ZENO, ARISTOTLE, THE RACETRACK AND THE ACHILLES:

A HISTORICAL AND PHILOSOPHICAL INVESTIGATION

By

BENJAMIN WILLIAM ALLEN

A Dissertation submitted to the

Graduate School-New Brunswick

Rutgers, The State University of New Jersey

in partial fulfillment of the requirements

for the degree of

Doctor of Philosophy

Graduate Program in Philosophy

written under the direction of

Robert Bolton

and approved by

____

____

____

____

New Brunswick, New Jersey

October 2008

ABSTRACT OF THE DISSERTATION

Zeno, Aristotle, the Racetrack and the Achilles:

A Historical and Philosophical Investigation

by BENJAMIN WILLIAM ALLEN

Dissertation Director:

Robert Bolton

I reconstruct the original versions of Zeno's Racetrack and Achilles paradoxes, along with Aristotle's responses thereto. Along the way I consider some of the consequences for modern analyses of the paradoxes.

It turns out that the Racetrack and the Achilles were oral two-party question-and-answer dialectical paradoxes. One consequence is that the arguments needed to be comprehensible to the average person, and did not employ theses or concepts familiar only to philosophical specialists. I rely on this fact in reconstructing the original dialectical versions of the paradoxes. I show that both paradoxes rely for their success on forcing the dialectical answerer to reflect on his own potentially unending experience of imagination, and show that this renders the most popular contemporary critique of each paradox unworkable in an ancient dialectical context.

In responding to the Racetrack, Aristotle seeks to replace the answerer's first-person experience with something more objective, a visible diagram. He then argues that the Racetrack involves an equivocation, an equivocation resulting from the fact that one

and the same visible diagram can be interpreted in two ways. He frames his charge of equivocation in standard dialectical fashion, but his use of a diagram is his own innovation.

While first employing his response to the Racetrack to construct a response to the somewhat different Achilles paradox, Aristotle later proceeds to offer a revised critique of the Racetrack itself. His revised critique is heavily influenced by his reaction to a variant pre-Aristotelian version of the paradox, a version involving counting. This leads him to reflect on the possibility of mentally experiencing an infinite collection. He also reflects on his earlier diagrammatic methodology, and on the conditions that individuate points, especially halfway-points. He concludes that points which are individuated in diagrams need not represent points that are individuated in reality. His revised critique of the Racetrack hinges on the distinction between actual points and potential points, and I show that this critique maintains the dialectical form of its predecessor, while constituting a reflection on the potentially misleading nature of diagrams.

ACKNOWLEDGEMENTS

I would like to thank Rob Bolton, who has served as my dissertation advisor for many years, and has read and commented on a large swath of writing. Years ago, shortly before I took a leave of absence from Rutgers, with no clear intention of returning, he indicated that he would be interested in working on a dissertation with me, and I doubt whether I would ever have returned had he not expressed such interest. Although things have taken longer than expected, I do not regret my decision.

Thanks is also due to Peter Klein, who for a number of years has been meeting with me and Rob Bolton to go over drafts of my writing. His comments have proved quite significant inasmuch as they have helped me to understand what is interesting and comprehensible to philosophers not specializing in ancient philosophy.

Pierre Pellegrin read and discussed with me some very early versions of some of this material, and has recently read the whole thing while serving on my dissertation committee. I thank him for his efforts.

Dean Zimmerman has graciously offered, at a fairly late date, to serve on my dissertation committee, and thus to read a fairly large chunk of writing. I thank him.

I am also grateful to Alvin Goldman for serving on my dissertation proposal committee.

I have presented some portions of this dissertation to audiences at Rutgers University, Wesleyan University and Wellesley College. I am grateful for the comments I received.

All of my professors at Yale, the University of Kentucky, and Rutgers have helped to develop my understanding of philosophy, and I have often found myself thinking back to ideas from earlier courses while undertaking the current research.

This entire dissertation is organized around things I discovered while in the process of writing. The need to present my ideas to others has proved an invaluable spur in forcing me to recognize the inadequacy of some of my initial formulations. I have repeatedly rethought things after having discussed them, and the current dissertation differs greatly from my original writings. Even so, I have received many valuable comments that are not reflected in this present piece of writing. I hope to take account of some of these comments in future versions.

Dedication

Dedicated to

Richard Edgar Allen

Table of Contents

Abstract ii
Acknowledgements iv
Dedication vi
Table of Contents vii
Introduction 1
Chapter 1: Zeno the Dialectician 3
Chapter 2: Disambiguating a Word 38
Chapter 3: Disambiguating a Diagram 71
Chapter 4: The Original Racetrack 114
Chapter 5: The Achilles 152
Chapter 6: These Things Unlimited 188
Chapter 7: The Counting Racetrack 225
Chapter 8: Aristotle Analyzes the *Logoi* 247
Chapter 9: Potential Points 282
Chapter 10: Some Conclusions and Ideas 311
Appendix: Further Observations on Zeno in the *Phaedrus* 323
Bibliography 327
Curriculum Vitae 334

Introduction

You are a runner, with a racetrack before you. You will run to the end of the track. Before you get to the end, you must reach the halfway point. And before you reach the halfway point, you must get to the point halfway to it. And before you get to that point, you must reach the point halfway to it. And before you get to that point, you must reach the point halfway to it. There is an infinity of these halfway points. You cannot pass through an infinity. You will not reach the end.

There is definitely something wrong here, since a runner can certainly reach the end of a track. I call this paradox the Racetrack. It was created by Zeno of Elea in the 5th century B.C., and has engaged philosophers for twenty-five centuries. It especially perplexed Aristotle, and his writings of a century later provide our earliest written evidence of the puzzle. Zeno himself did produce some paradoxical writing, but little of it survives and none of that concerns the Racetrack. Aristotle connects the Racetrack with a second paradox, the Achilles, named for the fastest Greek hero at Troy.

You are the fastest man alive. You are pitted in a race against a tortoise. The tortoise is given a head start. He will be up ahead when you start running, but soon you will catch him. Before you catch him, you need to reach the place where he was when you started. By the time you get there he has moved on ahead. Still, you can catch him, but first you need to get to where he is now. By the time you get there, he has moved on ahead. Again, you can catch him, but first you must reach the place where he is now. By the time you get there he has moved on ahead. Still you can catch him. Again, before you catch him you need to get to the place where he is now. By the time you get there he has moved on ahead. It is hopeless. The tortoise will always be ahead. You will never catch him.

The Racetrack and Achilles are among the stranger creations of humanity. Zeno and his paradoxes are widely known today, but Zeno himself often seems a legendary figure, even on a par with the hero Achilles. And yet the truth is that Zeno and his paradoxes had a kind of concrete reality, very much a part of space and time. In the pages to come, we will attempt to uncover something of their earliest history.

The way we will do so is by examining the historical evidence. This seems obvious, but it is easy to miss, as the Racetrack and Achilles are so much better known than their history. As much as possible, we will need to put aside our preconceptions and simply try to see what the evidence tells us. Since most of the evidence is from Aristotle, we will need to learn a good deal about Aristotle in the process. And a convoluted process it will be. Eventually we may reach our goal.

Chapter 1
Zeno the Dialectician

Zeno today may be the most famous of presocratic philosophers. Both popular and expert discussions of philosophy, mathematics and physics feature Zenonian paradoxes in one form or another. Among them, the Racetrack and Achilles may be the most celebrated.

To many people, the paradoxes will seem to raise the deepest questions about the nature of reality and reason. Some think these questions have answers; others do not. Cutting across different approaches to the paradoxes is often the sense that they are the sort of thing one must think about with the utmost concentration. Only the deepest contemplation will suffice, and thinking about such things is liable to seem the ultimate private activity.

This is not exactly wrong, and in a way, it is very much correct. Apart from our private thinking, in the public world of action, Zeno's paradoxes are liable to seem so much silly nonsense. No one who does not think privately will likely learn anything from them at all. And of course one can seemingly learn all kinds of other things without such contemplation. The value of the paradoxes seems indissolubly twined with private thought.

And yet, strangely, this perspective will likely lead us astray as we follow a historical quest for the Achilles and Racetrack. For what we will find is that the Achilles and Racetrack were born as public and interactive paradoxes, a part of the ancient art of dialectic. To understand their existence, we must comprehend their dialectical life.

Dialectic was a form of two-party debate. One person acted as a questioner and a second was an answerer. The questioner would begin by proposing a thesis which the answerer had to accept or deny. Regardless of the answerer's choice, the questioner then set out to prove him wrong. To do so, he would propose a series of propositions, trying to gain the answerer's approval. The questioner's goal was to construct an argument concluding with the contradiction of the answerer's original claim, an argument that would plausibly appear sound to the answerer and any audience. If the questioner succeeded, the answerer would seem to have refuted himself, since he himself had acceded to every step. By contrast, the answerer sought to prevent this from happening.

The history of dialectic has yet to be written. This is not surprising: we possess no videotapes from antiquity, so our knowledge of ancient oral debate must be reconstructed in a roundabout manner from written sources. This is no simple task, and even basic facts are in dispute. What we do know is intriguing: dialectic played a central role in the lives of Socrates, Plato and Aristotle. And Aristotle tells us that Zeno was "the discoverer of dialectic."[1]

We will soon examine Aristotle's claim, and a variety of other evidence about Zeno and dialectic. What we discover is that Zeno was indeed a dialectician, and the motion paradoxes were dialectical arguments. Indeed, these facts underly everything Aristotle says about the Racetrack and Achilles. Before we turn to a detailed study of how Aristotle analyses the paradoxes in the *Physics*, we thus need to first examine the evidence that shows their dialectical nature, and make sure that we understand what this means.

---

1 Diogenes Laertius, *Lives of the Philosophers*, VIII.57 and IX.25. See below for discussion.

While most people may not see Zeno as a dialectician, scholars have long been aware of the evidence linking Zeno with dialectic. In 1936, H.D.P. Lee wrote:

> It was as "founder of dialectic" that Zeno's influence was most widely and generally felt; Aristotle has summed up in a phrase what must have seemed to the educated public of Athens to be the real effect of Zeno's argumentation.[2]

Lee was a leading Zeno authority, being the editor of the only English edition of ancient Zeno texts. He follows this remark with a ten page discussion of Zeno's influence on dialectic, ten pages of a thirteen page conclusion devoted to Zeno's ancient influence.

Oddly, Lee's judgement has been largely ignored by subsequent Zeno scholars. In fact, the study of dialectic has played almost no role in the study of the ancient Zeno evidence. Indeed, in this regard, Lee himself is no exception. While he aptly assesses Zeno's *influence* on dialectic, Lee is strangely reticent to admit that Zeno actually *was* a dialectician. In fact, his piece by piece examinations of ancient Zeno evidence do not make use of the possibility, and even less, the fact, that Zeno was a dialectician.

In this respect, the present study differs from earlier scholarship. Most of this investigation will focus on the discussions of the Racetrack and Achilles that are found in Aristotle's *Physics*. But the framework for our examination of the *Physics* will first be established by a variety of evidence from elsewhere, evidence often well-known but not well understood. This evidence makes it clear that Zeno was a dialectician. Moreover, we find that the motion paradoxes in particular were dialectical arguments, and that Aristotle consistently takes this for granted when discussing them.

While some of the basic facts about Zeno and dialectic have often been recognized, if unappreciated, it does not seem that scholars have ever grasped the

---

2 Lee, pg. 113.

essentially dialectical character of the motion paradoxes as Aristotle saw them. Most have presumed that Aristotle was familiar with the paradoxes not via oral debate, but rather via some text written by Zeno. Yet this view suffers from well-known problems.

The most likely candidate for a text containing the motion paradoxes is a composition by Zeno that plays a role in Plato's dialogue *Parmenides*. The first part of the *Parmenides* consists of a discussion among Zeno, his mentor Parmenides, and a young Socrates.[3] As Plato depicts the discussion, it opens with Zeno reading aloud this very text. Thereupon, Socrates and Zeno discuss the nature of the treatise.[4] From their discussion, it appears that the treatise consisted of nothing but a series of distinct arguments designed to refute the rather abstract, if commonsensical, thesis that: "Many things exist." Indeed each argument began by assuming this very thesis, and then went on to derive seemingly absurd consequences from it. Zeno explains that the treatise is directed against those who scoffed at Parmenides, who had famously asserted that "All is one." The scoffers found this absurd, but Zeno aimed to show the equal absurdity of their own more "obvious" view that "Many things exist."

Zeno apparently did compose such a book. It is mostly lost, but both Proclus and Simplicius seem to provide us with information about this book, information that derives from a source independent of Plato, perhaps Zeno's text itself, and yet meshes well with

---

3 But actually, Plato presents the entire *Parmenides* as a narration of Cephalus (126a4), who is recalling the words of Antiphon, who is recalling the words of Pythodorus, who can supposedly recall from memory the Zeno/Parmenides/Socrates discussion. (126b8-c3) It is not a simple matter to say what this means for the literary structure of the *Parmenides*, nor is it a simple matter to say what it means for our understanding of the *Parmenides* as a source of evidence concerning Zeno.
Among Platonic dialogues, this sort of problem is not unique to the *Parmenides*, and it is in many ways well-known among scholars who study presocratic philosophy, regarding which Plato is an essential, if challenging, source of evidence.

4 See 127c5-128e4.

Plato's depiction.[5] This plurality text is the only piece of writing by Zeno that is distinctly described by the earliest authorities, and so many scholars have surmised that it is the only thing that Zeno wrote.

But this poses a problem. In *Physics* Z.9, Aristotle describes four Zenonian paradoxes involving motion, among them the Racetrack and the Achilles. No naive reader would ever suppose that they are intended to show the absurdity of the claim that "Many things exist." So there is an inherent problem in supposing that the motion paradoxes were present in the plurality book. Nonetheless, assuming that Zeno composed no other texts, but that the motion paradoxes were promulgated in writing, various scholars have contrived ways to fit them into the plurality book.[6] Yet no proposal commands widespread support. Indeed there remain many open questions about the plurality book, and more generally, about Zeno's use of writing.

Aristotle may or may not have possessed written versions of Zeno's motion paradoxes. But we will find that to understand how he portrays them, any written versions are largely beside the point, since Aristotle depicts the paradoxes as living pieces

5 In his commentary on the *Parmenides*, Proclus, writing the the 5th centuryA.D., refers multiples times to the Zenonian text mentioned in the dialogue. Dillon (1987, pp. xxxviii-xliii) surveys these references and concludes that Proclus is not simply extrapolating from Plato, but possessed a text that he himself believed to be the Zenonian original. Dillon adds that this was "probably, though not certainly, the same document that was available to Simplicius a century later." (pg. xliii) What is certain is that in his commentary on Aristotle's *Physics*, Simplicius makes several references to some Zenonian text. Moreover, he provides fragments and paraphrases from this text that would seem to fit well with the description of the plurality book given in the *Parmenides*. (139,5-19; 140,27-141,8) Also see below, footnote 17.

6 Vlastos, for instance, suggests that Plato misinterpreted what I am calling Zeno's "plurality" book, and erroneously assumed that each of its *logoi* was implicitly directed at the thesis "Many things exist", whereas, according to Vlastos, this was not actually the case, some of the *logoi*, including the motion arguments, being meant to support other Parmenidean ideas. (Vlastos (1985), pp. 275-279)) Owen claims that the motion paradoxes "play an essential part in the attack on plurality" (pg. 141), arguing that they constitute an exhaustive attack intended to show the absurdity of every possible pluralistic conception of space and time. (pg. 140) Matson, in "Zeno Moves!", has also recently argued that the motion paradoxes should be construed as part of the attack on plurality, while emphasizing, as his title suggests, that Zeno should not thereby be seen as himself rejecting the existence of motion, this absurdity being charged to his pluralistic opponents.

of oral dialectic. In discussing them, he sees himself not as analyzing a text of the dead Zeno, but as facing a living dialectical opponent. Understanding this central fact leads to numerous clues about how to understand Aristotle's detailed analyses of the paradoxes, clues that have typically gone unnoticed by scholars. If we want to grasp the *Physics*, we need to understand the background to the *Physics*. And so we need to first consider the evidence that Aristotle saw Zeno as a dialectician and the motion paradoxes as dialectical arguments.

Aristotle's Reference to Zeno as the Discoverer of Dialectic

Diogenes Laertius wrote a series of biographies of philosophers, likely in the 3rd century A.D. These included a *Life* of Zeno, and a *Life* of Zeno's rough contemporary, Empedocles. In the *Life* of Empedocles, Diogenes reports that Aristotle, in his dialogue *The Sophist*, now lost, called Empedocles the discoverer of rhetoric, and Zeno the discoverer of dialectic. (VIII.57) Later, in his *Life* of Zeno, Diogenes again repeats that Aristotle called Zeno the discoverer of dialectic and Empedocles the discoverer of rhetoric. (IX.25)[7] Diogenes has no obvious reason of his own for twice mentioning Zeno and Empedocles together in this way. So he is likely just following Aristotle, who himself must have paired the two claims in *The Sophist*.[8]

The Zeno claim is well-known to scholars. But they have typically ignored the fact that Aristotle pairs it with the Empedocles claim. In fact, this pairing is crucial to understanding what Aristotle means in depicting Zeno as the discoverer of “dialectic.” And so it is crucial to deciding what conclusions about Zeno we may legitimately derive

7 Additionally, in the Prologue to the *Lives* (I.18), Diogenes himself says that dialectic goes back to Zeno.

8 Diogenes is known as someone who reliably repeats what he reads, without subjecting it to much analysis. Barnes (1982, pg. 588) calls him a "scissors and paste historian of philosophy"

from Aristotle's depiction.

What we find is that Aristotle is depicting Zeno as the discoverer of the very sort of *oral two-party* question-and-answer debate that we have thus far been calling "dialectic." Interestingly, among scholars who have examined this passage, the most prominent view holds that what Aristotle here calls "dialectic" need not be oral *or* two-party. Instead, scholars think that Aristotle is referring to a general argumentative technique. Using this technique, an arguer begins by considering some thesis of an opponent, and logically deduces from it a contradiction, thus apparently revealing that the original thesis was false. Of course, the questioner does this very thing in oral two-party dialectic. But others may do so as well. Someone might use the maneuver orally to attack a thesis, without any actual opponent answering questions or even being present. Or someone might do the same thing in writing. In fact, Zeno himself did this in writing, in the plurality book described by Plato. So it might seem reasonable that scholars should interpret Aristotle's dialectic remark in light of this written work of Zeno.[9]

If the prevailing interpretation were correct, then Aristotle might be depicting Zeno as the discoverer of dialectic simply in virtue of Zeno having composed the plurality book. Hence the Zeno remark would provide no basis for us to conclude that Zeno practiced oral two-party dialectic. But the standard view is mistaken, and Aristotle is clearly insinuating that Zeno practiced and initiated a sort of oral two-party debate. This becomes clear when we consider the often ignored comparison between dialectic and rhetoric, which are invoked as a pair in Aristotle's *Sophist*.

9 Lee, pp. 117-8, seems to originate this view. Ryle (1968) pp. 44-5, concurs. By contrast, some scholars do seem to recognize that Aristotle ascribes an oral role to Zeno. For instance, Kirk, Raven and Schofield, pg. 278, note that Aristotle is likening Zeno to Socrates. But they do not significantly develop the idea.

Aristotle's joint invocation of dialectic and rhetoric is part of a substantial philosophical tradition that regards the two as distinct but related oral practices. The contrast between them is a significant theme in Plato's *Gorgias* and his *Phaedrus*.[10] Aristotle himself composed a work on each, the *Topics* on dialectic and the *Rhetoric* on rhetoric. The opening line of the *Rhetoric* affirms the relation between the two: "Rhetoric is the counterpart of dialectic." (1354a1) In the final chapter of the *Sophistical Refutations* (which is itself the final book of the *Topics*), Aristotle again contrasts the pair. In a passage well-known for its immodesty, Aristotle praises himself as the first to develop a *theory*, or *art*[11], of dialectic, as opposed to merely being a practitioner. He contrasts the situation with that of rhetoric, noting that the art of rhetoric had developed bit by bit over time. By contrast, with dialectic there had been no prior development of the art. Instead, he recalls how earlier "teachers" had merely passed out written dialectical arguments for their students to memorize, just as the sophist Gorgias had distributed written speeches when "teaching" rhetoric. (183b26-184a1)

In each of these comparisons, there is no doubt that dialectic and rhetoric are being contrasted as oral practices. One of the key distinctions between them is that while rhetoric involves one speaker, dialectic is an interactive process requiring two speakers. When Aristotle in *The Sophist* conjoins dialectic and rhetoric, we should naturally presume that he means what he normally means by *dialectic* and *rhetoric*. As he

10 In the *Gorgias*, the term διαλεκτική (dialectic) is not used, although the cognate verb διαλέγω (discuss) is commonly used to invoke the typical Socratic question-and-answer discussion that leads one party to accept his own refutation. This sort of interactive discussion is contrasted with rhetoric. (Cf. 449b4-8, 457c4-458b3, 471e1-472d4, 473d6-474b5.) The *Phaedrus* continues the contrast, and explicitly introduces the term διαλεκτική. As the *Phaedrus* contains important Zeno evidence in its own right, we will discuss some of the relevant passages below.

11 The Greek word is τέχνη (*techne*), which can refer either to the theoretical groundwork useful for engaging in practical activities, or to a handbook that presents such a theory, a handbook such as the *Topics* itself.

elsewhere regards dialectic as a sort of two-party question-and-answer debate, we should presume that Aristotle is depicting Zeno as the discoverer of this very sort of question-and-answer debate.

We might certainly wonder what exactly Aristotle means in depicting Zeno as the *discoverer* of dialectic, and likewise what historical facts might motivate this depiction. These are difficult questions, and especially so when we consider that if Aristotle's dialogues were like Plato's, they might well have taken many liberties with historical facts, using them for highly theoretical ends.[12] But we may still learn a great deal without addressing such issues. For if Aristotle so much as *portrays* Zeno as the *discoverer* of dialectic, which he does, then Aristotle thereby portrays Zeno as a *practitioner* of dialectic, for certainly it is most natural to assume that the discoverer of a practice actually practiced it.

Since Aristotle seems to portray Zeno as a practitioner of dialectic, it most likely follows that Aristotle himself actually *believed* Zeno to be a practitioner of dialectic. Now this is by no means a certain inference. But we have no specific reason to doubt it. In particular, we should not presume, as do many scholars, that the *Sophist* remark ought to be read as a comment on Zeno's writing. To do so is to impose a complicated interpretation, in the lack of any immediate evidence, where a a simpler explanation would suffice. So it seems most plausible that Aristotle indeed believed Zeno to be a practitioner of dialectic.

Since Aristotle, apparently, believed Zeno to be a practicing dialectician, the most

12 A good example of Plato doing lies at *Theaetetus* 152d2-e9, where Socrates depicts Protagoras, Heraclitus, Empedocles, Epicharmas, and Homer as proponents, contra Parmenides, of the theory that nothing ever *is*, but that everything is always *becoming*. Whatever the merits of this depiction, it is certainly not how most of these men would have *construed* themselves.

likely conclusion is that Zeno actually was. So Zeno, most likely, was an oral dialectician. We could be wrong, but the most natural reading of the *Sophist* remark suggests that Zeno was indeed an oral dialectician, and that Aristotle saw him as such.

Zeno in Plato's *Phaedrus*

Like Aristotle, Plato regarded Zeno as someone notable for his oral practice, most likely his oral practice of dialectic. This becomes clear when we examine a reference to Zeno in the *Phaedrus*. The passage in question is well-known to Zeno scholars, but as with the Aristotle passage, here too they have failed to examine the context in which it occurs.

The *Phaedrus* portrays Socrates in conversation with Phaedrus. At one point, Socrates makes an oblique reference to Zeno: "Do we not know the Eleatic Palamedes who speaks with art (*techne*[13]), so that the same things appear to his hearers to be like and unlike, also one and many, and again resting and moving?" Phaedrus replies: "Most certainly."[14] (261d6-9) Scholars agree that "the Eleatic Palamedes" is Zeno.[15] When we examine this passage carefully, we find that it involves both a reference to Zeno's plurality book *and* a reference to Zeno as a practitioner of some oral art.

---

13 Recall that *techne* may refer either to an art or craft itself, or to a written treatise describing that art. The whole section of dialogue surrounding the Zeno reference hinges on this ambiguity, which is difficult to capture in English. In the given passage, the Greek reads τέχνῃ, (dative of τέχνη) which can mean either artfully (i.e. with art), or using an *Art* (i.e. using a written treatise for the art). The ambiguity seems intentional. (Dušanić (1992) concurs, pg. 351.)

14 My translations from the *Phaedrus* largely follow Nehemas/Woodruff, with some modifications.

15 There is near universal agreement here. One novel exception is Dušanić (1992), who argues that τὸν Ἐλεατικὸν Παλαμήδην (the Eleatic Palamedes) is actually a double reference to Zeno of Elea *and* to the sophist Alcidamas of *Elaea*. His argument gets its initial motivation from the fact that Quintilian, the Roman writer on rhetoric, claims that Plato referred to Alcidamas as Palamedes. Dušanić marshals a good deal of additional evidence, but he does not, I think, sufficiently consider the issue of how this putative double reference is supposed to function within the literary structure of the dialogue. (But see pg. 352, ibid.) Moreover, he fails to consider some crucial clues about how the reference does function. See below, footnote 28. For additional details, consult his article, which constitutes one of the few attempts to seriously examine the Palamedes reference in its broader context, and contains many valuable references.

The potential complexity of Zeno's role is immediately suggested when we consider the phrase “to his hearers” (τοῖς ἀκούουσι). In Greek, this phrase has two crucial connotations that are missing in the English. On the one hand, a "hearer" of someone may be their student.[16] This usage obviously derives from the predominantly oral nature of teacher-student relationships. On the other hand, a "hearer" of someone may be a "reader" of their book.[17] In 4th century B.C. Athens, the main use of a written text was to be read aloud to an audience, silent and private reading gaining prominence much later. So the primary way in which anyone might experience the written text of another, and hence, in a way, "read" it, was by hearing it read aloud. Thus, as things stand, the "hearers" of the Eleatic Palamedes might be any casual hearers of Zeno himself, more committed students, or anyone hearing anyone read a text of Zeno, even after Zeno himself was dead, as he was when Plato wrote the *Phaedrus*.

If the reference to the hearers involves a text, then it would seem to be the plurality book discussed in the *Parmenides*.[18] The *Parmenides* makes clear that the first argument in the book aimed to show that if many things exist, then these “same things” are both “like and unlike”. (127d6-e5) This seems very similar to what the Eleatic Palamedes demonstrates to his hearers, at least according to the *Phaedrus*. Moreover, it seems that the rest of the arguments in the plurality book followed the same pattern of concluding that the “same things” share two contrary properties. This meshes well with the *Phaedrus* report.[19]

16 LSJ, ἀκούω, II.4.

17 LSJ, ἀκούω, I.4

18 Most scholars believe that the *Parmenides* was written after the *Phaedrus*.

19 Speaking of Zeno and, apparently, his plurality text, Simplicius writes: "Moreover, in the book of his having many arguments (ἐπιχειρήματα), he [Zeno] demonstrates in each [argument] that for someone asserting that many things exist, it follows that he asserts contrary things." (139, 5-7) Simplicius also provides evidence, including fragments, that, in addition to the three pairs mentioned above, the book

The clear consonance of the Zeno reference with what we otherwise know about the plurality book suggests that Plato does indeed intend to evoke the image of that very book being read aloud. What is much less obvious at first glance is that the passage also presupposes that Zeno is an oral dialectician. To see this, we need to consider how the reference fits into the broader context of the dialogue.

The Zeno reference occurs as part of an attempt by Socrates to investigate whether rhetoric is an art (*techne*). (260d3-261a6) Socrates asks: "Well, then isn't the rhetorical art, taken as a whole, a way of directing the soul by means of *logoi* (speeches/arguments), not only in lawcourts and on other public occasions but also in private?"[20] (261a7-9) He goes on to emphasize the unity of this art of rhetoric, regardless of topic. But Phaedrus demurs, noting that he has only encountered the art of rhetoric in the lawcourts and the Assembly. He is thus unfamiliar with the private rhetoric.

This reference to private rhetoric might sound odd. In Greek, as in English, the

---

contained arguments involving the pairs small-large (139, 7-19; 140, 34-141,8) and limited-unlimited (140, 27-34). These arguments are well-known, although their interpretation is difficult.
Rightly supposing that the *Phaedrus* reference involves Zeno's plurality book, and noting how Zeno is said to show that the same things are resting and moving, scholars have sometimes taken the *Phaedrus* passage as an indication that the plurality book contained the four motion paradoxes discussed by Aristotle in the *Physics*. But this is a hasty conclusion. The *Phaedrus* does not refer to any of these paradoxes, but rather to a different argument, paraphrased by Proclus in his commentary on Plato's *Parmenides*. (769, 23-770,1) Proclus reports the argument using neoplatonic language, so it would take some effort to extract Zeno's own formulation. But it is fairly evident that, unlike the motion paradoxes in the *Physics*, this resting-moving argument conforms to the structure that the *Phaedrus* ascribes to the plurality book arguments, beginning with the assumption that many things exist, and deriving the conclusion that the same things move and rest. (Cf. Dillon (1987), pg. xli.)
While it would not be an easy task to establish definitively that all of the plurality book arguments conformed to the aforementioned structure, the evidence from our best sources concerning this text, Plato, Proclus and Simplicius, suggests that it is plausible. This idea has not received as much scholarly attention as it seems to merit, no doubt, in part, due to efforts to include the four motion paradoxes in the plurality book, an idea which we will soon find to be seriously misguided. For our immediate purposes, all that really matters is that arguments of the given type played a prominent role in the plurality book.

20 I leave *logoi* untranslated to preserve an ambiguity. Rhetoric *per se* might well be described as an art of directing the soul by means of *speeches*. But since the "private rhetoric" turns out to be dialectic, we might rightly describe the art as directing the soul by means of *arguments*. (Nehemas/Woodruff translate "by means of speech".)

normal sense of “rhetoric” will not encompass such private rhetoric. But it is fairly evident that a practitioner of either the public or the private rhetoric will be someone actually *speaking*, not merely the author of a text *written* in a certain form, but *read* by someone else.[21]

Socrates then asks an absurd question: "Well, have you only heard the Arts (*technai*) of Logoi[22] by Nestor and Odysseus - those they wrote in their spare time in Troy? Haven't you also heard[23] the works of Palamedes?" (261b6-8) Nestor, Odysseus, and Palamedes are all heroes of the Trojan war, heroes noted for their speaking ability. Of course, the Homeric poems contain no references to any of them writing treatises, and scarcely any references to writing at all.[24]

The absurdity flies by Phaedrus, which is not unimportant. One of the themes of the *Phaedrus* is to disparage the role of writing, for writing seems merely an imitation of speech and thought, an imitation fostering passivity.[25] Clearly, the fact that Nestor, Odysseus and Palamedes were brilliant speakers does not entail any association with writing, whether as students or as writers of treatises. But Phaedrus is infatuated with written speeches and does not see this.[26]

The question by Socrates must be related to the prior statement by Phaedrus. Phaedrus has affirmed his familiarity with the public, but not the private, use of rhetoric. Socrates immediately responds by asking whether he has heard only the treatises of

---

21 But as things stand so far, it seems that either sort of rhetorician might read their own text, and thereby practice their form of rhetoric.

22 Recall that *techne* has a double meaning. In the present context, it clearly refers to a written text, as Socrates refers to Nestor and Odysseus as writing *technai*.

23 Nehemas/Woodruff write "heard of." But Socrates is asking whether Phaedrus has literally heard the treatises themselves read aloud.

24 One exception is *Iliad* Z.160-180, a passage that highlights the esoteric nature of writing.

25 See 274e1-275b2.

26 See 227d6-228a4.

Nestor and Odysseus, but not those of Palamedes. Nestor and Odysseus are thus linked with the public rhetoric, and Palamedes with the private.

Phaedrus responds: "No, by Zeus, I haven't even heard Nestor's - unless by Nestor you mean Gorgias, and by Odysseus, Thrasymachus or Theodorus." (261c1-3) Gorgias, Thrasymachus and Theodorus were all well-known orators and teachers of rhetoric. Moreover their rhetoric was designed for the lawcourts and the Assembly. And they may well all have written treatises on rhetoric.[27] They surely considered their written treatises to be mere propaedeutics to actual speechmaking, whether by themselves or others. The treatises were static teaching aids that could scarcely convey the full dynamic reality of an actual speech being delivered, and their authors would hardly think otherwise. Just as importantly, all three rhetoricians *both* composed their treatises (if they did even that) *and* engaged in oral speechmaking, the activity taught by the treatises.

Shortly thereafter, Socrates makes the above-mentioned reference to the Eleatic Palamedes. Zeno is thus brought into the dialogue as the counterpart of Gorgias, Thrasymachus and Theodorus, so we must consider both the similarities and the differences that Plato intends for us to notice. Just as his heroic analogue Palamedes, Zeno is the representative of the private rhetoric. But Plato insinuates that Zeno, just like his contemporaries, both practices and composes a treatise on his variety of rhetoric.[28]

27 Certainly Gorgias did. Theodorus may have. (See Aristotle's *Rhetoric*, 1414b12-15.) About Thrasymachus we do not know. The precise nature of these treatises is not clear. See also *S.E.* 183b15-184b8.

28 Both Vlastos (1975) and Dušanić (1992) rightly note the comparison between Zeno and the rhetoricians. But neither discusses the contrast between public and private rhetoric, without which the introduction of Zeno loses its point. Instead, Vlastos (pp. 287-291) merely seeks to refute the contention of Cornford (cited in Vlastos, pg. 287) that the Zeno passage shows that Plato regards Zeno as a sophist, in a pejorative sense. In this regard, Vlastos is certainly correct. Dušanić uses the similarity between the so-called “Eleatic Palamedes” and the rhetoricians as part of his motivation for positing the sophist Alcidamas as a second unstated referent, along with Zeno. But he offers no explanation of how the Alcidamas/Zeno conglomerate is supposed to exemplify “private rhetoric”.

In fact, Zeno surely did *not* compose a treatise on rhetoric in any normal sense of the word. But given the absurdity of the original reference to the "heroic" treatises, it seems that Plato must be *insinuating* that Zeno, like his contemporary “public” rhetoricians, *did* somehow compose such a treatise. Indeed, Socrates seems to be asking whether Phaedrus has actually heard Zeno's treatise on private rhetoric. Earlier, we saw that Socrates seemed to be asking about Zeno's plurality book. So if we have rightly understood everything so far, it follows that the “treatise on private rhetoric” and the plurality book are one and the same.

At first this seems absurd. But in fact there *is* a way of construing the plurality book as a treatise on "private rhetoric", so long as we understand "private rhetoric" to be dialectic. Certainly, the plurality book will not be a dialectic treatise in any normal sense[29], but it may well be such a treatise in the sense demanded by the *Phaedrus*. In fact, one of the main themes of the dialogue is that writing cannot convey knowledge at all. Only by thinking do we learn.[30] So properly speaking, the only way in which anything can *be* a worthwhile treatise, on anything, is by spurring thought. No treatise promoting passive acceptance of doctrine can perform this function. And this is especially true in the case of a treatise on dialectic, which is necessarily interactive.

By contrast with a treatise presenting a definite doctrine, Zeno's plurality book, just as Plato's dialogues, may enable thinking, in Plato's view, precisely because it does not tell the audience what to think. It merely lays out the seemingly contradictory consequences of the doctrine that "Many things exist." Plato apparently sees the book as designed to provoke the audience to puzzle over it and to discuss dialectically exactly

29 Indeed, we have already seen that Aristotle's *Topics* was the first treatise on dialectic.
30 Cf. 274e1-275b2, 275d4-276a9.

what positive conclusion should be drawn. Indeed, this is exactly what Socrates and Zeno do in the *Parmenides* after Zeno has read the "treatise". Thus, the book serves as a natural catalyst for dialectic, and this is the one and only way in which it is a treatise “on” dialectic. But according to the *Phaedrus*, this is the only way anything *could* be a useful treatise on dialectic: passive absorption of dialectical rules will never suffice.[31]

Later in the *Phaedrus*, we find a much more explicit sign that, in some way, "private rhetoric" really is dialectic. To thoroughly establish this would divert us from our main line of inquiry, so I will withhold discussion until the Appendix. But we have already attained some definitive results: we have seen that Plato portrays Zeno *both* as a writer *and* as an oral practitioner of some sort. And given that Zeno's oral practice is *contrasted* with the typical rhetoric of well-known rhetoricians, it is at this point quite plausible that Plato is portraying Zeno as a practitioner of dialectic.

The Paradox of the Millet Seed

We have now seen evidence that both Plato and Aristotle regarded Zeno as a dialectician. A further sign comes from Simplicius, the 5th century A.D. neoplatonist philosopher whose commentary on Aristotle's *Physics* provides much of our best evidence on presocratic philosophy.

In the *Physics*, Aristotle makes a terse reference to Zeno: "διὰ τοῦτο ὁ Ζήνονος λόγος οὐκ ἀληθής, ὡς ψοφεῖ τῆς κέγχρου ὁτιοῦν μέρος." (On account of this the *logos* of Zeno is not true, that any part whatsoever of the millet makes a sound.) (250a19-21) Aristotle proceeds to analyze the problem with this “*logos* of Zeno”, but he says no more about the original *logos* itself. We will later examine the word *logos* in more detail. Here

31 Thus Plato would say that memorizing the *Topics* will scarcely make someone a knowledgeable dialectician. The dialectician needs to know how to *do* something.

"the *logos* of Zeno" seems to be some kind of argument, but the *Physics* leaves it open just what kind it is.

In fact, from Simplicius, we learn that the *logos* was actually a dialectical argument. In explaining the line from Aristotle, Simplicius recounts a dialectical encounter between Zeno and the sophist Protagoras. In normal dialectical fashion, Zeno begins by asking Protagoras to affirm or deny a given thesis, in this case, the thesis that an individual millet seed makes a sound when it falls. Protagoras denies it. Then Zeno proceeds to question him, gradually obtaining assent to the premises needed to prove that an individual seed *does* make a sound when it falls. First he asks whether a whole bushel of millet seeds makes a sound when it falls. Then he notes that there is a ratio between an individual seed and the whole bushel. Finally, as a conclusion, he extracts the concession that an individual seed will likewise make a sound in the same ratio to the sound of the whole. (1108, 18-28) Simplicius thus describes a paradigmatic dialectical encounter. While the details of his account merit scrutiny, we are, on the whole, well-justified in ascribing a dialectical origin to the millet seed *logos*.

Simplicius wrote more than nine hundred years after the supposed Zeno-Protagoras conversation. He reports no source for his story, but we can be sure he is not merely offering a speculative reconstruction of his own.[32] This is because Aristotle nowhere mentions Protagoras in conjunction with the millet seed. Simplicius would have little reason of his own to insert Protagoras into the story, so he must be relying on some

32 In fact, Simplicius often does offer reconstructions of early philosophical arguments, often without giving any overt sign that he is doing so, as opposed to relying on written evidence. Indeed, most scholars agree that he does this with the Racetrack and Achilles. (947.6-31, 1013.3-16, 1013.31-1015.2) This can easily mislead contemporary historians. But he does *not* reconstruct the motion paradoxes as dialectical arguments, which is an additional sign that his rendition of the millet seed paradox, which is dialectical, is based on some separate tradition.

source independent of Aristotle.

Of course, the original author of Simplicius' report might himself have been offering speculation or a dramatization. But even if this particular Zeno-Protagoras encounter never happened, this earlier author is surely not the inventor of the millet seed paradox itself. And this means that he is likely familiar with it as a dialectical argument, even if he does dramatize one occasion when it might have been used. In fact, if the *logos* was known in antiquity as a dialectical argument, it would have been used orally many times by many people, even if Zeno was the originator. Our best bet is thus that Zeno himself first promulgated the paradox in dialectical form. The alternative is that Zeno first produced the argument in some narrative written text, from which someone later created a dialectical version.[33] But we have no need for such a complex story. As both Plato and Aristotle regarded Zeno as a dialectician, we should naturally expect him to be making dialectical arguments. The evidence indicates that the millet seed paradox was one of them.

Zeno, Dialectic and the Motion *Logoi*

We have now seen considerable evidence that Zeno was an oral dialectician, and that this fact was central to his image in antiquity. Yet strangely, in recent times, Zeno's oral activity has generally gone unrecognized, and has certainly played little role in Zeno scholarship. There are multiple reasons for this neglect.

In the case of the Plato and Aristotle evidence, we have already seen that scholars

---

33 This is Hasper's view. He rightly notes that Simplicius' report matches the Aristotelian definition of dialectic. But then he suggests that Simplicius' source invented the Zeno-Protagoras conversation to mesh with Aristotle's description of Zeno as the inventor of dialectic. (Hasper (2003), pp. 19-21) But there is a simpler explanation: Aristotle himself regarded Zeno as the inventor of dialectic in part because he regarded Zeno as the inventor of many dialectical arguments, the millet seed among them.

have erred by considering seeming Zeno evidence in isolation from the texts in which it is found. In fact, this is a problem with much 20th century scholarship on presocratic philosophy. Most of our evidence on early philosophy is found scattered throughout the writings of various later authors, and scholars have often focused their attention solely on the lines that deal directly with the early philosopher they are studying. Often, they have thus missed valuable clues that are revealed by the manner in which a later author invokes a presocratic.[34] In later chapters, we will find that the study of Zeno and Aristotle can scarcely be separated, even if we must continually remember that they are very distinct individuals.

Others reasons for the neglect of Zeno's dialectic arise from the specifically oral nature of dialectic. For one thing, our evidence for ancient philosophy consists largely of written texts, which makes it much easier to think about written texts than about oral activity. This is especially true in the case of classical scholars, who have often construed their very job as the study of texts.[35] By contrast, while historians of philosophy have largely focused their attention on specific texts, they have typically not seen their focus primarily *as* the study of those texts, and certainly not as the study of speech, but rather as the study of thoughts and arguments. Indeed, since Aristotle, philosophers have often maintained that that purpose of language is the expression of thought.[36] On this view, the manner in which thoughts are expressed, whether written or oral, might seem rather insignificant. Historians of philosophy have often incorporated such ideas into their

34 Recently, many scholars have started to realize that this is shortsighted. For one work that places emphasis on reading presocratic evidence in its textual context, see Osborne (1987).

35 This view certainly dominated the 19th century, but it has gradually lost sway since then, especially under the influence of the work of Parry and Lord on the oral origin of the Homeric poems. See Lord (2000).

36 This view is especially prevalent among so-called "analytic" philosophers. Historians trained in analytic philosophy have largely dominated the recent study of Zeno.

methodology. They have hence devoted little attention to what may be philosophically significant differences between the speech and writing of philosophers. But this is a crucial issue in the study of Zeno.

As a further problem, Zeno has been the victim of his own success. Unlike much presocratic philosophy, Zeno's paradoxes often seem relevant to contemporary debates in the philosophy of science, mathematics and logic. Discussion of such issues is typically filled with logical and mathematical symbols, and Zeno-style arguments are often cast in partially symbolic form. These written symbols are not a part of natural language, and ordinary oral expression of modern symbolic arguments seems at best an imperfect analogue. While such modern discussions often abjure any pretense to historical accuracy, historians themselves often see Zeno in light of his contemporary relevance. And so the inherently modern form of modern Zeno-style arguments can easily obfuscate the importance of dialectic, an inherently oral activity.

While these various factors help to explain why Zeno's dialectical role has been largely ignored, none of them, of course, gives us any real reason to doubt that Zeno was a dialectician. In fact he was.

Aristotle distinguishes three varieties of what I am calling dialectic.[37] He usually restricts the term *dialectic* to the first of these varieties. As he uses the term, a *dialectical* argument will always employ reputable opinions as premises and will always use valid

37 See *S.E.* 2. It might seem that Aristotle here distinguishes four varieties. In fact, he does distinguish four varieties of question-and-answer argument, but the first sort, didactic or demonstrative argument, is interactive only in a minimal sense. In didactic arguments, the answerer will be a learner learning from a master and will always answer *Yes*. Moreover, unlike the other varieties, didactic arguments do not generally attempt to *refute* any thesis, but simply to prove a conclusion directly. (Although they can be used for refutation as well. See *S.E.* 170a23-26. But Aristotle, at *S.E.* 170a34-6, distinguishes such "didactic" refutations from refutations that are properly dialectical.) Didactic arguments are the subject of the *Prior* and *Posterior Analytics*.

inferences. (165b3-4) The second variety is *peirastic*, or examinational argument. In peirastic examination, a non-expert in some domain will examine a putative expert in that field, attempting to overturn some purported thesis of the given domain, in an effort to test the supposed expert. (165b4-7) This is familiar from Plato's *Apology*, where Socrates describes his examination of the politicians, poets and craftsmen. In fact, Aristotle regards peirastic as a sub-class of what he calls dialectic. (171b4-6) Finally, there is *eristic*. An eristic arguer seeks to win a debate at all costs, without regard for truth. As such, he typically employs *apparently* reputable opinions or *apparently* valid arguments. (165b7-8) In Plato's *Euthydemus*, the brothers Euthydemus and Dionysodorus are paradigmatic eristical arguers.

When studying the motion paradoxes, we will sometimes need to distinguish among these three sorts of debate. But for the most part, I will speak of “dialectic” as the practice encompassing all three varieties. The reason for this is that Aristotle's demarcation is highly theoretical, and hinges on the subtle assessment of the intentions of dialecticians and the normative evaluation of their arguments.[38] The theoretical distinction most likely postdates Zeno, being, no doubt, based on Aristotle's observations of the dialectic of his own day. While we are greatly dependent on Aristotle for our knowledge of how dialectic was practiced, we also need to be wary of exporting Aristotelian classifications back to Zeno himself.

In general, we should avoid making too many prejudgements about Zeno's role as the originator of dialectic. We know that he *did* practice dialectic, but beyond that we do

38 In the same way, most contemporary uses of the term “dialectic” involve some philosophical or normative component. By contrast, in this investigation, I use the term with a purely descriptive sense to describe an ancient style of debate.

not yet know much. In fact, a full examination of Zeno's role as a dialectician would take us well beyond the scope of this investigation. For us, evidence of Zeno's practice of dialectic is really just a starting point, and forms the backdrop against which we will try to understand two of his dialectical arguments, the Racetrack and Achilles.

In *Physics* Z.9, Aristotle writes: "τέτταρες δ'εἰσὶν οἱ λόγοι περὶ κινέσεως Ζήνωνος οἱ παρέχοντες τὰς δυσκολίας τοῖς λύοισιν... ." (And four [in number] are the *logoi* of Zeno about motion, which give ill-tempers to those who [try to] resolve [them]... .)[39] (239b9-11) Two of these *logoi* are the Racetrack and Achilles, and we will soon see evidence that all four were dialectical arguments.

The word *logos* is one of the most significant words in Greek philosophy, and, in principle, can mean many things: speech, discussion, definition, explanation, justification, ratio, the faculty of reason, etc. Throughout the *Topics* and *Sophistical Refutations*, it is also used in a largely technical sense to refer to dialectical arguments. But given the wide variety of uses, we can scarcely take for granted that this is what Aristotle means in *Physics* Z.9.

In fact, there has often seemed good reason to doubt this. For one thing, the word *logos* is the very word used in Plato's *Parmenides* to refer to the *written* arguments of Zeno's plurality book. As written arguments, they were not dialectical, at least in the sense that matters. But given the many senses of *logos*, we should not presume that all references to "*logoi*" of Zeno are references to entities of precisely the same sort.

At the same time, the nature of the *Physics* has often obscured the dialectical nature of the motion *logoi*. The *Physics* is not about dialectic, but rather nature and

39 We will later discuss several aspects of this line in more detail.

natural motion, so contemporary scholars, not being immersed in dialectic like Aristotle's audience, easily miss dialectical references.

In fact, we will soon see that the motion paradoxes were indeed dialectical arguments. In the future, I will typically refer to such arguments simply as dialectical *logoi*, or merely as *logoi*, as opposed to calling them *arguments*. By the word *logos*, I intend to mean exactly what Aristotle means by it in the *Topics* and *Sophistical Refutations*. The foreignness of the word is a benefit, since it reminds us that none of us has any firsthand knowledge of ancient dialectic. Whatever we say about dialectic will need to be carefully justified. It is not entirely wrong to call a *logos* an *argument*, and I will sometimes do so. But *argument* can be misleading, as the term is widely used in contemporary philosophy, and there are many contentious opinions regarding what an argument is or should be. An argument is rarely regarded as something interactive, and never as essentially so. By contrast, a dialectical *logos* is necessarily interactive. Henceforth, I will largely abandon *argument* in favor of *logos*. In studying how the motion paradoxes worked as *logoi*, we will also learn a good deal about what *logoi* actually were.

The evidence that the motion *logoi* were dialectical derives from two passages in the *Topics*, and one in the *Sophistical Refutations*. Each mentions the Racetrack in a context that presupposes that it is a dialectical *logos*. These references have been largely ignored by scholars writing on the motion *logoi*. When we examine them carefully, and compare them with the *Physics*, we find that Aristotle regarded not only the Racetrack, but all four motion paradoxes as dialectical *logoi*.

The Racetrack and *Duskolia*

Topics Θ.8 comes in the midst of a long section giving advice to dialectical answerers. In the passage that concerns us, Aristotle envisages that the answerer faces a *logos* that has a universal proposition as a premise, and that this premise is justified in turn by an inductive appeal to several particular propositions. Aristotle then writes:

> τὰ μὲν οὖν καθ' ἕκαστα πάντα θετέον, ἂν ᾖ ἀληθῆ καὶ ἔνδοξα, πρὸς δὲ τὸ καθόλου πειρατέον ἔνστασιν φέρειν· τὸ γὰρ ἄνευ ἐνστάσεως ἢ οὔσης ἢ δοκούσης κωλύειν τὸν λόγον δυσκολαίνειν ἐστίν. εἰ οὖν ἐπὶ πολλῶν φαινομένου μὴ δίδωσι τὸ καθόλου, μὴ ἔχων ἔνστασιν, φανερὸν ὅτι δυσκολαίνει. ἔτι εἰ μηδ' ἀντεπιχειρεῖν ἔχει ὅτι οὐκ ἀληθές, πολλῷ μᾶλλον ἂν δόξειε δυσκολαίνειν. (καίτοι οὐδὲ τοῦθ' ἱκανόν· πολλοὺς γὰρ λόγους ἔχομεν ἐναντίους ταῖς δόξαις, οὓς χαλεπὸν λύειν, καθάπερ τὸν Ζήνωνος ὅτι οὐκ ἐνδέχεται κινεῖσθαι οὐδὲ τὸ στάδιον διελθεῖν, ἀλλ' οὐ διὰ τοῦτο τἀντικείμενα τούτοις οὐ θετέον.) εἰ οὖν μήτ' ἀντεπιχειρεῖν ἔχων μήτ' ἐνίστασθαι μὴ τίθησι, δῆλον ὅτι δυσκολαίνει· ἔστι γὰρ ἡ ἐν λόγοις δυσκολία ἀπόκρισις παρὰ τοὺς εἰρημένους τρόπους, συλλογισμοῦ φθαρτική.
>
> Certainly all the particular propositions must be admitted, if they are true and reputable, but against the universal one must attempt to bring an objection. For to hinder the *logos* without an objection, either real or maintained, is to display ill-temper (*duskolia*). Therefore if someone does not grant the universal when justified by many [particulars], while not having an objection, it is clear that he shows ill-temper. Moreover, if he does not even have a counter-argument that it is not true, he appears ill-tempered to an even greater degree. (But even this is not sufficient. For we have many *logoi* [whose conclusions are] opposed to reputable opinions which are [nonetheless] difficult to resolve, such as the [*logos*] of Zeno that it is not possible to move or get through the *stadion*[40], but this does not render it unnecessary to accept the [conclusions] opposite to these [reputable opinions].) If then someone does not admit [a universal proposition], having neither a counter-argument nor an objection, it is clear that he shows ill-temper. For ill-temper in [dialectical] *logoi* [consists of] an answer contrary to the ways mentioned and destructive of the reasoning. (160a39-160b10)

In trying to understand this passage, we may temporarily ignore the parenthetical remark about Zeno.[41] Aristotle considers the case of an answerer who has accepted several particular theses, but does not wish to accept the corresponding universal claim

---

40 The *stadion* was a race of one *stadion* length. We will later examine this term in detail.

41 The parentheses are an addition of the the modern editor, Ross, but here they are appropriate.

that they are meant to justify. Aristotle says that the proper response is to offer an objection, i.e. a counterexample to the universal. If the counterexample is legitimate, this would definitively show that the universal claim is false. But if the answerer cannot think of a counterexample, Aristotle says that he should at least attempt to provide a deductive argument that the universal thesis is false. If the answerer nonetheless objects to the universal claim while providing *neither* of these rationales for his objection, Aristotle says that he exhibits δυσκολία (*duskolia*), which for simplicity I rendered above as *ill-temper*.[42]

The term δυσκολία and the verb δυσκολαίνειν (to exhibit *duskolia*) are quite common in the *Topics* and *Sophistical Refutations*. In these contexts, they are dialectical terms of art, and the given passage effectively explains their use.[43] In short, *duskolia* is the state exhibited by a dialectical answerer who refuses to play by the rules. We may contrast it with the state of a dialectician, questioner or answerer, who makes a logical error of some sort, whether knowingly or unknowingly. In effect, one who makes a logical error still seems to acknowledge the existence of dialectical rules, and at least appears to follow them. But the one who commits *duskolia* simply steps outside of the rules altogether. He commits an emotional or behavioral error, not an intellectual error. *Duskolia* is thus an aberrant psychological state that may arise in a dialectical context.

We may now consider the role of Zeno's *logos* in the passage. Aristotle first says that when rejecting the universal conclusion of an induction, one way to avoid *duskolia* is

---

42 Following Pickard-Cambridge, in the Oxford translation.

43 Smith, pg. 135, renders δυσκολία as "cantankerousness." He notes: "Its parent word *duskolos* means 'finicky', 'hard to please', 'quarrelsome' (the root sense is 'having a bad colon': cf. English 'dyspeptic'.) *Duskolos* has slightly comic connotations and is often associated with bad tempered, unsociable old men who find fault with everything and agree with nothing - in a word, curmudgeons (the title character in Menander's *Duskolos* is a good illustration.)."

to offer a counterexample. Then he suggests that a second way to avoid *duskolia* is to offer a counter-argument against the conclusion. The mention of Zeno occurs as Aristotle offers a caveat to this second option. Aristotle seems to allow that the answerer who offers a counter-argument is acting in good faith, but is nonetheless not always providing a dialectically sufficient response. Certainly the reference to the Racetrack makes this point well. The Racetrack somehow seems to demonstrate that a runner cannot run. We will later examine the details, but regardless of the precise structure or conclusion of the argument, it would scarcely be difficult to offer a solid dialectical counter-argument against its conclusion. In fact, Aristotle seems to suggest that to offer a counter-argument against the conclusion of the Racetrack is so trivial as to be dialectically pointless, and that the only way the answerer can legitimately avoid acknowledging the counterintuitive conclusion would be to directly object to the premises or the reasoning.[44]

This is confirmed by a passage in *Sophistical Refutations* 24:

> οὐδὲν δὲ κωλύει τὸν αὐτὸν λόγον πλείους μοχθηρίας ἔχειν, ἀλλ' οὐχ ἡ πάσης μοχθηρίας ἐμφάνισις λύσις ἐστίν· ἐγχωρεῖ γὰρ ὅτι μὲν ψεῦδος συλλελόγισται δεῖξαί τινα, παρ' ὃ δὲ μὴ δεῖξαι, οἷον τὸν Ζήνωνος λόγον, ὅτι οὐκ ἔστι κινηθῆναι. ὥστε καὶ εἴ τις ἐπιχειρεῖ συνάγειν ὡς δυνατόν, ἁμαρτάνει, κἂν μυριάκις ᾖ συλλελογισμένος· οὐ γάρ ἐστιν αὕτη λύσις· ἦν γὰρ ἡ λύσις ἐμφάνισις ψευδοῦς συλλογισμοῦ παρ' ὃ ψευδής.
>
> Nothing prevents the same *logos* from having many flaws, but the exposure of just any flaw is not a resolution. For it is possible for someone to show that a false [conclusion] has been syllogized, but to not show [the error] on account of which it has been syllogized, such as [may happen with] the *logos* of Zeno, [which concludes] that it is not possible to have moved. So that even if someone attempts to show that it is possible[45] [i.e. possible to have moved], he errs, even if

44 Smith, pp. 135-6, offers a different interpretation: "In a corrective aside, Aristotle suggests a third form of cantankerousness: refusing to grant an obvious premise just because an exotic argument exists against it." But Aristotle has mentioned not two forms of *duskolia*, but two ways to avoid it, and the mention of Zeno certainly does not introduce a third. Smith likely does not realize that Aristotle regards "the *logos* of Zeno" as a dialectical *logos*.

45 Pickard-Cambridge rejects Ross' reading of δύνατον for ἀδύνατον, and thereby translates "impossible". But this misconstrues the passage. Zeno and the users of his *logos* argue that motion is impossible. But

> he syllogizes ten thousand times. For this is not a resolution. For a resolution was an exposure of a false *sullogismos* [showing the error] on account of which it is false.[46] (179b17-24)

Here Aristotle again makes the point that it simply does not suffice for an answerer to prove that the questioner has produced a false conclusion. He also needs to expose the error in the questioner's argument. Here the Racetrack recurs, for the second time, as an example of a dialectical *logos* having an obviously false conclusion, but with an error difficult to discover.

From these passages, we see that Aristotle regards the Racetrack as a paradigmatic producer of *duskolia*. Now recall what Aristotle says in *Physics* Z.9: "And four [in number] are the *logoi* of Zeno about motion, which give *duskolia* to those [trying to] resolve [them] ... ." (239b9-11) In the *Topics*, *duskolia* is a psychological state arising in oral dialectic, and results from the humiliation of being publicly trapped in absurdity. Given that Aristotle cites the Racetrack itself in explaining *duskolia* in the *Topics*, we can be confident that he means the same thing by *duskolia* in the *Physics*. In the *Physics*, all four motion paradoxes are said to produce *duskolia*. This means that Aristotle takes all four motion paradoxes to be dialectical *logoi*.

This is not the way most scholars seem to have understood the *Physics* passage.[47] To a contemporary scholar reading *Physics* Z.9 in isolation, Aristotle will likely seem to be referring to the *intellectual* perplexity he faces in attempting to analyze the *logoi*, a perplexity he seems to indicate that others have shared. But *duskolia* is not intellectual. Indeed, it is a non-intellectual response to an intellectual problem. So Aristotle certainly

---

the referent of τις (someone) in 179b21 is not the questioner, but the answerer.

46 We will later examine the nature of a "resolution" (λύσις) and a *sullogismos*.

47 In fact, the issue is hardly discussed. But I know of no translations that convey the dialectical significance of the passage.

does not see himself exhibiting *duskolia* in dealing with the motion *logoi*, even though he is intellectually puzzled by them. When we read the *Physics* passage against the backdrop of the *Topics* and *Sophistical Refutations*, there can be little doubt that *duskolia* is something that arises in oral dialectic. And so there can be little doubt that Aristotle regards the four motion paradoxes as dialectical *logoi*.

The "Common" Nature of the Racetrack

In *Sophistical Refutations* 11, we find another mention of the Racetrack that confirms its dialectical form, at the same time providing valuable information on the content of the *logos*. The relevant passage occurs while Aristotle is explaining how a questioner may employ dialectic to examine whether a putative expert in some field indeed possesses the knowledge that he claims. This is the aforementioned "peirastic" use of dialectic[48], which Aristotle describes: "Peirastic is a certain [branch of] dialectic and considers not the man who knows, but the man who is ignorant and pretends [to know]." (171b4-6) The peirastic questioner may effectively examine a putative expert, even if the questioner himself is not an expert in the given domain (much as Socrates did). In doing so, the questioner will seek to refute a purported thesis of the given science, put forth by the putative expert. The goal of the questioner is to reveal that the answerer is ignorant of the very science for which he claims expertise.

Aristotle mentions the Racetrack while giving an example of an argument that does *not* constitute a successful peirastic examination. He writes:

> ἢ εἴ τις μὴ φαίη βέλτιον εἶναι ἀπὸ δείπνου περιπατεῖν διὰ τὸν Ζήνονος λόγον, οὐκ ἰατρικός· κοινὸς γάρ.

Or if someone should say that it is not better to walk after dinner, using the *logos*

48 Cf. 165b4-7.

> of Zeno, [this] is not [a] medical [demonstration]. For it [i.e. the *logos* of Zeno] is [of] common [application]. (172a8-9)

This is a very terse passage, and it is embedded in a context that is not altogether easy to understand. But it is not difficult to grasp why Aristotle invokes the Racetrack here, and to draw some significant conclusions.

Imagine a putative doctor who claims that it is good for one's health to walk after dinner. A peirastic dialectician might wish to reveal that this supposed doctor is actually ignorant of medicine, and one way to do this might be to force him to deny this very claim which he has just made. Aristotle's example envisions a case where the questioner succeeds in getting the doctor to deny his original claim, but nonetheless fails to reveal that the doctor is ignorant of medicine.

While we do not know the precise argument that Aristotle envisions the questioner employing, we can easily see its general nature. The questioner will naturally being by getting the supposed doctor to affirm that it is good for one's health to walk after dinner. Then he might ask whether this entails that it is *possible* to walk after dinner. At this point, the questioner may use a version of Zeno's Racetrack *logos* to force the answerer, via a series of questions and answers, to admit that it is *not* possible walk after dinner, or at any other time.[49] The answerer has now rejected an admitted consequence of his original thesis, and so the questioner can easily compel him to reject the original thesis as well.

The argument is no doubt something like this, and so we can easily see why Aristotle regards it as an *ineffective* peirastic examination. The questioner has merely

49 The conclusion of the original Racetrack *logos* was no doubt something like this. As Aristotle is here dealing with a "medicalized" version of the *logos*, the conclusion may differ somewhat from the original conclusion. We will later examine in detail what the original conclusion was.

taken Zeno's Racetrack and presented it in medical garb. All the real work of the argument is done by the Zenonian portion. We might well suppose that in being forced to grant that walking is impossible, the answerer has made *some* error. But it is scarcely a *medical* error, as the same Zenonian argument can be presented in other non-medical guises. The examiner thus fails to establish that the answerer is ignorant of medicine.

This example shows once again that Aristotle is thinking about the Racetrack in the context of dialectic, as he clearly envisions one person employing the *logos* orally against another. This confirms what we saw in the prior two passages.

More interesting is what we learn from Aristotle's description of the *logos* as κοινός (common). (172a9) For Aristotle, dialectical arguments do not merely have a certain form, they also have a certain content. In his view, a dialectical *logos* always has premises that are ἔνδοξα (reputable opinions). (163b3-4) Such opinions are believed "by all or by most or by the wise – either by all of them or most or by the most well-known and reputable." (100b21-3) Peirastic arguments employ a subclass of ἔνδοξα as premises, τὰ κοινά (common things).[50] Describing how peirastic functions as a subclass of dialectic, Aristotle writes:

> For it is possible even for one not knowing the thing [i.e. a given subject] to hold an examination of one not knowing [that same subject], if he [the examinee] grants [premises] not from those things he knows, nor from the peculiar [principles of that subject], but from the consequences, consequences of the sort that, being known, in no way hinder [the one knowing them] from *not* knowing the art [in question], but not being known, then necessarily, he [the one ignorant of the given consequences] is ignorant [of the art]. So it is clear that peirastic is not knowledge (ἐπιστήμη) of any definite [subject]. And so [peirastic] concerns all [subjects], for all arts (τέχναι) have need of some common things (κοινοῖς τισιν). So everyone, even the unlearned, in some way makes use of dialectic and peirastic, for to some extent, everyone attempts to examine those professing

50 Cf. 171b4-7, 172a17-172b4.

> [knowledge]. And these things [the aforementioned "consequences"] are the common things (τὰ κοινά), for they themselves [the unlearned] know these things no less [than the learned], even if they seem to speak very much without [knowledge]. (172a23-34)

Aristotle is thus telling us that "the common things" are matters of common knowledge, things known to everyone, even to those apparently and actually lacking specialized knowledge.[51] Indeed, the skilled peirastic examiner is precisely the one who can use common knowledge to expose the ignorance of a supposed expert, even when the examiner himself is ignorant regarding the science being debated. While dialectical arguments in general employ reputable opinions as premises, and the subclass of peirastic arguments employs common knowledge, Aristotle leaves it for *demonstrative* arguments to employ premises that may well be known only to experts in a specific domain (165b1-3, 8-9) While demonstrative arguments may be promulgated in question and answer form, they are not dialectical, as the answerer has no real choices to make. He merely needs to accept whatever premises are offered, since the questioner is presumed to be an expert.[52] Since Aristotle describes the Racetrack *logos* as based on what is κοινός (common), he clearly sees it as a *logos* in which there will be genuine interaction between questioner and answerer, rather than passive acceptance of premises.

It is easy for a modern reader to miss this fact. Aristotle's most extensive discussion of the Racetrack comes in the *Physics*, so we might at first suppose that he sees the argument as a purported demonstration falling within the scope of the science of physics. But now we see that as a “common” *logos*, the Racetrack cannot involve

---

51 See also *S.E.* 171b4-7. Bolton (1994), pp. 124-131, discusses the nature of "the common things" in more depth.

52 Aristotle writes: "Didactic [*logoi*] are those [arguing] from the principles peculiar to each science and not from the opinions of the answerer (for it is necessary for the one learning to rely on [the teacher, i.e. the questioner])." (*S.E.* 2, 165b1-3)

esoteric principles of any science. In later chapters, we will attempt to reconstruct the Racetrack based upon the evidence in the *Physics*. The “common” nature of the *logos* gives some valuable hints on how to go about this. Indeed we can now already see that all of the premises should be items of common knowledge, or at least appear to be so. The premises must be seemingly obvious to the average person.[53] This means that we should be skeptical of any overly “philosophical” premises. For instance, some philosophers might hold that the Racetrack involves the premise that “A line is infinitely divisible.” But a typical person, in Aristotle's day or our own, has no opinion regarding whether a line is infinitely divisible. Hence the Racetrack could not involve exactly *that* premise, even if it might somehow involve something similar.[54]

In similar fashion, when we reconstruct the Racetrack, we should refrain from giving any excessively subtle interpretations to the words that it contains. Aristotle's *Physics* has many words with technical meanings. Often they are normal Greek words that have acquired a specialized use. And certainly, some of the technical notions discussed in the *Physics*, of movement, infinity, etc. seem related to the Racetrack. But we need to refrain from inserting such specialized notions into the *logos* itself. As a “common” *logos*, it was completely comprehensible to non-specialists. And a non-specialist would certainly take the words of the *logos* to mean whatever they normally meant, not what some philosopher thought they meant. In reconstructing the Racetrack, we should thus refrain from inserting any sophisticated philosophical, scientific or

53 We should initially allow that the “common knowledge” might be merely apparent common knowledge, inasmuch as we know that the Racetrack is apparently somehow not an entirely sound argument, quite apart from the way in which it might be used against the supposed doctor.

54 In principle, of course, "A line is infinitely divisible" could be a premise so long as it was not a foundational premise, and was instead itself derived from other "common" premises.

mathematical concepts.

Thus far we have seen direct evidence that all four motion *logoi* are dialectical, but only the Racetrack is specifically described as based on what is “common”. Still, given that Aristotle seems to indicate a close affinity among the four, it would be a good bet that if one is based on what is “common”, all four are. In fact, Aristotle sees an especially close relationship between the Racetrack and the Achilles (239b14,18-9), so we should certainly take it as a working hypothesis that the Achilles is also a “common” *logos*, with premises that seem to be common knowledge. Hence we should apply the above guidelines in reconstructing both the Racetrack and the Achilles.

Reconstructing the Ancient Debates

We have now found that Zeno was regarded in antiquity as a practicing oral dialectician. We have also seen that the four Zenonian motion paradoxes discussed by Aristotle in the *Physics* were all dialectical *logoi*. And we have seen that the Racetrack in particular, and likely the Achilles as well, involved only premises that appeared to be common knowledge, eschewing any esoteric claims. In the rest of this investigation, we will largely focus our attention on the *Physics*, where Aristotle discusses the Racetrack and Achilles in more detail. But our understanding of the dialectical nature of the *logoi* will form the background against which Aristotle's later analyses can be seen to make sense.

In reading the *Physics*, whenever we find Aristotle referring to one of the motion *logoi*, we should always assume that he is referring to a dialectical *logos*. In some cases, this will scarcely be evident from the immediate context of the *Physics*. But we have seen the evidence elsewhere. Aristotle need not explicitly identify the *logoi* as dialectical since it may be obvious to his audience. Aristotle, and likely many of his original audience, would likely have heard the *logoi* used in person, no doubt multiple times, by various people, in slightly different ways. None of them would have heard the *logoi* used dialectically by Zeno, long dead. There is thus a sense in which the *logoi* would be seen as living items of concern, not relics of the dead Zeno. Each dialectician employing the *logos* would have offered it as his own, in the sense that *he* would ask the questions. Zeno might well be known as the originator, but any use of the *logoi* would be, first and foremost, an interaction between the living questioner and answerer.

Since the *logoi* were really paradigms for actual debates, our goal in reconstructing the *logoi* should then be to reconstruct paradigmatic transcripts of these very debates. We want to understand how the *logoi* functioned interactively, and most especially, how the questioner could succeed in getting the answerer to reject a seemingly obvious truth. The way to do this is to reconstruct, as accurately as possible, the words that were actually spoken in the debates.

Of course, we can scarcely hope for complete success. But we can compare our task to that of a paleontologist, trying to reconstruct an ancient skeleton from fossils found in many places. The resulting skeleton will not perfectly match that of any *particular* creature that ever lived. But it might provide great insight into the lives of a wide swath of creatures nonetheless. In the same way, we are really interested in the elements of a given *logos* that would have remained constant through many renditions. This will give us insight into how the *logos* really worked.

Our goal is thus to reconstruct paradigmatic transcripts. We can contrast this with two alternatives. First, we are not seeking to reconstruct any written text of Zeno. That much is clear. At the same time, we are not seeking simply to reconstruct what contemporary philosophers would call an *argument*. This has been the standard approach of most Zeno scholars. We might say that the goal of an argument is to produce conviction in the minds of the audience. But we can scarcely presume that this is the goal of the motion *logoi*. Instead, we can say that the dialectical questioner seeks to produce the *appearance* that the answerer has rejected his original claim.[55] Certainly, what we might call the "logical" features of a *logos* will be relevant to this goal, just as in an argument *per se*. But they need not be the only relevant features. Other things will likely matter too: word choice, the ordering of the questions, etc. In short, we should not begin with a predetermined scheme regarding *how* a dialectician is able to get his job done. Instead, we should first try to reconstruct *what* he did, and *then* try to understand it. Even if it is a useful fiction, we should thus take the attitude that we are trying to reconstruct a historical conversation. In this way we can best distinguish empirical questions about the actual structure of the *logoi* from philosophical questions about how they worked. In practice, the issues are intertwined, but we should keep them conceptually distinct.

In seeking to understand the Racetrack and Achilles, then, we now have a clear goal: we want to reconstruct plausible transcripts of ancient dialectical debates, and figure out how they worked. This will be our focus in the chapters ahead.

55 For Aristotle's own contrast between sophistical and authentic refutations, see *S.E.* 1, 164a24-b31. The appearance of refutation will often be an appearance to an audience. Aristotle himself does not explicitly analyze the role, or potential role, of the audience as subject to appearance. But he does point out that those taken in by faulty reasoning will often do so both as participants in dialectic and as listeners. See 165a13-17.

Chapter 2
Disambiguating a Word

The Motion *Logoi* in *Physics* Z.9

Physics Z is a highly technical and abstract discussion of continuous motion. In *Physics* Z.9, we find what seems intended as a comprehensive account of four Zenonian motion paradoxes. It fits well within the overall book. Aristotle opens the discussion with a line we have already encountered:

> τέτταρες δ'εἰσὶν οἱ λόγοι περὶ κινέσεως Ζήνωνος οἱ παρέχοντες τὰς δυσκολίας τοῖς λύοισιν... .
>
> And four [in number] are the *logoi* of Zeno about motion, which give *duskolia* to those [trying to] resolve [them]... .[56] (239b9-11)

We will be mainly concerned with two of these *logoi*. But to understand what Aristotle says about these two, we need to understand the broader framework in which he discusses them. In fact, what we discover is that Aristotle regarded the motion *logoi* as an ordered quartet. And in *Physics* Z.9, he sees his foremost task as the discussion of the full quartet, as opposed to the enumeration of individual *logoi*.

This becomes evident when we examine the overall structure of the Zeno discussion. Aristotle follows a rigid order as he proceeds through the *logoi*. His format is to first give an exposition of each *logos* and then to analyze it. But he twice deviates from form, and these deviations provide insight into Aristotle's conception of his overall task. For easy reference, here is Aristotle's discussion, continuing on from the opening

56 Most translations render this differently, beginning with something like: "And there are four *logoi*... ." But this is misleading. The word τέτταρες (four) is not an attributive adjective, but a predicate adjective. Aristotle is not asserting the existence of something, namely, a set of four *logoi*. Instead, he is saying that the *logo*i (of this kind) are *four* in number. Aristotle's usage thus suggests that his audience is already more or less familiar with *logoi* of the given kind. But they may not know the given set of four, as such.

line above, but omitting most of the extended analysis of the fourth *logos*:

> πρῶτος μὲν ὁ περὶ τοῦ μὴ κινεῖσθαι διὰ τὸ πρότερον εἰς τὸ ἥμισυ δεῖν ἀφικέσθαι τὸ φερόμενον ἢ πρὸς τὸ τέλος, περὶ οὗ διείλομεν ἐν τοῖς πρότερον λόγοις. δεύτερος δ’ ὁ καλούμενος Ἀχιλλεύς· ἔστι δ’ οὗτος, ὅτι τὸ βραδύτατον οὐδέποτε καταληφθήσεται θέον ὑπὸ τοῦ ταχίστου· ἔμπροσθεν γὰρ ἀναγκαῖον ἐλθεῖν τὸ διῶκον ὅθεν ὥρμησεν τὸ φεῦγον, ὥστε ἀεί τι προέχειν ἀναγκαῖον τὸ βραδύτερον. ἔστιν δὲ καὶ οὗτος ὁ αὐτὸς λόγος τῷ διχοτομεῖν, διαφέρει δ’ ἐν τῷ διαιρεῖν μὴ δίχα τὸ προσλαμβανόμενον μέγεθος. τὸ μὲν οὖν μὴ καταλαμβάνεσθαι τὸ βραδύτερον συμβέβηκεν ἐκ τοῦ λόγου, γίγνεται δὲ παρὰ ταὐτὸ τῇ διχοτομίᾳ (ἐν ἀμφοτέροις γὰρ συμβαίνει μὴ ἀφικνεῖσθαι πρὸς τὸ πέρας διαιρουμένου πως τοῦ μεγέθους· ἀλλὰ πρόσκειται ἐν τούτῳ ὅτι οὐδε τὸ τάχιστον τετραγῳδημένον ἐν τῷ διώκειν τὸ βραδύτατον), ὥστ’ ἀνάγκη καὶ τὴν λύσιν εἶναι τὴν αὐτήν. τὸ δ΄ ἀξιοῦν ὅτι τὸ προέχον οὐ καταλαμβάνεται, ψεῦδος· ὅτε γὰρ προέχει, οὐ καταλαμβάνεται· ἀλλ’ ὅμως καταλαμβάνεται, εἴπερ δώσει διεξιέναι τὴν πεπερασμένην. οὗτοι μὲν οὖν οἱ δύο λόγοι, τρίτος δ’ ὁ νῦν ῥηθείς, ὅτι ἡ ὀϊστὸς φερομένη ἕστηκεν. συμβαίνει δὲ παρὰ τὸ λαμβάνειν τὸν χρόνον συγκεῖσθαι ἐκ τῶν νῦν· μὴ διδομένου γὰρ τούτου οὐκ ἔσται ὁ συλλογισμός.
>
> τέταρτος δ΄ ὁ περὶ τῶν ἐν τῷ σταδίῳ κινουμένων ἐξ ἐναντίας ἴσων ὄγκων παρ’ ἴσους, τῶν μὲν ἀπὸ τέλους τοῦ σταδίου τῶν δ’ ἀπὸ μέσου, ἴσῳ τάχει, ἐν ᾧ συμβαίνειν οἴεται ἴσον εἶναι χρόνον τῷ διπλασίῳ τὸν ἥμισυν. ἔστι δ' ὁ παραλογισμός...
>
> First is the one [*logos*] about not moving since the moving thing must first reach the half before the end, about which we have drawn a distinction in our earlier discussions (*logoi*). And second is the so-called Achilles. And it is this, that the slowest, in running, will never be caught by the fastest. For necessarily, the pursuer must first reach [the place] from which the pursued started, so that necessarily, the slower will always keep somewhat ahead. But this is the same *logos* as with the bisecting, but it differs in dividing the added magnitude not *half*-way. By all means, not catching the slower follows from the *logos*, but it comes about on account of the same thing as the bisection (for in both [*logoi*] not arriving at the end follows, with the magnitude being somehow divided), but it is added in this [*logos*] that not even the fastest made famous in a tragedy [will reach the goal] in pursuing the slowest, so that necessarily the *lusis* is also the same. And moreover, maintaining that the one keeping ahead is not caught is a fallacy. For *when* he keeps ahead, he is not caught, but nonetheless he [i.e. the slower] is caught, if he [i.e. the original answerer] will grant that [he, i.e. the slower] passes through the limited [line]. So then these are the two *logoi*, and third is the one now presented, that the flying arrow rests. And it follows from holding that the time is composed of the nows. For with this not granted, the *sullogismos* will not exist.
>
> And fourth is the one about equal bodies moving in the *stadion* from opposite

> [beginnings[57]] alongside equals, some from an end of the *stadion* and some from a middle, at equal speed, in which [*logos*] he thinks it follows that an equal time with the double [time] is the half [time]. But the fallacy is ... . (239b11-240a2)

Aristotle begins the discussion by giving a one-line encapsulation of the Racetrack. ("First...the end", 239b11-13) Then comes the first deviation from form. Instead of analyzing the Racetrack, he notes that it has already been analyzed. ("about which...earlier discussions", 239b13-14) This earlier analysis is found in *Physics* Z.2, 233a21-31, a passage we will soon examine in detail.

The second *logos* is the Achilles. Aristotle first presents the *logos* ("And second... somewhat ahead", 239b14-18), and then analyzes it ("But this...limited [line]", 239b18-29), basing his analysis on the earlier analysis of the Racetrack.

Third is the Arrow, and the second deviation from format. Aristotle begins by saying that the third *logos* is the one that he has just discussed, referring to a passage immediately prior to his announcement of the four motion *logoi*, a passage in which he does indeed discuss the Arrow, giving a synopsis and analysis (239b5-9) Coupled with this backwards reference is a half-line account of the Arrow (239b30). This is followed by a summary of the earlier analysis, with a slight addition. ("And it...not exist", 239b31-33).

Fourth comes the Stadium paradox. Aristotle provides an exposition ("And fourth...half [time]", 239b33-240a1), followed by an analysis. ("But the...", 240a1-18)

We thus see that despite the two references to earlier discussions, Aristotle rigidly adheres to a format of first presenting and then analyzing each *logos*. If he does not need to do something since it has already been done, as with the analysis of the Racetrack, and

57 I suppose that the most plausibly Aristotelian subject for ἐναντίας would be ἀρχάς (beginnings), even if this sounds odd in English.

the exposition and analysis of the Arrow, he simply refers to the earlier discussion instead of repeating himself.

When we consult the two earlier passages to which Aristotle refers, 233a21-31 and 239b5-9, we find that Aristotle's analyses of Zeno are closely connected to Aristotle's own immediately prior discussions. The first analysis of the Racetrack occurs after a discussion of the infinite divisibility of magnitudes, an issue that plays a key role in Aristotle's analysis of the *logos*. The first mention of the Arrow occurs after a discussion regarding the possibility of motion in an instant, a key issue in the Arrow *logos*. From these facts we can infer Aristotle's motive for discussing each *logos*. In each case, he supposes that the prior discussion has provided him with the principles necessary for the analysis of a Zenonian *logos*, and so he brings up the *logos* to analyze it.

It is immediately following the earlier discussion of the Arrow (239b5-9) that Aristotle commences the comprehensive discussion of the four *logoi*. This suggests that Aristotle has been prompted by the Arrow discussion to recognize that he has not yet given a thorough treatment of the Zenonian *logoi* in their own right. So he proceeds to do so.

Why does Aristotle mention the Racetrack at all, given that he adds nothing to his earlier discussion? And why does he call the Arrow the third *logos* when he has actually discussed it first? Clearly, the set of four *logoi* must constitute an ordered set whose order is fixed prior to the composition of *Physics* Z.9. While he has earlier mentioned two *logoi* in discussions relevant to their content, now that he is discussing the full set, Aristotle feels compelled to mention each *logos* in its proper place.

The origin of the ordering is unclear. It probably originates in writing. It may or may not originate with Zeno. Regardless, this written ordering does not detract from the fundamentally dialectical, and hence oral, nature of the *logoi*. Aristotle gives every sign that the preparations for dialectic commonly involved writing. Recall, for instance, the lists of *logoi* distributed by early "teachers" of dialectic. (*S.E.* 183b34-184a1)

While the existence of an ordering raises interesting questions about the early history of dialectic, what is more important for us now is the way that it offers insight into Aristotle's exposition. Notice first that Aristotle clearly regards the Racetrack and Achilles as distinct members of the preexisting quartet. This is so despite the fact that he also calls them "the same *logos*." (239b18-19) Second, notice how Aristotle seems to be carefully distinguishing each of the four original *logoi* from his reflective comments about them.

These ideas set the stage for our ensuing investigation. While we ultimately want to understand how the Racetrack and Achilles were "the same *logos*", we need to first try and understand them as distinct *logoi*, just as Aristotle did. Likewise, we need to constantly try to distinguish the places where Aristotle seems to be reporting the *logoi* from the places where he seems to be doing reflective analysis. In the end, we want to understand the distinct dialectical Racetrack and Achilles, along with Aristotle's dialectical analyses of each.

Aristotle discusses the Racetrack and the Achilles in Z.9. But the Racetrack alone receives additional extended discussion, in Z.2 and Θ.8. As Aristotle's comments on the Racetrack are much more elaborate than his comments on the Achilles, our investigation of the Racetrack will be much more detailed than our study of the Achilles, although we will eventually see how Aristotle ingeniously analyzes them in a similar fashion.

Our study of the Racetrack will proceed in what might seem a counterintuitive fashion. It might seem that in examining Aristotle's discussion of the Racetrack, we should first set out the paradox as he sees it, and then proceed to a discussion of his analysis thereof. But remember that we are at the mercy of the evidence: what it most immediately tells us may not always be what we think we want to know. In fact, what we will find, in *Physics* Z.2 and Z.9, is that Aristotle provides us with a good deal of information about how he analyzes the *logos*, but does not, in these passages, ever set out the full *logos* itself. And what we will find is that we can indeed understand a good deal of what Aristotle says without actually having a full grasp of the *logos* he is critiquing. And so understanding the analysis will be our main task in Chapters 2 and 3. Later in Chapter 4, we will find that *Physics* Θ.8 provides a fuller recounting of the Racetrack *logos* itself. But our seemingly backwards order of exposition remains appropriate. Aristotle is our primary source of evidence concerning the motion *logoi*, and virtually any opinion anyone has today concerning the Racetrack or Achilles is somehow causally derivative of the *Physics* itself. If we are at all concerned to uncover the pre-Aristotelian, possibly Zenonian, versions of the *logoi*, as we are, then we need to rigorously put aside our preconceptions regarding *why* we are interested in each piece of evidence. Indeed, we need to examine each piece of evidence in its own right, as a piece of Aristotle's own text. Only by doing this, by first looking at the text of the *Physics* for what it is in itself, can we begin to consider the separate issue of how Aristotle's responses to the motion *logoi* are and are not really responses to *logoi* that exist quite apart from the *Physics*. In the rest of this chapter, we will begin with Aristotle's comments on the Racetrack in Z.9, and then examine the structure of Aristotle's dialectical response to the *logos* in Z.2.

The Exposition of the Racetrack

In *Physics* Z.9, Aristotle writes:

> πρῶτος μὲν ὁ περὶ τοῦ μὴ κινεῖσθαι διὰ τὸ πρότερον εἰς τὸ ἥμισυ δεῖν ἀφικέσθαι τὸ φερόμενον ἢ πρὸς τὸ τέλος, περὶ οὗ διείλομεν ἐν τοῖς πρότερον λόγοις.
>
> First is the one [*logos*] about not moving since the moving thing must first reach the half before the end, about which we have drawn a distinction in our earlier discussions (*logoi*). (239b11-14)

This is the full account of the Racetrack in Z.9. In examining this passage, we should recall the structure of the whole Z.9 discourse. Discussion of each *logos* is divided into two parts, an exposition and an analysis. Here the exposition begins with “First” and the analysis with “about which.” We will first consider the exposition.

It is immediately obvious that Aristotle does not present a full dialectical *logos*. But this need not surprise us. We have already seen several occasions on which Aristotle refers to the Racetrack in such a way that he clearly presumes a good deal of audience familiarity.[58] Moreover, we have seen that in *Physics* Z.9, Aristotle's goal is not so much to discuss the individual motion *logoi* in full, but to examine the full quartet of motion *logoi*. In order to be systematic, he discusses the Racetrack first, apparently its “proper” place. But his discussion can afford to be skimpy if he assumes that everyone already more or less knows the *logos*. Of course, we do not know the full *logos*, but for now we must simply make do with the information that Aristotle gives us.[59]

---

58 Recall *Topics* 160b8-9 and 172a9, and *Sophistical Refutations* 179b20-21. Also see *Posterior Analytics* 65b18-19.

59 Some scholars seem to assume that in Z.9, Aristotle is purporting to present the basic *argument* of the *logos*, however cursorily. Thus Ross writes: “The first argument is put (in vi.9) in the brief form 'the argument that motion does not take place because the moving body must get to the midway point before it gets to the end.” (pg. 72) But we need not assume that in Z.9, Aristotle is at all purporting to give the basic logical structure of the *logos*. Instead, he need only be recalling for his audience which *logos* is the *first* of the four motion *logoi*, assuming that they already basically know the *logos* in its own right.

Given the word διά (since), Aristotle clearly attributes a certain inferential structure to the *logos*. And so the line that supports the inference must be a premise: "the moving thing must first reach the half before the end." (239b12-3)

Additionally, Aristotle provides some information, but not much, about the conclusion of the *logos*. Clearly, if the *logos* is "the one about not moving *since*...", it must be that the *logos* is about not moving insofar as its conclusion is about not moving. But this does not tell us what the conclusion actually is. Again, this need not surprise us if Aristotle assumes that his audience is largely familiar with the *logos*.[60]

From the exposition in Z.9 we thus obtain only a single premise and a rather broad claim about the conclusion.

"Drawing a Distinction" in the Analysis of the Racetrack

Aristotle continues and concludes the discussion as follows: "...περὶ οὗ διείλομεν ἐν τοῖς πρότερον λόγοις." (...about which we have drawn a distinction in the earlier discussions (*logoi*)[61].) (239b13-14) The "earlier discussions" are in *Physics* Z.2

---

60 Most translations make it seem that Aristotle does provide a conclusion for the Racetrack in Z.9. McKirahan (1994), pg. 187, and Vlastos (1966), pg. 189, write "says that there is no motion"; Faris, pg. 7, "says that motion is impossible"; Ross, pg. 417, "says that movement is impossible"; and Hardie/Gaye, pg. 404 , "asserts the non-existence of motion". But these are not literal translations of Aristotle's Greek, which has a negative particle modifying an infinitive. Hence I translate "not moving", using a noun phrase that refers to the state of not moving. Unlike some of the translators, Aristotle does not use a negative quantifier or make a negative existential claim. I suspect that these scholars simply assume that Aristotle must be intending to offer *some* definite conclusion, and so they simply insert what they think the conclusion is. But Aristotle need not be presenting a definite conclusion at all.
Not only does Aristotle not offer a conclusion in Z.9, but the conclusion that is offered by the translators turns out to be the wrong conclusion. Most offer some variant of the claim that "Motion does not exist." In Chapter 4, we will examine in detail what the conclusion actually was, and we will find that it is nothing so abstract. Here translators seem to be following Philoponus (802.31-803.221) and Simplicius (1013.6), both of whom think that the *logos* refutes the claim that motion exists. But these commentators lived in the 6th century A.D., and both seem to be relying on the *Physics* itself for their interpretations, as opposed to using some independent source. (This is widely accepted in principle, if ignored in practice.) Hence their opinions have no independent weight, and we will need to evaluate the *Physics* evidence ourselves. For now we may defer consideration of the conclusion.

61 This is one of many non-dialectical uses of the word *logos*.

(233a21-31), which we will soon examine. But first we must consider what Aristotle means when he says he has "drawn a distinction". Understanding this notion turns out to be crucial to understanding Aristotle's analysis of the Racetrack. Indeed, in *Physics* Z.9, by telling us he has earlier *drawn a distinction*, Aristotle is providing us with a framework by which we can interpret the actual analysis earlier in *Physics* Z.2. So before we examine the analysis itself, we need to understand the framework.

The term διαρέω (draw a distinction) is actually a dialectical term of art, the meaning of which we will examine momentarily. But the dialectical sense of διαρέω has been missed by most scholars. Indeed, most translators seem to suppose that since Aristotle provides an incomplete report of the Racetrack in *Physics* Z.9, he must be referencing his Z.2 discussion simply to complete the story. Thus Hardie/Gaye; Barnes[62]; and Waterfield write "we have discussed"; Ross writes "we have dealt with"[63]; and Sachs writes "*we* have gone all the way through". But these translations are not justified by any evidence. While διαρέω does have many non-dialectical uses, the LSJ lexicon reports no meaning akin to “discuss.” Instead, even in non-dialectical contexts, διαρέω always has an active connotation such as "interpret," "analyze," or “divide” (either in a physical sense, or verbally, as when offering a definition by genus and difference, a sense closely related to the crucial dialectical sense of the word). Thus Aristotle must be referring to his earlier discussion not simply for more information on the *logos* itself, but rather for his analysis of the *logos*.

Aristotle is not only telling us *that* he has earlier analyzed the *logos*, he is also telling us *how* he has done so. This becomes clear when we understand the dialectical

62 Barnes (1982), pg. 261.
63 Ross, pg. 416.

sense of διαρέω (draw a distinction). To *draw a distinction* is one of two basic ways in which a dialectical answerer may provide a counterattack against a flawed *logos*.[64] In seeking to understand this term, we will first consider its basic dialectical meaning, and then consider its broader significance.[65]

We may begin with *Topics* Θ, where Aristotle offers general dialectical strategy. In Θ.1-3, he begins with a set of advice for questioners. Then in Θ.4-10, he follows with advice to answerers. Θ.6 and Θ.7 consider how the answerer should deal with individual questions. Θ.6 assumes that a question has a single distinct meaning[66], and advises various responses depending on the plausibility and relevance of the thesis proposed by the question. Θ.7 then considers how to deal with questions that do *not* have a single distinct meaning.[67] I quote the passage in full:

> Ὁμοίως δὲ καὶ ἐπὶ τῶν ἀσαφῶς καὶ πλεοναχῶς λεγομένων ἀπαντητέον. ἐπεὶ γὰρ δέδοται τῷ ἀποκρινομένῳ μὴ μανθάνοντι εἰπεῖν ὅτι "οὐ μανθάνω", καὶ πλεοναχῶς λεγομένου μὴ ἐξ ἀνάγκης ὁμολογῆσαι ἢ ἀρνήσασθαι, δῆλον ὡσ πρῶτον μέν, ἂν μὴ σαφὲς ᾖ τὸ ῥηθέν, οὐκ ἀποκνητέον τὸ φάναι μὴ συνιέναι· πολλάκις γὰρ ἐκ τοῦ μὴ σαφῶς ἐρωτηθέντας διδόναι ἀπαντᾷ τι δυσχερές. ἂν δὲ

64 At *Topics* Θ.10, 161a1-15, Aristotle sets out four defensive strategies. But three of them are basically delaying tactics directed against the questioner and not against his *logos*. What we will find is that drawing a distinction is one of two ways to execute the first of these defensive strategies. That is to say, it is one of two ways to provide a *lusis*. (161a13-14).

65 We will soon see that the term διαρέω conveys precise information about what Aristotle thinks he has done earlier in *Physics* Z.2. But when he uses the term at 239b13-4, Aristotle may also be making a joke. The root meaning of διαρέω is "divide" or "make a division". So in analyzing this *logos* that involves an infinite division, Aristotle has, appropriately enough, "made a division". In fact, just as the *logos* involves division of an earlier (πρότερον) half, so Aristotle has "made a division" "in the earlier (πρότερον) discussions." Any Greek will likely hear the pun immediately, since "making divisions" has been a significant topic in *Physics* Z, whereas there has been no discussion of "drawing distinctions" in the dialectical sense. Given the juxtaposition of two rather distinct and precise meanings, the humorous sense is likely intended by Aristotle. The succinctness of the joke may in part motivate the brevity of the discussion in Z.9.

66 Recall that a dialectical questioner asks questions in the hope that the answerer will affirm the propositions that are conveyed by the questions, propositions that can serve as premises in an argument. In principle, we might distinguish between these premises and the questions by which they are presented. But we should keep in mind that it is the questions themselves that an answerer faces, and this is reflected in Aristotle's discussion.

67 Again, we might distinguish between the questions and the propositions they present. But only the questions are actually uttered.

> γνώριμον μὲν ᾖ πλεοναχῶς δὲ λεγόμενον, ἐὰν μὲν ἐπὶ πάντων ἀληθὲς ἢ ψεῦδος ᾖ τὸ λεγόμενον, δοτέον ἁπλῶς ἢ ἀρνητέον, ἐὰν δ' ἐπὶ τὶ μὲν ψεῦδος ᾖ ἐπὶ τὶ δ' ἀληθές, ἐπισημαντέον ὅτι πλεοναχῶς λέγεται καὶ ὅτι τὸ μὲν ψεῦδος τὸ δ' ἀληθές· ὕστερον γὰρ διαιρουμένου ἄδηλον εἰ καὶ ἐν ἀρχῇ συνεώρα τὸ ἀμφίβολον. ἐὰν δὲ μὴ προΐδῃ τὸ ἀμφίβολον ἀλλ' εἰς θάτερον βλέψας θῇ, ῥητέον πρὸς τὸν ἐπὶ θάτερον ἄγοντα ὅτι "οὐκ εἰς τοῦτο βλέπων ἔδωκα ἀλλ' εἰς θάτερον αὐτῶν"· πλειόνων γὰρ ὄντων τῶν ὑπὸ ταὐτὸν ὄνομα ἢ λόγον ῥᾳδία ἡ ἀμφισβήτησις. ἐὰν δὲ καὶ σαφὲς ᾖ καὶ ἁπλοῦν τὸ ἐρωτώμενον, ἢ "ναί" ἢ "οὔ" ἀποκριτέον.
>
> In the same way[68] one must [as answerer] meet things said unclearly or with multiple senses. For since it is allowed for the answerer, if he does not understand, to say "I don't understand", and with things said with multiple senses [he is] not under compulsion to [either] agree or disagree, it is clear in the first place that if the thing said is not clear, [the answerer] must not hesitate to say that he does not understand. For often one may encounter some difficulty from saying "yes" to unclear questions. But if the question asked is intelligible and has multiple senses, and the thing said is either a truth or a falsehood with respect to all [the senses], one should simply grant it or deny it, but if with respect to some [sense] it is a falsehood and concerning some [sense] it is a truth, one must indicate that it is said [i.e. can be meant] in multiple ways and that one is a falsehood and one is a truth. For if he *draws the distinction* (διαιρουμένου) later, it will be unclear if he originally saw the ambiguity.[69] But if he does not foresee the ambiguity but agrees [to a proposition] relying on one [sense], then against [a questioner] who takes it in the other sense it is necessary to say that "I granted it considering not that [sense], but the other [sense]". For given that many things fall under the same name or phrase (*logos*), a dispute is likely. But if the question asked is both clear and simple, one should answer either "yes" or "no". (160a17-34, my emphasis)

Recall that in Θ.6, Aristotle assumed that a question, or a proposition introduced by a question, had a single distinct meaning.[70] Here in Θ.7 Aristotle supposes that it does not, and considers two other possibilities. Either a question has no distinct meaning, or it has two or more distinct meanings. In the first case, the question is simply unclear

68 Aristotle has just advised that the answerer should *not* affirm premises that are more implausible than the intended conclusion of the *logos*. Likewise, in the situations Aristotle is about to discuss, he also counsels against simple affirmation. As for what the answerer *should* do, the cases are different.

69 Here Aristotle uses the neuter adjectival form τὸ ἀμφίβολον (the ambiguous thing) to encompass all varieties of ambiguity. Later in *S.E.* 4 he distinguishes six varieties. But there he uses the feminine noun ἡ ἀμφιβολία to refer to one of them, grammatical ambiguity. (165b23-27, 166a6-14)

70 Aristotle does not, in Θ.6, explicitly *say* that he is considering questions with a single meaning. But this is implicitly evident, since he is considering questions that obviously have a determinate content. See 159b37-39.

(ἀσαφής). (160a17) Aristotle thus counsels that the answerer should ask for clarification.[71]

In the second case, the question is fully intelligible (γνώριμον), but has multiple senses (πλεοναχῶς λεγόμενον). (160a23-24) If all interpretations are true, or all false, Aristotle pragmatically advises the answerer to answer and not quibble. What concerns us is the case where the interpretations differ with regard to truth. Here the answerer must do two things. First he must point out that the proposition introduced in the question has multiple meanings. Then he must indicate that on one interpretation it is true and on one false. It is evident that this two-part activity constitutes *drawing a distinction*, as is clear from Aristotle's comment that drawing a distinction is better off done immediately, rather than later. (160a28-29)

We now see the basic nature of drawing a distinction, the notion that Aristotle employs in his analysis of the Racetrack in *Physics* Z.9. But we have not yet exhausted the significance of the idea. In fact, drawing a distinction is one of two basic ways in which, according to Aristotle, a dialectician can provide a λύσις (*lusis*), or *resolution*, for a *logos*.

In general, a *lusis* is what a dialectical answerer needs to provide in order to defeat the *logos* of the questioner. For lack of a better term, we might render *lusis* as *resolution*.[72] This conveys the idea that a *lusis* is an intellectually adequate response to a

---

71 Aristotle does not discuss the issue, but the result of demanding clarification will be a question that *does* have a distinct meaning (assuming the questioner plays along, which will not always be the case). If the questioner provides a question with a single distinct meaning, the answerer is then in the position discussed in Θ.6. Whereas if the "clarified" question has two or more distinct meanings, the answerer then faces the second situation discussed in Θ.7. At first it might seem unlikely that a question would be "clarified" into an ambiguity, but a questioner is not necessarily being forthright about his intentions. (In fact, *Topics* Θ repeatedly advises the questioner *not* to be forthright. See 155b23, 155b29-30, 156a11-2, etc.)

72 Pickard-Cambridge and Smith write *solution*. I will typically render *lusis* simply as *lusis*, refraining

problem. But as a translation, *resolution* is both too intellectual and too imprecise. In particular, it misses the fundamentally social and interactive character of a *lusis*.

To get a better idea of what a *lusis* is, consider that dialectical debates were quite commonly compared with athletic events such as wrestling or boxing, in which two contenders grapple for victory. They fight not with their bodies, but with words.[73] This athletic imagery provides a needed corrective to a highly intellectualized reading of *lusis*, for a *lusis* is an *action*.[74] Just as a wrestling or boxing match is a series of actions, so is a dialectical *logos*. And just as it does a fighter no good to *know* the proper move to make unless he actually employs it when needed, the same holds for the dialectician. Knowing a *lusis* is a matter of knowing what to say and when to say it, but what matters is the saying.[75] What we will find is that knowing a *lusis* involves knowing *why* a given *logos* is able to produce a false appearance, and knowing *how* to expose the falsehood for what it is.

Aristotle lays out the basic nature of a *lusis* in *Topics* Θ.10, shortly after the passage we considered above. He writes:

> Ὅσοι δὲ τῶν λόγων ψεῦδος συλλογίζονται, λυτέον ἀναιροῦντα παρ' ὃ γίνεται τὸ ψεῦδος· οὐ γὰρ ὁ ὁτιοῦν ἀνελὼν λέλυκεν, οὐδ' εἰ ψεῦδός ἐστι τὸ ἀναιρούμενον. ἔχοι γὰρ ἂν πλείω ψεύδη ὁ λόγος, οἷον ἐάν τις λάβῃ τὸν καθήμενον γράφειν,

from translation. But the related verb λύω, which often means, more or less, "employ a *lusis*", I will render as "resolve".

73 While such athletic imagery is not often prominent in the *Topics* and *Sophistical Refutations* (but see *Topics* VIII.5, 159a25-37), it would have been familiar to Aristotle's audience. Plato's dialogues abound in examples. For a particularly relevant instance, see *Euthydemus* 271c-272b43, where two eristic dialecticians are said to be *pancratiasts*. The *pankration* was a no-holds-barred wrestling event, and *pankratiast* literally means "all-around fighter." Unlike the regular pankratiasts, who use only their bodies, these dialecticians, described as being already skilled in bodily fighting, are now said to have added the finishing touch, learning to fight with *logoi*.

74 To be precise, a *lusis* is is a certain kind of objection to a *logos*. See 161a1-15. The word does come to have much more intellectualized senses, akin to its Latinate cognates *resolution* and *solution*. But Aristotle leaves little doubt that in dialectic, a *lusis* is an action.

75 It is especially easy for a contemporary reader of the *Physics* to miss this, as the dialectical origin of Zeno's motion *logoi* is not so obvious, and even less the competitive nature of dialectic.

> Σωκράτη δὲ καθῆσθαι· συμβαίνει γὰρ ἐκ τούτων Σωκράτη γράφειν. ἀναιρεθέντος οὖν τοῦ Σωκράτη καθῆσθαι οὐδὲν μᾶλλον λέλυται ὁ λόγος· καίτοι ψεῦδος τὸ ἀξίωμα. ἀλλ᾽ οὐ παρὰ τοῦτο ὁ λόγος ψευδής· ἂν γάρ τις τύχῃ καθήμενος μὲν μὴ γράφων δέ, οὐκέτι ἐπὶ τοῦ τοιούτου ἡ αὐτὴ λύσις ἁρμόσει. ὥστε οὐ τοῦτο ἀναιρετέον, ἀλλὰ τὸ τὸν καθήμενον γράφειν· οὐ γὰρ πᾶς ὁ καθήμενος γράφει. λέλυκε μὲν οὖν πάντως ὁ ἀνελὼν παρ᾽ ὃ γίνεται τὸ ψεῦδος, οἶδε δὲ τὴν λύσιν ὁ εἰδὼς ὅτι παρὰ τοῦτο ὁ λόγος, καθάπερ ἐπὶ τῶν ψευδογραφουμένων. οὐ γὰρ ἀπόχρη τὸ ἐνστῆναι, οὐδ' ἂν ψεῦδος ᾖ τὸ ἀναιρούμενον, ἀλλὰ καὶ διότι ψεῦδος ἀποδεικτέον...
>
> With the *logoi* that syllogize a falsehood [i.e. a false conclusion], one must resolve them by destroying the [premise[76]] on account of which the falsehood arises, for someone destroying any [premise] whatsoever has not resolved [the *logos*], not even if a falsehood [i.e. a false premise] is the thing destroyed. For a *logos* might have many falsehoods [i.e. many false premises], just as if someone holds that he who sits, writes, and that Socrates sits, for from these it follows that Socrates writes. Therefore, when the [premise] that Socrates sits is destroyed, in no way at all has the *logos* been resolved. And still, the proposition [that Socrates sits] is a falsehood. But not on account of this is the *logos* false. For if someone happens to be sitting and not writing, no longer will the same *lusis* apply in such a case. So it is not this [the premise that Socrates sits] that must be destroyed, but the [premise] that he who sits, writes. For not everyone who is sitting writes. Therefore, someone destroying the [premise] on account of which the falsehood [i.e. false conclusion] arises has completely resolved [the *logos*], and someone knowing that the [false] *logos* [exists] on account of *this* [false premise], knows the *lusis*, just as with falsely drawn diagrams. For it is not sufficient to [merely] object, even if the [premise] demolished is a falsehood, but one must also show why [there exists] a falsehood [i.e. false conclusion]... (160b23-37)

Here Aristotle considers the case of a *logos* that reaches a false conclusion via a *sullogismos*.[77] The text makes clear that when an answerer faces such a *logos*, he must provide a *lusis*. If he fails to do so, he will lose the debate. To provide a *lusis*, he must pinpoint the particular question (i.e. premise) at the root of the falsity, and “destroy” that question. We will shortly consider both *sullogismoi* and "destruction" in slightly more detail.[78]

---

76 I write “premise” for simplicity, but recall that premises are in fact posed as questions.

77 Aristotle does not actually use the word συλλογισμός in the passage, but he does use does use the associated verb συλλογίζονται (160b23), which ensures that the *logoi* under consideration involve *sullogismoi*.

78 The given passage in itself has interest beyond my immediate observations, but the issues involved need not concern us here.

In *Sophistical Refutations* 18, Aristotle offers further comments on the nature of a *lusis*:

> Ἐπεὶ δ'ἐστὶν ἡ μὲν ὀρθὴ λύσις ἐμφάνισις ψευδοῦς συλλογισμοῦ, παρ' ὁτοίαν ἐρώτησιν συμβαίνει τὸ ψεῦδος, ὁ δὲ ψευδὴς συλλογοσμὸς λέγεται διχῶς (ἢ γὰρ εἰ συλλελόγισται ψεῦδος, ἢ εἰ μὴ ὢν συλλογισμὸς δοκεῖ εἶναι συλλογισμός), εἴη ἂν ἥ τε εἰρημένη νῦν λύσις καὶ ἡ τοῦ φαινομένου συλλογισμοῦ παρ' ὅ τι φαίνεται τῶν ἐρωτημάτων διόρθωσις, ὥστε συμβαίνει τῶν λόγων τοὺς μὲν συλλελογισμένους ἀνελόντα, τοὺς δὲ φαινομένους δειλόντα λύειν.
>
> Since a proper *lusis* is an exposure of a false *sullogismos*, [displaying] the sort of questioning on account of which the falsehood comes about, while, on the other hand, a false *sullogismos* is so-called in two ways (either if a falsehood [i.e. a false conclusion] has been syllogized, or if, while there is no *sullogismos*, there seems to be a *sullogismos*) there may be the [sort of] resolution now spoken of and [also] that of the apparent *sullogismos*, a setting straight of the questions on account of which some [conclusion] is made to appear, so that it follows that, of *logoi*, [it is necessary] to resolve those having syllogized [by] destroying [questions], [and] those appearing [to syllogize] [by] *drawing distinctions*. (176b29-36, my emphasis)

The term *sullogismos* is commonly translated *deduction*, although it always has the sense of *valid* deduction.[79] Aristotle thinks that every dialectical *logos* involves an attempt by the questioner to employ a core *sullogismos*, although most *logoi* will also involve some combination of auxiliary *sullogismoi*, inductions, and questions that are logically irrelevant but psychologically significant.[80] Recall that a *lusis* is a counterattack on a flawed *logos*. Now we can see that it is a counterattack on a false *sullogismos*.

The idea of a false *sullogismos*, as Aristotle understands it here, has no precise modern counterpart. Here in *Sophistical Refutations* 18, we see that Aristotle distinguishes two varieties of false *sullogismos*, and two corresponding varieties of *lusis*.[81] By contrast, earlier in *Topics* Θ.10, Aristotle seemed to consider only one sort of

---

79 For our immediate purposes, the idea that a *sullogismos* is a valid deduction will suffice. But it is not strictly accurate. For further discussion see Bolton (1994), especially §4-7.

80 See *Topics* Θ.1.

81 In *Sophistical Refutations* 18, Aristotle clearly distinguishes two ways in which we might speak of a

lusis, and hence, one sort of *sullogismos*. There was no mention there of *drawing a distinction*. To reconcile these two passages, we may presume that in the initial *Topics* Θ.10 discussion, Aristotle is considering the basic notion of a *lusis*, which requires "destroying" questions. This sort of *lusis* is directed against the first sort of false *sullogismos*, and in *Sophistical Refutations* 18, Aristotle calls it a "proper *lusis*". (176b29-30) By contrast, the second sort of lusis, which involves *drawing distinctions*, is introduced in *Sophistical Refutations* 18. This interpretation is confirmed by the fact that Aristotle contrasts the second sort of *lusis* with the sort "now spoken of" (176b33), a reference that apparently refers back to the description of a proper *lusis* (176b29-30), a description that meshes with the description of a *lusis* in *Topics* Θ.10.

In *Topics* Θ.10 and *Sophistical Refutations* 18, Aristotle seems to depict the first variety of false *sullogismos* as involving a false conclusion. But since a *sullogismos* is a valid argument, if it has a false conclusion, it must have a false premise as well. In fact, just beyond the quoted passage, Aristotle makes clear that the first variety encompasses *sullogismoi* that have a false premise and a *true* conclusion. (176b36-177a6), which is certainly a possibility with a valid argument. Hence we may apparently define the first variety of false *sullogismos* as involving a false *premise*.[82] Since the conclusion in such a *sullogismos* really does follow from the premises, Aristotle allows that the *sullogismos* is

---

false *sullogismos*. By contrast, in *Topics* Θ.12, he speaks of four ways in which a *logos* might be called false. (162b3-15) The fourth of these ways corresponds to the first variety of false *sullogismos* in *S.E.* 18. The first of these ways involves an "eristic *sullogismos*", which seems akin to the second variety of "false" *sullogismos*. By contrast, the "falsehood" of the second and third varieties of false *logos* hinges not on the *sullogismos* itself, but on its use within the context of a dialectical refutation.

82 See also *Topics* Θ.12, 162b3, 11-15, which refers back to 162a8-11. False *sullogismoi* of the first variety will be what are now called valid but unsound arguments. But Aristotle's *concept* of such *sullogismoi* does not seem to mesh with the modern concept of a valid but unsound argument. See also footnote 87, below.

an actual, albeit false one.[83]

By contrast, the second sort of false *sullogismos* might be termed a fake, or apparent, *sullogismos*. A fake *sullogismos*, as Aristotle describes it in *Sophistical Refutations* 18, involves an ambiguous question, and so an ambiguous premise.[84] It is a fake *sullogismos* since the questioner has not constructed any actual deduction or *sullogismos* from premises knowingly affirmed by the answerer. Instead, the *sullogismos* the questioner has in mind will employ one of the alternative senses of the ambiguous question. If the answerer fails to recognize the ambiguity and merely answers *Yes* to the ambiguous question, he will *appear* to have affirmed the needed thesis, and hence the questioner will appear to have constructed an actual *sullogismos* from premises the answerer has affirmed.

We saw in *Topics* Θ.6 and Θ.7 that Aristotle laid out two different ways in which an answerer could avoid affirming a thesis. In Θ.6 he considered questions with clear meanings, but posing dubious theses, which the answerer was advised to deny. And in Θ.7 he considered ambiguous questions, against which the answerer needed to draw distinctions. Now we see in *S.E.* 18 how these two ways of finding fault with questions become the basis for two types of *lusis*.

If a question has a clear sense, but proposes a false thesis, then it will be a

---

83 There does seem to be some inconsistency in Aristotle's treatment of the first variety of false *sullogismos*, and of the the corresponding sort of *lusis*, inasmuch as he initially seems to regard a false conclusion as the defining feature of the false *sullogismos*, but later seems to regard the false premise as such. I have not figured out how to resolve this, but the issue does not seem to have any bearing on our broader investigation.

84 Interestingly, the current classification of false *sullogismoi* presupposes that a false *sullogismos* involves either a false premise or an ambiguous premise (actually an ambiguous question). We might contrast this with the modern classification of bad arguments as involving either false premises or invalid inferences. The difference between these modes of classification merits further study.

component of a false, but actual, *sullogismos*.[85] Against such a *sullogismos*, the answerer needs to first discern which question, or premise, involves a false thesis. He must then "destroy" the false thesis.[86]

Aristotle does not explicitly say what he means by ἀναιρέω, "destroy". But throughout the *Topics* and *Sophistical Refutations*, he uses the term with reference to questions and premises. It is clear that the destruction of a premise requires the rejection of that premise. But rejection does not seem to be enough; it seems the answerer must also provide some dialectical justification. Consider this passage from *Topics* Θ.12:

> Τὸ μὲν οὖν ψευδῆ τὸν λόγον εἶναι τοῦ λέγοντος ἁμάρτημα μᾶλλον ἢ τοῦ λόγου, καὶ οὐδὲ τοῦ λέγοντος ἀεί, ἀλλ' ὅταν λανθάνῃ αὐτόν· ἐπεὶ καθ' αὑτόν γε πολλῶν ἀληθῶν ἀποδεχόμεθα μᾶλλον, ἂν ἐξ ὅτι μάλιστα δοκούντων ἀναιρῇ τι τῶν ἀληθῶν. τοιοῦτος γὰρ ὢν ἑτέρων ἀληθῶν ἀπόδειξίς ἐστιν· δεῖ γὰρ τῶν κειμένων τι μὴ εἶναι παντελῶς, ὥστ' ἔσται τούτου ἀπόδειξις.
>
> Certainly the existence of a false *logos* is a fault of the speaker more than of the *logos*, but nor is it always [a fault] of the speaker, but only when [the false *logos*] escapes his notice, since we accept [a false *logos*] in itself more than many true [*logoi*], whenever from whatever [premises] are most accepted, it [i.e. the false *logos*] destroys (ἀναιρῇ) some one of the truths. For such [a *logos*] is a demonstration of other truths being so. For [in this case] it is necessary that some one of the [premises] laid down altogether *not* be so, so that there will be a demonstration of this [i.e. a demonstration that the falsehood is false]. (162b16-22)

The passage has two interesting elements. First, notice that Aristotle clearly speaks of a *logos* destroying a conclusion. In this case, a *logos* with plausible premises, at least one of them nonetheless false, destroys a true conclusion. (162b18-20) Now typically, the subject of ἀναιρέω is a dialectician himself, rather than a *logos*, but what this passage suggests is that a dialectician will be using some sort of *argument* to effect "destruction".

85 Here I assume, for simplicity, that the falsehood of the premise is the only flaw in the *logos*.

86 But recall from *Topics* Θ.10 that not any destruction of a false thesis will suffice; it must be of the right false thesis.

This interpretation is bolstered when we read further in the passage. Consider that the *logoi* under consideration, which refute truths, and hence have false conclusions, are of the sort that we have already seen require destruction of a premise.[87] But with the *logoi* he is now considering, Aristotle tells us that the false premises actually require a demonstration (ἀπόδειξις) that the falsehood is indeed false. This suggests that it is precisely the demonstration, an argument, that is effecting the destruction. All in all, the evidence suggests that destruction of a premise requires some sort of justification of the claim that the premise is indeed false. And hence, if the dialectical answerer provides such justification, he will have achieved a *lusis*, a *lusis* of the first sort, the "proper" sort.

While the first sort of *lusis* is directed against a *logos* with a question proposing a false premise, the second sort is directed against a *logos* with an ambiguous question. Such an ambiguous question will be a component of a fake *sullogismos*. There will be no *valid* argument unless the questioner interprets the ambiguous thesis in one specific way, not necessarily the way the answerer had in mind when affirming it. Against such a fake *sullogismos*, the answerer needs to discern the ambiguous premise and *draw a distinction*. He will do this in the manner described in *Topics* Θ.7.

Although Aristotle does not discuss the issue directly, the two varieties of *lusis* are not incompatible, and indeed, it seems that Aristotle reasonably describes the first variety as the "proper" *lusis*, inasmuch as the answerer, when applying the second sort of *lusis*, can typically take things a step further and apply the first sort as well. Recall that the second sort is appropriate when a "fake" sullogismos employs an ambiguous question, a question that really involves two alternate premise. In practice, one will typically be true

87 I here presume that the *logoi* Aristotle is considering involve a *sullogismos*.

and one false.[88] To employ the second sort of *lusis*, that is, to expose the "fake" *sullogismos*, the answerer simply needs to expose the ambiguity by drawing a distinction. But the goal of the questioner will often be to employ the false interpretation in an actual, but false, *sullogismos*, the sort against which the first and proper sort of *lusis* is needed. Hence if the falsehood that results from the disambiguation is not *obviously* false, it might be desirable or necessary for the answerer to "destroy" it. That is to say, *drawing a distinction* against a "fake" *sullogismos* may reveal an actual but false *sullogismos* against which destruction is necessary. Or again, application of the second sort of *lusis* may set the stage for application of the first and proper sort. Indeed, this is precisely how we will find Aristotle dealing with the Racetrack.

We may now return to *Physics* Z.9. Recall that Aristotle, discussing the Racetrack, wrote: "...about which we have drawn a distinction in the earlier discussions." (239b13-14) We now see that this passage provides us with valuable information. Since Aristotle claims to have drawn a distinction against the *logos*, he must think that it involves a false *sullogismos* of the second sort, a merely apparent *sullogismos*. This is the sort that involves an ambiguous question. Moreover, in saying that he has *drawn a distinction*, Aristotle is also claiming to have provided a lusis of the second sort.

As things stand, that is all we can directly conclude from the claim about *drawing a distinction*. But given the potential connection between the two varieties of *lusis*, we might wonder if Aristotle employs *drawing a distinction* as the first step in the application of a proper *lusis* for the Racetrack. If so, then he must first disambiguate the

88 Of course, there is no necessity that this be so. But the divergent truth values will typically underlie the usefulness of the ambiguity, as the questioner employs the plausibility of the true interpretation to gain putative assent to the false interpretation. See also the discussion of λαμβάνω below.

ambiguous question, and then claim that one of the alternative theses is true and one false. To complete the *lusis*, he will then destroy the falsehood, doing so by justifying the claim that the false thesis is indeed false. In fact, when we examine *Physics* Z.2, we will find that this is exactly what he does.

Before we turn to Z.2, we may sum up the results of examining Z.9. We discovered that the *logos* involves the premise that "the moving thing must first reach the half before the end." (239b12-13) Moreover, the conclusion is "about not moving". And finally, Aristotle thinks that, somewhere, the *logos* contains an ambiguous premise. We have made some progress, but much remains obscure. So we now proceed to *Physics* Z.2.

The Structure of Aristotle's Analysis in *Physics* Z.2

In *Physics* Z. 2, Aristotle is discussing the parallel structure of time and magnitude. In the midst of this discussion comes the passage in which he later claims to have drawn a distinction involving the Racetrack. He writes:

> διὸ καὶ ὁ Ζήνωνος λόγος ψεῦδος λαμβάνει τὸ μὴ ἐνδέχεσθαι τὰ ἄπειρα διελθεῖν ἢ ἅψασθαι τῶν ἀπείρων καθ' ἕκαστον ἐν πεπερασμένῳ χρόνῳ. διχῶς γὰρ λέγεται καὶ τὸ μῆκος καὶ ὁ χρόνος ἄπειρον, καὶ ὅλως πᾶν τὸ συνεχές, ἤτοι κατὰ διαίρεσιν ἢ τοῖς ἐσχάτοις. τῶν μὲν οὖν κατὰ τὸ ποσὸν ἀπείρων οὐκ ἐνδέχεται ἅψασθαι ἐν πεπερασμένῳ χρόνῳ, τῶν δὲ κατὰ διαίρεσιν ἐνδέχεται· καὶ γὰρ αὐτὸς ὁ χρόνος οὕτως ἄπειρος. ὥστε ἐν τῷ ἀπείρῳ καὶ οὐκ ἐν τῷ πεπερασμένῳ συμβαίνει διιέναι τὸ ἄπειρον, καὶ ἅπτεσθαι τῶν ἀπείρων τοῖς ἀπείροις, οὐ τοῖς πεπερασμένοις.
>
> On account of this the *logos* of Zeno takes as granted (λαμβάνει) a falsehood, that it is not possible to go through things unlimited (ἄπειρα)[89] or to individually touch things unlimited in a limited time. For in two ways both length and time are said to be unlimited, and in general everything continuous, either with respect to division or with respect to its ends. Certainly it is not possible to touch things unlimited (ἄπειρα) with respect to quantity in a limited time, but it is possible [to touch] those [unlimited] with respect to division. For the time itself is unlimited in this way. So it follows that in the unlimited [time] and not in the limited [time] [one] goes through the unlimited [magnitude], and [one] touches things unlimited

89 In Chapter 3, we will examine in detail the meaning of ἄπειρος and its plural forms.

with things unlimited, not things limited. (233a21-31)

When we read this text, we are immediately confronted with a puzzle. We saw that in *Physics* Z.9, Aristotle claims to have earlier drawn a distinction against the Racetrack. So we should expect that here in Z.2 we will find Aristotle exposing an ambiguous premise as the problem with the *logos*. But instead, we find Aristotle writing that "the *logos* of Zeno takes as granted a falsehood...". This suggests that the problem is not an ambiguous premise, but a false premise. Recall from *Sophistical Refutations* 18 that exposing a false premise and disambiguating an ambiguous premise were *contrasted* as two alternative methods for providing a *lusis*. So now it might seem that Aristotle is employing a strategy different from the one that he mentions in Z.9.

There is, however, no conflict here. The "falsehood" mentioned in Z.2 is actually one of two theses that result from disambiguating an initially ambiguous claim. Recall that in a proper *lusis*, the disambiguation of a question will be followed by the exposure of one of the disambiguated theses as a falsehood. Aristotle performs both of these steps in Z.2, so his methods are just what we should expect. To see this, we must first examine the meaning of λαμβάνω.

I have translated λαμβάνω with "take as granted", but the word has many uses. All cluster around the ideas of "take" or "receive". The term has many specialized senses in the works of Aristotle, both in discussions about dialectic and elsewhere. It's dialectical use alone has many nuances,[90] but for now we need only consider one of its

90 One good source of information on the dialectical use λαμβάνω is *Topics* Θ.1-3, where Aristotle provides advice to questioners. In this context, Pickard-Cambridge reasonably translates with *secure*. The goal of the questioner is to *secure* theses, first and foremost the desired refutatory conclusion, but also every thesis needed to secure that refutation. These include, of course, all logically necessary theses, but also various other theses that may play a psychological role in moving the argument along. (155b17-25) A questioner can only secure a thesis via the actions of the answerer. In the simplest case, the questioner secures a proposition when the answerer answers a yes-no question affirmatively. But in

senses.[91]

In the case that concerns us, the object of λαμβάνω is an assertion with a meaning that is somehow in dispute, for instance, an ambiguous assertion. Given an assertion with a questionable meaning, one person may "take it" one way, and another may "take it" another.

Sophistical Refutations 17 contains a passage that uses λαμβάνω in this way. This discussion deals with responses to merely apparent *sullogismoi*, i.e. those with ambiguous premises. Aristotle writes:

> ἔτι ὅταν τὸ καθόλου μὴ ὀνόματι ληφθῇ ἀλλὰ παραβολῇ, λεκτέον ὅτι οὐχ ὡς ἐδόθη οὐδ' ὡς προὔτεινε λαμβάνει...
>
> Moreover whenever the universal was taken [i.e. secured as a premise] not by name but by comparison [of instances], it is necessary to say that [the questioner] takes [it] not as it was granted, nor [does he take it] in line with [the instances] he proffered. (176a33-5)

This passage is rather terse, so some elaboration is needed. Aristotle considers a case

many cases, a proposition can and should be secured without asking a direct question. (155b29-34, 156a27-34, 156b27-30, 174a33-7) Still, it is clear that the questioner *cannot* just arbitrarily assume a thesis and take it as secured.

Although Aristotle does not say this directly, it seems that the questioner secures a thesis if two requirements are met. First, he must explicitly intend to secure it. (Aristotle envisions the questioner planning in advance all the key theses to be secured. (155b3-7)) Second, he *has* secured it if he is able to construct some (potentially defeasible) argument that the answerer has actually consented to the thesis. But this does not require that the answerer has *actually* consented. The word λαμβάνω permeates Aristotle's later writings, so a detailed study of it's dialectical origins would be well-warranted.

91 Hardie/Gaye, Ross, Sachs, and Waterfield all use some form of "assumes" in translating λαμβάνω in *Physics* Z.2. While λαμβάνω can indeed mean "assume," this is a poor translation when dealing with a dialectical *logos*: for a questioner to merely *assume* something is as good as nothing. The questioner needs to produce at least the appearance that the answerer has assented to all premises, even unstated premises.

Unstated premises are not the same thing as "hidden assumptions". Contemporary philosophers often suppose that one can make an argument that somehow relies on "hidden assumptions" that are hidden even from the arguer. But the unstated premises of a dialectical *logos* will never be hidden from the questioner himself, only from the answerer. A truly "hidden assumption" could play no causal role in convincing the answerer and the audience that the refutation has succeeded. By contrast, premises that are initially left unstated can, if necessary, later be brought up by the questioner in reply to the answerer's counterattack.

where the questioner asks for and receives confirmation of several particular theses. These are intended as the premises for an induction.[92] He proceeds to ask for confirmation of the universal conclusion, but in doing so he does not state the predicate over which he is universalizing. Instead he asks something like: "And it is the same in all such cases, is it not?" This is what Aristotle refers to in the first clause of his remark. Faced with this situation, Aristotle advises the answerer to simply answer the question affirmatively without comment: "Let it be so."[93] (176a23-4) At some point the questioner will then attempt to employ the putative universalization as a premise for some further conclusion, this time stating it outright. If the universal claim is objectionable, the answerer may now jump in, claiming it was not what he had intended to agree to, and is not justified by the particulars previously enumerated.

Here we see that λαμβάνω (take) is counterposed to δίδωμι (grant, give). The answerer claims that he *granted* one thesis, but that the questioner *took as granted* a different one. There is no dispute about which *utterance* is at issue, this being the simple affirmation "Let it be so." What is in dispute is the content or import of that utterance. Thus the object of λαμβάνω is a certain interpretation of the original utterance.

We find a similar usage in Plato's *Euthydemus*, although here λαμβάνω (take) has

---

92 This seems clear from context, despite the fact that Aristotle uses the term παραβολή (illustration) and not ἐπαγωγή (induction), which are normally distinguished. For a clear depiction of the phenomena Aristotle is considering, see *Topics* Θ.2 (157a21ff.) On παραβολή, see Slomkowski, pg. 36.

93 Aristotle motivates this advice by warning that if the answerer affirms the universal by himself clarifying which predicate he is universalizing, he may be opening himself up to attack from an unexpected angle. (176a24-25) In general, the less an answerer says, the less he needs to defend. One might wonder why the answerer is not advised to seek immediate clarification (as was the case with ambiguous premises). Aristotle does not explain this here, but it seems that it might do more harm than good, since the answerer may then be faced with a harmful but precise universal claim against which he has no ready objection. Instead, the quoted passage seems to envision that the answerer should wait until the universal is employed by the questioner, and then object that he never granted it. In this way the answerer avoids having to consider the universal claim on its merits, at least until he has a better sense of how it will be used.

a prefix, yielding ὑπολαμβάνω (take *up*), which does not greatly alter it's meaning. Socrates is being questioned by an eristic dialectician, and is asked an unclear question, whereupon he requests clarification. When he is chastised for asking questions of his own rather than directly answering, Socrates explains his concern: the questioner may ask the question understanding it in one way (ἄλλῃ...διανοούμενος), but Socrates may "take it up" in another (ἄλλῃ ὑπολάβω), and answer with his own interpretation in mind. (295c4-6, cf. 295b1-c9) Just as λαμβάνω was used in the Aristotle passage, here ὑπολαμβάνω is used with it's object being an utterance interpreted in a certain way.

We can now resolve the aforementioned puzzle.[94] From *Physics* Z.9 we determined that Aristotle would resolve the Racetrack by charging that it had an ambiguous premise. But in Z.2 he seemed to be saying that the problem was a false premise. Now we see that there is no conflict here. The object of λαμβάνω in Z.2 may indeed be an utterance that is in itself ambiguous. But under one of its interpretations, it may be a falsehood, and it is in the guise of this falsehood that it serves as the object of λαμβάνω.

We now have a clear sense of how Aristotle will find fault with the *logos*. He thinks it involves an ambiguous question, such that the answerer takes it one way, and the questioner another. Our next task is to examine the analysis itself. Since this will involve a profusion of textual details, it will help to have an overview of where things are going.

In *Sophistical Refutations* 4, Aristotle discusses various fallacies that all involve ambiguity of one sort or another. One of them is ὁμωνυμία (lexical ambiguity). In this

94 For yet another relevant occurrence of λαμβάνω, in a passage involving "the unlimited", see *Physics* Γ.6 (206a19).

fallacy, a single word or phrase is ambiguous.[95] (165b30-166a6) In virtue of this, a whole sentence or question will be ambiguous.[96]

What we will find is that Aristotle thinks the Racetrack *logos* involves a premise suffering from ὁμωνυμία. In this case, the ambiguous word is ἄπειρος. We can tell this because Aristotle analyzes ἄπειρος as having two distinct meanings, thus directly applying his definition of ὁμωνυμία from *S.E.* 4.[97] So when ἄπειρος occurs in a dialectical question, the answerer may take it as having one meaning, while the questioner may take it as having another. Hence the same will be true of the question as a whole.

For the time being, we will ignore the question of what ἄπειρος actually means, whether to Aristotle or anyone else. We will later examine this in Chapter 3. In my translations, I will render ἄπειρος as "unlimited", but for now, we can avoid confusion by simply regarding the meaning as unsettled. Our immediate focus will be on the *structure* of Aristotle's discussion, and what we will find is that this structure would be thoroughly appropriate for *any* dialectical *logos* involving ὁμωνυμία. In fact, this dialectically-based structure has been ignored by scholars, but understanding it is essential if we wish to later understand what the discussion is actually *about*.

What we will discover is that Aristotle methodically applies his own advice from *Topics* Θ.7, which we examined earlier. He sees himself facing a situation where "the question asked is intelligible and has multiple senses" (*Topics* 160a23-4) and "with respect to some [sense] it is false and concerning some [sense] it is true". (160a26) In

---

95 For discussion of ὁμωνυμία, see 165b30-166a6. Besides ὁμωνυμία, ambiguities in the broad sense include grammatical ambiguity (166a6-21), ambiguities that result from the fact that the syntax of a phrase can be altered by its context (166a23-38), and others as well.

96 Cf. 175b39-41.

97 Aristotle does not use the word ὁμωνυμία itself in *Physics* Z.2.

such a case, a dialectical answerer "must indicate that it [i.e. the question/premise] is said [i.e. meant] in multiple ways and that one is false and one is true." (160a26-8) This is exactly what we find Aristotle himself doing in *Physics* Z.2.

He first reports the putatively ambiguous line: “It is not possible to go through things unlimited (ἄπειρα) or to individually touch things unlimited in a limited time.” (*Physics* 233a21-3) Next, he charges that ἄπειρος has a double meaning. (233a24-6) This entails that the entire original premise also has a double meaning. Finally, he employs his disambiguation of ἄπειρος to produce a disambiguation of the entire ambiguous thesis. Just as he counsels, he asserts that on one interpretation the thesis is true, and on the other false. (233a26-28) Aristotle's practice thus accords with his theory. To see this, we will now consider each of the steps in turn.[98]

The Ambiguous Claim

We must first determine exactly what the ambiguous claim is supposed to be.[99] Aristotle reports it in indirect discourse. (233a21-3) If we transform the indirect statement into a declarative statement, the Greek reads: οὐκ ἐνδέχεται τὰ ἄπειρα διελθεῖν ἢ ἅψασθαι τῶν ἀπείρων καθ' ἕκαστον ἐν πεπερασμένῳ χρόνῳ. (It is not possible to go through things unlimited or to individually touch things unlimited in a limited time.)

The given claim is a negated disjunction, and so a conjunction. We may isolate it’s two conjuncts, transforming each into a full sentence. First: οὐκ ἐνδέχεται τὰ ἄπειρα

98 The next three sections focus closely on the Greek text. Readers who know Greek should consult Aristotle's text as they read mine. But a non-reader of Greek should be able to follow my argument, as it focuses mainly on logical and syntactical issues, and my English translations closely mimic the grammar of the Greek.

99 That is to say, we must first determine what Aristotle, in *Physics* Z.2, regards as the ambiguous claim. We will defer until later the question of how the Z.2 discussion relates to pre-Aristotelian versions of the *logos*.
Moreover, in examining Z.2, we will treat the ambiguous claim as a definite assertion, rather than as a question, since that is what Aristotle himself does.

διελθεῖν ἢ ἅψασθαι τῶν ἀπείρων καθ' ἕκαστον ἐν πεπερασμένῳ χρόνῳ. (It is not possible to go through things unlimited in a limited time.) Second: οὐκ ἐνδέχεται ἅψασθαι τῶν ἀπείρων καθ' ἕκαστον ἐν πεπερασμένῳ χρόνῳ. (It is not possible to individually touch things unlimited in a limited time.)

I have included the qualification "in a limited time" with both conjuncts. Although in Aristotle's initial wording it is applied only to the second, from his analysis, it is clear that he intends the modifier to apply to the first conjunct as well.[100] He mentions "going through" (recalling the verb in the first conjunct), but "in the unlimited [time] and not in the limited [time]". (233a28-30) This must be meant to contrast with "in a limited time", construed as part of the first conjunct.

We thus have a conjunction of two theses. There seems little doubt that Aristotle regards them as alternatives to each other, so that a given version of the *logos* will contain one or the other but not both. We thus have two ambiguous theses, not one. In *Physics* Z.2, Aristotle treats them in tandem, supposing that each is ambiguous for the same reason, namely, the presence of a form of the ambiguous word ἄπειρος. To be sure, at 233a26-8, Aristotle focuses attention only on the second. But he likely does not wish to say basically the same thing twice. At 233a28-31 he returns to a parallel treatment of the two theses. So we may reasonably suppose that regardless of which claim the *logos* involves, Aristotle will find the same problem with it.[101]

The Charge of Ambiguity

After presenting the ambiguous claim, Aristotle continues: "διχῶς γὰρ λέγεται

100 We will later discover that it is likely Aristotle himself who inserts "in a limited time" into the *logos*.

101 We will later find that these two theses derive from substantively different versions of the *logos*. (Cf. 263a4-6, 263a6-11.) For now we may ignore this.

καὶ τὸ μῆκος καὶ ὁ χρόνος ἄπειρον, καὶ ὅλως πᾶν τὸ συνεχές, ἤτοι κατὰ διαίρεσιν ἢ τοῖς ἐσχάτοις." (For[102] in two ways both length and time are said to be unlimited, and in general everything continuous, either with respect to division or with respect to its ends.) (233a24-6)

Here Aristotle meets the first of his requirements for drawing a distinction, charging that something is "said in multiple ways". (*Topics* 160a27) In this case, he charges that the word ἄπειρος is lexically ambiguous. In fact, his charge is limited to a specific use of ἄπειρος, the case where a continuous thing is called ἄπειρος.[103]

If a continuous thing, such as a length or a time, is called ἄπειρος, then ἄπειρος might mean one of two different things. It might mean ἄπειρος κατὰ διαίρεσιν (unlimited with respect to division). Or it might mean ἄπειρος τοῖς ἐσχάτοις (unlimited with respect to its ends).

Thus suppose Aristotle encounters the claim: τὸ μῆκος ἄπειρον. (The magnitude is unlimited.) He could then point out that the claim by itself does not have a single meaning. Instead it may mean one of two things. It might mean: τὸ μῆκος ἄπειρον κατὰ διαίρεσιν. (The magnitude is unlimited with respect to division.) Or it might mean: τὸ μῆκος ἄπειρον τοῖς ἐσχάτοις. (The magnitude is unlimited with respect to its ends.) For the moment we need not worry about exactly *what* these two claims mean. What matters is that each results from the straightforward replacement of a lexically ambiguous claim by an explication of it.[104]

102 For now we may ignore this connective, but its sense will become evident in Chapter 3.

103 In some cases, ἄπειρος will not be ambiguous. Thus if Aristotle describes ἀριθμός (number, i.e. number in general) as ἄπειρος (unlimited) (cf. 206a11-2), this will have only one sense, meaning, roughly, that there is no bound beyond which there is not another number.

104 In this case, each of the replacement terms includes the original word ἄπειρος. Consider that we might replace "Georgia" with "the State of Georgia" or "the Republic of Georgia". But this need not be true generally. We might replace "bank" with "river bank" or "financial institution," and we would be doing

The charge of ambiguity is not explicitly justified by Aristotle.[105] However, its truth is presupposed in the passage immediately prior to the invocation of Zeno. There Aristotle argues that both the time and the magnitude of a given motion will be ἄπειρος in precisely the same ways. Each will be ἄπειρος either "with respect to its ends" or "with respect to division" or both. (233a16-21) Clearly, to understand this earlier discussion, Aristotle's audience must already understand the difference between the two senses of ἄπειρος. Hence Aristotle reasonably invokes the distinction in analyzing the Racetrack.

The Disambiguation

Aristotle continues: "τῶν μὲν οὖν κατὰ τὸ ποσὸν ἀπείρων οὐκ ἐνδέχεται ἅψασθαι ἐν πεπερασμένῳ χρόνῳ, τῶν δὲ κατὰ διαίρεσιν ἐνδέχεται." (Certainly it is not possible to touch things unlimited (ἄπειρα) with respect to quantity in a limited time, but it is possible [to touch] things [unlimited] with respect to division.) (233a26-28)

If we examine this line carefully, we can see that Aristotle is straightforwardly applying his prior claim that ἄπειρος is ambiguous. He proceeds to disambiguate the entire thesis in which the term occurs, and claims that one of the disambiguated theses is true and one false. He thus meets the second requirement for drawing a distinction in the provision of a *lusis*. (*Topics* 160a27-8) In offering the disambiguation, Aristotle focuses attention on the second of the two alternate versions of the original premise. But we may suppose that, from his point of view, he could just as well have offered a disambiguation

the same thing. Hence it is better to speak of replacement rather than clarification.

105 For a discussion of related issues, see *Physics* Γ.6-7. There Aristotle distinguishes between things ἄπειρος κατὰ πρόσθεσιν (unlimited with respect to addition) and things ἄπειρος κατὰ διαίρεσιν (unlimited with respect to division). It seems that the term "unlimited with respect to its ends" serves as a restriction of the term "unlimited with respect to addition", a restriction appropriate for the case when the unlimited thing is continuous.

Physics Γ.4-8 contains a detailed discussion of "the unlimited". It is complicated, but Aristotle's Z.2 discussion does not hinge on the nuances of his own theory.

of the first.

In disambiguating, Aristotle makes a statement that is a logical conjunction. By removing particles relevant only to Aristotle's own discussion, and inserting into the second conjunct words that Aristotle does not explicitly repeat from the first, we can express the two conjuncts as follows. First: τῶν κατὰ τὸ ποσὸν ἀπείρων οὐκ ἐνδέχεται ἅψασθαι ἐν πεπερασμένῳ χρόνῳ. (It is not possible to touch things unlimited with respect to quantity in a limited time.) Second: τῶν κατὰ διαίρεσιν ἀπείρων ἐνδέχεται ἅψασθαι ἐν πεπερασμένῳ χρόνῳ. (It is possible to touch things unlimited with respect to division in a limited time.)

Recall the original thesis at issue, the second of the two alternative ambiguous claims: οὐκ ἐνδέχεται ἅψασθαι τῶν ἀπείρων καθ' ἕκαστον ἐν πεπερασμένῳ χρόνῳ. (It is not possible to individually touch things unlimited in a limited time.) Two minor deviations mask the linguistic affinity of this line with the two later conjuncts. First, the phrase καθ' ἕκαστον (individually), found in the original claim, does not recur in the disambiguated conjuncts. But having said it once, Aristotle may take it as having clarified the meaning of ἅψασθαι (touch). He may then use ἅψασθαι throughout the Z.2 discussion, and need not repeat his clarification. Second, in his charge of ambiguity, Aristotle took one meaning of ἄπειρος to be ἄπειρος τοῖς ἐσχάτοις (unlimited with respect to its ends). We do not find this phrase in the disambiguated conjuncts, but rather a form of ἄπειρος κατὰ τὸ ποσόν (unlimited with respect to quantity). The precise relation between these two phrases is somewhat subtle, and we will examine this issue in Chapter 3. But for now we may reasonably suppose that Aristotle construes these terms as applicable to the same things, which is not far from the truth.[106]

If we accept these two points, the relation of the two conjuncts to the original thesis becomes clear. Aristotle first substitutes the appropriate forms of the two disambiguations of ἄπειρος for the original term itself. The original claim contains the genitive plural τῶν ἀπείρων (things unlimited). Taking the genitive plurals of the disambiguating terms, we get τῶν κατὰ τὸ ποσὸν ἀπείρων (things unlimited with respect to quantity) and τῶν κατὰ διαίρεσιν ἀπείρων (things unlimited with respect to division), which indeed occur in the two conjuncts. Having done this, Aristotle negates the second of the two resulting theses. As the original thesis is itself negated, the double negation leads Aristotle to drop the οὐκ (not) from the second conjunct. Thus the first disambiguated conjunct, in line with the original claim, asserts: "It is not possible...". And the second asserts: "It is possible...".[107]

We can now see that Aristotle has precisely followed his own advice on drawing distinctions when faced with an ambiguous premise. Remember that the questioner "must indicate that it [i.e. the question/premise] is said [i.e. meant] in multiple ways and that in one it is false and in one it is true." (*Topics* 160a26-8) Now we find Aristotle charging that the Racetrack contains an ambiguous premise. This premise contains the word ἄπειρος, which he thinks is lexically ambiguous. He first disambiguates the word, and then inserts his disambiguations into one version of the original thesis, thus producing two disambiguations of the whole thesis. And he affirms that the first of these is true, and the second false. Aristotle has thus *drawn a distinction*.

---

106 Ross, pg. 643, asserts that the two terms do apply here to the same thing, and no one seems to deny it.

107 I think it very unlikely that Aristotle literally acted in such a mechanical manner as I have described in this paragraph. But what he did do was no doubt motivated by the fact that he had thoroughly absorbed his own principles for dealing with dialectical *logoi*, principles that are highlighted in my description.

We may now take stock of the situation. When we consulted *Physics* Z.9, we found some direct evidence about the Racetrack[108], but also an indication of how Aristotle thought he had earlier analyzed the *logos*. When we consulted his dialectical works, we found that *drawing a distinction* was the method of resolution suitable for a dialectical *logos* with an ambiguity. Now we have examined *Physics* Z.2, and found that Aristotle does indeed *draw a distinction* against the Racetrack. So we have a good grasp of the basic *structure* of his analysis. But we can scarcely claim to *understand* it. That will be our goal in Chapter 3. Since Chapters 2 and 3 are so inextricably connected, we may defer until the end of Chapter 3 any consideration of how other scholars have treated Aristotle's Z.2 analysis. At this point, we may simply note that Aristotle is unquestionably treating the Racetrack as a dialectical *logos*, just as we would have expected, given Chapter 1. And yet, while building on this dialectical framework, Chapter 3 will take things in an entirely new direction.

108 We will return to this evidence in Chapter 4.

Chapter 3
Disambiguating a Diagram

The Word Ἄπειρος

To understand Aristotle's Racetrack analysis, it seems we need to understand how Aristotle disambiguates the word ἄπειρος. In fact, this is true, but misleading. What we really need to understand is how he disambiguates the plural form ἄπειρα, the word that actually occurs in the *logos*. And what we will find is that in disambiguating ἄπειρα, Aristotle is not so much thinking about the word as he is thinking about the fact that one and the same diagram may be construed in two different ways. Indeed we will later find that the central innovation in Aristotle's critique of the Racetrack is his introduction of diagrams into the analysis. For now we may defer consideration of the full critique, as we have yet to reconstruct the full *logos*. But we will soon find, I contend, that in the passage we have been examining in *Physics* Z.2, Aristotle presumes that his disambiguation of ἄπειρα will be understood by means of a diagram. Indeed we will see that because Aristotle thinks a certain diagram is ambiguous, so the word ἄπειρα will be ambiguous. And, in turn, because the plural ἄπειρα is ambiguous, so will be the singular ἄπειρος. Once we understand the basis for Aristotle's disambiguation of ἄπειρος, we will be able to see how Aristotle construes his Racetrack analysis as a dialectical lusis. But to reach that point we will need to proceed through a number of seemingly disparate issues.

To get things started, we first need to get a basic grasp on the word ἄπειρος. Traditionally, scholars have rendered ἄπειρος as “infinite”, and contemporary philosophers commonly take “infinite” to mean “numerically infinite.” But as we will see, ἄπειρος originally meant “untraversable,” and this is not an inherently numerical idea.

The translation “infinite” has a historical basis, since "infinite" is just an Anglicized version of the Latin "infinitus". And in the ancient world, “infinitus” gained prominence as a word mainly because of it's use in translating the Greek word ἄπειρος, and to it's ensuing value in philosophical discussions that were self-consciously in the Greek tradition. But this etymology does not ensure that “infinite” means the same thing as ἄπειρος.

In fact, today, among people who have studied analytic philosophy, the word “infinite” is commonly taken simply to mean “numerically infinite.” This results from reading “infinite” against the backdrop of Georg Cantor. In the nineteenth century, in a very well-defined way, Cantor developed the idea that there can be such things as “infinite numbers”. But prior to Cantor, the phrase “infinite number” would have seemed an oxymoron.

In recent times, many scholars have recognized the possibly anachronistic connotations of “infinite”, and have sought philological accuracy by rendering ἄπειρος as “unlimited”. This translation is motivated by a particular construal of the etymology of ἄπειρος. The idea is that ἄπειρος is derived from the noun πέρας (limit), along with the α-privative prefix. From Aristotle, we can see that this account had supporters even in his day.[109] Indeed, the theory may well have influenced the way the word was actually used, especially by people like philosophers.

In fact, the etymology at the root of the translation is simply incorrect. It turns out

109 Aristotle sometimes insinuates that ἄπειρος is used in this way by others. See *Physics* 203b6-7, 203b20-2, 206a3-8, 206b33-207a15 (esp. 14-5). Interestingly, in the last of these passages, Aristotle himself appears to reject the usage. Charles Kahn (pg. 233, fn. 1) also attests to this usage by other philosophers.

that Charles Kahn presents convincing evidence from Homer and Hesiod that ἄπειρος originally meant “untraversable”. Most Zeno scholars seem unaware of this result, so I here present a long citation:

> ... [It] is not the noun πέρας which is negated by the α-privative, but the verbal root **per*- represented in πείρω, περάω, and περαίνω, as well as in a number of Indo-European adverbs and prepositions, all referring in some way to the direction "forward, in front" (Greek πρό, Latin *per*, *prae*, etc.). The verbal forms indicate a movement in this direction, and the group of περάω, πέραν, περαίνω, πεῖραρ envisages the point at which the forward motion comes to an end. Thus περάω (like περαιόω) is regularly used of passing over a body of water to reach the other side; the adverb πέραν refers precisely to what lies "across," as the Echinades islands lie over the water from mainland Greece:
>
> νήσων, αἳ ναίουσι πέρην ἁλὸς Ἤλιδος ἄντα. (B 626)[110]
>
> Πεῖραρ is the limit or goal of a given passage - the point at which the forward movement comes to an end. So the word regularly occurs in Homer together with a verb of motion. (With the usual vowel gradation, the same root appears in the nouns πόρος, πορθμός designating either the motion as such, or the passage through which one moves.) It is this basic verbal idea which is negatived in ἀπείρων, ἄπειρος, exactly as the synonymous ἀπέραντος is formed from the verbal stem of περαίνω (aorist ἐπέρανα). The true sense of ἄπειρος is therefore "what cannot be passed over or traversed from end to end." When earth and heaven are called ἀπείρων, there is no contradiction; both have πείρατα, but few mortals can travel like Hera "to the ends of the earth" (Ξ 200).[111]

Kahn's reasoning appears sound to me. So it follows that ἄπειρος originally meant not “unlimited”, but “untraversable”.

This fact poses some problems. In Aristotle's day, it seems that ἄπειρος might have meant either “unlimited” or “untraversable”. But if we want a single English word to stand in for ἄπειρος in all of our translations from the *Physics*, it seems “unlimited” is the best choice. This is because “unlimited” seems somewhat better able to encompass the meaning “untraversable” than vice versa. So when I offer translations, I will always write

110 This is *Iliad* B626, which translates: islands, which lie across the sea over against Elis.
111 Kahn (1960), pg. 232. “Ξ 200” is *Iliad* Ξ 200.

"unlimited" for ἄπειρος. But these translations are merely starting points: in looking at the Racetrack evidence we will always need to examine the use of ἄπειρος in some detail.

Even more important than the contrast between "untraversable" and "unlimited" is the contrast between "untraversable" and "infinite". We might well assimilate "unlimited" to "untraversable" and we might also assimilate "unlimited" to "infinite". But we are then pushing "unlimited" in two different directions, for clearly, "infinite" and "untraversable" do not mean the same thing in English. And this is especially true if we read "infinite" as "numerically infinite". In much of contemporary academic philosophy, "infinite" is a numerical quantifier, like "one" or "seven" or "many", or technical terms known to cognoscenti, like "countably infinite" or "uncountably infinite". But "untraversable" is certainly not a numerical quantifier, and if ἄπειρος means something like "untraversable", than neither is ἄπειρος. Our key result so far is thus that when we encounter ἄπειρος, our initial presumption should be that it means something like "unlimited/untraversable", in contrast to "unlimited/infinite".

I suspect that the reading of ἄπειρος as a roughly numerical term has posed considerable confusion to scholars seeking to understand the Racetrack analysis. This is because Aristotle is repeatedly using plural forms of ἄπειρος, in particular, forms of the neuter plural ἄπειρα. Logically speaking, the term ἄπειρα might mean one of two distinct things. It might refer to a collection of things, each one being ἄπειρος. Or it might refer to a collection that is ἄπειρος as a whole. But in this second case, if we read ἄπειρος as itself a numerical term, then the plural form of ἄπειρα seems redundant and disconcerting: if a collections is infinite, it is necessarily two or more, and hence plural. What then does the grammatical number signify? And yet it seems to signify *something*, since Aristotle generally uses the singular form of ἄπειρος when discussing “the unlimited”. In fact, most English translations simply obfuscate the fact that Aristotle is using the plural at all.

And yet a close analysis suggests that the plural ἄπειρα is central to Aristotle's dialectical *lusis*. Recall that in opening his discussion, Aristotle considers in tandem two alternate premises of the Racetrack *logos* (233a21-3), and claims that each is problematic for the same reason, namely, the ambiguity of ἄπειρος. In fact, each premise actually involves a plural form of ἄπειρος. Recall again that Aristotle goes on to disambiguate one of the two premises, yielding two disambiguations. (233a26-8) Each of these disambiguations likewise contains a plural form of ἄπειρος. We thus have four uses of the plural of ἄπειρα, and given the structural connections between them, each must involve a plural in the same sense. That is to say, either each plural refers to a collection of things that are each ἄπειρος, or each plural refers to a collection of things that is ἄπειρος as a whole.

Thus any reasonable interpretation of Aristotle's text must treat these four plural instances of ἄπειρος in the same way. In fact, I do not know of any scholarly examination of the passage that actually does this, so if we can come up with even one plausible reading, that will be progress.[112] What we will find is that in *Physics* Z.2, Aristotle is using ἄπειρα to refer to a collection that is untraversable as a whole. And this, of course, is the very sort of reading that seems bizarre if we regard ἄπειρος as a numerical term. But it is not.

112 Hasper seems rare among scholars in recognizing that , in *Physics* Z.2, the translation of ἄπειρα involves a stark choice between two alternatives. (pp. 53-4) But his ultimate translation does not work, since he interprets ἄπειρα in 233a22-3 as referring to a collection unlimited as a whole, and ἄπειρα in 233a26-8 as referring to a collection of things, each unlimited. The root problem with his analysis seems to be that he does not realize that the whole passage is organized around disambiguating an ambiguous premise.

Acquaintance with Ἄπειρα

Our ultimate goal is to understand how Aristotle uses a diagram to disambiguate the word ἄπειρα. What we will find is that Aristotle speaks as if the term ἄπειρα literally applies to the diagram in question, that is, he speaks as if the diagram literally *is* a collection untraversable as a whole. To make sense of this, we will first need to consider how Aristotle's usage fits naturally into a tradition of using ἄπειρα to refer to collections that are visibly present, a tradition that seems quite disparate from the ways in which people today use the word "infinite".

If we think about "infinite" as a numerical term, and we use it to refer to a certain infinite collection, then it seems that the members of that collection must each have a determinate identity as well, at least in an ontological sense. This just seems a general truth about numbers. If a collection is fifty-three in number, that is because it is composed of fifty-three individuals. And if a collection is infinite in number, that is because it is composed of infinitely many individuals. The number of the collection is ontologically dependent on the individuality of its members.

But if ἄπειρος means "untraversable" and not "infinite", then there may be some sense in speaking of a collection that is in some way determinate, and untraversable as a whole, and yet for which the identity of the members of that collection is somehow indeterminate. In fact, this is precisely what we will see when Aristotle construes diagrams as ἄπειρος. Indeed, his disambiguation of ἄπειρα is, in a way, a recognition that there are two interestingly different ways of bringing some determination to this indeterminacy.

Aristotle's use of diagrams is in many ways novel. But his novelty comes against a background in which the plural of ἄπειρος is routinely used to refer to collections that are untraversable as wholes, but for which the identity of the members is somehow indeterminate. Indeed, this is what we find in Homer. But to understand this Homeric use, we need to temporarily step aside from ἄπειρος and first consider how, quite generally, the way in which people think and talk about pluralities is liable to differ from the way that philosophers will often insist that those pluralities really are.

From an ontological perspective, it might well seem that the existence of a plurality depends on the determinate existence of each of it's members. But this is certainly not true when we consider epistemology and semantics. We can certainly *think* and *talk* about determinate pluralities even while leaving their quantity and the identity of their members as open questions.

Consider this scenario: I am talking to you, and suddenly I say "Look at the birds" and point towards the sky. There you see a flock of birds swooping this way and that, all *en masse*. Here the phrase "the birds" refers to the flock of birds. You understand this phrase because you perceive the flock of birds as a whole. But this does not require you to know how many birds there are, or even to be cognizant of *any* of the individual birds that make up this collection. That is to say, your experience is not of *this* bird, and *this* bird, and *this* one, etc. but rather simply of a flock.

To make more sense of this scenario, we can fit it into a broader philosophical framework. Philosophers often take a word or phrase, and ask how it ends up referring to whatever it actually does refer to. In more technical language, how does it's referent get fixed? We need not examine this much-debated question in it's full generality; we need only consider some of the factors that might enable a particular term to refer to a particular plurality.[113] One way is if the members of a collection are somehow enumerated, and then some term is applied as a name for this collection. For instance, a scholar might say: "By *Zeno's motion paradoxes*, I mean the Racetrack, the Achilles, the Arrow, and the Stadium." Alternatively, if we have a complex plural term, the referent of the whole term might somehow get picked out via the meaning of the individual words in the term. For instance, "complete published English translations of the *Physics*" refers to a collection that includes the works of Hardie and Gaye, of Joe Sachs, of Robin Waterfield, and various others.

I give the two prior examples mainly to contrast them with the bird example, which I think illustrates a very common scenario, albeit one seemingly neglected by philosophers. Actually, in one way, many philosophers are familiar with the bird scenario inasmuch as we can understand it via the philosophical idea of *acquaintance*, made famous by Bertrand Russell.[114] This philosophical idea derives from the normal idea of acquaintance: think how you can be acquainted with a person even if you do not really know who they are, in some sense. In the same way, you can be acquainted with many things in a way that precedes seemingly crucial linguistic or conceptual knowledge of what they are. For instance, consider a red spot on a shirt. You might see it and ask: "What is that?" The very possibility of this question suggests that you can be aware of something without, in some way, knowing what it is. In general, we might say you can be acquainted with something if it can have a perceptual salience that somehow does not reveal it's fundamental nature.

113 In raising this question, I am thinking about terms that actually *do* refer to pluralities, since their referents *are* pluralities. Such terms may be syntactically either singular or plural.

114 I do not claim to apply the idea of acquaintance in precisely the same way as Russell did, merely that his idea suggests my analysis.

Just as we can be acquainted with individual things, so we can also be acquainted with pluralities. Or looking at it another way, a plurality *is* a sort of individual thing with which we may be acquainted, but a thing composed of many things. We can be acquainted with a plurality because the plurality as a whole may have a perceptual salience that does not depend on the perceptual salience of its individual members. And so we can often recognize a plurality as a plurality, even if we cannot identify its individual members, or tell how many of them there are.

We can now make sense of what I will call reference by acquaintance. Recall the flock of birds. We want to know why “the birds” refers to the particular flock of birds that it does refer to. The answer involves two kinds of facts. First is the fact that someone uses “the birds” to refer to a flock of birds that is perceptually salient. Second is a collection of facts: for each bird, there is the fact that it is a part of the flock that is perceptually salient. Thus the connection between “the birds” and the actual flock (of individuals) is mediated by the acquaintance with the flock as a whole.

We can contrast this means of reference with the two I mentioned above. In each of those two cases, a plural term refers to a collection precisely because each item in the collection is *independently* part of the referent of the term. Thus, when I enumerate a list of “Zeno's motion paradoxes”, I might add or subtract items from the list at will, and the term (as I use it here) will refer to whatever collection results. In the case of “complete published English translations of the *Physics*”, we may think of every individual thing in the world as either being or not being a complete published English translation of the *Physics*, and the plural term then refers to the collection of those that are. So in each case, reference to a plurality is mediated by a semantic connection with the individuals that compose the plurality. In the first case the individuals are directly enumerated, and in the second case, the plural simply refers to the extension of a *singular* concept.

By contrast, in the case of reference by acquaintance, semantic connection to the individuals within a plurality is mediated by semantic connection with the *whole* plurality. In the prior two cases, the very plausibility of thinking or talking about the collections presupposes the determinate identity of the individuals that compose them. But this is not so in the latter case. In some cases, we can think and talk about collections insofar as we can *perceive* them as wholes. But this does not in itself entail that the members of the collection have any determinate identities at all. Of course, we might go on and reason that if we can perceive a collection as a whole, that collection must exist, and since a collection just *is* a collection of determinate members, those members must have determinate identities. But as reasonable as this seems, it is an extra philosophical assumption about the nature of reality. It is not required by the semantic possibility of reference by acquaintance.

The upshot of all this is that it is quite possible, and, I suspect, quite common, to coherently think and talk about pluralities in ways that do not require assumptions about the determinate identities of their members. This is quite interesting in it's own right, and, I think, quite commonly neglected by philosophers. But it is certainly neglected by philosophers interested in Zeno, who are liable to think that discussions of Zeno should involve discussions of infinite collections. And of course it seems quite absurd to talk about referring to infinite collections by acquaintance.. By contrast, reference to infinite collections will seem possible precisely because some concepts have infinite extensions: for instance, as applied to numbers, the concept *prime* has an infinite extension, and so “the prime numbers” refers to an infinite collection. But if ἄπειρος does not mean “infinite” then it need not involve reference to infinite collections. In fact, if we look at Homer, we find that plural forms of ἄπειρος were commonly used to refer to collections that were known by acquaintance, and we will later see that Aristotle is working against this backdrop.

While the precise origins of our written version of the *Iliad* are open to question, scholars agree that the *Iliad* was originally an oral poem. It was not memorized, but was rather created anew each time it was performed, with the poem following the same basic story line, even if the exact wording differed. One of the keys to this performance was what scholars call *Homeric formulae*, which were stock phrases of which the poet had a thorough grasp of the metrical properties, so that he could use them on the fly to construct an apt metrical line. One of these phrases was ἀπερείσι' ἄποινα, of which the unelided rendering is ἀπερείσια ἄποινα. This phrase occurs twelve times in our *Iliad*, and translators usually render it something like “countless ransoms”. The word ἀπερείσια may reasonably be considered the same word as ἄπειρα. It is the neuter plural of the form ἀπερείσιος, which is a Homeric variant of ἄπειρος.[115]

The first occurrence of ἀπερείσι' ἄποινα, at *Iliad* I.13, gives us a good example of it's use. Here the priest Chryses is trying to free his daughter from Agamemnon. In order to persuade Agamemnon, he brings ἀπερείσι' ἄποινα, “countless ransoms”. As he pleads with the king, it is clear that the collections of treasures is beside him, in full view of the king and his Greek allies.

This use of ἀπερείσιος fits well with the basic meaning “untraversable”. The

115 The Homeric poems often employ many variants of the same word, with no difference in meaning, since each will fit different metrical conditions.

treasures are “untraversable” in the sense that they are too many for Agamemnon or any of the Greek heroes to actually count, perhaps because it was beneath their dignity, or perhaps because the Homeric poems originated in any era when the Greeks had no no counting system that went very high.

Thus the very word ἀπερείσια conveys an indeterminacy in the exact number of treasures, and hence, even an indeterminacy in the identity of the individual treasures in the collection. But this indeterminacy is certainly compatible with the fact that the treasures are, evidently, *many*. And the indeterminacy is compatible with the fact that the collection is visually apprehensible: there is no doubt that Agamemnon can *see*, in one fell swoop, the ἀπερείσι' ἄποινα.[116] Thus Agamemnon is acquainted with a plurality, indeed a plurality that is untraversable.

Naturally, there is no necessity that Aristotle should use ἄπειρος in precisely the same sense as Homer, and indeed he does not. But the Homeric use was certainly well-known Aristotle and to nearly every Greek. And this means that Aristotle was quite familiar with the idea that the plural of ἄπειρος should refer to a plurality that is known by acquaintance, and, at the same time, to a plurality that is somehow ἄπειρος, that is, untraversable, as a whole.

Diagrams of Ἄπειρα

Like Homer, Aristotle in *Physics* Z.2 uses a plural form of ἄπειρος to refer to a plurality with which we may be acquainted as a whole. But for Aristotle, the plurality in question is a diagram. To see why this makes sense, we need to once again temporarily put aside any detailed consideration of what Aristotle *means* by ἄπειρος, and simply

116 Of course, neither the Homeric poet nor his audience can literally see the treasures, but they could if they were present with Agamemnon.

consider the way in which his diagram use meshes with the earlier use of the word. Once we see this, we can go on to consider how Aristotle is effectively disambiguating a diagram.

To learn about Aristotle's use of diagrams, we can start by considering a passage that comes shortly after the Racetrack discussion. Aristotle writes: “ἔστω γὰρ πεπερασμένον μέγεθος ἐφ' οὗ ΑΒ, χρόνος δὲ ἄπειρος ἐφ' ᾧ Γ· εἰλήφθω δέ τι τοῦ Χρόνου πεπερασμένον, ἐφ' ᾧ ΓΔ.” (For let there be a limited magnitude upon which are Α [and] Β, and an unlimited time upon which is Γ. And let some limited [time], upon which are Γ and Δ, be taken from the [unlimited] time. [233a34-b1] This is the start of an argument, and Aristotle would have presented it in conjunction with diagrams. We will shortly consider why this must be so. But first, eschewing detailed argument, I will present an interpretation of what, more or less, the diagrams associated with this passage probably looked like.

One part of the diagram setup was a line, next to which were the letters Α and Β. In fact, they were no doubt near the ends of the line.[117] If we read beyond the quoted passage (237b1-2), we find that there was also an Ε upon the line, between the Α and the Β. So the diagram probably looked like this:

<u>Α   Ε   Β</u>

It is this very line, or rather the similar, but long-gone line in ancient Athens, that Aristotle speaks of as the “limited magnitude”.

Just as Aristotle speaks of a “limited magnitude”, so he also speaks of an “unlimited time, upon which is Γ.” Then he speaks of a limited portion of this time,

117 I think that is fairly evident in this case. But in general, we cannot *assume* that letters are at the ends of lines. This issue does not really affect our discussion.

"upon which are Γ and Δ." From this we can infer that his diagram looked something like this[118]:

Just as the prior line *was* the limited magnitude, so this line *is* the unlimited time. Of course, the diagram line itself is not unlimited, that is, it is not infinite in extent. But does this matter? Neither is it a time. But this need not prevent Aristotle from unambiguously treating the diagram line as the referent of the phrase χρόνος ἄπειρος.[119]

The key factor that shows the importance of diagrams in Aristotle's argument is the way in which he speaks of letter being "upon" a magnitude and a time. This sort of language is almost invariably absent from English translations, but it is actually extremely common. It is especially pervasive in *Physics* Z. In general, the topic of *Physics* Z is continuous motion, and in discussing continuous motion, Aristotle discusses at length three varieties of continuous thing: magnitudes (that is, what people today call spatial magnitudes), motions, and times. If we browse through *Physics* Z, we find Aristotle routinely speaking as if letters are upon or next to these very things. In fact, when Aristotle speaks of letters being "on" or "next to" magnitudes, times, and various other things, he means quite literally that the given letters are next to diagrams of the very things he is talking about.[120] And with the magnitudes, motions, and times of *Physics* Z,

---

118 In fact, all we can tell for sure is that Γ and Δ were on a line, and that seemingly, they were not *both* at ends of the line, since they mark off a limited portion of a larger, indeed unlimited, time. But in principle, they might have come in the reverse order, there might be another letter, etc. The current line of reasoning does not hinge on such issues.

119 For additional arguments that employ diagrams of unlimited magnitudes and times, see *Physics* Z.7.

120 It is by no means obvious that *every* time refers Aristotle refers to a letter as being "upon" something, he is referring to a diagram, and nothing that I say hinges on that claim. But it is certainly true quite often, and especially in *Physics* Z, which is what matters here. What gives some pause is the fact that Aristotle, especially throughout the *Analytics*, often speaks of letters being "upon" things that are not so geometric, such as men and animals. But the "diagrams" in such cases may well consist of *words* referring to men and animals. This is a complicated issue. For a different view, see Netz (1999), pp.

it is fairly clear that the diagrams were simply lines.[121]

There are many, many questions about the details of the Aristotelian use of diagrams. Indeed, even to try and map out the scope of these questions would take us far afield. But scholars have long been aware that diagrams played some kind of role in Aristotle's writing, and especially in *Physics* Z. Indeed, you can see Ross trying to reconstruct a number of these diagrams in his commentary on the Greek text.[122] But on the whole, philosophically oriented readers of Aristotle, and of the *Physics* in particular, have tended to ignore the role of diagrams. This probably has something to do with the fact that one of the core themes in the history of analytic philosophy was the idea that reasoning was fundamentally something linguistic or conceptual, and does not, or should not, involve pictures.[123] But in recent times, Reviel Netz, in *The Shaping of Deduction in Greek Mathematics*, has greatly advanced our understanding of diagrams in Greek geometry.

Netz analyzes Euclidean and post-Euclidean mathematical treatises, all of which involve significatory letters of the type we see above, and shows how these letters are always used to enable a certain type of diagram use. Netz devotes his attention to mathematical treatises, which in their surviving form are fairly systematized, and largely

48-9.

121 That the diagrams of magnitudes and times were lines is more or less obvious to any reader of *Physics* Z. That motions were lines is much more counterintuitive to a modern reader accustomed to the representation of motion via a Cartesian graph. But *Physics* Z.1 (231b0232a22) makes fairly clear that motions were depicted as lines, just like magnitudes and times. This raises many interesting questions, but the current investigation does not hinge on them.

122 See, for instance, pp. 641-2.

123 Interestingly, the origins of analytic philosophy are closely tied to the 19th century reinterpretation of some of the very themes central to *Physics* Z. In particular, Frege and Russell, among others, were greatly influenced by the “rigorization” of the calculus, in other words, the effort to give a mathematical treatment of continuous motion (the very topic of *Physics* Z) that does not involve appeals to “intuition” of any sort, but instead proceeds from the acceptance of certain conceptualizable axioms.

postdate Aristotle.[124] So he does not purport to say much about Aristotle himself.[125] But if we consider non-"mathematical" writings from the period prior to and contemporaneous with the earliest "mathematical" treatises[126], we find that Aristotle's own writing seems to provide us with the largest surviving source of such letters. So we would do well to consider how Netz' conclusions might apply to Aristotle.

For our immediate purposes, what is most interesting about ancient letter use is the way in which it's semantic framework depends on visual acquaintance.[127] By contrast, even when freestanding letters occur in contemporary formal mathematics, or in all sorts of technical and academic discourse, as "variable" letters, understanding them does not require any sort of perceptual acquaintance. Suppose I say: "If x is divisible, then x/2 is also divisible." Here the letter x is not a referring term at all; there is no *thing* to which x refers. Instead, we can explain the semantic structure of the sentence by saying that x is really a quantified variable, so that the sentence is really making a claim about a certain universe of discourse, perhaps one specified implicitly, in this case, probably something like the set of continuous magnitudes. By contrast, in Greek mathematical texts, the letters are *always* referring terms. If someone like a geometer, indeed Aristotle, speaks of a "χρόνος ... ἄπειρος ἐφ' ᾧ Γ" (unlimited time upon which is Γ), then the Γ in the text is *most definitely* a referring term. It refers to another Γ that is in the diagram. Next to the

---

124 Netz tells us that, most likely, "among extant mathematical authors, Autolycus is the earliest", and he was "almost certainly active in the second half of the fourth century BC." Netz also reports the "establishment view that Euclid was active around 300 BC". (pg. 314) Aristotle lived 384-322 BC.

125 But see below.

126 I put "mathematical" in quotes, since I here use the term simply to refer to a certain type of text, that is, the type of the Euclidean *Elements*, etc. No surviving Aristotelian treatises are *texts* of that type. I leave aside the question of whether some of Aristotle's original arguments ought to be construed in some way or other as "mathematical".

127 The current paragraph is grounded in my reading of Netz, but I emphasize different issues than he does.

diagram Γ will be a line, or more generally, *something*, with which a viewer will be visually acquainted. Then, because this thing, the line, with which he is visually acquainted, is next to the diagram Γ, a viewer can take this very line as the referent of the whole phrase "χρόνος ... ἄπειρος ἐφ' ᾧ Γ" (unlimited time upon which is Γ). So the viewer sees one thing, a line, next to a Γ, and Aristotle speaks of a different thing, an unlimited time, as being next to a Γ. But the viewer has no problem taking the one to be the other, so long as the diagram admits of only one interpretation, given the wording.[128] On the whole, I see little reason to doubt that Aristotle's use of letters meshes well , as indeed I have presented it, with the sort of letter use found in mathematical treatises.

The precise nature and genesis of the surviving Aristotelian texts, as texts, is a much controverted issue. But scholars generally agree that, in one way or another, the original written versions of Aristotelian arguments would have been designed for oral presentation to an audience. And this is just because, in Aristotle's day, writing in general was seen as an aid to oral presentation. This means that in thinking about how Aristotle used diagrams, we need to think about how he used diagrams in oral presentation.

On this issue, Netz offers some valuable comments. He argues that Aristotle most likely would have lectured in conjunction with an array of diagrams that had been previously prepared in some fixed medium, most likely with the diagrams drawn upon board painted with a white background.[129] What really matters here is the fact of prior preparation: the Greeks did not seem to possess any sort of easily rewritable medium, such as a blackboard, that would have been easily visible to more than a handful of

128 As I have described things, it might seem that Greek geometrical arguments draw a conclusion only about one physical diagram. Indeed this is true, but the arguments are usually accompanied by a statement that the whole argument could be repeated with other geometrical objects of the relevant sort.
129 Netz, pp. 15-16.

people. This means that when Aristotle speaks of letters being upon things, we need to imagine Aristotle himself standing next to a board that contains these very letters next to various diagrams.

This image fits well with our discussion of acquaintance. Imagine that you are in the audience when Aristotle (or an assistant) sets up a diagram board that will be employed in the next series of arguments. The different diagrams on the board immediately present themselves as distinct *things*.[130] In short, you are visually acquainted with them. But you do not yet know *what* they are, that is to say, what they are *supposed* to be. You just see letters arrayed near lines.

In fact, even in later mathematical texts, diagrams were routinely presented, *in toto*, as uninterpreted visual objects. Only a reading of the accompanying text would reveal *what* these objects were. But this would only happen progressively, with elements of the diagram being identified and described as they were employed in the reasoning itself.[131] Indeed, in Aristotle as well, we routinely find that elements of the diagram are progressively identified in conjunction with the reasoning about the diagram.[132] But we should not forget that the diagram itself, as a visual object, is salient from the start.

We are now in a position to see what a diagram of ἄπειρα might look like and why. Once we see this we can go on to consider why Aristotle thinks such a diagram can be ambiguous.

---

130 This would *not* necessarily be true if Aristotle were employing the sort of complex diagrams used in mathematical treatises, in which there are many intersecting and overlapping lines and shapes, and which do not immediately visually decompose into parts. But in *Physics* Ζ, Aristotle typically uses diagram constructs involving several simple and distinct parts, often distinct lines with letters. Each of these will have a visual identity.

131 See Netz: "[S]pecifications in Greek mathematics are done, literally, *ambulando*." (pg. 25) More generally, in sections 2.1 and 2.2, Netz emphasizes the interdependence of text and diagram.

132 See, for instance, 233b1-2, from the unquoted part of the argument that we considered above. The identity of the line segment "upon which are Β [and] Ε" is fixed, *in part*, via an inference.

In fact, the word ἄπειρα does not occur in *Physics* Ζ apart from the Racetrack discussion, and we will later see that the relevance of the word to the *logos* derives from the original dialectical version. We will later examine this original use in more detail. But now we want to understand Aristotle's response to the *logos*, and we know that, from one perspective, this response is a standard dialectical disambiguation. But we also know that this disambiguation occurs in the the middle of *Physics* Ζ, a text built around the use of diagram arguments. So we need an interpretation that fits both the context of dialectic and the context of diagram arguments. In fact, we can make that work if we see Aristotle as disambiguating a diagram of ἄπειρα.

Now suppose Aristotle is faced with the word ἄπειρα, and wants to represent the ἄπειρα in a diagram. What will the diagram look like? In fact, it will look pretty much like this:

A　　　　　　B　　　　　　Γ　　　　　　Δ

That is to say, it will be substantively indistinguishable from the majority of the diagrams in *Physics* Ζ, being simply a line with several letters upon it.

Why can we construe this as a diagram of ἄπειρα? This is possible due to the combination of two logically distinct facts. First, we can construe the single diagram line as something ἄπειρος. Second, we can construe the diagram as a plurality.

We will shortly consider these facts in turn. But first consider how they might fit together. If something is ἄπειρος, it is untraversable, that is, untraversable as a whole. If that thing is composed of parts, then that collection of parts, being one and the same as the whole, is also untraversable as a whole. Hence the collection of parts may be termed ἄπειρα. So in this case, ἄπειρος and ἄπειρα refer to the same exact thing, but a thing construed in different ways. Now it is clear that the untraversability of a whole, as a whole, may be logically distinct form the division of the whole into parts. And hence we have the possibility that the word ἄπειρα can refer to a collection finite in number, so long as the division of the whole into parts is independent of the whole's untraversability This turns out to be the case with Aristotle's ἄπειρα. And notice a curious fact: the collection of parts may certainly be finite in number. And why not? The untraversability of the whole is logically distinct from its division into parts.

Now recall the sample diagram above. Why can it be ἄπειρος? Well, a line is the sort of thing that might be ἄπειρος, or untraversable, regardless of the precise meaning of ἄπειρος. That is a basic assumption of *Physics* Z. Depending on what exactly ἄπειρος does mean, the given diagram line might or might not actually be ἄπειρος. But that is no problem. It is part of the basic framework of diagram argument that Aristotle may denominate certain diagrams as things they are not. So the above diagram may *be* ἄπειρος, by supposition.

Now, why can the diagram be a plurality? Well, we can *see* that it is a plurality. While philosophers might debate the ontology of the plurality, this does not matter to our visual system. In the absence of some learned interpretation of what such a diagram is *supposed* to be, we are naturally going to *see* it as a collection of intervals. And recall that Aristotle's audience will always initially lack such an interpretation. They will first see the diagram, and then more detailed interpretation will be provided progressively as the argument unfolds. And the semantics of such arguments always presuppose the presence of the diagram as a real thing.

Now recall our question: what will a diagram of ἄπειρα look like? It will be a line with several letters on it. The line as a whole can “be” ἄπειρος just because Aristotle says it is. But the line is a plurality because we see that it is. There is no inherent connection between these facts.

We now see that in the context of *Physics* Z, it makes sense for Aristotle to represent ἄπειρα via a single line with multiple letters on it. Our next step is to consider why such a diagram is ambiguous.

Disambiguating a Diagram

There is a significant mismatch between the way that a Greek geometrical diagram initially presents itself and the way in which geometers speak about it. You can easily see this by looking at nearly any page of Heath's edition of the Elements. Each diagram immediately appears as a complicate visual object, albeit a static one. But the text speaks as if the diagram is progressively being created.

The same mismatch is present in Aristotle, although it is less obvious, since Aristotle is using diagrams that consist simply of one or several disconnected lines. Still, a diagram would first be visible in it's entirety to Aristotle's audience, and then he would speak *as if* he were progressively creating it. But in a diagram that consists only of disconnected lines, there are only two things Aristotle can do, that is, pretend to do, to diagrams. Taking a certain line as given, he can either take a piece away from it, or what is the same thing, divide it, taking one part from the other), or he can add a piece to it. There are no other options.

In fact, in one sense, each of these “actions” amounts to the same thing. In each case, Aristotle is specifying the identity of an additional letter already present in the diagram.[133] Hence, in one sense, there is only *one* thing you can do to a purely linear diagram: you can "add" letters to it.

Now consider how this meshes with the basic meaning of ἄπειρος, untraversable. Something is untraversable relative to a traverser. (Remember: Hera can traverse the oceans; mortals cannot.) But naturally, this depends on the sort of traversing a given agent might do with a given object. Well, what sort of traversing can Aristotle undertake with a linear diagram? Only one sort: he can "add" letters to it. And so what sort of diagram will be "untraversable"? A diagram to which Aristotle can keep adding letters.

We can now see that every diagram with three or more letters will be ambiguous in the same way. Consider a diagram with only three letters:

A B Γ

If we assume that two of these letters are initially used to establish a determinate length[134], and that the third is introduced via some "action", then there are two basically different ways to interpret the diagram. The argument might start by considering the line from A to Γ, and then either divide it in some ratio at B, or cut off a certain length, either that from A to B or from B to Γ. Alternatively, the argument might start by considering either of the lines from A to B or from B to Γ, and then add the other to it. In the first case, the third letter is specified via a subtraction, and in the second case, via an addition. And of course, the diagram itself does not tell us how the argument will proceed. So the diagram is ambiguous.

Of course, diagrams with even more letters will also be ambiguous, since every

133 Of course, in some cases, multiple letters may be newly specified at once.

134 In some cases, a single letter is used in the initial stipulation. This is common in Euclid when the given diagram line will merely function as an arbitrary measure to be employed in discussing other parts of the diagram. See, for instance *Elements* I.3.

additional letter will be specified by either an addition or a subtraction. Indeed, the possibilities increase exponentially as we consider diagrams with more and more letters. But, in general, this “ambiguity” poses no problem, since the arguer himself will tell us what he intends by each letter.

We can now understand the ambiguity of ἄπειρα. In the context of linear diagram argument, ἄπειρα will refer to a line with several letters to which we may, somehow, keep on adding more letters. But what does this really mean? “Adding a letter” is “really” either adding a line segment or subtracting one. So we might be able to go on adding line segments to those already given. Or we might be able to go on subtracting line segments from those already given.[135]

We can now connect this ambiguity with Aristotle's own verbal disambiguation. On the surface, Aristotle seems to be disambiguating the word ἄπειρος, the basic word of which ἄπειρα is a form. And he tells us that ἄπειρος means one of two things: either ἄπειρος κατὰ διαίρεσιν or ἄπειρος τοῖς ἐσχάτοις. [233a24-5]

Recall the setting of the ambiguity. By supposition, a diagram line is ἄπειρος, untraversable, whatever that means. And this diagram line has multiple letters on it, and so is visibly a plurality of segments. But we should assume that none of the letters demarcating the segments has been given a distinct identity.

If we can use two letters to establish the identity of a determinate segment, and can then go on, without end, adding segments to the segments already given, then the diagram of ἄπειρα will be a diagram of ἄπειρα τοῖς ἐσχάτοις (things unlimited with respect to the ends). The name fits because we are always adding segments (and letters)

135 Or both. But this possibility does not introduce any complications that matter to our immediate discussion.

beyond the ends of the segments (and letters) already given. Of course, we *cannot* actually go on adding segments, due to the physical limits of the diagram board. But it is certainly plausible that Aristotle *speaks* as if we can. The application of the term ἄπειρα τοῖς ἐσχάτοις may be grounded in the real human capacity for relating to an actual diagram, even if this terminology suggests something impossible, as it does.

By contrast, if we can use two letters to establish the identity of a determinate segment, and can then go on, without end, taking segments from the segments already given, or what is the same thing, dividing segments already given, then the diagram of ἄπειρα will be a diagram of ἄπειρα κατὰ διαίρεσιν (things unlimited with respect to division). As with the prior case, we *cannot* actually always go on dividing the diagram, but this does not preclude speaking as if we can.

In each of these two cases, Aristotle is considering a plurality of segments, and considering how we might change the composition of the plurality without thereby changing the identity of the plurality as a plurality. And in fact there are two basically different ways in which we can do this unendingly.

Because the plural terms ἄπειρα τοῖς ἐσχάτοις and ἄπειρα κατὰ διαίρεσιν apply to collections in virtue of the nature of these collections as wholes, in a way that is partially independent of the identity of their members, it is easy to see how the singular terms ἄπειρος τοῖς ἐσχάτοις and ἄπειρος κατὰ διαίρεσιν apply to these very same collections. In fact, a *single* thing, a *whole* collection, is ἄπειρος in a given way just in case the collection may be termed ἄπειρα in that same way.

In effect, the use of the two plural terms thus seems to define the use of the two singular terms. This fact helps to resolve a puzzle, but also helps to create a new one.

Consider the two singular terms. I have translated ἄπειρος τοῖς ἐσχάτοις as "unlimited with respect to the ends". But if I were unconcerned with the nuances of scholarship, I could say that it means, more or less, "infinite in extent". And likewise, I have translated ἄπειρος κατὰ διαίρεσιν as "unlimited with respect to division". But again, scholarship aside, I could say that it means "infinitely divisible". Now in contemporary English, there is simply no single phrase that might ambiguously possess either of these two meanings. And, in fact, the same is largely true of Greek as well. I see no sign that anyone prior to Aristotle every used ἄπειρος to mean ἄπειρος κατὰ διαίρεσιν. And, indeed, Aristotle himself, apart from situations where he is contrasting the two meanings of ἄπειρος, never seems to use ἄπειρος by itself to mean ἄπειρος κατὰ διαίρεσιν. Yet his dialectical *lusis* clearly treats ἄπειρος as lexically ambiguous.

We can explain this disparity if we see Aristotle as treating the singular terms as effectively defined by their plurals. And we have seen how the single word ἄπειρα can be ambiguous insofar as it refers to a diagram that is itself ambiguous. So: because the diagram is ambiguous, the plural is ambiguous, and because the plural is ambiguous, so is the singular.

And yet there is something of a puzzle: on this account, the singular terms seem like they *must* refer to things divided into parts. And yet it seems that maybe something might be "infinite in extent" and yet not composed of parts, and that something might be "infinitely divisible" and yet not divided into parts. In fact, we can temporarily ignore this issue. In a way, it is invisible within the context of diagram argument, which is the context of *Physics* Z. In a diagram argument, if you are going to *say* anything about a line, that line *must* be divided into parts. And this is because *all* conclusions are derived via the "subtracting" and "adding" of line segments. The *only* thing you can do with an "undivided" line is to stipulate it's existence as a determinate line.[136] But if we want a conclusion reached by inference, we need letters, and additions or divisions. So every line, and indeed every continuous magnitude, that is actually *reasoned* about is a collection of segments, since that is what it's diagram is.

Many readers will no doubt recognize that this issue somehow plays a key role in Aristotle's later discussion of the Racetrack, in *Physics* Θ. And so we will indeed discuss it later. But for now we seem to have a plausible understanding of Aristotle's disambiguation, at least in principle. What remains is to fit it back into the framework of the overall dialectical *lusis*, to see how it works.

Disambiguating a Thesis

Recall how Aristotle sets up the Racetrack discussion. He presents two alternate theses, each containing a form of the problematic term ἄπειρα. In setting out his dialectical *lusis*, Aristotle focuses attention on the second of these variants, apparently assuming that his conclusion can just as easily be applied to the first.[137] Recall how we reconstructed this second thesis: οὐκ ἐνδέχεται ἅψασθαι τῶν ἀπείρων καθ' ἕκαστον ἐν πεπερασμένῳ χρόνῳ. (It is not possible to individually touch things unlimited in a limited time.) The putative dialectical questioner wants Aristotle, the putative answerer, to accept this thesis as true. But Aristotle's response is to disambiguate the thesis, allowing that in one way it is true, but in one way it is false. Aristotle thinks that the thesis is ambiguous since the word ἄπειρα is ambiguous, and we have now seen how he

136 In fact, this often does happen in Euclid, when the given line is to be used simply as an arbitrary unit of measure. Again, see *Elements* I.3. But the conclusions of a proof will involve other parts of the diagram, constructions "obtained" by *using* this measure.

137 Again, recall that we will later examine this issue in more detail.

disambiguates the word. We now need to consider how the verbal disambiguation yields two disambiguated theses, or, what is not exactly the same thing, two assertions by Aristotle that reflect these disambiguated theses.

We earlier reconstructed the two disambiguated theses. Recall the first: τῶν κατὰ τὸ ποσὸν ἀπείρων οὐκ ἐνδέχεται ἅψασθαι ἐν πεπερασμένῳ χρόνῳ. (It is not possible to touch things unlimited with respect to quantity in a limited time.) It is not difficult to see what this means, so long as we first, temporarily, dispose of some problems.

The first problem is the word ἅψασθαι (touch). This word sounds out of place in English. However, in Greek, "touch" is routinely used in mathematical contexts, where it means something like "be in contact with".[138] Hence the word makes sense in the rather abstract context of *Physics* Z.2.

The second problem is the phrase τῶν κατὰ τὸ ποσὸν ἀπείρων, that is, a variant of ἄπειρα κατὰ τὸ ποσόν (things unlimited with respect to quantity). And the problem is that given how Aristotle disambiguates the singular ἄπειρος (233a24-6), we might reasonably expect the plural ἄπειρα τοῖς ἐσχάτοις (unlimited with respect to the ends). So: do these plural forms mean the same thing? And why does Aristotle substitute the one for the other?

For Aristotle, the phrase τὸ ποσόν (quantity) encompasses two varieties of quantity: magnitude (that is, geometrical magnitude) and number. The phrase ἄπειρος κατὰ τὸ ποσόν can then refer either to something unlimited in magnitude or to something unlimited in number.

We have already considered the phrase ἄπειρα τοῖς ἐσχάτοις. We saw that it

---

138 LSJ ἅπτω, A.III.8

would apply to a diagram in which we can always go on adding segments outside the determinate portion of the diagram. So we can endlessly add segments of *any* given determinate length, which means that these ἄπειρα τοῖς ἐσχάτοις will constitute a collection unlimited in magnitude, that is unlimited in measure, or length. And since there is no end to the number of these segments we may add, these ἄπειρα are also unlimited in number. Thus, on either interpretation of ἄπειρα κατὰ τὸ ποσόν, the ἄπειρα τοῖς ἐσχάτοις meet the definition.

Recall that we are trying to understand Aristotle's formulation of the first disambiguated thesis, which he presents at 233a26-27 and which we rendered: τῶν κατὰ τὸ ποσὸν ἀπείρων οὐκ ἐνδέχεται ἅψασθαι ἐν πεπερασμένῳ χρόνῳ. (It is not possible to touch things unlimited with respect to quantity in a limited time.) In the prior sentence, 233a24-6, Aristotle has made clear that he is doing a dialectical disambiguation. So when we see the unexpected form of ἄπειρα κατὰ τὸ ποσόν in the thesis disambiguation, we can take for granted that the referent of this new phrase, *in this case*, is also a collection of ἄπειρα τοῖς ἐσχάτοις . This is possible even if the terms do not *mean* the same thing, as in fact they do not.[139]

But why does Aristotle make the switch? Because the new term best conveys the plausibility of what Aristotle is claiming. Recall the claim: τῶν κατὰ τὸ ποσὸν ἀπείρων οὐκ ἐνδέχεται ἅψασθαι ἐν πεπερασμένῳ χρόνῳ. I have thus far offered a literalistic translation: It is not possible to touch things unlimited with respect to quantity in a limited time. But what this really means is: It is not possible to touch things unlimited in

---

139 Since quantity encompasses both magnitude and number, ἄπειρα κατὰ τὸ ποσόν can, in principle, refer to collections unlimited in number, yet which are not collections of magnitudes. By contrast, ἄπειρα τοῖς ἐσχάτοις clearly applies to collections of magnitudes.

number in a limited time. And this seems like an eminently plausible claim, especially if we recall the καθ' ἕκαστον (individually) that accompanied the original ambiguous thesis: the “touching” is supposed to take place sequentially. Aristotle is allowing that to touch, one after the one, an unlimited number of things would take forever.

This seems plausible enough, and perhaps even obvious. And in the context of dialectic, seemingly obvious is often good enough. Later, in *Physics* Θ.8, we will find that Aristotle reevaluates this claim. But for now, we may regard the first part of the disambiguation as a tactical retreat on the part of Aristotle. The dialectical questioner seeks an admission that: It is not possible to touch things unlimited. And Aristotle allows: Yes, you are right, it is not possible, in one way. But, he adds, in another way, it *is* possible. So the second part of the disambiguation is the crucial part.

Recall our reconstruction of the second disambiguated thesis: τῶν κατὰ διαίρεσιν ἀπείρων ἐνδέχεται ἅψασθαι ἐν πεπερασμένῳ χρόνῳ. (It is possible to touch things unlimited with respect to division in a limited time.) So: What does this mean? And why does Aristotle think it is true?

We can easily see what it means. We have seen how the plural ἄπειρα κατὰ διαίρεσιν is coreferential with the singular ἄπειρος κατὰ διαίρεσιν, so long as the latter term refers to something with parts. In fact, both terms apply to any determinate magnitude with parts. That is to say, both terms apply to any magnitude that is limited in extent, limited “with respect to the ends”, and has parts. And recall that diagram argument involves the tacit assumption that every magnitude has parts. So both terms, plural and singular, really apply to any limited magnitude. So what Aristotle is really saying is this: It is possible to touch the parts of a limited magnitude in a limited time.

We now understand what Aristotle is really saying in presenting the disambiguated theses. First he tells us: It is not possible to touch things unlimited in number in a limited time. And then he tells us: It is possible to touch the parts of a limited magnitude in a limited time.

The crucial statement is the second one, since it prevents the dialectical questioner from getting what he wants. To see why Aristotle affirms it, we need to step back and consider the overall structure of the Racetrack discussion, and the way in which it fits into the immediate context of *Physics* Z.2.

The Dialectical *Lusis* as a Part of *Physics* Z.2

Up to now, I have emphasize the ways in which the Racetrack discussion is very much a standard dialectical disambiguation, of the sort commonly employed in oral debate. But now we can consider one way in which it differs from any interactive disambiguations.

Recall that in drawing a distinction, the goal of a dialectical questioner is not merely to expose the question as being ambiguous. Rather, the goal is to prevent the questioner from employing a falsehood as a premise in his argument, thus trapping the answerer. Recall how Socrates in the *Euthydemus* (295c4-6) explained the danger of ambiguity: the questioner hopes the answerer will affirm an ambiguous claim after considering one, true, interpretation. But the answerer will then employ a different, false, interpretation in his *sullogismos*.

The goal of the answerer is to prevent the questioner from "securing" the falsehood for use in his argument, that is to say, from gaining the apparent acquiescence of the answerer to the falsehood. And so answerer draws a distinction. But remember that this has two parts: the answerer first exposes an ambiguity, and then establishes that on one interpretation, the ambiguous claim is indeed false.

With many *logoi*, drawing the distinction will be the hard work. This is because the fact that the false disambiguated thesis is indeed false will often be obvious. This happens quite often when the questioner is consciously employing verbal trickery to gain a dialectical victory. He employs an ambiguous question since this is the only way to gain the affirmation of a thesis that is obviously false.[140] But once the answerer draws a distinction and then denies the obvious falsehood, the questioner must accept the denial.

By contrast, with the Racetrack, Aristotle does not take the falsehood of the false thesis to be obvious. Instead, he must establish it by argument. In fact, the overall structure of the whole Racetrack discussion is determined by the need to establish the falsehood of the false thesis.

Recall again the Racetrack discussion, this time in conjunction with the argument immediately prior:

> ἔτι δὲ καὶ ἐκ τῶν εἰώθότων λόγων λέγσεθαι φανερὸν ὡς εἴπερ ὁ χρόνος ἐστὶ συνεχής, ὅτι καὶ τὸ μέγεθος, εἴπερ ἐν τῷ ἡμίσει χρόνῳ ἥμισυ διέρχεται καὶ ἁπλῶς ἐν τῷ ἐλάττονι ἔλαττον· αἱ γὰρ αὐταὶ διαιρέσεις ἔσονται τοῦ χρόνου καὶ τοῦ μεγέθους. καὶ εἰ ὁποτερονοῦν ἄπειρον, καὶ θάτερον, καὶ ὡς θάτερον, καὶ θάτερον, οἷον εἰ μὲν τοῖς ἐσχάτοις ἄπειρος ὁ χρόνος, καὶ τὸ μῆκος τοῖς ἐσχάτοις, εἰ δὲ τῇ διαιρέσει, τῇ διαιρέσει καὶ τὸ μῆκος, εἰ δὲ ἀμφοῖν, ἀμφοῖν καὶ τὸ μέγεθος.
>
> διὸ καὶ ὁ Ζήνωνος λόγος ψεῦδος λαμβάνει τὸ μὴ ἐνδέχεσθαι τὰ ἄπειρα διελθεῖν ἢ ἅψασθαι τῶν ἀπείρων καθ' ἕκαστον ἐν πεπερασμένῳ χρόνῳ. διχῶς γὰρ λέγεται καὶ τὸ μῆκος καὶ ὁ χρόνος ἄπειρον, καὶ ὅλως πᾶν τὸ συνεχές, ἤτοι κατὰ διαίρεσιν ἢ τοῖς ἐσχάτοις. τῶν μὲν οὖν κατὰ τὸ ποσὸν ἀπείρων οὐκ ἐνδέχεται ἅψασθαι ἐν πεπερασμένῳ χρόνῳ, τῶν δὲ κατὰ διαίρεσιν ἐνδέχεται· καὶ γὰρ αὐτὸς ὁ χρόνος οὕτως ἄπειρος. ὥστε ἐν τῷ ἀπείρῳ καὶ οὐκ ἐν τῷ πεπερασμένῳ συμβαίνει διιέναι τὸ ἄπειρον, καὶ ἅπτεσθαι τῶν ἀπείρων τοῖς ἀπείροις, οὐ τοῖς πεπερασμένοις.

140 Examples abound. See *S.E.* 4, 17, 19, or 33, or nearly anywhere in the *Euthydemus*.

> And moreover, it is clearly stated by familiar arguments that if the time is continuous, then also the magnitude [is continuous], if in the half time one goes through a half [magnitude], and quite simply if in a lesser time [one goes through] a lesser [magnitude]. For there will be the same divisions [i.e. divided segments] of the time and of the magnitude. And if one for the two is unlimited, so is the other, and in whatever way the one [is unlimited], so also is the other, so for instance, if the time is unlimited with respect to the ends, so also is the magnitude, but if [the time is unlimited] with respect to division, so also is the magnitude, but if [the time is unlimited] with respect to both, then also the magnitude is [unlimited] with respect to both.
>
> On account of this the *logos* of Zeno takes as granted a falsehood, that it is not possible to go through things unlimited or to individually touch things unlimited in a limited time. For in two ways both length and time are said to be unlimited, and in general everything continuous, either with respect to division or with respect to it's ends. Certainly it is not possible to touch things unlimited (ἄπειρα) with respect to quantity in a limited time, but it is possible [to touch] those [unlimited] with respect to division. For the time itself is unlimited in this way. So it follows that in the unlimited [time] and not in the limited [time] [one] goes through the unlimited [magnitude], and [one] touches things unlimited with things unlimited, not things limited. [233a13-31]

In the passage that precedes the quotation, Aristotle has offered a series of diagram arguments about motions, arguments that represent the magnitude traversed, and the time, of a given motion as distinct diagram lines. And the arguments hinges on the idea that, given certain facts about the motions themselves, the magnitudes of the motions allow us to deduce things about the times, and vice versa. This theme continues in the first quoted paragraph: Given a determinate motion, Aristotle is telling us, the magnitude of that motion and the time of that motion will be unlimited, if unlimited at all[141], in precisely the same ways.

We next come to the start of the Racetrack discussion itself: “On account of this the *logos* of Zeno takes as granted a falsehood... .” [233a21-2] And Aristotle proceeds to report the alternate ambiguous theses, in their ambiguous form. We earlier determined

141 In Aristotle's view, the magnitude and the time of a given motion will *always* be unlimited with respect to division. But this is not an assumption of the given argument.

that the "falsehood" is the falsehood of the false interpretation. We can now see the significance of the διὸ (on account of). Aristotle is appealing to the prior argument to justify his claim that "the *logos* of Zeno" involves a falsehood. That is to say, the prior argument serves to justify the claim that the false disambiguation is indeed false.

Aristotle proceeds to draw the distinction. And so he affirms that it is possible to touch things unlimited with respect to division in a limited time. And this is the same as saying that is is *false* that it is *not* possible to do this. So Aristotle is asserting the falsehood of the thesis that he thinks is crucial for the questioner.

Now just above, we saw that Aristotle seemed to appeal to the whole prior argument in justifying the claim of falsehood. And now having done the disambiguation, he returns to the justification. To understand this justification, we need to see Aristotle as taking the result of his dialectical disambiguation, and interpreting it in the light of the framework he has already established here in *Physics* Z.2.

The claim to be justified is: It is possible to touch things unlimited with respect to division (ἄπειρα κατὰ διαίρεσιν) in a limited time. And the justification begins: "For the time itself is unlimited in this way." [233a28] Now recall that a collection of ἄπειρα κατὰ διαίρεσιν is a single thing that is ἄπειρος κατὰ διαίρεσιν.

Recall again the argument about how a magnitude and a time will be unlimited in the same ways. [233a13-21] That argument takes as given the existence of a certain *motion*, and then compares the magnitude of that motion with the time of the that motion. *This* magnitude and *this* time will be unlimited in the same ways.

If Aristotle is to apply this argument to the Racetrack, then he must fir the crucial "falsehood" into this framework. And so he does. The falsehood involves touching ἄπειρα κατὰ διαίρεσιν. But, in Aristotle's view, these form a thing ἄπειρος κατὰ διαίρεσιν. And now he goes a step further, construing this thing that is ἄπειρος (κατὰ διαίρεσιν) as the magnitude of some motion. Well if that is true, then the given motion will also have a time. And following the earlier argument, this time will be ἄπειρος in whatever way the magnitude is, in this case ἄπειρος κατὰ διαίρεσιν.

This does not yet complete the justification. But right now we can see something quite interesting: Aristotle seems to be importing a good deal of ontology into the *logos*. He reads the *logos* as involving a certain thing, a motion, and it is this thing, the motion that ties together a certain magnitude and a certain time. But notice that the ambiguous thesis, as Aristotle reports it, mentions no thing like a motion, and does not overtly mention any thing like a magnitude. It does mention ἄπειρος, but it does not say *what* these are. It does mention a time, but we will later find that this is an insertion by Aristotle. In fact we will return to this issue much later. For now, we may simply observe the facts.

At this point, Aristotle has introduced us to the manner in which he will fit the “falsehood” into the framework of the prior time and magnitude argument. What remains is to show that the falsehood is false. And this he does, but to understand the line in which this happens, 233a28-31, we need to look carefully at how the language of the line is carefully constructed to merge the language of the original ambiguous Racetrack theses with the framework of the time and magnitude argument. In Aristotle's view, once we merge these frameworks, the falsehood of the falsehood, and the truth of it's negation, is evident. So let us look at the language.

First, notice that Aristotle four times uses a form of ἄπειρος, and twice a form of it's contrary, πεπερασμένος. Now ἄπειρος, of course, is ambiguous, and logically, this means that it's contrary πεπερασμένος is ambiguous as well. But Aristotle does not tell us which reading of these terms he intends.

In fact, *this* ambiguity is intentional, and fits with Aristotle's way of speaking in the earlier argument, where he wrote: "And if one of the two [i.e. the time or the magnitude] is unlimited (ἄπειρος), so is the other... ." [233a17] This ambiguity is not pernicious, since the entire discussion makes clear that two readings are possible. And so it is again. We can read the entire line, 233a28-31, as if ἄπειρος means ἄπειρος τοῖς ἐσχάτοις, and πεπερασμένος means πεπερασμένος τοῖς ἐσχάτοις. Or we can read the entire line as if ἄπειρος means ἄπειρος κατὰ διαίρεσιν and πεπερασμένος means πεπερασμένος κατὰ διαίρεσιν. And Aristotle intends that both readings are true.

Of course, the terms πεπερασμένος κατὰ διαίρεσιν (limited with respect to division) and πεπερασμένα κατὰ διαίρεσιν (limiteds with respect to division), seem rather awkward. But their meaning, in principles, is clear. They apply to the same type of things as the corresponding forms of ἄπειρος, that is, to linear diagrams, especially linear diagrams with parts. And being contraries, the forms of πεπερασμένος apply in just the cases when the forms of ἄπειρος do not apply. Now, of course, there are *no such cases*, since every magnitude is unlimited with respect to division. But that is no matter: these πεπερασμένος disambiguations still *mean* something. But they fail to apply to anything that actually exists.[142] As to why Aristotle employs this empty usage, the answer partially

142 So far as I know, no one, Aristotle included, apart from the present context, ever uses πεπερασμένος to mean πεπερασμένος κατὰ διαίρεσιν . In fact, the word is actually the perfect passive participle of περαίνω (finish, complete), but it comes to function somewhat independently as an adjective, and the accepted contrary of ἄπειρος. So the nuances of its meaning will fluctuate along with the nuances of ἄπειρος. But, of course, just as the normal use of ἄπειρος will be closer to ἄπειρος τοῖς ἐσχάτοις than to ἄπειρος κατὰ διαίρεσιν, so too πεπερασμένος will typically negate the former idea, not the latter.

lies in the fact that the word πεπερασμένος occurs in the original ambiguous thesis as Aristotle reports it. But we may defer the details momentarily.

Consider now the two parallel portions of the justification line. The first part involves the verb διιέναι (go through), and singular forms of ἄπειρος and πεπερασμένος. The second part involves the verb ἅπτεσθαι (touch), and the corresponding plural forms. These facts enable us to read the justification line as a highly constructed statement, meant to fit the Racetrack discussions into the framework of the prior argument.

Now the διιέναι (go through) is very similar to the verb διελθεῖν (go through), which is found in the first variant of the ambiguous thesis. And the verb ἅπτεσθαι (touch) occurs in the second variant of the thesis. In the justification line, Aristotle gives a highly abstract reading of these words, one that fits well with the time and magnitude argument. Recall that this argument takes as given a motion, and presumes that it thereby has a determinate magnitude and time. And this magnitude and time will be represented by linear diagrams, diagrams typically composed of parts. And so these diagrams may be construed as single things or as collections of parts. The justification line thus makes sense if we see Aristotle as thinking that what a given motion does is to "go through" its magnitude, considered as a single thing. But this "going through" is one and the same thing, indeed the same motion, as "touching" the part of this magnitude, that is, each member of the magnitude considered as a plurality. In short, Aristotle is now using words similar or identical to those in the ambiguous thesis, but making it clear that he is talking about one and the same scenario as in the parallelism argument.

We can now understand how Aristotle construes the problem with the original ambiguous theses. Although the two variants involve ἄπειρα, Aristotle takes these ἄπειρα to constitute a single ἄπειρος thing, that is a single unlimited (in whatever way) magnitude. And he sees this ἄπειρος magnitude as the magnitude of some motion that has a πεπερασμένος, limited (in whatever way), time. And so Aristotle sees the dialectical questioner as gaining assent to the crucial thesis via the seeming incompatibility of an ἄπειρος magnitude and a πεπερασμένος time.

Aristotle's solution is that the two terms are, in principle, both ambiguous, and that they are incompatible only if disambiguated in the same way, so as to make them contraries. But they are quite compatible if they are not contraries.

Now recall how Aristotle denies the “falsehood”: It is possible to go through things unlimited with respect to division in a limited time. In justifying this denial, Aristotle simply appeals to his earlier supposition that a given motion will have a certain magnitude and a certain time, and that these will be ἄπειρος, unlimited, in exactly the same ways. Once this is restated in the language of the *logos*, the falsehood of the “falsehood” is evident.

In a logical sense, the key unargued for premise in Aristotle's justification is the supposition that it *is* possible for there to be a motion through a magnitude that is unlimited with respect to division, and limited with respect to the ends. Aristotle assumes this, and then it follows that such a motion will also be through a time unlimited with respect to division. and limited with respect to the ends.

Of course, in a way, Aristotle does have a justification for the aforementioned premise, insofar as he assumes that motions do exist and that motions are through magnitudes. Moreover, he argues in *Physics* Z.1 that every magnitude is continuous, which amounts to being unlimited with respect to division. But Aristotle does not present theses arguments as a direct response to the Racetrack *logos*.

In the broadest sense, then Aristotle's resolution relies on the supposition that motions exist, and are through magnitudes. But what Aristotle means by these things, and whether he is right to believe them, are issues that would take us far beyond the Racetrack. In fact, as far as the Racetrack itself goes, we have for now done all we can do with *Physics* Z.2.

Observations and Conclusions

We have now spent a good deal of time examining the Racetrack discussion in *Physics* Z.2. We found that Aristotle presents two alternate premises from the dialectical *logos*. He thinks each is ambiguous since it contains a form of ἄπειρος. So Aristotle follows standard dialectical procedure, and draws a distinction, disambiguating the word. But the disambiguation of ἄπειρος is really a disambiguation of a certain kind of diagram, a lettered line divided into parts. Yet this disambiguation is not enough. Aristotle needs to show that one of the disambiguated theses is false. And he does this by rewording the conclusion to an argument he has just given, the argument that the time and the magnitude of a motion will be unlimited in the same ways.

To understand the discussion, we needed to situate it within three different contexts. First is the context of dialectic. Next is the general context of diagram argument. And finally, there is the immediate textual context of *Physics* Z.2.

What has proved most confusing is the combination of the first two of these contexts. The Racetrack discussion results, in part, from the mixture of two ancient intellectual practices, dialectic and diagram argument. Of course, no one today employs either of these practices, at least not in anything like the way anyone did in Aristotle's day. But to understand the Racetrack discussion, we need to understand both, and only then can we see how the discussion fits into Aristotle's text.

Up to now, I have not much discussed the views of other scholars concerning the Racetrack analysis. And the reason is that no one seems to have recognized that Aristotle was doing a dialectical disambiguation, and that this disambiguation is really the disambiguation of a diagram. But once we fill in the background about dialectic and diagrams, the structure of the entire Racetrack discussion seems to fall into place. So my ultimate disagreement with other scholars lies in the fact that they have not fully examined the intellectual background that is need to grasp the discussion. But now that we see this, we can consider how my conclusions differ from some of the conclusions offered by others.

One common claim is that Zeno assumes the infinite divisibility of magnitude while neglecting the corresponding infinite divisibility of time. Some commentators, Simplicius among them, see Aristotle as making this very claim against Zeno himself.[143] But we can immediately see one problem with this interpretation: Aristotle is not making *any* charge against *Zeno*. Instead, Aristotle is analyzing a dialectical *logos*. In a *logos*, the only person who makes any assumptions, or commits to any premises, is the *answerer*, even if it is the questioner who *provides* the premises via his questions. So if Aristotle were lodging the given charge, he would need to be lodging it against the answerer, not against Zeno. But Aristotle is not doing even that.

143 Simplicius writes: "Zeno took magnitude as actually divided into an infinite [number of things], but he did not further [take] time [that way], even though it is similar to magnitude." [948,4-5] (trans. Konston) Ross concurs: "Zeno in fact, according to Aristotle, is failing to note the complete parallelism [between time and magnitude] ... and as soon as that is pointed out the difficulty disappears." (pg 72-3).

Certainly the infinite divisibility of time *is* central to Aristotle's dialectical *lusis*, but not in the way described by Simplicius. Recall that the overall lusis has two parts. Aristotle first charges that the dialectical questioner employs an ambiguous thesis, and so Aristotle disambiguates. But then he argues that one of the disambiguated theses is false. Aristotle's claim that the time and the magnitude of the motion are unlimited in the same ways serves to justify this claim of falsehood. But the error of the dialectical answerer lies not in failing to recognize the falsehood, it lies, rather, in failing to recognize the initial ambiguity. Only when the ambiguity has been disambiguated does the "false" thesis even become available to be false.

As for the questioner, we have already seen that in many cases, a dialectical questioner employs an ambiguity precisely because he knows that the thesis he needs for his argument is false (or appears false). So he asks a question that blurs together an apparent falsehood with an apparent truth. Does Aristotle think that Zeno is aware of the given falsehood as false, or that any dialectician employing the Racetrack *logos* must be aware of it? In fact, at this point, we simply cannot tell: the actual *beliefs* of a questioner are not necessarily relevant to the resolution of a *logos*. What matters is whether the questioner can trap the answerer in apparent absurdity. Aristotle's *lusis* aims to avoid this. But at this point we are not licensed to draw any conclusion about what Aristotle thinks Zeno, or any questioner, actually believes.

Richard McKirahan provides a clear statement of another interesting view: "[Aristotle] accuses Zeno of confusing two distinct notions of infinity: what is infinite in respect of its extremities... and what is infinite in respect of it's divisibility... ."[144] Now

144 McKirahan (1994), pg. 188.

one problem here is that McKirahan, just as the other scholars we have considered, thinks that Aristotle is critiquing Zeno himself. But Aristotle simply does not presume that Zeno, or any dialectical questioner, is actually giving an argument. And hence Aristotle does not criticize such an argument. Aristotle may be criticizing the *practices* of the questioner, but if anyone goes wrong in making an argument, this can only be the answerer. But is McKirahan's assessment plausible if we try to apply it to the answerer? It depends on what McKirahan means. If the answerer, or anyone, can confuse two *concepts* by thinking that a *word* meaning one thing actually means another thing, then Aristotle certainly does charge that the dialectical questioner wants the answerer to confuse two concepts. But inasmuch as this sort of confusion involves a particular *word*, it is language dependent, and no non-Greek speaker can make the same mistake as Aristotle actually attributes to the answerer.

Yet this does not seem to be the sort of confusion that McKirahan has in mind. His reader is likely to suppose that Aristotle is charging Zeno with a purely intellectual confusion, a confusion of two concepts themselves, a sort of confusion that perhaps any human could make, or even any rational being. Now there may well be some way to work this into a coherent notion, and even perhaps to use it in illuminating the Racetrack paradox. But our examination of Aristotle's *lusis* has revealed no role for such ideas.

In fact, I suspect that McKirahan is exporting back to Aristotle a confusion that is itself common among historians of philosophy. Historians quite commonly charge that some intellectual figure has “confused” two concepts. But I suspect that these historians are confusing confusion with failure to distinguish. And the difference is that someone who confuses two concepts would seem to already possess the two concepts in some way, whereas someone who fails to distinguish them need not possess either.[145] And this distinction is crucial if we want to understand intellectual history.

On a final issue, notice that in interpreting the Racetrack discussion, I have nowhere found Aristotle referring to anything like indivisible geometrical *points*, or to any sort of *limits* of magnitudes or motions. But many scholars do, interpreting at least seem of the references to ἄπειρα as references to unlimited collections of points.[146] By contrast, I always interpret ἄπειρα as referring to a collection of continuous segments that make up a single continuous and ἄπειρος whole. And in fact this close connection is essential if we are to understand how Aristotle's overt disambiguation of the singular ἄπειρος is meant to apply to the plural forms that actually occur in the ambiguous theses as Aristotle reports them. So the problem with thinking that the ἄπειρα are geometrical points is that I see no way to mesh this with the overall dialectical structure of Aristotle's discussion.

Curiously enough, the many scholars who see the ἄπειρα as points do have a reason, indeed a very good reason, for doing so. In fact, in the original dialectical *logos*, the ἄπειρα *were* points, or at least point-like. They were *not* continuous linear segments. And in fact, in providing his *lusis*, Aristotle himself reinterprets the *logos* for his own purposes. But these are things that remain for us to discover.

At this point, what have we actually learned about the original dialectical *logos*? From *Physics* Z.2, we may surmise that the *logos* involved one of two variant premises. First: it is not possible to go through things unlimited in a limited time. Second: it is not possible to individually touch things unlimited in a limited time.

145 For discussion of related issues, see *Theaetetus* 191d-200d.
146 See Barnes (1982), pg. 262; Faris, pp. 8-9; Lee, pg. 43, pg. 67; and Ross, pg. 405.

From *Physics* Z.2, we learn *nothing* else about the oral *logos*. Every other piece of Aristotle's discussion is thoroughly explicable as a response to the given lines, and there is no sign that he is reacting to other features of the *logos*. Our working hypothesis should then be that Aristotle is familiar with oral versions of the *logos* that involve each of the variant premises. In fact, this hypothesis will need to be revised, but so far, so good.

Now recall *Physics* Z.9. There we discovered the premise that "the moving thing must first reach the half before the end." And we discovered that the conclusion of the *logos* was "about not moving." [239b12-3]

In our quest for the *logos*, we have now discovered one complete premise, two variants of a seemingly different premise, and a topic for the conclusion. This is not a complete *logos*. So we must turn to *Physics* Θ.8, where Aristotle discusses the Racetrack once again.

Chapter 4
The Original Racetrack

A Report of the *Logos*

At this point, we have determined that Zeno's Racetrack was a dialectical *logos*. This means that in reconstructing the *logos*, we need to reconstruct a conversation between two people. We have also examined Aristotle's references to the Racetrack in *Physics* Z.2 and Z.9. We found additional confirmation that the Racetrack was originally a dialectical *logos*, and we learned a good deal about Aristotle's comments on the *logos*. But we did not find enough information to reconstruct the *logos* itself. Now we turn to Aristotle's final discussion of the Racetrack, in *Physics* Θ.8, where he turns out to give us much of the evidence that we need.

Here is the key text:

> τὸν αὐτὸν δὲ τρόπον ἀπαντητέον καὶ πρὸς τοὺς ἐρωτῶντας τὸν Ζήνωνος λόγον[147], εἰ ἀεὶ τὸ ἥμισυ διιέναι δεῖ, ταῦτα δ'ἄπειρα, τὰ δ' ἄπειρα ἀδύνατον διεξελθεῖν, ἢ ὡς τὸν αὐτὸν τοῦτον λόγον τινὲς ἄλλως ἐρωτῶσιν, ἀξιοῦντες ἅμα τῷ κινεῖσθαι τὴν ἡμίσειαν πρότερον ἀριθμεῖν καθ' ἕκαστον γιγνόμενον τὸ ἥμισυ, ὥστε δειλθόντος τὴν ὅλην ἄπειρον συμβαίνει ἠριθμηκέναι ἀριθμόν· τοῦτο δ' ὁμολογουμένως ἐστὶν ἀδύνατον.

And in the same way one must reply to those [dialecticians] asking the *logos* of

147 Following λόγον, all manuscripts contain the words καὶ ἀξιοῦντας (and requiring). I agree with Ross in omitting them. Unlike Ross, I do not see any sign that Simplicius reads a text that omits them. We simply cannot tell what Simplicius is reading, since his quotation of this passage ends at λόγον, leaving it an open question whether the next word he read is καὶ or εἰ. (1288.30-2) But it is still reasonable to omit the words as they add nothing to the sense of the text, and they mangle the grammar, producing the nonsense phrase ἀξιοῦντας εἰ (requiring whether). Their addition is easily explained as the result of some editor or scribe intentionally or unintentionally trying to create a parallel with the second part of Aristotle's report, which does involve ἀξιοῦντες. (263a7) This omission seems totally innocuous. It might seem that the omission is not at all innocuous, inasmuch as the inclusion of "and requiring" tends to undermine the image of dialecticians *asking questions*, conjuring instead some arguer demanding acceptance of his own premises, possibly in some non-dialectical manner. But even if present, "and requiring" can scarcely *eliminate* the reference to questioning. Aristotle undeniably speaks of "those asking...whether" and "some others who differently ask." This language seems largely out of place in *Physics* Θ.8 and any analysis of the Racetrack passage needs to take account of it. This is true regardless of whether καὶ ἀξιοῦντας is authentically Aristotelian.

> Zeno, [who ask] whether it is always necessary to pass through the half[148], but these [halves] are unlimited, and it is impossible to pass through things unlimited, or as some others differently ask this same *logos*, requiring that together with moving the half-[motion], there is earlier counting individually the half-[way-point] coming to be, so that with the whole [motion] gone through, the result is having counted unlimitedly. But this is admittedly impossible. (263a3-11)

Aristotle's discussion of the Racetrack continues beyond the quoted passage, and he offers a novel analysis of the *logos* that is meant to supersede the resolution that he offered in *Physics* Z.2. But for now that can wait, and we will focus here on the given passage.

The significance of the passage is immediately evident. In reconstructing the *logos*, we are seeking to reconstruct a conversation, and here Aristotle seems to be reporting conversations he has personally heard, or even participated in. We can see this from his language. He mentions "those asking" the *logos* and "some others" who ask it differently. (263a4-5,7) Aristotle is directly recalling his personal experiences of the Racetrack *logos*. The passage thus stands in clear contrast to *Physics* Z.2 and Z.9, where Aristotle scarcely purports to recount the *logos* as he actually heard it used in dialectic.[149]

If we read beyond the quotation, we see that Aristotle overtly aims to provide an

---

148 In principle, this neuter adjectival form might be used generically, or it might have some intended referent. And in either case, it might refer to something like a halfway-point, or alternatively, to some sort of half divided by a half-way point.
In Chapter 7, we will find that Aristotle might have reasons for leaving the word ambiguous, and so in my translation I leave it that way as well, although we will also find that the original *logos* definitely involves halfway-points. Chapter 7 will also consider my unambiguous renderings of the two ensuing forms of ἥμισυς.
In English, "halfway-point" would normally be unhyphenated: *halfway point*. I will be hyphenating it, both to preserve some of the verbal resemblance to phrases like half-motion and half-magnitude, and also to subdue some of the potentially anachronistic connotations of the modern word "point".

149 In principle, in *Physics* Θ.8, Aristotle might be recounting merely the reports of others who have actually heard the *logos* used. But given Aristotle's obvious extensive practical experience with dialectic, and the number of times that he mentions the Racetrack in his writings, especially in dialectical contexts, it seems highly unlikely that he never either heard or participated in a performance of the *logos*. And so given that he is now clearly reporting on the oral performance of the *logos*, we can be reasonably confident that he is reporting on his own experiences of such performance.

authentic report of his own experiences. Immediately after recounting the *logos*, Aristotle continues: "We resolved [this *logos*] in the earlier discussions concerning motion... ." (263a11-2) This is a reference to his analysis in *Physics* Z.2, which he now briefly recapitulates. Then he adds: "But this resolution (*lusis*) holds up satisfactorily against the questioner ... but regarding the fact and the truth [it does] not [hold up] satisfactorily." (263a15-6, 17-8) He then proceeds to offer a new resolution. Roughly speaking, Aristotle claims that his original resolution is sufficient *given the way the questioner actually asks the logos*. But he grants that it somehow misses the deeper issue raised by the *logos*, so that a new resolution is needed. We will examine the details later. For now we need simply note that when recounting the *logos* in *Physics* Θ.8, Aristotle will be especially keen to report the *logos* as it was actually given, in order to best display the features that supposedly render his initial analysis suitable.[150]

We thus see that Aristotle is reporting the *logos* as he has heard it in oral dialectic, and he has a special motivation to get the words right. So the Θ.8 passage is our best evidence as we seek to reconstruct the dialectical Racetrack. In fact, we will find that as an account of the original *logos*, it is not entirely pellucid or complete, so we will need to analyze it carefully and supplement it with evidence from elsewhere. But it is nonetheless the only sound starting point for a solid reconstruction of the *logos*.

Ironically, the very features that make this passage so valuable have often led to it's neglect by scholars.[151] It is easy to see why. Scholars have typically assumed that

150 Incidentally, we see here yet more evidence that the Racetrack actually *was* a dialectical *logos*. Aristotle has a special need to report the Racetrack accurately. Does he appeal to a quote from a written text? No, he recounts the words of the dialecticians that he and likely many of his audience had actually heard in debate. Even if Aristotle possesses some written version of the *logos*, as the numbering of the motion *logoi* in *Physics* Z.9 might suggest, he does not regard it as "the real thing". The real Racetrack is the *logos* of oral debate.

151 Kirk, Raven and Schofield (1983) do not print it in their collection of Zeno texts. Barnes (1982) does

Aristotle possessed some text, written by Zeno, which contained the motion *logoi*. But they rightly realize that in *Physics* Θ.8, Aristotle reports the *logos* as given by questioners of his own day, long after Zeno's death, and not by Zeno himself. So the Θ.8 report appears to be offering mere second-hand information about Zeno. This supposedly contrasts with *Physics* Z.2 and Z.9, where Aristotle might seem to write with a Zeno scroll in hand. *Physics* Z.2 and Z.9 thus appear to be the key texts for the reconstruction of the argument. But in fact we have no sign that Zeno *wrote* the Racetrack at all.[152] Instead he created an oral *logos*, and in *Physics* Θ.8 Aristotle reports this very *logos* as he has heard it. While the Z.2 and Z.9 texts will turn out to be valuable, we must fit them into the basic framework provided by the Θ.8 passage, our best guide to the dialectical Racetrack.

Two Versions of the Racetrack

If we now take a close look at the quoted passage, we can see that Aristotle recalls two different versions of the *logos*. He first mentions "those [dialecticians] asking the *logos* of Zeno." (263a4-5) Then he adds that "some others differently ask this same *logos*" (263a7) He seems to regard the first version as the standard version, since he initially gives no sign that it is one of two. Then he describes the second as a *different* way of doing the *same* thing which *some* use. So in Aristotle's mind the second version somehow has a secondary status.[153]

---

not use it in his reconstruction of the argument. (pp. 161-2) Recently both Hasper (2003) and McKirahan (2002) have given it more attention, but neither realizes that the Racetrack was originally a dialectical *logos*, and so neither grasps the importance of the fact that the Θ.8 passage is a direct report of the *logos* as Aristotle *heard* it.

152 And we have no sign that he didn't. In fact, the role of writing in Zeno's life raises many interesting and subtle questions. But we do not need to answer them in order to learn a great deal from Aristotle about the oral Racetrack *logos*.

153 On the surface, the existence of two versions might seem unsurprising, especially given that the Racetrack is a dialectical *logos*. For every rendition of a *logos* produces a new version. Even when a

But what does this mean? In what way is the second version secondary? We might suppose that Aristotle thinks the first version is somehow logically superior or more worthy of discussion. But why bother reporting two different versions if one is clearly better than the other? In fact, Aristotle does not attribute any sort of logical primacy to the first version, as we can see by considering Aristotle's two analyses of the *logos*, in *Physics* Z.2 and Θ.8. On both occasions, he considers both versions without emphasizing any philosophical difference between them.

First recall Z.2, where Aristotle charges that the *logos* "takes [as granted] a falsehood, that it is not possible to go through (διελθεῖν) things unlimited or to individually touch things unlimited in a limited time." (233b21-2) Here the two alternate versions of this thesis seem to derive from the two alternate versions of the *logos*.

Consider first that in Θ.8, the first version of the *logos* uses the verb διιέναι (pass through), while the first thesis in Z.2 involves the verb διελθεῖν (go through). Both verbs are constructed from the prefix διά (go through) added to a verb of motion, and in fact, we have already seen that in Z.2 itself, Aristotle seems to regard the two as interchangeable. (233a22, a29)

The second version of the *logos* in Θ.8 involves counting, while the second thesis in Z.2 involves individually touching the "things unlimited". These ideas are not

---

questioner seeks to use a *logos* he has heard before, he must actively consider the precise wording and the ordering of the questions, and determine what auxiliary questions will serve a logical or psychological purpose. Aristotle discusses this in *Topics* Θ.1 (155b3-28).
The differences between versions will often prove crucial to the dialectical success or failure of a particular rendition. But they will often do so in a way that has little to do with the particular content of the *logos*, much as skill in writing might help or hinder a text irrespective of subject. As the author of the *Topics* and *Sophistical Refutations*, Aristotle himself is quite interested in the effective conduct of dialectic. But this is not his concern in *Physics* Θ.8. Instead, he is interested in the content of the Racetrack *logos*. So the fact that he notices and reports two distinct versions is a sign that the distinction between the two has a greater than normal psychological salience.

identical, but each does involve some kind of discrete contact with each item in a series. Indeed, the pseudo-Aristotelian treatise *On Indivisible Lines* presents a version of the Racetrack that overtly assimilates the two notions. (968a19-b5) This suggests that the second version of the Z.2 thesis was read in antiquity as invoking the second version of the *logos* mentioned in Θ.8.

In fact, the actual relationship of the Z.2 discussion to the original oral *logoi* involves some subtle issues that we will later examine in detail. For now we have at least a plausible case that Aristotle intends to invoke both versions of the *logos* in Z.2. The burden of proof lies with anyone who claims that Z.2 deals only with the first, and not the second.

As for *Physics* Θ.8, the relevance of the second version is even more evident. First, recall again how in Θ.8, Aristotle is particularly keen to show how the original Z.2 analysis works against the *logos* as it was actually given. Since he recounts two versions, he likely sees both versions as relevant to his case. And in fact, his Θ.8 discussion invokes the second version three times, inasmuch as he three times mentions counting, a key component of the second version, but not the first.[154] Again, we will later examine these issues in detail. For now our findings are negative: we find no sign that Aristotle directs his analyses against the first version of the *logos* as opposed to the second. Instead he treats them in tandem.

This leaves us with our original question: why does Aristotle seem to grant some sort of primacy to the first version of the *logos*? I suggest that Aristotle is simply aware

154 See 263a16-7, 263a25, and 263a29-b3. On the first two occasions, Aristotle appears to treat the two versions of the *logos* in parallel, just as in Z.2. But the third time he focuses solely on the second version.

that the first version came first. It is the original from which the second was derived. Aristotle is well-known for his attention to detail, and knowing this historical fact, perhaps even having observed the evolution of the *logos* in his life, he reflects the historical order in his writing. The best argument for this view lies in the fact that the second version incorporates much of the first, while adding new features. But it will be some time before we can see this. For now, we can take the historical ordering as a reasonable hypothesis.

In fact, the historical primacy of the first version is widely accepted by scholars. But this issue is easily conflated with two others. First is the question of whether the first Θ.8 version is Zeno's own original version. Second is the issue of whether Aristotle intends the first Θ.8 version, to the exclusion of the second, as the target of his analyses in *Physics* Z.2 and Θ.8. In fact, confusion results from failing to disentangle these three distinct issues.

We can begin with the second issue. We have just seen some fairly plausible evidence that in both Z.2 and Θ.8, Aristotle's analyses are intended to deal with both versions of the *logos*. The details merit attention later, but at present we have no reason to doubt it. The evidence suggests *both* that Aristotle intends to analyze the two versions in tandem, *and* that he regards the first version as historically preceding the second. There is no incompatibility between these two apparent facts.

Things become more confusing if we suppose that Aristotle regards the first Θ.8 version as the version of Zeno himself, and the second as a derivative created later. We might then think that Aristotle will naturally wish to criticize the actual argument of the famous Zeno, rather than the creation of some later interloper. And it might seem that this is exactly what Aristotle purports to do. In *Physics* Z.2 (263a21), and in recounting the first version in Θ.8 (263a5), Aristotle refers to the Racetrack as "the *logos* of Zeno." Certainly the naive reader of Z.2 receives no sign that Aristotle is doing anything other than making an attribution to Zeno himself. Later in Θ.8, when Aristotle claims that his earlier resolution was satisfactory "against the questioner" we might again suppose that the questioner is Zeno himself. So Aristotle might seem to report the first version as being Zeno's own version, with the second version being mentioned only because Aristotle in Θ.8 is now delving into the deeper philosophical issues raised by the *logos*.

In fact, this view is flatly contradicted by Aristotle's account of the two versions in Θ.8. Aristotle opens his discussion by writing: "And in the same way one must reply to those [dialecticians] asking (τοὺς ἐρωτῶντας) the *logos* of Zeno... ." (263a4-5) Aristotle refers to those doing the asking with a *plural* participial phrase. In analyzing the Racetrack, Aristotle is replying to this plurality, not to Zeno alone, a single man long dead. *All* of "those asking" the *logos* ask "the *logos* of Zeno", not merely Zeno himself, and Aristotle is replying to all of them. Introducing the second version, Aristotle adds: "Or as some others (τινὲς) differently ask *this same logos*... ." (263a6-7) Here Aristotle refers to a second plurality of questioners who ask "this same *logos*", that is, "the *logos* of Zeno". Aristotle's novel analysis of the *logos* in Θ.8 is explicitly directed against these two groups of questioners, all of whom ask "the *logos* of Zeno". Zeno himself has no special status among them, and Aristotle certainly does not single out the first version alone as "the *logos* of Zeno". And if both versions instantiate the *logos* of Zeno in Θ.8, then we certainly have no grounds for assuming that Aristotle singles out any specific version for analysis earlier in Z.2.[155]

All of this makes perfect sense once we realize that the Racetrack is a dialectical *logos*. As the inventor of the *logos*, Zeno has his name attached to it. But a reply to the *logos* is not simply a reply to Zeno himself, but a reply to any dialectician who employs the *logos*.

As a matter of fact, the first Θ.8 version probably *is* fairly close to Zeno's version, and Aristotle himself probably believes this. But Aristotle does not *say* this. I believe it is close to Zeno's version only because it can be reconstructed with great simplicity. But this is a conclusion and not a starting point of our investigation. What matters now is that even though Aristotle likely regards the first version as more Zenonian, this fact is simply irrelevant to him. So the greater authenticity of the first version should play no role in our assessment of Aristotle's evidence. It's historical precedence does not license us to conclude that Aristotle aims to analyze the first version alone.

We may now sum up our results so far. In the Θ.8 passage, Aristotle aims to report the *logos* as he has heard it. He reports two distinct versions, and insinuates that

155 Various scholars offer views akin to those I have rejected. McKirahan (2002), pp. 480-1, explicitly interprets the Θ.8 passage in the manner I have critiqued, claiming that Aristotle reports the first, but not the second, version of the *logos* as Zeno's own version. Lee (1936), pg. 37, claims that the report of the first version "clearly re-states, without variation, the argument as we have already met it." In fact, we have seen that neither in Z.2 nor in Z.9 does Aristotle even purport to present a full recounting of the Racetrack *logos*. According to Barnes (1982), pg. 263, Aristotle "explicitly distinguishes between Zeno's paradox" and the second version, which Barnes claims is clearly "a vulgarization of Zeno's argument." In fact, both versions are versions of "the *logos* of Zeno" and the second will turn out to be conceptually more sophisticated. Barnes goes on to deny that the "touching" mentioned in Z.2 (263a23) should be correlated with the "counting" of the second Θ.8 version, on the grounds that Z.2 deals with Zeno's argument, while the counting version is non-Zenonian. But this is confused, as Aristotle does not purport to deal with Zeno himself in Z.2. Hasper (2003), more than anyone, seems to recognize that Aristotle is intellectually engaged with multiple versions. He nonetheless errs in supposing that Aristotle in Z.2 intends to make attributions to Zeno himself, in supposed contrast to his report of the two versions in Θ.8. (pg. 47) But given Aristotle's use of the term "the *logos* of Zeno" in Θ.8 , we have no reason to assume that in Z.2 the term is intended as part of a historical attribution to Zeno himself. It refers rather to a *logos* that might be given by anyone.

the first version historically precedes the second. But we have no sign that he regards the first version as any more important. Still, for us it is quite important. If we want to come as close as possible to Zeno's version of the *logos*, and understand how the second version may have evolved from the first, and understand how Aristotle's analyses are meant to deal with both, we should naturally begin by trying to reconstruct the first version in it's own right. This will be our focus for the rest of the chapter.

The Thesis of the *Logos*

Here again is how Aristotle recounts the first version of the *logos*:

> τὸν αὐτὸν δὲ τρόπον ἀπαντητέον καὶ πρὸς τοὺς ἐρωτῶντας τὸν Ζήνωνος λόγον, εἰ ἀεὶ τὸ ἥμισυ διιέναι δεῖ, ταῦτα δ'ἄπειρα, τὰ δ' ἄπειρα ἀδύνατον διεξελθεῖν...
>
> And in the same way one must reply to those [dialecticians] asking the *logos* of Zeno, [who ask] whether it is always necessary to pass through the half, but these [halves] are unlimited, and it is impossible to pass through things unlimited... (263a4-6)

From this we need to reconstruct a dialectical debate.

For the most part, scholars have not devoted much attention to the reconstruction of actual dialectical debates. This is not to say that they have not discussed dialectic and dialectical arguments. Indeed, it would be impossible to discuss most of Plato and much of Aristotle without doing so.[156] But scholars have not often reconstructed dialectical debates as entities in their own right, as things independent of the texts by which we gain access to them. And so there are not really conventions about how one should go about doing this. There is no established opinion regarding exactly what is important in reconstructing such a debate, and what is not.

In fact, I think that the way to discover such things is not by laying down

156 But scholars have often varied as far as the significance they attach to the dialectical character of dialectical exchanges.

strictures *a priori*, but rather by actually trying to reconstruct dialectical debates, and figuring out what matters and what does not. And so this is what I will try to do with the Racetrack *logos*. But in some ways, we can say what reconstructing a dialectical debate is *not*. It is not the same thing as translating a text. And it is not the same thing as reconstructing an argument, if we mean by *argument* what 20th century analytic philosophers mean by *argument*. But it may not even be the same thing as reconstructing a "dialectical argument", if by that we mean what someone like Aristotle *thinks* a dialectical argument is supposed to be. Instead, if we want to understand a *logos* as an element of dialectical practice, we need to consider how a dialectical debate would actually function as an interchange between two people. And we can only do this by trying to do the reconstruction.

To start things off, we can convert Aristotle's wording into question and answer form. In doing so, we should adhere as closely as possible to Aristotle's text. We get:

> Is it always necessary to pass through the half? Yes.
> Are these halves things unlimited? Yes.
> Is is possible to pass through things unlimited? No.

This is clearly not a full dialectical interchange.

One thing that is missing is a thesis.[157] Recall that every *logos* begins with the questioner gaining the answerer's assent to some thesis. The questioner then aims to lead the answerer into denying this very thesis. The negation of the initial thesis thus serves as

---

157 I use the English word "thesis" in accord with it's English meaning. In fact, the Greek word *thesis* is a dialectical term of art and does not mean exactly what we mean by "thesis". In fact, a *thesis* in the technical dialectical sense is a possible starting point for a *logos*, but it will only be termed a *thesis* if it is a proposition contrary to general opinion. (Cf. *Topics* 104b18-28; Slomkowski (1997), pp. 17-8.) So not every *logos* will start with a *thesis* in this sense, and in fact this includes the Racetrack. For whatever the precise claim that begins the Racetrack *logos*, it is surely very much *in accord* with general opinion, while it's negation is highly counterintuitive. As the precise meaning of *thesis* is not relevant to the Racetrack, I ignore this issue.

the conclusion of the argument.

Aristotle provides no thesis in *Physics* Θ.8. And we discovered no thesis in *Physics* Z.2 or Z.9, although Z.9 suggested that the conclusion was "about not moving". (239b11-2) In several other places Aristotle does provide conclusions for the *logos*: "it is not possible to move" (*Prior Analytics* 65b18-9)[158], "it is not possible to move or get through the *stadion*[159] (διελθεῖν τὸ σταδίον)" (*Topics* 160b8-9), and "it is not possible to have moved." (*Sophistical Refutations* 179b20-1) However, none of these conclusions is linked directly with any particular version of the *logos*. Since we are now seeking the precise thesis for the first version, we must approach this evidence with caution.

One of the given conclusions includes the word *stadion*, often rendered *stadium*. In fact, the exact meaning merits scrutiny. Generally speaking, public athletic events played a large role in classical Greece, and a number of athletic fields existed, bordered by embankments from which spectators might view the action. Running was the most prominent event, and races occurred on a prepared track called the *dromos*. The track was always straight and ran parallel to the spectator area. The original race was one length of the track, but later there were also races of two or four lengths in which the runners needed to reverse direction after looping around an endpost. In this setting, the word *stadion* might have several meanings. David Romano writes:

> [I]t would seem a good possibility that the word *stadion* came to be used first for the area where spectators stood bordering the *dromos* on low embankments; later the word was applied to the name of the athletic event held there; and finally the

158 This is the one Aristotelian reference to the Racetrack that I do not discuss in detail. Aristotle refers to a proof of the incommensurability of the side and the diagonal of a square, a proof that apparently relies on Zeno's Racetrack *logos*. This is an extremely interesting reference in it's own right, but the issues involved in assessing it are so complex and contentious that it is not liable to help us in the current investigation.

159 The term *stadion* is discussed at length below.

> term was applied to the specific length of the racecourse. By association, the word came to mean the combination of the spectator area and the competition area. ... By the fifth century B.C., ... the Greek word *stadion* had come to have three meanings: a structure for athletic contests; a footrace one length of the structure; and a linear distance always equal to 600 feet.[160]

With it's third meaning, *stadion* breaks loose from the athletic arena and can be used as a measure of distance regardless of context. We can thus divide uses of *stadion* into two classes. First are those where *stadion*, whatever it's exact sense, is clearly used in an athletic context. Second are those where the athletic connotation is absent, and *stadion* refers only to an abstract measure of length. In fact, it seems that Aristotle intends to evoke the athletic context. He writes not merely "*stadion*" but "the *stadion*." This definite phrase refers to something that is somehow *particular*. This could be a particular athletic event, or it could be the particular stretch of ground covered in this race.[161] By contrast, there would be no reason for Aristotle to use an article if the *logos* asked whether it is possible *in general* to go through a span of one *stadion*. It must have asked about some kind of particular *stadion*, and the obvious candidates are the race or the span thereof.[162]

Aristotle's own analyses of the *logos* never involve athletics, and neither does the context of *Topics* 160b8-9, where the word *stadion* occurs. So Aristotle himself would have no incentive to introduce the word, as he might if he were reinterpreting the *logos* to fit his own framework. It follows that some pre-Aristotelian version of the *logos* must

---

160 Romano (1993), pg. 16. He adds (fn. 25) that his etymological ordering differs from the order found in Liddell and Scott, who list the standard of length as the primary and seemingly original meaning. Ironically, *stadion* appears to derive from ἵστημι (to stand). (*Ibid*., pg. 14) (Notice that the English derivative *stand* may also refer to a viewing stand.) Consult Romano for additional information on *stadia*.

161 In principal, it could also be the athletic arena itself or the viewing stand. But the *logos* surely did not involve someone like a hot dog vendor going through the bleachers.

162 Of course, these particulars are themselves universals. The *logos* need not be about one particular running of the race, nor about one particular racetrack.

have used the word *stadion* in the opening thesis, in a way that clearly evoked the athletic arena. In this context, there would surely be an explicit mention of a runner. Thus some version of the *logos* opened with a question like: "Can a runner get through a *stadion*?" In fact, it was very likely the original version.

To see why this is plausible, consider that such a *logos* would involve concrete particulars, such as the runner. Apart from the one mention of the *stadion*, Aristotle's own discussions of the Racetrack never involve concrete particulars. In *Physics* Z.9, for instance, he describes the moving object not as a runner, but generically as "the moving thing". (239b12-3) At first glance this might seem evidence against a reconstruction involving a runner, but careful examination reveals that Zeno's original version of the *logos* would have likely involved concrete particulars *and* that Aristotle might well systematically omit them from his discussions.

First, consider that as Aristotle reports the other three motion *logoi* of Zeno, all three involve concrete particulars. The Achilles and the Arrow involve Achilles and an arrow. (238b14, 239b29-30) The fourth *logos*, often called the Stadium, involves, of course, a *stadion*. (239b33-240a1) In fact, in this *logos*, the *stadion* serves not as an abstract measure of length, but rather as the fixed setting for some moving objects, which makes all the more plausible our earlier interpretation of *stadion* in the Racetrack. More generally, given that the other three motion *logoi* involved concrete particulars, it is extremely plausible that concrete particulars also figured in Zeno's original version of the Racetrack.

Of course, in discussing the Racetrack, Aristotle only mentions the *stadion* once, and never the runner. But Aristotle himself gives us a reason for these omissions. In the

Topics, he gives this advice to dialecticians: "It is also necessary to keep the records of *logoi* in universal form, even if one has argued concerning particulars. For then it will be possible to make the one [*logos*] into many... . But [a questioner] himself should keep far away from having his deductions (*sullogismoi*) bear the universal." (164a3-5, 6-7)

Aristotle has likely observed and participated in many dialectical debates, and apparently the most effective *logoi* do not have overly abstract questions. This makes sense. A questioner wants a question where the answer seems obvious, so the answerer has little room to object. Questions about particular familiar things are much more likely to have seemingly obvious answers than questions about abstractions. Abstract questions invite counterexamples. By contrast, particular questions often seem decided by the evidence of the senses alone. If the Racetrack begins, "Can a runner get through a *stadion*?" then the *logos* is off to a good start: huge crowds have seen runners do this very thing.

By contrast, when writing about dialectic, Aristotle has good reason to record universal theses. Students of dialectic who study these universals will be able to apply them to construct a multitude of seemingly different *logoi*. We should then expect that Aristotle will often follow his own advice and record *logoi* in universal form even when they were typically presented with particulars. This is likely his practice with the Racetrack. In the *Physics*, Aristotle is concerned with basic principles, and will naturally suppose that no matter how often the runner and the *stadion* were mentioned in the oral *logos*, they are simply logically irrelevant, so he omits them. In fact, notice that in the one place where he does mention the *stadion*, he is not analyzing the *logos* per se, but discussing *duskolia*, a common feature of the Racetrack's performance. (*Topics*

160b2-10) Here the *stadion* will seem quite relevant, as the answerer will more likely become discomfited if he is forced to deny an observed fact than if he is forced to deny an abstract thesis.

We thus have good reason to suppose that an originally concrete Racetrack *logos* became abstract in the written works of Aristotle. In principle, it might seem that the reverse migration is possible: perhaps Zeno created abstract arguments from which concrete versions developed later.[163] A lecturer such as Aristotle might create them for illustrative purposes, much as lecturers do today. Or a dialectician might craft originally abstract arguments into concrete versions suitable for dialectic. In fact, if we consider the evidence concerning Zeno's written treatise, it seems that he did create some arguments that are highly abstract, and would likely need to be reworked if they were to prove effective as dialectical *logoi*.[164] But we can be fairly sure that Aristotle was not familiar with any abstract authentically Zenonian written versions of the motion *logoi*, for he surely would have discussed them directly, given his preference for universal principles. Now someone might think that Aristotle does this very thing in *Physics* Θ.8. But there,

163 Hasper (2003) appears to hold a view something like this, although he never fully spells it out. (See pp. 43-8.) He distinguishes between the “constructions” of the *logoi* and the “arguments”. His distinction appears to be based upon a key feature of Euclidean style proofs. Recall that such proofs adhere to the conceit that the reader constructs a diagram which constitutes a particular instance of the universal involved in the proposition to be proved. “Construction” thus becomes a technical term referring to the portion of a written proof in which the construction of the diagram is discussed. (For a specimen of a construction within the context of a proof, see Netz (1999), pg. 10.) Hasper thus appears to regard the runner, Achilles, etc., as components of an argument having purely illustrative value, and separable from the logical structure of the “argument,” just as the identity of the particular diagram that gets constructed in a geometrical proof is inessential to the logical structure of the proof. But this assimilation of the motion *logoi* to written Euclidean style proofs has no evidence to support it: the motion *logoi* were oral dialectical *logoi*. In fact, Hasper offers some of the most thoughtful analysis of recent Zeno discussions, but his analysis of the Racetrack is permeated by this unhistorical construction/argument distinction.

164 For some of this evidence see Kirk, Raven and Schofield (1983), §314-316. In fact, despite their complex verbal nature, I suspect that the power of Zeno's written *logoi* often depended on highly concrete visualization by his audience, so that there is an interesting sense in which they might not be quite so abstract, or at least, so “logical”, as they typically seem to contemporary scholars. But this issue takes us well beyond the present investigation.

while the Racetrack is seemingly abstract, we have seen that Aristotle is clearly talking about oral, and not written, versions of the *logos*. We thus have every reason to assume that Zeno himself originated the motion *logoi* in their concrete form.

To sum things up, we have now seen evidence that some pre-Aristotelian dialectical version of the Racetrack began with the question about the runner and the *stadion*. Indeed, given the use of concrete particulars in the other motion *logoi*, and the inherent value of concrete particulars in dialectical *logoi*, this is likely to have been the opening question of the original version. Aristotle's tendency to discuss the *logoi* in a more abstract form can be explained by his stated preference for universalizing dialectical *logoi* in written discussions. We may thus reasonably suppose that the first version of the *logos* discussed in *Physics* Θ.8 began along the lines: "Can a runner get through a *stadion*?"

Of course, there still seems to be an ambiguity in this question: does *stadion* refer to a race or to the length thereof? In fact, it is not clear exactly what it would mean to try and decide this issue. Put yourself in the mind of the answerer facing this question, the opening question of the *logos*. You do not hear language that cries out for semantic analysis. You imagine naked men running before cheering crowds. You have been to the games or at least heard reports about them. The answer to the question is obviously "Yes".

Inasmuch as the question almost surely asked about a runner, and was intended to evoke an athletic context, we may say that *stadion* clearly does have the sense of *stadion*-race. At the same time, running a *stadion*-race is exactly the same thing as running through a certain *stadion*-length. So if *stadion* has the sense of race, it also has the sense of length.[165] As both the race and the length are liable to be relevant to the *logos*, I will refrain from translating *stadion*. In the opening question, it refers to the race, but in a way that encompasses the length.

Where Does the Infinite Come From?

We may now add our results to our earlier rendition of the *logos*. This yields:

> Can a runner get through a *stadion*? Yes.
> Is it always necessary to pass through the half? Yes.
> Are these halves things unlimited? Yes.
> Is is possible to pass through things unlimited? No.
> And so you say that the runner cannot get through the *stadion*!

Here I have added the question proposing the initial thesis, as well a corresponding conclusion asking for the denial of this thesis.[166] Naturally, we know that the answerer will affirm the thesis in the beginning, but is forced to deny it in the end.

With the questions as given, the answers need to be what they are in order for the *logos* to succeed. So much is clear. But as a check on our reconstruction, we also need to consider the point of view of the answerer. We need to make sure that the answerer seems more or less forced to answer as we claim he does. If we seem to be ascribing an implausible answer, we know something is wrong with the reconstruction. In fact, this issue commonly goes unnoticed in discussions of the motion *logoi*, where scholars assume that premises get to be premises simply because *Zeno* happens to believe them.

As it stands, the second question, about always needing to go through the half, seems to pass the test. In principle, the "half" might be either a halfway-point or something like a half-distance, and we will soon need to consider this issue. But either

---

165 The converse is not true. Most references to a *stadion*-length will not be intended to evoke the athletic event.

166 From *Topics* 158a7-13 we learn that a questioner will often state the final conclusion outright, not asking for the answerer's approval. This avoids giving the answerer a final chance to object.

way, it does seem that we have a question for which the appropriate answer is *Yes* rather than *No*.

By contrast, the third question poses an obvious problem. The answerer affirms that the halves are unlimited in number.[167] But why? A racetrack has only *two* halves, and *one* halfway-point. We can scarcely doubt that the answerer *does* affirm the question, but this means that something is amiss. We need to fix things so that the questions themselves put the answerer in a position where the answer to the *unlimited*-question seems to be *Yes*.

Consider the first two questions of the *logos* as I have written them: Can a runner get through a *stadion*? Yes. Is it always (ἀεί) necessary to pass through the half? Yes. Here ἀεί (always) appears to be a universal quantifier, and it seems that it's scope is determined by the prior question. So *always* seems to mean, *always, when a runner is going through a stadion*. But no matter who the runner is, and no matter when and where he runs the *stadion*, the *stadion* will have only two halves, and one halfway-point. And so the answerer has no reason to affirm the *unlimited*-question.

It might seem that we can resolve this problem by reinterpreting the *always* in the *always*-question. As things stand, it seems to be a universal quantifier, with it's scope determined by the prior question. But perhaps it is a universal quantifier, and has its scope determined explicitly by the *always*-question itself. Indeed, suppose the *logos* starts like this:

Can a runner get through a *stadion*? Yes.

167 We have seen that ἄπειρα does not need to refer to a collection unlimited in number. But in this question, *Are these halves ἄπειρα?*, even as Aristotle reports it in *Physics* Θ.8, the natural reading of ἄπειρα is to construe it as referring to a collection unlimited in number. And in a dialectical *logos*, the natural reading is what matters.

> When a runner goes through a distance, must he always go through half the distance? Yes.
> Are these halves things unlimited? Yes.

In this version, the "halves" will obviously be half-distances, and not halfway-points.

This rendition might seem appealing, since it might seem that if the *always*-thesis is true, then the ensuing unlimited-halves-thesis must also be true. In fact, it might seem that in one stroke, the *always*-thesis makes a claims not only about distances of one *stadion*, but also about half-*stadia*, and half-half-*stadia*, and so on. Indeed every half-distance is itself a distance, and a smaller distance than that of which it is a half. With some knowledge of formal logic, we could even prove that the always-thesis can be true only if the distances, and the half-distances, constitute an infinite set.

In fact, this does nothing to resolve our problem. It does not matter what we might prove with formal logic. What matters is whether the answerer in a dialectical debate will feel compelled to affirm the *unlimited*-question. And we have not given him a reason to do so. Consider: When a runner goes through a distance, must he always go through half the distance? Yes. Are these halves things unlimited? No, surely not. A distance, any distance, has only *two* halves. The answerer will rightly reject this thesis if he quite reasonably construes "these halves" as referring to the two halves of a particular distance. There are no linguistic grounds for him to suppose that "these halves" refers instead to some series of a half-*stadion*, a half-half-*stadion*, and so on.

As things stand, our *logos* is seriously flawed, since one of the answers we are ascribing to the answerer is totally unwarranted. In principle, we might press ahead, changing or adding things until the *logos* works. But the more we deviate from Aristotle's report, the less confidence we will have in our reconstruction. I suggest, then, that we return to our original wording and reassess our situation.

Our task is to reconstruct an oral *logos* starting from Aristotle's written text. We might well make progress if we examine, quite apart from the Racetrack, the ways in which Aristotle's written reports of dialectical *logoi* are likely to differ from the *logoi* themselves. We have already seen that in writing, Aristotle systematically universalizes *logoi*, even though he counsels dialecticians to employ particulars. Are there other systematic deviations from the spoken *logos*?

In fact, it seems that there are. In *Topics* Θ.1, Aristotle is giving advice to dialectical questioners. He writes:

> It is also necessary to speak about the arrangement of the questioning, determining how many are the premises which must be obtained besides the necessary [premises]. Those [premises] on account of which the *sullogismos* comes about are called necessary. Those [premises] being accepted besides these are four [in kind]. [They are] either [those] of an induction (ἐπαγωγή) for the sake of the granting of a universal, or [those] to [add] weight to the *logos*, or [those] for concealment of the conclusion, or [those added] for the *logos* to be more clear. Besides these one should accept no premise, but through these one must attempt to augment [the *logos*] and structure the questioning. (155b17-25)

Here Aristotle draws a clear distinction between the questions that establish the premises of the *sullogismos*, which is the deductive core of the *logos*[168], and every other question that a dialectician must ask in order to secure a dialectical victory. It seems likely that in an oral *logos*, the bulk of the actual verbiage would consist of questions from the ancillary categories that Aristotle describes. At the same time, Aristotle will likely omit questions of these sorts when writing about *logoi*. Instead he will report what he regards as the deductive core of the *logos*, the *sullogismos*.[169] Certainly this is the impression we

168 For the deductive nature of a *sullogismos*, see *Topics* 100a25-7.

169 In the *Topics* and *Sophistical Refutations*, the term *sullogismos* encompasses *reductio* arguments. (See 157b34-158a2, esp. 157b37-8.) By contrast, in the *Analytics*, a *reductio* is not considered a *sullogismos*. (See Ross (1936), pp. 37-8) But this is merely a terminological shift, and a *reductio* is

get from reading the *Topics* and the *Sophistical Refutations*, where we rarely find anything like a realistic transcript of a *logos*, which would typically include at least some of these ancillary questions.

If we consult Aristotle's report in *Physics* Θ.8, it seems that he will assuredly construe the three premises he reports as the key deductive premises of the *logos*.[170] It is likely that if he is omitting any questions, they fall into one of the ancillary categories mentioned above.[171] In fact, I suggest that Aristotle is omitting what he construes as premises of an induction.

In the *Topics*, Aristotle defines induction:

> Induction is the passage from the particulars to the universal. For instance, if the knowledgeable pilot is best, and [the knowledgeable] charioteer [is best], then in general the most knowledgeable [practitioner] in each [domain] is best."[172] [105a13-6]

In general, an induction presents a conclusion that purports to be a universalization of some particular premises.[173] As a rule, Aristotle will find it philosophically uninteresting which particular premises are typically used to support a universal conclusion, and in fact

---

still considered a valid deductive argument. What matters for us is that Aristotle, in reporting *logoi*, will typically report what he sees as the crucial deductive premises.

170 In fact, his report at least superficially conforms to his standard schema for a *reductio*, which involves a two line syllogism, along with a third premise rejecting the conclusion of the syllogism. (See Ross (1936), pg. 31)

171 In principle, any premise of the central *sullogismos* may itself be justified by an ancillary *sullogismos*. (See *Topics* 155b35-156a38.) But it seems more likely that Aristotle would omit from his report seemingly non-deductive premises than that he should omit an ancillary *sullogismos*.

172 See also 156a4-7. In fact, Aristotle uses the quoted passage to contrast induction with *sullogismos* (deduction) as two varieties of dialectical *logoi*. (105a10-2) But among skilled dialecticians, it is clear that an induction would rarely stand alone (cf. 105A16-9), serving more naturally as the aid to a *sullogismos*, just as we saw above.

173 What I am now calling induction is different from what is today called "mathematical induction". A mathematical induction is a *deductive* argument, and always employs a *universal* premise. I see no sign that Aristotle ever employs or reports a mathematical induction, although perhaps someone could dispute this. In principle, mathematical induction might seem relevant to the issues raised by Zeno-style arguments, since it can be used to prove, with a finite proof, that some property applies to every member of an infinite set.

this will likely vary greatly among dialecticians, even when the same conclusion is at issue. Although the induction I will propose for the Racetrack will have a degree of systematicity unusual for inductions, Aristotle may nonetheless omit it from his report if he construes it "merely" as an induction.

Recall our problem. We need to explain why the answerer affirms the *unlimited*-question, and we want to do so without postulating unexplained differences between Aristotle's report and the original *logos*. If we can do so simply by adding some seemingly inductive premises, which would plausibly be omitted by Aristotle from his account, then this would seem a good option. So I suggest the following reconstruction:

> Can a runner get through a *stadion*? Yes.
> Particular Half Question #1
> Particular Half Question #2 (Iteration of prior question)
> Particular Half Question #3 (Iteration of prior question)
> Is it always necessary to pass through the half? Yes.
> Are these halves things unlimited? Yes.
> Is is possible to pass through things unlimited? No.
> And so you say that the runner cannot get through the *stadion*!

The only change to the earlier version is that I have now added some inductive premises. In this version, the *always*-question is the conclusion of an induction. Roughly speaking, the *logos* goes: This, this, this, and it's *always* like that. We earlier ran into problems when we tried to specify the scope of *always*. But that may have been a mistaken effort, as there might well be no overt specification of the scope of *always*, so long as it appears to be generalizing from particular premises.

Following the *always*-question comes: Are the halves unlimited? We know that the answerer does says *Yes*, and in fact he will have a reason to say *Yes* if the inductive premises, *en masse*, seem to be steps in an infinite division, with each new premise introducing some new “half”, in a way that can seemingly be repeated endlessly. It probably does not matter exactly how many inductive premises there are, so long as there are enough. And it is still not clear exactly what sort of “halves” are involved. But it does seem quite likely that each inductive premise after the first should somehow be constructed out of the prior, as this seems the way to continue the process unendingly.

This schema will likely be more convincing if we can work out the details, but we can now recap our situation. Our reconstruction of the *logos* began with Aristotle's own report. To this we added the thesis question and the corresponding conclusion. It is fairly clear what answers the answerer must give to make the *logos* work. But we scarcely have a plausible reconstruction unless those answers seem plausible in the dialectical context, which means we need to somehow either add questions or revise Aristotle's wording. Since induction was often used dialectically to support a universal conclusion, but rarely reported by Aristotle, adding inductive premises seems a reasonable way to revise the reconstruction, if we can do so in a way that makes the answerer give answers that are psychologically plausible, and even psychologically necessary. In fact, it seems that we can. Our next task is to determine, as best we can, what the inductive premises actually were.

The First Inductive Premise

We have now seen that the *logos* quite likely involved an induction, even if Aristotle himself does not tell us that in *Physics* Θ.8. But in reconstructing the induction, we need to stay as close to the textual evidence as possible. Ideally, Aristotle would actually tell us what the inductive premises were. In fact, I think that he does indeed provide us with the first inductive premise.

Recall how Aristotle described the Racetrack in *Physics* Z.9: πρῶτος μὲν ὁ περὶ τοῦ μὴ κινεῖσθαι διὰ τὸ πρότερον εἰς τὸ ἥμισυ δεῖν ἀφικέσθαι τὸ φερόμενον ἢ πρὸς τὸ τέλος... ." (First is the one [*logos*] about not moving since the moving thing must first reach the half before the end... ." (239b11-13) We earlier decided that this line contained a premise of the *logos*: "the moving thing must first reach the half before the end". We have so far paid little attention to it, but I suggest that it is the first premise of the induction.

In assessing this proposal, we must first decide whether the premise contains authentic language from the oral *logos*. In *Physics* Θ.8, Aristotle seems keen to report the Racetrack as he has heard it. But in *Physics* Z, Aristotle gives us no such indication, and there he may have much more of an incentive to reword things to fit the tight structure of his reasoning therein.

Consider also that in Θ.8, Aristotle reports two versions of the *logos*, and it seems that in his analyses in Z.2 and Θ.8, he aims to treat of both versions in tandem. But the premise in Z.9 is not linked to either version. So we might wonder whether the Z.9 premise is some kind of Aristotelian abstraction, culled from both versions. In this case, we might be skeptical of the wording, which could result more from Aristotelian interpretation than from verbal recall of the oral *logos*.

One sign that we may, perhaps, trust the premise as Aristotle reports it is the fact that neither of his two analyses of the *logos*, in Z.2 or Θ.8, focuses on the given line. In fact, neither seems to mention it at all. This suggests that Aristotle has no great motivation to reword it.

But a much more powerful argument for the authenticity of the Z.9 premise derives from the fact that it is quite different from what we might expect of an abstract Aristotelian rewording. The premise asserts: "the moving thing must first reach the half before the end." Naturally, some elements of this *are* abstractions. For instance, the original *logos* surely involved not a generic "moving thing", but a runner. But focus attention on "the half". It is clear that "the half" is a halfway-point, the sort of thing one might *reach* in the same way as one might reach "the end." It is manifestly *not* any sort of half-magnitude, one of two "halves" that compose a whole. One does not "reach" that sort of thing.

Why is this significant? Recall that when we examined Aristotle's analysis of the Racetrack in *Physics* Z.2, we found no reference to any sort of half-way points. In fact, Aristotle's analysis did not overtly mention "halves" of any kind, be they halfway-points or half-magnitudes. And careful examination showed that Aristotle's analysis did not directly involve any specific unstated assumptions about half-way points. By contrast, his analysis did focus on the manner in which a line might be divided into a series of smaller lines. Naturally, one way in which this might happen is by a line being successively divided in half. This suggests that Aristotle is construing the runner as needing to *pass through* an "unlimited" series of *half-magnitudes*. He is not construing the runner as needing to *reach* an unlimited series of *halfway-points*.

At first, it might seem that the difference here is minor. But in fact, Aristotle's dialectical *lusis* works only if the runner is seen as passing through half-magnitudes. Recall that the *lusis* hinges on disambiguating the singular ἄπειρος, and the plural ἄπειρα, and that Aristotle represents ἄπειρα via linear diagrams. In fact, he tells us that ἄπειρος is ambiguous *when it is applied to a continuous entity such as a line*. (233a24-6) The line may be "unlimited with respect to the ends" or "unlimited with respect to division". And the collection of its parts, the ἄπειρα, may likewise be unlimited in either way. By contrast, it seems that a collection of half-way points can be called ἄπειρος or ἄπειρα in only *one* way, when ἄπειρος means, roughly, "unlimited with respect to quantity".[174] For Aristotle's critique to work, he must therefore construe the word ἄπειρα, which occurs in his rendition of the premise he critiques (233a21-3), as referring to a collection of half-magnitudes. He cannot construe it as referring to a set of halfway-points.

Of course, Aristotle might somehow employ his Z.2 analysis against an argument that involves halfway-points, but only if he first "translates" it into an argument involving half-magnitudes. In fact, we will later find that this is exactly what he does, which suggests that his charge of lexical ambiguity might appropriately be brought against himself. But we may defer this issue. What matters for now is that Aristotle has no philosophical motivation for reporting the Z.9 premise as involving a halfway-point. Indeed, the premise would mesh much better with his Z.2 analysis if it said something like: "the moving thing must first pass through the half-magnitude before the whole."

174 Recall that the meaning "unlimited with respect to quantity" is akin to "unlimited with respect to the ends", but is of broader scope.

But in fact it involves a halfway-point. This suggests that Aristotle is reporting the Z.9 premise in roughly the language that he heard it.

If Aristotle heard the Z.9 premise used in the dialectical Racetrack, then he must have heard it used in one or both of the versions that he reports in *Physics* Θ.8. Given that he does not say which version it is from, and given that in Z.2 and Θ.8, he seems to treat both versions with equal seriousness, our best bet is that the premise occurs in both versions. And given that in Z.9, he offers the premise as a shorthand gesture at the full *logos*, it likely had a position of prominence in both versions. So I suggest that in both versions it was the first actual premise. That is, it was presented as the second question, the first step towards refuting the thesis offered in the opening question. This would give the premise psychological prominence, allowing it to stand in for the full *logos* in Z.9. And this opening position is perfectly compatible with Aristotle's failure to mention the premise in Θ.8. As the standard first premise of the induction, he will easily remember it as the first step of the argument, but he will regard it as logically insignificant compared with the conclusion of the induction, which he does report.

To recap the argument: we need an induction, and we need to stick the Z.9 premise somewhere, and Aristotle's manner of reporting the *logos* presents no obstacle to having the Z.9 premise begin the induction. So the *logos* might well begin like this:

> Can a runner get through a *stadion*? Yes.
> Must he first reach the halfway-point before the end? Yes.
> Particular Half Question #2 (Iteration of prior question)
> Particular Half Question #3 (Iteration of prior question)
> Is it always necessary to pass through the halfway-point? Yes.
> Are these halfway-points things unlimited? No.

In rendering these questions, I have systematically replaced the word "half" with "halfway-point". As we saw earlier, the first inductive premise involves a "half" that is a halfway-point. And so this entails that the "halves" in the ensuing questions are halfway-points as well.[175] We will later examine in detail the relation of the induction to the *always*-question and the *unlimited*-question. But for now it seems we have a plausible setup.

Progression and Regression

As things stand, our induction has an opening premise. But how did the induction proceed from there? Naturally, we cannot reconstruct the induction verbatim, but this might be a mistaken goal anyway, since the *logos* would have differed in various ways every time it was used. What we really want to discover are the systematic commonalities between individual askings of the *logos*. And on this issue we can make some progress.

We can begin by considering two fundamentally different options. Perhaps the *logos* began something like this:

> Can a runner get through a *stadion*? Yes.
> Must he first reach the halfway-point before the end? Yes.
> But before he reaches *this* halfway-point, must he first reach the halfway-point before *it*? Yes.
> And before he reaches *this* halfway-point, must he first reach the halfway-point before *it*? Yes.
> Is it always necessary to pass through the halfway-point? Yes.

175 This contradicts the reading of the many scholars who regard the "half" in the *always*-question as some kind of half-magnitude. (For examples, see Faris (1996), pg. 7; McKirahan (2002), pg. 478; Ross (1936), pg. 448; and the translations of Hardie/Gaye, Lee, and Waterfield. By contrast, Graham's translation involves "halfway-points".) I am not certain what motivates the half-magnitude reading, but there might seem to be an argument in it's favor. Consider that "passing through the half" would seem to be a process that takes time. Now suppose, as is common in contemporary kinematics, that a moving object is a sizeless particle, and that a midpoint is also sizeless. Then the moving object will take no time in passing through the midpoint, but will do so instantaneously. But the point particle will take time in passing through a half-magnitude, which might explain the common reading. Of course, this is anachronistic, since a runner is not a sizeless particle, and will indeed take time in passing through a halfway-point, even a sizeless halfway-point.

Alternatively, it might go:

> Can a runner get through a *stadion*? Yes.
> Must he first reach the halfway-point before the end? Yes.
> But after he reaches *this* halfway-point, and before he reaches the end, must he first reach the halfway-point in between? Yes.
> And after he reaches this halfway-point, but before he reaches the end, must he first reach the halfway-point in between? Yes.
> Is it always necessary to pass through the halfway-point? Yes.

In each of these versions, successive inductive premises each introduce a new halfway-point that the runner needs to reach. And of course, any potential induction will need to do this.

The two variants differ on a key issue: does the temporal framework of the runner *agree* or *disagree* with the temporal framework of the argument itself? In the first version, later and later premises introduce halfway-points that the runner needs to reach earlier and earlier. By contrast, in the second version, later and later premises introduce halfway-points that the runner needs to reach later and later. While either one of these reconstructions might be tweaked in various ways, it is clear that any possible induction will need to fall distinctly into one camp or the other.

In one sense, this contrast is well-known to scholars, and they typically call the first variety of Racetrack argument *regressive*, and the second variety *progressive*. There has been little consensus regarding which version Zeno used, and many scholars seem to regard the two variants as more or less interchangeable.[176] In fact, while scholars do

176 Barnes (1982), pg. 262, is representative of those who regard the distinction as unimportant. Hasper (2003), pg. 44, presents two common, and initially plausible, arguments that the Racetrack was progressive. The first rests on the fact that Aristotle calls the Racetrack "the same *logos*" as the Achilles *logos* (239b18-19), a *logos* that involves a runner getting closer and closer to his target, in this case another runner. In our later discussion of Aristotle's response to the Achilles, we will consider this claim in detail. The second argument for the progression relies on the second variant of the Racetrack that Aristotle reports in *Physics* Θ.8, the version that involves counting. On the surface, it might seem easier to construe the counting argument as involving a progression. But we will later examine the counting *logos*, and will find that it actually involved a regression.

recognize the regression/progression distinction, they do not realize that this distinction amounts to a difference between two different ways of formulating a dialectical induction. In fact, it turns out that the progressive induction simply would not work in a dialectical *logos*, and that the original was certainly regressive.

Consider first the progression. Successive inductive premises introduce later and later halfway-points for the runner to reach. Then comes the inductive conclusion: Is it always (ἀεί) necessary to pass through the halfway-point? The problem lies in the fact that there is a pernicious ambiguity in the temporal adverb *always*. In the progression, the temporal framework of the runner lines up with the temporal framework of the answerer. Both move into the future together. So, in principle, the answerer might construe the *always* as applying to the runner, or he might construe the *always* as applying to himself. And each choice yields a different answer to the given question.

If the answerer takes the *always* as applying to himself, then it functions something like a universal quantifier, as it typically does in an induction. The *always*-question literally asks: Is it *always* necessary to pass through the halfway-point? But when this question comes right after the inductive premises, and the answerer hears the *always* as applying to himself, then the question effectively becomes: Is it *always* necessary to answer *Yes* to questions like these? If the Racetrack is to be a viable *logos*, which it was, then we know that the answerer must reply *Yes*
to the concluding question as well.

The problem for the progression is that there is an alternative way to construe the

---

By contrast, both Simplicius (1013,7-10) and Philoponus (81,8-12) regard the Racetrack as regressive. This is interesting, since if these commentators were doing speculative reconstructions of their own, based solely on the text of the *Physics*, we might well expect them to favor the progression, for the reasons just given. So there may well have been an ancient tradition that the Racetrack was regressive.

always-question. The inductive premises involve the runner reaching halfway-points later and later in time. Next comes the question: Is it *always* necessary to pass through the halfway-point? If the answerer takes the *always* as applying not to himself, but to the runner, then he can reply: No, the runner doesn't need to pass through any more halfway-points once he reaches the end. And here the questioner is thwarted. The process of reaching halfway-points does *not* go on forever. It reaches an end once the runner reaches the end of the *stadion*. And remember that the *logos* establishes, in the very first question, that the runner does indeed reach the end of the *stadion*, so there is nothing wrong with the answerer adverting to this fact.

It might seem that there is an obvious way to fix this problem. Perhaps if the *always* were more directly presented as a universal quantifier then there would be no opportunity for the answerer to take it as a temporal adverb applying to the time of the runner. And the way to do this would be to explicitly fill in the scope of the *always*. So the *always*-question would ask something like: Is it *always* necessary, when a runner is starting from one place and trying to get to another, for the runner to pass through the halfway-point?

But we have already seen why this does not work. Remember that the inductive conclusion is followed by the question: Are the halfway-points unlimited? And we saw earlier that if this *unlimited*-question comes right after a question that carefully specifies the scope of *always*, then the answerer will easily reply that there is only *one* halfway-point that divides two halves. Indeed, it was precisely to avoid this scenario that we concluded the induction was necessary. But if we now explicitly fill in some scope for *always*, we nullify the effect of the induction, allowing the answerer to easily evade the *unlimited*-question. In fact, in some way, it seems that it is the very open-ended nature of *always* that makes the induction valuable, and it is this same open-ended nature that dooms the progression. It seems that *always* is a curious word, and we will later go on to examine it in more detail. But for now, it seems quite likely that the progressive Racetrack would make a miserable *logos*.[177]

By contrast, the regression faces no such problems. The inductive premises present halfway-points that the runner needs to reach earlier and earlier. Hence there is no way to hear the *always* as applying to the time of the runner, and so no way to construe the *always*-question as asking if the runner must *endlessly continue* reaching halfway-points, a question to which the answerer has a ready rejoinder. Instead, the *always* can apply only to the time of the argument itself, where it will function like a universal quantifier. So the inductive premises ask about halfway-points, and then comes the question: Is it *always* necessary to pass through the halfway-point? And here, in the absence of some philosophically acute objection, the answerer seems forced to say *Yes*.

So the regressive induction seems to work, and the progressive induction does not. It follows that the original Racetrack must have involved a regressive induction, which would accord with the apparent ancient tradition that the *logos* was regressive.[178]

The Original Racetrack?

At this point, we do know that the induction was regressive. However, we have not paid careful attention to the actual wording of the regressive premises. Since we lack direct evidence, I have thus far simply concocted one seemingly plausible regression,

---

177 Some readers might wonder if my argument here is too strong, inasmuch as the Achilles *logos* does indeed resemble the progressive Racetrack. But the crucial difference is that the Achilles involves a runner pursuing a *moving target*. We will examine the Achilles in Chapter 5.

178 See above, footnote 179.

with each new premise iterating the prior in a straightforward fashion. But I suspect this will not work.

The problem lies in the way that each new halfway-point is introduced. As things stand, each inductive premise after the first introduces a halfway-point different from the halfway-point introduced in the prior premise. But it is not at all clear that the existence of this new halfway-point should be taken as unproblematic.

Consider that there is a large difference between a scholar interested in the Racetrack today, and a dialectical answerer facing the *logos* for the first time. To a scholar today, it is fairly obvious that "the Racetrack paradox" involves some infinitude of halfway-points. By contrast, for a novice answerer, there is no such thing as "the Racetrack paradox". Instead, there is simply a series of questions about an athletic event, an event that no one would naturally think involves more than one halfway-point. Indeed, why should anyone think that anything involves more than one halfway-point? Now consider the question at issue:

> But before he reaches *this* halfway-point, must he first reach the halfway-point before *it*?

To a novice answerer, I suspect it would be less than obvious what this halfway-point before the halfway-point even is.

We might consider two alternate ways of making the transition from the initial halfway-point to the new halfway-point. First, there is the way we are already considering:

> Must he first reach the halfway-point before the end? Yes.
> But before he reaches *this* halfway-point, must he first reach the halfway-point before *it*? Yes.

In this setup, the very existence of the new halfway-point is introduced at the same time as the answerer is forced to make a judgement *about* the new halfway-point. Alternatively, we might suppose that the existence of the new halfway-point is first adverted to in a separate question. So we might have a series like this:

> Must he first reach the halfway-point before the end? Yes.
> Now in between the start of the *stadion* and the halfway-point, is there another halfway-point? Yes.
> And must the runner reach the earlier halfway-point before the latter? Yes.

In contrasting these renditions, we can temporarily set aside questions about historical justification and about precise wording. Indeed, focus on the question of whether or not the new halfway-point is introduced in the same question as that which forces a judgement about it.

We have already seen that there is something potentially unnatural about speaking of the new halfway-point as if it's existence is obvious. But the situation seems quite different if the answerer is forced to *explicitly* consider the existence of the new halfway-point. If he is actually *asked* whether there is a halfway-point between the other points, then it seems that he has little choice but to say *Yes*. It certainly seems that he would need an acute objection to maintain otherwise.

It thus seems likely that each question, other than the first, about reaching a halfway-point, will be preceded by a question about the existence of that halfway-point. In principle, there seems little inherent reason to doubt the existence of these additional premises. Indeed, we have already seen that *logoi* likely involved many questions beyond those that Aristotle would construe as the logical core, the sort he typically discusses. Our current additions, for instance, might reasonably be described as added "for the *logos* to be more clear." (*Topics* 155b23-4)

Indeed, this interpretation will gain even more plausibility when we examine the Achilles *logos*. There we will find that the Achilles involved a series of inductive questions about reaching places, and that each was very likely preceded by a separate question involving the existence of that place. And there the existence of the prior questions will be a near certainty, even if, using reasoning much like that employed here, we will need to justify the claim that these questions invoke the existence of a new place.

On the whole, it seems that in addition to the inductive questions about reaching halfway-points, there is good reason to believe in additional inductive questions about the existence of halfway-points. But there is little certainty regarding the exact nature of these additional questions.[179]

Not only are we ignorant, but we are ignorant of the extent of our ignorance. In principle, the precise formulation of these additional inductive questions might not matter much at all. Remember that every performance of the Racetrack would have differed from the others. So perhaps the precise formulation of these additional questions was among the things that varied. But that is not something that we should decide *a priori*.

Consider one way in which we might be going astray. As I am writing the existence premises, they involve halfway-points that are *between* other points, and they do not involve any mention of half-segments, such as half-magnitudes or half-motions, that are actually getting *divided*. Now I think that this is a plausibly conservative rendition, since the rest of the *logos* does involve points standing in relation to other points, and does not explicitly involve any half-segments. But the truth is that I am

179 Zeno fragment DK29B3 has some affinity with the line of questioning I propose here, inasmuch as it emphasizes the existence of "beings" between other "beings", *ad infinitum*.

significantly inclined towards this rendition since I think that much of the later Aristotelian emphasis on half-segments is a result of the influence of diagrams, which seem to play no explicit role in the original Racetrack. Now I may be right about this, but there is still no direct evidence for my reconstruction of the existence premises, and I have been wrong many times.

Despite our ignorance, I think that we should still try to produce something that looks like a full psychologically plausible dialectical *logos*. A *logos* is not a chain of propositions, but a pattern of human interaction, and we cannot really get a sense of this interaction if we omit steps because we do not know all the details. At the same time, when we go on to draw conclusions from the reconstruction, we must be careful to base our arguments on the elements of the reconstruction most grounded in the evidence.

With these caveats in mind, I suggest that the Racetrack *logos* looked much like this:

> Can a runner get through a *stadion*? Yes.
> Must he first reach the halfway-point before the end? Yes.
> Now in between the start of the *stadion* and the halfway-point, is there another halfway-point? Yes.
> And must the runner reach the earlier halfway-point before the latter? Yes.
> And in between the start of the *stadion* and this new halfway-point, is there another halfway-point? Yes.
> And must the runner reach the earlier halfway-point before the latter? Yes.
> And in between the start of the *stadion* and this new halfway-point, is there another halfway-point? Yes.
> And must the runner reach the earlier halfway-point before the latter? Yes.
> Is it always necessary to pass through the halfway-point? Yes.
> Are these halfway-points things unlimited? Yes.
> Is it possible to pass through things unlimited? No.
> And so you say that the runner cannot get through the *stadion*!

I have here simply incorporated my version of the existence premises into the earlier reconstruction.

On the whole, it seems that we have a plausible and powerful *logos* here. If we read through it, line by line, and imagine ourselves in the position of a dialectical answerer, it does indeed seem that the apparent answer to each question is precisely the answer that the questioner needs. And that is the sign of an effective dialectical *logos*.

So we have now more or less completed the process of reconstruction. What remains, among other things, is to examine this thing that we have reconstructed, the *logos* itself. What does it tell us? How does it work? We will shortly take up these questions, but not quite yet. Instead, we will now turn our attention to the Achilles, a *logos* in many ways like the Racetrack. As it turns out, the study of the Achilles will help us to understand how the Racetrack works.

Chapter 5
The Achilles

The Achilles and the Racetrack

As we begin the examination of the Achilles, we might reasonably ask why we should undertake it at all. Or at least, why we should undertake it now, in the midst of a long study of the Racetrack. The reasons are threefold.

First, we are now in a position where we can rather easily reconstruct the Achilles. As is well-known, Aristotle calls the Achilles “the same *logos*” (239b18-19) as the Racetrack, a claim that we will later need to examine in detail. But we will soon find that the Achilles indeed involves an induction, an induction in many ways like the induction we have now discovered in the Racetrack. As the issues involved in reconstructing the two *logoi* are similar, the Achilles reconstruction benefits greatly if we do it now.

Still, we might wonder whether the Achilles will help us to understand the Racetrack. Indeed it will. What we will find is that the Achilles *logos* depends for its dialectical success on the temporal experiences of the answerer. The same thing turns out to be true of the Racetrack as well, but with the Racetrack, this is not initially so evident as it is with the Achilles. Thus, our examination of the answerer's temporal experience in the Achilles will prepare the groundwork for our examination of the same issue as regards the Racetrack.

In a broader sense, the study of the Achilles belongs in the midst of the study of the Racetrack so long as we are concerned with the way in which Aristotle himself critiques the two *logoi*, which we are. As we have already seen, Aristotle's initial critique of the Racetrack, in *Physics* Z.2, relies heavily on diagrams. We will later examine this critique in light of our past and future study of the Racetrack itself. Indeed, it will eventually turn out that this initial Racetrack critique will figure in interestingly different ways in two later portions of the *Physics*. In particular, it will figure in both the critique of the Achilles, found in *Physics* Z.9, and in the revised critique of the Racetrack, found in *Physics* Θ.8. What we will find is that in the Achilles critique, Aristotle sees himself as employing a technical method, and indeed, he seems captivated by it. By contrast, in the revised critique of the Racetrack, we will find Aristotle undertaking what amounts to an examination of the ontological implications of this method itself. So in the Achilles critique, he employs the method, ingeniously, if uncritically, but in the revised Racetrack critique he steps back to examine the procedure itself. At the very least, this contrast is fascinating, and so on the whole, I think it is worth our while to take up the Achilles even as we still study the Racetrack.

Aristotle's only discussion of the Achilles is found in *Physics* Z.9. Recall that Aristotle reports the Racetrack as the first of four Zenonian motion *logoi*. Then he continues:

> δεύτερος δ’ ὁ καλούμενος Ἀχιλλεύς· ἔστι δ’ οὗτος, ὅτι τὸ βραδύτατον οὐδέποτε καταληφθήσεται θέον ὑπὸ τοῦ ταχίστου· ἔμπροσθεν γὰρ ἀναγκαῖον ἐλθεῖν τὸ διῶκον ὅθεν ὥρμησεν τὸ φεῦγον, ὥστε ἀεί τι προέχειν ἀναγκαῖον τὸ βραδύτερον. ἔστιν δὲ καὶ οὗτος ὁ αὐτὸς λόγος τῷ διχοτομεῖν, διαφέρει δ’ ἐν τῷ διαιρεῖν μὴ δίχα τὸ προσλαμβανόμενον μέγεθος. τὸ μὲν οὖν μὴ καταλαμβάνεσθαι τὸ βραδύτερον συμβέβηκεν ἐκ τοῦ λόγου, γίγνεται δὲ παρὰ ταὐτὸ τῇ διχοτομίᾳ (ἐν ἀμφοτέροις γὰρ συμβαίνει μὴ ἀφικνεῖσθαι πρὸς τὸ πέρας διαιρουμένου πως τοῦ μεγέθους· ἀλλὰ πρόσκειται ἐν τούτῳ ὅτι οὐδε τὸ τάχιστον τετραγῳδημένον ἐν τῷ διώκειν τὸ βραδύτατον), ὥστ’ ἀνάγκη καὶ τὴν λύσιν εἶναι τὴν αὐτήν. τὸ δ’ ἀξιοῦν ὅτι τὸ προέχον οὐ καταλαμβάνεται, ψεῦδος· ὅτε γὰρ προέχει, οὐ καταλαμβάνεται· ἀλλ’ ὅμως καταλαμβάνεται, εἴπερ δώσει διεξιέναι τὴν πεπερασμένην.
>
> And second is the so-called Achilles. And it is this, that the slowest, in running, will never be caught by the fastest. For necessarily, the pursuer must first reach

> [the place[180]] from which the pursued started, so that necessarily, the slower will always keep somewhat ahead. But this is the same *logos* as with the bisecting, but it differs in dividing the added magnitude not *half*-way. By all means, not catching the slower follows from the *logos*, but it comes about on account of the same thing as the bisection (for in both [*logoi*] not arriving at the end follows, with the magnitude being somehow divided), but it is added in this [*logos*] that not even the fastest made famous in a tragedy [will reach the goal] in pursuing the slowest, so that necessarily the *lusis* is also the same. And moreover, maintaining that the one keeping ahead is not caught is a fallacy. For *when* he keeps ahead, he is not caught, but nonetheless he [i.e. the slower] is caught, if he [i.e. the original answerer] will grant that [he, i.e. the slower] passes through the limited [line]. (239b14-29)

In our earlier discussion of *Physics* Z.9, we saw that in discussing the four motion *logoi*, Aristotle follows a format of first recounting a *logos*, and then proceeding to provide an analysis. And we certainly see this in the Achilles passage, Aristotle's analysis apparently commencing with the claim that the Achilles is “the same *logos*” as the Racetrack. (239b18-19)

This division between the report and the analysis provides the framework for an interpretive strategy. We concluded earlier that the Achilles, like the Racetrack, was a dialectical *logos*. So we should use the first part of Aristotle's discussion (239b14-18) as evidence in reconstructing the actual dialectical structure of the Achilles, much as we reconstructed the Racetrack. We have also seen that Aristotle's analysis of the Racetrack hinged on interpreting the *logos* in ways quite foreign to it's original use, in particular, on rendering it amenable to understanding via diagrams. So if Aristotle thinks the Achilles and the Racetrack are “the same *logos*”, and require the same *lusis* (239b25-6), then we might well expect that he will be analyzing the Achilles via diagrams, much as he did the Racetrack. So in reconstructing the original Achilles, we should be skeptical of relying

180 I discuss this insertion below.

on the things that Aristotle says in his analysis. (239b18-29) But once we have done the reconstruction, we can then go on to consider how Aristotle submits the *logos* to his method. In this chapter we will begin with the first task, the task of reconstructing the Achilles.

Reconstructing the *Logos*

In reconstructing the Achilles *logos*, we can follow the same basic strategy as we did with the Racetrack. There, we started with Aristotle's report in *Physics* Θ.8 and then converted it into a series of questions and answers. And then we examined these questions and answers, modifying them until we obtained a viable *logos* that meshed with the evidence. We can do the same once again.

To begin, we can break up Aristotle's report into two parts. He first reports a conclusion: "that the slowest, in running, will never be caught by the fastest." (239b15-16) Then he seems to justify this conclusion by appeal to an inference that itself contains a premise and a conclusion: "For necessarily, the pursuer must first reach [the place] from which the pursuer started, so that necessarily, the slower will always keep somewhat ahead." (239b16-18)

If we convert this to dialectical form, with the conclusion coming at the end, being attributed to the answerer by the questioner, then we get two questions and an assertion. But we also know that the negation of the conclusion needs to begin the *logos*, so we get:

> Will the slowest, in running, be caught by the fastest? Yes.
> Must the pursuer first reach the place from which the pursued started? Yes.
> So the slower will always keep somewhat ahead? Yes.
> And so you say that the slowest, in running, will never be caught by the fastest!

In rendering this *logos*, I have tried to convert Aristotle's assertions into questions as directly as possible. But in the second question, the word "place" is not derived from Aristotle's own text. In fact he does not tell us *what* it is "from which" (ὅθεν) the pursued started. To produce readable English, translators often insert some substantive word into the translation, and plausible options include place, point, spot, and position.[181] But the particular choice of English word seems to have little significance for the interpretation of Aristotle's own discussion.

Things may be more interesting when we consider the actual dialectical *logos*. We need to ask: From what sort of thing did the pursued start? Ultimately, we need to answer not with an English word but a Greek word. And here the most likely option seems to be τόπος (place). We have evidence that Zeno employed an argument involving τόπος, and it seems to hinge upon a clever distinction between a thing and its place.[182] And roughly speaking, this distinction seems to be at work here again in the Achilles. We have no certainty here, but lacking evidence for distinct alternatives, it does seem that τόπος was the word most likely used by Zeno, and most likely also by the dialecticians of Aristotle's day.[183]

Thus far, my wording of the *logos* closely follows Aristotle's text. But as we saw with the Racetrack, there may be differences between the way that the *logos* was actually

181 Sachs uses "place", whereas Ross (416), Faris (26), and Hardie/Gaye use "point". Simplicius (1014, 11-13) uses πέρας (limit) in recounting his version of the *logos*, but he seems to be doing reconstruction of his own.

182 Aristotle writes: "For Zeno's puzzle demands some explanation: For if every being is in a place, clearly there will be a place of the place, and so on *ad infinitum*." (*Physics* 209a23-25, cf. 210b22-24).

183 Either σημεῖον or στίγμα, both meaning "point", might seem alternatives to τόπος. But there is little evidence that Zeno, in the mid-fifth century, would have used the word to mean anything like "geometrical point". In fact, σημεῖον originally meant "sign", as it does in Parmenides (DK 8.1-4), and στίγμα originally meant "mark", i.e. *visible* mark. Each word acquired the sense "geometrical point" because each was used to refer to the *letters*, i.e. the signs and marks, that stand next to points in geometrical diagrams, so that each word eventually came to refer to the points themselves. (This if fairly clear from reading Netz, although he does not discuss the issue directly. See pp. 46, 97, 109, 113.) But recall that Aristotle is our earliest source for lettered diagrams, and it is quite unlikely that the aforementioned semantic shifts had occurred in Zeno's day.

used in dialectic and the way that Aristotle reports it. In fact, we have seen that Aristotle systematically converts concrete *logoi* into abstract form when he writes, and he is certainly doing this with the Achilles.

The Achilles *logos* was evidently named for Achilles the hero, who is repeatedly called "swift-footed Achilles" in the *Iliad*[184], and might reasonably be called "the fastest made famous in a tragedy". (239b24-25) But in one famous episode, his speed is put to the test: Achilles three times chases Hector around the walls of Troy. In fact, Hector is only "caught" because Athena, in disguise, persuades him to turn and fight.[185]

The identity of Achilles as "the fastest" (239b16) is widely accepted.[186] But comparison with the *Iliad* also suggests something that is not so widely recognized: the *logos* involves not a race, but a chase. The difference is that in a race, one runner seeks to traverse some fixed distance, or to reach some fixed endpoint, before another. But in a chase, one simply needs to catch the other. In fact, Aristotle directly speaks of the pursuer (τὸ διῶκον) and the pursued (τὸ φεῦγον), using the same words as Homer.[187] Aristotle's audience will surely hear this Achilles reference against the backdrop of the famous pursuit of Hector.

By contrast, when Aristotle first reports the *logos* (239b14-18), there is no hint of a race. And yet scholars often treat the *logos* as if it does involve a race.[188] They are no

184 The first epithet of this sort is at I.84.
185 Cf. *Iliad* XXII.224-231, 247-253.
186 One possible exception is Aquinas, who thinks the *logos* is called *Achilles* due to the strength of the *logos* itself. (864) But there may be some truth to this as well.
187 *Iliad* XXII.157
188 This interpretation is found in Themistius [200,1-8] and Simplicius [1014,23-1015,2], who use an identical scenario in which Achilles and a tortoise are running a *stadion*, the tortoise has a half-*stadion* head start, and Achilles has a speed ten times that of the tortoise. But each commentator makes clear that he is setting up a hypothetical scenario, and neither attributes this setup to Aristotle or Zeno. (But each *does* make a historical claim about the tortoise itself. See below.) Among modern scholars see Barnes, pp. 273-4, and Faris, pg. 26 *et seq*.

doubt assimilating the Achilles to the Racetrack, which involves a fixed racetrack, and doing so because Aristotle claims that the two *logoi* are the same. (239b18-19) We will later examine what Aristotle actually means by this. But for now, it seems clear that when we consider Aristotle's report of the *logos* (239b14-18), as opposed to his analysis (239b18-29), and read the report in light of the *Iliad*, we must conclude that the *logos* involves Achilles chasing someone, and not racing.

Aristotle does not tell us who Achilles chases, and indeed no one does until Themistius in the 4th century A.D. He has Achilles chasing a tortoise[189], and we find this same scenario in Simplicius.[190] Each commentator is clearly making a historical attribution.[191]

Given the late date of our evidence, we might be skeptical about the combination of Achilles and the tortoise. But I see little reason to doubt that the *logos* as Aristotle knew it typically involved these very characters. In fact, the place of Achilles is secure, and the only question is who he chases. Certainly not Hector, as Achilles *already* has difficulty catching Hector. And were Achilles to chase some other named hero, or even a named creature, this would place the *logos* in the realm of legendary speculation, and thus open it to irrelevant objections. So he must chase someone or something anonymous, and

---

189 199, 24-25

190 1014, 3-9 *et seq*. And see 1014, 22-23, where Simplicius compares Achilles' unsuccessful pursuit of Hector with his unsuccessful pursuit of the tortoise.

191 In fact, both Themistius and Simplicius seem to make a historical attribution to Zeno in particular. Much of what they attribute to Zeno is misguided, inasmuch as neither attributes to Zeno a dialectical *logos*. But each does distinguish his own version of the abstract structure of the argument from the fact that “the slowest” and “the fastest” are the tortoise and Achilles. And in making these identifications, each commentator uses a verb that shows that the identification came from some earlier source. Themistius uses “he says” (φησίν) (199,24), and Simplicius writes: “he establishes (κατασκευάζει) ... taking (λαμβάνων) the tortoise as slowest ... and Achilles as fastest.” (1014,4-7) That Zeno himself was the source of this pairing may well be conjecture, however reasonable, on the part of the commentators. But it seems a simple fact that each regarded the pairing as originating from someone prior to themselves.

since humans have names, most plausibly an anonymous animal, the slower the better. In fact, the tortoise is a paradigmatically slow creature in Greek folklore.[192]

Many philosophers no doubt regard Achilles and the tortoise as logically irrelevant. But they are quite interesting if we recall the social setting of dialectic. Today, philosophers interested in paradoxes are likely to see Achilles as a hapless character in a topsy-turvy world, indeed like the Achilles of Lewis Carroll.[193] But in the *Iliad*, Achilles is a proud hero, beloved of Zeus, and justice in the world is for Achilles to get his due. When he is slighted by Agamemnon, the wrath of Achilles makes countless heroes a feast for dogs, and that is the *whole story* of the *Iliad*. To suggest, even to *prove*, that Achilles cannot catch a *tortoise*... . That is ludicrous. Such a *logos* is no mere intellectual puzzle; it has a visceral resonance. Plato depicts Zeno as human and witty[194], and the combination of Achilles and the tortoise fits that image well.[195]

---

192 Aesop's fable of the tortoise and the hare is well-known. See also footnote 16.

193 In Carroll's dialogue, *What the Tortoise Said to Achilles*, the tortoise befuddles Achilles with infinite regress arguments. Douglas Hofstadter's widely read *Gödel, Escher, Bach* follows in this tradition,containing a whole series of dialogues involving Achilles and the tortoise, who dwell in an Escherian world.

194 See especially *Parmenides* 28a4-e4.

195 In Plutarch, we find an Achilles-style argument that might seem to cast doubt on the pairing of Achilles and the tortoise. In Plutarch's version, a tortoise outruns, not Achilles, but "the fast horse of Adrastus". (*On Common Conceptions*, 1082E) Of course, inasmuch as the Achilles, in Aristotle's day, was called the *Achilles*, we should presume that the *logos* in that era typically did involve Achilles and not a horse.

In fact, Plutarch does not present himself as offering a historical argument. Instead, he himself employs the Achilles-variant for direct polemical effect, claiming that it shows the absurd consequences of the Stoic belief that distances are infinitely divisible. His choice of characters has it's roots in Greek folklore, inasmuch as a race (not a chase) between a tortoise and a horse figures in one version of the well-known Aesopic fable of the tortoise and the hare. (*Corpus Fabularum Aesopicarum* I; Book 2; 130,17-131-21)

As Plutarch is employing the Achilles-variant for polemical effect, and not as a dialectical *logos*, it would make sense for him (and perhaps some earlier philosophers) to swap the incongruous pairing of Achilles and a tortoise for the more natural and familiar pairing of two animals. The "fast horse of Adrastus" figures in many mythological contexts, among them the *Iliad* (23.346-7), and actually had a name, Arion. Plutarch's failure to mention his name suggests that Arion is simply being inserted into the stock tortoise-horse pairing as an instance of a particularly fast horse.

On the whole, the incongruity of the tortoise-Achilles pairing would be most relevant in a social dialectical setting, whereas the more generic tortoise-horse pairing would fit better with a scholastic appropriation of the *logos*, which is what we find in Plutarch.

Considering the evidence so far, I suggest the following revision of the *logos*:

If a tortoise runs from Achilles, can Achilles catch him? Yes.
Must Achilles first reach the place from which the tortoise started? Yes.
So the tortoise will always keep somewhat ahead? Yes.
And so you say Achilles will never catch him!

This rendition incorporates three changes: I have replaced references to the fastest and the slowest, the pursuer and the pursued, by references to Achilles and the tortoise. I have made the scenario more evidently a chase, as opposed to a generic "running". And I have changed references to the tortoise being caught, or not, by references to Achilles, the dominant personality, catching him, or not.

We now need to consider whether this *logos* actually works, that is, whether the answerer will seem compelled to give the answer we know he must give. Looking at the first question, it does seem that Achilles can catch the tortoise. And then, in doing so, it does seem that he must first reach the place from which the tortoise set out.

Now ignore, for the moment, the question of whether the answerer should affirm the third question, admitting that the tortoise is always ahead. Suppose, instead, that he does admit this. It does then seem plausible, and perhaps even necessary, to maintain that since the tortoise is always ahead, Achilles will never catch him, just as the final question demands. Of course, someone might wish to object to this, as indeed Aristotle himself does (239b26-29), suggesting that the *always* needs to be understood in some restricted sense. We will later examine this issue in some detail. But for now, the very fact that Aristotle brings an objection like this is a sign that the original *logos* did involve such a transition form the *always*-question to the *never*-question.

Note that in explaining the Achilles-tortoise pairing, Simplicius cites the tortoise-horse fable as the source for the tortoise, alongside Homer as the source for Achilles. (1014,3-9)

So far, three out of four questions pose no trouble. But the remaining question, the *always*-question itself, is more of a problem.

Another Induction

In our *logos* as it stands, the answerer affirms the second question and admits that Achilles, before he catches the tortoise, must first reach the place from which the tortoise set out. And then the questioner responds: "So the tortoise will always keep somewhat ahead?" We know that the answerer needs to respond, *Yes*. But there seems no reason why he should. Indeed, it seems that an intelligent answerer will reply: No, first Achilles reaches the place from which the tortoise set out, and then he catches the tortoise. And thus the questioner has failed to establish that the tortoise is always ahead.

This need not surprise us, as we faced a quite similar problem in reconstructing the Racetrack. There, as here, we found that the answerer was not plausibly compelled to affirm an *always*-question, even thought we knew he must. But we eventually decided that the *always*-question would function properly as the conclusion of an induction. The same is likely to be true with the Achilles, so we need to figure out just how the induction works.

Our putative inductive conclusion speaks of the tortoise *always* being ahead. So the inductive premises that support this conclusion ought to involve the tortoise being ahead in various particular instances. But as things stand, the only preceding premise involves Achilles reaching the place that the tortoise has left, seemingly a different issue. This leaves us with two options: We can either extend the Achilles-premise so that it also asks about the tortoise, or we can add a second distinct question.

In fact, the first option will not work. Consider this schema:

> Must Achilles first reach the place from which the tortoise set out, at which time [the tortoise has moved ahead]? Yes.

For the moment, I wish to leave it an open question exactly how the part about the tortoise might be worded, and so I enclose it in brackets. Regardless of the wording, if this combined question is to work, then the event of the tortoise moving forward needs to be temporally aligned with the event of Achilles reaching the tortoise's starting place. But, on account of the "first", the given movement of Achilles admittedly happens *before* Achilles catches the tortoise. And thus the corresponding movement of the tortoise also happens before Achilles catches the tortoise. If ensuing inductive premises emulate this one, as they should, then they will also involve events that admittedly happen before Achilles catches the tortoise. And so these premises would easily support the inductive conclusion that the tortoise is *always* ahead, *before* Achilles catches him. But it would seem quite easy for the answerer to avoid the broader conclusion that the tortoise is *always* ahead. In fact, we have already seen that Aristotle himself differentiates these issues, and yet he also indicates that many answerers find difficulty with the Achilles. (239b9-11) This suggest that most answerers do *not* find Aristotle's analysis to be obvious, and this means that there is a problem with our reconstruction.[196]

What we need is to avoid explicitly qualifying the tortoise's moving ahead as something that happens only before Achilles catches him. And so it might seem best to separate the two issues verbally, leaving the original Achilles question as it is, but adding a second question about the tortoise. Naturally, this means that each question will then be

196 Consider further that Aristotle himself seems to regard the *logos* as more difficult than the current rendition allows, inasmuch as his aforementioned reply is only a portion, indeed a subsidiary portion, of a broader analysis that involves comparing the Achilles to the Racetrack. As things stand, this broader analysis would seem superfluous.

iterated as the induction proceeds. So we get something like this:

> Must Achilles first reach the place from which the tortoise set out? Yes.
> [By then, has the tortoise moved ahead]? Yes.
> [Must Achilles reach the tortoise's place]? Yes.
> [By then, has the tortoise moved ahead]? Yes.
> [Must Achilles reach the tortoise's place]? Yes.
> [By then, has the tortoise moved ahead]? Yes.

As before, I am enclosing in brackets segments where I am making no attempt to get the details of the wording right, and this includes the second and ensuing inductive premises in their entirety. Indeed, at the moment, the precise wording of these premises seems subject to great uncertainty. But these details aside, the general strategy of the induction seems very likely correct.

Given that the third and the fifth inductive questions, and maybe others of the same sort, are modeled on the first, we should presume that they will emulate the first as closely as possible. But some differences are required. The first inductive question begins: Must Achilles *first* reach ... ? Here this *first* makes sense because the inductive question follows the opening question of the *logos*: If a tortoise runs from Achilles, can Achilles catch him? So the *first* really means *first, before Achilles catches the tortoise*. If the questions that emulate this initial inductive question are to properly emulate it, then they need to convey that the events at issue happen before Achilles catches the tortoise. Hence the induction will proceed something like this:

> Must Achilles first reach the place from which the tortoise set out? Yes.[197]
> [By then, has the tortoise moved ahead]? Yes.
> But before Achilles catches the tortoise, must Achilles first reach [the tortoise's place]? Yes.
> [By then, has the tortoise moved ahead]? Yes.
> But before Achilles catches the tortoise, must Achilles first reach [the tortoise's

197 In fact, it is quite possible that the initial inductive question, like those that emulate it, begins: Before Achilles catches the tortoise... .

place]? Yes.
[By then, has the tortoise moved ahead]? Yes.

So now we can ask: What exactly is it that Achilles needs to reach in these new questions? In the initial variant, it is "the place from which the tortoise set out". But in the initial question, this place is the tortoise's *starting* place. And if the exact same phrase is repeated in ensuing questions, it will still refer to the tortoise's starting place, his initial starting place, at least in the eyes of a competent answerer. But the questioner needs Achilles to be reaching a new place every time. To avoid this problem, each ensuing Achilles question needs to employ a phrase that clearly refers to a place different from any of the prior places. I suspect that we can actually obtain a highly plausible and interesting answer as to what this phrase might be, but it will take some detailed analysis to get there.

We can begin by dividing referring expressions into three categories: names, definite descriptions, and deictic expressions. In fact, we can define these terms so that the options are exhaustive. First, a name, I say, is a term that refers to it's referent independent of any other fact about the semantics of the language in which it occurs. Thus, "Zeno" is a name that refers to Zeno, and this reference is independent of any other semantic facts about English. By contrast, definite descriptions and deictic expressions only succeed at referring to their referents on account of independent semantic facts involving their component parts. If a term can succeed at referring to something independently of whether it's user is acquainted with the referent, then I say it is a definite description. For instance, "the discoverer of dialectic" is a definite description, and it refers to whatever it does refer to in part due to the meanings of "the", "discoverer", "of" and "dialectic". And in fact it apparently refers to Zeno, but, in principle, it would still refer to Zeno even if nobody knew who Zeno was. By contrast, deictic expressions, which include words like "this", "that", "you" and "I", also depend for their reference on the semantic properties of their parts, but they depend in addition on the fact that their user is acquainted with their referent. For instance, "this essay" refers to this essay, and this fact depends on the meanings of "this" and "essay", but also on the fact that I, the user of the phrase, am acquainted with this essay. I could not refer to an unknown essay in this way. Since successful reference either does or does not depend on additional semantic facts about the referring term, and either does or does not depend on a user's acquaintance with the referent, these options are the only ones available.[198] So we now face the question: which type of expression did successive premises of the *logo*s use to refer to the successive places that Achilles needs to reach?

First consider names. At first this option might seem ludicrous. Certainly the ancient Greek language contained names for many places: Ἀθῆναι, Δελφοί, Αἴγυπτος, etc. But it certainly did not contain names for whatever random places Achilles needed to reach, the places the tortoise got to first.

And yet things might seem more complex if we consider that most modern renditions of the Achilles do use names to refer to the series of places or points that

198 My definitions may or may not mesh with other treatments of these issues, but it does not matter so long as I use the terms consistently. In fact, indexicals like "you" and "I" are often distinguished from deictic terms. But as normally used, they clearly fall under the definition I have given for deictic expressions. One interesting exception might seem to be the word "you" as it occurs in media, such as writing, that reach persons unknown to the user. Now we might say that the "you" in question is a plural "you" that refers to some "audience" with which I am *somehow* acquainted. But if we insist that the persons are genuinely unknown, then "you" in such cases is not a referring deictic expression as I define it. This does not matter, since we could then construct theories under which it is a sort of placeholder name, or a definite description, or simply a non-referring term. In short, the classification of particular expressions does not affect the tripartite distinction I have set up.

Achilles needs to reach. They often do so by directly setting up a new system of names, for instance, C_1, C_2, C_3, ... , each of which refers directly to a point. Or else they somehow presume the existence of a well-known type of naming system, a coordinate system, which names points by n-tuples of real numbers.[199] Of course, English itself, like Greek, contains no names for whatever points Achilles needs to reach. And this means that anyone giving one of these modern renditions of the paradox needs to somehow *establish* that they are using a certain naming convention.[200]

In principle, it might then seem that the dialectical Achilles could also use names to refer to the places that Achilles needs to reach, so long as the *logos* likewise establishes that the names do indeed refer to the relevant places. But it does not seem to do this. Thus far, we have a plausible reconstruction of the *logos* derived from the evidence that Aristotle provides. And we have a series of slots into which we need to insert terms referring to places. We might insert names into those slots, but then we would additionally need to somehow add a whole new element to the *logos*, an element that somehow establishes what the names refer to, an element for which we possess no evidence at all.[201] So the lack of evidence weighs heavily against names.

In fact, the use of names is inherently implausible on other grounds as well. A

199 Barnes (274) exploits the first method, naming the target points C_1, C_2, ... and not relying, in his argument setup, on the existence of a coordinate system. By contrast, Vlastos (201-2) employs the second method. He introduces no overt names for the target points, but his use of numerical variables makes geometrical sense only in a context where the distance in question has *some* coordinate system assigned to it.

200 Naming systems, including coordinate systems, will often involve syntactically complex names. So the question of what thing a name does refer to may depend on the structure of the name itself. Hence a naming system might itself seem to have a certain semantics, which might seem to undermine the idea that it is a *naming* system. But this semantic system will be utterly closed, and unable to interact with the broader natural language, say English, in which the naming system is employed. It is this disconnection between the semantics of the naming system and the semantics of the natural language that ensures that it really is a naming system.

201 Indeed, it is questionable whether anyone in the fifth or fourth centuries BC *ever* set up a naming system. Remember that diagram letters were not names, and certainly not in Aristotle.

modern writer or speaker can simply posit that names mean whatever they need to mean to make an argument work. By contrast, this will not do in a dialectical *logos*. A questioner cannot simply posit anything: he always needs to gain the assent of the answerer. But if a questioner puts forth the proposition that some potential name refers to something that it does not in fact normally refer to, there is no reason for the answerer to agree. On the whole, it thus seems rather unlikely that names of places played any role in the dialectical Achilles.

If not names, what about deictic expressions? The expressions in question need to refer to places, so the most plausible deictic expression would be something like "this place". In this scenario, premises about the tortoise would need to specifically mention some place that the tortoise reaches. And then the ensuing premises about Achilles would use "this place" to refer back to that very place. For instance:

> Must Achilles first reach the place from which the tortoise set out? Yes.
> But by then, hasn't the tortoise moved to some place up ahead? Yes.
> But before Achilles catches the tortoise, must Achilles first reach this place? Yes.

If we are being charitable, we might well allow that "this place" refers to the place that the tortoise reached. But dialectical answerers are not charitable, and in fact there is another option: perhaps "this place" refers to the place at which Achilles catches the tortoise. And this permits an easy answer: No, Achilles reaches the *place* where he catches the tortoise *at the same time* as he catches him, not before.

The only way to avoid this problem would be to add qualifications to "this place" so that it clearly refers only to the place mentioned in the prior premise, and not to the place most recently, if tacitly, evoked. Thus we might have "this place the tortoise has just reached". But now we are effectively turning our deictic expression into a definite description, so we ought to consider definite descriptions on their own merit.

A definite description will pick out it's referent via the properties of the referent. So we can ask: what properties are available for the Achilles-question to invoke in a definite description that refers to a place? In fact, only two properties have any plausibility: First, the desired place would seem to be the place that the tortoise *has reached*. Second, the same place would seem to be the place that the tortoise *is at*.

Actually, this is not strictly true. We might allow for all kinds of second-order variants of these properties. For instance, the place in question is *the place that the tortoise admittedly reached in the prior question*, and likewise *the place at which he is, ex hypothesi, located*, and so on. But it would surely be a foolish questioner who employed such circumlocutions if he could avoid them.

So now consider the two possible sorts of Achilles-questions. We might have something like:

> But before Achilles catches the tortoise, must Achilles first reach the place that the tortoise just reached? Yes.

Alternatively, we might have:

> But before Achilles catches the tortoise, must Achilles first reach the place where the tortoise is now? Yes.

Obviously, there might be variants of each question, but the fundamental difference is clear. The first question involves a place that the tortoise has reached, and the second a place where he is. Thus the first question involves the past, and the second involves the present.

Now if we look at time from some objective standpoint, the past, the present, and the future are not much different. They are merely different regions along some sort of timeline, or something like that. But is this the natural way of thinking? It seems rather that this is something you need to force yourself to do, if you are going to do it at all. For most people, for most of the time, the past, present and future are qualitatively different. In particular, the past is the realm of memory, and the future the realm of possibility.

In both variants of the Achilles-question, the real issue concerns the future. The answerer needs to decide whether one possible event must happen before another. The past is irrelevant. And yet the first variant question introduces the past by talking about what the tortoise *has done*. If the answerer, and the audience, are to make sense of the question, they need to think about some temporal sequence that involves the past and the future together. That is, they need to think about the sequence in an abstract, detached manner, and not think about the past and future as *their* past and future. Now I suspect this would not be a natural standpoint for a typical audience in Zeno's day, or even Aristotle's, but for now maybe that is a speculative claim. More convincing, I think, is the fact that it is totally against the interest of the questioner to have anyone thinking about the chase from a temporally objective standpoint. If we think about things like this, then it seems obvious that Achilles gets closer and closer, and then *catches the tortoise*. A *logos* that allows us to avoid temporal objectivity would seem much more effective.

In fact, the second variant question allows just that. It situates the answerer, and the audience, in the present, and asks them to consider some possibilities. There is no need to objectify these possibilities and align them with the hard facts about the past. The answerer and the audience can view the future as *their* future, an open realm in which things might or might not happen as expected, in which Achilles might or might not catch the tortoise.

So: does the *logos* enforce a standpoint of temporal objectivity? Or does it require only a personal view of time? Certainly the second claim seems preferable. Thus far the case rests significantly on the undesirability of the objective view. But we will soon see that the personal temporal standpoint is actually essential in both the Achilles and the Racetrack. Indeed, it is with a view towards this upcoming discussion that I have chosen to render the Achilles-question in precisely the manner as I did, involving "the place where the tortoise is *now*". Of course, we have no direct evidence of the actual Greek wording here, which may well have varied. But I think that the word "now" (νῦν) is even more suggestive of the *actual* present then is the unadorned present tense.[202]

To complete our reconstruction of the *logos*, we still need to face one comparatively minor issue. So far we have been examining the Achilles-questions of the induction. But each of these Achilles-questions (again, ignoring the first) follows a tortoise question. So what do these tortoise- questions look like? There seem two basic options:

> But by then, hasn't the tortoise moved some amount ahead? Yes.
> But before Achilles catches the tortoise, must Achilles first reach the place where the tortoise is now? Yes.

Alternatively:

> But by then, hasn't the tortoise moved to some place up ahead? Yes.
> But before Achilles catches the tortoise, must Achilles first reach the place where the tortoise is now? Yes.

That is to say, the tortoise questions either do or do not overtly mention some place that the tortoise reaches.

---

202 In fact, the word νῦν plays an essential role in Zeno's Arrow *logos*. (*Physics* 239b5-9) Jonathan Lear (1981) is one of the few scholars to argue, correctly I think, that the argument hinges on the presentness of νῦν. By contrast, many scholars thinks νῦν, in the Arrow, simply refers to an instant, present or not.

In examining these two alternatives, we can recall what we found in the Racetrack. There we decided that each question (after the first), about reaching a halfway-point, was preceded by a question about the existence of that halfway-point. We can view the paired tortoise and Achilles questions along the same lines. In each of the alternative pairs, the initial tortoise-question introduces a new place for the tortoise, in one case implicitly, in the other explicitly. And then the ensuing Achilles-question makes that place a goal for Achilles. So just as halfway-points were introduced as halfway-points and then made end-points, so the tortoise-questions likewise introduce entities (places) that will each immediately be employed in a way different from that in which it was introduced. Given that this entity will undergo a shift in guise from one question to the next, it is crucial that the existence of the underlying entity not be in dispute. So it follows that the places should be introduced overtly, as in the second alternative, and not tacitly, as in the first.

If this reasoning is correct, then we end up with the following *logos*:

> If a tortoise runs from Achilles, can Achilles catch him? Yes.
> Must Achilles first reach the place from which the tortoise started? Yes.
> But by then, hasn't the tortoise moved to some place up ahead? Yes.
> But before Achilles catches the tortoise, must Achilles first reach the place where the tortoise is now? Yes.
> But by then, hasn't the tortoise moved to some place up ahead?
> But before Achilles catches the tortoise, must Achilles first reach the place where the tortoise is now? Yes.
> But by then, hasn't the tortoise moved to some place up ahead?
> So the tortoise will always keep somewhat ahead? Yes.
> And so you say that Achilles will never catch him!

And so it now seems quite plausible that we have reconstructed the Achilles in something close to its original form.

Always

When we examine the reconstructed *logos*, there might seem to be a problem. The tortoise-questions in the induction each ask about a particular place. But the putative inductive conclusion, that is, the *always*-question, asks whether the tortoise is always ahead, mentioning no place at all. This might seem a problem, in that the *alway*-question will not seem to provide an actual universalization of the inductive tortoise-questions, which it is seemingly supposed to do.

In one sense, this critique is surely accurate: the inductive conclusion is not a strict universalization of the tortoise-premises. But this need not be a problem. We can understand the relation of the inductive conclusion to the prior inductive premises not primarily as a logical relationship, but as a temporal relationship. Indeed, the inductive conclusion will seem quite fitting if we consider the actual temporal experience of the answerer.

It might seem that any attempt to grasp the experience, temporal or otherwise, of the answerer, must be highly speculative, and indeed misguided. But we must make this attempt if we are to have any hope of understanding the *logos*. And we can actually point with confidence to some ways in which we know the answerer's experience of the Achilles must have differed from our own experience of the same *logos*.

For one thing, when we consider the Achilles *logos*, we are liable to see it *in toto*. Indeed, I always clearly demarcate it as a single unified thing by printing it isolated in the middle of the page. We are liable to try and understand it as a while by jumping back and forth between different lines. By contrast, in the actual performance of the *logos*, questions and answers arise and disappear sequentially, never to reappear. Reference to prior utterances is possible only via new utterances.

But it is not only the *logos* that appears to us as a unified whole. It is likewise with the individual questions therein. We can see each question on the page as a distinct thing. Indeed, we can see this without even reading the question. And indeed, looking at the written *logos*, the individual questions have more salience than do Achilles and the tortoise. By contrast, for an answerer who is being asked the questions, the questions themselves, as distinct entities, are most naturally invisible. Instead, what is most salient to the answerer are the contents of the question. So when the answerer is confronted with questions about Achilles and the tortoise, his natural response will be to think about Achilles and the tortoise, as opposed to thinking about a question about Achilles and the tortoise.

It follows that when the answerer confronts the first question, which is about Achilles chasing and catching the tortoise, he will think about Achilles chasing and catching the tortoise.

Next comes the induction. In some respects it is difficult to say what exactly an answerer would think about as he contemplated the inductive questions. How, for instance, would he think about the anticipated capture of the tortoise, admitted in the first question, and acknowledged in each Achilles-question. How exactly would the answerer imagine the "places" that Achilles needs to reach? I do not know. Perhaps the answerers differed in their imaginative styles. But two things seem undeniable. Each Achilles question involves Achilles, but not the tortoise, moving forward. And each tortoise question involves the tortoise, but not Achilles, moving forward. It thus seems that in answering Achilles-questions, the answerer will typically think about Achilles, but not the tortoise, moving forward. And in answering tortoise-questions, the answerer will typically think about the tortoise, but not Achilles, moving forward.

It follows that the experience of the answerer in confronting the induction is something like this: He imagines Achilles moving towards the tortoise. Then he imagines the tortoise moving away. Then he imagines Achilles moving towards the tortoise. Then he imagines the tortoise moving away. And this proceeds for as long as the questioner repeats the inductive questions.

When the answerer thinks about Achilles and the tortoise in this alternating pattern, the sequence is likely to seem like a sequence of attempted captures and successful escapes. Each time Achilles tries to capture the tortoise, the tortoise succeeds in getting away. Of course, the questions do not literally mention anything of the sort. Instead, it is the temporal pattern of the answerer's own imagination that sets up this scenario.

To the answerer the paired attempted captures and escapes will naturally appear as part of the same event, at least once the sequence has become established in his mind. Each new question will seem to be about an event of this sort, an attempted-capture-and-escape. So it is not the questions that seem to be repeating themselves; rather, the attempted-captures-and-escapes seem to be repeating themselves.

Once the induction has been well-established, the answerer confronts the *always*-question:

So the tortoise will always keep somewhat ahead? Yes.

The answerer is already confronting a series of attempted-captures-and-escapes, and with each, the tortoise indeed "keeps somewhat ahead." To the answerer, it will perhaps seem quite natural that this sequence will continue, and that the tortoise will always keep ahead. Indeed, it seems that anyone would be hard-pressed to deny it.

Now recall our original problem. It seemed that the *always*-question did not involve a proper universalization of any inductive premise, casting doubt on the suitability of the *always*-question as an inductive conclusion. But now we see that the *always*-question does, at least, yield its needed answer. It does so by seeming to ask about the continuation of a sequence that has already been established in the answerer's mind. The fact that the inductive conclusion seems misaligned with the inductive premises does not seem to matter.

At this point, two curious facts are evident. First, we see that *always* most definitely has a temporal sense. Second, it is the answerer's own imagination that sets the groundwork for his understanding of the always. Achilles is not actually repeatedly trying and failing to catch the tortoise. Indeed, Achilles is not actually doing anything. But the answerer is, actually, in the midst of the *logos*, imagining that Achilles repeatedly tries and fails to catch the tortoise. This imaginative experience is actually repetitive, and in some sense, clearly can continue indefinitely, and perhaps, always and forever. Although it might seem to the answerer that in answering the *always*-question, he is "thinking about" Achilles and the tortoise, he is actually reflecting on the potential unendingness of his own imaginative experience.

As it turns out, the fact that the answerer reflects on his own immediate temporal experience in answering the *always*-question seems to pose a serious problem for what has often been regarded as the most plausible line of critique against the Achilles.

The Non-ambiguity of *Always*

On the whole, critiques of the Achilles have tended to fall into one of two main categories (even if there are, perhaps, many exceptions). Both types of critique might seem have their roots in Aristotle's own critique in *Physics* Z.9, although we will later see why his critique is rather more complex than either. For now, we will simply consider why each of these two lines of critique, considered independently, faces a serious problem.

The first of these broad lines of critique involves assimilating the Achilles to the Racetrack. There will then be as many strategies for critiquing the Achilles as there are for critiquing the Racetrack. We will later examine two of these strategies in some detail, and consider why they are inherently problematic, even as critiques of the Racetrack. But as critiques of the Achilles, these strategies face a more fundamental problem: the Achilles is *not* like the Racetrack.

To be more precise, the Achilles is not a trivially different variant of the Racetrack. This is most evident when we consider the concluding questions, the questions that follow the inductions. In the Achilles, two questions follow the induction. By contrast, in the Racetrack, three questions follow the induction. Moreover, the final two questions of the Racetrack involve ἄπειρα, things unlimited, whereas ἄπειρα are not mentioned at all in the Achilles. It follows that the two *logoi* are not trivial variants of each other.

Of course, Aristotle himself says that the two *logoi* are "the same *logos*." (239b18-19) And I myself have already suggested that both *logoi* somehow depend on the temporal experience of the answerer. We will examine each of these issues in due course. But what should not be obvious is that any approach which immediately assimilates the two *logoi* is simply inaccurate.

The second broad strategy of critique focuses much more directly on the Achilles itself, and hence merits much more of our immediate attention. The critique hinges on the idea that *always* is ambiguous. On the one hand, *always* might mean *always, when Achilles is trying to catch the tortoise*. On the other hand, *always* might mean *forever*. The critique says that the *logos* involves a confusion between the two meanings. Intuitively, it is not difficult to see why this type of critique might seem to pose a problem for the *logos*, but of course, everything hinges on the details.

Aristotle might seem to offer the first known version of this objection (239b26-29), although we will see in Chapter 8 that this is not exactly correct. But even if we suppose that Aristotle does attempt to offer a variant of this objection, we ought to be surprised, for Aristotle does *not* offer a dialectical disambiguation. He does not *draw a distinction*. And that is surprising, since the objection, at least as I have framed it, involves an ambiguity, and would seem to mandate drawing a distinction as the correct response, just as with the Racetrack.

Curiously, what we will find is that the ambiguity-objection simply will not work as a dialectical objection. Our immediate concern will be to examine why this is so. But it might seem that I am critiquing a straw man: I do not know of anyone who has ever formulated the ambiguity-objection as a dialectical objection. And yet that is not my problem, it is the problem of the critics. The Achilles, as we have now discovered, *is* a dialectical *logos*. Hence, if the ambiguity-objection is to work at all, if it is to hit the right target, then it seems that it should work as a dialectical objection. But it does not.

In Chapter 8, we will more directly examine Aristotle's own objection. And we will shortly consider a variety of quite specific problems faced by other ancient and modern critics. But for now we may put aside all of these commentators, and directly examine the issue of whether the ambiguity-objection can be formulated as dialectical critique.

Recall the words of Socrates in the *Euthydemus*. Socrates is afraid that he will answer a question, "taking it" one way, while the questioner will employ the question "taking it" in another way.[203] (295c4-6) We found that this was precisely the situation for which Aristotle prescribed "drawing a distinction". In other words, if a *logos* employs an inherently ambiguous question, a problem arises for the answerer if the answerer answers it assuming one interpretation, but the questioner employs it assuming another. As we have seen, the remedy for the answerer is to distinguish the interpretations.

Is this the situation that we have with the Achilles? Consider the final lines:

So the tortoise will always keep somewhat ahead? Yes.

And so you say that Achilles will never catch him!

If the *never*-claim is to follow from the *always*-claim, then it seems the *always* in the *always*-claim must mean something like *forever*. So we might say that the questioner employs the *always*-claim so that *always* means *forever*.

But what about the answerer? What does he mean by *always* when he answers the *always*-question? The *always*-question comes after the inductive premise. And each of the inductive premise pairs involves an attempt by Achilles to catch the tortoise. So naturally it seems that the *always* in the *always*-question will mean *always, when*

203 Recall that Socrates uses the word ὑπολαμβάνω, literally "take up".

Achilles is trying to catch the tortoise. If we temporarily ignore any potential objections arising from our earlier discussion of *always*, we can certainly allow that the answerer does seem to construe *always* as meaning *always, when Achilles is trying to catch the tortoise*.

So the answerer answers the *always*-question taking *always* to mean *always, when Achilles is trying to catch the tortoise*. But the questioner uses the question as grounds for a premise that takes *always* to mean *forever*. So this might seem a perfect opportunity for *drawing a distinction*.

But this is simply not the case. The *logos* breaks down only if *always, when Achilles is trying to catch the tortoise* fails to mean *forever*. That is to say, it breaks down only if these two temporal modifiers range over different spans of time. And hence it breaks down only if *always, when Achilles is trying to catch the tortoise* ranges over some finite time, in contrast to the unlimited time of *forever*.

But now we see the problem. The *logos* does not actually involve the complex phrase *always, when Achilles is trying to catch the tortoise*. Instead, the *always*-question simply involves the word *always* (ἀεί). And we have seen that the answerer, in answering this question, reflects on the unendingness of his own imaginative process. Indeed, he imagines Achilles pursuing the tortoise into the future, unendingly. He most certainly does not construe *always* in such a way that it could, even in principle, encompass merely a finite time.

The ultimate problem with the ambiguity-critique, as I have formulated it, is that it construes *always* as a universal quantifier. On this interpretation, *always* might mean *always, when Achilles is trying to catch the tortoise*, or it might mean *always, forever*.

On this interpretation, we can say that the actual *meaning* of *always* is the same in both cases: in both cases its meaning is constituted by the fact that it is a universal quantifier. What distinguishes the cases is simply the scope of the quantifier. And this is merely a semantic distinction. The scope of *always* in the first case is given by the words *when Achilles is trying to catch the tortoise*, and in the second case by *forever*, and certainly these phrases do not *mean* the same thing. But in principle, they might apply to the same time span, or they might not, depending on whether Achilles is or is not forever chasing the tortoise. So in this scenario, the actual scope of *always, when Achilles is trying to catch the tortoise*, whether a finite time or an infinite time, is mediated by the fact that *always* is a universal quantifier that ranges over *whatever* time within which Achilles is trying to catch the tortoise.

The problem with this approach is that in the actual *logos*, there is no such linguistic mediation. The answerer does not construe *always* via some phrase which might or might not apply to an infinite time. Instead, he interprets *always* via his own immediate temporal experience. The *always*-question is asking him whether the temporal repetition which he is *currently* experiencing can always continue. The unendingness of the *always* is thus immediate, not mediated, and is inherent in the answerer's own understanding of the *always*-question.

What this means is that a dialectical disambiguation of *always* seems inherently problematic. The answerer himself, in answering the *always*-question, construes *always* as meaning *forever*. But this is precisely the meaning that the questioner needs if the *always*-question is to serve as the ground for the ensuing *never*-question, which proffers the conclusion of the *logos*. It thus seems that *always* (ἀεί) is not ambiguous in standard dialectical fashion, and hence that the *logos* cannot be resolved by drawing a distinction, at least not against *always*.

A Finite Time?

We have seen that the ambiguity-critique fails because *always* is not a universal quantifier that could even potentially range over a finite time. Instead *always* always, in the *logos* at least, means *forever*.

Of course, it does not seem that anyone has ever actually formulated the ambiguity-objection in precisely the version that I have critiqued. And this is not surprising, since the Achilles is generally not recognized as a dialectical *logos*. But we might well put aside the details of the dialectical critique, and attempt to consider the ambiguity challenge from a broader perspective. We might ask: Is there any way in which the *logos* might somehow involve a confusion concerning the scope of a universal quantifier? More precisely, is there any way in which the *logos* might hinge on the confusion of a finite scope and an infinite scope? Indeed, does the *logos*, in this way, involve the confusion of a finite time with an infinite time?

If the *logos* were to somehow "confuse" a finite time with an infinite time, then one thing is clear: the *logos* must somehow actually *involve* a finite time. It obviously cannot confuse two things unless it somehow involves both. So: in what sense doe the *logos* involve a finite time?

It certainly does not do so explicitly. Indeed, the *logos* makes no references to time spans at all.

This leaves the possibility that the *logos* somehow implicitly involves a finite time. Here, of course, everything hinges on the details. One possibility is that the very initial admission that Achilles does catch the tortoise amounts to an admission that he does so in a finite time. Now this might seem straightforward, but the inference is by no means trivial, involving as it does, a shift in ontology, the apparent introduction of such things as "times." Since Aristotle himself approaches the *logos* along these lines, we may defer discussion of the issue until Chapter 8.

I do not see how the *logos*, as it actually existed, might be said to implicitly involve a finite time. But if we put aside what now appears as the historical *logos*, we can easily see one way in which the Achilles might implicitly involve a finite time. This would be the case if the *logos* were to assume that Achilles and the tortoise ran with uniform speeds. If they do so, and if Achilles is faster, then Achilles will certainly catch the tortoise in a finite time. And if we know their speeds, or the ratio between them, then we can calculate, in various ways, the actual finite time in which Achilles will catch the tortoise.

The assumption of uniform speeds plays a central role in one influential variety of critique of the Achilles, the critique that suggests that the mathematical notion of a convergent series enables a resolution of the apparent paradox. This sort of critique typically involves an assimilation of the Achilles to the Racetrack, and ultimately arises from a misreading of Aristotle's analysis of the Achilles. We will therefore again defer discussion until Chapter 8.

Historically, the idea that the *logos* does involve uniform speeds dates back at least to Themistius[204] and Simplicius. But we simply do not find this idea in the *logos* itself. Indeed, we find no reference whatsoever to any sort of measurable quantities.

204 200,1-8

(And we certainly find no reference to times, distances, or motions, the three varieties of continuous quantity discussed by Aristotle in *Physics* Z.)

It even seems unlikely that the *logos* referred to anyone using comparative or superlative terms like "fastest" or "slowest." Aristotle himself uses such terms in recounting the Achilles (239b14-25), but we have already seen why such terms likely result from Aristotle's own generalization of a concrete *logos* involving Achilles and the tortoise. There is no reason for the *logos* to mention that Achilles is faster, as that is obvious. And we have seen that Aristotle counsels questioners against employing such generalizations in the actual performance of a *logos*. (*Topics* 164a6-7)

As for uniform speeds, not only is there no evidence for them, it is fairly evident that they would be absurdly out of place in a dialectical *logos*. Remember that a questioner cannot simply assume what he wants: he needs to gain the assent of the answerer. But there is no reason for an answerer to grant that Achilles and the tortoise move at uniform speed: what in actual life moves at uniform speed? Even if the answerer tried to slip the assumption in without explicitly asking about it, perhaps by building it into the opening question, the answerer would still have grounds for objection. He could simply point out that the questioner is incorporating an absurdity into his question. In a contentious contest before an audience, this would effectively stop the *logos* before it even began.

On the whole, there can be little doubt that the dialectical Achilles involved no uniforms speeds: there is no evidence for them, the *logos* works well enough without them, and their employment would actually be detrimental to the questioner. The introduction of such notions into the Achilles most likely results from the theoretical appropriation of originally dialectical arguments, although it is an open question as to who first introduced them and when.

The fact that the Achilles does not posit uniform speeds means that the uniform speeds cannot be used to determine a finite time, and hence that this finite time cannot serve as the scope of *always*. J. A. Faris presents a recent version of the ambiguity-critique that succumbs to precisely this problem.[205] He assumes that the Achilles does require uniforms speeds, and shows how the existence of uniform speeds can be used to determine a finite time. And he charges that this finite time is the proper scope of *always*, but that the Achilles illicitly construes *always* as meaning *always, at any time*. But, clearly, this critique breaks down if the Achilles requires no uniform speeds.

Many commentators, among them Faris himself, have actually recognized that the Achilles seems to work without any assumption of uniform speeds.[206] Their discussions bring to light a surprising fact: Achilles does not actually need to catch the tortoise, even though he is faster. If we temporarily put aside issues of historical and physical absurdity, we cans easily demonstrate this mathematically.[207] Suppose that Achilles takes one second to advance one meter to the tortoise's starting position, during which time the

205 See Faris (30-33). Curiously, Faris presents three reconstructions of the Achilles, this being the second. Faris himself seems to think that he is presenting them in order of decreasing plausibility as historical reconstructions. But I think that precisely the reverse is true. In his first reconstruction, he requires the impossibility of going through an infinite series, being influenced by Aristotle's comparison of the Achilles to the Racetrack. (29-30) With his second version, Faris, rightly, in my view, ignores Aristotle's extended analysis of the Achilles (*Physics* 239b18-29), which assimilates the two *logoi*, and bases his reconstruction on Aristotle's initial exposition. (239b14-18) So the second version abandons the infinite, but nonetheless retains the unhistorical supposition of uniform speeds. On Faris' third version, see footnote 28.

206 Faris' third version of the Achilles, which he does not develop as thoroughly as his prior versions, is closest to my version and involves no uniform speeds. (33-34) In proposing this version, Faris follows Zinkernagel (1971) (cf. Zinkernagel (1965)) and Grotton-Guiness (1974), who himself discusses multiple influences on his proposals, among them some oral comments of Karl Popper. These philosophers present some stimulating thoughts about the Achilles, but it would take us too far afield to fit their ideas into our current historical investigation.

207 This illustration is based on Faris, pp. 33-34.

tortoise advances one-half meter. Then suppose Achilles takes one second to move one-half meter to the tortoise's new location, while the tortoise proceeds another quarter meter. And thus Achilles always takes one second to reach his target, while the tortoise, in the same time, covers half the distance of Achilles. Since Achilles always covers a greater distance in an equal time, Achilles will always be faster. But he will nonetheless never catch the tortoise. In this scenario, Achilles covers as many meters as 1 + 1/2 + 1/4 + 1/8 + ... Since this series converges, or, we might say, sums, to the limit 2, Achilles spends forever pursuing the tortoise over a finite distance.

We can modify the given scenario so that Achilles spends forever pursuing the tortoise over an infinite distance. Suppose that, just as in the prior scenario, Achilles moves one meter in the first second, with the tortoise moving one-half meter. But then in the second second, Achilles advances one-half meter, with the tortoise advancing one-third meter. In the third second, Achilles advances one-third meter, and the tortoise one quarter meter. And thus Achilles covers as many meters as 1 + 1/2 + 1/3 + 1/4 + 1/5 + ... Unlike the earlier series, which converges, this new series diverges, and has, we might say, an infinite sum. Thus Achilles runs forever, and covers an infinite distance, never catching the tortoise even though he is always faster and continually getting closer.

This latest scenario enables us to see why there is a flaw in the critique of the Achilles offered by Jonathan Barnes.[208] Barnes correctly notes that the Achilles does not, and need not, involve a supposition of uniform speeds.[209] But he presents a critique that is a variant of the charge that *always* is ambiguous. In his version, however, what is ambiguous is not a universal quantifier ranging over times, but rather a universal

208 Barnes (1982), pp. 274-5.
209 *Ibid*., pp. 273-4.

quantifier ranging over points along the trajectory of the runners. And just as other versions of the ambiguity-critique fail because they illicitly suppose the existence of a finite time that is nowhere sanctioned by the *logos*, so does Barnes' critique fail because it illicitly supposes the existence of a finite distance. Barnes simply assumes, without warrant, that there is a point beyond any point reached by Achilles, a point that may then be outside the limited scope of the universal quantifier. But as we now see, this is not licensed by the *logos*. Since the *logos* does not require the runners to run at uniform speeds, they may very well spend forever traversing an infinite space, eventually reaching every point therein.

The dialectical Achilles, as we have seen, involved no measurable quantities, and hence no numerical series, convergent or divergent. But the mathematical scenarios, inasmuch as they are compatible with the dialectical Achilles, nonetheless confirm a surprising result: it really is not inevitable that Achilles will catch the tortoise.

Of course, whether this result should seem odd depends significantly on our initial presumptions. It is commonly believed that the Achilles is a paradox, and hence that it "proves" as false what is actually and obviously true. So once we now recognize that, even by the presuppositions of the *logos*, Achilles need not actually catch the tortoise, it might seem that the paradoxical nature of the *logos* is undermined.

But this is highly misguided. The Achilles was not a "paradox" in the abstract, but a dialectical *logos*. So it does not even *purport* to prove as false an obvious truth. That is to say, it does not purport to do so in the abstract. Rather, what it purports to do, and typically *does* do, is to force the answerer to seemingly deny something that he has already affirmed. Whether Achilles actually catches the tortoise, or actually would, if he actually chased the tortoise, is not immediately relevant. What matters is that the answerer affirms the capture and then denies it. And the *logos* successfully brings this about.

At this point, we can recap what we have and have not accomplished. We have reconstructed a fairly plausible rendition of the dialectical Achilles. But we have not as yet found any plausible dialectical defense strategy. What is perhaps the most popular variety of contemporary critique (apart from those critiques that assimilate the Achilles with the Racetrack), the critique that charges that the *logos* involves a confusion between a finite and an infinite scope for the universal quantifier *always*, fails because the *always* in the *logos* is not a universal quantifier at all. It is a temporal adverb, which the answerer understands by way of his own potentially unending temporal experience.

We are by no means done with the Achilles. We will later examine the way in which Aristotle himself attempts to resolve the *logos*. But first we will return to the Racetrack. Just as the answerer's temporal experience plays a central role in the Achilles, so it does in the Racetrack, but in a much less obvious fashion.

Chapter 6
These Things Unlimited

The Racetrack and the Achilles

In examining the Achilles, we discovered that the *logos* depended in an essential way on the answerer's own temporal experience. And this proved to pose a problem for one of the most popular contemporary critiques of the argument, the critique claiming that *always* is ambiguous. We found that this critique could not easily be formulated as a dialectical response. The Racetrack turns out to be quite similar to the Achilles in this respect. Like the Achilles, the Racetrack also depends on the answerer's own temporal experience. And just as with the Achilles, the most popular critique of the Racetrack seems to face an inherent difficulty in being formulated as a dialectical response, the difficulty once again arising from the role of the answerer's temporal experience.

Recall once again the Racetrack *logos*:

> Can a runner get through a *stadion*? Yes.
> Must he first reach the halfway-point before the end? Yes.
> And in between the start of the *stadion* and the halfway-point, is there another halfway-point? Yes.
> And must the runner reach the earlier halfway-point before the latter? Yes.
> And in between the start of the *stadion* and this new halfway-point, is there another halfway-point? Yes.
> And must the runner reach the earlier halfway-point before the latter? Yes.
> And in between the start of the *stadion* and this new halfway-point, is there another halfway-point? Yes.
> And must the runner reach the earlier halfway-point before the latter? Yes.
> Is it always necessary to pass through the halfway-point? Yes.
> Are these halfway-points things unlimited? Yes.
> Is it possible to pass through things unlimited? No.
> And so you say that the runner cannot get through the *stadion*!

Now recall as well the final steps of the Achilles:

> So the tortoise will always keep somewhat ahead? Yes.
> And so you say Achilles will never catch him!

I suggest that the best way to understand the relationship between the *logoi* is to compare the *always*-question of the Achilles with the *unlimited*-question of the Racetrack. More specifically, we should compare the way in which the *always*-question supports the ensuing *never*-thesis with the way in which the *unlimited*-question supports the ensuing impossibility-thesis.

In some intuitive sense, it is easy to see why there might be a similarity between the paired lines of the two *logoi*. In both the *always*-question of the Achilles and the *unlimited*-question of the Racetrack, there is an affirmation of some kind of unlimitedness. Then each *logos* follows by inferring the negative consequences of this unlimitedness as concerns some sort of action. In some sense, the connection is clear. But there also seems to be a significant difference between the cases. The *always* and *never* lines of the Achilles both seem, at least implicitly, to involve Achilles. By contrast the *unlimited* and *possibility* lines of the Racetrack do not seem to involve the runner of the *stadion*. But this difference turns out to be something of an illusion: what really matters in each case is what the answerer himself actually does in the midst of the *logos*. And what he does, in each case, is to engage in a repetitive process of imagination that he can continue unendingly. In some sense, the question of what this imagination is supposed to be *about* turns out to be irrelevant.

Imagining Halfway-Points

As we did with the Achilles, here with the Racetrack we may begin by considering how the answerer would think about the induction. And again, as with the Achilles, we should again acknowledge the inherent difficulty: we may not know for certain what answerers typically imagined, and it may well be that answerers had different imaginative styles. And yet if the Racetrack depended essentially on forcing the answerer to consider his own imagination, as I think it did, then we must likewise consider such imagination or else abandon discussion of the Racetrack. In truth, we may actually draw some fairly firm, and surprising, conclusions about what the answerer must have imagined.

With the opening question, the answerer most certainly imagines a runner running the *stadion*. Indeed the runner is probably competing with other runners before a crowd. This is beyond doubt what any novice answerer would imagine, before the Racetrack has become institutionalized. It is certainly *not* what most philosophers today would first imagine when recalling the Racetrack paradox.

Next comes:

Must he first reach the halfway-point before the end? Yes.

On an actual *dromos*, a racetrack, it seems that both the end and the halfway-point would have been visibly marked, typically by wooden posts.[210] Given the prominence of public festivals in Greek life, it seems likely that most early participants in the *logos* would have seen an actual *stadion* race.[211] Hence when an answerer heard the *logos* asking whether the runner must reach the halfway-point before the end, he would naturally imagine a runner reaching a wooden halfway-post before a wooden end-post. For anyone who had seen a *stadion* race, the question would not be merely theoretical, but would rather be asking for a recollection of something one has actually seen.

210 Romano (p. 64, cf. 49-65) depicts such wooden markers in his reconstruction of a Corinthian *dromos* from c. 500 B.C.

211 As is well-known, Plato's *Parmenides* depicts Zeno and Parmenides attending the Great Panathenaea festival (127a7-b1), which included athletics.

In the case of someone watching an actual *stadion* race, the halfway-post and end-post are quite literally visible things. And likewise for someone who recalls a *stadion* race that they have seen in the past: they recall the halfway-point and the end-point as visible things. And the same goes for anyone, like you or I, who has presumably never actually seen a *stadion* race, but merely imagines it: we imagine the halfway-point and end-point as visible things.

It is rather difficult to say whether *every* early participant in the *logos* would haves seen an actual *stadion* race. But this ultimately does not matter, since consideration of actual *stadia* has pointed us towards a more general truth: it seems that anyone who imagines a runner reaching a halfway-point before an end-point will imagine the halfway-point and the end-point as visible things, be they wooden posts or not. And the reason is that there simply is no way to imagine anything other than as a visible thing. Imagining just *is* imagining visible things.

This claim might seem somewhat contentious, but this is partly due, I think, to terminological ambiguity. In contemporary parlance, the verb "imagine" is commonly used in an abstract fashion, to mean something like "suppose" or something like "think, but for misguided or non-rational reasons." But this is not how I use the term here. Instead, I think I may fairly coherently speak of the concrete experience of imagination, an experience otherwise like the experience of perceiving, but not produced directly via perception. Philosophers and scientists may debate the nature and significance of imagination, but it seems fairly clear that many, if not most, or all, non-blind members of our society imagine things routinely. (As do some of the blind.)

It is certainly not obvious that the dialectical Racetrack involved the imagination of visible halfway-points and end-points. But it is, I think, historical fact. What makes this likely is the affinity between imagining and remembering. There does not seem to be any sharp distinction between the experience of imagining and the experience of remembering, apart, of course, from the question of whether one actually experienced the event in question. But this distinction is only relevant if we are concerned with the epistemological status of memory and imagination, whereas here we are concerned with the experiences themselves.

Given that the *stadion* was an actual athletic event, it is the sort of thing that many dialectical answerers would remember seeing. But even if someone had never seen it, he *would* have heard about it, and so would have already imagined it in ways akin to remembering. Thus every answerer participating in the dialectical Racetrack would initially construe it as being about an actual sort of event that he had *already* seen or imagined. And this is manifestly different from modern presentations of the Racetrack. Today, oral and written presentations of the Racetrack typically occur in the midst of some broader discussion of philosophy, mathematics, or physics. These discussions are certainly not about reality as it is naturally perceived, and so one ends up seeing the Racetrack first and foremost *as* some theoretical illustration or puzzle. And so the ways in which one thinks about it are shaped by the ways in which one is thinking about the broader context in which it is embedded.

While it may well be true that imagination, or as some might say, visualization, is essential to the practice of modern mathematics and to any modern theoretical discipline, it is also widely accepted that the *content* of modern mathematical claims is not tied in any direct way to imagination. One can often visualize one and the same mathematical claim in multiple different ways, and skill in theoretical disciplines often depends on developing this very ability. And this differs markedly from ancient mathematics, wherein, as we have seen, the very meaning of mathematical claims is directly dependent on visible diagrams.[212] In general, it is the linguistic and logical structure of modern mathematics that takes it, for better or worse, beyond what can be achieved by mathematics dependent on direct visualization.

All this goes to show that today, the Racetrack, while not necessarily always presented as an inherently mathematical issue, is almost invariably presented in a mathematics-laden context, and hence in a context where the separation of content and imagination is firmly entrenched. (And this is true no matter how any writer or lecturer actually employs diagrams in their presentation of the Racetrack.) By contrast, a dialectical *logos* is always presented *as* decontextualized.[213] And hence the Racetrack *logos* appears to be about exactly what it appears to be about: a runner and a racetrack. And so the answerer thinks about the runner and the racetrack in much the way he would normally think about a runner and a racetrack, by imagining them as real things.

Recall that the Racetrack *logos*, unlike many modern presentations of the argument, needs to be easily comprehensible to the typical person. The use of

---

212 By no means do I wish to make the false claim that seeing the same thing in multiple ways is a modern phenomenon (even if it may well be a learned and culturally dependent phenomenon). To a careful reader of the dialogues, it is clear that a central part of Plato's genius consists of doing just that. Likewise, Euclidean proofs depend essentially on repeatedly re-envisioning one and the same diagram, chopping it up in different ways. Where the difference lies is in the objects of mathematics. With Euclid, a triangle looks like a triangle, a circle like a circle, and straight lines like straight lines (although there may be exceptions in *reductio* proofs.) But modern mathematics deals with unfamiliar objects, like "functions", which will often be visualized, but for which there is no necessary mode of visualization. And likewise modern mathematics will routinely involve the visualization of familiar objects in unnatural ways. As is well-known, "straight" lines in certain non-Euclidean spaces are routinely imagined as curved.

213 More precisely, competitive *logoi* were certainly presented as decontextualized, and this is what Zeno's motion *logoi* were. But certainly oral *logoi* could have been embedded in broader discussion contexts, much as Plato often portrays.

imagination helps to make this plausible. To imagine a runner and a racetrack requires no training of any sort. And hence it is easier than consciously trying to represent the elements of the *logos* by means of any sort of symbols or diagrams. And it is easier than trying to isolate the seemingly relevant theoretical components of the *logos* for abstract discussion. But notice that this ease places the answerer in a potentially dangerous position. On the one hand, he is not focused on abstractions or representations, things evidently different from what they are abstractions or representations of. On the other hand, he is not focused on an actual runner and an actual racetrack. Instead, he focuses his attention on his imagination of the runner and the racetrack, and hence on something that inherently seems to be what it is not.

Since the answerer thinks about the runner and the racetrack as real visible things, then he will surely likewise think about the halfway-point and the end as visible things within the visual setting of the racetrack. This is undoubtedly true with the end and the initial halfway-point, which could literally be seen on an actual racetrack. But now it seems fairly clear that as the answerer passed through the induction, he would imagine each new halfway-point as a visible thing. And the reason we can be sure of this is that as the induction proceeds, new halfway-points are contrasted with old halfway-points. So the answerer is forced to think about each new halfway-point in the same way as the most recent old halfway-point. And since he is visually imagining the old halfway-points, he will then visually imagine the new halfway-points.

As for how the runner himself figures into the answerer's imagination, we can allow for a degree of ignorance. We might even say that the ultimate effect of the runner is to contrast with the concrete goals he is pursuing, the visible halfway-points. Indeed, it is the runner questions of the induction for which imagination is particularly crucial. With the halfway-point questions, the answerer could possibly answer rather unreflectively, without clearly imagining anything at all, on the grounds that, of course, there is a halfway-point between anything and anything. But with the runner questions, one goal for the runner is contrasted with another, and here some sort of imagination is essential.

In saying that halfway-points are visibly imagined, we need make no presumption concerning how. While the initial halfway-point may well be a wooden post, we should certainly not presume this is so with the others, since there is obviously not a whole series of wooden halfway-posts in reality. Indeed the imagined halfway-points need not be particularly distinct. Nor need the answerer at all reflect on the nature of the things he is imagining. Indeed, we should presume he does not. In particular, we should presume he does not reflect on the ontological distinction between halfway-points themselves, whatever they are, and markers that mark halfway-points. Remember that the answerer's focus is on the runner, a man, and to imagine a runner running to a halfway-point, and to imagine a runner running to some marker that visually marks a halfway-point is to do one and the thing.

As counterintuitive as it may seem, one thing that we can say for sure is that, inasmuch as halfway-points are imagined as visible, they are imagined as possessing size. And this is true despite what anyone, including the answerer, might *think* about the nature of halfway-points. But again, this is not likely an issue an answerer is thinking about at all. We may thus conclude that each new runner question of the induction introduces a new visible and sizable halfway-point to the runner's imagination.

At the conclusion of the induction, the answerer faces the *always*-question. His situation is much as with the *always*-question of the Achilles. The induction has set in motion a repetitive imaginative process. So to the answerer, what the *always*-question appears to be asking is whether this repetitive process can always continue. The *always* is thus a temporal adverb that the answerer understands via his own immediate temporal experience.

We should not be misled by the apparent difference between the *always*-questions in the two *logoi*. In the Achilles, successive inductive questions might seem to place Achilles later and later in his time frame. By contrast, in the Racetrack, successive inductive questions seem to place the answerer earlier and earlier in his time frame. So the temporal *always*, which seems to signify an unlimited *future*, might seem appropriate in the case of the Achilles, but not the Racetrack. But this is utterly mistaken. What matters in each *logos* is the temporal experience of the *answerer*. He interprets the *always* in light of what is actually happening to *him*. He is the one being questioned. The *logos* does not ask him to take the perspective of Achilles or the *stadion* runner, and so there is no reason why we should expect him to do this of his own accord.

There is, however, a difference between the *always*-questions. In the Achilles, the *always*-question is the key to the whole *logos*. By contrast, in the Racetrack, the crucial question is the *unlimited*-question, for which the *always*-question sets the stage.

Recall the *unlimited*-question:

Are these halfway-points things unlimited (ἄπειρα)? Yes.

For the moment, let us put aside the obviously important word ἄπειρα, and instead consider the seemingly banal phrase "these halfway-points".

Of course, being English, the phrase "these halfway-points" did not occur in the *logos*. In Greek, this phrase would likely read ταῦτα τὰ ἡμίσεια. In his possibly abbreviated report of the question, Aristotle simply writes ταῦτα (these [things]) (263a6), obviously referring to a collection of halfway-points. Perhaps the *logos* sometimes did the same. Or perhaps the *logos* omits the demonstrative, simply mentioning τὰ ἡμίσεια (the halfway-points). Whatever the wording of the phrase itself, the *unlimited*-question clearly does contain a phrase the referent of which is a collection of halfway-points. And hence we can ask: how does this referent get established? How does it get attached to the phrase that refers to it?

We earlier considered three ways in which the referent of a plural term might get established: by enumeration, by the extension of a sense, and by acquaintance. It is obvious that the referent of the Greek "these halfway points" is not established via enumeration. This is true despite the fact that a collection of halfway-points has been enumerated, in the induction. But this enumerated collection of halfway-points is obviously a finite collection. And we know that the answerer admits that the halfway-points are "things unlimited." This means that he does not take the referent of "these halfway-points" to be simply the enumerated finite collection.

More curious is the fact that the referent of the Greek version of "these halfway-points" is not established via the extension of some sense. That is to say, the Greek phrase, whatever it was, clearly did not have a sense that in itself established an extension that was infinite in number. While expressions like "these halfway-points", "the halfway-points", and "these", *might* refer to an infinite collection, they need not. And hence such expressions are quite different from expressions like "the prime numbers" or "{x:x=1/2^n, n=1..∞}". These expressions pick out their infinite extensions directly via their senses. Many modern versions of the Racetrack involve expressions of a similar sort. But it seems clear that the dialectical Racetrack involved no such thing.

While "these halfway-points" certainly did not pick out an infinite extension via its sense, we might wonder whether the dialectical *logos* actually contained some longer phrase that actually did, a phrase that somehow specified precisely which halfway-points were involved. Consider, for instance, "the halfway-points the runner needs to reach in running the *stadion*". This phrase does seem to pick out a specific referent via its sense. But on any natural reading of the phrase, the collection it picks out seems to consist of *one* halfway-point, *the* halfway-point of the *stadion*. In a sense, we have already encountered this issue earlier. Any phrase that captures the full infinite collection of halfway-points will somehow need to do so by referring to the full scope of the runner's task in running the *stadion*. But by turning the answerer's attention back to the full *stadion*, the effect of the induction is undermined. The series of "halfway points" is no longer a series of *half*way-points, that is, halfway-points of the original *stadion*. It follows that the imaginative induction is much more plausible than the reference via infinite extension.

To a modern philosopher, it might not seem to matter much that the infinite collection be a collection of halfway-points. Perhaps, it might seem, what really matters is that the runner needs to go through *some* infinite collection, be it an infinite sequence of points, of segments, of tasks, etc. On this view, the collection of *half*way-points will simply be a means to establishing that some infinite collection exists. And so the *unlimited*-question will read something like:

> And so when running the *stadion*, the runner must go through points unlimited?
> Yes.

Notice that on this view, the problem of picking out a particular infinite collection simply disappears, since there *is* no demonstrative or definite reference to the collection. Instead the question simply requires the existence of *some* infinite collection.

As many modern Racetrack-style arguments involve formulations of this sort, we will soon examine them in more detail. But our examination will focus on how they differ from the dialectical Racetrack. For it seems quite likely that, as a matter of historical fact, the Racetrack did *not* involve reference to an infinite collection of "points", "segments", "tasks", or any collection of entities not identified as a halfway-entities. Indeed, Aristotle, in *Physics* Θ,8 (263a5-6), seems to indicate fairly clearly that the collections which turns out to constitute "things unlimited" is a collection of *halfway*-points. And so we should do our best to reconstruct the *logos* on this basis.

If the Greek version of "these halfway-points" does occur in the *logos*, as it seems to, and refers to its referent not via enumeration, nor via the extension of its sense, then this leaves the possibility of reference by acquaintance. In our earlier discussion of reference by acquaintance, we found plural terms referring to pluralities by way of a listener's acquaintance with a plurality as a whole, irrespective of which particular members composed the plurality. In the examples we considered, the pluralities were real external pluralities that one could *see*. But this is certainly not the case with the Racetrack *logos*. No one is *looking* at a collection of halfway-points. But if one can visually imagine a collection of halfway-points, as one can, then one can be acquainted with this imagined collection in much the same way as one can be acquainted with an external collection. And hence the imagined plurality can serve as the apparent referent for a plural term. And this is exactly what seems to happen with the *unlimited*-question.

Recall that at this point in the *logos*, the answerer has already encountered an initial sequence of halfway-points, each of them introduced individually. Then, with the *always*-question, the answerer is asked whether the runner will always need to go through the halfway-point. In effect, he is being asked whether *he* will always need to agree that the runner goes through the halfway-points. And this forces him to consider whether *he* will go on affirming that the runner goes through halfway-points. And in the process he will consider some additional halfway-points.

When the *unlimited*-question comes, there is thus no doubt that "these halfway-points" has a referent. The answerer has indeed imagined, in succession, a collection of halfway-points. But the indeterminacy of the members of "this" collection is also established, inasmuch as the answerer takes it to include additional halfway-points that he may need to consider. Hence the answerer's situation is much the same situation as with normal cases of reference by acquaintance: he is acquainted with a plurality that has and identity more salient than the identity of its individual members.

The *unlimited*-question asks whether "these halfway-points" are "things unlimited." The key to understanding why the answerer must apparently answer *yes* is to consider the reference by acquaintance in conjunction with the answerer's own temporal experience. In effect, since "these halfway-points" refers by acquaintance, we might say that it means, to the answerer, "these halfway-points that I am imagining as part of the *logos*." But since the answerer has acknowledged that the runner goes through the next (in the context of the answerer) and prior (in the context of the runner) halfway-point, the answerer has tacitly acknowledged that there need be no end to the context of the *logos*. That is to say, there is no end to the potential inductive questions. And hence there is, in principle, no end to the halfway-points the answerer will need to imagine. What the unlimited-question does is to turn the answerer's attention directly to this fact: "these halfway-points" refers to nothing other than the collection he is now imagining, but since he does not take the identity of the collection to be constituted by the few particular points he has actually imagined, he can go on imagining members of this collection forever. Hence the collection itself appears to be a collection of "things unlimited."

The answerer's affirmation of the unlimitedness of the halves depends essentially on his own internal experience precisely because the very constitution of the collection of halves is an artifact of his experience. In his mind, the collection seems to have a reality that precedes the reality of its members. But the truth is that he will construe a halfway-points as part of this collection precisely insofar as he imagines it as such. And hence, if he can indeed go on imagining halfway-points forever, as he can, then the halfway-points in "this" collection that he is imagining will indeed constitute "things unlimited."

The *unlimited*-question is followed immediately by the impossibility-question:

Is it possible to pass through things unlimited? No

It is not difficult to see why the answerer seems compelled to answer *no*.

To being with, we should recall the actual meaning of ἄπειρος. For simplicity, I have been rendering it as *unlimited*. But we saw earlier that the meaning is much better understood as *unlimited/untraversable*, and that we should sharply contrast this meaning with the meaning *unlimited/infinite*. If something is untraversable, or if a collection is untraversable as a whole, then it seems trivially true that it cannot be traversed. And "passing through" seems much the same thing as traversing, so the claim that it is impossible to pass through things unlimited/untraversable seems merely a truism. Certainly this is how the typical dialectical answerer would hear things.

The only possible room for debate might seem to lie in the question of who exactly is doing the "traversing" and how. But here the fact that the impossibility-question follows the *unlimited*-question proves crucial. With the *unlimited*-question, the person who is doing the traversing is the answerer himself. And the way in which he is traversing is by imagining. In this context, the impossibility-thesis seems assured. It is precisely the answerer's ability to go on imagining the halfway-points that assures the unlimitedness of these "things unlimited." But if the answerer can go on imagining the halfway-points, then he will indeed never pass through them.

We now see that the situation here in the Racetrack is very much akin to the situation in the Achilles. With the Achilles, the answerer imagined Achilles moving ahead into his own, the answerer's, future, always pursuing the tortoise. In the Racetrack, he imagines an unending series of halfway-points, likewise moving into his own future. Then, in each case, the ensuing question asks whether there is a conclusion to this unending process. And in each case, the answer is *no*. Achilles never catches the tortoise, and the answerer does not pass through the things unlimited.

In the Achilles, of course, the *never*-claim concludes the *logos*. But in the Racetrack, the impossibility-claim is merely a means to an end. Inasmuch as the *logos* seems to have yielded a general absurdity via the answerer's affirmations, the questioner appears entitled to announce the rejection of the initial thesis, concluding that the runner cannot indeed get through the *stadion*. This fact is itself somewhat curious, since the

Racetrack may well be the only surviving Zenonian argument with this structure, and it may also be the earliest surviving argument of this type by anyone. But to explore these questions would take us well beyond the present discussion.

What is clear is that the *unlimited*- and impossibility-lines of the Racetrack differ in significant way from the prior *always*-line and the ensuing conclusion. We have already seen that these two interior lines parallel the *always*- and *never*-lines of the Achilles. But notice further that while the *always*-question and the conclusion both involve the runner, the *unlimited*-question and the impossibility-question do not. This makes sense, for we have seen - as odd as it seems to say so - that these two lines are not really about the runner at all; they are actually about the answerer. The answerer is the agent who cannot finish imagining the halfway-points; *he* is the one who cannot pass through the "things unlimited". It is the two interior lines "about" the answerer that serve as the core of the *logos*, and enable the parallel with the Achilles. Just as with the Achilles, they involve the answerer moving into his own future. By contrast, the *always*-question and the conclusion frame this core, and situate it in a *logos* that is seemingly about a runner.

Supertasks?

We now have a fairly plausible understanding of how the Racetrack actually works, how it forces the answerer to seemingly contradict himself. This is not to say we fully understand the Racetrack as a whole; it is still quite perplexing. But we have a sense of how it flows. The Racetrack was, of course, a dialectical *logos*. And so, as a way of coming to understand it better, we might consider how potential objections to the *logos* would or would not work as dialectical objections, much the same as we did with the Achilles, where we considered the dialectical ramifications of the popular charge that *always* is ambiguous.

Here with the Racetrack, we will once again focus attention on a highly popular critique, albeit one that is not today formulated overtly as a dialectical critique. This critique hinges on the idea that, contrary to the supposition of the Racetrack *logos*, it is not actually impossible to pass through an infinity. In what is perhaps the most prevalent version of this objection, the charge is that the notion of passing through an infinity involves an equivocation. We will soon examine the details, but it is clear that if the critique holds up, and the impossibility-thesis is false, then the *logos* will deduce no absurdity, and hence will not legitimately proceed to the final step of rejecting the original thesis. Naturally, what we will need to consider is whether there is any plausible way to formulate this critique as a dialectical response, and if not, why not.

As with the *always*-critique of the Achilles, it is once again Aristotle who provides us with our earliest known version of the charge that the impossibility-thesis is ambiguous. In fact, he provides us with two versions of this objection. We have already examined elements of his critique in *Physics* Z.2, and we will later reconsider it in light of the dialectical Racetrack as we have reconstructed it. But Aristotle's revised critique of the Racetrack in *Physics* Θ.8 is actually a revised version of the charge that the impossibility-thesis is ambiguous, and we will later examine this new critique in Chapter 9. For now, we may ignore Aristotle, and instead consider whether contemporary variants of this objection can actually be formulated into plausible dialectical critiques.

Among contemporary writers, the charge that the impossibility-thesis is ambiguous is typically presented in conjunction with one particular two-premise analysis of what the Racetrack argument actually involves. In order to best grasp the contemporary literature, we will first consider how the rejection of the impossibility-thesis is associated with this two-premise analysis. Then we will proceed to the charge of equivocation. Finally, we will consider whether this critique can be applied in a dialectical context.

Today, both historians and more systematically minded philosophers commonly critique the Racetrack by analyzing it as a two-premise argument, and then critiquing the second premise, the impossibility-premise. According to Jonathan Barnes, in his historical assessment, the paradox involves two key premises:

(1) If anything moves, it performs infinitely many tasks.
(2) Nothing can perform infinitely many tasks.[214]

Barnes himself ultimately critiques the argument by rejecting the second premise.[215] In his evaluation, Barnes is influenced by Vlastos, also aiming at historical accuracy, who offers an analogous two-premise analysis, and likewise critiques the impossibility-claim.[216]

Apart from historians, this two-premise analysis of the Racetrack is significant in its own right, in that it sets the stage for much contemporary discussion of Racetrack-style arguments, in particular, the extensive genre of literature dealing with so-called "supertasks". A *supertask* is simply an infinite sequence of tasks. Philosophers of science Earman and Norton write:

214 Barnes (1982), pg. 263.
215 *Ibid.*, pg. 273.
216 Vlastos (1995), pg. 191 *et seq*. Barnes cites an earlier version of this article as "the best scholarly study" of the Racetrack and the Achilles. (*Ibid*., pg. 664)
Faris (1996), also aiming at historical accuracy, offers a similar two-premise analysis, but focuses his critique on the first of the two premises. (pp. 10-11 *et seq*.)

> "The archetype of the supertask is Zeno's celebrated "Dichotomy." [i.e. the Racetrack] ... The standard resolution simply accepts as unobjectionable Zeno's notion that to complete a journey from *A* to *B*, a runner must complete an infinite number of subjourneys - from *A* to the midpoint of *AB*, then from there to the three-quarter point, etc. but claims that this of itself does not prevent completion of the journey.[217]

Earman and Norton seem quite accurate in their description of the "standard resolution", but thus far, I suggest that we consider the standard resolution as involving two distinct elements. First is the construal of the Racetrack as essentially a two-premise argument, the premises demanding, respectively, the necessity and the impossibility of a supertask. Second is the assertion that a supertask is actually possible.

The contemporary discussion of supertasks appears to originate with Max Black, although he did not use the term. Black himself actually rejected the possibility of supertasks, his main target being the popular idea that the mathematical theory of limits enabled resolutions of the Racetrack and the Achilles.[218] In rejecting the possibility of supertasks, Black critiques the Racetrack and the Achilles by arguing that they err in requiring the necessity of supertasks.[219] It is James Thomson, the coiner of *supertask*, who seems to first clarify that what is actually involved in Black's argument is the sort of two-premise analysis of the paradoxes that we have seen above.[220] Thomson offers support for Black's basic contention that supertasks are impossible, but that the Racetrack

217 Earman and Norton (1996), pg. 232.

218 Black (1950-51). Entitling his article "Achilles and the Tortoise", Black ostensibly focuses his attention on the Achilles paradox, not the Racetrack, and many other modern discussions follow suite. This focus on the Achilles is grounded in the assimilation of the Achilles to the Racetrack, and the belief that the Achilles thus involves a premise along the lines that it is impossible to go through an infinity. But as we have seen, this is simply not the case. Hence the supertask literature is more profitably considered in comparison with the historical Racetrack, which does involve such an impossibility-claim.
The assimilation of the two logoi has its roots in a misreading of Aristotle's analysis of the Achilles. Ironically, this misreading also grounds the well-known "mathematical" resolution of the paradox, which Black rightly critiques. We will hence examine this issue in more detail in Chapter 8 when we consider Aristotle's critique of the Achilles.

219 *Ibid*., pp. 80-81.

220 Thomson (1954-5), pp. 89-90.

does not legitimately require one. In response, offering reasons we will consider below, Benacerraf critiques Thomson's arguments against supertasks[221], and Thomson ends up largely agreeing that he was mistaken.[222]

Today many philosophers accept that supertasks are possible, even if counterintuitive, and moreover that the Racetrack falters precisely in rejecting their possibility. Earman and Norton see Benacerraf as laying the groundwork for this consensus.[223] But the influence of the supertask literature extends even to historians. Vlastos and Barnes, two of the most characteristic representatives of the analytic style of history of philosophy, show great familiarity with the modern supertask literature, and as we have seen, their historical assessments line up neatly with what Earman and Norton describe as the "standard resolution" of the Racetrack. Indeed, it seems most plausible to say that this standard resolution was forged by Vlastos and Benacerraf in conjunction, as each, in their respective highly influential articles, cite repeated conversation with each other as being particularly formative in the development of their ideas.[224]

While the nature and influence of the standard resolution now seems clear, we should note what it is missing: an *argument* that supertasks are actually possible. Benacerraf himself admits that he has no such argument, and limits himself to critiquing certain arguments against supertasks. Indeed, he acknowledges that he is not sure, even in principle, how to argue that supertasks are possible.[225] Barnes takes a similar view. While he critiques several distinct arguments in favor of the impossibility-thesis, he

221 Benacerraf (1962), pp. 765-784.
222 Thomson (1970), pp. 130-138.
223 Earman and Norton, *loc. cit.*
224 Vlastos (1995, originally 1966), fn. 18; Benacerraf (1970), pg. 103.
225 Benacerraf (1970), pp. 121-2.

admits that he has no actual arguments against it.[226] On the whole, it seems that contemporary literature offers few if any positive arguments against the impossibility-claim.

What we do find is a pervasive claim that the impossibility-thesis involves an equivocation. This charge has several varieties. Propounding one version, Earman and Norton write:

> Black's fallacy [in endorsing the impossibility-claim] lies in confusion of two sense of "incompletable" and its allure lies in the ease with which we can slide between the two senses.[227] An infinite sequence of acts is incompletable in the sense that we can nominate no last act, the act that completes it. An infinite sequence of acts may also be incompletable in the sense that we cannot carry out the totality of all its acts, even though each act individually may be executable. ... An infinite sequence of acts cannot be completed in the first sense, but that certainly does not entail that it cannot be completed in the second sense.[228]

Earman and Norton thus contrast the incompletability of some *last* act of the supertask with the incompletability of *all* the acts of the supertask. Barnes offers a similar critique.[229]

A slightly different charge of equivocation contrasts reaching the *last* task with reaching the *end* of the tasks. David Bostock writes:

> The argument which I supplied for Aristotle earlier - viz., that it must be impossible to come to the end of that which has no end - need not detain us for long, since it is fairly evident that it rests on an equivocation on the phrase 'come to the end of'. Certainly an infinite series has no last member, and therefore it is indeed impossible to *come to the last member* of the series. But it by no means follows that it is impossible to *finish* the series, *i.e.*, to come to a state in which no member remains outstanding. From the fact that there is no *last* member it does not follow that we cannot perform *every* member.[230]

226 Barnes (1982), pg. 264-273.
227 Earman and Norton here refer for discussion to the original version of Vlastos (1995, orig. 1967), pp. 248-251.
228 Earman and Norton (1996), pg. 233.
229 Barnes (1982), pp. 267-9.
230 Bostock (1972-3), pg. 46.

Vlastos[231] offers a similar account. But both Bostock and Vlastos make clear that the two variant charges of equivocation are actually equivalent. To complete *all* of a series of tasks is to reach a point in time at which no more remain to be completed. That is, to complete all is to reach the *end* of the supertask, so long as we understand the end as the point in time at which all tasks have been completed. Without loss of generality, we may thus say that the equivocation charge contrasts completing the last of the tasks with reaching the end of the supertask.

Benacerraf offers a critique that on the surface seems to differ. Recall that he is critiquing Thomson. Thomson had set up a series of thought experiments involving supertasks. One of them involves a lamp:

> There are certain reading lamps that have a button in the base. If the lamp is off and you press the button the lamp goes on, and if the lamp is on and you press the button the lamp goes off. So if the lamp was originally off, and you pressed the button an odd number of times, the lamp is on, and if you pressed the button an even number of times the lamp is off. Suppose now that the lamp is off, and I succeed in pressing the button an infinite number of times, perhaps making one jab in one minute, another jab in the next half minute, and so on, according to Russell's recipe. After I have completed the whole infinite sequence of jabs, i.e. at the end of the two minutes, is the lamp on or off? It seems impossible to answer this question. It cannot be on, because I did not ever turn it on without at once turning it off. It cannot be off, because I did in the first place turn it one, and thereafter I never turned it off without at once turning it on. But the lamp must be either on or off. This is a contradiction.[232]

Thomson thus argues that because the end state of the supertask is paradoxical, the supertask itself is impossible. He offers additional arguments of similar form.

In response, Benacerraf argues, correctly it seems, that Thomson errs in supposing the nature of the end-state to have been specified by the specification of the supertask

231 Vlastos (1995, orig. 1966), p. 193; Vlastos (1995, orig. 1967), pp. 248-9.
232 Thomson (1970, orig. 1954-5), pg. 94-5. Benacerraf (1970, orig. 1962), p. 107, quotes the same passage, with slight omissions.

itself.[233] In the case of the lamp, Thomson's description has specified the state of the lamp during the performance of the supertask, but has entailed nothing concerning the state of the lamp when the supertask *has been* completed.

We can now see that Benacerraf's critique is a variant of the equivocation charge. Thomson assumes that the end-time of the supertask, the time at which the supertask has been completed, is also the end-time of one of the individual tasks, indeed the end-time of the "last" individual task. Thus, according to Benacerraf, Thomson is confusing the completion of the last task with the completion of all the tasks and the reaching of the end-state. And so Benacerraf offers a variant of the equivocation charge.[234]

We can use Benacerraf's critique to formulate a quite general version of the equivocation charge. Thomson had presumed that the numerical end of an infinite series of tasks coincided with the temporal end of the same series. And since the numerical end did not exist, he concluded that neither did the temporal end, and hence, that the supertask could not be completed, that is, could not be completed *at any time*. But Benacerraf pointed out that the numerical end did *not* need to coincide with the temporal end.

Benacerraf is thus distinguishing two sorts of end, and correspondingly, two sorts of endlessness. On the one hand we have numerical ends, and numerical endlessness.[235] A numerical end is simply the member of a series that is ordinally last. On the other

233 Benacerraf (1970, orig. 1962), pp. 107-8.

234 Benacerraf considers some additional scenarios that might not seem to so easily accord with my analysis, inasmuch as they involve a genie that disappears at the moment of completing a supertask. (pp. 117-120) So it seems that the agent of the supertask, the genie, does not reach the end-time, inasmuch as he does not *exist* then. But he nonetheless *has* performed the supertask *at* the end-time (and no earlier) even if he (oddly) does no longer exist.

235 Alternatively, we might speak of ordinal ends, and ordinal endlessness, to allow for the possibility of unusual infinite orderings, orderings that have members in the ωth position and beyond. Barnes considers such a case (1982, pg. 268) But as these scenarios take us far from the historical Racetrack, I will not here examine them further.

hand, we have temporal ends, and temporal endlessness. A temporal end is simply the first point after which there is no more of something. The supertask literature concerns, naturally, supertasks, that is, infinite series of tasks. And it is easy to see how the two notions of endlessness apply to an infinite series of tasks. Assuming it is ordered like the natural numbers, such a series is numerically endless. But such a series may or may not have a temporal end, depending on how the tasks are situated in time.[236]

I suggest that the different presentations of the equivocation critique often appear more different than they are due to what might seem a quirk of contemporary English. In analytic philosophy, the word "infinite" is typically taken to mean "mathematically infinite," and so it does.[237] Hence, to speak of an "infinite series of tasks" is *ipso facto* to speak of a *numerically* infinite series of tasks. And so even if the equivocation critique charges a confusion of numerical endlessness and temporal endlessness, the word "infinite" is surely *not* ambiguous. And hence the ambiguity gets foisted on some different phrase, e.g. "incompletable", "come to the end of", etc. But we can simplify things if we simply avoid the use of the word "infinite" in formulating supertask arguments, and use instead a word that is more plausibly ambiguous, for instance, *unlimited*, the word we have been using to render ἄπειρος.

Recall the standard modern two-premise version of the argument, as given by Barnes:

236 Sorabji (1983, pg. 327) offers yet another version of the equivocation charge. He says that the Racetrack hinges on the claim that "the task of traversing an infinite series is never-ending." But he charges that the 'never-ending' is ambiguous. On the surface, he seems to be clarifying the ways in which a series of half-distances may be never-ending. But closer analysis, I suggest, reveals that he is actually focusing his attention on an infinite collection of tasks.

237 A. W. Moore (1990) discusses at great length how the notion of infinity has historically fluctuated between the two poles of, on the one hand, mathematical infinity and, on the other, metaphysical infinity, or infinity as perfection, the sort of infinity instantiated by the God of the ontological argument.

(1) If anything moves, it performs infinitely many tasks.
(2) Nothing can perform infinitely many tasks.[238]

We can render this using *unlimited*. We get:

(1) If anything moves, it performs an unlimited series of tasks.
(2) Nothing can go through an unlimited series of tasks.

We now see how to apply the equivocation critique. We can grant that the first premise is true if we interpret *unlimited* as meaning *unlimited with respect to number*. And we can grant that the second premise is true if by *unlimited* we mean *unlimited with respect to time*. If the tasks take forever, they cannot all be performed. But the second premise is false if by *unlimited* we mean *unlimited with respect to number*, since it *is* possible to perform a collection unlimited with respect to number so long as this collection is limited with respect to time, that is, so long as there comes a time when no more of the tasks remain. Or at least this is the view of many philosophers, so I shall grant it for the sake of argument. The result is that the second premise is true in only one case, and the argument is only valid in the other case. And hence the argument seems inherently flawed.

Two things make the equivocation charge interesting. First is its prominence. A wide variety of authors offer some variety thereof. Second, the equivocation critique has at least some initial historical plausibility. Given that the Racetrack *logos* does involve an impossibility-claim, it is initially plausible that some ancient figure, Zeno or otherwise, might knowingly or otherwise confuse two interpretations of the claim. By contrast, as we have seen, many other contemporary critiques of the impossibility claim are effectively critiques of arguments *for* the impossibility claim, arguments for which, whatever their intrinsic merit, we have no historical evidence. Our task now is to

238 Barnes (1982), pg. 263.

determine whether this critique would have any plausibility in the dialectical context.

A Dialectical Resolution?

Recall the final four lines of the Racetrack *logos*:

> Is it always necessary to pass through the halfway-point? Yes.
> Are these halfway-points things unlimited? Yes.
> Is it possible to pass through things unlimited? No.
> And so you say that the runner cannot get through the *stadion*!

The equivocation-critique contends that there is an ambiguity in the thesis:

> Nothing can go through an unlimited series of tasks.

This thesis seems roughly akin to the thesis involved in the impossibility question of the *logos*. And hence we ought to consider whether the impossibility-question is likewise ambiguous, and whether the answerer may respond to this question, and to the *logos*, by *drawing a distinction*.

In the dialectical *logos*, the ambiguous term will be ἄπειρα, "things unlimited". Following the equivocation critique, it might mean *things unlimited with respect to number*. Or it might mean *things unlimited with respect to time*. So it might seem that the answerer should respond to the impossibility-question by drawing a distinction, asserting that:

> It is not possible to pass through things unlimited with respect to time, but it is possible to pass through things unlimited with respect to number.

Hence it might seem that the answerer here prevent the questioner from gaining the needed affirmation of impossibility.

But this is not true. The disambiguation stops the questioner if the thesis he required was:

> It is not possible to pass through things unlimited with respect to number.

But it obviously does not stop the questioner if the thesis that he required was:

It is not possible to pass through things unlimited with respect to time.

Indeed the answerer grants this very thesis. So we must ask: which of these theses did the questioner actually require?

The answer, of course, is that the questioner requires an impossibility-thesis, and an impossibility-question, in which the meaning of ἄπειρα matches the meaning of ἄπειρα in the prior *unlimited*-question. The *logos* proceeds validly if the two uses of ἄπειρα correspond in meaning, and it breaks down if they do not. Indeed this is precisely the case with any potentially ambiguous term. A *logos* will use a term twice, in different lines, and if the two uses match in meaning, the *logos* proceeds, but if they do not match, the *logos* equivocates.

So recall the *unlimited*-question:

Are these halfway-points things unlimited? Yes.

We may reasonably suppose that the answerer does regard these "things unlimited" as *things unlimited with respect to number*.[239] And so if the *logos* actually asked about precisely this issue, then the *logos* would equivocate. But the *logos* merely uses the unadorned term ἄπειρα, and so we need to ensure that the answerer does not also regard the ἄπειρα as *things unlimited with respect to time*. But this we cannot do. He *does* think that the ἄπειρα are *things unlimited with respect to time*.

When we earlier examined the unlimited-question, we found that the answerer construed "these halfway-points" as the halfway points he was imagining in the context of

239 I grant this for the sake of argument. But in reality, the situation is less obvious than it might seem, given that the Greeks, moreso than our contemporaries, would have associated *numerical* properties closely with the act of counting, something that is not mentioned in *this* version of the *logos*.

the *logos*. And he saw that there need be no end to this process of imagination. This is precisely why he affirmed that the halfway-points were things unlimited. But the non-existent end to his process of imagination is clearly a *temporal* end. He can go on imagining the halfway-points forever. Thus he himself is admitting that "these halfway-points" are *things unlimited with respect to time*.

This means that the equivocation-critique unequivocally fails in the dialectical context. In answering *yes* to the unlimited-question, the answerer effectively affirms that:

These halfway-points are things unlimited with respect to time.

But then, in the putative disambiguation of the impossibility-question, he admits that:

It is not possible to pass through things unlimited with respect to time.

This gives the questioner everything he needs, and the answerer is defeated.

What does this ultimately amount to? What have we now determined? In what sense has the modern critique failed?   As far as the dialectical *logos* is concerned, we have now seen that the modern critique is an unequivocal failure insofar as it cannot provide a dialectical answerer with a workable dialectical response. For historians who have been promulgating the modern critique, this is certainly relevant news. Insofar as they purport to analyze the historical Racetrack argument, their analyses have missed the argument. But for non-historians, the news might seem of merely antiquarian interest. After all, people today are not actually practicing dialectic (at least in the relevant sense), and hence, whatever today inspires interest in the Racetrack, it is not a desire to actually win dialectical contests.

Yet the failure of the modern critique in the ancient contest might well have broader interest. Consider a quite plausible model of what is going on in a dialectical exchange. We might suppose that the questioner proposes a proposition, and then the answerer affirms or denies that proposition. Now it might seem that I have just described what literally *does* happen, but not so. In reality, the questioner simply asks questions and the answerer simply answers. To interpret the answerer as passing judgement on *propositions* is to interpret dialectic via an abstract model. Propositions are abstract entities that may or may not be instantiated in the questions of the questioner or the mind of the answerer. In any case, we have a model of what is going on, but a rather plausible model that I suspect many people accept.

Following this theory, if the answerer ends up affirming the denial or his original thesis, then he must have somewhere committed an error. Since it is evidently possible to run the *stadion*, we know that the propositions the answerer has affirmed do *not* constitute a sound argument. Hence one of two things must have happened: The answerer might have affirmed as true a premise he should have rejected as false. Or the answerer might have accepted as valid an inference he should have rejected as invalid. But in dialectic, this amounts to the same thing as affirming or rejecting a proposition, since the answerer will accept or reject an inference insofar as he accepts or rejects the conclusion purportedly following from the inference. Thus it follows that if the answerer ends up rejecting his initial thesis, we know that somewhere along the way he must have made an error and accepted a proposition that he should have rejected. And so we might well take it as our task to discover what this error is.

Notice that this model of what is going on in dialectic permits a close analogy between dialectical and non-dialectical arguments. In each case, the arguer – in the case of dialectic, this being the *answerer* – will contemplate propositions individually, affirming or denying them. His argument will then consist of the sequence of propositions affirmed, with the caveat that some propositions need to be marked as inferentially justified.[240] The only difference between dialectical and non-dialectical arguments will then seem to be the source of the propositions. In the case of dialectic, they come from the questioner, but with non-dialectical arguments, they come from the arguer. We have seen that contemporary historians do not recognize the dialectical nature of the Racetrack, but given our current model of dialectic, we now see that this might not matter. Many historians do see their task as determining, for apparently unsound arguments, which premises or inferences were illicitly accepted by their historical answerer.

We now see that we have failed to determine what the error of the Racetrack actually is. And so this entails one of two things. On the one hand, there might be some undiscovered error. And on the other hand, there might be no error. And if there is no error, then the argument might be sound, which is absurd, or else there might be something fundamentally wrong with our model of dialectic that requires an error.

Ultimately, we wish to account for the answerer's final answer, and we have supposed that the way to do so is to find an error in the provision of one of the prior answers. And what makes this reasonable is that we have been supposing that, in giving each answer, what the answerer has been doing is rendering judgement on a proposition that has been provided him by the questioner. But perhaps this is not what is going on at all.

---

240 I here presume that the affirmation *that* a given proposition follows from accepted premises is tacitly equivalent to the affirmation of the given proposition itself. This does seem correct, even if the propositions themselves are logically distinct. This might, of course, pose problems, as in “What the Tortoise Said to Achilles”.

To determine whether this is so, we can compare the modern variety of two-premise supertask argument with the dialectical *logos*. They are actually more similar than we have seen thus far, but the similarities serve to highlight the crucial differences. Recall the version offered by Barnes:

> (1) If anything moves, it performs infinitely many tasks.
> (2) Nothing can perform infinitely many tasks.[241]

These two premises correspond roughly to three questions of the *logos*:

> Is it always necessary to pass through the halfway-point? Yes.
> Are these halfway-points things unlimited? Yes.
> Is it possible to pass through things unlimited? No.

The impossibility premise of the modern version is roughly akin to the impossibility question of the *logos*. But the necessity premise of the modern argument seems to encapsulate in a single premise what the *logos* does in two questions.

This difference is not so great as it might seem, as we can divide the modern necessity-premise into two distinct sub-premises. Consider how Barnes explains the necessity-premise:

> Were he to travel from A to B, b [that is, the mover] would perform infinitely many tasks: there are infinitely many a_i's between A and B, each of which b must touch; there are infinitely many distinct propositions of the form 'b touches a_i', and if b reaches B every one of these propositions has been made true.[242]

Note in particular, and in reverse order, the two claims following the semicolon.[243] Barnes tells us that by the time the mover reaches the ultimate goal, he has made true *every* member of a certain set of propositions. But Barnes also tells us that this collection is infinite in number. We can transform these two claims into the language of *tasks*,

241 Barnes (1982), pg. 263.
242 Barnes (1982), pg. 263.
243 Faris (1996), pg. 11, likewise supports the necessity-premise with two sub-premises.

employing the term *unlimited*:

> Before a mover completes a simple task, he must complete *all* of such-and-such a series of tasks.
> This series of tasks is unlimited.

Here the 'such-and-such' might be expanded in various ways. But it is clear that the truth of the two premises depends on some precise specification of what the 'such-and-such' is.

We now see a difference between the modern argument and the *logos*. In the modern argument, if the “this series...” refers to the “such-and-such a series...”, then the two given premises can form a syllogism. They yield the necessity-premise of modern supertask arguments:

> Before a mover completes a simple task, he must perform an unlimited series of tasks.

Hence the modern arguments lose nothing by employing the compressed form.

The situation is different with the corresponding questions of the *logos*. In both the modern argument and the *logos*, we find, on the *unlimited*-line, a demonstrative reference to a plurality. And in the modern argument, on the prior line, we find a term that is co-referential with the demonstrative, namely, the 'such-and-such' term, a definite description. It is precisely because these terms are co-referential that the two premises can combine syllogistically. By contrast, in the *logos*, the *always*-question does *not* contain any term that is co-referential with the ensuing “these halfway-points”. Indeed, it contains no term referring to any plurality, employing only a singular reference to a halfway-point. There can thus be no simple syllogism, as there is no middle term.

There is a further affinity between the dialectical *logos* and modern presentations of the Racetrack. We have already seen that in the *logos*, the *always*-question comes at the conclusion of an induction. But in modern presentations as well, one of the two sub-premises for the necessity premise is typically presented as the conclusion to an induction. Consider, for instance, the version of Faris:

> (1) There is an infinite sequence Q_p of points between S and F (namely,
> the point half-way from S to F,
> the point half-way from that point to F,
> the point half-way from that point to F
> and so on).
> (2) If A moves from S to F, when it reaches F it has touched one by one in a finite time all the points in Q_p.

These two premises correspond roughly to the other cases of sub-premises that we have been considering, although Faris presents them in the opposite order. But here we see that Faris clearly supports the infinity-premise via an induction. And the same thing is true in other cases as well.[244]

There can be no doubt that modern presentations of the Racetrack commonly *do* involve inductions. But there might well be confusion as to how these inductions are supposed to function. In the case of Faris, we see that he uses the induction to build plausibility for the infinity-premise in the mind of the reader. The same thing is true of Barnes and Vlastos, who use inductions while laying out diagrams meant to illustrate the Racetrack scenario, diagrams that they present *before* offering any formal premises. But in each case, while the practical significance of the induction is crucial, it is just as clear that none of these authors regard the induction itself as a component of their actual Racetrack argument. Instead, they see it merely as an illustrative tool.

In fact, there is a fundamental mismatch between the way that modern authors typically present the Racetrack argument and the way that they construe what they are

244 Barnes (1982), pg. 262-3; Vlastos (1995), pg. 191.

doing. Consider again the case of Faris, and his term 'Q_p'. What does this term refer to? Does it refer to an infinite sequence of points? If you look up 'Q_p' in a dictionary, you will find no indication that it does? Nor does Faris ever define 'Q_p'. Instead, in Faris' English text, 'Q_p' functions as a demonstrative term referring to the collection, seemingly infinite, that he indicates via an induction. Indeed, in Faris' actual text, the 'Q_p' functions in precisely the same way as does the "these halfway-points" of the dialectical *logos*. And similar things are true of other authors as well.

But this is not the way in which these authors actually construe their arguments. We have already seen that modern arguments typically compress the all- and infinity-premises into the single necessity-premise. And naturally, the syllogistic compression depends on the middle term actually *being* a middle term and having a common sense and referent in both cases. And this will not likely be the case if one of the instances refers, quite literally, to a collection imagined by the *reader*. Instead, it is clear that no matter what symbolic construction contemporary authors actually *write* in the position of the middle term, they regard this textual object as merely a placeholder for *some* definite description that would unambiguously pick out a collection of points, indeed an infinite collection.

It follows that there is a mismatch between the way that the infinity premise is typically presented in modern Racetrack arguments, and the way in which it is construed. It is *presented* as the conclusion of an induction, with the universal term *not* clearly defined. But it is construed as involving a well-defined definite description.

Whether this mismatch poses any fundamental problem for contemporary authors unconcerned with history, I do not know. But it does show us directly how the dialectical

Racetrack differs fundamentally from modern versions thereof.

We earlier examined the modern equivocation critique of the Racetrack and saw that it failed in the dialectical context because in the *unlimited*-question, ἄπειρα did not simply mean *numerically unlimited*, as the critique required, but also meant *temporally unlimited*. If the modern argument were likewise to rely solely on demonstrative reference to a collection established by induction, then this collection would likewise be unlimited with respect to number *and* to time, the time, that is, of the reader. The equivocation critique would thus fail. But modern philosophers avoid this problem by presuming that, however necessary an induction may be for illustrative purposes, the argument itself, and in particular the infinity premise, has a semantic determinacy wholly independent of the reader's understanding thereof.

This is manifestly *not* the case with the *logos*, and in particular with the *unlimited*-question. Recall the question:

Are these halfway-points things unlimited? Yes.

We have seen that the answerer understands “these halfway-points” as a demonstrative reference to the plurality that he is actually imagining. And because his imagining is potentially endless, he will see the collection itself as unlimited with respect to time.

We now see the difference between the modern argument and the ancient. In the modern case, each premise has a semantic identity independent of the experience of any speaker, listener, arguer or audience. Barnes, Vlastos, Faris, etc. may construe themselves as pointing to propositions by means of their written sentences, and obviously our experience as readers may determine whether we rightly identify those propositions or not. But our experience does not determine *what* those propositions are. By contrast, in the unlimited-question of the *logos*, *no* proposition is passed to the answerer. The term "these halfway-points" has a referent *only* via the answerer's imagination, something to which the questioner has no access.

This result might seem surprising. We might have thought that in critiquing the modern equivocation critique, we were critiquing an approach that apparently works against a prominent modern version of the Racetrack, but fails to work against what seems to be the original version. And so we might then consider how this ancient version differs, as an argument, from the modern version.[245] In fact, we can do nothing of the sort, at least if by *argument*, we mean what most contemporary philosophers mean by argument. As the term is usually understood, an argument consists of propositions, or at least sentences, that have an identity that is independent of the audience of the sentences or the propositions expressed therein. But this is not the case with the Racetrack *logos*.

One of the major themes of analytic philosophy has been the idea that propositions and arguments can be identified and evaluated quite irrespective of the psychological states of anyone. But now we see that the Racetrack depends in an essential way on the psychology of the answerer, not merely for its reception, as is often the case, but as an element of its own identity.

Although we started by critiquing a particular modern critique, the equivocation critique, we now see that the problems inherent in this critique will be faced by *any* critique that interprets the Racetrack as a collection of propositions and inferences. Whatever is going on in the *logos*, we will clearly not understand it by adverting to the study of these abstract entities.

245 In discussion, Alvin Goldman suggested something along these lines.

We may now take stock of our situation. We have reconstructed the Racetrack, and have a plausible sense of how it functions in the mind of the answerer. And we have examined a widely popular critique of Racetrack style arguments, and found that it fails in the dialectical context. Moreover, it does not enable us to discover any *error* committed by the answerer, despite his fall into absurdity. And fundamentally, the problem with the modern critique lies in the supposition that the answerer can understand the *logos* and the questions therein independently of his own experience of understanding the *logos* and its questions.

If the modern critique fails, does Aristotle's critique, or rather, either one of his critiques, succeed? Is Aristotle able to provide a dialectical solution to a dialectical logos? This question does not have a simple answer. But we will eventually find that Aristotle arrives at a critique of the Racetrack logos that does recognize the central role of the answerer's imagination in yielding the absurdity, a fact that has been largely missed by scholars. As a first step towards understanding this revised critique, we will need to examine the counting version of the Racetrack, a version that explicitly introduces the notion of mental activity into the logos.

Chapter 7
The Counting Racetrack

The Counting Racetrack

In *Physics* Θ.8, Aristotle discusses two versions of the Racetrack, the second being a modification of the first, a modification that somehow involves counting. We have seen evidence that Aristotle's two analyses of the Racetrack, in *Physics* Z.2 and Θ.8, are both directed against both the original *logos* and the second version, which I will call the *counting Racetrack*. We must thus examine the counting Racetrack before we proceed to a discussion of Aristotle's critiques.

Recall Aristotle's report of the two versions:

> τὸν αὐτὸν δὲ τρόπον ἀπαντητέον καὶ πρὸς τοὺς ἐρωτῶντας τὸν Ζήνωνος λόγον, εἰ ἀεὶ τὸ ἥμισυ διιέναι δεῖ, ταῦτα δ'ἄπειρα, τὰ δ' ἄπειρα ἀδύνατον διεξελθεῖν, ἢ ὡς τὸν αὐτὸν τοῦτον λόγον τινὲς ἄλλως ἐρωτῶσιν, ἀξιοῦντες ἅμα τῷ κινεῖσθαι τὴν ἡμίσειαν πρότερον ἀριθμεῖν καθ' ἕκαστον γιγνόμενον τὸ ἥμισυ, ὥστε δειλθόντος τὴν ὅλην ἄπειρον συμβαίνει ἠριθμηκέναι ἀριθμόν· τοῦτο δ' ὁμολογουμένως ἐστὶν ἀδύνατον.
>
> And in the same way one must reply to those [dialecticians] asking the *logos* of Zeno, [who ask] whether it is always necessary to pass through the half, but these [halves] are unlimited, and it is impossible to pass through things unlimited, or as some others differently ask this same *logos*, requiring that, at the same time as moving the half-[segment], there is earlier counting individually the halfway-point coming to be, so that with the whole [segment] gone through, the result is having counted an unlimited number. But this is admittedly impossible. (263a3-11)

If we carefully examine his words, it seems that Aristotle is reporting the two versions somewhat differently. In reporting the first version, Aristotle initially makes it clear that he is *reporting* a *question*. But then he lapses into the voice of the questioner himself, a questioner who seems to be directly laying down propositions. (263a6) Now contrast this with the way that Aristotle reports the second version. Here Aristotle seems to maintain his distance, carefully speaking *about* the claims proposed by the questioner. Notice how the report is structured by the terms ἀξιοῦντες (requiring), ὥστε (so that), and συμβαίνει (the result it). There is a sense of detachment here that had disappeared in the first version.

The difference may be subtle, but it seems to be real, and there is a plausible explanation for it. In reporting the first version, Aristotle may simply see himself as reporting the *logos* directly. By contrast, in discussing the second version, Aristotle is not directly reporting a second version. Instead, he is telling us how the second version *differs* from the first. And of course, this would require a more detached form of discourse, since Aristotle would need to be thinking of both versions at once.

If this interpretation is correct, then we have a framework with which to reconstruct the second version. We start with the original version, indeed the dialectical Racetrack that we have already reconstructed. Aristotle does, of course, describe the second version as the same *logos*. (263a7) But then we somehow try to modify the original so that we take account of the reported differences in the new version.

There is no guarantee that this strategy will work. But if we can construct an account that meshes with the text, that will be progress. In fact, the counting Racetrack has been almost entirely ignored by scholars, and I do not know of any attempt to reconstruct the historical counting argument, in dialectical form or otherwise.[246]

Half-motions, Half-lines, and Halfway-points

In reconstructing the counting Racetrack, the most difficult task lies in

246 Racetrack-style arguments involving counting enjoy greater prominence in non-historical discussion. Max Black, for instance, develops the influential notion of an *infinity machine* in conjunction with a counting argument. (Black 1970, pp. 72-75)

interpreting the first clause of Aristotle's report: "requiring that, at the same time as moving the half-[segment], there is earlier counting individually the halfway-point coming to be... ." (263a7-9) This is no simple task. But we can begin by examining the two references to “halves”.

Aristotle first refers to some "half" using the feminine: τὴν ἡμίσειαν. Then he refers to another "half" using the neuter: τὸ ἥμισυ. While the neuter might be used generically, Aristotle would only use the feminine if he were motivated by some unstated feminine referent.

There are two obvious possible referents for the feminine "half". One is κίνησις (motion), which occurs multiple times in *Physics* Θ.8, both in the discussion that precedes the reconsideration of the Racetrack, and in the Racetrack discussion itself. (262a12-263b9) The second is γραμμή (line), which occurs twice in the discussion of the Racetrack (263a27, b8) But γραμμή is also the obvious unstated referent of the feminine form of ευθύς (straight), which occurs multiple times in the prior discussion. (262a12-263a3)

Both κίνησις and γραμμή share a similar feature: if either is the referent of τὴν ἡμίσειαν, then this "half" will obviously be a sort of half-segment rather than a halfway-divider. And so the given "half" obviously *is* a half-segment, even if it remains unclear exactly what it is half of.

In my translation, I have rendered τὴν ἡμίσειαν as half-[segment], since I think that Aristotle does not really care whether we construe the referent as κίνησις or γραμμή. In the ensuing discussion, he draws attention to the close relation between a motion and a line. (263a26-28) Now if we look back to our τὴν ἡμίσειαν, we see that someone or something is moving through it (263a8), and that some sort of counting is going on either at the same time (ἅμα) (263a8) or earlier (πρότερον) (263a8). So what really matters for Aristotle is the *time* of the moving through, a time that gets correlated with the time of the counting. So in this context, it doesn't seem to matter to Aristotle whether the object *getting* moved through is a motion (κίνησις) or a line (γραμμή). To allow for this ambiguity, my translation uses half-[segment], which might refer to either.

Of course, this "segment" is not intended as a translation of any Greek word, and for the time being, we should refrain from drawing any conclusion about the wording of the counting *logos* itself. This is not our immediate task. Instead, we are simply trying to understand what Aristotle means by *his* words in describing the counting *logos*.

Given that the feminine τὴν ἡμίσειαν is a half-segment, the contrasting neuter τὸ ἥμισυ in the same sentence must refer to something different. And it certainly refers to a halfway-point. Indeed Aristotle twice uses the neuter σημεῖον (point) in the ensuing discussion. (263a24,31) But even if σημεῖον were not the intended grammatical referent, it would be fairly clear that our τὸ ἥμισυ *is* a halfway-point, since half-segments and halfway-points, in one form or another, are the only sorts of halves there are.[247]

Counting and Moving

Aristotle tells us that dialecticians ask the counting Racetrack "requiring that, at the same time as moving the half-[segment], there is earlier counting individually the half-[way-point] coming to be... ." (263a7-9) But who or what is moving and counting? If the counting Racetrack is "the same *logos*" (263a7) as the original, then absent

247 Inasmuch as the neuter can be used generically, with no intended referent, we cannot assume that *any* use of the neuter of ἥμισυς will intend σημεῖον as the referent. In fact, the very next occurrence of the neuter, the plural at 263a23, clearly refers to two half-segments, as do ensuing uses.

evidence to the contrary, we should presume that the mover is a runner, and indeed that he is running the *stadion*.

If the mover is the runner, then the counter would seem to be the runner as well, inasmuch as there seem to be explicit temporal connections between the acts of moving and the acts of counting. The most plausible alternative would be that the counter is the answerer himself. But this requires that the counting acts of the answerer somehow be placed in explicit temporal alignment with the moving acts of the runner. We should not rule this out *a priori*, and we have, of course, seen that both the Achilles and the original Racetrack exploit the real temporal awareness of the answerer. But in neither case did the answerer *appear* to be part of the *content* of the *logos*. Instead, the questioners ask about the runner and Achilles and the tortoise, and the appeal to the answerer's own awareness is a subtle effect. To now insert the answerer directly into the Racetrack would be a significant change. If we want to keep the counting Racetrack as close as possible to the original, then our first choice should be that the counter is the runner, and not the answerer. And we will find that this choice works.[248]

So the runner is moving half-segments, and he is counting halfway-points. Moreover, Aristotle seems to be saying that each act of moving a half-segment will somehow be connected with the act of counting a halfway-point. So now we must ask: which half-segments are connected with which halfway-points, and how are they connected?

We may presume that the counting Racetrack, like the original, is regressive, not

248 Besides the runner and the answerer, there seem no reasonable choices for the counter. (Surely not the questioner.) But we might suppose that the act of counting is mentioned impersonally, with no particular subject. But we have already seen that effective *logoi* employ particulars.

progressive. This might seem surprising, and even incomprehensible. Scholars invariably presume that the counter must start counting at the midpoint of the whole, and count towards infinity as he moves towards the end. So it seems. But for now I will defer discussion of this issue. For now, it is clear that we should *presume* that the counting Racetrack is regressive. We found that the original was regressive, and we found that in *Physics* Z.2 and Θ.8, Aristotle treats the two versions in tandem, while his report of the counting version fails to note any switch from regression to progression. We must therefore opt for the regression, and we will find that it works.

If the counting Racetrack is indeed regressive, then we know that it involves a regressive series of halfway-points, the series extending to the beginning. But which half-segment is associated with each halfway-point? On the surface, Aristotle's wording is so confusing that any of three options might seem plausible: a halfway-point might be associated with the halfway-segment that it divides in half, it might be associated with the halfway-segment of which it is the beginning, or it might be associated with the halfway-segment of which it is the end.

If we had to decide this issue on the basis of Aristotle's immediate wording, we might seem to face a hopeless task. But it turns out that we have two other sources of information to help us. First is a passage, alluded to in Chapter 4, from the pseudo-Aristotelian treatise *On Indivisible Lines*. And second is a diagram puzzle that Aristotle analyzes shortly before the Racetrack discussion of *Physics* Θ.8. If we read these passages in conjunction, we can get a plausible counting Racetrack.

On Indivisible Lines is a text from the traditional Aristotelian corpus, although scholars have long agreed that it was not composed by Aristotle. The text begins by rehearsing several arguments, later to be refuted, towards the conclusion that there exist indivisible lines. Here is one of the arguments:

> Ἔτι δὲ κατὰ τὸν τοῦ Ζήνονος λόγον ἀνάγκη τι μέγεθος ἄμερὲς εἶναι, εἴπερ ἀδύνατον μὲν ἐν πεπερασμένῳ χρόνῳ ἀπείρων ἅψασθαι, καθ᾽ ἕκαστον ἁπτόμενον, ἀνάγκη δ᾽ ἐπὶ τὸ ἥμισυ πρότερον ἀφικνεῖσθαι τὸ κινούμενον, τοῦ δὲ μὴ ἀμεροῦς πάντως ἔστιν ἥμισυ. εἰ δὲ καὶ ἅπτεται τῶν ἀπείρων ἐν πεπερασμένῳ χρόνῳ τὸ ἐπὶ τῆς γραμμῆς φερόμενον, τὸ δὲ θᾶττον ἐν τῷ ἴσῳ χρόνῳ πλεῖον διανύει, ταχίστη δ· ἡ τῆς διανοίας κίνησις, κἂν ἡ διάνοια τῶν ἀπείρων ἐφάπτοιτο καθ᾽ ἕκαστον ἐν πεπερασμένῳ χρόνῳ, ὥστε εἰ τὸ καθ᾽ ἕκαστον ἅπτεσθαι τὴν διάνοιαν ἀριθμεῖν ἐστίν, ἐνδέχεται ἀριθμεῖν τὰ ἄπειρα ἐν πεπερασμένῳ χρόνῳ. εἰ δὲ τοῦτο ἀδύνατον, ἔιη ἄν τις ἄτομος γραμμή.
>
> Moreover, according to the *logos* of Zeno, it is necessary that there be some partless magnitude, if indeed it is impossible first to touch things unlimited in a limited time, touching [them] individually, and it is necessary for the moving thing to first reach the half, and a half belongs in no way to the partless. But if the thing moving on the line touches things unlimited in a limited time, and the faster [thing], in the equal time, traverses more [of the line], and [the] fastest [motion] is the motion of thought, then even thought might touch things unlimited individually in a limited time, so that if thought touching [things] individually is counting, it is possible to count things unlimited in a limited time. If this indeed is impossible, there must exist an indivisible line. (968a19-b5)

It is not altogether clear whether this passage is treating the entire given argument as "the *logos* of Zeno" or merely a portion of it. But this does not matter for us right now. What is clear is that this argument employs some variant of the Racetrack. Moreover, the language is highly reminiscent of Aristotle's discussions in *Physics* Z.2 and Z.9. And now we have mixed in some curious claim about counting.

Somewhat later, our Aristotelian author proceeds to reject the given argument. Once again, we can see the influence of the *Physics*. The author first faults the reasoning on the ground that a moving thing need not touch things unlimited in a limited time, since time and magnitude will be limited and unlimited in the same ways. (969a27-30) Here we see a straightforward application of Aristotle's initial resolution of the Racetrack in

Physics Z.2.

The author then follows this with a new critique, this time focusing on the idea that counting consists of thought touching things individually while moving along them. And the ultimate problem here is that counting will somehow need to involve stopping: “For counting is something with stoppings.” (969b4-5) This critique seems highly reminiscent of Aristotle's new critique of the Racetrack in *Physics* Θ.8. Aristotle's critique also, somehow, employs the idea that a counter will necessarily cause a stop in some motion. (263a29-b1) We will later examine this issue in detail, but for now the resemblance between the two texts is clear.

Our pseudo-Aristotle thus seems quite familiar with Aristotle's own discussion of the Racetrack and relies on it heavily. But for pseudo-Aristotle, the idea of counting as a motion of thought seems essential to understanding the Racetrack. So naturally we should ask whether this idea might somehow play a role in Aristotle's own thinking as well.

It does not seem that Aristotle ever directly describes the idea that counting points on a line is a process of moving the soul along the line. But the idea certainly fits well within the broad framework of Aristotelian thought. As is well known, Aristotle quite generally emphasizes the necessity of imagery in thinking.[249] And he quite specifically considers the idea that the soul is drawing diagrams, and hence, apparently, moving along them.[250] It is no simple matter to decide exactly how these ideas fit into Aristotle's own

249 He says that "the soul never thinks without an image." (*On the Soul* 431a16-17) See also *On Memory and Recollection* 449b30-450a1.

250 Aristotle compares the motion in the soul to the act of tracing an outline of a perceptible thing. (*On Memory and Recollection* 450a30-32) He also tells us that we perceive distant things via "similar diagrams and motions" in us. (*On Memory and Recollection 452b9-13)*

theory of the soul. But that does not matter for now. What is clear is that the idea of counting as being a motion of thought, along a line, is akin to other ideas that Aristotle considers and employs.

Now recall that our task is to understand the counting Racetrack, which has a runner both moving and counting. If we suppose that counting is itself a motion of thought, then the runner will actually be engaged in two motions, two parallel motions, at the same or roughly the same time. And this is quite interesting inasmuch as Aristotle explicitly compares his new resolution of the Racetrack to his prior resolution of a separate puzzle that involves two motions at the same time.

In *Physics* Θ.8, Aristotle begins the Racetrack discussion by saying: "And in the same way one must reply... ." (263a4) He then recounts the two versions of the *logos* and offers his new reply. (263a4-b9) Aristotle is thus saying that his reply to the Racetrack is like some prior reply to something. If we look at the prior discussion in *Physics* Θ.8, we find exactly one occasion, at 262b8-21, where Aristotle first sets up a puzzle and then proceeds to resolve it, just as he does with the Racetrack. So he must be comparing his reply to the Racetrack with his reply to this prior puzzle.

The earlier puzzle will figure considerably in our analysis of Aristotle's new resolution, so in Chapter 9 we will examine it in some detail. For now, we need only consider a few features of the puzzle setup. Aristotle presents the puzzle using diagrams.[251] He describes two lines of equal length. On each line, an object starts from one end, and moves towards the other, the two objects being of equal speed and starting

251 Of course, there are no actual diagram in our text. What we do find are the sort of letters that we earlier saw were indicative of the presence of diagrams.

at the same time.[252] On the first line, there is a named point somewhere in the middle. The crux of the puzzle hinges on the idea that the first object is at (ἐπί) (262b11) this midway-point at the same time (ἅμα) (262b11) as the second object is moving from the start of its line towards the end.[253]

If we suppose that counting is some motion of thought, wherein the counter moves along a line in thought, touching points, then it is easy to see how this puzzle setup might relate to the counting Racetrack. The first diagram will be the diagram of the counting motion. And the second diagram will be the diagram of the runner himself, progressing through the *stadion*. Just as in the puzzle, these two motions will need to begin and end at the same time, and will, somehow, need to proceed with the same "speed".

If this interpretation is correct, then when Aristotle begins to describe the counting Racetrack, he will be drawing attention to the way in which it resembles the prior puzzle. Remember that Aristotle describes the questioners as "requiring that, at the same time (ἅμα) as moving the half-[segment], there is first counting individually the half-[way-point] coming to be... ." (263a7-9) In the puzzle, he described the presence of one object at a midway-point as occurring *at the same time* as the motion of a second object through a line. And now we seem to have the counting of a halfway-point occurring *at the same time* as motion through some half-segment. We will somehow need to reconcile the ἅμα (at the same time) with the πρότερον (first), but it certainly seems that the ἅμα in the account of the *logos* is meant to draw a parallel with the ἅμα in the description of the diagram puzzle.[254] (262b11)

252 Aristotle does not explicitly say that the two objects start at the same time, but that is obviously what he intends.
253 For now we need not consider what the actual puzzle *is*.
254 In principle, the ἅμα of the counting *logos* description (263a8) might not be saying that the counting

Recall that we were trying to discover which half-segment was associated with a given halfway-point: the encompassing segment, the prior segment, or the following segment. We now have an answer. If Aristotle is construing the diagram puzzle as the framework for the counting Racetrack, then the *logos* must be pairing segments with their own interior halfway-points. In the puzzle, we are associating the first motion with the counting motion, and the second with the running motion, and the two are supposedly parallel. But the first object is at some midway-point in the middle of its motion, and hence in the middle of the associated motion. If the *logos* is like this, then counting the halfway-point will occur in the middle of the associated motion through a segment, and not at its beginning or end.

We have seen that the author of *On Indivisible Lines*, who is obviously influenced by Aristotle's discussions of the Racetrack, considers some variant of the Racetrack that crucially involves the idea that counting is a motion of thought. Moreover, Aristotle himself associates his new reply to the Racetrack with an earlier diagram argument involving two motions. If we construe counting as a motion, this makes it clear how the diagram argument is relevant, and allows us to read the pairing of the two motions in the diagram argument as a guide to understanding how the counting *logos* pairs the counting and the moving. The importance of the diagram argument will become much more evident when we go on to analyze Aristotle's new resolution. But for now we have a

---

and the moving occur "at the same time". Instead, it might simply apply to the dialectical questioners. In this case, they will be *requiring* (ἀξιοῦντες) two things at the same time. Not only will they be requiring that the runner moves a half-segment, as they also do in the original *logos*, even if no "segments", lines or motions, are explicitly mentioned, they will also require, "at the same time", that the runner count a halfway-point. This reading might seem attractive, since then the counting can happen before (πρότερον) the moving, eliminating the apparent conflict between ἅμα and πρότερον. But this will destroy the obvious parallel with the diagram puzzle, which Aristotle himself has told us is somehow relevant.

quite plausible start on understanding the counting *logos*. Our next task is to actually proceed with the reconstruction.

The Dialectical Counting *Logos*

Our strategy is to begin with the original *logos*, and then make as few modifications as possible while still accommodating Aristotle's description of the counting version. So the *logos* might begin as follows:

> Can a runner get through a *stadion*? Yes.
> Must he first reach the halfway-point before the end? Yes.
> And when he reaches the halfway-point, can he count it? Yes.
> Now in between the start of the *stadion* and the halfway-point, is there another halfway-point? Yes.
> And must the runner reach the earlier halfway-point before the latter? Yes.
> And when he reaches the halfway point, can he count it? Yes.
> And in between the start of the *stadion* and this new halfway-point, is there another halfway-point? Yes.
> And must the runner reach the earlier halfway-point before the latter? Yes.
> And when he reaches the halfway point, can he count it? Yes.
> And in between the start of the *stadion* and this new halfway-point, is there another halfway-point? Yes.
> And must the runner reach the earlier halfway-point before the latter? Yes.
> And when he reaches the halfway point, can he count it? Yes.

I have here started the *logos* as in the original, and retained the original induction. But now after each question about reaching a halfway-point, I have added a question about counting the halfway-point.

Unfortunately, this rendition yields an unworkable *logos*. The problem lies in the way that the counting gets started. Remember that, unlike us, the answerer is not viewing the *logos* all at once, he is encountering the questions one-by-one as they are asked. Now consider simply the first three questions as given above:

> Can a runner get through a *stadion*? Yes.
> Must he first reach the halfway-point before the end? Yes.

And when he reaches the halfway-point, can he count it? Yes.

At this point in the *logos*, there is only *one* halfway-point, obviously only one halfway-point, and no reason for a naive answerer to suspect otherwise. Hence it would seem absurd to be asked to count the only halfway-point.

While this seems implausible enough in English, in Greek the problem is even more salient. The verb for count, ἀριθμέω, is connected with the word for number, ἀριθμός. An ἀριθμός, a number, is what gets counted. But in Greek, an ἀριθμός is necessarily a *plurality*, which would perhaps be a better translation than *number*.[255] An individual thing might get "counted" in the sense that it is counted-off while the plurality of which it is a part is being counted.[256] But an individual thing, not *being* a plurality, cannot be counted *qua* plurality. The notion of ἀριθμός is thus quite different from the modern notion of *number*, which encompasses the number *one*.[257] The result is that it seems quite impossible for the answerer to be asked whether the runner can count what is obviously *not* a plurality. Or rather, he can be asked this, but can then easily reply: No, the runner cannot count the halfway-point, there is only one.

As a solution to this problem, I suggest that the induction did *not* involve any counting, and indeed proceeded precisely as in the original Racetrack.[258] The first

---

255 Cf. Klein (1968), pg. 46; Pritchard (1995), pp. 64-65.

256 Cf. Klein, *loc. cit*.

257 But notice that the English *number* often *does* mean *plurality*: Zeno concocted a *number* of paradoxes... .

258 There might seem a way to modify the "counting" so that it remains in the induction. The idea would be that, instead of having the inductive counting premises ask about counting *per se*, they should simply ask whether the runner can somehow mentally acknowledge each halfway-point when he reaches it. The *logos* would then need some later question that converts the series of mental notations into actual counting. This approach might seem to accord with the depiction, in *On Indivisible Lines*, of counting as "thought touching [things] individually." (968b2-3) But this seems a highly theoretical approach to the *logos*, an unlikely approach if the counting Racetrack were employed in real dialectical encounters, as I think we should presume it was, and not merely used as an object of theoretical discussion.

mention of counting can then come after the *always*-question, the conclusion of the induction. So we might have:

> Can a runner get through a *stadion*? Yes.
> Must he first reach the halfway-point before the end? Yes.
> And in between the start of the *stadion* and the halfway-point, is there another halfway-point? Yes.
> And must the runner reach the earlier halfway-point before the latter? Yes.
> And in between the start of the *stadion* and this new halfway-point, is there another halfway-point? Yes.
> And must the runner reach the earlier halfway-point before the latter? Yes.
> And in between the start of the *stadion* and this new halfway-point, is there another halfway-point? Yes.
> And must the runner reach the earlier halfway-point before the latter? Yes.
> Is it always necessary to pass through the halfway-point? Yes.
> When the runner reaches each halfway-point, can he count that halfway-point? Yes.

This scenario avoids the prior problem. As we have seen, when the answerer confronts the *always*-question, he has imagined a plurality, a number, of halfway-points, indeed a somewhat indefinite plurality. Hence there seems nothing wrong with someone, indeed the runner, counting them. While the question asks about counting them individually, this only makes sense because the answerer by this point construes them as members of some plurality.

The current reconstruction seems to fit Aristotle's description. We saw that Aristotle is telling us that while the runner is moving a given half-segment, he needs to count the interior halfway-point. And now in the reconstruction, we have the runner required to count halfway-points when he reaches them. This means that, if the halfway-points are halfway-points of half-segments, as all but one of them are, then the runner will, for each half-segment, be counting its interior halfway-point while moving the half-segment.

This might seem to leave a problem, in that the first halfway-point of the induction is not, of course, the halfway-point of any half-segment, but rather the halfway-point of the whole *stadion*. But notice that this does *not* contradict Aristotle's depiction. He never claims that each halfway-point is paired with a half-segment, but rather the reverse. And so our reconstruction does not contradict Aristotle's claim, but renders it true.

The reconstruction thus far seems to fit Aristotle's description, even if the *language* of the induction differs from Aristotle's language. In the reconstruction, there is no *mention* of any half-segments, be they half-lines, half-magnitudes, or half-motions. And there are certainly no "motions of thought". But such things were not mentioned in the original Racetrack either, supposedly "the same *logos*". Our goal is to have a *logos* as close as possible to the original, while still respecting Aristotle's language. And it certainly seems plausible, and even reasonable, that Aristotle should *describe* the given reconstruction as requiring that *while* the runner moves each half-segment, he counts the interior halfway-point.

The given setup allows us to reconcile an apparent conflict in Aristotle's description. Remember that the questioners are "requiring that at the same time as (ἅμα) moving the half-[segment], there is earlier (πρότερον) counting the halfway-point coming to be... ." (263a7-9) We may ask: Is the counting going on earlier than the moving, or at the same time? Now we can answer: *both*.

The counting does occur while the runner is *in the process* of moving. The *logos* does not say exactly that, but Aristotle can infer it if the counting occurs *when* the runner reaches the halfway-point, and if he reaches that point in the midst of moving the broader half-segment. It is Aristotle himself who emphasizes this coincidence inasmuch as he wants to show the parallel between the counting *logos* and the aforementioned diagram argument.

While the counting occurs *while* the moving is in process, the counting nonetheless also occurs *before* the moving of the half-segment is completed. Hence the πρότερον (earlier) seems appropriate. The πρότερον, of course, is derived directly from the *logos*. If the runner reaches one halfway-point before (πρότερον) the next, and counts the first one when he is at it, then he will also count it before he reaches the next one.

Thus the πρότερον (earlier) comes from the *logos*, and the ἅμα (at the same time as) comes from Aristotle's goal of linking the *logos* to the diagram puzzle, but they do not seem to be in conflict.

We also now have a way of understanding the phrase γιγνόμενον τὸ ἥμισυ (263a9), which I render "the halfway-point coming to be". Prior to the Racetrack discussion in *Physics* Θ.8, Aristotle has been discussing midway-points of motion and explains that they only exist in actuality, as opposed to existing in potentiality, so long as the moving thing reaches them and stops, and then begins to move again. He goes on: "And thus the middle becomes (γίγνεται) a beginning and an end... ." (262a21-5) But being a beginning and an end is precisely what being a middle *is*. (262a19-21) So Aristotle is really saying that a midway-point comes into actual existence as a midway-point *when* something reaches it (and stops). Aristotle tells us that the runner is counting the point that is "coming to be". But in Aristotle's view, a halfway-point "come to be" if and only if, and when, something reaches it. So the runner counts the points when he reaches them, and this is how it is in the reconstruction.[259]

At this point, we can fairly easily fill in the rest of the counting *logos*:

> Can a runner get through a *stadion*? Yes.
> Must he first reach the halfway-point before the end? Yes.
> And in between the start of the *stadion* and the halfway-point, is there another halfway-point? Yes.
> And must the runner reach the earlier halfway-point before the latter? Yes.
> And in between the start of the *stadion* and this new halfway-point, is there another halfway-point? Yes.
> And must the runner reach the earlier halfway-point before the latter? Yes.
> And in between the start of the *stadion* and this new halfway-point, is there another halfway-point? Yes.
> And must the runner reach the earlier halfway-point before the latter? Yes.
> Is it always necessary to pass through the halfway-point? Yes.
> When the runner reaches each halfway-point, can he count that halfway-point? Yes.
> Are these halfway-points things unlimited? Yes.
> So when the runner has gone through the whole *stadion*, has he counted an unlimited number? Yes.
> But is it possible to count an unlimited number? No.
> And so you say that the runner cannot get through the *stadion*!

Notice the *unlimited*-question remains just as it was in the original *logos*. But recall that in the original, the ensuing question was:

> Is it possible to pass through things unlimited? No.

Now this question is replaced by two questions, one about the runner having counted an unlimited number, and the second about the impossibility of doing so.

Overall, this yields a fairly conservative rendition of the counting *logos*. Recall that when Aristotle tells us how the counting version differs from the original, he employs

---

259 The present use of γιγνόμενον does not hinge on any distinction between the runner reaching the halfway-point by running, or "reaching" it via the "motion of thought". Indeed, Aristotle's ultimate resolution will claim that the *logos* is inconsistent regarding this issue. But interestingly, he does sometimes speak of things "coming to be" for the soul. He writes: "εἰ δὴ ἐστιν ἡ φαντασία καθ΄ ἣν λέγομεν φάντασμα ἡμῖν γίγνεσθαι ..." (But if imagination is that by which we say images come to be for us ...), making it clear that the answer is *yes*. (*On the Soul* 428a1-2) So Aristotle's wording will seem to fit the *logos* even if the halfway-points really only "come to be" for the runner via the mental "motion" of counting.

three distinct clauses. (263a6-11) Now we have added three new questions to the *logos*. And we have eliminated only the original impossibility-question, which is obviously rendered superfluous by the new absurdity.

The given rendition seems to work as a *logos*. The answerer does seem hard-pressed to give anything other than the required answers. One thing that might seem to give pause is the insertion of the counting-halves-question between the *always*-question and the *unlimited*-question. Earlier I emphasized the dependence of the *unlimited*-question on the answerer's experience of the *always*-question. But it seems plausible that the unlimited-question will still work, even with the intervening question.

A second potential problem is the apparent mismatch between *can* and *has*. First the questioner asks if the runner *can* count each halfway-point. But then he asks whether the runner *has* counted an unlimited number. Of course, logically speaking, from the fact that the runner *can* do something, nothing follows about what he *has* done.

Actually, this same apparent problem exists earlier in the *logos*. Consider that the opening question asks whether the runner *can* get through the *stadion*. Then ensuing questions ask about what he *must* do. But I think that there is no real problem here. In affirming that the runner *can* get through the *stadion*, the answerer *imagines* a runner who actually *does*. The ensuing questions then make sense to the answerer because he understands them as being about the imaginary runner. In the same way, when asked whether the runner *can* count each halfway-point, the answerer imagines the imaginary runner counting at least one halfway-point, and tacitly acknowledges that *this* runner can count each of them. He thus answers the ensuing questions on the assumption that the runner *does* do the counting.

There is indeed something odd going on here, but it is something we have already encountered; it is the answerer's use of his imagination. We already know that the answerer is employing his imagination in answering the questions, as opposed to subjecting them to detached logical analysis. And it might well seem that this is the ultimate source of his difficulties. But it need not force us to modify our reconstruction. We can understand the "can" questions as adding to the answerer's imagination, and the other questions as seeming to be about what is imagined. Although we cannot hope for perfection, it seems we now have a reasonable reconstruction of the counting *logos*.

Thinking of Things Unlimited

The existence of the counting Racetrack raises many questions: When did it develop from the original, and who developed it? Why did it develop? Was it perceived as having some potential advantage over the original, perhaps because some line of objection had gained traction? These are inherently interesting questions, and further study may yield insight into the answers. But I will not attempt answers here, largely because our immediate evidence is so minimal. Instead, to gain insight into the counting *logos*, we may now consider how the modern equivocation critique, which failed in response to the original Racetrack, might fare against the counting version.

Recall that the equivocation-critique is directed against an argument of the following form:

(1) If anything moves, it performs an unlimited series of tasks.
(2) Nothing can go through an unlimited series of tasks.

The critique charges that the two instances of *unlimited* have different senses, with the first meaning *unlimited with respect to number*, and the second meaning *unlimited with* *respect to time*. If so, then the argument equivocates.

In one sense, the counting Racetrack involves an argument even closer to the target of this critique than did the original. This is because the final two questions of the counting version offer theses closely akin to the premises that the critique attacks:

> So when the runner has gone through the whole *stadion*, has he counted an unlimited number? Yes.
> But is it possible to count an unlimited number? No.

But it seems fairly evident that whatever is wrong with the *logos*, it is not an equivocation involving ἄπειρος (unlimited) in these two premises. For clearly, in both questions, ἄπειρος means *unlimited with respect to number*.

But now we do see something odd. In the original *logos*, the equivocation critique failed for a similar and yet different reason. There, we found that both forms of ἄπειρα (things unlimited) involved the sense *things unlimited with respect to time*. But now we find two instances of ἄπειρος, each meaning *unlimited with respect to number*.

Now recall that the counting *logos* retains all of the questions of the original, up until and including the *unlimited*-question. So it seems that in the counting *logos*, just as in the original, the ἄπειρα (things unlimited) in the *unlimited*-question should have the sense *things unlimited with respect to time*. And if the ensuing question, the unlimited-number-question, drops this sense in favor of *unlimited with respect to number*, it seems the *logos* might somehow involve an equivocation nonetheless.

Certainly there is something odd going on. Recall that the ἄπειρα, the things unlimited, are "these halfway-points", the collection of halfway-points that the *answerer* is imagining. But then, in the unlimited-number-question, the *runner* is presumed to have counted this same unlimited collection. But the runner does not have access to the answerer's imagination. So how can the runner count the halfway-points that the answerer is imagining?

The fact that the runner is himself imaginary does not help. It would help, perhaps, if the runner had only a finite set of halfway-points to count. Indeed in answering the counting-halves-question, the answerer likely does imagine the runner counting a few particular halfway-points. But the unlimited-number-question, unlike the counting-halves-question, intrinsically involves the intentionality of the runner in a way that is inaccessible to the imagination of the answerer. Certainly the runner does not, and cannot, literally imagine that the runner has counted an unlimited number of halfway-points. In considering this thesis, the runner considers a thesis that is beyond the scope of his imagination. But in order for the thesis to even be meaningful, the runner's mind must somehow have access to the unlimited collection.

We now see that the counting *logos* inherently involves two different minds somehow making contact with the same unlimited collection: the mind of the runner and the mind of the answerer. Of course, in retrospect, this might also seem true of the original, which involves the questioner and the answerer. Indeed there, as here, we discovered that the two different minds were not in fact making contact with the same unlimited collection, even if it seemed to the answerer as if they were. The key difference between the two versions seems to be that in the counting *logos*, a second mind is explicitly incorporated into the content of the *logos*.

It seems that both versions of the *logos* hinge on the difficulty of achieving shared mental access to the unlimited. So it is perhaps quite reasonable that Aristotle's initial resolution of these *logoi* will be grounded in a fundamentally new way of achieving such shared access, the method of diagrams. So our next task will be to examine this Aristotelian resolution.[260]

260 At this point, one curious fact merits mention. The counting *logos* may involve one of the earliest references ever to the notion of a *particular* unlimited number. There are a number of cases in Plato and Aristotle where a form of ἄπειρος (unlimited) modifies a form of ἀριθμός (number). However, examination reveals that in most of these cases, the word ἀριθμός refers not to a particular number, but to the category of number in general. By contrast, the counting Racetrack seems to invoke the counting of a particular unlimited number. Investigation of this issue awaits further study.

Chapter 8
Aristotle Critiques the *Logoi*

Aristotle's First Resolution and the Dialectical Racetrack

Earlier, in Chapters 2 and 3, we examined Aristotle's first analysis of the Racetrack in some detail. However, at the time, we focused primarily on understanding the structure of Aristotle's own text. We had not yet reconstructed any version of the dialectical Racetrack, and so were not in a position to assess Aristotle's analysis in conjunction with the *logos* he was actually analyzing. But now we do have a reconstruction of the *logos*, indeed two of them, and so we can return to *Physics* Z.2 to examine Aristotle's critique in conjunction with its intended target.

Our present analysis will proceed in three steps. First we will consider two ways in which Aristotle's resolution seems directed at a target different from either of the two dialectical versions of the Racetrack. Second, we will consider the charge that these disparities should be taken as a sign of problems in our reconstruction, and we will consider, and reject, one way to fix things. Finally, we will seek to account for the differences between the original *logos* and Aristotle's apparent target.

Recall that both the original Racetrack and the counting version begin as follows:

> Can a runner get through a *stadion*? Yes.
> Must he first reach the halfway-point before the end? Yes.
> And in between the start of the *stadion* and the halfway-point, is there another halfway-point? Yes.
> And must the runner reach the earlier halfway-point before the latter? Yes.
> And in between the start of the *stadion* and this new halfway-point, is there another halfway-point? Yes.
> And must the runner reach the earlier halfway-point before the latter? Yes.
> And in between the start of the *stadion* and this new halfway-point, is there another halfway-point? Yes.

> And must the runner reach the earlier halfway-point before the latter? Yes.
> Is it always necessary to pass through the halfway-point? Yes.

Then the original version continues:

> Are these halfway-points things unlimited? Yes.
> Is it possible to pass through things unlimited? No.
> And so you say that the runner cannot get through the *stadion*!

By contrast, the counting version continues:

> When the runner reaches each halfway-point, can he count that halfway-point? Yes.
> Are these halfway-points things unlimited? Yes.
> So when the runner has gone through the whole *stadion*, has he counted an unlimited number? Yes.
> But is it possible to count an unlimited number? No.
> And so you say that the runner cannot get through the *stadion*!

In *Physics* Z.2, in response to these *logoi*, Aristotle wrote:

> On account of this the *logos* of Zeno takes as granted a falsehood, that it is not possible to go through things unlimited (ἄπειρα) or to individually touch things unlimited in a limited time. For in two ways both length and time are said to be unlimited, and in general everything continuous, either with respect to division or with respect to its ends. Certainly it is not possible to touch things unlimited (ἄπειρα) with respect to quantity in a limited time, but it is possible [to touch] those [unlimited] with respect to division. For the time itself is unlimited in this way. So it follows that in the unlimited [time] and not in the limited [time] [one] goes through the unlimited [magnitude], and [one] touches things unlimited with things unlimited, not things limited. (233a21-31)

When we examined this passage earlier, we concluded that Aristotle viewed the target of his critique as an argument containing one of two variant theses. The first was:

> It is not possible to go through things unlimited in a limited time.

And the second:

> It is not possible to individually touch things unlimited in a limited time.

According to Aristotle, the argument required that one of these two theses be true. But Aristotle charges that both are ambiguous, inasmuch as they contain an ambiguous term, ἄπειρα (things unlimited). And he claims that once we disambiguate, it is evident that both theses are actually false in the sense required by the *logos*.

If we temporarily ignore the part about the limited time, it is evident that the first variant thesis derives from the original version of the *logos*. It is simply the impossibility-question-and-answer converted into declarative form.

Parallel to the impossibility-question of the original *logos* is the impossibility-question of the counting *logos*. Counting does involve a sort of serial discrete contact, so it is reasonable to suppose that the second variant thesis, about the impossibility of touching things individually, is based upon the question about the impossibility of counting. We need not suppose that Aristotle regards this touching as any sort of mental touching, or in particular, that this “mental touching” is counting, as we might surmise from reading *On Indivisible Lines*. (968b2-4) Instead, in *Physics* Z.2, Aristotle seems to be abstracting serial contact, in whatever form, as one of the central elements of the impossible-counting-question.

It now seems evident that Aristotle regards the two variant theses as derived, respectively, from the the impossibility-questions of the two dialectical versions of the *logos*. But the truth is that each of these two theses differs in two crucial ways from the original questions.

First, each of the two theses involves the qualification "in a limited time". But no reference to a limited time, or indeed to any sort of time, occurs in either version of the *logos*. Recall why this conclusion is warranted: any reference to a limited time would occur, if it did occur, in the impossibility-question of the two versions of the *logos*. But when Aristotle most directly concerns himself with reporting these questions as dialectical questions (263a3-11), he fails to mention any limited times. And we have so far seen no need to insert any such references into our reconstructed *logoi*. So we should presume that they were not there. Of course, given the centrality of the limited time to Aristotle's own resolution, we might find this odd. We will soon consider, and reject, an argument that the limited time *must* have been mentioned, but for now, the evidence fails to show this.

While the limited time seems completely absent from the original *logos*, things are somewhat different with the counting *logos*. To be sure, as compared with the original, there are only three new questions, all about counting, and none of these directly invoke a limited time. But there is a sense in which the unlimited task of the counter and runner is measured against some external temporal framework: he *has* counted the unlimited number *when* he has gone through the whole *stadion*. This suggests that in some fashion, Aristotle's invocation of the limited time is based upon his reflection on the counting *logos*. But it remains to discover the precise manner in which this is so.

Besides the limited time, there is a second difference between the two variant theses and the two impossibility-questions from which they derive, a difference not visible if we simply look at the wording of the theses. To see it we need to consider the actual referent of the word ἄπειρα (things unlimited).

Recall from Chapter 3 that Aristotle's resolution undeniably construes ἄπειρα (things unlimited) as referring to a plurality of segments. And indeed it is only a plurality of continuous segments, a plurality coextensive with the whole that the segments compose, that can be both extended unlimitedly and divided unlimitedly, thus permitting Aristotle's disambiguation of *things unlimited* into *things unlimited with respect to the* *ends* and *things unlimited with respect to division*.

By contrast, in both dialectical versions of the *logos*, the word ἄπειρα (things unlimited) clearly refers to a collection of halfway-*points*. Recall that in both versions of the *logos*, the existence of the things unlimited is ultimately established via the induction. And recall that the induction introduces new halfway-points by setting them up as new sub-goals analogous to the *end* of the whole *stadion*. So any reference to segments would be quite out of place.

Notice that Aristotle's interpretation of the things unlimited as constituting a collection of half-segments is in no way a component of his disambiguation. Instead it is a presupposition thereof.

We now see two ways in which Aristotle's two theses differ from the two questions from which they derive. First, each involves the performance of some unlimited task *in a limited time*, whereas the original questions mention no limited time. Second, each of these unlimited tasks is the traversal, in some way, of a collection of spatial *segments*, whereas the originals had involved the traversal, in some way, of unlimited collections of *points*.

Recall now that Aristotle's resolution hinges on the coextension of a plurality of spatial segments and the whole that they constitute. And recall that Aristotle's disambiguation will emphasize that the *logos* involves pluralities of segments *limited* with respect to their ends. So, in effect, each variant impossibility thesis will involve a limited spatial magnitude.

Aristotle's two changes to the impossibility-thesis thus show a striking resemblance. He introduces both a limited spatial magnitude and a limited time.

Now recall Aristotle's discussion of the truth and falsehood of the disambiguated theses. He explains that it is possible to go through an "unlimited" collection of spatial magnitudes if and only if the collection of temporal segments corresponding to this traversal is unlimited in precisely the same ways, by extension or by division. So Aristotle's use of his disambiguation emphasizes the parallelism between the time and the magnitude of a given motion. But now we see that Aristotle's innovation runs much deeper than acknowledgement of[261] this parallelism. Indeed, he himself is the one who introduces both the time and the magnitude that he now declares to be parallel to each other.

Traversing Things Untraversable

At this point we face a challenge: we need to explain why Aristotle's analysis critiques a target that appears to differ from the *logos* that actually existed. But perhaps we are doing Aristotle a disservice in claiming this, perhaps we have somehow gone astray in our reconstruction. In principle, of course, we might indeed have erred. But the burden lies with anyone making this claim. I shall consider and reject on argument that we are in error.

One might well suppose that the original *logos* so obviously needs to refer to a limited time that Aristotle has no need to mention it. Actually, of course, he does mentions it – in his *analysis*. And so if it is obviously necessary in the *logos*, we should simply presume it was there. But is this limited time obviously necessary?

Here is an argument that might convince modern philosophers. I have rendered

261 To be sure, both versions of the dialectical *logos* involve the *stadion*. But they do so in a way that does not clearly distinguish between the *stadion* as race, indeed as action or movement, and the *stadion* as magnitude. Moreover, neither invokes the *stadion* in the impossibility-question, our immediate concern.

the impossibility-question:

Is it possible to go through things unlimited?

Suppose we instead construe it in line with the modern renditions:

Is is possible to perform infinitely many tasks?

This question might seem to demand clarification. Suppose we render it:

Is it possible to perform infinitely many tasks in an infinite time?

I suppose many modern philosophers would say *Yes* (presuming the existence, of course, of an agent who can exist forever). On the other hand, suppose we ask:

Is it possible to perform infinitely many tasks in a finite time?

Here opinions will diverge. The standard modern response to the Racetrack says *Yes*, due to the mathematical possibility of infinite, yet bounded series. But certainly the answer *No* has great initial plausibility, especially to someone unschooled in 19th century mathematics. And so the modern analysis seeks to resolve the paradox by converting this *No* to a *Yes*. Of course, on this understanding, the paradox never even gets traction unless the supertask *needs* to occur in a *finite* time, rendering the *No* at least plausible. Hence we might conclude that the original version must have mentioned a finite time.

The problem with this reasoning is that ἄπειρος did not mean "infinite". Thus far, I have been rendering it as "unlimited". But recall that the root meaning was *untraversable*, so we should understand the typical meaning of the term as unlimited/untraversable. We saw earlier that we can contrast this with unlimited/infinite. It is precisely the sense *untraversable* that reveals the irrelevance of a finite time in the original impossibility-question.

The difference between *untraversable* and *infinite* (as *infinite* is normally understood in supertask discussions), is that *untraversable* is an agent-relative modal term, while *infinite* is an objective numerical term. The word *untraversable* conveys the impossibility of an action, while *infinite* has nothing to do with action. What follows from this is that the original Greek impossibility-question is nearly a logical contradiction in a way that the modern variant is not. I have been writing:

Is it possible to go through things unlimited?

But we might as well write:

Is it possible to go through thing untraversable?

Or even:

Is it possible to traverse things untraversable?

It is this question that is at least suggested by the Greek, and clearly the answer here is *No*. To add “in a finite time” would be as pointless as adding any other qualification

By contrast, the modern word “infinite”, as typically used, involves nothing of action, or of time. Hence there is nothing in this word itself to counterpose the performance of a given collection of tasks. Instead, “infinite” merely tells us the number of tasks, and absurdity only seems to arise when we further specify how the tasks are supposed to be situated temporally.

The upshot is that we see no reason to insert “in a limited time” into the original *logos*. Moreover,we now see that Aristotle's resolution, which in one sense seems to miss the mark, is in another sense strikingly *apropos*. Suppose, indeed, that the impossibility-question really does involve a near contradiction. It might then seem inescapable. But this holds only if the opposing terms are opposing *in the same sense*. If instead there are two senses of *going through*, or *traversing*, involved, then there will be no contradiction.

And we have indeed seen that Aristotle's disambiguation of ἄπειρος does involve a distinction between two senses of traversing.

Comprehending Things Unlimited

While we have now seen that Aristotle's basic disambiguation strategy seems plausible, we still need to explain why his resolution critiques a target that differs from the original *logos*. To do so, we will focus on the word ἄπειρα (things unlimited).

We have already seen that ἄπειρος, which in its earliest form means *untraversable*, involves the notion of action. Hence if we wish to clarify the sense of a use of ἄπειρος, we might ask for clarification in two different respects: it seems that something will be untraversable with respect to some agent and with respect to some sort of traversal.[262]

Given any particular use of any form of ἄπειρος, we might ask, first, what sort of agent is involved, and second, what sort of traversal is involved. For now, I suggest we put Aristotle aside and ask these questions about both the original Racetrack and the counting version. Since Aristotle's critique focuses on the impossibility-question, we will consider how ἄπειρα functions in the impossibility-question of each of the dialectical *logoi*.

We need to further specify our task, since in asking about the meaning of ἄπειρα, we need to ask: the meaning for whom? In principle, the questioner and the answerer might attribute different meanings to the word. But recall that the goal of the questioner is to lead the answerer into absurdity. We are concerned with the case when this happens. So we need to understand how the answerer understands ἄπειρα in precisely the cases

262 By contrast, it seems the term "unlimited" requires, or permits, specification in only one dimension.

when the *logos* works as intended, leading him into absurdity.

Consider now the original Racetrack. Recall that the induction leads the answerer to start imagining "halfway-points". Then comes the question:

Are these halfway-points ἄπειρα (things unlimited/untraversable)?

We know the answerer says *Yes*. But why? The answerer considers that he can go on imagining "halfway-points". In answering the question, he thus construes *himself* as the traverser, and *imagining* as the mode of traversal.

To modern philosophers, this might seem extremely odd. But I suggest that the real oddity lies, once again, in the difference between "untraversable" and "infinite". In the corresponding premise of modern Racetrack arguments, there will be *no* hint of agency:

The halfway-points are infinite in number.

But once we recognize that reference to "things untraversable" *does* involve agency, it is clear that the agent is the answerer. For clearly the runner is *not* involved in this particular question.

Next comes the impossibility-question:

Is it possible to go through ἄπειρα (things unlimited/untraversable)?

The answerer has already tacitly accepted that the relevant sense of untraversability involves himself imagining. And this new question gives him *no* reason to question that. It fails to mention the runner.

Things are somewhat different with the counting *logos*. The unlimited-question works much the same way as in the original *logos*, with the answerer focusing on himself imagining. But then come the two questions about counting an unlimited/untraversable number. Here it is clear that counting is the relevant mode of traversal, and so the agent is a counter. And indeed this counter is explicitly identified as the runner.

As concerns the answerer's understanding of ἄπειρα in the impossibility-question, the difference between the two *logoi* is intriguing. In the original *logos*, he understands the untraversability of the things untraversable from a first-person perspective. He himself is the imaginer imagining. Of course he does not *conceive* of himself as thinking this way, but it is what he *does*. The *logos* does not return overt focus to the runner until the assertion of the conclusion, at which point the answerer has failed.

By contrast, when the answerer confronts the impossibility-question of the counting *logos*, he is forced to understand the untraversability from a third-person perspective. *Someone* else is the counter counting.

In one sense, it is clear that this shift to a third-person perspective heavily influences Aristotle in his analysis. In particular, the reference to a limited time is inappropriate when the answerer considers his actions from a first-person perspective, but seems, perhaps, relevant when we consider the runner from a third-person standpoint.

But this shift from the first to the third-person does not by itself explain the novelty of Aristotle's response. Thus far we have only considered the issue of how to understand the *untraversability* of the untraversable plurality. But we also need to consider the question of how to consider the *identity* of the untraversable collection. Every person who hears either version of the *logos*, or Aristotle's resolution thereof, hears references to an untraversable collection. And to understand such references, these listeners need to understand *two* different things. They need to somehow grasp what collection is being referred to. And they need to somehow grasp the sense in which this collection is supposed to be untraversable. As a means of understanding Aristotle's response, we should consider each of these issues with respect to the original *logos*, to the counting *logos*, and to Aristotle's response.

We have already discovered the answers as concern the original *logos*. As for *what* the ἄπειρα (things untraversable) *are*, the answerer takes them to be the collection of halfway-points that he is imagining. Recall why this is so: the word ἄπειρα refers to the same thing as “these halfway-points”. But we saw earlier that the answerer construes “these halfway-points” as a demonstrative term referring to a plurality, an imaginary plurality.[263] As for the untraversability of the things untraversable, we have seen that the answerer takes himself as the traverser and the moder of traversing as imagining. Thus both the identity and the untraversability of the ἄπειρα are understood in a private fashion, as no one else can have access to the answerer's imagination.

Things are different when we confront the counting *logos*. Initially, of course, the *logoi* are similar. The answerer faces the question:

Are these halfway-points ἄπειρα (things unlimited/untraversable)?

And he understands this question much as he does in the original, so that the identity and the untraversability of the ἄπειρα are understood in a private fashion. But then comes a question about the runner counting the ἄπειρα, so the answerer needs to understand the untraversability of the ἄπειρα as involving the runner counting, and not as involving himself imagining. But notice that there is *no* revision as concerns the identity of the ἄπειρα. This means that the word ἄπειρα refers the imaginary collection of the *answerer*. But how can the runner count the imaginary collection of the answerer? How does the

263 Recall that the members of this plurality need not be determinate, since the demonstrative attaches itself to the plurality via acquaintance.

mind of the runner have access to the imaginary collection of the answerer?

We have already encountered this puzzle in Chapter 7. It would be easy to miss its significance. Someone might say: there is no problem of correlating the mind of the runner with the collection that he counts, since *both* are imagined by the answerer. But this is not true. The answerer does *not* imagine the mind of the runner, since minds are not visible. This objection hinges on the idea that what the answerer is really doing is postulating some imaginary world, and discoursing about this. In fact, the answerer simply answers questions as they come, imagining as need be. This may lead to inconsistencies. What we have now is a new way of describing one such inconsistency. When the answerer confronts the impossibility question of the counting *logos*, he understands the untraversability of the untraversable collections from a third-person standpoint. But the very identity of the collection supposedly being traversed is accessible only to the answerer himself. He thus understands the identity and the untraversability of the untraversable collection in incompatible ways.

What Aristotle does is remove this mismatch. He seeks to reconfigure both versions of the *logos* so that we can understand both the identity and the untraversability from a public, not merely a private, standpoint.

Thus far we have focused our attention on how the dialectical answerer would construe the words of the dialectical questioner. But if we wish to understand Aristotle's analysis in *Physics* Z.2, we must change our focus. This is because *Physics* Z.2 is not a dialectical *logos*, but something like a lecture to an audience. So if we wish to understand Aristotle's own references to untraversable collections, we must understand how he intended *his* audience to understand them.

We have already examined this issue in Chapter 3. When Aristotle speaks the word ἄπειρα (things unlimited/untraversable), *what* this word refers to is a diagram, indeed a diagram looking something like this:

A B Γ Δ

Such a diagram visibly *is* a plurality of segments. It follows that, for Aristotle's audience, the identity of the untraversable collection is established in an inherently public and third-person fashion.

To get a better sense of the difference between Aristotle's approach and the situation in the *logoi*, we can consider how the perspective of Aristotle's audience differs from the perspective of an audience hearing the dialectical *logoi*. Every member of Aristotle's audience hears Aristotle say the word ἄπειρα (things unlimited/untraversable). And every member of this audience understands that this word refers directly to the diagram visible in front of them. They all hear the same word and see the same diagram.

With a dialectical audience, the situation is different. In general, we can presume that each member of the audience will hear and understand things in the same manner as the answerer. Naturally, the different natures of their emotional engagement will render this not strictly true, but while the audience members are not answerers, they are indeed listeners, hearers of the questioner, in much the same manner as the actual answerer. What this entails is that each will, other things being equal, proceed through the same processes of imagination as does the actual answerer. As a result, in both versions of the *logos*, each audience member will take the questioner's reference to things untraverable as referring to the collection that he himself, he the listener, is imagining.

It follows that each person hearing a questioner ask either dialectical version of the Racetrack will understand the untraversable plurality to be something different, inasmuch as each takes it to be a collection that *he* imagines. By contrast, each person who hears Aristotle discuss this "same" untraversable collection will understand it to be one and the same thing, namely, the diagram that they all see. This might seem quite odd. It might rather seem that we should say that the diagram and the various interior images *represent* one and the same thing but in different ways. The problem with this view is that we then need some external, objective standard that attaches the diagram and the interior images to that which they represent. But, at least in the case of the dialectical *logoi*, that is precisely what we are lacking. The very referent of ἄπειρα, in each *logos*, *is* the imaginary collection. There is no independent fact that attaches this collection to some non-imaginary determinate collection. So Aristotle's public diagram line really is significantly different from the privately imagined pluralities of points.

Just as Aristotle's audience understands the identity of the untraversable collection in a public fashion, so it is with the untraversability thereof. We saw in Chapter 3 that the relevant sense of traversing a linear letter diagram would consist of adding letters to the diagram. Inasmuch as letters can always be added beyond the existing exterior letters, the diagram will be untraversable with respect to the ends. And inasmuch as letters can always be added between existing letters, the diagram will be untraversable with respect to division.[264] In this case the agent of traversal is anyone who might wish to add additional letters. And hence, just as in the counting *logos*, the untraversability of the untraversable collection is understood from a third-person perspective.

264 Of course, in neither way can letters actually be added unlimitedly, since the line would need to get too big, or the letters too small. But remember that, typically, once a diagram was constructed, no one *actually* went about adding any letters. Instead, speakers would reference a *given* diagram, but speak as if they were doing things to it.

We thus see that Aristotle takes the third-person perspective that the counting *logos* employs as concerns the untraversability of the untraversable collection, and, using a diagram, employs it also to understand the identity of the untraversable collection. And once a third person's object of traversal is understood as being, quite literally, a visible diagram, it seems a small step to construe his time of traversal in the same way, as Aristotle does.[265]

It seems we have thus accounted for the two novelties of Aristotle's reconfiguration of the two *logoi*. Starting with the third-person understanding of the untraversable collection that he finds at the end of the counting *logos*, he employs diagrams to reconfigure both *logoi* so that each involves an entirely third-person understanding of the things untraversable.

Having now examined what Aristotle has done, we might wish to consider the degree to which he is successful. For now, however, the issues involved in examining this issue would take us too far afield, since we would need to determine what standards of success we should actually apply. We will briefly return to this issue at the beginning of Chapter 10. But we are not yet done with Aristotle's critique of the Racetrack, for, as is well-known, Aristotle himself later critiques his own initial critique. In Chapter 9, we will examine this critique of a critique, and will find that Aristotle's second thoughts are in many ways on the mark.

What we will later find is that Aristotle's reconsideration effectively critiques the obfuscation of reality by diagrams, an obfuscation that might well be imputed to Aristotle's own initial critique. Indeed we will find Aristotle returning, rather astutely, to

265 To be more precise, it seems a small step *given* that Aristotle is *already* representing times via diagrams.

a consideration of what actually goes on in the imagination of the answerer. But curiously, we will also find that Aristotle's later critique of the original resolution is itself entirely dependent on diagrams, and that the insights he offers therein could scarcely have been attained had he not proceeded through his initial resolution.

Since Aristotle's later resolution is effectively a critique of the diagram resolution, it will serve us well to ensure that we have a grasp of this resolution in its own right. One way to do this is to examine how Aristotle applies it, indeed how he applies it in analyzing the Achilles. So we now turn to Aristotle's resolution of the Achilles.

Aristotle Analyzes the Achilles

We have already reconstructed the Achilles *logos*:

> If a tortoise runs from Achilles, can Achilles catch him? Yes.
> Must Achilles first reach the place from which the tortoise started? Yes.
> But by then, hasn't the tortoise moved to some place up ahead? Yes.
> Before Achilles catches the tortoise, must Achilles first reach the place where the tortoise is now? Yes.
> But by then, hasn't the tortoise moved to some place up ahead? Yes.
> Again, before Achilles catches the tortoise, must Achilles first reach the place where the tortoise is now? Yes.
> But by then, hasn't the tortoise moved to some place up ahead? Yes.
> So the tortoise will always keep ahead some amount? Yes.
> So Achilles will never catch him? Right.

And recall that Aristotle's only discussion of the Achilles comes in *Physics* Z.9:

> δεύτερος δ' ὁ καλούμενος Ἀχιλλεύς· ἔστι δ' οὗτος, ὅτι τὸ βραδύτατον οὐδέποτε καταληφθήσεται θέον ὑπὸ τοῦ ταχίστου· ἔμπροσθεν γὰρ ἀναγκαῖον ἐλθεῖν τὸ διῶκον ὅθεν ὥρμησεν τὸ φεῦγον, ὥστε ἀεί τι προέχειν ἀναγκαῖον τὸ βραδύτερον. ἔστιν δὲ καὶ οὗτος ὁ αὐτὸς λόγος τῷ διχοτομεῖν, διαφέρει δ' ἐν τῷ διαιρεῖν μὴ δίχα τὸ προσλαμβανόμενον μέγεθος. τὸ μὲν οὖν μὴ καταλαμβάνεσθαι τὸ βραδύτερον συμβέβηκεν ἐκ τοῦ λόγου, γίγνεται δὲ παρὰ ταὐτὸ τῇ διχοτομίᾳ (ἐν ἀμφοτέροις γὰρ συμβαίνει μὴ ἀφικνεῖσθαι πρὸς τὸ πέρας διαιρουμένου πως τοῦ μεγέθους· ἀλλὰ πρόσκειται ἐν τούτῳ ὅτι οὐδε τὸ τάχιστον τετραγῳδημένον ἐν τῷ διώκειν τὸ βραδύτατον), ὥστ' ἀνάγκη καὶ τὴν λύσιν εἶναι τὴν αὐτήν. τὸ δ' ἀξιοῦν ὅτι τὸ προέχον οὐ καταλαμβάνεται, ψεῦδος· ὅτε γὰρ προέχει, οὐ καταλαμβάνεται· ἀλλ' ὅμως καταλαμβάνεται, εἴπερ δώσει

> διεξιέναι τὴν πεπερασμένην.
>
> And second is the so-called Achilles. And it is this, that the slowest, in running, will never be caught by the fastest. For necessarily, the pursuer must first reach [the place] from which the pursued started, so that necessarily, the slower will always keep somewhat ahead. But this is the same *logos* as with the bisecting, but it differs in dividing the added magnitude not *half*-way. By all means, not catching the slower follows from the *logos*, but it comes about on account of the same thing as the bisection (for in both [*logoi*] not arriving at the end follows, with the magnitude being somehow divided), but it is added in this [*logos*] that not even the fastest made famous in a tragedy [will reach the goal] in pursuing the slowest, so that necessarily the *lusis* is also the same. And moreover, maintaining that the one keeping ahead is not caught is a fallacy. For *when* he keeps ahead, he is not caught, but nonetheless he [i.e. the slower] is caught, if he [i.e. the original questioner] will grant that [he, i.e. the slower] passes through the limited [line]. (239b14-29)

We earlier determined that the initial portion of this discussion, 239b14-18 ("And second...somewhat ahead"), is intended as report of the *logos*, whereas the ensuing portion, 239b18-29 ("But this...limited [line]"), is intended as an analysis. Since we have already reconstructed a version of the original *logos* itself, we may henceforth ignore Aristotle's report, and focus attention on his analysis.

In Chapters 2, 3 and 8, we have examined Aristotle's *Physics* Z.2 analysis of the Racetrack. We now see that our examination has consisted of two parts. In Chapter 8, we have considered how Aristotle's critique focused on a reinterpretation of the dialectical versions of the Racetrack, a reinterpretation involving diagrams. But earlier, in Chapters 2 and 3, we examined what, in some respects, came later in reality, Aristotle's framing of his response to this reconstructed *logos* as a dialectical response.

In examining the Achilles, we will need to confront the same two issues: Aristotle's reframing of the *logos* via diagrams, and his effort to ensure that his response is dialectically plausible. But we will find that, whereas with the Racetrack Aristotle uses diagrams fairly directly to enable what he construes as a dialectical response, with the Achilles Aristotle first reinterprets the entire Achilles *logos* via his diagrammatic understanding of the Racetrack. But then, since this diagrammatic understanding of the Achilles diverges considerably from the actual Achilles *logos*, Aristotle seeks to show that his reinterpretation *does* enable a *dialectical* response, albeit one that turns out to be significantly more complicated than his suggested response to the Racetrack.

We will find that Aristotle's Achilles analysis consists of three distinct parts, which we will examine in turn. In the first part, "But this ... not *half*-way" (234b18-20), Aristotle sets out a bold thesis about the similarity between the two *logoi*, a thesis he sees as far from obvious. We will find that he construes the two *logoi* as the same because they involve the same diagram. The second portion of the analysis, "By all means ... the same" (234b20-26), performs two distinct functions. First, Aristotle seeks to reconcile his bold claim about the similarity of the two *logoi* with the obvious fact that the *logoi* differ. Second, Aristotle makes the seemingly trivial claim that because the *logoi* are the same, they have the same *lusis*. But this creates a new problem. Even if Aristotle is correct in everything he has said so far, it is also clear that the Racetrack *lusis* simply will not work as a response to the dialectical Achilles. And this explains the third and final section of Aristotle's discussion, "And moreover ... the limited [line]" (239b26-29). Here Aristotle seeks to reconfigure the Racetrack *lusis* so that it might actually work against the dialectical Achilles. To recap: Aristotle begins by identifying the *logoi* via diagrams. He next proceeds to confront the apparent differences between the *logoi* themselves. And finally he confronts the apparent irrelevance of his Racetrack *lusis* to the Achilles.

Similar Diagrams, Same *Logos*

Aristotle begins: "But this [i.e. the Achilles] is the same *logos* as with the bisecting, but it differs in dividing the added magnitude not *half*-way." (239b18-20) Understanding this line is the key to understanding Aristotle's analysis, and the key to understanding this line lies in understanding how Aristotle assimilates the diagrams he uses to render the two *logoi*.

It is far from obvious that Aristotle is thinking of diagrams here. Certainly the text employs no diagram letters. We will later see that the final phrase of the analysis, the reference to "the limited [line]" (239b29), does seem to involve a direct reference to a diagram. But our strongest evidence for the centrality of a diagram is the very fact that Aristotle assimilates the two *logoi*. He tells us directly that the Achilles is "the same *logos*" (239b18) as the Racetrack, "so that necessarily the *lusis* is also the same." (239b25-26) But we have found that Aristotle's *lusis* for the Racetrack critiques his diagrammatic reinterpretation of the Racetrack. If the Achilles *lusis* is "the same" as *this* Racetrack *lusis*, then it must be critiquing a *logos* that is "the same" as the reinterpreted diagram Racetrack. Hence the Achilles, as Aristotle construes it, will need to involve a diagram.

We will shortly consider direct evidence that Aristotle's strategy for understanding the Achilles involves ignoring the tortoise, and simply considering Achilles as needing to traverse the path that he does need to traverse in the course of catching the tortoise. For now we will simply assume that this is true, and consider the implications concerning the assimilation of the two *logoi*.

We know, more or less, what diagram Aristotle thinks is relevant to the Racetrack.

Recall that it looks like:

Α Ε Δ Γ Β

As for the Achilles, Aristotle must be thinking of something like this:

Α Γ Δ Ε Β

In one sense, the diagram I have given is more or less exactly what a diagram for a *progressive* version of the Racetrack would look like, if there were such a thing. In another sense, if we ignore the spacing and the ordering of the letters, neither of which typically has any inherent diagrammatic meaning, then the second diagram is actually the same as the first. To understand the second diagram, we need to see, first, how it fits the Achilles, and second, how it relates to the first, Racetrack, diagram.[266]

We can construe the full diagram as the full path that Achilles traverses in seeking to catch the tortoise. Recall that the *logos* begins with the answerer acknowledging that Achilles does catch the tortoise. Hence, it might seem that there is some path that he traverses in doing so. We can suppose that the full diagram represents this path. In this case Achilles will start next to the Α, the tortoise will start next to the Γ, and Achilles will finally catch the tortoise next to the Β. As for the remaining letters, Δ marks the place the tortoise is at when Achilles reaches Γ, Ε marks the place the tortoise is at when Achilles reaches Δ, and so on for any additional letters that might be on the diagram. Hence Δ, Ε, etc., are also the ensuing targets for Achilles after he reaches Γ.

What Aristotle calls "the added magnitude" (τὸ προσλαμβανόμενον μέγεθος) (237b19-20) is always the full magnitude that remains for Achilles to traverse, after he

266 Heath (1949) displays diagrams for the two *logoi* that, notwithstanding some minor anachronisms, are quite similar to the diagrams I give above.

has already gone part of the way.[267] It is the magnitude that needs to be "added" to the magnitude that Achilles has already covered. (Remember that diagrams are always construed as involving the addition or division of segments.) But the added magnitude keeps getting "divided" because Achilles always has an earlier point to reach *before* he reaches the end.

Although I have here described the diagram by saying that the tortoise started at Γ, was caught at Β, etc., once the diagram has been established, we can just as easily see the tortoise as irrelevant. Indeed we can suppose that somehow Achilles is moving along the diagram itself, from Α to Β. But before he reaches Β, there is always some additional spot that he needs to reach. We can leave it an open question whether the tortoise is also moving along this diagram, and in what manner.[268]

If the Racetrack were progressive, then the introduction of the diagram and the elimination of the tortoise would, apart from possible differences in the ratio of division, have converted the Achilles into a version of the Racetrack. But the Racetrack is regressive, not progressive. And so there might seem to be a significant difference in the way we need to interpret the diagrams of the two *logoi*. In the Achilles diagram, the successive points of division mark points that Achilles is supposed to occupy later and later. But in the Racetrack diagram, the successive points of division mark points that Achilles is supposed to occupy earlier and earlier.

This may very well be true, but the differences disappears if we simply ignore the runners, and focus our attention on the diagrams in relation to us, or better, to Aristotle

267 Ross concurs. (pg. 76)

268 Ultimately, the tortoise will somehow need to be relevant, since he does appear in the actual *logos*. But for now we can ignore him.

and his audience. But in doing so, we can still invoke the standard conceit that diagram segments can be added or divided, even though this does not literally happen. What we find is that the two diagrams are functionally identical. Each begins as a segment, which is then divided. And then one of the parts resulting from the division is divided, and so on. The diagrams may differ in the ratio of division, but this is not something that diagrams themselves ever purport to represent accurately. Admittedly, the diagrams are, in some sense, mirror images. But if we are ignoring the temporal implications this has for the runner, as we are, then this reversal does not matter. Both diagrams "are" simply collections divided unlimitedly, and the precise manner of this division is irrelevant.

We thus see that diagrams may provide a way for Aristotle to identify the Racetrack and the Achilles. Aristotle supposes that each *logos* involves a runner traversing a fixed magnitude. And in each case, Aristotle construes this magnitude as a diagram, a diagram that is in actuality divided into several parts, but one that can be treated, and spoken of, as a diagram that is divided unlimitedly. In short, it is precisely the use of diagrams to represent magnitudes that enables Aristotle to assimilate the two *logoi* via their use of magnitudes.

Aristotle's use of a diagram, and the magnitude it represents, constitutes a significant deviation from the original Achilles *logos*. Recall that the Achilles *logos* itself certainly did not mention any fixed magnitude. And the *logos* concerned not a race, which would involve a fixed magnitude, but a chase. Moreover the *logos* does not require that Achilles and the tortoise have fixed speeds, or a fixed ratio between their speeds, which would, in some sense, allow the determination of a fixed magnitude. In short, nothing in the *logos* itself requires the supposition that the runners traverse a magnitude. Instead, it is Aristotle himself who introduces a magnitude via a diagram, and this is what enables him to see an affinity between the two *logoi*. Naturally, we will soon need to consider how this diagram might function in a dialectical response, but for now we have a plausible understanding of Aristotle's underlying assimilation.

The Diagram and the *Logos*

In the opening line of his analysis, Aristotle has told us that the Achilles is "the same *logos*" as the Racetrack. (239b18-20) And we have seen that Aristotle seems to make this claim due to the similar diagrams that he sees as relevant to the two *logoi*. But the truth is that the dialectical *logoi* themselves appear to be quite different, a fact that would be obvious to both Aristotle and his audience. Hence Aristotle needs to reconcile his claim about the similarity of the *logoi* with the obvious fact that they differ, and he proceeds to do just that.

Aristotle's analysis continues: "By all means, not catching the slower follows from the *logos*, but it comes about on account of the same thing as the bisection (for in both [*logoi*] not arriving at the end follows, with the magnitude being somehow divided), but it is added in this [*logos*] that not even the fastest made famous in a tragedy [will reach the goal] in pursuing the slowest, so that necessarily the *lusis* is also the same." (239b20-26) This complex sentence performs two distinct functions. First, it explains the immediately prior claim that the two *logoi* are the same. And second, it advances a new claim on the basis of the prior claim, a claim about the appropriate *lusis* for the Achilles. We will consider these issues in turn.

Aristotle begins by appearing to offer a concession. We might imagine someone objecting to Aristotle's identification of the two *logoi*, objecting that in the Achilles,

Achilles fails to catch the tortoise, whereas the Racetrack involves no chase at all. How then can the two *logoi* be the same? In response, Aristotle concedes that the conclusion of the Achilles, that Achilles cannot catch the slower tortoise, is different from the conclusion of the Racetrack. But he nonetheless claims that the derivation of this conclusion somehow resembles the bisection process that helps to yield the conclusion of the Racetrack.

Aristotle's challenge is to explain how a *logos* about a chase is somehow "really" about a runner traversing a magnitude. To render this plausible, Aristotle takes two distinct steps. First, he reinterprets "not catching the slower" (239b20-21) as "not arriving at the end" (239b23). This effectively removes the slower tortoise from the picture, as the generic "end" may simply be the *place* where the tortoise *does* get caught. Second, Aristotle makes clear that in his movement toward the "end", the runner Achilles is traversing a magnitude that the *logos* presents as repeatedly divided. (239b23-24)

What Aristotle does not overtly mention is the underlying presumption of both of these revisions: the presumption that Achilles is traversing a magnitude at all. But clearly this *is* a presumption. The "end" that the runner must reach, inasmuch as it is *not* the tortoise, must be the end of *something*, and indeed of *some* magnitude. Moreover, if the magnitude is being divided, it must exist. Now the dialectical *logos* clearly mentions no magnitude. So *what* magnitude is Aristotle talking about? He would seem to be talking about a visible magnitude that he is presenting on a diagram board. This fits the context of *Physics* Z.2, where it is routine for motions to be presented as involving the traversal of a diagrammed magnitude, and it certainly meshes with Aristotle's reinterpretation of the Racetrack.

We now understand Aristotle's identity claim. Yes, Aristotle admits, the Achilles does conclude that Achilles fails to catch the tortoise. But this conclusion, he says, somehow results from the argument that Achilles cannot reach the end of a fixed magnitude, inasmuch as that magnitude can be repeatedly divided. And in Aristotle's view, this is basically the same as what happens in the Racetrack. Aristotle concludes his explanation by suggesting that the theme of the Achilles pursuing the tortoise is simply an addition to the underlying Racetrack framework. (239b24-25)

We now see that Aristotle has opened his analysis with a bold and seemingly unjustified claim about the identity of the two *logoi*. (239b18-20) But then he proceeds to reconcile this claim with the evident fact that the *logoi* differ. (239b20-25) It is precisely in this attempt at reconciliation that we see the need for a visible diagram. Aristotle needs his audience to *not* realistically imagine Achilles pursuing the tortoise. Instead, he needs them to suppose that somehow, Achilles, or someone or something, is actually moving along the diagram that they see.

There might seem to be something dubious about Aristotle's procedure. He is saying that the conclusion of the Achilles, the actual conclusion of the Achilles, "come about on account of the same thing as the bisection." (239b21-22) But this is, strictly speaking, false. The questions of the two *logoi* are quite different, and Aristotle's ability to re-envision the Achilles via a diagram does not change that. But for the moment we may defer discussion of this issue, since it turns out that Aristotle himself is likely aware of the discrepancy.

Having now, in his view, explained the claim that the two *logoi* are identical, Aristotle proceeds to draw a conclusion from that claim. If the Achilles is the same *logos,* he says, then "necessarily the *lusis* is the same." (239b25-6) In itself, this claim seems fairly uncontroversial. Naturally, two *logoi* that are the same should have the same *lusis*.

Things become more complicated once we recall what the *lusis* for the Racetrack actually involves. Remember that according to Aristotle, the Racetrack employs the premise: It is impossible to go through things unlimited (ἄπειρα) in a limited time. (For simplicity, I here ignore the alternate version.) Aristotle contends that this thesis is ambiguous because the term ἄπειρα (things unlimited) is ambiguous. And hence the *lusis* for the Racetrack consists of disambiguating the term ἄπειρα and showing that, of the two disambiguated theses resulting from the original ambiguity, the version required by the Racetrack argument is false. This is Aristotle's *lusis* for the Racetrack, and since he says that the Achilles is the same *logos*, and has the same *lusis*, this is also Aristotle's *lusis* for the Achilles.

But now we see a problem. The Achilles *logos* contains nothing like the ambiguous theses that Aristotle finds in the Racetrack. Indeed it nowhere contains the word ἄπειρα (things unlimited) or indeed any form of ἄπειρος (unlimited). How then does Aristotle suppose that a dialectical answerer can apply a *lusis* that involves disambiguating ἄπειρα?

This problem seems to have been entirely missed by contemporary scholars. Indeed, the problem is only evident once we have recognized the precise dialectical structure of Aristotle's Racetrack *lusis*, something that scholars have also missed. But now we have seen that the Racetrack *lusis* does involve a dialectical disambiguation, and hence that it will work only against a *logos* that contains the term being disambiguated.

I suggest that Aristotle is actually quite aware of the dialectical implausibility of his claims thus far, and that this is precisely what accounts for the final portion of his analysis. He has thus far shown, via his use of a diagram, a novel way of thinking about the Achilles *logos*. But he has not yet shown how to employ this new interpretation to enable a plausible dialectical counterattack. So what he needs to do is to show a dialectical questioner how to actually use the diagrammatic reinterpretation to somehow render the Racetrack *lusis* plausible as a dialectical response.

A Dialectical Counterattack

We have seen that the Racetrack *lusis* clearly will not work against the Achilles as it is actually presented. But perhaps it would work if the questioner actually employed an argument that used the putatively ambiguous term ἄπειρα (things unlimited). So I suggest that the essence of Aristotle's dialectical strategy is to have the original answerer go on the offensive, and force the original questioner into actually giving an argument that does employ the term ἄπειρα. This new argument will be based on Aristotle's diagrammatic understanding of the Achilles, and will succumb, at least as well as the Racetrack, to the disambiguation that Aristotle employs against the Racetrack.

If my interpretation is correct, then Aristotle's Racetrack critique will differ significantly from his Achilles critique. With the Racetrack, Aristotle attacks an element of the *logos* that actually exists, and he seems to envision that the answerer performs his disambiguation in the midst of the original *logos*, right when the ambiguous thesis is used by the questioner. By contrast, with the Achilles, Aristotle has no response to the *logos* as it is presented. So the *logos* will proceed to the end, with the answerer apparently defeated. But at this point the answerer will launch a counterattack, seeking to implicate the questioner in an argument that *is* fallacious for the same reason as the Racetrack.

Aristotle begins the final portion of his analysis by recalling the actual conclusion of the Achilles: "And moreover, maintaining that the one keeping ahead is not caught is a fallacy." (234b26-27) Recall that in arguing for the affinity of the two *logoi*, Aristotle has depicted the actual conclusion of the Achilles as irrelevant, and focused his attention on Achilles traversing a magnitude. But now he turns his attention back to the Achilles as it actually exists, his goal being to provide a workable dialectical response.

Aristotle continues: "For when he keeps ahead, he is not caught... ." (239b27-28) This clause seems to be an acknowledgement that the aforementioned erroneous conclusion does indeed appear plausible. Certainly the tortoise is *not* caught *when* he is ahead. Aristotle quite commonly offers plausible justifications for conclusions that he proceeds to reject, and I suggest that he has a similar motivation here.

It might well seem that what Aristotle is doing is setting up a version of the *always*-critique that we examined earlier. Recall that this critique contends that the *logos* equivocates concerning the scope of *always* (ἀεί), insinuating that the tortoise is *always*, that is, *at all times*, evading capture, when in fact he is *always* evading capture only *when he is ahead*. But this interpretation suffers serious problems. First, Aristotle has already told us that the Achilles requires the same *lusis* as the Racetrack. And yet the *always*-critique is certainly irrelevant to the Racetrack. Second, we have already seen that the *always*-critique simply does not work against the dialectical Achilles. Recall that nothing in the *logos* ensures that there is a difference between the scopes encompassed by *always, at all times*, and *always, when the tortoise is ahead*.

Certainly, there is a sense in which Aristotle intends to draw a distinction between two times. But I suggest that Aristotle's real goal is to contrast the time of the pursuit as the answerer typically *imagines* it, and the time of the pursuit as it *can* be understood via a diagram. Typically, the answerer does imagine that the tortoise will be ahead forever. But if the pursuit is construed as occurring along, and ending at the end of, a fixed diagram line, then clearly there will be a difference between the time of pursuit and the time of capture. Indeed, I suggest that Aristotle invokes the contrast between these two times as a way of evoking the more fundamental contrast between the non-diagrammatic *imagination* of the chase, where the difference is *not* evident, and the visible diagrammatic *depiction*, where it is. And indeed, we will see that the core of Aristotle's dialectical counterattack will be the very *introduction* of the diagram that enables this distinction.

The analysis concludes: "... but nonetheless he [i.e. the slower] is caught, if he [i.e. the original questioner] will grant that [he, i.e. the slower] passes through the limited [line]." (239b28-29) This text is rather confusing as it stands, so we will need to carefully consider its literal meaning before we go on to examine its broader import. To begin with, the subject of καταλαμβάνεται (he is caught) is obviously "the slowest", in the original *logos*, the tortoise. And hence we should presume that the unstated subject of διεξιέναι (pass through) is likewise "the slowest".

The term δώσει (he will grant) is central to my understanding of the dialectical counterattack. Being a form of δίδωμι (grant, give), we should presume that it is intended in its dialectical sense, a fact that has been missed by scholars. The person doing the "granting" will be some dialectical questioner. But we know that the basic Achilles *logos* involves no question about the tortoise passing through anything "limited". Hence

Aristotle must be considering a new question that is introduced by one of the dialectical participants in order to bring about a *lusis*. And since the original answerer is the one who needs a *lusis*, he must now be the one now asking the question. And so the intended subject of δώσει (he will grant) will be the original questioner.

The final words of the analysis are τὴν πεπερασμένην, literally, "the limited." (239b29) This phrase is grammatically feminine, which indicates that Aristotle has a specific referent in mind, a referent with a feminine name. If he had no specific referent in mind, he would have used the neuter form, not the feminine. So we need some feminine word that refers to something like a magnitude.

In the Racetrack analysis of *Physics* Z.2, Aristotle construes the runner as traversing a μῆκος (magnitude), and in *Physics* Z .2 Aristotle tends to use μῆκος interchangeably with μέγεΘος (magnitude). Hence in *Physics* Z.9, we might presume that Aristotle finds Achilles, "the fastest", traversing a similar μῆκος or μέγεΘος, and the tortoise, "the slower", traversing the same μῆκος or μέγεΘος. In some sense this is true, and indeed Aristotle does use μέγεΘος in the Achilles analysis. But neither term is the referent of τὴν πεπερασμένην , since both μῆκος and μέγεΘος are neuter.

The only reasonable alternatives are κίνησις (motion) and γραμμή (line), both feminine. While κίνησις has prominent role in role in *Physics* Z, it plays no role in the Racetrack *logos* or the Achilles *logos* nor, thus far, in Aristotle's analyses of either. By contrast, we have already made the case that Aristotle employs linear diagrams, that is, visible lines, in both of his analyses, and he very likely has a linear diagram on display as he discusses the Achilles. Such a diagram, *being* a line (γραμμή), may thus serve as the intended referent of τὴν πεπερασμένην (the limited), forcing it into the feminine. Indeed, given the lack of alternative explanations, we may even regard the feminine gender of this phrase as independent evidence supporting the existence of a diagram, evidence to be added to our earlier indirect case based upon the parallel structure of the Racetrack and Achilles analyses.

Aristotle is thus telling us that the tortoise will be caught if the answerer grants that the tortoise passes through a given line on Aristotle's diagram board. But recall that Aristotle himself has already employed the given diagram line, making clear that he regards it as the very line that Achilles needs to traverse. Aristotle's audience will thus hear Aristotle demanding that the original questioner grant that the tortoise will pass through the very same visible line that Achilles also passes through. If we add the qualification that Achilles and the tortoise pass through this same line in the same time, then it does seem that Achilles indeed catches the tortoise.[269]

We now have a good grasp of what Aristotle is literally saying. But we have not yet considered how his comments lay the groundwork for a dialectical counterattack. How does the concession that the tortoise traverses a diagram line enable the Achilles to be resolved by the disambiguation of ἄπειρα? It is obvious both that Aristotle requires an answer to this question, and also that he does not fully explain his answer. So if we are to make sense of Aristotle's overall strategy, we must do some extrapolation.

I suggest that, in presenting, as the key to the counterattack, the original questioner's concession that the tortoise traverses a "limited line", what Aristotle is

269 Strictly speaking, it might seem that the tortoise should traverse a shorter line, inasmuch as he has a head start. But we might suppose that the tortoise starts from the position of Achilles, and then runs away, yielding a time gap before Achilles starts the pursuit. In this case, the time of the pursuit would consist of the whole time of the tortoise's running, with Achilles being at rest for the first portion of this time. In any case, it seems fairly clear that Aristotle's analysis hinges on thinking of Achilles and the tortoise as independent traversers of the same path.

actually envisioning is the *introduction* of a diagram line into a *logos* that heretofore involved no such thing. Aristotle envisions the original answerer gaining the original questioner's assent to the introduction of this line. And then the original answerer proceeds to implicate his opponent in a diagrammatic version of the Racetrack argument. But in Aristotle's view, *this* argument can be exposed as fallacious by the disambiguation of the term ἄπειρα. And if the original answerer does this, he will produce the impression that it is the original questioner who has been wrong all along.

We have already determined that when Aristotle discusses the Achilles in *Physics* Z.9, he does so in conjunction with a diagram that looks like this:

A Γ Δ E B

Hence Aristotle is also thinking about the Achilles counterattack in conjunction with this diagram, even though he knows perfectly well that the *logos* involved no such diagram.

As Aristotle envisions it, the counterattack begins right when the Achilles seems to have run its course, with the answerer defeated. But then the answerer becomes questioner, proceeding, in Aristotle's scenario, roughly as follows:

> If the tortoise gets caught by Achilles, will the tortoise traverse a magnitude while running? Yes.
> Then let that be the line upon which are A and B. So if the tortoise gets caught when he traverses the line upon which are A and B, then will Achilles traverse the same line in the same time? Yes.
> And you say that Achilles must reach the Γ before he reaches the end? Yes.
> And you say that once he reaches the Γ, he must reach the Δ before he reaches the end? Yes.
> And you say that before he reaches the Δ, he must reach the E before he reaches the end? Yes.
> And so you say he must go through things unlimited if he is to reach the end? Yes.
> And so you say that if he cannot go through things unlimited, then he cannot reach the end? Yes.
> Well, I grant that it is not possible to go through things unlimited with respect to

> the ends, but it is possible to go through things unlimited with respect to division. And hence you are in error: Achilles will catch the tortoise.

In seeking to understand Aristotle's strategy, perhaps the central oddity lies in the fact that for Aristotle and his audience, the crucial diagram is present throughout the discussion. Moreover, they all understand the diagram and its elements before Aristotle ever suggests how it can be used dialectically. By contrast, the strategy of the counterattack itself clearly involves the original answerer in the introduction and progressive explication of the diagram letters. In fact, when Aristotle, with the diagram fully present, envisions that the original answerer progressively presents the diagram, Aristotle is fully in accord with normal diagrammatic practice. We have already seen that diagram arguments invariably begin with the diagram fully present, but that the arguer proceeds *as if* he is repeatedly *doing* things to it. So we should not be surprised that Aristotle envisions a dialectician doing things to a diagram that, for Aristotle, is already present.

I do not know whether anyone ever actually tried to employ the dialectical counterattack that I am attributing to Aristotle. But this does not matter, as our goal is to understand *Aristotle's* own analysis. And I think that we now have a plausible reconstruction of the *sort* of strategy that Aristotle has in mind, even if we should not place too much credence in my rendering of the individual questions.[270]

Ultimately, my reconstruction of the counterattack is grounded in two key pieces of evidence. First, we know that Aristotle envisions the original questioner going on the offensive and questioning the answerer, forcing him to grant that the tortoise traverses a diagram line, the same diagram line that Achilles traverses. And indeed Aristotle

270 By contrast, my rendering of the actual *logoi* are aiming for strict historical accuracy.

presents this very concession as the key to understanding why the tortoise does get caught. (239b28-29) Second, we know that Aristotle sees the disambiguation of the word ἄπειρα as the key to the resolution of the Achilles, just as it is the key to the resolution of the Racetrack. My reconstruction of Aristotle's counterattack is simply an attempt to bridge the gap between these two pieces of evidence, in a manner that Aristotle would find dialectically plausible.

We have now examined both Aristotle's Physics Z.2 analysis of the Racetrack and his Physics Z.9 analysis of the Achilles. Both are couched in the language of dialectic, and directed at dialectical logoi. But each functions by bringing diagrams into the dialectical setting. The Racetrack analysis employs a diagram to fundamentally reinterpret the nature of the ἄπειρα (things unlimited). And the Achilles analysis reinterprets the nature of the entire Achilles logos, reinterpreting it as a version of the reinterpreted diagrammatic Racetrack logos. The diagrammatic interpretation of the Racetrack is thus the foundation of both analyses. But we will find that, in Physics θ.8, Aristotle himself ultimately critiques his own diagrammatic interpretation. So we turn there now.

Chapter 9
Potential Points

Reexamining the Racetrack

In *Physics* θ.8, Aristotle returns to the Racetrack and offers a new resolution. The new resolution is well-known to scholars, but its precise nature and relation to the earlier resolution has long proved puzzling: why, indeed, does Aristotle offer a new resolution? What, if anything, does the new resolution have to do with the original?

Ross offers an answer that is initially plausible, and has proved popular. He proposes that Aristotle's first answer was an *ad hominem* response to Zeno, who had supposedly recognized the infinite divisibility of magnitude, but not of time.[271] But now in *Physics* θ.8, Aristotle seeks to address the "real underlying difficulty"[272], and proposes a resolution that relies on the distinction between points existing in actuality and points existing in potentiality. But Ross' proposal suffers from two problems. First, we have seen that Zeno's *logos* in no way relied on magnitude to the detriment of time. Second, Aristotle's initial resolution is in no way an *obvious* response to any error of Zeno or anyone, but instead involves a complex reinterpretation of the argument via diagrams. This makes it all the more surprising that Aristotle should seem to abandon his approach in favor of one seemingly disparate.

In truth, I contend that Aristotle does no such thing. His new resolution, like the original, relies on diagrams. But now Aristotle uses the diagrams in a self-referential way that reveals the *danger* of relying on diagrams uncritically. And his critique of the

271 Cf. Ross, pp. 73-74.
272 *Ibid*., pg. 74.

uncritical use of diagrams has an easy analogue in a critique of the uncritical use of the imagination, the very issue that seemed to lie at the core of the original Racetrack. Thus we will find that Aristotle reflecting on his own initial resolution in a way that leads him into a deeper reflection on the original Racetrack itself.

The remainder of this chapter will have four sections. In the fourth and final section, we will see how Aristotle's new resolution has precisely the same form as the original: it is a disambiguation of the word ἄπειρα (things unlimited), only this time the disambiguated terms become *things unlimited existing in actuality* and *things unlimited existing in potentiality*. It might seem possible to understand the disambiguation in isolation, without reference to the rest of this chapter. But then it would be a mystery how the new resolution relates to the old (other than being a variant of it).

To fit the new resolution into the broader context of Aristotle's engagement with the Racetrack, we will need to progress through three main ideas. The first idea has two parts: I say that Aristotle's new resolution is derived from his reflection on the counting *logos*, but a version of the counting *logos* in which time replaces magnitude. Second, as we saw in Chapter 7, but did not fully explore, Aristotle's analysis of this new variant is based on a separate diagram argument that occurs in *Physics* θ.8 prior to the Zeno discussion. The original diagram argument involves two parallel motions, and Aristotle adapts it so that the parallel motions will be the counter's “traversal” of a time and his counting of temporal halfway-points. Third, we will see that Aristotle relies on the idea that a point (halfway or otherwise) is actual only if something stops at it. I contend that Aristotle's discussion of this issue should be understood as a critique of the uncritical use of diagrams. In particular, Aristotle is reflecting on the differences between the way in which a point is individuated in a diagram and the way in which it is individuated in reality. Once we have considered each of these ideas in turn, we will consider how they lead Aristotle to his new disambiguation.

Counting Halfway-points of Time

In a passage we have now encountered several times, Aristotle recounts the two dialectical versions of the Racetrack. He then immediately explains that he will need to reexamine the Racetrack resolution. Beginning with the recollection of the *logoi*, Aristotle writes:

> τὸν αὐτὸν δὲ τρόπον ἀπαντητέον καὶ πρὸς τοὺς ἐρωτῶντας τὸν Ζήνωνος λόγον, εἰ ἀεὶ τὸ ἥμισυ διιέναι δεῖ, ταῦτα δ'ἄπειρα, τὰ δ' ἄπειρα ἀδύνατον διεξελθεῖν, ἢ ὡς τὸν αὐτὸν τοῦτον λόγον τινὲς ἄλλως ἐρωτῶσιν, ἀξιοῦντες ἅμα τῷ κινεῖσθαι τὴν ἡμίσειαν πρότερον ἀριθμεῖν καθ' ἕκαστον γιγνόμενον τὸ ἥμισυ, ὥστε δειλθόντος τὴν ὅλην ἄπειρον συμβαίνει ἠριθμηκέναι ἀριθμόν· τοῦτο δ' ὁμολογουμένως ἐστὶν ἀδύνατον.
>
> ἐν μὲν οὖν τοῖς πρώτοις λόγοις τοῖς περὶ κινήσεως ἐλύομεν διὰ τοῦ τὸν χρόνον ἄπειρα ἔχειν ἐν αὑτῷ· οὐδὲν γὰρ ἄτοπον εἰ ἐν ἀπείρῳ χρόνῳ ἄπείρα διέρχεταί τις· ὁμοίως δὲ τὸ ἄπειρον ἔν τε τῷ μήκει ὑπάρχει καὶ ἐν τῷ χρόνῳ. ἀλλ' αὕτη ἡ λύσις πρὸς μὲν τὸν ἐρωτῶντα ἱκανῶς ἔχει (ἠρωτᾶτο γὰρ εἰ ἐν πεπερασμένῳ ἄπειρα ἐνδέχεται διεξελθεῖν ἢ ἀριθμῆσαι), πρὸς δὲ τὸ πρᾶγμα καὶ τὴν ἀλήθειαν οὐχ ἱκανῶς· ἂν γάρ τις ἀφέμενος τοῦ μήκους καὶ τοῦ ἐρωτᾶν εἰ ἐν πεπερασμένῳ χρόνῳ ἐνδέχεται ἄπειρα διεξελθεῖν, πυνθάνηται ἐπ' αὐτοῦ τοῦ χρόνου ταῦτα (ἔχει γὰρ ὁ χρόνος ἀπείρους διαιρέσεις), οὐκέτι ἱκανὴ ἔσται αὕτη ἡ λύσις, ἀλλὰ τὸ ἀληθὲς λεκτέον, ὅπερ εἴπομεν ἐν τοῖς ἄρτι λόγοις.
>
> And in the same way one must reply to those [dialecticians] asking the *logos* of Zeno, [who ask] whether it is always necessary to pass through the half, but these [halves] are unlimited, and it is impossible to pass through things unlimited, or as some others differently ask this same *logos*, requiring that together with moving the half-[motion], there is earlier counting individually the half-[way-point] coming to be, so that with the whole [motion] gone through, the result is having counted unlimitedly. But this is admittedly impossible.
>
> So then, in the earlier discussions about motion, we resolved [this *logos*] on account of the time having, in itself, things unlimited. For there is nothing absurd if in an unlimited time someone goes through things unlimited. And in the same way does the unlimited belong in the magnitude and in the time. But on the one

> hand this same *lusis* measures up adequately against the one questioning (for he asked whether, in something limited, it is possible to go through or count things unlimited), yet concerning the fact and the truth it does not [measure up] adequately. For if someone, leaving aside the magnitude and the asking whether in a limited time it is possible to go through things unlimited, inquires about these things concerning the time itself (for the time has unlimited divisions), still it is necessary to speak the truth, the very thing we said in the discussions just now. (263a4-23)

Aristotle's opening words, “And in the same way one must reply...”, (263a4) link his new resolution to the prior diagram argument. Before we examine the earlier argument, we need to consider the problem that Aristotle's adaptation of this argument is intended to solve.

After recounting the two versions of the *logos*, Aristotle recalls his earlier critique, "So then ... adequately." (263a11-18) For the most part, this rehearsal conforms precisely to our earlier reconstruction of Aristotle's resolution. But there is one minor textual puzzle. Aristotle says, "for he asked whether, in something limited (ἐν πεπερασμένῳ), it is possible to go through things unlimited." (263a16-17) I have rendered ἐν πεπερασμένῳ so that the precise nature of the "something limited" is unclear, although it certainly must be either a time or a magnitude. And I think this ambiguity is probably intentional, since Aristotle's goal here is to recall how the *parallel* unlimitedness of a time and a magnitude enables the resolution of the Racetrack as it was asked. And so in recalling that the *logos* concerns a limited time *or* a limited magnitude, Aristotle is *ipso facto* recalling that it concerns *both*. And since, from Aristotle's standpoint, the *logos* does concern both, the ambiguity is quite reasonable.

Most scholars presume that the "something limited" is actually a limited time, and they could well be correct.[273] This might seem to pose a problem for our earlier

273 For instance, see Ross, pg. 448.

conclusion that it is Aristotle himself who introduces the limited time to the *logos*. But recall the unlimited-number-question of the counting *logos*:

> So when the runner has gone through the whole *stadion*, has he counted an unlimited number? Yes.

As we saw in Chapter 7, it is quite plausible that Aristotle should regard this question as being *about* a limited time, even though no such thing is mentioned. And the addition of the new terminology can be explained by Aristotle's need to link the dialectical *logos* with his earlier resolution. So although Aristotle might well construe the "something limited" specifically as a limited time, this gives us no reason to revise any of our earlier conclusions.[274]

Having recounted his earlier analysis, Aristotle next raises a problem for it. I think we can understand this problem if we focus on two hints in the text. First, Aristotle suggests that the new problem somehow results from swapping the magnitude of the original problem for a time. Second, the new problem somehow derives from the counting *logos*. We will consider these issues in turn.

That Aristotle somehow replaces magnitude with time is fairly evident from the text. Aristotle tells us that the new problem results "if someone, leaving aside the magnitude ... inquires about these things concerning the time itself." (263a18-20) The question is how exactly we should understand this switch.

As for the counting *logos*, there are two key signs that it is central to the new problem. First is the very fact that the counting *logos* is recounted at all. Up to this point, Aristotle has not seen fit to recount any version of the Racetrack, treating it as well-

274 It is also possible that Aristotle omits an overt reference to the time as a concession to the indirect nature of his attribution to the *logos*.

known. Now he recounts both versions, and the likely point of this is to highlight the distinctive nature of the counting version. Second, when he recalls the impossibility-question in explaining the new problem, he makes a point of incorporating the counting *logos*: "for he asked whether, in something limited, it is possible to go through or count things unlimited." (263a16-17)

We saw in Chapter 8 that Aristotle by no means ignored the counting *logos* in *Physics* Z.2. But there he significantly reinterpreted it. Recall how he rendered the putatively ambiguous thesis:

> It is not possible to go through things unlimited or to individually touch things unlimited in a limited time.

Here, Aristotle's "individually touching" has been derived from the counting of the dialectical counting *logos*. But Aristotle has transformed the mental activity of counting into the physical activity of touching, focusing his attention simply on the discrete serial nature of both counting and touching.

By contrast, in *Physics* θ.8, I suggest that Aristotle returns the focus to counting as a mental activity. It is precisely the mental *experience* of counting that poses a problem.

To see why, consider the crucial impossibility-question. Following *Physics* 263a16-17, but ignoring the "in something limited", we might render the question as:

> Is it possible to go through or count things unlimited (ἄπειρα)?

A dialectical questioner needs the answer *no*. In *Physics* Z.2, Aristotle showed how the answer could be *yes* if the "things unlimited" constitute a magnitude nonetheless limited in extent. But what if the "things unlimited" are the divisions of a time that is *ex hypothesi* limited in extent? Can they be "gone through" by way of counting? This seems impossible, as there is now way for the counter to experience this counting.[275]

Aristotle's earlier solution, which hinged on the existence of a limited time, separate and parallel to the magnitude, will no longer work, since in this case the time itself seems unlimited in extent. Aristotle does seem right in finding this a problem.

Although Aristotle has found a problem with his initial resolution, this doesn't actually undermine the significance of the earlier analysis. Indeed, Aristotle discovers the new problem by way of the original solution. Recall that the original *logos* deals only with a *stadion*; it is Aristotle himself who introduces the parallel magnitude and time. Only once these abstractions have been introduced can one be swapped for the other, creating a new puzzle. It would thus be quite a mistake to regard Aristotle's initial solution as a curiosity merely adequate *ad hominem*. Indeed, without it, there would be no new solution precisely because there would be no new problem.

Notwithstanding the way in which Aristotle actually develops the new problem, by reflection on the counting *logos* and his original diagram analysis, there is a sense in which the "new" problem is actually a return to the problem of the original *logos*. We saw that, in the original *logos*, the answerer really could not get through the imaginary halfway-points in his own mind. In the original *logos*, this was not the overt focus of the *logos*, but merely the actual source of the difficulty. But now, in considering the problem of counting temporal divisions, Aristotle is overtly considering the problem of getting through an unlimited collection in the mind. And this time his analysis will hinge

275 Here I differ from Graham (pg. 143), who suggests that Aristotle's problem with his original resolution is that "it does not reveal the nature of time." He adds "Zeno's paradox depends on a correspondence between time and space continua. But we can raise the question whether time (or space) by itself is continuous." I think this misses much of the significance of Aristotle's new resolution. The problem is that Graham neglects Aristotle's focus on the counting *logos*. If we consider both the focus on the counting *logos* and the focus on time, we see that Aristotle is engaged in a more subtle inquiry concerning our *thinking* about time.

precisely on the *imaginary* nature of the mental “things unlimited.”

Two Misaligned Motions

Now that we grasp the new problem in *Physics* Θ.8, we must try to grasp Aristotle's solution. He gives us the key to the solution right as he begins his rehearsal of the two versions of the *logos*, writing: “And in the same way, one must reply... .” (263a4) We saw in Chapter 7 that Aristotle here refers to an immediately prior resolution of a diagram puzzle, a puzzle involving two parallel motions. The key to understanding Aristotle's new resolution lies in seeing how this diagram argument can be construed as involving the parallel “motions” of counting and running.[276]

Presenting the diagram puzzle and its resolution, Aristotle writes:

> διὸ καὶ πρὸς τὴν ἀπορίαν τοῦτο λεκτέον· ἔχει γὰρ ἀπορίαν τήνδε. εἰ γὰρ εἴη ἡ τὸ Ε τῇ Ζ ἴση καὶ τὸ Α φέροιτο συνεχῶς ἀπὸ τοῦ ἄκρου πρὸς τὸ Γ, ἅμα δ’ εἴη τὸ Α ἐπὶ τῷ Β σημείῳ, καὶ τὸ Δ φέροιτο ἀπὸ τῆς Ζ ἄκρας πρὸς τὸ Η ὁμαλῶς καὶ τῷ αὐτῷ τάχει τῷ Α, τὸ Δ ἔμπροσθεν ἥξει ἐπὶ τὸ Η ἢ τὸ Α ἐπὶ τὸ Γ· τὸ γὰρ πρότερον ὁρμῆσαν καὶ ἀπελθὸν πρότερον ἐλθεῖν ἀνάγκη. οὐ γὰρ ἅμα γέγονε τὸ Α ἐπὶ τῷ Β καὶ ἀπογέγονεν ἀπ΄ αὐτοῦ, διὸ ὑστερίζει. εἰ γὰρ ἅμα, οὐχ ὑστεριεῖ, ἀλλ’ ἀνάγκη ἔσται ἵστασθαι. οὐκ ἄρα θετέον, ὅτε τὸ Α ἐγένετο κατὰ τὸ Β, τὸ Δ ἅμα κινεῖσθαι ἀπὸ τοῦ Ζ ἄκρου (εἰ γὰρ ἔσται γεγονὸς τὸ Α ἐπὶ τοῦ Β, ἔσται καὶ τὸ ἀπογενέσθαι, καὶ οὐχ ἅμα), ἀλλ’ ἦν ἐν τομῇ χρόνου καὶ οὐχ ἐν χρόνῳ.
>
> And on account of this, we must say the following about the puzzle. For the puzzle goes like this: For if the [line upon which is] the E is equal to the [line upon which is the] Z, and the A is moved continuously from the end [where the E is] to the Γ, and at the same time as the A is at the point B, also the Δ is being moved from the Z end [of the line][277] to the H, uniformly and with the same speed as the A, the Δ will be at the H earlier than the A is at the Γ. For the first setting

---

276 Simplicius construes “in the same way” merely as a reference to the proposition: “everything that is continuous contains in itself potentiality, not in actuality, the things of which it can be cut – the line <contains> points and the time interval <contains> instants.” (1240,3-4, trans. McKirahan) He seems to derive this claim from *Physics* 262a21-25. This idea does indeed play a role in Aristotle's new resolution, but it is much more natural to read “in the same way” as referring to Aristotle's use of the diagram argument to resolve a puzzle (ἀπορία) (262b8-22) The resolution of the prior puzzle is then analogous to the resolution of the Zenonian puzzle.

277 The adjectival phrase τῆς Ζ ἄκρας is feminine, which means that Aristotle sees it as modifying a feminine noun. The obvious choice is γραμμή (line). So Aristotle is really saying something like: “from the headmost line, where the Z is”.

> out and leaving is necessarily first to arrive. For not at the same time has the A come to be at the B and come to be away from it, on account of which it [i.e. the coming to be away from B] comes later. For if [the coming to be at B and the coming to be away from it happen] at the same time, it [i.e. the coming to be away from B] is not later, but [if it does come later[278]] there is a need to stop [in between]. Therefore one must not suppose that when the A was coming to be alongside the B, at the same time the Δ was being moved from the Z end (for if there will happen the A having come to be at the B, there will also happen [the A] coming to be away from [the B], and not at the same time), but it [i.e. the A coming to be alongside the B] was in a cut of time and not in a time. [262b8-21]

Notice that now once again, Aristotle opens his discussion by referring back to a prior passage: "And on account of this..." (262b8) We will later examine this backwards reference, but for now we will simply examine how the diagram puzzle relates to the counting *logos.*

The diagram puzzle requires a diagram that resembles the following:

E A B Γ

Z Δ H

As is often the case, Aristotle here uses letters both to mark spots on the diagram and to represent objects moving along the diagram. In this case, A and Δ are moving objects.

Aristotle begins by setting up the basic puzzle, "For if ... at the Γ." (262a10-14) He supposes that the lines are of the same length, and that the A and Δ move at the same speed from the E and the Z , respectively, to the Γ and the H. Although he does not state it directly, he clearly supposes that the motions start at the same time. He also supposes, crucially, that "at the same time" (263b11) as the A is at the B, the Δ *is* moving from the Z to the H. And somehow this scenario is supposed to entail that the Δ reaches the H *before* the A reaches the Γ. And this is puzzling because the motions started at the same

278 Here I follow Ross (pg. 713).

time and proceeded with the same speed.

Aristotle next proceeds to explain why this paradoxical conclusion comes about, "For the first ... [in between]." (262b14-17) Aristotle thinks that while the Α and the Δ started the full motion at the same time, the Δ passed the point opposite the Β *before* the Α left the Β. And if this is true, then the Α *does* reach the end first. So everything hinges on why Aristotle thinks the Α passes the Β later. And the explanation for this is that Aristotle thinks the Α does not arrive at the Β and depart from the Β at the same time. And hence he thinks that the Α must rest at the Β in between the arrival and the departure. But this means that the Α will be resting at the Β while the Δ is in the process of moving. And this explains why the Α leaves the Β after the Δ has already gone beyond the Β.

The supposition that the Α needs to rest at the Β is liable to seem thoroughly misguided to modern thinkers, who routinely suppose that a moving object can pass through a point instantaneously. We will soon examine Aristotle's view in more detail, and will find that it is more subtle than is commonly supposed. For now, however, it is clear that if Aristotle's supposition is correct, then his reasoning is sound, and the Δ will indeed arrive at the Η before the Α arrives at the Γ, despite the fact that they started together.

Aristotle's final task is to dissolve the paradox: "Therefore one ... in a time." (262b17-21) His solution is to simply deny that the Α was ever *at* the Β when the Δ was moving. If the Α was not at the Β, then it did not arrive at the Β and leave the Β, and so there was no need for it to rest there. Aristotle's resolution depends on an ontological distinction between two ways in which something might be somewhere. We will soon consider this issue in more detail. But for now we can grant that if his distinction makes sense, that is to say, if there is some way for the moving A to be alongside the B without arriving there and leaving there at separate times, then Aristotle will have resolved his puzzle.

Our immediate concern is the manner in which this puzzle relates to the counting *logos*. Just as the diagram involves two parallel motions, so does Aristotle construe the counting *logos* as involving two parallel motions, the physical motion of running and the mental motion of counting. These two motions are supposed to start and end at the same time. The second diagram line represents the path of the physical motion, and on *this* line there are no intermediate parts marked, just as, in reality, there are typically no intermediate parts marked along a span to be traversed.[279] The first diagram line represents the path of the mental motion. Here the mind moves along a line and counts points. The line in question, that is, the line *represented* by *Aristotle's* diagram line, is thus not a physical line, but a mental image. The mind of the counter does not literally move along the line that the counter runs, but along an image thereof.

The B on the first diagram line marks one of the halfway-points that the counter needs to count. Naturally, his mind will need to be *at* this imaginary intermediate point at the same time as his body is in the process of running the span of which the point is a division. So we can now easily see how Aristotle finds the diagram puzzle relevant to the counting *logos*.

Up to this point, we have focused on the counting logos as it is normally understood, with the counter counting points of the magnitude being run. But we have seen that Aristotle envisions a counting *logos* in which the counter counts temporal

279 We may here ignore the halfway-point that typically is marked in a *stadion*. Aristotle is here thinking more abstractly.

points. We will later consider how this revision fits with the diagram. But first we need to consider an issue we have so far deferred, the question of why Aristotle thinks that being at an intermediate point requires stopping.

Stopping in the Middle

Aristotle's discussion of stopping in the middle, and indeed the entire Zeno interlude, comes in the middle of a broader discussion about the nature of perpetual continuous motion. Aristotle wishes to establish that perpetual motion in a circle may be continuous (261b27-28), whereas perpetual motion along a line, that is, a limited line (the only kind that actually exists, in Aristotle's view) will necessarily be discontinuous, since, when an object reaches the end, it must stop before reversing direction and continuing the motion. (261b12-15)

Considering a moving object that reaches the end of a path and reverses direction, Aristotle writes:

> ὅτι δ΄ ἀνάγκη ἵστασθαι, ἡ πίστις οὐ μόνον ἐπὶ τῆς αἰσθήσεως ἀλλὰ καὶ ἐπὶ τοῦ λόγου. ἀρχὴ δὲ ἥδε. τριῶν γὰρ ὄντων, ἀρχῆς μέσου τελευτῆς, τὸ μέσον πρὸς ἑκάτερον ἄμφω ἐστίν, καὶ τῷ μὲν ἀριθμῷ ἕν, τῷ λόγῳ δὲ δύο . ἔτι δὲ ἄλλο ἐστὶν τὸ δυνάμει καὶ τὸ ἐνεργείᾳ, ὥστε τῆς εὐθείας τῶν ἐντὸς τῶν ἄκρων ὁτιοῦν σημεῖον δυνάμει μέν ἐστι μέσον, ἐνερείᾳ δ΄ οὐκ ἔστιν, ἐὰν μὴ διέλῃ ταύτῃ καὶ ἐπιστὰν πάλιν ἄρξηται κινεῖσθαι· οὕτω δὲ τὸ μέσον ἀρχὴ γίγνεται καὶ τελευτή, ἀρχὴ μὲν τῆς ὕστερον, τελευτὴ δὲ τῆς πρώτης (λέγω δ΄ οἷον ἐὰν φερόμενον τὸ Α στῇ ἐπὶ τοῦ Β καὶ πάλιν φέρηται ἐπὶ τὸ Γ). ὅταν δὲ συνεχῶς φέρηται, οὔτε γεγονέναι οὔτε ἀπογεγονέναι οἷον τε τὸ Α κατὰ τὸ Β σημεῖον, ἀλλὰ μόνον εἶναι ἐν τῷ νῦν, ἐν χρόνῳ δ΄ οὐδενὶ πλὴν οὗ τὸ νῦν ἐστιν διαὶρεσις, ἐν τῷ ὅλῳ (εἰ δὲ γεγονέναι τις θήσει καὶ ἀπογεγονέναι, ἀεὶ στήσεται τὸ Α φερόμενον· ἀδύνατον γὰρ τὸ Α ἅμα γεγονέναι τε ἐπὶ τοῦ Β καὶ ἀπογεγονέναι. ἐν ἄλλῳ ἄρα καὶ ἄλλῳ σημείῳ χρόνου. χρόνος ἄρα ἔσται ὁ ἐν μέσῳ. ὥστε ἠρεμήσει τὸ Α ἐπὶ τοῦ Β. ὁμοίως δὲ καὶ ἐπὶ τῶν ἄλλων σημείων· ὁ γὰρ αὐτὸς λόγος ἐπὶ πάντων. ὅταν δὴ χρήσηται τὸ φερόμενον Α τῷ Β μέσῳ καὶ τελευτῇ καὶ ἀρχῇ, ἀνάγκη στῆναι διὰ τὸ δύο ποιεῖν, ὥσπερ ἂν εἰ καὶ νοήσειεν.) ἀλλ᾽ ἀπὸ μὲν τοῦ Α σημείου ἀπογέγονε τῆς ἀρχῆς, ἐπὶ δὲ τοῦ Γ γέγονεν, ὅταν τελευτήσῃ καὶ στῇ.

> The belief that it is must stop [depends] not only upon perception, but also upon reason. And the source is this: For given that there are three things, beginning,

> middle and end, the middle is both with respect to each, and is one in number, but two in relation. And moreover the [middle] in potentiality is different from the [middle] in actuality, so that any point whatsoever of the line within the extremities is a middle in potentiality, but not in actuality, unless something divides [the line] in this way and stopped [there], again begins to move. And thus the middle becomes a beginning and an end, a beginning of the upcoming line, and an end of the prior line (I mean, for instance, if the moving A stops at the B and again moves to the Γ.) But whenever something moves continuously, it neither came to be at nor has come to be away from [the middle point], as does the A alongside the B point, but merely *is* [there] in the now, and not in any time except that of which the now is a division, [that is] within the whole [time of the motion]. (And if someone will claim it has come to be at [the middle point] and has come to be away from the [middle point], the moving A will always rest. For it is impossible for the A, at the same time, to have come to be at the B and to have come to be away from it. Therefore there will be a time in the middle. So that the A will rest at the B. And likewise at the other points, for the same reasoning applies to all. In fact, whenever the moving A must use the middle B as both an end and a beginning, it is necessary to stop in order to make [the one middle] two, just as if one also does so in thinking.) But [in fact] it has come to be away from the A point in the beginning, and has come to be at the Γ, whenever it comes to an end and stops. [262a17-b8]

This passage obviously requires a diagram of this sort:

A B Γ

The passage as a whole might initially seem somewhat confusing, since while Aristotle's overall goal is to establish that a moving object must stop and reverse direction upon reaching the *end* of a line, the actual argument concerns a midpoint *within* a line. Aristotle does not return to the case of an object that reaches the end of a line until 262b21-263a3, the passage between the exposition of the diagram puzzle and the invocation of the Racetrack, a passage we may here ignore, as it does not figure in the analysis of the *logos*.

To modern scholars, the idea that a moving thing needs to stop in order to be at a point is liable to seem quite mistaken. But I think that we can go a long way towards making sense of Aristotle's text if we see it as involving a reflection on the relation between fixed diagrams and actual moving objects.

The key to this analysis, and, I think, to the whole passage, is the line: "And moreover ... begin to move." (262a21-25) I suggest that Aristotle sees letters on a diagram line, that is to say, points on a diagram line, as representing, in their own right, only points that exist in potentiality. By contrast, he sees points that exist in actuality as existing *only* because they are or have been the limits of genuine motions. Each of these facts has an independent plausibility.

Recall first that diagrams are typically presented to an audience *in toto*, before they are invested with any meaning. Then letters will typically be invested with meaning via a supposed action, either the drawing of some line that begins or ends at the letter, or the division of a line at the letter. This is the near universal case in mathematical reasoning, although Aristotle himself often uses letters in other ways, such as represent, and to “be”, moving objects. There are certainly cases in the *Physics* where Aristotle uses letters as point markers without explicitly identifying any action that yields the given point. But this is perhaps to be expected, given the difference between pure mathematics and Aristotle's applications of diagrams to physics. In mathematics, diagrams themselves are regarded as instances of the very objects of study. The actions that invest letters with meaning are supposedly performed *by us* on these objects. By contrast, in the *Physics*, Aristotle is first and foremost studying motions themselves (and certainly not *our* motions *per se*). And so he often has need to refer, via diagrams, to the putative times and magnitudes of these motions in a way independent of his reference to the motions themselves. And this is true even if, in an ontological sense, the existence of the time and magnitude is supposed to be dependent on the existence of the motion. I see Aristotle's current argument as an attempt to step back from his profligate use of diagrams and ground them in reality.

As for reality itself, Aristotle thinks that it consists of matter and form, in such a way that the existence of anything *else*, that exists in any way, depends for its existence on enformed matter. Points will thus be attributes of enformed matter. Indeed they must be the limits of other things on whose reality they depend. They might be the limits of motions, that is, of *natural* motions, the motions via which all other motions must be understood. (And since motions are themselves attributes of bodies, the limits of a body's motion will ontologically depend on the body.) It might also seem that points, like lines and surfaces, can depend on bodies by being *their* limits, regardless of whether the bodies are moving or not. But for Aristotle, all terrestrial bodies are created bodies.[280] Hence the limits of these bodies will be brought into being by the motions of expansion or division. It follows that even if a point now exists as a limit of a body at rest (or more precisely, as a limit of a linear boundary of a planar boundary thereof), the limits of this body will have been brought into being via natural motions. So for Aristotle, it follows that *all* limits, points included, have been brought into being as the limits of natural motions, even if these motions are no longer occurring.

We can now easily see why diagram letters would initially seem to pose a problem for Aristotle, but also how he can offer a solution. As depicted on a diagram line, letters seem to mark points that are really there. (Indeed the letters themselves *are* really there.) And so the points seem to have an existence independent of whatever motions an arguer proceeds to pretend will begin or end at these points. But this seems to conflict with

280 I will leave aside the issue of heavenly bodies.

Aristotle's basic view of reality, wherein points must be brought into existence by serving as the limits of motions. Aristotle's solution, as I see it, is simply to remind us that diagrams, in their representative capacity, must always be understood via the motions that invest the specific letters with meaning.

The disparity between diagrams and reality is particularly salient when we think about our manner of *speaking*. We have seen that typical Greek diagram practice involves speaking as if the diagram entities literally are the things they are supposed to represent. In pure geometry this may pose no inherent problem, since the diagrams do closely resemble what they are supposed to be: a diagrammatic "circle" may not be perfectly circular, but it will be fairly close. But when Aristotle uses diagrams as instances of things significantly different, such as times, motions, or the paths of motions *other* than the motion of drawing the diagram itself, then problems arise. In these cases, since the diagram is fully present, we can easily *refer* to any point marked by a letter, e.g. "the point upon which is the Δ." But this point may have no identity at all in the situation that Aristotle is using the diagram to represent.

It would be easy for modern readers to greatly misunderstand the role of points in Aristotle's diagrams. Today, under the influence of modern mathematics, scholars are liable to see Aristotle's diagrams as *pictures* that depict an underlying mathematical reality. So consider the diagram:

A B Γ

In the modern view, the point at the B will have a mathematical identity quite independent of the picture. This is because mathematical lines are understood to *be* sets of points, sets of points on which various relations are defined.[281] So to refer to the point at the B is to refer to one of these mathematical points, which is presumed to have an existence independent of the picture itself. Indeed this mathematical point is distinct both from any element of the diagram *and* from any point in physical reality, or indeed any aspect of physical reality. In fact, philosophers of widely different persuasions concerning physical reality will use one and the same mathematical understanding of a line composed of points.

If we erroneously suppose that letters on Aristotle's diagrams mark mathematical points, then it will easily seem that Aristotle is saying that *some* mathematical points exist in actuality, whereas some exist only in potentiality. And this does indeed seem somewhat absurd, inasmuch as the entire mathematical structure of the line would seem to be needed to make each of the points, "actual" and "potential", the very point that it is. But since there *are* no "mathematical" points for Aristotle, he does not face this problem. For him, the only points are the *visible* points on the diagram, and the actual points that are individuated by being the limits of a motion. To speak of potentiality existing points is merely to speak of potential motions.

The difference between Aristotle's standpoint and the modern standpoint is easiest to understand if we recall that the points on Aristotle's diagrams do not even have a *location* apart from their diagrammatic location. It is only by invoking motion that Aristotle is able to assign a location that the points represent. By contrast, modern diagrams will typically be construed as representing an underlying spatial structure that is independent of the motions that may or may not occur therein.

281 At least this might be regarded as the "standard" approach. Typically, geometry is supposed to have been "reduced" to non-geometrical mathematics.

Now recall Aristotle's words: "And moreover...begins to move." (262a21-25) I suggest that Aristotle is here simply recalling the conditions by which we are able to make sense of diagrams. To be sure, there is no overt sign that his comments are intended to apply *only* to actual diagrams. But his words make sense if we see them as applying to any *depiction* of reality, be it via visible diagrams or via imagination. So Aristotle's attention is focused on a line, visible or imaginary, that is regarded as being something it literally is not, namely the path of a motion (of course, a motion other than that which actually constructed the line). And he considers some point marked on the line. If points have actuality only by being the endpoints of motion, then to imagine or visualize a point is to imagine that a motion *can* begin or end there. But of course this does not mean that any motion actually *does*.

I think that we thus have a simple and fairly plausible interpretation of Aristotle's remarks. But the modern reader is liable to be confused by Aristotle's emphasis on *stopping*. It might seem that we can grant Aristotle everything I have said, and yet maintain that stopping is still not needed to actualize a point. For it might seem that an object can move to a point, concluding one motion, and then immediately begin moving elsewhere. And indeed this would seem to lead to the downfall of Aristotle's account, for it seems that any continuous motion can be arbitrarily construed as a composition of sub-motions in infinitely many ways, thus actualizing "every" point in the path of the motion.

The flaw in this reasoning is that it neglects Aristotle's distinction between natural and artificial motion. A natural motion, of course, is one in which the mover moves of its own accord, whereas in an artificial motion, the moving thing is moved by something else. All artificial motions must be explained in terms of natural motions, and natural motion is the very topic of Aristotle's *Physics*. Indeed the word φύσις (*phusis*) is typically translated *nature*. What this means is that when Aristotle requires that stopping is necessary to actualize a point, he means that a *natural* motion must stop at a point to actualize it. Now this might seem like a wholly arbitrary distinction that would leave many obviously "real" limit points "unactualized". But Aristotle's view does have some plausibility if we keep in mind that ultimately there *are* no artificial motions. Motions are only artificial with respect to a certain moving object. If we understand their relation to their natural mover, then we can see how their limit points are the limits of natural motions. For instance, in Aristotle's view, if I draw a line on a chalkboard, we cannot construe this as due to the nature of the chalk. It is not in the nature of the chalk to move to the start of the line, stop, and then proceed straight to the end of the line, and then stop. So it might seem that the start and the end of the line, while obviously somehow real, remain unactualized by Aristotle's seeming dogma. But of course we must understand the motion of the chalk as derivative of *my* natural motion. I am the one who actualizes the limits of the chalk line.

It follows that it is simply false to say that we can arbitrarily interpret *any* interior point as the end of one motion and the start of another. From a modern, strictly mathematical standpoint, this is possible. But for Aristotle, something is a genuine motion only if it can be understood via a natural motion.

We now, I think, have a plausible grasp of why Aristotle thinks that a motion must stop in order to actualize a point: it is because points only *ever* exist inasmuch as they are the limits of natural motions. It is only uninterpreted diagrams, or more generally, imagery disconnected with that which it is supposed to image, that suggests otherwise.

And this, I suggest, is what Aristotle has in mind in what I construe as the core of the passage given above, the line "And moreover...to move." (262a21-25)

If we take this interpretation as our starting point, we can understand Aristotle's ensuing remarks, which might seem cryptic in their own right. Consider the line: "But whenever...[time of the motion]." (262a28-31) If we read this line in isolation, it might seem that Aristotle is employing some metaphysical legerdemain to countenance some way of being at a point without really actualizing it. Indeed it might seem that Aristotle is granting that a moving thing is at "all" of the interior points of its path, at each of them "in the now". Now something like this *is* true in a modern mathematical framework. But it is not true in Aristotle's framework, since Aristotle by no means quantifies over any and all potential interior points. Instead, he considers a *single* interior point, the point at the B, a point already given on the diagram. If there is such a determinate point at all, then *something* must have made it be, some motion, and indeed some motion other than the motion of the continuously moving object under discussion, which, *ex hypothesi*, moves *past* the given point. So Aristotle is considering the natural question of how a continuous moving thing relates to the already existing points that it passes in the course of its motion.

Aristotle's response makes sense if we see him as discussing *natural* motion. The *external* demarcation of a point along the path of a natural motion does not disrupt the nature of the natural motion itself. It does not force the naturally moving thing to stop there, arriving and then departing. Instead, the mover's presence at the given point is wholly accidental. Ultimately, Aristotle's claim seems plausible if we keep in mind that his very *reference* to a point presupposes its individuation by a motion other than the continuous motion under discussion.

The long parenthetical argument near the end of the quoted passage, "And if...in thinking" (262a31-262b7), should also be understood in terms of Aristotle's distinction between points in diagrams and points in reality. The argument *presupposes*, for the reasons already given, that a motion must stop in order to actualize a point. From this supposition Aristotle concludes that a moving thing does *not* arrive at and leave every potential middle point, since this would involve resting at every potential middle point. The import of this argument, I think, is to highlight the dependence of the notions of arriving and leaving on the more basic idea of motion.

It is easy to be confused by the given argument, since it involves an inference that might seem straightforwardly fallacious if viewed in isolation. Aristotle writes: "For it is impossible for the A, at the same time, to have come to be at the B and to have come to be away from it." (262a32-262b3) If we see this as Aristotle's argument *for* the conclusion that stopping is needed to actualize points, then it does seem fallacious. For the argument would then seem to presuppose that there is some *first* moment of having left a point behind. But this viewpoint seems incompatible with the possibility of continuous motion, and is even rejected by Aristotle himself. (236a7-27) But Aristotle's argument is quite plausible if he *presupposes* that stopping is needed to actualize a point. He will then be drawing the thoroughly valid conclusion that arriving and leaving will require a period of rest in between.

The overall point of the given argument is to show the absurd consequences of the idea that a moving thing arrives at and leaves every potential middle point. *Since* each arrival and departure will require rest in between, a moving thing will rest at *every* potential middle point. But this prevents continuous motion, and perhaps any motion. And so there is no arrival and departure, except at the beginning and end of natural motion. All in all, what this argument shows is that *if* we accept Aristotle's prior argument concerning the actualization of middle points, *then* we must deny that moving things arrive at and depart from just any arbitrary middle points. By contrast, Aristotle is *not* offering a new argument that stopping is needed to actualize middle points.

It follows that we should interpret the final line of the parenthetical passage not as a conclusion, but rather as a reiteration of the point that *explains* the rejection of arrival and departure: "In fact, whenever the moving A must use the middle B as both an end and a beginning, it is necessary to stop in order to make [the one middle] two, just as if one also does so in thinking." (262b5-7) Here Aristotle emphasizes the incompatibility between continuous motion (Aristotle's overall concern) and the possibility of arrival and departure at an interior point. Here the reference to thinking is worth noting, given how Aristotle will proceed to assess the counting Racetrack. Aristotle seems to be saying that a middle point has reality on an imaginary line only if the imaginer pauses in the mental construction of the line, thus individuating a point.[282]

We have now seen why Aristotle thinks a middle point is actualized only when a moving object stops there: it is because points are only *ever* individuated by being the limits of actual motions. And we thus understand the diagram argument that Aristotle has suggested will be the basis for his reconsideration of the Racetrack. What remains is for

282 Aristotle's supposition does seem plausible if we imagine actual moving things. But problems seem to arise if we imagine diagrams themselves, representations of moving things, so that our imagination will be constructing a second-order representation. In this case, it seems we can imagine, in one swoop, a full diagram with several points marked on it. No mental "rest" seems to be required at each point. And of course, once we start imagining diagrams, the difference between imagining motions and imagining representations of motions starts to blur.

us to directly consider this reconsideration itself.

The New Resolution

We now understand the two key elements that will enable our exegesis of Aristotle's new resolution. First, we know that Aristotle focuses on a version of the counting logos in which time replaces magnitude. Second, we know how Aristotle will represent the *standard* counting *logos* via a pair of diagram lines, one for running and one for counting. What remains is to consider how these elements combine to enable Aristotle's resolution.

Consider how Aristotle will use diagrams to represent counting and running:

E B Γ

Z H

These are the very diagrams from the earlier diagram puzzle, but I have removed the letters standing for the movers, since, in this case, one mover will be the runner's mind. The line from the Z to the H will be the runner's physical path. By contrast, the line from the E to the Γ will be the runner's image of that same path. His mind will move along this path, counting interior points like the point at the B.

Now suppose that the two parallel “motions” traverse not a magnitude, but a time, as Aristotle seems to envision. Then the diagram will look *exactly the same*:

E B Γ

Z H

The only difference lies in what the diagram is supposed to represent. The line from the Z to the H will be some determinate *time* that the counter needs to pass through, just as in the original it was a magnitude he needed to traverse. And likewise, the line from the E to the Γ will be this counter's image of that very time, with a middle point marked since he will be counting halfway-points as he progresses. But there is now a twist that as missing in the original scenario. If the top line is the counter's image of the bottom line, and if the counter, *ex hypothesi*, traverses both lines in the same time, then the top line is *also* an image of the very time the counter takes in traversing *that* image. Aristotle's critique will hinge on a breakdown between the requirements of the counting itself and the requirements of the representational function of the image.

Recall that Aristotle aims to address the question:

> Is it possible to go through or count things unlimited?

And recall his response:

> ἐὰν γάρ τις τὴν συνεχῆ διαιρῇ εἰς δύο ἡμίση, οὗτος τῷ ἑνὶ σημείῳ ὡς δυσὶ χρῆται· ποιεῖ γὰρ αὐτὸ ἀρχὴν καὶ τελευτήν. οὕτω δὲ ποιεῖ ὅ τε ἀριθμῶν καὶ ὁ εἰς τὰ ἡμίση διαιρῶν. οὕτω δὲ διαιροῦντος οὐχ ἔσται συνεχὴς οὔθ’ ἡ γραμμὴ οὔθ’ ἡ κίνησις· ἡ γὰρ συνεχὴς κίνησις συνεχοῦς ἐστιν, ἐν δὲ τῷ συνεχεῖ ἔνεστι μὲν ἄπειρα ἡμίση, ἀλλ’ οὐκ ἐντελεχείᾳ ἀλλὰ δυνάμει. ἂν δὲ ποιῇ ἐντελεχείᾳ, οὐ ποιήσει συνεχῆ, ἀλλὰ στήσει, ὅπερ ἐπὶ τοῦ ἀριθμοῦντος τὰ ἡμίσεα φανερόν ἐστιν ὅτι συμβαίνει· τὸ γὰρ ἓν σημεῖον ἀνάγκη αὐτῷ ἀριθμεῖν δύο· τοῦ μὲν γὰρ ἑτέρου τελευτὴ ἡμίσεος τοῦ δ’ ἑτέρου ἀρχὴ ἔσται, ἂν μὴ μίαν ἀριθμῇ τὴν συνεχῆ, ἀλλὰ δύο ἡμισείας. ὥστε λεκτέον πρὸς τὸν ἐρωτῶντα εἰ ἐνδέχεται ἄπειρα διεξελθεῖν ἢ ἐν χρόνῳ ἢ ἐν μήκει, ὅτι ἔστιν ὡς, ἔστιν δ’ ὡς οὔ. ἐντελεχείᾳ μὲν γὰρ ὄντα οὐκ ἐνδέχεται, δυνάμει δὲ ἐνδέχεται· ὁ γὰρ συνεχῶς κινούμενος κατὰ συμβεβηκὸς ἄπειρα διελήλυθεν, ἁπλῶς δ΄ οὔ· συμβέβηκε γὰρ τῇ γραμμῇ ἄπειρα ἡμίσεα εἶναι, ἡ δ’ οὐσία ἐστὶν ἑτέρα καὶ τὸ εἶναι.
>
> For if someone should divide the continuous [line] into two halves, he uses the one point as two. For he makes the same [point] a beginning and an end. And thus do both the one counting and the one dividing into halves. But being so divided neither the line nor the motion will be continuous. For the continuous motion is of something continuous, but in the continuous thing are present unlimited halves, but not in actuality, rather in potentiality. But if he should make [them] in actuality, he will not make a continuous [motion], but will stop, which indeed is something that clearly comes about in the case of the one counting the halves. For it is necessary for him to count the one point as two. For there will be an end of one half, and a beginning of another, if he should count not one continuous [line], but two halves. So that it is necessary to say, against the one

> asking if it is possible to go through things unlimited either in a time or in a magnitude, that in one way it is, but in another it is not. For it is not possible with the [things unlimited] existing in actuality, but it is possible [with the things unlimited existing] in potentiality. For the one moving continuously has incidentally gone through things unlimited, but has not done so in the strictest sense. For it just so happens that the line *is* unlimited halves, but the reality and the essence [of the line] are different. (263a23-b9)

The key to understanding Aristotle's argument lies in recognizing that the counter must not only count temporal halfway-points, he must also individuate those very halfway-points. And he must do this via his act of counting, since, in this purely mental scenario, he is not engaged in any other activity. In effect, Aristotle's discussion concerns *thinking* about interior halfway-points and the conditions under which it is possible to do so. And since the interior halfway-points are *interior* points, interior points of continuous wholes, they do not, in Aristotle's view, actually exist. So Aristotle is examining the ways in which it is possible to think about certain things that do not actually exist.

In his opening lines, "For if ... into halves" (263a23-26), Aristotle begins by focusing his attention on a continuous line. But recall that Aristotle is considering a continuous diagram line that represents a continuous *time* (which would naturally be the time of a continuous motion). To *think* of an interior point of a continuous time or motion as a dividing point, and thus to think of such an interior point at all, requires thinking of this point in two different ways. One must think of it as the end of one time (and motion), and as the beginning of the other. This follows from two facts. First, it is only by being the limit of a motion that points are *ever* individuated. Second, because the initially given time and motion are *ex hypothesi* continuous, there is motion before *and* after the given interior points.

It would be quite easy to become confused by Aristotle's contention that the thinker must think of "one point as two". (263a24) Is it not merely necessary to think of *one* point in two *ways*? In fact this is not sufficient. This is because Aristotle is not considering interior points of magnitudes *qua* magnitudes, in which case the objection seems accurate. Instead, Aristotle is thinking of diagram lines that represent times (and motions). And given the fact that it is *natural* motions that individuate points, the same temporal point *cannot* mark the end of one motion and the start of another, for the same mover. This is because a mover will rest upon completion of its first natural motion. So the apparent single interior point must be *two* interior points.[283]

I construe Aristotle's analogy between counting and dividing into halves as emphasizing the relation between reality and the diagram that represents it. We have already seen that we, Aristotle's audience, may "divide into halves" a visible line representing a time, but that this requires *thinking* of two points, even if that is not how it *looks* on the diagram. The same will be true of a counter counting temporal interior points. For he will need to be *imagining* a temporal line. And thus his relation to an imagined temporal halfway point will be analogous to our relation to a visible "temporal" halfway-point. In each case, Aristotle says, to truly think of the midpoint *as* a *temporal* midpoint *requires* thinking of it as two points. (But Aristotle is not saying the counter *can* do this. Indeed we will see that he cannot.)

Now recall our diagram:

E__________________B____________________________Γ

283 I take the current argument as an additional confirmation that Aristotle really is thinking of the "line" in 263a23-b9 as a time. Of course he has directly informed us in the prior passage that he will be thinking about a time (263a18-23), but since the ensuing analysis sometimes mentions a line (263a27, 263b8), not a time, the discussion can be confusing. Given that Aristotle has informed us that he is relying on diagrams here (263a3), we should not presume that a "line" must be a magnitude, especially given that Aristotle once explicitly refers to a magnitude (263b4) when he does mean this.

Z ____ H

The bottom line is the time that the counter traverses, and the top line is his image of that time. Aristotle's comments have just informed us that if this top line is indeed the counter's image of the time, then he must be *thinking* of the one temporal midpoint as two.

The ensuing comments, "But being so ... in potentiality" (263a26-29) , draw out the consequences for the bottom line, the time itself. The inference concerning the top line starts with the counter's *image* of the time, and draws a consequence for the runner's *thought* concerning that time. But the bottom line represents that very time, and so we can apply the consequences of the prior inference to the line. The line appears, and is, continuous. But it will not be continuous if it really is as the runner *must* think it is, as divided. And hence no motion through this line will be continuous.[284] As for the concluding comment concerning actual and potential halves, I construe this as explanatory, but not as playing a direct role in the argument.

Thus far, Aristotle's argument has proceeded through two steps. He started with the counter's image of a time, and drew a conclusion concerning how the runner must *think* of the time. From this, he drew a consequence concerning how the counter must think of the *motion* through this time. Finally, we will see that Aristotle next applies this consequence concerning the motion to the counter himself.

Aristotle concludes his argument: "But if he should make ... two halves." (263a29-263b3) Here Aristotle admits to the fact that the time and the motion the counter

284 This particular inference works if the given “line”, that is, the given diagram line, represents either a time or a magnitude. This may explain Aristotle's use of the ambiguous “line”. But for the inference to function in the broader resolution of the Racetrack, the diagram line must represent a time.

has been "thinking" about are not some arbitrary time and motion, they are the very time and motion *of* his counting. And so if the motion under consideration involves a stop, then the counter *himself* must stop. The time portion of this argument is simply a reiteration of the initial portion, but now the emphasis is on the fact that the counter is counting the points of his *own* time.

Aristotle's argument has thus started with the supposition that a counter may count an interior point of a time *as* he progresses through that time in a continuous motion. But he has shown that this leads to the consequence that the counter is not engaged in a continuous motion. So the original supposition is an impossibility. The argument is framed in terms of temporal points, since the mismatch between the imagined time and the experience time is what creates a contradiction. But given that temporal limit points always correspond to limit points of motions and magnitudes, it would be easy to construct a derivative argument that shows the impossibility of counting interior points of a magnitude in a continuous motion.

At this point, Aristotle has concluded his main argument. That is, he has concluded the inquiry that specifically concerns the temporalized counting *logos*. What remains is to use his result to obtain a more general critique of the Racetrack. He proceeds to raise the question of whether “it is possible to go through things unlimited either in a time or in a magnitude,” (263b3-4) and then offers his answer. (263b4-9) Aristotle thus sees himself as addressing once again the impossibility-question of the original *logos*:

Is it possible to go through things unlimited?

He supposes that the original *Physics* Z.2 analysis sufficed for the case when the “things unlimited" were "in a magnitude". But now, his investigation of the temporal case enables a resolution that he sees as fully general.

Aristotle's response has the same dialectical form as his original response to the Racetrack: he proceeds to draw a distinction, disambiguating the term ἄπειρα (things unlimited). This term might mean: *things unlimited existing in actuality*. Or it might mean: *things unlimited existing in potentiality*. When we insert these phrases into the thesis proffered by the *logos*, we end up with two disambiguated theses. First: it is possible to go through things unlimited existing in actuality. Second: it is possible to go through things unlimited existing in potentiality. Aristotle denies the first thesis, but affirms the second.

As we might expect, Aristotle concludes by offering an explanation as to why it *is* possible to go through things unlimited existing in potentiality. This is the thesis that he needs to block the *logos*, as it contradicts the thesis that the questioner needs. Aristotle's explanation simply reiterates that a line is not, by its very nature, *composed* of the divisions into which it *can* be divided.

We thus see that despite Aristotle's excursus through the counting *logos* and the diagrams thereof, Aristotle's revised critique of the Racetrack ultimately returns to offer a direct alternative to his original critique, a dialectical disambiguation of the term ἄπειρα (things unlimited).

At this point we have a reasonable grasp of Aristotle's revised critique, and we have now completed an exegesis of the Aristotelian texts concerning the Racetrack. What remains is to assess the overall path that we have traversed.

Chapter 10
Some Conclusions and Ideas

Aristotle's Critique of the Racetrack

It would be natural, at this point, to consider whether Aristotle's final resolution of the Racetrack is actually successful. But it is not a trivial issue to say what this question even means. We might ask whether Aristotle's first or his second critique would actually work in a dialectical setting. And we might ask whether either of Aristotle's proposed dialectical responses would actually *require* the introduction of representational diagrams into dialectic, and whether this would fundamentally change the nature of dialectic. After all, dialectic is supposed to rely on things that are "common". But should the use of diagrams to represent things that they, the diagrams, are not, be considered "common"? The answer is not obvious. Indeed, we might wonder what it says about our cognitive powers if the resolution of the Racetrack really does require the sort of pretense that is involved in the use of diagrams.

Putting aside the question of the practical efficacy of Aristotle's resolutions, we might ask whether he himself attains some sort of genuine insight into the puzzle. And we might further ask whether Aristotle attains some insight that the modern analysis misses.

We might go even further and contrast Aristotle's reframing of the Racetrack with modern reframings. Aristotle, of course, reframes the *logos* using diagrams. By contrast, the standard modern critique, which asserts that it is possible to traverse an unlimited collection, depends on reinterpreting the "racetrack" as an actual collection of points.

And, as we have seen, it also depends on reinterpreting the questions of the *logos* as propositions. How are these reinterpretations like and unlike? Must we necessarily reinterpret the *logos* in order to resolve it? If so, why?

It might even seem that, having devoted a great deal of attention to the Racetrack, I should say something about what it all means. But for now this will need to wait. For now we will simply consider some of the questions that arise naturally from our discoveries so far.

Zeno

We have spent much of our time examining Aristotelian texts. But the Racetrack and the Achilles were created by Zeno, in the fifth century BC, and our discoveries are perhaps most significant as concerns Zeno himself.

The ancient evidence concerning Zeno is extremely limited, particularly the evidence that concerns the actual content of his "philosophy". We know, more or less, of the four motion paradoxes, several plurality paradoxes, the paradox of the millet seed, and the paradox of place. And we have now discovered that the motion paradoxes, and most likely the paradox of the millet seed, were oral dialectical *logoi*. Hence a good portion of Zeno's known philosophical activity would seem to concern dialectical *logoi*. And this raises a host of questions.

To begin with, consider the relation between the motion paradoxes and the plurality paradoxes. Indeed, this relation has long posed the central problem of Zeno scholarship: how do the motion paradoxes fit into a Zenonian *text* containing the plurality paradoxes? We have now definitely answered this question: they don't. The motion paradoxes were interactive *logoi*, whereas the plurality paradoxes were in a text that would be read aloud by one person. But if we have sundered the motion *logoi* from the plurality text, we nonetheless face the question of the the relation between them.

It might now even seem that our puzzles have multiplied: not only do we have two bodies of paradoxes, separated by content, but we now find that the difference of content is accompanied by a difference of form. But why? Why should motion paradoxes be dialectical, and plurality paradoxes written?

More generally, we might wonder about the relation between Zeno's two *forms* of argument. Zeno was no doubt the originator of each. But which came first? And how exactly did the second variety depend for its development on the first, as we might reasonably suppose that it did? We might suppose that the plurality arguments came later, inasmuch as their abstract and formal language is an element absent from the motion paradoxes. On the other hand, their content renders them more akin to the poem of Zeno's mentor Parmenides. I do not know the answer, but I do suspect that in seeking the answer, we will need to pay close attention to the ways in which oral and written speech might be influencing each other.

In general, we may also wonder about Zeno's relation to Parmenides. The difference in method between Zeno and Parmenides could scarcely be more striking. Parmenides composed an epic poem, whereas Zeno composed free-standing arguments. Is there some spark in Parmenides that ignites the Zenonian conflagration of dialectic? If so, what is it?

Since Zeno was both an oral dialectician and a writer, we may wonder whether his different activities spawned disparate lines of influence. Did Zeno initiate, for instance, a chain of oral dialectic that extended to Socrates? It seems quite likely that he did. How did he influence the Megarian philosophers, fascinated by paradox? And what of Gorgias, and Zeno's influence on paid teachers of rhetoric?

When we confront the question of Zeno's influence on Socrates, we thereby confront the question of his influence on Plato. Typically Socrates and Parmenides have been accounted perhaps the two chief influences on Plato. But now it seems likely that there could have been no Socrates without Zeno, and thus no Socrates without Parmenides. So in a way, Parmenides is not only a major direct influence on Plato, but an indirect influence as well, in an entirely separate fashion. So the influence of Parmenides on world history is perhaps even greater than has been recognized.

But Zeno himself likewise reaches Plato in at least two ways: via Socrates and as a speaker (indirectly) and writer in his own right. And in considering Zeno's direct influence, we see some tantalizing possibilities. The relation between the two parts of the Parmenides has often seemed one of the most difficult questions of Plato scholarship. But the dialogue itself suggests that that the second half is derived via *reflection* on Zeno. Can we then use *Plato's* own reflection on Zeno as a tool in helping us to understand this dialogue? As for the Phaedrus, Plato's other "Zenonian" dialogue, scholars have often been puzzled regarding the relation between the discussions of love and the discussions of writing. I do not know whether the investigation of Zeno's role in the dialogue will help address this question, but it is clear that we have thus far discovered a much more prominent role for "the Eleatic Palamedes" than has hitherto been recognized, and the evidence suggests things are even more complex than they seem. What, for instance, is the relation between the Eleatic Palamedes and Thoth, the Egyptian god who plays a well-known role in the latter part of the dialogue? The *legendary* Palamedes was often described as having the very attributes attributed to Thoth in the dialogue.[285] And what of the fact that, in antiquity, it was regarded as a great puzzle why Homer did not *mention* Palamedes in the *Iliad*, despite his apparent significance in the story? Is Plato somehow modeling the non-reference to Zeno on the *Iliad*'s non-reference to Palamedes?

While the questions I ask are not entirely new, I think it is clear that our awareness of Zeno as a dialectician forces us to ask many questions at least in new ways.

Aristotle

We have spent a good deal of time closely examining some Aristotelian texts, and in doing so, we have obtained results by reading against the backdrop of oral dialectic and the backdrop of diagram argument. It is likely that both dialectic and diagrams can play a role in helping to understand other Aristotelian texts.

While the scholarly literature is filled with discussions of Aristotelian "dialectic", this dialectic is often seen as some theoretical construction, and there is no doubt something write about this. But dialectic in no sense originated as theoretical, and we have now seen the clear value of reading at least some of Aristotle's texts against the backdrop of *actual* oral dialectic. Other scholars have also seen the value of this approach. Whitaker, for instance, has recently argued that *On Interpretation* is about the nature of contradictory statements in actual oral dialectic.[286] I suspect that the value of

285 Dušanić notes the affinity.
286 C.W.A. Whitaker, *Aristotle's* De Interpretatione*: Contradiction and Dialectic*

reading Aristotle against the backdrop of actual dialectical practice will be seen in other cases as well.

Of particular interest is the relation between dialectical practice as Aristotle knew it, Aristotle's theoretical account of dialectic, and the philosophical use to which Aristotle puts the account. In examining the Racetrack and the Achilles, we have, with effort, succeeded in disentangling the *logoi* as Aristotle knew them from his reinterpretations thereof. This suggests that similar disentanglement may be possible in other cases as well.

In contrast to dialectic, the significance, for Aristotle, of diagrams has largely gone unrecognized, even if their presence in his original "works" has often been granted. But in examining the discussions of the motion *logoi*, we have found that Aristotle's reasoning simply cannot be understood unless we consider the concrete ways in which Aristotle would have employed his diagrams in giving his lectures. Naturally, this raises the question of how widespread his use of diagrams will be. Does every occurrence of a freestanding letter signify that Aristotle is employing a diagram?

This question becomes particularly interesting when we consider the *Analytics*, which are filled with letters, but obviously involve no geometrical diagrams. I suspect that they actually involve verbal diagrams, which would raise the question of the relation between such verbal diagrams and the more geometrical diagrams. Did Aristotle's development of logic derive from the insight that he could treat visible written words as objects much like visible lines?

The study of the diagrams in Aristotle will illuminate not merely Aristotle himself, but the history of diagram argument, that is to say, of Greek mathematics. The Aristotelian corpus provides our earliest body of diagram arguments. And while Netz has demonstrated the systematic manner in which diagrams are used in later mathematical texts, Aristotle's own use of diagrams *may* well be much less systematic. Indeed, Aristotle himself may well play a key role in developing techniques of diagram argument, and in particular, his works may play a key role in determining how diagram arguments got represented in writing. And of course, this may well be merely another way of describing the development of the "logical" structure of "mathematics". While oral mathematical arguments, using constructed diagrams, of course, preceded Aristotle, the very existence of written representations of such arguments may begin to shape the way the arguments are perceived. Further examination of Aristotle's diagrams may thus yield insight both into diagrammatic practice and into metadiagrammatic practice.

In discovering that certain Aristotelian texts make sense only if we immerse ourselves in the ancient practice of dialectic and the ancient practice of diagram argument, we might well be unsurprised. For we might well have suspected that these were the very practices in which Aristotle himself was immersed in the Academy, at least if Plato's academicians resembled the guardians of the Republic, who climbed first through mathematics, that is, reasoning via diagrams, and then through dialectic. While we should be extremely wary of simple-minded readings of Plato, it does not seem ludicrous to suppose, as many scholars have, that dialectic and mathematics were Plato's two most valued intellectual practices, and that Aristotle, "the Mind" of the Academy, become very good at them. Indeed, if we wonder how Aristotle is possible, how one person can produce ingenious works on so many topics, on a scale never subsequently matched, we would do well to ask how, precisely, Aristotle differed from the prior mass of humanity. And the starting point of an answer would seem to be his Academic training in dialectic and mathematics (coupled, perhaps, with his apparently voluminous reading). Certainly the question merits investigation, since the question of how Aristotle is possible may to a large extent be the question of how any one of us is possible, as the sort of intellectual beings that we are, given the multifarious ways in which so much of our intellectual culture is causally derivative of Aristotle.

In seeking to understand Aristotle via his education, we in no way cast aspersion on Aristotle's own innovation. Indeed, we have found Aristotle to be not merely a practitioner of known methods, but an originator of new applications and a reflector on his own techniques. It is Aristotle himself who uses diagrams to construct a dialectical response to the dialectical Racetrack, who reworks it to resolve the rather different Achilles, and who then critiques his own resolution by reflecting on his own diagrammatic techniques.

The History of Philosophy

Contemporary historians of philosophy often presume that arguments are the focus of their study. They often seek to discern which arguments a philosopher produced, and to assess those arguments. In focusing on arguments, historians of philosophy often seek to distinguish themselves from historians of ideas.

Arguments, in some sense, have played a central role in my investigation as well, but perhaps not in quite the same way as is often the case. While I initially supposed that arguments were the ultimate focus of my study, I have always treated *inscriptions* as the phenomena to be investigated. By inscriptions I mean concrete recorded speech. If we are to regard the history of philosophy as an empirical discipline, then it seems we must regard inscriptions as the starting point of our reasoning.[287]

In speaking of inscriptions, I mean to draw a contrast with "writings" or "texts", which seem to be abstract entities. By contrast, inscriptions are concrete particulars: books, manuscripts, etc.[288] I, for instance, own a blue book that contains Aristotle's *Physics*, in a text reconstructed by Ross. The pages of book Z are falling out. Most of my reasoning about Aristotle and Zeno has its starting point in my beliefs about this blue book. But since I trust that Ross is a good editor, and since I accept commonly held beliefs about manuscript transmission from antiquity to the present, I believe that there existed manuscripts in antiquity that resemble very closely, in their text, my blue edition of the *Physics*. Generally speaking, I regard the ancient manuscripts as the real starting point of my investigation. Nearly every contemporary belief about the Racetrack and the Achilles is causally derivative of these ancient manuscripts.

Up to this point, I suppose that few would object. But in my treatment of these ancient manuscripts, I may differ from many scholars. I have sought to avoid the presumption that these texts are fundamentally logical objects, *in* which we might seek out arguments. Instead, I have asked: *what* brought these manuscripts into being? In particular, what *actions* of Aristotle, Zeno and others played a causal role in the eventual

287 I have here understood the history of philosophy as an empirical discipline, but there may well be interesting alternatives.

288 Today, inscriptions include recordings of oral speech, but we may ignore these and the various complexities they raise.

production of the ancient *Physics* manuscripts? My reasoning ends up positing the very actions that seem to best explain the known result, the ancient *Physics*.

By seeking concrete actions, as opposed to abstract arguments, I have left open the possibility that quite a *variety* of verbal actions, interacting in various ways, contributed to the production of the eventual inscription. Some of these verbal actions may well constitute arguments of various sorts. But these arguments need not be present *in* the inscription itself. And we have indeed discovered a variety of verbal actions, some of them arguments: Zeno created and employed question-and-answer arguments, these were orally transmitted to Aristotle's time, Aristotle used diagram arguments in his lectures on the motion *logoi*, he reflected on how to apply the reasoning in actual dialectic, and he reflected on what he himself was doing in giving diagram arguments. If I am right, all of these verbal actions *actually happened*, and their happening plays a key role in explaining the eventual inscriptions.

The study of Zeno suggests a way out of a dilemma that has often afflicted historians of philosophy. On the one hand, a historian can seek to render a philosopher's arguments in a up-to-date fashion, rendering them relevant to contemporary philosophers, but at the cost of truth. On the other hand, a historian can cloak historical arguments in antique garb, preserving truth, but rendering them of interest only to specialists. As a way out, I suggest that what is often *interesting* about philosophy is the way in which it stands at the juncture of more than one *type* of speech.

Consider, for instance, the Racetrack. Upon discovering that the Racetrack was a dialectical *logos*, we might have supposed that we could simply view dialectic as the external form, and then extract the meaning that the questioner passes to the answerer. But not so. We discovered that this seemingly interactive *logos* actually requires the answerer to give meaning to certain words via his own imagination. The *logos* arises in some bizarre way from the confluence of interactive dialectic and from the answerer's ability to speak, in an obviously inherently private fashion, about his own imagination.

Next consider Aristotle's initial response to the Racetrack. Here we most evidently see the confluence between two modes of speaking. We might have expected Aristotle to offer a purely dialectical solution. But instead he incorporates into his resolution a fundamentally different verbal practice, the practice of diagram argument.

Finally, consider Aristotle's revision of his initial response to the Racetrack. We have seen that Aristotle's revision arises from his reflection on the ontological presuppositions of diagram argument. But if we think about what this "reflection" actually *is*, we see that it involves the confluence of two sorts of speech. First is diagram argument itself. And second is Aristotle's internal discourse *about* diagram argument.

The junctures between modes of speech will often interest us, I suggest, because they are *active*. Different tributaries of speech collide against each other. We are not viewing static pictures of the past, but motion. We see the speech moving in out minds. And we know that in our own time, while we may not have precisely the same types of speech, we do have *some* discernible types of speech. And seeing how others somehow discover new *things* simply by diverting flows of speech into each other, we hope and wonder if we can do the same.

In producing this research, I seem to have been instantiating the very sort of juncture between modes of speech that I have just been outlining. In my case, these two modes of speech are two varieties of my own internal discourse, varieties that have been clearly distinct, but thoroughly intertwined. On the one hand, I have discussed, in my mind, inscriptions, and the patterns of speech that end up inscribing them, as objects of my visual imagination. When I internally discuss "speech" that I see in this way, I do not "understand" it, I merely see it. On the other hand, I have to understand, and speak to myself, the speech that was actually spoken by various ancients, but especially Aristotle and the practitioners of the Racetrack and the Achilles. To the extent that I have succeeded in discovering anything, I suspect it is in large part due to my moving back and forth between these two modes of discourse.

The history of philosophy is not a history of ideas; it is the history of the speech of philosophers. It is the history of the ways in which this speech touches other *things*: other speech, other people, writing, symbols, imagination, and who knows what else. If speech is what makes us human, then the history of philosophy is a vast preserve in which we might see something of human nature. The history of philosophy will always be with us. So long as we think, we may examine it.

Appendix:
Further Observations on Zeno in the *Phaedrus*

In Chapter 1, we saw that Plato, in the *Phaedrus*, portrayed Zeno both as the composer of a treatise on "private rhetoric" and as a practitioner of this art. I suggested that this "treatise" is none other than Zeno's plurality text, and that "private rhetoric" may be understood as dialectic. Later in the dialogue, we indeed find more explicit signs of this identification.

Recall that Socrates has earlier introduced the idea that rhetoric is an art. (261a7-11) The reference to Zeno as a representative of the private practice of this art (261c10-e4) is then followed by a discussion of the supposed art, whereby Socrates sums up the chief elements of the art. (265d3-266b1) These have to do with rules for the practice of Plato's method of collection and division. Socrates notes that he is uncertain of the name, but thus far he has always called practitioners of *this* art "dialecticians". (266b7-9) Then he questions Phaedrus: "Or is it just that art (*techne*) of *logoi* (speeches/ arguments) which Thrasymachus and the others use in becoming skilled at speaking themselves, and in making others skilled as well, whoever is willing to bring gifts [i.e. fees] to them, as to kings?" (266c2-5) Phaedrus replies: "They may behave like kings, but they certainly lack the knowledge you're talking about. No, it seems to me that you are right in calling the sort of thing you mentioned dialectic; but, it seems to me, rhetoric still eludes us." (266c6-9)

On the surface, it might seem unclear whether we should construe Zeno as included among "Thrasymachus and the others," who have apparently been practicing rhetoric, but not dialectic. But if we read further, it becomes evident that Zeno, the

Eleatic Palamedes who was introduced as the representative of "private rhetoric", is certainly *not* one of these non-dialectical rhetoricians. Immediately following the prior remark of Phaedrus, Socrates responds: "What are you saying? Could there be anything valuable which is independent of the methods I mentioned and is still grasped by art? If there is, you and I must certainly honor it, and we must say what part of rhetoric it is that has been left out." (266d1-4) Socrates thus insinuates that one portion of rhetoric has been ignored when he set forth the "art of rhetoric." Earlier, we saw that "the rhetorical art" had been partitioned into two parts, the public and the private, the division of which occasioned the introduction of Zeno. (261a7-d9) Naturally, if one part of the art of rhetoric has been left out of the art of rhetoric, i.e. dialectic, it must be either the public part or the private part.

It soon becomes evident that it is the public part that has not been encompassed by dialectic. In answer to Socrates' question regarding whether some part of the art of rhetoric has been missed, Phaedrus replies: "Well there's quite a lot, Socrates: everything, at any rate, written up in the books on the art of *logoi*[289]." (266d5-6) So Socrates has asked whether some remaining part of rhetoric is covered by an art, but Phaedrus responds by recalling what is in the *books* on the "art of rhetoric." As we have already seen, these books are books by the "public" rhetoricians. By contrast, we saw that the "treatise" on "private rhetoric" was nothing other than Zeno's plurality book. Socrates proceeds to recall a whole host of elements of the "art" of rhetoric as presented in rhetorical handbooks, in the process recalling the names of many famous as rhetorical innovators. (266d7-267d9) These include the three "public" rhetoricians earlier

289 Nehemas/Woodruff write "the art of speeches".

contrasted with Zeno, namely, Theodorus, Gorgias, and Thrasymachus (referred to as "the mighty Chalcedonian"). Zeno is not mentioned. Concluding the recitation of the elements of rhetoric, Socrates proposes to examine "the power of the art these things produce". (268a1-2) To which Phaedrus replies: "A very great power, Socrates, especially in front of a crowd." (268a2-4)

There can thus be no doubt that what Socrates and Phaedrus are now discussing is "public rhetoric." Hence public rhetoric must be the part of rhetoric that got "left out" when the earlier discussion focused on the "art of rhetoric" that turned out to be dialectic. It follows that this earlier art of rhetoric must actually be the art of private rhetoric. Hence "private rhetoric" seems to be dialectic. And so if Zeno is the representative of private rhetoric, he is likewise the representative of dialectic.

It would certainly be a mistake to take every claim and insinuation of the *Phaedrus* as an expression of historical fact. In particular, it is quite unlikely that Zeno conceived of himself as practicing the method of collection and division, which Socrates ends up calling dialectic. But it is fairly clear that the dialogue takes for granted that Zeno was a practitioner of interactive oral argumentation, and, indeed, of what I have been calling, in a broad sense, dialectic. Recall that Zeno's plurality book could be construed as a “treatise on dialectic.” But it is clear that this “treatise” is significant only insofar as Zeno is also a *practitioner* of dialectic. Just like the public rhetoricians, Zeno must be “skilled at speaking... and in making others skilled as well... .” (266c3-4) While his treatise may help to make others skilled, Zeno himself must *be* a dialectician if the analogy is to hold. And he cannot be a dialectician merely by writing: he must speak interactively. Here dialectic differs crucially from rhetoric, since one might be a rhetorician by reading a prepared speech. Later in the *Phaedrus*, Socrates famously chastises written texts for their inability to respond to questions (275d4-5), a crucial component of dialectic. And he directly contrasts the activity of writing with the activity of dialectic (276d1-277e5), calling dialectic "nobler". If the *Phaedrus* depicts Zeno as a dialectician, which in some sense it surely does, then it undoubtedly depicts Zeno not merely as a writer, but as an actual oral dialectician. That is to say, it depicts him as a practitioner of some sort of interactive oral argumentation. And Plato apparently bases this much on historical fact, quite apart from his own development of the *concept* of dialectic, and from the literary role of Zeno in the dialogue.

This discussion of the *Phaedrus* has likely raised more questions than it answers. In fact, the complexity of the Zeno reference has been almost entirely ignored both by Zeno scholars and by just about everyone else. Certainly the role of Zeno in the *Phaedrus*, and more generally, his influence on Plato, merit further attention.[290] But at this point all signs indicate that Plato singles out Zeno not merely as a practitioner, but even as a heroic and paradigmatic practitioner of oral dialectic.

290 Here is one additional observation. We have seen that in the *Parmenides*, the conversation begins with Zeno reading his plurality book. Then in the second portion of the dialogue, Parmenides leads a highly formal dialectical exchange, the format of which he establishes by appeal to what Zeno has said in his book. (135d8-e4) The dialectic does not merely debate the content of the book, nor does it merely mimic orally the written form of the text. Instead, the written book is employed as a catalyst in the development of a *novel* oral form. I have not seen this discussed in the literature on the *Parmenides*. There are signs that the role of Zeno in the *Phaedrus* may be even more complex, and significant, than I have indicated, but these await further study.

Bibliography

Ancient Texts and Translations

Aristotle. *Aristotle's Physics: A Guided Study*. Trans. Sachs, Joe. New Brunswick, NJ: Rutgers University Press, 1995.

Aristotle. *Aristotle's Physics: A Revised Text with Introduction and Commentary*. Ed. Ross, Sir David. Oxford: Oxford University Press, 1936.

Aristotle. *Aristotle's Physics: Books III and IV*. Trans. and comm. Hussey, Edward. Oxford: Oxford University Press, 1983.

Aristotle. *Aristotle's Prior and Posterior Analytics: A Revised Text with Introduction and Commentary.* Ed. Ross, W. D. Oxford: Oxford University Press, 1949.

Aristotelis. *Ars Rhetorica*. Ed. Kassel, Rudolfus. Berlin: Walter de Gruyter & Co., 1976.

Aristotle. *The Complete Works of Aristotle*. Ed. Barnes, Jonathan. Princeton University Press, Princeton, 1984.

Aristotle. *Aristotle's Metaphysics: A Revised Text with Introduction and Commentary*. 2 vols. Corrected ed. Ed. Ross, W. D. Oxford: Oxford University Press, 1953.

Aristotle. *Minor Works*. Ed. Hett, W. S. Cambridge: Harvard University Press, 1936.

Aristotle. *Physics*. Trans. Waterfield, Robin. Oxford: Oxford University Press, 1996.

Aristotle. *Physics: Book VIII*. Trans. and comm. Graham, Daniel W. Oxford: Clarendon Press, 1999.

Aristotelis. *Topica et Sophistici Elenchi*. Ed. Ross, W. D. Oxford: Oxford University Press, 1958.

Aristotle. *Topics Books I and VIII with excerpts from related texts*. Trans. and comm. Smith, Robin. Oxford: Clarendon Press, 1997.

Diogenes Laertius. *Lives of Eminent Philosophers*. 2 vols. Ed. Hicks, R. D. London: William Heinemann, 1925.

Euclid. *Elementa*. 4 vol. Ed. Heiberg, I. L. Leipzig: BSB B. G. Teubner Verlagsgesellschaft, 1969

Euclid. *The Thirteen Books of the Elements*. 3 vol. 2nd ed. Trans. Heath, Sir Thomas L. New York: Dover Publications, 1956.

Homer, *Iliad*. 2 vol. With translation by A.T. Murray, revised by William F. Wyatt. Cambridge, MA: Harvard Univ. Press, 1999.

Plato. *Complete Works*. Ed. John Cooper. Indianapolis: Hackett Publishing Company, 1997.

Plato. *The Euthydemus of Plato*. Ed. Gifford, Edwin Hamilton. New York: Arno Press, 1973.

Plato. *Gorgias: A Revised Text with Introduction and Commentary*. Ed. Dodds, E. R. Oxford: Clarendon Press, 1959.

Plato. *Platonis Opera*. 4 vols. Ed. Joannes Burnet. Oxford: Clarendon Press, 1901.

Philoponus, John. *Ioannis Philoponi In Aristotelis Physicorum Libros Quinque Posteriores Commentaria*. Ed. Vitelli, Hieronymus. Berlin: Academiae Litterarum Regiae Borussicae, 1837.

Proclus. *Procli Commentarium in Platonis Parmenidem*, in *Procli Opera Inedita*, ed. Cousin, Victor. pp. 617-1258. Paris: Unveranderter Nachdruck der Ausgabe, 1864.

Proclus. *Commentary on Plato's* Parmenides. Trans. Morrow, Glenn R. and Dillon, John, M. Princeton: Princeton University Press.

Sextus Empiricus. *Sextus Empiricus*. 4 vols. Trans. Bury, R. G. Cambridge: Harvard University Press, 1987.

Simplicius. *Simplicii In Aristotelis Physicorum Libros Quattuor Posteriores Commentaria*. Ed. Diels, Hermannus. Berlin: Academiae Litterarum Regiae Borussicae, 1895.

Simplicius. *Simplicii In Aristotelis Physicorum Libros Quattuor Priores Commentaria*. Ed. Diels, Hermannus. Berlin: Academiae Litterarum Regiae Borussicae, 1882.

Simplicius. *On Aristotle's* Physics *6*. Trans. Konstan, David. Ithaca: Cornell University Press, 1989.

Simplicius. *On Aristotle's* “Physics *8.6-10*.” Trans. McKirahan, Richard. Ithaca: Cornell University Press, 2001.

Themistius. *In Physica paraphrasis*. Ed. Schenkl, H. Berlin: Academiae Litterarum Regiae Borussicae, 1900.

Zeno. *Zeno of Elea: A Text, with Translation and Notes*. Ed. Lee, H. D. P. London: Cambridge University Press, 1936.

Modern Literature

Bolton, Robert. “Aristotle's Conception of Metaphysics as a Science.” pp. 321-354 in *Unity, Identity, and Explanation in Aristotle's Metaphysics*. Ed. Scaltsas, T., Charles, D. and Gill, M. L. Oxford: Clarendon Press, 1994.

Bolton, Robert. “The Epistemological Basis of Aristotelian Dialectic.” pp. 57-105 in *From Puzzles to Principles? Essays on Aristotle's Dialectic*. Ed. Sim, May. Lanham, MD: Lexington Books, 1999.

Bolton, Robert. “The Problem of Dialectical Reasoning (Συλλογισμός) in Aristotle.” *Ancient Philosophy* (1994): 99-132.

Bostock, David. “Aristotle, Zeno and the Potential Infinite.” pp. 39--53 in Moore, *Infinity.*

Boyer, Carl B. *The History of the Calculus and It's Conceptual Development*. New York: Dover Publications, 1949.

Cajori, Florian. “The History of Zeno's Arguments on Motion.” in *The American Mathematical Monthly* XXII (1915): 1-6, 39-47, 77-82, 109-115, 143-149, 179-186, 215-220, 253-258, 292-297.

Dehaene, Stanislas. *The Number Sense: How the Mind Creates Mathematics*. New York: Oxford University Press, 1997.

Dillon, John M. “Proclus and the Forty Logoi of Zeno.” *Illinois Classical Studies* 11 (1986): 24-41.

Dušanić, Slobodan. “Alcidamas of Elea in Plato's *Phaedrus*.” *Classical Quarterly* 42 (ii) (1992): 347-357.

Faris, J. A. *The Paradoxes of Zeno*. Avebury: Aldershot, England, 1996.

Fowler, David. *The Mathematics of Plato's Academy: A New Reconstruction*. 2nd ed. Oxford: Clarendon Press, 1999.

Furley, David J. *Two Studies in the Greek Atomists*. Princeton: Princeton University

Press, 1967.

Grunbaum, Adolf. *Modern Science and Zeno's Paradoxes*. Middletown, CT: Wesleyan University Press.

Hasper, Pieter Sjoerd. "The Metaphysics of Continuity: Zeno, Democritus and Aristotle." Diss. Rijksuniversiteit Groningen, 2003.

Hasper, Pieter Sjoerd. "Zeno Unlimited." in *Oxford Studies in Ancient Philosophy*, Volume XXX, Summer 2006. Ed. David Sedley. Oxford: Oxford University Press, 2006.

Hardie, R. P. and Gaye, R. K. Translators of Aristotle's *Physics* in Barnes.

Heath, Sir Thomas. *Mathematics in Aristotle*. Bristol, UK: Thoemmes Press, 1949.

Kahn, Charles. *Anaximander and the Origins of Greek Cosmology*. Corrected edition. Indianapolis: Hackett Publishing Company, 1985.

Kirk, G. S., Raven, J. E., and Schofield, M. *The Presocratic Philosophers: A Critical History with a Selection of Texts*. 2nd edition. Cambridge: Cambridge University Press, 1983.

Klein, Jacob. *Greek Mathematical Thought and the Origin of Algebra*. Trans. Brann, Eva. Cambridge, MA: The M.I.T. Press, 1968.

Kneale, William and Kneale Martha. *The Development of Logic*. Oxford: Clarendon Press. 1962.

Knorr, Wilbur Richard. *The Evolution of the Euclidean Elements: A Study of the Theory of Incommensurable Magnitudes and Its Significance for Early Greek Geometry*. Dordrecht, Holland: D. Reidel Publishing Co., 1975.

Knorr, Wilbur R. "Infinity and Continuity: The Interaction of Mathematics and Philosophy in Antiquity." in Kretzmann.

Kretzmann, Norman, ed. *Infinity and Continuity in Ancient and Medieval Thought*. Ithaca: Cornll University Press, 1982.

Lear, Jonathan. "Aristotelian Infinity." pp. 55-78 in Moore, *Infinity*.

Liddell, Henry George, and Scott, Robert, compilers. *A Greek-English Lexicon*. 9th ed, with supplement. Rev. by Jones, Sir Henry Stuart. Oxford: Clarendon Press, 1996.

Lord, Albert B. *The Singer of Tales*. 2nd ed. Eds. Mitchell, Stephen and Nagy, Gregory. Cambridge, MA: Harvard Univ. Press: 2000.

Matson, W. I. "Zeno Moves!" in *Essays in Ancient Greek Philosophy VI: Before Plato*. Ed. Preus, Anthony. Albany: SUNY Press, 2001.

McKirahan, Richard D. Jr. *Philosophy Before Socrates: An Introduction with Texts and Commentary*. Indianapolis: Hackett Publishing Company, 1994.

McKirahan, Richard D. Jr. "La Dichotomie de Zenon chez Aristote" in *Qu'est-ce que la Philosophie Prâesocratique?* Ed. Laks, Andre. *Cahiers de philologie*, v. 20. Villaneuve-d'Ascq: Presses universitaires du Septentrion, 2002.

Moore, A. W. *Infinity*. Dartmouth: Aldershot, 1993.

Moore, A. W. *The Infinite*. London: Routledge, 1990.

Moraux, Paul. "La joute dialectique d'apres le huitième livre des Topiques." pp. 277-311 in Owen, *Aristotle on Dialectic*.

Nehamas, Alexander and Woodruff, Paul, translation of Plato's *Phaedrus* in Cooper.

Netz, Reviel. *The Shaping of Deduction in Greek Mathematics: A Study in Cognitive History*. Cambridge: Cambridge University Press.

Owen, G. E. L. *Aristotle on Dialectic: The* Topics. Oxford: Clarendon Press, 1968.

Owen, G. E. L. "Zeno and the Mathematicians." pp. 1-24 in Moore, *Infinity*.

Pickard-Cambridge, W. A. Translator of Aristotle's *Sophistical Refutations* in Barnes, pp. 278-314.

Pickard-Cambridge, W. A. Translator of Aristotle's *Topics* in Barnes, pp. 167-277.

Robb, Kevin. *Literacy and Paideia in Ancient Greece*. New York: Oxford University Press, 1994.

Romano, David Gilman. *Athletics and Mathematics in Archaic Corinth: The Origins of the Greek* Stadion. Philadelphia: American Philosophical Society, 1993.

Russell, Bertrand. *Principles of Mathematics*. 2nd ed. New York: W. W. Norton & Co., 1938.

Ryle, Gilbert. "Dialectic in the Academy" pp. 39-68 in *New Essays on Plato and*

Aristotle. Ed. Bambrough, Renford. London: Routledge & Kegan Paul, 1965.

Ryle, Gilbert. "Dialectic in the Academy" in Owen, pg. 69-79.

Salmon, Wesley C., ed. *Zeno's Paradoxes*. Indianapolis: The Bobbs-Merrill Company, 1970.

Schwayder, David S. "Achilles Unbound." in *The Journal of Philosophy* LII, No. 17 (1955): 449-459.

Slomkowski, Paul. *Aristotle's* Topics. Leiden: Koninklijke Brill, 1997.

Smyth, Herbert Weir. *Greek Grammar*. Rev. by Messing, Gordon M. Cambridge, MA: Harvard University Press, 1920.

Sorabji, Richard. *Time, Creation and the Continuum: theories in antiquity and the early middle ages*. Ithaca: Cornell University Press, 1983.

Stokes, Michael C. *One and Many in Presocratic Philosophy*. Washington, D.C.: The Center for Hellenic Studies, 1971.

Szabo, Arpad. *The Beginnings of Greek Mathematics*. Dordrecht, Holland: D. Reidel Publishing Co., 1978.

Tarrant, Harold. "*More on Zeno's* Forty Logoi." *Illinois Classical Studies* 15 (1990): 23-37.

Vlastos, Gregory. "Plato's Testimony Concerning Zeno of Elea." in *Studies*, pp. 264-300.

Vlastos, Gregory. *Studies in Greek Philosophy, Volume 1: The Presocratics*. Ed. Graham, Daniel W. Princeton: Princeton University Press, 1995.

Vlastos, Gregory. "Zeno of Elea." in *Studies*, pp. 241-263.

Vlastos, Gregory. "Zeno's Race Course: With an Appendix on the Achilles." in *Studies*, pp. 189-204.

Waschkies, Hans-Joachim. "Mathematical Continuum and Continuity of Movement" pp. 151-179 in *La Physique d'Aristote et les Conditions d'une Science de la Nature*. Ed. De Gandt, François and Souffrin, Pierre. Paris: Librairie Philosophique J. Vrin, 1991.

Waterfield, Robin. *The First Philosophers: The Presocratics and the Sophists*. Oxford: Oxford University Press, 2000.

White, Michael J. *The Continuous and the Discrete: Ancient Physical Theories from a Contemporary Perspective*. Oxford: Clarendon Press, 1992.

Curriculum Vitae
Benjamin William Allen

Degrees

1993 BA (Philosophy), Yale
1995 MA (Philosophy), University of Kentucky

Academic Positions

1993-1995	University of Kentucky, Dept. of Philosophy, Teaching Assistant
1995-1998	Rutgers University, Dept. of Philosophy, Fellowship
1999-2003	Rutgers University, Dept. of Philosophy, Teaching Assistant
2004-2007	Rutgers University, Dept. of English, Teaching Assistant
2008	Tufts University, Dept. of Philosophy, Instructor