TEDDY BEAR OR TOOL: STUDENTS' PERCEPTIONS OF GRAPHING CALCULATOR USE

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ABSTRACT OF THE DISSERTATION

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Graphing calculators are mainstay in the U.S. high school mathematics curriculum and because of that considerable research has been done on the effect of graphing calculators in the math classroom. Until recently most of the research on graphing calculator use in mathematics education has either been quantitative in nature, focusing on student achievement and attitude, or qualitative focusing on the teaching and learning of a particular mathematical topic (Choi-Koh, 2003; Ellington, 2003; Forester & Mueller, 2002; Smith & Shotsberger, 1997, for example). In addition, there is a growing body of research on how students are adapting graphing calculator technology to their mathematical learning (Artigue, 2002; Drijvers, 2000; Guinn and Trouche, 1999). However, none of this work addresses how students use the graphing calculator when they are working in independent situations or their perceptions of how the graphing calculator impacts their mathematical experience. My work aims to attend to this gap in the research.

This dissertation reports on a mixed methods study with data consisting of survey data (n = 111) and in-depth interview data compiled from six case studies. The case study students participated in a task based interview and a stimulated response reflection interview. Particular attention was paid to both the affective and mathematical aspects of graphing calculator use. The data indicates that AP Calculus students value the ability to change the cognitive demand of tasks, the ability to engage in mathematical play, to check their written solutions, and to manage time effectively when doing mathematics. All of the students reported that using the graphing calculator in each of these ways provides them with both a mathematical and affective pay-off. Most surprising is that the ways in which the students value using their graphing calculators to solve problems does not coincide with their perceptions of what it means to 'do math' in a school setting. This result suggests that in the continuing discussion of how and if graphing calculators should be incorporated into school mathematics and assessment it is important to address this inconsistency.

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Dedication

To Ian, my true love and best friend, for making this possible

Table of Contents

List of Tables

List of Figures

Teddy bear or tool: Students' perspectives on graphing calculator use

Chapter 1: Introduction

I was sitting toward the front of the classroom near Mr. P.'s desk while the students were taking their exam. This particular test was designed by Mr. P. and had two parts. The second part, which the students were working on, allowed for the use of a graphing calculator. As I sat quietly at the front of the room I was struck by what I saw. Some students seemed to be working solely on their graphing calculators, writing almost nothing on paper until an answer was reached, while others appeared to be using the tool sparingly. Olivia seemed particularly interested in what appeared on her calculator screen when she whispered a quiet, "Yes!" in response to what she saw. In stark contrast to Olivia was Sarin whose calculator was no where to be seen.

The above vignette described a class that I observed during a pilot study. Olivia and Sarin were two of fourteen students in that class who participated in the pilot study. They had been in the same math class for four years and used a graphing calculator in all of these courses. It struck me that these two students appeared to use their graphing calculators in such different ways even though they shared common classroom experiences. When I spoke to each of them about their graphing calculator use, the differences were even more striking than I first thought. Olivia explained to me that the calculator is helpful because she can use it when she is not sure how to solve a problem. She said, "I'm definitely more comfortable with the calculator only because I know that if I really didn't know the problem at least I can plug things in… Like, I think it is more of a security net kind of thing, I feel more comfortable with the calculator next to me…" Sarin, on the other hand, says the calculator impacts his mathematical experiences in very different ways and that the calculator often complicates things. When he was describing a particular situation he offered, “I got caught up in trying to use the calculator and lost track of what I was doing… Using a calculator is kind of like stressful.”

The experiences that Olivia and Sarin describe in relation to graphing calculator use reveals how little we know about students’ use of the graphing calculator in independent situations. In a classroom interaction, often how the graphing calculator is used is determined by the mathematical discussions taking place among the people in that classroom. However, when students step away from that interaction and are working independently they make their own decisions about how, or even if, they use the graphing calculator. For example, reflecting on the opening vignette one might wonder: How is each student in that classroom actually using the tool? How do they feel about being asked or expected to use it? Does it help them in their problem solving or actually add more stress to the situation? These questions are impossible to determine from a test score or even from observing from afar. In response to these musings, my work aims addresses the following research questions:

- When and why do students say they choose to use the graphing calculator?
- What reasons do students give for choosing to use the graphing calculator in a particular way?
- What do students perceive are the ‘pay-offs’ associated with having a graphing calculator available when working independently?

Literature on graphing calculator use

Most of the research on graphing calculator use in mathematics education has either been quantitative in nature focusing on student achievement and attitude or qualitative studies on the teaching and learning of a particular mathematical topic (Choi-

Koh, 2003; Ellington, 2003; Forester & Mueller, 2002; Smith & Shotsberger, 1997, for example). In addition, there is a rapidly growing body of research on how students come to use graphing calculators while learning mathematics and the obstacles that they often experience (Artigue, 2001; Ball & Stacey, 2004; Doerr & Zangor, 2000; Drijvers, 2000; Guinn & Trouche, 1999; Graham, et al., 2003). However, none of this work addresses how students use the graphing calculator when they are working in independent situations or how they perceive the graphing calculator impacts their mathematical experience. I briefly describe these studies and how they frame my study in the following sections.

Research on the impact of graphing calculator use on student achievement and attitude. A majority of the research on graphing calculator use has been comparative and quantitative in nature, focusing on measures of student achievement and attitude toward mathematics (Ellington, 2003). Ellington's meta-analysis of calculator-based studies points out that most of calculator-use research studies over the last 30 years have been of this type. Her meta-analysis of 54 research studies used meta-analytical techniques outlined by Lipsey and Wilson (2001). Ellington included all studies on calculator use in mainstream K – 12 classrooms, with complete findings provided published between January 1983 and March 2002 in this analysis. The meta-analysis determined that within these studies the following holds true:

- When calculators were included in instruction but not testing, the ability to select appropriate problem-solving strategies improved;
- When calculators were included in both instruction and testing, problem – solving skills improved; and

- Students who used calculators while learning mathematics report more positive attitudes towards mathematics than students who did not use them on a regular basis.

These findings were conclusive for all types of calculators (four function, scientific, and graphing). When attitude was broken down by calculator type, graphing calculators had a more significant influence on attitude compared with other types of calculators.

To provide a better understanding of the types of studies that Ellington included in her meta-analysis I will briefly discuss a few of the most commonly cited studies of this type. One area of mathematics that has been dramatically affected by the introduction of the graphing calculator is the study of functions and their graphs. As a result, most achievement studies take place in algebra, precalculus, or calculus settings. Most of these studies involve a comparison of traditionally taught students to those that used the graphing calculator on common, traditional, exams either with or without the availability of the calculator (Ruthven, 1990; Smith, et. al., 1997; Quesada & Maxwell, 1994 for example). Ruthven (1990) looked at upper secondary mathematics classes in four schools. In each of the four schools there were parallel classes with the only difference being the availability of a graphing calculator. He found that the graphing calculator groups outperformed the traditional group on items requiring students to examine certain graphs and describe them algebraically on common exams for which graphing calculator use was not permitted. There was no difference in the other test items. Thus, he concluded that the graphing calculator was an important influence on these students learning.

Quesada and Maxwell (1994) conducted a similar study at the collegiate precalculus level. Seven hundred and ten students over three semesters were enrolled in one of two precalculus courses. The experimental treatment consisted of the use of a graphing calculator in conjunction with a textbook that was written to be used with a graphing utility. The control groups were taught in more traditional mode and were permitted to use a scientific calculator. At the end of each semester a common final exam was administered. It was found that the experimental group scored on average 14.21 points higher on the final exam than the control group. However, the authors were careful to point out that the improvement of the experimental group may have been due to the different textbook, the more interactive presentation of topics, or even possibly the immediate feedback when checking solutions. Though it is unclear what aspect of the experiment caused these students improvement, the authors do conclude that participating in a graphing calculator enriched curriculum was an important contributor to these students significant improvement.

Another noteworthy comparative study was conducted by Smith and Shotsberger (1997). They did a quantitative study involving 114 students in four classes. The classes were taught by two instructors. Each of the instructors taught both a section with the graphing calculator using a technology intensive curriculum as well as a traditional curriculum in which no graphing calculator was allowed. The achievement means on a common exam were consistently higher for both of the calculator sections. The authors concluded that the graphing calculators did not hinder the students' learning of mathematics.

These studies are just three examples of such studies on achievement, it is generally believed that the use of graphing calculators does not hurt any student and may in fact help quite a few (Dunham, 2000).

Research on how graphing calculators are used to mediate learning. Though the literature over the last 30 years provides some evidence that there might be a correlation between graphing calculator use and achievement at least for some groups of students, there is very little research that describes how it mediates learning. The few studies that exist are qualitative in nature and aim to describe how the graphing calculator impacts the learning and teaching of particular topics in addition to how students come to know how to use them.

Choi-Koh (2003) conducted a case study of one 10^{th} grade student's study of trigonometry by engaging in graphing calculator based activities. The study took place over seven weeks, with the student working on one specially designed task per week. The tasks all involved building an understanding of the role of the coefficients in general trigonometric functions such as "The role of a in $y = a\sin(bx + c) + d$". The data consisted of observations of the instructional tasks and clinical interviews.

Choi-Koh found that this particular student was able to use the graphing calculator to work through the given tasks and develop an understanding of the role of the coefficients for the trigonometric function families. Furthermore, the student showed evidence of moving through all of the stages of Bloom's taxonomy (Bloom, 1956). Interestingly, the graphing calculator was most helpful to this student in the lower stages of Bloom's taxonomy, specifically the comprehension, application, and analysis stages.

Once the student moved to stages that were identified as the synthesis and evaluation stage he was working more often with paper and pencil.

This study was very limited in its scope, but it did provide strong evidence that the graphing calculator might be very helpful to some students in the early stages of Bloom's taxonomy with respect to understanding graphs of function families. Choi-Koh provided sufficient evidence for how it was determined that the student had reached the different stages in Bloom's taxonomy. However, there was very little description of what the student actually did on the graphing calculator over this seven week period, what the student was prompted to do or chose to do on his own, or even how the student perceived the calculator impacted his learning of this topic.

Graham and Thomas (2000) conducted a classroom-based study using an experimental design on a specific intervention designed to help students understand the idea of variable. "Tapping into Algebra" was an author designed module the goal of which was to use the graphing calculator's lettered stores as a model of a variable. One hundred and forty-seven experimental students and 42 control students took part in the study. The experimental group used the Tapping into Algebra module. The control group received corresponding algebra work but was taught using their normal algebra teaching program.

The experimental students were asked to engage in activities that required them to assign values to letters using the graphing calculator's store function, change those values, and to operate on the letters. One of the more novel aspects of the module was the use of 'screensnaps'. Students were given a screen view and asked to reproduce it on their calculator screen. The students would do so by assigning various values to the variables and predicting their outcomes. The students were provided a graphing calculator and generally worked collaboratively in pairs or small groups throughout the three week module.

Students in both the experimental and control groups were given a pre-test and a post-test comprising of 68 questions. Three of the experimental classes were matched with three control classes at their same schools. The results show that in the two schools with control groups the experimental groups performed significantly better on the post-test compared to no difference at the pre-test stage. The authors conclude that students can obtain improved understanding of the use of letters as variables by working through the Tapping into Algebra module. The report of this study also included some commentary by both the teachers and students who participated in this project. The students report that using the graphing calculators was “fun” and that others would benefit form using them as well. This study provided an excellent description of a specific intervention that appears to improve performance on an important algebraic idea, but it does not begin to explain the students’ experiences with that intervention or with the technology itself.

There is a growing body of research on how students are adapting graphing calculator technology to their mathematical learning. This research is focused on understanding the process that students go through as they learn how to use the technology itself and then how they learn to use it in support of their mathematical learning (Artigue, 2002; Drijvers, 2000; Guin and Trouche, 1999). Most of these studies have looked specifically at graphing calculators with computer algebra system (CAS) capabilities[1].

Guin and Trouche (1999) suggest that the interaction between a student and an artifact, like a graphing calculator, is necessary before the artifact becomes an instrument for the student. They call this interaction between a student's knowledge and the potential and constraints of an artifact *instrumental genesis*. Artigue (2002) reports on a study focused on the instrumental genesis of the TI-92 calculator. Defouad, the researcher on this study, identified several phases in the instrumentation process for a small group of students in a grade 11 mathematics course in France.[2] These phases were defined by the connections that the students were making between the graphical, numerical, and symbolic registers of the calculator. In the context of examining variation in functions, he found that there were three phases that students go through. First, the students are strongly attached to the graphical representation and to a lesser extent, tables of numerical values. In this phase, students considered a graph of a function and its derivative as the main tools for conjecturing and verification. Once the student hits the second or 'intermediate phase' the graphical representation still plays an important role, but HOME (the home screen, where CAS is used) is beginning to emerge as a tool. The connections between the graphic and the symbolic representations are beginning to develop. The final phase is referred to as the 'calculus phase', the student has "developed specific and efficient instrumented schemes for framing and variation analysis, by connection the symbolic and graphical applications of the calculator" (Artigue, 2002, p. 258). These phases are not necessarily linear. A student who faced an unfamiliar

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[1] Computer algebra systems (CAS) will be described in full in chapter 2.
[2] The specific number of students was not reported.

function would often revert back to the first phase and then quickly move through the stages until understanding was reached. Though the students in the study did reach a point in which they were consistently making connections between representations and moving back and forth between them with ease, the process was slow and circuitous.

Artigue and Defouad's comments on the instrumentation process for students studying variation provide a framework for documenting how students might progress through the learning of a new tool while learning mathematics. In addition, this study provides evidence that students use the technology in different ways as they become more familiar with both the calculator and the mathematics. Unfortunately, this study was presented as an overview by Artigue in a reflection paper that she wrote about the instrumentation process and she did not provide much information about the design of the study. Thus, its impact is limited.

The goal of work by Artigue, Guin and Trouche, and Defouad is to understand how students come to be proficient users of CAS graphing calculators and came to use them to mediate their mathematical learning. These studies did not take into account the role of the teacher in creating the meaning for the tool and how it might be used to build mathematical understanding. Drijvers (2000), who has reported on common obstacles that students encounter when they are using CAS graphing calculators, has suggested that a carefully designed instrumental genesis is worth considering. He points out that if the instrumental genesis was carefully planned out by the teacher then obstacles might be more easily overcome, the process might happen quicker, and students might be better informed about when using the technology will be helpful to them. This group of studies has begun to describe in great detail the stages that students go through as they learn to use a tool like the graphing calculator while learning mathematics and how to best foster this learning process. They have made clear the fact that instrumentation is an ongoing process that shifts between stages as new mathematics and new aspects of the tool are encountered. However, none of these studies have considered how students experience this instrumentation process. For example, how or why students choose to use particular modes of the graphing calculator when they are working on a challenging problem. What students perceive they gain from these decisions might impact the ways that they choose to use it.

Doerr and Zangor (2000) conducted a qualitative classroom-based study aimed at describing how students come to decide how to adapt graphing calculators as tools in the context of learning about linear, quadratic, exponential, and logarithmic functions. More specifically, the study was designed to reveal how a precalculus class and their teacher co-constructed the meaning of the graphing calculator in the context of their class and how this particular tool was used to construct mathematical meaning. It is the first study in which the notion of the graphing calculator as a tool for which a shared upon role must be created in the classroom is explored.

This study described how one teacher's knowledge and beliefs about graphing calculator were reflected in her pedagogical strategies and how the teacher along with her students created meaning for the graphing calculator. The study was set in two pre-calculus classes taught by the same teacher. The classes were observed over three units of study over a 21 week period. The units of study included: (1) linear functions, (2) exponential functions, and (3) trigonometric functions. The data consisted of field notes from observers, transcriptions of audio-taped group work, transcriptions of video taped whole class discussions, and interviews and planning session with the teacher. The data was analyzed in two phases. The purpose of the first phase was to identify the modes in which both the students and the teacher used the graphing calculator. This initial coding was then used to develop a profile of the teacher. The intention of the second phase of analysis was to refine the teacher profile by re-examining the data.

The teacher was found to have very clear beliefs about how the calculator should be used as a pedagogical tool and as a problem solving tool. These beliefs impacted ways in which the graphing calculator was typically used in both whole class discussions and in small group work. The researchers reported that this particular class used the graphing calculator in five different modes: (1) computational tool, (2) transformational tool, (3) data collection and analysis tool, (4) visualizing tool, and (5) checking tool (see table 1). The authors carefully described the use of the graphing calculator in these five ways as well as any complications that typically arose with each mode of use. One of the most interesting revelations was that the students often used regression equations to solve problems, but these solutions were not valued by most students because of the teacher's consistent insistence of meaning making. Thus, the teacher's beliefs impacted the students' use of the graphing calculator and encouraged interpretation and exploration with the tool.

Table 1: Doerr and Zangor (2000), patterns and modes of graphing calculator use

Role of the graphing calculator	Description of student actions
Computational tool	Evaluating numerical expressions, estimating and rounding
Transformational tool	Changing the nature of the task
Data collection and analysis tool	Gathering data, controlling phenomena, finding patterns
Visualizing tool	Finding symbolic functions, displaying data, interpreting data, solving equations

Checking tool	Confirming conjectures, understanding multiple symbolic forms

Doerr and Zangor's description of this teacher and her class provides an excellent basis for beginning to think about how students and teachers work together to create the norms for how a tool, like the graphing calculator, is used for creating mathematical meaning. Though the patterns of student graphing calculator use are well defined and described, it is unclear whether or not these categories are intended to be discrete. The mode that I envision overlaps many others is the use of the graphing calculator as a checking tool. For example, I imagine students using the graphing calculator as a visualizing tool in order to confirm a conjecture. The modes of use are of genuine interest to me. I am particularly interested in how these modes of graphing calculator use play out in different settings. For example, if the teacher were not as knowledgeable about the graphing calculator would the same classroom norms come to be? Furthermore, do these students continue to work within this co-constructed meaning of the calculator as a tool when they are doing mathematics independently? These are items that will be addressed in future research.

The literature has addressed the impact of graphing calculator use on achievement and attitude. In addition, it has begun to describe how students and teachers use graphing calculators in tutoring or classroom situations to mediate learning either as a new tool or as an established tool. However, none of the research addresses how or why students choose to use graphing calculators in independent situations. In addition, none of the graphing calculator literature has focused on the students' voices and their perceptions of graphing calculator use. As we learn more about the effects that the adoption of graphing calculator technology seems to be having on the assessment, attitude, and learning of particular topics it becomes apparent that we need to know more about what students are actually doing with their calculators when they work independently and how that use impacts their mathematical experience.

Setting the stage

This dissertation reports on an empirical study designed to investigate students' experiences with graphing calculator use in independent situations. The idea to focus on the student's voice is fairly new. As identified above, previous studies have focused on test scores (Ellington, 2000; Quesada & Maxwell, 1996; Ruthven, 1990; Smith et al., 1997, for example), observing student actions (Artigue, 2002; Choi-Koh, 2003; Drijvers, 2000, for example) and even teacher-student interactions (Artigue, 2002; Doerr & Zangor, 2000; Drijvers, 2000, for example), but few, if any, have focused on the students' own voice regarding perceptions of graphing calculator use. In order to 'give voice' to the students, their experiences with the graphing calculator and their reflections on those experiences will be described in detail. If you listen closely to the students voices you hear four different things: that they value the graphing calculator for changing the cognitive demand of a task, engaging in playful mathematical activities, checking written work, and managing time when doing mathematics. These categories have been under described in the literature, in part because of methodological approaches chosen, and in part because they are hidden parts of the mathematical process. This thesis looks between the lines and between the keystrokes of students' experiences with graphing calculators and provides some insight into how they perceive the graphing calculator impacts their experiences.

In the analysis of the data for this study particular attention was paid to both the students' mathematical and affective experiences related to their graphing calculator use. From a mathematics perspective I am concentrating on how students use the graphing calculator and why. It ends up that for these students often the graphing calculator provides a means for changing the cognitive demand of a problem and in turn allows for more playful mathematical activity, something that has not been explored in the graphing calculator literature. In the data chapters I will carefully define and describe both cognitive demand and mathematical play as they relate to this study.

Unlike describing mathematical experiences, the idea of looking at affect is relatively new and thus warrants a bit of clarification. With respect to affect, I focused specifically on the notion of *local* affect. Local affect is defined as "the rapidly changing (and possibly very subtle) states of feeling that occur during problem solving – emotional states, with all their nuances" (Goldin, 2000, p. 210) Pilot studies indicated that students sometimes have very strong feelings not only about the mathematics that they are engaging with, but often about graphing calculator use. These feelings might be related to simply whether or not a graphing calculator is available or possibly related to a solution that they have produced using the calculator. Either way, these feelings are sometimes intense and seem to impact their mathematical experiences.

Goldin (2000) has pointed out that students use emotions to provide useful information, to facilitate monitoring and to evoke heuristic processes. He suggests that affect is not inessential, but critical to the structure of competencies that account for success or failure in problem solving. An example of an affective pathway follows:

> In an (idealized) model, the initial feelings are of curiosity. If the problem has significant depth for the solver, a sense of puzzlement will follow, as it proves impossible to satisfy the curiosity quickly. Puzzlement does not in itself have unpleasant overtones – but bewilderment, the next state in the sequence, may. The latter can include disorientation, a sense of having "lost the thread of the argument" of being "at sea" in the problem…If independent problem solving continues, a lack of perceived progress may result in frustration, where the negative affect becomes more powerful and more intrusive. This is associated with the occurrence of an impasse. However, there is still the possibility that a new approach will move the solver back to the sequence of predominately positive affect. Encouragement can be followed by pleasure as the problem begins to yield, by elation as major insights occur, and by satisfaction with the sense of a problem well solved and with learning that has occurred (p. 211).

This idealized model illustrates how local affect might influence the heuristics employed by a problem solver. In the context of this study it is important to consider how the availability of a tool like the graphing calculator might further influence an affective pathway like the one described above. For example, if a student is facing feelings of bewilderment or disorientation it is possible that the introduction of a useful tool might invoke feelings that are of a more positive sequence. Previous studies on graphing calculator use that have addressed affect have only considered how using the graphing calculator might impact students' attitudes toward mathematics in a global sense (Dunham, 2000; Ellington, 2000) none have addressed affect from a local perspective.

Organization of this paper

The details of this study will be presented in six chapters. The specifics of the design will be presented in chapter two. The findings are arranged in four chapters. Each will present the data on one of the four categories in which the students' perceive acting with the graphing calculator impacts their mathematical experiences: changing the cognitive demand of a task, engaging in mathematical play, checking written work, and managing time when doing mathematics. The data chapters will be followed by a discussion of the limitations of this study, how this data answers the research questions, and how the findings may inform future research and practice.

Chapter 2: Methods

To understand a broad range of ways that calculus students perceive the graphing calculator impacts their mathematical experiences when they are working independently both qualitative and quantitative research methods were used. Initially a survey was used to identify the range of ways that students report that they use graphing calculators. Next, case study methods enabled an in-depth understanding of the participant perspective on the ways the graphing calculator is used and the influences that it can have on one's mathematical experience.

The choice of design methods for this study was influenced by a series of three pilot studies that I conducted in the spring of 2004, fall of 2005 and spring of 2005. The pilot studies all took place in the Advanced Placement Calculus AB class of an upper class suburban high school in the northeast United States. Three lessons that were learned from these experiences influenced the design of this study. First, students with different math backgrounds appear to have different perceptions of the usefulness of the graphing calculator. Therefore the settings for this study were chosen carefully to increase the possibility of identifying unique individual graphing calculator stories. Second, students have many different purposes for actually using the graphing calculator. Third, task selection is very important to the development of meaningful conversations about graphing calculator use. The tasks selected for the in-depth portion of this study were chosen based on the responses the students provided in the pilot studies. These tasks will be described in detail in this chapter.

Data for this study was collected in two phases. First, a survey was used to identify six unique individuals with respect to their graphing calculator use. Second, a series of two interviews with each of the individuals chosen from the survey was used to construct six case studies of individual graphing calculator use. The methods used in the two phases of this study are explained in detail in the following sections.

Setting and Sample

Students in AP Calculus in the United States vary widely in many aspects including their socioeconomic status, ethnicity, and the way that graphing calculators are used in the classroom. AP Calculus classes were chosen as the focus for this study because the curriculum and expectation of calculator use is relatively consistent nationwide since it is set by The College Board. To ensure that the population of students was as diverse as possible this study was set in four high schools in the northeastern United States. These high schools were purposefully selected based on access, presence of an AP Calculus program that uses graphing calculator technology, and their ethnic and socio-economic status (SES) make-up.

High School A. High School A is located in a low-income urban community. It serves approximately 2000 students in grades 9 through 12. The student population is 73% African American and 26% Hispanic with 62% percent of the students participating in a free or reduced lunch program.

High School B. High School B serves approximately 2800 students in grades 9 through 12 from both suburban and rural communities. The student population is 96% White. Approximately 3% of the students at High School B participate in a free or reduced lunch program.

High School C. High School C serves approximately 1100 students in grades 9 through 12 in an affluent suburban community. The student population is 76% white and 20% Asian. Less than 1% of the students participate in a free or reduced lunch program.

High School D. High School D serves approximately 1700 students in grades 9 through 12 in a middle class suburban community. The student population is 44% white, 23% black, 17% Asian, and 16% Hispanic. Less than 10% of the students participate in a free or reduced lunch program.

All four of these schools have provided the AP Calculus students a graphing calculator to use at home and at school. High schools A, B, and D provide their students with a TI-83+, while high school C provides the TI-89 (which has Computer Algebra System, or CAS, capabilities). Students may be using the calculator provided by the school or possibly their own personal calculator. For the purposes of this study, the term graphing calculator refers to both calculators with and without CAS capabilities. Any time student calculator use is described, the specific calculator in use will be identified.

The students at these four schools participated in a survey and then a small subset was chosen for case studies. The data collection process for both of these phases is described below.

Data Collection

As described above, the data collection process was completed in two phases. First a survey was used to detect the differences in ways students perceive they use the graphing calculator. Second, from this survey six students were chosen to be the focus of six individual case studies. The following sections describe the data collection process in detail.

Survey Data. The first phase of this study used survey data to understand how AP Calculus students in high schools A, B, C, and D perceive that they use the graphing calculator. This data was used to identify six students to participate in the second phase of the study.

The pilot studies done in 2004 and 2005 revealed that students use the graphing calculator as a tool in four different ways: as a numerical computational tool, as a symbolic computational tool, as a visualization tool, and as a data collection and analysis tool. Pilot studies also revealed that students are prompted to use their graphing calculators for different reasons. These findings were used to develop the survey instrument for this study (see Appendix A). In particular, the survey provided data on student demographics, mathematical achievement, frequency of graphing calculator use in school, modes of graphing calculator use, and triggers for graphing calculator use.

The collection of survey data took place over a three week period. I visited each school twice. The first visit was used to introduce the study and distribute consent and assent forms, the second to collect the signed forms and have the students complete the survey. All surveys were photo copied and data entered into SPSS the same day that the data was collected. The surveys were then filed, organized by school, in a locked file cabinet.

Participants. Every student in every AP Calculus class at high schools A, B,C, and D was asked to participate in the survey phase of this study (n = 111). From this rich pool of data six students that were that differed from one another in their description of their calculator use were asked to participate in an in-depth interview.

As anticipated, the survey data indicated that there are some differences between weaker and stronger students, as rated by their teacher, and their reported practices pertaining to graphing calculator use. Most significant was that stronger students report preferring to use a graphing calculator on a test as opposed to working with a student of their choice while the opposite is true for weaker students ($\chi^2(2, n = 109) = 9.978, p = .007$). Based on this data I chose six students that were representative of the extremes of the responses in these categories while also being representative of the types of schools participating in the study. These students are introduced below and in Table 2.

Aaron is a white male that attends a very wealthy suburban high school. Aaron's teacher considers him to be an average calculus student compared to his peers. His mathematics grades throughout high school indicate that he has been very successful in his course work. Aaron says that he is confident in his calculus abilities, but does not like when he is expected to use a graphing calculator. Graphing calculators have been integrated into the curriculum of all of his high school math classes, but he admits that he has not taken the time to learn how to use it.

Enoch is a black male that attends a traditionally lower income urban high school. He is considered to be one of the strongest math students at his school. However, Enoch is not very confident in his calculus abilities and says that he relies heavily on the graphing calculator. He explained that his previous coursework did not prepare him for calculus and the graphing calculator helps him with skills that he should have perfected in those courses.

Maryanne is a white female that attends a large rural county high school. Her calculus teacher identified her as one of his strongest calculus students. She is the only junior in this high school of over 2500 students that is taking calculus. Maryanne's parents are both scientists and have influenced her views on graphing calculator use significantly. She feels strongly that students should not rely on the graphing calculator. Maryanne explained that she rarely uses her graphing calculator because it is very important for her to be able to do everything without one if she is going to be successful in college.

Melissa is a white female that attends a very wealthy suburban high school. She was identified by her teacher as one of the weaker calculus students at her school. Melissa is not confident in her calculus abilities and says that she relies heavily on her graphing calculator. It is important to point out that though she is weak compared to the calculus students at her school; she is not a weak student. This was evident by her AP exam score, which she shared with me through an informal conversation, and in her ability to conceptualize each of the four tasks in the study Melissa uses her calculator to graph and check quite often, but does not use the CAS capabilities because she likes to do the manipulations on her own. Though she is not very confident in her abilities, she has shown that she is a capable calculus student.

Rudy is a black male that attends a traditionally lower income large urban high school. He is one of only six students enrolled in calculus at his school. However, he is not a very strong math student. Rudy is very aware that his less than proficient algebra skills are making learning calculus difficult. He points out that his high school did not offer traditional algebra and geometry courses and he feels that the courses that were offered did not prepare him well for upper level mathematics. Rudy uses his graphing calculator often and in very creative ways. He says that he would not have been able to do any calculus without it.

Shemika is a black female who attends a traditionally lower income large urban high school. Her calculus teacher says that she is by far his strongest student. Shemika is very busy both in school and after school and feels that the graphing calculator is a necessary tool given her other commitments. Shemika is very knowledgeable about how the graphing calculator actually works its strengths and limitations. She uses it very precisely, which is possible because of her deep understanding of the mathematics.

Table 2: Students chosen for case studies

	Gender	Teacher Rated Ability	School	G.C. Type	Preference to Work With (graphing calculator or a known student)
Aaron	M	average	C	TI-89	Graphing calculator
Enoch	M	strong	A	TI-83+	Graphing calculator
Maryanne	F	strong	B	TI-83+	Graphing calculator
Melissa	F	weak	C	TI-89	Another student
Rudy	M	weak	A	TI-83+	Another student
Shemika	F	strong	A	TI-83+	Graphing calculator

Interviews. To better understand the findings in the survey data the case study students were each asked to participate in two interviews. The first interview was a task-based interview. The second was a stimulated response reflection interview. Each of these interviews will be described in detail below.

The interview guides for both interviews are based on a revised protocol I used in a pilot study conducted in the spring of 2005. The purpose of the task-based interview was to put the student into a situation in which the graphing calculator may be used. This interview provided initial insight into the students' perceptions of graphing calculator use in mathematics and how the students' actually use the graphing calculator. This interview was both audio and video taped. In addition, video of the students' calculator screen was also collected.

In order to capture video of the graphing calculator screen the TI Presenter was used. The TI Presenter is a USB video capture card and video editing software made by Texas Instruments. The TI Presenter video interface is designed to allow any TI calculator to connect to the video input of a conventional TV, VCR, or projector. Using a USB video capture card, such as the Pinnacle 500, the TI Presenter video interface can be fed directly into the video editing software of any PC. Once all connections are complete, video capture may begin when the graphing calculator is in use.

When conducting task-based interviews, students were provided with the graphing calculator that they use regularly, for example the TI-83+ or the TI-89. A standard video camera on a tripod was used to capture students as they work. It was important that this camera picked up the students gestures when the graphing calculator was being used as well as capture the student's written work. At the same time the TI Presenter and video editing software was capturing all of the graphing calculator work.

The final step was to put these two pieces of video together side-by-side on a single PowerPoint slide. When running the slideshow both videos can be activated and thus providing a real time recreation of the written and calculator work. This side-by-side video slide was then shown to students. As students viewed the video slide, they are asked to recall their work using the video as a stimulus. This type of stimulated response interview provided student reflections on the use of the graphing calculator, what they attended to on the calculator screen, and what they gained from its use.

The tasks used in the task-based interviews were all designed so that they could be solved using mathematics that is typically taught before the second semester of AP Calculus AB, when the students were taking part in this study. However, they were not necessarily familiar problem types. The hope was that these tasks would challenge the students and require them to use problem solving methods that were not used regularly in their calculus courses. This work would then serve as a stimulus for a conversation about their perceptions of how the graphing calculator impacts their mathematical experience. The four tasks are described and possible non-calculator solutions presented in Table 3.

Table 3: Description of the tasks

The Task	Characteristics of the Task	Possible Solution
Task #1: Find a rational function that satisfies the given conditions: a. It has a vertical asymptote at x = 3 b. It has a horizontal asymptote at y = -2	The student must have a concept of what horizontal and vertical asymptotes are.	Given that the solution is a rational function such that $f(3) \notin \Re$ and $\lim_{x \to \pm\infty} f(x) = -2$, one possible solution is $f(x) = \frac{-2x}{x-3}$.
Task #2: Find the maximum rate of change of the graph of $y = -x^3 + 3x^2 + 9x - 27$.	The student must first recognize that they are being asked to find maximum slope of the first derivative and then have a method for finding that solution.	The task asks for the maximum rate of change of the derivative, $y' = -3x^2 + 6x + 9$. Using the first and second derivative test on could determine that the maximum rate of change of $y = -x^3 + 3x^2 + 9x - 27$ is 12 and occurs at $x = 1$.
Task #3: For what values of x is $-2 < \|k - x\| < 5$.	The student must make sense of the compound inequality and consider the influence of different values of *k* on the solution set for *x*.	The absolute value will always be greater than -2, so that leaves $-5 < k - x < 5$ to solve. The solution is $k - 5 < x < k + 5$.
Task #4: Give an example of a function for	The student must decode the equality and consider	Possible solutions include

which $\lvert f(x)\rvert = f(\lvert x\rvert)$.	functions that would make the statement true.	$f(x) = x^2$ and $f(x) = \cos x + 1$

Within three days after the task-based interview was complete I had transcribed the interview and had identified questions for the follow-up interview. In addition, I had constructed the PowerPoint slide show with video clips of both the students' calculator work and the student working on paper for each of the tasks a calculator was used on as described above. The use of this slide during the reflection interview allowed for the student to simultaneously view the written work and calculator work in real-time for each task.

The purpose of the reflection interview was to use the students' experiences in the task-based interview as a basis for discussion about graphing calculator use. Video clips, of both written and calculator work, from each task were used to prompt the students to reflect on the tasks and the role of the graphing calculator in their solution strategies. Students were asked to reflect on how they used the graphing calculator, why they chose to use the graphing calculator, and whether they could have completed the task without the graphing calculator.

All interview data was transcribed verbatim. The student work on the tasks appears in Appendix B. Participants were invited to review and edit the transcription to ensure that the information is accurate. This process, called member checking, adds to the validity of the study (Creswell, 1998). No changes in the transcript were requested by the participants. For each student both a computer and a hardcopy file have been kept. Each of the participants completed file includes: complete transcriptions of interviews, video data, audio data, cleaned up field notes, student work artifacts, and the students' survey document.

Data Analysis

Data analysis for this study took place in three phases. The first phase was an analysis of the quantitative data on the survey. The second phase was the analysis of the interview data which included constructing case studies of each of the six students. The third phase was the analysis of the qualitative survey data. Throughout this process I was constantly comparing the data types to one another. Each phase of analysis will be described in detail below.

Phase one: Quantitative survey data. All responses from the quantitative items on the survey were entered into a SPSS file. For the descriptive data frequencies were calculated and all meaningful correlations were analyzed using the Pearson Chi Square statistic. These statistics were used to choose the case study participants and to describe how these cases fit into the larger sample.

Phase Two: Interviews. The task-based interview and the reflection interview were analyzed using both a deductive and an inductive coding scheme. The purpose of the initial coding was to get a feel for the data and to identify how the students used their calculators on these particular tasks and what triggered them to do so. The codes for calculator use as a tool were adapted from a study on graphing calculator use in the context of the classroom (Doerr & Zangor, 2001). The codes for triggers were developed during my pilot studies (McCulloch, 2005). Descriptions and examples of the deductive coding scheme are provided in Table 4.

Table 4: Deductive code description

Code	Description	Example	Reason for code
Calculator use as a tool			
NC	*Numerical computational tool*: evaluating numerical expressions, estimating, rounding.	Melissa used entered $2\text{^}2+3\cdot 2+1$ on the homesreen of her g.c. to evaluate the function $y=x^2+3x+1$ at $x=2$.	She used the g.c. to compete a numerical computation.
SC	*Symbolic computational tool*: using the CAS capabilities of the g.c.	Aaron used the derive function, *derive*(*x*^2+3*x*+2, *x*, *x*) to find the derivative of the function $y=x^2+3x+2$.	He used the CAS capabilities of the g.c. to find the derivative.
DA	*Data collection and analysis tool*: gathering data, finding patterns	Rudy entered different integral values for k in $y1=\lvert k-x\rvert$ on the *Y*= screen. H then scrolled up and down the table where he determined that there would always be 6 integer solutions for the inequality $-2<\lvert k-x\rvert<5$.	Rudy used the table of values created for the function that he entered to find a pattern in the data.
V	*Visualization tool*: displaying a function graph, using a graph to solve a problem	Shemika used the math 8 command on the TI-83+ to find the graph of the derivative of a function.	She produced a graph and then proceeded to use it to find a solution.
Triggers			
SS	*Skipping a step*: the student knows what needs to be done, but does not know how to do it by hand.	Melissa: "Sometimes I use it for, like, regular calculus that I sometimes forget. Like, I don't know, derivatives or integrals or something."	Melissa is talking about using the calculator to do something that she doesn't remember how to do by hand, but knows needs to be done.
GO	*Getting oriented*: the student uses the g.c. as a starting point with the purpose of getting a feel for the task.	Maryanne read a task and then graphed the function in the task on her g.c.. When asked about this she said, "Um, I just, I like to see what the function looks like to begin with."	Maryanne looked at the graph of the function to get a feel for what she was dealing with.
ST	*Saving time*: the student knew how to complete the task, but chose to use the g.c. to save time.	Enoch: "So instead of doing the numbers and thinking about it, I knew it was gong to take forever so I just plugged it into the calculator, graphed it and got the maximum."	Enoch states that he recognized what needed to be done, but chose to use the g.c. to save time.
NA	*New Approach*: the student was having difficulty with the task and chose to use the g.c. to try another approach.	Rudy: "First I did it in my head, I worked that out and it didn't work right…and then I tried to see if I plugged in x what would y be on the table."	Rudy is talking about trying part of the task in his head. He stated that this approach didn't work so then he decided to use the table on his g.c. to try the task a new way.

C	*Checking*: the student is using the g.c. to confirm results or conjectures, understanding multiple symbolic forms.	Melissa: "I just wanted to see if I like moved it correctly. Sometimes I forget the formula, so I just wanted to see if I moved it correctly, that's why I did it on the calculator."	Melissa explains that she graphed a function on her g.c. to check and see if she had done a translation correctly on her paper.

Once deductive coding was complete the data was put back together and reread looking for emerging codes relating to graphing calculator use and the problem solving experience. Through the initial deductive coding and this rereading it became evident that the students' perceptions of the impact that the graphing calculator had on their mathematical experience fell into four broad categories or actions: the ability to change the cognitive demand of the task, the ability to engage in mathematical play, the ability to check their work, and the ability to manage time how time is spent on mathematical tasks. These categories will be defined in later chapters.

The four categories in which students indicated that the use of the graphing calculator impacted their mathematical experiences were broken into some more specific codes. These codes initially included both positive and negative aspects within each category to make sure that both possibilities were considered in the data. For example, in addition to assigning codes for situations in which students stated they felt more secure, I also included codes for situations in which students felt less secure. However, if after carefully coding the data no negative statements were identified the code was then deleted from the coding scheme. The final set of inductive codes along with examples of each is presented in Table 5.

Table 5: Inductive code description

Code	Description	Example	Reason for code
Impact on mathematical security			
S1	Statement of feeling more secure because of the availability of the g.c. to check work.	Aaron: "I think I could have gotten to and equally good answer if I didn't have a graphing calculator. With the calculator I know this is right. The other way I probably would have spent a lot more time on it."	Aaron points out that he is confident and didn't need to spend more time on the problem because he is able to check his solution on the g.c.
S2	Statement of feeling more secure because of the availability of the g.c. to look up information.	Melissa: "Sometimes I go to the calculator when I'm not sure about something. Like…arcsine and stuff like that."	Melissa points out that she can use her calculator when she forgets a fact and this relieves her feelings of uncertainty.
S3	Statement of feeling more secure because the g.c. is available to use.	Enoch: "It is important that its there just so I know, like, if I need it, ok, it's right there."	Enoch points out that it makes him more comfortable to know that the g.c. is there just in case he needs it.
S4	Statement of feeling more secure because the g.c. is always correct.	Rudy: "I retry the problem…I mean, I check it on there (pointing to the g.c.)…then if it is wrong I retry it until I get the answer in there (pointing at the g.c.)."	Rudy explains that he checks his work and if his work disagrees with the g.c. solution he will redo his work until it matches the solution he got on the g.c.
S5	Statement of feeling less secure because of the availability of the g.c.	Maryanne: "I'm always afraid that something might happen or maybe I'll have it in the wrong mode and I'll get all wrong answers."	Maryanne points out that sometimes she feels less secure when she uses a g.c. because she might input something incorrectly.
ST1	Student states that the g.c. saves time in testing situations.	Enoch: "In a test…if it is coming down to the last minute then I run to the calculator."	Enoch explains that using the g.c. during a test helps him get more done when he has just a little time left to work.
ST2	Student states that the g.c. saves time in non-testing situations.	Shemika: "I do a lot of things and all the course work we have and everything else. It is easier to have the calculator because it saves time…"	Shemika points out that saving time by using the g.c. is important to her because she is busy with course work and other commitments.
ST3	Student states that the g.c. does not save time.	Aaron: "I knew I had to plug in an integral, take the integral of something. So, like, I'd have to spend like extra time trying to remember how to plug that in…"	Aaron points out that sometimes having to use the g.c. on a test takes more time for him than if he wasn't expected to use it.
Impact on mathematical play			

P1	Statement that playing with the g.c. is helpful when problem solving.	Enoch: "…it's like a little toy. You share your experiences with it, experiment with it and things like that."	Enoch points out that he uses the g.c. to experiment and try things out.
Impact on cognitive demand			
C1	Student was capable of solving the task without the g.c., but chose to use it to intentionally lessen the cognitive demand of the task.	Shemika: "When the equation is given and I know the answer can be found using the graph and its lots of work, then I'll use the graph instead."	Shemika notes that she is capable of solving a problem of this type, but will often choose to use her g.c. to lessen the cognitive demand.
C2	Student was not able to solve the task without the g.c. and chose to use the g.c. to lessen the cognitive demand of the task.	Enoch: "I suppose I used the calculator as a scapegoat. I mean…I basically got the answer from the calculator. I couldn't, like, get it on my own."	Enoch states that he did not know how do find the answer on his own so he used the g.c. instead.
C3	Using the g.c. increased the cognitive demand of a task.	Aaron: "Well, its just like…you have to put solve, and then you have to put the equation and then you have to have comma x, comma this, comma that…it's just a lot of comma…it's a lot to memorize."	Aaron explains that using the g.c. actually adds more to what he needs to remember how to do.

After the interview transcripts were coded with the inductive codes, all pieces of data coded within the same category (changing cognitive demand, mathematical play, mathematical security when checking, and time management) were then copied and pasted into a separate word document. These documents were then reread, codes were collapsed into broader categories then cases and vignettes were constructed (Stake, 1995). Throughout this phase I constantly compared the results between cases and the cases to the survey data.

Phase Three: Qualitative survey data. After completion of the coding of the interview data the written responses on the 111 surveys were coded for the same overarching categories: impact on their mathematical security, impact on the cognitive demand of the task, and impact on their mathematical play, and impact on their use of time. There were two questions on the survey that required a written response and that also provide some additional information about how the students perceive the graphing calculator impacts their mathematical experience. They were the following:

12. If you were given the choice to:
    a. Take a test with another student (of your choosing) or
    b. Use a graphing calculator on the test, which situation would you choose and why?

13. Imagine the following situation: You solved a problem on your own and then used your graphing calculator to check your solution. The calculator gave you a different solution than the one you got when you worked on the problem on your own. Which answer do you trust? Why?

The student responses to these questions were read and coded according to the broad categories developed in the inductive coding schemes used on the interview data. Next, the responses were reread and more specific codes, relevant to the question, within each category were defined (see Table 6 & 7). Once all of the written responses were coded they were entered into SPSS. Finally, frequencies and correlations with gender and math ability were calculated.

Table 6: Inductive codes for survey question #12

Code	Description	Example	Reason for code
Sh	Mathematical security – helpful	"I'm very comfortable w/ a graphing calculator, and I know my own math capabilities; another student would be unlikely to have the skills that could complement my deficiencies, while I'm sure a calculator can provide some of those necessary functions."	Student writes that the g.c. is more helpful than another student would be.
Sch	Mathematical security – check	"Usually I would prefer a calculator because then I could check my answers."	Student writes that he/she prefers the g.c. because it can be used to check work.
Sr	Mathematical security –	"Use a graphing calculator	Student writes that the g.c. is

	reliable	because it is reliable."	reliable.
So	Mathematical security – others	"I would use the calculator b/c it doesn't have an opinion, have to be placated, or argued with."	Student writes that they chose g.c. because they don't want to work with others.
Sc	Mathematical security – correct	"Use graphing calc. since it is def. right, where as a person may be wrong."	Student writes that the g.c. is always correct.
St	Mathematical security - saves time	"Because the calculator helps consume time and makes problems much easy."	Student writes that the g.c. saves time.
C	Lessens cognitive demand	"I would take the graphing calculator because I am still doing my own work, but the calculator assists me with hard functions I cannot do with paper and pencil."	Student writes that using the g.c. lessens the cognitive demand of some tasks.

Table 7: Inductive codes for survey question #13

Code	Description	Example	Reason for code
Sc	Mathematical Security – Calculator is correct	"I would trust the calculator, because unlike humans, calculators don't make computational mistakes for no reason."	Student states that the calculator must be correct.
Sw	Mathematical Security – I'm wrong	"I would trust the calculator because I tend to frequently make careless errors while computating an answer."	Student states that their work must be wrong.

Reliability and validity: Many steps were taken to ensure both the reliability and validity of this study. Data was collected from three different sources, surveys, task-based interviews, and reflective interviews. These three pieces of data will provide triangulation, thus confirming any emerging findings (Merriam, 1998). To ensure that the participants were represented accurately, member checks were performed. Finally, through out the analysis process colleagues were asked to comment on the findings of the study. Two colleagues were also helpful in verifying my coding scheme. They each read and coded one task-based interview and one follow-up interview. Any disagreements in coding were discussed until all were in agreement. Upon completion of this process we were in agreement on 100% of the codes.

The following chapters will describe in detail the findings of this study. The findings are arranged in four chapters. Each will present the data on one of the four categories in which the students' perceive the graphing calculator impacts their mathematical experiences: impact on the cognitive demand of the task, impact on their mathematical play, impact on their mathematical security while checking, and impact on time management. The data chapters will be followed by a discussion of the limitations of this study, how this data answers the research questions, and how the findings may inform future research and practice.

Chapter 3: Students' perspectives of the impact of graphing calculator use on the cognitive demand of the tasks

"…you don't have to do all the computation. It just does it for you. And like a point of intersection, you don't have to set the equations equal to each other; you can just plug it into the calculator." ~ Aaron

Defining cognitive demand

To many of the students in this study, the graphing calculator provides a way to change the *cognitive demand* of a task. In the context of this study *cognitive demand* refers to how much thinking is involved in solving a particular task. As Henningsen and Stein (1997) have pointed out, a task for which students use only routine memorization to solve is considered to have required low cognitive demand, no matter how advanced the mathematics content. On the other hand, being aware of and explaining the underlying concepts and meaning of a task, even a basic one, requires high cognitive demand.

The use of the graphing calculator can greatly change the intended cognitive demand of some tasks (Hershkowitz & Kieran, 2001). As a matter of fact, it is often assumed that using the graphing calculator must necessarily lessen the cognitive demand of a task (Dunham, 2000; Hershkowitz & Kieran, 2001). The ways in which the students in this study perceive they have used the graphing calculator to impact the cognitive demand of the tasks will be discussed in detail. In addition, the students' perceptions of how having the ability to impact the cognitive demand of a task impacts their mathematical experiences will also be presented.

Students' perspectives on using the graphing calculator to lessen the cognitive demand of tasks

As Aaron's quote at the beginning of this chapter points out, students often think of the graphing calculator as a tool with which they can greatly change the amount of mathematics they actually have to do in order to solve a problem. For many, it is a way of getting around something they either have forgotten how to do, or never learned how to do. For example, consider Enoch's solution to task #2.

Task #2: Enoch
Enoch begins this task by writing down the derivative of *y* and setting it equal to zero. He proceeds to try to solve this equation, but sets the equation equal to 9, rather than 0 (he initially writes down a 0 and then quickly changes it to a 9) and tries to factor (see Figure 1). "I don't think I'm doing this right…there's something wrong…I need a calculator," he says as he reaches for his graphing calculator. He then enters the original function into *y1* and looks at the graph of the function. In order to get a picture of the important sections of the function, Enoch changes his window so that he can see everything between $x = -10$ and $x = 10$. Enoch then uses the maximum tool and finds the maximum of the original function to be at $x = 3$. After looking at his solution for a few moments, Enoch wrote the second derivative, $-6x+6$ on his paper and proceeded to graph it. Then he traced the graph of $-6x + 6$ and wrote his solution, $x = 1$ on his paper.

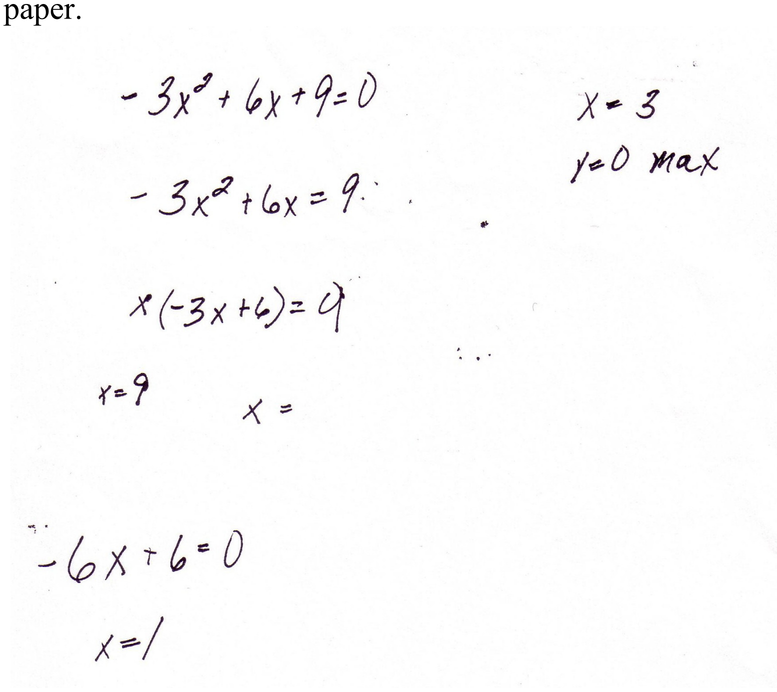


Figure 1: Enoch's written work for task #2

I asked Enoch why he decided to use the calculator when he did, he said "To find the maximum value…I knew I did something wrong and I thought back to what we do in class, why we use a calculator with this problem and just plugged it in and found the maximum." As described above, Enoch used the graph and the maximum tools on his calculator to find the correct solution. Next I asked him if this is how he typically uses the graphing calculator. Our conversation follows:

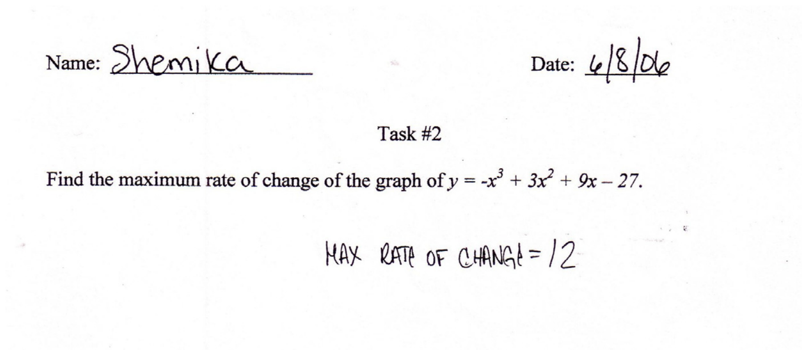
> Enoch: Yeah. That's how we…first of all we work on it as, um, we do it mentally right on paper. Then to see the graph to get a visual of it, you go to the calculator. So that's how we use it. I don't know I just…I just forgot.
>
> A: How would you describe how you used the calculator on this particular problem?
>
> Enoch: I s'pose I used the calculator as a scapegoat.
>
> A: Can you tell me what you mean by that.
>
> Enoch: I mean I used it like, so I could basically I got the answer from the calculator. I couldn't, like, get it on my own. I don't know.
>
> A: Why does that bother you so much?
>
> Enoch: I don't know. It just makes me feel like I'm not using my brain a lot. Yeah. I just, I have the brain power, I'm not using it I'm being lazy. I don't get what that is, being lazy.

Enoch's use of the graphing calculator might have changed the intended cognitive *load* of the task, but he knew enough about the underlying concepts to be able to construct a second approach to the problem and ultimately find the correct solution. However, Enoch's remarks above suggest *he* believes that by using the graphing calculator that he significantly lessened the cognitive demand. He used the graphing calculator to try things that he remembered doing in class on problems similar to this when he couldn't solve it himself by hand. When he began to use the graphing calculator he was using it to find the maximum of the original function, not the maximum of the derivative. When the solution found using the original function didn't make sense to him he decided to try something different. He said, "I don't know…I got confused between which derivative gives me the maximum. That's why I started to graph that. I saw the graph and realized that wasn't what I was looking for."

Statements such as, "I mean I used it like, so I could basically I got the answer from the calculator. I couldn't, like, get it on my own." may imply that Enoch thinks that he skipped over doing the mathematics. He believes that he has lessened the cognitive demand of this task because he had to use the calculator as a "scapegoat" to find the solution. He does not seem to see the powerful mathematics that he must have understood in order to use the calculator in the way that he did.

One might argue that Enoch's solution to task #2 above is an example of the use of a graphing calculator lessening the cognitive demand of a task because he was using a somewhat indirect solution method and he admitted to not knowing how to solve the problem by hand. However, the data in this study indicates that graphing calculators can be used in such sophisticated ways that the intended cognitive demand of a task remains unaltered or even increased. An excellent example of this is Shemika's work on task #2.

> **Task #2: Shemika**
> Shemika read the problem and knew exactly what to do. She picked up her graphing calculator and immediately went to the y = screen and entered nderive($-x^3+3x^2+9x-27$). "Well, the rate of change, I know that is the derivative so I just put it in the calculator", she explained. She glanced at the graph and decided to adjust the graph window to frame the area of the graph that contained the maximum. Once she had the maximum in view she used the maximum tool in the graphing mode of the calculator to approximate the maximum value. The tool said that the maximum was at $x=1$ and the maximum value was 11.9999. Though she assumed the maximum value was 12, Shemika recognized that this was an approximate value and tested it by evaluating the function at $x=1$. "When I get an answer like that I just put one in to see if it will get 12", she explained. At this point she writes on her paper for the first time, "max rate of change = 12".


Name: Shemika Date: 6/8/06

Task #2

Find the maximum rate of change of the graph of $y = -x^3 + 3x^2 + 9x - 27$.

MAX RATE OF CHANGE = 12

Figure 2: Shemika's work on task #2

When asked what it is about having the graphing calculator for this sort of problem that she liked Shemika said, "I didn't have to do the work, the calculator did it all." She did know how to solve the problem by hand however. When asked what she would do if she had not had the calculator to use she was able to explain exactly how to solve the problem by hand.

Enoch and Shemika used the graphing calculator in very different ways. While Enoch used it to do something that he was not able to do at that moment, Shemika used it in a very precise way, to do something she clearly did know how to do. Shemika would not have been able to construct such a precise solution with the graphing calculator unless she was aware of the underlying concepts and meaning of the task. Using the graphing calculator provided her with a means to skip over the general procedures, but still required her to think about the underlying concepts.

Though I believe Shemika didn't actually lessen the cognitive demand of the task, I believe it was her intention to do so. She specifically stated that she chose to use the graphing calculator on this task because she wouldn't have to do the work. She often spoke about the ways that she uses the calculator to avoid doing work, even when she knows how to do it.

A: What types of problems do you really like to have it for?

Shemika: Derivatives or integrals or both. I mean I can solve them without the calculator, or with the calculator. With the calculator math 9 or math 8 and I'm done. Put in the function and it calculates it and pops up. But, without it I can do it the same way…Just the calculator has been a big part of the calculus experience and it's helped a lot. Without it we could still do the work, but with it we can do it easier…

Both Enoch and Shemika intentionally used their graphing calculators to make their work easier, Enoch because he needed to and Shemika because she wanted to. I believe Shemika's work is an important example. It shows that the graphing calculator can be used in ways that do not necessarily impact the intended cognitive demand of a task. To use the graphing calculator in such a thoughtful way requires a deep understanding of the mathematics involved.

Perceptions about the ability to change the cognitive demand of tasks

Though Shemika may not have drastically impacted the intended cognitive demand of task #2, it is clear by her statements above that she perceives that the task took far less effort with the use of the graphing calculator. It appears that the availability of the graphing calculator to change the cognitive demand, or the perceived cognitive demand, of tasks is important to many of the participants in this study. Of the overwhelming number of the survey participants (81%) that said they would prefer to have a graphing calculator available on a test rather than working with another student, 30% gave reasons that reflected that it was important to them that the calculator could provide a way to lesson the cognitive demand on a task. The types of comments made by the survey participants were very similar to those made by the case study students. Both groups report using the graphing calculator to intentionally change the cognitive demand, or perceived cognitive demand, of problems for four different reasons: (1) because they forget the simple things, (2) to use a graph produced by the calculator, (3) to avoid tedious computations, and (4) because they don't know how to solve the problem by hand. These four reasons will be discussed below.

Skipping the simple math. Each of the case study students used the graphing calculator to do 'simple math' like evaluating a function, multiplying or dividing at least once during their task-based interview. Enoch mentioned that it is "just habit" and that he prefers to have it available when he is doing things like that. It was pointed out by a few students that it is easy to forget a procedure that you don't use very often and the graphing calculator is very helpful in those situations. When Melissa was reflecting on using her calculator in this way she said, "I like to use it for simple math or regular calculus that I sometimes forget." Similarly, Rudy mentioned that the calculator was important to have because, "Well, with sine and cosine and tangent, they're hard to remember, if you don't keep doing it every day. With the help of the graphing calculator I don't have to remember them." When Maryanne was speaking about her experiences on the AP Calculus exam she said:

> Maryanne: When I took the AP test, it was harder for me with the non-calculator section because it was all about 'I've got to remember just how to do simple stuff'… and it's just, it was kind of more difficult because when you have to multiply something, when you get to my level of math you just pretty much punch it in and get it out (laughs). And then you start thinking, 'I have to go back and do long division on this problem'. It was really frustrating. I know how to do this, I've done this before, and I know exactly how to do this, it's just taking a while. Because I haven't done it for such a long time.

She says that she often uses her calculator to do the 'simple stuff' without even thinking about it. Having it available helps her stay focused on the important mathematics in the problem, what she considers to be most important at her level. It seems that being able to use the graphing calculator when the 'simple things' aren't easily recalled is very important to many of the students.

Constructing graphs. The students also pointed to the importance of graphs in the changing of the cognitive demand of tasks. The production of a graph to avoid graphing by hand, or even to use a graph instead of another procedure, often makes a problem much easier to solve. We saw that Shemika's work on task #2 relied heavily on a graph that she produced using the graphing calculator. Shemika says she uses graphs as often as possible, "When the equation is given and I know that the answer can be found using the graph and its lots of work, then I'll use the graph." All six of the students produced a graph on their calculator at least once during the task-based interview. In addition, they all made some reference to the graph making the problem easier in their reflection interviews. For example, Melissa stated, "Sometimes just seeing it on the calculator is a lot easier." Similar statements were made by many of the students on the survey. Statements such as, "It helps me to visualize the problems better with graph." and "I like my graphing calculator because it is almost impossible to visualize many graphs without a calculator… Some math is too difficult to do" indicate that graphs produced on the graphing calculator are often perceived by the students to lessen the work load of a problem.

Avoiding tedious work. All of the case study students commented at least once on how the graphing calculator allows them to avoid doing some 'tedious' mathematics.

By ‘tedious’ they are referring to procedures that they have memorized and are time consuming. These tedious tasks are also often where students make careless mistakes when they are doing them by hand. For example, when Aaron was talking about how he uses his graphing calculator he gave a few examples of what he perceives to be tedious work, he said:

Aaron: I use it for complex integrals or complex derivatives. So you don’t have to do all the computation. It just does it for you. And like a point of intersection. You don’t have to set the equations equal to each other, you can just plug it in....And, well, I think its more that it’s like tedious, like fifteen sevenths x to the seven halves, its just tedious to multiply all that out. It would just be easier to plug it in. I mean obviously, if you have like, I don’t know…something like a trig function that you have to take the derivative of you have to do a trig substitution for, obviously it’s a lot easier just to use the calculator.

Shemika explained that the reason she used the graphing calculator on task #2 was to relieve herself of some of the more tedious work. Our conversation follows:

A: Can you talk me through what you did here? [As Shemika is watching a video of herself working on task #2]

Shemika: Well, um rate of change I know that is the, uh, the derivative and the derivative for like…I didn’t like writing out the derivative so I just put it in the calculator.

A: Ok.

Shemika: Um, I knew already that math 8 is the command 8 is the derivative and then you just put in this function and it calculates the derivative and graphs it.

[Shemika continues to explain each step she took on the graphing calculator.]

A: Is this typical, do you usually attack a problem like this?

Shemika: Yes. Unless they say write out the derivative and then calculate the maximum rate of change.

A: What is it about having the tool for that sort of thing that you like?

Shemika: I didn't have to do the work. The calculator did it all.

It appears that sometimes being able to get through the work without as much effort is important to these students. Melissa made this point very clear when she spoke about when she likes to use her graphing calculator, "If it took like a product rule, quotient rule, or chain rule, those get annoying and I'd rather use a calculator."

No other way. There are sometimes when students use their graphing calculators to change the cognitive demand of a task because they feel that they have no other choice. Enoch's work on task #2, described earlier, is a good example of this. Enoch did not know how to solve the problem by hand, but with the graphing calculator he was able to come up with a solution. Enoch says he uses his calculator in this way quite often, especially on tests. When describing how he used his graphing calculator on the AP exam he said, "I did it on my own and if anything was like not making sense to me I did it on the calculator."

Many of the students mentioned that there were often times that they had to use their graphing calculators in ways similar to what Enoch described. Melissa said, "I really like the calculator. I can't do a lot of problems without it." Rudy said that he had to use his calculator for hard problems. When I asked him to give me examples he explained that he didn't know how to find slope on a graph and some integrals or derivatives without the calculator. Many survey participants also used this as an explanation for why they would prefer to use a graphing calculator over working with a partner on a test. They provided statements such as:

> I would take the graphing calculator because I am still doing my own work but the calculator assists me with hard functions I cannot do with paper and pencil;
>
> I need the calculator to evaluate some problems

Statements such as these provide evidence that the students believe that they need the graphing calculator to change the thought involved to solve some problems because they would not be able to solve them otherwise.

Whether it be to avoid doing simple math that they've forgotten, to create a graph, to avoid doing tedious procedures, or to do a procedure that they would not be able to do without the graphing calculator, the data provides evidence that many of the students in this study perceive the graphing calculator to be a tool that is important because of its ability to lessen the thinking involved in solving a problem. Do all students feel this way? Is it possible that using the graphing calculator might actually increase the intended cognitive demand of a task for some students? These questions will be addressed in the following section.

Increasing the intended cognitive demand of tasks

The previous sections in this chapter considered the different ways that students perceive that using the graphing calculator changes the cognitive demand of a task. All of the examples were from students who felt that the ability to use a graphing calculator to change the cognitive demand of a task typically meant lessening the cognitive demand. Though this aspect of graphing calculator use was clearly important to many of the students involved in both the case studies and the survey, it was not an opinion held by all. There was one student, Aaron who was quite resolute about the graphing calculator actually adding more to the amount of thought required to solve a problem. Aaron uses the TI-89 graphing calculator in his calculus class. The calculator was provided to him by the school. It has been his to take home and use freely since the first day of school. When I asked Aaron in our first interview about some of his responses on the survey he began to share his feelings about being expected to use the graphing calculator on assessments. Our conversation follows.

Aaron: Uh, yea…we have like calc quizzes and noncalculator quizzes. I kind of prefer the non calculator quiz cuz I think like when you get the calculator its just like something else you have to know. You know what I mean? If you have the non calculator quiz you just have to know your rules, you have to know integrating, derivatives all that stuff.

A: So, you're more comfortable in that situation?

Aaron: Well, yea…if you know it you know it, and if you don't know it you don't know it. With the calculator you have to also know how to use the calculator with the problem so…like there have been times when I know how to do the problem, but I don't know how to plug it into the calculator necessarily.

When I asked Aaron to elaborate on this and to give some examples of situations where using the graphing calculator on a problem was more difficult than doing the problem by hand he explained that for him the graphing calculator was another thing he had to learn.

Aaron: Well, its just like… you have to put solve, and then you have to put the equation and then you have to have comma x, comma this, comma that…it's just a lot of comma…it's just like memorizing, you can't really figure out what to put in…For example, absolute value you have to type 'a-b-s' instead of like just putting those bars. So, the answers just like plain and clear, but I guess it's just hard to do.

Research has shown that there are many obstacles that come with learning to use calculators with CAS capabilities, such as the TI-89 (Drijvers, 2000; Guin & Trouche, 1999). Aaron was very quick to admit that he never took the time to learn his graphing calculator. He tried to follow along with his teacher, but often gave up when he fell behind on keystrokes, he explained:

> Aaron: Um, well, I'd be following him and then eventually it'd be like so many steps that he'd be doing that eventually I'd be like uh…there's no way I'd remember what he was doing. Unless I like studied, and I wasn't really willing to do that.

Aaron made many comments that lead me to believe that he found the graphing calculator to be a powerful and important tool, though it was difficult to use. When I asked him why he wasn't willing to take the time to study and learn how to use it he very eloquently compared it to learning how to use his new cell phone.

> Aaron: Yeah, I've never read my manual for my cell phone, you just look on it and figure it out. [He pulled his cell phone out of his pocket.]
>
> A: What is it that makes that easier, do you think?
>
> Aaron: Um, maybe it's like there's not as much. But I think its more like…like, uh, you go to menu, and then you go like, you scroll down to what you're looking for [shows on his cell phone]…
>
> A: So, it's organized in a way that's not as intimidating.
>
> Aaron: Yeah, yeah. The most confusing thing on this cell phone is how to get to speaker phone. Cuz there wasn't a way to go menu, setting, speaker phone so you had to just figure it out.
>
> A: How did you handle that?
>
> Aaron: Someone told me. And it wasn't even that hard, you just had to hold down this button. So, even that's not that hard.

A: Do you think you were willing to go look at things on that more because its organized really well?

Aaron: It was organized really well, and it's *my* cell phone. So I want to know what to do. It's not like…I care more about what I know on the cell phone than I do on my calculator.

Aaron's comparison of learning how to use the graphing calculator to learning how to use his cell phone is compelling. At some point in time Aaron was just as unfamiliar with his cell phone as he is with the graphing calculator. However, he was willing, and possibly even eager, to become familiar with the cell phone because it was *his*, part of his world. In contrast, the graphing calculator that Aaron uses does not belong to him, it was issued to him to use for his calculus course by the school. It is possible that ownership has a significant impact on the type of relationship one develops with a piece of technology.

Aaron stated that cared more about what he knows on his cell phone than what he knows on his graphing calculator, but he also said that it was a powerful tool, one that he should have taken the time to learn. When asked whether or not the calculator is an important part of the calculus course he replied, "Yea, definitely. I mean, it's just like such an advantage to have it. It just makes everything so much easier. If you have a really good understanding of the calculator you can do a lot more than you'd be able to if you didn't have a calculator or know how to use it that well." In addition, at the conclusion of our second interview together I asked Aaron if he had any advise for incoming calculus students. His response follows.

A: I guess it's about time to wrap up. But first of all, do you have any advise for incoming calculus students?

Aaron: Uh, get to know the basic things to do on the calculator.

A: Is that something that you would do differently?

Aaron: Yea, I probably would have just memorized how to do this, how to do the derivative, this is how you solve for a point of intersection…just like the five or six things that you repeatedly need to know.

In conclusion, the data from both the case studies and the survey suggest that calculus students perceive that the use of a graphing calculator can greatly change the cognitive demand of a task. Students who reported that they perceived that they had intentionally lessened the cognitive demand of a problem gave four different reasons for doing so: 1) to do simple math that they have forgotten; 2) to produce a graph; 3) to avoid tedious procedures; 4) and to do work that they can not do by hand. It is interesting that those that use the calculator to do 'simple math' and 'tedious procedures' seem to think of these actions as not being part of the math that is important at this stage in their course work. There was only one student that suggested that the graphing calculator might increase the cognitive demand of a problem, but even he pointed out that if he had taken the time to learn how to use his graphing calculator he would have been able to avoid many tedious procedures.

It is somewhat surprising that the students use their graphing calculators in ways that they often attribute to lessening the cognitive demand of the tasks in which they are engaged, yet they do not recognize the actions that they are choosing to take on their graphing calculators, or the thought involved with making the decision to do so, as "doing mathematics". Note that the reasons that the students gave for intentionally lessening the cognitive demand of a task were all related to *not* doing math. However, it is clear when you look at examples such as Enoch and Shemika's work on task #2 that a lot of mathematics was involved in simply deciding what actions to take with the graphing calculator. It seems as if they perceive their calculator actions to be completely separate from the mathematics. When, in fact, their decisions about how and when to use the calculator are clearly mathematical as well as affective.

Chapter 4: Impact of graphing calculator use on students' mathematical play

> *"It's like a little toy. You share your experience with it, experiment with it and things like that."* ~ Enoch

The graphing calculator is a relatively small piece of technology. It is similar in size to many hand-held gaming systems. Students often report that they have stored games on their graphing calculators and use them to play when they are bored in class. Melissa even explained that she was motivated to learn how to use her graphing calculator for just that reason. She said, "I used to have a calculator that had games too, so I wanted to learn it faster…cuz also I wanted to learn where I could hide stuff and where I could play with stuff." With that in mind, it is probably not surprising that students refer to the graphing calculator as a toy. Aaron even went as far as calling it a Game Boy[3]. However, a closer look at what the students were often referring to when they spoke about playing on their graphing calculator revealed that they were not referring to playing programmed games, but actually engaging in playful mathematical activities.

A lot has been published about the role of play in learning over the last 100 years (Holton et al., 2001). Dewey (1916) wrote about the natural engagement in play when people, of any age, come into contact with new materials. He stated that at this first stage of contact the student must engage in trial and error type activities in order to get to know the material. Similarly, Piaget (1962) suggested that children expand their understanding of the world and themselves by engaging in play. According to Piaget, children collect bits of information about an object as they interact with that object. This information is then assimilated into their already existing knowledge to expand the child's knowledge of

[3] A Game Boy is a hand-held gaming system produced by Nintendo.

that particular object. Though Piaget wrote extensively on the importance of play in the cognitive development of young children, according to him play was not as important as children mature. Like Piaget, Vygotsky believed that play was an important aspect of learning for children. According to Vygotsky, play allowed for children to develop abstract meaning separate from the objects with which they are interacting (Wertsch, 1985). In other words, they can continue the action of play even in the absence of the object that they originally interacted with. This might be done by using a different object in place of the original, or simply by imagining the action. Though some of histories greatest educational theorists have suggested that play is an important aspect of cognitive development, they did not consider its role in learning and doing mathematics.

More recently researchers such as Davis (1996) and Holton et al. (2001) took the idea of play a bit further and considered its value with respect to mathematics. In discussing the role of play in doing mathematics Davis (1996) says:

> Put simply, play is not so much an activity as it is an acceptance of uncertainty and a willingness to move. Play is thus the antithesis of the modern ideals of certainty, predictability, and linear progress. But it is not an abandonment of our quest for structure, order, pattern, and comprehensibility. Quite the opposite, these are the ends of play.
> But these ends are revealed only in the playing, for play is not simply random activity. Rather, by opening the door to the as yet inexperienced, to the possible, play reveals what is not yet known as it simultaneously offers space to support learning (p. 222).

Like Piaget and Vygotsky, Davis suggests that play is a way of making order in one's world, in this case one's mathematical world. "The acceptance of uncertainty and a willingness to move" is an action that has both affective and mathematical implications. The feelings that accompany uncertainty are often powerful feelings, possibly resulting in frustration and defeat or possibly in curiosity. A student that is frustrated and feeling defeated will likely stop working on the mathematical task at hand. In contrast, a student that is curious might make a move to explore the task. The data from this study indicate that the presence of a tool like the graphing calculator might possibly make the difference between accepting defeat and the willingness to make a move. In the context of this study, actions that students take that show that they are excepting uncertainty and yet are willing to move I refer to as engaging in *playful mathematical activity*.

Davis's discussion about play in mathematics does not go as far as formally defining *mathematical play*. Mathematical play in its formal sense is different from "playing around" when you don't know what to do. It is a bit more complex than simply excepting uncertainty and making a move. Formal mathematical play is what mathematicians describe that they do when they are exploring a problem for the very first time (Holton, 2001; Polya, 1945; Schoenfeld, 1994). To identify formal mathematical play I have chosen to adopt a definition provided by Holton et al. (2001). They define mathematical play in the following way:

> By mathematical play we mean that part of the process used to solve mathematical problems, which involves both experimentation and creativity to generate ideas, and using the formal rules of mathematics to follow any ideas to some sort of a conclusion. Mathematical play involves

> pushing the limits of the situation and following thoughts and ideas where ever they may lead. Hence there are no obvious short-term goals for mathematical play; it is designed to allow complete freedom on the part of the solver to wander over the mathematical landscape. However, there is a long-term goal and that is the solution of the problem at hand (p. 403).

Particular to this study is the use of the graphing calculator as a tool to wander the mathematical landscape. In other words, using the graphing calculator to generate ideas when one is in a completely unfamiliar situation and to push the limits of that situation as it is currently understood. This definition for mathematical play was chosen to help determine what to look for in the data; however it has been used in a more relaxed sense. In the analysis of the data it was not necessary that the students had reached the long-term goal, the solution of the problem at hand, but instead that they were clearly headed in that direction.

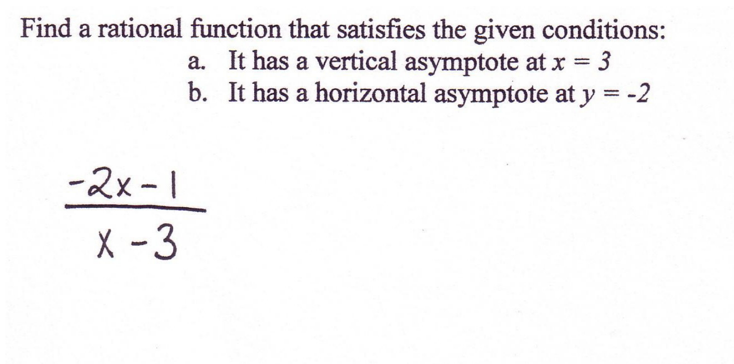
I think the most important difference between formal mathematical play and playful mathematical activity lies in the short-term goals. There are no short-term goals when one is engaging in mathematical play; you are free to try anything. This type of play often occurs when a student is trying to understand a problem, produce a conjecture, or possibly producing a generalization or extension. In contrast, playful mathematical activity is characterized by a short-term goal that is well defined; the knowledge that a particular way of exploring will most likely lead to further understanding and possibly a solution. Another term that is often used to describe playful mathematical activity is *exploration*. For example, a student that is exploring the impact that the leading coefficient has on a graph of a linear function by changing that coefficient and considering the corresponding changes in a graph of that function is engaging in playful mathematical activity.

Few, if any, of the research on graphing calculator use discuss the ways in which graphing calculators allow students to play with mathematics. As I observed students working on tasks for this study I would describe many of their actions as exploratory and playful. In addition, the students often talked about the benefits of *playing around* on their graphing calculators. In this chapter I will describe what is meant by *playful mathematical activity* and the more formal *mathematical play* in the context of using a graphing calculator. Furthermore, examples of play from the case studies will be presented and student perceptions of play with a graphing calculator will be discussed.

Playful mathematical activity

The students in this study often used their graphing calculators to explore or play with the goal of better understanding a mathematical situation with which they were facing uncertainty. They even spoke about the importance of being able to do so in testing situations. For example Shemika said, "If it is in a test situation then I'll do everything else I know how to do first and then if I have time I'll go back and start playing around with it on the calculator." When Melissa was talking about a particular problem on the AP exam that she was not able to solve I asked her if she thought she would have been able to solve it if she had her graphing calculator, to which she responded, "Possibly. Cuz I would be playing more with it, just plugging in and stuff."

Often as the students in this study were confronted with uncertainty on the four tasks they were well aware of what type of actions on their graphing calculator might provide them with what they need to better understand the situation. Since those students had a specific short-term goal for their play, the action was categorized as engaging in playful mathematical activity. Maryanne's work on task #1 provides an example of this. Maryanne seems to understand the task, but is unsure about how the characteristics of the numerator of the function would affect its graph. She used her graphing calculator to explore and test things in an area that she is not very comfortable, graphs. Maryanne's work on task #1 appears below.


Find a rational function that satisfies the given conditions:

a. It has a vertical asymptote at $x = 3$
b. It has a horizontal asymptote at $y = -2$

$\frac{-2x-1}{x-3}$

Figure 3: Maryanne's work on task #1

Task #1: Maryanne

Maryanne first read the task and wrote down $\frac{\quad}{x-3}$. She looked at it for a few minutes and then left the problem to work on another task. She returned to the task after taking a first run through each of the four tasks. She looked at what she had written and then picked up her graphing calculator and entered $y1$=1/(x-3) and graphed it. Next she changed the numerator in $y1$ to (2x – 1) and examined the graph. Finally, she changed the numerator one more time, to (-2x – 1) and examined the graph. She nodded as she looked at this final graph and wrote her solution, $\frac{-2x-1}{x-3}$ on her paper.

Maryanne had a plan for exploring this problem when she went to her graphing calculator. She used the calculator as a means to try out some mathematical ideas that she thought might lead her to a solution. Our conversation about the process she used to determine her solution to this task follows.

Maryanne: Well first I just put one over x minus three to make sure that there was an asymptote there.

A: You first did two x minus one and you took a look at that graph. And then you went back and did negative two x.

Maryanne: Yeah. I was checking to see where asymptotes were when I changed things.

A: Would you say this is characteristic of the way you tend to use the calculator?

Maryanne: Yeah.

A: Do graphs play an important role for you when you are thinking about ways that things make sense?

Maryanne: Um, I sometimes have a really hard time visualizing graphs…I'm a visual person and it bothers me sometimes that I can't always see it in my mind. So I do like to be able to see it at some point. It's just a lot easier to know what you're dealing with instead of just having a bunch of numbers.

Maryanne had a sense of the formal rules of mathematics that were associated with this particular task, a restricted domain and range. She was not sure exactly how they would be used, but did have a sense that the expression in the numerator was important, so she used the calculator to explore it. Maryanne's short-term goal, determining the affect of the numerator on the graph of the function, was very well defined. Though she was uncertain when she began the task, through her playful mathematical activity on the graphing calculator she was able to come up with a correct solution.

Melissa used her graphing calculator in ways similar to Maryanne when she was working on the four tasks. When she spoke about how she solved task #4 she pointed out that she often explores mathematics using her graphing calculator.

A: What made you go to the calculator on this one?

Melissa: I didn't know what to do.

A: Do you do that a lot?

Melissa: Yeah, you just play around with the calculator.

A: Do you find playing around to be a helpful thing?

Melissa: Yeah.

A: How often do you do that?

Melissa: When I'm stumped. (laugh)

A: Any time you're stumped?

Melissa: Yeah. Sometimes when you graph its like easier to just understand the problem.

Both Maryanne and Melissa referred to the graphing modes of their graphing calculators as being powerful ways to explore situations in which they are uncertain. They even used the language of "play" to describe their behaviors. They both suggested that it is easier to understand a problem when they have a graph to consider rather than just a symbolic representation. Their comments also suggest that having the graphing calculator makes exploring using a graph an easy thing to do. It is possible that they would be less likely to engage in playful mathematical activities if they did not have a graphing calculator available to them to create the graphs that they deem so helpful. It is important to note that all of the case study students used the graphing calculator in this way at least once. Each of these instances resulted in the student making mathematical progress on the task. Therefore, there is evidence that the students in this study value being able to engage in playful mathematical activities for both mathematical and affective reasons; mathematical because they make progress in understanding or solving a task, and affective because it provides a means for them to move past feelings like those of uncertainty. Davis provided a sense of the importance of this kind of play in doing mathematics, and these students have shown that "playing around" with mathematics is indeed important to them.

Formal Mathematical play

In addition to the large amount of playful mathematical activity that was observed in the students work, there were instances of formal mathematical play as well. An excellent example of mathematical play in the context of graphing calculator use is Rudy's work on task #3. Rudy did not understand the task initially. His long-term goal was to find a solution for the problem, but his short-term goal was not obvious. He used the graphing calculator to explore the mathematics and had very creative ideas about what might help him to do so. Rudy's work on task #3 appears below.

Task #3: Rudy

Task #3: For what values of x is $-2 < |k - x| < 5$.

Rudy read this task for quite a while. He asked, "Can k be anything?" To which the researcher replied it could be any real number. Rudy then wrote down $7 - x =$ and picked up his calculator. He entered $y1 = 7 - x$, graphed it, and quickly went to the table. He scrolled up and down the table of values between $x = 2$ and $x = 9$ and paused for a moment. Next he returned to the $Y =$ screen and changed the function from $y1 = 7 - x$ to $y1 = 6 - x$ and went back to the table. On the table he scrolled between $x = 1$ and $x = 8$. Then he wrote on his paper, "Depending on k there are 6 numbers that make x greater than -2 and less than 5."

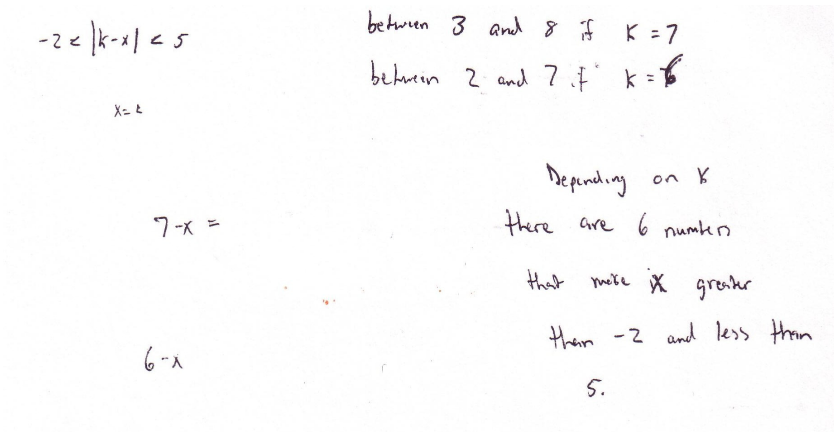


Figure 4: Rudy's written work for task #3

Rudy was unsure where to begin on the problem so he picked a simpler problem to think about. With a simpler problem in mind he then used his graphing calculator to explore what solutions might work. He noticed that there were six integral solutions when he chose *k* to be 7. When the same was true for *k* = 6 he realized he had found a pattern. I asked Rudy about his work on this problem. Our conversation follows:

A: So on this particular problem, let's see what you did here…you went to the calculator a few times…you decided to use 7…

Rudy: I went to the table.

A: Ok, and what were you looking for on the table?

Rudy: On the table here I was looking for the numbers that were between, that was less than five and bigger than negative 2. I saw that there were six numbers, three, four, five, six, seven, and eight. And it's the same for, there are six numbers…depending on the k, that's what I noticed.

A: What made you decide to try this on the calculator?

Rudy: At first I didn't understand it. I was trying to figure out what they meant.

Though he didn't meet the long-term goal of solving the problem, he did take an interesting stroll over the landscape. He chose a simpler problem to work on and used the table mode of the graphing calculator to test some numbers while trying to build an understanding of the role of the inequalities and the absolute value in the task. Rudy's short-term goal was not well defined. When he was exploring the table it was not in the hope of solving the task, but in hope of better understanding the task. When I asked him if he could have solved this problem without his graphing calculator he said, "Well, maybe if I had understood it from the beginning." However, it was through play on the graphing calculator that he began to build an understanding of the inequality in the task and thus the task itself.

Like Maryanne and Melissa I believe Rudy most likely would not have attempted this problem at all if he did not have a graphing calculator available. The availability of the graphing calculator changed the situation from one in which likely no mathematics would have taken place to one in which somewhat sophisticated mathematics was engaged in. Most importantly, was that by playing while doing mathematics, whether it is in the formal sense or less formal as described earlier, the graphing calculator provided an easily accessible way to engage in the mathematics when the students were facing uncertainty.

Playing with representations

Like Maryanne, Melissa and Rudy, many of the students mentioned that being able to produce and explore a new representation associated with a task, such as a graph or table, provides another way to think about a problem that they otherwise were not able to solve. For example, Aaron said, "If I'm trying to figure out a problem, seeing a graph helps me figure out a problem. So then I might like type in the function and then look at the graph and think about it and it will be more clear how to solve the problem." Similarly, Melissa said, "Yeah. I can actually see stuff on a graph and that really helps a lot. I'm a visual learner." All of the case study students used graphs produced on their calculators to explore a task at least once in their task based interview.

When students produce graphs on a graphing calculator for the sole purpose of gaining a better understanding of the task it is a form of playful mathematical activity, and sometimes formal mathematical play. In many cases the graphing calculator acts as a 'more capable peer' for the student to build ideas with. The students in this study were very clear about the fact that they find this kind of playful activity invaluable to their mathematical experiences. This is evidenced not only by the actions and statements made by the case study participants, but also by the survey responses.

Survey participants were asked to provide the reason they most often use the graphing calculator to produce a graph on their graphing calculator. Though this question was not intended to tease out issues of play, given the case study results it is very revealing. Over three-fourths of the students responded that they most often produced a graph because doing so was helpful (see Fig. 5). The same question was asked about the use of tables, over half of the students responded that they used tables because it was helpful (see Fig. 6). The number of students that used tables was fewer than those that used graphs most likely because they have not used tables as regularly in their classroom experiences. This is suggested by the percentage of students that responded that they use tables because it was suggested by their teacher or textbook. When the case study students stated that a graph or table was helpful to them, they were referring to using it to explore mathematical ideas that they were uncertain about. Given the similarities in the ways that the case study students used graphs and tables as a means to explore and play, it is likely that the survey participants who refer to using graphs and tables because they are helpful are also referring to using them in ways to that help them overcome uncertainty. If that is the case with the survey participants, then there is evidence that the student use the graphing calculator to engage in playful mathematical activity. This suggests that future research should be designed to purposefully tease out issues related to play in mathematics.

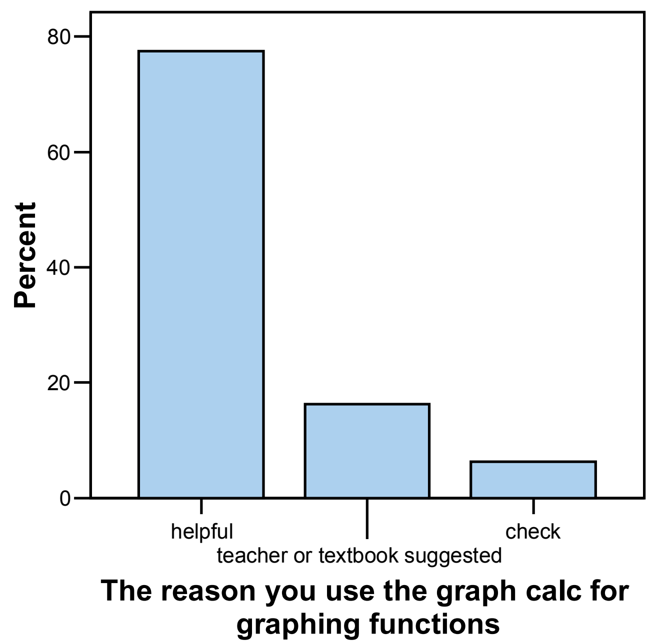


Figure 5: Survey responses to #10c: Please provide the reason you most often use the graphing calculator to produce a graph

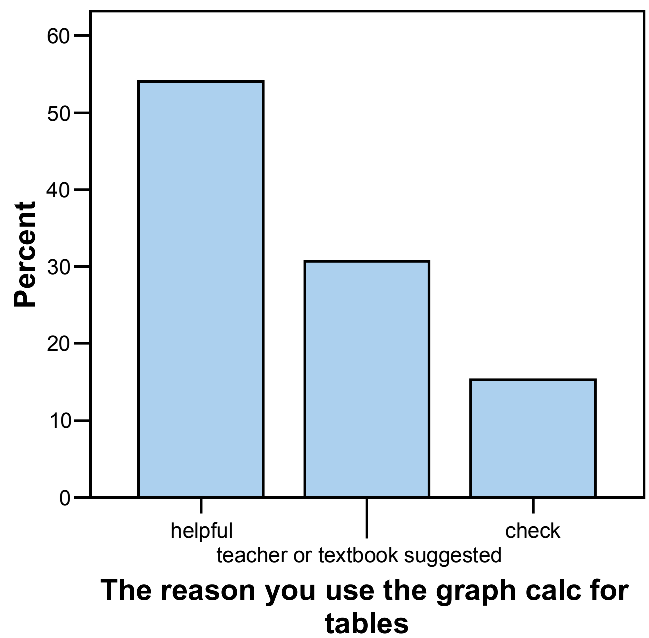


Figure 6: Survey responses to #10d: Please provide the reason you most often use the graphing calculator to produce a table

It strikes me that the most powerful example of use of the graphing calculator for formal mathematical play came from Rudy on task #3. According both Rudy and his teacher, Rudy is not a very strong calculus student. He came into calculus feeling unprepared due to his previous school math experiences. However, he expressed that the graphing calculator has provided him a means for doing a lot of calculus that he would not have been able to do otherwise. His work on task #3 shows that he is not using his graphing calculator to merely do something that he can not do on his own, but he is using it to engage in doing mathematics in ways that are similar to the ways that professional mathematicians work (Holton, 2001; Poly, 1945; Schoenfeld, 1994). In the case of task #3, he chose a simpler problem and tried to use that to tease out what the task was asking. The graphing calculator didn't just provide him with a means for making a move when he was uncertain, but also with a means for wandering the mathematical landscape. Through mathematical play Rudy began to build an understanding of inequality and absolute value, something that he had failed to do in his previous math experiences.

It isn't very surprising that the weakest of the case study students engaged in the most powerful mathematical play given that he most likely was the only student in a situation for which he didn't have any idea what short-term goals he should have to be successful in finding a solution. However, it is surprising that a student who declares that he is weak and unprepared is willing to engage in mathematics in such a powerful way. Would he have done that if he didn't have a graphing calculator available? Would the students who had engaged in playful mathematical activity on any of the tasks have done so if they did not have a graphing calculator available? Rudy, and others, said that the graphing calculator provides a fast and easy way to explore mathematical ideas. In addition, the graphing calculator provides a non-threatening environment for testing ideas. This is especially important when you consider the type of play that Rudy was engaged in. He was not sure what actions he should even try to take in order to possibly learn enough about the situation to be successful in finding a solution. He had to be willing to take a move with out knowing if it would be fruitful or not. The graphing calculator provided him with a quick and private means for playing with the mathematics involved. Doing mathematics seems to involve a private space where people can work out their ideas and a public space where those ideas are tested against some standard of social or absolute truth (Raman, 2003). For Rudy the graphing calculator is a safe and private space for him to test his ideas before they are made public and compared to the solution deemed correct by the community in which he is working (i.e. his teacher, textbook, or classmates). Furthermore, an attempt to engage in play without the graphing calculator as a tool adds another layer of complexity to a problem. It is likely that the cognitive demand required to engage in play without the graphing calculator would be a detractor.

The students in this study greatly value the graphing calculator as a tool used for engaging in playful mathematical activities, even those as complex as formal mathematical play. By using the graphing calculator in this way they are able to make mathematical progress even when they are unsure. Furthermore, they are often able to avoid the nonproductive affective pathways that can accompany uncertainty. As Enoch stated at the opening of this chapter when he referred to his graphing calculator, “It’s like a little toy. You share your experience with it, experiment with it and things like that.” It is that experience of experimentation and playfulness with the graphing calculator when they are uncertain that these students seem to appreciate most.

Chapter 5: Impact of checking on students' mathematical experiences

> *"I think I could have gotten an equally good answer if I didn't have a graphing calculator. I wouldn't have been sure of my answer. With the graphing calculator I know this is right. The other way I probably would have spent a lot more time on it. Like just thinking about it. Is this right? Is this not right?"* ~ Aaron

Some researchers have suggested that one possible reason for some students' improvement on assessments when a graphing calculator is permitted is simply that those students are more comfortable or confident when they have a graphing calculator to use (Dunham, 2000; Ellington, 2003). Based on this study it appears that there might be some truth the assertion that having a graphing calculator available might boost one's confidence or make one more comfortable. Students participating in the survey indicated overwhelmingly that when they use their graphing calculators for either numeric or symbolic computations it is for the purpose of checking. Furthermore, when they wrote about the importance of being able to check their work on the graphing calculator they often used words such as "reliable" and "comfortable" to describe their feelings about using the calculator in this way. In this chapter I will discuss the impact that the students perceive checking has on their mathematical experiences, particularly their feelings of mathematical security. I will share what they say they gain from checking their work, how they reconcile their solutions with calculator produced solutions, and their perceptions of mathematical authority when working with a graphing calculator. In the context of this study, the term mathematical security refers to how comfortable or secure a student feels about their mathematical ability at a particular moment in time. A student's feelings of mathematical security may be impacted by knowing that a solution is correct. Those feelings may also be affected by simply knowing that a tool, like the graphing calculator, is available to use if it is needed.

Perceptions of security: The availability of a tool to check

Doerr and Zangor (2000) found that in the context of a precalculus classroom, the graphing calculator is often used as a checking tool. They indicated that it was a mode of tool use that had been agreed upon by both the student and the teacher in the classroom. My previous studies have found that students often use the graphing calculator to check their work outside of class as well (McCulloch, 2005). The fact that students use the tool to check their work is not surprising. However, the literature has not considered what checking buys them. Both the survey participants and the case study students in this study indicated that the ability to check was very important to them. In addition, they often tied the ability to use the graphing calculator as a checking tool to specific feelings that they have when they are doing mathematics.

Seventy – eight percent of the survey participants indicated that they when they use their graphing calculators for computations they are doing so to check a solution (See Fig. 7 & 8). My field notes from visits to the mathematics classes of the survey participants show that checking was a commonly shared practice in all of the classes, much like in the class described by Doerr & Zangor. Given that graphing calculators are commonly used as checking tools in the classroom, it is not surprising that such a high percentage of students indicated that they were motivated to use them in this way.

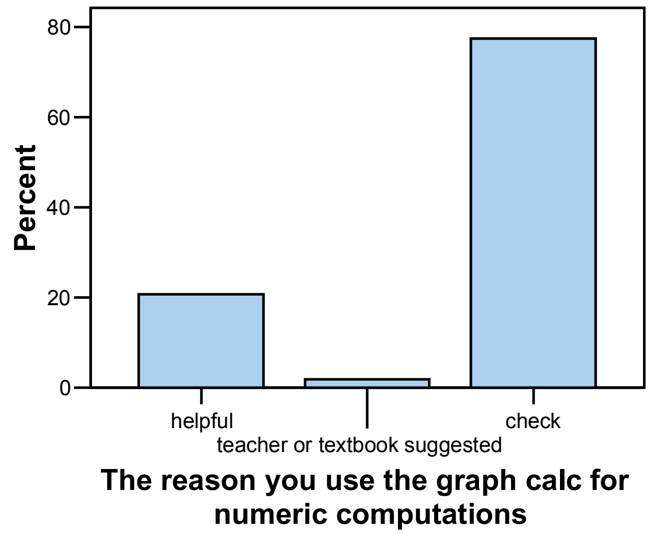


Figure 7: Survey responses: Students' reasons for using the graphing calculator for numeric computations

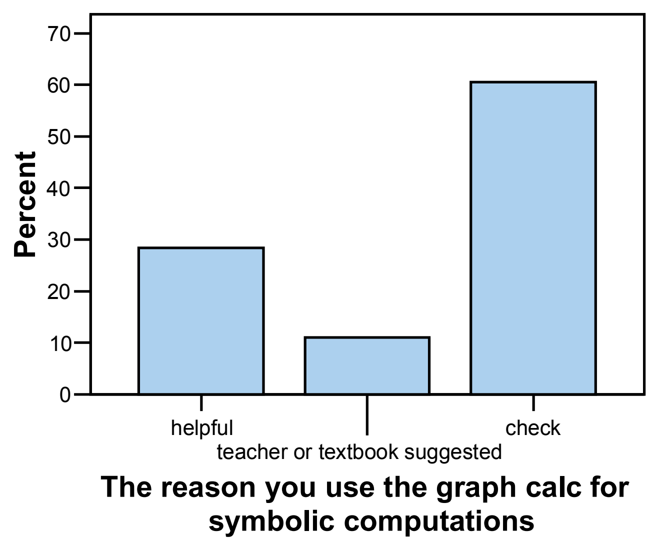


Figure 8: Survey responses: Students' reasons for using the graphing calculator for symbolic computations

The initial deductive coding of the case study data revealed a lot of student talk about checking and graphing calculator use. As a result, checking became one of the major themes for the inductive coding phase. All of the case study students used their graphing calculators to check at least once during the task-based interviews. Furthermore, the reflective interviews brought to light how these students, and possibly others, perceive checking impacts their problem solving experience.

When Aaron was talking about his use of the graphing calculator on task #4 he pointed out the importance of being able to check his work. Though Aaron is not very comfortable using his graphing calculator, he finds great value in being able to use it in this manner. Aaron's work and our conversation about task #4 follows.

Task #4: Give an example of a function for which $|f(x)| = f(|x|)$.

Task #4: Aaron
After reading through the task Aaron immediately wrote down x^2 as a solution. The researcher then asked him to come up with a few more examples, which prompted him to write down sin x. Next he picked up his calculator, a TI-89, and entered $\sin\left(\frac{\pi}{3}\right)$ on the home screen and got $-\frac{\sqrt{3}}{2}$ as a solution. Then he entered $\sin\left(\frac{-\pi}{3}\right)$ and got $-\frac{\sqrt{3}}{2}$. He stared at this for a long while. After a few moments he drew a 30-60-90 triangle on his paper and labeled the lengths of the sides. Once again, he paused. He then picked up his calculator and calculated both $\cos\left(\frac{\pi}{3}\right)$ and $\cos\left(\frac{-\pi}{3}\right)$ and got $\frac{1}{2}$ as an answer both times. He then wrote down sin x and cos x as solutions. Finally, he calculated both (3)^3 and (-3)^3 on his calculator and wrote x^3 as a solution as well.

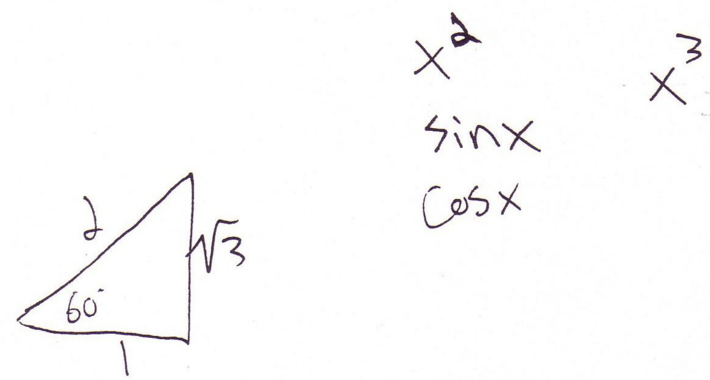


Figure 9: Aaron's written work for task #4

Aaron's actions indicate that he was aware that it was important that the functions that he identified as solutions work for values both greater than and less than zero. However, his work also indicates that he was considering only individual values and not whether or not the equality would hold true for all real numbers. However, after having some time to think about the task and reflect on his work Aaron seemed to have come to realize the limitations to his solution method as well as the incorrectness of his solution.

Aaron: I actually, after I left the room I was thinking about like this problem and I think if I had, I think if the camera wasn't on me and I had more time just to think about it I would have probably just plugged y equals absolute value of like x to the third and then y2 equals like the absolute value of x and then to the third. And I would graph both of them and if the graphs were the same then I would have known that was a function that worked. I think once I left the room actually I was thinking I should have done it that way. I would have used the graphing calculator…Then I would have done the absolute value of sine x and the sine of absolute value of x and see if there is only one graph.

A: Would you like to try it real quick and see if either of those work? Do you have any idea if they will work?

Aaron: Uh, which ever one is an even function. I don't remember. [He tries each of the functions he had originally given as solutions.]

A: Especially after you walked away and thought about it a little bit more, do you feel like the graphing calculator was a helpful thing on a problem like this? Did you find your calculator to be an important tool to have to think about this problem?

Aaron: Um, yeah. I mean if it had dawned on me to do it that way it would have been much easier. But, I kind of, even if I didn't have a graphing calculator I think I knew I was thinking uh, that the positive and negative value has to be the same number. And if it wasn't the same number then it wouldn't really work. So…

A: Do you think you would have been able to think across all the real numbers if you didn't have a graphing calculator use? Or do you think doing what you did here would have been really important to you?

Aaron: Well, that would have been the best [referring to the graphing calculator], but I think I could have gotten to like an equally good answer if I didn't really have a graphing calculator. I wouldn't have been sure of my answer. With the graphing calculator I know this is right. The other way I probably would have spent a lot more time on it. Like just thinking about it. Is this right? Is this not right?

I shared part of this discussion with Aaron as an opening to this chapter as well because it is very similar to discussions I have had with many students. Aaron implies that when his solution matches one he produces on his graphing calculator he feels much more confident about his solution. We spoke a little more about checking later in the interview. Aaron mentioned that he checks "all the time" especially on tests. "I'll do the work and then, like, check it, because, like…I'm a little OCD[4] on tests sometimes. You know what I mean?" When Aaron has checked his work on the calculator and the solutions match he feels a sense of security that allows him to move on and not worry about whether or not he is right or wrong.

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[4] Obsessive – compulsive disorder (OCD) is a commonly used phrase among students at this school. It is used to indicate a behavior that one feels needs to be done in order to move beyond a particular moment.

Knowing whether or not a solution is correct appears to be important to many of the students. When I was speaking with Enoch about his survey answers he mentioned that he uses the calculator to check quite often. Here is what he had to say.

> A: How do you use your graphing calculator most often?
>
> Enoch: To challenge myself.
>
> A: What do you mean?
>
> Enoch: To see how like my mind compares to a calculator. To see if I can get the answers right and then like I can pat myself on the back. If I get it wrong…it is just motivation. Competing with the computer.

In my field notes I have described at least two episodes in which I observed Enoch working on a task, checking his work on the graphing calculator, and then actually reaching up over his shoulder and giving himself a pat on the back. Being able to give himself a 'pat on the back' suggests that Enoch feels more secure about his mathematical work when he can check it. Even if the answers don't match he says that it is motivating. It is motivating because he knows he needs to work harder and go figure out where his error is.

I observed many instances during the task based interview in which a student checked a solution and found that the calculator produced answer did not match the students' written solution. Interestingly, many times students felt much like Enoch mentioned above – motivated. A few even pointed out how this checking process often helps them determine where errors are in their work. Shemika states, "It tells me when I think I know that I'm doing the right thing, that I'm doing it wrong. I need to know that. The calculator helps me to bring to my attention what I'm doing wrong." Mayanne's work on task #2 provides an excellent example of how checking can provide insight into errors.

Task #2: Maryanne

Maryanne began this task by working on paper. She first wrote down $y' = 27 +$ and then crossed it out. Next she wrote the first derivative, set it equal to zero, and factored it (incorrectly). Directly below that work she sketched a number line with what she thought were the roots of the derivative marked off and the values within each section denoted above the number line (See Fig. 10). She looked at this for a moment and then crossed out the number line. Next she wrote down the second derivative, a second number line denoting x = -1, the root of the second derivative. She then began to evaluate the first derivative at -1, looked at it for a while, and then left it and went to work on another task for a while.

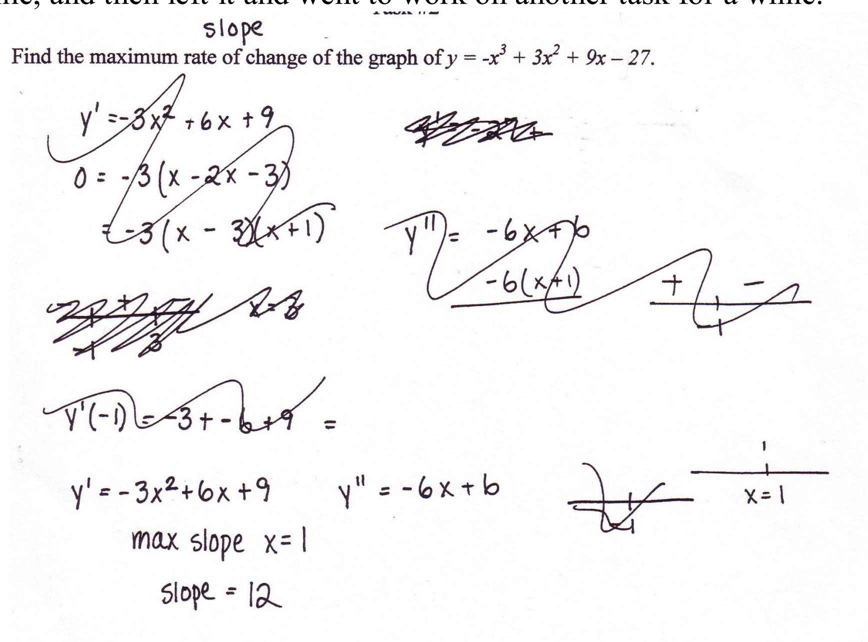


Figure 10: Maryanne's written work on task #2

When Maryanne returned to the task she immediately crossed off her previous work. She went back up and read the problem, and above the phrase 'rate of change' she wrote 'slope', then began to write again. She began by rewriting the first and second derivative and a number line with x = -1 denoted on it. Then she picked up her graphing calculator, a TI-83+, and graphs the original function, $y1 = -x^3 + 3x^2 + 9x - 27$. After changing the window using the zoom standard command she looks at the graph for a while. Next she returns to the y= screen and inserts nderiv in front of the original function she had entered, changes her mind and instead enters nderiv(y1, x, x) in y2 which commands the calculator to sketch the graph of the original function, y1, on the same screen. After looking at the graph of both functions together, she zooms out so that she

> can see the functions in a larger window. She follows this inspection of the graph with the use of the max tool to find the maximum of y2, the derivative. The calculator determines the maximum to be 12 at x = 1. Maryanne considers this for awhile, returns to her written work where she crosses out the number line she had drawn with x = -1. She writes down her solution, maximum slope at x = 1 is 12. Finally, she finishes by sketching a number line one more time with x = 1 denoted.

When we discussed what she did on this problem Maryanne pointed out when she realized what her error on paper was and how the calculator helped her determine that.

A: There's your original function [watching herself graph the functions on the video]…What did you do here?

Maryanne: I found the maximum of the slope just using the maximum function. Then I looked back and it said it was at one and I was like that's where I made my simple algebra mistake!

A: So this allowed you to help you find your mistake in here.

Maryanne: Yeah. Cuz I was like, I know this isn't right.

A: Does that happen a lot? Where you're kind of double checking yourself and you realize where your error was on the paper?

Maryanne: Well, it's happened before where you see the graph and realize there's something wrong in my work. I should go back and look at it…I'm a visual person and it bothers me sometimes that I can't always see it in my mind.

It is clear that the graphing calculator was instrumental in Maryanne's being able to identify where her error was on this task. She mentioned that it bothered her that she couldn't visualize the function and determine where the error was, though she was sure that she had made an error. Having the calculator available to create a new representation of her work provided Maryanne with a sense of security about the situation. She knew she had a tool that would help her determine what her error was; which for Maryanne was very important. It is unclear how students like Shemika and Maryanne feel about their mathematical experience at the moment in time that they check their work on a graphing calculator and find an error. However, we do know that when they reflect on that moment they perceive that checking provided them with a means for identifying that error. They describe the experience as being helpful and motivating.

The students typically noted that being able to check was a reassuring action for them, however a few of them also noted circumstances in which checking a solution was associated with feelings of insecurity. As mentioned above, Enoch identified checking as being very important to his mathematical experiences because he is motivated by knowing whether or not his answers are correct. However, on one occasion, he checked his work on the graphing calculator and the solutions did not match. When he spoke about that particular task he said:

> "I actually get to the point where I feel like I have my answer and then I'll just plug into my calculator and just get a different answer or something, then I know I did something wrong because I know the calculator wouldn't lie. So that right there, it makes me feel bad."

In this case, Enoch may know where he stands, which might still be comforting, but it also resulted in his feeling bad about his mathematical experience at that point in time. When I asked him if this was important for learning he said, "Yeah…I'll check anytime I have doubts about a problem." So, for Enoch, knowing where he stands is more important than how it makes him feel. Checking may bring out either feelings of security or insecurity, but he continues to check any time he is unsure because he sees this checking action as part of the learning process. Ultimately, if Enoch develops a deep understanding of the mathematics involved in a task over time he would begin to be correct more often than not. One could imagine that this would leave him feeling very secure about the mathematics in the end.

The ability to check appears to provide a powerful means through which students can take further control of their mathematical success. This is realized either by knowing that a solution is correct, or by knowing that it is not correct and reacting to the situation in a way that makes the student comfortable. It is clear from the data that the ability to check provides these students with a sense of security about their mathematical ability.

Student perceptions on the reconciliation of written work and calculator work when checking

As noted earlier the graphing calculator is often promoted by teachers as a tool for checking. The data in this study show that this mode of tool use is also embraced by students when they are working independently. I have shown that these students perceive checking to be very important to their mathematical experience. It is important however to consider the dilemma that students face when they use their calculators as a checking tool and are confronted with two different solutions, one that they produced on paper and a different one from the calculator. In this situation a student must make a decision about how to proceed. This decision is often very difficult, especially when you consider that the student may be experiencing feelings of frustration or even failure at that moment in time.

This predicament is one that I observed many students in my pilot studies struggle with. To better understand this situation, I included the following survey question:

> **Question 13:** Imagine the following situation: You solved a problem on your own and then used your graphing calculator to check your solution. The calculator gave you a different solution than the one you got when you worked the problem on your own. Which answer do you trust? Why?

Sixty-two percent of the students responded that they would trust their calculator produced solution. When the data was broken down by teacher rated ability level, it was revealed that 76.5 % of the weaker students chose the calculator produced solution over their written solution (see Table 8). An analysis of the participants written responses provided further explanation for their decisions.

Table 8: Survey responses to question #13.

	Written solution	Calculator produced solution (% of total)
Teacher rated weak	4	13 (76.5%)
Teacher rated average	17	24 (58.5%)
Teacher rated strong	18	27 (60%)
Total	64	103 (62 %)

Survey responses to question #13 were surprisingly specific. The responses revealed that many students have developed specific routines for reconciling differing solutions. For example, one student wrote "First I would double check what I did. If it didn't change, I'd go with the calculator; it's been programmed to do those problems without error." Another student wrote, "I recheck both then take the calculator's." Students who did not automatically defer to the calculator solution also provided some insight to their methods for reconciliation. For example, this student trusts both answers equally, but also has a method for thinking through the dilemma.

> "Well, I would compute the answer twice with each method. Then I identify what I did wrong on paper/calculator screen. Sometimes I write wrong signs, forget numbers, etc. on paper, but I also forget parentheses and other such items on the calculator so I trust the two answers equally."

When I posed the question to Aaron he said, "pretty much if I get the right answer in the calculator like twice, I'll trust the calculator." When I asked him why he assumed that the calculator answer was correct over his own he said, "I know the calculator's not thinking about it the wrong way, you know. I could just be thinking about it the wrong way." Surprisingly, many of the survey responses were similar to Aaron's in that their decision of which answer to trust had to do with where they place the authority in the situation, with themselves or with the graphing calculator.

The issue of authority was an unexpected one that played out in two different ways among the students that claimed that they would trust their graphing calculator over themselves. There were students who simply claimed that they were usually wrong, so they would trust the graphing calculator. There were others that instead of pointing out their own weaknesses pointed out the calculators merits and used those as a justification for placing the authority with the calculator. What all of these students had in common was that they were deferring the authority in the situation to the graphing calculator, and not placing it with themselves. The only students who seemed to recognize their role in the production of the graphing calculator solution were those who chose to trust their own solutions over the graphing calculator's. Many of their responses indicated that they either recognized that they may have put incorrect information into the graphing calculator accidentally, or possibly had chosen an incorrect procedure for solving the problem

Like Aaron above, many students reported that they did not have confidence in their own work thus they placed the authority in the situation with the graphing calculator. Some wrote very plainly which solution they trust, "The calculator – I have no confidence in my math abilities", or "The calculator. It is better at algebra than me." Of the students that chose the graphing calculator solution over their own, 65% gave reasons that indicated the choice was because they assumed that their written work was wrong.

Rudy, who often told me that he wasn't confident in his calculus ability, explained how he reconciled different solutions in the following way, "I retry the problem and if I approach it, I don't know…I mean, I check it on there [points to his calculator], I put the equation in, then if it is wrong I retry it until I get the right answer in there." To clarify I asked if he tries to match his work to what is on the calculator to which he responded, "yes, … if you don't come up with the right answer you don't know what you did wrong so it's helpful to back you up." Other survey participants reported similar actions. One student wrote, "I'd try to change my work to match the calculator answer." Others suggested that they would assume that the calculator solution was correct and then work backwards and look for their errors in the written work. For example, "I would trust the calculator because everyone makes mistakes, so I would use that proposed answer to work back and see my mistake and fix it." Another wrote "I won't turn in a test until my own answers match the calculator ones."

The remainder of the students that chose a graphing calculator produced solution over their written work stated that they made this decision based on their perceptions of the infallibility of the graphing calculator. A few examples of survey responses follow:

> "I would trust the calculator, because unlike humans, calculators don't make computational mistakes for no apparent reason."
>
> "I would probably trust the calculator more because the calculator can't give me the wrong solution."
>
> "The calculator because it can't make mistakes."
>
> "I trust the calculator's answer b/c I know that factors like lack of study/sleep may influence the correctness of my procedure and answer, but a calculator won't make the same mistakes. Thus I will trust the calculator b/c it doesn't err."

I found statements such as these surprising because I have observed the teachers in these classrooms point out particular situations in which the graphing calculator did not produce a correct solution. Only one student alluded to the possibility of the graphing calculator possibly not producing a correct solution, he wrote "I trust my own answer. Sometimes the graphing calculator comes up with weird answers with trig functions or does not find the right answer."

Given the shared meaning of the graphing calculator as a checking tool in the classroom, it is important to better understand the dilemmas that checking can lead to. The data here suggests that students often place the authority in a mathematical problem solving situation in which a calculator is used as a checking tool with the calculator. I believe they are prompted to do this because of the need they feel to reconcile different solutions. It is likely that it goes against common practice in the classroom to leave a solution unchanged if it is not consistent with another produced solution. However, it is important to note that not all students feel the need to reconcile their solutions when faced with this dilemma. Shemika's response to question #13 provides an example of a student who handles the dilemma of two differing solutions quite differently.

Shemika states that she often checks her work; however she doesn't feel the need to reconcile her solution with the graphing calculator. Being able to check and knowing that her answer is correct appears to be just as important to her, but she reacts quite differently to any differences that appear.

A: How often do you tend to check your work on the calculator?

Shemika: Um…whenever, like not on a test. If we're doing homework or classwork and it says don't use a calculator I'll do it and then use a calculator just to check my answer.

I won't change it if it's wrong…just to see if I got the right answer.

A: What is your reason for doing that?

Shemika: Just to make sure I know what I'm doing.

A: What do you do when your answer does not match what is on the calculator?

Shemika: Go back and do it again. Look at every step I did, see where I went wrong. If I can't figure out where I went wrong, I'll just leave the wrong answer.

As was pointed out earlier, Shemika views the ability to check and know whether she is right or wrong as part of the learning process. If her answer does not match she then spends more time trying to better understand the mathematics involved and does not give up control of the situation. She also mentions that she does not do this on tests, just on homework, where she expects learning to take place. Shemika says this provides her a sense of security because she goes to class knowing whether or not she needs more help on the mathematics when otherwise she would assume that she did not need to ask questions.

It strikes me that even Shemika, who does not feel the need to reconcile her solution with the one she produced on the graphing calculator, is assuming that she is wrong. There seems to be a perception that the graphing calculator is an 'other' and is doing things on its own. The students give the impression that they are over-looking their role in using the calculator, when in actuality they are making conscious decisions about what modes to use and what information to input. This separation of self from the tool might be problematic when one considers the many different ways with which the students value the use the graphing calculator. It is certainly an issue that needs further research.

As Aaron's quote at the beginning of this chapter points out, students' may be able to complete a task just fine without a graphing calculator, but with it they may be more sure of their abilities and more comfortable in their circumstances. The students in this study perceive that the graphing calculator is a valuable checking tool. Many indicate that using the graphing calculator in this way often provides them with a sense of security about their mathematics ability. The data from this study also suggests that many students have predetermined routines for making decisions when they produce different solutions with and without the graphing calculator. These decisions are based on routine and are often grounded in where they assign authority when working with a graphing calculator, with themselves or with the technology.

Chapter 6: The impact of the graphing calculator on students' use of time when doing mathematics

"It's a good tool to have, you know. I know like my grandfather was a math major. And like he, he loves it. He says we're like so lucky we have the calculator. Cuz I mean just for sine and cosine functions they had to go through like a little table or something…I don't know. He said everything just took like ten times longer, so." ~ Aaron

When a graphing calculator is used as a tool in a problem solving situation, its use will definitely affect the way that time is spent on that task. Thus, it is foreseeable that the use of a graphing calculator might greatly affect the ways that students choose to spend their time when doing mathematics. In fact, the students in this study often spoke about their use of time as it relates to the availability of a graphing calculator. Take Shemika's work on task #2 for example.[5] Shemika used the graphing calculator very efficiently to find the correct solution and wrote only the solution on her paper. She was able to explain exactly how she would solve the task if she had not had her graphing calculator available. I asked her what made her decide to use the calculator on this particular task and she replied, "I knew that it knew how to do it and I wouldn't have to do it." When I asked if she would have done anything different on a test she said no, but "in a test situation I would have done it to save time, but if I just had a problem to do it would be the fact that I didn't have to do it." So, Shemika recognizes that sometimes using the graphing calculator on a task saves her time and there are times, such as on tests, where it is important to manage her time well.

Like Shemika, all of the case study students mentioned the importance of time in their mathematical experiences and the impact that the graphing calculator sometimes has on their use of time. The data indicates that these students perceive that using a graphing

[5] Shemika's work on task #2 is presented in full in chapter 3.

calculator impacts their decisions about how to use their time in both testing situations and non-testing situations. Furthermore, the data also indicates that working quickly on mathematical tasks is valued by these students. This chapter will present the students' perceptions on how they use the graphing calculator to manage time in both testing and non-testing situations and their perceptions of the value of time in the context of doing mathematics.

Students' perspectives on the impact of the graphing calculator on time management in testing situations

Most testing situations that AP Calculus students take part in are restricted by time in some way. During the data collection period of this study many of these students had recently taken the SAT test and the AP Calculus test, both of which allow for the use of the graphing calculator on at least half of the questions. These tests both have time restrictions and are viewed by the students to be high stakes exams. In addition, to these standardized tests the students had also been expected to perform well on classroom assessments. The classroom assessments were always expected to be completed within the class period, thus time was restricted in these situations as well. Classroom assessments for all of the schools involved typically consisted of both calculator and non-calculator portions, much like the standardized tests. Given the nature of these assessments it is not surprising that the students perceived that the decisions that they made about how to use the graphing calculator within the restricted time allotment were important.

The survey used in this study was not designed specifically to identify how the students perceive the graphing calculator impacts the ways that they manage their time.

This theme emerged from the case study participants. It was not surprising that the students said that using the graphing calculator often saves them time, but it was surprising that this benefit was so important to them. Although the survey was not designed to address this issue, question #12 did offer some insight into how the students feel about this issue in relation to testing. Question 12 asked whether students would prefer to take a test with another student or a graphing calculator. Those that chose the graphing calculator sometimes explained their choice by pointing out that the calculator might help them work more quickly on a test. For example, here are a few of the written responses:

> "I think I would prefer to use a graphing calculator because you can solve and get an answer much faster using the graphing calculator."
>
> "A graphing calculator because two brains will not be better than a fast calculator."
>
> "Calculators are efficient tools in solving problems quickly."

It is important to note that none of the written responses indicated that the calculator might take more time, however this is likely explained by the fact that students that felt that way would have chosen a student to work with over a graphing calculator. These statements are not intended to show that all students feel that the graphing calculator necessarily makes problem solving faster, but that time is important to students especially in testing situations.

The case study students provide some insight into the importance of time and the role the calculator plays in the use of time in testing situations. Once again, time was not something I overtly asked about. However, it was often brought up by the students at which time I would ask them to elaborate. Many of the students indicated that the graphing calculator helped them save time in testing situations, time that could then be used on other problems. For example, when Enoch and I were discussing his work on task #2 he pointed out that he might have decided to use the graphing calculator even sooner had he been in a testing situation. Our conversation follows.

A: If you had been doing this problem at home, would you have used the calculator in the same way?

Enoch: Uh, huh.

A: How about if it was on a test?

Enoch: If it was on a test and there was a time limit…um…I would have used it the same exact way, but I may have decided to use it faster.

A: Do you think you would have gotten a solution here if you had not had the calculator available?

Enoch: No, I would have been stuck at this point right here [he pointed to his written work where the function was set equal to nine]. I would have been thinking and thinking…

For Enoch, the calculator allowed for him to complete the problem quicker and for this particular task he indicates that he would have used the calculator sooner if it had been on a test. This suggests that Enoch sees the calculator as a tool that can save him time. As a matter of fact, on his survey Enoch wrote that he would chose the graphing calculator over a student because "the calculator helps consume[6] time." He mentioned in both his first interview and the reflective interview that the graphing calculator is a particularly important tool to have in the last few minutes of a testing situation. In his first interview I asked how often he used his graphing calculator on tests or exams and he replied, "on tests…if it is coming down to the last minute then I run to the calculator." He repeated

[6] Enoch repeatedly used the word *consume* in this way. It is clear from the context of our conversations that by *consume* he meant *save*.

this sentiment in a later interview when we were discussing the AP exam. Our conversation follows.

A: In what ways does your teacher ask you to use it [the graphing calculator]?

Enoch: The teacher asks us to use to um see if everyone gets the same answer, to show the proper way how to do it. He was preparing us for the AP exam, so we know how to do it to consume time. Um, he's just basically teaching us to show us all the different things the calculator can do just in case we need to go back to it.

A: Was it important for you to have on the AP exam? Did it make a difference for you?

Enoch: If I wasn't properly taught, then it would have made a difference, but everything he taught us was on the AP exam, so I was well prepared. I just…

A: Did you use it very often during the part that you could use it?

Enoch: I used it as far as on the [undistinguishable word] problems. So yeah, I used it.

A: What was your reason for using it on those problems?

Enoch: To consume time. If I was running out of time and I had to get the answer I would put my pride aside and go.

For Enoch, the graphing calculator was a tool that helped him make the most of his time when he was in a situation when there wasn't much time remaining to do the work he needed to do by hand.

Not all students feel that the calculator actually saves them time. Some, like Aaron and Melissa, find that using the graphing calculator sometimes actually takes more time. Given the time constraints in testing situations, for these students a graphing calculator is not necessarily an efficient tool to be used on a test.

As was explained in chapter 3, Aaron does not consider himself a proficient calculator user. He never took the time to learn how to use his TI – 89 calculator and often felt that this placed him at a disadvantage. It is not surprising then, that taking the time to remember how to use his calculator in a testing situation might be frustrating for him. Aaron spoke about how the calculator affected his use of time on tests. Our conversation follows.

A: Since it [using the graphing calculator] was an important part of this class, did you ever feel like you were at a disadvantage?

Aaron: Yes, definitely.

A: Can you tell me a little about that?

Aaron: Well, when we'd have like a calculator test there were some problems where, like, I'd be struggling cuz I knew I had to plug in an integral, take the integral of something. So like I'd have to spend like extra time trying to remember exactly how to plug in the integral of this, to this, to this, for x value, this number, to this number, to this...it was like extra to deal with.

For Aaron, using the calculator would take more time because he was not a proficient calculator user. Aaron did note at another time that using his calculator for simple computations was definitely quicker at times than doing it by hand, but as we see above his knowledge of the graphing calculator prevents him from using the more complex modes in an efficient way.

The only other student that mentioned that using the graphing calculator is not always an efficient use of her time was Melissa who is also the only other student that uses the TI – 89. Melissa points out that she uses the calculator for derivatives that she has forgotten, and for small parts of a larger problem simply because it is faster. However, she also points out that her decisions about when to use her calculator depend on the problem and the situation. She says that she often bases her decisions on time constraints. When we were discussing her work on task #1, I asked her how she would have worked differently if this task had been on a test. She said, "It depends on time constraints. If there were a lot of time I would have used it." Melissa considers the CAS capabilities of her calculator to be more time consuming than working a problem out by hand, much like Aaron. She went on to say that sometimes it is faster to "just plug numbers in on the calculator" (referring to simple arithmetic). However, on her AP exam she didn't use it nearly as often as she thought she would. Our conversation follows.

A: Did you take the AP exam this year? Can you tell me a little bit about it?

Melissa: I thought it was hard. Only cuz I don't like calc. I'd be happy if I got a three. It's just a lot of information you've got to know throughout the year. And I've been struggling with calc for like ever.

A: Was the calculator important to you on that day?

Melissa: Well, yeah.

A: What kinds of things did you use it for?

Melissa: Well, I didn't really use it that much cuz I didn't really see where it would come into play, where it would help me. Or it would just slow me down.

Melissa appears to be aware that for her there are some modes of the graphing calculator that it would take her longer to figure out how to do than if she were to just do it by hand, specifically the CAS modes. However, as with Aaron, she also seems to feel that the calculator will save her time on some simpler computations.

Whether students perceive the graphing calculator will be helpful on a test when time is running out or only on particular types of problems, it is clear that time management in testing situations is very valuable to them. Furthermore, having a graphing calculator available for use on a test greatly impacts the decisions they make about how to manage their time.

Students' perspectives on the impact of the graphing calculator on time management in non-testing situations

It makes sense that students are concerned about their use of time in testing situations given the nature of most assessments that they encounter as calculus students. However, the students in this study indicated that they were concerned about time outside of testing situations as well. All six of the students mentioned that they appreciate that the calculator often saves them time when doing mathematics in non-testing situations.

These students perceive much of the mathematics that they do in non-testing situations to be mathematics that they need to "get through" and to do so in a timely fashion is preferred. For example, when I asked Enoch if he uses his graphing calculator when he is working outside of class as much as he does in class he responded, "I really don't use it unless I'm really trying to get through the work." He explains that when he is doing his homework he is usually just trying to get it done as fast as possible, so the graphing calculator is an important tool.

There are many possible reasons that students would value saving time when doing mathematics. Enoch was never explicit about why he wanted to complete his homework as fast as possible, but Shemika was. Shemika is a senior in high school, she is taking at least three other Advanced Placement (AP) courses. Shemika also works three different after school jobs and is expected to help care for her siblings when she is not at work. When I asked her if she had any general thoughts about the graphing calculator that she'd like to share she responded, "Well, it is good to know the things it can do. Know how to do them yourself, but when it is easier to save time and things like that… cuz I do a lot of things and all the course work that we have and everything else. It is easier to have the calculator because it saves time…" It is easy to imagine that there are many students who, though they many not be quite as busy as Shemika, do have many out of school commitments. For these students it makes sense that a tool that saves time on homework would be quite valuable.

Student perspectives on the value of time when doing mathematics

Whether they were using the calculator to manage their time on tests or to get through a homework assignment faster, these students clearly value time. Research has shown that speed in mathematics is often valued in the classroom (Garafalo, 1989). Even so, I was particularly struck by the importance that these students placed on doing mathematics quickly. Though they speak about finishing homework quicker or having more time to spend on difficult tasks on a test, data suggests that for some students working quickly is valued because they perceive it is an indicator of mathematical success.

To put this in context it is important to understand that typically AP Calculus students are near the top of their graduating class. They are constantly competing for the highest test scores, for the highest class ranking, and to get into the best colleges the next year. They are competing with their classmates as well as with other calculus students worldwide. I think that the competitive environment that these students are in might help explain why working quickly might be valued. When I was speaking with Enoch about how he typically uses his graphing calculator in class he said that he was "competing in class", when I asked him to clarify this statement he said;

Enoch: Like, Rudy, if Rudy…[unintelligible] we like to compete and see who gets the answers first. So I just go to the calculator and 'ok I got the answers blah, blah, blah' you know. I use it to compete.

A: Is competing important?

Enoch: Yeah.

A: Why is that?

Enoch: I realize that in college everybody is just as smart as me. So, I don't want to be the last person to get the answer. I want to be the first person.

Like Enoch, Rudy spoke about why he valued working quickly in class. He said that at home he doesn't use his calculator as often, "well, time-wise, at home you can work it out as long as you want."

Rudy and Enoch are classmates, and it is very possible that working quickly is valued in their particular classroom more than in others. This would explain why Enoch was so explicit about his beliefs about how doing mathematics quickly would set him apart from his equally talented classmates. It seems that for some students the graphing calculator buys them a way of working at a pace that they perceive is highly valued in the classroom.

The graphing calculator has dramatically changed way that calculus students spend their time. As Aaron said at the beginning of this chapter, when his grandfather was in school he had to look up values for trig functions on a table and "everything took ten times longer". The data in this study indicates that time is very important to these students and it impacts the decisions that they make about how and when to use their graphing calculators. They use the graphing calculator to manage their time on assessments and homework. On assessments they do this to manage their limited time. On homework they use their calculators to work quickly because of the need to save time for other commitments, such as other course work and jobs. In addition, some use the graphing calculator as a competitive tool, one that allows them to work quicker than their peers in an environment in which they perceive speed is the key to winning. Though previous literature has shown that speed is often valued in the mathematics classroom, none has considered how the graphing calculator might impact the playing field.

Chapter 7: Discussion

In this chapter I will present the limitations of this study, how the findings inform the initial research questions, the implications of these findings for teaching, and suggestions for future research.

Limitations of the study

It is important to remember that the sample of this study was quite small. Only 111 students participated in the survey and six in the case studies. However, these students do provide a basis from which we can begin to understand how students use the graphing calculator in independent situations and what they perceive they gain from having it available. Given the nature of the themes that emerged in the data and the design of the study, there were times when triangulation between data types was not possible. For example, there was nothing in the survey that addressed the use of the calculator as a tool for mathematical play. Even though the survey was not designed for this purpose, there still did end up being some data in the free response questions that indicates that the larger population had similar experiences with their graphing calculators as the case study participants.

One of the most important components in the design of this study was the stimulated-response reflective interview that took place a few days after the task based interview. The purpose of this interview was to have the students reflect on their mathematical experience with the graphing calculator in the task based interview. In the reflective interview the students often reported emotions related to the mathematics they remember doing. Some might argue that these emotions may not actually represent what the student felt at the origin of the mathematical experience. However, they do represent what the student recalls feeling at the time. Therefore, at the very least this recall reveals what emotional memories the student is taking with them from the mathematical experience. For this reason, I believe that this design feature was actually quite useful for gathering data about the students' perceptions of the emotional pay-offs that are associated with graphing calculator use.

Addressing the initial research questions

The overarching goal of this study was to understand how and why students use the graphing calculator in independent situations. Specifically, what reasons do students give for choosing to use the graphing calculator in a particular way; and what are the 'pay-offs' that students perceive are associated with having a graphing calculator available to use? The data indicates that these students decide to use the graphing calculator to perform the following four actions; (1) to change the cognitive demand of a task, (2) to engage in playful mathematical activities, (3) to check their written solutions, and (4) to manage time.

It appears that this value that the students have placed on all four actions is linked to both a mathematical and affective pay-off. As described in the introduction a mathematical pay-off is not surprising, however the attention to affect revealed that all actions that the students valued being able to take on the graphing calculator also had an affective pay-off. Feelings of security are only a small range of the emotions that students reported feeling and that I observed associated with graphing calculator use. Across all four modes of calculator use these students showed evidence of emotions such as frustration, annoyance, worry, happiness and even elation, for example. Sometimes the emotion was exhibited physically, like with Enoch who actually gave himself a pat on the back when he checked is work and found that his written solution matched his calculator produced one. Other times these emotions were exhibited verbally.

I believe there were many incidents in which I observed an affective pay-off in response to an action taken on the graphing calculator that the students themselves might not be aware of and thus did not speak directly about. For example, when Rudy used his graphing calculator to explore task #3 he said he used it because "At first I didn't understand it. I was trying to figure out what they meant." In watching Rudy negotiate this problem and use the table on his graphing calculator to understand the effect of varying values of *k*, I saw him become more confident and pleased with his work. As he began to see a pattern with different values of *k* he also began to sit up straighter and grin a bit. Rudy did not talk about an emotional response, but I believe that his physical actions provide evidence that his work on the graphing calculator evoked an emotional pay-off that complemented the mathematical pay-off.

Goldin (2006) has argued that it is important to understand students' affective pathways when problem solving so that we can understand how to help students on negative affective pathways deal with frustration and impasse and turn those into more positive emotions like curiosity, bewilderment, motivation and maybe even elation. It seems as if the graphing calculator often provides a means for these students to deal with frustration and impasse and move on to a more productive affective pathway. In chapter three we saw how Enoch had reached an impasse on task #2 and used the graphing calculator to change the nature of the problem and eventually successfully solve the problem. Similarly, in chapter four we saw Rudy engage in mathematical play on task #3 to move past his feelings of frustration and confusion. Rudy indicated that he most likely would have given up on the task if he had not had his graphing calculator. The choice of using a graphing calculator changed the situation from one of defeat and frustration to one in which he was curious because he had a tool to easily explore with. I believe when students are working independently one of the most important roles of the graphing calculator, no matter what actions are being taken, is to help maintain a productive affective pathway that supports their mathematical success.

Three of the students that participated in the case studies attended an urban school: Enoch, Rudy, and Shemika. All three of the students indicated in their first interview that they felt unprepared for AP Calculus. Their high school had adopted an integrated curriculum that they were required to enroll in and thus had not taken typical algebra, geometry, or algebra two courses. Their AP Calculus course consisted of 21 students at the beginning of the school year, but when I arrived in April only 6 students that remained. It is important to acknowledge that for these students the graphing calculator was a tool that they believed allowed them access to a mathematics course that they felt unprepared to take. Studies are needed to better understand how a tool like the graphing calculator might provide access to higher mathematics for students who have not traditionally taken those courses. It is important to consider both the limitations and affordances of such situations.

Implications for teaching

Even though the students use graphing calculators for things that we might find natural (to save time or check, for example), they are doing so in a way that hides from them the very fact that they are doing mathematics. There were many instances throughout the data collection process that I witnessed students making decisions about how to use their graphing calculators and in doing so demonstrated deep mathematical understanding. However, when they spoke about their calculator use they did not recognize their work as 'doing mathematics'. As a matter of fact, some explicitly stated that they had avoided doing any mathematics. Take Enoch's work on task #2 that was presented in Chapter 3. He essentially took an algebraic task and transformed it into a geometric task in order to find a solution. However, when asked to describe how he used the graphing calculator he said, "I got the answer from the calculator. I couldn't, like, get it on my own. I don't know."

The values the students perceive accompany actions that they perform on the graphing calculator are not all rooted in commonly suggested uses for the graphing calculator in most curricula. I reviewed three commonly used AP Calculus textbooks to determine how these texts address the incorporation of technology into the calculus curriculum (Hughes – Hallet, et. al., 1998; Larson & Hostetler, 1994; Stewart, 1998). The authors of all three of the texts addressed the importance technology in the calculus curriculum up front and in similar ways. They assume that the students have either graphing or CAS technology available. To incorporate these tools into the curriculum they have incorporated a few exercises in each section that either uses the technology as a means for discovery, to reinforce concepts, or as an efficient problem-solving tool. However, the merits of the graphing calculator as a means for changing the cognitive demand of a task, as a means for mathematical play, or even as a means for checking are not explicitly stated in these commonly used texts. I believe this is problematic because it influences the students' notion of what is valued as 'doing mathematics'.

There were many instances throughout the data collection process that I witnessed students making decisions about how to use their graphing calculators and in doing so demonstrated deep mathematical understanding. However, when they spoke about their calculator use they did not recognize their work as 'doing mathematics'. As a matter of fact, some explicitly stated that they had avoided doing any mathematics. Take Enoch's work on task #2 that was presented in Chapter 3. He essentially took an algebraic task and transformed it into a geometric task in order to find a solution. However, when asked to describe how he used the graphing calculator he said, "I got the answer from the calculator. I couldn't, like, get it on my own. I don't know."

Given the importance that the students placed on being able to change the nature of a task and to engage in mathematical play, it seems as if these skills might be incorporated into the curriculum. Though it is not explicitly encouraged in most texts, these types of actions are how mathematicians use technology when they are problem solving (Schoenfeld, 1992). As a matter of fact, the students did not seem to recognize their work on the graphing calculator as 'doing mathematics' when in truth actions like these are actually more similar to the work that mathematicians do than the rote repetition of algorithms on paper that they seem to associate with 'doing mathematics' (Polya, 1945; Schoenfeld, 1992). I can imagine a curriculum in which graphing calculator use is encouraged especially when a student encounters a moment of impasse. A curriculum of this type would also make clear that these sorts of actions which promote understanding and risk taking, is 'doing mathematics' and is valued in the community of mathematicians.

When Enoch spoke about his graphing calculator use, he always did so with a bit of trepidation. Though his decision to use the graphing calculator often resulted in success, he did not feel good about his decision. For example, when I first met Enoch he would avoid using his calculator as long as possible. I asked Enoch why he tried to avoid using his calculator and he responded, "It makes me feel stupid." When I asked him to explain this feeling he said, "Because I'm not using my brain." During our last meeting Enoch reflected on his work on the graphing calculator and we discussed the mathematical understanding that it required to be able to use the graphing calculator in the way that he did he. The last thing I asked him was if he had any final thoughts that he'd like to share. He responded, "Just that this is a good experience. It really helped me look at myself. I mean, I might even use the calculator more now (laughs)." I think that it is necessary to point out the mathematical understanding that is involved when one uses a graphing calculator to change the nature of a task or even to engage in mathematical play. These are both methods that mathematicians use on a regular basis (Polya, 1945; Schoenfeld, 1994) and should be valued in the calculus classroom as well.

If we listen to the students voices we hear that they value using the graphing calculator in ways very similar to the ways that mathematicians use tools of all types. However, if we listen more closely we also hear that valuing the graphing calculator in these ways presents a conflict for them. It is a conflict because they do not perceive that one should use tools in these ways when doing school mathematics. Their voices also reveal that they value both the affective and mathematical pay-offs that come with graphing calculator use. This is conflicted by their perception that in school affect is unacknowledged with respect to doing mathematics. Listening to the students' voices when creating a curriculum in which the graphing calculator is valued as a tool may result in students being less hesitant to use them in even more sophisticated ways.

Future research

Future studies should be designed to better understand how students, at all levels of secondary and undergraduate mathematics, use the graphing calculator in the four ways that these students valued most. For example, the students in this study have informed us that they value checking, but are conflicted when their answers differ from that on the graphing calculator. In addition, they have revealed the importance that they place on speed in mathematics. Given the common promotion of the graphing calculator as a checking tool and a tool to save time in the classroom, I think it is particularly important to understand the students' perspectives on these modes of use on a larger scale. In contrast, I think further qualitative studies are needed on the use of the graphing calculator to change the cognitive demand of tasks and to engage in mathematical play. We are just beginning to understand calculator use in this respect and more in-depth data is necessary to build on what has begun here.

Conclusion

This study has provided the students' voice to the impact that the graphing calculator has on their mathematical experiences. The data indicates that AP Calculus students value the ability to change the cognitive demand of tasks, the ability to engage in mathematical play, to check their written solutions, and to manage time effectively when doing mathematics. More importantly, the students perceive that using the graphing calculator in each of these ways provides them with both a mathematical and affective pay-off. However, they perceive that this pay-off doesn't seem to have a place in their perceptions of school math. Most striking is that the ways in which students say that they value using their calculators to solve problems does not coincide with their perceptions of what it means to 'do math'. However, the data indicates that their actions are actually very much aligned with what mathematicians would define as mathematical problem solving (Polya, 1945; Schoenfeld, 1992). In the ongoing conversation about if and how graphing calculators should be incorporated into curriculum and assessments we need to listen to these students' voices and consider the mathematical and affective pay-offs that they perceive to be associated with its use.

Appendix A: Survey Instrument

School ___________________________Name ________________________________

1. Year (circle one): Freshman Sophomore Junior Senior
2. Gender (circle one): Female Male
3. Please indicate the group(s) in which you would include yourself:

☐African American or Black
☐Asian, Asian American, or Pacific Islander
☐Mexican or Mexican American
☐Puerto Rican
☐Latin American, South American, Central American, Hispanic American, or Latin
☐White
☐Other / Interracial

4. Have you ever attended school outside of the United States (circle one)? Yes No

   If so, where? ______________When did you begin school in the U.S.? __________

5. Do you have a graphing calculator to use outside of school (circle one)? Yes No

   If so, which one (circle one)? TI – 83 TI – 89 Other ___________

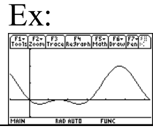
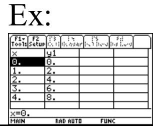
What grade did you receive in each of the following courses?

	Course	A (90 – 100)	B (80 – 89)	C (70 – 79)	D (60 – 69)	F (< 60)
6a.	Algebra I					
6b.	Geometry					
6c.	Algebra II					
6d.	Precalculus					
6e.	Calculus – 1st semester					

Did you use the graphing calculator as a part of these courses? If so, how often?

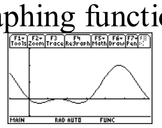
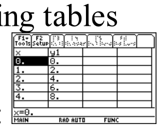
	Course	Yes / No	Calculator Name	Never	Seldom	About half the time	Frequently	Always
7a	Algebra							
7b	Geometry							
7c	Algebra 2							
7d	Precalc							
7e	Calculus							

Rate how comfortable you are using your graphing calculator in the following ways:

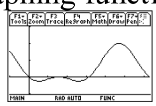
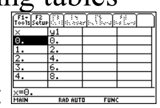
	Mode	Not at all Comfortable	Somewhat Uncomfortable	Neutral	Somewhat Comfortable	Very Comfortable
8a.	Numeric computations Ex: 2 + 3					
8b.	Symbolic computations Ex: solve(2x + 1 = 4, x)					
8c.	Graphing functions Ex: 					
8d.	Using tables Ex: 					
8e.	Statistics Ex: edit list					

	& stat plot					

Rate how often you use your graphing calculator in the following ways:

	Mode	**Never**	**Seldom**	**About half the time**	**Frequently**	**Always**
9a.	Numeric computations Ex: 2 + 3					
9b.	Symbolic computations Ex: solve(2x + 1 = 4, x)					
9c.	Graphing functions Ex: 					
9d.	Using tables Ex: 					
9e.	Statistics Ex: edit list & stat plot					

Please provide the reason you <u>most often</u> use the graphing calculator in the following ways:

	Mode	I think it will be helpful in solving a problem	Either my teacher or textbook suggests it	Just to check something
10a.	Numeric computations Ex: 2 + 3			
10b.	Symbolic computations Ex: solve(2x + 1 = 4, x)			
10c.	Graphing functions Ex: 			
10d.	Using tables Ex: 			
10e.	Statistics Ex: edit list & stat plot			

11. If you were given the choice to:
    a) take a test with another student (who you do not know) *<u>or</u>*
    b) use a graphing calculator on the test, which situation would you choose and why?

12. Would your answer to #11 change if you were able to choose the student for option a)?
    If so, why?

13. Imagine the following situation: You solved a problem on your own and then used your graphing calculator to check your solution. The calculator gave you a different solution than the one you got when you worked the problem on your own. Which answer do you trust? Why?

Appendix B: Student graphing calculator work on the tasks

Student graphing calculator solutions for task #1

Maryanne and Melissa were the only two students to use their graphing calculator on task #1. Shemika did not attempt the task, stating she didn't remember what asymptotes were. Enoch, Rudy, and Aaron all attempted to solve the problem without the graphing calculator, however none were successful. Maryanne and Melissa both used the graphing calculator to graph rational functions before determining their solutions. Maryanne's work is described below.

Task #1: Maryanne

Maryanne first read the tasks and wrote down $\frac{\quad}{x-3}$. She looked at it for a few minutes and then left the problem to work on another task. She returned to the task after taking a first run through each of the four tasks. She looked at what she had written and then picked up her graphing calculator, a TI-83+, and entered *y1=1/(x-3)* and graphed it. Next she changed the numerator in *y1* to *(2x – 1)* and examined the graph. Finally, she changed the numerator one more time, to *(-2x – 1)* and examined the graph. She nodded as she looked at this final graph and wrote her solution, $\frac{-2x-1}{x-3}$ on her paper.

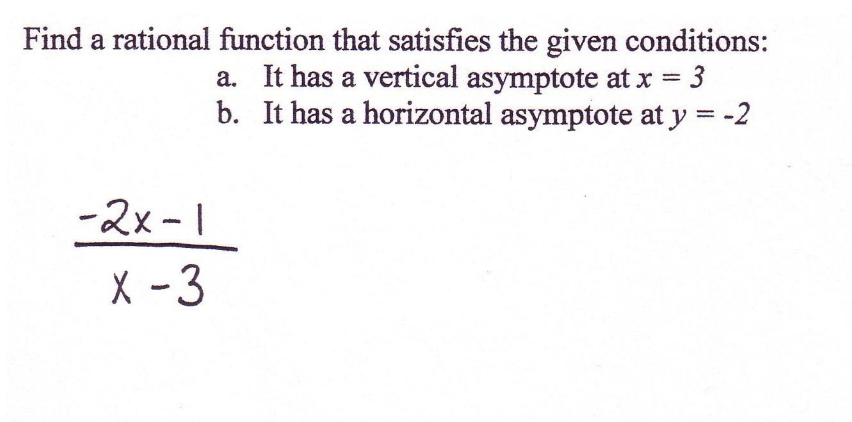


Figure 11: Maryanne's written work for task #1

Maryanne used the graphing calculator to try out a few possibilites before determining her final solution. Melissa used the graphing screen on her calculator as well in her solution, but in a different way. Note that Melissa is using a graphing calculator with CAS capabilities, though she doesn't use them. Melissa's graphing calculator and written work follow.

Task #1: Melissa
Melissa begins this task by turning on her graphing calculator, setting it aside, and then sketching a coordinate plane with a dotted line at $x = -2$ to represent an asymptote. She quickly scratches out this graph, presumably because she realized that she drew the asymptote vertical instead horizontal. She immediately sketches a second coordinate plane with dotted lines at $x = 3$ and $y = -2$.

Next she picked up her TI-89 calculator and looked at the graph of $\frac{1}{x}$. Then she changed $\frac{1}{x}$ to $\frac{1}{x-3} - 2$ and examined the new graph. The window was set in such a way that $x = 3$ was not in the viewing screen, so she adjusted the viewing screen and traced along the graph. After tracing along the graph both to the right and left of $x = 3$ and along the extreems, she wrote her answer down on her paper, $y = \frac{1}{x-3} - 2$.

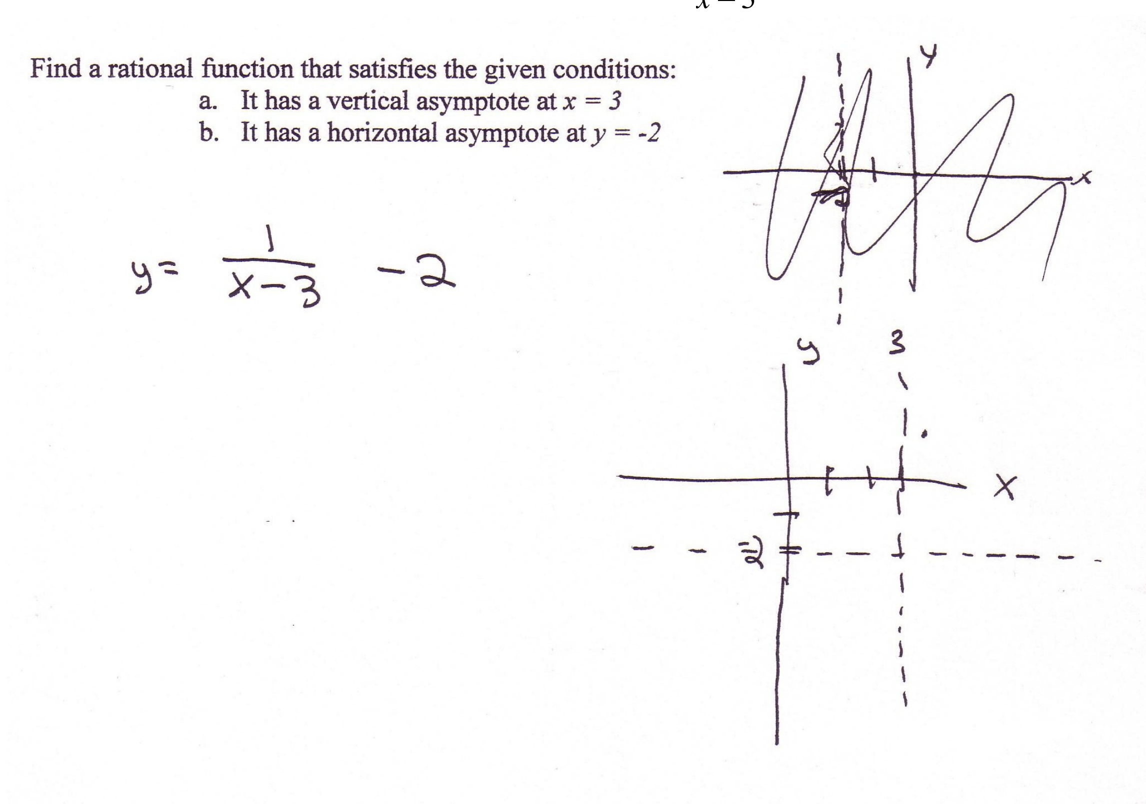


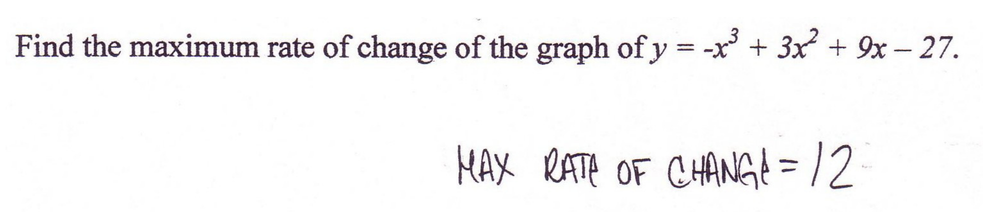
Figure 12: Melissa's written work for task #1

Both girls used their graphing calculators effectively and came up with correct solutions. Melissa had much more interaction with the graphs she produced than Maryanne, by tracing them and changing the viewing screen, though both girls appear to have used the graphing calculator to construct graphs instead of considering the algebraic work involved in determining the solution without a graphing calculator.

Student graphing calculator solutions for task #2

Task #2 encouraged graphing calculator use more than any other task. This may have been because it was similar to typical calculus problems they might have encountered in their calculus courses. Four of the six students used their graphing calculator on this task in some way. The only exceptions were Melissa and Aaron, who attend the same school, who worked entirely by hand and presented incorrect solutions. The other students' solutions are presented below.

> **Task #2: Shemika**
> Shemika read the problem and immediately picked up her graphing calculator and went to the y = screen where she entered *nderive*$(-x^3+3x^2+9x-27)$. She glanced at the graph and decided to adjust the graph window to frame the area of the graph that contained the maximum. Once she had the maximum in view she used the maximum tool in the graphing mode of the calculator to approximate the maximum value. The tool said that the maximum was at $x=1$ and the maximum value was 11.9999. She then evaluated the function at x = 1 and got 12. At this point she writes on her paper for the first time, "max rate of change = 12".


Find the maximum rate of change of the graph of $y = -x^3 + 3x^2 + 9x - 27$.

MAX RATE OF CHANGE = 12

Figure 13: Shemika's written work on task #2

Shemika's solution is very concise. There is no work on her paper, and yet she used the graphing calculator in a very purposeful way. As you will see, her solution method was far different from the other students.

Maryanne used similar tools on her graphing calculator as Shemika did, however she used them in a very different context. Her work is described below.

Task #2: Maryanne
Maryanne began this task by working on paper. She first wrote down $y' = 27 +$ and then crossed it out. Next she wrote the first derivative, set it equal to zero, and factored it (incorrectly). Directly below that work she sketched a number line with what she thought were the roots of the derivative marked off and the values within each section denoted above the number line (See Fig. 14). She looked at this for a moment and then crossed out the number line. Next she wrote down the second derivative, a second number line denoting x = -1, the root of the second derivative. She then began to evaluate the first derivative at -1, looked at it for a while, and then left it and went to work on another task for a while.

When Maryanne returned to the task she immediately crossed out her previous work. She went back up and read the problem, and above the phrase 'rate of change' she wrote 'slope', then began to write again. She began by rewriting the first and second derivative and a number line with x = -1 denoted on it. Then she picks up her graphing calculator, a TI-83+, and graphs the original function, $y1 = -x^3 + 3x^2 + 9x - 27$. After changing the window using the zoom standard command she looks at the graph for a while. Next she returns to the y= screen and inserts *nderiv* in front of the original function she had entered, changes her mind and instead enters *nderiv(y1, x, x)* in *y2* which commands the calculator to sketch the graph of the original function, *y1*, on the same screen. After looking at the graph of both functions together, she zooms out so that she can see the functions in a larger window. She follows this inspection of the graph with the use of the max tool to find the maximum of *y2*, the derivative. The calculator determines the maximum to be 12 at x = 1. Maryanne considers this for awhile, returns to her written work where she crosses out the number line she had drawn with x = -1. She writes down her solution, maximum slope at x = 1 is 12. Finally, she finishes by sketching a number line one more time with x = 1 denoted.

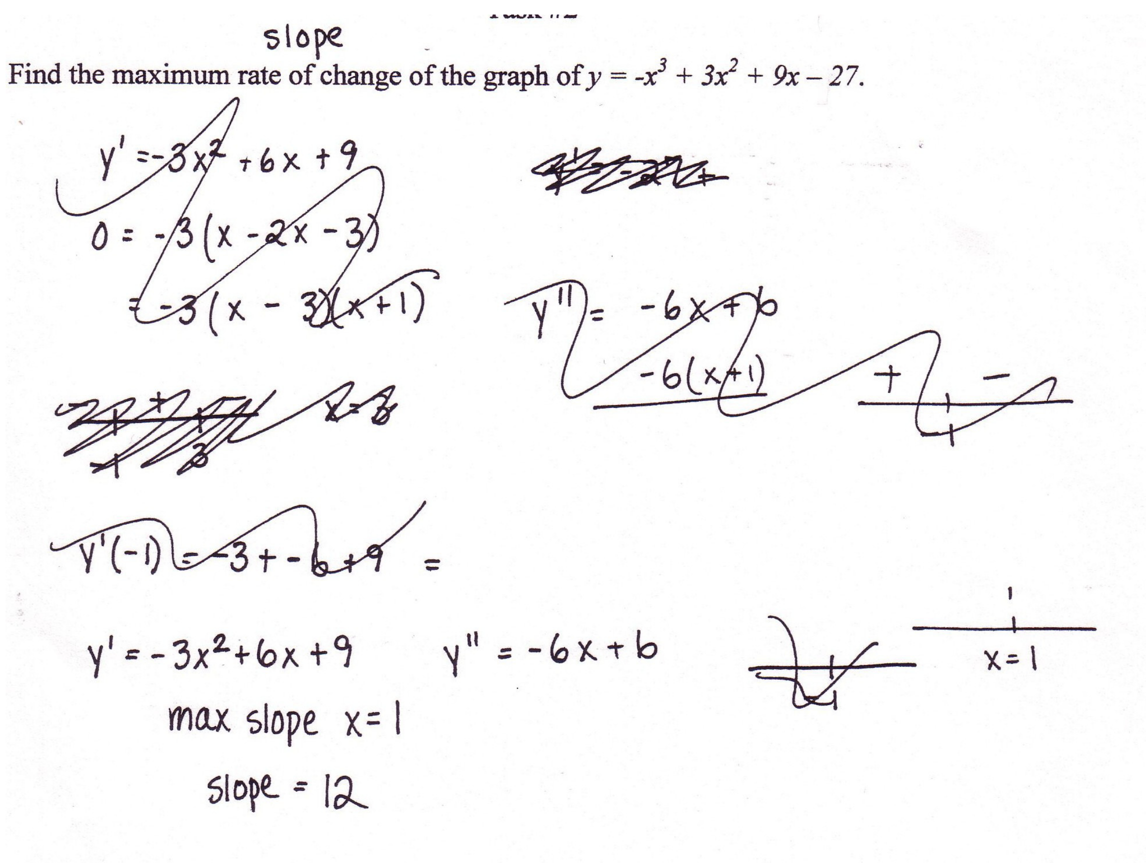


Figure 14: Maryanne's written work on task #2

Maryanne also used the graphing screen and maximum tool of the graphing calculator, however her purpose appears to be quite different from Shemika's who used the graphing calculator so precisely. Maryanne was effectively determined a correct solution to the problem both on her graphing calculator and with her calculations done by hand.

Enoch's use of the graphing calculator is described next. Enoch presents yet another way that the same tools on the graphing calculator might be used to solve this task.

> **Task #2: Enoch**
> Enoch begins this task by writing down the derivative of *y* and setting it equal to zero. He proceeds to try to solve this equation, but mistakenly sets the equation equal to 9 and tries to factor (see Figure 15). "I don't think I'm doing this right…there's something wrong…I need a calculator," he says as he reaches for his graphing calculator. He then enters the original function into *y1* and looks at the graph of the function.

In order to get a picture of the important sections of the function, Enoch changes his window so that he can see everything between x = 10 and x = -10. Enoch then uses the maximum tool and finds the maximum of the original function to be at x = 3. After looking at his solution for a few moments, Enoch wrote the second derivative, $-6x+6$ on his paper and proceeded to graph it. Then he traced the graph of $-6x + 6$ and wrote his solution, $x = 1$ on his paper.

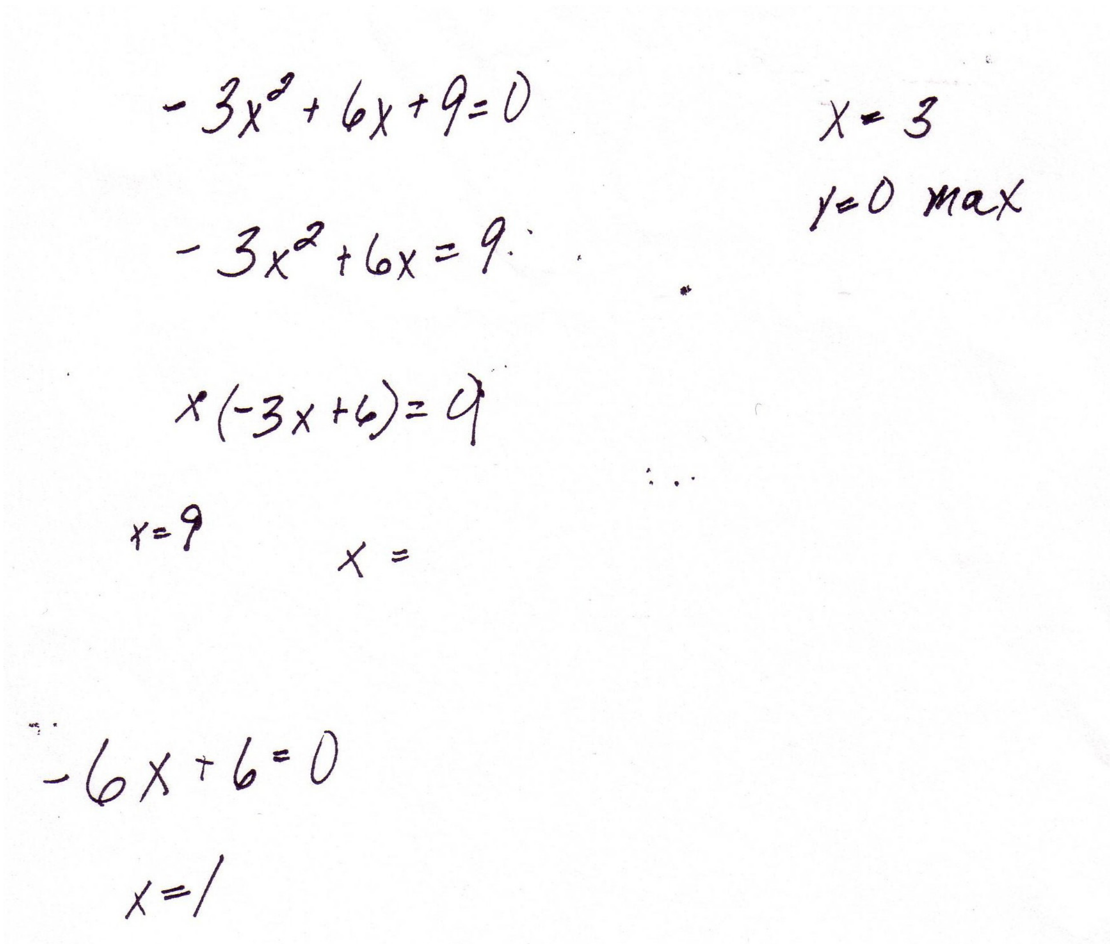


Figure 15: Enoch's written work on task #2

Though he attempted to solve this task by hand, Enoch was not able to do so. He made some algebra errors early on that prevented him from being able to find the maximum rate of change. He was able to use the calculator to find a solution however. He is correct that the maximum rate of change occurs at x = 1, though he did not write down what that rate of change is. Enoch used the same tools as both Shemika and Maryanne; however he certainly did not use them in the same ways.

Rudy attacked this task in a way that was similar to Enoch, however when he had difficulty solving in by hand he used his graphing calculator very differently than the previous student examples. Rudy's work on task #2 is described below.

Task #2: Rudy

Rudy began by reading the task and writing down the function given in the problem, $y = -x^3 + 3x^2 + 9x - 27$. He then set this function equal to zero and tried to solve for x, incorrectly. After quite a few algebra errors Rudy arrived at $x = \sqrt{1/2}$ (see Figure #). He then picked up his TI-83+ calculator and entered $nderive(-x\char`^3 + 3x\char`^2 + 9x - 27, x)$ on the home screen. This produced an error message. He tried to fix this error by adding a '0' between the 27 and *x* in the original command, but this also produced an error message. Next Rudy went to the *Y*= screen and entered the original function, $y = -x^3 + 3x^2 + 9x - 27$, in *y1* and proceeded to the table screen. On the table Rudy scrolled up and down the *x* values for a few moments. (It is important to note that he never scrolled to values of *x* < 3). He then changed the table settings so that he could enter *x* values and entered $\sqrt{1}$. At this point Rudy paused for quite a while. After the long pause Rudy returned to the *Y*= screen and entered $-3x^2 + 6x + 9$ in *y1*. He then scrolled through the table. He scrolled up and down the table repeatedly between *x* values as low as 3 and as high as 112. Once again, he pauses. When he resumes work, he scrolls again and this time scrolls down to 0 and stops. He laughs and says "There it is" and writes his answer down on a separate sheet of paper.

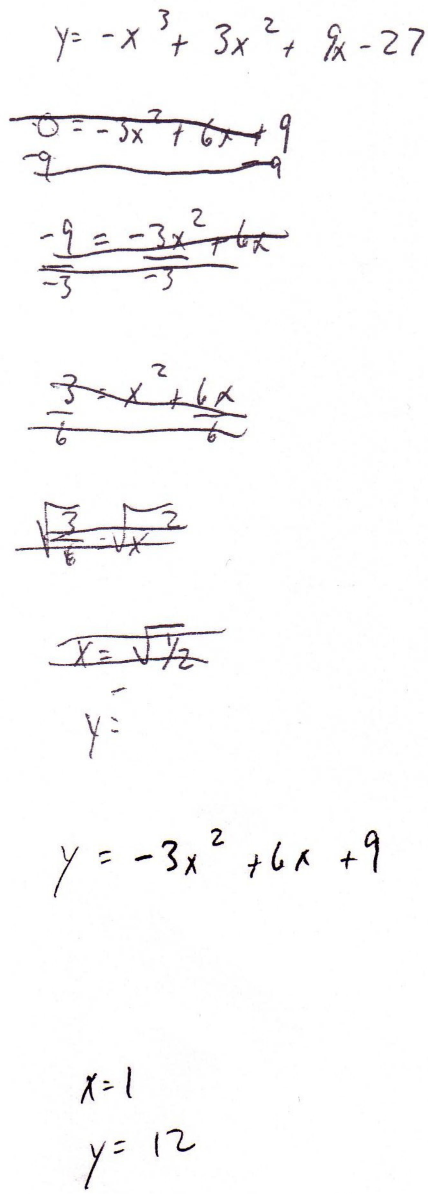


Figure 16: Rudy's written work on task #2

Rudy ran into similar problems as Enoch in his written work, but used the table to explore the function and ultimately determine his answer.

Student graphing calculator solutions for task #3

Task #3 proved to be a challenge for all six students. No one was able to correctly solve the problem. Most of the students opted to substitute values for *k* to get a specific solution, but were not successful with that either. Rudy was the only one who chose to use his graphing calculator to explore this problem. A description of his work follows:

> **Task #3: Rudy**
> Rudy read this task for quite a while. He asked, "Can k be anything?" To which the researcher replied it could be any real number. Rudy then wrote down $7 - x =$ and picked up his calculator. He entered $y1 = 7 - x$, graphed it, and quickly went to the table. He scrolled up and down the table of values between $x = 2$ and $x = 9$ and paused for a moment. Next he returned to the $y =$ screen and changed the function from $y1 = 7 - x$ to $y1 = 6 - x$ and went back to the table. On the table he scrolled between $x = 1$ and $x = 8$. Then he wrote on his paper, "Depending on *k* there are 6 numbers that make *x* greater than -2 and less than 5. "

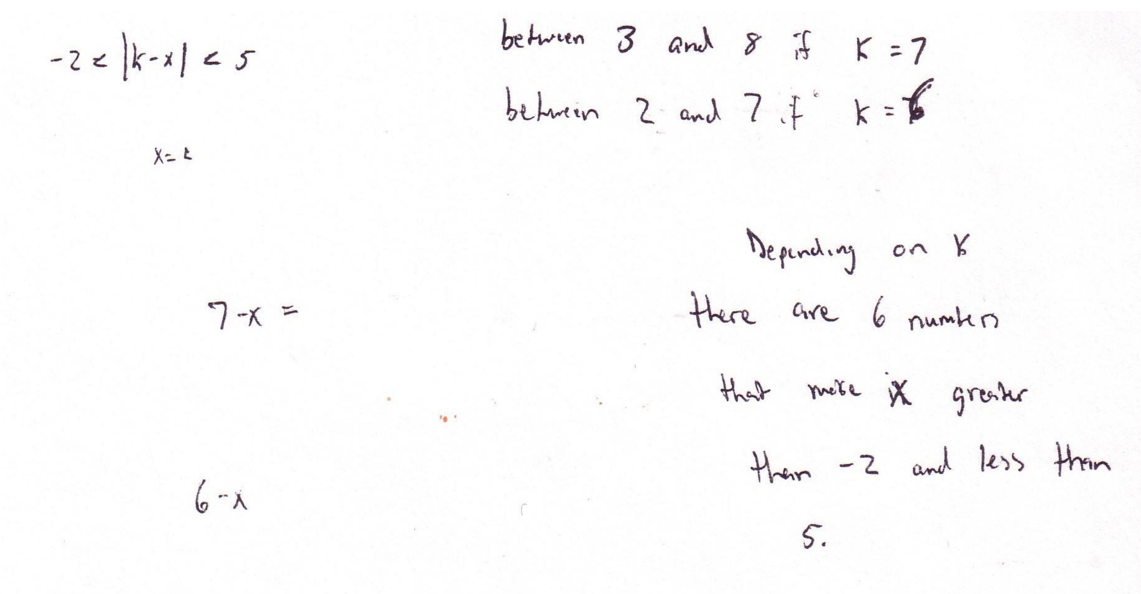


Figure 17: Rudy's written work on task #3

It must be noted that Rudy did not solve the problem correctly. However, he did explore the problem in a very meaningful way using the graphing calculator. This exploration allowed him to make a claim about the solution set for the inequality. Rudy came closer to a solution for this problem than any of the other students.

Student graphing calculator solutions for task #4

Task #4 was quite different in structure than the other tasks. It simply asks for an example of a function that satisfies the equality. Many of the students were able to come up with an example quite easily. However, if they did I challenged them to come up with more possible solutions. Four of the students chose to use their graphing calculators when working on this task. Aaron, Enoch and Rudy had similar strategies in using their graphing calculators. Their work is presented below.

Task #4: Aaron
After reading through the task Aaron immediate wrote down x^2 as a solution. The researcher then asked him to come up with a few more examples, which prompted him to write down sin *x*. Next he picked up his calculator, a TI-89, and entered $\sin\left(\frac{\pi}{3}\right)$ on the home screen and got $-\frac{\sqrt{3}}{2}$ as a solution. Then he entered $\sin\left(\frac{-\pi}{3}\right)$ and got $-\frac{\sqrt{3}}{2}$. He stared at this for a long while. After a few moments he drew a 30-60-90 triangle on his paper and labeled the lengths of the sides. Once again, he paused. He then picked up his calculator and calculated both $\cos\left(\frac{\pi}{3}\right)$ and $\cos\left(\frac{-\pi}{3}\right)$ and got $\frac{1}{2}$ as an answer both times. He then wrote down sin *x* and cos *x* as solutions. Finally, he calculated both (3)^3 and (-3)^3 on his calculator and wrote x^3 as a solution as well.

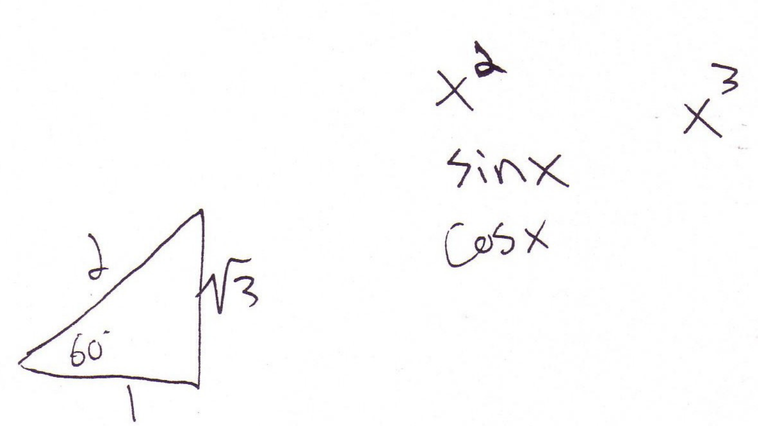


Figure 18: Aaron's written work for task #4

Task #4: Enoch

After reading the problem Enoch wrote down $-x^2$ and $y = x^2$. The researcher asked him if he could come up with a few more examples and he wrote down $\int_{3}^{-3} \cos x dx$. He then picks up his TI-83+ calculator and enters $\cos(1)$ and $\cos(-1)$ on the home screen and says, "It works." The researcher asked, "Does it work for everything?" Enoch turned to his calculator again and entered cos(-2), cos(2), cos(3), and cos(-3). Enoch then stopped and asked a few clarifying questions about the task. He then stated that functions with an even power will work. The researcher asked if $y = x^2 - 2$ would work. Enoch entered this function into the *Y*= screen and looked at the table. He scrolled up and down the table and then said, "yep, it works." He finished by stating that, "basically any function that is squared or has an even power will work", and wrote four examples on his paper (see Fig. 19).

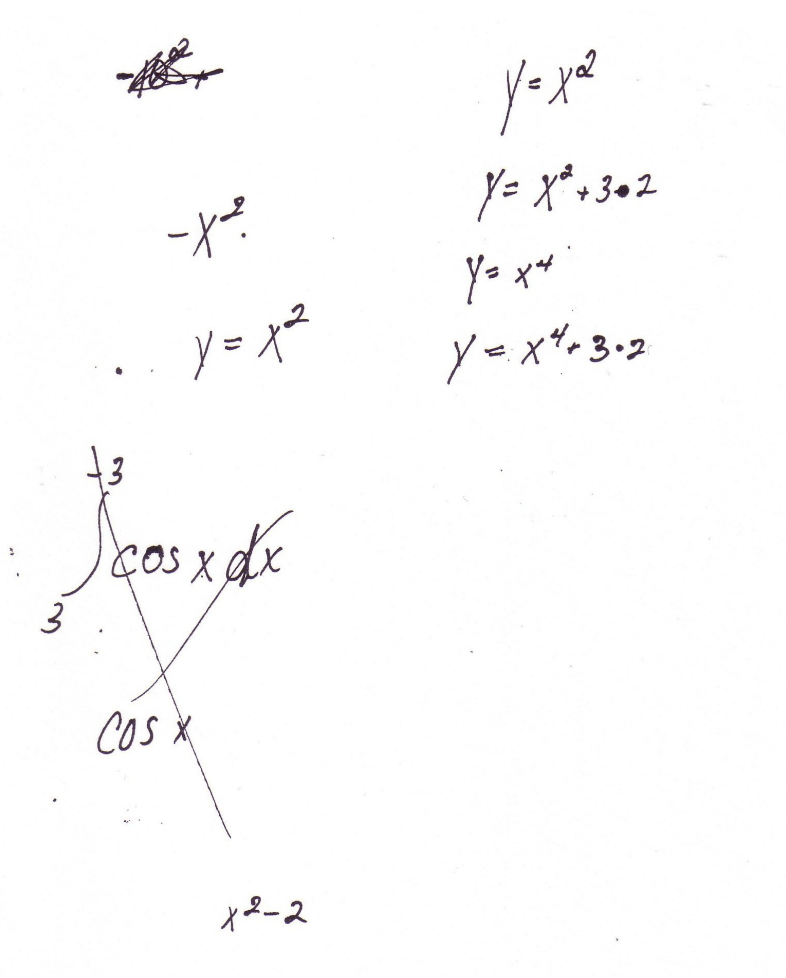


Figure 19: Enoch's written work for task #4

Task #4: Rudy
Rudy first wrote down $6x-3$ and then asked for clarification on what $|f(x)| = f(|x|)$ means. After an explanation Rudy pulled out a scratch sheet of paper and wrote down $4x^2+3x+5$ and then used his calculator, a TI-83+, to evaluate the expression at x = 2. He then scratched out the expression and wrote down a simpler function, $y=(4x^2+5)$. Rudy used his calculator to evaluate the function at x = 2 and x = -2 and then determined that the function did work. Next he followed the same evaluating as testing procedure to determine that $y=3x^2+16$ and $y=x^4+3x^2-2$ were also solutions.

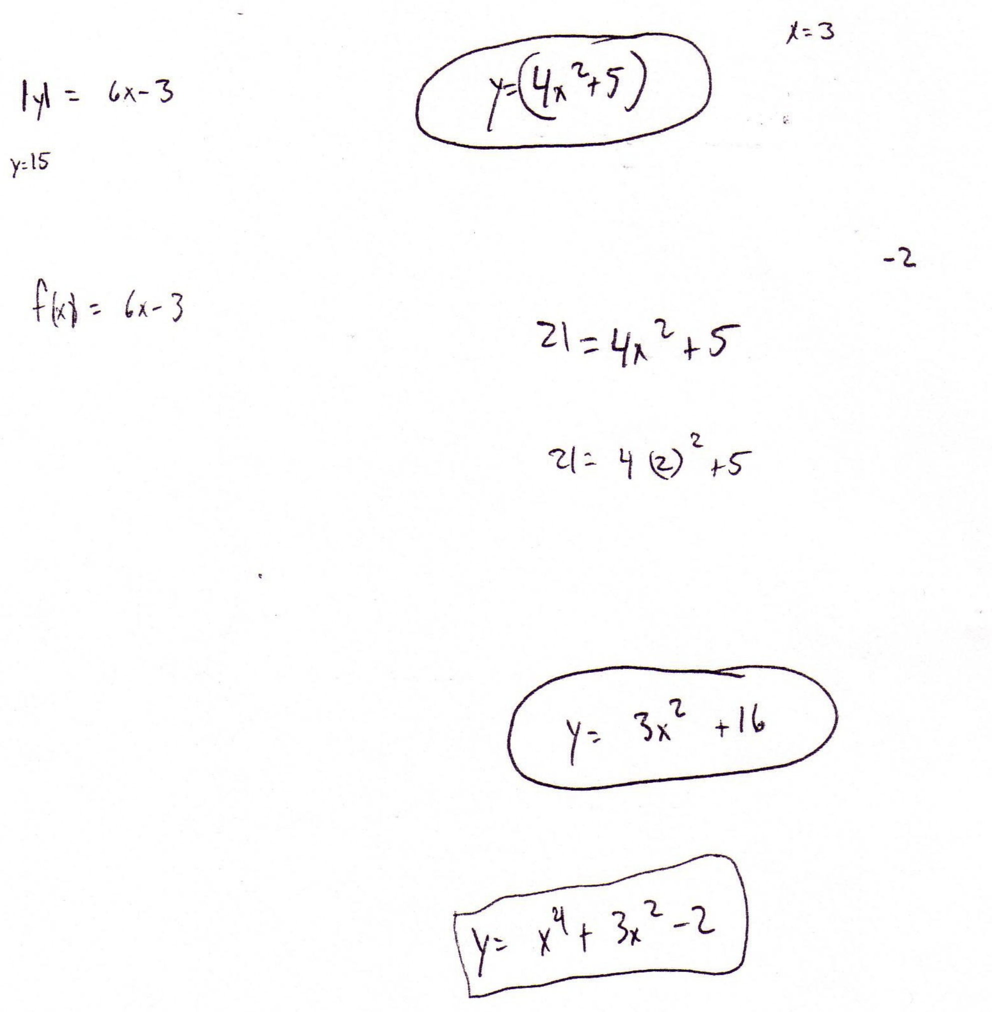


Figure 20: Rudy's written work for task #4

Notice that all three students used a point-wise method to determine whether or not a function was a solution. This method was problematic given the small range of values that they decided to test. Both Enoch and Aaron ran into problems with the function $y = \cos x$. They both tested values of x for which the statement is true, but never tested values in the domain which would have revealed that the function is not a solution.

Melissa used her graphing calculator on this task to examine possible functions in a more global manner. Her work is described next.

> **Task #4: Melissa**
> Melissa read the problem and wrote down $y = \cos x$ and $y = 5x$ relatively quickly. The researcher asked her how she knew they were correct at which time she picked up her TI-89 calculator. She graphed $y1 = abs(\cos(x))$ and $y2 = \cos(abs(x))$ on the same screen. After looking at the graphs, she went back to the *Y*= screen and looked at the functions again. Next she looked at each graph, *y1* and *y2*, independently. Without saying or writing anything down, she went on to enter $y1 = abs(5x)$ and $y2 = 5abs(x)$ and looked at each graph. She then entered $y1 = abs(x^\wedge 2+1)$ and $y2 = abs(x^\wedge 2)+1$ and looked at each graph. Finally, she wrote down her solutions, $y = 5x$ and $y = x^2 + 1$ and scratched out her original response, $y = \cos x$.

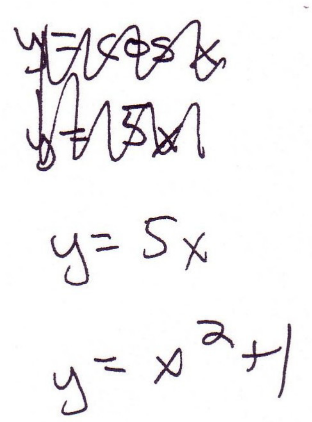


Figure 21: Melissa's written work for task #4

Melissa's methods proved to be quite efficient. Not only did she verify graphically that $y = 5x$ and $y = x^2 + 1$ are solutions, but she also determined correctly that $y = \cos x$ is not.

Appendix C: Interview Transcripts

Aaron: Transcript of 1st interview (6/13/2006)

Did you use the calc in algebra or algebra 2?

A: just for division or something like that…big numbers.

What kind of things do you use it for in calculus – in class?

A: Um, graphs, we do a lot of graphing so we could, we know what kind of derivative to take...stuff like that…I don't know. I did derivatives on it, I do integrals on it. I do like uh, graph things to find like points of intersection. If I want to find like the volume of something, rotate it around the axis. I graph, find the points of intersection, take the integral.

How does Mr. Phillhower use it in class?

A: He uses it for everything. He loves the calculator.

What do you feel about that?

A: Uh…I don't know…I don't use – I don't know the calculator as well as a lot of my classmates really. I don't know why…I just never really played around with it on my own or anything.

Do you know the things that Mr. Phillhower asks you to do on it?

A: Yeah. I pretty much know what to do…but…not like 100%, I've got to look at it a while and figure it out.

What sort of things give you trouble with the symbolic computations? (he said not comfortable on the survey)

A: well, its just like… you have to put solve, and then you have to put the equation and then you have to have comma x, comma this, comma that…it's just a lot of comma…it's just like memorizing, you can't really figure out what to put in.

How often do you use it?

A: I'll do work and then like just check it, because like…I'm a little OCD on test sometimes. You know what I mean?

So, do you check everything?

A: No, not everything, but … if I have time.

How do you decide whether you're going to check something or not?

A: Um…if it's a lot of like, computation, or a lot of numbers you're playing around with…then I'll use it. Just to make sure I didn't like make one mistake.

Do you check your derivatives and integrals as well?

A: Yea, if I have time. If I have the calculator I'll just use the calculator for that. If it's an easy derivative…like x squared or something like that, then I do it myself, but if it's like a pretty complex derivative, then …yea, I wouldn't even bother.

You said here that you usually use the calculator because your teacher or textbook suggests it. Is that a fair assessment?

A: Uh, yea…we have like calc quizzes and noncalculator quizzes. I kind of prefer the non calculator quiz cuz I think like when you get the calculator its just like something else you have to know. You know what I mean? If you have the non calculator quiz you just have to know your rules, you have to know integrating, derivatives all that stuff.

So, you're more comfortable in that situation?

A: Well, yea…if you know it you know it, and if you don't know it you don't know it. With the calculator you have to also know how to use the calculator with the problem so…like there have been times when I know how to do the problem, but I don't know how to plug it into the calculator necessarily, and a…

Are you expected to use the calculator on that? You can't do it without?

A: Uh…yea, you pretty much need to use the calculator. It's really complex equations.

On test question – chose calc…can you talk to me a little bit more about that?

A: Um…I think the calculator can do things another student can't do, pretty much. And I think if I study or look over my notes or something I, I'm pretty confident that I know it as well as any other student would know it…so I don't think they'd be helping me that much…I think I can figure it out on my own and then if I have something I want to check or something that's complex or something that would be easier if I just use the calculator I always have it with me.

What happens when you're working on a problem and you go to check it and your answer does not match the calculator? What do you do?

A: I plug it back into the calculator to make sure I plugged it in right. Then if it comes out like that again I check my work. Then…pretty much if I get the right answer in the calculator like twice, I'll trust the calculator. But my first instinct is to make sure I plugged everything in right.

After that if you still didn't match the calculator?

A: I probably just stay on the problem until I figure out what I did wrong.

How do you typically use it when you are doing hw or on exams…what type of things do you use it for most often?

A: Um…pretty much, just, like uh complex integrals or complex derivatives. So you don't have to do all the computation. It just does it for you. And like a point of intersection. You don't have to set the equations equal to each other, you can just plug it in to the calculator.

Do you think the graphing calc is an important part of the calculus course?

A: Yea, definitely. I mean, it's just like such and advantage to have it. It just makes everything so much easier. If you have a really good understanding of the calculator you can do a lot more than you'd be able to if you didn't have a calculator or know how to use it that well.

Are there things that you've done in calc that you understand better b/c you had the calculator?

A: I don't think so, no.

Do you use your calc differently when you are working by yourself than you do in class?

A: Not really. He'll go through and show us what to do on the calc, so I just follow him.

Do you do it the same way when you're at home?

A: If I could remember what he did. Cuz sometimes you have to press a lot of buttons to do something. Sometimes I can't remember.

Is it important to you to have the graphing calculator available when you are working on hw or in a testing situation?

A: Yea, I like to have it. It's definitely very helpful.

Do you have any general thoughts about graphing calculator use that you'd like to share.

A: It's a good tool to have, you know. I know like my grandfather was a math major. And like he, he loves it. He says we're like so lucky we have the calculator. Cuz I mean just for sine and cosine functions they had to go through like a little table or something…I don’t know. He said like everything just took ten times longer, so. My grandpa just plays with it like it’s a gameboy or something.

*** starting tasks ***

Aaron: Transcript of follow-up interview (6/15/2006)

So we’re going to start by looking at what you did on the calculator the other day. So first we’re going to look at task #2…so as we’re watching this, could you just kind of talk through what you were doing, why you decided to do it, if there was anything about it that was helpful to you…

Why did decide to go to the calculator?

A: I just wanted to check my work, just adding. I multiplied negative six times three plus six. Then I … I was checking my math.

So you had taken your x equals three and plugged it into the second derivative? Is that what you did there?

A: Uh, huh.

Had you done that by hand first?

A: Yeah. I did it and then…I had picked up the calculator for this part so I figured I might as well check this part. I had it in my hand so it only took a few seconds.

Why did you decide to do that part?

A: On the calculator? Um, I guess at first I just saw the pi exponents. Like to the third power and squared so, I wasn’t thinking like, ‘oh negative one’s easy to do’, I actually just plugged it in. I guess if it was like nine or eight or something I wouldn’t want to do like 8 to the third plus three times 8 squared.

Do you do this sort of thing a lot?

A: Well, yeah. I always do, that’s why I did it, it’s just kinda like habit. Even when I could just do it by hand.

When you decide to do something like this, why do you do it?

A: I’m trying to avoid making an error.

(mumbles)…what were you doing at this point?

A: I put in three to see what I would get. I was thinking, uh, that it wouldn't really make sense if the maximum rate of change is a negative number. Uh…then that's why…

What made you decide to try the three next?

A: Uh, I actually didn't know for the second derivative I forgot like if it was positive and then it was maximum or if it was negative for a minimum. So, I didn't remember. So then I got six…
Here you all of a sudden crossed everything out and wrote down negative 12. Where did that come from?

A: Um, I think I was like thinking of maximum rate of change would be when the first derivative…x equals zero…or when the first derivative equals zero. So that would be like…so then, I think, the first derivative equals zero when x is -1 or x is 3. and then a -1 was a local maximum I think maybe, so then I came up with -1 that's 1 and -3, I definitely plugged in 3.

You had plugged in -1 already and then you did 3…

A: I don't really remember what I did, I just got confused. I think, uh, I definitely thought that … If you plug in 1 the maximum is 12.

The way you used the calculator there, well first of all you used it just to kind of double check yourself and here you …was this the typical way you used your calculator? Was this helpful to you with this problem.

A: Do I need a calculator? No I don't.

Would you have done the same thing if you were sitting at home working?

A: Probably.

Ok, task #4…on this one you were looking for what absolute values of the function are equal to the function of the absolute value. And you really quickly wrote down x squared. You knew right away that was going to work…I skipped over some time here.

A: But uh, I actually, after I left the room I was thinking about like this problem and I think if I had, I think if the camera wasn't on me and I had more time just to think about it I would have probably just plugged y equals absolute value of like x to the third and then y2 equals like the absolute value of x and then to the third. And I would graph both of them and if the graphs were the same then I would have known that was a function that worked. I think once I left the room actually I was thinking I should have done it that way. I would have used the graphing calculator.

(talk about design of study and stressful situation)

A: then I would have done the absolute value of sine x and the sine of absolute value of x and see if there is only one graph.

Would you like to try it real quick and see if either of those work? Do you have any idea if they will work?

A: Uh, which ever one is an even function. I don't remember. Cosine?...[working on problem with g.c.]…uh oh, the syntax is wrong…

Absolute value is 'abs'

A: So like this…yep. And then I'd graph for cosine.

Ok. You don't have to do that if you don't want to. I was just curious about how you would go about it. What were you trying to do here instead? What were you looking at?

A: I was looking at the positive of the number. And um, and the negative of the number, cuz the numbers were the same. That's when I knew I wasn't like thinking clearly about it and I just gave up. I knew x to the third wasn't going to work.

Especially after you walked away and thought about it a little bit more, do you feel like the graphing calculator was a helpful thing on a problem like this?

A: Actually x to the third I wrote here, but I knew it didn't work. That's why it's in a different column. Cuz you see, 27, negative 27, that's why.

Did you find your calculator to be an important tool to have to think about this problem?

A: Um, yeah. I mean if it had dawned on me to do it that way it would have been much easier. But, I kind of, even if I didn't have a graphing calculator I think I knew I was thinking uh, that the positive and negative value has to be the same number. And if it wasn't the same number then it wouldn't really work. So…

Do you think you would have been able to think across all the real numbers if you didn't have a graphing calculator use? Or do you think doing what you did here would have been really important to you?

A: Well, that would have been the best (referring to the graphing calculator), but I think I could have gotten to like an equally good answer if I didn't really have a graphing calculator. I wouldn't have been sure of my answer. With the graphing calculator I know this is right. The other way I probably would have spent a lot more time on it. Like just thinking about it, is this right, is this not right?

If you were sitting at home and working on this problem and you had the graphing calculator available, would you have picked it up and used it?

A: Yeah, definitely.

If you were not in a situation with the camera staring at you when you were sitting and working yesterday, do you think that …would you have been as willing to spend as much time on without the graphing calculator as you would have been with the graphing calculator?

A: Um, I actually think that if I had the graphing calculator it probably would have gone faster. I would have been … I wasn't sure if I was thinking about it the right way. But with the graphing calculator I would have been sure that this is the same graph, the same values, it's the same function.

So having a visual…

A: because, like, I know the graphing calculators not thinking about it the wrong way, you know, I could just be thinking about it the wrong way that's why…

When you were talking about things you find helpful on the graphing calculator, one of the things you mentioned was graphing, but you didn't actually do any graphing on these problems yesterday. Do you look at graphs quite often…

A: I think I almost always draw like a graph of the function (pointing to calculator)…For this problem, wouldn't this be like, lets say, the absolute value of the inside so…that will always be positive on the inside, wouldn't they both always be positive?

Not necessarily. If you had negative three x

A: negative three x…if x equals negative three?

[conversation about examples for which the task situation is not true.]

You mentioned yesterday that Mr. Philhower uses the graphing calculator all the time in class. How do you feel about that?

A: I guess, its good cuz if you know how to use the calculator.

You feel like you don't know the calculator?

A: No, I don't know the calculator that well. I just didn't really feel like taking the time to get to know the calculator cuz there's like so much that you have to know. For example, absolute value you have to type 'a-b-s' instead of like just putting those bars. So, the answers just like really plain and clear, but I guess it's just hard to do.

I know that you just said that its good if you know how to use the calculator, but being you…how did you feel in that situation?

A: Um, well, I'd be following him and then eventually it'd be like so many steps that he'd be doing that eventually I'd be like uh…there's no way I'd remember what he was doing. Unless I like studied, and I wasn't really willing to do that.

Since it was an important part of his class, did you ever feel like you were at a disadvantage?

A: Yes, definitely.

Can you tell me a little about that?

A: Well, when we'd have like a calculator test there were some problems where, like, I'd be struggling cuz I knew I had to plug in an integral, take the integral of something. So like I'd have to spend like extra time trying to remember exactly how to plug in the integral of this, to this, to this, for x value um, this number, to this number, to this…it was like extra to have to deal with.

How did that make you feel?

A: Kind of annoyed I guess. I don't know.

How do you feel when you know there's a way to do something on the calculator, but you can't remember how to do it?

A: It's frustrating.

Does that happen to you pretty often?

A: Yep.

Before this year you used the TI-83. Were you more comfortable with that one?

A: Nope.

How about, if you think about other pieces of technology that you use, cell phone or some new program on maybe your computer at home, are those sort of things to pick up on where all the little things are that you're looking for?

A: Cell phone. Yeah, I've never read my manual for my cell phone, you just look on it and figure it out. (He pulled his cell phone out of his pocket.)

What is it that makes that easier, do you think?

A: Um, maybe it's like there's not as much. But I think its more like…like, uh, you go to menu, and then you go like, you scroll down to what you're looking for (shows on his cell phone)…

So, it's organized in a way that's not as intimidating.

A: Yeah, yeah. The most confusing thing on this cell phone is how to get to speaker phone. Cuz there wasn't a way to go menu, setting, speaker phone so you had to just figure it out.

How did you handle that?

A: Someone told me. And it wasn't even that hard, you just had to hold down this button. So, even that's not that hard.

Do you think you were willing to go look at things on that more because its organized really well?

A: It was organized really well, and it's my cell phone. So I want to know what to do. It's not like…I care more about what I know on the cell phone than I do on my calculator.

One comment you made was that you mentioned sometimes you're a little OCD on tests. You like to check everything.

A: I check all the time. Or I'd say, always in my mind. If I'm like, alright, sine of 60 degrees…and like, I always like to kind of think about it twice, but if I have time to go back and like draw a triangle…If I had a calculator, I wouldn't draw a triangle, I would just say sine of 60 and there's the answer. It's easy.

Do you check most things if you have time?

A: Yeah. I'll use the calculator.

Do you check your derivatives and integrals if you have time?

A: Uh, it's probably easier, I mean if it is a simple one like this, it's easier to just kind of look through it twice then actually, uh go through the process of typing in the equation and derivative and comma x comma this comma that…that's like…

If you were to decide to check it on the calculator, what would make you decide to do that?

A: If it was really hard. Like, uh, weird exponents or 15/7 was like one of the numbers. That's annoying.

How do you feel when you see some of those annoying complex functions that your going to have to find the derivative or integral of and you don't have your calculator available?

A: Uh, like when you have to do like a ‘u’ substitution or something like that?

Yeah.

A: Is it because the numbers are complicated or the ideas are complicated?

Um…it depends on what makes it complicated to you. If it was something that you were looking at that to you it looks like ok this is complicated…

A: Well, I think its more that it’s like tedious like 15/7 x to the 7/2, it’s just tedious to multiply that all out. It would just be easier to just plug it in. I mean obviously, if you have like, I don’t know…something like a trig function that you have to take the derivative of you have to do a trig substitution for, obviously it’s a lot easier just to use the calculator. It’s almost like too easy. You just plug it in and you don’t even know how you’re getting the answer.

What if it was one of those trig substitutions, but it was just a function within a long word problem, you weren’t being tested on can you find that, but you needed to find it to do the problem.

A: Then I’d definitely type that kind in.

Would have done it yourself first?

A: Just on the calculator.

A: It’s kind of unnecessary really.

What do you mean?

A: You’re always going to have a calculator available in life, right.

How are you going to feel if you show up on campus next year and you have a calculus class and the professor doesn’t believe them?

A: I wouldn’t really mind that much I guess. It wouldn’t really bother me that much. I wouldn’t agree with it, obviously. You’re always going to have a calculator, why wouldn’t you use it in that class. It’s something you need to know, you should know. I guess it wouldn’t bother me that much.

Did you take the AP exam this year?

A: Yes.

How important…

A: I felt more comfortable on the noncalculator section than on the calculator section. Definitely.

Why was that?

A: Um, because, like sometimes I think the calculator just confuses things if you don't know how to use it for that.

Did you get confused on some problems?

A: Yea, or like I wouldn't use the calculator in the way that the problem wanted you to use the calculator. Does that make sense? And I'd do it by hand and just do the easier things on the calculator, I didn't even need the graphing calculator necessarily.

Was there anything on the calculator part that you felt like you really needed it for?

A: No, there were some problems that I just didn't…like, I think I left a couple of problems blank. On the free response you like needed the calculator or you couldn't have done it at all…

Really? Why?

A: Uh, there were like, you'd have to find points of intersection so you could have done it, but it would have just taken forever…and also like, it would have been impossible to do, I mean the numbers were really…

You felt like it was really important to have for that part of the exam?

A: Yeah.

Was the exam what you expected?

A: Yeah, I guess so.

When you use the calculator, do you go with a reason in mind…

A: I would never use the calculator to try to figure out a problem. I never said I don't know how to solve this and looking at the calculator made me figure out how to solve it.

Do you ever just go look at a graph just to get an idea of what is going on there?

A: If I'm trying to figure out a problem, seeing a graph helps me figure out a problem. So then I might like type in the function and then look at the graph and think about it and it will be more clear how to solve the problem. So, in that way I do use the calculator to help solve it with the actual graphs.

What types of problems do you do that with?

A: I can see like, there is one example that always pops out is…if you want to find like the volume of multiple functions, rotated and…its easier to see the actual function to be able to visualize how do it. Do I need to take slices, or discs, or do I have to do the donut-type shape thing…

When you sit down to work on problems is it important to you to have the calculator there with you?

A: Um, if the work is going to be tedious, then it is. Otherwise, it's not such a big deal.

I guess it's about time to wrap up. But first of all, do you have any advise for incoming calculus students?

A: Uh, get to know the basic things to do on the calculator.

Is that something that you would do differently?

A: Yea, I probably would have just memorized how to do this, how to do the derivative, this is how you solve for a point of intersection…just like the five or six things that you repeatedly need to know.

Thank you!

Enoch: Transcript of 1st interview (6/08/2006)

Geometry grade – you are obviously a very, very strong math student. What happened?

E: That year…I had Mr. P…you know every student claims a teacher has something against him, but I think this time it really was like, it really was the teacher not liking the student, and I didn't know…I don't know, I don't understand it…

You mentioned you've used the graphing calculator for a long time. Can you talk to me just a little bit about how you typically used it in algebra? And do the same for alg 2, precalc, and calc.

E: In algebra we basically used it for the table. We would be using it, we would plug in the equation, put it into the calculator and it shows the values on the tables and stuff. So we would use it that way. Algebra 2, we used it for proportions, geometry was, I can't remember what geometry was. We did something with cookies…I know it was algebraic expressions.

In algebra and algebra 2 did you pull it out just for certain units, or did you use it all the time?

E: We didn't use it all the time. We used the IMP program…and so…

What did you think of that program?

E: Hated it.

Do you feel like it prepared you for calculus?

E: No. The only class that prepared me for calculus was precalculus.

IMP, was that through algebra 2?

E: Yea. So like we only took, um…well, my first three years I had IMP. But prior to coming to this school district I was in Jersey City public school system. We did traditional, so I was basically, I had most the students. So I guess I excelled that way. Just added on to that with going to precalculus and then to AP. That was traditional, that's my style. I prefer that.

Do you have any friend that preferred IMP over traditional?

E: Nah.

What was it about the IMP that bothered you?

E: It was um…it was too wordy. I like the numbers. Math is about numbers. I know you can get all the information in the word problems, but like the majority of the program was just like reading and writing. English is not my, I really don't like English like that. I do what I do to pass it, but …I was just reading and getting my own numbers, draw my own conclusions from it.

Well, I see here that you started using the graphing calculator even more in precalc and calculus. Can you talk to me about how typically use it there and how your teacher typically used it?

E: We used graph in trig and on trig functions. In precalculus the teacher had us graph to see how the sine graph looked and tangent, cosine, cosecant, secant, and cotangent. She was showing us how that worked. This year we were graphing derivatives, um…integrals, we also graphed…trapezoid rule, Simpson's rule.

Did you have special programs for that?

E: We just used the regular graph mode.

What things on your calc do you use most often?

E: I use graph, trace…y equals…I use…programs, like the integral program, derivative…and just the basic stuff.

Do you use it as much when you are working outside of class as you do in class?

E: I really don't use it unless I'm really trying to get through the work. Otherwise, I just do it by myself. You get more brain power that way.

When you were taking the AP exam, did you use it very much or did you…

E: I did it on my own and if anything was like not making sense to me I did it on the calculator.

When you were talking about the ways you most often use it you said either it was because your teacher or text book suggested it or you thought it would be helpful. Which place do you think you fall most often?

E: Because it helpful. Like, if there's an equation that I can just put in like get it like that I'll just put it in my mind, I won't use the calculator. If I know it is going to be a lot of decimals and a lot of dividing I'll use the calculator.

Excuse me?

E: To challenge myself.

What do you mean?

E: To see how like my mind compares to a calculator. To see if I can get the answers right and then like I can pat my self on the back. If I get it wrong…it is just motivation. Competing with the computer.

Do you do that often? Do you always check your work?

E: Yeah.

What happens if you don't agree with the calculator?

E: I figure um…I did something wrong. I didn't put the numbers in right or I copied the wrong numbers from the problem or if I'm reading a problem from a piece from a piece of paper maybe I'm not drawing the right conclusion from it. I always put it on myself. I can't put it on the calculator. I have to put it on myself. So I think it is something I did wrong.

Where do you look first? What tends to be the most common situation?

E: In the problem. I read the problem over and over. If I'm positive I'm not getting the right information then I ask somebody else. What do you draw from this problem? I usually take a second opinion over mine.

Here you mentioned…when I asked if you would prefer to take a test with another student or with a graphing calculator – you picked the calculator. Just a second ago you preferred another opinion. So, is this (the calc) like another opinion to you?

E: Yeah, cuz it is never going to lie to me. So I mean…I put the numbers in, it is only going to give me information on the numbers I put in so…If I put the right numbers in, it's going to give me the right answer as opposed to a student. I really don't know…how do I know if I ask you this question you're going to give me the right answer. I can't trust a student like that.

Do you have a graphing calc to use outside of school?

E: yeah. I have one at home.

Do you use it as often when you are doing your homework and stuff as you use it in school?

E: No. I usually like, things I learned from class I usually try to, if I do use the calculator I want to plug this in and see what the answer would be…I used it a couple of times, I didn't want to, but I, you know, to make sure that I was right I used it. Usually I just use it to check things.

How about on like tests or exams?

E: Test…if it is coming down to the last minute then I run to the calculator.

Is having the graphing calculator available to use important to you?

E: As far as graphing. That's basically it. As far as the other work, I can do it my own. It is just the graphing part.

Would it bother you if you didn't have the graphing calculator available to take a test?

E: No. I would do the graphing first and get that out of the way. Cuz I know I can do the other work faster so…

Do you have any general thoughts about graphing calculator use that you'd like to share?

E: It comes in handy.

Do you think it is an important part of the curriculum?

E: yeah. A lot more students learn how to use a calculator like this…it basically…it consumes a lot of time…and it also, its like a little toy. You share your experience with it, experiment with it and things like that.

Do you experiment on there a lot?

E: Yeah. I take what Mr. Garcia teaches and I experiment on there, put in different functions that I don't know anything about, so…I use it.

**** doing the four tasks now ****

E: I'm disappointed. I didn't think as much. I wasn't getting it like (snap, snap).

You did great…

E: I like to challenge myself by doing it mentally, by writing it down. I'm teaching myself at the same time. I'm learning then…I give my own self a pat on the back.

Did you feel like you used the calculator too much?

E: Yeah.

Do you prefer to avoid using it?

E: Yeah. Only time I want to use it when like, its really a big problem or it's a graph. I use it for graphs.

Why is it that you try to avoid using it?

E: Makes me feel stupid.

Why?

E: Because I'm not using my own brain. And it's a computer.

A lot of the things you did on there – you weren't asking it to do anything for you.

E: I was using it for information.

That's something that you shouldn't feel bad about. Are you encouraged to use it in class?

E: Yeah.

Do you feel like in made any difference when you took the AP exam?

E: Um…no…cuz Mr. Garcia, what we actually do in class, we do brain first, write it on a piece of paper or the board and then go back and try to answer it with the calculator. That's what we do in his class.

Did you do the same thing when you took the exam?

E: Yeah.

Did you always check or just sometimes?

E: Oh sometimes, I really didn't need it. It was, the questions we did…well some of them I forgot so I was like…ok, the calculator can't tell me so it is my fault. I don't want to put the blame on anybody else, only on myself.

Enoch: Transcript of follow-up interview (6/12/2006)

Here is your work from the other day…we're going to be talking about that so go ahead and take a look at it.

The first one was task #2. I think this was the first problem you decided to do. And on here…(looking at the video)…if you could just kind of talk to me about what you remember about this problem, what you were struggling with, when you used the calculator what you were doing there.

E: The reason I decided to do this was cuz um, this is what we've been doing like now basically. The, uh, the maximum rate of displacement and velocity and stuff and like I remembered it, it came to my mind and I know that this is displacement and you had to find the velocity so you had to find the first derivative and to get the maximum you have to do the second derivative? So instead of doing the numbers and thinking about it I knew it was going to take forever so I just plugged it the calculator, graphed it and got the maximum result.

You go along here and right about here, you kind of got stuck.

E: I messed up, instead of keeping it equal to zero, I put it equal to nine.

So you decided at that point…

E: That's what it was…it was equal to nine…cuz if I had left the nine over here I would have gotten to factor everything out and it would be easier.

Why did you decide to use the calculator at this point?

E: To find the maximum value.

What was your reason for doing that?

E: I knew I did something wrong and I thought back to what we do in class why we use a calculator with this problem and just plug it in and find the maximum.

Ok, you just entered the function in and you got that…

E: oh yeah, I remember this wasn't right.

That was the graph of this function right here that you just graphed…this is where you couldn't find the cursor – it's way down there.

E: I didn't the x value, I mean the y value would be that big.

So, right here you're doing a left bound right bound, what function are you using on the calculator?

E: I was um, finding the maximum between the left and the right.

Where on here are you going to take it once you find it?

E: Oh it should be like right in here.

Ok.

E: That's the left bound, right bound it'll be right in there.

There it is…it took a while, this isn't in synch with this…you decided to graph this, what changed your mind?

E: I changed my mind to graph this? Let me try to think on that…I wanted to make sure I had the zeros right.

What is this?

E: That's supposed to be the zero of the second derivative. This is the first, this is the second. I don't know…I got confused between which derivative gives me the maximum. That's why I started to graph that. I saw the graph and realized that wasn't what I was looking for.

Which one was not what you were looking for.

E: The second derivative.

E: Then once I got the answer I realize that's what I was looking for. I was looking for the zeroes.

So, what is the correct answer on that?

E: Oh, um, y equals zero.

Read carefully what it says there.

E: All you had to do was plug this in the calculator. Get the maximum from this. Well the maximum from this is what you have right here.

Well, the maximum of this is what you have right here. That's what this was. That was the maximum of this particular graph which was what you were doing in this section. Then you went back and you read this much more carefully and you said 'Wait a minute…I don't want the maximum of this' you said this is displacement, then velocity, acceleration…I want to know the maximum of the velocity."

E: The solution…the second derivative is the um…I had thought about the class and how he did it in class that's why…

So right here you found the maximum of the original function. So that's the maximum displacement. And then, after thinking about it a little bit more you had written this down as the second derivative and you took a look at the graph of that….what are you using right here? Are you tracing?

E: yeah.

Do you usually trace like that?

E: yeah, we usually trace like that when you're tying to find the value of something.

Even though you're not even on the function? So, you're not actually tracing, you're just kinda…

E: Yea, just kinda moving around, yea.

The way you used the calculator here, is that kind of typical of how you use it?...you started on paper, got stuck, and then went to the calculator.

E: Yeah. That's how we…first of all we work on it as, um, we do it mentally right on paper. Then to see the graph to get a visual of it, you go to the calculator. So that's how we use it. I don't know I just…I just forgot.

How would you describe how you used the calculator on this particular problem?

E: I s'pose I used the calculator as a scapegoat.

Can you tell me what you mean by that.

E: I mean I used it like, so I could basically I got the answer from the calculator. I couldn't, like, get it on my own. I don't know.

Why does that bother you so much?

E: I don't know. It just makes me feel like I'm not using my brain a lot. Yeah. I just, I have the brain power, I'm not using it I'm being lazy. I don't get what that is, being lazy.

On this particular problem, what made you decide to use the calculator?

E: So I could get a visual to see the graph. I can actually see the um, the um, change in the maximum value and minimum value.

Here this is the graph of the second derivative. What can you see in this graph that is helpful to you?

E: I see where the x and y intercepts are. It is a linear function in shape…steady increase or decrease in this case…

How did you get x = 1 from this?

E: That's for the zero. Which is right there.

The x-intercept…why is the zero important.

E: the zeroes, um, give you the maximum.

So, in this case, did the graph actually help you find what you needed?

E: They helped me double check. So I sorta knew it, but I just wanted to see the visual to double check.

Is that something that you do often?

E: Yeah, yeah.

Does that bother you when you double check?

E: If I don't get the same answer, then yeah.

Are you saying that you feel like when you go to the calculator to do the problem you said you don't like to do that. When you go to the calculator to check your work, do you feel the same way? Or are you ok w/ it?

E: If it's right then I give myself a pat on the back. But if I'm wrong then…oh well, I messed up again. I just continue to study, do harder.

Is that a good thing to have to be able to check your work?

E: Yeah. It's a good thing. It's a bad thing if I use it as a scapegoat.

If you had been doing this problem at home, would you have used the calculator in the same way?

E: Uh, huh.

How about if it was on a test?

E: If it was on a test and there was a time limit…um,

First of all, how about if there was no time limit?

E: Yeah, I would use it the same exact way.

How about if there was a time limit?

E: I may have decided to use it faster.

You may have gone to it sooner?

E: Yeah…yeah I would have.

Do you think you would have gotten a solution here if you had not had the calculator available?

E: No, I would have been stuck at this point right here. I would have been thinking and thinking….I don't know how I screwed that up.

Ok, lets take a look at task #4…first of all, do you remember working on that one?

E: Uh, huh. I was confused about this.

Tell me about that.

E: This particular problem the answer had to be the same as the function, I mean…the answer had to be the same as the variable you plugged in right…my mind went every where…I mean

Does this notation bother you?

E: It didn't bother me too much. It bothered me enough to have me thinking like ok well where do I start…first I was thinking about the simplest function to do. Then I started thinking about big numbers and something small. I don't want to use something that everybody used. I want to be different. So, I tried to go to the integral thing.

Would you have gone about it the same way with the same goal of being different if it had been a test question?

E: yeah.

Why is that?

E: I don't like to be the same. I like to be different. I like to stand out…I mean in a good way, not in a bad way.

Let's take a look and see what you did here…at this point as you can see you kind of looked along here and then you decided to do something on the calculator. Can you talk to me about what you were looking at here…

E: I was trying to figure, uh, the cosine of one and negative one that equals…that give me the same answer. So, we like either way we use it the absolute value of either or will give me .54. I was trying it for cosine of two and negative two. I thought which ever one…well I wasn't thinking about the whole function. Absolute value of the whole function will give me the same answer as that.

Then you went to…this time you were checking out your x squared minus two…

E: This would be the same thing. Um, whether a plug a negative three or a three will that give me the same answer, so…I started with the four (?)…

So the first time you did it by just sort of plugging in some values. The second time you did it by going through the table.

E: It was faster. There are different ways to use the calculator.

The way you were using it here, to test some things out. Is this a way you use it often?

E: Yeah. Here it basically using the table, it is much faster than just plugging it in and pressing equals and getting all the values. I was being lazy. Um, …

Do you use the table a lot?

E: Whenever I want to get the values of negative and positive for like the same function, that's when I use it. I usually do the independent variable when you just plug in the numbers and press equals. So sometimes I test out the work we learned in class on the computer, I mean calculator. Just have a little fun with it. Yeah.

You use that word fun and play with it a lot. Do you find yourself using it more to play around and test things or more with a goal in mind, like a procedure. I'm going to go and find the derivative…

E: Right now, I just play around.

Is that typical all year?

E: No, no. When I was competing in class…

What do you mean by competing?

E: Like, Rudy, if Rudy …[can't hear what he said here] we like to compete and see who get the answers first. So I just go to the calculator and 'ok I got the answers blah, blah, blah' you know. I use it to compete.

Is competing important?

E: Yeah.

Why is that?

E: I realize that in college everybody is just as smart as me. So, I don't want to be the last person to get the answer. I want to be the first person. That's as far as with the calculator, but when I'm home then the speed is not important to me. So, I don't get (?) writing it down or whatever.

Ok. Do you use the calculator differently at school than you do at home?

E: yeah.

Can you tell me a little bit about that?

E: Um…school I use it differently, I use it to teach. I actually use it competing, uh, at home I don't have to put up with neither one of those things so that's why I use it less at home.

In what ways does your teacher ask you to use it? And how often?

E: The teacher asks us to use to um see if everyone gets the same answer, to show us the proper way how to do it. He was preparing us for the AP exam, so we know how to do it to consume time. Um, he's just basically teaching us to show us all the different things the calculator can do just in case we need to go back to it.

Was it important to you to have it on the AP exam? Did it make a difference for you?

E: If I wasn't properly taught, then it would have made a difference, but everything he taught us was on the AP exam, so I was well prepared. I just…

Did you use it very often during the part that you could use it?

E: I used it as far as on the ? problems. So yeah. I used it.

In those situations…what was your reason for using it on those problems?

E: To consume time. If I was running out of time and I had to get the answer I would put my pride aside and go.

In a testing situation like that, did you use your calculator to check at all?

E: Uh, I didn't try to check, I just I get the answer I go on.

Do you use it to check very often when you're doing homework or a test in class.

E: Yeah. I use it to check. Often. I don't use it to get the answer, but I use it to check.

Was the calculator important to you on this problem?

E: yeah, it helped me like get the answer. It's um, put these values in quickly. Put the uh, negative input and positive input like that.

Do you think you would have been able to do that without a calculator?

E: Yeah, I would have. It'd just take more time.

What made you decide to use the calculator on this problem?

E: I was being lazy, I didn't feel like writing any more. Sometimes I get the calculator and just do it faster.

[Conversation off topic]

Ok, um…I've been thinking a lot about what you said the other day about the IMP program. Do you feel like it prepared you for precalculus or calculus?

E: No.

Why not?

E: Because, it was word problems you get to ambushed by word problems, but precalculus and calculus was strictly numbers. This is how you do this, how you do that, plug it into your calculator or write it down. Or work it out with formulas. IMP you have to read, you have to draw out all the information.

Did you know everything you needed to know to do precalc?

E: Um, some stuff I knew prior to coming to Plainfield. So, like the traditional background I had in the other public school system I was in actually helped me prepare for calculus, precalculus also. And precalculus had didn't really like the IMP program either, she was teaching traditional. That actually helped too.

Did you feel like you were prepared for the AP exam this year?

E: Yes. Yeah, I was very.
Was the exam what you expected.

E: Uh, actually I thought it was going to be hard. It came out to be easy. And uh, that's from being well prepared by the teacher. We went over everything that was on there. Everything.

Was the calculator important when taking the exam?

E: It was uh, there was uh important parts.

Do you think the calculator is important to learning calculus in general?

E: Yeah. As far as plugging big formulas in and integrals and derivatives and just getting the answer faster than working it out. It take you like 15 minutes to work it out…yeah…

Were there things that you learned in calculus that you think you understand better because you had the graphing calculator?

E: Well, the graphing wise, it helped me understand the graph better the visual, the maximum and minimum, the change in direction, um, as far as getting the answer I'd rather work it out and see what I'm doing instead of just plugging it in and get like the answer right there. But how did I get the answer, I want to see how I got the answer.

Do you feel like you're prepared for college math next year?

E: Mentally, um, that's a good question. I know there's a lot of competition out there. I feel prepared, I should be I believe, but…I don't get that

Why are you worried?

E: Cuz I don't know what's out there. You think you are at the top of your game, but then you see someone else that's better then you, like ok well…I just got my game some more. That's what it is, I really don't know what's out there.

If you take all of the other people out of the equation, do you feel like you're prepared to do the math that's asked of you?

E: Yeah. Yeah.

How will you feel if you happen to walk in to a calculus course…how important to you is it that you have a professor that allows the use of the graphing calculator?

E: Well, with problems like this where you have to find the asymptotes or the maximum and minimum, then it would be important. As far as just regular, traditional we just work it out then nah, you really don't need it.

[talking about the small class size in his calc class]

I was wondering if you could talk to me a little about what are some of the challenges about being a high achieving math student here at Plainfield. And maybe some of the rewards for being a high achieving math student.

E: Um. The class started out with about 15 people and a lot of people dropped, some moved. You know, my personal goal was to stay in there and not drop. I love math. I have the teacher to go with it, that was just a plus. The teacher he was just, he would really talk to you and help you out and explain everything to you…the thing about being great, it feels like nobody can tell you anything. You're like the top notch group. When I first went into there I was like, alright I'm here with a whole bunch of brainwaves and smart students and I if I do something wrong I'm going to look really dumb. I was going in with the attitude with just like, I was quiet. Then I got used to the teacher and the teacher was like, you know there's no dumb students in here. You made it here, everybody is on the same level. Ok, so then I started asking some more questions and stuff and got more confident. So he actually helped me get up that hump.

You mentioned a few times that you preferred not to use the graphing calculator unless you're trying to save time. Can you talk to me a little bit more about that?

E: I just like to um see how I got my answer instead of just plug it in and just getting a number. I like to work it out and see how I got the answer. The steps. Just in case I'm taking a test or something and the batteries die or something. Then I have a back up, I don't have batteries but I know how to do it on my own. So that's the way it is.

You also mentioned you like to use it to challenge yourself, to compare yourself to the calculator.

E: Yeah. I actually I get to the point where I feel like I have my answer and then I'll just plug into my calculator and just get a different answer or something then I know I did something wrong because I know the calculator wouldn't lie. So that right there, it makes me feel bad.

Is that important for learning?

E: Yeah.

How often do you check your answers?

E: I check it when I have doubts about a problem.

Are there any particular types of functions or problem types that it is really important to you to have the calculator?

E: Graphs. It really comes in handy. Um, integrals, maybe to find the integrals of big functions. When you have like a lot of values for the trapezoid rule, Simpson rule, and you have to just plug everything in. That's a good time. Trig functions of course. A lot of trig functions. Sometimes I get lazy and I don't feel like getting on my own. I guess those are basically it. And graphing trig functions and things like that.

When you are sitting down to take a test or work on something challenging, how important is it that you have the calculator available? Whether you use it or not.

E: It is important that it's there just so I know like if I need it, ok, it's right there. Don't be afraid to use it. I need to get over that 'I'm too good for a calculator' thing. Cuz then I miss out a lot like that. It is important that it is available for me when I need it, when I need to use it.

What is it about having it there that makes a difference for you?

E: I know that when I'm not too sure about something that I can always run to that as a scapegoat.

What advise do you have for incoming calculus students next year?

E: Study everyday, take a lot of notes, there's a lot of tricks on the calculator to use just to get ahead. A lot of things you do on the calculator use save it on the calculator. Stay organized, cuz it is a lot of work. And just love it! If you don't love it, get out of it.

Any final thoughts you'd like to share?

E: Just this is a good experience. It really helped me look at myself. I mean, I might even use the calculator more now (laughs).

Maryanne: Transcript of 1st interview (6/12/2006)

Can you talk to me a little bit about how you used the calc in your early math courses – up through calculus.

M: Um…algebra I remember a lot of the times that we used the calculator our books usually have some sort of corresponding thing on how to put it into the calculator. I remember we did like, plotting points and getting like the best fit line. That was also used in analysis. Um…that was along time ago

Did you have it to use on a daily basis? Or just for certain lessons?

M: Daily basis. They emphasized, everyone always have a calculator.

Can you tell me about the typically ways you use it in class and also your self when you're outside of class?

M: Um…in class we use it for the derivatives, functions, and integrals. I usually use it to check my work because I always like to use my method first before relying on the calculator because I'm always worried I might input something wrong. For calculus we used it for, Mr. Wasserman, he used the calculator a lot, but he always wanted to make sure we knew how to do it without the calculator. Because sometimes parts of our tests, especially when preparing for the AP exam we wouldn't have the calculator to use. So, I usually prefer not to use a calculator. I use it to check my work, is the only thing I do.

Do you use it if it is available, do you check most everything?

M: I'll check somethings. Like when I've used it for calculation and I've gotten a decimal I'll check to make sure I get the same decimal. But if it's a problem I know and it's just standard I won't use it.

Does it bother you at all when you are walking into a testing situation and it's not available to use?

M: Not so much if I know the material that's going to be thrown at me. When I took the AP test, it was harder for me with the non calc section because it was all about I've got to remember just how to do simple stuff.

Was it more of a situation where you were more worried because it wasn't there, or you really didn't remember?

M: Yea, I was a little worried. But I wasn't completely freaking out.

What kinds of things do you use the graphing and table modes for?

M: Um, graphing…well, in calculus whenever we would have to figure out how a graph would appear, where it was increasing and decreasing, um and actually before that in analysis before how you used calculus to do those kind of things I would always use it in analysis because I was never comfortable with just drawing based on what I had a general idea of.

Which category (given on the survey) categorizes what you usually do?

M: I usually check. I don't like relying on my calculator because something might happen or maybe I'll have it in the wrong mode and I'll get all the wrong answers.

If you were given the choice to take a test with a student or graphing calc which would you choose and why? You chose the graphing calc over the student. You said you prefer working a lone rather than with a student. Would you mind talking about that a little bit?

M: Um, I find that with collaborative work it can be more confusing. Especially if one of the people has a different idea of how to do a problem then it can be more confusing for the other person if you are going at the problem two different ways, that might lead to someone … you might not think its correct the way the other person is doing it and I'd just rather…I just don't like relying on other people for that kind of thing.

When you're working in that kind of situation where maybe its and unfamiliar problem and its just you and your calculator, what role do you see the calculator in that situation?

M: Um, it wouldn't play that much of a role because I really like my kind of thought. And in a situation like that, like I took physics and in physics I always relied on my classmates and not my calculator because with physics there are so many ways to approach a problem that its almost better to have a person with you.

You don't see math

M: I don't see math in the same way.

You mentioned you use it a lot to check. In the situation where your answer does not match the calculators, how do you handle that situation?

M: First I go back to my computations and check how I, like I'll go through line by line, cuz my work is very systematic, and if I can't find a problem I'll check the calculator and where I entered the data and see if I might have missed a key or something because usually that's what happens when I have a discrepancy.

Say you've done all that and you don't find an error in either place, what do you do in that situation? Has that ever happened?

M: It's happened and actually I have not trusted the calculator and gone with what I thought and it actually turned out right so…

Can you describe how Mr. Wasserman typically uses the calc in class?

M: When ever he's giving a lecture he usually doesn't use it. When he wants to show something cool that you can do with the calculator to illustrate and integral or something he'll so a problem on the board. Like doing exponential growth and that kind of thing, he'll show us how to do it on the calculator and why it shows up the way it does.

Does he encourage you to use it in any particular ways at all?

M: No, no he's assuming that we already know how to make it work and he doesn't like relying on the calculator so he doesn't encourage the students to do so.

Do you think the graphing calc is an important tool to have in a calculus course?

M: I think it is important, but I think that its' important to also do, there's a section of the test that you're not allowed to have the calculator on, I think it is ok when you're just doing simple numeric computations just to get through it faster. But I that if you're letting the calculator do all the work for you, you aren't going to learn anything. Then when you forget how to use the calc then you're really sunk and a lot of colleges don't even let you use them.

Is there anything that you maybe understand better because you had this as a tool to use while you were learning that particular thing?

M: Definitely figuring out how the shape of a graph will be and um…doing a kind of best fit of an equation.

How would you say you typically use the calc when you're just doing homework?

M: checking answers, but even then I don't use it that much for checking answers I use it for simple numeric computations, but even with checking our book has answers in the back so…

Do you use it differently when you're working alone then when you're in class?

M: Not really. When I'm working in class I try to use my knowledge first because um I just trust it more than the calculator especially because a lot of the ways our teacher explains stuff in class its just easier to, you know you really want to remember that instead of oh I know how to do this on the calculator.

Do you have any thoughts about graphing calculator use in general?

M: Well, I know my mom's from the time of slide rules and when she went to school the calculator was really expensive and she was an engineer. I think that when calculators are used properly they're fine, but when people rely on them to do everything and its more about training you how to use the calculator in a certain way than actually learning something then there is a real problem.

**** going to do tasks now ****

Maryanne: Transcript of follow-up interview (6/13/2006)

The first on you got to was this one (task 1) and um, lets take a look at what you did here. Ok, so you write that down, and then you took a break and went and did some other things. Ok, now here you're back and you spent a little more time on it. So here we are…at that point you went to the calculator. As it is going if you could maybe talk about what you were thinking about, what you were trying, and why you were doing it…

M: Ok, I remember I was going a little rusty, I remember this was just basic graphing. I always immediately remember that um, you know the asymptote, in order for the x asymptote to be three there would have to be, in the denominator it would have to be a x minus three in the denominator. And then I was trying to remember what would make the y asymptote. And then I just remember that it had to be, like you know, the x and the y in the numerator. The numerator had to have…

Wait, I'm not showing you your work. I'm sorry! [got out the correct folder – oops! I had grabbed Melissa's.] Ok, there we go…

M: I just remembered that um, that with limits the way that you would get y to approach something was to put the negative two, have the uh, the limits it would be approaching it based on the degree of x…[she's watching the video and has her hand over her mouth]…

Let's go back and look at what you did here.

M: Well first I just put one over x minus three to make sure that there was an asymptote there.

So, the reason there was just checking the asymptote.

M: Yeah…and the other one, I don't think, I think I just checked when I put this in.

Do you remember, did you already have this written down?

M: No, I didn't…Um, I wrote down this immediately, and then after I looked at my graph again, I went, I looked at the graph and tried to go through my head…

You first did two x minus one and you took a look at that graph. And then you went back and did negative two x.

M: Yeah.

So at that point were you testing some things out to see where it would go?

M: I was just checking. I knew it would work.

Would you say this is characteristic of the way you tend to use the calculator?

M: Yeah.

Do graphs play an important role for you when you are thinking about ways that things make sense?

M: Um, I sometimes have a really hard time visualizing graphs. So, it does help. Um, since I took calculus I am better at doing basic graphs. Um, like especially learning how to use derivatives and stuff it helped me a lot with finding out how a graph will behave, but um, prior to calculus I was always using a calculator for a graph.

Ok, so when you say you have a hard time visualizing, does that mean when you see a function you're not sure what it should look like so you decide to take a look at the graph or…

M: Yeah.

When you look at a graph is that ever confusing to you?

M: Um, usually like I know the basic stuff, like the way a third order will behave. Um, when we would work with some of the higher ones, like when we would get into fifth degree ones I always had to look because I just didn't know. Especially because with some of the graphs the basic shape would be different. Getting to third and fourth order graphs I wouldn't remember whether it would start from the bottom or whether it would start from the top. It was that kind of guessing and checking.

When the calculator is available to you, I know a lot of the time it's not, but when it is and you are asked to analyze a function and come up with a graph would you ever try to picture it before hand, or just start working through your first and second derivative test or would you take a look at it ahead of time and see if you could match it or would you do it yourself and then check it? What kind of order would you do that in?

M: Ok, I do the derivative test first and then I try to visualize it and then I use the calculator to check.

That would come in last?

M: Yes, see how I measured up (laughs).

[talked a bit about typical calc problems which lead to talking about solids of revolution]

M: My calculus teacher did a really good job of explaining that, but we did always use a graphing calculator in order to first be able to see it because it will determine…seeing the graph usually determines how you approach it.

[more talk about solids]

Do you think you would have used your calculator in another way, for this particular problem, if you were in another setting? Other than me staring at you (laughs), if you were working at home would you have gone about it the same way?

M: Um, probably. This is probably a problem I wouldn't have a calculator for. I was just a little bit rusty on how asymptotes worked.

If it were available to you?

M: I'd check, I'd just check.

Ok, task #2…this is another one you started, left, and then came back to.

M: Yeah, I know I was scratching stuff out. I remembered first of all reading through the question they were asking to find the maximum slope, so I knew that it was going to involve derivatives and I knew that it was going to involve the first order derivative and the second order derivative. So, um, I think I just got confused on the first part, so I crossed it out and decided to start over. So, when I went back I did the y prime. And I went and did my second derivative and with my second derivative I did the y prime chart, well the little graph where you put in. You know for the second derivative, the max its going to have a slope of zero at one. So, you check when it is increasing and decreasing. And I found that at one it is a maximum, so…

This actually you wrote down after you wrote down your answer.

M: Yeah, because…

You were looking at a graph over here with this…lets see, let me show you it

M: That was because um, I was on the fritz and actually wrote down negative one (laughing)

I know I saw that, you did that up here and you were stuck with it and it kind of threw you for a loop. We all do that…So the first thing you did was graph the original function. What were you looking for here?

M: Um, I just, I like to see what the function looks like to begin with.

Ok.

M: Just the shape of it. So I know, like, how much of a parameter I'm going to be working in.

Ok, so what did you get from this? What did you decide once you saw the general shape here?

M: Well, I knew that the slope is clearly the steepest in this area (points to screen) so I could tell that to begin with.

[we are watching the video]

It looks like you changed your mind about something there.

M: Yeah, it should have been on the next line. Because I wanted to graph the derivative next to the original function.

Do you like to do that? Get them both on the same graph?

M: Yeah.

What do you get from that when you look at it?

M: Um, it's just an easier way to compare. Because I just feel like if you forget what the first function is, then its just harder.

It kind of keeps it in context for you?

M: Yes.

There's your original function (still watching her actions on the video)…What did you do here?

M: I found the maximum of the slope just using the maximum function. Then I looked back and it said it was at one and I was like that's where I made my simple algebra mistake!

So this allowed you to help you find your mistake in here.

M: yeah. Cuz I was like, I know this isn't right.

Does that happen a lot? Where you're kind of double checking yourself and you realize where your error was on the paper?

M: Well, it's happened before where you see the graph and realize there's something wrong in my work. I should go back and look at it. It doesn't happen that often.

Back at the beginning when you kind of looked at the graph and said that's where it is steepest so I know that's where my answer's going to be. At that point did you kind of go wait a second?

M: Yeah, that's when I crossed this out.

Ok, so this sort of thing that you did here, where you had your work and the graph to go back and forth to look at is that relatively typical?

M: Yeah.

Is that something that is pretty important to you in situations when you are being asked about a graph?

M: Uh, yeah. I'm a visual person and it bothers me sometimes that I can't always see it in my mind. So I do like to be able to see it at some point. It's just a lot easier to know what you're dealing with instead of just having a bunch of numbers.

Would you have done this one pretty much the same way if you were outside of this situation?

M: Outside of the class I think this would probably be a no calculator problem also. I just…um, if I were doing this in homework when learning the problem I think I would probably do it the way that I did, but if I was a little fresher on it I probably wouldn't have.

[of topic conversation]

Did you have any other things about these problems that you wanted to talk about at all?

M: No. No, not really.

Let me see here…you mentioned that you didn't typically use the calculator at all for standard type problems, and these problems I would not necessarily characterize them as standard type of problems, so do you think you used it differently here than you normally would have?

M: I think I might have just because I haven't really touched a math book in a month. So…

Do you think it was representative of the way you would have used it if it hadn't been a month since you've had a math class and you were faced with some unfamiliar problems?

M: Yeah, It kind of did jostle my memory, which is what I would have used it for.

Do you find it helpful to do that?

M: Um, yeah. Sometimes, but I just never want to rely on it completely because I mean when you have a noncalculator section you just can't be completely dependent on it.

I can understand that…you mentioned the other day that you were a little worried when you sat down to take the noncalculator part of the AP exam because you had to remember the simple stuff, I think you called it. Can you talk to me a little more about that?

M: Um, well there were more noncalculator problems than there were calculator problems. And it's just, it was kind of more difficult because when you have to multiply something, when you get to my level of math you just pretty much punch it in and get it out (laughs). And then you start thinking I have to go back and do long division on this problem it was really frustrating. I know how to do this, I've done this before, and I know exactly how to do this, it's just taking a while. Because I haven't done it for such a long time.

That's understandable…another thing we talked about yesterday when we talked about the questions on the back of the survey….working with other students or working by yourself. You mentioned that you prefer to work by yourself in math. Then you talked about physics, and physics you liked working with other students because you saw it as problems where, there's more than one way to go about those problems. Math doesn't seem that way to you though?

M: Well, the kind of math that we were doing in calculus compared to physics…calculus we weren'y encouraged to work in groups when ever we had classwork, and all the math classes that I've had in high school I have had maybe one group project, ever. So, its usually on your own. So I'm used to working like that. Um, the thing with physics, you know with math you're given a polynomial. With physics your given a word problem that, I just feel like it's much more indepth and the way the people interpret the problem, first you have to figure out how you're going to interpret it. So sometimes it's a lot easier to talk out loud with someone who can be like, no this is the way that you're supposed to be interpreting it and you can work off each other.

That makes sense…you mentioned that your mom's an engineer, during the time of the slide rule I think you said…

M: Yeah, she was an engineer, she's actually a librarian here now. She was an engineer and my dad's a research chemist. She, they both worked for Exxon. So, she was a chemical engineer for a while.

So you have two very science and math oriented parents that went through this at a time that was very different from you, obviously. They see you being handed this technology in these classes, have they ever expressed any thoughts about that?

M: My mom doesn't, really does not like teaching to the calculator. She knows my teachers haven't done that, but she does talk about how she really does not like seeing kids…um, I once had to tutor a girl that was using a Casio instead of a TI and I knew nothing about how to use her calculator, but it was hard to explain to her about the actual concept because she was like, well in the book they show the calculator doing this and…so I will say that through my high school, elementary school and middle school they have always used TI devices and my mom also doesn't like that because she as a Hewlit-Packer calculator that's like 30 years old, and it still works. So she definitely sees that since the textbooks have a specific calculator that they work with….

You're the first person that has mentioned the actual textbooks and the calculator pages in them. Did many of your teachers really pull those into class?

M: Um, in my analysis class we actually did use the calculator pages for some of them. But actually some of the textbooks were outdated, so the TI instructions were a little behind the times. But, um, my teachers did sometimes use the calculator instructions, but it was more like…just a supplement to the material, it wasn't part of the material, ever.

Do you feel like most of your teachers integrated the calculators in a way that…

M: It was mostly, I felt like most of the time they would want the graphing function and the only other thing they would want is simple math.

When you go to the calculator do you typically go to it with a purpose? Or do you tend to use it to kind of explore.

M: Usually I have a procedure in mind. I look at a problem and I know there is a certain way to solve it and it is just checking it. I don't usually use it for exploratory purposes. I do know some people that are amazing at figuring out the ins and outs of their calculator. I don't know how to do any of that.

By exploratory I mean more of exploring a problem. Like I have a problem, I'm not sure where to start, lets play around over here and get some ideas….

M: I prefer usually just right in my mind.

I guess just to wrap up, do you have any advise for incoming calculus students?

M: Um, I would definitely say to incoming calculus students that if you're taking the AP exam make sure you prepare for noncalcualtor. I mean if your teacher is not giving you noncalculator sections on your tests, you should try as much as possible to just get used to not having a calculator. In college, most colleges won't let you use it. Some of the higher colleges just won't let you use it at all.

Melissa: Transcript of 1st interview (5/22/2006)

Melissa – transcript of first interview – 5/22/06

I read the questions from the survey and we talked. Here's what she said about #11,12…

M: Sometimes you may not exactly approach a problem correctly. Like, you may start going in one direction, but if you have like another student with you, you could see there is an easier solution to the problem. That's a lot more useful than the calculator itself. The calculator only does what you input into it.

#13…

M: Redo the problem. Most likely I made some kind of careless mistake. As long as I input the information to the calculator correctly the mistake is probably me.

Do you use the graphing calc in your calculus class?

M: Usually

Can you tell me about how it is typically used in class.

M: Um…a lot of times for graphing. If we're trying to do a function or something like that…and wanted to see…comparing like two lines I guess on a graph. Other times I use it just for like simple math or like regular calculus that I sometimes forget. Like, I don't know, derivatives or integrals sometimes I like to use that on the calculator, but usually the only times I use the calculator in class as a whole is when he's trying to teach us how to use a certain function…or, using a certain button.

Are you encouraged to use it on tests?

M: Depends what type of test it is. The last test we had was integration by parts. And we obviously couldn't use the calc for that b/c the calc could solve the problem in like two seconds. And if we wanted to see if we knew how to do the steps, we weren't aloud to use the calc for that test.

Did that bother you not having your calc for that test?

M: Well, if we used the calc for the test we would all get 100's.

What type of things are you typically encouraged to use it on?

M: Well, it depends…it depends how hard a problem is…like if something the calc can solve in two seconds for us and he really just wants to see if we get the basic understanding of the problem he'd let us use the calculator. Sometimes for graphs he lets us use it.

What types of things do you do on there most often?

M: Um…well, the whole value button. Working in that screen most of the time.

Do you think graphing calc are important in a calculus course?

M: yes (laughs).

Why?

M: I forget how to do like simple stuff. Like derivatives of sines sometimes. Or integrals of tan. So, I like to use the graphing calc to double check that. It's easier than flipping through my book and all my notes for stuff like that. Or if I'm trying to do like, graphs sometimes and we have two line values and we want to know what's in between it sometimes just seeing it on the calculator is a lot easier than like plotting it yourself.

You used the TI-83 until this year, right?

M: TI-83 plus addition

Ok. Did you have any trouble switching from the 83 to the 89?

M: It's different. Like the screens. At first I hated the 89 because I didn't know where any of the buttons were. But then like after maybe a few weeks I hate the 83 now. I gave that one to my sister to use. Like x to the three halfs I didn't know how to put that on the calculator.

Do you use your calc when you are alone than you do when you are working in class?

M: Not really. It's about the same.

Is having a graphing calculator available to use when you're solving math problems important to you?

M: Like on test, or on just homework.

Either way. Why don't you talk about both situations.

M: On homework I use it like all the time. Cuz, its ours for the entire year so I can bring it home and stuff. Um…

Is it important to you to have it there at home when your working?

M: I like to. It's kind of like a safety kind of option. Like even on the tests that we're not allowed a graphing calc for, he lets us use like a normal scientific calc, even though you know you don't need to use the calc at all, unless you want to do like three plus two equals five…he still lets us use like a scientific calc

Do you have anything thoughts about graphing calculator use in general that you'd like to share?

M: Some people are great with calculators. Other people just don't use there's at all. They hate it, they don't know what to do with it. It's kind of funny.

Where do you put yourself in there?

M: I really like the calculator. I can't do a lot of problems without it.

What is it about the calculator that allows you to do some of those problems that you don't think you'd be able to do without it?

M: It's just like, more like confidence I guess. Like you can plug in complicated problems that you aren't sure you go right or not and you can see on the calc and then if you plug it in correctly you can see like which steps you messed up on and stuff like that.

Did you use your calc when you took the SAT and the AP exams?

M: That's kind of a funny story about that. I was taking the Princeton review…and they and a lot of mock SAT tests. The day I forgot my calc though I scored better for the math section which was odd…in my opinion.

Why do you think that is?

M: Well, it was only like 20 – 30 points higher. I know like the SAT math section is designed so you can take it w/ out a calc so that's probably why.

*** begins tasks ***

Melissa: Transcript of follow-up interview (6/23/2006)

So what we're going to look at here is um…you already started graphing at that point. If you could talk to me a little bit about what you are doing and why you decided to do it while you see that.

M: I used the calculator? I probably was just trying to check it.

Not this part, you don't have anything written down yet…

M: Hm (she is quiet for a long time)

M: I think I'm just checking it.

What were you checking?

M: Uh…probably to see if I like moved it right, if the three should have been here and the two should have been here. I'm pretty sure that is what I was trying to do. The one over x, that is just one of the standard models that you just memorize. Like, one over x you memorize or like x squared and x cubed.

So you knew what it was. Then what did you try?

M: I just wanted to see if I like moved it correctly. I just like, sometimes I forget the formula. So I just wanted to see if I moved it correctly so that's why I did it on the calculator.

Would you have been able to do that problem without the calculator?

M: I got the right answer. It's just I double checked it kinda. Cuz wrote it first and then did that.

What made you decide to use the calculator on this problem?

M: Um…to check my answer.

Would you have felt ok about it if this had been a test question and you didn't have a calculator to use?

M: It depends on time constraints, like how much time I have left over, if I had like plenty of time I would probably use the calculator.

How would you have felt about that problem if it hadn't been available to use?

M: I could have plotted points. That'd be a pain, but you can always do that. (laugh)

Would you have?

M: Uh, huh. It depends on time constraints. If there's a lot of time and I was worried or lazy I would have.

How about if this was a HW question, would that have changed what you did?

M: I probably would not have used the calculator and would have just trusted my answer.

Would you say this is pretty typical of the types of things you do on the calculator? Do you tend to do the work first?

M: Usually I would do it all by hand because its just faster. Sometimes I would go to the calculator if I wasn't sure about something, like a couple times a derivative I forget, like arcsine and stuff like that so I just go to the calculator because it's faster. It just depends on the problem.

Ok. Let's look at task #4.

M: On this one I wasn't quite sure what the problem was asking. Cuz I'm not very good with absolute values and stuff. So when I graphed it it was kind of wrong.

You had y equals five x on your paper first and then you decided to take a look at the graph. What were you going to check on the graph when you looked on there?

M: I was trying to find, absolute value, like pretty much plug and chug.

Ok. If it worked, what did you expect to see?

M: Same thing.

You tried cosine first…[mumbles]…do you find the calculator to be a helpful tool when you run into a problem like this where you are not really sure what its asking?

M: Yeah.

What does it help you with in those kind of situations?

M: Um, for this I just knew since it is equal that they had to have the same graph. So, all I had to do was like plug it in to see if I could find a matching graph.

[mumbles]

If you hadn't had your calculator would you have just left those answers?

M: Probably…

What made you go to the calculator on this one?

M: I didn't know what to do.

Do you do that a lot?

M: Yeah, you just play around with the calculator.

What kinds of things do you do when you are just playing around?

M: I just go to the home menu and just see what could possibly work. Sometimes you actually just plug in the whole problem into the calculator.

Do you find playing around to be a helpful thing?

M: Yeah.

How often do you do that?

M: When I'm stumped. (laugh)

Any time you're stumped?

M: Yeah.

Let's take a look at the next one, which was task #2…you had already done down to this point and now you are going back to it again. What did you notice when you came back to it?

M: This one right here I just plugged in the x right here and one of them I got zero, so I got the right one. So I put that one in and it worked.

What was this problem asking for?

M: The first derivative pretty much. And like just take that and plug it in.

Ok. What does rate of change mean?

M: Um…change of distance over change of time…so that's the first derivative usually.

Ok. You actually found the maximum of the original function. This is asking for the maximum rate of change.

M: That's when it would equal zero.

You found the maximum displacement.

M: Hmmm…oh well.

So what would..

M: So, I needed the second derivative…so that would be…

You don't have to do it. But could you tell me how you would go about doing it?

M: I might have used the calculator. It depends on how easy it was. (laughs) Because sometimes it's easier to just do it by hand. If it took like product rule, quotient rule, or chain rule those get annoying and I'd rather use the calculator.

So, if it had been something more complicated like that do you typically do it first or…

M: Sometimes I have to do it by hand b/c that's how he grades it. We don't usually get a calculator for those types of problems.

What if it were just a small part of a word problem?

M: Then I would probably use the calculator cuz it's just faster.

You use the words faster and lazy a lot (laugh)

M: Well, sometimes, a lot of people I know do this too…so like its faster to do it on the calculator just because you don't feel like thinking.

Right here you actually decided you were going to take the values you found and set it equal to zero, which you solved on your own. Do you do this sort of thing a lot?

M: Like plugging in?

Yeah.

M: It's faster to plug in numbers on the calculator because you don't have to actually…it's just more convenient I guess.

If you look at a lot of what you did, especially on the first two, um, I noticed you used the graphs a lot to just sort of think about the problem. Is that a typical way you like to start a problem?

M: Uh, huh. Sometimes when you graph its like easier to just understand the problem. A lot of the rotating around the x axis or rotating around the y axis things like that I definitely need a graph for or I would be like what? What is this?

You mentioned that sometimes, especially when functions are really complex, that you like to use your calculator to find derivatives and integrals. How do you feel when you are in a situation where you are expected to find the integral of something complex and you forgot your calculator or you don't have it for some reason.

M: Well, then I would just do it by hand. Like as long as I know the method I can, and I have it memorized, I would just do it.

You talked about how sometimes you forget the simple stuff, I think that's what you called it. How do you feel when that happens and you don't have the calculator?

M: Well, sometimes you just have to figure out the equation on the spot. Like for the last test, the last quiz that we took, some people didn't know what arctan was and that made the problem ten times harder for them. So you just switch x and y and you like derive it right there on the spot.

You're ok with doing that when it happens?

M: Well, I didn't have to do that. Well, I actually got confused between two of them so I did it both ways. And then one of them turned out to be an undefined number and I kind of figured that there wasn't likely to be no solution so I just kinda took a guess. (laughs)

Did you take the AP exam this year? Can you tell me a little about it?

M: I thought it was hard. Only cuz I don't like calc. I'd be happy if I got a three. It's just a lot of information you've got to know throughout the year. And I've been struggling with calc for like ever.

Was the calculator important to you on that day?

M: Well, yeah.

What kind of things did you use it for?

M: Well, I didn't really use it that much cuz I didn't really see where it would come into play, where it would help me. Or it would just slow me down I guess.

On the part where you were not allowed to use it, were you wishing you had it?

M: Some questions yea, cuz I was like I don't understand the question. I think I left like one of the free response questions probably entirely blank because I didn't know what to do.

Do you think if you had your calculator there you would have gotten it?

M: Possibly. Cuz I would be playing more with it, just plugging in and stuff. But I probably wouldn't have gotten the right answer anyway because I don't think I knew how to solve it.

Just playing with numbers and trying things out, does that work out pretty well for you?

M: sometimes yeah.

Do you think you would have been as successful in calculus this year if the graphing calculator hadn't been part of the curriculum?

M: Oh, I definitely needed the graphing calculator for some things.

What do you think about math next year? Do you feel like you're prepared for college calculus?

M: Well, I'm taking calc one again because it's better to take the math and get it out of the way while it is still fresh in your head. So, I took the math. It's like a little more challenging than regular calc, but not like extremely. I'm hoping it's going to be a repeat of this year so I can do better, cuz this year was like tough.

How would you feel if you showed up the first day and you had a professor that said you were not allowed to use the graphing calculator?

M: Hmmm…I guess I'll just have try it.

Would that bother you?

M: Kind of…cuz, I mean I paid a lot of money for this calculator.

Are there certain kinds of problems or certain kinds of functions that you really feel better if you've got a calculator to use?

M: Arcsine, arctan…well, it used to be like sine and cosine, but after a while it just, I started memorizing them through the year. And like a long time ago it would be like sine for pi over three and you'd be like what is that? And it was just easier to do it on the calculator. I don't memorize things, I like to derive them, so I'd have to do that triangle thing and I like the calculator because its just faster.

Are there anythings from calculus this year that you feel like you understand better because you had the graphing calculator when you were learning it?

M: Yeah. I can actually see stuff on a graph and that really helps a lot. I'm a visual learner.

Would you say that when you go to the calculator you tend to do it in a way that is more exploratory or I go because I know exactly what I'm going to do?

M: Half and half.

Do you have any advise for incoming calculus students next year?

M: Learn your calculator. I know some people who just, some people don't need a calculator cuz they're just smart and they can do it in their head. While other people are afraid of their calculator because they never played around with it to begin with and they don't know where the buttons are and that…and some people…I heard this one story, that our teacher loves to tell us, that one girl defined the variable x equals some number, so she couldn't use her calculator for an entire AP test because every single time she plugged in stuff x would equal something and she didn't notice that. So she ended up taking her entire AP without the calculator and she got a 5 any way…

M: I used to have a calculator that had games to, so I wanted to learn it faster…cuz also I wanted to learn where I could hide stuff and where I could play with stuff.

Do you have any final thoughts that you'd like to share?

M: No…I'm glad I had an 89. If I had like my 83 for calculus it would be much harder.

Why do you think that?

M: Because the 83 can't do as much as the 89.

Do you feel like you had a big advantage over others on the AP exam?

M: yea.

Rudy: Transcript of 1st interview (6/12/2006)

Starting out, looking at your survey here, have you gone to Plainfield schools all through high school?

R: Yeah.

Ok. So you had the IMP?

R: Yeah.

(laugh) Everyone makes that same kind of face. So you had it in 9th, 10th and 11th grade and then 11th grade you took precalc as well?

R: Yeah.

Ok, thank you. I was just making sure that I understand the whole program. What kind of role did the calculator play in the IMP program?

R: Uh, it plays a big role. It help us answer equations mainly, and in and out tables, which is x and y really. It helped with that.

Did you use it pretty often then? Or only on certain problems?

R: Yes, pretty often. Even though, I'm good with math I really don't need it. For hard problems I used it.

What kind of problems? Can you give me an example?

R: Like slope on a graph or if you have an equation and you put x is like 2 and you put it in the calculator and graph it and it shows you what the y is.

And then you said you used it all the time in precalculus and a little less frequently in calculus. So, what sort of things have you been using it for in precalc and calc?

R: Mainly sine and cosine. And, um, yeah just like the trig functions and all that.

You're very comfortable with everything on the calculator?

R: Yeah.

Do you have a calculator of your own to use at home?

R: Yes, well, they gave us one to use and then we have to give it back.

So, you said you never use it for numeric computations. Is that right?

R: Yeah. I use it often for graphing functions, tangents…
(looking through the survey still)

One of the questions I asked and I was wondering if you could talk about it a little bit more, I asked if you were given a choice to take a test w/ another student or to have the calc available to you, which would you choose?

R: I'd use the calculator?

Why?

R: Because you can't depend on other people. And the calculator you can double check your work with a person you really can't b/c you don't know if they really know it or not.

When you do double check, how do you handle it when your answer is different than what you get on there?

R: I retry the problem and if I approach it, I don't know…I mean, I check it on there, I put the equation in, then if it is wrong I retry it until I get the right answer in there.

So you try to match your work to what's on the calculator?

R: Yes.

How would you say you typically use the calculator, on a day to day basis, like if you're doing homework? Or first of all, do you use it the same when you are working on your own at home as you do in class?

R: No, I use it more in class actually.

Why is that?

R: Uh, well, time wise, at home you can work it out as long as you want. I use it at home too, but not as much as in class.

You mentioned that you like being able to check your answers. Is that something you always do or only in certain situations or…?

R: Yeah, I like doing it always.

Why is that important to you?

R: Cuz the calculator is not going to be there all the time. So, I try to do that.

Does it bother you when you go into a situation and you know you're not going to have the calculator?

R: Uh, somewhat and then again, not. For all like questions, you really don't need the calculator, but for most just to make sure you're right. Cuz it's easy to work out the problem, but if you don't come up with the right answer you don't know what you did wrong so it's helpful to back you up.

Do you think graphing calculators are important for a calculus course?

R: Yeah, they're really important.

Why?

R: Because, well, with sine and cosine and tangent cuz those they're hard to remember if you don't keep doing it every day and with the help of the calculator…

How about outside of trig functions? Do you think its important for anything else?

R: Yeah, graphing, tables and equations. Some of them, I think, like the newer ones they do derivatives, the TI-89, those are good too. Then again if you use those you really can't show your work.

How does Mr. Garcia typically use that (the g.c.) in class? Or expect you to use it in class?

R: Uh, we really, I don't know…let me think…we use it really when we're doing work and then sometimes he gives us a test without it to see if we know the work and then for other parts he don't let us use it. So it's sometimes we use it for different problems and sometimes we don't.

Do you have any general thoughts about the graphing calc.? It's use in general, or in school or on tests that you'd like to share?

R: No not really.

[Starts the tasks. Rudy did the whole interview in one day b/c of scheduling difficulties. So I used the video of his calc use w/out actually cutting it together.]

Rudy: Transcript of follow-up interview (6/12/2006)

[Rudy's interviews all took place on the same day due to scheduling difficulty. I used unedited video of his calc screen, but did not have time to choose video clips of his work and screen to show to him simultaneously. I also did not have time to sit and think about what I wanted to ask. I feel like the interview still went well, but it is not the quality of the other 5 participants.]

So, you did these pretty much in order. So, lets start with the first on here (task 2) and um, can you talk to me a little bit about what you were trying to do and why you decided to go to the calculator…

R: For this part, I went to the derivative so I could get the rate of change, but in order to do that I think, I believe you have to have x and I thought … oh, that's the first thing I thought. And I've already explained I kind of got the picture.

Ok…that didn't work out then.

R: No, that didn't work out.

Why did you decide to try the calculator here to begin with?

R: To find the…well, integrals, I thought it worked the same as derivative cuz if you put a number between each number and comma it gives you the number…that's what I thought.

So, you're trying to get the derivative?

R: Yeah.

Why did you decide to do that on the calculator?

R: Uh, well actually I knew that, I just thought it would give you the number, the maximum rate of change. It don't give you that.

But you went there to try to get that…

R: First I did it in my head, I worked that out and that didn't work right. And then I tried to see if I plugged in x what would y be for the graph.

For another way to try it, since that didn't work out. So then we see here, then you went to graph it…can you tell me about what you're doing here?

R: Uh, I plugged in the function. First I plugged in the regular function, I looked at the graph. And then I plugged in the derivative function after that.

What made you decide to put the derivative in?

R: Cuz that's what it asks for, the maximum rate of change.

From there you went to the table, and what were you looking for on the table?

R: The highest number out of the number.

So, the reason you went to the calculator on this problem to begin with was because it wasn't working on paper. Is that typical of the way you do things? Try it on paper first?

R: Yeah.

Would you have used the calculator the same way on this problem if you were working in class or even at home?

R: Yeah, pretty much.

Is this the typical kind of thing you do on the calculator?

R: Uh, yes. I do integrals and all that.

Could you have done this problem without the calculator?

R: Well, if I would have spent some more time to think it out I probably would have.

Do you use the table a lot?

R: Not a lot. (his cell phone rings and he answers it and tells the caller he'll be done in ten minutes)

So do you use the table a lot when you are looking for maximum and minimums, things like that?

R: Yeah.

Why do you prefer the table?

R: Cuz it shows you the numbers instead of you going to the graph. Or sometimes on the graph you can actually put like, left bound, right bound and show you the max. And then for the min you can do the same thing.

So you could have done it either way?

R: yeah. It seems faster.

So in this case, this was a situation where you were trying it out on paper and it didn't work out and you went to the calculator and you're saying that's a pretty typical way for you to go about things. (he nods) Ok. The next one you did was number three, which was this one. So on this particular problem, let's see what you did here…you went to the calculator a few times…you decided to use 7…

R: I went to the table.

Ok, and what were you looking for on the table?

R: On the table here I was looking for the numbers that were between, that was less than five and bigger than negative 2. I saw that there were six numbers, three, four, five, six, seven, and eight. And it's the same for, there are six numbers…depending on the k, that's what I noticed.

In this situation you used it a little differently. First of all, what made you decide to try this on the calculator?

R: At first I didn't understand it. I was trying to figure out what they meant. When you explained that's when I decided to use the calculator to see what numbers.

Do you tend to use the calculator to test things like you did here? Or is this an unusual thing to do?

R: Uh, I use it to test things often.

Would you have been able to do this problem without the calculator?

R: Most likely, um, yea basically. If I would have understood it from the beginning.

So far I'm noticing the graph and the table seem to be really useful tools for you on here. What do you use most often?

R: Well, tables and graphs. I guess because since that's what we learned off of IMP…

Really, that's a big focus?

R: Yeah.

Alrighty, from there we went to task number four. On this one I think you used it mostly for some calculations.

R: Yeah, for some quick calculations.

That was something you said on your survey that you never do.

R: Well, you put me under pressure. (laughs) You know really I don't, but I guess since I was under pressure.

I'm not meaning that in a bad way (laughs), I'm just asking about it because I was surprised when you said you never do. I could tell you were thinking about it first and then just came here to see if it was right. Is that something you do a lot?

R: Yes.

What is your reason for doing that?

R: To make sure my answer is right. Cuz sometimes you think faster than you write and you can write something else and then you need to check it and make sure that you're right.

Do you check stuff pretty often?

R: Yeah.

Are there typical things that you check more regularly than others?

R: Uh, most like fraction numbers. That's probably what I mostly check.

Very good. And this one you came up with some great functions there, and you were just checking values to check that.

R: Yeah. That's the one I was stuck on.

If I asked you to classify the reason you went to the calculator on this problem, what would you say?

R: To show the graphs, and then to make sure the function was the same. Because…first…I don't know let me think?...to check my numbers and make sure I was right.

Could you have done this one without the graphing calculator?

R: Yeah. This one I could have more easier than the other two.

The way you used it here, checking those values, is that…

R: yeah, yeah…

Ok, and then the last one which you did not like at all. Here you kept trying functions. You were looking at this…What was your reason for going over here?

R: To see which function doesn't touch a point. Cuz usually when they don't touch they show like a 'v' or what ever you want to call it.

Ok, so you knew what you were looking for…in this situation you were

R: Testing…

Would you have used it the same why if you were working outside of class?

R: Yeah, mostly…

You were getting close, do you think you would have gotten any nearer without the calculator? Or do you think the calculator was really important to that problem?

R: I think it was important to this problem. Just so I could find that.

[we talked briefly about the problem and the pressure of being video taped]

All of you have been talking about the IMP program. Do you feel like it prepared you?

R: No it didn't, it didn't prepare me at all. Cuz when I took the SAT, it wasn't there.

What was it that you think was lacking in IMP?

R: It didn't show you like functions, straight up. It gave you problems, like word problems. It's mainly to help on the HESPA, cuz a lot of kids failed. It just…

Do you feel like doing those problems you had enough of the math to do precalc and calculus?

R: Uh, no. I should have took Trigonometry instead of taking IMP 3.

Was that a choice?

R: Um, it was a choice like precalculus.

Do you feel like you were prepared for the AP exam?

R: Yeah, I was prepared. You know the book was different too.

Was the exam what you expected?

R: I didn't use the calculator.

You didn't use the calculator for any part of the exam? Not even when it was allowed?

R: No, it didn't have it.

There were two parts, part with and part without.

R: Um, I don't know. Maybe I didn't have a calculator on this. I don't remember.

Do you wish you did?

R: Yeah!

Would it have made a difference if you did?

R: Most likely.

In what way?

R: It would have helped me check my answers. Even though I know I worked it out right, I would have checked my answers to get full credit.

Do you tend to check your answers all the time or just sometimes…

R: Just when I think the problem is hard.

Do you feel like you're prepared for college?

R: Um, yes. I think so…I hope so…

When you get there, is it important to you to have a professor that allows graphing calculators?

R: Uh, yeah.

How would you feel if you didn't?

R: I guess lost (laughing). I don't think I'd feel good…I don't know really to be honest.

I guess one of the things that I was really curious about is that when I walk into class there are only six of you, and it's a really big high school. And a big part of that was because in order to get to where you want to go you had to double up on some math classes. You are obviously one of the very talented math students in this high school. Are there any challenges? [tape stopped!]

Are there any special things that you'd say are rewarding about that?

R: Yeah, you get recognition.

What kind of recognition.

R: Like from the teachers and all that. They ask you what class you're in like calculus and all that and "oh that's good" and you know they don't hassle you or anything.

Are there any particular types of functions or problems that you really always want to have your calculator with you when you're solving.

R: Yeah, like graph functions, I need tables.

When you sit down to solve problems, whether on homework or on a test, is it important to you to have a calculator available?

R: Uh, yeah.

Can you tell me a little bit more about that?

R: Like well, on my homework to make sure I got the problems right and to check my work.

Do you have any advise for incoming calculus students for next year?

R: Study hard!

Study hard (laughs) anything else?

R: Try to get the TI-89.

Do you think that would make a big difference? Are you familiar with that one?

R: Some of my friends, they taught me a little about it.

Ok, do you think it is more powerful than the 83?

R: Yeah.

Do you have any final thoughts at all on any of this?

R: Not at all (laughs)

Shemika: Transcript of 1st interview (6/8/2006)

Did you use the calculator much in your other math courses?

S: um, we used it in precalculus almost as much as in calculus, probably about half the time in precalc. Um, we used it in all the other math classes, but not a lot.

Did you have it to take home every day to use?

S: I think 9th and 10th grade it was passed out to us when we used it. 11th and this year we got to take it home.

What kind of things did you use it for in algebra?

S: Well, we were doing IMP. Which is basically a lot of word problems. When ever they asked for a graph or anything of that type, we used the calculator. Other than that we just worked out the problem.

Were they just put away?

S: Yeah.

In precalculus you started using it a lot, what kind of things did you use it for in there?

S: Um, we started doing a lot of trig functions and looking at the graphs of them on the calculator. Solving equations, using the calculator. We started learning how to use the calculator really in precalculus.

So, you feel like after precalculus you really knew what you were doing on there?

S: uh, huh.

How did you solve equations on there?

S: plugged the equation in on the y equals and then either go to the graph of it and find the answer or go to the table.

How about calculus? Did you use it in any different ways in calculus?

S: it was basically the same thing, but we did more advanced things.

Like what?

S: Like finding solutions that were not visible in the first quadrant. Changing the window so that we could see the solutions that were way off and finding answers to the problems w/out equations. Like plotting graphs without, or creating the equation and plotting the graph using the equation we created. Finding more answers on the table. How to find more advanced answers.

Are there a lot of things that you do on this calculator that you wouldn't be able to do on a scientific calculator? Do you think having this made a big difference for you in calculus?

S: Yeah, just, basically for seeing the graph. It really helped when we could just find the answer on the graph.

Do you use it as much when you are doing your homework at home, or taking a test, or on the AP exam?

S: yes. About the same amount.

You mentioned that when you are graphing functions and using the table you usually decide to do that because either your teacher or textbook suggests it. Do you ever use those sort of things when you are away from class?

S: Yes. When the equation is given and I know that the answer can be found using the graph and its lots of work, then I'll use the graph.

So, it really saves you a lot of work sometimes it sounds like.

S: yes.

When you're working on your own, how important would you say it is to you to have that (the calc) available?

S: Very.

What do you do most often on there?

S: find the answers using the graph.

Do you have any general thoughts about the graphing calc at all that you'd like to share?

S: Well, it is good to know the things that it can do. Know how to do them yourself, but when it is easier to save time and things like that cuz I do a lot of things and all the course work that we have and everything else it is easier to have the calculator because it saves time and it helps you double check your work. Or you can do the problem find the answer and then do it yourself so you know that you're doing it right. Or do it the other way, just to check your answer. So it is very useful.

Would you say you use it more often to check answers or more often to find answers?

S: about the same amount.

Do you ever use it in any of your other courses?

S: no.

***** She did the 4 tasks at this point *****

Shemika: Transcript of follow-up interview (6/12/2006)

Of these problems, the only one that you used the graphing calculator on was this problem, which was task number 2. This asked you to …. Lets take a look here just to remind you what you did and then we're going to talk about it. [showed video clip]

You read the problem through really carefully…you asked me at one point what work do I have to show…now here, as you can see, this is the only work that you wrote down and then you started working on the calculator…here's what you did on the calculator…

Can you kind of talk me through what you did here?

S: Well, um rate of change I know that is the, uh, the derivative and the derivative for like…I didn't like writing out the derivative so I just put it in the calculator.

Ok.

S: Um, I knew already that math 8 is the command 8 is the derivative and then you just put in this function and it calculates the derivative and graphs it.

Does it actually give you the derivative when you do that? Or just the graph of it?

S: The graph of the derivative.

So you've got the graph here

S: and where the maximum of the graph is that's where the derivative is the greatest, so…I had to change the window because it wasn't showing exactly where the maximum is.

Ok... What function were you using here? Do you remember?

S: Um, yeah…

The maximum function?

S: yeah, to find the maximum, yeah…

S: When it gives an answer like that I just put in one, or … to see if it will get 12, or sometimes it is 12 and it will say 11.9999, which is the case here.

Which, you assumed that was 12.

S: Yeah.

Did you try it again just to double check?

S: huh, huh…

Is that typical? Do you always double check?

S: Yeah, to see if I'm rounding right.

Is this typical, do you usually attack a problem like this?

S: Yes. Unless they say write out the derivative and then calculate the maximum rate of change.

What is it about having the tool for that sort of thing that you like?

S: I didn't have to do the work. The calculator did it all.

If you didn't have the calculator what would you have done differently?

S: Um, actually calculated the derivative and wrote that out and probably um, no, um…then I would have set the derivative equal to zero and found what x equals when the derivative is zero and that would get the maximum.

What made you decide to use the calculator on this particular problem?

S: I knew that it knew how to do it and I wouldn't have to do it.

If I gave you a list of choices would it be just to kind of skip doing a step, just cuz you didn't want to, or more just to save time, or you knew how to do it, so it wasn't because you didn't know how to do it. So, which one of those do you think would best characterize your choice?

S: In a test situation it would be time, but if I just had a problem to do it would be the fact that I didn't have to do it. So it's both, but in a test situation it would be to save time.

Homework situations, and that sort of thing, do you do this sort of thing often? Whether it be with derivatives or integrals or anything, if it doesn't specifically say show your work, do you look for ways to use the calculator?

S: Yes. While I do, um, do the work myself to make sure I know how to do it and to make sure that I'm doing it right. But I do that on a separate paper, on my own paper.

So you don't even put it on your homework assignment?

S: No.

How often do you do both? Just when you're learning it, just to make sure you know how to do it? Or do you always do it both ways?

S: Just to make sure I know how to do it, when I'm first learning it.

Would it bother you turning in something like this on a test, with no work shown?

S: If it doesn't say show your work, then I don't show my work.

How would Mr. Garcia have gone about a problem like this in class?

S: Solving it or grading it?

Both.

S: Um…in grading, if it doesn't say show your work, if your answer is right, it is right. In solving it, he would probably do the same thing, just use the calculator. Cuz, when he teaches first he shows us how to do it. He shows us the most difficult way and then at the end he'll show us the easiest way which is using the calculator, so…we'd end up using the calculator.

On a test does he include both? Does he sometimes ask you to show your work and sometimes not?

S: uh, huh. The ones he want us to show our work we don't use the calculator on.

You're not allowed to have at all?

S: We're still allowed to have it.

It sounds like you use the calculator quite a bit to get things done that you don't have to show your work for, especially if it doesn't tell you to show your work. Was that a handy thing to be able to do when you were taking your AP exam?

S: Yeah.

Did you do that a lot?

S: As much as I could. (laughs)

You could have done this problem without the graphing calculator, so would you say it is typical of you to turn in homework, tests, AP exams, things like that with simply your answer there, having done your work on a calculator?

S: Uh, huh.

Did you have any questions about any of these problems that you did here?

S: Just this one…

Do you know what a vertical asymptote is?...

[conversation about task #1]

You obviously know how to use that calculator in ways that are very, very effective. Do you ever use it to just kind of play around with things to see what happens?

S: When I have nothing else to do (laughs).

Do you do it with math problems?

S: Yeah.

What kind of things? Can you give me an example?

S: Like if there's a lesson that we didn't get to in the book and it says there is a way to use the calculator to do it, I'll try to figure out using the calculator first. And then go back and actually read it and then figure out how to solve it myself.

So, when you're doing that are you looking for what procedure will allow me to answer this particular problem?

S: Uh, huh.

Do you ever use it just to kind of, in this sort of situation, where you're unsure of an answer even though there isn't an exact procedure how to do it on the calculator, but you just kind of start playing with things until you find out an answer that works for that problem?

S: Um, if we're learning how to do something like that, I'll just ask. If it is in a test situation then I'll do everything else I know how to do first and then if I have time I'll go back and start playing around with it.

Would you play around with it on paper?

S: on the calculator.

You mentioned that you guys used the IMP program here in ninth and tenth grade. What role did the calculator play in that? Did that curriculum ever ask you to use the calculator at all?

S: What ever problems say use your calculator…there weren't many though.

What did you think of that program?

S: I didn't like it at all.

No? Why?

S: Um, cuz it seemed like they focused too much on your explanation and whether or not your answer was right they didn't really care, it was just if you made it sound like it made sense then you got points. But a person that got the wrong answer but gave a good explanation would get the same amount of points as the person who gave the right answer. That didn't make sense to me. Cuz in math the right answer is the right answer. It seems like they were putting too much English in it. I didn't like that.

Was there a lot of writing and reading?

S: Uh, huh.

Do you feel like those – was it just ninth and tenth grade?

S: No, we had it 11th too, but I also took precalculus my 11th grade year.

So you took both. Oh…is that what most of you did?

S: Yes.

Do you feel like the IMP prepared you for calculus or precalculus?

S: No. No.

What was missing?

S: Math.

Math (laughs)

S: and um, it seemed like the stuff we did in IMP was elementary. It was stuff that we knew already, they just put it in word problems that were confusing.

So you really didn't learn anything in those courses that you didn't learn in middle school?

S: No.

[conversation about middle school program]

S: IMP was just word problems all the time. No regular problems, just word problems. It would be one problem that you would spend like eight weeks on. (more about this)

Thinking about precalc and calc then, do you feel like those courses prepared you for the AP exam?

S: Yes.

What did you think about the AP when you took it?

S: Um, it wasn't what everybody else, everybody else said it was so hard. If you knew the material then you knew it. If you didn't, you just didn't.

Was the calculator important to you on that day?

S: Yes.

If you had to guess percentage-wise what percent of the exam would you say you used your calculator on?

S: Of what we could use it? I used it every time I could use it.

During the portion that you couldn't use it, did it bother you that you didn't have it?

S: No, not really because those were problems that you could solve, well not easily, but you could solve by hand. So…

What is it about having the calculator available that is important to you?

S: It saves time.

The time factor, that's it?

S: uh, huh.

Do you feel like you're prepared for math courses you're going to take in college?

S: Uh, huh.

What do you expect those courses to be like?

S: The same thing it is now. Mr. Garcia teaches calculus at Rutgers, so I feel like I'm getting the same class that they're getting. He tell us that we're getting the same thing that they're getting. If its the same as what it is now, then I'll be alright.

I noticed that there are only six of you in that room…this is a big high school. What are some of the challenges that come along with being a high achieving math student at Plainfield High School?

S: None. There is nothing extra it's always if you can do it, then you do do it. Those who think they can't, they don't even try. If you try to do it, then you can. If you put some kind of time and effort into doing it then you can, but there's nothing along with being in that class, there's no additional challenges along with being in the class.

What are some of the rewards that come with taking AP calculus?

S: Just being prepared for the next level.

Is that what your focus is going in there?

S: Yes. I feel like the people who aren't in that class are not going to be ready. I'm ready.

I didn't realize that you had to double up in math in 11th grade in order to get to that point…going back to the calculator, are there certain types of functions that you deal with that you're really uncomfortable with unless you've got the calculator?

S: No.

I asked that just because you mentioned that you use it a lot for trig functions in precalc. You're fine with trig functions either way at this point?

S: Uh, huh.

What types of problems do you really like to have it for? What types of problems do you definitely want it?

S: Derivatives or integrals or both. I mean I can solve them without the calculator, or with the calculator. With the calculator math 9 or math 8 and I'm done. Put in the function and it calculates it and pops up. But, without it I can do it the same way.

How often do you tend to check your work on the calculator?

S: Um…whenever, like not on a test. If we're doing homework or classwork and it says don't use a calculator I'll do it and then use a calculator just to check my answer. I won't change it if it's wrong…just to see if I got the right answer.

What is your reason for doing that?

S: Just to make sure I know what I'm doing.

What do you do when your answer does not match what is on the calculator?

S: Go back and do it again. Look at every step I did, see where I went wrong. If I can't figure out where I went wrong, I'll just leave the wrong answer.
We've kind of hit this already, but when you are problem solving when you are taking a test, when you're doing homework, is it important to you to have the calculator there?

S: Yes.

Can you tell me a little bit about that.

S: It's just easier. It's easier and it saves time. Like if it says don't use a calculator, then I'll do it myself and then just go back and check my answer.

Is it important to you to be able to check your answer?

S: Yes. It tells me when I think I know that I'm doing the right thing, that I'm doing it wrong. I need to know that. The calculator helps me to bring to my attention what I'm doing wrong.

Does it bother you when you check your work on a test and you've got something wrong.

S: No.

Are there any final thoughts on calculator use at all that you'd like to share?

S: Just the calculator has been a big part of the calculus experience and it's helped a lot. Without it we could still do the work, but with it we can do it easier and get more problems done. Where as a lot of students don't finish all the problems because they focus too much on trying to solve it by hand. With the calculator it saves a lot of time and it gives you the right answer. So…

What would you say to an incoming calculus or precalculus student? What advise would you give them about…what they need to be successful in those courses?

S: Pay attention during the class and don't expect the calculator to show you how to solve the problem just, it just gives you the answer. If you want to know how to solve the problem yourself, then learn how to solve the problem. The calculator just gives you the right answer.

Is it important to you that your first math course in college allows you to use the calculator? How would you feel if you walked into a calculus course and you've got a professor that doesn't allow calculators?

S: It would be a shock at first, because I'm used to having the calculator. But as long as they are good teachers and they help us with learning how to solve the problems ourselves then it will be alright. Once I know how to solve it, I'll be fine.

That seems to be something that I hear you saying a lot, correct me if I'm wrong. Although you use the calculator a lot, you do know what you're doing.

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Curriculum Vita

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Education

October, 2007	Rutgers, The State College of New Jersey, New Brunswick, NJ Mathematics Education, Ph.D.
August, 2000	Sam Houston State University, Hunstville, TX Mathematics, M.A.
May, 1992	Purdue University, West Lafayette, IN Mathematics, B.S.

Professional Experience

2002 – present	Post Secondary Teaching: Taught undergraduate and graduate courses at Rutgers University in both the Mathematics Department and the Graduate School of Education. Courses included mathematics content for pre and in-service secondary mathematics teachers, for liberal arts majors, and research on teaching mathematics in urban environments. In addition, taught algebra, pre-calculus, and calculus at the community college level.
1993 - 2003	High School Teaching: Taught high school mathematics for ten years, nine of which were in Texas. Predominately taught Algebra for at risk students and AP Calculus. Wrote district curriculum for Algebra and Calculus courses. Chaired department content teams. Coached cross country and track and field. Received a district teaching award and a national teaching with technology award.
1992	Middle School Teaching: Taught sixth grade mathematics for one year. Assisted in the development of new district curriculum for the eighth grade. Represented the district as a participant in the Rice University Middle School Mathematics Project, which was designed to promote leadership in middle school mathematics education.

Publications

McCulloch, A.W. (2007). *Insight into graphing calculator use: Methods for capturing the student's voice*. Paper submitted to appear in the Proceedings of the 8th International Conference on Technology in Mathematics Teaching, Hradec Králové, Czech Republic.

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