SUPERSYMMETRY BREAKING IN GAUGE THEORIES AND STRING THEORY

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ABSTRACT OF THE DISSERTATION

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We study aspects of supersymmetry breaking in gauge theories and string theory. On the gauge theory side, we explore metastable vacua in a SQCD-like model with an extra sector connected by a singlet. The model combines dynamical supersymmetry breaking with an O'Raifeartaigh mechanism in terms of confined variables. On the string theory side, we study the dynamics of non-supersymmetric magnetized D-brane configurations on Calabi-Yau spaces. We also study the stabilization of the supersymmetry breaking runaway quiver.

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Dedication

This thesis is dedicated to Atreyi and Rohini

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Introduction

1.1 Supersymmetry and its Motivations

The Standard Model (see [1] for a recent review) is a very successful description of all known phenomena in particle physics, and has survived all experimental tests with a high degree of accuracy. As the energy frontiers push into the TeV range, there have been no direct clues about additional structure. Certainly, a new framework arises near the Planck scale $= (8\pi G_{\rm Newton})^{-1/2} = 2.4 \times 10^{18}$ GeV, when gravity becomes important. However, even before that, one can expect new phenomena to occur. One of the most actively studied such extensions has been supersymmetry.

From a purely phenomenological point of view, supersymmetry gives us a way to address the notorious hierarchy problem. This problem is manifest in the Higgs sector of the Standard Model. The Higgs field is a complex scalar H with a classical potential

$$V_{Higgs} = m_H^2 |H|^2 + \lambda |H|^4 \,. \tag{1.1.1}$$

The VEV of H is non-zero at the minimum of the potential. This will occur if $\lambda > 0$ and $m_H^2 < 0$, which implies that $\langle H \rangle = \sqrt{-m_H^2/2\lambda}$. Experimentally, we know that $\langle H \rangle$ is approximately 174 GeV, from measurements of the properties of the weak interactions. Thus, m_H^2 should roughly be of order $-(100 \text{ GeV})^2$. The problem is that m_H^2 receives quantum corrections from every particle that couples, directly or indirectly, to the Higgs field.

For example, consider the Higgs coupling to a Dirac fermion f with mass m_f through a term in the Lagrangian $-\lambda_f H f f$. Then, a diagram with external Higgs legs and the fermion running around in a loop gives the following contribution to the Higgs mass:

$$\Delta m_H^2 \;=\; -\frac{|\lambda_f|^2}{8\pi^2}\Lambda_{\mathrm{UV}}^2 + \ldots. \tag{1.1.2}$$

Here Λ_{UV} is an ultraviolet and should be interpreted as the energy scale at which new physics enters. The ellipses represent terms proportional to m_f^2, which grow at most logarithmically with Λ_{UV}.

Every lepton and quark in the Standard Model can play the role of f. If Λ_{UV} is around , then the quantum correction to m_H^2 is 30 orders of magnitude larger than the required value of $m_H^2 \sim -(100\text{ GeV})^2$. This is a direct disaster for the Higgs mass. Moreover, the quarks, leptons and gauge bosons Z^0, $W^\pm$ of the Standard Model all obtain masses from $\langle H\rangle$, so the entire spectrum of the Standard Model is directly or indirectly sensitive to this issue. This is the core of the hierarchy problem.

To address the issue, there have been various avenues of research. One could get rid of the elementary Higgs field and replace it with a bound state of fermions, leading to the much-studied Technicolor [4] and composite Higgs theories. Other avenues include large extra dimensions [2], in which the fundamental scale is much lower; TeV scale supersymmetry, which naturally cancels fermionic corrections with bosonic ones; Little Higgs models [3] in which fermionic and bosonic loops cancel separately; or split supersymmetry [5], which realizes supersymmetry breaking at a high scale and opts for the fine-tuning option to solve the Higgs hierarchy.

Of these theories, TeV scale supersymmetry has received the most attention and is particularly interesting. It is part of a larger vision of physics, not just a model-building solution. The Coleman-Mandula theorem [6] singles out supersymmetry as the unique extension of Poincare invariance in quantum field theory in more than two spacetime dimensions. Essentially, the theorem tells us that in more than $1+1$ dimensions, the only possible conserved quantities that transform as tensors under the Lorentz group are the four momenta P_μ and the generators $J_{\mu\nu}$ of Lorentz transformations, apart from internal symmetries. Thus, any extension of Poincare invariance must come from generators with spinor charges, that is, supersymmetry.

Supersymmetry is also essential for another unifying structure of physics: Grand Unification

[7]. The idea behind Grand Unified Theories (GUTs) is that the Standard Model gauge group structure $SU(3) \times SU(2) \times U(1)$ is embedded in a larger group, like $SU(5)$ or $SO(10)$. This organizes a single Standard Model generation into a $\overline{5} \oplus 10$ of $SU(5)$, or a 10 of $SO(10)$. Apart from this highly non-trivial organization, GUTs are important for a number of reasons. The neutrino mass scale given by Grand Unification, $m_\nu \sim M_W^2/M_{GUT} \sim 10^{-2}eV$ has turned out to be correct. The scale of unified gauge theories, M_{GUT}, is relatively close to the Planck scale, and high enough to be consistent with proton decay. Fluctuations in the cosmic microwave background can be naturally explained by an inflationary stage near the GUT scale. GUTs fit in nicely with string theory.

The reason supersymmetry seems to be inevitable in such a scheme is as follows. The unified group, say $SU(5)$, is spontaneously broken to the Standard Model group at a high energy scale we have called M_{GUT}, in analogy with electroweak symmetry breaking to the electromagnetic U(1) at the (much lower) electroweak scale. Above M_{GUT}, then, there is the single coupling of $SU(5)$ only, and this must therefore be the unified coupling constant of all the gauge group factors of the Standard Model. The values of the coupling constants are a function of the energy scale - this is the basic idea of the renormalization group. The beta-function tells us how quickly the coupling constants change with respect to the energy scale. The beta function receives contributions from one-loop and two loop diagrams (higher diagrams may be neglected at thelevel of accuracy of current experiments). The running of the coupling of each gauge group is affected by all particles that carry charges under that group. The Standard Model coupling constants, suitably normalized at low energy, may be extrapolated to high energies using the beta function. If the running is calculated in a nonsupersymmetric GUT setting, whose particle spectrum resembles the Standard Model at energies between M_{GUT} and low energies, it is found that the three lines don't meet at the same point. Each pair of lines among the three gauge groups intersects, but not three together. This is a severe setback in the scheme of Grand Unification. On the other hand, for a supersymmetric GUT, the three lines do indeed meet at a single point. Given that the chance of three lines intersecting on a plane is low, this points to

Names		spin 0	spin 1/2	$SU(3)_C,\ SU(2)_L,\ U(1)_Y$
squarks, quarks	Q	$(u_L\ \ d_L)$	$(u_L\ \ d_L)$	$(\,\mathbf{3},\ \mathbf{2}\,,\ \frac{1}{6})$
(×3 families)	u	u_R^*	$u_R^\dagger$	$(\,\overline{\mathbf{3}},\ \mathbf{1},\ -\frac{2}{3})$
	d	d_R^*	$d_R^\dagger$	$(\,\overline{\mathbf{3}},\ \mathbf{1},\ \frac{1}{3})$
sleptons, leptons	L	$(\nu\ \ e_L)$	$(\nu\ \ e_L)$	$(\,\mathbf{1},\ \mathbf{2}\,,\ -\frac{1}{2})$
(×3 families)	e	e_R^*	$e_R^\dagger$	$(\,\mathbf{1},\ \mathbf{1},\ 1)$
Higgs, higgsinos	H_u	$(H_u^+\ \ H_u^0)$	$(H_u^+\ \ H_u^0)$	$(\,\mathbf{1},\ \mathbf{2}\,,\ +\frac{1}{2})$
	H_d	$(H_d^0\ \ H_d^-)$	$(H_d^0\ \ H_d^-)$	$(\,\mathbf{1},\ \mathbf{2}\,,\ -\frac{1}{2})$

Table 1.1: Chiral supermultiplets in the Minimal Supersymmetric Standard Model. The spin-0 fields are complex scalars, and the spin-1/2 fields are left-handed two-component Weyl fermions.

Names	spin 1/2	spin 1	$SU(3)_C,\ SU(2)_L,\ U(1)_Y$
gluino, gluon	g	g	$(\,\mathbf{8},\ \mathbf{1}\,,\ 0)$
winos, W bosons	$W^\pm\ \ W^0$	$W^\pm\ \ W^0$	$(\,\mathbf{1},\ \mathbf{3}\,,\ 0)$
bino, B boson	B^0	B^0	$(\,\mathbf{1},\ \mathbf{1}\,,\ 0)$

Table 1.2: Gauge supermultiplets in the Minimal Supersymmetric Standard Model.

a highly non-trivial compatibility between supersymmetry and the unification scheme.

On a more phenomenological but very important note, the proton lifetime and the unification scale are too low without incorporating supersymmetry.

One of the drawbacks of supersymmetry is that it does not simplify the Standard Model - superpartners do not occur within the known spectrum. Standard Model bosons and fermions do not pair up to give supersymmetric multiplets - rather, the spectrum has to be doubled. The spectrum is depicted in tables 1.1 and 1.2. The Standard Model together with supersymmetry forms a model that is called the Minimal Supersymmetric Standard Model, or the MSSM.

None of the superpartners in the tables has been observed - thus, supersymmetry must be broken in our world. One of the major thrusts of research in supersymmetry has been to understand how it is spontaneously broken.

The basic approach has been to extend the MSSM to include a hidden sector called the SUSY-breaking sector, which has fields that implement supersymmetry breaking and are uncharged under the Standard Model. Supersymmetry breaking communication to the MSSM has been broadly studied under two mechanisms: gravity mediation and gauge mediation. We will mainly be interested in gauge mediation. In this case, there is a messanger sector which has fields that are charged under both the supersymmetry-breaking sector and the MSSM. The MSSM superpartner masses then come from loop diagrams containing the messengers and Standard Model gauge bosons.

Various supersymmetry breaking models have been studied in the hidden sector. Below, we discuss some generic features of such models.

1.2 Supersymmetry Breaking in Gauge Field Theories

One of the most attractive mechanisms of supersymmetry breaking is called Dynamical Supersymmetry Breaking (DSB) [60]. The Witten index for a supersymmetric theory, $\mathrm{Tr}(-1)^F$, counts the number of supersymmetric ground states. The index doesn't change in perturbation theory. Therefore, supersymmetry is either broken at tree level or by non-perturbative dynamics. To solve the hierarchy problem, the SUSY-breaking scale is much lower than the Planck scale. Tree-level breaking would then require very small parameters, which may be contrived. However, for theories which exhibit dimensional transmutation, for example asymptotically free non-Abelian gauge theories, the dynamical scale can be hierarchically smaller than the Planck scale. This occurs naturally, and hence the interest in DSB.

It can be shown that in global supersymmetry, a SUSY-preserving ground state necessarily has zero energy, while a SUSY-breaking ground state has positive energy. The scalar potential is given by

$$V(\phi, \phi^*) = F^{*i} K^{ij} F_j + \frac{1}{2} \sum_a D^a D^a = W_i^* K^{ij} W_j + \frac{1}{2} \sum_a g_a^2 (\phi^* T^a \phi)^2. \tag{1.2.1}$$

In this equation, the first contribution is called the F-term, while the second is called the D-term. W is the superpotential of the theory, and it is highly constrained by symmetries and holomorphy. K is called the Kahler potential. The F and D terms do not interfere - one may thus set the D-term to zero, and work on the resulting space which is called the D-flat moduli space. The reason one does this is that D-term breaking can only proceed through $U(1)$ factors, and in the case of the Standard Model, the $U(1)_Y$ would not work. The squarks and sleptons do not have superpotential mass terms - they can thus relax away from a zero vev and cancel the $U(1)_Y$ Fayet-Iliopoulos term. So, we work on the D-flat moduli space and look for non-zero F-terms.

The D-flat moduli space exhibits a number of very interesting properties. The gauge symmetry generally exhibits a definite pattern of breaking, $G \to H$ on the moduli space. The fields of the Lagrangian break up into heavy fields which are eaten by the super-Higgs mechanism, and light fields which parameterize the moduli space. It can be shown that the classical moduli space of D-flat directions is in one to one correspondence with gauge invariant combinations of the original fields. These parametrize the space.

While in the analysis of the F-terms, the superpotential W is fixed by symmetries and holomorphy, that is not the case with the Kahler potential K. Both are required for a calculable model of supersymmetry breaking, as is evident from the expression for the potential. In fact, one only has control over the Kahler potential in two regimes: far out in moduli space, the gauge group is highly Higgsed, and due to asymptotic freedom one is in a calculable weak-coupling limit. The Kahler potential is then canonical in the elementary fields. Near the origin of moduli space, the Kahler potential receives non-calculable contributions. However, the dominant contribution to the Kahler potential may still be calculable in this strong-coupling limit. This happens at points of enhanced symmetry, where the gauge group is enhanced and the classical Kahler potential becomes singular. Quantum mechanically, however, certain non-perturbative degrees of freedom become massless to saturate global anomalies. In terms of these degrees of freedom, the Kahler potential is canonical, provided the quantum theory is infrared-free. Identification of the correct non-perturbative degrees of freedom and the correct superpotential then gives one control over the theory. Seiberg duality, which is used extensively in this thesis, occurs in such a context.

Certain general conditions for supersymmetry breaking have emerged over the years. Most of them have also been circumvented, but only in non-generic scenarios. These conditions are that the model: (1) have chiral matter (2) have a non-perturbative superpotential generated over the moduli space (3) have a tree-level superpotential that lifts the classical moduli space (4) have an $U(1)_R$ symmetry. The reasons one needs these conditions and the exceptions are discussed below.

For theories with vector-like matter, the massive fields may be integrated out so that one is left with pure super Yang-Mills at low energy. However, super Yang-Mills has supersymmetric vacua. That means that the original theory one began with has supersymmetric vacua, since the Witten index cannot change between the two regimes. Thus, vector theories are ruled out in the context of supersymmetry breaking. The exception comes in theories where the supersymmetric vacua reside on the boundary of field space, where some fields may blow up. Then the counting of the Witten index is more subtle.

Supersymmetry breaking requires a non-zero potential, and typically this is provided by a non-perturbative contribution coming from instantons or gaugino condensation. By itself, however, such a term leads to runaways in the moduli space. Thus, a tree level supoerpotential that lifts flat directions is also needed. Generating a dynamical superpotential puts constraints on the matter - large matter representations do not work.

Supersymmetry may be broken without a dynamically generated superpotential in a number of ways, all of which rely on non-perturbative physics. One is confinement - as mentioned before, at singular points on the moduli space the Kahler potential may be canonical in terms of certain non-perturbative degrees of freedom. While the original superpotential may not have had linear terms, the one in terms of the new degrees of freedom may. This leads to a tree-level breaking in terms of confined degrees of freedom. Another is supersymmetry breaking by quantum deformation of the moduli space. This proceeds as follows. The original theory has a supersymmetric vacuum at the origin, but quantum effects deform the moduli space away from the origin, thus breaking supersymmetry.

The connection between R-symmetry and supersymmetry breaking is as follows [9]. For n

fields, the equation $F_\phi = \partial_\phi W = 0$, amount to n equations in n unknowns. Generically this system has a solution, and thus supersymmetry is unbroken. However, if the theory has a $U(1)_R$ symmetry (under which the superpotential has charge 2) and there is a field ϕ_i with non-zero R-charge which has a non-zero expectation value, one can redefine the superpotential by

$$W = \phi_i^{2/R_i} W(\phi_j / \phi_i^{R_j/R_i}). \tag{1.2.2}$$

We then have n equations in only $n-1$ unknowns. Generically this system does not have a solution, and supersymmetry is broken. A non-R-symmetry, on the other hand, would reduce both the number of equations and unknowns.

1.3 Metastable Vacua and SQCD

One of the main paradigm shifts of recent times in this context has been to focus on metastable supersymmetry breaking. This method circumvents the stringent conditions mentioned above, without appealing to subtle and non-calculable models. For example, one often deals with theories with vector matter and R-symmetry breaking terms. The supersymmetric vacua lie far away in field space, and come in as R-symmetry is restored or the theory becomes massless.

Metastable breaking is generic, and has been found in the simple theory of Supersymmetric QCD (SQCD). SQCD is also the setting of Seiberg duality. The main features of SQCD are given below [15], [14], [13], [12], [11], [10].

SQCD is a theory with gauge group $SU(N_c)$ and global symmetry $SU(N_f)$. Various interesting scenarios arise as one varies the number of flavors N_f. These have been elucidated in the last twenty years.

We consider N_f $(< N_c)$ flavors of matter fields in the fundamental Q and antifundamental

$\overline{Q}$ representations:

$$
\begin{array}{cccccc}
 & SU(N_f)_L & \times\ SU(N_f)_R & \times\ U(1)_B & \times\ U(1)_R \\
Q & N_f & 1 & 1 & \frac{N_f-N_c}{N_f} \\
\overline{Q} & 1 & \overline{N}_f & -1 & \frac{N_f-N_c}{N_f}
\end{array}
\tag{1.3.1}
$$

The D-flat directions can be parameterized by the vev's of the gauge invariant mesons $M_{ij} = Q_i\overline{Q}_j$. These degrees of freedom give a weakly coupled description near the origin of moduli space where the theory confines.

In this model a unique nonperturbative superpotential called the Affleck-Dine-Seiberg superpotential is allowed by the symmetries

$$
W_{dyn} = \left(\frac{\Lambda^{3N_c-N_f}}{\det(Q\overline{Q})}\right)^{\frac{1}{N_c-N_f}} \ , \tag{1.3.2}
$$

where Λ is the renormalization group invariant scale of the theory. It can be proved that this superpotential is in fact generated by instanton effects for $N_f = N_c - 1$, and gaugino condensation in all other cases. The classical flat directions are lifted by adding a mass term to the superpotential

$$
W_{tree} = m_{ij}Q_i\overline{Q}_j \ . \tag{1.3.3}
$$

This superpotential explicitly breaks the $U(1)_R$ symmetry, a signal of unbroken supersymmetry, as argued previously. In terms of the meson fields the supersymmetric vacua are given by

$$
M_{ij} = \left(\det(m)\Lambda^{3N_c-N_f}\right)^{1/N_c}\left(\frac{1}{m_{ij}}\right) \ . \tag{1.3.4}
$$

For $N_f = N_c$, the flat directions can be parameterized by fields with the quantum numbers of baryons $B = Q^N$ and antibaryons $= \overline{Q}^N$. In these equations, flavor and gauge indices are understood to have been contracted. Bose statistics of the superfields force the gauge invariant polynomials to obey the following classical constraint

$$
\det M - B = 0 \ . \tag{1.3.5}
$$

Seiberg showed that this constraint is modified quantum mechanically to

$$\det(M) - B = \Lambda^{2N} \ . \tag{1.3.6}$$

To enforce this quantum mechanical constraint one may introduce a Lagrange multiplier term in the superpotential

$$W = A(\det(M) - B - \Lambda^{2N}) + m_{ij}M_{ij} \ . \tag{1.3.7}$$

The validity of this superpotential may be verified by holomorphic decoupling - some of the matter fields may be made heavy and decoupled from the low energy theory, and integrating them out leads to the Affleck-Dine-Seiberg superpotential.

Classically, the Kahler potential of the $N_f = N_c$ theory is singular at the origin. This corresponds to the fact that the full gauge group $SU(N_c)$ is restored at the origin and additional degrees of freedom become massless. The origin is an enhanced symmetry point. The quantum moduli space, in contrast, is smooth since the singularity is removed by the constraint. The Kahler potential in terms of confined degrees of freedom is non-singular. In the infrared regime mesons and baryons give a good description of the theory - a nontrivial check of that is the 't Hooft anomaly matching conditions, which is frequently used in this context. Far from the origin the quantum moduli space asymptotes to the classical one and the elementary quarks give a weakly coupled description of the theory.

In the case of $N_f = N_c + 1$, there are N_f baryons and antibaryons transforming under the global $SU(N_f)_L \times SU(N_f)_R$ as $(N_f, 1)$ and $(1, \bar{N}_f)$ respectively. Classically, the baryons and mesons obey

$$\begin{gathered} \det(M) - B_i M_{ijj} = 0 \ , \\ B_i M_{ij} = M_{ijj} = 0 \ . \end{gathered} \tag{1.3.8}$$

In contrast to the previous case, these constraints are not modified quantum mechanically. By addig a mass term to the superpotential, one can show that the meson and baryon vevs can take any values on the moduli space.

In terms of the quarks, the Kahler potential is singular at the origin. In terms of confined degrees of freedom the Kahler potential is regular, giving a weakly coupled description of the theory. As before, 't Hooft anomaly matching conditions are satisfied by this description. The constraints can be implemented by the superpotential

$$W = \frac{1}{2N_c - 1}\left(B_i M_{ijj} - \det M\right) \ . \tag{1.3.9}$$

Holomorphic decoupling leads to the $N_f = N_c$ superpotential.

We next consider the case $N_f > N_c + 1$, which is the setting of Seiberg duality. We start from the case $\frac{3}{2}N_c < N_f < 3N_c$. The theory flows to an infrared fixed point. Seiberg provided a description of the theory near the fixed point in terms of a dual "magnetic" theory. The global symmetries, being physical, remain the same between the original "electric" description and the dual magnetic one. The gauge group of the dual is different, however. The dual theory has gauge group $SU(N_f - N_c)$ with N_f flavors of q and $\overline{q}$ transforming as fundamentals and antifundamentals respectively, as well as gauge-singlet fields M, corresponding to the mesons of the original ("electric") theory. The q and $\overline{q}$ are called magnetic quarks, while the gauge singlet M is an entirely new field. The global-symmetry charges are given by

$$\begin{array}{ccccccccc} & SU(N_f)_L & \times & SU(N_f)_R & \times & U(1)_B & \times & U(1)_R \\ q & \overline{N}_f & & 1 & & \frac{N_c}{N_f - N_c} & & \frac{N_c}{N_f} \\ \overline{q} & 1 & & N_f & & -\frac{N_c}{N_f - N_c} & & \frac{N_c}{N_f} \\ M & N_f & & \overline{N}_f & & 0 & & 2\frac{N_f - N_c}{N_c} \end{array} \tag{1.3.10}$$

The magnetic theory also flows to a fixed point. In the magnetic theory a tree level superpotential is allowed by symmetries

$$W = Mq\overline{q}. \tag{1.3.11}$$

The dictionary between the electric and magnetic theories is

$$\begin{aligned} M_{ij} = Q_i\overline{Q}_j \quad &\rightarrow \quad M_{ij} \ , \\ W = m_{ij}M_{ij} \quad &\rightarrow \quad W = m_{ij}M_{ij} + M_{ij}q_i\overline{q}_j \ , \\ b,\overline{b} \quad &\rightarrow \quad B,\overline{B} \ . \end{aligned} \tag{1.3.12}$$

The scales of the electric and the magnetic theories are related by

$$\Lambda^{3N_c-N_f}\tilde{\Lambda}^{3(N_f-N_c)-N_f} = (-1)^{N_f-N_c}\mu^{N_f} \ , \tag{1.3.13}$$

where the scale μ is needed to map the composite electric meson $Q\overline{Q}$ into an elementary magnetic meson M by dimensional analysis. The electric meson and the magnetic gauge singlet have the same dimension at the infrared fixed point, but different dimensions in the ultraviolet.

For $N_c + 1 < N_f < \frac{3}{2}N_c$ a similar dual description holds. The electric description is asymptotically free while the magnetic dual is infrared free.

In Chapter 2, we construct a model of metastable supersymmetry breaking in an SQCD-like theory, with various desirable phenomenological features.

1.4 Supersymmetry in String Theory: A Top-down Approach

We have outlined some of the bottom-up reasons for studying supersymmetry. It may be noted that while supersymmetry solves sweeping fundamental issues like the hierarchy problem and gauge coupling unification, it has more localized model-building problems, like reproducing precision electroweak measurements and the Higgs mass of $m_H \geq 113 GeV$. Thus, models built just for the hierarchy problem, which are less ambitious and "fundamental" in character, are also in the running as of now, as is the original "desert" scenario. Nevertheless, the bottom-up approach has motivated research in supersymmetry for over two decades.

It is possible to look at supersymmetry in a different way - the top-down approach, from the viewpoint of string theory. In this case, it emerges as a simplifying calculational tool. In many compactifications, supersymmetry greatly simplifies the computation of the four dimensional effective Lagrangian, since powerful physical and mathematical tools can be used. Supersymmetry also makes it far easier to prove whether a given vacuum is a local minimum of the potential. Essentially, all one has to show is that $|M_{Fermi}| >> M_{SUSY}$, where M_{SUSY} is an energy scale related to the scale of supersymmetry breaking. The calculational motivations for supersymmetry in the top-down approach mentioned above do not have the hierarchy issue built into them. Does string theory then predict low scale supersymmetry?

To address this question, one has to introduce the concept of stringy naturalness, as opposed to usual naturalness (avoiding fine-tuning) which serves as a phenomenological motivation to low-scale supersymmetry. Stringy naturalness is based on distributions of vacua with different properties in the string landscape. Many classes of string vacua have been considered which break supersymmetry at a high scale. One must thus ask the question whether these classes of vacua are more natural, that is, entropically favored over ones which break supersymmetry at low scale. The question has been addressed statistically [164], [166], [162]. The manner in which one proceeds is as follows. The set of string vacua is taken to consist of elements with label i. To each vacuum i, a probability $P(i)$ is associated, giving the probability that the vacuum was produced by an early cosmology theory. The supersymmetry breaking scale associated with i is given the label F_i, while the electroweak scale for that vacuum is called $M_{EW,i}$. Then, a joint probability distribution may be defined over the landscape. For $M_{EW} \sim 100$ and TeV scale supersymmetry, we obtain the probability that supersymmetry be discovered at the LHC to be

$$P_{susy} = \sum_{F_i \leq F_{exp}, M_{EW,i}=100} P(i). \tag{1.4.1}$$

A high probability would mean that supersymmetry has been derived from a top-down approach, in the sense that it is natural from a string point of view. The current status on this, and one of the most important results about compactifications and supersymmetry, is that TeV scale supersymmetry is not an inevitable prediction of string theory.

1.5 Four Dimensional Supergravity, Flux Vacua, and the Landscape

We now turn to the actual implementation of supersymmetry and its breaking within string theory. The details are very model-specific, and we consider one such model in Chapter 3. However, certain general theoretical frameworks form the foundations of all such efforts. We refer to [19] and references therein.

We consider a compactification which has our four-dimensional world times a six-dimensional manifold called M. Demanding that our compactification preserve $d = 4, \mathcal{N} = 1$ supersymmetry implies the existence of covariantly constant spinors on M, which is determined by its holonomy group $Hol(M)$. The number of supersymmetries in $d = 4$ is equal to the number of supercharges in the higher dimensional theory, divided by 16, and multiplied by the number of singlets in the decomposition of $\mathbf{4}$ of $SO(6)$ under $Hol(M)$. In general, $Hol(M) \cong SO(6)$ but then that gives zero supersymmetries in $d = 4$. Thus, one needs $Hol(M) \subset SO(6)$, that is, the manifold should have special holonomy.

All possible special holonomy groups have been classified. For dim $M = 6$, the special holonomy groups are $U(3)$ and $SU(3)$, and their subgroups. The only choice of $Hol(M)$ for which the spinor of $SO(6)$ contains a unique singlet is $SU(3)$. Spaces which admit a metric with this special holonomy are known as Calabi-Yau manifolds. One is left with $\mathcal{N} = 2$ supersymmetric theories, called Type II theories. To get to $\mathcal{N} = 1$ from here seemed insurmountable before the discovery of D-branes.

As illustrated below, compactifications on Calabi-Yau's are plagued by the so-called moduli problem. Moduli are four-dimensional scalar fields which correspond to Kahler and complex structure deformations of the Calabi-Yau. It is necessary to give them a potential to prevent unobserved long-range forces. One way of doing so is by turning on fluxes, and the framework is called flux compactification. The resulting theory is a four-dimensional supergravity theory with moduli stabilization, endowed with a definite Kahler potential and a flux-induced superpotential. This supergravity theory gives a scalar potential; critical points of this potential are the string vacua. By changing the parameters of the theory, such as the number of flux quanta, one

Cohomology group	basis	
$H^{(1,1)}$	w_a	$a = 1, .., h^{(1,1)}$
$H^{(0)} \oplus H^{(1,1)}$	$w_A = (1, w_a)$	$A = 0, .., h^{(1,1)}$
$H^{(2,2)}$	$\tilde{w}^a$	$a = 1, .., h^{(1,1)}$
$H^{(2,1)}$	χ_k	$k = 1, .., h^{(2,1)}$
$H^{(3)}$	(α_K, β^K)	$K = 0, .., h^{(2,1)}$

Table 1.3: Basis of harmonic forms in a Calabi–Yau manifold.

obtains a landscape of such vacua, each with different properties, for example SUSY-breaking scale. General predictions may then be gleaned from the landscape, as outlined in the previous section. Below we give some details of the whole procedure.

To obtain the four dimensional effective theory, one performs a Kaluza-Klein (KK)reduction of the ten-dimensional type II supergravities on an internal manifold, keeping only the massless modes. The massless modes for each supergravity field (metric g, dilaton ϕ and B-field B_2 in the NS sector, and RR potentials C_n in the RR sector) correspond to harmonic forms on the internal manifold.

With no fluxes turned on, the four-dimensional effective corresponds to an $= 2$ ungauged supergravity, whose matter content depends on whether we are in type IIA or type IIB.

The Hodge diamond of a Calabi-Yau contains one harmonic 0-form (a constant), one (3,0)-form Ω, one (0,3)-form $\bar{\Omega}$, and one (3,3)-form, the volume. Additionally, there are $h^{(1,1)}$ harmonic (1,1) and (2,2)-forms and $h^{(2,1)}$ harmonic (2,1) and (1,2) forms. The total number of harmonic 3-forms is $2h^{(2,1)} + 2$. There are no harmonic 1 and 5-forms. Table 1.3 gives a basis of harmonic forms.

The following are the expansions for the deformations of the fields in the NS sector. The

gravity multiplet	1	$(g_{\mu\nu}, C_1^0)$
vector multiplets	$h^{(1,1)}$	(C_1^a, v^a, b^a)
hypermultiplets	$h^{(2,1)}$	$(z^k, \xi^k, \tilde{\xi}_k)$
tensor multiplet	1	$(B_2, \phi, \xi^0, \tilde{\xi}_0)$

Table 1.4: Type IIA moduli arranged in $\mathcal{N} = 2$ multiplets.

fields are functions of space-time x and internal manifold coordinates y.

$$\phi(x,y) = \phi(x)\,, \tag{1.5.1}$$

$$g_i(x,y) = iv^a(x)(\omega_a)_i(y)\,, \qquad g_{ij}(x,y) = i\bar{z}^k(x)\left(\frac{(\bar{\chi}_k)_{i\bar{k}\bar{l}}\,\Omega^{\bar{k}\bar{l}}{}_j}{|\Omega|^2}\right)(y)\,, \tag{1.5.2}$$

$$B_2(x,y) = B_2(x) + b^a(x)\omega_a(y)\,. \tag{1.5.3}$$

All the x-dependent fields are the moduli of the 4D theory. In the NS sector we get a total of $2\,(h^{(1,1)}+1) + h^{(2,1)}$ moduli.

In the RR sector, we perform the following expansions

$$C_1(x,y) = C_1^0(x)\,, \tag{1.5.4}$$

$$C_3(x,y) = C_1^a(x)\,\omega_a(y) + \xi^K(x)\alpha_K(y) - \tilde{\xi}_K(x)\beta^K(y) \tag{1.5.5}$$

for type IIA, and

$$C_0(x,y) = C_0(x)\,, \tag{1.5.6}$$

$$C_2(x,y) = C_2(x) + c^a(x)\omega_a(y)\,, \tag{1.5.7}$$

$$C_4(x,y) = V_1^K(x)\,\alpha_K(y) + \rho_a(x)\tilde{\omega}^a(y) \tag{1.5.8}$$

for type IIB. These moduli arrange into the $= 2$ multiplets shown in Tables 1.4 and 1.5.

Inserting the above expansions in the ten-dimensional actions and integrating over the Calabi-Yau, one obtains a standard four-dimensional $\mathcal{N} = 2$ ungauged supergravity action. An orientifold projection onto invariant fields gives $\mathcal{N} = 1$. The Kahler potential for type IIA is given by

gravity multiplet	1	$(g_{\mu\nu}, V_1^0)$
vector multiplets	$h^{(2,1)}$	(V_1^k, z^k)
hypermultiplets	$h^{(1,1)}$	(v^a, b^a, c^a, ρ_a)
tensor multiplet	1	(B_2, C_2, ϕ, C_0)

Table 1.5: Type IIB moduli arranged in $\mathcal{N} = 2$ multiplets.

$$K_{=} - \ln\left[\frac{4}{3}\int J \wedge J \wedge J\right] = -\ln\left[\tfrac{i}{6}\mathcal{K}_{abc}(t-\bar{t})^a(t-\bar{t})^b(t-\bar{t})^c\right] = -\ln \tfrac{4}{3}\mathcal{K}\ , \tag{1.5.9}$$

where $\mathcal{K}$ is six times the volume of the Calabi-Yau manifold, and $\mathcal{K}_{abc}$ are the intersection numbers defined by

$$\begin{aligned} \mathcal{K}_{abc} &= \int \omega_a \wedge \omega_b \wedge \omega_c\ , \qquad & \mathcal{K}_{ab} = \int \omega_a \wedge \omega_b \wedge J = \mathcal{K}_{abc}v^c \\ \mathcal{K}_a &= \int \omega_a \wedge J \wedge J = \mathcal{K}_{abc}v^b v^c\ , \qquad & \mathcal{K} = \int J \wedge J \wedge J = \mathcal{K}_{abc}v^a v^b v^c\ . \end{aligned} \tag{1.5.10}$$

Here, t is given by the complexified Kahler deformations

$$B + iJ = (b^a + iv^a)\,\omega_a \equiv t^a\,\omega_a\ . \tag{1.5.11}$$

In type IIB, the Kahler potential is given by

$$K = -\ln\left[i\int \Omega \wedge \bar{\Omega}\right] \tag{1.5.12}$$

In type IIB, the flux induced superpotential, given by Gukov, Vafa, and Witten, is

$$W_{O3/O7} = \int G_3 \wedge \Omega. \tag{1.5.13}$$

The supergravity potential V is given in terms of the superpotential W, the Kahler potential, and the D-terms D_α by

$$V = e^K\left(K^{I\bar{J}}D_I W D_{\bar{J}}\bar{W} - 3|W|^2\right) + \tfrac{1}{2}\,(\mathrm{Re}\,f)^{-1\ \alpha\beta}D_\alpha D_\beta\ , \tag{1.5.14}$$

where covariant derivatives are defined as

$$D_I W = \partial_I W + W\partial_I K\,. \tag{1.5.15}$$

Critical points of this potential correspond to a landscape of vacua.

1.6 Outlines of Specific Models

In Chapter 2, we consider supersymmetry breaking in gauge theories. The work is done within the context of metastable breaking, which, as mentioned in section 1.3, is generic and evades the "classic" constraints of supersymmetry breaking. The setting is SQCD and Seiberg duality. Specifically, we construct a model with long-lived metastable vacua in which all the relevant parameters, including the supersymmetry breaking scale, are generated dynamically by dimensional transmutation. Our model consists of two sectors coupled by a singlet and combines dynamical supersymmetry breaking with an O'Raifeartaigh mechanism in terms of confined variables. The metastable vacua appear along a pseudo-runaway direction near a point of enhanced symmetry as a result of a balance between non-perturbative and perturbative quantum effects. We show that metastable supersymmetry breaking is a rather generic feature near certain enhanced symmetry points of gauge theory moduli spaces.

In Chapter 3, we consider a scenario of moduli stabilization and non-supersymmetric in a string theory setting. We explore the dynamics of magnetized nonsupersymmetric D5-brane configurations on Calabi-Yau orientifolds with fluxes. We show that supergravity D-terms capture supersymmetry breaking effects predicted by more abstract Pi-stability considerations. We also examine superpotential interactions in the presence of fluxes, and investigate the vacuum structure of such configurations. Based on the shape of the potential, we argue that metastable nonsupersymmetric vacua can be in principle obtained by tuning the values of fluxes.

In Chapter 4, we develop mathematical tools to find the tree-level superpotential for D-brane configurations on Calabi-Yau orientifolds. Our method is based on a systematic implementation of the orientifold projection in the geometric approach of Aspinwall and Katz. In the process we lay down some ground rules for orientifold projections in the derived category.

In Chapter 5, we study the supersymmetry breaking runaway quiver in string embeddings. Calculations are performed in four dimensional effective supergravity. Constraints on closed string fields in a type IIA construction are given. The particular case of stabilization by stringy instanton effects in a type IIB model is considered.

Chapter 2

Metastable Dynamical Supersymmetry Breaking near Points of Enhanced Symmetry

2.1 Introduction

The idea that our universe may be in a long-lived metastable state in which supersymmetry is broken has recently led to an increased interest in developing models of supersymmetry breaking. This has opened many new possibilities in constructing field theory and string theory models.

On the field theoretic side, the work of Intriligator, Seiberg and Shih (ISS) [20] presented calculable metastable vacua using Seiberg duality. This motivated related field theory constructions, involving gauge mediation [21, 22, 23, 243], generalized O'Raifeartaigh models [25], retrofitting [26], adjoint matter [27], applications to particle physics [28, 29, 30], etc. Similar developments have been seen in string theory based on a number of different tools, such as intersecting or wrapping branes [31, 32, 33, 34], flux compactifications [35, 276, 37, 38, 39], Calabi-Yau's with particular geometric properties [275, 41, 42, 261], IIa/M-theory configurations [277, 45, 46] and others. Statistical analyses of the supersymmetry breaking scale on the landscape of effective field theories were done, for instance, in [164, 165, 49].

The ISS model consists of supersymmetric QCD (SQCD) in the free magnetic range, and metastable vacua appear after taking into account one-loop corrections that lift the pseudo-moduli. Their work suggests that nonsupersymmetric vacua are rather generic, if one requires them to be only local, rather than global, minima of the potential. The construction still contained relevant couplings in the form of masses for the quarks though, and the search for models with all the relevant parameters generated dynamically has proven difficult; see [50, 51, 52, 256] for recent work in this direction.

One lesson from ISS is that certain properties of moduli spaces can hint at the existence of metastable vacua. In their case, it was the existence of supersymmetric vacua coming in from infinity that signaled an approximate R-symmetry. Here we will point out that one should also look for another feature, namely, enhanced symmetry points, which are defined by the appearance of massless particles. We claim that if the moduli space has certain coincident enhanced symmetry points, metastable vacua with all the relevant couplings arising by dimensional transmutation may be obtained.

Let us motivate this claim. In order to generate relevant couplings dynamically, a gauge sector is required, which gives nonperturbative contributions to the superpotential. However, in general this leads to a runaway behavior. We will show that starting with two gauge sectors, the runaway may now be stabilized by one loop effects from the additional gauge sector, but only around enhanced symmetry points where quantum corrections are large enough. Such runaways which are stabilized by perturbative quantum corrections will be called 'pseudo-runaways'. Surprisingly, the gauge theories where this occurs turn out to be generic.

The model considered here consists of two SQCD sectors, each with independent rank and number of flavors, coupled by a singlet. It involves only marginal operators with all scales generated dynamically. At the origin of moduli space, the singlet vanishes and the quarks of both sectors become massless simultaneously. There are thus two coincident enhanced symmetry points at the origin. While one of the SQCD sectors is in the electric range and produces a runaway, the other has a magnetic dual description as an O'Raifeartaigh-like model. Near the enhanced symmetry point, the Coleman-Weinberg corrections stabilize the nonperturbative instability producing a long-lived metastable vacuum. A feature of our model is that it may be possible to gauge parts of its large global symmetry to obtain renormalizable, natural models of direct gauge mediated supersymmetry breaking with a singlet. R-symmetry is broken both spontaneously and explicitly in our model.

The plan of this chapter is as follows. In Section 2, our model is introduced and its supersymmetric vacua are studied. In Section 3, we analyze in detail the non-supersymmetric vacua and argue that they are parametrically long-lived. In Section 4, we give a detailed analysis of the particle spectrum and the R-symmetry properties. In Section 5, we argue that such metastable vacua may be generic near points of enhanced symmetry in the landscape of effective field theories. In Section 6, we give our conclusions.

2.2 The Model and its Supersymmetric Vacua

We consider models with two supersymmetric QCD (SQCD) sectors characterized by (N_c, N_f, Λ) and (N_c', N_f', Λ'), respectively, that are coupled to the same singlet field Φ. The field Φ provides the mass of the quarks in both sectors. In Section 2.1, the general properties of such models will be discussed and their global symmetries analyzed. In Section 2.2, we analyze the supersymmetric vacua. Section 2.3 will discuss for which range of the parameters (N_c, N_f, Λ) and (N_c', N_f', Λ') metastable vacua will be shown to exist. The upshot will be that one sector has to be taken in the electric range and the other sector in the free magnetic range.

2.2.1 Description of the Model

The matter content of the models considered here consists of two copies of supersymmetric QCD, each with independent rank and number of flavors, and a single gauge singlet chiral superfield:

$$\begin{array}{cccl} & SU(N_c) & SU(N_c') & \\ Q_i & \square & 1 & i=1,\ldots,N_f \\ \overline{Q}_i & \overline{\square} & 1 & \\ P_{i'} & 1 & \square & i'=1,\ldots,N_f' \\ \overline{P}_{i'} & 1 & \overline{\square} & \\ \Phi & 1 & 1 & \end{array} \tag{2.2.1}$$

The most general tree-level superpotential with only relevant or marginal terms in four dimensions for the matter content (2.2.1) with N_c, $N_c' \geq 4$ is

$$W = (\lambda_{ij}\Phi + \xi_{ij})Q_i\overline{Q}_j + (\lambda'_{i'j'}\Phi + \xi'_{i'j'})P_{i'}\overline{P}_{j'} + w(\Phi)\,, \tag{2.2.2}$$

where $w(\Phi)$ is a cubic polynomial in Φ. Remarkably, we shall find metastable vacua even in the simplest case of $w(\Phi) = 0$, which we assume from now on. The general situation is discussed in Section 5 (in [52], the case $w(\Phi) = \kappa\Phi^3$ was used to stabilize Φ supersymmetrically).

At the classical level, the superpotential with $w(\Phi) = 0$ has an $U(1)_R \times U(1)_V \times U(1)'_V$

global symmetry under which the fields transform as

$$
\begin{array}{cccc}
 & U(1)_R & U(1)_V & U(1)'_V \\
 & & & \\
Q_i & +1 & +1 & 0 \\
\overline{Q}_i & +1 & -1 & 0 \\
P_{i'} & +1 & 0 & +1 \\
\overline{P}_{i'} & +1 & 0 & -1 \\
\Phi & 0 & 0 & 0 \\
\Lambda^{3N_c-N_f} & 2N_c & 0 & 0 \\
\Lambda'^{3N'_c-N'_f} & 2N'_c & 0 & 0
\end{array}
\tag{2.2.3}
$$

where the normalizations of the $U(1)_V \times U(1)'_V$ charges are arbitrary. In the quantum theory the $U(1)_R$ symmetry is anomalous with respect to the $SU(N_c)$ and $SU(N'_c)$ gauge dynamics. The theta angles θ and θ' transform inhomogenously under $U(1)_R$, and the holomorphic dynamical scale,

$$
(\Lambda/\mu)^{3N_c-N_f} = e^{-8\pi^2/g^2(\mu)+i\theta}\,, \tag{2.2.4}
$$

and likewise for $\Lambda'^{3N'_c-N'_f}$, transform with charges given in (2.2.3). The $U(1)_R$ symmetry is broken explictly by the anomalies to the anomaly free discrete subgroups $Z_{2N_c} \subset U(1)_R$ and $Z_{2N'_c} \subset U(1)_R$, respectively. The largest simultaneous subgroup of both Z_{2N_c} and $Z_{2N'_c}$ which is left invariant by the superpotential (2.2.2) which couples the two gauge sectors through Φ interactions is $Z_{\mathrm{GCD}(2N_c,2N'_c)} \subset U(1)_R$, where $\mathrm{GCD}(2N_c, 2N'_c)$ is the greatest common divisor of $2N_c$ and $2N'_c$.

In the $SU(N_f)_V \times SU(N'_f)_V$ global symmetry limit the superpotential (2.2.2) (with $w(\Phi) =$

0) reduces to

$$W = (\lambda\Phi + \xi)\mathrm{tr}(Q\overline{Q}) + (\lambda'\Phi + \xi')\mathrm{tr}(P\overline{P})\,. \tag{2.2.5}$$

This superpotential has the same $U(1)_R \times U(1)_V \times U(1)'_V$ global symmetry as (2.2.2), as well as a $Z_2 \times Z_2$ conjugation symmetry under which $Q_i \leftrightarrow \overline{Q}_i$ and $P_i \leftrightarrow \overline{P}_i$, respectively. The form of the superpotential (2.2.5) may be enforced for any N_c and N'_c by weakly gauging the $SU(N_f)_V \times SU(N'_f)_V$ symmetry. One of the masses, ξ or ξ', may always be absorbed into a shift of Φ. For $\xi = \xi'$ both masses may simultaneously be absorbed into a shift of Φ, and the tree level superpotential in this case reduces to

$$W = \lambda\Phi\ \mathrm{tr}(Q\overline{Q}) + \lambda'\Phi\ \mathrm{tr}(P\overline{P})\,. \tag{2.2.6}$$

This form agrees with the naturalness requirement that there be no relevant couplings. $\Phi = 0$ is an enhanced symmetry point for both sectors, where the respective quarks become massless. The case $\xi \neq \xi'$ is analyzed in Section 5.

At the classical level this superpotential has an $U(1)_R \times U(1)_A \times U(1)_V \times U(1)'_V$ global symmetry

$$\begin{array}{ccccc} & U(1)_R & U(1)_A & U(1)_V & U(1)'_V \\ Q_i & +1 & -\frac{1}{2} & +1 & 0 \\ \overline{Q}_i & +1 & -\frac{1}{2} & -1 & 0 \\ P_{i'} & +1 & -\frac{1}{2} & 0 & +1 \\ \overline{P}_{i'} & +1 & -\frac{1}{2} & 0 & -1 \\ \Phi & 0 & +1 & 0 & 0 \\ \Lambda^{3N_c-N_f} & 2N_c & -N_f & 0 & 0 \\ \Lambda'^{3N'_c-N'_f} & 2N'_c & -N'_f & 0 & 0 \end{array} \tag{2.2.7}$$

where the normalizations of the $U(1)_A \times U(1)_V \times U(1)'_V$ charges are arbitrary. The $U(1)_R$ charges are only defined up to an addition of an arbitrary multiple of the $U(1)_A$ charges. In the quantum theory both the $U(1)_R$ and $U(1)_A$ symmetries are anomalous. With the classical charge assignments (2.2.7) the $U(1)_R$ symmetry is broken explictly by the $SU(N_c)$ and $SU(N'_c)$ gauge dynamics to the anomaly free discrete subgroup $Z_{\mathrm{GCD}(2N_c,2N'_c)} \subset U(1)_R$ as described above. Likewise, the $U(1)_A$ symmetry is broken explicitly by $SU(N_c)$ and $SU(N'_c)$ gauge dynamics to anomaly free discrete subgroups $Z_{N_f} \subset U(1)_A$ and $Z_{N'_f} \subset U(1)_A$, respectively. The largest simultaneous subgroup of both Z_{N_f} and $Z_{N'_f}$ which is left invariant by the superpotential (2.2.6) is $Z_{\mathrm{GCD}(N_f,N'_f)} \subset U(1)_A$. The form of the potential (2.2.6) may be enforced by gauging the non-anomalous discrete $Z_{\mathrm{GCD}(N_f,N'_f)}$ symmetry if it is non-trivial, along with weakly gauging the $SU(N_f)_V \times SU(N'_f)_V$ symmetry. This forbids the presence of a polynomial dependence

$w(\Phi)$.

The marginal tree-level superpotential (2.2.6) is, up to irrelevant terms, of rather generic form within many UV completions of theories with moduli dependent masses. It requires only that the masses of the flavors of both gauge groups are moduli dependent functions, and that all flavors become massless at a single point in moduli space, here defined to be $\Phi = 0$. Importantly for the discussion of metastable dynamical supersymmetry breaking below, the superpotential (2.2.6) contains only marginal terms, so that any relevant mass scales must arise from dimensional transmutation. Generalizations to other gauge groups and matter contents in vector-like representations with the superpotential (2.2.6) are straightforward.

The classical moduli space for the theory (2.2.1) with superpotential (2.2.6) depends on the gauge group ranks and number of flavors. For $\lambda = \lambda' = 0$ the moduli space is parameterized by Φ, meson invariants $M_{ij} = Q_i\overline{Q}_j$ and $M'_{i'j'} = P_{i'}\bar{P}_{j'}$ and for $N_f \geq N_c$ and/or $N'_f \geq N'_c$ baryon and anti-baryon invariants $B_{i_1 i_2 \ldots i_{N_c}} = Q_{[i_1}Q_{i_2}\cdots Q_{i_{N_c}]}$, $\overline{B}_{i_1 i_2 \ldots i_{N_c}} = \overline{Q}_{[i_1}\overline{Q}_{i_2}\cdots\overline{Q}_{i_{N_c}]}$, and/or $B'_{i_1 i_2 \ldots i_{N'_c}} = P_{[i_1}P_{i_2}\cdots P_{i_{N'_c}]}$, $\overline{B}'_{i_1 i_2 \ldots i_{N'_c}} = \bar{P}_{[i_1}\bar{P}_{i_2}\cdots\bar{P}_{i_{N'_c}]}$ respectively. For $\lambda, \lambda' \neq 0$ the superpotential (2.2.6) lifts all the moduli parameterized by the mesons. The remaining moduli space has a branch parameterized by Φ. For $\Phi \neq 0$ the flavors are massive and the baryon and anti-baryon directions are lifted along this branch. For $N_f \geq N_c$ and/or $N'_f \geq N'_c$ there is a second branch of the moduli space parameterized by the baryons and anti-baryons with $\Phi = 0$. The two branches touch at the point where all the moduli vanish.

2.2.2 Supersymmetric Vacua

The classical moduli space of vacua is lifted by nonperturbative effects in the quantum theory. Since the metastable supersymmetry breaking vacua discussed below arise for $\Phi \neq 0$, only this branch of the moduli space will be considered in detail. On this branch, holomorphy, symmetries, and limits fix the exact superpotential written in terms of invariants, to be

$$\begin{aligned} W &= \lambda\Phi \ \mathrm{Tr}M + (N_c - N_f)\left[\frac{\Lambda^{3N_c-N_f}}{\det\ M}\right]^{1/(N_c-N_f)} \\ &\quad + \lambda'\Phi \ \mathrm{Tr}M' + (N'_c - N'_f)\left[\frac{\Lambda'^{3N'_c-N'_f}}{\det\ M'}\right]^{1/(N'_c-N'_f)} \end{aligned} \tag{2.2.8}$$

For gauge sectors in the free magnetic range, the nonperturbative contribution refers to the Seiberg dual. Since the meson invariants are lifted on this branch, they may be eliminated by equations of motion, $\partial W/\partial M_{ij} = 0$ and $\partial W/\partial M'_{i'j'} = 0$, to give the exact superpotential in terms of the classical modulus Φ

$$W = N_c\left[(\lambda\Phi)^{N_f}\Lambda^{3N_c-N_f}\right]^{1/N_c} + N'_c\left[(\lambda'\Phi)^{N'_f}\Lambda'^{3N'_c-N'_f}\right]^{1/N'_c} . \tag{2.2.9}$$

The supersymmetric minima are given by stationary points of the superpotential, $\partial W/\partial\Phi = 0$, for which

$$N_f\left[(\lambda\Phi)^{N_f}\Lambda^{3N_c-N_f}\right]^{1/N_c} + N'_f\left[(\lambda'\Phi)^{N'_f}\Lambda'^{3N'_c-N'_f}\right]^{1/N'_c} = 0\,. \tag{2.2.10}$$

Physically distinct supersymmetric vacua are distinguished by the expectation value of the superpotential.

2.2.3 Parameter ranges for the gauge sectors

Under mild assumptions we thus end up considering two SQCD sectors, characterized by (N_c, N_f, Λ) and (N'_c, N'_f, Λ'), respectively, and superpotential couplings (2.2.6). Different choices may be considered here; to restrict them, it is important to note that calculable quantum corrections can be generated in two different limits.

For $\lambda_i\Phi \gg \Lambda_i$, with $\Lambda_i = \Lambda$ or Λ', the corresponding gauge group is weakly coupled and hence generates small calculable corrections to the Kähler potential. Integrating out the massive quarks, for energies below Φ, leads to gaugino condensation, which gives nonperturbative contributions as in (2.2.9).

On the other hand, for $\lambda_i\Phi \ll \Lambda_i$, the corresponding gauge sector becomes strongly coupled. The calculable case corresponds to having the gauge theory in the free magnetic range. For concreteness, we choose this sector to be $SU(N_c)$ (the unprimed sector), so that $N_c+1 \leq N_f < \frac{3}{2}N_c$.

For the (N_c', N_f', Λ') (primed) sector, the interesting case arises for $N_f' < N_c'$ and $\lambda'\Phi \gg \Lambda'$. Although the classical superpotential pushes Φ to zero, the primed dynamics generates a nonperturbative term which makes the potential energy diverge as $\Phi \to 0$, in agreement with the fact that $\Phi = 0$ corresponds to an enhanced symmetry point where P and $\bar{P}$ become massless. Balancing the primed and unprimed contributions leads to a runaway direction in moduli space which will be lifted by one loop corrections. This stabilizes Φ at a nonzero value. Calculability demands working in the energy range $E \gg \Lambda'$ and $E \ll \Lambda$ so the dynamically generated scales must satisfy $\Lambda' \ll \Lambda$.

The semiclassical limit corresponds to energies $E \gg \Lambda, \Lambda'$, where both sectors are weakly coupled. Since $\Lambda' \ll \Lambda$, $SU(N_c)$ confines first when flowing to the IR. For $\Lambda' \ll E \ll \Lambda$, the primed sector is weakly interacting while the unprimed sector has a dual weakly coupled description [54] in terms of the magnetic gauge group $SU(\tilde{N}_c)$ with $\tilde{N}_c = N_f - N_c$, N_f^2 singlets M_{ij}, and N_f magnetic quarks $(q_i,\, \tilde{q}_j)$. In terms of this description, the full non-perturbative superpotential reads

$$\begin{aligned} W &= m\Phi \mathrm{tr} M + h\mathrm{tr} qM\tilde{q} + \lambda'\Phi \mathrm{tr} P\bar{P} + (N_c' - N_f')\left(\frac{\Lambda'^{3N_c'-N_f'}}{\det P\bar{P}}\right)^{1/(N_c'-N_f')} \\ &\quad + (N_f - N_c)\left(\frac{\det M}{\tilde{\Lambda}^{3N_c-2N_f}}\right)^{1/(N_f-N_c)}. \end{aligned} \tag{2.2.11}$$

Hereafter, $M_{ij} = Q_i\bar{Q}_j/\Lambda$, and $m := \lambda\Lambda$. The magnetic sector has a Landau pole at $\tilde{\Lambda} = \Lambda$.

In this description, the meson M and the primed quarks $(P,\, \bar{P})$ become massless at $\Phi = 0$. $M = 0$ is also an enhanced symmetry point since here the magnetic quarks $(q,\, \tilde{q})$ become massless.

2.3 Metastability near enhanced symmetry points

In this section, metastable vacua near the origin of moduli space will be shown to exist for the theory with superpotential (2.2.11). In Section 3.1, we analyze the branches of the moduli space and determine where Coleman-Weinberg effects may lift the runaway. Next, in 3.2, we focus on the region containing metastable vacua. In 3.3, we argue that other quantum corrections are under control and do not affect the stability of these vacua. Finally, in Section 3.4 the metastable vacua are shown to be parametrically long-lived.

2.3.1 Exploring the moduli space

Starting from the superpotential (2.2.11), the discussion is simplified by taking the limit $\tilde{\Lambda} \to \infty$, while keeping m fixed. The nonperturbative $\det M$ term is only relevant for generating supersymmetric vacua, as discussed in (2.2.9), and not important for the details of the metastable vacua that will arise near $M = 0$. Thus, for $M/\tilde{\Lambda} \to 0$ and $\Phi/\tilde{\Lambda} \to 0$, it is enough to consider the superpotential

$$W = m\Phi \; \mathrm{tr} \; M + h \; \mathrm{tr} \; qM\tilde{q} + \lambda'\Phi \, \mathrm{tr} \, P\bar{P} + (N_c' - N_f') \left(\frac{\Lambda'^{3N_c'-N_f'}}{\det P\bar{P}} \right)^{1/(N_c'-N_f')} . \tag{2.3.1}$$

In this limit all the fields are canonically normalized and the classical potential is

$$V = V_D + V_D' + \sum_a |W_a|^2 \tag{2.3.2}$$

where $W_a = \partial_a W$, and a runs over all the fields. V_D and V_D' are the usual D-term contributions from $SU(\tilde{N}_c)$ and $SU(N_c')$. Since both gauge sectors are weakly coupled, it is enough to consider the F-terms on the D-flat moduli space, parametrized by the chiral ring. This restriction has no impact on the analysis of the metastable vacua.

Let us study the regime $P\bar{P} \to \infty$. Then nonperturbative effects from $SU(N_c')$ may be neglected, and the classical superpotential

$$W_{cl} = m\Phi \; \mathrm{tr} \; M + h \; \mathrm{tr} \; qM\tilde{q} + \lambda'\Phi \, \mathrm{tr} \; P\bar{P} \tag{2.3.3}$$

is recovered. Setting

$$W_{M_{ij}} = m\Phi\delta_{ij} + hq_i\tilde{q}_j = 0 \, , \tag{2.3.4}$$

we obtain $\Phi = 0$ and $hq\tilde{q} = 0$. This implies $W_{\mathrm{tr}P\bar{P}} = W_q = 0$. The locus $W_\Phi = 0$ then defines a classical moduli space of supersymmetric vacua.

Let us keep $P\bar{P}$ large, but include the non-perturbative effects from $SU(N_c')$. Then $W_{\mathrm{tr}P\bar{P}} = 0$ sets $P\bar{P} \to \infty$ and $W_\Phi = 0$ implies $M \to \infty$. Therefore the model does not have a stable vacuum in the limit $\tilde{\Lambda} \to \infty$. As discussed above, for $\tilde{\Lambda}$ finite and M large enough, the nonperturbative det$M$ term introduces supersymmetric vacua as in (2.2.9).

All the F-terms are small in the limit $M \to \infty$, $\Phi \to 0$, which thus corresponds to $M_F^2 \gg |F|$. The one-loop corrections give logarithmic dependences on the fields (Φ, M) and these cannot stop the power-law runaway behavior.

Thus we are led to consider the region near the enhanced symmetry point $M = 0$. As we shall see below, this still has a runaway. Crucially, it turns out that one-loop corrections stop this runaway (this novel effect is characterized as a "pseudo-runaway"). The reason for this is that the Coleman-Weinberg formula [55]

$$V_{CW} = \frac{1}{64\pi^2}\,\mathrm{Str}\,M^4\,\ln\,M^2 \tag{2.3.5}$$

will have polynomial (instead of logarithmic) dependence. This will be explained next.

A global plot of the potential is provided in Fig. 2.1, where M has been expanded around zero as below in equation (3.8). In the graphic, the 'drain' towards the supersymmetric vacuum corresponds to the curve $W_\Phi = 0$.

2.3.2 Metastability Along the Pseudo-Runaway Direction

In the region $\Phi \neq 0$, $(P, \bar{P})$ may be integrated out by equations of motion provided that $\Lambda' \ll \lambda'\Phi$. This is a good description if we are not exactly at the origin but near it, as given by $\Phi/\tilde{\Lambda} \ll 1$. Taking, as before, $\tilde{\Lambda} \to \infty$ and m fixed, the superpotential reads

$$W = m\Phi\,\mathrm{tr}\,M + h\,\mathrm{tr}\,qM\tilde{q} + N_c'\big[\lambda'^{N_f'}\,\Lambda'^{3N_c'-N_f'}\,\Phi^{N_f'}\big]^{1/N_c'}. \tag{2.3.6}$$

This description corresponds to an O'Raifeartaigh-type model in terms of magnetic variables but with no flat directions.

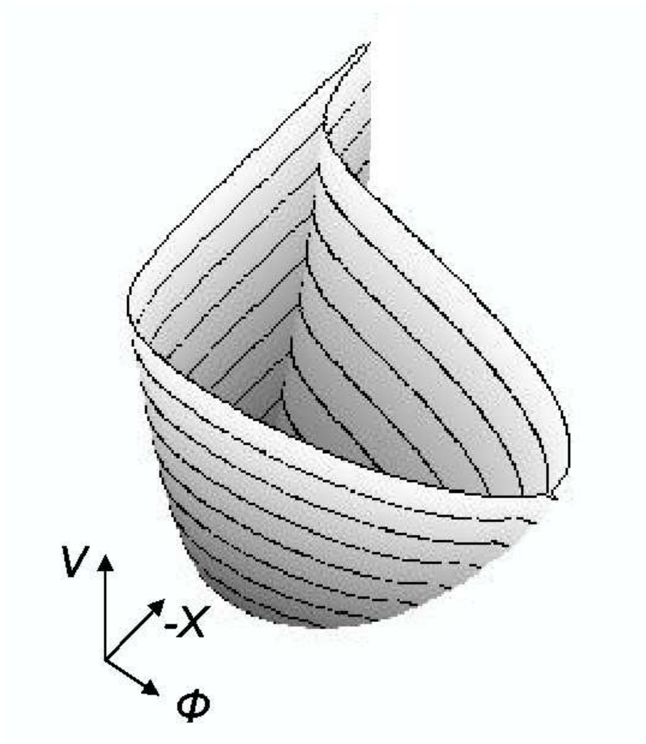

Figure 2.1: A plot showing the global shape of the potential. M has been expanded around zero as in equation (3.8). Note the runaway in the direction $X \to -\infty$ and $\phi \to 0$. The singularity at $\phi = 0$ and the "drain" $W_\phi = 0$ are clearly visible. Also visible is the Coleman-Weinberg channel near $X = 0$ and ϕ large, discussed later. This plot was generated with the help of [56].

Given that $\phi = \langle \Phi \rangle \neq 0$, we will expand around the point of maximal symmetry

$$q = \begin{pmatrix} q_0 & 0 \end{pmatrix} \ , \ \tilde{q} = \begin{pmatrix} \tilde{q}_0 \\ \\ 0 \end{pmatrix} \ , \ M = \begin{pmatrix} 0 & 0 \\ & \\ 0 & 0 + X \cdot I_{N_c \times N_c} \end{pmatrix} . \tag{2.3.7}$$

Here q_0 and $\tilde{q}_0$ are $\tilde{N}_c \times \tilde{N}_c$ matrices satisfying

$$h q_{0i} \tilde{q}_{0j} = -m\phi\, \delta_{ij} \ , \ i, j = \tilde{N}_c + 1, \ldots N_f \ , \tag{2.3.8}$$

and the nonzero block matrix in M has been taken to be proportional to the identity; indeed, only $\operatorname{tr} M$ appears in the potential. This minimizes W_M and sets $W_q = W_{\tilde{q}} = 0$. The spectrum of fluctuations around (2.3.7) is studied in detail in Section 4, where it is shown that the lightest degrees of freedom correspond to (ϕ, X) with mass given by m. The effective potential derived from (2.3.6) is

$$V(\phi, X) = N_c m^2 |\phi|^2 + \left| m N_c X + N_f' \lambda'^{N_f'/N_c'} \left(\frac{\Lambda'^{3N_c' - N_f'}}{\phi^{N_c' - N_f'}} \right)^{1/N_c'} \right|^2 + V_{CW}(\phi, X) \,, \qquad (2.3.9)$$

where the second term comes from W_ϕ. The Coleman-Weinberg contribution will be discussed shortly.

As a starting point, set $X = 0$ and $V_{CW} \to 0$. Minimizing $V(\phi, X = 0)$ gives

$$|\phi_0|^{(2N_c' - N_f')/N_c'} = \sqrt{\frac{N_c' - N_f'}{N_c N_c'}} \, N_f' \frac{\lambda'^{N_f'/N_c'}}{m} \Lambda'^{(3N_c' - N_f')/N_c'} \,, \qquad (2.3.10)$$

and since $W_{\phi\phi} \sim m$, $V(\phi_0 + \delta\phi, X = 0)$ corresponds to a parabola of curvature m. The nonperturbative term only affects ϕ_0 but not the curvature m; this will be important in the discussion of subsection 3.4.

Next, allowing X to fluctuate (but still keeping $V_{CW} \to 0$), $V(\phi_0, X)$ gives a parabola centered at

$$X_{W_\phi = 0} = -\sqrt{\frac{N_c'}{N_c(N_c' - N_f')}} |\phi_0| \qquad (2.3.11)$$

and curvature m. In other words, $X = 0$ is on the side of a hill of curvature m and height $V(\phi_0, 0) \sim m^2 |\phi_0|^2$.

To create a minimum near $X = 0$, V_{CW} should contain a term $m_{CW}^2 |X|^2$, with $m_{CW} \gg m$; this would overwhelm the classical curvature. As explained in Section 4, the massive degrees of freedom giving the dominant contribution to V_{CW} come from integrating out the massive fluctuations along q_0 and $\tilde{q}_0$. The result is

$$V_{CW} = N_c b h^3 m |\phi| |X|^2 + \ldots \qquad (2.3.12)$$

with $b = (\log 4 - 1)/8\pi^2 \tilde{N}_c$ [20], and '...' represent contributions that are unimportant for the present discussion. In this computation, X and ϕ are taken as background fields. It is crucial to notice that the quadratic X dependence appears because $X = 0$ is an enhanced symmetry point.

In order to be able to produce a local minimum, the marginal parameters (λ, λ') will have to be tuned to satisfy

$$\epsilon \equiv \frac{m^2}{m_{CW}^2} = \frac{m}{bh^3|\phi|} \ll 1\,. \tag{2.3.13}$$

In this approximation, the value of ϕ at the minimum is still given by (2.3.10); also, X is stabilized at the nonzero value

$$X_0 = -e^{-i\frac{N_c'-N_f'}{N_c'}\alpha_\phi}\,\frac{N_f'}{bh^3}\lambda'^{N_f'/N_c'}\left(\frac{\Lambda'^{3N_c'-N_f'}}{|\phi_0|^{2N_c'-N_f'}}\right)^{1/N_c'}\,. \tag{2.3.14}$$

The phases of ϕ and X are thus related by

$$\alpha_X + \frac{N_c'-N_f'}{N_c'}\alpha_\phi = \pi\,. \tag{2.3.15}$$

Inserting (2.3.10) into (2.3.14) gives

$$|X_0| = \sqrt{\frac{N_c N_c'}{N_c'-N_f'}}\,\frac{m}{bh^3}\,. \tag{2.3.16}$$

At the minimum, (2.3.13) gives

$$(m/\Lambda')^{3N_c'-N_f'} \ll (bh^3)^{(2N_c'-N_f')/N_c'}\lambda'^{N_f'} \tag{2.3.17}$$

so the Yukawa coupling λ in $m = \lambda\Lambda$ must be taken small for the analysis to be self-consistent. The calculability condition $\Lambda' \ll \lambda'\Phi$ follows as a consequence of this. At the minimum, $X_0 \ll \phi_0$. The F-terms are given by

$$W_\phi \approx \sqrt{\frac{N_c N_c'}{N_c'-N_f'}}m\phi_0 \sim W_X\,. \tag{2.3.18}$$

and from (2.3.10) the scale of supersymmetry breaking is thus controlled by the dynamical scales of both gauge sectors. In the next subsection, the vacuum will be shown to be long-lived if (2.3.13) is satisfied.

Thus the model has a metastable vacuum near the origin, created by a combination of quantum corrections and nonperturbative gauge effects. The pseudo-runaway towards $X =$

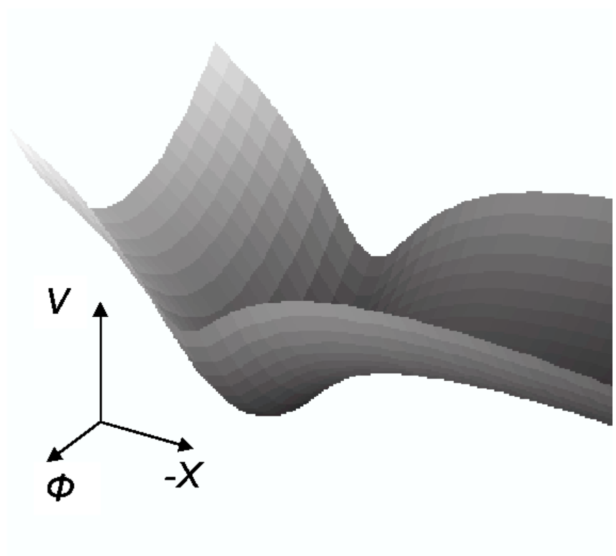

Figure 2.2: A plot showing the shape of the potential, including the one-loop Coleman-Weinberg corrections, near the metastable minimum. In the ϕ-direction the potential is a parabola, whereas in the X-direction it is a side of a hill with a minimum created due to quantum corrections. This plot was generated with the help of [56].

$X_{W_\phi=0}$ has been lifted by the Coleman-Weinberg contribution, as anticipated. This is the origin of the $1/b$ dependence in (2.3.16). The local minimum is depicted in Fig. 2.2.

2.3.3 Stability under other quantum corrections

The metastable vacuum appears from a competing effect between a runaway behavior in the primed sector and one loop corrections for the meson field X. One is naturally led to ask if, under these circumstances, other quantum effects are under control. These include higher loop terms from the massive particles producing V_{CW} as well as perturbative g' corrections.

Let us first study higher loop contributions from the massive fields in $(q, \tilde{q})$. They can correct the potential by additive terms of the form X^n, $n > 2$; these are automatically subleading, because $|X_0|^2 \ll m|\phi_0|$. They can also produce higher ϕ powers. However, such quantum corrections can only depend on the combination $m\phi$, and thus will be suppressed by powers of the UV cutoff Λ_0. For instance, a quartic term would appear as $(m\phi)^4/\Lambda_0^4$. We conclude that all these effects are subleading to (2.3.12).

Furthermore, since nonperturbative effects from $SU(N_c')$ were used, we should make sure that perturbative g' effects are not important. First note that the nonperturbative term in (2.3.9) is of the same order as the classical height of the potential $m^2|\phi|^2$ (see eq. (2.3.18)). It thus suffices to show that g' perturbative corrections to this height are subleading. A simple argument for this is as follows. Loops generate typical quartic terms in the Kähler potential

$$\delta K = \frac{\alpha}{\Lambda_0^2}(\Phi^*\Phi)^2 \tag{2.3.19}$$

which change the scalar potential by

$$\left[\frac{\alpha}{\Lambda_0^2}\,|\phi|^2\right](m^2|\phi|^2)\,. \tag{2.3.20}$$

The prefactor is parametrically small, making these contributions negligible.

2.3.4 Tunneling Out of the Metastable Vacuum

This section will show that the metastable non-supersymmetric vacuum can be made parametrically long-lived by taking the parameter $\epsilon \equiv \frac{m}{bh^3|\phi_0|}$ sufficiently small. The lifetime of the metastable vacuum may be estimated using semiclassical techniques and is proportional to the exponential of the bounce action, e^B [57].

First, the direction of tunneling in field space needs to be determined. Recall that the metastable vacuum in the $(|\phi|, X)$ space lies at

$$|\phi_0|^{\frac{2N_c'-N_f'}{N_c'}} = \sqrt{\frac{N_c'-N_f'}{N_cN_c'}}\,N_f'\,\frac{\lambda'^{\frac{N_f'}{N_c'}}}{m}\Lambda'^{\frac{3N_c'-N_f'}{N_c'}}\,,\quad X_0 = -\sqrt{\frac{N_cN_c'}{N_c'-N_f'}}\,\frac{m}{bh^3}\,. \tag{2.3.21}$$

(The phase of ϕ, not of qualitative importance for the present discussion, has been chosen to be zero. This fixes X to be real - see equation (2.3.15).) For fixed X the potential has a minimum at $|\phi| = |\phi_0|$; while quantum corrections may change this value by an order one number, corrections to the curvature of the potential in the $|\phi|$ direction are negligible. This curvature is positive, and thus the potential increases as $|\phi|$ moves away from $|\phi_0|$. The field therefore does not tunnel in the $|\phi|$ direction (see (2.2)). Along the X direction, however, the potential without quantum corrections near the enhanced symmetry point is like the side of a hill. For fixed $|\phi| = |\phi_0|$, the potential decreases in the negative X direction, and the classical curvature at $X = 0$ is m.

Quantum corrections are qualitatively important when $|X|$ is sufficiently small. For $|X|^2 \ll |W_X|$, their size grows quadratically as a function of X and they are sufficient to change the slope of the classical potential enough to introduce a minimum. For $|X|^2 \simeq |W_X|$, the growth of the quantum corrections is only logarithmic, and the slope of the classical potential again starts to dominate. Hence, the total potential has a peak that parametrically may be estimated to lie near

$$X_{\text{peak}} \simeq -\sqrt{|W_X|} = -\sqrt{N_c m|\phi_0|}. \tag{2.3.22}$$

For $|X| > |X_{\text{peak}}|$, the potential decreases as X becomes more negative until X reaches the 'drain' $W_\phi = 0$,

$$X_{W_\phi=0} = -\sqrt{\frac{N_c'}{N_c(N_c' - N_f')}}|\phi_0|. \tag{2.3.23}$$

The direction in field space to tunnel out of the false vacuum is towards negative X with fixed $|\phi| = |\phi_0|$. It thus suffices to consider the tunneling in the one-dimensional potential, $V(X) \equiv V(|\phi_0|, X)$. Note that parametrically $|X_0| \ll |X_{\text{peak}}| \ll |X_{W_\phi=0}|$ as $\epsilon \to 0$.

For negative X, using equations (2.3.9) and (2.3.21), the one-dimensional potential may be written as

$$V(X) = \left(\frac{2N_c' - N_f'}{N_c' - N_f'}\right) N_c\, m^2\, |\phi_0|^2 + N_c^2\, bh^3\, m^2\, |\phi_0|^2\, f\left(\frac{-|X|}{bh^3|\phi_0|}\right). \tag{2.3.24}$$

In the region $|X| \ll |X_{\text{peak}}|$, the function $f(x)$ is dominated by quantum corrections and may be approximated by

$$f(x) \simeq \frac{bh^3}{N_c\, \epsilon}\, x^2\,, \tag{2.3.25}$$

where a constant piece coming from the quantum corrections, again not important for the calculation of the bounce action, has been neglected. On the other hand, in the region $|X_{\rm peak}| \ll |X| \ll |X_{W_\phi=0}|$, the constant slope of the classical potential dominates. The potential in this region may be approximated by the classical potential plus a constant contribution from the quantum corrections whose size is roughly given by the height of the potential barrier. The height of the potential barrier is, from (2.3.25), of order $f(X_{\rm peak}/bh^3|\phi_0|) = 1$, and it is thus loop-suppressed compared to the overall magnitude of the potential near the metastable minimum. The potential in this region will be parametrized by a straight line

$$f(x) \simeq 1 - 2\sqrt{\frac{N_c'}{N_c(N_c' - N_f')}}\,(x - x_{\rm peak}). \tag{2.3.26}$$

In order to estimate the bounce action it is not appropriate to use the thin-wall approximation [57]. Instead, the potential may be modeled as a triangular barrier [58]. Using the results of [58], the value to which the field tunnels to is

$$\tilde{X} \sim -\,b\,h^3|\phi_0|. \tag{2.3.27}$$

Note that parametrically $|X_0| \ll |X_{\rm peak}| \ll |\tilde{X}|$ as $\epsilon \to 0$, and that $|\tilde{X}|$ is loop-suppressed compared to $|X_{W_\phi=0}|$. The bounce action scales as

$$B \sim \frac{\tilde{X}^4}{V(X_{peak}) - V(X_0)} \sim b\,h^3\,\frac{1}{\epsilon^2}. \tag{2.3.28}$$

Therefore $B \to \infty$ as $\epsilon \to 0$, and the metastable vacuum is parametrically long-lived.

The total potential $V(X)$, including the full one-loop Coleman-Weinberg potential computed numerically with the help of [56], is shown in Fig. 2.3. The program of [56] also allowed us to check numerically the previous tunneling properties.

2.4 Particle Spectrum and R-symmetry

In this section, we discuss in more detail the particle spectrum of the model and comment on the R-symmetry properties.

The fluctuations of the fields around the metastable minimum may be parametrized following

ISS,

$$\phi = \phi_0 + \delta\phi\ ,\ M = \begin{pmatrix} Y_{\tilde{N}_c \times \tilde{N}_c} & Z^T_{\tilde{N}_c \times (N_f - \tilde{N}_c)} \\ \\ \tilde{Z}_{(N_f - \tilde{N}_c) \times \tilde{N}_c} & X_0 + X_{(N_f - \tilde{N}_c) \times (N_f - \tilde{N}_c)} \end{pmatrix} \tag{2.4.1}$$

$$q = \begin{pmatrix} q_0 + \chi_{\tilde{N}_c \times \tilde{N}_c} \\ \\ \rho_{(N_f - \tilde{N}_c) \times \tilde{N}_c} \end{pmatrix}\ ,\ \tilde{q} = \begin{pmatrix} \tilde{q}_0 + \tilde{\chi}_{\tilde{N}_c \times \tilde{N}_c} \\ \\ \tilde{\rho}_{(N_f - \tilde{N}_c) \times \tilde{N}_c} \end{pmatrix}\ , \tag{2.4.2}$$

where $q_0\tilde{q}_0 := -m\phi_0/h$. All fields are complex; ϕ_0 and X_0 are the values at the metastable minimum.

The relevant mass scales are

$$M^2 = 0,\ m^2,\ m^2_{CW} = bh^3 m|\phi_0|,\ hm|\phi_0|\,. \tag{2.4.3}$$

The particles may be divided into three 'sectors' with small mixing amongst themselves. Up to quadratic order, the superpotential is

$$\begin{aligned} W \quad = \quad & W_{\phi\phi}\delta\phi\delta\phi + mN_c\delta\phi(X_0 + X) + m\delta\phi\sum_{\alpha=1}^{\tilde{N}_c} Y_{\alpha\alpha} \\ & + mN_c\phi_0(X_0 + X) + h\sum_{f=1}^{N_c}\left[q_0(\tilde{\rho}Z^T)_{ff} + \tilde{q}_0(\rho\tilde{Z}^T)_{ff} + X_0(\rho\tilde{\rho}^T)_{ff}\right] \\ & + h\sum_{\alpha=1}^{\tilde{N}_c}\left[q_0(\tilde{\chi}Y)_{\alpha\alpha} + \tilde{q}_0(\chi Y)_{\alpha\alpha}\right]. \end{aligned} \tag{2.4.4}$$

The first line is related to the new dynamical field $\delta\phi$; unlike ISS, now X is not a pseudo-flat direction. The second and third lines are as in ISS.

Consider the case $N_f = N_c + 1$; the spectrum of classical masses is shown in Fig. 2.1, and the spectrum of the masses including one-loop CW corrections is shown in Fig. 2.2. The fields are grouped in sectors of $\mathrm{STr}M^2 = 0$.

The fields $(Y, \chi, \tilde{\chi})$ form three chiral superfields, with supersymmetric masses, and hence do not contribute when integrated out at one loop. The Coleman-Weinberg potential is generated by the fields $(Z, \tilde{Z}, \rho, \tilde{\rho})$, which are the heaviest in the spectrum. Including such quantum corrections, $\mathrm{tr}\,X$ acquires a mass m^2_{CW}, while the mass of ϕ is not modified. Interestingly, at the classical level there is no massless goldstino, since the expansion is not around a critical point of the classical potential. Including quantum corrections, one of the massive fermions in the $(\phi, \mathrm{tr}\, X)$-sector becomes massless, as may be seen in Fig. 2.2. A similar situation, in the opposite limit of small supersymmetry breaking, has been discussed recently in [59].

The case $\tilde{N}_c = N_f - N_c > 1$ can be similarly analyzed, and is shown in Fig. 2.3.

The Standard Model gauge group can be embedded inside the global symmetry group of this model. In this way, renormalizable models of direct gauge mediated supersymmetry breaking may be constructed.

2.4.1 Breaking the R-symmetry

To have gaugino masses, any R-symmetry must be broken, explicitly and/or spontaneously [20], [59]. The low energy superpotential 2.3.6 has the following $U(1)_R$ symmetry:

$$R_\phi = 2\frac{N'_c}{N'_f}\ ,\ R_X = 2\frac{N'_f - N'_c}{N'_f}\ ,\ R_q = R_{\tilde{q}} = \frac{N'_c}{N'_f}\ . \tag{2.4.5}$$

Since the VEV's of these fields are nonzero in the metastable vacuum, the R-symmetry is spontaneously broken, and there is an R-axion a. In terms of the phase of the i-th field, the axion is

$$\phi_i = \frac{1}{\sqrt{2}}\frac{f_R}{R_i}\, e^{iR_i(a/f_R)}\ , \tag{2.4.6}$$

where the decay constant f_R is defined as

$$f_R = \Big[\sum_i \big(\sqrt{2}R_i|\langle\phi_i\rangle|\big)^2\Big]^{1/2} \tag{2.4.7}$$

and R_i is the R-charge of ϕ_i. In [25] it was pointed out that if R-symmetry is broken spontaneously in an O' Raifeartaigh model, then the theory should contain a field with R-charge different than 0 or 2. This is also the case in the present situation, although our model does not contain the linear O' Raifeartaigh term.

For finite $\tilde{\Lambda}$, the det X contributions need to be taken into account, and the $U(1)_R$ symmetry becomes anomalous. Adding this term induces a tadpole for Y, which now acquires an expectation value of order

$$Y \sim \left[\frac{X_0}{\tilde{\Lambda}}\right]^{\frac{3N_c-2N_f}{N_f-N_c}} X_0\,, \tag{2.4.8}$$

so that $|Y| \ll |X_0|$. Then the mass of the R-axion follows from

$$|W_X|^2 \sim \left| m\phi + cX_0^2 \left[\frac{X_0}{\tilde{\Lambda}}\right]^{2\,\frac{3N_c-2N_f}{N_f-N_c}} \right|^2 . \tag{2.4.9}$$

Deriving twice the cross-term, which is proportional to $\cos(a/f)$, yields the axion mass

$$m_a^2 \sim m^2 \left(\left[\frac{\lambda}{bh^3}\right]^{2\,\frac{3N_c-2N_f}{N_f-N_c}} \frac{\epsilon}{bh^3} \right) \ll m^2\,, \tag{2.4.10}$$

where λ is the Yukawa coupling appearing in $m = \lambda\Lambda$. Thus, R-symmetry is both spontaneously and explicitly broken.

2.5 Meta-Stability Near Generic Points of Enhanced Symmetry

In this section, the existence and genericity of metastable vacua near enhanced symmetry points is explored. Statistical analyses of the supersymmetry breaking scale up to date have not taken into account loop quantum effects ([164], [165], [49]) as these corrections are hard to evaluate on an ensemble of field theories. However, metastable vacua introduced by the Coleman-Weinberg potential, with all the relevant parameters generated dynamically, may change such results. Before considering the general case, let us analyze (2.2.5).

2.5.1 Non-coincident enhanced symmetry points

Consider two gauge sectors as in (2.2.5), with enhanced symmetry points at $\Phi = 0$ and $\Phi = \xi$, respectively. The free magnetic sector is taken to be massless at $\Phi = 0$; integrating over the other primed sector gives

$$W = m\Phi\, \mathrm{tr}\; M + h\, \mathrm{tr}\, qM\tilde{q} + N_c'\left[\lambda'^{N_f'}\, \Lambda'^{3N_c'-N_f'}\, (\Phi+\xi)^{N_f'}\right]^{1/N_c'} . \tag{2.5.1}$$

Since metastable vacua were shown to exist for $\xi = 0$, here the discussion is restricted to the limit of ξ much bigger than all the energy scales in the problem. This is consistent with the fact that naturalness demands any relevant coupling to be of order the UV cutoff.

Introducing the notation

$$\alpha = N'_f/N'_c\ ,\ K = N'_c\lambda'^{N'_f/N'_c}\,\Lambda'^{(3N'_c-N'_f)/N'_c}\,, \tag{2.5.2}$$

the equations of motion for ϕ and X give

$$N_c m^2\phi = \alpha^2(1-\alpha)\frac{K^2}{\xi^{3-2\alpha}}\,. \tag{2.5.3}$$

$$|X| = \frac{N_c}{\alpha(1-\alpha)}\,\frac{m^2\xi^{2-\alpha}}{K}\,. \tag{2.5.4}$$

Without fine-tuning m or K, X tends to be driven away from the origin as ξ increases. The fine-tuning may be seen, for instance, from the requirement $m_{CW} \gg m$, which implies

$$m^3 \ll bh^3\frac{K^2}{\xi^{3-2\alpha}}\,. \tag{2.5.5}$$

Although this resembles the calculability condition (2.3.17), now there are powers of the large scale ξ in the denominator. For ξ of order the UV cutoff, this represents a big fine-tuning, either on the coefficient K or on the small mass parameter m.

The conclusion is that, while metastable vacua can occur for far away enhanced symmetry points, this situation is not generic and requires fine-tuning. This is to be expected, once relevant parameters are allowed to appear in the superpotential.

2.5.2 General Analysis

A generic structure in the landscape of effective field theories corresponds to a gauge theory with vector-like matter and mass given by a singlet, whose dynamics is related to another sector. The superpotential may be written as

$$W = f(\Phi) + \lambda\,\Phi\mathrm{tr}(Q\bar{Q})\,. \tag{2.5.6}$$

Here, $(Q,\bar{Q})$ are N_f quarks in $SU(N_c)$ SQCD; $f(\Phi)$ may be generated, for instance, from a flux superpotential, by nonrenormalizable interactions [26], or, as in the case studied in this work, by another gauge sector. Next, it is required that the SQCD sector be in the free magnetic range; this is still a generic situation. The dual magnetic description is weakly coupled near the enhanced symmetry point $\Phi = 0$, where the superpotential reads

$$W = f(\Phi) + m\Phi \, \mathrm{tr}\, M + h \, \mathrm{tr}\, qM\tilde{q}\,. \tag{2.5.7}$$

The question that will be addressed here is: what restrictions need to be imposed on $f(\Phi)$, so that the one loop potential V_{CW} can create a metastable vacuum near $M = 0$? Since we are interested in the novel effect of pseudo-runaway directions we will demand $f'(\Phi) \neq 0$. The case $f'(\Phi) = 0$ is standard in such analyses, see e.g. [52].

As discussed in Section 3, this is possible only if

$$m_{CW}^2 := N_c b h^3 m|\phi| \gg m^2 \tag{2.5.8}$$

where ϕ denotes the expectation value of Φ at the metastable vacuum. Further, one needs to impose that

$$h^2|X|^2 \ll m|\phi| \tag{2.5.9}$$

in order for the Taylor expansion of V_{CW} around $X = 0$ to converge. Evaluating the potential as in (2.3.9),

$$V = N_c m^2 |\phi|^2 + \left| f'(\phi) + m N_c \, X \right|^2 + m_{CW}^2 |X|^2\,. \tag{2.5.10}$$

The rank condition, an essential ingredient in the discussion, just follows from having SQCD in the free magnetic range. This fixes the first term, which comes from W_M, and the block structure of the matrix M; X was defined in (2.3.7).

Extremizing $V(\phi, X = 0)$ leads to

$$N_c m^2 \phi = -f'(\phi)\, f''(\phi)^*\,. \tag{2.5.11}$$

On the other hand, minimization with respect to X in the approximation $m_{CW}^2 \gg m^2$, gives the metastable vacuum

$$m_{CW}^2 \, X = -N_c m f'(\phi)\,. \tag{2.5.12}$$

Notice that $m_{CW}^2 \gg m^2$ makes this value parametrically smaller than the position of the 'drain' $f'(\phi) + m N_c \, X = 0$. This ensures the stability of the nonsupersymmetric vacuum. Replacing

(2.5.11) in (2.5.12) (with $m_{CW}^2 = N_c b h^3 |\phi|$) yields

$$|X| = \frac{N_c m^2}{bh^3} \frac{1}{|f''(\phi)|} \,. \tag{2.5.13}$$

It is possible to combine the conditions (2.5.8) and (2.5.9) with the values at the metastable vacuum (2.5.11), (2.5.13), to derive constraints on $f(\phi)$: (2.5.8) now reads

$$\frac{|f'(\phi) f''(\phi)|}{m^3} \gg \frac{1}{bh^3} \,, \tag{2.5.14}$$

while (2.5.9) gives

$$h^2 |f'(\phi)|^2 \ll m (bh^3)^2 |\phi|^3 \,. \tag{2.5.15}$$

Summarizing, the necessary conditions to have metastable vacua near $X = 0$ are (2.5.14) and (2.5.15). As illustrated in §2.5.1, they require fine-tuning the coefficients of $f(\phi)$, except in the case of coincident enhanced symmetry points, where there are no relevant scales.

2.6 Conclusions

We constructed a model with long-lived metastable vacua in which all the relevant parameters, including the supersymmetry breaking scale, are generated dynamically by dimensional transmutation. The model consists of two $N = 1$ supersymmetric QCD sectors with flavors whose respective masses are controlled by the same singlet field. One of the gauge sectors is in the free magnetic range while the other is in the electric range. The metastable vacua are produced near a point of enhanced symmetry by a combination of nonperturbative gauge effects and, crucially, perturbative effects coming from the one-loop Coleman-Weinberg potential.

The model has the following desirable features: an explicitly and spontaneously broken R-symmetry, a singlet, a large global symmetry, naturalness and renormalizability.

There are two points that have to be stressed. First, a salient feature of the model is the existence of pseudo-runaway directions. They correspond to a runaway behavior that is lifted by one loop quantum corrections. This has not been observed before, the closest analog corresponding for example to the pseudo-moduli of [20]. It is quite plausible that this phenomenon appears in other models as well. The criterion is that the height of the potential has to be parametrically larger than the curvature, as quantified in Section 3. The strength of the quadratic Coleman-Weinberg corrections is set by this height, thus introducing a local minimum of high curvature in the (otherwise) runaway potential.

In dynamical supersymmetry breaking models ([60], [61], [62], [63], [64], [65]), nonsupersymmetric vacua generally arise due to competing effects between a nonperturbative runaway and a classical term in the superpotential, as in the (3,2) model [66]. Our analysis shows that it is possible to stabilize such runaways even without tree-level terms, provided that one is close to certain enhanced symmetry points.

The second feature worth emphasizing is the connection between enhanced symmetry points in gauge theory moduli spaces and metastable dynamical supersymmetry breaking. There are reasons to believe that such vacua are generic. At the field theory level this is associated to the fact that a nonzero Witten index [67] may still allow an approximate R-symmetry [68]. While dynamical ISS models are not hard to construct, in general these mechanisms involve discrete R-symmetries [26]. This is very suppressed in the landscape of string vacua, correponding to a high codimension locus in the flux lattice [69]. On the other hand, the construction presented here does not suffer from the previous difficulty. Therefore, it would be interesting to study how statistical estimates of the scale of supersymmetry breaking change, once the model is embedded in string theory.

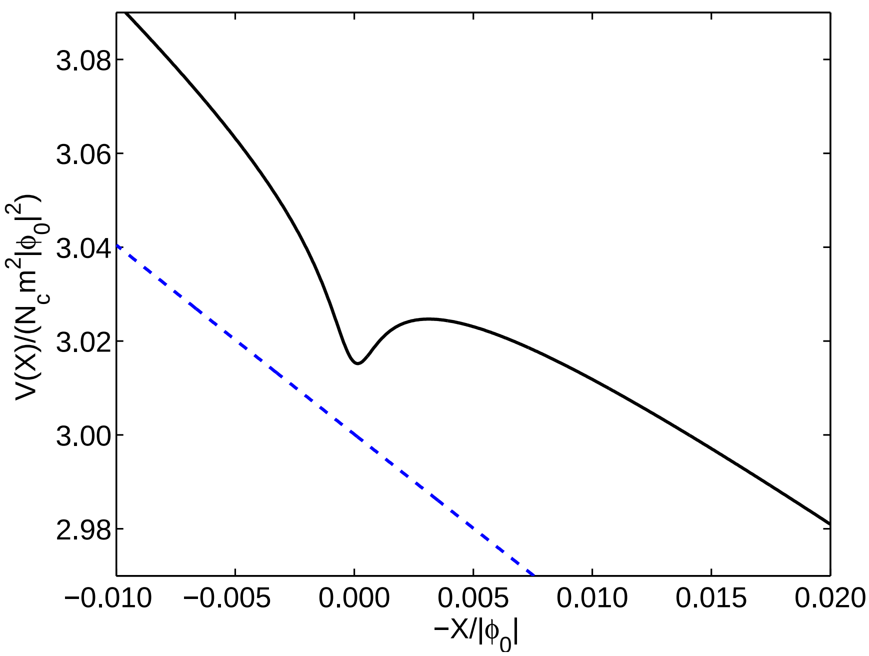

Figure 2.3: A plot of the classical potential (dashed line) and the total potential including one-loop corrections (solid line) for fixed $|\phi| = |\phi_0|$, where $|\phi_0|$ is the position of the metastable minimum in the ϕ-direction, defined in (2.3.21). In the figure, $N_f = 3$, $N_c = 2$, $N_f' = 1$ and $N_c' = 2$. The values were scaled so that the position of the "drain", $W_\phi = 0$, equals 1 on both axes. In these units, the position of the metastable minimum is on the order of 10^{-4}. This plot was generated with the help of [56].

	Fermions			Bosons		
	Weyl mult.	mass^2	$U(N_f-1)$	Real mult.	mass^2	$U(N_f-1)$
ϕ, tr$X$	2	$\mathcal{O}(m^2)$	1_0	1	0	1_0
				3	$\mathcal{O}(m^2)$	1_0
$X_{ij}-\text{tr}X$	$(N_f-1)^2-1$	0	Adj$_0$	$2((N_f-1)^2-1)$	0	Adj$_0$
Y, $\chi\ \tilde{\chi}$	1	0	1_0	1	0_{GB}	1_0
				1	0_{NCGB}	1_0
	2	$\mathcal{O}(hm\vert\phi_0\vert)$	1_0	4	$\mathcal{O}(hm\vert\phi_0\vert)$	1_0
Z,$\tilde{Z}$, ρ, $\tilde{\rho}$	$2(N_f-1)$	$\mathcal{O}(hm\vert\phi_0\vert)$	$\square_1+\overline{\square}_{-1}$	$2(N_f-1)$	0_{GB}	$\square_1$
				$2(N_f-1)$	$\mathcal{O}(hm\vert\phi_0\vert)$	$\overline{\square}_{-1}$
	$2(N_f-1)$	$\mathcal{O}(hm\vert\phi_0\vert)$	$\square_1+\overline{\square}_{-1}$	$2(N_f-1)$	$\mathcal{O}(hm\vert\phi_0\vert)$	$(\square_1+$
				$2(N_f-1)$	$\mathcal{O}(hm\vert\phi_0\vert)$	$\overline{\square}_{-1})$

Table 2.1: Table showing the classical mass spectrum, grouped in sectors of $\text{Str}\, M^2 = 0$ for $N_f = N_c + 1$. The $O(m^2)$ fields in (ϕ, tr X) are not degenerate. Although supersymmetry is spontaneoulsy broken, there is no goldstino at the classical level.

	Fermions			Bosons		
	Weyl mult.	mass^2	$U(N_f-1)$	Real mult.	mass^2	$U(N_f-1)$
ϕ, tr$X$	1	0	1_0	1	0	1_0
	1	$\mathcal{O}(m^2)$	1_0	1	$\mathcal{O}(m^2)$	1_0
				2	$\mathcal{O}(m^2_{\mathrm{CW}})$	1_0
$X_{ij}-\mathrm{tr}X$	$(N_f-1)^2-1$	0	Adj$_0$	$2((N_f-1)^2-1)$	$\mathcal{O}(m^2_{\mathrm{CW}})$	Adj$_0$
Y, χ $\tilde{\chi}$	1	0	1_0	1	0_{GB}	1_0
				1	$\mathcal{O}(m^2_{\mathrm{CW}})$	1_0
	2	$\mathcal{O}(hm\vert\phi_0\vert)$	1_0	4	$\mathcal{O}(hm\vert\phi_0\vert)$	1_0
Z,$\tilde{Z}$, ρ, $\tilde{\rho}$	$2(N_f-1)$	$\mathcal{O}(hm\vert\phi_0\vert)$	$\square_1+\overline{\square}_{-1}$	$2(N_f-1)$	0_{GB}	$\square_1$
				$2(N_f-1)$	$\mathcal{O}(hm\vert\phi_0\vert)$	$\overline{\square}_{-1}$
	$2(N_f-1)$	$\mathcal{O}(hm\vert\phi_0\vert)$	$\square_1+\overline{\square}_{-1}$	$2(N_f-1)$	$\mathcal{O}(hm\vert\phi_0\vert)$	$(\square_1+$
				$2(N_f-1)$	$\mathcal{O}(hm\vert\phi_0\vert)$	$\overline{\square}_{-1})$

Table 2.2: Table showing the mass spectrum, including one-loop corrections, grouped in sectors of $\mathrm{Str}\, M^2 = 0$ for $N_f = N_c + 1$. Notice the appearance of the goldstino in the $(\phi, \mathrm{tr}\, X)$ sector. The $O(m^2)$ fields in $(\phi,\, \mathrm{tr}\, X)$ are not degenerate; here $m^2_{CW} = bh^3 m|\phi_0|$.

	Fermions				Bosons			
	Weyl mult.	mass^2	$U(N_f - \tilde{N}_c)$	$SU(\tilde{N}_c)_D$	Real mult.	mass^2	$U(N_f - \tilde{N}_c)$	$SU(\tilde{N}_c)_D$
ϕ, tr$X$	2	$\mathcal{O}(m^2)$	1_0	1	1	0	1_0	1
					3	$\mathcal{O}(m^2)$	1_0	1
$X_{ij} - \mathrm{tr}X$	$(N_f - \tilde{N}_c)^2 - 1$	0	Adj$_0$	1	$2((N_f - \tilde{N}_c)^2 - 1)$	0	Adj$_0$	1
Y, χ $\tilde{\chi}$	$\tilde{N}_c^2$	0	1_0	Adj	$\tilde{N}_c^2$	0_{GB}	1_0	Adj
					$\tilde{N}_c^2$	0_{NCGB}	1_0	Adj
	$2\tilde{N}_c^2$	$\mathcal{O}(hm\|\phi_0\|)$	1_0	Adj	$4\tilde{N}_c^2$	$\mathcal{O}(hm\|\phi_0\|)$	1_0	Adj
Z,$\tilde{Z}$, ρ, $\tilde{\rho}$	$2\tilde{N}_c(N_f - \tilde{N}_c)$	$\mathcal{O}(hm\|\phi_0\|)$	$\square_1+\overline{\square}_{-1}$	$\square+\overline{\square}$	$2\tilde{N}_c(N_f - \tilde{N}_c)$	0_{GB}	$\square_1$	$\overline{\square}$
					$2\tilde{N}_c(N_f - \tilde{N}_c)$	$\mathcal{O}(hm\|\phi_0\|)$	$\overline{\square}_{-1}$	$\square$
	$2\tilde{N}_c(N_f - \tilde{N}_c)$	$\mathcal{O}(hm\|\phi_0\|)$	$\square_1+\overline{\square}_{-1}$	$\square+\overline{\square}$	$2\tilde{N}_c(N_f - \tilde{N}_c)$	$\mathcal{O}(hm\|\phi_0\|)$	$(\square_1+$	$(\overline{\square}+$
					$2\tilde{N}_c(N_f - \tilde{N}_c)$	$\mathcal{O}(hm\|\phi_0\|)$	$\overline{\square}_{-1})$	$\square)$

Table 2.3: Table showing the classical mass spectrum, grouped in sectors of $\mathrm{Str}\, m^2 = 0$, for $N_f > N_c + 1$. After gauging $SU(\tilde{N}_c)$, the traceless goldstone bosons from $(\chi, \bar{\chi})$ are eaten, giving a mass $m_W^2 = g^2 m|\phi_0|/h$ to the gauge bosons. Further, from $V_D = 0$, the noncompact goldstones also acquire a mass m_W^2. Including CW corrections, $\mathrm{tr}\, X$ acquires mass m_{CW}^2 and one of the fermions becomes massless.

Chapter 3

A D-brane Landscape on Calabi-Yau Manifolds

3.1 Introduction

Magnetized branes in toroidal IIB orientifolds have been a very useful device in the construction of semirealistic string vacua [70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80]. A very attractive feature of magnetized brane systems is Kähler moduli stabilization by D-term effects [81, 82, 83, 84, 85, 86, 87]. By turning on background fluxes, one can stabilize the complex structure moduli as well, obtaining an interesting distribution of isolated vacua in the string theory landscape. These are typically supersymmetric vacua because magnetized brane configurations are supersymmetric for special values of the toroidal moduli. Note however, that nonsupersymmetric vacua have also been found in [83, 85, 86] as a result of the interaction between D-term and nonperturbative F-term effects.

The purpose of the present work is to explore the landscape of magnetized brane configurations on Calabi-Yau manifolds. The starting point of this investigation is the observation that certain Calabi-Yau orientifolds exhibit a very interesting class of metastable D-brane configurations. As opposed to toroidal models, these brane configurations are not supersymmetric for any values of the moduli, but the supersymmetry breaking parameter is minimal at the Landau-Ginzburg point in the underlying $N = 2$ moduli space. In this chapter we investigate the dynamics of these brane configurations from the point of view of the low energy effective supergravity action. We compute the D-term contribution to the potential energy and show that it agrees with more abstract Π-stability considerations. A similar relation between supergravity D-terms and the perturbative part of Π-stability was previously found in [88]. We also develop a generalization of the flux superpotential in the presence of magnetized branes. Then we argue that the interplay between D-term effects and the flux superpotential can in principle give rise to a landscape of metastable nonsupersymmetric vacua. Note that different aspects of the open string landscape have been recently studied in [89, 90, 91, 92].

Let us briefly outline of our construction. We will consider IIB orientifolds of Calabi-Yau manifolds with $h^{1,1} = 1$ which have only space-filling O3 planes. Our main example, described in detail in section two, is an orientifold of the octic hypersurface in weighted projective space $WP^{1,1,1,1,4}$. The D-brane configuration consists of a D5-brane wrapping a holomorphic curve C and an anti-D5-brane wrapping the image curve C' under the orientifold projection. Both C, C' are rigid and do not intersect each other. We also turn on worldvolume $U(1)$ magnetic fluxes so that each brane has p units of induced D3-brane charge.

Such configurations are obviously nonsupersymmetric, at least for generic values of the Kähler moduli, since D5-branes and O3 planes do not preserve the same fraction of supersymmetry. The supersymmetry breaking parameter can be taken to be the phase difference between the central charges of these objects in the underlying $N = 2$ theory. This phase can be computed using standard Π-stability techniques, and depends on the complexified Kähler moduli of the $N = 2$ theory. We will perform detailed computations for the octic orientifold example in section three and appendix A. The outcome of these computations is that this system is not supersymmetric anywhere on the real subspace of the $N = 2$ Kähler moduli space preserved by the orientifold projection. However the supersymmetry breaking parameter reaches a minimum at the Landau-Ginzburg point. This is a new dynamical aspect which has not been encountered before in toroidal orientifolds.

In flat space we would expect this system to decay to a supersymmetric configuration of space-filling D3-branes. The dynamics is different on Calabi-Yau manifolds since the curves C, C' are rigid, which means that the branes have no moduli. This can be viewed as a potential barrier in configuration space opposing brane anti-brane annihilation. If the branes are sufficiently far apart, so that the open string spectrum does not contain tachyons and the attractive force is weak, we will obtain a metastable configuration. The system can still decay, but the decay has to be realized by tunelling effects.

This construction already poses a problem since the $N = 1$ dynamics is very hard to control in a nongeometric phase of the Kähler moduli space. Ideally one would like to describe the theory in terms of a large volume compactification so that the α' corrections are small. This can be achieved in the present context using orientifold mirror symmetry [93, 94, 95, 274]. Since the supersymmetry breaking phase is independent on complex structure moduli, we can take the IIB Calabi-Yau manifold to be near the large complex structure limit point. In this regime, the theory has an a alternative description in terms of a large volume IIA compactification, which will allow us to control the dynamics. Taking this limit, we will be able to compute the D-term effects in section three. We will also show that the results agree with the Π-stability analysis.

Moduli stabilization in this system can be achieved by turning on IIA fluxes as in [97, 98, 99, 100, 101, 277, 274, 103, 104] Since we also have branes in the picture, it turns out that the most convenient description of the flux superpotential involves a combination of IIB and IIA variables. This is a special case of the bi-period superpotentials introduced in [105], except that we have to take into account the D-brane superpotential as well. The F-term effects in the presence of branes and fluxes are described in section four, together with some general aspects of the D-brane configuration space. Our discussion of the brane-flux superpotential builds on previous work on this subject [176, 177, 108, 109, 178, 111], emphasizing the relation between the geometry and the light open-string spectrum.

Finally, in section five we investigate the vacuum structure of the D-brane landscape. We analyze the shape of the potential energy, and formulate sufficient conditions for the existence of nonsupersymmetric metastable vacua. Then we argue that these conditions can be in principle satisfied by tuning the values of background fluxes. In principle this mechanism can give rise to either de Sitter or anti de Sitter vacua, providing an alternative to the existing constructions of de Sitter vacua [276, 81, 113, 114, 115, 116, 117, 118] in string theory.

Note added. When this chapter was ready for submission, two new papers appeared [119, 120] which have partial overlap with our D-term and F-term computations in sections 3 and 4.

3.2 A Mirror Pair of Calabi-Yau Orientifolds

In this section we review some general aspects of Calabi-Yau orientifolds and present our main example. We will first describe the model in IIB variables and then use mirror symmetry to write down the low energy effective action in a specific region in parameter space.

Let us consider a $N = 2$ IIB compactification on a Calabi-Yau manifold X. Such compactifications have a moduli space $\mathcal{M}_h \times \mathcal{M}_v$ of exactly flat directions, where $\mathcal{M}_h$ denotes the hypermultiplet moduli space and $\mathcal{M}_v$ denotes the vector multiplet moduli space. It is a standard fact that $\mathcal{M}_h$ must be quaternionic manifold whereas $\mathcal{M}_v$ must be a special Kähler manifold. The dilaton field is a hypermultiplet component, therefore the geometry of $\mathcal{M}_h$ receives both α' and g_s corrections. By contrast, the geometry of $\mathcal{M}_v$ is exact at tree level in both α' and g_s. The hypermultiplet moduli space $\mathcal{M}_h$ contains a subspace $\mathcal{M}_h^0$ parameterized by vacuum expectation values of NS-NS fields, the RR moduli being set to zero. At string tree level $\mathcal{M}_h^0$ has a special Kähler structure which receives nonperturbative α' corrections. These corrections can be exactly summed using mirror symmetry.

Given a $N = 2$ compactification, we construct a $N = 1$ theory by gauging a discrete symmetry of the form $(-1)^{\epsilon F_L}\Omega\sigma$ where Ω denotes world-sheet parity, F_L is left-moving fermion number and ϵ takes values $0, 1$ depending on the model. $\sigma : X \to X$ is a holomorphic involution of X preserving the holomorphic three-form Ω_X up to sign

$$\sigma^*\Omega_X = (-1)^\epsilon\Omega_X.$$

We will take $\epsilon = 1$, which corresponds to theories with O3/O7 planes. In order to keep the technical complications to a minimum, in this chapter we will focus on models with $h^{1,1} = 1$ which exhibit only O3 planes. More general models could be treated in principle along the same lines, but the details would be more involved.

According to [219], the massless spectrum of $N = 1$ orientifold compactifications can be organized in vector and chiral multiplets. For orientifolds with O3/O7 planes, there are $h_-^{2,1}$ chiral multiplets corresponding to invariant complex structure deformations of X, $h_+^{1,1}$ chiral multiplets corresponding to invariant complexified Kähler deformations of X, and $h_-^{1,1}$ chiral multiplets parameterizing the expectation values of the two-form fields $(B, C^{(2)})$. Moreover, we have a dilaton-axion modulus τ. Note that the real Kähler deformations of X are paired up with expectation values of the four-form field $C^{(4)}$ giving rise to the $h_+^{1,1}$ complexified Kähler moduli. Note also that for one parameter models i.e. $h^{1,1} = 1$, we have $h_-^{1,1} = 0$, hence there are no theta angles $(B, C^{(2)})$.

The moduli space of the $N = 1$ theory must be a Kähler manifold. For small string coupling and large compactification radius the moduli space is a direct product between complex structure moduli, complexified Kähler moduli and a dilaton-axion factor. The Kähler geometry of the moduli space can be determined in this regime by KK reduction of ten dimensional supergravity [219].

For more general values of parameters, the geometry receives both α' and g_s corrections which may not preserve the direct product structure. In particular, we expect significant α' corrections in nongeometric regions of the Kähler moduli space such as the Landau-Ginzburg phase. There is however a different regime in which the geometry of the moduli space is under control, although the Kähler parameters take nongeometric values. This follows from mirror symmetry for orientifolds [93, 94, 95, 274].

Mirror symmetry relates the IIB $N = 2$ compactification on X to a IIA $N = 2$ compactification on the mirror Calabi-Yau manifold Y. The complex structure moduli space $\mathcal{M}_v$ of X is identified to the Kähler moduli space of Y. In particular, there is a special boundary point of $\mathcal{M}_v$ – the large complex structure limit point (LCS) – which is mapped to the large radius limit point of Y. Therefore if the complex structure of the IIB threefold X is close to LCS point, we can find an alternative description of a large radius IIA compactification on Y. This is valid for any values of the Kähler parameters of X, including the region centered around the LG point, which is mapped to the LG point in the complex structure moduli space of Y.

Orientifold models follow the same pattern. Orientifold mirror symmetry relates a Calabi-Yau threefold (X, σ) with holomorphic involution to a threefold (Y, η) equipped with an anti-holomorphic involution η. As long as the holomorphic involution preserves the large complex limit of X, we can map the theory to a large radius IIA orientifold on Y which admits a supergravity description. At the same time, we can take the Kähler parameters of X close to the LG point, which is mapped to the LG point in the complex structure moduli space of Y. This is the regime we will be mostly interested in throughout this chapter.

In this limit, the moduli space of the theory has a direct product structure [274]

$$\mathcal{M} \times \mathcal{K} \tag{3.2.1}$$

where $\mathcal{M}$ is the complex structure moduli space of the IIB orientifold (X, σ) and $\mathcal{K}$ parameterizes the complex structure moduli space of the IIA orientifold (Y, η) and the dilaton. $\mathcal{M}$ can also be identified with the Kähler moduli space of the IIA orientifold, but the description in terms of IIB variables will be more convenient for our purposes. We discuss a specific example in more detail below.

3.2.1 Orientifolds of Octic Hypersurfaces

Our example consists of degree eight hypersurfaces in the weighted projective space $WP^{1,1,1,1,4}$. The defining equation of an octic hypersurface X is

$$P(x_1, \ldots, x_5) = 0 \tag{3.2.2}$$

where P is a homogeneous polynomial of degree eight with respect to the $\mathbb{C}^*$ action

$$(x_1, x_2, x_3, x_4, x_5) \to (\lambda x_1, \lambda x_2, \lambda x_3, \lambda x_4, \lambda^4 x_5).$$

This is a one-parameter model with $h^{1,1}(X) = 1$ and $h^{2,1}(X) = 149$.

In order to construct an orientifold model, consider a family of such hypersurfaces of the form

$$Q(x_1, \ldots, x_4) + x_5(x_5 + \mu x_1 x_2 x_3 x_4) = 0 \tag{3.2.3}$$

where $Q(x_1,\ldots,x_4)$ is a degree eight homogeneous polynomial, and μ is a complex parameter. We will denote these hypersurfaces by $X_{Q,\mu}$. Consider also a family of holomorphic involutions of $WP^{1,1,1,1,4}$ of the form

$$\sigma_\mu : (x_1,x_2,x_3,x_4,x_5) \to (-x_3,-x_4,-x_1,-x_2,-x_5-\mu x_1x_2x_3x_4) \tag{3.2.4}$$

Note that a hypersurface $X_{Q,\mu}$ is invariant under the holomorphic involution σ_μ if and only if Q is invariant under the involution

$$(x_1,x_2,x_3,x_4) \to (-x_3,-x_4,-x_1,-x_2). \tag{3.2.5}$$

We will take the moduli space $\mathcal{M}$ to be the moduli space of hypersurfaces $X_{Q,\mu}$ with Q invariant under (3.2.5). A similar involution has been considered in a different context in [122].

One can easily check that the restriction of σ_μ to any invariant hypersurface $X_{Q,\mu}$ has finitely many fixed points on $X_{Q,\mu}$ with homogeneous coordinates

$$\left(x_1,x_2,\pm x_1,\pm x_2,-\frac{\mu}{2}x_1x_2x_3x_4\right)$$

where (x_1,x_2) satisfy

$$Q(x_1,x_2,\pm x_1,\pm x_2)-\frac{\mu^2}{4}x_1^4x_2^4=0.$$

Moreover the LCS limit point $\mu\to\infty$ is obviously a boundary point of $\mathcal{M}$. This will serve as a concrete example throughout this chapter.

Mirror symmetry identifies the complexified Kähler moduli space $\mathcal{M}_h^0$ of the underlying $N=2$ theory to the complex structure moduli space of the family of mirror hypersurfaces Y

$$x_1^8+x_2^8+x_3^8+x_4^8+x_5^2-\alpha x_1x_2x_3x_4x_5=0 \tag{3.2.6}$$

in $WP^{1,1,1,1,4}/\left(\mathbb{Z}_8^2\times\mathbb{Z}_2\right)$ [123, 124, 125]. At the same time the complex structure moduli space $\mathcal{M}_v$ of octic hypersurfaces is isomorphic to the complexified Kähler moduli space of Y. Orientifold mirror symmetry relates the IIB orientifold (X,σ) to a IIA orientifold determined by (Y,η) where η is a antiholomorphic involution of Y.

For future reference, let us provide some details on the Kähler geometry of the moduli space following [274]. Let z^i, $i=1,\ldots,h_-^{1,2}(X)$, be algebraic algebraic coordinates on the complex structure moduli space $\mathcal{M}$. The Kähler potential for $\mathcal{M}$ in a neighborhood of the large complex structure is given by

$$K_{\mathcal{M}} = -\ln\left(i\int_X \Omega_X \wedge \overline{\Omega}_X\right) \tag{3.2.7}$$

where Ω_X is the global holomorphic three-form on X. This expression is naturally a function of algebraic coordinates on the IIB complex structure moduli space. If we express it in terms of special coordinates adapted to the LCS limit, we will obtain the tree level Kähler potential for the IIA Kähler moduli space [274] plus α' corrections which are exponentially small near the large radius limit.

The second factor $\mathcal{K}$ parameterizes complex structure moduli of IIA orientifold and the dilaton. The corresponding moduli fields are [274] the real complex parameters of Y and the periods of three-form RR potential $C^{(3)}$ preserved by the antiholomorphic involution plus the IIA dilaton.

The antiholomorphic involution preserves the real subspace $\alpha = \overline{\alpha}$ of the $N = 2$ moduli space. This follows from the fact that the IIB B-field is projected out using the mirror map

$$B + iJ = \frac{1}{2\pi i}\ln(z) + \ldots$$

where $z = \alpha^{-8}$ is the natural coordinate on the moduli space of hypersurfaces (3.2.6) near the LCS point.

According to [274] (section 3.3), the Kähler geometry of $\mathcal{K}$ can be described in terms of periods of the three-form Ω_Y and the flat RR three-form C_3 on cycles in Y on a symplectic basis of invariant or anti-invariant three-cycles on Y with respect to the antiholomorphic involution. We will choose a symplectic basis of invariant cycles $(\alpha_0, \alpha_1; \beta^0, \beta^1)$ adapted to the large complex limit $\alpha \to \infty$ of the family (3.2.6). Using standard mirror symmetry technology, one can compute the corresponding period vector $(Z^0, Z^1; \mathcal{F}_0, \mathcal{F}_1)$ near the large complex structure limit by solving the Picard-Fuchs equation. Our notation is so that the asymptotic behavior of the periods as $\alpha \to \infty$ is

$$Z^0 \sim 1 \qquad Z^1 \sim \ln(z) \qquad \mathcal{F}_1 \sim (\ln(z))^2 \qquad \mathcal{F}_0 \sim (\ln(z))^3.$$

Moreover, we also have the following reality conditions on the real axis $\alpha \in \mathbb{R}$

$$\mathrm{Im}(Z^0) = \mathrm{Im}(\mathcal{F}_1) = 0 \qquad \mathrm{Re}(Z^1) = \mathrm{Re}(\mathcal{F}_0) = 0. \tag{3.2.8}$$

This reflects the fact that (α_0, β^1) are invariant and (α^1, β_0) are anti-invariant under the holomorphic involution. The exact expressions of these periods can be found in appendix A. Note that the reality conditions (3.2.8) are an incarnation of the orientifold constraints (3.45) of [274] in our model. In particular, the compensator field C defined in [274] is real in our case, i.e. the phase $e^{-i\theta}$ introduced in [274] equals 1.

The holomorphic coordinates on the moduli space $\mathcal{K}$ are

$$\begin{aligned} \tau &= \frac{1}{2}\xi^0 + iC\mathrm{Re}(Z^0) \\ \rho &= i\widetilde{\xi}_1 - 2C\mathrm{Re}(\mathcal{F}_1) \end{aligned} \tag{3.2.9}$$

where $(\xi^0, \widetilde{\xi}_1)$ are the periods of the three-form field $C^{(3)}$ on the invariant three-cycles (α_0, β^1)

$$C^{(3)} = \xi^0 \alpha_0 - \widetilde{\xi}_1 \beta^1. \tag{3.2.10}$$

Mirror symmetry identifies (τ, ρ) with the IIB dilaton and respectively orientifold complexified Kähler parameter [274], section 6.2.1. A priori, (τ, ρ) are defined in a neighborhood of the LCS, but they can be analytically continued to other regions of the moduli space. We will be interested in neighborhood of the Landau-Ginzburg point $\alpha = 0$, where there is a natural basis of periods $[w_2\ w_1\ w_0\ w_7]^{tr}$ constructed in [124]. The notation and explicit expressions for these periods are reviewed in appendix A. For future reference, note that the LCS periods $(Z^0, \mathcal{F}_1)$ in equation (3.2.9) are related to the LG periods by

$$\begin{bmatrix} Z^0 \\ Z^1 \\ \mathcal{F}_1 \\ \mathcal{F}_0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{3}{2} & -\frac{3}{2} & -\frac{1}{2} \\ -1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} w_2 \\ w_1 \\ w_0 \\ w_7 \end{bmatrix} \tag{3.2.11}$$

Note that this basis is not identical to the symplectic basis of periods computed in [124]; the later does not obey the reality conditions (3.2.8) so we had to perform a symplectic change of basis.

The compensator field C is given by

$$C = e^{-\Phi} e^{K_0(\alpha)/2} \tag{3.2.12}$$

where $e^{\Phi} = e^{\phi}\mathrm{vol}(Y)^{-1/2}$ is the four dimensional IIA dilaton, and

$$\begin{aligned} K_0(\alpha) &= -\ln\left(i \int_Y \Omega_Y \wedge \overline{\Omega}_Y \right)\Bigg|_{\alpha=\overline{\alpha}} \\ &= -\ln\left[2\left(\mathrm{Im}(Z^1)\mathrm{Re}(\mathcal{F}_1) - \mathrm{Re}(Z^0)\mathrm{Im}(\mathcal{F}_0)\right)\right] \end{aligned} \tag{3.2.13}$$

is the Kähler potential of the $N = 2$ complex structure moduli space of Y restricted to the real subspace $\alpha = \overline{\alpha}$. The Kähler potential of the orientifold moduli space is given by [274]

$$\begin{aligned} K_{\mathcal{K}} &= -2\ln\left(2\int_Y \mathrm{Re}(C\Omega_Y) \wedge *\mathrm{Re}(C\Omega_Y) \right) \\ &= -2\ln\left[2C^2\left(\mathrm{Im}(Z^1)\mathrm{Re}(\mathcal{F}_1) - \mathrm{Re}(Z^0)\mathrm{Im}(\mathcal{F}_0)\right)\right]. \end{aligned} \tag{3.2.14}$$

Note that equations (3.2.9), (3.2.12) define $K_{\mathcal{K}}$ implicitly as a function of (τ, ρ). The Kähler potential (3.2.14) can also be written as

$$K_{\mathcal{K}} = -\ln(e^{-4\Phi}) \tag{3.2.15}$$

where Φ is the four dimensional dilaton. Let us conclude this section with a discussion of superpotential interactions.

3.2.2 Superpotential Interactions

There are several types of superpotential interactions in this system, depending on the types of background fluxes. Since the theory has a large radius IIA description, an obvious option is turning on even RR fluxes $F^A = F_2 + F_4 + F_6$ as well as NS-NS flux H^A on the manifold Y. In principle one can also turn on the zero-form flux F_0 as in [277, 103], but we will set to zero throughout this chapter.

Even RR fluxes give rise to a superpotential for type IIA Kähler moduli of the form [126, 127, 274, 99]

$$W_{\mathcal{M}}^{A} = \int_{Y} F^{A} \wedge e^{-J_Y}, \tag{3.2.16}$$

where J_Y is the Kähler form of Y. The type IIA NS-NS flux is odd under the orientifold projection, therefore it will have an expansion

$$H^{A} = q_1\alpha^1 - p^0\beta_0. \tag{3.2.17}$$

According to [274], this yields a superpotential for the IIA complex structure moduli of the form

$$W_{\mathcal{K}}^{A} = -2p^0\tau - iq_1\rho. \tag{3.2.18}$$

The superpotential (3.2.16) can be given a IIB interpretation using mirror symmetry. Recall that in large volume IIB compactifications, one usually has a flux induced superpotential [126]

$$W^{B} = \int_{X} \Omega_X \wedge F^{B} \tag{3.2.19}$$

where F^B is the three-form RR flux on X. For a comprehensive review of IIB flux compactifications with a complete list of references see [128]. Based on the nonrenormalization result of [129], this superpotential does not receive perturbative α' or g_s corrections. Therefore this superpotential formula should still be valid for small values of the IIB Kähler modulus, although we may not have a clear microscopic description of the fluxes. Then the superpotential (3.2.16) can be regarded as a IIB superpotential of the form (3.2.19), where F^B is the IIB RR flux related by mirror symmetry to F^A. Using the mirror map, one can show that the two expressions agree near the LCS point of the IIB moduli space up to exponentially small corrections. For us, it will be more convenient to use the IIB expression, keeping in mind that this is just a reformulation of the large radius IIA superpotential.

In principle, one could also turn the IIB NS-NS flux H^B, but the IIA description of the theory would be more involved. According to [130], the mirror type IIA theory would be a compactification on a manifold with a half-flat $SU(3)$ structure. We will not review this conjecture in detail here. It suffices to note that granting this conjecture one can reformulate the IIB superpotential

$$-\int_X \Omega_X \wedge \tau H^B$$

in IIA variables [130]. More details can be found in [100, 101, 103]. In this chapter we will not turn on IIB NS-NS flux, but it may be helpful to keep in mind that we also have this option.

In conclusion, in the absence of branes, we will have a total superpotential of the form

$$W = W^B + W^A_{\mathcal{K}}. \tag{3.2.20}$$

This formula has to be modified in the presence of magnetized branes. We will discuss the necessary modifications in section 4.

We would like to conclude this section with a remark about tadpole cancellation. Since we have set the IIB NS-NS flux H^B and the type IIA zero-form flux F_0 to zero, the only sources for RR tadpoles are the orientifold planes and the background D-branes. Magnetized D5-branes can also contribute to the tadpole because they carry induced D3-brane charge. Therefore the tadpole cancellation condition can be written as

$$N_{D3} + N_{O3} + p = 0, \tag{3.2.21}$$

where p is the induced D3-brane charge of magnetized D5-branes. As explained in the next section, the best option for us is to saturate this condition by taking $N_{D3} = 0$, i.e. no background D3-branes. Let us turn now to magnetized brane configurations.

3.3 Magnetized Branes on Calabi-Yau Orientifolds

In this section we study the dynamics of magnetized D5-branes wrapping holomorphic curves in Calabi-Yau threefolds. We will analyze their dynamics both from the world-sheet and low energy supergravity point of view. The world-sheet analysis is based on Π-stability considerations in the underlying $N = 2$ theory [131, 202, 133]. Using mirror symmetry, we will show that the world-sheet aspects are captured by D-term effects in the IIA supergravity effective action. Similar computations have been performed for Type I D9-branes in [88], for IIB D3 and D7-branes on Calabi-Yau orientifolds in [134, 135, 136, 137, 138], and for D6-branes in toroidal models in [81, 82, 83, 84, 85, 86, 87]. In particular, a relation between the perturbative part of Π-stability (μ-stability) and supergravity D-terms has been found in [88]. D6-brane configurations in toroidal models have been thoroughly analyzed from the world-sheet point of view in [139, 140]. Earlier work on the subject in the context of rigid supersymmetric theories includes [141, 142, 143, 144]. Our setup is in fact very similar to the situation analyzed in [142], except that we perform a systematic supergravity analysis. Finally, a conjectural formula for the D-term potential energy on D6-branes has been proposed in [145, 146] based on general supersymmetry arguments. We will explain the relation between their expression and the supergravity computation at the end of section 3.2. Let us start with the Π-stability analysis.

3.3.1 Π-stability and magnetized D-branes

From the world-sheet point of view, a wrapped D5-brane is described by a boundary conformal field theory which is a product between an internal CFT factor and a flat space factor. Aspects related to Π-stability and superpotential deformations depend only on the internal CFT part and are independent on the rank of the brane in the uncompactified four dimensions. For example the same considerations apply equally well to a IIB D5-brane wrapping C or to a IIA D2-brane wrapping the same curve. The difference between these two cases resides in the manner of describing the dynamics of the lightest modes in terms of an effective action on the uncompactified directions of the brane. Since the D5-brane is space filling the effective action has to be written in terms of four dimensional supergravity as opposed to the D2-brane effective action, which reduces to quantum mechanics. Nevertheless we would like to stress that in both cases the open string spectrum and the dynamics of the system is determined by identical internal CFT theories; only the low energy effective description of these effects is different. Keeping this point in mind, in this section we proceed with the analysis of the internal CFT factor.

Although our arguments are fairly general, for concreteness we will focus on the octic hypersurface in $WP^{1,1,1,1,4}$. Other models can be easily treated along the same lines. Suppose we have a D5-brane wrapping a degree one rational curve $C \subset X$. Note that curvature effects induce one unit of spacefilling D3-brane charge as shown in appendix A. In order to obtain a pure D5-brane state we have to turn on a compensating magnetic flux in the $U(1)$ Chan-Paton bundle

$$\frac{1}{2\pi}\int_C F = -1.$$

However for our purposes we need to consider states with higher D3-charge, therefore we will turn on $(p-1)$ units of magnetic flux

$$\frac{1}{2\pi}\int_C F = p-1$$

obtaining a total effective $D3$ charge equal to p. The orientifold projection will map this brane to a anti-brane wrapping $C' = \sigma(C)$ with $(-p-1)$ units of flux, where the shift by 2 units is again a curvature effect computed in appendix A.

We will first focus on the underlying $N=2$ theory. Note that this system breaks tree level supersymmetry because the brane and the anti-brane preserve different fractions of the bulk $N=2$ supersymmetry. The $N=1$ supersymmetry preserved by a brane is determined by its central charge which is a function of the complexified Kähler moduli. The central charges of our objects are

$$Z_+ = Z_{D5} + pZ_{D3} \qquad Z_- = -Z_{D5} + pZ_{D3} \tag{3.3.1}$$

where the label $\pm$ refers to the brane and the anti-brane respectively. Z_{D5} is the central charge of a pure D5-brane state, and Z_{D3} is the central charge of a D3-brane on X. The phases of Z_+, Z_- are not aligned for generic values of the Kähler parameters, but they will be aligned along a marginal stability locus where $Z_{D5} = 0$. If this locus is nonempty, these two objects preserve identical fractions of supersymmetry, and their low energy dynamics can be described by a supersymmetric gauge theory. If we deform the bulk Kähler structure away from the $Z_{D5} = 0$ locus, we expect the brane world-volume supersymmetry to be broken. Ignoring supergravity effects, this supersymmetry breaking can be modeled by Fayet-Iliopoulos couplings in the low energy gauge theory. We will provide a supergravity description of the dynamics in the next subsection. This effective description is valid at weak string coupling and in a small neighborhood of the marginal stability locus in the Kähler moduli space. For large deformations away from this locus the effective gauge theory description breaks down, and we would have to employ string field theory for an accurate description of D-brane dynamics.

Returning to the orientifold model, note that the orientifold projection leaves invariant only a real dimensional subspace of the $N = 2$ Kähler moduli space, because it projects out the NS-NS B-field. As explained in section 2.1, the IIB complexified Kähler moduli space can be identified with the complex structure moduli space of the family of mirror hypersurfaces (3.2.6). The subspace left invariant by the orientifold projection is $\alpha = \overline{\alpha}$.

Therefore it suffices to analyze the D-brane system along this real subspace of the moduli space. Note that orientifold $O3$ planes preserve the same fraction of supersymmetry as D3-branes. Therefore the above $D5 - \overline{D5}$ configuration would still be supersymmetric along the locus $Z_{D5} = 0$ because the central charges (3.3.1) are aligned with Z_{D3}. Analogous brane configurations have been considered in [243] for F-theory compactifications.

A bulk Kähler deformation away from the supersymmetric locus will couple to the world-volume theory as a D-term because this is a disc effect which does not change in the presence of the orientifold projection. This will be an accurate description of the system as long as the string coupling is sufficiently small and we can ignore higher order effects. Note that the $Z_{D5} = 0$ locus will generically intersect the real subspace of the moduli space along a finite (possibly empty) set.

To summarize the above discussion, the dynamics of the brane anti-brane system in the $N = 1$ orientifold model can be captured by D-term effects at weak string coupling and in a small neighborhood of the marginal stability locus $Z_{D5} = 0$ in the Kähler moduli space. Therefore our first concern should be to find the intersection between the marginal stability locus and the real subspace $\alpha = \overline{\alpha}$ of the moduli space. A standard computation performed in appendix A shows that the central charges Z_{D3}, Z_{D5} are given by

$$Z_{D3} = Z^0 \qquad Z_{D_5} = Z^1.$$

in terms of the periods $(Z^0, Z^1; \mathcal{F}^1, \mathcal{F}^0)$ introduced in section 2.1. Then the formulas (3.3.1) become

$$Z_+ = pZ^0 + Z^1, \qquad Z_- = pZ^0 - Z^1. \tag{3.3.2}$$

In appendix A we show that the relative phase

$$\theta = \frac{1}{\pi}\left(\mathrm{Im}\ln(Z_+) - \mathrm{Im}\ln(Z_{D3})\right) \tag{3.3.3}$$

between Z_+ and Z_{D3} does not vanish anywhere on the real axis $\alpha = \overline{\alpha}$ and has a minimum at the Landau-Ginzburg point $\alpha = 0$. The value of θ at the minimum is approximatively $\theta_{min} \sim 1/p$. For illustration, we represent in fig 1. the dependence $\theta = \theta(\alpha)$ near the Landau-Ginzburg point for three different values of p, $p = 10, 20, 30$. Note that the minimum value of theta is $\theta_{min} \sim 0.12$, therefore we expect the dynamics to have a low energy supergravity description.

It is clear from this discussion that the best option for us is to take the number p as high as possible subject to the tadpole cancellation constraints (3.2.21). This implies that there are no background D3-branes in the system, and we set $p = N_{O3}$. In fact configurations with background D3-branes would not be stable since there would be an attractive force between D3-branes and magnetized D5-branes. Therefore the system will naturally decay to a configuration in which all D3-branes have been converted into magnetic flux on D5-branes.

In order for the above construction to be valid, one has to check whether the D3-brane and D5-brane are stable BPS states at the Landau-Ginzburg point. This is clear in a neighborhood of the large radius limit, but in principle, these BPS states could decay before we reach the Landau-Ginzburg point. For example it is known that in the $\mathbb{C}^2/\mathbb{Z}_3$ local model the D5-brane decays before we reach the orbifold point in the Kähler moduli space [148]. The behavior of the BPS spectrum of compact Calabi-Yau threefolds is less understood at the present stage. At best one can check stability of a BPS state with respect to a particular decay channel employing Π-stability techniques [133, 131, 202], but we cannot rigorously prove stability using the formalism

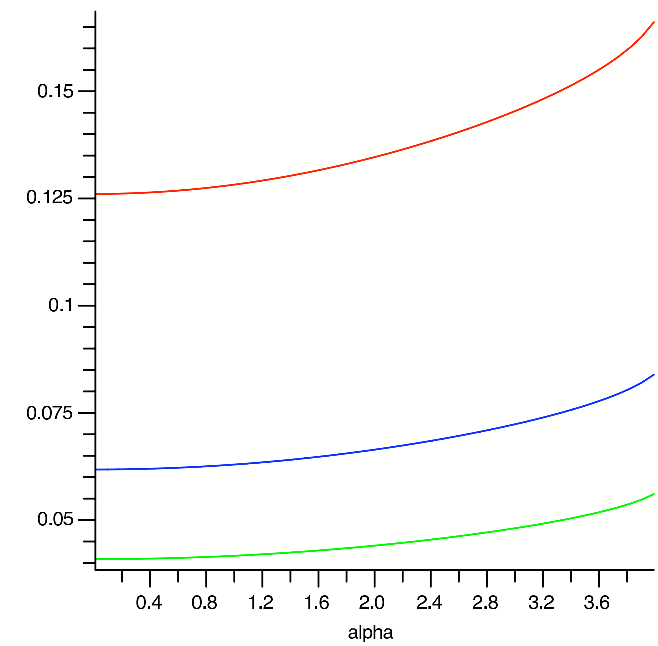

Figure 3.1: The behavior of the relative phase θ near the LG point for three different values of p. Red corresponds to $p = 10$, blue corresponds to $p = 20$ and green corresponds to $p = 30$.

developed in [149, 150]. In appendix A we show that magnetized D5-branes on the octic are stable with respect to the most natural decay channels as we approach the Landau-Ginzburg point. This is compelling evidence for their stability in this region of the moduli space, but not a rigorous proof. Based on this amount of evidence, we will assume in the following that these D-branes are stable in a neighborhood of the Landau-Ginzburg point. Our next task is the computation of supergravity D-terms in the mirror IIA orientifold described in section 2.1.

3.3.2 Mirror Symmetry and Supergravity D-terms

The above Π-stability arguments are independent of complex structure deformations of the IIB threefold X. We can exploit this feature to our advantage by working in a neighborhood of the LCS point in the complex structure moduli space of X. In this region, the theory admits an alternative description as a large volume IIA orientifold on the mirror threefold Y. The details have been discussed in section 2.1. In the following we will use the IIA description in order to compute the D-term effects on magnetized branes.

Open string mirror symmetry maps the D5-branes wrapping C, C' to D6-branes wrapping special lagrangian cycles M, M' in Y. Since C, C' are rigid disjoint $(-1,-1)$ curves for generic moduli of X, M, M' must be rigid disjoint three-spheres in Y. The calibration conditions for M, M' are of the form

$$\mathrm{Im}(e^{i\theta}\Omega_Y|_M) = 0 \qquad \mathrm{Im}(e^{-i\theta}\Omega_Y|_{M'}) = 0. \tag{3.3.4}$$

where Ω_Y is normalized so that the calibration of the IIA orientifold O6-planes has phase 1. The phase $e^{i\theta}$ in (3.3.4) is equal to the relative phase (3.3.3) computed above, and depends only on the complex structure moduli of Y. The homology classes of these cycles can be read off from the central charge formula (3.3.2). We have

$$[M] = p\beta^0 + \beta^1, \qquad [M'] = p\beta^0 - \beta^1 \tag{3.3.5}$$

where $[M], [M']$ are cohomology classes on Y related to M, M' by Poincaré duality.

Taking into account $N = 1$ supergravity constraints, the D-term contribution is of the form

$$U_D = \frac{D^2}{2\mathrm{Im}(g)} \tag{3.3.6}$$

where g is the holomorphic coupling constant of the brane $U(1)$ vector multiplet. The holomorphic coupling constant can be easily determined by identifying the four dimensional axion field a which has a coupling of the form

$$\int aF \wedge F \tag{3.3.7}$$

with the $U(1)$ gauge field on the brane. Such couplings are obtained by dimensional reduction of Chern-Simons terms of the form action.

$$\int C^{(3)} \wedge F \wedge F + C^{(5)} \wedge F$$

in the D6-brane world-volume action. Taking into account the expression (3.2.10) for $C^{(3)}$, dimensional reduction of the Chern-Simons term on the cycle M yields the following four-dimensional couplings

$$p \int \xi^0 F \wedge F + \int D^1 \wedge F. \tag{3.3.8}$$

Here ξ^0 is the axion defined in (3.2.10) and D^1 is the two-form field obtained by reduction of $C^{(5)}$

$$C^{(5)} = D^1 \wedge \alpha^1.$$

Equation (3.3.8) shows that the axion field a in (3.3.7) is ξ^0. Then, using holomorphy and equation (3.2.9), it follows that the tree level holomorphic gauge coupling g must be

$$g = 2p\tau. \tag{3.3.9}$$

The second coupling in (3.3.8) is also very useful. The two-form field D^1 is part of an $N = 1$ linear multiplet L^1 whose lowest component is the real field $e^{2\Phi}\mathrm{Im}(Z^1)$, where Φ is the four dimensional dilaton [274]. Moreover, one can relate L to the chiral multiplet ρ by a duality transformation which converts the second term in (3.3.8) into a coupling of the form

$$\int A_\mu \partial^\mu \widetilde{\xi}_1.$$

The supersymmetric completion of this term determines the supergravity D-term to be [151, 152, 153, 154]

$$D = \partial_\rho K_{\mathcal{K}}. \tag{3.3.10}$$

Note that using equation (B.9) in [274], the D-term (3.3.10) can be written as

$$D = -2e^{2\Phi}\mathrm{Im}(CZ^1) \tag{3.3.11}$$

where C is the compensator field defined in equation (3.2.12). Using equations (3.2.9) and (3.2.15), we can rewrite (3.3.11) as

$$\begin{aligned} D &= -2e^{K_\mathcal{K}/2}\mathrm{Im}(CZ^1) \\ &= -\frac{1}{C}\frac{\mathrm{Im}(Z^1)}{\mathrm{Im}(Z^1)\mathrm{Re}(\mathcal{F}_1)-\mathrm{Re}(Z^0)\mathrm{Im}(\mathcal{F}_0)} \\ &= -\frac{1}{\mathrm{Im}(\tau)}\frac{\mathrm{Re}(Z^0)\mathrm{Im}(Z^1)}{\mathrm{Im}(Z^1)\mathrm{Re}(\mathcal{F}_1)-\mathrm{Re}(Z^0)\mathrm{Im}(\mathcal{F}_0)}. \end{aligned} \tag{3.3.12}$$

Then, taking into account (3.3.9), we find the following expression for the D-term potential energy

$$U_D = \frac{1}{4p\mathrm{Im}(\tau)^3}\left[\frac{\mathrm{Re}(Z^0)\mathrm{Im}(Z^1)}{\mathrm{Im}(Z^1)\mathrm{Re}(\mathcal{F}_1)-\mathrm{Re}(Z^0)\mathrm{Im}(\mathcal{F}_0)}\right]^2. \tag{3.3.13}$$

This is our final formula for the D-term potential energy.

In order to conclude this section, we would like to explain the relation between formula (3.3.13) and the Π-stability analysis performed earlier in this section. Note that the Π-stability considerations are captured by an effective potential in the mirror type IIA theory which was found in [145, 146]. According to [145, 146], the D-term potential for a pair of D6-branes as above should be given by

$$V_D = 2e^{-\Phi}\left(\left|\int_M \widehat{\Omega}_Y\right| - \int_M \mathrm{Re}(\widehat{\Omega}_Y)\right) \tag{3.3.14}$$

where $\widehat{\Omega}_Y$ is the holomorphic three-form on Y normalized so that

$$i\int_Y \widehat{\Omega}_Y \wedge \overline{\widehat{\Omega}}_Y = 1.$$

Recall that Φ denotes the four dimensional dilaton.

In the following we would like to explain that this expression is in agreement with the supergravity formula (3.3.13) for a small supersymmetry breaking angle $|\theta| << 1$. For large $|\theta|$ the effective supergravity description of the theory breaks down, and we would have to employ string field theory in order to obtain reliable results.

Note that one can write

$$\widehat{\Omega}_Y = e^{K_0/2}\Omega_Y \tag{3.3.15}$$

where K_0 is has been defined in equation (3.2.13), and Ω_Y has some arbitrary normalization. The expression in the right hand side of this equation is left invariant under rescaling Ω_Y by a nonzero constant.

Formula (3.3.14) is written in the string frame. In order to compare it with the supergravity expression, we have to rewrite it in the Einstein frame. In the present context, the string metric has to be rescaled by a factor of $e^{2\phi}(\mathrm{vol}(Y))^{-1} = e^{2\Phi}$ [155], hence the potential energy in the Einstein frame is

$$V_D^E = 2e^{3\Phi}\left(\left|\int_M \widehat{\Omega}_Y\right| - \int_M \mathrm{Re}(\widehat{\Omega}_Y)\right). \tag{3.3.16}$$

Taking into account equations (3.3.5) and (3.3.15) we have

$$\int_M \widehat{\Omega}_Y = e^{K_0/2}(p\mathrm{Re}(Z^0) + i\mathrm{Im}(Z^1)) = e^{K_0/2} Z_+$$

where Z_+ is the central charge defined in equation (3.3.1). For small values of the phase, $|\theta| << 1$, we can expand (3.3.16) as

$$V_D^E \sim e^{3\Phi} e^{K_0/2} \frac{\mathrm{Re}(Z_0)}{p} \left[\frac{\mathrm{Im}(Z^1)}{\mathrm{Re}(Z^0)}\right]^2. \tag{3.3.17}$$

Now, using equations (3.2.9) and (3.3.11) in (3.3.6), we obtain

$$U_D = Ce^{4\Phi} \frac{\mathrm{Re}(Z^0)}{p} \left[\frac{\mathrm{Im}(Z^1)}{\mathrm{Re}(Z^0)}\right]^2 = e^{3\Phi} e^{K_0/2} \frac{\mathrm{Re}(Z^0)}{p} \left[\frac{\mathrm{Im}(Z^1)}{\mathrm{Re}(Z^0)}\right]^2 \tag{3.3.18}$$

Therefore the supergravity D-term potential agrees indeed with (3.3.14) for very small supersymmetry breaking angle. This generalizes the familiar connection between Π-stability and D-term effects to supergravity theories. In order to complete the description of the dynamics, we will focus next on superpotential interactions.

3.4 Fluxes, Branes and Superpotential Interactions

In this section we study superpotential interactions of magnetized branes in Calabi-Yau orientifolds with background fluxes. Brane-flux superpotentials have been first discussed in [108, 109]. Our treatment is based on the same idea, although our treatment of compact Calabi-Yau situations will be closer to [178, 111].

Let us first consider magnetized branes in the absence of fluxes. The fluxes will naturally enter the picture at a later stage. Our first task is to identify the lowest lying modes which govern the low energy physics in the presence of D-branes. The massless fields correspond to marginal deformations of the internal bulk-boundary CFT. Suppose we have a D-brane wrapping a holomorphic curve C in a Calabi-Yau threefold X. The marginal deformations of the bulk-boundary CFT are in one-to-one correspondence with deformations of the pair (X, C). The infinitesimal deformations of X are classified by $H^1(X, T_X)$. Using a standard spectral sequence, one can show that the space $\mathbb{H}$ of infinitesimal deformations of the pair (X, C) fits in an exact sequence the form

$$0 \longrightarrow H^0(C, N_{C/X}) \longrightarrow \mathbb{H} \longrightarrow H^1(X, T_X) \xrightarrow{f} H^1(C, N_{C/X}) \qquad (3.4.1)$$

where $N_{C/X}$ is the normal bundle to C in X. The map $f : H^1(X, T_X) \to H^1(C, N_{C/X})$ is induced by the natural projection $T_X \to N_{C/X}$.

From a physical point of view the first term in (3.4.1), $H^0(C, N_{C/X})$ parameterizes marginal boundary operators. The third term $H^1(X, T_X)$ parameterizes marginal deformations of the bulk CFT in the absence of boundaries. It is important to note that not all these marginal operators remain marginal in the bulk-boundary CFT. In fact the exact sequence (3.4.1) shows that only those deformations in $H^1(X, T_X)$ which map to zero in $H^1(C, N_{C/X})$ are marginal deformations of the bulk-boundary theory.

In our case the Calabi-Yau threefold X is equipped with a holomorphic involution σ, and and the magnetized branes are wrapped on two disjoint curves $C, C' = \sigma(C)$ on X. Then the infinitesimal deformations are captured by the invariant part of (3.4.1) with respect to σ

$$0 \longrightarrow H^0(C, N_{C/X}) \longrightarrow \mathbb{H}_+ \longrightarrow H^1(X, T_X)_+ \xrightarrow{f_+} H^1(C, N_{C/X}). \qquad (3.4.2)$$

Let us denote by $\mathcal{N}$ a connected component of the moduli space of data (X, σ, C, C'). Note that there is a natural forgetful map $\rho : \mathcal{N} \to \mathcal{M}$, where $\mathcal{M}$ is a connected component of the moduli space of Calabi-Yau threefolds (X, σ) with involution. At a generic point in $\mathcal{N}$, C, C'

are $(-1,-1)$ curves on X, hence

$$H^0(C, N_{C/X}) = H^0(C', N_{C'/X}) = 0.$$

The only low energy light modes near such a point in the moduli space correspond to deformations of X which preserve (σ, C, C'). The map $\rho : \mathcal{N} \to \mathcal{M}$ is locally finite-to-one near such a point. However, the curves C, C' may have nontrivial normal deformations in X for special values of the complex structure moduli. These normal deformations yield new light fields which have to be taken into account in the low energy effective action. This behavior is similar in spirit with the Seiberg-Witten solution of $N = 2$ gauge theories. Around each point, the low energy theory will have an effective superpotential which is a holomorphic function of the lightest fields in the spectrum near that point.

The local expression of the superpotential on $\mathcal{N}$ is given by a three-chain period of the holomorphic three-form on X [176, 177]. More precisely, the space $\mathcal{N}$ can be locally identified near each point (X, σ, C, C') with an open set $\mathcal{U}$ in the linear space

$$H^0(C, N_{C/X}) \oplus \mathrm{Ker}(f_+).$$

Let us pick a three chain Γ_0 interpolating between C, C' on X i.e.

$$\partial\Gamma = C' - C.$$

Then we can extend Γ_0 to a multivalued family of three-chains Γ_u, $u \in \mathcal{U}$ so that

$$\partial\Gamma_u = C_u - C'_u$$

for each $u \in \mathcal{U}$ [178]. This extension is obtained by transporting the three-chain Γ_0 to any point in $\mathcal{U}$ using the Gauss-Manin connection. The superpotential is a holomorphic function on $\mathcal{U}$ given by

$$W = \int_{\Gamma_u} \Omega_{X_u} \tag{3.4.3}$$

where Ω_{X_u} is the global holomorphic three-form on X_u. Since the overall normalization of Ω_{X_u} is not fixed, (3.4.3) actually defines a local section of the line bundle $\rho^*\mathcal{L}$ over $\mathcal{U}$.

Note that the expression (3.4.3) is ambiguous since the chain Γ_0 is only defined up to a shift

$$\Gamma_0 \to \Gamma_0 + \gamma. \tag{3.4.4}$$

where γ is a closed three-cycle on X. This is not a problem from a mathematical point of view since one can show that the critical set of W is independent of the choice of Γ_0. Nevertheless this ambiguity has a very natural physical interpretation because we can interpret a shift of the form (3.4.4) as a shift in the background RR flux. More precisely, note that the shift (3.4.4) changes the superpotential (3.4.3) by

$$\Delta W = \int_{\gamma_u} \Omega_{X_u}$$

where γ_u is again a family of three-cycles obtained by parallel transport with respect to the Gauss-Manin connection. Therefore, using Poincaré duality, we can identify the ambiguity in the choice of Γ_0 with a shift

$$F \to F + \eta$$

in the background RR flux F on X, where $\eta \in H^3(X, \mathbb{Z})$ is the Poincaré dual of γ. This identification is natural since in the presence of D-branes, the RR flux is not well defined as an element of $H^3(X, \mathbb{Z})$; an element of $H^3(X, \mathbb{Z})$ only determines a shift in the background flux, but the overall value of the flux depends on the choice of a trivialization of the D-brane charge [156]. More formally, the RR fluxes take values in a torsor over $H_3(X, \mathbb{Z})$.

We are therefore led to the conclusion that in superstring compactifications, the superpotential (3.4.3) should be interpreted as a combined brane – RR flux superpotential. There is no natural way of splitting this formula in separate brane and respectively RR flux contributions, but changes in the background flux are captured by shifts of the form (3.4.4). Although in this section we have used IIB variables, (3.4.3) can be equally interpreted as a IIA superpotential using open string mirror symmetry.

We conclude this discussion with a few remarks. In the next section we will investigate the vacuum structure of magnetized branes in the octic orientifold taking into account both F-term and D-term effects.

(i) The superpotential (3.4.3) depends only on the complex structure deformations of X which preserve the curve C. In general these deformations span a proper closed subspace of the moduli space $\mathcal{M}$. We argued that the remaining complex moduli of X are generically massive and do not appear in the low energy effective action. This argument is in principle correct at generic points in the moduli space, but it may fail at special points in the moduli space where the fields we have integrated out become light. Such effects can be taken into account extending the superpotential (3.4.3) to a local function of all complex structure moduli. Let us consider an open subset $\mathcal{V}$ of

$$H^0(C, N_{C/X}) \oplus H^1(X, T_X)$$

containing $\mathcal{U}$ as a closed subset. Then we can use the Gauss-Manin connection to extend the three-chain Γ_0 to a family of three-chains Γ_v labeled by points in $\mathcal{V}$ and define the extension of W to be

$$W = \int_{\Gamma_v} \Omega_{X_v}. \tag{3.4.5}$$

The main difference with respect to the previous case is that the boundary of Γ_v is no longer a holomorphic cycle on X_v if v is not in $\mathcal{U}$.

(ii) The low energy theory may contain extra light open string fields at points in the moduli space $\mathcal{N}$ where the two curves C, C' coincide. Then we will have additional superpotential interactions involving these fields as well.

(iii) The expression (3.4.3) is very similar to the flux superpotential (3.2.19). In particular they have the same tree level dependence on the dilaton multiplet τ and they are subject to the same axion shift symmetries. Therefore, using the same low energy arguments as [129] one can show that (3.4.3) is subject to the same nonrenormalization result. This means that this formula is reliable at small IIB volume.

(iv) In general situations, the superpotential (3.4.3) cannot be canonically split into a brane contribution and a flux contribution of the form (3.2.19). However, in special cases, this is possible using specific features of the geometry. For example suppose the threefold X contains a connected family of holomorphic curves interpolating between C, C'. Then one can choose the three-chain Γ to be swept by a real one-parameter family of holomorphic curves in X. It is known that the period of Ω_X on such three-chains vanishes. Therefore if we make such a choice, the superpotential (3.4.3) will be identically zero. Then a shift of the form (3.4.4) will produce a superpotential of the form (3.2.19).

3.5 The D-Brane Landscape

In this section we explore the magnetized D-brane landscape in the octic orientifold model introduced in section 2. We compute the F-term and D-term contributions in a neighborhood of the Landau-Ginzburg point in the IIA complex structure moduli space $\mathcal{K}$. For technical reasons we will not be able to find explicit solutions to the critical point equations. However, given the shape of the potential, we will argue that metastable vacuum solutions are statistically possible by tuning the values of fluxes.

Throughout this section we will be working at a generic point in the configuration space where all open string fields are massive and can be integrated out. Following the reasoning of the previous section, this is the expected behavior for D-branes wrapping isolated rigid holomorphic curves in a Calabi-Yau threefold. One should however be aware of several possible loopholes in this assumption since open string fields may become light along special loci in the moduli space.

In our situation, one should be especially careful with the open string-fields in the brane anti-brane sector. According to the Π-stability analysis in section 3, there is a tachyonic contribution to the mass of the lightest open string modes proportional to the phase difference θ. At the same time, we have a positive contribution to the mass due to the tension of the string stretching between the branes. In order to avoid tachyonic instabilities, we should work in a region of the moduli space where the positive contribution is dominant. Since the curves are isolated, the positive mass contribution is generically of the order of the string scale, which is much larger than the tachyonic contribution, since θ is of the order 0.05. Therefore we do not expect tachyonic instabilities in the system as long as the moduli are sufficiently generic.

This argument can be made more precise in the mirror IIA picture. As discussed in section 3.2, the IIA description of the system involves two disjoint special lagrangian cycles M, M' on the Calabi-Yau manifold Y. The position of M, M' in Y is determined by the calibration conditions (3.3.4), which are invariant under a rescaling of the metric on Y by a constant $\lambda > 1$. Such a rescaling would also increase the minimal geodesic distance between Y, Y', which determines the mass of the open string modes. Therefore, if the volume of Y is sufficiently large, we expect the brane anti-brane fields to have masses at least of the order of the string scale.

Even if the open string fields have a positive mass, the system can still be destabilized by the brane anti-brane attraction force. Generically, we expect this not to be the case as long as the brane-brane fields are sufficiently massive since the attraction force is proportional to θ and it is also suppressed by a power of the string coupling. We can understand the qualitative aspects of the dynamics using a simplified model for the potential energy. Suppose that the effective dynamics of the branes can be described in terms of a single light chiral superfield Φ. Typically this happens when we work near a special point X_0 in the complex structure moduli space where the curves C, C' belong to a one parameter family $\mathcal{C}$ of holomorphic curves. The field Φ corresponds to normal deformations of the brane wrapping C, which are identified with normal deformations of the anti-brane wrapping C' by the orientifold projection. A sufficiently generic small complex deformation of X away from X_0 induces a mass term for Φ. Therefore we can model the effective dynamics of the system by a potential of the form

$$m(r - r_0)^2 + c \ln\left(\frac{r}{r_0}\right)$$

where r parameterizes the separation between the branes. The quadratic terms models a mass term for the open string fields corresponding to normal deformations of the branes in the ambient manifold. The second term models a typical two dimensional attractive brane anti-brane potential. The constant $c > 0$ is proportional to the phase θ and the string coupling g_s. Now one can check that if $c << mr_0$, this potential has a local minimum near $r = r_0$, and the local shape of the potential near this minimum is approximatively quadratic. In our case, we expect m, r_0 to be typically of the order of the string scale, whereas $c \sim g_s\theta \sim 10^{-2}$ therefore the effect of the attractive force is negligible.

Since it is technically impossible to make these arguments very precise, we will simply assume that there is a region in configuration space where destabilizing effects are small and do not change the qualitative behavior of the system. Moreover, all open string fields are massive, and we can describe the dynamics only in terms of closed string fields. This point of view suffices for a statistical interpretation of the D-brane landscape. By tuning the values of fluxes, one can in principle explore all regions of the configuration space. The vacuum solutions which land outside the region of validity of this approximation will be automatically destabilized by some of these effects. Therefore there is a natural selection mechanism which keeps only vacuum solutions located at a sufficiently generic point in the moduli space.

Granting this assumption, we will take the configuration space to be isomorphic to the closed string moduli space $\mathcal{M}\times\mathcal{K}$ described in section 2.1. As discussed in section 2.2, we will turn on only RR fluxes $F^A = F_2 + F_4 + F_6$ and NS-NS flux H^A. In the presence of branes, the NS-NS flux H^A must satisfy the Freed-Witten anomaly cancellation condition [157], which states that the the restriction of H^A to the brane world-volumes M, M' must be cohomologically trivial. Taking into account equations (3.2.17), (3.3.5), it follows that the integer q_1 in (3.2.17) must be set to zero. Therefore the superpotential does not depend on the chiral superfield ρ. This can also be seen from the analysis of supergravity D-terms in section 3.2. The $U(1)$ gauge group acts as an axionic shift symmetry on ρ, therefore gauge invariance rules out any ρ-dependent terms in the superpotential [158]. The connection between the Freed-Witten anomaly condition and supergravity has been observed before in [100].

The total effective superpotential is then given by

$$W = \int_\Gamma \Omega_X - 2p^0\tau, \qquad (3.5.1)$$

where Γ is a three-chain on X interpolating between the two curves C, C'. As explained in remark (i) section 4, this expression makes sense over the entire moduli space $\mathcal{M}$ although some complex structure deformations may not preserve the curves C, C'. This only means that some complex structure moduli fields are actually massive, and their mass terms are encoded in W. Alternatively, one can take the configuration space to be of the form $\mathcal{N} \times \mathcal{K}$ by integrating out the massive fields, but the two points of view are equivalent, at least generically.

The F-term contribution to the potential energy is

$$U_F = e^K \left(g^{i\bar{j}} (D_i W)(D_{\bar{j}} \overline{W}) + g^{a\bar{b}} (D_a W)(D_{\bar{b}} \overline{W}) - 3|W|^2 \right). \tag{3.5.2}$$

where $i, j, \ldots$ label complex coordinates on $\mathcal{M}$ and and $a, b = \rho, \tau$ label complex coordinates on $\mathcal{K}$. The D-term contribution is given by equation (3.3.13). We reproduce it below for convenience

$$U_D = \frac{1}{4p\text{Im}(\tau)^3} \left[\frac{\text{Re}(Z^0)\text{Im}(Z^1)}{\text{Im}(Z^1)\text{Re}(\mathcal{F}_1) - \text{Re}(Z^0)\text{Im}(\mathcal{F}_0)} \right]^2 .$$

Since the moduli space of the theory is a direct product $\mathcal{K} \times \mathcal{M}$, the Kähler potential K in (3.5.2) is

$$K = K_{\mathcal{K}} + K_{\mathcal{M}}.$$

Note that we Kähler potentials $K_{\mathcal{K}}$, $K_{\mathcal{M}}$ satisfy the following noscale relations [274]

$$g^{i\bar{j}} \partial_i K_{\mathcal{M}} \partial_{\bar{j}} K_{\mathcal{M}} = 3 \qquad g^{a\bar{b}} \partial_a K_{\mathcal{K}} \partial_{\bar{b}} K_{\mathcal{K}} = 4. \tag{3.5.3}$$

Using equations (3.2.9) and (3.2.14), we have

$$e^{K_{\mathcal{K}}} = \frac{1}{4\text{Im}(\tau)^4} \left[\frac{\text{Re}(Z^0)^2}{\text{Im}(Z^1)\text{Re}(\mathcal{F}_1) - \text{Re}(Z^0)\text{Im}(\mathcal{F}_0)} \right]^2 .$$

Now we have a complete description of the potential energy of the system. Finding explicit vacuum solutions using these equations seems to be a daunting computational task, given the complexity of the problem. We can however gain some qualitative understanding of the resulting landscape by analyzing the potential energy in more detail.

First we have to find a convenient coordinate system on the moduli space $\mathcal{K}$. Note that the potential energy is an implicit function of the holomorphic coordinates (τ, ρ) via relations (3.2.9). One could expand it as a power series in (τ, ρ), but this would be an awkward process. Moreover, the axion $\widetilde{\xi}_1 = \text{Im}(\rho)$ is eaten by the $U(1)$ gauge field, and does not enter the expression for the potential. Therefore it is more natural to work in coordinates (τ, α) where

α is the algebraic coordinate on the underlying $N = 2$ Kähler moduli space. As explained in section 2.1, α takes real values in the orientifold theory.

There is a more conceptual reason in favor of the coordinate α instead of ρ, namely α is a coordinate on the Teichmüller space of Y rather than the complex structure moduli space. Since in the Π-stability framework the phase of the central charge is defined on the Teichmüller space, α is the natural coordinate when D-branes are present.

Next, we expand the potential energy in terms of (τ, α) using the relations (3.2.9). Dividing the two equations in (3.2.9), we obtain

$$\frac{\rho + \overline{\rho}}{\tau - \overline{\tau}} = 2i \frac{\mathrm{Re}(\mathcal{F}_1)}{\mathrm{Re}(Z^0)} \tag{3.5.4}$$

Let us denote the ratio of periods in the right hand side of equation (3.2.9) by

$$R(\alpha) = \frac{\mathrm{Re}(\mathcal{F}_1)}{\mathrm{Re}(Z^0)}. \tag{3.5.5}$$

Using equations (3.5.4) and (3.5.5), we find the following relations

$$\frac{\partial \alpha}{\partial \rho} = \frac{1}{2i} \frac{1}{\tau - \overline{\tau}} \left(\frac{\partial R}{\partial \alpha} \right)^{-1} \qquad \frac{\partial \alpha}{\partial \tau} = -\frac{R}{\tau - \overline{\tau}} \left(\frac{\partial R}{\partial \alpha} \right)^{-1}. \tag{3.5.6}$$

Now, using the chain differentiation rule, we can compute the derivatives of the Kähler potential as functions of (τ, α). Let us introduce the notation

$$V(\alpha) = \frac{\mathrm{Im}(Z^1)\mathrm{Re}(\mathcal{F}_1) - \mathrm{Re}(Z^0)\mathrm{Im}(\mathcal{F}_0)}{\mathrm{Re}(Z^0)^2}.$$

Then we have

$$\begin{aligned}
\partial_\tau K_{\mathcal{K}} &= -\partial_{\overline{\tau}} K_{\mathcal{K}} = -\frac{2}{\tau - \overline{\tau}} \left[2 - R \frac{\partial_\alpha V}{V} (\partial_\alpha R)^{-1} \right] \\
\partial_\rho K_{\mathcal{K}} &= \partial_{\overline{\rho}} K_{\mathcal{K}} = \frac{i}{\tau - \overline{\tau}} \frac{\partial_\alpha V}{V} (\partial_\alpha R)^{-1} \\
\partial_{\tau\overline{\tau}} K_{\mathcal{K}} &= -\frac{2}{(\tau - \overline{\tau})^2} \left[2 - R \frac{\partial_\alpha V}{V} (\partial_\alpha R)^{-1} - R \partial_\alpha \left(R \frac{\partial_\alpha V}{V} (\partial_\alpha R)^{-1} \right) (\partial_\alpha R)^{-1} \right] \\
\partial_{\tau\overline{\rho}} K_{\mathcal{K}} &= -\partial_{\rho\overline{\tau}} K_{\mathcal{K}} = -\frac{i}{(\tau - \overline{\tau})^2} \left[\frac{\partial_\alpha V}{V} (\partial_\alpha R)^{-1} + R \partial_\alpha \left(\frac{\partial_\alpha V}{V} (\partial_\alpha R)^{-1} \right) (\partial_\alpha R)^{-1} \right] \\
\partial_{\rho\overline{\rho}} K_{\mathcal{K}} &= \frac{1}{2(\tau - \overline{\tau})^2} \partial_\alpha \left(\frac{\partial_\alpha V}{V} (\partial_\alpha R)^{-1} \right) (\partial_\alpha R)^{-1}
\end{aligned} \tag{3.5.7}$$

Using equations (3.5.7), and the power series expansions of the periods computed in appendix A, we can now compute the expansion of the potential energy as in terms of (τ, α). The D-term contribution takes the form

$$U_D = \frac{1}{p\mathrm{Im}(\tau)^3}(0.03125 - 0.00178\alpha^2 + 0.00005\alpha^4 + \ldots). \tag{3.5.8}$$

We will split the F-term contribution into two parts

$$U_F = U_F^{\mathcal{M}} + U_F^{\mathcal{K}}$$

where

$$U_F^{\mathcal{M}} = e^{K_{\mathcal{K}}+K_{\mathcal{M}}}\left(g^{i\bar{j}}(D_iW)(D_{\bar{j}}\overline{W}) - 3|W|^2\right)$$

$$U_F^{\mathcal{K}} = e^{K_{\mathcal{K}}+K_{\mathcal{M}}}\left(g^{a\bar{b}}(D_aW)(D_{\bar{b}}\overline{W})\right).$$

We will also write the superpotential (3.5.1) in the form

$$W = W_0(z^i) + k\tau$$

where $k = -2p^0$. The factor $e^{K_{\mathcal{K}}}$ and the inverse metric coefficients $g^{a\bar{b}}$ can be expanded in powers of α using the equations (3.5.7) and formulas (3.6.5) in appendix A. Using the noscale relations (3.5.3), we find the following expressions

$$\begin{aligned} U_F^{\mathcal{M}} =& \frac{e^{K_{\mathcal{M}}}}{4\mathrm{Im}(\tau)^4}(0.0625 - 0.00357\alpha^2 + 0.00004\alpha^4 + \ldots) \\ &\left(g^{i\bar{j}}(\partial_iW_0)(\partial_{\bar{j}}\overline{W}_0) + g^{i\bar{j}}[(\partial_iW_0)(\partial_{\bar{j}}K_{\mathcal{M}})(\overline{W}_0 + k\overline{\tau}) + (\partial_{\bar{j}}\overline{W}_0)(\partial_iK_{\mathcal{M}})(W_0 + k\tau)]\right) \end{aligned} \tag{3.5.9}$$

$$\begin{aligned} U_F^{\mathcal{K}} =& \frac{e^{K_{\mathcal{M}}}}{\mathrm{Im}(\tau)^4}\Big[\mathrm{Im}(\tau)^2(0.03125 - 0.00073\alpha^2 + 0.00001\alpha^4 + \ldots)k^2 \\ &- \mathrm{Im}(\tau)(0.03125 - 0.00178\alpha^2 + 0.00002\alpha^4 + \ldots)(2k^2\mathrm{Im}(\tau) + 2k\mathrm{Im}(W_0)) \\ &+ (0.0625 - 0.00357\alpha^2 + 0.00004\alpha^4 + \ldots)(k^2\tau\overline{\tau} + k\tau\overline{W}_0 + k\overline{\tau}W_0 + |W_0|^2)\Big] \end{aligned} \tag{3.5.10}$$

Let us now try to analyze the shape of the landscape determined by the equations (3.5.8) and (3.5.9), (3.5.10). We rewrite the contribution (3.5.9) to the potential energy in the form

$$U_F^{\mathcal{M}} = \frac{e^{K_{\mathcal{M}}}}{4\mathrm{Im}(\tau)^4}(0.0625 - 0.00357\alpha^2 + 0.00004\alpha^4 + \ldots)\left(P + kM\mathrm{Re}(\tau) + kN\mathrm{Im}(\tau)\right) \tag{3.5.11}$$

where

$$\begin{aligned} P &= g^{i\bar{j}}(\partial_iW_0)(\partial_{\bar{j}}\overline{W}_0) + g^{i\bar{j}}[(\partial_iW_0)(\partial_{\bar{j}}K_{\mathcal{M}})\overline{W}_0 + (\partial_{\bar{j}}\overline{W}_0)(\partial_iK_{\mathcal{M}})W_0] \\ M &= g^{i\bar{j}}[(\partial_iW_0)(\partial_{\bar{j}}K_{\mathcal{M}}) + (\partial_{\bar{j}}\overline{W}_0)(\partial_iK_{\mathcal{M}})] \\ N &= (-i)g^{i\bar{j}}[(\partial_iW_0)(\partial_{\bar{j}}K_{\mathcal{M}}) - (\partial_{\bar{j}}\overline{W}_0)(\partial_iK_{\mathcal{M}})] \end{aligned} \tag{3.5.12}$$

Then the α expansion of the F-term potential energy can be written as

$$U_F = U_F^{(0)} + \alpha^2 U_F^{(2)} + \dots$$

where

$$\begin{aligned} U_F^{(0)} &= 0.0156 \frac{e^{K_{\mathcal{M}}}}{\mathrm{Im}(\tau)^4}[P + k(N + 4\mathrm{Im}(W_0))\mathrm{Im}(\tau) + 2k^2\mathrm{Im}(\tau)^2 \\ &\qquad + 4|W_0|^2 + k(M + 8\mathrm{Re}(W_0))\mathrm{Re}(\tau) + 4k^2\mathrm{Re}(\tau)^2] \end{aligned} \tag{3.5.13}$$

$$\begin{aligned} U_F^{(2)} &= -0.00178 \frac{e^{K_{\mathcal{M}}}}{2\mathrm{Im}(\tau)^4}[P + k(N + 4\mathrm{Im}(W_0))\mathrm{Im}(\tau) + 0.82k^2\mathrm{Im}(\tau)^2 \\ &\qquad + 4|W_0|^2 + k(M + 8\mathrm{Re}(W_0))\mathrm{Re}(\tau) + 4k^2\mathrm{Re}(\tau)^2] \end{aligned} \tag{3.5.14}$$

The critical point equations resulting from (3.5.8), (3.5.9) and (3.5.10) are very complicated, and we will not attempt to find explicit solutions. We will try to gain some qualitative understanding of the possible solutions exploiting some peculiar aspects of the potential. Note that all contributions to the potential energy depend on even powers of α. Then it is obvious that $\alpha = 0$ is a solution to the equation

$$\partial_\alpha U = 0$$

where $U = U_D + U_F^{\mathcal{M}} + U_F^{\mathcal{K}}$. Moreover we also have

$$(\partial_i \partial_\alpha U)_{\alpha=0} = (\partial_\tau \partial_\alpha U)_{\alpha=0} = 0.$$

This motivates us to look for critical points with $\alpha = 0$. Then, the remaining critical point equations are

$$(\partial_i U)_{\alpha=0} = (\partial_\tau U)_{\alpha=0} = 0 \tag{3.5.15}$$

plus their complex conjugates.

The second order coefficient of α in the total potential energy is

$$U_F^{(2)} - 0.00178 \frac{1}{p\mathrm{Im}(\tau)^3}. \tag{3.5.16}$$

Since the mixed partial derivatives are zero at $\alpha = 0$, in order to obtain a local minimum, the expression (3.5.16) must be positive. This is a first constraint on the allowed solutions to (3.5.15).

Next, let us examine the τ dependence of the potential for $\alpha = 0$ and fixed values of the Kähler parameters. Note that $U_F^{(0)}$ given by equation (3.5.13) is a quadratic function of the axion $\mathrm{Re}(\tau)$. For any fixed values of $\mathrm{Im}(\tau)$ and the Kähler parameters, this function has a minimum at

$$\mathrm{Re}(\tau) = -\frac{8\mathrm{Re}(W_0) + M}{8k}. \tag{3.5.17}$$

Therefore we can set $\mathrm{Re}(\tau)$ to its minimum value in the potential energy, obtaining an effective potential for the Kähler parameters and the dilaton $\mathrm{Im}(\tau)$. Then equations (3.5.13), (3.5.14) become

$$\begin{aligned} U_F^{(0)} = 0.0156\frac{e^{K_{\mathcal{M}}}}{\mathrm{Im}(\tau)^4}\Big[&P + k(N + 4\mathrm{Im}(W_0))\mathrm{Im}(\tau) + 2k^2\mathrm{Im}(\tau)^2 \\ &+ 4|W_0|^2 - \frac{1}{16}(M + 8\mathrm{Re}(W_0))^2\Big] \end{aligned} \tag{3.5.18}$$

$$\begin{aligned} U_F^{(2)} = -0.00178\frac{e^{K_{\mathcal{M}}}}{2\mathrm{Im}(\tau)^4}\Big[&P + k(N + 4\mathrm{Im}(W_0))\mathrm{Im}(\tau) + 0.82k^2\mathrm{Im}(\tau)^2 \\ &+ 4|W_0|^2 - \frac{1}{16}(M + 8\mathrm{Re}(W_0))^2\Big] \end{aligned} \tag{3.5.19}$$

Now let us analyze the dependence of $U_F^{(0)}$ on $\mathrm{Im}(\tau)$. It will be more convenient to make the change of variables

$$u = \frac{1}{\mathrm{Im}(\tau)}$$

since u is proportional to the string coupling constant. Then $U_F^{(0)}$ becomes a quartic function of the form

$$U_F^{(0)} = Au^2 - Bu^3 + Cu^4 \tag{3.5.20}$$

where

$$\begin{aligned} A &= 2k^2 \\ B &= -kN - 4k\mathrm{Im}(W_0) \\ C &= P + 4|W_0|^2 - \frac{1}{16}(M + 8\mathrm{Re}(W_0))^2. \end{aligned} \tag{3.5.21}$$

The behavior of this function for fixed values of the Kähler parameters is very simple. For positive A, this function has a local minimum away from the origin if and only if the following inequalities are satisfied

$$B > 0 \qquad C > 0 \qquad \text{and} \qquad 9B^2 > 32AC. \tag{3.5.22}$$

The minimum is located at

$$u_0 = \frac{3B + \sqrt{9B^2 - 32AC}}{8C}. \tag{3.5.23}$$

Therefore, in order to construct metastable vacua, we have to find solutions to the equations (3.5.15) satisfying the inequalities (3.5.22). Moreover, we would like u_0 to be small in order to obtain a weakly coupled theory. The conditions (3.5.22) translate to

$$\begin{aligned} &9(N + 4\mathrm{Im}(W_0))^2 > 64\left(P + 4\mathrm{Im}(W_0)^2 - M\mathrm{Re}(W_0) - \frac{M^2}{16}\right) > 0 \\ &k\,(N + \mathrm{Im}(W_0)) < 0 \end{aligned} \tag{3.5.24}$$

where P, M, N are given by (3.5.12). This shows that we need a certain amount of fine tuning of the background RR fluxes in order to obtain a metastable vacuum. Note that in our construction the fluxes are not constrained by tadpole cancellation conditions, therefore we can tune them at will. Statistically, this improves our chances of finding a solution with the required properties.

Finally, note that we have to impose one more condition, namely the second order coefficient (3.5.16) in the α expansion of the potential should be positive. Assuming that we have found a solution of equations (3.5.15) which stabilizes u at the value $0 < u_0 < 1$, let us compute this coefficient as a function of (u_0, A, B, C). Note that equation (3.5.19) can be rewritten as

$$U_F^{(2)} = 0.00178\frac{e^{K_{\mathcal{M}}}}{2}(Bu_0^3 - 0.4Au_0^2 - Cu_0^4). \tag{3.5.25}$$

Equation (3.5.23) yields

$$B = \frac{4}{3}Cu_0 + \frac{2}{3}\frac{A}{u_0} \tag{3.5.26}$$

Substituting (3.5.26) in (3.5.25), and adding the D-term contribution, the coefficient of α^2 becomes

$$0.00178\left[e^{K_{\mathcal{M}}}(\frac{2}{15}Au_0^2 + \frac{1}{6}Cu_0^4) - \frac{1}{p}u_0^3\right] \tag{3.5.27}$$

Since $C > 0$, a sufficient condition for (3.5.27) to be positive is

$$\frac{2p}{15}Ae^{K_{\mathcal{M}}} > u_0 \quad \Rightarrow \quad \frac{4pk^2}{15} > u_0\mathrm{vol}(Y). \tag{3.5.28}$$

Here we have used

$$e^{K_{\mathcal{M}}} = \frac{1}{\mathrm{vol}(Y)}.$$

This condition reflects the fact that the F-term and D-term contributions to the potential energy must be of the same order of magnitude in order to obtain a metastable vacuum solution. If the volume of Y is too large, there is a clear hierarchy of scales between the two contributions, and the D-term is dominant. This would give rise to a runaway behavior along the direction of α. On the other hand, we have to make sure that the volume of Y is sufficiently large so that the IIA supergravity approximation is valid. Therefore some additional amount of fine tuning is required in order to obtain a reliable solution.

In conclusion, metastable nonsupersymmetric vacua at $\alpha = 0$ can be in principle obtained by tuning the IIA RR flux $F^{(A)}$ and NS-NS flux $H^A = k\beta^0$ so that conditions (3.5.24), (3.5.28) are satisfied at the critical point. A more precise statement would require a detailed numerical analysis, which we leave for future work.

We would like to conclude this section with a few remarks.

(i) In this chapter we have taken a conservative approach towards fluxes, avoiding half flat structures in the IIA theory, which correspond to IIB NS-NS flux H^B. If one is willing to consider compactifications of this form, we have additional terms in the superpotential. In IIB variables, these terms would read

$$-\tau \int_X \Omega_X \wedge H^B.$$

One can also turn on additional flux degrees of freedom as advocated in [159, 160]. Such terms may be helpful in the above fine tuning process.

(ii) We have also restricted ourselves to singly wrapped magnetized D5-branes. One could in principle consider multiply wrapped D5-branes as long we can maintain the phase difference θ sufficiently small. If this is possible, we would obtain an additional nonperturbative contribution to the superpotential of the form

$$be^{-a\tau}.$$

Such terms may be also helpful in the fine tuning process.

(iii) Finally, note that we could also allow for a nonzero background value of the RR zero-form F_0, which was also set to zero in this chapter. Then, according to [277], there is an additional contribution to the RR tadpole cancellation condition, which becomes

$$p - km_0 - |N_{O3}| = 0.$$

If we choose k, m_0 so that $km_0 > 0$, it follows that p can be larger than $|N_{O3}|$. In fact it seems that there is no upper bound on p, hence we could make the supersymmetry breaking D-term very small by choosing a large p. This may have important consequences for the scale of supersymmetry breaking in string theory.

(iv) Note that the vacuum construction mechanism proposed above can give rise to de Sitter or anti de Sitter vacua, depending on the values of fluxes. In particular, it is not subject to the no-go theorem of [161] because the magnetized branes give a positive contribution to the potential energy. In principle we could try to employ the same strategy in order to construct nonsupersymmetric metastable vacua of the F-term potential energy (3.5.2) in the absence of magnetized branes. Then we have several options for RR tadpole cancellation. We can turn on background F_0 flux as in [277] or local tadpole cancellation by adding background D6-branes. It would be interesting to explore these alternative constructions in more detail.

(v) Since it is quite difficult to find explicit vacuum solutions, it would be very interesting to attempt a systematic statistical analysis of the distribution of vacua along the lines of [162, 163, 164, 165, 166, 167, 168].

(vi) In our approach the scale of supersymmetry breaking is essentially determined by the total RR tadpole $p = |N_{O3}|$ of the orientifold model. While this tadpole is typically of the order of 32 in perturbative models, it can reach much higher values in orientifold limits of F-theory. It would be very interesting to implement our mechanism in such an F-theory compactification, perhaps in conjunction with other moduli stabilization mechanism [169, 276, 170, 171, 172, 173, 174]. Provided that the dynamics can still be kept under control, we would then obtain smaller supersymmetry breaking scales.

3.6 Appendix: Π-Stability on the Octic and $N = 2$ Kähler Moduli Space

In this appendix we analyze the $N = 2$ Kähler moduli space and stability of magnetized branes for the octic hypersurface. Recall [124] that the mirror family is described by the equation

$$x_1^8 + x_2^8 + x_3^8 + x_4^8 + x_5^2 - \alpha x_1 x_2 x_3 x_4 x_5 = 0. \tag{3.6.1}$$

in $WP^{1,1,1,1,4}/(\mathbb{Z}_8^2 \times \mathbb{Z}_2)$. The moduli space of the mirror family can be identified with a sector in the α plane defined by

$$0 \leq \arg(\alpha) < \frac{2\pi}{8}.$$

The entire α plane contains eight such sectors, which are permuted by monodromy transformations about the LG point $\alpha = 0$. In this parameterization, the LCS point is at $\alpha = \infty$, and the conifold point is at $\alpha = 4$.

A basis of periods for this family has been computed in [124] by solving the Picard-Fuchs equations. For our purposes it is convenient to write the solutions to the Picard-Fuchs equations in integral form

$$\begin{aligned}
\Pi_0 &= \frac{1}{2\pi i} \int ds \frac{\Gamma(1+8s)\Gamma(-s)}{\Gamma(1+s)^3\Gamma(1+4s)} e^{i\pi s} (\alpha)^{-8s} \\
\Pi_1 &= -\frac{1}{(2\pi i)^2} \int ds \frac{\Gamma(1+8s)\Gamma(-s)^2}{\Gamma(1+s)^2\Gamma(1+4s)} (\alpha)^{-8s} \\
\Pi_2 &= \frac{2}{(2\pi i)^3} \int ds \frac{\Gamma(1+8s)\Gamma(-s)^3}{\Gamma(1+s)\Gamma(1+4s)} e^{i\pi s} (\alpha)^{-8s} \\
\Pi_3 &= -\frac{1}{(2\pi i)^4} \int ds \frac{\Gamma(1+8s)\Gamma(-s)^4}{\Gamma(1+4s)} (\alpha)^{-8s}.
\end{aligned} \tag{3.6.2}$$

as in [133]. All integrals in (3.6.2) are contour integrals in the complex s-plane. The contour runs from $s = -\epsilon - i\infty$ to $-\epsilon + i\infty$ along the imaginary axis and it can be closed either to the left or to the right. If we close the contour to the right, we obtain a basis of solutions near the LCS limit $\alpha = \infty$, while if we close the contour to the left, we obtain a basis of solutions near the LG point $\alpha = 0$. Near the large radius limit it is more convenient to write the solutions in terms of the coordinate $z = \alpha^{-8}$.

Note that there is a different set of solutions at the LG point [124] of the form

$$w_k(\alpha) = \Pi_0(e^{2\pi k i}\alpha), \qquad k = 0, \ldots, 7. \tag{3.6.3}$$

In particular we have an alternative basis $[w_2\ w_1\ w_0\ w_7]^{tr}$ near $\alpha = 0$. The transition matrix between the two bases is

$$\begin{bmatrix} \Pi_0 \\ \Pi_1 \\ \Pi_2 \\ \Pi_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ 0 & -1 & -2 & -1 \\ -1 & -\frac{1}{2} & \frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} w_2 \\ w_1 \\ w_0 \\ w_7 \end{bmatrix} \tag{3.6.4}$$

In section 2 we have used a third basis of periods $[Z^0\ Z^1\ \mathcal{F}_1\ \mathcal{F}_0]^{tr}$ compatible with the orientifold projection. The relation between the orientifold basis and the LG basis $[w_2\ w_1\ w_0\ w_7]^{tr}$ is given in equation (3.2.11). The power series expansion of the orientifold periods at the LG point is

$$\begin{aligned} \mathrm{Re}(Z^0) &= -0.37941\alpha + 0.00541\alpha^3 + 0.00009\alpha^5 + \ldots \\ \mathrm{Im}(Z^1) &= -0.53656\alpha + 0.00766\alpha^3 - 0.00012\alpha^5 + \ldots \\ \mathrm{Re}(\mathcal{F}_1) &= 1.29538\alpha - 0.00317\alpha^3 - 0.00005\alpha^5 + \ldots \\ \mathrm{Im}(\mathcal{F}_0) &= 0.31431\alpha - 0.02615\alpha^3 + 0.00043\alpha^5 + \ldots \end{aligned} \tag{3.6.5}$$

Now let us discuss some geometric aspects of octic hypersurfaces required for the Π-stability analysis. For intersection theory computations, it will be more convenient to represent X as a hypersurface in a smooth toric variety Z obtained by blowing-up the singular point of the weighted projective space $WP^{1,1,1,1,4}$. Z is defined by the following $\mathbb{C}^\times \times \mathbb{C}^\times$ action

$$\begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & u & v \\ 1 & 1 & 1 & 1 & -4 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{array} \tag{3.6.6}$$

with forbidden locus $\{x_1 = x_2 = x_3 = x_4 = 0\} \cup \{u = v = 0\}$. The Picard group of Z is generated by two divisor classes η_1, η_2 determined by the equations

$$\eta_1 : \ x_1 = 0 \qquad \eta_2 : \ v = 0. \tag{3.6.7}$$

The cohomology ring of Z is determined by the relations

$$\eta_2^4 = 64 \qquad \eta_2(\eta_2 - 4\eta_1) = 0. \tag{3.6.8}$$

The total Chern class of Z is given by the formula

$$c(Z) = (1 + \eta_1)^4(1 - 4\eta_1 + \eta_2)(1 + \eta_2) \tag{3.6.9}$$

and the hypersurface X belongs to the linear system $|2\eta_2|$. Using the adjunction formula

$$c(X) = \frac{c(Z)}{(1 + 2\eta_2)} \tag{3.6.10}$$

one can easily compute

$$c_1(X) = 0 \qquad c_2(X) = 22\eta_1^2 \qquad \mathrm{Td}(X) = 1 + \frac{11}{6}\eta_1^2. \tag{3.6.11}$$

Note that the divisor class $\eta_2 - 4\eta_1$ has trivial restriction to X, therefore the Picard group of X has rank one, as expected. A natural generator is η_1, which can be identified with a hyperplane section of X in the weighted projective space $WP^{1,1,1,1,4}$. Then we will write the complexified Kähler class as $B + iJ = t\eta_1$. For future reference, note that we will denote by $E(p)$ the tensor product $E \otimes \mathcal{O}_X(p\eta_1)$ for any sheaf (or derived object) E on X.

Employing the conventions of [175], we will define the central charge of a D-brane E in the large radius limit to be

$$Z^\infty(E) = \int_X e^{B+iJ}\mathrm{ch}(E)\sqrt{\mathrm{Td}(X)}. \tag{3.6.12}$$

This is a cubic polynomial in t. Using the mirror map

$$t = \frac{\Pi_1}{\Pi_0} \tag{3.6.13}$$

and the asymptotic form of the periods

$$\begin{aligned}
\Pi_1 &= t + \dots \\
\Pi_2 &= t^2 + t - \frac{11}{6} + \dots \\
\Pi_3 &= \frac{1}{6}t^3 - \frac{13}{12}t + \dots
\end{aligned} \tag{3.6.14}$$

we can determine the exact expression of the period Z_E as a function of the algebraic coordinate α. The phase of the central charge is defined as

$$\phi(E) = -\frac{1}{\pi}\arg(Z(E)) \tag{3.6.15}$$

and is normalized so that it takes values $-2 < \phi(E) \leq 0$ at the large radius limit point.

As objects in the derived category $D^b(X)$, the magnetized branes are given by

$$\underline{\mathcal{O}_C}(p-1) \qquad \underline{\mathcal{O}_{C'}}(-p-1)[1] \tag{3.6.16}$$

where C, C' are smooth rational curves on X conjugated under the holomorphic involution. Given a coherent sheaf E on X, we have denoted by $\underline{E}$ the one term complex which contains E in degree zero, all other terms being trivial. In order to compute their asymptotic central charges using formula (3.6.12), we have to use the Grothendieck-Riemann-Roch theorem for the embeddings $\iota : C \to X$, $\iota' : C' \to X$. Since the computations are very similar, it suffices to present the details only for one of these objects, for example the first brane in (3.6.16).

Given a line bundle $\mathcal{L} \to C$, the Chern character of its pushforward $\iota_*(\mathcal{L})$ to X is given by

$$\mathrm{ch}(\iota_*(\mathcal{L}))\mathrm{Td}(X) = \iota_*(\mathrm{ch}(\mathcal{L})\mathrm{Td}(C)). \tag{3.6.17}$$

In our case (3.6.17) yields

$$\mathrm{ch}_0(\iota_*(\mathcal{L})) = \mathrm{ch}_1(\iota_*(\mathcal{L})) = 0 \qquad \mathrm{ch}_2(\iota_*(\mathcal{L})) = [C] \qquad \mathrm{ch}_3(\iota_*(\mathcal{L})) = (\deg(\mathcal{L}) + 1)[pt] \tag{3.6.18}$$

where $[C] \in H^{2,2}(X)$ denotes the Poincaré dual of C and $[pt] \in H^{3,3}(X)$ denotes the Poincaré dual of a point on X. The shift by 1 in $\mathrm{ch}_3(\iota_*(\mathcal{L}))$ represents the contribution of the Todd class of C

$$\mathrm{Td}(C) = 1 + \frac{1}{2}c_1(C)$$

to the right hand side of equation (3.6.17). From a physical point of view, this can be thought of as D3-brane charge induced by a curvature effect. Using formulas (3.6.12), (3.6.18) it is easy to compute

$$Z^{\infty}\left(\underline{\mathcal{O}_C}(p-1)\right) = t + p \qquad Z^{\infty}\left(\underline{\mathcal{O}_{C'}}(-p-1)[1]\right) = -t + p. \tag{3.6.19}$$

The exact expressions for the central charges are

$$Z\left(\underline{\mathcal{O}_C}(p-1)\right) = \Pi_1 + p\Pi_0 \qquad Z\left(\underline{\mathcal{O}_{C'}}(-p-1)[1]\right) = -\Pi_1 + p\Pi_0. \tag{3.6.20}$$

Taking into account the transition matrices (3.2.11), (3.6.4), it is clear that these formulas are identical with (3.3.2) in the main text. In order to study the behavior of their phases near the LG point, we have to rewrite the central charges (3.6.20) in terms of the basis $[w_2\ w_1\ w_0\ w_7]^{tr}$ using the transition matrix (3.6.4). We find

$$\begin{aligned} Z\left(\underline{\mathcal{O}_C}(p-1)\right) &= \frac{1}{2}(w_2 + w_1 - w_0 - w_7) + pw_0 \\ Z\left(\underline{\mathcal{O}_{C'}}(-p-1)[1]\right) &= -\frac{1}{2}(w_2 + w_1 - w_0 - w_7) + pw_0. \end{aligned} \tag{3.6.21}$$

Note that the central charge of a single D3-brane is

$$Z(\underline{\mathcal{O}_{pt}}) = w_0. \tag{3.6.22}$$

Then, using the expansions (3.6.5) we can plot the relative phase

$$\theta = \phi\left(\underline{\mathcal{O}_C}(p-1)\right) - \phi(\underline{\mathcal{O}_{pt}}) \tag{3.6.23}$$

near the LG point, obtaining the graph in figure 1.

In the remaining part of this section, we will address the question of stability of magnetized brane configurations near the LG point. As explained below figure 1, we will analyze stability with respect to the most natural decay channels from the geometric point of view. We will show below that the objects (3.6.16) are stable with respect to all such decay processes, which is strong evidence for their stability at the LG point. Since all these computations are very similar, it suffices to consider only one case in detail. For the other cases we will just give the final results.

Decay channels in the Π-stability framework are classified by triangles in the derived category [133]. In our case, the most natural decay channels are in fact determined by short exact sequences of sheaves. For example let us consider the following short exact sequence

$$0 \to \mathcal{J}_C(p-1) \to \mathcal{O}_X(p-1) \to \mathcal{O}_C(p-1) \to 0 \tag{3.6.24}$$

where $\mathcal{J}_C$ is the ideal sheaf of C on X. The first two terms represent rank one D6-branes on X with lower D4 and D2 charges. All three terms are stable BPS states in the large volume limit. The mass of the lightest open string states stretching between the first two branes in the sequence (3.6.24) is determined by the relative phase

$$\Delta\phi = \phi\left(\underline{\mathcal{O}_X}(p-1)\right) - \phi\left(\underline{\mathcal{J}_C}(p-1)\right). \tag{3.6.25}$$

If $\Delta\phi < 1$, the lightest state in this open string sector is tachyonic, and these two branes will form a bound state isomorphic to $\underline{\mathcal{O}_C}(p-1)$ by tachyon condensation. In this case $\underline{\mathcal{O}_C}(p-1)$ is stable. If $\Delta\phi > 1$, the lightest open string state has positive mass, and it is energetically favorable for $\underline{\mathcal{O}_C}(p-1)$ to decay into $\underline{\mathcal{J}_C}(p-1)$ and $\underline{\mathcal{O}_X}(p-1)$. In this case $\underline{\mathcal{O}_C}(p-1)$ is unstable. Therefore we have to compute the phase difference $\Delta\phi$ as a function of α in order to find out if this decay takes place anywhere on the real α axis. For the purpose of this computation it is more convenient to denote $q = p-1$, and perform the calculations in terms of q rather than p.

We have

$$\begin{aligned} Z^\infty\left(\underline{\mathcal{O}_X}(q)\right) &= \int_X e^{(t+q)\eta_1}\sqrt{\mathrm{Td}(X)} \\ &= \frac{1}{3}(t+q)^3 + \frac{11}{6}(t+q) \\ Z^\infty\left(\underline{\mathcal{J}_C}(q)\right) &= \int_X e^{(t+q)\eta_1}\sqrt{\mathrm{Td}(X)} - Z^\infty\left(\underline{\mathcal{O}_C}(q)\right) \\ &= \frac{1}{3}(t+q)^3 + \frac{5}{6}(t+q) - 1. \end{aligned} \tag{3.6.26}$$

Using the asymptotic form of the periods (3.6.14) and formulas (3.6.26), we find the following expressions for the exact central charges

$$\begin{aligned} Z\left(\underline{\mathcal{O}_X}(q)\right) &= 2\Pi_3 + q\Pi_2 + (q^2 - q + 4)\Pi_1 + \left(\frac{1}{3}q^3 + \frac{11}{3}q\right)\Pi_0 \\ Z\left(\underline{\mathcal{J}_C}(q)\right) &= 2\Pi_3 + q\Pi_2 + (q^2 - q + 3)\Pi_1 + \left(\frac{1}{3}q^3 + \frac{8}{3}q - 1\right)\Pi_0 \end{aligned} \tag{3.6.27}$$

In terms of the LG basis of periods, these expressions read

$$
\begin{aligned}
Z\left(\underline{\mathcal{O}_X}(q)\right) &= \left(\frac{1}{2}q^2 - \frac{1}{2}q\right) w_2 + \left(\frac{1}{2}q^2 - \frac{3}{2}q + 1\right) w_1 \\
&\quad + \left(\frac{1}{3}q^3 - \frac{1}{2}q^2 + \frac{13}{6}q - 1\right) w_0 + \left(-\frac{1}{2}q^2 - \frac{1}{2}q\right) w_7 \\
Z\left(\underline{\mathcal{J}_C}(q)\right) &= \left(\frac{1}{2}q^2 - \frac{1}{2}q - \frac{1}{2}\right) w_2 + \left(\frac{1}{2}q^2 - \frac{3}{2}q + \frac{1}{2}\right) w_1 \\
&\quad + \left(\frac{1}{3}q^3 - \frac{1}{2}q^2 + \frac{7}{6}q - \frac{3}{2}\right) w_0 + \left(-\frac{1}{2}q^2 - \frac{1}{2}q + \frac{1}{2}\right) w_7
\end{aligned}
\tag{3.6.28}
$$

Substituting the expressions (3.6.2) in (3.6.27), (3.6.28), we can compute the the relative phase (3.6.25) at any point on the real axis in the α-plane except the conifold point $\alpha = 4$. The conifold point can be avoided following a circular contour of very small radius ϵ centered at $\alpha = 4$.

The graph in fig. 2 represents the dependence of $\Delta\phi$ as a function of $z = \alpha^{-8}$ in the large radius phase $0 < z < 4$ for $p = 10$. Note that it decreases monotonically from 0.0075 to 0.0044 as we approach the conifold point. Using formulas (3.6.28), we find that in the LG phase $0 < \alpha < 4$, $\Delta\Phi$ also decreases monotonically until it reaches the value 0.027 at the LG point. One can also calculate the values of $\Delta\phi$ along a small circular contour surrounding the conifold, confirming that it varies continuously in this region. Since $\Delta\phi < 1$, everywhere on the real axis, we conclude that the magnetized brane $\underline{\mathcal{O}_C}(q)$ is stable with respect to the decay channel (3.6.24).

The analysis of other decay channels is very similar. Another decay channel is given by the following short exact sequence

$$
0 \to \mathcal{O}_D(-C)(q) \to \mathcal{O}_D(q) \to \mathcal{O}_C(q) \to 0 \tag{3.6.29}
$$

where D is a divisor on X in the linear system η_1 containing C. Then, an analogous computation yields a similar variation of $\Delta\phi$ on the real axis, except that the maximum value is approximatively 0.015 and it decreases monotonically to 0.008 at the LG point. Therefore the magnetized brane is also stable with respect to the decay (3.6.29). In principle there could exit other decay channels, perhaps described by more exotic triangles in the derived category. A systematic analysis would take us too far afield, so we will simply assume that the magnetized branes are stable at the LG point based on the evidence presented so far. A rigorous proof of

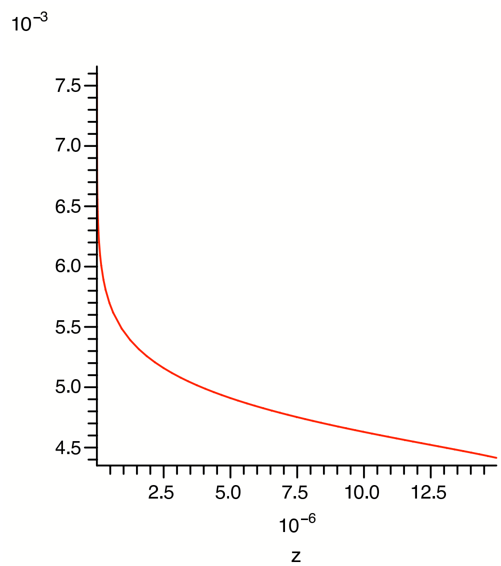

Figure 3.2: The behavior of the relative phase $\Delta\phi$ in the geometric phase for $p = 10$.

stability is not within the reach of current Π-stability techniques.

Chapter 4

D-brane Superpotentials on Calabi-Yau manifolds

4.1 Introduction

D-branes in Type IIB orientifolds are an important ingredient in constructions of string vacua. A frequent problem arising in this context is the computation of the tree-level superpotential for holomorphic D-brane configurations. This is an important question for both realistic model building as well as dynamical supersymmetry breaking.

Various computational methods for the tree-level superpotential have been proposed in the literature. A geometric approach which identifies the superpotential with a three-chain period of the holomorphic $(3,0)$-form has been investigated in [176, 177, 178, 179, 180]. A related method, based on two-dimensional holomorphic Chern-Simons theory, has been developed in [181, 182, 183, 184]. The tree-level superpotential for fractional brane configurations at toric Calabi-Yau singularities has been computed in [185, 186, 187, 188, 189, 190]. Using exceptional collections, one can also compute the superpotential for non-toric del Pezzo singularities [191, 192, 193, 194]. Perturbative disc computations for superpotential interactions have been performed in [195, 196, 197]. Finally, a mathematical approach based on versal deformations has been developed in [198] and extended to matrix valued fields in [199].

A systematic approach encompassing all these cases follows from the algebraic structure of **B**-branes on Calabi-Yau manifolds. Adopting the point of view that **B**-branes form a triangulated differential graded category [200, 201, 202, 203, 204, 205] the computation of the superpotential is equivalent to the computation of a minimal A_∞ structure for the D-brane category [206, 207, 208, 209, 210].

This approach has been employed in the Landau-Ginzburg D-brane category [211, 212, 213], and in the derived category of coherent sheaves [214, 215]. These are two of the various phases that appear in the moduli space of a generic $\mathcal{N} = 2$ Type II compactification. In particular, Aspinwall and Katz [214] developed a general computational approach for the superpotential, in which the A_∞ products are computed using a Čech cochain model for the off-shell open string fields.

The purpose of the present chapter is to apply a similar strategy for D-branes wrapping holomorphic curves in Type II orientifolds. This requires a basic understanding of the orientifold projection in the derived category, which is the subject of section 4.2. In section 4.3 we propose a computational scheme for the superpotential in orientifold models. This relies on a systematic implementation of the orientifold projection in the calculation of the A_∞ structure.

We show that the natural algebraic framework for deformation problems in orientifold models relies on L_∞ rather than A_∞ structures. This observation leads to a simple prescription for the D-brane superpotential in the presence of an orientifold projection: one has to evaluate the superpotential of the underlying unprojected theory on invariant on-shell field configurations. This is the main conceptual result of the chapter, and its proof necessitates the introduction of a lengthy abstract machinery.

Applying our prescription in practice requires some extra work. The difficulty stems from the fact that while the orientifold action is geometric on the Calabi-Yau, it is *not* naturally geometric at the level of the derived category. Therefore, knowing the superpotential in the original theory does not trivially lead to the superpotential of the orientifolded theory. To illustrate this point we compute the superpotential in two different cases. Both will involve D-branes wrapping rational curves, the difference will be in the way these curves are obstructed to move in the ambient space.

The organization of the chapter is as follows. Section 2 reviews the construction of the categorical framework in which we wish to impose the orientifold projection, as well as how to do the latter. Section 3 describes the calculation of the D-brane superpotential in the presence of the projection. Finally, section 4 offers concrete computations of the D-brane superpotential for obstructed curves in Calabi-Yau orientifolds.

4.2 D-Brane Categories and Orientifold Projection

This section will be concerned with general aspects of topological **B**-branes in the presence of an orientifold projection. Our goal is to find a natural formulation for the orientifold projection in D-brane categories.

For concreteness, we will restrict ourselves to the category of topological **B**-branes on a Calabi-Yau threefold X, but our techniques extend to higher dimensions. In this case, the D-brane category is the derived category of coherent sheaves on X [200, 202]. In fact, a systematic off-shell construction of the D-brane category [203, 204] shows that the category in question is actually larger than the derived category. In addition to complexes, one has to also include twisted complexes as defined in [216]. We will show below that the off-shell approach is the most convenient starting point for a systematic understanding of the orientifold projection.

4.2.1 Review of D-Brane Categories

Let us begin with a brief review of the off-shell construction of D-brane categories [216, 203, 204]. It should be noted at the offset that there are several different models for the D-brane category, depending on the choice of a fine resolution of the structure sheaf $\mathcal{O}_X$. In this section we will work with the Dolbeault resolution, which is closer to the original formulation of the boundary topological **B**-model [217]. This model is very convenient for the conceptual understanding of the orientifold projection, but it is unsuitable for explicit computations. In Section 4 we will employ a Čech cochain model for computational purposes, following the path pioneered in [214].

Given the threefold X, one first defines a differential graded category $\mathcal{C}$ as follows

$$\mathrm{Ob}(\mathcal{C})\colon \text{holomorphic vector bundles } (E,\overline{\partial}_E) \text{ on } X$$

$$\mathrm{Mor}_{\mathcal{C}}\left((E,\overline{\partial}_E),(F,\overline{\partial}_F)\right) = \left(\oplus_p A_X^{0,p}(\mathscr{H}om_X(E,F)),\ \overline{\partial}_{EF}\right)$$

where we have denoted by $\overline{\partial}_{EF}$ the induced Dolbeault operator on $\mathscr{H}om_X(E,F)$-valued $(0,p)$ forms.[1] The space of morphisms is a $\mathbb{Z}$-graded differential complex. In order to simplify the notation we will denote the objects of $\mathcal{C}$ by E, the data of an integrable Dolbeault operator $\overline{\partial}_E$ being implicitly understood.

The composition of morphisms in $\mathcal{C}$ is defined by exterior multiplication of bundle valued differential forms. For any object E composition of morphisms determines an associative algebra structure on the endomorphism space $\mathrm{Mor}_{\mathcal{C}}(E,E)$. This product is compatible with the differential, therefore we obtain a differential graded associative algebra structure (DGA) on $\mathrm{Mor}_{\mathcal{C}}(E,E)$.

At the next step, we construct the *shift completion* $\widetilde{\mathcal{C}}$ of $\mathcal{C}$, which is a category of holomorphic vector bundles on X equipped with an integral grading.

$$\mathrm{Ob}(\widetilde{\mathcal{C}})\colon \text{pairs } (E,n), \text{with } E \text{ an object of } \mathcal{C} \text{ and } n\in\mathbb{Z}$$

$$\mathrm{Mor}_{\widetilde{\mathcal{C}}}((E,n),(F,m)) = \mathrm{Mor}_{\mathcal{C}}(E,F)[n-m].$$

The integer n is the boundary ghost number introduced in [202]. Note that for a homogeneous element

$$f\in \mathrm{Mor}^k_{\widetilde{\mathcal{C}}}((E,n),(F,m))$$

we have

$$k = p + (m-n)$$

where p is the differential form degree of f. The degree k represents the total ghost number of the field f with respect to the bulk-boundary BRST operator. In the following we will use the notations

$$|f| = k, \qquad c(f) = p, \qquad h(f) = m-n.$$

The composition of morphisms in $\widetilde{\mathcal{C}}$ differs from the composition of morphisms in $\mathcal{C}$ by a sign, which will play an important role in our construction. Given two homogeneous elements

$$f\in \mathrm{Mor}_{\widetilde{\mathcal{C}}}((E,n),(E',n')) \qquad g\in \mathrm{Mor}_{\widetilde{\mathcal{C}}}((E',n'),(E'',n''))$$

[1] $\mathscr{H}om_X(E,F)$ is the sheaf Hom of E and F, viewed as sheaves.

one defines the composition

$$(g \circ f)_{\widetilde{\mathcal{C}}} = (-1)^{h(g)c(f)} (g \circ f)_{\mathcal{C}} \,. \tag{4.2.1}$$

This choice of sign leads to the graded Leibniz rule

$$\overline{\partial}_{EE''}(g \circ f)_{\widetilde{\mathcal{C}}} = \left(\overline{\partial}_{E'E''}(g) \circ f\right)_{\widetilde{\mathcal{C}}} + (-1)^{h(g)} \left(g \circ \overline{\partial}_{EE'}(f)\right)_{\widetilde{\mathcal{C}}} \,.$$

Now we construct a pre-triangulated DG category Pre-Tr($\widetilde{\mathcal{C}}$) of twisted complexes as follows

$$\mathrm{Ob}\left(\text{Pre-Tr}(\widetilde{\mathcal{C}})\right): \quad \begin{array}{l} \text{finite collections of the form} \\ \left\{(E_i, n_i, q_{ji}) |\; q_{ji} \in \mathrm{Mor}^1_{\widetilde{\mathcal{C}}}((E_i, n_i), (E_j, n_j))\right\} \\ \text{where the } q_{ji} \text{ satisfy the Maurer-Cartan equation} \\ \overline{\partial}_{E_iE_j}(q_{ji}) + \sum_k (q_{jk} \circ q_{ki})_{\widetilde{\mathcal{C}}} = 0. \end{array}$$

$$\mathrm{Mor}_{\text{Pre-Tr}(\widetilde{\mathcal{C}})}((E_i, n_i, q_{ji}), (F_i, m_i, r_{ji})) = \left(\bigoplus_{i,j} \mathrm{Mor}_{\widetilde{\mathcal{C}}}((E_i, n_i), (F_j, m_j)),\; Q\right)$$

where the differential Q is defined as

$$Q(f) = \overline{\partial}_{E_iF_j}(f) + \sum_k (r_{kj} \circ f)_{\widetilde{\mathcal{C}}} - (-1)^{|f|} (f \circ q_{ik})_{\widetilde{\mathcal{C}}}, \qquad f \in \mathrm{Mor}_{\widetilde{\mathcal{C}}}((E_i, n_i), (F_j, m_j)).$$

$|f|$ is the degree of f in $\mathrm{Mor}_{\widetilde{\mathcal{C}}}((E_i, n_i), (F_j, m_j))$ from above. For each object, the index i takes finitely many values between 0 and some maximal value which depends on the object. Note that $Q^2 = 0$ because $\{q_{ji}\}, \{r_{ji}\}$ satisfy the Maurer-Cartan equation. Composition of morphisms in Pre-Tr($\widetilde{\mathcal{C}}$) reduces to composition of morphisms in $\widetilde{\mathcal{C}}$.

Finally, the triangulated D-brane category $\mathcal{D}$ has by definition the same objects as Pre-Tr($\widetilde{\mathcal{C}}$), while its morphisms are given by the zeroth cohomology under Q of the morphisms of Pre-Tr($\widetilde{\mathcal{C}}$):

$$\begin{aligned} &\mathrm{Ob}\,(\mathcal{D}) = \mathrm{Ob}\left(\text{Pre-Tr}(\widetilde{\mathcal{C}})\right) \\ &\mathrm{Mor}_{\mathcal{D}}((E_i, n_i, q_{ji}), (F_i, m_i, r_{ji})) = H^0\left(Q,\, \mathrm{Mor}_{\text{Pre-Tr}(\widetilde{\mathcal{C}})}((E_i, n_i, q_{ji}), (F_i, m_i, r_{ji}))\right). \end{aligned} \tag{4.2.2}$$

The bounded derived category of coherent sheaves $D^b(X)$ is a full subcategory of $\mathcal{D}$. To see this consider the objects of the form (E_i, n_i, q_{ji}) such that

$$n_i = -i, \qquad q_{ji} \neq 0 \;\Leftrightarrow\; j = i - 1. \tag{4.2.3}$$

Since $q_{ji} \in \mathrm{Mor}^1_{\widetilde{\mathcal{C}}}((E_i, n_i), (E_j, n_j))$, the second condition in (4.2.3) implies that their differential form degree must be 0. The Maurer-Cartan equation for such objects reduces to

$$\overline{\partial}_{E_i E_{i-1}} q_{i-1,i} = 0, \qquad (q_{i-1,i} \circ q_{i,i+1})_{\widetilde{\mathcal{C}}} = 0.$$

Therefore the twisted complex (E_i, n_i, q_{ji}) is in fact a complex of holomorphic vector bundles

$$\cdots \longrightarrow E_{i+1} \xrightarrow{q_{i,i+1}} E_i \xrightarrow{q_{i-1,i}} E_{i-1} \longrightarrow \cdots \tag{4.2.4}$$

We will use the alternative notation

$$\cdots \longrightarrow E_{i+1} \xrightarrow{d_{i+1}} E_i \xrightarrow{d_i} E_{i-1} \longrightarrow \cdots \tag{4.2.5}$$

for complexes of vector bundles, and also denote them by the corresponding Gothic letter, here $\mathfrak{E}$.

One can easily check that the morphism space (4.2.2) between two twisted complexes of the form (4.2.3) reduces to the hypercohomology group of the local Hom complex $\mathscr{H}om(\mathfrak{E}, \mathfrak{F})$

$$\mathrm{Mor}_{\mathcal{D}}((E_i, n_i, q_{ji}), (F_i, m_i, r_{ji})) \simeq \mathbb{H}^0(X, \mathscr{H}om(\mathfrak{E}, \mathfrak{F})). \tag{4.2.6}$$

As explained in [205], this hypercohomology group is isomorphic to the derived morphism space $\mathrm{Hom}_{D^b(X)}(\mathfrak{E}, \mathfrak{F})$. Assuming that X is smooth and projective, any derived object has a locally free resolution, hence $D^b(X)$ is a full subcategory of $\mathcal{D}$.

4.2.2 Orientifold Projection

Now we consider orientifold projections from the D-brane category point of view. A similar discussion of orientifold projections in matrix factorization categories has been outlined in [218].

Consider a four dimensional $N = 1$ IIB orientifold obtained from an $N = 2$ Calabi-Yau compactification by gauging a discrete symmetry of the form

$$(-1)^{\epsilon F_L} \Omega\, \sigma$$

with $\epsilon = 0, 1$. Employing common notation, Ω denotes world-sheet parity, F_L is the left-moving fermion number and $\sigma\colon X \to X$ is a holomorphic involution of X satisfying

$$\sigma^* \Omega_X = (-1)^{\epsilon}\, \Omega_X, \tag{4.2.7}$$

where Ω_X is the holomorphic $(3,0)$-form of the Calabi-Yau. Depending on the value of ϵ, there are two classes of models to consider [219]:

1. $\epsilon = 0$: theories with $O5/O9$ orientifolds planes, in which the fixed point set of σ is either one or three complex dimensional;

2. $\epsilon = 1$: theories with $O3/O7$ planes, with σ leaving invariant zero or two complex dimensional submanifolds of X.

Following the same logical steps as in the previous subsection, we should first find the action of the orientifold projection on the category $\mathcal{C}$, which is the starting point of the construction. The action of parity on the K-theory class of a D-brane has been determined in [220]. The world-sheet parity Ω maps E to the dual vector bundle $E^\vee$. If Ω acts simultaneously with a holomorphic involution $\sigma\colon X \to X$, the bundle E will be mapped to $\sigma^*(E^\vee)$. If the projection also involves a $(-1)^{F_L}$ factor, a brane with Chan-Paton bundle E should be mapped to an anti-brane with Chan-Paton bundle $P(E)$.

Based on this data, we define the action of parity on $\mathcal{C}$ to be

$$\begin{gathered} P\colon E \mapsto P(E) = \sigma^*(E^\vee) \\ P\colon f \in \mathrm{Mor}_{\mathcal{C}}(E,F) \mapsto \sigma^*(f^\vee) \in \mathrm{Mor}_{\mathcal{C}}(P(F),P(E)) \end{gathered} \tag{4.2.8}$$

It is immediate that P satisfies the following compatibility condition with respect to composition of morphisms in $\mathcal{C}$:

$$P((g\circ f)_{\mathcal{C}}) = (-1)^{c(f)c(g)}\,(P(f)\circ P(g))_{\mathcal{C}} \tag{4.2.9}$$

for any homogeneous elements f and g. It is also easy to check that P preserves the differential graded structure, i.e.,

$$P(\overline{\partial}_{EF}(f)) = \overline{\partial}_{P(F)P(E)}(P(f)). \tag{4.2.10}$$

Equation (4.2.9) shows that P is not a functor in the usual sense. Since it is compatible with the differential graded structure, it should be interpreted as a functor of A_∞ categories [221]. Note however that P is "almost a functor": it fails to satisfy the compatibility condition with composition of morphisms only by a sign. For future reference, we will refer to A_∞ functors satisfying a graded compatibility condition of the form (4.2.9) as *graded functors.*

The category $\mathcal{C}$ does not contain enough information to make a distinction between branes and anti-branes. In order to make this distinction, we have to assign each bundle a grading, that is we have to work in the category $\widetilde{\mathcal{C}}$ rather than $\mathcal{C}$. By convention, the objects (E, n) with n even are called branes, while those with n odd are called anti-branes.

We will take the action of the orientifold projection on the objects of $\widetilde{\mathcal{C}}$ to be

$$\widetilde{P}\colon (E, n) \mapsto (P(E), m - n) \tag{4.2.11}$$

where we have introduced an integer shift m which is correlated with ϵ from (4.2.7):

$$m \equiv \epsilon \bmod 2. \tag{4.2.12}$$

This allows us to treat both cases $\epsilon = 0$ and $\epsilon = 1$ in a unified framework.

We define the action of P on a morphisms $f \in \mathrm{Mor}_{\widetilde{\mathcal{C}}}((E, n), (E', n'))$ as the following graded dual:

$$\widetilde{P}(f) = -(-1)^{n' h(f)} P(f), \tag{4.2.13}$$

where $P(f)$ was defined in (4.2.8).[2] Note that the graded dual has been used in a similar context in [218], where the orientifold projection is implemented in matrix factorization categories.

With this definition, we have the following:

Proposition 4.2.1. *$\widetilde{P}$ is a graded functor on $\widetilde{\mathcal{C}}$ satisfying*

$$\widetilde{P}((g \circ f)_{\widetilde{\mathcal{C}}}) = -(-1)^{|f||g|} (\widetilde{P}(f) \circ \widetilde{P}(g))_{\widetilde{\mathcal{C}}} \tag{4.2.14}$$

for any homogeneous elements

$$f \in \mathrm{Mor}_{\widetilde{\mathcal{C}}}((E, n), (E', n')), \qquad g \in \mathrm{Mor}_{\widetilde{\mathcal{C}}}((E', n'), (E'', n'')).$$

Proof. It is clear that $\widetilde{P}$ is compatible with the differential graded structure of $\widetilde{\mathcal{C}}$ since the latter is inherited from $\mathcal{C}$.

[2] There is no a priori justification for the particular sign we chose, but as we will see shortly, it leads to a graded functor. A naive generalization of (4.2.8) ignoring this sign would not yield a graded functor.

Next we prove (4.2.14). First we have:

$$\begin{aligned}\widetilde{P}((g\circ f)_{\widetilde{\mathcal{C}}}) &= -(-1)^{n''h(g\circ f)}P((g\circ f)_{\widetilde{\mathcal{C}}}) \qquad \text{by (4.2.13)}\\ &= -(-1)^{n''h(g\circ f)+h(g)c(f)}P((g\circ f)_{\mathcal{C}}) \qquad \text{by (4.2.1)}\\ &= -(-1)^{n''h(g\circ f)+h(g)c(f)+c(f)c(g)}(P(f)\circ P(g))_{\mathcal{C}} \qquad \text{by (4.2.9)}\end{aligned}$$

On the other hand

$$\begin{aligned}(\widetilde{P}(f)\circ\widetilde{P}(g))_{\widetilde{\mathcal{C}}} &= (-1)^{n'h(f)+n''h(g)}(P(f)\circ P(g))_{\widetilde{\mathcal{C}}} \qquad \text{by (4.2.13)}\\ &= (-1)^{n'h(f)+n''h(g)}(-1)^{h(P(f))c(P(g))}(P(f)\circ P(g))_{\mathcal{C}} \qquad \text{by (4.2.1)}\end{aligned}$$

But

$$h(g\circ f)=h(f)+h(g),\quad h(P(f))=h(f),\quad c(P(g))=c(g).$$

Now (4.2.14) follows from

$$n''(h(f)+h(g))-n'h(f)-n''h(g)=(n''-n')h(f)=h(g)h(f)$$

and

$$|f||g|=(h(f)+c(f))(h(g)+c(g)).$$

$\square$

The next step is to determine the action of P on the pre-triangulated category Pre-Tr$(\widetilde{\mathcal{C}})$. We denote this action by $\widehat{P}$. The action of $\widehat{P}$ on objects is defined simply by

$$(E_i,n_i,q_{ji})\mapsto(P(E_i),m-n_i,\widetilde{P}(q_{ji})) \tag{4.2.15}$$

Using equations (4.2.10) and (4.2.14), it is straightforward to show that the action of $\widehat{P}$ preserves the Maurer-Cartan equation, that is

$$\overline{\partial}_{E_iE_j}(q_{ji})+\sum_k(q_{jk}\circ q_{ki})_{\widetilde{\mathcal{C}}}=0\ \Rightarrow\ \overline{\partial}_{P(E_j)P(E_i)}\widetilde{P}(q_{ji})+\sum_k(\widetilde{P}(q_{ki})\circ\widetilde{P}(q_{jk}))_{\widetilde{\mathcal{C}}}=0,$$

since all q_{ji} have total degree one. Therefore this transformation is well defined on objects. The action on morphisms is also straightforward

$$\begin{aligned}&f\in\oplus_{i,j}\mathrm{Mor}_{\widetilde{\mathcal{C}}}((E_i,n_i),(F_j,m_j))\\ &\qquad\mapsto\widehat{P}(f)=\widetilde{P}(f)\in\oplus_{i,j}\mathrm{Mor}_{\widetilde{\mathcal{C}}}((P(F_j),m-m_j),(P(E_i),m-n_i)).\end{aligned} \tag{4.2.16}$$

Again, equations (4.2.10), (4.2.14) imply that this action preserves the differential

$$Q(f) = \overline{\partial}_{E_i F_j}(f) + \sum_k (r_{kj} \circ f)_{\widetilde{\mathcal{C}}} - (-1)^{|f|}(f \circ q_{ik})_{\widetilde{\mathcal{C}}}$$

since $\{q_{ji}\}$, $\{r_{ji}\}$ have degree one. This means we have

$$\widehat{P}(Q(f)) = \overline{\partial}_{P(F_j)P(E_i)}\widetilde{P}(f) + \sum_k (\widetilde{P}(q_{ik}) \circ \widetilde{P}(f))_{\widetilde{\mathcal{C}}} - (-1)^{|\widetilde{P}(f)|}(\widetilde{P}(f) \circ \widetilde{P}(r_{kj}))_{\widetilde{\mathcal{C}}} \tag{4.2.17}$$

For future reference, let us record some explicit formulas for complexes of vector bundles. A complex

$$\mathfrak{E}\colon \qquad \cdots \longrightarrow E_{i+1} \xrightarrow{d_{i+1}} E_i \xrightarrow{d_i} E_{i-1} \longrightarrow \cdots$$

in which E_i has degree $-i$ is mapped to the complex

$$\widehat{P}(\mathfrak{E})\colon \qquad \cdots \longrightarrow P(E_{i-1}) \xrightarrow{\widetilde{P}(d_i)} P(E_i) \xrightarrow{\widetilde{P}(d_{i+1})} P(E_{i+1}) \longrightarrow \cdots \tag{4.2.18}$$

where $\widetilde{P}(d_i)$ is determined by (4.2.13)

$$\widetilde{P}(d_i) = (-1)^i\, \sigma^*(d_i^\vee)$$

and $P(E_i)$ has degree $i - m$. Applying $\widehat{P}$ twice yields the complex

$$\widehat{P}^2(\mathfrak{E})\colon \qquad \cdots \longrightarrow E_{i+1} \xrightarrow{\widetilde{P}^2(d_{i+1})} E_i \xrightarrow{\widetilde{P}^2(d_i)} E_{i-1} \longrightarrow \cdots \tag{4.2.19}$$

where

$$\widetilde{P}^2(d_i) = (-1)^{m+1} d_i\,.$$

Therefore $\widehat{P}^2$ is not equal to the identity functor, but there is an isomorphism of complexes $J\colon \widehat{P}^2(\mathfrak{E}) \to \mathfrak{E}$:

$$\begin{tikzcd} \cdots \arrow[r] & E_{i+1} \arrow[r, "\widetilde{P}^2(d_{i+1})"] \arrow[d, "J_{i+1}"] & E_i \arrow[r, "\widetilde{P}^2(d_i)"] \arrow[d, "J_i"] & E_{i-1} \arrow[r] \arrow[d, "J_{i-1}"] & \cdots \\ \cdots \arrow[r] & E_{i+1} \arrow[r, "d_{i+1}"] & E_i \arrow[r, "d_i"] & E_{i-1} \arrow[r] & \cdots \end{tikzcd} \tag{4.2.20}$$

where

$$J_i = (-1)^{(m+1)i}\chi\, \mathrm{Id}_{E_i}.$$

and χ is a constant. Notice that $J^{-1}\colon \mathfrak{E} \to \widehat{P}^2(\mathfrak{E})$, and that $\widehat{P}^4 = \mathrm{Id}_{D^b(X)}$ implies that also $J\colon \widehat{P}^2(\widehat{P}^2(\mathfrak{E})) = \mathfrak{E} \to \widehat{P}^2(\mathfrak{E})$. Requiring both to be equal constrains χ to be $(-1)^\omega$ with $\omega = 0, 1$.

This sign cannot be fixed using purely algebraic considerations, and we will show in section 4.4 how it encodes the difference between SO/Sp projections. In functorial language, this means that there is an isomorphism of functors $J : \widehat{P}^2 \to \mathrm{Id}_{D^b(X)}$.

We conclude this section with a brief summary of the above discussion, and a short remark on possible generalizations. To simplify notation, in the rest of the chapter we drop the decorations of the various P's. In other words both $\widehat{P}$ and $\widetilde{P}$ will be denoted by P. Which P is meant will always be clear from the context.

1. The orientifold projection in the derived category is a graded contravariant functor $P\colon D^b(X) \to D^b(X)^{op}$ which acts on locally free complexes as in equation (4.2.18). Note that this transformation is closely related to the derived functor

$$\mathbf{L}\sigma^* \circ \mathbf{R}\mathscr{H}om(-, \mathcal{O}_X)[m].$$

   The difference resides in the alternating signs $(-1)^i$ in the action of P on differentials, according to (4.2.18). From now on we will refer to P as a graded derived functor.

2. There is an obvious generalization of this construction which has potential physical applications. One can further compose P with an auto-equivalence $\mathcal{A}$ of the derived category so that the resulting graded functor $P \circ \mathcal{A}$ has its square isomorphic to the identity. This would yield a new class of orientifold models, possibly without a direct geometric interpretation. The physical implications of this construction will be explored in a separate publication.

In the remaining part of this section we will consider the case of D5-branes wrapping holomorphic curves in more detail.

4.2.3 $O5$ models

In this case we consider holomorphic involutions $\sigma\colon X \to X$ whose fixed point set consists of a finite collection of holomorphic curves in X. We will be interested in D5-brane configurations supported on a smooth component $C \simeq \mathbb{P}^1$ of the fixed locus, that are preserved by the orientifold projection. We describe such a configuration by a one term complex

$$i_* V \tag{4.2.21}$$

where $V \to C$ is the Chan-Paton vector bundle on C, and $i\colon C \hookrightarrow X$ is the embedding of C into X.

Since $C \simeq \mathbb{P}^1$, by Grothendieck's theorem any holomorphic bundle V decomposes in a direct sum of line bundles. Therefore, for the time being, we take

$$V \simeq \mathcal{O}_C(a) \tag{4.2.22}$$

for some $a \in \mathbb{Z}$. We will also make the simplifying assumption that V is the restriction of a bundle V' on X to C, i.e.,

$$V = i^* V'. \tag{4.2.23}$$

This is easily satisfied if X is a complete intersection in a toric variety Z, in which case V can be chosen to be the restriction of bundle on Z.

In order to write down the parity action on this D5-brane configuration, we need a locally free resolution $\mathfrak{E}$ for $i_* V = i_* \mathcal{O}_C(a)$. Let

$$\mathfrak{V}: \qquad 0 \longrightarrow \mathcal{V}_n \xrightarrow{d_n} \mathcal{V}_{n-1} \xrightarrow{d_{n-1}} \cdots \xrightarrow{d_2} \mathcal{V}_1 \xrightarrow{d_1} \underline{\mathcal{V}_0} \longrightarrow 0 \tag{4.2.24}$$

be a locally free resolution of $i_* \mathcal{O}_C$.[3] The degree of the term $\mathcal{V}_k$ to be $(-k)$, for $k = 0, \ldots, n$. Then the complex $\mathfrak{E}$

$$\mathfrak{E}: \qquad 0 \longrightarrow \mathcal{V}_n(a) \xrightarrow{d_n} \mathcal{V}_{n-1}(a) \xrightarrow{d_{n-1}} \cdots \xrightarrow{d_2} \mathcal{V}_1(a) \xrightarrow{d_1} \underline{\mathcal{V}_0(a)} \longrightarrow 0 \tag{4.2.25}$$

is a locally free resolution of $i_* \mathcal{O}_C(a)$.

The image of (4.2.25) under the orientifold projection is the complex $\mathcal{P}(\mathfrak{E})$:

$$\begin{gathered} 0 \longrightarrow \sigma^* \mathcal{V}_0^\vee(-a) \xrightarrow{-\sigma^* d_1^\vee} \sigma^* \mathcal{V}_1^\vee(-a) \xrightarrow{\sigma^* d_2^\vee} \cdots \\ \cdots \xrightarrow{(-1)^{n-1} \sigma^* d_{n-1}^\vee} \sigma^* \mathcal{V}_{n-1}^\vee(-a) \xrightarrow{(-1)^n \sigma^* d_n^\vee} \sigma^* \mathcal{V}_n^\vee(-a) \longrightarrow 0 \end{gathered} \tag{4.2.26}$$

The term $\sigma^* \mathcal{V}_k^\vee(-a)$ has degree $k - m$.

[3]We usually underlined the 0th position in a complex.

Lemma 4.2.2. [4] *The complex* (4.2.26) *is quasi-isomorphic to*

$$i_* (V^\vee \otimes K_C)[m-2], \tag{4.2.27}$$

where $K_C \simeq \mathcal{O}_C(-2)$ *is the canonical bundle of* C.

Proof. As noted below (4.2.18), (4.2.26) is isomorphic to $\sigma^*(\mathfrak{E}^\vee)[m]$. Since C is pointwise fixed by σ, it suffices to show that the dual of the locally free resolution (4.2.24) is quasi-isomorphic to $i_* K_C[-2]$. The claim then follows from the adjunction formula:

$$i_* V = i_*(V \otimes \mathcal{O}_C) = i_*(i^* V' \otimes \mathcal{O}_C) = V' \otimes i_* \mathcal{O}_C \tag{4.2.28}$$

and the simple fact that $i^*(V'^\vee) = V^\vee$.

Let us compute $(i_*\mathcal{O}_C)^\vee$ using the locally free resolution (4.2.24). The cohomology in degree k of the complex

$$\mathfrak{V}^\vee: \quad 0 \to (\mathcal{V}_0)^\vee \to (\mathcal{V}_1)^\vee \to \cdots \to (\mathcal{V}_n)^\vee \to 0 \tag{4.2.29}$$

is isomorphic to the local Ext sheaves $\mathscr{E}xt^k_X(i_*\mathcal{O}_C, \mathcal{O}_X)$. According to [222, Chapter 5.3, pg 690] these are trivial except for $k = 2$, in which case

$$\mathscr{E}xt^2_X(\mathcal{O}_C, \mathcal{O}_X) \simeq i_*\mathcal{L},$$

for some line bundle $\mathcal{L}$ on C.

To determine $\mathcal{L}$, it suffices to compute its degree on C, which is an easy application of the Grothendieck-Riemann-Roch theorem. We have

$$i_!(\mathrm{ch}(\mathcal{L})\mathrm{Td}(C)) = \mathrm{ch}(i_*\mathcal{L})\mathrm{Td}(X).$$

On the other hand, by construction

$$\mathrm{ch}_m(i_*\mathcal{L}) = \mathrm{ch}_m(\mathfrak{V}^\vee) = (-1)^m \mathrm{ch}_m(\mathfrak{V}) = (-1)^m \mathrm{ch}_m(i_*\mathcal{O}_C).$$

Using these two equations, we find

$$\deg(\mathcal{L}) = -2 \ \Rightarrow \mathcal{L} \simeq K_C.$$

---

[4] *We give an alternative derivation of this result in Appendix 4.5.1. That proof is very abstract, and hides all the details behind the powerful machinery of Grothendieck duality. On the other hand, we will be using the details of this lengthier derivation in our explicit computations in Section 4.4.*

This shows that $\mathfrak{V}^\vee$ has nontrivial cohomology i_*K_C only in degree 2.

Now we establish that the complex (4.2.29) is quasi-isomorphic to $i_*K_C[-2]$, by constructing such a map of complexes. Consider the restriction of the complex (4.2.29) to C. Since all terms are locally free, we obtain a complex of holomorphic bundles on C whose cohomology is isomorphic to K_C in degree 2 and trivial in all other degrees. Note that the kernel $\mathcal{K}$ of the map

$$\mathcal{V}_2{}^\vee|_C \to \mathcal{V}_3{}^\vee|_C$$

is a torsion free sheaf on C, therefore it must be locally free. Hence $\mathcal{K}$ is a sub-bundle of $\mathcal{V}_2{}^\vee|_C$. Since $C \simeq \mathbb{P}^1$, by Grothendieck's theorem both $\mathcal{V}_2{}^\vee|_C$ and $\mathcal{K}$ are isomorphic to direct sums of line bundles. This implies that $\mathcal{K}$ is in fact a direct summand of $\mathcal{V}_2{}^\vee|_C$. In particular there is a surjective map

$$\rho\colon \mathcal{V}_2{}^\vee|_C \to \mathcal{K}.$$

Since $H^2(\mathfrak{V}^\vee|_C) = K_C$ we also have a surjective map $\tau\colon \mathcal{K} \to K_C$. By construction then $\tau \circ \rho\colon \mathfrak{V}^\vee|_C \to K_C[-2]$ is a quasi-isomorphism. Extending this quasi-isomorphism by zero outside C, we obtain a quasi-isomorphism $\mathfrak{V}^\vee \to i_*K_C[-2]$, which proves the lemma. □

Let us now discuss parity invariant D-brane configurations. Given the parity action (4.2.27) one can obviously construct such configurations by taking direct sums of the form

$$i_*V \oplus i_*(V^\vee \otimes K_C)[m-2] \tag{4.2.30}$$

with V an arbitrary Chan-Paton bundle. Note that in this case we have two stacks of D5-branes in the covering space which are interchanged under the orientifold projection.

However, on physical grounds we should also be able to construct a single stack of D5-branes wrapping C which is preserved by the orientifold action. This is possible only if

$$m = 2 \quad \text{and} \quad V \simeq V^\vee \otimes K_C. \tag{4.2.31}$$

The first condition in (4.2.31) fixes the value of m for this class of models. The second condition constrains the Chan-Paton bundle V to

$$V = \mathcal{O}_C(-1).$$

Let us now consider rank N Chan-Paton bundles V. We will focus on invariant D5-brane configurations given by

$$V = \mathcal{O}_C(-1)^{\oplus N}.$$

In this case the orientifold image $P(i_*V) = i_*(V^\vee \otimes K_C)$ is isomorphic to i_*V, and the choice of an isomorphism corresponds to the choice of a section

$$M \in \mathrm{Hom}_C(V, V^\vee \otimes K_C) \simeq \mathcal{M}_N(\mathbb{C}). \tag{4.2.32}$$

where $\mathcal{M}_N(\mathbb{C})$ is the space of $N \times N$ complex matrices. We have

$$\begin{aligned}\mathrm{Hom}_C(V, V^\vee \otimes K_C) &\simeq H^0(C, S^2(V^\vee) \otimes K_C) \oplus H^0(C, \Lambda^2(V^\vee) \otimes K_C)\\ &\simeq \mathcal{M}_N^+(\mathbb{C}) \oplus \mathcal{M}_N^-(\mathbb{C})\end{aligned}$$

where $\mathcal{M}_N^\pm(\mathbb{C})$ denotes the space of symmetric and antisymmetric $N \times N$ matrices respectively. The choice of this isomorphism (up to conjugation) encodes the difference between SO and Sp projections. For any value of N we can choose the isomorphism to be

$$M = \mathrm{I}_N \in \mathcal{M}_N^+(\mathbb{C}), \tag{4.2.33}$$

obtaining $SO(N)$ gauge group. If N is even, we also have the option of choosing the antisymmetric matrix

$$M = i \begin{bmatrix} 0 & \mathrm{I}_{N/2} \\ -\mathrm{I}_{N/2} & 0 \end{bmatrix} \in \mathcal{M}_N^-(\mathbb{C}) \tag{4.2.34}$$

obtaining $Sp(N/2)$ gauge group. This is a slightly more abstract reformulation of [223]. We will explain how the SO/Sp projections are encoded in the derived formalism in sections 4.3 and 4.4.

4.2.4 $O3/O7$ Models

In this case we have $\epsilon = 1$, and the fixed point set of the holomorphic involution can have both zero and two dimensional components. We will consider the magnetized D5-brane configurations introduced in [224]. Suppose

$$i\colon C \hookrightarrow X \qquad i' : C' \hookrightarrow X$$

is a pair of smooth rational curves mapped isomorphically into each other by the holomorphic involution. The brane configuration consists of a stack of D5-branes wrapping C, which is related by the orientifold projection to a stack of anti-D5-branes wrapping C'. We describe the stack of D5-branes wrapping C by a one term complex i_*V, with V a bundle on C.

In order to find the action of the orientifold group on the stack of D5-branes wrapping C we pick a locally free resolution $\mathfrak{E}$ for i_*V. Once again the orientifold image is obtained by applying the graded derived functor P to $\mathfrak{E}$.

Applying Prop. 4.5.1, we have

Lemma 4.2.3. *$P(\mathfrak{E})$ is quasi-isomorphic to the one term complex*

$$i'_*(\sigma^*(V^\vee) \otimes K_{C'})[m-2]. \tag{4.2.35}$$

It follows that a D5-brane configuration preserved by the orientifold projection is a direct sum

$$i_*V \oplus i'_*(\sigma^*(V^\vee) \otimes K_{C'})[m-2]. \tag{4.2.36}$$

The value of m can be determined from physical arguments by analogy with the previous case. We have to impose the condition that the orientifold projection preserves a D3-brane supported on a fixed point $p \in X$ as well as a D7-brane supported on a pointwise fixed surface $S \subset X$.

A D3-brane supported at $p \in X$ is described by a one-term complex $\mathcal{O}_{p,X}$, where $\mathcal{O}_{p,X}$ is a skyscraper sheaf supported at p. Again, using Prop. 4.5.1 one shows that $P(\mathfrak{V})$ is quasi-isomorphic to $\mathcal{O}_{p,X}[m-3]$. Therefore, the D3-brane is preserved if and only if $m = 3$.

If the model also includes a codimension 1 pointwise-fixed locus $S \subset X$, then we have an extra condition. Let V be the Chan-Paton bundle on S. We describe the invariant D7-brane wrapping S by $\mathfrak{L} \simeq i_*(V)[k]$ for some integer k, where $i\colon S \to X$ is the embedding.

Since S is codimension 1 in X, Prop. 4.5.1 tells us that

$$P(\mathfrak{L}) \simeq i_*(V^\vee \otimes K_S)[m-k-1]. \tag{4.2.37}$$

Therefore invariance under P requires

$$2k = m-1 \qquad V \otimes V \simeq K_S. \tag{4.2.38}$$

Since we have found $m=3$ above, it follows that $k=1$. Furthermore, V has to be a square root of K_S. In particular, this implies that K_S must be even, or, in other words that S must be spin. This is in agreement with the Freed-Witten anomaly cancellation condition [225]. If S is not spin, one has to turn on a half integral B-field in order to cancel anomalies.

Returning to the magnetized D5-brane configuration, note that an interesting situation from the physical point of view is the case when the curves C and C' coincide. Then C is preserved by the holomorphic involution, but not pointwise fixed as in the previous subsection. We will discuss examples of such configurations in section 4.4. In the next section we will focus on general aspects of the superpotential in orientifold models.

4.3 The Superpotential

The framework of D-brane categories offers a systematic approach to the computation of the tree-level superpotential. In the absence of the orientifold projection, the tree-level D-brane superpotential is encoded in the A_∞ structure of the D-brane category [206, 207, 208, 210].

Given an object of the D-brane category $\mathcal{D}$, the space of off-shell open string states is its space of endomorphisms in the pre-triangulated category Pre-Tr$(\widetilde{\mathcal{C}})$. This carries the structure of a $\mathbb{Z}$-graded differential cochain complex. In this section we will continue to work with Dolbeault cochains, and also specialize our discussion to locally free complexes $\mathfrak{E}$ of the form (4.2.5). Then the space of off-shell open string states is given by

$$\mathrm{Mor}_{\text{Pre-Tr}(\widetilde{\mathcal{C}})}(\mathfrak{E},\mathfrak{E}) = \oplus_p A^{0,p}(\mathscr{H}om_X(\mathfrak{E},\mathfrak{E}))$$

where

$$\mathscr{H}om^q_X(\mathfrak{E},\mathfrak{E}) = \oplus_i \, \mathscr{H}om_X(E_i, E_{i-q}).$$

Composition of morphisms defines a natural superalgebra structure on this endomorphism space [226], and the differential Q satisfies the graded Leibniz rule. We will denote the resulting DGA by $\mathcal{C}(\mathfrak{E},\mathfrak{E})$.

The computation of the superpotential is equivalent to the construction of an A_∞ minimal model for the DGA $\mathcal{C}(\mathfrak{E},\mathfrak{E})$. Since this formalism has been explained in detail in the physics literature [208, 214], we will not provide a comprehensive review here. Rather we will recall some basic elements needed for our construction.

In order to extend this computational framework to orientifold models, we have to find an off-shell cochain model equipped with an orientifold projection and a compatible differential algebraic structure. We made a first step in this direction in the previous section by giving a categorical formulation of the orientifold projection. In section 4.3.1 we will refine this construction, obtaining the desired cochain model.

Having constructed a suitable cochain model, the computation of the superpotential follows the same pattern as in the absence of the orientifold projection. A notable distinction resides in the occurrence of L_∞ instead of A_∞ structures, since the latter are not compatible with the involution. The final result obtained in section 4.3.2 is that the orientifold superpotential can be obtained by evaluating the superpotential of the underlying unprojected theory on invariant field configurations.

4.3.1 Cochain Model and Orientifold Projection

Suppose $\mathfrak{E}$ is a locally free complex on X, and that it is left invariant by the parity functor. This means that $\mathfrak{E}$ and $P(\mathfrak{E})$ are isomorphic in the derived category, and we choose such an isomorphism

$$\psi\colon \mathfrak{E} \to P(\mathfrak{E}). \tag{4.3.1}$$

Although in general ψ is not a map of complexes, it can be chosen so in most practical situations, including all cases studied in this chapter. Therefore we will assume from now on that ψ is a quasi-isomorphism of complexes:

$$\begin{tikzcd}
\cdots \arrow[r] & E_{m-i+1} \arrow[r, "d_{m-i+1}"] \arrow[d, "\psi_{m-i+1}"] & E_{m-i} \arrow[r, "d_{m-i}"] \arrow[d, "\psi_{m-i}"] & E_{m-i-1} \arrow[r] \arrow[d, "\psi_{m-i-1}"] & \cdots \\
\cdots \arrow[r] & P(E_{i-1}) \arrow[r, "P(d_i)"] & P(E_i) \arrow[r, "P(d_{i+1})"] & P(E_{i+1}) \arrow[r] & \cdots
\end{tikzcd} \tag{4.3.2}$$

We have written (4.3.2) so that the terms in the same column have the same degree since ψ is a degree zero morphism. The degrees of the three columns from left to right are $i-m-1$, $i-m$ and $i-m+1$. For future reference, note that the quasi-isomorphism ψ induces a quasi-isomorphism of cochain complexes

$$\psi_*\colon \mathcal{C}(P(\mathfrak{E}), \mathfrak{E}) \to \mathcal{C}(P(\mathfrak{E}), P(\mathfrak{E})), \qquad f \mapsto \psi \circ f. \tag{4.3.3}$$

The problem we are facing in the construction of a viable cochain model resides in the absence of a natural orientifold projection on the cochain space $\mathcal{C}(\mathfrak{E}, \mathfrak{E})$. P maps $\mathcal{C}(\mathfrak{E}, \mathfrak{E})$ to $\mathcal{C}(P(\mathfrak{E}), P(\mathfrak{E}))$, which is not identical to $\mathcal{C}(\mathfrak{E}, \mathfrak{E})$. How can we find a natural orientifold projection on a given off-shell cochain model?

Since $\mathfrak{E}$ and $P(\mathfrak{E})$ are quasi-isomorphic, one can equally well adopt the morphism space

$$\mathcal{C}(P(\mathfrak{E}), \mathfrak{E}) = \mathrm{Mor}_{\text{Pre-Tr}(\widetilde{\mathcal{C}})}(P(\mathfrak{E}), \mathfrak{E})$$

as an off-shell cochain model. As opposed to $\mathcal{C}(\mathfrak{E}, \mathfrak{E})$, this morphism space has a natural induced involution defined by the composition

$$\mathcal{C}(P(\mathfrak{E}), \mathfrak{E}) \xrightarrow{P} \mathcal{C}(P(\mathfrak{E}), P^2(\mathfrak{E})) \xrightarrow{J_*} \mathcal{C}(P(\mathfrak{E}), \mathfrak{E}) \tag{4.3.4}$$

where J is the isomorphism in (4.2.20). Therefore we will do our superpotential computation in the cochain model $\mathcal{C}(P(\mathfrak{E}), \mathfrak{E})$, as opposed to $\mathcal{C}(\mathfrak{E}, \mathfrak{E})$, which is used in [214].

This seems to lead us to another puzzle, since a priori there is no natural associative algebra structure on $\mathcal{C}(P(\mathfrak{E}),\mathfrak{E})$. One can however define one using the quasi-isomorphism (4.3.1). Given

$$f^p_{q,k} \in A^{0,p}(\mathscr{H}om_X(P(E_k), E_{m-k-q})) \qquad g^r_{s,l} \in A^{0,r}(\mathscr{H}om_X(P(E_l), E_{m-l-s}))$$

we define

$$g^r_{s,l} \star_\psi f^p_{q,k} = \begin{cases} (-1)^{sp} g^r_{s,l} \cdot \psi_{m-k-q} \cdot f^p_{q,k} & \text{for } l = k+q \\ \\ 0 & \text{otherwise.} \end{cases} \tag{4.3.5}$$

where $\cdot$ denotes exterior multiplication of bundle valued differential forms.

With this definition, the map (4.3.3) becomes a quasi-isomorphism of DGAs. The sign $(-1)^{sp}$ in (4.3.5) is determined by the sign rule (4.2.1) for composition of morphisms in $\widetilde{\mathcal{C}}$. This construction has the virtue that it makes both the algebra structure and the orientifold projection manifest. Note that the differential Q satisfies the graded Leibniz rule with respect to the product $\star_\psi$ because ψ is a Q-closed element of $\mathcal{C}(P(\mathfrak{E}),\mathfrak{E})$ of degree zero.

Next we check two compatibility conditions between the involution (4.3.4) and the DGA structure.

Lemma 4.3.1. *For any cochain $f \in \mathcal{C}(P(\mathfrak{E}),\mathfrak{E})$*

$$J_* P(Q(f)) = Q(J_* P(f)). \tag{4.3.6}$$

Proof. Using equation (4.2.18), the explicit expression for the differential Q acting on a homogeneous element $f^p_{q,k}$ as above is

$$Q(f^p_{q,k}) = \overline{\partial}_{P(E_k)E_{m-k-q}}(f^p_{q,k}) + (d_{m-k-q} \circ f^p_{q,k})_{\widetilde{\mathcal{C}}} - (-1)^{p+q}(f^p_{q,k} \circ P(d_k))_{\widetilde{\mathcal{C}}}.$$

According to equation (4.2.17), we have

$$\begin{aligned} P(Q(f^p_{q,k})) =& \overline{\partial}_{P(E_{m-k-q})P^2(E_k)}(P(f^p_{q,k})) + (P^2(d_k) \circ P(f^p_{q,k}))_{\widetilde{\mathcal{C}}} \\ & - (-1)^{|P(f)|}(P(f^p_{q,k}) \circ P(d_{m-k-q}))_{\widetilde{\mathcal{C}}} \end{aligned} \tag{4.3.7}$$

The commutative diagram (4.2.20) shows that

$$J \circ P^2(d_k) = d_k \circ J.$$

Then, equation (4.3.7) yields

$$\begin{aligned} J_*P(Q(f^p_{q,k})) =& \overline{\partial}_{P(E_{m-k-q})E_k}(J_*P(f^p_{q,k})) + (d_k \circ J_*P(f^p_{q,k}))_{\widetilde{\mathcal{C}}} \\ & - (-1)^{|f|}(J_*P(f^p_{q,k}) \circ P(d_{m-k-q}))_{\widetilde{\mathcal{C}}} \end{aligned}$$

which proves (4.3.6). □

Lemma 4.3.2. *For any two elements $f, g \in \mathcal{C}(P(\mathfrak{E}), \mathfrak{E})$*

$$J_*P(g \star_\psi f) = -(-1)^{|f||g|} J_*P(f) \star_\psi J_*P(g). \tag{4.3.8}$$

Proof. Written in terms of homogeneous elements, (4.3.8) reads

$$J_*P(g^r_{s,l} \star_\psi f^p_{q,k}) = -(-1)^{(r+s)(p+q)} J_*P(f^p_{q,k}) \star_\psi J_*P(g^r_{s,l}) \tag{4.3.9}$$

where $l = k+q$. Using equations (4.2.13), (4.3.5) and the definition of (4.2.20) of J, we compute

$$\begin{aligned} J_*P(g^r_{s,l} \star_\psi f^p_{q,k}) &= (-1)^{(m-s-l)(m+1)+\omega}(-1)^{(s+q)(m-s-l)+1}(-1)^{sp}\, \sigma^*(g^r_{s,l} \cdot \psi_{m-k-q} \cdot f^p_{q,k})^\vee \\ &= (-1)^{(m-s-l)(m+1)+\omega}(-1)^{(s+q)(m-s-l)+1}(-1)^{sp}(-1)^{rp} \\ &\quad \sigma^*(f^p_{q,k})^\vee \cdot \sigma^*(\psi^\vee_{m-k-q}) \cdot \sigma^*(g^r_{s,l})^\vee \\ &= (-1)^{(m-s-l)(m+1)+\omega}(-1)^{(s+q)(m-s-l)+1}(-1)^{sp}(-1)^{rp} \\ &\quad (-1)^{(m-k-q)(m+1)+\omega}(-1)^{q(q+k-m)+1}(-1)^{(m-s-l)(m+1)+\omega}(-1)^{s(s+l-m)+1} \\ &\quad J_*P(f^p_{q,k}) \cdot \sigma^*(\psi^\vee_{m-k-q}) \cdot J_*P(g^r_{s,l}) \end{aligned}$$

$$-(-1)^{(r+s)(p+q)} J_*P(f^p_{q,k}) \star_\psi J_*P(g^r_{s,l}) = -(-1)^{(r+s)(p+q)}(-1)^{qr} J_*P(f^p_{q,k}) \cdot \psi_l \cdot J_*P(g^r_{s,l})$$

These expressions are in agreement with equation (4.3.9) if and only if ψ satisfies a symmetry condition of the form

$$J^*P(\psi_{m-l}) = -\psi_l \quad \Leftrightarrow \quad \sigma^*(\psi_{m-l})^\vee = (-1)^{(m+1)l+\omega}\psi_l \tag{4.3.10}$$

□

We saw in the last proof that compatibility of the orientifold projection with the algebraic structure imposes the condition (4.3.10) on ψ. From now on we assume this condition to be satisfied. Although we do not know a general existence result for a quasi-isomorphism satisfying (4.3.10), we will show that such a choice is possible in all the examples considered in this chapter. We will also see that symmetry of ψ, which is determined by $\omega = 0, 1$ in (4.3.10), determines whether the orientifold projection is of type SO or Sp.

Granting such a quasi-isomorphism, it follows that the cochain space $\mathcal{C}(P(\mathfrak{E}), \mathfrak{E})$ satisfies all the conditions required for the computation of the superpotential, which is the subject of the next subsection.

4.3.2 The Superpotential

In the absence of an orientifold projection, the computation of the superpotential can be summarized as follows [209]. Suppose we are searching for formal deformations of the differential Q of the form

$$Q_{\text{def}} = Q + f_1(\phi) + f_2(\phi) + f_3(\phi) + \ldots \tag{4.3.11}$$

where

$$f_1(\phi) = \phi$$

is a cochain of degree one, which represents an infinitesimal deformation of Q. The terms $f_k(\phi)$, for $k \geq 2$, are homogeneous polynomials of degree k in ϕ corresponding to higher order deformations. We want to impose the integrability condition

$$(Q_{\text{def}})^2 = 0 \tag{4.3.12}$$

order by order in ϕ. In doing so one encounters certain obstructions, which are systematically encoded in a minimal A_∞ model of the DGA $\mathcal{C}(P(\mathfrak{E}), \mathfrak{E})$. The superpotential is essentially a primitive function for the obstructions, and exists under certain cyclicity conditions.

In the orientifold model we have to solve a similar deformation problem, except that now the deformations of Q have to be invariant under the orientifold action. We will explain below that this is equivalent to the construction of a minimal L_∞ model.

Let us first consider the integrability conditions (4.3.12) in more detail in the absence of orientifolding. Suppose we are given an associative $\mathbb{Z}$-graded DGA $(\mathcal{C}, Q, \cdot)$, and let H denote the cohomology of Q. In order to construct an A_∞ structure on H we need the following data

(i) A $\mathbb{Z}$-graded linear subspace $\mathcal{H} \subset \mathcal{C}$ isomorphic to the cohomology of Q. In other words, $\mathcal{H}$ is spanned in each degree by representatives of the cohomology classes of Q.

(ii) A linear map $\eta : \mathcal{C} \to \mathcal{C}[-1]$ mapping $\mathcal{H}$ to itself such that

$$\Pi = \mathbb{I} - [Q, \eta] \tag{4.3.13}$$

is a projector $\Pi \colon \mathcal{C} \to \mathcal{H}$, where $[\quad,\quad]$ is the graded commutator. Moreover, we assume that the following conditions are satisfied

$$\eta|_{\mathcal{H}} = 0 \qquad \eta^2 = 0. \tag{4.3.14}$$

Using the data (i), (ii) one can develop a recursive approach to obstructions in the deformation theory of Q [209]. The integrability condition (4.3.12) yields

$$\sum_{n=1}^{\infty} [Q(f_n(\phi)) + B_{n-1}(\phi)] = 0 \tag{4.3.15}$$

where

$$\begin{aligned} B_0 &= 0 \\ B_{n-1} &= \phi f_{n-1}(\phi) + f_{n-1}(\phi)\phi + \sum_{\substack{k+l=n \\ k,l\geq 2}} f_k(\phi) f_l(\phi), \quad n \geq 2 \end{aligned}$$

Using equation (4.3.13), we can rewrite equation (4.3.15) as

$$\sum_{n=1}^{\infty} [Q(f_n(\phi)) + ([Q,\eta] + \Pi) B_{n-1}(\phi)] = 0. \tag{4.3.16}$$

We claim that the integrability condition (4.3.15) can be solved recursively [209] provided that

$$\sum_{n=1}^{\infty} \Pi(B_{n-1}) = 0. \tag{4.3.17}$$

To prove this claim, note that if the condition (4.3.17) is satisfied, equation (4.3.16) becomes

$$\sum_{n=1}^{\infty} (Q(f_n(\phi)) + [Q,\eta] B_{n-1}(\phi)) = 0. \tag{4.3.18}$$

This equation can be solved by setting recursively

$$f_n(\phi) = -\eta(B_{n-1}(\phi)). \tag{4.3.19}$$

One can show that this is a solution to (4.3.19) by proving inductively that

$$Q(B_n(\phi)) = 0.$$

In conclusion, the obstructions to the integrability condition (4.3.15) are encoded in the formal series

$$\sum_{n=2}^{\infty} \Pi\Big(\phi f_{n-1}(\phi) + f_{n-1}(\phi)\phi + \sum_{\substack{k+l=n \\ k,l\geq 2}} f_k(\phi) f_l(\phi)\Big) \tag{4.3.20}$$

where the $f_n(\phi)$, $n \geq 1$, are determined recursively by (4.3.19).

The algebraic structure emerging from this construction is a minimal A_∞ structure for the DGA $(\mathcal{C}, Q)$ [227, 228]. [228] constructs an A_∞ structure by defining the linear maps

$$\lambda_n \colon \mathcal{C}^{\otimes n} \to \mathcal{C}[2-n], \qquad n \geq 2$$

recursively

$$\begin{aligned} \lambda_n(c_1, \ldots, c_n) =& (-1)^{n-1}(\eta\lambda_{n-1}(c_1, \ldots, c_{n-1})) \cdot c_n - (-1)^{n|c_1|} c_1 \cdot \eta\lambda_{n-1}(c_2, \ldots, c_n) \\ & - \sum_{\substack{k+l=n \\ k,l\geq 2}} (-1)^r [\eta\lambda_k(c_1, \ldots, c_k)] \cdot [\eta\lambda_l(c_{k+1}, \ldots, c_n)] \end{aligned} \tag{4.3.21}$$

where $|c|$ denotes the degree of an element $c \in \mathcal{C}$, and

$$r = k + 1 + (l-1)(|c_1| + \ldots + |c_k|).$$

Now define the linear maps

$$\mathfrak{m}_n \colon \mathcal{H}^{\otimes n} \to \mathcal{H}[2-n], \qquad n \geq 1$$

by

$$\begin{aligned} \mathfrak{m}_1 &= \eta \\ \mathfrak{m}_n &= \Pi\lambda_n. \end{aligned} \tag{4.3.22}$$

The products (4.3.22) define an A_∞ structure on $\mathcal{H} \simeq H$. If the conditions (4.3.14) are satisfied, this A_∞ structure is a minimal model for the DGA $(\mathcal{C}, Q, \cdot)$. The products $\mathfrak{m}_n$, $n \geq 2$ agree up to sign with the obstructions $\Pi(B_n)$ found above.

The products $\mathfrak{m}_n$ determine the local equations of the D-brane moduli space, which in physics language are called F-term equations. If

$$\phi = \sum_{i=1}^{\dim(\mathcal{H})} \phi^i u_i$$

is an arbitrary cohomology element written in terms of some generators $\{u_i\}$, the F-term equations are

$$\sum_{n=2}^{\infty} (-1)^{n(n+1)/2} \mathfrak{m}_n(\phi^{\otimes n}) = 0. \tag{4.3.23}$$

If the products are cyclic, these equations admit a primitive

$$W = \sum_{n=2}^{\infty} \frac{(-1)^{n(n+1)/2}}{n+1} \langle \phi, \mathfrak{m}_n(\phi^{\otimes n}) \rangle \tag{4.3.24}$$

where

$$\langle \quad , \quad \rangle \colon \mathcal{C} \to \mathbb{C}$$

is a bilinear form on $\mathcal{C}$ compatible with the DGA structure. The cyclicity property reads

$$\langle c_1, \mathfrak{m}_n(c_2, \ldots, c_{n+1}) \rangle = (-1)^{n|c_2|+1} \langle c_2, \mathfrak{m}_n(c_3, \ldots, c_{n+1}, c_1) \rangle.$$

Let us now examine the above deformation problem in the presence of an orientifold projection. Suppose we have an involution $\tau \colon \mathcal{C} \to \mathcal{C}$ such that the following conditions are satisfied

$$\begin{aligned} \tau(Q(f)) &= Q(\tau(f)) \\ \tau(fg) &= -(-1)^{|f||g|} \tau(g) \tau(f) \end{aligned} \tag{4.3.25}$$

As explained below equation (4.3.12), in this case we would like to study deformations

$$Q_{def} = Q + f_1(\phi) + f_2(\phi) + \ldots$$

of Q such that

$$\tau(f_n(\phi)) = f_n(\phi) \tag{4.3.26}$$

for all $n \geq 1$.

In order to set this problem in the proper algebraic context, note that the DG algebra $\mathcal{C}$ decomposes into a direct sum of τ-invariant and anti-invariant parts

$$\mathcal{C} \simeq \mathcal{C}^+ \oplus \mathcal{C}^-. \tag{4.3.27}$$

There is a similar decomposition

$$H = H^+ \oplus H^- \tag{4.3.28}$$

for the Q-cohomology.

Conditions (4.3.25) imply that Q preserves $\mathcal{C}^{\pm}$, but the associative algebra product is not compatible with the decomposition (4.3.27). There is however another algebraic structure which is preserved by τ, namely the graded commutator

$$[f, g] = fg - (-1)^{|f||g|}\, gf. \tag{4.3.29}$$

This follows immediately from the second equation in (4.3.25). The graded commutator (4.3.29) defines a differential graded Lie algebra structure on $\mathcal{C}$. By restriction, it also defines a DG Lie algebra structure on the invariant part $\mathcal{C}^+$. In this context our problem reduces to the deformation theory of the restriction $Q^+ = Q|_{\mathcal{C}^+}$ as a differential operator on $\mathcal{C}^+$.

Fortunately, this problem can be treated by analogy with the previous case, except that we have to replace A_∞ structures by L_∞ structures, see for example [229, 208, 209]. In particular, the obstructions to the deformations of Q^+ can be systematically encoded in a minimal L_∞ model, and one can similarly define a superpotential if certain cyclicity conditions are satisfied.

Note that any associative DG algebra can be naturally endowed with a DG Lie algebra structure using the graded commutator (4.3.29). In this case, the A_∞ and the L_∞ approach to the deformation of Q are equivalent [208] and they yield the same superpotential. However, the L_∞ approach is compatible with the involution, while the A_∞ approach is not.

To summarize this discussion, we have a DG Lie algebra on $\mathcal{C}$ which induces a DG Lie algebra of Q. The construction of a minimal L_∞ model for $\mathcal{C}$ requires the same data (i), (ii) as in the case of a minimal A_∞ model, and yields the same F-term equations, and the same superpotential. In order to determine the F-term equations and superpotential for the invariant part $\mathcal{C}^+$ we need again a set of data (i), (ii) as described above (4.3.13). This data can be naturally obtained by restriction from $\mathcal{C}$ provided that the propagator η in equation (4.3.13) can be chosen compatible with the involution τ i.e

$$\tau(\eta(f)) = \eta(\tau(f)).$$

This condition is easily satisfied in geometric situations, hence we will assume that this is the case from now on. Then the propagator $\eta^+ : \mathcal{C}^+ \to \mathcal{C}^+[-1]$ is obtained by restricting η to the invariant part $\eta^+ = \eta|_{\mathcal{C}^+}$. Given this data, we construct a minimal L_∞ model for the DGL algebra $\mathcal{C}^+$, which yields F-term equations and, if the cyclicity condition is satisfied, a superpotential W^+.

Theorem 4.3.3. *The superpotential W^+ obtained by constructing the minimal L_∞ model for the DGL $\mathcal{C}^+$ is equal to the restriction of the superpotential W corresponding to $\mathcal{C}$ evaluated on τ-invariant field configurations:*

$$W^+ = W|_{H^+}. \tag{4.3.30}$$

In the remaining part of this section we will give a formal argument for this claim. According to [229], the data (i), (ii) above equation (4.3.13) also determines an L_∞ structure on $\mathcal{H}$ as follows. First we construct a series of linear maps

$$\rho_n \colon \mathcal{C}^{\otimes n} \to \mathcal{C}[2-n], \qquad n \geq 2$$

by anti-symmetrizing (in the graded sense) the maps (4.3.21). That is the recursion relation becomes

$$\begin{aligned}\rho_n(c_1, \ldots, c_n) = &\sum_{\sigma \in Sh(n-1,1)} (-1)^{n-1+|\sigma|} e(\sigma)[\eta\rho_{n-1}(c_{\sigma(1)}, \ldots, c_{\sigma(n-1)}), c_{\sigma(n)}] \\ &- \sum_{\sigma \in Sh(1,n)} (-1)^{n|c_1|+|\sigma|} e(\sigma)[c_1, \eta\rho_{n-1}(c_{\sigma(2)}] \\ &- \sum_{\substack{k+l=n \\ k,l\geq 2}} \sum_{\sigma \in Sh(k,n)} (-1)^{r+|\sigma|} e(\sigma)[\eta\rho_k(c_{\sigma(1)}, \ldots, c_{\sigma(k)}), \eta\rho_l(c_{\sigma(k+1)}, \ldots, c_{\sigma(n)})]\end{aligned} \tag{4.3.31}$$

where $Sh(k,n)$ is the set of all permutations $\sigma \in S_n$ such that

$$\sigma(1) < \ldots < \sigma(k) \quad \text{and} \quad \sigma(k+1) < \ldots < \sigma(n)$$

and $|\sigma|$ is the signature of a permutation $\sigma \in S_n$. The symbol $e(\sigma)$ denotes the Koszul sign defined by

$$c_{\sigma(1)} \wedge \ldots \wedge c_{\sigma(n)} = (-1)^{|\sigma|} e(\sigma) c_1 \wedge \ldots \wedge c_n.$$

Then we define the L_∞ products

$$\mathfrak{l}_n \colon \mathcal{H}^{\otimes n} \to \mathcal{H}$$

by

$$\mathfrak{l}_1 = \eta, \qquad \mathfrak{l}_n = \Pi \rho_n. \tag{4.3.32}$$

One can show that these products satisfy a series of higher Jacobi identities analogous to the defining associativity conditions of A_∞ structures. If the conditions (4.3.14) are also satisfied, the resulting L_∞ structure is a minimal model for the DGL algebra $\mathcal{C}$.

Finally, note that the A_∞ products (4.3.22) and the L_∞ products (4.3.32) are related by

$$\mathfrak{l}_n(c_1, \ldots, c_n) = \sum_{\sigma \in S_n} (-1)^{|\sigma|} e(\sigma) \mathfrak{m}_n(c_{\sigma(1)}, \ldots, c_{\sigma(n)}). \tag{4.3.33}$$

In particular, one can rewrite the F-term equations (4.3.23) and the superpotential (4.3.24) in terms of L_∞ products [208, 209].

The construction of the minimal L_∞ model of the invariant part $\mathcal{C}^+$ is analogous. Since we are working under assumption that the propagator η^+ is the restriction of η to $\mathcal{C}^+$, it is clear that the linear maps $\rho_n^+(c_1, \ldots, c_n)$ are also equal to the restriction $\rho_n|_{(\mathcal{C}^+)^n}$. The same will be true for the products $\mathfrak{l}_n^+$, i.e.

$$\mathfrak{l}_n^+ = \mathfrak{l}_n|_{(H^+)^n}.$$

Therefore the F-term equations and the superpotential in the orientifold model can be obtained indeed by restriction to the invariant part.

Now that we have the general machinery at hand, we can turn to concrete examples of superpotential computations.

4.4 Computations for Obstructed Curves

In this section we perform detailed computations of the superpotential for D-branes wrapping holomorphic curves in Calabi-Yau orientifolds.

So far we have relied on the Dolbeault cochain model, which serves as a good conceptual framework for our constructions. However, a Čech cochain model is clearly preferred for computational purposes [214]. The simple prescription found above for the orientifold superpotential allows us to switch from the Dolbeault to the Čech model with little effort. Using the same definition for the action of the orientifold projection P on locally free complexes $\mathfrak{E}$, we will adopt a cochain model of the form

$$\mathcal{C}(P(\mathfrak{E}),\mathfrak{E}) = \check{C}(\mathfrak{U}, \mathscr{H}om_X(P(\mathfrak{E}),\mathfrak{E})) \tag{4.4.1}$$

where $\mathfrak{U}$ is a fine open cover of X. The differential Q is given by

$$Q(f) = \delta(f) + (-1)^{c(f)}\mathfrak{d}(f) \tag{4.4.2}$$

where δ is the Čech differential, $\mathfrak{d}$ is the differential of the local Hom complex and $c(f)$ is the Čech degree of f.

In order to obtain a well-defined involution on the complex (4.4.1), we have to choose the open cover $\mathfrak{U}$ so that the holomorphic involution $\sigma\colon X \to X$ maps any open set $U \in \mathfrak{U}$ isomorphically to another open set $U_{s(\alpha)} \in \mathfrak{U}$, where s is an involution on the set of indices $\{\alpha\}$. Moreover, the holomorphic involution should also be compatible with intersections. That is, if $U_\alpha, U_\beta \in \mathfrak{U}$ are mapped to $U_{s(\alpha)}, U_{s(\beta)} \in \mathfrak{U}$ then $U_{\alpha\beta}$ should be mapped isomorphically to $U_{s(\alpha)s(\beta)}$. Analogous properties should hold for arbitrary multiple intersections. Granting such a choice of a fine open cover, we have a natural involution J_*P acting on the cochain complex (4.4.1), defined as in (4.3.4).

According to the prescription derived in the previous section, the orientifold superpotential can be obtained by applying the computational scheme of [214] to invariant Q-cohomology representatives. Since the computation depends only on the infinitesimal neighborhood of the curve, it suffices to consider local Calabi-Yau models as in [214]. We will consider two representative cases, namely obstructed $(0,-2)$ curves and local conifolds, i.e., $(-1,-1)$ curves.

4.4.1 Obstructed $(0,-2)$ Curves in $O5$ Models

In this case, the local Calabi-Yau X can be covered by two coordinate patches (x, y_1, y_2), (w, z_1, z_2) with transition functions

$$
\begin{aligned}
w &= x^{-1} \\
z_1 &= x^2 y_1 + x y_2^n \\
z_2 &= y_2.
\end{aligned}
\tag{4.4.3}
$$

The $(0,-2)$ curve is given by the equations

$$
C\colon \quad y_1 = y_2 = 0 \qquad \text{resp.} \quad z_1 = z_2 = 0 \tag{4.4.4}
$$

in the two patches. The holomorphic involution acts as

$$
\begin{aligned}
(x, y_1, y_2) &\mapsto (x, -y_1, -y_2) \\
(w, z_1, z_2) &\mapsto (w, -z_1, -z_2)
\end{aligned}
\tag{4.4.5}
$$

This is compatible with the transition functions if and only if n is odd. We will assume that this is the case from now on. Using (4.2.31), the Chan-Paton bundles

$$
V_N = \mathcal{O}_C(-1)^{\oplus N}. \tag{4.4.6}
$$

define invariant D-brane configurations under the orientifold projection.

The on-shell open string states are in one-to-one correspondence with elements of the global Ext group $\mathrm{Ext}^1(i_*V_N, i_*V_N)$. Given two bundles V, W supported on a curve $i\colon C \hookrightarrow X$, there is a spectral sequence [206]

$$
E_2^{p,q} = H^p(C, V^\vee \otimes W \otimes \Lambda^q N_{C/X}) \;\Rightarrow\; \mathrm{Ext}_X^{p+q}(i_*V, i_*W) \tag{4.4.7}
$$

which degenerates at E_2. This yields

$$
\mathrm{Ext}^1(i_*\mathcal{O}_C(-1), i_*\mathcal{O}_C(-1)) \simeq H^0(C, N_{C/X}) = \mathbb{C},
$$

since $N_{C/X} \simeq \mathcal{O}_C \oplus \mathcal{O}_C(-2)$. Therefore a D5-brane with multiplicity $N = 1$ has a single normal deformation. For higher multiplicity, the normal deformations will be parameterized by an $(N \times N)$ complex matrix.

In order to apply the computational algorithm developed in section 4.3 we have to find a locally free resolution $\mathfrak{E}$ of $i_*\mathcal{O}_C(-1)$ and an explicit generator of

$$\mathrm{Ext}^1(i_*\mathcal{O}_C(-1), i_*\mathcal{O}_C(-1)) \simeq \mathrm{Ext}^1(P(\mathfrak{E}), \mathfrak{E})$$

in the cochain space $\check{C}(\mathfrak{U}, \mathscr{H}om(P(\mathfrak{E}), \mathfrak{E}))$. We take $\mathfrak{E}$ to be the locally free resolution from [214] multiplied by $\mathcal{O}_C(-1)$, i.e.,

$$0 \longrightarrow \mathcal{O}(-1) \xrightarrow{\begin{pmatrix} y_2 \\ -1 \\ x \end{pmatrix}} \begin{matrix} \mathcal{O}(-1) \\ \oplus \\ \mathcal{O} \\ \oplus \\ \mathcal{O} \end{matrix} \xrightarrow{\begin{pmatrix} 1 & y_2 & 0 \\ -x & 0 & y_2 \\ -y_2^{n-1} & -s & -y_1 \end{pmatrix}} \begin{matrix} \mathcal{O} \\ \oplus \\ \mathcal{O} \\ \oplus \\ \mathcal{O}(-1) \end{matrix} \xrightarrow{(\, s \ y_1 \ y_2 \,)} \mathcal{O}(-1) \tag{4.4.8}$$

The quasi-isomorphism $\psi\colon \mathfrak{E} \to P(\mathfrak{E})$ is given by

$$
\begin{tikzcd}[column sep=huge, row sep=huge]
\begin{matrix}\mathcal{O}(-1)\\ \oplus \\ \mathcal{O}^{\oplus 2}\end{matrix} \arrow[r, "{\begin{pmatrix} 1 & y_2 & 0 \\ -x & 0 & y_2 \\ -y_2^{n-1} & -s & -y_1 \end{pmatrix}}"] \arrow[d, "{\begin{pmatrix} 0 & x & 1 \end{pmatrix}}"'] & \begin{matrix}\mathcal{O}^{\oplus 2}\\ \oplus \\ \mathcal{O}(-1)\end{matrix} \arrow[r, "{\begin{pmatrix} s & y_1 & y_2 \end{pmatrix}}"] \arrow[d, "{\begin{pmatrix} 0 & y_2^{n-1} & -x \\ -y_2^{n-1} & 0 & -1 \\ x & 1 & 0 \end{pmatrix}}"'] & \mathcal{O}(-1) \arrow[d, "{\begin{pmatrix} 0 \\ x \\ 1 \end{pmatrix}}"] \\
\mathcal{O}(1) \arrow[r, "{\begin{pmatrix} s \\ y_1 \\ y_2 \end{pmatrix}}"] & \begin{matrix}\mathcal{O}^{\oplus 2}\\ \oplus \\ \mathcal{O}(1)\end{matrix} \arrow[r, "{\begin{pmatrix} 1 & -x & -y_2^{n-1} \\ -y_2 & 0 & s \\ 0 & -y_2 & y_1 \end{pmatrix}}"] & \begin{matrix}\mathcal{O}(1)\\ \oplus \\ \mathcal{O}^{\oplus 2}\end{matrix}
\end{tikzcd}
\tag{4.4.9}
$$

Note that ψ satisfies the symmetry condition (4.3.10) with $\omega = 0$, which in this case reduces to

$$\sigma^*(\psi_{2-l})^\vee = (-1)^l \psi_l. \tag{4.4.10}$$

We are searching for a generator $\mathsf{c} \in \check{C}(\mathfrak{U}, \mathscr{H}om(P(\mathfrak{E}), \mathfrak{E}))$ of the form $\mathsf{c} = \mathsf{c}^{1,0} + \mathsf{c}^{0,1}$ for two homogenous elements

$$\mathsf{c}^{p,1-p} \in \check{C}^p(\mathfrak{U}, \mathscr{H}om^{1-p}(P(\mathfrak{E}), \mathfrak{E})), \qquad p = 0, 1.$$

The cocycle condition $Q(\mathsf{c}) = 0$ is equivalent to

$$
\begin{gathered}
\mathfrak{d}\mathsf{c}^{0,1} = \delta \mathsf{c}^{1,0} = 0 \\
Q(\mathsf{c}^{0,1} + \mathsf{c}^{1,0}) = \delta \mathsf{c}^{0,1} - \mathfrak{d}\mathsf{c}^{1,0} = 0
\end{gathered}
\tag{4.4.11}
$$

A solution to these equations is given by

$$
\mathsf{c}^{1,0} := \begin{tikzcd}
\mathcal{O}(1) \arrow[r] \arrow[d, "\begin{pmatrix} x^{-1} \\ 0 \\ 0 \end{pmatrix}_{01}"] & \begin{matrix} \mathcal{O}^{\oplus 2} \\ \oplus \\ \mathcal{O}(1) \end{matrix} \arrow[r] \arrow[d, "\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -x^{-1}y_2^{n-2} \end{pmatrix}_{01}"] & \begin{matrix} \mathcal{O}(1) \\ \oplus \\ \mathcal{O}^{\oplus 2} \end{matrix} \arrow[d, "(\, x^{-1} \; 0 \; 0 \,)_{01}"] \\
\begin{matrix} \mathcal{O}(-1) \\ \oplus \\ \mathcal{O}^{\oplus 2} \end{matrix} \arrow[r] & \begin{matrix} \mathcal{O}^{\oplus 2} \\ \oplus \\ \mathcal{O}(-1) \end{matrix} \arrow[r] & \mathcal{O}(-1)
\end{tikzcd} \tag{4.4.12}
$$

$$
\mathsf{c}^{0,1} := \begin{tikzcd}
\mathcal{O}(1) \arrow[r] \arrow[d, "\begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}_0 + \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}_1"] & \begin{matrix} \mathcal{O}^{\oplus 2} \\ \oplus \\ \mathcal{O}(1) \end{matrix} \arrow[d, "(\, 0 \; 1 \; 0 \,)_0 + (\, -1 \; 0 \; 0 \,)_1"] \\
\begin{matrix} \mathcal{O}^{\oplus 2} \\ \oplus \\ \mathcal{O}(-1) \end{matrix} \arrow[r] & \mathcal{O}(-1)
\end{tikzcd}
$$

These satisfy the symmetry conditions

$$
J_* P(\mathsf{c}^{p,1-p}) = -(-1)^{\omega} \mathsf{c}^{p,1-p}, \qquad p = 0, 1, \tag{4.4.13}
$$

For multiplicity $N > 1$, we have the locally free resolution $\mathfrak{E}_N = \mathfrak{E} \otimes \mathbb{C}^N$. The quasi-isomorphism $\psi_N \colon \mathfrak{E}_N \to P(\mathfrak{E}_N)$ is of the form $\psi_N = \psi \otimes M$, where $M \in \mathcal{M}_N(\mathbb{C})$ is an $N \times N$ complex matrix. Note that ψ_N induces the isomorphism (4.2.32) in cohomology. Moreover, we have

$$\sigma^*(\psi_{N,m-l})^\vee = (-1)^{l+\omega}\psi_{N,l}.$$

Referring back to (4.4.10), we see that this last equation constrains the matrix M:

$$\omega = \begin{cases} 0, & \text{if } M = M^{tr} \\ 1, & \text{if } M = -M^{tr} \end{cases} \tag{4.4.14}$$

The first case corresponds to an $SO(N)$ gauge group, while the second case corresponds to $Sp(N/2)$ (N even). This confirms the correlation between the symmetry of ψ_N and the SO/Sp projection, as we alluded to after (4.3.10).

The infinitesimal deformations of the D-brane are now parameterized by a matrix valued field

$$\phi = \mathsf{C}(\mathsf{c}^{1,0} + \mathsf{c}^{0,1})$$

where $\mathsf{C} \in \mathcal{M}_N(\mathbb{C})$ is the $N \times N$ Chan-Paton matrix. Taking (4.4.13) into account, invariance under the orientifold projection yields the following condition on C

$$\mathsf{C} = -(-1)^\omega \mathsf{C}^{tr}. \tag{4.4.15}$$

For $\omega = 1$, this condition does not look like the usual one defining the Lie algebra of $Sp(N/2)$ because we are working in a non-usual basis of fields, namely $\mathcal{C}(P(\mathfrak{E}_N), \mathfrak{E}_N)$. By composing with the quasi-isomorphism ψ_N, we find the Chan-Paton matrix in $\mathcal{C}(P(\mathfrak{E}_N), P(\mathfrak{E}_N))$ to be $M\mathsf{C}$. By performing a change of basis in the space of Chan-Paton indices, we can choose M to be

$$M = \begin{cases} \mathrm{I}_N, & \text{if } \omega = 0 \\ i\begin{pmatrix} & \mathrm{I}_{N/2} \\ -\mathrm{I}_{N/2} & \end{pmatrix}, & \text{if } \omega = 1 \end{cases}$$

and so the Chan-Paton matrices satisfy the well-known conditions [223]

$$
\begin{aligned}
(M\mathsf{C})^{tr} &= -(M\mathsf{C}), && \text{for } \omega = 0,\\
(M\mathsf{C})^{tr} &= -M(M\mathsf{C})M, && \text{for } \omega = 1.
\end{aligned}
$$

The superpotential is determined by the A_∞ products (4.3.22) evaluated on ϕ. According to Theorem 4.3.3, the final result is obtained by the superpotential of the underlying unprojected theory evaluated on invariant field configurations. Therefore the computations are identical in both cases ($\omega = 0, 1$) and the superpotential is essentially determined by the A_∞ products of a single D-brane with multiplicity $N = 1$.

Proceeding by analogy with [214], let us define the cocycles

$$\mathsf{a}_p \in \check{C}^1(\mathfrak{U}, \mathscr{H}om^0(P(\mathfrak{E}), \mathfrak{E})) \qquad \mathsf{b}_p \in \check{C}^1(\mathfrak{U}, \mathscr{H}om^1(P(\mathfrak{E}), \mathfrak{E}))$$

as follows

$$
\mathsf{a}_p := \quad
\begin{tikzcd}
\mathcal{O}(1) \arrow[r] \arrow[d, "(0)_{01}"] & \begin{matrix}\mathcal{O}^{\oplus 2}\\ \oplus \\ \mathcal{O}(1)\end{matrix} \arrow[r] \arrow[d, "\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -x^{-1}y_2^p \end{pmatrix}_{01}"] & \begin{matrix}\mathcal{O}(1)\\ \oplus \\ \mathcal{O}^{\oplus 2}\end{matrix} \arrow[d, "(0)_{01}"] \\
\begin{matrix}\mathcal{O}(-1)\\ \oplus \\ \mathcal{O}^{\oplus 2}\end{matrix} \arrow[r] & \begin{matrix}\mathcal{O}^{\oplus 2}\\ \oplus \\ \mathcal{O}(-1)\end{matrix} \arrow[r] & \mathcal{O}(-1)
\end{tikzcd}
\tag{4.4.16}
$$

$$
\mathsf{b}_p := \begin{tikzcd}
\mathcal{O}(1) \arrow[r] \arrow[d, "\begin{pmatrix} 0 \\ 0 \\ x^{-1}y_2^p \end{pmatrix}_{01}"] & \begin{matrix} \mathcal{O}^{\oplus 2} \\ \oplus \\ \mathcal{O}(1) \end{matrix} \arrow[d, "\begin{pmatrix} 0 & 0 & -x^{-1}y_2^p \end{pmatrix}_{01}"] \\
\begin{matrix} \mathcal{O}^{\oplus 2} \\ \oplus \\ \mathcal{O}(-1) \end{matrix} \arrow[r] & \mathcal{O}(-1)
\end{tikzcd}
$$

One shows by direct computation that they satisfy the relations

$$
\begin{aligned}
\mathsf{b}_p &= Q(\mathsf{a}_{p-1}) \\
\mathsf{b}_p &= \mathsf{c} \star_\psi \mathsf{a}_p + \mathsf{a}_p \star_\psi \mathsf{c}
\end{aligned}
\tag{4.4.17}
$$

Moreover, we have

$$
\begin{aligned}
\mathsf{c} \star_\psi \mathsf{c} &= \mathsf{b}_{n-2} \\
\mathsf{b}_p \star_\psi \mathsf{b}_p &= 0
\end{aligned}
\tag{4.4.18}
$$

for any p. Therefore the computation of the A_∞ products is identical to [214]. We find only one non-trivial product

$$
\mathfrak{m}_n(\mathsf{c}, \ldots, \mathsf{c}) = -(-1)^{\frac{n(n-1)}{2}} \mathsf{b}_0. \tag{4.4.19}
$$

If we further compose with c we obtain

$$
\mathsf{b}_0 \star_\psi \mathsf{c} := \begin{tikzcd}
\mathcal{O}(1) \arrow[d, "(-x^{-1})_{01}"] \\
\mathcal{O}(-1)
\end{tikzcd}
$$

which is a generator of $\mathrm{Ext}^3(i_*\mathcal{O}_C(-1), i_*\mathcal{O}_C(-1))$. Therefore we obtain a superpotential of the form

$$W = \frac{(-1)^n}{n+1}\,\mathsf{C}^{n+1}$$

where C satisfies the invariance condition (4.4.15).

4.4.2 Local Conifold $O3/O7$ Models

In this case, the local Calabi-Yau threefold X is isomorphic to the crepant resolution of a conifold singularity, i.e., the total space of $\mathcal{O}(-1)\oplus\mathcal{O}(-1)\to\mathbb{P}^1$. X can be covered with two coordinate patches (x, y_1, y_2), (w, z_1, z_2) with transition functions

$$\begin{aligned} w &= x^{-1} \\ z_1 &= xy_1 \\ z_2 &= xy_2. \end{aligned} \tag{4.4.20}$$

The $(-1,-1)$ curve C is given by

$$x = y_1 = y_2 = 0 \qquad w = z_1 = z_2 = 0 \tag{4.4.21}$$

and the holomorphic involution takes

$$\begin{aligned} (x, y_1, y_2) &\mapsto (-x, -y_1, -y_2) \\ (w, z_1, z_2) &\mapsto (-w, z_1, z_2). \end{aligned} \tag{4.4.22}$$

In this case we have an $O3$ plane at

$$x = y_1 = y_2 = 0$$

and a noncompact $O7$ plane at $w = 0$. The invariant D5-brane configurations are of the form $\mathcal{E}_n^{\oplus N}$, where

$$\mathcal{E}_n = i_*\mathcal{O}_C(-1+n)\oplus i_*(\sigma^*\mathcal{O}_C(-1-n))[1], \quad n\geq 1. \tag{4.4.23}$$

We have a global Koszul resolution of the structure sheaf $\mathcal{O}_C$

$$0 \longrightarrow \mathcal{O}(2) \xrightarrow{\begin{pmatrix} -y_2 \\ y_1 \end{pmatrix}} \mathcal{O}(1)^{\oplus 2} \xrightarrow{\begin{pmatrix} y_1 & y_2 \end{pmatrix}} \mathcal{O} \longrightarrow 0 \tag{4.4.24}$$

Therefore the locally free resolution of $\mathcal{E}_n$ is a complex $\mathfrak{E}_n$ of the form

$$\begin{tikzcd}[column sep=large]
\sigma^*\mathcal{O}(1-n) \arrow[r, "\left(\begin{smallmatrix} 0 \\ y_1 \\ y_2 \end{smallmatrix}\right)"] & \begin{matrix} \mathcal{O}(1+n) \\ \oplus \\ \sigma^*\mathcal{O}(-n)^{\oplus 2} \end{matrix} \arrow[r, "\left(\begin{smallmatrix} -y_2 & 0 & 0 \\ y_1 & 0 & 0 \\ 0 & y_2 & -y_1 \end{smallmatrix}\right)"] & \begin{matrix} \mathcal{O}(n)^{\oplus 2} \\ \oplus \\ \sigma^*\mathcal{O}(-1-n) \end{matrix} \arrow[r, "\left(\begin{smallmatrix} y_1 & y_2 & 0 \end{smallmatrix}\right)"] & \mathcal{O}(-1+n)
\end{tikzcd} \tag{4.4.25}$$

in which the last term to the right has degree 0, and the last term to the left has degree -3. The quasi-isomorphism $\psi\colon \mathfrak{E}_n \to P(\mathfrak{E}_n)$ is given by

$$\begin{tikzcd}[column sep=large, row sep=huge]
\sigma^*\mathcal{O}(1-n) \arrow[r, "\left(\begin{smallmatrix} 0 \\ y_1 \\ y_2 \end{smallmatrix}\right)"] \arrow[d, "1"] & \begin{matrix} \mathcal{O}(1+n) \\ \oplus \\ \sigma^*\mathcal{O}(-n)^{\oplus 2} \end{matrix} \arrow[r, "\left(\begin{smallmatrix} -y_2 & 0 & 0 \\ y_1 & 0 & 0 \\ 0 & y_2 & -y_1 \end{smallmatrix}\right)"] \arrow[d, "\left(\begin{smallmatrix} & 1 & \\ & & 1 \\ 1 & & \end{smallmatrix}\right)"] & \begin{matrix} \mathcal{O}(n)^{\oplus 2} \\ \oplus \\ \sigma^*\mathcal{O}(-1-n) \end{matrix} \arrow[r, "\left(\begin{smallmatrix} y_1 & y_2 & 0 \end{smallmatrix}\right)"] \arrow[d, "\left(\begin{smallmatrix} & & 1 \\ 1 & & \\ & 1 & \end{smallmatrix}\right)"] & \mathcal{O}(-1+n) \arrow[d, "1"] \\
\sigma^*\mathcal{O}(1-n) \arrow[r, "\left(\begin{smallmatrix} y_1 \\ y_2 \\ 0 \end{smallmatrix}\right)"] & \begin{matrix} \sigma^*\mathcal{O}(-n)^{\oplus 2} \\ \oplus \\ \mathcal{O}(1+n) \end{matrix} \arrow[r, "\left(\begin{smallmatrix} y_2 & -y_1 & 0 \\ 0 & 0 & -y_2 \\ 0 & 0 & y_1 \end{smallmatrix}\right)"] & \begin{matrix} \sigma^*\mathcal{O}(-1-n) \\ \oplus \\ \mathcal{O}(n)^{\oplus 2} \end{matrix} \arrow[r, "\left(\begin{smallmatrix} 0 & y_1 & y_2 \end{smallmatrix}\right)"] & \mathcal{O}(-1+n)
\end{tikzcd} \tag{4.4.26}$$

and satisfies $\sigma^*(\psi_{3-l})^\vee = \psi_l$, that is, the symmetry condition (4.3.10) with $\omega = 0$. The on-shell open string states $\mathrm{Ext}^1_X(\mathfrak{E}_n, \mathfrak{E}_n)$ are computed by the spectral sequence (4.4.7):

$$\begin{aligned}
\mathrm{Ext}^1_X(\mathcal{O}_C(-1+n), \mathcal{O}_C(-1+n)) &= 0 \\
\mathrm{Ext}^1_X(\sigma^*\mathcal{O}_C(-1-n)[1], \sigma^*\mathcal{O}_C(-1-n)[1]) &= 0 \\
\mathrm{Ext}^1_X(\mathcal{O}_C(-1+n), \sigma^*\mathcal{O}_C(-1-n)[1]) &= \mathbb{C}^{4n} \\
\mathrm{Ext}^1_X(\sigma^*\mathcal{O}_C(-1-n)[1], \mathcal{O}_C(-1+n)) &= \mathbb{C}^{2n+1},
\end{aligned} \tag{4.4.27}$$

where in the last two lines we have used the condition $n \geq 1$.

To compute the superpotential, we work with the cochain model $\check{C}(\mathfrak{U}, \mathscr{H}om(P(\mathfrak{E}_n), \mathfrak{E}_n))$. The direct sum of the above Ext groups represents the degree 1 cohomology of this complex with respect to the differential (4.4.2). The first step is to find explicit representatives for all degree 1 cohomology classes with well defined transformation properties under the orientifold projection. We list all generators below on a case by case basis.

a) $\mathrm{Ext}^1(\sigma^*\mathcal{O}_C(-1-n)[1], \mathcal{O}_C(-1+n))$

We have $2n+1$ generators $\mathsf{a}_i \in \check{C}^0(\mathfrak{U}, \mathscr{H}om^1(\mathcal{P}(\mathfrak{E}_n), \mathfrak{E}_n)$, $i = 0, \ldots 2n$, given by

$$\mathsf{a}_i := x^i \mathsf{a}, \tag{4.4.28}$$

where

$$
\mathsf{a} := \begin{tikzcd}
\sigma^*\mathcal{O}(1-n) \arrow[r] \arrow[d, "\left(\begin{smallmatrix}1\\0\\0\end{smallmatrix}\right)"] & \begin{matrix}\sigma^*\mathcal{O}(-n)^{\oplus 2}\\ \oplus \\ \mathcal{O}(1+n)\end{matrix} \arrow[r] \arrow[d, "\left(\begin{smallmatrix}0&1&0\\-1&0&0\\0&0&0\end{smallmatrix}\right)"] & \begin{matrix}\sigma^*\mathcal{O}(-1-n)\\ \oplus \\ \mathcal{O}(n)^{\oplus 2}\end{matrix} \arrow[d, "\left(\begin{smallmatrix}1&0&0\end{smallmatrix}\right)"] \\
\begin{matrix}\mathcal{O}(1+n)\\ \oplus \\ \sigma^*\mathcal{O}(-n)^{\oplus 2}\end{matrix} \arrow[r] & \begin{matrix}\mathcal{O}(n)^{\oplus 2}\\ \oplus \\ \sigma^*\mathcal{O}(-1-n)\end{matrix} \arrow[r] & \mathcal{O}(-1+n)
\end{tikzcd} \tag{4.4.29}
$$

Note that we have written down the expressions of the generators only in the U_0 patch.[5] The transformation properties under the orientifold projection are

$$J_*P(\mathsf{a}_i) = -(-1)^{i+\omega}\mathsf{a}_i, \qquad 0 \leq i \leq 2n. \tag{4.4.30}$$

b) $\mathrm{Ext}^1(\mathcal{O}_C(-1+n), \sigma^*\mathcal{O}_C(-1-n)[1])$

We have $4n$ generators $\mathsf{b}_i, \mathsf{c}_i \in \check{C}^1(\mathfrak{U}, \mathscr{H}om^0(P(\mathfrak{F}_n), \mathfrak{F}_n)$, $i = 1, \ldots, 2n$ given by

$$\mathsf{b}_i := x^{-i}\mathsf{b}, \qquad \mathsf{c}_i := x^{-i}\mathsf{c} \tag{4.4.31}$$

[5]The expressions in the U_1 patch can be obtained using the transition functions (4.4.20) since the a_i are Čech closed. They will not be needed in the computation.

where

$$
\mathsf{b} := \begin{tikzcd}
\begin{matrix}\sigma^*\mathcal{O}(-n)^{\oplus 2} \\ \oplus \\ \mathcal{O}(1+n)\end{matrix} \arrow[r] \arrow[d, "\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix}_{01}"] & \begin{matrix}\sigma^*\mathcal{O}(-1-n) \\ \oplus \\ \mathcal{O}(n)^{\oplus 2}\end{matrix} \arrow[d, "\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}_{01}"] \\
\begin{matrix}\mathcal{O}(1+n) \\ \oplus \\ \sigma^*\mathcal{O}(-n)^{\oplus 2}\end{matrix} \arrow[r] & \begin{matrix}\mathcal{O}(n)^{\oplus 2} \\ \oplus \\ \sigma^*\mathcal{O}(-1-n)\end{matrix}
\end{tikzcd} \tag{4.4.32}
$$

$$
\mathsf{c} := \begin{tikzcd}
\begin{matrix}\sigma^*\mathcal{O}(-n)^{\oplus 2} \\ \oplus \\ \mathcal{O}(1+n)\end{matrix} \arrow[r] \arrow[d, "\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}_{01}"] & \begin{matrix}\sigma^*\mathcal{O}(-1-n) \\ \oplus \\ \mathcal{O}(n)^{\oplus 2}\end{matrix} \arrow[d, "\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}_{01}"] \\
\begin{matrix}\mathcal{O}(1+n) \\ \oplus \\ \sigma^*\mathcal{O}(-n)^{\oplus 2}\end{matrix} \arrow[r] & \begin{matrix}\mathcal{O}(n)^{\oplus 2} \\ \oplus \\ \sigma^*\mathcal{O}(-1-n)\end{matrix}
\end{tikzcd} \tag{4.4.33}
$$

The action of the orientifold projection is

$$
J_* P(\mathsf{b}_i) = (-1)^{i+\omega}\mathsf{b}_i, \qquad J_* P(\mathsf{c}_i) = (-1)^{i+\omega}\mathsf{c}_i. \tag{4.4.34}
$$

For multiplicity $N \geq 1$, we work as in the last subsection, taking the locally free resolution $\mathfrak{E}_{n,N} = \mathfrak{E}_n \otimes \mathbb{C}^N$, together with the quasi-isomorphism $\psi_N \colon \mathfrak{E}_{n,N} \to P(\mathfrak{E}_{n,N})$; $\psi_N = \psi \otimes M$. Again, M is a symmetric matrix for $\omega = 0$ and antisymmetric for $\omega = 1$. A general invariant degree one cocycle ϕ will be a linear combination

$$\phi = \sum_{i=0}^{2n} \mathsf{A}^i \mathsf{a}_i + \sum_{i=1}^{2n} (\mathsf{B}^i \mathsf{b}_i + \mathsf{C}^i \mathsf{c}_i) \tag{4.4.35}$$

where A^i, B^i, C^i are $N \times N$ matrices satisfying

$$(\mathsf{A}^i)^{tr} = -(-1)^{i+\omega} \mathsf{A}^i \qquad (\mathsf{B}^i)^{tr} = (-1)^{i+\omega} \mathsf{B}^i \qquad (\mathsf{C}^i)^{tr} = (-1)^{i+\omega} \mathsf{C}^i. \tag{4.4.36}$$

In the following we will let the indices $i, j, k, \ldots$ run from 0 to $2n$ with the convention $\mathsf{B}^0 = \mathsf{C}^0 = 0$.

The multiplication table of the above generators with respect to the product (4.3.5) is

$$\begin{aligned} &\mathsf{a}_i \star_\psi \mathsf{a}_j = \mathsf{b}_i \star_\psi \mathsf{b}_j = \mathsf{c}_i \star_\psi \mathsf{c}_j = 0 \\ &\mathsf{b}_i \star_\psi \mathsf{c}_j = \mathsf{c}_i \star_\psi \mathsf{b}_j = 0. \end{aligned} \tag{4.4.37}$$

The remaining products are all Q-exact:

$$\mathsf{a}_i \star_\psi \mathsf{b}_j = Q(f_2(\mathsf{a}_i, \mathsf{b}_j))$$

$$\mathsf{b}_i \star_\psi \mathsf{a}_j = Q(f_2(\mathsf{b}_i, \mathsf{a}_j))$$

as required in (4.3.15). Let us show a sample computation.

$$
\mathsf{b}_i \star \mathsf{a}_j = \begin{tikzcd} \sigma^*\mathcal{O}(1-n) \arrow[r] \arrow[d, "x^{-i+j}\begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}_{01}"] & \begin{matrix} \sigma^*\mathcal{O}(-n)^{\oplus 2} \\ \oplus \\ \mathcal{O}(1+n) \end{matrix} \arrow[d, "x^{-i+j}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}_{01}"] \\ \begin{matrix} \mathcal{O}(1+n) \\ \oplus \\ \sigma^*\mathcal{O}(-n)^{\oplus 2} \end{matrix} \arrow[r] & \begin{matrix} \mathcal{O}(n)^{\oplus 2} \\ \oplus \\ \sigma^*\mathcal{O}(-1-n) \end{matrix} \end{tikzcd} \tag{4.4.38}
$$

For $j \geq i$,

$$
f_2(\mathsf{b}_i, \mathsf{a}_j) = \begin{tikzcd} \sigma^*\mathcal{O}(1-n) \arrow[r] \arrow[d, "x^{-i+j}\begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}_{0}"] & \begin{matrix} \sigma^*\mathcal{O}(-n)^{\oplus 2} \\ \oplus \\ \mathcal{O}(1+n) \end{matrix} \arrow[d, "x^{-i+j}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}_{0}"] \\ \begin{matrix} \mathcal{O}(1+n) \\ \oplus \\ \sigma^*\mathcal{O}(-n)^{\oplus 2} \end{matrix} \arrow[r] & \begin{matrix} \mathcal{O}(n)^{\oplus 2} \\ \oplus \\ \sigma^*\mathcal{O}(-1-n) \end{matrix} \end{tikzcd} \tag{4.4.39}
$$

For $j < i$,

$$
f_2(\mathsf{b}_i, \mathsf{a}_j) = \begin{tikzcd}
\sigma^*\mathcal{O}(1-n) \arrow[r] \arrow[d, "(-1)x^{-i+j+1}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}_1"] & \begin{array}{c} \sigma^*\mathcal{O}(-n)^{\oplus 2} \\ \oplus \\ \mathcal{O}(1+n) \end{array} \arrow[d, "(-1)x^{-i+j+1}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix}_1"] \\
\begin{array}{c} \mathcal{O}(1+n) \\ \oplus \\ \sigma^*\mathcal{O}(-n)^{\oplus 2} \end{array} \arrow[r] & \begin{array}{c} \mathcal{O}(n)^{\oplus 2} \\ \oplus \\ \sigma^*\mathcal{O}(-1-n) \end{array}
\end{tikzcd} \tag{4.4.40}
$$

$$
\mathsf{a}_i \star \mathsf{b}_j = \begin{tikzcd}
\begin{array}{c} \sigma^*\mathcal{O}(-n)^{\oplus 2} \\ \oplus \\ \mathcal{O}(1+n) \end{array} \arrow[r] \arrow[d, "x^{i-j}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix}_{01}"] & \begin{array}{c} \sigma^*\mathcal{O}(-1-n) \\ \oplus \\ \mathcal{O}(n)^{\oplus 2} \end{array} \arrow[d, "x^{i-j}\begin{pmatrix} 0 & -1 & 0 \end{pmatrix}_{01}"] \\
\begin{array}{c} \mathcal{O}(n)^{\oplus 2} \\ \oplus \\ \sigma^*\mathcal{O}(-1-n) \end{array} \arrow[r] & \mathcal{O}(-1+n)
\end{tikzcd} \tag{4.4.41}
$$

For $i \geq j$,

$$f_2(\mathsf{a}_i, \mathsf{b}_j) = \begin{array}{ccc} \begin{array}{c} \sigma^*\mathcal{O}(-n)^{\oplus 2} \\ \oplus \\ \mathcal{O}(1+n) \end{array} & \longrightarrow & \begin{array}{c} \sigma^*\mathcal{O}(-1-n) \\ \oplus \\ \mathcal{O}(n)^{\oplus 2} \end{array} \\ \Big\downarrow {\scriptstyle x^{i-j}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix}_0} & & \Big\downarrow {\scriptstyle x^{i-j}\begin{pmatrix} 0 & -1 & 0 \end{pmatrix}_0} \\ \begin{array}{c} \mathcal{O}(n)^{\oplus 2} \\ \oplus \\ \sigma^*\mathcal{O}(-1-n) \end{array} & \longrightarrow & \mathcal{O}(-1+n) \end{array} \tag{4.4.42}$$

For $i < j$,

$$f_2(\mathsf{a}_i, \mathsf{b}_j) = \begin{array}{ccc} \begin{array}{c} \sigma^*\mathcal{O}(-n)^{\oplus 2} \\ \oplus \\ \mathcal{O}(1+n) \end{array} & \longrightarrow & \begin{array}{c} \sigma^*\mathcal{O}(-1-n) \\ \oplus \\ \mathcal{O}(n)^{\oplus 2} \end{array} \\ \Big\downarrow {\scriptstyle x^{i-j+1}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}_1} & & \Big\downarrow {\scriptstyle x^{i-j+1}\begin{pmatrix} 0 & 1 & 0 \end{pmatrix}_1} \\ \begin{array}{c} \mathcal{O}(n)^{\oplus 2} \\ \oplus \\ \sigma^*\mathcal{O}(-1-n) \end{array} & \longrightarrow & \mathcal{O}(-1+n) \end{array} \tag{4.4.43}$$

Since all pairwise products of generators are Q-exact, it follows that the obstruction $\Pi(B_1(\phi)) =$

$\Pi(\phi \star \phi)$ vanishes. Moreover, the second order deformation $f_2(\phi)$ is given by

$$f_2(\phi) = \sum_{i,j} \left(\mathsf{A}^i \mathsf{B}^j f_2(\mathsf{a}_i, \mathsf{b}_j) + \mathsf{B}^i \mathsf{A}^j f_2(\mathsf{b}_i, \mathsf{a}_j) + \mathsf{A}^i \mathsf{C}^j f_2(\mathsf{a}_i, \mathsf{c}_j) + \mathsf{C}^i \mathsf{A}^j f_2(\mathsf{c}_i, \mathsf{a}_j) \right). \tag{4.4.44}$$

Following the recursive algorithm discussed in section 4.3 we compute the next obstruction $\Pi(\phi \star f_2(\phi) + f_2(\phi) \star \phi)$. For this, we have to compute products of the form

$$\alpha_i \star f_2(\alpha_j, \alpha_k), \qquad f_2(\alpha_j, \alpha_k) \star \alpha_i.$$

Again we present a sample computation in detail. For $i \geq j$,

$$-\mathsf{a}_k \star f_2(\mathsf{b}_j, \mathsf{a}_i) = \begin{tikzcd} \sigma^*\mathcal{O}(1-n) \arrow[r] \arrow[d, "x^{i-j+k}\begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}_0"] & \begin{array}{c} \sigma^*\mathcal{O}(-n)^{\oplus 2} \\ \oplus \\ \mathcal{O}(1+n) \end{array} \arrow[d, "x^{i-j+k}\begin{pmatrix} 0 & -1 & 0 \end{pmatrix}_0"] \\ \begin{array}{c} \mathcal{O}(n)^{\oplus 2} \\ \oplus \\ \sigma^*\mathcal{O}(-1-n) \end{array} \arrow[r] & \mathcal{O}(-1+n) \end{tikzcd} \tag{4.4.45}$$

and, for $i < j$,

$$-\mathsf{a}_k \star f_2(\mathsf{b}_j, \mathsf{a}_i) = \begin{tikzcd} \sigma^*\mathcal{O}(1-n) \arrow[r] \arrow[d, "(-1)^{n+1}x^{i-j+k-2n+1}\begin{pmatrix}0\\1\\0\end{pmatrix}_1"] & \begin{matrix}\sigma^*\mathcal{O}(-n)^{\oplus 2}\\ \oplus \\ \mathcal{O}(1+n)\end{matrix} \arrow[d, "(-1)^{n+2}x^{i-j+k-2n+1}\begin{pmatrix}0 & 1 & 0\end{pmatrix}_1"] \\ \begin{matrix}\mathcal{O}(n)^{\oplus 2}\\ \oplus \\ \sigma^*\mathcal{O}(-1-n)\end{matrix} \arrow[r] & \mathcal{O}(-1+n) \end{tikzcd} \tag{4.4.46}$$

For $k \geq j$,

$$-f_2(\mathsf{a}_k, \mathsf{b}_j) \star \mathsf{a}_i = \begin{tikzcd} \sigma^*\mathcal{O}(1-n) \arrow[r] \arrow[d, "x^{i-j+k}\begin{pmatrix}0\\1\\0\end{pmatrix}_0"] & \begin{matrix}\sigma^*\mathcal{O}(-n)^{\oplus 2}\\ \oplus \\ \mathcal{O}(1+n)\end{matrix} \arrow[d, "x^{i-j+k}\begin{pmatrix}0 & 1 & 0\end{pmatrix}_0"] \\ \begin{matrix}\mathcal{O}(n)^{\oplus 2}\\ \oplus \\ \sigma^*\mathcal{O}(-1-n)\end{matrix} \arrow[r] & \mathcal{O}(-1+n) \end{tikzcd} \tag{4.4.47}$$

and, for $k < j$,

$$
-f_2(\mathsf{a}_k, \mathsf{b}_j) \star \mathsf{a}_i = \begin{tikzcd} \sigma^*\mathcal{O}(1-n) \arrow[r] \arrow[d, "(-1)^{n-1}x^{i-j+k+1-2n}\begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}_1"] & \begin{matrix} \sigma^*\mathcal{O}(-n)^{\oplus 2} \\ \oplus \\ \mathcal{O}(1+n) \end{matrix} \arrow[d, "(-1)^{n}x^{i-j+k+1-2n}\begin{pmatrix} 0 & -1 & 0 \end{pmatrix}_1"] \\ \begin{matrix} \mathcal{O}(n)^{\oplus 2} \\ \oplus \\ \sigma^*\mathcal{O}(-1-n) \end{matrix} \arrow[r] & \mathcal{O}(-1+n) \end{tikzcd} \tag{4.4.48}
$$

Then the third order products are the following. For $k < j \leq i$,

$$
\mathfrak{m}_3(\mathsf{a}_k, \mathsf{b}_j, \mathsf{a}_i) = \begin{tikzcd} \sigma^*\mathcal{O}(1-n) \arrow[r] \arrow[d, "x^{i-j+k}\begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}"] & \begin{matrix} \sigma^*\mathcal{O}(-n)^{\oplus 2} \\ \oplus \\ \mathcal{O}(1+n) \end{matrix} \arrow[d, "x^{i-j+k}\begin{pmatrix} 0 & -1 & 0 \end{pmatrix}"] \\ \begin{matrix} \mathcal{O}(n)^{\oplus 2} \\ \oplus \\ \sigma^*\mathcal{O}(-1-n) \end{matrix} \arrow[r] & \mathcal{O}(-1+n) \end{tikzcd} \tag{4.4.49}
$$

and, for $i < j \le k$,

$$\mathfrak{m}_3(\mathsf{a}_k, \mathsf{b}_j, \mathsf{a}_i) = \begin{tikzcd} \sigma^*\mathcal{O}(1-n) \arrow[r] \arrow[d, "x^{i-j+k}\left(\begin{smallmatrix}0\\1\\0\end{smallmatrix}\right)"] & \begin{matrix}\sigma^*\mathcal{O}(-n)^{\oplus 2}\\ \oplus \\ \mathcal{O}(1+n)\end{matrix} \arrow[d, "x^{i-j+k}(\,0\;1\;0\,)"] \\ \begin{matrix}\mathcal{O}(n)^{\oplus 2}\\ \oplus \\ \sigma^*\mathcal{O}(-1-n)\end{matrix} \arrow[r] & \mathcal{O}(-1+n) \end{tikzcd} \tag{4.4.50}$$

According to [214], the corresponding terms in the superpotential can be obtained by taking products of the form $\mathfrak{m}_2(\mathfrak{m}_3(\alpha_i, \alpha_j, \alpha_k), \alpha_l)$, which take values in $\mathrm{Ext}^3(\mathcal{E}_n, \mathcal{E}_n)$. For $k < j \le i$ and $i - j + k - l = -1$ we have

$$\mathfrak{m}_2(\mathfrak{m}_3(\mathsf{a}_k, \mathsf{b}_j, \mathsf{a}_i), \mathsf{c}_l) = \begin{tikzcd} \begin{matrix}\sigma^*\mathcal{O}(-n)^{\oplus 2}\\ \oplus \\ \mathcal{O}(1+n)\end{matrix} \arrow[d, "x^{i-j+k-l}(\,0\;0\;1\,)_{01}"] \\ \mathcal{O}(-1+n) \end{tikzcd} \tag{4.4.51}$$

The expression obtained in the right hand side of equation (4.4.51) is a generator for

$$\mathrm{Ext}^3(\sigma^*\mathcal{O}_C(-1-n)[1], \sigma^*\mathcal{O}_C(-1-n)[1]) = \mathbb{C}. \tag{4.4.52}$$

For $i < j \leq k$ and $i - j + k - l = -1$,

$$\mathfrak{m}_2(\mathfrak{m}_3(\mathsf{a}_k, \mathsf{b}_j, \mathsf{a}_i), \mathsf{c}_l) = \begin{array}{c} \sigma^*\mathcal{O}(-n)^{\oplus 2} \\ \oplus \\ \mathcal{O}(1+n) \\ \Big\downarrow {\scriptstyle x^{i-j+k-l}(\,0\;0\;-1\,)_{01}} \\ \mathcal{O}(-1+n) \end{array} \tag{4.4.53}$$

Note that the expression in the right hand side of (4.4.53) is the same generator of (4.4.52) multiplied by (-1). The first product (4.4.51) gives rise to superpotential terms of the form

$$\mathrm{Tr}(C^l A^k B^j A^i)$$

with

$$(i+k) - (j+l) = -1, \qquad k < j \leq i.$$

The second product (4.4.53) gives rise to terms in the superpotential of the form

$$-\mathrm{Tr}(C^l A^k B^j A^i)$$

with

$$(i+k) - (j+l) = -1, \qquad i < j \leq k.$$

If we consider the case $n = 1$ for simplicity, the superpotential interactions resulting from these two products are

$$\begin{aligned} W = \mathrm{Tr}(&C^1A^0B^1A^1 - C^1A^1B^1A^0 + C^2A^0B^1A^2 - C^2A^2B^1A^0 \\ &+ C^1A^0B^2A^2 - C^1A^2B^2A^0 + C^2A^1B^2A^2 - C^2A^2B^2A^1). \end{aligned} \tag{4.4.54}$$

4.5 Appendix: An alternative derivation

In this appendix we give an alternative derivation of Lemma 4.2.2. This approach relies on one of the most powerful results in algebraic geometry, namely Grothendieck duality. Let us start out by recalling the latter. Consider $f\colon X \to Y$ to be a proper morphism of smooth varieties[6]. Choose $\mathcal{F} \in D^b(X)$ and $\mathcal{G} \in D^b(Y)$ to be objects in the corresponding bounded derived categories. Then one has the following isomorphism (see, e.g., III.11.1 of [230]):

$$\mathbf{R}f_*\mathbf{R}\mathscr{H}om_X(\mathcal{F},\, f^!\mathcal{G}) \cong \mathbf{R}\mathscr{H}om_Y(\mathbf{R}f_*\mathcal{F},\, \mathcal{G}). \tag{4.5.1}$$

Now it is true that $f^!$ in general is a complicated functor, in particular it is *not* the total derived functor of a classical functor, i.e., a functor between the category of coherent sheaves, but in our context it will have a very simple form.

The original problem that lead to Lemma 4.2.2 was to determine the derived dual, a.k.a, Verdier dual, of a torsion sheaf. Let $i\colon E \to X$ be the embedding of a codimension d subvariety E into a smooth variety X, and let V be a vector bundle on E. We want to determine $\mathbf{R}\mathscr{H}om_X(i_*V, \mathcal{O}_X)$. Using (4.5.1) we have

$$\mathbf{R}\mathscr{H}om_X(i_*V, \mathcal{O}_X) \cong i_*\mathbf{R}\mathscr{H}om_E(V, i^!\mathcal{O}_X), \tag{4.5.2}$$

where we used the fact that the higher direct images of i vanish. Furthermore, since V is locally free, we have that

$$\mathbf{R}\mathscr{H}om_E(V, i^!\mathcal{O}_X) = \mathbf{R}\mathscr{H}om_E(\mathcal{O}_E, V^\vee \otimes i^!\mathcal{O}_X) = V^\vee \otimes i^!\mathcal{O}_X, \tag{4.5.3}$$

where $V^\vee$ is the dual of V on E, rather than on X. On the other hand, for an embedding

$$i^!\mathcal{O}_X = K_{E/X}[-d]\,, \tag{4.5.4}$$

where $K_{E/X}$ is the relative canonical bundle. Now if we assume that the ambient space X is a Calabi-Yau variety, then $K_{E/X} = K_E$. We can summarize this

Proposition 4.5.1. *For the embedding $i\colon E \to X$ of a codimension d subvariety E in a smooth Calabi-Yau variety X, and a vector bundle V on E we have that*

$$\mathbf{R}\mathscr{H}om_X(i_*V, \mathcal{O}_X) \cong i_*\left(V^\vee \otimes K_E\right)[-d]. \tag{4.5.5}$$

---

[6] The Grothendieck duality applies to more general schemes than varieties, but we limit ourselves to the cases considered in this chapter.

Chapter 5

Stabilizing the Runaway Quiver in Supergravity

5.1 Introduction

Fractional branes at singularities can give rise to quiver gauge theories with a variety of IR properties. Broadly, such brane configurations have been classified into three categories - (1) $N = 2$ fractional branes: the field theory on these configurations has flat directions along which the dynamics reduces to an $N = 2$ theory. (2) Deformation branes: non-perturbative effects in the gauge theory lead to quantum deformation of the moduli space - in terms of the supergravity dual, the geometry undergoes a complex deformation that smoothes the singular geometry. The gauge theory in such scenarios goes through a number of duality cycles, and the IR end of the cascade, probed by D3 branes, reveals a deformed moduli space. Many examples have been studied in the literature, including F_0, dP_2 and dP_3, the SPP singularity, etc. For details, we refer to [234] and references therein. (3) Dynamical Supersymmetry Breaking (DSB) branes: the IR behavior of the gauge theory shows DSB as opposed to quantum deformation. In terms of the supergravity dual, supersymmetry breaking is due to obstructed complex deformations. Although the most studied geometry in this category is dP_1 ([231], [235], [242]), various other scenarios including the SPP singularity, higher del Pezzos, and applications to string phenomenology have been explored [236], [237], [263], [238], [239], [240], [241], [244], [243].

In this chapter we will be concerned with DSB branes in the dP_1 geometry, which in fact turns out to have runaway directions in field space, and hence does not break supersymmetry in the desired way. This was pointed out from a field theory analysis in [245], and subsequently studied in [246], [241], [247], [248], [249], [250], [251], [252], [253], [254], [255], [256], [257], [258], [259], [260]. Most constructions in the literature have relied on open string effects such as the addition of extra flavors to stabilize the runaway and produce supersymmetry-breaking string phenomenological models.

As pointed out in [235] and [243], the runaway is essentially due to dynamical FI terms in the action, which allow otherwise bounded field directions to relax to infinity. Thus, consistent closed string moduli stabilization in realistic embeddings of the supersymmetry breaking quiver in string theory is required. Such embeddings have been done in both type IIA [261] and type IIB [262].

In this chapter, we perform a general supergravity analysis of the quiver, extracting conditions on fluxes and instanton effects such that one obtains stabilization. The general procedure is to couple string moduli to the quiver fields and perform an extremization of the supergravity potential. Both IIA and IIB examples are then considered.

In the type IIA case, moduli stabilization is performed by RR and NS flux [277], [264], [265], [268], [269], [270], [271], [272], [273]. Consistent orientifolding and the Freed Witten anomaly cancellation condition introduce various constraints on the Calabi Yau and the quiver locus. We analyze a variety of IIA toy models in supergravity. The conclusion is that under mild conditions on the Kahler potential and with proper choices of flux or instanton contributions to the superpotential, the quiver gauge theory is indeed stabilized. Comments on the possible uplift to dS vacua are made. One expects these basic features to be true in a full-blown IIA computation, although extremization in such a scenario would be technically difficult.

In the type IIB case, we consider the embedding constructed in [262], where various instanton effects have been explicitly calculated and a procedure for extremization of the supergravity potential has been given. We show that the potential in fact does display a stabilized minimum, albeit in a non-calculable regime of field space.

The plan of the chapter is as follows. In section 2, we summarize the runaway in terms of the quiver gauge theory, as originally given in [245]. In section 3, we work out the general supergravity stabilization conditions, and apply them in IIA scenarios. In section 4, we discuss stabilization in a type IIB construction, treating the example of [262].

5.2 The Runaway Quiver : Field Theory Description

The gauge theory of M D5 branes on the complex cone over F_1 is given by $SU(3M)\times SU(2M)\times SU(M)$. For the purpose of this chapter, we consider the case $M=1$. The various fields transform as follows [245]:

$$\begin{array}{ccccccc} & SU(3M) & SU(2M) & SU(M) & [SU(2) & U(1)_F & U(1)_R] \\ Q & \mathbf{3M} & \overline{\mathbf{2M}} & \mathbf{1} & \mathbf{1} & 1 & -1 \\ \overline{u} & \overline{\mathbf{3M}} & \mathbf{1} & \mathbf{M} & \mathbf{2} & -1 & 0 \\ L & \mathbf{1} & \mathbf{2M} & \overline{\mathbf{M}} & \mathbf{2} & 0 & 3 \\ L_3 & \mathbf{1} & \mathbf{2M} & \overline{\mathbf{M}} & \mathbf{1} & -3 & -1 \end{array} \tag{5.2.1}$$

The gauge invariant fields are defined as

$$Z=\det_{fj}Q^f\overline{u}_j,\quad X_{ia}=Q\overline{u}_iL_a,\quad V^a=\frac{1}{2}L_bL_c\epsilon^{abc}. \tag{5.2.2}$$

The low energy spectrum of the system consists of the fields V^i, $i=1,2$ and $V^3\equiv V$ after all other fields have satisfied their SUSY equations of motion. The dynamical superpotential is

$$W \;=\; 3\,(V\,\Lambda_3^7)^{1/3}. \tag{5.2.3}$$

The Kahler potential far out in V moduli space is given by

$$K_{eff} \approx K_{cl} = 2\sqrt{T} = 2\sqrt{VV^\dagger + V^i V^{i\dagger}} \tag{5.2.4}$$

This leads to a runaway in field space of the form:

$$V_{eff} \approx 2|\Lambda_3^7|^{2/3}(VV^\dagger)^{-1/6} \tag{5.2.5}$$

with $V_i = 0$.

5.3 Stabilization Conditions and Type IIA examples

In embeddings of the above quiver gauge theory in type IIB string theory, the runaway in field space is caused by a lack of moduli stabilization mechanism at the string level. Closed string Kahler moduli are typically stabilized by instanton effects. However, in the present context, such instantons develop extra zero modes due to their interaction with the fractional branes, which can lead to cancellations in the effective superpotential. Some progress has been made recently in that direction, for example in [262], where instanton effects are explicitly calculated. In the next section, we consider stabilizations in such a scenario.

An alternate embedding begins with the observation that such quiver gauge theories occur at non-geometric phases in the Kahler moduli space, and hence can be treated in a type IIA scenario by using mirror symmetry. Supergravity methods can be used in the mirror picture. A full blown IIA embedding of the runaway quiver consists of generic NS and RR flux stabilizing complex structure and Kahler moduli of the Calabi Yau Y respectively. The quiver is realized by D6 branes wrapping special Lagrangian cycles. In compact models, an orientifold projection is introduced. A number of conditions on the quiver locus and the geometry of the IIB mirror Calabi Yau X have to be imposed - (i) X should contain a pair of disjoint del Pezzos (S, S') which don't intersect the fixed point locus X^σ of the orientifold projection and (ii) the holomorphic involution of the orientifold projection should be compatible with the large complex structure limit in the complex structure moduli space of X, so that computations can be done in the supergravity limit of the mirror IIA scenario.

In the mirror IIA construction, the superpotential gets the following flux contributions

$$W_K \;=\; \int_Y F \wedge e^{-J_Y}, \quad W_H(x^k, t_\lambda) \;=\; -2x^k g_k - i t_\lambda h^\lambda \tag{5.3.1}$$

where W_K is the superpotential contribution to the Khaler moduli of Y, J_Y is the Kahler class of Y, F is the RR flux and is given by $F = F_0 + F_2 + F_4 + F_6$, W_H is the superpotential contribution to the complex structure moduli of Y, (g_k, h^λ) are the NS flux, and x^k, t_λ are $h^3_+ = h^{2,1} + 1$ holomorphic coordinates on the $N = 1$ complex structure moduli space [274], [275]. The coordinates x^k and t_λ are given by

$$x^i \;=\; 1/2 \int_Y \Omega^c_Y \wedge \beta^i, \quad t_\lambda \;=\; \int_Y \Omega^c_Y \wedge \alpha_\lambda \tag{5.3.2}$$

where $(\alpha_\lambda, \beta^i)$ form a symplectic basis of three-cycles on Y and Ω^c_Y is a linear combination of the RR three-form $C^{(3)}$ and the real part of the holomorphic three-form of Y. A specific choice of symplectic basis for explicit calculations demands more constraints on the construction [261] - (iii) the natural push-forward maps $H_2(S) \to H_2(X)$ and $H_2(S') \to H_2(X)$ have rank one, and (iv) under the orientifold projection, the anti-invariant subspace $H^{1,1}_-(X)$ is one-dimensional and is spanned by the difference $S - S'$ between the divisor classes of the conjugate del Pezzos S and S'.

In general, W_K is enough to stabilize all Kahler moduli. On the other hand, NS flux is subject to the Freed-Witten anomaly cancellation condition, which can hinder moduli stabilization by hindering generic flux. Requiring F-flatness of the superpotential then requires additional conditions due to the non-appearance of certain complex structure moduli due to the anomaly cancellation condition. In [261], explicit embeddings of the quiver gauge theory have been constructed taking into account all the above constraints in the case of certain quintic threefolds.

In this section, we perform an effective four-dimensional supergravity analysis of the quiver. We take general closed string contributions to the Kahler potential and superpotential, and couple them to the open string sector. Our strategy is to begin with a supersymmetric vacuum on the closed string side, and stabilize the open string field $\psi = \kappa^2 V_3$ in that vacuum. A self-consistent analysis is performed for $\psi \ll 1$, which allows independent stabilization of the closed string sector and removes higher order corrections to the open string sector coming from $U(1)$ D-terms. Comments on the possible uplift to dS vacua are made.

We work out specific examples for the case of a single complex modulus x in a type IIA context, without taking into account the complications introduced by the Freed Witten anomaly. The Kahler potential is taken to be a power series in x, while the superpotential is considered to be either a flux contribution like (5.3.1), or a typical instanton effect.

Our general result is that in the case of a flux superpotential, tuning the value of flux enables stabilization in the region of calculability and possible uplift to small positive cosmological constant. In the case of an instanton superpotential, consistent stabilization without strong constraints on the Kahler potential or superpotential requires a hierarchy of scales between the two sectors. Uplift to dS vacuum is correspondingly more difficult to achieve. The stabilization procedure in both cases puts mild conditions on the Kahler potential, and in the second case, on the instanton contribution. It is expected that in a full IIA calculation, these basic features would be maintained.

5.3.1 General Analysis

We take the following Kahler potential and superpotential:

$$\kappa^2 K = (\psi\bar{\psi})^{1/2}\,(1 + \gamma \sum p_i) + \sum f_i \tag{5.3.3}$$

$$W = \Lambda^3 \psi^{1/3}\,(1 + \sigma \sum q_i) + \Lambda_1^3 \sum g_i \tag{5.3.4}$$

where $p_i = p(x_i, \bar{x}_i)$, $f_i = f(x_i, \bar{x}_i)$, $q_i = q(x_i)$, $g_i = g(x_i)$, the x_i being an arbitrary number of closed string moduli. We consider the simple case where different closed string moduli x_i and x_j are decoupled. γ and σ parametrize the strength of the coupling between the open and closed string sectors in the Kahler potential and superpotential respectively. In particular, we will be working to first order in these parameters. Also, $\psi = \kappa^2 V_3 \ll 1$, so that the field V_3 is stabilized below the Planck scale.

The Kahler metric may be inverted, and to first order in γ one obtains

$$K^{\psi\bar{\psi}} = 4\kappa^2\,(1 - \gamma \sum p_i)\,|\psi|$$

$$K^{x_i\bar{\psi}} = (1/2)\kappa^2\,\gamma\,\partial_i p_i\,(\partial_i\bar{\partial}_i f_i)^{-1}\,\psi$$

$$K^{x_i\bar{x}_i} = \kappa^2\left[(\partial_i\bar{\partial}_i f_i)^{-1} \,-\, \gamma\,(\partial_i\bar{\partial}_i p_i)\,(\partial_i\bar{\partial}_i f_i)^{-2}\,|\psi|\right] \tag{5.3.5}$$

while $K^{x_i\bar{x}_j}, i \neq j$ starts at order γ^2.

All the contributions to the supergravity scalar potential can be computed, and we keep terms upto order $|\psi|^{1/3}$. The resulting stabilization places constraints on the functions f_i, g_i, p_i, q_i. Generally, the potential is of the form

$$V = e^{\kappa^2\,\Sigma f}\left[A|\psi|^{1/3} \;+ B|\psi|^{-1/3}\right] + V_{\text{closed}} \tag{5.3.6}$$

where A, B can be expressed in terms of f_i, g_i, p_i, q_i.

The non-Abelian D-term contributions to the potential are set to zero by working on the D-flat moduli space defined by (5.2.2). The $U(1)$ D-term contributions in general introduce new open-closed mixing terms, since the gauge coupling is a holomorphic function of the closed string moduli. However, these mixings begin at order $|\psi|$, and we neglect them.

We study some limiting cases of the parameters γ and σ, and work out some examples with a single complex structure modulus. The functions f and p in the Kahler potential are taken to be power series expansions in the complex structure modulus. The superpotential term is taken to be a flux contribution or a typical instanton contribution.

We note that similar supergravity calculations have been performed (see [266], [267], for example) in the context of uplifting the KKLT AdS vacuum by coupling it to a SUSY breaking sector such as an O'Raifeartaigh or ISS model.

5.3.2 $\gamma = \sigma = 0$

In this case, one obtains

$$A = \sum_i \kappa^2 \Lambda^3 \Lambda_1^3 \left[(2/3)\Sigma\, \bar{g} \, + \, [\partial_i f_i \bar{\partial}_i \bar{g}_i + \partial_i f_i \bar{\partial}_i f_i \Sigma\, \bar{g}]/\partial_i \bar{\partial}_i f_i \;\; - 3\Sigma\, \bar{g} \right] e^{i\theta/3} + c.c.$$

$$B = (4/9)\kappa^2 \Lambda^6 \tag{5.3.7}$$

Here, θ is the phase of ψ. We work out the case of a single complex structure modulus x, with $f = f_0 \, + \, \alpha_1(x + \bar{x}) \, + \, \alpha_2(x\bar{x}) + ...$

(i) Taking a single complex structure modulus, we have $g = g_0 x$. For $|\psi| \ll 1$ we can stabilize the closed string sector independently. A stable supersymmetric solution is located at

$$x_{\text{min}} \; = \; (\alpha_1/2\alpha_2)\big[-1 + \sqrt{1 - (4\alpha_2/\alpha_1^2)}\big], \tag{5.3.8}$$

Stabilizing the open string sector, one obtains

$$|\psi|_0^{1/3} \; = \; [(4/9)(\Lambda/\Lambda_1)^3]^{1/2} \, g_0^{-1/2} \, J^{-1/2} \tag{5.3.9}$$

where

$$J \; = \; (4/3 \, - \, \alpha_1^2/\alpha_2)(x_{\text{min}}) \, - \, \alpha_1/\alpha_2 \, - \, 4\alpha_1 x_{\text{min}}^2 \, g_0 > 0. \tag{5.3.10}$$

Note that (5.3.10) places constraints on the coefficients appearing in the Kahler potential. The self-consistency condition $|\psi| \ll 1$ can be obtained by tuning the flux g_0 to be large, without assuming a hierarchy of scales between Λ and Λ_1. On the other hand, the value of the potential at the minimum is

$$V_{\rm min} \;=\; 2e^{\kappa^2 K}\kappa^2\Lambda^{9/2}\Lambda_1^{3/2}g_0^{1/2}J^{1/2} \;-\; 3\kappa^2 e^{\kappa^2 K}g_0^2 x_{\rm min}^2\Lambda_1^6. \tag{5.3.11}$$

Tuning the flux such that $J \sim g_0 x_{\rm min}^2$, one can potentially lift to a dS vacuum.

On the other hand, assuming a hierarchy of scales $\Lambda/\Lambda_1 \ll 1$ without tuning the flux automatically satisfies $|\psi| \ll 1$, but in this case uplift to a dS vacuum is difficult to achieve.

(ii) Taking a typical instanton correction to the superpotential sets $g = \beta e^{-\alpha x}$. For $|\psi| \ll 1$, the stabilization of the closed string sector is decoupled from the open string sector. We start with a stable closed sting vacuum satisfying $D_x W = 0$, located at

$$x_{\rm min} \;=\; (1/\alpha_2)(\alpha - \alpha_1) \tag{5.3.12}$$

Minimizing the open string sector with respect to $|\psi|$ sets

$$|\psi|_0^{1/3} \;\sim\; (4/21)^{1/2}(\Lambda/\Lambda_1)^{3/2}\,\beta^{-1/2}\exp[(\alpha/2\alpha_2)(\alpha-\alpha_1)] \tag{5.3.13}$$

The condition $|\psi| \ll 1$ can be achieved by having $\beta \gg 1$ and $\Lambda \ll \Lambda_1$.

At the minimum, we obtain

$$\begin{aligned} V \;\sim\; & (112/27)^{1/2}\kappa^2\,\Lambda^3\,(\Lambda\Lambda_1)^{3/2}\,\beta^{1/2}\exp[\mathrm{f}_0 - (\alpha-\alpha_1)^2/\alpha_2]\times \\ & \times\exp[(3\alpha/2\alpha_2)(\alpha-\alpha_1] \;-\; 3\kappa^2\Lambda_1^6\beta^2\exp[\mathrm{f}_0 - (\alpha-\alpha_1)^2/\alpha_2] \end{aligned} \tag{5.3.14}$$

In the regime of calculability $|\psi| \ll 1$, the vacuum remains close to the closed string AdS vacuum, and there isn't much uplift.

5.3.3 $\gamma = 0, \sigma \neq 0$

In general, apart from flux contributions to the superpotential, instanton corrections coming from the closed string sector can couple to the open string fields. In that case, $\sigma \neq 0$, $q \;=\; \beta e^{-\alpha x}$.

Such corrections will also lead to open-closed coupling in the Kahler potential, but as a limiting case we set $\gamma = 0$ here. We take the case of a single IIA complex structure modulus and consider two cases - where the pure closed string contribution g is a flux effect, and where g is also due to an instanton effect.

In section 5.4, we study a type IIB embedding scenario where such couplings have been explicitly calculated.

For $\gamma = 0, \sigma \neq 0$, we obtain

$$A = \sum_i \kappa^2 \Lambda^3 \Lambda_1^3 \Big[(\sigma \bar{\partial}_i \bar{g}_i \partial_i q_i)/\partial_i \bar{\partial}_i f_i \, + \, (2/3)\Sigma\bar{g} \, + \, [\partial_i f_i \bar{\partial}_i \bar{g}_i (1 + \sigma\Sigma q) +$$

$$+ \sigma \bar{\partial}_i f_i \partial_i q_i \Sigma\bar{g}]/\partial_i \bar{\partial}_i f_i \, + \, (1 + \sigma\Sigma q)\partial_i f \bar{\partial}_i f \Sigma\bar{g}/\partial_i \bar{\partial}_i f_i \, - \, 3(1 + \sigma\Sigma q)\Sigma\bar{g} \Big] e^{i\theta/3} + c.c.$$

$$B = (4/9)\kappa^2 \Lambda^6 \, (1 + \sigma\Sigma q) \tag{5.3.15}$$

(i) We consider a flux contribution to the superpotential as before $g = g_0 x$ and take $q = \beta e^{-\alpha x}$. For $|\psi| \ll 1$ and σ such that $|\psi|^{-1/3}\sigma \ll 1$, the open-closed mixing in the potential is small, and the closed string sector can be stabilized independently as before. We obtain a supersymmetric minimum, where the value of x is given by (5.3.8). Stabilization on the open string side gives ψ as a function of the coefficients $\alpha, \alpha_1, \alpha_2$. For small x, this simplifies and we get

$$|\psi|^{1/3} \; = \; (2/3)(\Lambda/\Lambda_1)^{3/2} \, g_0^{-1/2} \, [\alpha_2(1 + \sigma\beta)]^{1/2} [\sigma\beta\alpha - (1 + \sigma\beta)\alpha_1]^{-1/2}. \tag{5.3.16}$$

The calculability condition can be satisfied by taking large values of g_0. We also note that reality of $|\psi|$ sets the condition $\sigma\beta\alpha - (1 + \sigma\beta)\alpha_1 \; > \; 0$. The value of the potential at the minimum is given by

$$V_{\rm min} \; = \; 2 e^{\kappa^2 K} \kappa^2 \Lambda^{9/2} \Lambda_1^{3/2} \, g_0^{1/2} \, [\alpha_2(1 + \sigma\beta)]^{1/2} [\sigma\beta\alpha - (1 + \sigma\beta)\alpha_1]^{1/2} \, -$$

$$- \, 3\kappa^2 \, \Lambda_1^6 \, e^{\kappa^2 K} \, g_0^2 \, x_{\rm min}^2. \tag{5.3.17}$$

In principle, it is possible to uplift the AdS vacuum by controlling the flux g_0 such that $(g_0^{3/2} x_{\min}^2)^{-1} \sim 1$.

(ii) We now consider the case where the pure closed string contribution to the superpotential is also an instanton effect. In this case, $g = \beta_g\, e^{-\alpha x}$, $q = \beta_q\, e^{-\alpha x}$. The supersymmetric minimum of the closed string sector is given by (5.3.12). For small x, the open string sector is stabilized at

$$|\psi|^{1/3} \;=\; (2/3)(\Lambda/\Lambda_1)^{3/2}\,[1+\sigma\beta_q]^{1/2}\,J^{-1/2}, \tag{5.3.18}$$

where $J \;=\; \beta_g[7/3 + (\alpha\alpha_1/\alpha_2) - \alpha_1^2/\alpha_2] \;+\; \beta_g\beta_q[3\sigma + 2\sigma\alpha\alpha_1/\alpha_2 - \sigma\alpha^2/\alpha_2 - \sigma\alpha_1^2/\alpha_2] > 0$ is a condition that can be satisfied if $\alpha \,>\, \alpha_1$, for example. Also, $|\psi| \ll 1$ requires the hierarchy of scales $\Lambda/\Lambda_1 \ll 1$.

As in (5.3.14), the minimum of the system remains close to the AdS.

5.3.4 $\gamma \neq 0, \sigma = 0$

In the limit where open and closed string contributions may be taken to be decoupled in the superpotential, the Kahler potential of the system will in general still contains couplings between the two sectors. Considering $\gamma \neq 0, \sigma = 0$, we get

$$A = \sum_i\; \kappa^2\Lambda^3\Lambda_1^3\left[(2/3)\Sigma\,\bar{g} \;+\; [\partial_i f_i \bar{\partial}_i \bar{g}_i + \partial_i f_i \bar{\partial}_i f_i \Sigma\,\bar{g}]/\partial_i\partial_{\bar{i}} f_i \;-\; 3\Sigma\,\bar{g} - \right.$$
$$\left. -\,(2/3)\,\gamma\,\partial_i\bar{g}_i\partial_i p_i/\partial_i\bar{\partial}_i f_i\right] e^{i\theta/3} + c.c$$

$$B = (4/9)\kappa^2\Lambda^6(1-\gamma\Sigma p) \tag{5.3.19}$$

(i) We take $g = g_0 x$, $f = f_0 \,+\, \alpha_{1f}(x+\bar{x}) \,+\, \alpha_{2f}(x\bar{x}) + ...$, and $p = p_0 \,+\, \alpha_{1p}(x+\bar{x}) \,+\, \alpha_{2p}(x\bar{x}) + ...$ The closed string sector is stabilized at the supersymmetric vacuum given by (5.3.8). For $x \to 0$, the open string sector is stabilized at

$$|\psi|^{1/3} = (2/3)(\Lambda/\Lambda_1)^{3/2}\, g_0^{-1/2}\, [1-\gamma p_0]^{1/2}\, [2/3\gamma\alpha_{1p}/\alpha_{2f} - \alpha_{1f}/\alpha_{2f}]^{-1/2}. \qquad (5.3.20)$$

This gives the constraint $2/3\gamma\alpha_{1p} > \alpha_{1f}$. As before, $|\psi| \ll 1$ can be achieved by $g_0 \gg 1$, while an uplift of the AdS vacuum can be achieved by tuning g_0 such that $(g_0^{3/2} x_{\min}^2)^{-1} \sim 1$.

(ii) For an instanton-like contribution $g = \beta e^{-\alpha x}$, the closed string supersymmetric minimum lies at (5.3.8), while for small x, the open string field is stabilized at

$$|\psi|_0^{1/3} = (4/3)(\Lambda/\Lambda_1)^{3/2}\, \beta^{-1/2}\, (1-\gamma p_0)^{1/2}\, J^{-1/2} \qquad (5.3.21)$$

where $J = (7/3) - (\alpha_{1f}^2/\alpha_{2f}) + \alpha\alpha_{1f}/\alpha_{2f} - (2/3)\gamma\alpha\alpha_{1p}/\alpha_{2f}$. We require $J > 0$.

5.4 Stabilization with Stringy Instantons in IIB

Following [262], we consider the quiver gauge theory on a singular dP_1 geometry, with added Euclidean D3 brane instantons. The D3's which intersect the singularity will in general also give rise to Ganor strings stretching from the occupied nodes of the quiver. Denoting quiver fields generically by ψ_i, the superpotential of the system is deformed by effects of the form

$$\Delta W \sim f(\psi_i)\exp(-\mathrm{Vol}) \qquad (5.4.1)$$

where Vol is the volume of the D3. The scalar potential can be stabilized to obtain metastable vacua.

Concretely, the complex cone over dP_1 can be described in terms of toric data as follows. The non-trivial two-cycles in dP_1 are denoted by f and C_0. A basis of branes is given by

$$[\,\mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3, \mathcal{L}_4\,] = [\mathcal{O}_{F_1}, \mathcal{O}_{F_1}(C_0+f), \overline{\mathcal{O}_{F_1}(f)}, \overline{\mathcal{O}_{F_1}(C_0)}\,]. \qquad (5.4.2)$$

Denoting the $\mathbb{P}^1$ fibrations over f and C_0 by D_2 and D_3, and the the dP_1 base by D_5, one obtains the nonzero triple intersections

$$D_5^3 = 8,\ D_5D_2D_3 = 1,\ D_5^2D_2 = D_5D_3^2 = -1\,. \tag{5.4.3}$$

Various instanton effects can be calculated in this geometry. This requires knowledge about the topology of the D3 brane and its spectrum of Ganor strings. The Euclidean D3's and the quiver nodes wrap a surface S on the del Pezzo cone, and carry different line bundles $\mathcal{L}_A$ and $\mathcal{L}_B$ over S. The most general bundle for the instanton is $X_{ab} = \overline{\mathcal{O}_{D_5}(aC_0 + bf)}$. Computing the number of fermionic zero modes between X_{ab} and $\mathcal{L}_{1,2,3}$ gives

$$n_{\text{ferm}}(X_{ab}, \mathcal{L}_{1,2,3}) = (a + 2b,\ \ -3 + a + 2b,\ 2 - a - 2b)\,. \tag{5.4.4}$$

An important instanton effect one can have in this geometry is the Affleck-Dine-Seiberg (ADS) instanton effect. In this case, $a = 0$, $b = 1$; that is, the instanton wraps the bundle $\mathcal{L}_3$. It turns out that the instanton contribution in this case leads to the superpotential

$$W_{ADS} = \frac{\Lambda^7}{Z} e^{-S_1}. \tag{5.4.5}$$

Here, S_1 is given by

$$\mathrm{Re}(S_1) = (1/2(8r_5^2 - r_3^2 - 2r_3r_5 - 4r_2r_5 + 2r_2r_3) + r_3 - 2r_5. \tag{5.4.6}$$

where r_5, r_2, r_3 parametrize the Kahler form J in terms of the toric data in the following way:

$$J = r_5D_5\ + r_2D_2\ + r_3D_3\,. \tag{5.4.7}$$

Volumes are measured in string units $\alpha' = (2\pi)^{-1}$.

One can have stringy deformations of the above field theory superpotential in the case $b > 1$ or $b \leq -1$. The superpotential in this case is

$$W_{\text{stringy}} = \frac{\Lambda^7}{M_s^6} V_3 \sum_{b>1\&b\leq -1} f(b)\, e^{-S_1+(b-1)S_2}. \tag{5.4.8}$$

where S_2 is given by

$$\mathrm{Re}\,(S_2) = 3r_3 - 2r_2 \tag{5.4.9}$$

and W_{stringy} is valid near the quiver locus $|\mathrm{Re}(S_2)| \ll 1$.

Apart from these contributions to the superpotential, there is the usual term W_{flux} responsible for fixing complex structure moduli, and $W_{\text{gaugino}} \sim \Lambda_{SO(8)} e^{-S_3}$ arising from gaugino condensation in pure $SO(8)$ gauge theory on a divisor D_6 at infinity. D_6 does not intersect D_5, and thus there is no mixture between quiver fields and instanton effects in W_{gaugino}. Here, α is a number less than one, and S_3 is given by

$$\mathrm{Re}\,(S_3) = r_2 r_3 - (1/2) r_3^2\,. \tag{5.4.10}$$

The superpotential is a sum of all these effects. Denoting $x_a = 2\mathrm{Re}(S_a)$, the regime of validity of this superpotential is

$$x_3 \gg x_1 \gg 1, \quad |S_2| \ll 1, \quad \text{or equivalently,} \quad r_2 \sim (3/2) r_3 \gg r_5 \gg 1 \tag{5.4.11}$$

To simplify the analysis of the vacuum structure of this system, we set $r_2 = (3/2) r_3$, and only consider the contribution from instantons with $b = 1$. The superpotential of the system, after integrating out the fields Z and X_{ia}, is

$$W_{\text{eff}} = W_{\text{flux}} + 3\Lambda^{7/3} \kappa^{-2/3} \psi_3^{1/3} e^{-S_1/3} + \Lambda_{SO(8)}^3 e^{-\alpha S_3} \tag{5.4.12}$$

where

$$\psi_a = \kappa^2 V_a, \quad \kappa^2 = M_{pl}^{-2} \tag{5.4.13}$$

In our regime of validity, we can use the standard large radius expression for the Kahler potential:

$$\kappa^2 K = -2\log\Big(f_1 + f_2 \sqrt{\psi_a \psi_a^*}\Big)\,. \tag{5.4.14}$$

where f_1 is the volume of the threefold, and f_2 is the volume of the divisor D_5, in string units.

Under our approximations, we obtain f_1 and f_2 in terms of the fields x_1 and x_3 as follows:

$$f_1 = (1/4\sqrt{2})x_3^{1/2}x_1^{1/2}[x_3^{1/2} - x_1^{1/2}] \tag{5.4.15}$$

and

$$f_2 = (1/2)x_1 \tag{5.4.16}$$

Equipped with W and K, we have the supergravity scalar potential

$$V = \exp(\kappa^2 K)\left(K^{i\bar{j}}W_{eff;i}W^*_{eff;\bar{j}} - 3\kappa^2 W^*_{eff}W_{eff}\right) + \frac{1}{2g_X^2}\sum_{a=1}^{3}(D_a)^2 \tag{5.4.17}$$

where the $U(1)$ D-terms are given by:

$$D_1 = -D_2 = -2\Big(\psi^a\partial_{\psi^a}K + \partial_{x_1}K\Big), \quad D_3 = 0.$$

First, we perform an analysis to minimize V with respect to the fields ψ_1 and ψ_2. Since these fields do not appear in the superpotential or its derivatives, their contribution to the F-term comes from the inverse Kahler metric and derivatives of the Kahler potential. We study the region of field space where $\alpha_1 = \psi_1\bar{\psi}_1 \ll \psi_3\bar{\psi}_3$ and $\alpha_2 = \psi_2\bar{\psi}_2 \ll \psi_3\bar{\psi}_3$. V_F as a function of α_1 and α_2 takes the form:

$$V_F(\alpha_1,\alpha_2) = \kappa^2\left(\frac{J_1\alpha_1 + J_2\alpha_2 + J_3\alpha_1\alpha_2 + J_4}{J_5\alpha_1 + J_6\alpha_2 + J_7\alpha_1\alpha_2 + J_8}\right) - 3\kappa^2 W\bar{W}. \tag{5.4.18}$$

The J_i are functions of $\psi_3\bar{\psi}_3$ and S_1, S_3. In writing the J_i, we have used the approximation $\psi_a\bar{\psi}_a \sim \psi_3\bar{\psi}_3$. We see that J_1, J_2, J_3, J_4 have mass dimension six, and consist of products of W and its derivatives. In the limit of $W_{\rm flux} \gg W_{\rm correction}$ where $W_{\rm correction} = 3\Lambda^{7/3}\kappa^{-2/3}\psi_3^{1/3}e^{-S_1/3} + \Lambda^3_{SO(8)}e^{-\alpha S_3}$, we can write

$$V_F(\alpha_1,\alpha_2) \sim \kappa^2 W^2_{\rm flux}\left(\frac{J_1\alpha_1 + J_2\alpha_2 + J_3\alpha_1\alpha_2 + J_4}{J_5\alpha_1 + J_6\alpha_2 + J_7\alpha_1\alpha_2 + J_8} - 3\right). \tag{5.4.19}$$

where $J_i, i = 1$ to 4 have been redefined, and are now dimensionless.

On the other hand, the D-term contribution is

$$V_D = \kappa^{-4} g^{-2} [J_9 \alpha_1 + J_{10} \alpha_2 + J_{11}]^2 \tag{5.4.20}$$

For $g^2 \kappa^6 W_{\text{flux}}^2 \ll 1$, the D-term dominates over the F-term, and we can argue that the potential is minimized at $\alpha_1 = \alpha_2 = 0$. Since the F-term contribution is essentially monotonic as a function of α_1 and α_2, the minimum will again be decided by the D-term in the regime where the F-term and D-terms are comparable. For $g^2 \kappa^6 W_{\text{flux}}^2 \gg 1$, the F-term dominates, and the minimum will be decided by whether it is monotonically rising or falling in the regime of validity. Since the J_i in the numerator and denominator are comparable, this rise or fall is essentially flat, and we can set $\alpha_1 = \alpha_2 = 0$. This also matches with the result in the case of global supersymmetry.

The Kahler metric then simplifies into block diagonal form, and in particular the inverse entries in ψ_3, S_1, S_3 space are unaffected by the ψ_1 and ψ_2, and a direct analytical treatment becomes tractable.

We work in the regime where the F-term dominates over the D-term. Then, the scalar potential becomes (neglecting pure $W_{\text{correction}}$ terms)

$$\begin{aligned} V \ \sim e^{\kappa^2 K} \Bigg[& \Big(-3\kappa^2 + K^{i\bar{j}} \partial_i (\kappa^2 K) \partial_{\bar{j}} (\kappa^2 K) \Big) |W_{\text{flux}}|^2 \\ & + K^{i\bar{j}} \left[\partial_i (\kappa^2 K) W_{\text{flux}} \partial_{\bar{j}} (\bar{W}_{\text{correction}}) + cc \right] \Bigg] \end{aligned} \tag{5.4.21}$$

Taking the open string field V_3 to be stabilized below M_{Planck} we get $|\psi_3| \ll 1$. Also, the regime of validity of the model is $x_3 \gg x_1$.

Evaluating the inverse Kahler metric and keeping to lowest powers of $|\psi_3|$ and x_1/x_3, one obtains

$$e^{\kappa^2 K} \kappa^4 (\partial_{S_1} K)^2 K^{S_1 \bar{S}_1} W_{\text{flux}}^2 \ \sim \ e^{\kappa^2 K} \kappa^2 (.33) W_{\text{flux}}^2 \, ,$$

$$e^{\kappa^2 K}\kappa^4(\partial_{S_1}K)(\partial_{S_3}K)K^{S_1\bar{S}_3}W_{\rm flux}^2 + c.c. \ \sim \ e^{\kappa^2 K}\kappa^2(.5)W_{\rm flux}^2\,,$$

$$e^{\kappa^2 K}\kappa^4(\partial_{S_3}K)^2K^{S_3\bar{S}_3}W_{\rm flux}^2 \ \sim \ e^{\kappa^2 K}\kappa^2(.25)W_{\rm flux}^2\,,$$

while other contributions to $e^{\kappa^2 K}K^{i\bar{j}}\partial_i(\kappa^2 K)\partial_{\bar{j}}(\kappa^2 K)W_{\rm flux}^2$ contain positive powers of $|\psi_3|$ and x_1/x_3 and are thus further suppressed.

One thus obtains

$$e^{\kappa^2 K}\Big(-3\kappa^2 + K^{i\bar{j}}\partial_i(\kappa^2 K)\partial_{\bar{j}}(\kappa^2 K)\Big) \ < \ 0\ . \tag{5.4.22}$$

On the other hand, $K^{i\bar{j}}\left[\partial_i(\kappa^2 K)W_{\rm flux}\partial_{\bar{j}}(\bar{W}_{\rm correction}) + cc\right]$ gives

$$\begin{aligned} e^{\kappa^2 K}\Big[&\Lambda^{7/3}\kappa^{4/3}|\psi_3|^{1/3}e^{-x_1/6}(1+2(x_1/x_3)+...)\cos(\theta/3 - {\rm Im}S_1/3) \\ &+\kappa^2\Lambda_{SO(8)}^3\alpha\, e^{-\alpha x_3}x_3\cos(\alpha{\rm Im}S_3)\Big]W_{\rm flux} \end{aligned} \tag{5.4.23}$$

where θ is the phase of ψ_3. Setting $\theta/3 - {\rm Im}S_1/3 = \pi$ and $\alpha{\rm Im}S_3 = \pi$, we get a negative contribution from this term also.

One thus obtains a negative scalar potential in the regime of calculability of the theory. As x_3 and x_1 grow large, $e^{\kappa^2 K} \sim x_3^{-2}x_1^{-1}$ damps out the scalar potential, and V goes to zero. Since the potential is also bounded below as long as the model is well-defined, one obtains an AdS minimum. We note that the metastable minimum of the system may lie outside the regime of calculability.

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Publications

1. Duiliu-Emanuel Diaconescu, Alberto Garcia-Raboso, **Kuver Sinha**, "A D-brane landscape on Calabi-Yau manifolds", JHEP 0606:058, 2006, hep-th/0602138.

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