UTILITY-BASED POWER CONTROL FOR PACKET-SWITCHED WIRELESS NETWORKS

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ABSTRACT OF THE DISSERTATION

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Efficient management of radio resources is crucial in maintaining peak performance of cellular wireless networks under resources constraints. As wireless networks evolve toward 3G/4G and beyond with broadband multimedia services, managing radio resources to satisfy diverse Quality-of-Service requirements is becoming even more critical. Battery energy conservation for portable terminals on the move is another important aspect in wireless networks. Due to the limit on battery life-time, each unit of battery energy saved directly translates to an increase in the value of communication for a subscriber.

In this work, we investigate a uplink power control problem for packet-switched data services, with a focus on energy efficiency for mobile terminals. Packet-switched data differ fundamentally from circuit-switched data in the burstiness of traffic and the connectionless nature of communications. Based on a utility-maximization approach from microeconomics, we define a probabilistic utility model as a performance metric for a wireless data user, which takes into account both the traffic burstiness and average packet delay requirement. Game-theoretic approach is then utilized to study a distributed power control strategy to simultaneously maximize the utility for each individual user in the system.

In general, the problem is mathematically intractable. Using several approximation methods, the problem is reduced into tractable format and is studied both analytically and by simulations. Results show that the proposed power control scheme converges to a unique Nash equilibrium which depends on mobile's location, average packet delay requirement, traffic burstiness, and the mean and variance of the interference and background noise at mobile's base station receiver. The scheme can be easily extended to multi-class user traffic environments.

For performance evaluation, we establish two idealized slot-by-slot based power control strategies as performance benchmarks. It's shown that the performance achieved by the proposed scheme is close to those by benchmark schemes. Generally, this work provides a new approach based on approximation techniques for the investigation of uplink power control problem in packet-switched systems. The results and insights generated by this study provide guidance on the efficient management of transmit powers for energy-efficient packet-switched data services in current and future generations of wireless networks.

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Dedication

To Ximei, Justin and Madeline, as well as my Parents and Brothers.

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2G/3G/4G	Second/Third/Fourth-Generation Wireless Networks
3GPP	The Third-Generation Partnership Project
3GPP2	The Third-Generation Partnership Project 2
A-DPC	Average Distributed Power Control
AMPS	Advanced Mobile Phone System
AWGN	Additive White Gaussian Noise
BER	Bit Error Rate
BFSK	Binary Frequency-Shift Keying
BS	Base Station
cdf	cumulative distribution function
CDMA	Code Division Multiple Access
cdma2000	A Third-Generation Air-Interface Standard based on IS-95 CDMA Evolution
DPC	Distributed Power Control
DS/SS	Direct Sequence/Spread Spectrum
ETPR	Earned Throughput-to-Power Ratio
EVDO	Evolution-Data Optimized or Evolution-Data Only (IS-856), as part of the cdma2000 family of standards by 3GPP2
FTP	File Transfer Protocol
GSM	Groupe Speciale Mobile, The Pan-European Digital Cellular Standard
IEEE	Institute of Electrical and Electronics Engineers
IP	Internet Protocol
IS-136	The North American Digital Cellular Standard based on TDMA

IS-95	The North American Digital Cellular Standard based on CDMA
LHS	Left-Hand Side
LTE	3GPP Long Term Evolution, a 4G technology proposed by 3GPP
MIMO	Multiple-Input and Multiple-Output
MS	Mobile Station
MSC	Mobile Switching Center
NAPC	Network-Assisted Power Control
OFDM	Orthogonal Frequency Division Multiplexing
PCS	Personal Communications Systems
pdf	Probability Density Function
pmf	Probability Mass Function
PER	Packet Error Rate
PSR	Packet Success Rate
QoS	Quality-of-Service
RHS	Right-Hand Side
RF	Radio Frequency
SBS	Slot-By-Slot
SBS-DPC	Slot-By-Slot Distributed Power Control
SBS-NAPC	Slot-By-Slot Network-Assisted Power Control
SIR	Signal-to-Interference Ratio
TDMA	Time Division Multiple Access
UMB	Ultra Mobile Broadband, a 4G technology proposed by 3GPP2
UMTS	Universal Mobile Telecommunications System
UTRAN	Universal Terrestrial Radio Access Network
W-CDMA	A Third-Generation Air-Interface Standard based on Wideband CDMA Technology adopted in UMTS
WiMAX	Worldwide Interoperability for Microwave Access, a 4G technology based on the IEEE 802.16 standard.

Chapter 1

Introduction

The concept of cellular wireless network communications was first conceived in 1947 at AT&T Bell Laboratories [99]. After about three-decade's advancement of computer and signal processing technologies, the first-generation of cellular wireless networks (e.g. AMPS [100, 103]) was finally made commercially available in the late 1970s and early 1980s to provide wireless voice services to mobile telephone subscribers. In the early 1990s, with increasing demand for a higher system capacity and more robust communications, the first-generation systems evolved into the second-generation (2G) wireless networks based on digital transmission techniques that support both voice and low-speed data services (e.g. GSM in Europe, IS-136 TDMA and IS-95 CDMA in North America [100, 101, 102]).

In recent years, wireless networks have been evolving toward the third-generation (3G) with various 3G wireless services being currently deployed. The primary goals for 3G wireless networks are universal adoption of an inter-operable set of wireless network standards, facilitation of global roaming using a common multimode terminal, and efficient support of a wide range of data/multimedia services [88, 89]. Major standards adopted for 3G air interface are based on Wideband CDMA technologies (e.g. cdma2000/EVDO and W-CDMA in UMTS [88, 89, 90, 91, 92]).

While the evolution to 3G is being commercialized, efforts have being undertaken to break the limitations of 3G systems and further evolve wireless networks toward fourth-generation (4G). The main targets for 4G evolution is to provide wireless broadband services in meeting the demand of rapid growth in Internet traffic over wireless. With the adoption of Orthogonal Frequency Division Multiplexing (OFDM), Multiple-Input and Multiple-Output (MIMO) technologies, and a simplified All-IP based network architecture, 4G systems will significantly increase the data rates and improve the spectrum efficiency while reducing the latency in data transmission [94, 95]. There are three main standards in competition for 4G: (1) the Long Term Evolution (LTE) proposed by the Third-Generation Partnership Project (3GPP) [96]; (2) Ultra Mobile Broadband (UMB) by the Third-Generation Partnership Project 2 (3GPP2) [97], and (3) Worldwide Interoperability for Microwave Access (WiMAX) based on the IEEE 802.16 standard [98]. All those 4G technologies are focusing on wireless data services.

With the evolution toward wireless broadband networks, wireless access to the Internet are becoming an essential part of the networks. As the predominant type of traffic, packet-switched data from the Internet and multimedia services will need to be supported in wireless networks with the same speed and quality that are expected from their wireline counterparts. How to achieve this goal is an important area that requires research. In the study of this work, we will focus on one of the fundamental aspects in this area — the radio resource management for efficient transmission of packetized data over wireless channels. Specifically, we will investigate some issues concerning the allocation of mobile transmit powers in wireless packet-switched data network environment.

With the convergence of wireless and the Internet, the extensive information resources on the Internet will eventually be set free from the wires of fixed networks by the mobility of wireless broadband networks. From the birth of the concept of cellular communications to the realization of wireless broadband networks, more than half a century's human endeavor to make information mobile is becoming a reality - information will be accessible *anytime, anywhere, via any media* and *for any device.*

1.1 Value of Communication

People are always fascinated by the newly-invented or discovered ways for conveying information. We all love efficient and reliable communication services and devices because they facilitate the information exchange among people and make people's lives much easier. With those ever-renewing tools of communications equipped, we lead to a better-connected society. Nowadays, people are becoming increasingly dependent on electronic and optical communication tools and devices. Telephone is a very good example. It is hard to imagine the life without phones in a modern society.

Wireless communications have experienced phenomenal growth worldwide in the recent decades. Wireless communications give people the freedom of mobility for anywhere and anytime information exchange. The much-needed convenience of wireless attracts people to subscribe to the wireless services. As soon as the prices of wireless services and wireless terminals or handsets became affordable as we observed, the growth in subscription to wireless services has grown rapidly and extensively.

For a wireless subscriber, an important factor in choosing a handset is the talk time or the maximum length of operation without the recharging of the handset battery. Nowadays, an handset can have various functions and features to support multiple types of wireless services. While voice is still the primary service for wireless, but other non-voice services such as Email, Web Browsing, Instant Messaging and Video are getting increasingly popular. Different services require different levels of processing power, which results in different levels of energy consumption for the handset battery.

From the perspective of a wireless subscriber, it is always preferred if a handset can do more by burning each unit of energy in handset battery. The more a handset can do per unit of battery energy, the higher value the handset presents to the subscriber. Therefore, in terms of the *Value of Communication*, it all boils down to a performance/cost ratio in choosing a wireless communication device that makes the most efficient use of the mobile battery energy in providing various services to the subscriber. We use the following performance and cost definitions in the study of this work:

- **Performance**: Information throughput of individual wireless user in supporting of various wireless services
- **Cost**: Battery energy consumption of individual mobile terminal

In this study, we will concern ourselves with how to achieve the best performance while minimizing the battery energy consumption of mobile terminals. We will consider the value of wireless communication from the perspective of an individual mobile user - an individual user's information throughput and its battery energy consumption through the use of a wireless communication device.

1.2 Radio Resource Management for Packet-Switched Wireless Data Services

Radio resources in cellular wireless networks such as spectrum, bandwidth, transmit powers, channels and base stations are generally limited due to physical, cost and regulatory restrictions as well as the interference-limited nature of cellular wireless network architecture. Efficient management of radio resources is crucial in maintaining cellular wireless networks at their peak performance under limited resources constraints. In 2G/3G wireless networks, radio resources allocation has been primarily optimized for voice services. As wireless networks evolve toward 3G/4G and future generations that are capable of supporting high-speed data/multimedia services, managing radio resources to satisfy diverse quality-of-service (QoS) requirements is becoming even more critical to the success of wireless broadband services. From the perspective of wireless subscribers, another important aspect in wireless networks is the battery energy conservation for portable terminals on the move. As pointed out before, to a portable terminal, since a battery with a limited life-time before recharging is usually the only energy resource available, each unit of battery energy saved directly translates to an increase in the value of communication for the subscriber.

In this study, we investigate a radio resource allocation problem - the uplink transmit power control for wireless packet-switched data services, with a focus on energy efficiency for mobile terminals. Radio resource allocation for data services differs from that for voice services, simply because they have very different Quality-of-Service (QoS) requirements. Data require a bounded delay for error-free transmission while voice demands a target signal-to-interference ratio (SIR) to be maintained. Particularly the power control problem for wireless voice has been extensively studied [15, 16, 17, 18, 22, 23, 24, 25, 26], but power control for wireless data is a relatively new area. There are basically two different types of communication systems for wireless data, one is circuit-switched and the other is packet-switched. While power control for circuit-switched data services has been studied extensively as in [13, 14, 38, 39, 40, 41, 42, 43, 44, 45, 46], power control for packet-switched data services is still a challenging area that requires research.

We choose to focus our study on the uplink for mobile terminals because we are more interested in the behaviors of individual mobile users in terms of radio resource allocation. Uplink power control is generally managed by mobile users and is usually being implemented with distributed algorithms. While in comparison, downlink power control is generally scheduled by base stations and is usually being implemented with centralized algorithms. Distributed power control algorithms for uplink will be investigated in this research.

Because packet-switched systems have the advantage with built-in statistical multiplexing capability, they are the natural choice for current and future generations of wireless multimedia services. Fundamentally, packet-switched data systems differ from circuit-switched data systems in the burstiness of packet traffic and the connectionless nature of communications. Because a wireless link in packet-switched networks is statistically shared among users on demand basis, the presence of packet traffic on a specific channel is random. As a result, the interference introduced to any user in the system is random in nature.

Based on a utility-maximization approach in microeconomics [116] and prior work for wireless circuit-switched data services [13, 14, 38, 39], we define a *probabilistic utility model* as a performance metric for a wireless packet-switched data user, which measures the average information throughput over the air link powered by each unit of the mobile's battery energy [bits/Joule]. This model takes into account both effect of traffic burstiness and QoS requirements in terms of average packet delay tolerance for mobile users. Based on the above model, we study a distributed strategy for the allocation of user transmit powers. The objective of the strategy is to simultaneously maximize the value of the utility for each individual user in the system. Owing to the mutual-dependence of user performance in wireless networks, a game-theoretic approach is taken to formulate the power control problem as a non-cooperative game. For easy demonstration of the strategy, as an example, we use a cellular slotted packet-switched CDMA network model in the work.

In general, the uplink power control problem is mathematically intractable. Utilizing several approximation techniques, the problem is reduced into analytically-tractable format and is studied both analytically and by simulations. Our investigation shows that the proposed uplink transmit power control scheme converges to a unique Nash equilibrium solution. At the equilibrium, the allocation of transmit power for a mobile user depends on: (1) location of the mobile; (2) average packet delay requirements of the mobiles; (3) burstiness of packet traffic; (4) mean and variance of the interference and background noise as seen by the receiver at the mobile's serving base station.

The characteristics of the proposed power control scheme are further studied both analytically and by simulations. It is shown that, given equal probabilities of traffic burstiness and equal user packet delay requirements, the power control scheme converges to a unique Nash equilibrium with equal signal-to-interference ratios (SIRs) for all users in the system. For unequal packet delay requirements from different users, the scheme converges to a unique Nash equilibrium with mixed SIRs which correspond to user's unequal delay requirements. The increase of traffic burstiness or the decrease of the delay tolerance will cause the system to converge to a higher equilibrium SIR. If a user tightens its packet delay requirement, transmission of its packets will require a higher SIR and a higher transmit power, which adversely affects all other users by lowering their SIRs, increasing their transmit powers and decreasing their utilities. On the other hand, relaxing a user's delay tolerance will do the exact opposite, benefiting all other users by reducing their transmit powers and increasing their utilities.

In addition, it is shown that the proposed scheme can be easily extended to multi-class user traffic environments where mobile terminals can communicate via various types of media with different QoS requirements.

The proposed power control scheme is an *average* strategy in the sense that it is driven only by local measurements of the mean and variance of bursty interference. We also consider an ideal scenario where the level of interference is measured by base station on a slot-by-slot basis and fed back instantly to users. We establish two idealized instantaneous power control strategies which assign optimum transmit power for each user in each time-slot. These two ideal strategies are used as benchmarks for performance evaluation on the proposed power control scheme. Analysis shows that the performance achieved when each user adapts its power level to the average interference is not much lower than when it adapts to the ideal instantaneous interference on slot-by-slot basis.

In general, this research work provides a new approach based on approximation techniques to model and investigate uplink transmit power control problem in packet-switched wireless networks. The results and insights generated by this study provide guidance on the efficient management of transmit powers for energy-efficient packet-switched data services for mobile terminals in current and future generations of wireless networks.

1.3 The Reference System Model

In general, we assume a generic wireless network model with interference-limited channels carrying packetized data traffic as the reference system model that we use for this research. Figure 1.1 shows the basic structure of this system model.

Some details about the reference system model are as follows. Data messages are packetized with the same fixed length packet. Time is divided into slots of duration T_p equal to the transmission of one packet. There are N users in the system. In each time slot, due to the burstiness of packet traffic, user i has a random number of interferers at its base station receiver which depend on the random arrivals of packets from all other users. We assume that the packet arrivals from user i, $\forall i$ into a time slot are Bernoulli-distributed.

When a packet arrives at a terminal, its transmission is started immediately. If the transmission is not successful, the packet is retransmitted until its successful reception at its base station receiver. We assume that our system uses a powerful error-detecting channel code that can detect almost all the errors in a packet. Thus the probability of undetected transmission errors can be neglected. We assume instantaneous error-free feedback, i.e., a user learns whether the transmission has succeeded before the beginning of the next time slot.

Generally, in packet-switched wireless networks, we always assume that there are a large number of mobile users in each cell, each transmitting for only a small fraction of time. Below is a summary of general assumptions that we make in this study:

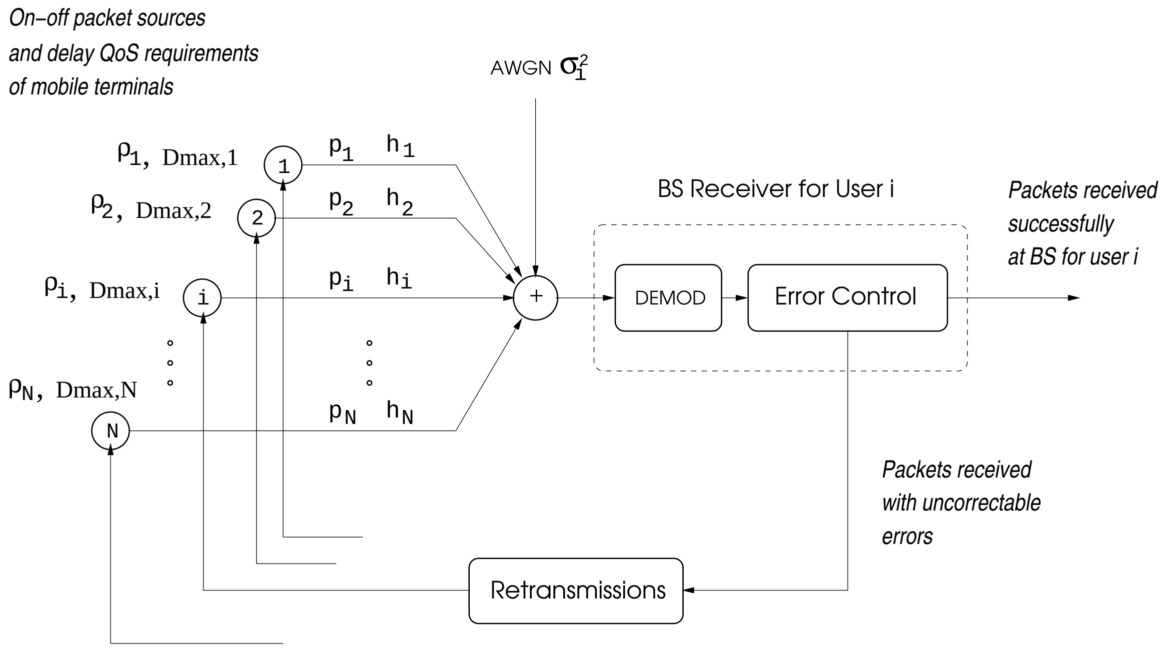

Figure 1.1: The reference system model — a wireless packet-switched data system with mobile terminals characterized as on-off stochastic packet traffic sources with their respective packet delay QoS requirements.

- Generic, single-cell interference-limited wireless network with N number of mobile users.
- Uplink transmit power control (from mobile user to base station).
- Users are stationary. No mobility is involved.
- No fading, only propagation loss is considered.
- All packets generated by users have the same fixed length of M bits carrying information payload of L bits.
- Packet transmissions are time-synchronized and slotted with time-slot interval T_p which is equal to the duration of a packet.
- The burstiness of packet traffic sources are modeled as random on-off stochastic processes with packet activity probability denoted by ρ_i, $\forall\, i \in [1, N]$.

- QoS requirement of data users studied here is the maximum tolerance of average packet transmission delay denoted by $D_{max,i}$, $\forall\, i \in [1, N]$.
- Only the packet transmission delay is considered. No queueing delay is involved.

Although the above reference system model is very general and not associated with any specific type of air interface technologies, we would like to use a concrete network example for analytical investigation as well as for easy demonstration of the uplink power control strategy studied in this work.

A typical example of the interference-limited wireless networks is the well-known CDMA-based systems in which multiple users all share the same frequency spectrum [100, 101]. We use a cellular CDMA network as a concrete example by considering a single-cell synchronous (or slotted) packet-switched data CDMA system [75, 101] in this study.

1.4 Review of Recent Research on Utility-Based Power Control

Most notably, a good summary on application of game theory in communication systems can be found in a recent publication of a Special Issue on "Game Theory in Communications Systems" in IEEE Journal on Selected Area in Communications (IEEE JSAC) [1]. This special issue contains most recent research works on applying game theory in understanding and designing of more efficient and reliable algorithms and protocols in communications.

A study reported in this IEEE JSAC Issue [2] presented a game-theoretic framework on the power and rate control problem for IEEE 802.11 Wireless LANs (WLANs). Specific utility functions are defined and problems are formulated as non-cooperative games with Nash Equilibrium established. Another research reported in this IEEE JSAC Issue [3] studies the cooperation between rational users in wireless networks based on coalitional game theory. Rate achieved by a user is defined as its utility to achieve stable coalition structure. Transmitter and receiver cooperation in an interference channel is studied as an illustrative cooperative model to determine the stable coalitions.

Specifically, there has been extensive research in the area of utility-based power control for wireless networks. Below, we review some of those research works in the literature.

A study in [4] presents a utility-based power control scheme by using a softened SIR requirement (utility) and adding a penalty on power consumption (cost). Utility is defined based on a Sigmoid function, and the the goal is to maximize the net utility, defined as utility minus cost.

A paper in [5] presents a utility-based power control for a two-cell CDMA data network which is a study on downlink. Utility is defined as the users willingness to pay for a received SIR. The objective is to allocate the transmitted power to maximize the total utility summed over all users subject to power constraints in each cell. Optimization is achieved by a pricing scheme in which each base station announces a price per unit transmitted power to the users, and each user requests power to maximize individual surplus (utility minus cost). Coordination between the two cells is needed to achieve the maximum utility.

A study in [6] presents a utility-based joint power and rate allocation scheme for downlink CDMA with blind multiuser detection. A sigmoid utility function as a function of SIR is used. A hierarchical rate allocation scheme is proposed, which together with the utility-based power control and the opportunistic fair scheduling scheme, maximizes the instantaneous weighted system throughput and the number of feasible users, and at the same time, guarantees the fairness among all users.

A CDMA uplink distributed power control proposed in [7] is based on a non-cooperative game formulation. The study addresses not only the power control problem, but also pricing and allocation of a single resource among several users. A cost function is introduced as the difference between the pricing and utility functions, and the existence of a unique Nash equilibrium is established. Simulations are used to study the convergence properties and robustness of each algorithm.

A game-theoretic approach is used to study energy-efficient power and rate control problem with QoS constraints in [8]. Utility is defined as the ratio of a user goodput to its transmit power (bits/Joule), and system model is based on an M/G/1 queue. Objective is to maximize each user's own utility and at the same time satisfy its QoS requirements which are specified in terms of the average source rate and average delay. The utility function considered here measures energy efficiency and the delay includes both transmission and queueing delays. The Nash equilibrium solution for the proposed non-cooperative game is derived and a closed-form expression for the utility achieved at equilibrium is obtained.

A study [9] proposes a centralized power control algorithm for CDMA data networks based on game formulation, subject to SIR constraints imposed by users. Non-cooperative case is examined first, and then is compared to its cooperative counterpart (through the Nash bargaining solution). Utility function is defined as the ratio of a user goodput to its transmit power (bits/Joule), but "efficiency function" [38] is used. It was shown that the use of the cooperative scheme results in significant reduction in the transmit power of the mobiles, while the achieved QoS is slightly compromised, compared to the non-cooperative scheme.

From the review of the above recent development of research in the area of utility-based power control for wireless networks, it seems there has been no research conducted on utility-based uplink power control specifically for packet-switched wireless networks. This research intends to contribute in this area by focusing on the modeling and investigating special characteristics of uplink power control in packet-switched wireless network environments.

1.5 Outline and Organization of the Dissertation

This dissertation consists of ten chapters, plus one appendix that provides supplementary materials to support the studies in the chapters. The ten chapters are organized as follows:

In Chapter 1, we introduce the research topic in general, and describe the system model and main assumptions that we use in this study. A summary of the main parameters and notations used throughout this thesis work is presented in two tables at the end of the chapter for easy reference.

In Chapter 2, we introduce the concept of utility as a performance metric for wireless data services. The behavior of a mobile user is basically to maximize the value of its utility whenever possible. In reference to a previously-proposed utility model for circuit-switched data systems, a delay-dependent utility model specifically designed for packet-switched data systems is proposed which serves as the basis of this study.

In Chapter 3, we derive the analytical expression for the packet-data utility model with the help of two main approximation methodologies. One is Gaussian interference approximation, and the other is the approximation of the expectation of the function of a random variable.

In Chapter 4, we formulate the distributed uplink transmit power control problem based on a non-cooperative game played among the users of the system. We study the equilibrium solution of the power control game, and derive the formula for the optimum solution of the non-cooperative power control game. We provide the proof for the existence and uniqueness of the equilibrium solution of the utility-maximizing power control game. Closed-form expression for the optimum allocation of transmit powers is presented.

In Chapter 5, we analytically investigate the characteristics of the proposed power control scheme under both equal and unequal delay QoS requirements from users in the system. Closed-form results such as the average packet SIR, transmit and received powers at the equilibrium of the game are derived and discussed. Particularly, the capacity issues and engineering concerns for packet-switched data systems are addressed, and the importance of the variance of interference is discussed.

In Chapter 6, we present the results of simulation studies on the packet-data power control scheme. Numerical results obtained include the equilibrium SIRs, utilities and transmit powers at equilibrium, effects of traffic burstiness, effects of packet delay requirements, and effects of the distance from users to their serving base station. Examples are presented to show the effects of unequal delay requirement on the solution of the power control game. We show that the results of simulation study support the results of the analytical study obtained in previous chapters.

In Chapter 7, we extend our study to a multimodal collaboration environment where multimedia traffic substreams with diverse timing and delay requirements are originated from a user and carried over a same wireless link. Two typical types of traffic substreams are considered. One of them is delay-sensitive and the other is delay-tolerant. Two strategies for transmitting the traffic substreams are studied, depending on whether to orthogonalize the traffic substreams or not.

In Chapter 8, we formulate and construct two idealized slot-by-slot (SBS) power control schemes for packet-data systems. One scheme is based on the Network-Assisted Power Control (NAPC) scheme proposed previously and is named as SBS-NAPC, and the other is based on the circuit-data Distributed Power Control (DPC) scheme proposed previously and is named as SBS-DPC. Using those two ideal schemes as performance benchmarks, we evaluate the performance of the packet-data power control scheme proposed in this work by comparing their optimum average utilities achieved. We show that the utility achieved when each user adapts its power level to the average interference is not much lower than when it adapts to the instantaneous interference on slot-by-slot basis. Practical value of the proposed scheme is discussed and compared to the other two idealized slot-by-slot schemes.

In Chapter 9, we discuss the implementation issues and practical aspects of the proposed transmit power control scheme for wireless packet-switched data services.

In Chapter 10, we conclude this thesis by summarizing the study results and indicating directions for future work.

1.6 Summary of Parameters and Notations

For easy reference to the materials presented in this work, the definitions of main parameters and notations used throughout this dissertation are summarized and listed here in the following tables.

Table 1.1: List of main parameters and notations used in the study.

Param.	**Unit**	**Description**
W	Hz	bandwidth of the wireless channel
N		number of mobile users in a CDMA cell
s		number of simultaneous interfering packets in a time-slot
M	bits	total number of bits in a data packet
L	bits	number of payload bits in a data packet
x_i	$\in \{0, 1\}$	on-off random variable for packet arrival activity of user i
ρ_i	$\in [0, 1]$	packet arrival activity probability of user i
k		number of transmissions needed to send a packet correctly
T_p	sec	packet length in time
d_i	sec	average transmission delay of a data packet from user i
$D_{max,\, i}$	sec	max. average packet transmission delay tolerated by user i
$\mathcal{P}_D$	$\in [0, 1]$	probability of delay requirement satisfaction $Prob\{d \leq D_{max}\}$
R	bits/sec	bit transmission rate of the wireless channel
R_p	packets/sec	packet transmission rate of the wireless channel
h_i		path gain from user i to its serving BS
p_i	Watts	uplink transmit power of user i
$\mathbf{p}_{-i}$	Watts	notation for transmit power vector without ith element
$P_{min,\, i}$	Watts	minimum transmit power required from user i
$P_{max,\, i}$	Watts	maximum transmit power allowed from user i
$P_{rec,\, i}$	Watts	received power from user i at its BS receiver
σ_i^2	Watts	AWGN background noise power at user i's BS receiver
Y_i	Watts	total interference and noise at user i's BS receiver (a *r.v.*)
μ_{Y_i}	Watts	mean of the total interference and noise Y_i
$\sigma_{Y_i}^2$	Watts^2	variance of the total interference and noise Y_i

Table 1.2: List of main parameters and notations used in the study (continued).

Param.	**Unit/Formula**	**Description**
$f_G(\cdot\|\mu,\sigma)$		Gaussian pdf with mean μ and variance σ^2
$F_G(\cdot\|\mu,\sigma)$	$\in [0,1]$	Gaussian cdf with mean μ and variance σ^2
$\Phi(\cdot)$	$\in [0,1]$	standard Gaussian cdf ($\mu = 0$, $\sigma^2 = 1$)
b_i	$= \frac{W}{R} \frac{h_i\, p_i}{\gamma_{min,\,i}}$	interference threshold for user i (event $\{Y_i \leq b_i\}$)
α_i	$= \frac{\mu_{Y_i}}{\sigma_{Y_i}}$	notation for the relative value of μ_{Y_i} over σ_{Y_i}
β_i	$= \frac{b_i}{\sigma_{Y_i}}$	notation for the relative value of b_i over σ_{Y_i}
Γ_i		user i's SIR in circuit-switched systems (*deterministic*)
γ_i	$= \frac{W}{R} \frac{h_i\, p_i}{Y_i}$	user i's SIR in packet-switched systems (*r.v.*)
$\overline{\gamma}_i^0$	$= \frac{W}{R} \frac{h_i\, p_i}{\mu_{Y_i}}$	notation for γ_i with Y_i replaced by its mean μ_{Y_i}
$\overline{\gamma}_i^1$	$= \frac{W}{R} \frac{h_i\, p_i}{\mu_{Y_i} + \sqrt{3}\,\sigma_{Y_i}}$	notation for γ_i with Y_i replaced by $\mu_{Y_i} + \sqrt{3}\,\sigma_{Y_i}$
$\overline{\gamma}_i^2$	$= \frac{W}{R} \frac{h_i\, p_i}{\mu_{Y_i} - \sqrt{3}\,\sigma_{Y_i}}$	notation for γ_i with Y_i replaced by $\mu_{Y_i} - \sqrt{3}\,\sigma_{Y_i}$
$\gamma_{min,\,i}$		min. SIR required by user i to achieve $\{d_i \leq D_{max,\,i}\}$
$\overline{\gamma}_{pkt,\,i}^*$	$= \frac{W}{R} \frac{h_i\, p_i^*}{\sigma_{Y_i}^*\; g\,(\mu_{Y_i}^*/\sigma_{Y_i}^*)}$	Avg. Packet SIR, a definition to facilitate the analysis
γ_{opt}		optimum target SIR for SBS-NAPC power control
$BER\,(\cdot)$	$\in [0,1]$	bit error rate (BER) or bit error probability
$f(\cdot)$	$\in [0,1]$	packet success rate (PSR)
$f_e(\cdot)$	$\in [0,1]$	efficiency function (a modified version of PSR)
$u_i(\cdot)$	bits/Joule	packet-data utility function of user i
$\overline{u}_i(\cdot)$	bits/Joule	expected packet-data utility function of user i
a_0		offset parameter of estimated linear solution function
a_1		slope parameter of estimated linear solution function
P_{succ}	$\in [0,1]$	notation for average packet success probability
P_{out}	$\in [0,1]$	outage probability for SBS-DPC power control

Chapter 2

Modeling of Utility Function for Packet-Switched Data Systems

2.1 Concept of Utility

The term *utility* that we adopt here is a concept widely used in the utility-maximization theory of microeconomics [116]. Its microeconomic definition is:

Definition 1 (Utility in Microeconomics [116]) *Utility is the level of satisfaction that a user gets from consuming a good or undertaking an activity.*

Applying this concept of utility to wireless mobile communications, the user is now the mobile subscriber and the good is the energy stored in the battery of the mobile terminal device. The subscriber consumes the battery energy to gain information throughput. The utility now measures how much information is delivered by consuming a basic unit of energy. Of course, the more, the better the subscriber would be satisfied.

As typical examples in wireless voice/data communications, Figure (2.1) shows two different types of utilities: one for a voice user and the other for a data user. The utilities are drawn against two important parameters: (1) the signal-to-interference ratio (SIR) — the quality metric of the wireless channels; and (2) the mobile-to-base transmit power — the main source of battery energy consumption in mobile terminals.

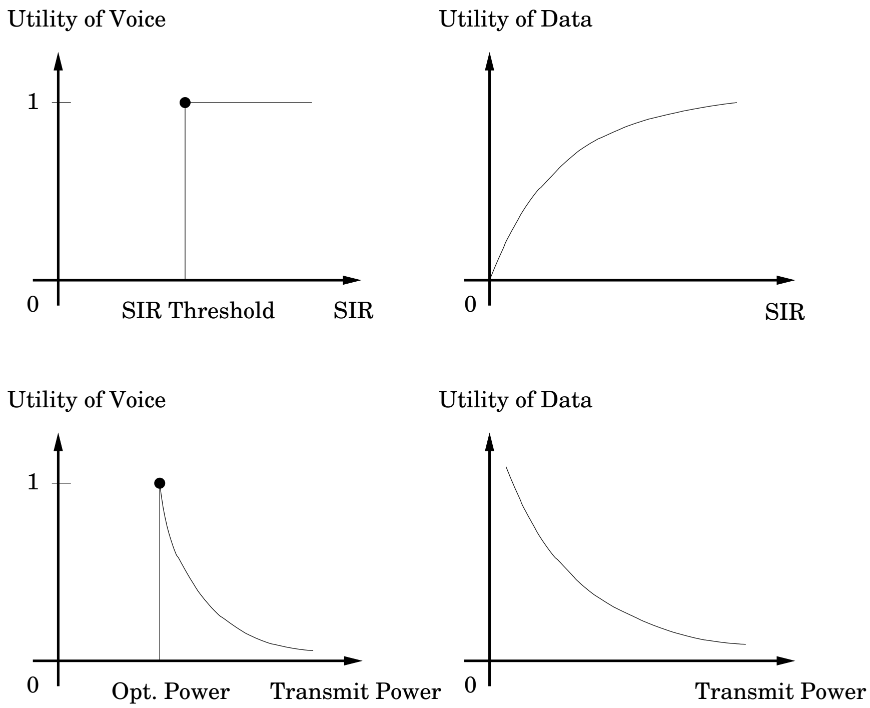

Figure 2.1: Conceptual utilities for a wireless voice user and for a wireless data user as functions of their respective signal-to-interference ratio (SIR) and transmit power.

For wireless voice service, the utility of communication is a binary function of SIR. The utility is zero if SIR is below a SIR threshold, which describes the situation that voice quality is beyond recognition if the signal is received with its SIR below the threshold. But the utility stays at one after SIR increases beyond the threshold, meaning that any further SIR improvement does not help the perception of voice for a listener. In the other aspect, given a fixed value of SIR that is equal or larger than the SIR threshold, the voice utility is a discontinuous function of transmit power. The utility achieves its maximum right at the point where the critical SIR threshold is attained with just enough transmit power (i.e., the optimum power). With powers lower that that, voice quality is unacceptable and thus utility is zero. The utility drops quickly as the power increases beyond the optimum point, indicating a growing dissatisfaction of the user due to the increasing waste in battery energy.

However, for wireless data service, the utility of communication is a continuously-increasing function of SIR. Any improvement in SIR gives a higher level of satisfaction for a data user. This is essentially because retransmission techniques are adopted in data communications to ensure error-free data reception. Hence, with a higher SIR due to a better wireless link, or lower interference, less errors are made in the transmission of data packets. This leads to less retransmissions of the same packet and thus achieves a higher data throughput. In the other aspect, given a fixed SIR, the data utility is a continuously-decreasing function of transmit power. This reflects the user preference for the conservation of its battery energy. The lower the transmit power, the less the battery energy consumption. Additionally, with a lower transmit power, less interference is created by the user to other users in the system.

In this thesis, we are considering radio resource allocation problems for wireless data services. Therefore, we will focus on the utility models for wireless data users in the following discussions.

2.2 Utility for Wireless Data in Circuit-Switched Systems

In data communications, there are two fundamentally different systems. One is called circuit-switched data systems, and the other is called packet-switched data systems.

In circuit-switched data systems, a mobile terminal tries to seize an available wireless link when it has data to send. After the successful set-up of a data call, it occupies the wireless link throughout the lifetime of the call, no matter whether it has data packets to send or is just idle with nothing to send. Therefore, at any specific time, a circuit-switched data system can only support a fixed number of user terminals that equals to the total number of available channels in the system. Circuit-switched data systems are good for high-speed and high-volume data transactions that need constant connections. However, radio channel resources are not efficiently shared among simultaneous users in a cell, as compared to packet-switched data systems, resulting in waste of expensive radio resources.

In this study, we assume that our system uses a channel code with powerful error-detecting capability which can detect almost all the errors in a packet. Thus the probability of undetected transmission errors can be neglected.

Let $f(\Gamma_i)$ represent the probability of a successful transmission of the packet, i.e., the packet success rate (PSR), for user i with Γ_i as its average SIR received at its base station. Under the assumption of independent bit errors in each packet of M bits, we have PSR as:

$$f(\Gamma_i) \;=\; [\,1 - BER(\Gamma_i)\,]^M \tag{2.1}$$

where $BER(\Gamma_i)$ represents the probability of bit errors or bit error rate (BER). For example, assuming that a non-coherent Binary Frequency-Shift Keying (BFSK) modem is used in our system, the BER performance expression of non-coherent BFSK [104]:

$$BER\,(\Gamma_i) \;=\; \frac{1}{2}\,e^{-\frac{1}{2}\,\Gamma_i} \tag{2.2}$$

With the assumption of constant and continuous transmissions of packets from all users in a circuit-switched data system, a utility model was developed [13, 14, 40, 38] as defined below:

<u>**Definition**</u> **2 (Circuit Data Utility Model)** *The utility of a wireless data user is the total number of correct bits that a user can transmit per unit of its battery energy [bits/Joule].*

$$u_i(p_i,\, \mathbf{p}_{-i}) \;\triangleq\; \frac{L\,R}{M\,p_i}\, f_e(\Gamma_i) \qquad \text{bits/Joule} \tag{2.3}$$

where p_i is the transmit power of user i, and $\mathbf{p}_{-i}$ denotes a power vector without the ith element. L is the payload of a packet in bits, and M is packet length in bits. R is bit rate of the radio channel [bits/sec]. $f_e(\Gamma_i)$ is defined as *"Efficiency Function"* which measures the efficiency of the transmission protocol [38]. $f_e(\Gamma_i)$ is a modified version of the packet success rate $f(\Gamma_i)$:

$$f_e(\Gamma_i) \;\triangleq\; [\,1 - 2 \times BER(\Gamma_i)\,]^M \tag{2.4}$$

Where BER_i is user i's bit error rate (BER). The purpose of insertion of a 2 into PSR formula in front of BER is to achieve a well-behaved efficiency function for the convenience of closed-form analytical study [38]. Γ_i, the average SIR received at BS for user i, is defined as:

$$\Gamma_i \;=\; \frac{W}{R}\, \frac{h_i\, p_i}{\sum_{j=1, j\neq i}^{N} h_j\, p_j \;+\; \sigma_i^2} \tag{2.5}$$

where p_i and h_i, $i = 1, \cdots, N$, are user i's transmit power and path gain to its serving BS. W is system bandwidth [Hz]. σ_i^2 is background noise power at user i's receiver.

2.3 Improvement on the Modeling of Utility Function

The efficiency function $f_e(\Gamma_i)$ (2.4) introduced in the modeling of the above utility function captured the essential behaviors of wireless data throughput. However, it

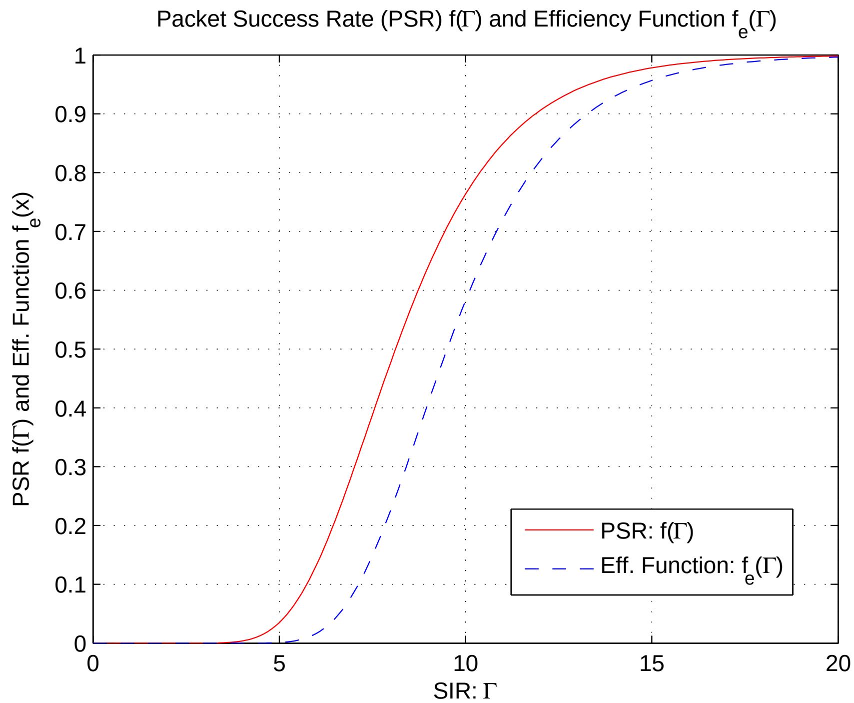

Figure 2.2: Illustration of the difference between Packet Success Rate (PSR) $f(\Gamma_i)$ and Efficiency Function $f_e(\Gamma_i)$.

has its shortcomings. Figure 2.2 shows the curve of Packet Success Rate (PSR) $f(\Gamma_i)$ in comparison to that of Efficiency Function $f_e(\Gamma_i)$.

$f_e(\Gamma_i)$ was essentially introduced to remedy the zero-power issue of the throughput-to-power ratio, $\frac{f(\Gamma_i)}{p_i}$, where $f(\Gamma_i)$ is the PSR (2.9). The misbehavior occurs when $p_i \to 0$, thus $\Gamma_i \to 0$, $f(\Gamma_i)$ remains a positive number. Therefore the throughput-to-power ratio, $\frac{f(\Gamma_i)}{p_i}$, becomes unbounded as $p_i \to 0$.

The use of the efficiency function $f_e(\Gamma_i)$ to replace PSR $f(\Gamma_i)$ makes the *modified* throughput-to-power ratio $\frac{f_e(\Gamma_i)}{p_i}$ mathematically tractable, since $\lim_{p_i \to 0} f_e(\Gamma_i) \to 0$. However, the physical meaning of the modified ratio becomes vague. A study in [11] showed that the power control algorithms designed with this efficiency function can deviate far from the optimal solution.

Radio resource management study based on the utility maximization for wireless data users in [10, 11, 12] has pointed to a new approach to model the utility function. The method is to introduce a new concept of *"earned* throughput-to-power ratio" or ETPR, which is defined as below [10]:

$$\text{ETPR} \triangleq \frac{[f(\Gamma_i) - f(0)]}{p_i} \tag{2.6}$$

Or

$$\text{ETPR} \triangleq \frac{1}{p_i}\left[\frac{f(\Gamma_i) - f(0)}{1 - f(0)}\right] \tag{2.7}$$

The difference between the two definitions above is that equation (2.7) is normalized so $[\frac{f(\Gamma_i)-f(0)}{1-f(0)}]$ behaves like a probability density function.

The numerator of the ratio, $[f(\Gamma_i) - f(0)]$, represents the incremental throughput achieved beyond the trivial throughput floor purely due to noise and interference. The incremental throughput achievement is the result of dispensing terminal's transmit power [10].

ETPR is shown to exhibit good mathematical behavior, and it has a clearer physical interpretation since it is tightly-related to the throughput or PSR. Detailed discussions on ETPR are in [10, 11, 12].

In our study, we will utilize ETPR definition in Equation (2.7) to represent the throughput-to-power ratio in the utility modeling in subsequent discussions.

With this improvement on utility modeling, the utility function for circuit-switched data CDMA systems (2.3) becomes:

$$u_i(p_i,\ \mathbf{p}_{-i}) \ \triangleq\ \frac{L\,R}{M\,p_i}\left[\frac{f(\Gamma_i)-f(0)}{1-f(0)}\right] \qquad \text{bits/Joule}\,. \tag{2.8}$$

2.4 Utility for Wireless Data in Packet-Switched Systems

In wireless packet-switched data systems, a data terminal occupies a wireless link only if it has data packets to send. In the silent periods with no packets to be sent, the link is released and can be seized by any other user who needs to send packets. Because each data user only actively occupies a link whenever it needs to, radio channel resources can be efficiently shared by all the users in the system. This is called the “statistical multiplexing” property of packet-switched systems. This fundamental difference from the circuit-switched systems leads to the following characteristics:

- Support of a larger number of users than that of circuit-switched systems — more than the total number of channels available in the system.
- Efficient accommodation of heterogeneous traffic with various quality-of-service (QoS) requirements.
- Random discontinuous transmissions over radio channels due to bursty packet traffic sources.
- The interference, SIR and packet success rate (PSR) are all random in nature, due to the random discontinuous transmissions.
- Due to the nature of randomness in packet-switched systems, statistical and probabilistic measures are utilized to evaluate performance, which are different from the ones used in circuit-switched systems.

2.4.1 Modeling to Capture the Characteristics of Packet-Switched Data Systems

In data communications, delay is a very important performance measure and delay requirement is one of the key specifications for QoS. In our study of wireless data systems, we want to introduce the concept of delay into our utility model. Here we only consider the transmission delay of a packet over a wireless channel.

The packet transmission delay is caused by retransmissions of a packet if the packet is received with errors. The more number of packet retransmissions that is needed to receive a packet error-free, the longer delay it introduces for the delivery of that packet.

In the utility modeling, as before, we assume that the system uses a channel code with powerful error-detecting capability which can detect almost all the errors in a packet. Hence, the probability of undetected transmission errors can be neglected. Also, bit errors in a packet are assumed to be statistically independent.

Let $f(\gamma_i)$ represent PSR for user i in a packet-switched system, where γ_i is the SIR for the user. Notice that here we use an lower-case γ_i to denote SIR in packet-switched systems, in contrast to the upper-case Γ_i (2.5) for circuit-switched systems in prior discussions. The key difference is that SIR γ_i is a *random variable* while SIR Γ_i is *deterministic*.

For a data packet of M bits, we have PSR as:

$$f(\gamma_i) \; = \; [\, 1 - BER\,(\gamma_i)\,]^M \tag{2.9}$$

Again, we assume that a non-coherent Binary Frequency-Shift Keying (BFSK) modem is used in our system. So the BER is described by [104]:

$$BER\,(\gamma_i) \;=\; \frac{1}{2}\, e^{-\frac{1}{2}\,\gamma_i} \tag{2.10}$$

Notice that *BER* is a function of a random variable, the SIR γ_i.

Now, the SIR for packet-switched data γ_i is defined as:

$$\gamma_i \;\triangleq\; \frac{W}{R}\, \frac{h_i\, p_i}{\sum_{j=1, j\neq i}^{N}\; x_j\, h_j\, p_j \;+\; \sigma_i^2} \tag{2.11}$$

where $x_j \in \{0,\,1\}$ is a on-off random variable to indicate the packet activity of the traffic generated by user j. Here $x_j,\ \forall\, j \neq i$, is used to characterize the random burstiness of packet traffic generated by all mobile users other than user i. This stochastic on-off traffic model is described by the following:

$$x_j \;=\; \begin{cases} 1 & \text{with probability} \quad \rho_j \\ 0 & \text{with probability} \quad 1-\rho_j \end{cases} \quad , \qquad \forall\, j \neq i \tag{2.12}$$

where ρ_j represents the *packet activity probability*, or simply *activity probability* of user j.

In this study, we assume that the value of x_j is constant over the entire duration of a time-slot or in the entire interval of a packet (the length of a time-slot and the length of a packet are equal in this study).

Let k denote the number of transmissions (transmission and retransmissions) needed for a serving BS to correctly receive a packet from a user. Due to the randomness of the interference, k is a random number and it is geometrically distributed.

Another simplifying assumption that we make here is that over the k transmissions of the same packet from user i, the packet activity of all other users x_j remains the same. This assumption is necessary to make our study analytically tractable. The assumption may not be realistic in actual systems. However, knowing that k usually has a small range in practical systems and also the continuity of traffic sources, assuming stable traffic activities from all other users within the window of k transmissions of a packet is not too far-fetched.

Similarly, transmit powers of all users in the system are assumed to be constant over the k transmissions of a packet. Because of the above assumptions, SIR γ_i is constant over the k transmissions of a packet of user i. γ_i changes randomly only between respective transmissions of different packets originated from user i.

Conditioned on γ_i, we can write the geometric probability mass function (pmf) of $k|\gamma_i$ as:

$$P_{k|\gamma_i}(k|\gamma_i) = \begin{cases} f(\gamma_i)\,[1 - f(\gamma_i)]^{k-1} & k = 1, 2, 3, \cdots \\ 0 & \text{otherwise} \end{cases} \tag{2.13}$$

with its mean as [108]:

$$E[k|\gamma_i] = \frac{1}{f(\gamma_i)} \tag{2.14}$$

Denote d_i [sec] as the average transmission delay of data packets originated from user i, and denote $D_{max,\,i}$ [sec] as the maximum delay tolerated by user i or the delay tolerance of user i. d_i can be expressed as

$$d_i = T_p\,E[k|\gamma_i] = \frac{T_p}{f(\gamma_i)} \qquad (sec.) \tag{2.15}$$

where $T_p = M/R$ is packet length in time [sec]. $R_p = 1/T_p$ is the packet transmission rate of the wireless channel [packets/sec].

2.4.2 Packet Value and Energy Cost for Wireless Data Users

Let us introduce the concept of *Packet Value* adopted in this study with the following definition:

<u>Definition</u> 3 (Packet Value) *The value of a data packet is the number of its payload bits, if the average transmission delay of the packet is less or equal to a specific average delay constraint. The packet value is zero if the average delay requirement of the packet is violated.*

$$\textbf{packet value} \;=\; \begin{cases} L \quad \textit{bits} & \text{with prob.} \quad Prob\,\{\, d_i \leq D_{max,\,i} \,\} \\ 0 & \text{otherwise.} \end{cases} \tag{2.16}$$

That is, a packet is useful *only if*, on average, it arrives at its destination error-free before a deadline $D_{max,\,i}$. Otherwise, the packet is considered useless by the user.

This $D_{max,\,i}$ can be specified by the system or agreed upon by end-users, based on the needs to support a specific application service. For example, for wireless voice or video services, $D_{max,\,i}$ needs to be small. But for wireless Email or FTP services, $D_{max,\,i}$ can be very large.

As in the previous study in [13, 14], we define the *Energy Cost* of packet transmission similarly:

<u>Definition</u> 4 (Energy Cost) *The energy cost of packet transmission is the number of battery Joules consumed by the transmission and retransmissions of a data* *packet for its successful reception at its BS receiver.*

$$\textbf{energy cost} \;=\; k\,(\,p_i \,\times\, T_p\,) \;=\; k\,\frac{p_i\,M}{R} \tag{2.17}$$

For a wireless data user, we need to find the average packet value and the average energy cost to achieve that packet value. Taking conditional expectations and using Equations (2.14) and (2.15), we can write the expected packet value and the expected energy cost, conditioned on the SIR γ_i, as follows:

$$E[\textbf{packet value}|\gamma_i] \;=\; L\,Prob\{\,T_p\,E[k|\gamma_i] \le D_{max,\,i}\,\} \tag{2.18}$$

$$E[\textbf{energy cost}|\gamma_i] \;=\; \frac{p_i\,M}{R}\,E[k|\gamma_i] \tag{2.19}$$

2.4.3 Packet-Data Utility Model - The Expected Utility

Based on the average packet value and average energy cost, we define a utility function as a function of the random variable, *i.e.*, SIR γ_i, as below:

$$\begin{aligned} u_i(\gamma_i) \;&\triangleq\; \frac{E[\textbf{packet value}|\gamma_i]}{E[\textbf{energy cost}|\gamma_i]} \\ &=\; \frac{L\,R}{M\,p_i}\,f(\gamma_i) \times Prob\left\{\frac{T_p}{f(\gamma_i)} \le D_{max,\,i}\right\} \end{aligned} \tag{2.20}$$

Looking at this utility function $u_i(\gamma_i)$, it resembles the one for circuit-data discussed in Section 2.2 except it is no longer a deterministic function. Besides, it has an extra discounting factor in form of a probability. The probabilistic discounting factor, $Prob\left\{\frac{T_p}{f(\gamma_i)} \le D_{max,\,i}\right\}$, shows the level of satisfying the delay requirement.

Notice that if the maximum delay tolerance $D_{max,\,i} \to \infty$, then the discounting factor $Prob\left\{\frac{T_p}{f(\gamma_i)} \le D_{max,\,i}\right\} = 1$, and the utility model returns to the same format as the one in circuit-data scenario:

$$\lim_{D_{max,\,i} \to \infty} u_i(\gamma_i) \;=\; \frac{L\,R}{M\,p_i}\,f(\gamma_i) \tag{2.21}$$

The scenario with $D_{max,\,i} \to \infty$ and $Prob\left\{\frac{T_p}{f(\gamma_i)} \le D_{max,\,i}\right\} = 1$ means that there is no concern on packet delay, or there is no restriction on the number of packet retransmissions, as long as the packet is received at BS correctly.

Understandably, the packet-data utility model introduced above share the same shortcoming as its circuit-data counterpart on zero-power issue as discussed in Section 2.3, because they originated from the same idea on utility modeling for radio resource management.

In the utility modeling for circuit-switched data networks, we encountered the zero power issue with the throughput-to-power ratio $\frac{f(\Gamma_i)}{p_i}$, where Γ_i is the signal-to-interference ratio for circuit-data systems. In the utility modeling for packet-switched data networks, we face the same issue with the throughput-to-power ratio as $\frac{f(\gamma_i)}{p_i}$. Both models have the similar functional structure. The only difference is that Γ_i is deterministic while γ_i is random in nature due to the bursty traffic in packet-switched data systems. The randomness in γ_i contributes to the uncertainty in utility.

To summarize the above discussions, we have two issues with the utility definition in (2.20):

1. **Zero-Power Issue**: Abnormality of utility model at zero-power due to the fact that $\dfrac{f(\gamma_i)}{p_i}$ grows unbounded as $p_i \rightarrow 0$, since $f(\gamma_i) \neq 0$.

2. **Uncertainty Issue**: Uncertainty in utility model due to randomness in signal-to-interference ratio γ_i, attributed to the burstiness of packet-switched data traffic.

To resolve those two issues in utility modeling for packet-data systems, two respective approaches in the following are utilized:

- <u>On the Zero-Power Issue</u>:

  To remedy the zero power issue, we follow the approach discussed in Section 2.3 to improve the utility model. The normalized *earned* throughput-to-power ratio (ETPR) in Equation (2.7) will be adopted in place of the regular throughput-to-power ratio:

$$\frac{f(\gamma_i)}{p_i} \quad \Longrightarrow \quad \frac{1}{p_i}\left[\frac{f(\gamma_i) - f(0)}{1 - f(0)}\right] \tag{2.22}$$

- <u>On the Uncertainty Issue</u>:

  To compute the value of utility under conditions of uncertainty, we follow the concept of *expected utility* in Microeconomics [116]. By taking expectation on utility model in (2.20), we will arrive at a probabilistic measure of average utility. Thus uncertainty introduced due to randomness of γ_i is accommodated in the utility modeling.

Putting the above discussions all together, then formally, we define the packet-data utility function model that characterizes both the packet value and energy cost of packet-switched data networks as follows:

<u>**Definition**</u> **5 (Expected Packet-Data Utility Model)** *The expected utility of a wireless data user in a packet-switched system is the **average** total number of useful bits that the user can transmit per unit of its battery energy [bits/Joule], discounted by the probability of failing to satisfy a packet delay requirement.*

Mathematically, the utility function for user i is derived by taking expectation on Equation (2.20) with respect to the burstiness of data traffic and by using earned throughput-to-power ratio (ETPR) as in (2.22). Thus we have:

$$\begin{aligned} \overline{u}_i(\,p_i,\ \mathbf{p}_{-i}\,) &= E[u_i(\gamma_i)] \\ &= \frac{L\,R}{M\,p_i}\;E\left[\frac{f(\gamma_i)-f(0)}{1-f(0)}\right]\;Prob\left\{\frac{T_p}{f(\gamma_i)} \le D_{max,\,i}\right\} \\ &= \frac{L\,R}{M\,p_i}\left[\frac{E[f(\gamma_i)]-f(0)}{1-f(0)}\right]\;Prob\left\{\frac{T_p}{f(\gamma_i)} \le D_{max,\,i}\right\} \quad bits/Joule \end{aligned} \tag{2.23}$$

Notice that during calculation of the expectation, the probability discounting factor $Prob\{\,\frac{T_p}{f(\gamma_i)} \le D_{max,\,i}\,\}$ is outside the expectation. This is because of the fact that $Prob\{\,\frac{T_p}{f(\gamma_i)} \le D_{max,\,i}\,\}$ is a function of $D_{max,\,i}$ given T_p, and the resulting value of the probability is a deterministic number.

The physical interpretation of the packet-data utility definition in equation (2.23) is essentially a trade-off between two important but conflicting aspects of wireless data communications initiated from a mobile terminal to its serving base station (uplink communications), as described below:

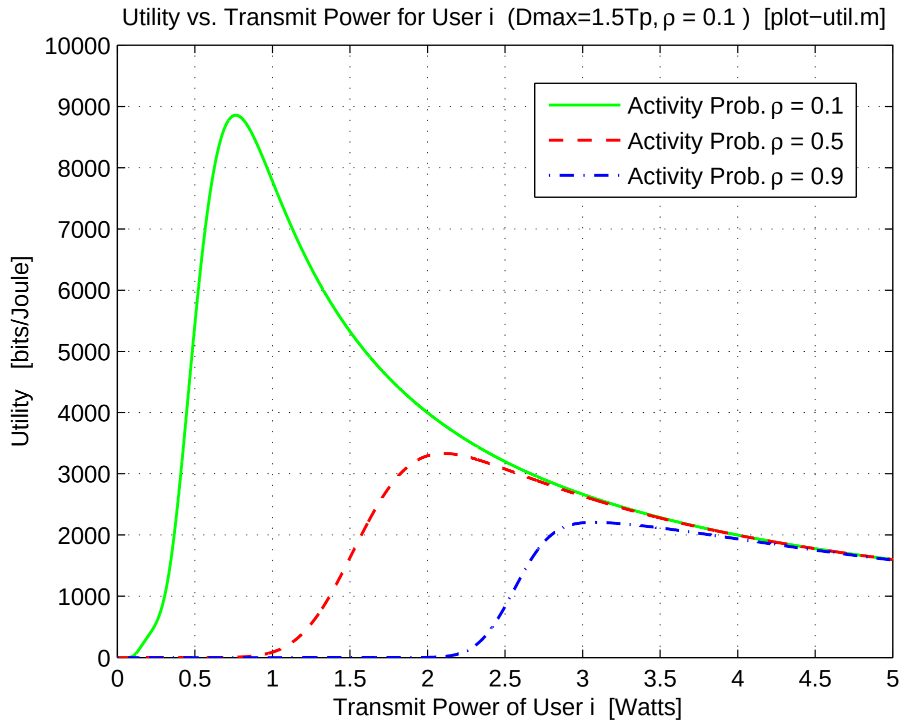

Figure 2.3: Utility model of user i vs. its transmit power p_i for different activity probability $\rho_j = \rho$, for all $j \neq i$ and packet delay tolerance $D_{max,i} = 1.5\,T_p$, given some fixed amount of interference powers from all other users in the system.

1. Energy conservation of portable battery in the mobile terminal — utility decreases with the increase of transmit power: a low transmit power is preferred to save more battery energy and to prolong the lifetime of mobile battery.

2. Average data throughput requirement of the mobile terminal — utility increases with the increase of transmit power: a high transmit power is preferred to increase the average PSR and to reduce the average packet transmission delay d_i. Hence for a given delay tolerance $D_{max,i}$, the average packet value is also increased.

Figure (2.3) illustrates this trade-off by plotting the utility of a user as a function of its transmit power, given some known interference from all other users. From viewpoint of the user, it doesn't want to burn up its battery too quick by using a transmit power that is too high, and meanwhile it also doesn't want to ruin its average data throughput by using a transmit power that is too low. The balance of these two aspects gives rise to an optimum transmit power which corresponds to the peak of the utility function. Higher values of the activity probability ρ signifies an increased average interference from all other users. Therefore, higher transmit powers have to be applied to overcome the increased interference, which results in lower utilities.

2.5 Evaluation of the Packet-Data Utility Model

In the packet-data utility model defined above in Equation (2.23), there are two key components in the model that need to be evaluated before we can go further to use the model to assist our study. They are:

1. the probability of delay requirement satisfaction $Prob\left\{\frac{T_p}{f(\gamma_i)} \leq D_{max,\,i}\right\}$;
2. the expected packet success rate (PSR) $E[f(\gamma_i)]$.

To demonstrate the difficulty in the analytical computation of the packet-data utility model, we take the calculation of the expected PSR $E[f(\gamma_i)]$ as an example in the following.

First, let us simplify the probability discounting factor by using the notation:

$$\mathcal{P}_D \triangleq Prob\left\{\frac{T_p}{f(\gamma_i)} \leq D_{max,\,i}\right\} \tag{2.24}$$

By writing out the expected PSR expression, the exact expression for the expected utility in Equation (2.23) for user i in a system with N users can be derived as follows:

$$\overline{u}_i(\,p_i,\ \mathbf{p}_{-i}\,) \;=\; \frac{L\,R}{M\,p_i}\left[\frac{\mathcal{P}_D}{1-f(0)}\right]\left\{\sum_{s=0}^{N-1}\left[\sum_{l=1}^{\binom{N-1}{s}} f[\gamma_i(s,\,l)]\right]\rho^s(1-\rho)^{(N-1)-s} \;-\; f(0)\right\} \tag{2.25}$$

where

$$\gamma_i(s,\,l) \;=\; \frac{W}{R}\,\frac{h_i\,p_i}{\sum_{j=1,j\neq i}^{N}\,x_j\,h_j\,p_j\;+\;\sigma_i^2} \tag{2.26}$$

and

$$\sum_{j=1,j\neq i}^{N} x_j \;=\; s \tag{2.27}$$

where s denotes the number of active interferers (number of other users who are actively transmitting packets), and l represents the lth combination in a group of s active interferers ($l \in [1,\ \binom{N-1}{s}]$). All users have the same activity probability, i.e., $\rho_j = \rho,\ \forall\, j \neq i$ is assumed here for simplicity.

To show the complexity of the above computation, let us consider a system with only a small number users, say $N = 5$, as an example. In this system, to a particular user (say user 1), there are 16 possibilities of interference from the other four remaining users. Therefore, there 16 terms in the expected utility model for user 1 which can be written out as follows:

$$
\begin{aligned}
E[u_1] \;=\; & \frac{L\,R}{M\,p_i} \times \left[\frac{\mathcal{P}_D}{1-f(0)}\right] \times \left\{ f\left(\frac{W}{R}\,\frac{h_1\,p_1}{\sigma_i^2}\right)(1-\rho)^4 \right. \\
& + \left[f\left(\frac{W}{R}\,\frac{h_1\,p_1}{h_2\,p_2+\sigma_i^2}\right) \;+\;\cdots\;+\; f\left(\frac{W}{R}\,\frac{h_1\,p_1}{h_5\,p_5+\sigma_i^2}\right)\right]\rho(1-\rho)^3 \\
& + \left[f\left(\frac{W}{R}\,\frac{h_1\,p_1}{h_2\,p_2+h_3\,p_3+\sigma_i^2}\right) \;+\;\cdots\;+\; f\left(\frac{W}{R}\,\frac{h_1\,p_1}{h_4\,p_4+h_5\,p_5+\sigma_i^2}\right)\right]\rho^2(1-\rho)^2 \\
& + \left[f\left(\frac{W}{R}\,\frac{h_1\,p_1}{h_2\,p_2+h_3\,p_3+h_4\,p_4+\sigma_i^2}\right) \;+\;\cdots \right. \\
& \qquad\qquad \left. \cdots\;+\; f\left(\frac{W}{R}\,\frac{h_1\,p_1}{h_3\,p_3+h_4\,p_4+h_5\,p_5+\sigma_i^2}\right)\right]\rho^3(1-\rho) \\
& \left. + \; f\left(\frac{W}{R}\,\frac{h_1\,p_1}{\sum_{j=2}^{5} h_j\,p_j+\sigma_i^2}\right)\rho^4 \;-\; f(0) \right\} \qquad (2.28)
\end{aligned}
$$

Obviously, as the number of users in the system grows, the evaluation of the exact value of the average utility becomes exponentially tedious. As N grows, the increasing complexity of the model essentially prevents us from getting analytically-tractable solutions of the problem. Needless to say the extra troubles arising from the evaluation of the probability of delay requirement satisfaction $\mathcal{P}_D = Prob\left\{\frac{T_p}{f(\gamma_i)} \leq D_{max,\,i}\right\}$, we absolutely need a different and better approach to tackle the problem.

Instead of immediately resorting to numerical methods, alternatively in the following chapters, we will use a couple of approximation approaches to reduce the complexity of the packet-data utility model without much loss of accuracy. Then we will be able to analytically investigate the utility-maximization problem for wireless packet-switched data systems based on packet-data utility model.

Besides numerical studies, by obtaining closed-form solutions whenever possible, we will gain much more insights into the transmit power resource allocation problem for wireless data services. This will be shown through the studies in the subsequent Chapters.

Chapter 3

Approximation of Interference and Estimation of Expected Packet Success Rate

3.1 From Delay Requirement to Interference Requirement

The study of the packet-data utility function defined in (2.23) requires us to know the probability distribution of the average packet delay d_i as in (2.15). If we know d_i's distribution, we can then calculate the probability of delay requirement satisfaction $\mathcal{P}_D = Prob\{\, d_i \leq D_{max,i} \,\}$. Unfortunately, a probability distribution as such is difficult to find.

To make the problem analytically tractable, we first transform the delay requirement inequality $d_i \leq D_{max,i}$ into its basic form as an *interference requirement inequality*. We then characterize the distribution of the interference. And finally we approximate the probability of delay requirement satisfaction $Prob\{\, d_i \leq D_{max,i} \,\}$ based on Gaussian approximation of the total interference received at user i's BS receiver.

3.1.1 Interference Requirement Inequality

From previous discussions on the packet delay and packet-data utility model as in Equations (2.15) and (2.23), the delay requirement inequality, or the probability event $\{\, d_i \leq D_{max,i} \,\}$ can be expressed equivalently as:

$$\{ d_i \leq D_{max,i} \} \iff \left\{ \frac{T_p}{f(\gamma_i)} \leq D_{max,i} \right\} \iff \left\{ f(\gamma_i) \geq \frac{1}{R_p\, D_{max,i}} \right\} \quad (3.1)$$

where $R_p = \dfrac{1}{T_p}$ which is the packet transmission rate.

Inserting the PSR expression (2.9), and the BER expression of non-coherent BFSK in (2.10), (3.1) is continued as below:

$$\iff \left\{ BER(\gamma_i) \leq 1 - \frac{1}{(R_p\, D_{max,i})^{\frac{1}{M}}} \right\} \iff \{ \gamma_i \geq \gamma_{min,i} \} \qquad (3.2)$$

with $\gamma_{min,i}$ as the *minimum SIR requirement* for user i which is defined as below:

$$\gamma_{min,i} \triangleq 2 \ln \left\{ \frac{(D_{max,i}\, R_p)^{\frac{1}{M}}}{2\left[(D_{max,i}\, R_p)^{\frac{1}{M}} - 1\right]} \right\} \qquad (3.3)$$

Again, using γ_i expression (2.11), we can finally express the delay requirement inequality as below. Those three equivalent inequalities actually represent the same probability event.

$$\{ d_i \leq D_{max,i} \} \iff \{ \gamma_i \geq \gamma_{min,i} \} \iff \{ Y_i \leq b_i \} \qquad (3.4)$$

where Y_i is defined as the total interference and background noise seen by the receiver of user i at BS, and b_i is defined as an *interference threshold parameter* for user i.

$$Y_i \triangleq \sum_{j=1, j\neq i}^{N} x_j\, h_j\, p_j \;+\; \sigma_i^2 \tag{3.5}$$

$$b_i \triangleq \frac{W}{R}\,\frac{h_i\, p_i}{\gamma_{min,\,i}} \tag{3.6}$$

Note that b_i is a function of the user's transmit power p_i and its minimum SIR requirement $\gamma_{min,\,i}$. b_i increases with p_i and decreases with $\gamma_{min,\,i}$. Given that other parameters are the same, with higher $\gamma_{min,\,i}$, a bigger p_i is required if b_i is kept the same.

Here $\{\, Y_i \le b_i \,\}$ is called *interference requirement inequality*. Our derivation shows that it is equivalent to the original *packet delay requirement inequality* $\{\, d_i \le D_{max,\,i} \,\}$. Therefore, we can now calculate the probability $Prob\,\{\, Y_i \le b_i \,\}$, instead of calculating the original probability $Prob\,\{\, d_i \le D_{max,\,i} \,\}$ which is hard to do since the distribution of d_i is unknown.

3.1.2 Minimum SIR Requirement vs. Maximum Packet Delay Tolerance

To better understand how the requirement on average packet transmission delay tolerance $D_{max,\,i}$ translates into the requirement on the minimum SIR $\gamma_{min,\,i}$ as described by Equation (3.3). Figure (3.1) plots $\gamma_{min,\,i}$ as a function of $D_{max,\,i}$ for various packet size M. From the plots, we observe that

$$D_{max,\,i} \;\le\; 2^M\, T_p \quad\Longrightarrow\quad \gamma_{min,\,i} \;\ge\; 0 \;\;^{\S} \tag{3.7}$$

§ Note that care needs to be taken while using Equation (3.3). $\gamma_{min,\,i}$ could fall into negative region with large $D_{max,\,i}$. To prevent this irregularity, restriction $\gamma_{min,\,i} \ge 0$ needs to be imposed.

with $\gamma_{min,\,i} = 0$ at $D_{max,\,i} = 2^M\,T_p$. Note that $T_p = 1/R_p$ is the packet length in time which is equal to the interval of one packet transmission, or the duration of a time-slot.

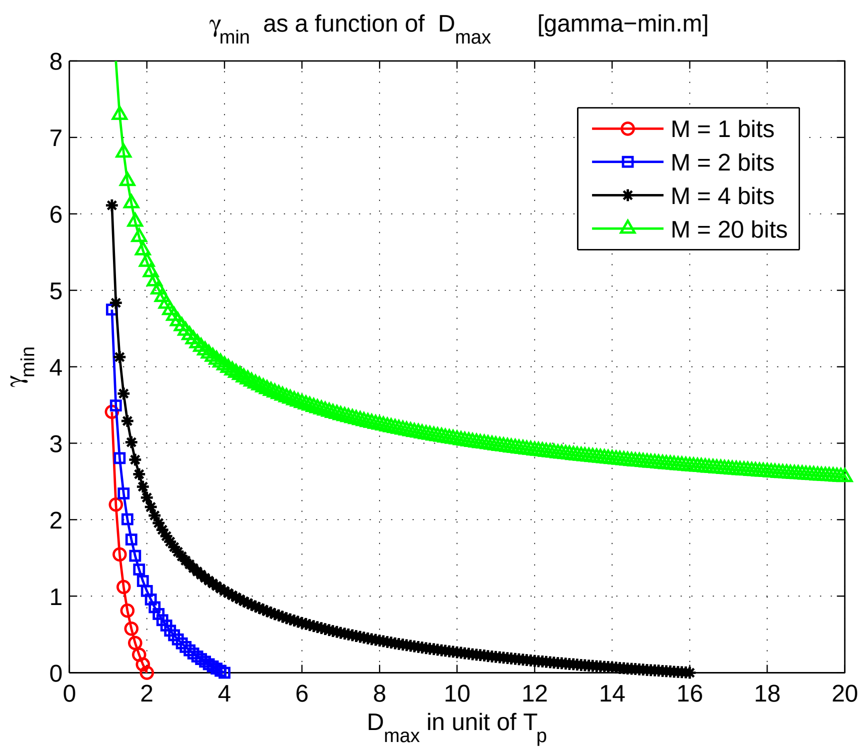

Figure 3.1: Relationship between the minimum SIR requirement $\gamma_{min,\,i}$ and the maximum tolerance of packet transmission delay $D_{max,\,i}$ (in units of T_p) for various packet size M.

It is interesting to note that the x-axis of Figure 3.1 corresponds to zero $\gamma_{min,\,i}$ requirement ($\gamma_{min,\,i} = 0$), which in turn corresponds to the worst case bit error rate (BER) requirement (BER $= 0.5$, the probability of random guessing). Take a simple case of 1 bit packet ($M = 1$) for example: if the transmission delay requirement is 2 transmissions ($D_{max,\,i} = 2\,T_p$), then the minimum SIR requirement is $\gamma_{min,\,i} = 0$. This is equivalent to no requirement on $\gamma_{min,\,i}$, and random guessing is good enough to satisfy this $D_{max,\,i}$ requirement. For any delay requirement with $D_{max,\,i} < 2\,T_p$, the system has to do better than just guessing randomly (BER < 0.5).

Another property that is worth noticing in Figure 3.1 or in Equation (3.3) is that, as $D_{max,i} \to T_p$, $\gamma_{min,i} \to \infty$. This indicates that it is impossible to receive a packet error-free in just *one* shot or a single transmission on average. That kind of perfection is unrealistic to achieve, given that there are always interference and noise in the system!

3.2 Interference Characterization with Gaussian Approximation

We now characterize Y_i in (3.5), the sum of the random interference and background noise appearing at the input of user i's BS receiver. The key to describe Y_i is to know its probability distribution. If the analytical expression of the distribution is known, we can then study analytically the probability $Prob\{\, Y_i \le b_i \,\}$ as a function of user i's transmit power p_i. Unfortunately, closed-form expression for the probability distribution function (pdf) of Y_i is unavailable. However, the numerical characteristics of Y_i are obtainable. Based on Y_i's numerical characteristics, we can still describe the pdf of Y_i analytically by means of approximation approaches.

From the description of the on-off activity random variables $\{\, x_j \,\}$ in (2.12), we can calculate their means and variances as [108]:

$$
\begin{aligned}
E[x_j] &= \rho_j \,, &\quad (3.8)\\
Var[x_j] &= \rho_j \,(1-\rho_j)\,. &\quad (3.9)
\end{aligned}
$$

Because the component interference powers of Y_i are from $N-1$ user terminals in the system, they are mutually independent due to the interference-limited wireless system assumption. And the interference powers are also independent from the

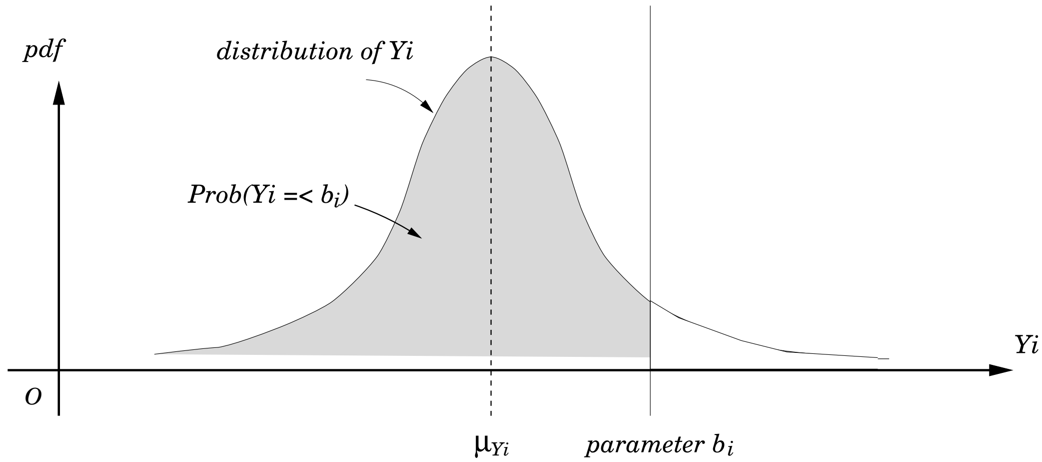

Figure 3.2: Illustration of the distribution of the total interference plus noise Y_i with mean as μ_{Y_i} and the probability of the event $\{Y_i \leq b_i\}$: $Prob\,\{Y_i \leq b_i\}$.

AWGN background noise power σ_i^2 which is a constant here. Due to these independence properties, (3.8) and (3.9), the mean and variance of Y_i can be directly computed as below:

$$\mu_{Y_i} = E[Y_i] = \sum_{j=1, j\neq i}^{N} \rho_j\, h_j\, p_j + \sigma_i^2\,, \tag{3.10}$$

$$\sigma_{Y_i}^2 = Var[Y_i] = \sum_{j=1, j\neq i}^{N} \rho_j\,(1-\rho_j)\, h_j^2\, p_j^2 \tag{3.11}$$

Figure 3.2 shows visually the probability distribution of the interference and noise Y_i, the interference threshold parameter b_i, and the probability event $\{\, Y_i \;\leq\; b_i \,\}$ which is equivalent to the probability event $\{\, d_i \;\leq\; D_{max,\,i} \,\}$.

Basically, Y_i in (3.5) is the sum of an AWGN noise power σ_i^2 and $(N-1)$ interference powers $\{x_j\, h_j\, p_j,\;\; \forall\, j \neq i \text{ and } j = 1, 2, \cdots, N\}$ that are mutually independent variables with different distributions. We assume that random variables

$\{x_j\, h_j\, p_j,\ \ \forall\, j \neq i$ and $j = 1, 2, \cdots, N\}$ are uniformly bounded, which is true realistically with an upper bound for transmit powers. Since packet-switched systems are designed to carry a large number of bursty users, we can assume naturally that the number of users in the system N is a large number.

In probability theory, there is a Lindeberg Theorem [107] which we state in the following without proof:

<u>**Theorem**</u> **1 (Lindeberg Theorem [107])** *Every uniformly bounded sequence $\{\mathcal{X}_k\}$ of mutually independent random variables obeys the Central Limit Theorem, provided that $s_n = \sum_{k=1}^{n} Var(\mathcal{X}_k) \rightarrow \infty$.*

According to the above theorem, the random interference power sequence $\{\, x_j\, h_j\, p_j \mid j = 1, 2, \cdots, N,\ \ j \neq i \,\}$ obeys **Central Limit Theorem** [107], as $N \rightarrow \infty$. Therefore, the distribution of Y_i can be approximated by a Gaussian or Normal distribution with mean μ_{Y_i} and variance $\sigma^2_{Y_i}$:

$$Y_i \ \ \approxeq \ \ \mathcal{N}(\, \mu_{Y_i},\ \sigma_{Y_i}\,). \tag{3.12}$$

Define Z_i as:

$$Z_i \ \ \triangleq \ \ \frac{Y_i - \mu_{Y_i}}{\sigma_{Y_i}}, \tag{3.13}$$

then Z_i conforms to the standard Gaussian or Normal distribution:

$$Z_i \ \ \approxeq \ \ \mathcal{N}(\, 0,\ 1\,). \tag{3.14}$$

We can now calculate the probability of delay requirement satisfaction $\mathcal{P}_D =$

$Prob\{\, d_i \leq D_{max,i} \,\} = Prob\{\, Y_i \leq b_i \,\}$ approximately based on the above Gaussian approximation of Y_i. Figure (3.2) illustrates the approximated pdf of Y_i and the probability $Prob\{\, Y_i \leq b_i \,\}$.

3.3 Approximation of the Expected Packet Success Rate — The Holtzman's Approach

To approximate the expected PSR, $E[f(\gamma_i)]$, we utilize Holtzman's Approximation Formula [28, 29]. Basically, Holtzman's approximation method says that the expectations of functions of random variables can be accurately approximated by a sum of three terms, if the mean and variance of the random variable are known.

Suppose $Q(Y_i)$ is a function of the random variable Y_i, and since the mean and variance of Y_i are known as μ_{Y_i} and $\sigma^2_{Y_i}$, then the expectation $E[\, Q(Y_i)\,]$ can be approximated by the following formula [28]:

$$E[Q(Y_i)] \;\approx\; \frac{2}{3}\, Q(\mu_{Y_i}) \;+\; \frac{1}{6}\,[\, Q(\mu_{Y_i} + \sqrt{3}\,\sigma_{Y_i}) \;+\; Q(\mu_{Y_i} - \sqrt{3}\,\sigma_{Y_i})\,] \tag{3.15}$$

According to the studies in [28, 29], the above approximation formula is very accurate if the random variable Y_i is Gaussian. However, the Gaussian assumption is *not* necessary. The above approximation is fairly robust to non-Gaussian distributions.

In previous Section, we showed that the random interference Y_i can be practically approximated as Gaussian-distributed. The close-to-Gaussian nature of Y_i further justifies the use of Holtzman's formula, because of the formula's increased accuracy in the approximation.

We use the approximation formula in Equation (3.15) to compute the average packet success rate $E[f(\gamma_i)]$ in the new packet-data utility model. We know the SIR γ_i is random because of the randomness of the interference Y_i:

$$\gamma_i \;=\; \frac{W}{R}\,\frac{h_i\,p_i}{Y_i} \tag{3.16}$$

Therefore, the average packet success rate $E[f(\gamma_i)]$ can be expressed as:

$$\begin{aligned} E[f(\gamma_i)] \;&\approx\; \frac{2}{3}\,f\left(\frac{W}{R}\,\frac{h_i\,p_i}{\mu_{Y_i}}\right) \\ &+\; \frac{1}{6}\left[f\left(\frac{W}{R}\,\frac{h_i\,p_i}{\mu_{Y_i}+\sqrt{3}\,\sigma_{Y_i}}\right) \;+\; f\left(\frac{W}{R}\,\frac{h_i\,p_i}{\mu_{Y_i}-\sqrt{3}\,\sigma_{Y_i}}\right)\right] \end{aligned} \tag{3.17}$$

For convenience, let us use the simplifying notations below:

$$\overline{\gamma}_i^0 \;\triangleq\; \frac{W}{R}\,\frac{h_i\,p_i}{\mu_{Y_i}} \tag{3.18}$$

$$\overline{\gamma}_i^1 \;\triangleq\; \frac{W}{R}\,\frac{h_i\,p_i}{\mu_{Y_i}\;+\;\sqrt{3}\,\sigma_{Y_i}} \tag{3.19}$$

$$\overline{\gamma}_i^2 \;\triangleq\; \frac{W}{R}\,\frac{h_i\,p_i}{\mu_{Y_i}\;-\;\sqrt{3}\,\sigma_{Y_i}} \tag{3.20}$$

Then the approximation of the expected PSR $E[f(\gamma_i)]$ can be written as a function of three parameters $\overline{\gamma}_i^0$, $\overline{\gamma}_i^1$ and $\overline{\gamma}_i^2$:

$$E[f(\gamma_i)] \; \approx \; \frac{2}{3}\, f\left(\overline{\gamma}_i^0\right) \; + \; \frac{1}{6}\, \left[\, f\left(\overline{\gamma}_i^1\right) \; + \; f\left(\overline{\gamma}_i^2\right) \right]. \tag{3.21}$$

The above approximation formula is utilized in the derivation of the analytical expression for the expected packet-data utility model.

3.4 Analytical Expression for the Expected Packet-Data Utility Model

Let $f_G(\, x \,|\, \mu,\ \sigma\,)$ represent Gaussian probability density function (pdf) with mean μ and variance σ^2, i.e.,

$$f_G(\, x \,|\, \mu,\ \sigma\,) \;=\; \frac{1}{\sqrt{2\,\pi}\,\sigma}\, e^{-\frac{(x-\mu)^2}{2\,\sigma^2}} \tag{3.22}$$

And let $F_G(\, x \,|\, \mu,\ \sigma\,)$ represent the Gaussian cumulative distribution function (cdf) with mean μ and variance σ^2, then

$$F_G(\, x \,|\, \mu,\ \sigma\,) \;=\; \int_{-\infty}^{x} f_G(\, t \,|\, \mu,\ \sigma\,)\, dt \tag{3.23}$$

$$=\; \frac{1}{\sqrt{2\,\pi}} \int_{-\infty}^{\frac{x-\mu}{\sigma}} e^{-\frac{1}{2}\,t^2}\, dt\,. \tag{3.24}$$

Let $\Phi(x)$ represent the standard Gaussian cdf, we have

$$F_G(\, x \,|\, \mu,\ \sigma) \ = \ \Phi \left(\frac{x - \mu}{\sigma} \right) . \tag{3.25}$$

Based on the previous discussions and by replacing the probability of delay requirement satisfaction $\mathcal{P}_D = Prob\,\{\, d_i \leq D_{max,\,i}\,\}$ and the expected packet success rate $E[f(\gamma_i)]$ with their respective estimates, the packet-data utility function (2.23) can be expressed and approximated in the following formulas. Those expressions are equivalent with different variations for $Prob\,\{\, d_i \ \leq \ D_{max,\,i}\}$ (with various expressions for $Prob\,\{\, d_i \ \leq \ D_{max,\,i}\}$ plugged in) as below:

$$\begin{aligned}
\overline{u}_i(\, p_i,\ \mathbf{p}_{-i}\,) \ &\triangleq \ \frac{L\,R}{M\,p_i} \left[\frac{E[f(\gamma_i)] - f(0)}{1 - f(0)} \right] Prob\,\{\, d_i \ \leq \ D_{max,\,i}\,\} && (3.26) \\
&= \ \frac{L\,R}{M\,p_i} \left[\frac{E[f(\gamma_i)] - f(0)}{1 - f(0)} \right] Prob\,\{\, \gamma_i \ \geq \ \gamma_{min,\,i}\,\} && (3.27) \\
&= \ \frac{L\,R}{M\,p_i} \left[\frac{E[f(\gamma_i)] - f(0)}{1 - f(0)} \right] Prob\,\{\, Y_i \ \leq \ b_i\,\} && (3.28) \\
&\approxeq \ \frac{L\,R}{M\,p_i} \left[\frac{E[f(\gamma_i)] - f(0)}{1 - f(0)} \right] F_G\,(\, b_i \,|\, \mu_{Y_i},\ \sigma_{Y_i}) && (3.29) \\
&= \ \frac{L\,R}{M\,p_i} \left[\frac{E[f(\gamma_i)] - f(0)}{1 - f(0)} \right] \Phi \left(\frac{b_i - \mu_{Y_i}}{\sigma_{Y_i}} \right) && (3.30)
\end{aligned}$$

Taking a close look at the above expressions of the expected packet-data utility model, we recognize that some minor adjustment needs to be done to achieve a well-behaved utility function.

Because Y_i defined in (3.5) is a *strictly positive* random variable, its approximated pdf and cdf should not include the part corresponding to the non-positive argument in $(\,-\infty,\ 0\,]$. We apply truncation and normalization to make both the Gaussian cdf $F_G\,(\,b_i \,|\, \mu_{Y_i},\ \sigma_{Y_i})$ and the standard Gaussian cdf $\Phi\left(\frac{b_i - \mu_{Y_i}}{\sigma_{Y_i}}\right)$ account for that.

After the modification with the approximated $E[f(\gamma_i)]$ (3.21) inserted, Equations (3.29) and (3.30) become:

$$\overline{u}_i(\,p_i,\ \mathbf{p}_{-i}\,) \ \approxeq\ \frac{L\,R}{M\,p_i}\left[\frac{E[f(\gamma_i)] - f(0)}{1 - f(0)}\right]\ \frac{F_G\,(\,b_i \,|\, \mu_{Y_i},\ \sigma_{Y_i}) - F_G\,(\,0 \,|\, \mu_{Y_i},\ \sigma_{Y_i})}{1 - F_G\,(\,0 \,|\, \mu_{Y_i},\ \sigma_{Y_i})} \tag{3.31}$$

$$= \ \frac{L\,R}{M\,p_i}\left[\frac{\dfrac{2}{3}\,f\,(\overline{\gamma}_i^0) + \dfrac{1}{6}\,[\,f\,(\overline{\gamma}_i^1) + f\,(\overline{\gamma}_i^2)\,] - f(0)}{1 - f(0)}\right]\frac{\Phi\left(\dfrac{b_i - \mu_{Y_i}}{\sigma_{Y_i}}\right) - \Phi\left(-\dfrac{\mu_{Y_i}}{\sigma_{Y_i}}\right)}{1 - \Phi\left(-\dfrac{\mu_{Y_i}}{\sigma_{Y_i}}\right)} \tag{3.32}$$

Practically speaking, in packet-switched data networks, the effect of the above modification on the utility function is negligible. Because:

1. the modification does not change the basic structure of the utility model;
2. in packet-switched systems, we always assume that there are a large number of mobile users in each cell, each transmitting for only a small fraction of time (large N and small ρ). For a given value of ρ, a large N contributes to a high value of μ_{Y_i}/σ_{Y_i} because the large N increases both the mean μ_{Y_i} and

the standard deviation σ_{Y_i}, but the mean μ_{Y_i} is increased faster. The large value of μ_{Y_i}/σ_{Y_i} makes $\Phi(-\mu_{Y_i}/\sigma_{Y_i})$ negligible.

To simplify the mathematical formulas, we define the following notations:

$$\alpha_i \triangleq \frac{\mu_{Y_i}}{\sigma_{Y_i}}, \tag{3.33}$$

$$\beta_i \triangleq \frac{b_i}{\sigma_{Y_i}}. \tag{3.34}$$

Recall the expression of b_i in (3.6) as:

$$b_i \triangleq \frac{W}{R} \frac{h_i\, p_i}{\gamma_{min,\,i}}.$$

From those definitions, we can re-write the expressions for $\overline{\gamma}_i^0$, $\overline{\gamma}_i^1$ and $\overline{\gamma}_i^2$ in (3.18), (3.19) and (3.20) as functions of α_i, β_i and $\gamma_{min,\,i}$, as below:

$$\overline{\gamma}_i^0 = \frac{W}{R} \frac{h_i\, p_i}{\mu_{Y_i}} = \frac{\beta_i}{\alpha_i} \gamma_{min,\,i} \tag{3.35}$$

$$\overline{\gamma}_i^1 = \frac{W}{R} \frac{h_i\, p_i}{\mu_{Y_i}} \left(\frac{\mu_{Y_i}}{\mu_{Y_i} + \sqrt{3}\,\sigma_{Y_i}} \right) = \frac{\beta_i}{\alpha_i + \sqrt{3}} \gamma_{min,\,i} \tag{3.36}$$

$$\overline{\gamma}_i^2 = \frac{W}{R} \frac{h_i\, p_i}{\mu_{Y_i}} \left(\frac{\mu_{Y_i}}{\mu_{Y_i} - \sqrt{3}\,\sigma_{Y_i}} \right) = \frac{\beta_i}{\alpha_i - \sqrt{3}} \gamma_{min,\,i} \tag{3.37}$$

Using notations in (3.33) and (3.34), the expected packet-data utility function for user i can be finally written as below, from its original definition to its approximated expression:

$$\overline{u}_i(\, p_i,\ \mathbf{p}_{-i}\,) \ \triangleq \ \frac{L\,R}{M\,p_i}\left[\frac{E[f(\gamma_i)] - f(0)}{1 - f(0)}\right] \ Prob\,\{\, d_i \ \le \ D_{max,\,i}\,\} \tag{3.38}$$

$$\approx \ \frac{L\,R}{M\,p_i}\left[\frac{\dfrac{2}{3}\,f\,(\overline{\gamma}_i^0) + \dfrac{1}{6}\,\left[\,f\,(\overline{\gamma}_i^1) + f\,(\overline{\gamma}_i^2)\,\right] - f(0)}{1 - f(0)}\right] \frac{\Phi\,(\beta_i - \alpha_i) \ - \ \Phi\,(-\alpha_i)}{1 \ - \ \Phi\,(-\alpha_i)} \tag{3.39}$$

with the two main components $E[f(\gamma_i)]$ and $Prob\,\{\, d_i \ \le \ D_{max,\,i}\,\}$ being replaced by their respective approximations:

$$\begin{aligned} Prob\,\{\, d_i \le D_{max,\,i}\,\} \ &= \ Prob\,\{\, Y_i \le b_i\,\} \\ &\approx \ \frac{\Phi\,(\beta_i - \alpha_i) \ - \ \Phi\,(-\alpha_i)}{1 \ - \ \Phi\,(-\alpha_i)} \ = \ \frac{\displaystyle\int_{-\alpha_i}^{\beta_i - \alpha_i} e^{-\frac{1}{2}\,u^2}\,du}{\displaystyle\int_{-\alpha_i}^{\infty} e^{-\frac{1}{2}\,u^2}\,du} \end{aligned} \tag{3.40}$$

$$E[f(\gamma_i)] \ \approx \ \frac{2}{3}\,f\left(\overline{\gamma}_i^0\right) \ + \ \frac{1}{6}\,\left[\,f\left(\overline{\gamma}_i^1\right) \ + \ f\left(\overline{\gamma}_i^2\right)\,\right] \tag{3.41}$$

Observe that because parameters $\overline{\gamma}_i^0$, $\overline{\gamma}_i^1$ and $\overline{\gamma}_i^2$ in (3.35), (3.36) and (3.37) depend on the parameters α_i, β_i and $\gamma_{min,\,i}$, the expected packet-data utility function $\overline{u}_i(\, p_i,\ \mathbf{p}_{-i}\,)$ is a function of the parameters $M,\ L,\ R,\ p_i,\ \alpha_i,\ \beta_i$ and $\gamma_{min,\,i}$.

Given information about structure of the data packets and rate of transmission channels ($M,\ L$ and R), and QoS requirement $D_{max,\,i}$ (contained in $\gamma_{min,\,i}$), as well

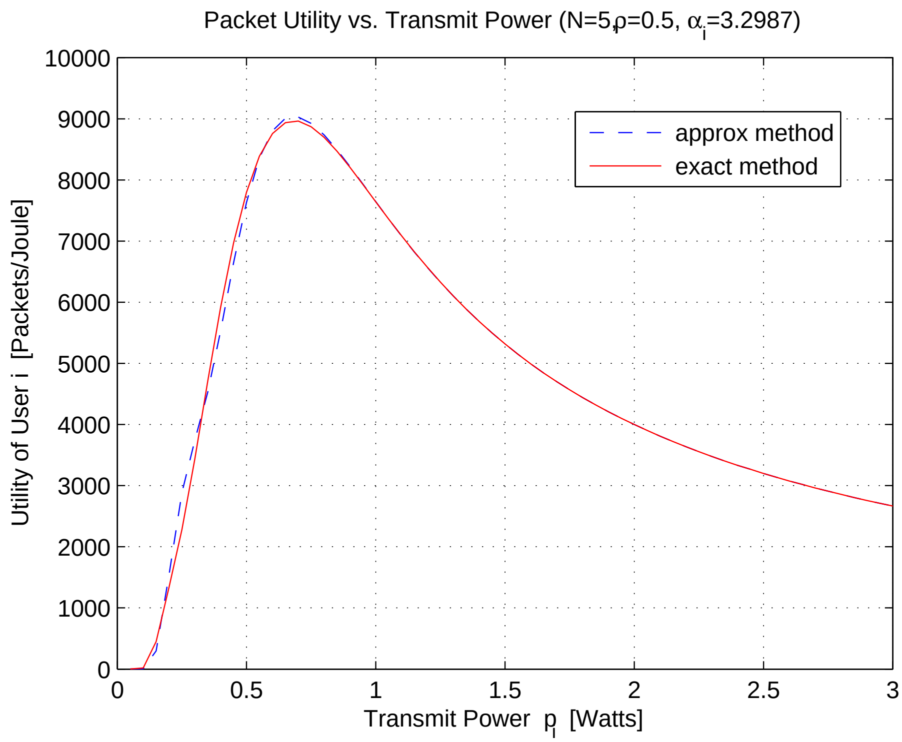

Figure 3.3: Comparison of utility functions based on (1) the approximation approach, and (2) the exact calculation, in a $N = 5$ user system with $\rho_i = 0.5$ and $Prob\{\, d_i \leq D_{max,i}\,\} = 1$.

as interference statistics (contained in α_i and β_i), utility model $\overline{u}_i(\, p_i,\ \mathbf{p}_{-i}\,)$ solely depends on user transmit power p_i (note that β_i is also a function p_i).

3.4.1 Exact Calculation vs. Approximation

Finally, let us look at the curves of the utility function obtained from the two different approaches:

1. Approximation methods - based on the approximation of the interference and the estimation of the expected PSR, as presented above.
2. Exact calculation - based on the exact expression of the expected utility as in Equation 2.25.

Due to the complexity in computing the utility with the exact method, let us consider a system with a small number users with $N = 5$ as an example. The utility based on exact calculation is expressed in Equation 2.28. For easy comparison, let us assume $\mathcal{P}_D = Prob\{\, d_i \;\leq\; D_{max,\,i}\,\} = 1$, and $\rho_i = 0.5$ in both cases.

Figure 3.3 shows the plots of the utility functions. Clearly, the utility curve based on approximation methods closely resembles the one based on exact calculation. This shows the value and beauty of approximation methods in reducing a complex and intractable problem into a mathematically-solvable form without losing any critical contents of the original problem.

Chapter 4

Non-Cooperative Game-Based Uplink Power Control Problem and Its Solution

In previous chapters, we developed a probabilistic utility model as a performance metric for a wireless data user, which measures the average information throughput over the air link powered by each unit of the mobile's battery energy [bits/Joule]. The model takes into account both the effect of traffic burstiness and the effect of the average packet delay requirement of the mobile user in a packet-switched data CDMA network. With this packet-data utility model readily available, we are now in a good position to address the uplink transmit power control problem for wireless packet-switched data services.

In this chapter, based on the utility model developed, we propose and study a distributed strategy for the allocation of transmit powers of wireless user terminals. The objective of the strategy is to simultaneously maximize the value of the utility model for every individual user through user's independent adjustment of its transmit power. Because of the conflict of interests among individual users who are competing for the same limited radio resources in the system, a game-theoretic approach is taken to formulate the transmit power allocation problem as a non-cooperative game. This game-based power control formulation best describes the mutual-dependence of user performance in cellular wireless networks.

4.1 Game Theory and Distributed Power Control Problem

4.1.1 Basics of Game

Game theory is the study of interacting decision makers [116]. It has been widely used in economics to clarify the nature of strategic interaction in economic models. Basically, the description of a game consists of three sets: (1) a set of *players* of the game, (2) a set of *strategies*, the choices that each player can make, and (3) a set of *payoffs* that indicate the utility that each player receives if a particular combination of strategies is chosen [114, 115, 116]. In this study, we use the following standard notation for a game:

$$[\, N,\ \{\, S_i \,\},\ \{\, u_i(\cdot) \,\}\,] \tag{4.1}$$

where

- N is the number of game players. In this study, N is the number of active mobile users in the system;
- S_i is the strategy set for user i. The strategy space of the game $S = S_1 \times S_2 \times \ldots S_N$;
- $u_i(\cdot)$ is the payoff of user i, or the utility function (utility model) of user i as defined in the previous chapters.

A *non-cooperative game* [115] is a game within which each player is out to maximize its own payoff, and each player makes its decisions independently of the other players in the game. That is, there is no coordinated actions among the players of the game. The non-cooperative game is the foundation of our study here on the efficient allocation of transmit powers among user terminals for wireless packet-switched data services.

4.1.2 Game-Based Uplink Transmit Power Control Problem

Efficient allocation of radio resources and interference management have a crucial importance in cellular wireless systems, due to the scarcity of RF spectrum and the interference-limited nature of the systems. Uplink transmit power control is one of the key techniques adopted in cellular wireless systems, particularly in cellular CDMA systems, to reduce interference, mitigate the near-far problem, and save battery energy of mobile terminals.

The power control strategy that we consider in the work is based on the maximization of utilities for individual users. By maximizing utilities, users achieve the highest possible energy-efficiency in the transmission of their data packets, given some QoS requirements.

In the process of utility maximization, each user of the system is assumed to be independent and selfish in this study. A user will try every possible opportunity to maximize its utility by adjusting its own transmit power p_i. However the problem is that the individual decision (power adjustment) of user i depends on the decisions of all other users in the system. When a user changes its transmit power, its interference to others changes accordingly. This will cause the utilities of all other users to change. Then other users have to re-adjust their transmit powers to regain the peak of their utilities, this in turn changes the utility of user i.

This inter-dependence of the interference among all the users in the system and decision conflicts of power adjustments among users can be best modelled by a game-theoretical approach as introduced above. Each mobile user is then a player in this "utility-maximizing" power control game, making its own decision independently and expecting good returns on the value of its own utility function.

Because each user adjusts its transmit power independently to compete in the game, we formulate the power control problem as a non-cooperative game $[N, \{S_i\}, \{\overline{u}_i(\cdot)\}]$ as follow:

$$\max_{p_i} \overline{u}_i(p_i, \mathbf{p}_{-i}), \quad p_i \in S_i, \ \ \forall\, i = 1, \cdots, N \tag{4.2}$$

where S_i is the strategy set for user i with $S_i = [\, P_{min,\,i}, \ P_{max,\,i}\,]$.

- $P_{min,\,i}$ — Minimum power constraint as $P_{max,\,i}$ for mobile user i
- $P_{max,\,i}$ — Maximum power constraint as $P_{max,\,i}$ for mobile user i

In this study, we assume $P_{max,\,i}$ as the upper bound for p_i, and further assume $P_{max,\,i}$ is same for all users in the system, that is:

$$P_{max,\,i} \ = \ P_{max} \quad \forall\, i \in [1, 2 \cdots N] \tag{4.3}$$

The determination on the value of this P_{max} parameter may be influenced by considering the factors such as:

(1) the rate of energy consumption of mobile battery,

(2) the requirement on minimum talk time to sustain at worst RF condition, and

(3) the regulatory restriction on the maximum for RF energy radiation per specific technology (to limit unnecessary interference and excessive RF radiation).

On the other hand, $P_{min,\,i}$, the lower bound for p_i is determined by the following analysis. By nature, the total interference plus noise Y_i (3.5) is always greater than and equal to σ_i^2, the Gaussian background noise power (always positive apparently):

$$Y_i \;=\; \sum_{j=1, j\neq i}^{N} x_j\, h_j\, p_j \;+\; \sigma_i^2 \;\;\geq\;\; \sigma_i^2 \tag{4.4}$$

To make our problem meaningful, there is a basic inequality that needs to be satisfied as below:

$$b_i \;>\; \mu_{Y_i} \tag{4.5}$$

which says that the interference threshold parameter b_i (3.6) has to be greater than μ_{Y_i} (3.10), the mean of the total interference plus noise Y_i. This fact will be shown in a later discussion ($\beta_i \;>\; \alpha_i \;\Rightarrow\; b_i \;>\mu_{Y_i}$). The relationship between b_i and μ_{Y_i} is illustrated in Figure (3.2).

The above inequality leads to:

$$b_i \quad > \quad \min\,\{\mu_{Y_i}\} \;=\; \sigma_i^2\,, \tag{4.6}$$

where "min $\{\mu_{Y_i}\}$" happens when there are zero transmit powers from all users except user i, that is, $p_j = 0,\; \forall\, j \neq i$.

Finally, using the expression for b_i as in Equation (3.6), we have

$$\frac{W}{R}\,\frac{h_i\, p_i}{\gamma_{min,\,i}} \;>\; \sigma_i^2 \quad\Longleftrightarrow\quad p_i \;>\; \frac{R}{W}\,\frac{\gamma_{min,\,i}}{h_i}\,\sigma_i^2\,. \tag{4.7}$$

Therefore

$$P_{min,\,i} \triangleq \frac{R}{W}\frac{\gamma_{min,\,i}}{h_i}\,\sigma_i^2\,. \tag{4.8}$$

Clearly, $P_{min,\,i}$ is the minimum transmit power that is needed for user i to fight against only the background noise σ_i^2. This corresponds to the scenario when user i is the only user that is active in the system, and thus the interference from all other users is absent.

4.1.3 Nash Equilibrium of Non-Cooperative Power Control Game

When a user plays a game to adjust its transmit power in an attempt to increase its utility, it can win (utility increased) or lose (utility decreased) or stay the same. By repeating this process to update the individual transmit powers iteratively and user-by-user in a round robin fashion, the power control vector (the decision vector or strategy vector) $\mathbf{p} = (p_1, p_2, \cdots, p_N)$ will converge to *an equilibrium solution*, if there is one and if the utility functions possess certain characteristics. In the following study, we will show that certain nature and properties of our packet-data utility model guarantee the existence of an equilibrium solution to the power control game.

The solution to a non-cooperative game is called Nash equilibrium [51, 116] as defined below in the context of our power control game:

Definition 6 (Nash Equilibrium) *A Nash equilibrium for the non-cooperative power control game is a power vector* $\mathbf{p}^* = (p_1^*, p_2^*, \cdots, p_N^*)$ *such that no user can improve its utility by varying its power unilaterally.*

Mathematically, a power vector $\mathbf{p} = (p_1, \cdots, p_N)$ in a system of N users is said to be a Nash equilibrium of the non-cooperative game $[N, \{S_i\}, \{\overline{u}_i(\cdot)\}]$, if for every $i \in N$,

$$\overline{u}_i(\, p_i,\ \mathbf{p}_{-i}\,) \ \geq\ \overline{u}_i(\, p_i',\ \mathbf{p}_{-i}\,) \qquad \forall\, p_i' \in S_i \tag{4.9}$$

where $\{S_i\}$ is the strategy space for the power vector $\mathbf{p} = (\, p_1,\, p_2,\ \cdots\ ,\, p_N\,)$. Clearly, condition (4.9) holds for every single user in the system of N users.

The packet-data utility function $\overline{u}_i(\, p_i,\ \mathbf{p}_{-i}\,)$ is assumed to be continuous and differentiable as we constructed. Therefore, the necessary condition or the first-order condition for the existence of a Nash equilibrium can be found by solving the following N equations of the first-order derivative simultaneously:

$$\frac{\partial\, \overline{u}_i(\, p_i,\ \mathbf{p}_{-i}\,)}{\partial\, p_i} \ =\ 0, \quad \forall\, i = 1, \cdots, N \tag{4.10}$$

Notice that user i's utility $\overline{u}_i$ defined in (3.38) is a function of user i's transmit power p_i. This is obvious from the elaborated expression of utility function in (3.39), observing that parameters β_i and $(\overline{\gamma}_i^0,\ \overline{\gamma}_i^1,\ \overline{\gamma}_i^2)$ are all functions of p_i, the transmit power of user i.

Starting with the utility definition (3.38), the first-order condition for the existence of a Nash equilibrium can be found as:

$$\begin{aligned}
\frac{\partial\,\overline{u}_i(\,p_i,\;\mathbf{p}_{-i}\,)}{\partial\,p_i} &= \frac{L\,R}{M\,[1-f(0)]}\Bigg(-\frac{1}{p_i^2}\,Prob\,\{\,d_i\;\leq\;D_{max,\,i}\,\}\,[E[f(\gamma_i)]-f(0)] \\
&+ \frac{1}{p_i}\,\frac{\partial\,Prob\{\,d_i \leq D_{max,\,i}\}}{\partial\,p_i}\,[E[f(\gamma_i)]-f(0)] \\
&+ \frac{1}{p_i}\,Prob\{\,d_i \leq D_{max,\,i}\}\,\frac{\partial\,E[f(\gamma_i)]}{\partial\,p_i}\Bigg) \\
&= 0\,, \qquad (4.11)
\end{aligned}$$

which can be further written as below (see Appendix A.1 for the derivation):

$$[E[f(\gamma_i)]-f(0)]\times Prob\{\,d_i \leq D_{max,\,i}\} \;=$$

$$\left\{\frac{\partial\,E[f(\gamma_i)]}{\partial\,\beta_i}Prob\{\,d_i \leq D_{max,\,i}\} + [E[f(\gamma_i)]-f(0)]\,\frac{\partial\,Prob\{\,d_i \leq D_{max,\,i}\}}{\partial\,\beta_i}\right\}\times\beta_i \qquad (4.12)$$

4.2 Quasi-Concavity of the Expected Packet-Data Utility Function

In this Section, we will investigate an important characteristic of the expected packet-data utility function – its quasi-concavity. Quasi-concavity is critical in this study, and it establishes the basis for the existence and uniqueness of the Nash equilibrium solution to the power control game.

4.2.1 Quasi-Concavity

As a generalization of concave/convex functions, a family of functions are classified as quasi-concave/quasi-convex functions. Let us look at quasi-concavity specifically for this study. Generally speaking, any single-peaked (unimodal) function is quasi-concave. Mathematically, a quasi-concave function is defined as below [117, 118, 119, 120]:

<u>**Definition**</u> **7 (Quasi-Concave Function)** *Function $F(x)$ is quasi-concave if and only if, for any $x,\, y \in \mathbf{X}$ and for all $\lambda \in [0,\, 1]$,*

$$F(\lambda\, x + (1-\lambda)\, y) \;\geq\; \min\,\{F(x),\; F(y)\}. \tag{4.13}$$

$F(x)$ is strictly quasi-concave if and only if, for any $x,\, y \in \mathbf{X}$ with $x \neq y$, and for all $\lambda \in (0,\, 1)$,

$$F(\lambda\, x + (1-\lambda)\, y) \;>\; \min\,\{F(x),\; F(y)\}. \tag{4.14}$$

Notice that any concave function is obviously quasi-concave. However, a quasi-concave function is not necessarily concave.

In addition to likely being single-peaked, a quasi-concave function may also contain convex portions or line segments, or even horizontal line segments (”flats”) [119]. However, for a *strictly* quasi-concave function, although it may still contain convex or linear portions, condition in (4.14) makes it impossible to have linear segments that are horizontal. Therefore, any two adjacent points on the curve of a function will never have equal values, if the function is strictly quasi-concave (or strictly quasi-convex).

Notice that strict quasi-concavity is a subset of quasi-concavity. So a strictly quasi-concave function is always quasi-concave, but the reverse is not necessarily true.

One property about quasi-concave functions is that quasi-concavity is preserved by any monotonically-nondecreasing transformation, as rephrased in the following theorem [120].

Theorem 2 (Transformation of Quasi-Concavity) *Suppose function $\xi : \boldsymbol{X} \to \mathbb{R}$ is quasi-concave and function $\phi : \boldsymbol{X} \to \mathbb{R}$ is nondecreasing. Then $\phi \circ \xi$ is quasi-concave. If ξ is strictly quasi-concave and ϕ is strictly increasing, then $\phi \circ \xi$ is strictly quasi-concave.*

Proof of the theorem follows directly from the definition of quasi-concavity. For completeness, we include the proof from [120] here.

Proof. Consider any $x, y \in \mathbf{X}$ and any $\lambda \in [0, 1]$. If ξ is quasi-concave, then $\xi(\lambda x + (1-\lambda) y) \geq \min\{\xi(x), \xi(y)\}$. Therefore, ϕ nondecreasing implies

$$\phi(\xi(\lambda x + (1-\lambda) y)) \geq \phi(\min\{\xi(x), \xi(y)\}) = \min\{\phi(\xi(x)), \phi(\xi(y))\}.$$

If $\phi(x)$ is strictly quasi-concave, then for any $x \neq y$ and for all $\lambda \in (0, 1)$, we have $\xi(\lambda x + (1-\lambda) y) > \min\{\xi(x), \xi(y)\}$. Therefore, if ϕ is strictly increasing, we have

$$\phi(\xi(\lambda x + (1-\lambda) y)) > \phi(\min\{\xi(x), \xi(y)\}) = \min\{\phi(\xi(x)), \phi(\xi(y))\}.$$

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4.2.2 Analysis of the Expected Utility Model for Quasi-Concavity

Note that the expected packet-data utility function $\overline{u}_i(\,p_i,\ \mathbf{p}_{-i}\,)$ (3.26) is assumed to be continuous and differentiable as we constructed. These are important characteristics to be used in the analysis later.

Let us investigate the expected packet-data utility function in the following, as given in (3.38):

$$\overline{u}_i(\,p_i,\ \mathbf{p}_{-i}\,) = \frac{L\,R}{M\,[1 - f(0)]}\ \underbrace{\left[\frac{E[f(\gamma_i)] - f(0)}{p_i}\right]}_{\mathcal{A}} \times \underbrace{Prob\,\{\,d_i\ \leq\ D_{max,\,i}\,\}}_{\mathcal{B}} \tag{4.15}$$

There are two main components in above utility expression which are labeled as $\mathcal{A}$ and $\mathcal{B}$. In the following analysis, we will show that:

(1) $\mathcal{A}$ is a strictly quasi-concave function of p_i, and

(2) $\mathcal{B}$ is a strictly and monotonically-increasing function of p_i .

First of all, Point (2) is straight-forward. From previous discussions, we know

$$Prob\,\{\,d_i \leq D_{max,\,i}\,\}\ =\ Prob\,\{\,Y_i \leq b_i\,\}\ \approx\ \Phi\left(\frac{b_i - \mu_{Y_i}}{\sigma_{Y_i}}\right) \tag{4.16}$$

where $\Phi(\cdot)$ is standard Gaussian cdf. It is basically an integral of standard Gaussian pdf function with a variable upper limit which is a function of b_i.

From Calculus, it is evident that, if a function is positive in its domain, its integral with a variable upper limit is a strictly increasing function of the variable upper limit. Because standard Gaussian pdf function is positive, therefore, $\mathcal{B} = Prob\{\, d_i \leq D_{max,\,i} \,\} = Prob\{\, Y_i \leq b_i \,\}$ is a strictly increasing function of b_i. Since b_i is a linear function of p_i, so $\mathcal{B}$ is a strictly increasing function of p_i.

With the truth of Point (2) on $\mathcal{B}$, based on **Theorem 2** on the transformation of quasi-Concavity, the quasi-concavity of $\overline{u}_i(\, p_i,\ \mathbf{p}_{-i}\,)$ then depends solely on the quasi-concavity of component $\mathcal{A}$ of the expected utility expression in (4.15).

In the following discussions, we will make use of a proposition from [110] about the quasi-concavity of a function, which we also repeat here without proof:

Proposition 1 (Property of Quasi-Concavity) *A function is quasi-concave if and only if every local maximum is a global maximum.*

An immediate corollary that follows from **Proposition 1** above is resulting from applying the proposition to *strictly* quasi-concave functions.

Corollary 1 (Property of Strict Quasi-Concavity) *A function is strictly quasi-concave if and only if its local maximum is its global maximum.*

The proof of the above corollary is straight-forward. Because strict quasi-concavity is a subset of quasi-concavity, it naturally carries all properties that belong to quasi-concavity. However, for a strictly quasi-concave function, its maximum or peak cannot be more than one. So if existed, the maximum can only be either a unique one or none.

This result is well stated in a theorem from [117] as below. For details on proof of this theorem that are omitted here, refer to [117].

<u>**Theorem**</u> **3** *Suppose $\xi : \boldsymbol{X} \to \mathbb{R}$ is strictly quasi-concave where the domain $\boldsymbol{X}$ is convex. Then, any local maximum of ξ on $\boldsymbol{X}$ is also a global maximum of ξ on $\boldsymbol{X}$. Moreover, the set* $\arg\max\{\,\xi(x)\,|\,x \in \mathbf{X}\}$ *of maximizers of ξ on $\boldsymbol{X}$ is either empty or a singleton.*

As commented in [117], the most significant part of **Theorem 3** for optimization problems with quasi-concavity is that strict quasi-concavity implies uniqueness of the solution, exactly as with strict concavity.

4.2.3 Proof of Strict Quasi-Concavity for the Expected Utility Function

Based on **Corollary 1** above on the property of strict quasi-concavity, we now prove that our packet-data utility model expressed in Equation (4.15) is indeed strictly quasi-concave. We summarize this conclusion in following proposition.

<u>**Proposition**</u> **2 (Quasi-Concavity of Expected Utility Model)** *The expected packet-data utility model for user i $\overline{u}_i(p_i,\ \mathbf{p}_{-i})$ defined in Equation (4.15) is strictly quasi-concave in p_i, the transmit power of user i.*

<u>**Proof**</u>. The quasi-concavity of $\overline{u}_i(p_i,\ \mathbf{p}_{-i})$ (4.15) is solely determined by the quasi-concavity of component $\mathcal{A}$ in (4.15). This is a conclusion that we arrived at among the preceding discussions. Hence, let us focus on confirming the quasi-concavity of $\mathcal{A}$ of $\overline{u}_i(p_i,\ \mathbf{p}_{-i})$ expression in (4.15).

Assuming component $\mathcal{B} = Prob\,\{\,d_i \leq D_{max,\,i}\,\} = 1$, we will then be left mainly with component $\mathcal{A}$ in $\overline{u}_i(p_i,\ \mathbf{p}_{-i})$ expression (4.15).

With $\mathcal{B} = 1$, let us re-examine the first-order condition for the existence of Nash equilibrium, the equality $\frac{\partial \overline{u}_i(p_i,\, \mathbf{p}_{-i})}{\partial p_i} = 0$ (4.10) or the detailed version of the equality in (4.12). Clearly, with $\mathcal{B} = 1$, equality (4.12) reduces simply to:

$$E[f(\gamma_i)] - f(0) \;=\; \frac{\partial\, E[f(\gamma_i)]}{\partial\, \beta_i}\, \beta_i \tag{4.17}$$

Make use of the approximation formula for $E[f(\gamma_i)]$ in (3.41) (repeated here for easy reference) and its partial derivative with respect to β_i as below:

$$E[f(\gamma_i)] \;\approx\; \frac{2}{3}\, f\left(\overline{\gamma}_i^0\right) \;+\; \frac{1}{6}\,\left[\, f\left(\overline{\gamma}_i^1\right) \;+\; f\left(\overline{\gamma}_i^2\right)\,\right], \tag{4.18}$$

$$\frac{\partial\, E[f(\gamma_i)]}{\partial\, \beta_i} \;=\; \frac{2}{3}\, f'\left(\overline{\gamma}_i^0\right) \frac{\gamma_{min,\,i}}{\alpha_i} \;+\; \frac{1}{6}\left[f'\left(\overline{\gamma}_i^1\right) \frac{\gamma_{min,\,i}}{\alpha_i + \sqrt{3}} \;+\; f'\left(\overline{\gamma}_i^2\right) \frac{\gamma_{min,\,i}}{\alpha_i - \sqrt{3}}\right]. \tag{4.19}$$

After re-arranging terms, the first-order condition equality (4.17) can be re-written in the form below:

$$\frac{4\, f(\overline{\gamma}_i^0) \;+\; f(\overline{\gamma}_i^1) \;+\; f(\overline{\gamma}_i^2) \;-\; 6\, f(0)}{4\, f'(\overline{\gamma}_i^0)\, \frac{\gamma_{min,\,i}}{\alpha_i} \;+\; f'(\overline{\gamma}_i^1)\, \frac{\gamma_{min,\,i}}{\alpha_i + \sqrt{3}} \;+\; f'(\overline{\gamma}_i^2)\, \frac{\gamma_{min,\,i}}{\alpha_i - \sqrt{3}}} \;=\; \beta_i\,. \tag{4.20}$$

Because

$$\beta_i \;=\; \frac{b_i}{\sigma_{Y_i}} \;=\; \frac{W}{R}\, \frac{h_i\, p_i}{\gamma_{min,\,i}\; \sigma_{Y_i}}$$

is a linear function of p_i, we will look at β_i instead of p_i, which will not change the basic properties of the function involved. Let us examine the characteristics of the

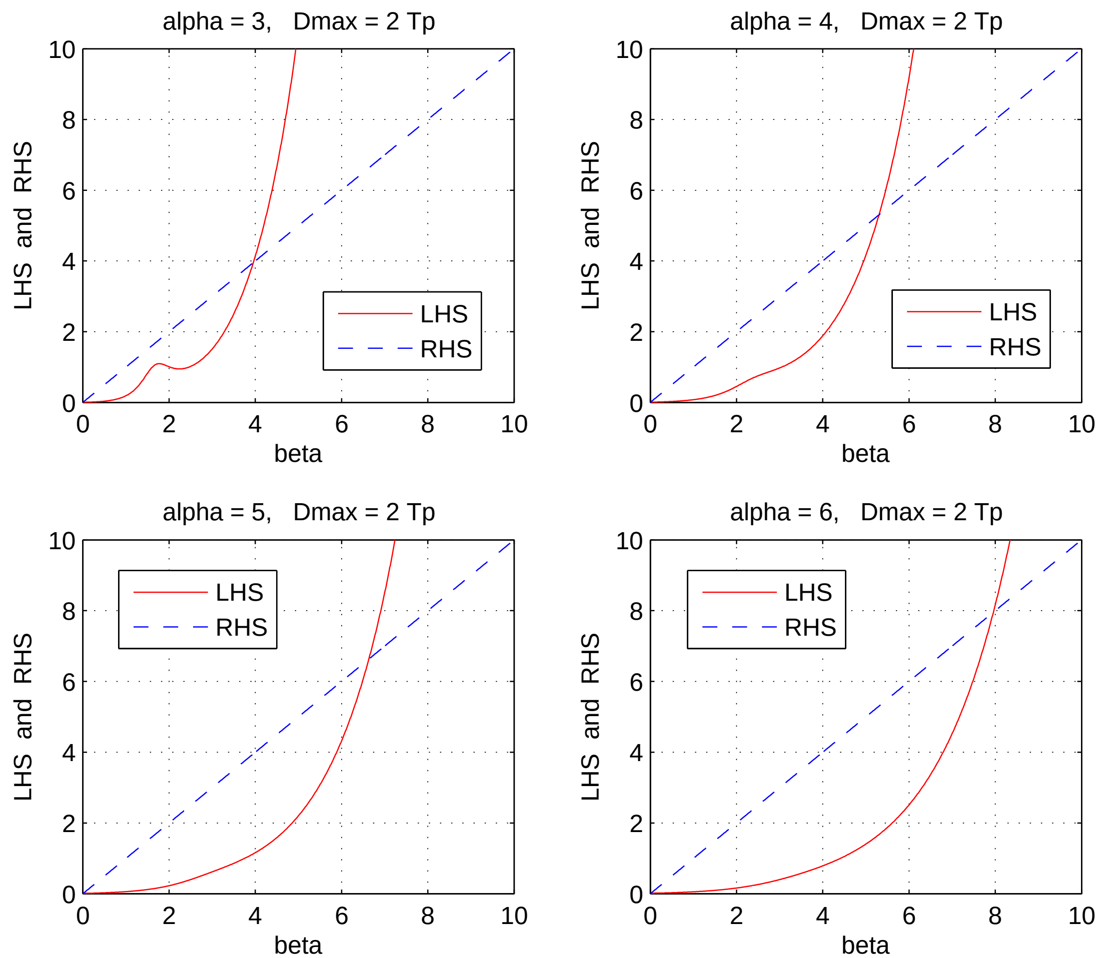

Figure 4.1: Demonstration of quasi-concavity in component $\mathcal{A}$ of the expected packet-data utility model, through examining LHS and RHS of the first-order condition in equation (4.20), given $\alpha_i = 3$, 4, 5 and 6 respectively, and $D_{max,i} = 2\,T_p$ (additional assumptions: processing gain $W/R = 100$, and data packets of size $M = 80$ bits of which payload $L = 64$ bits).

above equation by looking at both sides (LHS and RHS) of the equation. To assist our investigation, we will plot both LHS and RHS of Equation (4.20) with respect to β_i for some given values of α_i.

RHS of Equation (4.20) is trivial, because it is simply a straight line of 45° originated from Origin. Let us look at the characteristics of the LHS of the equation more closely.

In Figure 4.1, both sides of equation (4.20) are plotted for several values of α_i and for a fixed $D_{max,i}$. Notice that, given a value of α_i, the solution for the optimum value of β_i, namely β_i^*, corresponds to the crossover point of LHS and RHS curves.

It is interesting to observe that there is an irregularity in curvature of LHS plot when α_i is around the value of 3 or less in Figure 4.1. This irregularity at small α_i is traced back to the approximation methods that we adopted in deriving the closed-form expression for the expected packet-data utility model. We observe that there is more restriction on the conditions of using the approximation methods in utility modeling.

Basically, the restriction is that $\mu_{Y_i} \geq 3\,\sigma_{Y_i}$ ($\alpha_i \geq 3$) is required in application of both Holtzman's formula for expected PSR $E[f(\gamma_i)]$ and Gaussian approximation for total interference Y_i specifically in this study. The discussions on this is deferred to the next subsection.

Inspecting the curves in Figure 4.1, it is obvious that there are only two places where β_i could possibly have solutions:

(1) $\beta_i = 0$, or equivalently $p_i = 0$, and

(2) β_i corresponding to the nonzero crossover point of LHS and RHS.

Clearly, both LHS and RHS curves start at zero, so there is a possible solution at $\beta_i = 0$, or $p_i = 0$. As β_i increases, the LHS curve extends forward but it is always below the 45° straight line of RHS. After several changes in its convexity at small values of α_i, the LHS curve finally grows exponentially. As a result, it must meet from below and cross the 45° straight line exactly once, which represents the solution $\beta_i = \beta_i^*$, or equivalently $p_i = p_i^*$.

As indicated previously, there is a lower boundary for p_i as $p_i = P_{min,i} > 0$. Hence, the possibility to have a solution at $\beta_i = 0$ ($p_i = 0$) is outside the domain of consideration. So consequently (1) above is rejected as a possible solution. Therefore, only (2) above is left as the valid and unique solution for the first-order condition for component $\mathcal{A}$ (4.17) of the expected utility function (4.15).

By inspection, with this unique solution $\beta_i = \beta_i^*$ or $p_i = p_i^*$, component $\mathcal{A}$ clearly attains its peak or maximum which is both its local and global maximum. So, *its local maximum is a global maximum.* According to **Corollary 1 (Property of Strict Quasi-Concavity)**, component $\mathcal{A}$ is then strictly quasi-concave.

Based on **Theorem 2 (Transformation of Quasi-Concavity)**, with $\mathcal{A}$ is a strictly quasi-concave function of p_i and $\mathcal{B}$ is a strictly and monotonically-increasing function of p_i, we conclude that the expected packet-data utility function $\overline{u}_i(p_i, \mathbf{p}_{-i})$ in (4.15) is strictly quasi-concave.

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4.2.4 Conditions on Using Approximation Methods in Utility Modeling

As mentioned above, in Figure 4.1, there is an irregularity in curvature of LHS plot when α_i is around the value of 3 or less. This irregularity is related to the restrictions on the approximation methods that we used in deriving the closed-form expression for the expected packet-data utility model. The explanation is twofold, as discussed below.

(1) Requirement for application of Holtzman's formula

Recall Holtzman's formula that we used to approximate the expected packet success rate (PSR) in (3.21), repeated below for easy reference:

$$E[f(\gamma_i)] \approx \frac{2}{3} f\left(\overline{\gamma}_i^0\right) + \frac{1}{6}\left[f\left(\overline{\gamma}_i^1\right) + f\left(\overline{\gamma}_i^2\right) \right]$$

where, from (3.35), (3.37) and (3.37), we have

$$\overline{\gamma}_i^0 = \frac{\beta_i}{\alpha_i}\, \gamma_{min,\,i}\,, \quad \overline{\gamma}_i^1 = \frac{\beta_i}{\alpha_i + \sqrt{3}}\, \gamma_{min,\,i}\,, \quad \overline{\gamma}_i^2 = \frac{\beta_i}{\alpha_i - \sqrt{3}}\, \gamma_{min,\,i}\,.$$

Notice that when α_i is small and approaching $\sqrt{3}$ from above, $\frac{1}{\alpha_i - \sqrt{3}}$ in $\overline{\gamma}_i^2$ will go unbounded. This will cause the derivative of $E[f(\gamma_i)]$ in (4.19) grow without bound, and similarly, cause the denominator of LHS of equation (4.20) shoot to infinity.

Additionally, if $\alpha_i < \sqrt{3}$, $\overline{\gamma}_i^2$ becomes negative, which renders the PSR expression $f(\gamma)$ meaningless. For $f(\gamma)$, γ is physically the signal-to-interference ratio (SIR) which is always a nonnegative quantity.

To summarize to above discussion, when $\alpha_i \leq \sqrt{3}$, the application of Holtzman's formula (3.21) to approximate PSR fails, which causes the abnormality in the expected utility model. Therefore, an important requirement for using Holtzman's formula to approximate a random PSR is basically $\alpha_i > \sqrt{3}$. Since $\alpha_i = \frac{\mu_{Y_i}}{\sigma_{Y_i}}$ (3.33), we have

$$\alpha_i > \sqrt{3} \quad \Longrightarrow \quad \mu_{Y_i} > \sqrt{3}\,\sigma_{Y_i}\,. \tag{4.21}$$

which indicates that the mean of Y_i has to be at least $\sqrt{3}$ times larger than its standard deviation.

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(2) Requirement for Gaussian approximation of total interference Y_i

From above discussion, we know $\mu_{Y_i} > \sqrt{3}\,\sigma_{Y_i}$ as a basic requirement for applying Holtzman's formula. However, we do not know that to what extent μ_{Y_i} has to be larger than $\sqrt{3}\,\sigma_{Y_i}$. The fact is that it is as disastrous for α_i to be in the neighborhood of $\sqrt{3}$ as $\alpha_i = \sqrt{3}$ exactly.

Results of our numerical analysis showed that $\alpha_i \geq 3$ is required to make the irregularity caused by small α_i under acceptable control. That is

$$\alpha_i \;\geq\; 3 \quad \Longrightarrow \quad \mu_{Y_i} \;\geq\; 3\,\sigma_{Y_i}\,. \tag{4.22}$$

The reason why $\alpha_i \geq 3$ is required happens to coincide with "$3\,\sigma$ **rule**" of Gaussian distribution. "$3\,\sigma$ **rule**" states that about 99.7% of values drawn from a Gaussian distribution are within three standard deviations (σ) away from the mean (μ).

In our study, this means that, to approximate the random interference Y_i using Gaussian distribution, the mean μ_{Y_i} has to be at least three times larger than the standard deviation σ_{Y_i}. With $\mu_{Y_i} \geq 3\,\sigma_{Y_i}$ ($\alpha_i \geq 3$), distribution of Y_i bears a close resemblance to Gaussian pdf. If $\mu_{Y_i} < 3\,\sigma_{Y_i}$, a significant portion of the area under the assumed Gaussian pdf, $Y_i \sim \mathcal{N}(\,\mu_{Y_i},\,\sigma_{Y_i})$, would fall in the domain of negative Y_i, which is physically meaningless and has to be truncated. The truncation would distort the distribution of Y_i to make it dissimilar to the assumed Gaussian pdf $\mathcal{N}(\,\mu_{Y_i},\,\sigma_{Y_i})$. Illustration in Figure 3.2 should help showing this point.

■

To summarize on (1) and (2), $\mu_{Y_i} \geq 3\,\sigma_{Y_i}$ ($\alpha_i \geq 3$) is a basic requirement for applying both Holtzman's formula for expected PSR $E[f(\gamma_i)]$ and Gaussian approximation for total interference Y_i specifically in this study.

Practically, $\mu_{Y_i} \geq 3\,\sigma_{Y_i}$ is a valid and natural requirement for packet-switched data systems. As discussed before, it is always assumed in this study that there are a large number of mobile users in each cell with each user transmitting for only a small fraction of time. So we have many users (large N) but all of them have a low activity level (small ρ). Therefore, for a given value of ρ, a large N results in a high value of ratio $\frac{\mu_{Y_i}}{\sigma_{Y_i}}$, because the large N increases both the mean μ_{Y_i} and the standard deviation σ_{Y_i}, but the mean μ_{Y_i} is increased faster.

Additionally, the above discussions clearly show that, although approximation methods are great in helping us simplify difficult problems so we would have a chance to solve them, cares have to be taken in using them. They are usually only valid and/or accurate under restricted conditions or for limited ranges of values for the parameters involved.

4.3 Existence and Uniqueness of Nash Equilibrium Solution

In Section 4.1, we formulated the uplink transmit power control problem as a non-cooperative game. We also discussed the first-order or necessary condition for the existence of a Nash equilibrium solution to the power control game. As we know, the operating points of a non-cooperative game are Nash equilibria. However, generally a non-cooperative game may or may not have an equilibrium solution. And if it does have an equilibrium solution, the number of solutions may not be unique. Therefore, we need to address the problems of how to determine the following:

1. Whether an equilibrium exists in our power control game;

2. Whether the equilibrium is unique in case that the equilibrium does exist.

In this section, we formally address the above issues concerning the existence and uniqueness of Nash equilibria in our non-cooperative power control game.

4.3.1 Proof of Existence and Uniqueness of the Solution

Following the studies in [13, 38], we use an important theorem contributed by Debreu, Fan and Glicksberg [48, 49, 50], which we repeat below without proof:

Theorem 4 (Equilibrium Existence Theorem) *Nash equilibrium exists for a non-cooperative game in which each user's utility is quasi-concave in its own strategy, and the strategy space of each user is non-empty, convex and compact.*

Similarly, the same issue is also addressed in **Theorem 3** from [117] as previously mentioned in Subsection 4.2.2. Furthermore, **Theorem 3** states that for a *strictly* quasi-concave function in its convex domain, the solution to the problem of maximizing the function, if existed, is *unique*.

Based the theorems above and the prior analysis on the expected packet-data utility function, we will formally prove the existence and uniqueness of the Nash equilibrium solution to the non-cooperative uplink power control game in (4.2). The result is summarized in the proposition below, followed by its proof.

Proposition 3 (Existence and Uniqueness of Nash Equilibrium Solution) *There exists a unique Nash equilibrium solution to the non-cooperative uplink power control game formulated in (4.2).*

Proof. Based on **Proposition 2 (Quasi-Concavity of Expected Utility**

Model), we know that the expected packet-data utility function $\overline{u}_i(\, p_i,\, \mathbf{p}_{-i}\,)$ is strictly quasi-concave in its own strategy space for the transmit power p_i.

As for the characteristics of the strategy space for p_i, in **Appendix A.2**, it is proved that the strategy space for p_i is *nonempty, convex and compact*. Therefore, according to **Theorem 4 (Equilibrium Existence Theorem)**, there exists a Nash equilibrium for the non-cooperative uplink power control game.

The proof on the uniqueness of this Nash equilibrium solution follows directly from **Theorem 3**. Because the expected utility function is strictly quasi-concave and its domain is convex (refer to **Appendix A.2**), the solution can only be either empty or singleton. From the prior analysis on the expected utility function, the solution is obviously nonempty. Therefore, the solution is unique.

In conclusion, it is clear that there exists a unique value of $\beta_i = \beta_i^*$ or $p_i = p_i^*$ with which the expected packet-data utility of each user in the power control game is maximized.

■

4.3.2 Consideration of Boundary Conditions

Because we study the noncooperative uplink power control game under a constrained range of domain $[\, P_{min,\,i},\, P_{max,\,i}\,]$ for the transmit power p_i, we need t investigate the effect of those boundary conditions on the optimum solution of the game.

As we know from above, the game has a unique solution as $\beta_i = \beta_i^*$ or $p_i = p_i^*$. A strictly quasi-concave function implies that the function is strictly increasing in $[\, P_{min,\,i},\, p_i^*\,]$ and strictly decreasing in $[\, p_i^*,\, P_{max,\,i}\,]$ [120]. Therefore, we the following properties for the expected packet-data utility function (3.38):

1. When $p_i = P_{min,\,i}$ (4.8), we have

$$\overline{u}_i(\,P_{min,\,i},\;\mathbf{p}_{-i}\,) \;=\; \min_{\{P_{min,\,i}\,\leq\, p_i\,\leq\, p_i^*\}} \overline{u}_i(\,p_i,\;\mathbf{p}_{-i}\,) \tag{4.23}$$

where p_i^* is the power with which the expected utility achieves its peak value.

2. When $p_i = P_{max}$ (4.3), we have

$$\overline{u}_i(\,P_{max},\;\mathbf{p}_{-i}\,) \;=\; \min_{\{p_i^*\,\leq\, p_i\,\leq\, P_{max}\}} \overline{u}_i(\,p_i,\;\mathbf{p}_{-i}\,)\,. \tag{4.24}$$

Since $p_i = P_{min,\,i}$ is close to zero, its effect on the optimum solution is insignificant. However, for $p_i = P_{max}$, the optimum solution could be impacted significantly. There are two scenarios for the relationship between p_i^* and P_{max} and the corresponding consequences as below:

(1) If $p_i^* \;<\; P_{max}$, the maximum of the expected utility function is

$$\overline{u}_i^*(\,p_i^*,\;\mathbf{p}_{-i}\,) \;=\; \max_{\{P_{min,\,i}\,\leq\, p_i\,\leq\, P_{max}\}} \overline{u}_i(\,p_i,\;\mathbf{p}_{-i}\,)\,, \tag{4.25}$$

(2) If $p_i^* \;>\; P_{max}$, the maximum achievable for the expected utility function is

$$\overline{u}_i^*(P_{max},\;\mathbf{p}_{-i}\,) \;=\; \max_{\{P_{min,\,i}\,\leq\, p_i\,\leq\, P_{max}\}} \overline{u}_i(\,p_i,\;\mathbf{p}_{-i}\,)\,. \tag{4.26}$$

Notice that for both scenarios (1) and (2), the solution is unique. However, in scenario (2), the maximum happens to be at the boundary $p_i = P_{max}$ which is the highest value achievable for $\overline{u}_i(\,p_i,\;\mathbf{p}_{-i}\,)$ in the constrained domain of $[\,P_{min,\,i},\;P_{max,\,i}\,]$.

In the subsequent studies, we will assume that P_{max} is chosen to be significantly large such that $p_i^* > P_{max}$ in scenario (2) would not happen. Hence, our remaining studies will focus on the usual case with $p_i^* < P_{max}$ as in scenario (1) above.

4.4 Analysis and Computation of the Solution to the Power Control Game

In this section, we will further investigate the characteristics of the solution to the noncooperative uplink power control game. We will explore the solution graphically first and then formulate the expression for the solution in closed-form.

4.4.1 First-Order Condition - Further Derivation

As previously discussed, the necessary condition or the first-order condition for a Nash equilibrium solution to the power control game can be found by solving N simultaneous equations as in (4.10) which leads to equation (4.12) as repeated here for easy reference:

$$[E[f(\gamma_i)] - f(0)] \times Prob\{\, d_i \le D_{max,\,i}\} \;=$$

$$\left\{ \frac{\partial\, E[f(\gamma_i)]}{\partial\, \beta_i} Prob\{\, d_i \le D_{max,\,i}\} \,+\, [E[f(\gamma_i)] - f(0)]\, \frac{\partial\, Prob\{\, d_i \le D_{max,\,i}\}}{\partial\, \beta_i} \right\} \times \beta_i \tag{4.27}$$

The transmit power p_i is strictly positive and $0 < P_{min,\,i} \le p_i \le P_{max}$ in this study. Based on the nature of the functions $Prob\{\, d_i \le D_{max,\,i}\,\}$ and $[E[f(\gamma_i)] -$

$f(0)]$ as expressed in equations (3.40) and (3.41), the following are true:

$$
\begin{aligned}
Prob\{\, d_i \le D_{max,i}\} \;&\ne\; 0\,, \qquad for \quad P_{min,i} \le p_i \le P_{max} && (4.28)\\
E[f(\gamma_i)] - f(0) \;&\ne\; 0\,, \qquad for \quad P_{min,i} \le p_i \le P_{max} && (4.29)
\end{aligned}
$$

Then Equation (4.27) can be simply re-written as:

$$
\left\{ \frac{\dfrac{\partial\, Prob\{\, d_i \le D_{max,i}\,\}}{\partial\, \beta_i}}{Prob\{\, d_i \le D_{max,i}\,\}} + \frac{\dfrac{\partial\, E[f(\gamma_i)]}{\partial\, \beta_i}}{E[f(\gamma_i)] - f(0)} \right\} \times \beta_i \;=\; 1, \qquad \forall\, i = 1, \cdots, N \tag{4.30}
$$

From the expressions of $Prob\{\, d_i \le D_{max,i}\,\}$ and $E[f(\gamma_i)]$ in (3.40) and (3.41), the partial derivatives of $Prob\{\, d_i \le D_{max,i}\,\}$ and $E[f(\gamma_i)]$ with respect to parameter β_i can be found as follows:

$$
\frac{\partial\, Prob\{\, d_i \le D_{max,i}\}}{\partial\, \beta_i} \;=\; \frac{e^{-\frac{1}{2}\,(\beta_i - \alpha_i)^2}}{\displaystyle\int_{-\alpha_i}^{\infty} e^{-\frac{1}{2}\,u^2}\, du}\,, \tag{4.31}
$$

$$
\frac{\partial\, E[f(\gamma_i)]}{\partial\, \beta_i} \;=\; \frac{2}{3}\, f'\left(\overline{\gamma}_i^0\right) \frac{\gamma_{min,i}}{\alpha_i} \;+\; \frac{1}{6}\left[f'\left(\overline{\gamma}_i^1\right) \frac{\gamma_{min,i}}{\alpha_i + \sqrt{3}} \;+\; f'\left(\overline{\gamma}_i^2\right) \frac{\gamma_{min,i}}{\alpha_i - \sqrt{3}} \right]. \tag{4.32}
$$

With the above expressions and equations as in (3.40) and (3.41), using notations $\overline{\gamma}_i^0$, $\overline{\gamma}_i^1$ and $\overline{\gamma}_i^2$ in (3.35), (3.36) and (3.37), Equation (4.30) can be written out in more detail as below:

$$\frac{\beta_i \, e^{-\frac{1}{2}\,(\beta_i \,-\, \alpha_i)^2}}{\displaystyle\int_0^{\beta_i} e^{-\frac{1}{2}\,(u \,-\, \alpha_i)^2}\, du} \;+\; \frac{4\, f'\,(\overline{\gamma}_i^0)\;\overline{\gamma}_i^0 \;+\; f'\,(\overline{\gamma}_i^1)\;\overline{\gamma}_i^1 \;+\; f'\,(\overline{\gamma}_i^2)\;\overline{\gamma}_i^2}{4\, f\,(\overline{\gamma}_i^0) \;+\; f\,(\overline{\gamma}_i^1) \;+\; f\,(\overline{\gamma}_i^2) \;-\; 6\, f(0)} \;=\; 1\,,$$

$$\forall\, i = 1, \cdots, N\,. \qquad (4.33)$$

The above expression of the first-order condition for the equilibrium solution of the power control game will be used later in the simulations and numerical studies.

4.4.2 Solution Function and Its Properties

In the above equality (4.33), as we know, parameters $\overline{\gamma}_i^0$, $\overline{\gamma}_i^1$, and $\overline{\gamma}_i^2$ depend only on α_i, β_i and $\gamma_{min,\,i}$ which in turn depend on $D_{max,\,i}$ (refer to (3.3), (3.35), (3.36) and (3.37)).

For each given value of α_i and $D_{max,\,i}$, equation (4.33) gives a unique solution for β_i (the reason why it is unique will be shown later in the subsequent discussion). Hence, given $D_{max,\,i}$, the solution to Equation (4.33) will not a fixed number but a fixed *solution function* defined by

$$\beta_i^* \;\triangleq\; g(\alpha_i^*)\,, \qquad (4.34)$$

where $(\beta_i^*,\ \alpha_i^*)$ denotes a solution point, i.e., a point on the curve of the function $g(\cdot)$. An example to illustrate this solution point with some given values of α_i and $D_{max,\,i}$ is shown in Figure (4.2).

The above solution function $g(\cdot)$ is *implicitly* expressed by equation (4.27) or equivalently by (4.33). We will show that this solution function exists and is

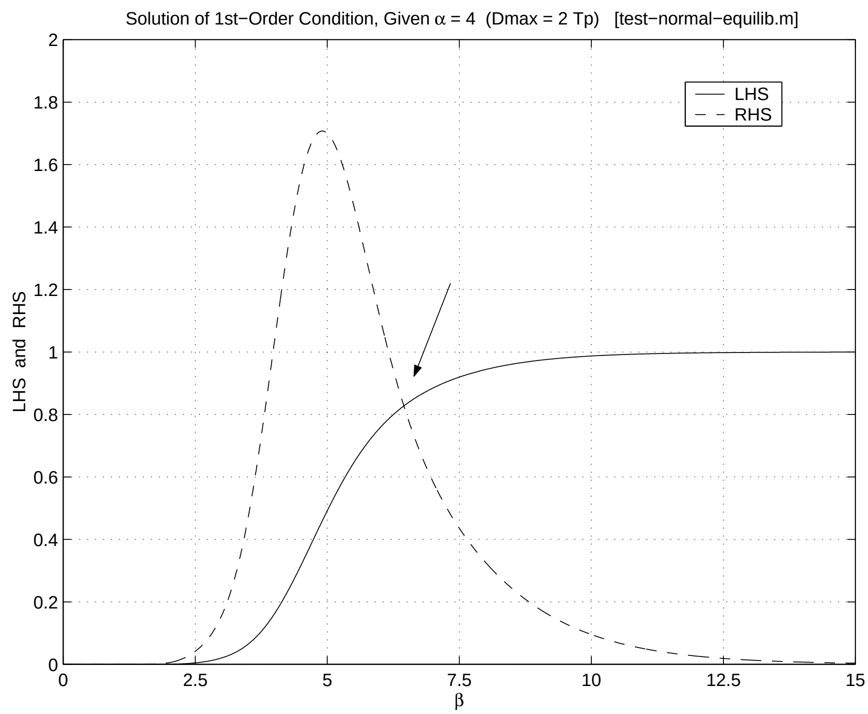

Figure 4.2: Illustration of a solution of β_i^* to the first-order condition expressed in Equation (4.27), given $\alpha_i = 4$ and $D_{max,i} = 2\,T_p$. The arrow-pointed crossover point of LHS and RHS curves represents the solution $\beta_i = \beta_i^*$.

uniquely defined by Equation (4.33). But before doing that, we demonstrate a basic fact about the necessary condition for game equilibrium in equation (4.33) in a lemma:

<u>Lemma</u> 1 (α_i^* and β_i^* Relationship) *For Gaussian-distributed interference Y_i with $\alpha_i = \frac{\mu_{Y_i}}{\sigma_{Y_i}}$ and $\beta_i = \frac{b_i}{\sigma_{Y_i}}$, given an arbitrary $\alpha_i^* > 0$, there always exists a unique β_i^* that satisfies Equation (4.27) or (4.33), and β_i^* is strictly larger than the given α_i^*, i.e.,*

$$\beta_i^* \;>\; \alpha_i^*. \tag{4.35}$$

<u>Proof</u>. The proof of the above lemma is best shown graphically in Figure (4.3). Inspecting the first-order condition in Equation (4.27), we observe an interesting and unique feature about the equation:

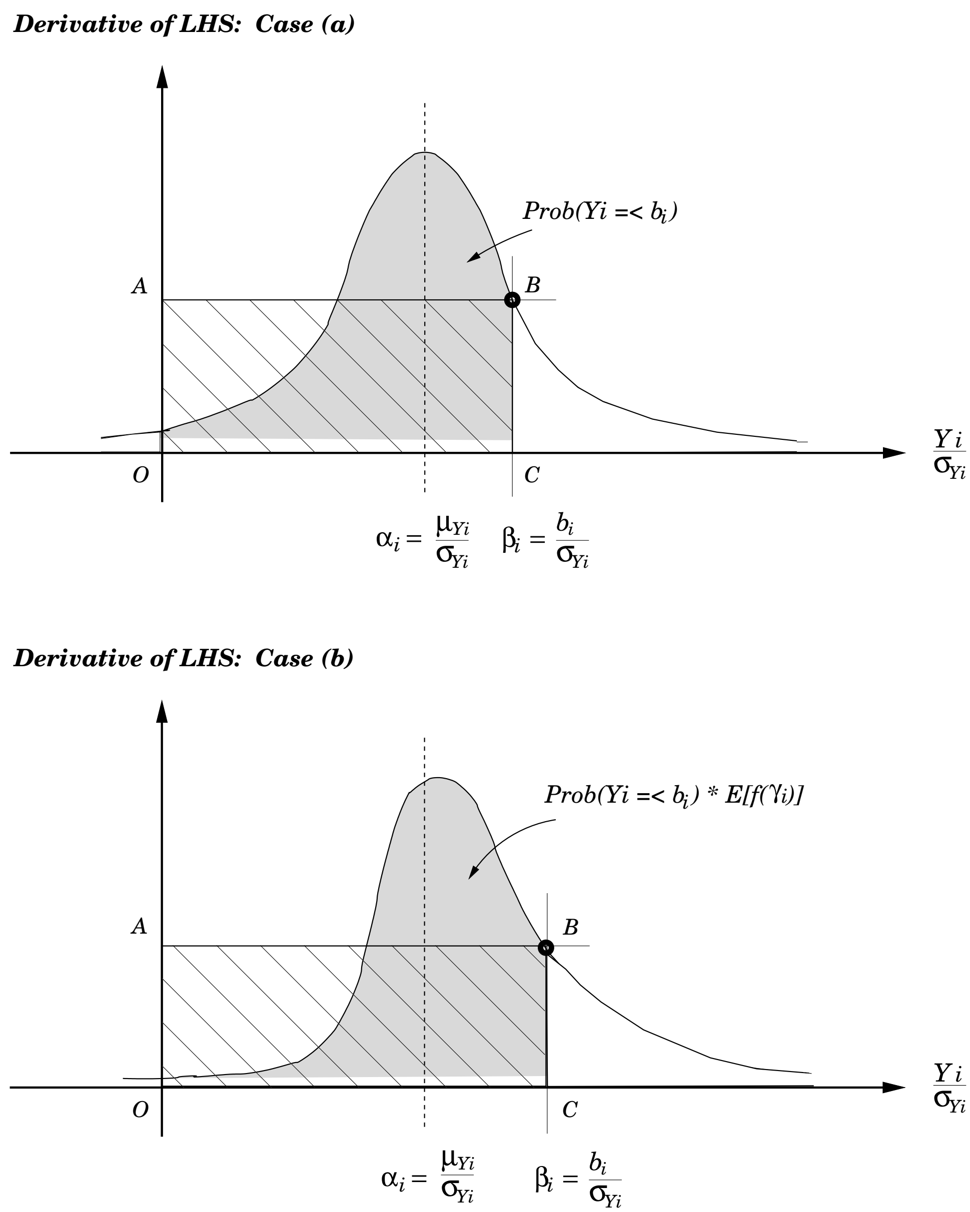

Figure 4.3: Geometrical demonstration of equation (4.27) – the first-order condition of game equilibrium: *the area under the curve of Derivative of LHS of equation (4.27) between O and β_i (<u>the shaded area</u>) must equal to the area of rectangle ABCO (<u>the hatched area</u>).* **Case (a)**: a special case assuming $\{E[f(\gamma_i)] - f(0)\} = 1$ in the utility model; **Case (b)**: general case with both $Prob(Y_i \leq b_i)$ and $\{E[f(\gamma_i)] - f(0)\}$ in the utility model.

- **LHS**: Product of the expected PSR and the probability of delay requirement,
- **RHS**: "**Derivative of LHS**" multiplied by β_i.

Therefore, **LHS** = **RHS** in Equation (4.27) means that **Derivative of LHS** times β_i equals exactly the **LHS** itself.

In Figure (4.3), Derivative of LHS is depicted as a single-peaked curve. As demonstrated in the figure, the value for LHS of equation (4.27) is represented as the *shaded area* under the curve of Derivative of LHS between the origin O and the parameter β_i. Obviously, the size of this shaded area grows with the increase of β_i.

On the other hand, the RHS of Equation (4.27) is a multiplication of the value of Derivative of LHS at β_i and the argument β_i itself. This is represented as the *hatched area* of rectangle $ABCO$. As the value of β_i changes, the size of this hatched area also changes.

Graphically, by Equation (4.27), LHS = RHS means that the two areas mentioned above (shaded vs. hatched) must be equal in size for some value of β_i.

For easy demonstration, two scenarios (a special case and the general case) are illustrated in Figure (4.3). Those two cases are described below:

- **Case (a)**: Assuming that the expected PSR $\{E[f(\gamma_i)] - f(0)\} = 1$, there is only the probability of delay requirement factor $Prob(Y_i \leq b_i)$ in the utility model. In this case, the distribution of interference Y_i is symmetrical about its mean μ_{Y_i} (or α_i) (Gaussian, bell-shaped).
- **Case (b)**: In general, we have both the probability of delay requirement factor $Prob(Y_i \leq b_i)$ and the expected PSR factor $\{E[f(\gamma_i)] - f(0)\}$ in the utility model. Insertion of the multiplication factor $E[f(\gamma_i)]$ makes the curve of resulting product unsymmetrical.

We know that the total interference Y_i is assumed to be Gaussian-distributed (single-peaked and bell-shaped pdf) and that the expected PSR $\{E[f(\gamma_i)] - f(0)\}$ is an nonnegative monotonically-increasing function of β_i. These facts guarantee the continuous and unimodal structure of the Derivative of LHS curve.

The effect of multiplying $\{E[f(\gamma_i)]-f(0)\}$ to the probability $Prob\{\, d_i \leq D_{max,i}\} = Prob(\, Y_i \leq b_i\,)$ reduces the value of the product diminishingly as β_i increases, since $0 \leq \{E[f(\gamma_i)]-f(0)\} \leq 1$. As a result, the peak of the Derivative of LHS is pushed toward a higher value of $\frac{Y_i}{\sigma_{Y_i}}$, as shown in **Case (b)** in Figure (4.3). Notice that the mean of interference μ_{Y_i} (or $\alpha_i = \frac{\mu_{Y_i}}{\sigma_{Y_i}}$) is given and fixed in Figure (4.3).

Because of the continuity and unimodality of the Derivative of LHS curve, to satisfy Equation (4.27), there exists *only one possibility* with $\beta_i^* > \alpha_i^*$ (i.e., $\frac{b_i^*}{\sigma_{Y_i}^*} > \frac{\mu_{Y_i}^*}{\sigma_{Y_i}^*}$), such that the part of the shaded area above line AB and the part of the hatched area above the Derivative of LHS curve can compensate with each other. This is the only way to meet the requirement LHS = RHS, or equivalently *shaded area = hatched area.*

Observe that in Figure (4.3), there is a point $\boldsymbol{B}$ on the Derivative of LHS curve that corresponds to the optimal β_i^*. As a matter of fact, inequality $\beta^* > \alpha^*$ is only a necessary condition for LHS = RHS. Strictly, point $\boldsymbol{B}$ must be off the peak of the Derivative of LHS curve on the right-hand side to satisfy the above statement on *shaded area = hatched area.*

As a counter-example, let us suppose that point $\boldsymbol{B}$ is either at the peak or on the left-side of the peak on the Derivative of LHS curve. From this assumption, geometrically, an immediate result will follow, that is: *shaded area < hatched area.* This violates the geometry implied by equality LHS = RHS, and hence cannot be true.

■

Using definitions of α_i (3.33), β_i (3.34) and b_i (3.6), the inequality (4.35) in **Lemma 1 (α_i^* and β_i^* Relationship)** can be shown equivalently as the following:

$$\frac{b_i^*}{\sigma_{Y_i}^*} > \frac{\mu_{Y_i}^*}{\sigma_{Y_i}^*} \quad \Longleftrightarrow \quad \frac{W}{R}\,\frac{h_i\,p_i^*}{\gamma_{min,\,i}} > \mu_{Y_i}^* \quad \Longleftrightarrow \quad \frac{W}{R}\,\frac{h_i\,p_i^*}{\mu_{Y_i}^*} > \gamma_{min,\,i}\,. \tag{4.36}$$

Basically, the above inequality demands that a user's SIR in fighting against the average interference μ_{Y_i} be strictly larger than its target $\gamma_{min,\,i}$. In addition, this demand on SIR has to leave some room for fighting against the part of interference caused by the interference variability. Because, in packet-switched data systems, the random interference can be characterized essentially by two statistics: the mean of interference and the variance of interference. Both of them are the key measures on strength and behavior of the interference.

Note that in the discussion that follows, the subscript i in notations such as α_i and β_i will be omitted if the notations in the discussion are the *same* for every user (*same* for every i) in the system.

For convenience, we use the following simplifying notations in the discussions. Notice that subscript i is omitted in these notations.

$$\mathcal{P}_D \;=\; Prob\{\,d \le D_{max}\} \tag{4.37}$$

$$E[f] \;=\; E[f(\gamma)] \tag{4.38}$$

Now we discuss the characteristics of the solution function defined in (4.34) for the first-order condition of game equilibrium. The statement below summarizes the result, followed by its proof:

Proposition 4 (Characteristics of Solution Function) *The solution function*

$\beta^* = g(\alpha^*)$ to Equation (4.27) or (4.33) of the first-order condition for Nash equilibrium exists and is uniquely defined. And $g(\cdot)$ is a continuous and strictly monotonic function.

Proof. Consider a new function $F(\alpha,\,\beta)$ below by moving all the terms in (4.27) to the left-side:

$$F(\alpha,\,\beta) \;\triangleq\; \mathcal{P}_D\,[E[f]-f(0)] \;-\; \left\{\frac{\partial\,\mathcal{P}_D}{\partial\,\beta}[E[f]-f(0)] \;+\; \mathcal{P}_D\,\frac{\partial\,E[f]}{\partial\,\beta}\right\}\beta \;=\; 0\,. \tag{4.39}$$

The solution function $\beta^* = g(\alpha^*)$ is implicitly defined by function $F(\alpha,\,\beta) = 0$ above. Assume that $(\alpha^*,\ \beta^*)$ is an arbitrary solution of Equation (4.39), then we have $F(\alpha^*,\,\beta^*) = 0$. From the structure of the function and the nature of $Prob\{\,d \le D_{max}\}$ and $E[f(\gamma)]$, we can show that function (4.39) is continuous in both α and β.

The partial derivatives of $F(\alpha,\,\beta)$ with respect to α and β exist and can be found as below:

$$F'_\alpha \;=\; \frac{\partial F}{\partial\alpha} \;=\; \frac{\partial\mathcal{P}_D}{\partial\alpha}\,[E[f]-f(0)] \;+\; \mathcal{P}_D\,\frac{\partial E[f]}{\partial\alpha}$$

$$-\;\left\{\frac{\partial^2\mathcal{P}_D}{\partial\beta\,\partial\alpha}[E[f]-f(0)] \;+\; \frac{\partial\mathcal{P}_D}{\partial\beta}\,\frac{\partial E[f]}{\partial\alpha} \;+\; \frac{\partial\mathcal{P}_D}{\partial\alpha}\,\frac{\partial E[f]}{\partial\beta} \;+\; \mathcal{P}_D\frac{\partial^2 E[f]}{\partial\beta\,\partial\alpha}\right\}\beta, \tag{4.40}$$

$$F'_\beta \;=\; \frac{\partial F}{\partial\beta} \;=\; -\left\{\frac{\partial^2\mathcal{P}_D}{\partial\beta^2}[E[f]-f(0)] \;+\; 2\,\frac{\partial\mathcal{P}_D}{\partial\beta}\,\frac{\partial E[f]}{\partial\beta} \;+\; \mathcal{P}_D\frac{\partial^2 E[f]}{\partial\beta^2}\right\}\beta\,. \tag{4.41}$$

From $\dfrac{\partial \mathcal{P}_D}{\partial \beta}$ and $\dfrac{\partial E[f]}{\partial \beta}$ expressions in (4.31) and (4.32), we find:

$$\frac{\partial^2 \mathcal{P}_D}{\partial \, \beta^2} \;=\; \frac{-\,(\beta - \alpha)\, e^{-\frac{1}{2}\,(\beta-\alpha)^2}}{\displaystyle\int_{-\alpha}^{\infty} e^{-\frac{1}{2}\,u^2}\, du}\,, \tag{4.42}$$

$$\frac{\partial^2 E[f]}{\partial \, \beta^2} \;=\; \gamma_{min}^2 \left\{ \frac{2}{3}\, \frac{f''(\overline{\gamma}_i^0)}{\alpha^2} + \frac{1}{6} \left[\frac{f''(\overline{\gamma}_i^1)}{(\alpha + \sqrt{3})^2} + \frac{f''(\overline{\gamma}_i^2)}{(\alpha - \sqrt{3})^2} \right] \right\} \tag{4.43}$$

Now look at F'_β expression in (4.41). From $\mathcal{P}_D$ and $E[f]$ expressions in (3.40) and (3.41), the first-order partial derivatives in (4.31) and (4.32), and the second-order partial derivatives in (4.42) and (4.43), and using **Lemma 1**, we find that the three terms inside the braces of Equation (4.41) will not be zero for $0 < \beta < \infty$. Since β itself will not be zero ($0 < \beta < \infty$), we then have:

$$F'_\beta(\alpha^*,\ \beta^*) \neq 0\,. \tag{4.44}$$

With above conditions satisfied, according to **Implicit Function Theorem** in mathematical analysis [112], implicit function $g(\cdot)$ exists and is uniquely defined with $F[\alpha^*,\ g(\alpha^*)] = 0$. Also $g(\cdot)$ is a continuous and strictly monotonic function.

■

Again, using definitions for α_i , β_i and b_i in (3.33), (3.34) and (3.6), consider the solution function $\beta^* = g(\alpha^*)$ in the following forms:

$$\frac{b_i^*}{\sigma_{Y_i}^*} = g\left(\frac{\mu_{Y_i}^*}{\sigma_{Y_i}^*}\right) \quad \Longleftrightarrow \quad \frac{W}{R}\frac{h_i\,p_i^*}{\gamma_{min,\,i}} = \sigma_{Y_i}^*\,g\left(\frac{\mu_{Y_i}^*}{\sigma_{Y_i}^*}\right)$$

$$\Longleftrightarrow \quad \frac{W}{R}\frac{h_i\,p_i^*}{\sigma_{Y_i}^*\,g\left(\frac{\mu_{Y_i}^*}{\sigma_{Y_i}^*}\right)} = \gamma_{min,\,i}\,. \tag{4.45}$$

Recall the general uplink SIR expression in CDMA systems. The quantity $\sigma_{Y_i}^*\,g\left(\frac{\mu_{Y_i}^*}{\sigma_{Y_i}^*}\right)$ in the denominator of Equation (4.45) above resembles the amount of interference in the packet-switched data CDMA system that user i has to overcome to achieve a target SIR.

For the convenience of analysis, let us call this quantity as "*Equivalent Interference*" denoted as $\overline{I}_{equ}^*$, because it represents the overall effect contributed by both the mean and variability of the random interference in packet-switched data systems. The effect of this "Equivalent Interference" is also similar to that of deterministic interference in circuit-switched data systems.

With the idea of "Equivalent Interference", i.e., $\overline{I}_{equ}^*$, let us define a new SIR specifically for packet-switched data systems, namely *Average Packet SIR*:

$$\overline{\gamma}_{pkt,\,i}^* \triangleq \frac{W}{R}\frac{h_i\,p_i^*}{\overline{I}_{equ}^*} = \frac{W}{R}\frac{h_i\,p_i^*}{\sigma_{Y_i}^*\,g\left(\frac{\mu_{Y_i}^*}{\sigma_{Y_i}^*}\right)}\,. \tag{4.46}$$

Hence, the solution function (4.45) indicates a basic requirement that can be simply written as:

$$\overline{\gamma}_{pkt,\,i}^* = \gamma_{min,\,i}\,, \qquad \forall\, i = 1, \cdots, N\,. \tag{4.47}$$

Obviously, the above equation shows that the necessary condition for a Nash equilibrium (4.10) gives a fixed-point solution. The solution requires the *Average Packet SIR* $\overline{\gamma}^*_{pkt,\,i}$ of each user to be exactly same as the user's minimum SIR requirement $\gamma_{min,\,i}$ as specified in (3.3).

From Equations (4.46) and (4.47), we can express the optimum transmit power p_i^* as a function of the mean of interference $\mu^*_{Y_i}$, the variance of interference $\sigma^*_{Y_i}$, the target SIR requirement $\gamma_{min,\,i}$, and other system parameters, as follows:

$$p_i^* \;=\; \frac{R\,\gamma_{min,\,i}}{W\,h_i}\,\sigma^*_{Y_i}\,g\left(\frac{\mu^*_{Y_i}}{\sigma^*_{Y_i}}\right), \qquad \forall\, i = 1, \cdots, N\,. \tag{4.48}$$

This transmit power for user i is optimum in the sense of Nash equilibrium achieved by the power control game. To find its relationship to the interference in terms of $\mu^*_{Y_i}$ and $\sigma^*_{Y_i}$, the solution function $g(\cdot)$ needs to be well understood, which is the topic of next subsection.

4.4.3 Computation for the Solution Function and the Optimum Transmit Powers

Owing to the nonlinearity of solution function $g(\cdot)$, it is analytically intractable to solve for a closed-form expression for $g(\cdot)$. So we have to turn to numerical approaches for help on investigating the characteristics of solution function $\beta^* = g(\alpha^*)$.

- Firstly, we will compute a significant number of data pairs $(\beta^*,\ \alpha^*)$ on the curve of the solution function. For each pair of data points, utility maximization is performed to solve for the optimum β^* under given value of α^*.

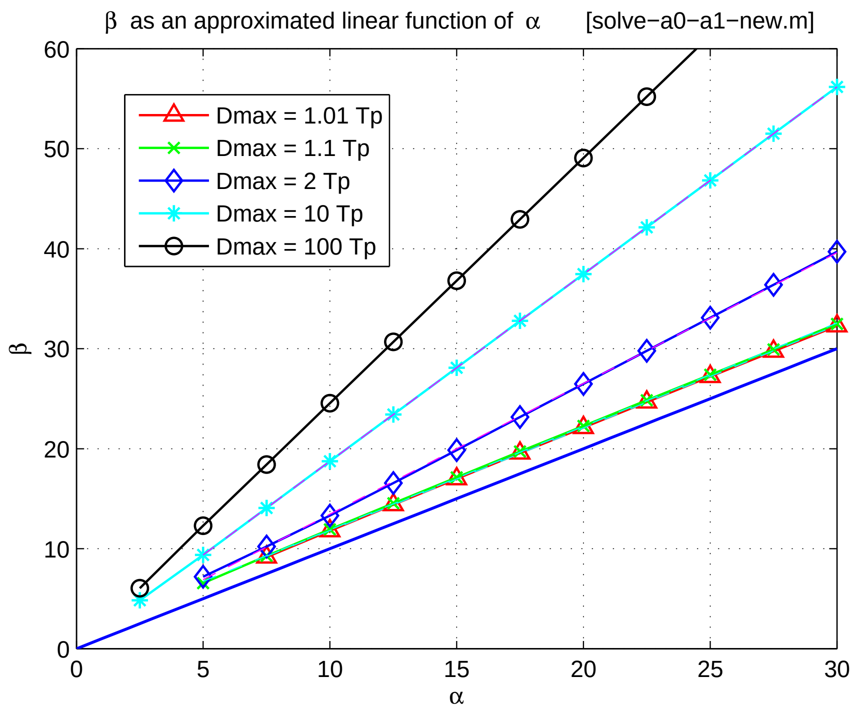

Figure 4.4: The linearly-approximated solution function of the equilibrium equation $\beta^* = g(\alpha^*)$ for different values of D_{max}. Notice that $\beta^* > \alpha^*$ is always true for all feasible D_{max} as stated in **Lemma 1 (α_i^* and β_i^* Relationship)**. The 45° line (bottom line) is also plotted for easy comparisons.

- Secondly, we will use statistical data-fitting techniques to estimate the key parameters of solution function $g(\cdot)$ in order to approximate $g(\cdot)$ with analytically-trackable functions.

Figure (4.4) plots solution function $\beta^* = g(\alpha^*)$ for different values of D_{max} based on numerical calculations. Visually, it seems that given D_{max}, the solution function can be well approximated by a linear function. Suppose that it is a good fit to approximate the solution function as linear, let us make the following assumption:

$$\beta_i^* \;=\; g(\alpha_i^*) \;\approxeq\; a_0 \;+\; a_1\,\alpha_i^*\,. \tag{4.49}$$

where a_0 is the offset and a_1 is the slope. Those are the two parameters required to define a linear function.

D_{max} (unit of T_p)	a_0 (offset)	a_1 (slope)
1.01	1.5426	1.0267
1.10	1.5528	1.0336
1.50	1.1858	1.1234
2.00	0.3640	1.3078
2.50	0.1743	1.4131
5.00	0.0310	1.6658
10.0	0.0229	1.8718
100.0	0.0296	2.4519

Table 4.1: Estimated values of the offset parameter a_0 and the slope parameter a_1 by first-order polynomial curve fitting of the solution functions to the equilibrium equation.

Observe that in Figure (4.4) all the feasible lines (corresponding to $T_p \leq D_{max} < \infty$) are above the 45° line (the bottom line in Figure (4.4)), which implies $\beta^* > \alpha^*$. This is a fact stated and proved in **Lemma 1 (α_i^* and β_i^* Relationship)**.

Due to the quasi-linear nature of the solution function, for simplicity, first-order polynomial curve-fitting (i.e., linear line-fitting) method is utilized to approximate the solution function. Then the linear parameters a_0 and a_1 are estimated in a least-squares sense. With this approach, in Figure (4.4), the lines of the linearly-approximated solution function are also plotted in addition to the ones based on numerical calculation. We see a very good match between them – the linearly-approximated versions overlap with the respective numerical versions perfectly for the same values of D_{max}.

Given a set of values for D_{max}, Table (4.1) shows the corresponding results of the a_0 and a_1 estimation for the linear approximation of the solution function.

Based on the results of the above numerical study and analysis, we approximate the solution function $\beta_i = g(\alpha_i)$ with a linear function for analytical tractability. That is, in the studies hereafter, we will assume the following:

$$\beta_i^* \;=\; a_0 \;+\; a_1\, \alpha_i^*, \qquad \forall\; i = 1, \cdots, N\,. \tag{4.50}$$

With definitions of α_i (3.33) and β_i (3.34), the equation above means

$$b_i^* \;=\; a_1\, \mu_{Y_i}^* \;+\; a_0\, \sigma_{Y_i}^*\,. \tag{4.51}$$

With definition of b_i (3.6), we have

$$\frac{W}{R}\,\frac{h_i\, p_i^*}{\gamma_{min,\,i}} \;=\; a_1\, \mu_{Y_i}^* \;+\; a_0\, \sigma_{Y_i}^*\,. \tag{4.52}$$

Therefore, the previously-defined *Equivalent Interference* as $\overline{I}_{equ}^* = \sigma_{Y_i}^*\, g^*\left(\frac{\mu_{Y_i}^*}{\sigma_{Y_i}^*}\right)$ and the *Average Packet SIR* defined in (4.46) are now:

$$\overline{I}_{equ}^* \;=\; \sigma_{Y_i}^*\, g^*\left(\frac{\mu_{Y_i}^*}{\sigma_{Y_i}^*}\right) \;=\; a_1\, \mu_{Y_i}^* \;+\; a_0\, \sigma_{Y_i}^*\,, \tag{4.53}$$

$$\overline{\gamma}_{pkt,\,i}^* \;=\; \frac{W}{R}\,\frac{h_i\, p_i^*}{a_1\, \mu_{Y_i}^* \;+\; a_0\, \sigma_{Y_i}^*}\,, \qquad \forall\; i = 1, \cdots, N\,. \tag{4.54}$$

Hence Equation (4.47) becomes

$$\frac{W}{R}\,\frac{h_i\, p_i^*}{a_1\, \mu_{Y_i}^* \;+\; a_0\, \sigma_{Y_i}^*} \;=\; \gamma_{min,\,i}\,, \qquad \forall\; i = 1, \cdots, N\,. \tag{4.55}$$

And Equation (4.48) for the expression of the optimum transmit power of user i is now:

$$p_i^* \quad = \quad \frac{R\,\gamma_{min,\,i}}{W\,h_i}\,(\,a_1\,\mu_{Y_i}^* \;+\; a_0\,\sigma_{Y_i}^*\,)\,, \qquad \forall\, i = 1, \cdots, N\,. \tag{4.56}$$

Based on the above discussions, the equilibrium condition of the power control game indicates that user i's transmit power p_i^* depends on radio bandwidth W, data bit rate R and the minimum SIR requirement $\gamma_{min,\,i}$ as well as the mobile's location h_i (location-dependent attenuation). And it is a function of the mean value of the interference received ($\mu_{Y_i}^*$) and the variance of the interference received ($\sigma_{Y_i}^*$).

Inserting the expression of $\gamma_{min,\,i}$ as defined in equation (3.3), we have the following formula for the optimum transmit power of user i as a function of packet delay requirement $D_{max,\,i}$:

$$p_i^* \quad = \quad \frac{2\,R}{W\,h_i}\,\ln\left\{\frac{(D_{max,\,i}\,R_p)^{\frac{1}{M}}}{2\,\left[(D_{max,\,i}\,R_p)^{\frac{1}{M}} - 1\right]}\right\}(\,a_1\,\mu_{Y_i}^* \;+\; a_0\,\sigma_{Y_i}^*\,), \qquad \forall\, i = 1, \cdots, N\,. \tag{4.57}$$

Based on the above result, the uplink transmit powers of individual mobile users can be allocated to optimize the utilities for all users in the system. However, we have not considered the practical aspects the uplink power control scheme, which will be discussed later in Chapter 9.

Chapter 5

Characteristics of the Utility-Based Packet-Data Power Control Strategy

In previous chapter, we established the existence and uniqueness of the Nash equilibrium solution to the non-cooperative uplink power control game, and we investigated the mathematical expression for the power control strategy based on the solution. In this chapter, we explore the characteristics and various aspects of this equilibrium solution for the game-based power control strategy.

As discussed before, the first-order condition of the game equilibrium in Equation (4.12) implicitly defines a linear solution function:

$$\beta_i^* \;=\; a_0 \,+\, a_1\,\alpha_i^*, \qquad \forall\, i = 1, \cdots, N\,.$$

where a_0 and a_1 are fixed coefficients that depend only on the transmission delay tolerance $D_{max,\,i}$ if other system and user parameters are given.

In general, as discussed previously, at the equilibrium of the power control game, we have the following equilibrium equation (4.56) as repeated below for easy reference:

$$p_i^* \;=\; \frac{R\,\gamma_{min,\,i}}{W\,h_i}\,(\,a_1\,\mu_{Y_i}^* \,+\, a_0\,\sigma_{Y_i}^*\,)\,, \qquad \forall\, i = 1, \cdots, N\,.$$

which is applicable to any given values of $\rho_i, \quad \forall\, i = 1, \cdots, N$ and $D_{max,i}, \quad \forall\, i = 1, \cdots, N$.

For analytical simplicity, we assume a condition with *equal traffic activities* from all user terminals in the system. That is, in this study, we only consider the scenarios with data packet activity probability $\rho_i = \rho, \quad \forall\, i = 1, \cdots, N$. Hence, all users in the system have the same packet-data traffic pattern in terms of the probability of being active in packet transmission.

5.1 Properties with Equal Packet Delay Requirements

In this section, we investigate the properties of the solution of the power control game when all mobile users have a uniform packet delay requirement, *i.e.*, the same D_{max}.

5.1.1 Equal Average Packet SIR and Equal Received Power

The proposition below summarizes the results followed by the proof.

Proposition 5 (Equal Average Packet SIR and Equal Received Power)
At the equilibrium of the non-cooperative power control game, given

1. *equal activity probability:* $\rho_i \;=\; \rho, \qquad \forall\, i = 1, \cdots, N,$
2. *equal average delay tolerance:* $D_{max,i} \;=\; D_{max}, \qquad \forall\, i = 1, \cdots, N.$

Then, we have

1. *equal average packet SIRs for all users:*

$$\overline{\gamma}^*_{pkt,\,i} \;=\; \gamma_{min}\,, \qquad \forall\, i = 1, \cdots, N\,; \tag{5.1}$$

2. equal received powers of all users at the base station:

$$h_i\, p_i^* \;=\; h_j\, p_j^* \;=\; P_{rec}^*\,, \qquad \forall\, i,\; j = 1, \cdots, N\,. \tag{5.2}$$

Proof. Note that from $\gamma_{min,\,i}$ definition in Equation (3.3), it is clear that equal $D_{max,\,i}$ implies equal $\gamma_{min,\,i}$. That is:

Given $D_{max,\,i} = D_{max},\ \forall\, i = 1, \cdots, N$, we have

$$\gamma_{min,\,i} \;=\; \gamma_{min}\,, \qquad \forall\, i = 1, \cdots, N\,. \tag{5.3}$$

Then the first result of the proposition follows trivially from the equation of the fixed-point solution expressed in equation (4.47).

The second result of the proposition can be derived as follows: from the linear solution function in (4.51), we can simultaneously solve the following group of equations for the relationship between user i and user j's parameters (β_i^* and β_j^*, or α_i^* and α_j^*):

$$\begin{cases} b_i^* \;=\; a_0\, \sigma_{Y_i}^* \;+\; a_1\, \mu_{Y_i}^* \\ b_j^* \;=\; a_0\, \sigma_{Y_j}^* \;+\; a_1\, \mu_{Y_j}^* \end{cases} \tag{5.4}$$

From previous discussion, when $D_{max,\,i} = D_{max},\ \forall\, i = 1, \cdots, N$, *i.e.*, $D_{max,\,i}$ is given as a specific value, there exists a linear solution function defined by two fixed parameters a_0 and a_1. Those two parameters are the same for all users since they all have equal $D_{max,\,i}$. Then from (5.4) above, we can write the following:

$$\frac{b_i^* - a_1\,\mu_{Y_i}^*}{\sigma_{Y_i}^*} \;=\; \frac{b_j^* - a_1\,\mu_{Y_j}^*}{\sigma_{Y_j}^*}\,, \qquad \forall\; i \neq j \tag{5.5}$$

$\Longrightarrow$

$$\frac{\sigma_{Y_i}^*}{\sigma_{Y_j}^*} \;=\; \frac{b_i^* - a_1\,(\,\mu_t^* - \rho\,P_{rec,\,i}^*\,)}{b_j^* - a_1\,(\,\mu_t^* - \rho\,P_{rec,\,j}^*\,)}\,, \tag{5.6}$$

where $P_{rec,\,i}^* = h_i\,p_i^*$ is the received power for user i and $P_{rec,\,j}^* = h_j\,p_j^*$ is the received power for user j, at base station receiver. And μ_t^* is defined as the mean of the *total* received power (useful power + interference + noise, all together) at base station:

$$\mu_t^* \;\triangleq\; \sum_{k=1}^{N} \rho\,h_k\,p_k^* + \sigma_i^2\,. \tag{5.7}$$

$\Longrightarrow$

$$\frac{\sigma_{Y_i}^*}{\sigma_{Y_j}^*} \;=\; \frac{P_{rec,\,i}^* - Q}{P_{rec,\,j}^* - Q}\,, \tag{5.8}$$

where Q is common element which is defined below for simplicity:

$$Q \;\triangleq\; \frac{a_1\,\mu_t^*}{\dfrac{W}{R}\,\dfrac{1}{\gamma_{min}} + a_1\,\rho}\,. \tag{5.9}$$

On the other hand, from Equation (5.4), we can also write:

$$\frac{b_i^*}{b_j^*} \;=\; \frac{P_{rec,\,i}^*}{P_{rec,\,j}^*} \;=\; \frac{\dfrac{a_0}{a_1}\sigma_{Y_i}^* + \mu_{Y_i}^*}{\dfrac{a_0}{a_1}\sigma_{Y_j}^* + \mu_{Y_j}^*} \;=\; \frac{\dfrac{a_0}{a_1}\sigma_{Y_i}^* + \mu_t^* - \rho\,P_{rec,\,i}}{\dfrac{a_0}{a_1}\sigma_{Y_j}^* + \mu_t^* - \rho\,P_{rec,\,j}}\,, \tag{5.10}$$

$\Longrightarrow$

$$\frac{P^*_{rec,\,i}}{P^*_{rec,\,j}} \;=\; \frac{\dfrac{a_0}{a_1}\sigma^*_{Y_i} + \mu^*_t}{\dfrac{a_0}{a_1}\sigma^*_{Y_j} + \mu^*_t}\,. \tag{5.11}$$

Solve Equations (5.8) and (5.11) simultaneously, we have:

$$\frac{P^*_{rec,\,i}}{P^*_{rec,\,j}}\left(\frac{a_0}{a_1}\sigma^*_{Y_j} + \mu^*_t\right) \;=\; \frac{a_0}{a_1}\left(\frac{P^*_{rec,\,i} - Q}{P^*_{rec,\,j} - Q}\right)\sigma^*_{Y_j} + \mu^*_t\,, \tag{5.12}$$

$\Longrightarrow$

$$P^*_{rec,\,i}\,P^*_{rec,\,j}\,\mu^*_t - P^*_{rec,\,i}\,Q\left(\frac{a_0}{a_1}\sigma^*_{Y_j} + \mu^*_t\right) \;=\; (P^*_{rec,\,j})^2\,\mu^*_t - P^*_{rec,\,j}\,Q\left(\frac{a_0}{a_1}\sigma^*_{Y_j} + \mu^*_t\right), \tag{5.13}$$

$\Longrightarrow$

$$P^*_{rec,\,i}\left[P^*_{rec,\,j}\,\mu^*_t - Q\left(\frac{a_0}{a_1}\sigma^*_{Y_j} + \mu^*_t\right)\right] \;=\; P^*_{rec,\,j}\left[P^*_{rec,\,j}\,\mu^*_t - Q\left(\frac{a_0}{a_1}\sigma^*_{Y_j} + \mu^*_t\right)\right]. \tag{5.14}$$

Canceling out the common factors on both sides of above equation, we can thus conclude that the received powers at base station from all users in the system are the same at the equilibrium of the game. That is,

$$P^*_{rec,\,i} \;=\; P^*_{rec,\,j} \;=\; P^*_{rec}, \qquad \forall\, i \neq j\,. \tag{5.15}$$

■

5.1.2 Equal Basic Parameters, Expected PSRs and Probabilities of Packet Delay Requirement

In this subsection, we discuss two corollaries that follow directly from **Proposition 5 (Equal Average Packet SIR and Equal Received Power)** discussed previously.

Corollary 2 (Equal Basic Parameters) *At the equilibrium of the non-cooperative power control game, given $\rho_i = \rho,\ \forall\ i$ and $D_{max,i} = D_{max},\ \forall\ i$, we have the following properties for the basic parameters of the power control game:*

$$b_i^* = b_j^*\,, \qquad \forall\, i,\, j\ \in \{1, 2 \cdots, N\}\,; \tag{5.16}$$

$$\mu_{Y_i}^* = \mu_{Y_j}^*\,, \qquad \forall\, i,\, j\ \in \{1, 2 \cdots, N\}\,; \tag{5.17}$$

$$\sigma_{Y_i}^* = \sigma_{Y_j}^*\,, \qquad \forall\, i,\, j\ \in \{1, 2 \cdots, N\}\,. \tag{5.18}$$

And consequently:

$$\alpha_i^* = \alpha_j^* = \alpha^*\,, \qquad \forall\, i,\, j\ \in \{1, 2 \cdots, N\}\,; \tag{5.19}$$

$$\beta_i^* = \beta_j^* = \beta^*\,, \qquad \forall\, i,\, j\ \in \{1, 2 \cdots, N\}\,. \tag{5.20}$$

where

$$\alpha^* = \sqrt{\frac{(N-1)\,\rho}{1-\rho}} + \frac{\sigma^2}{\sqrt{(N-1)\,\rho\,(1-\rho)}\,P_{rec}^*} \tag{5.21}$$

$$\beta^* = \frac{W}{R}\,\frac{1}{\gamma_{min}\,\sqrt{(N-1)\,\rho\,(1-\rho)}} \tag{5.22}$$

Notice that in Equation (5.21), σ^2 is the background noise power at base station receiver which is assumed to be none different from user to user. That is, $\sigma_i^2 = \sigma^2,\ \forall\ i = 1, \cdots, N$.

<u>**Proof**</u>. The proofs are generally trivial, by using the result on equal received powers of **Proposition 5 (Equal Average Packet SIR and Equal Received Power)** and the definitions of b_i, μ_{Y_i} and σ_{Y_i} in (3.6), (3.10) and (3.11), as well as the definitions of α_i and β_i in (3.33) and (3.34).

Specifically, looking at Equation (5.8), it is obvious that equal received powers implies equal variance of interference $\sigma^*_{Y_i}$. Furthermore, based on definitions of mean and variance of interference in (3.10) and (3.11), equal received powers implies equal $\mu^*_{Y_i}$ as well as equal $\sigma^*_{Y_i}$, as below:

$$\mu^*_{Y_i} \;=\; \sum_{j=1, j\neq i}^{N} \rho_j \, h_j \, p^*_j \;+\; \sigma_i^2 \;=\; \rho\,(N-1)\,P^*_{rec} \;+\; \sigma^2\,, \tag{5.23}$$

$$\sigma^{*\,2}_{Y_i} \;=\; \sum_{j=1, j\neq i}^{N} \rho_j\,(1-\rho_j)\,h_j^2\,p_j^{*\,2} \;=\; \rho\,(1-\rho)\,(N-1)\,P^{*\,2}_{rec}\,. \tag{5.24}$$

From definition of b_i (3.6), equal received powers implies equal b^*_i, as below:

$$b^*_i \;=\; \frac{W}{R}\,\frac{h_i\,p^*_i}{\gamma_{min,\,i}} \;=\; \frac{W}{R}\,\frac{P^*_{rec}}{\gamma_{min}}\,. \tag{5.25}$$

Therefore, results on α^* and β^* in (5.21) and (5.22) follow directly from above analysis, using definitions of α_i and β_i in (3.33) and (3.34).

$\blacksquare$

<u>**Corollary**</u> **3 (Equal Expected PSRs and Equal Delay Probabilities)**
At the equilibrium of the non-cooperative power control game, given $\rho_i = \rho$, $\forall\ i$ and $D_{max,\,i} = D_{max}$, $\forall\ i$, we have the following properties:

1. *equal average packet success rate (PSR) for all users:*

$$E^*[f(\gamma_i)] \;=\; E^*[f], \qquad \forall\, i = 1, \cdots, N\,; \tag{5.26}$$

2. *equal probabilistic guarantee of delay requirement for all users:*

$$Prob^*\{\, d_i \leq D_{max,\,i}\,\} \;=\; \mathcal{P}_D^*, \qquad \forall\, i = 1, \cdots, N\,. \tag{5.27}$$

Proof. Both $E^*[f(\gamma_i)]$ and $Prob^*\{\, d_i \leq D_{max,\,i}\,\}$ are functions of α_i and β_i, given ρ and D_{max}. Therefore, applying the results in **Corollary 2 (Equal Basic Parameters)**, the above results can be shown easily by looking at the formulas for $E^*[f(\gamma_i)]$ and $Prob^*\{\, d_i \leq D_{max,\,i}\,\}$ as below.

The formula for the expected or average packet success rate as in (3.21):

$$E[f(\gamma_i)] \;\approx\; \frac{2}{3}\, f\left(\overline{\gamma}_i^0\right) \;+\; \frac{1}{6}\,\left[\, f\left(\overline{\gamma}_i^1\right) \;+\; f\left(\overline{\gamma}_i^2\right)\,\right]$$

where $\overline{\gamma}_i^0$, $\overline{\gamma}_i^1$ and $\overline{\gamma}_i^2$ are again functions of α_i, β_i and $\gamma_{min,\,i}$ as in (3.35), (3.36) and (3.37).

The expression of the probabilistic guarantee of delay requirement is a function of α_i and β_i as in (3.32) and (3.40):

$$Prob^*\{\, d_i \leq D_{max,i}\} \;=\; \frac{\Phi\left(\dfrac{b_i^* - \mu_{Y_i}^*}{\sigma_{Y_i}^*}\right) \;-\; \Phi\left(-\dfrac{\mu_{Y_i}^*}{\sigma_{Y_i}^*}\right)}{1 \;-\; \Phi\left(-\dfrac{\mu_{Y_i}^*}{\sigma_{Y_i}^*}\right)} \;=\; \frac{\Phi\left(\beta_i^* - \alpha_i^*\right) \;-\; \Phi\left(-\alpha_i^*\right)}{1 \;-\; \Phi\left(-\alpha_i^*\right)}.$$

Notice that the arguments in $E^*[f(\gamma_i)]$ formula and $Prob^*\{\, d_i \le D_{max,\,i}\,\}$ expression all depend on α_i, β_i and $\gamma_{min,\,i}$. Since those parameters are the same for all users, $E^*[f(\gamma_i)]$ and $Prob^*\{\, d_i \le D_{max,\,i}\,\}$ are the same for all users in the system at the equilibrium of the power control game.

$\blacksquare$

5.2 Capacity and Engineering of Packet-Switched Data CDMA Systems

In this section, we will look at the characteristics of the power control scheme proposed in this work from the perspective of system performance. We define a *Packet-Data Capacity* for the system and investigate the properties of this capacity metric. Observations and special consideration in engineering of a packet-switched data CDMA system are also presented.

5.2.1 Capacity of CDMA for Packet-Data Services

In a single-cell packet-switched data CDMA network, we study its uplink capacity under the assumption of equal traffic activity and equal packet delay tolerance for all users in the network. With the power control strategy based on the equilibrium solution of the non-cooperative game, we can then evaluate the upper limit of the system performance by a *capacity metric*. This metric is conceptually similar to the one that is widely used in circuit-switched data CDMA systems, but technically different. The following theorem states a result of this capacity metric.

Theorem 5 (Uplink Capacity of Packet-Switched Data CDMA Systems) *Given equal traffic activity ρ and equal delay tolerance D_{max} for all users in a single-cell packet-switched data CDMA system, then the uplink capacity of the cell – the* *maximum number of active users that can be supported simultaneously to transmit data from users to base station is*

$$N \; < \; 1 \; + \; \frac{1}{\rho} \left\{ \frac{W}{R} \frac{1}{a_1 \, \gamma_{min}} \; + \; \frac{(\frac{a_0}{a_1})^2 \, (1-\rho)}{2} \right.$$
$$\left. - \; \frac{a_0}{a_1} \sqrt{(1-\rho) \; \left[\frac{W}{R} \frac{1}{a_1 \, \gamma_{min}} \; + \; \frac{(\frac{a_0}{a_1})^2 \, (1-\rho)}{4} \right]} \right\},$$
$$\forall \, \rho \; \in (0, \; 1] \; . \tag{5.28}$$

<u>Proof</u>. From **Proposition 5 (Equal Average Packet SIR and Equal Received Power)**, we have the property of equal received powers as $h_i \, p_i^* = h_j \, p_j^* = P_{rec}^* \, , \; \forall \, i, \, j$. This leads to the properties of equal values for the parameters $\mu_{Y_i}^*$, $\sigma_{Y_i}^*$ and b_i^* as discussed in (5.23), (5.24) and (5.16).

From Equation (4.54), we have

$$\begin{aligned} \overline{\gamma}_{pkt,\,i}^* \;\; &= \;\; \frac{W}{R} \frac{P_{rec}^*}{a_1 \, \mu_{Y_i}^* \; + \; a_0 \, \sigma_{Y_i}^*} \\ &= \;\; \frac{W}{R} \frac{P_{rec}^*}{a_1 \, [(N-1) \, \rho \, P_{rec}^* \; + \; \sigma_i^2] \; + \; a_0 \, \sqrt{(N-1) \, \rho \, (1-\rho)} \, P_{rec}^*} \\ &= \;\; \gamma_{min} \; , \end{aligned} \tag{5.29}$$

which leads to

$$P^*_{rec} \left\{ 1 \; - \; \frac{R}{W} \gamma_{min} \left[a_1 \, (N-1) \, \rho \; + \; a_0 \, \sqrt{(N-1) \, \rho \, (1-\rho)} \right] \right\} \;\; = \;\; \frac{R}{W} \, a_1 \, \gamma_{min} \; \sigma_i^2 \, . \tag{5.30}$$

Since received power P^*_{rec} can never be negative and RHS of above equation is a positive quantity, we thus require

$$\frac{W}{R} \; - \; \gamma_{min} \left[a_1 \, (N-1) \, \rho \; + \; a_0 \, \sqrt{(N-1) \, \rho \, (1-\rho)} \right] \;\; > \;\; 0 \, . \tag{5.31}$$

$\Longrightarrow$

$$\left[\sqrt{(N-1) \, \rho} \; + \; \frac{\frac{a_0}{a_1} \sqrt{1-\rho}}{2} \right]^2 \; - \; \frac{(\frac{a_0}{a_1})^2 \, (1-\rho)}{4} \;\; < \;\; \frac{W}{R} \, \frac{1}{a_1 \, \gamma_{min}} \, . \tag{5.32}$$

Solving the above inequality for N gives the uplink capacity expression for packet-data CDMA systems (5.28).

■

It is interesting to note that the uplink cell capacity expression in (5.28) bears general features for packet-switched data CDMA systems. The capacity expression clearly shows the effects of traffic activity ρ, packet delay tolerance D_{max} (via γ_{min}). Note that a_0 and a_1 also depend on D_{max} .

Capacity expression (5.28) is distinctively different from the *classic* capacity expression for circuit-switched data systems with constant and continuous transmissions. However, with $\rho \equiv 1$ (i.e., no burstiness, constant transmissions from all users), it reduces to the following form that is similar to *classic* capacity formula:

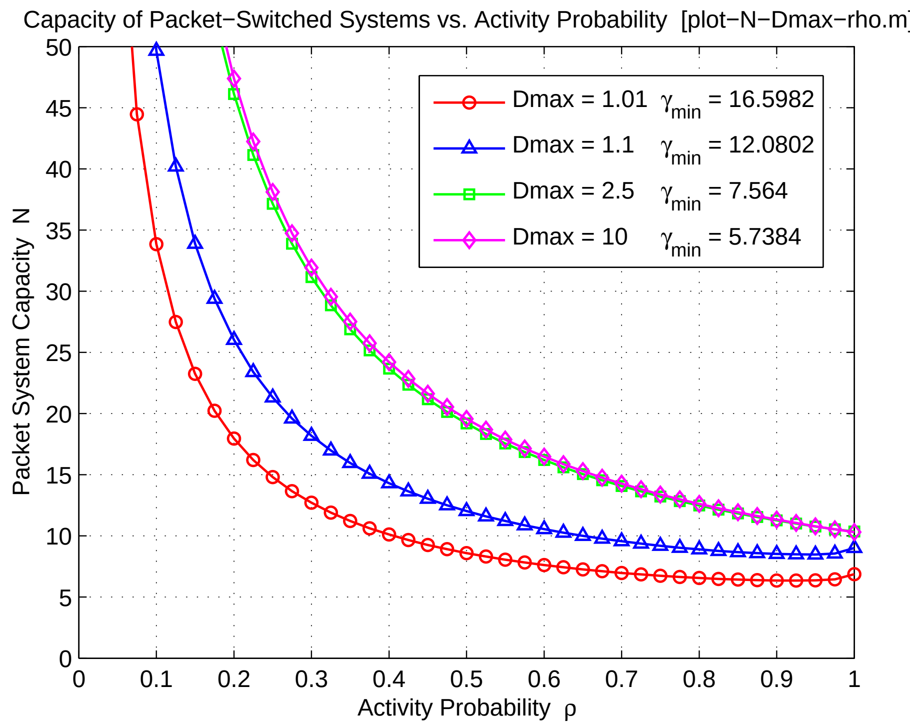

Figure 5.1: Single-cell uplink capacity of a packet-switched data CDMA system with respect to traffic activity ρ. Given $W/R = 100$ and $D_{max} = 1.01,\ 1.1,\ 2.5,\ 10\,T_p$.

$$N \;<\; 1 \;+\; \frac{W}{R}\,\frac{1}{a_1\,\gamma_{min}}\,. \tag{5.33}$$

As we know, packet-switched data networks can support many more users than their circuit-switched counterparts due to statistical multiplexing based on the bursty nature of packet traffic. Figure 5.1 plots the curves of the uplink packet-data capacity (5.28) as functions of traffic activity ρ for various packet delay tolerances D_{max}. As shown in the figure, generally speaking, in packet-switched data systems, the number of users that can be supported increases as their traffic activities reduces. Meanwhile relaxed delay requirement (larger D_{max}) also helps to increase the capacity.

However, it is very counter-intuitive to observe a *unusual* portion on the packet-data capacity curves of Figure 5.1. Notice that the lowest capacity values occur in

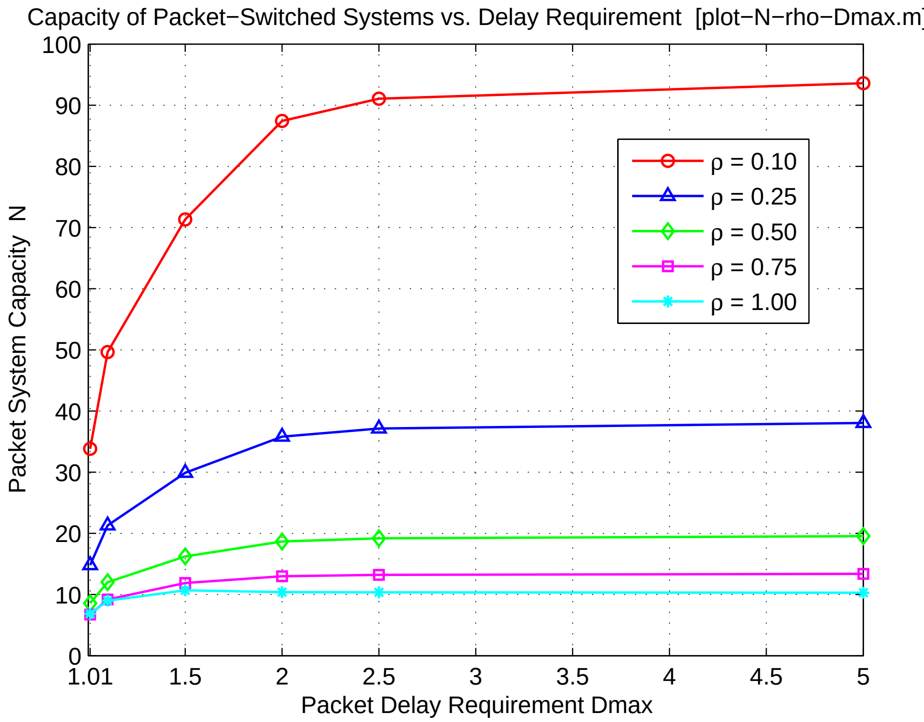

Figure 5.2: Single-cell uplink capacity of a packet-switched data CDMA system with respect to packet delay requirement D_{max}, given $W/R = 100$ and various ρ.

the high ρ neighborhood, close to $\rho = 1$, but may *not* at $\rho = 1$ as what we would usually expect. The capacities of circuit-switched systems which corresponds to the cases with $\rho = 1$ may *not* always be the worst scenario as compared to those of their packet-switched counterparts! There exist some special cases with certain values of $\rho \neq 1$ which achieve the worst uplink capacity performance for packet-switched networks, as shown in Figure 5.1.

This special characteristic of the capacity curves demonstrates a fact that it is complex to engineer the performance of a packet-switched data network in scenarios with traffic behaviors which closely resemble, but not exactly circuit-switched types of traffic. This is attributed to the variability of the interference.

Figure 5.2 shows the uplink packet-data capacity (5.28) as a function of the packet delay requirement D_{max} for various values of traffic activity ρ. From the plots, the general trending is clear that the capacity curves increase with D_{max} and decrease with ρ.

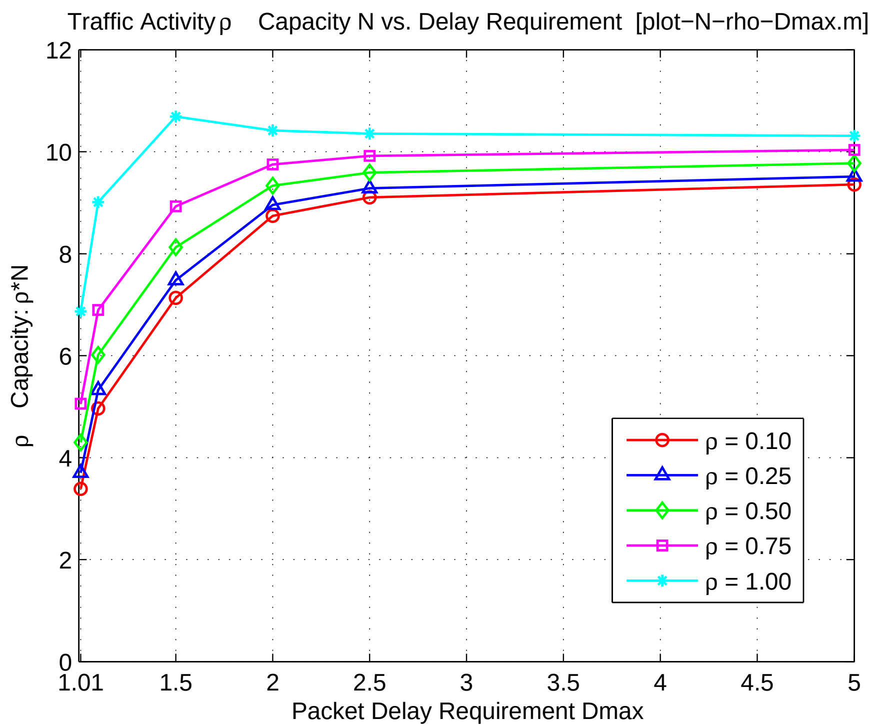

Figure 5.3: Combined effect of traffic activity ρ and system capacity N: $\rho \times N$, with respect to packet delay requirement D_{max}, given $W/R = 100$ and various ρ.

Figure 5.3 shows the uplink packet-data capacity from a different perspective by plotting $\rho \times N$ vs. packet delay requirement D_{max}. This result shows the *average* capacity of the system, or the *mean* value of capacity, by taking into account of the weighted effect of traffic activity. Comparing to the plots in Figure 5.2, generally, the trending is that the mean value of the capacity increases with D_{max} and also increases with ρ. There is one exception to the trending with respect to D_{max} in case of $\rho = 1$ which has a slight peak up around $D_{max} = 1.5\, T_p$. With $\rho = 1$, what we are actually plotting is the formula below:

$$N \; < \; 1 \; + \; \frac{W}{R} \frac{1}{a_1 \, \gamma_{min}} \, . \tag{5.34}$$

This corresponds to the scenario with no traffic burstiness and all users in the system transmit packets to their base station constantly.

5.2.2 On the Engineering of Packet-Switched Data CDMA Systems

As discussed above, because of the variance of the interference, special care has to be taken in the engineering of a packet-switched data CDMA system. We continue and elaborate on this point with the discussion on the received powers of the packet-switched data system in the following corollary:

Corollary 4 (Received Power at Equilibrium) *In the non-cooperative uplink power control game, assuming equal traffic activity and equal packet delay tolerance for all users in a single-cell CDMA network, the received power at base station from any user at equilibrium of the game is*

$$P^{*}_{rec} \;=\; \frac{\sigma_i^2}{\dfrac{W}{R}\,\dfrac{1}{a_1\,\gamma_{min}} - \left[(N-1)\,\rho + \dfrac{a_0}{a_1}\sqrt{(N-1)\,\rho\,(1-\rho)}\right]}\,. \qquad (5.35)$$

Proof. The proof of the above corollary is trivial. It follows directly from Equation (5.30) in the proof of **Theorem 5 (Uplink Capacity of Packet-Switched Data CDMA Systems)**.

■

Generally speaking, to maintain a constant target SIR in a cellular wireless network, as the total interference increases, received powers have to be increased to overcome the increased interference. However, from our study, there is an interesting point to observe, which is contradictory to the general belief. This point is described below.

Figure 5.4 shows three curves of the received power P_{rec}^* at equilibrium with respect to ρ. As we know, a bigger ρ means a higher activity of packet traffic which indicates a stronger level of interference. In Figure 5.4, the first and third curves go up as ρ increases. The first curve shoots up around $\rho = 0.5$ because the capacity is approached earlier due to the tighter packet delay requirement (refer to the capacity curves in Figure 5.1, with $D_{max} = 1.01\,T_p$ and $N = 8$).

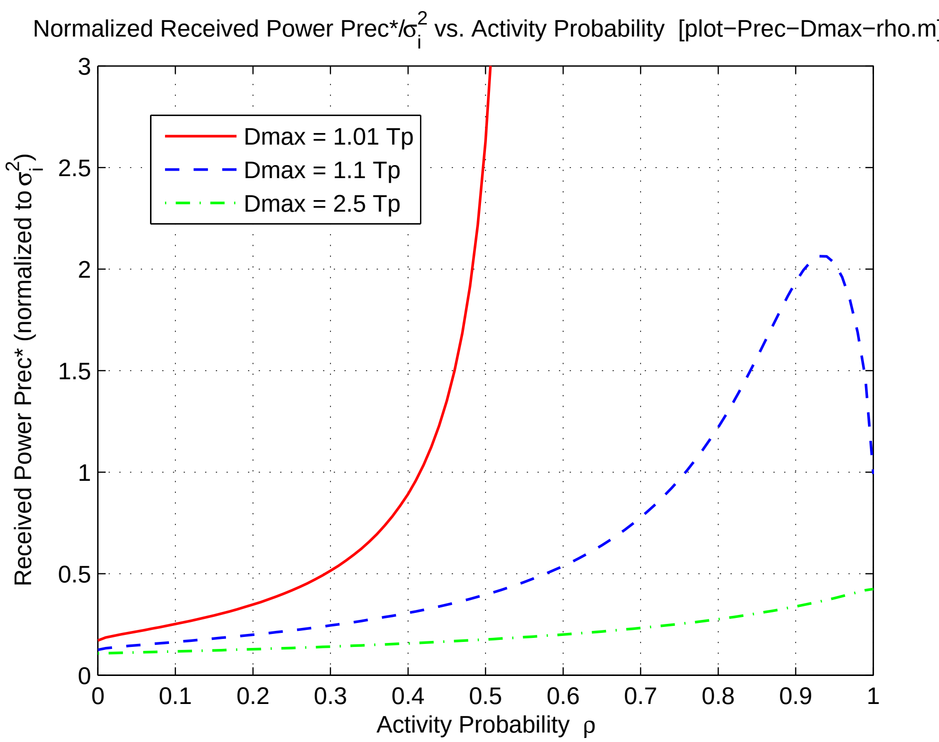

Figure 5.4: Received power P_{rec}^* normalized to background noise power σ_i^2 with respect to traffic activity ρ. Given $W/R = 100$, $N = 8$, and $D_{max} = 1.01,\ 1.1,\ 2.5\,T_p$.

The second curve demonstrates an interesting fact about the received power at equilibrium P_{rec}^* in packet-switched data systems. The fact is that the highest received power may not necessarily occur at $\rho = 1$ which was usually taken as the highest interference state, because all users are actively on, busy transmitting packets. In packet-switched data systems, the interference is random in nature due to the burstiness of traffic sources. At $\rho = 1$, the mean of the random interference is highest, but the variance of interference is actually zero. Hence, the combined effect of mean and variance produces this special phenomenon on the second curve in Figure 5.4 with P^*_{rec} peaking up around $\rho = 0.93$ and falling back afterward as ρ increases.

Generally we know that large mean of the random interference is harmful, but sometimes it is actually the *variability*, not the mean of the random interference that *critically impacts* the system. Particularly this phenomenon happens when the total received power approaches the limit up-to which the system capacity can barely handle, as is shown in Figure 5.4 (also refer to the capacity curves in Figure 5.1, with $D_{max} = 1.1\,T_p$ and $N = 8$).

In conclusion, the results of this Section clearly demonstrates that special care has to be taken in the design and engineering of packet-switched wireless data systems. Technically, performance and traffic engineering for a packet-switched data network should carefully consider not only the effect of the *average strength* or the *mean* of interference, but also the effect of the *variability* or the *variance* of interference. With those considerations taken care of in the system design, the system can then be engineered to operate even at the worst scenario of interference if required. The critical impact of interference variability together with interference strength is one of the key differences between the engineering of a wireless circuit-switched data system and that of a wireless packet-switched data system.

5.3 Properties with Unequal Packet Delay Requirements

In previous sections, we studied the characteristics of the game-based power control strategy under a specific assumption of equal packet delay requirements.

More generally in this section, we will analyze scenarios with unequal packet delay requirements from different individual users or groups of users. As we will show shortly, at equilibrium of the non-cooperative power control game, the power control scheme still converges to a unique Nash equilibrium. However, the equilibrium solution will be associated with unequal parameters for users with unequal delay requirements.

The main properties of the game-based power control scheme given unequal delay requirements are summarized in the following proposition:

Proposition 6 (Properties with Unequal Packet Delay Requirements) *At equilibrium of the non-cooperative power control game, given that*

1. *equal activity probability:* $\rho_i \;=\; \rho, \quad \forall\, i = 1, \cdots, N,$
2. *users are classified into different groups with unequal packet delay requirements. But within the same group, the packet delay requirements of users are the same:*

$$D_{max,i} \;\neq\; D_{max,j} \tag{5.36}$$

if user i *and user* j *belong to two different groups with packet delay requirements represented by* $D_{max,i}$ *and* $D_{max,j}$ *respectively.*

Then, we have the following properties:

1. *unequal average packet SIRs for users from different groups. But within the same group, the average packet SIRs of users are equal:*

$$\overline{\gamma}^{*}_{pkt,i} \;=\; \gamma_{min,i}\,, \quad \forall\, i = 1, \cdots, N\,; \tag{5.37}$$

2. *unequal received powers at base station from users belonging to different groups. But the received powers from users within the same group are equal. The relationship between the received powers from two users belonging to different groups is*

$$\frac{P^*_{rec,\,i}}{\gamma_{min,\,i}\,(a^i_1\,\mu^*_{Y_i} + a^i_0\,\sigma^*_{Y_i})} = \frac{P^*_{rec,\,j}}{\gamma_{min,\,j}\,(a^j_1\,\mu^*_{Y_j} + a^j_0\,\sigma^*_{Y_j})} \quad \dagger \tag{5.38}$$

where user i and user j are from two groups with different values of D_{max}, and $P_{rec,\,i} \neq P_{rec,\,j}$.

3. *unequal average packet success rate (PSR) for users belonging to different groups. But within the same group the average PSRs are equal:*

$$E^*[f(\gamma_i)] \;\neq\; E^*[f(\gamma_j)] \tag{5.39}$$

4. *unequal probabilities of packet delay requirements for users belonging to different groups. But within the same group the probabilistic guarantees are equal:*

$$Prob^*\{\, d_i \leq D_{max,\,i}\,\} \;\neq\; Prob^*\{\, d_j \leq D_{max,\,j}\,\} \tag{5.40}$$

Proof. The proof of the first point is trivial, because the power control game converges with the Average Packet SIR $\overline{\gamma}^*_{pkt,\,i}$ of each user equal to the user's minimum SIR requirement $\gamma_{min,\,i}$ as in (4.47). Specifying different $D_{max,\,i}$ to different users, their corresponding $\gamma_{min,\,i}$ will be different. Hence their $\overline{\gamma}^*_{pkt,\,i}$ will be different, depending on the respective values of $D_{max,\,i}$.

---

†Note: a_0 and a_1 are also functions of $D_{max,\,i}$. Thus, a^i_0 and a^i_1 denote the values of a_0 and a_1 when delay requirement $D_{max,\,i}$ is given, while a^j_0 and a^j_1 denote the values of a_0 and a_1 when $D_{max,\,j}$ is given.

For the second point, the relationship between received powers from different groups in Equation (5.38) can be easily shown below. Using the equilibrium Equation (4.56), two received powers from two users (user i and user j) in two groups with different D_{max} can be found:

$$P^*_{rec,i} \;=\; \frac{R}{W}\,\gamma_{min,\,i}\;(\,a_1^i\,\mu^*_{Y_i}\;+\;a_0^i\,\sigma^*_{Y_i}\,)\,, \qquad \forall\, i\,\in\,[\,1,\cdots,N\,|\,D_{max,\,i}\,]\,, \tag{5.41}$$

$$P^*_{rec,j} \;=\; \frac{R}{W}\,\gamma_{min,\,j}\;(\,a_1^j\,\mu^*_{Y_j}\;+\;a_0^j\,\sigma^*_{Y_j}\,)\,, \qquad \forall\, j\,\in\,[\,1,\cdots,N\,|\,D_{max,\,j}\,]\,. \tag{5.42}$$

Taking the ratio of those two received powers above, we obtain their relationship as in (5.38). Because γ_{min}, a_0 and a_1 all depend on D_{max}, a different D_{max} leads to a different received power.

If two users are within the same group, then they will share the same D_{max} and the effects of the combined interference from outside the group are always the same to those two users. Therefore, using the same logic and analysis as in the proof of **Proposition 5 (Equal Average Packet SIRs and Equal Received Powers)** for the equal received powers, it can be shown that the received powers from users within a same group are equal.

The proof for the third and fourth points are straight-forward based on the characteristics of the received powers in unequal D_{max} scenarios. As we know from the definitions and previous discussions, all the main parameters b_i^*, $\mu^*_{Y_i}$ and $\sigma^*_{Y_i}$ and hence α_i^* and β_i^* are functions of the received powers as in (5.23), (5.24) and (5.16). Because of the property of the received powers in unequal D_{max} scenarios as discussed above, those parameters mentioned here possess a same general property which is "unequal if from different groups but equal if within a same group". This property makes $E^*[f(\gamma_i)]$ and $Prob^*\{\,d_i \leq D_{max,\,i}\,\}$ behave in the same way as the received power does, since they are functions of b_i^*, $\mu^*_{Y_i}$ and $\sigma^*_{Y_i}$ or α_i^* and β_i^*.

■

From **Proposition 6 (Properties with Unequal Packet Delay Requirements)** and its proof above, we see that there exists a common property for the game-based power control scheme when users are classified into groups with different delay QoS requirements represented by $D_{max,\,i}$.

This common property is that the main quantities such as received powers $P^*_{rec,\,i}$, average PSRs $E^*[f(\gamma_i)]$ and probabilistic guarantee of delay requirements $Prob^*\{\, d_i \le D_{max,\,i}\,\}$ all satisfy the statement *"unequal if from different groups but equal if within a same group"*. This is an important property that is very useful in understanding the behaviors of the power control scheme and in explaining the results of the simulation studies performed on the power control scheme in next chapters.

5.4 Packet-Switched Systems vs. Circuit-Switched Systems

In previous sections, we studied the non-cooperative game-based uplink power control scheme for cellular packet-switched data CDMA systems in a single-cell environment. In this section, we will do a comparison study on how these results relate to those from circuit-switched counterpart.

We will show that, although the studies are meant for packet-switched data CDMA systems, the main results produced in the analysis and investigation are general and also applicable to circuit-switched data CDMA systems. Because with the activity probability $\rho \equiv 1$, the assumption of traffic burstiness is gone, and the whole study returns to the classical scenario with continuous and constant transmissions as in circuit-switched data systems.

Specifically, with $\rho \equiv 1$ (no burstiness, continuous and constant transmissions from all users in the system), then we have the following changes for the relevant quantities and parameters in the study:

$$\mu_{Y_i} \;=\; E[Y_i] \;=\; \rho \sum_{j=1, j\neq i}^{N} h_j\, p_j \;+\; \sigma_i^2 \;=\; \sum_{j=1, j\neq i}^{N} h_j\, p_j \;+\; \sigma_i^2\,, \tag{5.43}$$

$$\sigma_{Y_i}^2 \;=\; Var[Y_i] \;=\; \rho\,(1-\rho) \sum_{j=1, j\neq i}^{N} h_j^2\, p_j^2 \;=\; 0. \tag{5.44}$$

and

$$\overline{\gamma}_i^0 \;=\; \frac{W}{R}\,\frac{h_i\, p_i}{\mu_{Y_i}} \;=\; \frac{W}{R}\,\frac{h_i\, p_i}{\sum_{j=1, j\neq i}^{N} h_j\, p_j \;+\; \sigma_i^2} \;=\; \Gamma_i \tag{5.45}$$

$$\overline{\gamma}_i^1 \;=\; \frac{W}{R}\,\frac{h_i\, p_i}{\mu_{Y_i} \;+\; \sqrt{3}\,\sigma_{Y_i}} \;=\; \frac{W}{R}\,\frac{h_i\, p_i}{\sum_{j=1, j\neq i}^{N} h_j\, p_j \;+\; \sigma_i^2} \;=\; \Gamma_i \tag{5.46}$$

$$\overline{\gamma}_i^2 \;=\; \frac{W}{R}\,\frac{h_i\, p_i}{\mu_{Y_i} \;-\; \sqrt{3}\,\sigma_{Y_i}} \;=\; \frac{W}{R}\,\frac{h_i\, p_i}{\sum_{j=1, j\neq i}^{N} h_j\, p_j \;+\; \sigma_i^2} \;=\; \Gamma_i \tag{5.47}$$

From above results, Holtzman's Approximation Formula introduced in Chapter 3 (3.21) now reduces to the following form:

$$E[f(\gamma_i)] \;=\; \frac{2}{3}\, f\left(\overline{\gamma}_i^0\right) \;+\; \frac{1}{6}\,\left[\, f\left(\overline{\gamma}_i^1\right) \;+\; f\left(\overline{\gamma}_i^2\right) \right] \;=\; f(\Gamma_i). \tag{5.48}$$

which describes exactly the packet success rate (PSR) of circuit-switched data systems with a *deterministic* SIR Γ_i.

With $\rho \rightarrow 1$ assumption, the probabilistic guarantee of packet delay requirement as expressed in Equation (3.40) reduces to:

$$\lim_{\rho \to 1} \; Prob\{\, d_i \leq D_{max,\,i} \,\} \;\; = \;\; \lim_{\rho \to 1} \; \frac{\Phi\,(\beta_i - \alpha_i) \; - \; \Phi\,(-\alpha_i)}{1 \; - \; \Phi\,(-\alpha_i)} \quad \longrightarrow \quad 1\,, \tag{5.49}$$

because as $\rho \to 1$, both $\alpha_i \to \infty$ and $\beta_i \to \infty$, and it can be shown easily that $\lim_{\rho \to 1} (b_i - \mu_{Y_i}) > 0$. Therefore, we have

$$\lim_{\rho \to 1} \, (\beta_i - \alpha_i) \;\; = \;\; \lim_{\rho \to 1} \, \left(\frac{b_i - \mu_{Y_i}}{\sigma_{Y_i}} \right) \;\; \longrightarrow \;\; \infty\,. \tag{5.50}$$

Using above results, Limit (5.49) can be shown easily.

The physical meaning of the above result in (5.49) is that, as the randomness of interference disappears, the *probabilistic* guarantee of delay requirement becomes a *deterministic* guarantee with probability of *one*.

Therefore, with $\rho \to 1$, our packet-data utility model proposed in this study (3.39) reduces to the circuit-data utility model (2.3) introduced in the previous studies as in [13, 38, 39, 40] and the improved version in (2.8) in Chapter 2:

$$u_i(p_i,\, \mathbf{p}_{-i}) \;\; = \;\; \frac{L\,R}{M\,p_i} \left[\frac{f(\Gamma_i) - f(0)}{1 - f(0)} \right] \qquad \text{bits/Joule}\,. \tag{5.51}$$

Besides the uplink cell capacity formula for packet-switched data systems discussed in **Theorem 5 (Uplink Capacity of Packet-Switched Data CDMA Systems)**, other definitions and formulas can also be shown that with $\rho \equiv 1$, we return to the expressions used in circuit-switched data systems as in [13, 38, 39, 40]. We summarize those results below:

1. The *Average Packet SIR* defined in (4.54):

$$\overline{\gamma}^{*}_{pkt,\,i} \;=\; \frac{W}{R}\,\frac{h_i\,p_i^*}{a_1\,\mu^*_{Y_i} \;+\; a_0\,\sigma^*_{Y_i}} \tag{5.52}$$

$\Longrightarrow$

$$a_1 \times \overline{\gamma}^{*}_{pkt,\,i}\,\Bigg|_{\rho\,=\,1} \;=\; \Gamma_i^* \;=\; \frac{W}{R}\,\frac{h_i\,p_i^*}{\sum_{j=1,j\neq i}^{N} h_j\,p_j^* \;+\; \sigma_i^2}\,. \tag{5.53}$$

2. The uplink cell capacity formula for packet-switched data CDMA systems as defined in (5.28):

$$N \;<\; 1 \;+\; \frac{1}{\rho}\left\{\frac{W}{R}\,\frac{1}{a_1\,\gamma_{min}} \;+\; \frac{(\frac{a_0}{a_1})^2\,(1-\rho)}{2} \right.$$
$$\left. -\;\frac{a_0}{a_1}\sqrt{(1-\rho)\,\left[\frac{W}{R}\,\frac{1}{a_1\,\gamma_{min}} \;+\; \frac{(\frac{a_0}{a_1})^2\,(1-\rho)}{4}\right]}\;\right\} \tag{5.54}$$

$\Longrightarrow$

$$N\,\Bigg|_{\rho\,=\,1} \;\leq\; 1 \;+\; \frac{W}{R}\,\frac{1}{a_1\,\gamma_{min}}\,. \tag{5.55}$$

3. The received power formula in (5.35):

$$P^*_{rec} \;=\; \frac{\sigma_i^2}{\frac{W}{R}\,\frac{1}{a_1\,\gamma_{min}} \;-\; \left[(N-1)\,\rho \;+\; \frac{a_0}{a_1}\sqrt{(N-1)\,\rho\,(1-\rho)}\right]} \tag{5.56}$$

$\Longrightarrow$

$$P^*_{rec}\,\Bigg|_{\rho\,=\,1} \;=\; \frac{\sigma_i^2}{\frac{W}{R}\,\frac{1}{a_1\,\gamma_{min}} \;-\; (N-1)}\,. \tag{5.57}$$

Notice that in the above expressions a_1 is a parameter which associates with the SIRs (such as $\overline{\gamma}^*_{pkt,\,i},\ \gamma_{min,\,i}$).

For power control game in circuit-switched data CDMA networks studied in [13, 38, 39, 40] with utility model defined in (2.3), at equilibrium of the power control game, as we know, the SIR converges to a common target value – a specific fixed number:

$$\Gamma_i^* \;=\; 12.42, \qquad \forall\, i = 1, \cdots, N\,.\ ^{\dagger} \tag{5.58}$$

With the same system parameters, let us consider the improved version of the utility function as in (2.8) which is adopted in this study. At equilibrium of the power control game, the SIR converges to a number that is different from above and can be found as below:

$$\Gamma_i^* \;=\; 10.745, \qquad \forall\, i = 1, \cdots, N\,. \tag{5.59}$$

This target SIR is lower than the one with "Efficiency Function" based utility model. Apparently, the improved version of the utility model helps reduce the common target SIR from 12.42 to 10.745 for all users in the system. This reduction in target SIR translates directly to savings of unnecessary transmit powers, thus extending battery life for the mobile terminals.

For packet-switched data systems, at equilibrium of the power control game, we have Equation (4.47) which says

$$\overline{\gamma}^*_{pkt,\,i} \;=\; \gamma_{min,\,i}, \quad \forall\, i = 1, \cdots, N\,.$$

† Result obtained when Efficiency Function is assumed in place of PSR in the utility modeling.

At $\rho = 1$, from Equation (5.53), we have the following relationship:

$$a_1 \times \overline{\gamma}^*_{pkt,\,i} \;=\; a_1 \times \gamma_{min,\,i} \;=\; \Gamma^*_i \;=\; 10.745, \quad \forall\, i = 1, \cdots, N\,.^{\ddagger} \tag{5.60}$$

Again, the above equation shows that as $\rho \to 1$, the common target SIR of the game-based power control for packet-switched data systems is exactly the same as that of game-based power control for circuit-switched data systems.

In conclusion, the above analysis indicates that when $\rho \to 1$, meaning that traffic activities for all users approach the "always-on" packet transmission, the power control scheme for packet-switched data CDMA systems studied here converges to that for circuit-switched data CDMA systems as presented in previous research work in [13, 38, 39, 40].

‡ Since the improved utility model is adopted in this research, target SIR $\Gamma^*_i = 10.745$ for circuit-switched data network is used in comparison with that for packet-switched data network.

Chapter 6

Simulation Studies

In prior chapters, we employed both analytical and numerical approaches to study the non-cooperative game based power control scheme for packet-switched wireless data CDMA networks. With the help of several approximation methodologies, we were able to achieve the closed-form solution for the utility-maximization problem formulated as a non-cooperative uplink power control game.

In this chapter, we will study the utility-maximizing problem further, not analytically, but through computer simulations. We will use the results from simulation studies to verify and to reinforce and the results from the prior analytical studies. In addition, analytical and simulation studies will complement each other to give us a more complete picture of the utility-based uplink power control problem.

According to previous analysis, a unique equilibrium solution to the noncooperative uplink power control game exists. We will simulate the power control game by computer programs and find the equilibrium solution of the game through iteration algorithms. In addition, we will explore various characteristics of the equilibrium solution of the game under different traffic and load conditions. Particularly we will examine SIR, transmit powers and user utilities at equilibrium of the game with respect to traffic activity probability, average packet delay tolerance and user's distances to their serving base station.

6.1 Simulation Model and Approach

We assume a single-cell cellular CDMA network environment in the simulation. System parameters that we use are similar to IS-95 CDMA systems [101] in PCS band with carrier frequency $f_c = 1.9$ GHz, system bandwidth $W = 1$ MHz, data rate $R = 10$ kbps, packet length $M = 80$ bits with payload $L = 64$ bits.

Antenna height of base station is assumed as $H_{BS} = 100$ m, and mobile antenna height $H_{MS} = 3$ m. Modulation scheme adopted is non-coherent binary FSK (BFSK). Hata's propagation model [102] for urban environment is used to simulate the signal propagation loss between mobiles and their serving base station (BS). In the simulated system, mobile users are distributed at various locations with different distances away from their serving BS.

Table 6.1 summarizes specific values of simulation parameters for easy reference.

With system parameters given in Table 6.1, for a circuit-switched data CDMA system, if we design the uplink power control scheme based on the utility model as defined in (2.3), the scheme will converge to common target SIR $\Gamma_i^* = 12.24$ [13, 38]. Then the single-cell capacity of the CDMA system is $N = 9$ users . That is, the system can support the maximum of 9 users simultaneously.

If, instead, the improved version of utility model based on ETPR as in (2.8) is used in the design, the power control scheme will converge a different common target SIR $\Gamma_i^* = 10.745$. This was also discussed previously in Chapter 5, Equation (5.59). In this case, the single-cell capacity of the CDMA system increases to $N = 10$ users due to the lower value of Γ_i^*. Scenarios with $N = 10$ users are included in the simulation studies.

For a packet-switched data system, the capacity is defined differently (5.28). Because of the random on-off transmissions from user terminals, the packet-switched data system can support many more number of users in a system, given the same

Table 6.1: List of simulation parameters.

System model	Single-cell CDMA (IS-95)
Carrier frequency	$f_c = 1.9$ GHz (PCS band)
Bandwidth of wireless channel	$W = 1$ MHz
Number of users in system	$N = 10,\ 20$ users (two scenarios)
Number of bits in a data packet	$M = 80$ bits
Payload bits in a data packet	$L = 64$ bits
Bit rate of wireless channel	$R = 10^4$ bits/sec
Chip rate	$R_c = 10^6$ chips/sec
Packet rate of wireless channel	$R_p = 125$ packets/sec
Packet length in time	$T_p = 8$ msec
Noise power at user i's BS receiver	$\sigma_i^2 = 5 \times 10^{-15}$ Watts
Base station antenna height	$H_{BS} = 100$ m
Mobile antenna height	$H_{MS} = 3$ m
Modulation	Non-Coherent Binary FSK
Propagation model	Hata model in urban environment

system parameters specified above. In the simulation studies, we assume that the total number of users in the system $N = 20$ users.

Simulation studies are conducted under two user loading conditions with $N = 10$ and $N = 20$ number of users in the system. The results of both scenarios are compared and discussed.

In the simulation of the game-based power control scheme, given a set of initial values for the transmit powers, each user in the system takes turns to modify its transmit power with a goal to maximize its own utility. Newton's method or Newton-Raphson method in numerical analysis, nonlinear programming and optimization [109, 113] is utilized for iterative updates on transmit powers of mobile users in the system.

Newton's method is basically a root-finding iterative algorithm using the derivative of a function. In case of this study, the root of $u_i'(\,p_i,\;\mathbf{p}_{-i}\,) = 0$ needs to be located, which corresponds to the maximum value of utility $u_i(\,p_i,\;\mathbf{p}_{-i}\,)$. The algorithm for Newton's method in context of this study is expressed as below:

$$p_i^{(m+1)} \;=\; p_i^{(m)} - \frac{u_i'(\,p_i^{(m)},\;\mathbf{p}_{-i}^{(m)}\,)}{u_i''(\,p_i^{(m)},\;\mathbf{p}_{-i}^{(m)}\,)}\,. \quad \dagger \tag{6.1}$$

Obviously, transmit power for user i at step $m+1$ depends on the result from the computation involving quantities p_i, the first and the second derivatives of the utility function at step m. All users update their transmit powers according to the iterative algorithm of Newton's method step-by-step to achieve the equilibrium of the power control game represented by condition $u_i'(\,p_i,\;\mathbf{p}_{-i}\,) = 0$. Due to the uniqueness of the equilibrium as proved previously, if the equilibrium is reached, then the only optimum solution is found.

When the power control game converges to its equilibrium, a set of optimum transmit powers $\{\,p_i^*,\;\forall\, i = 1, 2, \cdots, N\,\}$ are achieved while utilities for individual users are maximized. We know from previous discussions that p_i^* at equilibrium is a function of the mean and variance of its interference and background AWGN noise (4.56). The SIR at equilibrium for user i is evaluated using the following SIR formula:

$$\gamma_i^* \;=\; \frac{W}{R}\,\frac{h_i\,p_i^*}{\sum_{j=1, j\neq i}^{N} h_j\,p_j^* \;+\; \sigma_i^2}\,, \qquad \forall\, i = 1, 2, \cdots, N\,. \tag{6.2}$$

† Note: m denotes the iteration index of the power updates.

In the simulation studies, for simplicity, we assume that the activity probabilities of traffic sources are the same, i.e., $\rho_i = \rho,\ \forall\, i = [1, 2, \cdots, N]$.

Four aspects of the non-cooperative power control game are focused through the simulation studies. They are:

1. The effects of traffic activity probability ρ on the SIR ${\gamma_i}^*$, the expected PSR $E^*[f(\gamma_i)]$, the probability of delay requirement $Prob^*\{\, d_i \,\leq\, D_{max,\,i}\,\}$, the transmit powers and user utilities at equilibrium of the game;

2. The effects of average packet delay tolerance $D_{max,\,i}$ on ${\gamma_i}^*$, the expected PSR $E^*[f(\gamma_i)]$, $Prob^*\{\, d_i \,\leq\, D_{max,\,i}\,\}$, the transmit powers and user utilities at equilibrium of the game;

3. The effects of the distance between mobile users and their base station on users' transmit powers and utilities at equilibrium of the game;

4. The effects of *unequal* packet delay tolerance $D_{max,\,i}$ on ${\gamma_i}^*$, transmit powers and user utilities at equilibrium of the game.

6.2 Simulation Results and Discussions

In this section, we present the results of simulation studies by plotting the SIR, expected PSR, probability of delay requirement, transmit powers and user utilities against user parameters such as the packet activity probability ρ, the packet delay tolerance D_{max} as well as users' distance to their basestation. Simulations of the power control game are run for each given set of system and user parameters. At the convergence of the power control game to its equilibrium, numerical data about SIR, expected PSR, probability of delay requirement, transmit power and utility for each individual user are collected, formatted and plotted. We will discuss the results of simulation studies in those plots, and provide the physical interpretations of those results whenever possible.

6.2.1 Effect of ρ on Equilibrium SIR, Transmit Powers and Utilities

For two user population scenarios of $N = 10$ and $N = 20$, simulation results shown in Figure 6.1 is the SIR at equilibrium of the power control games with respect to the packet traffic activity probability ρ with a given delay tolerance ($D_{max} = 2.5\, T_p$). As shown in the figure, the SIR at equilibrium is an increasing function of ρ. Obviously, this is because the interference is increased as traffic activity intensifies as signified by a high value of ρ. Hence, users have to increment their transmit powers to fight more interference in an attempt to maintain their target SIR, which clearly results in a higher SIR at equilibrium.

For a same packet activity probability ρ, the equilibrium SIR for 20 user scenario is much lower than that for 10 user scenario, again, because more interference is introduced when user population is increased.

Notice that in those plots for simulations with 20 user scenarios, results are available only roughly between $0 < \rho < 0.5$. This is due to the fact that the system approaches its *capacity limit* ($C = 10$ users) as $\rho \to 0.5$ in a system with 20 users. In actual simulations, the attempt to run the power control game fails as the value of ρ approaches 0.5. Clearly, the power control algorithm does not converges in those scenarios.

The capacity limit for a single-cell packet-switched data CDMA network has been discussed in analytical studies in previous chapter, as shown in **Theorem 5 (Uplink Capacity of Packet-Switched Data CDMA Systems)**. Actually, given the number of users N and packet delay tolerance D_{max}, the point of capacity

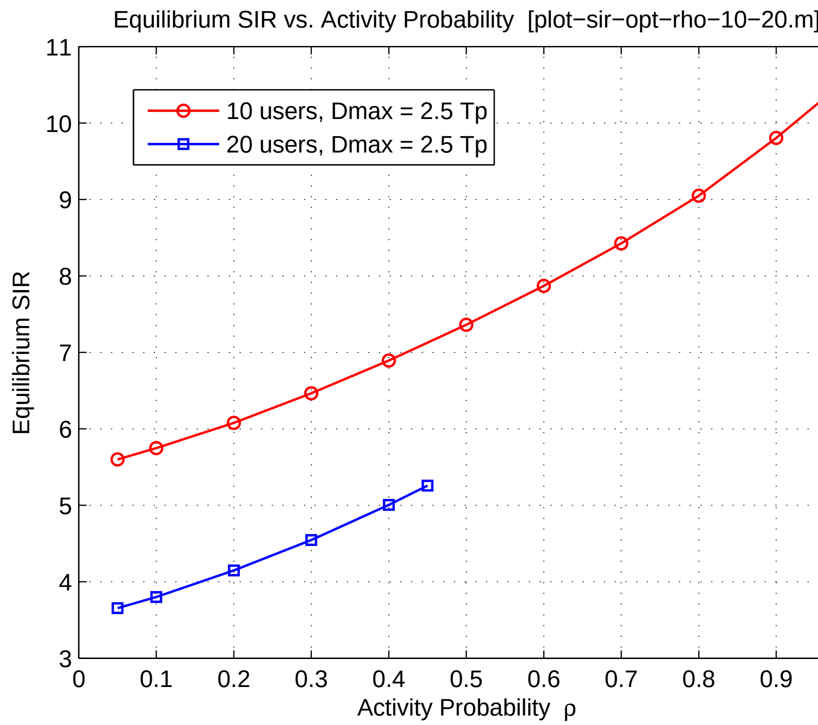

Figure 6.1: SIR γ^* at equilibrium vs. activity probability ρ, given $D_{max} = 2.5\,T_p$.

limit can be easily identified on one of the curves of packet-data capacity plots in Figure 5.1. For example, given $N = 20$ and $D_{max} = 2.5\,T_p$, it is clear immediately that the system has to operate with $\rho < 0.5$ to be within the limit of capacity.

In Figures 6.2 and 6.3, the expected PSR and the probability of delay requirement at equilibrium are plotted with respect to activity probability ρ, for both $N = 10$ and $N = 20$ user population scenarios. Observe that the variations for both quantities with the change of ρ are very limited. But the patterns of change are interesting – it appears that the changes of $E^*[f(\gamma_i)]$ and $prob\{d_i \leq D_{max,i}\}$ are against each other. For scenarios with $N = 10$ users, it seems that both $E^*[f(\gamma_i)]$ and $prob\{d_i \leq D_{max,i}\}$ are symmetrical about $\rho = 0.5$.

In Figures 6.3, for the same ρ, the probability of delay requirement at equilibrium for 20 user scenario is lower than that for 10 user scenario due to more

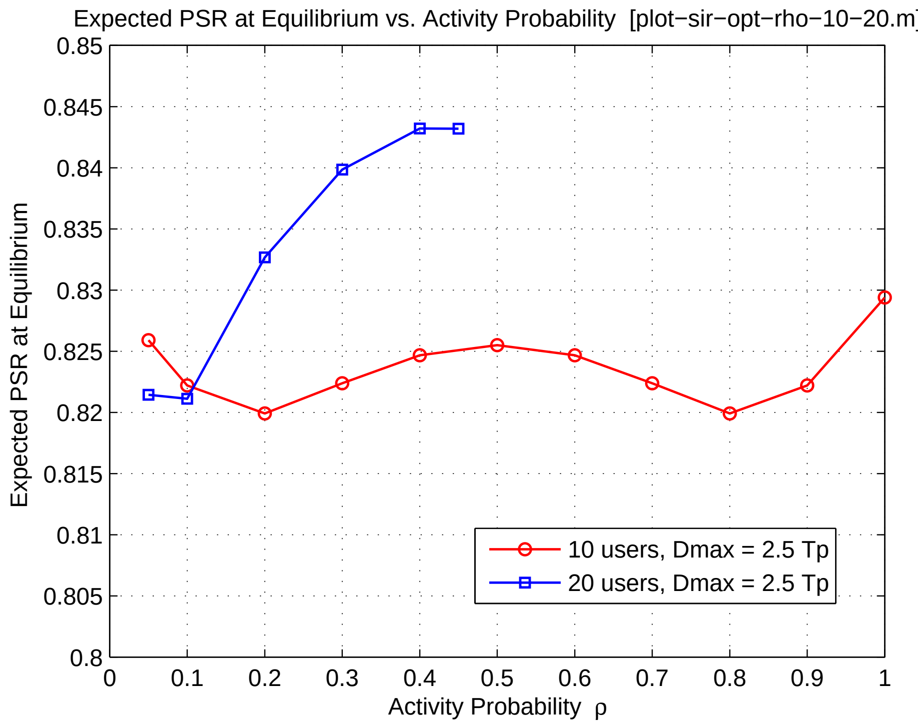

Figure 6.2: Expected PSR at equilibrium $E^*[f(\gamma_i)]$ vs. ρ, given $D_{max} = 2.5\,T_p$.

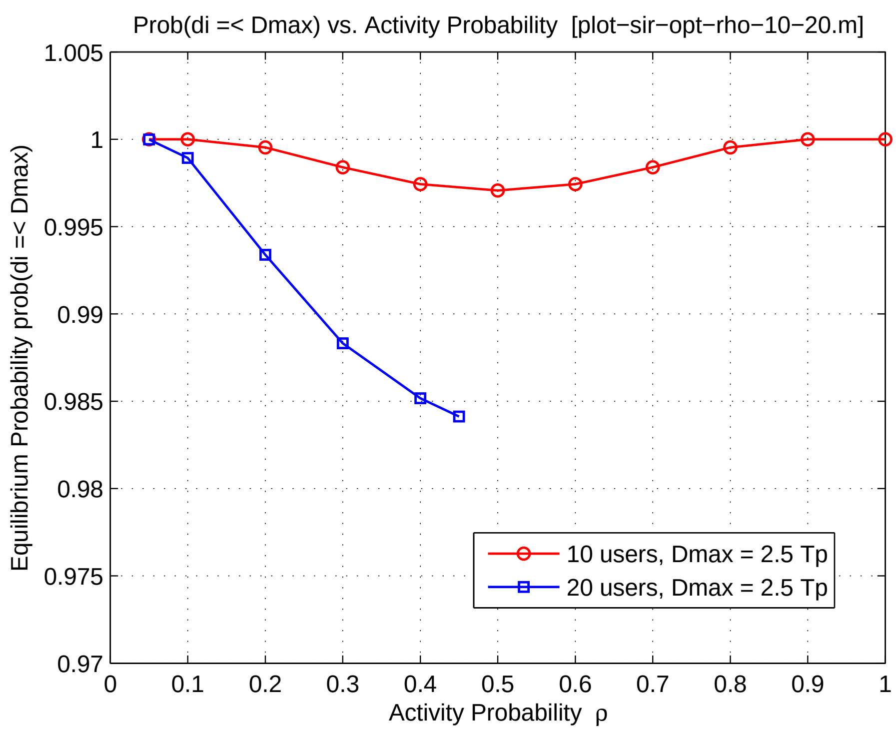

Figure 6.3: Probability of delay requirement $prob\{d_i \leq D_{max,i}\}$ at equilibrium vs. activity probability ρ, given $D_{max} = 2.5\,T_p$.

interference from a larger user population. The difference between the two scenarios increases as ρ moves from 0 to 0.5.

Particularly, the effect of *interference variance* can best be seen in Figure 6.3. Probability of delay requirement $prob\{d_i \leq D_{max,i}\}$ is at its lowest value when $\rho = 0.5$ which corresponds the peak of interference variance. It is interesting to observe that $prob\{d_i \leq D_{max,i}\}$ improves as the interference variability reduces. This is shown in the figure that the probability of delay requirement increases as ρ moves away from $\rho = 0.5$ toward either $\rho = 0$ or $\rho = 1$, which corresponds to the two cases of zero interference variance.

This result indicates that it is increasingly difficult to satisfy a packet delay requirement probabilistically in a packet-switched data system as the *burstiness* of the packet traffic sources increases.

Figures 6.4 and 6.5 plot transmit powers and utilities at equilibrium versus ρ for three user terminals located at three different distances away from the base station. As ρ increases, transmit powers rise while utilities drop. Obviously, closest user (for example, User 20) gets the best benefits from the system with highest utility achievement and lowest energy consumption owing to the lowest transmit power. Notice that the capacity limit phenomenon mentioned above can also be seen in Figures 6.4 and (6.5). The power increases and the utility decreases very rapidly as ρ grows to approach the capacity limit which is close to $\rho = 0.5$.

For easy comparisons between 20 user and 10 user scenarios, Figures 6.6 and 6.7 plot transmit powers and utilities at equilibrium versus ρ for the same three users in a $N = 10$ user system. Compared to the 20 user scenario, the powers and utilities both have similar patterns. However, the points of capacity limits are quite different for the two scenarios. With the assumed system and user parameters, as mentioned before, $N = 10$ is the capacity limit for the circuit-switched data system. In packet-switched data system with 10 users, the capacity limit phenomenon would occur at $\rho = 1$. This is when the packet-switched data system changes into a constant

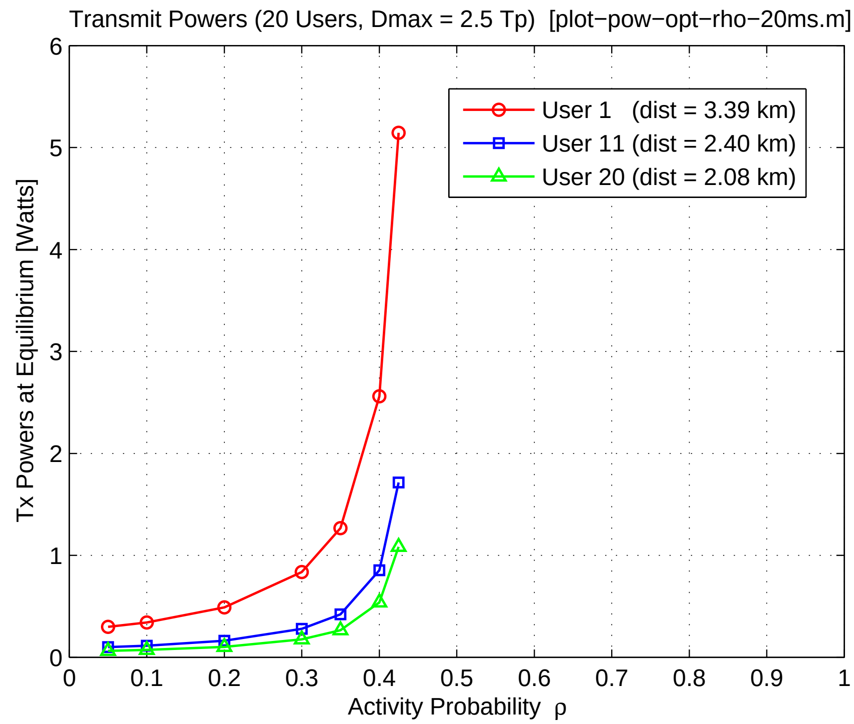

Figure 6.4: Transmit powers at equilibrium vs. ρ ($D_{max} = 2.5\,T_p\,,\ N = 20$ users).

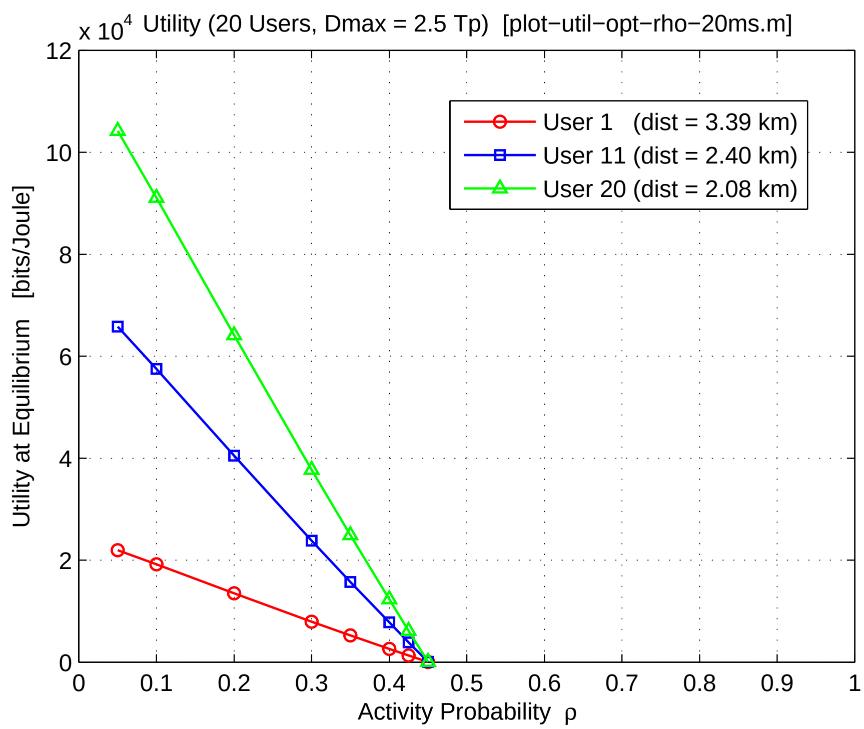

Figure 6.5: Utilities at equilibrium vs. ρ ($D_{max} = 2.5\,T_p\,,\ N = 20$ users).

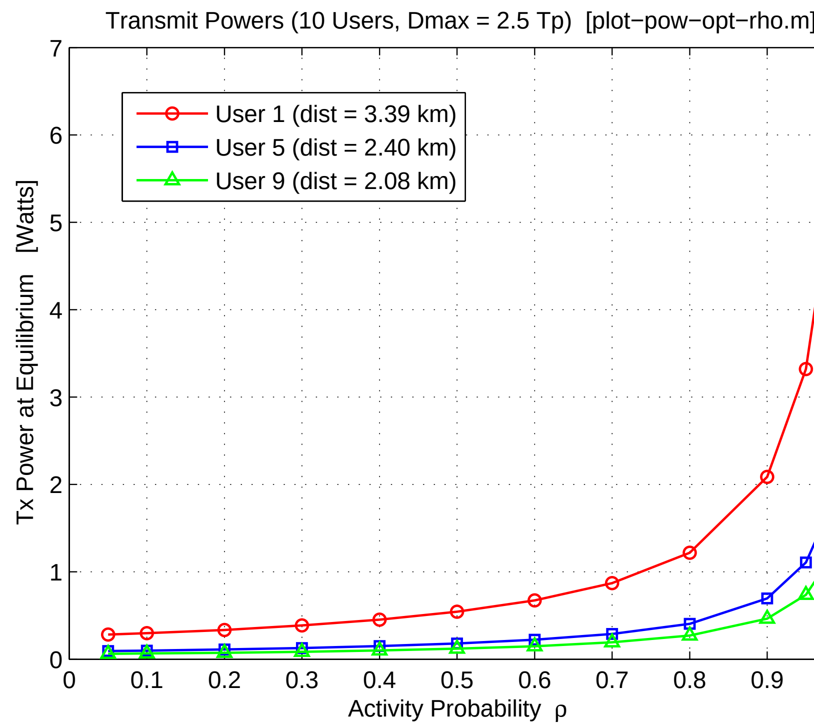

Figure 6.6: Transmit powers at equilibrium vs. ρ ($D_{max} = 2.5\,T_p\,,\ N = 10$ users).

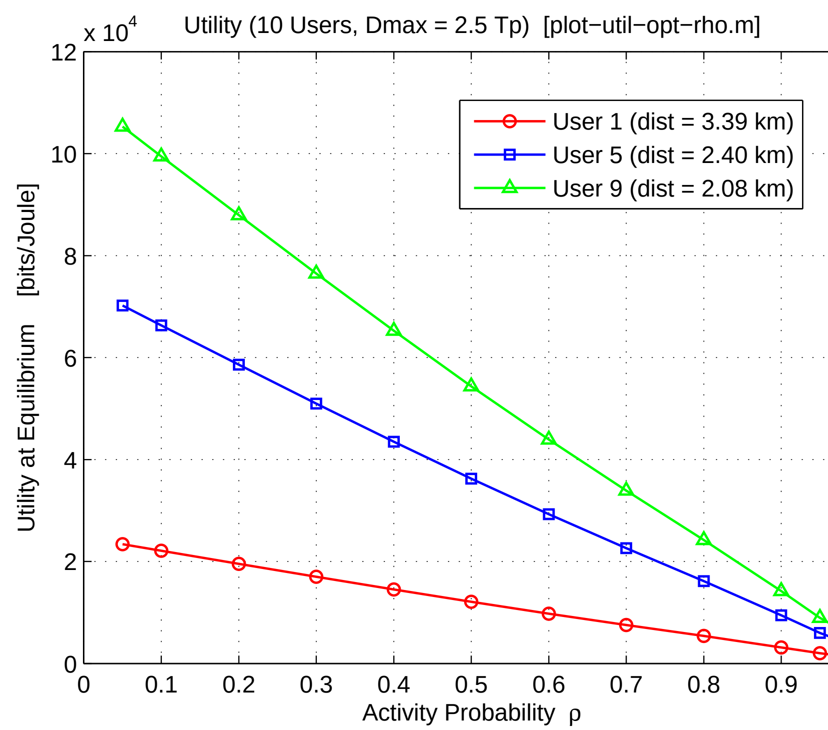

Figure 6.7: Utilities at equilibrium vs. ρ ($D_{max} = 2.5\,T_p\,,\ N = 10$ users).

packet data transmission system exactly like its circuit-switched counterpart. That is, at $\rho = 1$, both systems are equivalent.

6.2.2 Effect of $D_{max,i}$ on Equilibrium SIR, Transmit Powers and Utilities

Figure 6.8 plots equilibrium SIR as functions of delay tolerance $D_{max,i}$ for $N = 20$ user scenarios, given three values of ρ. Observe in the figure that a stringent delay requirement $D_{max,i}$ drives up the equilibrium SIR quickly. Because a tighter $D_{max,i}$ means a higher minimum SIR requirement $\gamma_{min,i}$. Thus a higher target SIR is needed to satisfy the basic requirement: $\gamma_i \geq \gamma_{min,i}$.

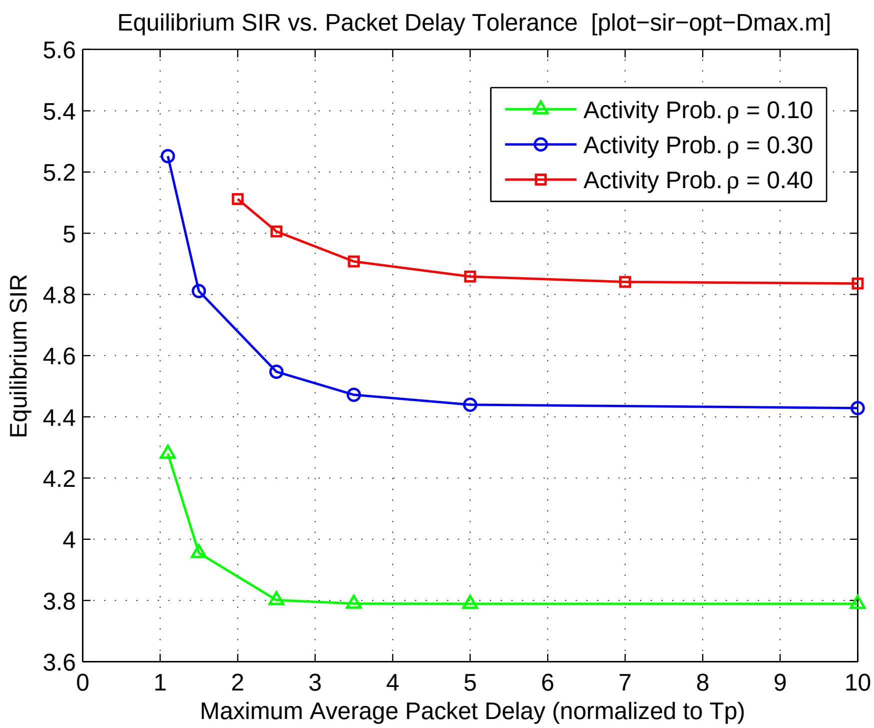

Figure 6.8: Equilibrium SIR vs. $D_{max,i}$ ($N = 20$ users).

Also as shown in Figure 6.8, for the same D_{max}, a lower ρ leads to a lower operating SIR. As explained previously, as ρ reduces, interference decreases and

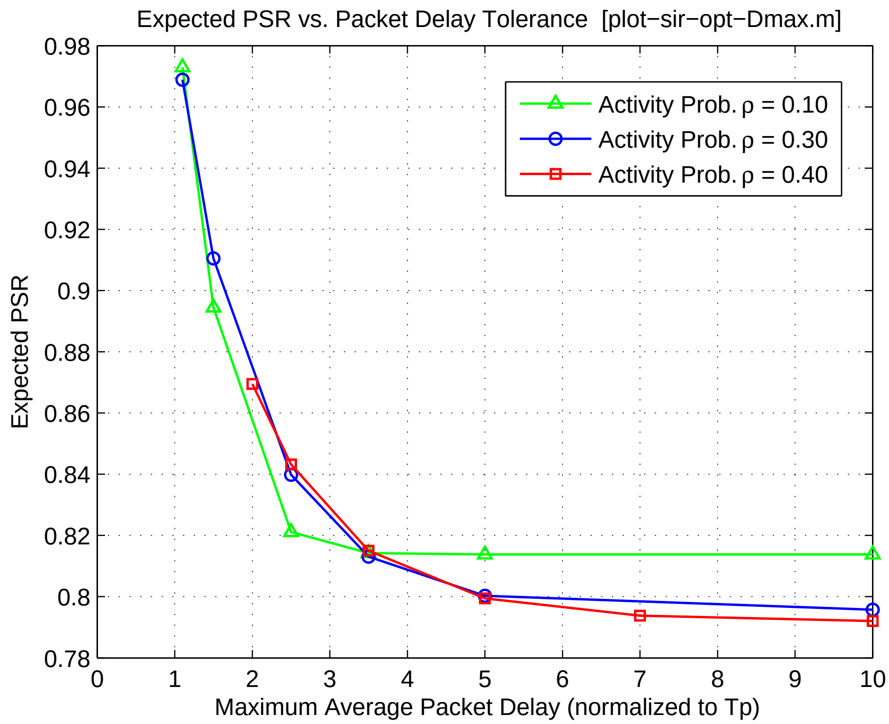

Figure 6.9: Expected PSR at equilibrium vs. $D_{max,i}$ ($N = 20$ users).

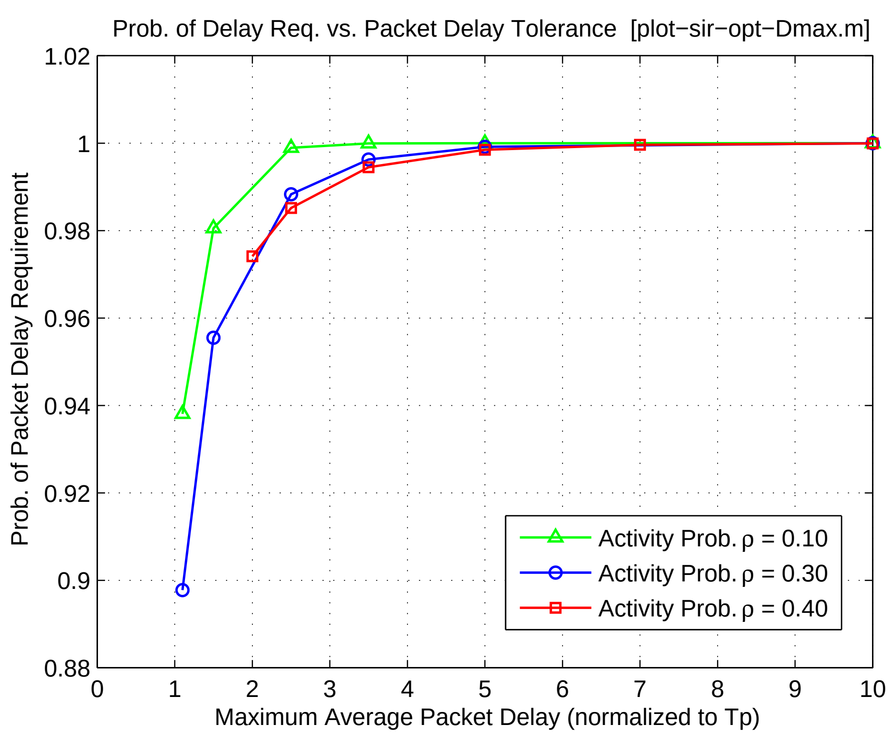

Figure 6.10: Probability of packet delay requirement at equilibrium vs. $D_{max,i}$ ($N = 20$ users).

increasingly the background AWGN noise would become a dominant factor in determine the target SIR.

In Figures 6.9 and 6.10, expected PSR and probability of delay requirement at equilibrium are plotted with respect to D_{max}. Again, we observe those two quantities behave oppositely as D_{max} varies. For example, a tigher D_{max} drives up the expected PSR while driving down the probability of delay requirement.

Figures 6.11 and 6.12 plot transmit powers and utilities at equilibrium as functions of packet delay requirement $D_{max,\,i}$ for three users at three distances away from the base station. A traffic source with a small $D_{max,\,i}$ requirement demands a high transmit power allocation and thus results in a low utility, i.e., a low energy-efficiency for the transmission of its packets.

Observe in Figures 6.11 and 6.12 that for the same amount of reduction on delay tolerance $D_{max,\,i}$, user 1 (farthest user in the system) has the most severe power increase. However, for the same amount of delay tolerance reduction, user 1's utility is subject to the least decrease. As we know, In a multi-cell environment, users at cell border have the most destructive influence on system performance because they produce the highest interference to neighboring cells. If a user at cell boundary demands a tighter requirement on packet delay, the user will have a reduced return on utility as a result. What is more is that the user will make the system even worse by introducing an extra huge amount of interference to other cells.

Therefore, for the sake of overall performance of a multi-cell system, users located at cell border should be somehow requested or disciplined, or be influenced or even compensated for not exerting tight delay requirements. Otherwise, "power war" between neighboring cells would be difficult to avoid, due to the interdependence of interference between cells in cellular systems.

Because $D_{max,\,i}$ is directly related to the promptness of packet delivery, while

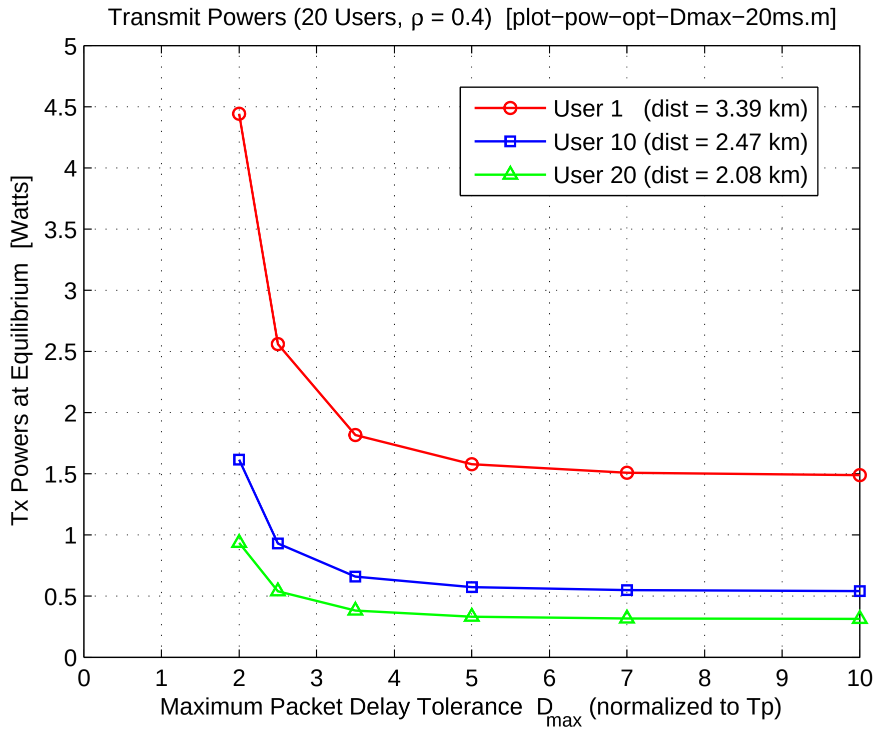

Figure 6.11: Transmit Powers at equilibrium vs. $D_{max,i}$ ($\rho = 0.4,\ N = 20$ users).

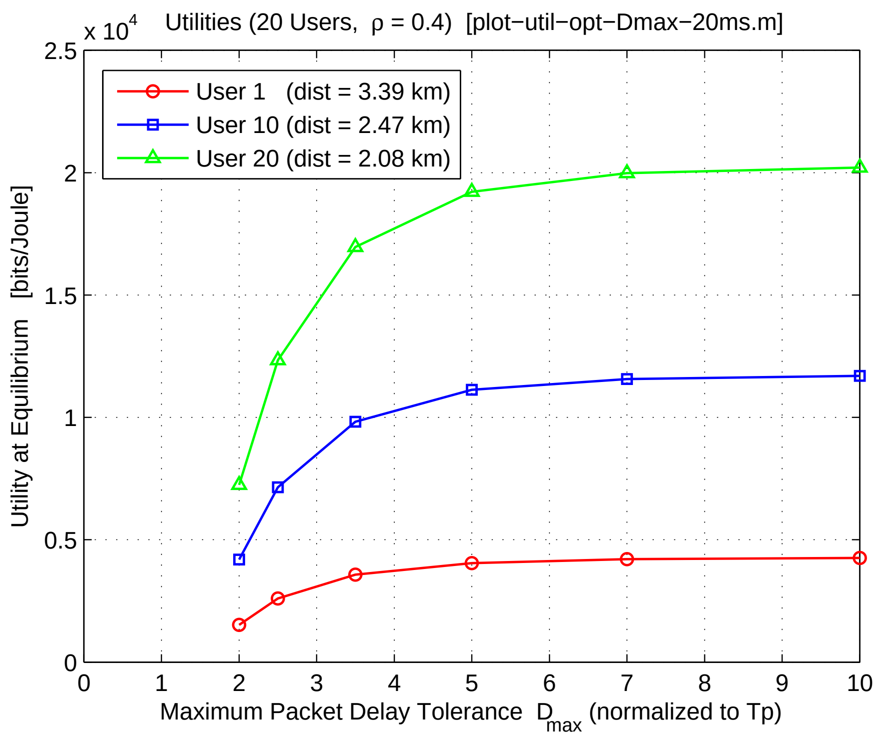

Figure 6.12: Utilities at equilibrium vs. $D_{max,i}$ ($\rho = 0.4,\ N = 20$ users).

utility is directly related to the energy-efficiency of packet transmissions, we actually encounter an interesting trade-off here between the promptness of information delivery and the energy efficiency of information delivery. The choice of packet delay requirement $D_{max,\,i}$ can be used as a leverage to balance the trade-off as below:

- $D_{max,i} \uparrow \;\longrightarrow\; \gamma_i^* \downarrow \;\longrightarrow\; p_i^* \downarrow \;\longrightarrow\; u_i^* \uparrow$
  - **Gain**: energy efficiency (bits/Joule)
  - **Loss**: promptness of packet delivery
- $D_{max,i} \downarrow \;\longrightarrow\; \gamma_i^* \uparrow \;\longrightarrow\; p_i^* \uparrow \;\longrightarrow\; u_i^* \downarrow$
  - **Gain**: promptness of packet delivery
  - **Loss**: energy efficiency (bits/Joule)

How to balance these two conflicting interests is user-dependent and traffic-dependent. For some users with time-critical information to send, they may not care about the energy conservation problem. On the contrary, for some other energy-sensitive users, to save battery energy and prolong the lifetime of their device before next re-charge, may overpower the need to send packets as quick as possible.

6.2.3 Effect of User Distance on Transmit Powers and Utilities

Figures 6.13, 6.14 and 6.15 plot SIR, transmit powers and utilities at equilibrium with respect to user locations. Clearly, at equilibrium, a fixed SIR is shared by all users, given ρ and D_{max}. SIR at equilibrium is location-independent, meaning it does not change with user location or user's distance to base station.

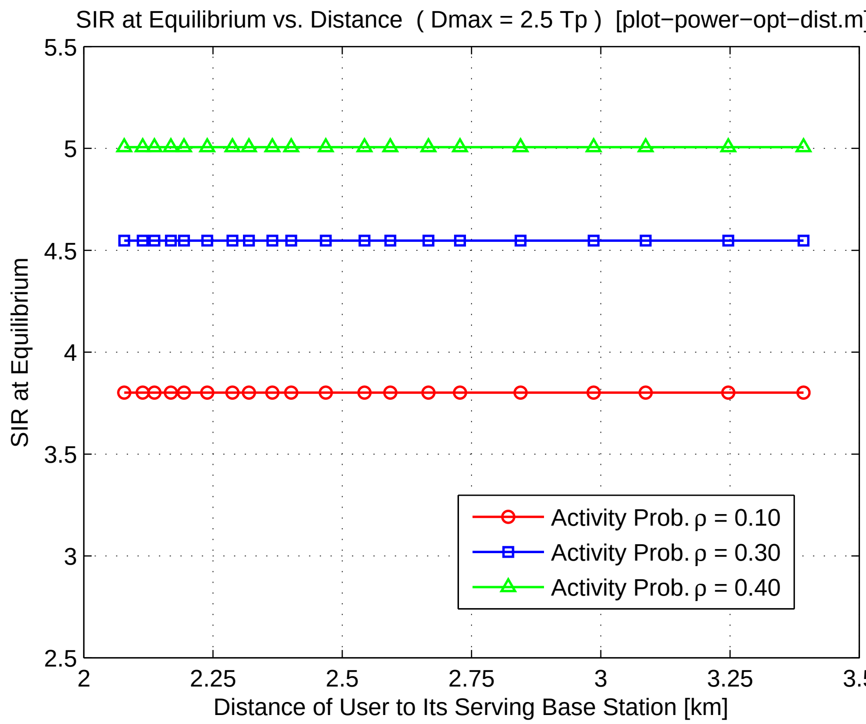

Figure 6.13: SIR at equilibrium vs. distance ($D_{max} = 2.5\,T_p\,,\ N = 20$ users).

Figures 6.14 and 6.15 show the unfairness of this power control scheme in the sense that users close to base station get high utilities but need only to radiate a little energy. While a far user radiates a lot of energy but gets only little utilities in return. Therefore, with equal traffic activity ρ and equal packet delay tolerance D_{max}, relative locations of users differentiate them from each other in terms of transmit powers allocated and utilities achieved.

6.2.4 Effect of Unequal Delay Tolerance $D_{max,\,i}$ on the Power Control Solution

In previous simulation studies, we assumed equal traffic activity ρ and equal packet delay tolerance D_{max} for all users. Now we consider the power control scheme with still the same traffic activity ρ for all users, but with *unequal* packet delay tolerance

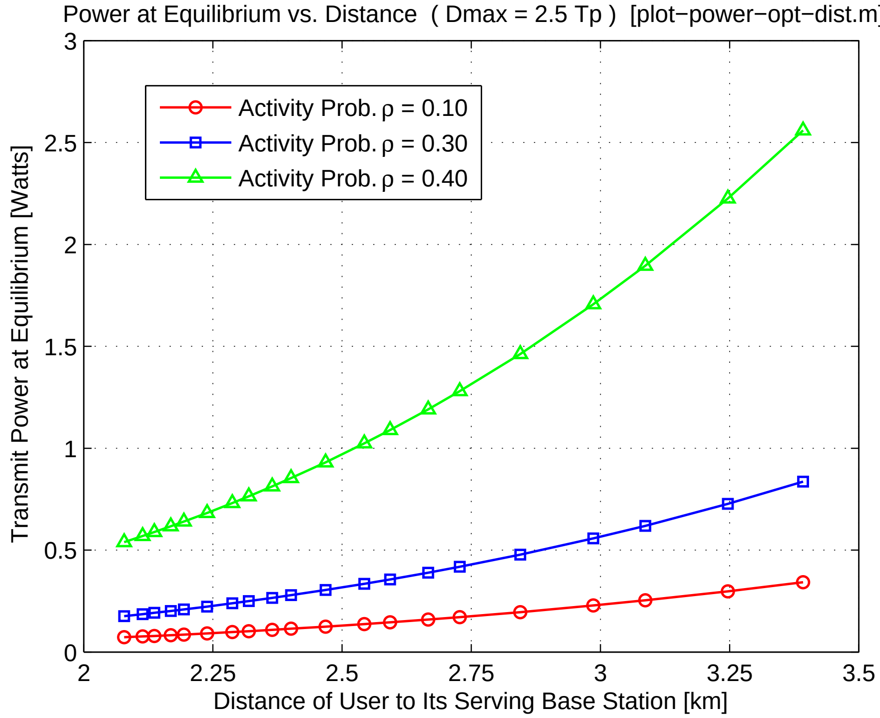

Figure 6.14: Transmit powers at equilibrium vs. distance ($D_{max} = 2.5\,T_p\,,\ N = 20$ users).

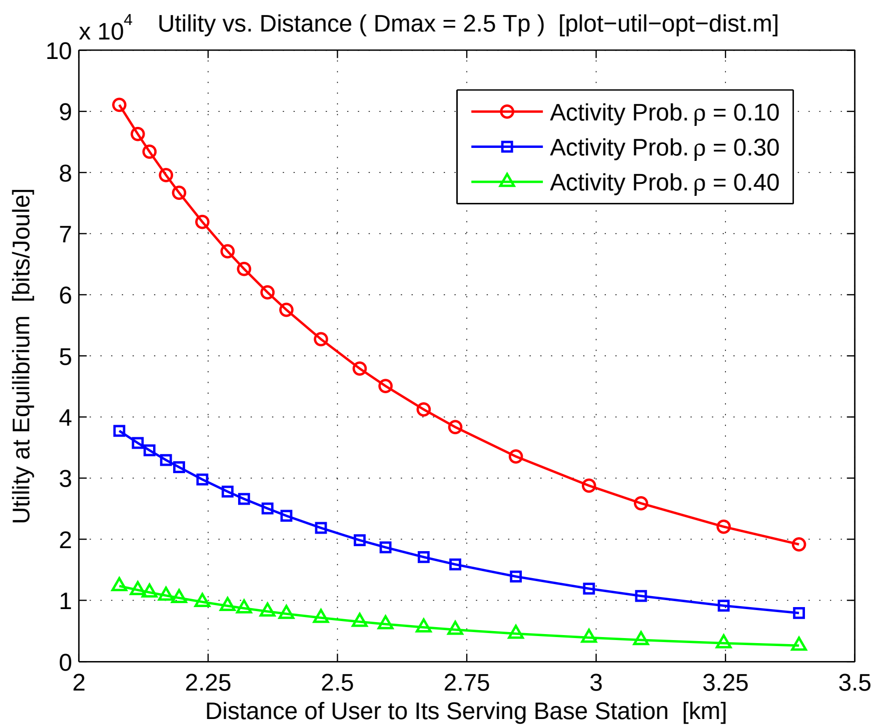

Figure 6.15: Utilities at equilibrium vs. distance ($D_{max} = 2.5\,T_p\,,\ N = 20$ users).

D_{max} among users.

In the simulations, we divide users into two groups with different delay tolerance: one group with $D_{max} = 5.0\,T_p$ and the other with $D_{max} = 2.0\,T_p$. For easy comparisons, we also plotted the results from two reference cases: one is when all users are as one group with $D_{max} = 5.0\,T_p$, and the other is when all users are as one group with $D_{max} = 2.0\,T_p$. Let us look at the problem through the following three examples.

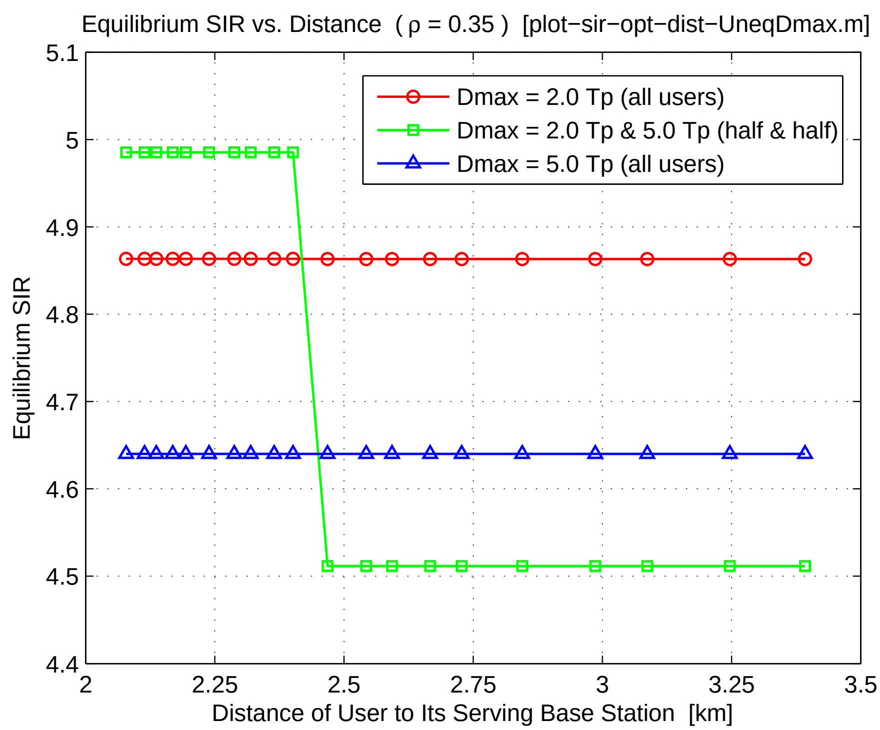

Figure 6.16: **Example 1**: SIR at equilibrium vs. distance for two groups of users with unequal $D_{max,i}$. $D_{max,i} = 2\,T_p$ for the 10 inner users (close to the serving BS) and $D_{max,i} = 5.0\,T_p$ for the 10 outer users. ($\rho = 0.35,\ N = 20$ users).

- **Example 1.** Divide users into two groups of equal size: half of them (far users: user 1 to user 10) with $D_{max} = 5.0\,T_p$, and the other half (close users: user 11 to user 20) with $D_{max} = 2.0\,T_p$.

1. Figure 6.16 shows that the SIRs achieved are the same if users are within

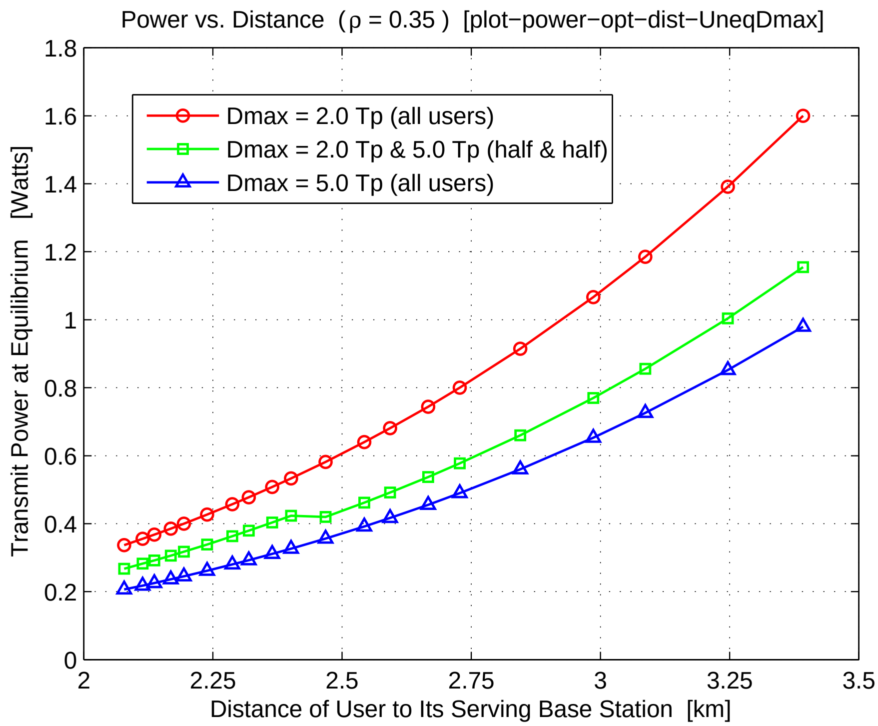

Figure 6.17: **Example 1**: Powers at equilibrium vs. distance. Two equal-sized groups of users with unequal $D_{max,i}$ ($\rho = 0.35,\ N = 20$ users).

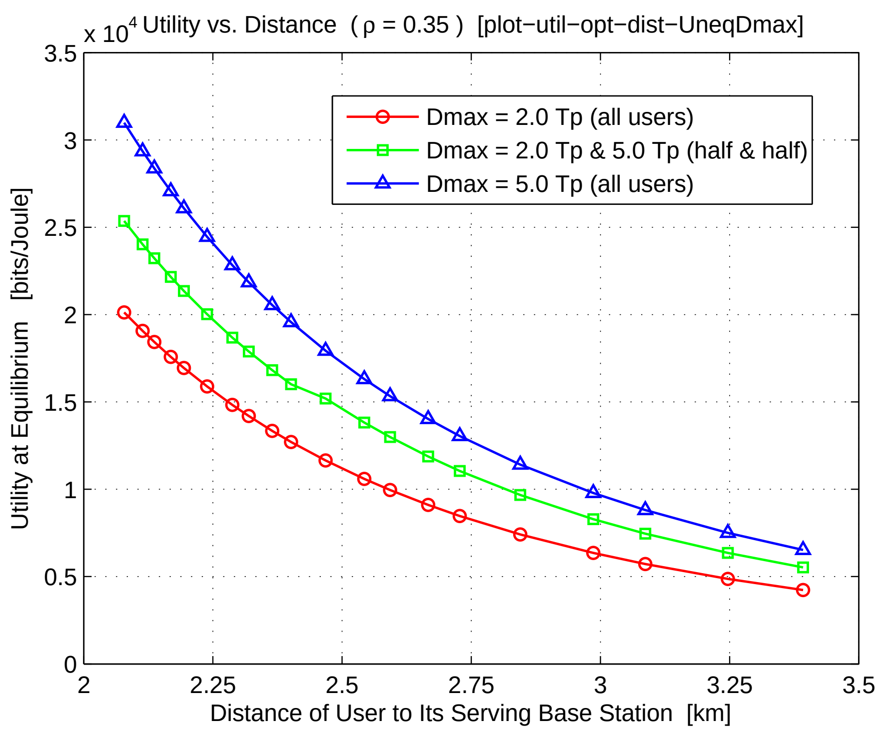

Figure 6.18: **Example 1**: Utilities at equilibrium vs. distance. Two equal-sized groups of users with unequal $D_{max,i}$ ($\rho = 0.35,\ N = 20$ users).

the same groups (same D_{max}), but SIRs are unequal if users are from different groups.

2. Figure 6.17 shows that the transmit powers at equilibrium are between the transmit powers of the two reference cases. Specifically, the transmit powers for the tighter D_{max} group ($D_{max} = 2.0\,T_p$) are relatively higher as shown in the figure.

3. Figure 6.18 shows that the utilities at equilibrium are between the utilities of the two reference cases. Specifically, the utilities for the tighter D_{max} group ($D_{max} = 2.0\,T_p$) are relatively lower as shown in the figure.

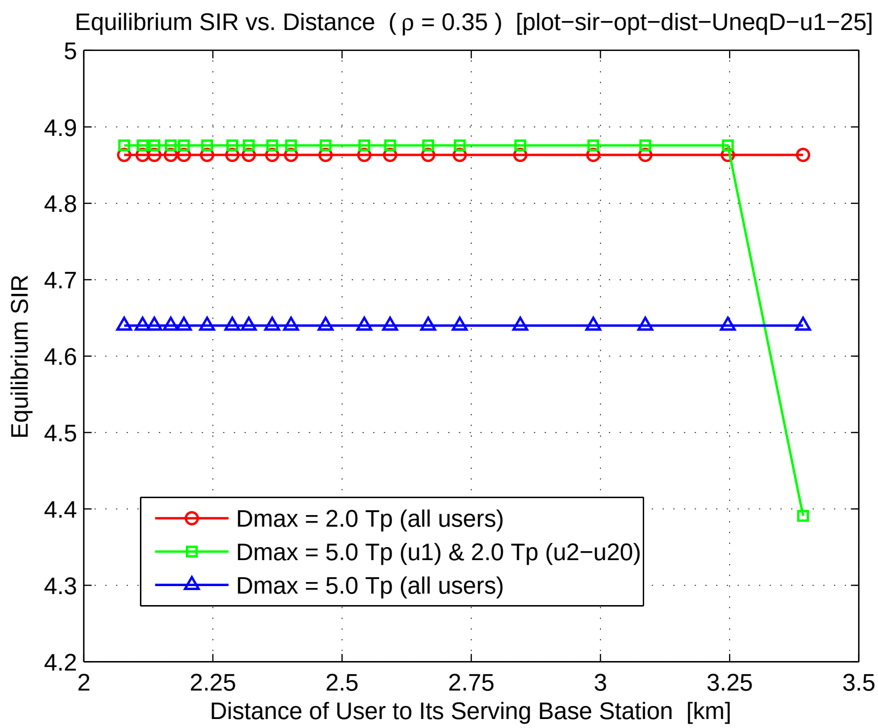

Figure 6.19: **Example 2**: SIR at equilibrium vs. distance. User 1 relaxes its delay tolerance from $D_{max,1} = 2\,T_p$ to $D_{max,1} = 5.0\,T_p$ ($\rho = 0.35,\ N = 20$ users).

- **Example 2.** Generally, when a user relaxes its QoS requirement, for example, relax its packet delay requirement ($D_{max,i} \uparrow$), it benefits all the users in the system by reducing powers and increasing utilities. This example shows this

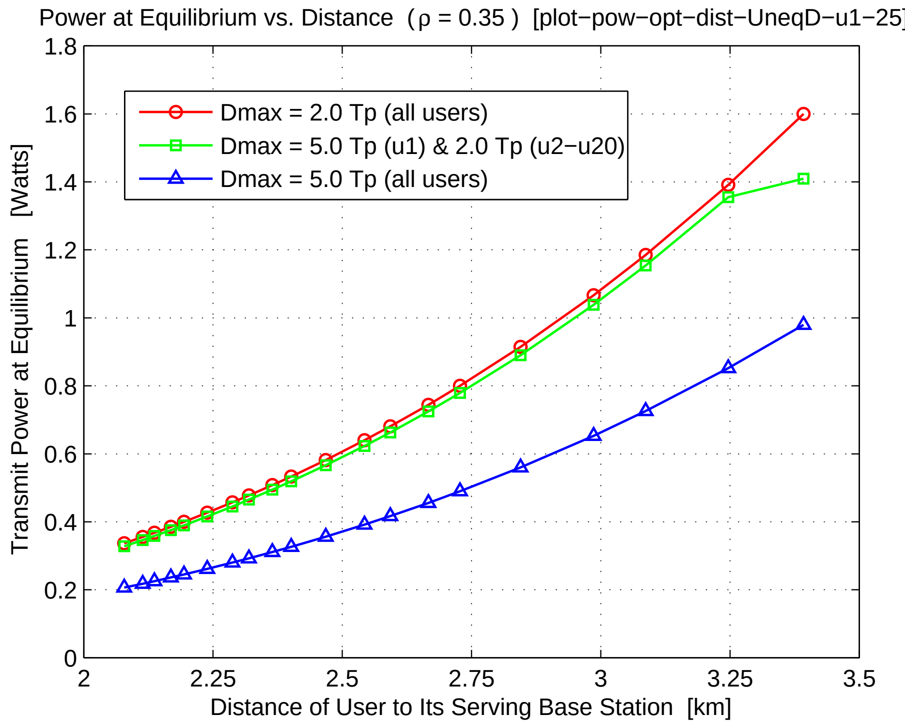

Figure 6.20: **Example 2**: Powers at equilibrium vs. distance. User 1 relaxes its delay tolerance from $D_{max,1} = 2\,T_p$ to $D_{max,1} = 5.0\,T_p$ ($\rho = 0.35,\ N = 20$ users).

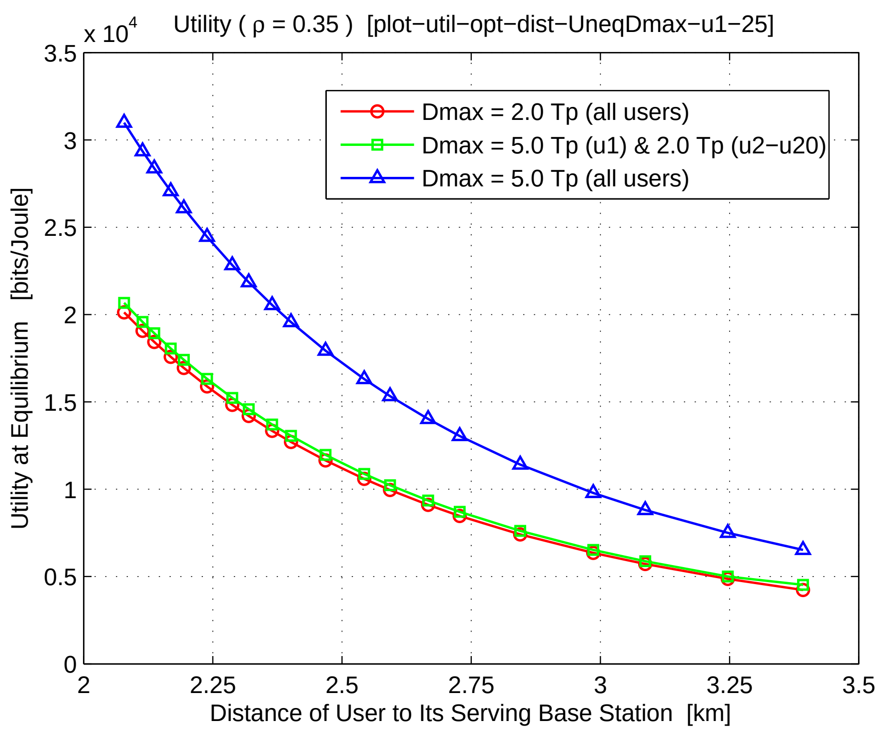

Figure 6.21: **Example 2**: Utilities at equilibrium vs. distance. User 1 relaxes its delay tolerance from $D_{max,1} = 2\,T_p$ to $D_{max,1} = 5.0\,T_p$ ($\rho = 0.35,\ N = 20$ users).

effect when user 1 (user at cell border) relaxes its delay tolerance by increasing its $D_{max,1}$ from $D_{max,1} = 2\,T_p$ to $D_{max,1} = 5.0\,T_p$.

1. Figure 6.19 shows that the SIR of user 1 drops considerably. As a result, the SIRs of the remaining users are actually raised a little, comparing to the reference case with $D_{max,1} = 2\,T_p$.

2. Figure 6.20 shows the transmit powers at equilibrium. User 1's relaxing of its delay tolerance reduces the powers of all the remaining users, comparing to the reference case with $D_{max,1} = 2\,T_p$.

3. Figure 6.21 shows the utilities at equilibrium. User 1's relaxing of its delay tolerance increases the utilities of all the remaining users, comparing to the reference case with $D_{max,1} = 2\,T_p$.

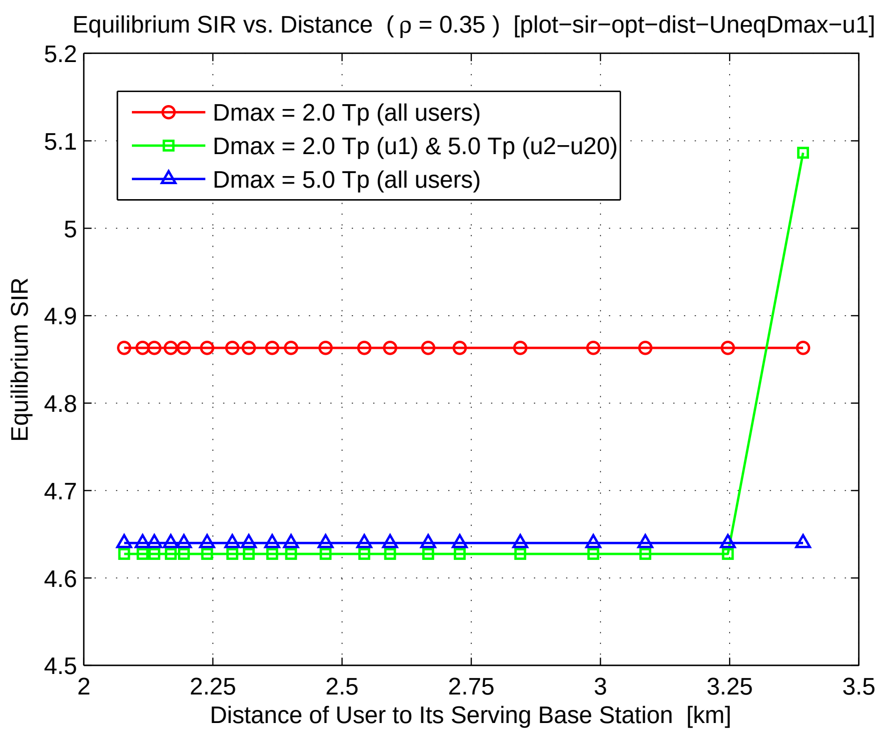

Figure 6.22: **Example 3**: SIR at equilibrium vs. distance. User 1 tightens its delay tolerance from $D_{max,1} = 5.0\,T_p$ to $D_{max,1} = 2.0\,T_p$ ($\rho = 0.35,\ N = 20$ users).

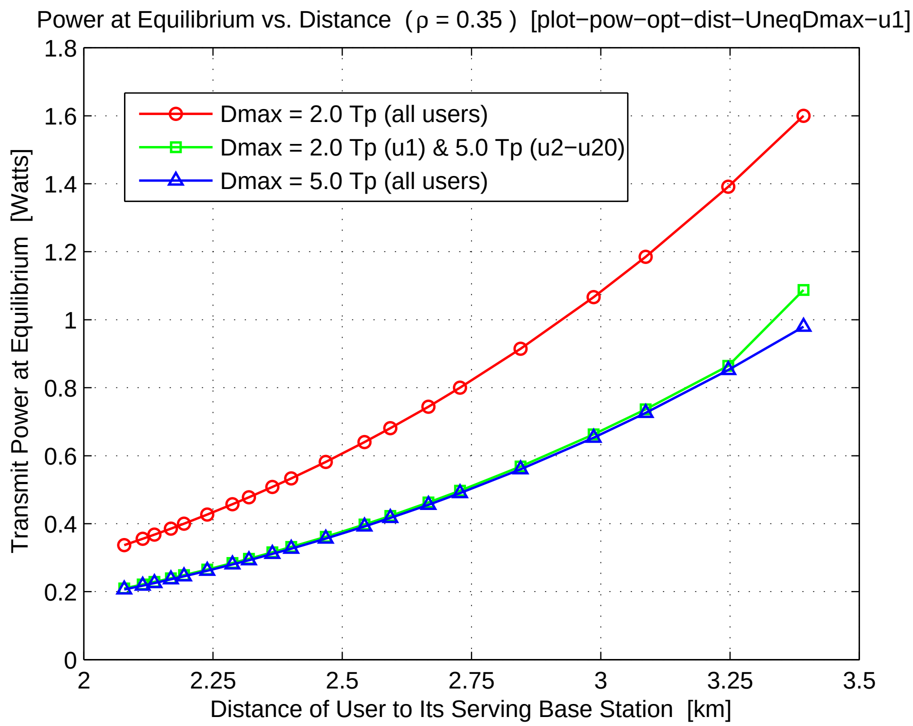

Figure 6.23: **Example 3**: Powers at equilibrium vs. distance. User 1 tightens its delay tolerance from $D_{max,1} = 5.0\,T_p$ to $D_{max,1} = 2.0\,T_p$ ($\rho = 0.35,\ N = 20$ users).

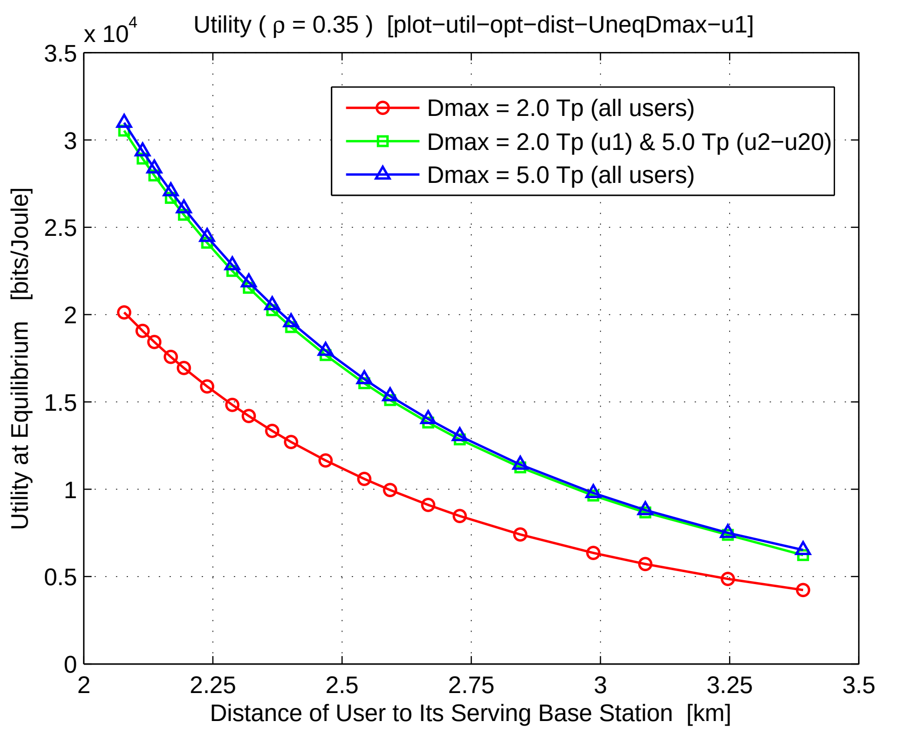

Figure 6.24: **Example 3**: Utilities at equilibrium vs. distance. User 1 tightens its delay tolerance from $D_{max,1} = 5.0\,T_p$ to $D_{max,1} = 2.0\,T_p$ ($\rho = 0.35,\ N = 20$ users).

- **Example 3.** Generally, when a user tightens its QoS requirement ($D_{max,\,i}\downarrow$), it hurts all the users in the system by increasing powers and lowering utilities. This example shows this effect when user 1 tightens its delay tolerance by reducing its $D_{max,\,1}$ from $D_{max,\,1} = 5.0\,T_p$ to $D_{max,\,1} = 2.0\,T_p$.

  1. Figure 6.22 shows that the SIR of user 1 is raised considerably and the SIRs of the remaining users drop a little, comparing to the reference case with $D_{max,\,1} = 5.0\,T_p$.

  2. Figure 6.23 shows the transmit powers at equilibrium. User 1's tightening of its delay tolerance increases the powers of all the remaining users, comparing to the reference case with $D_{max,\,1} = 5.0\,T_p$.

  3. Figure 6.24 shows the utilities at equilibrium. User 1's tightening of its delay tolerance decreases the utilities of all the remaining users, comparing to the reference case with $D_{max,\,1} = 5.0\,T_p$.

6.3 Analysis vs. Simulations

Up to now, we have investigated the uplink power control problem for packet-switched wireless data network both analytically and by simulations. Now, we would like to know if the results from analysis and from simulations support each other.

For easy comparison, we assume that all users in the system have a same packet delay requirement $D_{max} = 2.5\,T_p$, same packet activity probability $\rho = 0.4$, and there are $N = 20$ users in the system who are located at different distances to their serving base station.

Figure 6.25 shows the plots of the results on utilities as a function of user distance from both the analysis and simulation studies. The general trending of the curves from both cases are similar. But for this specific scenario, the utility values from analysis are higher than those from simulation studies while they are not too far apart.

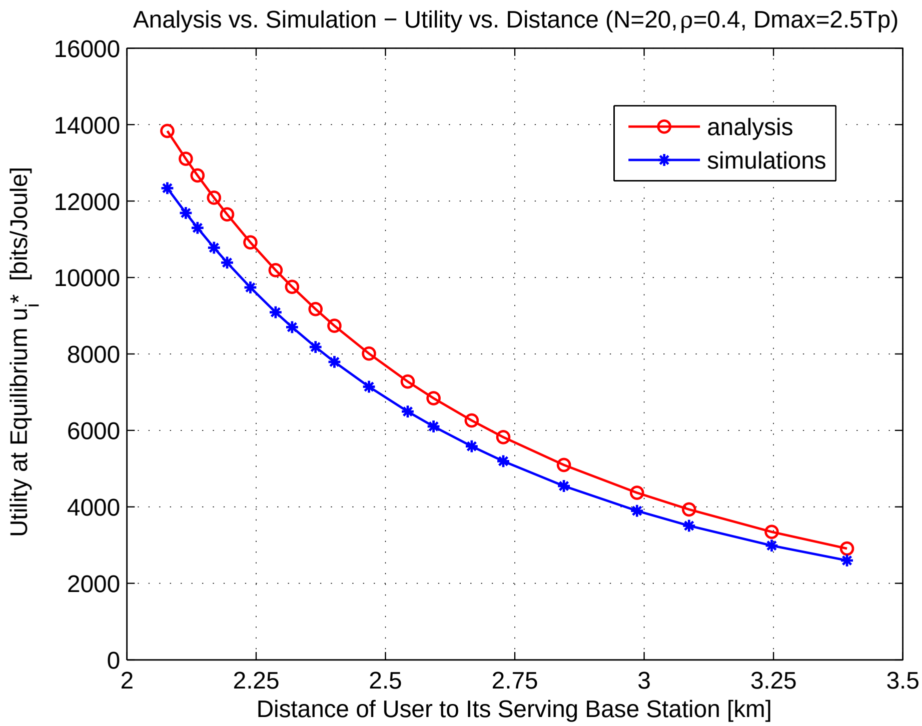

Figure 6.25: Comparison of the analytical and simulation results on the utilities at equilibrium vs. user distance to its serving base station ($N = 20$ users, $\rho = 0.4$ and $D_{max} = 2.5\,T_p$).

The reason why those two curves are not matching each other perfectly is due to the fact that we went through with one more approximation in the analysis to obtain the closed-form solution as discussed in Subsection 4.4.3. We used first-order polynomial curve-fitting method to approximate the solution function $\beta^* = g(\alpha^*)$ and thereby introduced two linear parameters a_0 and a_1 which were estimated in a least-squares sense. This extra step of approximation in analysis may have contributed to the discreppancy between those curves, because this step was not involved in the simulation studies.

Chapter 7

Application to Multi-Class Communications in Multimodal Collaboration Networks

In this Chapter, as an application example, we use the study results from previous chapters and extend them to a multi-class wireless packet-switched data communications environment — a wireless multimodal collaboration system.

In an environment of multimodal collaboration over wired and wireless networks, the multimedia traffic (voice, data, text, image, video, etc.) generated by multimodal collaboration terminals contains tight timing requirements and strong correlations among component modes of traffic. Since the resources over the wireless links are extremely scarce compared to wired ones, it is far more difficult to maintain the exact timing relationships among the traffic components over a wireless link than a wired one. Wireless links are essentially the bottlenecks of the multimodal collaboration systems. The success of the multimodal collaboration over wired and wireless networks strongly depends on the wireless part. Based on the delay-dependent utility model and the game optimization approach developed above, we consider radio resource allocation and delay performance issues in wireless networks from the perspective of multimodal collaboration.

We assume that a slotted packet data CDMA system is adopted as the air interface for the wireless links in a multimodal collaboration environment. As we know from the previous study, in packet-switched systems, the packet transmission delay over a wireless link for a mobile user is a random variable due to the random interference generated from all the other mobile users in the system. In multimodal collaboration, a user has several types of packet traffic streams to transmit, which may have different QoS requirements in terms of average or maximum tolerance on the mean or variance of packet delay.

In this work, we focus on the uplink power control problem for multimodal collaboration terminals. Our objective is still the same as before — to maximize the total number of useful data packets transmitted from an individual user per unit of the user's battery energy (packets/Joule), but now under diverse delay requirements among the users' traffic substreams. We investigate how to allocate transmit powers to different traffic substreams originated from a same user according to its substreams' delay requirements, traffic activities and levels of interference.

We also study the effect of different transmission strategies for the traffic substreams generated from a user. Particularly we evaluate the effect of an *orthogonalizing transmission strategy*. With that strategy, the transmissions of different substreams from a same user are orthogonalized, such that the transmit powers from different substreams of the same user won't interfere with each other.

7.1 Reformulation of the Utility-Maximization Problem for Multi-Class Traffic

Consider a multimodal collaboration system where each user terminal generates two types of traffic (type "v" and type "d") with different delay requirements as follows:

1. Type "v" traffic is *delay-sensitive*, and uses $D^v_{max,i}$ as its average packet delay requirement. This type of packet-data traffic represents applications with focus on "real-time" requirements. Typical examples for this type of traffic

Table 7.1: Summary of main parameters and notations for the study of the multi-modal traffic scenarios with two types of packet delay QoS requirements.

Param.	**Unit**	**Description**
type "v"		*"delay-sensitive"* traffic (e.g., voice, video, ...)
type "d"		*"delay-tolerant"* traffic (e.g., data, text, ...)
N		total number of users (type "v" and type "d") in system
x_i^v	$\in \{0,1\}$	on-off $r.v.$ for packet activity of user i, type "v"
x_i^d	$\in \{0,1\}$	on-off $r.v.$ for packet activity of user i, type "d"
ρ_i^v	$\in [0,1]$	packet activity probability of user i, type "v"
ρ_i^d	$\in [0,1]$	packet activity probability of user i, type "d"
$D_{max,\,i}^v$	sec	max. packet transmission delay tolerated by user i, type "v"
$D_{max,\,i}^d$	sec	max. packet transmission delay tolerated by user i, type "d"
p_i^v	Watts	uplink transmit power of user i, type "v"
p_i^d	Watts	uplink transmit power of user i, type "d"
$\mathbf{p}^v$	Watts	vector formed by all transmit powers of all type "v" users
$\mathbf{p}^d$	Watts	vector formed by all transmit powers of all type "d" users
$\mathbf{p}_{-i}^v$	Watts	power vector formed by all type "v" but ith users
$\mathbf{p}_{-i}^d$	Watts	power vector formed by all type "d" but ith users
γ_i^v		SIR of user i, type "v" in packet-switched systems
γ_i^d		SIR of user i, type "d" in packet-switched systems
$\gamma_i^{v,\,\mathrm{orth}}$		γ_i^v with orthogonalization
$\gamma_i^{d,\,\mathrm{orth}}$		γ_i^d with orthogonalization
$\overline{u}_i^v(\cdot)$	bits/Joule	expected packet data utility of user i, type "v"
$\overline{u}_i^d(\cdot)$	bits/Joule	expected packet data utility of user i, type "d"
$\overline{u}_i^{v,\,\mathrm{orth}}(\cdot)$	bits/Joule	$\overline{u}_i^v(\cdot)$ with orthogonalization
$\overline{u}_i^{d,\,\mathrm{orth}}(\cdot)$	bits/Joule	$\overline{u}_i^d(\cdot)$ with orthogonalization

are voice and video.

2. Type “d” traffic is *delay-tolerant*, and uses $D^{d}_{max,i}$ as its average packet delay requirement. This type of packet-data traffic represents applications with focus on ”error-free” requirements. Typical examples for this type of traffic are Email and FTP file transfer.

We want allocate a pair of transmit powers $(p_i^v,\ p_i^d)$ to the two substreams of traffic for user i to maximize the individual utility of each substream respectively. We study two different transmission strategies for the traffic substreams originated from a user terminal in terms of the following two scenarios:

- **Scenario 1**: “Without orthogonalization” — assume transmitting signals for different traffic substreams are not orthogonalized. Therefore, all all data substreams generated from all traffic sources are independent from each other, and interfering with each other.
- **Scenario 2**: “With orthogonalization” — assume transmitting signals for different traffic substreams from a same traffic source are orthogonalized. Therefore, the two traffic substreams transmitted from a same user will not interfere with each other.

7.1.1 Problem Formulation without Orthogonalization

Because of the structural change in the nature of the problem, we need to re-define the utility models for specific traffic types, taking into account the new interference pattern of a multimodal collaboration communications system.

In the formulation of this problem, we treat all traffic substreams generated by all users as independent sources. Therefore, all the substreams from all users interfere with each other. Even any two substreams originated from a same user terminal interfere with each other. So, it is a "full" interference system, and there is no elimination of any type of interference by the nature of design.

Based on previous work on the utility modeling as in equation (3.27), we have the following two definitions of utility model for two respective types of traffic substreams produced by a user terminal.

1. **Utility model for traffic substream of type "v" for user** i:

$$\overline{u}_i^v(p_i^v,\ \mathbf{p}_{-i}^v,\ \mathbf{p}^d) \ \triangleq\ \frac{L\,R}{M\,p_i^v}\left[\frac{E[f(\gamma_i^v)] - f(0)}{1 - f(0)}\right] Prob\{\,\gamma_i^v\ \geq\ \gamma_{min,\,i}^v\,\}$$

bits/Joule (7.1)

where γ_i^v is SIR for user i's type "v" traffic which is expressed as below:

$$\gamma_i^v \ =\ \frac{W}{R}\,\frac{h_i\,p_i^v}{\sum_{j=1, j\neq i}^{N} h_j\,(x_j^v\,p_j^v\ +\ x_j^d\,p_j^d)\ +\ x_i^d\,h_i\,p_i^d\ +\ \sigma_i^2}, \tag{7.2}$$

and $\gamma_{min,\,i}^v$ is related to $D_{max,\,i}^v$ in equation (3.3) as below:

$$\gamma_{min,\,i}^v \ =\ 2\,\ln\left\{\frac{(D_{max,\,i}^v\,R_p)^{\frac{1}{M}}}{2\left[(D_{max,\,i}^v\,R_p)^{\frac{1}{M}} - 1\right]}\right\}. \tag{7.3}$$

Notice that the interference is now coming from all the other traffic substreams including substreams from all other users and the other substream of type "d" from the same user - user i itself.

In the above equation, x_j^v is an activity $r.v.$ for user j's type "v" traffic which is described below:

$$x_j^v \;=\; \begin{cases} 1 & \text{with prob.} \quad \rho_j^v \\ 0 & \text{with prob.} \quad 1-\rho_j^v \end{cases} \tag{7.4}$$

where ρ_j^v is the activity probability for user j's type "v" traffic.

x_j^d is activity $r.v.$ for user j's type "d" traffic which is described below:

$$x_j^d \;=\; \begin{cases} 1 & \text{with prob.} \quad \rho_j^d \\ 0 & \text{with prob.} \quad 1-\rho_j^d \end{cases} \tag{7.5}$$

where ρ_j^d is the activity probability for user j's type "d" traffic.

2. **Utility model for traffic substream of type "d" for user** i:

$$\overline{u}_i^d(p_i^d,\ \mathbf{p}_{-i}^d,\ \mathbf{p}^v) \;\triangleq\; \frac{L\,R}{M\,p_i^d}\left[\frac{E[f(\gamma_i^d)]-f(0)}{1-f(0)}\right] Prob\,\{\,\gamma_i^d \;\geq\; \gamma_{min,\,i}^d\,\} \quad \text{bits/Joule} \tag{7.6}$$

where γ_i^d is SIR for user i's type "d" traffic as below:

$$\gamma_i^d \;=\; \frac{W}{R}\,\frac{h_i\,p_i^d}{\sum_{j=1,j\neq i}^{N} h_j\,(x_j^d\,p_j^d \;+\; x_j^v\,p_j^v) \;+\; x_i^v\,h_i\,p_i^v \;+\; \sigma_i^2} \tag{7.7}$$

with x_j^v and x_j^d as defined in Equations (7.4) and (7.5).

Again, $\gamma^d_{min,\,i}$ in above equation is related to $D^d_{max,\,i}$ in Equation (3.3) as below:

$$\gamma^d_{min,\,i} \;=\; 2\,\ln\left\{ \frac{(D^d_{max,\,i}\,R_p)^{\frac{1}{M}}}{2\left[(D^d_{max,\,i}\,R_p)^{\frac{1}{M}}-1\right]} \right\}. \tag{7.8}$$

7.1.2 Problem Formulation with Orthogonalization

In the formulation of this problem, we treat only traffic substreams produced by *different* users as independent sources. Therefore, all the substreams from *different* users interfere with each other. However, since we build orthogonalization on the two substreams of traffic originated from a same user, any two substreams from a same user will not interfere with each other.

Again, based on utility modeling in Equation (3.27), we have the following two definitions of utility model for two respective types of traffic substreams produced by a user terminal.

1. **Utility model for traffic substream of type "v" for user i (with orthogonalization)**:

$$\overline{u}_i^{v,\,\mathrm{orth}}(p_i^v,\;\mathbf{p}_{-i}^v,\;\mathbf{p}^d) \;\triangleq\; \frac{L\,R}{M\,p_i^v}\left[\frac{E[f(\gamma_i^{v,\,\mathrm{orth}})]-f(0)}{1-f(0)}\right] Prob\{\,\gamma_i^{v,\,\mathrm{orth}}\;\geq\;\gamma^v_{min,\,i}\}$$

$$\text{bits/Joule} \tag{7.9}$$

where $\gamma_i^{v,\,\mathrm{orth}}$ is SIR of user i's type "v" traffic with orthogonalization as below:

$$\gamma_i^{v,\,\mathrm{orth}} \;=\; \frac{W}{R}\,\frac{h_i\,p_i^v}{\sum_{j=1,j\neq i}^{N}\,h_j\,(x_j^v\,p_j^v\;+\;x_j^d\,p_j^d)\;+\;\sigma_i^2} \tag{7.10}$$

and $\gamma_{min,\,i}^v$ as in Equation (7.3).

2. **Utility model for traffic substream of type "d" for user** i **(with orthogonalization)**:

$$\overline{u}_i^{d,\,\mathrm{orth}}(p_i^d,\;\mathbf{p}_{-i}^d,\;\mathbf{p}^v)\;\triangleq\;\frac{L\,R}{M\,p_i^d}\left[\frac{E[f(\gamma_i^{d,\,\mathrm{orth}})]-f(0)}{1-f(0)}\right]Prob\{\,\gamma_i^{d,\,\mathrm{orth}}\;\geq\;\gamma_{min,\,i}^d\}$$

$$\text{bits/Joule} \tag{7.11}$$

where γ_i^d is SIR of user i's type "d" traffic with orthogonalization as below:

$$\gamma_i^{d,\,\mathrm{orth}} \;=\; \frac{W}{R}\,\frac{h_i\,p_i^d}{\sum_{j=1,j\neq i}^{N}\,h_j\,(x_j^d\,p_j^d\;+\;x_j^v\,p_j^v)\;+\;\sigma_i^2} \tag{7.12}$$

and $\gamma_{min,\,i}^v$ as in Equation (7.8).

Notice that, with the assumption of orthogonalization, the interference patterns in (7.10) and (7.12) are different from the scenarios without orthogonalization as in (7.2) and (7.7).

7.2 Simulation Studies for the Scenarios of Two-Class Communications

Based on the above formulations, and using the same system parameters used in the prior simulation studies as in Table 6.1, we investigate the uplink power control game in two-class traffic scenarios by simulations.

We assume that there are $N = 10$ user terminals with multimedia capability in the system. Each of those multimedia terminals generates two specific traffic substreams, one of which is "v" type while the other is "d" type. Therefore, there are actually 20 traffic substreams in the system, but they are collocated in pairs to resemble single user terminal that generates two types of multimedia traffic.

We also assume $D^v_{max,\,i} < D^d_{max,\,i}$ with $D^v_{max,\,i} = 1.5\,T_p$ and with $D^d_{max,\,i} = 5.0\,T_p$. The activity probabilities for both "v" and "d" types of packet traffic are assumed the same for all users for simplicity. That is $\rho^v_i = \rho^d_i = \rho,\ \forall\, i$ is assumed in the simulations.

7.2.1 Scenario 1: Traffic Sources without Orthogonalization

Simulation results are plotted in Figures 7.1, 7.3, 7.5, 7.7 and 7.9 for SIR, expected PSR, probability of delay requirement, received powers, transmit powers and utilities at equilibrium respectively.

From the results in Figure 7.1 and 7.3, "v" substreams have a higher SIR, higher expected PSR and lower probability of delay requirement. This is due to the tighter delay requirement for "v" traffic as compared to "d" traffic.

From the results in Figure 7.5, we observe that the received powers are different for different traffic types. But within the same traffic type, the received powers are the same. This is what we expected based on our previous analytical studies. And a tighter D_{max} requirement demands a higher received powers.

Figure 7.7 shows us how to allocate transmit powers to different traffic substreams. We observe that for the "v" traffic type with a tighter D_{max}^{v} requirement, we have to allocate a higher transmit power to the substreams which is an increasing function of the distance from the base station. And Figure 7.9 shows that the utilities achieved for the "v" traffic type is lower than those achieved for "d" traffic type. Obviously the utilities decreases with the increase of the distance from base station.

7.2.2 Scenario 2: Traffic Sources with Orthogonalization

Simulation results are plotted in Figures 7.2, 7.4, 7.6, 7.8 and 7.10 for SIR, expected PSR, probability of delay requirement, received powers, transmit powers and utilities at equilibrium respectively.

From the results in Figure 7.2 and 7.4, similarly, "v" substreams have a higher SIR, higher expected PSR and lower probability of delay requirement. SIR at equilibrium is higher as compared to the scenario without orthogonalization.

From the results in Figure 7.6, similarly, the received powers are different for different traffic types. But within the same traffic type, the received powers are the same. Received powers at equilibrium are much lower as compared to the scenario without orthogonalization.

Figure 7.8, similarly, shows transmit powers as increasing functions of user's distance to the serving base station. Transmit powers for both "v" and "d" types of traffic are much lower as compared to the scenario without orthogonalization. As shown in Figure 7.10, the utilities achieved for both "v" and "d" traffic substreams are much higher as compared to the scenario without orthogonalization.

7.2.3 Summary of Comparisons

Comparing simulation results for scenarios with and without orthogonalization, in general, orthogonalization increases SIRs at equilibrium, substantially reduces transmit powers and significantly increases user utilities achieved at equilibrium. All of these are attributed to the elimination of the interference between the traffic substreams originated from a same user terminal.

In more details, the improvements achieved by utilizing orthogonalization technologies are:

1. Orthogonalization improves SIRs by roughly 25% for "v" type of traffic, and 40% for "d" type of traffic.
2. Orthogonalization reduces transmit powers substantially by around 40%. The reduction of transmit powers increases with user's distance to serving base station. This helps to mitigate the destructive effects of users close to cell borders.
3. Orthogonalization increases user utilities substantially by roughly 50% to 70%. The improvement on utilities decreases with user's distance to serving base station. So this improvement benefits the users close to base station the most.

Furthermore, because of above results, with orthogonalization, additional users can be supported by the same system. Or, more modes from multimodal collaboration terminals can be supported in multimodel collaboration wireless netwroks.

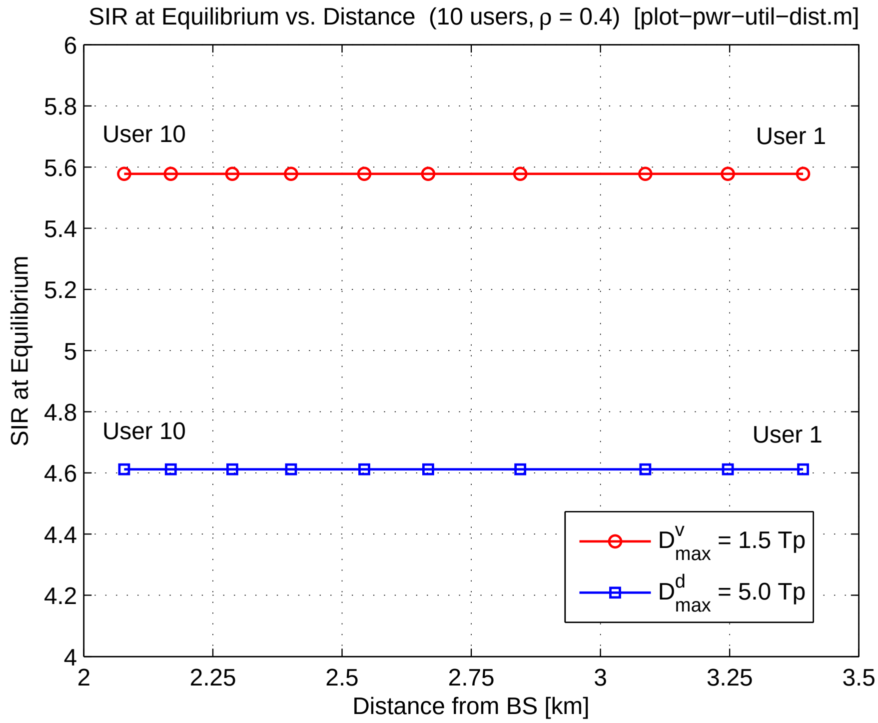

Figure 7.1: SIR at equilibrium in multimodal collaboration environment: scenario without orthogonalization.

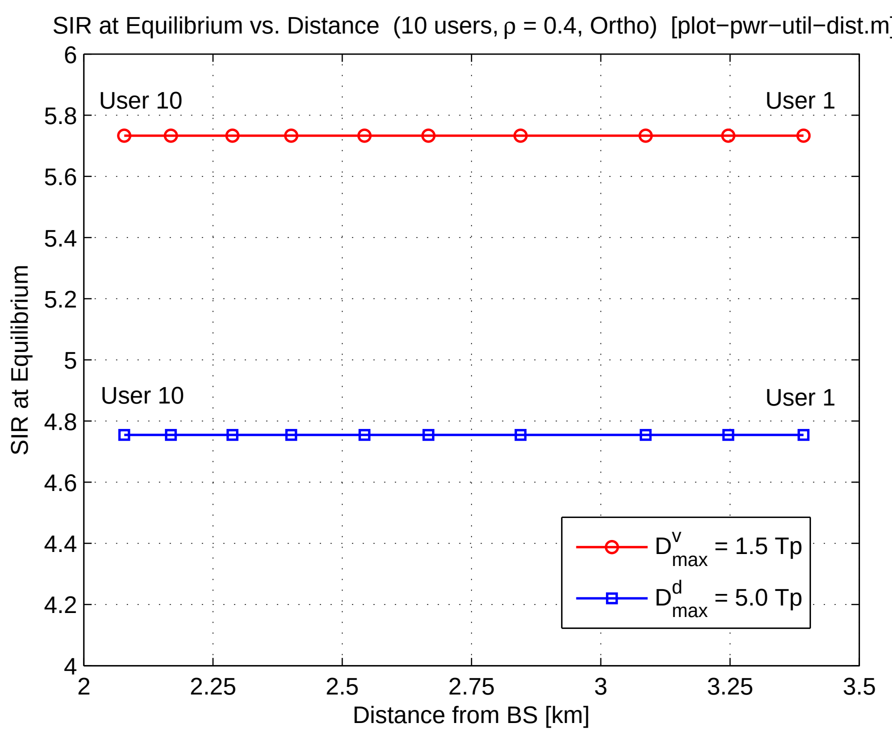

Figure 7.2: SIR at equilibrium in multimodal collaboration environment: scenario with orthogonalization.

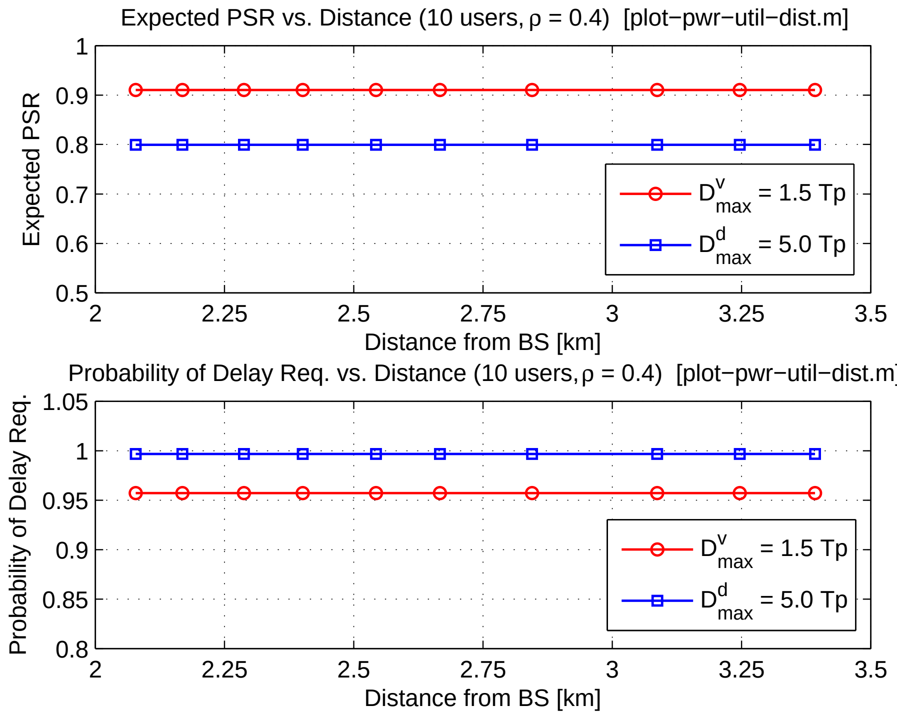

Figure 7.3: Expected PSR and Probability of Delay Requirement at equilibrium in multimodal collaboration environment: scenario without orthogonalization.

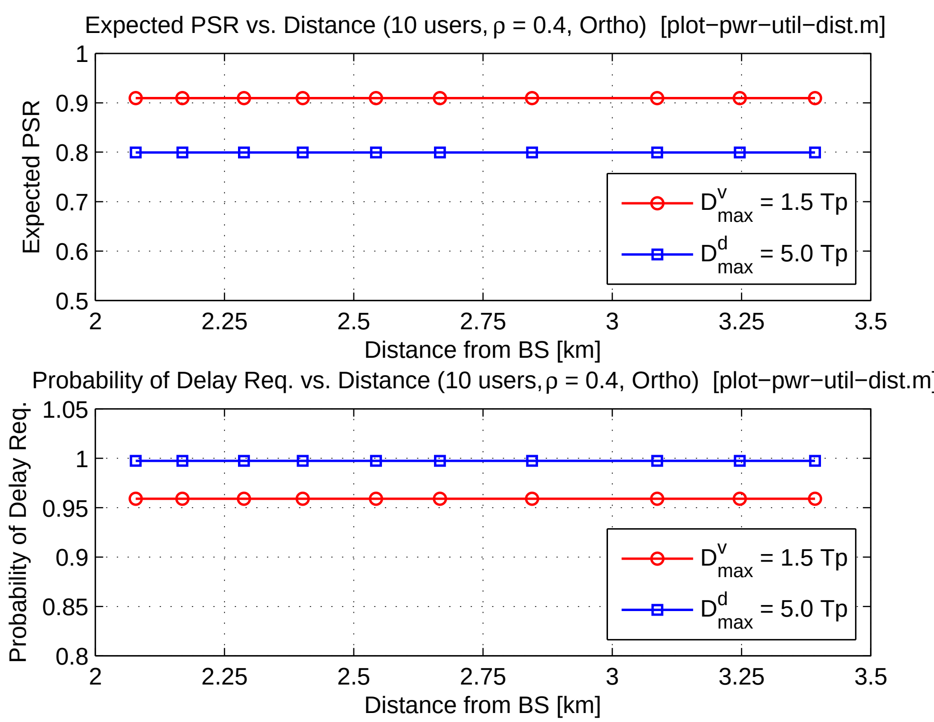

Figure 7.4: Expected PSR and Probability of Delay Requirement at equilibrium in multimodal collaboration environment: scenario with orthogonalization.

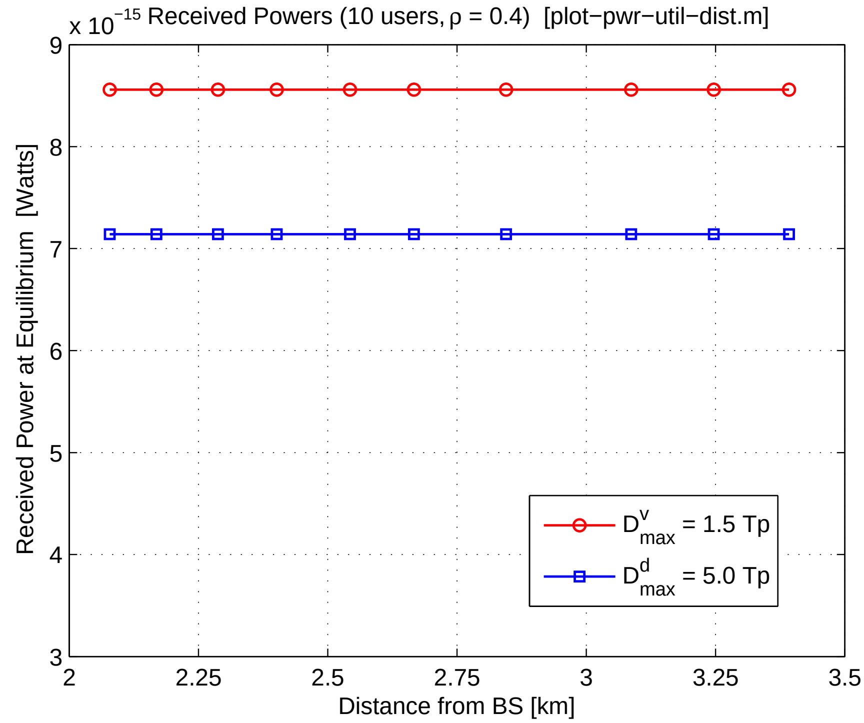

Figure 7.5: Received powers in multimodal collaboration environment: without orthogonalization.

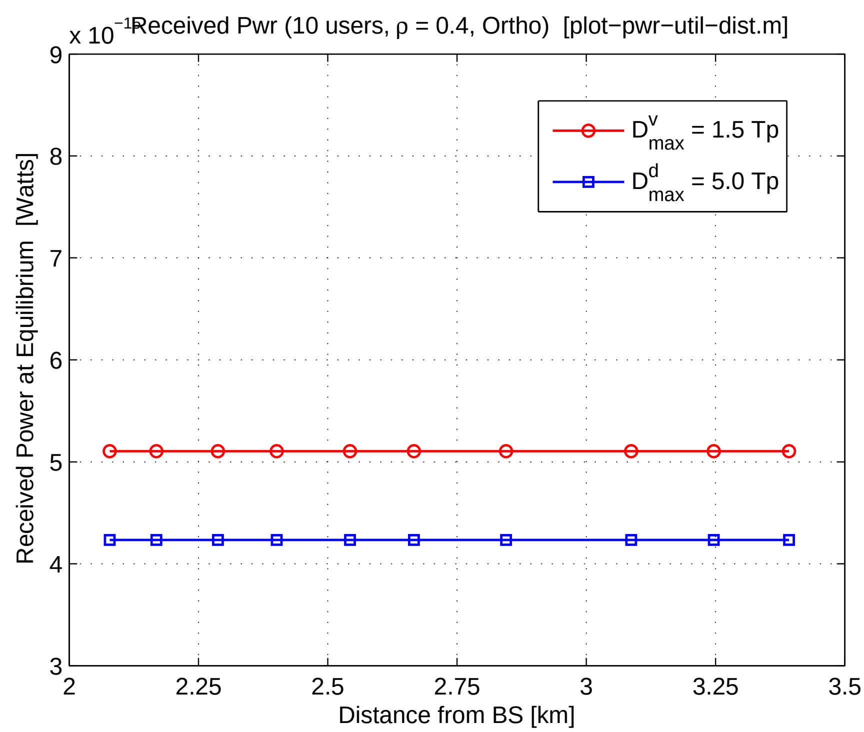

Figure 7.6: Received powers in multimodal collaboration environment: with orthogonalization.

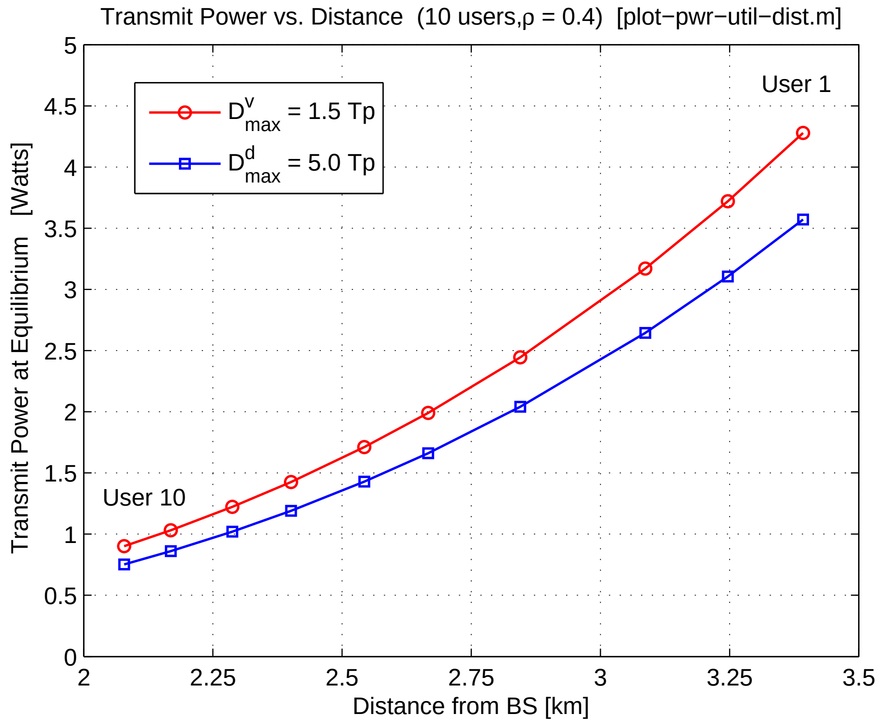

Figure 7.7: Transmit powers in multimodal collaboration environment: without orthogonalization.

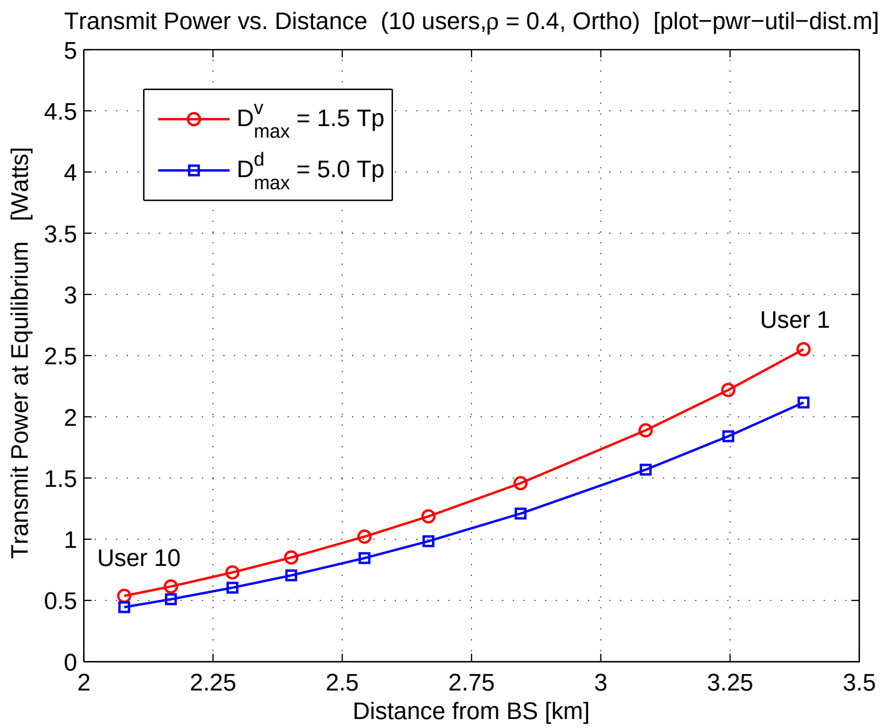

Figure 7.8: Transmit powers in multimodal collaboration environment: with orthogonalization.

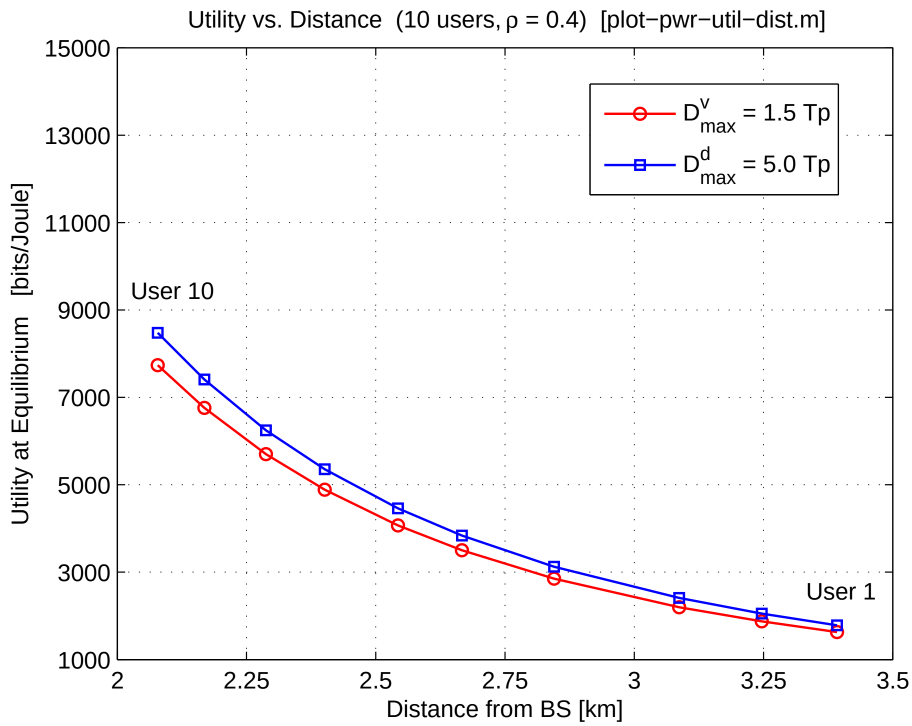

Figure 7.9: Utilities in multimodal collaboration environment: without orthogonalization.

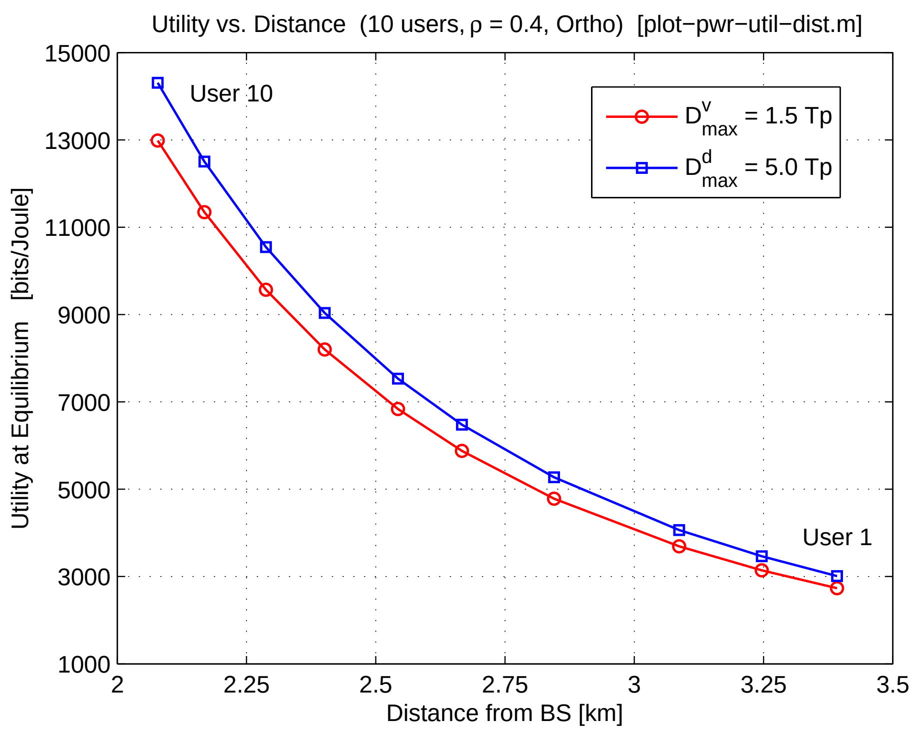

Figure 7.10: Utilities in multimodal collaboration environment: with orthogonalization.

Chapter 8

Performance Analysis with Slot-By-Slot Power Control Benchmarks

In this chapter, we will evaluate the performance of the proposed uplink power control scheme through comparative studies with respect to *performance benchmarks* – two idealized uplink power control strategies.

The power control scheme proposed in this work is basically an *average strategy*, meaning that the allocation of transmit power for a user is driven only by the local measurements of the mean and variance of the random interference produced by bursty traffic from all other users. The performance of the proposed scheme is best in statistical sense that the *expected utilities* are maximized for every individual user in the network. Because the proposed uplink power control scheme for packet-switched wireless data services is relatively new, it is hard to find an appropriate reference scheme to use as a basis for comparison on performance.

In an attempt to establish a reference scheme, in this chapter, we first design an idealized power control strategy for packet-switched wireless data CDMA networks. In contrast to the *average strategy*, this newly-created scheme is an *instantaneous strategy*. Technically, the transmit powers are adjusted on a time-slot by time-slot basis in this instantaneous strategy, hence the scheme is named as a *Slot-By-Slot* (SBS) power control strategy. Based on this ideal SBS strategy, the utility performance, or (the *average utility* of two power control schemes proposed in previous research is evaluated and serves as performance benchmarks. Finally the performance of the average power control strategy proposed in this work is compared against those two SBS performance benchmarks.

8.1 The Slot-By-Slot Power Control Strategy

Let us take a close look at what we mean by *Slot-By-Slot (SBS) power control strategy*. Basically, by SBS, we mean that signal measurements can be done at time-slot level or within each packet interval. Note that the size of a time-slot is assumed to be equal to the length of a packet in this study. Thus interference information such as the mean and variance on a wireless link is instantly available to each mobile terminal in every time-slot. Therefore, decisions on the allocation of transmit power in response to current interference can be made by each user in current time-slot.

Ideally and precisely, we made following main assumptions for SBS strategy:

1. Base station can measure the strength of the received signals, interference and noise in a time-slot in real-time, i.e., within current time-slot, on slot-by-slot basis.
2. Results of measurement by base station can be fed back to users instantaneously, i.e., within current time-slot, on slot-by-slot basis. The results are transmitted back to users error-free.
3. Network knows the exact number of simultaneous packets on a wireless link in every time-slot (with some sort of network intelligence or magic!).

The above assumptions are obviously unrealistic. But they are made for the purpose of establishing the SBS reference scheme for performance analysis only.

Under the above idealized assumptions, *perfect* transmit powers can be allocated to each user of the system to achieve a specific QoS target. Those perfect powers change from packet to packet (from time-slot to time-slot) as the random on-off packet traffic from different users comes in and goes out on wireless links. This impeccable power control scheme would show us what a power control strategy could do at its best in the extreme if technology permits. Therefore, the study of this idealized strategy will establish a baseline for performance evaluation of the *average power control strategy* for packet-switched wireless data systems, such as the one that we are proposing in this work.

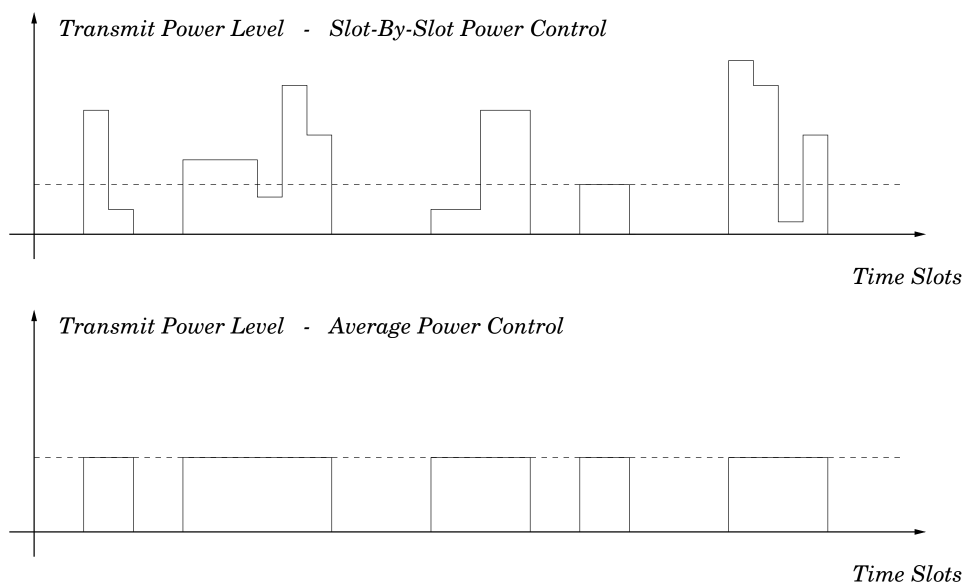

Figure 8.1: Conceptual illustration of the slot-by-slot power control strategy and the average power control strategy for packet-switched data systems — transmit power levels vs. time-slots.

Figure 8.1 is a conceptual illustration of both a slot-by-slot power control strategy and an average power control strategy in an arbitrary measurement window for the average strategy. As shown in the figure, the slot-by-slot power control strategy reacts to the instantaneous bursty interference from time-slot to time-slot. While the average power control strategy allocates transmit powers for users that reacts only to the *mean* and *variance* of the total interference and background noise received at base station.

Practically, it is not possible to achieve an acceptable accuracy if this type of measurements are done only within a single time-slot. A window of considerable time-slots are usually required to accumulate the observation on a random process before statistical parameters of the process can be estimated with confidence.

In the analysis of the SBS power control strategy, an important quantity is the number of interfering packets, or the number of simultaneous packets from *other* users, on a wireless link in a time-slot. Clearly, this number is random in nature. So, it is equally important to understand the probability distribution of this number which is defined as below:

Definition 8 (Number of Interfering Packets) *The number of interfering packets is the total number of packets produced simultaneously by all other user terminals in the system in a same time-slot:*

$$s \triangleq \sum_{j=1,\, j\neq i}^{N} x_j \tag{8.1}$$

where x_j is the on-off random variable for the packet activity of user j, and i denotes the index of target user who is actively transmitting. N is the total number of mobile users in the system.

Hence, the quantity s is the sum of $N-1$ on-off random variables. Clearly, it conforms to Binomial probability distribution [108] as below:

$$Prob(s) = \binom{N-1}{s} \rho^s \, (1-\rho)^{(N-1)-s} \quad \S \tag{8.2}$$

§ Recall that it is assumed that the packet activity probabilities of all users in the system are the same in this study, i.e., $Prob(x_j = 1) = \rho$ and $Prob(x_j = 0) = 1 - \rho$, $\forall\, j \in [1, \cdots, N]$.

with the mean and variance of random variable s as

$$\mu_s = (N-1)\,\rho\,, \tag{8.3}$$

$$\sigma_s^2 = (N-1)\,\rho\,(1-\rho)\,. \tag{8.4}$$

In the following sections, particularly, we apply the slot-by-slot strategy concept described above to two power control algorithms for wireless data services proposed in research published previously. Then we study the utility performance of those two idealized SBS power control schemes. The two power control schemes that we considered here are:

- **SBS-NAPC** — slot-by-slot power control based on the "Network-Assisted Power Control" algorithm proposed in [14].
- **SBS-DPC** — slot-by-slot power control based on the "Distributed Power Control" algorithm introduced in [13].

For easy reference and simple notations, relating to above abbreviations (**SBS-DPC**, **SBS-NAPC**), we name the power control scheme studied in this work as **A-DPC** which stands for *Average Distributed Power Control*.

- **A-DPC** — average power control based on the "Distributed Power Control".

The optimum utilities achieved with slot-by-slot strategies - **SBS-NAPC** and **SBS-DPC** will be used as baselines to compare with those achieved by **A-DPC** – the average strategy studied in this work.

In subsequent discussions, notice that we have some common assumptions as below:

1. all system parameters are assumed to have the same values as in Table 6.1, the list of Simulation Parameters.

2. to compare the performance of SBS-NAPC, SBS-DPC and A-DPC on the same ground, we assume that the improved version of utility model based on ETPR (Earned Throughput-to-Power Ratio) as in (2.23) is utilized for all three power control strategies.

3. performance analysis and studies are all based on the equilibrium results from the utility-maximizing power control games after their convergence.

8.2 Slot-by-Slot Strategy based on Network-Assisted Power Control

The concept and algorithm of Network-Assisted Power Control (NAPC) for wireless data are introduced in research [14]. A similar utility model as in Equation (2.3) is utilized in NAPC while replacing "Efficiency Function" with the regular PSR. NAPC algorithm maximizes the utilities for users while maintaining equal SIR for all users. Transmit powers are controlled via SIR balancing with the assistance of the network that broadcasts the common SIR target [14].

In NAPC, the optimum target SIR γ_{opt} is obtained via the optimization of the utility over the choice of the target SIR, assuming that the powers received at base station from all users are the same. The optimum target SIR γ_{opt} is derived as a decreasing function of the number of users in the system, N, which is assumed to be known [14]. Thus with NAPC, the system can take as many users as possible until the target SIR γ_{opt} drops below certain level such that the performance degradation make it unbearable for users to keep staying in the system.

8.2.1 The Design of SBS-NAPC Strategy

To apply the slot-by-slot (SBS) power control strategy concept to NAPC, we assume ideally that the network knows the *exact* number of simultaneous packets (i.e., the number $s+1$) in *every* time-slot. Then an optimum target SIR γ_{opt} can be calculated for each time-slot, which decreases as the number of simultaneous packets increases.

Given a number for s, specifically, the value of γ_{opt} can be found by optimizing the utility at equilibrium with respect to the received power P_{rec} which is the same for all users at equilibrium. The solution gives the nonlinear equation below for the computation of γ_{opt}:

$$2\, e^{\frac{\gamma_i^*}{2}} \;=\; 1 \,+\, \frac{M}{2} \left(\gamma_i^* \,-\, s\,\frac{R}{W}\,{\gamma_i^*}^2 \right) . \quad ^{\S} \tag{8.5}$$

Derivation of Equation (8.5) is omitted here, which can be found in [14]. Notice that γ_i^* denotes the SIR at equilibrium of the power control game.

For the case involving the improved version of utility model with ETPR, the equation for computing γ_{opt} can be derived as below:

$$f(\gamma_i^*) - f(0) \;=\; \frac{d\,[f(\gamma_i^*)]}{d\,\gamma_i^*}\,\frac{d\,\gamma_i^*}{dP_{rec}^*}\,P_{rec}^* \quad ^{\P} \tag{8.6}$$

where P_{rec}^* is the received power at base station at equilibrium of the power control game. It is equal for all users and can be expressed as a function of γ_i^* and s as below:

§ This equation for the solution of γ_{opt} is based on the utility model with TPR (Throughput-to-Power Ratio), *i.e.*, $\frac{f(\gamma_i)}{p_i}$.

¶ This equation for the solution of γ_{opt} is based on the improved utility model with ETPR (Earned Throughput-to-Power Ratio), *i.e.*, $\frac{f(\gamma_i)-f(0)}{[1-f(0)]\,p_i}$.

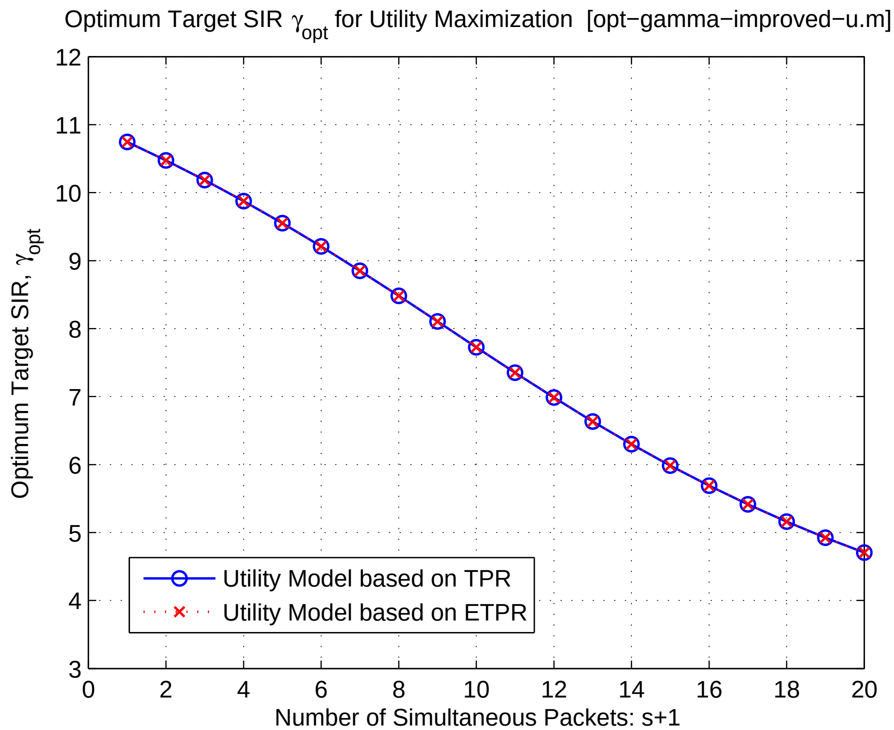

Figure 8.2: The optimum target SIR γ_{opt} in SBS-NAPC as a function of the number of simultaneous packets in a time-slot. The difference is trivial in results of utility model with TPR (Throughput-to-Power Ratio) and that with ETPR (Earned TPR).

$$P^*_{rec} \;=\; \frac{\sigma_i^2\,\gamma_i^*}{W/R - s\,\gamma_i^*}\,. \tag{8.7}$$

Figure 8.2 plots the numerical solutions for the optimum SIR target γ_{opt} for SBS-NAPC as a function of the total number of simultaneous packets in a time-slot, i.e., $s+1$. Both cases based on different definitions of utility model (with TPR and with ETPR) are shown in the figure. The optimum SIRs γ_{opt} from both cases appear to differ trivially.

In the study on SBS power control strategies, there are two aspects of the parameter s, the number of interfering packets in a time-slot, that should be emphasized here:

1. Within a time-slot, s is assumed to be known to the network. Hence, s is a *deterministic number* in a time-slot;

2. From a time-slot to next time-slot, s is changing randomly, depending on the packet on-off activities of user traffic sources. Hence, s is a *random number* from time-slot to time-slot.

Given the number s and the optimum target SIR γ_{opt}, with the assumption on equal received powers from all users at base station, the received power can be found by the equation below:

$$P^*_{rec}(s) \;=\; \frac{\sigma_i^2}{\dfrac{W}{R}\dfrac{1}{\gamma_{opt}(s)} - s}\,. \tag{8.8}$$

where notation $\gamma_{opt}(s)$ indicates that the optimum target SIR γ_{opt} is a function of s, and received power $P^*_{rec}(s)$ is hence also a function of s.

8.2.2 Utility Achievable by SBS-NAPC

Based on the utility definition for circuit-switched data systems as in (2.3), with the optimum target SIR γ_{opt} and received power $P^*_{rec}(s)$ (8.8), the optimum utility achieved for SBS-NAPC in a time-slot is given by

$$u_i^{\textit{SBS-NAPC}}(s) \;=\; \frac{L}{M}\, R\, \frac{f(\gamma_{opt}(s)) - f(0)}{1 - f(0)}\, \frac{h_i}{P^*_{rec}(s)}\,. \;^{\ddagger} \tag{8.9}$$

which is obviously a function of s.

‡ Note that the improved version of utility function with ETPR is always used in performance analysis.

This utility function for SBS-NAPC is *random* on a slot-by-slot basis, because the number of interfering packets s in a time-slot is a random number that changes from time-slot to time-slot.

Finally, using the probability distribution of the number s in Equation (8.2) and $P^*_{rec}(s)$ expression in (8.8), the *expected* optimum utility for SBS-NAPC scheme can be calculated by averaging the random optimum utility in a time-slot in (8.9) over all the possibilities of the number of simultaneous interfering packets as follows:

$$\overline{u}_i^{\textit{SBS-NAPC}} = \sum_{s=0}^{N-1} \binom{N-1}{s} \; u_i^{\textit{SBS-NAPC}}(s) \;\; \rho^s \, (1-\rho)^{(N-1)-s} \tag{8.10}$$

$$= \sum_{s=0}^{N-1} \binom{N-1}{s} \frac{L}{M} \; R \; \frac{f(\gamma_{opt}(s)) - f(0)}{1 - f(0)} \; \times$$

$$\times \; \frac{h_i}{\sigma_i^2} \left[\frac{W}{R} \frac{1}{\gamma_{opt}(s)} \; - \; s \right] \rho^s \, (1-\rho)^{(N-1)-s} \; . \tag{8.11}$$

In general, the value of this expected optimum utility represents the best utility achievable on average in any time-slot by the SBS-NAPC power control strategy. This utility will be used as one of the benchmarks for performance evaluation.

8.3 Slot-by-Slot Strategy based on Distributed Power Control

Based on the utility-based power control scheme for wireless circuit-switched data services proposed in a previous work [13, 38, 39, 40], we establish a slot-by-slot (SBS) power control scheme for wireless packet-switched data services. We name this SBS scheme as SBS-DPC, where DPC stands for Distributed Power Control.

Because in essence, the utility-based power control introduced in [13, 38, 39, 40] is a distributed power control scheme based on a non-cooperative game formulation.

Similarly as in the discussion of SBS-NAPC scheme in previous section, it is still assumed that the network knows the exact number of simultaneous interfering packets in every time-slot, the number s. In SBS-DPC strategy, in each time-slot, there are $s+1$ number of packets competing with each other for network resources. Given that, we play a power control game to determine what transmit power to allocate for each mobile terminal that is actively transmitting a packet in that time-slot (one of the $s+1$ simultaneous packets in that time-slot). Only two possibilities for the outcome for playing the power control game: either the game converges or diverges.

8.3.1 The Design of SBS-DPC Strategy

In the study of the utility-based circuit-switched wireless data power control [13, 38, 39, 40], an *interference threshold* model is actually adopted behind the scene. This model indicates that as long as the number of users is less than and equal to the uplink capacity of a single CDMA cell, packets sent by users can be received correctly at base station via transmission/retransmissions. However, when the number of users exceeds the capacity, all the packets transmitted by users are considered lost due to unrecoverable errors from data corruption with too-much interference in a over-congested system.

The capacity mentioned above is defined here as the maximum number of users supported by the uplink of a single CDMA cell to achieve a common target SIR Γ^* for all users. This capacity is denoted as C in this study.

Let us consider the utility-based circuit-switched data power control in the perspective of a SBS strategy. In a time-slot, when the total number of packets $(s+1)$ is less than and equal to the capacity C, the power control game always

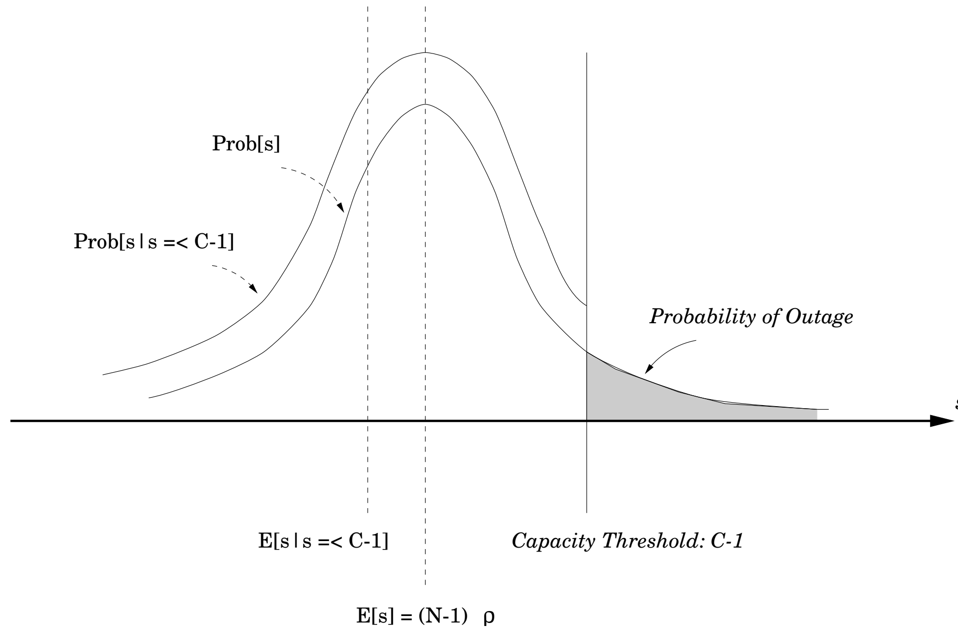

Figure 8.3: Illustration on the distribution of the number of simultaneous interfering packets s in a time-slot, and its relationship to capacity of the system (denoted as C) and probability of outage.

converges to a fixed optimum SIR $\Gamma^* = 12.42$ [§] or $\Gamma^* = 10.745$ [¶], and users achieve their optimum utilities in the sense of Nash equilibrium [38].

Whenever the number $(s+1)$ is larger than the capacity C, the power control game will never converge — it is then a disaster for both users and system since no one in the system can ever get any utility out of the system. This represents an *outage* state for the scheme in which all users in the system get a zero utility.

Figure 8.3 depicts the distribution of the random number s and the *probability of outage* P_{out}, assuming the interference is Gaussian-distributed as we always did before. Illustrated in Figure 8.3 is also the conditional distribution of the quantity s conditioned on the non-outage event $(0 \leq s \leq C-1)$ which is actually a truncated version of the probability $Prob(s)$:

[§] When Efficiency Function $f_e(\gamma_i)$ is used in the utility model.

[¶] When ETPR, $\frac{f(\gamma_i)-f(0)}{[1-f(0)]\,p_i}$, is used in the improved utility model.

$$Prob(\, s \,|\, 0 \leq s \leq C-1 \,) \;=\; \frac{\binom{N-1}{s} \, \rho^{s} \, (1-\rho)^{(N-1)-s}}{\sum_{s=0}^{C-1} \binom{N-1}{s} \, \rho^{s} \, (1-\rho)^{(N-1)-s}} \,. \tag{8.12}$$

From above discussions, we know that the packet success rate (PSR) in a time-slot for the SBS-DPC scheme can have two possibilities as below:

$$PSR \;=\; \begin{cases} f(\Gamma^{*}) & when \;\; s \leq C-1 \,, \\ 0 & when \;\; s > C-1 \,. \end{cases} \tag{8.13}$$

For the improved version of utility model based on ETPR, corresponding to the above relationship, we have similar expression for ETPR:

$$ETPR \;=\; \begin{cases} \dfrac{f(\Gamma^{*}) - f(0)}{[1-f(0)]} \, \dfrac{1}{p_i} & when \;\; s \leq C-1 \,, \\ 0 & when \;\; s > C-1 \,. \end{cases} \tag{8.14}$$

This leads to the following utility definition for user i in a time-slot for the SBS-DPC scheme:

$$u_i^{\textit{SBS-DPC}}(s) \;=\; \begin{cases} \dfrac{L\,R}{M\,p_i^{*}(s)} \, \dfrac{f(\Gamma^{*}) - f(0)}{1-f(0)} & for \;\; s \leq C-1, \quad with\ prob.\ (1-P_{out}) \\ 0 & for \;\; s > C-1, \quad with\ prob.\ P_{out} \end{cases} \tag{8.15}$$

where $p_i^{*}(s)$ is the transmit power for user i given that the number of interfering packets is s. The expression for the probability of outage is

$$P_{out} \;=\; Prob\,(\,s > C-1\,) \;=\; \sum_{s=C}^{N-1} \binom{N-1}{s} \rho^{s}\,(1-\rho)^{(N-1)-s}\,. \tag{8.16}$$

Notice that although the system would get into the outage state with some probability, the packets lost in the outage state in a time-slot would eventually come back in later time-slot for retransmissions until they are successfully received at the base station.

Taking the effect of outage event into consideration, the average packet success probability, denoted as P_{succ} ($P_{succ} = E[PSR]$), is now:

$$\begin{aligned} P_{succ} &= 1-(PER+P_{out}\,)-PER\times P_{out} \\ &= (1-PER)\times(1-P_{out}\,) \\ &= f(\Gamma^{*})\times(1-P_{out}). \end{aligned} \tag{8.17}$$

where PER is the packet error rate and $(1-PER\,) = PSR$ is the original packet success rate which is represented by $f(\Gamma^{*})$.

Similarly, for the improved version of utility model based on ETPR, corresponding to the above relationship, we have similar expression in the case for ETPR:

$$P_{succ}^{\mathrm{EPTR}} \;=\; \frac{f(\Gamma^{*})-f(0)}{[1-f(0)]}\times(1-P_{out}). \tag{8.18}$$

As we know from the earlier discussion on the modeling of packet-data utility, the number of transmission/retransmissions needed to send a packet successfully, the number k, is geometrically distributed:

$$p_k(k) \; = \; (1 - P_{succ})^{k-1} \, P_{succ} \, . \tag{8.19}$$

Hence, considering the packets lost in outages, the average transmission delay of a user packet, i.e., the average number of transmissions required to get a packet through, is

$$E[k] \; = \; \frac{1}{P_{succ}} \; = \; \frac{1}{f(\Gamma^*) \times (1 - P_{out})} \, . \tag{8.20}$$

8.3.2 Utility Achievable by SBS-DPC

The average optimum utility of user i for SBS-DPC power control strategy can be found by averaging the random optimum utility in a time-slot, $u_i^{\textit{SBS-DPC}}(s)$ as defined in Equation (8.15), over the possibility of outage and over all the possibilities of the number of interfering packets, as in the following formula: [§]

$$\overline{u}_i^{\textit{SBS-DPC}} \; = \; E_s[\; E_{out\,|s}[\; u_i^{\textit{SBS-NAPC}}(s) \mid s \;]\;] \tag{8.21}$$

$$\begin{aligned} &= \; E_s\left[\frac{L}{M} \; R \; \frac{f(\Gamma^*) - f(0)}{[1 - f(0)]} \, (1 - P_{out}) \; \frac{h_i}{P^*_{rec}(s)} \right] \\ &= \; \frac{L}{M} \; R \; \frac{f(\Gamma^*) - f(0)}{[1 - f(0)]} \, (1 - P_{out}) \; h_i \; E_s\left[\frac{1}{P^*_{rec}(s)} \right] \end{aligned} \tag{8.22}$$

§ $E_s[\cdot]$ denotes the expectation taken with respect to s, and $E_{out\,|s}[\cdot|s]$ denotes the conditional expectation taken with respect to the outage event and conditioned on s.

where the received power $P^*_{rec}(s)$ given s for SBS-DPC is

$$P^*_{rec}(s) = \frac{\sigma_i^2}{\frac{W}{R}\frac{1}{\Gamma^*} - s} . \tag{8.23}$$

Note that Γ^* is a fixed number in SBS-DPC scheme.

Using the truncated probability distribution of s given $(0 \le s \le C-1)$ in Equation (8.12), the expectation on the factor of the reciprocal of the received power $P^*_{rec}(s)$ with respect to s in Equation (8.24) can be obtained.

Finally, plugging the expression of $P^*_{rec}(s)$ in Equation (8.23), the formula to calculate the expected optimum utility for SBS-DPC power control strategy can be written as below:

$$\begin{aligned} \overline{u}_i^{\textit{SBS-DPC}} &= \frac{L}{M}\, R\, \frac{f(\Gamma^*) - f(0)}{[1 - f(0)]}\,(1 - P_{out})\; h_i \;\times \\ & \frac{1}{\sigma_i^2}\left\{ \frac{W}{R}\frac{1}{\Gamma^*} - \sum_{s=0}^{C-1}\left[\frac{s\binom{N-1}{s}\rho^s\,(1-\rho)^{(N-1)-s}}{\sum_{s=0}^{C-1}\binom{N-1}{s}\rho^s\,(1-\rho)^{(N-1)-s}}\right]\right\} . \end{aligned} \tag{8.24}$$

where the last term inside the braces is actually the conditional mean of random number s conditioned on $(0 \le s \le C-1)$, which can be simply written as $E_{s\,|\,0 \le s \le C-1}[\,s\,|\,0 \le s \le C-1\,]$.

The relative positions of the the mean and the conditional mean of the random number s are also illustrated in Figure 8.3 as dotted lines.

In general, the value of the above average optimum utility $\overline{u}_i^{SBS\text{-}DPC}$ in Equation (8.24) represents the best utility achievable on average in any time-slot by the SBS-DPC power control strategy. This utility will be used as one of the benchmarks for performance evaluation.

8.4 Performance Evaluation: Slot-by-Slot Strategies vs. Average Strategy

In this section, we look at the average optimum utilities (bits/Joule) achieved by the SBS power control strategies (the SBS-NAPC and SBS-DPC strategies) for packet-switched wireless data systems as discussed in previous sections. We then compare these average optimum utilities with those achieved by A-DPC - the Average Distributed Power Control strategy, for packet-switched wireless data systems, i.e., the power control scheme proposed and studied in this work.

After presenting the results, we will discuss them and draw conclusions based on those results.

8.4.1 Results of Numerical and Simulation Studies

For the slot-by-slot strategies, SBS-NAPC and SBS-DPC, numerical studies are conducted based on the analysis done in previous sections.

For A-DPC strategy, to be able to compare with SBS strategies on the value of utilities, we assume that there is no packet delay constraint (*i.e.*, $D_{max} \to \infty$). Therefore, the delay probability factor in the packet-data utility model for A-DPC as in Equation (2.23) becomes 1, and so the factor vanishes.

In A-DPC scheme, the transmit power is allocated according to the measurements of the mean and variance of the total interference plus background AWGN noise, as well as the burstiness of packet traffic sources in terms of ρ. The *expected* PSR is approximated by Holtzman's formula as described in Equation (3.17), and interference distribution is approximated as Gaussian. The packet-data utility model is then built with ETPR for improvement. The optimum values of the *expected* utilities are computed via simulations of the utility-maximizing power control game.

Both the numerical and simulation studies are conducted under two system loading scenarios with 10 users and 20 users. Study results on the maximum average utilities achieved are plotted with respect to the activity probability for user's packet transmission, *i.e.*, $\rho_i = \rho$, $\forall i \in [1 \cdots N]$. Note that as we did before, for simplicity, ρ_i is assumed to be the same for all users.

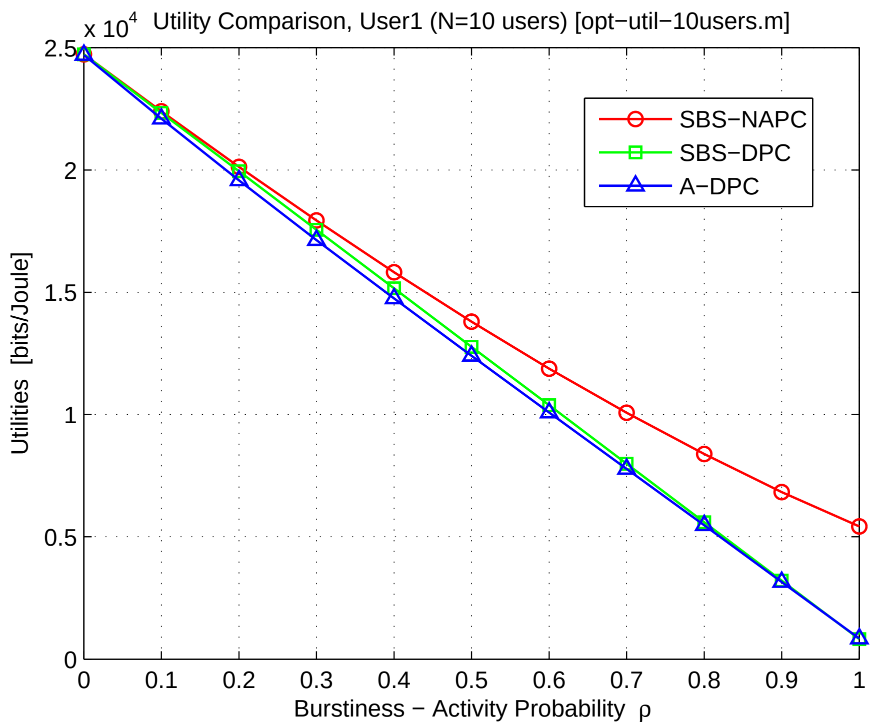

Figure 8.4: Comparisons of Utilities achieved by three strategies (**SBS-NAPC**, **SBS-DPC** and **A-DPC**) for user 1 (the farthest user) in an $N = 10$ user system.

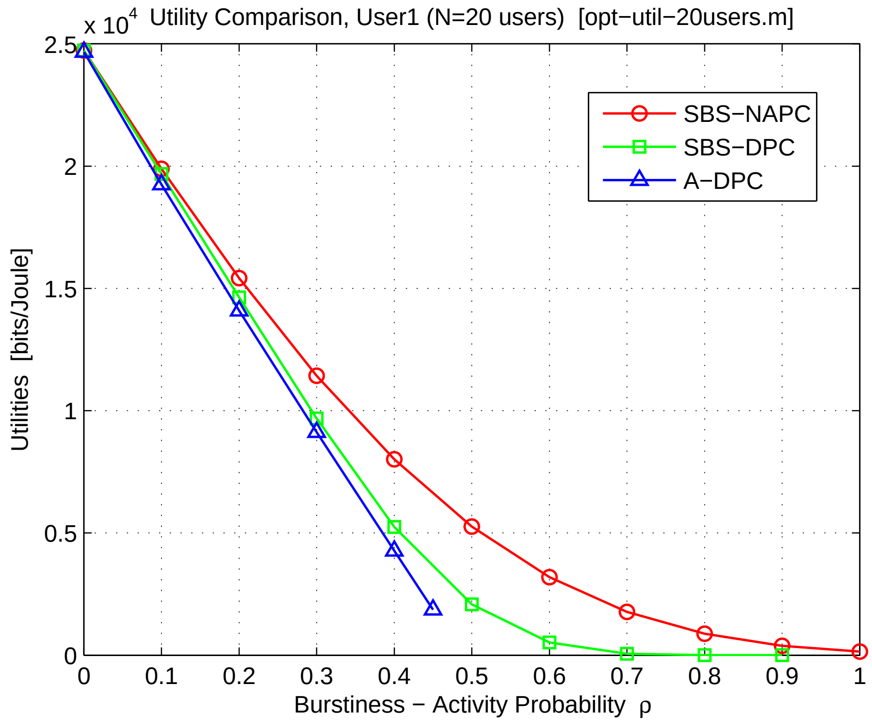

Figure 8.5: Comparisons of Utilities achieved by three strategies (**SBS-NAPC**, **SBS-DPC** and **A-DPC**) for user 1 (the farthest user) in an $N = 20$ user system.

Figures 8.4 and 8.5 ¶ summarize the study results by plotting the maximum average utilities achieved by the three power control strategies (SBS-NAPC, SBS-DPC and A-DPC) with respect to activity probability of packet traffic, respectively for $N = 10$ and $N = 20$ user population scenarios. Those results are discussed in next subsection.

8.4.2 Comparisons and Discussions of Three Strategies

Generally, as we expected, Figures 8.4 and 8.5 show that slot-by-slot *"instantaneous"* strategies (SBS-NAPC and SBS-DPC) performs better than the average strategy (A-DPC). Additionally, centralized strategy (SBS-NAPC) performs better

¶ For fair comparisons among three strategies, EPTR is assumed in the utility modeling for all three strategies. Therefore, EPTR is used in the computing of the utilities in those figures.

than the distributed strategies (SBS-DPC and A-DPC).

Specifically, as shown in Figures 8.4 and 8.5, the utility performance of A-DPC is very close to that of SBS-DPC.

Those results are purely from the value of utility point of view. However, from practical point of view, distributed strategies are easier to implement, since they do not need any system intervention and coordination effort. In more detail, we compare and discuss the three power control strategies and their average utility performance as presented in Figures 8.4 and 8.5 in the following aspects:

1. From the plots in Figures 8.4 and 8.5, we observe that SBS-NAPC strategy always outperforms the other two with the best average utilities among the three. In 10 user case, as the traffic activity increases, the advantage of SBS-NAPC over others grows accordingly. While for the 20 user case, as activity probability ρ moves toward 0.5 either from 0 or from 1, the advantage of SBS-NAPC also increases. It appears that at $\rho = 0.5$, SBS-NAPC achieves it best performance over the other two.

2. The advantage of SBS-NAPC is attributed to the fact that SBS-NAPC is a scheme that makes important information about operating status of the system available to all users in each time-slot, such as the number of simultaneous packets $(s + 1)$ and the optimum target SIR γ_{opt}. This type of intervention by system helps to promote the better coordination among users and system, resulting in more efficient allocation of radio resources. In this respect, SBS-NAPC looks like a centralized power control strategy that achieves better bits/Joule performance with assistance from the network side.

3. For SBS-DPC strategy, it is a distributed scheme because it is based on a non-cooperative power control game in each time-slot. Seemingly, there is no centralized control or coordination among users for this strategy. However, in

the design of SBS-DPC, the network has to know the number of simultaneous packets $(s+1)$ in each time-slot in order to play the power control game in that time-slot. Therefore, the network has to keep track of the current $(s+1)$ from time-slot to time-slot. SBS-DPC is a distributed strategy by its name only because this SBS strategy originally comes from a distributed power control scheme.

4. Figures 8.4 and 8.5 show that the average utility performance of the SBS-DPC strategy is between that of SBS-NAPC and that of A-DPC. Observe that for low packet burstiness (small ρ), all three strategies (SBS-NAPC, SBS-DPC and A-DPC) have small differences in their average utility performances. As we know, packet-switched data systems are usually designed to service a large number of users with low transmission activities. The closeness of the utility performances of the three strategies at small ρ region indicates the potential value of the proposed A-DPC power control strategy in practical application for wireless packet-switched data services.

5. It is also interesting to note that at $\rho = 0$ in particular, the performances of the three strategies are equal (an interesting case mathematically, but a useless situation practically, since $\rho = 0$ means that nobody in the system is actually transmitting anything). For $N = 10 \leq C$ scenario, the performances of SBS-DPC and A-DPC strategies also converges at $\rho = 1$. This is because at $\rho = 1$, packet-switched data systems behave in the same way as circuit-switched data systems do, as analyzed in Chapter 5, Section 5.4.

6. Keep in mind that both SBS-NAPC and SBS-DPC are idealized power control schemes which are designed only for the purpose of performance evaluation. With performance baselined by SBS-NAPC and SBS-DPC, so we know where A-DPC stands in terms of packet-data utilities.

Based on the above discussions and the results presented in Figures 8.4 and 8.5, we draw the following conclusions on the practical value of the average distributed power control (A-DPC) strategy proposed in this work:

- The comparisons on the average utility performances indicate that the utility achieved when each user adapts its power level to the average interference is not much lower than when it adapts to the instantaneous interference on slot-by-slot basis. This result shows the usefulness of the A-DPC power control strategy since it has practical value for implementation as compared to the other two idealized slot-by-slot strategies. This is an encouraging result because in practical systems, it would be difficult, if not impossible, for a mobile terminal to respond to the rapidly fluctuating interference of packet-switched data systems.

- The value of A-DPC also resides in its purpose. Since it is specifically designed for wireless packet-switched data CDMA networks, the results from this study would provide insights and guidance in the practical engineering of packet-switched wireless data networks.

Chapter 9

Implementation Issues

The adaptation of transmit powers in Average Distributed Power Control (A-DPC) proposed in this work is driven by the local measurement of the mean and variance of interference, $\tilde{\mu}_{Y_i}$ and $\tilde{\sigma}_{Y_i}$†. As indicated in Equation (4.56), the power control scheme also needs information on the average packet delay tolerance requirements of users, $D_{max,i}\ \ \forall\ i$. Given $D_{max,i}$, the offset and slope parameters for the linear approximation of the solution function, a_0 and a_1, can be computed. Based on Equation (4.56), the optimum allocation of transmit powers can then be determined.

As discussed in the previous chapter on performance study with slot-by-slot (SBS) strategies, the proposed power control scheme is an *average strategy*, meaning it only tracks the average interference in the systems. Quick variation of interference is smoothed out by the local measurement. But if the interference variation is small, this average scheme can do the job as good as the slot-by-slot power control schemes. This is clearly shown in Figures 8.4 and 8.5. For example, in a 10 users system, when the packet traffic burstiness — the activity probability $\rho \to 0$ and $\rho \to 1$ (corresponds to the zero interference variance scenario), the utilities achieved by the average strategy (i.e., A-DPC scheme) and by the slot-by-slot distributed scheme (i.e., SBS-DPC scheme) are the same. As the variance increases, the gap between the utilities achieved by the two schemes expands. The biggest difference

†Note: $(\tilde{\mu}_{Y_i},\ \tilde{\sigma}_{Y_i})$ denote the measured quantities of the mean and variance of interference $(\mu_{Y_i},\ \sigma_{Y_i})$.

between the utilities occurs at the peak of interference variance which corresponds to $\rho = 0.5$.

On the other hand, if the measurement of the mean and variance of interference can be done fast enough, then it is also possible that the A-DPC average power control scheme adjusts user transmit powers according to the measurement results on a slot-by-slot basis. Thus, the A-DPC power control scheme can track the quick variations of the interference.

Additionally, for the A-DPC power control scheme, if the number of users in the system N, the burstiness parameter ρ and the delay requirement $D_{max,i}$, $\forall\, i \in [1, \cdots, N]$ are all available via data collection, and if the user profile data collected possess certain regular patterns (e.g., daily, monthly, or geographically), then the interference measurements can be done with much less effort, because a regular pattern is much easier to track.

As for the complexity of the power control schemes, the A-DPC average scheme is much easier to implement than the SBS-NAPC and SBS-DPC slot-by-slot schemes as discussed in the previous chapter. For slot-by-slot schemes, to achieve the utility performance demonstrated in Figures 8.4 and 8.5, the network has to monitor the *exact* number of simultaneous packets (the number, $s+1$) on a wireless link in *every* time-slot and has to send the measurement results back to every user. In reality, this is a none trivial task. However, for the A-DPC average scheme, the network needs only to know the total number of mobile users in the system, the number N, which is trivial.

Furthermore, A-DPC needs to know the traffic activity patterns of mobile terminals in order to design the activity probability ρ_i for each individual user. This could be accomplished by monitoring user's activity patterns, analyzing user behaviors and correlating network and social events.

If it is not impossible, the implementation of the slot-by-slot schemes would demand a considerable amount of processing power to do the instant measurement of interference and feedback to all users in the system. The heavy overhead with a time-slot would definitely deem it impractical.

Finally, we should emphasize on the assumptions that we made in the study of this A-DPC packet-data power control scheme. We assumed that a BS receiver for a target user can make measurement on the mean and variance of the interference from all other interfering users. And we assumed that a large number of users are in the system. This assumption is used to justify the application of Gaussian approximation method in the modeling of the packet-data utility model. And we assumed that all users in the system become active similarly. This assumption is used to justify the same on-off activity probability, i.e., the same ρ, for all users. We should keep those assumptions in mind when we consider the practical aspects of the proposed power control schemes.

Chapter 10

Conclusion and Future Work

10.1 Conclusion

In this work, we investigated a radio resource allocation problem - the uplink transmit power control for wireless packet-switched data services, with a focus on energy efficiency for mobile terminals. Packet-switched data systems differ fundamentally from circuit-switched data systems in the burstiness of packet traffic and the connectionless nature of communications. Based on a utility-maximization approach in Microeconomics and prior work for wireless circuit-switched data services, we defined a probabilistic utility model as a performance metric for a wireless packet-switched data user, which takes into account both the effect of traffic burstiness and average packet delay QoS requirement for mobile users. Based on the above model, we studied a distributed power control strategy to simultaneously maximize the value of the utility for each individual user in the system.

Owing to the mutual-dependence of user performance in interference-limited wireless networks, a game-theoretic approach was taken to formulate the problem as a non-cooperative game. In general, the problem is complex and mathematically intractable. Utilizing several approximation techniques, the problem was reduced into tractable form and was studied both analytically and by simulations. We investigated the existence and uniqueness of the equilibrium solution of the power control game, and studied various characteristics of the power control scheme. For easy demonstration of the scheme, we used a CDMA network model as an example in the work.

The proposed power control scheme is an average strategy in the sense that it is driven by local measurements of the mean and variance of bursty interference. To evaluate the performance of the proposed power control scheme, we considered an ideal scenario where the level of interference is measured by base stations on a slot-by-slot basis and available instantly to users. We established two idealized slot-by-slot based power control strategies as performance benchmarks for the performance evaluation of our scheme.

In addition, we extended our study to multi-class communications in a wireless multimodal collaboration environment.

Our study on the uplink transmit power control for wireless packet-switched data services is concluded with the following points:

1. The proposed power control scheme is a *average strategy* that allocates transmit power to a mobile user according to:

    - the location of the mobile user;
    - the maximum tolerance of average packet transmission delay of user traffic;
    - the burstiness or on-off activities of packet traffic sources in the system;
    - the total number of mobile users in the system.
    - the *mean* and *variance* of the interference and background noise received at the user's base station receiver.

2. The non-cooperative game-based power control problem is shown to have a *unique* Nash equilibrium analytically and by simulations. The Nash equilibrium solution depends on both the user's traffic activity and user's QoS delay requirements.

- Under identical traffic burstiness and user delay requirements, the power control scheme converges to an unique Nash equilibrium — an unique signal-to-interference ratio (SIR) for all users in the system.
- For unequal delay requirements from different users, the scheme converges to an unique Nash equilibrium with unequal SIRs which correspond to the unequal delay requirements of users.

3. If a packet from a user has a tighter delay requirement, transmission of the packet will require a higher SIR and a higher transmit power, which adversely affects all other users by lowering their SIRs, increasing their transmit powers and decreasing their utilities. On the other hand, a user's increase of its delay tolerance will benefit the system by reducing the powers and increasing the utilities of remaining users.

4. The study on the uplink capacity of a packet-switched data CDMA system reveals one of the key differences between the engineering of a circuit-switched system and that of a packet-switched system. Special care has to be taken for packet-switched wireless systems to consider the impact of the *interference variability*, such that the system is engineered to work even at the worst scenario of interference.

5. The study can be easily applied to multi-class user system environment. As an application example, the study is extended to a multimodal collaboration environment where mobile terminals can communicate via two types of traffic substreams with two different QoS requirements, e.g., voice and data traffic. The optimum transmit powers are allocated to the traffic substreams originated by same mobile user with different packet delay requirement for each substream.

6. Performance analysis shows that the performance achieved when each user adapts its power level to the average interference is not much lower than

when it adapts to the instantaneous interference on slot-by-slot basis. This indicates practical value of the proposed uplink transmit power control scheme for wireless packet-switched data services.

In general, this work provides a new approach based on approximation techniques for the investigation of uplink transmit power control problem in packet-switched wireless networks. The insights generated by this study should help us better understand the radio resource allocation problem in packet-switched data network environments. The results of this research provide guidance on the efficient management of transmit powers for energy-efficient packet-switched data services in current and future generations of wireless networks.

10.2 Future Work

For future directions on this topic, the following are possible topics that need further research:

1. This study considers only the average packet transmission delay and the queueing delay of data packets in user terminal buffer is neglected for simplicity. Future study should consider the queueing delay aspect of the problem which is complicated by the inter-dependence between packet source activities and packet retransmissions.

2. Study on the Pareto improvement of the scheme — introduction of a penalty function or a pricing function [41, 42, 43] into the packet data utility model that takes into account of both transmit power and average packet delay requirement (p_i, $D_{max,i}$).

3. Further extension to consider the mobile multimedia traffic, such as voice, data, text, fax, image and video. Different utility behaviors and diverse QoS

requirements need to be integrated into user utility model. More complex timing and synchronization issues need to be studied when considering the collaboration among different modes of traffic originated from a same user terminal.

4. Extension of the study to multi-cell cellular network environments.

5. Effect of user mobility and channel fading on the performance of the scheme.

Appendix A

Supplementary Derivation and Proof

A.1 Derivation of Equation (4.12) — Necessary Condition for the Existence of Nash Equilibria

From the definition of the necessary condition for the existence of Nash equilibria:

$$\frac{\partial\,\overline{u}_i(\,p_i,\ \mathbf{p}_{-i}\,)}{\partial\,p_i} \;=\; 0, \quad \forall\, i = 1, \cdots, N \tag{A.1}$$

which requires that the N first-order partial derivatives of the utility function with respect to transmit power of each of the N users in the system be zero simultaneously. Using the utility definition of Equation (3.26), Equation (A.1) can be found as below:

$$
\begin{aligned}
\frac{\partial\, \overline{u}_i(\, p_i,\ \mathbf{p}_{-i}\,)}{\partial\, p_i} \;&=\; \frac{L\, R}{M\,[1 - f(0)]} \left(-\frac{1}{p_i^2}\, Prob\,\{\, d_i \;\leq\; D_{max,\,i}\,\}\, [E[f(\gamma_i)] - f(0)] \right. \\
&+\; \frac{1}{p_i}\, \frac{\partial\, Prob\{\, d_i \leq D_{max,\,i}\}}{\partial\, p_i}\, [E[f(\gamma_i)] - f(0)] \\
&+\; \left. \frac{1}{p_i}\, Prob\{\, d_i \leq D_{max,\,i}\}\, \frac{\partial\, E[f(\gamma_i)]}{\partial\, p_i} \right) \\
&=\; 0\,, \qquad \text{(A.2)}
\end{aligned}
$$

which gives rise to the following expression:

$$
\begin{aligned}
[E[f(\gamma_i)] - f(0)] \times Prob\{\, d_i \leq D_{max,\,i}\} \;=&\; \left\{ \frac{\partial\, E[f(\gamma_i)]}{\partial\, p_i}\, Prob\{\, d_i \leq D_{max,\,i}\} \right. \\
&+\; \left. [E[f(\gamma_i)] - f(0)]\, \frac{\partial\, Prob\{\, d_i \leq D_{max,\,i}\}}{\partial\, p_i} \right\} \times p_i\,. \qquad \text{(A.3)}
\end{aligned}
$$

From Equations (3.40) and (3.41) in Chapter 3, we have the approximation formulas for $Prob\{\, d_i \leq D_{max,\,i}\,\}$ and $E[f(\gamma_i)]$ as below:

$$
Prob\,\{\, d_i \leq D_{max,\,i}\,\} \;\approx\; \frac{\Phi\,(\beta_i - \alpha_i) \;-\; \Phi\,(-\alpha_i)}{1 \;-\; \Phi\,(-\alpha_i)} \;=\; \frac{\displaystyle\int_{-\alpha_i}^{\beta_i - \alpha_i} e^{-\frac{1}{2}\,u^2}\, du}{\displaystyle\int_{-\alpha_i}^{\infty} e^{-\frac{1}{2}\,u^2}\, du}\,, \qquad \text{(A.4)}
$$

$$
E[f(\gamma_i)] \;\approx\; \frac{2}{3}\, f\left(\overline{\gamma}_i^{\,0}\right) \;+\; \frac{1}{6}\, \left[\, f\left(\overline{\gamma}_i^{\,1}\right) \;+\; f\left(\overline{\gamma}_i^{\,2}\right)\,\right]. \qquad \text{(A.5)}
$$

Obviously $Prob\{\, d_i \leq D_{max,\,i}\,\}$ and $E[f(\gamma_i)]$ are both functions of parameter β_i ($\overline{\gamma}_i^0$, $\overline{\gamma}_i^1$, $\overline{\gamma}_i^2$ are all functions of β_i, refer to Equations (3.35), (3.36) and (3.37)). And β_i in turn is a function of p_i. From the definitions of β_i (3.34) and parameter b_i (3.6), we have the following *linear* relationship between β_i and p_i:

$$\beta_i \;=\; \frac{b_i}{\sigma_{Y_i}} \;=\; \frac{W}{R}\,\frac{h_i\,p_i}{\gamma_{min,\,i}\,\sigma_{Y_i}}\,. \tag{A.6}$$

Therefore, it is easy to see that

$$\frac{\partial\,\beta_i}{\partial\,p_i} \;=\; \frac{W}{R}\,\frac{h_i}{\gamma_{min,\,i}\,\sigma_{Y_i}} \;=\; \frac{\beta_i}{p_i}\,. \tag{A.7}$$

Because

$$\frac{\partial\,Prob\{\,d_i \leq D_{max,\,i}\}}{\partial\,p_i} \;=\; \frac{\partial\,Prob\{\,d_i \leq D_{max,\,i}\}}{\partial\,\beta_i} \times \frac{\partial\,\beta_i}{\partial\,p_i}\,, \tag{A.8}$$

$$\frac{\partial\,E[f(\gamma_i)]}{\partial\,p_i} \;=\; \frac{\partial\,E[f(\gamma_i)]}{\partial\,\beta_i} \times \frac{\partial\,\beta_i}{\partial\,p_i}\,, \tag{A.9}$$

thus, inserting the above expressions (A.8) and (A.9) into Equation (A.3) and using the property in (A.7), Equation (A.3) can be finally written as:

$$[E[f(\gamma_i)] - f(0)] \times Prob\{\, d_i \leq D_{max,\,i}\} \;=$$

$$\left\{ \frac{\partial\, E[f(\gamma_i)]}{\partial\, \beta_i} Prob\{\, d_i \leq D_{max,\,i}\} \,+\, [E[f(\gamma_i)] - f(0)]\, \frac{\partial\, Prob\{\, d_i \leq D_{max,\,i}\}}{\partial\, \beta_i} \right\} \times \beta_i \tag{A.10}$$

which is the basic expression that we use in the discussion of the necessary condition for the existence of Nash equilibria as in Equation (4.12) in Chapter 4.

■

A.2 Strategy Space for the Transmit Power of Each User: Nonempty, Convex and Compact

As discussed previously in Chapter 4, Section 4.1, the strategy set for user i's transmit power p_i is in $[\,P_{min,\,i},\ P_{max,\,i}\,]$. Since we have strictly $P_{max,\,i} > P_{min,\,i} > 0$, the strategy space is obviously nonempty.

In slotted packet-switched data systems, the number of simultaneous packets in a transmitting time-slot generated by users is a random number which can run from 1 to N (N is the total number of users in the system). Therefore, the SIR γ_i (2.11) is a random variable as discussed in the modeling of the expected packet-data utility model (Chapter 2, Section 2.4). To generate useful packet value by transmitting a data packet, as defined in the expected packet-data utility model, we require that the average transmission delay of the packet has to be less or equal to a delay tolerance specification:

$$d_i \ \leq \ D_{max,\,i} \tag{A.11}$$

which corresponds to (refer to (3.4))

$$\gamma_i \ \geq \ \gamma_{min,\,i}\,. \tag{A.12}$$

Therefore, we have the following condition that needs to be satisfied:

$$\frac{W}{R}\,\frac{h_i\,p_i}{\sum_{j=1,j\neq i}^{N}\ x_j\,h_j\,p_j\ +\ \sigma_i^2}\ \geq\ \gamma_{min,\,i}\,. \tag{A.13}$$

When there are no simultaneous interfering packets in a time-slot , i.e., $x_j = 0,\ \forall\, j,\ j \neq i$, then we have

$$p_i \;\geq\; \frac{R}{W}\frac{\gamma_{min,\,i}}{h_i}\,\sigma_i^2 \;=\; P_{min,\,i} \tag{A.14}$$

which indicates the lower bound of the transmitting power p_i.

Considering also the upper bound of p_i

$$p_i \;\leq\; P_{max,\,i}\ , \tag{A.15}$$

we can now address the issue of convexity about the subspace enclosed by the above three conditions in (A.13), (A.14) and (A.15).

Generally, a set is a convex set if and only if all the convex combinations of the points in the set are also contained in the set.

A general theorem in the theory of convex sets states that if $q(\mathbf{x})$ is a concave function, then the set $\mathcal{S} = \{\,\mathbf{x}\,|\,q(\mathbf{x}) \;\geq\; c\,\}$ is a convex set [109]. Applying this theorem to our case, the line segment enclosed by $p_i \;\geq\; P_{min,\,i}$ and $p_i \;\leq\; P_{max,\,i}$ can be viewed as concave because a line segment can be defined as either concave or convex.

The condition $\gamma_i \;\geq\; \gamma_{min,\,i}$ expressed in (A.13) can be re-written as follows

$$h_i\,p_i \;-\; \gamma_{min,\,i}\,\frac{R}{W}\sum_{j=1,j\neq i}^{N} x_j\,h_j\,p_j \;\;\geq\;\; \gamma_{min,\,i}\,\frac{R}{W}\;\sigma_i^2\ . \tag{A.16}$$

The LHS of the above inequality represents also a straight line in a multi-dimensional space and the number of dimensions is equal to the number of simultaneous packets in a time-slot. Although the number of this dimensions is changing randomly from time-slot to time-slot, the characteristic of a straight line for the LHS of (A.16) stays the same. Thus, condition $\gamma_i \geq \gamma_{min,\,i}$ is also concave. Therefore, the strategy space of p_i enclosed by the above three conditions indeed forms a convex set.

By definitions, the compactness of a set means that the set is closed and bounded, and a set is closed if it contains all the boundary points [109, 111]. In our case, the boundary points of the convex set are $P_{max,\,i}$ and a power level given by the condition $\gamma_i \geq \gamma_{min,\,i}$ which are all contained in the convex set. Hence the set is closed. Since the transmit power level is constrained to finite values in $[\,P_{min,\,i},\ P_{max,\,i}\,]$, the set is clearly bounded. Therefore, the strategy space of p_i indeed forms a compact set.

In conclusion, the strategy space of transmit power p_i for user i is non-empty, convex and compact.

$\blacksquare$

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