:

$F_s(t)$ = system unreliability at time t

$F_i(t)$ = unreliability of component i at time t

Two other types of measures that are commonly used to rank the importance of components in a system (Vasseur & Llory, 1999) are the Reliability Reduction Worth (RRW) and the Reliability Achievement Worth (RAW). It is important to note that these are defined in terms of reliability, and not risk. The RRW considers the impact of a loss of reliability, and alternatively, RAW considers the impact of an increase in reliability. In other literature, RAW and RRW can also be used to denote risk achievement worth and risk reduction worth.

The Reliability Achievement Worth (RAW) is the ratio of the actual system reliability obtained when element *i* is always in perfect functioning ($R_i(t)=1$) to the actual value of the system reliability. This measure quantifies the maximum possible percentage increase in system reliability generated by a particular component. It is defined as:

$$\mathrm{RAW}_i = \frac{R_s(t; R_i(t)=1)}{R_s(t)} \tag{5.3}$$

The Reliability Reduction Worth importance measure, as defined by Levitin *et al*. (2003), is the ratio of the actual system reliability to the value of the system reliability when element *i* is considered always failed ($R_i(t)=0$). This measure is an index measuring the potential damage caused to the system by a particular component. The expression for RRW is given as:

$$\mathrm{RRW}_i = \frac{R_s(t)}{R_s(t; R_i(t)=0)} \tag{5.4}$$

Fussell & Vesely (1975) proposed an alternative measure. According to this measure, the importance of a component in the system depends on the number and on the order of the cut-sets in which it appears. It quantifies the maximum decrement in system reliability caused by a particular component. There are different forms of the Fussell-Vesely metric (Elsayed, 1996; Levitin *et al.*, 2003).One version of the FV expression is defined by:

$$I_i^{FV}\left(t\right) = \frac{R_s(t) - R_s(t; R_i(t)=0)}{R_s(t)} \tag{5.5}$$

where:

I_i^{FV} = Fussell-Vesely importance of component i

The previously introduced criticality importance measures (Birnbaum, Criticality Importance, RRW, RAW and Fussell-Vesely) are functionally different. They measure subtly different properties of the system behavior, and thus, one can infer different information from each one of them. There is no general consensus that one measure is the "best" and none of them can be directly applied to ETDS without the selection of some mission time, t, which would be somewhat arbitrary given the indefinite need for electricity.

5.3 Custom Criticality Measures

Traditional importance measures were reviewed in the previous section. None can be applied directly to ETDS because they are not properly characterized by probability of failure or success for specified mission times. ETDS have more specific measures such as outage rates, repair time and system downtime. In this section, the previously discussed measures have been transformed and applied to ETDS.

The newly developed metrics for the evaluation of the importance of components for the ETDS were developed, based on the original criticality measures, by using the individual component sustained outage rates (λ_i) instead of probability of failure at a specific time, $F_i(t)$. The new metrics are related to an increase or decrease of the total system unavailability (U_s) rather than system reliability at an specified time.

All of the developed equations require a lower limit ($\lambda_i = l_i$) and an upper limit ($\lambda_i = u_i$) outage rate specification for each individual component in the system. Selection of these values depends on each specific component. There is the possibility for selecting

∞ and 0; however, these two extremes may not be realistic. Often there is past data and knowledge to select realistic bounds In all of our examples, we have selected as maximum outage rate of 100 outages/year, and as minimum outage rate of 0 outages/year. In practice, lower and upper limits should be based on the highest and lowest conceivable outage rates for a particular application. Past experience or data may be useful to make this selection.

The new criticality importance measures are presented in the remainder of this section. Examples of applications of the developed measures for some common electric configurations are shown in Section 5.4.

Transformed Birnbaum Importance Measure

$$I_i^{ETS-B} = U_s(\boldsymbol{\lambda},\mathbf{r} \mid \lambda_i = u_i) - U_s(\boldsymbol{\lambda},\mathbf{r} \mid \lambda_i = l_i) \tag{5.6}$$

where:

$U_s(\boldsymbol{\lambda},\mathbf{r})$ = system unavailability

l_i = lower limit for sustained outage rate of component *i* (outages/year)

u_i = upper limit for sustained outage rate of component *i* (outages/year)

λ_i = sustained outage rate of component *i*

$U_s(\boldsymbol{\lambda},\mathbf{r} \mid \lambda_i = l_i)$ = system unavailability when the sustained outage rate of component *i* is l_i

$U_s(\boldsymbol{\lambda},\mathbf{r} \mid \lambda_i = u_i)$ = system unavailability when the sustained outage rate component *i* is u_i

$\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \ldots, \lambda_i)$

$\mathbf{r} = (r_1, r_2, \ldots, r_i)$

The transformed Birnbaum importance measure represents the maximum change in system unavailability when component *i* switches from the condition of highest possible availability, $U_s(\boldsymbol{\lambda},\mathbf{r} \mid \lambda_i = l_i)$ to the condition of lowest possible availability, $U_s(\boldsymbol{\lambda},\mathbf{r} \mid \lambda_i = u_i)$. In order to explicitly consider the actual component outage rate, we have also proposed the following transformed Criticality Importance measure.

Transformed Criticality Importance Measure

$$I_i^{ETS-CR} = \left[U_s(\boldsymbol{\lambda},\mathbf{r} \mid \lambda_i = u_i) - U_s(\boldsymbol{\lambda},\mathbf{r} \mid \lambda_i = l_i)\right]\left(\frac{\lambda_i}{U_s(\boldsymbol{\lambda},\mathbf{r})}\right) \tag{5.7}$$

Transformed Reliability Reduction Worth/ Reliability Achievement Worth

Two other commonly used criticality importance measures have been extended to be directly applied in the ETDS area. The Transformed Reliability Reduction Worth measure and the Transformed Reliability Achievement Worth measure are given by <u>Transformed Reliability Reduction Worth (RRW)</u>:

$$I_i^{ETS-RRW} = \frac{U_s(\boldsymbol{\lambda},\mathbf{r} \mid \lambda_i = u_i)}{U_s(\boldsymbol{\lambda},\mathbf{r})} \tag{5.8}$$

<u>Transformed Reliability Achievement Worth (RAW):</u>

$$I_i^{ETS-RRW} = \frac{U_s(\boldsymbol{\lambda},\mathbf{r})}{U_s(\boldsymbol{\lambda},\mathbf{r} \mid \lambda_i = l_i)} \tag{5.9}$$

Transformed Fussell-Vesely (F-V):

Finally the Transformed Fussell-Vesely importance measure is defined as follows:

$$I_i^{ETS-RRW} = \frac{U_s(\boldsymbol{\lambda},\mathbf{r} \mid \lambda_i = u_i) - U_s(\boldsymbol{\lambda},\mathbf{r})}{U_s(\boldsymbol{\lambda},\mathbf{r})} \tag{5.10}$$

5.4 Numerical examples

The different importance measures were applied to several commonly used ETDS. The configurations are the breaker-and-a-half, breaker-and-a-third and DESN.

5.4.1 Example 1

The DESN configuration is a common type of configuration used in the power industry. It has two different load points (8, 9) as shown in Figure 5.1 The components labeled 12 and 13 represent two lines; components 1 and 2 represent the transformers, and circuit breakers are represented by components 3, 4, 5, 10, 11 and 20. Finally, components 6 and 7 represent the buses. Table 5.1 shows the component data used for each component in the DESN configuration. Typical outage rates and repair rates are given. In the examples, they are varied for similar components so the effect of different outage rates can be observed in the transformed criticality measures. For each component, the maximum sustained outage rate (u_i) was set to 100 outages/year and the minimum was set to 0. In practice, these values should be selected based on a particular problem, and the highest and lowest conceivable outage rates for that application.

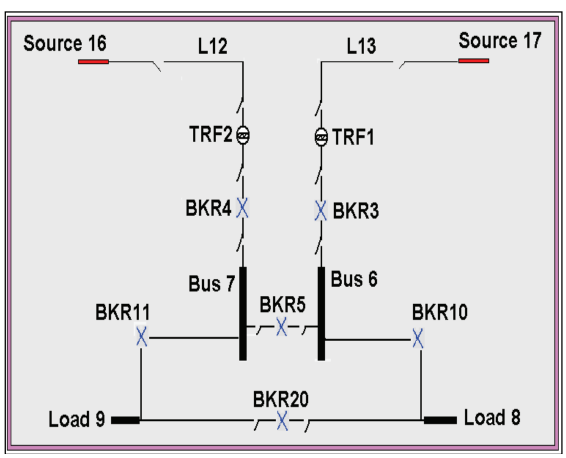

Figure 5.1 DESN Configuration

Table 5.1 Outage Rates and Repair Times for Components - DESN

Component	Sustained outage rate, λ_i (outages / year)	Repair duration, r_i (hours / outage)
Line 12	0.758	12
Line 13	0.657	14
TRF 1	0.165	116
TRF 2	0.178	145
Bus 6	0.100	2.9
Bus 7	0.120	3.5
BKR 3	0.060	157
BKR 4	0.062	134
BKR 5	0.070	176
BKR 10	0.090	160
BKR 11	0.060	168

All minimal cut-sets (up to fourth order) were obtained for failures at load 8, load 9 and simultaneous failures at both loads. For failures at load 8, load 9, and loads 8 & 9, there are 17, 17 and 19 minimal cut sets, respectively that lead to system failure. The minimal cut sets are shown in Appendix A. Reliability importance measures were computed for each component and for each new criticality measures. The components were ranked according to their importance based on their respective metric values as given in Tables 5.2, 5.3 and 5.4.

In Table 5.2, we can see that for failures at load 9, in all the different metrics, the most important component is the breaker 11, the second ranked component is bus 7 and the least important component is breaker 10, which has no impact at all for failure at load 9.

Table 5.2. Component Rankings and Metric Values for Failure at Load L9

Component	Birnbaum		CI		RAW		RRW		F-V	
	Rank	Value	Rank	Value	Rank	Value	Rank	Value	Rank	Value
Line 12	9	6.89973	6	0.48658	6	1.00489	9	1.63706	9	0.63706
Line 13	8	6.90635	9	0.42215	9	1.004239	8	1.63832	8	0.63832
TRF 1	7	57.224	4	0.878447	4	1.008862	7	6.31514	7	5.31514
TRF 2	4	83.3717	3	1.380676	3	1.014	4	8.7428	4	7.7428
Bus 6	10	1.4306	10	0.01331	10	1.000133	10	1.13296	10	0.13296
Bus 7	2	350	2	3.907533	2	1.040664	2	33.5237	2	32.5237
BKR 3	5	77.4497	8	0.432339	8	1.004342	5	8.20133	5	7.20133
BKR 4	6	77.0469	7	0.444427	7	1.004464	6	8.16373	6	7.16373
BKR 5	3	86.8226	5	0.565437	5	1.005687	3	9.07202	3	8.07202
BKR 10	11	0	11	0	11	1	11	1	11	0
BKR 11	1	16800	1	93.78078	1	16.0792	1	1563.08	1	1562.08

Table 5.3 shows the component rankings for the different components in the DESN configuration for failures at load 8. In all the different metrics the component ranked highest is breaker 10, and the component ranked the lowest is breaker 11. This is as expected, because the breaker 11 has no influence in failures at load 8.

Table 5.3. Component Rankings and Metric Values for Failure at Load L8

Component	Birnbaum		CI		RAW		RRW		F-V	
	Rank	Value	Rank	Value	Rank	Value	Rank	Value	Rank	Value
Line 12	9	5.17233	8	0.26258	8	1.00263	9	1.34379	9	0.34379
Line 13	8	8.94242	6	0.39348	6	1.00395	8	1.59497	8	0.59497
TRF 1	5	74.0943	3	0.81879	3	1.00826	5	5.9542	5	4.9542
TRF 2	6	62.499	4	0.74507	4	1.00751	6	5.17835	6	4.17835
Bus 6	2	290	2	1.94224	2	1.01981	2	20.403	2	19.403
Bus 7	10	1.5086	10	0.01212	10	1.00012	10	1.10092	10	0.10092
BKR 3	3	100.283	5	0.40298	5	1.00405	3	7.71231	3	6.71231
BKR 4	7	57.7577	9	0.23983	9	1.0024	7	4.86586	7	3.86586
BKR 5	4	75.8608	7	0.35565	7	1.00357	4	6.07714	4	5.07714
BKR 10	1	16000	1	96.4425	1	28.1096	1	1071.62	1	1070.62
BKR 11	11	0	11	0	11	1	11	1	11	0

In the analysis for simultaneous failures at both loads, the components that are the most important are the breaker 3 for the Birnbaum, RRW and F-V, and the transformer 2 is ranked highest in criticality importance for RAW and Criticality Importance.

Table 5.4. Component Rankings and Metric Values for Failure at Loads 8 & 9

Component	Birnbaum		CI		RAW		RRW		F-V	
	Rank	Value	Rank	Value	Rank	Value	Rank	Value	Rank	Value
Line 12	8	5.21205	5	19.07528	5	1.23572	8	25.9745	8	24.97452
Line 13	7	6.97347	4	22.12112	4	1.28405	7	34.4487	7	33.44868
TRF 1	4	57.7802	2	46.03155	2	1.85293	4	279.519	4	278.5187
TRF 2	2	62.979	1	54.12631	1	2.1799	2	304.539	2	303.5391
Bus 6	10	1.7782	10	0.858567	10	1.00866	10	9.57708	10	8.577082
Bus 7	9	2.09553	9	1.214135	9	1.01229	9	11.1056	9	10.10565
BKR 3	1	78.2025	3	22.65502	3	1.29291	1	378.357	1	377.3572
BKR 4	3	58.2013	6	17.42276	6	1.21099	3	281.838	3	280.838
BKR 5	11	0	11	0	11	1	11	1	11	0
BKR 10	6	19.1781	7	8.333748	7	1.09091	6	93.5139	6	92.51387
BKR 11	5	28.1726	8	8.161518	8	1.08887	5	136.944	5	135.9437

5.4.2 Example 2

Figure 5.2 represents a functional diagram of a breaker-and-a-half configuration with two diameters. In this diagram, components 13 and 16 represent the supply lines, components 14 and 15 are the load lines, the circuit breakers are defined by components 3, 4, 5, 6, 7, and 8, and finally, the buses are represented with components 1 and 2. The minimal cut sets (up to 4th order) for this configuration are shown in Appendix A.

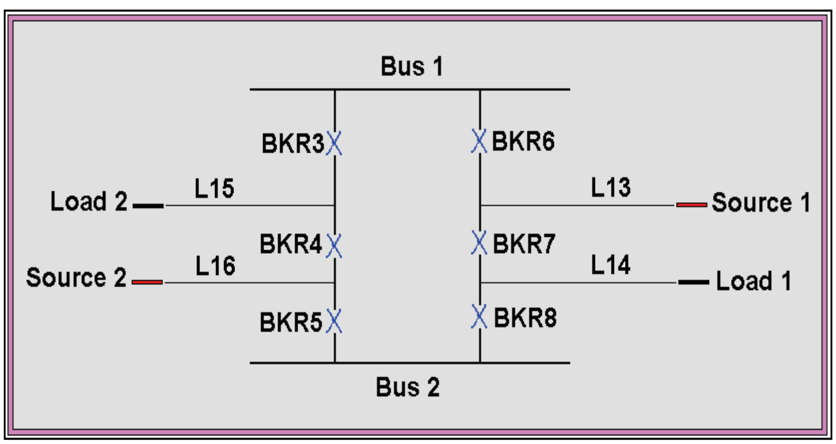

Figure 5.2 Breaker-and-a-Half Configuration – Two Diameters

The individual component sustained outage rates and repair times used in this example are shown in Table 5.5. The sensitivity analysis is performed considering the total unavailability of the system at the different load points of the breaker-and-a-half configuration. Table 5.5 shows the component data used for each component in the breaker-and-a-half configuration. Similar to the previous example, the lines have the maximum sustained outage rates, while the breakers have the minimum outage rates in the system.

Table 5.5. Outage Rates and Repair Times for Components - Breaker-and-a-Half

Component	Sustained outage rate, λ_i (outages / year)	Repair duration, r_i (hours / outage)
Line 13	0.930	15
Line 14	0.860	12
Line 15	0.780	18
Line 16	0.880	10
Bus 1	0.200	2.7
Bus 2	0.180	3.2
BKR 3	0.090	150
BKR 4	0.076	167
BKR 5	0.023	159
BKR 6	0.070	176
BKR 7	0.034	168
BKR 8	0.056	146

In this electric configuration, there are 21 minimal cut sets (up to fourth order) that lead to system failure at load point 1, 21 minimal cuts for failure at load 2, and for simultaneous failure at both loads, there are a total of 41 minimal cut sets. Based on these minimal cuts and by using the recursive equations (Coit *et al.*, 2005) to obtain the total system downtime, the five new criticality measures were applied to determine the importance of each component within the system. Tables 5.6, 5.7 and 5.8 show the metric values and ranking of the different components in the breaker-and-a-half configuration for failures at load 1, load 2 and failures at load 1 and load 2, respectively.

In Table 5.6 we can see that for all metrics, the most important component is line 14, because the outage rate of this component definitely impacts the unavailability at load 1. The second ranked component in importance is the breaker 7 by using transformed Birnbaum, RRW and F-V, while by using the transformed CI and RAW, it is line 13. The least important component in the system is line 15, as would be expected because it is a load and not a source point. Furthermore, we can consider bus 1 as the second least important component for failures at load 1.

Table 5.6 Component Rankings and Metric Values for Failures at Load L1

Component	Birnbaum		CI		RAW		RRW		F-V	
	Rank	Value	Rank	Value	Rank	Value	Rank	Value	Rank	Value
Line 13	6	1.51632	2	0.13635	2	1.0013654	6	1.145251	6	0.145251
Line 14	1	1200	1	99.78516	1	465.47318	1	116.0314	1	115.0314
Line 15	12	0	12	0.00000	12	1	12	1	12	0
Line 16	5	1.59537	3	0.13575	3	1.0013593	5	1.152901	5	0.152901
Bus 1	11	0.00079	11	0.00002	11	1.0000002	11	1.000076	11	7.58E-05
Bus 2	7	0.21093	7	0.00367	7	1.0000367	7	1.020358	7	0.020358
BKR 3	10	0.04366	8	0.00038	8	1.0000038	10	1.004218	10	0.004218
BKR 4	9	0.04861	9	0.00036	9	1.0000036	9	1.004697	9	0.004697
BKR 5	3	10.4805	6	0.02331	6	1.0002331	3	2.013142	3	1.013142
BKR 6	8	0.05123	4	0.00035	4	1.0000035	8	1.00495	8	0.00495
BKR 7	2	23.8733	10	0.07848	10	1.0007855	2	3.307551	2	2.307551
BKR 8	4	9.62365	5	0.05211	5	1.0005214	4	1.930000	4	0.93

Table 5.7 shows all of the metric values for the developed criticality importance measures, for failure at load 2. Line 15 is the most important component and breaker 4 is the second most important component in the system when considering failures at load 2. The least important components in the electric configuration are the line 14 and the bus number 2.

Table 5.7 Component Rankings and Metric Values for Failures at Load L2

Component	Birnbaum		CI		RAW		RRW		F-V	
	Rank	Value	Rank	Value	Rank	Value	Rank	Value	Rank	Value
Line 13	6	1.511345	6	0.099739	6	1.0009984	6	1.106249	6	0.106249
Line 14	12	0	12	0	12	1	12	1	12	0
Line 15	1	1800	1	99.62886	1	269.43998	1	127.733	1	126.733
Line 16	5	1.598690	5	0.099831	5	1.0009993	5	1.112446	5	0.112446
Bus 1	7	0.391752	7	0.00556	7	1.0000556	7	1.027743	7	0.027743
Bus 2	11	0.001705	11	2.18E-05	11	1.0000002	11	1.000121	11	0.000121
BKR 3	4	21.76404	3	0.138995	3	1.0013919	4	2.543003	4	1.543003
BKR 4	2	50.30752	2	0.271309	2	1.0027205	2	4.567145	2	3.567145
BKR 5	9	0.084749	10	0.000138	10	1.0000014	9	1.006012	9	0.006012
BKR 6	3	25.53648	4	0.126846	4	1.0012701	3	2.810819	3	1.810819
BKR 7	8	0.089546	9	0.000216	9	1.0000022	8	1.006352	8	0.006352
BKR 8	10	0.077819	8	0.000309	8	1.0000031	10	1.005519	10	0.005519

In the analysis for failures at both load points, we see from the results shown in Table 5.8, that the lines are the four most important components for all different metrics, while the buses are the least important components for all the different criticality importance measures.

Table 5.8 Component Rankings and Metric Values for Failures at Loads L1 and L2

Component	Birnbaum		CI		RAW		RRW		F-V	
	Rank	Value	Rank	Value	Rank	Value	Rank	Value	Rank	Value
Line 13	4	1.509938	3	45.80344	3	1.845136	4	49.79298	4	48.79298
Line 14	2	1.928525	1	54.09781	1	2.178546	2	63.36346	2	62.36346
Line 15	1	2.122223	2	53.99351	2	2.173607	1	69.68253	1	68.68252
Line 16	3	1.594433	4	45.76622	4	1.843869	3	52.54941	3	51.54940
Bus 1	12	0.00064	12	0.004172	12	1.000042	12	1.020818	12	0.020818
Bus 2	11	0.001078	11	0.006331	11	1.000063	11	1.035112	11	0.035111
BKR 3	10	0.035529	6	0.1043	6	1.001044	10	2.157846	10	1.157845
BKR 4	5	0.09692	5	0.240261	5	1.002408	5	4.158927	5	3.158926
BKR 5	7	0.053526	10	0.040155	10	1.000402	7	2.745505	7	1.745504
BKR 6	9	0.041688	8	0.095183	8	1.000953	9	2.358811	9	1.358810
BKR 7	6	0.08903	7	0.098735	7	1.000988	6	3.90299	6	2.902989
BKR 8	8	0.04915	9	0.089776	9	1.000899	8	2.602262	8	1.602261

5.4.3 Example 3

The breaker-and-a-third design is another common ETDS configuration. It can be analyzed for failures at three different load points and any possible combinations, e.g., failure at loads 1 & 2, failure at loads 1, 2 & 3, etc. The simplified diagram illustrated in Figure 5.3 is composed of six lines which are labeled 13 through 16, 27 and 28; components 3, 4, 5, 6, 7, 8, 23 and 24 represent eight circuit breakers, and components 1 and 2 represent the buses. This configuration has 35 different minimal cut sets of components that guarantee system failure at load 1, 37 sets of components for system failure at load 2, 37 sets of components for failure at load 3, and 98 minimal cut sets that guarantee system failure at loads 1, 2 & 3 simultaneously. All the of the minimal cut sets (up to fifth order) for this configuration are shown in Appendix A.

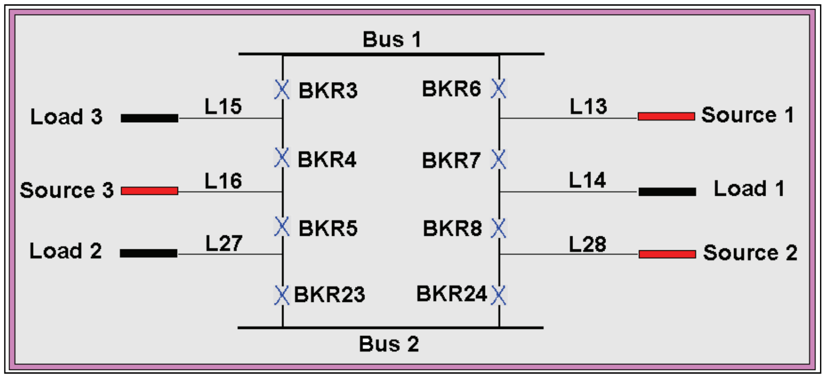

Figure 5.3 Breaker-and-a-Third Configuration

Table 5.9 shows the component data used for each component in the breaker-and-a-third configuration. The components were ranked according to their importance with respect to system downtime for overlapping component sustained outages at load 1, load 2, load 3 and failure at all loads. The results are shown in Tables 5.10, 5.11, 5.12 and 5.13, respectively.

Table 5.9 Outage Rates and Repair Times for Components - Breaker-and-a-Third

Component	Sustained outage rate, λ_i (outages / year)	Repair duration, r_i (hours / outage)
Line 13	0.98	20
Line 14	0.76	28
Line 15	0.50	12
Line 16	0.88	21
Line 27	0.69	16
Line 28	0.56	19
Bus 1	0.30	4.5
Bus 2	0.20	3.8
BKR 3	0.10	187
BKR 4	0.23	176
BKR 5	0.09	190
BKR 6	0.20	154
BKR 7	0.12	125
BKR 8	0.09	178
BKR 23	0.20	186
BKR 24	0.40	169

As indicated by the criticality metrics, the most important component for failures at load 1 is line 14, since changes in the outage rate of this component have a direct impact in the unavailability of load point 1. In this case, the components ranked second and third are breaker 8 and breaker 7, which are the breakers closest to the line 14. The least important for all of the metrics are the lines 15 and 27 as shown in Table 5.10.

Table 5.11 shows the ranking for the different components for failure at load 2. In this case, the most important component is line 27 and the second most important component is breaker 5. When considering failures at load 3, the most important component is line 15. Breaker 4 is the second most important and the least important components in this case are lines 15 and 27.

Table 5.10 Component Rankings and Metric Values for Failures at Load L1

Component	Birnbaum		CI		RAW		RRW		F-V	
	Rank	Value	Rank	Value	Rank	Value	Rank	Value	Rank	Value
Line 13	8	0.04372	4	0.002011	4	1.0000201	8	1.002032	8	0.002032
Line 14	1	2800	1	99.86805	1	757.87857	1	131.4067	1	130.4067
Line 15	15	0	15	0	15	1	15	1	15	0
Line 16	12	0.005873	11	0.000243	11	1.0000024	12	1.000273	12	0.000273
Line 27	16	0	16	0	16	1	16	1	16	0
Line 28	7	0.055216	5	0.001451	5	1.0000145	7	1.002577	7	0.002577
Bus 1	13	0.00186	13	2.62E-05	13	1.0000003	13	1.000087	13	8.7E-05
Bus 2	14	0.000804	14	7.55E-06	14	1.0000001	14	1.000038	14	3.77E-05
BKR 3	4	0.07731	10	0.000363	10	1.0000036	4	1.003625	4	0.003625
BKR 4	5	0.072762	6	0.000785	6	1.0000079	5	1.003407	5	0.003407
BKR 5	9	0.040219	12	0.00017	12	1.0000017	9	1.001886	9	0.001886
BKR 6	6	0.063667	8	0.000598	8	1.000006	6	1.002982	6	0.002982
BKR 7	3	23.07251	3	0.129936	3	1.0013011	3	2.081505	3	1.081505
BKR 8	2	30.89585	2	0.130496	2	1.0013067	2	2.448652	2	1.448652
BKR 23	10	0.039372	9	0.00037	9	1.0000037	10	1.001844	10	0.001844
BKR 24	11	0.035774	7	0.000672	7	1.0000067	11	1.001672	11	0.001672

Table 5.11 Component Rankings and Metric Values for Failures at Load L2

Component	Birnbaum		CI		RAW		RRW		F-V	
	Rank	Value	Rank	Value	Rank	Value	Rank	Value	Rank	Value
Line 13	14	0.005353	11	0.000466	11	1.0000047	14	1.000471	14	0.000471
Line 14	15	0	15	0	15	1	15	1	15	0
Line 15	16	0	16	0	16	1	16	1	16	0
Line 16	9	0.269831	5	0.02111	5	1.0002111	9	1.023777	9	0.023777
Line 27	1	1600	1	98.14635	1	53.947748	1	142.2596	1	141.2596
Line 28	12	0.022336	10	0.001112	10	1.0000111	12	1.001975	12	0.001975
Bus 1	13	0.011446	14	0.000305	14	1.0000031	13	1.001014	13	0.001014
Bus 2	5	0.750145	6	0.013338	6	1.0001334	5	1.066555	5	0.066555
BKR 3	6	0.475654	9	0.004229	9	1.0000423	6	1.042244	6	0.042244
BKR 4	7	0.447663	7	0.009153	7	1.0000915	7	1.039706	7	0.039706
BKR 5	2	229.0366	2	1.832536	2	1.0186674	2	21.34318	2	20.34318
BKR 6	8	0.391705	8	0.006965	8	1.0000697	8	1.034753	8	0.034753
BKR 7	11	0.030682	13	0.000327	13	1.0000033	11	1.002724	11	0.002724
BKR 8	10	0.043691	12	0.00035	12	1.0000035	10	1.003881	10	0.003881
BKR 23	3	36.71762	4	0.652844	4	1.0065713	3	4.257693	3	3.257693
BKR 24	4	33.36171	3	1.186352	3	1.0120059	4	3.954015	4	2.954015

Table 5.13 shows the metric values for simultaneous failures at loads 1, 2 and 3. We can see that the most important components are the lines, which range from first to sixth in importance, followed by the breakers and the least important components are the buses.

Table 5.12 Component Rankings and Metric Values for Failures at Load L3

Component	Birnbaum		CI		RAW		RRW		F-V	
	Rank	Value	Rank	Value	Rank	Value	Rank	Value	Rank	Value
Line 13	12	0.038218	10	0.006004	10	1.00006	12	1.006067	12	0.006067
Line 14	15	0	15	0	15	1	15	1	15	0
Line 15	1	1200	1	96.18878	1	26.238335	1	192.4157	1	191.4157
Line 16	9	0.276505	6	0.039008	6	1.0003902	9	1.043938	9	0.043938
Line 27	16	0	16	0	16	1	16	1	16	0
Line 28	14	0.009326	13	0.000837	13	1.0000084	14	1.001487	14	0.001487
Bus 1	5	2.086782	5	0.100363	5	1.0010046	5	1.333538	5	0.333538
Bus 2	13	0.010727	14	0.000344	14	1.0000034	13	1.001716	13	0.001716
BKR 3	3	86.96534	4	1.394182	4	1.0141389	3	14.92787	3	13.92787
BKR 4	2	102.3045	2	3.772209	2	1.0392008	2	17.36319	2	16.36319
BKR 5	6	0.536358	9	0.007739	9	1.0000774	6	1.085909	6	0.085909
BKR 6	4	71.61851	3	2.296299	3	1.0235027	4	12.45853	4	11.45853
BKR 7	11	0.130252	12	0.002506	12	1.0000251	11	1.020856	11	0.020856
BKR 8	10	0.18548	11	0.002676	11	1.0000268	10	1.029708	10	0.029708
BKR 23	7	0.525066	8	0.016835	8	1.0001684	7	1.084007	7	0.084007
BKR 24	8	0.431535	7	0.027673	7	1.0002768	8	1.068905	8	0.068905

Table 5.13 Component Rankings and Metric Values for Failures at Loads 1, 2 & 3

Component	Birnbaum		CI		RAW		RRW		F-V	
	Rank	Value	Rank	Value	Rank	Value	Rank	Value	Rank	Value
Line 13	3	0.00526	2	69.27615	2	3.2548	3	70.9972	3	69.9971
Line 14	5	0.00297	4	30.35908	4	1.435937	5	40.6426	5	39.6425
Line 15	4	0.00375	6	25.21382	6	1.337145	4	51.1755	4	50.1755
Line 16	2	0.00619	1	73.23427	1	3.736121	2	83.4884	2	82.4884
Line 27	6	0.0028	5	25.97107	5	1.350823	6	38.3795	6	37.3795
Line 28	1	0.00905	3	68.10235	3	3.135027	1	121.93	1	120.930
Bus 1	15	3.6E-05	15	0.146356	15	1.001466	15	1.48639	15	0.4863
Bus 2	16	1.6E-05	16	0.042741	16	1.000428	16	1.21328	16	0.21327
BKR 3	7	0.00151	10	2.028313	10	1.020703	7	21.2628	7	20.2628
BKR 4	13	0.00051	11	1.561591	11	1.015864	13	7.77391	13	6.77391
BKR 5	12	0.00066	13	0.802383	13	1.008089	12	9.90735	12	8.90734
BKR 6	9	0.00124	8	3.340763	8	1.034562	9	17.6704	9	16.6704
BKR 7	14	0.00023	14	0.368883	14	1.003702	14	4.07034	14	3.07033
BKR 8	8	0.00127	12	1.541488	12	1.015656	8	18.1122	8	17.1122
BKR 23	10	0.00078	9	2.093283	9	1.02138	10	11.4455	10	10.4454
BKR 24	11	0.00071	7	3.801619	7	1.039519	11	10.466	11	9.46603

5.5 Conclusions

In all previous examples, it was observed that, among the five new transformed importance measures, two different groupings of solutions were obtained. Each group gave the same ranking of components. The measures contained in the first group are: Birnbaum, Fussell-Vesely and RRW. In the second group, the measures which gave the same rankings for the different components in the system are: RAW and Criticality Importance. Therefore, we are just obtaining two different sets of rankings, and thus, we recommend that only two importance measures need to be further considered. The two criticality importance measures recommended are the RAW and the RRW, because these two measures require less computation to be calculated and are conceptually easier to understand.

The two metrics can be applied for any number of studies or analyses that require an objective quantitative ranking of the components. RAW is based on the upper limit on component sustained outage rate. This metric describes the potential for improvement and would be useful for studies to determine which components should be upgraded to a newer and more reliable replacement. Alternatively, RRW is based on the lower limit for component sustained outage rate. This is a more appropriate metric if no upgrades or improvements are under consideration, but it may be important to understand where further deterioration is more critical, and perhaps, inspections are to be determined and prioritized.

Reliability importance measures are valuable in establishing direction and prioritization of actions in relation with upgrading in the functioning of the system, or suggesting the most efficient way to operate and maintain system status. In the present work, based on popular importance measures, new criticality importance metrics, Birnbaum, Fussell-Vesely, reliability achievement worth (RAW), reliability reduction worth (RRW) and Criticality Importance (CI), have been developed for the power industry to be directly applied in the ETDS. These new metrics pertain to outage rates and system unavailability instead of a specific probability of failure. They have been applied to several common electric configurations such as the breaker-and-a-half, breaker-and-a-third and DESN. The ranking of components provided by the applications of the metrics can provide insightful meaning in where investments to increase the availability of the system should be made.

Chapter 6

Component Replacement Analysis

Many of the electric utilities in the USA were built in early 1960s, and although highly reliable, the U.S. electricity infrastructure is old and aging (Brown & Willis, 2006; Li & Jiachun, 2006; Li *et al*., 2006; Endrenyi & Anders, 2006). Aging infrastructure has higher costs to operate and maintain, and, more importantly, lower reliability. As equipment ages, the component outage rates increase having an impact on the total system downtime and leading to an increased cost of unmet demand. Typically, network upgrades and improvements have been made in an ad-hoc manner given yearly budgets. Therefore, there is a need to develop models to systematically, and optimally, upgrade the Electricity Transmission and Distribution Systems (ETDS) grid.

This chapter presents a new component replacement optimization model to solve replacement analysis problems for systems designed with a set of heterogeneous assets subject to annual budget constraints. In these problems, the objective is to plan for replacements over an extended planning horizon to minimize cost. This is a unique problem since previous methods only consider systems composed of sets of homogeneous assets (Hartman, 2000; Karabakal *et. al*., 1994; Hartman, 2001; Hartman & Rogers, 2005, etc.). This new method can potentially be applied to obtain component replacement policies for systems composed with a large number of components, with different reliability behavior and different costs associated with a loss of power. These new methods represent a new research contribution in the replacement analysis area. The method developed is applied for ETDS configurations.

6.1 Introduction

Replacement analysis is designed to minimize operating costs by identifying the optimal time periods to replace aging assets with a new or refurbished replacement. Capital assets are required to produce goods and/or provide services on a daily basis for industry and the government. As these assets are utilized over time, they age, become worn and lead to increased operating and maintenance expenditures. The timely replacement of these assets is necessary to assure economically efficient operations. Determining minimum cost replacement schedules requires the analysis of current and future costs over some time horizon.

The performance of many components within most operating systems deteriorate with age. Aging components associated with ETDS include transformers, power lines, circuit breakers, etc. Although routine maintenance can keep the equipment working, there comes a point when the repairs occur too frequently and are too expensive, and it becomes economically prudent to a replace or completely refurbish the old component or system.

As equipment ages, the component outage rates increase which impacts the total system downtime and lead to an increased cost of unmet demand. Therefore, there is a need for developing capital replacement methods in order to obtain optimal policies for systematically upgrading old and aging equipment.

Owners of the electric transmission systems are required to maintain their electric systems at a specific performance standard. Meeting these standards can be a challenging task for transmission owners or providers. The present work proposes a method for solving capital replacement problems for a set of heterogeneous assets within ETDS systems subject to annual budget constraints. The modeling of two different radial configurations is presented, and this will later be extended to more complex ETDS system configurations in the next chapter. The main application of the replacement model is to identify where and when investments should be made in a more effective and efficient way in order to minimize the total system cost over a finite planning horizon.

6.2. Replacement analysis policies

Economic replacement analysis is a well-known and studied research area. Given a level of output or service required from an asset over time, a decision is made periodically, to keep or replace the asset, as it wears with the aging process. This sequence of keep and replace decisions over the given time horizon is determined, such that some total cost function is minimized. Costs include capital or replacement costs (purchase costs and salvage revenues), operating and maintenance costs (O&M) costs, and cost of unmet demand (opportunity costs).

A replacement problem can be categorized as either serial or parallel replacement. Serial replacement problems consider a single asset or multiple independent assets. In serial replacement problems, it is assumed that there is no economic interdependence among the assets that provide the service together. Therefore, their replacement decisions can be made separately.

Parallel replacement analysis considers assets that are economically interdependent and operate in parallel. Economic interdependence may result from system-level budget constraints, demand constraints or service requirements. For parallel replacement problems, the desired solution includes keep and replace decisions for each individual asset over the horizon, resulting in a difficult combinatorial optimization problem as the replacement of groups of assets must be analyzed.

6.3 Dynamic programming modeling of replacement analysis

Dynamic programming provides a framework for studying problems where a sequential decision over time has to be made, as well as algorithms for computing optimal decision policies (Bertsekas, 1995). Dynamic programming has been often used to solve capital replacement problems. Bellman (1955) presented a dynamic programming formulation to solve the finite horizon equipment replacement problem with general costs. In his formulation, he considered a single challenger in each decision period. Later, Wagner (1975) solved a replacement analysis problem by using a dynamic programming formulation in which the state of the system is the time period and the main decision is the number of periods to retain an asset. Later, Oakford *et al*. (1984) generalized Wagner's dynamic programming formulation to find a optimal sequence of replacements when there are one or more challengers. They modeled technological change with a variety of continuous functions. Sethi & Chand (1979) developed a dynamic programming forward algorithm and provided planning horizon results for several machine replacement models under an improving technological environment over time.

Hartman & Rogers (2005a, 2005b) presented two dynamic programming formulations, which were extensions of Wagner (1975) and Bellman (1985) models to solve the equipment replacement problem. Their formulation considered that assets available for purchase evolve over a finite time horizon according to continuous and discontinuous functions of technological change. The main problem when using dynamic programming is that the state space grows exponentially with the number of components in the system, and therefore, an approximation to the final solution may be required to find the recommended solution.

Unfortunately, the overwhelming computational requirements of these algorithms render them inapplicable or inefficient for many realistic problems. An alternative to solve difficult combinatorial problems, that is closely tied to the framework of dynamic programming, is neuro-dynamic programming (Bertsekas & Tsitsiklis, 1996). This approach makes use of ideas from artificial intelligence involving simulation-based algorithms and functional approximation techniques such as neural networks. The outcome is a methodology for approximating dynamic programming solutions without demanding the associated computational requirements. Recently, neuro-dynamic programming methods have been successfully applied. Some examples are an elevator dispatcher (Crites & Barto, 1996), a program that plays backgammon at the world champion level (Tesauro, 1992) and an approach to job shop scheduling (Zhang & Dietterich, 1996).

ETDS are heterogeneous assets because of the different components in the system, but also, even when the same component is used, the objective functions are different because of the opportunity costs considered (outage in New York City vs. rural Pennsylvania)

In this work, a dynamic programming formulation is developed to obtain a component replacement policy over a finite horizon. A MATLAB code is implemented to obtain the expected cumulative net present value (NPV) of cost given by the minimum policy in the planning horizon.

6.4 Method proposed to solve the replacement problem for heterogeneous assets

A method has been developed to solve equipment replacement problems for systems composed of sets of heterogeneous assets subject to annual budgetary constraints over a finite planning horizon. The proposed methodology is based on an integrated iterative dynamic programming and integer programming model. This methodology can potentially be applied in any replacement problem composed of sets of heterogeneous components subject to constraints imposed on the system. This is a new solution methodology that offers distinct benefits to previous methods which only pertained directly to a system composed of homogeneous assets.

Figure 6.1 present an overview of the proposed methodology. First, a dynamic programming algorithm is solved for each individual component in the system analyzed without consideration of the other components. Then, two different integer programming models are applied. The first one is used to check whether a feasible solution can be obtained and to identify infeasibilities for the original problem, while the second integer programming model finds the solution with the minimum cumulative discounted cost. As shown in Figure 6.1, it is often necessary to iteratively repeat the dynamic programming and the first integer programming model until enough component replacement profiles are generated to provide a feasible system-level replacement schedule.

The dynamic programming algorithm is developed and applied to the system components to obtain the optimal replacement policy for each asset in the system separately. The objective is to minimize the Net Present Value (NPV) of all component costs over the planning horizon. Once the time periods are identified where replacements should be made, all the different solutions obtained from the dynamic programming model are used as inputs to the integer program 1 (IP1) to determine whether the individual replacement schedules can collectively form a feasible policy.

The IP1 model checks if the replacement policies obtained satisfy the actual budget constraints for each time period. This is done by defining the sum of all constraints violations as the objective function, in the case that a constraint is violated. If constraint violation exists, then there is no feasible solution to the original problem and additional component replacement schedules are needed. Therefore, the dynamic programming models are run again. However, it is now forbidden to make replacements in the periods where there are constraint violations due to exceeded annual budgets. This process is repeated until the optimal objective function for the IP1 is zero, meaning that there are possible schedules with no constraint violations.

The second integer programming program (IP2) uses then, all the information generated from all the different replacement schedules created. IP2 is solved to determine for the solution with the minimum NPV of the total system replacement analysis cost. If the first iteration of IP1 yields an objective function of zero, then IP2 provides the global optimal solution to the original problem.

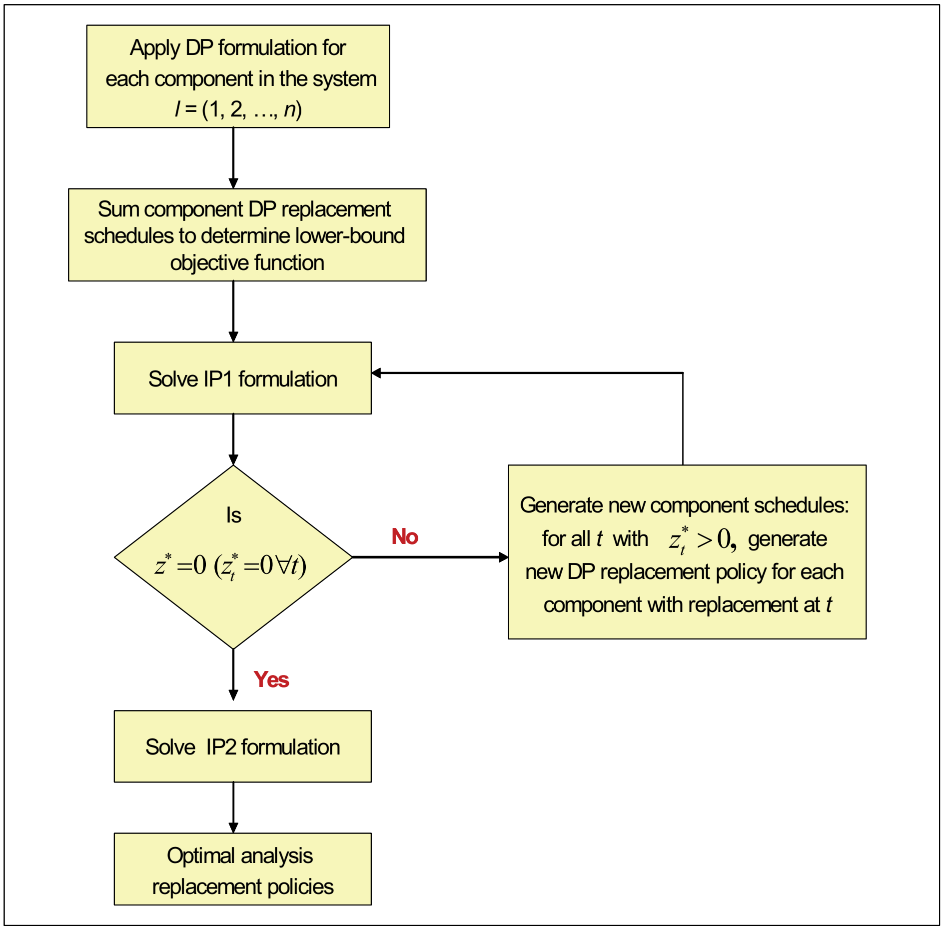

Figure 6.1 Replacement analysis process

6.4.1 Dynamic programming formulation

A dynamic programming formulation was developed to solve the optimal replacement for all of the components in the system individually. The objective function for each individual component is to obtain the minimum cost policy such that the NPV of maintenance, purchase and opportunity costs is minimized over the entire time horizon. The program is solved for each component in the system and the final result is the cumulative cost from all periods considered in a radial configuration system. If the individual component replacement schedules are considered together, this can often lead to an unsatisfactory system-level plan because replacement cost expenditures may exceed acceptable levels in some time periods. This is then resolved by subsequent integer programming problems that follows the dynamic programming. As a result of these integer programming problem, additional dynamic programming analyses maybe required.

In the dynamic programming formulation, the stages are the time or decision periods. At each time period (t = 0, 1, 2, ..., T), the decision-maker has the option to keep using the existing assets, or replace it with the purchase (or total refurbishment) of a new asset. A general diagram for each time period is shown in Figure 6.2. In this figure, the black upwards arrow indicates a decision of keeping the asset one more year while a red arrow indicates a decision to replace the asset at the beginning of the time period. The objective is to find the minimum cost policy over the entire planning horizon of T time periods. In the figure, each node is labeled as $f_t(i)$ representing the NPV of all costs up until period t for an asset that is of age i at time t (but may have been replaced periodically prior to t). At the beginning of the analysis, the asset has an age of τ. The minimum cost is defined to be the NPV of all costs over the T periods.

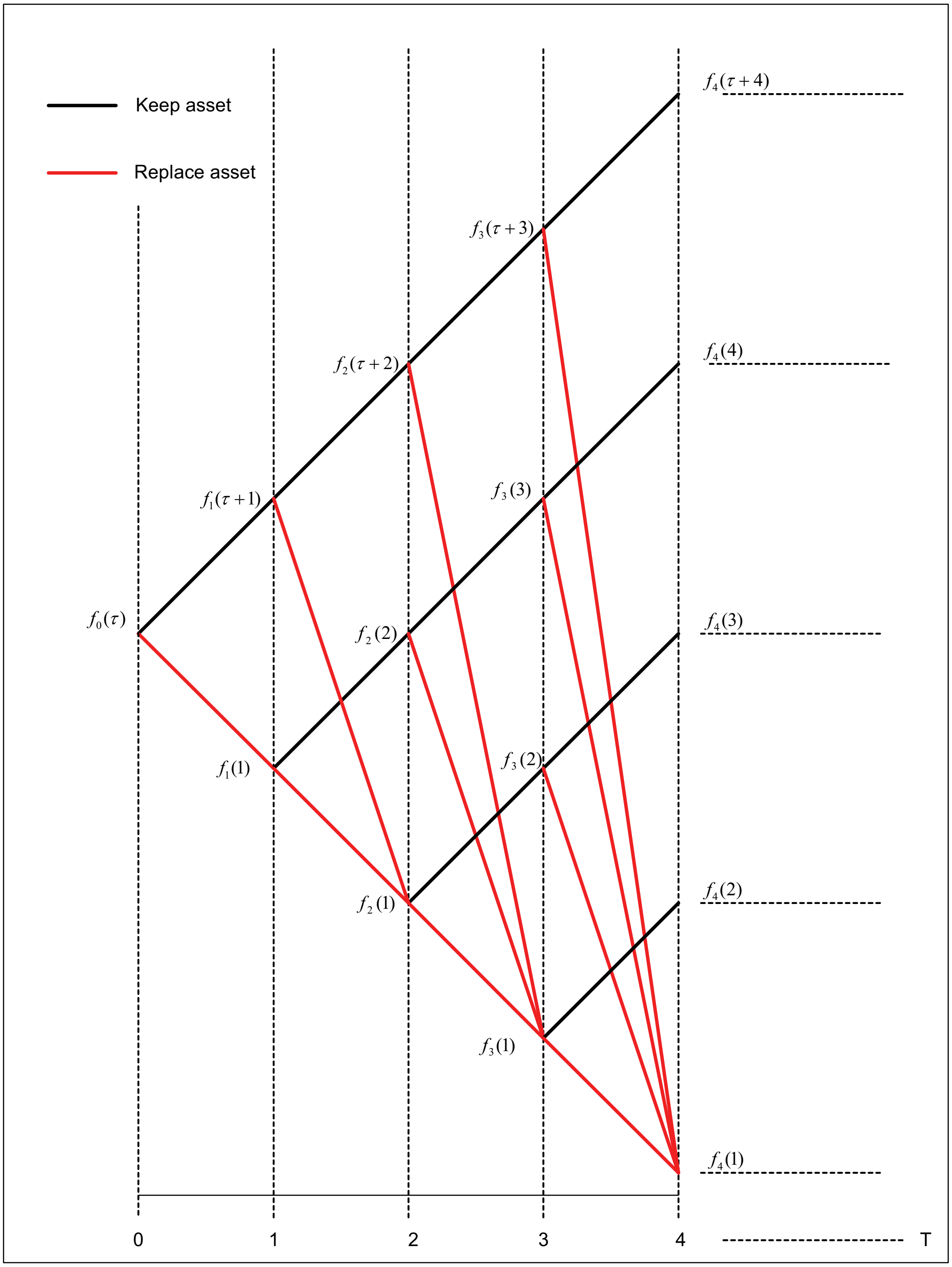

Figure 6.2 General diagram for an asset for a *T* year planning horizon

To obtain the replacement schedule with the minimum cost, a dynamic programming program was developed, and the following notation is used. The same formulation is used for each component within the system and a subscript (or other notation) denoting the specific component type is omitted. It is also important to note that in our formulation, period t takes place between t - 1 and t, e.g., period 1 is from t = 0 to t = 1.

Dynamic programming notation:

$\tau =$ Initial age of an asset at beginning of planning horizon (when $t = 0$)

$i =$ Asset age in use during period t

$m_t(i) =$ Maintenance costs associated with an asset of age i in use during period t

$T =$ Time horizon

$f_t(i) =$ Minimum cumulative NPV at the end of period t for an asset of age i

$CU_t(i) =$ Expected opportunity costs associated with an asset of age i in use during period t

$g_t(i) =$ NPV of maintenance and opportunity costs of an asset of age i in period t

$$g_t(i) = (m_t(i) + CU_t(i))\,(1+\iota)^{-t}$$

$\iota =$ Interest rate or minimum acceptable rate of return (MARR)

$P_t(0),\ P_t^{NPV}(0) =$ Purchase cost and NPV of purchase cost of a new asset at beginning of period t ($P_t^{NPV}(0) = P_t(0)\,(1+\iota)^{-t}$)

The dynamic program proceeds as follows. At the beginning of the replacement analysis study ($t = 0$) the total cumulative cost is $f_0(\tau) = 0$.

At the beginning of the planning horizon ($t = 0$), there are two different replacement options, which are either to keep the existing τ years old asset in service, or replace it with a new one. If the component is not replaced, then the cumulative cost at the end of the first year is the previous cost ($f_0(\tau) = 0$) plus the cost incurred during the first year of an asset aged τ years ($g_1(\tau)$). Alternatively, if the asset is replaced at the beginning of the planning horizon, then the cost is the previous cost ($f_0(\tau) = 0$), plus the replacement cost ($P_0^{NPV}(0)$), and the cost associated with a new asset ($g_1(0)$). Therefore, the possible cumulative costs at the end of the first year are as follows.

$$f_1(\tau+1) = f_0(\tau) + g_1(\tau)$$

$$f_1(1) = f_0(\tau) + P_0^{NPV}(0) + g_1(0)$$

At the end of year 2, there are now three different possibilities (nodes in Figure 6.2 for $t = 2$). One refers to the total cost of keeping the existing τ year old asset for two additional years, $f_2(\tau+2)$. Another possibility is to replace at the very beginning (t = 0), the aged asset with a new one, and keep it during the first and second year $f_2(2)$. The third involves the replacement of the asset after year 1. The minimum cost for this third possibility requires the comparison of two different options (two arcs entering the same node). The costs for each possibility are given by,

$$f_2(\tau+2) = f_1(\tau+1) + g_2(\tau+1)$$

$$f_2(2) = f_1(1) + g_2(1)$$

$$f_2(1) = \min\left\{\begin{matrix} f_1(1) + g_2(0) + P_1^{\mathrm{NPV}}(0) \\ f_1(\tau+1) + g_2(0) + P_1^{\mathrm{NPV}}(0) \end{matrix}\right\} = \min\left\{f_1(1), f_1(\tau+1)\right\} + g_2(0) + P_1^{\mathrm{NPV}}(0)$$

At the end of year 3, there are four different possibilities (distinct nodes in Figure 6.2 for $t = 3$),

$$f_3(\tau+3) = f_2(\tau+2) + g_3(\tau+2)$$

$$f_3(3) = f_2(2) + g_3(2)$$

$$f_3(2) = f_2(1) + g_3(1)$$

$$f_3(1) = \min\left\{\begin{matrix} f_2(1) + g_3(0) + P_2^{\mathrm{NPV}}(0) \\ f_2(2) + g_3(0) + P_2^{\mathrm{NPV}}(0) \\ f_2(\tau+2) + g_3(0) + P_2^{\mathrm{NPV}}(0) \end{matrix}\right\} = \min\left\{f_2(1), f_2(2), f_2(\tau+2)\right\} + g_3(0) + P_2^{\mathrm{NPV}}(0)$$

At the end of year 4, there are five different possibilities (distinct nodes in Figure 6.2 for $t = 4$),

$$f_4(\tau + 4) = f_3(\tau + 3) + g_4(\tau + 3)$$

$$f_4(4) = f_3(3) + g_4(3)$$

$$f_4(3) = f_3(2) + g_4(2)$$

$$f_4(2) = f_3(1) + g_4(1)$$

$$f_4(1) = \min\left\{\begin{array}{c} f_3(1) + g_4(0) + P_3^{\mathrm{NPV}}(0) \\ f_3(2) + g_4(0) + P_3^{\mathrm{NPV}}(0) \\ f_3(3) + g_4(0) + P_3^{\mathrm{NPV}}(0) \\ f_3(\tau + 3) + g_4(0) + P_3^{\mathrm{NPV}}(0) \end{array}\right\} = \min\left\{f_3(1), f_3(2), f_3(3), f_3(\tau + 2)\right\} + g_4(0) + P_3^{\mathrm{NPV}}(0)$$

This continues until the end of period T when the different possibilities are,

$$f_T(\tau + T) = f_{T-1}(\tau + T - 1) + g_T(\tau + T - 1)$$

$$f_T(T) = f_{T-1}(T - 1) + g_T(T - 1)$$

$$f_T(T - 1) = f_{T-1}(T - 2) + g_T(T - 2)$$

.

.

.

$$f_T(2) = f_{T-1}(1) + g_T(1)$$

$$f_T(1) = \min\left\{f_{T-1}(1), f_{T-1}(2), \ldots, f_{T-1}(T - 1), f_{T-1}(\tau + T - 1)\right\} + g_T(0) + P_T^{NPV}(0)$$

Then to determine the minimum cost replacement schedule for the entire planning horizon, it is necessary to consider all possibilities and the minimum cost is given by

$$f^* = \min_i f_T(i)$$

To determine the minimum cost schedule associated with f^*, it is necessary to re-visit the recursive dynamic programming stages from the optimal objective function.

General equations can also be written to express the dynamic programming recursive equations as follows. For an asset originally aged τ years, to develop a replacement schedule over a T year planning horizon, it is necessary to sequentially compute the following, for $t = 0$ to T, and for $i = 1$ to t, and $i = \tau + t$.

<u>for $t = 0$</u>

$$f_0(\tau) = 0$$

<u>for $t = 1$</u>

$$f_1(\tau + 1) = f_0(\tau) + g_1(\tau)$$

$$f_1(1) = f_0(\tau) + g_1(0) + P_0^{NPV}(0)$$

<u>for $t = k$ ($1 < k < T$)</u>

for $i = \tau + k$, $\quad f_k(\tau + k) = f_{k-1}(\tau + k - 1) + g_k(\tau + k - 1)$

for $1 < i \leq k$, $\quad f_k(i) = f_{k-1}(i-1) + g_k(i-1)$

for $i = 1$,

$$f_k(1) = \min\{f_{k-1}(1), f_{k-1}(2), \ldots, f_{k-1}(k-1), f_{k-1}(\tau + k - 1)\} + g_k(0) + P_{k-1}^{NPV}(0)$$

<u>for $t = T$</u>

for $i = \tau + T$, $\quad f_T(\tau + T) = f_{T-1}(\tau + T - 1) + g_T(\tau + T - 1)$

for $1 < i \leq T$, $\quad f_T(i) = f_{T-1}(i-1) + g_T(i-1)$

for $i = 1$,

$$f_T(1) = \min\{f_{T-1}(1), f_{T-1}(2), \ldots, f_{T-1}(T-1), f_{T-1}(\tau + T - 1)\} + g_T(0) + P_{T-1}^{NPV}(0)$$

and

$$f^* = \min\{f_T(1), f_T(2), \ldots, f_T(T), f_T(T + \tau)\}$$

The dynamic programming model is applied to each component to determine a replacement schedule. For each component l (l = 1, 2, …, n), the following is computed to be used in IP1 and IP2. These are the first replacement profiles (j = 1).

c_{l1} = Total NPV of first replacement profile (j = 1) for component l

f^* for each l

γ_{l1t} = NPV of maintenance and replacement costs incurred in period t for the first replacement profile (j = 1) for component l

In the solution of the integer programming programs, the term γ_{ljt} (NPV of maintenance and replacement costs incurred in period t for the j^{th} replacement profile for component l) does not include the opportunity costs, since this is not an actual cost for the company, i.e., an example charged to company accounts. The opportunity costs are included in the dynamic program to represent the potential amount of money a particular company is not earning given the demand is not fully satisfied. Of course if a utility wanted to minimize total costs without considering these opportunity costs, then they simply could be set equal to zero, $CU_t(i) = 0$.

Additionally, for each of the initial profiles, it is necessary to determine the sets, I_{l1}, which are the sets of all time periods where a component replacement is made. For example, for a particular transformer (l = 1), the optimal replacement schedule may specify replacements in years 12, 24, and 36, and for a particular line (l = 2), there is a scheduled replacement in year 40 only. Therefore, I_{11}={12, 24, 36} and I_{21}={40}. If another replacement schedule is needed for the transformer, then this will require the determination of I_{12} and so on.

If the initial component replacement profiles are not collectively feasible when aggregated together, then it will be necessary to generate additional profiles (j > 1) and determine the corresponding c_{lj}, γ_{ljt} and I_{lj}.

A lower-bound for the original problem can be determined by summing the NPV of all initial component replacement schedules. This is useful to evaluate the integrity of the final produced schedules given budget constraints. The lower bound is given as,

$$LB_{\mathrm{IP2}} = \sum_{l=1}^{n} c_{l1}$$

6.4.2 Integer programming formulations

Two integer programming formulations need to be solved, after the initial component replacement policies are determined for each asset in the system. The purpose of the first integer programming formulation (IP1) is to determine whether a feasible solution exists and to indicate at which time periods t the available budget is exceeded (if any). For those time periods, new component replacement profiles are then obtained using the dynamic programming model for those components with a replacement at time t, i.e., $t \in I_{lj}$. The additional component replacement profiles are generated by forbidding replacements in the time periods with the budget infeasibilities. The process is repeated until the optimal solution for IP1 indicates no violations in the budget available for each period, i.e., z = 0. Once this is achieved, the second formulation (IP2) is solved to select the recommended component replacement policies for the components in the system. The specific formulations are as follows:

Integer program 1 (IP1)

For IP1, the objective is to determine whether a feasible system schedule can even be determined given the component profiles thus far generated. The objective function is the sum of the constraint violations for each time period. If there are no constraint violations, then the algorithm can proceed to IP2. If there are constraint violations, then it is necessary to generate alternative component replacement profiles. The formulation is,

$$\min z = \sum_{t=1}^{T} z_t \tag{6.1}$$

Subject to:

$$\sum_{l=1}^{n} \sum_{j=1}^{k} \gamma_{ljt} y_{lj} - z_t \le B_t \quad \forall t \tag{6.2}$$

$$\sum_{j=1}^{k} y_{lj} = 1 \quad \forall l \tag{6.3}$$

$$y_{lj} \in \{0,1\} \tag{6.4}$$

$$z_t \ge 0 \tag{6.5}$$

Where:

z_t = Constraint violation (costs in excess of B_t) for period t

B_t = Available budget for period t

γ_{ljt} = Replacement and maintenance cost for the j^{th} profile for component l in period t

$$y_{lj} = \begin{cases} 1, & \text{if the } j^{th} \text{ profile is selected for component } l \\ 0, & \text{otherwise} \end{cases}$$

In IP1, the objective function (6.1) minimizes the sum of the constraint violations. The first set of constraints (6.2) attempt to limit the annual expenditures for maintenance and replacements to within a specified budget (B_t). The excess variables (z_t) are added so that there will be always a feasible solution to IP1 even if the actual costs are greater than B_t. The optimal solution to IP1 has the smallest summed constraint violation. The second set of constraints (6.3) assures that exactly one replacement profile is selected for each component. The replacement profiles (j = 1, 2, …, k) are those profiles which are generated from each return to the dynamic programming model.

After solving IP1, an objective function $z > 0$ means that there are no feasible solutions to the actual planning problem, and new component policies are needed. Where $z_t^* > 0$, new dynamic programming profiles are needed for all components with a replacement in time t, i.e., $t \in I_{lj}$. If IP1 yields an objective function of zero, then the algorithm proceeds to IP2.

Integer program 2 (IP2)

From IP1, it is known that a feasible solution is available. IP2 considers all the individual component replacement schedules generated and is used to determine the recommended system-level replacement schedule.

This solution produces the optimal schedule from among all component schedules generated. If the first IP1 produces an objective function of zero (z = 0), then IP2 produces a global optimal solution. The objective function (6.6) is the minimization of the NPV for all replacement, purchase and opportunity costs for all components in the system over a finite planning time horizon. The IP2 formulation is given as,

$$\min \sum_{l=1}^{n} \sum_{j=1}^{k} c_{lj} y_{lj} \tag{6.6}$$

Subject to:

$$\sum_{l=1}^{n} \sum_{j=1}^{k} \gamma_{ljt} y_{lj} \le B_t \quad \forall t \tag{6.7}$$

$$\sum_{j=1}^{k} y_{lj} = 1 \quad \forall l \tag{6.8}$$

$$y_{lj} \in \{0,1\} \tag{6.9}$$

Where:

B_t = Available budget for period t

c_{lj} = NPV of all costs for the j^{th} profile for component l over all time periods

γ_{ljt} = NPV of replacement and maintenance cost for j^{th} profile for component l in period t

$$y_{lj} = \begin{cases} 1, & \text{if the } j^{th} \text{ profile is selected for component } l \\ 0, & \text{otherwise} \end{cases}$$

The first set of constraints (6.7) limit the annual expenditures within a specified budget. The second set of constraints assures that exactly one replacement profile is selected for each component. The solution for the second integer program model provides the final recommended system-level component replacement schedule.

6.5 Radial system configuration

Models for system reliability metrics were presented in Chapter 3. For radial systems, where all the components are arranged in series, Equations 6.10 to 6.12 are used to obtain the total system outage rate, the average repair time and expected system downtime respectively. The outage rates are in outages per year, the repair rates are in hours per outage, and the expected system downtime is in hours per year.

$$\lambda_{s,t} = \sum_{l=1}^{n} \lambda_{l,t} \tag{6.10}$$

$$r_{s,t} = \frac{\sum_{l=1}^{n} \lambda_{l,t} r_{l,t}}{\lambda_{s,t}} \tag{6.11}$$

$$U_{s,t} = \lambda_{s,t} r_{s,t} \tag{6.12}$$

Where:

$\lambda_{l,t}$ = Outage rate of component l during period t

$r_{l,t}$ = Repair time of component l during period t ($r_{l,t} = r_l$ when repair times do not change with t)

$\lambda_{s,t}$ = System Outage rate during period t

$r_{s,t}$ = System repair time during period t

$U_{s,t}$ = System downtime during period t

6.5.1 Opportunity costs

Opportunity costs represent the costs associated with losses to businesses and consumers when they are without power. These opportunity costs can be estimated using average interruption rate cost metrics, for a particular municipality, location or business type. These relate the proportional loss of business revenue and depend on the geographic location of the distribution system and the type of businesses located there. For a radial system, these opportunity costs for time t are estimated simply as the sum of the estimated downtime for each component multiplied by average interruption rate cost (IC_t).

Equation 6.13 can be used to determine the total system opportunity cost due to unsupplied energy associated with each time period in the radial system. In the present example, the customer interruption costs, IC_t, for period t are in dollars per hour. The opportunity cost associated with each component in the system for each time period can be obtained by using Equation 6.14.

$$SOC_t = U_{s,t}\ IC_t \tag{6.13}$$

$$CU_{l,t} = (\lambda_{l,t} r_{l,t})\ IC_t \tag{6.14}$$

Where:

$CU_{l,t}$ = Opportunity cost for component l in period t

SOC_t = Total system opportunity cost due to unsupplied demand in period t

(associated with the system unavailability)

$U_{s,t}$ = Expected system downtime during period t

IC_t = Customer interruption costs ($/hour) during period t

$\lambda_{l,t}$ = Outage rate for component l in period t

$r_{l,t}$ = Average repair time for component l in period t

By using Equations 6.10 through 6.14, the associated opportunity costs for each individual component are obtained, and this information is then used in the dynamic programming model.

6.5.2 Modeling of component outage rates

Non-homogeneous Poison Process (NHPP) models for repairable systems in which the intensity function is non-constant have been used in the reliability literature (Ansell & Phillips, 1994; Beiser & Rigdon, 1997; Gilardoni & Colosimo, 2007). In the present work, the Crow/AMSAA model is used to model the aging (increasing outage rates) of the different components in the ETDS. The component repair times are assumed to remain constant through all the periods in the study. The components in the system follow a minimal repair policy, which means that once the components have an outage they are restored to the condition they were just before the outage. Let $N(t)$ be the number of failures in the time interval (0, t]. A process $\{N(t)\colon\ t \geq 0\}$ having independent increments and with $N(0)=0$ is said to be a Poisson process with intensity $\mu(\cdot)$ if, for any t, the random variable $N(t)$ follows a Poisson distribution with mean $M(t) = \mathrm{E}(N(t)) = \int_0^t \mu(u)du$. The NHPP is a Poisson process for which the intensity function $\mu(\cdot)$ is non constant. One of the most popular parametric forms for the intensity function is the Crow/AMSAA (Army Materiel System Analysis Activity) model. This model is used in the replacement analysis to model the aging of the different components in the system and to estimate the expected number of outages.

The Crow/AMSAA model has the following intensity function,

$$\mu(t) = \lambda\beta t^{\beta-1} \tag{6.15}$$

The expected number of failures by time t is given by:

$$E[N(t)] = \int_0^t \lambda\beta t^{\beta-1}dt = \frac{\lambda\beta t^{\beta-1+1}}{\beta+1-1} = \lambda t^{\beta} \tag{6.16}$$

Therefore, the expected number of failures on any given interval is given by,

$$E[N(t_{i+1}) - N(t_i)] = \int_{t_i}^{t_{i+1}} \lambda\beta t^{\beta-1}dt = \lambda(t_{i+1}^{\beta} - t_i^{\beta}) \tag{6.17}$$

The expected number of failures for the component is considered in the replacement analysis model. The maintenance costs associated with an aging asset used in the dynamic programming (Equation 6.17) considering the NHPP model are,

$$m_t(i) = C_{mrl}E[N(t_{i+1}) - N(t_i)] = C_{mrl}\lambda(t_{i+1}^{\beta} - t_i^{\beta})$$

Where:

$m_t(i)$ = Maintenance costs for component l of age i in period t (dynamic programming formulations are for each component individually and the "l" subscript is often omitted)

$C_{mrl}(i)$ = Cost of minimal repair associated with component l

6.6 Example 1

To demonstrate the replacement analysis model consider the radial system shown in Figure 6.3. In this example, nine components are considered which are, three lines, three circuit breakers, one switch, one transformer and one bus. The problem parameters as presented in Table 6.1. Outage rates and other data were taken from various sources. The replacement models are effective for any values selected. In practice, a utility or the organization would use data collected specifically for the systems being studied. The estimated customer interruption costs (IC_t) at the load point analyzed are 1,500 \$/hour. All the costs are discounted back to time 0 using the NPV considering an interest rate, ι. An interest rate of 10% ($\iota = 0.1$) was used in all sample calculations.

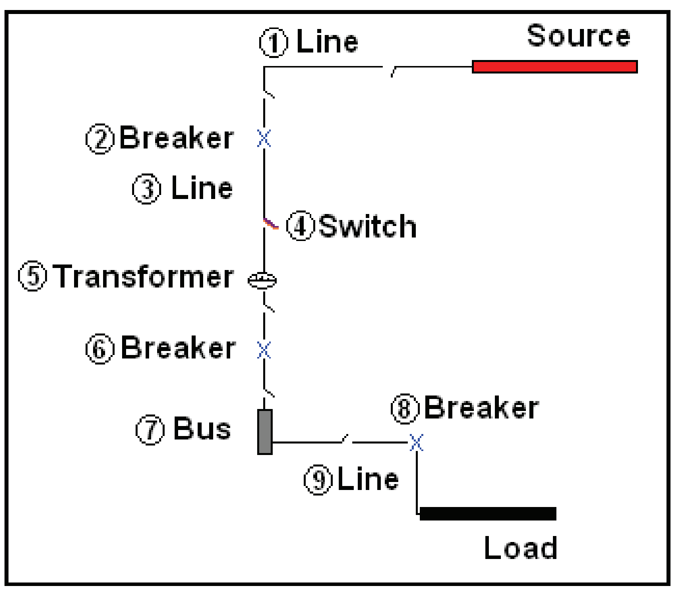

Figure 6.3 Radial configuration – Example 1

Table 6.1. Example 1 data

Component	Asset initial age (τ)	λ_l (outages/year)	β_l	r_l (hours/outage)	C_{mrl} ($/outage)	$P_0(0)$ ($)
Line 13.8 kV	20	1.9560	1.25	1.32	1500	45000
Breaker 13.8 kV	10	0.0036	1.6	83.12	1000	35000
Line 600ft	40	0.0055	1.8	26.51	1900	33300
Switch	35	0.0061	1.85	5.60	700	10000
Transformer	30	0.0030	1.55	342	3000	45000
Breaker 480V	25	0.0027	1.95	20	900	16000
Bus	47	0.0044	1.85	44	3000	30000
Breaker 480 V	10	0.0027	1.95	20	900	16000
Line 300 ft	47	0.0045	1.8	20.5	1900	17500

For the replacement analysis model, the decision maker needs to make a decision; whether to keep a component or replace it immediately with a new one at the beginning of each time period t for $1 \le t \le T$. The objective function is to minimize the total NPV including investment, maintenance, and opportunity costs less the salvage value (which is assumed to be 0 for this problem) over the time horizon. Since most power utilities equipment were built in early 1960s, most of the components in the systems are old and aging. The initial age of the different components in the system, for this example, is shown in Table 6.1.

Step 1, iteration 1. The dynamic programming algorithm is applied to generate initial replacement schedules for each component in the system. Figure 6.4, shows the optimal replacement policies for each component in the system. A downward arrow represents a replacement decision; while an upward arrow means to keep the asset. As shown in Figure 6.4, the red line corresponding to Line 13.8 Kv begins going upward which means that the asset is kept during the first 4 periods, then at beginning of the 5th period the asset is replaced and it is kept in service during 23 years and so on. The individual replacement decisions obtained for each component in the system are shown in Table 6.2. Appendix B contains all the information used in the present example.

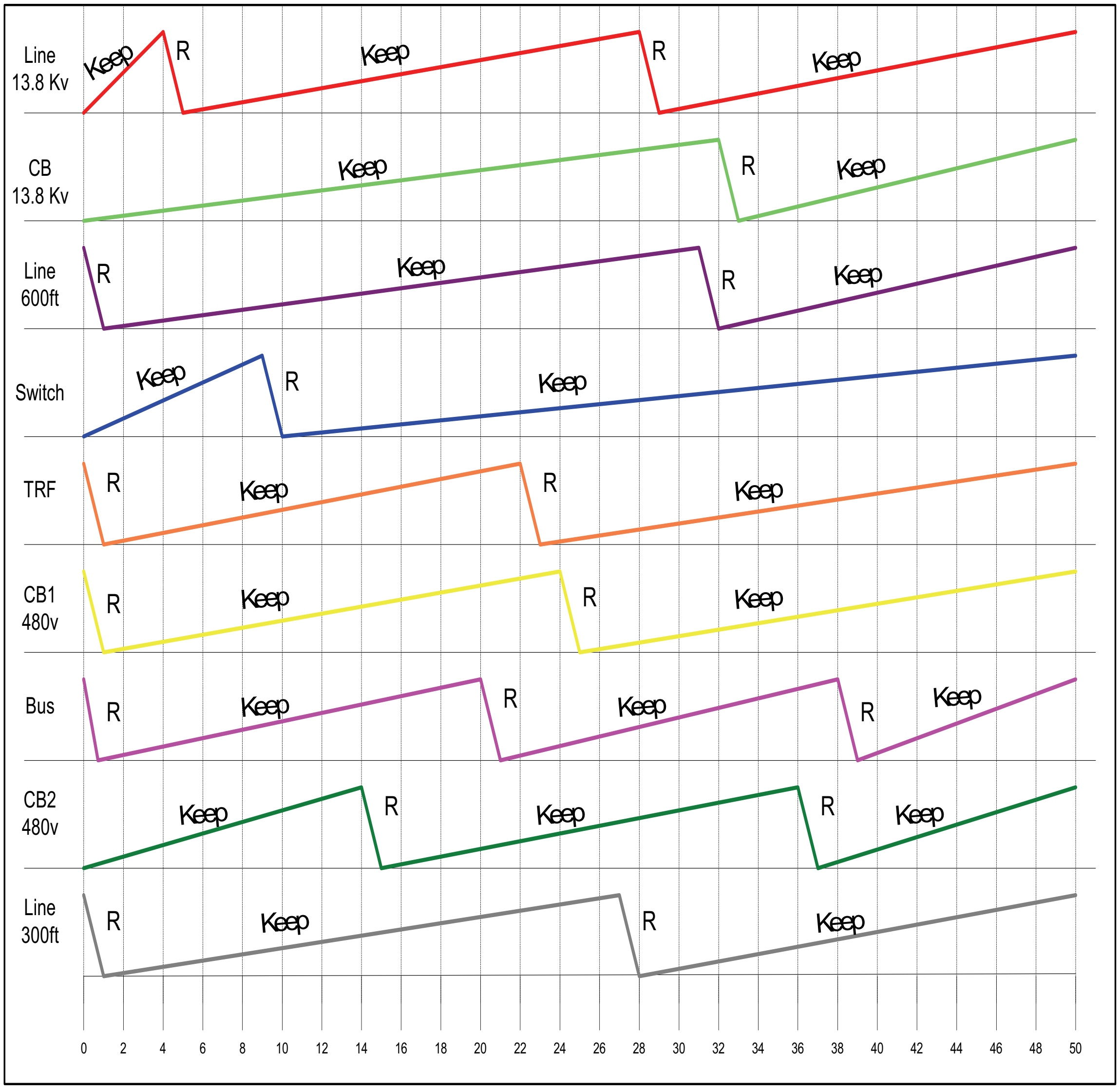

Figure 6.4. Component replacement policies

Table 6.2. Replacement analysis policies

Period	Line 13.8Kv	CB 13.8Kv	Line 600ft	Switch	Transformer	CB1 480v	Bus	CB2 480v	Line 300ft
1	Keep	Keep	Replace	Keep	Replace	Replace	Replace	Keep	Replace
2	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
3	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
4	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
5	Replace	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
6	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
7	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
8	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
9	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
10	Keep	Keep	Keep	Replace	Keep	Keep	Keep	Keep	Keep
11	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
12	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
13	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
14	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
15	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Replace	Keep
16	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
17	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
18	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
19	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
20	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
21	Keep	Keep	Keep	Keep	Keep	Keep	Replace	Keep	Keep
22	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
23	Keep	Keep	Keep	Keep	Replace	Keep	Keep	Keep	Keep
24	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
25	Keep	Keep	Keep	Keep	Keep	Replace	Keep	Keep	Keep
26	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
27	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
28	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Replace
29	Replace	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
30	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
31	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
32	Keep	Keep	Replace	Keep	Keep	Keep	Keep	Keep	Keep
33	Keep	Replace	Keep	Keep	Keep	Keep	Keep	Keep	Keep
34	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
35	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
36	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
37	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Replace	Keep
38	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
39	Keep	Keep	Keep	Keep	Keep	Keep	Replace	Keep	Keep
40	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
41	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
42	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
43	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
44	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
45	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
46	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
47	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
48	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
49	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
50	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
Total cost	1,642,553	33,995	527,223	19,465	103,713	26,685	57,744	19,926	29,733

Step 2: iteration 1. Solving the integer programming formulation 1

The IP1 was solved using the data shown in Appendix B, Step 2, Iteration 1. In this case, in the final solution we found that $z_1 > 0$, which means that there is a constraint violation in period 1. New component replacement policies need to be generated for the specific components with replacements scheduled in time 1. In the present example, the components with replacement scheduled in period 1 are the 600 ft line, bus, circuit breaker 1, transformer and the 300 ft line. Therefore, additional replacement profiles were generated for each of these components. In order to do so, Step 1 is repeated again without allowing the components to be replaced in period 1 with the aim of finding new additional component replacement profiles.

Step 1, iteration 2. New additional replacement schedule policies were generated for components in the system with replacement in period 1. The new replacement profiles are obtained by prohibiting replacement in period 1. Table 6.3 shows the new additional component replacement policies obtained The components are forced to not make replacement in period 1. Five new additional replacement policies are created, they are shown in Figure 6.5.

Table 6.3. Additional replacement schedules for components with replacement in period 1

Beginning	end	Line 600ft	Transformer	CB1 480v	Bus	Line 300ft
0	1	Keep	Keep	Keep	Keep	Keep
1	2	Replace	Replace	Replace	Replace	Replace
2	3	Keep	Keep	Keep	Keep	Keep
3	4	Keep	Keep	Keep	Keep	Keep
4	5	Keep	Keep	Keep	Keep	Keep
5	6	Keep	Keep	Keep	Keep	Keep
6	7	Keep	Keep	Keep	Keep	Keep
7	8	Keep	Keep	Keep	Keep	Keep
8	9	Keep	Keep	Keep	Keep	Keep
9	10	Keep	Keep	Keep	Keep	Keep
10	11	Keep	Keep	Keep	Keep	Keep
11	12	Keep	Keep	Keep	Keep	Keep
12	13	Keep	Keep	Keep	Keep	Keep
13	14	Keep	Keep	Keep	Keep	Keep
14	15	Keep	Keep	Keep	Keep	Keep
15	16	Keep	Keep	Keep	Keep	Keep
16	17	Keep	Keep	Keep	Keep	Keep
17	18	Keep	Keep	Keep	Keep	Keep
18	19	Keep	Keep	Keep	Keep	Keep
19	20	Keep	Keep	Keep	Keep	Keep
20	21	Keep	Keep	Keep	Keep	Keep
21	22	Keep	Keep	Keep	Replace	Keep
22	23	Keep	Keep	Keep	Keep	Keep
23	24	Keep	Replace	Keep	Keep	Keep
24	25	Keep	Keep	Keep	Keep	Keep
25	26	Keep	Keep	Replace	Keep	Keep
26	27	Keep	Keep	Keep	Keep	Keep
27	28	Keep	Keep	Keep	Keep	Keep
28	29	Keep	Keep	Keep	Keep	Replace
29	30	Keep	Keep	Keep	Keep	Keep
30	31	Keep	Keep	Keep	Keep	Keep
31	32	Replace	Keep	Keep	Keep	Keep
32	33	Keep	Keep	Keep	Keep	Keep
33	34	Keep	Keep	Keep	Keep	Keep
34	35	Keep	Keep	Keep	Keep	Keep
35	36	Keep	Keep	Keep	Keep	Keep
36	37	Keep	Keep	Keep	Keep	Keep
37	38	Keep	Keep	Keep	Keep	Keep
38	39	Keep	Keep	Keep	Keep	Keep
39	40	Keep	Keep	Keep	Replace	Keep
40	41	Keep	Keep	Keep	Keep	Keep
41	42	Keep	Keep	Keep	Keep	Keep
42	43	Keep	Keep	Keep	Keep	Keep
43	44	Keep	Keep	Keep	Keep	Keep
44	45	Keep	Keep	Keep	Keep	Keep
45	46	Keep	Keep	Keep	Keep	Keep
46	47	Keep	Keep	Keep	Keep	Keep
47	48	Keep	Keep	Keep	Keep	Keep
48	49	Keep	Keep	Keep	Keep	Keep
49	50	Keep	Keep	Keep	Keep	Keep
Total profile cost $s		53,763	105,642	26,822	63,459	31,293

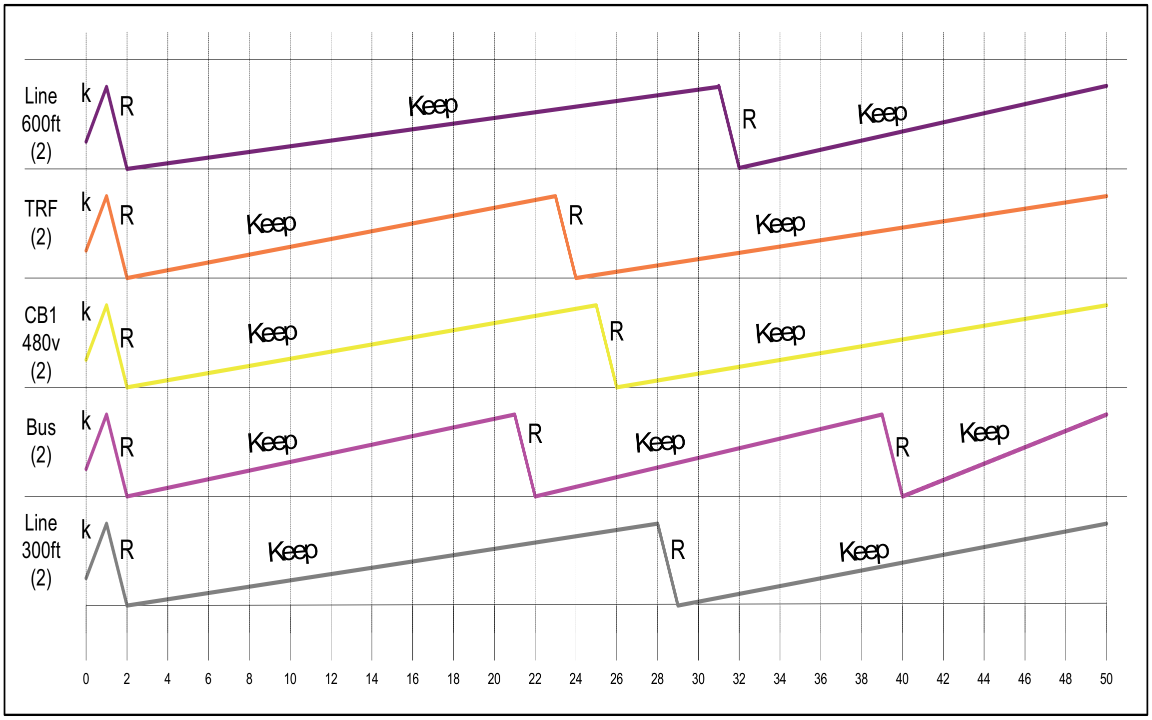

Figure 6.5 Additional replacement policies for components

Considering all the component replacement schedules generated so far (i.e., 14 profiles), the integer programming formulation 1, shown in Appendix B, is solved again to check for budget violations. In the final solution, z_1 and z_2 were greater than zero, which means we have now budget violations in periods 1 and 2. Therefore, we need to apply again the dynamic programming model to generate new component replacement policies for components with replacement in period 1 (CB1 480v, Bus and 300 ft Line) and period 2 (600 ft Line and Transformer).

Step 1, iteration 3. New component replacement schedules need to be obtained for the components with replacement in periods 1 and 2. The new replacement profiles are obtained by prohibiting replacements for those components having a replacement in periods 1 and 2. Table 6.4 shows the new additional component replacement policies obtained by the dynamic programming formulation. Figure 6.6 shows the new component replacement schedules obtained in the third iteration.

Table 6.4. Additional replacement analysis policies created in iteration 3

Beginning	end	CB1 480v	Bus	Line 300ft	Line 600ft	Transformer
0	1	Keep	Keep	Keep	Keep	Keep
1	2	Replace	Replace	Replace	Keep	Keep
2	3	Keep	Keep	Keep	Replace	Replace
3	4	Keep	Keep	Keep	Keep	Keep
4	5	Keep	Keep	Keep	Keep	Keep
5	6	Keep	Keep	Keep	Keep	Keep
6	7	Keep	Keep	Keep	Keep	Keep
7	8	Keep	Keep	Keep	Keep	Keep
8	9	Keep	Keep	Keep	Keep	Keep
9	10	Keep	Keep	Keep	Keep	Keep
10	11	Keep	Keep	Keep	Keep	Keep
11	12	Keep	Keep	Keep	Keep	Keep
12	13	Keep	Keep	Keep	Keep	Keep
13	14	Keep	Keep	Keep	Keep	Keep
14	15	Keep	Keep	Keep	Keep	Keep
15	16	Keep	Keep	Keep	Keep	Keep
16	17	Keep	Keep	Keep	Keep	Keep
17	18	Keep	Keep	Keep	Keep	Keep
18	19	Keep	Keep	Keep	Keep	Keep
19	20	Keep	Keep	Keep	Keep	Keep
20	21	Keep	Keep	Keep	Keep	Keep
21	22	Keep	Replace	Keep	Keep	Keep
22	23	Keep	Keep	Keep	Keep	Keep
23	24	Keep	Keep	Keep	Keep	Keep
24	25	Keep	Keep	Keep	Keep	Replace
25	26	Replace	Keep	Keep	Keep	Keep
26	27	Keep	Keep	Keep	Keep	Keep
27	28	Keep	Keep	Keep	Keep	Keep
28	29	Keep	Keep	Replace	Keep	Keep
29	30	Keep	Keep	Keep	Keep	Keep
30	31	Keep	Keep	Keep	Keep	Keep
31	32	Keep	Keep	Keep	Keep	Keep
32	33	Keep	Keep	Keep	Replace	Keep
33	34	Keep	Keep	Keep	Keep	Keep
34	35	Keep	Keep	Keep	Keep	Keep
35	36	Keep	Keep	Keep	Keep	Keep
36	37	Keep	Keep	Keep	Keep	Keep
37	38	Keep	Keep	Keep	Keep	Keep
38	39	Keep	Keep	Keep	Keep	Keep
39	40	Keep	Replace	Keep	Keep	Keep
40	41	Keep	Keep	Keep	Keep	Keep
41	42	Keep	Keep	Keep	Keep	Keep
42	43	Keep	Keep	Keep	Keep	Keep
43	44	Keep	Keep	Keep	Keep	Keep
44	45	Keep	Keep	Keep	Keep	Keep
45	46	Keep	Keep	Keep	Keep	Keep
46	47	Keep	Keep	Keep	Keep	Keep
47	48	Keep	Keep	Keep	Keep	Keep
48	49	Keep	Keep	Keep	Keep	Keep
49	50	Keep	Keep	Keep	Keep	Keep
Total profile cost $s		26,822	63,459	31,293	54,815	107,583

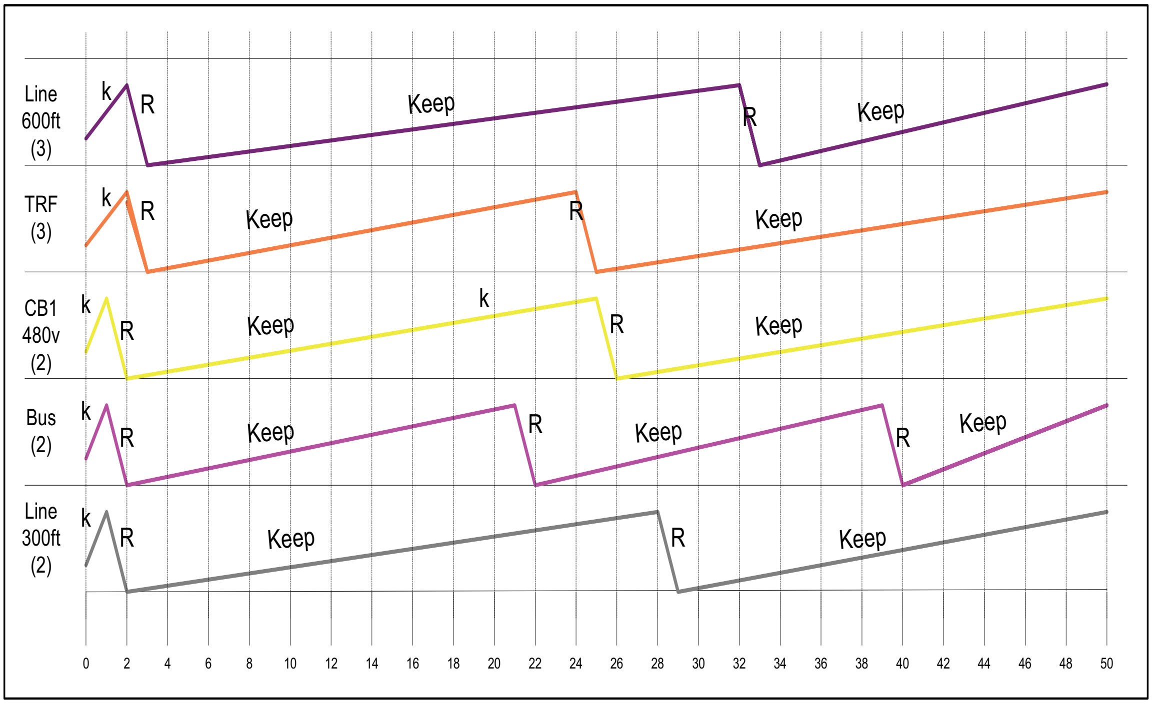

Figure 6.6 Additional replacement analysis policies generated in iteration 3

Considering all the different component replacement schedules generated so far (i.e., 16 profiles), the integer programming formulation 1, is applied again to check budget violations, the formulation is shown in Appendix B. The problem was solved using LINDO, and this time in the final solution, all $z_t > 0$, which means there are no more annual budget violations. Therefore, the second integer programming formulation (IP2) is used to find the recommended replacement analysis policy among all the different component replacement profiles generated.

Step 3. The second integer program formulation (IP2) is applied using all the data generated from all the profiles created in the previous steps. The complete formulation is shown in Appendix B. The problem was solved using LINDO. Table 6.5 shows the final results obtained for the solution to the replacement problem, the total NPV replacement cost is \$513,291. Table 6.6 has the specific information about the keep/replace decisions for each time period. Figure 6.7 shows the optimal system level replacement policies. The lower bound of the optimal solution is $508,243. The solution is within 0.98% of the lower bound, and therefore, this solution has less than 0.98% variation from the unknown optimal solution.

Table 6.5. Final results

Component	**Profile number**	**Replacement cost**
Line 13.8Kv	1	164,256
CB 13.8Kv	1	33,996
Switch	1	53,764
Bus	1	19,466
CB2 480v	1	107,584
Line300ft	1	26,822
Line 600ft	2	57,745
CB1 480v	2	19,926
Transformer	3	29,734
Total replacement cost		$513,293

Table 6.6. Replacement analysis policies

Beginning	end	Line 13.8Kv	CB 13.8Kv	Line 600ft	Switch	Transformer	CB1 480v	Bus	CB2 480v	Line 300ft
0	1	Keep	Keep	Keep	Keep	Keep	Keep	Replace	Keep	Replace
1	2	Keep	Keep	Replace	Keep	Keep	Replace	Keep	Keep	Keep
2	3	Keep	Keep	Keep	Keep	Replace	Keep	Keep	Keep	Keep
3	4	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
4	5	Replace	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
5	6	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
6	7	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
7	8	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
8	9	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
9	10	Keep	Keep	Keep	Replace	Keep	Keep	Keep	Keep	Keep
10	11	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
11	12	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
12	13	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
13	14	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
14	15	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Replace	Keep
15	16	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
16	17	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
17	18	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
18	19	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
19	20	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
20	21	Keep	Keep	Keep	Keep	Keep	Keep	Replace	Keep	Keep
21	22	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
22	23	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
23	24	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
24	25	Keep	Keep	Keep	Keep	Replace	Keep	Keep	Keep	Keep
25	26	Keep	Keep	Keep	Keep	Keep	Replace	Keep	Keep	Keep
26	27	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
27	28	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Replace
28	29	Replace	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
29	30	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
30	31	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
31	32	Keep	Keep	Replace	Keep	Keep	Keep	Keep	Keep	Keep
32	33	Keep	Replace	Keep	Keep	Keep	Keep	Keep	Keep	Keep
33	34	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
34	35	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
35	36	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
36	37	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Replace	Keep
37	38	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
38	39	Keep	Keep	Keep	Keep	Keep	Keep	Replace	Keep	Keep
39	40	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
40	41	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
41	42	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
42	43	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
43	44	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
44	45	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
45	46	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
46	47	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
47	48	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
48	49	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
49	50	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep

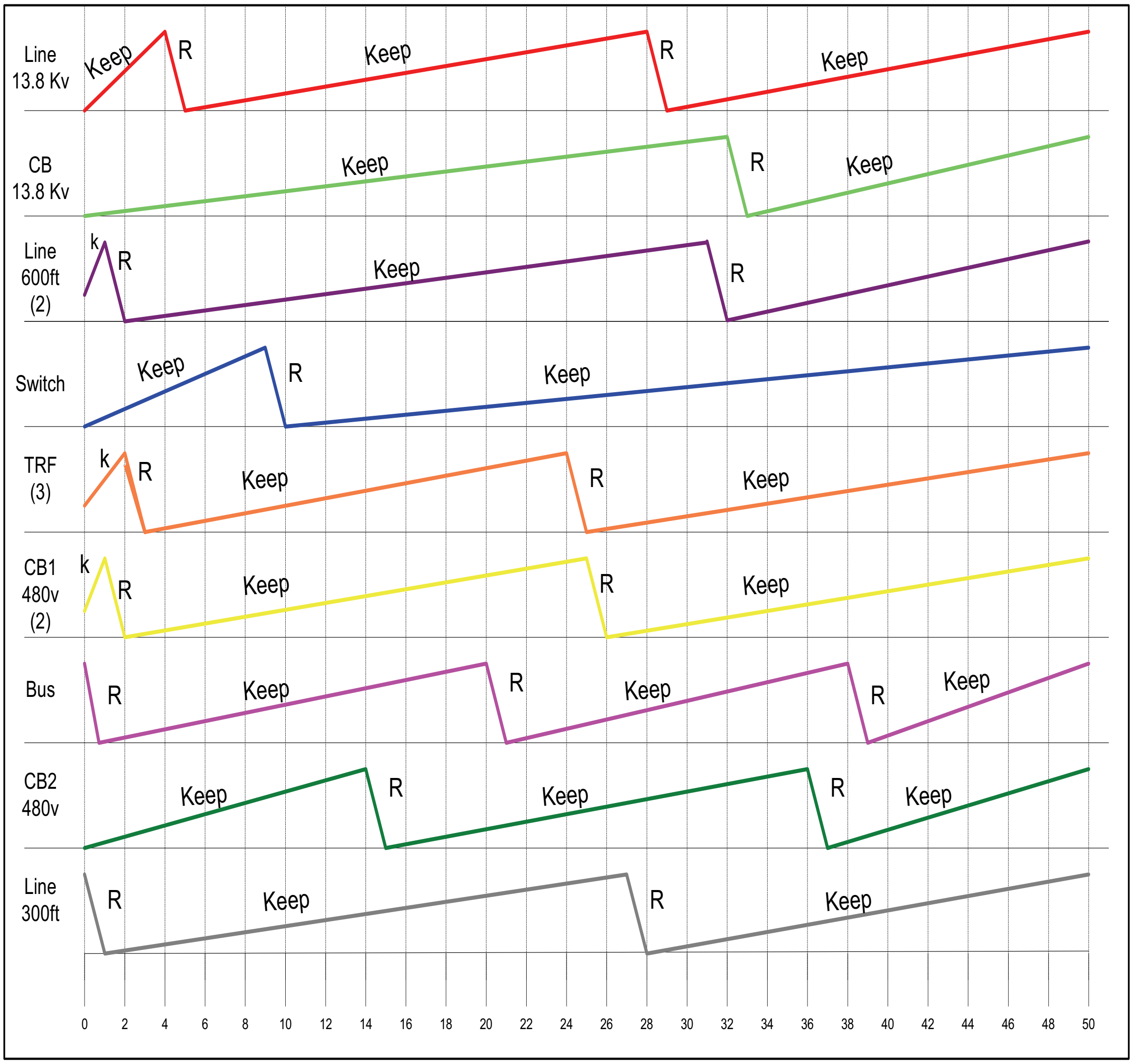

Figure 6.7 Recommended system-level replacement schedule for example 1

6.7 Example 2

The second example considers a radial system composed of six components which are three lines and three circuit breakers. This system was originally presented by Billinton & Li (1994). This is shown in Figure 6.8. The parameter values for the problem are presented in Table 6.7. The estimated customer interruption costs (IC_t) at the load point analyzed are 600 $/hour. All the costs are discounted back to time 0 using the NPV considering an interest rate, ι. An interest rate of 10% ($\iota = 0.1$) was used in all sample calculations.

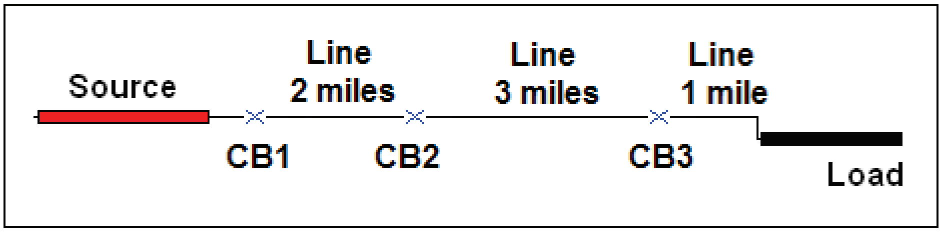

Figure. 6.8 Radial configuration –Example 2

Table 6.7. Example 2 data

Component	length	Asset initial age (τ)	λ_l (outages/year)	β_l	r_l (hours/outage)	C_{mrl} ($/outage)	$P_0(0)$ ($)
L1	1 mile	18	0.25	1.55	1	5500	45000
L2	2 miles	24	0.2	1.58	3	6500	66000
L3	3 miles	22	0.3	1.60	0.5	7200	75000
CB1	--	27	0.036	1.45	54.23	4500	35000
CB2	--	21	0.047	1.40	46.21	4800	39000
CB3	--	25	0.059	1.47	35.67	4700	34000

Step 1, Iteration 1. The dynamic programming model is run for each component in the system to find each individual component replacement schedules. Figure 6.9 shows the results for the replacement decisions for the components in the system. The lower bound in this example is $703,523. The replacement schedule number for each component is indicated in parentheses.

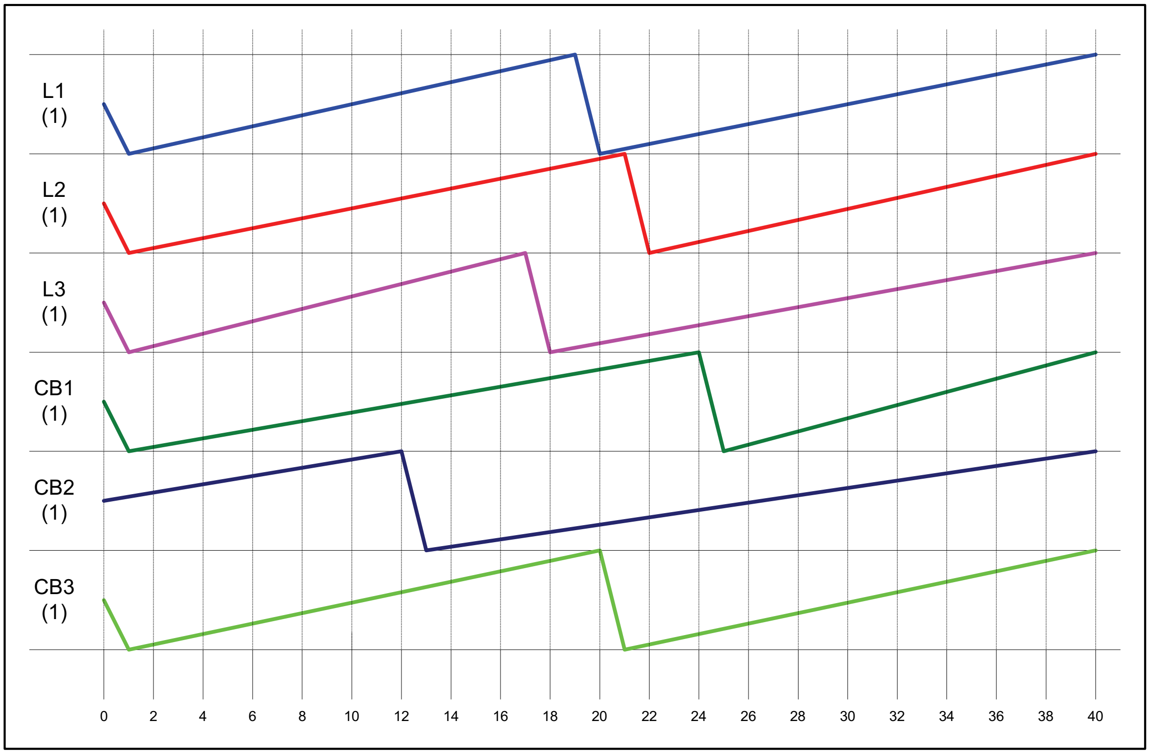

Figure 6.9 Component replacement schedules first iteration

Step 2, Iteration 1. The data generated in the previous step is used in the IP1 model to check for possible budget violations. The problem is solved using LINDO. In the final solution, all z_t values are equal to zero, except for period 1 ($z_1>0$), which means that new additional replacement schedules need to be generated for those components with replacement in period 1, in this case, components (L 1mile, L 2miles, L 3miles, CB1 and CB3).

Step 1, Iteration 2. Five new replacement schedules are generated after applying the dynamic programming model for the components with replacement in period 1 in the previous iteration. Figure 6.10 shows the new additional profiles generated.

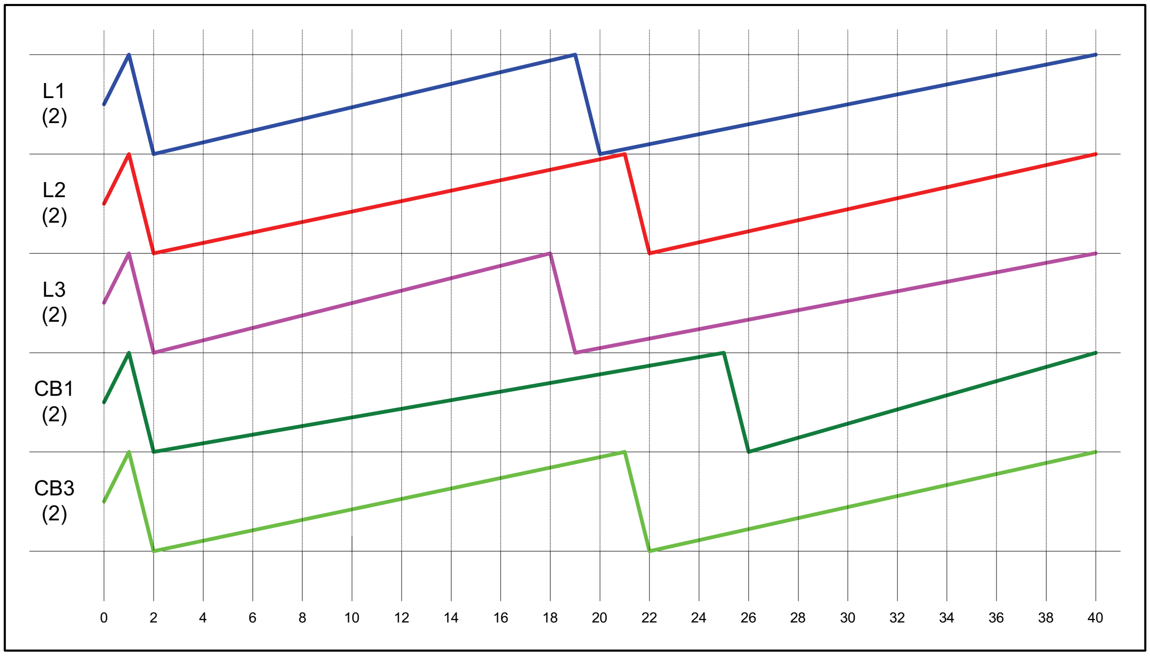

Figure 6.10 Additional replacement schedules second iteration

Step 2, Iteration 2. All the replacement profiles (i.e., 11 profiles) generated up to this step are considered in the IP1 model to check for possible budget violations. In the final solution, for the model z_1 , z_2 and z_{22} are greater than zero. Therefore new profiles have to be generated for the components with replacement in those periods.

Step 1, Iteration 3. Figure 6.11 shows all the new additional component replacement profiles generated in the third iteration.

Step 2, Iteration 3. Again the IP1 model is run to check for possible budget violations. It considers all the 18 replacement profiles generated up to this iteration. In the final solution, there is a budget violation in period 3 ($z_3 > 0$). Therefore, new profiles have to be generated for those components with replacement in period 3.

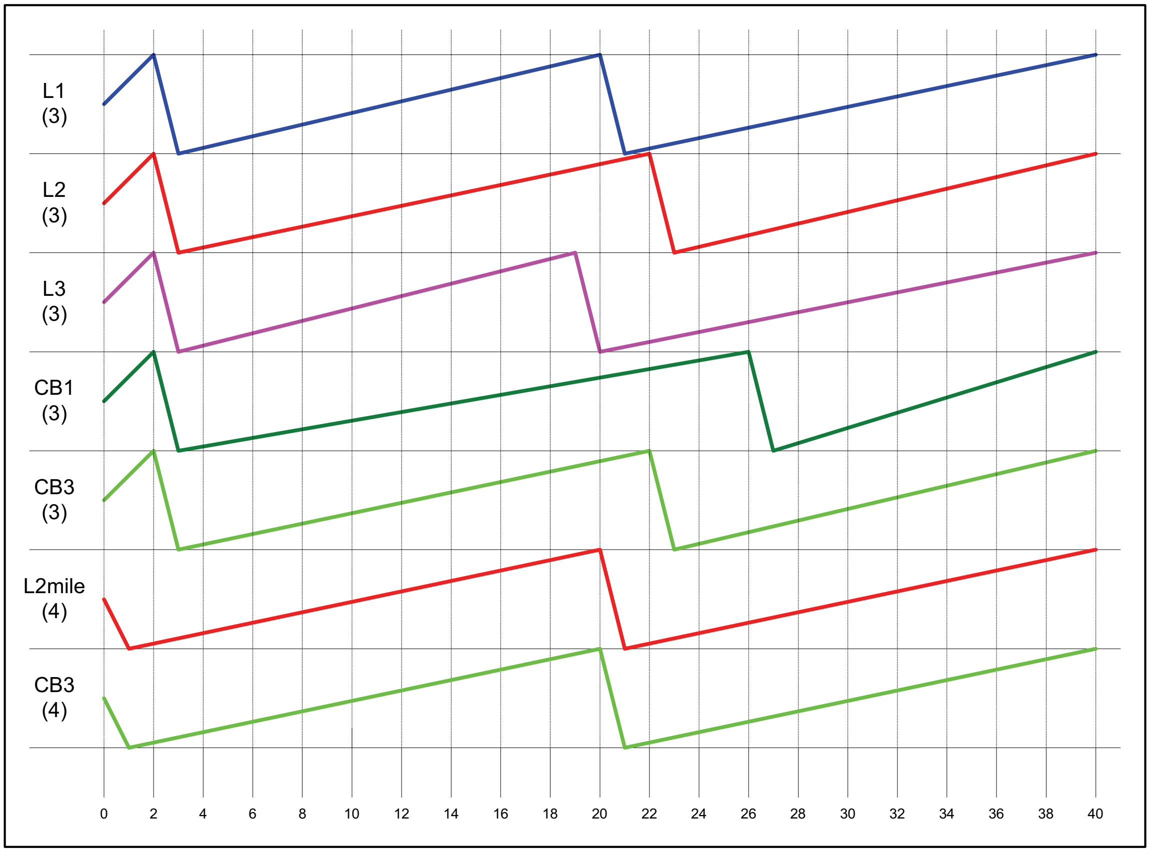

Figure 6.11 New replacement schedules third iteration

Step 1, Iteration 4. The new additional profiles generated in the fourth iteration are shown in Figure 6.12. In this figure, it can be seen that component Line 2 miles has now 5 different replacement profiles, while the CB1 has 4 different replacement schedules.

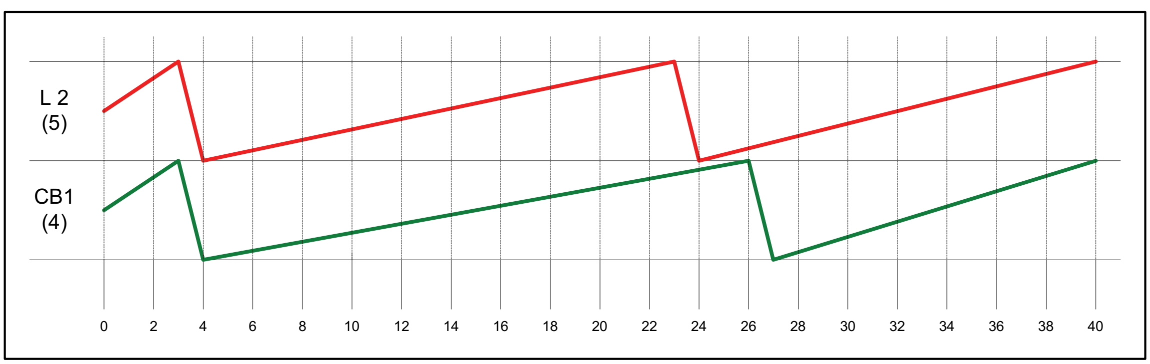

Figure 6.12 New replacement schedules fourth iteration

Step 1, Iteration 4. The IP program is solved again considering all the 20 different replacement schedules generated. In the final solution, all the z_t values are equal to zero. There are not annual budget violations and the program can now proceed to IP2.

Step 3. The IP2 model is applied to obtain the optimal system-level schedule with the minimum cost. Figure 6.13 graphically shows the optimal solution found in the present example. The component profile number chosen is also indicated. For example, the profile selected for the CB3 is the schedule number 3, while the replacement schedule selected for the Line with length of two miles is the profile number 5.

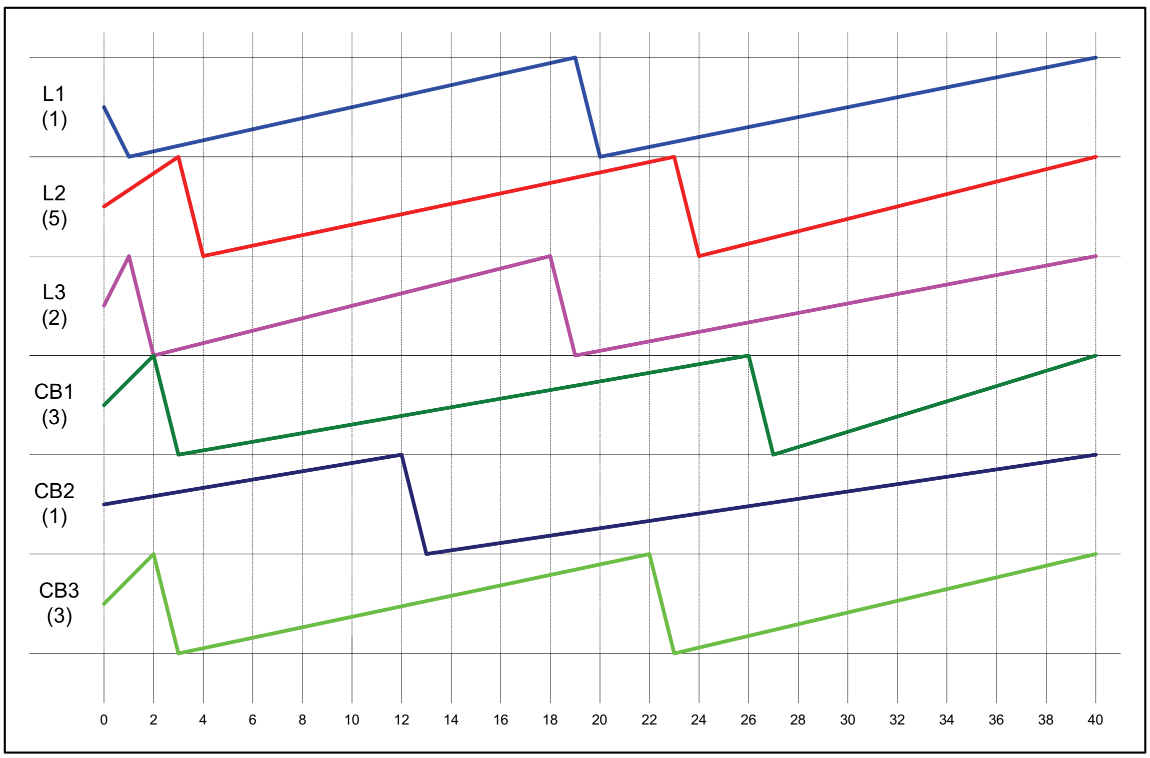

Figure 6.13 System level replacement schedule

Table 6.8 shows the detailed keep and replace decisions to be made over the entire time horizon and Table 6.9 indicates the total system-level replacement cost of the optimal policy obtained. The total cost is $714,590, which compares to the lower-bound of $703,523. The solution from the new algorithm is within 1.55% of the lower bound, and therefore, less than 1.55% of the unknown optimal solution.

Table 6.8. Keep/Replace decisions for the components in the system

Period	L1	CB2	L3	CB1	CB3	L2
1	Replace	Keep	Keep	Keep	Keep	Keep
2	Keep	Keep	Replace	Keep	Keep	Keep
3	Keep	Keep	Keep	Replace	Replace	Keep
4	Keep	Keep	Keep	Keep	Keep	Replace
5	Keep	Keep	Keep	Keep	Keep	Keep
6	Keep	Keep	Keep	Keep	Keep	Keep
7	Keep	Keep	Keep	Keep	Keep	Keep
8	Keep	Keep	Keep	Keep	Keep	Keep
9	Keep	Keep	Keep	Keep	Keep	Keep
10	Keep	Keep	Keep	Keep	Keep	Keep
11	Keep	Keep	Keep	Keep	Keep	Keep
12	Keep	Keep	Keep	Keep	Keep	Keep
13	Keep	Replace	Keep	Keep	Keep	Keep
14	Keep	Keep	Keep	Keep	Keep	Keep
15	Keep	Keep	Keep	Keep	Keep	Keep
16	Keep	Keep	Keep	Keep	Keep	Keep
17	Keep	Keep	Keep	Keep	Keep	Keep
18	Keep	Keep	Keep	Keep	Keep	Keep
19	Keep	Keep	Replace	Keep	Keep	Keep
20	Replace	Keep	Keep	Keep	Keep	Keep
21	Keep	Keep	Keep	Keep	Keep	Keep
22	Keep	Keep	Keep	Keep	Keep	Keep
23	Keep	Keep	Keep	Keep	Replace	Keep
24	Keep	Keep	Keep	Keep	Keep	Replace
25	Keep	Keep	Keep	Keep	Keep	Keep
26	Keep	Keep	Keep	Keep	Keep	Keep
27	Keep	Keep	Keep	Replace	Keep	Keep
28	Keep	Keep	Keep	Keep	Keep	Keep
29	Keep	Keep	Keep	Keep	Keep	Keep
30	Keep	Keep	Keep	Keep	Keep	Keep
31	Keep	Keep	Keep	Keep	Keep	Keep
32	Keep	Keep	Keep	Keep	Keep	Keep
33	Keep	Keep	Keep	Keep	Keep	Keep
34	Keep	Keep	Keep	Keep	Keep	Keep
35	Keep	Keep	Keep	Keep	Keep	Keep
36	Keep	Keep	Keep	Keep	Keep	Keep
37	Keep	Keep	Keep	Keep	Keep	Keep
38	Keep	Keep	Keep	Keep	Keep	Keep
39	Keep	Keep	Keep	Keep	Keep	Keep
40	Keep	Keep	Keep	Keep	Keep	Keep

Table 6.9. System-level replacement schedule cost

Component	Profile selected	Cost
L1	1	113535
CB2	1	79455
L3	2	193546
CB1	3	82322
CB3	3	92071
L2	5	153661
Total system-level replacement cost		714590

6.8 Conclusions

In the present chapter, a novel model for solving the replacement schedules for sets of heterogeneous components subject to annual budget constraints is presented. The method is demonstrated in the replacement analysis of a radial distribution system which is commonly used in rural areas. Extensions of the present model are developed in the next chapter where complex ETDS systems are considered.

Chapter 7

Component replacement analysis for complex ETDS

The previous model was extended so it could be readily applied to any complex ETDS system. For many systems in congested areas, more complex networks are required to provide redundancy and the needed reliability. This is a complex nonlinear, non-separable optimization problem. An accurate approximation was made to the objective function so the previous analysis approach could be extended to consider complex ETDS.

The model developed in the previous chapter was applied for radial distribution systems where all the components are arranged in series. The method cannot be readily applied to complex ETDS systems because the system outage rate is a nonlinear and non-separable model and estimation of each component opportunity costs cannot be directly obtained. In the present chapter, a Taylor series expansion model is used to approximate the associated opportunity cost for each component in the system. An example is shown for the component replacement schedule for the Dual Element Spot Network (DESN).

7.1 Approximating system unavailability/downtime for complex systems

The objective function minimized in the dynamic programming used in the determination of each individual component replacement schedule considers the minimization of the opportunity costs. In radial systems the computation can be directly obtained because the opportunity cost calculation depends on the total system downtime which is proportionally dependent on each component outage rate and repair time. The previous dynamic programming model required a separable function, one that could be expressed as the sum of the contribution of each individual component to the total system unavailability. Unfortunately, that is not the case for complex systems like DESN, breaker-and-a-half, etc. Therefore, a Taylor series expansion (TSE) model was used to estimate the total system unavailability for complex ETDS. The approximation was developed to determine the costs of unmet demand associated with each component in the system. Equation 7.1 is a general formulation of the TSE. This method can be used to approximate complex objective functions (Coit & Jin, 2001).

$$\begin{aligned} f(\mathrm{x}) = f(\mathrm{x}_0) + \sum_{i=1}^{n}(x_i - x_{0i})\frac{\partial}{\partial x_i} f(x)\Big|_{x=x_0} + \sum_{i=1}^{n}\frac{(x_i - x_{0i})^2}{2}\frac{\partial^2}{\partial x_i^2} f(x)\Big|_{x=x_0} \\ + \sum_{i=1}^{n}\sum_{j>i}^{n}\frac{(x_i - x_{0i})(x_i - x_{0j})}{2}\frac{\partial^2}{\partial x_i \partial x_j} f(x)\Big|_{x=x_0} + \ldots \end{aligned} \tag{7.1}$$

In the complex ETDS, the decomposition of the opportunity costs is based on a TSE considering the total system unavailability $U(\mathbf{u})$ around the point $\mathbf{u} = \mathbf{0}$. The approximation is based on the first terms of the TSE model, as shown on Equation 7.2.

$$U_{s,t} = U(\mathrm{u}) \approx U(0) + \sum_{l=1}^{n}\left(\mu_{l,t} - 0\right)\frac{\partial U}{\partial u_{l,t}} + \ldots = \sum_{l=1}^{n}(\lambda_{l,t} r_l - 0)\frac{\partial \lambda_{l,t}}{\partial u_{l,t}}\frac{\partial U}{\partial \lambda_{l,t}} = \sum_{l=1}^{n}\lambda_{l,t} r_l \frac{1}{r_l}\frac{\partial U}{\partial \lambda_{l,t}} \tag{7.2}$$

Each component opportunity cost in complex configurations can be estimated by using Equation 7.3

$$U_{s,t} = \sum_{l=1}^{n}\lambda_{l,t}\frac{\partial U}{\partial \lambda_{l,t}} \tag{7.3}$$

By applying Equation 7.3, we now have a linear model where we can separate the total system unavailability into the approximate contribution of each component. Using Equation 7.4, the individual contribution to the system unavailability for each component in the system ($\phi_{l,t}$) can be determined.

$$\phi_{l,t} = \lambda_{l,t} \frac{\partial U}{\partial \lambda_{l,t}}$$

(7.4)

7.2 Opportunity costs

In the previous chapter, the opportunity costs for each component in radial systems was determined by using the interruption costs times the annual downtime of the system attributed to the component being analyzed by,

$$CU_{l,t} = (\lambda_{l,t} r_l)\ IC_t \tag{7.5}$$

In the present chapter, the component opportunity cost is obtained using the results obtained by using the TSE, for each time period ($\phi_{l,t}$).

$$CU_{l,t} = \phi_{l,t}\ IC_t \tag{7.6}$$

Where:

IC_t = Customer interruption costs ($/hour) during period t

$\phi_{l,t}$ = Component l contribution to the system unavailability in period t

$CU_{l,t}$ = Opportunity cost for component l in period t

By using Equation 7.6, the associated opportunity costs for each individual component are obtained, and this information is then used in the dynamic programming model.

7.3 DESN configuration

The DESN configuration has been adopted (by Hydro One) as a basic design for supplying electricity to significant load areas. Figure 7.1 presents a functional diagram of a DESN configuration. Components 12 and 13 representing supply lines, components 1 and 2 are the power transformers, buses are defined by components 6 and 7, and finally, breakers are component numbers 3, 4, 5, 10, 11 and 20. Breaker number 20 is normally open.

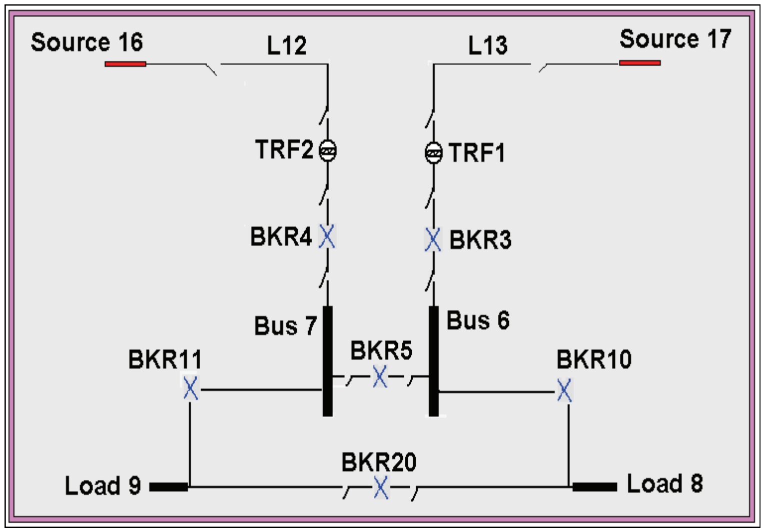

Figure 7.1: DESN Configuration

The minimal cut-sets for outages at either load 8 or load 9 are shown in Table 7.1. The outage rate (outages per year) and repair times (hours per outage) associated with each minimal cut-set are presented in Table 7.2.

Table 7.1 DESN minimal cut-sets for outages at either load 8 or load 9

{BUS 7}	{BKR 11}	{BUS 6}	{BKR 10}
{LINE 13, BKR 4}	{TRF 1, BKR 4}	{TRF 2, BKR 5}	{ LINE 13,TRF 2}
{LINE 13, TRF 2}	{TRF 1, TRF 2}	{LINE 12, BKR 5}	{LINE 13, BKR 5}
{LINE 13, LINE 12}	{TRF 1, LINE 12}	{BKR 4, BKR 3}	{ TRF 1, TRF 2}
{BKR 4, BUS 6}	{LINE 12, BKR 3}	{TRF 2, BKR 3}	{ TRF 1, BKR 5}
{TRF 2, BUS 6}	{LINE 12, BUS 6}	{BKR 3, BUS 7}	{BKR 3, BKR 5}
{LINE 12, LINE 13}	{LINE 13, BUS 7}	{LINE 12, BKR 3}	{TRF 1, BUS 7}
{BKR 4, BKR 5}	{LINE 13, BKR 4}	{LINE 12, TRF 1}	{ TRF 1, BKR 4}
{BKR 3, TRF 2}	{BKR 3, BKR 4}		

Table 7.2 Outage rate and repair times of cut sets for outages at either load 8 or load 9

Cut sets	Outage rate	Repair time	Unavailability
{BUS 7}	λ_7	r_7	$\lambda_7\, r_7$
{BKR 11}	λ_{11}	r_{11}	$\lambda_{11}\, r_{11}$
{BUS 6}	λ_6	r_6	$\lambda_6\, r_6$
{BKR 10}	λ_{10}	r_{10}	$\lambda_{10}\, r_{10}$
{LINE 13, TRF 2}	$\lambda_{13}\lambda_2\,(r_{13} + r_2)$	$(r_{13}\, r_2)\,/\,r_{13} + r_2$	$\lambda_{13}\lambda_2\,(r_{13}\, r_2)$
{TRF 2, BUS 6}	$\lambda_2\lambda_6\,(r_2 + r_6)$	$(r_2\, r_6)\,/\,r_2 + r_6$	$\lambda_2\lambda_6\,(r_2\, r_6)$
{BKR 4, BUS 6}	$\lambda_4\lambda_6\,(r_4 + r_6)$	$(r_4\, r_6)\,/\,r_4 + r_6$	$\lambda_4\lambda_6\,(r_4\, r_6)$
{LINE 12, LINE 13}	$\lambda_{12}\lambda_{13}\,(r_{12} + r_{13})$	$(r_{12}\, r_{13})\,/\,r_{12} + r_{13}$	$\lambda_{12}\lambda_{13}\,(r_{12}\, r_{13})$
{BKR 3, BKR 4}	$\lambda_3\lambda_4\,(r_3 + r_4)$	$(r_3\, r_4)\,/\,r_3 + r_4$	$\lambda_3\lambda_4\,(r_3\, r_4)$
{TRF 2, BKR 5}	$\lambda_2\lambda_5\,(r_2 + r_5)$	$(r_2\, r_5)\,/\,r_2 + r_5$	$\lambda_2\lambda_5\,(r_2\, r_5)$
{LINE 12, BUS 6}	$\lambda_{12}\lambda_6\,(r_{12} + r_6)$	$(r_{12}\, r_6)\,/\,r_{12} + r_6$	$\lambda_{12}\lambda_6\,(r_{12}\, r_6)$
{TRF 1, LINE 12}	$\lambda_1\lambda_{12}\,(r_1 + r_{12})$	$(r_1\, r_{12})\,/\,r_1 + r_{12}$	$\lambda_1\lambda_{12}\,(r_1\, r_{12})$
{LINE 13, BUS 7}	$\lambda_{13}\lambda_7\,(r_{13} + r_7)$	$(r_{13}\, r_7)\,/\,r_{13} + r_7$	$\lambda_{13}\lambda_7\,(r_{13}\, r_7)$
{LINE 12, BKR 5}	$\lambda_{12}\lambda_5\,(r_{12} + r_5)$	$(r_{12}\, r_5)\,/\,r_{12} + r_5$	$\lambda_{12}\lambda_5\,(r_{12}\, r_5)$
{BKR 3, BUS 7}	$\lambda_3\lambda_7\,(r_3 + r_7)$	$(r_3\, r_7)\,/\,r_3 + r_7$	$\lambda_3\lambda_7\,(r_3\, r_7)$
{ TRF 1, BKR 5}	$\lambda_1\lambda_5\,(r_1 + r_5)$	$(r_1\, r_5)\,/\,r_1 + r_5$	$\lambda_1\lambda_5\,(r_1\, r_5)$
{LINE 12, BKR 3}	$\lambda_{12}\lambda_3\,(r_{12} + r_3)$	$(r_{12}\, r_3)\,/\,r_{12} + r_3$	$\lambda_{12}\lambda_3\,(r_{12}\, r_3)$
{LINE 13, BKR 5}	$\lambda_{13}\lambda_5\,(r_{13} + r_5)$	$(r_{13}\, r_5)\,/\,r_{13} + r_5$	$\lambda_{13}\lambda_5\,(r_{13}\, r_5)$
{BKR 3, BKR 5}	$\lambda_3\lambda_5\,(r_3 + r_5)$	$(r_3\, r_5)\,/\,r_3 + r_5$	$\lambda_3\lambda_5\,(r_3\, r_5)$
{ TRF 1, TRF 2}	$\lambda_1\lambda_2\,(r_1 + r_2)$	$(r_1\, r_2)\,/\,r_1 + r_2$	$\lambda_1\lambda_2\,(r_1\, r_2)$
{TRF 1, BUS 7}	$\lambda_1\lambda_7\,(r_1 + r_7)$	$(r_1\, r_7)\,/\,r_1 + r_7$	$\lambda_1\lambda_7\,(r_1\, r_7)$
{BKR 4, BKR 5}	$\lambda_4\lambda_5\,(r_4 + r_5)$	$(r_4\, r_5)\,/\,r_4 + r_5$	$\lambda_4\lambda_5\,(r_4\, r_5)$
{LINE 13, BKR 4}	$\lambda_{13}\lambda_4\,(r_{13} + r_4)$	$(r_{13}\, r_4)\,/\,r_{13} + r_4$	$\lambda_{13}\lambda_4\,(r_{13}\, r_4)$
{LINE 12, TRF 1}	$\lambda_{12}\lambda_1\,(r_{12} + r_1)$	$(r_{12}\, r_1)\,/\,r_{12} + r_1$	$\lambda_{12}\lambda_1\,(r_{12}\, r_1)$
{ TRF 1, BKR 4}	$\lambda_1\lambda_4\,(r_1 + r_4)$	$(r_1\, r_4)\,/\,r_1 + r_4$	$\lambda_1\lambda_4\,(r_1\, r_4)$
{BKR 3, TRF 2}	$\lambda_3\lambda_2\,(r_3 + r_2)$	$(r_3\, r_2)\,/\,r_3 + r_2$	$\lambda_3\lambda_2\,(r_3\, r_2)$

The expected system downtime for outages at either load 8 or load 9 ($U_{L8\cup L9}$) can be obtained by using Equation 7.4.

$$\begin{aligned} f(U_{L8\cup L9}) &= \lambda_6 r_6 + \lambda_{10} r_{10} + \lambda_7 r_7 + \lambda_{11} r_{11} + \lambda_4\lambda_5(r_4 r_5) + \lambda_{13}\lambda_4(r_{13} r_4) \\ &+ \lambda_1\lambda_4(r_1 r_4) + \lambda_2\lambda_5(r_2 r_5) + \lambda_{13}\lambda_2(r_{13} r_2) + \lambda_1\lambda_2(r_1 r_2) + \lambda_{12}\lambda_5(r_{12} r_5) \\ &+ \lambda_{13}\lambda_{12}(r_{13} r_{12}) + \lambda_1\lambda_{12}(r_1 r_{12}) + \lambda_4\lambda_3(r_4 r_3) + \lambda_4\lambda_6(r_4 r_6) + \lambda_{12}\lambda_3(r_{12} r_3) \\ &+ \lambda_2\lambda_3(r_2 r_3) + \lambda_2\lambda_6(r_2 r_6) + \lambda_{12}\lambda_6(r_{12} r_6) + \lambda_3\lambda_7(r_3 r_7) + \lambda_3\lambda_5(r_3 r_5) \\ &+ \lambda_{13}\lambda_7(r_{13} r_7) + \lambda_{13}\lambda_5(r_{13} r_5) + \lambda_1\lambda_7(r_1 r_7) + \lambda_1\lambda_5(r_1 r_5) \end{aligned} \tag{7.7}$$

Using Equation 7.5, the individual contribution to the system unavailability for each component in the system ($\phi_{l,t}$) can be determined. Results for the DESN configuration are shown in Equations 7.8 through 7.20.

$$\phi_1 = \lambda_1\left[\lambda_4(r_1 r_4) + \lambda_2(r_1 r_2) + \lambda_{12}(r_1 r_{12}) + \lambda_7(r_1 r_7) + \lambda_5(r_1 r_5)\right] \tag{7.8}$$

$$\phi_2 = \lambda_2\left[\lambda_5(r_2r_5) + \lambda_{13}(r_{13}r_2) + \lambda_1(r_1r_2) + \lambda_3(r_2r_3) + \lambda_6(r_2r_6)\right] \tag{7.9}$$

$$\phi_3 = \lambda_3\left[\lambda_4(r_3r_4) + \lambda_{12}(r_{12}r_3) + \lambda_2(r_2r_3) + \lambda_7(r_3r_7) + \lambda_5(r_3r_5)\right] \tag{7.10}$$

$$\phi_4 = \lambda_4\left[\lambda_5(r_4r_5) + \lambda_{13}(r_{13}r_4) + \lambda_1(r_1r_4) + \lambda_3(r_4r_3) + \lambda_6(r_4r_6)\right] \tag{7.11}$$

$$\phi_5 = \lambda_5\left[\lambda_2(r_5r_2) + \lambda_{12}(r_{12}r_5) + \lambda_3(r_3r_5) + \lambda_{13}(r_{13}r_5) + \lambda_1(r_1r_5)\right] \tag{7.12}$$

$$\phi_6 = \lambda_6\left[r_6 + \lambda_4(r_4r_6) + \lambda_2(r_2r_6) + \lambda_{12}(r_{12}r_6)\right] \tag{7.13}$$

$$\phi_7 = \lambda_7\left[r_7 + \lambda_3(r_3r_7) + \lambda_{13}(r_{13}r_7) + \lambda_1(r_1r_7)\right] \tag{7.14}$$

$$\phi_8 = 0 \tag{7.15}$$

$$\phi_9 = 0 \tag{7.16}$$

$$\phi_{10} = \lambda_{10}r_{10} \tag{7.17}$$

$$\phi_{11} = \lambda_{11}r_{11} \tag{7.18}$$

$$\phi_{12} = \lambda_{12}\left[\lambda_5(r_{12}r_5) + \lambda_{13}(r_{13}r_{12}) + \lambda_1(r_1r_{12}) + \lambda_3(r_{12}r_3) + \lambda_6(r_{12}r_6)\right] \tag{7.19}$$

$$\phi_{13} = \lambda_{13}\left[\lambda_4(r_{13}r_4) + \lambda_2(r_{13}r_2) + \lambda_{12}(r_{13}r_{12}) + \lambda_7(r_{13}r_7) + \lambda_5(r_{13}r_5)\right] \tag{7.20}$$

For example, ϕ_1 is used to obtain the estimated contribution to the total system unavailability associated with component 1 (transformer) in the DESN configuration.

7.4 Model Testing

The model has been tested for different values of outage rate and average repair times for the different components in the DESN system. Tables 7.3 to 7.5 show the results for different scenarios for the different components in the system. Table 7.3 considers low outage rates, Table 7.4 considers medium outage rates and Table 7.5 considers high outage rates, the repair rates are varied from low to high. The results obtained show the accuracy of the estimation of the total expected system downtime by the TSE method developed. In the examples the percent error ranged from 0.054% to 1.33%. This is an important result because the TSE model is separable; that is, the system downtime can be expressed as a linear sum of contributions attributed to individual components.

Table 7.3 Low component outage rates and increasing repair rates

Case 1					
Component	Outage rate	Average repair time	Component	Outage rate	Average repair time
Line 12	0.0025	0.5	BKR 3	0.056	1
Line 13	0.0044	0.5	BKR 4	0.043	1.5
TRF 1	0.0023	0.12	BKR 5	0.078	0.3
TRF 2	0.0078	0.6	BKR 10	0.023	0.8
Bus 6	0.023	0.29	BKR 11	0.087	0.5
Bus 7	0.065	0.18			
Unavailability	**Expected system downtime**	**Taylor series expansion**			
Load 8 or Load 9	**0.08027**	**0.08035**			
Case 2					
Component	Outage rate	Average repair time	Component	Outage rate	Average repair time
Line 12	0.0025	24	BKR 3	0.056	13
Line 13	0.0044	23	BKR 4	0.043	12
TRF 1	0.0023	21	BKR 5	0.078	18
TRF 2	0.0078	26	BKR 10	0.023	17
Bus 6	0.023	28	BKR 11	0.087	15
Bus 7	0.065	21			
Unavailability	**Expected system downtime**	**Taylor series expansion**			
Load 8 or Load 9	**3.7051**	**3.7072**			
Case 3					
Component	Outage rate	Average repair time	Component	Outage rate	Average repair time
Line 12	0.0025	2.5	BKR 3	0.056	1.3
Line 13	0.0044	2.8	BKR 4	0.043	1.7
TRF 1	0.0023	1.9	BKR 5	0.078	2.4
TRF 2	0.0078	1.5	BKR 10	0.023	2.6
Bus 6	0.023	1.7	BKR 11	0.087	1.9
Bus 7	0.065	1.6			
Unavailability	**Expected system downtime**	**Taylor series expansion**			
Load 8 or Load 9	**0.3682**	**0.3684**			
Case 4					
Component	Outage rate	Average repair time	Component	Outage rate	Average repair time
Line 12	0.0025	40	BKR 3	0.056	18
Line 13	0.0044	35	BKR 4	0.043	21
TRF 1	0.0023	23	BKR 5	0.078	16
TRF 2	0.0078	20	BKR 10	0.023	19
Bus 6	0.023	35	BKR 11	0.087	24
Bus 7	0.065	32			
Unavailability	**Expected system downtime**	**Taylor series expansion**			
Load 8 or Load 9	**5.4099**	**5.4127**			

Table 7.4 Medium component outage rates and increasing repair rates

Case 1					
Component	Outage rate	Average repair time	Component	Outage rate	Average repair time
Line 12	0.59	0.5	BKR 3	0.45	1
Line 13	0.75	0.5	BKR 4	0.87	1.5
TRF 1	0.23	0.12	BKR 5	0.97	0.3
TRF 2	0.43	0.6	BKR 10	0.99	0.8
Bus 6	0.95	0.29	BKR 11	0.96	0.5
Bus 7	0.98	0.18			
Unavailability	**Expected system downtime**	**Taylor series expansion**			
Load 8 or Load 9	**1.7240**	**1.7268**			
Case 2					
Component	Outage rate	Average repair time	Component	Outage rate	Average repair time
Line 12	0.59	2.5	BKR 3	0.45	1.3
Line 13	0.75	2.8	BKR 4	0.87	1.7
TRF 1	0.23	1.9	BKR 5	0.97	2.4
TRF 2	0.43	1.5	BKR 10	0.99	2.6
Bus 6	0.95	1.7	BKR 11	0.96	1.9
Bus 7	0.98	1.6			
Unavailability	**Expected system downtime**	**Taylor series expansion**			
Load 8 or Load 9	**7.5824**	**7.5913**			
Case 3					
Component	Outage rate	Average repair time	Component	Outage rate	Average repair time
Line 12	0.59	24	BKR 3	0.45	13
Line 13	0.75	23	BKR 4	0.87	12
TRF 1	0.23	21	BKR 5	0.97	18
TRF 2	0.43	26	BKR 10	0.99	17
Bus 6	0.95	28	BKR 11	0.96	15
Bus 7	0.98	21			
Unavailability	**Expected system downtime**	**Taylor series expansion**			
Load 8 or Load 9	**78.477**	**78.516**			
Case 4					
Component	Outage rate	Average repair time	Component	Outage rate	Average repair time
Line 12	0.59	40	BKR 3	0.45	18
Line 13	0.75	35	BKR 4	0.87	21
TRF 1	0.23	23	BKR 5	0.97	16
TRF 2	0.43	20	BKR 10	0.99	19
Bus 6	0.95	35	BKR 11	0.96	24
Bus 7	0.98	32			
Unavailability	**Expected system downtime**	**Taylor series expansion**			
Load 8 or Load 9	**106.52**	**106.59**			

Table 7.5 High component outage rates and increasing repair rates

Case 1					
Component	Outage rate	Average repair time	Component	Outage rate	Average repair time
Line 12	3.455	0.5	BKR 3	15.463	1
Line 13	3.254	0.5	BKR 4	16.378	1.5
TRF 1	9.487	0.12	BKR 5	18.215	0.3
TRF 2	9.591	0.6	BKR 10	16.185	0.8
Bus 6	17.82	0.29	BKR 11	17.216	0.5
Bus 7	16.403	0.18			
Unavailability	**Expected system downtime**	**Taylor series expansion**			
Load 8 or Load 9	**29.7505**	**30.0810**			
Case 2					
Component	Outage rate	Average repair time	Component	Outage rate	Average repair time
Line 12	3.455	2.5	BKR 3	15.463	1.3
Line 13	3.254	2.8	BKR 4	16.378	1.7
TRF 1	9.487	1.9	BKR 5	18.215	2.4
TRF 2	9.591	1.5	BKR 10	16.185	2.6
Bus 6	17.82	1.7	BKR 11	17.216	1.9
Bus 7	16.403	1.6			
Unavailability	**Expected system downtime**	**Taylor series expansion**			
Load 8 or Load 9	**131.60**	**132.46**			
Case 3					
Component	Outage rate	Average repair time	Component	Outage rate	Average repair time
Line 12	3.455	24	BKR 3	15.463	13
Line 13	3.254	23	BKR 4	16.378	12
TRF 1	9.487	21	BKR 5	18.215	18
TRF 2	9.591	26	BKR 10	16.185	17
Bus 6	17.82	28	BKR 11	17.216	15
Bus 7	16.403	21			
Unavailability	**Expected system downtime**	**Taylor series expansion**			
Load 8 or Load 9	**1387.80**	**1388.72**			
Case 4					
Component	Outage rate	Average repair time	Component	Outage rate	Average repair time
Line 12	3.455	40	BKR 3	15.463	18
Line 13	3.254	35	BKR 4	16.378	21
TRF 1	9.487	23	BKR 5	18.215	16
TRF 2	9.591	20	BKR 10	16.185	19
Bus 6	17.82	35	BKR 11	17.216	24
Bus 7	16.403	32			
Unavailability	**Expected system downtime**	**Taylor series expansion**			
Load 8 or Load 9	**1859.26**	**1884.31**			

It can be seen from Tables 7.3 through 7.5 that the approximation used to estimate the total expected system downtime provides accurate results. The approximation is more accurate for components with low outage rates and low repair times (as in the case of real power systems components).

7.5 Example

To demonstrate the replacement analysis model, consider the DESN configuration shown in Figure 7.1. It has two incoming lines, two transformers, two buses and six circuit breakers; component number 20 is a normally open breaker.

The outage rates for components in the system are assumed to increase as a NHPP model for repairable components. In the dynamic programming formulation for each individual component, the outage rate for the specific component under consideration is modeled according to a NHPP assumed, but for the rest of the components an average outage rate over a specified period of time was used because the age of these components at each tome is unknown at this stage of the analysis. Once all replacement decisions have been made, the system outage rate can be more accurately estimated given the component outages based on the planned replacements.

The component outage rates and repair times are presented in Table 7.1. In practice, a power company can use data collected specifically for the components in the system under consideration. The estimated customer interruption costs (IC_t) at the load point analyzed are 5,500 $/hour. All the costs are discounted back to time 0 using the NPV considering an interest rate, ι. An interest rate of 10% ($\iota = 0.1$) was used in all sample calculations.

Table 7.6. DESN Example data

Component	Asset initial age (τ)	λ_l (outages/year)	β_l	r_l (hours/outage)	Cmr_l ($/outage)	$P_0(0)$ ($)
Line 12	35	1.8394	1.25	6.0177	5000	85000
Line 13	38	2.2453	1.28	9.14320	5500	105000
Transformer 1	28	0.2042	1.55	97.6910	6000	120000
Transformer 2	31	0.3478	1.75	103.690	7000	140000
Bus 6	25	0.1982	1.73	5.97490	3000	95000
Bus 7	29	0.2275	1.76	4.89640	3500	88000
Breaker 3	31	0.1820	1.75	157.520	2000	75000
Breaker 4	19	0.1560	1.72	132.940	1800	78000
Breaker 5	32	0.0620	1.75	100.180	2500	76000
Breaker 10	25	0.0058	1.52	43.82010	1800	56000
Breaker 11	30	0.0066	1.59	48.11010	2000	48000

Step 1, Iteration 1:

The dynamic programming formulation is run for each component in the system in order to obtain their individual optimal replacement schedule. In the first iteration, 11 replacement schedules are obtained. Figure 7.2 graphically shows the replacement schedules for the components in the system. If there are no budget constraints, then this is the optimal system-level replacement schedule, but if there are annual budget constraints, then the first integer program (IP1) is applied to identify where those violations occur (if any) and the components with replacement at those time periods.

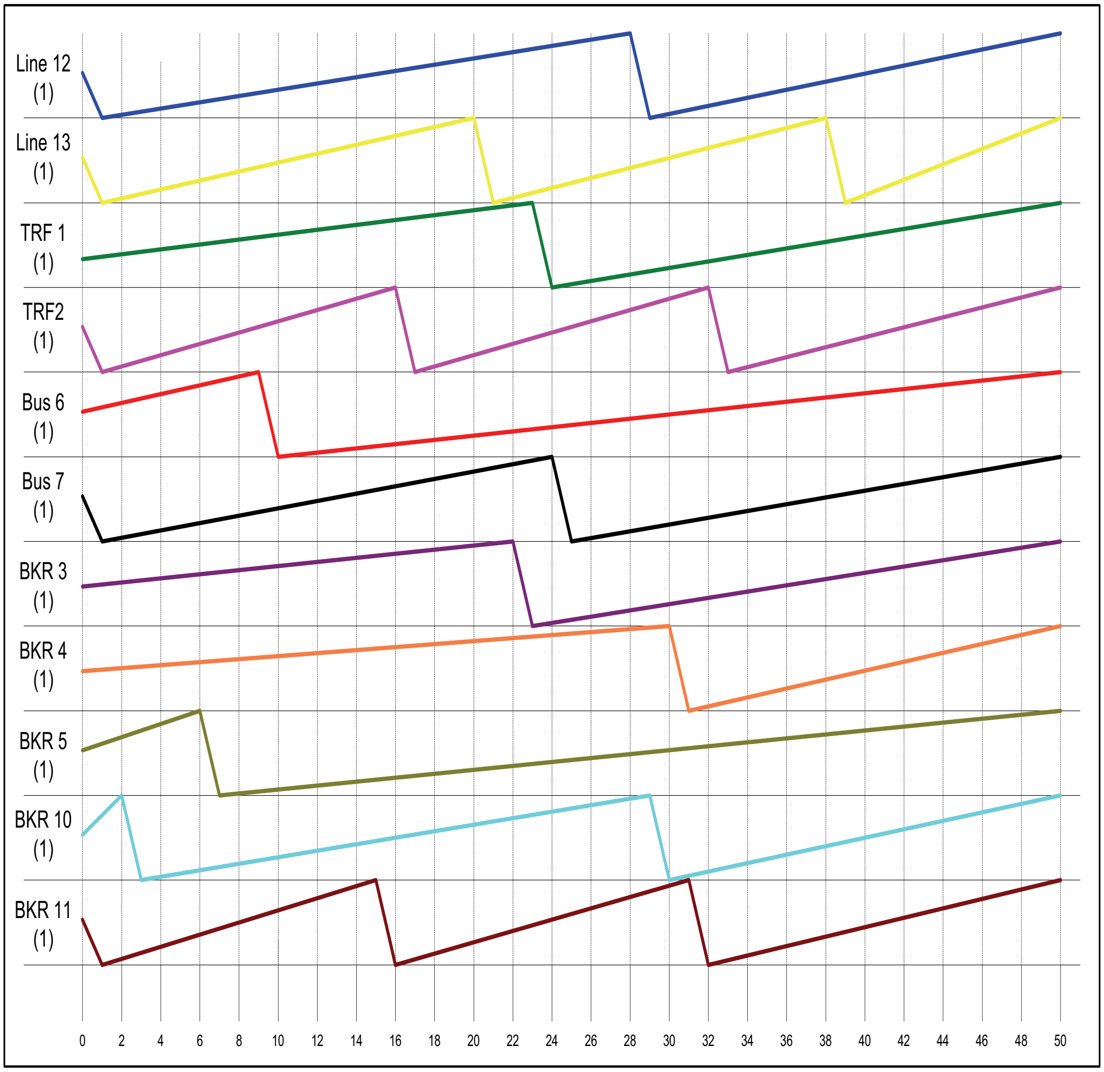

Figure 7.2. Replacement schedules first iteration

Step 2, Iteration 1. LINDO was used to run the first IP program is run to check for budget violations. In the final solution, z_1 was greater than zero. Therefore new replacement schedules need to be created for components with replacement in the first period.

Step 1, Iteration 2. Additional replacement schedules need to be obtained for the components with replacement in period 1, which are the lines 12 and 13, transformer 2, the bus 7, and the breakers 3 and 11. Figure 7.3 shows the profiles generated in the second iteration.

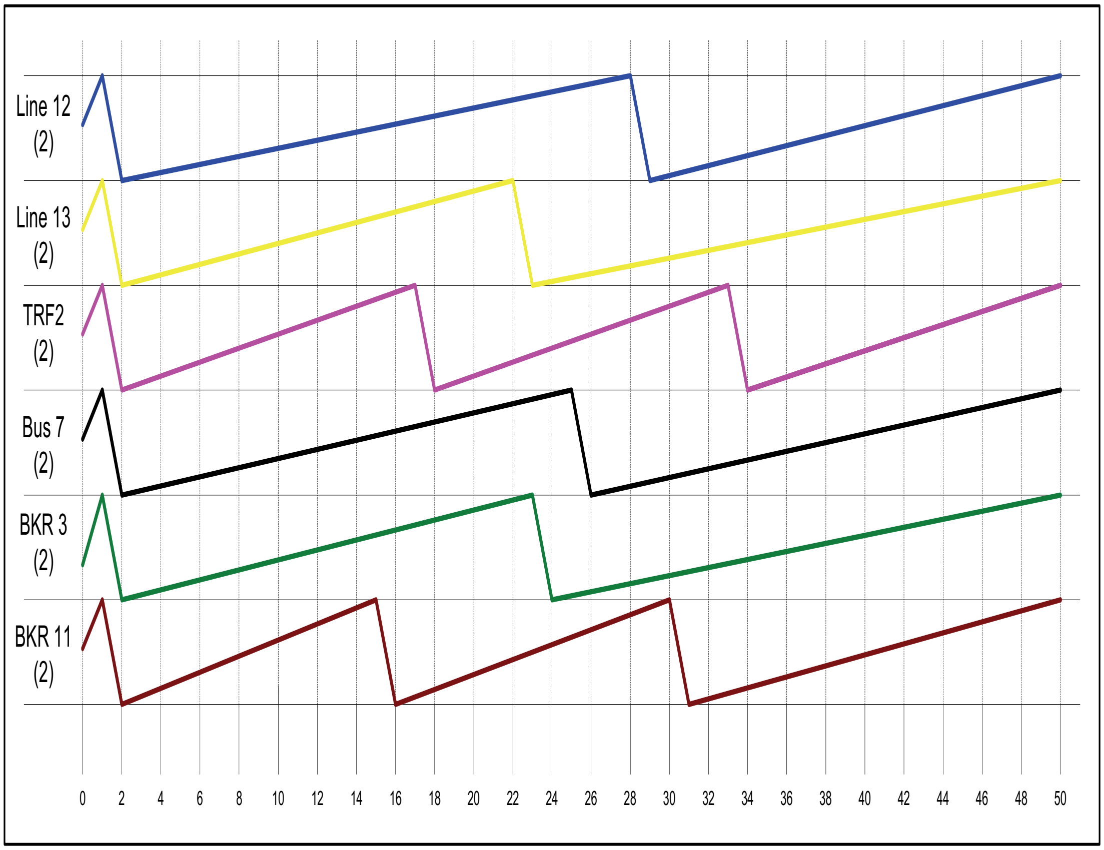

Figure 7.3. Replacement schedules second iteration

Step 2, Iteration 2. The first IP program is applied again to check for potential budget violations. It considers all the component replacement profiles generated up to this point (17 replacement schedules). Again, the problem is solved used LINDO and in the final solution it is found that there are budget violations in periods 1, 2, 17 and 34. The program returns again to Step 1 to generate additional replacement profiles for the components with replacement in those periods.

Step 1, Iteration 3

Additional profiles are generated for the components with replacement in periods 1, 2, 17 and 34. Figure 7.4 shows the new 10 component replacement schedules obtained for the different components in the system. As it can be seen, there are some components with 3 or 4 different profiles generated so far. The program proceeds again to the first IP program to check for budget violations.

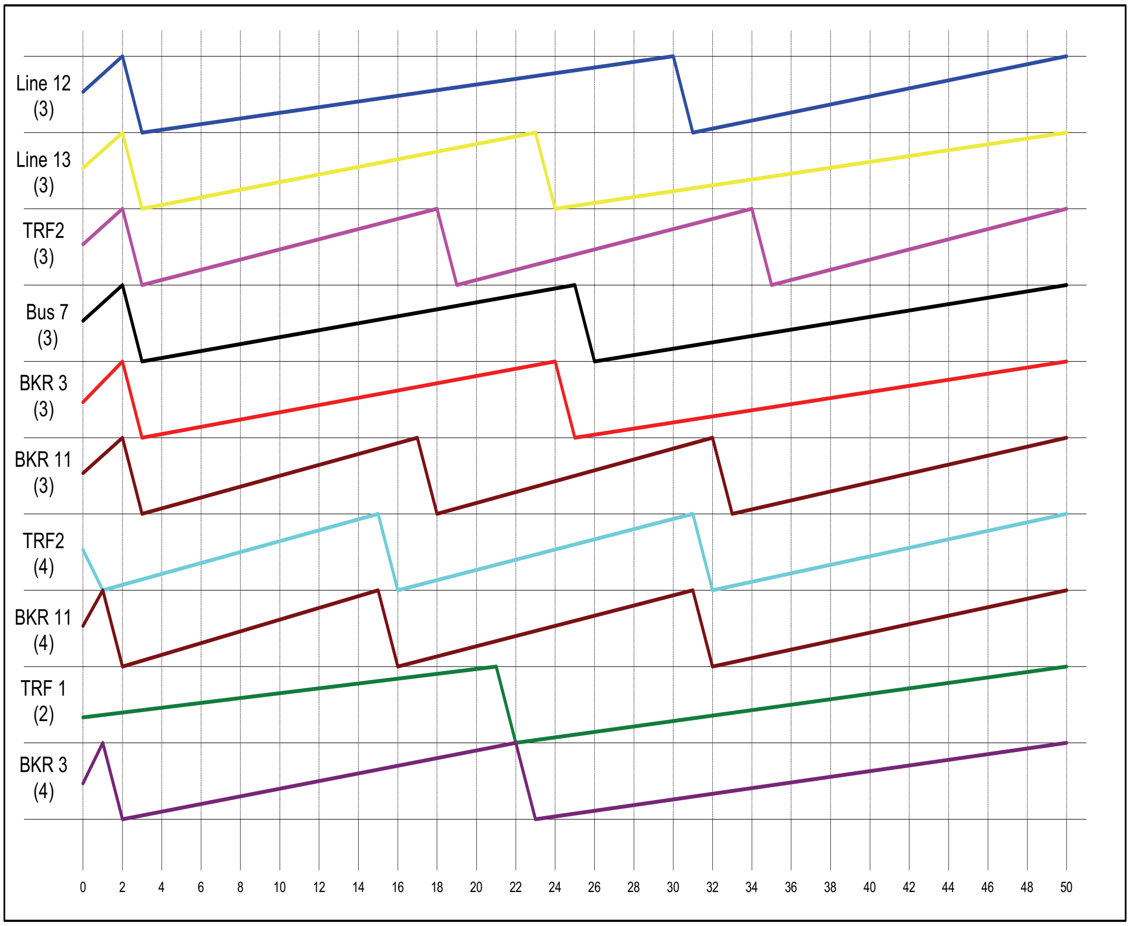

Figure 7.4 Replacement schedules third iteration

Step 2, Iteration 3. The integer program IP1 is applied considering all the component replacement profiles generated up to this point, which are 27. In the final solution there are budget violations in periods 1, 2 and 3. The program goes back again to the first step to create new replacement profiles for the components with replacement in the first 3 periods.

Step 1, Iteration 4. The dynamic program is run again. Figure 7.5 graphically shows the new component replacement schedules generated.

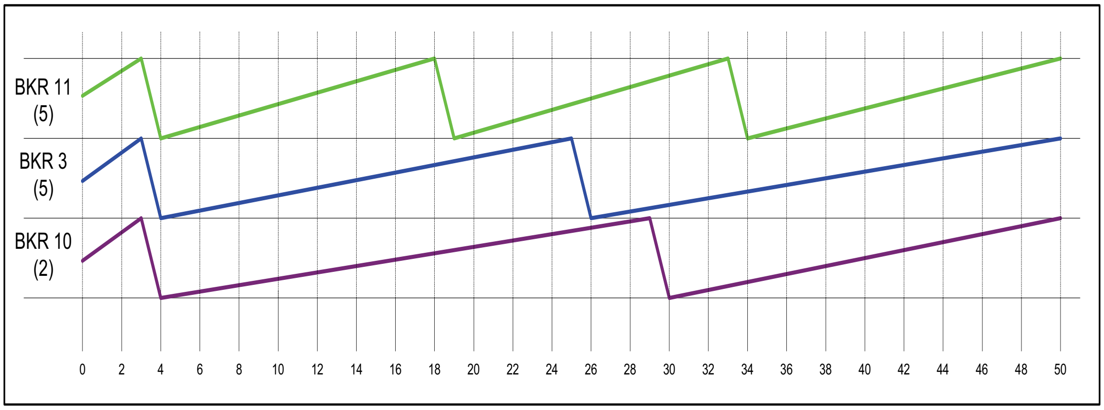

Figure 7.5. Replacement schedules fourth iteration

Step 2, Iteration 4

The IP1 is solved again using LINDO considering 30 replacement profiles generated. In the final solution there is a budget violation in period 34. New component replacement schedules need to be generated by using the dynamic programming model.

Step 1, Iteration 5

The dynamic programming model is run again and the new additional replacement schedules are shown in Figure 7.6.

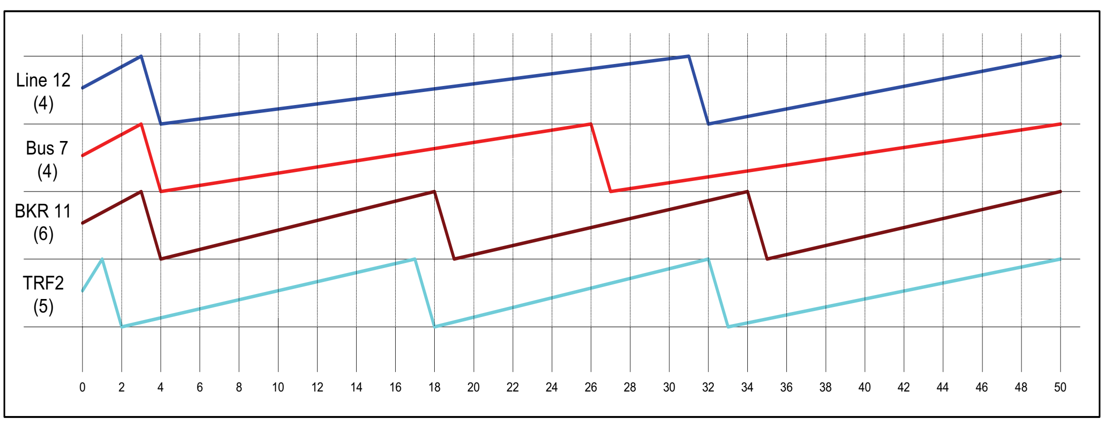

Figure 7.6. Additional replacement schedules fifth iteration

Step 2, Iteration 5. The first integer program is solved again using LINDO to check if there still exist budget violations. It considers all the 34 replacement profiles generated so far. In the final solution all the z_t are equal to zero which means that there are no more budget violations. The program proceeds to the second integer programming model to find the optimal system-level replacement schedule.

Step 3, Iteration 5. The second integer programming model is applied considering all the individual replacement profiles generated. Figure 7.7 shows the optimal system-level replacement schedule, and Table 7.7 presents the information for the specific decisions to be made at each time period. The total replacement cost for the DESN configuration of the selected replacement schedule is shown in Table 7.8.

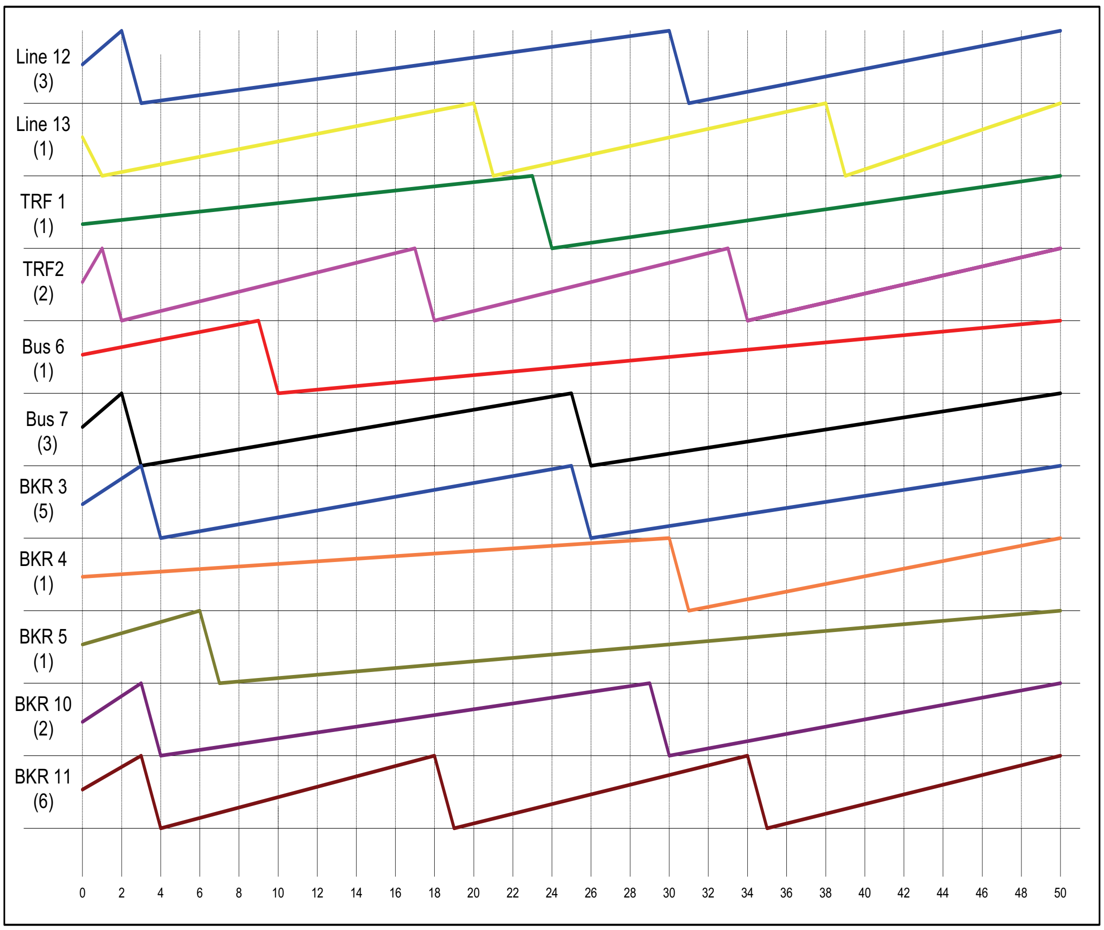

Figure 7.7. System-level replacement component replacement schedule

Table 7.7. System-level replacement decisions for DESN

Period	Bus 6	BKR 4	BKR 5	TRF 1	Line 13	TRF 2	Line 12	Bus 7	BKR 3	BKR 10	BKR 11
1	Keep	Keep	Keep	Keep	Replace	Keep	Keep	Keep	Keep	Keep	Keep
2	Keep	Keep	Keep	Keep	Keep	Replace	Keep	Keep	Keep	Keep	Keep
3	Keep	Keep	Keep	Keep	Keep	keep	Replace	Replace	Keep	Keep	Keep
4	Keep	Keep	Keep	Keep	Keep	keep	Keep	Keep	Replace	Replace	Replace
5	Keep	Keep	Keep	Keep	Keep	keep	Keep	Keep	Keep	Keep	Keep
6	Keep	Keep	Keep	Keep	Keep	keep	Keep	Keep	Keep	Keep	Keep
7	Keep	Keep	Replace	Keep	Keep	keep	Keep	Keep	Keep	Keep	Keep
8	Keep	Keep	Keep	Keep	Keep	keep	Keep	Keep	Keep	Keep	Keep
9	Keep	Keep	Keep	Keep	Keep	keep	Keep	Keep	Keep	Keep	Keep
10	Replace	Keep	Keep	Keep	Keep	keep	Keep	Keep	Keep	Keep	Keep
11	Keep	Keep	Keep	Keep	Keep	keep	Keep	Keep	Keep	Keep	Keep
12	Keep	Keep	Keep	Keep	Keep	keep	Keep	Keep	Keep	Keep	Keep
13	Keep	Keep	Keep	Keep	Keep	keep	Keep	Keep	Keep	Keep	Keep
14	Keep	Keep	Keep	Keep	Keep	keep	Keep	Keep	Keep	Keep	Keep
15	Keep	Keep	Keep	Keep	Keep	keep	Keep	Keep	Keep	Keep	Keep
16	Keep	Keep	Keep	Keep	Keep	keep	Keep	Keep	Keep	Keep	Keep
17	Keep	Keep	Keep	Keep	Keep	keep	Keep	Keep	Keep	Keep	Keep
18	Keep	Keep	Keep	Keep	Keep	Replace	Keep	Keep	Keep	Keep	Keep
19	Keep	Keep	Keep	Keep	Keep	keep	Keep	Keep	Keep	Keep	Replace
20	Keep	Keep	Keep	Keep	Keep	keep	Keep	Keep	Keep	Keep	Keep
21	Keep	Keep	Keep	Keep	Replace	keep	Keep	Keep	Keep	Keep	Keep
22	Keep	Keep	Keep	Keep	Keep	keep	Keep	Keep	Keep	Keep	Keep
23	Keep	Keep	Keep	Keep	Keep	keep	Keep	Keep	Keep	Keep	Keep
24	Keep	Keep	Keep	Replace	Keep	keep	Keep	Keep	Keep	Keep	Keep
25	Keep	Keep	Keep	Keep	Keep	keep	Keep	Keep	Keep	Keep	Keep
26	Keep	Keep	Keep	Keep	Keep	keep	Keep	Replace	Replace	Keep	Keep
27	Keep	Keep	Keep	Keep	Keep	keep	Keep	Keep	Keep	Keep	Keep
28	Keep	Keep	Keep	Keep	Keep	keep	Keep	Keep	Keep	Keep	Keep
29	Keep	Keep	Keep	Keep	Keep	keep	Keep	Keep	Keep	Keep	Keep
30	Keep	Keep	Keep	Keep	Keep	keep	Keep	Keep	Keep	Replace	Keep
31	Keep	Replace	Keep	Keep	Keep	keep	Replace	Keep	Keep	Keep	Keep
32	Keep	Keep	Keep	Keep	Keep	keep	Keep	Keep	Keep	Keep	Keep
33	Keep	Keep	Keep	Keep	Keep	keep	Keep	Keep	Keep	Keep	Keep
34	Keep	Keep	Keep	Keep	Keep	Replace	Keep	Keep	Keep	Keep	Keep
35	Keep	Keep	Keep	Keep	Keep	keep	Keep	Keep	Keep	Keep	Replace
36	Keep	Keep	Keep	Keep	Keep	keep	Keep	Keep	Keep	Keep	Keep
37	Keep	Keep	Keep	Keep	Keep	keep	Keep	Keep	Keep	Keep	Keep
38	Keep	Keep	Keep	Keep	Keep	keep	Keep	Keep	Keep	Keep	Keep
39	Keep	Keep	Keep	Keep	Replace	keep	Keep	Keep	Keep	Keep	Keep

Table 7.7. System-level replacement decisions for DESN (cont'd)

Period	Bus 6	BKR 4	BKR 5	TRF 1	Line 13	TRF 2	Line 12	Bus 7	BKR 3	BKR 10	BKR 11
40	Keep	Keep	Keep	Keep	Keep	keep	Keep	Keep	Keep	Keep	Keep
41	Keep	Keep	Keep	Keep	Keep	keep	Keep	Keep	Keep	Keep	Keep
42	Keep	Keep	Keep	Keep	Keep	keep	Keep	Keep	Keep	Keep	Keep
43	Keep	Keep	Keep	Keep	Keep	keep	Keep	Keep	Keep	Keep	Keep
44	Keep	Keep	Keep	Keep	Keep	keep	Keep	Keep	Keep	Keep	Keep
45	Keep	Keep	Keep	Keep	Keep	keep	Keep	Keep	Keep	Keep	Keep
46	Keep	Keep	Keep	Keep	Keep	keep	Keep	Keep	Keep	Keep	Keep
47	Keep	Keep	Keep	Keep	Keep	keep	Keep	Keep	Keep	Keep	Keep
48	Keep	Keep	Keep	Keep	Keep	keep	Keep	Keep	Keep	Keep	Keep
49	Keep	Keep	Keep	Keep	Keep	keep	Keep	Keep	Keep	Keep	Keep
50	Keep	Keep	Keep	Keep	Keep	keep	Keep	Keep	Keep	Keep	Keep

Table 7.8. System-level replacement costs

Component	Profile selected	Cost
Bus 6	1	140,917
BKR 4	1	53,635
BKR 5	1	67,681
TRF 1	1	157,202
Line 13	1	387,379
TRF 2	2	352,530
Line 12	3	283,984
Bus 7	3	170,033
BKR 3	5	119,349
BKR 10	2	117,821
BKR 11	6	154,484
Total replacement cost		2,005,014

7.6 Conclusions

A new replacement analysis model was developed for complex systems. A Taylor series expansion model was developed to determine the impact of each individual component on system opportunity costs. The approximation was tested in several examples for different values of outage rates and repair times and proved to provide good estimates of the total system downtime. The Taylor series expansion model was incorporated in the previously developed replacement analysis method. Using this approach, any complex network can be analyzed. An example is presented in the replacement schedule for the different components in the DESN configuration.

Chapter 8

Research extensions

Extensions to this research involve further enhancement of the ETDS reliability estimation and replacement models so that they can provide practical results for difficult problems. New component cost-based criticality importance measures will be developed to consider reliability metrics used by the power industry, such as the system average interruption frequency index (SAIFI) and the customer average interruption duration index (CAIDI).

8.1. Extension of criticality metrics

Existing criticality metrics have been developed. However they do not directly address the primary concerns and metrics used by power utilities. New metrics could be developed to more accurately meet industry needs.

<u>A) Cost-based</u>

New criticality measures could be developed to include cost. The components in the systems are going to be ranked according to their impact in the total system cost. Given the actual system outage rates and repair times for each component l in the system, the total system expected downtime is determined. Then, for each component in the system, the outage rate is going to be set to 0 (perfect functioning of the component) and the corresponding outage rate for the system is determined. Each system unavailability is then multiplied by the interruption costs (*IC*), divided by the total sum of the improvement differences. The result is the estimated opportunity cost for component l. This is shown in Equation 8.1.

$$I_l^{CU} = \frac{(U_s(\boldsymbol{\lambda}, \mathbf{r} \mid \lambda_l = a_l) - U_s(\boldsymbol{\lambda}, \mathbf{r} \mid \lambda_l = 0))\ IC}{\sum_{l=1}^{n}((U_s(\boldsymbol{\lambda}, \mathbf{r} \mid \lambda_l = a_l) - U_s(\boldsymbol{\lambda}, \mathbf{r} \mid \lambda_l = 0))} \tag{8.1}$$

where:

a_l = actual sustained outage rate of component *l* (outages/year)

$U_s(\boldsymbol{\lambda}, \mathbf{r} \mid \lambda_l = 0)$ = system unavailability when sustained outage rate of component *l* is 0

$U_s(\boldsymbol{\lambda}, \mathbf{r} \mid \lambda_l = a_l)$ = system unavailability when sustained outage rate of component *l* is a_l

I_l^{CU} = criticality cost based on opportunity cost for component *l*

λ_l = component *l* outage rate

r_l = component *l* average repair time

B) Reliability indices-based: SAIFI & CAIDI

Power companies measure electric utility service performance mainly according to two reliability indices: (1) the system average interruption frequency index (SAIFI) and (2) the customer average interruption duration index (CAIDI) (Warren *et al*., 1999; Billinton *et al*., 2002; Billinton & Billinton, 1989).

SAIFI measures the number of service interruptions experienced by all customers of a given utility. It is obtained by dividing the total number of service interruptions in a year by the number of delivery customers, thereby deriving the number of times a customer experienced service interruption during that year.

$$SAIFI = \frac{\text{Total number customers interruptions}}{\text{Total number customers served}}$$

CAIDI measures how long it takes a utility to restore service after an interruption. It is obtained by adding up the durations of each service interruption in a year and dividing the total by the total number of customer service interruptions, thereby deriving the average outage duration for that year.

$$CAIDI = \frac{\sum \text{customers interruption durations}}{\text{Total number customers interuptions}}$$

The use of the SAIFI and CAIDI quality measures enables power companies to determine how frequently there have been service interruptions and how long the interruptions have lasted on average. The criticality measures are going to be extended such that the components in the system affecting SAIFI and CAIDI measures can be ranked according to their impact in such measures.

8.2. Multi-objective power systems optimization

Most real-world engineering optimization problems involve the achievement of several objectives, normally conflicting with each other. These problems are called "multi-objective," "multi-criteria," or "vector" optimization problems, and were originally studied in the context of economics. However, scientists and engineers soon realized the importance of solving multi-objective optimization problems, and the development of techniques to model and solve such problems became an important area within operations research.

Within this aspect, the development and application of a multi-objective evolutionary algorithm to the power systems area could be considered. The objectives to be considered can potentially be the multi-state stationary availability, the expected multi-state capacity, the expected unsupplied demand, and the loss of load probability. A multi-state power system consisting of elements combined into a series-parallel structure could be created from a set of elements with different reliability and performance and cost available from the market. The Universal Generating Function (UGF) approach will be used to determine the total system's multi-state availability.

Appendix A

Minimal Cut Sets for Breaker-and-a-Half, Breaker-and-a-Third and DESN

A1: Breaker-and-a-Half configuration (minimal cut-sets up to fourth order)

Table A1.1: Minimal Cut Sets for Breaker-and-a-Half for Failure at Load 1

{LINE 14}	{LINE 13, BKR 4, BKR 5}	{LINE 13, BKR 3, BKR 8}
{BKR 7, BKR 8}	{LINE 13, BKR 4, BUS 2}	{LINE 13, BUS 1, BKR 5}
{BKR 7, BKR 5}	{LINE 13, BKR 4, BKR 8}	{LINE 13, BUS 1, BUS 2}
{BKR 7, BUS 2}	{LINE 13, BKR 3, BKR 5}	{LINE 13, BUS 1, BKR 8}
{LINE 13, LINE 16}	{LINE 13, BKR 3, BUS 2}	{LINE 13, BKR 5, BKR 6}
{LINE 16, BKR 6, BKR 7}	{LINE 16, BKR 3, BKR 7}	{LINE 13, BUS 2, BKR 6}
{LINE 16, BUS 1, BKR 7}	{LINE 16, BKR 4, BKR 7}	{LINE 13, BKR 8, BKR 6}

Table A1.2: Minimal Cut Sets for Breaker-and-a-Half for Failure at Load 2

{LINE 15}	{LINE 16, BKR 6, BKR 7}	{LINE 16, BKR 6, BUS 2}
{BKR 4, BKR 6}	{LINE 16, BUS 1, BKR 7}	{LINE 16, BUS 1, BUS 2}
{BKR 4, BKR 3}	{LINE 16, BKR 3, BKR 7}	{LINE 16, BKR 3, BUS 2}
{BKR 4, BUS 1}	{LINE 16, BKR 6, BKR 8}	{LINE 16, BKR 6, BKR 5}
{LINE 13, LINE 16}	{LINE 16, BUS 1, BKR 8}	{LINE 16, BUS 1, BKR 5}
{LINE 13, BKR 4, BKR 5}	{LINE 16, BKR 3, BKR 8}	{LINE 16, BKR 3, BKR 5}
{LINE 13, BKR 4, BUS 2}	{LINE 13, BKR 4, BKR 8}	{LINE 13, BKR 4, BKR 7}

Table A1.3: Minimal Cut Sets for Breaker-and-a-Half for Failure at Load 1 & Load 2

{LINE 14,LINE 15}	{BKR 3, BKR 4, LINE 14}	{LINE 13, LINE 15, BKR 3, BKR 5}
{LINE 13,LINE 16}	{BUS 1, BKR 4, LINE 14}	{LINE 13, LINE 15, BKR 3, BUS 2}
{BKR 4, BKR 5, LINE 13}	{BKR 6, BKR 4, LINE 14}	{LINE 13, LINE 15, BKR 3, BKR 8}
{BKR 4, BUS 2, LINE 13}	{BKR 7, BKR 8, BKR 4, BKR 6}	{LINE 13, LINE 15, BKR 5, BUS 1}
{BKR 4, BKR 8, LINE 13}	{BKR 7, BKR 8, BKR 4, BKR 3}	{LINE 13, LINE 15, BUS 2, BUS 1}
{BKR 7, BKR 6, LINE 16}	{BKR 7, BKR 8, BKR 4, BUS 1}	{LINE 13, LINE 15, BKR 8, BUS 1}
{BKR 7, BUS 1, LINE 16}	{BKR 7, BKR 5, BKR 4, BKR 6}	{LINE 13, LINE 15, BKR 6, BKR 5}
{BKR 7, BKR 3, LINE 16}	{BKR 7, BKR 5, BKR 4, BKR 3}	{LINE 13, LINE 15, BKR 6, BUS 2}
{BKR 7, BKR 8, LINE 15}	{BKR 7, BKR 5, BKR 4, BUS 1}	{LINE 13, LINE 15, BKR 6, BKR 8}
{BKR 7, BUS 2, LINE 15}	{BKR 7, BUS 2, BKR 4, BKR 6}	{LINE 14, LINE 16, BKR 6, BKR 8}
{BKR 7, BKR 5, LINE 15}	{BKR 7, BUS 2, BKR 4, BKR 3}	{LINE 14, LINE 16, BUS 1, BKR 8}
{LINE 14, LINE 16, BKR 6, BUS 2}	{BKR 7, BUS 2, BKR 4, BUS 1}	{LINE 14, LINE 16, BKR 3, BKR 8}
{LINE 14, LINE 16, BUS 1, BUS 2}	{LINE 14, LINE 16,BKR 6, BKR 5}	{LINE 14, LINE 16, BKR 3, BKR 5}
{LINE 14, LINE 16, BKR 3, BUS 2}	{LINE 14, LINE 16, BUS 1, BKR 5}	

A2: Breaker-and-a-Third System (minimal cut-sets up to fifth order)

Table A2.1: Minimal Cut Sets for Breaker-and-a-Third for Failure at Load 1

{LINE 14}	{BKR 7, BKR 8}	{LINE 13, LINE 16, LINE 28}
{LINE 28, BKR 7, BUS 2}	{LINE 28, BKR 7, BKR 24}	{LINE 28, BKR 7, BKR 23}
{LINE 28, BKR 7, BKR 5}	{LINE 13, BKR 8, BKR 6}	{LINE 13, BKR 8, BKR 3}
{LINE 13, BKR 8, BUS 1}	{LINE 13, BKR 8, BKR 4}	{LINE 13, BKR 8, LINE 16, BKR 23}
{LINE 13, BKR 8, LINE 16, BKR 5}	{LINE 13, BKR 8, LINE 16, BUS 2}	{LINE 13, BKR 8, LINE 16, BKR 24}
{LINE 13, LINE 28, BKR 6, BKR 24}	{LINE 13, LINE 28, BUS 1, BKR 24}	{LINE 13, LINE28, BKR 4, BKR 24}
{LINE 13, LINE 28, BKR 3, BKR 24}	{LINE 13, LINE 28, BUS 1, BUS 2}	{LINE 13, LINE 28, BKR 6, BUS 2}
{LINE 13, LINE 28, BKR 4, BUS 2}	{LINE 13, LINE 28, BKR 3, BUS 2}	{LINE 13, LINE 28, BUS 1, BKR 23}
{LINE 13, LINE 28, BKR 6, BKR 23}	{LINE 13, LINE 28, BKR 3, BKR 23}	{LINE13, LINE28, BKR4, BKR 23}
{LINE 13, LINE 28, BKR 6, BKR 5}	{LINE13, LINE 28, BUS 1, BKR 5}	{LINE 13, LINE 28, BKR 3, BKR 5}
{LINE 13, LINE28, BKR4, BKR 5}	{LINE 28, LINE 16, BKR 7, BKR 4}	{LINE28, LINE 16, BKR7, BKR 3}
{LINE 28, LINE 16, BKR 7, BUS 1}	{LINE28, LINE 16, BKR7, BKR 6}	

Table A2.2: Minimal Cut Sets for Breaker-and-a-Third for Failure at Load 2

{LINE 27}	{BKR 5, BKR 23}	{BKR 5,BUS 2}
{BKR 5,BKR 24}		
{LINE 28, BKR 5, BKR 8}	{LINE 28, BKR 5, BKR 7}	{LINE 16, BKR 24, BKR 6}
{LINE 16, BKR 23, BKR 4}	{LINE 16, BKR 23, BKR 3}	{LINE 16, BKR 23, BUS 1}
{LINE 16, BKR 23, BKR 6}	{LINE 16,BUS 2,BKR 3}	{LINE 16,BUS 2,BUS 1}
{LINE 16,BUS 2,BKR 4}	{LINE 16,BUS 2,BKR 6}	{LINE 13, LINE 16, LINE 28}
{LINE 16,BKR 24,BKR 4}	{LINE 16,BKR 24,BKR 3}	{LINE 16,BKR 24,BUS 1}
{LINE 13, BKR 6, LINE 28, BKR 5}	{LINE 28, BKR 5, LINE 13, BUS 1}	{LINE 13, BKR 3, LINE 28, BKR 5}
{LINE 13, LINE 28, BKR 4, BKR 5}	{LINE 13, LINE 16, BKR 7, BKR 24}	{LINE 13, LINE 16, BKR 8, BKR 24}
{LINE 13, LINE 16, BKR 23, BKR 8}	{LINE 13, BKR 23, LINE 16, BKR 7}	{LINE 28, LINE 16, BKR4, BKR 8}
{LINE 16,BUS 2,BKR 7,BKR 3}	{LINE 16,BUS 2,BKR 8,BKR 3}	{LINE 28,LINE 16,BKR 6,BKR 7}
{LINE 28,LINE 16,BUS 1,BKR 7}	{LINE 28,LINE 16,BKR 3,BKR 7}	{LINE 28,LINE 16,BKR 4,BKR 7}
{LINE 28,LINE 16,BKR 6,BKR 8}	{LINE 28,LINE 16,BUS 1,BKR 8}	{LINE 28,LINE 16,BKR 3,BKR 8}

Table A2.3: Minimal Cut Sets for Breaker-and-a-Third for Failure at Load 3

{LINE 15}	{BKR 3, BKR 4}	{BKR 4, BUS 1}
{BKR 6, BKR 4}	{LINE 13, LINE 16, LINE 28}	{LINE 13, BKR 8, BKR 4}
{LINE 16, BKR 3, BKR 24}	{LINE 16, BKR 24, BKR 6}	{LINE 13, BKR 7, BKR 4}
{LINE 16, BKR 3, BKR 23}	{LINE 16, BUS 2, BKR 6}	{LINE 13, LINE 16, BKR 7, BKR 24}
{LINE 16, BKR 3, BKR 5}	{LINE 16, BKR 23, BKR 6}	{LINE 13, LINE 16, BKR 7, BKR 23}
{LINE 16, BKR 3, BUS 2}	{LINE 16, BKR 5, BKR 6}	{LINE 13, LINE 16, BKR 7, BKR 5}
{LINE 16, BKR 24, BUS 1}	{LINE 13, LINE 16, BKR 8, BKR 24}	{LINE 13, LINE 16, BKR 7, BUS 2}
{LINE 16, BKR 24, BUS 2}	{LINE 13, LINE 16, BKR 8, BKR 23}	{LINE 13, LINE 28, BKR 4, BKR 5}
{LINE 16, BKR 24, BKR 23}	{LINE 13, LINE 16, BKR 8, BKR 5}	{LINE 13, LINE 28, BKR 4, BKR 23}
{LINE 16, BKR 24, BKR 5}	{LINE 13, LINE 16, BKR 8, BUS 2}	{LINE 13, LINE 28, BKR 4, BKR 24}
{LINE 16, LINE 28, BKR 7, BKR 6}	{LINE 16, LINE 28, BKR 8, BKR 6}	{LINE 13, LINE 28, BKR 4, BUS 2}
{LINE 16, LINE 28, BKR 7, BUS 1}	{LINE 16, LINE 28, BKR 8, BKR 3}	
{LINE 16, LINE 28, BKR 7, BKR 3}	{LINE 16, LINE 28, BKR 8, BUS 1}	

Table A2.7: Minimal Cut Sets for Breaker-and-a-Third for Failure at Loads 1, 2 and 3

{LINE 13, LINE 16, LINE 28}	{LINE 14, LINE 15, LINE 27}	{LINE 16, LINE 28, BKR 7, BKR 6}
{LINE 14, LINE 15, BKR 5, BKR 23}	{LINE 15, LINE 27, BKR 7, BKR 8}	{LINE 16, LINE 28, BKR 7, BKR 3}
{LINE 14, LINE 15, BKR 5, BKR 24}	{LINE 14, LINE 16, BKR 6, BKR 24}	{LINE 16, LINE 28, BKR 7, BUS 1}
{LINE 14, LINE 15, BKR 5, BUS 2}	{LINE 14, LINE 16, BKR 3, BKR 24}	{LINE 13, LINE 28, BKR 4, BKR 5}
{LINE 14, LINE 27, BKR 3, BKR 4}	{LINE 14, LINE 16, BUS 1, BKR 24}	{LINE 28, LINE 15, BKR 5, BKR 7}
{LINE 14, LINE 27, BKR 6, BKR 4}	{LINE 14, LINE 16, BUS 1, BUS 2}	{LINE 13, LINE 27, BKR 4, BKR 8}
{LINE 14, LINE 27, BUS 1, BKR 4}	{LINE 14, LINE 16, BKR 6, BUS 2}	{LINE 13, LINE 16, BKR 8, BKR 24}
{LINE 14, LINE 16, BKR 23, BUS 1}	{LINE 14, LINE 16, BKR 6, BKR 23}	{LINE 13, LINE 16, BKR 8, BUS 2}
{LINE 14, LINE 16, BKR 3, BUS 2}	{LINE 14, LINE 16, BKR 3, BKR 23}	{LINE 13, LINE 16, BKR 8, BKR 23}

{LINE 14, LINE 27, LINE 13, BKR 8, BKR 4}	{LINE 15, LINE 27, LINE 28, BKR 5, BKR 7}
{LINE 14, LINE 27, LINE 13, BKR 7, BKR 4}	{LINE 15, LINE 27, LINE 28, BKR 23, BKR 7}
{LINE 14, LINE 27, LINE 16, BKR 3, BKR 24}	{LINE 15, LINE 27, LINE 28, BUS 2, BKR 7}
{LINE 14, LINE 27, LINE 16, BKR 3, BKR 23}	{LINE 15, LINE 27, LINE 28, BKR 24, BKR 7}
{LINE 14, LINE 27, LINE 16, BKR 3, BKR 5}	{LINE 15, LINE 27, LINE 13, BKR 8, BKR 4}
{LINE 14, LINE 27, LINE 16, BKR 3, BUS 2}	{LINE 15, LINE 27, LINE 13, BKR 8, BKR 3}
{LINE 14, LINE 27, LINE 16, BUS 1, BKR 24}	{LINE 15, LINE 27, LINE 13, BKR 8, BUS 1}
{LINE 14, LINE 27, LINE 16, BKR 24, BKR 6}	{LINE 15, LINE 27, LINE 13, BKR 8, BKR 6}
{LINE 14, LINE 27, LINE 16, BUS 2, BKR 6}	{BKR 7, BKR 8, BKR 5, BKR 23, LINE 15}
{LINE 14, LINE 27, LINE 16, BKR 23, BKR 6}	{BKR 7, BKR 8, BKR 5, BUS 2, LINE 15}
{LINE 14, LINE 27, LINE 16, BKR 5, BKR 6}	{BKR 7, BKR 8, BKR 5, BKR 24, LINE 15}
{LINE 14, LINE 15, LINE 28, BKR 5, BKR 8}	{BKR 5, BKR 23, BKR 3, BKR 4, LINE 14}
{LINE 14, LINE 15, LINE 28, BKR 5, BKR 7}	{BKR 5, BUS 2, BKR 3, BKR 4, LINE 14}
{LINE 14, LINE 15, LINE 16, BKR 23, BKR 4}	{BKR 5, BKR 24, BKR 3, BKR 4, LINE 14}
{LINE 14, LINE 15, LINE 16, BKR 23, BKR 3}	{BKR 5, BKR 23, BUS 1, BKR 4, LINE 14}
{LINE 14, LINE 15, LINE 16, BKR 23, BUS 1}	{BKR 5, BUS 2, BUS 1, BKR 4, LINE 14}
{LINE 14, LINE 15, LINE 16, BKR 23, BKR 6}	{BKR 5, BKR 24, BUS 1, BKR 4, LINE 14}
{LINE 14, LINE 15, LINE 16, BUS 2, BKR 4}	{BKR 5, BKR 23, BKR 6, BKR 4, LINE 14}
{LINE 14, LINE 15, LINE 16, BKR 24, BKR 4}	{BKR 5, BUS 2, BKR 6, BKR 4, LINE 14}
{LINE 14, LINE 15, LINE 16, BKR 24, BKR 6}	{BKR 5, BKR 24, BKR 6, BKR 4, LINE 14}
{LINE 14, LINE 15, LINE 16, BKR 24, BUS 1}	{BKR 7, BKR 8, BKR 3, BKR 4, LINE 27}
{LINE 14, LINE 15, LINE 16, BKR 24, BKR 3}	{BKR 7, BKR 8, BUS 1, BKR 4, LINE 27}
{LINE 14, LINE 15, LINE 16, BUS 2, BKR 6}	{BKR 7, BKR 8, BKR 6, BKR 4, LINE 27}
{LINE 14, LINE 15, LINE 16, BUS 2, BUS 1}	{LINE 14, LINE 28, LINE 16, BKR 6, BKR 8}
{LINE 14, LINE 15, LINE 16, BUS 2, BKR 23}	{LINE 14, LINE 28, LINE 16, BUS 1, BKR 8}
{LINE 14, LINE 15, LINE 13,LINE 28,LINE 16}	{LINE 14, LINE 28, LINE 16, BKR 3, BKR 8}
{LINE 14, LINE 13, LINE 16, BKR 23, BKR 8}	{LINE 14, LINE 13, LINE 16, BKR 23, BKR 7}
{LINE 27, LINE 16, LINE 28, BKR 7, BKR 6}	{LINE 15, LINE 13, LINE 28, BKR 4, BKR 5}
{LINE 27, LINE 16, LINE 28, BKR 7, BUS 1}	{LINE 15, LINE 13, LINE 28, BKR 3, BKR 5}
{LINE 27, LINE 16, LINE 28, BKR 7, BKR 3}	{LINE 15, LINE 13, LINE 28, BUS 1, BKR 5}
{LINE 27, LINE 13, LINE 28, BKR 4, BKR 5}	{LINE 15, LINE 13, LINE 28, BKR 6, BKR 5}
{LINE 27, LINE 13, LINE 28, BKR 4, BKR 23}	{LINE 15, LINE 16, LINE 28, BKR 7, BKR 6}
{LINE 27, LINE 13, LINE 28, BKR 4, BUS 2}	{LINE 15, LINE 16, LINE 28, BKR 7, BUS 1}
{LINE 27, LINE 13, LINE 28, BKR 4, BKR 24}	{LINE 15, LINE 16, LINE 28, BKR 7, BKR 3}
{LINE 15, LINE 13, LINE 16, BKR 8, BKR 24}	{LINE 15, LINE 16, LINE 28, BKR 7, BKR 4}
{LINE 15, LINE 13, LINE 16, BKR 8, BKR 23}	

A3: DESN configuration

Table A3.1: DESN Minimal Cut Sets for Load 1

{BUS 7}	{BKR 11}	{BKR 4, BKR 5}
{LINE 13, BKR 4}	{TRF 1, BKR 4}	{TRF 2, BKR 5}
{LINE 13, TRF 2}	{TRF 1, TRF 2}	{LINE 12, BKR 5}
{LINE 13, LINE 12}	{TRF 1, LINE 12}	{BKR 4, BKR 3}
{BKR 4, BUS 6}	{LINE 12, BKR 3}	{TRF 2, BKR 3}
{TRF 2, BUS 6}	{LINE 12, BUS 6}	

Table A3.2: DESN Minimal Cut Sets for Load 2

{LINE 12, LINE 13}	{BUS 6}	{BKR 10}
{ LINE 13,TRF 2}	{LINE 13, BKR 4}	{LINE 13, BUS 7}
{LINE 13, BKR 5}	{LINE 12, TRF 1}	{LINE 12, BKR 3}
{ TRF 1, TRF 2}	{ TRF 1, BKR 4}	{TRF 1, BUS 7}
{ TRF 1, BKR 5}	{BKR 3, TRF 2}	{BKR 3, BKR 4}
{BKR 3, BUS 7}	{BKR 3, BKR 5}	

Table A3.3: DESN Minimal Cut Sets for Loads 1 and 2

{BUS 7, BUS 6}	{TRF 2, BKR 3}	{BUS 6, BKR 4}
{BUS 7, BKR 10}	{TRF 2, TRF 1}	{BUS 6, TRF 2}
{BKR 11, BUS 6}	{TRF 2, LINE 13}	{BUS 6, LINE 12}
{BKR 11, BKR 10}	{LINE 12, BKR 3}	{BUS 7, BKR 3}
{BKR 4, BKR 3}	{LINE 12, TRF 1}	{BUS 7, TRF 1}
{BKR 4, TRF 1}	{LINE 12, LINE 13}	{BUS 7, LINE 13}
{BKR 4, LINE 13}		

Appendix B

Replacement analysis: Radial configuration example

Step 1, iteration 1. By applying the dynamic programming model for each component in the configuration, the initial component replacement schedules for each component in the system were obtained and they are shown in Table B.1.

Table B.1. Replacement analysis policies

Beginning	**end**	**Line 13.8Kv**	**CB 13.8Kv**	**Line 600ft**	**Switch**	**Transformer**	**CB1 480v**	**Bus**	**CB2 480v**	**Line 300ft**
0	**1**	Keep	Keep	Replace	Keep	Replace	Replace	Replace	Keep	Replace
1	**2**	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
2	**3**	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
3	**4**	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
4	**5**	Replace	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
5	**6**	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
6	**7**	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
7	**8**	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
8	**9**	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
9	**10**	Keep	Keep	Keep	Replace	Keep	Keep	Keep	Keep	Keep
10	**11**	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
11	**12**	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
12	**13**	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
13	**14**	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
14	**15**	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Replace	Keep
15	**16**	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
16	**17**	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
17	**18**	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
18	**19**	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
19	**20**	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
20	**21**	Keep	Keep	Keep	Keep	Keep	Keep	Replace	Keep	Keep
21	**22**	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
22	**23**	Keep	Keep	Keep	Keep	Replace	Keep	Keep	Keep	Keep
23	**24**	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
24	**25**	Keep	Keep	Keep	Keep	Keep	Replace	Keep	Keep	Keep
25	**26**	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
26	**27**	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
27	**28**	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Replace
28	**29**	Replace	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
29	**30**	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
30	**31**	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
31	**32**	Keep	Keep	Replace	Keep	Keep	Keep	Keep	Keep	Keep
32	**33**	Keep	Replace	Keep	Keep	Keep	Keep	Keep	Keep	Keep
33	**34**	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
34	**35**	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
35	**36**	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
36	**37**	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Replace	Keep
37	**38**	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
38	**39**	Keep	Keep	Keep	Keep	Keep	Keep	Replace	Keep	Keep
39	**40**	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
40	**41**	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
41	**42**	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
42	**43**	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
43	**44**	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
44	**45**	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
45	**46**	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
46	**47**	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
47	**48**	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
48	**49**	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
49	**50**	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
Total profile cost $s		1642553	33995	527223	19465	103713	26685	57744	19926	29733

Figure B.1 is a graphical representation of the different component replacement profiles generated in the first iteration. In the Figure, an upward arrow means to keep the asset while a downward arrow means to replace the asset. The keep/replace decisions are made at the beginning of each period.

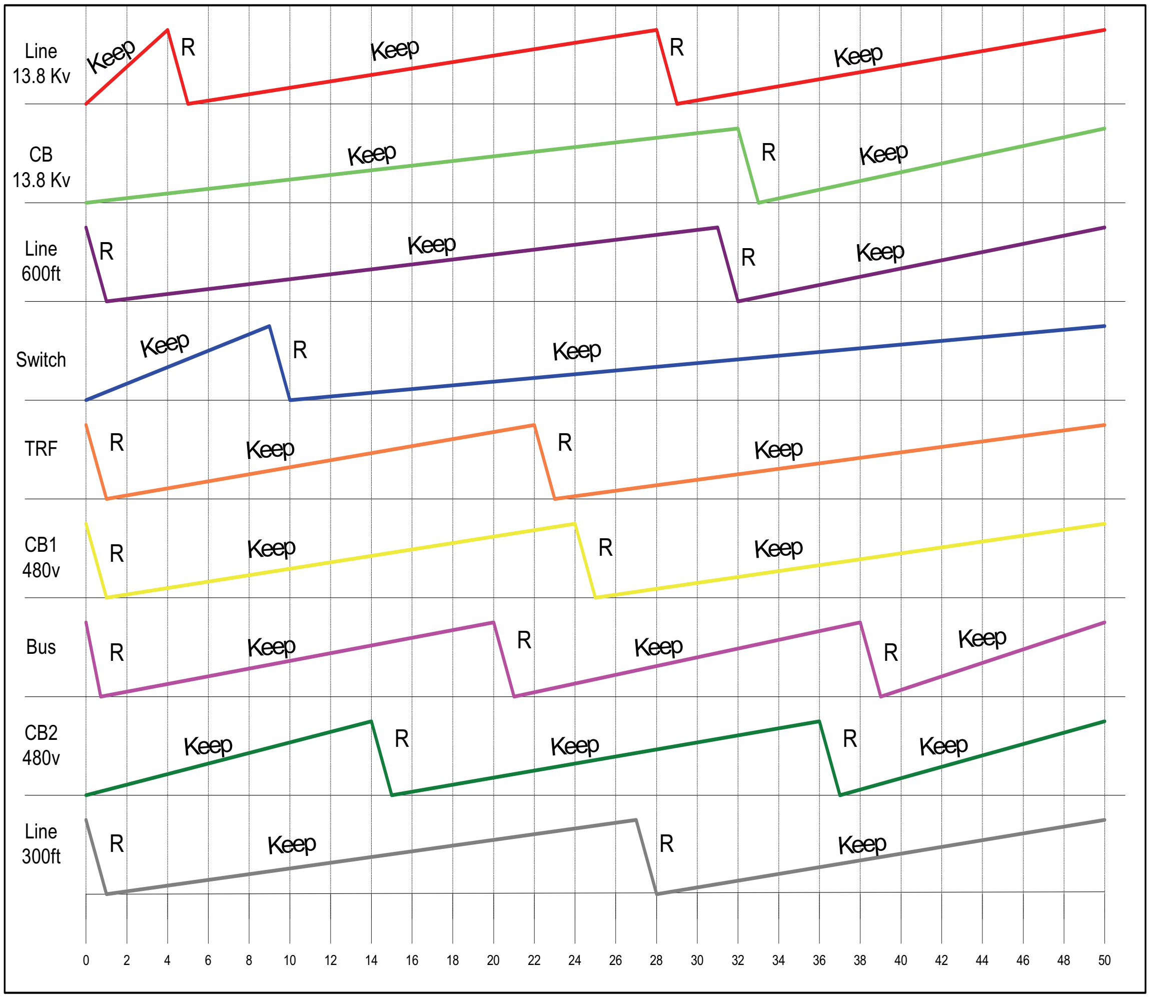

Figure B.1. DP Replacement policies

Step 2, iteration 1. The individual optimal replacement schedules generated from the dynamic program are used as input data to the first integer program (IP1) to check if for possible constraint violations (excess in budget expenditures). The IP1 program was solved using LINDO.

Integer programming formulation 1

$$\min \sum_{t=1}^{50} z_t$$

Minimize
1Z1+1Z2+1Z3+1Z4+1Z5+1Z6+1Z7+1Z8+1Z9+1Z10+1Z11+1Z12+1Z13+1Z14+1Z15+1Z16+1Z17+1Z18+1Z19+1Z20+1Z21+1Z22+1Z23+1Z24+1Z25+1Z26+1Z27+1Z28+1Z29+1Z30+1Z31+1Z32+1Z33+1Z34+1Z35+1Z36+1Z37+1Z38+1Z39+1Z40+1Z41+1Z42+1Z43+1Z44+1Z45+1Z46+1Z47+1Z48+1Z49+1Z50

C1:7094.272X11+21.46367X21+33309.5X31+149.2448X41+45008.18X51+16002.21X61+30012X71+40.21335X81+17507.77X91-1Z1<=60000

C2:6526.601X11+20.60739X21+21.4372X31+138.9189X41+14.34177X51+5.751158X61+28.4182X71+39.85767X81+17.53953X91-1Z2<=54545.45

C3:6001.102X11+19.6953X21+29.38304X31+129.225X41+17.31972X51+8.498998X61+39.94344X71+39.22129X81+24.04067X91-1Z3<=49586.78

C4:5515.187X11+18.75116X21+34.98126X31+120.1348X41+18.96155X51+10.63811X61+48.35528X71+38.36034X81+28.62103X91-1Z4<=45078.89

C5:32557.39X11+17.79341X21+38.89127X31+111.6201X41+19.79958X51+12.27967X61+54.43757X71+37.32269X81+31.82013X91-1Z5<=40980.81

C6:2282.884X11+16.83632X21+41.5171X31+103.6523X41+20.10339X51+13.50831X61+58.69775X71+36.149X81+33.96854X91-1Z6<=37255.28

C7:2363.512X11+15.89089X21+43.14218X31+96.20338X41+20.03649X51+14.39261X61+61.50615X71+34.87375X81+35.29815X91-1Z7<=33868.44

C8:2338.661X11+14.96545X21+43.97895X31+89.24573X41+19.70774X51+14.98968X61+63.14882X71+33.52608X81+35.98277X91-1Z8<=30789.49

C9:2264.493X11+14.06626X21+44.19261X31+82.7523X41+19.19369X51+15.34768X61+63.85364X71+32.13049X81+36.15759X91-1Z9<=27990.44

C10:2164.819X11+13.1979X21+43.91458X31+6363.11X41+18.55008X51+15.50751X61+63.80553X71+30.70751X81+35.93011X91-1Z10<=25445.86

C11:2052.1X11+12.3636X21+43.25083X31+3.898669X41+17.81838X51+15.50399X61+63.15622X71+29.27423X81+35.38704X91-1Z11<=23132.6

C12:1933.583X11+11.56549X21+42.28759X31+5.479806X41+17.02989X51+15.36686X61+62.03105X71+27.84475X81+34.59894X91-1Z12<=21029.63

C13:1813.731X11+10.80484X21+41.09541X31+6.63382X41+16.20844X51+15.1215X61+60.53402X71+26.43064X81+33.62352X91-1Z13<=19117.85

C14:1695.375X11+10.08226X21+39.73217X31+7.468245X41+15.37218X51+14.78957X61+58.75152X71+25.04126X81+32.50814X91-1Z14<=17379.86

C15:1580.325X11+9.397766X21+38.2454X31+8.052695X41+14.535X51+14.38951X61+56.7554X71+4213.882X81+31.29169X91-1Z15<=15799.88

C16:1469.725X11+8.750985X21+36.67407X31+8.437977X41+13.7074X51+13.93701X61+54.60533X71+1.51446X81+30.00606X91-1Z16<=14363.52

C17:1364.27X11+8.141195X21+35.05013X31+8.663333X41+12.89728X51+13.44534X61+52.35075X71+2.238052X81+28.67738X91-1Z17<=13057.75

C18:1264.348X11+7.567423X21+33.39961X31+8.760026X41+12.1105X51+12.92575X61+50.03249X71+2.801347X81+27.32696X91-1Z18<=11870.68

C19:1170.132X11+7.028506X21+31.74364X31+8.753427X41+11.35127X51+12.38769X61+47.68413X71+3.233622X81+25.97207X91-1Z19<=10791.53

C20:1081.646X11+6.523145X21+30.09921X31+8.664348X41+10.62253X51+11.83908X61+45.33309X71+3.557159X81+24.6266 3X91-1Z20<=9810.479

C21:998.8085X11+6.049943X21+28.47991X31+8.509988X41+9.926171X51+11.28648X61+4461.093X71+3.790023X81+23.3017 4X91-1Z21<=8918.618

C22:921.4664X11+5.60745X21+26.8964X31+8.30461X41+9.263318X51+10.73536X61+4.224185X71+3.947251X81+22.00615X9 1-1Z22<=8107.834

C23:849.4178X11+5.194176X21+25.35697X31+8.060071X41+5529.074X51+10.19016X61+5.937338X71+4.041524X81+20.7466 1X91-1Z23<=7370.758

C24:782.4299X11+4.808627X21+23.86786X31+7.786226X41+1.761829X51+9.65449X61+7.187704X71+4.083611X81+19.52825 X91-1Z24<=6700.689

C25:720.2502X11+4.449315X21+22.43364X31+7.491259X41+2.127658X51+1624.634X61+8.091798X71+4.082684X81+18.3548 X91-1Z25<=6091.536

C26:662.617X11+4.114776X21+21.05749X31+7.181955X41+2.32935X51+0.5838897X61+8.725047X71+4.046574X81+17.22886 X91-1Z26<=5537.76

C27:609.2654X11+3.803579X21+19.74144X31+6.863915X41+2.432298X51+0.8628658X61+9.142498X71+3.981965X81+16.152 09X91-1Z27<=5034.327

C28:559.9326X11+3.514333X21+18.48655X31+6.541746X41+2.46962X51+1.08004X61+9.386669X71+3.894557X81+1335.452X 91-1Z28<=4576.661

C29:3305.408X11+3.245698X21+17.29312X31+6.219208X41+2.461402X51+1.246701X61+9.491436X71+3.789209X81+1.33787 5X91-1Z29<=4160.601

C30:231.7711X11+2.996383X21+16.16079X31+5.899353X41+2.421016X51+1.371439X61+9.484285X71+3.670049X81+1.83376 7X91-1Z30<=3782.365

C31:239.957X11+2.765156X21+15.08872X31+5.584626X41+2.357867X51+1.461218X61+9.387769X71+3.540579X81+2.183146 X91-1Z31<=3438.513

C32:237.4339X11+2.550838X21+1735.381X31+5.276966X41+2.278802X51+1.521836X61+9.220521X71+3.403755X81+2.42716 6X91-1Z32<=3125.921

C33:229.904X11+1657.84X21+1.11685X31+4.977881X41+2.188916X51+1.558183X61+8.997996X71+3.262067X81+2.591041X9 1-1Z33<=2841.746

C34:219.7845X11+0.2862554X21+1.530818X31+4.688519X41+2.092054X51+1.574409X61+8.733039X71+3.117599X81+2.6924 61X91-1Z34<=2583.406

C35:208.3407X11+0.3546029X21+1.822478X31+4.409722X41+1.991141X51+1.574051X61+8.436329X71+2.972084X81+2.7446 83X91-1Z35<=2348.551

C36:196.3082X11+0.3947942X21+2.026184X31+4.142082X41+1.888411X51+1.56013X61+8.116735X71+2.826955X81+2.75801 7X91-1Z36<=2135.046

C37:184.1402X11+0.4174512X21+2.162986X31+3.885975X41+1.785566X51+1.53522X61+7.781605X71+517.6584X81+2.74066 6X91-1Z37<=1940.951

C38:172.124X11+0.4281286X21+2.247651X31+3.641605X41+1.683899X51+1.50152X61+7.437011X71+0.1860453X81+2.69924 2X91-1Z38<=1764.501

C39:160.4434X11+0.4302817X21+2.291245X31+3.409031X41+1.584379X51+1.460904X61+802.3667X71+0.2749356X81+2.639 127X91-1Z39<=1604.092

C40:149.2147X11+0.42626X21+2.302377X31+3.188193X41+1.487726X51+1.414963X61+0.7597567X71+0.3441342X81+2.5647 24X91-1Z40<=1458.265

C41:138.5083X11+0.4177473X21+2.287892X31+2.978936X41+1.394458X51+1.365046X61+1.067882X71+0.3972374X81+2.479 646X91-1Z41<=1325.696

C42:128.3637X11+0.4059904X21+2.253311X31+2.781029X41+1.304934X51+1.312295X61+1.292772X71+0.4369827X81+2.386 858X91-1Z42<=1205.178
C43:118.7984X11+0.3919324X21+2.203128X31+2.59418X41+1.21939X51+1.257668X61+1.455381X71+0.465589X81+2.288793 X91-1Z43<=1095.616

C44:109.8148X11+0.3762965X21+2.141017X31+2.418053X41+1.137961X51+1.201969X61+1.569276X71+0.4849039X81+2.187444X91-1Z44<=996.0148

C45:101.4046X11+0.3596416X21+2.069994X31+2.252273X41+1.060706X51+1.145867X61+1.644359X71+0.496485X81+2.084437X91-1Z45<=905.468

C46:93.55242X11+0.3424013X21+1.992535X31+2.096444X41+0.9876201X51+1.089913X61+1.688275X71+0.5016552X81+1.981089X91-1Z46<=823.1527

C47:86.23766X11+0.3249125X21+1.910671X31+1.950151X41+0.9186538X51+1.034562X61+1.707118X71+0.5015413X81+1.878462X91-1Z47<=748.3207

C48:79.43666X11+0.3074359X21+1.826066X31+1.812971X41+0.8537201X51+0.9801779X61+1.705832X71+0.4971054X81+1.777403X91-1Z48<=680.2915

C49:73.12383X11+0.2901719X21+1.740076X31+1.684473X41+0.7927048X51+0.9270554X61+1.688473X71+0.4891683X81+1.678578X91-1Z49<=618.4468

C50:67.27259X11+0.2732731X21+1.653802X31+1.56423X41+0.7354735X51+0.875424X61+1.658392X71+0.4784307X81+1.582503X91-1Z50<=562.2244

C51:1X11=1
C52:1X21=1
C53:1X31=1
C54:1X41=1
C55:1X51=1
C56:1X61=1
C57:1X71=1
C58:1X81=1
C59:1X91=1

X_{ij}>=0 for all ij

In the final solution, all z_t's were equal to zero except for:

$$z_1 = 89144.857$$

in the final solution z_1 was greater than zero, which means we have a budget violation in period 1. Therefore we need to apply again the dynamic programming model to generate new component replacement policies for components with replacement in period 1 (Line 600 ft, Transformer, CB1 480v, Bus and Line 300 ft).

Step 1, iteration 2. New component replacement schedules need to be obtained for the components with replacement in period 1. The new replacement profiles are obtained by prohibiting replacement in period 1. Table B.2 shows the new additional component replacement policies obtained. Figure B.2 shows the new component replacement schedules obtained in the second iteration.

Table B.2. New replacement analysis policies for iteration 2

Beginning	end	Line 600ft	Transformer	CB1 480v	Bus	Line 300ft
0	1	Keep	Keep	Keep	Keep	Keep
1	2	Replace	Replace	Replace	Replace	Replace
2	3	Keep	Keep	Keep	Keep	Keep
3	4	Keep	Keep	Keep	Keep	Keep
4	5	Keep	Keep	Keep	Keep	Keep
5	6	Keep	Keep	Keep	Keep	Keep
6	7	Keep	Keep	Keep	Keep	Keep
7	8	Keep	Keep	Keep	Keep	Keep
8	9	Keep	Keep	Keep	Keep	Keep
9	10	Keep	Keep	Keep	Keep	Keep
10	11	Keep	Keep	Keep	Keep	Keep
11	12	Keep	Keep	Keep	Keep	Keep
12	13	Keep	Keep	Keep	Keep	Keep
13	14	Keep	Keep	Keep	Keep	Keep
14	15	Keep	Keep	Keep	Keep	Keep
15	16	Keep	Keep	Keep	Keep	Keep
16	17	Keep	Keep	Keep	Keep	Keep
17	18	Keep	Keep	Keep	Keep	Keep
18	19	Keep	Keep	Keep	Keep	Keep
19	20	Keep	Keep	Keep	Keep	Keep
20	21	Keep	Keep	Keep	Keep	Keep
21	22	Keep	Keep	Keep	Replace	Keep
22	23	Keep	Keep	Keep	Keep	Keep
23	24	Keep	Replace	Keep	Keep	Keep
24	25	Keep	Keep	Keep	Keep	Keep
25	26	Keep	Keep	Replace	Keep	Keep
26	27	Keep	Keep	Keep	Keep	Keep
27	28	Keep	Keep	Keep	Keep	Keep
28	29	Keep	Keep	Keep	Keep	Replace
29	30	Keep	Keep	Keep	Keep	Keep
30	31	Keep	Keep	Keep	Keep	Keep
31	32	Replace	Keep	Keep	Keep	Keep
32	33	Keep	Keep	Keep	Keep	Keep
33	34	Keep	Keep	Keep	Keep	Keep
34	35	Keep	Keep	Keep	Keep	Keep
35	36	Keep	Keep	Keep	Keep	Keep
36	37	Keep	Keep	Keep	Keep	Keep
37	38	Keep	Keep	Keep	Keep	Keep
38	39	Keep	Keep	Keep	Keep	Keep
39	40	Keep	Keep	Keep	Replace	Keep
40	41	Keep	Keep	Keep	Keep	Keep
41	42	Keep	Keep	Keep	Keep	Keep
42	43	Keep	Keep	Keep	Keep	Keep
43	44	Keep	Keep	Keep	Keep	Keep
44	45	Keep	Keep	Keep	Keep	Keep
45	46	Keep	Keep	Keep	Keep	Keep
46	47	Keep	Keep	Keep	Keep	Keep
47	48	Keep	Keep	Keep	Keep	Keep
48	49	Keep	Keep	Keep	Keep	Keep
49	50	Keep	Keep	Keep	Keep	Keep
Total profile cost $s		53763	105642	26822	63459	31293

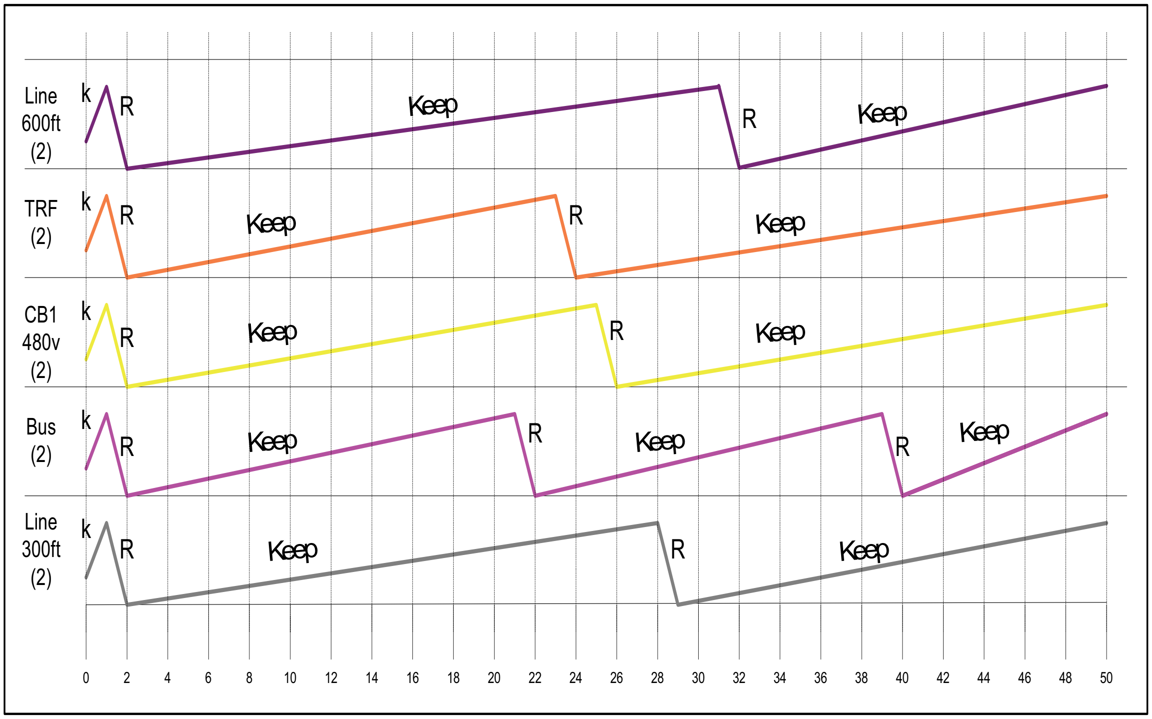

Figure B.2 Additional replacement analysis policies

Considering all the component replacement schedules generated so far (14), the integer programming formulation 1, is applied again to check budget violations, the formulation is as follows:

Integer programming formulation 1

$$\min \sum_{t=1}^{50} z_t$$

s.t.

C1:7094.272X11+21.46367X21+33309.5X31+149.2448X41+45008.18X51+16002.21X61+30012X71+40.21335X81+17507.77X91 +330.3379X32+83.08863X52+93.42434X62+590.9374X72+307.0432X92-1Z1<=60000

C2:6526.601X11+20.60739X21+21.4372X31+138.9189X41+14.34177X51+5.751158X61+28.4182X71+39.85767X81+17.53953X91+30281.36X32+40916.53X52+14547.46X62+27283.64X72+15916.16X92-1Z2<=54545.45

C3:6001.102X11+19.6953X21+29.38304X31+129.225X41+17.31972X51+8.498998X61+39.94344X71+39.22129X81+24.04067X91+19.48837X32+13.03797X52+5.228325X62+25.83473X72+15.94503X92-1Z3<=49586.78

C4:5515.187X11+18.75116X21+34.98126X31+120.1348X41+18.96155X51+10.63811X61+48.35528X71+38.36034X81+28.62103X91+26.71186X32+15.7452X52+7.726361X62+36.31222X72+21.85516X92-1Z4<=45078.89

b:32557.39X11+17.79341X21+38.89127X31+111.6201X41+19.79958X51+12.27967X61+54.43757X71+37.32269X81+31.82013X91+31.80114X32+17.23777X52+9.671009X62+43.95934X72+26.01912X92-1Z5<=40980.81

C6:2282.884X11+16.83632X21+41.5171X31+103.6523X41+20.10339X51+13.50831X61+58.69775X71+36.149X81+33.96854X91+35.3557X32+17.99962X52+11.16334X62+49.4887X72+28.92739X92-1Z6<=37255.28

C7:2363.512X11+15.89089X21+43.14218X31+96.20338X41+20.03649X51+14.39261X61+61.50615X71+34.87375X81+35.29815X91+37.74282X32+18.27581X52+12.28028X62+53.3616X72+30.88049X92-1Z7<=33868.44

C8:2338.661X11+14.96545X21+43.97895X31+89.24573X41+19.70774X51+14.98968X61+63.14882X71+33.52608X81+35.98277X91+39.22017X32+18.21499X52+13.08419X62+55.91468X72+32.08923X92-1Z8<=30789.49

C9:2264.493X11+14.06626X21+44.19261X31+82.7523X41+19.19369X51+15.34768X61+63.85364X71+32.13049X81+36.15759X91+39.98086X32+17.91612X52+13.62698X62+57.40802X72+32.71161X92-1Z9<=27990.44

C10:2164.819X11+13.1979X21+43.91458X31+6363.11X41+18.55008X51+15.50751X61+63.80553X71+30.70751X81+35.93011X91+40.1751X32+17.44881X52+13.95244X62+58.04876X72+32.87054X92-1Z10<=25445.86

C11:2052.1X11+12.3636X21+43.25083X31+3.898669X41+17.81838X51+15.50399X61+63.15622X71+29.27423X81+35.38704X91+39.92234X32+16.8637X52+14.09773X62+58.00502X72+32.66374X92-1Z11<=23132.6

C12:1933.583X11+11.56549X21+42.28759X31+5.479806X41+17.02989X51+15.36686X61+62.03105X71+27.84475X81+34.59894X91+39.31894X32+16.19852X52+14.09453X62+57.41474X72+32.17004X92-1Z12<=21029.63

C13:1813.731X11+10.80484X21+41.09541X31+6.63382X41+16.20844X51+15.1215X61+60.53402X71+26.43064X81+33.62352X91+38.44326X32+15.48172X52+13.96987X62+56.39186X72+31.45358X92-1Z13<=19117.85

C14:1695.375X11+10.08226X21+39.73217X31+7.468245X41+15.37218X51+14.78957X61+58.75152X71+25.04126X81+32.50814X91+37.35947X32+14.73494X52+13.74682X62+55.03093X72+30.56684X92-1Z14<=17379.86

C15:1580.325X11+9.397766X21+38.2454X31+8.052695X41+14.535X51+14.38951X61+56.7554X71+4213.882X81+31.29169X91+36.12016X32+13.97471X52+13.44507X62+53.41047X72+29.55286X92-1Z15<=15799.88

C16:1469.725X11+8.750985X21+36.67407X31+8.437977X41+13.7074X51+13.93701X61+54.60533X71+1.51446X81+30.00606X91+34.76854X32+13.21364X52+13.08138X62+51.59582X72+28.44699X92-1Z16<=14363.52

C17:1364.27X11+8.141195X21+35.05013X31+8.663333X41+12.89728X51+13.44534X61+52.35075X71+2.238052X81+28.67738X91+33.34007X32+12.46127X52+12.67X62+49.64121X72+27.27824X92-1Z17<=13057.75

C18:1264.348X11+7.567423X21+33.39961X31+8.760026X41+12.1105X51+12.92575X61+50.03249X71+2.801347X81+27.32696X91+31.86376X32+11.7248X52+12.22304X62+47.59159X72+26.07035X92-1Z18<=11870.68

C19:1170.132X11+7.028506X21+31.74364X31+8.753427X41+11.35127X51+12.38769X61+47.68413X71+3.233622X81+25.97207X91+30.36329X32+11.00955X52+11.75069X62+45.48409X72+24.84269X92-1Z19<=10791.53

C20:1081.646X11+6.523145X21+30.09921X31+8.664348X41+10.62253X51+11.83908X61+45.33309X71+3.557159X81+24.62663X91+28.85785X32+10.31934X52+11.26154X62+43.34921X72+23.61097X92-1Z20<=9810.479

C21:998.8085X11+6.049943X21+28.47991X31+8.509988X41+9.926171X51+11.28648X61+4461.093X71+3.790023X81+23.30174X91+27.36292X32+9.656841X52+10.7628X62+41.2119X72+22.38785X92-1Z21<=8918.618

C22:921.4664X11+5.60745X21+26.8964X31+8.30461X41+9.263318X51+10.73536X61+4.224185X71+3.947251X81+22.00615X91+25.89083X32+9.023792X52+10.26044X62+4055.539X72+21.1834X92-1Z22<=8107.834

C23:849.4178X11+5.194176X21+25.35697X31+8.060071X41+5529.074X51+10.19016X61+5.937338X71+4.041524X81+20.74661X91+24.45128X32+8.421199X52+9.759416X62+3.840168X72+20.00559X92-1Z23<=7370.758

C24:782.4299X11+4.808627X21+23.86786X31+7.786226X41+1.761829X51+9.65449X61+7.187704X71+4.083611X81+19.52825X91+23.05179X32+5026.431X52+9.263778X62+5.39758X72+18.86056X92-1Z24<=6700.689

C25:720.2502X11+4.449315X21+22.43364X31+7.491259X41+2.127658X51+1624.634X61+8.091798X71+4.082684X81+18.3548X91+21.69805X32+1.601663X52+8.77681X62+6.534276X72+17.75295X92-1Z25<=6091.536

C26:662.617X11+4.114776X21+21.05749X31+7.181955X41+2.32935X51+0.5838897X61+8.725047X71+4.046574X81+17.22886X91+20.39422X32+1.934234X52+1476.94X62+7.35618X72+16.68618X92-1Z26<=5537.76

C27:609.2654X11+3.803579X21+19.74144X31+6.863915X41+2.432298X51+0.8628658X61+9.142498X71+3.981965X81+16.15209X91+19.14318X32+2.117591X52+0.5308089X62+7.931861X72+15.6626X92-1Z27<=5034.327

C28:559.9326X11+3.514333X21+18.48655X31+6.541746X41+2.46962X51+1.08004X61+9.386669X71+3.894557X81+1335.452X91+17.94677X32+2.21118X52+0.7844234X62+8.311361X72+14.68372X92-1Z28<=4576.661

C29:3305.408X11+3.245698X21+17.29312X31+6.219208X41+2.461402X51+1.246701X61+9.491436X71+3.789209X81+1.337875X91+16.80596X32+2.24511X52+0.9818549X62+8.533336X72+1214.048X92-1Z29<=4160.601

C30:231.7711X11+2.996383X21+16.16079X31+5.899353X41+2.421016X51+1.371439X61+9.484285X71+3.670049X81+1.833767X91+15.72101X32+2.237638X52+1.133365X62+8.628578X72+1.21625X92-1Z30<=3782.365

C31:239.957X11+2.765156X21+15.08872X31+5.584626X41+2.357867X51+1.461218X61+9.387769X71+3.540579X81+2.183146X91+14.69163X32+2.200924X52+1.246763X62+8.622077X72+1.667061X92-1Z31<=3438.513

C32:237.4339X11+2.550838X21+1735.381X31+5.276966X41+2.278802X51+1.521836X61+9.220521X71+3.403755X81+2.427166X91+1735.381X32+2.143516X52+1.32838X62+8.534335X72+1.984678X92-1Z32<=3125.921

C33:229.904X11+1657.84X21+1.11685X31+4.977881X41+2.188916X51+1.558183X61+8.997996X71+3.262067X81+2.591041X91+1.11685X32+2.071638X52+1.383487X62+8.382292X72+2.206515X92-1Z33<=2841.746

C34:219.7845X11+0.2862554X21+1.530818X31+4.688519X41+2.092054X51+1.574409X61+8.733039X71+3.117599X81+2.692461X91+1.530818X32+1.989923X52+1.41653X62+8.179996X72+2.355492X92-1Z34<=2583.406

C35:208.3407X11+0.3546029X21+1.822478X31+4.409722X41+1.991141X51+1.574051X61+8.436329X71+2.972084X81+2.744683X91+1.822478X32+1.901867X52+1.431281X62+7.939126X72+2.447692X92-1Z35<=2348.551

C36:196.3082X11+0.3947942X21+2.026184X31+4.142082X41+1.888411X51+1.56013X61+8.116735X71+2.826955X81+2.758017X91+2.026184X32+1.810128X52+1.430956X62+7.66939X72+2.495166X92-1Z36<=2135.046

C37:184.1402X11+0.4174512X21+2.162986X31+3.885975X41+1.785566X51+1.53522X61+7.781605X71+517.6584X81+2.740666X91+2.162986X32+1.716737X52+1.4183X62+7.37885X72+2.507288X92-1Z37<=1940.951

C38:172.124X11+0.4281286X21+2.247651X31+3.641605X41+1.683899X51+1.50152X61+7.437011X71+0.1860453X81+2.699242X91+2.247651X32+1.623242X52+1.395654X62+7.074186X72+2.491514X92-1Z38<=1764.501

C39:160.4434X11+0.4302817X21+2.291245X31+3.409031X41+1.584379X51+1.460904X61+802.3667X71+0.2749356X81+2.639127X91+2.291245X32+1.530817X52+1.365018X62+6.760919X72+2.453856X92-1Z39<=1604.092

C40:149.2147X11+0.42626X21+2.302377X31+3.188193X41+1.487726X51+1.414963X61+0.7597567X71+0.3441342X81+2.564724X91+2.302377X32+1.440345X52+1.328094X62+729.4243X72+2.399206X92-1Z40<=1458.265

C41:138.5083X11+0.4177473X21+2.287892X31+2.978936X41+1.394458X51+1.365046X61+1.067882X71+0.3972374X81+2.479646X91+2.287892X32+1.352479X52+1.28633X62+0.690688X72+2.331568X92-1Z41<=1325.696

C42:128.3637X11+0.4059904X21+2.253311X31+2.781029X41+1.304934X51+1.312295X61+1.292772X71+0.4369827X81+2.386858X91+2.253311X32+1.267689X52+1.240951X62+0.9708021X72+2.254224X92-1Z42<=1205.178

C43:118.7984X11+0.3919324X21+2.203128X31+2.59418X41+1.21939X51+1.257668X61+1.455381X71+0.465589X81+2.288793X91+2.203128X32+1.186304X52+1.192995X62+1.175247X72+2.16987X92-1Z43<=1095.616

C44:109.8148X11+0.3762965X21+2.141017X31+2.418053X41+1.137961X51+1.201969X61+1.569276X71+0.4849039X81+2.187444X91+2.141017X32+1.108537X52+1.143335X62+1.323074X72+2.080721X92-1Z44<=996.0148

C45:101.4046X11+0.3596416X21+2.069994X31+2.252273X41+1.060706X51+1.145867X61+1.644359X71+0.496485X81+2.084437X91+2.069994X32+1.03451X52+1.092699X62+1.426615X72+1.988586X92-1Z45<=905.468

C46:93.55242X11+0.3424013X21+1.992535X31+2.096444X41+0.9876201X51+1.089913X61+1.688275X71+0.5016552X81+1.981089X91+1.992535X32+0.9642779X52+1.041697X62+1.494871X72+1.894943X92-1Z46<=823.1527

C47:86.23766X11+0.3249125X21+1.910671X31+1.950151X41+0.9186538X51+1.034562X61+1.707118X71+0.5015413X81+1.878462X91+1.910671X32+0.8978364X52+0.9908305X62+1.534795X72+1.80099X92-1Z47<=748.3207

C48:79.43666X11+0.3074359X21+1.826066X31+1.812971X41+0.8537201X51+0.9801779X61+1.705832X71+0.4971054X81+1.777403X91+1.826066X32+0.8351399X52+0.9405106X62+1.551926X72+1.707693X92-1Z48<=680.2915

C49:73.12383X11+0.2901719X21+1.740076X31+1.684473X41+0.7927048X51+0.9270554X61+1.688473X71+0.4891683X81+1.678578X91+1.740076X32+0.7761092X52+0.8910708X62+1.550756X72+1.615821X92-1Z49<=618.4468

C50:67.27259X11+0.2732731X21+1.653802X31+1.56423X41+0.7354735X51+0.875424X61+1.658392X71+0.4784307X81+1.582503X91+1.653802X32+0.7206407X52+0.8427776X62+1.534975X72+1.52598X92-1Z50<=562.2244

C51:1X11=1, C52:1X21=1; C53:1X31+1X32=1; C54:1X41=1; C55:1X51+1X52=1

C56:1X61+1X62=1, C57:1X71+1X72=1, C58:1X81=1, C59:1X91+1X92=1

In the final solution all z_t's were equal to zero except for z_1 and z_2 (z_1= 1124.60 and z_2= 23430.1). In the final solution z_1 and z_2 were greater than zero, which means we have a budget violation in periods 1 and 2. Therefore we need to apply again the dynamic programming model to generate new component replacement policies for components with replacement in period 1 (CB1 480v , Bus, Line 300 ft) and period 2 (Line 600 ft,Transformer).

Step 1, iteration 3. New component replacement schedules need to be obtained for the components with replacement in periods 1 and 2, the new replacement profiles are obtained by prohibiting replacements for those components having a replacement in periods 1 and 2. Table B.3 shows the new additional component replacement policies obtained by the Dynamic Programming formulation. Figure B.3 shows the new component replacement schedules obtained in the third iteration.

Table B.3. Additional replacement analysis policies

Beginning	end	CB1 480v	Bus	Line 300ft	Line 600ft	Transformer
0	1	Keep	Keep	Keep	Keep	Keep
1	2	Replace	Replace	Replace	Keep	Keep
2	3	Keep	Keep	Keep	Replace	Replace
3	4	Keep	Keep	Keep	Keep	Keep
4	5	Keep	Keep	Keep	Keep	Keep
5	6	Keep	Keep	Keep	Keep	Keep
6	7	Keep	Keep	Keep	Keep	Keep
7	8	Keep	Keep	Keep	Keep	Keep
8	9	Keep	Keep	Keep	Keep	Keep
9	10	Keep	Keep	Keep	Keep	Keep
10	11	Keep	Keep	Keep	Keep	Keep
11	12	Keep	Keep	Keep	Keep	Keep
12	13	Keep	Keep	Keep	Keep	Keep
13	14	Keep	Keep	Keep	Keep	Keep
14	15	Keep	Keep	Keep	Keep	Keep
15	16	Keep	Keep	Keep	Keep	Keep
16	17	Keep	Keep	Keep	Keep	Keep
17	18	Keep	Keep	Keep	Keep	Keep
18	19	Keep	Keep	Keep	Keep	Keep
19	20	Keep	Keep	Keep	Keep	Keep
20	21	Keep	Keep	Keep	Keep	Keep
21	22	Keep	Replace	Keep	Keep	Keep
22	23	Keep	Keep	Keep	Keep	Keep
23	24	Keep	Keep	Keep	Keep	Keep
24	25	Keep	Keep	Keep	Keep	Replace
25	26	Replace	Keep	Keep	Keep	Keep
26	27	Keep	Keep	Keep	Keep	Keep
27	28	Keep	Keep	Keep	Keep	Keep
28	29	Keep	Keep	Replace	Keep	Keep
29	30	Keep	Keep	Keep	Keep	Keep
30	31	Keep	Keep	Keep	Keep	Keep
31	32	Keep	Keep	Keep	Keep	Keep
32	33	Keep	Keep	Keep	Replace	Keep
33	34	Keep	Keep	Keep	Keep	Keep
34	35	Keep	Keep	Keep	Keep	Keep
35	36	Keep	Keep	Keep	Keep	Keep
36	37	Keep	Keep	Keep	Keep	Keep
37	38	Keep	Keep	Keep	Keep	Keep
38	39	Keep	Keep	Keep	Keep	Keep
39	40	Keep	Replace	Keep	Keep	Keep
40	41	Keep	Keep	Keep	Keep	Keep
41	42	Keep	Keep	Keep	Keep	Keep
42	43	Keep	Keep	Keep	Keep	Keep
43	44	Keep	Keep	Keep	Keep	Keep
44	45	Keep	Keep	Keep	Keep	Keep
45	46	Keep	Keep	Keep	Keep	Keep
46	47	Keep	Keep	Keep	Keep	Keep
47	48	Keep	Keep	Keep	Keep	Keep
48	49	Keep	Keep	Keep	Keep	Keep
49	50	Keep	Keep	Keep	Keep	Keep
Total profile cost $s					54815	107583

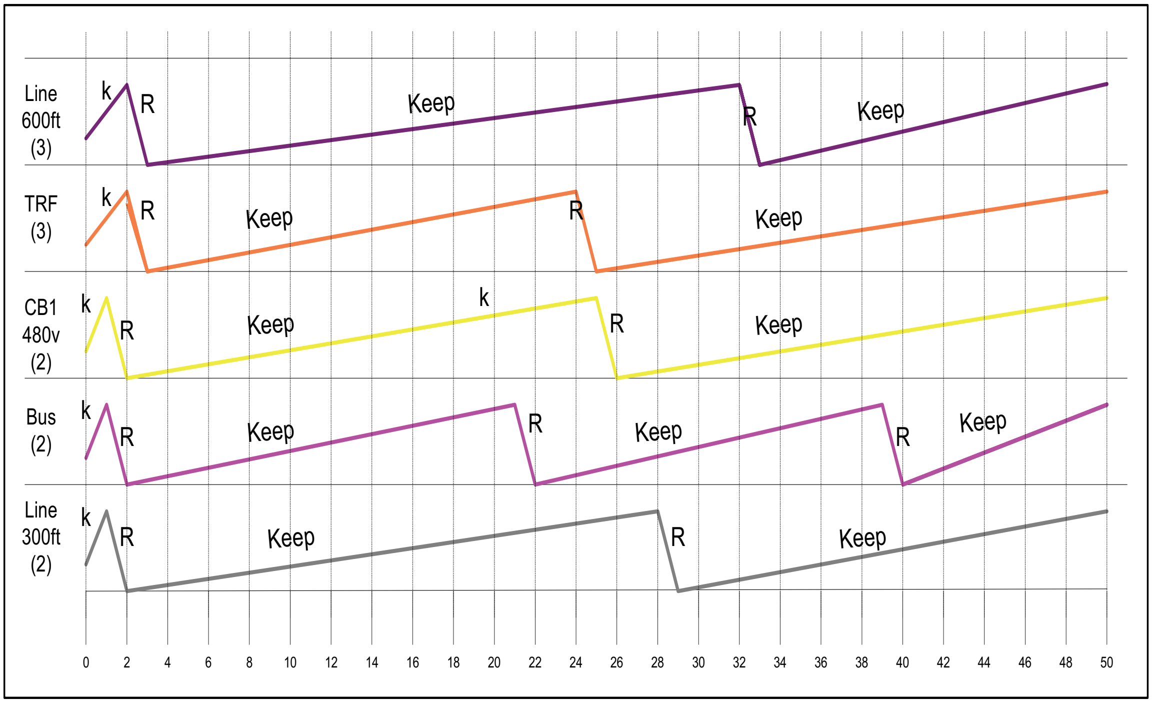

Figure B.3 Additional replacement analysis policies generated in iteration 3

Considering all the different component replacement schedules generated so far (16), the integer programming formulation 1, is applied again to check budget violations, the formulation is as follows:

Integer programming formulation 1

$$\min \sum_{t=1}^{50} z_t$$

s.t.

C1:7094.272X11+21.46367X21+33309.5X31+149.2448X41+45008.18X51+16002.21X61+30012X71+40.21335X81+17507.77X91+330.3379X32+83.08863X52+93.42434X62+590.9374X72+307.0432X92+330.3379X33+83.08863X53-1Z1<=60000

C2:6526.601X11+20.60739X21+21.4372X31+138.9189X41+14.34177X51+5.751158X61+28.4182X71+39.85767X81+17.53953X91+30281.36X32+40916.53X52+14547.46X62+27283.64X72+15916.16X92+306.2247X33+76.88739X53-1Z2<=54545.45

C3:6001.102X11+19.6953X21+29.38304X31+129.225X41+17.31972X51+8.498998X61+39.94344X71+39.22129X81+24.04067X91+19.48837X32+13.03797X52+5.228325X62+25.83473X72+15.94503X92+27528.51X33+37196.84X53-1Z3<=49586.78

C4:5515.187X11+18.75116X21+34.98126X31+120.1348X41+18.96155X51+10.63811X61+48.35528X71+38.36034X81+28.62103X91+26.71186X32+15.7452X52+7.726361X62+36.31222X72+21.85516X92+17.7167X33+11.8527X53-1Z4<=45078.89

C5:32557.39X11+17.79341X21+38.89127X31+111.6201X41+19.79958X51+12.27967X61+54.43757X71+37.32269X81+31.82013X91+31.80114X32+17.23777X52+9.671009X62+43.95934X72+26.01912X92+24.28351X33+14.31382X53-1Z5<=40980.81

C6:2282.884X11+16.83632X21+41.5171X31+103.6523X41+20.10339X51+13.50831X61+58.69775X71+36.149X81+33.96854X91+35.3557X32+17.99962X52+11.16334X62+49.4887X72+28.92739X92+28.91013X33+15.6707X53-1Z6<=37255.28

C7:2363.512X11+15.89089X21+43.14218X31+96.20338X41+20.03649X51+14.39261X61+61.50615X71+34.87375X81+35.29815X91+37.74282X32+18.27581X52+12.28028X62+53.3616X72+30.88049X92+32.14155X33+16.36329X53-1Z7<=33868.44

C8:2338.661X11+14.96545X21+43.97895X31+89.24573X41+19.70774X51+14.98968X61+63.14882X71+33.52608X81+35.98277X91+39.22017X32+18.21499X52+13.08419X62+55.91468X72+32.08923X92+34.31165X33+16.61437X53-1Z8<=30789.49

C9:2264.493X11+14.06626X21+44.19261X31+82.7523X41+19.19369X51+15.34768X61+63.85364X71+32.13049X81+36.15759X91+39.98086X32+17.91612X52+13.62698X62+57.40802X72+32.71161X92+35.65469X33+16.55908X53-1Z9<=27990.44

C10:2164.819X11+13.1979X21+43.91458X31+6363.11X41+18.55008X51+15.50751X61+63.80553X71+30.70751X81+35.93011X91+40.1751X32+17.44881X52+13.95244X62+58.04876X72+32.87054X92+36.34624X33+16.28738X53-1Z10<=25445.86

C11:2052.1X11+12.3636X21+43.25083X31+3.898669X41+17.81838X51+15.50399X61+63.15622X71+29.27423X81+35.38704X91+39.92234X32+16.8637X52+14.09773X62+58.00502X72+32.66374X92+36.52282X33+15.86255X53-1Z11<=23132.6

C12:1933.583X11+11.56549X21+42.28759X31+5.479806X41+17.02989X51+15.36686X61+62.03105X71+27.84475X81+34.59894X91+39.31894X32+16.19852X52+14.09453X62+57.41474X72+32.17004X92+36.29304X33+15.33064X53-1Z12<=21029.63

C13:1813.731X11+10.80484X21+41.09541X31+6.63382X41+16.20844X51+15.1215X61+60.53402X71+26.43064X81+33.62352X91+38.44326X32+15.48172X52+13.96987X62+56.39186X72+31.45358X92+35.74449X33+14.72593X53-1Z13<=19117.85

C14:1695.375X11+10.08226X21+39.73217X31+7.468245X41+15.37218X51+14.78957X61+58.75152X71+25.04126X81+32.50814X91+37.35947X32+14.73494X52+13.74682X62+55.03093X72+30.56684X92+34.94842X33+14.07429X53-1Z14<=17379.86

C15:1580.325X11+9.397766X21+38.2454X31+8.052695X41+14.535X51+14.38951X61+56.7554X71+4213.882X81+31.29169X91+36.12016X32+13.97471X52+13.44507X62+53.41047X72+29.55286X92+33.96315X33+13.3954X53-1Z15<=15799.88

C16:1469.725X11+8.750985X21+36.67407X31+8.437977X41+13.7074X51+13.93701X61+54.60533X71+1.51446X81+30.00606X91+34.76854X32+13.21364X52+13.08138X62+51.59582X72+28.44699X92+32.83651X33+12.70429X53-1Z16<=14363.52

C17:1364.27X11+8.141195X21+35.05013X31+8.663333X41+12.89728X51+13.44534X61+52.35075X71+2.238052X81+28.67738X91+33.34007X32+12.46127X52+12.67X62+49.64121X72+27.27824X92+31.60776X33+12.0124X53-1Z17<=13057.75

C18:1264.348X11+7.567423X21+33.39961X31+8.760026X41+12.1105X51+12.92575X61+50.03249X71+2.801347X81+27.32696X91+31.86376X32+11.7248X52+12.22304X62+47.59159X72+26.07035X92+30.30915X33+11.32843X53-1Z18<=11870.68

C19:1170.132X11+7.028506X21+31.74364X31+8.753427X41+11.35127X51+12.38769X61+47.68413X71+3.233622X81+25.97207X91+30.36329X32+11.00955X52+11.75069X62+45.48409X72+24.84269X92+28.96705X33+10.65891X53-1Z19<=10791.53

C20:1081.646X11+6.523145X21+30.09921X31+8.664348X41+10.62253X51+11.83908X61+45.33309X71+3.557159X81+24.62663X91+28.85785X32+10.31934X52+11.26154X62+43.34921X72+23.61097X92+27.60299X33+10.00868X53-1Z20<=9810.479

C21:998.8085X11+6.049943X21+28.47991X31+8.509988X41+9.926171X51+11.28648X61+4461.093X71+3.790023X81+23.30174X91+27.36292X32+9.656841X52+10.7628X62+41.2119X72+22.38785X92+26.23441X33+9.381217X53-1Z21<=8918.618

C22:921.4664X11+5.60745X21+26.8964X31+8.30461X41+9.263318X51+10.73536X61+4.224185X71+3.947251X81+22.00615X91+25.89083X32+9.023792X52+10.26044X62+4055.539X72+21.1834X92+24.87538X33+8.778946X53-1Z22<=8107.834

C23:849.4178X11+5.194176X21+25.35697X31+8.060071X41+5529.074X51+10.19016X61+5.937338X71+4.041524X81+20.74661X91+24.45128X32+8.421199X52+9.759416X62+3.840168X72+20.00559X92+23.53712X33+8.203447X53-1Z23<=7370.758

C24:782.4299X11+4.808627X21+23.86786X31+7.786226X41+1.761829X51+9.65449X61+7.187704X71+4.083611X81+19.52825X91+23.05179X32+5026.431X52+9.263778X62+5.39758X72+18.86056X92+22.22843X33+7.655635X53-1Z24<=6700.689

C25:720.2502X11+4.449315X21+22.43364X31+7.491259X41+2.127658X51+1624.634X61+8.091798X71+4.082684X81+18.3548X91+21.69805X32+1.601663X52+8.77681X62+6.534276X72+17.75295X92+20.95617X33+4569.482X53-1Z25<=6091.536

C26:662.617X11+4.114776X21+21.05749X31+7.181955X41+2.32935X51+0.5838897X61+8.725047X71+4.046574X81+17.22886X91+20.39422X32+1.934234X52+1476.94X62+7.35618X72+16.68618X92+19.7255X33+1.456057X53-1Z26<=5537.76

C27:609.2654X11+3.803579X21+19.74144X31+6.863915X41+2.432298X51+0.8628658X61+9.142498X71+3.981965X81+16.15209X91+19.14318X32+2.117591X52+0.5308089X62+7.931861X72+15.6626X92+18.5402X33+1.758395X53-1Z27<=5034.327

C28:559.9326X11+3.514333X21+18.48655X31+6.541746X41+2.46962X51+1.08004X61+9.386669X71+3.894557X81+1335.452X91+17.94677X32+2.21118X52+0.7844234X62+8.311361X72+14.68372X92+17.40289X33+1.925083X53-1Z28<=4576.661

C29:3305.408X11+3.245698X21+17.29312X31+6.219208X41+2.461402X51+1.246701X61+9.491436X71+3.789209X81+1.337875X91+16.80596X32+2.24511X52+0.9818549X62+8.533336X72+1214.048X92+16.31524X33+2.010164X53-1Z29<=4160.601

C30:231.7711X11+2.996383X21+16.16079X31+5.899353X41+2.421016X51+1.371439X61+9.484285X71+3.670049X81+1.833767X91+15.72101X32+2.237638X52+1.133365X62+8.628578X72+1.21625X92+15.27814X33+2.041009X53-1Z30<=3782.365

C31:239.957X11+2.765156X21+15.08872X31+5.584626X41+2.357867X51+1.461218X61+9.387769X71+3.540579X81+2.183146X91+14.69163X32+2.200924X52+1.246763X62+8.622077X72+1.667061X92+14.29183X33+2.034217X53-1Z31<=3438.513

C32:237.4339X11+2.550838X21+1735.381X31+5.276966X41+2.278802X51+1.521836X61+9.220521X71+3.403755X81+2.427166X91+1735.381X32+2.143516X52+1.32838X62+8.534335X72+1.984678X92+13.35602X33+2.00084X53-1Z32<=3125.921

C33:229.904X11+1657.84X21+1.11685X31+4.977881X41+2.188916X51+1.558183X61+8.997996X71+3.262067X81+2.591041X91+1.11685X32+2.071638X52+1.383487X62+8.382292X72+2.206515X92+1577.619X33+1.948651X53-1Z33<=2841.746

C34:219.7845X11+0.2862554X21+1.530818X31+4.688519X41+2.092054X51+1.574409X61+8.733039X71+3.117599X81+2.692461X91+1.530818X32+1.989923X52+1.41653X62+8.179996X72+2.355492X92+1.015318X33+1.883308X53-1Z34<=2583.406

C35:208.3407X11+0.3546029X21+1.822478X31+4.409722X41+1.991141X51+1.574051X61+8.436329X71+2.972084X81+2.744683X91+1.822478X32+1.901867X52+1.431281X62+7.939126X72+2.447692X92+1.391653X33+1.809021X53-1Z35<=2348.551

C36:196.3082X11+0.3947942X21+2.026184X31+4.142082X41+1.888411X51+1.56013X61+8.116735X71+2.826955X81+2.758017X91+2.026184X32+1.810128X52+1.430956X62+7.66939X72+2.495166X92+1.656798X33+1.72897X53-1Z36<=2135.046

C37:184.1402X11+0.4174512X21+2.162986X31+3.885975X41+1.785566X51+1.53522X61+7.781605X71+517.6584X81+2.740666X91+2.162986X32+1.716737X52+1.4183X62+7.37885X72+2.507288X92+1.841986X33+1.645571X53-1Z37<=1940.951

C38:172.124X11+0.4281286X21+2.247651X31+3.641605X41+1.683899X51+1.50152X61+7.437011X71+0.1860453X81+2.699242X91+2.247651X32+1.623242X52+1.395654X62+7.074186X72+2.491514X92+1.966351X33+1.56067X53-1Z38<=1764.501

C39:160.4434X11+0.4302817X21+2.291245X31+3.409031X41+1.584379X51+1.460904X61+802.3667X71+0.2749356X81+2.639127X91+2.291245X32+1.530817X52+1.365018X62+6.760919X72+2.453856X92+2.043319X33+1.475675X53-1Z39<=1604.092

C40:149.2147X11+0.42626X21+2.302377X31+3.188193X41+1.487726X51+1.414963X61+0.7597567X71+0.3441342X81+2.564724X91+2.302377X32+1.440345X52+1.328094X62+729.4243X72+2.399206X92+2.08295X33+1.391652X53-1Z40<=1458.265

C41:138.5083X11+0.4177473X21+2.287892X31+2.978936X41+1.394458X51+1.365046X61+1.067882X71+0.3972374X81+2.479646X91+2.287892X32+1.352479X52+1.28633X62+0.690688X72+2.331568X92+2.09307X33+1.309404X53-1Z41<=1325.696

C42:128.3637X11+0.4059904X21+2.253311X31+2.781029X41+1.304934X51+1.312295X61+1.292772X71+0.4369827X81+2.386858X91+2.253311X32+1.267689X52+1.240951X62+0.9708021X72+2.254224X92+2.079902X33+1.229526X53-1Z42<=1205.178

C43:118.7984X11+0.3919324X21+2.203128X31+2.59418X41+1.21939X51+1.257668X61+1.455381X71+0.465589X81+2.288793X91+2.203128X32+1.186304X52+1.192995X62+1.175247X72+2.16987X92+2.048465X33+1.152445X53-1Z43<=1095.616

C44:109.8148X11+0.3762965X21+2.141017X31+2.418053X41+1.137961X51+1.201969X61+1.569276X71+0.4849039X81+2.187444X91+2.141017X32+1.108537X52+1.143335X62+1.323074X72+2.080721X92+2.002844X33+1.078458X53-1Z44<=996.0148

C45:101.4046X11+0.3596416X21+2.069994X31+2.252273X41+1.060706X51+1.145867X61+1.644359X71+0.496485X81+2.084437X91+2.069994X32+1.03451X52+1.092699X62+1.426615X72+1.988586X92+1.946379X33+1.007761X53-1Z45<=905.468

C46:93.55242X11+0.3424013X21+1.992535X31+2.096444X41+0.9876201X51+1.089913X61+1.688275X71+0.5016552X81+1.981089X91+1.992535X32+0.9642779X52+1.041697X62+1.494871X72+1.894943X92+1.881813X33+0.940464X53-1Z46<=823.1527

C47:86.23766X11+0.3249125X21+1.910671X31+1.950151X41+0.9186538X51+1.034562X61+1.707118X71+0.5015413X81+1.878462X91+1.910671X32+0.8978364X52+0.9908305X62+1.534795X72+1.80099X92+1.811395X33+0.8766163X53-1Z47<=748.3207

C48:79.43666X11+0.3074359X21+1.826066X31+1.812971X41+0.8537201X51+0.9801779X61+1.705832X71+0.4971054X81+1.777403X91+1.826066X32+0.8351399X52+0.9405106X62+1.551926X72+1.707693X92+1.736974X33+0.8162149X53-1Z48<=680.2915

C49:73.12383X11+0.2901719X21+1.740076X31+1.684473X41+0.7927048X51+0.9270554X61+1.688473X71+0.4891683X81+1.678578X91+1.740076X32+0.7761092X52+0.8910708X62+1.550756X72+1.615821X92+1.66006X33+0.759218X53-1Z49<=618.4468

C50:67.27259X11+0.2732731X21+1.653802X31+1.56423X41+0.7354735X51+0.875424X61+1.658392X71+0.4784307X81+1.582503X91+1.653802X32+0.7206407X52+0.8427776X62+1.534975X72+1.52598X92+1.581887X33+0.7055538X53-1Z50<=562.2244

C51:1X11=1, **C52**:1X21=1, **C53**:1X31+1X32+1X33=1, **C54**:1X41=1, **C55**:1X51+1X52+1X53=1
C56:1X61+1X62=1, **C57**:1X71+1X72=1, **C58**:1X81=1, **C59**:1X91+1X92=1

The problem was solved using LINDO, and this time in the final solution all $z_t > 0$, which means there are no annual budget violations. Therefore, the second integer programming formulation (IP2) is used to find the optimal replacement analysis policy among all the different component replacement profiles generated.

Step 3. The second integer program formulation (IP2), is applied using all the data generated from all profiles created in the previous steps. The full IP2 is as follows:

Minimize

164255.9X11+33995.88X21+52722.71X31+19465.52X41+103713.2X51+26685.21X61+57744.7X71+19926.47X81+29733.64X91+53763.73X32+105641.9X52+26821.99X62+63458.53X72+31292.94X92+54815.63X33+107583.6X53

C1:7094.272X11+21.46367X21+33309.5X31+149.2448X41+45008.18X51+16002.21X61+30012X71+40.21335X81+17507.77X91+330.3379X32+83.08863X52+93.42434X62+590.9374X72+307.0432X92+330.3379X33+83.08863X53<=60000

C2:6526.601X11+20.60739X21+21.4372X31+138.9189X41+14.34177X51+5.751158X61+28.4182X71+39.85767X81+17.53953X91+30281.36X32+40916.53X52+14547.46X62+27283.64X72+15916.16X92+306.2247X33+76.88739X53<=54545.45

C3:6001.102X11+19.6953X21+29.38304X31+129.225X41+17.31972X51+8.498998X61+39.94344X71+39.22129X81+24.04067X91+19.48837X32+13.03797X52+5.228325X62+25.83473X72+15.94503X92+27528.51X33+37196.84X53<=49586.78

C4:5515.187X11+18.75116X21+34.98126X31+120.1348X41+18.96155X51+10.63811X61+48.35528X71+38.36034X81+28.62103X91+26.71186X32+15.7452X52+7.726361X62+36.31222X72+21.85516X92+17.7167X33+11.8527X53<=45078.89

C5:32557.39X11+17.79341X21+38.89127X31+111.6201X41+19.79958X51+12.27967X61+54.43757X71+37.32269X81+31.82013X91+31.80114X32+17.23777X52+9.671009X62+43.95934X72+26.01912X92+24.28351X33+14.31382X53<=40980.81

C6:2282.884X11+16.83632X21+41.5171X31+103.6523X41+20.10339X51+13.50831X61+58.69775X71+36.149X81+33.96854X91+35.3557X32+17.99962X52+11.16334X62+49.4887X72+28.92739X92+28.91013X33+15.6707X53<=37255.28

C7:2363.512X11+15.89089X21+43.14218X31+96.20338X41+20.03649X51+14.39261X61+61.50615X71+34.87375X81+35.29815X91+37.74282X32+18.27581X52+12.28028X62+53.3616X72+30.88049X92+32.14155X33+16.36329X53<=33868.44

C8:2338.661X11+14.96545X21+43.97895X31+89.24573X41+19.70774X51+14.98968X61+63.14882X71+33.52608X81+35.98277X91+39.22017X32+18.21499X52+13.08419X62+55.91468X72+32.08923X92+34.31165X33+16.61437X53<=30789.49

C9:2264.493X11+14.06626X21+44.19261X31+82.7523X41+19.19369X51+15.34768X61+63.85364X71+32.13049X81+36.15759X91+39.98086X32+17.91612X52+13.62698X62+57.40802X72+32.71161X92+35.65469X33+16.55908X53<=27990.44

C10:2164.819X11+13.1979X21+43.91458X31+6363.11X41+18.55008X51+15.50751X61+63.80553X71+30.70751X81+35.93011X91+40.1751X32+17.44881X52+13.95244X62+58.04876X72+32.87054X92+36.34624X33+16.28738X53<=25445.86

C11:2052.1X11+12.3636X21+43.25083X31+3.898669X41+17.81838X51+15.50399X61+63.15622X71+29.27423X81+35.38704X91+39.92234X32+16.8637X52+14.09773X62+58.00502X72+32.66374X92+36.52282X33+15.86255X53<=23132.6

C12:1933.583X11+11.56549X21+42.28759X31+5.479806X41+17.02989X51+15.36686X61+62.03105X71+27.84475X81+34.59894X91+39.31894X32+16.19852X52+14.09453X62+57.41474X72+32.17004X92+36.29304X33+15.33064X53<=21029.63

C13:1813.731X11+10.80484X21+41.09541X31+6.63382X41+16.20844X51+15.1215X61+60.53402X71+26.43064X81+33.62352X91+38.44326X32+15.48172X52+13.96987X62+56.39186X72+31.45358X92+35.74449X33+14.72593X53<=19117.85

C14:1695.375X11+10.08226X21+39.73217X31+7.468245X41+15.37218X51+14.78957X61+58.75152X71+25.04126X81+32.50814X91+37.35947X32+14.73494X52+13.74682X62+55.03093X72+30.56684X92+34.94842X33+14.07429X53<=17379.86

C15:1580.325X11+9.397766X21+38.2454X31+8.052695X41+14.535X51+14.38951X61+56.7554X71+4213.882X81+31.29169X91 +36.12016X32+13.97471X52+13.44507X62+53.41047X72+29.55286X92+33.96315X33+13.3954X53<=15799.88

C16:1469.725X11+8.750985X21+36.67407X31+8.437977X41+13.7074X51+13.93701X61+54.60533X71+1.51446X81+30.00606X91+34.76854X32+13.21364X52+13.08138X62+51.59582X72+28.44699X92+32.83651X33+12.70429X53<=14363.52

C17:1364.27X11+8.141195X21+35.05013X31+8.663333X41+12.89728X51+13.44534X61+52.35075X71+2.238052X81+28.67738X91+33.34007X32+12.46127X52+12.67X62+49.64121X72+27.27824X92+31.60776X33+12.0124X53<=13057.75

C18:1264.348X11+7.567423X21+33.39961X31+8.760026X41+12.1105X51+12.92575X61+50.03249X71+2.801347X81+27.32696X91+31.86376X32+11.7248X52+12.22304X62+47.59159X72+26.07035X92+30.30915X33+11.32843X53<=11870.68

C19:1170.132X11+7.028506X21+31.74364X31+8.753427X41+11.35127X51+12.38769X61+47.68413X71+3.233622X81+25.97207X91+30.36329X32+11.00955X52+11.75069X62+45.48409X72+24.84269X92+28.96705X33+10.65891X53<=10791.53

C20:1081.646X11+6.523145X21+30.09921X31+8.664348X41+10.62253X51+11.83908X61+45.33309X71+3.557159X81+24.62663X91+28.85785X32+10.31934X52+11.26154X62+43.34921X72+23.61097X92+27.60299X33+10.00868X53<=9810.479

C21:998.8085X11+6.049943X21+28.47991X31+8.509988X41+9.926171X51+11.28648X61+4461.093X71+3.790023X81+23.30174X91+27.36292X32+9.656841X52+10.7628X62+41.2119X72+22.38785X92+26.23441X33+9.381217X53<=8918.618

C22:921.4664X11+5.60745X21+26.8964X31+8.30461X41+9.263318X51+10.73536X61+4.224185X71+3.947251X81+22.00615X91+25.89083X32+9.023792X52+10.26044X62+4055.539X72+21.1834X92+24.87538X33+8.778946X53<=8107.834

C23:849.4178X11+5.194176X21+25.35697X31+8.060071X41+5529.074X51+10.19016X61+5.937338X71+4.041524X81+20.74661X91+24.45128X32+8.421199X52+9.759416X62+3.840168X72+20.00559X92+23.53712X33+8.203447X53<=7370.758

C24:782.4299X11+4.808627X21+23.86786X31+7.786226X41+1.761829X51+9.65449X61+7.187704X71+4.083611X81+19.52825X91+23.05179X32+5026.431X52+9.263778X62+5.39758X72+18.86056X92+22.22843X33+7.655635X53<=6700.689

C25:720.2502X11+4.449315X21+22.43364X31+7.491259X41+2.127658X51+1624.634X61+8.091798X71+4.082684X81+18.3548X91+21.69805X32+1.601663X52+8.77681X62+6.534276X72+17.75295X92+20.95617X33+4569.482X53<=6091.536

C26:662.617X11+4.114776X21+21.05749X31+7.181955X41+2.32935X51+0.5838897X61+8.725047X71+4.046574X81+17.22886X91+20.39422X32+1.934234X52+1476.94X62+7.35618X72+16.68618X92+19.7255X33+1.456057X53<=5537.76

C27:609.2654X11+3.803579X21+19.74144X31+6.863915X41+2.432298X51+0.8628658X61+9.142498X71+3.981965X81+16.15209X91+19.14318X32+2.117591X52+0.5308089X62+7.931861X72+15.6626X92+18.5402X33+1.758395X53<=5034.327

C28:559.9326X11+3.514333X21+18.48655X31+6.541746X41+2.46962X51+1.08004X61+9.386669X71+3.894557X81+1335.452X91+17.94677X32+2.21118X52+0.7844234X62+8.311361X72+14.68372X92+17.40289X33+1.925083X53<=4576.661

C29:3305.408X11+3.245698X21+17.29312X31+6.219208X41+2.461402X51+1.246701X61+9.491436X71+3.789209X81+1.337875X91+16.80596X32+2.24511X52+0.9818549X62+8.533336X72+1214.048X92+16.31524X33+2.010164X53<=4160.601

C30:231.7711X11+2.996383X21+16.16079X31+5.899353X41+2.421016X51+1.371439X61+9.484285X71+3.670049X81+1.833767X91+15.72101X32+2.237638X52+1.133365X62+8.628578X72+1.21625X92+15.27814X33+2.041009X53<=3782.365

C31:239.957X11+2.765156X21+15.08872X31+5.584626X41+2.357867X51+1.461218X61+9.387769X71+3.540579X81+2.183146X91+14.69163X32+2.200924X52+1.246763X62+8.622077X72+1.667061X92+14.29183X33+2.034217X53<=3438.513

C32:237.4339X11+2.550838X21+1735.381X31+5.276966X41+2.278802X51+1.521836X61+9.220521X71+3.403755X81+2.427166X91+1735.381X32+2.143516X52+1.32838X62+8.534335X72+1.984678X92+13.35602X33+2.00084X53<=3125.921

C33:229.904X11+1657.84X21+1.11685X31+4.977881X41+2.188916X51+1.558183X61+8.997996X71+3.262067X81+2.591041X91+1.11685X32+2.071638X52+1.383487X62+8.382292X72+2.206515X92+1577.619X33+1.948651X53<=2841.746

C34:219.7845X11+0.2862554X21+1.530818X31+4.688519X41+2.092054X51+1.574409X61+8.733039X71+3.117599X81+2.692461X91+1.530818X32+1.989923X52+1.41653X62+8.179996X72+2.355492X92+1.015318X33+1.883308X53<=2583.406

C35:208.3407X11+0.3546029X21+1.822478X31+4.409722X41+1.991141X51+1.574051X61+8.436329X71+2.972084X81+2.744683X91+1.822478X32+1.901867X52+1.431281X62+7.939126X72+2.447692X92+1.391653X33+1.809021X53<=2348.551

C36:196.3082X11+0.3947942X21+2.026184X31+4.142082X41+1.888411X51+1.56013X61+8.116735X71+2.826955X81+2.758017X91+2.026184X32+1.810128X52+1.430956X62+7.66939X72+2.495166X92+1.656798X33+1.72897X53<=2135.046

C37:184.1402X11+0.4174512X21+2.162986X31+3.885975X41+1.785566X51+1.53522X61+7.781605X71+517.6584X81+2.740666X91+2.162986X32+1.716737X52+1.4183X62+7.37885X72+2.507288X92+1.841986X33+1.645571X53<=1940.951

C38:172.124X11+0.4281286X21+2.247651X31+3.641605X41+1.683899X51+1.50152X61+7.437011X71+0.1860453X81+2.699242X91+2.247651X32+1.623242X52+1.395654X62+7.074186X72+2.491514X92+1.966351X33+1.56067X53<=1764.501

C39:160.4434X11+0.4302817X21+2.291245X31+3.409031X41+1.584379X51+1.460904X61+802.3667X71+0.2749356X81+2.639127X91+2.291245X32+1.530817X52+1.365018X62+6.760919X72+2.453856X92+2.043319X33+1.475675X53<=1604.092

C40:149.2147X11+0.42626X21+2.302377X31+3.188193X41+1.487726X51+1.414963X61+0.7597567X71+0.3441342X81+2.564724X91+2.302377X32+1.440345X52+1.328094X62+729.4243X72+2.399206X92+2.08295X33+1.391652X53<=1458.265

C41:138.5083X11+0.4177473X21+2.287892X31+2.978936X41+1.394458X51+1.365046X61+1.067882X71+0.3972374X81+2.479646X91+2.287892X32+1.352479X52+1.28633X62+0.690688X72+2.331568X92+2.09307X33+1.309404X53<=1325.696

C42:128.3637X11+0.4059904X21+2.253311X31+2.781029X41+1.304934X51+1.312295X61+1.292772X71+0.4369827X81+2.386858X91+2.253311X32+1.267689X52+1.240951X62+0.9708021X72+2.254224X92+2.079902X33+1.229526X53<=1205.178

C43:118.7984X11+0.3919324X21+2.203128X31+2.59418X41+1.21939X51+1.257668X61+1.455381X71+0.465589X81+2.288793X91+2.203128X32+1.186304X52+1.192995X62+1.175247X72+2.16987X92+2.048465X33+1.152445X53<=1095.616

C44:109.8148X11+0.3762965X21+2.141017X31+2.418053X41+1.137961X51+1.201969X61+1.569276X71+0.4849039X81+2.187444X91+2.141017X32+1.108537X52+1.143335X62+1.323074X72+2.080721X92+2.002844X33+1.078458X53<=996.0148

C45:101.4046X11+0.3596416X21+2.069994X31+2.252273X41+1.060706X51+1.145867X61+1.644359X71+0.496485X81+2.084437X91+2.069994X32+1.03451X52+1.092699X62+1.426615X72+1.988586X92+1.946379X33+1.007761X53<=905.468

C46:93.55242X11+0.3424013X21+1.992535X31+2.096444X41+0.9876201X51+1.089913X61+1.688275X71+0.5016552X81+1.981089X91+1.992535X32+0.9642779X52+1.041697X62+1.494871X72+1.894943X92+1.881813X33+0.940464X53<=823.1527

C47:86.23766X11+0.3249125X21+1.910671X31+1.950151X41+0.9186538X51+1.034562X61+1.707118X71+0.5015413X81+1.878462X91+1.910671X32+0.8978364X52+0.9908305X62+1.534795X72+1.80099X92+1.811395X33+0.8766163X53<=748.3207

C48:79.43666X11+0.3074359X21+1.826066X31+1.812971X41+0.8537201X51+0.9801779X61+1.705832X71+0.4971054X81+1.777403X91+1.826066X32+0.8351399X52+0.9405106X62+1.551926X72+1.707693X92+1.736974X33+0.8162149X53<=680.2915

C49:73.12383X11+0.2901719X21+1.740076X31+1.684473X41+0.7927048X51+0.9270554X61+1.688473X71+0.4891683X81+1.678578X91+1.740076X32+0.7761092X52+0.8910708X62+1.550756X72+1.615821X92+1.66006X33+0.759218X53<=618.4468

C50:67.27259X11+0.2732731X21+1.653802X31+1.56423X41+0.7354735X51+0.875424X61+1.658392X71+0.4784307X81+1.582503X91+1.653802X32+0.7206407X52+0.8427776X62+1.534975X72+1.52598X92+1.581887X33+0.7055538X53<=562.2244

C51:1X11=1, C52:1X21=1, C53:1X31+1X32+1X33=1, C54:1X41=1
C55:1X51+1X52+1X53=1, C56:1X61+1X62=1, C57:1X71+1X72=1, C58:1X81=1, C59:1X91+1X92=1

The problem was solved using LINDO and Table B.4 shows the final results obtained for the solution to the replacement problem, the total replacement cost is $513,291.

Table B.3. Final results

Component	Profile number	Replacement cost
Line 13.8Kv	1	164,256
CB 13.8Kv	1	33,996
Switch	1	53,764
Bus	1	19,466
CB2 480v	1	107,584
Line300ft	1	26,822
Line 600ft	2	57,745
CB1 480v	2	19,926
Transformer	3	29,734
Total replacement cost		**$513,293**

Table B.5 has the specific information about the keep/replace decisions made. Figure B.4 shows the optimal system level replacement policy replacement policies.

Table B.5. Replacement analysis policies

Beginning	end	Line 13.8Kv	CB 13.8Kv	Line 600ft	Switch	Transformer	CB1 480v	Bus	CB2 480v	Line 300ft
0	1	Keep	Keep	Keep	Keep	Keep	Keep	**Replace**	Keep	**Replace**
1	2	Keep	Keep	**Replace**	Keep	Keep	Replace	Keep	Keep	Keep
2	3	Keep	Keep	Keep	Keep	Replace	Keep	Keep	Keep	Keep
3	4	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
4	5	Replace	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
5	6	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
6	7	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
7	8	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
8	9	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
9	10	Keep	Keep	Keep	Replace	Keep	Keep	Keep	Keep	Keep
10	11	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
11	12	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
12	13	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
13	14	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
14	15	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Replace	Keep
15	16	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
16	17	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
17	18	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
18	19	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
19	20	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
20	21	Keep	Keep	Keep	Keep	Keep	Keep	Replace	Keep	Keep
21	22	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
22	23	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
23	24	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
24	25	Keep	Keep	Keep	Keep	Replace	Keep	Keep	Keep	Keep
25	26	Keep	Keep	Keep	Keep	Keep	Replace	Keep	Keep	Keep
26	27	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
27	28	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Replace
28	29	Replace	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
29	30	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
30	31	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
31	32	Keep	Keep	Replace	Keep	Keep	Keep	Keep	Keep	Keep
32	33	Keep	Replace	Keep	Keep	Keep	Keep	Keep	Keep	Keep
33	34	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
34	35	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
35	36	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
36	37	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Replace	Keep
37	38	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
38	39	Keep	Keep	Keep	Keep	Keep	Keep	Replace	Keep	Keep
39	40	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
40	41	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
41	42	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
42	43	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
43	44	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
44	45	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
45	46	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
46	47	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
47	48	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
48	49	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep
49	50	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep	Keep

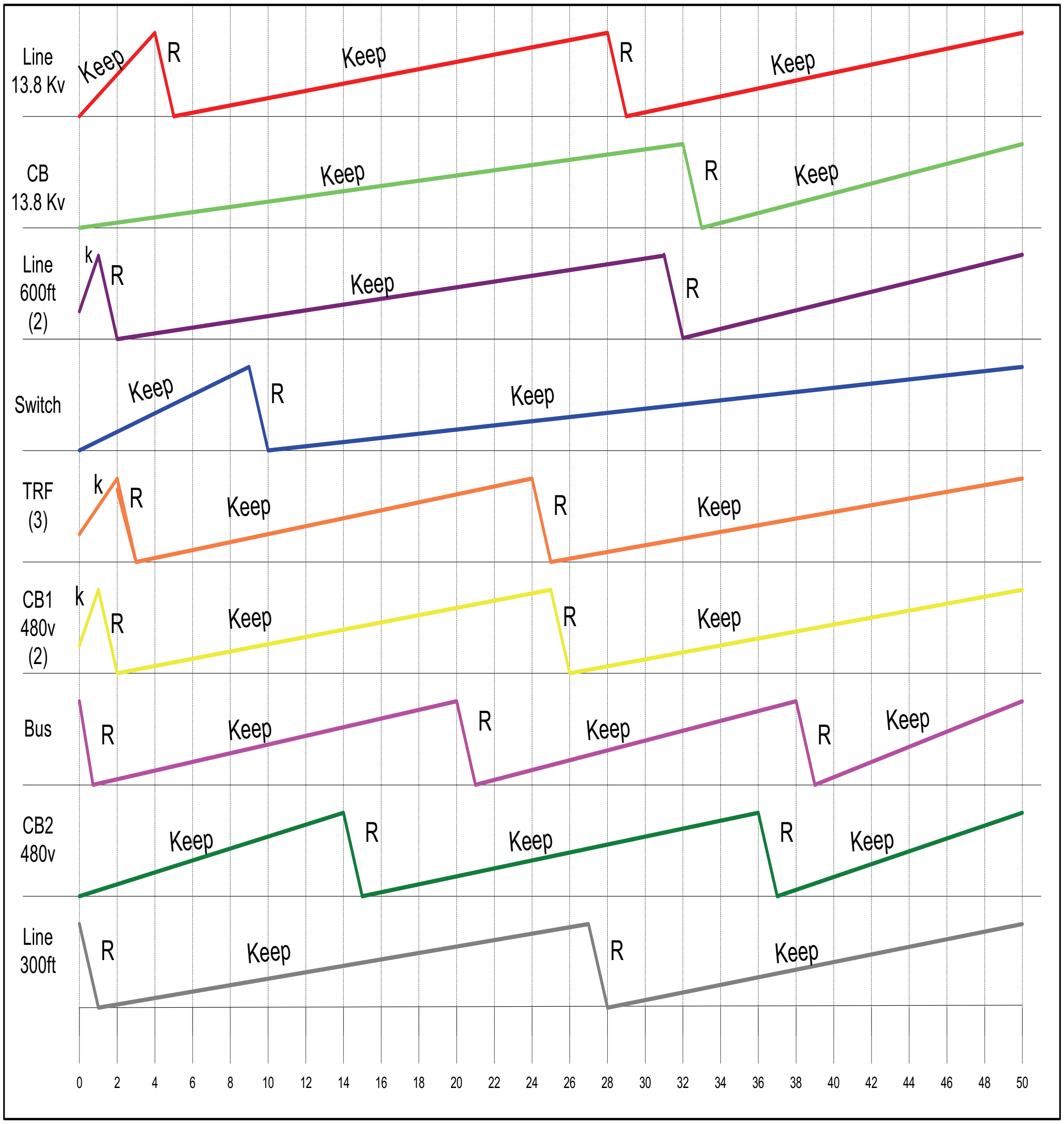

Figure B.4 Recommended system-level replacement schedule for example 1

References

1. Allan, R. N., Billinton, R. and Goel, L. (1991). A reliability test system for educational purpose-basic distribution system data and results. *IEEE Transactions on Power Systems*, 6:813–820.

2. Ansell, J. I. and Phillips, M. J.(1994). Practical Methods for Reliability Data Analysis: Oxford University Press.

3. Ayyub, B. M. (2003). Risk Analysis in Engineering and Economics. Chapman and Hall/CRC Press.

4. Barker, P., Leksan, T., and Smith, D. (1998). Integration of Distributed Resources in Electric Power Systems*:* Current Interconnection Practice and Unified Approach*.* EPRI TR-111489 Report, September 30.

5. Beiser, J. A. and Rigdon, S. E. (1997). Bayes prediction for the number of failures of a repairable systems. *IEEE Transactions on Reliability*, 46:291–295.

6. Bellman, R. E. (1955). Equipment replacement policy. *Journal of the Society for the Industrial Applications of Mathematics*, 3:133-136.

7. Bertsekas, D. P. (1995). Dynamic Programming and Optimal Control. Athena Scientific, Bellmont, MA. 1995.

8. Bertsekas, D. P. and Tsitsiklis, J. N. (1996). Neuro-Dynamic Programming. Athena Scientic, Bellmont, MA.

9. Billinton, R., and Bollinger K. E.. (1968). Transmission system Reliability Evaluation Using Markov Processes. *IEEE Transactions on Power Apparatus Systems*, 87(2):538-547.

10. Billinton, R. and Allan, R. (1983). Reliability Evaluation of Engineering Systems. Plenum Press, London.

11. Billinton, R. and Allan, R. (1984). Reliability Evaluation of Power Systems. Plenum Press, London.

12. Billinton, R. and Vohra, P. K. (1987). Station - Initiated Outages in Composite - System Adequacy Evaluation. *IEE Proceedings*, 134(1):10-16.

13. Billinton, R. and Allan R. N. (1988). Reliability assessment of large electric power systems. Kluwer Academic Publishers, Boston, MA.

14. Billinton, R. and Billinton, J. E. (1989). Distribution system reliability indices. *IEEE Transactions on Power delivery*, 4(1):561-568.

15. Billinton, R. and Li, W. (1994). Reliability Assessment of Electric Power Systems Using Monte Carlo Methods. Plenum Press, New York.

16. Billinton, R. and Satish, J. (1995). Reliability cost/reliability worth assessment of station configurations. *IEEE Conference Proceedings WESCANEX 1995. Communications, Power and Computing,* 1:175–180.

17. Billinton, R. and Stish J. (1995). Minimum cost analysis of station configurations. *IEEE Transactions on Power Delivery*, 10(4).

18. Billinton, R. and Wang, P. (1998). Distribution system reliability cost/worth analysis using analytical and sequential simulation techniques. *IEEE Transactions on Power Systems*, 13:1245–1250.

19. Billinton, R. and Zhang, W. (2000). State extension for adequacy evaluation of composite power systems-applications-. *IEEE Transactions on Power Systems*, 15(1),427-432.

20. Billinton, R., Cui L. and Pan Z. (2002). Quantitative reliability considerations in the determination of performance-based rates and customer service disruption payments. *IEEE Proceedings Generation, Transmission and Distribution*, 149(6):640-644

21. Birnbaum, Z. W. (1969). On the importance of different components in a multicomponent system. Multivariate Analysis 2, New York, Academic Press.

22. Broadwater, R. P. and Shaalan, H. E (1994). Distribution system reliability and restoration analysis, *Electric Power System Research*, 29:203-211.

23. Brown, R. E., Gupta, S., Christie, R. D., Venkata, S. S., and Fletcher, R. (1995). Distribution system reliability assessment using hierarchical Markov modeling. *IEEE Transactions on Power Delivery*, 10:1929–1934.

24. Brown, R. E. and Ochoa, J. R. (1998). Distribution system reliability: Default data and model validation. *IEEE Transactions on Power Systems*, 13:704–709.

25. Brown, R. E. and Taylor, T. M. (1999). Modeling the impact of substations on distribution reliability. *IEEE Transactions on Power Systems*, 14:349–354.

26. Brown, R. E. and Willis, H. L. (2006). The economics of aging infrastructure. *IEEE Power and Energy Magazine*, 4(**3**):36-43.

27. Chan, G. K. and Asgarpoor, S. (2005). Optimum maintenance policy with Markov processes. Electric Power Systems Research, in press.

28. Chowdhury, A. and Koval, D. O. (1998). Value-based distribution system reliability planning. *IEEE Transactions on Industry Applications*, 34:23–29.

29. Coit, D.W., Espiritu, J., Prakash, U. and Ramirez-Marquez J. (2004). Impact of Parameter Uncertainty on Asset Criticality and System Reliability, Hydro One, Inc. Toronto, Ontario. Rutgers University Technical Report 2004.

30. Coit, D.W. and Jin T. (2001). Prioritizing system-reliability prediction improvements. *IEEE Transactions on Reliability*, 50(**1**):17-25.

31. Crites, R. H. and Barto, A. G. (1996). Improving Elevator Performance Using Reinforcement Learning. Advances in Neural Information Processing Systems 8, MIT Press, Cambridge, MA.

32. Dalton, J. G., Garrison, D. L. and Fallon, C. M. (1996). Value-based reliability transmission planning. *IEEE Transactions on Power Systems*, 11(3):1400–1408.

33. Elsayed, E. A. (1996). Reliability Engineering. Reading, MA. Addison Wesley.

34. Endrenyi, J. and Anders G. J. (2006). Aging, maintenance and reliability. *IEEE Power and Energy Magazine*, 4(**3**):53-59.

35. Espiritu, J., Coit D.W., Prakash, U. and Ramirez-Marquez, J. (2005). Reliability Modeling of Electricity Transmission Systems: An Extension of Traditional Reliability Methods. *Proceedings of the Industrial Engineering Research Conference* (IERC), Atlanta, GA, May 2005.

36. Espiritu, J., Coit, D. and Prakash, U. (2007). Component Criticality Importance Measures for the Power Industry. *Electric Power systems Research*, 77(5-6).

37. Espiritu, J. and Coit, D. (2007). Component Replacement Analysis for Electricity Transmission and Distribution Systems with Heterogeneous Assets Subject to Annual Budget Constraints - a Short Communication. *International Journal of Performability Engineering*, 3(2).

38. Fussell, J. (1975). How to calculate system reliability and safety characteristics. *IEEE Transactions on Reliability*, 24(3), 169-174.

39. Gandini, A. (1990). Importance and sensitivity analysis in assessing system reliability. *IEEE Transactions on Reliability*, 39(1):61-70.

40. Gaver, D. P., Montmeat, F. E. and Patton, A. D. (1964). Power system reliability: I-Measures of reliability and methods of calculation. *IEEE Transactions on Power Apparatus Systems*, 83:727-737.

41. Gilardoni, G. L. and Colosimo, E. A. (2007). Optimal maintenance time for repairable systems. *Journal of Quality Technology*, 39(**1**):48-53.

42. Gonen, T. (1986). Electric Power Distribution System Engineering. NewYork: McGraw-Hill, pp. 174–187.

43. Hamoud, G., Chen, R-L., Bradley, I. (2003). Risk assessment of power systems SCADA. *IEEE Power engineering society general meeting,* 2:758-764.

44. Hamoud, G., Lee, L., Toneguzzo, J. and Watt, G. (2004). Assessment of component criticality in customer delivery systems. *8th International Conference on Probabilistic Methods Applied to Power Systems.* Iowa State University, Ames, Iowa.

45. Hartman, J. C. (2000). The parallel replacement problem with demand and capital budgeting constraints. *Naval Research Logistics*, 47:40-55.

46. Hartman, J. C. (2001). A economic replacement model with probabilistic asset utilization. *IIE Transactions*, 33:717-727.

47. Hartman, J. C. and Rogers, J. L. (2006). Dynamic Programming approaches for equipment replacement with continuous and discontinuous technological change. IMA Journal of Management Mathematics, 17(**2**):143-158.

48. Hilber, P. and Bertling, L. (2004). Monetary importance of component reliability in electrical networks for maintenance optimization. *Proceedings, 8th International Conference on Probabilistic Methods Applied to Power Systems, PMAPS 2004.* Ames USA.

49. Hoyland A. and Rausand M. (1994). System Reliability Theory: models and statistical methods. New York. Wiley.

50. Karabakal, M., Lohmann, J. R. and Bean, J. C. (1994). Parallel replacement under capital rationing constraints. *Management Science*, 40(**3**):305-319.

51. Kasztenny, B., Sevov, L. and Jaques, A. (2004). New algorithm for low-impedance restricted earth fault protection. *Eight IEEE International Conference on Developments in Power System Protection*, 1:372-375.

52. Levitin, G., Podofillini, L. and Zio, E. (2003). Generalized importance measures for multi-state elements based on performance level restrictions. *Reliability Engineering and System Safety*, 82:63-73.

53. Li, Z. and Jiachun, G. (2006). Wisdom about age -aging electricity infrastructure-. *IEEE Power and Energy Magazine*, 4(**3**):44-51.

54. Medicherla, T. K. P., Chau, M., Zigmund, R. E. and Chan, K. (1994). Transmission Station Reliability Evaluation. *IEEE Transactions on Power System*, 9(1):295 - 304.

55. Meng F. C. (1995). Some further results on ranking the importance of system elements. *Reliability Engineering and System Safety*, 47:97-101.

56. Meng F. C. (1996). Comparing the importance of system elements by some structural characteristics. *IEEE Transactions on Reliability*, 45(1):59-65.

57. Montmeat, F. E., Patton, A. D., Zemkowski, J. and Cumming, D. J. (1965). Power system reliability II—Applications and a computer program. *IEEE Transactions on Power Apparatus Systems*, 84:636-643.

58. Oakford, R.V., Lohmann J. R. and Salazar, A. (1984). A dynamic replacement economy decision model. IIE Transactions, 16:65–72.

59. Ramakumar, R. (1993). Engineering Reliability: Fundamentals and Applications. Prentice Hall, Englewood Cliffs, New Jersey.

60. Ramamoorty, M. and Gopal B. (1970). Block diagram approach to power system reliability. *IEEE Transactions on Power Apparatus and Systems*, May-June 1970: 802-811.

61. Ramirez-Marquez, J. E. and Coit, D. (2005). Composite importance measures for multi-state systems with multi-state components. *IEEE Transactions on Reliability*. 54(3):517-529.

62. Resnick, I. S. (2002). Adventures in stochastic processes. Birkhauser, third edition.

63. Rogers, J. L. and Hartman, J. C. (2005). Equipment replacement under continuous and discontinuous technological change. *Journal of Management Mathematics*, 16:23-36.

64. Ross, M. S. (2002). A first course in probability. Academic Press, 6th edition, 2002.

65. Ross, M. S. (2003). Introduction to probability models. Academic Press, 8th edition.

66. Sand, K., Seljeseth, H. and Samdal, K. (2004). Quality of supply regulation - status and trends. Nordic distribution and asset management conference.

67. Satish, J. and Billinton, R. (1995). Minimum cost analysis of station configurations. *IEEE Transactions on Power Delivery*. 10(4):1799-1805.

68. Vasseur, D. and Llory, M. (1999). International survey on PSA figures of merit. *Reliability Engineering and System Safety*, 66:261-274.

69. Sethi, S. and Chand S. (1979). Planning horizon procedures for machine replacement models. Management Science, 25:140–151.

70. SGI Federal (2003). Enabling Effective Power Management Solutions for Today's Changing Power Environment. http://www.sgi.com/pdfs/2999.pdf

71. Terborgh G. (1949). Dynamic Equipment Policy: A MAPI Study. McGraw-Hill, New York.

72. Tesauro, G. J. (1992). Practical Issues in Temporal-Difference Learning. Machine Learning, 8.

73. Vojdani, A. F., Williams, R. D., Gambel, W., Li, W. Eng, L. and Suddeth, B. N. (1996). Experience With Application Of Reliability And Value Of Service Analysis In System Planning, *IEEE Transactions on Power Systems*, 11(3):1489-1496.

74. Wagner, H. M. (1975). Principles of operations research. Englewood Cliffs, NJ. Prentice Hall.

75. Wang, P. and Billinton, R. (2002). Reliability cost/worth assessment of distribution systems incorporating time-varying weather conditions and restoration resources. *IEEE Transactions on Power Delivery*, 17:260–265.

76. Warren, C. A., Ammon, R. and Welch, G. (1999). A survey of distribution reliability measurement practices in the U.S. *IEEE Transactions on Power Delivery*, 14(1):250–257.

77. Wendai, W., Loman, J. and Vassiliou, P. (2004). Reliability importance of components in a complex system. *IEEE Proceedings of Annual Reliability and Maintainability Symposium*, Los Angeles California.

78. Wenyuan, Li., Vaahedi, E. and Choudhury, P. (2006). Power system equipment aging. *IEEE Power and Energy Magazine*, 4(**3**):52-58.

79. Williams, C. and Ochoa, R. (1995). Distribution reliability analysis for international drive. Presented at the 1995 IEEE Summer Power Meeting, Portland, OR, July 23–27.

80. Zhang, W., and Dietterich, T. G. (1995). A Reinforcement Learning Approach to Job Shop Scheduling. *Proceedings of the Fourteenth International Joint Conference on Artificial Intelligence*, pp. 1114-1120.

Curriculum Vita

José Francisco Espiritu Nolasco

2007 Rutgers, The State University of New Jersey. New Brunswick, NJ.
January 2003 – July 2007. Ph D. in Industrial & Systems Engineering

2005 Rutgers, The State University of New Jersey. New Brunswick, NJ.
January 2003 – January 2005. MS. in Industrial & Systems Engineering

2002 Instituto Tecnológico de Celaya. Guanajuato, México.
September 2000 – June 2002. MS. in Industrial Engineering

2000 Instituto Tecnológico de Zacatepec. Morelos, México.
September 1994 – January 2000. B.S. in Biochemical Engineering

Publications

2007 Jose F. Espiritu, David W. Coit & Upyukt Prakash. (2007). Component Criticality Importance Measures for the Power Industry. *Electric Power Systems Research*, 77(**5-6**).

2007 Jose F. Espiritu & David W. Coit. (2007). Component Replacement Analysis for Electricity Transmission and Distribution Systems. *International Journal of Performability Engineering,* 3(**2**).

2007 Heidi Taboada, Jose F. Espiritu & David W. Coit. (2007). MOMS-GA: A Multiobjective Multi-State Genetic Algorithm for System Reliability Optimization Design Problems. *IEEE Transactions on Reliability,* (in print).

2007 Heidi Taboada, José Espiritu, David Coit and Gregory Levitin. (2007). A multi-objective evolutionary algorithm for determining optimal configurations of multi-task production systems. *European Journal of Operational Research* (under review).

2007 Jose F. Espiritu and David W. Coit. (2007). Electric Power Systems Reliability & Optimization . *Proceedings of the 5th International Conference on Quality and Reliability* (ICQR). Chiang Mai, Thailand, November 2007 (to appear).

2006 Jose F. Espiritu & David W. Coit. (2006). Customized Component Reliability Importance Measures for Electricity Transmission Systems. In *Proceedings of the Industrial Engineering Research Conference* (IERC). Orlando, FLA.

2005 Jose F. Espiritu, David W. Coit, Upyukt Prakash & Jose Ramirez-Marquez. (2005). Reliability Modeling of Electricity Transmission Systems: An Extension of Traditional Reliability Methods. *Proceedings of the Industrial Engineering Research Conference* (IERC). Atlanta, GA.