

**UTILITY AND PROFIT MAXIMIZATION IN DYNAMIC
SPECTRUM ALLOCATION**

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ABSTRACT OF THE DISSERTATION

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Demand driven, short term allocation of spectrum will be important for future wireless systems. Engineering and economics will jointly determine optimal ways to operate such systems. In this thesis, we characterize two operating principles of dynamic spectrum access: decentralized commons and centralized property right.

In decentralized commons, co-located devices sense spectrum for vacant bands to transmit. Assuming an OFDM based physical layer, this means that a device can transmit in non contiguous tones. We analyze how symbol timing synchronization can be achieved using cyclic prefix based algorithms. For different spectral occupancies of the transmitter and fading conditions, we identify scenarios where synchronization algorithms yield satisfactory results and scenarios where they do not.

For the centralized property rights regime, we develop a two tiered spectrum allocation model where spectrum is first allocated to service providers (SPs) by a broker and then to customers by SPs.

First we assume that the users transmit to the SPs in the uplink after spectrum allocation, who maximize the sum utility of the users. We derive optimal allocation for different system parameters. We introduce a spectrum price and use it to demonstrate several key results about spectrum allocation. The spectrum price proves to be the regulatory mechanism that brings about coordination amongst the SPs with minimal control messaging. Our approach thus strikes a balance between a total and no central coordination.

Next we consider a downlink scenario where SPs sell spectrum to users and then transmit data. The SPs operate to maximize their profits. Each SP transmits at a specific power spectral density which is an indicator of the modulation and coding technology used for transmission. When there is only one SP, it can act as a monopolist and when there are multiple SPs, they compete. We characterize the customer to SP interactions in monopoly and SP price competition. We derive the prices charged and profits made by the SPs and show how they vary with provider efficiencies and spectrum costs charged by the broker. We show that an SP should invest in better technology if the broker cost of spectrum is high.

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Dedication

To the future Mrs. Acharya. And to my parents Mr. Jayanta Kumar Acharya, Mrs. Chameli Acharya and my sister Ms. Chandrika Acharya.

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Chapter 1

Introduction

1.1 Technology Trends: Dynamic Spectrum Access

We are witnessing a large growth in the scope of wireless communications services. New broadband technologies such as 3G (third generation) and B3G (beyond third generation) WiMAX and LTE and the upcoming 4G systems, as per ITU's IMT-Advanced specifications [1] are being designed and deployed that will co-exist with traditional technologies such as WLAN and 2G cellular. In the future we are likely to experience a plethora of wireless devices belonging to different technologies in the same geographic region. Spectrum allocation among different transmit-receive device pairs is thus important for ensuring fairness and efficiency for end-to-end applications.

The traditional regulatory process for spectrum allocation has been largely non responsive to the application requirements of network subscribers. Spectrum is auctioned to operators for relatively long periods, over large regional areas, often with a mandate for deploying specific services. This has resulted in a market environment where only a handful of large service providers own spectrum and the smaller players face significant barriers for market entry. Another consequence of this approach has been the under-utilization of spectrum since it is not possible to predict the spectrum demand of user applications at the time the allocation is made to the providers who serve them. This artificially restricts more users from obtaining service and also reduces QoS to the users who are being serviced. As an example, in the 1950s, the FCC sold licenses for 330 MHz of spectrum for UHF television in USA. This experiment never succeeded leading to considerable bands of unused spectrum between VHF and UHF broadcast channels from 54 to 865 MHz [2]. Other instances of underutilization of spectrum has been reported in [3].

This has motivated the development of dynamic spectrum allocation (DSA) techniques that take into account the application requirements, presence of other devices in the region and

link gains between the transmit-receive pairs. In this context, the term *cognitive radio* is often used [4, 5, 6] for referring to devices that could enable DSA by sensing the surrounding region, dynamically determining what spectrum to transmit on and adapting its modulation/coding strategies accordingly.

In the recent past, the Govt. and spectrum regulatory bodies have taken increased cognizance of the future importance of DSA and have initiated several efforts at understanding the basics of a DSA system with the ultimate goal being a full scale practical deployment. In 2003, the US Federal Communications Commission (FCC) issued a NPRM [7] to seek comments on ways to encourage spectrum sharing and remove regulatory impediments to the deployment of cognitive radio technologies. The next year the FCC issued a more specific NPRM [8], on the utilization of unused spectrum in the VHF and UHF TV bands between 54 MHz and 862 MHz by license-exempt devices. The aim was to offer wireless broadband services in rural areas that were not well served by alternatives such as cable or DSL. The cellular operators are considering to acquire spectrum for deploying in-home base stations called Femtocells [9] for improving coverage, which provides an opportunity for spectrum sharing across operators. Each operator could potentially use spectrum licensed out to competing operators through a sub-lease arrangement. Thus some sort of coordination and dynamic sharing amongst operators is assumed. However there could be other applications where the spectrum sharing is not coordinated by a common protocol. There could be a primary licensee of spectrum such as TV broadcasters in the 54 to 862 MHz band and secondary systems such as IEEE's 802.22 based cognitive radio WRANs [10, 11] could operate in the vacant bands of this spectrum to provide broadband access in rural areas. The US Defense Advanced Research Projects Agency (DARPA) also joined the movement by establishing the NeXt Generation Communications (XG) program [12], to develop a standard for cognitive radio with dynamic spectrum access for military communications.

The ventures mentioned [7, 8, 9, 11, 12] are at the rudimentary stages of designing a full fledged DSA network. The question of how best to operate such a network, i.e. how best to allocate spectrum to the communicating devices, is still open. This problem is being actively researched by both communication engineers and economists/policy makers [13].

1.1.1 Engineering Solutions

Communication engineers and information theorists have several models for DSA. In comparing and contrasting between various models, the following features stand out:

1. *The level of centralized control over the communicating devices:* The devices could be fully coordinated by a central base station as in a OFDMA based cellular network in the downlink [14, 15, 16, 17], there could be partial control [18] or the devices could be fully distributed [19, 20].
2. *Whether the devices are strategic:* Strategic devices can bid for spectrum from central entity [21, 22] or in a decentralized case, can greedily try to maximize their objectives [20] by transmitting at higher power. Non strategic devices can be simple price takers [18] or follow a distributed spectrum etiquette protocol [23, 24] such as 802.11.

1.1.2 Economic Policies

The DSA policies espoused by economists either belong to *property rights* or *spectrum commons* regime. In the property rights regime [25, 26] spectrum is owned by individuals or companies who can buy, sell and trade in spectrum just like any other commodity. The belief is that such a spectrum market will lead to an efficient allocation and evaluation of spectrum. Others have favored a commons regime where spectrum is unlicensed [27] and is shared by smart communicating devices who are able to cooperate and co-exist, without creating excessive interference for each other. Though the broad principles of both these regimes are distinct, the lack of precise modeling and details of implementation can lead to confusion [28, 29] when trying to classify a specific system as belonging to either of the two.

1.2 Broad Direction of the Thesis

As a result of the apparent dichotomy in the models and taxonomy of the communications engineer and the economist, there have been several efforts [13, 30, 31] to show how they relate to each other. As a result members of both communities now accept that there are several fundamental technical and market questions that have to be resolved before the full potential of DSA based networks is realized. Accordingly, in this thesis, we will focus on the interplay between radio technology and market dynamics for DSA. We first state two regimes of DSA

that combines the work of both camps and then discuss their features and potential research issues. These two regimes are

1.2.1 Distributed Systems/ Spectrum Commons

In this regime there is no central control and users can either be non strategic and follow a spectrum etiquette protocol [23, 24] or be strategic and greedily maximize their spectrum and power allocations to maximize their utilities [19, 20, 32, 33]. We do not study the former type of systems in this thesis.

The Nash equilibrium of the latter systems can be very inefficient [20, 32] leading to poor resource utilization. Additionally the lack of central control can adversely affect the physical layer timing synchronization. The de facto physical layer of future generation wireless systems is based on OFDM which, in contrast to single-carrier systems, is particularly sensitive to synchronization errors like carrier frequency offset and symbol timing errors, which leads to increased inter carrier interference (ICI). Thus to ensure reliable communication, extensive work has been done in designing robust algorithms that estimate the carrier frequency and symbol timing with high accuracy [34, 35, 36]. All these works assume that there is some total bandwidth which is utilized by a single user. However for systems with no coordination and strategic users, multiple users in a geographical region will sense a common pool of spectrum for the presence of vacant frequency bands to transmit in. In the OFDM context, this means that a user may transmit in non contiguous tones (termed as Non-Contiguous OFDM or NC-OFDM). It is not clear how the existing synchronization algorithms will perform in this situation and till date this important problem has been largely ignored. In this thesis, we analyze the performance of existing synchronization schemes for NC-OFDM [37, 38] and propose new schemes that take cognizance of the non contiguous nature of transmission.

1.2.2 Centralized Systems/ Property Rights

We believe that the majority of the DSA networks in the future will belong to this regime. One reason are the problems with the alternative as mentioned in Section 1.2.1. The other reason has to do with economic incentives. Market trends indicate that a cellular service provider raises much more revenue from monthly subscriptions fees of the customers than a maker of WLAN access points does by selling them. Accordingly, we examine DSA mechanisms that can be

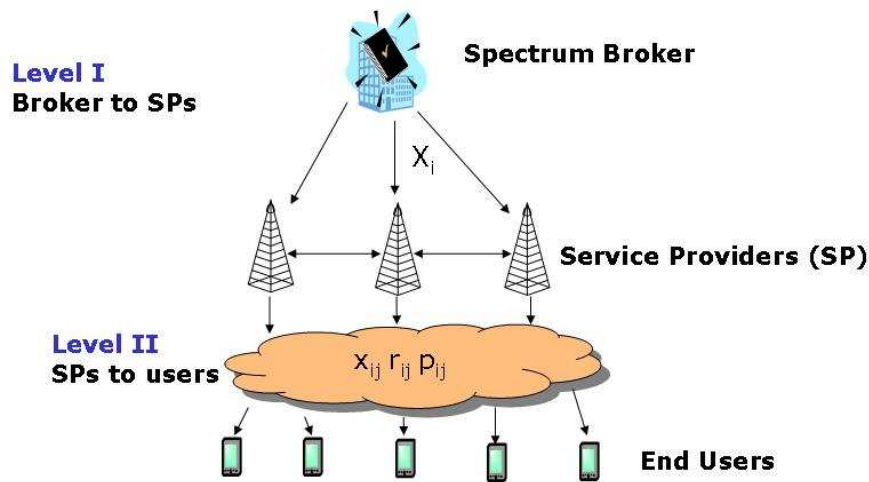


Figure 1.1: The network topology

categorized into two steps as shown in Figure 1.1

- S1) Spectrum is allocated between co-located Service Providers (SP) by a Spectrum Broker. Example of co-located SPs could be TV stations and 802.22 WRAN systems [10] or cellular and femtocell operators [9].
- S2) SPs allocate this spectrum to their customers (denoted by end users) who subsequently transmit/receive over this spectrum.

In Step S1, the Broker allocates short term spectrum licenses to a SP, typically for a session duration after which the spectrum may be allocated to another SP. Such spectrum with short term ownership could correspond to the coordinated access bands (CAB) as introduced in [39]. Possible strategies for Step S1 have been studied and espoused by both engineers and economists. However, in these strategies, the details of the physical transmission characteristics are broadly abstracted into simple parameters such as width of spectrum band and transmit power which define a spectrum license. However to understand how spectrum allocation is different from generic resource management we need to consider Step S2, which deals with the characteristics

of the communication channel that influences the user demand of spectrum and consequently the amount of spectrum the SPs need from the broker.

In this thesis, we consider the joint performance of a spectrum broker to SP and SP to user spectrum allocation. A logical question is *What should be the operating principle of the SPs?*. They can act as a social planner and maximize the sum utility of users. The SPs could also operate the network in an attempt to maximize their profits. We examine both these approaches in the thesis.

1.3 Specific Contributions of the Thesis

This thesis has the following three components,

1.3.1 Timing Acquisition for enabling Distributed DSA

We study the performance of cyclic prefix correlation based symbol timing acquisition algorithms for NC-OFDM transmission. We first derive the ML estimator when the channel is frequency non-selective and show that it has high computational complexity. Consequently we study the performance of low complexity, sub-optimal approaches both for frequency non-selective and frequency selective channels. Our simulations indicate that in some likely situations such as the users occupying multiple non-contiguous sub-bands and having large differences in the timing offsets between their transmitters and receivers, cyclic prefix based timing acquisition algorithms can perform quite poorly. This points to the need for better algorithms of reasonable complexity, or entirely different approaches to symbol timing acquisition, for example based on the periodic transmission of known sequences.

1.3.2 User Utility Maximization for centralized DSA

We develop and analyze a model for dynamic spectrum allocation, that is applicable for a broad class of practical systems. We consider multiple service providers (SPs), in the same geographic region, that share a fixed spectrum, on a non-interference basis. This spectrum is allocated to their customer end users for transmission to the SPs. Assuming that a user can obtain service from all the SPs, this work develops an efficient algorithm for spectrum allocation. The quality of service depends on system parameters such as number of users and SPs, the channel conditions between the users and SPs and the total transmit power of each user.

The SPs have different efficiencies of reception. We adopt a user utility maximization framework to analyze this system. We develop the notion of spectrum price that enables a simple distributed spectrum allocation with minimal coordination among the SPs and users. Given the user utility functions and the system parameters, we characterize the spectrum price and the users' optimal bandwidth allocations. Our work provides theoretical bounds on performance limits of practical operator to user based dynamic spectrum allocation systems and also gives insights to actual system design.

1.3.3 Service Provider Profit Maximization for centralized DSA

The user utility maximization framework provides a baseline case for understanding the change in spectrum price and allocation when the SPs have profit motives of their own. Since a SP pays the Broker for obtaining spectrum licenses, it is natural to assume that it operates to maximize its profits by charging users for the spectrum allocation. We apply principles of microeconomics [40, 41] to explore SP pricing for profit maximization. We model the SP profit as a function of the cost it has to pay the Broker, the revenues it accrues from the users and possible price competition of other SPs in the region. An SP seeks to maximize its profits by choosing its price. The users consider the SP prices and their applications to determine which SP to obtain service from and the amount of spectrum to obtain. In this work, we characterize the SP prices and user spectrum allocations. We show that the pricing structure changes from a single SP network to a network with multiple SPs in price competition. Our model also demonstrates when it is in the interest of the SP to opt for a more efficient but costly transmission technology.

1.4 Other Issues

There are other important areas about a future DSA system that we have not considered in this thesis. Sensing a common pool of spectrum, for the presence of licensed and unlicensed users is an important problem which has been studied in [42, 43]. Then there are important network layer aspects related to protocol stacks and network architecture design. Prominent amongst them are WINLAB's NSF-funded network-centric cognitive radio project [44]. There are RF issues in designing tunable wideband radios for the cognitive radio front end. Finally before cognitive radio devices are produced commercially, a prototype has to be developed. GNUs

open-source software defined radio project [45] supports a hardware platform using the Universal Software Radio Peripheral (USRP), which is a low cost, high speed USB 2.0 peripheral for the construction of software radios. Vanu Inc. [46] provides solutions for communication between disparate wireless devices and frequencies and is the a preliminary version of a cognitive radio. For ongoing work in this area also refer to [47] and references therein.

1.5 Organization of the Thesis

Chapter 1 introduces the main topics to be covered in this thesis. The timing synchronization problem of non strategic and uncoordinated users has been covered in Chapter 2. The resource allocation of non strategic users from user utility maximization has been covered in Chapter 3 and from service provider profit maximization in Chapters 4, 5 and 6. Specifically Chapter 4 establishes the notation and analytical foundations for profit maximizing systems and then Chapters 5 and 6 respectively deal with single SP monopoly and multiple SP price competition. The reader interested in the work about resource allocation for centralized DSA can skip Chapter 2 without loss of continuity. Finally Chapter 7 summarizes the various findings of the thesis and looks at future extensions.

Chapter 2

Timing Acquisition for enabling Distributed DSA

2.1 Introduction

As mentioned in Section 1.2.1, when devices engage in DSA they transmit in non contiguous spectrum bands. The physical layer technology that is well suited for such a transmission is OFDM, where a device transmits only in the tones corresponding to the vacant spectrum. This allows for tighter usage of spectrum compared with traditional FDM and is more efficient. The spectrum corresponding to the other tones could be used by other devices. Such a transmission scheme is called NC-OFDM in [37, 38] and differs from conventional OFDMA systems as the devices are uncoordinated.

For such a transmission scheme, it is important to study the symbol timing acquisition performance at the receiver. This can be explained as follows: assume that a NC-OFDM symbol is $N + L$ samples long with the first L samples, called the cyclic prefix (CP), being the same as the last L samples. Let the transmitted NC-OFDM samples be $s(k)$. The received samples $r(k)$, in presence of timing offset θ between transmitter and receiver, is given by

$$r(k) = \sum_{l=0}^{N_l} h(l)s(k - \theta - l) + s_I(k) + n(k), \quad (2.1)$$

where $h(l)$ represents a N_l tap frequency selective channel and $s_I(k)$ the signals from other users that interfere at the receiver. Timing acquisition is about estimating the OFDM symbol boundary by estimating θ at the receiver.

For NC-OFDM, the signals $s(k)$ and $s_I(k)$ will occupy non-overlapping set of tones. However, since the practical pulse-shaping filters are not ideally band-limited, part of the symbol energies will spill over to the adjacent bands causing interference. Hence, the performance of acquisition algorithms will improve with wider guard bands between the signals of the different users. For systems where the spectrum is licensed to a primary user and secondary users opportunistically use it, presence of wide guard bands may be assumed to protect the primary

users from interference. However if the spectrum is unlicensed and a group of uncoordinated devices attempt to access it, then the spectrum can become tightly packed to maximize its usage leading to loss in timing acquisition performance.

2.1.1 Related Work

Acquisition performance is well understood for a single user system, i.e. $s_I(k) = 0$ in (2.1) and when the transmitter occupies the entire spectrum [34, 35]. Even in the presence of multiple users, single-user algorithms are often used as acquisition is the first step at the receiver and at this stage there are usually no signal processing methods to *distinguish* the signal from the interferer. The single user OFDM acquisition algorithms can be broadly classified as,

Cyclic Prefix Correlation

The optimal ML estimator of symbol timing for a frequency non-selective channel is derived in [35]. The CP introduces correlations in the OFDM samples and that is used to perform a sliding window correlation between two L length sample blocks, placed N samples apart.

Pilot Symbol Correlation

The authors in [34, 48] postulate the transmission of two specially designed OFDM symbols to achieve symbol timing synchronization. The idea is to introduce known correlations in the samples of the OFDM symbol which could be tracked by the receiver.

Joint Cyclic Prefix and Pilot Symbol Correlation

Symbol timing recovery by transmitting pilot symbols and extending the CP correlation based approach of [35] is proposed in [49].

Blind acquisition Methods

Such methods do not rely on cyclic prefix or pilot symbol correlation. In [50], a method for achieving symbol acquisition is proposed by constructing certain autocorrelation matrices from the received signal and minimizing their rank. This method is shown to perform well even in frequency selective channels.

The presence of multiple users affects the acquisition performance. If the users are coordinated, as in the uplink of an OFDMA system, joint timing acquisition for all users can be performed [51, 52]. For such *cellular based* systems, the uplink performance is also helped by the fact that the users are already synchronized to a common system timing during initialization using the downlink signal. This phenomenon will be discussed in Section 2.4.1.

If there is no coordination amongst the interfering users, such as in an ad-hoc network, then the acquisition performance deteriorates. The interfering users could be OFDM transmitters themselves with different delays from the intended user. For example, the term $s_I(t)$ in (2.1) could be another OFDM sample stream $s'(k - \theta')$ with $\theta' \neq \theta$. The receiver can incorrectly estimate θ' as the timing instead of the correct instant θ . The presence of a narrowband interferer is studied in [53, 54] where one user occupies the entire bandwidth and uses a pilot based acquisition algorithm as in [34]. Distributed timing acquisition amongst different interfering devices can also be realized in the MAC layer if all the devices are assumed to follow a common MAC protocol for example 802.11 in ad-hoc mode [55].

2.1.2 Our Contribution

In this work, we consider a different scenario where a cognitive transmitter, employing NC-OFDM, only transmits in tones corresponding to the vacant spectrum and the receiver has to acquire the timing of the delayed signal. In fact depending on what fraction of the spectrum is vacant, the user could be narrowband instead of the interferer. The transmit power of the interfering users could be higher than that of the desired user. The user data could be in non-contiguous tones and it is not clear upfront as to how this would affect the acquisition performance. Also the different devices participating in Dynamic Spectrum Access could come from different networks and employing different technologies to transmit over the spectrum and so the case of their following a common MAC protocol is improbable.

As a starting point, we consider the relevance of single user OFDM acquisition algorithms of Section 2.1.1 for NC-OFDM. For pilot based correlation approaches there has to be some initial signaling for the receiver to know the pilot sequences or the receiver and transmitter should follow some pre-decided link level protocol. This can be ruled out for Dynamic Spectrum Access applications. The pilot based scheme in [34] requires a user to transmit in all tones in order to generate a symbol with symmetric samples after the IFFT which is ruled out

for NC-OFDM. Thus we focus on CP based correlation, which only assumes that the transmission structure is OFDM based. A correlation based timing acquisition unit can be easily implemented at a receiver. Blind acquisition methods lead to wastage of subcarriers which could have been used for data transmission, besides when the presence of the CP guarantees a correlation in the OFDM samples, it seems natural to exploit it for purposes of synchronization. CP correlation based acquisition algorithms have been implemented in many practical systems and they yield satisfactory performance even in channels for which they are not optimal, for example in frequency selective channels [35]. This further motivates us to study performances of CP based algorithms. Thus in this chapter we try to answer the following question,

Do CP based acquisition algorithms by themselves or with realizable enhancements, suffice to yield satisfactory timing acquisition performance for a NC-OFDM transmission?

To answer this question, we first derive the ML estimator for CP correlation based acquisition for NC-OFDM transmission in a frequency non-selective channel and show that it has high computational complexity. Consequently we consider the performance of low complexity, sub-optimal approaches such as using the ML estimator of frequency non-selective OFDM transmission [35] for NC-OFDM and also introducing a band pass filter at the receiver before the acquisition phase to filter out the interference from the other users. As a result of our simulations, we have been able to identify situations in the CP correlation based acquisition algorithms deliver satisfactory results and situations in which they do not. The results of this chapter was published in [56].

The rest of the chapter is organized as follows: In Section 2.2 we study the performance of CP based timing acquisition in a channel impaired with Gaussian noise and also derive the optimal ML algorithm. In Section 2.3 we introduce frequency selective fading and in Section 2.4 we also consider the presence of an interfering user for studying the performance of CP based acquisition algorithms. In this chapter we will not investigate other forms of acquisition such as carrier frequency offset correction or frame synchronization.

2.2 CP based Timing acquisition for Frequency Non-Selective Channels

We reproduce the main result of [35] for OFDM symbol timing acquisition in channels impaired with Gaussian noise when one user occupies the entire bandwidth, i.e. $h(k) = \delta(k)$ and $s_I(k) = 0$ in (2.1). The optimal estimate that minimizes the mean square error is the ML

estimate when the Tx signal can be modeled as a white Gaussian sequence. This is a reasonable assumption for full spectral occupancy, since the number of guard tones typically used is small relative to the FFT length. It is shown in [35] that the optimal ML estimate of θ is given by

$$\text{CPCorr} : \hat{\theta}_{\text{ML}} = \arg \max_{\theta} \{\text{Re}(\gamma(\theta)) - \rho\Phi(\theta)\} \quad (2.2a)$$

$$\gamma(m) = \sum_{k=m}^{m-L+1} r(k)r^*(k+N) \quad (2.2b)$$

$$\Phi(m) = \frac{1}{2} \sum_{k=m}^{m-L+1} |r(k)|^2 + |r(k+N)|^2, \quad (2.2c)$$

where $E[s^2(k)] = \sigma_s^2$, $E[n(k)^2] = \sigma_n^2$ and $\rho = \sigma_s^2/(\sigma_s^2 + \sigma_n^2)$. Define $\text{snr} = \sigma_s^2/\sigma_n^2$. The quantity θ is modeled as deterministic but unknown and thus the mean square error in estimating θ is given by

$$\text{mse}(L, \text{snr}) = E \left(\theta - \hat{\theta}_{\text{ML}} \right)^2, \quad (2.3)$$

where the expectation is over the statistics of the estimate. We can interpret $\gamma(m)$ as the operator which captures the correlation energy between two L sample blocks separated N samples apart with the first sample of first block taken at time $k = m$. We term this algorithm as CPCorr. Note that for CPCorr to work, the receiver needs to know ρ a priori before the acquisition phase. Though this is not practical, we can assume that the transmit power and the receiver noise characteristics stay constant over the transmission interval and the receiver can obtain a good estimate of ρ based on past history. Also for moderate/high snr regimes, $\rho \sim 1$ irrespective of the actual value of snr.

The following situation could arise in NC-OFDM,

Scenario 1. Consider a system with available bandwidth W Hz and the user of interest transmits in some parts of the entire band, and the remaining parts of the band are not occupied by other users. The channel is frequency non-selective and is impaired only by Gaussian noise. This could correspond to a channel with a strong line of sight component.

2.2.1 The Optimal ML Algorithm

For Scenario 1, we derive the optimal ML estimator. Consider that the FFT length is N and the set of tones in which the desired user transmits be \mathcal{T} . Let the information symbol vector be $\mathbf{x} = [x(1), \dots, x(N)]$ such that $x(j) = 0$ if $j \notin \mathcal{T}$. The transmitted symbol vector $\mathbf{s} =$

$[s(1), \dots, s(N)]$ is generated through IFFT of information vector \mathbf{x} ,

$$\mathbf{s} = \mathbf{Q}\mathbf{x}, \quad (2.4)$$

where $\mathbf{Q} = [\mathbf{q}_1, \dots, \mathbf{q}_N]$ is the IFFT matrix. We show that the transmitted symbols of \mathbf{s} , at two different time instants j and k are correlated even if the vector \mathbf{x} has uncorrelated entries.

The correlation between OFDM samples j and k is

$$\begin{aligned} E[s(j)s(k)] &= E[\mathbf{q}_j^H \mathbf{x} \mathbf{x}^H \mathbf{q}_k] = \mathbf{q}_j^H E[\mathbf{x} \mathbf{x}^H] \mathbf{q}_k \\ &= \mathbf{q}_j^H \mathbf{W} \mathbf{q}_k = \rho^{jk}, \end{aligned} \quad (2.5)$$

where \mathbf{W} has 1's in diagonal positions given by \mathcal{T} and zeros elsewhere. Thus the correlation is non-zero. We now use (2.5) to calculate the correlation matrix of the received signal vector. The receiver collects a $2N + L$ sample block \mathbf{r} , as this is sure to contain a single complete $(N + L)$ sample OFDM symbol which starts after θ samples. For a generic user, the received OFDM symbol is given by $\mathbf{r} = \mathbf{s} + \mathbf{n}$ with $\mathbf{s} = [\mathbf{u} | \mathbf{v} | \mathbf{u}]$. Samples $\mathbf{u} = [s(\theta), \dots, s(\theta + L - 1)]$ are the prefix symbols and $\mathbf{v} = [s(\theta + L), \dots, s(\theta + N - 1)]$ are the data symbols. We define the following matrices,

$$\mathbf{X} = E[\mathbf{u}\mathbf{u}^H], \quad \mathbf{Y} = E[\mathbf{u}\mathbf{v}^H], \quad \mathbf{Z} = E[\mathbf{v}\mathbf{v}^H]. \quad (2.6)$$

After some algebra, it can be shown that the correlation matrix of \mathbf{r} , $\tilde{\mathbf{C}} = E[\mathbf{r}\mathbf{r}^H]$ is given by,

$$\tilde{\mathbf{C}} = \begin{pmatrix} \sigma_n^2 \mathbf{I}_L + \mathbf{X} & \mathbf{Y} & \mathbf{X} \\ \mathbf{Y}^H & \sigma_n^2 \mathbf{I}_{N-L} + \mathbf{Z} & \mathbf{Y}^H \\ \mathbf{X} & \mathbf{Y} & \sigma_n^2 \mathbf{I}_L + \mathbf{X} \end{pmatrix}. \quad (2.7)$$

Let \mathbf{C}_θ be the actual correlation matrix of the received $2N + L$ sample window. It is given by,

$$\mathbf{C}_\theta = \text{diag} \left[(P + \sigma^2) \mathbf{I}_\theta, \tilde{\mathbf{C}}, (P + \sigma^2) \mathbf{I}_{N-\theta} \right]. \quad (2.8)$$

The optimal ML estimate of θ is thus given by

$$\arg \min_{\theta} \log (|\mathbf{C}_\theta|) + \frac{1}{2} \mathbf{r}^H \mathbf{C}_\theta^{-1} \mathbf{r}. \quad (2.9)$$

To compare the performance of the algorithm CPCorr and the ML algorithm, we simulate the MSE performance when the total spectrum is W and the desired user transmits in bands $[0, f_a] \cup [f_b, W]$ with $0 < f_a < f_b < W$ and does not transmit in band $[f_a, f_b]$. We call

| | |
|--|---|
| FFT Size, N | 256 |
| snr (dB) | 4, 10, 16 |
| $[f_a, f_b]$ (MHz) (bands not occupied) | $[0.25W, 0.35W]$ 90% occupancy $[0.15W, 0.6W]$ 55% occupancy |

Table 2.1: Simulation parameters for testing the performance of CPCorr and ML algorithm for a single user with partial spectral occupancy in a frequency non-selective channel

this situation as *partial spectral occupancy* and the spectral occupancy is $1 - (f_b - f_a)/W$. CPCorr might not perform well in this situation as it assumes IID signal samples, whereas partial bandwidth occupancy causes significant correlations (increasing with the fraction of unoccupied bandwidth). The simulation parameters are shown in Table 2.1. Note that CPCorr metrics could be calculated for each of several OFDM symbols, added and then the sum be used for finding the best delay. In fact the the higher snr values like 16 dB can be regarded as an approximation of what would happen if we accumulated across OFDM symbols as mentioned.

An appropriate metric for the acquisition performance is normalized mean square error,

$$\text{nmse}(L, \text{snr}) = \frac{\text{mse}(L, \text{snr})}{L^2}, \quad (2.10)$$

where mse is defined in (2.3). This is because symbol timing errors up to L do not result in intersymbol interference. Normalizing this way allows us to compare performance at different values of L . Thus for frequency non-selective channels as long as $\text{nmse}(L, \text{snr}) < 1$, the acquisition performance is satisfactory. For frequency selective channels, since the CP also has to provide immunity against the delay spread of the channel, we will consider a lower threshold of $\text{nmse}(L, \text{snr})$ than unity.

Observation 1. *The simulation results for 90% and 55% spectral occupancy are shown in Figures 2.1 and 2.2. The following observations can be made*

1. *The ML algorithm yields a lower $\text{nmse}(L, \text{snr})$ than CPCorr for all values of L and snr.*
2. *The relative loss of performance of CPCorr over the ML algorithm is more for lower spectral occupancies as CPCorr is optimal for full spectral occupancy.*
3. *With respect to the criterion $\text{nmse}(L, \text{snr}) < 1$, CPCorr performs satisfactorily for moderate/high values of snr.*

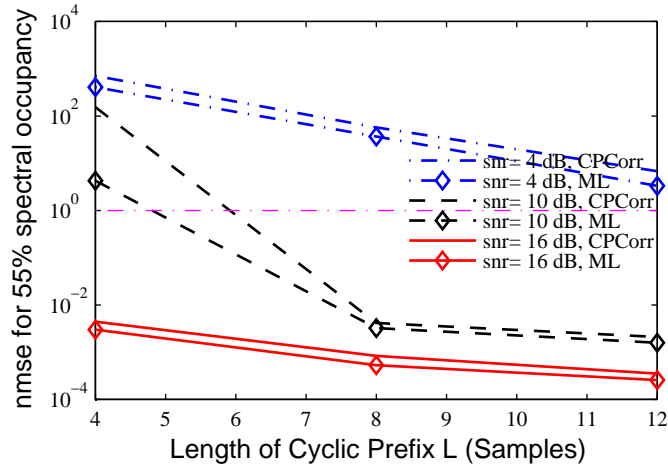


Figure 2.1: Performance of CPCorr and ML algorithm for a single user frequency non-selective channel with partial spectral occupancy of 90% of total bandwidth W , with no transmission in $[f_a, f_b] = [0.25W, 0.35W]$ MHz for different values of snr

Conclusion 1. *Algorithm CPCorr satisfactorily acquires symbol timing for a single user transmission with partial spectral occupancy in a frequency non-selective channel.*

2.2.2 A note about Sample Correlation for Partial Spectral Occupancies

The reason behind correlations in the OFDM samples when the user did not occupy all the tones is that the bandwidth of the signal was less than W but it was being oversampled at W . This can be avoided by carefully sampling the analog NC-OFDM signal at the correct rate depending on its bandwidth. However this depends on what is the spectral occupancy of the NC-OFDM signal which is a dynamic quantity. Also the transmitted signal might be in multiple non-contiguous sub-bands (instead of one contiguous sub-band as considered in Figures 2.1 and 2.2) and in that case, careful sampling over multiple sub-bands is needed. Implementing a fixed sampling rate of W is the simplest working algorithm.

Even in OFDM, there is partial spectral occupancy (and hence correlations) due to the guard tones at the ends of the band, but these are usually slight, i.e. the oversampling factor is typically small to cause noticeable degradation in acquisition performance.

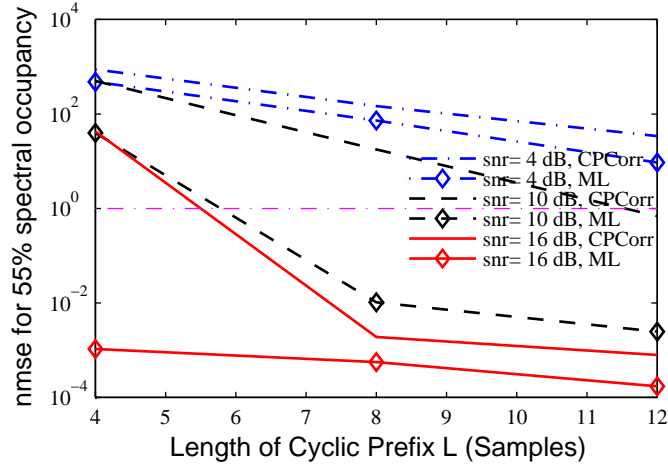


Figure 2.2: Performance of CPCorr and ML algorithm for a single user frequency non-selective channel with partial spectral occupancy of 55% of total bandwidth W , with no transmission in $[f_a, f_b] = [0.15W, 0.6W]$ MHz for different values of snr

| | | | | | | |
|----------------------|---|------|------|------|------|------|
| Delay (μs) | 0 | 0.31 | 0.71 | 1.09 | 1.73 | 2.51 |
| Power (dB) | 0 | -1 | -9 | -10 | -15 | -20 |

Table 2.2: The power delay profile of the Vehicular A channel model

2.3 Timing acquisition for Frequency Selective Channels

Even in the case of regular OFDM, deriving the optimal ML algorithm is difficult because of the lack of channel statistics during the acquisition step. Also as noted in (2.9), the optimal ML is computationally intense as the FFT size N grows. Since we want low complexity acquisition algorithms, we'll focus only on the performance of CPCorr from now on.

Example 1. To illustrate how frequency selectivity affects CPCorr, consider a channel with $N_l = 2$ in (2.1) and $s_I(k) = 0$. From (2.1), we substitute for $r(k)$ in the expression for $\gamma(m)$ in (2.2b) and look at the correct timing instant $m = \theta$ to understand how it is affected in the presence of multiple paths. Define $k' = k - \theta$, so that $s(k' + i) = s(k' + i + N)$ for

$i = 0, \dots, L - 1$. The following components are present in $\gamma(\theta)$

$$\begin{aligned}
\gamma(\theta) \sim & \underbrace{|h(0)|^2 \sum_{k'=0}^{L-1} s(k')s^*(k' + N)}_{\mathcal{S}_1} \\
& + \underbrace{|h(1)|^2 \sum_{k'=0}^{L-1} s(k' - 1)s^*(k' + N - 1)}_{\mathcal{S}_2} \\
& + \underbrace{h(0)h^*(1) \sum_{k'=0}^{L-1} s(k')s^*(k' + N - 1)}_{\mathcal{J}_1^S} \\
& + \underbrace{h(1)h^*(0) \sum_{k'=0}^{L-1} s(k' - 1)s^*(k' + N)}_{\mathcal{J}_2^S}.
\end{aligned} \tag{2.11}$$

The following observations can be made

- a) Components \mathcal{S}_1 and \mathcal{S}_2 capture the correlation in the received signal. In \mathcal{S}_1 all L terms in the summation contribute toward the correct correlation but for \mathcal{S}_2 the first term is the product of two uncorrelated variables $s(-1)$ and $s(N - 1)$. We'll call $s(-1)s^*(N - 1)$ as *self interference*. For $N_l > 2$ the subsequent \mathcal{S}_j terms where $2 < j \leq N_l$ have more self interference components in them but since they are weighted by $|h_j|^2$ which is usually decreasing in magnitude, their effect is less significant.
- b) In the components \mathcal{J}_1^S and \mathcal{J}_2^S , all terms act as self interference as all the products are amongst uncorrelated variables. The quality of the estimate deteriorates as the self interference increases.

Note that this method only gives an indication of the effects of frequency selective fading; for a complete analysis, we would have to investigate how it affects the timing instants other than the true value at $m = \theta$. However for those values, most terms would be products of uncorrelated variables with or without fading and thus looking only at $\gamma(\theta)$ is sufficient for qualitative purposes.

Scenario 2. For simulating a frequency selective channel, we consider the Vehicular A model which is given in Table 2.2. The OFDM parameters are shown in Table 2.3. Note that the length of the CP in samples is less than 15% of the FFT size to minimize the spectrum and

| | |
|----------------------|----------------|
| Bandwidth, W (MHz) | 5 |
| FFT size, N | 512 |
| CP, L | 30, 40, 51, 76 |

Table 2.3: OFDM parameters for simulation. The CP lengths L corresponds to 6%, 8%, 10% and 15% of FFT length N

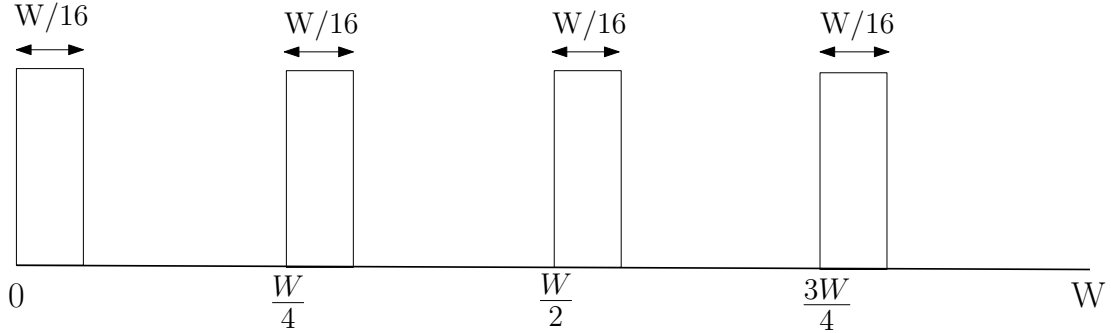


Figure 2.3: Four sub-band *non-contiguous* spectral occupancy of the first user in the band $[0, W]$

power overhead. To test the robustness of *CPCorr*, we chose an unfavourable (but possible) scenario where the user has a 25% spectral occupancy and the vacant bands are split into four subbands as shown in Figure 2.3.

Observation 2. Figure 2.4 shows the performance of *CPCorr* for a frequency selective channel characterized by a Vehicular A power delay profile. We see that there is a penalty when there is frequency selectivity but by increasing snr to $\text{snr} = 16$ dB, and/or L , the performance can be made satisfactory. A threshold of $\text{nmse}(L, \text{snr}) < 0.7$ has been shown in Figure 2.4.

Conclusion 2. Algorithm *CPCorr* satisfactorily acquires symbol timing for a single user transmission with partial spectral occupancy in a frequency selective channel.

2.4 Timing acquisition in presence of a second user

In this section we consider how the presence of a second user affects the acquisition performance. We assume that the second user also transmits OFDM signals with the same symbol duration. One important observation that we will make is that, if the timing delays of both users

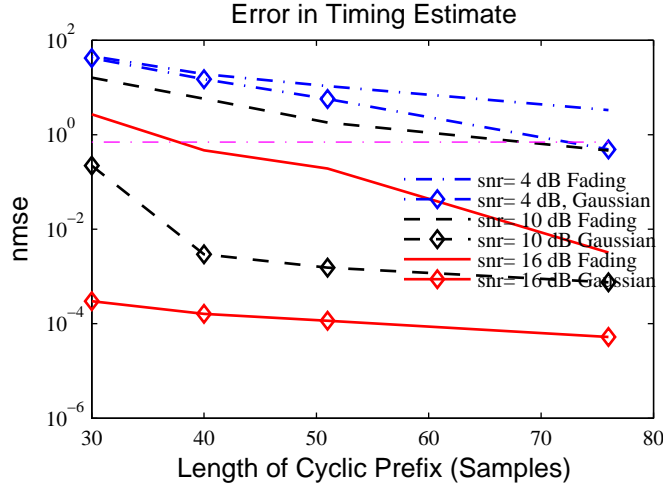


Figure 2.4: Performance of CPCorr in a single user frequency selective channel with partial spectral occupancy of 25% as given in Fig 2.3 for different values of snr

are similar, then the performance of the timing acquisition is enhanced as the signals from the two users reinforce each other and appear a single high power signal to the receiver. But if the timing delays of the two users are far apart, the receiver of user one might end up acquiring the timing of user two. We formalize this in the following example,

Example 2. Consider a two user system with the timing delays given by θ_1 and θ_2 respectively. Thus the signal of the desired user is $s_1(k - \theta_1)$ and $s_I(k) = s_2(k - \theta_2)$ in (2.1). Assume without loss of generality that $\theta_1 < \theta_2$ and let $\tilde{\theta} = \theta_2 - \theta_1$. For simplification, consider that the channel is impaired only with Gaussian noise. Thus $h(k) = \delta(k)$ ($N_l = 1$) in (2.1). From (2.1), we substitute for $r(k)$ in the expression for $\gamma(m)$ in (2.2b) and look at the correct timing instant $m = \theta_1$ to understand how it is affected in the presence of the second user. Define $k' = k - \theta_1$, such that $s_1(k' + i) = s_1(k' + i + N)$ for $i = 0, \dots, L - 1$. The following components are present in $\gamma(\theta)$

$$\gamma(\theta) \sim \underbrace{\sum_{k'=0}^{L-1} s_1(k')s_1^*(k' + N)}_{\mathcal{S}} + \underbrace{\sum_{k'=0}^{L-1} s_2(k' + \tilde{\theta})s_2^*(k' + \tilde{\theta} + N)}_{\mathcal{J}^M} \quad (2.12)$$

The following observations can be made,

1. Component \mathcal{S} yield the sum of the correlation energies of the first user.

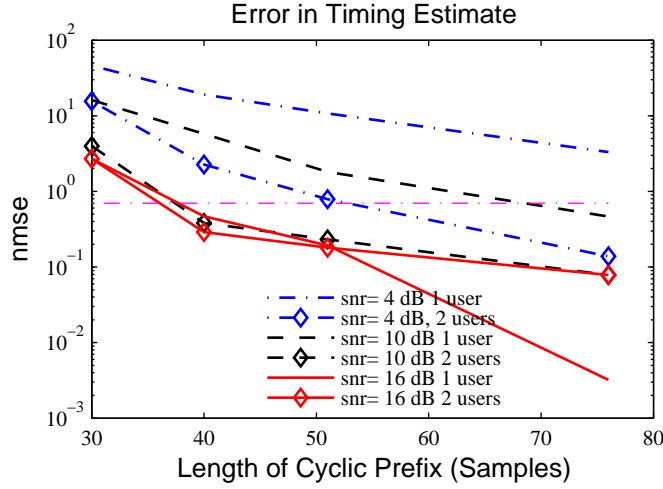


Figure 2.5: Performance of CPCorr in a two user frequency selective channel. The first user has partial spectral occupancy of 25% as given in Fig 2.3 and the second user transmits in the remaining 75% of the bands. The timing delays are chosen as per case a) of Scenario 3 ($\tilde{\theta} < L$) for different values of snr

2. If $\tilde{\theta} < L$, then $\gamma(\theta)$ contains $L - \tilde{\theta}$ terms of the second user's signal that are repeated, i.e. the terms $s_2(k' + \tilde{\theta}) = s_2(k' + \tilde{\theta} + N)$ for $k' = 0, \dots, L - \tilde{\theta} - 1$. Thus these terms add to the correlation energy. However, the last $\tilde{\theta}$ terms of the second user's signal contribute as multiple access interference (MAI) as the terms $s_2(k' + \tilde{\theta}) \neq s_2(k' + \tilde{\theta} + N)$ for $k' = L - \tilde{\theta}, \dots, L$.
3. If $\tilde{\theta} > L$, then the entire correlation energy of the second user constitutes as MAI.

Scenario 3. Consider a frequency selective channel with a Vehicular A power delay profile as given in Table 2.2. Let there be two users with orthogonal spectral occupancies in $[0, W]$. Let the occupancy of the first user be given in Figure 2.3. Let the transmit power and thus transmit snr and the CP lengths of the two users be same. The OFDM parameters for both users are given in Table 2.3. We will consider the following cases for simulation,

- a) $\theta_1 = 25, \theta_2 = 51$ and thus $\tilde{\theta} < L$ for all of L . Note that the probability of $\tilde{\theta} < L$ is roughly L/N .
- b) $\theta_1 = 10, \theta_2 = 150$ and thus $\tilde{\theta} > L$ for all L . Note that the probability of $\tilde{\theta} > L$ is roughly $1 - L/N$. Thus this is more probable than event a).

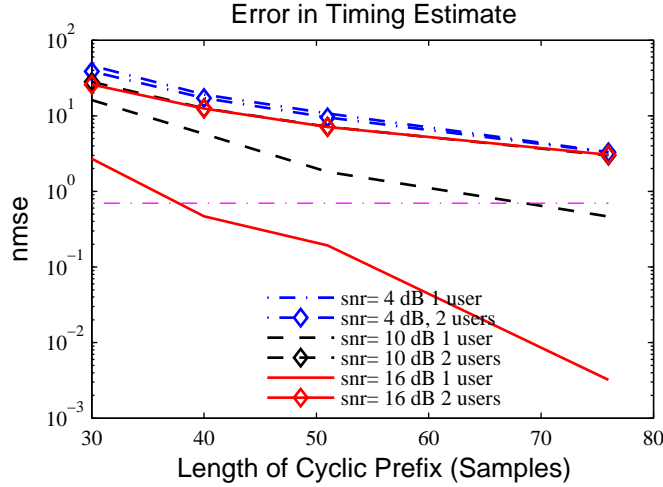


Figure 2.6: Performance of CPCorr in a two user frequency selective channel. The first user has partial spectral occupancy of 25% as given in Fig 2.3 and the second user transmits in the remaining 75% of the bands. The timing delays are chosen as per case b) of Scenario 3 ($\tilde{\theta} > L$) for different values of snr

Observation 3. *Figures 2.5 and 2.6 show the performance of CPCorr for cases 3a) and 3b) respectively. It is seen that*

1. *In case 3a), the contribution of useful correlation dominates over MAI and thus the presence of the second user helps. In case 3b), the presence of the second user degrades the performance for all values of L and snr.*
2. *Since in our model, the second user has the same received signal power as the desired user, accumulating energy from multiple symbols to increase received snr does not help in enhancing the performance, as the received power of the second user is also increased. This effect is more acute for 3b), as the entire signal of the second user is MAI.*

Conclusion 3. *In presence of another user, with the same transmit power, algorithm CPCorr satisfactorily acquires symbol timing for the desired user, with a partial spectral occupancy in a frequency selective channel, only when the differential timing delay between the two users is within the length of the cyclic prefix. If the differential delay is much larger than the cyclic prefix, the performance is not satisfactory irrespective of snr.*

2.4.1 A Note about Timing Acquisition in Cellular

To put our work in proper context, we examine the timing acquisition in OFDM based cellular systems such as WiMAX where the same scenario of multiple transmissions during the acquisition phase is prevalent. Based on Conclusion 3, our main claim is that algorithms based on CPCorr, will give satisfactory performance for cellular systems. To see this we examine the downlink and uplink separately,

Downlink

Here the problem is users synchronizing to the BS. When a mobile is first turned on, it will receive multiple transmissions from interfering BSs and will end up associating with, and acquiring the timing of, the strongest BS. This is different from Scenario 3 where for a given transmitter, the receiver was fixed and there was an equal power interferer (which can arise for ad-hoc networks engaging in DSA). For cellular networks, the mobile to BS association process ensures that signals from interfering BSs have significantly lesser power than the associated BS and thus CPCorr will work.

Uplink

Here the problem is the BS synchronizing to the transmissions from different mobiles, who may be simultaneously transmitting. All those mobiles would have already been synchronized to the timing of the BS when they had first turned on. Due to differential path lengths from mobiles to the BS, there might be some difference in the timings of the signals from these mobiles when they reach the BS, but they will be close. Thus this is similar to the case considered in Scenario 3a) and by Conclusion 3, algorithms like CPCorr will yield satisfactory performance.

2.4.2 Effect of Filtering

The reason for Conclusion 3 is that CPCorr performs energy capture from correlations and can't distinguish between the signal of the desired user from that of others. If the receiver of the first user knows the spectral occupancy of the transmitted signal, one possible way is to filter out the second user's signal before performing CPCorr. In this section, we explore this possibility. Let a band-pass filter $d(k)$ be applied to filter out the second user's signal. The

| Filter | $A_{\text{stop}}(\text{dB})$ | $A_{\text{pass}}(\text{dB})$ | $\delta(\text{MHz})$ | Filter Lengths |
|--------|------------------------------|------------------------------|----------------------|------------------|
| A | 15 | 5 | 0.2 | [25, 15, 15, 15] |
| B | 15 | 5 | 0.1 | [49, 27, 27, 27] |
| C | 20 | 1 | 0.1 | [61, 61, 61, 61] |

Table 2.4: Parameters for band pass filters used in Figure 2.7

received signals without filtering, $r(k)$ and with filtering, $r_f(k)$ are respectively given by,

$$\begin{aligned}
r(k) &= h_1(k) \otimes s_1(k - \theta_1) + h_2(k) \otimes s_2(k - \theta_2) + z(k) \\
r_f(k) &= \underbrace{d(k) \otimes h_1(k)}_{\text{more correlation}} \otimes s_1(k - \theta_1) \\
&\quad + \underbrace{d(k) \otimes h_2(k) \otimes s_2(k - \theta_2)}_{\text{suppression of } s_2(k)} + z(k).
\end{aligned} \tag{2.13}$$

As seen in (2.13), filtering suppresses the signal of the second user but makes the desired user's signal pass through the effective channel, $d(k) \otimes h_1(k)$ which has a longer *effective delay spread*. This increases the self interference as noted in Example 1a). Thus roughly speaking, the introduction of a filter introduces a trade-off between suppressing multi-user interference and suppressing self interference. It would be insightful to analytically characterize this trade-off, for given channel parameters and snr. In this chapter, we however perform extensive simulations that enables us to identify some of the trends in the trade-off and decide if use of the filter makes the subsequent use of CPCorr satisfactory.

Scenario 4. Consider the system mentioned in Scenario 3. Let the receiver of user one knows its spectral occupancy. The receiver extracts the signal of the first user using band-pass filters, whose impulse responses are generated by Kaiser windowing technique of FIR filter generation [57, Chapter 10]. This allows to specify the stop-band attenuation, A_{stop} which corresponds to the bands occupied by other users, maximum allowable pass-band ripples, A_{pass} and the transition width between stop band and pass band, δ . The parameters used to generate the three such filters are shown in Table 2.4 and their magnitude responses are given in Figure 2.7. Filter A has the least stringent specifications for MAI suppression but also the smallest length filters leading to least self-interference. The opposite is true for Filter C.

Observation 4. Figures 2.8 and 2.9 compare the performance of CPCorr with and without filtering for $\tilde{\theta} < L$ and $\tilde{\theta} > L$ respectively. In general, as the length of CP increased, there

is a cross-over point beyond which the performance with filtering becomes better than without filtering. Specifically when $\tilde{\theta} < L$, from Figure 2.8 we see that

- a) For high value of L , the effective extra delay spread introduced due to the filter (self-interference) is less than L and hence having a filter is better due to MAI suppression.
- b) For low L , the effective extra delay spread introduced due to the filter is significant compared to L and hence having a filter leads to worse performance.

When $\tilde{\theta} > L$, all three filters fail to restore the performance of CPCorr to acceptable levels.

As a note, we conducted simulations with a variety of other channel power delay profiles and filters and the general trends in Observation 4 continue to hold.

2.4.3 Single Sun-band Spectral Occupancy

Filtering did not help to improve performance in the four sub-band spectral occupancy case as suppression of narrow bands required longer filters which increased the self-interference. However, if the spectral occupancies were not divided into such narrow bands as in Figure 2.3, then MAI could be reduced with shorter filters which would cause much less self-interference. Intuitively, the performance of CPCorr should improve.

Scenario 5. Consider a frequency selective channel with a Vehicular A power delay profile as given in Table 2.2. Let there be two users with orthogonal spectral occupancies in $[0, W]$. Let the occupancy of the first user be 25% and the occupancy be in a single contiguous subband as shown in Figure 2.10. Let $W_0 = 2$ MHz. The rest of the parameters are same as in Scenario 3.

For the uniform occupancy case we'll only consider the adverse situation of $\tilde{\theta} > L$, as our aim is to establish the shortcomings of CPCorr (if any) and for $\tilde{\theta} < L$, CPCorr performed satisfactorily, even for the non-contiguous spectral occupancy case as noted in Conclusion 3. First we compare the performance of CPCorr, in absence of filtering for the four sub-bands vs the single sub-band spectral occupancy. This is shown in Figure 2.11. The performances are almost same which means that the exact nature of the spectral occupancy is not important for CPCorr. Finally we consider the effects of filtering for uniform spectral occupancy. The filters used have parameters as shown in Table 2.5 and their magnitude responses are plotted in Figure 2.12. The performance of CPCorr, when $\tilde{\theta} > L$ is shown in Figure 2.13.

| Filter | $A_{\text{stop}}(\text{dB})$ | $A_{\text{pass}}(\text{dB})$ | $\delta(\text{MHz})$ | Filter Lengths |
|--------|------------------------------|------------------------------|----------------------|----------------|
| A | 10 | 5 | 0.2 | 7 |
| B | 15 | 5 | 0.2 | 15 |
| C | 15 | 5 | 0.1 | 27 |

Table 2.5: Parameters for band pass filters used in Figure 2.12

Observation 5. *The following observations can be made,*

- a) *Filtering satisfactorily restores the performance of algorithm CPCorr for all filters. Performance of filter B is almost similar to that of filter A and is not shown in Figure 2.12 for purposes of clarity.*
- b) *Since all filter lengths are less than the length of the prefix, choosing the longest filter (Filter C) gives best performance due to MAI suppression.*

Conclusion 4. *From Observations 4 and 5, we conclude that for a two user transmission when the differential delay is much larger than the length of the cyclic prefix, then for a given fraction of occupied bandwidth, filtering becomes less helpful as that occupied bandwidth is split between more and more contiguous parts.*

The reason behind Conclusion 4 is that to suppress the interferer's signal, a smaller length filter is required in the case of contiguous spectrum occupancy. This also implies that, even in the case of non-contiguous occupancy, if there are guard bands in between different users, then the requirements on filter are relaxed leading to smaller length filters and the acquisition performance can be improved. Presence of guard bands can be assumed if the spectrum is licensed to a primary user and secondary users opportunistically access it but must leave guard bands to minimize interference to the primary user.

2.5 Conclusion

We considered a scenario where co-located cognitive systems would dynamically share a given spectrum by transmitting in non-overlapping and possibly non-contiguous bands and studied the performance of a practically implementable, cyclic prefix correlation based algorithm for OFDM symbol timing acquisition. Since the algorithm is optimal only for a single user transmission in a frequency non-selective channel, we developed a concept of self and multiple

access interference to derive important insights into the working of the algorithm for multi-user transmissions for general frequency selective channels. We found that for a two user system, when the differential delays of the two users are much larger than the cyclic prefix, the performance of the algorithm, even with filtering, deteriorates as the occupied bandwidth is split into several pieces, and in some realistic cases becomes quite poor. If the receiver of the desired user is aware of the bands in which the transmitted signal lies, we showed that it could filter its intended signal and partially restore the performance. The single-user results point to the limits on acquisition performance imposed by occupying a small fraction of the band (e.g., cognitive radio over 100 MHz, with each user occupying only a few MHz), esp. when the occupied bandwidth is split into multiple pieces.

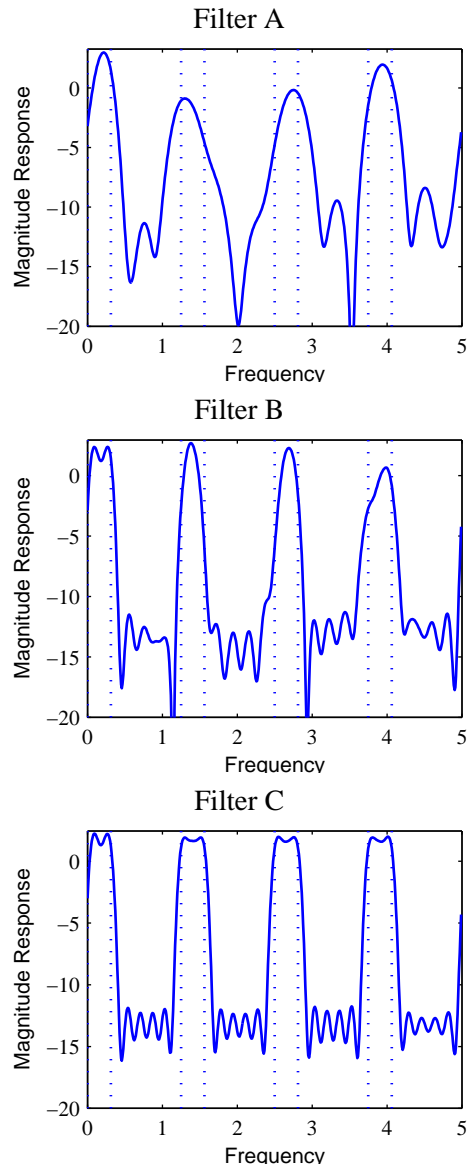


Figure 2.7: Magnitude response of band pass filters used to filter out the second user's signal with spectral occupancy of the first user given in Figure 2.3. The corresponding parameters are given in Table 2.4

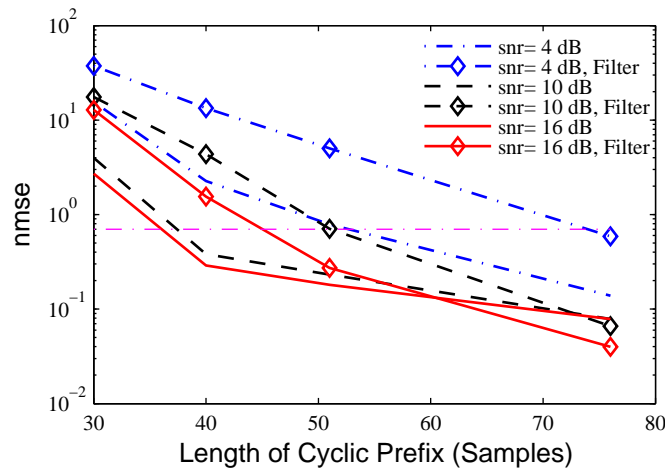


Figure 2.8: Performance of CPCorr in a two user frequency selective channel. The first user has partial spectral occupancy of 25% as given in Fig 2.3 and the second user transmits in the remaining 75% of the bands. The timing delays are chosen as per case a) of Scenario 3 ($\tilde{\theta} < L$) for different values of snr. Band-pass filter B shown in Figure 2.7 has been used for filtering out the signal of user one prior to CPCorr.

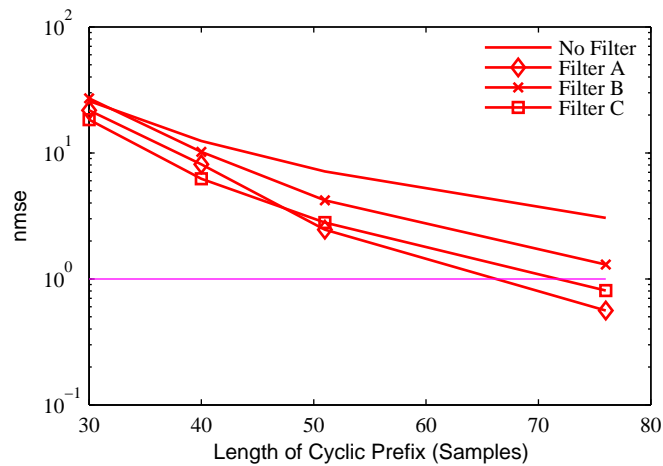


Figure 2.9: Performance of CPCorr in a two user frequency selective channel. The first user has partial spectral occupancy of 25% as given in Fig 2.3 and the second user transmits in the remaining 75% of the bands. The timing delays are chosen as per case b) of Scenario 3 ($\tilde{\theta} > L$). The snr is fixed at 16 dB. All three band-pass filters shown in Figure 2.7 have been used for filtering out the signal of user one prior to CPCorr.

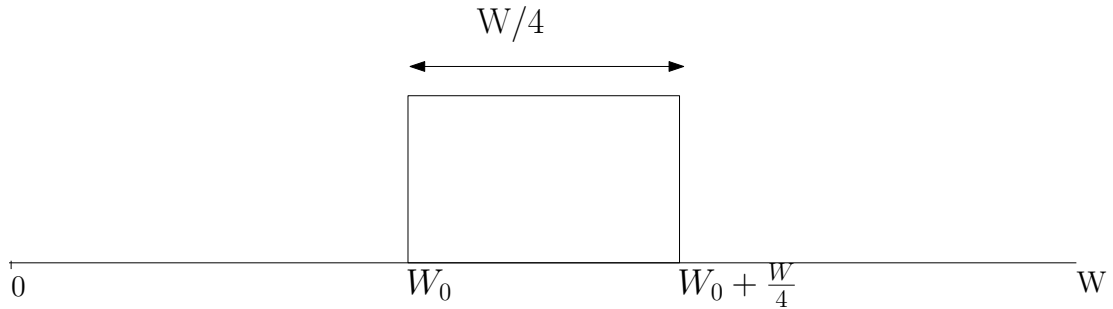


Figure 2.10: Single sub-band *uniform* spectral occupancy of the first user in the band $[0, W]$

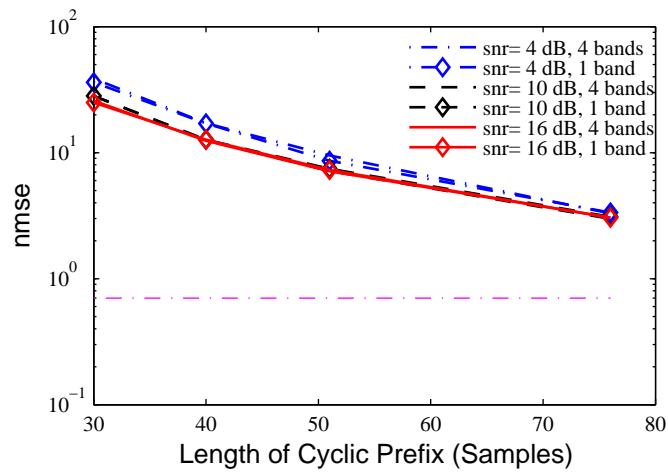


Figure 2.11: Performance of CPCorr for a two user system with the first user having a 25% spectral occupancy. The single band vs four sub-band spectral occupancy has been compared.

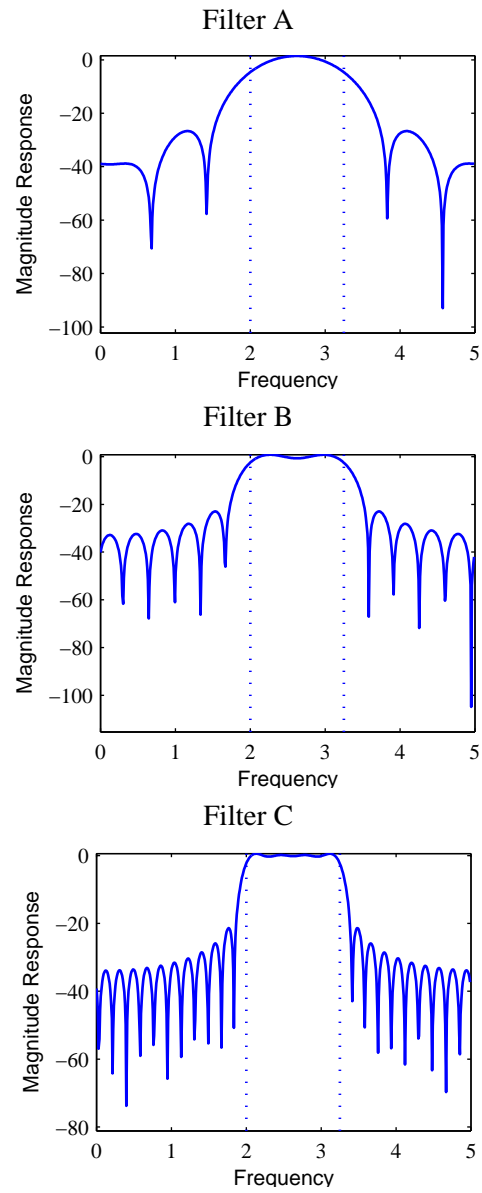


Figure 2.12: Magnitude response of band pass filters used to filter out the first user's signal with spectral occupancy as given in Figure 2.10. The corresponding parameters are given in Table 2.5

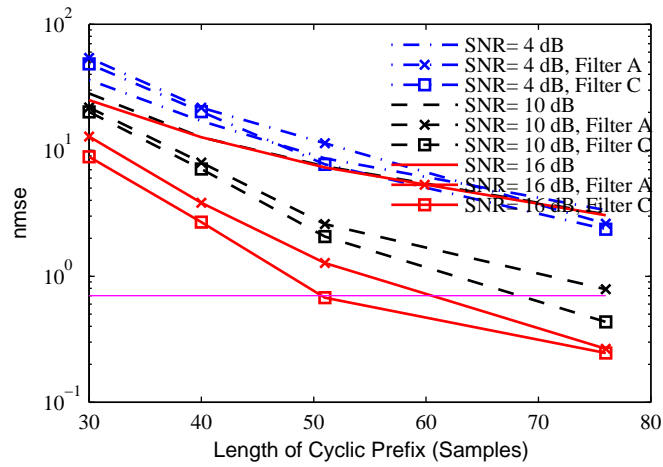


Figure 2.13: Performance of CPCorr in a two user frequency selective channel. The first user has partial spectral occupancy of 25% as given in Fig 2.10 and the second user transmits in the remaining 75% of the bands. The timing delays are chosen as per case b) of Scenario 3 ($\tilde{\theta} > L$). Performances of two band-pass filters shown in Figure 2.12 have been used for filtering out the signal of user one prior to CPCorr.

Chapter 3

User Utility Maximization for centralized DSA

3.1 Introduction

As mentioned in Section 1.1, the IEEE had set up a working group to develop the 802.22 cognitive radio standard that would employ the unused spectrum in the VHF and UHF TV bands to offer wireless broadband services in areas that were not well served by alternatives such as cable or DSL [11]. In the 802.22 draft, it has been decided that fixed wireless access will be provided in these bands [10] by professionally installed Wireless Regional Area Network (WRAN) base stations to WRAN user terminals. A service provider (SP) operating a base station will not have to pay any licensing fees. It will share the total spectrum with other SPs in the region and further allocate this spectrum to users efficiently. Since the exact band of spectrum, within 54 to 865 MHz, where the TV broadcasters operate is variable, the WRAN SPs will have to sense their and each other's presence before deciding how to share the spectrum among themselves. Fundamental issues about cognitive radios can be found in [4, 42, 58]. Motivated by the SP-user model of 802.22, in this chapter we propose and analyze a dynamic spectrum allocation algorithm based on *limited coordination amongst devices* and the notion of a *spectrum price*. The authors in [24, 39, 59] have also espoused similar design principles.

3.1.1 Our Contribution

While [24, 39, 59] have mostly focused on the network architecture and protocol signaling, in this chapter we study the distributed spectrum allocation problem with limited cooperation from an analytical standpoint. We consider a two tiered spectrum allocation scheme as shown in Chapter 1, Figure 1.1. There is a total spectrum W available in a geographic area which is allocated to the users through the SPs. The users are permitted to obtain spectrum from all the SPs. We assume that the users obtain non-overlapping chunks of spectrum from the SPs to avoid interference. Assuming that each user application has an associated utility which is

concave and increasing as a function of spectrum obtained, we adopt an utility maximization framework [60, 61] to analyze the system. Given user utility functions, channel coefficients between users and SPs and user power constraints, our aim is to derive how much spectrum should a user obtain from an SP and what power should he allocate for sending his information to the SPs. We allow for simple SP coordination to share the spectrum W where the spectrum utilized by a SP depends on how much spectrum it has to allocate to the users. This is facilitated by a spectrum clearing house (SCH), akin to an FCC-controlled regional spectrum broker [39]. Note that our model is not specific to 802.22 systems and is applicable for any user-SP based spectrum allocation system.

Prior work had mostly considered a single SP with fixed frequency bins (OFDM tones) whereas in our work we allow for multiple SPs and treat spectrum as a continuous resource. Treating spectrum as continuous is justified for systems where the subcarrier spacing is small and the number of subcarriers is large. An example system is LTE which can operate with 15 KHz spacing and 2048 subcarriers [62]. In addition, we also allow the different SPs to have different efficiencies which is defined as the fraction of Shannon capacity that the SP can reliably deliver.

Based on our analysis we propose a simple spectrum allocation protocol based on the notion of *spectrum price*. In the first part of the protocol the SPs broadcast the spectrum price to the users. Given the spectrum price and local parameters such as its utility and link gains to the SPs, each user can decide on its own how much spectrum to use. In the second part of the protocol this value is conveyed back to the SPs who update the spectrum price. The SPs do not need to know the link gains or user utilities and hence we claim that there is partial coordination in the system and not total.

Our model precludes spectrum overlap as we assume that the various users are close to each other and to the SPs and thus spectral overlap would cause significant interference. For centralized networks if users are allowed to share spectrum, the situation is similar to finding the capacity of frequency selective interference channels. But apart from certain specific bounds, the optimal signaling is unknown even for simple interference channels [63, 64, 65]. In distributed networks [20], for regimes when the cross gains between transmit receive pairs are stronger than direct gains, only orthogonal spectrum allocation guarantees Pareto efficiency. Because of orthogonal allocation, our model has similarities with network flow control models [61, 66]. The notion of SP efficiency and the usage of the Shannon rate function (defined

later) in the user utilities distinguishes our work. Practical transmitters might employ directional antennae to achieve frequency reuse but in this work, we limit ourselves to finding the fundamental limits on gain possible with only bandwidth allocation. Our results serve as a baseline case for understanding the additional benefits if multiple antennae are deployed.

Note that our spectrum price based approach [18, 67, 68] is not the only way of dealing with non strategic users with partial coordination. The authors in [69] consider discrete frequency bins and allow users to overlap but charge each user based in the interference that it creates for others. They show that the resulting rates are higher than those obtained by iterative water-filling.

3.2 System Model

The network topology was shown in Chapter 1, Figure 1.1. There are N SPs and L end users and a central Spectrum Broker. Based on the demand for spectrum, Service Provider i provides X_i units to the L users or a subset of them. Let x_{ij} be the amount of spectrum obtained by user j from SP i . The users and SPs are assumed to be capable of transmitting and receiving over any spectrum band x_{ij} which lies within W . This could be achieved using non-contiguous OFDM technology [37]. Subsequently, user j transmits his data to SP i over spectrum x_{ij} at rate r_{ij} and with power p_{ij} . Each user has a total transmit power constraint. The user j to SP i link gain is h_{ij} , which remains constant during the period of spectrum allocation and subsequent transmission to the SP. We assume that h_{ij} is flat over frequency and hence is same no matter in which band x_{ij} lies. The coefficients h_{ij} are assumed to be known by both users and SPs. The background additive Gaussian noise is assumed of unit power spectral density. We first introduce some notations. A source transmitting with power p , over a flat channel of bandwidth x and link gain h has signal to noise ratio $\text{snr}(x, p, h) = hp/x$ and achieves the rate

$$r(x, p, h) = x \log(1 + \text{snr}(x, p, h)). \quad (3.1)$$

In terms of $r(x, p, h)$ the rate r_{ij} is given by

$$r_{ij} = \eta_i r(x_{ij}, p_{ij}, h_{ij}), \quad (3.2)$$

where η_i is the fraction of the Shannon capacity that can be reliably guaranteed by SP i to a user. A possible example would be SP i , who has invested in a better decoder (a Turbo decoder

with more iterations or better interleaver design) has a higher η_i than an SP with a conventional Viterbi decoder. Thus the total rate at which user j can transmit reliably is

$$R_j = R(\mathbf{x}_j, \mathbf{p}_j, \mathbf{h}_j) = \sum_{i=1}^N r_{ij}, \quad (3.3)$$

where $\mathbf{x}_j = [x_{1j} \cdots x_{Nj}]$, $\mathbf{h}_j = [h_{1j} \cdots h_{Nj}]$ and $\mathbf{p}_j = [p_{1j} \cdots p_{Nj}]$.

There is a utility function $U_j(R_j)$ associated with user j which is concave and increasing in R_j . The operating principle of the network is to maximize social welfare or the sum utility of the users. The optimization problem is

$$\max_{x_{ij} \geq 0, p_{ij} \geq 0, X_i \geq 0} \sum_{j=1}^L U_j(R_j) \quad (3.4a)$$

$$\text{s.t.} \quad \sum_{j=1}^L x_{ij} \leq X_i, \quad 1 \leq i \leq N, \quad (3.4b)$$

$$\sum_{i=1}^N p_{ij} \leq P_j, \quad 1 \leq j \leq L, \quad (3.4c)$$

$$\sum_{i=1}^N X_i \leq W. \quad (3.4d)$$

As shown in (3.4b), X_i is the spectrum utilized by SP i which is equal to the spectrum it has to allocate to the users. User j transmits with power p_{ij} to SP i and as (3.4c) shows there is a constraint P_j on the total transmit power. The total amount of available spectrum is W . User j optimizes over x_{ij} and p_{ij} . In Appendix A, we show that the objective is concave in these variables and since the constraints are linear, the problem can be solved efficiently.

3.2.1 Distributed Solution and Pricing

In this section we give a distributed implementation of the spectrum allocation problem (3.4). First we relax the constraints (3.4b) and (3.4d) in the objective function to form the partial Lagrangian \mathcal{L} [70]

$$\begin{aligned} \mathcal{L}(x_{ij}, p_{ij}, X_i, \boldsymbol{\lambda}, \mu) = & \sum_{j=1}^L U_j(R_j) + \sum_{i=1}^N \lambda_i \left(X_i - \sum_{j=1}^L x_{ij} \right) \\ & + \mu \left(W - \sum_{i=1}^N X_i \right), \end{aligned} \quad (3.5)$$

where $\boldsymbol{\lambda} = [\lambda_1, \dots, \lambda_N]^T$. The stationarity conditions w.r.t. X_i can be expressed as,

$$\frac{\partial \mathcal{L}}{\partial X_i} = \lambda_i - \mu \leq 0, \quad (3.6)$$

with equality holding *iff* $X_i > 0$. Interpreting μ as the price the broker charges to the SPs and λ_i as the price that SP i charges to its users [61] we see that each SP i that provides non-zero spectrum ($X_i > 0$) charges the same price $\lambda_i = \mu$. This is because the SPs have no objectives of their own.

Thus we form the Lagrangian

$$\mathcal{L}(x_{ij}, p_{ij}, \mu) = \sum_{j=1}^L U_j(R_j) - \mu \sum_{i=1}^N \sum_{j=1}^L x_{ij} + \mu W \quad (3.7)$$

for the new optimization problem and the dual

$$\begin{aligned} D(\mu) &= \max_{x_{ij} \geq 0, p_{ij} \geq 0} \mathcal{L}(x_{ij}, p_{ij}, \mu) \\ \text{s.t. } &\sum_{i=1}^N p_{ij} \leq P_j, \quad 1 \leq j \leq L. \end{aligned} \quad (3.8)$$

The spectrum price μ is set jointly by the SPs and the broker by minimizing the dual

$$\min_{\mu > 0} D(\mu). \quad (3.9)$$

From (3.7) the optimization in (3.8) decomposes into separate optimization problems for the users [70]. The optimization subproblem for user j is

$$\mathcal{U}_j = \max_{x_{ij} \geq 0, p_{ij} \geq 0} U_j(R_j) - \mu \sum_{i=1}^N x_{ij} \quad (3.10a)$$

$$\text{s.t. } \sum_{i=1}^N p_{ij} \leq P_j. \quad (3.10b)$$

Quantity \mathcal{U}_j , called the *user surplus* in microeconomics [41, Chapter 14], is the residual utility of user j after paying the spectrum cost. In the context of a spectrum price, this is the payment in terms of the utility function that has to be given to user j to persuade him to give up his consumption of spectrum. The price μ is set by a distributed price update for (3.9)

$$\mu(t+1) = \left[\mu(t) - \alpha_\mu(t) \left(W - \sum_{i=1}^N \sum_{j=1}^L x_{ij}(\mu(t)) \right) \right]^+ \quad (3.11)$$

where $x_{ij}(\mu(t))$ is the spectrum obtained by user j from SP i , for a given value of $\mu(t)$ and $\alpha_\mu(t)$ is a positive step size. From (3.11) we see that if the spectrum is underutilized, $W -$

| Distributed Spectrum Allocation Mechanism |
|--|
| 1) At time t , SPs broadcast price $\mu(t)$. |
| 2) Each user j solves (3.10) and calculates $x_{ij}(\mu(t))$ and $p_{ij}(\mu(t))$ for all i SPs. |
| 3) All users pass $x_{ij}(\mu(t))$ to each SP i . |
| 4) The SPs calculate $\mu(t+1)$ from (3.11). |

Table 3.1: Distributed update of spectrum and power

$\sum_{i=1}^N \sum_{j=1}^L x_{ij}(\mu(t))$ is positive and thus the price decreases to facilitate greater utilization of spectrum. Similarly if spectrum is over-utilized, the price increases. This is summarized in the following theorem

Theorem 1. *The global spectrum price μ charged by all the SPs is set such that the entire spectrum is utilized.*

The distributed spectrum allocation mechanism is given in Table 3.1. From [71, Proposition 3.4], $\mu(t)$ converges to the equilibrium price μ for proper choice of step size $\alpha_\mu(t)$.

3.3 Characterizing the Spectrum Allocation

We will denote the first and second derivatives of the utility function by

$$\dot{U}_j(R_j) \triangleq \frac{\partial U_j}{\partial R_j}, \quad \ddot{U}_j(R_j) \triangleq \frac{\partial^2 U_j}{\partial R_j^2}. \quad (3.12)$$

The derivatives of the rate function $r(x, p, h)$, in (3.1), are

$$\Gamma_p(x, p, h) \triangleq \frac{\partial r}{\partial p} = \frac{hx}{x + hp} = \frac{h}{1 + \text{snr}(x, p, h)} \quad (3.13a)$$

$$\begin{aligned} \Gamma_x(x, p, h) &\triangleq \frac{\partial r}{\partial x} = \log\left(1 + \frac{hp}{x}\right) - \frac{hp}{x + hp} \\ &= \log(1 + \text{snr}(x, p, h)) - \frac{\text{snr}(x, p, h)}{1 + \text{snr}(x, p, h)} \end{aligned} \quad (3.13b)$$

It follows from (3.2) and (3.3) that the derivatives of R_j wrt x_{ij} and p_{ij} can be expressed as,

$$\frac{\partial R_j}{\partial p_{ij}} = \eta_i \Gamma_p(x_{ij}, p_{ij}, h_{ij}), \quad (3.14a)$$

$$\frac{\partial R_j}{\partial x_{ij}} = \eta_i \Gamma_x(x_{ij}, p_{ij}, h_{ij}). \quad (3.14b)$$

To arrive at the optimal solution for the user subproblem (3.10), we first write its Lagrangian

$$\mathcal{L}_j = U_j(R_j) - \sum_{i=1}^N \mu x_{ij} + \gamma_j \left(P_j - \sum_{i=1}^N p_{ij} \right), \quad (3.15)$$

where all Lagrange multipliers are positive. The stationarity conditions for the Lagrangian are

$$\frac{\partial \mathcal{L}_j}{\partial x_{ij}} \equiv \eta_i \dot{U}_j(R_j) \Gamma_x(x_{ij}, p_{ij}, h_{ij}) \leq \mu, \quad (3.16a)$$

$$\frac{\partial \mathcal{L}_j}{\partial p_{ij}} \equiv \eta_i \dot{U}_j(R_j) \Gamma_p(x_{ij}, p_{ij}, h_{ij}) \leq \gamma_j, \quad (3.16b)$$

with equality holding for users with $x_{ij} > 0$ and $p_{ij} > 0$ respectively.

Theorem 2. *In the optimal solution of (3.10) only one SP is active per user almost surely.*

Proof. Consider user j and SP i and assume $x_{ij} > 0$ and $p_{ij} > 0$. Thus (3.16a) and (3.16b) are satisfied with equality. Dividing (3.16a) by (3.16b) and after some manipulation we obtain,

$$\left(1 + \frac{h_{ij} p_{ij}}{x_{ij}}\right) \log \left(1 + \frac{h_{ij} p_{ij}}{x_{ij}}\right) - \frac{h_{ij} p_{ij}}{x_{ij}} = \kappa_j h_{ij}, \quad (3.17)$$

where $\kappa_j = \mu/\gamma_j$. Now consider the function $\Psi(\text{snr}) = (1 + \text{snr}) \log(1 + \text{snr}) - \text{snr}$, which can be shown to be one-to-one and increasing in snr. Substituting for $\text{snr} = h_{ij} p_{ij}/x_{ij} = \Psi^{-1}(\kappa_j h_{ij})$ in (3.13a) and then substituting for $\Gamma_p(\cdot)$ in (3.16b) we obtain

$$\eta_i \dot{U}_j(R_j) \left[\frac{h_{ij}}{1 + \Psi^{-1}(\kappa_j h_{ij})} \right] = \gamma_j. \quad (3.18)$$

Let user j obtain spectrum from SPs i and k . From (3.18)

$$\frac{\eta_i h_{ij}}{1 + \Psi^{-1}(\kappa_j h_{ij})} = \frac{\eta_k h_{kj}}{1 + \Psi^{-1}(\kappa_j h_{kj})}. \quad (3.19)$$

Since h_{ij} is a continuous random variable the probability of (3.19) is zero. This is a contradiction. Thus each user obtains spectrum from one SP *almost surely*. \square

Various flavors of Theorem 2 are also observed in [72, 73]. If instead of a net spectrum constraint ((3.4b) and (3.4d) together) there were individual spectrum constraints at each SP (only (3.4b)), then the problem of *ties* would occur [14, 74].

Let the active SP of user j be denoted by i_j^* . Denote $x_{i_j^* j}$, $h_{i_j^* j}$ and $\eta_{i_j^*}$ by x_j^* , h_j^* and η_j^* respectively. The user optimization in (3.10) can be re-written by considering only $i = i_j^*$. The rate R_j given in (3.3) has contribution only from $r_{i_j^* j}$ and is denoted by $R_j^* = \eta_j^* x_j^* \log \left(1 + h_j^* P_j / x_j^*\right)$.

3.3.1 Insights to SP User Assignment

Since user j is attached to SP i_j^* , from (3.4), it means that if it were allocated the optimum spectrum x_j^* from any other SP $i \neq i_j^*$, it would have still obtained a lower utility. Since the utility $U_j(R_j)$ is an increasing function of R_j this implies that user j obtains the highest rate from SP i_j^* , for given spectrum x_j^* . Define the signal to noise ratio, $\text{snr}_{ij} = h_{ij}P/x_j^*$. Thus

$$i_j^* = \arg \max_i \eta_i x_j^* \log(1 + \text{snr}_{ij}) \quad (3.20a)$$

$$= \arg \max_i (1 + \text{snr}_{ij})^{\eta_i}. \quad (3.20b)$$

Actually if user j were to be associated with SP $i \neq i_j^*$, the allocated spectrum would be different from x_j^* , but this does not affect our result.

Observation 6. *The following observations can be made*

- a) Low snr_{ij} regime: Use $(1 + x)^n \simeq 1 + nx$ in (3.20b) to obtain $i_j^* = \arg \max_i \eta_i h_{ij}$.
- b) High snr_{ij} regime: Use the approximation $(1 + x)^n \simeq x^n$ in (3.20b). For a better insight consider 2 SPs with SP 1 being more efficient. Thus $\eta = \eta_1/\eta_2 > 1$. The condition for which SP 1 is the active SP for user j turns out to be

$$x_j^* < P \left(\frac{h_{1j}^\eta}{h_{2j}} \right)^{1/(\eta-1)}. \quad (3.21)$$

Thus user j attaches to the more efficient SP when the optimal bandwidth allocation x_j^ is less than a threshold. That is, user j will use the more efficient SP when bandwidth becomes scarce.*

Corollary 1. *If all SPs have the same efficiency, then each user obtains spectrum from the SP to which it has the highest link gain.*

Proof. Follows from condition (3.20b) with $\eta_i = \eta$. □

Lemma 1. *The following facts hold,*

- a) $\dot{U}(R)$ for $R = r(x, P, h)$, as defined in (3.1), is a decreasing function of x .
- b) $\Gamma_x(x, P, h)$ is a strictly decreasing function of x and is positive for all values of $\mathbf{v} = [x, P]$ for fixed h .

Proof. a) Since $U(R)$ is concave, $\ddot{U}(R) < 0$. This means $\dot{U}(R)$ is decreasing in R . But R increases in x from (3.1). Combining we get the desired result.

b) It can be verified that $r(x, P, h)$ is concave and increasing in $\mathbf{v} = [x, P]$ for fixed h and thus concave and increasing in x for fixed P and h . From concavity of $r(x, P, h)$ wrt x , $\Gamma_x(x, P, h)$ is monotonic decreasing in x and since $R(x, P, h)$ is increasing, we conclude that $\Gamma_x(x, P, h) > 0$. \square

In the next Theorem, we verify that each user obtains a strictly positive spectrum allocation. Intuitively this makes sense as if a user is not allocated spectrum then the potential increase to the sum utility due to his transmit power is wasted. The proof appearing in the Appendix C, Section C.1 shows that when a new user $L + 1$ joins the system of L users, a new allocation in which each of the original L users forfeits spectrum ϵ and user L obtains spectrum $L\epsilon$ provides higher sum utility for small ϵ .

Theorem 3. *In the optimal allocation each user j obtains spectrum $x_j^* > 0$.*

3.3.2 Dependence on Marginal Utility and Received Power

Theorem 4. *When two users have the same channel gains, transmit powers and active SP efficiencies, the optimal allocation of spectrum favors the user with a higher marginal utility of spectrum i.e. whose utility function has a higher rate of increase with spectrum.*

Proof. Consider users j and k with utility functions satisfying $\dot{U}_j(R) > \dot{U}_k(R)$ for all R and for whom $h_k^* = h_j^* = h$, $P_k = P_j = P$ and $\eta_k^* = \eta_j^*$. Let the allocated spectrum for users j and k be x_j^* and x_k^* respectively. We have to show that $x_j^* > x_k^*$.

Assume the contrary i.e. $x_k^* \geq x_j^*$. Now consider (3.16a) for both users

$$\dot{U}_j(R_j^*)\Gamma_x(x_j^*, P, h) = \dot{U}_k(R_k^*)\Gamma_x(x_k^*, P, h) = \mu. \quad (3.22)$$

Consider $x_k^* \geq x_j^*$. Let $R_j^* = R_j(x_j^*, P, h)$ and $R_k^* = R_j(x_k^*, P, h)$. This implies

1. $\dot{U}_j(R_j^*) \stackrel{(a)}{>} \dot{U}_k(R_j^*) \stackrel{(b)}{\geq} \dot{U}_k(R_k^*)$ where (a) is given in the statement of the problem and (b) is true from Lemma 1(a)
2. $\Gamma_x(x_j^*, P, h) \geq \Gamma_x(x_k^*, P, h)$ from Lemma 1(b)

Thus $\dot{U}_j(R_j^*)\Gamma_x(x_j^*, P, h) > \dot{U}_k(R_k^*)\Gamma_x(x_k^*, P, h)$ from 1) and 2), which contradicts (3.22). \square

This is because a unit of spectrum Δx yields a higher contribution to sum utility when allocated to user j than to user k . This has also been observed in [66] for a network flow control problem.

We can illustrate this phenomenon with the class of exponential utilities given by

$$U_j(R_j) = \Gamma_j \left(1 - e^{-R_j/\Gamma_j}\right), \quad (3.23)$$

where Γ_j is the *target rate* of user j . For example, $\Gamma_j = 10^6$ b/s might be appropriate for a file transfer while $\Gamma_j = 10^4$ b/s would be adequate for a voice application. Since $\dot{U}_j(R_j) = e^{-R_j/\Gamma_j}$ is increasing in Γ_j for all R_j , the high target rate users are allocated more spectrum than those with low ones. As $R_j \rightarrow \infty$, these utilities become flat, i.e. $U_j(R_j) \rightarrow \Gamma_j$.

Another class of utilities used to model elastic applications are α utilities [75], given by

$$U_\alpha(R) = \frac{1}{\alpha} R^\alpha, \quad 0 < \alpha \leq 1, \quad U_0(R) = \log(R). \quad (3.24)$$

$\alpha = 1$ gives rate as the utility and for lower values of α , the utility increases sub-linearly for rates above a threshold. Thus high α models applications with high rate requirements.

Lemma 2. *For α utilities, $\dot{U}(R)\Gamma_x(x, P, h)$ is a strictly increasing function of P for fixed x .*

Proof. Refer to Appendix C, Section C.2. □

Note that $\dot{U}(R(x, P, h))$ is actually *decreasing* in P while $\Gamma_x(x, P, h)$ is increasing in P . For α utilities, we show, in Appendix C, Section C.2, that their product increases with P . This need not be true for any arbitrary increasing concave function, such as the exponential utilities in (3.23) as they flatten out at Γ_j .

Theorem 5. *If all users have α utilities and the received power of one user increases and user to SP assignments remain the same or the user switches to a SP with same efficiency, then that user obtains more spectrum and the spectrum price increases.*

Proof. Consider user j and let $k \neq j$ be any other user. Let the price be μ and users j and k obtain spectrum x_j^* and x_k^* . Let user j increase his power from P_j to $\tilde{P}_j > P_j$. Let the new allocations be \tilde{x}_j^* and \tilde{x}_k^* for users j and k . The spectrum price changes from μ to $\tilde{\mu}$ and the rates from R_j^* and R_k^* to \tilde{R}_j^* and \tilde{R}_k^* for users j and k . By Theorem 3, all spectrum allocations

are strictly positive and relation (3.16a) holds with equality for the old and new allocations and

$$\eta_j^* \dot{U}_j(R_j^*) \Gamma_x(x_j^*, P_j, h_j) = \eta_k^* \dot{U}_k(R_k^*) \Gamma_x(x_k^*, P_k, h_k) = \mu. \quad (3.25a)$$

$$\eta_j^* \dot{U}_j(\tilde{R}_j^*) \Gamma_x(\tilde{x}_j^*, \tilde{P}_j, h_j) = \eta_k^* \dot{U}_k(\tilde{R}_k^*) \Gamma_x(\tilde{x}_k^*, P_k, h_k) = \tilde{\mu}. \quad (3.25b)$$

We have to show that $\tilde{x}_j^* > x_j^*$. Assume the contrary that the event $\mathcal{A} \equiv \tilde{x}_j^* \leq x_j^*$ holds. Since there is a sum spectrum constraint, $\mathcal{A} \Rightarrow \mathcal{B}$, where $\mathcal{B} \equiv \tilde{x}_k^* \geq x_k^*$ for some user $k \neq j$.

From (3.25a), the old allocation for user k satisfies $\eta_k^* \dot{U}_k(R_k^*) \Gamma_x(x_k^*, P_k, h_k) = \mu$. From \mathcal{B} , $\tilde{x}_k^* \geq x_k^*$ and applying Lemma 1 we obtain,

$$\eta_k^* \dot{U}_k(\tilde{R}_k^*) \Gamma_x(\tilde{x}_k^*, P_k, h_k) \leq \mu. \quad (3.26)$$

For user j , there are two changes: a decrease in allocated spectrum and an increase in transmit power. Let us see their effects in isolation. First keep transmit power unchanged. From \mathcal{A} , $\tilde{x}_j^* \leq x_j^*$ and using Lemma 1 we get

$$\underbrace{\eta_j^* \dot{U}_j(R_j(\tilde{x}_j^*, P_j, h_j)) \Gamma_x(\tilde{x}_j^*, P_j, h_j)}_{\partial U_j / \partial \tilde{x}_j^*} \geq \mu. \quad (3.27)$$

Next we keep the spectrum fixed and consider the increase in transmit power. From Lemma 2

$$\underbrace{\eta_j^* \dot{U}_j(R_j(x_j^*, \tilde{P}_j, h_j)) \Gamma_x(x_j^*, \tilde{P}_j, h_j)}_{\partial U_j / \partial \tilde{P}_j} > \mu. \quad (3.28)$$

Recall that $\tilde{R}_j^* = R_j(\tilde{x}_j^*, \tilde{P}_j, h_j)$. Since $U_j(\cdot)$ is jointly concave in x_j^* and P_j from Appendix A we conclude from (3.27) and (3.28) that,

$$\eta_j^* \dot{U}_j(\tilde{R}_j^*) \Gamma_x(\tilde{x}_j^*, \tilde{P}_j, h_j) > \mu, \quad (3.29)$$

But (3.26) and (3.29) taken together contradict (3.25b). Hence our original assumption, events \mathcal{A} and \mathcal{B} are wrong. Thus $\tilde{x}_j^* > x_j^*$ which implies $\tilde{x}_k^* < x_k^*$ for some user $k \neq j$. Hence,

$$\tilde{\mu} \stackrel{(a)}{=} \eta_k^* \dot{U}_k(\tilde{R}_k^*) \Gamma_x(\tilde{x}_k^*, P_k, h_k) \stackrel{(b)}{>} \mu, \quad (3.30)$$

where (a) follows from relation (3.25b) and (b) follows from Lemma 1. Hence proved. \square

Thus user j demands more spectrum as his transmit power increases. This leads to a higher price and all other users obtain less spectrum.

Corollary 2. *The user with increased power derives a higher utility and surplus and the sum utility also increases.*

Proof. $U_j(R_j^*)$ increases as it is an increasing function of both P_j and x_j^* . In Appendix C, Section C.3 we show that the surplus, $\mathcal{U}_j(x_j^*) = U_j(R_j^*) - \mu x_j^*$ increases with x_j^* . The increase in sum utility can be proved indirectly as follows: consider the suboptimal allocation where each user l is retained at x_l^* . Since the power of user j increases, this allocation will still increase the utility of user j and thus the sum utility. The optimal utility can not be worse. \square

3.3.3 Dependence on number of SPs and Users

Theorem 6. *As more users are added to the system, the spectrum price increases.*

Proof. Assume that the system is in equilibrium with L users who have been allocated spectrum and user $L + 1$ user joins in with link gain h_{L+1}^* and transmit power P_{L+1} . From Theorem 3, in the new equilibrium, he is allocated non-zero spectrum. This will reduce the allocated spectrum for all other users j , $1 \leq j \leq L$. Since h_j^* and P_j stay the same, this means that the price of spectrum goes up from Lemma 1 and (3.16a) considered with equality at the new price. A new user increases the demand for spectrum thus raising the price. \square

Theorem 7. *If all SPs are equally efficient and users have α utilities then the addition of an SP either increases the spectrum price or keeps it unchanged.*

Proof. Assume that the system is in equilibrium and SP $N + 1$ joins in the system. If it offers no better channel to any of the users than their existing ones, i.e. if $h_j^* > h_{(N+1)j}$ for all j , then no user engages itself to the SP and the optimal solution (spectrum price, spectrum allocated etc) is the same as before.

However, if for user j , the new SP provides a better channel coefficient, i.e. $h_j^* < h_{(N+1)j}$, then user j engages itself to SP $N + 1$ and adjusts its engaged SP index to $i_j^* = N + 1$ and channel coefficient to $h_j^* = h_{(N+1)j}$. Thus user j 's channel condition to his active SP has improved and as per Theorem 5, the price goes up. \square

As more SPs join the system, a subset of them offer better link gains to users resulting in *better access* to the spectrum. This increases demand for spectrum and hence the price

increases. To understand this consider an analogy from beachfront property: There exist beach-houses (analogous to spectrum) and they are in demand from vacationers. If good roads are built so that these houses become easily *accessible* (analogous to improving link gains or transmit power) then their demand goes up and so do their prices.

3.4 Linear Utility Functions, $U_j(R_j) = R_j$

This is the sum rate maximization problem and gives an indication of the capacity of the user-SP vector channel. We present the results and the reader is referred to [18] for the details.

Theorem 8. *For given link gain h_j^* , power P_j and efficiency η_j^* , user j operates at a unique signal to noise ratio, snr_j^* which is given by the solution of*

$$\Phi(snr_j^*) = \log(1 + snr_j^*) - \frac{snr_j^*}{1 + snr_j^*} = \frac{\mu}{\eta_j^*}. \quad (3.31)$$

From (3.31) we can also interpret SP efficiency as a scaling factor of spectrum price μ , i.e. a SP with higher efficiency has a smaller *effective* price μ/η_j^* .

Corollary 3. *If all SPs are equally efficient, allocated spectrum and user surplus are given by*

$$x_j^* = \frac{h_j^* P_j}{\sum_{k=1}^L h_k^* P_k} W, \quad (3.32a)$$

$$\mathcal{U}(x_j^*) = \frac{h_j^* P_j}{1 + \sum_{k=1}^L h_k^* P_k / W}. \quad (3.32b)$$

It can be shown that (3.32b) is an increasing function of h_j^* thus validating Theorem 5. From (3.32a), the spectrum allocation is directly proportional to the received signal power and hence can be very unfair if the users have wide variations in link gains and transmit powers. The use of exponential and α utilities mentioned in Section 3.3 lead to more fair allocation of spectrum as the allocation now depend on the marginal utilities which have a lesser variation than the link gains. We will explore this in Section 3.5 via numerical experiments.

3.5 Numerical Results

The spectrum allocation algorithm has the following basic steps

1. *SP selection by users:* The atomic setting is a network with one user and two SPs with different efficiencies.

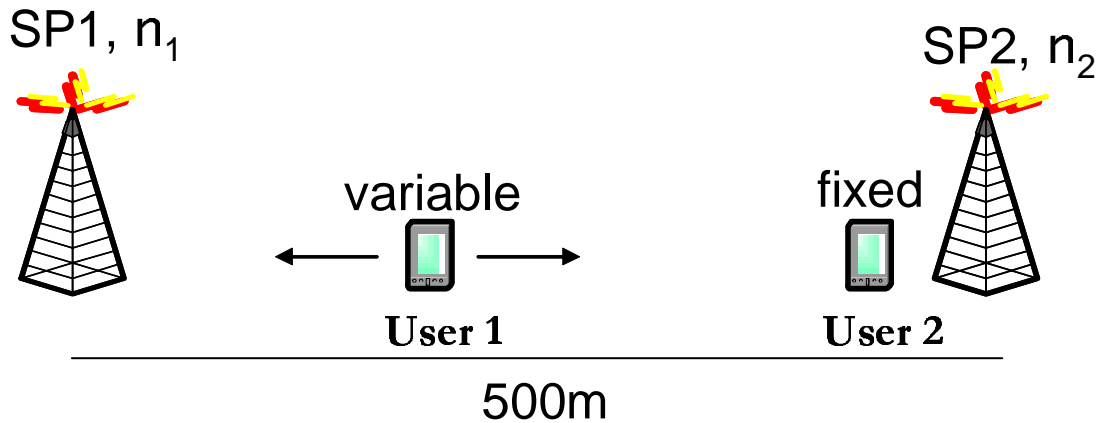


Figure 3.1: The linear network with two SPs and two users.

2. *Spectrum allocation to users*: The atomic setting is a network with one SP and two users with different received powers.

We consider a network of two users and two SPs which incorporates both steps. We believe that insights from this network will be applicable to bigger networks as well. We consider two SPs in a linear cell with inter-base distance of 500 meters as shown in Figure 3.1. For path loss, we choose the COST-231 propagation model for outdoor WiMAX environments [76] at an operating frequency of 2.4 GHz. Let the noise power spectral density of $N_0 = -174$ dBm/Hz. Denote the SP i to user j distance by d_{ij} and the link gain, that incorporates N_0 , by h_{ij} ,

$$h_{ij,dB} = P_{\text{loss}} - N_0 = -31.5 - 35 \log(d_{ij}) - N_0. \quad (3.33)$$

The distances are measured with SP 1 located at the origin. User 2 is fixed at a distance of $d_{22} = 100$ m from the SP 2 and the location of user 1 is varied from $d_{11} = 1$ m to $d_{11} = 499$ m from SP 1 in steps of 1 m. The total spectrum is 50 KHz. The following classes of utilities are considered based on the required rates of a user,

- a) *low* required rate: α utilities $U(R) = \log(R)$; exponential utilities $\Gamma = 1$ Kbps.
- b) *high* required rate: α utilities $U(R) = R$; exponential utilities $\Gamma = 1$ Mbps.

We first consider the spectrum allocation for users with exponential utilities. Let user 2 have a *high* required rate. SP efficiency ratios of $\eta_2/\eta_1 = 1$ and 10 are considered. Figure 3.2 shows the fraction of the spectrum allocated to user 1. It also indicates the active SP of user

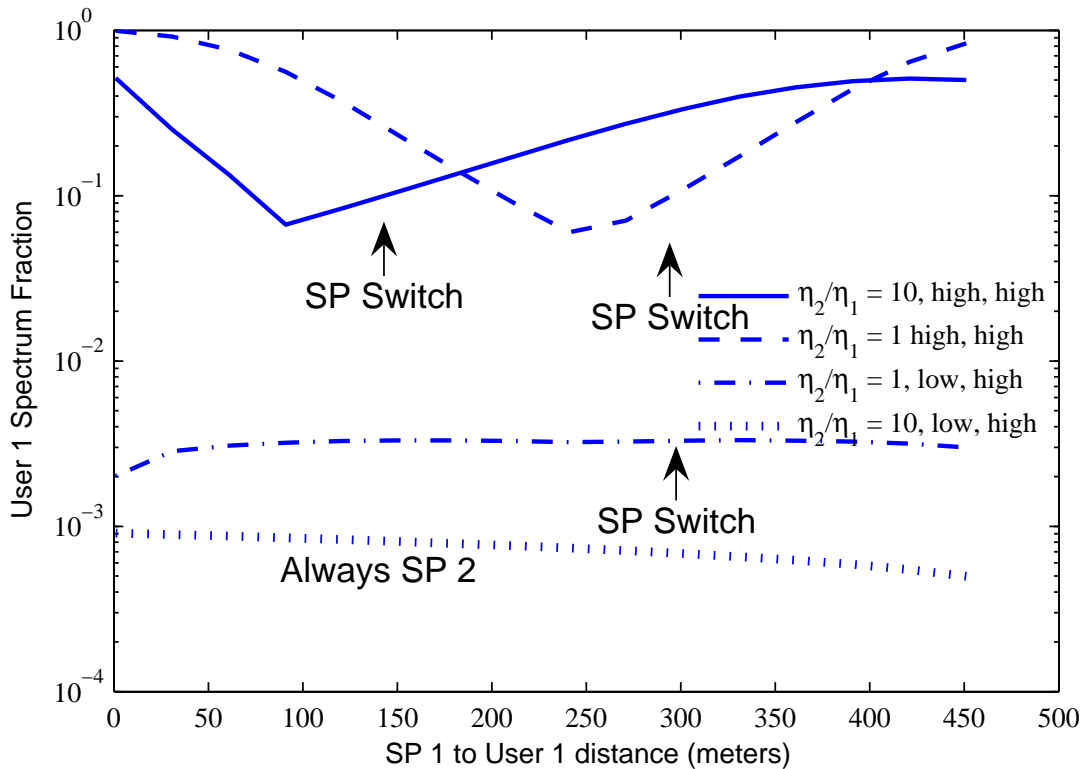


Figure 3.2: Fraction of total spectrum allocated to user 1 as a function of distance for different target rates and SP efficiencies. Both users have exponential utilities and user 2 is fixed at 100m from SP 2

1. The term *SP Switch* at distance $d = d_S$ means that for $d < d_S$ user 1 is attached to SP 1 and for $d > d_S$ it switches to SP 2. First consider that user 1 has a *high* required rate. Note that the switch to SP 2 occurs earlier when it is more efficient. The spectrum ratio is mostly increasing in the link gain to the active SP, h_1^* , as the rate function in (3.1) is increasing in h_1^* and spectrum x_1^* and if h_1^* improves then the rate achieved is increased even more by allocating more spectrum. Also an increase in R for low/medium R increases the utility $U(R)$. However x_1^* becomes constant in the region \mathcal{V} defined by $\eta_2/\eta_1 = 10$ and $d_{21} > 400$ m. This is because the exponential utility $U_1(R)$ flattens near the value of Γ_1 at high R . In region \mathcal{V} user 1 has a very high h_1^* (to SP 2) and SP 2 is more efficient. So user 1 achieves a high rate and his utility is near Γ_1 . This can be seen in Figure 3.4. Thus as user 1 gets closer to SP 2, any extra spectrum would increase its rate but not its utility. Another way to interpret this is to look at the prices in Figure 3.3. For region \mathcal{V} both users are close to the flat regions of utilities and hence demand for additional spectrum is less. Consequently the prices are initially constant and then

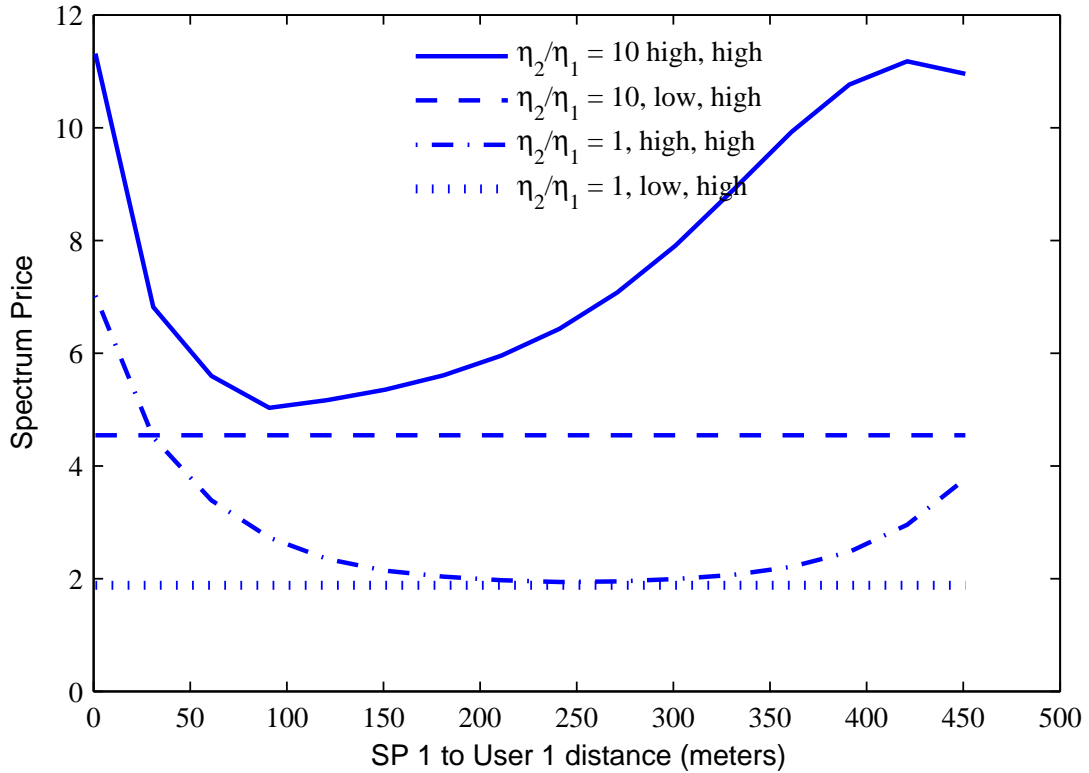


Figure 3.3: The spectrum price μ as a function of user 1 distance from SP 2 for different target rates and SP efficiencies. Both users have exponential utilities and user 2 is fixed at 100m from SP 2

falls slightly.

Figures 3.2-3.4 also show results when user 1 has *low* required rate. Allocation x_1^* is much less as per Theorem 4. However x_1^* is enough to satisfy user 1's utility. Since x_1^* is less, user 1 always attaches to the more efficient SP as per observation 6(b). The prices are almost invariant to changes in d_{21} . This is because user 2 gets majority of the spectrum and thus sets the demand. Since it is stationary the prices change only with SP efficiencies.

The corresponding results when users have α utilities are shown in Figures 3.5-3.7. The same trends of exponential utility results are observed but the disparities between the users in terms of spectrum allocated and utilities are much more severe for dissimilar link gains. Comparing Figures 3.2 and 3.5 we see that when user 1 has *low* required rate, allocation x_1^* for α utilities is significantly less than x_1^* for the exponential utilities. The user j with a stronger h_j^* has a much larger impact on the prices for α utilities. From Figures 3.3 and 3.6 we see that

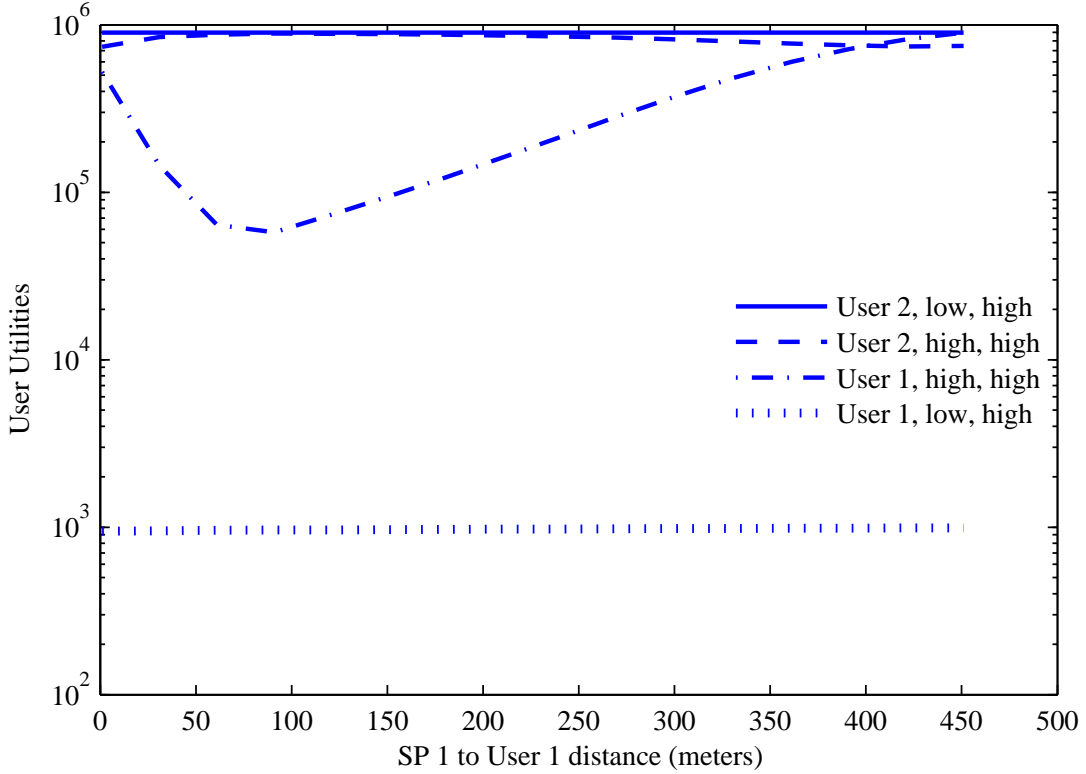


Figure 3.4: The utilities for both users as a function of distance for different target rates. Both users have exponential utilities and user 2 is fixed at 100m from SP 2. The efficiency ratio is $\eta_2/\eta_1 = 10$

when $h_2^* > h_1^*$, the α prices vary much less with d_{21} than the exponential prices. The unbounded nature of α utilities also mean that there is always demand for spectrum. Accordingly Figures 3.3 and 3.6 for the (high,high) case show that α prices in region \mathcal{V} keeps on increasing unlike the exponential prices. Overall exponential utilities yield more equitable spectrum allocation than α utilities.

3.6 Discussions and Conclusion

Dynamic spectrum allocation is important both for centralized broadband access networks and decentralized cognitive radio systems. Efficient networks are often designed for non-strategic behavior either by a central command and control plane or by adherence to a distributed protocol. In this chapter we have developed and analyzed a two tier allocation system for non-strategic users who obtain spectrum from multiple SPs. We model the system from user welfare

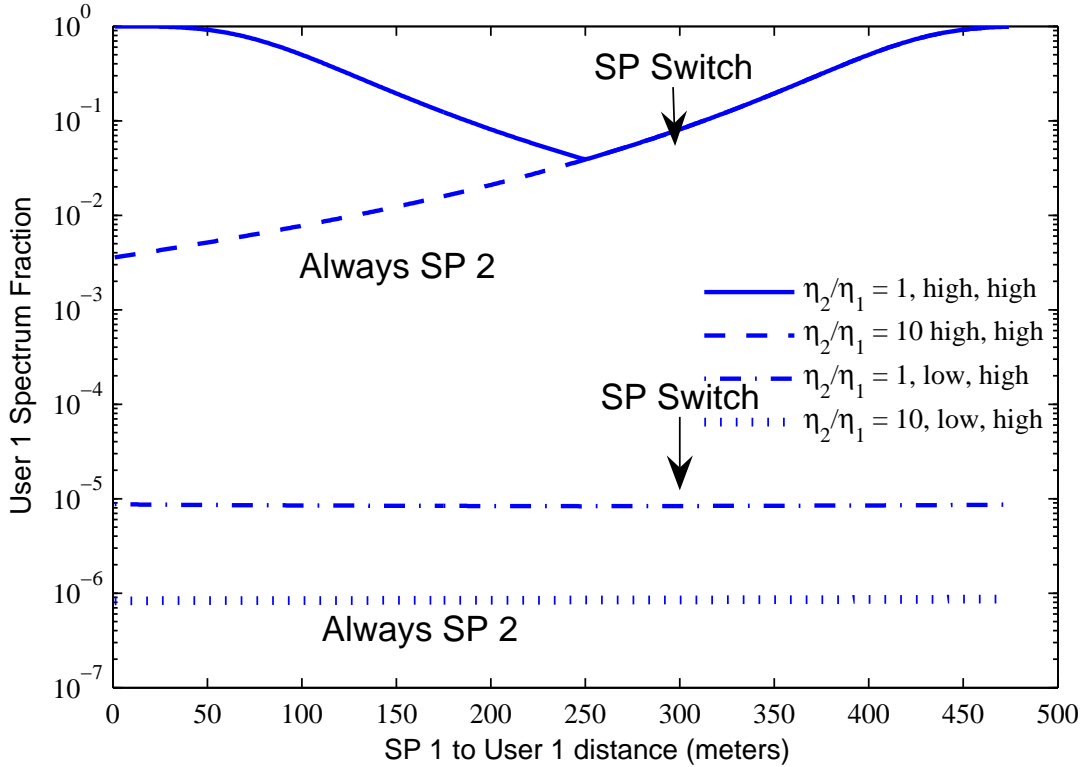


Figure 3.5: Fraction of total spectrum allocated to user 1 as a function of distance for different target rates and SP efficiencies. Both users have α utilities and user 2 is fixed at 100m from SP 2

maximization framework. We show that in the optimal policy each user obtains spectrum only from one service provider given by a function of the link gains and provider efficiency. Based on our analysis we develop the notion of a spectrum price to facilitate distributed allocation. For two general classes of concave utility functions namely exponential and α , we analytically characterize the spectrum allocation and price. We show that our results are consistent with basic economics principles. Our work provides theoretical bounds on performance limits of practical operator to user based dynamic spectrum allocation systems and also gives insights to actual system design.

We have assumed that h_{ij} , N and L stay constant during the optimization and transmission process. Whenever they change the optimization needs to be re-done. While we have not addressed such timescale issues, the proposed price based allocation is ideal for static outdoor settings with a strong Line-of-Sight component between users and SPs. For more mobile environments the average values of link gains can be used to derive reasonable allocations.

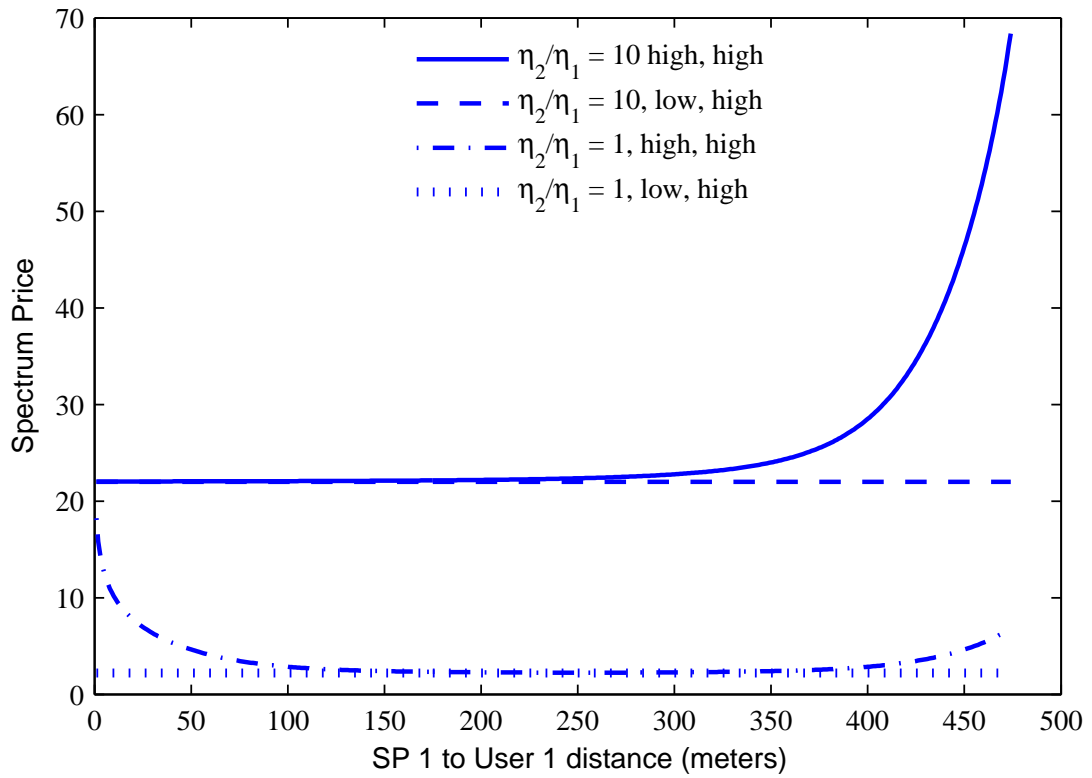


Figure 3.6: The spectrum price μ as a function of user 1 distance from SP 2 for different target rates and SP efficiencies. Both users have α utilities and user 2 is fixed at 100m from SP 2

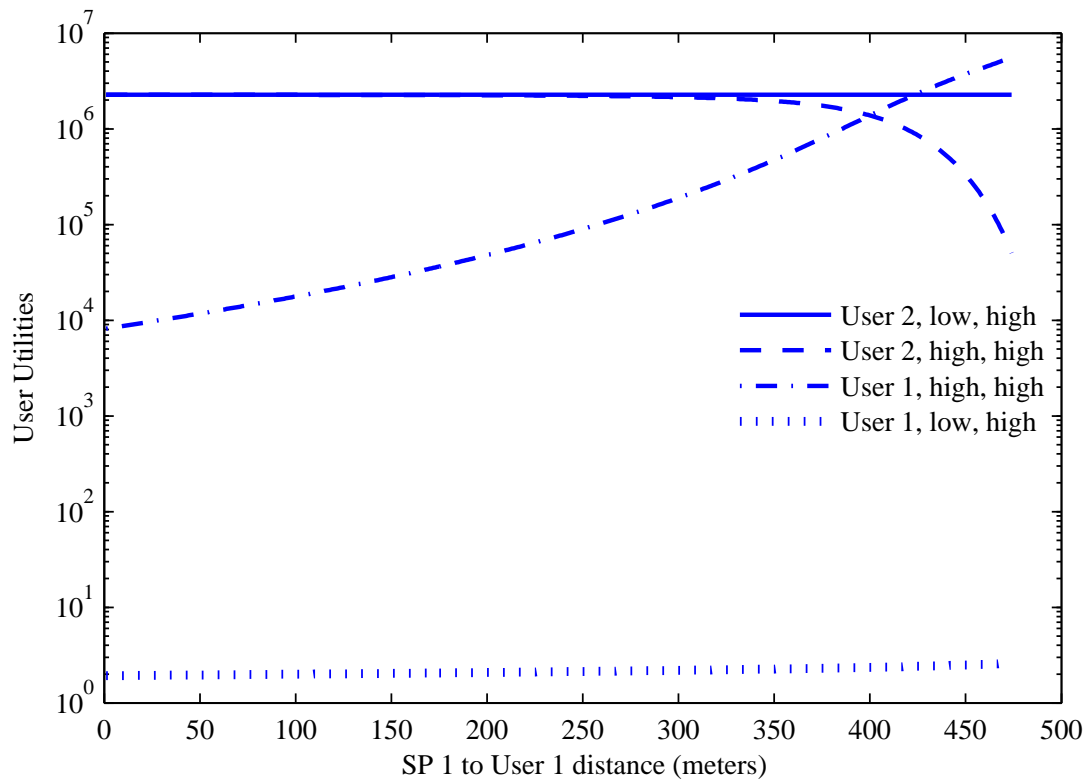


Figure 3.7: The utilities for both users as a function of distance for different target rates. Both users have α utilities and user 2 is fixed at 100m from SP 2. The efficiency ratio is $\eta_2/\eta_1 = 10$

Chapter 4

Service Provider Profit Maximization for centralized DSA

4.1 Introduction

In Chapter 3 we considered that the SPs act as a social planner and maximize the sum utility of users. However since a SP pays the broker for obtaining spectrum licenses, it can also operate to maximize its profits by charging users for the spectrum allocation. In this paper, we apply principles of microeconomics [40, 41] to explore SP pricing for profit maximization. We model the SP profit as a function of the cost it has to pay the broker, the revenues it accrues from the users and possible price competition from other SPs in the region. An SP seeks to maximize its profits by choosing its price. The users consider the SP prices and their link gains to the SPs to determine the SP to obtain service from. In this work, we characterize the SP prices and user spectrum allocations. In this chapter we establish the basic terminology and notations of profit maximizing SP networks. Specific situations such as monopolistic SPs and price competition among SPs are dealt in subsequent chapters 5 and 6.

4.1.1 Related Work and Our Contribution

Pricing for profit maximization has been studied under various contexts. For wireless applications [13] considers a two SP, multiple user model where the SPs offer fixed prices and rates to the users, who decide which SP to obtain service from. In [77, 78] profit maximizing pricing strategies are considered for multi-rate CDMA applications. In [79] price competition equilibria are considered for a two SP, multiple user network for simplistic demand functions that are linear in the SP to user distance have been considered. However none of [13, 77, 78, 79] consider the full range of relationships between spectrum prices, costs and user demands are not established. On the other hand, several works in microeconomics have considered pricing for profit maximization [40, 41] but for very generic user demand functions and costs. The authors in [80] consider various non-wireless flow control problems. Though some fundamental results

stay same across models, our focus is more on capturing the problems specific to the wireless model. In this work, we have applied some of these principles specifically to a wireless setting and evaluated the prices and characterized the behavior of the allocation. The focus and system model of [81] is similar to our work but they do not include the effect of link gains and variable spectrum usage based broker cost, in determining SP prices.

In Chapter 3 we had considered that the users were constrained by their total transmit power. This is a valid assumption in the uplink as mobile devices have limited battery lives. In this chapter we consider the downlink where the transmitting base station does not have a total power constraint. Rather we assume that the transmission from the SP to the users is constrained by a total power spectral density. This is also consistent with the spectral mask requirements of the FCC. Thus total transmit power scales with bandwidth, which is a unique feature of our work. The choice of power spectral density, also leads to an equivalent modulation technology and is a DSA policy of the SPs.

4.1.2 Spectrum and Spectral Efficiency

Let us assume that the spectrum allocated by a SP be X and it transmits at a power spectral density of K , when it's appropriate the use Shannon capacity to represent the rate, $R = \log_2(1 + \text{snr})$ where snr is the signal to noise ratio of the received signal. The transmit rate is thus given by,

$$R = KX. \quad (4.1)$$

A SP who wants to increase its rate to its customers can increase either K or X as per (4.1). However the associated costs and benefits to the SP are quite different. A SP buys spectrum from a spectrum broker or Govt. agency such as FCC who decide the cost per unit spectrum X . On the other hand, the costs associated with increasing K has to do with fixed hardware costs of installing equipment such as base stations or encoding blocks and variable costs proportional to the transmit power. The user payments are proportional to what the SP charges for unit spectrum and the rate R that it is able to provide. Whether the SP decides to buy more spectrum or invest in increasing its spectral efficiency should be decided based on a joint optimization over the costs and benefits of K and X to the SP. Our spectrum allocation model, though simple, captures all these essential features and proposes the optimal allocation and prices.

4.2 System Model

Let there be N SPs and L users. SP i allocates spectrum x_{ij} to user j and transmits to users in the downlink with power spectral density of ν_i Watts/MHz. We assume that all bands of spectrum x_{ij} are non-overlapping. Let the link gain between SP i and user j be given by h_{ij} . We assume a frequency non-selective channel and thus h_{ij} does not depend on the width of the band x_{ij} . The spectral efficiency in the transmission from SP i to user j is given by

$$K_{ij} = \log \left(1 + \frac{\nu_i h_{ij}}{N_0} \right). \quad (4.2)$$

Note that the spectral density ν_i is thus also an indicator of the *transmission technology* of the SP i . Increasing K_{ij} by increasing ν_i implies that SP i must support higher rates through more complex modulation and coding. Thus the SP i and user j communication is characterized by the following rates and transmit powers

$$r_{ij} = r(\nu_i, x_{ij}) = K_{ij} x_{ij} \quad (4.3a)$$

$$p_{ij} = p(\nu_i, x_{ij}) = \nu_i x_{ij}. \quad (4.3b)$$

Thus the total achieved rate by user j , R_j , the total transmit power of SP i , P_i , the total spectrum allocated by SP i , X_i^S and the total spectrum allocated to user j , X_j^U are given by

$$R_j = \sum_{i=1}^N r_{ij} = \sum_{i=1}^N K_{ij} x_{ij}, \quad (4.4a)$$

$$P_i = \sum_{j=1}^L p_{ij} = \sum_{j=1}^L \nu_i x_{ij}, \quad (4.4b)$$

$$X_i^S = \sum_{j=1}^L x_{ij}, \quad X_j^U = \sum_{i=1}^N x_{ij}. \quad (4.4c)$$

The assumption that different users are allocated different continuous bands of spectrum is similar to OFDMA where different users are each allocated a number of discrete tones, as per their application requirements. If the tone spacing is narrow as compared to the total bandwidth, we can model the frequency variable to be continuous. An example system is LTE which can operate with 15 KHz spacing and 2048 subcarriers [62].

4.2.1 The User Cost Function

The SPs charge users for the spectrum that is allocated to them. This could consist of a fixed connection fee and/or an spectrum usage dependent cost. This results in *user cost* $\rho_i(x_{ij})$ that

user j pays to SP i . In the beginning of an allocation interval, which could correspond to the beginning of file download sessions for all users in the system, SP i announces this cost function $\rho_i(\cdot)$ to all users.

4.2.2 The User Optimization Problem

Given the SP cost functions and the spectral efficiencies to SPs, each user decides what fraction of its total spectrum requirement to obtain from each SP. User j 's application is characterized by a utility function $U_j(R_j)$ which is increasing and concave in R_j . Let the set of user cost functions $\{\rho_1(\cdot), \dots, \rho_N(\cdot)\}$ be denoted by $\boldsymbol{\rho}(\cdot)$. After payment of the spectrum cost to all SPs, user j has the following residual utility

$$S_j(X_j^u, \boldsymbol{\rho}(\cdot)) = U_j(R_j) - \sum_{i=1}^N \rho_i(x_{ij}). \quad (4.5)$$

In the microeconomic literature [41, Chapter 14] S_j is called the *user surplus* and is the amount of money necessary to persuade user j to give up its consumption of spectrum. Given the cost functions $\rho_i(x_{ij})$, each user j solves the optimization problem

$$\max_{x_{1j}, \dots, x_{Nj}} S_j(X_j^u, \boldsymbol{\rho}(\cdot)), \quad (4.6)$$

to choose the spectrum allocations x_{ij} to maximize its surplus. It is possible that some of the x_{ij} are zero implying that the user does not obtain service from certain SPs.

4.2.3 User Utility Functions

We consider the following utility functions to model user applications,

Logarithmic Utilities

This is similar to the α utilities considered in Chapter 3 that model elastic applications where the utility is unbounded. Specifically we consider utility functions of the form,

$$U_j(R_j) = \log(1 + R_j). \quad (4.7)$$

Exponential Utilities

Exponential utilities were considered in Chapter 3, to model applications where user j has a target rate Γ_j . We reproduce (3.23) here

$$U_j(R_j) = \Gamma_j \left(1 - e^{-R_j/\Gamma_j}\right). \quad (4.8)$$

4.2.4 The SP Cost Function

From (4.4c), SP i has to provide the users a total spectrum of X_i^s which it purchases from the Spectrum Broker for duration of the communications session. The Broker charges SP i for the spectrum purchase. Note that in Chapter 3, we had assumed that there is a net spectrum constraint W which had led to a *shadow* cost of μW for the SPs. However in this chapter we do not assume any such fixed constraint at the broker. SP i has to pay two types of *real* spectrum costs to the broker:

Spectrum Cost

The spectrum cost $C(X_i^s)$ is the license fees paid by SP i to the Spectrum Broker for securing the right to offer services in the spectrum band X_i^s and collect revenues from the users. As mentioned in [82], a reasonable cost model is linear with amount of spectrum purchased, with the constant of proportionality C depending on the geographical location, duration for which spectrum would be used and availability of spectrum at the broker. Also as in [13, 81], cost C can also be determined by a bidding process between the SPs and the broker. Thus

$$C(X_i^s) = CX_i^s. \quad (4.9)$$

Power Cost

The SP also incurs a cost proportional to its total transmit power $P_i = \nu_i X_i^s$. A part of this could be the electricity costs. Additionally the Broker can also charge SP i proportional to P_i as a measure of the cost due to interference. This is because transmission with a higher power spectral density ν_i could cause increased interference to other systems potentially operating in that band. Out of band interference in adjacent bands is also an important issue. For example, the SP may be an 802.22 transmitter [10] operating a secondary system in the TV bands. To

safeguard the primary TV transmitters, the Broker may decide to charge the SP more if it transmits with more power. In short, the SP incurs a cost

$$F(\nu_i, X_i^s) = T\nu_i X_i^s, \quad (4.10)$$

where T is the constant of proportionality. From (4.9) and (4.10) the total spectrum cost is $(C + T\nu_i)X_i^s$ and thus the *spectrum price* that SP i pays the spectrum broker is,

$$C_i^s = C + T\nu_i. \quad (4.11)$$

4.2.5 The SP Optimization Problem

Given C_i^s , each SP i designs a user cost function $\rho_i(\cdot)$, to maximize its profit

$$\Pi_i = \sum_{j=1}^L \rho_i(x_{ij}) - C_i^s X_i^s. \quad (4.12)$$

We assume that each SP i is aware of how user j chooses x_{ij} via the optimization (4.6). Thus the profit Π_i in (4.12) is just a function of $\rho_i(\cdot)$ of all SPs i and can be expressed as $\Pi_i(\boldsymbol{\rho}(\cdot))$. Thus each SP i maximizes its profit by solving

$$\max_{\rho_i(\cdot)} \Pi_i(\boldsymbol{\rho}(\cdot)) \quad (4.13)$$

to chose the optimal $\rho_i(\cdot)$ to induce a particular user behavior. Such SP-user interaction is an instance of a *Stackelberg Game* [83, Ex. 97.3].

This chapter describes the basic principles and terminology for profit maximizing SPs. For a single SP, (4.13) describes a monopolistic pricing framework which will be analyzed in Chapter 5. For multiple SPs, solution of (4.13) involves a price competition game and is analyzed in Chapter 6.

Chapter 5

A Single SP monopoly

5.1 Introduction

In this chapter we consider a centralized network consisting of a single service provider (SP) that allocates orthogonal chunks of spectrum to its customers dynamically, based on their demand. It then transmits to these users over their allocated spectrum. The user demand of spectrum depend on the received rate which is different for users due to the variations in the link gain. The SP purchases the amount of spectrum needed by its customers, from the broker. The SP has to pay the broker for the purchased spectrum and in turn charges the users to recover its costs. In this work we model the dynamic allocation as a SP profit maximization problem and derive the optimal values of the prices [84].

5.2 Monopolistic Two part tariff

For sake of clarity we will drop the SP index i from equations and summations of Chapter 4 as there is only one SP. This SP charges a two part tariff [40] from user j , consisting of a fixed connection price κ and a price μ charged per unit of spectrum. The user does not have to pay the connection price κ if he is not receiving any service from the SP. Thus the cost function $\rho(x_j)$ for user j as defined in Section 4.2.1 is

$$\rho(x_j) = \begin{cases} \mu x_j + \kappa, & x_j > 0 \\ 0, & x_j = 0. \end{cases} \quad (5.1)$$

5.2.1 Solutions to User and SP Optimizations

The SP initially announces a price pair (μ, κ) . Given this, the optimization problem (4.6) for user j becomes

$$S_j^* = \max_{x_j} U_j(R_j) - \mu x_j - \kappa. \quad (5.2)$$

Note that if the price pair (μ, κ) is *high*, some of the users may refuse service and hence $x_j = 0$ for these users. After all users perform this optimization, they inform the SP about how much spectrum they desire. If user j receives nonzero spectrum, then $S_j^* \geq 0$. It can be easily verified that (5.2) is concave in x_j . Taking derivatives

$$\mu = \frac{\partial U(R_j)}{\partial x_j}. \quad (5.3)$$

The graph of (5.3) is called the *demand function* [41] which shows how the demand for resource x_j varies with price μ .

The Marginal User Principle

Let the system requirements be such that the SP has to serve all the users. To maximize its profit, the SP will raise its prices (μ, κ) to the point that the surplus of some user m is equal to κ . After paying the connection fee κ , user m 's residual utility is zero. If the prices are raised any further, user m will decide not to obtain service from the SP. User m with zero surplus is said to be *indifferent* from obtaining the service [41]. Mathematically this can be restated from (5.2) as $S_m^* = 0$. The SP optimization from (4.13) is given by

$$\Pi^* = \max_{\mu, \kappa} \mu X^s + \kappa L - C^s X^s \quad (5.4)$$

The first order condition of (5.4) is

$$X^s + L \frac{\partial \kappa}{\partial \mu} + (\mu - C^s) \frac{\partial X^s}{\partial \mu} = 0. \quad (5.5)$$

Let x_m be the spectrum allocated to the marginal user. It was shown in [82] that

$$\partial \kappa / \partial \mu = -x_m. \quad (5.6)$$

The elasticity of demand, [41] is given by

$$\epsilon = -\frac{\partial X^s / X^s}{\partial \mu / \mu} = -\frac{\mu}{X^s} \frac{\partial X^s}{\partial \mu}. \quad (5.7)$$

The elasticity gives the relationship between percentage change in demand to the percentage change in price. Note that $\epsilon > 0$ as $\partial X^s / \partial \mu < 0$ which is to say that demand reduces with price. Defining $\chi = x_m / X^s$, the fraction of the spectrum allocated to the marginal user, we can show that the (5.5) can be re-written to solve for μ as

$$C^s = \mu \left[1 - \frac{1 - L\chi}{\epsilon} \right]. \quad (5.8)$$

5.2.2 Logarithmic User Utilities

For logarithmic utilities given in (4.7), the demand function, (5.3), for user j is given by

$$\mu = \frac{K_j}{1 + K_j x_j}. \quad (5.9)$$

The maximum value of the RHS of (5.9) is K_j (for $x_j = 0$) and thus for a feasible allocation,

$$\mu < \min_j K_j = K_{\min}. \quad (5.10)$$

The intuition behind (5.10) is that if $\mu > K_{\min}$ then μ , which is the marginal cost of a user for purchasing an extra unit of spectrum, always exceeds the marginal utility that the user obtains by purchasing that unit of spectrum and hence no transaction takes place. A plot of the demand function is given in Figure (5.1).

Lemma 3. *For logarithmic utilities, the user with the weakest link gain is the marginal user.*

Proof. Let the optimal value of the spectrum price be μ^* . The marginal user is the user who has the least surplus. Substituting for x_j^* from (5.9) in (4.5), the surplus of user j is given by

$$S_j = \log \left(\frac{K_j}{\mu^*} \right) - 1 + \frac{\mu^*}{K_j}. \quad (5.11)$$

Taking derivatives of S_j w.r.t. K_j and using relation (5.10), it can be shown that $\partial S_j / \partial K_j > 0$ i.e. S_j increases in K_j . So the user, m with least surplus S_m is given by $m = \arg \min_j K_j \stackrel{(a)}{=} \arg \min_j h_j$. Relation (a) follows from (4.2). \square

A graphical proof for a similar system was given in [82]. Substituting for x_j from (5.9) in (5.8) and after some algebraic manipulation, we can show that the optimal value of the price $\mu = \mu^*$ satisfies the following quadratic equation

$$\left(\frac{1}{K_m} - \frac{K_s}{L} \right) \mu^2 - \mu + C_e = 0 \Big|_{\mu=\mu^*} \quad (5.12a)$$

$$\text{where } K_s = \sum_{j=1}^L \frac{1}{K_j}. \quad (5.12b)$$

The optimal values of spectrum, x_j^* are given by substituting for $\mu = \mu^*$ in (5.9). Denote $X^* = \sum_j x_j^*$.

Lemma 4. *For users with logarithmic utilities, the spectrum is overpriced, i.e. $\mu^* > C_e$.*

Proof. Relation (5.12a) can be rewritten as

$$\left(\frac{1}{K_m} - \frac{K_s}{L}\right)\mu^{*2} = \mu^* - C_e. \quad (5.13)$$

Using (5.10), we can prove that $(1/K_m - K_s/L) > 0$ and hence the LHS of (5.13) is positive. Thus the RHS has to be positive which yields the desired result. \square

An intuitive explanation is that, the logarithmic utility function is unbounded from above and there is always demand even for high prices. So the SP exploits this to maximize its profits by keeping μ above C_e . However this does not mean that the SP can arbitrarily overprice spectrum. To understand this, we first calculate the value of the elasticity, ϵ^* for the given optimal values of X^* and μ^* . Using the value of X^* from (5.9) in (5.7) we can show that

$$\epsilon^* = \frac{L}{L - \mu^* K_s} > 1. \quad (5.14)$$

Lemma 5. *For logarithmic utilities, the aggregate demand function for all users is elastic.*

Proof. This follows from (5.14) [41, Chapter 15]. \square

For elastic demands, the percentage change in spectrum demanded is greater than that percentage change in price. Hence, when the optimal price μ^* is increased, percentage decrease in spectrum demand is higher and the total *revenue* of the SP given by $\mu^* X^*$ decreases.

Now let us look at the SP profit. From Chapter 4, equation (4.12)

$$\Pi^*(\nu, \mu^*) = \underbrace{(\mu^* - C_e)X^*}_{\Pi_U^*} + \underbrace{\kappa^* L}_{\Pi_C^*}. \quad (5.15)$$

Note that Π_U^* and Π_C^* are the profits from the usage cost and the subscription fees respectively. We now want to investigate how $\Pi^*(\nu, \mu^*)$ changes as a function of spectrum price μ^* .

Lemma 6. *The following results hold about the SP profit functions.*

- a) *The profit from subscription, Π_C^* decreases with cost μ^* .*
- b) *The profit from usage, Π_U^* increases with μ^* for $C_e < \mu^* < \sqrt{C_e L / K_s}$*

Proof. Refer to Appendix D. \square

The fact that there is a maximum threshold on μ^* , occurs because of the elastic nature of spectrum demand, i.e. the increase in μ^* is more than offset by the decrease in X^* .

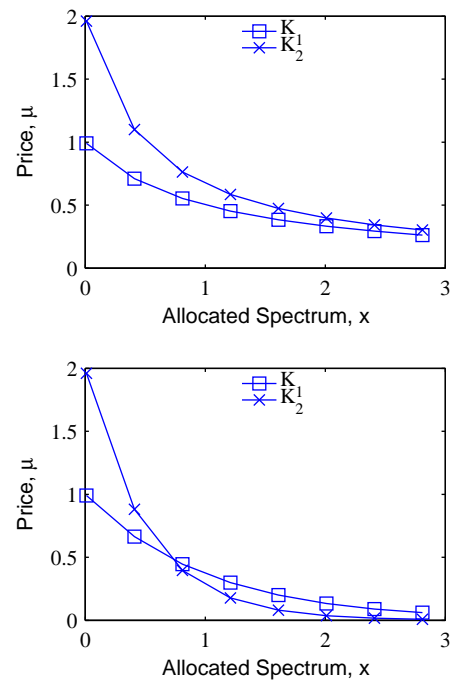


Figure 5.1: Demand functions for logarithmic (top) and exponential utilities (bottom) with two users with spectral efficiencies $K_1 = 1$ and $K_2 = 2$ and $\Gamma = 1$ for the exponential utility target rate

5.2.3 Exponential User Utilities

For exponential utilities given in (4.8) the demand function, (5.3), for user j is given by

$$\mu = K_j e^{-(K_j/\Gamma_j)x_j}. \quad (5.16)$$

Being of exponential dependence, the demand function decreases sharply at high values of spectrum as the utility function flatten at a value of Γ_j for high spectrum and in that regime there is little demand for spectrum. The inverse demand function, derived from (5.16) is

$$x_j = \frac{\Gamma_j}{K_j} \log\left(\frac{K_j}{\mu}\right). \quad (5.17)$$

Substituting for x_j from (5.17) in (5.8) and after some algebraic manipulation, we can show that the optimal price μ^* is given by the positive solution of

$$C^s = (1 - C_1)\mu + C_2\mu \log(\mu), \text{ where} \quad (5.18a)$$

$$C_1 = \left(\sum_{j=1}^L \frac{\Gamma_j}{K_j}\right)^{-1} \left[\sum_{j=1}^L \frac{\Gamma_j}{K_j} \log(K_j) - \frac{L\Gamma_m}{K_m} \log(K_m) \right] \quad (5.18b)$$

$$C_2 = \left(\sum_{j=1}^L \frac{\Gamma_j}{K_j}\right)^{-1} \left[\sum_{j=1}^L \frac{\Gamma_j}{K_j} - \frac{L\Gamma_m}{K_m} \right]. \quad (5.18c)$$

Lemma 7. *When all users have equal target rates, the user with the weakest link gain to the SP is the marginal user. When all users have same link gains, the use with the least target rate is the marginal user.*

Proof. Assume that the optimal price from (5.18) is μ^* . Substitute for x_j^* from (5.17) in expression of user surplus in (4.5) to obtain,

$$S_j = \Gamma_j \left(1 - \frac{\mu^*}{K_j}\right) - \mu^* \frac{\Gamma_j}{K_j} \log\left(\frac{K_j}{\mu^*}\right). \quad (5.19)$$

For the first part of the proof take $\Gamma_j = \Gamma$ for all users j . We can show that $\partial S_j / \partial K_j > 0$ and the remainder follows. For the second part of the proof take $K_j = K$ and using the identity $e^{x-1} > x$ for $x > 0$, it can be shown that $\partial S_j / \partial \Gamma_j > 0$. \square

No specific relationship exists when Γ_j and K_j are both different. Since S_j must be non-negative, from (5.19) we infer that

$$\mu^* < \min_j K_j = K_{\min}, \quad (5.20)$$

which turns out to be the same condition as (5.10).

Definition 1. The transition prices μ_d and μ_u are defined as $\mu_d = K_{\min}/e$ and $\mu_u = K_{\max}/e$, where K_{\min} is defined in (5.20) and $K_{\max} = \max_j K_j$.

Lemma 8. The following facts hold for $\Gamma_j = \Gamma$ for all users

- a) If price $\mu^* < \mu_d$, a user with lower link gain obtains more spectrum.
- b) If price $\mu^* > \mu_u$, a user with higher link gain obtains more spectrum.

Proof. Both assertions follow from (5.17) by taking derivatives of x_j w.r.t. K_j and showing that x_j is decreasing in K_j when $\mu^* < \mu_d$ and is increasing when $\mu^* > \mu_u$. \square

This also means that the marginal user obtains the maximum spectrum in case a). Intuitively for low μ^* , enough spectrum can be bought by each user j to make R_j high enough such that $U_j(R_j) \rightarrow \Gamma$. But marginal user m would have to purchase most spectrum to obtain high R_m (as $R_m = K_{\min}x_m$).

To calculate the elasticity substitute for x_j^* from (5.17) in (5.7)

$$\epsilon^* = \frac{\sum_{j=1}^L \frac{\Gamma_j}{K_j}}{\sum_{j=1}^L \frac{\Gamma_j}{K_j} \log\left(\frac{K_j}{\mu^*}\right)}. \quad (5.21)$$

Lemma 9. For exponential utilities, the aggregate demand function of spectrum is inelastic, i.e. $\epsilon^* < 1$ when $\mu^* < \mu_d$ and elastic when $\mu^* > \mu_u$.

Proof. From (5.21) we can show that a sufficient condition for elasticity/inelasticity is

$$\epsilon^* \geq 1 \Leftrightarrow \log(K_j/\mu^*) \geq 1 \text{ for all } j. \quad (5.22)$$

The rest follows from the definitions of K_{\min} and K_{\max} . \square

Intuitively, at low prices, below μ_d , each user has adequate spectrum to be in the flat region of the exponential utility. Even if price μ^* changes, users have little incentive to alter their purchased spectrum. Hence the demand is inelastic.

Lemma 10. The following results hold about the SP profit functions.

- a) The profit from subscription, Π_C^* , decreases with cost μ^* .
- b) The usage profit, Π_U^* , increases when $\mu_d > \mu^* > C_e$.

Proof. Refer to Appendix D. \square

5.2.4 Note on SP Transmit Power

In this chapter, we have fixed the SP transmit power spectral density and varied the transmit power for reasons mentioned in Chapter 4. For sake of completeness, we have also studied the spectrum allocation and pricing problem when the SP has a fixed transmit power constraint [82]. The results are reproduced in Appendix B.

5.3 Numerical Results

We consider a linear network with one SP and $L = 10$ users. For path loss, we choose the COST-231 propagation model [76], at an operating frequency of 2.4 GHz. Let the distance of user j from the SP be d_j . Thus the link gain is given by

$$h_{j,dB} = -31.5 - 35 \log(d_j). \quad (5.23)$$

Consider an user arrangement where the vector of user distances are $\mathbf{d}^{(1)} = [d_1, \dots, d_L] = [10, 20, \dots, 10L]$. We consider users with exponential utilities having $\Gamma_j = 1$ Mbps because the exponential utility allocation results present more possible variations as seen in Section 5.2.3. The total SP profit is given in Figure (5.2) when the power cost is 10 times the total transmit power. We see that the profit reduces with spectrum cost C . Also when cost C increases, the SP has an incentive to switch to higher transmit power spectral density ν as the effective cost $C_e = C + T\nu$ is dominated by C and is invariant of ν , but the user utilities and hence payments increase with ν . If C is above a threshold, then the profits reduce to zero as no spectrum is purchased. Similarly for low C regimes, SP cost is dominated by $T\eta$ and the SP has no incentive to transmit at high ν . Though the results in Figure (5.2) are for $\mathbf{d}^{(1)}$, the general trends hold for other user placements.

Figure (5.3) plots the SP effective cost C_e and spectrum price μ together. We see that for most portions the spectrum is underpriced. The effect of this is also seen in Figure (5.4) which plots the breakup of the usage and the connection profits, Π_U^* and Π_C^* respectively. We see the the places where spectrum is underpriced, Π_U^* is a loss. Another thing to note is that the SP profit comes predominantly from the connection fee. These effects are also described in [85]. The intuition is to look at the demand function in (5.16). Since it extends to infinity, there is a demand even at large amounts of spectrum. However the demand decays exponentially. So the users want a large amount of spectrum but have low willingness to pay usage fees for large

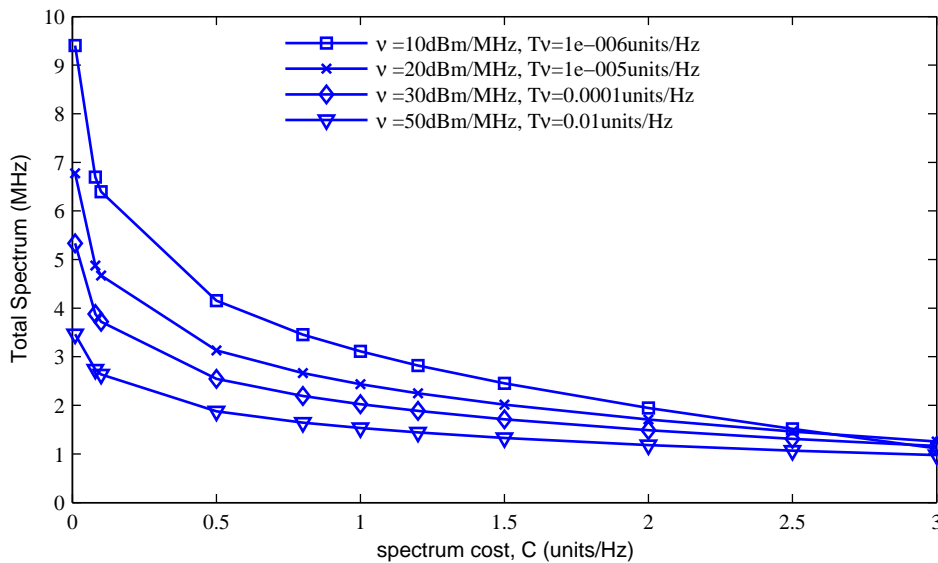


Figure 5.2: Total SP profit as function of efficiency and spectrum costs user when each user is homogeneous and has exponential utility with $\Gamma = 1$ Mbps and $T = 10$

amounts. So the SP can't hope to gain from the usage fees. It thus reduces the spectrum price (underpricing it in the process) so that users purchase a lot of spectrum and the SP can make use of their increased utility by extracting their increased surplus as the connection fee.

Lastly we consider that the SP can operate with 10% user outage. For $L = 10$ users, let link gains satisfy $h_1 > \dots > h_{10}$. Thus user 10 is the marginal user. The SP can choose to serve 9 users by raising prices to make user 10 refuse service. User 9 would be the new marginal user. The loss of revenue from user 10 can be made up by the increased revenue from the other users. The results are shown in Figure (5.5). For high values of ν , the profits are more for $L = 10$. For low ν , it is slightly advantageous to serve 9 users when C is high. Recall from Figure (5.3) that in that regime, $\mu \sim C_e$ and the profits are mostly due to the connection fees. So the deciding factor is the relative differences in the surpluses of users 10 and 9.

5.4 Conclusion

In this work, we have considered a network where a single service provider allocates spectrum to its customers in the downlink. We propose a dynamic allocation scheme based on SP profit maximization. The SP uses two part monopolistic pricing, consisting of a fixed connection fee and a variable usage cost. We showed that for a broad range of concave user utilities, the user

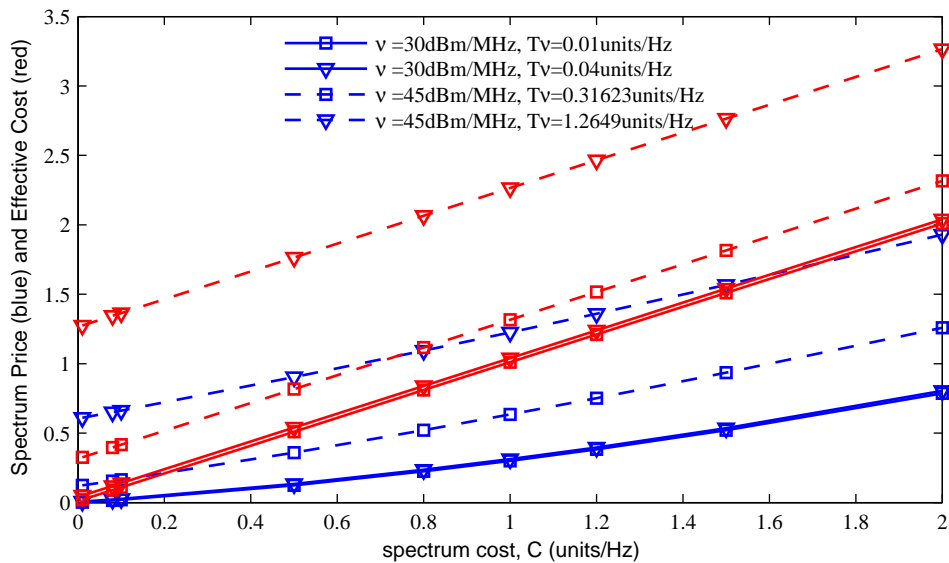


Figure 5.3: The effective cost $C_e = C + T\nu$ and spectrum price μ as a function of spectrum cost C when each user is homogeneous and has exponential utility with $\Gamma = 1$ Mbps and $T = 10$

with the weakest link gain decided the connection fee. We characterized the spectrum allocation and derived values for various prices involved. We showed that for logarithmic utilities, the spectrum was overpriced relative to the costs of the SP and the demand was elastic. In contrast, for users whose applications have exponential utilities, the demand could be inelastic. Numerically we illustrated some key analytical ideas and also tested the performance of the allocation algorithm with user outage. We conclude that the microeconomic model gave us an instructive framework to study the profit maximization problem.

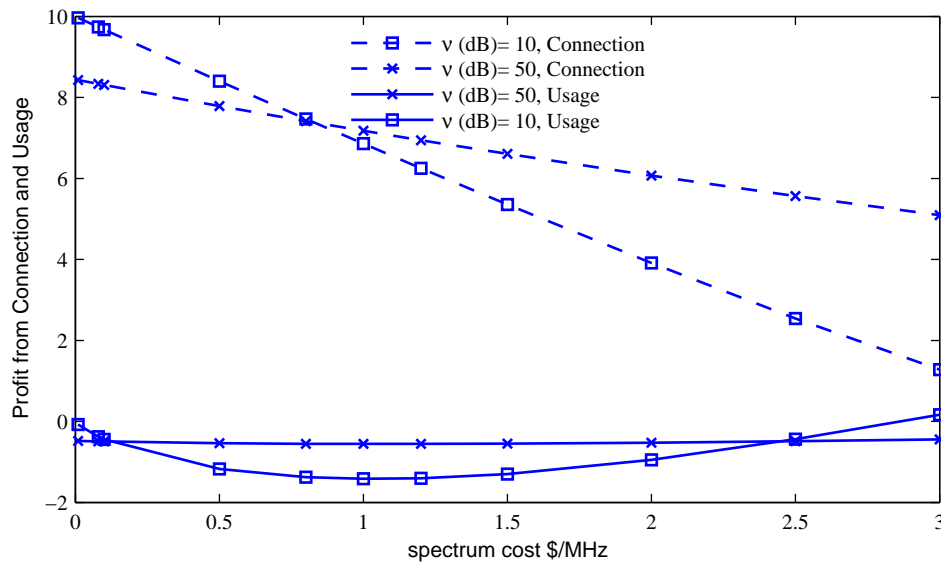


Figure 5.4: Breakup of SP profit, Π^* as total usage cost, Π_U^* and total connection fee, Π_C^* as a function of efficiency and spectrum costs user when each user is homogeneous and has exponential utility with $\Gamma = 1$ Mbps and $T = 10$

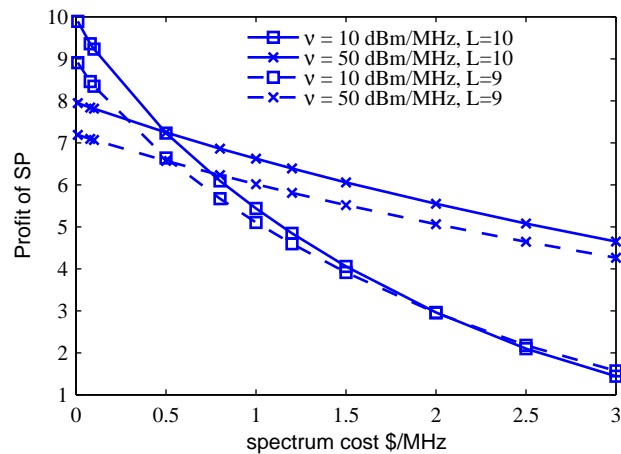


Figure 5.5: A example plot of the SP profits for $L = 10$ users and with $L = 9$ obtained by removing the marginal user from the original population.

Chapter 6

Price Competition between Service Providers

6.1 Introduction

In this chapter we consider a spectrum allocation model consisting of multiple service providers (SP) that compete to obtain the services of end users. The result of the competition is a user base for each provider with the providers selling orthogonal chunks of spectrum to its users by setting a price per unit spectrum. Then the SPs transmit to their users over their allocated spectrum. Based on their spectrum demand and spectrum price, the users decide how much spectrum to buy. The user demand for spectrum depends on the received data rate which is different for different users due to the variations in the link gain and modulation technologies of the SPs. Each SP purchases the amount of spectrum needed by its customers, from a regional spectrum broker [31]. The SP has to pay the broker for the purchased spectrum. The SPs choose their spectrum prices and modulation technologies to attract users and maximize their profits [86].

6.2 Features of the Spectrum Allocation

The connection fee κ was the key feature of a monopolistic price structure in Chapter 5 as it was used to extract the remaining user surplus, once the user had paid the usage cost $\mu_i x_i$. Thus κ depended upon μ_i , x_i and the user utilities. Such a usage price dependent connection fee that extracts the entire user surplus is inadmissible due to the SP competition. Thus for SP price competition we assume that each SP i only charges a usage price μ_i per unit of spectrum. Thus cost function for user j to SP i is $\rho_i(x_{ij}) = \mu_i x_{ij}$.

The spectrum allocation, to be described in detail in this chapter, has three components. These are described qualitatively as follows

- a) Given a particular price and spectral efficiency pair (μ_i, K_{ij}) for SP i , each user j calculates x_{ij} , the spectrum to obtain from SP i . This is explored in Section 6.3.
- b) The SPs are assumed to be aware of the user preferences as mentioned in a). SP i can optimize its price μ_i to induce a desired user allocation x_{ij} that will maximize its profits. The single SP profit maximization is explored in Section 6.4. We show that to maximize profits, SP i can choose to serve a subset of the users.
- c) When there are multiple SPs in the system, each SP competes to obtain the services of the users. We show in Section 6.5 that as a result of this competition each SP ends up with its own user base, in which it is the sole provider, free of competition. Then the SP prices its users in the way mentioned in case b).

The spectrum allocation can be done periodically, typically at the start of a session or as channels vary with time.

6.3 User Optimization Problem for Given SP Prices

Given price μ_i from SP i , user j maximizes its *surplus* \mathcal{S}_j to decide how much spectrum to purchase from SP i as mentioned in (4.6),

$$\mathcal{S}_j = \max_{x_{ij}} U_j(R_j) - \sum_{i=1}^N \mu_i x_{ij}. \quad (6.1)$$

It is possible that for user j and SP i , $x_{ij} = 0$, which implies that user j does not obtain service from SP i . For finding the optimal solution of (6.1), first note that the optimization for each user is independent. Let us focus on user j and write the Lagrangian by introducing the slack variables α_{ij} to account for the constraints $x_{ij} \geq 0$,

$$\mathcal{L}_j = U_j(R_j) - \sum_{i=1}^N \mu_i x_{ij} + \sum_{i=1}^N \alpha_{ij} x_{ij} \quad (6.2)$$

Let us denote $\dot{U}_j(R_j) \triangleq \partial U_j / \partial R_j$. The first order conditions of the Lagrangian (6.2) are

$$\frac{\partial \mathcal{L}_j}{\partial x_{ij}} = \dot{U}_j(R_j) K_{ij} - \mu_i + \alpha_{ij} = 0. \quad (6.3)$$

Let k and l be two SPs from which user j obtains service with spectrum allocations $x_{kj} > 0$ and $x_{lj} > 0$. Thus $\alpha_{kj} = 0$, $\alpha_{lj} = 0$ and from (6.3) we conclude that

$$\frac{\mu_k}{K_{kj}} = \frac{\mu_l}{K_{lj}} = \dot{U}_j(R_j). \quad (6.4)$$

If user j obtains service from SP k but not from SP l , then $x_{kj} > 0$ and $x_{lj} = 0$ and from (6.3), we can show that

$$\frac{\mu_i}{K_{kj}} < \frac{\mu_k}{K_{lj}}. \quad (6.5)$$

Definition 2. We define the effective spectrum price between SP i and user j as $p_{ij} = \mu_i/K_{ij}$.

Note that the units of p_{ij} is dollars per bits/sec. This is because of the actual price μ_i is measured in terms of dollars/Hz but the utility $U_j(R_j)$ is measured in terms of bits/sec. Thus p_{ij} is the translation of a price per unit of spectrum to a price per unit rate. Thus (6.5) can also be stated as,

Lemma 11. *The user connects to the SP with the lowest effective price.*

To attract users, SPs can lower their effective prices by either lowering actual prices or by raising efficiency, to attract users. This is a part of the SP competition as explored this in Section 6.5.

Definition 3. Define the set of users to whom SP i provides service as \mathcal{S}_i . Define the net spectrum that SP i has to provide as $X_i = \sum_{j \in \mathcal{S}_i} x_{ij}$. Define $\boldsymbol{\mu} = [\mu_1, \mu_2, \dots, \mu_N]$. We will sometimes denote X_i by $X_i(\boldsymbol{\mu})$ to explicitly indicate the relationship between spectrum and the vector of prices as captured in (6.1).

For exponential utilities defined in Section 4.8, the demand function and the *inverse demand function* are given by (6.4)

$$\mu_i = K_{ij} e^{-(K_{ij}/\Gamma_j)x_{ij}} \quad (6.6a)$$

$$x_{ij} = \frac{\Gamma_j}{K_{ij}} \log \left(\frac{K_{ij}}{\mu_i} \right) = \frac{\Gamma_j}{K_{ij}} \log \left(\frac{1}{p_{ij}} \right). \quad (6.6b)$$

Note that (6.6b) implies that $p_{ij} < 1$ and we discuss the reason in Section 6.4.1.

6.4 Analysis for Single SP Network

We first consider a network with a single SP, free of any competition. This is because spectrum allocation and pricing results of this case will be needed for the general case, when a competing SP is present. This analysis is different from the monopolistic pricing structure of Chapter 5 because

1. The SPs do not charge an usage cost dependent subscription fee κ .
2. The SPs can chose to serve a subset of the total number of users to maximize their profits.

6.4.1 Calculation of Optimal Price

The SPs are assumed to be aware of (6.6b), i.e. how the users react to a price vector. They make use of this information while maximizing profits by choosing the right prices to induce a particular spectrum demand from the users. Thus the SP to user interaction is an instance of a *Stackelberg Game* [87, Section 6.2], where one player is the leader and the other players are followers. First the leader makes a move which is then observed by the followers in deciding their own action. The leader knows of this behavior and uses this to design his move in order to induce a desired follower action. Let the SP index be i . The profit maximization function is given by

$$\Pi_i(\mu_i) = (\mu_i - C_i^s) \sum_{j=1}^N x_{ij}(\mu_i), \quad (6.7a)$$

$$\text{where } x_{ij}(\mu_i) = \left\{ \frac{\Gamma_j}{K_{ij}} \log \left(\frac{K_{ij}}{\mu_i} \right), 0 \right\}^+ \quad (6.7b)$$

Lemma 12. *SP i buys spectrum at price C_i^s but sells spectrum to its users at a higher price $\mu_i > C_i^s$.*

Proof. This follows from (6.7a), in order to achieve non-negative profit. \square

Lemma 12 says that the SPs *overprice* spectrum when selling to the end users. This need not be true for other price structures as seen for the monopoly pricing for users with exponential utilities in Section 5.3. Also note that if $\mu_i > K_{ij}$, for some user j then $x_{ij}(\mu_i) = 0$, meaning that the SP does not serve these users. Thus we conclude that SP i can charge price μ_i to serve user j if

$$K_{ij} > \mu_i > C_i^s. \quad (6.8)$$

The interpretation of the result is as follows: Note from (6.6a) that the maximum value of $\dot{U}_j(R_j)$, the marginal utility of user j , is K_{ij} . Thus the upper bound in (6.8) says that for feasible allocation, the price charged should be less than the maximum value of the marginal utility or else the user can never have non zero surplus. From Definition 2, an alternate way of representing (6.8) is

$$1 > p_{ij} > \frac{C_i^s}{K_{ij}}. \quad (6.9)$$

Thus the minimum possible value of the effective price for user j is given by $p_{ij}^{\min} = C_i^s / K_{ij}$. Note that (6.7) is not a convex optimization problem as the function $x_{ij}(\mu_i)$ is not concave.

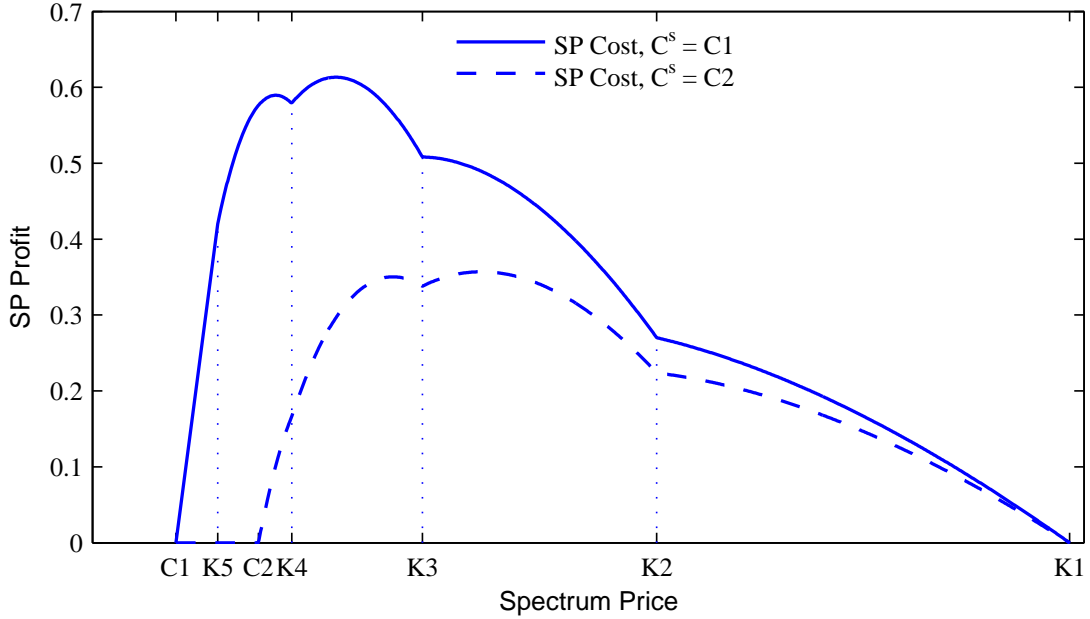


Figure 6.1: Illustration of how price is related to spectral efficiencies and spectrum cost of the SP. Consider one SP and 5 users and for ease of illustration drop the SP index i . When SP cost, $C^s = C1 < K_5$, the SP can serve all users. If it sets $K_1 > \mu_i \geq K_2$, it serves the first user who is nearest to it. If it decreases price to a range $K_2 > \mu_i \geq K_3$, it serves the first two users and so on. When SP cost is increased to $C^s = C2 > K_5$, the SP can't serve user 5 as doing so would make profits negative.

We now show that (6.7) can be reduced to an equivalent optimization problem, that can be easily solved. For simplicity in illustration consider the case shown in Figure 6.1 where the users are arranged linearly with user 1 the closest and user L the farthest for $L = 5$. Thus $K_{i1} > \dots > K_{iL}$. The following cases can arise

$$K_{iL} > C_i^s$$

The SP can serve all the L users. Consider the case when the SP serves exactly the first $M < L$ users. From (6.8), this will happen if $K_{iM} > \mu_i > K_{i(M+1)}$. The corresponding profit of SP i , denoted by $\Pi_i^{(M)}$, is given by

$$\Pi_i^{(M)} = \max_{\mu_i} (\mu_i - C_i^s) \sum_{j=1}^M \frac{\Gamma_j}{K_{ij}} \log \left(\frac{K_{ij}}{\mu_i} \right), \quad (6.10a)$$

$$\text{s.t. } K_{iM} > \mu_i > K_{i(M+1)}, \quad (6.10b)$$

For the last user, $M = L$, constraint (6.10b) is modified to $K_{iL} > \mu_i > C_i^s$. It can be verified that (6.10) is a convex optimization problem.

$$C_i^s > K_{iL}$$

Assume that $K_{i1} > \dots > K_{iM_{\max}} > C_i^s > K_{i(M_{\max}+1)} > \dots > K_{iL}$. This means that SP i can serve at most M_{\max} users. To serve $M_{\max} + 1$ users, it would have to set $\mu_i < C_i^s$, which is not allowed. For serving exactly M_{\max} users, constraint (6.10b) is modified to $K_{iM_{\max}} > \mu_i > C_i^s$. To keep notations consistent, define $M_{\max} = L$ for case $K_{iL} > C_i^s$. Thus the solution to the SP optimization problem (6.7) is given by the integer problem

$$\max_{M \leq M_{\max}} \Pi_i^{(M)}, \quad (6.11)$$

where $\Pi_i^{(M)}$ for all M can be calculated by solving a convex optimization problem. Optimization (6.11) can be solved by exhaustive search for low to medium values of L .

6.5 Multiple SP Interaction

Assume that each SP i communicates cost C_i^s and efficiencies K_{ij} to all users j in the system to the Spectrum Broker who mediates the interaction between the SPs. Thus the Broker has global knowledge but an SP does not obtain the private informations of other SPs. Similar assumptions have been made in [13]. The SPs engage in a game with the aim to maximize their profits. The SP competition can be modeled as an *extensive game with perfect information* [87, Chapter 5] represented by $G = [N_S, \{M_i\}, \Pi_i]$, where N_S is the set of the N SPs, M_i is the strategy space available to the i^{th} SP which is the number of users that it attempts to serve. The optimization of SP i depends on the prices charged by the other SPs which determines how many users obtain service from SP i . We explore these ideas in Sections 6.5.1 and 6.5.2.

6.5.1 Single User System

Consider that initially there is one SP, serving one user by charging a profit maximizing price derived in Section 6.4.1. Let us denote the price by $p_{1j}(0)$, where $p_{ij}(n)$ is the effective price of SP i to the user j which corresponds to a price $\mu_i(n)$ charged by SP i at time instant n . Assume that SP 1 profit is $\Pi_1^{(1)}$, as defined in (6.10a). Assume that at time instant $n = 1$, a second SP

enters the network. Let the user index be j . Let K_{1j} and K_{2j} be the spectral efficiencies of the two SPs and C_1^s and C_2^s , their spectrum costs. SP 2 can take one of the following two actions,

1. *Compete*: If SP 2 decides to compete, it broadcasts its intention of providing service to the user. From Lemma 11, it does so by broadcasting a price $\mu_2(1)$ which leads to a lower effective price $p_{2j}(1) < p_{1j}(0)$.
2. *Withdraw*: If SP 2 decides to withdraw, it broadcasts that it would not provide service to the user. SP 1 can charge the same price as before when it was the only SP in the system.

If SP 2 decides to compete at time $n = 1$, then the user will obtain spectrum from SP 2 and SP 1 profit will be driven to zero. Thus at time $n = 2$, SP 1 can also compete and broadcast a reduced price $\mu_1(2)$ which leads to $p_{1j}(2) < p_{2j}(1)$ or it may decide to withdraw. Thus at each time instant, the competing SP will cut prices to so that it becomes the sole provider to the user. Assuming that each SP knows the link gains to the users and efficiencies of its own and the other SP, this can be modeled as an extensive game with perfect information [87, Chapter 5]. The game will continue till one of the SPs hits the lower bound on the price that it can charge and still achieve non-negative profit. This is given by (6.9). As an example, consider $p_{1j}^{\min} < p_{2j}^{\min}$. If at time n , SP 1 decides to compete and chooses a price $\mu_1(n)$ that leads to an effective price, $p_{1j}(n) < p_{2j}^{\min}$, then SP 2 can not compete in time $n + 1$ and he withdraws leading to the termination of the game. Thus the profit of SP 2 is zero. Hence if SP 2 decides to compete initially, SP 1 will eventually undercut him and reduce his profits to zero. Thus SP 2 faces no loss in profit by deciding to withdraw initially. This is a subgame perfect equilibrium [87, Section 5.4] of the extensive game. Thus in this equilibrium, SP 1 will charge an effective price $p_{1j}(0)$, as in the single SP case. This is because the threat that SP 2 will undercut him in pricing is not *credible*.

Note that this result could be generalized for more than two SPs. The reason is that since there is only one user, SPs must obtain its service to get nonzero profit. So each SP will reduce the price till all but one have reached the lower bound on their prices. We state this in the following theorem

Theorem 9. *For a single user system, the SP k with $k = \arg \min_i p_{ij}^{\min}$ obtains the services of the user j and sets a price, assuming no competition from the other SPs.*

From Theorem 9, the user optimization problem (6.1) can be recast as

$$\mathcal{S}_j = \max_{x_{kj}} U_j(K_{kj}x_{kj}) - \mu_k x_{kj}, \quad (6.12a)$$

$$\text{s.t. } k = \arg \min_i \frac{C_i^s}{K_{ij}}. \quad (6.12b)$$

Let us now reflect on the relationship between Theorem 9 and Lemma 11, which forms its basis. Lemma 11 states that SP i with a lower effective price p_{ij} will attain service of user j at the expense of the other SPs. Theorem 9 says that SP i with the lowest value of minimum effective price to user j , p_{ij}^{\min} will compete and successfully attain the service of user j as per Lemma 11. But could there be situations in which SP i with the lowest value of p_{ij}^{\min} , decides against competing for attaining the services of user j , even though it would have been successful in attaining the services of the user if it had done so? For multiple users this could be true as shown in Section 6.5.2.

6.5.2 Multiple User System

To develop an understanding of multiuser systems, we assume for the moment a system with two SPs and make the simplifying assumption that the users $j = 1, \dots, L$ are arranged such that

$$h_{11} > h_{12} > \dots > h_{1L} \text{ and } h_{21} < h_{22} < \dots < h_{2L}. \quad (6.13)$$

One possible arrangement for (6.13) is shown in Fig 6.2. Though not essential for the algorithm to be presented for multiuser pricing, (6.13) simplifies it.

Theorem 10. *If user positions satisfy (6.13), there is a user u^* such that all users from $1, \dots, u^*$ obtain service from SP 1 and all users from $u^* + 1, \dots, L$ obtain service from SP 2.*

Proof. Refer to Appendix E. □

Definition 4. *Consider a two SP network with L users as per arrangement (6.13). Define the game $\Gamma_h(L_1, L_2, L)$ as the extensive game where SP 1 can either Compete for the L_1 users closest to it, $1, \dots, L_1$ (set a price to obtain the services of all of them) or Withdraw (decide not to serve the users) and SP 2 decides to either compete for the L_2 users closest to it, $L - L_2 + 1, \dots, L$, or withdraw.*



Figure 6.2: The network topology for two SPs and L users. For a single SP, assume either SP 1 or SP 2 is present

Note that the game in Section 6.5.1 is an instance of the game $\Gamma_h(1, 1, 1)$. If $L_1 + L_2 \leq L$, then there is no conflict, i.e. there is no user whom both SPs want to serve. In this case each SP sets a price as per Section 6.5.1, assuming that it is the only SP in the subsystem consisting of the users that it wants to serve. To understand the multiuser case, let us first consider a two user example:

Example 3. *The two user extensive game is depicted in Figure 6.3. SP 1 starts the game and its strategy is to compete for exactly 1 user or both users. Then SP 2 acts with the same strategy. The notation $\tilde{\pi}_1^{(L_1, L_2)}$ and $\tilde{\pi}_2^{(L_1, L_2)}$ indicate the payoffs (profits) of SP 1 and SP 2 respectively, when SP 1 competes for the L_1 users closest to it and SP 2 competes for the L_2 users closest to it. If $L_1 = L_2 = 1$, then there is no conflict. In this case SP 1 sets its price $K_{11} > \mu_1 > K_{12}$ as explained in Section 6.4.1 for which user 2 will never attach itself to SP 1. Similarly SP 2 sets its price $K_{22} > \mu_2 > K_{21}$. Recalling the notation of single SP profit from (6.10a), the profits in the two SP case are thus*

$$\tilde{\pi}_1^{(1,1)} = \Pi_1^{(1)}, \quad \tilde{\pi}_2^{(1,1)} = \Pi_2^{(1)}. \quad (6.14)$$

If SP 1 decides to compete for the first user and SP 2 for both users, then we have an instance of the game $\Gamma_h(1, 2, 2)$. Similarly the other two histories of the extensive game leads to the games $\Gamma_h(2, 1, 2)$ and $\Gamma_h(2, 2, 2)$.

The solution to the game depend on which of the following three conditions are satisfied

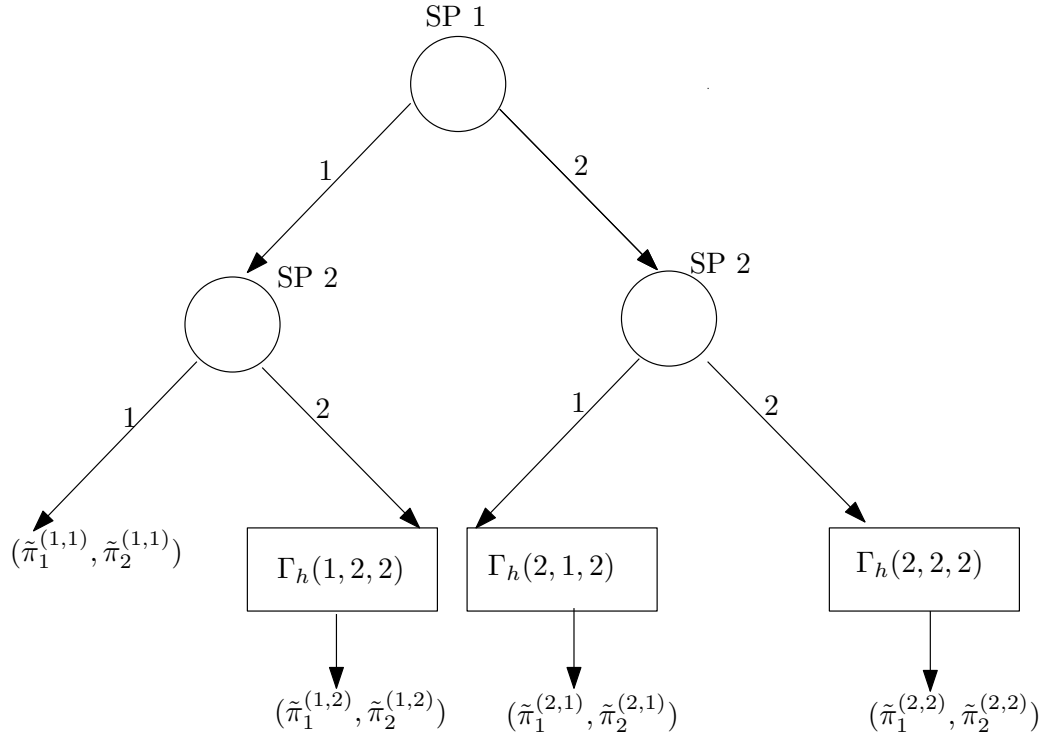


Figure 6.3: A two SP two user extensive game. Action $l = \{1, 2\}$ for a SP means that the SP decides to compete with the other one to attain the service of the l users closest to it.

SP 1 can compete and successfully attain the services of user 1 and the SP 2 can do the same for user 2

Thus $p_{11}^{\min} < p_{21}^{\min}$ and $p_{12}^{\min} > p_{22}^{\min}$. Now consider the game $\Gamma_h(1, 2, 2)$. SP 2 competes for both users but SP 1 can always undercut it in a price game to attain the services of user 1. So in the equilibrium each SP will end up with one user, the one closest to it. Thus following the notation of Figure 6.3, we should have $\tilde{\pi}_1^{(1,2)} = \tilde{\pi}_1^{(1,1)}$ and $\tilde{\pi}_2^{(1,2)} = \tilde{\pi}_2^{(1,1)}$. However, assume that the broker charges an SP a penalty for competing unsuccessfully, i.e. if SP i , decided to compete for L_i users and ended up obtaining $K < L_i$ users, then it pays a penalty of $(L_i - K)\Delta$. This is because if SP i bids for the extra $L_i - K$ users, then there is a price competition game as mentioned in Section 6.5.1 but the final user allocations do not change. This is because the threat that SP i will undercut the other SP is not credible. The broker

however has to allocate resources and computing power to coordinate this game on behalf of the SPs and so it charges SP i , who initiated the game. Thus we have,

$$\tilde{\pi}_1^{(1,2)} = \Pi_1^{(1)}, \quad \tilde{\pi}_2^{(1,2)} = \Pi_2^{(1)} - \Delta. \quad (6.15)$$

Similarly we can show that for the games $\Gamma_h(2, 1, 2)$ and $\Gamma_h(2, 2, 2)$,

$$\tilde{\pi}_1^{(2,1)} = \Pi_1^{(1)} - \Delta, \quad \tilde{\pi}_2^{(2,1)} = \Pi_2^{(1)} \quad (6.16)$$

$$\tilde{\pi}_1^{(2,2)} = \Pi_1^{(1)} - \Delta, \quad \tilde{\pi}_2^{(2,2)} = \Pi_2^{(1)} - \Delta. \quad (6.17)$$

It is thus easy to see that in the subgame perfect equilibrium of two user game each SP will compete for serve the user closest to it.

SP 1 can compete and successfully attain the services of both users at the expense of SP 2

This happens when $p_{11}^{\min} < p_{21}^{\min}$ and $p_{12}^{\min} < p_{22}^{\min}$. Now consider the game $\Gamma_h(1, 2, 2)$. In this case, since SP 1 bids only for the first user, he obtains it and SP 2 obtains the other user though it had competed for both. The profits are same as (6.15). Consider the games $\Gamma_h(2, 1, 2)$ and $\Gamma_h(2, 2, 2)$. SP 1 competes for both users and successfully obtains them. SP 2 does not obtain any user but has to pay the penalty of competing unsuccessfully. Thus,

$$\tilde{\pi}_1^{(2,1)} = \Pi_1^{(2)}, \quad \tilde{\pi}_2^{(2,1)} = -\Delta \quad (6.18)$$

$$\tilde{\pi}_1^{(2,2)} = \Pi_1^{(2)}, \quad \tilde{\pi}_2^{(2,2)} = -2\Delta. \quad (6.19)$$

Now if the SP 1 profit from serving two users is greater than that from serving one, i.e. if $\Pi_1^{(2)} > \Pi_1^{(1)}$, then in the subgame perfect equilibrium, SP 1 will serve both users. Else each SP will again serve only the user closest to it.

SP 2 can compete and successfully attain the services of both users at the expense of SP 1

This is the reverse situation to case 3. Arguing similarly we conclude that if SP 2 profit from serving two users is greater than one, then in the subgame perfect equilibrium, SP 2 will serve both users. Otherwise each SP will serve only the user closest to it.

Thus each SP serves the users for which it has the minimum effective price, only if doing so increases its profits when it sets a price assuming no competition. This gives us the intuition to design the algorithm for the multiuser case,

1. Each SP i identifies the set of users j for whom $i = \arg \min_l p_{lj}$. Let these sets be $\mathcal{V}_1 = [1, \dots, u^*]$ and $\mathcal{V}_2 = [u^* + 1, \dots, L]$.
2. For each SP $i = 1, 2$ do
 - (a) if $|\mathcal{V}_i| \leq 1$ exit
 - (b) else retain all \mathcal{V}_i users and calculate the profit from (6.7). Then remove the user with the weakest spectral efficiency (users u^* for SP 1; $u^* + 1$ for SP 2) and calculate the profits. Keep doing this to identify the optimal set of users for which the profit is maximized. Call these new sets as $\tilde{\mathcal{V}}_i$.
3. The following cases arise
 - (a) $\{\tilde{\mathcal{V}}_1 = \mathcal{V}_1 \text{ and } \tilde{\mathcal{V}}_2 \neq \mathcal{V}_2\}$: Thus there exists $M > 0$ such that SP 2 has raised prices to exclude users $u^* + 1, \dots, u^* + M$. SP 1 will include these users one by one, starting with user $u^* + 1$ and reoptimize its price to see if including them increases its profit.
 - (b) $\{\tilde{\mathcal{V}}_1 \neq \mathcal{V}_1 \text{ and } \tilde{\mathcal{V}}_2 = \mathcal{V}_2\}$: SP 1 has raised prices to exclude some users. SP 2 will reoptimize its price to see if including some of these users increases its profit similar to above.
 - (c) $\{\tilde{\mathcal{V}}_1 = \mathcal{V}_1 \text{ and } \tilde{\mathcal{V}}_2 = \mathcal{V}_2\}$: Each SP achieves maximum profit by serving all users that they can out-bid the other SP in a price competition. Thus the prices and allocations will not change.
 - (d) $\{\tilde{\mathcal{V}}_1 \neq \mathcal{V}_1 \text{ and } \tilde{\mathcal{V}}_2 \neq \mathcal{V}_2\}$: Each SP i has excluded some users. SP i would not want to include users that the other SP has excluded as they would be at weaker link gains than the ones that he had excluded (because of assumption (6.13)) and thus no change in prices and allocations will take place.

It is interesting to note that the exclusive attachment of each user to one SP had also been observed in Chapter 3, Theorem 2 for a different framework where the SPs did not have profit motives and jointly set a single spectrum price μ to maximize the sum of user utilities in the uplink. Under certain assumptions about SP efficiencies, it was shown that the user attached itself to the SP to which it had the best link gain. In this work, if $\nu_i = \nu$ for all SPs, then the SP with the lowest effective price is the SP with the best link gain to the user as per (4.2). Hence

the conclusions are along similar lines and seem to be applicable for a wide variety of dynamic spectrum allocation systems. Also, because of this result, it seems that SPs have economic incentives to vary their spectral density ν_i . Section 6.6 about numerical results demonstrate that this is indeed true.

6.5.3 Discussion about Game Formulation

In this chapter we have formulated a game where the SP strategy is to first look at the users that it can serve without competition from the other SP and then decide to serve a subset of them. If the other SP had rejected some users, it might redo the optimization. Thus the control variable in SP optimization is the number of users that a SP decided to serve. There have been other formulations where the SP control variable is price. For example the authors in [21] consider a system where the SPs broadcast a price and rate pair which determines the probability that a user will obtain service from it. Based on this probability the SP may alter its price. While this may seem a natural way to set up the price competition game, it is difficult to jointly optimize prices when the user demand functions are captured in the SP optimization via a Stackleberg interaction. Our model, which is based on the non credible threat principle succeeds in capturing this.

Setting price as a control variable is also the norm for most classical problems of industrial organization [88]. However the problems there are somewhat simplified as each SP considers an aggregate demand function that is often a continuous distribution defined over their physical separation. In this chapter we consider discrete number of users and the analysis is simplified if the number of users to be served is the control variable.

We believe that our model is relevant in situations when a SP who wants to enter a market first studies the user profile and presence of other SPs in the region. Based on the prices that the other SP is offering, it can use the credible threat analysis to determine what percentage of customers it is likely to end up with. This could influence its decision whether or not to enter the market.

6.6 Numerical Results

In this section, we consider specific network topologies and evaluate the corresponding spectrum allocation and prices. We consider a linear network with two SPs and $L = 100$ users

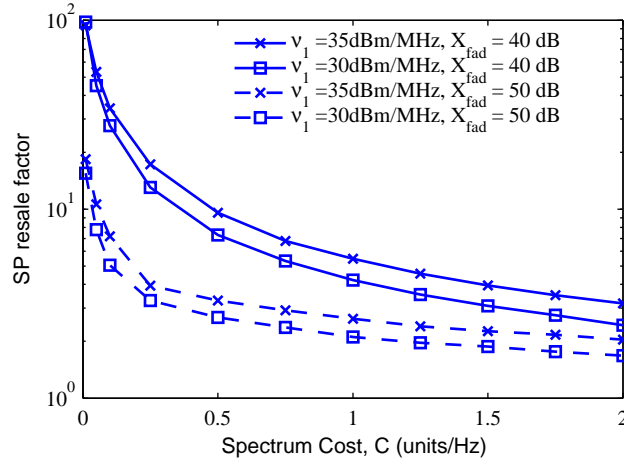


Figure 6.4: Ratio of price μ_1 that SP 1 charges from users to C_1^s , the price that SP 1 pays to the broker for different values of ν_1 . The target rate is $\Gamma = 1$ Mbps. For SP 2 $\nu_2 = 37.5$ dBm/MHz.

arranged symmetrically between the SPs as in Fig 6.2 with inter-user distance $d = 1.5$ m. For path loss, we choose the COST-231 propagation model [76], at an operating frequency of 2.4 GHz. Let the distance of user j from the SP i be d_{ij} . Thus the link gain is given by

$$h_{ij,dB} = -31.5 - 35 \log(d_{ij}) - X_{\text{fad}}, \quad (6.20)$$

where X_{fad} is a constant margin, which is a design allowance that provides for sufficient system gain or sensitivity to accommodate expected fading, for the purpose of ensuring that the required quality of service is maintained. For no fading $X_{\text{fad}} = 0$. The minimum separation between a SP and its nearest user is $d_{\min} = 35$ m as per the COST-231 model [76]. In Fig 6.4, we plot the *resale factor* for SP 1 which is defined as μ_1/C_1^s : the ratio of the price at which the SP sells spectrum to its purchase price from the broker. We see that the factors are quite high. This can be intuitively understood from the following relationship,

$$\min_j K_{1j} > \mu_1 > \max C_1^s, \quad (6.21)$$

which is a direct consequence of (6.8). For the values considered in Figure 6.4, (6.21) becomes $55 > \mu_1 > 2$ for $\nu_1 = 30$ dBm/MHz. Since $K_{1j} \gg C_1^s$, the value of μ_1 which lies in between these two values, can be much greater than C_i^s .

Thus from the broker perspective it could raise the value of C to collect more revenue from the SPs. Also from Figure 6.4, the price increases with increasing efficiency and decrease with

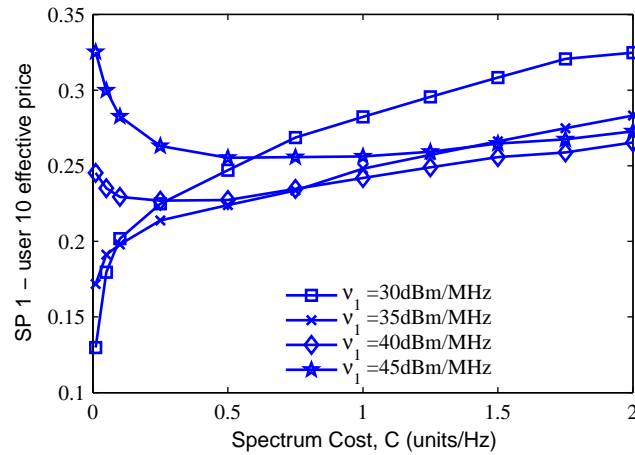


Figure 6.5: Effective price of SP 1 to user 10 as a function of spectrum cost C for different values of ν_1 . Other parameters are a) Target rate, $\Gamma = 1$ Mbps b) SP 2 psd, $\nu_2 = 37.5$ dBm/MHz c) Fading Margin, $X_{\text{fad}} = 40$ dB.

fading as the spectral efficiencies increase and decrease respectively.

To understand the effect of prices and spectral efficiencies, we plot the effective price of SP 1 for user 10, $K_{i,10}$ in Figure 6.5 for different values of ν_1 and C . The efficiency of SP 2 is fixed at $\nu_2 = 37.5$ dBm/MHz. When broker spectrum cost C is low, the lowest effective price is obtained by choosing a lower efficiency while the opposite holds when broker spectrum cost is high. This effect also shows up in Figures 6.6 and 6.7 which respectively plot the fraction of total users who obtain services from SP 1 and the profit of SP 1.

It is seen that both the number of users attached and profits are maximized by choosing a low value for ν_1 when C_1^s is low and a high value of ν_1 when C_1^s is high. Since a SP that transmits at a high value of ν , has to support higher order modulation and coding, increasing ν is an indicator of the SP employing a better technology. Thus we observe that a SP should invest in a better technology to transmit to its users only if it has to pay high spectrum cost to the broker. Else it should purchase maximum spectrum from the broker and transmit at the lowest possible spectrum efficiency.

In Figure 6.8, we compare the profit of SP 1 with and without competition. For high efficiency ν_1 , the gap in the two profit values is more than for low ν_1 . This is because in absence of competition, SP 1 was serving all users when it had high ν_1 but only a fraction of them (who are closer to it) for low ν_1 . Thus when a competing SP comes, the number of customers lost

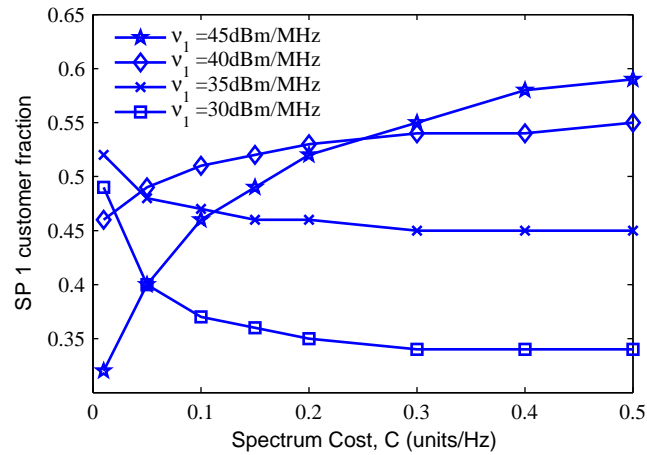


Figure 6.6: Fraction of total number of users who are customers of SP 1 as a function of spectrum cost C for different values of ν_1 . Other parameters are a) Target rate, $\Gamma = 1$ Mbps b) SP 2 psd, $\nu_2 = 37.5$ dBm/MHz c) Fading Margin, $X_{\text{fad}} = 40$ dB.

is less when SP 1 had a low ν_1 . Figure 6.9 plots the fraction of users who are better off due to competition. This is calculated as follows

1. Assume that the system has only SP 1. Calculate the surplus as defined in (6.12) for all L users. Call them $\mathcal{S}_j^{\text{nc}}$ for $1 \leq j \leq L$.
2. Assume that the system has both SPs. Calculate the surplus as defined in (6.12) for all L users. Call them \mathcal{S}_j^{c} for $1 \leq j \leq L$.
3. Calculate the number of users j for whom $\mathcal{S}_j^{\text{c}} > \mathcal{S}_j^{\text{nc}}$.

There are two qualitative points that explain Figure 6.9. When a second SP is added, the following can happen

1. Some users who didn't receive service from the first SP, now do so from the second.
2. Some users who had received service from the first SP, may have a reduced surplus due to competition. Due to competition, SP 1 serves a smaller set of users in which user who is farthest away from the SP has a better link gain than the corresponding one in the original set of users when there was no competition. Recall that SP price is determined by the link gain to the farthest user as discussed in Section 6.4.1. Thus in the SP re-optimization, the price μ_1 increases and the user surpluses decrease.

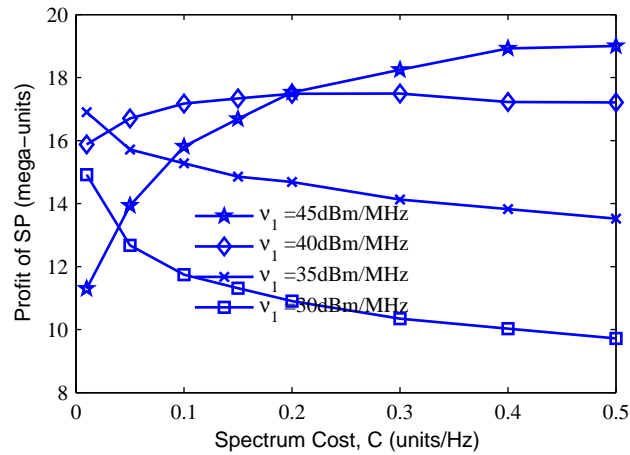


Figure 6.7: The profit of SP 1 as a function of spectrum cost C for different values of ν_1 . Other parameters are a) Target rate, $\Gamma = 1$ Mbps b) SP 2 psd, $\nu_2 = 37.5$ dBm/MHz c) Fading Margin, $X_{\text{fad}} = 40$ dB.

For example consider the case when the SP transmits at low efficiency at high C . There are more users who are better off if there is competition. This is because at low efficiency and high C , which means increased price, the users who were far from SP 1 are not being served but they obtained service from SP 2 when there was competition. This was also noted in Figure 6.8.

6.7 Discussion and Conclusion

In this chapter, we have considered a network where multiple service providers compete with each other in securing the service of users in the downlink. Each provider sets a spectrum price and the users decide which provider to obtain service from based on their price and the link gains. We propose a dynamic allocation scheme based on SP profit maximization. We developed the concept of an effective spectrum price and showed how it lead to equilibrium solutions of the SP price competition game. We showed that in the optimal allocation each user obtained service from only one SP, each provider has its own customer base and it can price them assuming no competition. Further insights were obtained from the numerical results, which showed that a SP should invest in better technology only if the cost of spectrum is high.

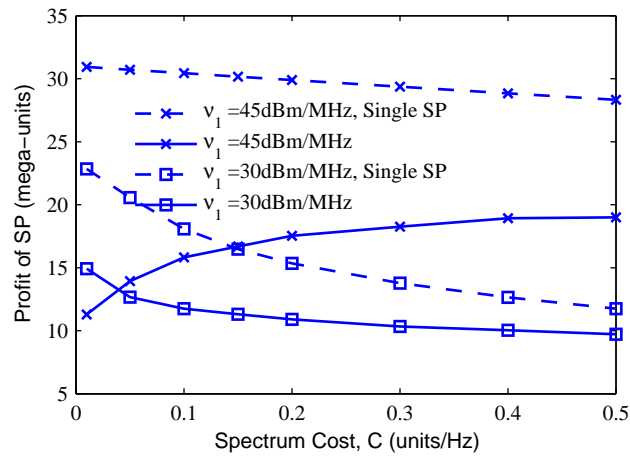


Figure 6.8: Comparison of the profit of SP 1 with and without competition as a function of spectrum cost C for different values of ν_1 . Other parameters are a) Target rate, $\Gamma = 1$ Mbps b) SP 2 psd, $\nu_2 = 37.5$ dBm/MHz c) Fading Margin, $X_{\text{fad}} = 40$ dB.

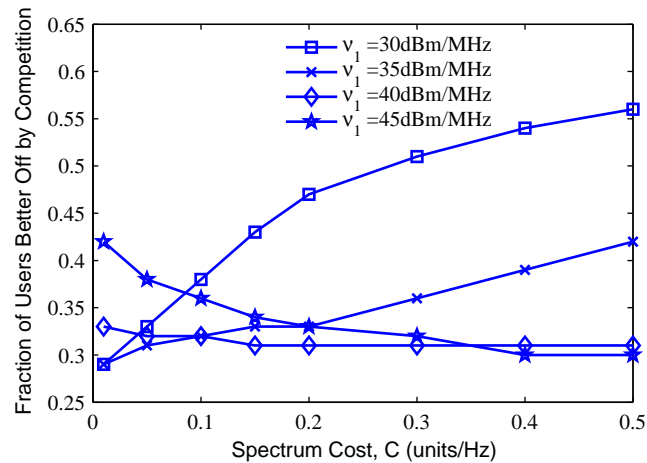


Figure 6.9: Fraction of total number of users whose surpluses are higher when there are two SPs competing for service as compared to the case when there is a single SP. Different values of ν_1 are considered. Other parameters are a) Target rate, $\Gamma = 1$ Mbps b) SP 2 psd, $\nu_2 = 37.5$ dBm/MHz c) Fading Margin, $X_{\text{fad}} = 40$ dB.

Chapter 7

Conclusion

Under current allocation principles, the broker or a Govt. agency allocates long term spectrum licenses to service providers who in turn allocate it to their customers. We envisage that, in the future, spectrum would be allocated on much shorter time scales, typically on the duration of a session and would be demand driven. The user who needs more spectrum and is willing to pay for it will receive a larger chunk. In this thesis we characterize two regimes of dynamic spectrum allocation: a centralized, property rights regime and a decentralized commons.

For the decentralized commons regime, we studied the timing acquisition performance of devices that engage in distributed dynamic spectrum access. Since the physical layer of most upcoming standards is based on OFDM, we studied a practically implementable, cyclic prefix correlation based algorithms for OFDM symbol timing acquisition. Since the algorithm is optimal only for a single user transmission in a frequency non-selective channel, we developed a concept of self and multiple access interference to derive important insights into the working of the algorithm for multi-user transmissions for general frequency selective channels where users might occupy discontinuous bands due to dynamic allocation. We found that for a two user system, when the differential delays of the two users are much larger than the cyclic prefix, the performance of the algorithm, even with filtering, deteriorates as the occupied bandwidth is split into several pieces, and in some realistic cases becomes quite poor. Assuming that the receiver of the desired user is aware of the bands in which the transmitted signal lies, we showed that it could filter its intended signal and partially restore the performance. We feel that this work provides a systematic study of non-contiguous OFDM timing acquisition and has important implications in the design of future dynamic spectrum access based systems. There is ample scope for future work, notably in determining acquisition algorithms when the user transmits in multiple narrow sub-bands and also in investigating practical methods by which the receiver may infer the spectral occupancy of its transmitter during the acquisition phase.

The major part of the thesis is focused on centralized, property rights based dynamic allocation of spectrum as we believe this to be predominant regime in the years to come. To understand the interplay of technology and economics that characterize this regime, we develop and analyze a two tiered dynamic spectrum allocation model consisting of a spectrum broker, service providers and end users. We believe that such a model is relevant for many upcoming wireless systems.

User utility maximization was the first operating principle that we considered. For this we show that in the optimal policy each user obtains spectrum only from one service provider given by a function of the link gains and provider efficiency. Based on our analysis we develop the notion of a spectrum price to facilitate distributed allocation. For two general classes of concave utility functions namely exponential and α , we analytically characterize the spectrum allocation and price. We show that our results are consistent with basic economics principles. Our work provides theoretical bounds on performance limits of practical operator to user based dynamic spectrum allocation systems and also gives insights to actual system design.

Service provider profit maximization was the next operating principle considered in this thesis where a network of multiple service providers compete with each other in securing the service of users in the downlink. Each provider sets a spectrum price and the users decide which provider to obtain service from based on their price and the link gains. We propose a dynamic allocation scheme based on SP profit maximization. We developed the concept of an effective spectrum price and showed how it lead to equilibrium solutions of the SP price competition game. We showed that in the optimal allocation each user obtained service from only one SP, each provider has its own customer base and it can price them assuming no competition. The loss in profit due to competition may drive SPs to operate in markets where they are the sole provider. We analyzed this situation through a monopoly framework and showed the difference in the optimal pricing structure. Further insights were obtained from the numerical results, which showed that a SP should invest in a better modulation and coding technology only if the cost of spectrum is high. If the broker charges minimal cost for spectrum, an SP is better off in buying abundant spectrum and transmitting at a reduced spectral efficiency.

We established the relationship between the spectrum prices that the broker charges to the SPs and what the SPs charge to their customers. We showed this depends on the user demand functions. From a network operator standpoint, if the application requirements of the customers are known, our work gives useful insights about setting the optimal price values.

There are interesting questions that arise from our work. From equation (6.8), we show that a design principle for the broker could be to choose the value of spectrum price C close to the spectral efficiency values of the SPs. A second interpretation is that raising the broker cost forces the operators to operate at higher spectral efficiency. It is not clear if this is good for the users since higher costs can preclude some types of applications and services. If we view the broker as the FCC and the users as taxpayers, the key question is to what extent does lowering the broker cost increase the profitability of the SPs vs providing lower costs to the users.

The issue of timescale has not been dealt rigorously in this thesis. We have asserted that our allocations are valid for duration of a communication session in which the system parameters such as number of users and SPs and link gains stay constant. A session could correspond to the time taken to download a file and could be of the order of seconds or minutes. We have not analyzed the dynamics of price change and the convergence of update algorithms such as (3.11) in this thesis, choosing instead to focus on given snapshots of the system. The dynamics of distributed update algorithm is an important issue but we take encouragement from the fact that many update algorithms designed for system snapshots seem to work in practical dynamic environments. An example is the convergence of power control in cellular CDMA [89, 90].

We believe that our work is the first attempt to jointly understand the broker to SP and SP to user spectrum allocation from a joint SP cost and user utility framework. Though our model is simple, it reveals interesting trends about the interactions between spectrum allocation and pricing to end users, choice of technology by a SP and spectrum costs charged by a broker. There is ample scope for future work. In particular, the linear model for spectrum and power costs can be replaced by more realistic models that are closer to practical systems. Also consideration of two dimensional and random network topologies will yield a more complete understanding of the properties of spectrum allocation and pricing than the linear arrangement of users and SPs considered in this chapter.

Appendix A

Properties of the Utility Function

In this section we establish various properties of the utility function considered in this paper.

Lemma 13. *If $U(R)$ is an increasing and concave function in R then $U(\eta x \log(1 + hP/x))$ is an increasing concave function of the vector $\mathbf{v} = [x, P]$*

Proof: Lets first prove that $R(x, P) = \eta x \log(1 + hP/x)$ is a concave function of the \mathbf{v} . The direct way is to evaluate the Hessian and prove that it is negative definite. A shorter proof is given in [72] where the authors show that it is sufficient to test for concavity along any line which lies in the domain of the function, i.e a line of the form $P = ax + b$, thus restricting our attention to a single variable x . Here we present an even shorter proof which uses the idea perspective of a function [70]. We state the following lemma and refer the interested reader to Section 3.2.6 of [70] for the proof.

Lemma 14. *If $f : \mathbf{R}^n \rightarrow \mathbf{R}$, then the perspective of f is the function $g : \mathbf{R}^{n+1} \rightarrow \mathbf{R}$ defined by,*

$$g(\mathbf{x}, t) = tf(\mathbf{x}/t),$$

with domain $\mathbf{dom} g = \{(\mathbf{x}, t) | \mathbf{x}/t \in \mathbf{dom} f, t > 0\}$. The perspective function preserves convexity. If $f(\mathbf{x})$ is concave in \mathbf{R}^n then $g(\mathbf{x}, t)$ is concave in \mathbf{R}^{n+1} .

Now consider the function $\tilde{R}(P) = \eta \log(1 + hP)$ which is concave in P . We see that the function $R(x, P) = \eta x \log(1 + hP/x)$ is the perspective of $\tilde{R}(P)$ w.r.t. variable x and is thus concave in $\mathbf{v} = [x, P]$.

Thus $R(x, P)$ is a concave function of \mathbf{v} and $U(\cdot)$ is an increasing concave function. Thus from [70], $U(R(x, P))$ is an increasing concave function of \mathbf{v} .

Lemma 15. *If $U_j(R_j)$ is an increasing concave function in R_j then the following is an increasing concave function of the vector $\mathbf{v}^{(\mathbf{N})} = [\mathbf{v}_1, \dots, \mathbf{v}_N]$ where $\mathbf{v}_i = [x_{ij}, p_{ij}]$,*

$$U_j \left(\sum_{i=1}^N \eta_i x_{ij} \log(1 + h_{ij} p_{ij} / x_{ij}) \right)$$

Proof. Consider the function $R_j(\mathbf{v}^{(\mathbf{N})}) = \sum_{i=1}^N r_{ij}(\mathbf{v}_i)$ where each $r_{ij}(\mathbf{v}_i) = \eta_i x_{ij} \log(1 + h_{ij} p_{ij} / x_{ij})$ depends only on x_{ij} and p_{ij} it can be shown that the Hessian of R_j has a block diagonal structure. Hence $\nabla^2 R_j = \text{diag}[D_1, \dots, D_N]$ where from Lemma 13, each 2×2 matrix D_i is negative definite. Now consider any vector $\mathbf{z} \in \mathbf{R}^{2N}$. From the block diagonal structure of $\nabla^2 R_j$ it can be shown that $\mathbf{z}^T (\nabla^2 R_j) \mathbf{z} = \sum_{i=1}^N \mathbf{z}_i^T D_i \mathbf{z}_i$ where $\mathbf{z}_i = [z_{2i-1}, z_{2i}]$. Since each D_i is negative definite, the sum is also negative for any vector \mathbf{z} . \square

Appendix B

Monopoly Pricing under Downlink Transmit Power Constraint

B.1 System Model

We consider that the SP has a total power constraint P and allocates power P_j to the transmission of user j . The SP is thus performing a joint power and spectrum allocation. The spectral efficiency obtained by user j is now given by, $\nu(x_j) = \log(1 + h_j P_j / x_j)$, where we have assumed unit power spectral density of background AWGN. We assume that h_j is flat over frequencies and thus no matter what bands x_j lies, h_j is same. We consider linear user utilities, i.e. $U_j(R_j) = R_j$. The SP cost function is assumed to be $C(X) = CX$.

We consider two different types power allocations,

1. The SP serves all the L users with equal power allocation. Thus $x_j \neq 0$ and $P_j = P/L$ for all j .
2. The SP optimizes P_j to maximize his profit. Thus some users not accept service i.e. $x_j = 0, P_j = 0$ because of prices (μ, κ) . The loss of revenue from these users can be made up by increased revenue from other users.

Usually the demand function of a user does not depend on number of users L . However in our case the demand function of spectrum as per Chapter 5, equation (5.3) is,

$$\mu = \eta \log \left(1 + \frac{h_j P_j}{x_j} \right) - \frac{\eta h_j P_j}{x_j + h_j P_j}, \quad (\text{B.1})$$

which depends on P_j . Since there is a total transmit power constraint P , the demand functions of the users are in fact dependent on each other.

SP serves all users

From (5.8) the elasticity of demand, ϵ equation is given by

$$C = \mu \left(1 - \frac{1 - Ls}{\epsilon} \right). \quad (\text{B.2})$$

where $s = x_m/X$ and ϵ is the elasticity of demand.

Lemma 16. *The function*

$$\mu = f(z) = \log\left(1 + \frac{1}{z}\right) - \frac{1}{1+z} \quad (\text{B.3})$$

is decreasing and convex for positive z .

Proof: For $z > 0$, $f'(z) < 0$ and $f''(z) > 0$.

Lemma 17. *The user with the weakest channel to the SP is the marginal user.*

Proof. The demand function of user j is obtained by substituting $P_j = P/L$ in (B.1),

$$\mu = \log\left(1 + \frac{h_j P}{Lx_j}\right) - \frac{h_j P}{Lx_j + h_j P}. \quad (\text{B.4})$$

Assume WLOG that $h_1 > h_2 > \dots > h_L$. Consider users L and k , where $k < L$. Fix $x_L = x_k = x$. In terms of $z_L = Lx/h_L P$ and $z_k = Lx/h_k P$, (B.4) reduces to (B.3). Now $h_k > h_L \Rightarrow z_k < z_L$ and hence by Lemma 16, $f(z_k) > f(z_L)$. This is illustrated in Figure B.1 which shows that the graph for user k lies to the right of the graph for user L for all k .

Let us recollect equation (5.6) from Chapter 5 which stated that $\partial\kappa/\partial\mu = -x_m$. This implies that

$$\kappa = \int_{\mu}^{\infty} x_m(\mu) d\mu. \quad (\text{B.5})$$

Let the optimal price be μ^* . The surplus of user k is the area of the region DAC while that of user L is the area of region DBC as these are the values of the integral in (B.5) evaluated for x_k and x_L . Since area of DAC $>$ area of DBC user L has the least surplus amongst all users and is thus the marginal user. \square

Recall that the same result was observed in Chapter 5, Lemma 3 and 7.

Lemma 18. *At the optimal solution spectrum price μ is more than the marginal cost of production $C'(X)$.*

Proof: Let the solution of (B.3) be $z = f^{-1}(\mu)$. Since $f(z)$ is decreasing and convex, it is one-to-one and thus $f^{-1}(\mu)$ is also one-to-one. Let the optimal price be $\mu = \mu^*$. Since $z_k = Lx_k/h_k P$, we obtain,

$$x_k = \frac{h_k P}{L} f^{-1}(\mu^*). \quad (\text{B.6})$$

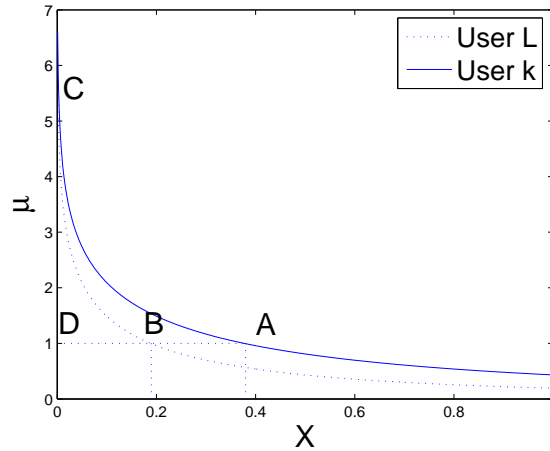


Figure B.1: The demand function for two users

Thus in (B.2) we can evaluate,

$$Ls = \frac{Lx_L}{X} = \frac{Lh_L(P/L)f^{-1}(\mu^*)}{\sum_{k=1}^L h_k(P/L)f^{-1}(\mu^*)} = \frac{Lh_L}{\sum_{k=1}^L h_k} < 1, \quad (\text{B.7})$$

as $h_L = \min(h_1, h_2, \dots, h_L)$. Substituting for this in (B.2), we obtain the desired result. ■

The relationship $\mu > C'(X)$ means that the spectrum cost charged by spectrum regulatory body of a country like FCC to the SP is increased and passed on directly to the customers by the SP.

Substituting for x_k from (B.6) in (B.2) and after some algebraic manipulations we obtain,

$$C = \mu + \frac{1 - Lh_L / \sum_{k=1}^L h_k}{(1 + f^{-1}(\mu))^2}. \quad (\text{B.8})$$

Thus given a set of L users (B.8) can be solved and the optimal value of μ determined. In practice we can assume that before the actual spectrum allocation takes place, each user sends beacon packets to the SP and thus the SP is aware of number of users L and the channel coefficients $h_1 \dots h_L$. The SP can then solve (B.8).

From (B.5) and (B.6) we can show that

$$\kappa^* = \frac{h_L P}{L} \int_{\mu^*}^{\infty} f^{-1}(\mu) d\mu. \quad (\text{B.9})$$

Observe from (B.8) and (B.9) that for $C(X) = CX$, μ^* does not depend on transmit power P while κ^* does. In fact (B.9) shows that κ^* increases linearly with P . Thus it is profitable for the SP to increase his transmit power. Of course we haven't factored in the cost incurred by the

SP for increasing his transmit power (for e.g. batteries draining out faster) which might have given the optimal value of P .

SP optimizes the transmit powers

In this section we return to the general demand function as given in (B.1) and ask the following question: *if there are L users and the SP has a total transmit power constraint how should he optimally allocate power to their transmissions so as to maximize his profits?* We mention at this juncture that power allocation has traditionally been used for objectives such as user sum capacity maximization but to the best of our knowledge has never been explored for the SP profit maximization.

First assume that in the optimal solution the SP serves only the first N out of L users. Then the user N is the marginal user. Then (B.6) is modified to $x_k = h_k P_k f^{-1}(\mu^*)$ and (B.9) is modified to $\kappa^* = h_N P_N \int_{\mu^*}^{\infty} f^{-1}(\mu) d\mu$. Recall that SP profit $\pi(N) = (\mu^* - C)X + N\kappa^*$. Hence the power allocation problem for profit maximization can be expressed as,

$$\begin{aligned} \max_N \max_{P_1, \dots, P_N} & (\mu^* - C) f^{-1}(\mu^*) \sum_{k=1}^N h_k P_k + N\kappa^* \\ \text{s.t. } C &= \mu^* + \frac{1}{(1 + f^{-1}(\mu^*))^2} \left(1 - \frac{N h_N P_N}{\sum_{k=1}^N h_k P_k} \right) \\ \kappa^* &= h_N P_N \int_{\mu^*}^{\infty} f^{-1}(\mu) d\mu \\ P_1 + \dots + P_N &= P \\ P_1, \dots, P_N &> 0. \end{aligned}$$

This is a complicated non-convex problem. However we can arrive at the optimal solution indirectly. Let us first consider 2 users with $h_1 > h_2$ and derive the optimal power allocation. The result can be generalized to the case when more users are present. From definition of $\rho(x_j)$, the SP revenue from user j (see (5.1)) we can write the SP objective function is,

$$\pi(2) = \rho(x_1) + \rho(x_2) - C(x_1 + x_2) \tag{B.10}$$

Recall from the definition of the marginal user in Section 5.2.1, the entire surplus of the second user is extracted by the SP. Thus utility $V_2(x_2) = \rho(x_2)$. This is true even if no spectrum is allocated to user 2 as then $V_2(x_2) = 0$ and $\rho(x_2) = 0$ by definition. However the first user still has some surplus left and hence $V_1(x_1) > \rho(x_1)$. Thus for all x_1, x_2

$$\pi(2) < V_1(x_1) + V_2(x_2) - C(x_1 + x_2). \quad (\text{B.11})$$

Thus,

$$\max_{\substack{x_1 \geq 0, x_2 \geq 0 \\ P_1 \geq 0, P_2 \geq 0 \\ P_1 + P_2 = P}} \pi(2) < \max_{\substack{x_1 \geq 0, x_2 \geq 0 \\ P_1 \geq 0, P_2 \geq 0 \\ P_1 + P_2 = P}} \sum_{k=1}^2 V_k(x_k) - Cx_k. \quad (\text{B.12})$$

But the optimization problem in the RHS of (B.12) is similar to the sum utility maximization problem considered in Chapter 3 with the shadow price being replaced by the spectrum cost C . It has been shown in Chapter 3, Corollary 1 that the solution of the optimization problem is achieved by allocating all power to the user with the best channel. Since $h_1 > h_2$, the optimal power vector is $[P, 0]$. But for this power vector both the LHS and RHS optimization problems in (B.12) become the same problem. This is a optimizing x_1 for $P_1 = P$. Thus for $[P, 0]$ the optimization of $\pi(2)$ touches the maximum value of its upper bound and hence this is the optimal power allocation strategy.

Thus to maximize profits the SP maximizes the sum utility of the system as he can then extract dollar revenues proportional to the sum utility.

B.1.1 Numerical Results

We now numerically evaluate how the SP profit varies with the number of users served under a uniform power allocation policy. Let the SP serve N users with the largest value of channel gains. The users are indexed from 1 to N and each transmission takes place at power P/N . Cases considered in Sections B.1 and B.1 correspond to $N = L$ and $N = 1$ respectively.

We assume $L = 15$ users are distributed in a cellular area of 1 km. The channel coefficients originate from a distance based path loss model. We also assume that due to shadow fading the received SNR is reduced by a constant fading margin. As mentioned earlier the the channel coefficients are flat over frequencies and depend only upon the user locations. The values of μ and κ are calculated from (B.8) and (B.9) respectively. The simulation parameters are explained in Table B.1. The SP profit depends on how the function $g(N) = Nh_N / \sum_{k=1}^N h_k$, varies with N . Now $g(1) = 1$ and for $N_1, N_2 > 1$, $g(N_1), g(N_2) < 1$, but their relative order depends

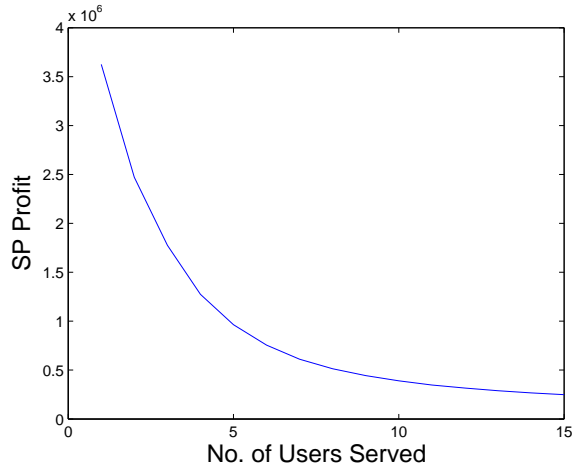


Figure B.2: The SP profit as the number of served users varies

| | |
|--------------------|--------|
| No. of Users, L | 15 |
| Path Loss Coeff | 3.7 |
| Reference Distance | 100m |
| Cell Radius | 1km |
| Transmit Power | 5 Watt |
| Efficiency, η | 0.08 |
| Fading Margin | 44 dB |

Table B.1: Parameters for Spectrum Allocation

on the values of the coefficients h_k . Thus for each $N > 1$, we generate 10000 instances of channel vector $[h_1, \dots, h_N]$ and calculate the average value of SP profit. The result is shown in Figure B.2. We see that the SP profit decreases as the number of users increase. It is most profitable for the SP to serve only one user as was proved in Section B.1.

Figure B.3 plots the breakup of the SP revenue from subscription cost κ and usage cost μx for the marginal user and the user with the best channel to the SP (referred to as the best user), for $C = 0.5$. The majority of the revenue comes from the usage cost of the best user. Least revenue comes from the subscription cost. The demand function graphs as given in Figure B.1 give us the intuition that for lower values of C , the spectrum purchased and the surplus κ is more. This is observed in Figure B.4 where the value of C is lowered to 0.05. Lastly Figure B.5 shows that the SP profit reduces exponentially with production cost C .

Note that the absolute values of the various parameters shown in the figures should not

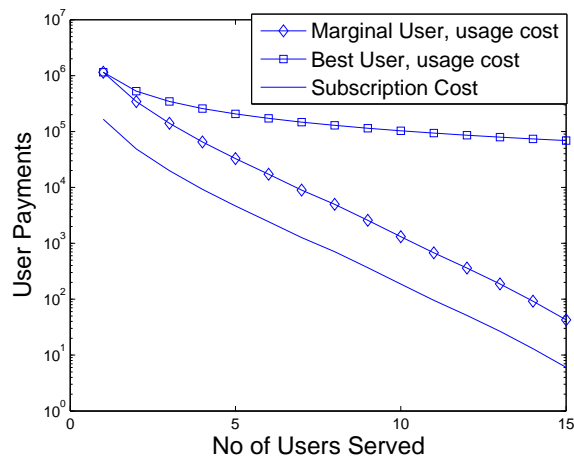


Figure B.3: The SP revenue from different users for $C = 0.5$

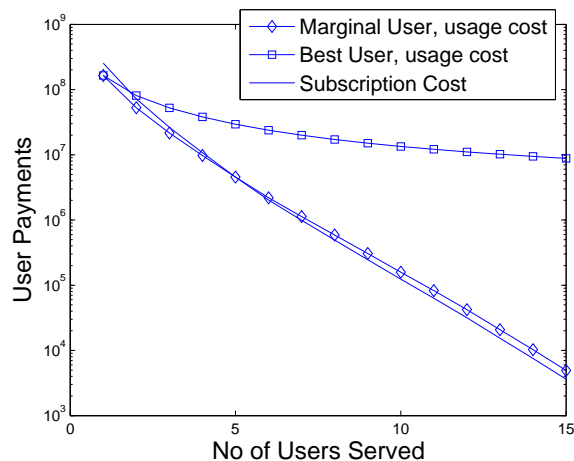


Figure B.4: The SP revenue from different users for $C = 0.05$

be interpreted literally. For e.g. we have $C = 0.5, 0.05$ but profit values which are bigger by several orders of magnitude. This is because for simulation purposes, the various systems equations like $\mu = V'(x) = C'(x)$ haven't been normalized. One *dollar*, the unit of $C'(x)$ is not equivalent to one *bps*, the unit of $V'(x)$. The results in this paper are true within bounds of proper scaling.

In passing we note that the role of the marginal user in profit maximizing pricing strategies have also been studied in [91] for a communications system with only fixed subscription costs κ and where the SP allocates power to a group of downlink nodes. The results are slightly

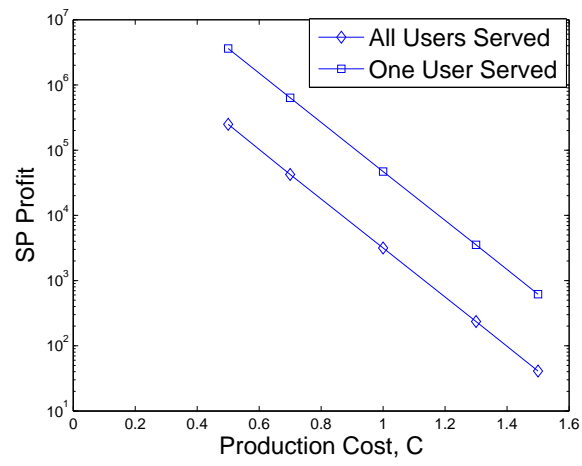


Figure B.5: The SP profit as spectrum production cost C varies

different due to the non inclusion of usage based cost in the problem formulation.

Appendix C

Miscellaneous Proofs of Chapter 3

C.1 Proof of Theorem (3)

Define $\nabla U_j(x_j^*) = \partial U_j / \partial x_j^* = \dot{U}_j(R_j) \Gamma_x(x_j^*, P, h)$ and $M = \max_j \nabla U_j(x_j^* - \epsilon)$ for some $\epsilon > 0$. From Lemma 1, $\nabla U_j(x_j^*)$ is decreasing. The decrease in sum utility is

$$\Delta U_{dec} = \sum_{j=1}^L \int_{x_j^* - \epsilon}^{x_j^*} \nabla U_j(x) dx \leq \sum_{j=1}^L \nabla U_j(x_j^* - \epsilon) \epsilon < ML\epsilon. \quad (\text{C.1})$$

However the utility of user $L + 1$ is

$$\Delta U_{inc} = \int_0^{L\epsilon} \nabla U_{L+1}(x) dx \geq \nabla U_{L+1}(L\epsilon) L\epsilon. \quad (\text{C.2})$$

From (C.1) and (C.2), we have to show existence of $\epsilon > 0$ such that $\nabla U_{L+1}(L\epsilon) > M$. Now M is increasing in ϵ while $\nabla U_{L+1}(L\epsilon)$ is decreasing in ϵ . As $\epsilon \rightarrow 0$, $\nabla U_{L+1}(L\epsilon) \rightarrow \infty$ due to Γ_x while $M \rightarrow \max_j \nabla U_j(x_j^*)$. Thus at $\epsilon = 0$, the decreasing function is above the increasing function and so they are sure to intersect at some $x = x_s$. So for $0 < \epsilon < x_s$ sum utility increases by allocating spectrum to user $L + 1$.

C.2 Proof of Lemma 2

We have to show that $\dot{U}(R(x, p, h)) \Gamma_x(x, p, h)$ is a strictly increasing function of p for fixed x when $U(R) = R^\alpha / \alpha$ for $0 \leq \alpha \leq 1$. Alternatively substituting $z = hp/x$ we have to show that the following is strictly increasing in z ,

$$\begin{aligned} & (x \log(1+z))^{\alpha-1} \left[\log(1+z) - \frac{z}{1+z} \right] \\ &= x^{\alpha-1} (\log(1+z))^\alpha \left[1 - \frac{z/(1+z)}{\log(1+z)} \right]. \end{aligned} \quad (\text{C.3})$$

Since $(\log(1+z))^\alpha$ is strictly increasing in z , a sufficient condition is to show that $f(z) = (1+z) \log(1+z)/z$ is strictly increasing in z , which is proved by evaluating $\dot{f}(z)$ and using the fact that $z - \log(1+z) > 0$ for all $z > 0$.

C.3 User Surplus in Corollary 2

We have to show that $\mathcal{U} = U(R(x, p, h)) - \mu x$ for $\mu = \dot{U}(R(x, p, h))\Gamma_x(x, p, h)$ is increasing in p . A sufficient condition is to show that $\mathcal{U}(x, p, h)$ is increasing in both x and p for fixed h , since Theorem 5 proved that increasing p increases x . Define $R_{xx}(x, p, h) = \partial\Gamma_x/\partial x$. We can show

$$\frac{\partial\mathcal{U}}{\partial x} = -x \left[\dot{U}(R)R_{xx}(x, p, h) + \ddot{U}_j(R)\Gamma_x^2(x, p, h) \right]. \quad (\text{C.4})$$

Since $U(R)$ is increasing and concave, $\dot{U}(R) > 0$ and $\ddot{U}(R) < 0$. Since $R(x, p, h)$ is concave in x , $R_{xx}(x, p, h) < 0$. Using all these we can show that $\partial\mathcal{U}/\partial x > 0$. Differentiating

$$\frac{\partial\mathcal{U}}{\partial p} = \dot{U}(R) \left[\Gamma_p - x \frac{\partial\Gamma_x}{\partial p} \right] - x\ddot{U}(R)\Gamma_p\Gamma_x.$$

It can be shown that,

$$\Gamma_p - x \frac{\partial\Gamma_x}{\partial p} = \frac{hx^2}{(x + hP)^2} > 0. \quad (\text{C.5})$$

With this information we can also show that $\partial\mathcal{U}/\partial p$ is positive.

Appendix D

Miscellaneous Proofs of Chapter 5

D.1 Proof of Lemma 6

a) From (5.6) we have $\partial\Pi_C^*/\partial\mu^* = -Lx_m^* < 0$.

b) Taking derivatives

$$\frac{\partial\Pi_U^*}{\partial\mu^*} = X^* + (\mu - C_e)\frac{\partial X^*}{\partial\mu^*} \stackrel{(a)}{=} X^* \left[1 - \epsilon^* \left(\frac{\mu^* - C_e}{\mu^*} \right) \right], \quad (\text{D.1})$$

where equality (a) follows from (5.7). From (D.1), it can be shown $\partial\Pi_U^*/\partial\mu^* > 0$ holds when

$$\epsilon^* \left(\frac{\mu^* - C_e}{\mu^*} \right) < 1 \quad (\text{D.2})$$

Substitute for ϵ^* from (5.14) in (D.2) and after some manipulations we obtain $\mu^* < \sqrt{C_e L / K_s}$.

D.2 Proof of Lemma 10

a) Proof is same as proof of Lemma 6a)

b) Similar to the proof of Lemma 6b), we have to show that the elasticity satisfies condition (D.2). Now $(\mu^* - C_e)/\mu^* < 1$. And it was proved in Lemma 9 that for $\mu^* < \mu_0$, elasticity, $\epsilon^* < 1$.

Appendix E

Miscellaneous Proofs of Chapter 6

E.1 Proof of Theorem 10

We have to show that if users positions satisfy (6.13) and user j obtains service from SP 1, so does user $j - 1$. Since user j is attached to SP 1, the user optimization (6.12) yields, $\max_x U_j(K_{1j}x) - \mu_1x > \max_x U_j(K_{2j}x) - \mu_2x$. Now since the user locations satisfy (6.13)

$$U_{j-1}(K_{1j-1}x) > U_j(K_{1j}x) \text{ for all } x \quad (\text{E.1})$$

$$U_{j-1}(K_{2j-1}x) < U_j(K_{2j}x) \text{ for all } x \quad (\text{E.2})$$

From (E.1), we obtain $U_{j-1}(K_{1j-1}x) - \mu_1x > U_j(K_{1j}x) - \mu_1x$ for all x . Let $x^* = \arg \max_x U_j(K_{1j}x) - \mu_1x$. Thus $U_{j-1}(K_{1j-1}x^*) - \mu_1x^* > \max_x U_j(K_{1j}x) - \mu_1x$. But $\max_x U_{j-1}(K_{1j-1}x) - \mu_1x > U_{j-1}(K_{1j-1}x^*) - \mu_1x^* > \max_x U_j(K_{1j}x) - \mu_1x$. Similarly we can prove that $\max_x U_{j-1}(K_{2j-1}x) - \mu_2x < \max_x U_j(K_{2j}x) - \mu_2x$. Combining we get the desired result.

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