TRACING MIDDLE SCHOOL STUDENTS' UNDERSTANDING OF PROBABILITY:

A LONGITUDINAL STUDY

by

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ABSTRACT OF THE DISSERTATION

TRACING MIDDLE SCHOOL STUDENTS' UNDERSTANDING OF PROBABILITY: A LONGITUDINAL STUDY

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Dr. Carolyn A. Maher

This study traces the probabilistic reasoning of five students from an urban middle school who attended an after-school mathematics enrichment program through grades 6, 7, and 8. Case study methodology is used to describe the ways of thinking and development of ideas of these students as they were presented with open-ended tasks intended to engage them in building ideas about chance. The tasks called for the students to investigate dice games to determine whether or not they were fair, and to devise strategies to make the games fair. Students were encouraged to discuss their ideas and justify their conjectures in small groups and with the whole class.

The data for this study come from videotape records of seven after-school sessions and interviews in the Rutgers *Informal Mathematics Learning* project (IML) during the spring of 2004 and 2005, when the students were in grade 6 and 7. The video data were transcribed and analyzed along with student work according to the model for studying the development of mathematical thinking proposed by Powell, Francisco, and Maher (2003).

Analysis of the data revealed that students exhibited the use of common judgmental heuristics such as representativeness, availability, and the equiprobability bias. At least three of the students combined the representativeness heuristic with the outcome approach to create what I call the *hybrid heuristic for chance events*. The application of this heuristic to assessing the fairness of games is the belief that if either player is able to win a game, then the game must be fair.

All of the students studied came to reject the idea that dice sums are equally likely. They reached conclusions based on both classical and experimental approaches. Each student produced a sample space or worked with a partner who did. Though small samples were used, all of the students used experimental data to inform or provide support for their conjectures about fairness.

In grade 7, the question of whether permutations of dice outcomes should be counted as different events was raised repeatedly, and, despite persistent challenges and questions by graduate interns, the students did not change their beliefs about this issue.

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DEDICATION

for Fannie, Dottie, Dana, and Amy

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1.1 The Importance of Learning to Reason Probabilistically

In 1989 the National Council of Teachers of Mathematics, NCTM, issued its *Curriculum and Evaluation Standards for School Mathematics* and recommended an increased emphasis on probability and statistics, quoting Huff and Greise (1959): "Probability theory is the underpinning of the modern world. Current research in both the physical and social sciences cannot be understood without it. Today's politics, tomorrow's weather report and next week's satellites all depend on it" (NCTM, 1989, p. 109). Now, nearly fifty years after Huff and Greise's pronouncement, society's reliance on probability theory and statistical methods has grown to include nearly all walks of life.

Today, understanding probability is essential for all informed citizens. The language of probability and statistics is commonplace in the news, in government reports, and in advertising. An appreciation for probability and statistics is necessary not only to understand the constant stream of information, but to make informed decisions about a myriad of things – such as health choices, finances, purchasing, education, and voting. According to Shaughnessy (1992, p. 466), "there is perhaps no other branch of mathematical sciences that is as important for <u>all</u> students, college bound or not, as probability and statistics."

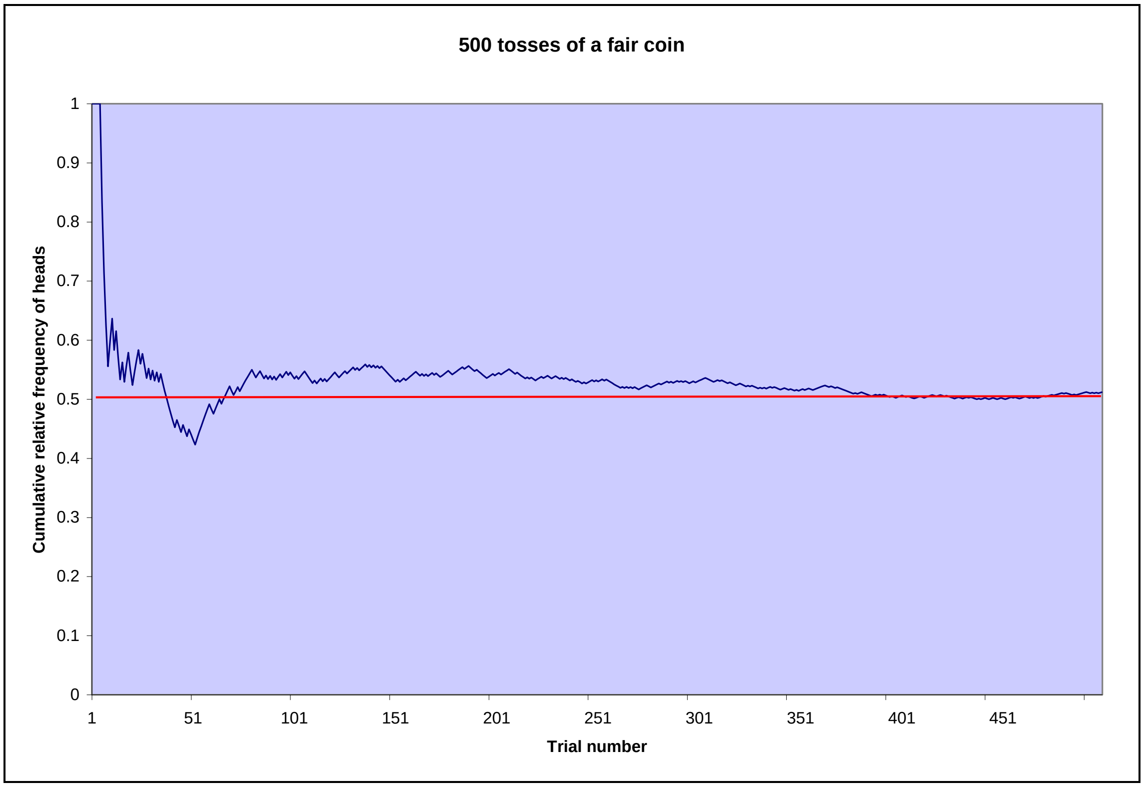
As the need for probabilistic literacy has grown, probability and statistics have emerged from being peripheral, often optional, high-school topics to become mainstream subjects in the K-12 curriculum in the United States and abroad (Jones & Thornton, 2005). In 2000, the NCTM renewed its appeal for an increased emphasis on probability and data analysis in the K-12 curriculum, naming these topics as one of five major content strands in school mathematics. The NCTM asserted, “The kind of reasoning used in probability and statistics is not always intuitive, and so students will not necessarily develop it if it is not included in the curriculum” (NCTM, 2000, p. 48). As Shaughnessy (1992) wryly noted, “people are going to use it, and abuse it – perhaps more than any other branch of mathematics – whether or not we teach it to them” (p. 467).

1.2 Conceptions of Probability

Hawkins and Kapadia (1984) define four different ways of thinking about probability.

1. *A priori* (also called classical or theoretical) probability requires prior knowledge of the set of all possible outcomes of a chance event. The set of possible outcomes is called the *sample space*. If all outcomes in the sample space are equally likely, the probability of an event is obtained from the fraction $\frac{\textit{number of outcomes favorable to the event}}{\textit{number of outcomes in the sample space}}$.
2. *A posteriori* (also called frequentist or experimental) probability requires that an experiment is repeatable many times. The observed relative frequency of an event after many repeated trials approximates the probability of the event. The *Law of Large Numbers* holds that as the number of trials increases, the relative frequency of an event approaches its true probability, as illustrated in Figure 1 on the following page.

Figure 1 – *As the number of trials increases, the cumulative relative frequency of heads approaches the theoretical probability of heads, 0.5.*



3. *Subjective and intuitive* probabilities are described as one's personal degree of belief that an outcome will occur. Subjective probability might be applied to a unique event (Kahneman & Tversky, 1996), such as judging the chances that Rutgers will be invited to play in the Rose Bowl next season, or it might derive from a basic intuition about chance. In the subjectivist perspective, probability is not inherent in the event but is an expression of the personal beliefs, intuitions, or experiences of the person estimating it. In this view, probabilities can be updated based on new experiences; the probability of an event is subject to change. Subjective probability "may be a fundamental precursor for the formal probability taught in schools" (Hawkins & Kapadia, 1984).

4. *Formal,* or axiomatic, probability is based on mathematical axioms, definitions, and theorems. While this approach to probability can exist entirely in the abstract, formal probability provides a structure for any conception of chance. For example, coherence to Kolmogorov's axioms is necessary :

   i. Probabilities are non-negative: For any event E, $P(E) \geq 0$.
   ii. Something must occur: $P(S) = 1$ for sample space S.
   iii. For a set of disjoint events $E_1, E_2, \ldots$, the probability of their union is the sum of the individual probabilities: $P(E_1 \cup E_2 \cup \ldots) = \sum_i P(E_i)$.

At the outset, students hold on to the subjectivist point of view. Watson and Moritz (2003) found that students come to school with their own subjective beliefs about probability, including "beliefs that God, fate, or mental powers determine dice outcomes" (p. 271), and students may hold onto these beliefs throughout their years of schooling. In fact, students may hold multiple and opposing beliefs about probability in a given situation (Konold, 1995).

A goal for instruction is for students to replace incorrect intuitions about chance with beliefs that are consistent with the objectivist perceptions of probability. Fischbein and Schnarch (1997) asserted:

> In learning probability, students must create new intuitions. Instruction can lead students to actively experience the conflicts between their primary intuitive schematas and the particular types of reasoning specific to stochastic situations. If students can learn to analyze the causes of these conflicts and mistakes, they may be able to overcome them and attain a genuine probabilistic way of thinking (p. 104).

1.3 The Problem

Learning to think probabilistically is not a simple matter. The deterministic nature of school mathematics (Fischbein, 1975), the classroom culture of teacher telling

(Metz, 1997), cultural or religious beliefs that a divine power controls all events (Batanero & Sanchez, 2005; Watson & Moritz, 2003), and people's erroneous instincts about chance (Kahneman, Slovic, & Tversky, 1982) are all hindrances to probabilistic reasoning. Researchers have found that for many students, incorrect reasoning is resistant to instruction (Jones & Thornton, 2005), and so misconceptions and biases may continue into adulthood. A famously illustrative example is the public outcry over a probability problem and its solution in Marilyn vos Savant's "Ask Marilyn" column in *Parade* magazine. Some 10,000 readers sent letters to Ms. vos Savant, most of them decrying her (correct) solution to the "Monty Hall problem". Nearly 1,000 of the letters that criticized Ms. vos Savant's solution were from Ph.D. mathematicians and scientists (Tierney, 1991). Indeed, the history of probability abounds with examples of mathematicians making errors, even in simple circumstances (Hawkins & Kapadia, 1984).

Unlike much of school mathematics, probability requires a way of thinking that does not consist of procedures to be followed to reach a predetermined solution (Fischbein & Schnarch, 1997). Correct probabilistic reasoning is often counterintuitive. According to Fischbein (1975), it may be impossible to modify one's faulty intuitions "once the basic cognitive schemas of intelligence have stabilized (after 16-17 years of age)" (p. 12). For this reason, it is especially important for students to develop an understanding of probability prior to the high school years. But how can this understanding be achieved?

With the recently increased emphasis on probability in the K-12 curriculum, there has been a growing body of research into the teaching and learning of probability at the pre-college level. However, there remain many unanswered questions. There is little research on how probabilistic intuitions evolve during instruction (Jones, 2005) or on students' ability to make connections between experimental and theoretical probability. A need for "clinical teaching experiments that carefully document changes in students' stochastic conceptions, beliefs, and attitudes over long periods of time" (Shaughnessy, 1992, p. 489) has been cited. Furthermore, studies with students of different social and cultural backgrounds have been recommended (Powell & Wilkins, 2006).

My research contributes to and expands the existing research base in a number of ways. It provides a rich level of detail about students' reasoning, strategies, and cognitive models as they engage in probability tasks over a two-year period of time. The tasks in this study were utilized in previous research settings, and this allows for comparisons across studies. The students in my sample were from an urban, economically depressed school district, representing a demographic that has not received sufficient attention in the literature.

1.4 Purpose of the Study and Research Questions

The purpose of this study is to trace the probabilistic reasoning of five students from an urban middle school who attended an after-school mathematics enrichment program through grades 6, 7, and 8. Using case study methodology, I describe the ways of thinking and development of ideas of these students as they engage in carefully designed open-ended probability tasks during class sessions and interviews in grades 6 and 7.

The following questions guide my research:

1. What understandings about probability (particularly fairness, sample space, probability of an event, probability comparisons) do the students exhibit?
2. How do these understandings change through the course of the after-school sessions?
3. What connections, if any, do the students make between experimental and theoretical probability?

1.5 Significance and Limitations

As a qualitative study, this research brings to light the evolution of probabilistic understanding over a two-year period as students explore and revisit thoughtfully designed open-ended problems in an informal setting. It reveals classroom practices that foster understanding as well as circumstances that can impede it. Such information can inform curriculum and lesson design.

The results of a small qualitative study are not generalizable, and the informal after-school setting may not readily translate to a typical classroom. However, these limitations are outweighed by the deep insight to be gained into the development of probabilistic reasoning of these five case-study students.

CHAPTER 2 - THEORETICAL FRAMEWORK AND LITERATURE REVIEW

2.1 Theoretical Framework

The framework for this study is based on a constructivist theory of learning. The basic principle of this theory is that "knowledge is not passively received either by the senses or by way of communication; knowledge is actively built upon by the cognizing subject" (von Glasersfeld, 1995, p. 51). In a constructivist learning environment, "the task of the educator is not to dispense knowledge but to provide students with opportunities and incentives to build it up" (von Glasersfeld, 2005, p. 7). My research is set in such an environment.

2.1.1 Rutgers Longitudinal Study

The setting for my study is the Rutgers *Research on Informal Mathematics Learning* (IML) project[1], which was built upon by a prior 20-year longitudinal study at Rutgers[2]. In the first study, researchers worked with students in classrooms and later after school, providing well-defined, open-ended tasks with minimal involvement of teachers or investigators (Maher, 2005). The salient features of what Benko (2006) dubbed "The Rutgers Method" include (Benko, 2006; Francisco & Maher, 2005; Maher & Powell, 2002):

- Carefully selecting tasks that build upon students' prior understanding.
- Allowing extended time for ideas to develop, often revisiting ideas after a prolonged break.

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- Encouraging students to discuss and justify their problem-solving strategies in small groups and with the whole class.
- Providing appropriate tools for student learning.
- Deferring closure of problems so that students can come to their own understanding.

In the Rutgers Method, the classroom serves as a community in which students are comfortable to openly share and discuss their ideas. As Fostnot has recommended, "the learners (rather than the teacher) are responsible for defending, proving, justifying, and communicating their ideas to the classroom community" (Fosnot & Perry, 2005, p. 34).

2.1.2 The Growth of Mathematical Understanding

The growth of mathematical knowledge is a process by which the learner builds mental representations (Davis, 1984; Davis & Maher, 1990) that can be "carried forth and used, and revisited and modified, in the light of new experiences" (Maher, 2002, p. 34).

Davis and Maher (1990) stated that thinking about a mathematical situation necessitates cycling through a number of steps, perhaps more than once. First, students must build a representation of the input data. This is typically a mental representation of the situation, though it may be enhanced with the use of physical materials. Second, from this *data* representation, the student must search his or her personal inventory of mental representations to retrieve or construct a representation of relevant knowledge that can be used in solving the problem or otherwise going further with the task. This step is not effortless and automatic, but may require careful reflection. The third step is to construct a mapping between the *data* representation and the *knowledge* representation. Making this mapping and checking its suitability may lead to a rethinking of the representations. Next, the student must check this mapping and these constructions to see if they seem to be correct. When learners are challenged to explain their ideas, they might modify, reject, or extend their original knowledge representation and make convincing arguments to support their generalizations. As they cycle among representations and justifications, they construct new knowledge. However, the growth of understanding in probability may be especially problematic in the building of new representations, as outlier data may support inappropriate inferences and lead to the construction of faulty schemes. Indeed, research on probabilistic reasoning has shown that children, like adults, are prone to misconceptions that are difficult to overcome (e.g., Kahneman & Tversky, 1972; Konold, Polletsek, Well, Lohmeier, & Lipson, 1993; Lecoutre, 1992; Rubel, 2006).

When learners are confronted with a mathematical task, they do not simply build upon what they already know. Instead, they "fold back" to an earlier level of understanding, where they can reflect on and reorganize earlier ideas in light of new information and experiences (Pirie & Kieren, 1994).

Understanding is the process of making connections between new ideas and previously learned concepts. This understanding is advanced by giving students interesting and challenging tasks that cause them to draw upon their prior knowledge to conceive new solutions.

2.2 Literature Review

The research on probabilistic reasoning comes from the fields of cognitive psychology and mathematics education, and covers four major themes. While cognitive psychologists have focused on describing the developmental stages of probabilistic reasoning and identifying commonly held misconceptions about probability, mathematics educators have looked at the effects of instruction and how to assess probabilistic reasoning. I will discuss the major research in each of these four areas.

2.2.1 The Development of Probabilistic Reasoning

The seminal texts on the development of probabilistic reasoning come from cognitive psychologists Piaget and Inhelder (1975, originally published in French in 1951) and Fischbein (1975).

2.2.1.1 Piaget and Inhelder's Stages of Development

Piaget and Inhelder's work was based on interviews with 20 children, ages 4 to 15. Though it is unlikely that this is a representative sample or that interviews with 20 students can be generalized, Piaget's work was profoundly influential. One of his findings, that children could not reason probabilistically before reaching the stage of formal operations, had an enormous impact on education. "Piaget and Inhelder's claim about the need for formal operations in dealing with probability was a powerful deterrent in limiting the study of probability to high school and college mathematics for more than three decades" (Jones & Thornton, 2005, p. 69).

Piaget and Inhelder interviewed their subjects through a variety of tasks such as random mixture and coin tossing. In each case, they identified three stages of development.

1. *Preoperational* (age 4 – 7) – In this stage, children had difficulty distinguishing between what is certain to occur and what is possible. They had no method for enumerating a sample space, but rather they did this in a haphazard way. They had little sense of the Law of Large Numbers and did not show a clear understanding of randomness.
2. *Concrete operational* (age 7 – 11) – In this stage, students were aware of the difference between certainty and uncertainty. Their intuitions about chance appeared. They had a global sense of probability but did not understand different degrees of it. They were more successful at enumerating a sample space than the preoperational children, though they did not have a consistent method for doing so. The Law of Large Numbers was not recognized.
3. *Formal operational* (age 11 and up) – It is during this stage of intellectual development that proportional reasoning arrives and with it, an understanding of probability, according to Piaget and Inhelder. Randomness and the Law of Large numbers were understood by the interview subjects, and the subjects were able to use principles of combinatorics to systematically enumerate a sample space.

The conclusions of Piaget and Inhelder, though influential, have come under considerable criticism. "[M]any workers disagree with Piaget's approaches, feeling that his work is too lacking in rigorous experimental controls to enable unambiguous interpretations to be derived" (Hawkins & Kapadia, 1984, p. 353). Piaget and Inhelder have also been criticized for considering only a classical approach to probability, ignoring subjective or frequentist perspectives. Many of the tasks used in their research relied on proportional reasoning and might be viewed as exercises in comparing fractions more than reasoning about chance (Garfield & Ahlgren, 1988).

Subsequent research has contradicted Piaget's assertions that children spontaneously develop probabilistic reasoning as they reach the stage of formal operations and cannot benefit from instruction before that time. Though the understanding of ratios and part-whole relationships is essential for a deep understanding of probability, supporting Piaget's premise, Shaughnessy (2003) reported that

> Research seems to suggest that (1) young children do indeed have some intuitions about probability prior to instruction, and (2) young children can learn more about probability in the context of particular instructional settings, and in some cases, can even change their thinking from their prior intuitions (p. 218).

2.2.1.2 Fischbein's Theory of Intuitions

Even as Piaget (1975) held the position that children do not possess the cognitive skills needed to learn probability before the stage of formal operations at age 11 or later, Fischbein (1975) contended that youngsters have early intuitions about probability and randomness that <u>can</u> be modified and developed through instruction. Before they begin school, children develop *primary intuitions* about chance based upon their own experiences with chance events. Fischbein characterized a primary intuition as a cognitive belief that arises from experience, not systematic instruction. It is "a global, synthetic, non-explicitly justified evaluation or prediction . . . [that is] felt by the subject as being self-evident, self-consistent, and hardly questionable" (Fischbein & Gazit, 1984, p. 2). It is also sometimes erroneous. *Secondary intuitions* are cognitive beliefs that are gained through instruction. Fischbein found that, in many cases, young students replaced erroneous primary intuitions with correct secondary intuitions after a brief period of instruction. He reached his conclusions after performing several experimental lessons with children in various age groups, from preschool to grade 8, with anywhere from 20 to 60 students at each level.

One study, reported in the appendix of his text (Fischbein, 1975), involved a teaching experiment in which students were shown a tree diagramming technique to represent permutations and combinations. Subjects were asked to estimate the number of permutations of 3, 4, and 5 objects both before and after instruction. Prior to instruction, the students, ages 10 to 14, performed poorly on the task, countering Piaget's claim that combinatorial techniques arise spontaneously around age 11. However, after instruction these students were successful in enumerating the numbers of permutations, lending support to Fischbein's assertion that primary intuitions can be built upon or replaced through instruction, even before the stage of formal operations.

Like Piaget, Fischbein suggested three developmental stages in probabilistic reasoning. Jones and Thornton (2005, p. 73) summarize these stages as follows:

1. *Preschool* (before age 7) – In this stage, children have a limited notion of chance but they will adjust their predictions based on experimental data. Instruction is not effective at changing their primary intuitions. Given concrete materials, they show some ability to consider the number of possible outcomes in a sample space.
2. *Concrete operational* (age 7 – 12) – For children at this level, "chance becomes an organized conceptual structure" but misconceptions begin to form. Learners

respond to instruction and develop strategies to compare probabilities. Using trial-and-error, they are somewhat successful at enumerating a sample space.

3. *Formal operational* (beyond age 11 or 12) - In this phase, a "fuller concept of probability" is developed. Students are responsive to the reinforcement of their predictions by experimental data. They also respond to instruction in constructing probabilities. Though their combinatorial skills are not fully developed, they respond to instruction in this area as well.

While Piaget emphasized *a priori* approaches to probability, Fischbein considered both theoretical and experimental approaches. Also, while Piaget was concerned with the spontaneous development of probability concepts, Fischbein took the effects of instruction into account. Through his experimental lessons, Fischbein "derived many principles for the design of effective teaching of probability" (Greer, 2001, p. 19). He noted, "What seems to us most important is that *practical experience with probabilities provides an ideal way of familiarizing children with the fundamental concepts of science, such as prediction, experiment and verification, chance and necessity, laws and statistical laws, knowledge through induction, and so on"* (Fischbein, 1975, p. 93, italics in original). Today's NCTM recommendations for teaching probability (NCTM, 2000) show Fischbein's influence.

Additional research related to instruction will be discussed in a later section (beginning on page 25). Next, I will discuss the research on misconceptions in probabilistic reasoning.

2.2.2 Misconceptions

Cognitive psychologists Kahneman and Tversky conducted many studies on the "psychology of uncertainty" with hundreds of students from high school through graduate school and concluded "that people do not follow the principles of probability theory in judging the likelihood of uncertain events. . . . Apparently, people replace the laws of chance by heuristics which sometimes yield reasonable estimates and quite often do not" (Kahneman & Tversky, 1982, p. 32).

Kahneman, Tversky, and others identified several *judgmental heuristics*, which are fairly consistent, incorrect strategies used by naïve and experienced learners to make judgments under uncertainty. I discuss the research around some of these heuristics below.

2.2.2.1 Representativeness

Representativeness is the belief that a sample, no matter how small, should be representative of the larger population. Using the representativeness heuristic, one judges the probability of an event by how closely it mirrors the parent population and exhibits the process that generates it (Kahneman & Tversky, 1972). For example, the representativeness heuristic might lead one to believe the outcome HTTHT is more likely than HHHHH when a fair coin is flipped 5 times. This heuristic manifests itself in the *gambler's fallacy*, where a person will predict that an outcome is due because it has not occurred lately (*negative recency*), as if a random generator must compensate over the short run for overlooked events. The opposite of this is *positive recency*, the belief that a chance outcome can be "hot" and therefore will keep occurring. (Jones & Thornton, 2005)

In their study of representativeness, Kahneman and Tversky gave a short questionnaire to approximately 1,500 students in grades 10 to 12 at college-preparatory Israeli high schools. Each questionnaire contained only 2 to 4 questions; the questions and their ordering were varied. A sample question is:

> All families of six children in a city were surveyed. In 72 families the *exact order* of births of boys and girls was G B G B B G.
>
> What is your estimate of the number of families surveyed in which the *exact order* of births was B G B B B B? (Kahneman & Tversky, 1982, p. 34)

Though both of these sequences are equally likely, 75 of 92 students judged B G B B B B to be less likely than G B G B B G, which shows an equal number of girls and boys, as would be expected in the parent population. In a similar question, B B B G G G was judged less likely than G B B G B G, which shows a mixed order of girls and boys and appears more random.

Another manifestation of the representativeness heuristic is the failure to recognize the effect of sample size. Though the Law of Large Numbers calls for very large samples to be representative of their parent population, Tversky and Kahneman found that “people’s intuitions about random sampling appear to satisfy the law of small numbers, which asserts that the law of large numbers applies to small numbers as well” (Tversky & Kahneman, 1982c, p. 25). The researchers posed a question regarding significance levels and sample size at meetings of the Mathematical Psychology Group and the American Psychological Association. The professionals at these meetings made serious overestimates of the significance of a test with small sample size. Kahneman and Tversky concluded, “the same type of systematic errors that are suggested by considerations of representativeness can be found in the intuitive judgments of sophisticated scientists. Apparently, acquaintance with the theory of probability does not eliminate all erroneous intuitions concerning the laws of chance" (1982, p. 46).

Hirsch and O'Donnell (2001) found confirming evidence of this when they gave a test to measure use of the representativeness heuristic to 263 undergraduate and graduate students. Though the proportion of students using this heuristic decreased according to the number of statistics courses the students had taken, 37.5% of the subjects who had two or more statistics courses were found to have this misconception.

2.2.2.2 Availability

Another judgmental heuristic, availability, occurs when one decides the probability of an event by how easily he or she can recall instances of that event (Tversky & Kahneman, 1982b). For example, a traveler who has been pick-pocketed while on a trip to Rome will give a higher estimate of the rate of pick-pocketing incidents in Rome.

In one study, subjects were asked whether a word in an English text is more likely to start with the letter K or have K as the third letter. Since it is easier to recall words that start with K, subjects who use the availability heuristic would choose these words as more likely. However, "a typical text contains twice as many words in which K is in the third position than words that start with K" (Tversky & Kahneman, 1982a, p. 167). Nonetheless, 105 of 152 subjects believed that the first position was more likely.

2.2.2.3 Conjunction Fallacy

With this misconception, one assigns a higher probability to the intersection of two events (A & B) than to either individual event (Tversky & Kahneman, 1982d). To test for the conjunction effect, subjects were given fictitious personality sketches, such as:

> Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination

> and social justice, and also participated in anti-nuclear demonstrations. (Tversky & Kahneman, 1982d, p. 92).

Subjects were asked to rank a number of statements from most probable to least probable, including

1. Linda is a bank teller. (T)
2. Linda is a bank teller and is active in the feminist movement. (T & F)

Overwhelmingly, subjects ranked the conjunction T & F more probable than the simple event, T. The subjects in this study included statistically naïve undergraduates, graduate students who had taken several courses in probability, and graduate students who had taken advanced courses in probability. For the Linda question, 89% of the undergraduates, 90% of the intermediate graduate students, and 85% of the advanced graduate students exhibited the conjunction fallacy. It seems that knowledge of probability had little if any effect on this misconception.

Kahneman and Tversky are not without their critics, as some have suggested that semantics, more than cognitive errors, may have caused subjects to misinterpret questions and thus give incorrect responses (Gigerenzer, 1996). I agree that the Linda question above, and others like it, bring certain stereotypes to mind and may not be viewed as questions about a chance event. It is possible that Kahneman and Tversky exaggerated the incidence of certain judgmental heuristics. However, there is substantial empirical evidence of the existence of faulty judgments under uncertainty.

Kahneman's and Tversky's misconception research has stimulated many additional studies to look for the use of judgmental heuristics, to examine their durability, and to measure the effects of instruction on correcting them. Research by Konold and his colleagues (Konold, Polletsek, Well, Lohmeier, & Lipson, 1993) uncovered a judgmental heuristic, the outcome approach, that was not previously catalogued by Kahneman and Tversky.

2.2.2.4 Outcome Approach

Using the outcome approach, one views each trial of an experiment as an individual phenomenon instead of as one of many possible outcomes. This approach leads one to interpret a probability task as needing to correctly predict an outcome instead of recognizing what is likely to occur. Konold et al. discovered this phenomenon with a question similar to Kahneman and Tversky's GBGBBG query. Subjects were asked,

> Part 1. Which of the following is the most likely result of 5 flips of a fair coin?
> a) HHHTT
> b) THHTH
> c) THTTT
> d) HTHTH
> e) all 4 sequences are equally likely
>
> Part 2. Which of the above sequences would be least likely to occur?
> (Konold et al., 1993, p. 397)

The subjects in this study included 16 high school students in a summer math program, 25 undergraduates in remedial mathematics, and 47 students in a statistics methods course. Seventy-two percent of the students correctly chose option e for Part 1; only a small percentage (9.3%) chose b, indicating use of the representativeness heuristic. The answers to Part 2 were surprising. Only 38% of the students said that all four sequences were equally *unlikely*. About half of the students who correctly answered Part 1 named one of the sequences to be least likely. Konold et al. reasoned that students using the outcome approach viewed the two parts of the problem with different perspectives. For Part 1, they tried to predict what *would* happen.

> Since the 50% probability associated with coin flipping suggests to them that no prediction can be made, they choose the answer 'equally likely'. In this context,

> equally likely does not mean that the sequences have the same numeric probability of occurrence, but that there is no basis for making a prediction of what will happen. (Konold et al., 1993, p. 399)

For Part 2, which was not interpreted as a question of what *would* happen, students identified a particular sequence that they believed was unlikely. This study, which was replicated with 20 undergraduates, showed that students can be inconsistent in reasoning about probability. It also showed that a correct response to a multiple choice question does not necessarily indicate that a student's reasoning is correct.

Rubel (2007) included questions like those from the Konold et al. (1993) study in a probability inventory given to 173 boys in grades 5, 7, 9, and 11 attending a private school in New York City. Unlike Konold, she found very few instances of inconsistencies between the "most likely" and "least likely" versions of the coin toss question.

Another misconception about probability, the equiprobability bias, was described by Lecoutre (1992).

2.2.2.5 Equiprobability bias

With this misconception, one believes that all outcomes of a chance event have the same probability. For example, the view that all sums of a pair of dice, 2 through 12, are equally likely is an instance of this bias. In fact, a sum of 2 has only a $\frac{1}{36}$ probability; a sum of 7 has probability $\frac{6}{36}$.

A problem used in Lecoutre's research is:

> Two dice are simultaneously thrown, and the following two results are obtained: R1 "a 5 and a 6 are obtained" and R2 "a 6 is obtained twice." The question asked is, "Do you think the chance of obtaining each of these results is equal? Or is

> there more chance of obtaining one of them, and if so, which, R1 or R2? (Lecoutre, 1992, p. 557)

Since R1 can occur two ways, 5-6 or 6-5, and R2 can occur only one way, the correct response is that R1 has a greater chance to occur.

In studies of over 600 subjects with varying backgrounds in probability, Lecoutre reported equiprobability responses by at least half of all the subjects at any level of expertise. "Even a thorough background in the theory of probability did not lead to a notable increase in the proportion of correct responses" (Lecoutre, 1992, p. 560). An analysis of students' justifications for saying that the two events were equally likely led Lecoutre to conclude that students with this misconception believe that all random events are naturally equiprobable.

In a later experiment, she tried a different question to mask its chance nature. Instead of dice, three cards were used: two showing an isosceles triangle and the third, a square. Subjects were shown how the two triangles could be placed together to form a rhombus, while the square and a triangle could form a house. Subjects were asked to compare the chances of obtaining a rhombus and a house if two cards were randomly selected. Lecoutre found that a greater proportion of subjects (75%) gave the correct response to this question. Lecoutre suggested that masking the chance nature of a problem can induce students to use appropriate probabilistic models. However, the transfer of the correct model to a subsequent standard probability problem does not always occur.

2.2.2.6 50/50 Approach

Rubel (2006) identified a misconception related to the outcome approach and equiprobability bias, which she called the 50/50 approach. In her study of 173 boys in grades 5, 7, 9, and 11 in private school, students were given a Probability Inventory in which they responded to ten probability questions. Follow-up interviews were conducted with 33 of the students. One of the questions involved the probability of getting one "heads" and one "tails" when two coins are tossed. Though somewhat more than half of the students correctly answered ½, a substantial number of them justified this answer by generalizing the probability of getting "heads" or "tails" on a single coin toss. Rubel cited an interview with one student that further illustrates this misconception. When asked the probability of getting all tails when three coins are tossed, the student said 50 percent, explaining that "unless something affects the way the quarters come down, it's still going to be equal" (p. 52). In fact, this student maintained that the probability is 50 percent that 100 coins, even 100,000 coins, would all land on "tails." Overall, 40% of her sample used the 50/50 approach on at least two questions in the Probability Inventory.

The research on misconceptions shows that both novices and experts are prone to incorrect reasoning. Next, I will discuss a study that intended to reveal differences in the incidence of misconceptions at various ages.

2.2.2.7 Misconceptions Across Different Age Groups

A widely cited study by Fischbein and Schnarch (1997) sought to describe the evolution of probabilistic misconceptions across several age groups. To do so, the researchers administered a 7-question written test to 20 students in each of grades 5, 7, 9, and 11, as well as to 18 undergraduate pre-service mathematics teachers. None of the students tested had any prior instruction in probability. The test questions were designed to reveal the common misconceptions identified by Kahneman, Tversky, and others.

The results were mixed. Though some misconceptions such as representativeness and negative recency "decreased with age" (p. 101), the misconception that sample size is not relevant "*developed* with age in a surprisingly regular manner" (p. 101, italics in original). An explanation for this observation may be that older students used equal ratios to conclude that the probability of more than 60% of births will be males is the same in a hospital with 15 births a day as in a hospital with 45 births a day.

Fischbein and Schnarch based their conclusions on the percentages of students at each grade level who either answered a question correctly or exhibited a common misconception. For example, a question that examined representativeness and the percentages of responses in each category are shown below.

Table 1 *Representativeness question and percentages of student answers.*
(Fischbein & Schnarch, 1997, p. 98)

	GRADES				
Problem	5	7	9	11	CS*
In a lotto game, one has to choose 6 numbers from a total of 40. Vered has chosen 1, 2, 3, 4, 5, 6. Ruth has chosen 39, 1, 17, 33, 8, 27. Who has a greater chance of winning?					
Vered has a greater chance of winning.	0	0	0	0	0
Ruth has a greater chance of winning. (Main misconception)	**70**	**55**	**35**	**35**	**22**
Vered and Ruth have the same chance to win. (Correct)	30	45	65	65	78

*College students

The decreasing percentage of incorrect responses and the increasing percentage of correct responses across the five age groups led Fischbein and Schnarch to conclude that representativeness, as measured with this sort of question, decreases with age. This conclusion seems questionable. In order to affirm that a misconception changed with age, it would be better to test the same students over several years, rather than to compare unrelated groups of students. Though Fischbein and Schnarch's conclusions from this research seem to be overstated, they do present interesting hypotheses that warrant further study.

Rubel (2007) performed a similar analysis with her sample of 173 boys in grades 5, 7, 9, and 11. She found comparable percentages of errors across the different grade levels, which led her to conclude that “most of the errors were stable across ages” (p. 553).

The misconceptions and faulty heuristics catalogued above “can appear to be a daunting list of potential roadblocks to students’ understanding of probability” (Shaughnessy, 2003). However, armed with this knowledge, teachers are better prepared to understand students’ thinking and to plan instructional activities accordingly. In the next section, I will discuss several studies about the effects of instruction on developing correct probabilistic reasoning.

2.2.3 Effects of Instruction

Despite the new prominence of probability and statistics in school curricula, there is limited research about instructional methods and their effects. This is an area where further study is warranted. Three overlapping themes for instruction have begun to emerge as offering promise to overcome misconceptions and foster understanding of probability. These are: 1) starting probability instruction in the early grades, 2) giving students ample opportunities to experiment, build models, and discover concepts through small group work, and 3) using technology to conduct probability simulations.

2.2.3.1 Probability in the Early Grades

A pivotal study showing that children as early as grade 3 can benefit from instruction in probability was conducted by Jones and his colleagues (Jones, Langrall, Thornton, & Mogill, 1999). The subjects were 37 third-grade students who underwent an instructional program of sixteen biweekly lessons. Students were divided into two groups: one group was taught during the fall semester, the other in the spring. Each of the lessons began with a whole-class discussion that was followed by tasks that the students worked on in pairs, mentored by teacher-education students. The problem tasks related to the constructs of sample space, probability of an event, comparison of probabilities, and conditional probability. Using a cognitive framework (Jones, Langrall, Thornton, & Mogill, 1997, see page 50 of this paper for an expanded version) that identifies four levels of thinking in each these constructs – subjective, transitional, informal quantitative, and numerical – the researchers assessed the students' probabilistic thinking prior to instruction and at the end of the fall and spring semesters. Three assessments permitted researchers to use the delayed instruction group as a control for the early instruction group at the end of the fall semester, and to use the early instruction group to assess more long-term effects of instruction at the end of the spring term. In addition, four students were targeted for case study analysis.

While there were no students at the informal quantitative level (level 3) prior to instruction, seven of the 18 students in the early instruction group and 12 of the 19 in the delayed instruction group advanced to this level by the final assessment. Comparison of the early and delayed instruction groups at mid-year supported the claim that advances were due to instruction and not maturation. Five students, however, did not advance beyond the subjective level (level 1) after instruction. Analysis of the case study students' learning showed that

> a) misconceptions in sample space, when they exist, can be deep-seated and appear to be fueled by subjective judgments;
> b) the application of part-part reasoning is crucial to students' quantifying probability situations in any meaningful way;
> c) the application of both part-part and part-whole relationships in probability situations is the key to producing growth in probabilistic thinking; and
> d) the use of invented or conventional language to describe part-whole relationships provides scaffolding for coherent probabilistic thinking. (Jones, Langrall et al., 1999, p. 502)

The researchers acknowledge that by working in pairs with a mentor, the students in this study benefited from what amounted to individualized instruction, which would not be possible to replicate in the classroom. However, as we will see below, several studies have shown that small groups working with carefully designed tasks can develop correct probabilistic reasoning with minimal intervention.

In another study with young students, Aspinwall and Tarr administered a five-day instructional program to a sixth-grade class of 23 students. The researchers were interested in learning whether probability experiments influenced students' understanding of the role of sample size in experimental probability. Like the Jones et al. study, lessons comprised whole-class discussions and small group work. Students worked on a series of probability tasks that required them to use random generators and draw inferences from the resulting data. The data for all students was combined for class discussions.

Students were given task-based interviews one week before and again several days after instruction, and their levels of probabilistic thinking were assessed using a version of the framework used in the Jones et al. study which was expanded to include experimental probability (Jones, Thornton, Langrall, & Tarr, 1999, see page 38 of this paper). A Wilcoxon signed ranks test was used to compare the pre- and post-instruction levels ($z = 2.03$, $p < .05$), and a qualitative analysis was performed with six case-study students. Overall, the results of the study were uneven. The qualitative analysis showed evidence that the students could relate sample size to experimental probability, but their understanding was largely limited to realizing that it is more likely to get unusual results with small samples. Also, the results of some atypical simulations tended to reinforce misconceptions for some students.

One of the tasks used during instruction was called *To Sum it Up: A Dice Game*. This is a game for two players that involves rolling a pair of dice. The class was divided into two groups by distributing a white or yellow card to each student. The rules of the game are:

> WHITE: Scores one point if the sum of the dice is 2, 3, 4, 9, 10, 11, or 12.
> YELLOW: Scores one point if the sum of the dice is 5, 6, 7, or 8.
>
> Students were then asked to predict which color would win if the game were played in each of the following formats:
>
> - The first player to score one point is the winner;
> - The winner is the player leading after three rolls;
> - The winner is the player leading after 11 rolls;
> - The winner is the player leading after 21 rolls. (Aspinwall & Tarr, 2001, p. 240)

Initially, most students believed that white was the most likely winner because there were more sums favoring white. (This is incorrect. The probability that white will score a point is 16/36, while yellow has a 20/36 chance.) Nearly all the students agreed that regardless of color choice, the probability of winning was greatest with the largest number of rolls. (This is true for yellow, but not for white.) As students played the game in pairs, they were asked to hold up their color card if they were winning at various points in the game. In the beginning, the whites and yellows were fairly even, but after 21 rolls, only one white card holder was a winner. In the class discussion that followed the game, a few students held on to the belief that white had a better chance to win, all evidence to the contrary. One student worked out the theoretical probability distribution and shared it with the class, and students were asked to work in pairs to confirm it. In the end, students agreed that yellow has a better chance to score points, but with a small number of trials white can win the game.

2.2.3.2 Dice Games

The dice game used by Tarr and Aspinwall is a variation of one that has been used by the Working Group for the Complexity of Learning to Reason Probabilistically of the North American Chapter of the International Group for the Psychology of Mathematics Education, PME-NA (Maher & Speiser, 1999). Through this working group, researchers were invited to explore two dice games with different student populations. The games are described as follows:

> *Game 1, a game for two players.* Roll 1 die. If the die lands on 1, 2, 3, or 4, Player A gets one point (and Player B gets 0). If the die lands on 5 or 6, Player B gets one point (and Player A gets 0). Continue rolling the die. The first player to get 10 points is the winner. Is this game fair? Why or why not?
>
> *Game 2, another game for two players.* Roll two dice. If the sum of the two is 2, 3, 4, 10, 11, or 12, Player A gets one point (and Player B gets 0). If the sum is 5, 6, 7, 8, or 9, Player B gets one point (and Player A gets 0). Continue rolling the dice. The first player to get 10 points is the winner. Is this game fair? Why or why not? (Maher & Speiser, 1999, p. 183)

These tasks were developed for sixth-graders in the longitudinal study where researchers from Rutgers University worked with students in the Kenilworth, NJ, school district from grade one through high school and, in fact, they were used in my study. In the original study using these tasks, Maher (1998) was interested in the representations that students built to analyze the dice games and how these representations changed over the course of two days of instruction. Students worked in small groups with no teacher intervention, playing the games and hypothesizing about whether or not they were fair. They were asked to prepare overhead transparencies to present their findings to the entire class. Three video cameras recorded the students at work, and a qualitative analysis of the class sessions was performed.

The first game gave little challenge to the students, as they readily agreed it was unfair and set about modifying it to make it fair. There was considerable disagreement about game 2. As in Aspinwall and Tarr's study, some students thought that Player A had an advantage because there were 6 sums that gave A a point while only 5 sums awarded a point to B. (In this game, the probability that A will score a point is 12/36; B's chances are 24/36.) Other students concluded that the game might be fair because some of B's numbers were easier to get, thus making up for the deficit in possible sums. Some thought that even sums were more likely than odd sums, or high numbers more likely than low numbers. After playing the game a number of times, several students recognized that B seemed to win more often than A. As the first session on this task ended, students were asked to think about the game, play it as often as they'd like at home, and return to the next class ready to discuss their conjectures or conclusions about the game.

On the second day with this task, students agreed that B had the advantage, but they were largely divided into two camps: one which believed there were 36 equally likely outcomes and the other claiming 21 outcomes, treating symmetric pairs as a single outcome. (In the case of 36 outcomes, B's probability of scoring a point is 24/36. With

21 outcomes assumed to be equiprobable, it would be 13/21, still more than half.) With no intervention from the teacher except to ask students to explain their reasoning, the students were able to resolve the issue among themselves and convince each other that there were 36 equally likely outcomes. This study provided a powerful example of how students, working together with carefully designed tasks, can develop probabilistic understanding and make sense out of conflicting evidence. The social interactions that occurred in this class were an essential component to learning.

Vidakovic, Berenson, and Brandsma (1998) used the same two dice games with a class of 16 eighth-grade students in an urban school district. The researchers were interested in students' initial intuitions about fairness and chance, and whether faulty intuitions could be challenged and modified in a non-threatening, game-playing context. Instruction took place over a four day period in which the students initially worked in pairs and then in larger groups to share the results of their investigations. Sessions were videotaped, and qualitative methods were used to analyze the sessions.

As with the students in Maher's (1998) study, students readily agreed that game 1 (rolling a single die with $P(A) = 4/6$) was not fair. However, there was considerable disagreement over how to modify the game to make it fair. Many students believed that giving points to player A for a roll of 1, 2, or 3 would not make the game fair because these numbers are more likely to occur than 4, 5, and 6. Using a limited number of trials, these students believed that the evidence supported this view. Though other students argued that giving half of the numbers to player A and half to player B would make the game fair, it was not clear that all of the class was convinced.

Game 2 also ended in disagreement for these eighth-grade students. Like the students in Maher's (1998) study, they did not agree about whether symmetric pairs should be counted as one outcome or two. One student argued that if the dice were two colors, say green and white, a six on the green die and a one on the white was a different outcome than a one on the green and a six on the white. "[T]he class was not ready to accept this interpretation as many students still argued that it does not matter" (Vidakovic et al., 1998, p. 72), and so the researchers chose to leave the class undecided about this issue and return to discuss it at a later date.

Vidakovic's subjects, who were two years older and had two more days of instruction than Maher's subjects, did not advance as far in their development with respect to the concepts of fairness and sample space. However, Maher's subjects had an advantage in that her students were accustomed to a classroom culture of working together and constructing convincing arguments for their theories that was a part of the Rutgers-Kenilworth project since grade one. The Kenilworth students had previously worked on a variety of tasks that included combinatorial reasoning, which made them better prepared for the probability tasks.

Speiser and Walter (1998) used the dice games as part of an instructional unit for undergraduate elementary education majors. They reported on a focus group of five students in the second semester of a mathematics course designed for preservice teachers. The students played the games themselves and then watched video of the Kenilworth students doing the same activity. Speiser and Walter's focus was on how education students build mathematical ideas through this kind of investigation. The researchers wanted to know what disagreements would emerge among their students and how the disagreements would be resolved. What kind of evidence would be needed to convince the students, and what theories would they develop?

Like the youngsters in Maher's and Vidakovic's studies, the undergraduate students disagreed about the number of equally likely outcomes in the sample space when two dice are rolled. Some students were very tentative in their arguments, one of them saying, "I'm wondering . . . I don't know if I'm right. I don't even think I'm right but I don't know. If this [(1,2)] has one chance, and if this [(2,1)] has one chance, because they each have 50-50 chances of happening. Right? . . . But, . . . so *together* are they just one chance or two different chances?" (Speiser & Walter, 1998, pp. 63-64).

The students made lists and charts to enumerate both the 21-outcome and the 36-outcome sample spaces, and they constructed a map from the larger sample space to the smaller one. Once this map was constructed and understood, the probabilities were easily computed.

One can only hope that preservice teachers everywhere have opportunities like this to work through problems and confront their misconceptions, lest they bring these misconceptions into the classroom.

The dice games were also used by Amit (1998) with 62 fifth- and sixth-graders in Israel. Amit's purpose was to study how "children (and teachers) think, develop and use probability concepts to make decisions about fairness and chances to win" (p. 45). As in the Vidakovic et al. study, students worked in pairs, sessions were videotaped, and qualitative methods were used for analysis. Some of the initial misconceptions noted by Amit were:

1. Some students believed that the player to roll first would win. They resolved this by taking turns.
2. Students who were familiar with Backgammon, a game in which doubles are favored, initially thought that the player who rolled doubles had a better chance to win. Others explained that doubles have higher status in Backgammon because they are harder to get.
3. A teacher expressed concern that if a player with a lower probability of winning actually won a game, students would be confused and their understanding of probability ruined. However, students accepted the unpredictability of events and were not confused.

Amit did not provide much detail about the discussions that took place, but she made a general claim that students developed “rules for fair games and sophisticated strategies to prove their justice” (p. 47).

As an extension of the dice games discussed above, students in the Rutgers-Kenilworth project were asked to analyze games involving the sum of three dice in grade 7. Pyramidal dice were introduced as a way for students to test their conjectures about the sample space with a smaller number of outcomes. Two studies, one focused on effective teacher questioning (Dann, Pantozzi, & Steencken, 1995) and the other on student representations (Benko & Maher, 2006), demonstrate how the Kenilworth students made and justified conjectures about the sample space for rolls of two, three and four pyramidal dice. Students created original graphs and charts to systematically generate the sample space, and they discovered a general rule to determine the number of outcomes in *y* tosses of an *x*-sided die. My study also uses pyramidal dice games in grade 7, with very different results.

In questions concerning the fairness of dice games, there is an underlying assumption that students have a common understanding of what it means for dice to be fair. Watson and Moritz (2003) showed that this may not be the case and, further, that the strategies students use to determine whether dice are fair may not be consistent with their beliefs. The researchers conducted interviews with 108 students in grades 3 through 9, and re-interviewed 44 of these students a few years later using the same protocol. In the interview sessions, students were given some dice, at least one of which was "loaded", and asked to decide whether or not each die was fair. The researchers identified four different levels of beliefs about the fairness of dice:

1. Ikonic – Students believe dice are unfair in that certain numbers are more likely to occur than others. Students may have inconsistent beliefs that, although some numbers are more likely, all numbers have an equal chance.
2. Unistructural – Students believe that dice are fair despite experimental evidence to the contrary.
3. Multistructural – Students believe that dice are fair <u>if</u> they are rolled in a particular unbiased way.
4. Relational – Students believe that dice are fair in the long run, though short-term results may not appear so.

Additionally, Watson and Moritz noted four levels of strategies to determine fairness:

1. Ikonic – Students rely on intuitive beliefs, such as lucky numbers.

2. Unistructural – With the belief that dice are inherently fair, students do not see a need to test for fairness.
3. Multistructural – Students observe the physical features of a die – checking that all numbers are present and that the cube is symmetrical. They do not use data to draw conclusions.
4. Relational – Students roll the dice, record the outcomes of many trials, and compare the relative frequencies of each outcome.

Surprisingly, the researchers found little evidence of a correspondence between the students' beliefs about the fairness of dice and their strategies for assessing fairness ($r = .28$, $p < .005$). This lack of association did not change in the subsequent interviews a few years later ($r = .29$, $p < .005$). An important implication is that a student's beliefs about fairness based on theoretical probability may be "quite divorced from the empirical approach of judging probability based on long-term relative frequency" (Watson & Moritz, 2003, p. 298).

2.2.3.3 Making Inferences With Limited Data

The issue raised by the teacher in Amit's study, that the occurrence of improbable outcomes in a small number of trials might confuse students, can be a legitimate concern when students try to make inferences from a limited amount of data. A paper, of questionable merit to me, called "The Effects of Instruction on Likelihood Misconceptions" (Ayres & Way, 2001) illustrates this point. The study was conducted with 24 sixth-grade girls of above-average mathematical ability (as measured by a state numeracy test) in Australia. The purpose was, as the title suggests, to examine the effects of instruction with small-group, hands-on activities on the decision-making strategies of these students. The instruction consisted of two one-hour sessions over two days followed by a test session at a later date.

On the first day of instruction, students were randomly assigned to groups of four. Each group was given a bag containing ten tiles of differing ratios of green: yellow: blue. Though the ratios varied from 5:3:2 to 7:2:1, green was the predominant color in each bag. The students were made aware that the bags contained tiles of these three colors, but they did not know the counts. The activity, presented as a game, was to have students predict the color of a tile before it was drawn from the bag. Each game consisted of only five predictions. Four games were played, and the winner was the student with the most correct predictions. Students were asked to think about the winning strategies and consider how they might have improved their predictions. I do not understand the logic of using such a small number of trials. In my view, the researchers were misguided in this approach. The conclusions about winning strategies by some students bear this out. The winning student in one group adjusted her prediction on the basis of whose turn it was to withdraw a tile from the bag. Her misconception was reinforced because, coincidentally, her guesses were correct. She said,

> I worked out a theory. The teacher (researcher) is English, and he pulled out a yellow tile. My dad's English and I also pulled out a yellow tile. Alison's dad is Australian and Australia is on the opposite side of the world to England, therefore she would pull out a blue tile and she did. Maria's dad is Greek, therefore she should pull out a green tile and she did. (Ayres & Way, 2001, p. 76)

Despite this result, Ayres and Way claim, without providing further evidence, that "overall, quantitative and qualitative data revealed that most students demonstrated a good understanding of likelihood in this domain" (p. 76).

2.2.3.4 A Quantitative Study With Middle School Students

Much of the research on the effects of instruction in probability is qualitative in nature. According to Shaughnessy (1992, p. 476), "Clinical methodologies seem most appropriate for mathematics educators interested in exploring students' cognitive and affective processes on stochastic tasks." Breaking from that mold is a study by Fischbein and Gazit (1984), who did a large-scale analysis of the effects of instruction on students in grades 5, 6, and 7. For their study, 285 students were given an instructional program in probability that included hands-on activities with random devices such as dice and marbles. An emphasis was placed on relating *a priori* probabilities and experimental frequencies. Fischbein and Gazit posited that "new intuitive attitudes can be developed only through the personal involvement of the learner in a practical activity" (1984, p. 2). For comparison, a control group of 305 students had no instruction in probability.

Two questionnaires were developed, Questionnaire A, which was a test of the concepts and procedures that had been taught, was given only to the students who had instruction. Questionnaire B, which tested for the indirect effect of instruction on misconceptions, was given to both groups. Fischbein and Gazit provided a question-by-question analysis of the two questionnaires, listing the percentages at each grade level and in each group who answered questions correctly. I was surprised that with all this quantitative data, no statistical analysis was performed.

The results of Questionnaire A revealed that the concepts taught were too difficult for the fifth graders. The sixth and seventh graders did better, leading Fischbein and Gazit to conclude that probability instruction should begin in grade 6 or 7. Given the success with younger students found in other studies, I do not agree. As for the effects of instruction on misconceptions, the researchers concluded that "in grades six and seven the teaching programme has had an indirect positive effect on" the representativeness bias, the positive recency effect, and superstitious beliefs (p. 22). A surprising result was that on the two questions related to proportional reasoning, the control group outperformed the group of students who received instruction in probability. Fischbein and Gazit hypothesized that "probabilistic thinking and proportional reasoning are based on two distinct mental schemata." Though ratios are involved in the computation of probability, "probability, as a specific mental attitude, does not, necessarily, imply a formal understanding of proportion concepts" (p. 23). This seems to refute Piaget's contention that proportional reasoning is necessary to understand probability.

2.2.3.5 Studies With Older Students

In a study with high school students, Kiczek and Maher (2001) reported on further effects of the Rutgers-Kenilworth project on students' probabilistic thinking. In this study, the researchers focused on the development, stability, and durability of ideas about probability. Some of the same students from the Maher (1998) study, now in 11th grade and attending after-school problem solving sessions, were challenged with two tasks:

> *The World Series Problem:* In a World Series two teams play each other in at least four and at most seven games. The first team to win four games is the winner of the World Series. Assuming that the teams are equally matched, what is the probability that a World Series will be won: (a) in four games? (b) in five games? (c) in six games?
> (d) in seven games?
>
> *The Problem of Points:* Pascal and Fermat are sitting in a café in Paris and decide to play a game of flipping a coin. If the coin comes up heads, Fermat gets a point. If it comes up tails, Pascal gets a point. The first to get ten points wins. They each ante up fifty francs, making the total pot worth one hundred francs. There are, of course, playing "winner takes all." But then a strange thing happens. Fermat is winning, 8 points to 7, when he receives an urgent message that his child is sick and he must rush to his home in Toulouse. The carriage man who

> delivered the message offers to take him, but only if they leave immediately. Of course, Pascal understands, but later, in correspondence, the problem arises: how should the 100 Francs be divided? (Kiczek & Maher, 2001, p. 427)

The students worked for several hours on these tasks over four sessions – three of which occurred in consecutive weeks in January and February, and the fourth in August. During these sessions, as was the norm in this project, students worked collaboratively to invent strategies, build representations, recognize patterns, and justify results (p. 426). The teacher/researcher did not give any instruction. The sessions were videotaped and analyzed using qualitative research methods.

The World Series problem was solved on the first day, as students employed combinatorial strategies that they had learned in earlier sessions to determine the number of ways a Series could be won in 5, 6, or 7 games. In checking that $P(4) + P(5) + P(6) + P(7) = 1$, the students found and corrected an error they had made. In the next session, the students used an area model of probability to explain why probabilities of a given sequence of wins and losses should be multiplied, and they generalized the problem to a situation in which the teams are not equally matched. Without relying on formulas, the students showed deep conceptual understanding.

To test the stability of the students' reasoning, the researchers presented them with an alternative (incorrect) solution that a group of graduate students had suggested. All but one of the students was convinced that their own solution was correct and saw the flaw in the graduate students' reasoning. Similar to the dice game for two players, at issue was whether or not the outcomes in the sample space were equally likely. It was not until the fourth session, some months later, that the unconvinced student resolved the discrepancy in World Series Problem as he explained it to another student.

In the third and fourth sessions, the students solved the Problem of Points, which they recognized was isomorphic to the World Series Problem. What is remarkable for me about this study is the fact that students solved these challenging problems with no formal instruction, relying instead on the rich experiences they'd had over the years of the Rutgers-Kenilworth project and the culture of social interaction and sense making.

Kiczek (2000) and Benko (2006) documented the growth of probabilistic understanding over several years in two cohorts of students in the Rutgers-Kenilworth project. Both studies showed the success of the instructional methodology that allowed students to work collaboratively on carefully chosen problems that challenged their intuitions and biases and to build durable conceptual foundations prior to any formal instruction.

A quantitative study by Shaughnessy (1977) involved a controlled experiment with 80 undergraduate students to compare the effects of small-group, activity-based instruction to traditional lecture classes in overcoming the representativeness and availability heuristics. Four of seven sections of a finite math course were randomly selected, then two of the sections were randomly assigned to experimental, activity-based classes; the other two sections received traditional lectures in probability. Though the content in both types of classes was similar, the experimental classes used a "problem-solving and model-building approach" (p. 299) in which students worked in small groups on tasks meant to develop their understanding of sample space, theoretical probability, counting rules, and the effect of sample size.

All students were given a pretest and a posttest to measure their use of the representativeness and availability heuristics. Using a contingency table analysis, Shaughnessy found that students in the experimental classes were "more successful at overcoming reliance upon representativeness ($p < .05$, $df = 2$) and tended to be more successful at overcoming reliance on availability ($p < .19$, $df = 2$)" (p. 308).

Though the equiprobability bias was not targeted by Shaughnessy in this study, classroom observations of the experimental sections revealed the presence of this misconception. Students experimented with tossing a thumbtack and estimated P(Up) = 2/3. However, when asked to construct a mathematical model for tossing 3 tacks, each outcome was assumed equally likely – for example, P(UUU) = P(DDD) = P(UDU) = 1/8. Despite experimental evidence to the contrary, the students insisted that the eight outcomes should be equally likely and suggested that there was a flaw in the thumbtacks.

2.2.3.6 Studies With Educational Technology

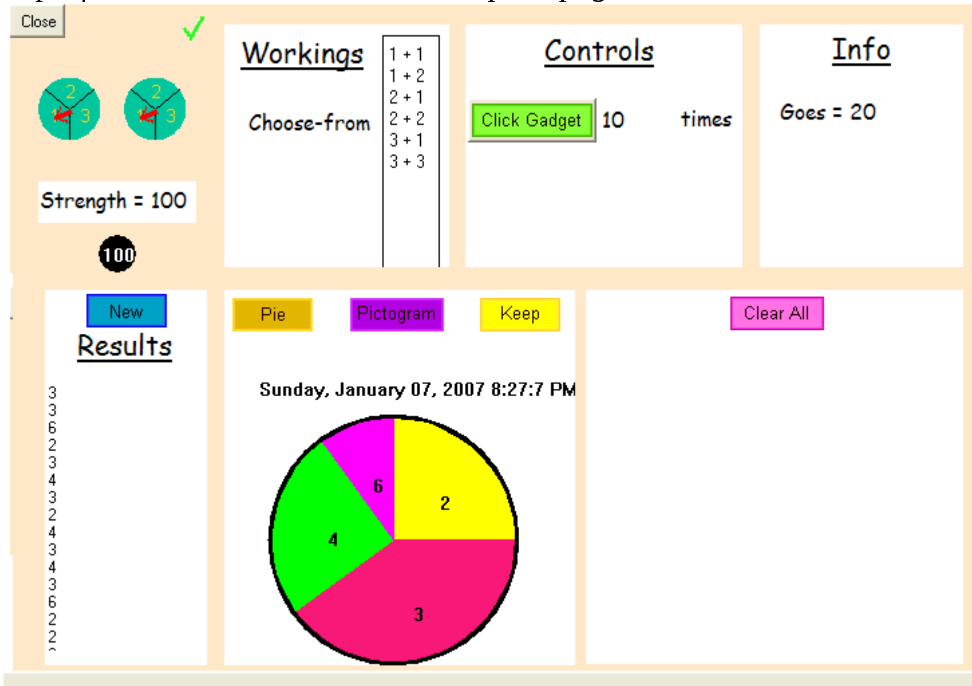
Another theme in the research on instruction is the use of technology to perform probability simulations. Computer and calculator programs that allow students to collect and summarize large amounts of data in a short amount of time have the potential to forge a link between theoretical and experimental conceptions of probability. As students compare their predictions to the distribution of outcomes, they can try to resolve the source of any inconsistencies.

Garfield and delMas (1989) used a program called *Coin Toss* with 57 undergraduates in an introductory statistics class. The Coin Toss program simulated as many as 10,000 tosses of a fair coin and illustrated the variability of samples, the effect of sample size on sampling distributions, independence, and randomness. The students were given a *Reasoning About Chance Events* pretest on the first day of class to identify their misconceptions and biases. Students used a workbook with the software in which they recorded their predictions, experimental outcomes, and observations. A full class discussion was held after all the students had used the software, and then the students were given Reasoning About Chance Events once more as a posttest.

On the pretest, only a handful of the students showed correct and stable conceptions about variability, and “a larger number had conceptions that were stable, but incorrect and resistant to change” (Garfield & delMas, 1989, p. 194). Stable and incorrect conceptions about the effect of sample size were also held by many of the students. However, after using the Coin Toss software, a majority of the students’ misconceptions did change.

Chance-Maker (Pratt, 1998) is another educational program that provides a selection of *gadgets* such as coins, dice, and spinners that emulate their real-world counterparts. Figure 2 shows a Chance-Maker screen in which the sum of the numbers on two spinners was simulated for 20 trials. Each spinner is divided into three equal sections, numbered 1, 2, and 3. To the right of the spinners is a box labeled *Workings* which intentionally shows only part of the sample space. Students are able to edit the Workings box to include or delete outcomes. A pie chart displays the distribution of sums. In this example, the sums 1+3, 2+3, and 3+2 were omitted from the Workings box and so these sums were not possible. The sum of 5 did not appear at all.

Figure 2 - *A screen shot of Chance-Maker, downloaded from http://fcis1.wie.warwick.ac.uk/~dave_pratt/page13.html*



Pratt (2000) reported on a case study in which two ten-year old girls worked with Chance-Maker to make sense of the total of two spinners and of two dice. Starting with the two-spinners gadget as depicted in Figure 2, the girls were instructed that they needed to determine if the gadget was working properly and to fix it if it wasn't. At the onset, the girls exhibited the equiprobability bias, as they expressed the belief that all totals, 2 through 6, had an equal chance. One of the girls said, "There's a 50-50 chance of getting any total" (p. 612).

After running 50 trials with the default Workings as shown in Figure 2, the girls noted that the pie chart was not "even", and so they decided to run 1,000 trials. When 5 still did not appear, they adjusted the Workings box to include 2+3 and 3+2. They did not insert the other missing pair, 1+3. Perhaps they didn't notice its absence. The girls readily identified that 2+3 and 3+2 were different outcomes: the first term associated with the first spinner and the second term with the second spinner. Since the pie chart showed a smaller area for a sum of 2, they decided to put an additional 1+1 in the box. This seemed logical to them, as 2+3 and 3+2 were different, why not 1+1 and 1+1? There is a tension here between the girls' desire to see a uniform distribution of sums and their attempt to fix the Workings box correctly. It was only after some strong suggestions from Pratt that the girls withdrew the extra 1+1 and inserted 1+3. In my view, the researcher gave too much away and did not allow the girls to resolve the issues for themselves. I also think that a bar graph display of the data, in addition to the pie chart, would have been helpful so that the students could see the part-part relationships. After 1,000 trials with the correct sample space, the girls noted that 4 was an easier sum to obtain, while 2 and 6 were harder. Had they overcome their equiprobability bias?

No. After the spinners, the girls went on to the two-dice gadget. The following conversation ensued (Pratt, 2000, p. 618):

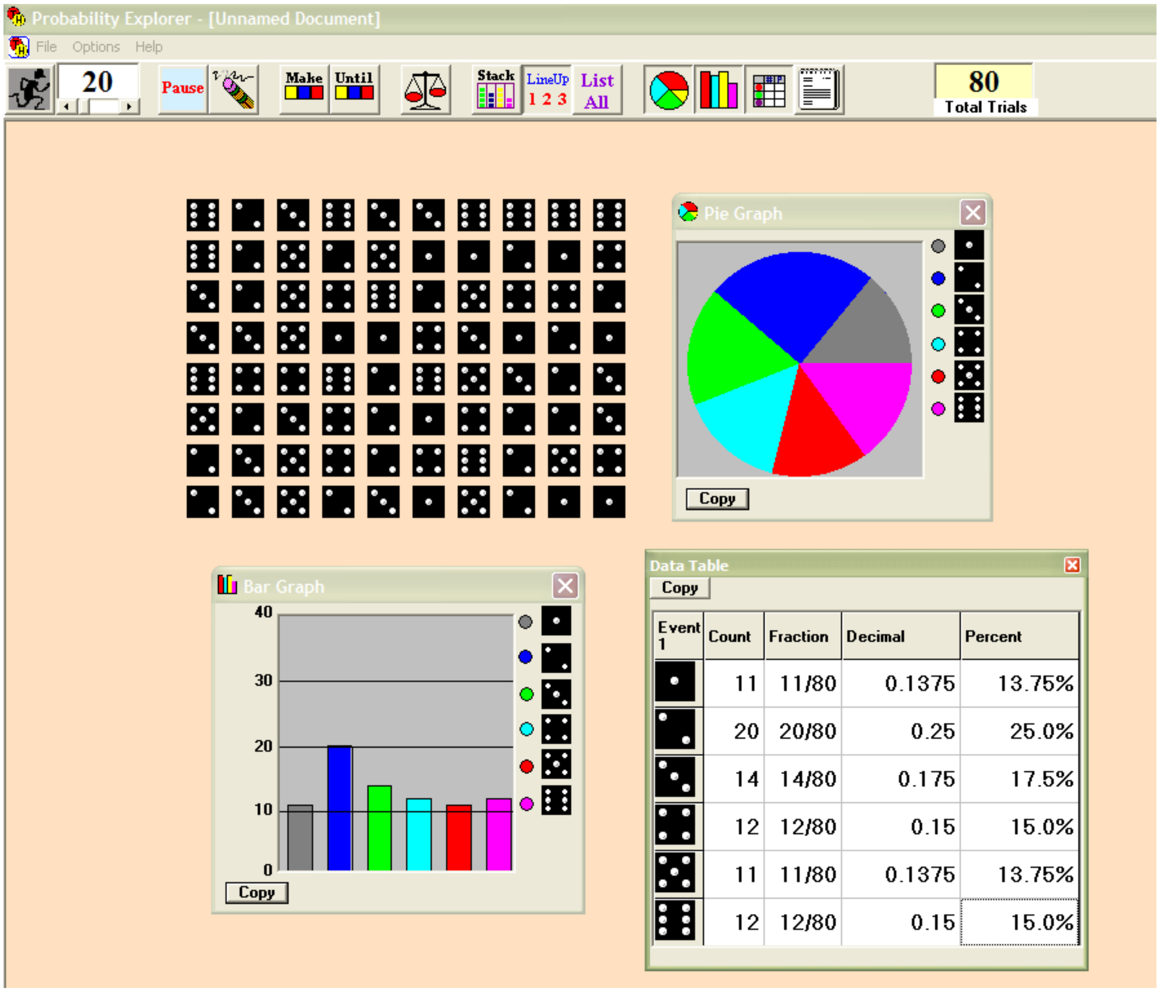
> *Researcher*: If we were shaking two real dice, do you think all the totals you could get are just as easy, just as hard, or do you think some totals are easier than others, harder than others?
> *Rebecca:* Fifty-fifty chance of getting them. [Anne agreed.]
> *Researcher:* So you think they are all about the same chance?
> *Both:* Yes.

As with the two-spinners gadget, the Workings box was missing several outcomes. After 1,000 trials, sums of 7 and 11 had not occurred once. With some coaching from the researcher, "aimed at helping them to be systematic" (p. 619), the girls completed the sample space in the Workings box. Again, I believe that the researcher's interference tainted any conclusions that might be drawn from this study.

In my opinion, Chance-Maker shows potential for creating useful activities in which students can confront their misconceptions and possibly resolve them. I like the possibility of editing the sample space. Two things that would have made this a better study are less interference by the researcher and the addition of bar charts to the graphical display.

Probability Explorer is another interactive program written by Stohl (1999-2005). Like Chance-Maker, this program simulates a number of random events. The standard events include flipping coins, tossing dice, and choosing marbles from a bag. Students also have the option to create other simulations with a number of available icons. The outcomes can be weighted so that they are not necessarily equally likely. Figure 3 shows a screen shot displaying 80 tosses of a fair die. A bar graph, pie graph, and data table are available to display the results.

Figure 3 – *A screen shot of Probability Explorer.*



In one study, Stohl and Tarr (2002) used Probability Explorer as the centerpiece of a 12-day instructional unit with a class of 23 sixth-grade students in an urban middle school. Students spent two days working in pairs on each of six tasks designed around the concepts of fairness, randomness, sampling, variation, and sample size. The researchers' focus was to explore how the students might come to understand the link between theoretical probability, experimental probability, and sample size, and how they might use the computer data to justify their judgments.

Stohl and Tarr (2002) presented a case-study analysis of two boys in the class. Their data sources included video recordings of the computer monitor, audio recordings of the students' conversation, written class work, and homework. For the final task, *Schoolopoly*, the students were asked to investigate whether or not a die was fair. The particular "die" that the case study boys were given was weighted 2-3-2-3-2-3.

Initially the boys believed the die to be fair. They simulated varying numbers of tosses: 51, 500, 50, and 300. Though they noted that the distribution was not uniform, they concluded, "Every single thing doesn't have to be even, man, it's the luck. They are pretty much close" (p. 332). However, a run of 1,500 trials gave the boys pause to consider that the die might not be fair. They concluded, on the basis of comparing the relative frequencies of different outcomes, that the die was unfair.

In preparing a poster to present their findings to the class, the boys used their original, small-sample data as an example of how the results of small samples can lead to incorrect inferences. The researchers concluded that "The fact that they used their initial hypothesis as a counterexample demonstrated they understood the interplay between empirical and theoretical probability and that sample size was the connecting link between these concepts" (p. 334).

I think that Probability Explorer shows a good deal of promise for students. The tasks that Stohl and Tarr developed are conceptually rich and hold the students' interest.

In recent years, more attention has been paid to the assessment of probabilistic understanding. In the next section, I will discuss a framework that has been developed to assess probabilistic reasoning.

2.2.4 Assessment

In much of mathematics instruction and assessment, too much attention is paid algorithms and procedural knowledge. "Instruction and assessment in statistics and probability have frequently constituted an extreme example of a focus on procedures to the neglect of underlying concepts and big ideas" (Metz, 1997, p. 1). Assessing probabilistic reasoning is especially problematic because students can have multiple and contradictory beliefs about the same chance situation (Konold, 1995).

Jones, Langrall, Thornton, and Mogill (1997) developed a framework that serves as a rubric to assess probabilistic thinking in young children. The framework was developed and validated through interviews and teaching experiments with third-grade students at a university laboratory school. The original framework describes four levels of thinking across the constructs of sample space, theoretical probability of an event, probability comparisons, and conditional probability. Subsequently the constructs of experimental probability of an event and independence were added (Jones, Thornton et al., 1999), and the framework was tested and validated with students through the middle grades.

For the validation process, the researchers "sought to (a) refine the initial descriptions of the four levels of probabilistic thinking; (b) examine the profiles and consistency of children's thinking levels over the . . . constructions prior to and following exposure to an instructional program; and (c) illuminate the distinguishing characteristics of each level within the framework" (Jones et al., 1997, p. 107).

The assessment framework is reproduced on the following page.

Table 2 - *A framework for describing students' probabilistic reasoning.*
(Jones, Thornton et al., 1999, p. 150)

CONSTRUCT	Level 1 Subjective	Level 2 Transitional	Level 3 Informal Quantitative	Level 4 Numerical
SAMPLE SPACE	●lists an incomplete set of outcomes for a one-stage experiment	●lists a complete set of outcomes for a one-stage experiment and *sometimes* for a two-stage experiment.	●consistently lists the outcomes of a two-stage experiment using a partially generative strategy	●adopts and applies a generative strategy that enables a complete listing of the outcomes for two-and three-stage cases
EXPERIMENTAL PROBABILITY OF AN EVENT	●regards data from random experiments as irrelevant and uses subjective judgments to determine the most or least likely event ●indicates little or no awareness of any relationship between experimental and theoretical probabilities	●puts too much faith in small samples of experimental data when determining the most or least likely event; believes that any sample should be *representative* of the parent population. ●may revert to subjective judgments when experimental data conflict with preconceived notions.	●begins to recognize that more extensive sampling is needed for determining the event that is most or least likely. ●recognizes when a sample of trials produces an experimental probability that is markedly different from the theoretical probability.	●collects appropriate data to determine a numerical value for the experimental probability. ●recognizes that the experimental probability determined from a large sample of trials approximates the theoretical probability. ●can identify situations in which the probability of an event can be determined only experimentally.
THEORETICAL PROBABILITY OF AN EVENT	●predicts most/least likely event on the basis of subjective judgments ●recognizes *certain* and *impossible* events	●predicts most/least likely event on the basis of quantitative judgments but may revert to subjective judgments	●predicts most/least likely events on the basis of quantitative judgments. ●uses numbers informally to compare probabilities	●predicts most/least likely events for one- and simple two-stage experiments. ●assigns a numerical probability to an event (either a real probability or a form of odds)
PROBABILITY COMPARISIONS	●uses subjective judgments to compare the probabilities of an event in two different sample spaces. ●cannot distinguish "fair" probability situations from "unfair" ones.	●makes probability comparisons on the basis of quantitative judgments – not always correctly. ●begins to distinguish "fair" probability situations from "unfair" ones.	●uses valid quantitative reasoning to explain comparisons and invents own way of expressing the probabilities. ● uses quantitative reasoning to distinguish "fair" and "unfair" probability situations.	●assigns numerical probability and makes a valid comparison.
CONDITIONAL PROBABILITY	●following one trial of a one-stage experiment, does not always give a complete listing of possible outcomes for the second trial. ●uses subjective reasoning in interpreting with and without replacement situations.	●recognizes that the probabilities of *some* events changes in a without replacement situation; however, recognition is incomplete and is usually restricted to events that have previously occurred	●recognizes that the probability of all events changes in a without replacement situation. ●can quantify changing probabilities in a without replacement situation.	●assigns numerical probabilities in with replacement and without replacement situations. ●uses numerical reasoning to compare the probability of events before and after each trial in with replacement and without replacement situations.
INDEPENDENCE	●has a predisposition to consider that consecutive events are always related. ● has a pervasive belief that one can control the outcome of an experiment.	●begins to recognize that consecutive events may be related or unrelated. ● uses the *distribution* of outcomes from previous trials to predict the next outcome (representativeness).	● can differentiate independent and dependent events in with and without replacement situations. ● may revert to strategies based on representativeness.	● uses numerical probabilities to distinguish independent and dependent events.

The four levels of thinking represent a continuum from subjective to numerical. Students in level 1 have a limited perception of probability. Rather than considering all possible outcomes of a chance event, they are inclined to focus on the most likely outcome, often applying subjective reasons for its occurrence, such as, "I think 6 will come up because it's my favorite number." Level 2 students make weak connections between sample space and probability and they may revert to subjective thinking. These students are prone to the representativeness misconception. Students at level 3 use quantitative reasoning and recognize the variation among samples. At level 4, students are able to enumerate a sample space, understand the Law of Large Numbers, and use numerical reasoning in all chance situations.

The researchers who developed this framework view it as a vehicle to "nurture" probabilistic reasoning. Teachers can use it in planning lessons by constructing tasks that fit their students' level of reasoning. During instruction, teachers might use the framework "as a filter for analyzing and classifying students' oral and written responses" (Jones, Thornton et al., 1999, p. 153). It may also be used to evaluate the effects of instruction, as teachers can measure students' growth from one level to the next.

2.2.5 Directions for Future Research

An important theme for future research is *connections* (Jones, 2005; Powell & Wilkins, 2006). Some open questions are:

- How do students make connections between experimental and theoretical probability?

- How do students make connections between probability and statistical concepts such as variation, sample size, sampling distributions, and inference?
- What classroom practices are effective in forging these connections?
- What is the role of technology in facilitating these connections?

Research that traces individual and group thinking during instruction will give insight into the evolution of probabilistic intuitions and misconceptions.

Research on teachers' content knowledge and pedagogical knowledge in this area must be explored, along with the effects of professional development (Jones, 2005).

The learning and teaching of probability is a complex process that, despite a substantial research base, is not well understood. Now that probability is an important part of every student's education, we must strive to make it understood.

CHAPTER 3 – METHODOLOGY

As the purpose of this study is to examine the development of probabilistic thought from students' perspective and provide a rich description of their mathematical behavior, a case study design has been employed. A case study is an examination of a bounded system over a specific period of time through the use of detailed data collected from a variety of sources (Creswell, 1998). This study is bounded over the duration of the Rutgers *Informal Mathematics Learning* project (IML), from September, 2003, to June, 2006.

3.1 Setting

The IML project took place in Plainfield, NJ. Plainfield is an urban, economically depressed city of about 48,000 in central New Jersey. In 1997, the N.J. Supreme Court identified the Plainfield K-12 school district as one of 30 Abbott districts in the state, in need of state funding to improve its educational programs and outcomes. Plainfield's graduation rate was 81.1% in 2006, compared to the state average of 92.5%. More than half of the graduates achieved their diplomas by way of an alternative exam. At the time of this study, the percentages of students deemed proficient in mathematics according to statewide tests were considerably below the state averages, as shown in Table 3. (Education Law Center, 2006)

Table 3 - *Percentages of students passing standardized mathematics exams.*

Grade Level	Plainfield	New Jersey
4	52.0%	80.2%
8	32.7%	61.2%
11	34.6%	75.7%

In the most recent report, only 22.2% of eighth graders passed the standardized mathematics exam (Education Law Center, 2008).

At the time of this study, the student population was 99% minority, with 61.8% African American and 37.2% Latino. Sixty-six percent of Plainfield students were eligible for free or reduced-price lunch. Statewide, this figure was 26.1%. (Education Law Center, 2006)

The IML project was a three-year venture that began in the fall of 2003. With NSF funding[3], a team from Rutgers University provided after-school mathematics enrichment classes several times during the school year and for two weeks during the summers of 2004 and 2005. The school-year sessions took place in a classroom at Hubbard Middle School, one of two middle schools in Plainfield, serving grades 6, 7, and 8.

The IML project sought to provide an enrichment experience for students that is unlike the typical mathematics classroom. The project was designed to provide a nurturing environment in which students were invited to work together on challenging, open-ended tasks, free of the school constraints and stressors of grading and testing. Students in IML were encouraged to discuss their ideas and to offer arguments to justify their conjectures. Ideas were not judged as correct or incorrect, but were open for discussion, review, and revision. The mathematical topics that were explored in these sessions were not part of the grade-level curriculum, so that students' work would not be influenced by classroom instruction. There were three mathematical content strands for the project: combinatorics, probability, and algebraic thinking. My study focuses on a

[3] National Science Foundation Grant REC0309062, directed by C. A. Maher, A. B. Powell, and K. H. Weber.

series of lessons and interviews in the probability strand. The timetable for these sessions is depicted in Table 4. The specific tasks are provided in Appendix A.

Table 4 – *IML probability sessions and interviews. The shaded sessions will be analyzed in this study*.

Grade 6 2003 - 2004			Grade 7 2004 – 2005			Grade 8 2005 - 2006	
fall	spring	summer	fall	spring	summer	fall	
IML begins	3 sessions	8 sessions		4 sessions	4 sessions		2 sessions
	dice games	Probability Explorer – experimental probability		dice games	Probability Explorer - experimental probability	in-class unit on probability	Prob. Explorer revisited

3.2 Sample

Sixth-grade students at Hubbard School were invited to participate in the IML project, and all who applied were accepted. There were initially 28 sixth-grade students in the first cohort. That number varied as several students dropped out or moved away and a few new students joined. I purposefully chose five students, three girls and two boys, for my case study sample who consistently attended the IML sessions throughout the prior two years and who were present for the summer sessions on probability. Their attendance records for the IML probability sessions are documented in Appendix B. All of these students are articulate and provide a good window into their thinking as they discuss their solutions to problems.

My analysis also includes other students who worked in groups or pairs with any of the focus students.

3.3 Data Collection

In keeping with a case study design, several methods were used to collect data to document the students' mathematical behavior. These include observations using videotape, documents, and interviews.

3.3.1 Observations

Though I was present at many of the IML sessions, I did not attend all of the probability lessons. However, there are videotape records of all the sessions. Cameras positioned around the room captured the discourse and work of students working in small groups, while a roving camera captured whole-class discussions. All of the video data have been digitized and stored on CD-ROMs.

3.3.2 Documents

Throughout the course of the project students were encouraged to document their mathematical thinking through the creation of papers and overhead transparencies that put forth their arguments and provided evidence for them. These papers have been collected and digitized, and I have integrated key documents into the transcript.

3.3.3 Interviews

Some of the focus students were interviewed by members of the research team outside of the classroom. The interviewers discussed with the students the same tasks that were used in the classroom sessions. The interview format provided an opportunity to probe the students' reasoning in greater depth. Two cameras were used for the interviews in grade 6: one focused on the students and the other on their written work. Again, all the video data has been digitized and stored on CD-ROMs.

3.4 Data Analysis

3.4.1 Video analysis

Much of the data for this study was recorded on videotape and then digitized and stored on compact discs. In order to describe the problem-solving strategies, ways of thinking, and development of ideas, my analysis of the video data of IML probability sessions was adapted from the model for studying the development of mathematical thinking proposed by Powell, Francisco, and Maher (2003). This model includes seven interacting, non-linear steps which are described below.

The first step in the analysis is to view the videos several times to become acquainted with the data. Though the model suggests the first viewing take place without making notes, I felt the need to jot down ideas from the start. A few of the videos have been viewed and described by other graduate students. After my second viewing of the video data, I read the descriptions to confirm my own impressions and to see if there were any areas where my views were at odds with what others had described. I did not note any areas of disagreement.

Next, I synthesized all that I watched and read as I wrote more detailed descriptions of the video data broken down into short time intervals, generally about five minutes each. (For the interviews, I've found that a full transcription is necessary, and so this step was skipped.) I also referred to any documents that were created during these sessions to obtain a complete picture of the students' mathematical behavior. At this stage I had a descriptive summary of the students' mathematical activity and a good sense of what they were doing. The repeated viewings of the video discs, consultation with others' descriptions, and examination of written work provides triangulation in my analysis.

The next step in the analysis is to identify *critical events*, which are significant moments in the students' mathematical behavior. The identification of critical events is key to my study. A critical event shows a significant change in comprehension, a moment of insight, or a cognitive obstacle (Powell et al., 2003). In playing the dice games, for example, a critical event may be a student's expressed realization that a game is unfair, a decision about what evidence to use in support of his or her inferences, or the realization that all sums are not equally likely.

Maher (2002) described the role of critical events in data analysis as follows:

> The analysis begins with the identification of critical events. The mathematical content of each critical event is identified and described, taking into account the context in which the event appears, the identifiable student strategies and/or heuristics employed, earlier evidence for the origin of the idea, and subsequent mathematical developments that follow its emergence. (p. 35)

For each critical event that was identified, a timeline was established in which events leading up to and following the critical event were examined. In this way, the flow of ideas can be described and the parts of a storyline will begin to emerge. The critical events provide a framework for the bigger picture of what occurred as mathematical ideas developed.

Once critical events were identified, I followed the protocol of Powell et al. (2003) and transcribed the critical-event timelines. My transcriptions include spoken words, gestures, and inscriptions. In many cases actual student work or a reproduction of it is inserted into the transcript. Once my transcriptions were complete, the videos were viewed and the transcripts verified by graduate students. The transcripts were used in coding, and parts of them appear in my final narrative in order to accurately represent the students' development.

3.4.2 Coding

Following the transcription of critical events, I coded each one for themes related to probabilistic understanding. Preliminarily, I expected to code for misconceptions and for levels of probabilistic reasoning and of reasoning about fairness as identified in the literature. After viewing the videos, I realized that many of my preliminary codes were not a good fit for the data, and so I followed the advice of Charmaz (2006) and used a grounded-theory approach. Initially I identified broad themes in the data, and within these themes I created codes for subcategories that I observed. I was fortunate to have the assistance of Anoop Ahluwalia, a fellow doctoral student, who helped me to verify and refine the codes over several iterations. The coding scheme is presented below.

Table 5 - *Coding scheme*

The notion of chance (CD)	
• Outcomes can be controlled in some way	CD-D
• All outcomes in the sample space are possible	CD-A
• "Lucky" outcomes are more likely (subjective reasons)	CD-L
• Some outcomes are more likely (objective reasons)	CD-M
• Representativeness (any sample will mirror population)	CD-R
• Outcome approach (focus on predicting a single outcome)	CD-O
Determining fairness/unfairness (F)	
• *A priori*	
○ A player has more possible outcomes (unfair)	F-B-M
○ Lists sample space and counts outcomes for each player	F-SS
• *A posteriori*	
○ A player has more frequent outcomes after n rolls (unfair)	F-A-F(n)
○ Game is fair because either player can win (after playing n games)	F-A-W(n)
Sample Space (SS)	
• Complete sample space showing all possible outcomes	SS-C
• Partial sample space, omitting permutations of sums	SS-P

• Incomplete or incorrect sample space, omitting some combinations as well as permutations, or containing some errors	SS-I
Making a game fair (MF)	
• Number of outcomes believed to be even	
○ Gives both players the same number of events (not necessarily equally likely events)	MF-S
○ Divides number of simple outcomes in half and gives each player that number of outcomes	MF-H
• Number of outcomes believed to be odd	
○ Eliminates one outcome and divides the others	MF-E
○ Divides the odd outcome so that it goes to Player A half the time and Player B the other half	MF-DO
• Weighs the outcomes to make expected point values equal	MF-W
• Game can be made fair in more than one way	MF-M
• Other	MF-O
Does color or order of dice matter when considering the sum?	
• Color doesn't matter	C-N
• Color matters	C-Y
• Order doesn't matter	O-N
• Order matters	O-Y
• It's the same concept either way (whether or not color or order is considered)	C/O-S
Probability comparisons (PC)	
• Non-numerical (as in: A has more than B)	PC-N
• Attends to subsets of the sample space (as in: A has 4, B has 6) or the numbers of combinations for each sum (as in: A's numbers have 1 combination, B's numbers have 2)	PC-S
• Part-to-whole (fractions)	PC-W
Theoretical probability (TP)	
• x/n based on correct sample space	TP-C
• x/n based on partial sample space	TP-P
• x/n based on incomplete or incorrect sample space	TP-I
• x/n based on equiprobability assumption	TP-E
• 1/x based on x ways for event to occur	TP-1/x
Experimental probability (EP)	
• relative frequency based on n trials	EP (n)
• availability (based on recall)	EP-A
Connecting experimental and theoretical probability (ET)	
• expresses belief that frequency of an event reflects its likelihood	ET-F
• experimental data support theoretical ideas	ET-S
• experimental data contradict theoretical ideas	ET-C
• unsure or makes no connection	ET-U

3.4.3 Reporting Results

Once the coding was complete, there was an enormous amount of data to process. I looked for the major themes within each activity and developed tables charting the mathematical activity throughout the sessions. When tables were not suitable, I wrote analytical memos in which I collected all the critical events related to a certain concept. For each student individually, I organized the critical events to reconstruct his or her experiences during the probability sessions into a cohesive storyline that was developed into the written narrative. Key parts of the transcripts are included in the narrative so that the reader can "hear" the students' voices.

3.5 Validity

Ensuring the trustworthiness of this study is of great importance to me, and it is inherent in the procedures for data collection and analysis. Creswell (1998) recommends that qualitative researchers use at least two verification procedures to guarantee the credibility of their conclusions. The verification procedures that I employed are (1) the triangulation of information through multiple sources of data (video recordings, written work, and others' descriptions) in order to corroborate evidence in support of themes I put forth in the narrative, (2) my persistent observation in the field (having been with the IML project since its inception in 2003) in order to know the participants and have a sense of the culture of the IML sessions, (3) peer review, through transcript verification and collaborative code building, and (4) rich, thick description of the data that will allow readers to establish recognizability and transferability.

CHAPTER 4 - RESULTS

The purpose of this study is to trace the development of probabilistic reasoning in urban middle-school students who attended the IML after-school program from September, 2003, to June, 2006. There were three mathematical content strands in the IML project: combinatorics, probability, and algebraic thinking. The probability strand included after-school lessons and interviews in April and May of grades 6, 7, and 8, as well as one-to-two-week summer institutes in August following grades 6 and 7. This study focuses on the after-school sessions and interviews during the first two years: three sessions in April and May of 2004 and four sessions in May of 2005. In these sessions, students were presented with open-ended tasks intended to engage them in building ideas about chance by investigating dice games to determine whether or not they were fair, and to devise strategies to make the games fair. The tasks include existing successful tasks from previous research and new tasks that built upon them.

The research questions guiding the study are:

1. What understandings about probability (particularly fairness, sample space, probability of an event, probability comparisons) do the students exhibit?
2. How do these understandings change through the course of IML sessions?
3. What connections, if any, do the students make between experimental and theoretical probability?

In order to address these questions, transcripts of the video-taped after-school sessions and interviews, students' written work, and video-taped debriefing sessions were analyzed to trace the development of the probabilistic ideas mentioned above. Transcripts were coded using categories related to notions of chance, determining fairness, making a game fair, sample space, whether color or order of dice matters, probability comparisons, and experimental and theoretical probability.

The following sections are organized chronologically by tasks and separated into episodes that exhibit the various types of probabilistic reasoning. Numbers written in parentheses refer to specific lines in the transcript, Appendix D. When quoting the transcript, I use the following conventions: numerals, rather than words, are used for dice outcomes, an ellipsis within a quote indicates that the speaker paused or was interrupted, and an ellipsis inside brackets indicates that I have omitted a word or words to make the quote more readable – without changing its meaning. The names of researchers, graduate students, and teachers are omitted. Instead, these members of the research team are designated by the letters R, G, and T, respectively.

4.1 Probability Sessions and Interviews in Grade 6

4.1.1 Activity 1- A Game With One Die

R2 begins the first probability session by introducing the task, a game for two players. In this game, a single die is rolled. Player A gets a point if the die lands on 1, 2, 3, or 4, while Player B gets a point if the die lands on 5, or 6. The first player to get 10 points wins the game. [Note: The game favors Player A with a $\frac{2}{3}$ probability of winning a point and a probability $\sum_{k=0}^{9}\binom{k+9}{k}\left(\frac{1}{3}\right)^{k}\left(\frac{2}{3}\right)^{10} \approx .935$ of winning a game.]

R2 demonstrates how the game is to be played and tells the class to think about whether or not the game is fair. Various students call out their ideas, some claiming that the game is not fair and others claiming that it is (63). After the whole-class discussion, students are separated into five groups. Within the groups, students will work in pairs. Each group has a researcher or graduate student assigned to observe them and support their being on task.

4.1.1.1 Is the One Die Game Fair?

All of the students in this study recognized that the game is unfair because Player A has more outcomes than Player B. Some of their comments follow.

Jerel: "We already knew it was unfair because Player A had more choices to choose from than Player B" (143-144).

Justina: "Most likely the die was going to drop on the um the numbers that Player A had because Player A had so many, and Player B didn't have that many numbers. So the die wasn't going to really drop on those, that little amount of numbers" (2317-2320).

Danielle: "It's not fair because the way the points is like set up" (860).

Danielle and Chanel: "'Cause it's like 1, 2, 3, 4, and then it's only 5 and 6" (864-865).

Kori: "I think it's unfair because Player A has 1, 2, 3, *and* [italics added to indicate vocal emphasis] 4 to get a point, and Player B only has 5 and 6. And I have . . . four opportunities to get a chance and you only have two (788-790).

4.1.1.2 If You Think the One Die Game Is Unfair, How Could You Change It to Make It Fair?

Most of the students create a fair game by giving three outcomes to each player. The exception is Kianja, who suggests making the game fair by weighting the outcomes:

> … and Player B, every time they got 5 or 6, they made it instead of one point, if they gave 'em two points, would it be even? (591-592). . . . I think it would work.

> It would be even because they have four points, right? They can have four points. Say the game goes up to four. If they get all of their numbers they have four. If you get both of your numbers, you have four, too. So it's a tie (599-602).

For the other students, assigning half the outcomes to each player makes the game fair:

Jerel: "Same amount of choices, like three and three" (154).

Chris: "You gotta have like three choices to win. Like Player A had to get 1, 2, or 3 to get a point, and then Player B had to get 4, 5, or 6 to get a point" (169-171).

Justina: "So we changed it. She got 1, 2, and 3, and I got 4, 5, and 6. And then we mixed it up. I went, I got 1, 3, and 5, and then she got 2, 4, and 6. And that's the way we made it even" (499-508).

Chanel: "It should be like 4, 5, 6 and 1, 2, 3" (864-866).

Kori: "I think that they should move 4 to Player B so it'd be even. 1, 2, and 3 for A and 4, 5, and 6 for B" (790-792).

4.1.1.3 Does It Matter Which Numbers Are Assigned to Each Player?

In making the game fair, initially the students do not express concern for how the outcomes are divided, as long as each player gets three outcomes. Justina and her partner try two different arrangements and are satisfied that both are fair: Justina says, "It still is fair because it doesn't really matter whether the number is high or low because the dice might still roll on the low numbers as much as it rolls on the high numbers" (527-529).

Chris and Jerel are interviewed about this activity the following week and are asked whether the game would still be fair if Player A got a point for 2, 3, or 4, and Player B for 1, 5, or 6 (1913-1914). Chris responds, "Yeah, that would've been fair, too.

Or if he got odd and even numbers" (1915-1916). Jerel explains that what makes the game fair is that each player gets three numbers (1921). Later during that same interview, however, Chris offers an explanation why certain sums of two dice are more likely than others, and he attributes this to the fact that the "large" numbers 4, 5, and 6 are more likely to appear than 1, 2, and 3 (2121-2128). When R2 points out the inconsistency between the large number – small number claim and the fair game that gives 1, 2, and 3 to Player A and 4, 5, and 6 to Player B, Jerel changes the rules: "I can make that a fair game. We give somebody 1, 4, and 5, and give the other person 2, 3, and 6. That'd be fair. You got two low numbers and one high number" (2264-2266).

Danielle also expresses the belief that the larger numbers on the die are more likely. After she and Chanel play two rounds of the revised game (1, 2, 3 against 4, 5, 6), both times with a close score, Chanel asserts that the new game is fair (991). Danielle disagrees: "Oh no. To me it wasn't because the 1, 2, 3 numbers, it's pro-, it's halfway impossible to get 'em sometimes" (992-993). Chanel counters, "Nuh-uh!" (994) . . . "It's 50-50, girl!" (1003).

Kori, who originally believed that 1, 2, 3 against 4, 5, 6 was a fair split, changes her opinion after playing this game. She invents the term *common rollers* to describe outcomes that are more likely. She says, "1, 2, 3, and 4 were common rollers. . . . And you will usually get 5 and 6 like, one out of a blue moon" (1337-1339). Her approach to making the game fair is to redistribute the outcomes so that each player has two common rollers: Player A gets **2**, **4**, and 6, and Player B gets **1**, **3**, and 5 (1239-1243). (The common rollers are indicated in boldface.)

4.1.1.4 How Are Experimental Data Used as Evidence in the One Die Game?

Chris judges the original game to be unfair. Asked whether the results of playing the game support his answer, Chris writes, "Yes, because Player A won 10 to 2" (151). He explains to R2 why the revised game (1, 2, 3, against 4, 5, 6) is fair: "'Cause, uh, the first game, since it was 10 to 2, that was a kill by eight points, but in the second game it was only a kill by four points" (1857-1858).

Later, when Chris and Jerel claim that large numbers are more likely, R2 suggests that they test their assertion. The boys roll a die 22 times and record the results: the "large" numbers come up 10 times and the "small" numbers 12 times (2223-2227). Though the data do not support their claim, Chris seems uneasy about renouncing it (2235-2245). Jerel is also uncertain, saying "I don't want to say nothin'" (2273). The interview concludes with the question unresolved and the boys agreeing to think more about it.

R4 asks Justina and Adanna how they knew that their revised game (1, 2, 3 against 4, 5, 6 or 1, 3, 5 against 2, 4, 6) was fair (2341). Adanna replies, "Because she won, then I won. Then she won, then I won" (2343). Justina adds, "It was even. It was even" (2344).

Chanel cites the close score of 10 to 8 in a game of 1, 2, 3 against 4, 5, 6 as evidence of fairness. "Because when it was fair um she got like close to mine" (956-957). Of three games played, Player A (1, 2, 3) wins the first two, and Chanel attributes this to luck (952). She declares the game "totally fair" (970). Her partner Danielle, however, is not sure. Contrary to the data, which have Player A in the lead, Danielle asserts that 1, 2 and 3 are "halfway impossible" to roll.

Kori and Nia express conviction, based on their data, that certain numbers are common rollers and others occur once in a blue moon. As they play a game with 2, 4, 6 against 1, 3, 5, Kori remarks, “ Yeah, this game is better [than 1, 2, 3 against 4, 5, 6]. It gives you a better chance of winning” (1295).   She cites the close score of 8 to 6 as evidence that this split is fair (1302-1303). Nia contrasts this to the 10 to 1 score of their first attempt at a fair game (1308), which they say is unfair.

4.1.1.5 Notions of Probability Expressed During the One Die Game

In an interview with Chris and Jerel, R2 elicits some thoughts about probability with regard to Activity 1. When Jerel asserts that there’s a “higher percentage” that the die will land on Player A’s numbers (1820), R2 asks whether the boys can say how likely it is for Player A to get a point. Both Chris and Jerel say yes (1828), and Chris explains, “The probability of getting is 4 out of 6, ‘cause there’s 6 numbers on the dice and he has 4 chances of getting it” (1832-1833).

4.1.1.6 What Might Happen in Repeated Trials of the One Die Game?

R2 asks Chris and Jerel which player they think would win the original game if it were played six times (1864-1865). They answer that Player A would win all (Jerel, 1871), or almost all (Chris, 1872) six games. If the game were played 60 times, Chris says that Player A would win most of the games (1876), while Jerel says 59 of the 60 games would be won by Player A (1878). If 100 games were played, Jerel thinks Player A would win 99 of them (1881).

Justina and Adanna, interviewed by R4, are also asked whether Player B could win any of six games (2501, 2503). Both girls agree that Player A would win every time (2504-2506). R4 asks, “even if you played a hundred times, you don’t think that Player B could ever win?” (2512-2513). The girls decide that Player B might win one or two games out of a hundred “’cause Player B only had two numbers, and Player A had four” (2514-2520).

Asked about their fair game, Jerel explains that “it’s a 50-50 chance of Player A or Player B winning” (1893-1894). If 100 fair games were played, Chris says the two players would win “probably 50 each” (1897), while Jerel says maybe 40 games for one player and 60 for the other (1899).

4.1.1.7 Summary of Activity 1

The students readily conclude that the player with more outcomes has the advantage and, with the exception of Kianja’s weighting scheme, determine that a fair game would give three outcomes to each player. There is not general agreement, however, about how the outcomes should be divided between the players. The initial consensus is that the assignment of any three outcomes to each player will make the game fair. Kori changes her mind when faced with experimental data that seem to indicate otherwise. Chris, Jerel, and Danielle decide, despite evidence to the contrary, that 1, 2 and 3 are less likely to occur than 4, 5, and 6. Perhaps they are relying on primary intuitions or using the availability heuristic.

Chris, Kori, and Nia use scores that are far apart as evidence that a game is unfair. Chris, Chanel and Nia use close scores to support their belief that a game is fair. Adanna and Justina note that Players A and B alternated winning the fair game. In some instances (992-993, 2234-2245), students disregard the data and tentatively hold on to unsubstantiated beliefs.

Chris and Jerel demonstrate an understanding of the probability of a simple event when they state that Player A has probability "4 out of 6" to win a point. They appear to use a combination of the outcome approach and the representativeness heuristic when judging the number of games either player might win in many repeated trials. In the unfair game, Jerel expects Player A to win *n-1* out of *n* times. This judgment seems to take the outcome approach – that Player A is expected to win the next game – and extend it to nearly all the trials. However in a fair game, where anything can happen, Jerel finds a 40-60 split to be reasonable. Justina and Adanna also use this combined heuristic in their judgment that Player B might win only once or twice in a hundred games.

4.1.2 Chris' Game

Upon completing the first activity, Chris and Jerel invent their own games. In Chris' game, two dice are rolled. Player A gets a point for rolling an odd sum, and Player B gets a point for an even sum (202-204). [Note: This is a fair game.] Chris and Jerel play the game, and Chris wins as Player A.

4.1.2.1 Is Chris'Game Fair?

G2 asks the boys if they believe the game is fair. Chris answers, "Yeah, 'cause it was 10 to 9" (213). Jerel adds, "Yeah, and because I was losing and . . . it wasn't like the first game where, like he, when I was Player A it wasn't like he, he couldn't come back or like I couldn't come back" (214-216).

G2 asks Chris and Jerel if they think that the number of chances for an odd roll or an even roll is the same, and both boys answer affirmatively (219). Chris explains that there are six even numbers and six odd numbers from 1 to 12 (220-221). When G2 points out that a sum of 1 can't be obtained with two dice, Jerel declares the game unfair and accuses Chris of cheating him (227-228). Chris points out that Jerel, as Player B, is able to roll each of the even sums, but Chris, as Player A, cannot roll a sum of 1 (232-234). Chris attributes his win to "skills" (246).

4.1.2.2 Summary of Chris' Game

Chris and Jerel both cite the close score as evidence that the game is fair. It is interesting that, for these boys, a score of 10 to 9 suggests that odd and even numbers are equally likely, but a score of 10 to 12 does not convince them that small numbers and large numbers are equally likely.

Chris and Jerel exhibit the equiprobability bias in their assertion that odd and even sums have the same chance because there are six of each. Jerel's accusation of cheating will recur throughout the IML sessions whenever he loses a game. Chris' boast about skills may reflect a deterministic view of dice outcomes, or he may simply be joking.

4.1.3 Activity 2- A Game With Two Dice

As each group completes the first activity, they are given a second game to analyze. In this game, two dice are rolled. If the sum is 2, 3, 4, 10, 11, or 12, Player A gets a point. Player B gets a point for a sum of 5, 6, 7, 8, or 9. The first player to get 10 points wins the game. Again, the students are asked to decide whether or not the game is fair, to justify their answer, and to play the game to see whether the results support their answer. [Note: The game favors Player B with a $\frac{2}{3}$ probability of winning a point and a probability $\sum_{k=0}^{9}\binom{k+9}{k}\left(\frac{1}{3}\right)^{k}\left(\frac{2}{3}\right)^{10} \approx .935$ of winning a game.]

4.1.3.1 Is the Two Dice Game Fair?

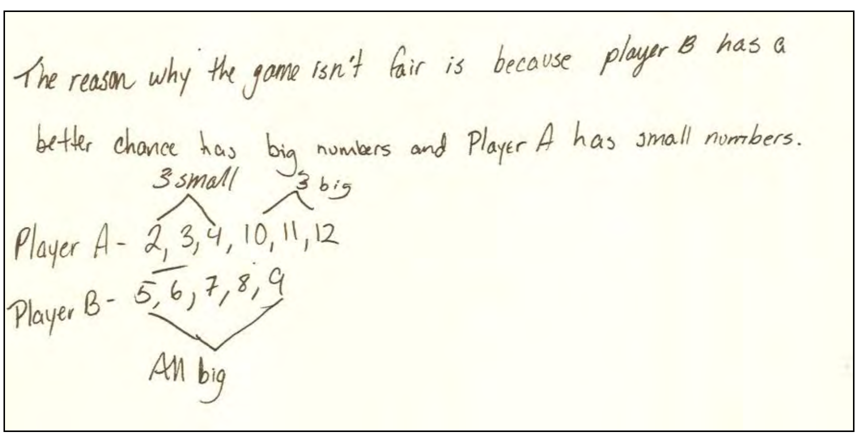
Justina and Adanna initially judge the game to be unfair, with Player A having the advantage, because Player A has more outcomes than Player B (653-658). They play one game and Player B (Adanna) wins with a score of 10 to 2 (695-697). Justina indicates that she wants to remain Player A for the next game (700-701), possibly believing that Player A is due to win. The following week, Justina questions her original prediction because Player B has won all the games (1416 – 1421). Justina creates the sample space with 21 outcomes and concludes, “Anyway, the um, amount of total ways for Player B was 13, and […] the amount for Player A was only 8. So this was not fair because um Player B had […] 13 ways, which was more than 8 ways Player A has” (1574-1578).

Chanel also starts out believing that the game is unfair, citing six chances for Player A and five for Player B (1043-1048). However, after playing one game with Player B winning 10 to 5, Chanel says, “I told you. I knew it was fair. I think it’s fair” (1102). Chanel explains that some numbers are “usual to pop up”, but 11 and 12 are not (1104-1107). Chanel and Danielle play the game a second time, and Player B wins again. Chanel concludes that the game is fair, saying, “But I do think it is fair for a sec.

Because, because she won" (1131). She goes on to explain that single-digit numbers are more likely than 11 or 12 (1132-1136). "But see, see we keep rolling it but 12 or 11 doesn't pop up that much" (1171-1172). Asked why 11 and 12 don't pop up much, Danielle says, "Because we don't roll it" (1174).

When Chris and Jerel begin this activity, Chris notes that "Player B got more chances, but I got, he got better ones to play," making a distinction between which player has small numbers or big numbers (1713-1714, 1716-1718). Jerel wants to play the game before deciding about fairness (1724-1725), while Chris says, "We gotta find out how many ways you can get each number" (1742-1743). In their interview with R2, both Chris and Jerel say that initially they thought the game was unfair (1944-1945). Chris explains, "'Cause Player A it had like, it had 3 small numbers, which are 2, 3, and 4, and you really can't get 'em" (1947-1948). Chris' written explanation is shown in Figure 4.

Figure 4. Chris' explanation of why the game is not fair.



Chris elaborates, "Because after we played the game we realized that um Player B had, since it had larger numbers it had more chance of getting 'em" (1984-1985).

Like Justina, Chris also lists the sample space with 21 outcomes and shows that Player B has 13 ways to get his numbers while Player A has 8 (1996-2002). He says that they expected Player A to win, "but after you played the game we saw that Player B started winning, so we just, um, thought that it was unfair and we figured it out" (2008-2010).

Kianja also constructs the sample space with 21 outcomes and determines that the game is unfair by comparing the probabilities for each player to get a point. She explains, "I added up all of the, I added up all of the combinations, right? The um number sentences, and I got 21. So, on this one it's 8 out of 21 chances for the Player B to win and there's 13 chances out of 21 for Player A to win" (619-622). Kianja was not filmed consistently for this task, so it is not known whether she had an initial opinion that she changed. Unlike other students who note 8 chances for Player A and 13 for B, Kianja compares the players' chances using part-to-whole relationships.

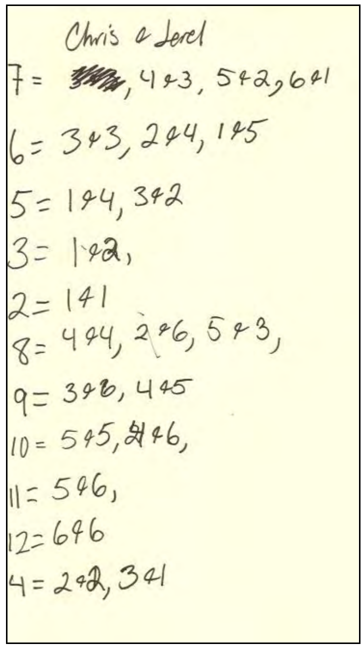
4.1.3.2 What Is the Sample Space for the Sum of Two Dice?

In this activity researchers encourage the students to record the outcome of each roll of the dice. In doing so, many students spontaneously begin to write down the number of ways to obtain each sum. Of the students studied, Chanel and Danielle are the only ones who do not write out the sample space.

All of the students who enumerate the sample space find 21, rather than 36 outcomes, as they do not consider symmetric pairs as different events. However, the students do not all take the same approach.

Chris and Jerel's sample space is shown in Figure5. The sums are written in no particular order, with Player A's and Player B's numbers mixed together. The final entry for 4 was written during the interview with R2.

Figure 5. Chris and Jerel's sample space for the sum of two dice.

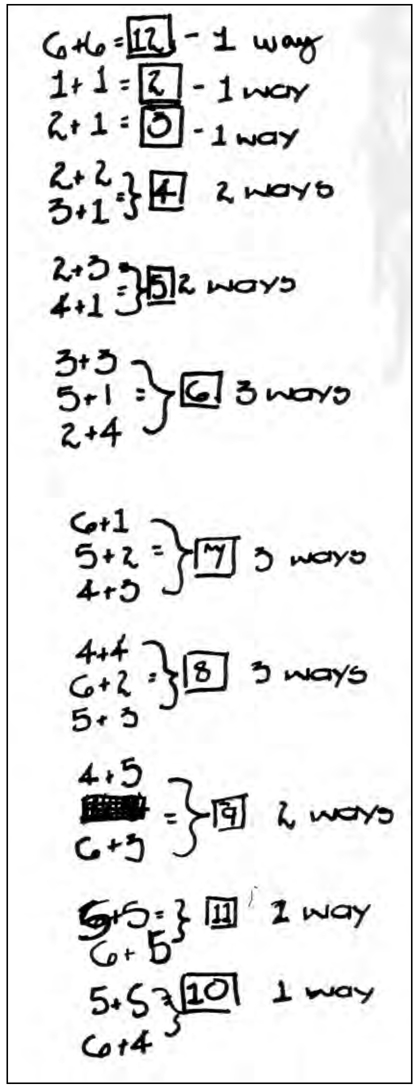

Chris & Jerel
7 = , 4+3, 5+2, 6+1
6 = 3+3, 2+4, 1+5
5 = 1+4, 3+2
3 = 1+2,
2 = 1+1
8 = 4+4, 2+6, 5+3,
9 = 3+6, 4+5
10 = 5+5, 4+6,
11 = 5+6,
12 = 6+6
4 = 2+2, 3+1

Justina's sample space, reproduced in Figure 7, emphasizes the number of ways to obtain each sum. Her sums are also not written in any particular order. Adanna summarizes Justina's sample space by partitioning it according to the number of ways each sum can be formed (2464-2465). Figure 6 is a reproduction of Adanna's chart, which indicates that the sums 2, 3, 11, and 12 can each be obtained one way; 4, 5, 9, and 10 can each be obtained two ways; and 6, 7, and 8 can each be obtained three ways.

Figure 6. Reproduction of Adanna's chart showing the number of ways to obtain each sum.

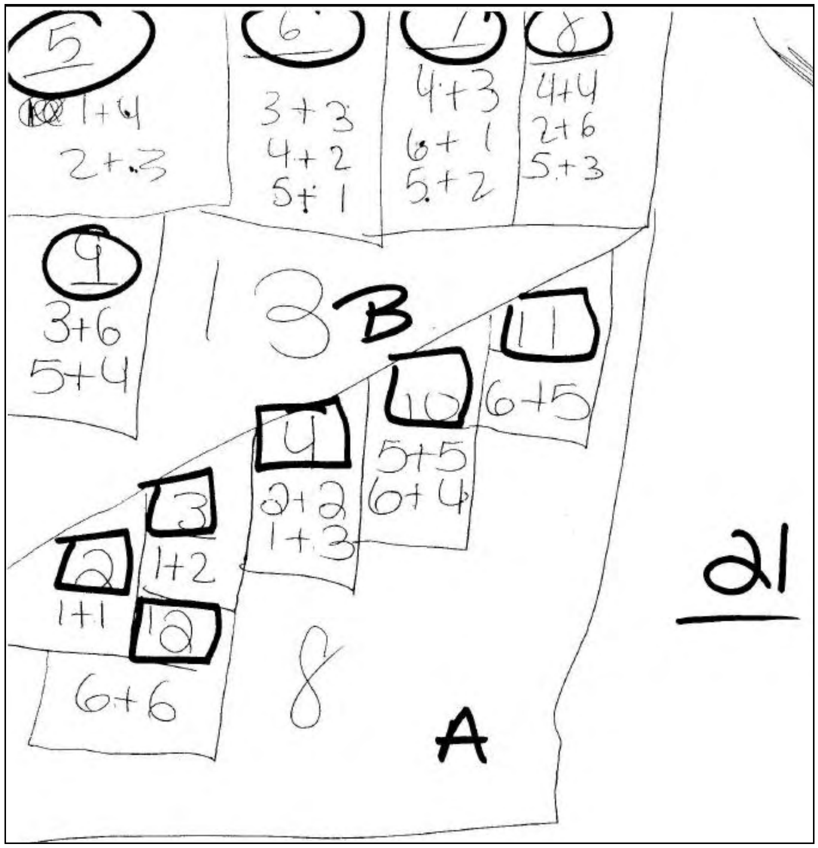
1	2	3
2 | 3 | 11| 12|	4 | 5 | 9 | 10 |	6 7 8

Figure 7. Justina's sample space for the sum of two dice.



Kianja separates her sample space into the outcomes favoring Player A and the outcomes favoring Player B. She writes the total number of outcomes for each player, as well as the total number of outcomes in the sample space (see Figure 8). On a separate paper she writes, "$\frac{13}{21}$ probability of winning" for Player B and "$\frac{8}{21}$ probability of winning" for Player A.

Figure 8. Kianja's sample space for the sum of two dice.



4.1.3.3 If You Think the Two Dice Game Is Unfair, How Could You Change It to Make It Fair?

Justina is the only student in this group who is filmed creating a fair game. She counts 21 outcomes in all, "but 21 is an odd number and I can't get, um I can't make it even with an odd number because this is dice, and the dice doesn't have one-half on it. Okay?" (1580-1583). Justina explains that she took away the sum of 12, now leaving Player A with seven outcomes comprising five sums, and Player B with thirteen outcomes comprising five sums. With a total of 20 outcomes, Justina gives each player 10 outcomes to make the game fair (1590-1597). In order to distribute 10 outcomes to each player, Justina makes a chart that shows the number of ways to obtain each sum. Her notations appear as follows: (1631-1632)

Figure 9. Reproduction of Justina's notations.

A:	$3^{(8)}$	$2^{(10)}$	$1^{(11)}$	$3^{(1)}$	$6^{(3)}$
B:	$3^{(7)}$	$2^{(9)}$	$1^{(2)}$	$4^{(2)}$	$5^{(2)}$

The number in parentheses is meant to be the dice sum, and the number before it is the number of ways to obtain that sum. As Justina explains her notation to R4, she realizes that she reversed the notation for $3^{(1)}$, $4^{(2)}$, $6^{(3)}$, and $5^{(2)}$ (1666-1687).

Justina explains, "I was just trying to even it out and decide which numbers should go to um different players" (2560-2562). "And then I started mixing up the numbers a little in order to get tens for both of us" (2584-2585). Her fair game gives Player A a point for 3, 6, 8, 10, and 11 (2604). Player B gets a point for 2, 4, 5, 7, and 9 (2607). Neither player scores with a roll of 12. [Note: Assuming Justina's sample space with 21 outcomes, this scheme gives 10 chances to each player. In actuality, P(A's point) $= \frac{17}{36}$ and P(B's point) $= \frac{18}{36}$. If Player A were also given a point for rolling 12, the game would be fair.]

4.1.3.4 How Are Experimental Data Used as Evidence in the Two Dice Game?

After playing the game, Adanna notes that 11 and 12 appear infrequently, while 2, 3, and 4 are "hard to get." She says that the sums 5 through 10 come up most often (1431-1433). Justina explains that Adanna's observations are consistent with the sample space: "those numbers that she's talkin' about is 5, 6, 7, um they have more, um many more ways to get them than the other ones do, like 11, is only one way to get 11. So you're really not likely to get that as much as you would, say, 6" (1521-1524).

Justina also uses experimental data to confirm that the game she devised is fair. In an interview, R4 asks Justina, "How many times do you think you need to play the game to test whether it's fair or not?" (2664-2665). Justina replies, "At least twice" (2666). She indicates that she's not quite sure that her game is fair because, although she gave the same number of outcomes to each player, the game "went from Player B always winning to Player A always winning" (2669-2670). As she and Adanna play the game again, Justina remarks on the close score, 3 to 3, as evidence that the game is fair (2679). When Player B wins the game, R4 asks whether the girls think it's fair. Justina answers, "Yeah, I do, because um at first A won, and then now B won" (2699-2700).

R4 asks Justina and Adanna what sum they would choose in a sudden death game in which winning depends on one roll of the dice (2773-2776). Both girls refer to their data and choose 6 because it was the most frequent sum (2779-2780, 2783-2785). Asked to choose between 7 and 8, the girls pick 8 for the same reason – it was more frequent than 7 (2790, 2804). Neither girl refers to the sample space to answer these questions; their sample space shows 6, 7, and 8 as equally likely.

Chris and Jerel observe that 7 appeared frequently in their games (2022, 2030). Asked why, Jerel explains, "Oh because it had a better chance, because it had three ways to get it" (2033).

4.1.3.5 Probability Comparisons With Two Dice

According to Chris and Jerel's sample space, a sum of 6 can also be obtained three ways, so R2 asks about this outcome (2059-2062). The boys acknowledge that 6 did not occur as often as 7 (2069, 2074). R2 probes, what might happen if the game were played 10 times – would 7 still occur more than 6? (2090-2091, 2096-2097). Together, Chris and Jerel say, "Seven would still come up more often" (2098). R2 expresses his confusion – if both sums have the same number of chances, why would 7 be more frequent? (2100, 2104-2106). Jerel quietly concedes that he "never thought about that" (2107), while Chris introduces his theory about large and small numbers (2110-2112). He explains that the "small" numbers on a die, 1, 2, and 3, are less likely than the "large" numbers, 4, 5 and 6 (2122-2128). Jerel concurs (2129). Since the pairs that make a sum of 6 contain two large numbers (3 and 3, 2 and **4**, 1 and **5**), while the pairs that make a 7 contain three large numbers (**4** and 3, **5** and 2, **6** and 1) [boldface added to indicate large numbers], Chris maintains that 7 is more likely than 6 (2135-2139). Asked how he knows that the larger numbers are more likely (2140-2141), Chris demonstrates by rolling a die (2145). In his first few rolls, the larger numbers prevail (2145-2146).

Chris and Jerel decide to corroborate Chris' theory by rolling a die 10 times (2157-2158). Losing track of the count, they roll 12 times and find that 1 came up five of the 12 times (2167). Chris and Jerel agree that so far the data do not support the large-small number theory (2174-2176). Jerel suggests that perhaps the outcome depends on whether or not they roll the die on a mat (2180-2181). As they roll the die 10 more times, Jerel whispers to Chris, "It's still low numbers" (2190). A roll of 1 that misses the mat is not counted (2189), but a roll of 5 off the mat is (2191). Even so, 4 of the rolls counted were small numbers and 6 were large. The combined results of 22 rolls show that the small numbers occurred 12 times and the large numbers, 10. Chris and Jerel are uncertain about how to reconcile this with their theory. Jerel concludes “that the big numbers don’t always show up” (2247).

4.1.3.6 What Might Happen in Repeated Trials of the Two Dice Game?

R4 asks Justina and Adanna about the game they analyzed and found 8 chances for Player A and 13 for Player B. R4 asks, if the game were played 10 times, would Player A ever win? (2722). Adanna says yes (2728), and Justina agrees, but “just once” (2729). Adanna explains that Player A did win the game once, but Player B won most of the time (2730).

R4 asks what might happen in 20 plays of the fair game (2734-2735). Adanna answers that it’s possible that each player would win 10 games, or that one player would win five games and the other, 15 (2740-2741). If the game were played 100 times, Justina says, “You can’t be sure about that. ‘Cause dice is dice and it just rolls on whatever number” (2751-2752). Adanna predicts that in 100 games the score might be 50 to 50 (2759); Justina adds that it could be 60 to 40 (2765). Justina seems to allow for much more variability in the outcomes of a fair game than an unfair game.

4.1.3.7 Summary of Activity 2

Many of the students begin this activity with the belief that Player A, with 6 sums to B’s 5 sums, is favored to win the game. Chris seems to question this assumption from the start, as he talks about numbers that are “better ones to play”, and Jerel is reluctant to decide about fairness before playing the game. Eventually, all the students in this study except Chanel provide evidence that the game is unfair in Player B's favor.

Chanel, like many others, begins with the belief that Player A is favored, but she is convinced after B wins the game twice that Player A's presumed advantage is neutralized by having two numbers that are difficult to get, 11 and 12. Chanel and her partner Danielle do not investigate why 11 and 12 are difficult: they simply observe that these numbers are not rolled often.

All of the other students studied create the sample space with 21 outcomes and conclude that Player B has a better chance to win. Kianja emphasizes the relative probabilities: $\frac{13}{21}$ to $\frac{8}{21}$. Chris and Jerel note that Player B has 13 chances while Player A has 8. They also focus on the idea that Player B's numbers are all "large" and therefore more likely, while only half of Player A's numbers are large. Justina and Adanna attend to the number of ways each sum can be obtained, and they partition the sample space accordingly.

Justina and Adanna are the only ones in this group who are recorded making a fair game. They do so in a reasonable way, first making the total number of outcomes even by omitting one outcome, and then giving half to each player. When they play the fair game, the fact that A and B each win a game is sufficient evidence for them that the game is indeed fair.

The reliance on a small number of trials in this instance and others, as in Activity 1, shows that the representativeness heuristic is readily used by these students. However, when faced with data that do not support their intuitions, Chris, Jerel, and Danielle remain somewhat dubious about the weight of experimental evidence.

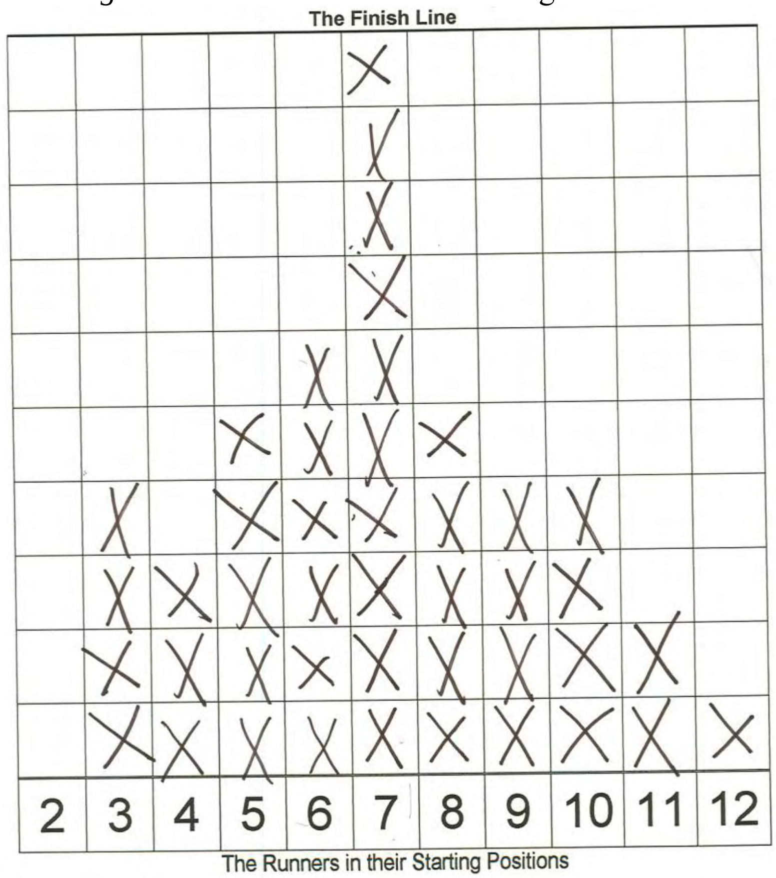
4.1.4 Racing Game With Two Dice

One more activity was performed off camera on the third and final day of the sixth-grade probability sessions. While the cameras were in use recording interviews, students were given the following task:

> Below, numbered 2 to 12, are the starting positions of eleven runners lined up for a race. Roll two dice. On each roll, the runner whose number equals the sum of the dice advances 1 square toward the finish line. The other runners do not advance forward. Continue to play the game until a runner reaches the finish line. The first to reach it wins. (1) Is this a fair game? Why or why not? If it is not fair, which runners are more likely to win and why? (2) Play the game with your partner. Do the results of playing the game support your prediction? Explain. (3) If you think the game is unfair, how could you change it so that it would be fair?

Though video of students playing this game was not obtained, documents indicate that Chris, Jerel, David, and Ian all played the game a few times. As they played, they marked an X in each square that a runner advanced. Upon completion of the game, their paper showed the distribution of outcomes. Figure 10 shows the results of one of Chris and David's games. It is typical of the others on file.

Figure 10. Chris and David's Racing Game sheet.



4.1.5 Summary of Grade 6 Results

The IML probability activities in grade 6 may be the first probability experiments encountered by these students, as the subject was not a part of their school curriculum at this grade level. However, the students arrive with their own ideas about chance. These ideas range from subjective intuitions that large numbers on a die are more likely than small numbers, to the correct application of *a priori* probability.

Most of the students appear to believe that outcomes having more chances to occur will occur more frequently. Their level of reasoning about experimental probability, though, is largely transitional, as they rely on small samples to make inferences and, in the cases of Danielle, Chris, and Jerel, revert to subjective reasoning when the data do not match their expectations.

Justina, Adanna, Chris, and Jerel were asked to consider what might happen in many repeated trials, and their answers are quite similar. In the case of an unfair game, these students assert that the player who has the advantage will win almost all games, even if a hundred are played. However, for a fair game, the students allow for much more variability in the outcomes, citing possible scores of 15 to 5 or 60 to 40. When two events are equiprobable, the perception is that anything can happen; but, when one event is more likely, that event is expected to prevail almost exclusively.

Like the students in other studies using these dice games, most IML students react to Player B's unexpected wins in game 2 by looking for an explanation in the sample space. Though no one uses all 36 equally likely outcomes, the partial sample space of 21 outcomes is sufficient to answer the question of fairness.

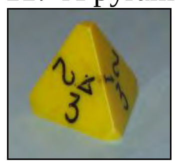
In the summer after grade 6, IML students attend a two-week institute in which probability experiments are performed using *Probability Explorer* software. The current study joins them the following year, in the spring of grade 7, for four more after-school sessions in which they analyze dice games using pyramidal dice.

4.2 Probability Sessions and Interviews in Grade 7

4.2.1 Activity 3- A Game With Two Pyramidal Dice

R2 opens the discussion by asking students to describe the difference between pyramidal dice and six-sided dice (2858). Students talk about the different shapes, numbers of faces, and colors of the dice (2859, 2861, 2863, 2866). Pyramidal dice are distributed to all the students, and R2 asks them to determine how to read the outcome of a roll (2884). Since each face of the die shows three numbers from the set {1, 2, 3, 4} (see Figure 11), the answer is not obvious to everyone.

Figure 11. A pyramidal die.



Kianja quickly determines that "whatever's facing at the bottom" (2887) is the number to read. Dante and Ian, though, spend about five minutes deciding how to read the dice outcomes with intermittent help from R2 (3463-3523, 3558-3581). R4 helps Chanel by demonstrating that when the die lands, the same number appears on the bottom of the three upright faces (3552-3553). Once all the students are confident about how to read the dice, R2 introduces the task, a game for two players.

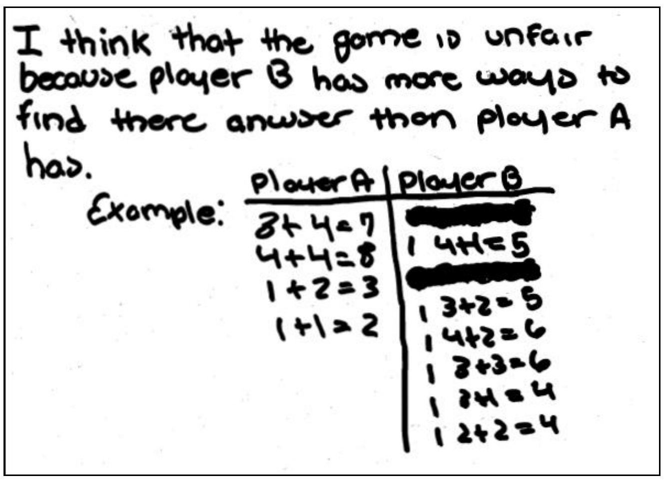
In this game, two pyramidal dice are rolled. If the sum is 2, 3, 7, or 8, Player A gets a point. Player B gets a point for a sum of 4, 5, or 6. The first player to get 10 points wins the game. As before, students are asked to determine whether or not the game is fair and to justify their answers. [Note: The game favors Player B with a $\frac{5}{8}$ probability of winning a point and a probability $\sum_{k=0}^{9}\binom{k+9}{k}\left(\frac{3}{8}\right)^{k}\left(\frac{5}{8}\right)^{10} \approx .869$ of winning a game.]

4.2.1.1 Is the Two Pyramidal Dice Game Fair?

Upon hearing the rules of the game, Dante immediately tell the class that, like last year's game, this one is unfair because Player A has more chances than Player B (2945-2953). Other students agree with Dante's assessment (2976-2981).

After playing the game a few times, Chanel comes to disagree with Dante’s claim that Player A has the better chance (3403-3406). She constructs the sample space with 10 outcomes (ignoring symmetric pairs), and notes that Player A has just one way to obtain each sum, while Player B has two ways for each of his sums, making the game unfair (3390-3398). She counts four outcomes for Player A and six for Player B (3707-3710). Her written explanation is shown in Figure 12.

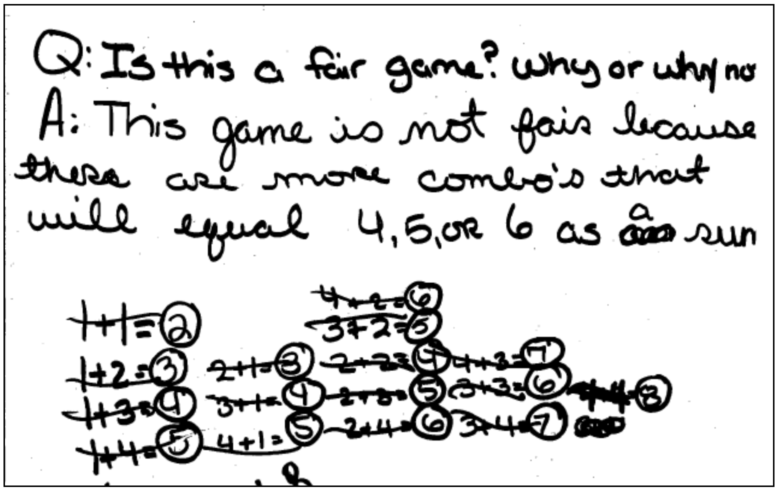
Figure 12. Chanel’s explanation of why the game is not fair.



While the class is discussing whether or not the game is fair, Kianja writes out the sample space showing 10 outcomes (not including symmetric pairs) in an organized way (2969-2974). She determines that Player B is going to win (3025), and says that “this game is not fair because there are more combos that will give you 4, 5, or 6” (3066-3067, Figure 13). Like Chanel, she counts 6 sums that favor Player B and 4 that favor Player A (3089-3091).

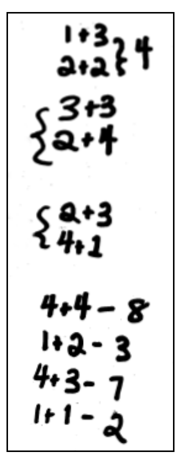
After a brief intervention where G4 asks Kianja to consider 1+2 and 2+1 by reversing the dice (3109-3122), Kianja alters her sample space to show all 16 outcomes (3123-3142). She and her partner Brionna conclude that the game is still unfair in Player B’s favor (3143-3146). They have not yet played the game.

Figure 13. Kianja's explanation of why the game is not fair.



Justina, before playing the game, says that "Player A has more of an advantage" because he "has more numbers" (4198, 4200). After Player B wins a game with a score to 10 to 1, Justina changes her opinion (4224-4225). She determines the sample space with 10 outcomes (see Figure 14), and concludes that "this game is unfair because Player B's sum of numbers has two different ways, has two different combinations, and Player A's sum of numbers only have one different combination" (4435-4438).

Figure 14. Justina's sample space.



David, Ian, and Jerel work together on the task. David says that the game is unfair. “Man, look, they got, it’s 4 numbers right there and he only got 3 numbers. So he got four chances of getting’ ‘em and he only got three of getting ‘em” (4624-4626). Initially Jerel says Player B will win (4567). As he plays the game, with Player B in the lead, he momentarily suggests that the game is fair (4721) “‘cause I’m winnin’” (4725). Apparently he was under the mistaken impression that he was Player A (4729-4731). Once he realizes that it is Player B who is ahead, he declares that the game is not fair (4734) because 1+1 and 1+2 are “very hard to get” (4741), while “7 and 8 is like a good number to get” (4744-4745). He explains that Player A’s numbers have only “one or two combinations” (4779), “and the other ones got like, they got like 2, 3, 4 …” (4785). These assertions are made without writing down the sample space.

During a second playing of the game, Jerel decides the contest is fair (4892). “Because, I changed to Player A and . . . I’m gettin’ the same amount of rolls with my numbers comin’ up as Player B. Yeeess!” (4897-4899). He cites the tied score of 4 to 4 as evidence of fairness (4908). Player B eventually wins the game 10 to 8, and Jerel accuses his partner of cheating (4940). The boys play a third and a fourth time, and Jerel wins both games as Player A (4984, 5096). In a presentation to the class, Jerel explains that originally he thought the game was unfair because Player B’s numbers had more combinations (5269-5273). “And then, when I started playin’ the game, I changed my mind because . . . [Player A] has just as good a chance as B” (5273-5277).

Jerel disputes Kianja and Brionna’s claim that Player B has a better chance to win based on the sample space. He comments, “But look, you said that uh Player B has more combinations, oh, but uh Player A has more numbers” (5178-5179). Jerel points out that

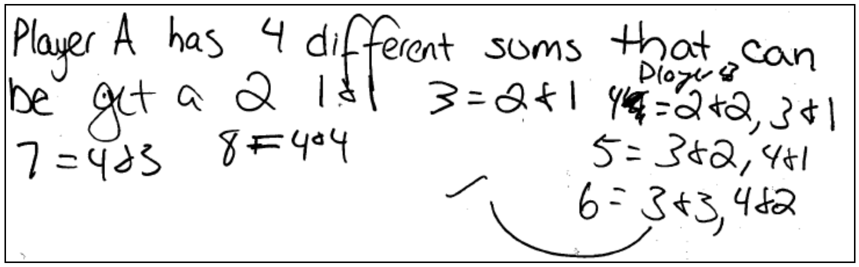
Player A can win. He won as Player A (5187). Last year, Jerel indicated that in an unfair game the favored player would win almost all the time, 99 out of 100 games. Apparently he still holds this belief. The fact that Player A can win a game is evidence for Jerel that Player B is not favored.

Jerel may be relying on primary intuitions or past experience with six-sided dice when he states that 1+1 and 1+2 are much harder to roll than 7 and 8. Though he speaks about the numbers of combinations for each sum, he does not construct the sample space. He is convinced that the game is fair because each player won two games, and he uses this as evidence to refute Kianja's claim that Player B has the advantage.

Jerel's partner Ian agrees with Kianja that the game favors Player B (4368-4371). Ian's sample space has 4 combinations for Player A and 6 for Player B (4377). David maintains his original position that A has 4 chances and B has 3 (4359-4360).

Chris is given the task in an interview with R4 and G6. Before playing the game, Chris, like David and Justina, says it is unfair in Player A's favor because A has four numbers and B has three (5368-5371, 5381-5387). Chris plays a game and Player A wins, 10 to 3 (5445). [Note: The probability of Player A winning with this score is .003. This unlikely occurrence supports Chris' assertion that Player A has an advantage.] Player B wins the second game, 10 to 6 (5466), and the third game is close, with a possible scoring error (5489-5490, 5497). Chris begins to talk about the number of ways to get each sum (5530-5543), and he concludes that Player B has 6 possible outcomes to Player A's 4 (5547-5552). "So it still isn't fair, so Player B will win" (5552). He also notes that Player B has two ways to obtain each of his sums, while Player A has only one (5557-5558).

Figure 15. Chris' sample space.



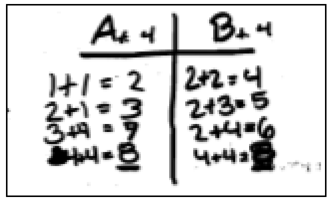
In summary, three approaches to assessing fairness are seen with this task.

1. Equiprobability: Dante, Justina, David, and Chris start with the assumption that all sums are equally likely and judge the game to be unfair in favor of Player A. David does not budge from this position.
2. Sample space: Kianja immediately sets out to construct the sample space, and once she has done so, she determines that the game is unfair in favor of Player B. Chanel, Justina, and Chris do the same, but only after playing the game. Ian also uses the sample space to show that the game is unfair.
3. Reliance on experimental results: Jerel seems to have an intuition that Player B's numbers are easier to get, but when Player A and Player B each win two games, he decides that the game is fair.

4.2.1.2 If You Think the Two Pyramidal Dice Game Is Unfair, How Could You Change It to Make It Fair?

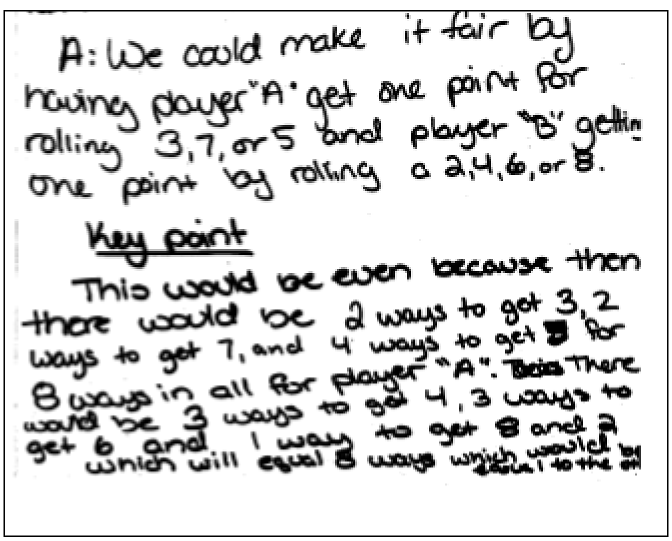
Despite having developed the sample space with 16 outcomes and having determined the number of ways to obtain each sum, Kianja and Brionna initially divide the seven possible sums so that Player A would get a point for 2, 3, or 7, and B would get a point for 4, 5, or 6. Either player would get a point for rolling 8 (3157-3170). Kianja makes a chart that omits several outcomes and indicates how points are to be assigned in the fair game (Figure 16). [Note: This game is not fair. The probability that Player A will get a point is $\frac{6}{16}$, and Player B's probability is $\frac{11}{16}$.]

Figure 16. Point allocation for Kianja and Brionna's "fair" game.



When asked by G4 to explain why the new game is fair, Kianja exclaims, "It's still unfair, Brionna. Sugar!" (3245). Several minutes later, she says, "Oh great! I know how to make the game even" (3317). Working alone, Kianja writes the rules for a fair game, correctly assigning 8 outcomes to Player A and 8 outcomes to Player B. Her explanation that each player would have eight ways to win a point is shown in Figure 17.

Figure 17. Kianja's second (correct) attempt to make the game fair.



Chanel tries a unique approach to make the game fair. She considers modifying the dice by adding zero as an outcome on each die (3685-3690), which she says gives Player A two ways to get each of his sums and Player B three ways (3703-3705, 3731).

She determines, however, that Player B would still have more ways to win: "So , I don't think that Player A would ever have as much as Player, like Player B would always have two more than Player A" (3870-3872). She later suggests altering the dice by replacing 1 with 0 (3885 – 3889), but finds that without redistributing the numbers, Player B would still have more outcomes (3920-3921). Finally, she makes what she believes is a fair game with her revised dice (0, 2, 3, 4) by taking the 10 outcomes in the sample space (symmetric pairs omitted), eliminating a sum of 4 (2+2 or 4+0), and dividing the remaining eight outcomes so Player A gets a point for 0, 2, 3, or 6 and B gets a point for 5, 6, 7, or 8 (4033-4046). In Chanel's sample space each sum other than 4 or 6 can be obtained only one way. [Note: By eliminating 4 and giving each player a point for 6, her game appears to give each player a five chances to win a point. In actuality, it is a fair game with each player having probability $\frac{8}{16}$ of winning a point.]

Justina does not modify the dice like Chanel to make the game fair, but she does eliminate one sum, 6. She creates a fair game by assigning 2, 7, and 4 to Player A and 3, 5, and 8 to Player B, explaining that each player has two numbers with one combination and one number with two combinations. Neither player gets a point for rolling 6 (4459-4464). [Note: Using Justina's sample space, each player appears to have four chances to win a point. In actuality, the game is not fair. Player A's probability of winning a point is $\frac{6}{16}$ and Player B's probability is $\frac{7}{16}$.]

Before playing the game, Chris suggests a way to make it fair: "since you got only 7 numbers, you could say if either one gets 3 different numbers, 3 different numbers, and that one number maybe nobody gets a point" (5395-5397). Later, after deriving the sample space with 10 outcomes, Chris suggests keeping the same numbers for Players A and B, but splitting the two ways to roll 6 so that A gets a point for 3,3 while B gets a point for 4,2, giving each player 5 chances (5684-5686). [Note: In actuality, this game is not fair. Player A's probability of winning a point is $\frac{7}{16}$ and Player B's probability is $\frac{9}{16}$.]

Jerel believes the original game to be fair, and so he does not modify it. His partners Ian and David play the game competitively but do not attempt to revise it to make it fair.

In summary, three approaches to making the game fair are seen:

1. Equiprobability: Initially Kianja, Brionna, and Chris give three numbers to each player, and either give both players a point for the remaining number or omit the remaining number. They later abandon this approach.
2. Sample space: Kianja and Brionna, Justina, and Chris divide the outcomes in their sample space between the two players, but in different ways:
   a. Kianja and Brionna divide 16 outcomes so that each player has eight. Player A gets a point for 3, 5, or 7, and Player B for 2, 4, 6, or 8.
   b. Justina does not speak about the total number of outcomes, but rather that some sums can be obtained only one way while others can be obtained two ways. In her fair game, each player has one sum that can be obtained two ways and two that can be obtained one way. She omits the sum of 6.
   c. Chris, using the same sample space as Justina, modifies the original rules by giving a point to Player A for rolling a 6 as 3 and 3, and a point to B for

rolling a 6 as 4 and 2.

3. Chanel tries to modify the dice by adding 0 as a dice outcome, and later by removing 1 and replacing it with 0. When Player B continues to have an advantage, she eliminates one sum (4) and shares another (6) between the players.

Though Kianja and Brionna use the sample space to determine fairness, at first they ignore it as they create a fair game. Upon questioning, Kianja realizes that her game is not fair and correctly devises a fair game using the sample space.

4.2.1.3 How Are Experimental Data Used as Evidence During the Two Pyramidal Dice Game?

When first asked whether the game is fair, Dante tells the class that, like last year's game, this one is unfair "because Player 1 gets more chances than Player 2" (2947). The following day in her presentation to the class, Chanel says that "at first . . ., Dante's reason was kinda sounding good, but until we started playing the game more . . ." (3397-3398). Chanel reports: "I played the game three times, and out of all those times, Player B came out to winning" (3406-3408). "When I went and looked at it, … there were actually two different ways to find all [of Player B's sums], . . . but only one way to find [Player A's sums]" (3404-3406). For Chanel, the experimental data causes her to question her original intuition about which player was more likely to win the game and to seek answers in the sample space.

Justina has a similar reaction. After playing and losing one game with a score of 10 to 1, she tells T6 that she no longer believes that Player A has the advantage "'Cause you kept winning" (4227). Like Chanel, Justina looks for an explanation in the sample space.

Chris also starts to consider the sample space after playing the game a few times. His original prediction that Player A is more likely to win is supported by his first game, in which, against the odds, Player A wins 10 points to 3. Player B wins the second game, 10 to 6, and the third game is close. Such results might suggest that the game is fair. On R4's suggestion, Chris records not only the sums but the individual dice outcomes for each roll. Perhaps it is this representation of the data, more than the results of playing the game, that causes Chris to consider the sample space and determine that Player B has the advantage. Chris explains to R4 that experimental data can be difficult to interpret: "Well you could say like Player A wins 5 games and Player B only wins 1 game. Right there you're gonna know that it's not fair. Or you never know because Player B might be able to win other games too" (5403-5406).

After Chris determines that Player B has more chances to win, R4 asks, "What about your experiment?" (5555). Chris responds, "But Player 1 [*sic*] only won once. And Player B has six diff-, well, two for each. Two different ways to get each number. And Player A only has one for each" (5556-5558). He appears to give more weight to the sample space than to experimental outcomes.

Chris further demonstrates this tendency in a discussion with R4 about which is more likely: a sum of 2 or a sum of 3 with two dice (5588-5592). He asserts that "both of em have the same probability, which is only one way you could get it" (5590-5591). When he shows some uncertainty about this, saying, "I don't really know" (5591-5592), R4 suggests playing a game in which Player A gets a point for a roll of 2 and Player B gets a point for a roll of 3 (5602-5603). Chris plays this game with G6 using two dice of different colors. On R4's suggestion, he records not only the outcome of 2 or 3, but also which number came up on each die, white or green. After many rolls, 2 has appeared five times and 3 has appeared 10 times (5657-5658). Chris says, “I really still think it’s the same thing” (5660). His sample space shows one way to obtain each sum, and the experimental data, along with the white outcome/green outcome representation, do not sway his opinion.

Jerel’s opinion is more readily influenced by experimental data. Though he considers that Player A’s numbers have fewer “combinations” (4779) than Player B’s numbers in the original game, a tied score of 4 to 4 causes Jerel to change his mind and proclaim that the game is fair (4889-4895). He explains that Player A’s and Player B’s numbers are “gettin’ the same amount of rolls” (4898). During the class presentations, where Kianja, Chanel, Justina, and others demonstrate with the sample space that Player B has the advantage, Jerel insists that the game is fair because he won as Player A (5277-5279). Throughout the grade 7 activities when a game doesn’t go their way, Jerel and his partners frequently accuse one another of cheating by “scuffing the dice” (e.g. 4942). This may reflect the boys’ competitive nature rather than evidence of their beliefs about the fairness of the dice game.

Kianja and Brionna stand out as the only two students in this study who do not use data to develop or support their argument. As soon as the task is described, Kianja begins to enumerate the sample space, and she successfully completes the activity without a roll of the dice.

4.2.1.4 Notions of Probability Expressed During the Two Pyramidal Dice Game

Very little is said about probability per se during this activity. A brief discussion among Kianja, Brionna, and G5 is worth noting, however. G5 asks Brionna how many opportunities Player A has to win the game (3264), and Brionna answers, "Six. One out of six" (3267). Struggling a bit with her explanation, Brionna asks Kianja to join the conversation (3270). Kianja elaborates, "It's six ways that A could score a point, right? So it's one out of six chances that A would score a point" (3290-3291). G5 asks about Player B's chances (3292), and Kianja replies, "One out of ten. Because it's ten chances, it's, there's ten possible ways for B to score a point, so it'd be one out of ten" (3293-3294). Using $\frac{1}{x}$ instead of $\frac{x}{n}$ to describe the players' chances may have been a momentary lapse for Kianja. A year earlier, she correctly stated probabilities based on her sample space.

4.2.1.5 What Is the Sample Space for the Sum of Two Pyramidal Dice?

With the exception of Kianja and Brionna, each of the students who enumerates the sample space for this activity finds ten distinct outcomes, four favoring Player A and six favoring Player B. (See, for example, Figures 12, 14, and 15.) Kianja starts out with ten outcomes as well, which she writes in an organized way, as shown in Figure 18 (2969-2973).

Figure 18. Reproduction of Kianja's initial sample space.

1	2	34	
1+1	1+2	2+2	
1+3	2+3	3+3	
1+4	2+4	3+4	4+4

During a discussion with G4 (3091-3134), Kianja modifies her sample space and adds the remaining six outcomes. The discussion begins as G4 asks Kianja whether she has found all the sums.

G4	Do you think these are the only ways in which you can do it?
Kianja	Yes.
G4	There are no other ways?
Kianja	Well, if you use addition. 'Cause there's only 4 numbers on here. I mean, it's only numbers from 1 to 4.
G4	Okay. So …
Kianja	So if you get a 1, right …
G4	Um humh, Um humh.
Kianja	Say you rolled a 1 and then you rolled a 1 on this die, . . .
G4	Okay, so, so, suppose you got 1 and 1.
Kianja	It'd be 1 + 1.
G4	So which one is that?
Kianja	Right here. [Points at "1+1" on her paper.]
G4	Suppose we got 1, 1. Okay.
Kianja	It'd be 1+1.
G4	All right. And if you get this, 2 and 2.
Kianja	2 and 2, it would be 4.

Next, G4 asks Kianja to show him the outcomes 1, 2 and 2, 1 in her sample space. While Brionna insists that 1+2 and 2+1 are the same thing, Kianja begins to write the missing outcomes on her paper.

G4	Okay, I'll ask you a question. Which one is this? 1, 2.
Kianja	Right here. [Points at "1+2" on her paper.]
G4	1, 2 is this one?
Kianja	Yes.
G4	Okay. Now let me change this, okay. This is 2, this is 1. [Reverses the dice.]
Brionna	It's 3.
Kianja	This. [Points at "1+2" on her paper.]
G4	No.
Kianja	It'd be 3.
G4	Yeah.
Brionna	2+1
Kianja	See?
G4	Yeah.
	[Kianja writes "2+1=3".]
G4	This is 2+1, right?

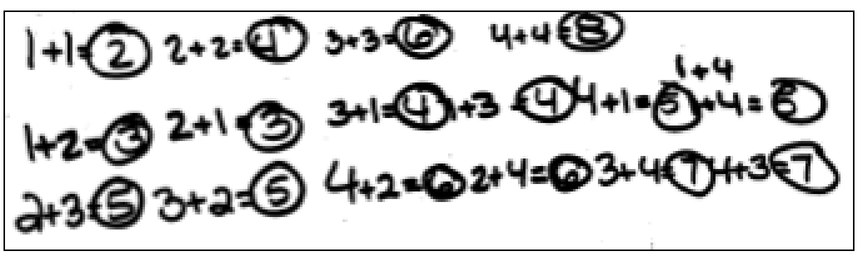
Brionna Yeah, it equals 3.
G4 Yeah, and this is 1+2.
Brionna 1+2. That's the same thing, 3.
[Kianja writes "3+1 =4", "4+1 = 5".]
G4 Um humh. What is this here you're writing? [Points at Kianja's paper.]
[Kianja continues writing, "3+2=5", "4+2 = 6".]
Brionna [quietly] You still get the same answer.

While Kianja appears to accept G4's suggestion that symmetric pairs are different outcomes, it may be the case that she is doing so in order to mollify him. Her words "If you wanted to do that" imply that counting these outcomes or not is a matter of choice.

Kianja If you wanted to do that, then it would only be [writes "4+3=7"], then it would be 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 [counting up the outcomes for Player B she had circled on her paper].

The next day, when students present their findings to the class, Kianja and Brionna show their sample space with 16 outcomes, as shown in Figure 19.

Figure 19. Kianja and Brionna's sample space for the sum of two pyramidal dice.



However, they do not disagree with other students who display only 10 outcomes. On the contrary, Kianja insists that "it's the same concept" (4376). The following exchange takes place after R2 points out that Ian's and Kianja's sample spaces appear to be different (4379, 4387-4399).

Kianja He had six, which I had first. But then we had switched some of the numbers around like 2+1 we did, I mean 1+2, we had changed it to 2+1 which gave us another combination. That kind of thing.

R2	Right. So you had ten, he had six [outcomes for Player B].
Kianja	Yeah.
R2	He did not count 2+1 and 1+2 as different events.
Kianja	Right.
R2	But you did.
Kianja	He counted them as the same thing. We counted them as one, I mean, different things, but he counted them as one. That's why we didn't get the same numbers.
R2	Right. So I think that we …
Kianja	But it's still the same. I mean, it's the same concept.

A few minutes later, Justina presents her analysis of the game to the class. She points out that there are two ways to get each of Player B's sums, but only one way for each of Player A's sums. Kianja interrupts, and the following conversation ensues (4446-4457).

Kianja	Oh, wait. Can I say, wait, can I say what I think you're saying? Um, you saying that um, each, each number on Player A has only one combination that can get to that sum, and then on Player B, each number has two? Okay.
Justina	Um humh. That's why I had the greater advantage.
Kianja	Okay.
Justina	That's why I think it's unfair. And, for my game, …
R2	I'm sorry. Do you agree with that point of hers, Kianja? Kianja, do you agree with her point?
Kianja	Yes.
R2	That the numbers for player A each have just one combination?
Kianja	Um humh. I know. I know what she's talking about. Yeah.

Justina continues to describe how she made the game fair by eliminating 6 and dividing the remaining outcomes so that each player has one number with two combinations and two numbers with one combination (4459-4464). R2 asks Kianja for her opinion (4467), and she replies, "I think she's right" (4468). Brionna concurs (4471). R2 points out that Justina says a sum of 4 can be made two ways, and he asks Kianja how many ways she found to make a sum of 4 (4484-4487). Kianja names three ways: "It would be 1+3, 3+1, and 2 + 2" (4491-4492). Justina insists that "1+3 is the same thing"

(4493) since “1+3 and 3+1 would still equal 4” (4495). Kianja does not challenge this claim, and the session ends with R2’s suggestion that perhaps the class will return to discuss this next week (4498-4499).

Though Kianja and Brionna, after a brief suggestion from G4, have developed the sample space with all 16 outcomes, they do not dispute the work of other students. In the debriefing that follows this session, T5 conjectures that Kianja did not want to “entertain the argument” because her personality is not confrontational. T5 asserts, however, that he believes Kianja has convinced herself that she is correct. Of course, Kianja’s non-confrontational nature may also explain why she readily adopts without argument G4’s suggestion to reverse addends. As we will see in the next section, other students do not give in to the strong suggestions and repeated questions of adults over the issue of sample space.

4.2.1.6 Does Color or Order of Dice Matter With Sums of Two Pyramidal Dice? Interventions and Conversations

During this activity and the next one, some of the teachers and graduate students challenge students to support their assertions about the sample space. Some try to scaffold student learning by demonstrating ways of representing dice outcomes. As the following excerpts illustrate, these efforts are not always successful.

As Chanel explains her sample space to T5 and R2, T5 asks whether 1+2 and 2+1 are the same (3777). Chanel replies (3782-3787):

Chanel	This, yes, I think these 2+1 is the same thing as 1+2. It’s the same thing, just reversed.
R2	The same thing because they both equal 3?
Chanel	Exactly. But they’re just switched around in reverse. So two’s

over here [holds up left hand] plus one [holds up right hand], still gonna equal three.

T5 asks whether it would matter if the dice were different colors (3792-3793). Chanel says, “It’s gonna be the same thing” (3794). Chanel takes a yellow die and a green die to demonstrate (3799). The following conversation takes place (3800-3809).

T5	So can you show me what 1+2 would look like with those dice?
Chanel	1+2?
T5	You can manipulate them if you’d like.
Chanel	1+2 [places the dice to show this]
T5	And could you show me what 2+1 would look like?
Chanel	Same thing.
T5	But what would happen if I got a, a, ‘cause this is, okay, so you’re saying one plus 2 [points to one die and then the other]. But what if I said [changes the outcomes of the dice], is that the same roll?
Chanel	Yes.

T5 asks Chanel, “So when you’re now figuring out the possibilities, do you think that if that were different it would affect the outcomes?” (3814-3815), and Chanel says yes, it would (3816). T5 asks Chanel to explain why she thinks 1+2 and 2+1 are the same outcome. She says, “Because it’s, it’s they all have the same numbers on ‘em, the same amount on each side. So this is like saying 1 minus 2, but [waves her hand]” (3823-3825). Since Chanel has raised the idea of subtracting dice outcomes, T5 asks her whether the 1-2 and 2-1 are the same (3827). Chanel determines that they are not. The conversation continues (3837-3858).

T5	So they’re not the same during subtraction.
Chanel	No.
T5	But they are the same during addition.
Chanel	Exactly.
T5	And is it, and the reason why?
Chanel	Because this is, like it’s the same number. It just being twisted around, so. It’s the, it’s the same thing, just in reverse. But if you’re doing subtraction, then the, if you’re doing 2 minus 3 it’s always gonna be, it’s gonna be the same number but one is gonna be a negative and one is gonna be a positive.

T5	Okay. So, because you get a different answer, that's the only way that it can be different. But if you don't get the same, if you get the same answer it's the same.
Chanel	If you get the same answer, 2 + 3, same. But go like that, 3 + 2. It's the same thing. It's just being twisted around. So if you're doing 3 – 2, 3 – 2 is, I had to think on that, oh, one. And then 2 – 3 is gonna be negative one. It's the same thing, it's just one is negative and one is positive.
T5	Would they count as two different opportunities when rolling dice, or would they count as the same opportunity?
Chanel	They count as the same opportunity 'cause you're adding, not subtracting.

At another table, G5 speaks with Kianja and Brionna. They have already developed the sample space with all 16 outcomes, G5 asks them, "Is 4 + 2 the same, like 2 + 4?" (3317-3318). Brionna responds that "even though it's like the same answer you still have to do it […] because you also have 2 + 4 and 4 + 2" (3321-3323). Kianja and Brionna's sample space shows both of these outcomes. The conversation continues (3324-3329):

G5	They are the same? Is the same chance or different chance?
Brionna	It's the same thing.
G5	It's the same thing?
Brionna	[nods] It's just that it, it's worded differently.
G5	Oh. So how about 3+4 and 4+3?
Brionna	It's the same thing.

G5 continues to ask Brionna about other sums, 4+1 and 1+4, 3+1 and 1+3, 1+2 and 2+1, are they the same? (3341, 3345, 3349). Brionna replies that sums with the same addends are the same because "you get the same answer no matter which way you put it" (3346-3347). Though G5 may be attempting to determine whether Brionna views 4+1 and 1+4 as different experimental outcomes, Brionna appears to interpret the question differently, answering that 4 + 1 and 1 + 4 are the same sum.

G5 tries a different approach, asking, “What if we use subtraction?” (3353). Perhaps G5 overheard Chanel talk about subtracting dice outcomes. Brionna notes that 4 - 1 and 1 - 4 are opposites (3372-3373). G5 asks, “What would, is the same chance if we use subtraction?” (3376). Brionna says, “It would be the opposite. Like it would come out to 3 no matter what but it would be like a negative or a positive” (3377-3378). The conversation ends as R2 announces that students will begin making presentations to the class (3379-3380).

The exchange between G5 and Brionna illustrates the importance of using precise language that both participants understand. It may be that because of the lack of a shared understanding of G5’s questions, Brionna does not make it clear that she considers permutations of sums to be different outcomes, as her sample space shows. Or, it is possible that Brionna is not convinced that permutations of addends are different events.

The next day, when Ian reports a roll of 2 and 1, R2 asks whether it was 1 and 2 (4928-4929). Jerel says, “It’s the same thing, he just mixin’ it up” (4933). Several minutes later, G1 asks Jerel and his partners whether 4 and 3 is the same thing as 3 and 4 (5041). Jerel replies, “Yeah” (5042). Eager to continue the competition, Jerel does not elaborate on his answer.

R2 also asks Kianja whether 2 and 1 is the same as 1 and 2 (4253). Yesterday, Kianja listed these as two different outcomes in her sample space. Today, she says, “It is the same” (4254). R2 suggests that Kianja and Brionna try a new game in which Player A gets a point for rolling a sum of 2 with two dice, and Player B gets a point for rolling a sum of 3 (4262-4264). Before the girls begin the game, R2 asks whether it is a fair game (4268-4277).

R2	Hold on. Now who's gonna win? Is this a fair game that I'm just introducing?
Kianja	I mean, Player B gonna win.
R2	Why?
Kianja	'Cause there's only one possible way that you can get 2.
R2	Okay. So let's, let's try. Okay?
	[Kianja holds up her paper and looks at it.]
Kianja	Only one way to get both of 'em, so …
R2	So it's a fair game, right?
Kianja	[looks at R2 and tilts her head but does not answer]

The camera moves away from Kianja and Brionna, who play the game with T3 looking on. During the debriefing session after the students leave, T3 talks about the game:

> The 2-3 game was interesting. It took, it took a while for them to be able to articulate to me the fact that you have the, you know, that 3 has multiple combinations. And my thing to them was 2 and 1, 1 and 2, what's the difference? So now, they have to process. I said, "Well, if it's a 2 here and a 1 there, it's 3. So what, if I say 1 and 2, does it change anything?" And it took a while for them to realize that, well it could be 2 on this one die and 1 on this one, or vice versa. That's when the connection finally came through, I think, and uh once they realized that they were able to take it from there.

During Chris' interview with R4, the subject of the order of addends is raised (5562-5570).

Chris	A 7 is a 4 and a 3 [turns dice to show 4 and 3].
R4	Uh huh. Okay, if I rolled, and this one turned out 4 and this one turned out 3, is that different from the one you just showed me?
Chris	No. It's still the same thing. You're still gonna get the same sum.
R4	And you only have one chance to get a seven?
Chris	[nods]
R4	When you're rolling. If, if I did it this way [rolls a green and a white die, instead of two green dice], and it was a 4 and a 3 …
Chris	It's still the same thing. 'Cause you have the same sum.

R4 further pursues the topic (5583-5592).

R4	And if you had a white 1 and a green 2, or a green 1 and a white 2, those are not different ways?

Chris	[shakes head] It's, even though it could be different dice, different colored dice, different, maybe a 2 and a 1 or a 1 and a 2, it's still gonna add the same.
R4	Okay. If I was gonna bet you $100 that you would roll a 2 before I rolled a 3…
Chris	Umm, both of 'em have the same probability, which is only one way you could get it, well, [looks down, takes a breath] I don't really know.

R4 suggests that Chris and G6 play the game in which Player A gets a point for a sum of 2 and Player B for a sum of 3 (5602-5603). When asked, Chris says he believes this is a fair game (5606-5607). R4 gives Chris and G6 each a white die and a green die, and she suggests that Chris record the outcomes according to the dice colors (5613). Chris writes "W&G" at the top of his column (5613-5614) and takes care to write the outcomes in the correct order (5626-5627). Player B wins the game with a score of 5 to 2 (5632). Chris reacts (5634-5642):

> It's the same, it's the same thing. It uh, it doesn't really matter which player wins it, but it's the same thing because it had two different numbers, and both dice have the same kind of numbers. And, so if you get 3 and a 1, or 2 and a 1, in either one, it's still gonna get a 3. If you get a 1 and a 2 or, no, I mean a 1 and a 1 on the other dice, it's still the same thing. So you could get a 1 here and a 1 here [holding one die in each hand], it's still gonna be 2. And you get a 2 [right hand], 1 [left hand], or a 2 [left hand], 1 [right hand], it's still the same thing.

Chris and G6 play a second game, and Player B wins again, this time with a score of 5 to 3 (5656). R4 points out that with the scores of both games combined, Player A has only five points and Player B has ten (5657-5658). Chris maintains, "I really still think it's the same thing" (5660). Unlike Kianja and Brionna, Chris is not convinced that order or color makes a difference.

4.2.1.7 Summary of Activity 3

Though some students (Dante, Chanel, Justina, David, and Chris) start this task with the assumption that all sums are equally likely and Player A has the advantage, all but one (David) become convinced by experimental data and/or by the sample space that this is not the case. Kianja and Brionna are the only students who construct the sample space with all 16 outcomes, but they do not dispute other students' presentations of a 10-outcome sample space. They do not demonstrate a strong conviction that symmetric pairs are different events, as they are willing to go along with either interpretation of the sample space. Chanel, Justina, and Chris show strong convictions that symmetric pairs should not be counted as different events. They are not influenced by questions or suggestions from the research team or, in Chris' case, by experimental data that suggest otherwise.

Like last year, students who use experimental data to make inferences do so with a small number of trials. Justina decides after just one game that Player B must have the advantage, while Chanel and Jerel are convinced within a few games. At one point Jerel cites a score of 4 to 4 as evidence of a fair game. Chris, who was somewhat distrusting of experimental data last year, remains so this year. While his trials seem to suggest that the game might be fair, Chris rejects this evidence and uses the sample space to make inferences.

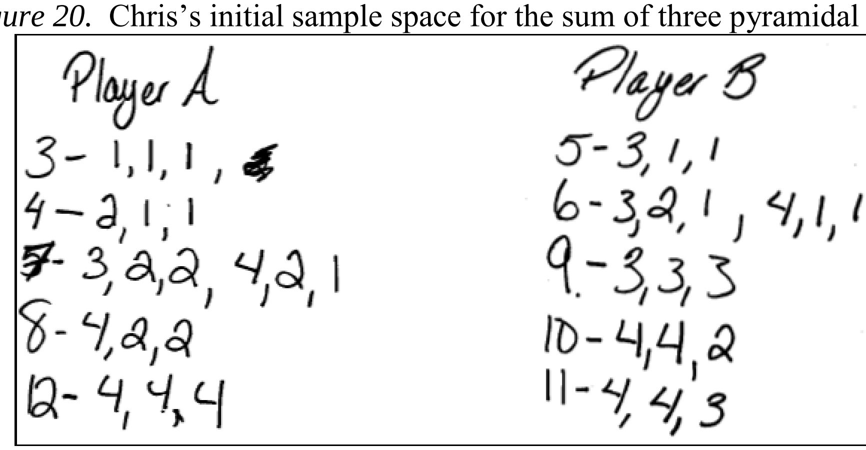
4.2.2 Activity 4- A Game With Three Pyramidal Dice

The following week, R1 introduces a new game using three pyramidal dice. In this game Player A gets a point if the sum is 3, 4, 7, 8, or 12, and Player B gets a point for a sum of 5, 6, 9, 10, or 11. The first player to get 10 points wins the game. As before, students are asked to determine whether or not the game is fair and to justify their answers. [Note: Though both players have the same number of sums, the game favors Player B with a $\frac{35}{64} \approx .547$ probability of winning a point and a probability $\sum_{k=0}^{9} \binom{k+9}{k} \left(\frac{29}{64}\right)^{k} \left(\frac{35}{64}\right)^{10} \approx .661$ of winning a game.]

4.2.2.1 Is the Three Pyramidal Dice Game Fair?

Chris and Terrill are partners for this activity. As they get started, Chris says, "Hold on, brother. I've gotta see if it's fair" (5748). He begins to write down combinations that give each of the possible sums. T7 suggests that they start playing the game, but Chris insists, "Hold on, bro" (5757). Terrill explains to T7, "He counting up the possibilities of going to those numbers. If he finds all the possibilities then whichever one has more possibilities is um, better, it's fairer for um that one" (5762-5764). Chris finds 12 outcomes in the sample space, six for each player, as shown in Figure 20. He remarks, "They're both equal, they're equal" (5768).

Figure 20. Chris's initial sample space for the sum of three pyramidal dice.



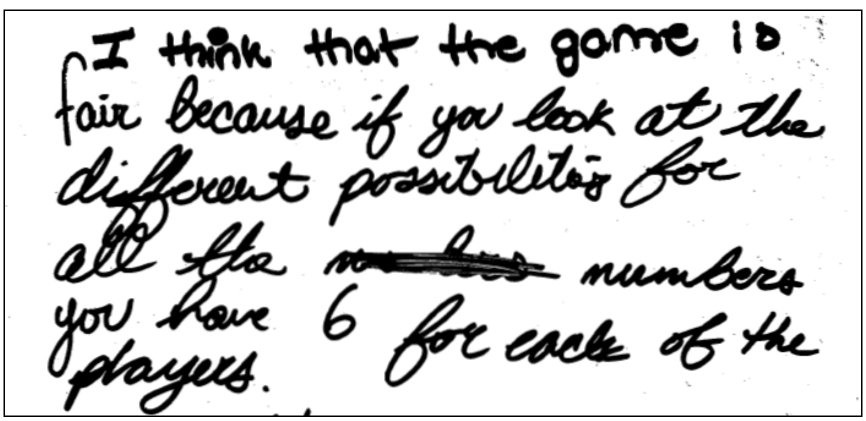
Chris and Terrill play a game, with Player A consistently in the lead (5784, 5785, 5790). Ultimately Player A wins with a score of 10 to 8 (5796). In their second game, Terrill is careless in his scorekeeping (5820, 5824-5826), but Chris, as Player A, is declared the winner again (5833). T7 asks whether the boys still believe the game is fair (5835), and they answer affirmatively (5841-5844):

Chris	I say it's fair.
Terrill	The game is fair.
T7	Why?
Terrill	Because it has the same amount of chances to um …

Terrill abruptly changes the subject as he attends to some excitement in the classroom (5844-5845).

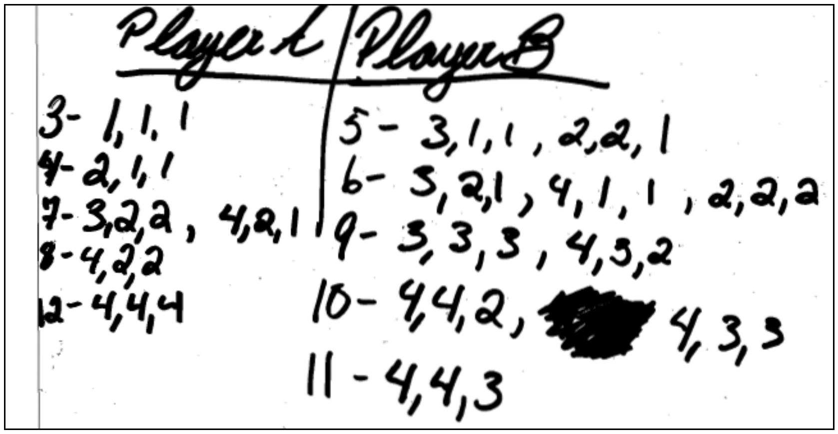
The following day, Chris tells G4 that the game is fair (7632) and shows him the sample space with six outcomes for each player as evidence (7667). Chris writes his conclusion on a transparency, Figure 21.

Figure 21. Chris' explanation of why the game is fair.



However, as Chris copies his sample space to the transparency, he discovers four more combinations for Player B. His sample space now has a total of 6 outcomes for Player A and 10 for Player B (see Figure 22).

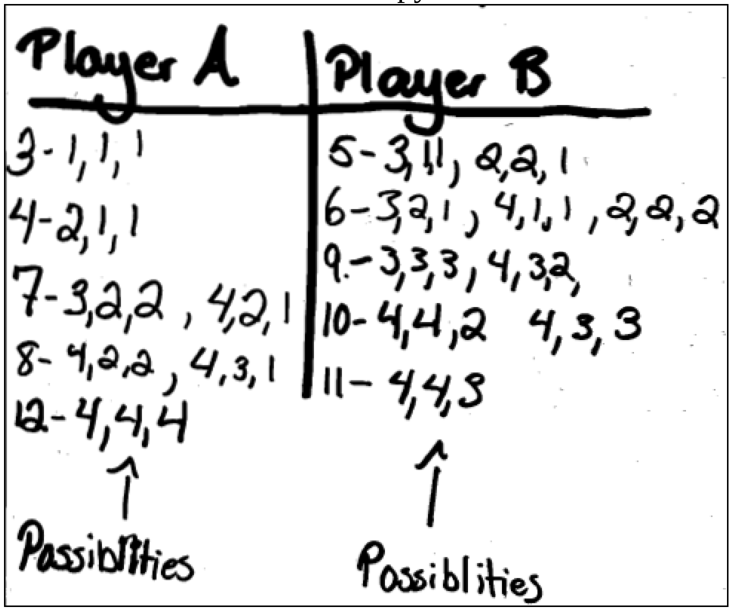
Figure 22. Chris' revised sample space for the sum of three pyramidal dice.



Chris tells G4 that he now believes the game is not fair (7751), and he writes up a new transparency to this effect (7764-7765). Chris shows Terrill his new sample space, saying, "You know it's not fair, right?" (7770). Terrill is not convinced, and he suggests that they play the game to "see if it's actually fair" (7783). Terrill explains to G4, "You have to play it first to see if it's really fair" (7790-7791).

Chris continues to write the sample space and finds one additional outcome for Player A. His sample space now shows 7 outcomes favoring Player A and 10 favoring Player B (see Figure 23).

Figure 23. Chris' second revision of the sample space for the sum of three pyramidal dice.



Chris and Terrill begin a game and Player A takes the lead, 3 to 1 (7809). Terrill taunts Chris about this (7811-7815):

> Shouldn't Player B be winning, since um I got more possibilities? Huh, huh? See how dumb you are without me, huh? Now, if we wouldn't 've played the game, we'd 've known that he was right, he was wrong. But we still do.

As the play continues, G4 asks Chris his opinion with each roll of the dice (7818, 7825, 7828, 7832, 7835). When the score becomes tied (7828) Chris concedes, "Yeah, I think it is fair. It's just about how they roll [shaking his hand in a dice-tossing motion]. People sometimes get lucky" (7830-7831). Player B finally wins the game by two points (7837), and Terrill agrees that "Player B has more um ways to get their answer than Player A" (7852-7853). Though they are not recorded saying so, it appears that Chris and Terrill have come to believe that the game is unfair because they work to devise a fair game (7857-7887).

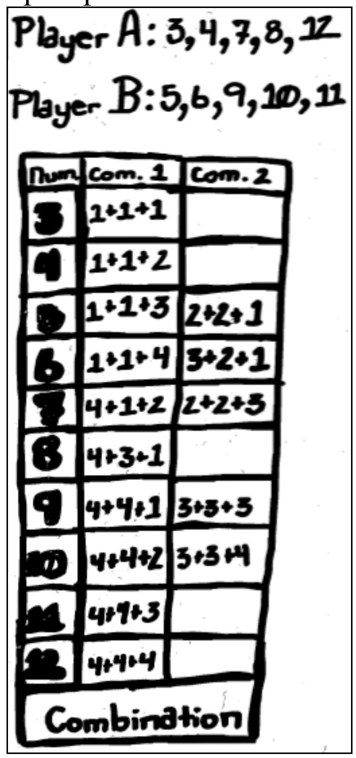
Jerel and Ian are partners for this activity. At the outset, R3 asks the boys whether they think the game is fair (6075). Ian answers succinctly, “No” (6076), and, when asked why not, “Because” (6078). Jerel is noncommittal as they begin to play (6080), though he is quick to accuse Ian of cheating when the roll does not go his way (6083, 6086). After about 10 minutes of play, R1 stops by to ask the boys’ opinion about the game. Ian asserts that Player B has the advantage because he “has a better range of numbers” (6136) with “more multiples” (6150). When pressed to explain what he means by this (6154-6155), Ian simply says that “B has better numbers” (6156). Jerel agrees: “Oh yeah, he is right. It’s like not, not a very fair game” (6157-6158). Ian explains that “this time they got the same amount of numbers, but B got the more multiples” (6159-6160).

The boys continue playing, and about 23 minutes later they have the following discussion (6378-6387):

Ian	Jerel, this game fair to you?
Jerel	Yeah. I think. No.
Ian	No. No. Well yeah yeah yeah yeah. 1, 2, 3, 4, 5, 6, 7, no, 1, 2, 3, 4, 5, 6, 1, 2, 3, 4, 5, 6, 7. [counting the outcomes in his sample space]. The game’s not fair. Seven has more ways than … [holds his hands out, palms facing Jerel].
Jerel	But Player B can still win.
Ian	That’s what I just said.
Jerel	It’s fair.
Ian	But it’s not fair. B has more ways than A-town.

A short time later, Ian’s sample space shows six outcomes favoring Player A and nine favoring Player B (Figure 24).

Figure 24. Ian's sample space for the sum of three pyramidal dice.



Several minutes later, T3 asks Ian and Jerel whether or not the game is fair. They have played two games and the score is tied (6507-6509). Ian still believes the game is unfair, based on his sample space, while Jerel claims that it's fair, based on the tied score. T3 asks Ian how he knows the game is unfair, and an animated discussion ensues (6510-6540):

T3	It was a tie? You guys say that when you played, it was a tie? Huh?
Ian	Yeah.
T3	Then how do you know it's unfair?
Ian	'Cause we played twice.
Jerel	I thought it was fair.
T3	So because you won and because he won, it's fair?
Jerel	Yeah.
T3	Is that what you're saying?
Jerel	Yep, basically.
T3	Wow. But he just said it was unfair.
Jerel	He thinks it's unfair.
T3	What makes it unfair?
Jerel	Ian, Ian, you won!

Ian	I just told you.
Jerel	But you won once.

Ian raises his voice and leans forward with his palms on the desk.

Ian	It doesn't matter!
Jerel	[expletive], it's basically what I said.
T3	You need to justify for me why you think it's unfair. On your end [Jerel], you think it's fair because you won once and he won once.
Ian	All right, look. I'm gonna explain it one last time.
T3	OK, I'm listening.
Ian	All right A, Player A, which is red, you gotta see that right there [Ian has color coded his sample space], all right 1, 2, 3, 4, 5, 6, 6 combinations , that's it. Now, blue, blue, all right, 1, 2, 3, 4, 5, 6, 7, 8, 9, 9 combinations. That's why it's unfair. Got more combinations.
T3	But you just told me it was fair 'cause you won and he won.
Jerel	But you won!

Ian stands up and slams his palms on the desk.

Ian	It don't matter.
Jerel	Well yes it do!

The boys agree to play one more game in order to settle their argument (6542). Jerel will be Player A (6552). When the camera rejoins them, the game is tied, 6 to 6, and Jerel is accusing Ian of cheating (5930-5932). Jerel ultimately wins the game (5936), and T3 asks if that is evidence enough of a fair game (5939-5951).

T3	Is the fact that Player A won sufficient for you to say it's fair?
Jerel	Whatever player I am is always wins. Right? We just learned that.
T3	So what does the fact that whichever player you are wins, that makes it fair automatically?
Jerel	'Cause look, Player B has more, look, you sayin' Player B has better chance of gettin' them numbers, but look, I just proved to you that Player A can still win.
Ian	[inaudible] But doesn't on the chart, doesn't it look fair?
Jerel	Yes.
Ian	On the chart.
Jerel	It looks, it looks unfair on the chart. But look, we, I just proved that Player A can win.

Jerel seems to be holding on to the notion that in a fair game either player can win, but in an unfair game the favored player will win almost all of the time. After one more game, which Player B wins (5980), Ian backs away from his opinion based on the sample space and declares, “Yeah, it’s fair. They each have enough of a chance to get …” (5984).

Kianja and Brionna decide to each tackle a different part of the task. While Kianja works on developing the sample space, Brionna rolls the dice and keeps score (6096-6099). Kianja lists the numbers for Player A and for Player B separately and begins writing the possible addends for each sum (6104-6105). Her paper shows permutations of addends as different events (6110, 6114-6115).   As Kianja writes the sample space, R3 and R4 ask whether she has found all the sums for a particular number (6176, 6181, 6196-6197), and R4 suggests addends that Kianja hasn’t considered (6185, 6188, 6232, 6235).   With a little help from R3 and R4, Kianja finds a total of 58 outcomes in the sample space, 26 favoring Player A and 32 favoring Player B (6249-6256). She is missing just three outcomes for each player. Kianja concludes, “So B is gonna win” (6257), “and I have an example” (6260), indicating Brionna’s score with Player B in the lead.

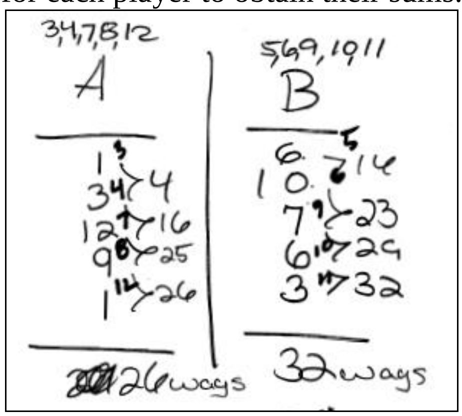
Later, T3 stops by and asks Kianja why she wrote out the sample space. The following conversation ensues (6389-6399):

T3	Now why did you do that, though? What was the purpose of doing that? [writing the sample space]
Kianja	So I could know who, who …
Brionna	Who can win.
Kianja	Yeah, who will win. And I added it up. So, these numbers [pointing to her paper], Player A has 26 ways to win, Player B has 32 ways to win.
T3	That’s a lot of numbers.

Kianja   Yes, it really is. Set, it's all set.
T3   Are you sure?
Kianja   Yes, I'm very sure.

Kianja shows T3 that Player B has the advantage in this game (6404) and Brionna explains that, although the two players have the same number of sums, there are more ways to obtain Player B's numbers (6406-6407).   Kianja writes the sums on her chart to the right of the number of ways to obtain them (6410), as shown in Figure 25.

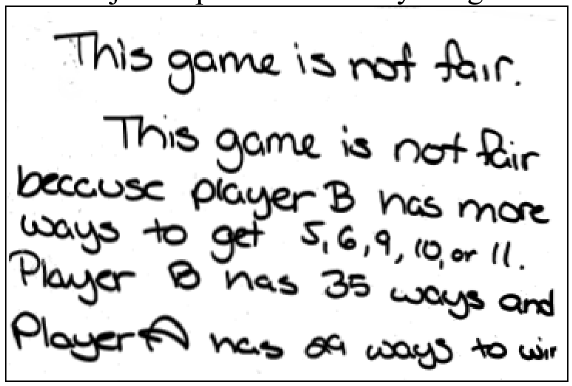
Figure 25. Kianja writes the number of ways for each player to obtain their sums.



Kianja writes on her transparency: "This game is not fair. This game is not fair because player B has more ways to get 5, 6, 9, 10, or 11. B has 32 ways and A has 26 ways" (6500-6503).

The next day, G6 asks Kianja how she knows that she's found all the possible outcomes (7333-7335). As she copies the sample space onto a transparency, Kianja realizes that she missed some outcomes yesterday. She lists the complete sample space with 64 outcomes (7340-7341), saying "I shoulda known it was wrong" (7342), pointing out the symmetry in the distribution (7342-7344).   Kianja rewrites her transparency as shown in Figure 26.

Figure 26. Kianja's explanation of why the game is not fair.



Justina and Adanna are partners for this activity, and they are joined by Alia on the second day. While Adanna, who was not present for Activity 3, spends much of the time talking about other topics, Justina does her best to stay on task. R4 asks the girls to predict whether or not the game is fair (6774-6775), and Justina suggests that they "look at the possibilities for getting each number" (6779). However, the girls start playing before making a prediction (6810).

Justina wins the first game as Player B with a score of 10 to 8 (6842), and she declares, "I guess it's a fair game. You had a close chance of winnin'" (6843-6844). The girls look over their data and determine that 8 was the most frequent sum, occurring 6 times, while 7, 9, 10, and 11 came up only once each (6848-6853). Justina states that "the highest numbers didn't come up that much" (6853-6854). Adanna tells T8 that the game is fair (6868), while Justina deliberates:

> Most of the high numbers have, um did not come up that much, and the lowest numbers came up more often. No, wait. Even though Player B had the lowest numbers, I mean high numbers, it still won. Maybe it's a fair game. (6871-6874).

Justina suggests that they play another game, switching roles as Players A and B (6882). While Adanna speaks about other topics, Justina tries to keep the game going

(6883-6885). When the score reaches 5 to 1 in Player B's favor, Justina says, "Um, I don't think it's fair" (6899). Justina predicts that Player B will win the game (6908), but ultimately Player A is the winner, with a score of 10 to 9 (6920). Justina remarks that each player has won a game (6925), and the girls tell T9 that they believe the game is fair (6928-6929). Justina explains, "Because each player has um a good, yeah, each player could win" (6931).

Adanna writes on her paper (6965-6968):

> Yes it's a fair game because in the first game Player B won and on the second game Player A won. If it wasn't fair Player A will have kept on winning like the last dice game when Player A had even numbers while Player B had odd numbers.

T9 encourages the girls to play again (6969), and they do, switching roles again (6974). Adanna wins as Player A (7023) with a score of 10 to 5. The session ends with both girls agreeing that the game is fair.

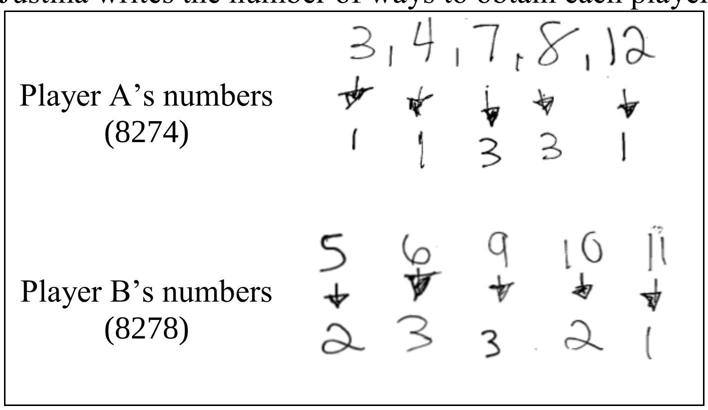
The next day Justina explains the game to G8, who wasn't present for the prior probability session. Justina remarks that 8 and 6 came up more often than the other numbers when they played (8037-8038, 8040), and G8 notes that each of those numbers goes to a different player (8046-8048). This prompts Justina to respond, "Maybe it's a fair game" (8049). Adanna concurs (8054). While Adanna and Alia play the game, which Player A wins 10 to 7 (8123), Justina writes the sample space on her paper (8113-8114, 8169-8170). G8 asks her if she still believes the game to be fair (8116-8117), and Justina says "Yeah. . . . But maybe not a fair game" (8118, 8120). Shortly later, she explains:

> I'm just tryin' to see, um the different ways of each number to come up (8147-8148). . . . Because last time when I played this game, like some numbers they

came up, like they had different ways of, they had different ways to come up more than others did (8150-8152).

When G8 asks how they might use this information (8195-8196), Adanna explains that "The ones with the most combinations are gonna come out more than the less combinations" (8197-8198). With a bit of coaching from G8 (8218, 8223, 8227-8228, 8242, 8261), Justina finds all 20 combinations of addends to form the sample space (8272). She counts up and records the number of ways to obtain each sum, separating Player A's and Player B's numbers, as shown in Figure 27.

Figure 27. Justina writes the number of ways to obtain each player's numbers.



With some prodding from G8 (8313-8316), Adanna determines that there are 11 combinations favoring Player B and nine favoring Player A (8317, 8319). As G8 asks how these numbers might be interpreted (8322-8323), Justina opens the folder holding yesterday's papers and takes one out, looking at it (8325). The following conversation ensues (8326-8330):

Justina	This game we played, and Player A won. And this one Player B won.
G8	Uh huh. So you played only twice. [inaudible] What do the sums tell us? 11 that we got here and the 9 that we got here.
Justina	Player B has more of a chance of winning than Player A does.

It appears that Justina has determined that the game is not fair.

Chanel is briefly filmed explaining her thoughts about the game to G7. She says that at first she thought the game was fair “because it has the same amount of numbers” [for each player] (7123). But then, she continues, she decided that the game was not fair because Player B’s numbers are less likely to occur than Player A’s numbers using three dice (7126-7133). Asked how she determined that Player A’s numbers were more likely (7134-7135), Chanel explains that certain numbers, such as 8, 5, and 10, can be obtained two ways, while other numbers, such as 4 and 6, can only be obtained one way (7136-7152). Two of the three numbers Chanel named as more likely belong to Player B, so G7 asks, “Which one did you say again is easier to get, this list or this list?” (7157-7158). Chanel indicates Player A’s list at first, and then says, “Well actually no, I think this [Player B’s] list” (7160-7162). G7 asks Chanel to make a list of all the possible sums, telling her that she’s “off to a good start” (7163-7164, 7168). The camera leaves Chanel at this point.

In summary, two approaches to assessing fairness are seen with this task: using the sample space and reliance on experimental results. Chanel briefly entertains the equiprobability assumption, but she abandons it as she begins to consider the different combinations for each sum. Kianja and Brionna are the only students studied who seem certain that the game is unfair. Their evidence is based largely upon the sample space, with the support of a small number of experimental trials. Other students, i.e. Chris, Terrill, Jerel, Ian, Justina, and Adanna, are indecisive, as they vacillate between declaring the game fair and saying that it is unfair. Chris, Ian, and Justina are inclined to give the sample space more weight in assessing fairness, yet their opinions are swayed when experimental data contradict their conclusions. Terrill, Jerel, and Adanna attend more to the results of playing the game than to the sample space. The tension between the theoretical and experimental approaches is played out in Jerel and Ian's heated exchange about whether or not the game is fair (6510-6540).

4.2.2.2 If You Think the Three Pyramidal Dice Game is Unfair, How Could You Change It to Make it Fair?

On the first day of this task, Kianja develops the sample space showing 26 outcomes for Player A and 32 for Player B. In her first attempt to make the game fair, she writes two columns of numbers showing the number of ways each player has to obtain their sums (6567-6572):

1	6
3	10
12	7
9	6
1	3
26	32

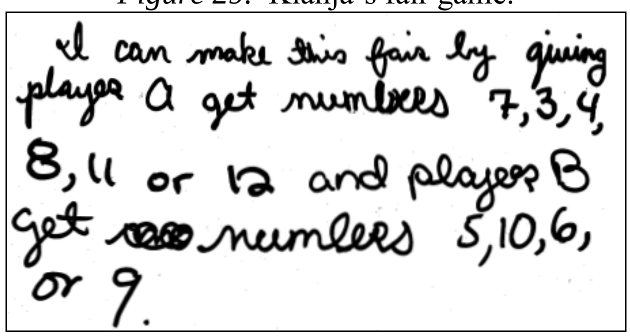
She then matches pairs of numbers in the first column with pairs in the second column that have the same sum: 1 and 12 in column A with 10 and 3 in column B;   3 and 9 in column A with 6 and 6 in column B (6573-6576).   Her efforts are interrupted when T5 stops by to ask her about her progress (6577).

When Kianja resumes the task, she notes that there are 58 outcomes in all (6654) and asks Brionna to tell her what half of 58 is (6661). Brionna answers "24 and 34" (6665), but Kianja corrects her, saying "it'd be 29 plus 29" (6670). It what appears to be a triumphant gesture, Kianja says, "Oh yes!" and raises her arms above her head (6682). She partitions the outcomes into two sets of 29, as shown in Figure 28 (6687-6688).

Figure 28. Kianja partitions the sample space to make the game fair.

Laughing and excited, Kianja says, “Yes, yes. . . . It’s 29 and 29. 29 ways and 29 ways! . . . You know what? I can make this game fair” (6689, 6693, 6697). She writes the rules for a fair game on a transparency, as shown in Figure 29 (6704).

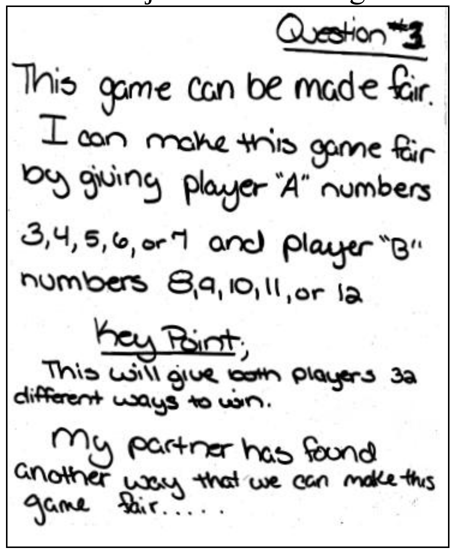
Figure 29. Kianja’s fair game.



Although Kianja’s sample space is missing six outcomes, this partition does produce a fair game.

The next day, Kianja discovers the missing outcomes and revises her sample space. Independently, she and Brionna create fair games using different partitions. Brionna gives the numbers 3, 5, 8, 9, and 11 to Player A and 4, 6, 7, 10, and 12 to Player B (7404-7405). Figure 30 exhibits Kianja’s fair game.

Figure 30. Kianja's second fair game.



Question #3
This game can be made fair.
I can make this game fair by giving player "A" numbers 3, 4, 5, 6, or 7 and player "B" numbers 8, 9, 10, 11, or 12
Key Point;
This will give both players 32 different ways to win.
My partner has found another way that we can make this game fair.....

Chris and Terrill are also filmed in their attempt to create a fair game. Chris' sample space shows seven outcomes favoring Player A and ten favoring Player B (Figure 23). Terrill suggests that they "take away one of Player B's numbers, like 11" (7871). This would leave Player B with nine outcomes, still more than Player A has, but Terrill says, "give him 11 and it'll be tied up" (7873). Chris counts "nine, eight and nine", indicating that the two would not be tied (7875). Together, Chris and Terrill work out how to make the game fair (7877-7883):

Terrill	Give him 11 and …
Chris	And whoever gets 10 …
Terrill	Give him 11 and then take out, just take out …
Chris	One of the tens, one of the tens. [The sample space shows two outcomes for 10.]
Terrill	Give him 11 …
Chris	Like either one of the tens.

The strategy the boys devise is similar to what Chris has done in previous tasks: with an odd number of outcomes, one is removed and the others are split between the two players. Chris' sample space shows that 10 can be achieved with a roll of 4, 4, 2 or a roll of 4, 3, 3. He proposes to omit one of these outcomes and move 11 to Player A's column. This is a reasonable strategy, but it does not produce a fair game because Chris and Terrill have not found all 64 equally likely outcomes in the sample space.

4.2.2.3 How Are Experimental Data Used as Evidence in the Three Pyramidal Dice Game?

Of all the games played, this one is the most closely matched, with Players A and B having probabilities 45% and 55% , respectively, of winning a point. Therefore, conclusions based on a small number of trials are especially unreliable.

However, as documented in section 4.2.2.1, students use limited amounts of experimental data to form conclusions and to justify arguments. Of the students studied, Jerel and Terrill appear to rely most heavily on empirical data, giving it more weight than the *a priori* arguments put forth by their partners. As in the previous activity, Jerel asserts that the game must be fair after each player has won one game (6515-6517), even though Ian argues forcefully that Player B has three more combinations than Player A (6533-6536). Surprisingly, Ian comes to agree with Jerel after Player B wins the third game, saying, " Yeah, it's fair. They each have enough of a chance . . ." (5984).

When Chris shows Terrill his sample space with more outcomes favoring Player B, Terrill insists that they play the game to determine whether it's fair (7790-7791), and he taunts Chris when Player A takes the lead (7811-7815). Like Ian, Chris, who had been convinced that Player B was favored, changes his opinion when the score becomes tied and proclaims the game to be fair (7829). It is possible that Chris was influenced by G4's frequent questioning: "What do you think, Chris, because A is winning more?" (7817). "Is this game fair, Chris? It's becoming equal now. Do you think it's fair?" (7827-

7828). It is likely that G4's intention was to engage Chris to talk about his experimental results. However, asking these questions after just a few rolls of the dice begs for a conclusion to be drawn before it is appropriate to do so. It is in response to G4's persistent questioning that Chris says the game is fair. Just a moment later, G4 asks, "What do you think, Chris? What do you think about this now? B, B has one more. So what do you think?" (7831-7832). Ultimately, Chris and Terrill agree that the game is not fair.

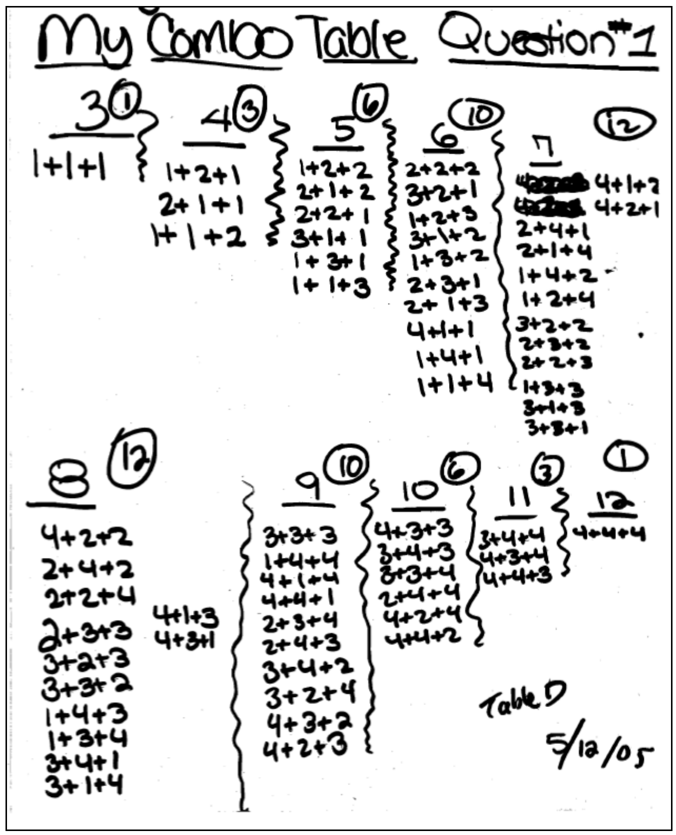
Justina initially wants to "look at the possibilities for getting each number" (6779), but her partner Adanna begins rolling the dice before Justina has the opportunity to do so. After one game, with a close score of 10 to 8, Justina concludes that the game is fair (6843-6844). Justina will change her opinion with each shift in the experimental data: when Player B leads the next game 5 to 1, she states that the game is not fair (6899), but moments later when Player A wins with a score of 10 to 9, she decides that the game is fair (6929). T9 may have contributed to Justina's frequent change of opinion, as he, like G4 with Chris, asks Justina to make judgments on the basis of a small amount of data: "Okay, so Player A gets only one? Player B gets 5? So, who gonna win, you think?" (6906-6907). "So Player A wins, all riiight. OK. So what do you think, it's fair or not fair?" (6926-6927). In the end, Justina determines that the game is unfair on the basis of her sample space (8330).

From the start of this activity, as in the previous one, Kianja develops the sample space to make a decision about fairness. However, she does cite the results of just 9 rolls of the dice as corroboration that her conclusion is correct (6355).

4.2.2.4 What Is the Sample Space for the Sum of Three Pyramidal Dice?

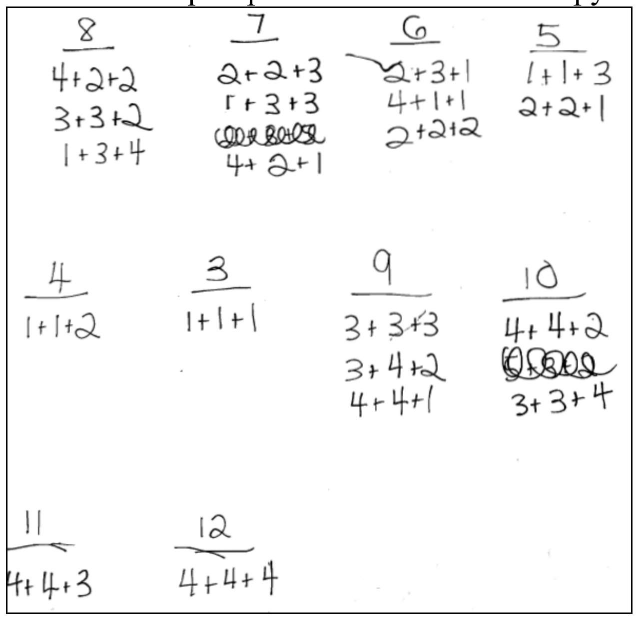
When three pyramidal dice are tossed, the 64 possible equally likely outcomes include 20 distinct combinations. Over the two days of this activity, Kianja is the only student who discovers all 64 outcomes. Her sample space is shown in Figure 31.

Figure 31. Kianja's sample space for the sum of three pyramidal dice.



Justina is the only other student studied who finds all 20 combinations. Because she believes that the order of addends "doesn't matter" (8354), she does not list permutations as different events. Justina's sample space is shown in Figure 32.

Figure 32. Justina’s sample space for the sum of three pyramidal dice.



Both Kianja and Justina benefitted from interventions by researchers who suggested that they seek outcomes they may have missed. The following examples illustrate these interventions:

> R3 (to Kianja): Are you sure you got all of them for 8? (6176)
> R4 (to Kianja): Can you get 9 using twos? (6185)
> R4 (to Kianja): Why can’t you do 3, 3, and 1 for 7? (6235)
>
> G8 (to Justina): So any ideas for the 8? Or is that all? (8218)
> G8 (to Justina): Are we missing anything for 7? (8223)
> G8 (to Justina): Is this all you can do for 10? (8261)

Though Justina and Kianja organized their sample spaces by listing addends under each sum, it is not evident from their discussion or from their written work that either girl used a strategy other than guess-and-check to generate addends. In fact, there was no evidence of a generative strategy by any of the students studied.

Chanel’s approach does not exhibit any organization. Asked by G7 to make a list of all the possible outcomes (7163-7164), Chanel begins writing “4 + 3 + 3 = 10, 2 + 1 + 4 = 7" (7169-7170). G7 leaves Chanel to work on her own. She indicates that she will come back to check on Chanel's progress (7167-7168).

Chanel is not filmed for the remainder of this session, but her papers are on file. One paper appears to be the one she started with G7 present, as it begins with the sums 4+ 3+3 and 2+1+4. It shows that she enumerated 20 outcomes for the sum of three pyramidal dice (Figure 33). She does not show sums of 4 or 5 on this list, but she has told G7 that there is just one way to get a sum of 4: 1+2+1, and two ways to get a sum of 5: 2+2+1 and 3+1+1 (7141-7149). Combining these stated outcomes with what she has written, Chanel has 22 distinct outcomes (4+1+3 is listed twice). Her list includes some permutations for sums of 7, 8, and 9, but only combinations for the other sums. She is missing the combinations (1, 3, 3), (2, 2, 3), (2, 3, 3), and (2, 3, 4). It is of interest that she has written the outcomes in no particular order.

Figure 33. Chanel enumerates some outcomes for the sum of three pyramidal dice.

$4+3+3=10$
$2+1+4=7$
$4+4+3=11$
$1+3+2=6$
$4+2+2=8$
$3+1+4=8$
$4+1+3=8$
$4+1+1=6$
$4+2+3=9$
$2+2+2=6$
$1+1+1=3$
$3+3+3=9$
$4+4+4=12$
$4+2+1=7$
$4+3+1=8$
$4+1+2=7$
$4+1+3=8$
$4+4+1=9$
$4+2+4=10$
$4+1+4=9$

Chris' sample space (Figure 23) shows 17 combinations of addends. He has overlooked (1, 3, 3), (2, 3, 3), and (1, 4, 4). Chris believes that different arrangements of addends are "the same thing" (7686) and so he does not include permutations in his sample space.

Jerel's partner Ian finds 15 combinations in his sample space (Figure 24). Missing are (2, 2, 2), (1, 3, 3), (2, 3, 3), (2, 2, 4), and (2, 3, 4). Like Chris and Justina, Ian believes that permutations of addends are the "same thing" (6469) and so he does not include them in his sample space.

When discussing the number of opportunities for each player to obtain a point, all the students treat the outcomes in their sample space as equally likely.

4.2.2.5 Does Color or Order of Dice Matter With the Sum of Three Pyramidal Dice? Interventions and Conversations

As in the previous activity, members of the research team challenge students to support their assertions about the sample space. Some try to scaffold student learning by demonstrating ways of representing dice outcomes and, in this activity, some are persistent in their questioning. As before, these efforts are met with mixed results.

On the first day of this activity, R1 asks Chanel to imagine a television game show in which a player can win a million dollars if certain numbers come up on three dice. She asks Chanel which option she'd prefer: that the numbers had to come up on specific colored (white, red, and blue) dice, or that it didn't matter on which dice the numbers appear (5986-5989). Chanel says that there's a better chance of winning if the numbers are not required to appear on specific dice (5992-5993). She starts to explain (5998-6002):

> Because, um, it makes a better chance because if you, if you were to have 4, 2, and 3 and you had to get 'em in the same, exact way they put it, then that means you have to exactly get 4,2,3, like say if you switched it around and you had 2,4,3, then, on the other hand you could win the million dollars even if it's like …

R1 suggests that Chanel think about how much better the chance to win would be if the colors of the dice didn't matter (6003-6013). She advises Chanel to write the different ways to obtain a roll of 4, 2, and 3 on her paper (6016-6018), and proposes that Chanel keep track by writing the heading "white, red, blue" (6020-6025):

> How do you, how are you gonna keep track? This one is white, this one is red, this one is blue. You could get a 4,2,3 on white, red, and blue, right? [aligns the dice in this way] So why don't you write "white, red, blue" up there. Well, just the letter's good enough. R, B. Okay. Now, now when you, now is that the only way you could get a 4, 2, 3? Write all the ways.

Before the camera moves on to another table, Chanel is seen writing the following, reproduced in Figure 34 (6033-6041).

Figure 34. Chanel shows different arrangements of 4, 2, and 3 (reproduction).

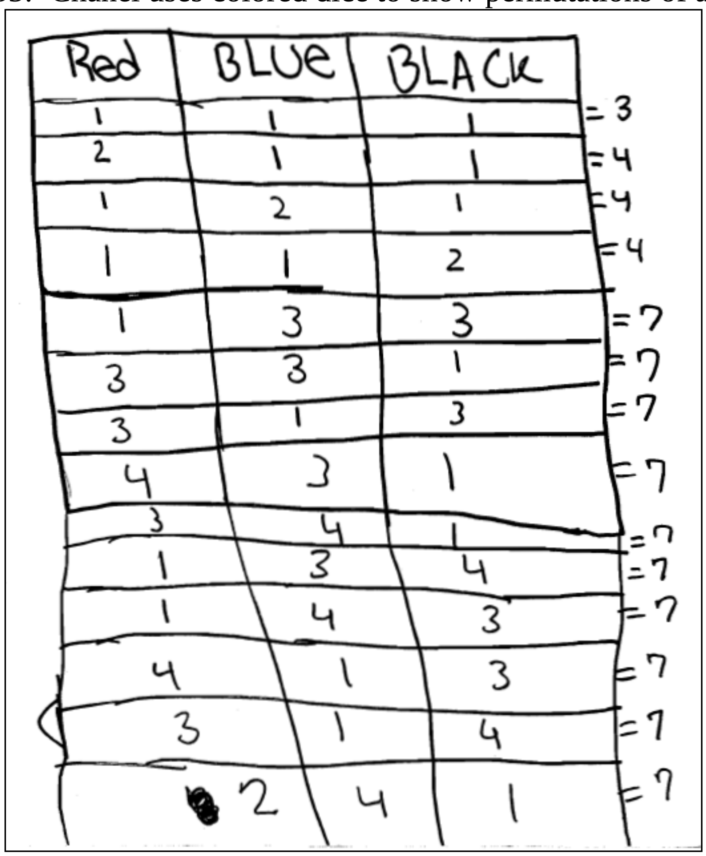
white	R	B
4	2	3
W	B	R
4	3	2
R	W	B
2	4	3
B	R	
3	2	4

Although Chanel has written the numbers in different orders, in each case she shows 4 on the white die, 2 on the red die, and 3 on the blue die. Though the numbers were permuted, they remain associated with the same colors. Just before the session adjourns for the day, R1 asks Chanel to think about the number of different ways to get a sum of 10 with three pyramidal dice (6044-6045).

The following day, Chanel tells G7 that there is just one way to get a sum of 4: 1+2+1, two ways to get a sum of 5: 2+2+1 and 3+1+1, and one way to get a sum of 6: 2+3+1 (7141-7151). She does not state permutations as different events. It appears that at this time Chanel has not made the connection that R1 attempted to foster. However, when G7 asks Chanel to make a list of all the possible outcomes (7163-7164) Chanel produces a list that includes some, but not all, permutations (Figure 33).

Unfortunately Chanel is not filmed for the remainder of this activity, and so the events surrounding her next paper are undocumented. This paper shows that she used red, blue and black dice to demonstrate a number of permutations of addends for sums of 4 and 7. Perhaps Chanel adopted this approach based upon her earlier conversations with researchers.

Figure 35. Chanel uses colored dice to show permutations of addends.



Red	BLUE	BLACK	
1	1	1	= 3
2	1	1	= 4
1	2	1	= 4
1	1	2	= 4
1	3	3	= 7
3	3	1	= 7
3	1	3	= 7
4	3	1	= 7
3	4	1	= 7
1	3	4	= 7
1	4	3	= 7
4	1	3	= 7
3	1	4	= 7
2	4	1	= 7

Kianja and Brionna also list permutations of dice sums as distinct outcomes. At the end of Activity 3 Kianja listed permutations in her sample space, but she did not demonstrate a strong conviction that symmetric pairs are different events. She concurred with other students who presented the sample space without them. At the start of this activity, however, Kianja immediately writes permutations of sums as different events when she enumerates the sample space. During the first 35 minutes of doing so, Kianja is visited by R1, R3, and R4. No one questions her decision to include permutations, and this might be viewed as tacit acknowledgment that Kianja is correct. Finally, though, R3 raises the question, asking Kianja why she shows three ways to obtain a 4 but only one way to obtain a 3 (6277-6278). She begins to defend her decision, referring back to the two-dice game (6279-6293), but she quickly defers to R3's implied suggestion that there is only one way to obtain a sum of 4 and adjusts her counts accordingly (6294-6300):

R3	But isn't there only 2, 1, and 1 to get 4?
Kianja	[brief pause] Well, yeah, but we switched them around, so. We will divide it by 3 if you want. All right, so then it would be …
R3	Oh no, no, no. Don't change it.
Kianja	No, I'm just sayin', no, I'm sayin' if we didn't want to add the little things in there. So that'd be 1, 1, 4, 3, 1 [revising the number of ways to obtain each of Player A's numbers: 3, 4, 7, 8, and 12].

As in the previous activity, Kianja expresses a willingness to accept either interpretation of the sample space. When R3 asks her, "Which way is a better way of counting?" Kianja points to the list without permutations (6305-6308).

A few minutes later R1 returns to speak with Kianja. As a result of their discussion, Kianja reverts to including permutations in her sample space (6314-6323).

R1	[to Kianja] What's the sum of these? [pointing to a pair of dice] Is there another way I could get that?
Kianja	[rearranges the dice]
R1	No, that's still the same. I just moved the dice around. I got a 4 on this [white] die, just moved it, and a 3 on the black.
Kianja	[changes the dice to show 3 on white, 4 on black]
R1	Ah, now you've got it. That's different, isn't it? You got a 4 on there. So they're different, aren't they?
Kianja	Um humh.
R1	Don't let somebody talk you out of that.

Kianja still expresses some uncertainty, however, as she allows for the alternative interpretation, while R1 offers encouragement for Kianja's approach (6324-6334):

Kianja	I don't know. I was saying, I was saying if you wanted to do it this way … [taps her paper]
R1	Yes.
Kianja	Then that's how you would do it. But I didn't do it this way. This is the way I did it.
R1	So tell me the way you did it again.
Kianja	[points to her original sample space] See, I switched all of 'em. 4+2+2 and 2+4+2 and then …
R1	You saw them all as different.
Kianja	Yes.
R1	Okay. Very good.

Later that day, T5 asks Kianja why she believes that permutations are distinct events. He points out that other people don't seem to think so. This time, Kianja does not change her opinion (6599-6609):

T5	I've been talkin' with some other people who don't think these [different arrangements] are the same, so could you, how could you convince me that they are different?
Kianja	They different, to me, if it's on a different dice it is different.
T5	Okay. Is that, is that, is that all you think about it? Is there anything else you think? Is there anything else you could do to convince me besides they're on different dice so they're different?
Kianja	'Cause it really depends on the die that it's on.
T5	It depends on the die that it's on? So that 1, 4, 2, …
Kianja	1, 4, 2, this would be different if this was a 4, this was a 1, and this was a 2. [demonstrates with 3 dice]

The next day, Kianja's partner Brionna tells G6 why she believes that permutations are different events (7214 -7222):

G6	Now, now here's somethin' I wondered, if you could explain to me. So you've got a 3+ 2 + 1. Now isn't that the same thing as 1 + 2 + 3?
Brionna	It is, but on the dice, on the dice, you could write this one, this could be 3, this could be 1, and this could be 2 [turns the dice to demonstrate]. 'Cause they come up different on each dice.

G6	Okay. Okay. So the order in which you write it, you're sayin' that makes it different.
Brionna	Yeah.

While Kianja, Brionna and, to some degree, Chanel have somewhat haltingly come to the conclusion that the order of addends makes a difference, the other students studied hold fast to the conviction that it doesn't matter, despite the interventions of research team members.

T5 uses colored dice to suggest to Terrill that permutations are different events (5850-5854):

T5	Is 4, 4, 3 the same as 3, 4, 4?
Terrill	Yeah [inaudible].
T5	Even if I have different color dice?
Terrill	If you had different color dice [inaudible] it would be the same numbers on each of 'em.

When Terrill dismisses T5's suggestion, T5 proposes another way to think about the outcomes: as three-digit numbers or sums of money where place value is determined by the die's color – red, white, or blue. Though Terrill clearly understands the difference between $241 and $412, he does not make the connection between this representation and 2+4+1 or 4+1+2 on the dice, and he asserts that the sample space would have the same twelve outcomes that his partner Chris enumerated earlier (5858-5877). Even so, T5 challenges Terrill's assertion and continues promoting the place-value representation. The following dialogue illustrates this intervention (5876 – 5892):

T5	You think that it's gonna be the same amount of outcomes.
Terrill	Yes, because you're using the same numbers.
T5	But here I see you've listed um 1, 1, 4, right? Now, if I'm, if I'm talking about roll the dice and you get this amount of money, right, what one, which one do you want to roll? Do you want to roll it as a 1, 1, 4? Let's say I always …
Terrill	4, 1, 1

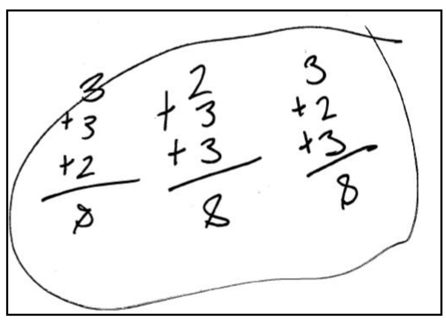
T5 Oh, you want 4, 1, 1. Okay. So let's say it depends on the number, uh, the color of the dice, right? So if I say that the blue always has to be in the hundreds position, the red always has to be in the tens position, and the white always in the ones. Right? What, what's gonna happen if, if I can only, let's say this is, this is the order that they have to be recorded in with the table: blue, red and white. And I'm just writing down the outcome. What's on the die. So I roll it now [rolls 3 dice]. This time it's a blue 4, a red 3, and a white 2. So that's four thirty-two. Right?

Terrill Uh huh.

Terrill appears to go along with T5's argument, but he counters by demonstrating that any permutation of 3, 3, and 2 will give a sum of 8 (5895-5898, Figure 36):

> All the um, all the thing, no matter where you put it, no matter if, all right, take 3, 3, 2. What's 3 + 3 + 2? [writes this sum in a column] Eight, right? Okay, 8. What's 2 + 3 + 3? Eight. What's 3 + 2 + 3? Eight. So it doesn't matter how you put it.

Figure 36. Terrill shows that different permutations yield the same sum.



T5, however, appears unwilling to yield on this point. He compliments Terrill for writing the numbers in different sequences (5902-5904), and asks him to make a table recording "what's on the blue dice, what's on the red dice, red die, and white die" (5909-5910). A few minutes later, Terrill announces, "All right, I'm done" (5922). His table, showing no permutations, is reproduced in Figure 37.

Figure 37. Reproduction of Terrill's table showing outcomes on blue, red, and white dice.

Blue	Red	White
4	4	3
4	4	4
2	1	3
3	3	3

The intervention by T5 with Terrill lasted about 15 minutes. In the end, Terrill maintained his original belief that, despite different colors on the dice or place-value considerations, the order of addends does not affect the sum of the dice and therefore should not be considered when enumerating possible outcomes.

Elsewhere in the room, T3 uses three colored dice to show Jerel and Ian different ways to obtain a sum of 4 (6460-6461). The boys insist that the permutations are not different "because all you did was switch 'em around" (6464). T3 then suggests rolling the dice one at a time and asks if that would make a difference (6474-6475). Jerel and Ian maintain that the order in which the numbers appear does not matter.

Over the two days spent on this activity, Justina develops the sample space showing all 20 combinations. On the first day, T9 asks Justina about the number of ways to obtain a sum of 4 (6957-6961).

T9	You think 1, 1, 2 is the only three number you can get 4?
Justina	I thought so.
T9	Okay. Good. Even if you have different colors?
Justina	Different colors don't mean anything.
T9	Doesn't mean nothing? [*sic*] Okay.

The following day, another member of the research team, G8, raises the same question, but she is not as willing as T9 was to accept Justina's response (8351-8358).:

G8	And it's still the numbers 1, 1, and 2, right? But would you

	consider this a different way, 'cause you know, you see, I uh, I just changed positions [inaudible].
Justina	It doesn't matter.
G8	How come it doesn't matter? I mean, now the white one is a 1 and, and this one is a 2.
Justina	But we're not focusing on the colors. We're just focusing on the numbers. 2+1+1 still equals 4.

Though Justina has clearly stated her opinion, G8 continues (8359-8362):

G8	Correct, but [unclear] you could just focus on the numbers and not focus on the colors?
Justina	Well it's not based on the color.
G8	Are you sure?

G8 appears unwilling to give up the argument, and so she turns to Justina's partners, Adanna and Alia, and continues this line of questioning (8369-8374, 8378-8384, 8388):

G8	But look, this is one way to get a 4, right? 2, 1, 1, yeah? But now look, if I make this change and put the 1 here, and the 2 here, this is still a combination for 4. But this is in a way different because now the blue is a 1, and this is a 2. So should we make a difference between these two ways of getting a 4? [. . .]
Adanna	That's the same thing.
G8	Well it's still the same numbers, but should we pay attention to the, to the way they come up? I mean do, does the 1 come up on this one or this one? Does the 2 come on this or this? Do they, should we care about that?
Adanna	[shakes head]
G8	No? [. . .]
Adanna	It's the same numbers, 'cept different combination of ways.

At this point, Justina appears to have tuned out the discussion. She rests her head on her arm on the desk and doodles with her pen (8385-8387). Both Justina and Adanna have told G8 that they don't believe the colors of the dice make a difference. G8 goes on (8397-8416):

G8	So, so this is the challenge that I'm throwing at you. Should we pay attention to where each number appears apart from what combination of numbers we have? So we have the combination 1,

	1, and 2, but where does the 1 appear, where does the 2 appear, and so on? Should we pay attention to that? I mean, does it have anything to do with chance and probability?
Adanna	I don't think it do.
G8	You don't think it should. Okay. [to Justina] What do you think?
Adanna	Justina!
Justina	[lifts her head from the desk] Huh?
G8	What do you think? Should we pay attention to the fact that, you know we can get the sum of 4 in those, at least those two different ways that I showed you. We still have the numbers 1, 1, and 2 but you know, these are showing different things.
Justina	[shrugs]
G8	I know, I know that in the problem it doesn't say anything about colors, but if you're thinking about it in terms of how likely it is for such combination to pop up, you know, does that make any difference?
Adanna	No.

At this point, G8 has asked ten times whether different arrangements of the addends should be considered as different events, and each time Justina or Adanna has answered no. G8 continues her questioning, asking whether a sum of 4 and a sum of 3 have the same chance to occur (8417-8418). Adanna says that she doesn't know (8419), while Justina and Alia indicate that these two sums are equally likely (8423, 8432). G8 asks, "What I just showed you before, that doesn't make any difference?" (8433). Alia shakes her head to indicate "no" and replies "They're just a different color combination" (8434).

As G8 continues to confront the girls on this issue, they tune out and stop responding. Despite G8's repeated insistence, the girls are not influenced to change their minds.

A similar, if not as lengthy, conversation is had by G4 and Chris. Again, a question is asked and answered, then asked again, repeatedly (7684-7694):

G4	If you get 2, 1, 1, and if you get 1, 2, 1, that's like, say [reaches across desk] …
Chris	It's the same thing.
G4	Say it's uh, say this yellow one is the first, okay? So let's say this is 1, this is, let's make it a 2, and this is 1, okay? [arranges the dice in this order] Look at this, 2, 1, 1, right? And if I, if I made this as 1, 2, 1 …
Chris	Same thing
G4	Do you think it's the same thing?
Chris	They both add, they both add up to the same thing.
G4	So why do you think it is the same thing?

Not only has Chris answered twice that 2, 1, 1, and 1, 2, 1 are the "same thing", he has explained why he thinks so: because they add to the same sum. When G4 asks again why Chris believes this, Chris explains again (7695-7707):

Chris	Because they both add up. Either way it's gonna add up to …
G4	Because they both add up to …
Chris	Four.
G4	Um humh. But, but, but do you think if this yellow one [die] is 2 and this green one is 1, and then this yellow one becomes 1, and this green one becomes 2 …
Chris	It's the same thing.
G4	Still it's the same thing?
Chris	Yeah.
G4	So you don't find any difference between the two?
Chris	[shakes head]
G4	Absolutely no difference?
Chris	[looking down, rubbing his arm, shakes head]

Both G8 and G4 seem so eager for their charges to recognize permutations as different events that they do not appear to attend to the students' answers. And, like Justina and her partners, Chris seems to tune out from the questioning as he looks away and shakes his head.

At the end of the day, Kianja and Brionna are the only students who have clearly come to accept, after some vacillation, the idea that permutations of addends should be counted as distinct events. Chanel's understanding is difficult to assess because her later work was not videotaped. Though her paper shows some permutations, her reasoning is not clear. Terrill, Jerel, Ian, Justina, Adanna, Alia, and Chris are all presented with different ways of representing dice sums, but they are not convinced that the color or order of the dice makes any difference.

4.2.2.6 Summary of Activity 4

Of all the students studied, Chanel is the only one who initially assumes the game is fair because each Player has the same number of sums. The other students have come to expect that they need to explore the sample space (Kianja, Justina, Chris, Ian) or play the game (Jerel, Adanna, Terrill) before declaring that the game is fair or unfair. Soon Chanel also realizes that some sums are more likely than others, and she, too, explores the sample space.

Kianja and her partner Brionna say they are certain that the game is unfair, with a sample space that shows 29 outcomes favoring Player A and 35 favoring Player B. The scant experimental evidence they obtain, a score of 6 to 3 for Player B, confirms their belief. Chris, Terrill, Jerel, Ian, Justina, and Adanna are not as certain. They change their opinions frequently. Chris, Ian, and Justina are inclined to give the sample space more weight in assessing fairness, yet their opinions are swayed when a few rolls of the dice disagree with their expectations. Terrill, Jerel, and Adanna take the frequentist approach and give little regard to the sample space created by their partners.

Kianja finds all 64 permutations that make up the equally likely events in the sample space; she is the only student to do so. Justina finds all 20 combinations and, despite being repeatedly challenged by G8, does not abandon her belief that permutations of addends amount to the same thing and therefore should not be counted as different events. Chris, Ian, and Chanel also attempt to enumerate the sample space, but they do not succeed in finding all the possible combinations. Chanel lists some permutations, but does not do so consistently. It appears that all of the students studied used a guess-and-check strategy to write the outcomes. The two girls who have complete lists received some assistance from members of the research team.

Kianja and Brionna each create a fair game by partitioning the 64 outcomes so that each player gets a point for 32 of them. Terrill and Chris, with 17 outcomes in their sample space, also try to make the game fair by removing one of the sums and reassigning another to Player A. Because their sample space is incomplete, the game they devise is not fair.

In this activity, the question of whether permutations of dice outcomes should be counted as different events is raised repeatedly, and it is remarkable that the students begin and end the activity with their beliefs about this issue unchanged. Kianja, who had accepted permutations as different events in Activity 3, begins Activity 4 with this opinion. She is temporarily sidetracked by a question from R3, but she recovers after a brief discussion with R1 and then maintains her opinion with conviction. Chris, Jerel, Justina, Adanna, Terrill, and Ian do not believe at the outset that permutations count as distinct events. Their beliefs are challenged and questioned by members of the research team, but they do not change.

As in previous activities, students who use experimental data to make inferences do so with a small number of trials. In this game, where the two players are more closely matched than in the other games, each change in score may lead to a change of opinion about fairness. Jerel continues to uphold that the game must be fair if both players can win.

4.2.3 Racing Games With Three Pyramidal Dice

For the last half hour of the final day of this session, the final IML session in grade 7, R3 gives the students a new racing game to play. Each group has a grid with the numbers 1 to 14 written across the bottom. A marker is placed in each of the 14 spaces in the bottom row. The students as a team are to pick five numbers, and they will play against a research team member, who gets the remaining numbers. For each turn, three pyramidal dice are rolled and the marker corresponding to the sum of the three dice is moved forward one square on the grid. The first marker to cross the finish line is the winner (8476-8481). Ice cream bars will be awarded to the winners (8471-8472).

At first, students at some of the tables play a variation of this game, choosing only two numbers instead of five. Perhaps the instructions were misunderstood (7511-7515). Jerel chooses 4 and 11 as his numbers for the first round (7437) and 11 wins the game (7472). Jerel is not videotaped explaining why he chose these numbers -- Ian's sample space shows only one way to get each of 3, 4, 8, 11, and 12, but two ways to get 5, 6, 7, 9, and 10 (Figure 24). For the next round, Jerel and Ian choose 4, 5, 6, 7, and 11 (7548). Jerel indicates that "we don't want 8" (7545). It appears that their choices are not entirely based upon Ian's sample space, which shows more combinations for 9 and 10 than for 4 and 11.

Kianja and Brionna choose 7 and 9 because "7 won and 9 won" (7485). It is interesting that the girls chose 9 rather than 8, which has a slightly higher theoretical probability. Their choices appear to be based on the frequency of occurrence and not on the number of favorable outcomes in the sample space. For the next round, however, the girls choose 5, 7, 8, 9, and 11 (7539). Kianja tells G7 that "I thought 7 and 8 would be the top numbers because they had the most [possible outcomes], right?" (7568-7569). Given the choice of any four numbers, Kianja says she would "pick 7, 9, 8, and 6" (7577), which are in fact the optimal choices according to the sample space.

Chris and Terrill pick 6 and 8 for the first round (7945, 7948). In Chris' sample space (Figure 23), 6 is the most likely sum. He smiles at his choice, saying "they got three [outcomes, more than any other sum]" (7948). For the next round, Terrill claims 5, 6, 7, 8, and 9 (7988). These numbers fared best in the first round of play (7980-7981).

Justina chooses 8 (8510), for which her sample space shows three possible outcomes (Figure 32). Her partners Adanna and Alia choose 4 and 10, respectively (8550, 8552), though there are other sums in Justina's sample space showing a greater likelihood than 4 and 10. For the next round, the girls as a team choose 5, 6, 7, 10, and 11 (8621-8622, 8626). While Justina appears to pick numbers that have been rolled frequently (8576, 8579) and numbers that her sample space shows are more likely, Adanna's choices (8603) seem more subjective.

The session ends with students excitedly declaring victory in anticipation of their reward (7557, 7559, 7563, 7564).

4.2.4 Summary of Grade 7 Results

The subjective intuitions that some students exhibited in grade 6 are no longer evident in grade 7. Several students (Chanel, Justina, Chris, Dante, David, and others), though, show signs of the equiprobability bias at the start of the grade 7 probability activities, as they assert that all sums rolled by a pair of pyramidal dice are equally likely. By examining their experimental data and/or the sample space, however, they conclude (with the exception of David) that some sums are more likely than others. At the start of the second activity, Chanel is the only student studied who briefly entertains the notion of equiprobability, but she abandons this idea rather quickly. By the end of the grade 7 sessions, it seems that all the students studied realize that dice sums are not equally likely. Each student has produced a sample space for the sums of two and three pyramidal dice or s/he has worked with a partner who has done so.

Two students, Kianja and Brionna, make exhaustive lists of all the equally likely outcomes for both games. Chanel, Justina, Chris, and Ian find all 10 possible combinations of addends for the sum of two pyramidal dice, but only Justina finds all 20 combinations of three dice. These students do not believe that permutations of addends should be counted as different events. None of the students appears to have used a strategy other than guess-and-check to develop the sample space for the sum of three pyramidal dice.

Though Justina, Adanna, Chris, Jerel, Ian, and Terrill are challenged by members of the research team to consider permutations of addends as distinct events, the students hold their ground and their opinions are not swayed by these interventions.

Like last year, students who use experimental data to make interferences do so with a small number of trials. Their level of reasoning about experimental probability is still in the transitional stage.

CHAPTER 5 - FINDINGS

In this chapter I will discuss the findings from this study that address the research questions:

1. What understandings about probability (particularly fairness, sample space, probability of an event, probability comparisons) do the students exhibit?
2. How do these understandings change through the course of IML sessions?
3. What connections, if any, do the students make between experimental and theoretical probability?

The chapter begins with a brief discussion of the overall findings. Following that, I will trace the development in the above-named areas of each of the five focus students: Chanel, Chris, Jerel, Justina, and Kianja, as well as their partners for some of the activities.

5.1 Overall Findings

From the start of the grade 6 activities, students exhibit a shared understanding of fairness, or actually unfairness, in claiming that the player with more outcomes has the advantage in a game with one die. Though at least five of the sixth-grade students contend that certain numbers on a die are more likely than others, this misconception is not apparent the following year.

To determine whether or not a game is fair when two or three dice outcomes are summed, several of the students start with the assumption that all sums are equally likely and then, after playing the game, begin to explore the sample space. For the games involving two dice, all the students who attempt to write the sample space are successful in finding all possible combinations of addends and correctly assessing that the game is not fair. For the game in which three pyramidal dice are summed, students use primitive strategies to generate outcomes and so they may not discover all the possible combinations. Kianja is the only student who finds the complete sample space for both the two- and three-pyramidal dice games. She counts permutations of addends as different events.

In grade 7, graduate interns demonstrate ways of representing dice outcomes with the intended result that students would recognize permutations of addends as distinct outcomes. Their efforts are largely unsuccessful.

A few students take the frequentist approach to determine whether a game is fair, however their judgments are based upon a small number of trials. In addition to the representativeness and availability heuristics, at least three students use a hybrid of the outcome approach and representativeness to decide that a game is fair if it is possible for either player to win.

5.2 Determining Fairness

All four of the dice games analyzed during the IML sessions are unfair. For reference, the games are summarized in Table 6.

Table 6.
Summary of IML Dice Games

Activity	Grade	Dice used	Player A's numbers	Player B's numbers	P(B wins point)	P(B wins game)
1	6	1 cube	1,2,3,4	5,6	1/3	.065
2		2 cubes	2,3,4,10,11,12	5,6,7,8,9	2/3	.935
3	7	2 pyramids	2,3,7,8	4,5,6	5/8	.869
4		3 pyramids	3,4,7,8,12	5,6,9,10,11	35/64	.661

As students grapple with the question of whether or not a game is fair, they sometimes reveal not only their views about fairness, but also their thinking about the likelihood of an event, probability comparisons, sample space, and experimental probability. In the sections that follow, any references to levels of probabilistic reasoning are based upon the framework developed by Jones et al. (1999), which is discussed in Chapter 3 and summarized in Table 2 on page 50. Briefly, the framework is based on four developmental levels of reasoning (subjective, transitional, informal quantitative, and numerical) across various probability constructs, including the ones mentioned above.

5.2.1 Tracing Chanel's Assessments of Fairness

Like many of the other sixth-grade students, Chanel quickly recognizes that the game in Activity 1 is unfair. She explains, "'Cause it's like 1, 2, 3, 4, and then it's only 5 and 6" (864-865). She makes the game fair by assigning 4, 5, and 6 to one player and 1, 2, and 3 to the other (864-866).

In the second activity, Chanel initially asserts that the game is unfair in Player A's favor because Player A has six sums to Player B's five. She applies the equiprobability bias in assuming that all 11 sums are equally likely. After playing one game, however, which Player B wins with a score of 10 to 5, Chanel decides that the game is fair (1102). Abandoning equiprobability, she notes that 11 and 12 are not "usual to pop up" (1106-1107), and so Player's A's presumed advantage is offset by having these two numbers. When Player B wins a second game, Chanel continues to claim that the game is fair (1131, 1192), explaining that "single numbers" like 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10 are "usually […] the ones who really pop up the most" (1132-1134). She explains that 11 and 12 "have two different numbers or […] two of the same numbers. And two of the same numbers don't really pop up" (1135-1137). Perhaps Chanel is applying the availability heuristic and recalling dice games in which doubles are special events. She seems to exhibit a deterministic view of dice outcomes when she tells G1, “see if I go like this [cupping the dice in her hands and shaking] and I drop it, it’s gonna be a 6 and 4”(1148-1150), two different numbers rather than two of the same. While noting that “we keep rolling it but 12 or 11 doesn’t pop up that much” (1171-1172), Chanel does not provide a quantitative rationale for the infrequency of 11 and 12. Her partner Danielle simply states that these numbers don’t come up “because we don’t roll it […], it doesn’t come” (1174).

In grade 6, Chanel shows evidence of operating at the subjective level of reasoning about probability comparisons. Her incorrect conclusion that the game is fair is based upon personal judgment rather than a quantitative argument.

A year later, in grade 7, Chanel appears to have advanced to the transitional level of probabilistic reasoning. At the start of Activity 3, Chanel agrees with Dante’s explanation, based on equiprobability, that the game is unfair in Player A’s favor because Player A has four numbers and Player B has three (2975-2978), but she changes her opinion after playing the game three times and finding Player B to be the winner (3396-3397, 3405-3406). She constructs the sample space showing 10 combinations of addends and determines that Player B has “six chances” while Player A “only ha[s] four” (3712). She writes that “the game is unfair because player B has more ways to find there [*sic*] answer than player A has” (Figure 12). In her presentation to the class, Chanel does not discuss six chances vs. four chances, but emphasizes that Player B’s numbers can each be obtained two different ways while Player A’s numbers can only be obtained one way (3394-3395). Her quantitative part-to-part comparison and her focus on the number of ways each player has to obtain his sums seem to fall in the transitional category, just shy of informal quantitative reasoning.

Just one week later, given Activity 4, Chanel briefly returns to the equiprobability assumption, declaring the game to be fair because each player "has the same amount of numbers" (7123). As with the previous activity, Chanel changes her opinion after playing the game. She considers some of the ways to obtain certain sums, but she does not make an organized list of the outcomes in the sample space. At first, she tells G7 that Player A's numbers are more likely than Player B's numbers (7132-7133), but as she talks about some of the ways to obtain sums of 5 and 10, she says that the game favors Player B (7162). Chanel's reasoning about sample space, at the transitional level, may be an impediment for her to assess the fairness of this game using quantitative judgments. Consequently, she appears to have slipped into less precise quantitative reasoning about probability comparisons than she exhibited with the previous activity.

In summary, for each of the games involving the sum of two dice, Chanel begins the task assuming that the sums are equiprobable and later changes her opinion upon playing the game. In grade 6 she relies on personal beliefs and perhaps the availability heuristic to incorrectly conclude that the game is fair. In grade 7, once she rejects the equiprobability assumption, her reasoning becomes more advanced as she uses the sample space to argue that the game in Activity 3 is unfair. However, she does not immediately transfer her strategies from Activity 3 to the next activity. That she revisits the equiprobability assumption, if only briefly, in Activity 4, shows some instability in her understanding. Her difficulties in enumerating the sample space for the sum of three pyramidal dice prevent her from making a reasonable judgment about the fairness of that game.

5.2.2 Tracing Chris' Assessments of Fairness

Chris acknowledges that the game in Activity 1 is unfair, explaining that "you gotta have like three choices to win" (169). He states that the probability Player A will get a point "is 4 out of 6, 'cause there's six numbers on the dice and he has four chances of getting it" (1831-1832). After playing the game with one die, Chris invents a new game in which two dice are rolled. Player A gets a point for rolling an odd sum, and Player B gets a point for an even sum (202-204). He determines that his game is fair for two reasons: a score of 10 to 9 (213), and the fact that there are six odd and six even numbers from 1 to 12 (220-221). These reasons suggest both the representativeness heuristic and the equiprobability bias. When G2 points out that there are only 11 possible sums when two dice are rolled, Chris attributes his win to "skills" (246).

Chris tells R2 that before playing the game with two dice for Activity 2, he thought it was unfair "'cause Player A it had like, it had 3 small numbers, which are 2, 3, and 4, and you really can't get 'em" (1946-1947). After playing the game, which Player B won with a score of 10 to 3 (2020), Chris decides to enumerate the sample space "because after we played the game we realized that um Player B had, since it had larger numbers it had more chance of getting 'em" (1983-1984). Though Chris lists thirteen outcomes favoring Player B and eight favoring Player A (1997, 2001), he explains Player B's advantage in terms of having more big numbers than Player A has (Figure 4).

Chris' theory that big numbers are more likely pertains not only to the sum of two dice, but to an individual die as well. He explains that although a sum of 6 and a sum of

7 each have three possible sets of addends, 7 is more likely than 6 "'cause it takes more smaller numbers to make up, um the 6. And for 7 it takes like most, more large numbers to make […] it up" (2109-2111). He contends that the numbers 4, 5, and 6 are more likely than 1, 2 and 3 on the roll of a single die (2121-2127). This belief may be the result of an application of the availability heuristic in which Chris recalls that 4, 5, and 6 are more likely than 1, 2, and 3 when the sum of two dice is considered. Chris attempts to illustrate his theory by rolling a die, but the smaller numbers prevail in 12 of 22 rolls (2225-2226).

It is difficult to classify Chris' level of probabilistic reasoning in grade 6, as he demonstrates characteristics of the subjective level in his large/small number theory, the transitional level in his use of representativeness and equiprobability, and the informal quantitative level by stating the numerical probability of a simple event. Some of Chris' statements are contradictory: he tells R2 that the one die game would be fair with any allocation of three numbers to each player (1910-1915), yet he later says that 4, 5, and 6 are more likely than 1, 2, and 3 (2121-2127).

In grade 7 Chris begins Activity 3, the game with two pyramidal dice, by applying the equiprobability bias when he calls the game unfair because Player A has four numbers and Player B has three (5385-5387). His theory about large and small numbers is not transferred to pyramidal dice, or perhaps he no longer believes it. He explains that this game would be fair if each player had three different numbers and no one got a point for the remaining number (5395-5397).

Chris plays one game, which has the unlikely result (probability .003) that Player A wins with a score of 10 to 3. Player B wins the second game, 10 to 6, and the third game is close. Though these scores might suggest that the game is fair, Chris has, on R4's suggestion (5475-5477), recorded the individual dice outcomes and so he begins to consider the number of ways to obtain each sum. He finds four sets of addends favoring Player A and six favoring Player B (5547-5552). He says, "so it still isn't fair, so Player B will win" (5552). He also notes that Player B has two ways to obtain each of his sums, while Player A has only one (5557-5558). Now, to make the game fair, Chris suggests dividing the two ways to get a sum of six between the two players and leaving the other numbers as they were originally assigned (5684-5686). This strategy would equally partition the 10 outcomes that Chris has identified.

To analyze the game with three pyramidal dice in Activity 4, Chris immediately begins to write the sample space (5748). It seems he has abandoned the equiprobability bias. Initially, he finds six outcomes favoring Player A and six favoring Player B (Figure 20) and notes, "they're both equal, they're equal" (5768). He plays two games with Terrill, and Player A wins both of them. Chris maintains that the game is fair because each player has six chances to win a point (7662-7666). Later, Chris discovers additional outcomes that give Player B the advantage, and he changes his opinion about the fairness of the game (7751). Terrill insists that "you have to play it first to see if it's really fair" (7790-7791), and as they play the game Player A takes the lead. While Terrill taunts Chris about Player B falling behind (7811-7815), G4 asks for an update of Chris' opinion after each roll of the dice (7818, 7825, 7828, 7832, 7835). When the score becomes tied (7828), Chris succumbs and says, "Yeah, I think it is fair" (7830). Perhaps Chris' earlier assessment that the game is unfair is vindicated when Player B ultimately wins the game by two points (7837). Apparently Chris and Terrill have come to believe that the game is unfair because they work to devise a fair game (7857-7887).

Over the two years of IML probability activities, Chris has progressed from subjective judgments to making decisions about fairness on the basis of the sample space, albeit an incomplete one. He appears to be approaching the informal quantitative level of reasoning.

5.2.3 Tracing Jerel's Assessments of Fairness

In grade 6, Jerel readily states that the one-die game of Activity 1 is unfair: "We already knew it was unfair because Player A had more choices to choose from than Player B" (143-144). He notes that Player A's chances have a "higher percentage" (1819), and proposes making the game fair by giving each player the "same amount of choices, like three and three" (154). Jerel participates with Chris in the interview where Chris reveals his large number-small number theory, and Jerel does not dispute Chris' claim. In fact, he suggests that the one-die game can be made fair by redistributing the numbers to each player so that each player gets "two low numbers and one high number" (2265). [Note: Since Chris has named three numbers as large and three as small, Jerel's strategy would not be feasible.]

An interesting conception of fairness and unfairness that Jerel will hold throughout the IML sessions is revealed in his grade 6 interview. R2 asks which player might win if the unfair one-die game were played six times. Jerel says that Player A would win all six games (1870). If the game were played 60 times, Jerel expects Player A to win "59 out of 60" (1877), and in 100 games, Player A would win "99 out of 100" (1880). On the other hand, in a fair game played 100 times, a score of 40 to 60 might occur (1898). Jerel appears to have combined the outcome approach, in which one attempts to predict the outcome of the next trial of an experiment, with the representativeness heuristic, where one believes that each sample should be representative of the larger population. This combination results in what I will call the *hybrid heuristic for chance events*. In the unfair game, Player A is expected to win the next trial (outcome approach), and that result becomes representative of all, or all but one, of the trials. However, in a fair game, either player might win the next trial, and so Jerel allows for much more variability in repeated plays of the game. Jerel applies the hybrid heuristic in the following way: if either player is able to win a game, then the game must be fair.

At the start of Activity 2, the game with two ordinary dice, R2 asks Jerel to write down the reason why he and Chris think the game is unfair before they play the game (1721-1722). Jerel balks at this suggestion, saying, "Wait, we didn't even play the game yet. How do you know Player B won't win?" (1723-1724). While Chris analyzes the sample space and declares the game to be unfair, Jerel tacitly goes along but does not express a strong opinion of his own.

The following year, during Activity 3 using two pyramidal dice, Jerel initially declares that Player B will win (4567). While playing the game and thinking momentarily that he is Player A, Jerel finds himself in the lead and decides that the game is fair "'cause I'm winnin'" (4721, 4725). However, when he becomes aware that it is Player B that is winning, Jerel again calls the game unfair (4734). Jerel names sums of 2 and 3 as "hard to get" (4741-4742), and 7 and 8 as "good number[s] to get" (4744-4745). Since all of these are Player A's numbers, it is not clear whether Jerel is implying an advantage or a disadvantage for Player A. Without writing the sample space, Jerel concludes that Player B has more combinations to get his numbers, making the game unfair (4766-4767, 4784-4785). However, after two games, Jerel changes his opinion again (4892). He explains that the game is fair because as Player A "I'm getting' the same amount of rolls with my numbers comin' up as Player B" (4897-4899). During the next game, as the score reaches 4 to 4, Jerel again declares that the game is fair because Player A "has just as good of a chance as B" (4910).

Jerel's partner Ian has enumerated the sample space showing four combinations favoring Player A and six favoring Player B. Also, Jerel sees Kianja and Brionna present their sample space to the class, showing six permutations for A and ten for B. Jerel is not influenced by any argument based on the sample space. He insists that because he won the game as Player A (5178), it is a fair game (5274).

Jerel's belief that if either player can win, then the game is fair carries over into Activity 4, the game with three pyramidal dice. Jerel's partner Ian lists six outcomes in the sample space favoring Player A and nine outcomes favoring Player B (Figure 24), and he tells Jerel that this makes the game unfair (6387). Jerel disagrees (6515) because each player has won one game (6516-6517). He tells his partner, "Ian, Ian, you won!" (6523). When Ian replies, "It don't matter", slamming his palms on the desk (6539), Jerel rejoins, "Well yes it do!" (6540). The boys play another game, which Jerel wins by a score of 10 to 9 (5934, 5936). Jerel tells Ian, "Look, you sayin' Player B has better chance of gettin' them numbers, but look, I just proved to you that Player A can still win" (5943-5945). Ian tells Jerel to look at a his chart showing the sample space. Jerel says, "It looks unfair on the chart. But look, we, I just proved that Player A can win" (5950-5951).

Throughout the IML sessions, Jerel shows an awareness that the outcomes of a sum of two or three dice are not equiprobable. He refers to certain sums as being hard to get or having more combinations than others. Though he does not construct the sample space himself in any of the activities, one of his partners does. However, when faced with evidence in the sample space that contradicts his beliefs about fairness, Jerel disregards the theoretical evidence. His level of probabilistic reasoning is best described as transitional because of his tendency to revert to subjective judgments and his reliance on small samples.

5.2.4 Tracing Justina's Assessments of Fairness

Justina says that the game in Activity 1 is unfair because "Player A had so many, and Player B didn't have that many numbers" (2317-2320). She and her partner Adanna make the game fair two different ways, each time allocating three numbers to both players (506-508). Asked whether it makes a difference if one player has all the high numbers and the other player has all the low numbers, Justina contends that the game would still be fair, since the die might just as likely land on the high numbers as on the low numbers (527-529). Justina and Adanna play their revised games and find them to be fair because, as Adanna says, "she won, then I won. Then she won, then I won" (2342).

Like Jerel, Justina and Adanna invoke the hybrid heuristic when R4 asks them what might happen if the unfair game were played repeatedly. In six rounds of play, the girls claim that Player A would win every game (2502-2505). In 100 rounds, Adanna says that Player B might win just two games (2513). Justina gives her opinion:

> I don't think Player B would really win, because Player um, Player A had the majority of the numbers. Well, yeah, in a hundred maybe, I agree with Adanna, maybe one or two times, but not really that much, 'cause Player B only had two numbers, and Player A had four. (2515-2519)

Later, referring to Activity 2, R4 asks Justina and Adanna whether Player B would ever win the original, unfair game if it were played 10 times (2725-2726). Since the girls have played that game and Player B won once, they concede that Player B can win, but "just once" (2728). R4 asks what might happen if the revised, fair game were played 20 times (2732-2734), and the girls agree that the players might win 10 games each, or one might win 5 games and the other 15 (2739-2742). And if the fair game were played 100 times, Justina says, "you can't be sure about that. 'Cause dice is dice and it just rolls on whatever number" (2751-2751). The score might be 50 to 50, or 60 to 40 (2758, 2764).

Like Jerel, the girls appear to combine the outcome approach and the representativeness heuristic. Unlike Jerel, however, this belief does not tend to dominate their judgments about fairness or unfairness, particularly in the case of Justina.

For Activity 2, both girls begin the game assuming all sums are equally likely. Justina states that Player A has an advantage over Player B (653). Adanna explains: "Player B has like five, and Player A has six. So Player A should […] get most of the points" (657-658). Playing the game, however, causes them to question their original opinion. Justina tells R4, "She kept beating me, and she was Player B and she had less numbers" (1420-1421). As they play the game again, writing down the outcome of each roll, they note that some numbers are more likely than others. Justina tells R4:

> [T]hose numbers that she's talkin' about is 5, 6, 7, um they have more, um many more ways to get them than the other ones do, like 11, is only one way to get 11. So you're really not likely to get that as much as you would, say, 6. (1521-1524)

Justina enumerates the sample space showing 21 outcomes. She concludes, "So this was not fair because um Player B had […] 13 ways, which was more than 8 ways Player A has" (1576-1578). She makes the game fair by eliminating the sum of 12 and dividing the remaining 20 outcomes between the two players (1590-1597).

The following year, before playing the game in Activity 3, Justina again applies the equiprobability bias and states that the game is not fair because Player A has more numbers than Player B (4191-4193). After playing one game, which Player B wins with a score of 10 to 1, Justina no longer believes that Player A has an advantage over Player B (4220-4224). She constructs the sample space showing ten outcomes, four favoring Player A and six favoring Player B (Figure 14). She tells the class that "this game is unfair because Player B's sum of numbers has two different ways, has two different combinations, and Player A's sum of numbers only have one different combination" (4434-4437). Like Chanel, Justina emphasizes the number of ways to obtain each sum rather than the total number of outcomes for each player.

Justina's sample space has 10 outcomes, an even number. To make the game fair, she eliminates a roll of 6, which she believes can be obtained two ways, and assigns 2, 4, and 7 to Player A and 3, 5, and 8 to Player B. She explains that each player has two numbers with one combination and one number with two combinations (4459-4463). Justina appears to be more attentive to the number of combinations for each sum than to the total number of outcomes or the fraction of outcomes favoring each player.

In the final week of grade 7 activities, Justina no longer exhibits the equiprobability bias when she discusses the game for Activity 4. She immediately suggests "look[ing] at the possibilities for getting each number" (6779) in order to determine whether or not the game is fair. Before she has an opportunity to do so, Adanna starts the game (6810). Player B wins the first game with a score of 10 to 8, and Justina says, "I guess it's a fair game. You [Player A] had a close chance of winnin'" (6843-6844). The girls review the outcomes they wrote down as they played the game and note that 8 was the most frequent sum, while 7, 9, 10, and 11 came up just once each (6848-6853). Justina remarks that Player B won the game, even though she had many of the infrequent high numbers, so "maybe it's a fair game" (6872-6874). The girls begin a second game, and when the score reaches 5 to 1 for Player B, Justina says that she thinks the game is unfair (6899). The game concludes with A as the winner, though. The score is 10 to 9. Justina remarks that "Player B won last time and now this time, Player A wins. […] I think it's fair." (6925, 6929). She goes on to say that "each player could win" (6931), perhaps invoking the hybrid heuristic.

The next day, Justina reviews the data from the previous day's play and notes that 8 and 6 were the most frequent sums rolled (8040). Since 8 is assigned to Player A and 6 to Player B, Justina again suggests, "Maybe it's a fair game" (8049). While her partners Adanna and Alia play the game, Justina writes on her paper. She says:

> I'm just tryin' to see, um, the different ways of each number to come up. […] Because last time when I played this game, like some numbers they came up, like they had different ways of, they had different ways to come up more than others did. (8147-8148, 8150-8152)

With a bit of coaching from G8, Justina develops the sample space showing all 20 combinations of addends. She concludes, “Player B has more of a chance of winning than Player A does” (8330).

Justina exhibits progress in the development of her probabilistic reasoning over the course of the IML sessions. In Activities 2 and 3, she begins with the equiprobability assumption but discards it on the basis of experimental data. She then uses the sample space to make inferences about fairness. For Activity 4, she indicates that the sums may not be equally likely, but she does not immediately investigate the sample space because she and her partner start to play the game. Consequently, Justina’s opinion is influenced by a small amount of experimental data, and her opinion changes with each shift in the data. Once she has completed the sample space, she is assertive in her conclusion that the game is unfair. She has progressed from a transitional level of reasoning about probability comparisons to the informal quantitative level.

5.2.5 Tracing Kianja’s Assessments of Fairness

Kianja stands out as very different from the other students in this study with regard to her conceptions about probability. Starting with the game in Activity 1, which Kianja recognizes as unfair, she suggests a unique approach to create a fair game: keep the assigned numbers as they are, but award Player B two points whenever 5 or 6 is rolled, and Player A one point for a roll of 1, 2, 3, or 4.

Within the first five minutes of Activity 2, Kianja creates the sample space with 21 outcomes and states, “this one is 8 out of 21 probability of winning” (612-613). She explains that she found 21 combinations of addends, and that “it’s 8 out of 21 chances for the Player B to win and there's 13 chances out of 21 for Player A to win" [*sic* – she has reversed the players' probabilities] (621-622).

The following year, Kianja similarly determines the sample space for the sum of two pyramidal dice for Activity 3 and declares the game unfair:

> See, there's one, two three, four, five, six, six [outcomes] that equal 4, 5, or 6. And then we have 2, 8, 3, and 7. One, two, three, four. Four [outcomes] that equal 2, 3, 7, 8. You see how I came to my conclusion? (3088-3090)

After a brief intervention by G4, Kianja decides to include permutations of addends in her sample space, showing ten outcomes favoring Player B and six favoring Player A.

In grade 6, when Kianja discussed the probabilities of either player winning a point, she noted 8 chances out of 21 and 13 chances out of 21, based on her sample space. When the question of probability of an event arises in grade 7, Kianja answers in a different vein. In a conversation with Brionna, G5 asks how many opportunities Player A has to win the game (3264), and Brionna answers, "Six. One out of six" (3267). Struggling a bit with her explanation, Brionna asks Kianja to join the conversation (3270). Kianja elaborates, "It's six ways that A could score a point, right? So it's one out of six chances that A would score a point" (3290-3291). G5 asks about Player B's chances (3292), and Kianja replies, "One out of ten. Because it's ten chances, it's, there's ten possible ways for B to score a point, so it'd be one out of ten" (3293-3294). Using $\frac{1}{x}$ instead of $\frac{x}{n}$ to describe the players' chances may have been a momentary lapse for Kianja, as she was occupied writing her results at the time and may not have been carefully attending to the discussion. The question of numerical probability does not come up again.

To make the game of Activity 3 fair, Kianja muses to her partner, “Let’s see, how could we make this fair, Brionna? There’s only seven numbers” (3156-3157). Brionna suggests that each player might get four numbers (3159), but Kianja reminds her that there are only seven numbers in all (3160). Brionna proposes, “So they both don’t get or get 8” (3164), and Kianja writes the rules for a “fair” game with two pyramidal dice: Player A gets a point for 2, 3, or 7; Player B gets a point for 4, 5, or 6; and whoever rolls a sum of 8 gets a point (3165-3168). In devising this game, Kianja and Brionna assume that the seven sums are equally likely, even though Kianja’s sample space reveals otherwise.

G4 asks Kianja to explain why the new game is fair, and Kianja suddenly remarks, “It’s still unfair, Brionna. Sugar!” Approximately seven minutes later, Kianja announces, “Oh great! I know how to make the game even” (3316). She correctly partitions the sample space of 16 outcomes, giving Player A a point for 3, 5, or 7 and Player B a point for 2, 4, 6, or 8 (3428).

At the start of Activity 4, Kianja begins enumerating outcomes in the sample space, while she gives Brionna the task of rolling the dice and keeping score. Her list of outcomes includes permutations of addends. On the first day of the task, Kianja finds a total of 58 outcomes: 26 favoring Player A and 32 favoring Player B. She concludes that the game is not fair, and to make the game fair she redistributes the outcomes so that each player has 29 of them. The next day, Kianja discovers the six missing outcomes and revises her fair game to give each player 32 outcomes.

Kianja’s level of reasoning with regard to probability comparisons and theoretical probability appears to be the fourth level, numerical, from the start of the IML activities. She is very consistent in reasoning about fairness by way of the sample space. Her use of $\frac{1}{x}$ instead of $\frac{x}{n}$ in grade 7 was perhaps a slip due to a lack of attention. It is curious that she briefly entertained the equiprobability bias in making a fair game during Activity 3 just after she enumerated the sample space. As we will see in the next sections, Kianja's levels of reasoning about experimental probability, and initially about sample space, are not as advanced.

5.2.6 Other Students' Assessments of Fairness

In addition to the five focus students, other students who worked with or nearby them were filmed during the IML sessions. These students may not have been present at all times, and so I can only provide snapshots of the probabilistic reasoning they exhibited when they were filmed.

Kori and Nia are seated at the table next to Chanel and her partner Danielle for Activity 1. The girls recognize that the one-die game is unfair. Kori says, "I have four opportunities to get a chance and you only have two" (789-790). To make the game fair, Kori suggests that they "move 4 to Player B so it'd be even" (791). Kori and Nia play their revised game, but Kori notes that "it still wasn't fair 'cause I still won because I kept on rollin' and it got just 1, 2, and 3" (1238-1239). Kori decides, based upon the games she played, that 1, 2, 3, and 4 are more likely to come up than 5 or 6 (1249-1255). Nia explains, "'Cause it doesn't really pop up that, it doesn't really pop up that, like usually" (1257-1258). The girls demonstrate this to R2 by rolling the dice a few times and obtaining outcomes of 2, 3, and 4 (1261, 1264). Kori says, "Then one out a blue moon you get a 5" (1264). They decide to revise the game again, this time giving a point to

Player A for a roll of 1, 3, or 5 and a point to Player B for a roll of 2, 4, or 6. Kori explains that this new assignment still gives three numbers to each player, but this time "each of us has two common rollers and each of us has one, one out of the blue roller. So it kind of makes us even" (1346-1347).

Using the availability and representativeness heuristics, Kori and Nia have decided that the six outcomes of the roll of a die are not equally likely. Given this belief, their revision of the game to make it fair is quite reasonable. They appear to operate at the transitional level of probabilistic reasoning.

Chanel's partner for Activity 1, Danielle, also indicates that the six outcomes of the roll of a die are not equiprobable. In her case, however, she has experimental data that indicate otherwise. Though she says that rolls of 1, 2, and 3 are "halfway impossible to get" (993), she has played three games in which Player A, who has the numbers 1, 2, and 3, won twice against Player B, who has 4, 5, and 6. Danielle's probabilistic reasoning would be characterized as subjective, as she uses personal judgment rather than quantitative evidence to decide that the game is not fair (992).

In grade 7, when Activity 3 is introduced to the class, Dante is the first to announce that the game is unfair because Player A has more chances than Player B (2946). Many other students in the class agree with Dante initially (2979-2980). Several students, such as Chanel, Chris, Justina, Ian, and Dante, decide to investigate the sample space after playing the game, and they determine that Player B, not Player A, has the advantage. David, however, maintains throughout the activity that Player A is favored, based upon his assumption that all sums are equally likely (4637-4643, 4358-4359).

Though other students have demonstrated that the game favors Player B, David does not agree. He remains at the subjective level of reasoning.

In Activity 4, Terrill, who is Chris' partner, emphasizes the need to play the game before deciding on fairness (7789-7790). However, he recognizes that Chris' sample space will also inform his judgment, as he tells T7, "He counting up the possibilities of going to those numbers. If he finds all the possibilities then whichever one has more possibilities is um, better, it's fairer for um that one" (5762-5764). Ultimately Terrill and Chris decide that the game is unfair after Player B wins one game (out of three played) and Chris' sample space shows more combinations favoring Player B.

Ian, who is Jerel's partner for Activity 4, finds fifteen outcomes in the sample space and notes that Player B has the advantage. He and Jerel spar over whether or not the game is fair. Jerel argues that the game is fair is because each player won one game, while Ian insists that the sample space shows more possibilities for Player B. T3 plays one more game against Jerel and Ian, and Player B wins (5962, 5976). Ian and T3 have the following conversation (5979-5984):

T3	Do you still think it's fair? A won, B won.
Ian	I didn't ever think it was fair! I still don't. 'Cause look, B won.
T3	Okay, but accord-, but according to your game, though …
Ian	Yeah, it is. [looks at his papers]
T3	According to your game, the outcomes of your game …
Ian	Yeah, it's fair. They each have enough of a chance to get …

Inexplicably, Ian has changed his opinion. (The camera cuts away at this point.)

5.3 What Is the Sample Space for the Sum of Dice Outcomes?

The students in this study exhibit three ways of thinking about the sum of a number of dice outcomes:

1. Each sum is a separate event, so that if there are *n* possible sums, then there are *n* possible events in the sample space.
2. The different combinations of addends that make up the sums are counted as different events. Changing the order of addends within a combination does not create another outcome. So, for example, with two dice a sum of 4 has two combinations, each a separate event: 1+3 and 2+2.
3. The different permutations of addends that make up the sums are counted as different events. For example, with two dice a sum of 4 has three permutations, each a separate event: 1+3, 2+2, and 3+1.

Any of these conceptions is correct as long as the outcomes are properly weighted. However, the students studied do not weigh the outcomes. With the exception of Chris' subjective theory about large and small numbers, the students treat the outcomes as equally likely. In that case, the conception that allows for all permutations of addends to be counted is the correct one. Combinations will suffice, however, for the purpose of determining whether or not these games are fair, without regard for the actual probabilities of either player winning a point. For reference, the numbers of sums, combinations, and permutations for Activities 2, 3, and 4 are summarized in Table 7.

Table 7.
Number of Sums, Combinations, and Permutations for Activities with Two or More Dice.

	Activity 2 two ordinary dice	Activity 3 two pyramidal dice	Activity 4 three pyramidal dice
sums	11	7	10
combinations	21	10	20
permutations	36	16	64

5.3.1 Tracing Chanel's Notions of Sample Space

As discussed above, Chanel begins each of Activities 2, 3, and 4 under the assumption that the *n* sums are equally likely. Playing the game causes her to doubt her initial intuition. For Activity 2 in grade 6, Chanel does not attempt to write the sample space for the sum of two dice. Though she notes that sums of 11 and 12 are not frequent, she does not provide a quantitative justification of her claim.

For Activity 3 in grade 7, however, Chanel decides to write the sample space after she plays the game and finds Player B a three-time winner. She lists all ten combinations of addends. T5 and R2 question Chanel about whether 1+2 and 2+1 are the same outcome, and Chanel says that they are, "just reversed" (3782-3783). T5 uses two different colored dice and asks Chanel to show him 1+2 and 2+1, and Chanel maintains that they are the "same thing" (3805). She volunteers that if the outcomes were subtracted rather than added, then the results would be different (3824, 3837-3838). However, in this game, 2+3 and 3+2 "count as the same opportunity 'cause you're adding, not subtracting" (3857-3858).

On the first day of Activity 4, R1 asks Chanel to think about all the ways that the outcome 4, 2, 3 can occur using white, red, and blue dice (6025). Chanel writes the numbers 4, 2, and 3 in four different orders, but in each case she shows 4 on the white die, 2 on the red die, and 3 on the blue die (Figure 34). Though she permutes the numbers, they remain associated with the same colors.

On the second day of this activity, Chanel tells G7 that certain sums are more likely to occur than others because there are more ways to obtain those sums (7136-7152). G7 suggests to Chanel that she make a list of the possible sums, and Chanel complies. Her written list (Figure 33) shows 19 distinct outcomes in no particular order and includes some of the permutations for sums of 7, 8, and 9 but only combinations for the other sums. Seven of the 20 possible combinations are missing from Chanel's written list. Since one combination, 4+1+3, is listed twice, it is possible that Chanel's inclusion of some permutations was also unintentional.

Unfortunately the roving camera did not film Chanel for the remainder of this session. A paper in Chanel's file from this day shows that she used red, blue, and black dice to demonstrate permutations of addends for sums of 4 and 7 (Figure 35). We do not know the circumstances surrounding this paper. Could a breakthrough have occurred? Based upon Chanel's comments and other written work during the grade 7 activities, it is not likely.

5.3.2 Tracing Chris' Notions of Sample Space

Chris begins Activity 2 with the notion that some numbers are "better ones to play" (1713), though he may be applying his big number – small number theory (1715-1717) and not referring to the number of ways that each sum can be obtained. He says, however, that "we gotta find out how many ways you can get each number" (1741-1742). In fact, Chris does list the sample space for the sum of two dice (Figure 5). He shows all 21 combinations in no particular order, with Player A's and Player B's numbers mixed together. Though he identifies eight combinations favoring Player A and thirteen favoring Player B (1995-1996, 2001), the reason he gives for the game being unfair is based on his big number – small number theory (Figure 4).

In grade 7, Chris begins the game for Activity 3 assuming the sums are equally likely. He plays three games with G6, and after the second game R4 suggests that Chris keep a record not only of the sums rolled but also of how they were obtained (5475-5477). As the third game concludes, with some encouragement from R4, Chris begins to talk about the different combinations that make the sums (5529-5543). He writes the sample space for the sum of two pyramidal dice showing ten combinations (Figure 15).

Chris demonstrates with the dice that 7 is obtained with a 4 and a 3 (5562), and R4 asks whether it would be a different outcome if the numbers on the dice were reversed. Chris says, “No. It’s still the same thing. You’re still gonna get the same sum” (5565). R4 tries again, using a green and a white die instead of two green dice (5568-5569), and Chris maintains that it is still the same sum (5570). R4 asks, “And if you had a white 1 and a green 2, or a green 1 and a white 2, those are not different ways?” (5583-5584). Chris replies that even with different colored dice, the sum will be the same (5585-5587). R4 makes one more effort to challenge Chris to think about permutations: She suggests a game in which Player A gets a point for a sum of 2 and Player B for a sum of 3 (5602-5603). Chris indicates that both sums have the same probability since there is only one way to get each, but he hesitates momentarily and says, “I don’t really know” (5590-5592). Chris and G6 play the game twice, and Player B wins both times with scores of 5 to 2 and 5 to 3. Chris does not change his opinion, however. He says, “I really still think it’s the same thing” (5660).

The following week for Activity 4, Chris immediately begins to write down combinations that give each of the possible sums of three pyramidal dice (5748). Unlike the previous activity, Chris does not begin with the equiprobability assumption. He uses a guess-and-check method to generate combinations, and he does not find all of them. Initially he finds six combinations for each player, and so he determines that the game is fair (5768). Player A wins two games in a row, and Chris still calls the game fair (5841, Figure 21). The next day, Chris finds more outcomes in the sample space, ultimately listing seven outcomes favoring Player A and ten favoring Player B (Figure 23), and he tells Terrill that the game is unfair (7770). Though his conclusion about fairness is correct, he does not have all 20 combinations, and he does not consider any permutations.

Like last week with R4, Chris is questioned by G4 about whether different arrangements of the dice outcomes count as different events. Chris repeatedly says any arrangement, even with different colored dice, amounts to the “same thing” because they “add up to the same thing” (7691, 7693). Despite some rather insistent questioning by adults, Chris is firm in his position that permutations of addends do not count as different events.

5.3.3 Tracing Jerel’s Notions of Sample Space

For each of the activities, Jerel works with a partner who uses the sample space to determine fairness. Jerel does not write the sample space for himself, nor does he seem to give it much weight. If the sample space and experimental data lead to conflicting conclusions, Jerel will side with the experimental data and his hybrid heuristic.

In grade 6, Jerel partners with Chris for Activity 2. When Chris presents his theory about large numbers being more likely than small numbers, Jerel agrees (2128). However, when the boys roll a die and the small numbers come up 12 times out of 22, Jerel remarks, “The big numbers don’t always show up” (2246).

In grade 7 during Activity 3, Jerel calls some of the outcomes “very hard to get” (4741) and others “a good number to get” (4745), but he does so without referring to the sample space. Ian suggests, “Maybe you should make a multiple chart, Jerel” (4752), but

Jerel does not make a chart. Still, he claims that Player A's numbers have one, two or three combinations while Player B's numbers have two, three, or four combinations (4784-4785). Despite this claim, and the sample space that his partner Ian shows him, Jerel decides that the game is fair because each player has won two games. Similarly, during Activity 4, Jerel ignores Ian's sample space and argues that the game is fair. He says, "It looks unfair on the chart. But look, we, I just proved that Player A can win" (5950-5951).

The question of permutations is raised with Jerel during Activities 3 and 4, and Jerel says that different arrangements of the addends are "the same thing, he just mixin' it up" (4933).

5.3.4 Tracing Justina's Notions of Sample Space

Justina begins Activity 2 with the equiprobability bias, saying that the game is unfair because Player A has more outcomes than Player B (655). After playing a few games, she remarks that Player B keeps winning (1411-1412). R4 suggests that Justina and Adanna play some more, and she asks them to record the individual dice outcomes as well as the sums (1442). A few minutes later, R4 asks the girls about how certain sums were obtained (1498-1503):

R4	What did you do to get the 11?
Justina	We rolled a 5 and a 6.
R4	Okay. How many ways did you, how many, what did you do to get the 6?
Justina	I rolled a 3 and a 3, a 4 and 2, and [pause] a 6, I mean a 5 and a 1.
R4	Um humh. [pause] Does that matter?

Adanna remarks that some of the numbers are "easier to get" (1510) while others are "hard to get" (1513), and Justina explains that the easier numbers have "many more ways to get them than the other ones do" (1522-1523). R4 encourages the girls to keep a record of the number of ways to obtain each sum (1553), and so Justina develops the sample space showing all 21 combinations of addends (Figure 7). As R4 requested, her list emphasizes the number of ways to obtain each sum.

A year later, Justina begins Activity 3 once again with the equiprobability bias, stating that Player A has an advantage because she "has more numbers" (4199). After just one game, however, which Player B wins with a score of 10 to 1, Justina questions her intuition and writes the sample space with 10 combinations (Figure 14). When she presents her analysis to the class, she emphasizes that there are two ways to get each of Player B's sums, but only one way for each of Player A's sums (4441-4444).

At the start of Activity 4, Justina wants to "look at the possibilities for getting each number" (6779) in order to determine whether or not the game is fair, but her partner Adanna starts playing the game before Justina has the chance to do so. On the second day of this activity, Justina reviews the data from the games she and Adanna played, and remarks, "when I played this game, like some numbers they came up, like they had different ways of, they had different ways to come up more than others did" (8150-8152). Justina begins to list the combinations for each sum. Adanna says, "The ones with the most combinations are gonna come out more than the less combinations" (8197-8198). G8 reviews Justina's list and asks if she might have missed any combinations (8210, 8218, 8221, 8223, 8227-8229, 8241-8242, 8261-8262, 8269), and Justina discovers some more. She has used a guess-and-check approach to listing the sums. In the end, Justina has all 20 combinations (Figure 32). On a separate paper, she lists Player A's numbers in a row and below them writes the number of combinations for each sum. She does the same with Player B's numbers (Figure 27), and she concludes that "Player B has more of a chance of winning than Player A does" (8330).

At this point, G8 begins to challenge Justina and her partners to consider permutations of addends as different outcomes. For about 10 minutes, G8 repeatedly asks the girls whether it makes a difference if the same numbers appear on different dice. Justina says, "It doesn't matter" (8354), and "We're not focusing on the colors. We're just focusing on the numbers" (8357-8358). She is not influenced to change her mind.

5.3.5 Tracing Kianja's Notions of Sample Space

In all of the IML activities involving dice sums, Kianja immediately begins writing the sample space in order to assess fairness. Unlike many of the other students, she does not exhibit the equiprobability bias.

For Activity 2, Kianja writes the sample space showing all 21 combinations within the first five minutes of the activity. She writes Player A's and Player B's sums separately and indicates the *a priori* probabilities that either player will score a point (Figure 8).

The following year, she similarly begins Activity 3 by writing the 10 possible combinations of two pyramidal dice. G4 asks Kianja whether there are other ways to write the outcomes (3093), and he demonstrates 1+2 and 2+1 as different outcomes on the dice (3108, 3112). Instantly, Kianja begins to write the additional permutations (3122, 3127, 3130, 3132). She says, "If you wanted to do that, then it would only be" 10 outcomes for Player B (3132-3134) and 6 outcomes for Player A (3141). "So it would still be more" for Player B (3141). Kianja is willing to go along with G4's suggestion to include permutations in the sample space, but she is equally willing to agree with other students in the class, such as Ian and Justina, who show only combinations. She says, "It's the same concept" (4375, 4399, 4402).

The next day, R2 asks Kianja whether 2 and 1 is the same as 1 and 2 (4253). This is the same question that G4 asked the previous day that prompted Kianja to write permutations. This time Kianja says, "It is the same" (4254). R2 suggests that Kianja and Brionna try a new game in which Player A gets a point for rolling a sum of 2 with two dice, and Player B gets a point for rolling a sum of 3 (4262-4264). He asks whether this game is fair. Initially, Kianja says that Player B will win because there's just one way to roll a sum of 2 (4270, 4272). Then she adds, "Only one way to get both of 'em, so . . ." (4275). Kianja and Brionna play this game off camera with T3. During the debriefing following this session, T3 reports that after a while the girls realized that the numbers can appear on different dice and that 2 and 1 is a different outcome than 1 and 2.

The following week, for Activity 4, Kianja once again sets out to write the sample space at the start of the session. She lists the numbers for the two players separately and begins to write the possible addends for each sum, showing permutations as different events (6109-6110, 6114-6115). Her work shows organization in permuting each combination that she finds, but she does not exhibit a strategy to generate combinations of three addends other than guess and check. Despite some helpful suggestions from R3 and R4 (for example, 6176, 6181, 6185, 6235), Kianja misses six of the outcomes on the first day of the activity (6654). She discovers the missing outcomes on her own the next day. She notes the symmetry in the distribution and says, "I shoulda known it was wrong" (7342).

Kianja is briefly thrown off course by a question from R3. He asks why she shows three ways to obtain a sum of 4 but only one way to obtain a sum of 3 (6277-6278). Kianja begins to explain that she “switched them around”, but then says, “We will divide it by three if you want” (6295-6296). She adjusts the list showing the number of ways to obtain each sum, omitting permutations. R3 asks, “Which way is a better way of counting?” Kianja points to the list without permutations (6305-6308).

Kianja’s willingness to go back and forth about permutations and combinations may indicate some instability in her understanding of sample space, or it may be a consequence of her non-confrontational personality, as T5 has suggested. Kianja said during Activity 3, “It’s the same concept”, which might imply that the same conclusion about fairness would be reached whether or not permutations are counted. Therefore, either interpretation works for her.

R1 returns to speak with Kianja, and Kianja admits that she saw permutations as different events, but “if you wanted to do it this way [using combinations only,] […] then that’s how you would do it. But I didn’t do it this way” (6324, 6327). R1 replies, “Okay. Very good” (6334) and goes on to ask Kianja whether she’s sure that she has all the outcomes. From this point forward, Kianja uses permutations in her sample space. She explains to T5, “If it’s on a different dice [*sic*] it is different” (6602).

Kianja began the IML probability sessions at the transitional level of reasoning about sample space and progressed to the informal quantitative level, which is the highest level achieved by any of the students studied. She has not yet reached the numerical level, as she does not demonstrate the use of a strategy that will generate all the outcomes.

5.3.6 Other Students' Notions of Sample Space

Adanna is partnered with Justina for many of the IML probability sessions. In grade 6 the two girls contribute equally to working on the tasks. For the game with two dice in Activity 2, Adanna partitions Justina's sample space of 21 combinations in a unique way (Figure 6), separating the sums according to the number of ways they can be obtained. She notes that sums of 2, 3, 11 and 12 can be obtained one way, 4, 5, 9, and 10 can be obtained two ways, and 6, 7, and 8 have three ways (2450-2452). Apparently looking for a pattern, she notes that each partition contains two even numbers (2457-2461). R4 briefly entertains Adanna's observation (2466-2467) and then steers the conversation in another direction (2489-2490).

In grade 7, Adanna is less focused on the tasks and spends much of her time talking about other topics. She does make her opinion known during Activity 4 when G8 questions the girls about whether permutations should be counted as different events. Adanna answers five times, indicating that she does not think so (8378, 8383, 8388, 8403, 8416).

Brionna, Kianja's partner, is soft spoken and tends to follow Kianja's lead during the activities. While Kianja works on the sample space, Brionna rolls the dice and keeps score (3216, 6096-6097). On one occasion, she quietly disagrees with Kianja, and that occurs during Activity 3 when G4 suggests considering 1+2 and 2+1 as different outcomes. Kianja has inserted "2+1=3" into her sample space, which already shows "1+2=3". G4 speaks with Brionna (3123-3131):

G4	This is 2+1, right?
Brionna	Yeah, it equals 3.
G4	Yeah, and this is 1+2.
Brionna	1+2. That's the same thing, 3.

	[Kianja inserts "3+1 =4", "4+1 = 5" into the sample space.]
G4	Um humh. What is this here you're writing? [Points at Kianja's paper.]
	[Kianja continues writing, "3+2=5", "4+2 = 6".]
Brionna	[quietly] You still get the same answer.

Despite her demure protest, Brionna adopts Kianja's position and helps her adjust the count of outcomes in the sample space to reflect the insertion of permutations (3135). Later, she tells G5 that there are six ways for Player A to get a point (3256-3257) and ten ways for Player B (3259). However, a conversation between Brionna and G5 reveals that either Brionna is not convinced about counting permutations or that she and G5 have difficulty communicating.

G5 asks whether 4+2 and 2+4 are the same (3317-3318), and Brionna responds that "even though it's like the same answer you still have to do it […] because you also have 2 + 4 and 4 + 2" (3321-3323). G5 goes on to ask about four additional pairs of addends: are they the same or different if the numbers are reversed? (3328, 3335, 3341, 3345, 3349). Brionna consistently replies, "The same." "You get the same answer no matter which way you put it" (3346-3347). It may be that because of the lack of a shared understanding of G5's questions, Brionna does not make it clear that she considers permutations of sums to be different outcomes, as Kianja's sample space shows. Or, it is possible that Brionna is not convinced that permutations of addends are different events.

During Activity 4, Brionna rolls the dice while Kianja writes the sample space showing permutations. She is present when R3's question prompts Kianja to revert to combinations only and when R1's intervention helps Kianja to recover from that misstep. By the final day, Brionna appears to agree that permutations count as different events.

She shows G6 how the same numbers can show up on different dice, which makes the outcomes different (7217-7219).

Because Brionna did not generate the sample space herself or suggest outcomes to Kianja, it is not possible to assess her level of reasoning about sample space.

Ian works with Jerel during the grade 7 activities. For the game with two pyramidal dice, he writes the sample space showing 10 combinations. As the boys play the game, R2 stops by and asks what the last roll was (4927). The brief dialogue that follows is the only instance during Activity 3 where Ian responds to a question about the order of dice outcomes (4928-4932).

Ian	He got 2 and 1. [1 and 1 is also said by someone]
R2	Not 1 and 2?
Ian	You asked me that yesterday.
R2	Well I'm asking that …
Ian	Don't, don't let him use psychology on you.
Jerel	It's the same thing, he just mixin' it up.

For Activity 4, Ian lists 15 combinations in the sample space (Figure 24). Demonstrating no particular strategy to generate outcomes, he misses five combinations. T3 asks Ian and Jerel whether 1, 1, and 2 is the only way to get a sum of 4 (6444-6446). Ian answers, "Yup" (6447). Using colored dice, T3 changes from black 1, yellow 1, green 2 to black 1, yellow 2, green 1, and he asks, "Is this different, is this different from that?" (6460). Ian says, "No" (6462), and Jerel adds, "Because all you did was switch 'em around" (6464).

Ian's level of reasoning about sample space is classified as transitional.

5.4 How Are Experimental Data Used as Evidence?

In grade 6, a few students (notably Chris, Jerel, and Danielle) make subjective judgments about the likelihood of an event and reassert their beliefs even after their data indicate otherwise. By grade 7, all of the students studied use experimental data to some degree in order to inform or provide support for their opinions about fairness. In every case, students make inferences based on a small number of trials.

Kianja, Chris, and Ian are more inclined to use the theoretical approach to assess whether or not a game is fair, while Jerel, Terrill, and Adanna tend to use the frequentist approach. Justina and Chanel try to balance the two, which sometimes results in a frequent reversal of opinion.

5.4.1 Tracing Chanel's Use of Experimental Data

During Activity 1, Chanel notes that the game with one die is unfair because "it should be like 4, 5, 6 and 1, 2, 3" (866). She plays the game with the new rules and Player A wins twice. Chanel still believes that the revised game is fair because the scores were close (956-957). She laughs and says, "Player A is lucky" (952). Player B wins the third game and Chanel declares, "It's fair" (991). Her partner Danielle disagrees, however, saying, "Oh no. To me it wasn't [fair] because the 1, 2, 3 numbers, it's […] halfway impossible to get 'em sometimes" (992-993). Chanel replies, "Nuh-uh!" (994). The girls roll dice to try to convince one another (998-1010) but reach no resolution. G1 asks whether they are convinced that the new game is fair (1023). Chanel answers "yes" and Danielle quietly says "no" (1024-1025).

During Activity 2, Chanel becomes convinced by the experimental data that the (unfair) game is fair. After playing one game, which Player B wins with a score of 10 to

5, Chanel declares that the game is fair. Player B wins a second game, and Chanel maintains her opinion. She explains that 11 and 12 "pop up" infrequently, thus offsetting Player A's presumed advantage of having more sums than Player B. Chanel uses subjective reasons to explain why 11 and 12 are infrequent, and she also notes that "we keep rolling it but 12 or 11 doesn't pop up that much" (1171-1172). Chanel does not write the sample space for the sum of two dice.

In grade 7, Chanel begins both of the activities assuming that the sums are equally likely. After playing the games and getting unexpected results, she is convinced by the experimental data to look at the sample space. For Activity 3, she plays three games and Player B wins each of them (3405-3407). This causes her to question her original intuition and seek an explanation for Player B's success. She writes the sample space showing all 10 possible combinations for the sum of two pyramidal dice. For Activity 4, Chanel discovers by rolling the dice that there are different ways to obtain some of the sums, making certain numbers "hard for you to get" (7141-7141). As a result, she begins to consider how the sums are obtained and writes some of the outcomes in the sample space.

Chanel appears to be at the transitional level of reasoning about experimental probability through all the IML sessions. She recognizes that there is a relationship between the frequency of an event and its likeliness, but she is willing to make inferences on the basis of small samples.

5.4.2 Tracing Chris' Use of Experimental Data

Throughout the IML probability sessions, Chris uses experimental data to corroborate his theoretical claims. However, when data seem contradictory to Chris' beliefs, he is reluctant to change his opinion. Chris refers to experimental results in order to contrast the original unfair game of Activity 1 to the revised fair game. Comparing the point spreads of the two games, he says, "'Cause, uh, the first game, since it was 10 to 2, that was a kill by eight points, but in the second game it was only a kill by four points" (1857-1858). Chris also refers to a score of 10 to 9 as evidence that his evens vs. odds game is fair (213).

Though, in Activity 1, Chris and Jerel had assigned outcomes of 1, 2, and 3 to Player A and 4, 5, and 6 to Player B and called this game fair (with a "kill" of only 4 points), Chris later asserts that the larger numbers 4, 5, and 6 are more likely to occur (2125-2127). R2 asks Chris and Jerel to roll a die and keep track of the outcomes (2151). In 22 rolls the smaller numbers come up 12 times (2225-2226). R2 asks, "So what about your theory? […] Do you still hold to that?" (2228, 2234). The following piece of transcript epitomizes Chris' uncertainty (2236-2244):

R2	Chris? You don't look like you're sure.
Chris	[Shakes head no]
R2	You're shaking your head meaning what?
Chris	Don't know [smiling].
R2	You don't know whether you want to revise your idea or whether you're going to stick with it?
Chris	[shrugs his shoulders and makes a small giggle]
R2	You're not sure?
Chris	[shakes head]

R2 does not push the issue, but suggests that the boys think more about the problem and perhaps return to talk about it another time (2273-2276).

In grade 7, R4 interviews Chris and asks what has to be true in order for a game to be fair (5401). Chris' reply is indicative of his uncertainty about experimental data (5402-5406):

> To be fair? Well then, um, not only one person could like, well you could say like Player A wins five games and Player B only wins one game. Right there you're gonna know that it's not fair. Or you never know because Player B might be able to win other games too.

Chris begins the game of Activity 3 believing that Player A is favored because he has four sums against the three for Player B. Defying the odds, Player A wins Chris' first game with a score of 10 to 3. Rather than claim this as evidence that his belief is correct, Chris says "I don't really know" and agrees to play another game (5449). R4 asks him who he expects to win the next game, and Chris indicates Player A (5457-5459). Instead, Player B wins with a score of 10 to 6, and the next game is close. If anything, these results might suggest that the game is fair. However, Chris has begun to note that the sums can be obtained in different ways and so, under R4's questioning, he finds ten combinations in the sample space and determines the game to be unfair in Player B's favor.

For Activity 4, Chris immediately begins to construct the sample space, and when he finds just six outcomes for each player, he declares the game fair (5768). Once again obtaining an unlikely result, Chris plays the game twice and Player A wins both games. G4 asks whether Chris still believes the game is fair (7662-7663) and Chris nods to indicate yes (7664). He shows G4 his sample space as justification (7666).

Later, Chris adds more outcomes to his sample space and decides that the game is unfair in Player B's favor. As he plays the game with Terrill, not only does Terrill tease him when Player A takes the lead, but G4 asks Chris to update his opinion after each roll of the dice. At one point the score becomes tied and Chris appears to give in and says, "Yeah, I think it is fair" (7830). In the end, Player B does win the game and it appears that Chris returns to his belief that the game is unfair.

Chris' level of reasoning about experimental probability is difficult to pin down. At times it seems that he regards data from experimental trials as irrelevant or untrustworthy, but this may reflect the recognition that larger samples are needed.

5.4.3 Tracing Jerel's Use of Experimental Data

Unlike Chris, Jerel relies heavily on experimental data to make judgments. He appears to regard the sample space as irrelevant when experimental results disagree with *a priori* predictions.

For the first activity with one die, Jerel knows from the start that the game is unfair because Player A has more numbers than Player B (142-144). Asked whether the results of playing the game support his prediction, Jerel cites a score of 10 to 2 as evidence that the game is unfair (150). Later, when he plays Chris' game of evens vs. odds, Jerel decides that this game is fair. He notes that either player could come back from losing to win the game (214-216). The notion that if either player can win then the game must be fair is a manifestation of Jerel's hybrid heuristic.

For Activity 2, Jerel is reluctant to make a prediction about fairness without playing the game. Though Player A has more sums, Jerel says, "How do you know Player B won't win?" (1723-1724). As Jerel and Chris play the game, they write down the dice outcomes, and this leads them to consider the number of ways each sum can be obtained (1959-1960). The boys tell R2 about their findings, and Jerel mentions repeatedly that "seven kept popping up" (1985, 1992, 2021). He explains why: "Oh because it had a better chance, because it had three ways to get it" (2032). Here he appears to make a clear connection between theoretical and experimental probability.

In grade 7, Jerel begins Activity 3 with the intuition that the game is unfair. Without writing the sample space, he contends that Player B's numbers have more combinations than Player A's numbers (4766-4767, 4784-4785). His partner Ian does write the sample space and arrives at the same conclusion. Jerel changes his opinion, however, after he plays the game. He decides that the game is fair because as Player A "I'm getting' the same amount of rolls with my numbers comin' up as Player B" (4898-4899). In another round of play, when the score reaches 4 to 4 Jerel again asserts that the game is fair (4906-4908). Though other students such as Ian and Kianja explain to the class, by way of the sample space, that the game is unfair in Player B's favor, Jerel insists that the game is fair because "as Player A, I had won" (5187).

A similar scenario occurs with Activity 4. Jerel and Ian play two games, and each player wins once. Jerel calls the game fair (6515). Ian shows Jerel his sample space with six combinations for Player A and nine for Player B. He says, "That's why it's unfair. Got more combinations" (6535-6536). Jerel argues (6538-6544):

Jerel	But you won!
Ian	It don't matter. [stands up, slamming his palms on the desk]
Jerel	Well yes it do!
T3	So why, how can we settle this? How can we settle it?
Jerel	Play one more game.
T3	Just one more game?
Jerel	Yeah.

Jerel indicates that one more game will provide enough evidence for him to prove his point. In this game the score remains close, and in the end Player A wins with a score of 10 to 9. Jerel insists that, although the sample space makes the game appear unfair, the fact that Player A can win makes it a fair game (5943-5945). This argument is consistent with the hybrid heuristic that Jerel has applied throughout the IML sessions: if either player can win, then the game is fair. Although he briefly makes a connection between experimental and theoretical probability during Activity 2, it seems that for Jerel a small amount of experimental data overrides any theoretical considerations.

5.4.4 Tracing Justina's Use of Experimental Data

Justina shows a tendency to use experimental data to support her judgments. However, when theory and data are not in agreement, Justina may change her predictions based upon a small amount of data.

For Activity 1, Justina expresses confidence that the original game is not fair. She and Adanna modify the game two different ways, each time giving three numbers to Player A and three to Player B. They play the new games and the results confirm their belief that these games are fair, with the two players alternating as the winner in four games. Justina says, "It was even. It was even" (2343).

Playing the game in Activity 2 gives Justina pause to question her prediction that Player A has an advantage. She tells R4 that Adanna "kept beating me, and she was Player B and she had less numbers" (1420-1421). As Justina and Adanna play another game, recording the outcomes, Justina makes a link between experimental and theoretical probability when she explains that certain numbers are easier to roll than others because there are more ways to roll the easier numbers (1521-1524). Based on her observations, she constructs the sample space showing 21 combinations.

Justina also uses experimental data to confirm that the new game she devised is fair. The first round goes to Player A, with a score of 10 to 3 (2657). R4 asks Justina, "How many times do you think you need to play the game to test whether it's fair or not?" (2663-2664). Justina replies, "At least twice" (2665). She indicates that she's not quite sure that her game is fair because, although she gave the same number of outcomes to each player, the game "went from Player B always winning to Player A always winning" (2668-2689). As she and Adanna play the game again, Justina remarks on the close score, 3 to 3, as evidence that the game is fair (2679). When Player B wins the game, R4 asks whether the girls think it's fair. Justina answers, "Yeah, I do, because um at first A won, and then now B won" (2698-2699).

R4 asks Justina and Adanna what sum they would choose in a sudden death game in which winning depends on one roll of the dice (2772-2775). Both girls refer to their data and choose 6 because it was the most frequent sum (2778-2779, 2782-2784). Asked to choose between 7 and 8, the girls pick 8 for the same reason – it was more frequent than 7 (2789, 2803). Neither girl refers to the sample space to answer these questions; their sample space shows 6, 7, and 8 as equally likely.

The following year for Activity 3, Justina retraces her steps from the previous probability session. She begins with the prediction that Player A is favored, but after playing a game, which Player B wins by a score of 10 to 1, she changes her opinion and begins to write the sample space. Again, experimental data have motivated Justina to look at the sample space for an explanation of why her prediction may be incorrect.

Justina's opinion changes frequently during Activity 4 as she relies on small amounts of data to make inferences. Before she makes a prediction about the game, she and Adanna begin to play. When Player B wins the first game with a score of 10 to 8, Justina decides that the game is fair (6844). When the score of the second game reaches 5 to 1 in Player B's favor, Justina says, "I don't think it's fair. 'Cause […] I only have one point" (6899). A few minutes later, Player A wins the game with a score of 10 to 9 and Justina observes: “Player B won last time and now this time, Player A wins. […] I think it’s fair.[…] Because each player has um a good, yeah, each player could win” (6925, 6929,6931). Here Justina appears to invoke the hybrid heuristic, claiming that the game is fair because either player can win. It is possible that T9 contributes to Justina’s frequent change of opinion, as he, like G4 with Chris, asks Justina to make judgments on the basis of a small amount of data as she plays the game (for example: 6906-6907, 6926-6927).

The following day, as Justina reviews the data from her previous games, she notes that 8 and 6 were the most frequently rolled sums. Again she determines that the game is fair because 8 is assigned to Player A and 6 to Player B (8049). Ultimately, Justina writes the sample space and finds the game to be unfair in Player B’s favor (8330).

Justina typifies the transitional level of reasoning about experimental probability since she gives too much weight to small samples. In fact, none of the students studied exhibit a more advanced level of reasoning.

5.4.5 Tracing Kianja’s Use of Experimental Data

Kianja does not appear to have much interest in experimental data, as she makes her judgments about fairness on the basis of the sample space. The only recorded instance of Kianja referring to data occurs during Activity 4 when she cites Brionna’s score of 6 to 3 for Player B as corroboration of her *a priori* conclusion that Player B is more likely to win the game (6260). When Jerel challenges Kianja’s conclusions about the game in Activity 3, telling her that he won the game as Player A, Kianja tells him, twice, “I don’t care if you won” (5190, 5195).

5.4.6 Other Students' Use of Experimental Data

In grade 6, Kori and Nia judge the numbers 1, 2, 3 and 4 on a single die to be more likely than 5 or 6 because they do not observe many occurrences of 5 or 6 when they roll the dice. They dub the numbers 1 to 4 *common rollers* as a result of their data. As they play a game with 2, 4, 6 against 1, 3, 5, Kori remarks, " Yeah, this game is better [than 1, 2, 3 against 4, 5, 6]. It gives you a better chance of winning" (1295). She cites the close score of 8 to 6 as evidence that this split is fair (1302-1303). Nia contrasts this to the 10 to 1 score of their first attempt at a fair game (1308), which they say is unfair.

Danielle, on the other hand, declares 1, 2, and 3 to be "halfway impossible to get" despite data to the contrary. While Kori and Nia form an opinion based on a small amount of data, Danielle deems the data to be irrelevant and makes a subjective judgment.

In grade 7, Terrill's frequentist approach complements Chris' tendency to make *a priori* decisions. Though Terrill comments on the relationship between the sample space and the expected outcome of the game (5762-5764), he declares, "you have to play it first to see if it's really fair" (7990-7991). He teases Chris when Player A unexpectedly takes the lead in a game.

Ian's classical approach complements his partner Jerel's tendency to disregard the sample space. Ian and Jerel have an animated discussion about whether or not the game in Activity 4 is fair, with each boy holding fast to his opinion. Surprisingly, after Player B wins two of three games, Ian reverses course and says, "Yeah, it's fair. They each have enough of a chance . . ." (5984).

5.5 Conclusions and Implications

The difficulties of learning to reason probabilistically have been well documented in the literature, and this study reinforces those findings. The learning of probability requires ways of thinking that often run counter to learners' natural intuitions and occurs in situations fraught with variable and sometimes conflicting evidence. In the informal and supportive environment provided by the IML project, all the students studied made some progress towards normative probabilistic reasoning, but their journey is far from complete.

The IML students had no formal instruction in probability before the project began. Some students, such as Chris and Danielle, came to the project with the intuition that large numbers on a die are more likely than small numbers. Chris, in particular, maintained two contradictory beliefs: that the game of 1, 2, 3 vs. 4, 5, 6 is a fair game, and that 4, 5, and 6 are more likely to occur than 1, 2, and 3 when a single die is rolled. Prior studies have documented that inconsistent beliefs about chance events often coexist in people's minds (Konold et al., 1993; Rubel, 2007; Watson & Moritz, 2003).

Other IML students exhibited the use of common judgmental heuristics. Chris' assertion that large numbers on a die are more likely than small numbers may well be an application of the availability heuristic in which one judges the likelihood of an event based on what he can easily recall (Tversky & Kahneman, 1982b). Chanel, too, may have used the availability heuristic to declare that 11 and 12 are unlikely outcomes for the sum of two dice. Kori and Nia's designation of 1, 2, 3 and 4 as *common rollers* seems to be an application of the availability and representativeness heuristics.

Representativeness is the belief that a sample, no matter how small, should be representative of the larger population (Kahneman & Tversky, 1972). All of the IML students demonstrated belief in the "law of small numbers" (Tversky & Kahneman, 1982c) when they made judgments about fairness and probability comparisons based on a small number of trials. Justina provides a good example of this in Activity 4 when she calls the game fair after a score of 10 to 8 and then, moments later, declares the same game unfair when the score reaches 5 to 1.

Another judgmental heuristic, the outcome approach (Konold et al., 1993), was seen in the questioning by some of the researchers and graduate interns. Using the outcome approach, one views each trial of an experiment as an individual phenomenon instead of as one of many possible outcomes. This approach leads one to interpret a probability task as needing to correctly predict an outcome instead of recognizing what is likely to occur. Many times in the course of the IML probability sessions, adults asked, "Who is gonna win the game?" (for example, 798-800, 4268, 5742, 6242-6243, 6951). On a few occasions, students volunteered their predictions (for example, 3024, 4270, 4652). An exchange between R2, Jerel, and Chris demonstrates how R2 deftly corrected this approach (2010-2014):

R2	So, so let me see if I understand. When you first read the game, you thought that that Player A …
Jerel	Was gonna win.
R2	Was more likely to win.
Chris	Um humh.

Though the adults more than the students in the IML sessions showed use of the outcome approach, at least three of the students combined the outcome approach with the representativeness heuristic to create what I have called the *hybrid heuristic for chance* *events*. Jerel, Justina, and Adanna were asked what might happen if an unfair game were played many times. The game in question gave one player a $\frac{2}{3}$ probability to win a point. The representativeness heuristic alone would prompt one to say that the player who had the advantage would probably win about two-thirds of the games. However, these students agreed that the favored player would likely win all, or all but one of the games, even if 100 games were played. My interpretation is that the students first applied the outcome approach to predict that the favored player would win the next game, and then extended this result to represent all possible games. More evidence of this way of thinking is found in the students' answer to what might occur if a fair game were played many times. In this case, the students allowed for much more variability, saying that scores of 15 to 5 or 40 to 60 were possible. In a fair game, each player is just as likely to win, and so the outcome approach is problematic. Extending the idea that "anything can happen" over time, students arrived at the suggestion of more divergent scores than the representativeness heuristic would indicate.

The application of the hybrid heuristic to assessing the fairness of games is the belief that if either player is able to win, then the game must be fair. Jerel exhibited this way of thinking throughout the IML sessions. Given a choice between applying the hybrid heuristic and making a judgment based on the sample space, Jerel consistently went with the former. Justina and Adanna also applied this heuristic to their judgments, but not to the exclusion of other ways of reasoning about fairness.

The equiprobability bias (Lecoutre, 1992) is another judgmental heuristic that many of the students used to judge the fairness of games. Applying this heuristic, one believes all outcomes of a chance event are equally likely. The IML tasks were designed in part to provide cognitive conflict about this bias. The games using two dice gave more sums to Player A, but more outcomes to Player B. A learning trajectory for many students was:

1. Assume that the sums are equally likely and therefore the game favors Player A.
2. Play the game a few times and find that Player B wins more points.
3. Explore the number of ways the various sums can be obtained.
4. List the outcomes in the sample space and see that the game favors Player B.

In prior studies that used these games (Amit, 1998; Benko, 2006; Kiczek, 2000; Maher, 1998; Speiser & Walter, 1998; Vidakovic et al., 1998) students often followed this trajectory and then reached a point where they tried to resolve whether symmetric pairs of addends should be counted as separate outcomes in the sample space.

It was interesting to see, in the case of the IML students, that some (Chanel, Justina, Adanna) who followed this trajectory in Activity 2 during grade 6 started Activity 3 in grade 7 back at the first step. Chris did not use the equiprobability bias in Activity 2 with two ordinary dice (perhaps because he had some familiarity with the outcomes), but he did in Activity 3 using pyramidal dice. Chanel began Activity 4 once again at the first step of the trajectory. The return to the equiprobability bias in subsequent activities may be an indication that the students' understanding was unstable, or perhaps represents an instance of "folding back" (Pirie & Kieren, 1994) to an earlier level of understanding.

Unlike the students in the prior studies referenced above, no one in this study considered a sample space beyond 21 outcomes for the sum of two dice. Further, it became clear during the game with three pyramidal dice that the IML students had not built schemes for systematically generating outcomes as did the students in the Rutgers-Kenilworth project (Benko, 2006; Benko & Maher, 2006; Dann et al., 1995). This is surely an unfair comparison, though, as the Kenilworth students had been exploring counting problems since third grade. For the IML students, there had been no exposure to combinatorics before the project began.

Determining the sample space for a compound event is difficult for learners. In the second year of the project some of the graduate interns attempted to help students recognize permutations of addends as different events by demonstrating ways to think about dice sums, for example, by using dice of different colors. Their efforts were met with much resistance and little success. One obstacle to student understanding may be the negative transfer of the commutative property of addition. Perhaps an intermediate activity to build the concept of sample space for the outcomes of tossing two or more dice – without adding – could be helpful for students to identify permutations as distinct outcomes. R1 discussed such an activity with Chanel (5986-5989); it is similar to one used in the Rutgers-Kenilworth study with very favorable results (Benko & Maher, 2006, p. 2):

> Contest 1: A hat contains 3 tetrahedral dice, one white, one black, and one green. You win $900 if you roll a white 1 and a black 2 and a green 3.
>
> Contest 2: A hat contains 3 tetrahedral dice all the same color. You win $900 if you roll a 1, a 2, and a 3.
>
> Is there a difference in your chance of winning for each contest? Why or why not? Explain.

In addition to sample space, another area from which these students need further development is experience with experimental probability. Though most of the students expressed an understanding that the outcomes in the sample space having the most combinations are the most likely to occur, they demonstrated no conception of the Law of Large Numbers. Indeed, each of the students studied used small samples to justify or support their judgments. Later in the project, beginning in the summer sessions of IML, students used computer simulations of random generators with *Probability Explorer* software (Stohl, 1999-2005) to investigate a variety of tasks. Research currently underway by Barbara Tozzi and others could provide insight into the development of the reasoning of these students about experimental probability as a result of these interventions and could possibly show the impact of gathering large samples and collecting data from multiple representations.

Through the course of the IML probability sessions, some of the graduate interns who were assigned to observe and record the mathematical activity of small groups nevertheless intervened in the student investigations. Sometimes, they asked questions to better understand students' reasoning. However, they sometimes also seized what they judged to be teachable moments and questioned and challenged students' findings about generating outcomes in the sample space. It seems that a pervasive belief among some of the graduate interns is that learning occurs when teachers are able to transmit their personal understanding of a concept to students. This belief is based on the idea that in showing and explaining based on one's own understanding, others can also learn. This may be encouraged by observing behaviors of students who exhibit the desired outcomes which could be obtained by imitation and without understanding. The students may produce outcomes in a way that suggests that they understand, but, in fact, do not. Consider Kianja's reaction when G4 suggested 1+2 and 2+1 as different outcomes. Though she followed G4's suggestion and modified her sample space as guided, her later comment suggested that there was no difference. She said that it was "the same concept" whether permutations were used or not. The following day, she told R2 that 2 and 1 is the same as 1 and 2. It was not until she was given a task that provided her the opportunity to build her own understanding that she came to count permutations as different events. Some of the interventions seen in this study illustrate that students' conceptions are not altered by being told what or how to think. However, suggestions – whether by an adult or a student – backed by experience can offer alternatives that might not otherwise be pursued.

The students in this study exhibited some growth in their probabilistic reasoning over the two years, as measured by the Jones et al. (1999) framework. Their progress was not uniform across constructs. Many of the students remained fixed at the transitional level of reasoning about experimental probability, for example, but advanced to the informal quantitative level of reasoning about probability comparisons. Through their game activities, students grappled with concepts such as assessing fairness, sample space, and probability comparisons for perhaps the first time. By the end of the grade 7 sessions, it seems that all of the students studied realized that dice sums are not equally likely. Each student produced a sample space for the dice sums or s/he worked with a partner who did so. And, though small samples were used, all of the students used experimental data to some degree in order to inform or provide support for their opinions about fairness. The challenge for researchers and teachers is to find those activities that make students aware of the conflicts between their judgmental heuristics and normative probabilistic reasoning. In resolving these conflicts, students may learn to abandon their faulty intuitions and build solutions based on more complete data and reliable evidence.

APPENDIX A - IML PROBABILITY TASKS

A Game for Two Players

Roll one die. If the die lands on 1, 2, 3, or 4, Player A gets one point (and Player B gets 0). If the die lands on 5 or 6, Player B gets one point (and player A gets 0). Continue rolling the die. The first player to get ten points is the winner. (1) Is this a fair game? Why or why not? (2) Play the game with a partner. Do the results of playing the game support your answer? Explain. (3) If you think the game is unfair, how could you change it so that it would be fair?

Another Game for Two Players

Roll two dice. If their sum is 2, 3, 4, 10, 11, or 12, Player A gets one point (and Player B gets 0). If their sum is 5, 6, 7, 8 or 9, Player B gets one point (and Player A gets 0). Continue rolling the dice. The first person to get ten points is the winner. (1) Is this a fair game? Why or why not? (2) Play the game with a partner. Do the results of playing the game support your answer? Explain. (3) If you think the game is unfair, how could you change it so that it would be fair?

A Racing Game

Below, numbered 2 to 12, are the starting positions of eleven runners lined up for a race. Roll two dice. On each roll, the runner whose number equals the sum of the dice advances 1 square toward the finish line. The other runners do not advance forward. Continue to play the game until a runner reaches the finish line. The first to reach it wins. (1) Is this a fair game? Why or why not? If it is not fair, which runners are more likely to win and why? (2) Play the game with your partner. Do the results of playing the game support your prediction? Explain. (3) If you think the game is unfair, how could you change it so that it would be fair?

A Pyramidal Dice Game

A pyramidal die has four sides. The number that is rolled is shown upright. Roll two dice. If the sum of the two dice is 2, 3, 7, or 8, Player A gets one point (and player B gets 0). If the sum is 4, 5, or 6, Player B gets one point (and Player A gets 0). Continue rolling the dice. The first person to get ten points is the winner. (1) Is this a fair game? Why or why not? (2) Play the game with a partner. Do the results of playing the game support your answer? Explain. (3) If you think the game is unfair, how could you change it so that it would be fair?

Another Pyramidal Dice Game

Roll three pyramidal dice. If the sum of the three dice is 3, 4, 7, 8, or 12, Player A gets one point (and Player Be gets 0). If the sum is 5, 6, 9, 10, or 11, Player B gets one point (and Player A gets 0). Continue rolling the dice. The first player to get ten points is the winner. (1) Is this a fair game? Why or why not? (2) Play the game with a partner. Do the results of playing the game support your answer? Explain. (3) If you think the game is unfair, how could you change it so that it would be fair?

APPENDIX B - ATTENDANCE AT IML PROBABILITY SESSIONS

Date/Activity	Chanel	Chris L.	Jerel	Justina	Kianja
4/29/04 Activities 1 and 2 Dice games	F	F	F	F	F
5/5/04 Activity 2 Dice game	P	P	P	P	
5/5/04 – 5/6/04 Interviews		F	F	F	
8/2/04 Sampling	F	F	F		
8/3/04 coins & marbles	F	F	F	N	F
8/4/04 10 marbles	F	F	F	F	F
8/5/04 100 marbles	F	F	F	F	F
8/9/04 100 marbles	F	F	F	F	F
8/10/04 100 marbles	F	F	F	F	F
8/11/04 Fish study	F	F	F	N	F
8/12/04 Fish study	F	F	F	F	F
5/4/05 Activity 3 Pyramidal dice game	F				F
5/5/05 Activity 3 Pyramidal dice game		F	F	F	F
5/11/05 Activity 4 Pyramidal dice game		F	F	F	F
5/12/05 Activity 4 Pyramidal dice game	F	F	F	F	F
8/1/05 Marbles		F	F	F	F
8/2/05 Gym class	F	F	F	N	F
8/3/05 Schoolopoly	F	F	F	N	F
8/4/05 Schoolopoly	F	F	F	F	F
9/ 14/05 Schoolopoly revisited	F		F	F	F
9/ 15/05 Schoolopoly revisited	F		F	F	F

F= filmed P= filmed as part of a large group N= present but not filmed [blank] = Absent

APPENDIX C – CD DATABASE

Grade	DATE	CD numbers	Focus students present	Activity
6	4/29/04	42a, 43a	Chris, Jerel	# 1 and 2 Games with ordinary dice
		42b, 43b	Justina, Kianja	
		42c, 43c	Chanel	
	5/5/04	44a, 45a	Justina	#2 Game with two ordinary dice
		44b	Chris, Jerel, Chanel	
		46a, 46b	Chris, Jerel	Interview
	5/6/04	49a, 49b	Justina (with Adanna)	Interview
7	5/4/05	119c, 120c	Kianja	#3 Game with two pyramidal dice
		119d, 120d	Chanel	
	5/5/05	121b, 122b	Kianja, Justina	
		121c, 122c	Jerel	
		122a	Chris	Interview
	5/11/05	123a, 124a	Chris	#4 Game with three pyramidal dice
		123b, 124b	Kianja, Jerel	
		123d, 124d	Justina	
	5/12/05	125a, 126a	Kianja (roving camera)	
		125c, 126c	Chris	
		125d, 126d	Justina	

APPENDIX D - COMPLETE TRANSCRIPT

Date: 29 April 2004 Grade 6
Location: Hubbard Middle School
CD: ROLE 042A-043A
Transcribed by: Kathleen Shay
Verified by: Christopher Beattys

Time	**Speaker**	**Transcription**
5:47	R2	Here is the problem. The problem is a game for two players. So you're gonna play this game in pairs. It says "Roll one die." Does everyone know what a die is?
	students	Yes. [chatter]
	R2	Why does it say die instead of dice?
	students	One. 'Cause it's one. Abbreviation. One. No.
	R2	Okay. So it just stands for one of them [holding up a die in his hand], right? If the die lands on 1, 2, 3, or 4, Player A gets one point and Player B gets zero. If the die lands on 5 or 6, Player B gets one point and Player A gets zero. Now we'd like for you to continue rolling the die, and the first player to get 10 points is the winner.
	student	Okay, you gonna give us some dice?
	R2	Okay? So that's the game. You will have paper and pens and markers so that you can, as someone said, keep score. And you might wanna think about very carefully what kind of information you wanna keep. What kind of information do you want to record as you play this game? Understand? Okay? So, just, Jelani, would you come up?
	Jelani	Why? Why me?
	R2	I want, I want, you and I are gonna just play … [Jelani gets up.]
	students	[chatter]
7:22	R2	Uh, Jelani, do you want to be Player A or Player B?
	Jelani	Player A. [sits down]
	R2	All right. Jelani decides to be Player A. And, Jelani, could you tell us why you want to be Player A?
	Jelani	I don't know. 'Cause I got an A in my name, I don't know.
7:40	R2	Uh huh. Okay. Well, come on, come on up. You're gonna roll the die. You're the first. You're gonna roll.
	Jelani	You got die?
	R2	Yeah.
	Jelani	Oh. [takes die from R2's hand] I was about to say, "How you gonna play dice when you ain't got a dice? How you gonna roll when you ain't got a dice?"
	R2	Okay? Just roll right on top of there [overhead projector]. Be careful it doesn't fall off.
	Jelani	[places die on top of the projector]

	R2	Did he roll it?
	students	No. [laughter]
	R2	No. Okay. I think you know how to roll the die, right? [gives die to Jelani]
	Jelani	[laughing, rolls die] Alright, alright. That’s it.
8:02	R2	All right. So what did you get?
	Jelani	2.
	R2	Who gets that point?
	students	A. A. A. B. A. C.
	R2	So he gets the point, huh? So I’m going to roll now [rolls die]. This was a 4. Who gets the point?
	students	A. Jelani. 6 points.
	R2	Jelani gets the point. So how many points does Jelani have so far?
	students	6. 2. 6.
	R2	He gets one point …
	student	Oh no, 2.
	R2	… when he wins the roll. Okay. So how many points does he have?
	students	2. 2.
	R2	And how many points do I have?
	students	2. 0.
8:40	R2	Okay. Now let’s just take a look at some other parts of the game that we’d like for you to think about. We’d like you to think about whether or not you believe this game is fair. Okay? Is the game fair? And, why or why not?
	students	It is not. Yes. It is. No, it’s not fair.
	R2	Okay. In your groups, and with your partner, you’ll discuss whether you think the game is fair and why or why not.
	student	It is not.
	R2	Okay? You’ll play the game with your partner and you’ll see whether or not the results that you obtain support what, how you responded to the first question. So when you start playing the game, you might write down on a piece of paper whether or not you and your partner think the game is fair. And then play the game and see whether or not the results that you obtain, do they support, do they support your, what you thought. Okay? And if you think the game is unfair after playing it some, see whether or not you can come up with a modification of the game so that you have a fair game. Okay? All right. So let me tell you where the groups are going to be. [R2 announces the groups and where they will be working. Each group will have a researcher assigned to work with them: R2, R3, R4, G1, and G2.]
		[The camera follows two groups to another room. The groups set up. Chris and Jerel sit in facing desks near the wall, with space between them and the next pair, Dante and David.]
15:55		[G2 gives a green die to Jerel and Chris. Chris asks about having

		another die.]
	G2	[to Chris] Do you want, like, when Jerel rolls he's gonna use a green one and when you roll you're gonna use a white one?
16:10	Chris	Yeah, yeah.
	G2	Is that what you're saying? Okay. But you only, you gotta take turns, though. [gives Chris a white die]
	Jerel	I'll go first. [rolls the green die onto the mat while Chris takes a trial roll off the mat]
	G2	Exactly. And you're gonna need to …
	Jerel	You gotta wait. They gotta give us paper, right?
	G2	Exactly. We need some paper to keep score on.
	Chris	[points to Jerel] You gonna keep score?
	Jerel	Yep. I guess.
16:33	G2	Here's some paper. Please put your names on them, okay?
17:00	Chris	Ready?
	Jerel	[rolls a 5]
	Chris	You got 5. I get a point.
	Jerel	Wait, no, no. You don't even have a copy of the [inaudible]. I was just playin'.
	Chris	1, 2, 3, or 4, you get a point. 5 or 6, I get a point. You don't get nuttin.
17:10	Jerel	Oh, this is a, this is an unfair game already.
	Chris	I know. [smiles]
	Jerel	So it's going to be easy. [rolls die off the mat] Wait, that don't count. [rolls again]
	Chris	6. My, I get a point. This is nice. [marks a point on his paper]
17:25		[An assistant places a paper with the task description on the desk.]
	Jerel	Okay.
	Chris	[reading upside down, aloud] Roll one die. If the die lands on 1, 2, 3, or 4 Player A gets the point.
	Jerel	I'm Player B?
	Chris	And Player B … No.
	Jerel	I'm Player A.
	Chris	Yeah. [reading aloud] On 5 and 6 Player B gets a point and Player A gets 0. So I get a point.
	Chris	[inaudible] Yeah. Point.
	Jerel	No, that's my point. If a die lands on 3 or 4, Player A, I'm Player A.
	Chris	I know, but Player B, it's now his turn to roll.
	Jerel	But wait, you got it all twisted.
	Chris	Oh, I get ya. Okay. I gotta roll again. [rolls]
18:10	Jerel	Oh that's me. [hands Chris the white die] I got green. [rolls the green die] 1. [Chris rolls] 2. Me. I'm killing you. You stink.
	Chris	I'm losing.
	Jerel	[rolls] Ooh! I'm scorching you. This game is unfair. It's just 'cause of my luck at gambling here. [rolls again several times –

		keeps missing the mat, so the rolls don't count]
	Chris	Wow, wow. 2 points. [referring to his score] [rolls, marks score] You got 8.
	Jerel	I know. [rolls, makes a gesture pulling his forearm across his chest] Oooh, son. Huh?
19:40	Chris	You won.
	R2	[announcing to class] Make sure you answer the first question, whether or not you think the game is fair, before you start playing the game. Don't tell me your answer.
	Chris	We already played.
	R2	Just make sure you discussed it and write down what your prediction is, whether the game is fair or not.
	Jerel	Well we already knew it was unfair from the [inaudible]. So, [dictates as Chris writes] we already knew it was unfair because Player A had more choices to choose from than Player B.
	R2	Are you also writing why you think it's fair or unfair? Did you guys do that?
	Jerel	Yeah.
	Chris	[reading aloud] Do the rules, results of playing the game support your answer?
	Jerel	Yes. That's because I beat you 10 to 6. I mean 10 to 2.
	Chris	[writes, "Yes, because Player A won 10 to 2."] [reads aloud] If you think the game is unfair, how could you change it?
	Jerel	You could change, oh, this is easy. You could change it by …
	Chris	Both having the same choices.
	Jerel	Same amount of choices like 3 and 3.
	Chris	[writes, "You can change it so both can have the same amount of choices like 1, 2, or 3 = Player A, 4, 5, or 6 = Player B."]
21:42	Jerel	Okay. The game is over. [Jerel rolls two dice.]
		[The boys discuss other dice games. Chris tells Jerel, "You've gotta get 7 or up." They play this recreational dice game, using different dice rolling techniques such as rubbing the dice between palms, blowing on the dice. They talk about bouncing dice off the wall.]
23:42		[An observer whispers to Jerel.]
	Jerel	We finished, though.
	Chris	Yeah. [rolls again]
	T1	[to Chris] So what did you guys decide to do to make the game fair?
	Chris	You gotta have like 3 choices to win. Like Player A had to get 1, 2, or 3 to get a point, and then Player B had to get 4, 5, or 6 to get a point.
	T1	Okay, well did you play with the new rules?
	Chris	No. Do we gotta play it with the new rules?
	T1	Well try it and see if it works …
	Jerel	You take white. [gives Chris the white die]

	Chris	[sets up another score chart on his paper] You start. You A, right? I'm B. Let's go.
	Jerel	What's my numbers – 1, 2 and 3?
	Chris	Yeah.
	Jerel	Wait, tis is unf…
	Chris	If you get 4, 5, or 6, I get a point.
	Jerel	[rolls] 3. Give me that point, boy! You not up on this!
25:00		[Chris & Jerel continue playing. The camera moves to Dante and David.]
31:58		[Camera moves to R2 with Michael.]
39:25		[Camera returns to Dante and David.]
46:10		[Camera returns to Chris and Jerel.]
		[Chris and Jerel are playing a game with two dice.]
46:23	G2	You're playing which game?
	Jerel	I'm winning.
	Chris	This, uh …
	G2	The game by Chris or the game by Jerel?
	Chris	Mine. [points to himself]
	Jerel	[shaking the dice] His.
	G2	Okay.
		[The boys roll 10 twice in a row.]
46:46	G2	I thought if you got more than 6 you got points.
	Jerel	No, that's made by Jerel.
	G2	I'm using the wrong game.
	Jerel	That's made by Jerel.
	G2	Oh, odd numbers and even numbers.
		[Chris has created a game in which Player A gets a point for rolling an odd number with two dice, Player B gets a point for rolling an even number. Jerel's game gives no points for a roll higher than 6. The boys continue playing Chris' game.]
	Jerel	10! I came back on you. And I'm about to win. It's just my hand.
	Chris	5.
	Jerel	Wait, I think you won.
	Chris	No. Oh yeah, I won! I won. Well, you should have never said nothin'. [looking up at G2] I won.
47:18	G2	Do you think this is a fair game?
	Jerel	Uh huh.
	Chris	Yeah, 'cause it was 10 to 9.
	Jerel	Yeah, and because I was losing and it wasn't like, it wasn't like the first game where, like he, when I was Player A it wasn't like he, he couldn't come back or like I couldn't come back.
	G2	But do you think there's an even number of chances of getting either an odd roll or an even roll?
	C&J	Yeah, uh huh.
	Chris	Yeah, 'cause the even numbers from 1 to 12 are 6. There are 6 of them. The odd numbers from 1 to 11 there are 6. 'Cause you can't

		get 12, it's an even number.
47:54	G2	I don't know. See, I'm, I'm not sure how you'd roll two dice and get a 1. A total … Right? If you're talkin' about the score on that.
	Chris	Uh
	Jerel	Cheating! He was cheating me! That's not fair. I can't get a, I can't get 1, 2, 3. [Chris smiles.]
	G2	Right, so what, what a …
	Chris	You can get a 2. You can get a 2.
	Jerel	I can't get 1 though, oh, wait.
	Chris	You can get a 2, you can get a 4, you can get a 6. You can get 8. And you can get 10. And you could get a 12. [counting with his fingers] I can't get a 1.
	G2	Here, maybe you should write that, maybe you could write that on the bottom of this. Here, right? So where, where, what rolls would Player A get a point?
48:30	Jerel	Well, he can't get 1.
	Chris	Oh well, I still won.
	T1	So even with the way the game was set up, uh, you couldn't get 1. You had odd numbers? And you still won?
	Jerel	Yep. But you, but you can get 2, though. You roll two 1's.
	G2	But if you played, maybe you need to play that more times and see if that's really fair. Maybe, maybe Chris just got lucky being um, the player with the odd numbers.
	Chris	It's skills.
	G2	Oh, skills? Oh, okay.
	Jerel	I'll take odd numbers. I'll take odd numbers and I'm gonna still win. Make up a new game board. Make up a new game board. I'm about to thrash you with my odd numbers.
	G2	Isn't it possible for Player A to get a point with, with a roll of 12? Over there? [no response]
	Chris	Got even.
	Jerel	I got odd.
49:28		[Chris and Jerel begin another game.]
50:04	Jerel	7, 7, son! You see them skills?
50:18		[Jerel makes some off-task remarks to Dante. The camera moves to Dante and David's table.]
53:50		[end of CD 042A]
		[begin CD 043A]
1:30		[Chris & Jerel write up their games on overhead transparencies. Chris' game gives Player A a point for rolling an odd number with two dice, while Player B gets a point for rolling an even number. Jerel's game gives no points for a roll higher than 6.]
8:30	Chris	All right, come on. [Inaudible] countdown. [Chris & Jerel take turns counting down. Their voices are heard off camera.]
9:50		[Camera returns to Chris and Jerel. They are playing a game with

		two dice.]
	Jerel	You’re not supposed to give me a point. [sounds like:] I’m not 1.
	G2	Did you finish writing up your games?
	Jerel	Yep. I made a mistake.
	G2	That’s okay. That happens sometimes. You got a big blue blob on yours, huh?
		[Chris and Jerel speak quietly.]
	Jerel	You didn’t give yourself a point?
	Chris	Huh? No, oh yeah. [writes on his paper]
	Jerel	Oh crap!
	Chris	I won.
	G2	Yeah? Which game?
	Chris	Evens. It’s the skills.
	Jerel	Now let’s play “Made by Jerel.” That one is better.
	Chris	I got skills.
	G2	Yeah? Your game is better?
	Jerel	Yeah.
	G2	Because it’s more fair?
	Jerel	Uh huh.
	G2	Or it’s more challenging, or …
	Jerel	It’s more challenging. It’s more fair, too. I’m gonna have to pick out a …
	Chris	[inaudible] If I’m gonna win, I want 50 cents.
		[Chris & Jerel play the game. They practice spinning a die. Jerel blows on it.]
12:19		[The camera moves to Dante and David.]
12:42		[The camera returns to Chris and Jerel. They are playing the game and talking about clothing.]
13:35	Chris	I’m beating you. I’m beating you.
		[Chris and Jerel continue playing. Chris keeps score.]
14:24	R2	May I have everyone’s attention? May I have everyone’s attention for a moment? I’m really sorry to interrupt. But, um, we have, we have a special treat. And so what we need to do in order to, uh, engage in it, I need you to be sure to put your name and the date and number the pages of all your work and MFP will come along and collect them by table. And once that’s done, then we can move into the next room. Okay? And next week when you come in on Wednesday, you’ll report to the rest of the group about your findings. Thank you.
		[Students prepare their papers and gather by the door.]
18:27		[end of CD 043A]

Date: 29 April 2004 Grade 6
Location: Hubbard Middle School
CD: ROLE 042B-043B
Transcribed by: Kathleen Shay
Verified by: Jeremy Milonas

Time	**Speaker**	**Transcription**
6:00		[Justina is seated alone with her arms crossed.]
11:30		[Adanna sits across from Justina at her desk.]
12:00		[Justina brings another desk adjacent to the first one. R4 gets the groups organized.]
13:50	R4	Does anybody think they understand what this game is, from what they said up there?
	Shanei	I think I do.
	R4	Okay, Shanei is gonna explain it. Okay.
	Shanei	We need to roll the dice and if it lands on the even numbers and the number 1 then that's Player A's point. If it lands on 5 [pause] and some other number then that's B's point.
	R4	Okay , okay. If, if I have a die like this one [holding a die in her open palm]. Hey, Shanei, and everybody. Did you know that if it's one, it's a die. If it's more than one, it's dice. Did you know that that was the plural? Uh, it is. Die, that's really, that's really just a word. If it's one of 'em, it's a die. Now, what the rules said, if I remember it and I have on her, is for each group of you, one of you's gonna be uh the A player and one's gonna be the B player. Can you decide between yourselves, or you want me to tell you? You want to be A or B?
15:00		[Camera is focused on Shanei and Shirelle.]
15:14	R4	Okay everybody. Listen up one more time. Okay. The rules for the game are, okay, it's noisy so we've got to really listen and look up to me. Shirelle, Shirelle can you look up? Uh for this game, who's A in each group? It was you, and it was you, it was you, and it was you [pointing to a member of each pair]. Okay, if uh when you roll a die, can I have this? [takes a mat from Shirelle and demonstrates rolling a die on a mat], okay, if it lands on 1, or 2, or 3, or 4, Player A gets a point. If it, that's what we're gonna think about. If it lands on, what's the other one, 5 or 6, then Player B gets a point. Now, uh, and you keep rolling and the first person, the first player to get 10 okay, uh, uh, the first player to get 10 points wins the game. Okay? Now, what you guys are gonna have to do is to keep a record of what you're doing so that you can prove it to us that you really won or didn't win. Shanei, you think it's not fair.
	students	It isn't. Me, too. It isn't.
	R4	Okay. We're gonna test it out [inaudible] and find out. Okay. So would everybody play one game, which is the first person who gets 10, and keep a record and see if it, if it lives up to your prediction

		that you think Player B's gonna win. Is that right?
17:20		[Camera is focused on Lorrin and Sha'Nae. Justina and Adanna are seated beyond them.]
19:10		[Camera moves toR4 talking to Justina and Adanna.]
	R4	Yeah, swap this time. That's only fair.
	Adanna	Oh, that's 10 points. I kept on going.
	R4	Okay, so 10 points makes a game. Would you this time keep a record of what number you rolled? Does that make sense?
	Adanna	Um humh.
	R4	Okay. [turns toward another pair of students]
	Justina	[rolls die] One. That's mine.
	Adanna	It was 4, 4, 5, 3. [rolls die]
	Justina	That's mine again.
	Adanna	I don't have any points.
	Justina	I know. I'm putting, I'm making a record of what number yours is[inaudible]. [rolls die] [To Adanna:] Roll.
20:14		[Camera moves to R4 talking to Shirelle and ShaNae]
	R4	It just makes it take a lot longer. Okay. okay. So you gonna do it again. And this time it doesn't matter who rolls. If it's 1, 2, 3, 4, it's yours [taps Shirelle's arm], if it's a 5, 6 it's yours [taps Shanei's arm]. [turns to another pair of students]
		[Camera remains on Shirelle & ShaNae.]
22:25		[Camera returns to Justina & Adanna.]
	R4	Which number comes up the most often?
	J&A	[inaudible]
	R4	No, I was just wondering if there's any one number you get more than any of the others. Or are they all about the same?
	Adanna	Yeah. No. You get this numbers the most [pointing at paper].
	R4	Oh, you think you get 1, 2, 3, 4 more than you get 5, 6.
	Adanna	Yeah, 'cause it's not fair. This person has 4, um, 4 numbers to score and only Player B has 2. It's to make it even …
	Justina	There's 6 numbers. To make it even give each person …
	Adanna	To make it even, no, Player A should get 8, um 3. And Player B should get 3.
	R4	But how would you do it?
	Justina	But that's not even still. [Stands up and reaches across to write on Adanna's paper – where draws circles around 1,2,3,4 and around 5,6.] If you want to make it even now you're only giving yourself two. You're giving me four.
	R4	I think that's what she … Is that what you were saying? Adanna, how would you make it even?
	Adanna	[Writes the numbers 1-6 in a column and draws a horizontal line separating 1, 2, 3 from 4, 5, 6. Writes "Player A" at the top and "Player B" at the bottom.]
	R4	Okay. So do you want to try it a coupld of more times and see if if if it's more fair now? Your new way?

	Justina	Okay. I'm Player A, which is 1, 2, 3.
23:36		[Camera follows R4 to Lorrin and Shanei]
25:00	R4	[To Adanna & Justina:] After you've tested it out by playing the new game a few times, uh, and make sure that what you say seems to be uh corroborated by your experience. Because that's the way it is with these kinds of things, if you really test them. And so keep records now for the new game and see if it's fair or not.
	Adanna	It's fair.
	R4	How would you know if it's fair?
	Adanna	Because she has 3 just like me. I have 3.
	R4	I got that. But what would you predict? Do you think Player A is going to [inaudible].
		[Camera is focused on Lorrin & Shanei, and their voices drown out the other conversation.]
26:50		[Camera moves to R4 with ShaNae & Shirelle.]
27:59		[After spinning around, camera comes to Justina & Adanna.]
	J&A	[After spinning around, camera comes to Justina & Adanna.]
	Justina	Wait, you went 5 times and I went [inaudible].
	Adanna	You went 4 times and …
	Justina	Wait [stands up and looks over at Adanna's paper.]
	Adanna	You went 5 times and I went 2 times.
	Justina	No, no. You went 5 times and I went twice. You spelled my name wrong. You put Justin. Justina. I have to pick up on 2, 3. You can't check with you because you already have 5. [rolls die]
	Adanna	Hold it. I'm all confused. Whose turn is it?
	Justina	Look, you went 5 times. I went like 3 times. Well, I have to catch up to you.
	Adanna	Start over please because I don't want to get confused.
	Justina	I just need to go. No. I just need to roll one more and then you'll roll again. I don't wanna start over. I'm not startin' over, Adanna, so let's just keep playing.
	Adanna	I might as well [unclear]. How am I suppose to understand [unclear] I was keeping the score.
	Justina	Okay, fine. Just go. [hands over the die]
	Adanna	[rolls the die]
	Justina	[rolls] Oh, that was a 2, Adanna. That's your point.
	J&A	[continue playing]
30:58		[Camera moves to R4 with ShaNae & Shirelle.]
32:23		[Camera returns to Justina & Adanna.]
	Adanna	[holding paper] All right, that's the rule. 1, 2, 3 Player A gets the point. And Player B, and [unclear] 4, 5, or 6, you get a point.
	T2	Okay, so why don't you see if you can mix up the numbers? Instead of doing 123 – 456, do something different and see what happens, what happens.
	Adanna	Do what different?
	T2	I'm asking, does the numbers matter? 456

	Justina	I'm getting 1, 3, 5. And I'm Player A.
	T2	She's using 1, 3, 5, which limits you to what numbers?
	Adanna	I need another paper.
	Justina	You could just use that space down there.
	Adanna	What'd you pick, 1, 3, 5?
	Justina	Yes. And I'm Player A.
	Adanna	We trying a different way, [R4]. She got 1, 3, 5, and I got 4, 6, 2.
	R4	Sort of the evens and the odds?
	Adanna	Yeah, even against the odd. Player A or B?
	Justina	I'm gonna go twice even and twice odd.
	Adanna	I know but player A or B?
	Justina	I'm A. [sets up her score sheet] [rolls]
	Adanna	She rolled a 1, so that's her point. I got 2 so it's my point. 4, my point.
	Justina	Wait, what numbers did you roll?
	J&A	[continue playing]
35:05		[Camera spins around. R4 brings Jarae to sit with J&A.]
36:00		[Camera on R4 with Shirelle & ShaNae.]
41:00	Justina	For each and every row, you get a certain amount of [inaudible]. The person with the most money wins. This row, I'm gonna write it down, too. [Writes on paper, preparing a game.] All right, so it goes 1, 5, 10, 15, 20, you know, and it goes by the fives. [unclear] Now, the person you go, you go 20, um [shakes head] 20, 10 times. And by all the 10 times, the person with the most money wins. All right? I'll go first. I don't even know if this is gonna work. [rolls die, looks over at the outcome] I got 10 now.
	Adanna	[unclear] But it landed right here?
	Justina	Oh wait. No, not the one where it rolls. I meant what, um, the little thing right here. And this is where you stay. This is where uh you stay right here until it tells you to roll. And every time you pass here, you get $5 extra.
	Adanna	1, 5, 10, 15, 20, 25
	Justina	You got $25. Right there, keep track.
	T3	What are you girls doing?
	Adanna	Making a game.
	T3	Making a game. Is that interesting? Is that what you say?
	Justina	[turns away from T3 and looks down, with her hand on her head]
43:05	T3	What are you initially supposed to do with the game?
	Justina	We were supposed to, um, just roll it. We were supposed to have different numbers. She had 1, 2, 3, 4 and I was supposed to have 5, 6. And then we rolled, and the person, the amount, the amounts of dots on it, which would equal 4, it would prob-, it would go for her. And then, and you went 10 times, the person who got 10 first, wins.
	T3	Okay. And who won?
	Justina	Well, [grabs paper] she won, wait, I won, I won, I won this one, I

		won.
	T3	You won all the time?
	Justina	[nods]
	T3	Which player were you?
	Justina	I was, no, she won once. I forgot that one. She won once. I was Player B for that one. I won for Player A, she lost. I was Player A again, she lost. I was Player B and she Player A, I won. Then I was Player A over here and she Player B, and I still won.
	T3	Okay. So do you think the way the game is set up if it's fair or not fair? Do you think it's fair the way it's set up?
	Justina	No, it wasn't fair, so we changed it. We changed it.
	T3	Now, why do you say it wasn't fair?
	Justina	Because it was uneven. She had, if, she had 4 numbers .
	T3	You said it wasn't even. What wasn't even?
	Justina	She had 4 numbers and I only had 2.
	T3	She had 4 numbers and you only had 2? Hmmm.
	Justina	Yeah. Yeah. And um the person with the most numbers, the dice is most likely to drop on the ones with the most numbers because you know, she just has the most and I only have a little bit, just um 5 and 6. So we changed it. She got 1, 2, and 3, and I got 4, 5, and 6. And then we mixed it up. I went, I got 1, 3, and 5, and then she got 2, 4, and 6. And that's the way we made it even.
	T3	Okay, so the second time you guys you guys made a change, right? So when you made the change you say she got 1, 2, and 3? And you got 4, 5, and 6? When you did it that way who won? Who won most of the time?
	Justina	Me.
	T3	[to Adanna] She won most of the time? Really? Why?
	Justina	It's a luck game.
	T3	It's a luck game? But you both…
	Adanna	Here, here I won. And when we, when we made it even, it was whoever wins gets the game.
	T3	So when you had 3 numbers and she had 3 numbers, did that make the game more of a fair game then?
	Adanna	Yeah. Because it allows whoever wins to win and whoever lose to lose. Um, here if she, if I win, it wouldn't be fair to her because um here I didn't roll none of her numbers.
	T3	Okay. I see here you have 1, 2, 3, right? And she has 4, 5, and 6. So she has all the high numbers? And you have all the low numbers? And that made the game fair?
46:08	Justina	It still is fair because it doesn't really matter whether the number is high or low because the dice might still roll on the low numbers as much as it rolls on the high numbers.
	T3	Ummm. Okay.
	Adanna	So it is anybody's game.
	T3	So then what happened when you mixed up the numbers?

Justina	It basically still stayed the same.
Adanna	You still won [inaudible].
T3	What were your numbers when you mixed up the numbers?
Adanna	I had evens and she had odds.
T3	Oh. So when you do odd and even, you got like, it’s a fair game still? Yeah? So do you win as many times as she won?
Justina	No. I won more than she did.
T3	How many times did you play the game?
Justina	We played 1, 2 …
T3	Whoa, whoa, when you mixed up the numbers, how many times did you play the game?
Justina	Once.
T3	Oh, well that’s not good enough.
Justina	[laughs]
T3	Try again. Like, 5 times? When you mixed up the numbers?
Adanna	Yes, because look.
Justina	This is the first time that we played it was unfair. And then we played, um then we changed it and we played a fair game 1,2,3, and had 4,5,6.
Adanna	It was 5.
Justina	No! And then we only played once for the odds and evens.
Adanna	Yes it is. ‘Cause look, here I had 1, 2, 3, 4, and here you had 1, 2,3, 4. [referring to her score sheet]
J&A	[more bickering about how many times they played]
T3	So why don’t we do this, right? Just for argument’s sake, I want to know if the game is really fair. Could you guys just play the game again with you guys mixing up the numbers the way you did to see if it’s fair or not? I just want to see who wins. All right? We just wanna experiment. Don’t throw that away [to Adanna, who has crumbled her paper]. Hang on to this one.
Justina	You waste paper.
Adanna	No I don’t.
Justina	You can use the back.
T3	Okay. That’s okay. All right. So who had the odd numbers, who had the even numbers in this one?
Adanna	I had the even numbers.
Justina	I had the odd.
T3	Okay.
Justina	I’m A, and you’re B.
T3	Okay, so Player A is odd and Player B is even?
Justina	Um humh. All right. I rolled a 5, that’s my point.
Adanna	Could you keep the score, because [unclear].
Justina	Roll. You asked me to keep the score. No need [for you] to keep the score. Just roll. You rolled a 5.
Adanna	That’s yours.
T3	That’s yours.

	Justina	I rolled a 3. That's my point. Okay. Your rolled a 5. That's my point.
		[Observer asks Justina about her numbers.]
	T3	She [pointing to Justina] has all the odd numbers …
	Adanna	1, 3, and 5. And I have 2, 4, 6.
49:10	Justina	You rolled a 4. She's got one point. You rolled a 4. [rolls] Five.
	Adanna	[rolls] In this game, in this game 5 is [inaudible].
49:35		[Camera moves to Lorrin & Shanei. A teacher is working with them, asking which number occurred the most. She tells them to make that number the wild number in their game.]
54:22		[end of CD 042B]
		[begin CD 043B]
0:25	Kianja	… and Player B, every time they got 5 or 6, they made it instead of 1 point, if they gave 'em 2 points, would it be even?
	ShaNae	[nods] Probably.
	R4	Why?
	Kianja	Because the score is that, is like having 4 numbers, but you only have 2.
	R4	Oh. We've got to try that one. But your notion is that if we did it that way, it would fair up the game as well?
	Kianja	I think it would work. It would be even because they have 4 points, right? They can have 4 points. Say the game goes up to 4. If they get all of their numbers they have 4. If you get both of your numbers, you have 4, too. So it's a tie.
	R4	So it'd be a tie. Yeah. That's really interesting. Yeah. I was just wondering if that might be a way to do it, too. So you're working on the second game?
	Kianja	[writes on her paper]
2:00		[Camera moves to Shirelle & ShaNae.]
5:26		[Camera on Kianja, with a teacher (T2).]
	T2	Okay, does it make a difference because we're, we're only comparing two players? So whether A, it doesn't really matter which is A.
	Kianja	[shrugs] It's okay. [writing] This one is 8 out of 21 probability of winning.
	T2	Why did you? Can you tell me what this means?
	Kianja	8 out of, 8 over 21?
	T2	So you wrote it as a fraction.
	Kianja	Right.
	T2	And what does the fraction represent?
	Kianja	[finishes writing] Well, I added up all of the, I added up all of the combinations, right? The um number sentences, and I got 21. So, on this one it's 8 out of 21 chances for the Player B to win and there's 13 chances out of 21 for Player A to win. So. [resumes writing]
	T2	So it's not even?

	Kianja	[shakes head]
6:52		[camera wanders off]
7:05		[camera on R4 with Shirelle, Shanei, Adanna, Justina]
	R4	You got 1? How do you get a 1? [with two dice]
	Student	Oh!
	R4	Okay. You told me you couldn't get a 1. Okay. And so do you think it's fair now?
	Adanna	No.
	R4	Which one do you think has the advantage?
	Adanna	Player A.
	R4	Why?
	Lorrin	Because there are 6, and there are 5.
	R4	Okay. Well I don't care who's Player A and Player B, you can take turns. But I want you to play now a few games. Can you put this one away for me? [moving over to Justina's desk] And I'm gon-, is this a blank? Okay, so Player A remember it is 2, 3, 4, 10, 11, and 12 [writing these on a paper], and Player B it is 5, 6, 7, 8, and 9. And so you guys predicted that Player A still has an advantage. Is that what you said? Justina?
	Justina	Yep.
	R4	They said they thought Player A was, had an advantage. And so I want you to play it a few times. Again, first person to get 10, wins. Okay?
	Justina	[smiling] I roll first.
	T3	So who's Player A and who's Player B?
	Justina	I'm Player A.
	T3	Well, well, before you start, before you start I wanna know why you wanna be Player A.
8:30	Justina	Because Player A has the advantage.
	T3	How do you know Player A has the advantage?
	Justina	Because Player A has more than Player B does.
	T3	More what?
	Adanna	Player B has like 5, and Player A has 6. So Player A should be, should get most of the points.
	T3	You really think so? You really believe that? I want to see this. Put it this way so that it doesn't roll all over the place.
	Justina	[rolls 2 dice]
	T3	All right, so the total is what?
	Adanna	7.
	T3	6. So that's Player B's points, right?
	Adanna	[rolls dice]
	Justina	5, 6, 7. [apparently adding on to the 5 die]
	T3	Player B again.
	Justina	[smiling, rolls] 5.
	T3	5, Player B.
	Justina	Okay. [laughs]

	T3	[after Adanna's roll] It's 8, Player B. It's 8, Player B. Okay? Okay, you go.
	Adanna	Man, you freezin'.
	Justina	Sh- [unclear] [rolls dice wildly, one falls off the table.]
	T3	Easy, easy, easy.
	Adanna	There's no way you could get up now.
	Justina	[laughs and rolls dice – perhaps placing them down without rolling]
	Adanna	Nah-ah, you cheated! [both girls laugh]
	T3	I see Justina trying to be slick over here.
	Justina	[rolls again] 5, 6, 7, 8 [apparently adding on to the 5 die]
	T3	Player B.
	Adanna	[rolls] 8.
	T3	8 again. Player B.
	Adanna	You don't even have one point yet.
	Justina	[rolls] 10.
	T3	Finally.
	Adanna	You, you lucky you be touched by an angel. [rolls] Ah no!
	T3	10 again.
	Adanna	Why he go and play me like that?
	Justina	[giggles]
	T3	5, that's 5.
	Adanna	My luck is back.
	T3	Player B. Player B. [after Adanna's roll] 6. Player B. [after Justina's roll] 8. Player B again. What's the score, 9, 9-2? [after a rolls misses the mat] Roll again. Roll again.
	Adanna	[rolls 9] I win! [R4]!
11:52		[Camera moves to Lorrin & Shanei.]
		[T3 is heard off camera talking with Justina & Adanna re: Player A has 6 numbers and Player B has 5. Justina wants to remain as Player A.]
12:43	R4	Everybody. Everybody. We have a special treat. Can anyone smell and tell me …
	student	Pizza. It's called P-I-Z-Z-A
		[students organize their papers for collection]
16:37	T3	So who do you think would've won this next game if you were to continue?
	Adanna	Me.
	Justina	I would win.
	Adanna	Me because my angel was on vacation.
	Justina	Well I guess it's gonna stay there a while, because I'm gonna beat you.
	T3	So what was the score when we left off? 3-1. It was 3-1? And who was favored?
	T2	Justina.
	T3	Justian was up to be goin'?

	Justina	You goin' down. This time I mean it, okay?
	Adanna	Oh, she dreamin'. I got [unclear].
	Justina	This is me close to the finish line, this is the finish line right here, this is me [pointing at the edge of her paper], this is you [pointing at the edge of her desk farthest from the "finish line"].
	Adanna	[grabs paper] Oh let me draw me kicking her butt.
17:30		[end of CD 043B]

Date: 29 April 2004 Grade 6
Location: Hubbard Middle School
CD: ROLE 042C-043C
Transcribed by: Kathleen Shay
Verified by: Christopher Beattys

Time	**Speaker**	**Transcription**
3:55	R2	Here is the problem. The problem is a game for two players. So you're gonna play this game in pairs. It says "Roll one die." Does everyone know what a die is?
		[The remainder of this introduction is transcribed with ROLE 042A.]
11:25		[Chanel, Nia, Danielle, and Kori set up to work with G1.]
11:27	G1	We're going to be rolling some dice today. So, um, let's set this up so it will be easier for you all to work together. Have you figured out who you want your partner to be?
	Student	Yeah, Chanel.
	Chanel	I'm so popular. [laughs]
		[The desks are rearranged so that Chanel & Danielle are partnered at one desk, and the other two girls at another desk.]
13:46	Chanel	Let's play rock, paper, scissors shoot to see who um, A or B.
	Danielle	I'm A.
	Chanel	No B I already got 'cause you know …
	Danielle	Why you want to be A for?
	Chanel	'Cause I got more!
	Danielle	I don't wann be …
	Chanel	You gonna be B?
	Danielle	I wanna be A.
	Chanel	Well, we gotta play rock paper scissors shoot for it.
	Danielle	No I'm A. I got the paper. And I got the red dice, which is the red dice if… I got mo' money. I got mo' money.
	Chanel	All right you could be A, who cares…
14:22	Chanel	I knew that you was gonna take my idea.
15:22		[Chanel and Danielle begin the game.]
	Chanel	That's my point.
	Danielle	No, that's my point.
	Chanel	No, that's my point. 'Cause if you have 1, 2, 3, 4, that's your

		point. If I have 5 or 6 it's my point.
	Danielle	I'm A, nah.
	Chanel	You A, but I'm B. [to G1] B's 5 and 6, right? Hers is 1, 2, 3, 4.
	G1	[reading] If the die lands on 1, 2, 3, or 4, Player A gets 1 point. If the die lands on 5 or 6, Player B gets one point.
	Danielle	Oh, that's my point.
	Chanel	No, that's my point.
	Danielle	No, I didn't know I had to start over so I got stuck. All right, this one.
	G1	Wait. Hold on a second. Let me see if there's a write-up of the problem for you so then that way you could read it. [brings a copy]
16:10	Danielle	[grabs the paper] I could read better. [reads aloud] Roll one die. If the die lands on 1, 2, 3, or 4, Player A gets 1 point and Player B gets 0. If the die lands on 5 or 6, Player B gets 1 point and Player A gets 0. Continue rolling the die. The first player to get 10 points is the winner. Is this a fair game? Why why not? Play the game with a partner. Do the results of playing the game support your answer? Explain. If you think the game is unfair, how could you change it so that it would be fair?
	G1	Okay. So what's the first thing we wanna do?
	Danielle	Roll the die.
	Chanel	Roll the die.
	G1	Well, what does the first question say?
	Danielle	Do you think it's fair or unfair?
	G1	Do you think it's fair or unfair?
	Nia	[at the next table] I think it's unfair.
	G1	You think it's unfair, why?
	Nia	Because, like, um, like, I don't know.
	Kori	I think it's unfair because …
	Nia	Because like if you roll the die …
	Kori	You say you don't know!
	Nia	I know now!
	Kori	No, my turn.
	Nia	I'll go after you.
	Kori	All right. I think it's unfair because Player A has 1, 2, 3, AND 4 to get a point, and Player B only has 5 and 6. And I have, I have 4 opportunities to get a chance and you only have 2. So I think that they should move 4 to Player B so it'd be even. 1, 2, and 3 for A, and 4, 5, and 6 for B.
17:30	G1	Okay, so who's Player A and who's Player B?
	Kori	I'm Player A, she's [pointing to Nia] Player B.
	G1	And you think it's unfair for who?
	Kori	For me to get, um, a number of chances, like 4 chances to get a point, and she only has 2.
	G1	Okay, so help me understand this. If you were to predict who's gonna win, just from reading the problem, who do you think is

gonna win?

Kori Me.

G1 And you are?

Kori Kori.

G1 Kori, and you’re Player?

Kori A.

G1 Player A. Okay. And Nia, what do you think?

Nia I think it’s unfair also because, like, I agree with Kori but I just like to add just because like it [picks up paper with instructions for the game] says something about Player gets 5 or, hmm 5 or, I have a question. Does it mean like if I have 5 or 6 and she has like 4 points, will that mean she loses all her points and gets 0 points?

G1 No. Kori, you’re shaking your head no. Why are you shaking your head no?

Kori Um because that doesn’t mean I’d lose all my points.

G1 So it means that if you roll, if you roll one die, so say we roll one die. Could you roll one for me? And what’s that?

Nia 4.

Kori I get a point and you don’t.

G1 So that means because since she’s Player A, if that lands on 1, or if it lands on 2, or if it lands on 3, or if it lands on 4, she’ll just get one point. But say that she rolled this and it landed on 6. Then you would get a point because it landed on 5 or 6. So you just keep accumulating points.

19:02 Nia ‘Cause like, just like, like there’s like, like ‘cause I don’t think it’s fair because like how come like she gets all, like I agree, she gets all these um, um chances [word suggested by G1] and I like I only get 2. Like if I was to change that I would get like, me like, like I would get, I would get like 4 chances. That’s like what Kori said and, she was getting 4. ‘Cause it wouldn’t be fair if I only have 2 chances. ‘Cause I might roll, it might land on 3 or 1 and like, like, it’s it’s like if I land on it, it’s not, I wouldn’t really, like, I don’t know how to say it like.

20:03 G1 Yeah, I think I get what you’re saying. So what you both are telling me is that it’s unfair because Player A has more chances than Player B. So this you’ve developed a hypothesis. So now that you’ve decided that the game is unfair and you told me why, you wanna go to number 2 and play it out and see if your, if it’s true?

Kori I have a question. When um, when we, like say if I roll it and it lands on 4, right?   Do I get 4 points?

G1 No, just one. All right. So that’s what it says when it says Player A gets one point and Player B gets zero points. Okay? So just start whichever color you like. And um, go ahead. Have fun, and I’ll be back.

20:32 G1 [to Chanel & Danielle] So, this first question – is the game fair?

	Chanel	No.
	Danielle	Yes it is. You just saying that ‘cause you lost.
	Chanel	No, it’s not fair. It’s not fair.
	G1	Did you think about it before you started playing?
20:40		[video of Kori and Nia’s game, audio of G1 with Chanel & Danielle in the background.]
	Chanel	Yeah. I thought about it.
	Danielle	And you said it was fair until you lost.
	Chanel	No. I said it’s not fair.
	G1	Okay. You think it’s fair, you think it’s not fair.
	Danielle	I think it’s fair and it’s not fair.
	G1	Why? Why is it fair? And why is it not fair?
	Danielle	It’s fair because it’s fun, and it’s not fair because …
	Chanel	I lost.
	Danielle	No, it’s not fair because the way the points is like set up.
	G1	Okay. So it’s fair because it’s fun but it’s not fair because of the way the points are set up. What do you mean by the way the points are set up?
	D&C	[speaking together] ‘Cause it’s like 1, 2, 3, 4, and then it’s only 5 and 6.
	Chanel	It should be like 4, 5, 6, and 1, 2, 3.
	G1	Okay. So who is it unfair for?
	Chanel	Me!
	G1	And you are?
	Chanel	B.
	G1	You are B. And you’re [Danielle] Player A. So you played the game and what happened?
	Chanel	I lost.
	G1	You lost. So what do you want to do next?
	Danielle	Wanna play again?
	G1	That sounds good.
	Chanel	Play the fair way.
	G1	How about, [to Danielle] why do you want to play again?
21:37	Danielle	I don’t know. No, I’m saying don’t play, don’t play again until…
	G1	Keep playing to see, to see, you said that it was unfair for B. So do a couple of runs.
	Danielle	So then I’ll be B. And you’ll be D.
	Chanel	And tell me if it’s not fair!
	Danielle	It’s fair.
	Chanel	[laughs] Don’t say that when you lose.
		[Chanel & Danielle set up their score sheets for a new game. They play a game.]
24:00		[Camera moves to G1 with Nia and Kori.]
27:29		[Camera returns to G1 with Chanel and Danielle.]
	G1	So tell me, what are you going to do right now?
	Chanel	I’m about to try um a new split ‘em up.

	G1	Split 'em up how?
	Chanel	Into um equal, like 1, 2, 3 and 4, 5, 6.
	G1	So who's gonna have 1, 2, 3 and who's gonna have 4, 5, 6?
	Danielle	Um, I'm gonna be A.
	Chanel	I'm gonna be B.
	G1	You're gonna be B. And A is gonna have what numbers?
	Danielle	A - 1,2,3. [writing to prepare the score sheet]
27:57	G1	[pointing at paper] Now these two trials, when you did this one, how was this split?
	Danielle	[writing] No that's A, I'm B.
	G1	How did they get points when you did these two?
	Danielle	Oh, um.
	G1	So when you ran these first two, who did you get points?
	Danielle	Oh. By, um, by rolling the dice and coming out with one of my numbers.
	G1	And what were your numbers? What were your numbers?
	Danielle	Mine was, oh 1, 2, 3, 4.
	G1	Okay. And then now when you do it this time, what are your Numbers going to be?
	Danielle	1, 2, 3.
	G1	Okay. And Player B's gonna be?
	Danielle	4, 5, 6.
	G1	All right. And you think this one's gonna make it fair or unfair?
	D&C	Fair.
	G1	All right. Let's see what happens.
		[C&D start to play the game.]
29:50	Danielle	Hold on. 1, 2, 3, 4, 5, 6, 7, 8.
	Chanel	And I got, I only got 5.
30:08		[Danielle – Player A- wins the game.]
	G1	So what happened there? So what happened there?
	Chanel	She won.
	G1	She won. And she was Player?
	C&D	A.
	G1	All right. So, are you itching to play another one to see who's gonna win again?
	C&D	Yeah.
	Chanel	Okay, this time I'm B.
	Danielle	And I got mine. Go first. No, I go first. I won the other one.
		[Chanel and Danielle start another game.]
30:45		[Camera moves to Kori and Nia.]
34:00		[With camera on Kori and Nia, audio picks up Chanel and Danielle with G1.]
	G1	So, did you finish your trials?
	C&D	Yes. Yes.
	G1	Who won?
	Chanel	I won this one, she won that one.

	G1	Who was Player A?
	Chanel	Me. This time on the second round …
34:20	Danielle	Hold up. We messed it up. ‘Cause, um, I was A and you was B.
	Chanel	Right. So now you’re B and I’m A.
	Danielle	Oh, I did it wrong. No wonder why.
	Chanel	But you did roll uh B.
	G1	And who won up here? [pointing to score sheet]
	Danielle	Me.
	G1	Player? What player?
	C&D	A.
	G1	And who won down here?
	C&D	A.
	G1	Player A. So again Player A. But this was supposed to be the fair one. So do you see anything fishy going on here?
34:51	Chanel	Yeah [laughs]. Player A is lucky.
	G1	Player A is lucky. You want to chalk it up to luck?
	Chanel	Let’s go one more time.
	Danielle	Player A is lucky. Yeah, A, A was winning.
	Chanel	Let’s go one more time. Because when it was fair um she got like close to mine.
	Danielle	Yeah, it was tied up.
	G1	Okay. What do you mean by close?
	Chanel	Like see how it was right here? It’s like 8, like 10 to 4 and right here is like 10 to what, 5, 6?
	Danielle	Yeah 5. 2, 4, 6, no 6.
35:16	G1	Okay. And then so what are you telling me that “close” means? Over here what happened?
	Danielle	[pointing to score sheet] No, down here was one that was close ‘cause it was tied up 8 to 8 and then she um she went she um she just went ahead.
	G1	So what does that mean? [pause] More fair? Less fair? Totally fair? Unfair?
	Chanel	Totally fair!
	G1	Totally fair. And you said you want to play another one to see?
	C&D	Yep. Yah.
	G1	All right. Play another one. Who’s gonna be A in this one?
	Danielle	Me.
	Chanel	I’m being B.
	Danielle	You’re B.
	G1	And you’re just rolling one die, right? Can I hold on to your other die for you?
36:00		[Chanel and Danielle begin another game.]
36:58	Danielle	No! You rolled twice.
	Chanel	Nuh-uh. You just rolled. And I didn’t. Then I just rolled.
	Danielle	Oh. [laughs]
37:22		[Danielle is poised to roll twice in a row.]

	Chanel	Gimme that, gimme that, nah nah nah!
	Danielle	[laughs]
37:50		[Chanel – Player B- wins the game.]
38:00	G1	So how what was the score?
	Chanel	I dunno. I think it was 10 [pause] 6, 7. [Danielle kept score by writing a mark for each point, e.g. //////////]
	G1	So what do you think about the fairness of the game?
	Chanel	It's fair …
	Danielle	Oh no. To me it wasn't because the 1,2,3 numbers, it's pro-, it's halfway impossible to get 'em sometimes.
	Chanel	Nuh-uh!
	Danielle	Yes it is!
	Chanel	What you mean it's halfway impossible? Every time I kept rollin' it, it oh … Everytime I kept rollin' …
	Danielle	Come on! [rolls a die, apparently landing on 4, 5, or 6] See?
	Chanel	So? [rolls die, perhaps landing on 1, 2, or 3 as she gestures to Danielle]
	G1	What do you mean it's "halfway impossible" to get?
	Danielle	It's, it's halfway impossible.
	Chanel	It's 50-50, girl! See, look at that.
	G1	What do you mean by 50-50?
	Danielle	[rolling die] 4 is you! See look 5 is you! Do it over. She drops it. Six is you.
	Chanel	1 is you.
	Danielle	Finally.
	Chanel	1 is you. 2 is you. 2 is you. Ha, you lucky you got that. Two is you again. Um hmm, 2.
	G1	So when you did this first round over here . . . [C&D are engaged in rolling the die and do not respond to G1.]
	G1	When you did the first round over here the way the problem was originally set up where Player A got points from 1, 2, 3, or 4, you said it was unfair and you did some trials. And then what happened?
39:17	Chanel	Then it got fair when we put it um 1, 2, 3 and 4, 5, 6.
	Danielle	Until she cheated.
	G1	So it got fair, say that again a little louder, it got fair when what?
	Chanel	When we put 1,2,3 and 4,5,6 together.
	G1	So that was the next set of trials you played?
	Chanel	Yeah.
	G1	Okay. So now do you, are you convinced that it's fair?
	Chanel	Yes.
	Danielle	[quietly] No.
39:39	G1	Okay. So hold on one second. [looking at paper] Let me get you another problem to work on. So now I'm going to give you two dice to work with. So are you ready? I'm going to ask you to roll the two dice together. And if the sum, the sum of the two, is equal

		to 2, 3, 4, 10, 11, or 12, Player A gets a point. And if the sum is 5, 6, 7, 8, or 9, Player B gets the point.
	Danielle	I'm A though.
	Chanel	No, I'm A. You always A. You was A the first time, so I'm B, I mean A.
	Danielle	Nah-hah. 'Cause you, B won, so that means I get to choose 'cause you B.
	Chanel	No. B won so, B won, right?
	Danielle	Okay. It doesn't matter.
40:47	Chanel	We got to roll 'em at the same time?
	G1	Yep. And you have to calculate the sum of them. And if the sum is 2, 3, 4, 10, 11, or 12, Player A gets it.
	Danielle	Come on, Chanel. Are you ready?
	G1	So let's have the same discussion. Do you think this is fair, or not?
	Chanel	No.
	G1	Why not?
	Chanel	Because this side got 1, 2, 3, 4, 5, 6, and this side only has …
	Danielle	12.
	Chanel	1,2,3,4,5, yeah, 5.
	G1	So Player A has …
	Chanel	6.
	G1	Player B has …
	C&D	5.
	G1	So you think it's fair or unfair?
	C&D	Unfair.
	G1	Unfair for who?
	Danielle	'Cause it's not even.
	Chanel	For Player B.
	G1	Unfair for Player B because it's … [to Danielle] What did you just say?
	Chanel	Because it's 6 for Player A and uh it's 5 for Player B.
	Danielle	It's odd.
	G1	Okay. So now, let's do the same thing we did before. Keep rolling it and see who wins.
41:42		[Chanel and Danielle begin to play. They roll 6 and disagree over whose point it is. They do not have a copy of the rules of the game. G1 indicates that she will go to the next table and take the paper with her. She repeats the rules while Chanel and Danielle sit quietly.]
42:26	G1	Do you want to write it down?
	Chanel	No, I think I could manage.
	G1	You got it?
	Danielle	You could manage to cheat, too.
	Chanel	[laughs]
	Danielle	1, 2, 3 [writes "A 1 2 3 4 10 11 12 B) / 5 6 7 8 9 "]

	G1	2, 3, 4.
42:54	Danielle	She has more of a probability of winning because of the numbers.
	G1	She has more of a probability of winning because of the numbers? What does that mean?
	Danielle	Yeah. That mean, the numbers aren't even.
	G1	What do you mean by "not even"?
	Danielle	Like, she has 1, [brief pause] 2, 3, 4, 5, 6, 7, [brief pause] 2, 3, 4, 10, 11, or 12. And I should get, she should and, I should get 1, no she should get 1, I should get 2 …
	G1	One? Can one ever happen here?
	Danielle	Yeah, a little bit.
	Chanel	No, remember we [unclear].
	Danielle	Oh yeah. It's 2.
	G1	Okay. So then tell me again what you mean about "even."
	Danielle	Oh, it's like 1, 2, 6 numbers up here and 5 numbers down here.
	G1	Okay. So it's gonna be unfair for who?
	Danielle	B.
	G1	For B. Let's do some trials and see if that's true or not.
43:43	Chanel	Okay.
	G1	How many games are you gonna play?
	Chanel	Um 2, 3. Ready set [rolls dice].
	Danielle	All right. I'm B.
		[Chanel and Danielle play the game, each rolling a die concurrently to get the sum of 2 dice.]
45:45		[Danielle – Player B- wins, 10-5.]
	Danielle	I won [laughs].
	Chanel	I told you. I knew it was fair. I think it's fair.
	Danielle	I didn't think it was.
	Chanel	Do you want me to tell you why? Because these numbers, these numbers right here, take out 11 and 10, I mean 12. These numbers are usual, are usual to pop up but 11 and 12, I don't think they usual to pop up, so.
	Danielle	Yeah, okay. 'Cause if it would've popped up for me you'd have been like ooh I told you it should've been. And you 11 did pop up for you.
	Chanel	But only like once.
	Danielle	It just now popped up once.
46:33		[Camera moves to G1 with Kori and Nia.]
50:20		[G1 and camera return to Chanel and Danielle.]
	G1	So where were you? So what were we working on?
	Chanel	[to someone else, not responding to G1] Yo class don't.
	G1	Do you guys know what we were workin' on? [no response] [pointing to paper] This was the first game you did?
	Danielle	No this [pointing elsewhere on the paper].
	G1	And who won?
	Danielle	B.

	G1	B won? And how about this one?
	Danielle	B, uh B won.
	G1	So twice B won. But before when I left you told me it was unfair because who was gonna win?
	Chanel	[apparently ignoring G1] It fell, it fell, it fell.
	Danielle	I know.
	G1	So before, before I left you told me it was unfair because ….
	Danielle	I won this one and she won this one, so see Chanel. [writes Chanel's name next to the game she won]
51:00	Chanel	But I do think it is fair for a sec. Because, because she won. 'Cause like 5, 5, 6, 7, 8 and 9, and 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 are, are single numbers so they like are usually, usually uh the ones who really pop up the most. And the 11 and 12 they uh, they don't really pop up because they have to have two different numbers or um two of the same numbers. And two of the same numbers don't really pop up.
	G1	What do you mean two of the same numbers don't pop up?
51:39	Chanel	Like, uh 5 and 5 for 10 'cause that 10 didn't pop up too much [unclear].
	G1	Could you show me on the dice what you're talking about?
	Chanel	Where's the dice?
	Danielle	I ain't got it, you got it.
	Chanel	No I don't. [Danielle drops the dice on the desk.] I knew you had it. 8.
	G1	So what are you talking about the two different numbers showing up?
	Chanel	These are, she dropped it and it was 4 and 4. But see if I go like this [cupping the dice in her hands and shaking] and I drop it it's gonna be 6 and 4.
	Danielle	Which is 11. 6, 7, 8, 9, 10, 11. [starts over] 6, 7, 8, 9, it's 10. [apparently counting on from the 6 die.]
	Chanel	[laughs]
	G1	So tell me a little bit more about what you were saying with two different numbers.
	C or D[?]	3. 6. 9.
	G1	So in respect to these numbers for A and B, how does that make it fair or unfair?
	Chanel	It makes it um, it makes it …
	Danielle	It makes it, it, it's fair. Well, it's sort of unfair because these numbers, okay, you know 5 and 6 and 7, 8, and 9 is gonna pop up, sure 'nuff, but …
	G1	Why is it gonna pop up sure 'nuff? [laughs] Sure 'nuff, you better tell me why.
	Danielle	[rolls dice] 6. See? 2.
	Chanel	But see if I go like this [tosses dice], 3 and 3 that's usually to pop up, 3 and 3. So …

	G1	How do you know it's usually gonna pop up? What makes it so special?
52:49	Danielle	Okay. That's 4 and 4, which is hers.
	Chanel	But see, see we keep rolling it but 12 or 11 doesn't pop up that much.
	G1	Why do you think they don't pop up that much?
	Danielle	Because we don't roll it. [laugh] It doesn't get, it doesn't come.
	G1	Do you think that's because it's fair, or it's unfair?
	Danielle	It's unfair.
	G1	So, you said it's unfair, for A or for B?
	Chanel	For, uh A. I say for A.
	Danielle	Aren't we playing another one? We said three.
	G1	So you said it's fair or unfair? It's unfair for who? [pause, no response] This is your second time you're doing this one?
	Danielle	Um humh.
	G1	And who won the first time?
	Danielle	Me.
	G1	And you're Player?
	Danielle	B.
	G1	Okay. And this is the second time you're doing it.
	Danielle	Uh huh. And she's Player B and she won.
	G1	Okay. And now this is the third time?
	Danielle	Yep. About to be the third time.
54:12	G1	So play until you convince yourself if it's fair or unfair.
	Chanel	I still say it's fair.
	G1	You still say it's unfair.
	Danielle	Even if you play a hundred times.
	Chanel	Yeah.
	G1	Unfair for who?
	Danielle	B.
	G1	For B, okay.
	Danielle	Right, A and B?
	Chanel	No, it was !.
	Danielle	No, it's B.
	G1	We're gonna do the same thing. I want you, if you think it's unfair, I want you to try to figure out how to make it fair. Then I'm gonna give you some transparencies so you can write it up on that.
	Danielle	[to Chanel] Girl if you don't give me my uh green [die]… Oh yeah, I wanna win.
	Chanel	Ready, set, and go. 6, 7, 8, 9, that's mine.
	Danielle	That's not you! I'm A.
	Chanel	Right, I'm B.
	Danielle	No, I'm B. 'Cause you was B last time.
	Chanel	Oh, well go ahead, take it. Take it. [waving her hand] Take it. Just take it.

	Danielle	Ain't that right?
	Chanel	No.
	Danielle	Yes you was. See, little cheater. [shows Chanel the score sheet] Yup, you was B last time. Little cheater. So I'm B. Oh what did you roll?
55:16	Chanel	Hmm?
	Danielle	Anyway, I know it was my point. What did you roll?
	Chanel	[quietly] I think I rolled a 3 and you rolled a 6.
	Danielle	[another roll] 6, 7, 8. That's you. No, that's me. See, it's the probability. Hello-o, Chanel. [Danielle is ready to roll, Chanel is staring blankly.]
	Chanel	Oh. [rolls]
	Danielle	3, 4, 5, 6, 7, 8. Don't even be tryin' to [juke?] my dice up.
		[The girls continue playing.]
56:05		[Kori is heard in the background while the camera is on Chanel and Danielle. Camera moves to Kori speaking to R2.]
	Kori	One out of a blue moon you would get 5 or 6. But 1,2,3,4… [inaudible] Right now [rolls die] I get 1. And if I keep on rolling, I would get 2 or 3 or 4. It what, that's why I say it's not fair because I have 4 opportunities to get a point and my, Nia only had 2. So it's not right. So that's why we um switched over 4 to um 5 and 6. So it'd be even. One, two, three, that's me, and 4, 5, and 6, would be Nia. But then I um I played. [to Nia, who has just returned] I was just um explaining something. So we played again and it still wasn't fair 'cause I still won because I kept on rollin' and it got just 1, 2, and 3. So then we figured we'd try, she gets 1, 3, 5 [tapping Nia's arm], [recoils her arm] and I get 2, 4, and 6 [tapping her chest]. That way it's still 3 numbers, but I don't think they're, they're um, all of them are common. So each of us have a common one and a non-common one.
57:24	R2	When you say "common one," what do you mean?
	Kori	Like it more likely to um it be um on the top.
	R2	Ah, more likely to roll to that number. So which numbers do you think are common, more likely to roll?
	Nia	I don't know.
	Kori	1, 2, 3, and 4.
	R2	[to Nia] You're not …
	Nia	Yeah, that's true.
	R2	Yeah? You think those are more likely to roll?
	Nia	Yes.
	R2	Uh huh. And they're more likely to roll than 5 and 6?
	Kori	Yes.
	R2	Why is that?
	Nia	'Cause it doesn't really pop up that, it doesn't really pop up that, like usually.
	R2	That often? Uh huh.

	Nia	Like, see? [rolls die]
	Kori	[also rolls a die] 2 and 3.
	R2	Okay, okay.
	Nia	Like and if I was to roll again, [rolls] see?
	Kori	4. [rolls] then one out of a blue moon you get a 5.
	R2	And did you, after coming up with your, what you think is a fair game, did you try it? Did you play it?
	Kori	Not yet. 'Cause we tried the other one.
	R2	Okay. Why don't you play your game, that you think is fair, and see what happens.
	Kori	Okay. Your are 2, 4, …
	Nia	6, 8?
	Kori	and 6. I am 1, 3, and 5.
	Nia	Yeah, I think that's [nods head].
		[The girls prepare to start the game.]
58:49		[end of CD 042C]
		[begin CD 043C]
0:00		[Kori and Nia are playing.]
	Kori	[to Nia] Was you Player A or B?
	Nia	I'm A. So A's this [tapping paper].
	Kori	Yeah.
	G1	So you think that's what's affecting it?
	Kori	Yeah. And 1 is 1, 3, and 5.
	Nia	[rolls] 2.
	Kori	So this is how we can really find out which which um number is *really* the most common roller. [rolls] 1. [Nia rolls.] 6. [Nia rolls again.] No, I get that.
	Nia	Scribble that out. Or you coulda kept that if I woulda got, I don't know.
	Kori	[rolls] 5.
	Nia	It's kinda even now. [rolls]
	Kori	6.
	Nia	We shoulda kept that. [perhaps referring to her "extra" roll]
	Kori	I know, but you told me to roll it out. [rolls] 6. [rolls again] 5
1:12	Nia	I like this too.
	Kori	Yeah, this game is better. It gives you a better chance of winning.
	Nia	'Cause at least you, you're like kind of close to me, like you got 4, you're like 2 points away.
1:42	Kori	Yeah. [the girls continue rolling] This ver- this um a better game than the last was because they are, it gives you a better chance at winning just …
	G3	Who won? Who won?
	Kori	Hmm? We don't won yet. The score is 1,5,6,7, 8 to 6. It gives you a better chance of having a shot of winning. Except for being like you're at 3 and the other person has 9.
	Nia	Even though you're losing it's still good like. I like how the points

		are other than how it was.
	Kori	They're close together, they're far apart like you have.
	Nia	And before I had like 1, you had 10. All right.
	Kori	Um humh.
	Nia	We're talking so much that we don't even know who goes.
	Kori	[rolls] 4, that's you. [Nia rolls] That 4? [Nia nods] She's 10. She won, I mean.
	Nia	I mean, but it's a good game.
	Kori	Yeah.
2:46	G3	Let me ask you this. Do you think this game is fairer as you have it set up this way with A being 2, 4, and 6 and B being 1, 3, and 5?
	K&N	Yes.
	G3	Or do you think as you did it before where A was 1, 2, and 3 and B was 4, 5, and 6 was fairer?
	Nia	I don't like that one.
	Kori	I don't, I don't think that one, that um last one was fair because …
	Nia	If you had this game, we, we could both be tied if we were to play this game. We could also be both tied. And like we're like close together. But like in the other game, you could have been like 1. Say I was B. I had 1 and Kori had 9. That's like very far apart. But this game was like it's pulled together, it's kinda pulled together.
	Kori	Yeah, it's more together.
	G3	Yeah, that's when you had the two dice. But I'm saying when you had the one die, …
3:33	Kori	In the other game? I don't think it was fair.
	G3	And you had, and when you rolled that it was A was 1, 2, and 3 and B was 4, 5, and 6. Do you think that game is fairer or less fairer than if A is 2, 4, and 6 and B is 1, 3, and 5?
	Kori	I think the game we are playing now is more fair because the last one, like I said before, 1, 2, and 3 were common rollers and 4, 5, and 6, well, 1, 2, 3, and 4 were common rollers. And I had 3 common rollers and Nia only had 1. And you will usually get 5 and 6 like, one out of a blue moon. So.
	Nia	That's why she has 5 and I have 6.
	Kori	Yeah.
4:20	Nia	And these numbers usually come up like [tosses die].
	Kori	We have like, each of us has …
	Nia	Three, and she has three, if I roll again [rolls], that's poppin' up, you know.
	Kori	And each of us has two common rollers and each of us has one, one out of blue roller. So it kind of makes us even.
4:37	G3	Okay. Let's play a game again as A is 1, 2, and 3 and B is 4, 5, and 6, and let's see what ha-
	Kori	4, 5, and 6?
	G3	A is 1, 2, and 3, okay, and B is 4, 5, and 6.

		[Kori and Nia prepare to play.]
5:15	Kori	1, 2, and 3; 4, 5, and 6. [rolls die] See what I mean? Three I'll get all the time, 2 I'll get all the time and 1 I'll get all the time. [Nia rolls a 3] See 3? Again. [Kori rolls a 2] Two.
	Nia	Who's A?
	Kori	1, 2, and 3; 4, 5, and 6.
	Nia	Yeah but that 2 belongs to me. Wait.
	Kori	1, 2, and 3 [pointing to herself]; 4, 5, and 6 [pointing to Nia].
	Nia	I'm A?
	Kori	Yes, I'm A.
	Nia	So, wait. If you're A how did you get 1, 3?
	Kori	No, we're playing the other game. 'Member 1, 2, and 3? 4, 5, and 6? Remember when we changed them? That's the game we playing.
	Nia	[points at paper] This one, right?
	Kori	No.
	Nia	Wait a minute.
	Kori	[holding up paper] 'Member when we did this, and I changed 4 to over here? That's the game we playing.
	Nia	Oh, okay. [rolls die off the table] Wait a minute.
	Kori	1. [rolls die] 6. [Nia rolls a 4, Kori marks it on the score sheet] Oh no, that's somethin'.
	Nia	Um um.
	Kori	4, 5, and 6.
		[The girls continue playing.]
6:52	G1	So what happened over here?
	Kori	Uh, that's 4. Oh yeah.
	Nia	Keep it. Just keep it 'cause you might get another one.
	G1	So what is this game?
	Kori	This is, this is um 1, 2, 3; 4, 5, and 6.
	G1	What's 1, 2, 3? Oh, the way you broke them down before?
	Kori	Yeah.
	G1	Okay.
	G3	They explained to [R2] that they thought it was fairer if A was 1, 3, 5 and B was …
	G1	Could you explain it to me, Kori? Could you explain to me what? how? Or Nia?
	Kori	How what? How we're playin'?
7:27	G1	Yeah.
	Nia	This one?
	Kori	No, this one, right.
	G1	Yeah, the one you're doing right now.
	Kori	We we saw this, and 'member?
	Nia	We're trying to prove our point that this one [pointing to paper] is not unfair.
	G1	That it's not unfair?

	Nia	I mean that it's unfair.
	G1	That it's unfair. Okay, so how are you proving your point?
	Nia	By just playing the game.
	Kori	By um changing 4 to 5 or 6 it w-
7:47	R2	[announces to the class that there will be a special treat. Asks students to organize their papers so they can move to the next room.]
		[Kori and Nia write their names on their papers.]
10:37		[end of CD 043C]

Date: 5 May 2004 Grade 6
Location: Hubbard Middle School
CD: ROLE 044A – 045A
Transcribed by: Kathleen Shay
Verified by: Judith Leonard

Time	**Speaker**	**Transcription**
1:54		[R4 is holding a paper, leaning over the desk and speaking to Justina and Adanna. Her speech is not audible. R4 puts the paper on the desk.]
		[The girls look at the paper between them.]
2:10	Justina	Well I thought, well me and her, we were playin' it and she kept pickin' up, she she kept beatin' me, and she was um Player B and this was …
	R4	Oh. Well maybe she just lucky. Maybe she just lucky. So why don't you play the game a few times and keep really good records.
	Justina	My first prediction, my first prediction was that it wasn't fair because …
	R4	That it would be Player A?
	Justina	That Player A would have the advantage because Player A had more numbers. But she kept beating me, and she was Player B and she had less numbers.
	R4	Okay, so that's where you were last time? Adanna, do you remember that as well?
	Justina	Yeah, and I was beatin' you on the third, by the third time.
2:50	R4	Okay, well why don't you play a little bit more and then decide whether you think it may be fair or whether you think either Player A or Player B has the advantage and why, okay?
3:23		[Camera leaves as Justina and Adanna are writing on their papers.]
25:38		[Camera returns to R4 with Justina and Adanna.]
	Adanna	It seems that like 5 to 8, 5 through 9, and 10 are the only ones are likely to appear, and 11 and 12 are few ones to appear, and 2, 3, and 4 are hard to get because um most, most of the numbers we be comin' up with was 5, 6, 7, 8 , 9, and 10, and not 1+1 or 1+2 or 1+3. So that's how it's hard to get.

	R4	Yeah. We might need to think a little bit more about why it might be easier to get these.
	Adanna	[to Justina] Roll it again please. Roll the dice again.
	R4	You could just play again. What do you want, me to roll the dice? [rolls] It is an 11.
	Adanna	Roll it again. Eleven, we came up with 11. [writes on her paper] Roll it again, please.
	R4	And d'you want to put down it was 5 and 6, too, if you don't mind. I'd like to keep that record. [inaudible] So we want to play again? Here's a 6.
		[Adanna has written on her paper: 11 = 5 + 6 6 = 4 +2]
	R4	Okay, these were B's [pointing to "6 = 4+2"] and these were A's [pointing to "11 = 5+6"]. Is that right?
		[Adanna writes "A" and "B" above these sums.]
	R4	Okay, Justina why don't you throw it for a few minutes and see if it's just me.
	Justina	[rolls] 6, 7, 8, 9.
	Adanna	9. Hers. Hold on. [Justina had started to pick up the dice before Adanna recorded the sum.]
	Justina	It was a 6 and I think a 3.
	R4	Who gets a point for 9?
	Justina	Me.
27:36	R4	But doesn't it go over there? [in B's column]
	Adanna	Hold on.
	Justina	[pointing to B on the paper] B is me.
	Adanna	You got 9, right?
	Justina	Yeah.
	Adanna	Which is? [crosses out the entry under A and rewrites it under B]
	Justina	A 6 and a 3. [rolls] Four. It's yours. 3 and 1.
	Adanna	Hold on, what was that? [writes "3+1=4" in A's column]
	Justina	[rolls] Five. That's mine. 3 and a 2.
		[Adanna writes "5 = 3+2" in B's column.]
	Justina	[rolls] Ten. That's yours. 5 and a 5.
		[Adanna writes "5" in B's column, then crosses it out and writes "5+5 = 10" in A's column.]
	Justina	Six. That's mine. 5 and 1.
		[Adanna writes "5+1=6" in B's column.]
	Justina	That's not visible. That's not visible. Just …
		[Adanna crosses out and rewrites "6= 5+1".]
	Justina	That's 7, and that would happen to be mine. That's 6 and 1.
		[Adanna writes "6+1=7" in B's column.]
29:31	Justina	Six. 3 and a 3.
	Adanna	It's cold. The window. [writes "3+3=6".]
	Justina	Six. 4 and a 2.
	R4	How many sixes have you gotten?
	Justina	1, 2, 3, …

	R4	No, that was 7.
	Adanna	1, 2, 3
	R4	And up here [pointing to paper]. Up here is 4. How did you get ‘em? I mean, were they all the same dice?
	Adanna	I think I see a …[pause] six
	R4	How many different ways did you get 6?
	Adanna	2 and 4, 3,3, 6, I mean 5 and 1, and 4 and 2 which is the same as …
30:37	R4	So you got it a lot of different ways. How many ways could you get 11?
	Adanna	One.
	R4	Does that make any difference?
	Adanna	[shakes her head no]
	R4	That doesn’t matter? You have one way to get this and you had a bunch of ways to get that, does that make it easier to get the other or not? [no response] What do you think, Justina? How many ways could you get an 11?
	Justina	Um, with the dice, I guess …
	R4	What did you do to get the 11?
	Justina	We rolled a 5 and a 6.
	R4	Okay. How many ways did you, how many, what did you do to get the 6?
	Justina	I rolled a 3 and a 3, a 4 and 2, and [pause] a 6, I mean a 5 and a 1.
	R4	Um humh. [pause] Does that matter? If I was gonna say …
	Adanna	These numbers are most likely to roll. [looking at what she has written on her paper]
	R4	Why? [pause – looking at Adanna’s paper] I think 8 is good. What about 8?
	Adanna	Huh? Where’s 8?
	R4	You just skipped it. You have 6, 7, 5, 9.
	Adanna	[unclear] numbers here. These numbers are easier to get which are here. [Adanna has written “10, 6, 9, 5, 7” on her paper. On the paper giving the rules of the game, she circled 2, 3, 4 and wrote “hard to get” upside down, she drew an arc over 11, 12 and wrote “few”, and she underlined the remaining numbers, 10, 3, 6, 7, 8, 9.]
	R4	Justina, do you have any ideas? Adanna just said that she thought she thought these numbers, 5, 6, 7, 8, and 9, are easier,10, maybe 10, are easier to get than 1 and 2 and 3 and 11 and 12. Do you agree or not?
	Justina	[makes a gesture off camera]
	R4	Why?
	Justina	Well um because those numbers that she’s talkin’ about is 5, 6,7, um they have more, um many more ways to get them than the other ones do, like 11, is only one way to get 11. So you’re really not likely to get that as much as you would, say, 6.
	R4	Okay … Oh, how many ways can you get 6?
	Justina	So far I’ve gotten three ways, four, no that’s three.

	Adanna	There's only one way to get 11.
	R4	Which is what?
	Adanna	Which was 5 and 6.
	R4	Yeah.
	Adanna	There's only one way to get 12.
	R4	Which would be?
	Adanna	6 and 6.
	R4	Could you do some record keeping for about that, and then maybe you can prove what you just said you thought. [to Justina] You could start workin' on it, too. I'd like to know what Adanna's saying, which is the different ways. She says you can only get 12 one way? [looking at Justina] You can always … [moves the dice closer to Justina]
		[Adanna asks R4 if she may close the window.]
33:57	Justina	Okay. Six and …
	Adanna	[speaks as she writes the following] There are one way …
	Justina	Six and six is one way.
	Adanna	to get 11.
	R4	Why don't you start working? Well you can do it right there, I'd like to know how many, to know what you think about all the different ways. What about 3? And 2? How many ways can you get a 2?
	Adanna	For 2 there's only one way.
	Justina	One and one.
		[Adanna has written, "There are one way to get 11 and 12 which is 5+ 6 and 6+6."]
	R4	Could you keep some records on that?
		[Justina and Adanna write on their papers. Justina has written "6+6 = 12 -- 1 way 1+1 = 2 – 1 way 2 +1 "]
	Adanna	There's gonna be many ways to get 12 with 3 dice.
	R4	Oh, but we only have two.
	Adanna	I know.
	R4	But you're absolutely right. If we did it, we could change the game and use 3 dice. That would be interesting. But let's finish with 2 first, and then we can play that other game. Okay, what numbers have you done so far?
	Adanna	3, no no no 2 and 1
		[Justina writes "2 + 1 = 3 – 1 way 2+2 } = } 4 2 ways 3+1 } "]
	Justina	[speaking to herself as she writes] Five. 2+3. 4 + [inaudible]
36:35		[Camera leaves Justina and Adanna.]
1:03:15	R4	Okay. Justina, explain it to Adanna and the camera.

	Adanna	And the camera. Talk!
	Justina	Okay. And don't talk to me like that. Anyway, the um, amount of total ways for Player B was 13, and um um the amount for Player A was only 8. So this was not fair because um Player B had [raises her voice, Adanna is speaking at the same time], Player B had 13 ways which was more than 8 ways Player A has. So, I had to, in order to make this right I had to add 13, which is Player B, and 8, which is Player A, together and I got 21. But 21 is an odd number and I can't get, um I can't make it even with an odd number because this is dice, and the dice doesn't have one-half on it. Okay? Okay? [waves her hand in front of Adanna's face] Were you listening?
	R4	So your problem is?
	Justina	So.
1:04:32		[end of CD 044A]
		[begin CD 045A]
10:34	R4	Justina says that she's gonna make it fair. And, can you explain it?
	Justina	All right. This is what I did to make it fair. I took away one of the numbers so that both of the, both of the players had 5 numbers, and then I just happened to take away 12. And then, so they, so then when I add-, then what I had left was Player A, which was with Player A that they came up to a total of 7, and then Player B still, I didn't take away anything away from Player B, so that was 13. And 7 + 13 = 20. So in order to make this even, each player had to have um the same amounts of ways. So, they each got 10.
	R4	Could you explain that again to [T4]?
	Justina	Um, for Player, what was it, for Player A, Player A used to have 8 points because um, they were, um the, the numbers that are one the side of here, those are the different ways that you can get them. That was a 12.
	T4	What do you mean? Give me an example.
	Justina	So, with the two dice you can only get 12 once.
	T4	How?
	Justina	6 and 6.
	T4	Okay. I understand.
	Justina	So, and that turned out to be an 8. And eight's, and then I, I went to Player B and then I found that all of these had [points to Player B's numbers] had wait, where's Player B? Where is it? Adanna, where's my Player B? You had Player B, I did Player A [inaudible]. Well, um, Player B ended up with 13, 'cause 13 all together. So I added 8 and 13 and it came to 21 but I found I couldn't' do 21 'cause 21 was an odd number. So I took um 12 away so that they both have 5, 5 um numbers, and I make, and so that, since I took away the 12, I only had 7, and I added the 7 onto the 13 and I got 20. And 20 is an even number so I can't split that up. So I gave both of the um, both of the members 10.

13:27	T4	So can you tell me how you assigned the numbers to each player?
	Justina	I was looking at Adanna's chart, and you probably can't see it anymore because we crossed things out. But this was 2 and that was 1. This was the number that they had and this was how many ways you could get it. [The part of the chart that is still readable shows: " | 2 | 3 | 4 | 5 | 6 | | 1 | 1 | 2 | 2 | 3 | "]
	Justina	So I did, I made sure, um first, um, the first, the first time I did it, um what did I do? These are the numbers in here, and these are the amounts of ways that you could get it. [Justina has written the following on her paper: "A: $3^{(8)}$ | $2^{(10)}$ | $1^{(11)}$ | $3^{(1)}$ | $6^{(3)}$ B: $3^{(7)}$ | $2^{(9)}$ | $1^{(2)}$ | $4^{(2)}$ | $5^{(2)}$ "]
	T4	I see, okay.
	Justina	So I did, um I did it so that we both have the same amount, and then it came to 4 ways, I got 4 ways [pointing to $4^{(2)}$ in the second row] and she got 3 ways [pointing to $3^{(1)}$ directly above $4^{(2)}$]. And I got 5 ways and she got 6 ways. And 3+2+1+3+6 equal, would equal 10. Ah-ite. These, uh, we both got, I made sure we both got the same amounts of um ways, but then I went over here and I gave her 3 ways and I gave myself 4 ways. And since I had gotten 4, she, um, I had, I had gotten 4, and she had, oh, I dunno [puts hand to her forehead].
	T4	Think about what you're saying.
	Justina	I had one more than her.
	T4	Right.
15:00	Justina	She could get one more than me.
	T4	Absolutely. I understand.
	Justina	So this all together equal 10 [waving her pen over the top row of her chart], and this all together [bottom row] equal 10.
	T4	Very nice.
	R4	I think that's pretty, could you explain it to me one more time? I got lost. Uh, you, what's this number represent? [pointing to $3^{(8)}$]
	Justina	This number represents how many ways you can get that number.
	R4	[in unison] you can get that number. So 3 ways to get an 8. And this one? [pointing to $3^{(7)}$]
	Justina	3 ways to get a 7.
	R4	Okay. And this one? [$2^{(10)}$]
	Justina	2 ways to get 10.
	R4	And this one? [$2^{(9)}$]
	Justina	2 ways to get 9.
	R4	And this one? [$1^{(11)}$]
	Justina	One way to get 11.
	R4	And this one? [$1^{(2)}$]
	Justina	One way to get 2.

	R4	But what about this one? [$3^{(1)}$]
	Justina	One way to get 3. Um. Oops!
	R4	You just switched them, didn't you?
	Justina	Yeah.
	R4	Could you change that so that it makes sense? So it's one way to get a 3. And what about this one? [$4^{(2)}$]
	Justina	I switched that, too.
	R4	It was 2 ways to get a 4, wasn't it?
	Justina	Um humh. Two ways to get a 4, and …
	R4	Okay. I think, [T4]… And so it was 3 ways to get what? There were two numbers, weren't there, that you had 3 ways to get?
	Justina	No, wait. I didn't think I mixed, I mixed this one up. Wait.
	R4	Okay.
	Justina	Adanna, right, this was um the amount of ways … [referring to the first chart]?
	R4	To get a 6.
	Justina	No, this was the number and this was the amount of ways?
	R4	Yeah.
	Justina	Okay, so I did switch that up.
	R4	Okay.
	Justina	Two ways to get a 5. But it would still be the same thing. [writes corrections on her chart]
	R4	Oh because it was the 2 and 2 and the 1 and 3. Okay, now go back and ex-, I wanna make sure because I may have to explain this to somebody af-, later. And so this one was 3 ways to get an 8?
	Justina	To get 8. Three ways to get a 7. Two ways to get a 10. Two ways to get a 9. One way to get 11. One way to get 2. One way to get 3. Two ways to get 4. Three ways to get, is that, two ways to get …
	R4	Is that 6?
	Justina	I think so. [The corrections, written over the original numbers, are difficult to read.]
	R4	Three ways to get 6.
	Justina	Yeah, three ways to get 6 and two ways to get 5.
	R4	Okay. Okay. And so then show me that it's 10. Ten points.
	Justina	This would equal 10. [Writes "=10" at the end of the first row.]
	R4	What does?
	Justina	A [underlines the first row]. This whole thing would equal 10.
	R4	Show me why. Just do it for me. 3+2, is that right?
	Justina	3 plus 2 is 5, plus 1 is 6, plus another is 7, plus 3 is 10.
	R4	Okay, and the bottom one?
	Justina	3 plus 2 is 5, plus 1 is 6, plus 2 is 8, and 2 again is 10. [writes "=10" at the end of the second row]
	R4	Okay. Okay, and so you can make it pretty tomorrow night, tomorrow. That's really very nice..
18:16		[end of CD 045A]

Date: 5 May 2004 Grade 6
Location: Hubbard Middle School
CD: ROLE 044B
Transcribed by: Kathleen Shay
Verified by: Christopher Beattys

Time	Speaker	Transcription
7:15		[R2 is seated with Chris and Jerel.]
	Chris	[looking at paper] Player B got more chances, but I got, he got better ones to play. [Hands paper to Jerel.]
	Jerel	[tapping paper with his pen] 1, 2, 3, 4, 5, 6.
	Chris	I know, but look, Player A got 2, and then he got [inaudible]. He got 3 small numbers and they got 3 big numbers. They also got, almost all of them are big numbers.
	Jerel	Yeah, that's cheating. That's cheating. Well, you can't get, you can't get … That's cheating, still, though.
	R2	Excuse me, Jerel. I'm going to go get some dice for you. What I'd like for you to do, I'd like you to write down the reason why you think it's unfair.
	Jerel	Okay. Wait, we didn't even play the game yet. How do you know Player B won't win?
	R2	Well, I just want you to write down what do you think. Then you'll play the game and see whether or not your prediction is correct. Okay?
	Chris	You player B, right?
		[Some down time as students get organized. Camera is roving to other tables.]
12:50	R2	[Speaking to Chris.] You need another column to keep track of what the roll is. [Inaudible] You have a column for one, a column for you, and then we need a column to show what the roll is. Do you know what I mean by the roll? When you roll [inaudible – R2 appears to demonstrate what he means by "the roll", showing two dice.] So, who's Player A?
	Jerel	[points to himself]
		[roving camera]
26:40		[R5 gives Pyramidal dice game to Chanel, Nia, Danielle, Kori.]
29:25		[Camera moves to R2 talking with Chris and Jerel.]
	Chris	[inaudible] 'Cause we gotta find out how many ways you can get each number.
	R2	Have you thought about that?
	Chris	[inaudible]
	R2	Have you written that down?
	Chris	No.
	R2	Why don't you write that down? I think that's an interesting idea, Okay? We've got some paper here, okay.
		[Camera moves on. In the distance, Chris and Jerel are seen doing

		some writing. After a few minutes they leave the room, taking their name cards with them.]
34:14		[R4 is at the girls' table rolling dice. He asks Nia if she is watching. Chanel and Danielle are looking down towards the floor.]
34:57	R4	Chanel, what number, if I roll the die, this one, what number came up?
	Chanel	1, 3, and 4.
	R4	No, but which one we going to count?
	Chanel	The, um, 4. No, 3.
	R4	3. It's the one that comes up here, right? 3.
	Chanel	No, I don't get how you do that.
	R4	I do this [rolling die]. Which number do you think is coming up?
	Chanel	4.
	R4	Yeah. Because these are facing [motions with his hands], they're not upright. Four and 1 are not upright. It's the number that's sitting on the base.
	Chanel	Oh, I didn't know that. So if you flip this way it's 4.
	R4	So in this case, wait, let me roll once. What number came up?
	Chanel	[lifting the die] 2 [smiles]
	R4	Let's go one more time, Chanel. [rolls die] What number came up?
	Chanel	2.
	R4	Good.
		[Some off topic chatter with Kori. R4 tries to get the girls on task.]
36:12	R4	So we want to know this one. Same question: Is it a fair game? Uh, do the results, uh, show it? And, uh, how to make it fair.
		[Nia wants to play the game with Chanel.]
36:47	Chanel	I'm A, so.
		[Camera focuses on Kori and Danielle. Some off-task chatter.]
44:32		[Camera moves to G2 with Jeffrey and Shamar.]
47:08		[Camera moves to T5 with Dante and David.]
55:50		[end of CD 044B]

Date: 5 May 2004 Grade 6
Location: Hubbard Middle School
CD: ROLE 046A-046B (two views of the same interview)
Transcribed by: Kathleen Shay
Verified by: Christopher Beattys

Time	**Speaker**	**Transcription**
0:30	R2	Jerel and Chris, how are you guys doing?
	C&J	Good.
	R2	Yeah, well thank you for coming down here. 'Cause I told you there are some things, uh, that I heard you talk about, some ideas

		that you have that I'm really interested in hearing more about. And since it's so noisy down at the other end of the room, and the hall, I thought we would, uh, chat here. Okay? So last week Thursday we started working on some dice games.
	C&J	Um humh. [in unison]
	R2	And do you remember the very first game that we worked on?
	C&J	Yeah. [in unison]
	Jerel	The one that was unfair.
	R2	The one that was unfair. Could you tell me about that first game? What was the rules of that first game?
	Jerel	The rules was that Player A got, uh, numbers 2 . . . [scratches his neck, then reaches for paper]. Ah, I can look at this one, it'll tell. Player A got 2, 3,…
	Chris	No, that's the one we did that today.
	R4	That's the second game. Do you want to see the rules of the first one? [To Chris]: Do you remember anything about the first one?
	Chris	[Shakes his head no.] Nope.
	R2	Okay. [Gives paper to Jerel.]
	Jerel	I remember that Player A had, uh, [pause – looking at and pointing on the paper], I remember that Player A had 1, 2, 3, or 4. And Player, if it landed on one of them A gets one point and Player B gets zero. And if the die had landed on 5 or 6, Player B gets one point. And then from there we knew it was unfair because Player A had more choices than Player B can. And Player B only had two.
2:02	R2	So you think that your, you think that that game is unfair because Player A has more choices than Player B?
	Chris	Yep.
	R2	Uh huh. And, um, would it matter, you're saying more choices or because of the numbers that they ?
	Chris	They got more choices.
	R2	They had more choices, okay.
2:20	Jerel	It's a higher percentage of, it 1,2, it landed on 1, 2, 3, or 4 than 5 or 6.
	R2	Uh huh. When you say it's a higher percentage, you know what percentage, or do you have any idea?
	Chris	[Shakes head no.]
	Jerel	Chance.
	R2	Chance, uh? Do you have any idea how likely it is for Player A to get a point than Player B?
	C&J	[Nod their heads to indicate yes.]
	Jerel	Uh huh.
	R2	Yeah? What can you say about that?
	C&J	[In unison] That … [Jerel indicates that Chris should speak.]
	Chris	The probability of getting is 4 out of 6, 'cause there's 6 numbers on the dice and he has 4 chances of getting it.

	R2	Um humh. And did you guys play the game?
	C&J	Yeah.
	R2	Uh huh. And what happened? Tell me about what happened when you played the game.
	Jerel	[grabs paper] All right this was the first game. I beat Chris 10 to 2.
	R2	And you were …
	C&J	Player A.
	R2	Player A. You were Player A. On the first game you received 10 points and Chris received 2.
	C&J	[Nod in agreement]
	R2	Okay. Did you play the game anymore?
	Jerel	Yeah. We played it one more time to see if it, we changed, we changed …
	Chris	sides rules.
	Jerel	Chris became Player B and I became, I mean Chris became Player A and I became Player B. And he beat me 5 to 6. I mean 10 to 6.
	R2	10 to 6.
	Chris	Um humh, ‘cause we had to change the rules. We put that Player A gets 3 choices 1, 2, and 3, and Player B got 4, 5, and 6.
	R2	Oh, I see. So that’s when, when you decided to change the rules of the game to make it, why did you change the rules?
3:45	C&J	[In unison] So it could be fair.
	R2	So you changed it so it could be fair.
	C&J	Uh huh.
	Chris	‘Cause, uh, the first game, since it was 10 to 2, that was a kill by 8 points, but in the second game it was only a kill by 4 points.
	R2	Okay. Well, let’s go back to the first game for a minute. Um, do you think that if you played the first game, right, where Player A receives a point if it receives, if it rolls 1, 2, 3, or 4, and Player B receives a point if the dice rolls, if the die rolls 5 or 6, do you think that that game, if you played it 6 times, would it be … who, who do you think might win?
	Chris	Player A.
	Jerel	Player B. Player A
	R2	You still think Player A might win.
	Jerel	[Nods in agreement.]
	R2	All 6 times? Or just once?
	Jerel	All 6 times.
	Chris	Almost all 6 times.
	R2	Yeah? Suppose you were to play the game 60 times.
	Jerel	Player A would still win.
	R2	Yeah? Do you have …
	Chris	Most of the games.
	R2	Most of the games? When you say most …
	Jerel	59 out of 60, yeah.
	R2	59 out of the 60 games Player A? What about 100 times?

	C&J	[smile]
	Jerel	99 out of 100
	R2	Yes. 99 out of 100. So it seems like Player B's chances goes down the longer, the more that you play the game. Is that right? Is that what you're saying?
	C&J	Um humh. Yep.
	R2	What about your fair game? Tell me about your fair game. What were the rules?
5:12	Jerel	Uh, that …
	Chris	The rules were that um Player A, if Player A rolled a 1, 2, or 3, it would got a point, it would get a point, and Player B woulda got zero. But if Player B rolled a 4, 5, or 6, it woulda got a point.
	R2	I see. So why is that fair?
	Jerel	Because, they, it's a 50-50, it's a 50-50 chance of Player A or Player B winning.
	R2	What do ya mean 50, you mean if you played a hundred times, what would you expect to happen?
	Chris	Probably 50 each.
	R2	They would each win 50 times?
	Jerel	Or 40, or 40-50. Or 40 or 50 or 40 se-…, no [laughs] 40-60. Somethin' like that.
	R2	Uh huh. 40-60. So you think, and 40-60, is that sort of close enough to be fair?
	C&J	Uh huh. Um humh.
	R2	Okay. Um, does it matter which numbers …
	Jerel	If you playin'
	R2	they can roll?
	Jerel	If you playin' with one dice, yeah. But if you was playin' with two, it would matter 'cause you can't get 1, you can't get 1 when you playin' with two dice 'cause 1 is the first number, you can't roll [rolls two dice] you can't get number 1 like that.
	R2	But like if you're only playing with one die, okay, would it matter whether you said Player A receives a point if, for example, Player A instead of getting 1, 2, or 3, got 2, 3, 4, and Player B had 1, 5, and 6?
6:42	Chris	Yeah, that would've been fair, too. Of if he got odd and even numbers.
	R2	That would, yeah? So what is it that's making it fair?
	Chris	The number of chances that you have to get the number.
	R2	Oh, and in this case it'd have to be, what do you think it would have to be?
	Jerel	3 and 3 people get 3 numbers and the other person gets 3 numbers.
7:05	R2	What about the second game? Do you remember the rules of the second game that you played?
	Chris	Yeah.
	Jerel	That we made up?

R2	Not the uh second game that you made up. You made up more than one fair game for the first game?
Chris	[Nods.] We made up two games. We made up two games.
R2	Okay. What was the second one?
Jerel	Oh no, not for this one [pointing at paper on the table], not for this one.
Chris	We made up our own.
R2	Oh, okay. What about for the game with two dice?
Jerel	Oh, two dice …
R2	Tell me, tell me about that game. Tell me what, as it was stated, what were the rules of that game?
Chris	It was, it was, the rules were um … [turns over paper].
Jerel	If the, if the dice…
Chris	landed on 2, 3, 4, 10, 11, or 12, Player A woulda got a point and Player B woulda got zero. And if the dice land on 5, 6, 7, 8, or 9, Player B woulda got a point.
R2	And what did you think before you started playing it? Was, did ya think that this game was fair or not?
Chris	Unfair.
Jerel	It was unfair.
R2	Unfair.
Chris	‘Cause Player A it had like, it had 3 small numbers, which are 2, 3, and 4, and you really can’t get ‘em. ‘Cause right here we made a chart after …
Jerel	[Nudges Chris and points to his paper.]
	[The paper says: “The reason why the game isn’t fair is because player B has a better chance has big numbers and Player A has small numbers.” It then lists the numbers for Player A, labeling 2, 3, and 4 as “3 small” numbers and 10, 11, 12 as “3 big” numbers. Player B’s numbers, 5, 6, 7, 8, and 9, are labeled as “all big”.]
Chris	that 3 got one chance to get it, 2 got one chance, and, oh I didn’t do 4.
R2	What? Let me see. Put you paper here just so I can see it. And explain to me what you’re, what the idea is.
Chris	Right here [pointing at paper], we put like how many times, how many ways can you get um each number.
	[The paper shows: 7 = 4+3, 5+2, 6+1 6 = 3+3, 2+4, 1+5 5 = 1+4, 3+2 3 = 1+2, 2 = 1+1 8 = 4+4, 2+6, 5+3, 9 = 3+6, 4+5 10 = 5+5, 4+6, 11 = 5+6,

		12 = 6+6]
	Jerel	Like for this …
	R2	How many ways there are to roll each number?
	C&J	Um humh. Yeah.
	Jerel	Like for 7 it was 4, 4 + 3 equals 7, 5 +2, and 6+1. For 6 it was 3+3, 2+4, and 1+5. For 5 it was 1+ 4, 3+2. For 3 it was 1+2, 1+1 for 2. Eight for, was 4+4, 2+6, and 5+3.
	R2	Um humh.
	Jerel	Nine was 3+6 and 4 + 5. Ten was 5+5, 4+6. Eleven was 5+6. Twelve was 6+6. And 4 was 2+2 and 3+1.
9:12	R2	And so why did you, why did you make this calculation? Why did you figure this out?
	Chris	Because after we played the game we realized that um Player B had, since it had larger numbers it had more chance of getting ‘em.
	Jerel	And 7 …
	R2	Since the numbers were larger.
	Chris	Um humh.
	R2	So what were the numbers that Player B on, would receive a point?
	Chris	5, 6, 7, 8, and 9.
	R2	5, 6, ,7 , 8, and 9.
	Chris	Uh huh. ‘Cause if you add up how many ways you can get ‘em …
	Jerel	[Interrupts.] Seven kept popping up.
	Chris	You got, for 5 you got 2, then you got, for 6 you had 3, then for 7 you had 3, for 8 you had 3, and for 9 you had 2 [writing these counts on the paper]. So if you add these up, you had 13 different ways to get your numbers.
	R2	So Player B had 13 different ways of winning on a roll.
	Chris	Yeah. And Player A had, for 2 you only had 1 chance, for 3 you had 1 chance of getting it. Four you had 2 chances, 10 you had 2 chances, 11 you have 1 chance and 12 you have 1 chance [writing the counts on the paper]. So you got 8.
	R2	So, and is that what you thought at first, when you first read the game?
10:29	Chris	I thought, when we first read the game, I thought …
	Jerel	I thought it was fair.
	Chris	We thought it was fair because Player A had, well, it was still unfair but Player A woulda got more, woulda won. But after you played the game we saw that Player B started winning, so we just, um, thought that it was unfair and we figured it out.
	R2	Uh huh. So, so let me see if I understand. When you first read the game, you thought that that Player A …
	Jerel	Was gonna win.
	R2	was more likely to win.
	Chris	Um humh.
	R2	Um humh. Then you played the game and you found out that B was winning.

11:00	C&J	Um humh.
	R2	Let’s see. Where are the games you played where …
	Chris	Right here. [C&J point at the paper.] For the first game, Player B won, won 10 to 3. And right here we put the rolls of each one.
	Jerel	Seven kept coming up.
	Chris	Uh huh. Seven came up. For Player B it came out 5 times and for Player A it came out 3 times.
	R2	So you’re saying when Player B rolled, 7 came up 3 times?
	Chris	Five times.
	R2	Five times. And when Player A rolled, 7 came up …
	Chris	Three times.
	R2	Three times.
	Chris	So 7 kept on popping up most of the games.
	R2	Why did 7 come up so much?
	Chris	‘Cause it …
11:38	Jerel	Oh because it had a better chance, because it had 3 ways to get it. And that’s why, if you can’t, if you added them together, that’s what kept coming.
	Chris	Um humh. So it’s 5, 6, no, I mean, 7, 6, 7, 8 had 3 different ways of getting the numbers.
	R2	I see, so that’s what you’re, you’re saying here. So that’s why you did this analysis is because you saw 7 came up so often?
	Chris	Um humh.
	R2	And you wanted, so you did this to try to understand why 7 came up that often?
	Chris	Yep.
	R2	And here you’re saying you can roll a 7 if you have a 4 or 3.
	Chris	Um humh
	R2	And, or a 5 and a 2, and a 6 and a 1.
	Chris	Um humh.
	R2	And those are the different ways that it’s po-, that you can obtain a 7 on a roll of two dice.
	Chris	Um humh.
	R2	Now, I see here [pointing at paper where Chris had just written the number of ways to get each sum] you’re saying that this 2 refers to the number of times, which number?
	Chris	5.
	R2	Five appears. And this 3?
	Chris	6.
	R2	And this one? [pointing at 3]
	Chris	7.
	R2	Ah hah. But you’re saying 6 is a, has 3 possibilities, and there are 3 possibilities of rolling a 7. Now, did you, did that come out for you experimentally when you played the game? That 6 also appeared…
	C&J	[Nod in agreement.]

	Jerel	Yeah.
	R2	More often? Did it appear as often as 7?
	Chris	No. [shakes head]
	R2	How often did 6 appear?
	Jerel	Uh not uh …
	Chris	Not as much as 7. ‘Cause when …
	Jerel	The first game it appeared twice on my side and once on his side.
13:12	Chris	And the second game it came out 1, 2, 2 times on his side and 1, 2, 3, 3 times on my side, uh on my side.
	R2	Uh huh.
13:21	Jerel	It wasn’t as consistent as 7 was. It didn’t come, it kept coming out like this [tosses dice, apparently rolling a 7]. See? [waving his hand over the dice and smiling]
	Chris	‘Cause 7 in the second game, it came out 1, 2, 3, 4, 5, 6, 7 times.
	R2	Um humh.
	Chris	And then, last time it came out 1, 2, 3 times.
	R2	The 6?
	Chris	Um humh.
	R2	Okay.
	Chris	No, the 7.
	R2	The 7. So you’re saying the 6 doesn’t come up quite as often as the 7.
	Chris	No.
	Jerel	Even though it has 3, uh, ways to get it.
	R2	Um humh.
	Jerel	Eight comes up a lot, though.
13:53	R2	If you were to play the game more often, say you played it 10 times, what do you think might happen in terms of the number of times 6 and 7 would come up?
14:01	Jerel	It’d, it’d be a lot more.
	Chris	Um humh.
	Jerel	15 to 20.
	R2	Would they, would it be about the same or would 7 still come up more often?
	C&J	Seven would still come up more often.
	R2	Seven still come up more often. So, Chris and Jerel, there’s something I don’t understand. I’m a little confused here. You said here you have 7, there are 3 possibilities for 7. And Chris you said here there are 6 possibilities for 6, 3 possibilities for 6?
	Chris	Um humh.
	R2	So if you say that the number of possibil-, number of possible ways to obtain a 6 and a 7 are both 3, why do you say that 7, it’s more likely for 7 to appear if you were to play the game often?
	Jerel	[very quietly] Never thought about that. [louder] Maybe because [rolls dice], wait, let me see that. That was 7, right? Maybe because it takes, [pause] I don’t know.

	Chris	'Cause it takes more smaller numbers to make up, um the 6. And for 7 it takes like most, more large numbers to make up, make it up.
	R2	I don't know what you mean. Will you explain that a little further?
	Chris	Like here, like say 1, 2, and 3 on the dice are the smallest numbers, like the smallest numbers or have the smallest. So 3 came out twice, 2 came out once, and 1 came out once. So you had two large numbers left.
	R2	Um humh.
	Chris	So, but for 7 it had 3, 2, 1, three of 'em, and then 3 large numbers, so it had more possibilities again.
15:42	R2	So you're, let me see if I understand. You're saying that the, for 7, you have a 1, a 2, and a 3, and you call those the small numbers.
	Chris	Um humh.
	R2	And they're more likely or less likely to appear over all?
	Chris	Less likely.
	R2	Less likely to appear. And the 4, 5, and 6 are larger numbers and they're more or less likely?
	Chris	More.
	Jerel	[Has had his head down during this exchange.] More.
	R2	More likely. Um, and so, tell me again about the 6 here.
	Chris	It had 3, 3, 2, and 1, which is four less likely to appear.
	R2	Oh, so those are four less likely to appear numbers because those are smaller.
	Chris	And then two, 4 and 5 were more likely to appear numbers.
16:34	R2	Um humh. So the 7 has how many likely pairs, to appear numbers that come up when you …
	Chris	Three.
	R2	Uh huh. And the 6?
	Chris	That's 2.
	R2	It's 2. That's interesting. So, and how do you know that the 4 and the 5, the 4, 5, and 6, are more likely to appear than the 1, 2, and 3? Or, is that on the roll of the die?
	Chris	[Nods]
	R2	You're saying that they're more likely to appear?
	Chris	See, 'cause if you roll [rolls one die], got a 5, a 5, 6, 3. See, that's only once. And if you keep rolling [rolls again] 4, 3, twice …
	Jerel	6
	Chris	Second time …
	Jerel	I can maybe 'cause…
	Chris	Third time, fourth time.
	Jerel	Seven got one even number…
17:27	R2	Wait. Let's keep track of this, okay? Let's take a sheet of paper and keep track of how they're coming up. [Gives the boys a paper.] Who's gonna roll and who's going to keep record?
	Jerel	[points to Chris] Roll.

	C&J	[Chris rolls die] 1, 4
	R2	How many times do you intend to roll?
	Chris	Uh, 10.
	R2	Okay.
	Chris	6, 2, 4, 1, 3, 1, 2, 6. [To Jerel] How much is that?
	Jerel	One is consistent. [Taps his pen on the paper as if pointing to and counting the rolls.]
	Chris	We did it 12 times
	Jerel	I know.
	R2	Um humh. Okay. So what does this tell you? What does this experiment tell you?
	Jerel	That 1 came up a lot. One came up 1, 2, 3, 4, 5 times.
	R2	Um humh.
	Jerel	And the other numbers came up 1, 2, 3, 7 times.
	R2	Which other ones?
	Jerel	Like, 6 came up twice.
	R2	Um humh.
	Jerel	Four came up twice. Three came up once and 2 came up twice.
	R2	Now, does this experiment corroborate your original idea?
	Chris	No. [shakes head no]
	Jerel	[shakes head no]
	R2	No. So, is it because of the way you threw the die, or …
	Jerel	Yeah, wait a minute . . .
	R2	Or maybe you have to throw it more times?
	Jerel	When it landed on here [lifts mat from the table] it kept rolling to 7. Look. Well it kept rolling to 6 or something like that. [Places die on the mat.] 5
	R2	Was that, do you call that a roll, what Jerel just did?
	Chris	No [laughs].
	R2	That seemed like placing it down to me.
	Jerel	[rolls die] 1
	Chris	[rolls die] 1
	R2	Are you keeping track?
	Chris	[rolls 1 off the mat and doesn't count it] 2, 6, 1
	Jerel	[whispers to Chris] It's still low numbers.
20:00	Chris	5, 5, 4, 6, 1, 5. [The 5 was rolled off the mat, but counted.] How many times is that?
	Jerel	[counting silently] 10
	Chris	It's fine [?]. Okay.
	Jerel	Well, all the numbers you can get 7 by. [Looks at R2 and smiles.] 'Cause 1+6, 2+ …
	Chris	Four.
	Jerel	Yeah, 2+4. No, wait. [Turns and looks at Chris.]
	Chris	Oh, 4+3
	Jerel	[To Chris] No, 5 + 2. There's 6+1, 5+2, 5+2, 4+3, 6+1, and 5+2. [taps paper with his pen]

R2	Oh, but I thought we were, you were talking about whether or not the 1, 2, or 3 is less likely to appear than 4, 5, 6.
Chris	[Reaches for paper] The 1 appears…
R2	So what about this idea?
Chris	[Circles the 1's and 2's on the paper. There were no 3's.] The 1, 2, or 3 appears 4 times, and the large numbers appear 6 times.
R2	So you have, you rolled the dice now, you rolled the die how many times so far altogether?
Chris	Ten. Oh. [Writes "large numbers = 6", later changes this to 10.]
Jerel	Oh, all 22.
R2	Okay, so what happened in this, these 22 trials?
Jerel	Ummm, [pointing at paper] the first time little numbers kept coming up.
Chris	Um humh. [Writes "small numbers = 10", later changes this to 12]
Jerel	The second time all the big numbers came, like …
R2	So if you combined this, if you combined the two trials?
Jerel	The little numbers showed up more.
R2	Is that true?
Chris	[writing on the paper] Let me check.
R2	And by little numbers you mean 1, 2, and 3?
Jerel	[speaking at the same time] 1, 2, or 3. [Nods in agreement.]
R2	So how many times did a 1, 2, or 3 show up?
Jerel	All together, the 1, 2, [inaudible] …
Chris	Ten, [inaudible] wait, counted wrong.
Jerel	[counting while tapping the paper] Twelve times. And the large numbers showed up 10 times.
Chris	Um humh.
R2	So what about your theory? The idea that you have.
Jerel	Well, what about when you roll with two dice?
R2	Before we go into the two dice situ-, two dice , what about this one die? Because you guys originally said that the lower numbers, 1, 2, and 3, were less likely to appear than the 4, 5, 6.
Jerel	Yeah, but that was …
R2	Do you still hold to that?
Jerel	No.
R2	Chris? You don't look like you're sure.
Chris	[Shakes head no]
R2	You're shaking your head meaning what?
Chris	Don't know [smiling].
R2	You don't know whether you want to revise your idea or whether you're going to stick with it?
Chris	[shrugs his shoulders and makes a small giggle]
R2	You're not sure?
Chris	[shakes head]
R2	So, what did this experiment tell you?
Jerel	That the big numbers don't always show up. Like, when we

		played, it don't always show up.
	R2	Um humh. So in the one, remember in the one die game? How did you make that game fair?
	Jerel	Um [laughs twice]
	R2	Do you remember, Chris, what you told me?
	Jerel	Oh yeah, we, we gave each person 3, 3 numbers.
	R2	Um humh. And which numbers did you give to Player A?
	Chris	Player A, 1, 2, and 3.
	R2	And to Player B?
	Chris	Player B, 4, 5, 6.
	Jerel	But that…
	R2	And you call that a fair game. But I thought, but by your theory, that 1, 2, and 3 are less likely to appear, then it's not a fair game.
	Jerel	What?
	Chris	[shakes head]
	R2	So I'm confused about what you're trying to tell me.
24:00	Jerel	Now [sighing and smiling]. All right. I can make that a fair game. We give somebody 1, 4, and 5, and give the other person 2, 3, and 6. That'd be fair. You got two low numbers and one high number.
	R2	Yep. That's fair. So it seems to me that this experiment somehow is causing you both to doubt your idea. Is that right?
	C&J	Yep.
24:30	R2	Uh huh. Is there something you want to say about that?
	Jerel	Uh, nah.
	Chris	[shakes head]
	Jerel	I don't want to say nothin'.
	R2	Well, you know maybe it would be interesting to think again about this problem involving both the one die and the two dice games so that you could come back maybe some other time to give me a better idea of what you're thinking about?
	Chris	[nods in agreement]
	R2	To see whether or not things have changed or whether or not you're still holding on to the same set of ideas that you now have.
	Chris	[nods]
	R2	Yeah?
	Chris	Um humh.
	Jerel	[nods]
	R2	Okay.

Date: 6 May 2004 Grade 6
Location: Hubbard Middle School
CD: ROLE 049A-049B (two views of the same interview)
Transcribed by: Kathleen Shay
Verified by: Christopher Beattys

Time	**Speaker**	**Transcription**
3:34	R4	In the last week or so we played a couple of games. Can you remember what any of ‘em were?
	Adanna	One of ‘em was to figure out if the game was fair because Player A had most of the numbers and Player B had few of the numbers and Justina and I thought it wasn’t fair because …
	Justina	Yeah …
	R4	Mm, it wasn’t fair?
	Justina	… when we played …
	Adanna	because they’re supposed to get the same equal amount of numbers but Player A got the most.
	Justina	Yeah but Player B kept winning.
	R4	Can you, can you, why don’t you, say that again?
	Justina	But Player B kept winning.
	R4	Oh, this was in that first game?
	Adanna	Second game.
	Justina	Oh. Oh.
	Adanna	The first game Player A kept winning, but the second game Player B kept winning.
	Justina	Yeah.
	R4	Oh, I got it. So neither one were fair?
	Adanna	Yeah.
	R4	Is that what you … Can you remem-, can you help me remember what the first game was?
	Adanna	The fi-, I think the numbers was 1, 2, 3, [pause] and 4, and the other one was like 5, 6.
	R4	Yeah, ‘cause those are the numbers on the dice? And so Player A got it if it was 1, 2, 3, 4, and Player B if it was 5 and 6?
	J&A	Yeah.
	R4	And you didn’t think it was fair?
	Adanna	Uh uh.
	Justina	No, because Player A had more numbers and it was only one die, and um most likely the die was going to drop on the um the numbers that Player A had because Player A had so many, and Player B didn’t have that many numbers. So the die wasn’t going to really drop on those, that little amount of numbers.
5:13	R4	Okay. You said the first one had 1, 2, 3, 4 and the second one had 5 and 6? Do you think Player B would ever get any points?

	Adanna	Player B had like 2, 3 points. And on the second game Player B had no points. Player A had 10 points and Player B had …
	R4	Oh you mean you're remembering when you were playing? And so …
	Adanna	Because we had, we um, Justina was Player B and I was Player A and I won, and I was Player B and she was Player A and she won. Then we made it fair, we made it 2, 4, 6. She got the even, I got the odds. And then she was, it was dependent on whoever win. It mostly was on luck, whoever wins gets the game. And then we did it um …
	R4	Oh, so what do you mean, dependent on luck?
	Adanna	Yeah. We did it differently.
	R4	How'd you do that?
	Adanna	She got 3 and I got, she got 1, 2, 3, and I got 4, 5, 6.
	R4	And was it still fair or was it not fair?
	Adanna	It was fair. I mean, eh, it depends on whoever wins the game gets the...
	R4	Yeah. That's what you mean by the "luck" kind of thing? But how did you know? Did, when, did you try it and it seemed more fair?
	Adanna	Yep. Because she won, then I won. Then she won, then I won.
	Justina	It was even. It was even.
	R4	Um humh. Um humh. Okay. And then the next game that you were playing, can you remember what it was?
	Adanna	Yeah. We used two dice. And again Player A got most of the numbers and Player B got few of the numbers.
	R4	Okay. Can you remember which numbers it was for Player A?
	Adanna	For B I remember it was 5, 9, 7 …
	Justina	No. For … nevermind.
	R4	No, say. What do you mean?
	Adanna	I think it was 5, 9, 7, and uh 10.
7:08	R4	This is after you made it fair or before you made it fair?
	Adanna	It was the game number 2. Game 2.
	R4	Yeah. But for game number 2, how do you remember it? Uh, I remember that Player A got a point and Player, for some numbers, and Player B got a point for some num-, other numbers.
	Adanna	Yeah.
	R4	And they couldn't, they didn't have any …
	Adanna	Usually Player B, usually Player B kept on winnin'. It wasn't, it wasn't fair because Player A has most of the numbers.
	R4	Player A had most of the numbers?
	Adanna	Um humh.
	R4	What's the smallest number, how did you do it with the two dice? You'd throw 'em …
	Adanna	We'd roll it and if it lands on the paper it counts but if it, if one of them lands out the number don't count.

	R4	I see, then you'd throw it again. okay, but what then would you do with the numbers? You added 'em? Did you add 'em together?
	Adanna	Yeah. Yeah.
	R4	And so you were counting up …
	Adanna	To see what number appears the most. And it was, I think it was 4, 6, and 8.
	R4	Good for you. Okay. What's the littlest number you could get when you …
	Adanna	Two.
	R4	Okay.
	Adanna	And that one was hard to get.
	R4	Two was hard to get?
	Adanna	Uh huh, because you have to depend on luck to get 1+1.
	R4	Oh. What do you think, Justina?
	Justina	I agree with her.
	R4	Okay. And so, if I remember, it was 2, 3, 4, and 10, 11, 12 for A. And B was the other numbers, the ones in the middle.
	Justina	Um humh. 5, 6, 7, 8, 9
	R4	[writing] 5, 6, 7, 8, and 9. Is that right?
	Justina	Um humh.
	R4	Okay. And so you played it and who, who got, who won the most for this game?
	Adanna	Player B
	R4	Even though they only had, they only had 5 numbers, and the other num-, the other one had 6 numbers?
8:59	Justina	Yeah because, um, Player B had many different ways to make um those numbers that it had.
	Adanna	Because it's easier …
	Justina	Player A had like one or two ways to make the numbers that it had, so that's why Player B kept winning.
	Adanna	These numbers [pointing at paper] was easy to get but Player A's number was a little hard because you have to …
	R4	You mean these numbers [pointing to paper] were a little hard?
	Adanna	Yeah. Two and 3 and mostly 4.
	R4	Yeah. Uh, what made 'em hard? Why was 2 and 3 hard to get?
	Adanna	Because um if you spin it you'll only get …
	R4	If you toss it, yeah.
	Adanna	There's only one time you could get that number. Like 1+1, 1+, and for 2 you have to get either, for 2 it's 1+1, for 3 it's 2 + …
	R4	Maybe just write this down so we could remember what you would do with it. I saved all the stuff yesterday.
		[Justina and Adanna write on their papers. Adanna writes: "For 2 it is 1+1 and for 3 it is 2+1."]
10:32	R4	Okay. For 2 it was 1 and 1, and for 3 it was 2 and 1.
	Adanna	And they had only one way. It was one way to get 2 and 3.
	R4	Yeah. Were there any other numbers that it was only one way to

		get?
	Adanna	Um. [pause] I think there was only 2 and 3.
	R4	Oh really?
	Adanna	Because for 4 it is 2+, 2+2 or 3+1.
	R4	Um humh.
		[J&A continue writing. Adanna writes: “It was one way to get 2 and three.”]
	R4	[inaudible] actually I have your stuff from yesterday. So I know you don’t want to, to write it again. Uh, um maybe I know, I know that you already got that way and uh Justina wrote it out this way [shows paper from her folder]. Do you want to just review that? Tell me, tell me what you were, what that meant, what you were writing there?
	Justina	Um, basically all I was writing, well for this section right here it was just keeping track of the games that we were playing. And over here it was when I was trying to figure out why Player B kept winning.
	Adanna	How many choices for each.
	Justina	Yeah. And so [inaudible]…
	R4	And so can you explain to me what all that means? All those numbers? You had one way for …
	Justina	Twelve. One way for 1, I mean for 2, one way for 3, two ways for 4, two ways for 5, three ways for 6, 3 ways for 7, 3 ways for 8, 2 ways for 9, one way for …
	R4	What were the two ways for 9?
	Justina	Um 4 and 5, and 6 and 3. Um one way for 11, and one way for 10. Now …
	R4	One way for 10?
	Justina	Um humh.
	Adanna	No it was two ways because she had messed up on 5+5.
	Justina	Oh, oh yeah it was two. [Justina writes 5+5 and beneath it 6+5 next to “=11”]
	R4	Now remember you all worked on this, too. [places another paper on the desk] That sort of helped you to figure it out. You were saying something when you, when you put ‘em this way about a pattern or something.
13:15	Adanna	It was just a [inaudible]. For 2 you only get 1 way, for 3 one way, 4 two way, 5 two way, 6 three way, 7 three way, 8 three way, 9 two way, 10 two way, 11 one way, 12 one way.
	R4	Um humh. And then, uh, let me ask you a question. For the sort of the f-
	Adanna	Oh yeah.
	R4	Yeah. What do you mean “oh yeah”?
	Adanna	For 1, 2 and 3, it seems like there are two even numbers in each. For 2, 3, 11, and 12, which was one way, there was two even numbers. For 4, 5, 9, and 10, for two it is two even numbers which

	is 4 and 10. For three there was two even numbers which are 6 and 8.
	[Adanna points to her chart: 1 2 3 2 | 3 | 11| 12| 4 | 5 | 9 | 10 | 6 7 8]
R4	Um, say that one again. I have a hard time understanding. What do you mean, “two even numbers”? For what numbers …
Adanna	Look, this one, this was the numbers that only has one way to go …
R4	Oh, I got it.
Adanna	And in each of them there seems that there are always two even numbers or two odd numbers.
R4	In this case they were 2 and 12.
Adanna	Yeah, two even numbers which was 2 and 12, and for this one it was 4 and 10, and for the other one it was 6 and 8.
R4	Oh, so there are always two even numbers, and over there there were how many odd numbers?
Adanna	Two.
R4	Okay. And so they were 3 and 11, and 5 and 9, …
Adanna	If there was 13 then it would go right here [points to the 6 7 8 section], I think.
R4	Maybe. Except you can’t do 13, can you? And so this one [pointing to paper], there were four that got you two? And four that got you one? That had only one way? And then there were three that had three ways?
Adanna	There was 8, 8 and 6 that had three ways.
R4	Oh. What were the three ways for, for 6?
Adanna	3+3 and 4+2 and, uh, 5+1.
R4	Um humh. Um humh. Hey, before we talk about how you changed it, let me, let’s go back. Remember that first game, and, did you play it a lot?
Justina	Oh yeah.
Adanna	We kept switchin’ the numbers obviously because the man who was there was like, “You have to play again.”
R4	omigod.
Adanna	Play again, so we had to play again.
R4	So if you played it the beginning, it was when Player A got a point for 1, 2, 3, 4. And Player B got a point for 5 and 6.
Adanna	Yeah, 5, 6, ,7 ,8.
R4	Um, do you think Player B could ever win?
Adanna	No. Ye-, no.
R4	Suppose you played it 6 times.
J&A	[shake heads no]
R4	Do you think Player A would win every time?
Justina	Yeah.

	Adanna	Umm, because if we were to play it right now, uh, Player A would win. And Player B would get mostly of the points, either she, either she gets um like 5 points or 6 or lower.
	R4	Um humh. And so it's impossible for Player …
	Adanna	B to get to 10.
	R4	Yeah. And so even if you played a hundred times, you don't think that Player B could ever win?
	Adanna	For a hundred times, I think that Player B could win like 2 times.
	R4	Umm. what do you think, Justina?
	Justina	I don't think Player B would really win, because Player um, Player A had the majority of the numbers. Well, yeah, in a hundred maybe, I agree with Adanna, maybe 1 or 2 times, but not really that much, 'cause Player B only had two numbers, and Player A had four.
	R4	Um humh. And you figured out on your paper over there uh how many opportunities Player A and Player B had for the new, for the new one. Do you remember that? You were adding those numbers up over there. What was that, do you remember, Adanna?
	Justina	These numbers, um, I think …
	Adanna	It was chances of either A or B winnin'.
	Justina	Yeah. Yeah, well these were the different chances
	R4	Okay. How many chances did Player A have to get a point? Player A was the, was the 1, 2, …
	Adanna	Player A was, let me see this [paper]…
	R4	Player A got a point if it was 2, 3, 4, 10, 11, 12. Okay? And you, and you figured out how many of those were …
	Adanna	Here it is [looking at paper]. Player A was 2, 3, 4, 10, 11, and 12, and Player B …
	R4	B was 5, 6, 7, 8, and 9? Yeah? And when you added 'em up over there, what was that 8 and 13, do you remember?
18:15	Justina	Oh, I was adding up um, what Adanna got. Right here, under it that was 8. I got the total of 8, and …
	R4	What did 8 represent?
	Justina	Eight represent the total of different ways that Player A could get …
	R4	Oh, I see. Uh huh. And you got that by adding up all these …
	Justina	Um humh. All the different words.
	R4	All these, all these things from here. Okay. What about Player B? What number would, would ….
	Justina	Player E came up …
	R4	E?
	Justina	[laughs] Oh B, Player B came up to 13. Um, when I added 8 and 13 up it became 21, and 21 was an odd number, and I couldn't really even that out without using a half, and there was no half on the dice.

	R4	That’s for sure.
19:05	Justina	So, I had to take um 12 away so that, 12 away from Player A so that ….
	R4	Yeah, I want you to maybe explain because isn’t, isn’t this [shows paper] where you were doing that stuff, you two were doing it? Okay, And so you took 12 away?
	Justina	Yeah, we took um, yeah. We took a number from Player A, which was 12. So um over here [pointing on her paper] I was just trying to even it out and decide which numbers should go to um different players. So the numbers in the parentheses, here, are the numbers …
	R4	[inaudible]
	Justina	right here
	R4	The number in the parenthesis is?
	Justina	Is the number that each player has.
	R4	Like if it was 8, that means you you were holding an 8.
	Justina	I mean, no no. Um, this is the, this is the number that I’m giving to that player, and the larger number down here is the amount of ways.
	R4	[whispering] Okay. I got it. [louder] And so, for instance, this [$3^{(8)}$] is 8. Eight is the number.
	Justina	8, and the different ways you could get that was 3.
	R4	I got it.
	Justina	And this one was 7, the different ways you could get that was 3.
	R4	Okay. So you gave one to A and one to B. Okay.
	Justina	And I kept going like that um three times, three, um two more times after that, and then because we both had …
	R4	Can you tell me what they were, just in case I can’t remember?
	Justina	Ten, you could do 10 twice. You could do 9 twice. You could do 11 once and you could do 2 once. And then, I think that’s a 3…
	Adanna	Yeah, that’s a 3.
	Justina	And then I started mixing up the numbers a little in order to get tens for both of us. So, for 3, I put you could get that once, and for 4, I put you could get that twice. But since I um, I had one, in the ones that I gave out I had one more than her, so …
	R4	Oh, I see, yeah.
	Justina	So I gave her 6 in the next one and I gave myself 5. And…
	R4	Why did you do that?
	Justina	Because I already gave myself one more than her over here. I gave myself a large number over here, she would end up with 9 and I would end up with 11. So I gave her a larger number and I gave myself a smaller number. And then with the, and then I checked the total, I added up the total, it came out to 10, and then I added up her total and it came up to 10. So, and that added up to 20, so I knew that it was …
	R4	Oh, I see. Yeah. Yeah. [asks someone to get 2 white and 2 green

		dice] Now okay, now could you put down here again, because I see that, but I need now to know, uh for Player A, which numbers? Because we've got to play again, I want to see …
	Justina	A had, wait [looking at paper] …
	R4	No, on your new game, on the new game [turns paper over].
22:29	Justina	Oh. Player A had 8, 10, 11, 3, and 6.
	R4	Okay. Could you write that down, Adanna? So we can, we can put it on another piece of paper.
	Justina	And Player B had the number 7, 9, 2, 4, and 5.
	R4	Okay. I need to write that down now here, too. What did, one more time? Player A was …
	Adanna	Was it this one? [picking up another paper]
	Justina	No.
	Adanna	Player A, 2, 3, 4, 10.
	R4	No that was the old one.
	Justina	We're doing the new one.
	R4	Here, this is the new one that you just made. What did you say, Justina? Player A …
	Justina	I said Player A has 8, 10, 11, 3, and 6.
	R4	3 and 6. Did you get that, Adanna? 8, 10, 11, 3, and 6.
	Adanna	Yeah.
	R4	And Player B?
	Justina	And Player B had 7, 9, 2, 4, and 5.
		[Note, P(A gets a point) = 17/35, and P(B gets a point) = 18/35. If Player A were given a point for rolling 12 also, the game would be fair.]
	R4	2, 4, and 5. So they each have the same number of numbers. What are you gonna do if you if you roll a 12? What happens if you roll a 12? You just roll again?
	Adanna	[to Justina] [asks question – unclear – ending with the word "twelve."]
	R4	What are you going to do if, if somebody rolls a 12?
23:41	Adanna	Do you think that you stopped on the number 10?
	Justina	Um, then it just, then it doesn't count, because 12 is already excluded from the game.
	R4	[inaudible] Okay.
	Justina	Yeah.
	Adanna	Why don't can't we just add one more? Oh, no no no because …
	Justina	No, no, because then it would be uneven.
	Adanna	Oh, yeah.
	R4	Because now, Player A you say gets 10 points, has, has 10 opportunities and Player B has 10 opportunities. Have you …
	Adanna	[to Justina] You want to be Player A or B?
	Justina	I guess I'll just be B.
	R4	Okay. Does it matter what kind of dice you use? Whether they're the same color?

	Adanna	No. They have the same numbers on that, so it doesn't matter …
	R4	So any two of 'em. Okay, Which, which ones do you want to use?
	Justina	Okay.
	Adanna	Green. [Justina takes 2 green dice from R4.]
	Justina	You're Player A, you roll first.
	R4	Okay. And what we're trying to test is to see if it's fair, is that right?
	Justina	Um humh.
	R4	Is somebody going to keep our score for us? You're doing it?
	Adanna	[rolls] 6. We're both doing it. Whose point is that? Oh that's my point.
24:50	R4	Okay. Maybe we can put not only just 6, but 5 and 1, too. Just so we can remember which way we got it.
		[The girls continue playing. Player A (Adanna) wins, 10-3. Six or eight came up on 7 of the 13 rolls.]
28:39	R4	She won. Does that make the game still not fair because she won?
	Justina	Um.
	Adanna	I think we should play again and I'll be Player B and she'll be Player A.
28:51	R4	Okay. Play it again. How many times do you think you need to play the game to test whether it's fair or not?
	Justina	At least twice.
	R4	Do you think it's fair from what you did? In terms of, of the scores?
	Justina	I'm not really sure because we did even it out, but yet it was, it went from Player B always winning to Player A always winning.
	R4	Yeah. So now you're gonna be Player B, Adanna, and Justina's gonna be Player A?
	Adanna	Yeah, pretty much. I think this still works.
		[J&A begin to play the game. After 4 rolls – two for Player A and two for Player B, Justina remarks:]
30:25	Justina	I think you just have good luck.
	R4	It's pretty even now, isn't it?
		[After 6 rolls, 3-3, Justina says:]
30:47	Justina	So far I think it's fair.
	R4	What makes you think it's fair?
	Adanna	Because we …
	Justina	She kept coming up. I just had bad luck in the first game.
		[The girls continue playing.]
34:10	Justina	Okay. See, it's even. Player A won the first one and Player B the second.
	R4	But you didn't win yet.
	Justina	But Player B is in the lead.
	Adanna	Eight, 8 to 7 [looking over at Justina's paper]
	R4	8 to 7. I promise.
	Justina	I thought, umm, I gave her an extra point, though.

	Adanna	What you mean?
	R4	A couple extra points. But no, isn’t it 8 to 7, Adanna?
	Adanna	Yes, because …
	Justina	I thought it was 9. Okay.
		[The girls complete the game.]
35:00	Adanna	I win.
	R4	Oh, 4 is yours, that’s right. Okay, does, does what we’ve just done make you think that it’s pretty fair?
	Justina	Yeah, it is. Yeah, I do, because um at first A won, and then now B won. [inaudible]
	R4	Uh huh. If you play it …
	Adanna	Hold up. When they got to 4+4, and 3+1 there was a tie. And then I got in the lead and then she caught up. And then that’s when she had taken the whole lead, and I had to catch up and I won.
	R4	Um humh. Yeah, sort of interesting, but it was pretty even, you think?
	Adanna	Yeah. Takin’ that one number made it even.
	R4	Um humh. Oh, takin’ out the 12?
	Adanna	[nods]
	R4	Is that the only thing that made it even?
35:45	Adanna	You could take out any number and it would still be even.
	Justina	No, I don’t think so.
	R4	What, what did, what else did you, what else did you have to do to make, what else did she do to make it even?
	Adanna	You could take out 11 and it’ll still be even.
	R4	What else did you have to do to make it even? From that first game?
	Justina	Oh, and uh, I had to sort out the different numbers to the different players.
	R4	Yeah, oh, okay. Well now, if you played the way it was the first time, when you say that it wasn’t fair, that B had the advantage, if you played it, um, 10 times, do you think B would ever win?
	Adanna	What was the numbers?
	R4	The way it was to begin with, with uh, this way [handing the paper to Adanna], where it was 8 chances for, 8 chances for A and 13 chances for B. If you played it 10 times, do you think B would, that A would ever win?
	Adanna	Umm umm. Yeah.
	Justina	Um, just once [holding up 1 finger].
	Adanna	Yeah, because she won one time and I won most of them.
	R4	Oh. But that’s the new one.
	Adanna	I know, but most of the games before [inaudible].
	R4	Yeah. Okay. But if you played, if you played the new game, the fair one, about 20 times, how many times do you think each, that you might win?
	Adanna	twenty, twenty

	Justina	Ten.
	R4	If you played 20 different games. Do you think you'd do it 10 times?
	Adanna	If there's a possible way she could win 10 and I could win 10 and there could be a possible way that she could win 5 and I could win …
	Justina	Fifteen.
	Adanna	Yeah. What she said.
	Justina	I said 15.
	R4	Oh. So it's not for sure?
	J&A	No. Uh uh.
	R4	…that it would come out. But it might be 10 and 10 or 15 whatever. What if you played it a ton of times, about a hundred times? Would, what do you think?
37:34	Justina	Um, you can't be sure about that. 'Cause dice is dice and it just rolls on whatever number.
	R4	It depends on the angel [laughs].
	Adanna	[rolls dice] Yeah, it is the way you roll.
	R4	Okay. And if, so if you played it a hundred times, what would you, what would you predict?
	Adanna	A hundred times?
	R4	Um humh. Played a hundred games.
	Adanna	50/50
	Justina	Um, maybe one player would get 60 points, one would get 50, or maybe 59 and one would, um, [pause] would get uh 40 or somethin'. Ew, my math is so off.
	R4	Um humh. But 50/50 is one possibility?
	J&A	Yeah it is.
	Justina	One player gets 60, one player gets 40.
	R4	Um humh. They have to add up. It has to add up to a hundred.
	Justina	Um humh.
	R4	Um. So. Whatever. I'm going to ask you one final question before you go back and play the racing game. Um, suppose we had a final game and everything was on one roll of the dice. And you could choose …
	Adanna	You mean if the game was tied and it was equal like …
	R4	Yeah. And and the first person, like a sudden death, you know, in a, in a ball game, uh the first person who, you'd roll, you'd roll the dice until a number that you had chosen came up. Um, which number would you choose?
	Adanna	[looks at her paper] I would choose 6.
	R4	You'd choose 6, why?
	Adanna	Because it seems on here [her paper] you could see 6, 6, 6, 6, 6. I'd choose 6.
	R4	What would you choose, Justina? You could choose, I mean would you choose 6 as well? Would you choose something else?

Justina	Oh I would. I would choose 6. How many times did 8 come up? Only twice. Yes, I really would pick 6. Six was the number that came up the most.
Adanna	So you [inaudible] on 6?
R4	[murmurs, inaudible] Okay. Suppose I’m gonna ask you this: suppose the two numbers you could choose from are either 7 or 8. Which one would you choose? Or does it matter?
Adanna	Eight. Because 8 appears here the most than, I don’t see 7 anyway.
R4	Um humh. So on your …
Justina	I used to see 7, ‘cause …
Adanna	I’m talkin’ about on the first game. And on the second game, I would choose …
Justina	Seven appeared 1, 2, …
Adanna	Three.
Justina	Yeah, three times.
R4	And 8?
Adanna	And 8 appeared 1, 2, 3, 3, 4, 5, 6 …
Justina	But you said we would …
Adanna	[inaudible – tapping paper with her pen]
R4	Okay. On any, but based on your games, you, you think you would choose which one? 7 or 8?
J&A	Eight.
R4	Um humh. Okay. Would you ever choose 12?
Justina	No. You can’t win with 12. Whenever you get 12, you have to roll again [according to the rules of the game she devised].
R4	What about 11?
Adanna	No.
Justina	No. Eleven only came up, let me see, here …
Adanna	One.
R4	So, so 11 would not be a good choice for you to play this one. Okay, the game you’re playin’ in the other room, with the race going up, does anything have to do with this? Is it different from this?
Adanna	So far, it’s the same because it’s still 12 numbers and the numbers startin’ with 2, and we’re still rollin’ with two dices, and we just seen that the most number that appears the most and it’s the same, it’s still the same because we tryin’ to see if the game is fair or not.
Justina	Yeah, but I don’t think it’s the same because um it, it isn’t really unfair.
Adanna	Everything that’s …
Justina	It is sorta um lucky, like a luck game.
Adanna	Everything is the same except the chart.
Justina	Because there is, um no player gets a cert-, okay, yeah, they do. All right. [smiles]
R4	What do you mean?
Justina	No, because I was thinking of a player, um the first runner gets this

		um like uh different numbers but I was thinking …
	R4	OK, but what is it, is it you're trying to see which position wins the race first?
	Justin	Oh wait, yeah, no, well, I'm thinking. Yeah I do agree with Adanna, the games are the same. Because some of the numbers appear more because they have more, more um different ways to get them.
	R4	Oh. So if you had to put your racer in any one of those 11 positions from 2 up to 12, where would you put it?
43:05	Justina	Um, who was the one that was winning?
	Adanna	Seven.
	Justina	Seven was the one that was winning?
	Adanna	Eight, and nine. I think 7, 8, and 5 or 4 or 9 was tied.
	Justina	No, it was 8. It was 8, and um eight is the one that's always in the lead.
	Adanna	Eight or seven because seven started bein' on the lead and then 8 caught up to 7 and they became tied.
	Justina	Yeah but 7 sometimes …Yeah but 7 always, um, um, is always left behind. No, first it was 8 that was left behind, and then 7 kept getting, um, left behind with the other numbers. Seven caught up to 8, but I'm sure 8 is gonna beat 7.
	R4	You really believe eight's gonna win? That's what you said here, too, wasn't it? Yeah. Um, okay! Thank you. Do you have any questions you'd like to ask me?
43:57		[Justina asks about seeing the video they made. They briefly discuss towers and Cuisenaire rods.]

Date: 4 May 2005 Grade 7
Location: Hubbard Middle School
CD: ROLE 119C-120C
Transcribed by: Kathleen Shay
Verified by: Jeremy Milonas

Time	**Speaker**	**Transcription**
5:01	R2	[to class] Ian has noticed that we have a different shaped dice on the table. These are, these are the dice that we used the last time, right? One at a time, you used the last time. And today we're going to work with this kind of dice. What's the difference?
	Kianja	It's a pyramid, it has 4 sides.
	R2	This one is a pyramid, and it has 4 sides. Chanel?
	Chanel	The other one is square and has 6 sides.
	R2	The other one is square? What name do we give to this shape?
	students	Cube. Cube.
	R2	It's a cube, and it has 6 sides. Okay. And, what else do you

		notice? Any other differences?
		[more discussion about the shape and color of the dice]
7:20	R2	Before I give each pair a pair of dice, I want to ask you a question about what do you remember about the dice game we played last year?
	male S	We had a mat to roll.
	R2	Okay. We used a mat to roll the dice on. What else do you remember about the game? Terrill wasn't here, so, what are some things … Brionna, nor Kiesha, so a good number of you weren't here. [chatter] When we rolled the dice, you had a pair of dice and, and you had to roll them, right? What, do you remember what we did with that roll? What, what happened?
		[coughing and inaudible speech]
	R2	Well, I don't know if we rolled that, but we certainly added the outcomes, right? We added the face values of what came up on the dice. Let's give out a pair.
		[dice are distributed to the class]
9:47	R2	I'd like for each pair just to roll the pair of dice that you have and tell me, what comes up? Look at the, look at the dice and determine how do you know what comes up?
	Kianja	[rolls] It's 44. [rolls again] 33. See, I know how to do it.
	R2	Which number comes up?
	Kianja	The one at the bottom. Whatever's facing at the bottom. [Kianja rolls dice and adds the outcomes.] That's what you do when you roll dice.
		[The task has not yet been given.]
14:11	G4	OK. What do you, what do you think here? Which number has a higher chance? Can I ask you a few questions? Which number do you think … [K&B are talking to one another and laughing.] Which number, which number do you think comes more times?
	Brionna	I say 3 and 2, because you always see 2.
	Kianja	You know the bottom … No, this is the answer. Look, this is the answer, Okay? [to] Wait, what did he ask you?
	Brionna	You know what …
	G4	Which, which number comes more times?
	Brionna	I think it's 2 and 3 because …
	G4	What do you think?
	Kianja	It's 2 because, wait …
	Brionna	2 and 3.
	Kianja	2? 3? Yeah, 2 and 3 because 2 is on here 3 times, see, 1 …
	Brionna	Three's on here 3 times, too.
	Kianja	I know, that's why I said 2 and 3.
	Brionna	That's what I'm sayin'.
	Kianja	1, 2, and 3. And then 3 is on there 3 times: 1, 2, and 3. And 1 is only on there twice.
	G4	One is twice.

	Kianja	See, 1 and 2, and 4 is twice. Oh wait a minute.
	G4	Just check it out.
	Kianja	Oh shoot! It’s on there all the time, Brionna.
	G4	What do you notice?
	Kianja	See, 1, 2, 3. 1, 2, and 3.
	Brionna	No because 2 is always closer to another 2.
	G4	What do you notice?
	Kianja	So is the other numbers.
	Brionna	See, no …
	G4	So which number comes more, then?
	Brionna	See 2 always comes near a 2. One …
	Kianja	I don’t know.
	Brionna	‘Cause [inaudible] the bottom.
15:40	R2	Can I have your attention? Every group has decided what a roll is, right? When you throw the dice … Excuse me, guys? Okay, here’s the problem. Let me show you the problem. [Turns on overhead projector.] I’ll read the problem to you. Each of you will get a statement of the problem, but here’s the task we’d like you to work on. It says, does everyone, can I have everyone’s attention? Kian- Keisha. Everyone’s attention here? But I don’t think she can see if you’re in the way there. Can’t see this. Would someone read what’s on the …
	Terrill	I wanna do it, I wanna do it. A pyramidal die has 4 sides …
	R2	Terrill, I called on Chanel.
	Chanel	A pyrami-, how do you say that? A pyramidal dice game. A pyramidal die has 4 sides. The number that is rolled is shown upright. Roll two die, dice. If the sum of the dice is 2, 3, 7, or 8, Player A gets one point and Player B gets zero. If the sum is 4, 5, or 6, Player B gets one point and Player A gets zero. Continue rolling the dice. The first person who, to get 10 points is the winner. 1) Is this a fair game? Why or why not?
		[Note: P(A gets a point) = 6/16; P(B) = 10/16]
	Students	No. No.
	R2	So you think it’s not a fair game?
	Dante	It’s like last year’s. It’s not a fair game.
	R2	Why?
	Dante	Because Player 1 gets more chances than Player 2.
	R2	Wait, I believe Player A, is that …
	Dante	Yeah, Player A.
	R2	When you say Player A gets more chances, what do you mean?
	Dante	Because it gets 2, 3, 7, and 8 and Player uh B only gets 4, 5 and 6. So Player B has a less chance of getting, of getting um, a point instead of Player A.
		[The camera is on Brionna. She and Kianja are talking and laughing.]
	R2	Does everyone understand what Dante, the point that he made?

	Students	Yeah.
	R2	Excuse me, Kianja? And Terrill? Did you hear what Dante said about why he thinks this game is unfair?
	Terrill	Yes.
		[Kianja looks down at the paper on her desk and does not answer.]
18:19	R2	Okay. Who could tell us what he said? All right, Terrill.
	Terrill	It's not a fair game because …
	Students	[chatter]
	R2	Terrill is going to tell us Dante's [inaudible over coughing]. OK?
	Terrill	Dante says it's not fair because, what'd you say it wasn't fair again? Oh he said it's not fair because all right, never mind. I don't even remember. I forgot.
18:54		[Camera shows Kianja is writing: " 1 2 34 1+1 1+2 2+2 1+3 2+3 3+3 1+4 2+4 3+4 4+4"]
	R2	Okay. Who could tell us what Dante's point was? Chanel?
	Chanel	Dante's point was that the game isn't fair because Player A gets 2, 3, 7 or 8 and that's 4 numbers, and Player B only gets 4, 5, and 6, 3 numbers, so Player A has a um better chance at getting what he wants than Player B.
	R2	Does everyone agree with Dante's point?
	Students	Yes.
	R2	Okay. Do you agree? Keisha? Do you have an opinion about this?
	Terrill	All right. Could somebody explain to me, say it like exactly why the game isn't, 'cause we just like going around in circles.
	Student	The game isn't fair because Player A has more chances
	R2	Hold on, Dante. Excuse me, Dante why don't you come up here for a minute?
		[chatter]
	R2	I think Terrill has asked a serious question. So we want Dante to explain again his opinion about why it's not fair.
	Terrill	Can you like um explain in one sentence, that means with no 'ands' and noth of that, none of that, why this game is unfair.
	Dante	The game is unfair because Player A gets more chances than Player B.
	Terrilll	Okay. That's what I needed to know.
		[Kianja & Brionna are passing notes to each other.]
	Students	[chatter] I'm Player A, then.
20:22	R2	So what we'd like for you to do is to play this game. One, one of you will be Player A, the other is Player B. Player B.
	Students	I'm Player A. I'm Player A.
	R2	Okay. Remember, what we're gonna try to do, we're gonna try to,

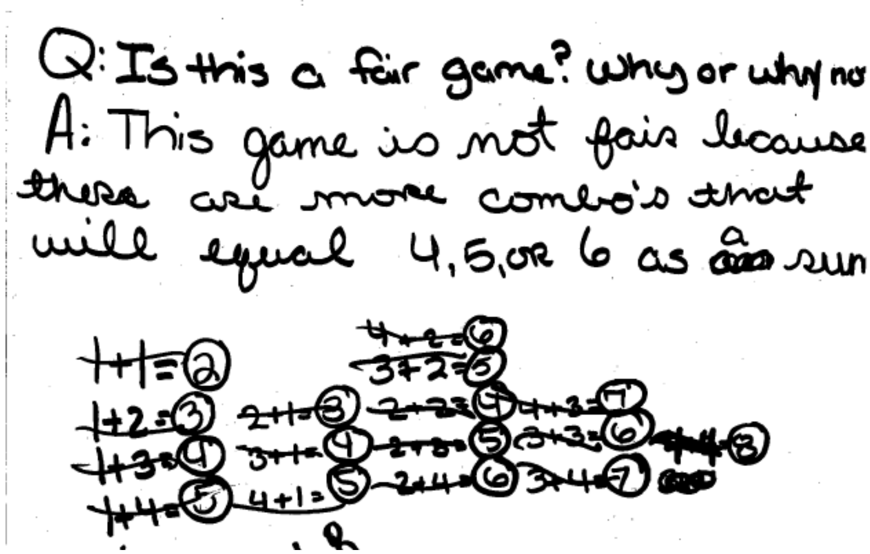
		excuse me, we're gonna try to determine whether or not the game is fair. So it doesn't matter who's Player A or Player B, because your task is to determine whether the game is fair. Oh, and I already see that Chanel has begun to make a little score card for keeping track of, of what?
21:16		[A copy of the problem is placed on Kianja & Brionna's desk. Kianja & Brionna are chatting off task.]
	G4	So, who's Player A?
	Kianja	We have to turn this [camera] off for a minute.
	G4	Who's Player A?
	Kianja	[to Brionna] Like I said, [unclear]. [Takes problem paper and moves it to her left.] I'm beat you, just so you know. Is this a fair game? Now let's see. This equals 2. So wait 2, … [writes "=2" next to 1+1, continues writing the total above each sum on her paper]
	G4	So Kianja, you are A or B?
	Brionna	I'm B and she A.
	G4	You're B?
	Brionna	Yeah, I'm B. B, A, B. It's 2, 3, 7, 8. B. 2, 3, 7, or 8.
		[Kianja makes tally marks on her paper]
	Brionna	I get 2, 3, 7, or 8. You want 2, 3, 7, or 8?
	G4	Okay. You want to throw the dice and [inaudible]?
	Kianja	I'm gonna win. I'm gonna win if I'm Player B. I am going …
	Brionna	I don't care.
	Kianja	to win.
	Brionna	I said I'm Player B, you A.
	Kianja	[to G4] Didn't she just say I'm 2, 3, 7 … Didn't she say I'm 2, 3, 7 and 8? 2, 3, 7, and 8 is A. Is that correct? Exactly! You just said you 2, 3, and 7.
	Brionna	I don't care. I'm only getting' B because it's part of my name.
	Kianja	So you gonna win. [rolls dice] This is 6, so you get, what's that point?
	G4	[to Brionna] Here, can you write on the top [inaudible] squares?
	Kianja	[rolls] This is 8, so I get a point. You get a point, too. [rolls] This is 3, so I get a point. [rolls] This is 4 so you get a point.
	R2	May, may I just make a suggestion? That in addition to keeping a tally, one second, in addition to keeping a tally, also indicate what the outcomes are. Okay? So for example …
	Kianja	All right. Okay. 1+4 is 5, yeah, all right.
	R2	Uh huh, but indicate what, what the outcomes were in addition to the sum.
	G4	So can you, can you write down, Brionna, can you write down here 2, 3.
	Kianja	But she don't know what it is, so we gotta start over.
	G4	What are the numbers, 2, 3, 7, 8? So then you [inaudible].
	Kianja	Isn't that right, Brionna?

	G4	What about this? B is 4, 5, 4, 5, 6. 4, 5, 6. Will you write that, 4, 5, 6? Okay. Now you can start. If it is 4, 5, or 6, B wins, right?
	Kianja	Well, Brionna, you know what you're doin', right?
	G4	If it is 2, 3, 7, 8, A wins, right?
	Brionna	Um humh. Ummm huh.
	Kianja	I'm going to work on number 3. [question 3 – how to make the game fair?]
	G4	Can we start rolling the dice? [Brionna rolls] What is that, 2?
25:15	student	Actually no it's not. It's not fair. It's not fair for Player A. Because there's more odd numbers on [inaudible].
	G4	Is the game fair? Why don't one of you throw? Kianja? Why don't you roll the dice?
	Kianja	I was going, I was gonna say that she can roll the dice and I can write question 3.
		[Kianja writes: "1. This game is Not fair because"]
	Kianja	This game is not fair. Why is it not fair, Brionna? I don't know. All right. Because there are more combos, more triple combos, see if we had 3 dice and [laughs]. [Back on task] This game is not fair because there are more combos that will give you 4, 5, or 6. Wait, this is, never mind okay.
		[Kianja & Brionna are heard saying: "Yeah, I thought it was 6." "Ten" "Six" "Not that way, that way." "I thought it was 10." [laugh] "Yeah, or 9. Point five! Nine and a half? Ya know? Oh, okay." "We'll find out."] [The speaker and the subject of the conversation are not clear.]
		[Kianja has written: " 1. This game is Not fair because there are more combos that will 4, 5, or 6 as an answer."]
28:52	G4	What can you say, Kianja? Let me see this.
	Kianja	This game is not fair because there are more combos that w-…
	G4	How can you say that?
	Kianja	equal [inserts the word "equal" on her paper] 4, 5, or 6 as an answer.
	G4	Um humh. How can you say that, more combos?
29:09	Kianja	Because look, I did it. I did it. See, you get 1+1 on the dice
	Brionna	and 2.
	Kianja	Shhh. 2+2 on the die. 3+3 on the die. No, a die. One is die, two is dice. 1+2, 1+3, 1+4, right? 2+3, 2+4, and 3+4, right? correct? [Kianja writes these sums in a column as she speaks.]
	Brionna	2, 4, 6, 8
	Kianja	So this [1+1] would be 2, [continues writing the total of each sum, and circles each total of 4, 5, or 6]. See, there's 1, 2, 3, 4, 5, 6, six that equal 4, 5, or 6. And then we have 2, 8, 3, and 7. 1, 2, 3, 4. Four that equal 2, 3, 7, 8. You see how I came to my conclusion?
30:56	G4	Do you think these are the only ways in which you can do it?
	Kianja	Yes.
	G4	There are no other ways?

Kianja	Well, if you use addition. ‘Cause there’s only 4 numbers on here. I mean, it’s only numbers from 1 to 4.
G4	Okay. So …
Kianja	So if you get a 1, right …
G4	Um humh, Um humh.
Kianja	Say you rolled a 1 and then you rolled a 1 on this die, . . .
G4	Okay, so, so, suppose you got 1 and 1.
Kianja	It’d be 1 + 1.
G4	So which one is that?
Kianja	Right here. [Points at “1+1” on her paper.]
G4	Suppose we got 1, 1. Okay.
Kianja	It’d be 1+1.
G4	All right. And if you get this, 2 and 2.
Kianja	2 and 2, it would be 4.
G4	Okay, I’ll ask you a question. Which one is this? 1, 2.
Kianja	Right here. [Points at “1+2” on her paper.]
G4	1, 2 is this one?
Kianja	Yes.
G4	Okay. Now let me change this, okay. This is 2, this is 1. [Reverses the dice.]
Brionna	It’s 3.
Kianja	This. [Points at “1+2” on her paper.]
G4	No.
Kianja	It’d be 3.
G4	Yeah.
Brionna	2+1
Kianja	See?
G4	Yeah.
	[Kianja writes “2+1=3”.]
G4	This is 2+1, right?
Brionna	Yeah, it equals 3.
G4	Yeah, and this is 1+2.
Brionna	1+2. That’s the same thing, 3.
	[Kianja writes “3+1 =4”, “4+1 = 5”.]
G4	Um humh. What is this here you’re writing? [Points at Kianja’s paper.]
	[Kianja continues writing, “3+2=5”, “4+2 = 6”.]
Brionna	[quietly] You still get the same answer.
Kianja	If you wanted to do that, then it would only be [writes “4+3=7”], then it would be 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 [counting up the outcomes for Player B she had circled on her paper].
Brionna	And this would be 12 [points to the 6 on Kianja’s paper].
Kianja	10 [Crosses out 6 and writes “10”.]
Brionna	10? How?
Kianja	‘Cause there’s only 4 more. ‘Cause you can’t [inaudible].
G4	What do you get here? [pointing at Kianja’s paper, where it says

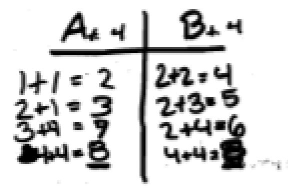
		“4 that = 2,3,7,8”.]
	Kianja	6. [changes the 4 to 6] So it would still be more.
	G4	So you mean to say the game is unfair?
	K&B	Yeah.
	G4	Okay, so who’s going to win?
	Brionna	B.
	G4	B will be winning? What was you idea at the origin? What do you thought first?
	Brionna	Huh?
	G4	What do you thought first, who would be winning?
	Kianja	B.
	G4	Before you start, who would the game go to?
	Kianja	B.
	G4	Um humh. You thought B would win?
	Kianja	Ya. I believe we’re done. Oh wait, dag.
	G4	Well, what made you think B would win?
	Kianja	[writes 3 (for question 3) on her paper] Let’s see, how could we make this fair, Brionna? There’s only 7 numbers.
33:18	R2	Is it still unfair? Do you still think it’s unfair?
	Brionna	Each number, everybody get, each, everybody get 4?
	Kianja	No, it’s only 7 numbers.
	Brionna	Now, we might get 3, and [unclear] 8. I don’t know. [Pointing at the paper with the rules of the original game] Like 4, 5, 6, or 8.
	Kianja	Oh wait. Here we go.
	Brionna	2, 3, 7, or 8. So they both don’t get or get 8.
		[Kianja begins writing on her paper: “If player A gets 2, 3, 7, or 8”, then she crosses out “or 8” and continues “ then Player A gets 1 pt. If player B gets 4, 5, 6 theN player B gets 1 pt. *Which every player gets 8 gets 1 pt.”] [Note: This game is not fair. P(A) = 6/16 and P(B) = 11/16.]
	Kianja	We’re done. Could I have another piece of paper? [She is given a transparency to write on.]
		[Kianja prepares the overhead transparency while Brionna sets up a score sheet showing a column for A (Kianja) with the numbers 2, 3, 7 and a column for B (Brionna) with the numbers 4, 5, 6.]
39:02	Kianja	Write the numbers at the top, Brionna, ‘cause you might wanna …
	Brionna	It’s right here.
	Kianja	You gotta put 8, too. Both. [Brionna writes 8 for A only.]
	G4	Are you trying to make it fair?
		[While Kianja continues writing on the transparency, Brionna rolls the dice and keeps score.]
40:34	G4	So who’s winning?
	Brionna	B. [The score is 0-5 for B.]
	G4	B has less numbers, right? 4, 5, 6. Why doesn’t A win?
	Brionna	Because, uh, finally! [she has rolled a 3, giving A a point.]
	G4	What do you think, Brionna? Why does it happen like this?

	Brionna	Because …
	G4	Can you roll it a few more times and see.
		[Brionna continues to roll the dice and keep score.]
	G4	So who’s winning more points?
41:52	Brionna	Yeah I am, B. [The score is 2-7.]
42:46		[Kianja writes the following on her transparency, showing the sample space with 16 outcomes. A’s sums are circled in red, B’s sums in black.]



		[Kianja taps each of the sums with her finger, as if counting.]
	Brionna	It is 12, right?
	Kianja	Huh? [slaps desk] No.
	Brionna	Then what is it?
	Kianja	It’s 10 like I said.
	Brionna	Didn’t you mess up?
		[Kianja does not respond. She starts writing on a new transparency.]
	Brionna	What is the new one? What is number 3?
	Kianja	Yeah, what is number 3?
	Brionna	Right here. [Passes some papers to Kianja.]
44:25	G4	Can you explain me these red circles and black circles, what is this? [pointing to Kianja’s transparency]
		[no response]
44:50	G4	Kianja, can I ask you a couple of questions?
	Kianja	Hold on, I gotta write this down.
		[Brionna prepares a new score sheet showing “A 2 37 8 B 4 5 6 8”]
46:02		[end of CD 119C]
		[begin CD 120C
0:21		[Brionna is rolling dice and keeping score. Kianja is preparing another transparency to explain why the game is not fair.]
3:53		[Brionna has completed one “game” on her score sheet, showing 4

		points for A (having rolled 8, 3, 2, 3) and 3 points for B (having rolled 6, 8, 5). She prepares a new score table on the same sheet.]
7:52	G4	Kianja, where is the paper? Did you, did you try to make the fair game?
	Kianja	It's right here.
	G4	Did you think of the formula [?].
	Kianja	This one?
	G4	No, where is the, where is white paper?
	Kianja	I'm making it. Right here.
	G4	[Points to paper with task instructions] If you think the game is unfair, how would you change it?
	Kianja	I'm writin' it down. I'm writin' it down.
	G4	[to Brionna] Are you trying to make it a fair game?
	Brionna	This one is the fair game [points to "A 2 37 8 B 4 5 6 8"] and this one [pointing at the second table on her paper] is, is the right one [original game].
		[Kianja has written "We could make it fair by having player "A" get one pt. for rolling a 2, 3, or 7 and player "B" getting one pt. for rolling a 4, 5, 6. *Which ever player rolls an 8 gets 1 point." She shows the table below, which omits several outcomes.]



12:12	G4	Do you think it will be a fair game? Explain this, Kianja [inaudible]. Do you think this will become fair?
	Kianja	Yeah!
	G4	Can you explain to that? How will that become fair?
	Kianja	It's still unfair, Brionna. Sugar! Hold on, all right. [gets up and walks away]
		[While Kianja is away from the desk, Brionna takes out her notebook and looks at (homework?) papers.]
13:50	G5	It's a fair game? Or non unfair game?
	Brionna	This one? [pointing at paper]
	G5	Yeah!
	Brionna	It's a non-, it's not fair because, here it is [Kianja's transparency]. Because, like there's more ways to, it's more ways to get 4, it's more ways to get 4, 5, and 6 than 2, 3, 7, or 8, because …
	G5	Okay. Why?
	Brionna	1+2 is 2. I'm gonna do A. 1+2 equals 2, then 1+2 equals 3, then 2+1 equals 3, 4+4 is 8, 4+3 is 7, 3+4 is 7, and that's it. It's only 1, 2, 3, 4, 5, 6 [ways for A to get a point].
14:56	Kianja	Oh you explained that to her? Don't explain this one.
	Brionna	And for B, for B you have 1, 2, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 ways to explain it.[inaudible].

	Kianja	Did you throw our other transparency away?
	Brionna	No.
	Kianja	Thought I lost my mind.
	G5	So, how many, how many opportunities to win for Player A? Can you just say how many? Yeah, how many? If Player A wanna, wanna win, how many opportunities he have, he has?
	Brionna	Six. One out of six.
	G5	Which, which six? What are the six you're talking about?
	Brionna	One out of six chances, like, like who gets it and one out of, no one out of, I don't know. Kianja [inaudible].
	Kianja	Chop chop chop. Chop chop chop.
16:09	R2	[announces to class] We'll take another two minutes to finish up whatever you're preparing.
	Kianja	We got another what?
	Brionna	How many chances …
	R2	Then we'll have the groups report, okay? Will two minutes be enough time for you?
	Kianja	No! Wait a minute, Brionna.
	Brionna	Well anyway, um, how many chances do A have to win? Roll to get some, right? To have, to get like, a point.
	G5	How many chances – are those total uh chances? [indicating the sample space on Kianja's transparency]
	Brionna	And I said 1 out of 6. One out of 6 chances to get one point.
	Kianja	Who had 1 out of 6 chances to get?
	Brionna	A.
	G5	So what are, what are the 6? How do you get 6?
	Kianja	A sixth.
	G5	How do you get the number 6?
	Brionna	Because that's how many times like 6 ways to get…
	Kianja	It's six ways that, It's six ways that A could score a point, right? So it's one out of six chances that A would score a point.
	G5	So how many's for, how many chances for Player B?
	Kianja	One out of ten. Because it's ten chances, it's, there's ten possible ways for B to score a point, so it'd be one out of ten.
	G5	One out of, one out of ten ways to get uh the Player B to win.
	Kianja	Brionna, it's right there. So, you acting like I'm telling on you.
	G5	Kianja, you gotta, you gotta help me out here. If I want Player A to win, how many, how many ways, how many numbers like we can have?
	Kianja	What do you mean?
	G5	If we want Player A to win, right, and then we throw the dice, how outcomes can see, how many total number, how many different total number we can see from through the dice? [no response] Now you have the 2, so you have 2, 3, 6, right? [pointing to Kianja's sample space] So 2, 3, 6, so these are 2, 3, …
	Kianja	Seven, shoot!

	Brionna	2, 3, 8, 7 [pointing to sums in the sample space].
	G5	So, yeah 6, right? How many are for Player B?
	Brionna	4, 5, 6
	G5	No, what are total different ways to show 4, 5, 6?
	Brionna	Ten. There are 10 ways.
	G5	Which ten ways? [pointing to sample space]
	Brionna	The [ones that are circled in] black. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 [tapping her pen on each sum].
	G5	4+4 is 8. Do you think 4+2 and 2+4 are the same or different?
19:01	Kianja	Oh great! I know how to make the game even.
	G5	Yeah how… Is 2+ 4 equal to 4 + 2 + 4? 4+2 equal, is 4 +2 the same, like 2+4?
	Brionna	Umm, ‘cause 2+4, if you go down like say you do 1+2, 2+2, 3+2. No, let’s say like you list all the ones to be adding on to 2+4, 4+4, even though it’s like it’s the same answer you still have to do it ‘cause, because, ‘cause you can’t like [inaudible] 4+2 and you can switch it because you also have 2+4 and 4+2.
	G5	They are the same? Is the same chance or different chance?
	Brionna	It’s the same thing.
	G5	It’s the same thing?
	Brionna	[nods] It’s just that it, it’s worded differently.
	G5	Oh. So how about 3+4 and 4+3?
	Brionna	It’s the same thing.
	G5	The same thing? So, if we don’t, if we count this two as one, if we count this two as one opportunity, and this one. So you mean these two can be the same thing, right? Is that what you said? 4+2 and 2+4 are the same thing?
	Brionna	Um humh.
	G5	And 3+4 and 4+3 are the same thing?
	Brionna	Yes.
	G5	So, this is one chance and one chance, right, same thing.
	Brionna	This one, this one [putting her finger over some outcomes], these two, these two, these two, these two, these two. So 1, 2, 3, 4, 5, 6 [counting outcomes]. Six ways.
	G5	And then also is the, 4+1 and 1+4 are they the same or different?
	Brionna	The same.
	G5	So you mean this two is the same, this two is the same, this two is the same, how about 3+1 and 1+3?
	Brionna	It’s the same. ‘Cause you said you get the same answer no matter which way you put it.
	G5	Oh, I see how why you put on here. So actually these two are the same [indicating 1+2 and 2+1] these two are the same, and these, these. Is that what you mean? Is that what you mean?
	Brionna	Yeah. It’s just that the numbers are put like, 2,4, you could have 4, 2. It’s put down differently.

	G5	What if we use subtraction? We use minus. If you, if you played a game if we use minus, would it be the same? If we don't use addition, we use subtraction. Do you want to try and roll once? Roll the dice once? Do you want to roll the dice and try to use subtraction?
	Brionna	[rolls dice, getting 4 and 1] That'd be 3.
	G5	Um, how much was that?
	Brionna	Three.
	G5	Three? Is it 4 – 1? The same like, how about 1 – 4?
	Brionna	Um um. [holding her head]
	G5	Is it 1 – 4?
	Brionna	You'll get minus 3. You get minus 3, right?
	G5	Minus 3 or, is it minus?
	Brionna	Yeah, I think so.
	G5	Or negative 3?
	Brionna	Negative 3.
	G5	Negative 3. How about 4 - 1?
	Brionna	4 – 1? You'll get positive 3.
	G5	So is, if we use subtraction here, is …
	Brionna	You got the opposite, right? Like it'd be the opposite of the other number, like like 1+4, I mean 1-4, it'd be 3 but do 4, no, 1- 4 would be negative, be negative, no it'd be 3 and 1 minus, I don't know.
	G5	What would, is the same chance if we use subtraction?
	Brionna	It would be the opposite. Like it would come out to 3 no matter what but it would be like a negative or a positive.
24:00	R2	[announces to class] Okay. I think we're ready to hear some reports from groups about what the found, and I'm going to let Chanel go first.
		[Kianja, who has been working alone while Brionna and G5 were talking, wrote: "We could make it fair by having player "A" get one point for rolling 3, 7, or 5 and player "B" getting one point by rolling 2, 4, 6, or 8."]
25:00	Chanel	Here, I said that I think the game is unfair because Player B has more ways to find their answer than Player A has. Ya'll stole it from me first. Okay. For example, Player A only has one way to find its answer. For example, 1+1 equals 2, 1+2 equals 3, 4+4 equals 8, 3+4 equals 7. But, Player B has 4+1 equals 5 and 3+2 equals 5. Uh and 4+2 equals 6, and uh right here is 3+3 equals 6. So that's two different ways to find a 6. 3+1 equals 4 and 2+2 equals 4. That's two different ways to find all of 'em. But over here it's only one different way to find, and that's why, that's why, at first Dante, Dante's reason was kinda sounding good, but until we started playing the game more …
	R2	What was Dante's reason? Remind us again.

Dante — Yeah, Player A had more options of numbers than Player B did, so therefore Player A had, Player A would have the better chance of um winning.

Chanel — But just because they had more reason, more answers, I mean more uh uh numbers, that doesn't mean because when I went and looked at it, there's, there were actually two different ways to find all of 'em. But only one way to find [inaudible]. I played the game three times, and out of all those times, Player B came out to winning. And uh I had a little thing, I guess, to say. Player A, and Player A and Player B had 0 to 4 numbers on their dice, Player A would have 2 ways to find their answer and Player B would have 3 ways. So, I don't think that Player A would ever have as much as Player, like Player B would always have two more than Player A. For example, 2+0 equals 2, 1+ 1 = 2.

27:35 R2 — I'm sorry. I have a question. Um, you're saying that if that, if you numbered the dice differently, and so you had, what, still a pyramidal dice?

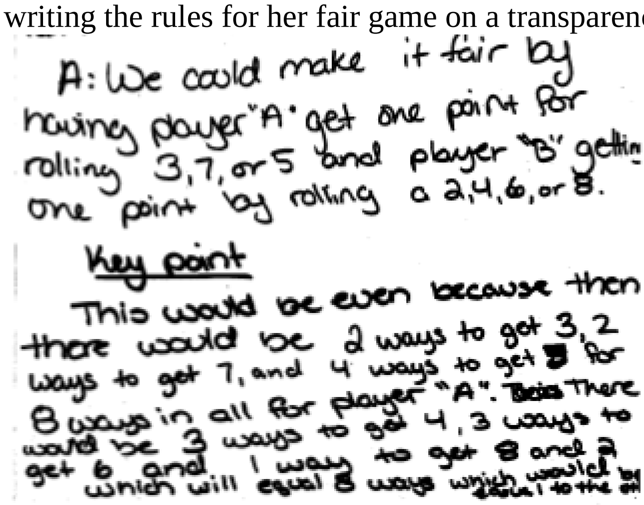
Chanel — Yes. I'd just put uh zero in the middle of it or on the side to make it still, it'd still have the same thing but it would just have a zero.

G2 — So zero could be one of the outcomes when you throw a die? And the pyramidal dice have how many sides?

Chanel — But no, I'm saying. Because a side, it has 3 sides, so you'd have to take one of the numbers away. So actually they wouldn't have two ways to find a number. 'Cause if I took the number 1, if I took off the 1, right here, [says something about the marker]

[Note: the remainder of Chanel's presentation is transcribed in ROLE 120 D.]

[While Chanel is presenting, the camera is on Kianja, who is writing the rules for her fair game on a transparency.



34:34 [end of CD 120C]

Date: 4 May 2005 Grade 7
Location: Hubbard Middle School
CD: ROLE 119D-120D
Transcribed by: Kathleen Shay
Verified by: Christoper Beattys

4:39	R2	[to class] Ian has noticed that we have a different shaped dice on the table. These are, these are the dice that we used the last time, right? One of the dice you used the last time [holding up a die]. And today we're going to work with this kind of dice. What's the difference?
	Kianja	It's a pyramid, it has 4 sides.
	R2	This one is a pyramid, and it has 4 sides. Chanel?
	Chanel	The other one is square and has 6 sides.
	R2	The other one is square? What name do we give to this shape?
	students	Cube. Cube.
	R2	It's a cube, and it has 6 sides. OK. And, what else do you notice? Any other differences?
		[more discussion about the shape and color of the dice]
6:58	R2	Before I give each pair a pair of dice, I want to ask you a question about what do you remember from the dice game we played last year?
	male S	We had a mat to roll.
	R2	Okay. We used a mat to roll the dice on. What else do you remember about the game? Terrill wasn't here, so, what are some things … nor Brionna, nor Kiesha, so a good number of you weren't here. [chatter] When we rolled the dice, you had a pair of dice and, and you had to roll them, right? What, do you remember what we did with that roll? What, what happened?
		[coughing and inaudible speech]
	R2	Well, I don't know if we did all that, but we certainly added the outcomes, right? We added the face values of what came up on the dice. Let's give out a pair.
		[dice are distributed to the class]
9:25	R2	I would like for each pair just to roll the pair of dice that you have and tell me, what comes up? Look at the, look at your dice and determine how do you know what comes up? You have to do something, roll the dice, and see what comes up.
	Dante	Uh, the triangle. The tip part of it.
	R2	What number comes up?
	Dante	Nuttin'. Nothing. Nothing.
	R2	How do you know when you roll it, when you roll it [rolls die], how do you know what comes up?
	Dante	The number facing towards you?
	Ian	You don't. It's the one on the bottom.
	R2	What number did you find on the bottom?

	Ian	[picks up die and looks at the bottom face] Four, six, seven.
	Dante	No – four, two, one.
	Ian	Which is seven.
	Dante	You’re not supposed to just say the number.
	R2	So which, which number came up?
	Dante	We don’t know.
	R2	Okay. You want to find one number that’s coming up. So how do you know? When you roll it [rolls die] …
	Dante	[picks up die and looks at the bottom] One, two four.
	Ian	One, two, four. They always land on that.
	R2	You want a single number to come up. So what number is it?
	Ian	Seven!
	Dante	Four, it’s four.
	Ian	Seven!
	Dante	No, it’s four. It’s four!
	R2	Show me. Roll it and see. Tell me what number you think comes up.
	Dante	Four right there, right?
	R2	But I see other numbers there. How do you know …
	Dante	Yeah four, but watch. Watch, I’m gonna roll it again.
11:00		[Dante rolls a die, picks it up and looks at the bottom, and looks askance.]
	Ian	[laughs] It changed.
	R2	All right. Roll it …
	Ian	Seven!
	R2	Don’t, don’t pick it up. And tell me, from what you see …
	Ian	Seven!
	R2	which number, what number is it that?
	Ian	No, eight. [loudly] Four, three, and one!
	Dante	Three, so far.
	R2	Hmm, but I don’t know, how do you know that? Like, if I roll this what number comes up? Don’t touch it, don’t touch it. Tell me what number.
	Dante	Four, two, one.
	R2	What number? One number. You told me several numbers.
	Dante	Four. Four.
	Ian	I said seven.
	R2	[to Ian] When you look at it, what number do you see?
	Ian	Six.
	Dante	How you going to see number six? There’s only, there’s three different kinds of …
	Ian	I see six.
	Dante	… numbers.
	Ian	I add all the numbers.
	Dante	You’re not supposed to add ‘em up, stupid, that’s not part of the game.

	R2	You have to make a decision about – before you can play the game, you first have to make a decision … [D&I are talking to each other.] Excuse me. Before we can play the game, Ian, before I can tell you what the game is, [pause] you guys not gonna listen. So what, we've gotta determine what, what the rule is.
	Ian	I don't know, I don't care.
12:09	Dante	[calls out to R2, who is walking away] It goes up by one.
		[Ian gets up and walks away from his desk. Camera moves to Chanel with R4.]
12:22	Chanel	I am so good at this.
	R4	You're so good.
13:10	R2	[to Chanel] Have you decided how to tell which …?
	Chanel	Yep. [nods]
	R2	So let's see. If [R4] rolls it …
	R4	Just one?
	R2	Yeah, one or two.
	R4	No, we were rolling two because we were talking about what were all of them. [rolls two dice]
	R2	So what …
	R4	What's on this one?
	Chanel	This one?
	R4	What did you just roll?
	Chanel	A two.
	R4	Um humh. And this one?
	Chanel	Another two. Two and two.
	R4	Oh, you turned it.
	R2	You turned it. Ha ha ha.
	R4	Here's the way it was. Now what was that?
	Chanel	Oh. Three.
	R4	[laughing] Yeah.
	R2	Okay. How do you know that a three was rolled here?
	Chanel	'Cause it's, it's at the bottom.
	R2	Uh huh. On all sides? All the visible sides?
	Chanel	No. I know this 'cause this is the biggest number over here [pointing to the side of the die facing her].
	R4	It's on the bottom here [pointing to a side of the die], it's on the bottom here [pointing to another side].
	R2	Is it on the bottom on that side?
	Chanel	Yeah. [turning the die on the table] The bottom over here and the bottom over here. And then the bottom here. When you turn it this way. It's gonna be one.
14:15	R2	[with Dante & Ian] Just, just here. [to teacher intern] So they're, they're trying to determine, like when they roll a die, when they roll one die, what number is it that was rolled?
	Ian	Four hundred twenty one.
	R2	What one number ?

	Ian	That is one number!
	R2	It has to be one of the numbers there.
	Ian	Four, two, one. Four hundred twenty one.
	Dante	But wait, it's at the bottom! The number at the bottom.
	R2	Ahhh, what number do you see at the bottom?
	Ian	[speaking loudly, unclear]
	Dante	No you said the number, you said the bottom of the pyramid.
	Ian	I said, only one number off the bottom.
	R2	So how do you know, what number here was rolled? Dante? Dante? What number was rolled there?
	Dante	One.
	R2	Let's roll this die.
	Dante	One.
	R2	Okay. Take them both and roll them again.
	Ian	I know how to do it.
	R2	What number was rolled there?
	Dante	Three and one.
	R2	Do you agree?
	Ian	Yeah.
	R2	Okay.
15:18	R2	[to class] May I have everyone's attention? Every group has decided what a roll is, right? When you throw the dice … Excuse me, guys? Okay, here's the problem. Let me show you the problem. [Turns on overhead projector.] I'll read the problem to you. Each of you will get a statement of the problem, but here's the task that I'd, we'd like you to work on. It says, does everyone, do I have everyone's attention? Kian- Keisha. Everyone's attention here? But I don't think she can see if you're in the way there. Can't see this. Would someone read what's on the …
	Terrill	I wanna do it, I wanna do it. A pyramidal die has 4 sides …
	R2	Terrill, I called on Chanel.
	Chanel	A pyrami-, how do you say that word? A pyramidal dice game. A pyramidal die has 4 sides. The number that is rolled is shown upright. Roll two die, dice. If the sum of two dice is 2, 3, 7, or 8, Player A gets one point and Player B gets zero. If the sum is 4, 5, or 6, Player B gets one point and Player A gets zero. Continue rolling the dice. The first person who, to get 10 points is the winner. 1) Is this a fair game? Why or why not? [Note: P(A gets a point) = 6/16; P(B) = 10/16]
	Ian,	Dante No. No.
	R2	So you think that it's not a fair game?
	Dante	Just like last year. It's not a fair game.
	R2	Why?
	Dante	Because Player 1 gets more chances than Player 2.
	R2	Wait, you mean Player A, is that …
	Dante	Yeah, Player A.

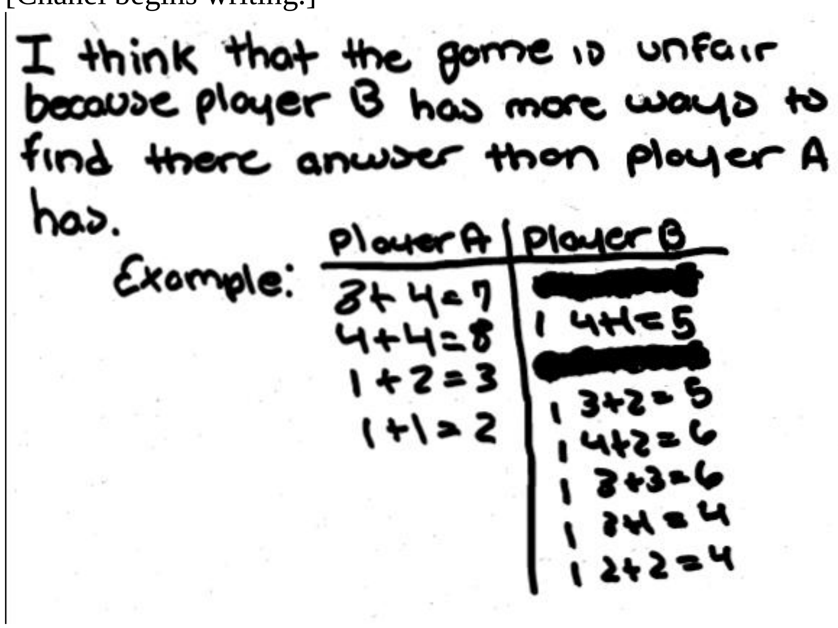
	R2	When you say Player A gets more chances, what do you mean?
	Dante	Because it gets 2, 3, 7, and 8 and Player uh B only gets 4, 5 and 6. So Player B has a lesser chance of getting, of getting um, a point instead of Player A.
	R2	Does everyone understand what Dante, the point that he made?
	students	Yeah.
	R2	Excuse me, Kianja? And Terrill? Did you hear what Dante said about why he thinks this game is unfair?
	Terrill	Yes.
	R2	Okay. Who could tell us what he said? All right, Terrill.
	Terrill	It's not a fair game because …
	students	[chatter]
	R2	Excuse me. Terrill is going to tell us Dante's [inaudible over coughing]. Okay?
	Terrill	Dante says it's not fair because, what'd you say it wasn't fair again? Oh he said it's not fair because all right, never mind. I don't even remember. I forgot.
18:31	R2	Okay. Who could tell us what Dante's point was? Chanel? [Chanel's arm is raised.]
	Chanel	Dante's point was that the game isn't fair because Player A gets 2, 3, 7 or 8 and that's 4 numbers, and Player B only gets 4, 5, and 6, 3 numbers, so Player A has a um better chance at getting what he wants than Player B.
	R2	Does everyone agree with Dante's point?
	students	Yes.
	R2	Yeah. Do you agree? Keisha? Do you have an opinion about this?
	Terrill	All right. Could somebody explain to me, say it like, okay, exactly why the game isn't, 'cause we just like going around in circles.
	student	The game isn't fair because Player A has more chances
	R2	Hold on, Dante. Excuse me, Dante why don't you come up here for a minute?
		[chatter]
	R2	I think Terrill has asked a serious question. So we want Dante to explain again his opinion about why it's not fair.
	Terrill	Can you like um explain in one sentence, that means with no 'ands' and noth of that, none of that, why this game is unfair.
	Dante	This game is unfair because Player A gets more chances than Player B.
	Terrilll	Okay. That's what I needed to know.
	students	[chatter] I'm Player A, then.
	R2	So what we'd like for you to do is to play this game. One, one of you will be Player A, the other is Player B. Play the game.
20:14	Chanel	I'm gonna be Player A.
	R2	Okay. Remember, what you're gonna try to do, you're gonna try

to, excuse me, we're gonna try to determine whether or not the game is fair. So it doesn't matter who's Player A or Player B, because your task is to determine whether the game is fair. Oh, and I already see that Chanel has begun to make a little score card for keeping track of, of what?

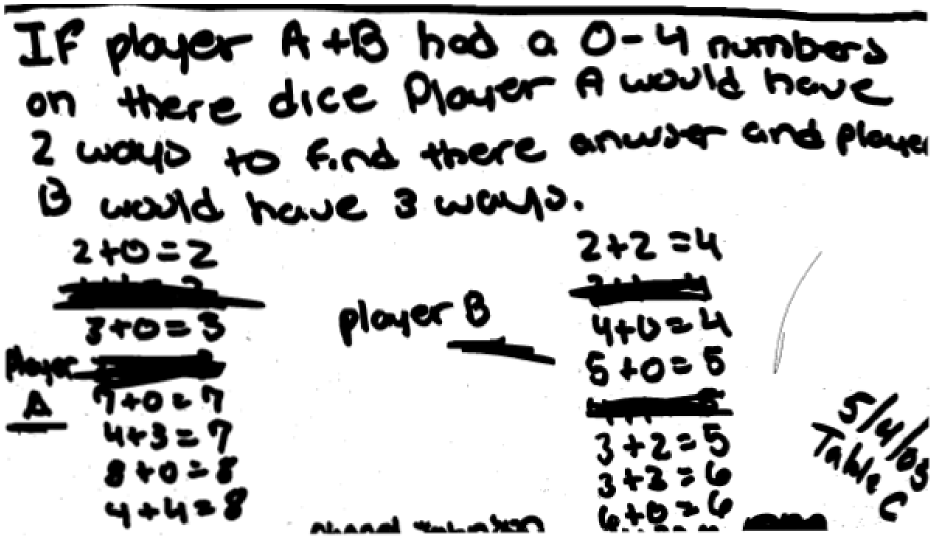
[Chanel's scorecard shows two columns labeled Player A and Player B.]

20:52	R4	It would help me if you put the numbers that Player A gets a point for.
	Chanel	Okay. A gets 2, 3, 7, and 8 [writes these numbers on her score sheet]. Five, sev-, 4, 5, and 6 [writes these numbers next to Player B]. So I roll first.
	R4	Okay.
	Chanel	[rolls] One, two. That's a three.
	R4	Okay, let's, I wanna remember what you got. So … [camera leaves this table].
39:13		[Camera returns to Chanel, now sitting with G5.]
	G5	When you first see this properly, do you think it is fair?
	Chanel	No [shakes her head].
	G5	Why not?
	Chanel	Because, like, they get all the numbers that, that don't, that you can't get. Like, I'm saying, they had a zero on there, then it's gonna be two diff-, then it'd be two different numbers for them. But for them, it'd be three different numbers.
	G5	Ohhhh! So when we played 3 times of this game do you think your answer is correct?
	Chanel	Yes.
	G5	Can you show me, what do you mean by uh this one needs 2 to get this number and this one needs 3 number to get? Can you show me why you say this?
	Chanel	Okay. For this, these already have two different numbers you can get to. 2+2 equals 4, and 3+1 equals 4. [writes these sums] And then, if they had zero, it'd be 4+0 equals 4. And that'd be 3 different ways. For 5 it'd be 5+0 equals 5, or 4+1 equals 5, or 3+2 equals 5. Then for 6 it'd be 3+3 equals 6, 4+2 equals 6, or 6+0 equals 6.
	G5	And how 'bout 2, 3, 7, and 8?
	Chanel	2, 3, 7, 8? It'd just be 2+0 equals 2, or 1+1 equals 2. For 3 it'd be 3+0 equals 3, or 2+1 equals 3. For 7 it'd be 7+0 equals, equals 7, or 3+4 equals 7. Then it'd be 8+0 equals 8 or 4+4 equals 8. [Note: the dice contain 1, 2, 3, and 4.]
	G5	But, do you see the 7 one. Don't you think that 1+6 is 7, too? And 2+5?
	Chanel	But these [holding dice] don't have 6 on it.
	G5	Oh. It only has 1, 2, 4, 1, 2, 3, 4, right?
	Chanel	Uh huh.

	G5	1, 2, 3, 4, 5. But they don't have a zero either. So we can't have this here.
	Chanel	No. But if they had a zero, then these would have two [pointing to the list of sums for Player A] and these would have 3 [pointing to the other list of sums].
	G5	Ohhhh.
	Chanel	But since they only, they don't have zero, these [Player A] have 1, 2, 3, 4 ways to find their answer. And if these didn't have zeros, they [Player B] would have 1, 2, 3, 4, 5, 6 ways to find their answer.
	G5	Oh, so you think, this is for Player B, right? So do, so you think which one has more chance to win?
	Chanel	These have six chances, these only have four.
	G5	[inaudible] Would you like, would you like to write an answer? So are you comfortable with your answer now?
	Chanel	[nods]
	G5	Would you write out your answer on the sheet? [gives Chanel an overhead transparency]
	Chanel	OK. Thank you.
43:00		[Chanel begins writing.]



45:47	Chanel	That's just I'm trying to show them basically how, what do I mean by them having more ways to find [?] than Player A has.
	G5	Do you also want to tell people that your rationale to find why, like this [shows paper], do you want to tell people like why, why Player B got more chance to win.
	Chanel	I could show, I could write that down at the bottom.
46:20		[Chanel adds the following to her transparency. The parts shown crossed out were crossed out later, when Chanel presented to the class.]



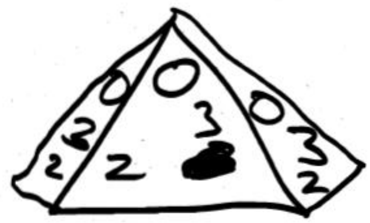
47:20	G5	Are you sure, are you showing the 1 to 4 game, dice game? What's the number on the game we played? What's the number on the dice? Are you sure it's 0 to 4?
	Chanel	No. I say, if a Player A, if Player A and B had a 0 to 4 num-, 0 to 4 numbers on their dice, Player A would have 2 ways to find this answer and Player B would have 3 ways. So I could show them what I did right here.
	G5	Could you also write [the sums] at the bottom?
		[Chanel continues writing. After she has written the first column of sums, G5 asks:]
	G5	So is that [pointing to column of sums] for Player 1, or A or B?
	Chanel	[writes] Player A. And that'd be for Player B [begins second column].
48:42		[Teacher T5 joins Chanel and G5.]
	G5	But later you can tell [T5] how did you find the answer.
	T5	How many dice were you playing with now?
	G5	We played three.
	T5	Two dice, or three?
	G5	Two dice, and we played three games. [Shows LP the score sheets.]
	T5	Oh, you recorded the, uh. Did they, did somebody tell you to do that, or did you do that on your own?
	Chanel	I did that on my own.
	T5	That's why you da bomb. Um, interesting. And was there one number that kept coming up more than others?
	Chanel	Yes. Player B has a better chance than Player A.
	T5	And why was that?
	Chanel	Because there's two different ways you could find the answer for Player B, and there's only one way you could find the answer for Player A. So…
	T5	Some of the, some of the sums? Or, or all of the numbers?

	Chanel	For all of the, for all of these right here. You could find two different ways to find them.
	T5	Okay. So 4 + 1 and 3+2.
	Chanel	3+2, and 4+2 for 6, 3+3 for 6. Four, 2+2 equals 4, and what was the other one? I couldn’t remember …
	G5	She wrote down there to show …
	T5	Okay. So you’re saying that …
	Chanel	Oh. 3+1 for 4 and 2+2 for 4.
	T5	There’s only one way to get 3?
	Chanel	Yeah.
	T5	There’s only one way to get 7?
	Chanel	Um humh.
	T5	There’s only one way to get 2, 8, 7. OK. And what do you, what do you think about um, this is an idea I’ve heard people talk about. Is 1+3 the same as, is 1+2 the same as 2+1?
	Chanel	Yes.
	R2	But you do have it differently here. [points to Chanel’s paper]
	Chanel	Oh, right there I was doing an example.
	T5	But do you, do you think these two are the same?
	Chanel	This, yes, I think these 2+1 is the same thing as 1+2. It’s the same thing, just reversed.
	R2	The same thing because they both equal 3?
	Chanel	Exactly. But they’re just switched around in reverse. So two’s over here [holds up left hand] plus one [holds up right hand], still gonna equal three. It’s the same thing, like I’m saying two minus one is two. But …
	T5	2-1 is 2?
	Chanel	I said 3. Oh, I didn’t say 3? Well, 3-1 is 2.
	T5	Sorry, I just, I, I knew you wouldn’t slip up like that, so it must have just been a, a verbal error. But, so you think, what if I had two different color dice?
	Chanel	[widens her eyes] It’s gonna be the same thing.
	T5	Still the same thing?
	Chanel	Um humh.
	T5	The guys, um, that uh made it to this, ‘cause a couple …
	R2	Why don’t you see if that’s really true.
51:32	Chanel	Two different dice. [grabs a yellow and a green die]
	T5	So can you show me what 1+2 would look like with those dice?
	Chanel	1+2?
	T5	You can manipulate them if you’d like.
	Chanel	1+2 [places the dice to show this]
	T5	And could you show me what 2+1 would look like?
	Chanel	Same thing.
	T5	But what would happen if I got a, a, ‘cause this is, OK, so you’re saying one plus 2 [points to one die and then the other]. But what if I said [changes the outcomes of the dice], is that the same roll?

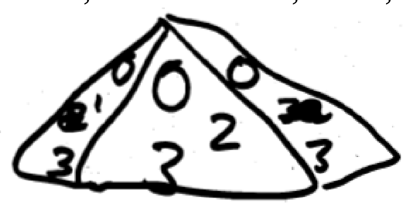
	Chanel	Yes. [camera is not on Chanel]
	T5	[nods his head left-to-right and up-and-down] That looked like a yes-no.
	Chanel	Yes, it is.
	T5	It is the same. So you don't think that there's two different things. So when you're now figuring out the possibilities, do you think that if that were different it would affect the outcomes?
	Chanel	If it was different, yeah, I think so.
	T5	It would, it would affect the outcomes if it was different?
	Chanel	[nods]
	T5	'Cause, um, we've been, we've been talkin' about it and some students, some students think it's the same, some students think it's different. That's why I was interested in your opinion on it, and why. So why was it again that you think it's the same?
	Chanel	Because it's, it's they all have the same numbers on 'em, the same amount on each side. So this is like saying 1 minus 2, but [waves her hand]…
52:50	T5	1 minus 2. So wait, actually, I'm interested in your thinking there. If I say 1 – 2, is 1 – 2 the same as 2 – 1?
	Chanel	I have to think on that one.
	T5	What about the, the answer?
	Chanel	Well, 1 minus 2 is …
	T5	Are both those differences the same?
	Chanel	No.
	T5	What would the answer to 1 – 2 be?
	Chanel	Uhhh, negative one I think.
	T5	And what would the answer to 2 – 1 be?
	Chanel	One.
	T5	So they're not the same during subtraction.
	Chanel	No.
	T5	But they are the same during addition.
	Chanel	Exactly.
	T5	And is it, and the reason why?
	Chanel	Because this is, like it's the same number. It just being twisted around, so. It's the, it's the same thing, just in reverse. But if you're doing subtraction, then the, if you're doing 2 minus 3 it's always gonna be, it's gonna be the same number but one is gonna be a negative and one is gonna be a positive.
	T5	Okay. So, because you get a different answer, that's the only way that it can be different. But if you don't get the same, if you get the same answer it's the same.
	Chanel	If you get the same answer, 2 + 3, same. But go like that, 3 + 2. It's the same thing. It's just being twisted around. So if you're doing 3 – 2, 3 – 2 is, I had to think on that, oh, one. And then 2 – 3 is gonna be negative one. It's the same thing, it's just one is negative and one is positive.

	T5	Would they count as two different opportunities when rolling dice, or would they count as the same opportunity?
	Chanel	They count as the same opportunity ‘cause you’re adding, not subtracting.
	T5	Oh, in this case. We’re adding, not subtracting. But if, so, you don’t, you don’t think that, that [reaches for the dice]. Let me grab another die.
	R2	Have you thought about a fair game?
55:05		[end of ROLE 119D]
		[begin ROLE 120D]
16:38		[Chanel prepares to discuss her findings with the class. However, the camera is not focused on Chanel at first.]
18:55	Chanel	[reads from her transparency] If Player A and Player B had had 0 to 4 numbers on their dice, Player A would have two ways to find their answer, and Player B would have 3 ways. So , I don’t think that Player A would ever have as much as Player, like Player B would always have two more than Player A. For example, 2+0 equals 2, 1+1 equals 2.
	R2	I’m sorry. I have a question. Um, you’re saying that if the, if you number the dice differently, huh, and so you have what, still a pyramidal dice?
	Chanel	Yeah. So just put uh zero like in the middle of it or on the side to make it [moves her hands up and down], still, it’d still have the same thing, but just have a zero.
	R2	So zero could be one of the outcomes when you, when you throw a die?
	Chanel	Exactly.
	R2	And the pyramidal dice have how many sides?
	Chanel	But wait, no wait. But I’m saying because a side is on 3 sides, so you have to take one of the numbers away.
	R2	Okay.
	Chanel	So actually they wouldn’t have two ways to find, they wouldn’t have two ways. ‘Cause if I took, if I took off 1, if I took off the 1, right here, aw, this ain’t no new marker. Well, if I took off [R2 gives Chanel a new marker], if I took off the 1, there’s only one way to find the 2. [crosses off “1+1=2”] So, if I took off this, there’d be only one way to come to 3. [crosses off “1+2=3”] And if I took off, if I took off [inaudible], there’d only be one way to find a 2, one way to find a 3, two ways to find, um, 7, two ways to find 8, so it’d be 3, 4, 6 …
20:48	Terrill	Um, excuse me, Chanel, you’re wrong because 8+0, there is no zero on the dice.
	Chanel	You didn’t let me finish it.
	Terrill	What are you talking about?
	R2	His question is, does the dice that you’re making have a zero on it?
	Chanel	No. Okay, let me show y’all.

		[R2 gets a blank transparency for Chanel to draw her new dice. She draws a pyramid with the numbers 0. 3, and 2 showing on three sides.]



22:00	Dante	Yo, excuse me.
	Chanel	Dante! Let me finish. Go ahead, go ahead Dante.
	Dante	How can you have um the same thing on every side of the dice?
	Terrill	I know.
	Chanel	But I'm trying to show y'all something. It's supposed to be two dice. Not, well not that. But I'm saying then on this side you have [drawing a second die] a zero, a three, and a two; a zero, a two, and a three; and the bottom, a zero, a two, and a three.



		Now that's B 3+2 equals 5. So, if the one, if the one, if I took the one off, it'd only be 2, 4, 6 ways to find, to get, um to get Player A There's only be 6 different ways out of all. And for Player B if there was no ones [crosses out sums involving 1] there'd be 2, 4, 6, 7 ways to find for Player A, for Player B. So, Player B would always have more that what Player A has. 'Cause Player B has, like, it's still two diff-, it's still two different ways to find the answer. On here it's not two different ways to find 2. It's not two different ways to find 3. So it'd make it one less than what Player B has.
23:42	R2	Does anyone have questions for Chanel?
	Terrill	How do you get zero?
	Chanel	Not even listening! I said …
	Dante	He listened. You made your own dice and all that other stuff. But how can you have, how can you have the same thing on every side of the dice?
	Chanel	You don't have, you don't have the same numbers on every side of the dice.
	Dante	You kept going zero, three, two, or zero, two, three.
24:06	Chanel	Dante, this, this is the dice, right? [holding up a die] What's at the bottom? Fours, right? What's on top, one and twos, right? Then

		it’s two and three, right? Well so what? But still, it’s still four on the bottom, right?
	R2	All right, so Chanel, Chanel, tell Dante what you just realized.
	Chanel	That it’s not, well right here, I [unclear] right here to put in to be 2, 3 and over here should be 3, 1. [makes changes on the die she drew] So, 1, 2, 3, 3 well I actually caught myself right there.
	R2	Okay. So maybe you need to think about that a little bit more. Uh, but Chanel’s trying to construct new dice in order to show us why she believes that Player B will always have more chances of winning than Player A. Is that right, Chanel?
	Chanel	[nods] But it’s still, out of all, Player, Player B has more chances than Player A has.
	R2	So maybe you can think carefully about how to construct your new dice. And maybe tomorrow when you come in you’ll …
	Chanel	I’m not going to be here tomorrow. No, I’m going to see a play.
	R2	All right, we have, we have 5 minutes. That clock is 5 minutes fast.
		[chatter]
	R2	What we’ll do is, we’ll resume these reports tomorrow.
26:20	R4	[privately to Chanel] Could you just explain that to me? You made two new dice. Is that right, or just one? OK, show me what, can you show me what the dice is?
	Chanel	This, well right here, it’s different, but I tried to get it right. See how it’s toward the bottom [handling a real die]. Well here it’s one and two, three and two, one and three, and here three and four, four and one.
	R4	Okay. Show me, let’s have a die. Okay. And you’re putting zeros on some of them?
	Chanel	If I replace the one with the zero, will they have the same amount, will they …
	R4	So you don’t have any ones anymore? You don’t have any ones anymore. You have zero, two, three, and four?
	Chanel	Yeah, and I’m saying, will Player B still have more than Player A?
	R4	Ahhh. I understand what you said. Okay. But the thing that really confused me was all these big numbers here. How could you ever get a five? Or a seven? You don’t have them on your dice, do you?
	Chanel	Oh my gosh, no.
	R4	Okay. So for a minute, so that we can start with this tomorrow, tell me what you have on your dice. Show me exactly. Show me exactly. You have a zero?
	Chanel	[writing] Zero, and then there’d be two on the bottom and three over here. And over here it’d be zero …
	R4	They have to be the same, don’t they?
	Chanel	Yeah.
	R4	It’s right here. [reaches for transparency] Okay. Now. So they

		look just like this [dice on transparency]. This is great. Uh, now what I want you to do, if you can give me just one more minute, uh, if you have two dice, okay, okay, suppose this one is a zero, what could this one be?
	Chanel	It could be a two.
	R4	It could be a zero, couldn't it?
	Chanel	Um humh.
	R4	Okay. And so what could this, this one could be, no, it couldn't be a one. It could be any one of those things [0, 2, 3, 4]. Okay, and this one could be [writing], okay is that right?
	Chanel	[nods]
	R4	Okay, so what, what could the sums be? What possible sums could you get?
	Chanel	Well, I could get 4, 6, and 8.
	R4	And zero.
	Chanel	Oh. Zero.

0 * 0 = 0
2 2 = 4
3 3 = 6
4 4 = 8

	R4	Okay. What else could you get? Couldn't you get this plus this [pointing at different pairs of numbers]?
	Chanel	It'd be this plus this.
29:09	R4	Okay. Write that over here.
	Chanel	0+2 equals 2, and then 0+3 equals 3, 0+4 equals 4.
	R4	Okay. Great. Now, and so you could've had, why don't you put those plusses down here.
	Chanel	I'll write it. I'm saying, if you had 2 + 2 equals 4, and then again you had 3 + 3 equals 6, and you had um 4 + 4 equals 8.
	R4	And you have 0 + 0 equals 0.
	Chanel	Yeah.
	R4	But these, okay, now let's do it.
	Chanel	2 + 3 equals 5. Oh, that's [inaudible]. [Chanel writes the sums as she speaks.]
	R4	And that's 2+4.
	Chanel	Equals 6.
	R4	Okay, and then, so that's all you can have with twos. Is that right?
	Chanel	Yeah.
30:06	R4	'Cause you already had 0 + 2 and 2 + 2.
	Chanel	That's all you can do.
	R4	And you already had 3 + 3 and 3 + 4. OK, how many are there?

30:15	Chanel	There's 1, 2, 3 ,4 , 5, 6, 7, 8, 9, 10.
	R4	Okay. So, how are you gonna make it fair? How much would each person have to get?
	Chanel	Five each.
	R4	Five of those? But these are both fours. So there's two ways to get a 4 still. So that means what?
	Chanel	There can't be any fours.
	R4	Well, so you're saying each person gets five, five chances. How can you even it up, because that means they've got two chances to get a 4?
	Chanel	Then you can, I think you should, whoever gets, like, no. Actually, you know what I think? I think they can just X the fours off. Then it'd be 3, and then over here 1, 2, 3, 4, 5, 6, 7, 8, 9.
	R4	No, you did two of 'em off.
	Chanel	Oh. 1, 2, 3, 4, 5, 6, 7, 8. So then everybody have four different chances, four each.
	R4	So what, what would I do? How would I get a point?
	Chanel	You can get a point if you pick 0,0, that's zero, and you can take 3+3, so now you have 1, 2, 3, 4 over here…
	R4	So if I get 6, 0, 2, or 3, …
	Chanel	That's Player A. And if you get 8, 5, 6 …
	R4	5, 6, 7, 8
	Chanel	That's Player B.
	R4	And you throw it out if you get a 4.
	Chanel	[nods]
	R4	Well, that is certainly one way. You wanna put your name on that? Do you think you can remember that so that you can talk about it after you get, what are you going to see tomorrow?
	Chanel	Six Flags
		[discuss Six Flags]
32:43		[end of CD ROLE 120D

Date: 5 May 2005 Grade 7
Location: Hubbard Middle School
CD: ROLE 121B-122B
Transcribed by: Kathleen Shay
Verified by: Judith Leonard

Time	**Speaker**	**Transcription**
3:00	R2	I wanna welcome all of you back today, those who were with us yesterday, and those who were not with us yesterday, I'm very happy to see you.
		[Camera is focused on 4 girls seated with facing desks arranged in a square. Kianja, Brionna, and Keisha are talking and giggling. Justina is sitting quietly with her eyes downcast.]

	R2	All right. Those you who were here yesterday, you wanna help bring the new people up to speed. And I want to find out who would like to say, to tell the others what did you work on yesterday without telling them how, the answers you've come up with? All right, Kianja, will you come up?
	Kianja	They can hear me [from her seat].
	R2	Okay. So, Kianja's going to talk about, going to tell you what we worked on yesterday. [Tells some of the boys to turn around and pay attention.] Kianja?
	Kianja	Yesterday we worked with that little, what's it called, what's that kind of dice?
	R2	Pyramidal.
	Kianja	Yeah, that one. That kind of dice and um we had to [chatter in class] and we had to [squints in Brionna's direction] …
	Brionna	We had to make the game fair.
	Kianja	Yeah, we had to um make the game [laughs] ...
	R2	Yes, we had to make the game fair. But Kianja, can you explain …
	Kianja	[laughing] We had to try to make the game fair.
	R2	Can you explain what the game was?
	Kianja	Um, [laughs] we had to try to make the game fair and the game was um if you rolled a certain number then you would get a point [laughs]. Shut up, Keisha. Stop distracting me. And then, and then we had to try to make the game fair, and that's it.
	R2	All right. Do you all remember from last year we played a game involving two dice? Okay. We're playing a game very similar to that one. I'm gonna show it to you on the transparency, and I'll ask Justina, would you like to read it.
	Justina	I don't want to read it.
	R2	No? Okay. David?
		[As David reads the problem aloud, Kianja, Brionna, and Keisha talk and giggle. Justina sits quietly, not smiling.]
	R2	Okay. So that's the problem that you worked on yesterday. I'm gonna hand each pair of you a copy of the problem. Now, some of you played the game yesterday, and some of you have not. So we're gonna give everyone a chance today to actually play the game and see what results you come up with. Now I know Kianja and Brionna started working on a presentation, right? So we'll get you those transparencies so they can continue.
		[more talk to get organized]
		As you're working, keep track not just your sums but also your outcomes.
		[Justina is the only one at the table who was not present yesterday and has not played the game before.]
8:30	R2	So Justina, do you have a copy of the problem?
	Justina	No. [Kianja places a paper in front of Justina.]

		[Someone off camera asks R2 a question that is unintelligible – presumably about having two different colors of dice.]
	R2	I don’t think, no, I want them to have two of the same color to start off. I can give you one, you want an extra color one?
	Kianja	Yeah.
	R2	So, Brionna and, you guys remember what you … You did a lot of keeping track of what you were finding with the game, didn’t you? [to Kianja] And you had some written ideas, if I remember, is that right?
	Kianja	Um humh. [nods]
	R2	Okay. So [G5]’s looking to get, to get those for you.
	Kianja	That’s wonderful. [seems sarcastic]
	R2	But, what I’d like for you to do is help Justina understand what the game is. [to assistant] Do you have those for them? Their work from yesterday?
	Keisha	[laughing] Who you talking to? Who you talking to? Who you talking to? Who was you talking to?
	R2	I meant to be talking to you.
	Keisha	No, you wasn’t. [laughs] You asked me that already and I, it seemed like you heard my answer.
	R2	Well, but you have to play the game and keep track of your outcomes, okay?
	Keisha	[has her face in her hands, leaning onto the desk, laughing, sits up] Can I go to the bathroom?
	R2	[speaking quietly to Kianja – inaudible]
		[to Keisha] Yeah, why don’t you do that, quickly.
		[to Brionna] Brionna, you played the game a lot, so you could help show Justina what to do, okay?
	Brionna	Okay.
		[G5 gives Kianja and Brionna their papers and transparencies from the previous day. They talk about which papers they want to use today. Neither girl shows or says anything to Justina.]
11:01	T6	So what all did you have down there? Did you put down the combinations? Is that what that is?
		[Kianja & Brionna do not respond to T6. They continue to look through their papers. They decide that they don’t need some of the papers and return these to G5.]
	T6	[quietly, to Kianja] You have to watch [wash?] that.
	Kianja	Huh?
	T6	You have to watch the other [inaudible] or else we have to watch it. [unclear] So did you all explain the game to Justina?
	Brionna	No, we just, um …
	T6	Well, let’s show her how we play.
	Kianja	I don’t like this game, though.
	T6	How many dice were you playing with, three or two?
	Kianja	Two.

	T6	Just playing with two? Okay.
	Brionna	[to Kianja] Player A or B?
	Kianja	[to Brionna] We playing a fair game, so it don't matter.
	G5	[sits in Keisha's seat, across from Justina] You know how to play this game?
	Justina	Yeah.
	G5	Now when, if you roll the dice [rolls dice] and then we add the bottom number 2 [pointing at one die] and the bottom number 1 [pointing at the other die] is 3, right? So with number 3 it's Player [inaudible] [pointing at paper with problem statement] point.
	Kianja	[speaking to Brionna at the same time that G5 speaks to Justina] You get 1, 3, and oh, wait, yeah. You get 3, 5, and 7, and I get 2, 4, 6, 8
	G5	[to Justina] But you got all the numbers 2, 3, 7, or 8, A got a point. Uh, 4, 5, 6 is Player B get a point.
		[at the same time, Brionna and Kianja discuss which player gets points for which numbers.]
	Brionna	That's not fair.
	Kianja	Yes it is.
12:47	T6	[T6 pulls up a chair next to Justina.] Would you like to be A or B?
	Justina	A. [slightly shaking her head from side to side]
	T6	You want to be A? OK. Throw the dice. And it's the number on the bottom that we'll use when we're counting. Okay, do you want to just throw the dice? [Justina rolls the dice.] Okay, so you got 4 and 4, so you got 8. And if we look on here, let's see who gets 8. [looks at problem sheet] Okay, Player A. So you get a point. Okay, then I be Player B. [rolls dice] I got 5. Let's see who gets points for 5. So Player B gets a point for 5. [Justina rolls] Okay 5, so I get a point. Okay your, no my turn. Okay 5 …
	Kianja	[to Brionna] Nobody gets it! If I rolled it and I, and that's not my number, I don't get a point.
	T6	[to Justina] I get another point. Your turn.
	Brionna	I don't get that.
	Kianja	1 and 1 is 2, so you get a point. [Writes "1+1=2" in the B column of her score sheet.] 3 plus 4 is 7. [Writes this sum in the A column.]
	T6	No, what we did not do, 'cause this is your first time playing it, right? Right, 'cause let's say Player A, Player A has 4 combinations they can get and Player B looks like they only have the 5, 6, 4, 5 and 6. And if you think the game is gonna be fair or not like this.
	Justina	No.
	T6	You don't? Okay. Why?
	Justina	Because, Player A has um more numbers than Player B does.
	T6	OK.
	G5	[to Justina] So do you think Player A or Player B got a more

		chance to win?
	Justina	Player A has more of an advantage.
	G5	Why? Because …
	Justina	Has more numbers.
	G5	Ohhh. Okay.
	T6	Yeah. She said because they have more numbers versus the 3 numbers that Player B has.
	G5	Okay. Then you can start to play the game with Mr.[T6] to see.
	T6	Yeah, I'm trying to think whose turn is it, uh.
	Justina	Your turn.
	T6	My turn. It's an odd number, so I guess it's gotta be mine. [rolls] Okay. Another 5. [Justina rolls.] 5 again. [Justina's paper shows the score is A-1, B-6.] [T6 rolls] 6, Okay, you get a point.
	Justina	No, Player B has um…
	T6	You're right, I'm sorry. Player B has 6 also. [Justina rolls] 6 again. [rolls] 4. Still think this game is fair?
16:49	Justina	[shrugs]
	G5	Who got a 10-point win?
	T6	Nobody yet. [Justina rolls] Well, now they do. So B just won.
	Justina	Can I just see that one more time [T6's score sheet]? [looks at the two score sheets side by side]
17:12		[Keisha returns]
	T6	Keisha, Keisha, Keisha, Keisha, let's just get back in. I just played one game with her.
	T3	[to Justina] My understanding is that your conjecture was that Player A was going to win, have the advantage 'cause they have more numbers?
	T6	Right. Do you still feel that way?
	Justina	[shakes her head no]
	T6	Why?
	Justina	'Cause you kept winning, and you got all the [money ?].
	T6	Why do you think that is, though, that with less numbers I was still able to win?
	Justina	[takes a die and examines it]
	T6	Oh, do you think we have loaded die? [to Keisha] Kiesha, did you get to play yesterday?
	Keisha	Yep.
	T6	Do you think the game was fair yesterday?
	Keisha	I don't know.
	T6	You say you do or you didn't think the game was fair?
	Keisha	I don't think it [camera moves away from Keisha and the rest of her statement is not heard].
	T3	[to Kianja] A was gonna win because he had more numbers, right?
	Kianja	No! Read this [hands him her paper from yesterday].
	T3	[speaking during a PA announcement] What do you think is the reason why B won so easily?

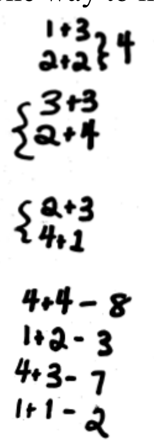
18:44	Justina	Maybe most of the sum of numbers comes up to …
	T3	Maybe most of the what?
	Justina	The sum of the numbers comes up to [shrugs] I don’t know.
	T3	Is there any way to find that out?
	Kianja	[nods yes]
	T3	What would you have to do to find that out, to figure that out?
	R2	Can I ask you guys a question? Remember yesterday there was, oh, I’m sorry, did I interrupt your play?
	Kianja	No.
	R2	Remember yesterday there was this question of whether or not 2 plus, whether a 2 and a 1 is the same as a 1 and a 2?
	Kianja	It is the same.
	R2	It is the same. Right?
	Kianja	Yes. It’s the same thing, just [inaudible].
	R2	But you remember that issue that came up?
	Brionna	Yes.
	R2	Okay. And do you remember …
	Brionna	1 minus 4 and 4 minus 1.
	R2	Um humh. Okay. Now we’re doing it in terms of the sums, right? Well this is what I’d like, I have a slightly different game I would like to introduce you to. Okay? This is the game. Throw two dice. If it’s, if the sum is 2, Player A gets a point. If the sum is 3, Player B gets a point. Okay? But those are the only possibilities for getting points. 2 and 3. Two goes to Player A, 3 goes to Player B.
	Kianja	[inaudible]
	R2	Hold on. Now who’s gonna win? Is this a fair game that I’m just introducing?
	Kianja	I mean, Player B gonna win.
	R2	Why?
	Kianja	‘Cause there’s only one possible way that you can get 2.
	R2	Okay. So let’s, let’s try. Okay?
		[Kianja holds up her paper and looks at it.]
	Kianja	Only one way to get both of ‘em, so …
	R2	So it’s a fair game, right?
	Kianja	[looks at R2 and tilts her head but does not answer]
	R2	All right. So let’s, let’s play. Who’s gonna, who’s making the first roll? Who’s gonna roll the dice first?
	Kianja	Me. [rolls] I don’t get no point.
	R2	Who gets a point? No one.
	Kianja	Nobody.
	R2	[to Brionna] Okay, You throw. You roll.
	Kianja	Who are you?
	R2	Roll, that’s 3.
	Kianja	That’s 6.
	R2	Oh, I’m sorry. Was it 3 and …

	Kianja	It was 4 and 2.
	R2	I'm sorry, go ahead.
	Kianja	[rolls, shakes her head]
	R2	[to T3] So they're playing this game, 2 and 3.
	T3	Ahhh. So if you get 2 you get a point, …
	Kianja	[loud] You know you cheatin'! Cheatin'! She cheatin'! She cheatin' 'cause she never told me what her number was.
	R2	She did, she did. She wrote it down.
	Kianja	She, no! But she never said whether she was B or A.
	Brionna	It don't matter, because whatever the sum is, it gives, it gives, if it's 3, it gives that person gets he said no matter whose turn it is, that person …. So no matter who we are …
	R2	So now you go.
	T6	[to Keisha] Well, is there a particular number that you think, or numbers, if they were changed between the two, A or B, that would make a difference, or that would make the game more balanced, more even, more fair?
	Keisha	No.
	T6	No particular number?
	Keisha	I don't know. Why you askin' me all these questions?
	T6	'Cause you're the only one that knows the answer.
	Keisha	No I don't.
	T6	About what you think, you are. I stopped reading minds about 10 years ago. It got to be too big, too heavy for me. Well, if you remember the game yesterday, what number seemed to come up more frequently?
	Keisha	What?
	T6	What, what are the numbers, like when we played today, it's like 5 kept coming up and then 6 came up a couple times and 4. What numbers do you recall coming up more often yesterday?
	Keisha	[has been using the markers to write her name in many colors – now folds up the paper and smiles at T6]
	T6	Want to play a game and find out?
	Keisha	I don't feel like doing it. I don't even know why I came. I shoulda just went home.
	T6	I'm glad you came, though.
23:56	Keisha	[Inaudible] And that microphone's always somewhere.
		[end of CD 121B]
		[begin CD 122B]
0:18	Kianja	[at the overhead projector] That's it. [She has made a presentation, on another CD, 121C.]
	R2	Any other questions? For Kianja? Kianja, there was something you had there, a key point. Do you want to talk about the key point?
	Kianja	[walking to her seat] I read that [holding transparency].
	T6	Kianja, you did a good job.

0:48		[Jerel begins his presentation, but the camera is not on Jerel.]
1:13	Jerel	All right. If you want me to put not fair because, not fair in favor of Player A. A has 4 chances and B has 3.
	R2	All right, so that was your, that's what they thought about the game, excuse me, Kianja. This was, this was their prediction before they started playing the game. Okay?
	Jerel	I put it's not fair, this game, because A has 1, 2, 3 combinations to get a number.
	Ian [?]	It don't make no sense!
	Jerel [?]	That's what you said!
	R2	Well what, what, make the correction, Jerel. Jerel. Jerel? Make the correction.
	Jerel	All right. This is what I originally said. I said it's not fair because [chatter]. I would not fair the game because Player A has, has to only get 1, 2, and 3 combinations. But then it uh Player B has to get more combinations in it. And then, when I started playing the game, I changed my mind because it's fair because, because what? [Unclear] has just as good a chance as B because when I was playing, and I was rolling the dice, I beat, I beat David, Player A.
	David [?]	You beat me once.
	Jerel	I did not beat Ian.
	David	You beat me once, though.
	T5	I can't understand a word that you're saying.
	David	All right, look. Let me explain. Let me explain. He'll know what I'm talking about. Look. David says it's not fair because Player A, A has 4 chances and B has 3. That's all I'm gonna say. But Jerel said that it's not fair because the number for A has 1, 2, 3 combinations to get A's numbers 2, 3, 7, and 8. But then he changed his mind 'cause I beat him. And, it's fair because A has just as good of a chance as B. That's it. But, [inaudible].
4:10	R2	All right. Do you want to explain this? I think Ian, Ian, Ian, you wanna explain this? [chatter] Excuse me, one second, hold on. I'm gonna ask David and Jerel to have a seat while Ian's explaining.
	Ian	All right, look. Player A got less, more numbers but less combinations, all right. Player B got less numbers but more combinations. That's why [inaudible]. But then, man, I just said what Kianja and them said!
	R2	Wait. Hold on, hold on. Ian. Ian. Ian. Put that back up, because I don't think you're saying exactly what they said.
	Ian	Yeah, I am.
	Kianja	Well, he is. It's the same concept. [gets up and asks permission to go to the bathroom – R2 asks her to wait]
	Ian	Player A got 4, right? And then Player B got 6. That's it! That's all you need to know.
5:23	R2	But now, for me these are two different, Kianja, Kianja, for me

	what Jerel is saying sounds different to me than what you …
Kianja	No, that’s not Jerel, that’s Ian. And Ian …
R2	All right, what Ian was saying …
Kianja	[standing] It’s the same thing. He just put different numbers in. I mean, like, ‘cause he didn’t do the [waves hands]. You know how I had 10 [outcomes in the sample space]?
R2	Um humh.
Kianja	He had 6, which I had first. But then we had switched some of the numbers around like 2+1 we did, I mean 1+2, we had changed it to 2+1 which gave us another combination. That kind of thing.
R2	Right. So you had 10, he had 6.
Kianja	Yeah.
R2	He did not count 2+1 and 1+2 as different events.
Kianja	Right.
R2	But you did.
Kianja	He counted them as the same thing. We counted them as one, I mean, different things, but he counted them as one. That’s why we didn’t get the same numbers.
R2	Right. So I think that we …
Kianja	But it’s still the same. I mean, it’s the same concept.
R2	Well, I don’t know. Maybe others … We’ll have to see whether or not Justina agrees.
Kianja	[turns to face Justina] I think it’s the same concept.
R2	[to Justina] Do you think also it’s the same concept?
Justina	Yep. [nods in agreement]
R2	Yeah? What is it that you’re agreeing to?
Justina	I wasn’t listening.
R2	Uh huh. [to Kianja] You want to explain again?
Kianja	[laughs] Do I what?
R2	Do you want to explain it again, because I think it’s a very important point.
Kianja	[laughing] I really don’t, but …
R2	It’s a very important point. Go ahead.
Kianja	I really don’t, but I guess. that’s what I’m here for. [reaches for her papers]
R2	You gonna show her on your transparency?
Kianja	[to Justina] You know how to read, right? OK. [hands her a paper]
R2	Well, why don’t you point it out? From the combinations that you’re indicated there …
Kianja	What is wrong with that child?
R2	Okay?
Kianja	[hands another paper to Justina] Hmm, read this paper still.
G5	I think Justina found a real good uh reason why it’s not a fair game. I think she’s ready to pre-…
R2	She’s ready to talk about it?

	G5	Yeah. She's ready. She has her own reasoning, yeah.
	R2	Would you like to talk about it, Justina?
	Justina	Um, okay.
	R2	Okay.
	G5	She found out a new, she created a new game, too. More fair than this one.
	R2	[as Justina gathers her transparencies] Ah, you've got 3 transparencies. Okay. Let's see what she has to say, Kianja.
8:00	Justina	[standing at the overhead projector] Okay. Well, I said that this game is unfair because Player B's sum of numbers has two different ways, has two different combinations, and Player A's sum of numbers only have one different combination. So the way I would make this game fair …
	R2	I'm sorry, can you explain a little bit by, when you say that Player B has two different combinations, what do you mean by that?
	Justina	Um, 1+3, 2+2, those are two different ways to get 4. 3+3, 2+4 are two different ways to make 6. And 2+3, 4+1 are two different ways to make 5. And for Player A's, 4+4 equals 8; there's only one way to make 8. 1+2 …



[Justina's sample space]

	Kianja	Oh, wait. Can I say, wait, can I say what I think you're saying? Um, you saying that um, each, each number on Player A has only one combination that can get to that sum, and then on Player B, each number has two? Okay.
	Justina	Um humh. That's why I had the greater advantage.
	Kianja	Okay.
	Justina	That's why I think it's unfair. And, for my game, …
	R2	I'm sorry. Do you agree with that point of hers, Kianja? Kianja, do you agree with her point?
	Kianja	Yes.
	R2	That the numbers for player A each have just one combination?
	Kianja	Um humh. I know. I know what she's talking about. Yeah.
	R2	Yeah? Um, okay. Go on. We might come back to this point.
9:45	Justina	Okay. Okay. Um, for my game, Player A would have 2, 7, and 4

		because they have two numbers that only, that have only one combination, and then they have 4, which has two combinations. And same for Player B – 3 and 8 only has one combination and 5 has two combinations, so it's the same. And 6 would just be zero. So no, no player gets that point.
	R2	So that would be your fair game?
	Justina	Yeah.
	R2	Okay. [turns to Kianja] What do you think?
	Kianja	I think she's right.
	R2	Brionna? Do you agree that the game that Justina's made is a fair game?
	Brionna	Yeah.
	R2	Yep?
	Brionna	Um humh.
	R2	Do you want to say why you think it is?
	Brionna	No.
	R2	[to Justina] Could you go back, could you go back, you have another transparency you wanted to show us? 'Cause I want to go back to your first one.
		[Justina puts her first transparency on the projector. This shows the score table for 3 runs of the original game, as well as the sample space she constructed showing the number of ways to obtain each sum.]
	R2	So, 4, you're saying you can make 4 in two different ways.
	Justina	Yes.
	R2	Well, I think that's different than what Kianja has. Kianja, on your paper, how many ways can you make 4?
	Kianja	[makes a noise, like nuh nuh nuh nuh, then raises her arm and holds up 3 fingers]
	R2	Three. What are they?
	Kianja	Reverse the 4 and 2. Oh wait, you said 4? It would be 1+3, 3+1, and 2 + 2.
	Justina	[turns to look at the screen] 1+3 is the same thing.
	R2	Same thing as what?
	Justina	1 + 3 and 3+1 would still equal 4, so …
	R2	Okay, so you saying those are the same.
	Justina	Yeah.
	R2	Okay. All right. Well, it's 5:00. We may have to come back to this question next week. But I think that this is an interesting point for us to stop because this is where I think that there's some disagreement. Okay? Thank you, Justina.
12:20		[end of CD 122B]

Date: 5 May 2005 Grade 7
Location: Hubbard Middle School
CD: ROLE 121C-122C
Transcribed by: Kathleen Shay

Time	**Speaker**	**Transcription**
2:07	R2	[Welcome and introduction of task. This part is transcribed on ROLE 121B.]
5:04	R2	David, read what's on the [another student offers to read]. David, he needs to learn it.
	David	A pyramidal die has 4 sides. The side that is rolled is shown upright. Roll 2 dices if the, if the sum of the 2 dices is 2, 3, 7, or 8, Player A gets one point and Player B gets zero. If the sum is 4, 5, or 6, Player B gets one point and Player A gets zero. Continue rolling the dice. The first person to get 10 points is the winner.
	R2	Okay. So that's the problem that you worked on yesterday. I'm gonna hand each pair of you a copy of the problem. Now, some of you played the game yesterday, and some of you have not. So we're gonna give everyone a chance today to actually play the game and see what results you come up with. [more talk to get organized] Ian, you and Dante worked on a presentation.
	Ian	No! Ian worked on a presentation.
	R2	All right, Ian wrote up a presentation. But Ian is going to take, he's agreed to take responsibility of helping the others, thank you, helping the others learn the game.
	Ian	All right. [to Jerel and David] I'll help everybody, 'cept David.
	R2	So that's what we'll be doing for a while, and as you're working, keep track not just your sums but also your outcomes.
		[While R2 speaks, the 3 boys draw designs on their papers.]
7:03	R2	[to the table] All right, so Jerel and David, Ian is gonna help to get you started.
		[R2 walks away, and the boys continue drawing and chatting.]
9:04	Ian	All right. I gotta tell y'all what to do. Okay, y'all man, stop. You know what, y'all do it yourself, get outta here.
	Jerel	All right, if you don't help me, I'm leavin'.
	Ian	You're leavin'? Nobody stopping you. [R2], Jerel's leavin'. [laughs]
		[G1 and Ian say to someone off camera that Jerel said he was going to leave if he didn't get help. Though it is not shown on camera, it appears that Jerel has left.]
	R2	Okay, well, David, you've read the problem and, Ian, Ian [Ian has gotten up from the table], maybe uh you could play with David, and David will keep track of the score…
	David	No, Ian's right there. [Ian returns to his seat, possibly with Jerel.]
	R2	So, Ian, before you start playing, before you start playing …
	Ian	All right, look, here, let me, let me describe it, 'cause it took us

	like half an hour just to figure out what this is. Lemme show you.
R2	There is something there he has to …
Ian	All right. All right, so you don't want me tellin' him?
R2	No, I want to tell him, go ahead.
Ian	All right, look. You see how when you roll the dice, right? You get, all you got, you see how all those same numbers are around? That's a number and that's a number, you gotta add that, then you get, figure out what number. That's, that's it.
David	What? How do you know which one it is, like?
R2	Ah, that's a good question.
Ian	Look. Look. Roll it then you get, you see a 4 all around on the bottom, right? 4, and then 2 all around. Then you add that, you get 6. And that goes to Player B.
David	Ohhh.
Ian	And you go up to 10.
Jerel	Ohhh.
R2	But before you start, do you think that this is gonna be a fair game?
boys	No.
R2	Why?
David	Because …
Jerel	Because you always get the same number around.
R2	Well, by fair … [Jerel gets into a dispute with Ian] Jerel, this is what I, guys …
Jerel	Play me, I will get 2, 4, 6. I'll get 2, 4, 6, that will be a tie. No, no. Player B will win.
R2	Throw the, throw the dice again.
	[Jerel cups the dice in his hands, blows on them, and tosses.]
R2	Ian now, now, don't touch the dice. Don't touch. Ian, I want you to verify something. Uh, Dante [looking at Jerel].
Jerel	I'm not Dante, bro!
R2	I know, you were sitting where Dante was. Jerel, what numbers came up?
Jerel	2 and 4.
R2	2 and 4. OK. Now, which Player gets that point?
Jerel	Uh, we both do.
Ian	No.
Jerel	What!?
Ian & David	B. B.
Jerel	Nuh uh!
David	4, if the sum is 4, 5, or 6, Player B gets one point.
Ian	4 and 6, you gotta add them. I just told you to add 'em.
Jerel	Shut up!
R2	He knew to have to add them. Ian, he knew what to do in terms of adding, but he didn't understand which player gets the point. Which player gets the point?
Jerel	Me.

	Ian	B.
	Jerel	Me.
	Ian	Player B. Are you B? Or are you A? [Jerel indicates that he is B.] Okay, then.
	R2	Okay. Are you gonna keep track?
	David	Yeah. I'm gonna keep track.
	Jerel	Yeah, keep track. I'm gonna go against Ian 'cause I'll rub it all in his face when I beat him.
	R2	No, you go against David. Ian is going, Ian worked on the problem yesterday, and he's going to, he's gonna watch you guys to see whether or not the same player … But before you, before you actually start, before you guys start, hold on one second, may I ask you a question? You have which numbers, the sums for Player A to win?
	Ian	0, 0, 5
	Jerel	5
	Ian	Oh. The sums for Player A …
	R2	Ian, this is a question just for them.
	Ian	2, 3, 7, 8.
	R2	Okay. That sum goes to, those points go to Player A.
	Ian	Here, I'll put it into kid language, Jerel. All you gotta do … I'm just saying …
	R2	Ian, Ian, no no, hold on. Player B, which sums go to him?
	Jerel	4, 6, I mean 4, 5, and 6. [slight pause] That's cheatin'.
	R2	[to David] Now, do you think this is gonna be a fair game?
	David	Oh no because this one got 4 and that one got 3. He's got three uh …
	Jerel	No, no 'cause you can't get this. You can't get this one whole number left.
	David	What, 5?
	Jerel	Yeah, you can't get 5.
	R2	Why do you say that you can't get a 5?
	Jerel	'Cause I gotta spit. [gets up and walks away]
13:36	David	Man, look, they got, it's 4 numbers right there and he only got 3 numbers. So he got 4 chances of getting' 'em and he only got 3 of getting' 'em.
	R2	So, is the game fair?
	David	No.
	R2	And it's in whose favor?
	David	A
	R2	It's in A's favor?
	David	Uh huh.
	Ian	I gotta write that down. David said it's not fair.
	R2	And write down why he says it's not fair. Say again, David, why you think it's not fair.
	David	Because …

	Ian	[writing] in favor of A, of Player A.
	R2	All right, so why isn't it fair?
	David	He's not fair because Player A got 4 chances of getting his number compared to 3 chances of getting a number.
	R2	And David, is it for you the case that each of these numbers are equally likely? That have the same likelihood of coming up with these sums as… And so that's why, for Player A, Player A has 4 numbers and Player B only has 3, it's in favor of Player A?
	David	Um humh.
	R2	Uh, Ian, when you played the game yesterday …
	Ian	It was challenging, it was stupid, and I liked it.
	R2	Um humh. But which player won more often?
15:01	Ian	I can't say that.
	R2	Okay. We're gonna hold that back.
	Ian	Hold up. It's ….
	R2	No, hold that back. That's a good idea. 'Cause we're gonna see whether or not their suggestion …
	Ian	So, David, which, which player you think gonna win?
	David	A.
	R2	So why don't the two of you start playing. Well, play against David while Jerel is out. And uh, David, you're gonna keep track, right? Okay. Oh, by the way, Ian, tell him, how should he keep track? What are the things that he has to …
	Ian	Oh. Add the um, you know how you got the um, you gotta add the addition sentences and the numbers that you get.
	David	What?
	Ian	Put D and I and put …
	David	D and I?
	Ian	Yeah, well make it long, like this. [Draws a long line on his paper.] So you got enough space to write the, so you got enough space to write the sentences like 4 + a equals 7.
	David	All right, I got it.
	R2	Now who's Player A and who's Player B?
	David	I'm Player A.
	Ian	I'll be B.
	R2	Okay. All right. So you, David, by your logic Player A should win, right?
	David	Uh huh.
	Ian	Yep. Oh, Jerel's back. Jerel's Player B.
	David	Oh well, we started the game already.
	Jerel	Champ is here. I get to play against Ian, right?
	David	Jerel, no.
	Jerel	I want to play against Ian.
	David	You got to wait.
16:50	David	[rolls the dice] Five.
	Jerel	Wait, how you getting' five? Two …

	David	2 + 3
	Ian	What’s 2 + 3, Jerel? I’d really like to know that.
	Jerel	2, 2, 6!
	Ian	That’s not a 6.
	Jerel	Y’all retarded, y’all.
	David	You gotta add one of them and one of them.
	Ian	One of them, one of them.
	Jerel	Oh, I thought you was addin’
	Ian	[pretends to slap Jerel] That’s for bein’ dumb.
	R2	No more, no more hands, no more hands on each other.
	Ian	No, David, that’s my point!
	R2	Is that right?
	Ian	Yeah, that’s right. That’s my point.
	R2	You’re Player A?
	Ian	[to Jerel] Don’t even think about getting close to me.
	R2	Okay, Ian. What did I, what did I ask you?
	Ian	He keeps tryin’ to hit me!
	R2	Ian, Ian, what did I ask you?
	Ian	All right, [inaudible]. All right, three. That’s your point, David.
	R2	So what came up? Tell him what numbers came up.
	Ian	2 and a 1. Here, Jerel, you could take my spot.
	R2	How are you, how are you reading it?
	Ian	[talking over a PA announcement] 2 and 1. Jerel, you could take my spot.
	David	[rolls dice] Ooh, ooh, ooh 6. That was 6.
	Ian	That’s Jerel’s point.
	Jerel	How’s that my point?
	David	That’s mine. That’s yours.
	Ian	If you add the sum, right, 4, 5, or 6 is Player B. It’s your point. [refers to the paper that states the problem]
		[The boys continue to play. Jerel gets a few points in a row.]
19:10	Ian	He’s killin’ you, boy.
	Jerel	I’m killin’ you, boy.
	David	I’m gonna come back, though.
20:00	Jerel	There’s no way you could win. I got 1, 2, 3, 4, 5, 6, 7.
	David	I could still catch up.
	Ian	Yeah, I caught up with Dante yesterday. He had 7 and I had 1. I came up and got back at 6, 7-6.
	David	7-6, look at that. 4.
	Ian	You ain’t gonna win. You ain’t comin’ back.
20:18	Jerel	I think this game is fair. [to Ian] It is fair, right?
	Ian	No.
	David	No it’s not. Do it look like it’s fair, Jerel? Jerel, does that look fair?
	Jerel	‘Cause I’m winnin’.
	Ian	Just ‘cause you winnin’ one game don’t mean you gonna win all of

		‘em.
	David	Jerel, Jerel, does this look fair?
	Jerel	Ye-, well you got, you got these numbers.
	Ian	No, you got them numbers!
	David	I’m A!
	Jerel	I got these numbers.
	Ian	Look, whose … Blue is you, no red is David, right?
	Jerel	Oh, it’s not fair.
	Ian	Then blue is who?
20:46	G1	So why did you change your mind to it’s not fair?
	Ian	‘Cause he understood the numbers.
	G1	Jerel, you tell me. Wait, don’t roll yet. So why’d you go from fair to not fair, Jerel? Jerel, Jerel. Why’d you change your mind from fair to not fair?
	Jerel	Because of, it’s very hard to get 1+2, I mean 1+1. It’s hard to get two ones or a 1 and a 2.
	G1	Why is it? Why?
	Jerel	I dunno. It’s just hard like that. But you can get 7 and 8. 7 and 8 is like a good number to get.
	G1	Why is it a good number to get?
	Ian	I’m not gonna say nuthin’.
	G1	What do you mean by good?
	Jerel	Because you can get 4 and 4, 3 and 4. No, no, no, no, ‘cause they only got one multiples. Yeah, son.
	G1	Wait, hold on a second. Can I ask you …
	Ian	Maybe you should make a multiple chart, Jerel.
	Jerel	I need your help, bro.
	Ian	Okay, fine.
	G1	Ian, Ian, Ian, Ian. Remember earlier how you wrote down what David’s prediction was? Would you write down what Jerel just said? So Jerel, Jerel, hey Jerel, Ian’s gonna write down your words because earlier he wrote down David’s words when you weren’t here. So say what you said so he can record it.
	Jerel	It’s not fair.
	Ian	All right, what else?
	Jerel	Not fair because, because they only got one multiple.
	Ian	Who’s they?
	Jerel	They, the number, uh, Team A only got …
	Ian	Player A.
	Jerel	Player A, bro, don’t correct me. Player A only has, uh, one combination, can you spell combination? [chatter] [Ian writes: “Jerel – Not fair ‘cause the number for A has 1 combination.”]
22:44	G1	All right, go ahead. So Jerel, is he getting your words down? So he says, is that it? What do you mean …
	Jerel	One combination to get these numbers.

	G1	To get what numbers? What's these?
	Ian	What's these?
	Jerel	Uh, Player A numbers.
	G1	Which are?
	Jerel	2, 3, 7, and 8.
	G1	Keep it in your own words.
	Jerel	Well, one or two combinations.
		[Ian adds on to what he has written: "Jerel – Not fair 'cause the number for A has 1 combination to get A numbers (2, 3, 7, 8).
	G1	Well that's good, Jerel? Jerel, what Ian just wrote for you, that's good? That represents your reasoning? What you just said.
	Jerel	Yeah. Put or 2 or 3; 1, 2, or 3 [pointing to where Ian has written "1 combination"] And the other ones got like, they got like 2, 3, 4 …
	Ian	All right, go ahead, keep playing. See if you're [inaudible]
	David	I got me my little comeback.
	Jerel	Go ahead. Do your comeback, sir.
	Ian	David, if you lose, I'm a laugh at you, 'cause you say you got [inaudible].
	Jerel	Uh, that's my piece.
	Ian	7. That's 7.
	Jerel	Oh, that's David's piece! Ah!
	Ian	That's yours.
	David	That's mine.
	Jerel	That's David's.
		[The boys continue playing the game, which they had left off with Jerel in the lead.]
24:01	Ian	Dang David, you aren't comin' back.
	Jerel	That's 9. That's 9.
	Ian	1, 2, 3, 4, 5, 6, 7, 8, 9.
	David	3+2?
	Ian	That's 9. 9, he need 1 more to win. David, you better hope your comeback …
	Jerel	Aw, Dave, you're about to give it to me. How much you got? I bet you, one dolla, one dolla, one dolla. Dave about to give it to me.
	David	7
	Jerel	Nah, that don't count, bro. That was off the board.
	Ian	Shut up, bro. Get outta here. That's 7!
		[After a minor dispute, Jerel rolls a 6 and wins the game.]
	Ian	Jerel, you very cocky, though.
	Jerel	You wanna play me?
	Ian	Yeah!
	Jerel	I get Player A.
	Ian	No.
	Jerel	All right, I get Player B.
	G1	So what happened?

	Ian	He got too cocky, so he lost his point.
	G1	Is the game over, or is it still goin'?
	Ian	No, they're still goin'.
	Jerel	Uh uh. I beat him.
	Ian	No you didn't.
	Jerel	I went up to 10, I won with 10.
	Ian	[points at score sheet] There you go.
	Jerel	[to Ian] So I'll play you. I'll get Player B.
	Ian	I don't want Player A.
	David	Player B won.
	Jerel	Y'all want Player B.
	Ian	No, I'm getting' tired, Jerel, so let's go.
	Jerel	[to David] Player A or B, for me or Ian.
	G1	You still stickin' with your prediction of if it's fair or unfair?
	Jerel	Yeah.
	Ian	I'm Player B.
	Jerel	I'm Player B.
	David	I-J, I-J. J-I.
	G1	Who's Player A, who's Player B?
	David	Jerel.
	Jerel	[rolls dice] Oh, that's my point. Give it to me, son.
	David	Oh, look. A-I. B-J. A-I, B-J.
25:34	Jerel	I got one point. I got one point.
	Ian	My point, my point.
	Jerel	It don't take me like 15,000 turns to get a point. That's not me! I got 1 + 2, bro.
	Ian	A-I. All right, look. [takes score sheet from David] I got Jer-, know what? I got 4 + 2. Oh look.
	David	No, it's supposed to go under Jerel's.
	Jerel	Nuh uh.
	David	Yes it is, 'cause it's 6.
	Jerel	You messed it up.
	Ian	You messed the whole thing up.
	Jerel	I'm Player A now. And I had a point.
	Ian	All right, Jerel had a point. He got 7, right? And then I had a point. I got 6. All right.
	Jerel	Dave, I remember when you do that. [inaudible] I-B-I
	David	Sorry.
	Jerel	You could have killed me, though. I didn't have 4 + 3.
	G1	So who's A and who's B here?
	Jerel	Ian A.
	Ian	You A!
	David	Why don't you put A first?
	G1	Wait, can I ask one more question? What about over here, who was A, who was B?

	Jerel	I was B. D-A, and I was J-B. [rolls dice] That's my point, give it to me.
	David	That's Jerelly's? What was it?
	Ian	3. 1 and 3. 1 and 2.
	Jerel	I can tell I'm going to Vegas when I grow up. That's my point, too, 1 and 2, give it to me. I tell I'm going to Vegas when I grow up, son! [rolls dice with a flourish] Ah, give it to me.
	Ian	5, my point.
	Jerel	Dang! You got lucky, y'all.
	Ian	Don't get too cocky.
	Jerel	All right, that's my point, that's my point.
	David	What is it? 3 + 4?
	Ian	7. You still think the game unfair, Jerel?
	Jerel	[rolls dice] Noooo! [perhaps in response to the outcome – not in his favor]
	Ian	You still think the game unfair?
27:31	Jerel	Unfair?
	Ian	You still think it's unfair?
	David	That's an Ian get. Pro at this.
	Ian	I ain't a pro.
	David	Yes you is.
	I & D	[unclear]
	Jerel	Ah, that's Ian's point. What score?
	G1	Ian, did you just ask Jerel a question?
	Ian	Yeah. Does he still think this game is unfair?
	David	4 – 4
	G1	What do you think, Jerel?
27:48	Jerel	I think it's fair.
	G1	You think it's fair?
	Ian	Now you think it's fair!
	G1	What happened? Why'd you change your mind?
	Ian	Again!
	Jerel	Because, I changed to Player A and I did, I'm gettin' as much, I'm gettin' as much number rolls, I'm gettin' the same amount of rolls with my numbers comin' up as Player B. Yeeess!
	G1	So Ian, do you want to change what, I mean do you want to change what Jerel said?
	Ian	No.
	G1	Jerel, you want him to change what you said?
	Ian	No, he keeps changing his mind.
	G1	You don't have to cross it out. Jerel, you just put change your mind. So Ian is documenting that. Now you think it's a fair game, because …
	Jerel	Because, I'm Player A now, and it's 4 to 4, and I got …
	Ian	has just as good of a chance as B.
	Jerel	Yeah. Has just as good a chance as B.

		[Ian writes: "Change – It's fair 'cause A has just as good of a
		[play continues]
29:03	R2	What have you guys come up with so far?
	Ian	Nothin'. Jerel's learnin' that it's fair.
	R2	What's fair? The game so far is fair?
	boys	Yeah.
	Ian	First he said it's not fair, then he said it's fair, then he said it's not fair, then he said it's fair again.
	David	Make up your mind.
	Jerel	Didn't I say it all that many times!
	Ian	You said fair, then not fair, then fair. He swears he's goin' to Vegas.
		[game continues]
	Ian	What's the score, Dave?
30:00	David	He got 7, you got 6.
	Jerel	Ah, that's my point. I got 8.
	R2	So what was that roll?
	Ian	He got 2 and 1. [1 and 1 is also said by someone]
	R2	Not 1 and 2?
	Ian	You asked me that yesterday.
	R2	Well I'm asking that …
	Ian	Don't, don't let him use psychology on you.
	Jerel	It's the same thing, he just mixin' it up.
		[game continues]
30:34	Ian	It's 8-8.
	Jerel	8-8! Dang, I gotta come back.
		[Ian wins the next roll. He blows on the dice and rolls, winning the game.]
	Ian	I win.
	Jerel	Nah, you cheated.
	R2	Which player?
	Jerel	You cheated! You must have scuffed the dice or somethin'. You cheated. You scuffed it.
	David	Player B won both times.
	R2	You want to try a different pair of dice?
		[Jerel continues to argue about cheating and scuffing the dice.]
	David	I'm playing against Jerel. I'm playing against Jerel.
	Jerel	All right. Come on, Ian!
	R2	That's all right.
	David	I'm playing against Jerel. Ian, I'm playing against Jerel.
	Jerel	All right. I'm takin' Player A.
	R2	Okay. You're Player A, and you'll be Player B?
	David	Yes.
	R2	Okay.
	Jerel	Ian cheatin'.
	R2	Who's keeping score? Ian, do you want to keep score? You want

		to write down their predictions?
	David	All right, Jerel. I'm Player B, Jerel. Jerel, Jerel, I'm Player B.
	Ian	If you want to beat him, if you want to beat him, just do like this. [off camera]
	Jerel	You scuffed the dice.
	Ian	I didn't sc-. All right, give me that dice. Give me the dice. Okay, then. Jerel's a sore loser.
	Jerel	No, you scuffed the dice.
	R2	By the way, before you start playing, let me say this. Remember that it's not really a competition.
	Ian	Yes it is.
	R2	What we're trying to do is understand, guys, we're trying to understand whether or not the game is fair or not, okay?
		[David and Jerel begin to play. Jerel continues to accuse Ian of scuffing the dice.]
33:02	Jerel	Both y'all be cheatin'. That's my point.
	Ian	Jerel, you're just a sore loser.
	David	Yes, Jerel, you just can't handle it.
	Jerel	I don't like losin'.
	Ian	You are a loser, you lost to me.
		[Play continues. Jerel accuses David of cheating.]
33:57	Jerel	I'm up one, right?
	Ian	Yeah.
	Jerel	All right, that's my point. Gimme that, young bro.
34:25		[The score is 7-5, in Jerel's favor (Player A).]
	Ian	Jerel, you just lucky. You rolled the same thing three times.
	David	How come you keep rollin' that, Jerelly?
35:00	Jerel	I won. I won. I won. The champ is here.
	David	How much I got?
	Ian	You got like 6. You can't be the champ.
	Jerel	I told you Ian scuffed the dice.
	Ian	I didn't scuff 'em. You kept rollin' the same thing like a cheater.
	David	I don't right how you kept getting all those 1 + 1's.
	Ian	[to G1] Look, he got the same thing, 1, 2, 3 …
	Jerel	David rolled 'em, bro.
	David	No, I didn't. You rolled after that. You rolled all the 1+1's.
	G1	So what happened? Wait a second, lemme, can I ask you guys some questions first? [chatter]
	Ian	Ask them some questions.
	G1	Can I ask all you guys some questions?
	Ian	Nah, I did this yesterday.
	G1	Okay, so in the first game, who won?
	Jerel	Ian.
	Ian	No, him [points to David].
	G1	No, tell me, A and B?
	David	B. B.

G1	Who won in the first one?
Jerel	You mean in the first one, the very first one?
David	B. B.
Ian	Me and Jerel.
David	It was me and Jerel.
Jerel	Yeah, and I beat David.
David	He won. That was B, he was B.
G1	How about in the second game?
David	B.
Ian	Me.
I & J	B. Player B.
G1	And then the third game?
David	A
Jerel	I won with A.
G1	So what do you think, is it fair or not fair?
Jerel	Yeah.
G1	You think it's fair? What do you think of all these numbers that are occurring here? Is the other side …
David	No, 'cause he kept getting 1+1.
Jerel	No, bro. Ian scuffed the dice. That's how he beat me.
Ian	Okay, get another pair of dice.
Jerel	No, we just switched the dice, bro. You trying to get to the same dice that you scuffed! [unclear]
G1	Wait, I have a couple more questions.
Ian	Were the last dice I had white? No.
Jerel	All right, change that to, uh, change that to [inaudible].
G1	Ian, Jerel, I have a couple more questions, is that okay?
David	Black and white. [he has one black and one white die]
G1	Could we … What do you think of all these numbers that are showing up here? All of these combinations.
Ian	You can answer those questions, 'cause I did this already.
G1	What do you think of them?
Jerel	They some good numbers.
G1	What do you mean by good numbers?
David	He was cheatin' 'cause he kept rollin' …
Jerel	They almost all got 4 in them.
Ian	Almost all, almost. [chatter]
G1	So what do you think about these combinations? How come you're always running … Is 4 and 3 the same thing as 3 and 4?
Jerel	Yeah.
G1	It's the same thing?
Jerel	Um humh.
G1	Okay. So you ready, you think it's still fair?
Jerel	I wanna play Ian.
Ian	No you don't.
Jerel	I wanna play Ian, that's who I wanna play.

	David	Huh, black and white. Pick black and white.
	G1	So who's playing this time?
	Jerel	Me and Ian.
	Ian	David and Jerel.
	Jerel	Me and Ian. Ian, I wanna play you.
	Ian	I'll beat you up. You can't retire until you become the best. I can retire.
	David	Me and Jerel, me and Jerel are playing.
		[The boys continue to argue about who will play. Jerel wins the argument; he and Ian will play.]
	G1	So who's Player A and B?
	Ian	I'm B.
	Jerel	Ian B, Ian B.
	G1	Why do you want to be B?
	Ian	Because B is rugged.
	G1	What do you mean by rugged?
	David	Better than A.
		[Ian and Jerel begin to play. Ian warns Jerel, "If you get two in a row, then you scuffed it."]
	R2	Why are you guys playing with two different colored dice?
	Jerel	They swore on the last one that I scuffed the dice and I beat David that bad.
		[The boys continue playing.]
38:23	R2	In about 5 minutes we're going to have presentations, so …
	Ian	They're not ready to present.
	R2	They're not ready. Okay.
	Ian	They didn't put it on no [reaches for a transparency] this. David, you continue playing while I write this down.
	Jerel	I'm about to finish against David?
	Ian	Yeah. I started you off well, David. If you can't beat him now, you suck. David, if you can't beat him now, you suck.
		[David and Jerel continue the game.]
39:24	Ian	David, you killin' him?
	Jerel	No, he only got …
	David	5-3, 5-3. [inaudible]
40:43	Jerel	Ah, I tied up with you, boy!
	David	[counts ups the score] Seven. [implication that it's a 7-7 tie]
	Ian	I don't know what to say, David.
		[Jerel and David continue to play.]
41:07	Ian	[as he writes on the transparency] Not fair and …
	Jerel	I didn't say not fair. I said it was fair!
	Ian	That's the first thing you said.
	Jerel	[after getting another point, to David] Who the champ? Say my name.
	Ian	[after Jerel gets another point, now 10-7] He becoming too cocky, David. You got to teach him a lesson.

		[Jerel and Ian argue]
	David	Jerel, the game is over. [Jerel, as Player A, wins.]
	Jerel	It is?
	Ian	Oh my God, you suck, David. You suck.
	Jerel	And what did you have, like a 5-2 lead?
43:25		[David and Jerel begin another game. David is Player B.]
44:44		[The boys discover some discrepancies in the scoring. Ian takes the score sheet and makes corrections.]
45:41	R2	Okay. I think we're ready now for presentations.
		[Ian offers to go first with his presentation, but Kianja and Brionna are selected. Kianja and Brionna go to the overhead.]
		[David & Jerel continue playing. They accuse one another of cheating. It appears that David (Player B) is winning, 8-4.]
46:54	R2	David, and Jerel. Jerel, all right. I want you guys to listen carefully to what, to what Brionna and Kianja have to say, okay?
	Jerel	Ian, you think it's unfair?
	Ian	No, I'm not tellin' you what I think. That's is what y'all think. This is the right paper right here.
	Jerel	Yeah, you think it's unfair!
	Ian	No.
	R2	Okay, we're ready to hear from Kianja and Brionna. [cut] We're all, I think we're all ready. David.
		[Brionna, off camera, reads the transparency to the class. It is difficult to hear her.]
48:16	Kianja	There are 10 combinations that Player B could win by. There are 6 combinations that Player A could win by.
	R2	Please, for one second, let's go back to that. Did everyone understand what they're saying here?
	Ian	Yeah, I do. I do. Me. [waving his arms over his head]
	R2	Hold on here. All right, what, Ian, Ian, Ian, you say you understand what they're talking about. Could you tell the rest of us what you understand from what they said.
	Ian	All they're saying is like Player A got 4 combinations and Player B got 6.
	R2	I don't think that's what they said. Is that what they said?
	Ian	Yeah that's what they said.
	Kianja	What's what we said?
	Ian	Player A got 4 combinations and Player B got 6. That's it.
	Jerel	No, no they said …
	Ian	[raising his voice] I didn't ask you!
	Kianja	No, that's not what we said.
	Jerel	They said 10, you dunce.
	R2	Listen carefully. That's why … Ian, Ian, I want you to listen carefully because I think that what they've come up with is different than yours. So you wanna hear what they have to say.

		All right. Would you go through that again, because I don't think everyone's understood.
49:21	Brionna	This game is not fair because there are more [inaudible] that will equal 4, 5, and 6. There are 10 combinations that Player B could win by and only 6 combinations that Player A could win by.
	R2	All right. This is, I think, very interesting what they're saying is that Player, there are how many combinations for Player A?
	Voices	6
		But you got to remember …
	Ian	And together they're 10.
	R2	No, they're saying that there are 6 for Player A …
	Ian	That's what I said!
	R2	And 10 for Player B. And I think you're [Ian] saying something different.
	Voice	You said 4 for A and 6 for …
	R2	Okay, so let's let, you'll go on, and then we'll hear from Ian. Go ahead.
	Ian	Huh? I could go?
	R2	No no. We're gonna let them continue.
50:19	Brionna	How could you make the game fair? We could make it fair by having Player A get one point for rolling 3, 7, or 5 and Player B getting one point by rolling a 2, 4, 6, or 8. This would be even because then there would be 2 ways to get 3, 2 ways to get, 2 ways to get 7, and 4 ways to get 5, for 8 ways in all. For Player A, there would be 3 ways to get 4, 3 ways to get 6, and 1 way to get [inaudible], 1 way to get 8, and so, which would equal 8 ways, which would be equal to Player B.
	R2	So they came up with a, a game that they say is fair, so that each Player, A and B, each have how many points? how many different combinations? Ian?
	Kianja	8
	R2	Ian, did you say 8? I didn't hear you.
		[Ian, Jerel, and David do not appear to be attentive. Jerel is squeezing his wrist and the other two are looking on.]
	Kianja	8
	R2	8 for each? Okay.
	Jerel	I have a question.
51:40	R2	Go ahead. You have a question. Go ahead, Jerel.
	Jerel	Oh. But look, you said that uh Player B has more combinations, oh, but uh Player A has more numbers.
	Ian	I've been sayin' that. You read my paper, didn't you?
	Jerel	No! Bro, I didn't read your paper. You wanna fight?
	Ian	Yeah!
	R2	Jerel, Kianja, Kianja, do us a favor. Would you repeat the question you think Jerel's asking you.
	Kianja	What you wanna know is, how is it that Player B is winning when

		Player A got more numbers?
	Jerel	As Player A, I had won.
	Kianja	Is that what you're saying?
	Jerel	I won. I won the champ-
	Kianja	I don't care if you won.
	David	You won once against me. You won once against me, Jerel.
	Ian	You're not the champ!
	R2	Jerel, Jerel, she's asking whether or not she understands your question.
	Kianja	I don't care if you won.
		[Jerel and David argue. Jerel waves his elbow toward David.]
	R2	Jerel, Jerel, let Kianja know whether or not she's understood you.
	Kianja	Is that your question?
	Jerel	What?
	Kianja	Okay, is your question, you wanna know why Player B won, right, Player B has the advantage and Player A has more numbers?
	Jerel	Not exactly.
	Kianja	Just say yes.
	Jerel	All right, yes, yes, yes.
	R2	Justina, [inaudible] do you understand the question now that Kianja's going to respond to?
	Voice	Yes.
	R2	[inaudible] repeat the question?
	Kianja	[shakes her head to indicate no]
	R2	Okay.
		[Jerel, Ian, and David are chatting.]
	Kianja	Are you gonna listen?
	Jerel	Yeah, I'm listenin'.
	Kianja	At all. All right. Um, they won 'cause, like I say, they won …
	Voices	Be quiet!
	Ian	That's not a good explanation.
	Kianja	I don't like that word.
	Ian	Not a good explanation. I don't like that.
	T5	Are you saying that because you don't understand it or because you're just [inaudible]?
	Ian	I understand it, but they said, she said …
	Kianja	He's trying to annoy me.
	Ian	She said 'cause they just went.
	Kianja	No I didn't. I was trying to explain, but you don't want to sit here and listen.
	R2	All right, just move off to the side a little bit so we can see your paper, okay?
54:11	Kianja	They won, um, they don't have a lot of ways to win. That's why …
	Jerel	But they got more numbers!
	Kianja	So what?

	Ian	Like, like she's tryin' to explain. Just chill!
	R2	She's gonna explain.
	Kianja	Like 8, right? 8 and 2, it's only two, I mean one way that you can get 8 and 2.
	Jerel	Hold up. But look, so you saying …
	Ian	Yeah, I gotta agree with you. [to Jerel] Just look at the chart, look at the chart. [shows Jerel his paper]
	Kianja	There's only one way you can get 8 and 2. 1+1 is 2 and 4+4 is 8, and that's it.
	Jerel	All right, all right, all right, whatever.
	Kianja	That's it.
	R2	Yeah, but I, you understood her?
	Jerel	Yeah, I understood it.
	R2	Any other questions? for Kianja?
55:02	Jerel	[gets out of his seat] I want to go up next. [Ian gets up.]
	R2	There's something you had here, a key point. Can you talk about the key point?
		[Kianja is off camera and her response is not seen or heard. Jerel, Ian, and David approach the overhead and put up Ian's transparency.]
	Jerel	All right, this is what Doobid put. Doobid put not fair …
	R2	I'm sorry, Jerel. I think you're standing in their way. Stand on this side. Jerel? If you stand on this side you won't be in anybody's way. [Jerel moves to the side.]
	Jerel	All right, Doobid put not fair because, not fair in favor of Player A. A has 4 chances and B has 3.
	R2	All right, so that was their, that's what they thought about the game … [chatter] Kianja, this was, this was their prediction before they started playing the game. OK. Continue.
	Jerel	I put not fair in the game because the numbers for A has 1, 2, 3 combinations to get A numbers 2, 3, 7, and 8. That don't make no sense! [claps his hands]
	Ian	That's what you said!
	Jerel	Oh.
	Ian	Don't step up to me.
	R2	Well what, what, make the correction, Jerel. Jerel, Jerel, make the correction.
	Jerel	All right. This is what I originally said. I said it's not fair because [walks over to Ian and shoves him] it was not fair the game because num- Player A has, has to only get 1, 2, and 3 combinations but then, oh, Player B had to get the more combinations in it. And then, when I started playin' the game, I changed my mind because, because it's fair because, because, what?! His just as good as …
	Ian	Has just as good a chance
	Jerel	You know what? has just as good a chance as B. Because, when I

Time	Speaker	Transcription
		was playin', and I was rollin' my dice, I beat, I beat David for Player A.
	David	He beat me once.
	Jerel	No, and then I beat Ian.
	David	He beat me once, though.
	LP	Jerel, I can't understand a word that you say.
	Ian	Let me explain. Let me explain. He'll know what I'm talkin' about. All right, look, David said it's not fair 'cause in favor of Player A. A has 4 chances and B has 3. So, that's why it's not fair. But Jerel said that it's not fair because the number for A has 1, 2, 3 combinations to get A's numbers 2, 3, 7, 8. But then he changed his mind 'cause I beat him, and he said it's fair because A had just as good of a chance as B. That's it.
		[banter and laughing between Ian and Jerel]
	G1	Do you have another slide, Jerel? Ian, do you have another slide you want to display?
	T5	Ian, Ian, another slide? No?
	R2	All right, do you want to explain this? Okay. Ian, Ian, Ian, do you want to explain this? Excuse me. One second. Hold on. [Ian has placed the slide he made with Dante the previous day on the overhead.]
	Kianja	Excuse me,[R2]. I can't see.
	R2	I'm gonna ask David and Jerel to have a seat while Ian's explaining.
59:15		[end of CD 121C]
		[NOTE: 122C duplicates 122B from another angle.]

Date: 5 May 2005 Grade 7
Location: Hubbard Middle School
CD: ROLE 122A
Transcribed by: Kathleen Shay
Verified by: Jeremy Milonas

Time	**Speaker**	**Transcription**
0:13	R4	This is an interview with Chris uh and uh [G6] as his partner. Um, about, um some of the probability. We'll see what we think about this, um and, and Chris, what I'm gonna let us do, as we're talkin' about these ideas, is also give you an opportunity to think about um what we were doing yesterday and see if you, do you remember playing any of the games with the dice last year?
	Chris	Yeah.
	R4	What do you remember?
	Chris	I remember, um, that we had to, I forgot, it was something about rollin' dice, but I don't really remember everything.
	R4	Um humh. It may come back to you as we think it … This is a

		different game. We used the regular kind of dice?
	Chris	Yeah. [nods]
	R4	How many sides, how many faces does it …
	Chris	Six.
	R4	Yeah. And so if you put a number on each side it'd be one, or there were dots, actually, it would be 1 through 6?
	Chris	[nods]
	R4	And so if you tossed two together, and we're thinking about the sum of the two, what sums could you get?
	Chris	You could get, the most you could get is 12.
	R4	And the lowest?
	Chris	The lowest you could get is 2, 2.
	R4	Um humh. Sure, and you could get everything in between. And if you remember, there was a game about that, uh, where we threw two dice and added 'em together. And, what if you were playing a game so that you got points, uh, and maybe let's, why don't you read this one for us so that, G6 has never seen this either. This time, where instead of using, instead of using the kind of dice we used last year, we're gonna use this kind of dice. What would you, how would you describe the dice?
	Chris	A pyramid.
	R4	A pyramid. Yeah, and so it has how many faces?
	Chris	Four.
	R4	Uh huh. And so can you tell, for instance, [rolls die] there, what's the number that I just tossed?
	Chris	[smiles and shrugs]
	R4	If you had to guess, G6, what do you think?
	G6	I would guess it's the number that's showing upright. It's the same on all three sides. On all three exposed sides. It's always a three.
	R4	So it's a three. And so [tosses another die] what's that one?
	Chris	Ummm, four.
	R4	Yeah. Uh, okay. And so if you tossed two of 'em [tosses two dice], and, and I asked you what is the sum of 'em, what would it be?
	Chris	[looks at dice] Um [shrugs]
	R4	[pointing at one die] What's on this one?
	Chris	It's four. Two, six.
	R4	Sure. Does that make sense?
	Chris	[nods] Um humh.
	R4	Okay. So how 'bout read the directions for the game, both for the camera and for [G6].
3:31	Chris	Okay. What's that word say? Pyra -
	R4	Pyramidal.
	Chris	A pyramidal die has four sides. The number that is rolled is shown upright. Roll two dice. If the sum of the two dice is 2, 3, 7, or 8, Player A gets one point and Player B gets zero. If the sum is 4, 5,

		or 6, Player B gets one point and Player A gets zero. Continue rolling the dice. The first person to get ten wins, points is the winner.
	R4	Okay. You know what, just because we have so little room, could you sort of figure out a way to, to keep records and to remember. [Chris starts a score sheet with two columns headed “Chris” and “[G6]”.]
	R4	Okay. And now who’s, uh, do you think it’s a fair game?
	Chris	No.
	R4	Uh, why not?
	Chris	Because Player A gots 4 different numbers to roll.
	R4	Okay. We’re gonna let Chris be Player A, you wanna put that down those 4 different, is it numbers or sums or what?
	Chris	It’s sums.
	R4	Oh, okay. And which sums did Player A get?
	Chris	Player A gets 2, 3, 7, or 8.
	R4	Okay. You wanna put that down just so that we, I don’t, I don’t wanna …
	Chris	[writes “Player A 2, 3, 7 8” next to his name] And, I guess 4, 5, or 6. [writes “Player B 4 5 6” next to G6’s name]
	R4	Okay. And so you’re saying, who, who, whom do you think has the advantage?
	Chris	Well, Player A does.
	R4	Because?
	Chris	Because they got four different numbers, so you could add four different numbers up. Well, you could add two numbers to get four different kinds of numbers. But Player B only gots three.
	R4	Uh huh. Okay. Okay and so then what we, what makes something fair?
5:10	Chris	‘Cause I did four ways [?] . . .
	R4	I mean, what has to be true for it to be fair?
	Chris	Uh. [shrugs, shakes his head] I don’t know.
	R4	I mean, just in general, for a game to be fair, what needs to be happening?
	Chris	You could say, since you got only 7 numbers, you could say if either one gets 3 different numbers, 3 different numbers, and that one number maybe nobody gets a point.
	R4	Okay. Because to be fair means that [pause] you’re always gonna win? [smiles]
	Chris	[shrugs] You never know.
	R4	But to be fair, what would have to be true?
	Chris	To be fair? Well then, um, not only one person could like, well you could say like Player A wins 5 games and Player B only wins 1 game. Right there you’re gonna know that it’s not fair. Or you never know because Player B might be able to win other games too.

	R4	Yeah. But it needs to be sort of evened out?
	Chris	Um humh.
	R4	Okay. What I'd like for you and [G6] to do is to play for a little bit and figure out, sort of keep a record of what you're doing. Okay?
	Chris	[nods] Okay.
	G6	All right. So we'll play to 10.
	R4	Yeah. First person gets 10, wins the game.
	G6	So, will we alternate who rolls? It doesn't matter to me.
	Chris	[shakes head] It doesn't matter.
	R4	Sure. I see that Chris co-opted A.
	G6	Oh, maybe, maybe I should do over.
	R4	So you might take turns. Just do whatever.
	G6	All right. Let's see. [rolls dice: 4 and 3] What's the sum?
	Chris	Seven.
	G6	Um humh. So that's one of yours.
	R4	Can we sort of keep a record of what you do? [Chris writes on his paper.] Uh, okay.
	G6	Okay. You just got a point, so …
	Chris	I don't, I don't know…
	R4	Who did?
	G6	Player, Player A, that's you.
	R4	Player A? Okay?
	Chris	So that's one point, and it was a seven. He rolled a seven. [His score sheet is in two parts: the first part has two columns to keep a point tally. The second part has two rows to indicate the sums rolled by each player.]
	R4	Okay, now wait just, oh.
	Chris	So that's one point, but he rolled a se-… I'm just trying to keep score …
	R4	Oh, that's, that's fine. Okay. And, Okay.
	Chris	[rolls 2 and 2] That's 4. And I think he gets a point. [G6 rolls 3 and 2.] Five, he gets a point. [Chris rolls 1 and 1.] Two. [G6 rolls 1 and 2.] Three. [Chris rolls 3 and 4.] Seven. [G6 rolls 1 and 1.] That's 2. [Chris rolls 4 and 3.] That's 7. [G6 rolls 1 and 2.] That's 3.
	R4	Sounds to me like you had a pretty good prediction. [Player A is ahead.]
8:42	Chris	[rolls 3 and 3] That's 6. [G6 rolls 1 and 1] That's 2. [Chris rolls 4 and 4] That's 8. [G6 rolls 1 and 1. Chris marks the score, 10-3, and the sum,2.] That's game.
	R4	Okay, so you won. You're not surprised? You think one game is enough?
	Chris	I don't really know.
	R4	Let's test it.
	Chris	Okay.
	R4	And would you mind, trading, and letting the other person be,

that's game 1, okay?

Chris: So he'll be Player A and I'll be Player B. All right. [G6 rolls 3 and 3.] Six. [Chris rolls 3 and 4.] That's 7. [G6 rolls 2 and 1.] That's 3.

R4: According to your prediction, who should win this time, you or [G6]?

Chris: G6. [Chris rolls and writes the sum, 4.] [G6 rolls.] That's 7. [Chris rolls.] That's 2. [G6 rolls.] Six. [Chris rolls 4 and 1.] That's 5. [G6 rolls 4 and 1.] That's 5. [Chris rolls 2 and 2.] Four. Me. [Chris' point. The score is now 6-4 in favor of Chris, Player B.] [G6 rolls 4 and 1.] Five. Sweet. [another point for Chris] [Chris rolls 2 and 1.] Three. [G6 rolls 3 and 4.] Seven. [Chris rolls 3 and 3.] Six. [G6 rolls 3 and 3.] Six. [Chris rolls 3 and 1.] Four. That's me. [Score: 10 -6]

R4: Did you cheat? Did you cheat?

Chris: No.

R4: Okay, so, so this time, maybe they're even. Who knows? But right now you're tied. So what do you think, should we play again?

Chris: Yeah. We could play again. [Chris sets up the score sheet indicating that he will be Player A and G6 will be Player B.] Okay, so I'll be Player A again. [rolls dice] That's 4.

R4: You know, what would help me is if, right beside the 4, instead of the 4, how did you get that 4? What were the things that gave it to you?

Chris: 3 and a 1.

R4: Okay. Could you put that, let's think about that, too. [Chris writes 3&1 beside the 4.] Okay.

Chris: [G6 rolls 3 and 3.] That's 6. That'll be …

R4: That was Player B as well, right? Who's Player B?

Chris: He is. [rolls 4 and 2] That's another 6. [G6 rolls.] That's 5. [Chris rolls 3 and 1.] That's 4. [G6 rolls 4 and 1.] That's 5. [Chris rolls 3 and 1.] Four. [The score is 7-0 in favor of Player B.] [G6 rolls 4 and 4.] Eight. [Chris rolls 4 and 2.] Six. [G6 rolls 4 and 3.] Seven. [Chris rolls 2 and 3.] It's five. [G6 rolls 3 and 4.] That's 7. [Chris rolls 3 and 4.] Seven. [G6 rolls 2 and 1.] Three. B. [G6 rolls 4 and 1.] Whoa, I went twice. [There is an equal number of entries in the two rows where Chris recorded the rolls.]

G6: You did? Oh, you did?

Chris: I did.

G6: Well, if you rolled twice …

R4: Does it matter? who rolls?

Chris: I don't know, but …

R4: But doesn't it come out even, I mean, what do you mean?

16:03 Chris: I don't know. I think I didn't write it, or I just wrote twice.

R4: 2, 3, 4, 5, 6, no! Well, let's count.

	Chris	[to G6] You rolled a 3?
	G6	I can’t remember. I probably did.
	R4	Somebody did. Somebody rolled a 3. Yeah.
	Chris	No, I rolled a 3. I rolled a 2 and a 1.
	R4	What do you wanna, who do you think should have rolled?
16:32	Chris	He did. He could just leave that [inaudible]. So I ought to take that point away, then. [Chris changes the score from 5 points for Player A to 4 points, and he writes over the 3 on his chart of outcomes.]
	R4	Count them. Oh, so that one shouldn’t have come? You shouldn’t have done the 3, is that what you’re thinkin’?
	Chris	That’s a 5, so it’s a 4 and a 1. And he would’ve got that point.
	R4	What do you think?
	Chris	‘Cause I think I did go twice.
	R4	Yeah. Doesn’t matter about that. What do you think about the game?
	Chris	I think they both probably have equal amounts. There could be two, either have two different poss-, well, probabilities of getting …
	R4	[inaudible]
	Chris	Well it could have, this could have four different numbers if you, if you add two different numbers and you get four you could add them up and get these four numbers.
	R4	Show me what you mean.
17:28	Chris	Like say …
	R4	Write, write that down over there.
	Chris	[writing] Player A has 4 different sums that can be [pause] well, I don’t know how to say it, but to me, they have 4 different numbers that if you take the dice and you roll them and you get those two numbers, then you if add ‘em ….
	R4	Okay, show me. How do you get a 2? Is that what you’re saying?
	Chris	Yeah, yeah. If you get a 2 you have a 1 and a 1.
	R4	Okay. Okay, so, so to get a 2 you can have …
	Chris	So to get a 2, you get 1 and a 1. [writing]
	R4	Okay.
	Chris	To get a 3, you have a 2 and a 1.
	R4	Show me.
	Chris	[turns the dice to show 2 and 1] 2 and 1.
	R4	Okay. To get, to get a …
	Chris	To get a 4, you have a 2 and a 2, a 3 and, or a 3 and a 1. And to get, ooh not 4, uh, that’s for Player B.
	R4	Oh. What else …
	Chris	7, you would get 4 and a 3. And that’s probably it.
	R4	And 8?
	Chris	And 8 you would get a 4 and a 4, a, that’s it.
	R4	Okay. So that’s what you were just sayin’, that there were four

		different opportunities. What about the other guy? What about Player B?
	Chris	Player B has, so this one has 1, 2, 3, 4. Player B has 2 and 2, a 3 and a 1, uh, for 5 he has a 3 and a 2 or 4 and a 1, and 6 has a 3 and a 3, a 4 and a 2, and that's really it. That's 2, 1, 2, 3, 4, 5, 6 [counting the sums for Player B]. 1, 2, 3, 4 [counting the sums for Player A]. Right. 1, 2, [taps his pen two more times]. So this one only has four. So [pause], so it still isn't fair, so Player B will win.

Player A has 4 different sums that can be get a 2 1+1 3=2+1 7=4+3 8=4+4

Player B 4=2+2, 3+1
5=3+2, 4+1
6=3+3, 4+2

	R4	What about your experiment?
	Chris	For, yeah that, but Player 1 only won once. And Player B has 6 diff-, well, two for each. Two different ways to get each number. And Player A only has one for each.
	R4	Show me about the 3. How do you get a …
	Chris	3 is only 2 and 1.
	R4	Okay. You got a 2 and a 1. [pointing to dice]
	Chris	A 7 is a 4 and a 3 [turns dice to show 4 and 3].
	R4	Uh huh. Okay, if I rolled, and this one turned out 4 and this one turned out 3, is that different from the one you just showed me?
	Chris	No. It's still the same thing. You're still gonna get the same sum.
	R4	And you only have one chance to get a seven?
	Chris	[nods]
	R4	When you're rolling. If, if I did it this way [rolls a green and a white die, instead of two green dice], and it was a 4 and a 3 …
	Chris	It's still the same thing. 'Cause you have the same sum.
	R4	You absolutely do have the same sum. But now, are you telling me then that if that's [pause], how many ways are there to make, make a 2? [places 2 green and 1 white die on the table]
	Chris	One.
	R4	Yeah.
	Chris	Which is a 1 and a 1.
	R4	Regardless of … And to make a 3?
	Chris	A 2 and a 1.
	R4	Okay. [arranges the dice so that a green die shows 1, the other green and the white show 2] And so there's just one way to make a 3?
		[Chris does not respond.] [10 seconds of silence]
21:59	R4	And if you had a white 1 and a green 2, or a green 1 and a white 2, those are not different ways?

	Chris	[shakes head] It's, even though it could be different dice, different colored dice, different, maybe a 2 and a 1 or a 1 and a 2, it's still gonna add the same.
	R4	Okay. If I was gonna bet you $100 that you would roll a 2 before I rolled a 3…
	Chris	Umm, both of 'em have the same probability, which is only one way you could get it, well, [looks down, takes a breath] I don't really know.
	R4	What do you mean?
	Chris	[pause 7 sec.] What's the …
	R4	[gets up and speaks to someone off camera]
		[apparent break in the action]
22:59	R4	[There are a white and a green die at one end of the mat, and a white and a green die at the other end.] And you can actually be rolling at the same time, if you want. And you can, but you gotta keep score, so maybe if you'll keep, here's another piece of paper. Okay. Now, um, it may take a little bit longer this time because we don't get to do anything else, but uh Player A only gets a point when you get a 2, Player B only gets a point when you get a 3. Okay? And the first person to get 5 points wins.
	Chris	Okay. So I'll be Player B. So I gotta get a 3?
	R4	You gotta get a 3. And you think this is fair?
	Chris	Um, yeah.
	R4	Because of what you just said?
	Chris	Um humh.
	R4	Okay.
		[G6 and Chris roll dice.]
	R4	I think you do … Help me with that. It was a white 2 and a green 1. Okay, so why we over here say white and green. [Chris writes W&G at the top of his column.] Okay, and so it was a, okay. Okay. He didn't ….
	G6	I haven't gotten it yet.
		[G6 and Chris roll again. G6 rolls a 3.]
	Chris	Do I get a point?
	R4	Yeah. Let's say you get that point.
	Chris	Do I write here [G6's column] or do I write on my side?
	R4	That's okay. That's fine. [Chris has written G6's 2, 1 roll in G6's column.]
		[G6 and Chris continue rolling.]
	R4	There's a 2!
		[Another 2 is rolled. The score is 2 – 2.]
		[G6 rolls a 3. Chris begins to write 2, 1, but corrects himself and writes 1, 2.]
	R4	What is it, 5 points?
	Chris	Um humh.
		[G6 rolls a 2, which Chris reads as 2 but records as 2, 1 and gives a

		point to Player B. After several more rolls, another 3 is tossed, and Player B has 5 points.]
	R4	I wonder why that happened.
26:00	Chris	It's the same, it's the same thing. It uh, it doesn't really matter which player wins it, but it's the same thing because it had two different numbers, and both dice have the same kind of numbers. And, so if you get 3 and a 1, or 2 and a 1, in either one, it's still gonna get a 3. If you get a 1 and a 2 or, no, I mean a 1 and a 1 on the other dice, it's still the same thing. So you could get a 1 here and a 1 here [holding one die in each hand], it's still gonna be 2. And you get a 2 [right hand], 1 [left hand], or a 2 [left hand], 1 [right hand], it's still the same thing.
	R4	So this just is luck?
	Chris	Uh huh.
	R4	That we got more 3's. Okay. Let's keep going. 'Til 10.
	Chris	Okay.
	R4	Okay, or another game of 5, okay?
26:54		[G6 and Chris roll dice. 4, 2, 4, 5, 5, 6, 6, 4, 4, 5, 6, 4, 6, 6, 5, 6, 5, 7, 2, 8, 4, 4, 7, 8, 4, 7, 2 – the score is now 3 – 0 for A.]
28:27	R4	It makes it sort of more even, doesn't it?
		[G6 rolls 2&1. More rolls: 5, 4, 8, 4, 7, 5, 6, 3 (2&1), 4, 8, 4, 6, 5, 5, 3(2&1). The score is now tied 3 – 3. More rolls: 5, 7, 7, 4, 7, 4, 4, 6, 6, 6, 6. 3 (2&1). Chris correctly recorded this as W2, G1.]
29:59	R4	That was the other way. It was white and green. [Chris changes his notation to 1, 2.]
		[More rolls: 4, 7, 6. 3(2, 1). Player B wins with a score of 5 – 3.]
30:18	R4	That's interesting. So that actually this player [A] only had 5 all together when that one had 10. [combining the scores of two games]
	Chris	I really still think it's the same thing.
	R4	Still think it's the same. And the other kind of, of dice, if, well, maybe it is. And so I know you have to go down and be the evil prince right now. So think about it and sort of catches you up to where we are, so if you can come join us next week.
	Chris	Yeah 'cause I don't have it next week, 'cause the teacher, she's going on vacation.
	R4	You mean the play person?
	Chris	Yeah, the teacher. [Chris will be able to come to IML next week because there are no play rehearsals.]
	R4	And so what you're saying is that you thought the first game …
	Chis	was …
	R4	was not fair.
	Chris	No. Because Player B woulda, um
	R4	What is it down there? [pointing to Chris' paper]
	Chris	Yeah Player B because Player B has 6 different, well, 2 for each, and Player A only had one for each.

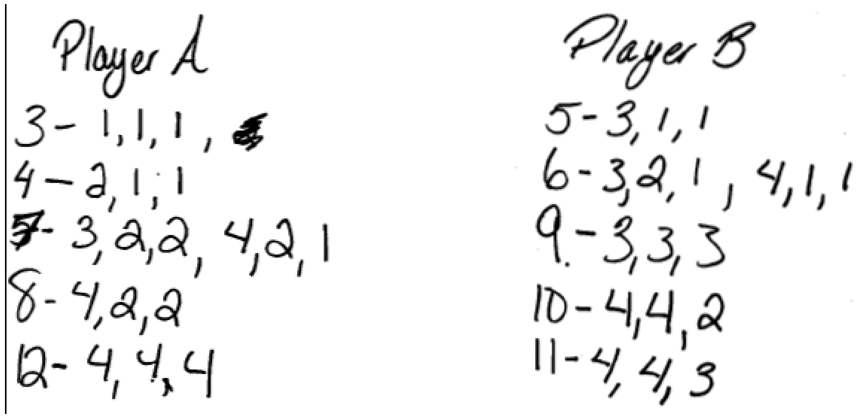
	R4	Oh, I see. And so, part of it you might think is how you, how would you make it fair?
	Chris	Ummm, [mumbles – sounds like 4 and 4]. Well, I'd say, say if either one had a 6 but Player A would have to have a 3 and a 3 and Player B had to have a 4 and a 2. Like, both of them could have got 6, but …
	R4	Show me what you mean.
	Chris	Like this, like you could just put this 3 and 3 over here [draws an arc from Player B's list to Player A] and keep this [4 & 2] here. So it would have 1, for 2, 3, 4, 5. 1, 2, 3, 4, and then 5.
	R4	Oh, I get it.
	Chris	1, 2, 3, 4, and then 5.
	R4	Did you understand that, [G6]?
	G6	Yeah.
	R4	Yeah. That's pretty logical. That's great.
32:30		[goodbyes. Camera films Chris walking down the corridor. End of CD.]

Date: 11 May 2005 Grade 7
Location: Hubbard Middle School
CD: ROLE 123A – 124A
Transcribed by: Kathleen Shay
Verified by: Judith Leonard

Time	**Speaker**	**Transcription**
1:49	R1	I'd like you to get started. You see the problem in front of you. You've played a game before, um, and this game is a little bit different, and there's an extra question on it. Now you notice you have 3, you have 3 dice, right? Does anyone know what the shape of this is called?
	voices	Triangle. A triangle. Pyramidal dice.
	R1	All right, pyr-, it's a pyramid, right? People call it another name for it, pyramid. Anyone else?
	voice	Pyramidal dice.
	R1	A little one. How many sides does it have?
	voices	3. I don't know, a lot. 4. 4. 4. No, 3! 4. 4. You didn't count the bottom.
	R1	Okay, there's another name for these, these dice. This is called, have you heard this before - a tetrahedron.
	voices	No. Yes.
	R1	You've heard tetrahedron?
	voice	Yes.
	CAM	Okay, so this is also called a tetrahedron. So, or a pyramid, or 4-sided. And now you're gonna play the game and you have 3 of them. It's very, very important, you have paper and pencils that

		when you play the game, before you play the game, I want you to read the question and I want you to guess what you think is gonna happen with your partner. And I want you to write down, before you play the game, what you think is going to happen and why. I want you to put your name on your paper right now. Everybody put your name on your paper right now, and today's date. Does anyone know what today's date is?
	Terrill	The 80th. May 11.
	R1	May 11, okay, and your name. And if you want your own sheet for the game, we have extra copies. We can give everybody one. Um, if you'd rather have your own copy, put your name on it. Okay. I want you to read it through. [chatter]
5:30	R1	[approaches Chris and Terrill] Can you roll the dice for me? Can you roll one of these for me? Terrill, roll one of them. [Chris rolls] Can you tell me what you're reading here?
	Chris	4 + 4 is 8, plus 1 is 9.
	R1	So you know how to read, you know how, what would you record here? On your paper.
	Chris	Uh 2, 1 for red, 1 for white, 1, white 1. [the outcome was red 4, white 4, white1]
	R1	What would you record for this one?
	Chris	Red 4, white 4, white 1.
	R1	So you're gonna keep track, you're gonna keep track of what you rolled, right?
	Chris	[coughs] I'm sick, so I can't talk now.
		[camera follows R1 to Jelani and Jeffrey's table]
7:09		[camera moves to show a male teacher, T7, sitting with Chris & Terrill]
	T7	Okay. Chris, who do you think is gonna win? Who do you think is gonna win?
	Chris	Hold on, I gotta see this.
		[Students discuss someone named Jasper who was in a fight. Later they talk about some girls who fought.]
	Terrill	I'm Player B, you Player A.
	Chris	Hold on, brother. I've gotta see if it's fair. [Chris begins to writes combinations that give each of the possible sums. The discussion of students fighting continues.]
9:03	T7	Okay. So let's do it, let's play the game. Who comes first?
	Chris	Uh, you go first.
		[Someone asks for the time. T7 shows his watch at 4:00.]
	T7	Come on, let's play. Who's recording the game?
	Terrill	Me, but this guy's just sitting here. [Chris is still writing combinations.]
	Chris	Hold on, bro.
	voice	Why don't you just throw 'em?
	Chris	That's the different possibilities to get, to get the numbers.

10:23	T7	[to Chris] So why you put only these numbers on the page?
	Chris	I don't know yet. Hold on, hold on.
	Terrill	He counting up the possibilities of going to those numbers. If he finds all the possibilities then whichever one has more possibilities is um, better, it's fairer for um that one.
		[Chris finishes writing the combinations.]



	Chris	1, 2, 3, 4, 5, 6. 1, 2, 3, 4, 5, 6. They're both equal, they're equal. [waving his hands]
	T7	Okay. So those equal? Okay, let's prove it. Let's prove it now.
	Terrill	They equal?
	Chris	Yeah.
	T7	Okay, let's play the game and see if equal.
		[Note: P(A) = 29/64 and P(B) = 35/64]
	Terrill	All right. That's 3. You get one point, game boy. 1, 1, 2.
		[Chris & Terrill continue to play.]
11:48	T7	So do you write the numbers or no?
	Chris	Huh?
	T7	Do you write the numbers? Like 3, 2, 1.
	Terrill	Yeah.
	T7	Okay.
12:54		[Terrill's paper shows some, but not all, of the outcomes written down.]
		[The score is currently A-5, B-3.]
14:05		[The score is A-7, B-4.]
14:25	Chris	I was just rollin' dice with my little brother, right, and I was like this, and it ran in the side of a car on the street. [Chris demonstrates how he rolled the dice.]
15:38	R1	Gentlemen, how are you doin'?
	Chris	Good, I'm winnin'. [The score is A-9, B- 7.] see? Look. 5 dollars, I betcha 5 dollars.
	R1	Who's gonna win? Who's gonna win, Chris?
	Chris	Find out. It don't matter.
	R1	It doesn't matter? Do you think it's fair to start with?
	Chris	Yeah.

15:56		[Chris wins, 10 – 8.]
	T7	Chris, think it’s a fair game?
		[Students are carrying on conversations across the room. Chris and Terrill are engaged in this off-task discussion.]
17:30		[The camera picks up on Chris and Terrill playing another game.]
	R1	Are you recording, wait, you just got those down, but you didn’t record. You need to record what you get.
	T7	Just write the number, so I know what you got. ya know,start from the beginning. Start from the beginning, scratch, from the beginning.
		[The dice are rolled, and Terrill marks the score. He lays his pen over the score and over the outcome 2, 3, 1 that he had written earlier. He does not write the outcome of this roll.]
18:40		[Chris leaves to get a wet paper towel for an itch. T7 takes his place while he is gone. As they play, he instructs Terrill to write the outcomes for each roll.]
19:38	Terrill	I can’t do this. [Puts down the dice and pen for a moment, then picks up the dice.]
	Chris	I’m sweatin’. [Chris has returned to his seat.]
	Terrill	It’s hot.
	T7	[picks up the dice and rolls them.] Write this: 3, 1, 2.
		[Terrill has crossed out part of the score; it looks like 3-4.]
	Chris	[points to Terrill’s paper] Seven here isn’t 1, 2, 3, 4. [Though 7 points were scored, only 4 outcomes are shown.]
	Terrill	I ain’t writing down some of them ‘cause I keep forgetting.
		[The boys continue to play, though they seem easily distracted by events in the room. Terrill tries juggling the dice.]
23:26	T7	I want to see who is winning. So far, what?
	Chris	You can’t tell who’s winning.
	T7	Why?
	Chris	Because of Terrill. [The score is difficult to read.]
	Terrill	He’s winning by 1.
24:41	Terrill	Come on, come on. I win! I win! I win! Ha ha. Now. Hold on, I gotta get one more.
	Chris	7 is me.
	Terrill	I need one more.
	Chris	7 is me!
	Terrill	Aw, shhh. You win.

25:01	T7	Okay, so what do you think now? Do you still think it was fair?
	Chris	It's fair.
	T7	Why? Why?
	Chris	[coughs and looks away]
		[Terrill is speaking to someone across the room.]
	T7	So why, why, why the game is fair?
	Chris	I say it's fair.
	Terrill	The game is fair.
	T7	Why?
	Terrill	Because it has the same amount of chances to um … y'all watchin' the fight? I'm done, man. I'm done. They say we play two games. We've played two games.
	26:20	[end of CD 123A]
		[begin CD 124A] [Some students have left for play rehearsal, so students have regrouped.]
0:35	T5	What did you say? Is 4, 4, 3 the same as 3, 4, 4?
	Terrill	Yeah [inaudible].
	T5	Even if I have different color dice?
	Terrill	If you had different color dice [inaudible] it would be the same numbers on each of 'em.
1:12		[camera moves to Jeffrey playing 2-dice game with R4.]
7:26		[camera returns to Terrill, Keisha & Chanel with T5.]
	Terrill	If 421 is the same number? It's the same number.
	T5	If you're gonna give me, if you're gonna give me 241 dollars or 412 dollars, I'm takin' 412. So are they the same thing? Do you think they're the same thing, then?
	Terrill	[sits up and gives a small smile, shakes his head]
	T5	So, so then I hear, I think I hear you say that they're different.
	Terrill	Yeah, they're different.
	T5	They're different. Um, what if I, what if I were to trade this one, right? What if I were to trade this one here, right? We're gonna be patriotic today. Red, white, and blue. So, can you guys, why don't we think about these as 3 different colors, right? Ladies and gentlemen. So if I were to record all the possibilities in a table and use the colors, is it possible that you can try and break down all the outcomes now, thinking about it this way. 'Cause you guys came up with 12.
	Terrill	Wait a minute.
	T5	Do you think that there's more outcomes if I say that they're different, or less than 12?
	Terrill	It's the same thing.
	T5	You think that it's gonna be the same amount of outcomes.
	Terrill	Yes, because you're using the same numbers.
	T5	But here I see you've listed um 1, 1, 4, right? Now, if I'm, if I'm

talking about roll the dice and you get this amount of money, right, what one, which one do you want to roll? Do you want to roll it as a 1, 1, 4? Let's say I always …

Terrill 4, 1, 1

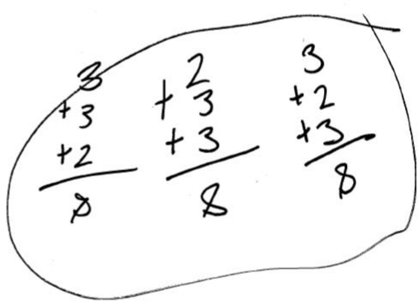
9:05 T5 Oh, you want 4, 1, 1. OK. So let's say it depends on the number, uh, the color of the dice, right? So if I say that the blue always has to be in the hundreds position, the red always has to be in the tens position, and the white always in the ones. Right? What, what's gonna happen if, if I can only, let's say this is, this is the order that they have to be recorded in with the table: blue, red and white. And I'm just writing down the outcome. What's on the die. So I roll it now [rolls 3 dice]. This time it's a blue 4, a red 3, and a white 2. So that's four thirty-two. Right?

Terrill Uh huh.

T5 Or it's 4 + 3 + 2, is the way we're thinkin' of it, but I'm sayin' 4, 3, 2. But I see the way you're writing it with a comma.

Terrill All the um, all the thing, no matter where you put it, no matter if, all right, take 3, 3, 2. What's 3 + 3 + 2? [writes this sum in a column] Eight, right? Okay, 8. What's 2 + 3 + 3? Eight. What's 3 + 2 + 3? Eight. So it doesn't matter how you put it.

[Terrill shows that the sums are equal.]



10:23 T5 It's true but if I'm being … I like the way, the way that you're recording this I think is good, right? Because you're thinking about the sequence of the numbers. But, I agree with you that they do equal the same sum. But if I, if I'm going to make a connection with these numbers, right, and I'm going to, prior to making the sum, right, that's the order they're in and I'm gonna say that's the number, 4, 3, and 2, right? Is that the same as if I had 2, uh, 1, and 3? 'Cause remember I'm sayin' I want to record what's on the blue dice, what's on the red dice, red die, and white die. So, um, I want you guys to experiment with that a little bit. Just, just record your sums. I want you to record your sums in a table similar to to this what I said, but I want you to keep track of which one is the blue dice, which one is the red die, and which one is the white die. 'Cause I hear both of you sayin' different things.

[Terrill asks for a ruler to prepare his table. One of the girls says she doesn’t like this. T5 says that he’d much rather learn math this way, and talks about his experiences as a math student.]

13:20 T5 Okay, Keisha. Um, since he’s uh recording, why don’t you uh, why don’t you roll the dice and then [interruption] .

13:33 [camera moves to R4 with Jeffrey]

14:19 Terrill All right, I’m done. [His table is shown below.]

[Blue Red White
4 4 3
4 4 4
2 1 3
3 3 3]

14:48 [Camera moves to R3 talking to a graduate student and R1.]

16:24 [Camera moves to Jerel and Ian.]

Jerel You’re cheatin’. That’s what I don’t like. Cheatin’, bro. cheater, cheater, cheater, 1, 2, 3, 4, 5, 6. 1, 2, 3, 4, 5, 6. [Their game is tied 6-6.] You’re a cheater, bro.

[more arguing about cheating]

18:22 Jerel It’s 9 up. Who’s gonna win?

Ian I don’t really care. It’s just like that cootie game.

Jerel Uh uh, I won.

Ian You see how cocky he is? I’m leaving. Jerel doesn’t agree with me.

T3 Is the fact that Player A won sufficient for you to say it’s fair?

Jerel Whatever player I am is always wins. Right? We just learned that.

T3 So what does the fact that whichever player you are wins, that makes it fair automatically?

19:05 Jerel ‘Cause look, Player B has more, look, you sayin’ Player B has better chance of gettin’ them numbers, but look, I just proved to you that Player A can still win.

Ian [inaudible – appears to be talking to T3 about his ID card] But doesn’t on the chart, doesn’t it look fair?

Jerel Yes.

Ian On the chart.

Jerel It looks, it looks unfair on the chart. But look, we, I just proved that Player A can win.

Ian Okay, you play him. No, you play him. [to T3]

Jerel No, I want to play Ian again.

T3 Do you want to be Player A again?

Jerel No, I want to be Player B this time.

T3 Why?

Jerel [shrugs]

T3 Okay.

Jerel You wanna be Player B?

T3 Who do you want to be, A or B?

Jerel It don’t matter. I’m still gonna win.

T3 Okay, so I’ll be B, then.

[Ian’s chart]

Player A: 3,4,7,8,12

Player B: 5,6,9,10,11

Num	Com. 1	Com. 2
3	1+1+1	
4	1+1+2	
5	1+1+3	2+2+1
6	1+1+4	3+2+1
7	4+1+2	2+2+3
8	4+3+1	
9	4+4+1	3+3+3
10	4+4+2	3+3+4
11	4+4+3	
12	4+4+4	
Combination		

20:10		[Jerel & T3 begin a game. The first point goes to T3.]
20:20	R1	Okay, I’d like you to start writing up your results, and if you’ve finished writing them on your paper, you might want to start writing them on overheads so that we could share uh what you think about the fairness of the game, and your findings and why. And if you think the game is fair, I need to know why. If you think the game is unfair, I need to know why. And I’d like you to make it fair if it’s unfair. Can you make it a fair game if you think it’s unfair.
		[Jerel and T3 continue playing.]
23:10	T3	What’s the score?
	Ian	You won. [it appears that Ian has taken Jerel’s place]
	T3	That’s 10?
	Ian	Yeah, no, yeah. That’s 10.
	T3	Do you still think it’s fair? A won, B won.
	Ian	I didn’t ever think it was fair! I still don’t. ‘Cause look, B won.
	T3	Okay, but accord-, but according to your game, though …
	Ian	Yeah, it is. [looks at his papers]
	T3	According to your game, the outcomes of your game …
	Ian	Yeah, it’s fair. They each have enough of a chance to get …
		[camera moves to R1 talking with Chanel]
23:40	R1	I don’t care which dice they came on. You get a million dollars. Would you, would you want to be the person that had to get them on white, red, and blue, or did you want to be the person that it didn’t matter what dice they came on, the numbers?

Chanel	That didn’t matter.
R1	Why?
Chanel	‘Cause, if it, if it doesn’t matter what numbers [inaudible] on then you can get um less a better chance of winnin’.
R1	Well how much of a better chance? That’s the important question. How much of a better chance?
Chanel	Uhhh.
R1	What makes it a better chance?
Chanel	Because, um, it makes a better chance because if you, if you were to have 4, 2, and 3 and you had to get ‘em in the same, exact way they put it, then that means you have to exactly get 4,2,3, like say if you switched it around and you had 2,4,3, then, on the other hand you could win the million dollars even if it’s like …
R1	Okay, so try to specifically tell me how much a better chance you get because, you know if you had a, you’re, supposed you’re in this television contest, right, and and the television contest, they told you you could pick it either way, [interruption]. Suppose you were at this television and they said to you you could win this money and you’d have to pick, what what why do you have an advantage one way? What is the advantage in particular? How many ways could it occur to get a 4,2,3 the second way rather than the first way. That’s the question. That’s the big question. ‘Cause that’s the question that’s gonna help you answer this question about fairness.
Chanel	[rearranges the red, white, and blue dice] Um, you get a 6, like, no, 2, 3, 4, and 2, okay.
R1	4,2,3 you had. 4,2,3. So you could get 4,2,3. Why don’t you write them down on the back of your paper? So write the different ways you could get a 4,2,3
Chanel	Okay. [starts writing] 3
R1	How do you, how are you gonna keep track? This one is white, this one is red, this one is blue. You could get a 4,2,3 on white, red, and blue, right? [aligns the dice in this way] So why don’t you write “white, red, blue” up there. Well, just the letter’s good enough. R, B. Okay. Now, now when you, now is that the only way you could get a 4, 2, 3? Write all the ways. [Chanel writes: white R B 4 2 3 W B R 4 3 2]
Chanel	Then it could be [writes the numbers 2 4 3 on the next line], red, white, blue [writes R W B above the numbers]. [white R B 4 2 3 W B R

		4 3 2
		R W B
		2 4 3]
	Chanel	3, 2, 4 [writes these numbers on the next line, then writes B R above them and pauses with her pen over the 4, as camera moves to Keisha rolling dice].
26:56	R1	Okay. So, so I see your point. Right? I'm beginning to see your point. So my question is, if a player can get 10, right, it's not just one way to get the number 10, is there? Right? What I want you to think about is how many different ways are there to get this.
	Chanel	It's only [taps her paper]
	R1	Do you have them all? OK, does that change your idea about which game is fair?
		[Chanel and Terrill are talking about something else.]
27:35		[session is adjourned]
28:20		[end of CD 124A]

Date: 11 May 2005 Grade 7
Location: Hubbard Middle School
CD: ROLE 123B – 124B
Transcribed by: Kathleen Shay
Verified by: Jeremy Milonas

Time	**Speaker**	**Transcription**
3:17	R1	I'd like you to get started. You see the problem in front of you. You've played a game before, um, and this game is a little bit different, and there's an extra question on it. Now you notice you have 3, you have 3 dice, right? Does anyone know what the shape of this is called? [The remainder of introduction is transcribed on ROLE 123 A.]
6:34	R1	[to Jerel & Ian] Would you roll the dice for me, please, and tell me how to, how to read what comes out? [Jerel rolls 3 dice.] Read what, what number came out here?
	Ian	3, 1, 3
	R1	How do you know that?
	Ian	3 is all around on the bottom.
	R1	You're so smart!
	Ian	I did this before.
	Jerel	Sure, bro, sure. You're smart.
7:07	Jerel	I'm Player A.
	Ian	No, no, no, no, hold up. I'm Player A.
	Jerel	No, I'm bro.
	Ian	No, I'm bro.
	Jerel	All right. I'm Player B, then.
7:30	R3	So Ian, do you think that you're gonna win because you're a better roller?
	Ian	Yeah. Jerel keep rollin' [inaudible].
7:51	R3	Do you think this game is fair?
	Ian	No.
	R3	Why not?
	Ian	Because.
	R3	What about you, Jerel? Is this fair?
	Jerel	I don't know, A and B …
	Ian	I don't care.
	R3	All right.
9:25	Jerel	… would have been a 1! You the cheater. I hate cheaters.
	Ian	Who's the sore loser! All right, go 'head. Oh no, it's my turn. [rolls dice]
	Jerel	Cheat. You a cheat.
		[Ian & Jerel continue playing. As Jerel gets some points, he stops accusing Ian.]
13:31	Kianja	[to R3] Excuse me.

	R3	What's up, Kianja?
	Kianja	I'm sorry for interrupting and eavesdropping [R3 and R1 were talking about two girls who were not present last week], but I also think that they should play the second game [inaudible] 'cause it will be easier for them to understand the third game.
	R3	Okay, that's what we'll try to do. How are you guys doin' here?
	Kianja	We're doin' good. I'm tryin' to answer questions and she's [Brionna] doin' that [rolling dice].
13:52	R3	What are you doing here? What is this?
	Kianja	[makes a rolling motion with her hand]
	R3	That'll help you answer the question?
	Kianja	Yeah.
	R3	That's an interesting way to do it. What does this, what does this mean?
	Kianja	Oh, these are the different ways that you could get these numbers [shows a paper with A's and B's numbers written separately].
	R3	So there's 3 ways to get a 4. Does that mean a 4 is easier than a 3?
	Kianja	Yes. Well, yes. Yeah. 'Cause there's more ways that you're gonna get it.
		[Kianja's paper shows one way to get a 3 : 1+1+1, and three ways to get a 4: 1+1+2, 2+1+1, and 1+2+1.]
	R3	Okay. All right. How many ways is there to get a 5?
	Kianja	I'm still counting.
	R3	Okay. I'll come back.
		[Kianja has six ways to get 5: 1+2+2, 2+1+2, 2+2+1, 3+1+1, 1+3+1, 1+1+3.]
	Kianja	[inaudible – talking under her breath] Okay. All right. I think I'm good with that. [starts a new column for 6]
		[While Kianja writes ways to get 6, Brionna rolls the dice and records outcomes. Jerel's voice is heard from across the room, accusing someone of cheating.]
17:05	Brionna	[to R1] Because it has different ways, like, it has different ways to get it.
	R1	Do you think B has different ways? What about A?
	Brionna	A, like, it has ways, but it doesn't, like [inaudible] is like 3, 4, 7, 8, 12. It doesn't have that many ways for 5, 6, 9, 10, 11.
	R1	So you think B has more ways?
	Brionna	And she's [Kianja] proving it.
	R1	She's proving it? OK, she's writing them all out?
	Brionna	Yes.
	R1	Okay. Are you checking her to be sure that she doesn't miss any?
	Brionna	Yes, when we finish.
	R1	Thank you.
17:45	R1	[to Ian & Jerel] Have you resolved the rolling problem?
	Jerel	Yeah.
	R1	Who has the advantage? You think B does, why?

	Ian	‘Cause B has a better range of numbers.
	R1	Why?
	Ian	He got a better range of numbers.
	R1	Better range of numbers? What does that mean, a better range of numbers?
	Ian	Well, his range is better.
	R1	What do you mean by better range?
	Jerel	He’s beatin’ me, but he can’t beat me when the thing is flat. It proves I’m a better player than him.
	R1	I wanna know what, why you think B has better numbers. Can you tell Ian why you think B has better numbers?
	Jerel	That’s mine [referring to the outcome of the dice roll].
	Ian	Aw, man. B is better. I don’t care.
	R1	Do you know why? Tell me why.
	Ian	The range of numbers has more multiples. Hey, I’m usin’ smart words.
	R1	Big words, but I don’t know what they mean.
	Ian	I don’t either!
	R1	Tell me, can you talk to me in a way that I can understand what you mean? You think B has a better chance.
	Ian	B has better numbers let, no, yeah, de-uh.
	Jerel	Not really, because you can get 4 by 2, oh yeah, he is right. It’s like not, not a very fair game.
	Ian	All right, this time they got the same amount of numbers, but B got the more multiples.
	Jerel	But you can get 5 with …
	Ian	Man, y’all some dumb crackhead, yo! [laughs]
	R1	What’s that?
	Ian	Ew, why you gonna write “Player Be”? [a typo]
	R1	That’s sad. Yeah. [fixes typo] Thank you. You’re gonna be an editor when you grow up.
	Ian	I wanna be a hustler.
		[Jerel & Ian prepare to resume play.]
	Ian	I just went.
	Jerel	You scuffed the dice.
	Ian	I didn’t scuff them.
	Jerel	That’s how you won last game.
21:16		[camera in vicinity of Kianja & Brionna]
	R3	[off camera, to Kianja] Sure you got all of them for 8?
	Kianja	Huh?
	R3	Are you sure you got all of them for 8?
	Kianja	8? No, I’m not sure. I think there is something else. [Continues to work on the sample space.]
23:30		[So far, Kianja has 1 sum for 3, 3 sums for 4, 6 sums for 5, 8 sums for 6, 9 sums for 7, 6 sums for 8, 4 sums for 9, and 3 sums for 10.]
23:47	R4	How else could you get 10?

		[Kianja writes 4+4+2, 4+2+4, 2+4+4 in the 10 column, now showing 6 sums for 10.]
	Kianja	I mean, that's it.
	R4	Can you get 9 using twos?
		[Kianja pauses for a moment, then writes 2+3+4, 3+2+4, 3+4+2 in the 9 column, now showing 7 sums for 9.]
24:42	R4	Can you get 8 using ones?
	Kianja	Huh?
	R4	Can you get 8 using ones?
		[Kianja writes 1+4+3, 4+1+3, 4+3+1 in the 8 column, now showing 9 sums for 8.]
25:27		[Camera leaves.]
27:30		[Camera returns to Kianja & Brionna. R4 is reading Kianja's sample space.]
28:05	R4	[pointing at the 6 column] Could you, for those, I think there are two more. See if you can think of them.
	Kianja	There is two more.
	R4	What are they?
		[Kianja writes 2+1+3, 2+3+1.]
	R4	So how many are there for 6?
	Kianja	Huh? What'd you say?
	AA	How many are there for 6?
	Kianja	1, 2, 3, 4, wait.
		[Kianja writes the number of sums above each column: 3(1), 4(3), 5(6), 6(10), 7(9), 8(9), 9(7), 10(6), 11(3), 12(1)] [Note: the correct numbers are: 3(1), 4(3), 5(6), 6(10), 7(12), 8(12), 9(10), 10(6), 11(3), 12(1)]
	R4	Hey Kianja, look at that one and that one. Ki-an-ja.
	Kianja	I'm sorry, I can't focus.
	R4	I know you're having a hard time. But look at this one and this one [indicating two of the sums in the 7 column]. Look at this one and this one, are they the same? This one and this one. [The sum 1+4+2 was written twice.]
	Kianja	I'm gonna show you what's the same. [Kianja begins to draw connecting lines.]
	R4	Those two are not the same. 4,2,1 and 2,4,1. These three go together?
	Kianja	Whoa, wait, say that again. You said these go together?
	R4	You put a little arrow there, why? [arrow grouping 3+2+2, 2+3+2, 2+2+3]
	Kianja	Yeah. 'Cause they're the same, just put in different places.
	R4	But what I'm saying to you is, this one and this one are really the same [1+4+2]. You can't have them both, so figure out what you should have instead. I agree there should be 6 there.
	Kianja	[changes one of the sums to 1+2+1]. Oh, wait.
	R4	It's 1, 2, 4.

	Kianja	Beautiful. [circles like sums in the 8 and 9 columns]
	R4	Why can’t you do, can I ask you a question?
		[Kianja is arguing with another student. R4 places her clipboard to block the view.]
	R4	Why can’t you do 4,4, and 1 for 9? Oh, I didn’t know where it is. You did.
	Kianja	Yeah.
	R4	Why can’t you, I’m sorry, why can’t you do 3,3, and 1 for 7?
		[Kianja writes 3+3+1 in the 9 column.]
	R4	Not for 9, for 7. This [3+3+1] goes over there [7 column].
		[Kianja makes adjustments to her column totals. She changes 9 to 7 sums. She changes 7 to 12 sums, and writes 3+3+1, 3+1+3, 1+3+3 in that column. She continues circling like sums in groups of three.]
32:46	R3	Kianja, that’s a nice table that you have. Do you have a prediction for who’s gonna win the game?
	Kianja	All right, hold on.
	R3	Or do you need some more time?
	Kianja	I gotta, I gotta tally up now.
	R3	Okay. Take your time.
		[On a new sheet of paper, Kianja writes:

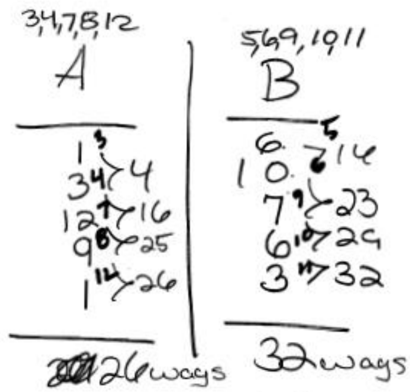
3,4,7,8,12	5,6,9,10,11
A	B
1	6
3	10
12	7
9	6
1	3
26	32

34:52	Kianja	So B is gonna win.
	Brionna	Oh, that’s what I said.
	R3	You think B is gonna win?
	Kianja	[nods] And I have an example [shows R3 Brionna’s data].
	R3	And B won?
	Kianja	Yes.
	R3	All right.
	Kianja	And I didn’t know she was playin’. She was rollin’ the dice.
	R3	All right. Can you explain why you think B is gonna win? I’m a little confused. I’m a little confused. Why is B gonna win?
	Kianja	Why is B gonna win?
	R3	Yeah.
	Kianja	Because their numbers are 5, 6, 9, 10, and 11, right?
	R3	Yeah.
	Kianja	So 5, 6, 9, 10, 11 [circles each column as she says the numbers].
	R3	So you added these.
	Kianja	Yeah. I added all the combinations. These are combinations. This

		is a combination. So I added a 6 combination to equal 5, well to get 5. So I put that 6, and then this 10, 7, 6, and 3. And then I added it up and that's 32 combinations.
	R3	So, so why, uh, I haven't played this before, but why would a 4 get 3 and a 3 only get a 1, because there's only one way to get …
	Kianja	Like in the game, in the two game, in the two game, right?
	R3	Yeah.
	Kianja	When there was two dice, right?
	R3	Yeah.
	Kianja	There was only one combination to get 2 because the lowest number was 1.
	R3	1 and 1
	Kianja	On the dice, so when it was 1 and 1 …
	R3	But wouldn't there only be one for 3, 'cause it's 2 and 1?
	Kianja	[talking at the same time] and since there's 3 dice … huh?
	R3	Okay, okay, sorry, go ahead.
	Kianja	And then on the 3 dice, on the third, if we add a third dice, then there's only 3 ways to get to um, I mean one way to get to 3.
	R3	I agree with that. 1, 1, and 1, right?
	Kianja	Yes.
	R3	But isn't there only 2, 1, and 1 to get 4?
	Kianja	[brief pause] Well, yeah, but we switched them around, so. We will divide it by 3 if you want. All right, so then it would be …
	R3	Oh no, no, no. Don't change it.
	Kianja	No, I'm just sayin', no, I'm sayin' if we didn't want to add the little things in there. So that'd be 1, 1, 4, 3, 1 [adjusting the number of sums for each total for Player A]. [For Player B, Kianja works out the math and writes 2, 4, 3, 2, 1.]
	R3	Oh, I see. Hmmm. 7, 8, 9, 10, 11, so that one's 12.
		[Kianja adds the numbers for A to get 10, and the numbers for B to get 12.]
	R3	Which one do you think is a better way of doing it? Which way is a better way of counting?
		[Kianja points to the more recent list – where permutations are not counted.]
	R3	You think that one's better? I don't know. I'm not sure. This one's starting to make more sense to me now [referring to Kianja's original list].
38:35		[end of CD 123B]
		[begin CD 124B]
0:33	R1	[to Kianja] What's the sum of these? [pointing to a pair of dice] Is there another way I could get that?
	Kianja	[rearranges the dice]
	R1	No, that's still the same. I just moved the dice around. I got a 4 on this [white] die, just moved it, and a 3 on the black.
	Kianja	[changes the dice to show 3 on white, 4 on black]

R1	Ah, now you’ve got it. That’s different, isn’t it? You got a 4 on there. So they’re different, aren’t they?
Kianja	Um humh.
R1	Don’t let somebody talk you out of that.
Kianja	I don’t know. I was saying, I was saying if you wanted to do it this way … [taps her paper]
R1	Yes.
Kianja	Then that’s how you would do it. But I didn’t do it this way. This is the way I did it.
R1	So tell me the way you did it again.
Kianja	[points to her original sample space] See, I switched all of ‘em. 4+2+2 and 2+4+2 and then …
R1	You saw them all as different.
Kianja	Yes.
R1	Okay. Very good. And you didn’t, you’re sure you didn’t miss any, right? You said there are 3 ways of getting this [pointing at 4+2+2, 2+4+2, 2+2+4]?
Kianja	Yes.
R1	Okay. And so you’re sure there are no more than 3 of getting this.
Kianja	Right.
R1	And likewise here and likewise here. So how many ways all together are for Player A …
Kianja	12, 9, 7, 6, 3, oh right here [points at her other paper, on which she had added the numbers of outcomes], for Player A 26 and B 32.
R1	So you’re saying, who has the advantage?
Kianja	B.
R1	Player B. ‘Cause you guys played and did B have an advantage?
Kianja	Yes.
R1	But does B have to win, necessarily?
Kianja	No, but it’s more likely for them to win.
R1	What does that mean, more likely?
Kianja	They have a better chance of winning.
R1	They have a better chance of winning. Okay. So how many times did you, did you two play the game?
Kianja	1, 2, 3, 4, 5, 6, 7, 8, 9 [counting individual rolls listed on Brionna’s paper].
R1	And what happened in the 9 times you played?
Kianja	[points to Brionna’s paper] B won.
R1	Okay. B won how many times?
Kianja	1, 2, 3, 4, 5, 6.
R1	And A won …
Kianja	3
R1	And it makes sense to you because of your analysis here, you’re saying?
Kianja	Yes
R1	Are you ready to share that with the class?

	Kianja	I could share it. I just have to make a paper.
	R1	Will you put it on an overhead and explain it? Good work, girls! Good work.
	Brionna	I don't want to explain it. I'm sitting down.
4:39	R1	[to Ian & Jerel] Are you ready to talk about …
	Jerel	Yeah.
	R1	You need to be ready to present.
	Jerel	Are you ready, come on, let's go. I'm presenting with Ian.
	R1	I want you to tell me, I want you to take your results of what you found and put it on the overhead, okay? Ready to present.
		[Jerel places a transparency over one of their score sheets and traces.]
5:32	Ian	Jerel, this game fair to you?
	Jerel	Yeah. I think. No.
	Ian	No. No. Well yeah yeah yeah yeah. 1, 2, 3, 4, 5, 6, 7, no, 1, 2, 3, 4, 5, 6, 1, 2, 3, 4, 5, 6, 7. [counting the outcomes in his sample space]. The game's not fair. 7 has more ways than … [holds his hands out, palms facing Jerel].
	Jerel	But Player B can still win.
	Ian	That's what I just said.
	Jerel	It's fair.
	Ian	But it's not fair. B has more ways than A-town [makes a hand motion]. I don't wanna work. [pounds his desk]
6:28	T3	[to Kianja] Now why did you do that, though? What was the purpose of doing that?
	Kianja	So I could know who, who …
	Brionna	Who can win.
	Kianja	Yeah, who will win. And I added it up. So, these numbers [pointing to her paper], Player A has 26 ways to win, Player B has 32 ways to win.
	T3	That's a lot of numbers.
	Kianja	Yes, it really is. Set, it's all set.
	T3	Are you sure?
	Kianja	Yes, I'm very sure.
	T3	So how, if you're sayin' it's unfair, who, who has the advantage there?
	Kianja	Huh?
	T3	Who has the advantage?
		[Kianja points to B's column on her paper.]
	T3	Do they have the same amount of numbers, though?
	Brionna	Yeah, but it's different when they play, like these numbers there's more ways, 'cause these are the numbers … Kianja, look, this is how many ways for each? I dunno.
	Kianja	For each number? Yeah. 5, 6, 9, 10, 11, 3, 4, 12, 8 and 12. [writing the sums next to the number of ways to obtain them]



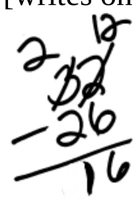
	T3	You have 9 ways to get 7? [speaking at the same time] We have 9 ways to get, 12 ways to get, oh, 1 way to get 12.
	Kianja	Yeah.
	T3	You have 7 ways to get 9?
	Kianja	Yeah. Which are sets of 3. 2 times 3 is 6. You should know that.
	T3	So does it matter [moves his hand in a twisting motion]?
	Kianja	It depends on what dices it's on. Die it's on.
	T3	So, if you were to make this a fair game, what would you do? How would you make it a fair game?
	Kianja	I don't know that yet.
	T3	You haven't figured that out. What are we writin' now, the reason why it's not fair?
	Kianja	Yes. [reading] This game is …
	T3	Are you going to include this [sample space] on your overhead?
	Kianja	I'll try. If it ain't [inaudible] somethin'. I don't know how I'm fit it on there.
		[Kianja and Brionna work on their transparencies.]
10:30	T3	[to Jerel & Ian] I'm saying would it make a difference how I make it [inaudible].
	Jerel	Nope.
	T3	It wouldn't?
	Jerel	Oh yeah yeah yeah. Wait.
	Ian	No. Now you're tryin' to confuse him.
	Jerel	[inaudible] same numbers.
	T3	I'm not trying to confuse him.
	Jerel	Yes you are, bro, nah.
	T3	Can I get a different die? I just wanna, I wanna see something. Do you have a different color one? Grab that white one. Grab a white one. [Jerel & Ian get another die.] So your statement is that the only way to get 4 is 1, 1, and 2, right? My question is, is that the only way?
	Jerel	Yeah.
	T3	See if I roll this [rolls 3 dice], does it matter, did I say 1, 1, and 2 [sets the dice to show these numbers], right, that is the only way I could get 4?
	Ian	Yup.

	T3	So it doesn’t matter. So if I have 1 here [turns the green die to 1], does it matter?
	Jerel	Nope
	Ian	Yup.
	T3	How? How so?
	Ian	They all 1’s. There ain’t no 2 in there, Jerel. [By changing the green die to show 1, all 3 dice now show 1.]
	Jerel	Dude, you not getting’ 4.
	T3	Okay, but if, does … Okay, so is …
	Ian	That’s what I just said. He said does it matter if he changes the number!
	Jerel	Oh yeah.
	T3	Is this different, is this different from that? [The dice show black 1, yellow 2, green 1.]
	Ian	No.
	T3	Why not?
	Jerel	Because all you did was switch ‘em around.
	Ian	[skip in CD] All you did was [skip] numbers.
	T3	What do you mean, I changed ‘em?
	Ian	This is what you’re tryin’ to say: 1, 2, 1. I put 1, 1, 2.
	Jerel	Yeah, it doesn’t matter, bro.
	Ian	It doesn’t matter. Same thing.
	Jerel	1, 1, 2, you still get 4.
	T3	Are you sure?
	Jerel	Yeah, bro. Also [unclear] 2 plus, all right, this is, this is just like 2+2.
	T3	Okay. Suppose I rolled this separately, right? Suppose I rolled the die separately. Let’s say I get a 2 on this one, right? That means I need to make sure I get what?
	Jerel	Uh, 4. Wait. You got 3 on here. Oh.
	T3	No, I’m sayin’ I rolled that separately, so that means if I roll the green one, what am I exp-, what would you expect me to get on the green one?
	Jerel	Not a 2. I mean not a 1. ‘Cause 1 is like …
	T3	But I have 2 already. Right?
	Jerel	You need another 1.
	T3	I need a 1, so I need a 1 on the green, right?
	Jerel	Yeah.
	T3	What would I need on the yellow?
	Jerel	You would need a 1.
	T3	I’ll need a 1. If I rolled the yellow and got 2, …
	Jerel	You would have 5.
12:50		[camera moves to R1 with Kianja]
	R1	Excuse me. What would you do, what, what might you do to make

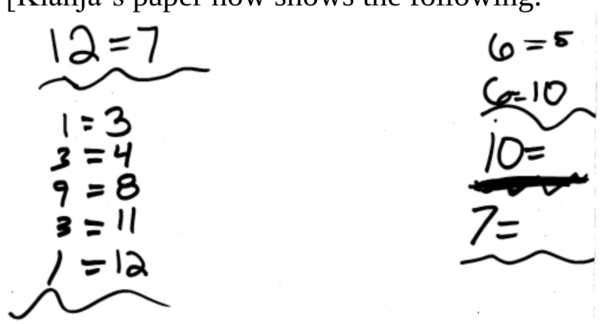
		it fair? That's the second question. I didn't ask anybody else that question. So I want to give you different paper. If you say it's not fair, how would you distribute …
	Kianja	I'm workin' on it. I'm workin' on it.
	R1	Okay. Great. If you need more paper or something, let me know, Okay? So how would you make it fair? Call me when you think you have an explanation.
		[Kianja has written on her transparency:
		"This game is not fair.
		This game is not fair because player B has more ways to get 5, 6, 9, 10, or 11.
		B has 32 ways and A has 26 ways."]
13:42	Ian	[to T3] Yo, this is the only combination you could get with these numbers?
	T3	That's question 3, right? You guys answered the first one. You played it several times. What was the outcome? Who won most of the time?
	Ian	It was a tie.
	T3	It was a tie? You guys say that when you played, it was a tie? Huh?
	Ian	Yeah.
	T3	Then how do you know it's unfair?
	Ian	'Cause we played twice.
	Jerel	I thought it was fair.
	T3	So because you won and because he won, it's fair?
	Jerel	Yeah.
	T3	Is that what you're saying?
	Jerel	Yep, basically.
	T3	Wow. But he just said it was unfair.
	Jerel	He thinks it's unfair.
	T3	What makes it unfair?
	Jerel	Ian, Ian, you won!
	Ian	I just told you.
	Jerel	But you won once.
	Ian	It doesn't matter! [leaning forward with his palms on the desk]
	Jerel	[expletive], it's basically what I said.
14:53	T3	You need to justify for me why you think it's unfair. On your end [Jerel], you think it's fair because you won once and he won once.
	Ian	All right, look. I'm gonna explain it one last time.
	T3	OK, I'm listening.
	Ian	All right A, Player A, which is red, you gotta see that right there [Ian has color coded his sample space], all right 1, 2, 3, 4, 5, 6, 6 combinations , that's it. Now, blue, blue, all right, 1, 2, 3, 4, 5, 6, 7, 8, 9, 9 combinations. That's why it's unfair. Got more combinations.
	T3	But you just told me it was fair 'cause you won and he won.

	Jerel	But you won!
	Ian	It don't matter. [stands up, slamming his palms on the desk]
	Jerel	Well yes it do!
	T3	So why, how can we settle this? How can we settle it?
	Jerel	Play one more game.
	T3	Just one more game?
	Jerel	Yeah.
	T3	Now remember, it says "play the game several times." Right?
	Jerel	Twice. [holds up 2 fingers]
	T3	No, twice is a couple of times.
	Jerel	No, twice is several, that's what she said.
	Ian	Several doesn't mean um a couple. Several don't mean 7.
	T3	So play it a few times and see what you, if the results are the same.
	Jerel	I'll go first. I'm Player B.
	Ian	Nah, you want to switch it up? You be Player A.
	Jerel	I, watch me still win. I'm just that talented.
	Ian	Man, he's so cocky.
	T3	He beat you once, that the only reason why you so cocky?
	Ian	I beat him like 5 times, he's still cocky.
	Jerel	How many times did I beat you?
	Ian	One!
16:10	Kianja	[to Ian and Jerel] Who Player B over there?
	Ian	Do it matter?
	Kianja	You? And who winnin'?
		[Jerel & Ian argue]
16:37	R1	[to Kianja & Brionna] So what did you decide? Did you come up with a way of making it …
		[Kianja has not yet finished writing up her results. She writes two columns of numbers: 1 6 3 10 12 7 9 6 <u>1</u> <u>3</u> 26 32]
20:11		[Kianja draws an arrow from the bottom 1 to the 12 in column A, and draws lines through 10 and 3 in column B.]
	Kianja	What's 6 and 6, 12. 9+3 is 12. [She draws lines through 3, 12, 9, and 1 in column A and some numbers (out of view) in column B.]
21:16	T5	[to Kianja] How are we makin' out with the problem? What are you doin' now?
	Kianja	[inaudible]
	T5	What's uh 6, what's 12 minus 7? What's this mean? [pointing to "12 – 7" written on Kianja's paper, below column A]. You're startin' to write it on your paper.
	Kianja	Oh, it's not minus 7.

T5	What’s that mean, 12 and 7?
Kianja	It’s 12 ways to get 7.
T5	There’s 12 ways to get 7?
Kianja	Um humh.
T5	Can you show me?
Kianja	All of them. [points to her sample space]
T5	Oh, wow. So, so you think that these are 3 different possibilities. [indicates 4+2+1, 4+1+2, 2+4+1, which are circled on Kianja’s paper]
Kianja	Brionna, it’s a scrap sheet of paper! Why does one have to be precise on a scrap sheet of paper? [takes the paper she was writing on back from Brionna. Brionna had her pen poised to change the 12 – 7 notation. Kianja changes it to 12 = 7, and under column B, 6= 5 and 6 = 10.]
T5	You’re just, you’re just recording your results here. But that’s interesting, so I’ve been talkin’ with some other people who don’t think these [different arrangements] are the same, so could you, how could you convince me that they are different?
Kianja	They different, to me, if it’s on a different dice it is different.
T5	Okay. Is that, is that, is that all you think about it? Is there anything else you think? Is there anything else you could do to convince me besides they’re on different dice so they’re different?
Kianja	‘Cause it really depends on the die that it’s on.
T5	It depends on the die that it’s on? So that 1, 4, 2, …
Kianja	1, 4, 2, this would be different if this was a 4, this was a 1, and this was a 2. [demonstrates with 3 dice]
T5	So if I’m talkin’ money here, which one would you prefer to have [pause] out of these ones, 421, 412, or 241? Not sums. If I were to say these three, these are three numbers that you’re rolling. You take 421?
Kianja	You talkin’ money?
T5	Yeah.
Kianja	421.
T5	So is 421 different than 241?
Kianja	Yes!
T5	Yes! Right?
Kianja	Yes it is more money.
T5	Uh huh. Even if we’re not, obviously if we’re talkin’ the sum they’re the same, right, but if I was talkin’ yeah, so you would agree with that statement?
Kianja	Yes.
T5	Okay. ‘Cause I, I, I’ve wondered about whether or not students think these are the same thing and, and some folks don’t think they’re the same thing.
Kianja	I think it’s different. That’s okay.
T5	I mean, some folks they’re all the same, but they’re on different

		dice and you would agree with the statement that those are different sums of money. If I were just to say these are digits.
	Kianja	Yes. [returns to writing on her paper with 2 columns] What's 10 plus 7 plus 3, 20 right? All riiight. [she has written this sum next to column B, and writes 1, 3, 9, 1 in a column next to column A]
	T5	You doing 4 dice now? Or 3 dice?
23:49		[Kianja holds up 3 fingers.]
	T5	3, okay. What's this, what are you sayin' 10 + 7 + 3?
	Kianja	Um, I put the sixes down here.
	T5	Oh, you're tryin' to make a fair game, then.
	Kianja	Yeah.
	T5	Okay. So how many total outcomes did you get?
	Kianja	32 for Player B, and 26 for Player A.
	T5	So this is or is not a fair game?
	Kianja	It's not a fair game. 32 for B and 26 for A.
	T5	Okay. So how many, how many total do you get, then, if you put them together?
	Kianja	[writes on her paper: ]
	T5	There's 16 in all?
		[Kianja realizes her subtraction error and changes the answer to 06.]
	Kianja	No, I said hold on.
	T5	All right. You figure your game out.
	Kianja	Okay. And, 58 in all.
	T5	You think there's 58 in all?
	R1	[announcing to the class] Okay, I'd like you to start writing up your results, and if you've finished writing them up on your paper … [continues giving instructions for students to prepare transparencies to share with the class and to justify their conclusions about the game]
	Kianja	Half 58, Brionna, half 58. Half of 58.
	Brionna	What's half of 58?
	T5	Did you write this all up on here? [T5 looks over the girls' transparencies and comments about them.]
25:42	Brionna	24 and 34.
	Kianja	[pause] That is not, Brionna, what's 5 + 4?
	Brionna	5 + 4, what you asking me?
	Kianja	9, right?
	Brionna	Yeah.
	Kianja	So then it'd be 29 plus 29.
	Brionna	What do you ask me?

	Kianja	Half of 58!
	Brionna	Ohhh! [laughs]
	Kianja	29 plus 29
	Brionna	I didn't know what you was askin' me.
	Kianja	Ding, how I know that?
		[Brionna goes on about not knowing what Kianja asked her.]
	Kianja	I did that fast, though, Brionna. That's a, that's a miracle.
		[Kianja and Brionna look at each other through the transparencies. Then, Kianja asks Brionna to copy the sample space onto a transparency.]
27:48	Kianja	Oh yes! [sits up and raises both arms]
		[On the bottom half of her paper, Kianja had written: 12 = 7 6 = 5 6 = 10
		[She adds to this, while making a variety of noises.]
29:10		[Kianja's paper now shows the following:



	Kianja	[laughing] Yes, yes.
29:30	Kianja	You know what? [puts her head down at the edge of the desk, holding her face in her hands] That's 29 and 29, Brionna.
	Brionna	Humh?
	Kianja	It's 29 and 29. 29 ways and 29 ways! [writes 29 at the bottom of each column.]You understand now? 'Cause you lookin' at me like dumb.
		[off-task conversation]
32:44	Kianja	You know what? I can make this game fair.
		[Kianja writes: " I can make this fair. I can make this fair by giving player A get"]
33:55	R1	[announcing to the class] Okay, guys, you need to finish up what you're doing.
		[Kianja continues writing:

I can make this fair by giving players A get numbers 7,3,4, 8,11 or 12 and players B get numbers 5,10,6, or 9.

35:30		[end of CD ROLE 124 B]

Date: 11 May 2005 Grade 7
Location: Hubbard Middle School
CD: ROLE 123D -124D
Transcribed by: Kathleen Shay
Verified by: Judith Leonard

Time	**Speaker**	**Transcription**
4:18	R1	I'd like you to get started. You see the problem in front of you. You've played a game before, um, and this game is a little bit different, and there's an extra question on it. Now you notice you have 3, you have 3 dice, right? Does anyone know what the shape of this is called? [The remainder of introduction is transcribed on ROLE 123 A.]
6:37	R1	[to Justina & Adanna] Do you know how to read what comes out on the dice when you roll it? Adanna, can you show me? Can you take one and roll it and tell me how do you know what comes out? [Adanna rolls a die.] Which number came out?
	Adanna	Nothin'.
	Justina	[laughs]
	R1	How do you know which number …
	Justina	I think the bottom one.
	R1	No, no leave it here, leave it here. This is important. Do you know what number to record?
	Adanna	Oh, the side thingy.
	Justina	The upright number. No, the bottom of the …
	R1	No, no, look at this [pointing closely at the die]. You can see numbers on all 3 sides, right? You can't read the bottom, so it's not the bottom. Now, if you look at all 3 sides, is there a number that's the same?
	Justina	2. You gotta read the bottom edge, you gotta read the bottom number.
	R1	Okay?
	Adanna	2.
	R1	So the outcome is a 2. Let's do it again. [rolls die]
	Adanna	2.

	R1	You roll it now, Adanna.
	Adanna	[rolls die] 3.
	R1	You got the idea? So do you know what to record when you read it?
	Adanna	Um humh.
		[off-task conversation]
11:21	Justina	We haven't even started our work yet. Okay.
	Adanna	So how do we put this. I put your name [on the paper], you roll, and then I put my name, I roll.
	R4	You all have played with these before? [R4 leans over Adanna's desk and directs most of her conversation to Adanna.]
	Justina	Yeah.
	R4	And so you know, when you toss this [rolls die], what number did I just toss.
	Adanna	1. That means you get, how many points you get?
	R4	Well, you gotta throw 'em all [shakes 3 dice in her hand].
	Adanna	[looking at the problem sheet] There's no 1 here.
	R4	Well, that's right. If you're throwing 3 [rolls 3 dice] and adding them together [inaudible].
	Adanna	2
	Justina	5
	R4	So 5, sure. Is there any way that you could get a sum of 1?
	Adanna	Ohhhh. That means you get 0.
	Justina	You can't get 1.
	Adanna	No, you can't get 1, 'cause there's no 0.
	Justina	Or 2.
	R4	What is the smallest sum you could get?
	Adanna	1. [shakes head] I mean 2. No.
	Justina	3.
	R4	You're tossing 3.
	Adanna	Yeah, 3.
	R4	Okay. What's the biggest one you could get?
	Adanna	11, 12, yeah 12.
	R4	What would you have to do to get 12?
	Adanna	Roll it.
	R4	You gotta get this, and this, and this [arranges the dice to get 12].
	Adanna	[nods]
	Justina	Okay. Come on, let's go.
	R4	Okay.
	R4	Okay. Okay now [skip in CD 12:32] that you've got to make a pre-, you've gotta read it carefully, and make a prediction as to whether you think it's fair or not before you start playing.
	Justina	Oh, okay.
	R4	Hey, can you do that, Adanna? Read it, and sort of figure out who gets, who gets points for what.
	Justina	Okay, well look at the possibilities for getting each number.

	Adanna	So I'm Player A? 'Cause my name starts with A.
	R4	Player A gets a point for what? [points to the problem sheet]
	Justina	3, 4, oh, 3, 4, 7, 8, 12.
	R4	Why don't you put those down here just to keep 'em so that [inaudible]. Okay, what about Player B?
	Justina	Okay. Okay.
	R4	Okay, you think it's fair?
	Adanna	No. [handling dice] 2 + 1 + 4, that's 7.
	Justina	How many, how many possibilities to get 5?
	Adanna	3 + 2 + 3 [handling dice], that's not fair.
13:54	Justina	Wait, let me see.
	Adanna	How do you earn a point? Oh.
	R4	How do you earn a point? Okay. Can we practice a minute? [rolls dice on Adanna's desk] Okay. What, what did we just do?
	Adanna	Just rolled the dice, and it came out 1+1+1, which is 3. So Player A gets 1 point.
	R4	And she doesn't get anything. Okay. [rolls dice on Adanna's desk] What about this time?
	Adanna	3 +, 3+3+3, which is 9.
	R4	So? [pointing at problem sheet]
	Adanna	She gets a point.
	R4	Okay. But we haven't started yet, but that's what it is. Do you think it's fair?
	Adanna	[shakes head] Because Player …
	Justina	3+1+1, 2+2+1
	Adanna	Player B has the highest number, and there's like makes it harder …
	R4	But Player A has 12.
	Justina	Okay. [Justina does not appear to be involved in the conversation, but she is talking to herself as she writes on her paper.]
	Adanna	I don't know. [pause] [to R4] Start playing?
	R4	Well, you want to answer that question first. And Justina was fiddling around there.
		[off-task conversation]
17:58	Adanna	Is the game fair? Why or why not?
	Justina	[inaudible] These stupid games! Don't make up these games anyway. They all stupid and all boring. Oh well.
		[off-task conversation]
		[camera briefly moves to R3 with Lorrin and Shanei]
19:24	Justina	Oh, yeah. [writes on her score sheet]
		[The girls play the game while Adanna talks about other topics.]
25:40	T8	So, I'm just watching you. You're playing, you're still playing the first game with these?
	Adanna	Um humh. Yeah, we're playing the first game.
	Justina	No, it's the new game.
	T8	Right, but this game with 3 dice, this is the first game with the 3

		dice that you're playing?
	Justina	Yeah
	T8	I think you have a prediction there, right? Question number 1, is this a [inaudible].
	Adanna	Not yet, because we didn't play it yet.
	T8	Oh.
	Justina	Well, we were supposed to do it before.
	Adanna	Whose turn is it?
	Justina	I don't know, just go.
		[The girls continue to play the game while Adanna talks about other topics.]
27:27	Adanna	8
	Justina	Ya, you keep on winnin'.
	Adanna	Hmmm, that's 9 to 8.
	Justina	Wait, do I have 8? I gotta keep a count. [rolls dice]
	Adanna	Huh, 8. That's the last game. [rolls dice, looks at the outcome, slams her hand on the desk] You won!
	Justina	Uh uh. [writes on her paper] Okay. [rolls dice] Okay. I won. I guess it's a fair game. You had a close chance of winnin'. But first [inaudible]. [Takes another sheet of paper and writes heading for each of the possible sums.]
	Adanna	What is the numbers that come up the most, 5, 6, and 9?
	Justina	Let's see. [looks at the outcomes she recorded] 8 is one of 'em. 8 came out, what, 5 times, no, 6 times. 7, 7 only came up one time. [writes on her paper: "8 came up 6 times. 7 came up 1 time."]
	Adanna	The highest numbers didn't come up, right?
	Justina	Let me see. 9 only came up once, and 9 is one of the high numbers. 11 and 10 [each came up once], yeah, that's true. The highest numbers didn't come up that much.
30:30	T8	Can I ask you ladies something else? You finished playin' the first game, and I just heard you making some observations about it and asking some questions, which number came out the most? Did the highest numbers come out …
	Adanna	Um, the lowest numbers.
	Justina	The highest number is 8 [most frequent], the lowest is 7, 7 and 9 and 10 [only came up once].
	T8	So now, if you can, if you consider the question, is this a fair game? You played one game, somebody won, and you asked yourself, looked at what actually came out. So, do you have some information now with which to make a prediction? 'Cause you're gonna play some more, right? What do you think?
	Justina	I predict that … [J&A begin talking over each other.]
	Adanna	Well, I think it's a fair game. I'm gonna change my mind.
	Justina	because …
	Adanna	because on the other game we played before this …
	Justina	Most of the high numbers have, um did not come up that much,

		and the lowest numbers came up more often. No, wait. Even though Player B had the lowest numbers, I mean high numbers, it still won. Maybe it's a fair game.
	Adanna	It's a fair game. Because you remember on the dice game last time we played it?
	Justina	Yeah.
	Adanna	The, they gave Player A all odd numbers and Player B all even numbers.
	Justina	No, but the last time the dice game, it wasn't fair.
	Adanna	But this one …
32:12	Justina	Okay, let's play again. I want to be Player A this time.
		[J&A set up their papers. Off-task conversation. Justina rolls the dice and points them out to Adanna, who is talking about other topics.]
34:05	Justina	You rolled?
	Adanna	Huh?
	Justina	You rolled?
	Adanna	No. Who's Player A this time?
	Justina	Me.
34:49	Adanna	[rolls the dice] 8.
	T8	Who got 8?
	Justina	I get 8.
	Adanna	You do.
		[J&A continue to play while conversing about other topics.]
39:00		[end of CD 123D]
		[begin CD 124D]
1:42	T9	So what do you think, guys? The game is fair?
	Justina	Um, I don't think it's fair. 'Cause Player B, I only have one point. Player B has …
	Adanna	2.
	Justina	No, 5. You ain't keepin' track.
	Adanna	How did we …
	Justina	Yeah.
	Adanna	I just put the numbers.
	T9	Okay, so Player A gets only one? Player B gets 5? So, who gonna win, you think?
	Justina	Player B.
	T9	Okay, let's finish it. See who's gonna win.
		[Justina & Adanna continue playing under T9's supervision.]
4:07	Adanna	For the first time, we see 11.
	Justina	I am so bored, I wanna go home.
	T9	It's your turn.
	Adanna	We got like 30 minutes.
	T9	Yeah. We have to, we have to play a little like 4 games or something.
		[Justina & Adanna continue playing under T9's supervision.]

7:34	Justina	We even now.
	T9	Yeah, you believe this? You're even.
	Justina	[rolls] 7. I win. [score 10-9 for Player A]
	Adanna	I hate her. She won! She, This is like the second time she won.
	Justina	Who won last time?
	T9	Who won last time?
	Adanna	But Player B …
	Justina	Player B won last time and now this time, Player A wins.
	T9	So Player A wins, all riiight. OK. So what do you think, it's fair or not fair?
	Adanna	Fair.
	Justina	I think it's fair.
	T9	Why?
	Justina	Because each player has um a good, yeah, each player could win.
	T9	It's fair either way?
	Justina	Like, any player, Player A and Player B, both have the equal, I can't, I forgot the word. they could just both, they are both able to win.
	T9	Why? I mean, what, why you think it's that? Why? [Justina looks at her score sheet but does not reply.]
	Adanna	Winnin' has 2 n's, right?
	T9	'Cause what?
	Adanna	Winnin' has 2 n's, right?
	Justina	Win?
	Adanna	Winnin'!
	Justina	[laughs] I cannot believe that you just asked that question.
	Adanna	No, seriously, like, one time …
	T9	So Adanna, why you think it's um it's fair game?
	Adanna	Huh?
	T9	Why you think it's fair?
	Adanna	Because they each had a chance to win one game.
	T9	Chance to what?
	Adanna	To win one game. If it wasn't fair, [unclear]
	T9	So maybe there will be another game, so who's gonna win? [no response]
9:35	T9	[to Justina] So why you writing 1, 1 plus 2? Is this the only way to get 4? 1, 1, 2?
	Justina	For 3?
	T9	4. What other number can get 4? With 3 dice. [pause, no response] You think 1, 1, 2 is the only 3 number you can get 4?
	Justina	I thought so.
	T9	Okay. Good. Even if you have different colors?
	Justina	Different colors don't mean anything.
	T9	Doesn't mean nothing? Okay.
		[While Adanna & Justina talk about Michael, Jackson, the camera shows Adanna's paper. She kept a tally score of the two games, 9-

		10 and 10-9, and wrote: "Yes it's a fair game because in the first game Player B won and on the second game Player A won. If it wasn't fair Player A will have kept on winning like the last dice game when Player A had even numbers while Player B had odd numbers."]
11:28	T9	Maybe we can play another 2 games, see if this is true or not. Somebody gonna win and somebody gonna lose. Let's play a game.
11:44	Adanna	This game is boring. Can we add like some Ludacris song into it?
	T9	Okay. Okay, who's Player A?
	Justina	You wanna be Player A?
	T9	She's Player A, OK. So let's start from beginning.
12:26	T9	[after 20 seconds] OK, great. We're gonna start. Justina wanna start?
	Justina	She's Player A.
	T9	Okay, you start, Adanna.
	Adanna	I started last time. No fair.
		[J&A play under T9's supervision. They continue their off-task conversation while playing.]
15:17		[T9 notes an error in Justina's scorekeeping – a sum of 10 should be a point for Player B, not A. Justina corrects it.]
15:40		[Justina goes to the rest room.]
16:10		[T8 takes Justina's seat, after asking J&A whether she can play in Justina's place.]
	T8	Okay, so is it Justina's turn?
	Adanna	It's my turn. [rolls] 7.
	T8	7, so that's …
	Adanna	That's my point.
	T8	And the, which player is that?
	Adanna	Player A.
	T9	7 get the A, yes.
	T8	Okay. So I'm gonna record just like she does. 3 + 3 + 1 = 7. [rolls dice, drops one] Oops, an illegal roll. [rolls again] 4, 2, and 2. 8.
	Adanna	That's my point.
	T8	Player A, uh, so that's 4 + 2 + 2 = 8.
	Adanna	[rolls] Oh. 3 + 2 + 2 [pause] 7.
	T8	Okay. 3 + 2 + 2 = 7.
		[Play continues, with Player A in the lead 7 – 2.]
17:53	Adanna	[rolls] 4 + 1 + 3
	T8	Can I ask you something? You said 4 + 1 + 3, and I'm just noticing what I wrote down. I wrote down 4 + 3 + 1. Is that the same thing?
	Adanna	Either way it's 8.
	T8	Either way it's 8? Okay. I just wanted to know.
		[Play continues.]

19:00	T8	Oh! [counting score in A's column] 1, 2, 3, 4, 5, 6, 7, 8, 9.
	T9	Almost there. Come on, Adanna.
	Adanna	I got 8.
	T8	Uh oh. Um, okay, well, we could cross check if you were writing down, I mean it's okay, because, um, Justina is recording. But, so you wanna, wanna trust it as 9? Okay, as long as you agree. 'Cause this one didn't count [a misplaced sum in A's column that was crossed out]. All right, my fault.
	Adanna	Your turn or my turn?
	T8	Um humh.
		[Another scoring discrepancy: Adanna has 9-6, T8 has 9-5. They agree on 9-5.]
		[Adanna rolls 4,3,1 and Player A wins.]
21:32	Adanna	I win! For the first time in my life I won.
	T8	Okay. So how many games in total have been played?
	Adanna	3.
	T8	3. So maybe this will give you additional information to rethink the question. Is it a fair game, and if so, why? [gets up as Justina returns]
	Adanna	Yes.
	Justina	[tells Adanna about her experience in the hallway]
	T8	[to Justina] I took your place, but I don't think that had anything to do with Player B losing.
	Justina	Player B.
		[J&A continue talking about what happened in the hallway and other topics.]
24:18	T8	Can I ask you something? Back in the first game, you were saying something about the highest numbers. What do you mean by that, when you say the highest numbers?
	Justina	The numbers that come up the most.
	T8	The numbers that come up the most. Okay. I still have to ask you a question.
	Adanna	[unclear] numbers come out the least.
	T8	Are those the numbers, are you talking about the numbers that are rolled on each of the dice, or the sum?
	Justina	The sum.
	T8	The sum. Okay. So …
24:48	R1	[announcing to class] Okay. I'd like you to start writing up your results, and if you've finished writing them on your paper, you might want to start writing them on overheads so that we could share, uh, what you think about the fairness of the game and your findings and why. And if you think the game is fair, I need to know why. If you think the game is unfair, I need to know why. And I'd like you to make it fair if it's unfair. Can you make it a fair game if you think it's unfair? Do you understand the question?
	T8	[to J&A] Just so I can understand what you're saying, does player,

		does one player have more high numbers than the other?
		[an argument erupts across the room]
	T8	There's a 12 here. 12 is the highest number that you can get at all, right? And that's over here. And then you've got 9, 10, and 11 over here, but then 8 and 7 is over here, so what is it about having high numbers makes it fair or not fair? Just something to think about as you're writing up your … Which is, is it fair? If so, why? Is it not fair? Why not? And how, what will make it fair?
	Justina	Okay.
		[J&A chat as Justina prepares her transparency. It shows examples of sums in Player A and Player B's columns, and has the incomplete sentence: "I think it is a fair game because both players have a".]
34:34	T8	Okay, so Justina, I think you should go ahead and finish recording that before you leave. Finish recording those.
34:39		[end of CD 124D]

Date: 12 May 2005 Grade 7
Location: Hubbard Middle School
CD: ROLE 125A-126A
Transcribed by: Kathleen Shay
Verified by: Jeremy Milonas

Time	**Speaker**	**Transcription**
3:08	R3	[to class] Can you go ahead and tell my friends what you did yesterday?
	voices	Some dice thingy. I was only here for a couple of minutes. You was here for like half an hour.
	Jerel	We played the die game.
	Ian	Oh yeah, I remember. We was playin' this dice game and then Jerel cheated, but then I won and then he won again.
	Jerel	I beat you, bro. Don't even say I cheated!
3:44	Chris	[talking to G4] … have to get like these certain numbers for Player A and certain numbers for Player B. That's all I remember. And after I had to leave, so …
	G4	How many, how many dice you have?
	Chris	We had 3 dice.
	G4	3 dice. And then what do you have to do?
	Chris	We had to like get certain numbers [coughs].
	G4	Guess the numbers, you mean?
	Chris	No, get certain numbers when you roll them. Uh, so the sums equaled up to um …
	G4	So you had to guess the number or you had to guess the sum?
	Chris	You had to get the sum.
	G4	Guess the sum.

	Chris	Yeah. [nods]
	G4	Uh huh. Then you had to, you kind of keep a record of that?
	Chris	You have to keep a record. That'd be like [inaudible] say you had two 1's and a 4, then you put 4, 1, 1, or …
	G4	Can you, can you show me like what you did like?
	Chris	All right. [reaches for dice]
	G4	Like say if, okay, give me an example.
	Chris	Like say I roll it, and it's 3, 2, 2. This is like Player A and Player B [writes these on his paper]. And say Player A got the point [places a tally mark under "Player A"]. And it'd be like 3, 2, 2 [writes these numbers on the side].
	G4	So how can you say Player A got the point?
	Chris	I don't know. I'm just saying, I forget the numbers that have to come up. Then you gotta do it again [rolls], it'd be 4, 3, 4. So then you get 4, 3, 4 [writes these numbers below the previous ones], like that.
	G4	So who, who gets the point here?
	Chris	I don't remember.
	G4	Okay, so is there any, any, any, any criteria for getting a point to Player A or Player B?
	Chris	What do you mean?
	G4	Is there any rule like if so much is the sum …
	Chris	You have to get a sum, and then you have to get the exact …[camera abruptly switches to Chanel]
5:20	Chanel	[to G7 and 2 young boys] … more than these, and I thought that wasn't fair because then Player A can win more times than Player B.
	G7	Okay. I'm gonna ask you to repeat that again so we can listen to it again later. You, you originally thought it was fair, right? Why did you think it was fair?
	Chanel	Because it has the same amount of numbers.
	G7	Okay. So they each have the same amount there. Okay. And you decided it might not be fair. Why was that, again?
	Chanel	I decided it wasn't fair because over here, on for Player B, they, it's, these numbers are most likely to come up, because now we have 3 dice.
	G7	Okay. So they are or are not more likely?
	Chanel	Aren't more likely.
	G7	Okay.
	Chanel	And over here [pointing to the problem sheet] Player A has, are most likely to come up, now that we have 3 dice.
	G7	How did you, how did you decide that these [Player A's numbers] were more likely to come up than these [Player B's numbers]?
	Chanel	Well, because yesterday , because yesterday when we played this game, it did like 2, 4, and 2 [points to an outcome on her score sheet], which is 8, and then they have another side where you can

		get 4 + 3 + 1. So, you can get 8 that way. Like, or you can do, or you can get 4 + 2 + 1 [shows with dice], well, 4 + 2 + 1 for um 7. And for like 4, for like 4, it's kinda hard, it's kinda hard for you to get 4 um 'cause you need um you need 3, you need 2 dice now that you do 1 + 2, and then [there is a 1 on the third die], but you're gonna get that 1. But you get 8 …
	G7	Okay. There's only one way you can get 4?
	Chanel	Yeah. Now for 5, over here, we use 2 + 2 + 1, but then you also use 3 + 1 + 1 and you, you'll get 5 [demonstrates with the dice].
	G7	So how many ways were there to get 5, then?
	Chanel	It's two ways to get 5. But, what I'm sayin' is that it's only, it's only, uh 6, uh, it's that way to get 6 [2+3+1], and it's, oh no, that's the only way to get 6. And for 10, like, it's 4, 8, 9, 10 [places dice 4, 4, 2]. Then you can do 3, 6 [places dice 3, 3, 4] …
	G7	Have you listed all the ways that you could possibly get each of these numbers?
	Chanel	[shakes her head to indicate no]
	G7	Like you did with the [inaudible]. So you've got a pretty good idea that one is probably easier to get than the other. Which one did you say again is easier to get, this list or this list? [pointing at the numbers listed on the problem sheet]
8:32	Chanel	I think this list is easier to get.
	G7	Okay, so you think Player A should win.
	Chanel	Well actually no, I think this list.
	G7	Player B should win. Okay. Can you make a list of all the possible things you could get there …
	Chanel	Okay.
	G7	'Cause we have, that might give us a better idea of what will actually happen. Okay, okay, and then I'll come back and look at that. Great job so far. You're off to a good start.
		[Chanel writes "4 + 3 + 3 = 10 2 + 1 + 4 = 7"]
9:18		[camera moves to G6 sitting down with Kianja & Brionna]
9:36	G6	I'm here to determine what you've been doing. Tell me what you've been doing.
	Kianja	Huh. What did you say? [writing her sample space]
	G6	So tell me what you've been doing. What's, what's the game here? What are you doing?
	Kianja	Brionna, explain what we doing.
	G6	Explain what, explain to me what you're doing.
	Brionna	What paper have you got?
	Kianja	Explain what we been doing.
	Brionna	I need the paper and the questions. [Kianja gives her the problem sheet.]
	G6	Okay. [reading] Roll 3 pyramidal dice. If the sum of the 3 dice is 3, 4, 7, 8, 12, Player A gets one point, Player B gets zero. If the

		sum is 5, 6, 7, 9, 10, 11. All right, so, what do you think? Is this a fair game?
	Brionna	No.
	G6	Why not?
	Brionna	On the paper over here [picks up one of the transparencies from yesterday].
	G6	[reading] This game is not fair because Player B has more ways to get 5, 6, 9, 10, 11.
	Brionna	[inaudible] [shows G6 the sample space that Kianja prepared yesterday]
	G6	Ah. Okay. These are the different ways, okay. Okay. So the ones you circled, why did you circle these? Why did you circle 5, 6, 7, 10, 11?
	Brionna	That's B.
	G6	'Cause that's …
	Brionna	B.
	G6	I see.
	Kianja	What he ask you?
	Brionna	Why you circled these. 5, 6, 9, 10, 11, yeah, that's B.
	G6	[referring to Kianja's sample space] So, you say there's 6 ways to roll a 5, 10 ways to roll a 6, …
	Brionna	7 ways to roll a 9, and 6 ways to roll a 10, 3 ways to roll 11.
	G6	All right. So, so let's see. So who's more likely to win the game?
	Brionna	[inaudible]
	G6	And, and you say it's because B has more ways.
	Brionna	Like it has like different ways like, where's the other dice? [looks around the desk for dice] Um, 3, 2, 1 [arranges the dice to with these outcomes]. That's one way. [inaudible] other ways [inaudible] There's 10 ways [unclear] to this. And also you can…
	G6	Now, now here's somethin' I wondered, if you could explain to me. So you've got a 3+ 2 + 1. Now isn't that the same thing as 1 + 2 + 3?
	Brionna	It is, but on the dice, on the dice, you could write this one, this could be 3, this could be 1, and this could be 2 [turns the dice to demonstrate]. 'Cause they come up different on each dice.
	G6	Okay. Okay. So the order in which you write it, you're sayin' that makes it different.
13:22	Brionna	Yeah.
	G6	So how do you know you've got all the possibilities? The ways to write, to get to 6?
	Brionna	Because the dice, each die goes to 4. 1, 2, 3, and 4 [turning a die in her hand]. So that's why I try to get, like each way to get 4, each way to do it. [coughs] You have, like, because the highest you go up to is one 4 [?], plus 1 for the other numbers.
		[PA announcement].
	G6	I'm getting' a little bit lost here. Let me say my question one more

time. I was wondering, so you've got, say that you've got 10 ways to roll a 6. Now how do you know you've got all of 'em? How do you know there isn't one combination that you're missin'?

Brionna Um [laughs].

G6 'Cause maybe when you were goin' through this, you at first left out a certain number and then realized you missed one, and then wrote down the new one, new combination. How do you know you're done? How do you know you have all of them?

14:98 Kianja We know this because, you took 4, right? Say 4. You know that we have all the ones for 4 because the highest number on there is 4, right? The highest number on the die is 4, right?

G6 Okay.

Kianja And the next highest number is 3, but we have 3 dice, so you can't use 3 as any of the, any numbers that you can use to [makes a hand motion].

G6 And why is that?

Kianja Because 3, anything you have to use a 1, and then, and then we don't have halves. So the only way you would use a 3 is 3 plus half plus half equals 4.

G6 And there are no halves.

Kianja Right.

G6 So the smallest number, if you wanted to use the 2 …

Kianja [talking at the same time] It would be, that's the largest number on this that you could use to create the sum of 4, would be 2. So we tried to use 2 in every one.

G6 So, one of these, if one die were to roll a 3, the other two, no matter what you roll, the very smallest they could be is what?

Kianja 5.

G6 5, and that's bigger than 4.

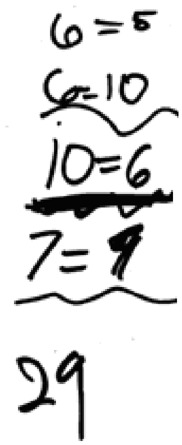
Kianja Right. That's what we did with the rest of 'em. We did that but 2, I mean 5, you know, the largest number would be 4. And you couldn't use a 4, so we did 3 and 2. 3, 2, and 1. And then 6, the highest number would be 5, but we don't have 5, so we did 4. And then we found ways to make that 5 and whatever, and we added whatever you needed to add.

G6 So you've worked your way down, in a way. Worked your way down. Let' see. [looks at Kianja's sample space] Okay. Okay. Interesting. So what's the, what's the total number of ways that uh Player B can win? What are the number of combinations here? How many ways.

[Kianja passes a paper to Brionna. Brionna says something inaudible.]

G6 So, oh, so you said B can, has more ways of winning than A. So how many ways is that? [Brionna points at the paper.] 32. And A has 26. Um. OK. Interesting. Interesting. Um, now did you try playin' this game against each other? Did your results come out

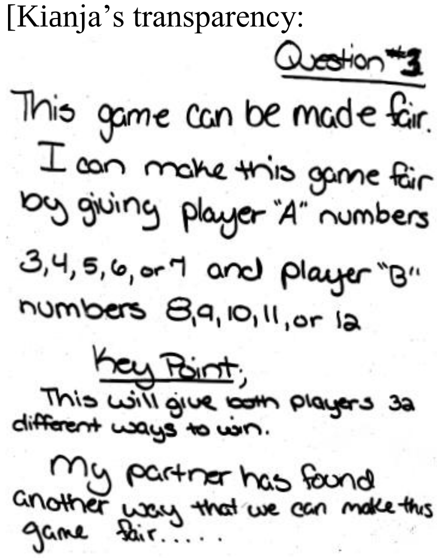
		and match this?
	Brionna	Yes. [Shows G6 another paper that Kianja just handed her.] B has 7 points and A has [inaudible].
	G6	7 total points 3. How many, how many games did you play?
	Brionna	We played 9 times, 10.
	G6	You played ‘til 10? Like for each, or, you rolled 10 times? Or you waited ‘till one person got 10 points?
	Kianja	[over a lot of background noise] You played the game 10 times.
	G6	So one game, one game involves playing to a score of 10? To a score of 5?
	Brionna	Like how many points add together.
	G6	Oh. Oh okay.
	Kianja	[circles individual outcomes of Player B on Brionna’s score sheet] I won this game, I won this game, this game, this game, this game. Look, those are all just [inaudible].
18:49	G6	Now, do you think it’s possible that, so Brionna you were Player A, do you think it’s possible that, playing this game, you know, playing 10 times, would it have been possible that you would have won? Could that have happened?
	Brionna	[inaudible]
	Kianja	It could have, but it’s not likely.
	G6	But it’s not as likely, okay. Okay. Let’s see. Okay. Question 3. If you think the game is unfair, which you do, how could you change it so it would be fair?
	Brionna	Where’s that paper at? [looks for paper] [to Kianja] Do you have the paper for it? The fair game? [Kianja passes a transparency to Brionna.]
	G6	It’s backwards. Can make it fair by giving Player A get the numbers 7, 3, 4, 8 [11 or 12]. Player B gets the numbers 5, 6, 5, 10, 6, or 9. Let me see that paper again, just for a moment. OK, so why, why would this make it fair? I guess this one will explain.
	Brionna	[refers to Kianja’s paper from yesterday] ‘Cause each of ‘em, [taps each number with her pen] together, is 29. And 6, there’s 5 ways, no 6 is 10 ways, 10 is 6 ways, 7 is 9 ways. And that equals [points to 29].
		[Brionna has misinterpreted Kianja’s notation.]



	G6	You've got, you've got a 6 and a 6 twice there. Is one of those just wrong?
		[Brionna looks at paper.]
	G6	10, 6, and 9. 5, oh, 5, Okay.
	Brionna	This is [unclear]: 5 is 6, 10 is 6, 6 is 10, and 9 is 7.
	G6	Right, right. So there are 7 ways to roll a 9, 6 ways to roll a 10, I'm sorry, 10 ways to roll a 6.
	Brionna	Put this way, write the number [writing headings for the two colums: "ways #"].
	G6	Yeah, maybe, maybe it would be good to write somethin' to distinguish so you don't get confused. Right. Okay.
	Brionna	And all of these add up to 29. And that's 29, and [runs her pen over the similar column for Player A, puts her hand over her face].
	G6	[to Kianja] So what are you doing right here, you're just writing up your final results? Is that what is goin' on?
	Kianja	[nods]
	G6	Is this so you can present it to the rest of the group? Okay?
	Kianja	[nods] [writing her sample space on a transparency]
24:42	G6	I think I'd still like to understand fully, uh, Kianja, what's your organization, what your scheme is here, to make sure you've got every single way to roll an 8.
	Kianja	[utters a few words, inaudible]
	G6	So you just discovered some new ways to roll 8?
		[inaudible, if any, response] [Kianja continues writing the sample space.]
27:46	Kianja	Oh, my gosh! [writes the number of combinations for each sum. She has found all 64.]
28:48	Kianja	I shoulda known it was wrong. You wanna know how? 1-1, 3-3, 6-6, 10-10, 12-12. [pointing out the numbers at equidistant from the center] I shoulda known it was wrong.
	G7	What was wrong? I missed. You're gonna have to fill me in. Do you have something that was different?

	Kianja	There were 3 more missing. There was 3 missing in this one, 3 missing in that one.
	G7	Okay. How'd you figure out which ones are missing?
	Kianja	I don't know.
	G7	You don't know. When did you decide there were some missing? When you started writing them here?
	Kianja	[nods, words unclear]
	G7	Okay. All right, so …
	Kianja	I gotta write this over 'cause I did it wrong 'cause you have to have 3 more, then the numbers are gonna change.
	G7	Well, let's take a look at this [inaudible] so far.
	Kianja	Ohhh! Oh wait wait wait wait wait. [looks at her paper]
	G7	Okay, can you, while she's counting up there can you tell me what you guys decided here? I didn't, I haven't been here so I didn't get filled in on this.
	Brionna	These numbers [shows G7 the list 6=5, 6 = 10, 10 = 6, 7=9] [inaudible] 5, 10 , 6, 9 …
	G7	Okay, so that's the number of ways to get each of those?
	Brionna	Yeah.
	G7	So you, so you decided the game was not fair. And who did you decide it was, who's gonna win?
	Brionna	B
	G7	B was always gonna win? And it was because of all these different ways [points to list].
	Brionna	Yeah. And here's all the games. [shows score sheet]
	G7	Oh, okay.
	Brionna	It's 7 and 3 [pointing at the score: 7 points for B, 3 for A]
30:26	G7	Okay, so what are you changing? Kianja, before you write further, what are you changing here?
	Kianja	Um, um.
	G7	Kianja, can you tell me what you're gonna change about this?
	Kianja	Brionna? Didn't you realize this was question 1 and I need to change question 3?
		[K&B talk about what transparencies must be reworked.]
31:55	G7	Before you even start writing, some of the stuff here you probably don't need to change. So question 1, this one was, is it a, is this game fair, why or why not. You said, you guys decided it was unfair because B has more ways to get its numbers. Okay, so B has how many ways?
	Kianja	Well actually 2 because
	G7	We can, we can look at these numbers, that list you just finished. All right, let's take a look at that. So …
	Kianja	What's the numbers? 5, 6, 6, 10, 9, 26, 32, 35 ways.
	G7	Okay. So you can just change that to a 35. [Kianja makes the change.] Okay, now count up the ways for A.
	Kianja	2, 2, 5, 17, 29.

	G7	Okay. And you can show that with your chart right here. Great. So number 3, then, is, if you think the game is unfair, how could you change it so it would be fair. So that's the one you were startin' to think about, right? How can you make this a fair game? Brionna, what do you think? How can you make it a fair game? [camera jumps around here]
	G7	Go ahead, what do you think? No, she's counting. What do you think? How could you, look at what you guys did here. [Brionna looks at the sample space. She does not appear to say anything.]
	G7	It wasn't fair before because B had 5, 6, 9, 10, and 11.
34:35		[Brionna makes a table with 2 columns: Column A has 8, 5, 9, 11, 3 and B has 7, 10, 6, 4, 12. She writes the numbers in pairs, first column A, then column B.]
35:21	G7	Okay. So tell me how you decided this.
	Brionna	'Cause each number [says some numbers as she points at different outcomes in the sample space], like she was sayin' before, so I'm sayin' if you know they equal up the same thing.
	G7	So how many chances of winning do they both have now?
	Brionna	Um, that's 12 and [begins writing].
	G7	So you both have 32 chances of winning a point now. Very good. Is she agreeing with you with what she wrote there? Let's see what she said. 3, you've got 3 and 4 on opposite lists.
	Kianja	I did it another way.
	G7	How did you do it?
	Kianja	I did it going across the top. Player A gets these [the first 5 sums] and Player B gets these [the last 5].
	G7	Oh, okay. And, okay, so, you know what? Let's write both of yours up. 'Cause it's the same idea. We'll show, we'll put these both together to show you both, you've got two ways. [inaudible]
		[Kianja's transparency:


Question #3

This game can be made fair.

I can make this game fair by giving player "A" numbers 3,4,5,6, or 7 and player "B" numbers 8,9,10,11, or 12

Key Point:

This will give both players 32 different ways to win.

My partner has found another way that we can make this game fair.....

40:00		[Kianja rewrites her transparency for Question 1.]

This game is not fair.

This game is not fair beccuse player B has more ways to get 5, 6, 9, 10, or 11. Player B has 35 ways and Player A has 09 ways to wir

43:00		[camera moves to Ian & Jerel's table, where a dice race mat is set up.]
43:23		[end of CD 125A]
		[begin CD ROLE 126A]
0:30		[Jerel is playing the dice race game with T3. The mat shows columns labeled 1 – 14.]
	T3	Whoever's blue got 3 and 12? [referring to blue markers on the mat] Is that how we're doin' it?
	Jerel	Yeah.
	T3	'Cause I wanna play, I wanna know what …
	Jerel	I want, I want, I want, uh I want these two, [markers on] 4 and 11.
	Ian	You can't have 4 and 11.
	Jerel	Why? Why can't I?
	Ian	[raises his hand] You can have 4 and 11. Yeah, that's the best, that's the best.
	Jerel	[to T3] And you got 3 and 12. Ready to play. [shakes the dice in his hand]
	Ian	You gonna lose. [not clear who he's talking to]
	T3	Whoa, whoa. 3 and 12. Why can't I get one of these numbers? Why don't we switch up? You get one of the higher …
	Ian	Because I already should have picked.
	Jerel	That's how the game go.
	T3	Oh, so one person gotta have 12 and 3?
	Ian	I asked him that, he said no.
	T3	So what is the objective? First person to get to what?
	Ian	This one right here.
	Jerel	He says this game is goin' to here. [Ian draws a line half way up the mat.]
	Ian	It wasn't that one, it was the next one.
	Jerel	Oh, well.
	T3	Let's say, okay, so first person to get to the fourth block? All right. I'm cool.
	Ian	I'm gonna make a line. [draws over the line and makes it darker]
	T3	Now, now if I win, you're not gonna say I'm cheatin' in there?
	Jerel	No.
	T3	All right.

	Jerel	7, 8, that's nobody's move.
		[Jerel & T3 take turns rolling 3 pyramidal dice.]
5:42	Jerel	[to camera] I'm winnin' [pointing his thumbs to his chest]. I'm the best, remember that. I'm, the champ is here.
	Ian	I retired. I'm too much of a champ.
		[The marker on 11 has moved up 4 spaces. Each of the others has moved 1 space.]
	T3	So whoever goes over the line? Is that what, is that the objective?
	Ian	Yeah.
7:17		[Jerel's marker on 11 crosses the finish line. In this game, P(3 or 12) = 2/64, P(4 or 11) = 6/64.]
7:20		[camera moves to Jeffrey's table, where they appear to be playing a variation of the race game]
12:55		[camera moves to Kianja & Brionna with G7, playing dice race game. There are markers on each number, 1-14. G7, Kianja, and Brionna take turns rolling three pyramidal dice and moving markers forward according to the sum.]
13:41	G7	[to Kianja] You have 7, right?
	Kianja	[to Brionna?] You got 9, right?
	G7	Yeah. 7 and 9.
14:08	G7	How come you keep pickin' 7?
	Kianja	Well, she picked 7 [inaudible]. Eight. Um, she picked 7 the first time, and then 9. Well, 7 won and 9 won. I told her to pick 9 'cause 7 won [inaudible].
	G7	Oh, okay.
18:33		[The marker on #7 reaches the finish line. Kianja wins.]
18:45	G7	Don't move anything [the markers] yet. We're gonna play again, but I want you to look at this here. First of all, why did, did 7 win twice?
	Kianja	Yes.
	G7	What else won?
	Kianja	9.
	G7	We had 8 win also?
	Kianja	Can we play one more time before we talk about it?
	G7	Sure.
		[The girls move all the markers back to the starting position. G7's words are not entirely clear, but she appears to tell K&B to each pick two numbers, taking turns.]
19:25		[R3 stops by the table and asks about the game.]
	G7	Kianja won. We went three-way. Kianja won.
	Kianja	She [Brionna] got ice cream bar, too. 'Cause we were both ahead of her, so …
	R3	What number did you have, [G7]?
	G7	I had 6.
	Kianja	I had 7, she [Brionna] had 9. [to G7] So, you go.
	G7	We started to talk about it. Kianja just said, could we play one

		more time before we talk about it. She's got a conjecture here, she wants to test it out.
	R3	Okay. That wasn't quite the game that I had in mind. They were supposed to get to pick 5 numbers …
	G7	Oh, we were getting ready to do that now.
	R3	… as a team …
	G7	Oh, I get it. I mis-, misunderstand it. We'll get it.
	R3	That's all right. It's all good.
	G7	As a team they pick 5 numbers. Gotcha.
	R3	Yeah. But it's all good if you already played.
	G7	Well we'll go again. We were, they were going to pick something out now, anyway. So let's do that. Pick 5 numbers between the two of you. Let's do that. You guys pick 5 numbers. Five numbers.
	Kianja	We gotta pick 5 numbers now?
	G7	What 5 numbers do you guys wanna pick?
		[Kianja writes 7, 9, 5, 11, 8]
	G7	8? I guess I pick 6.
	Kianja	You pick 6?
	G7	Yeah. Do I get 5 numbers also? So, if anything, you get those 5, and if it's anything other than those 5, are you listening? You guys picked these 5. If it's anything other than those 5, I win. Okay?
	Brionna	Okay.
	G7	All right. Who's going?
21:16	Kianja	Did you say anything other than those five? Wait a minute, let me see if that's fair.
	G7	Okay.
		[Kianja writes "1, 2, 3, 4, 5, 10, 11, 12".]
	G7	You guys have 11.
	Kianja	Oh, we have 11? [crosses out 11]
	G7	You have 5, 7, 8, 9, and 11. [Kianja writes these numbers.]
22:02		[camera moves to Ian & Jerel with R3. R3 challenges I&J to a dice race game.]
22:21	R3	You gotta get 5 numbers.
	Jerel	[points to the clear markers on the game mat as he counts] From 1, 2, 3, 4. [The markers are on 4, 5, 6, and 10] [R3 points between 8 and 9.] We don't want 8. [camera jumps] 1, 2, and 5, 4.
	R3	So what do you … So write them down.
	Jerel	All right.
	Ian	6, 4, 5, 7, and 11. [Jerel writes 4, 5, 6, 7, 11.]
	R3	I don't know, you sure you wanna give me 8?
	Ian	Yeah.
	R3	You sure?
	Jerel	Yeah, boy!
	R3	All right.
		[They take turns rolling 3 pyramidal dice and advancing markers

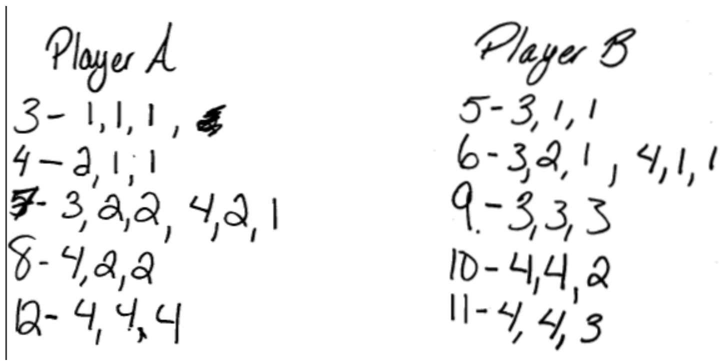
Time	Speaker	Transcription
		according to the sum. R3 calls for 8, which sometimes comes up.]
26:00	R3	[as 8 takes the lead] 8. You guys shoulda took 8.
26:23		[6 crosses the finish line. Jerel does a victory dance.]
27:29		[camera returns to Kianja & Brionna with G7.]
	Kianja	Yeah! [throws her arms overhead] I win.
	G7	All right. Enough, guys.
	Kianja	Look at our numbers. [The markers on the game mat are in a triangular arrangement.]
27:50		[Justina does a victory dance.]
28:09		[Chanel and Keisha do a victory dance.]
28:50		[camera returns to Kianja's sample space.]
	G7	What are you look at on your chart there? What did you expect to happen?
	Kianja	Because I thought 7 and 8 would be the top numbers because they had the most, right?
	G7	Okay.
	Kianja	But I guess it depends on odd numbers because it was 3 dice. So, the two top odd numbers are 7 and 9.
	G7	Okay. So if you could pick any 4 numbers to play the game again, [inaudible] odd numbers?
	Kianja	What 4 would I pick?
	G7	Yeah, what 4 numbers would you pick?
	Kianja	Well no, I wouldn't pick all odd numbers. I'd pick 7, 9, 8, and 6 [pointing to her sample space as she says this].
	G7	Okay. Very good.
30:27		[end of CD ROLE 126A]

Date: 12 May 2005 Grade 7
Location: Hubbard Middle School
CD: ROLE 125C-126C
Transcribed by: Kathleen Shay

Time	**Speaker**	**Transcription**
2:50	R3	[to class] Can you go ahead and tell my friends what you did yesterday?
	student	Some dice thingy.
	Chris	I was only here for a couple of minutes, so.
	student	You was here for like half an hour.
		[chatter]
3:13	Chris	We were playin' a dice game, and we had to try to, we had like different numbers, and I think …
	G4	What were the numbers?
	Chris	Uh those pyramid dice. So, we had to play them, and we had to get like these certain numbers for Player A and certain numbers for Player B. That's all I remember, and after that I had to leave, so.
	G4	So how many dice here?

	Chris	We had 3 dice.
	G4	Three dice. Okay, and then what do you have to do?
	Chris	We had to like get certain numbers [coughs].
	G4	Guess the numbers, you mean?
	Chris	No, get certain numbers. Roll them. So um, so the sums equaled up to um …
	G4	So you had to guess the number or you had to guess the sum?
	Chris	You had to get the sum.
	G4	Guess the sum, okay. Uh huh. Then you had to, did you kind of keep a record of that?
	Chris	Yeah, we had to keep a record. That would be like, just say like, say we had two 1s and a 4, then we'd put 4, 1, 1 and like …
	G4	Can you, can you show me like what you did, like?
	Chris	All right. [reaches for dice]
	G4	Let's say, okay, just give me a sample of what you did.
	Chris	I rolled a 3, 2, 2. Then this is like Player A, and Player, Player B [writing]. And, you know, say Player A got the point. And like 3, 2, 2, [writing] okay.
	G4	So how can you say Player A got the point?
	Chris	I don't know. I'm just sayin'. I forget the numbers. I had to sum 'em up and then you gotta do it again. You go 4, 3, 4. So then you do 4, 3, 4 [writing]. Like that.
4:45	G4	So who, who gets the point here?
	Chris	I don't know.
	G4	Okay. So is there any, any, any criteria for giving a point to Player A or Player B?
	Chris	What do you mean?
	G4	Is there any rule like, if so much is the sum …
	Chris	You have to get a sum, and then you have to get like different sums, like use the dice, that's all I remember.
	G4	So certain sum comes up then Player A gets a point.
	Chris	[nods in agreement]
	G4	So who won? Who won the ….?
	Chris	Yesterday I did.
	G4	Player A or Player B? Who won the game?
	Chris	Uh, yesterday we just played twice. Player A won both times.
	G4	Player A won. Okay. Do you have any reason why Player A won?
	Chris	[shakes head] It was, it was fair.
	G4	It was fair?
	Chris	Yeah.
	G4	So, what do you mean by fair?
	Chris	Like, like, I forget. Like you know when, you remember when we were playin' that and I had found different numbers that add up to them? Remember we had to roll like the number and some numbers that add up to them?
	G4	Um humh.

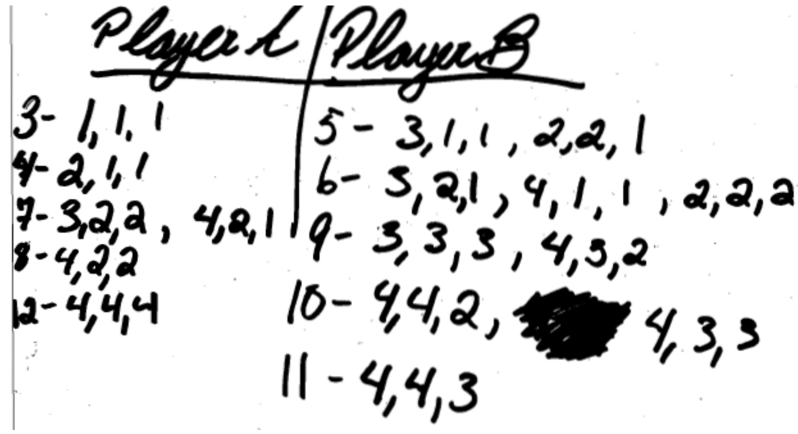
	Chris	Yeah, that's what I meant. You had different possibilities of getting those numbers.
	G4	You had some papers? You created something? Is this what you did? [reaches for papers]
	Chris	Here you go.
	G4	Can you, is this what you did?
	Chris	[holding paper] This is not mine. [picks up another paper] This is ours.
	G4	Is this yours, Chris?
	Chris	Yeah. Yeah, here you go, like different ways to get 'em. And they all had the same, 1, 2, 3, 4, 5, 6; 1, 2, 3, 4, 5, 6. And here we recorded them.



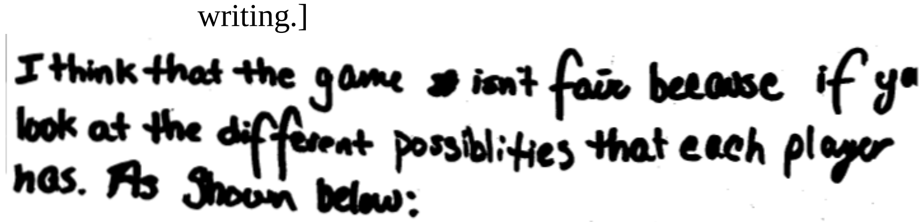
		But then this, and this, I don't know how she got those. I wasn't …[coughs]
	G4	Is this the game you played? Is this the game?
	Chris	[nods] Yeah.
	G4	Is this the game?
	Chris	Um humh. David, where Terrill at? Where Terrill at?
	G4	So Chris, what, what are the questions here? This question is, is this a fair game? Did you find it a fair game?
	Chris	Yeah. [nods]
	G4	Even though Player A both the times, still you feel it's, it's a fair game?
	Chris	[nods]
	G4	Um humh. What makes it a fair game? Is there any reason?
	Chris	I just did this [points to paper], that's all.
	G4	Uh huh. So play the game several times [reading] … So you mean to say the game is fair, there is no need for you to …
	Chris	Uh huh.
	G4	… change it to make it fair?
	Chris	[nods] Mr. [T5], where Terrill at? Mr. [T5]. Mr. [T5] [deep voice].
	G4	Chris, can we, can we talk about, can you tell me how many ways you can get this 3?
	Chris	Two, one.
	G4	One way? And how many ways you can get a four?
7:44	Chris	One.

	G4	Only one way?
	Chris	[nods]
	G4	So, which is that one way? There's 2, 1, 1?
	Chris	Yeah.
	G4	So, I would like to ask a question. If you get 2, 1, 1, okay?
	Chris	Mr. [T5]. Mr. [T5]. Mr. [T5]. Mr. [T5]. Yo!
	G4	If you get 2, 1, 1, and if you get 1, 2, 1, that's like, say [reaches across desk] …
	Chris	It's the same thing.
	G4	Say it's uh, say this yellow one is the first, okay? So let's say this is 1, this is, let's make it a 2, and this is 1, okay? [arranges the dice in this order] Look at this, 2, 1, 1, right? And if I, if I made this as 1, 2, 1 …
	Chris	Same thing
	G4	Do you think it's the same thing?
	Chris	They both add, they both add up to the same thing.
	G4	So why do you think it is the same thing?
8:44	Chris	Because they both add up. Either way it's gonna add up to …
	G4	Because they both add up to …
	Chris	Four.
	G4	Um humh. But, but, but do you think if this yellow one [die] is 2 and this green one is 1, and then this yellow one becomes 1, and this green one becomes 2 …
	Chris	It's the same thing.
	G4	Still it's the same thing?
	Chris	Yeah.
	G4	So you don't find any difference between the two?
	Chris	[shakes head]
	G4	Absolutely no difference?
	Chris	[looking down, rubbing his arm, shakes head]
	G4	And, and, and, what makes you this fair game? Is this like any, any, you did any counting? To be sure it's fair?
	Chris	[shakes head] I didn't do any counting.
	G4	How is that like, how do you decide that it's fair? I, I do not …
	Chris	I, I just did this and that's how I got it fair. [referring to his paper from yesterday]
	G4	Um humh. Is this you counted something?
	Chris	Yeah. 1, 2, 3, 4, 5, 6. [pointing to paper as he counts] And that's 6. 1, 2, 3, 4, 5, 6.
	G4	Okay. All right, so, so, so what makes you think it's fair? This is 6, okay.
	Chris	There's 6 different ways.
	G4	And, and what about this? What makes you think it is fair? Okay, I agree with you that this is 6 ways. These are 6 ways. But what makes you feel it's fair?
10:01	Chris	I dunno, it's just fair.

	G4	Okay.
	Chris	Mr. [T5]. Mr. [T5]. [stage whisper] Mr. [T5]. Mr. [T5]. Where's um Terrill? [off-topic chat]
11:10	G4	So would you like to write your observations here, Chris?
	Chris	[shrugs] I don't know if I have to leave [for play rehearsal] or not. [off-topic chat]
11:42	G4	Chris, would you like to write this on a transparency?
	Chris	All right. I need markers. [scratching his arm] Itch!
	G4	Would you like to right it.
	Chris	Yeah. I need a marker. David, where'd you get that marker? David, where'd you get that marker? [someone tosses a marker to Chris] [Chris writes:]
		I think that the game is fair because if you look at the different possibilites for all the ~~[illegible]~~ numbers you have 6 for each of the players.
13:25	Chris	Mr. [T5]. Mr. [T5]. Mr. [T5]. Mr. [T5].
	G4	What do you want, Chris?
	Chris	I wanna find out if I gotta go down [to rehearsal] or not. [off-topic chat]
		[Chris copies his sample space from yesterday's paper. For 5, he writes 3, 1, 1, and 2, 2, 1. The second outcome was not on yesterday's list.]
	G4	[points to 2, 2, 1] What's that?
	Chris	Yeah, I had forgot about that one yesterday. [Chris also writes an additional outcome for 6: 2, 2, 2.]
	G4	So what do you think now because of this?
	Chris	That Player B would probably have more possibilities.
	G4	Okay, so, so what, what does that mean? What are you thinking?
	Chris	It's not fair.
	G4	It's not fair? So what do you, what do you do?
	Chris	[continues to write, does not respond] Damn, I missed a lot.

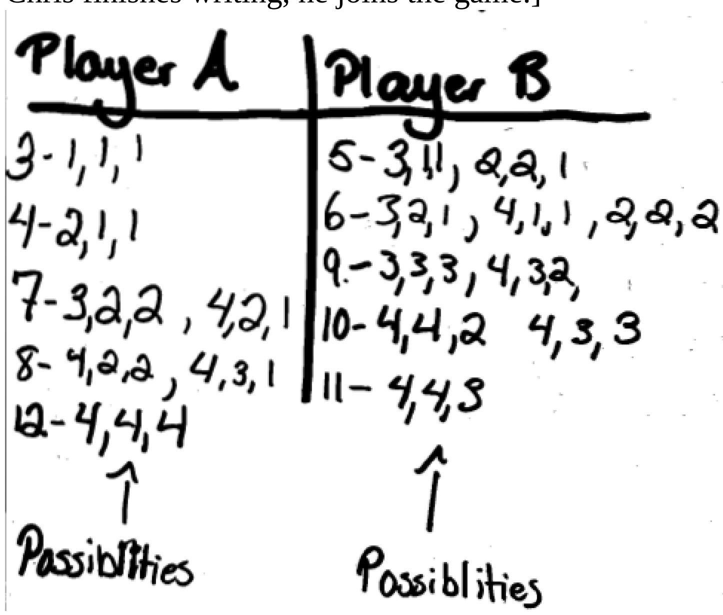


		[His sample space shows 6 outcomes for A and 10 for B.] I missed a lot.
	G4	Um humh. So what do you think? What do you, do you still think it's fair?
	Chris	I need another one [transparency].
	G4	Would you like to change it?
	Chris	Do you have another transparency?
18:08	G4	So do you think, uh, you need to change the game now to make it fair?
	Chris	[nods] [Someone hands Chris a new transparency, and he begins writing.]



		[Terrill comes in, and they have an off-task conversation.]
22:23	Chris	You know it's not, you know it's not fair, right?
	Terrill	Huh?
	Chris	You know it's not fair, right? Look. [shows Terrill his transparency]
	Terrill	I think that the game is not fair …
	Chris	Don't look at that, look at the bottom. You've gotta have that together so you could see it. Look at this. Player A, Player B. Look at the possibilities he got and look at all the possibilities the other person got.
	Terrill	Oh, this is a different game?
	Chris	Same retarded game. [unclear]
	Terrill	[unclear] Okay, 3. 1, 2, 3, 4, 5, 6, 7, 8, 9.
	G4	Chris, do you think you need to play the game, actually, to find out?
	Chris	I don't know. I gotta write the possibilities down.
	Terrill	Well, um, do you wanna play the game and see if it's actually fair?
	Chris	Hold on.
	Terrill	Stop being gay, let's just play the game.

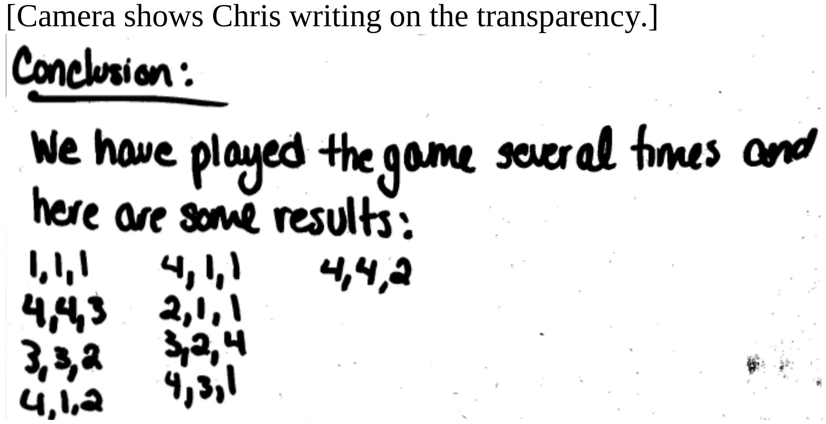
	G4	Do you think it's a good idea to actually play the game?
	Terrill	Yes, so you could actually see. Because like if Maria, like okay. Say if, like, Maria went downtown, I mean not like that, bro, but no, I'm saying like okay. How're you gonna just be like okay, I won't play this game 'cause it looks unfair. You have to play it first to see if it's really fair. That's what we would do.
	G4	So why don't you play the game and find out. Would you like to keep a record of that?
	Terrill	He's gonna keep the record.
	Chris	I'm not keepin' no record.
	Terrill	Yes you are keepin' a record. 'Cause [unclear] ask if you're gonna keep a record.
	G4	All right, why don't you just write it down.
		[Chris continues to write the sample space.]
	G4	So, so can we, can we write it down, Terrill? All right, this is A, this is B, okay? [starts 2 columns on the paper] If A wins you can write down the score here, all right? Go ahead.
	Terrill	All right. [rolls the dice] That's 3.
		[After some discussion, Terrill agrees to roll the dice and keep a record of results while Chris works on the sample space. When Chris finishes writing, he joins the game.]



[7 outcomes for A, 10 for B]

26:50	G4	What do you have to say? Who is winning?
	Terrill	A is winning. [score: 3 – 1]
	G4	You say that it's fair or not fair?
	Terrill	Player, yeah, now, see, look at you. Shouldn't Player B be winning, since um I got more possibilities? Huh, huh? See how dumb you are without me, huh? Now, if we wouldn't 've played the game, we'd 've known that he was right, he was wrong. But we still do.

		[The boys continue playing. A teacher comes by and they talk about roles in the play.]
28:49	G4	What do you think, Chris, because A is winning more. So what do you think, that these could be wrong? [pointing at Chris' sample space] Do you think that?
	Chris	[nods]
	Terrill	Of course, it's him.
	G4	So is there any [camera jumps] ?
		[The boys continue playing. At 29:40, the score is A: 5, B:4.]
31:25	G4	What do you think? Okay, Chris, what do you think? There's a 6 [unclear], what do you think?
		[Chris and Terrill are talking and do not respond.]
	G4	Is this game fair, Chris? It's becoming equal now. Do you think it's fair?
	Chris	Yeah, I think it is fair. It's just about how they roll. [shakes his hand in a dice-tossing motion] People sometimes get lucky.
32:56	G4	What do you think, Chris? What do you think about this now? B, B has one more. So what do you think?
	Terrill	I'm about to win, I need my prize.
	G4	So what's the conclusion? B is winning more times. [inaudible]
		[Chris rolls]
	Terrill	That's 6, which is me. Where's my prize? [Player B wins.]
		[off-topic conversation]
37:27		[T3 asks Terrill about the game.]
	Terrill	Okay. We gotta roll 3 dice, we gotta add up the bottom numbers. When we add up the bottom numbers, um, we get some, one of these numbers. When we get one of these numbers, either Player A or Player B gets a point. And whoever gets to 10 wins.
	T3	Is that the same as the game before you just did? Or it's different?
	Terrill	It's the same.
	T3	You guys didn't play this game yesterday?
	Terrill	We played it.
38:54		[end of ROLE 125C]
		[begin ROLE 126C]
0:30		[Camera shows Chris writing on the transparency.]


Conclusion:

We have played the game several times and here are some results:

1,1,1 4,1,1 4,4,2
4,4,3 2,1,1
3,3,2 3,2,4
4,1,2 4,3,1

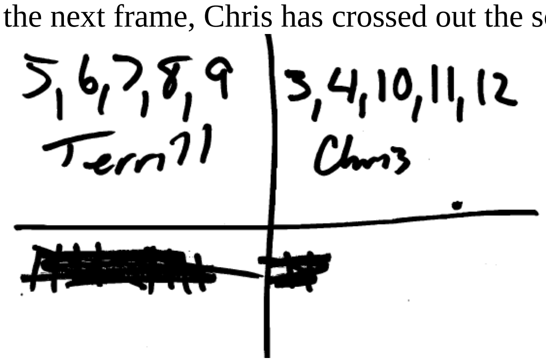
0:38	Terrill	Uhhh no, because um Player B has more um ways to get their

		answer than Player A.
	G4	Player B has more ways? Okay. So so what do you think, Terrill? What do you think? How can you [PA announcement – the rest is inaudible]?
	Terrill	Give, um, Player A one more number.
	G4	So, which number?
	Terrill	Give Player A, um, 13 or somethin'.
	G4	Thirteen?
	Terrill	Yeah, 13.
	Chris	You can't make 13. Wait a second. He's talkin' about one of these [points to his paper].
	Terrill	Oh yeah, you can't make 13.
	Chris	Yes you could. Oh no you can't.
	Terrill	You can't make 13.
	Chris	You gotta get one of these. 1, 2, 3, 4, 5, 6, 7 [tapping the individual outcomes listed for his sample space, continues tapping though he stops counting aloud].
	Terrill	Take away one of um Player B's numbers.
	Chris	You could take away …
	Terrill	Take away one of Player B's numbers, like 11.
	Chris	11
	Terrill	Give him 11 and 10 and they'll be, give him 11 and it'll be tied up.
	G4	So do you think it will become ….
	Chris	Nine. Eight and nine. [pointing to the Player A/Player B columns in his sample space]
	Terrill	Give him 11 and …
	Chris	And whoever gets 10 …
	Terrill	Give him 11 and then take out, just take out …
	Chris	One of the tens, one of the tens. [The SS shows two outcomes for 10.]
	Terrill	Give him 11 …
	Chris	Like either one of the tens.
	Terrill	Just keep, yo, listen to daddy, listen to daddy. Now, … [banter]
	G4	Can you make it fair?
	Chris	Yeah, I'm making it, yeah. I'll write it out for you.
		[off-topic conversation and laughter among Chris, Terrill, and others]
5:39	R3	Hey Chris, I got one more game for you. And, but aft-, in the class you gotta play [G4], you and um Terrill play [G4] in the game, and if you beat him, I'll give you an ice cream bar.
	Chris	Ice cream bar?
	R3	Yeah. [hands each boy a paper]
	Terrill	Oh! Come on, come on, let's go.
	R3	So I worked a good bit [unclear].
	R1	I'm gonna root for you boys, so …
	R3	I'm rooting for [G4].

	R1	I'm rooting for the boys. I'm rooting for them.
	R3	So why don't you guys take a look over and [G4], do you know how to play?
	R1	And bring up a chair, [G4].
	Terrill	Is this like Clobber? Oh, this one's like Clobber. I might be able, I was a champion of this. Lemme see somethin'. [reading aloud] Place a marker on the game board on each square with the number 1 to 14, 1 to 14. You and a partner each pick a number. Roll a few pyramidal dice, paramidal dice, whatever. Find the sum of 3 numbers of the dice. Move the marker that is on this one number one square toward the finish line. Uh, continue rolling the dice. If the marker crosses the finish line first, ohhhh, oh man!
	Chris	We need to place it on the line…
	Terrill	This don't make no sense. It means like put these right here. You roll the dice, and you move up to the finish line, but I don't know what they talkin' like.
	Chris	You gotta get your number? You gotta roll you number?
	Terrill	You gotta, when you roll a number you go that many spaces toward the finish line.
	Chris	I don't get it.
	G4	Well what you don't get?
	Chris	I don't get none of it.
	G4	Can you, you know what is this?
	Chris	I don't get from here [points to his head] [smiles and shakes his head].
	G4	You know what you have to do, Chris? You and your partner choose your numbers.
	Terrill	All right, all right, hold on. I got something here.
	G4	Would you like to put the markers here? How do you choose this? You wanna put the markers here? [Chris, Terrill, and G4 put the markers along the starting line.]
8:14	Chris	You can't put 'em all.
		[They continue placing markers. There are enough to go from 1 to 11.]
	Chris	So what if you got a roll?
	G4	Can you get 12? No, maybe. You cannot get 2, right? 1, 2. Should we put the markers here [points at 1 and 2]? Why not?
	Chris	You can't get it. You can't do a 1, a 1 and a 2.
	Terrill	I know how to play now. I win.
	G4	Shall we get going? Would you roll the dice? You take turns in rolling.
	Terrill	This one's just like, this reminds me of something.
	G4	2, 3, 1, is it 6? Okay, so we move it one, one point, okay?
		[Terrill rolls the dice and moves the #10 marker one space.]
10:10	Chris	Oh, we gotta pick a number! We gotta pick a number. He's gotta pick a number. Pick a number. Pick a number!

	G4	What’s your number, Chris?
	Terrill	8, I pick 8.
	G4	You pick 8. What do you pick?
	Chris	[looks at his sample space, which shows 2 outcomes for 8 and 3 outcomes for 6] I pick 6. [smiling] I pick 6. They got 3.
	Terrill	I pick 14.
	Chris	Nah, I pick 6.
	Terrill	I pick 14.
	Chris	Dumb day, ‘cause you can’t get 14 with three dices.
	Terrill	I don’t pick 6. I mean …
	Chris	I want 6.
	Terrill	I pick, oh, lemme see somethin’, lemme see somethin’.
	Chris	Chris [unclear] got 6. [writing] Chris, 6. Terrill, 1 and 700 and something.
	Terrill	No, I dunno which one I want yet. Where’s my paper? Where are the things that I listed at? That’s a [unclear]. Where our paper at? Remember what we just played?
		[Chris asks G4 to spell his name, which Chris writes on the paper.]
	G4	Can you take my number as 7?
	Terrill	I want 6.
	Chris	I already got 6.
	Terrill	Uh. [does a counting motion with his fingers. Chris shows him the sample space.] Ohhh [smiles at Chris]. You know what? I’m gonna get 5. I’m gonna get 5, watch. I want 5. [rolls dice]
	Chris	Who says you’re first? [takes dice]
	Terrill	Nah, I don’t care.
	Chris	Now if we, if you get a 6, a 5, or a 7, then you move. If you get any other than that, you still gotta move. [rolls an 8] Nobody’s, so you still gotta move it.
		[They play the game. Chris records the outcomes.]
17:57		[The score is Chris (6) – 1, Terrill (5) – 3, G4 (7) – 5.]
20:06		[The score is Chris (6) – 2, Terrill (5) – 6, G4 (7) – 7.]
20:57	Terrill	I wonder why 6 has the most ways to get it but he’s not moving anywhere.
23:33		[The markers for 5, 6, 7, and 8 are tied, 1 space from the finish line.]
24:35		[The markers for 5, 7, 8, and 9 are at the finish line; 6 is one space behind.]
24:45		[8 wins. No one had picked 8.]
	R3	Are you guys ready for the game? The big game for the ice cream. What I’ll let you guys go ahead and do is um, you guys can pick any 5 numbers and [G4] gets the rest. You guys get to pick the five. You guys might wanna talk about it, which five you want.
	Chris	Which number? Which numbers?
	Terrill	I want 7, I want 5, 6, 7, and 8. 5, 6, 7, 8, and 9.
	G4	5, 6, 7, 8, 9

Time	Speaker	Transcription
	R3	You sure about that? Why do you want them?
	Terrill	'Cause those are the ones that went the highest. So you're left with …
	R3	10, 11, 12
	Chris	[unclear][holds up one finger pointing towards G4, leans back, smiling] Ahhh.
	Terrill	So we play each other? I'm gonna win. You know I'm gonna win, right? I'm gonna win. You know I'll win, right? Huh?
		[Chris has set up the score sheet indicating 5, 6, 7, 8, 9 for Terrill and 3, 4, 10, 11, 12 for himself. He did not include G4 in the game. As the dice are rolled and the markers move up the game board, Chris also keeps score of the "points" each player gets and writes the sums in a column on his paper.]
28:50		[G4 points to Chris' score and asks why – as the camera skips. In the next frame, Chris has crossed out the scores.]



Time	Speaker	Transcription
29:14	G4	Do you think Chris this game is fair or something? Is it fair?
	Terrill	It's not fair because the lower numbers …
	Chris	8, 9, 10, yes! [it appears he was adding the outcomes to arrive at a sum of 10]
	Terrill	… the low-, you get the lower numbers no matter what.
31:45		[The markers for 5, 6, 7, 9, and 10 are tied, with 8 one ahead of them. 3, 4, 11, and 12 are far behind.]
32:46		[end of CD ROLE 126C, before the game concludes]

Date: 12 May 2005 Grade 7
Location: Hubbard Middle School
CD: ROLE 125D-126D
Transcribed by: Kathleen Shay

Time	**Speaker**	**Transcription**
4:05	R3	[to class] Can you go ahead and tell my friends what you did yesterday?
	student	Some dice thingy.
4:34	G8	[approaches Justina, Adanna, and Alia] I was not here yesterday. Can you tell me …
	Alia	Me either.
	G8	Oh, you were not. Were you here yesterday [to Adanna]? Yeah?

	Were you here [to Justina]? So none of you was here yesterday?
Justina	Yeah, I was here yesterday.
G8	Wanna tell us what happened yesterday? Make sure that they hear as well.
Justina	We was playin’ a pyramidal dice game.
G8	Okay. What was the purpose of it? What were you doing?
Justina	We were trying to figure out if it was a fair game or not.
G8	So what were the rules of the game?
Justina	Well, hold on a sec. [looks through her papers and puts one on top of the stack]
G8	Oh, so this is the one from yesterday. [looks at the paper] Okay, what did you figure out?
Justina	We didn’t get that far.
G8	Did you get to play it at all? Did you play it?
Justina	Oh yeah, we played.
G8	What did you notice when you’re …
Justina	There were a few numbers that came up more than other numbers did.
G8	Um humh. What were those, the ones that were coming up more?
Justina	[looks through her papers] 8 and 6
G8	Okay, 8 and 6 got more than the others.
Justina	Yep.
G8	All right. So then, does that help you in any way figuring out whether it’s a fair game? How would you use that?
Justina	[pause, looks down at her paper] I don’t know [inaudible].
G8	See, from here, 8 is among these numbers, right? For Player A. And 6 is here. [pointing to paper] So it’s for the other person, right? So then, …
Justina	Maybe it’s a fair game.
G8	Maybe it is a fair game? What do you guys think? [to Alia & Adanna] Did you hear what the game was last time?
Alia	[shakes head no, looks at paper] Ohhh. A, A will win. No, no, B will win because it got less numbers and it um …
Adanna	It’s a fair game.
Alia	[makes a face and shakes her head no] Shut up.
G8	Wait, you think it’s a fair game. You say that B’s gonna win.
Alia	[nods] Because there are more numbers than A does.
G8	So, what do you mean by more numbers? Show me on this one. [hands Alia a paper]
Alia	A got 3, 4, 7, and 8, and 12.
G8	So how many numbers is that?
Alia	Um, 5.
G8	Okay. And for Player B, we have …
Alia	And 5, 6, 9, 10, and 11.
G8	Okay. Which is how many numbers?
Alia	5.

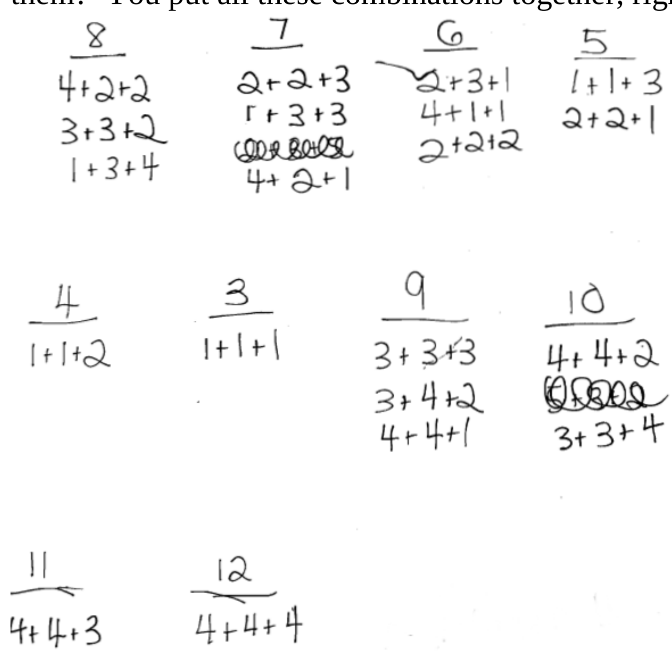
	G8	5. Okay. So then why would B get, why would B win?
	Alia	It would um win because it appear, the numbers appear more than A numbers appear.
	G8	Oh, so you noticed that from playing it?
	Alia	[nods]
	G8	But wait, you said you weren't here yesterday.
	Alia	I know. I played it last week.
	G8	Oh, you played more last week. Okay. Okay. So how, did you notice all these numbers up here for B, they all appear more often?
7:52	Alia	[nods]
	G8	Yeah? Okay. So then, so then you'd choose to be Player B if you were to play this game?
	Alia	[nods]
	G8	Okay. And what kind of um dice did you play last time? Did you play with something like this or something like that?
	Alia	Something like this [pyramidal dice], it was this dice.
	G8	This? Well then, when you, when you throw it, how do you read the answer?
	Alia	You read the bottom one.
	G8	Oh, only the bottom one? Okay. Oh but you have to do three at the same time in this one.
	Alia	[rolls 3 dice] 4 + 1 is 5, plus 3, 8. So that'd be Player A.
	G8	So you'd always the [unclear] to the bottom one and just add 'em up. I see. Okay. Then, but how do you know that that's gonna happen, next time you play Player B's gonna win again? What if it was only an accident that that happened when you did it? How many times did you play?
	Alia	Um, I don't …
	G8	You play to get to 10, okay. How many games up to 10 did you play?
	Alia	Um, 1, I don't know. Yeah, 1.
	G8	And Player B won?
	Alia	[nods]
	G8	Okay. So then how do you know that the next time you play it, it's still gonna be Player B?
	Alia	It would be Player A next time.
	G8	Oh, next time's gonna be Player A? Okay, can you go ahead and play it once and see if you're right? So you're saying that next time it will be Player A. And I'm just curious to see if that happens.
	Alia	All right. And we're gonna keep track of 'em?
	G8	Yeah, go ahead and play it so, [to Adanna] can you help her play it? You can be Player A and she's gonna be Player B. See who wins.
		[Adanna sets up the score sheet. She is Player A (3, 4, 7, 8, 12) and Alia is Player B (5, 6, 9, 10, 11).]

		[While Adanna and Alia play the game, Justina writes on her paper.]
13:51	G8	[to Justina] Do you agree with them that the next one is gonna be Player A that's gonna win? But your, you still say that it might be a fair game, right? Is that what you wrote last time?
	Justina	Yeah. [nods]
	G8	You're still sticking to that.
	Justina	[pause] But maybe not a fair game. 'Cause … [pauses, looks around]
14:33		[camera shows Adanna's score sheet: A-9, B-4.]
15:35		[Player A wins, 10- 7.]
	G8	Okay. So last time Player B won? Last time you guys played it, Player B won? Is that what is … So now you have a game where Player B won and a game where Player A won. So then, what does that tell us about the game? Can you draw any conclusions?
	Adanna	It's fair.
	G8	What if, what if you were to play it another 4 times and let's say one of them won 3 times and another one just once. Would that convince you otherwise?
	Adanna	What was the question?
	G8	If you were to play it another 4 times, and let's say one of the players wins 3 of those and the other player wins only once …
	Adanna	I think they're gonna win equal.
	G8	Okay. So you're saying probably that's not gonna happen.
	Adanna	Huh?
	G8	So you're saying probably that's not gonna happen, what I just said, that one of them wins 3 times and one of them 1 time. You say there's little chances …
	Adanna	It's a possibility. It's a possibility, but it's very short.
	G8	Okay. So then, what else can we do to decide whether this is a fair game? Is this enough, what we've done so far? [no response]
	G8	[to Justina] What are you trying to do? Are you doing the sums that they were doing, or are you trying something else? What is it, can you explain?
	Justina	I'm just tryin' to see, um, the different ways of each number to come up.
	G8	Oh, okay. How would that help you to figure out the [inaudible]?
	Justina	Because last time when I played this game, like some numbers they came up, like they had different ways of, they had different ways to come up more than others did.
	G8	Oh, okay. Did you guys hear what she said? Do you understand what she's doing? Would that make any sense for this game? What would be the reason for doing this?
		[A&A do not respond. They joke about Justina being "on the air."]
	G8	No, no, she did her explanation. Now you guys, does it make any

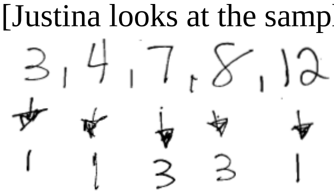
		sense to do what she's doing? Why would that be helpful? [pause, no response] How would that help her to decide whether it's a fair game?
		[Adanna & Alia have off-topic conversation]
18:35	G8	How about this: What if, what if you went to all the other tables? What if you were to go to the other tables and ask them, and ask them how many times did Player A won? How many times did Player B win? And they would tell you various numbers. Would you, do you think that those numbers are gonna be equal?
		[off-topic, no response]
19:30		[camera shows Justina's paper, where she is developing the sample space]
21:25	G8	So is that all that we can get? Are those all the sums? Are there more? By the way, what is, what is the maximum sum that you can get?
	Justina	12
	Adanna	10
	Justina	12
	Adanna	Yeah, 12.
	Alia	12
	G8	And the minimum?
	Adanna	10, or 9
	Justina	It's 3. The minimum is 3. Because there's only 3 dice.
	G8	Can you get 3? Can you get the sum of 3?
	Alia	[nods]
	G8	Can you get the sum of 2?
		[off-topic]
	G8	Can you also get any sum between 3 and 12? Can you get any sum between those?
	Alia	[shrugs her shoulders]
		[off topic]
23:00	G8	[to Justina] Are these the only ones [inaudible]?
		[no response – Justina is looking down at her paper, pen in hand]
		[room is very noisy]
23:25	G8	As soon as we finish this one we can move on to something more interesting. So let's figure this one out. So then, are we close to, are we close to figuring it out just by looking at those sums? How are we gonna use 'em?
	Adanna	The ones with the most combinations are gonna come out more than the less combinations.
	G8	Okay. So let's, so let's finish this. [to Alia] Do you agree with what she said?
	Alia	Yes. [nods]
	G8	Okay, let's see. Hey you guys, is this all the combinations for each

		of the numbers? Do you think she missed any? [referring to Justina's SS] 'Cause then if she missed any we're gonna be in trouble. All right? 'Cause then we're not gonna count …
	Adanna	[pointing at 8] 5 plus 2 …
	Justina	There is no 5.
	Adanna	Then why you write 5 here? [Justina had written 5+3+2 under 10. She scribbles over it.]
	G8	How about, is there anything, is there anything missing here? [pointing at sums for 8: 4+2+2 and 3+3+2]
		[Adanna talking off topic]
	G8	Hey Adanna, is there anything missing here?
	Adanna	4+4?
	G8	4+4? But you still need to read from all 3 dice.
	Adanna	Oh. 4+4-1.
	Alia	There's no minus.
	G8	So any ideas for the 8? Or is that all?
	Alia	Uh, I think that's all.
	Justina	1+3+4
	G8	She found 1+3+4, a different combination. Okay. Any other? How about here? [pointing at paper] Do you have anything [inaudible]? Okay, so what about for 7? Are we missing anything for 7?
		[Adanna and Alia are off topic. Justina rubs her head and looks away.]
27:41	G8	So what about, oh, you said 4+2+1. Uh huh. How about here, are you missing any here? Guys, what about 6? The sum of 6. Are we missing anything here? So she has 2, 3, and 1; 4, 1 and 1. Any other possible ways? [Justina writes] Oh! 2, 2, 2, all right.
	Adanna	3, 2, 2
	G8	3, 2, 2?
	Justina	No. That's 7.
	Adanna	Oh, 3, 2, 1.
	G8	3, 2, 1. Does she have that?
	Alia	Yeah, at top. [points at Justina's paper]
	G8	Well, she has 2, 3, 1.
	Adanna	Or 2, 1, 3.
		[Justina points at her paper – possibly at 2+3+1- and looks up at G8. G8 nods]
	G8	So, do you think we're done for the 6? Is that all, the 3 combinations? [no response] What about the 5? So far everything had 3 combinations on top here, right?
	Adanna	Can we play the game?
	G8	You want to play it again?
	Adanna	Like to, like 5:00.
	G8	Will that help you to figure out if it's a fair game?
	Adanna	Yeah. We could answer the question in August.

	G8	In August?
	Adanna	When I’m not here.
	G8	Nah, well, that’d be too late. But wait, you said that if you count these things, that it’s really gonna help you figure out whether it’s a fair game or not. So we just need to make sure that we have all the possible combinations. And if we have all of them correct, then, you know, you should have your answer, right?
	Adanna	4, 2, 3
	Justina	What?
	G8	For which one, for 12?
	Adanna	Oh that’s not right. There’re no more.
	G8	That’s it? Okay, so let’s assume that we have, you guys are saying that it’s all the combinations. How about for 10, is this all? Is this all you can do for 10?
	Alia	Let me see. [looks at paper and nods]
	Justina	3, 3, 4
	G8	Which one?
	Justina	Wait. 3, 3, 4.
	G8	3, 4, 4?
	Justina	3, 3, 4
	G8	Oh! All right. Is that [unclear]. Is that all? Okay, so then how do you use all these things? How do we count up, what do we do with them? You put all these combinations together, right?



31:25		[Justina looks at the sample space and begins writing.]



33:00		[Player A]

G8		So then how [unclear] with the sums? What should she do now? She has, let's say she has all the sums. [Adanna & Alia are off topic. Justina continues writing.]
		5 6 9 10 11 ↓ ↓ ↓ ↓ ↓ 2 3 3 2 1 [Player B]
	G8	Okay. So look what she did, guys. Guys, look what she did. So this is for Player, what player is this?
	Justina	That's Player A.
	G8	Player A, Player B [pointing to paper]. And now what are we doing with these numbers underneath?
	Justina	What we should do with the numbers …
	G8	Yes, what are we doing with these numbers down here? [Adanna & Alia are off topic.]
	G8	Okay, so we have this. For a sum of 3 only one combination, right? We have all these combinations here. What shall we do with them with this number under here in the second row? What should we do with them? Answer the question for the game.
	Adanna	Didn't we answer it?
	G8	Well, I don't know. How did you use these things to answer it? Did you use these in any way?
	Adanna	I think the lowest numbers are the ones that come up the most.
	Justina	This player …
	G8	The lowest numbers?
	Adanna	Uh huh.
	G8	The lowest sums, you mean?
	Adanna	But some of these come out like only once or two times in a game.
	Justina	This is Player B.
	G8	Wait, so this is according to the game that you played, you mean, right?
	Adanna	Yeah.
	G8	Okay. So now how can you use these combinations here?
	Justina	Look at Player B.
	G8	How can we use these numbers in the second row, which is in how many ways you can get a sum of 5, a sum of 6? [noisy distraction in the room]
35:23	G8	All right, so why [inaudible] numbers here? In order to be able to simplify … Oh, you're adding? Okay, let's add 'em. What do we get? [A&A off topic]
	G8	All right, so why did you say, you said we should add 'em? Is that what you're proposing? Adanna, did you say 2+3? Did you say 2+3? Is that what you said? All right, so do you want to add all them up [number of combinations for Player B], or just 2+3?
	Adanna	2+3+3+3+1 equals 11.

	G8	11, all right. What about here? [pointing to A's numbers]
	Adanna	1+1+3+3+1 equals 9.
	G8	Nine, okay so then what does that tell us? How do we interpret these sums? Did you hear what they said, they said the sum for B is, what did you say, 11? For this one? And 9 here. How do we interpret these sums?
		[A&A off topic. Justina looks at the paper with pen in hand. She takes a paper out of the folder from yesterday and looks at it.]
	Justina	This game we played, and Player A won. And this one Player B won.
	G8	Uh huh. So you played only twice. [inaudible] What do the sums tell us? 11 that we got here and the 9 that we got here.
	Justina	Player B has more of a chance of winning than Player A does.
	G8	Okay. You're saying that Player B, right? Guys, she's saying that Player B has more chances than Player A. Just based on the sums that she just calculated, you said that [camera spins around, inaudible chunk] that Player B would have more chances with these sums. Is that what that is telling you? How about this. Let me show you something. [arranges 3 dice] Okay. Guys, let's assume that you got this [unclear]. What is the sum here?
	Adanna	3.
	G8	You get a sum of 3, right? So, in how many ways can you get a sum of 3?
	Justina	1.
	G8	1, right? And according, and according to your little table here, there's only one way to get a 3 and only one way to get a 4, right?
	Justina	Yeah.
	G8	Now how about this? So for 4 you're saying 1, 1, and 2, right?
	Justina	Um humh.
	G8	Okay. So if I made this 2 and I do this this way, this is a way to get a 4, right? Yeah? But what if I do this [turns dice], is this a way to get a 4?
	Justina	[inaudible]
	G8	And it's still the numbers 1, 1, and 2, right? But would you consider this a different way, 'cause you know, you see, I uh, I just changed positions [inaudible].
	Justina	It doesn't matter.
	G8	How come it doesn't matter? I mean, now the white one is a 1 and, and this one is a 2.
40:02	Justina	But we're not focusing on the colors. We're just focusing on the numbers. 2+1+1 still equals 4.
	G8	Correct, but [unclear] you could just focus on the numbers and not focus on the colors?
	Justina	Well it's not based on the color.
	G8	Are you sure? I mean, hey guys, what do you think of this? Did

		you hear what we're discussing here? We're discussing the following thing. See, to get a 3, this is the only way to get a 3, right? [arranges dice] Yeah?
	Adanna	Yes.
	G8	Meaning the black one is 1, the white one is 1, and the blue one is 1. Now for 4, you have only one combination as well written here. Yeah? So, she has only one combination put down for 4. But look, this is one way to get a 4, right? 2, 1, 1, yeah? But now look, if I make this change and put the 1 here, and the 2 here, this is still a combination for 4. But this is in a way different because now the blue is a 1, and this is a 2. So should we make a difference between these two ways of getting a 4? I mean before, for getting a 3 it was obviously one way because I had to have three 1s. All right? There was no other way to change it. So this one look, I just showed you two ways. There are at least two ways …
	Adanna	That's the same thing.
	G8	Well it's still the same numbers, but should we pay attention to the, to the way they come up? I mean do, does the 1 come up on this one or this one? Does the 2 come on this or this? Do they, should we care about that?
	Adanna	[shakes head]
	G8	No?
		[Justina does not appear to be attending to this discussion. She has her head resting on her arm on the desk and is doodling with her pen.]
	Adanna	It's the same numbers, 'cept different combination of ways.
	G8	True, the same numbers. But look. When I throw this [holds dice], you know one way to turn out would be with the 1 down, all right, and one way, another way to turn, uh would be with the 2 down. And let's see that these other two come up in a way that the combination was still a 4. Right? So then isn't that two different ways that this came out?
		[G8 and Adanna speak at the same time – neither voice is clear. Alia asks to go to the restroom.]
	G8	So, so this is the challenge that I'm throwing at you. Should we pay attention to where each number appears apart from what combination of numbers we have? So we have the combination 1, 1, and 2, but where does the 1 appear, where does the 2 appear, and so on? Should we pay attention to that? I mean, does it have anything to do with chance and probability?
43:11	Adanna	I don't think it do.
	G8	You don't think it should. Okay. [to Justina] What do you think?
	Adanna	Justina!
	Justina	[lifts her head from the desk] Huh?
	G8	What do you think? Should we pay attention to the fact that, you

	know we can get the sum of 4 in those, at least those two different ways that I showed you. We still have the numbers 1, 1, and 2 but you know, these are showing different things.
Justina	[shrugs]
G8	I know, I know that in the problem it doesn't say anything about colors, but if you're thinking about it in terms of how likely it is for such combination to pop up, you know, does that make any difference?
Adanna	No.
G8	So then you are saying that the chances of getting a sum of 4 are the same as the chances of getting a sum of 3? Yeah?
Adanna	I don't know.
G8	[to Justina] Do you agree with that? So the chances of getting a 4 are the same as the chances of getting a sum of 3 at any given toss? Do you agree?
Justina	Um humh. [nods]
	[Alia returns]
G8	So Alia, this is the question that I've been asking them. The chances of getting a sum of 4 …
Alia	[shouts across the room at another student]
G8	Okay, so here's my question. From this thing [paper showing sample space], you see we have one combination for 4 and one combination for 3. Does this mean that the chances of getting a 4, a sum of 4, are the same as the chances of getting a 3?
Alia	[nods]
G8	What I just showed you before, that doesn't make any difference?
Alia	[shakes head] They're just a different color combination.
G8	Right, but just imagine, how about if you didn't throw them all at the same time but you throw, you threw them like this: one, two and three. Okay? And you'd read the sum of that after you do that way. Would it be a difference getting a, you know when I through this one this lands with a 1 down, 1 down, and the other one is gonna be a 2 down, so that's one way. And then let's say another time I throw it I get the first one that I throw has a 2 down, then the second one that I throw has a 1 down, and the third one that I throw has a 1 down. Wouldn't that be a different way of getting the 4?
Alia	[nods her head, as if to a beat, for several seconds]
G8	So then, doesn't that affect chance in any way?
Alia	[shrugs her shoulders and shakes her head]
G8	Well what does your intuition tell you? Just based on intuitions.
Alia	It's not fair.
G8	Okay, so at this point, you girls, if I were to ask you about your conclusion about this game, what would you say? That, based on all the sums that we did you stick to the conclusion that? What

		was the conclusion? What was the conclusion about the game? Based on all the sums that we did and everything.
		[Justina's head is turned away. Adanna is off camera.]
	Alia	You have different combinations in each um numbers.
	G8	Right, so what is, which one is more likely to win based on your combinations here.
	Alia	Ummmm [looks at paper].
	G8	They um, Adanna, do you want to help her? [no response] Well you need something here [points at paper].
	Alia	I'm [getting?] this one [points at paper]. B.
	G8	So you're saying B because, why?
	Alia	Because up here with most numbers like 2, 5, 2 up here with 5, 2 up here with 10, and 3 up here with 9, and 3 up here with 6.
	G8	All right, but how, you know, in what way do you compare this to this to say that the Player is …
	Alia	These have, uh, more numbers paired up than A. Oh no no no no no, this, I don't know, I don't get it. Adanna, figure it out.
	R3	I have one more game for you. But this is gonna be a good game for you to learn, because if you can beat G8 at the end of class, I'll give you an ice cream bar.
49:46		[end of ROLE 125D]
		[begin ROLE 126D]
0:33	G8	Okay. So let's see what this game is about. I don't know it.
	Alia	[reading] Place a marker on the game board in each square with a number 1 to 14. You and, you and your partner each choose one number. Roll three pyramid, pyramidal dice. Find, find the sum of the three numbers on the dice. Move the marker that is on the, this number one square towards the finish line. Continue rolling the dice if your marker cross the finish line first you win. If your partner marker reaches the finish line first then your partner wins. If any other marker cross, crosses the finish line first both you and your partner lose. Play several games. Write down the results. What number you choose and what number won. So place, place these, we all in the same.
	G8	Did you guys understand? Let's not start before everyone understands the rule.
	Alia	Don't, ain't all three of us on a team?
	Justina	[nods]
	Alia	Ain't all of us, ain't us three on a team?
	Adanna	I don't get it.
	G8	Well, can you explain to her what the rules are, because she's not getting it.
	Justina	You don't get it? Omigod. Look. We place a marker on the game board right here. 1 through 14. Both you and your partner choose a number. For example, 8. Put the marker here. And then roll …
	Adanna	Is this the marker?

Justina	We roll the dice. We roll it [rolls], find the sum, [whispers] put it there, would you move the number?
Adanna	That's 4.
G8	So wait, how is this, how are you guys …
Justina	If another marker gets to the finish line before you do, you lose. Okay?
Alia	Are these squares all one?
Adanna	So if you're right you move up?
Alia	You gotta place a marker on each square. So 1, 2, 3, …
G8	On each square with a number. So these have a number, these here.
Justina	I choose 8.
G8	So wait.
Alia	Are all three of us gonna choose?
Justina	No. Put my marker down.
Alia	What's that mean?
G8	So which way, so from the directions of the game do you think we should put a marker in each of the squares or each of us should choose a number and put? I mean 'cause otherwise how is this a game? If we put a marker in each of these things, then how are you supposed to beat me? I don't understand, what is the competition there? What are, is, do I have a marker of my own or what?
Alia	You get hers, or …
Justina	What are you talking about?
Alia	Ask that guy right there. I don't know.
Justina	Let's just play by the rules of the game.
Alia	All three of us on a team then we gotta beat her or something.
Justina	We beat her at this game [pointing at paper]. Yeah. Why we changing it?
Alia	I say all three of us is gonna play so [unclear].
G8	[returns to the table after stepping away briefly] So, from the very beginning we have to put a marker in each of the squares, right? But then each of us chooses a game, see [unclear, points at directions] each choose one number. Okay? Choose a number that you think is gonna, that you think is gonna win at the end. All right?
Alia	All right.
G8	So, let's start by putting the markers all in here. Adanna, can you help me? Put one in each of these things. All right, everyone clear with the rules of the game so far? So each of us has to choose, or maybe you should play it in teams, maybe huh? Because it says "you and your partner."
Justina	Adanna, I choose you.
G8	Okay. So then it says you and your partner each choose a number.

		So then you guys are a team and you choose a number and then she chooses a number. Right?
	Justina	Okay. I choose 8.
	Adanna	But you haven’t [unclear].
	G8	Let’s, yeah, let’s put all those markers there. [to Alia] And we have to choose a number each, too.
	Adanna	I pick 4.
	G8	So, I choose 7. What do you choose?
	Alia	10.
G8		Okay. [to Justina] Can we write that down just to be sure we remember what we each chose? [Justina writes.]
5:16	Justina	7 and what, 10?
	G8	7 and 10, yeah.
6:00		[Alia begins the game. The first 3 rolls are 10, 9, and 7. They advance the markers for 10 and 7, but not 9. G8-Alia team is ahead.]
6:43	G8	They might catch up at some point. Let’s not rush to conclusions.
13:50	R3	You guys should be moving them all up. [They had only moved their selected numbers, 4, 7, 8, 10.] You got a 9, that should be moving up. That’s okay.
	G8	Oh, I see. We were just moving, oh, I see, I see, I see. Oh yeah, that’s a good point.
16:00		[The positions of the markers are: G8(7) in row 8, Justina(8) in row 6, Alia(10) in row 4, and Adanna(4) in row 2.]
17:12		[G8’s marker (7) reaches the finish line.]
	G8	And I finished. I won, I won! Wait, wait, wait, let’s look at the position. Wait, wait, wait.
	Justina	Omigod, she won!
	G8	So I won, yeah?
	Adanna	She cheated. Don’t you know she got magical powers?
	G8	Yes, it’s mind power.
	Justina	Come on, let’s play another game, come on. I’m 7, 7.
	G8	So wait, record that. Can you record that?
	Adanna	Seven came up the most.
	Justina	Yep, 7 came up the most.
	G8	What numbers you chose and what numbers won. So let’s record 7 as the one winning and the numbers that we chose. Okay, can we do that um Justina? Okay. Let’s play one more time, yeah?
	Justina	Okay. Seven, I got 7, 7. [to Adanna] You you you, you choose 8, okay?
	G8	I choose 8.
	Adanna	I choose 7.
	Justina	I chose 7!
	Adanna	I chose 10.
	Alia	Nah! I choose …
	Adanna	I chose 10.

	Alia	Six.
	G8	You choose 6? Okay, so why don't we ….
18:08	R3	Are you guys starting another game?
	G8	Yeah.
	R3	Why don't we play for the ice cream …
	Adanna	We won first.
	R3	All right, if you win …
	Alia	No, we won first, so.
	Adanna	Don't believe them.
	R3	All right, look guys. If you guys can win this game against G8 I'll give you ice cream. And you guys get to pick 5 numbers.
	Justina	[jumps up] Okay.
	Adanna	Okay. I pick 1, 2, 3, 4, 5.
	R3	No, you all pick them together. And I don't think those are very good numbers. Try to think about it. Talk about it.
	Justina	I got 7 already.
	Alia	Hold up, hold up, sir. Hey sir. You said all of us, all 3 of us, pick one number.
	R3	The team gets to choose 5.
	Adanna	Three against one?
	R3	Yeah, but you get to choose 5 numbers and G8 gets the other 5.
	Alia	Three against one?
	G8	And they get to choose first?
	Adanna	So you play by yourself? She plays by herself?
	R3	Right. Right. They choose all 5. So you guys pick the best 5 numbers you can think of.
	G8	So you guys pick first, and I'm picking after you.
	Adanna	7, 10, how you write your name?
	Justina	Okay, okay. 7, 6, 11, …
	Adanna	Hold up!
	G8	Let's record stuff.
	Justina	Okay, we got 7, 6, I'm gonna write this. We gotta get 7, 6, 11, and 5 and …
	Adanna	No, use 7…
	Alia	No, we all pickin' numbers at the same time.
	Justina	These are the numbers, I think.
	Adanna	No, 10. Don't forget 10.
	Justina	Okay. This our numbers.
	G8	That's what you choose?
	Adanna	7, 6, 9, 5, 10
	G8	Okay, my turn, right?
	Justina	Um humh.
	G8	Let me think. Use my magical powers, right? So let's say, I'm gonna have 8, …
	Adanna	Huh! You forgot 8!
	G8	Eh, we're done, we're done, I'm sorry. So I have, uh what else do

		you guys have? 7, 6, 5, interesting. Uh, 9, you're writing down mine, yeah?
	Justina	8, 9
	G8	8, 8 is first. Or, it doesn't matter. Okay. 8, 9, what else do I have left? Um…
	Adanna	12
	Justina	[to Adanna] Don't help much.
	G8	No, I don't really want 12. 4, and how many do I have left?
	Justina	Two.
	G8	I could just choose the remaining numbers then. I have no choice?
	Justina	You've got all the good ones.
	G8	Oh, okay. So I'm just gonna be 8, 9, 4, and what are the remaining two?
	Adanna	13. 12, 13, 14?
	G8	Huh? Which one?
	Adanna	12, 13, 14, 9.
	G8	Whoa, whoa, whoa. Can you do, can you do 13 and 14?
		[Justina, Adanna, G8 all talking at once]
	G8	What did you say was the maximum, the maximum sum possible?
	Adanna	14, 13, 12, 9.
	Justina	She already got 9.
	G8	So I'm gonna choose um, I'm gonna choose uh 3 …
	Adanna	3, 2, and 1.
	G8	3 and 12. All right?
	Adanna	She gonna lose.
	Alia	She gets to uh, she gets to roll first.
	Justina	I wanna roll first.
	G8	I get to roll first?
	Alia	Yeah, 'cause you uh …
		[some discussion about who rolls first – Justina starts]
27:15		[With girls: 5, 6, 7, 10, 11 and G8: 3, 4, 8, 9, 12, G8 is in the lead with 8 and 9 tied 4 spaces from start. 6 and 7 are 3 spaces from start.]
28:23	Justina	We won! We won.
	Adanna	We won. We won. [It's not clear why the girls claimed victory. The leading number was 8.]
28:43		[end of ROLE 126D]

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Curriculum Vita

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Education

Undergraduate: Douglass College, Rutgers University, New Brunswick, New Jersey
A. B., Mathematics Education, 1971

Graduate: Rutgers University, New Brunswick, New Jersey
Ed. M., Mathematics Education, 1975

Rutgers University, New Brunswick, New Jersey
M. S., Statistics, 1986

Appointments

Middlesex County College, Edison, New Jersey

2007 – present	Professor of Mathematics
1987 – 2007	Associate Professor of Mathematics
1984 – 1987	Assistant Professor of Mathematics
1981 – 1984	Instructor of Mathematics

Rutgers University, New Brunswick, New Jersey

1983 – present	Visiting Part-Time Lecturer in Statistics

Douglass College, New Brunswick, New Jersey

1978-1981	Lecturer in Mathematics

North Plainfield High School, North Plainfield, New Jersey

1971-1978	Teacher of Mathematics

Publications

Shay, K. (1997) The TI-92, an excellent companion for differential equations reform. *The International Journal of Computer Algebra in Mathematics Education*, 4:1, 99-109.

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