TEACHER QUESTIONS THAT ENGAGE STUDENTS IN MATHEMATICAL

CONVERSATION

by

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ABSTRACT OF THE DISSERTATION

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Currently, mathematics educators argue that teachers should create classrooms where students are engaged in conversation about mathematical ideas. However, to achieve these goals, it is important that teachers understand how to engage students in discussion. I address this issue by describing questioning techniques that teachers can use to make students' reasoning public and encourage conversation. In this thesis, I examined two student-centered classrooms. The first was three sessions from a high school pre-calculus class, the second was three after school sessions from a longitudinal study in which students solved challenging open-ended mathematics problems (Maher, 2002). The common thread between both research environments was an emphasis on student conversation and thinking, which allowed for a rich data in order to answer my research questions.

The two main questions guiding my research are: What kinds of questions do these two mathematics teachers in student-centered settings ask; and to what extent and in what ways did these teachers' questions engage students in mathematical conversation? These research questions led me to identify teacher questions and student

responses, and examine how teachers used questioning to engage students in conversation.

In order to answer my first research question, I used inductive coding to describe teacher questions and student responses. To answer the second research question, I began with a quantitative approach to determine the frequencies of each question and response. Additionally, a frequency chart relating student responses that immediately followed teacher questions allowed insight into how teachers elicited student reasoning in conversation. For a descriptive account of how these teachers engaged students in mathematical conversation, I used inductive coding to examine patterns in teacher questioning. This coding process resulted in questioning themes that describe how the teachers used questioning to elicit reasoning and promote conversation in their classroom.

Dedication

- To my committee:

  – Carolyn for bringing me in the program when it started and always providing the support necessary to stay in the program,

  – Lara for the fun we had teaching when I started this process and helping me get through the qualifying exams,

  – Keith for reading all of my drafts, providing valuable feedback and helping me complete my work,

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- To Kristine, whose unrelenting love and support kept me moving forward through all the challenges of life.

- To my son, whose imminent arrival expedited the completion of my work.

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Introduction

Currently, influential theories in mathematics education, including constructivism, semiotics, and social learning theories, support the view that active participation in mathematical discussion is an important part of students' learning process (e.g., Ernest, 1996; Voigt, 1996; Pirie & Kieren, 1994; Davis & Maher, 1997; Watson & Chick, 2001; Doerr & Tripp, 1999). As a result, many influential mathematics educators call for classrooms to be student-centered, where the ideas in the classroom are developed by the students rather than handed down from the teacher. A student-centered environment can occur when the teacher refrains from lecturing on a consistent basis to allow more time for student discovery, classroom discussions, problem solving activities and group work.

One topic that is highlighted within student-centered classrooms is the role of discussion. Discussion, sometimes also referred to also as discourse, highlights the ways in which knowledge is constructed and exchanged in a classroom (Ball & Friel, 1991). Discussion allows students to share, create and justify meanings for their ideas and discoveries as well as build their understanding of mathematical concepts. Listening to a discussion may provide both students and teachers with insights into a student's reasoning, learning and problem solving, which makes it easier to communicate mathematically with this student. Additionally, conversations in which mathematical ideas are explored from multiple perspectives can help the participants make connections and develop different ways of representing the same mathematical idea (NCTM, 2000).

Research Supporting Student-centered Classrooms

In recent years, several mathematics education organizations have argued that the mathematics classroom should be a student-centered environment filled with discussion

about mathematics, arguments about solutions, and peer consensus about answers (e.g.

National Research Council, 1989; NCTM, 2000). Additionally, education groups

advocate engagement in higher-level problem-solving activities that involve discussion

and community consent for solutions to these problems (e.g., NRC, 1989; NCTM, 2000).

In student-centered classrooms, communication is essential in order for ideas to become

objects of reflection, discussion, and refinement as part of the process of organizing,

consolidating and giving meaning to these ideas (NCTM, 2000).

Many researchers endorse the ideas put forth by NCTM (2000) that the classroom

should be an environment that encourages problem-solving and problem-posing,

expression of the students' ideas, presentation of convincing arguments, and where

developing an approach to thinking about mathematics is valued over rote memorization

(Hiebert & Wearne, 1993; Maher & Alston, 1990; Pimm, 1987; Shifter, 1996).

Constructivist and social learning theories address how learners construct mathematical

knowledge. Interpretations of both theories emphasize that there is a social aspect of

children's learning and suggest that teachers be more attentive to student thinking by

emphasizing pedagogy that promotes active learning by students (e.g. Ernest, 1996;

Voigt, 1996). Although the importance of communication in the classroom is supported

by researchers, how teachers can elicit productive conversation is less clear. It would be

valuable to understand more about some of the difficulties students and teachers

encounter in student-centered classrooms.

Challenges of the Student-centered Classroom

Implementing a student-centered classroom centered on discussion is not an easy

task. In these settings, the teachers and students need to work together and negotiate

well-defined roles so that student work drives the conversation between the students and the teacher (cf., Yackel & Cobb, 1996). The ability to work together can be halted by difficulties of either the students or the teacher. For productive mathematical discussions to occur, the teacher needs to create a supportive environment where students feel free to share their mathematical ideas, provide students with interesting tasks that provoke discussion, be willing to devolve some control over the mathematical direction of the class to the students, and ask probing questions that elicit students' reasoning. Students, in turn, need to accept the responsibility of describing their work and justifying their claims, as well as carefully attending to and building upon the explanations of others. Students also need to develop the abilities to engage in sophisticated mathematical reasoning and to articulate this reasoning to others. The skills, actions, and dispositions required of teachers to create environments that support useful discussion are clearly not trivial, but they are also not delineated and under researched. The central goal of this thesis is to partially address this void by focusing on one important issue: What types of teacher questions are useful for encouraging discussion, in particular by eliciting students' reasoning?

Research Questions

The purpose of this study is to support the changing nature of the mathematics classroom and inform mathematics education practice by classifying and describing teacher questions that engage students in mathematical conversation within a student-centered classroom. The goal is not to create prescribed questions for selection by the teacher. Rather, the goal is create a classification of types of questions teachers can use to engage students in mathematical conversation. Then if teachers wish to engage

students in mathematical conversation, they can use these types of questions to help students become part of the conversation. By creating a classification, teachers can apply ideas about questioning to any mathematical content area and student-teacher interaction.

To investigate this issue, I examine two individual teachers in two specific student-centered environments. The first was a high school pre-calculus classroom where students were studying the area under a curve. The teacher in this setting was a 30 year veteran of the school system and held a doctorate in education. The second setting was a research environment, which is part of a larger longitudinal study conducted at Rutgers University, where students were studying the growth of a Placentaceras shell. The teacher/researcher, who was selected for the detailed analysis, in this setting was an experienced professor of mathematics and mathematics education at the university level. The common thread between both research environments was an emphasis on student conversation and thinking, which allowed for a rich data in order to answer my research questions.

The two main questions guiding my research are:

1. What kinds of questions do these two mathematics teachers in student-centered settings ask?

2. To what extent and in what ways did these teachers' questions engage students in mathematical conversation? Specifically, how did these teachers elicit students' mathematical reasoning?

These results can inform the more general issue of what types of questions can be used in a student-centered mathematics classroom to engage students in mathematical conversation and allow teachers' insight into how students are thinking and reasoning.

Literature Review

This study focuses on how teachers question students in mathematics educational settings. Historically, most of the studies published on teacher questioning focus on the actions of the teacher (Cotton, 1989; Clegg, 1987; Cunningham, 1987), but not the ways in which students respond to the teacher's questions.

In this thesis, I will focus on teacher actions. However, the analysis will emphasize the interactions between the teacher and the students. More specifically, I will examine the kinds of student responses and behaviors that are elicited by the teachers' questions. This topic has been the subject of research in non-mathematical educational settings (Wigle, 1999; Mewborn & Huberty, 1999). Although there has been a great deal of mathematics education research on teacher questioning of students (Maher & Davis, 1990; Maher, Davis & Alston, 1992; Maher & Martino, 1992; Martino & Maher, 1999; Martino & Maher, 1994; Vacc, 1993) there has not been a specific focus on questions that engage students in mathematical conversation.

Since this study examines teacher questions with respect to mathematical conversation, literature regarding discourse, conversation, and communication informs this study. Research in these areas has focused on the role of discourse in the classroom (Blanton & Stylianou, 2003; Cobb, Boufi, McClain & Whitenack, 1997; Manouchehri & Enderson, 1999; O'Connor & Michaels, 1993) and how questioning is one component for creating discussion in the classroom (Hufferd-Ackles, Fuson & Sherin, 2004; McCrone, 2005). However, while these studies promote questioning as important to conversation in the classroom, their results do not identify what types of teacher questions engage students in conversation.

Teacher Questioning and Questions

This literature review is divided into four main sections: the need for teacher questions in a student-centered environment, categories of questions that teachers ask, student thinking, and discourse and community. Before moving into these main sections, I briefly describe how teacher questions have been studied as part of effective teaching. Glenn (2001) defines effective teaching as "qualities that benefit students, improve instruction, and help an organization run more smoothly" (p. 19). Researchers describe many qualities of an effective teacher and include the questions a teacher asks as one component of effective teaching. When examining the cognitive level of questions, effective teachers tend to ask more "process" questions – that is, questions asking for explanations. Still the majority of questions asked by teachers were product questions, asking for a single response (Reynolds & Muijs, 1999). These studies suggest asking good questions is part of effective teaching as a way to keep students involved in the lesson and allow teachers to monitor students' understanding (Reynolds & Muijs, 1999). While determining teacher effectiveness is not the main goal of this study, teacher questions that engage students in mathematical conversation may be part of the effective teaching equation.

Need for Teacher Questions in a Student-Centered Environment. One difficulty that students often have is a limited ability to talk about mathematics. Kitchen (2004) conducted a study in a high-poverty, rural school examining the discourse in a mathematics classroom that illustrate potential obstacles to productive discussions. He found that students who have greater mathematical knowledge dominate a discussion,

students may not engage in higher-order thinking, and students may resist sharing mathematical ideas (Kitchen, 2004).

Students may also have a problem with contributing ideas to a classroom discussion because they have never experienced a classroom where discussion is valued. Instead, students may feel that the purpose of discussion is to give the teacher the opportunity to assess their mathematical knowledge. As a result, students may be reluctant to participate complex reasoning to the discussion, but would rather limit their contributions to more basic statements that they are certain are correct (Lubienski, 2000). Therefore, an important first step toward implementing a discussion filled classroom is for teachers and students to work together to establish a risk-free environment, where students feel comfortable to express their ideas (Mewborn & Huberty, 1999; NCTM, 2000). For discourse to be a meaningful part of the classroom, the rules of speaking, whether explicit, negotiated, or tacit, need to allow students the opportunity to be legitimate speakers in the classroom.

Yackel and Cobb (1996) define sociomathematical norms as the acceptable and valued mathematical activity during classroom discourse. Since sociomathematical norms are generated and modified through ongoing student and teacher interactions, student explanation can be established as one possible acceptable mathematical activity. Therefore, negotiating productive norms about mathematical argumentation, including what constitutes acceptable explanations, is an important part of allowing student participation to be a legitimate aspect of classroom discussion. Yackel and Cobb argue the teacher is central in establishing the mathematical quality of the classroom and norms for student activities, which are related to goals and beliefs about mathematical activity.

Creating an environment for students to express their mathematical ideas is not only a first step, but several researchers state this is the biggest challenge in creating student-centered classrooms (Manouchehri & Enderson, 1999; McCrone, 2005). Unless students are in an environment where they feel safe to speak and express their ideas, and unless they have a teacher who can establish this environment, a student-centered classroom will be difficult, perhaps impossible, to achieve.

Teachers may have trouble implementing a student-centered classroom because fostering discourse is an area of difficulty for most teachers, especially when they have not seen the importance of a dynamic classroom discussion (Van Zoest & Enyart, 1998). There are several possible reasons for a teacher's difficulty in establishing a student-centered classroom. The first may arise from lack of preparation during pre-service training or continued teacher development as an in-service teacher. Without continued support to understand the instructional demands of the mathematics classroom, teachers will continue to perpetuate teaching in the way they were taught when they become the instructional leader in their own classroom. Another difficultly may stem from the result that in a classroom driven by student ideas, the teacher no longer has full control of the minute to minute happenings in the class. When a teacher allows students to investigate and discuss mathematics in small groups, it is advantageous for the teacher to understand how students' mathematical knowledge is developing; this can be done by carefully listening to the students' ideas. The challenge for the teacher is to determine how to have students explain their reasoning, and then use the students' ideas, discussions, and approaches to the mathematics to ensure students are developing a deep conceptual and

correct understanding of the material. This requires a teacher to have strong content and pedagogical knowledge covering many possible topics.

Still another challenge in a student-centered classroom is when introducing a new concept, the teacher must also be prepared to discuss several other related or supporting concepts based on student responses and actions in the classroom (e.g., Yackel, 2002). Teachers need to be flexible, responsive, and adapt quickly to student verbal explanations and the mathematics discussed in their classroom (Manouchehri, 2007; Himmelberger & Schwartz, 2007). As a result, teachers may not feel sufficiently confident in their own mathematical knowledge to feel comfortable engaging in the open-ended conversations that may ensue.

Another difficulty for teachers might be that a student-centered classroom presents the teacher with several roles and decisions to make based on student responses and ideas. Creating mathematical discussions is a complex process that goes beyond setting the classroom environment. McCrone (2005) found that rich discussions also involve a choice of tasks, the nature of questions and growing the communicative competence of the participants. Another challenge put forth by Manouchehri and Enderson (1999) is that teachers must encourage all students to participate and be involved in the conversation. Once the teacher establishes a classroom environment and discussion occurs, the teacher's job is not finished. During a conversation a teacher should be

> observing students, listening carefully to their ideas and explanations, having mathematical goals, and using the information to make instructional decisions. Teachers who employ such practices motivate students to engage in mathematical

thinking and reasoning and provide learning opportunities that challenge students

at all levels of understanding. (NCTM, 2000)

Also, when students discuss mathematics unexpected interpretations may occur with the

language used during a discussion. Teachers or students may be using mathematical

terms that are not understood by all of the participants or they may use terms from

everyday language to describe mathematical objects. Kotsopoulos (2007) describes these

issues as two types of interference between discussing mathematics and understanding

mathematics. The first is teacher-talk interference, which results from the use of too

many mathematical terms, and the second is student-talk interference, which results from

students using everyday language. Both of these types of interference create a barrier to

understanding for the students.

Teachers need direction in order to have discussion play a role in their classroom.

Lampert, Rittenhouse, and Crumbaugh (1996) point out, "NCTM sidesteps the question

of what exactly teachers need to teach and students need to learn for this kind of talk to

been seen as an appropriate mode of public interactions among school children and their

teacher" (p. 16). One of the biggest difficulty teachers may face is a lack of resources to

help them create discussion in the classroom. Two recent studies provide suggestions for

establishing discussion in the classroom as part of establishing a discourse rich

classroom. Staples and Colonis (2007) highlight aspects of sharing and collaboration

discussions. Sharing conversation consists of students' expression of ideas and the

teacher valuing those ideas. Collaboration discussion is characterized by students sharing

ideas and building upon classmate thinking in order to extend the line of thinking.

During sharing, the teacher uses a student idea for comments by other students, asks

students for alternative ideas, and connects ideas together for the students. In a collaboration discussion, the teacher builds upon students' idea, generates discussion about these ideas and brings out connections through more student input. Truxaw and DeFranco (2007) propose a mixture of two types of discussion in conjunction with an inductive model of teaching to promote understanding. The authors developed an inductive model of teaching from analyzing a selected conversation and determining that the teacher used questioning and feedback to move students through recursive, inductive cycles rather than in linear steps (p.271). The first type of discussion consists of teachers asking questions and providing feedback to convey information to students from the teacher's point of view. The second type of discussion is a give-and-take communication in which the students are actively involved in the construction of meaning. Truxaw and DeFranco illustrate both types of discussion, and how they can be used in tandem, with an example from an 8th grade algebra course. In this example, the teacher revisits students' initial thoughts about a problem they are solving so that the students can established a shared meaning of the problem. He then guides students from a specific case to a more generalized theory to advance students' understanding.

These current studies focus on the role of the teacher within the classroom discourse, but they do not provide specific teacher talk that may encourage a conversation. Hence, these studies are valuable in the sense that they provide characteristics of the discussions that should occur in student-centered classrooms; however, they do not provide teachers guidance with specific things they can do to create these discussions. This thesis will partially address this gap by describing types of questions that teachers can ask to encourage discussion.

The Role of Questioning in Mathematical Discussion

Due to the many difficulties in implementing discussion in a student-centered classroom, organizing and promoting a mathematics classroom where students solve problems, discuss ideas, and build their own knowledge places new demands on teachers and students. While a range of pedagogical responses are possible, one possible way for the teacher to facilitate discussion is by asking questions. In student-centered classrooms, teachers are expected to build upon students' comments and their ideas. Hence, they will often have to improvise, and it may not be feasible for teachers to use prepared questions or highly prescribed strategies. If teachers are to take students' contributions seriously, their questions will sometimes need to be based on things students previously said. Questions following students' contributions can allow the teacher to engage students in mathematical thinking, gather information about their understanding, as well as many other objectives. The ability for a teacher to continue a conversation based upon student responses can be described as an *improvisational* move, which is made "'on the fly' in response to unanticipated developments in the discourse" (Springer & Dick, 2006, p. 106). Springer and Dick state the improvisational move is the most demanding challenge for a teacher within a discourse-rich classroom since it requires the teacher have adequate content and pedagogical knowledge. Therefore, understanding what types of questions engage students in mathematical conversations and having heuristics for generating such questions would be useful pedagogical knowledge for teachers in student-centered classrooms.

Questioning is a difficult pedagogical challenge when trying to promote productive mathematical discussions, but not extinguish original thought. For example,

when working in a student-centered classroom, "questions can be used for checking understanding, starting a discussion, inviting curiosity, beginning an inquiry, determining students' prior knowledge, and stimulating critical thinking" (Harris, 2000). Additionally, Manouchehri and Lapp (2003) state that when teachers design questions, they should consider the form, content and purpose.

Categories of Questions. One focus of research on questions concerns correlating the types of questions that teachers ask to students' subsequent achievement.

Cotton (1989) found the majority of researchers conducted dualistic comparisons about questioning. Examining nearly forty documents on questioning prior to 1989, Cotton found that researchers placed questions into higher and lower cognitive domain categories based on the types of student responses these questions were designed to elicit. Types of questions in the lower cognitive domain are referred to as fact, closed, direct, recall, and knowledge. Higher cognitive domain questions are called open-ended, interpretive, evaluative, inquiry, and synthesis. Woolfolk (1998) also suggests categorizing questions into divergent questions, which have many possible answers, or convergent questions, which have one right answer.

Cunningham (1987) provides a more extensive list of questions for teachers to ask based upon the cognitive level of the expected student responses. This research separates divergent and convergent questioning into high and low subcategories. Low-Convergent questions ask students to transfer information by comparing, contrasting or explaining, such as, "What is the meaning conveyed in this cartoon about the state election for governor?" (Cunningham, 1987, p. 73). High-Convergent questions encourage students to support their reasoning and draw conclusions, such as, "Why do you think violence on

television appeals to so many people?" (Cunningham, 1987, p. 73). Similar arguments are made about divergent questions and Cunningham names evaluative, perceiving and initiating action, valuing, and actualizing questions as additional ways to reach particular cognitive and affective domain levels.

Researchers in mathematics education have also formed categories of questions. When examining the types of questions asked by teachers in a second grade mathematics classroom, Hiebert and Wearne (1993) identify four types of questions: recall, describe strategy, generate problem, and examine underlying features. Vacc (1993) cites three categories of questions that occur in the mathematics classroom; factual, reasoning, and open based on a study by Barnes (as cited in Vacc, 1993) on questioning in classroom instruction. Vacc (1993) concludes that teachers asking factual questions will find out the specific facts their students know, but teachers who ask questions in the open category gain information about their students' understanding. It is worth noting that in all of these studies, the researchers generally did not focus on the types of responses that students actually gave to these questions, only the types of responses they expected these questions to elicit.

Another categorization scheme is a hierarchy. The most widely used hierarchy is Bloom's taxonomy, where questions are labeled from simple to complex cognitive objectives (Woolfolk, 1998). Wolf (1987) suggests a different hierarchy, which focuses solely on what the author considers challenging questions, from observations in the classroom. This hierarchy extends Bloom's to include five more categories of challenging questions: (a) Inference questions ask students to go beyond immediately available information; (b) Interpretation questions ask students to fill in missing

information and understand consequences of information; (c) Transfer questions ask students to take their knowledge to new places; (d) Questions about hypotheses ask students to think about what can be predicted and tested; and (e) Reflective questions ask students to ponder how they know what they know.

The questions identified in these studies have several names and describe student responses and cognitive levels using different descriptions. Due to numerous names and descriptions used by the authors, Table 2.1 summarizes the questions found by the authors and provides a brief description of the question's meaning in order to condense the information. Despite the different language and names for questions used by the authors there are similarities among the author's categories.

These studies developed their classification schemes or hierarchies based upon the anticipated or intended student responses, often by the type of cognitive reasoning needed to answer the question adequately. One difference between this literature and the study reported in this thesis is that the questions categories developed in this thesis will be correlated with the actual responses provided by students. Of course, there will be some overlap between the categories described in this thesis and the names of questions described in the Table 2.1. The overlap is a result of the notion that student conversation will include higher level cognitive reasoning and if teachers are seeking this result from students, the questions they ask may correlate to the research on question categories. Therefore, these categories provide a foundation for how to examine teacher questions in relation to student responses, but their definitions may differ because of this study's attempt to correlate teacher questions to actual student responses.

Table 2.1

Summary of Teacher Questions in Literature

Question	Author		
Comparison Type			
	Cotton	Cunningham	Woolfolk
Dualistic	Convergent:	Low - recall information	Convergent -
	Low - predictable transfer of information	High - manipulate information to	one right
	High - encourage reasoning	create an answer	answer
	Divergent:		Divergent -
	Low - think of alternative way to do		many
	something		answers
	High – encourage creative thinking		
	Vacc	Hiebert and Wearne	
Category	Factual - name specific information	Recall – give known information	
	Reasoning - develop one or more logically	Describe strategy – explain solutions	
	organized response	Generate problems – extend thinking to	

	Open - have a wide range of possible answers	new areas Examine underlying features – generalize ideas
	Bloom	**Wolf**
Hierarchical	Knowledge - recalling information	Interpretation - understand consequences of
	Comprehension – demonstrating	information
	understanding of	Inference – go beyond available
	information	information
	Application – use information to solve a	Hypothesis – predictive thinking
	problem	Transfer – take knowledge to new areas
	Analysis – making inferences	Reflective – explaining how you know
	Synthesis – divergent, original thinking	
	Evaluation – judge the merit of ideas	

Student Thinking. One reason for promoting discussion in the classroom is to have students make their thinking explicit; this provides students and teachers with the opportunity to discuss, organize, evaluate and refine their understanding of a mathematical concept. Ideally, revealing student thinking may be a method for starting conversation in the classroom.

One issue discussed in the literature is when teachers should pose questions to students with the goal of affecting student thinking through questioning (Cazden, 2001). Steele (2003) states a teacher who rushes in to tell students something they do not understand robs them of the opportunity to construct understanding for themselves; instead she advocates that teachers should avoid impeding on students' construction of knowledge.

In a study of two elementary age students solving the same problem at two different time periods as part of a larger longitudinal study, Martino and Maher (1999) explain how the timing of questions can influence a student's understanding of a mathematical idea. Like Steele, Martino and Maher argue that teachers should refrain from asking questions while students are developing their ideas, but ask probing questions afterwards.

These studies also identify specific types of questions teachers can ask to reveal students' thinking. In order to build upon student ideas, Martino and Maher propose asking students questions that encourage them to justify their reasoning. Similarly, Duckworth (cited in Cazden, 2001) advocates questioning that clarifies what the child is thinking. These questions call the learner's attention to their own reasoning and understanding and provide teachers with insight into students' ways of thinking.

Returning to Steele's (2003) idea of providing students with space to construct mathematical understanding, she describes, "I give them the opportunity to think. I am silent. I wait. I listen. I encourage them to test their ideas. I encourage them to talk to teacher other. I wait. I listen." (p. 59). The actions of Steele in her classroom focus on both the timing asking questions and using questions encouraging student interaction. Timing of questions is an important attribute because giving students time to think about mathematical ideas allows them to develop ideas. Once students have a chance to develop ideas, teachers who wish to establish conversation in the classroom can ask students to interact with each other as way to promote conversation. Several researchers provide suggestions for how to accomplish interaction through promoting and listening to student thinking.

Dann, Pantozzi and Steencken (1995) examined teacher questions in a seventh grade classroom where students where exploring ideas in combinatorics. They recommend that teachers ask questions that promote student interaction to help students extend their ideas and justify their conclusions. Like Steele (2003) and Martino and Maher (1999), Herbst (2002) also advocates that teachers should refrain from giving explicit hints or suggestions to students, since this may be counterproductive to the goals of having student build their own understandings. Instead in a study involving proof making of ninth grade geometry students, Herbst proposes "suggestion" as a form of questioning can be a natural part of a teacher's practice. One example of suggestion is for the teacher to ask interesting questions that lead students to make and prove conjectures (Herbst, 2002), which adheres to not providing explicit hints, but rather using questions to lead to students to draw their own conclusions.

Several other studies focus on the purpose that questions should serve. Brent Davis (1997) suggests that when teachers question students, they should not be doing so with the intention of evaluating the correctness of the students' explanation, but rather understanding students' reasoning. When the teacher listens to students, the questioning of the teacher focuses more on asking for elaboration, clarification, and explanation rather than following a logical pre-set sequence. Falle (2003) reports on a model for teaching and learning where teachers listen to student conversations and then interject questions to guide the conversation. Samples of questions involved include: "How do we know…?", and "Show me why…?" as part of the model to examine student responses for their mathematical understanding (Falle, 2003).

van Zee and Minstrell (1997) suggest the teacher ask an elaboration question rather than a question with a specific answer; elaboration questions are more likely to place students in a position to continue the conversation either by commenting or asking more questions. Using data from a suburban high school physics classroom, van Zee and Minstrell (1997) define a specific questioning technique they call a 'reflective toss'. This questioning pattern consists of a sequence of a student statement, a teacher question and additional student statements. A *reflective toss* is an utterance, which the teacher used to elicit further thinking by "catching" the meaning of the student's prior utterance and "throwing" responsibility for thinking back to the students (van Zee & Minstrell, 1997, p. 241). An example reflective toss analyzed in the study:

Student: You would add all of the numbers together and divide 'em by eight.

Teacher: Now what do you mean by "adding all the numbers"?

Student: You would add each separate number that everybody got; you wouldn't

just add one 107, you'd add all the 107s. (p. 251)

One goal of examining these utterances is to trace how a teacher's influence affected

what a student said. In the above example, the teacher's question led to further

clarification of a process for averaging by the student. Overall, a reflective toss serves

three goals: to help students clarify their ideas, to help students consider a variety of

views; and, to help students monitor their own thinking.

Mathematics education research shows that teacher questions play an important

role in accessing student thinking and understanding. Research in this area also supports

the idea that justification, elaboration, and clarification questions can encourage students

to talk about mathematics. Therefore, these types of questions might be used as way to

describe teacher questions that engage students in mathematical conversation. While the

previous section lists schemes for classifying teacher questioning, this section describes

specific techniques that teachers can use, some of which do not fit in the classification

schemes previously described.

One particularly study examined the role of questioning in unveiling student

understanding in a ninth grade algebra class. Manouchehri and Lapp (2003) argue that

teacher questions should allow students to communicate their ideas so teachers can gather

data about how students think. When making suggestions about what questions teachers

should ask the authors revert to the open or closed forms of questions based on cognitive

domain levels of student thinking. Therefore, the literature on the relationship between

teacher questioning and student thinking provides this study with some suggestions for

labeling questions that engage students in mathematical conversation, but may also

suggest the classification of questions in the previous section may help teachers develop questions that engage students in mathematical conversation.

   Discourse and Community. There is a large body of research in mathematics education that examines how teachers can use questions as a way to create an environment that encourages meaningful mathematical discourse in the classroom. While this thesis will examine teacher questioning in classrooms where student-centered learning environments are already in place, the difficulty of establishing this environment has been discussed in the first section of the literature review. Hence, this study can inform work connecting teacher questions and establishing a mathematical community based on discussion.

   Manouchehri and St. John (2006) support the idea from the previous section that teachers who use questions to reveal student thinking may be able to foster discussion. In a study of two geometry teachers with similar students, the authors found the teacher, who used questions to show students they needed to be responsible for sharing their understanding, created a valuable discourse about mathematics in the classroom. In a different line of research, White (2003) illustrates how questions that help all students become involved in the classroom discourse can have a positive influence on students' mathematical thinking. White studied the questions of two third-grade teachers, in an enhancement program, at different urban magnet schools in Washington, D.C., which both contained a majority of minority students. The teacher questions in this study followed four patterns: asking students what they noticed about and how to solve a problem; how they arrived at the answer; to share solution strategies; and, to interact with

each other. These questions valued student ideas and thinking and enabled teachers to develop a productive classroom discourse with their students.

Hufferd-Ackles, Fuson and Sherin (2004) provide a detailed description of community from a year-long study of one teacher in an urban classroom of Latino students. The authors describe a math talk community where teachers and students use discourse to support the mathematical learning of all participants. The community is made of four components: questioning, explaining mathematical thinking, source of mathematical ideas, and responsibility for learning. This research examined how the teacher moved students toward a student-centered environment in these communities. Questioning played a key role in order to find answers and uncover the mathematical thinking behind the answers. They also allowed student ideas to become a public part of the discourse; challenging the students' thinking led them to elaborate on their work (p. 92).

Similar research by Goos (2004) and Mewborn and Huberty (1999) also view teacher questioning as a way to move toward a community of inquiry. Inquiry mathematics is where students learn to speak and act mathematically by participating in mathematical discussion and solving new or unfamiliar problems (Richards, 1991 cited in Goos, 2004). The questions teachers used in these studies are similar to the questions from the studies on revealing student thinking. Goos, who reports on teachers from an Australian school, noticed teachers asked students to make more explicit statements and prompted reflection of their students. Mewborn and Huberty also cite that teachers can ask follow-up questions after student responses, ask other students to restate one student's ideas, or ask for alternate methods for solving a problem. Davis and Forster

(2003) provide specific context and content examples of open-ended questions from a Year 12 Calculus class in Australia that created communication in the classroom, which connects back to the categorization idea of questions from the first section. Open-ended questions about vector functions posed in this classroom promoted conversations that included student explanations, judgments of their responses, and debates. These studies not only reinforce the notion that teacher questioning can be used as a tool for revealing student thinking, but also that they may be useful in developing communities of learning in which classroom discussion becomes the norm.

Two studies refer to discourse strategy rather than questions as a method for examining how teachers can engage students in classroom discourse. O'Connor and Michaels (1993) and Springer and Dick (2006) state that revoicing is a move teachers can use to develop discourse. Springer and Dick describe *revoicing* "as a move made when one person repeats, summarizes, rephrases, translates, or recasts the contribution of another participant in the discourse" (p. 107). O'Connor and Michaels show how using revoicing helps teachers coordinate academic tasks and bring students into the process of intellectual socialization. Revoicing statements serve several purposes: clarifying student statements to the whole class, so they can evaluate it correctness of an idea; inducting students into the discourse community; and, giving voice to the contributions of quiet students. When teachers use revoicing they defer evaluation of a statement to students and change the expectations of the classroom discussions (Springer & Dick, 2006). If students are expected to respond to other student statements, then there is a mechanism in place for creating conversation. Thus, while it is not a specific type of question,

revoicing is similar to questioning in that it can be used as a tool to engage students in mathematical conversation.

The idea of using questions to shift responsibility to the students is also demonstrated in a study about undergraduate students' understanding in a one-year discrete mathematics course. In this study, Blanton and Stylianou (2003) focus on one teacher shifting the responsibility of construction of mathematical ideas to the student. In this view, teacher discourse includes questioning, summarizing, and elaborating. The questioning component is used to draw students into a discussion by requests for clarification, justification and elaboration by students. Summarizing establishes what is learned or spoken by the students. The summarizing stage has also been referred to reconceptualization where student ideas are rephrased into culturally mature terms. Elaboration by the teacher follows the student's original idea to lead them toward desired mathematical goals.

Similar to the idea of examining discourse instead of teacher questions, Cobb and his colleagues (1997) cite a model called reflective discourse, where earlier student and teacher actions become explicit objects of discussion. Teachers in this discourse model use questions to guide the shift from action and exploration to discussion so that students step back and reorganize the work they have already done. Hence, the idea of questions helping to create discussion is prevalent in several studies and this body of literature does provide some examples.

Providing two more explicit teacher actions that can create a community of communication, Lobato, Clarke, and Ellis (2005) provide two specific examples of teacher actions that can be used to foster communication in the classroom. They use data

from a single ninth grade student in an after-school teaching experiment to redefine traditional teacher-centered classroom actions of telling as initiating and eliciting. *Initiating* is the set of teaching actions that serve the function of stimulating students' mathematical constructions via the introduction of new mathematical ideas into a classroom conversation (p. 110). *Eliciting* is a teacher action intended to ascertain how students interpret the information introduced by the teacher (p. 111). In this perspective, questions are an important component of the initiating and eliciting processes as long as the teacher's questions allows students to explain, share, discuss and justify their understanding of mathematics.

The literature regarding discourse or communities based on communication provides this study with possible ideas for labeling questions teachers use to engage students in mathematical conversation. This literature is important because as mentioned in the introduction and described in Rittenhouse (1998), "students do not automatically begin talking about mathematics in a meaningful way simply because they are presented with appropriate tasks or are placed together in groups and told 'talk to each other'" (p. 169). Therefore, this study will narrow the lens described in this literature to focus on connecting questions to establishing mathematical conversation within the classroom. This narrowing may best be described by Rittenhouse's explanation of a teacher role called 'stepping in'. Magdalene Lampert played this role as a teacher/researcher in her fifth grade mathematics classroom by becoming part of a mathematical discussion with students in order to help them become mathematical competent. This study wishes to study the moment when the teacher 'steps in' to the work and thinking of students by looking at the questions teachers ask in order to establish mathematical discussion.

Summary

The topic of teacher questions has been studied extensively and from various viewpoints. Researchers have identified frameworks with which to categorize teachers' questions, provided guidelines or techniques for asking productive questions, and illustrated how questioning can be used as a tool to promote productive norms within the classroom. One underrepresented area concerns what types of questions are useful in eliciting students' reasoning in *mathematics* classrooms, and what types of responses these questions invoke in the students. Answering these questions is the subject of this thesis.

Theoretical Perspective

The environments selected for this study have an existing framework and norms that allow students and teachers to participate in a discussion about mathematics. The importance of discussion in supporting students' understanding is prevalent in many theories of learning, such as constructivist theory (e.g. Noddings, 1990), sociocultural theory (e.g. Forman, 1996), and linguistic theories (e.g. Sfard, 2008). In order to frame this study, the theoretical perspective examines the benefits of teacher questioning and class discussion within Piaget's constructivist theory of learning, Vygotsky's zones of proximal development, and more recent theories about sociomathematical norms and the participation metaphor of mathematical activity. The purpose of this section is to demonstrate that regardless of background theory of learning one may advocate the examination of teacher questions is relevant. Therefore, the review of each learning

theory is meant to provide a small glimpse into substantial bodies of research in order to demonstrate a connection to teacher questions.

Teacher's Role in the Construction of Knowledge

Several theories of learning argue for the active participation of students in the learning process (e.g. Confrey, 1990, Yackel & Cobb, 1996) and suggest a change in the role the teacher plays within the classroom. According to Piaget (1972/1995), the "role is less that of a person who gives 'lessons' and is rather that of someone who organizes situations that will give rise to curiosity and solution–seeking in the child" (p. 731). These suggestions shifted from the role of the teacher as transmitting information to the students to the teacher as building students' mental models by providing appropriate mathematical situations (Cobb & Steffe, 1983). Although the teacher may no longer be the centerpiece of the classroom in these frameworks, these theories emphasize that the teacher is an important part in helping students understand mathematics. This discussion shows how teacher questions, which can help students understand mathematics, relate to theories of learning.

Piaget's Theory of Learning. Piaget (1952) offers a theory of how intellect grows that includes three fundamental processes: assimilation, accommodation, and equilibrium. *Assimilation* is the process by which a person brings information into pre-existing cognitive structures. *Accommodation* is the process by which a person changes their pre-existing cognitive structures to accept new information. *Equilibrium* is the balance between assimilation and accommodation. Based upon these processes, the teacher's role is two-fold. The first role is to present mathematics in way that students

can assimilate the information. If a student does not have the necessary cognitive

structures for assimilation, the teacher's role is to create disequilibrium so students can

reorganize their existing cognitive structures in a way that allows them to reason

productively and assimilate new mathematical ideas. In order to decide how he or she

should proceed, the teacher must have an idea of what students' cognitive structures are.

Only then will the teacher know if students are ready to assimilate the new ideas that are

about to be investigated or if a situation evoking disequilibrium is necessary. One way

for the teacher to have knowledge of student thinking is for students to verbalize their

thoughts. Questions that elicit student reasoning are a tool the teacher can use so they

have a model of student thinking. In order to guide student learning, questions are

possible method for a teacher to determine if students are assimilating knowledge or if

the teacher must create disequilibrium to promote growth in understanding.

　　　　Zone of Proximal Development. By drawing from the work of Goos (2004), Cobb

and Bauserfield (1996) and van Oers (1996), three general themes can be identified in

Vygotsky's (1978) theory of learning. The first is that to understand mental

phenomenon, there needs to be a focus on the process of growth and change, not on the

product of development. The second theme is that individual learning is based upon

social interaction. The third theme is that mental processes are mediated by signs and

tools, such as language or algebraic symbols. Vygotsky viewed the zone of proximal

development as a place where the transformation from social phenomena to

psychological phenomena occurs (Goos, 2004). Vygotsky's original definition of the

zone of proximal development is the distance between a child's problem-solving

capability when working alone and with the assistance of someone more knowledgeable.

Vygotsky's idea of the zone of proximal development portrays the belief that learning can occur through carefully planned instructional tasks (Sfard, 2003). Also, students learn when faced with problems beyond their current level of competence, and when someone with more knowledge, such as a teacher, is present for learning to take place (Sfard, 2003). The implications for instruction are that the teacher should provide students with problems beyond their current abilities, but not so far beyond their abilities that teacher or classmate guidance cannot help them obtain a solution. Once selecting these problems, the teacher's role is to move students across the zone of proximal development from their need for assistance to independence. In order to determine where a student is in the zone of proximal development, the teacher should be aware of student reasoning. Teacher questioning is one possible way to assist with eliciting student reasoning and help the teacher move students toward independence.

Creating Student Agency. The learning theories of Piaget and Vygotsky include the idea that students interact in order to learn. An interactive perspective assumes the development of reasoning and sense-making cannot be separated from participation in mathematical activity (Yackel & Cobb, 1996). These views propose that students should be actively participating in many aspects of their learning process, not just the implementation of procedures which is sometimes the case in traditional classrooms. These constraints, which can be heavily influenced by the teacher's belief about the nature of mathematical activity, form students' beliefs about their role in learning mathematics.

One way to advocate student agency and define their role in the learning environment is to establish social and *sociomathematical norms* that promote active

participation by students. Social norms are the established classroom culture that defines the way students interact within the classroom. Yackel and Cobb (1996) take this notion one step further and refer to sociomathematical norms as "normative aspects of mathematics discussion specific to students' mathematical activity" (p. 461). Examples of social norms are that presented solutions should be accompanied with justifications and some of the ideas in an investigation should not only come from the teachers, but also the students. Examples of sociomathematical norms include understanding what counts as mathematically different, efficient and acceptable explanation and justification. When the students and the teacher negotiate social and sociomathematical norms within a classroom, students come to understand their responsibility in the classroom. One way to deflect the responsibility of mathematical argumentation, explanation and justification to students in order to establish sociomathematical norms is to ask questions. Questions and subsequent norms may allow students to recognize that they are responsible for sense-making and justifying, and thus can increase students' agency.

Participatory View of Learning. At the most fundamental levels, learning is described by two metaphors, acquisition and participation (Sfard, 1998). Whether by active or passive participation of the learner, the acquisition metaphor, in which knowledge is something that can be accumulated, is a common historical view of learning. However, some contemporary educational research refers to the participation metaphor as a shift that considers 'knowing' as the process of becoming a member of an established community (Sfard, 1998). When knowledge is viewed as a process instead of a product, communication and language become mathematical activities that coincide with learning (e.g., Sfard, 2001, 2008).

Studies emphasizing the participatory metaphor of learning, outlined in Lampert and Cobb (2003), see talking and writing as aspects of doing mathematics. In order to make mathematical talk a primary activity in the classroom, a teacher needs to support the development of abstract mathematical reasoning. When communicating about mathematics is considered a genuine mathematical activity, the teacher's roles include supporting discourse, managing mathematical words and definitions, establishing mathematical norms and helping students articulate meaning. However, the practice of talking about mathematics and negotiating mathematical meaning as a focal point of classrooms is not typical (Lampert & Cobb, 2003). As a result, students are given few chances to either communicate mathematically or communicate to learn mathematics. Classrooms where students discuss ideas rather than wait for the teacher to deliver information can increase student participation. Questions are one possible way to help the teacher increase participation and encourage students to communicate with each other.

Summary.

There are a wide variety of learning theories in mathematics education. This section illustrates how, within most mainstream contemporary theories of learning in mathematics education, the student is expected to play an active part in their learning and teacher questioning can play a pivotal role in this process. When learning is viewed as students taking an active part in the process of gaining knowledge, the teacher's role is no longer the conveyor of information. Rather the teacher is an important component of social interaction in a classroom and their participation in the learning process is

important to students. Whether providing students with more challenging problems than which they are cognitively capable or establishing sociomathematical norms, teachers need a way to understand student thinking. Teacher questioning can address the issue of knowing student reasoning in order to create disequilibrium, increase student agency or promote participation.

Methodology

The settings selected for data collection needed to have conversation as an established norm in order to examine how teachers and students interacted to create discussion. Access to data collected as part of a longitudinal study at Rutgers University provided one setting where teachers are listening to student ideas. This was the first set of data I chose for this study. Since this data is not collected from a classroom, a second setting of a high school classroom provided a comparison. I selected a high school classroom known to be a student-centered environment due to my previous employment in the district in order to provide me with a second data set. Additionally, the students and mathematical concepts in the classroom were similar in age and content to the Rutgers data. The consistent trait of conversation being valued in both settings allowed for an abundance of data for examination of teacher questioning, and their differences allowed for comparable analysis and possibly conclusions about teacher questions. Therefore, this study proposes two cases studies that examine two different settings in order to provide insight into questions teachers ask, and what questions engage students in mathematical conversation.

Participants and Settings

The first setting for data collection was an urban high school classroom. Thirty-five honors students worked on calculus problems. The students, who were sophomores and juniors, were seated at individual desks in rows throughout the data collection window. The teacher, Dr. G, observed was a 30 year veteran of the school system and held a doctorate in education. The primary role of the teacher in this setting was to pose

a question to the students and then have students take turns at the board working on the solution. The whole class participated in the process of solving the problems, which were typical definite integral problems selected from a calculus textbook. During the class, the teacher spent the majority of his time in the back of the class while students worked at the board. At times, the teacher went to the front of the class to write on the board or have a discussion with a student. The camera recorded the work of the students at the board and a microphone recorded the conversation of the participants.

The second setting was a component of a precalculus strand of the Rutgers longitudinal study of children's mathematical thinking and proof-making[1]. The research sessions were not teaching experiments, but rather unique environmental conditions that focused on determining the knowledge of students. In this learning environment, students were asked to attempt to solve open-ended, but well-defined mathematical problems, where collaboration and justification were encouraged. The Summer Institute, which is part of the precalculus strand of the longitudinal study, focused on the work of seventeen students with various ethnic and educational backgrounds, entering their fourth year of high school, and working in groups on an open-ended precalculus mathematics problem in a library of David Brearley high school. The students were assigned to groups and five teacher/researchers interact with the students. Each day the students worked for four hours on mathematical tasks that were different than any of their school mathematics experiences. Six students, seated at the same table (two males and four females), and the teachers and researchers who interacted with them were chosen for this

student. The teachers and researchers let the students, who usually worked in their group of six except for occasional interactions with other students present in the research setting, work independently after introducing the task. The interactions recorded by the camera occurred at the students' table and required a teacher or researcher to walk over to the group to question them.

One teacher, Mrs. W and two teacher/researchers, Dr. A and Dr. W, interacted with the students during the data collection. For the initial analysis of developing teacher question codes, I considered the interactions of these three people. When developing questions themes, I selected one teacher/researcher, Dr. A, an experienced professor of mathematics and mathematics education at the university level. For a more detailed examination of the longitudinal study, several publications provide more information (e.g. Maher, 2002, Maher, 2005, Francisco & Maher, 2005).

Data Collection

In order to obtain an accurate picture of the kinds of questions asked and provide evidence of student mathematical thinking, video recording is the method used for observations. Eighty-minute videotape observations of the high school classroom sessions and two-hour videotape sessions of the two-week institute, both from a consecutive three-day period, comprise the data for this study.

In the high school classroom, field notes describing ten-minute intervals supplement the video in order to account for events beyond the view of the camera. These notes also provide a summary of the class activity. Field notes for the longitudinal study were taken by researchers present and provide further context for the events recorded on

camera. The purpose of the field notes was to supplement video recordings and provide the researchers with journal recordings of the events captured on video. These journals provided a written summary of the video until the recordings could be summarized and transcribed.

Data Analysis

Based upon my research questions, the overarching goals of my analysis are to: (a) find out what types of questions teachers asked and how often they asked them, (b) find out what types of student reasoning these questions tended to elicit, and (c) develop an understanding of how specific types of questions could engage students to share their reasoning.

In order to accomplish goals (a) and (b), I used videotape recordings of each data setting to perform this analysis. Video recording of data provides several advantages to a researcher. The activity of the setting is permanently recorded in both visual and audio mediums. Videotape allows for repeated viewing to support analysis and can be used to generate a transcript of verbal phenomena.

The first step in analyzing types of teacher questions and student reasoning required getting a strong sense of the data. In order to do this I viewed the videotape, identified and described conversations, and transcribed the conversations for more in-depth analysis as described in Powell, Francisco and Maher (2003). Powell et. al. developed a methodology that allowed them "to investigate the nature of teacher intervention in the growth of student mathematical ideas" (p. 2) while viewing videotaped classroom data. These goals coincide with my analysis goals since this study

focuses on the ways teachers question students and how the students respond. After

transcribing the teacher-student conversation in the data, I began coding for teacher

questions and student responses. Conversations that did not involve teacher questioning

were not transcribed, but I carefully noted the content of the data in order to provide

myself with an understanding of the continuum of the data.

   Teacher Coding. The process of developing teacher codes occurred by starting

with a predetermined list of codes from Ilaria and Maher (2001) and Ilaria (2002) as an

initial framework for the coding of this data. I developed the initial list of codes from an

examination of data containing student-teacher conversation. When students and the

teacher were engaged in conversation, their words were examined for the questions

teachers asked. The teacher question codes from the previous studies are summarized in

table 3.1.

Table 3.1

Initial Code Framework

Code	Description
T(r)	Teacher asks a student to consider an old idea.
T(d)	Teacher asks a student to contribute to the ongoing discourse.
T(c)	Teacher asks the student to clarify his/her statements or ideas.
T(j)	Teacher asks the student to justify his/her statements or ideas.
T(con)	Teacher confirms the student and teacher both agree on what has been done or said.
T(f)	Teacher follows the student's idea or suggestion.

These codes are different from the taxonomies discussed earlier because the codes are not connected to cognitive or affective domains. Rather these codes directly connect teacher questions to student responses as part of a classroom mathematical conversation. For example, several authors refer to questions in a dualistic manner, convergent or divergent (Cotton, 1989; Cunningham, 1987; Woolfolk, 1998), but when a teacher asks another student to contribute to the ongoing discourse, T(d), the student can respond with a short, closed statement or can elaborate upon the ideas already shared in the classroom. According to Woolfolk (1998), a T(d) question would be divergent if there are many possible answers and convergent if there was only one answer. Since a student is contributing to an existing conversation, a possible response could be a single piece of information to help a student take the next step in solving a problem or the student can explain why the last step in the solution is mathematically valid. This would mean this question could be either a convergent or divergent depending upon the student's response. Therefore, this dualistic taxonomy does not provide enough flexibility to describe teacher questioning within a conversation and similar arguments can be made for other questions in this preliminary list.

While the codes were developed from the data in these preliminary studies, they do mimic types of questions mentioned in the literature that connected questions to student thinking and classroom discourse. Table 3.2 catalogs the literature review articles in which there are references to one or more of the questions in this coding list. Connections between the literature and teacher codes developed in this study will be discussed in more detail in the results section.

Table 3.2

Connection of Codes to Literature Review

Code	Authors	Connection to Teacher Code
T(r)	White (2003)	Teachers encourage use of student background knowledge
T(d)	van Zee & Minstrell (1997)	Reflective Toss – give students responsibility for thinking
	White (2003)	Teachers encourage student-to-student interaction
T(c)	Blanton & Stylianou (2002)	Teacher requesting clarification, elaboration, justification or assessment
	O'Conner & Michaels (1993)	Revoicing to clarify academic content
	Goos (2004)	The teacher calls on students to clarify, elaborate, critique, and justify their assertions
T(j)	Lobato, Clark, & Ellis (2005)	Eliciting actions
	Dann, Pantozzi, & Steencken (1995)	Teachers prompt students into explanations and justifications of their ideas
	Martino & Maher (1999)	Teacher questioning which enhances students' building of arguments
	Goos (2004)	The teacher calls on students to clarify, elaborate, critique, and justify their

		assertions
T(con)	O'Connor & Michaels (1993)	Revoicing to accept the student's response and allow the child to validate the teacher's inference
	Blanton & Stylianou (2002)	Teacher revoicing and/or confirming a student statement
T(f)	Mewborn & Huberty (1999)	Teacher asked a follow-up question to help the student amend their thinking

While a coding system exists for possible imposition on teacher questions, inductive analysis of the data drove the development of additional categories for teacher questions through line-by-line examination of the transcription of critical events. The codes emerging from Ilaria and Maher (2001) provided a starting point for examining the data, but the final codes only resemble the Ilaria and Maher codes in name. At times during coding, some utterances matched two or more coding categories and received multiple codes. Additionally, statements may have contained two or more distinct utterances that were describable by different categories. These statements received two codes as well, but the codes connect to different parts of the utterance.

Finally, I went through both videotapes and coded for each instance of teacher questioning. The final questions codes were developed from continual and repetitive reading of the transcript during the coding process. Each of these codes is defined more rigorously and illustrated in chapter four. A table summarizing these findings is also presented in chapter four.

After coding the data for teacher questions, I counted the number of teacher questions in each category. Single teacher utterances receiving two codes were minimal and ignored in this part of the analysis. Teacher utterances that received two different codes because the statement contained distinct utterances were included in this data by separating the teacher's statement into two separate lines of transcript without a new time code. The totals are presented in chapter five. This table is used to address the first goal, finding out what types of questions teachers asked and how often these types of questions were asked.

Student Coding. Addressing the second goal of my analysis, what types of student responses each category of questions elicited, required coding for the types of responses that students provided. This phase of coding focused on conversations and examined student responses. Using open coding as the fundamental analytic process, the analysis for student responses commenced by noting repeated responses types by students. These responses were compared in order to develop a category name the student responses.

After completing an initial coding of all of the data, I created a record of the number of times each code appeared. Due to a minimal frequency of certain codes, I revisited the data and searched for the specific codes in both data settings. After reading the surrounding transcript for context and mathematical content, I determined some codes could be eliminated by a tightening and/or redefining of other codes to include the identified utterances. The rejected question and student response codes are described in appendix A.

During this repeating viewing and counting, I compared coded responses to determine if the coding scheme needed further refinement. In comparing responses, I started by examining the codes that appeared the frequent amount of times. Once the leas frequent codes were identified, I looked for similarities between the definitions and the student statements. By examining student statements with two particular codes in mind, similarities became apparent if definitions of other codes were adjusted, tightened or broadened. As a result of reworking the definitions other codes, it was possible to combine codes serving similar purpose or eliminate codes as described in appendix A. After eliminating infrequent codes, the remaining codes were compared again to see if there was overlap in definitions by examining sample student responses. This process also led to recoding utterances in the data to reflect changes to definitions.

As an additional part of the refinement process, two sample transcripts without codes were selected from each data setting and given to two mathematics teachers, who were colleagues at the time, for coding. After each teacher coded the data, I compared my coded data to their coding results. We discussed each discrepancy and the teachers provided feedback regarding definitions and code changes. Based upon these discussions, I finalized list of student response codes, which is presented in detail in chapter four. The total number of each category of student codes is summarized in chapter five.

Teacher Question and Student Response Relationship. After completing the coding for both teacher questions and student responses, I developed a table that determined the frequency of each student response to each type of teacher question. A table of student responses that immediately followed a teacher's question provided

evidence of the responses elicited by teacher questions. The table can partially address the second questions, what responses each type of question tended to elicit.

This table provides a sense of what students said in response to questions, but not a rich understanding of how these questions elicited these responses or encouraged learning. Since the participants and settings were selected because of their already established conversational norms, qualitative analysis of conversations provided a way to achieve my third goal of developing an understanding of how specific types of questions could engage students to share their reasoning.

In order to provide a more qualitative description, I selected a representative conversation from each data setting in order to further illustrate how teacher questions produced mathematical conversation and eliciting student thinking. For the Old Bridge data, I divided the transcript by the problems students solved over the three-day period. I selected the third problem, which occurred on the second day, because it was the third time students were completing a similar process. Since students were familiar with the process, I hoped their thinking would be more mature and provide for an opportunity to explore their understanding. For the Kenilworth data, I divided the transcript by each time Dr. A visited the group. I selected a conversation where students were discussing their solutions to the problem they were working on over the first two days of data collection. This provided an opportunity to listen to student thinking about their work on the problem.

After selecting each conversation, I further divided the conversations into segments. I based the Old Bridge divisions on each time a new student went to the board to work on solving the problem and the Kenilworth data by the topic discussed within the

conversation. After this further division, I searched for patterns in the data to determine questions that engage students in conversation. The patterns found within each setting led to the development of themes that described how the teachers in the study used questions to engage students in conversation.

Developing Questioning Themes. Determining questioning themes occurred differently depending on the setting. For the Old Bridge setting, I divided the transcript into segments based upon a different student going to the board to lead the class discussion and record work on the board. Within each segment, I examined the student responses to find places where students were verbalizing their thinking. Evidence of student thinking occurred either by the student at the board or by other students in the classroom. By examining the questions that elicited student thinking, I separated the segments into times when the student at the board shared their thinking and times when questions elicited thinking from other students. This separation led to two different themes, initiating and inviting, which occurred in nearly every segment. By examining the remainder of the discussion, I recognized there were several times when students engaged each other in direct conversation. During these times, the teachers used questions in an alternative way that helped the discussion between students. This led the development of a third theme, called supporting, for this setting.

For the Kenilworth setting, I divided the transcript into segments based upon the different mathematical topics discussed within the selected conversation. Within each segment, I followed the same process as the Old Bridge by finding times when students verbalized their thinking. When the same pattern of who shared their thinking, either the first student questioned by the teacher or another student in the group, the initiating and

inviting themes from the Old Bridge setting were applied to this data. Once again I examined the rest of the conversation and noticed the teacher kept using a particular type of question to elicit student thinking. This led to the development of a third theme, revisiting, for this setting, which is a fourth theme overall.

Once the themes were set, I reviewed the entire data set for the study by examining each problem posed in the Old Bridge setting and each teacher intervention by Dr. A in the Kenilworth setting. While reviewing the data, I looked for the emergence of these themes and coded the data accordingly. I also looked for the possibility of additional themes while I reviewed the remainder of the data. An analysis of these findings is described in the chapter five. Additionally, the creation of questioning themes allowed me to generate a narrative describing these conversations with respect to each theme in order to demonstrate how each teacher used questions to elicit student thinking. The narratives are presented in chapter five and provide insight into how particular questions engage students in mathematical conversation.

Results

This chapter describes the codes for the teacher questions and student responses developed from the data. The two sections in this chapter provide an operational definition of each question and student code, along with several illustrative examples. The next chapter is a quantitative analysis of the frequency of each type of question and student response.

Teacher Questions

One question of this research study is to determine the kinds of questions teachers ask in a student-centered mathematics classroom. While the categories of questions described in this section occurred with different frequencies in each setting, they were present in both settings, (with the sole exception of the final category), and most were asked by the teachers included in the study.

The word "question" is an interesting choice when I participated in the coding process. Despite studies cited in the literature review stating the most frequent form of teacher-student interaction is through questioning, it became obvious that this is not always the case when there is genuine conversation in the classroom. However, by reading through the transcript multiple times, the storyline of the conversations showed the teachers were constantly inquiring about their students' knowledge, even in cases where their utterances were not in an interrogative form. Therefore, the teachers can be considered to portray the role of questioning throughout the observed time and their utterances categorized as questions. As a result of this view about the teacher and their

statements any utterances with "interrogative intent" (in my judgment) were classified as questions.

Therefore, the use of the term question for this study depicts the role of the teacher within the research setting as seeker of understanding the students' knowledge and reasoning about the mathematical concepts and therefore taking on the role of questioner within the respective setting. The question categories developed from the coding process of this study answer the research question about what questions teachers ask in a student-centered classroom are described in this section. The categories are discoursive, retracing, explanation, clarification, justification, confirmation, following, suggestion, procedural, and repeat. The questions are presented in the order in which the codes were developed during the inductive analysis. The illustrative examples were selected to portray the clearest representation of each code's definition.

Discoursive question. The first type of question to emerge from the data is a *discoursive question*. A teacher's utterance is coded as a *discoursive question*, T(d), when: a) the teacher specifically addressed a particular student or class as a whole; b) the student was not engaged in the conversation immediately before being addressed.

In the Old Bridge data, the teacher received a *discoursive question* coding by stating another student's name or posing a question and calling upon a student not part of the ongoing conversation to answer the question. Several examples of *discoursive questions* are shown in the provided transcript.

Kristen is at the board and has found the derivative of $f(x) = x^2 + 1$ in order to determine if the function is increasing or decreasing over the interval [1, 2].

06:12   Dr. G   T(s)   Now over the interval 1 to 2 inclusive. Two x, is it

positive or negative?

06:24	Kristen	S(a)	What was the question? Sorry
06:29	**Dr. G**	**T(d)**	**Jason repeat the question**
06:31	Jason	S(ans)	Is the derivative positive or negative?
06:33	Dr. G	T(f)	Over the interval
06:34	Jason	S(ans)	from one to two
06:37	Kristen	S(ans)	positive
06:38	Dr. G	T(j)	How do you know that?
06:51	Kristen	S(dnc)	Um, I have no clue. Just explain it to me.
06:53	**Dr. G**	**T(d)**	**Eric, how do you know it's positive or how do you know it's negative?**
06:58	Eric	S(pb)	Ah, I think because if you solve for x. and uh, I don't know it's positive
07:08	Dr. G	T(d)	Becca
07:09	Becca	S(pb)	Because the derivative is the slope, so it's like going up two and over one
07:20	**Dr. G**	**T(d)**	**Matthew**
07:21	Matthew	S(pb)	If you fill in any number between the interval between one and two, the answer is positive because graph is
07:31	Dr. G	T(f), T(p)	Graph y equals 2x between 1 and 2. Rather, I am going to change that slightly. Graph y equals 2 x over the interval 1 to 2 inclusive. [Kristen starts to graph 2x on the board]

07:55	Dr. G	T(p)	On a new graph. [Kristen draws a new coordinate axis and graphs y = 2x]
08:21	Dr. G	T(r)	Now, is the derivative always positive on that interval?
08:38	Kristen		Um
08:39	**Dr. G**	**T(d)**	**Time out. Repeat the question Brittany**
08:44	Brittany	S(ans)	Is that always positive in the interval?
08:47	Dr. G	T(c)	No pronouns
08:51	Brittany	S(c)	Is the derivative always positive on that interval?
08:53	**Dr. G**	**T(d)**	**Kristen**
08:55	Kristen	S(ans)	Yes
08:55	Dr. G	T(j)	How so? How is it we know this? How did you make that decision?
09:02	Dr. G	T(j)	By the look on your face that was a fifty-fifty guess type thing?
09:10	Kristen	S(dnc)	I don't, I don't, I have no clue
09:12	**Dr. G**	**T(d),T(c)**	**Alright. Rachel, explain what we mean by the derivative being positive over that interval.**
09:18	Rachel	S(pb)	Is it because of all the number that you put in for are going to be positive for x and when you multiply by 2, it is going to be positive
09:29	Kristen	S(a)	What?
09:30	Dr. G	T(rep)	Louder
09:31	Rachel	S(pb)	All the numbers you're putting in for x are positive. And

when you multiply a positive by a positive you are

going to get a positive.

09:41   Kristen   S(con)   okay

09:43   Dr. G   T(d)   Anyone want to say it another way?

No students responded to the final question so Dr. G moves the discussion to another

topic. These questions were coded as *discoursive questions* because they directly ask a

student, who did not contribute to the conversation immediately before the teacher called

upon them, to give input to the current discussion. Each *discoursive question* requires

input from a different student so seven different students become involved in determining

if the function is increasing or decreasing on the given interval.

In the Kenilworth data, a *discoursive question* occurred more often when the

teacher[2] wanted another student's input about what was being discussed between the

teacher and another student. In the transcript provided, Dr. A and Robert are discussing

why his model for the data points is a spiral sometimes and a non-spiral other times.

00:47:43:09   Dr. A   T(f)   Oh, so when you're throw theta in there instead of

                                    an x

00:47:45:29   Robert   S(pb),   It acts as x, because doesn't polar like go on

                        S(qs)   degrees or something?

00:47:49:08   Sherly   S(con)   Mm hmm.

00:47:50:16   Robert   Instead of

00:47:51:18   Dr. A   T(c)   So it circles it around.

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[2] Dr. A is the teacher/researcher studied in the research setting and is referred to in the methodology
section. For consistency throughout the document, she is referred to as a teacher.

00:47:53:03	Robert	S(con)	Yeah.
00:47:55:12	**Dr. A**	**T(d)**	**Angela does that make sense to you at all? Or not?**
00:47:59:04	Angela	S(ans)	Sort of. I like get lost with all this stuff. I hate this.
00:48:02:24	Dr. A	T(f)	What, what are the, what is the sort of question that throws you?
00:48:08:22	Angela	S(ans)	Like, I get like little bits and pieces of what he's explaining, but I don't really get all of it.
00:48:14:04	Sherly	S(qs)	What don't you understand?
00:48:16:29	Angela	S(ans)	I don't know.
00:48:18:24	Dr. A	T(con-t)	I think I hear her say, she doesn't even know what she's asking. Um what, what are you asking...in this?
00:48:28:11	Angela	S(ans)	I don't like have a specific question. I just don't like understand the whole, like everything you just explained. Like why that, the whole thing, like what you were saying like why it's like the spiral unraveled or something like that. Like I don't even know how to explain, the points and just trying to follow and I just didn't.
00:48:53:27	Robert	S(ans)	It's kind of hard to explain.
00:48:54:10	Sherly	S(qs)	Which one are you on?
00:48:56:00	Angela	S(a)	Huh?

00:48:57:07	Sherly	S(a)	No, I was talking to Michelle.
00:48:59:08	Angela	S(qs)	Do you get this?
00:48:59:08	Ashley	S(qs)	Do you understand?
00:49:01:09	Michelle	S(ans)	What he just said? Kinda.
00:49:06:18	**Dr. A**	**T(d),**	**Could you, could you try Michelle, to explain.**
		T(r)	**Cause every time one of you explains it, it helps me a little more. This is really just as foggy for me, Angela, as it is for you. I am even further away than you, from this stuff, because I don't understand the calculator either. So Michelle could you try it again.**
00:49:26:18	Michelle	S(pb)	Okay, um. Ooh. Alright if you took, let me draw a piece of the spiral, and you picked like certain points, whatever ones they were. Right? Robert?

These questions were coded *discoursive* because the teacher is asking another student to provide input about what is being discussed.

When a teacher uses a *discoursive question*, it appears they want to promote involvement of other students. Rather than have a two-way conversation between the teacher and student, a reasonable conclusion is the teacher wants to hear the thinking of other students. If other students are involved, then the teacher does not become the center of the conversation, and using these questions could be the teacher's way of placing themselves on the edge of a conversation. Additionally, the teacher can use these

questions to elicit the thinking of more students, which can promote more student-to-student conversation.

Retracing Question. This category developed from a pilot study involving another table of students from the Kenilworth data. The researcher, Bob Speiser, continually referred back to previous discussions each time he visited a table of students in order to provoke thinking about the group's solution. This type of question appeared again in this study. A teacher's utterance is coded as a *retracing question*, T(r), when: a) the teacher brought up a specific idea; b) there was verbal evidence that the idea was mentioned in the conversation prior to that point.

The examples of this question category from the Old Bridge data are shown in the transcript. The class had already found the area under the graph of the derivative of $f(x) = 2x^3 + x^2$ using several different techniques. The transcript starts after Carl has gone to the board and begins to write an expression for finding the area using an infinite number of rectangles. The teacher calls on Kristen, who had her hand raised.

00:51:40:20	Kristen	S(ta)	Shouldn't it be 2 over n cause that is how you did everything else?
00:51:46:09	Carl		(inaudible) [calls on a student to answer]
00:51:53:01	Matthew	S(ta)	It should be. That first expression should be n over 2, which that would be. So that top number which is n is four. So the number in the denominator is four.
00:52:03:25	Kristen	S(i)	But that.
00:52:05:02	Matthew	S(qs)	What?

00:52:06:00	Dr. G	T(r)	**Kristen, let us hear your argument again please.**
00:52:10:22	Kristen	S(pb)	Okay, it should be 2 over n. because like to get one half, you do two over four. Cause it was like you have four triangles and the area two. Two. So it was two over four which is one half and you had two over ten which is one fifth. And so on. So wouldn't it be two over n.
00:52:35:16	Dr. G	T(con-t), T(d)	Any disagreement? Everyone understand that. What'd she say Chris?
00:52:47:19	Chris	S(c)	She said that since those four triangles and those were split into two parts. I mean four rectangles. It was two over n, which was reduced to one half and that's how we got one half.
00:53:06:26	Dr. G	T(con-t)	Is that what you said Kristen?
00:53:09:27	Kristen	S(ans)	Uh, not really.
00:53:10:24	**Dr. G**	**T(r)**	**Try it again, Kristen. And then we'll back it up and try and have him run it through again.**
00:53:16:18	Kristen	S(pb)	Okay, I don't know how. I'm sure how to explain it. Yeah, You had okay. You had like two, one to three, three minus one is two. So, you have like two spots and you're doing it for four rectangles. So you did two over four is equal to one half and then you did two over ten is one fifth and so on. So to get it

for n, you get two over n. I think that's what I said.

00:53:48:01	Dr. G	T(d)	So, now you try and say it Chris.
00:53:51:07	Chris	S(c)	Yeah, fine. Uh, Since we're going from one to three, the top part is two. and since there's four rectangles, the bottom part is four.
			Dr. G praises Kristen's response. Over the next 14 minutes, the class continues to work on the same problem. A new problem is introduced giving the derivative function and determining the parent function: Area under the curve f(x) = 2x^2 + 3x over [0,1].
01:12:22	Dr. G	T(p)	Alright. Now it would be a good idea to set up the limit expression before we get out of here because that's one that probably the most difficult to set up. Mike it's all yours
01:13:04	Dr. G	T(e)	I want you to talk to us as you go.
01:13:26	Mike	S(seek)	So would I start with a sigma right here.
01:13:30	Dr. G	T(p)	Why don't you start with a rough graph?
01:13:34	Mike	S(a)	So I'm making the graphs.
01:13:35	Dr. G	T(con-s)	Yep.
01:13:36	Mike	S(con)	Okay.
			During the next minute of discussion, the Mike and

			Dr. G debate whether a graph will help.
01:14:43:04	Mike	S(a)	So just, uh, write the equation.
01:14:48:29	**Dr. G**	**T(r)**	**Now, why one over n. Awhile ago it was two over n.**
01:14:55:00	Mike	S(pb)	Because the uh, zero to one, not one to three.

The first two questions received a *retracing* code because they referred back to Kristen's original explanation of what the width value should be for Carl's expression. The third question received a *retracing* code because it referred to the idea of how to find the width of the rectangles in the discussion initiated by Kristen earlier.

In the Kenilworth data, Dr. A approaches the group to catch up with their progress since her last visit to the table. She inquires about what the group has done.

41:59:18	Victor	S(ta)	No I just took all the measurements that I had before and I added them up and got 35.3 and then divide that by 90. Had a new
42:07:22	Dr. A	T(j)	Why?
42:08:21	Victor	S(ta)	Instead of 13
42:10:00	Dr. A	T(j)	Why?
42:11:11	Sherly	S(ans)	Just to see
42:11:21	Victor	S(pb)	My thought was that, maybe it's just like the average of 90 degrees. Not the average and then the average again. You never take two averages of something. So I just took the average over every 90 degrees and got .3922

centimeters change. And then multiply that by total

degrees, which was1027.5 and got 47.

42:38:22	Dr. A	T(r)	**You have, you have to catch me up on that. You added up all these radiuses. R1 through R13.**
42:50:06	Victor	S(ans)	35.3
42:51:27	Dr. A	T(con-s)	35.3
42:53:12	Victor	S(con)	exactly
42:53:26	Dr. A	T(r)	**That was the number that was missing awhile ago. Why did you divide it by 90?**
43:01:04	Victor	S(pb)	let me think, um. let me see now. I lost the thought here. Okay. What I started to think was that maybe it's not. Cause what we did was take two averages, and I never heard in mathematics of taking two averages. Like take one average and then take another average of the one you have before. So we just took, instead of just taking the average.
43:27:19	Dr. A	T(r)	**Can, can you go back to the first time? I'm not sure. I wanna know what those two averages are. You said the first thing you did was to added up what you considered to be all those distances.**

The first two questions received a *retracing* code because they referred to ideas in

Victor's statement about why he measured the radii and divided by 90. The third

question refers to the averages Victor mentions in justification of why he divided by 90 (43:01:04) so the teacher's statement fits the *retracing question* category.

The possible intent of a *retracing question* is for the teacher to elicit repeated student thinking about an idea. Through these types of questions, the teacher can attempt to have the student reorganize their thinking each time the teacher takes them back to the idea. The first time the student verbalizes a mathematical idea may not be clear, so the teacher could use *retracing questions* to give the student multiple opportunities to explore their thinking and develop their understanding into a more sophisticated mathematical argument.

Explanation and Clarification Questions. These two questions categories are presented together because they both require a student to explain their thinking. However, they have different requirement for the type of student thinking expected. A teacher's utterance is coded as an *explanation question,* T(e), when: a) the teacher requested the student to verbalize their thoughts; b) the student had not made their thinking public prior to that point. A teacher's utterance is coded as a *clarification question*, T(c), when: a) the teacher sought more information from the student about a particular explanation or idea the student presented; b) the student had previously explained the idea verbally.

In order to demonstrate the difference between these question categories the transcript selected provides examples of both question types. In this part of the Old Bridge data, a student is simplifying an expression that represents the area under a curve. She writes $\Sigma((i^2(1/16)+1/2\ i\ +1)+1)$ on the board and is asked to simplify it.

| 35:12 | Dr. G | T(e) | **Can you do anything with the expression to further** |

simplify it on that line?

35:21	Ricci	S(seek)	Do I even need this parenthesis (points to last parenthesis in expression) Can I just combine those two? (point to the parenthesis at the end of the expression)
35:25	**Dr. G**	**T(c)**	**What do you think?**
35:27	Ricci	S(ans)	Yes
35:28	Dr. G	T(j)	Why? What purpose do they serve?

After a short discussion about the purpose of the parenthesis, the conversation continues:

37:08	**Dr. G**	**T(e)**	**I want to get a common factor as the fraction 1/16th and somehow move it to the left of the sigma notation. How does that happen?** **[Student writes: 1/16 sigma (i^2+1/32i]**
38:11	**Dr. G**	**T(e)**	**Now explain how that happened Ricci**
38:14	Ricci	S(ta)	Uh, I did 1/16th times 2. Well, not actually, I did. I figured if 1/16th was out here to get 1/2, it'd have to be 1/32
38:33	**Dr. G**	**T(c)**	**Show me how that worked? [Student writes 1/16*x = ½]**
38:56	Ricci	S(seek)	Like that
38:59	Dr. G	**T(c)**	**Well, I am still waiting for you to show me how it works**

The questions in the transcript received an *explanation* code because the teacher asked the student to explain how to simplify an expression after the student silently wrote some mathematics on the board. The student does not answer the second *explanation question* verbally so the next question is coded as an *explanation question* because the student's thinking is not public at this point of the conversation. The first question is coded as a *clarification question* because the question is requesting more information from the student about their ideas in the previous statement. The next two questions received a *clarification question* code because they are requesting more specific detail than the verbal or written response given by the student.

In the Kenilworth data, Dr. W visits the students and discusses the work they have done to this point.

03:14	Dr. W	T(d)	You mind if I join you for a few minutes.
03:16	Robert	S(con)	Sure
03:18	**Dr. W**	**T(e)**	**I've been looking around other places and I haven't been able to see what you guys are really doing, but what you have here looks very interesting and I wondered if you could maybe you could kind of walk, walk by me what you've been doing. You know what you've got it's very interesting what's developed here.**
03:45	Angela	S(ta)	We started off with we traced the spiral and then we drew the like you know the axis thingy and then um we measured the distance well from the center of these points where it you know intersects.

04:02	Dr. W	T(con-s)	Okay
04:03	Angela	S(ta)	We wrote it down there that's why it's letter because that's the original one
04:06	Dr. W	T(con-s)	Okay
04:07	Angela	S(ta)	Then that's like zero degrees, zero radians or something like that.
04:14	**Dr. W**	**T(c)**	**That's this (points to red axis on transparency)**
04:16	Angela	S(ta)	That's the original in there, which we have on the chart. Then we drew the first line and we had to figure out the angle. So we made a right triangle using a ruler like that
04:34	Dr. W	T(f)	So you just drew that line in
04:36	Angela	S(ans)	Anywhere
04:37	Dr. W	T(f)	Somewhere that looked interesting
04:38	Angela	S(ta)	Yeah where ever we felt like it. And then we measured the different distances and we figured out the angle in degrees using inverse cosine. And then we converted to radians and we have the measurements right up there (points to corner of transparency)
04:53	**Dr. W**	**T(f), T(c)**	**So how did you use the inverse cosine here?**
04:57	Angela	S(c)	We did the adjacent divided by the hypotenuse and then inverse cosine
05:04	Dr. W	T(con-t)	So you measured a couple of line segments. Is that true?
005:10	Angela	S(con)	Um hum

05:12   **Dr. W**   **T(c)**   **So which line segments?**

05:16   Angela   S(c), S(pb)   Well for the angle we did this to there. (pointing to transparency) and there to there. That's the hypotenuse. I mean we could have use um, what is that tangent, no? opposite over adjacent. yeah, tangent like for measuring that, we just did cosine. We could have used anything actually. Um, so we figured out the angle and converted it to radians and we have it right up there and there too(pointing to transparency) . Then we measured out the segments, you know where the points are. How far away from the center and that line. Then we did it on this one too (takes out second transparency)

The first question is coded as an *explanation question* because this is the first time Dr. W is asking the students to share what their work on the problem. For the first and third question coded that received a *clarification question* code, Dr. W is asking Angela to be more specific about the zero axis and the line segments she mentions. The second question coded as *clarification* received the code because Dr. W asked for more information about how the inverse cosine function was used.

It appears a teacher uses these questions as method for eliciting thinking from one student. This is a different intent than *discoursive questions*, because the thinking comes from the same student. The idea behind an *explanation question* may be to have a student verbalize their thinking because they are performing mathematics without talking.

For a *clarification question,* the intent may be to have the student verbalize their thinking further to provide more robust details. Both questions allow the teacher to better understand they way students are reasoning about the mathematics they are doing.

Justification Question: Understanding students mathematical thinking involves knowing more than a description of their mathematical actions. It also involves knowing what aspects of the mathematical situation made their actions valid and appropriate. Teachers in both studies sometimes sought justifications for how they knew certain facts or why certain actions were permissible. A teacher's utterance is coded as a *justification question,* T(j), when: a) the teacher requests the "because" of a mathematical idea already discussed; b) the wording in the question demonstrates a need for the student to provide a form of proving, convincing or providing an example to support reasoning.

The simplest form of this question is "Why?", which is asked several times by all the teachers. The examples below provide other questions that were identified in this category.

The Old Bridge transcript example provided for *discoursive questions* also contains several examples of questions in the *justification* category. At this point of the transcript, Kristen is at the board and has found the derivative of $f(x) = x^2 + 1$ in order to determine if the function is increasing or decreasing over the interval [1, 2].

06:12	Dr. G	T(s)	Now over the interval 1 to 2 inclusive. Two x, is it positive or negative?
06:24	Kristen	S(a)	What was the question? Sorry
06:29	Dr. G	T(d)	Jason repeat the question
06:31	Jason	S(ans)	Is the derivative positive or negative?

06:33	Dr. G	T(f)	Over the interval
06:34	Jason	S(ans)	from one to two
06:37	Kristen	S(ans)	positive
06:38	**Dr. G**	**T(j)**	**How do you know that?**
06:51	Kristen	S(dnc)	Um, I have no clue. Just explain it to me.
06:53	Dr. G	T(d)	Eric, how do you know it's positive or how do you know it's negative?
06:58	Eric	S(pb)	Ah, I think because if you solve for x. and uh, I don't know it's positive
07:08	Dr. G	T(d)	Becca
07:09	Becca	S(pb)	Because the derivative is the slope, so it's like going up two and over one
07:20	Dr. G	T(d)	Matthew
07:21	Matthew	S(pb)	If you fill in any number between the interval between one and two, the answer is positive because graph is
07:31	Dr. G	T(f), T(p)	Graph y equals 2x between 1 and 2. Rather, I am going to change that slightly. Graph y equals 2 x over the interval 1 to 2 inclusive. [Kristen starts to graph 2x on the board]
07:55	Dr. G	T(p)	On a new graph. [Kristen draws a new coordinate axis and graphs y = 2x]
08:21	Dr. G	T(r)	Now, is the derivative always positive on that

interval?

08:38	Kristen		Um
08:39	Dr. G	T(d)	Time out. Repeat the question Brittany
08:44	Brittany	S(ans)	Is that always positive in the interval?
08:47	Dr. G	T(c)	No pronouns
08:51	Brittany	S(c)	Is the derivative always positive on that interval?
08:53	Dr. G	T(d)	Kristen
08:55	Kristen	S(ans)	Yes
08:55	**Dr. G**	**T(j)**	**How so? How is it we know this? How did you make that decision?**
09:02	**Dr. G**	**T(j)**	**By the look on your face that was a fifty-fifty guess type thing?**
09:10	Kristen	S(dnc)	I don't, I don't, I have no clue
09:12	Dr. G	T(d),T(c)	Alright. Rachel, explain what we mean by the derivative being positive over that interval.
09:18	Rachel	S(pb)	Is it because of all the number that you put in for are going to be positive for x and when you multiply by 2, it is going to be positive

Dr. G. changes topics after hearing Kristen's response. These questions were coded as *justification questions* because they ask a student how they knew the information they gave in a previous answer.

In the Kenilworth data, Robert has been helping Ashley and Angela perform a cubic regression on their calculator. Victor agrees with the answer and explains what he has done to the group. Dr. A interacts with the table when Victor finishes.

34:34:07	Dr. A	T(r)	Can I ask one more time how you got this number?
34:37:01	Sherly	S(pb)	I don't know. Okay, Um. Oh yeah, cause this number this is degrees and in radians it's 6.71. So instead of dividing total distance of 47.5 in by degrees, I did it by radians.
34:56:20	**Dr. A**	**T(j)**	**I understand that. yes, that's what I thought you were saying, but. Can you show me?**
35:05:21	Sherly	S(ans)	Well it's already there. Do I have to do it over again?
35:07:03	Dr. A	T(f)	No, not if you talk me through it. Okay, So 6 point 71.
35:17:19	Sherly	S(ans)	That's how it just came up.
35:20:06	Dr. A	T(c)	But is the pi in the number.
35:21:20	Sherly	S(ans)	Well yeah, cause look it's right there.
35:24:14	Dr. A	T(c)	so its 47.5 times 6 point
35:26:16	Sherly	S(ta)	No divide by
35:29:10	Dr. A	T(c)	71. hmm.
35:33:25	Sherly	S(c)	See, that's what was in here.
35:36:09	**Dr. A**	**T(j)**	**My only question is. Why is that (whispering low) to know that you're dividing by it.**
35:41:26	Sherly	S(pb)	Because I typed it in and that's how it came out.
35:43:27	Dr. A	T(f)	Okay. I see. So. Can you just on the calculator show me what 6.71 times pi is.

35:56:24	Sherly	S(ans)	Maybe cause I don't if it sometimes, it doesn't sometime show up.
36:02:29	Victor	S(i)	A couple times that decimal will show up.
36:04:24	Sherly	S(con)	It will
36:06:20	Victor	S(ans)	Like 2.0
36:07:04	Sherly	S(con)	Okay
36:12:13	**Dr. A**	**T(j)**	**So it's 21 point 'oh' eight 'oh' one. Okay now can you show me**
36:19:05	Sherly	S(ta)	Divide 47.5 by that

These first and third questions were coded as *justification questions* because they asked the student to show the teacher proof of the student's idea. The second example is missing a few words, but the words 'why' and 'know that' ask the student to give the 'because' and reason for their division.

A possible reason for asking a *justification question* is to have students provide support for their reasoning. The teacher is seeking for a reason behind the student thinking that is mathematically supported. The main difference between a *justification* and *clarification question* is proof. There is an important distinction between a *clarification question* and a *justification question*. The former type of question seeks more detail about a description of a student action, while the latter seeks mathematical support for why the student took the action. The teacher's intent for asking a *justification question* may be to have the student ground their reasoning in the mathematics to help

students support their understanding from an external logical perspective rather than an internal personal one.

Confirmation Question. Due to the large number of students either participating or listening to the mathematical conversation in a classroom setting, it is important that the students comprehend the discussion. The prevalence of the teachers checking to make sure they agreed with the students as well as making sure the students were in agreement with each other led to the development of this question category. A teacher's utterance is coded as a c*onfirmation question*, T(con), when there is a request for agreement between the teacher and student. This question category is broken up into two sub-codes because of the bi-directional way agreement can occur between the teacher and student. A teacher's utterance is a coded as a *confirmation question – teacher*, T(con-t), when the teacher seeks agreement from a particular student or the class about ideas stated in the discussion. A teacher's utterance is coded as a *confirmation question – student*, T(con-s), when: a) the teacher confirms agreement with a student statement; b) the teacher provides an indication they are following the student response. Part b of the definition means the teacher's utterance may not be in an interrogative statement. The non-interrogative part of the category does not conform to teacher questioning, however; a basic generalization of this category can be considered making sure everyone involved is "on the same page".

The example from the Old Bridge data shows utterances coded as *confirmation questions* using either criteria. At this point of the transcript, Matthew is at the board and he is leading the class in a discussion about how to determine if the derivative previously

calculated is positive or negative on the interval [1, 2] without graphing or plugging in numbers.

31:37	Dr. G	T(c)	How is it that you choose only to do it with the x rather than with the base of two as well?
31:46	Matthew	S(ans)	I didn't realize that I can do both
31:49	**Dr. G**	**T(con-t)**	**Okay**
31:51	Matthew	S(ans)	I'm just going to put that there so I remember. Okay [Erases 2 in the denominator and changes exponent on 2 to (-1/2)]
32:03	**Dr. G**	**T(con-s)**	**Now what do we have there. Negative two to the negative one-half power times x to the negative three-halves power. Everybody agree with this?**
32:14	Chris	S(ans)	No
32:18	Dr. G	T(c)	Alright now you got to talk to them because they're not with you.
32:20	Matthew		Chris
32:21	Chris	S(ta)	Alright when you move the x squared up. You did what? You
32:27	Matthew	S(pb)	You subtracted. You have this and you subtracted two from it.
32:35	Chris	S(con)	okay.
32:40	**Dr. G**	**T(con-s)**	**Is everybody alright?**
32:41	Eric	S(ans)	No

32:42	Dr. G	T(d)	Eric. Talk to him.
32:43	Eric	S(qs)	I don't understand. I don't understand any of this. I don't know what you did.
32:45	Matthew	S(seek)	Alright. Should I show it on the side?
32:46	Dr. G	T(p)	Hang on a second.
			Class converses as Dr. G works with Eric on two exponent rules. Class is interrupted by an announcement. A student checks what Eric has written on the board. Eric fills in another rule.
34:25	**Dr. G**	**T(con-s)**	**Now Eric do those manipulative rules make sense to you?**
34:29	Eric	S(ans)	Uh. Yeah, I think. Yeah.
34:37	Dr. G	T(d)	Question Yla.
34:38	Yla	S(qs)	I don't get what happened to the two?
34:41	Dr. G	T(f)	Talk about the two
34:44	Matthew	S(pb)	Alright. It was just the same thing, because, but instead of a variable, it was just. Two is kind of like the x. So it's two to the one and that's two to the one-half. So then it would be two to the one-half over two to the one and then you do one-half minus one. Do you understand that?
35:06	Yla	S(con)	Yeah.
35:09	Dr. G	T(r)	Now what rule are you going to contemplate for your

derivative?

35:16	Matthew	S(ans)	Power Rule.
35:19	**Dr. G**	**T(con-t)**	**Alright.**
35:23	Matthew	S(ta)	I'm just a little confused. You can't do anything right now. You don't use. You just turn that into something more easier to see right. Cause you don't do the power.
35:38	Chance	S(a)	What? Two to the negative one-half.
35:39	Matthew	S(seek)	Cause it's a constant right?
35:41	Dr. G	T(f), T(c)	What's a constant? Where is your constant? Tell me what your constant is.
35:48	Matthew	S(ans)	Negative two to the negative one-half.
35:49	**Dr. G**	**T(con-t)**	**Correct**

There are six questions highlighted in the transcript. The second, third and fourth utterances were coded as *confirmation questions – student* because the teacher sought agreement about the mathematics discussed. The fourth question addressed an individual student, while the other two address the class as a whole. The other utterances are not interrogative statements, however; they were coded as a *confirmation question - teacher* because the teacher explicitly states they agree with the student's idea.

The example of utterances in this category of questions from the Kenilworth data shows both parts of the definition as well. In this transcript example, the students have

been talking with Dr. A about how they calculated an average from their data. The transcript starts as Mrs. W[3] enters the ongoing conversation.

18:38:02	Victor	S(ta)	We got a number and then we divided that by 13 and got the average. And then we divided that by 90 degrees and got the rate of change, per degree.
18:49:15	Dr. A	T(j)	You need to start. I don't understand how's that.
19:01:00	**Mrs. W**	**T(con-t)**	**I, I understand how you mean**
19:02:00	Victor	S(con)	You understand the box points.
19:03:18	Mrs. W	T(r)	I understand where these points came from. I'm cool with that. The thing that I, that then I get lost is...I understand you added them up.
19:13:23	Victor	S(con)	Okay, what we did was we added them up, right.
19:16:24	Mrs. W	T(j)	Why? Why?
19:18:00	Victor	S(pb)	So we can get an average, we are trying to find an average.
19:20:09	Mrs. W	T(c)	Average what?
19:21:15	Victor	S(ans)	Average distance per 90 degrees.
19:25:09	Dr. A	T(c)	Distance of what?
19:26:15	Victor	S(ans)	Of this whole thing.
19:29:17	**Mrs. W**	**T(con-s)**	**So, you're taking the distance from the origin to the points, and added all those distances up,**

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[3] Mrs. W. is a teacher present in the research setting. As noted in the methodology section, when other adults interacted with students their questions were counted; however, the detail analysis of questioning themes only used the interactions of Dr. A.

19:35:03	Victor	S(ta)	And divided by
19:35:03	**Mrs. W**	**T(con-s)**	**and so you're trying to find an average**
19:37:08	Victor	S(ta)	average.
19:37:17	**Mrs. W**	**T(con-s)**	**of those distances?**
19:39:04	Victor	S(con)	Yes. Average of (inaudible).
19:42:06	Victor	S(con)	Yes, an average.
19:42:20	Mrs. W	T(e)	That's what you did, but so you added up all the lengths, to get the, what number?
19:48:00	Victor	S(ta)	Average. Uh, I didn't write it down. But then we found out the average. Which was all these added up,
19:54:23	**Mrs. W**	**T(con-t)**	**Uh huh.**
19:54:23	Victor	S(ta)	divided by 13. We got 13 points. And we got the, that's, so 2.
20:00:08	**Dr. A**	**T(con-t)**	**That's the 2.36.**
20:01:04	Victor	S(ta)	2, 9, 2, 3. That's the average the, that's the average distance for every 90 degrees, of change from like
20:11:01	Victor	S(ta)	From this point to that point the average is about 90 degrees.
20:14:20	**Mrs. W**	**T(con-s)**	**And when, okay. So you're saying the average**
20:17:29	Victor	S(ans)	From point to point.
20:19:23	**Mrs. W**	**T(con-s)**	**of every 90 degrees, it's because each of those points are 90 degrees?**
20:24:27	Victor	S(ans)	Apart.

20:25:08   Mrs. W   T(con-t)   Apart.

20:26:29   Victor   S(con)   Okay.

Dr. A re-enters the conversation and with Mrs. W continues to question the students about the average they found. The are five utterances shown in the transcript because the three utterances from 19:29 to 19:37 and the two from 20:14 to 20:19 are a continuous flow of Mrs. W's questioning. These two groups of utterances were coded as *confirmation questions – student* because they were seeking agreement about the student's ideas. The remaining three utterances, coded T(con-t), are teacher statements of agreement about a student idea.

An intent of asking *confirmation questions* may be informal assessment by the teacher. The teacher sub-code indicates a teacher questioning a student for agreement and therefore assessing their understanding of the mathematics under discussion. The student sub-code indicates a teacher agreeing with a student response and therefore providing the student feedback about their understanding. The student assesses from the teacher their thinking is being understood and they can continue to verbalize their mathematical thoughts. Either form of this question can provide the participants with an informal assessment of where they stand with regards to understanding the ongoing discussion.

Following Question. When a teacher operates within a student-centered setting, they must respond to student's statements and thoughts. Within both research settings, a teacher based their questions on something the student said, which led to a code for this category of teacher questioning. A teacher's utterance is coded as a *following question*,

T(f), when a) the teacher speaks immediately after a student statement or action; b) the utterance is directly related to an idea in a student's statement or action.

In the Old Bridge data, Dr. G. writes $F(x) = 3x^2 + 2x$ and gives directions to graph the derivative of the function over the interval [1, 3] without using a calculator. A student has completed calculating the derivative and determining the value of the derivative at the endpoints of the given interval.

08:13	Dr. G	T(d)	Question.
08:13	Chris	S(ta)	Um, isn't this going to be a linear function because well, are we going to estimate the area the usual way?
08:25	Dr. G	T(c)	I don't understand the question.
08:28	Chris	S(c)	Well the function is a linear function right, the derivative?
08:32	Dr. G	T(con-s)	The derivative is a linear function, I agree.
08:34	Chris	S(ta)	Okay, then uh, like, perhaps I'm jumping too far ahead, but wouldn't it be simpler if we just used the area of a triangle and added the area of a rectangle on to it.
08:48	**Dr. G**	**T(f)**	**Could be. Let's draw the graph. [Michael graphs the derivative function 6x+2 on the interval 1 to 3.]**
09:46	Dr. G	T(r)	Alright now. We are attempting to find the area under that curve. Now, what is it you said about that region, Chris?
10:06	Chris	S(ta), S(c)	Um, you could simply take the area and split it up into two separate parts. One a right triangle.
10:19	**Dr. G**	**T(f)**	**Go show us this stuff. Thank you, Mike**

| 10:23 | Eric | S(i) | This is a good way to find the area beforehand and then we can do all that other crap. |
| 10:29 | Chris | S(ta), S(seek) | Yeah, exactly, yeah. We could split this area up into two separate parts. A right triangle, represented her and a rectangle represented here. Okay, so um. So, is there something specific you want me to explain? |

These utterances were coded as *following questions* because the teacher's statements are made based upon what Chris said in the previous line of dialogue. For the first question, the student has stated if geometric areas can be used and the teacher next step is to draw the graph. This was coded as a *following question* because a graph will allow the class to see if there are geometric figures to calculate an area. The second teacher utterance telling Chris to show the class was coded as a *following question* because it was a direct response to Chris stating how to break up the region in order to find the area.

Another sample transcript is provided from the Old Bridge data to show a question being coded as a *following question* because of an action of the student. In the transcript, Dr. G tells the student to draw a graph. The question was coded as a *following question* because the question is related to the student drawing a graph of the second derivative.

| 11:09 | Dr. G | T(p) | New graph. This time the second derivative. Same interval. [Chance draws a coordinate axis and the line y = 2.] |
| **11:28** | **Dr. G** | **T(f)** | **Very good. Now is the second derivative positive or** |

			negative over this interval?
11:35	Chance	S(pb)	Well it has the slope of 0. So. So. I mean, I know it's positive, like about the x axis but
11:43	**Dr. G**	**T(f)**	**Okay, Alright. Since it's positive. We know this because the graph exists totally above the x axis**
11:51	Chance	S(con)	Right
11:51	**Dr. G**	**T(f)**	**that tells us what about the original parent function?**

The first teacher question received a following code because the student drew the second derivative and the teacher's next question was based upon that drawing. The second question received a following code because the teacher uses the student's answer of the graph being positive.

In the sample transcript from Kenilworth, Dr. A has been discussing with the students the meaning of a number they found earlier in their work for the average rate of change. The group is trying to determine I this value is an average distance around the spiral and have measured the picture of the spiral using a string. Dr. A has been questioning the students about how they found their value for the average change from data point to data point.

46:24:11	Dr. A	T(con-s)	Okay, so now
46:25:24	Victor	S(c)	this number
46:27:06	Dr. A	T(c)	Which is?
46:28:02	Victor	S(pb)	This is the new rate of change. This is when we added all

the distances. All these up.

46:31:16	Dr. A	T(con-s)	Yeah
46:32:08	Victor	S(pb)	and 35.3. divided by 90 degrees. Instead of by 13 and then by 90, we just divided by 90. And I got 3.92 repeating.
46:41:23	**Dr. A**	**T(f)**	**Okay, you added them all up and you got 35.3 and then you divided by.**
46:49:21	Victor	S(ans)	90
46:50:18	**Dr. A**	**T(f)**	**By 90, and you got.**
46:53:12	Victor	S(dnc)	uh.

These two questions were coded as *following questions* because the teacher uses the student's previous statements to form the question.

The possible intent of asking a *following question* is for the teacher to use student ideas to move toward an agreement or solution or other goal. The teacher may be following the student's thinking and wants to use the student's ideas to move the conversation forward toward a goal. The goal may not be clearly stated, but the use of student ideas to reach that goal may be interpreted from the teacher questioning.

Suggestion Question. In contrast to asking a question based on what students think, there are times when the teacher provided a path for the students to follow. A teacher's utterance is coded as a *suggestion question*, T(s), when: a) the teacher interjects an idea into the conversation; b) the idea has not been discussed previously.

This question was combined from another category called *new*, which was originally defined to signify when the teacher asked the student to move in a new direction from the current conversation. Since giving ideas to students either based on their conversation or to move the conversation in a new direction, are both suggestive, the *new* question category changed to suggestion.

The transcript example provided from the Old Bridge data, occurs when the students have written an expression for one of four rectangles used to approximate the area under the curve $x^2 + 1$. Amanda is the student at the board.

23:50	Dr. G	T(c)	The constant width[points to 1/4] The value of x we use as input into the function.[points to (1+2(1/4)]. The output of the function equivalent to the height of the second rectangle.[points to ((1+2(1/4))^2+1)] Very nicely done. Finish that whole line while you're up there Amanda. [Student writes: 1/4((1 + 3(1/4))^2)+1) + 1/4((1+4(1/4)^2)+1)]
24:53	**Dr. G**	**T(s)**, T(j)	**Alright Amanda. Assuming you did all that already. Would our estimation of the area be too small or too large?** And Why?
25:06	Amanda	S(ta)	Um, too large
25:10	Dr G	T(j)	Too Large. Tell me why?
25:15	Amanda	S(pb)	Because these are higher. [points to rectangle on graph.]
25:19	**Dr. G**	**T(s)**	**So, we're going to go ahead and do these four and get an idea, a rough idea, for what the area is. Then we'll**

rework the same situation for an infinitely large

number of rectangles. So, thank you Amanda. Mikey,

summarize that line, will you?

The first teacher question received a *suggestion question* code because the teacher introduces the idea of over or under approximation when using rectangles to calculate the area under a curve. In the second question is coded as a *suggestion question* because the teacher mentions using an infinite number of rectangles to go beyond the "rough idea" for area using only four rectangles. Introducing this idea moves the mathematics of finding the area under a curve toward a more precise answer.

In the Kenilworth transcript provided, Dr. A is discussing with Sherly the method used for calculating the value for the variable alpha the group is using in their equation modeling the shell's growth.

36:54:16	Dr. A	T(p)	But it's because you're not dividing by it. Divide 47.5 by 6.71. Okay. And multiply the answer times pi. What do you get?
37:17:17	Sherly	S(ans)	22.2.
37:19:14	**Dr. A**	**T(s)**	**Yeah see you're not dividing. Your pi doesn't (?) the denominator. You understand what I am saying.**
37:24:14	Sherly	S(con)	Oh.
37:25:23	**Dr. A**	**T(s)**	**It's all in the wrong (?). So the number that you should be using isn't this one. It's that other one.**
37:34:07	Sherly	S(con)	What one?

37:35:27	**Dr. A**	**T(s)**	**It was this one.**
37:37:15	Sherly	S(con)	Oh, yeah.
37:38:23	Dr. A	T(con-t)	Do you understand what I am saying?
37:40:00	Sherly	S(con)	Okay.
37:41:11	Dr. A	T(p)	Now can you, can you, can you graph it. This one became. Is that what your A is?
37:50:10	Sherly	S(a)	My A.
37:51:14	Dr. A	T(con-s)	Uh, huh.
37:51:28	Sherly	S(seek)	should it be that then?
37:53:07	Dr. A	T(con-s)	Uh huh.
37:53:16	Sherly	S(con)	Okay.
37:54:01	Dr. A	T(con-t)	I mean let's see what happens. Do you understand what.
37:56:19	Sherly	S(con)	Yeah.
37:57:01	Dr. A	T(con-t)	what we're trying to do.
37:57:21	Sherly	S(con)	uh, huh. Hold on.
38:00:06	Dr. A	T(p)	Put it in for another function.
38:02:11	Sherly	S(con)	Okay, I just wanna.
38:08:07	Dr. A	T(p)	It's 21.08. Wait a sec.
38:15:21	Sherly	S(ta)	No I didn't put 7.
38:18:27	Dr. A	T(c)	21.08 and do 47.5, is that what you're saying?
38:25:15	Sherly	S(con)	Yeah.
38:26:07	Dr. A	T(con-s)	Okay.
38:54:19	**Dr. A**	**T(s)**	**It was 2.253.**

38:55:26	Sherly	S(ans)	Something like that. It just totally didn't work.
39:09:07	**Dr. A**	**T(s)**	**I think your window, your window's up too much.**
39:46:12	**Dr. A**	**T(s)**	**Can you get in between those two things?**
39:48:06	Sherly	S(con)	Oh.
39:51:10	Dr. A	T(con-t)	You know what I mean.
39:52:03	Sherly	S(con)	Yeah.

The first three questions were coded as *suggestion questions* because they suggest to the student their operations were incorrect and they number being used was not the intended value. Because of this idea, the students could move forward with graphing a model of the spiral on their calculator. The next question received a suggestion code because Dr. A provided Sherly with the number needed to attempt to graph a new spiral. The final two questions were coded as *suggestion questions* because the teacher is suggesting proper graphing windows to see the new function on the graphing calculator.

The intent of a *suggestion question* may be to help the students move forward with the mathematics. The teacher may have decided that without interjecting some information into the conversation, the students will not be able to move toward a solution, reach agreement about their understanding, or develop a justification for their thinking. The teacher's intended result of asking a *suggestion question* might be to allow students to consider the teacher's idea so they can continue to discuss the current mathematics or a new mathematical concept in order to reach the goal of the lesson or problem.

Procedural Question. When a student is working at the board or as part of a group in a student-centered classroom, they do not always know what steps to follow the

next. The teacher can be considered having an orchestrating role for deciding what students will lead the work at the board or what students will share their knowledge from a group. A teacher's utterance is coded as a *procedural question*, T(p), when: a) the teacher gives the student directions; b) the direction is related to an explicit/specific mathematical task. A *procedural question* is different from a *suggestion question* because the teacher's utterance requests the student to perform an action. *Suggestion questions* focus on information about the mathematics while *procedural questions* focus on directing mathematical actions, such as where to record mathematical work on the board.

This question does not always refer directly to the mathematical procedures student perform when solving a problem. The intent implied by "procedure" is a directing of student actions in the classroom. As a clarifying example, the teacher may ask the student to perform a mathematical procedure, such as, simplify the expression or the teacher may ask the student to go to the black board to point at an inscription. In the first example, the student completes a mathematical procedure, and, in the second example, the student completes a physical action. Both instances represent a *procedural question.*

In the Old Bridge, a student has been sent to the board to write the area under the curve already written by other students as the area of rectangles using summation notation. The students have discussed the first attempt and Mikey realizes he must include the variable i in his statement.

| 29:47 | Mikey | S(seek) | Can I start? |
| 29:48 | Dr. G | T(con-s) | Yep |

29:51	Mikey	S(seek)	Can I make some room somewhere?
29:54	**Dr. G**	**T(p)**	**Uh, if you want to do that part as an aside stick it up top there some place. Erase that stuff that I wrote. [Student writes (1+i(1/4))(1+i(1/4)).]**
30:05	**Dr. G**	**T(p)**	**Now he's just going to do the multiplication as an aside and then he'll insert.**
30:56	**Dr. G**	**T(con-t),**	[Student writes: (i(1/4))^2 +2(i(1/4)+1] Gretchen?
		T(p)	Everybody alright? **Keep going. Simplify it first. Up there.**
31:07	Mikey	S(seek)	Where? [Teacher erases some earlier work and points for student to continue simplifying in clear space. Student pauses and Dr. G repeats for the student to simplify his work in the space he just erased. Student continues to simplify problem and writes i^2(1//16)+(2/4)i+1. He then moves his work to the bottom of the board.]
32:57	Dr. G	T(con-t)	Everybody alright?
33:01			Several students mumble
33:05	Dr. G	T(d)	Molnar. Talk to him
33:10	Molnar	S(ta)	It says i squared times one over six, it should be one over 16.
33:19	Mikey	S(qs)	Would it be easier to write it the other way?
33:25	Molnar	S(c)	If you want to write it another way, you can just put i squared over 16.

33:29	Dr. G	T(d)	Wait a minute, Rachel
33:31	Rachel	S(ta)	It says at the end one half i shouldn't there be another one at the end
33:37	Student	S(ta)	There's a plus sign.
33:39	Rachel	S(qs)	Isn't it supposed to be plus one?
33:41	Student	S(ta)	No that was part of the (inaudible).
33:45	Kristen	S(con)	You mean there.
33:46	**Dr. G**	**T(p)**	**Walk up here Rachel. Show him. [Rachel goes to the board and points at the plus sign from Mikey's line above and then points to the line he has rewritten at the bottom of the board.]**
33:59	Dr. G	T(con-t)	Alright. Now everybody alright. Ricci next line.
34:06	Ricci	S(ta)	Uh, you could simplify like the.
34:10	**Dr. G**	**T(p)**	**Like, uh, walk up there and write the next line. Thank you Mikey. [Student writes: Σ((i^2(1/16)+1/2 i +1)+1)]**

These utterances were coded as *procedural questions* because in each statement the teacher give specific directions to student about where to write mathematics, how much more mathematics to write, and to physically demonstrate a mathematical point.

In the Kenilworth data, Dr. A is discussing the two different equations the table came up with to model the shell. The students know that the polar function will spiral

while the Cartesian function will not spiral and are determining how to make the spiral

graph model the shell more closely.

01:06:30:24   Dr. A   T(p)   Why don't you, show me what y verses 2 theta would

be.

01:06:35:29   Robert   S(con)   Alright.

01:06:37:05   Dr. A   T(e)   What's (inaudible) your graph?

01:06:39:04   Robert   S(ta)   Ah, I don't think it will be 2 theta will be too big,

because like, I put like 1 theta and it was like huge. So,

I had to put a .01 in front of it to make it fit.

01:06:44:29   Dr. A   T(p)   Oh, well then do point oh two.

01:06:54:13   Robert   S(ta)   You know, it kind of starts, it kind of goes out a little

farther.

These teacher utterances were coded as *procedural questions* because the teacher is

telling the student directly what mathematical task to perform.

One intent of *procedural questions* is for the teacher to make sure the

mathematics is displayed clearly and properly to the rest of the class. This would help

with clarity and provide students with the proper mathematical representations of their

discussions. The teacher may also be selecting students to record mathematics at the

board based on the teacher's knowledge of the students' abilities. If the teacher selects a

student who is better equipped to discuss the mathematics, the student can help the larger

classroom discussion flow in a coherent manner. *Procedural questions* may also be

necessary so the teacher can provide the student with the actions they must take in order

to convince others of their ideas. Students may not realize they can write down their mathematical ideas or use technology to support their ideas and the teacher questions them in order to have them perform the necessary actions.

Repeat Question. Another part of mathematical conversation is speaking clearly so everyone who is part of the conversation can hear each other. Due to students not speaking clearly or other students being focused on another part of the conversation, such as processing what has been written on the board or a previous response, not every statement is hear. A teacher's utterance is coded as a *repeat question*, T(rep), when a) the teacher asks the student to repeat their statement; b) the previous student response was not heard or clearly stated.

This question only appeared in the Old Bridge data. Prior to this point in the transcript, a student has written a numeric representation for the area of a rectangle under a curve. The teacher is asking students to connect the numeric representation to the graphic representation.

| 00:18:06:24 | Dr. G | T(e) | Point to that place on the x axis. Okay. Now notice how easy it was to mix up the words. Area, height, value of x, you can't just use these words interchangeably. You gotta think about the words you're gonna use. So, Chance, if we take that value of x, which you said was right here (points to place on x axis Chance just pointed to) and I wanna know how high this rectangle is, what do I do with that value of x to know how |

high this is?

00:18:49:01	Chance	S(ta)	You add it the the x, x variable.
00:18:51:25	Dr. G	T(d)	Andrew, what do I do (Interrupted by someone coming into class. Responds to the person) Andrew, what do I do?
00:19:05:04	Andrew	S(ta)	Uh. Multiply by the one-fourth. Multiply it by the one-fourth.
00:19:19:29	Dr. G	T(j)	Why?
00:19:21:20	Andrew	S(pb)	Cause one-fourth is supposed to represent the width.
00:19:23:23	**Dr. G**	**T(rep)**	**Louder.**
00:19:25:11	Andrew	S(c)	One-fourth represents the width of the rectangle.
00:19:28:07	Dr. G	T(f)	So, if I multiply it by one-fourth and the one-fourth is the width, you're telling me that is the.
00:19:39:15	Andrew	S(ta)	the height.
00:19:41:28	Maureen		(inaudible)
00:19:43:05	**Dr. G**	**T(rep)**	**I don't think he heard you.**
00:19:45:22	Maureen	S(ta)	It's a value of x.
00:19:47:21	Dr. G	T(f), T(d)	It's a value of x that does what Andrew.
00:19:59:17	Dr. G	T(d)	That does what, Nick
00:20:01:22	Nick	S(ta)	Moves quarterly.
00:20:04:18	**Dr. G**	**T(rep)**	**Louder.**
00:20:04:22	Nick	S(c)	Moves quarterly to the right.

These utterances were coded as *repeat questions* because the teacher indicated that a student response needed to be repeated since the response was not heard. The intent of these questions is obvious, but at times the teacher may be asking the student to repeat a response for emphasis on the importance of the student's response.

Table 4.1

Summary of Teacher Questions

Question Name	Code	Description
Discoursive	T(d)	a) the teacher specifically addressed a particular student or class as a whole; b) the student was not engaged in the conversation immediately before being addressed.
Retracing	T(r)	a) the teacher brought up a specific idea; b) their was verbal evidence the idea was mentioned in a conversation prior to that point.
Clarification	T(c)	a) the teacher sought more information from the student about a particular explanation or idea the student presented; b) the student had previously explained the idea verbally.
Explanation	T(e)	a) the teacher requested the student to verbalize their thoughts; b) the student had not made their thinking public prior to that point
Justification	T(j)	a) the teacher requests the "because" of a mathematical idea already discussed; b) the wording in the question demonstrates a need for the student to provide a form of proving, convincing or providing an example to support reasoning.

Confirmation - teacher	T(con-t)	the teacher seeks agreement from a particular student or the class about ideas stated in the discussion
Confirmation - student	T(con-s)	a) the teacher confirms agreement with a student statement; b) the teacher provides an indication they are following the student response.
Following	T(f)	a) the teacher speaks immediately after a student statement or action; b) the utterance is directly related to an idea in a student's statement or action.
Suggestion	T(s)	a) the teacher interjects an idea into the conversation; b) the idea has not been discussed previously.
Procedural	T(p)	a) the teacher gives the student directions; b) the direction is related to an explicit/specific mathematical task
Repeat	T(rep)	a) the teacher asks the student to repeat their statement; b) the previous student response was not heard or clearly stated.

Student Reponses

A classroom conversation requires student participation either in a dialogue with the teacher or amongst other students. In a student-centered setting, student ideas are a valued part of the verbal dialogue that occurs. The second question of this study is to identify teacher questions that engage students in mathematical conversation. A mathematical conversation occurs when students talk about mathematical ideas, either with the teacher or amongst themselves. In order to attempt to answer the second question of the study, student responses must be categorized to determine if there is a relationship between the teacher question and student response resulting in a verbalization of mathematical ideas.

The codes for student responses identify characteristics of a mathematical conversation and their relationship to teacher questions, discussed in the next chapter, provides insight into how teachers can engage students in mathematical conversation. This section identifies and describes the student responses present in the study. The responses are thinking aloud, proof building, answer, clarification, confirmation, attunement, questions student, seeking, and non-contribution. They are presented in order in which the codes were developed during the inductive analysis.

Thinking Aloud and Proof Building Responses. By focusing on student engagement in conversation as a research question, when students provided responses that were more than one sentence long, the response drew my attention. The longer the explanation by the student, the more they have a chance to verbalize their thinking to the teacher or classroom. From focusing on the longer trains of thought of the students, despite interruptions from the teacher, the first two response categories began to emerge.

A student's utterance is coded as a *thinking aloud response*, S(ta), when: a) a student is speaking publicly about mathematics; b) the explanation does not include a justification component. A student's utterance is coded as a *proof-building response*, S(pb), when: a) a student is speaking publicly about mathematics; b) the explanation includes evidence of a reason or justification of their mathematical thought. Distinguishing between these two responses is difficult at times, but the deciding factor for coding an utterance is the student providing a reason or reasons for their utterances describing their thinking.

In the Old Bridge data, students are on their second attempt to find the parent function or antiderivative of $f(x) = \sqrt{\dfrac{2}{x}}$. Dr. G suggested rewriting the function and the students are working with the function in the form $f(x) = \sqrt{2}x^{-\frac{1}{2}}$ so they can reverse the power rule.

16:15	Dr. G	T(p)	Now you can reverse the power rule.
16:26	Dr. G		Ah, ah, ah.
16:30	Vinny	S(a)	Should it be. [writes f(x) = sqrt 2 x^(-1/2)]
17:15	Dr. G	T(d)	How do you reverse it Stephanie?
17:17	**Stephanie**	**S(ta)**	**Uh, What. Oh. Um, The square root of two is like a constant so you don't really have to take that into account when you do the whole add one and divide by the exponent thing. So you can just leave it as the square of root of two.**
17:34	Vinny	S(con)	Okay so.

17:34	Stephanie	S(con)	I'm, I'm pretty sure this is correct.
17:36	Dr. G	T(con-s)	Correct. You're doing good.
17:36	**Stephanie**	**S(ta)**	**The square root of two times x to the one-half plus one, which is three-halves. So just change the exponent to three-halves.**
17:48	Vinny	S(qs)	This is to the negative one-half so do I just go one-half.
17:50	**Stephanie**	**S(ta)**	**Oh so yeah. Yeah okay. And then divide by one-half.**
17:58	Vinny	S(con)	Uh, Okay [Writes / under whole expression, then 1/2]
18:05	**Stephanie**	**S(ta)**	**But the just the x by the one-half not the whole line.**

The student responses were coded as *thinking aloud responses* because Stephanie explained her mathematical thinking about the process for reversing the power rule, but did not provide a justification for her thinking. In her first statement, Stephanie explains what should be done with the constant, but does not justify why the constant does not change or affect the anti-derivative function. Stephanie's second statement explains the process of reverse power rule for finding anti-derivatives or her terms "the whole one and divide by the exponent thing". Her next statements explain where the constant from the rule or the "divide by the exponent thing" should go in the expression. Each response demonstrated verbal thinking about the mathematics, but the student thinking was not sophisticated enough to include a justification for her reasoning about why the reverse power rule process works.

In the Kenilworth data, Mrs. W approaches the students to discuss what they have worked on as a group.

01:08:57:22	Mrs. W	T(d)	Now when I say "you" I mean everybody contribute to what brought you to the point that you're at.
01:09:03:03	Sherly		Let's start. [All three transparencies are layered together. Angela and Sherly explain process.]
01:09:09:28	Angela	S(a)	Me? Alright.
01:09:12:28	**Angela**	**S(ta)**	**We traced the spiral that we...(inaudible) first made the ray [**Angela retrieves the photocopy of the shell from a stack of papers in front of her. She refers to the photocopy by gesture]
01:09:19:25	**Angela**	**S(ta)**	**Alright, and then we measured how far, well, we traced the spiral too,**
01:09:24:28	**Angela**	**S(ta)**	**We measured, like, wherever the spiral intersects the ray, we measured how far from the center it was.** [Angela points to the photocopy and gestures with a spiraling motion of her hand and finger just above the photocopy as if reenacting the tracing of the spiral.]
01:09:30:28	**Angela**	**S(ta)**	**We have our measure in centimeters. So we do the second line, and we had to figure out what that was** [Angela points to the angle formed by the two rays when she says "that"]
01:09:37:14	**Angela**	**S(ta)**	**So we made that a right triangle and used a ruler so**

it might not be exactly accurate, but, um, [Angela

gestures with her finger on the photocopy to indicate

"that a right triangle]

01:09:45:28	**Angela**	**S(ta)**	**You know, like, we used cosine to figure out what the angle was and then we changed it into radians.**
01:09:51:19	Sherly	S(i)	We traced it (inaudible)
01:09:52:25	Angela	S(qs)	What?
01:09:53:11	Sherly	S(qs)	We didn't do all those yet, did we?
01:09:55:05	Angela	S(ans)	No, we did that for the first one.

01:09:56:29	**Angela**	**S(ta)**	**Now we just, um, we just traced it on a transparency so that we'd be able to present it, or whatever and show it better. [**Angela presents the spiral traced on the transparency, and sets the photocopy of the shell aside.]

01:10:06:27	**Angela**	**S(ta)**	**And um, we just, so far we just kinda made the line, well, we just, you know, did that and we measured how far the points were for this other one**

01:10:17:03	**Angela**	**S(ta)**	**And so we figured out what theta was**

01:10:19:11	**Angela**	**S(ta)**	**And we kind of wrote it in the color that we have it up there [**Angela points to the equations written in green or blue showing R with subscripts equal to measures in centimeters, and then gestures to the points, labeled with a green or blue R with subscripts, of intersection of the rays with the traced spiral.]

01:10:23:20 Angela S(ta) We're just doing that for the second line now, we measured the distances, and now we're just going to figure out what, um, the angle is. [Angela overlays another transparency.]

The students continue to discuss if they have all the measurements, then continue to draw rays and calculate measurements using inverse cosine. Angela continues to draw rays to find measurements for the spiral while other students perform calculations.

Angela's response to Mrs. W's questions were coded as *thinking aloud responses* because she is explaining what the group has done to collect data about the shell, but has not justified the steps they have taken. This displays a first attempt at verbalizing thinking, which could be considered a common theme for responses receiving this code.

For examples of *proof building responses,* in the Old Bridge data, Amy is at the board to draw a graph of the function $f(x) = 2x^3 + x^2$ in order to find the area under this curve.

22:33 Dr. G T(e) Very good. Talk to us Amy.

22:36 Amy S(pb) Okay, um. So you have to find the area of the derivative of the parent function. So to find out if it increases you have to find the derivative of this. So, the derivative of the derivative of the parent function would be 12x + 2. And then I should graph it. And then when it's one it's going to be, when it's one it's going to be 14. And then when its three it going to be, 36 times, 38. And then we

know that this function is linear because it's to the power of one and it's above the x axis so you know that it's increasing.

23:54	Dr. G	T(c)	Whoop, whoop, whoop, whoop, whoop. What did you say?
23:58	Amy	S(ta)	Wait, you know that it's above the x axis
24:02	Dr. G	T(c)	Wait. Time out. Remember this rule we have about pronouns. That they stink because we don't know what they mean half the time.
24:08	**Amy**	**S(c), S(pb)**	**Okay, you have these two points that, of the, of lower case f(x) and you have to figure out if the graph increases and the concavity of it. So you have to find the first derivative of the derivative to find out if this line, this linear function, is above the x axis or below the x axis and if it's above the x axis then you know that this graph is increasing.**
24:34	Dr. G	T(d), T(c)	What'd she say Mikey?
24:39	Mikey	S(c)	The derivative above the x axis increases.
24:43	Dr. G	T(rep)	A little louder Mikey, I almost caught it.
24:45	Mikey	S(c)	The derivative of the derivative is above the x axis it's increasing.
24:49	Dr. G	T(c)	You uses this word it's and that's where I lost you. Try again.
24:53	Mikey	S(c)	The original graph. The first derivative of the parent

function.

| 24:57 | Dr. G | T(rep) | So say it all again. |

| 25:00 | Mikey | S(c) | Okay. If the first derivative of the derivative of parent function is above the x axis then the derivative of the parent function is increasing. |

| 25:10 | Dr. G | T(con-s) | That's not bad. Go ahead. |

25:12 Amy   S(pb)   So, this line is above the x axis so you know that it's increasing

| 25:18 | Dr. G | T(c) | What's increasing? |

| 25:22 | Amy | S(c), S(ta) | The derivative is increasing. The derivative of the parent function is increasing. So then now you have find the concavity of it. So now you have to find the derivative of the derivative of the derivative of the parent function. |

| 25:40 | Eric | S(ans) | It's called the second derivative |

| 25:45 | Dr. G | | Now Amy, the next time you say something like that please look at the camera, because nobody would believe this. Alright. |

| 25:52 | Amy | S(ta) | Okay. So, I'm finding the second derivative. So then I use the thing. And then |

| 25:59 | Dr. G | T(c) | Uh, the thing. |

| 26:01 | Amy | S(c) | I use the power rule. |

| 26:03 | Dr. G | T(con-s) | Okay. |

26:04 Amy   S(pb)   And so it's just 12. So. (Draws the first quadrant of the

coordinate plane.) So then like. The domain is restricted

to one and three so you just need it at 12 between 1 and

3. And now, this graph it's above the x axis again, so

now you know the concavity of this because it's

increasing so you know that it concaves it up. So now

that we've determined that it increases that this, the

lowercase f(x) increases and it concaves up. So you can

draw the line.

The student responses were coded as *proof building responses* because Amy provides the

reasoning for why the function is increasing over her first four responses. In Amy's last

response, she justifies why the second derivative informs her the original function is

concave up. Her justification is partially correct since she states the graph is above the x-

axis, but her justification is because the second derivative is increasing. This error is

corrected in the next few lines of conversation through a student's question.

In the Kenilworth data, Robert and Sherly are tying to explain to Michelle what

their graph is trying to model and the different between why one graph spirals and the

other does not. Dr. A has been listening to the conversation when she questions the

students.

00:39:59:19 Dr. A   T(r)   Can you explain to me (inaudible) the two equations

you're working with?

00:40:05:07	**Robert**	**S(pb)**	**No, we were just saying like. To explain like why this becomes that. That's what we're, that's what we're trying to explain. Um, because this is like that, kind of like unraveled, un-spiraled. And like all the points would be the same. Like if I graph the points in polar it would be that.**
00:40:24:09	**Robert**	**S(pb)**	**If I graph it in a function, it would be like line. If you get an equation that goes through the points, that when we like spiral it or put it in polar, it will go through the points too. It will go through the points when they're circular if you just put it, if you get it to go through when it's a function. We just saw it like this, this kind of like that unraveled, like if you just took the spiral out and made it into a straight line. And then we um, and then we just kind of respiraled it.**
00:40:55:29	Dr. A	T(c)	Can you, could you use a pen and just give me a quick, easy explanation on the paper of what you just said, maybe not use all the points, but do something that helps me to understand it?
00:41:08:14	Robert		Um.
00:41:12:26	**Robert**	**S(pb)**	**Say we had three points that like here, here, and here.**

00:41:17:06	Dr. A	T(con-s)	Okay.
00:41:17:28	**Robert**	**S(pb)**	**And let's say like, this was point five, I don't know, like one point two and like two. I (inaudible).**
00:41:27:08	Dr. A	T(c)	Okay, okay, what do those numbers represent?
00:41:29:29	Robert & Sherly	S(c)	The distance away from the
00:41:31:21	Sherly	S(c)	center.
00:41:32:09	Robert	S(con)	Yeah.
00:41:33:11	Dr. A	T(con-t)	Okay, and so you've done the spiral and you've said that. Let me see what you've done. And so you had a point over here and maybe a point over here. And a point over here or whatever. And so the distance from the center is your, is your number, is your radius.
00:41:52:06	Robert	S(ans)	Yeah and then we figured.
00:41:52:20	Dr. A	T(con-s)	Okay.
00:41:53:14	**Robert**	**S(pb)**	**I don't know, I guess like then if you draw it on one of these.**
00:41:57:24	Dr. A		Mm hmm.
00:41:57:24	**Robert**	**S(pb)**	**I guess this would be the point number, like 1, 2, 3. Like this would be the first point the second point the third point.**

00:42:04:29	Dr. A		Mm hmm.
00:42:05:18	**Robert**	**S(pb)**	**And then you go over like, here I'll make this .5,**
			1.5, 2. So you go over to .5 and you go up one
00:42:07:23	Sherly		(Inaudible)
00:42:13:02	Dr. A		Uh, huh.
00:42:13:15	**Robert**	**S(pb)**	**And then you put a line here. And you go to 1.2,**
			up to 2, and 2 up to 3. And you want to find an
			equation.
00:42:19:24	Dr. A	T(c)	Okay, I'm pretty dumb about this. I need to know
			which numbers, what is, what are you're two
			coordinates at those points?
00:42:27:28	**Robert**	**S(pb)**	**Oh yeah right. This is distance around spiral.**

The student responses were coded as *proof building responses* because Robert's first responses provide a reason for why the function graph is another representation for the spiral. The remaining responses provide an example to further justify his thinking about the function versus spiral representations.

Answer Response. In order for an environment of talk to exist, students need to participate in the dialogue. Even though conditions may be in place for students to share their thinking, not all student responses will involve detailed or lengthy explanations.   At times throughout a conversation, student responses may be minimal with respect to verbalizing thinking. This may be due to the teacher's question or the student's understanding at the time of the response. A student's utterance is coded as an *answer*

response, S(ans), when: a) a student gives a short/closed-ended recall response; b) the student states a fact/piece of information and c) the student is responding to a question posed by the teacher or another student; d) the student does not provide an explanation for how they arrived at this answer or a justification for why this is correct. *Answer responses* do not necessarily to include mathematical information in the utterance. At times, such a response may indicate a simple "no" response.

From the Old Bridge data, in order to begin the class Dr. G needs some information from the students about last night's work. The questions only require the student give information rather than an explanation.

00:52	Dr G.	T(e)	Now. What's the function Andrew, for which you were to find the area under the curve?
01:15	Andrew	S(a)	Which problem?
01:16	Dr. G	T(f)	The one you worked on last night.
01:20	**Andrew**	**S(ans)**	**x squared plus one.**
01:35	Dr. G	T(e)	And what was the interval? Andrew?
01:41	**Andrew**	**S(ans)**	**Zero to one, no, I'm sorry, one to two.**
01:53	Dr. G	T(p)	Now, Kristen, the first thing I would see is a reasonable sketch of this function and all you need to do is give me two relevant points of interest. On a reasonable coordinate plane. [Kristen draws an xy axis and sketches the function.]
02:44	Dr. G	T(r)	Kristen, do you recall the instructions I gave you when you, before you went up there.
02:50	Kristen	S(con)	Yeah.

02:52   Dr. G   T(c)   What were they?

02:53   Kristen   S(ans)   Draw a sketch. Reasonable

02:55   Dr. G   T(p)   All you have to do is give me two points of interest

The student responses were coded as *answer responses* because the student provided the teacher with a piece of information about the assigned homework problem. Kristen's response received an *answer response* code because she gave a short recall response about the previous directions.

In the Kenilworth data, the students have given Michelle a recap of the work from the previous day. Dr. A converses with Victor about the values the students found and how the values relate to their work on the calculator.

00:10:36:02   Michelle   S(qs)   Did you copy down this equation?

00:10:39:28   Sherly   S(ans)   I don't know, (inaudible).

00:10:40:07   Victor   S(ans)   Yeah, I did.

00:10:40:08   Sherly   S(con)   Okay.

00:10:42:22   Dr. A   T(c)   And c1 was your...

00:10:45:08   Victor   S(c)   And then all we, and then c1 was our

                                     measurements that we actually measured off the

                                     thirteen points.

00:10:49:08   Dr. A   T(con-s)   Mm hmm.

00:10:51:08   Victor   S(pb)   And then instead of using x, we just used x of theta,

                                     and then we put it in our program, and that's how

                                     we got it.

00:11:03:13	Dr. A	T(j)	And so when you got that what do you have?
00:11:06:12	**Victor**	**S(ans)**	**We got a**
00:11:06:12	Michelle	S(i)	Wait, I lost the equation. I just pushed some button. I didn't lose it. I didn't lose it. I just like went off the screen, I don't know how to get out.
00:11:14:08	**Ashley**	**S(ans)**	**Second, second quit.**
00:11:16:27	Michelle	S(a)	Second quit? Okay there's a bunch of numbers that I don't get. I'm checking it.
00:11:26:12	**Michelle**	**S(ans)**	**I don't know how to work the calculator.** **[Michelle puts the calculator in front of Robert.]**
00:11:27:29	Victor	S(i)	Let me see. [Robert hands the calculator to Victor.]
00:11:31:24	Victor	S(a)	(inaudible) you want the equation again?
00:11:32:13	Mrs. W	T(s)	So do all of you have the information in your calculators? Do you all have c1 and c2?
00:11:38:02	**Sherly**	**S(ans)**	**No.**
00:11:39:15	Mrs. W	T(con-t)	You don't, you don't all have the c1 and c2?
00:11:40:17	**Victor**	**S(ans)**	**And this is the equation that, that group came up with, um the first group. [to Michelle]**
00:11:41:26	**Sherly**	**S(ans)**	**No.**

The first two student responses were coded as *answer responses* because the students responded to each other with a fact about their knowledge of writing down the equation. Ashley and Michelle's responses also stated information about their knowledge of the

calculator. Victor's responses to the teacher received an *answer response* since they were a short response about information related to the equation, particularly since Michelle interrupted the first response.

A student may give an answer response because the teacher has asked a question that requires only a small amount of feedback or because the student does not have a level of understanding that allows for a longer response. An *answer response* may or may not include mathematical information, but a possible intent of this response may be to give the teacher information the student knows at the time.

Clarification Response. During a conversation, the teacher may want a student to provide additional information about a response given earlier. A student's utterance is coded as a *clarification response,* S(c), when a student provides more detail to a previous statement without justifying their thinking.

In the Old Bridge data, Amy is working at the board to determine if the parent function $f(x) = 2x^3 + x^2$ is concave up or down on the interval [1,3].

23:58	Amy	S(ta)	Wait, you know that it's above the x axis
24:02	Dr. G	T(c)	Wait. Time out. Remember this rule we have about pronouns. That they stink because we don't know what they mean half the time.
24:08	**Amy**	**S(c),** **S(pb)**	**Okay, you have these two points that, of the, of lower case f(x) and you have to figure out if the graph increases and the concavity of it. So you have to find the first derivative of the derivative to find out if this line, this linear function, is above the x**

axis or below the x axis and if it's above the x axis

then you know that this graph is increasing.

24:34	Dr. G	T(d), T(c)	What'd she say Mikey?
24:39	**Mikey**	**S(c)**	**The derivative above the x axis increases.**
24:43	Dr. G	T(rep)	A little louder Mikey, I almost caught it.
24:45	**Mikey**	**S(c)**	**The derivative of the derivative is above the x axis it's increasing.**
24:49	Dr. G	T(c)	You uses this word it's and that's where I lost you. Try again.
24:53	**Mikey**	**S(c)**	**The original graph. The first derivative of the parent function.**
24:57	Dr. G	T(rep)	So say it all again.
25:00	**Mikey**	**S(c)**	**Okay. If the first derivative of the derivative of parent function is above the x axis then the derivative of the parent function is increasing.**
25:10	Dr. G	T(con-s)	That's not bad. Go ahead.
25:12	Amy	S(pb)	So, this line is above the x axis so you know that it's increasing
25:18	Dr. G	T(c)	What's increasing?
25:22	**Amy**	**S(c),** S(ta)	**The derivative is increasing. The derivative of the parent function is increasing.** So then now you have find the concavity of it. So now you have to find the derivative of the derivative of the derivative of the

parent function.

25:40	Eric	S(i)	It's called the second derivative
25:45	Dr. G		Now Amy, the next time you say something like that please look at the camera, because nobody would believe this. Alright.
25:52	Amy	S(ta)	Okay. So, I'm finding the second derivative. So then I use the thing. And then
25:59	Dr. G	T(c)	Uh, the thing.
26:01	**Amy**	**S(c)**	**I use the power rule.**

These student responses were coded as *clarification responses* because the student is rephrasing and providing clarity to a previous explanation. The first response is a clarification of why the function is above the x axis. Mikey's responses are *clarification responses* because he attempts to clarify the explanation of Amy. Amy's last two responses are providing more detail about what is increasing and the method she used to find the second derivative.

In the Kenilworth data, the students have checked to make sure they all have the correct measurements. Mrs. W asks if Robert can explain to her what is going on and the entire group of students join in to explain/recap the measurement sequence and color coding, they used cosine to get theta in degree and radian measurements.

36:10:00	Robert	S(ta)	Well we started by drawing a line up, in the center up, and then we took lines all over. We started going out this way. (pointing to the left) and then we measured

the length.

36:20:00	Mrs. W	T(c)	Going which way?
36:22:00	**Robert**	**S(c)**	**Well we started here and then we drew a line going diagonal this way.**
36:25:00	Mrs. W	T(con-t)	So you started on this red line going up and then this was your second line.
36:31:00	Robert	S(ta)	Yeah, and then we measured it.
36:34:00	Mrs. W	T(c)	Measured what?
36:36:00	**Robert**	**S(c)**	**Measured the all the, all the distances.**
36:37:00	Sherly	S(i)	The distances.
36:38:00	**Angela**	**S(i), S(c)**	**Like where the spiral intersects.**
36:42:00	Mrs. W	T(e)	Okay and you've recorded those where?
36:43:00	Robert	S(ans)	There (points to transparency) and the colored lines are the color it's written in.
36:47:00	Mrs. W	T(con-s)	Okay, Okay. So it's color coded. Okay.
36:51:00	Robert	S(ta)	Uh, and then uh we just used cosine to figure out the angle. Like we took all the measurements and divided them and then used cosine and all that and then put them into radians too; converted them to radians.
37:04:00	Ashley	S(i)	We made a ninety degree angle.
37:07:00	**Robert**	**S(con), S(c)**	**Yeah we made a 90 degree angle connecting these two lines.**
37:10:00	Mrs. W	T(c)	Where's the 90 degree angle?

37:12:00	Robert	S(c)	**See the dotted line.**
37:13:00	Mrs. W	T(con-s)	The dotted line.
37:15:00	Robert	S(ta)	Yeah, and then we just recorded it in degrees and radians for all our measurements and we just kept drawing lines.
37:22:00	Mrs. W	T(c)	And so when you said you used, you drew a right triangle and you used the right angle and you. What did you do with that?
37:31:00	Robert	S(c)	**Uh, well then after we drew the right angle**
37:33:00	Sherly	S(i)	We used cosine.
37:34:00	Robert	S(c)	**We used the right triangle to use cosine**
34:38:00	Sherly	S(i)	Adjacent over hypotenuse.
37:39:00	Mrs. W	T(c)	Okay, you used cosine to get what?
37:41:00	Sherly	S(c)	**The theta.**
37:45:00	Angela	S(con), S(c)	The theta thing**. In degrees and then we converted to radians.**

These student responses were coded as *clarification responses* because the students provided more details in subsequent responses. Robert's first two *clarification responses* provide more detail to his *thinking aloud* responses. Angela's response interrupts the Robert's responses, but provided more information about where the distances were measured. After Angela's response, Robert continues to clarify his original response at

36:51 after questioning by the teacher. Sherly and Angela also clarify Robert's explanation in response to Mrs. W's question.

A student may give a *clarification response* because they feel the questioner is not fully satisfied with their initial response. The feeling for having to provide more information can be a result of teacher questioning, not receiving teacher feedback, or student questioning.

Confirmation Response. In order to move a conversation forward and inform students of positive progress, the teacher made a statement in the *confirmation question* category. The student must also express agreement so there is a consensus for the conversation to move forward. A student's utterance is coded as a *confirmation response*, S(con), when a student expresses agreement with a previous statement.

In the Old Bridge data, Dr. G called Anita to the board in order to reverse the power rule for the function $f(x) = \sqrt{\dfrac{2}{x}}$.

02:06	Anita	S(seek)	Where can I erase?
02:08	Dr. G	T(p)	Anything you like.
02:09	**Anita**	**S(con)**	**Okay.**
02:11	Dr. G	T(p)	Except the graph. Leave the graph.
02:13	**Anita**	**S(con)**	**Alright.**
02:25	Anita	S(a)	What did you say Stephanie?
02:27	Stephanie	S(ta)	the first (inaudible) [Anita writes sqrt (2/x)]
02:28	**Anita**	**S(con)**	**Okay. Oops Sorry. Okay**
02:36	Stephanie	S(ta)	Equals to two over x to the one-half, quantity to the one-

half. [Anita writes = (2/x) ^ ½]

02:41	**Anita**	**S(con)**	**Alright**
02:45	Stephanie	S(ta)	And then you're supposed to add one to the exponent
02:49	**Anita**	**S(con)**	**Okay**
02:58	Dr. G	T(s)	Time out. What you now have is a false statement.
03:09	**Stephanie**	**S(con)**	**Yeah don't write it like that.**
03:10	**Anita**	**S(con)**	**Okay.**
03:13	Stephanie	S(ta)	Erase the plus one too. And erase the equals
03:23	Anita	S(a)	I'm going from the bottom.
03:24	**Stephanie**	**S(con)**	**Yeah.**
03:25	Anita		Chance
03:28	Chance	S(qs)	Is this like rule with the function thing or?
03:30	Dr. G	T(con-t)	Time out. I'll help you with that. Right now this is still a derivative right? [Dr. G adds 'f(x) =' in front of sqrt 2/x]
03:36	**Anita**	**S(con)**	**Right**

The student responses were coded as *confirmation responses* because each response denotes agreement by the student with either the teacher or another student who is speaking.

In the Kenilworth data, Dr. A is discussing with the students the meaning of their coordinates and how they relate to the lists on their calculator.

00:43:35:10	Dr. A	T(con-t)	But you got those points to plot on the scatter plot?
00:43:40:11	**Robert**	**S(con)**	**Yeah.**

00:43:40:22	Dr. A	T(r)	What were you plotting and what were the two coordinates then?
00:43:44:09	Robert	S(pb)	Ah, c2, so that would be like the degree here. Like the amount or something. And I guess like, compared to one circle, so if like you go 90 degrees around, this would be like 1/4, I guess, and then if you go like half way around the circle this would be 1/2. And then um 3/4's, if you go like 3/4's around the circle. [Robert shows this on the axes he drew.]
00:44:05:24	Dr. A	T(r)	Okay, I can't, Victor said something a minute ago about which was the x and which was the y? Which is the
00:44:12:04	Sherly	S(ans)	C2 is the x, and c1 was the y.
00:44:15:10	**Victor**	**S(con)**	**The degrees is c1, um wait yeah.** [Victor is looking down at his calculator in his lap.]
00:44:20:11	**Sherly**	**S(con)**	**C2 was the x and c1 was the y.**
00:44:22:08	Dr. A	T(c)	Okay, c2?
00:44:23:08	Sherly	S(ans)	Was the x.
00:44:24:08	**Victor**	**S(con)**	**Yeah and c2**
00:44:24:21	Dr. A	T(con-t)	Is the x.
00:44:25:28	**Victor**	**S(con)**	**Uh huh.**
00:44:26:08	Dr. A	T(c)	And what is c2?
00:44:27:29	Victor	S(ans)	That's, they're radians.

00:44:30:23   Sherly   S(con)   Yeah.

The student responses were coded as *confirmation responses* because each response

states agreement with the teacher's questions.

Most of the time a student's *confirmation response* is a simple "okay", "right" or

similar response. By giving such a response, the student's intent may be to signal to the

teacher they are ready to move ahead in the conversation. This signal is important for the

teacher to know when there is consensus among the participants in the conversation.

Attunement Response. Because of the nature of conversation, students may not

hear or understand the utterances of another person in the conversation. A student's

utterance is coded as an *attunement response*, S(a), when a student attempts to

comprehend another's utterance in the way that person intended.

In the Old Bridge data, the student's have just confirmed via calculator their work

with reversing the power rule did produce the correct answer to find the area under the

curve.

25:21	Dr. G	T(p)	Now Chris let's begin to set up this limit form which has to come out to four minus two root two. So go start. This is your primary job for the weekend. Make this limit question actually equal four minus two root two.
25:42	**Chris**	**S(a)**	**So find the limit of the derivative.**
25:45	Dr. G	T(r)	No, find the limit for the sum of the areas of an infinitely large number of rectangles? [Chris writes $\lim\limits_{n\to\infty}\sum\limits_{1}^{\infty} i$.]

26:02	Eric	S(qs)	What is that?
26:03	Student	S(ans)	Limit
26:03	Eric	S(qs)	I know, but what's the thing?
26:04	Dr. G	T(s)	An n. Eric you should know that
26:08	Eric	S(con)	No, I know it's an n, but it looks. Alright
26:26	Dr. G	T(d)	Comment Chance.
26:29	**Chance**	**S(a)**	**Me Comment?**
26:30	Dr. G	T(con-s)	Yeah
26:31	Chance	S(dnc)	I don't have a comment
26:32	Dr. G	T(rep)	Yeah you do.
26:33	**Chance**	**S(a)**	**Really.**
26:34	Dr. G	T(d)	Alright Charlie.

The student responses were coded as *attunement responses* because each response checks to make sure the student understands what was previously stated. Chris' response makes sure he understands the teacher's direction. For both of Chance's responses, she wants to make sure Dr. G means she should be commenting about the mathematics written on the board.

In the Kenilworth data, Robert is explaining to Mrs. W the work of another group.

00:12:02:22	Robert	S(ta)	Like they plotted all those points, and this, um kind of went through all of them and when they graphed it in polar it kind of went spiral.
00:12:11:09	Mrs. W	T(con-t)	Have you done that yet?

00:12:13:05	**Robert**	**S(a),**	**What?** No, I don't think so.
		S(ans)	
00:12:15:23	Victor	S(qs)	Did what?
00:12:17:06	Robert	S(ans)	A, what they did like plot all the points and then drew a line that go through all the points.
00:12:22:11	Victor	S(ans)	Oh yeah, we did that.
00:12:23:10	**Robert**	**S(a)**	**We did?**
00:12:24:12	Victor	S(con)	Mm hmm.
00:12:25:29	Mrs. W	T(e)	What did you see?
00:12:27:29	Robert	S(ans)	I don't remember.
00:12:28:19	Victor	S(ta)	Um, we just boxed em. We put em. Our thing was a box, and we saw a whole bunch of them like together and then some were just spread out. I could do it again, but
00:12:39:09	Mrs. W	T(f)	I'd like to see it.
00:12:40:09	Sherly	S(qs)	What did we call those points?
00:12:42:10	**Victor**	**S(a)**	**Points?**
00:12:43:03	**Sherly**	**S(a)**	**We called it points right?**
00:12:44:16	Victor	S(ans)	No I, I don't know. I think

The student responses were coded as *attunement responses* because each response checks for understanding between two participants in the conversation. Robert's initial response of "What?" attunes his thinking to Mrs. W's question before the rest of his response. The

remaining *attunement responses* are a check for understanding between two students to make sure they have a mutual understanding.

The intent of a student's *attunement response* may be making sure they are on the same page as the other person in the conversation. The student may need to ask it because they did not completely hear or understand the other participant's utterance. The possible goal of responding this way is so both participants have the same understanding about the topic of the discussion.

Questions Student Response. A positive result of promoting discussion in the classroom is when students begin to rely on themselves rather than the teacher to perform many aspects of mathematical reasoning. At several points throughout the data, particularly in the Old Bridge classroom, students are responsible for the work and leading a discussion. This characteristic of the settings allowed for students to feel comfortable making errors, sharing their thinking and asking other students for help. This response category is related to these ideas.

A student's utterance is coded as *questions student response,* S(qs), when a student asks another student for information either about how to proceed or for assistance understanding. When this utterance is made, the student is not responding to the teacher, but directs their response to another student who is either offering help or whose help is being requested.

In the Old Bridge data, the students are trying to rewrite a single sigma expression for the area under the curve as three separate sigma terms. The discussion is focused on how to find a common denominator and leading coefficient for the three terms.

02:15   Ryan   S(qs)   Okay, Chance you have question

02:16	**Chance**	**S(qs)**	**Yeah, um, what your doing know. What's the point of this?**
02:20	Ryan	S(ta)	What's the? I don't know what the point is. I would think that would be simpler, but
02:24	Chance	S(ta)	Wait, so like
02:26	Dr. G	T(s)	Oh, Okay. Hang on a second. This version is no calculator what so ever.
02:32	**Chance**	**S(qs)**	**Okay so. How did you know to bring the 1/8th. Multiply like. Like what did you. Multiply by the reciprocal of eight.**
02:42	Ryan	S(a)	What this?
02:43	Chance	S(c)	Yeah like how you have $1/8^{th}$
02:45	Ryan	S(pb)	I just brought the 8 out. Put 8 over 64. which is one eighth.
02:47	**Chance**	**S(qs)**	**Why?**
02:49	Ryan	S(ans)	Cause that's what he told me to do.

In the Kenilworth data, the students have talked with Dr. A about the value of their last coordinate and the total length of the spiral. Victor completes his explanation when Sherly and Angela question him.

51:44:07	Victor	S(ta)	Well, (inaudible)right. What I just did was take 47.5, hold on.
51:46:12	**Sherly**	**S(qs)**	**How is this the same number?**
51:49:09	Angela	S(ta)	Yeah isn't that the same number because.

51:53:22	**Sherly**	**S(qs)**	**Victor**
51:55:07	Victor	S(a)	Huh
51:55:21	**Sherly**	**S(qs)**	**How is this the same number?**
51:59:26	Victor	S(a)	What?
52:00:17	**Sherly**	**S(qs)**	**How are this and this the same number?**
52:02:27	Angela	S(i)	Cause they're the same thing.

The student responses in both data examples were coded as *questions student responses* because the students involved in the conversation are directly responding to each about the mathematics in their respective discussion.

The intent of *questions student responses* may be so the student giving the response can obtain any necessary information to continue working with the group. A student may have a lack of understanding about the mathematics being discussed and will turn to another student for help. The reasons for turning to another student may be because another student is offering assistance or the other student may have worked through the mathematics previously and can provide insights based on their common experience.

Seeking Response. The next student response can be considered the opposite of a *questions student response*. If students relying on each other to develop mathematical ideas is a goal of promoting conversation in the classroom, then a student turning to the teacher for answers would be the opposite of that goal. A student's utterance is coded a *seeking response,* S(seek), when a student requests feedback from the teacher.

In the Old Bridge data, Wendy has completed a numerical expression for the area of rectangles approximating the area under a curve.

32:12	Dr. G	T(r)	Back to the first rectangle. Show me the height. Now where are we going to determine the height. On the right hand side correct?
32:24	**Wendy**	**S(seek)**	**Right here?**
32:25	Dr. G	T(f)	So how high is that?
32:43	**Wendy**	**S(seek)**	**Is it one plus one times one times twelve?**
32:49	Dr. G	T(d)	Suppose I said. It's this high when x equals what Ronak.
33:02	Ronak	S(a)	Wait. Can you say that one more time?
33:04	Dr. G	T(f)	It's this high when x equals?
33:11	Ronak	S(ta)	x equals the. When x equals one and a half.
33:21	Dr. G	T(r)	Now you want to try it again?
33:22	**Wendy**	**S(seek)**	**Am I supposed to change this number?**
33:36	Dr. G	T(r)	Okay. Now now. Time out. Right now what you're trying to do is edit a number and you're not looking at the big picture. So let's take all this out here [erases Wendy's work]. Every one of those rectangles is how wide?

These questions were coded as *seeking responses* because Wendy is requesting feedback from Dr. G. about her ideas rather than explaining her thoughts as, for example, in a *thinking aloud response*.

In the Kenilworth data, the group is preparing their presentation. Robert and Michelle are discussing how to graph the spiral and Dr. A begins to question Michelle about what she knows regarding gathering the points recorded in their data lists.

01:17:55:24	Dr. A	T(c)	Where on the spiral, have you gone 90 degrees?
01:18:01:14	Michelle	S(ans)	I don't know.
01:18:03:00	Dr. A	T(c)	Where does this. Where does this spiral go?
01:18:04:06	Robert	S(i)	No, that's 180.
01:18:07:08	Michelle	S(ans)	This way.
01:18:08:05	Dr. A	T(con-s)	Uh, huh.
01:18:09:10	**Michelle**	**S(seek)**	**But like how do you know how far like when you reach 90 degrees?**
01:18:12:05	Dr. A	T(j)	How do you know?
01:18:13:20	**Michelle**	**S(seek)**	**I'm prob, it's probably really obvious isn't it?**

Michelle's responses were coded as *seeking responses* because she wants feedback from Dr. A about the work she trying to explain. The second line of transcript shows Michelle does not know the 90 degree mark on her spiral and she seeks help from Dr. A about that information in both *seeking responses*.

The intent of a *seeking response* may be for the student to get information from the teacher. The student could have reached a point of not being able to move forward in the conversation and needs help. A natural response is to turn to the teacher for advice and request information from the perceived expert in the classroom.

Non-Contribution Response. One possibility a teacher faces when trying to promote mathematical discourse is when the students do not respond to their questions at all. A student's utterance is coded as a *non-contribution response*, S(dnc), when a student does not participate in the current conversation.

In the Old Bridge data, the students are discussing how to simplify an expression of sigmas. Rachel asks a question of Charlie while Dr. G and two other students are in a discussion.

29:52	Rachel	S(qs)	So why he do. Never mind. So why is there an n squared on the second one?
30:03	Dr. G	T(c)	What's the question Charlie?
30:04	**Charlie**	**S(dnc)**	**I don't know.**
30:05	**Rachel**	**S(dnc)**	**Never mind.**
30:05	Dr. G	T(d)	Rachel again.
30:06	**Rachel**	**S(dnc)**	**Never mind.**

Charlie and Rachel's responses were coded as *non-contribution responses* because they did not continue the conversation Dr. G and the other students were having about clearing up the work done to this point.

In the Kenilworth data, Victor has been working individually on his calculator and makes some generally directed verbal statements when Dr. A questions him.

01:07:33:25	Dr. A	T(f)	What did you find?
01:07:34:27	**Victor**	**S(dnc)**	**I didn't find anything. Who said I found anything, man? Just go away. Move away from**

me. Go ahead cut that out.

01:07:43:10   Dr. A   I'll go away. (laugh).

Victor is left alone to continue working. Victor's response was coded as a *non-contribution* response since he does not engage Dr. A in conversation about his work.

A student may give a *non-contribution* response because they do not wish to engage in the conversation. This can be considered as a student's choice for not responding within the context of the discussion. The student may not have a choice when they do not respond because they do not know what to say due to a lack of knowledge about how to contribute to the conversation.

Table 4.2

Summary of Student Responses

Student Response	Code	Description
Thinking Aloud	S(ta)	a) a student is speaking publicly about mathematics; b) the explanation does not include a justification component.
Proof-Building	S(pb)	a) a student is speaking publicly about mathematics; b) the explanation includes evidence of a reason or justification of their mathematical thought.
Answer	S(ans)	a) a student gives a short/closed-ended recall response; b) the student states a fact/piece of information; c) the student is responding to a question posed by the teacher or another student; and, d) the student does not provide an explanation for how they arrived at this answer or a justification for why this is correct.
Clarification	S(c)	a student provides more detail to a previous statement without justifying their thinking.
Confirmation	S(con)	a student expresses agreement with a previous statement.
Attunement	S(a)	a student attempts to comprehend another's utterance in the way that person intended.

Questions Students	S(qs)	a student asks another student for information either about how to proceed or for assistance understanding.
Seeking	S(seek)	a student requests feedback from the teacher.
Non-Contribution	S(dnc)	a student does not participate in the current conversation.

Teacher Questions and Student Response Relationships

The previous chapter answered the first research question, describing the types of

questions that the two teachers asked in a student-centered problem-solving environment,

by categorizing and describing the utterances of the teacher and student to illustrate how

both groups of participants communicated during the classroom conversation. In order to

answer the second research question regarding how teachers engaged students in

mathematical conversation and elicited their thinking, the relationship between questions

and responses is examined in this chapter. By starting with a quantitative approach, the

frequency of each category of question and response provides a picture of the how often

the teacher and students were communicating in a particular form. After examining the

frequency of each code, the relationship between a particular teacher question and how

often a particular student response occurred should allow for insight into how the

teachers elicited student thinking. A vignette from each setting followed by a discussion

will provide a qualitative description of student talk and how teacher questioning

promoted talk.

Teacher Question Frequency

The frequency of teacher questions provides evidence of how often a teacher

asked a question type. The total number of questions asked by teachers in the Kenilworth

data is 558 and in the Old Bridge data is 604. Table 5.1 provides a frequency count of

the teacher questions asked in both settings and the total for each question category. It is

clear from the table that each setting had different conversations in some ways, such as

the large difference between procedural questions in the settings. A comparison of

questioning between settings is addressed in the analysis. However, the focus of this

discussion is not to address the global issues of conversation occurring in each setting.

Each question is discussed in relation to the literature in the following analysis section.

Table 5.1

Teacher Question Frequency/Percentage

Teacher Questions	OBHS		Kenilworth		Totals	
Discoursive	97	16.06%	30	5.38%	127	10.93%
Retracing	53	8.77%	76	13.62%	129	11.10%
Clarification	65	10.76%	145	25.99%	210	18.07%
Explanation	30	4.97%	19	3.41%	49	4.22%
Justification	22	3.64%	36	6.45%	58	4.99%
Confirmation - t	36	5.96%	87	15.59%	123	10.59%
Confirmation - s	67	11.09%	71	12.72%	138	11.88%
Following	69	11.42%	60	10.75%	129	11.10%
Suggestion	23	3.81%	22	3.94%	45	3.87%
Procedural	118	19.54%	12	2.15%	130	11.19%
Repeat	24	3.97%	0	0.00%	24	2.07%

Teacher Question Frequency Analysis. The most frequent question in the data

was a *confirmation question*, a question that seeks agreement among participants. There

were 261 questions (22% of all questions) of this type. This means the teacher provided

regular agreement feedback to the students and constantly checked that the teacher and

student were together about what was being stated in conversation. Agreement among the participants in a discussion is supported by education groups, such as National Research Council (1989) and NCTM (2000), but I did not review any studies that either directly or indirectly mention questions that requested agreement during conversation.

When comparing the frequency of *confirmation questions* asked in each setting, there were more questions where the teacher sought agreement from a student or the class in the Kenilworth setting. There were 87 questions (15.5% of Kenilworth questions) compared to 36 questions (6% of OBHS questions). This disparity may be a result of the one-to-one conversations occurring between the teacher and student rather than the whole class discussions occurring in the Old Bridge setting. Another possible reason may be individual teacher's checking for understanding preference, meaning that Dr. A preferred to determine agreement from the students on a more regular basis than Dr. G.

The second most frequent question was *clarification questions*, or questions that seek more information from students. There were 210 questions (18% of all teacher questions) of this type. Since interpreting students' mathematical explanations and descriptions can be challenging for teachers in student-centered classroom, as discussed in Manouchehri (2007), Himmelberger and Schwartz (2007) and Kotsopoulos (2007), clarifying questions may help lessen these concerns. Additionally, O'Connor and Michaels (1993) and Springer and Dick (2006) endorse having teachers ask for students to clarify their thoughts by asking "revoicing" questions.

When comparing the frequency of *clarification questions* asked in each setting, there were more questions in the Kenilworth setting. There were 145 questions (26% of Kenilworth questions) compared to 65 questions (11% of OBHS questions). This

disparity may also be a result of the nature of conversations occurring in each setting. Since Dr. A was not present during all of the student discussion, the students did not experience teacher input during their development of mathematical ideas. Therefore, Dr. A may have needed to ask more clarification questions in order to have students explain their thinking using more formal mathematical language.

The third most common type of question was *procedural questions*, or questions that directed students with a particular task. There were 130 questions (11% of all teacher questions) of this type. Though not directly addressed in the literature, the idea of a procedural question is embedded within the research of Truxaw and DeFranco (2007) and Staples and Colonis (2007) because they envision the teacher directing the discussion so students can make connections and development mathematical meaning. Since there are no specific details related to procedural question, I hypothesize this question is present for two reasons. The first reason is the teacher's role in a student-centered classroom is to guide the discussion, as mentioned in the two studies, toward the goals of the lesson. In doing so, the teacher needs to direct the students to perform certain mathematical tasks, such as, provide a graph, simplify an expression, or show the calculations performed. The second reason is the nature of the student's mathematical knowledge. Because students are exploring concepts and sharing their thoughts about their exploration, they may not have the understanding necessary to best present their arguments or to convince others of their beliefs, but may be capable of developing this knowledge with scaffolding. Therefore, as Sfard (2003) discusses about Vygotsky's perspective on learning, the teacher, as the expert in the room, may direct students to perform a task that helps them move forward with or demonstrates their thinking.

When comparing the frequency of *procedural questions* asked in each setting, there were more questions asking the Old Bridge setting. There were 118 questions (19.5% of OBHS questions) compared to 12 questions (2% of Kenilworth questions). The disparity may be a result of the environments. Since the Old Bridge setting was a classroom where whole class conversation was the persistent mode of discussion, the teacher was responsible for more direction of student actions. Throughout the attempts to solve each problem, different students went to the board to record inscriptions and lead the solution process. In contrast, the non-classroom Kenilworth setting involved six students seated at a table solving their problem. The teacher did not have to direct their actions unless directly engaged in conversation with the students and when the conversation occurred students were frequently discussing mathematical tasks that were already performed.

The fourth most frequent type of question was *following questions*, or questions that build upon a student statement. There were 129 questions (11% of all teacher questions) of this type. The presence of following questions is consistent with research stating the teacher should listen to student ideas and then ask questions based upon those ideas. Abdi (2007) found that teachers asked following questions when leading problem-solving sessions in an informal mathematical environment. Staples and Colonis (2007) mention discussion where the teacher builds connections and encourages more input based upon the student ideas and Mewborn and Huberty (1999) encourage teachers to ask follow-up questions.

When comparing the frequency of *following questions* asked in each setting, there was a small difference between each setting. There were 67 questions in the Old Bridge

setting (11% of OBHS questions) compared to 60 questions in the Kenilworth setting (11% of Kenilworth questions). This implies that both teachers asked questions that followed student ideas at approximately the same frequency.

Retracing questions, or questions that refer to a previous student idea, also appeared 129 times (11% of all teacher questions). They did not appear in the research reviewed for this study. However, the idea of questioning students about previous mathematical thoughts is similar to Pirie and Kieren's (1994) idea of folding back in order for students to develop a deeper understanding of a mathematical concept. Because the thoughts developed from the students and their initial thinking may change as the discussion moves forward, if a teacher retraces the thought process, students may be able to develop their thinking by addressing the idea multiple times in a discussion and structure new ideas based upon additional student input.

When comparing the frequency of *retracing questions* asked in each setting, there were more questions asking the Kenilworth setting. There were 76 questions (14% of Kenilworth questions) compared to 53 questions (9% of OBHS questions). The disparity may be a result of the nature of each setting's discussions. Since Dr. A let students work independently when student-to-student conversation occurred, when Dr. A stepped back into the conversation she may have needed to connect what students did independently to explanations in previous student-teacher interactions.

The fifth most frequent type of question was *discoursive questions*, or questions that the teacher asked to invite other students to share their thinking. There were 127 questions (11% of all teacher questions) of this type. In order to have students explain their thinking, teachers can invite students to contribute to the conversation, which is

advocated by Manouchehri and Enderson (1999). White's (2003) study of a magnet

school in Washington, DC also supports the need to include students into the

conversation to affect mathematical thinking. While not directly referring to discoursive

questions as means to allow students to share ideas, these two studies provide a research

foundation for this question.

When comparing the frequency of *discoursive questions* asked in each setting,

there were more questions asking the Old Bridge setting. There were 97 questions (16%

of OBHS questions) compared to 30 questions (5% of Kenilworth questions). The

disparity may also be a result of the nature of each setting's discussions. In the classroom

setting, a teacher is responsible for providing all students with the opportunity to learn,

participate and contribute to the desired mathematical goals of a lesson. This means Dr.

G would have to include as many students as possible in the conversation as part of his

obligation to help students learn the pre-determined curriculum. Another source of the

frequency inequality is that more students were involved in the conversation than in the

research setting. Therefore, Dr. G had more students available to draw out thinking when

other students were the primary speaker.

The remaining were questions were asked in a small numbers compared to the

other teacher questions. *Justifying questions*, or questions that ask students to provide a

form of proving or convincing, occurred 58 times (5% of all teacher questions).

Explanation questions, or questions that ask students to verbalize their thinking, occurred

49 times (4% of all teacher questions). Both of these questions can be drawn from Dann,

Pantozzi, and Steencken (1995), Martino and Maher (1999), and van Zee and Minstrell

(1997) as a way to encourage students to continue to talk about their mathematical thinking.

When comparing the frequency of *justification questions* asked in each setting, there was a small difference between each setting. There were 22 questions in the Old Bridge setting (4% of OBHS questions) compared to 36 questions in the Kenilworth setting (6% of Kenilworth questions). When comparing the frequency of *explanation questions* asked in each setting, there was also a small difference between each setting. There were 30 questions in the Old Bridge setting (5% of OBHS questions) compared to 19 questions in the Kenilworth setting (3% of Kenilworth questions). This implies that both teachers asked students to explain or justify their thinking at approximately the same frequency.

The least frequent question asked in both settings was *suggestion questions*, or questions that the teacher asked to inject ideas into the conversation. There are 45 questions (4% of all teacher questions) of this type. Suggestion questions are mentioned in Herbst (2002) and Falle (2003). When comparing the frequency of *suggestion questions* asked in each setting, there was a small difference between each setting. There were 23 questions in the Old Bridge setting (4% of OBHS questions) compared to 22 questions in the Kenilworth setting (4% of Kenilworth questions). This implies that both teachers asked questions that provided students with information at approximately the same frequency.

The only questions asked in one setting were *repeat questions*, or questions the teacher asked to have students repeat a previous statement. There were 24 questions (2% of all teacher questions) of this type. Repeat questions were not present in the literature

reviewed for this study. However, since these questions only appeared in the classroom setting, they may be the result of a large group discussion where students may not speak loud enough for others to hear or where all the participants may not be actively listening to the speaker.

Student Response Frequency

The frequency of student codes provides information about the type of responses students were giving to teacher questions. The quantity of a student response speaks to how often students were communicating in a particular form during mathematical conversation and can provide evidence to a teacher's eliciting student thinking. The frequency of student responses is given in Table 5.2. The total number of responses from Kenilworth students is 820 and OBHS students is 837.

Table 5.2

Student Response Frequency/Percentage

Student Responses	OBHS		Kenilworth		Totals	
Attunement	73	8.72%	40	4.88%	113	6.82%
Confirmation	103	12.31%	154	18.78%	257	15.51%
Clarification	30	3.58%	52	6.34%	82	4.95%
Thinking Aloud	146	17.44%	126	15.37%	272	16.42%
Proof Building	70	8.36%	100	12.20%	170	10.26%
Answer	257	30.70%	273	33.29%	530	31.99%
Seeking	79	9.44%	14	1.71%	93	5.61%

Questions Student	56	6.69%	46	5.61%	102	6.16%
Does Not Contribute	23	2.75%	15	1.83%	38	2.29%

Student Response Frequency Analysis. The first observation of the frequency of student responses is the totals are larger than the totals for the teacher questions. In Kenilworth, the ratio of student to teacher utterances is 820 to 558. In OBHS, the ratio is 837 to 604. The greater amount of student responses codes than teacher question codes demonstrates that students were participating in a majority of the classroom conversation. Also note, this does not include the student discussion that occurred in the Kenilworth data without the presence of a teacher.

The literature on teacher questions encourages teachers to ask questions requiring students to share their thinking about mathematics (Cazden, 2001; Falle 2003; Steele, 2003; van Zee & Minstrell, 1997). However, the research examined for this study does not specifically address student responses to teacher questions. Therefore, student responses are not connected to the research literature.

The most frequent student response was *answer responses* or responses where students gave a short recall response containing a fact or piece of information. There were 530 responses (32% of all student responses) of this type. Since answer responses were the most frequent, students in both settings were most often responding to questions with small pieces of information. When comparing the frequency of *answer responses* in each setting, there was little difference between each setting. There were 257 responses in the Old Bridge setting (31% of OBHS responses) compared to 273 responses in the Kenilworth setting (33% of Kenilworth responses).

The second most frequent student response was *thinking aloud responses*, or responses where students explained their thinking verbally. There were 272 responses (16% of all student responses) of this type. When teachers try to engage students in conversation, a thinking aloud response provides evidence of student mathematical thinking. By being the second most frequent response type, there is evidence that teacher questions elicited student thinking as part of the conversation. When comparing the frequency of *thinking aloud responses* in each setting, students provided this response at approximately the same frequency. There were 146 responses in the Old Bridge setting (18% of OBHS responses) compared to 126 responses in the Kenilworth setting (15% of Kenilworth responses).

The third most frequent student response was *confirmation responses*, or responses where students expressed agreement. There were 257 responses (15.5% of all student responses) of this type. In order to move toward the objective of a mathematical conversation, such as determining the solution to a problem, students need to provide feedback to teachers about their understanding of the current topic of discussion. The frequency of the confirmation response is evidence that students were provided with opportunities to express agreement throughout the conversation, which allowed the participants to move forward in their conversation.

When comparing the frequency of *confirmation responses* in each setting, there were more responses in the Kenilworth setting. There were 154 responses (19% of Kenilworth responses) compared to 103 responses (12% of OBHS responses). Since Dr. A asked more confirmation questions seeking student agreement, this may have contributed to the disparity.

The fourth most frequent student response was *proof building responses*, or responses where students gave a reason for their mathematical thought. There were 170 responses (10% of all student responses) of this type. When students can justify their thinking to others, they demonstrate a deeper understanding of the mathematics they are discussing. Within a conversation, proof building responses can provide other participants an opportunity to evaluate the validity of an argument or understand the reasoning behind another student's thinking.

When comparing the frequency of *proof building responses* in each setting, there was slightly more responses in the Kenilworth setting. There were 100 responses (12% of Kenilworth responses) compared to 70 responses (8% of OBHS responses). The additional proof building responses in the Kenilworth setting may be a result of students performing most of their independently of the teacher. Therefore, when asked to explain their work, students were required to provide justifications more often than the students in the Old Bridge setting because the teacher was not part of the development of their ideas.

The fifth most frequent student response was *attunement responses*, or responses where students attempted to check they understood someone else's utterance. There were 113 responses (7% of all student responses) of this type. The presence of attunement responses within a conversation may indicate that students are not always fully engaged in the conversation. Students may be developing an internal understanding of the mathematics or be at different levels of understanding compared to other participants. Attunement responses give students the opportunity for mutual understanding before making a contribution to the conversation.

When comparing the frequency of *attunement responses* in each setting, there was slightly more responses in the Old Bridge setting. There were 73 responses (9% of OBHS responses) compared to 40 responses (5% of Kenilworth responses). The additional attunement responses in the Old Bridge setting may be because of the classroom setting. In a classroom, more activities, such as side conversations, are taking placing simultaneously. These activities could contribute to additional need of students to make sure they understand what another student has said.

The sixth most frequent student response was *question student responses*, or responses where students asked a question of another student. There were 102 responses (6% of all student responses) of this type. When students question other students in a conversation, the teacher becomes a secondary participant. Student-to-student conversation can allow students to develop mathematical understanding independently of teacher intervention. When comparing the frequency of *questions student responses* in each setting, students provided this response at approximately the same frequency. There were 56 responses in the Old Bridge setting (7% of OBHS responses) compared to 46 responses in the Kenilworth setting (6% of Kenilworth responses).

The seventh most frequent student response was *seeking responses*, or responses where the student requested feedback from the teacher. There were 93 responses (5.6% of all student responses) of this type. The frequency of this response shows how often students relied on the teacher for information during the conversation. When students request information directly from the teacher, the students may not have the mathematical understanding necessary for further thinking based upon the current information. A

seeking response could provide teachers with information about a student's level of understanding.

When comparing the frequency of *seeking responses* in each setting, there were more responses in the Old Bridge setting. There were 79 responses (9% of OBHS responses) compared to 14 responses (2% of Kenilworth responses). Similar to the attunement responses, the additional seeking responses may be a result of the classroom setting. Since students were interacting with the teacher throughout the solution process, there were more opportunities for them to question the teacher for more information. The students still had access to the teacher's knowledge unlike in the non-classroom Kenilworth setting where the teacher stepped away when students were involved in finding a solution.

The eighth most frequent student response was *clarification responses*, or responses where students gave more detail to a previous response without justifying their thinking. There were 82 responses (5% of all student responses) of this type. When a student provides more detail about a previous response, the other participants may gain a better understanding of the student's thinking. Since clarification responses do not include a justification of thinking, students still need to provide reasoning for their thinking and the other participants will need to elicit that reasoning. When comparing the frequency of *confirmation responses* in each setting, there were more responses in the Kenilworth setting. There were 52 responses (6% of Kenilworth responses) compared to 30 responses (3.5% of OBHS responses). Since Dr. A asked more clarification questions, this may have contributed to the disparity.

The least frequent student response was *does not contribute responses*, or responses where the student does not participate in the conversation. There were 38 responses (2% of all student responses) of this type. If a student is incapable of responding to a question, they may not have the information necessary to contribute or they may be choosing not to share their thinking. When a student does not contribute to the conversation because of a gap in their understanding of the mathematics being discussed, the teacher may need to directly intervene with the student to allow them to participate positively. When comparing the frequency of *does not contribute responses* in each setting, students provided this response at approximately the same frequency. There were 23 responses in the Old Bridge setting (3% of OBHS responses) compared to 15 responses in the Kenilworth setting (2% of Kenilworth responses).

Teacher Question and Student Response Relationship

The frequency of questions and responses during a conversation provides only part of the picture of how teacher questions engage students in mathematical conversation and elicit student thinking. In order to propose what kinds of questions elicited student thinking through conversation, the relationship between the teacher question and the type of student responses is important to examine. Table 5.3 summarizes how many times a student response immediately followed a teacher question. Table 5.4 summarizes the percentage of times a student response immediately followed a teacher question.

The first observation of tables 5.3 and 5.4 shows that each category of teacher questions resulted in various types of student responses. Some questions also had more than one common student response. This documents the unsurprising claim that no type

Table 5.3

Number of Times a Student Response Immediately Followed a Teacher Question

Teacher Questions						Student Responses			
	Attunement	Confirmation	Clarification	Thinking Aloud	Proof Building	Answer	Seeking	Questions Student	Does Not Contribute
Discoursive	8	4	2	40	11	31	1	4	5
Retracing	8	17	3	9	16	45	9	1	-
Clarification	5	20	22	19	11	93	6	4	5
Explanation	2	3	-	9	5	17	4	-	3
Justification	3	5	1	6	13	12	1	-	6
Confirmation - t	1	45	4	11	7	36	4	1	-
Confirmation - s	2	14	4	18	17	18	18	6	1
Following	3	9	5	21	8	52	8	4	7
Suggestion	5	10	1	2	1	10	4	3	2
Procedural	13	21	2	6	3	13	18	2	1
Repeat	1	-	7	1	4	5	3	-	1

Note: Dashes indicate the response did not immediately follow the teacher question

Table 5.4

Percentage of Times a Student Response Immediately Followed a Teacher Question

Teacher Questions						Student Responses			
	Attunement	Confirmation	Clarification	Thinking Aloud	Proof Building	Answer	Seeking	Questions Student	Does Not Contribute
Discoursive	8	4	2	38	10	29	1	4	5
Retracing	7	16	3	8	15	42	8	1	-
Clarification	3	11	12	10	6	50	3	2	3
Explanation	5	7	-	21	12	40	9	-	7
Justification	6	11	2	13	28	26	2	-	13
Confirmation - t	1	41	4	10	6	33	4	1	-
Confirmation - s	2	14	4	18	17	18	18	6	1
Following	3	8	4	18	7	44	7	3	6
Suggestion	13	26	3	5	3	26	11	8	5
Procedural	16	27	3	8	4	16	23	3	1
Repeat	5	-	32	5	18	23	14	-	5

Note: Dashes indicate the response did not immediately follow the teacher question

of teacher question automatically elicited a particular kind of student response; there was a high degree of variation in response to each question. In particular the lack of correlation between specific questions and student responses is also interesting. Clarification questions would appear to require clarification responses by the nature of their definitions. However, the most frequent response was answer responses (50% of all responses to clarification questions). This means that when directly asked by the teacher to provide more information about their mathematical students, half of the time students provided only short/recall responses. Similarly, explanation questions would appear to require thinking aloud responses based upon their definitions. However, once again, the students most frequently gave answer responses (39.5% of all responses to explanation questions). The analysis of the relationship between teacher questions and student responses examines each question and discusses the most frequent student response. Additional information about other student responses is provided for some questions.

The most frequent response to *discoursive questions* was *thinking aloud responses*. There were 39 responses (38% of all responses) of this type. Nearly half of the responses to discoursive questions were *thinking aloud* or *proof-building* responses, illustrating that attempts to have more student ideas injected into the conversation led to frequent eliciting of student reasoning. No other question type elicited a combination of thinking aloud or proof-building responses as often. When students were invited to participate in the conversation their first response involved sharing their thinking, which provides the participants with an additional explanation of the concepts. There were also 31 answer responses (29% of all responses) to discoursive questions. Answer responses can be considered at the opposite end of the conversation spectrum to thinking aloud

responses because a thinking aloud response is the sharing of student ideas and an answer response is short recall statement. Students giving answer responses to discoursive questions provided additional information to the conversation, but their contribution did not include thinking for other students to consider.

The most frequent response to *retracing questions* was *answer responses*. There were 45 responses (42% of all responses) of this type. When teachers asked student consider a previous idea, the question elicited a short recall statement as the first response. This could mean that the retracing questions referred to previous knowledge that students only needed to recall instead of asking students to revisit their previous thinking and elaborate upon earlier ideas. It also suggests that a retracing question might bring a particular piece of information into the conversation where subsequent questions could have students reflect on these questions in more detail.

The most frequent response to *clarification questions* was *answer responses*. There were 92 responses (50% of all responses) of this type. The correlation appears to be reasonable if the questions referred to previous information given by the students. However, clarification questions could have elicited thinking in the form of thinking aloud response if the questions asked student to provide more detail to their thinking.

The most frequent response to *explanation questions* was also *answer responses*. There were 17 responses (40% of all responses) of this type. When teachers asked students to explain their thinking, the student's first response was to provide a short recall response. The could mean the teacher needed to question students further to elicit their thinking, particularly since there were only nine thinking aloud responses (21% of all responses) as an immediate response to explanation questions. It is worth noting that

31% of the responses to explanation questions were thinking aloud and proof-building responses, indicating that they sometimes were effective at eliciting student thinking.

The most frequent response to *justification questions* was *proof building responses*. There were 13 responses (28% of all responses) of this type. When teachers asked a student to provide some form of proof for their thinking, students most often provided some form of justification as a first response. This result supports the correspondence between justification questions and proof building responses. However, as with discoursive questions, students gave almost as many answer responses. There were 12 responses (26% of all responses) of this type, which could be considered the opposite of a teacher asking a student to validate their ideas. Without students justifying their thinking, the participants can not discuss and determine the validity of an argument.

Similar to the agreement between teacher justification questions and student proof building responses, the most frequent response to *confirmation questions* seeking student agreement, T(con-t), was *confirmation responses*. There were 45 responses (41% of all responses) of this type. This is not surprising. Since this response indicates the teacher and student had reached a mutual understanding, the response matched the reason for the question.

The most frequent response to *confirmation questions* where teachers indicate agreement with student, T(con-s), was *thinking aloud, answer* and *seeking responses*. Each response occurred 18 times (18% of all responses) immediately after a confirmation question. If students immediately followed the teachers agreement with a thinking aloud response, they may have been sharing their thinking previously and continued to share their thinking after teacher approval. When the first student responses were answer

responses, the students may have provided the teacher with another piece of information or could have stated their conclusion after the teacher agreed with their reasoning. If a student followed the teacher's agreement with a seeking response, they may have reached the point where their own thinking halts their ability to contribute to the conversation.

The most frequent response to *following questions* was *answer responses*. There were 51 responses (44% of all responses) of this type. When the teacher asked a question based on students' statements, a short recall response may be the student giving one more piece of information about their previous statement. An answer response may indicate the student is not capable at that point to provide share their thinking or provide a reason for their ideas.

The most frequent response to *suggestion questions* was *confirmation* and *answer responses*. Each response occurred nine times (26% of all responses). Students who gave a confirmation response were most likely agreeing with the teacher's input. Students who gave answer responses most likely figured out a small piece of information needed to continue solving a problem.

The most frequent response to *procedural questions* was *confirmation responses*. There were 21 responses (27% of all responses) of this type. When teachers directed students to perform a specific mathematical task, the students were most likely agreeing with the instructions of the teacher. Students also responded with a seeking response 18 times (23% of all responses). When students looked for more information from the teacher as their first response to a procedural question, they were most likely asking for more help than offered by the teacher.

Finally, the most frequent response to *repeat questions*, which only occurred in the Old Bridge setting, was *clarification responses*. There were seven responses (32% of all responses) of this type. When teachers asked students to repeat a statement that was not heard, students most often followed the question with more detail without direct prompting. This indicates that simply asking students to restate their ideas will sometimes prompt them to do so in more detail.

Teacher Questions and Student Thinking. The myriad of student responses to each teacher question does not provide a complete picture of when students were explaining their thinking. For example, the two responses where students verbalize their mathematical reasoning are thinking aloud and proof building. According to table 5.3, discoursive questions and confirmation questions that indicate agreement with students resulted in students explaining or justifying their thinking more frequently than other types of questions. There were 50 and 35 responses, respectively, of these types combined. The questions that resulted in the greatest percentage of thinking aloud and proof building responses were discoursive and justification questions. These responses occurred 48% and 40% of the time, respectively. Based upon this information, one could say teachers should ask discoursive and justification questions and make statements that agree with the student in order in an attempt to elicit student thinking. These questions would not be expected to produce the type of response from students that involve sharing thinking. This implies that engaging students in mathematical conversation in order to elicit their thinking is more complex than asking specific types of questions.

When examining the teacher questions asked by the two teachers is this study another way to elicit student thinking is to ask explanation and justification questions.

These two questions imply that the teacher is requesting the student to verbalize their thinking by either sharing or providing a form of convincing for their ideas. However, these questions accounted for only nine percent of all questions asked by both teachers individually and when considering all questions in both settings. With such a small occurrence of requests by the teacher for students verbalize their thinking, further analysis is necessary in order to determine how teachers used questions to create a conversation and elicit student thinking.

While tables 5.3 and 5.4 were helpful to see how students immediately responded to teacher questions, another view of the data may provide a fuller view of the conversation. Therefore, in order to probe deeper into how these teachers elicited student thinking, I will use a sample vignette from each setting to describe the student talk and how the teacher questions led to that talk.

Themes of Questioning. As described in the methodology chapter, I developed the themes by viewing each data set as smaller pieces based on appropriate divisions, students going to the board in the Old Bridge setting and mathematical topics in the Kenilworth setting, in order to look for patterns with respect to teacher questions. Within the sub-sections of the data, three themes of questioning appeared. The first two themes were common among both the whole class discussion in the Old Bridge setting and the group work interactions in the Kenilworth setting. The third and fourth theme were distinct to the Old Bridge and Kenilworth settings respectively.

Initiating. Rittenhouse's (1998) comment that "students do not automatically begin talking about mathematics in a meaningful way simply because they are presented with appropriate tasks or are placed together in groups and told 'talk to each other'" (p.

169), implies the teacher must play a role in encouraging students to explain their reasoning. In order for students to explain their thinking in a classroom, the teacher can use questioning to elicit the reasoning from students. The initiating theme is defined as the teacher using questions to make student thinking part of the public discourse in the classroom.

Inviting. When a student's thinking becomes part of the public discourse, their reasoning becomes the primary information disseminated to the class. The teacher can continue to question the student to elicit more thinking, but in order to develop a conversation among other members of the mathematical community additional voices and ideas should be included. When other students include their thinking as part of the public discourse, the discussion can focus on the validity and understanding of student ideas. The inviting theme is defined as the teacher using questions to include other community members to become participants in the public discourse.

Supporting. When students interject ideas and questions into the conversation, without teacher prompting, they are participating in a student-to-student dialogue. Even though the teacher has not initiated student thinking or invited other students to contribute to a conversation, the teacher often assumes the responsibility for determining if the ideas in the conversation are correct and the arguments in the conversation are mathematically valid. In order to work within the constraints of a classroom, such as time limits or curriculum goals, the teacher can use questioning to make sure the student-to-student conversation is productive. The supporting theme is defined as the teacher using questions to assist student-to-student conversation.

Revisiting. When students share and discuss ideas, their discussion may not lead to agreement among the participants or be mathematically valid. During a discussion, the teacher makes pedagogical decisions about which student ideas to follow. At some point, it can be useful for the community to return to ideas that are crucial to mathematical understanding or the teacher must be sure students have the correct understanding of the mathematical ideas discussed. The revisiting theme is defined as the teacher using questions to reconsider student ideas.

Discussion

In order to answer the third goal of this study, which is to develop an understanding of how specific types of questions could engage students to share their reasoning, a sample conversation from each setting is analyzed. Each conversation is divided into the sections used to develop the questioning themes and the themes are used to demonstrate how each teacher elicited student reasoning and engaged students in mathematical conversation. For the Old Bridge setting, I selected the conversation because it discussed the entire process of solving one problem. For the Kenilworth setting, I selected the conversation because it represented the entire discussion about the students' two solutions to the task. Both settings also demonstrate each theme multiple times within each of the divided sections.

Old Bridge Setting. The conversation selected from the Old Bridge data comes from the second day of data collection and comprises the second problem discussed during the class. In this conversation, the students are asked to find the area under the derivative curve $2x^3 + x^2$. The students draw the curve using the first and second

derivative and then develop an estimate of the area under the curve using four rectangles. After developing an equation for the area using sigma notation for four rectangles, students use a computer algebra system capable calculator to determine an approximation using more rectangles. Next, students use the fundamental theorem of calculus to find the exact area. The homework assignment to write the area using an infinite number of rectangles is the final part of the conversation.

The transcript presented contains four parts based upon the student leading the class at the board. For part one, Andrew leads the class, the transcript shows the initiating theme. For part two, Amy leads the class, the transcript shows the follow themes: initiating, inviting and supporting. For part three, Wendy leads the class, the transcript shows the following themes: initiating, supporting, inviting, initiating, inviting and supporting. For part four, Carl leads the class, the transcript shows the inviting and supporting theme simultaneously followed by the supporting theme.

Andrew Leads the Class. Dr. G presents the problem to the class by writing it on the board and explaining the problem to the class. Andrew is the first student called to the board and he begins the process of finding the solution to the area problem.

Initiating Theme

18:37	Dr. G	T(d)	So. Andrew, you're up. Now you can erase everything but my question. [Andrew begins to write on the board.]
19:57	Dr. G	T(e)	Now a little verbalization, Andrew would help.
20:01	Andrew	S(ans)	Alright, the derivative of the parent function.

20:02	Dr. G		Remember when your mother watches this on public access channel. She wants to hear you talk.
20:10	Andrew	S(ta)	Alright. The derivative of the parent function would be $6x^2 + 2x$, which is the power rule. And then to draw the graph, just plug in one, two and three to get the points.
20:23	Dr. G	T(p)	Let's just use one and three. Just the endpoints.
20:26	Andrew	S(con)	Okay. [Plugs in one and three and writes points down]
21:18	Dr. G	T(p)	Now the graph can be rough. Doesn't have to be finely tuned with lots of tick marks. Okay. Just start with those two points and Amy will take it from there. [Andrew draws the first quadrant of the coordinate plane and plots the points (1,8) and (3,60)]
21:57	Andrew	S(seek)	Should I put any tick points or just?

At 19:57, Dr. G asks an explanation question to make Andrew's thinking about his written inscriptions public for the class. This question is an example of the initiating theme because Dr. G wants Andrew to share his thinking with the class. Andrew's thinking aloud response (20:10) provides information about how he found the derivative and how he is going to produce the graph. This part of the discussion also contains an inviting theme question. At 21:18, Dr. G directs Andrew about the quality of the graph and then gives instructions for Amy to continue the work. Dr. G has continued the conversation by inviting another student to share their thinking.

Amy Leads the Class. Amy is now at the front of the class and continues the work of Andrew, who plotted two points on the graph at (1,8) and (3, 60). Amy draws a vertical line on the graph.

Initiating Theme

22:10	Dr. G	T(c)	Alright. Amy. Now what are you about to do Amy?
22:16	Amy	S(ans)	I am going to start the rectangles.
22:18	Dr. G		No.
22:20	Amy	S(a)	Why?
22:21	Dr. G	T(f)	Oh, I don't want to Amy.
22:23	Amy		Geez.
22:24	Dr. G	T(f)	What else could you do?
22:26	Amy	S(ta)	Oh Uh, I have to do the thing to figure out the increases, concave up.
22:33	Dr. G	T(e)	Very good. Talk to us Amy.
22:36	Amy	S(pb)	Okay, um. So you have to find the area of the derivative of the parent function. So to find out if it increases you have to find the derivative of this. So, the derivative of the derivative of the parent function would be $12x + 2$. And then I should graph it. And then when it's one it's going to be, when it's one it's going to be 14. And then when its three it going to be, 36 times, 38. And then we know that this function is linear because it's to the

power of one and it's above the x axis so you know that it's increasing.

23:54	Dr. G	T(c)	Whoop, whoop, whoop, whoop, whoop. What did you say?
23:58	Amy	S(ta)	Wait, you know that it's above the x axis
24:02	Dr. G	T(c)	Wait. Time out. Remember this rule we have about pronouns. That they stink because we don't know what they mean half the time.
24:08	Amy	S(c), S(pb)	Okay, you have these two points that, of the, of lower case f(x) and you have to figure out if the graph increases and the concavity of it. So you have to find the first derivative of the derivative to find out if this line, this linear function, is above the x axis or below the x axis and if it's above the x axis then you know that this graph is increasing.

Shift to Inviting Theme

24:34	Dr. G	T(d), T(c)	What'd she say Mikey?
24:39	Mikey	S(c)	The derivative above the x axis increases.
24:43	Dr. G	T(rep)	A little louder Mikey, I almost caught it.
24:45	Mikey	S(c)	The derivative of the derivative is above the x axis it's increasing.

24:49	Dr. G	T(c)	You uses this word it's and that's where I lost you. Try again.
24:53	Mikey	S(c)	The original graph. The first derivative of the parent function.
24:57	Dr. G	T(rep)	So say it all again.
25:00	Mikey	S(c)	Okay. If the first derivative of the derivative of parent function is above the x axis then the derivative of the parent function is increasing.
25:10	Dr. G	T(con-s)	That's not bad. Go ahead.
25:12	Amy	S(pb)	So, this line is above the x axis so you know that it's increasing
25:18	Dr. G	T(c)	What's increasing?
25:22	Amy	S(c), S(ta)	The derivative is increasing. The derivative of the parent function is increasing. So then now you have find the concavity of it. So now you have to find the derivative of the derivative of the derivative of the parent function.
25:40	Eric	S(ans)	It's called the second derivative
25:45	Dr. G		Now Amy, the next time you say something like that please look at the camera, because nobody would believe this. Alright.
25:52	Amy	S(ta)	Okay. So, I'm finding the second derivative. So then I use the thing. And then

25:59	Dr. G	T(c)	Uh, the thing.
26:01	Amy	S(c)	I use the power rule.
26:03	Dr. G	T(con-s)	Okay.
26:04	Amy	S(pb)	And so it's just 12. So. (Draws the first quadrant of the coordinate plane.) So then like. The domain is restricted to one and three so you just need it at 12 between 1 and 3. And now, this graph it's above the x axis again, so now you know the concavity of this because it's increasing so you know that it concaves it up. So now that we've determined that it increases that this, the lowercase f(x) increases and it concaves up. So you can draw the line.
26:59	Dr. G	T(con-s)	Alright. Very Good

Shift to Supporting Theme

27:01	Chris	S(qs)	Amy can I ask a question?
27:03	Amy		Yeah
27:04	Chris	S(a)	Because the uh second derivative of the original derivative is above the x axis, it means that the first derivative is concave up.
27:16	Amy	S(ta)	The derivative, yeah. The derivative of the parent function is concave up. Which is this
27:19	Chris		Alright

27:20	Dr. G	T(con-t)	Alright so now we have a reasonable analysis of what the graph looks like over the interval for which we're interested.
27:31	Amy	S(ans)	Correct.
27:32	Dr. G	T(f)	Now what are we gonna do?
27:33	Amy	S(ans)	Rectangles

This part of the discussion begins with questions in the initiating theme. At 22:10, Dr. G asks a clarification about Amy drawing a vertical line on the board. This question leads to a back and forth exchange about the next step in the process. Amy's thinking aloud response (22:26) indicates she will find the shape of the graph between the two points plotted by Andrew. Dr. G's explanation question (22:33) initiates Amy into explaining her thinking about determining the shape of the graph. Amy's next three responses (22:36, 23:58, and 24:08) share her justifications and thinking about how to use the first and second derivative to find the shape of the graph.

Once Amy's ideas are made public for the class, Dr. G's questioning changes to the inviting theme. At 24:34, Dr. G asks a discoursive question to Mikey, which is also asking Mikey to clarify Amy's utterances because of the use of pronouns. Mikey contributes to the conversation with a clarification response (25:00) that gives a clearer explanation of why a function is increasing and Dr. G agrees with Mikey's utterance.

After Mikey's input, Amy continues to share her thinking about the concavity of the graph. Since Amy is already initiated into the discussion, Dr. G only asks clarification questions during her following utterances. Students continue to be engaged in the conversation without intervention from Dr. G when Chris asks a question directly

of Amy (27:01). Dr. G is now questioning in the supporting theme since Chris and Amy

are engaged in a student-to-student conversation. A confirmation question (27:20)

indicates Dr. G's agreement with their discussion so the class can move forward. Next,

Dr. G asks a following question (27:32) to determine where the process should go. These

two questions support student conversation by showing agreement with Chris and Amy's

reasoning and allowing the students to provide information about how the process should

continue.

     Wendy Leads the Class. After Amy answers the next step in the process is

rectangles, Dr. G invites Wendy to participate in the discussion. At the beginning of this

part of the conversation, Dr. G. uses several procedural questions to frame how the class

is going to use rectangles to calculate the area.

Initiating Theme

27:35	Dr. G		Alright, at that point. Wendy. [interrupted by announcement on public address system]
27:49	Dr. G	T(p)	Now Wendy. Um, let's start with four
27:54	Wendy	S(con)	Okay.
27:55	Dr. G	T(p)	and then we'll build our way up. I'm going to assign certain groups other numbers of rectangles and then you can create some sort of chart for us. Alright
28:05	Wendy	S(con)	Alright.
28:07	Dr. G	T(p)	So Wendy is going to do four and then we'll break this up into some teamwork. [Makes a table on the board.]
28:45	Dr. G	T(p)	So you go ahead and do four and we'll fill this in later

28:49	Wendy		Um.
28:50	Dr. G	T(e)	Now Wendy I'm really really interested in hearing you talk about how you put this together. With sigma notation.
29:01	Wendy	S(seek)	Wait should I do the one-fourth?
29:05	Dr. G	T(p)	Let's see if we can go straight to sigma and see if everybody buys it.
29:10	Wendy	S(seek)	Just write it.
29:11	Dr. G	T(p)	Straight to sigma. [Wendy writes sigma, n = 1 to 4, (2(1/4(1+1/4)^2+( ]
29:57	Dr. G	T(d)	Time out. Now talk to us.
30:02	Wendy	S(ta)	Um. Since this is divided into four rectangles so to get from. To figure out the area of the first rectangle is going to be one-fourth times um. This is the starting value which is one. Can I answer a question?
30:25	Dr. G	T(f)	Sure can.

Shift to Supporting Theme

30:26	Wendy		Mike.
30:27	Mike	S(pb)	Yeah, you should only divide it into one half cause even though it's four rectangles it's over a space of two. So it would be one-half instead of one-fourth
30:36	Wendy	S(qs)	Wait why is?

30:37	Dr. G	T(c)	Wendy. What did he say?
30:38	Wendy		I can't hear him.
30:39	Dr. G		I couldn't either.
30:39	Mike	S(c)	You would divide it into one-half instead of one-fourth.
30:42	Wendy	S(qs)	Why?
30:42	Mike	S(pb)	Cause even though you are dividing it into four rectangles it's over an interval of two from one to three. So. So each distance would be one-half.
31:04	Wendy	S(a)	I'm not sure I understand
31:06	Dr. G	T(p)	Okay. Put your hands around the width of the first rectangle.
31:13	Wendy	S(seek)	This.
31:14	Dr. G	T(r)	Yeah, now Mikey how wide is that?
31:17	Mike	S(ans)	one-half.
31:19	Dr. G	T(e)	How did he get that?
31:20	Wendy	S(ans)	By saying it's one half of this interval.
31:22	Mike	S(c)	No, it's one half with the first rectangle. Yeah.
31:28	Dr. G	T(c)	Now Mikey, try to explain how you arrived at that conclusion.
31:31	Mike	S(pb)	Alright see from one to two.
31:33	Wendy	S(con)	Uh huh.
31:33	Mike	S(pb)	Halfway between that. Yeah. So from one to that point is one- half.

31:39	Wendy	S(con)	Uh huh. Then you still start from one.
31:41	Mike	S(con)	Yeah.
31:43	Wendy	S(a)	You still start from one right.
31:45	Dr. G	T(f)	Now wait a minute. Show me the second rectangle.
31:50	Wendy	S(ans)	Here.
31:51	Dr. G	T(f)	How wide is that one?
31:52	Wendy	S(ans)	one-half
31:53	Dr. G	T(con-s)	They're all one half.
31:53	Wendy		Okay
31:54	Dr. G	T(p)	Now show me where you put that one-half.
31:58	Wendy	S(ans)	Here. [Points to 1/4 inside (1+1/4)^2]
32:00	Dr. G	T(f)	Where else?
32:01	Wendy	S(ans)	Here. [Points to first 1/4 in expression]
32:03	Dr. G	T(j)	Why there?
32:09	Wendy	S(dnc)	I don't know. Why?
32:12	Dr. G	T(r)	Back to the first rectangle. Show me the height. Now where are we going to determine the height. On the right hand side correct?
32:24	Wendy	S(seek)	Right here?
32:25	Dr. G	T(f)	So how high is that?
32:43	Wendy	S(seek)	Is it one plus one times one times twelve?

Shift to Inviting Theme

32:49	Dr. G	T(d)	Suppose I said. It's this high when x equals what Ronak. [Points to original function.]
33:02	Ronak	S(a)	Wait. Can you say that one more time?
33:04	Dr. G	T(f)	It's this high when x equals? [Points to original function.]
33:11	Ronak	S(ta)	x equals the. When x equals one and a half.

Shift to Initiating Theme (Starting the problem over again)

33:21	Dr. G	T(r)	Now you want to try it again?
33:22	Wendy	S(seek)	Am I supposed to change this number?
33:36	Dr. G	T(r)	Okay. Now now. Time out. Right now what you're trying to do is edit a number and you're not looking at the big picture. So let's take all this out here [erases Wendy's work]. Every one of those rectangles is how wide?
33:45	Wendy	S(ans)	one -half.
33:45	Dr. G	T(f)	One-half. So every length or height is going to be multiplied by the same factor of?
33:55	Wendy	S(ans)	One-half.
33:56	Dr. G	T(c)	One-half. So we could write the one-half one time if we want because it's a factor of all these areas. So. Where do you want to write it?

34:09	Wendy	S(ans)	All the way at the beginning.
34:11	Dr. G	T(con-s)	Fine by me.
34:18	Wendy	S(seek)	I just multiply. [Writes sigma (1/2)]
34:19	Dr. G	T(r)	Close. Because remember that's a common factor right.
34:21	Wendy	S(con)	Okay.
34:22	Dr. G	T(r)	Now hang on a second. Now here's the tricky part. Let's concentrate just on the first rectangle. What value of x are you going to use in the function as input? [Draws [after (1/2)]
34:41	Wendy	S(ans)	One.
34:43	Dr. G	T(f)	Plus.
34:45	Wendy	S(seek)	Should I write it? [writes 1 +]
34:46	Dr. G	T(r)	Go ahead. Now how many widths do you have walk to get to that value of x that's interesting?
34:55	Wendy	S(ans)	One.
34:57	Dr. G	T(c)	Where is that one already accounted for?
35:04	Wendy	S(ans)	Here. This one. [Points to 1 in expression]
35:09	Dr. G	T(c)	Where is that one already addressed?
35:17	Wendy	S(a)	What do you mean?

Shift to Inviting Theme

35:24	Dr. G	T(d)	Yla, where is that one already addressed?
35:27	Yla	S(ta)	In the summation.

35:29	Dr. G	T(con-t)	In the summation counting numbers. Okay?
35:31	Wendy	S(con)	Okay. [adds 2(1 to expression]
35:44	Dr. G	T(c)	Time out. Pretend for a minute. You know that value of x. What do you do with it?
35:59	Wendy	S(ta)	Plug it in.
36:01	Dr. G	T(con-t)	Is that what you're doing?
36:05	Wendy	S(ans)	No.
36:07	Dr. G	T(r)	What's the function start with?
36:08	Wendy	S(ans)	Two.
36:09	Dr. G	T(c)	So is that what you're doing?
36:11	Wendy		Oh.
36:12	Dr. G	T(c)	Didn't that come a little late? Yeah. Now what's the first thing you see in that function?
36:26	Wendy	S(ans)	Two.
36:26	Dr. G	T(con-s)	Two.
36:28	Anthony		Wait.
36:33	Dr. G	T(d)	Carl.
36:35	Carl	S(ta)	Shouldn't you? Aren't we using the derivative of the function not the parent function? So shouldn't it be 6 times?
36:45	Dr. G	T(c)	[to Amy]We're using the wrong one?
	Dr. G		[Inaudible conversation with Wendy.]
37:20	Dr. G	T(r)	Alright now. What does the function start with?

37:21	Wendy	S(ans)	Six.
37:23	Dr. G	T(c)	Okay. Now we need a value of x. How do we get that value of x?
37:37	Wendy	S(ans)	One plus one-fourth. Is it one-fourth or is it one-half?
37:44	Dr. G	T(con-s)	One-half. It's always one-half.
37:46	Wendy		Okay.
37:58	Dr. G	T(r)	How many one-halves? [Inaudible conversation with Wendy as she finishes writing the expression.]

Shift to Inviting Theme (Inviting comments from entire class)

38:36	Dr. G	T(d)	Now comments. Anthony. Question.
38:40	Anthony	S(ans)	You're missing one parenthesis.
38:47	Chance	S(ta)	I still don't know where the one-half comes from. Like I'm totally confused about that. Like. Cause all this time I thought it was four like there's four intervals so it would be one-fourth.
38:58	Wendy	S(pb)	Because this point is one.
39:00	Chance	S(con)	Right.
39:01	Wendy	S(pb)	and one-half.
39:02	Chance	S(con)	Right.
39:03	Wendy	S(pb)	So the width is going to be one-half. One to one-half.
39:08	Chance	S(con)	Oh. Okay. Okay. So you have to look at the.
39:10	Wendy		Amy.

| 39:11 | Amy | S(ta) | I think, but I'm not sure. But I think you need parenthesis around the whole thing cause. I think. I don't know if it's necessary. |

Shift to Supporting Theme

39:18	Dr. G	T(c)	What whole thing Amy?
39:20	Wendy		She wants the whole.
39:20	Amy	S(ans)	The whole expression.
39:21	Dr. G	T(s)	If it was a plus sign in there some place yes. That's a product so we're okay.
39:26	Kristen	S(c)	You know how he talked to you and was like okay one-half is the common factor and the six is where you started.
39:30	Wendy	S(con)	Yeah.
39:32	Kristen	S(a)	What is the one after that. Like, what does that stand for?
39:34	Wendy	S(ans)	You start at one.
39:36	Kristen	S(con)	Okay.
39:38	Dr. G	T(con-t)	Alright. Now. Jason are you ready.
39:41	Jason	S(con)	Yeah.
39:42	Dr. G	T(p)	Let's get that in the calculator at least one time and let's see what happens.

The questioning at the beginning of this part of the conversation mirrors the previous two sections. Dr. G begins by using questions in the initiating theme so Wendy's thinking is made public to the class. At 28:50, Dr. G asks an explanation question. Wendy's seeking responses do not share her thinking, but rather indicate she needs more information from Dr. G. At 29:57, Dr. G asks a discoursive question so Wendy can rejoin the conversation after her silence and explain her written inscriptions. Wendy gives a thinking aloud response about how to find the area of the first rectangle to Dr. G's second initiating question.

In Wendy's thinking aloud response she wants to answer a question from a classmate. This places Dr. G's questioning the supporting theme since Wendy and Mike begin a student-to-student conversation. Dr. G's questions assist the two students as they determine the correct width for each rectangle. During their exchange, Dr. G supports their thinking by assisting when Wendy states she does not understand (31:04). He asks a retracing question to have Mike give the width of the rectangle again and then asks Wendy to explain the answer. These questions and the clarification question (31:28) attempt to have the students reach agreement about the width and understand why the value is correct. Dr. G follows the students thinking (31:45) and engages Wendy directly to determine the width and how her expression should change.

Dr. G changes his question theme to inviting (32:49) when he asks Ronak a discoursive question about the height of the rectangle. After Ronak's response, Dr. G's questioning changes again to initiating when he asks a retracing question (33:21) to have Wendy start over with writing an expression for the area of the four rectangles. Wendy's seeking response demonstrates she still needs more information before being able to

explain her reasoning about the expression for the area. Dr. G's questioning becomes more direct and frequent as he tries to draw her thinking out in smaller segments. At 33:36, Dr. G erases her work and proceeds to ask questions (to 34:46) that focuses on a single part of the expression.

The questions in this section were more specific than similar initiating theme questions in previous parts of the discussion. The students in earlier parts of the discussion held a more sophisticated understanding of the work they discussed compared to Wendy's understanding of this part of the solution. For example, Amy was able to explain how to use derivatives to determine if a function is increasing or decreasing well enough so that only a few clarification questions moved the class from Amy's thinking to a more formal mathematical statement of the process. In this section, Wendy has replied with several seeking responses to Dr. G's questioning demonstrating she needs more information before being able to proceed. Due to Wendy's need for more assistance, Dr. G asked more questions in order to elicit her reasoning and continue the conversation.

In addition to more direct questioning within the initiating theme, Dr. G also asks questions in the inviting theme. At 35:24, Dr. G asks a discoursive of Yla, who shares her thinking, when Wendy can not answer his clarification question (35:09). As Wendy begins to write the expression again (35:31), Dr. G continues to question her and asks Carl to provide input also (36:33). Carl's thinking shows the class has been using the wrong function when trying to write an area expression. Dr. G works one-on-one with Wendy at the board and they start the process over again with the correct function.

When Wendy finishes her expression, Dr. G's questions return to the inviting theme. At 38:36, he asks a discoursive question for anyone to comment about the result.

This question leads to a student-to-student conversation as Chance questions Wendy directly. Since the one inviting question led to a conversation between the students, Dr. G's questions change to the supporting theme. Dr. G asks Amy a clarification question and then provides a suggestion to her response. He completes his support by confirming (39:38) the class is ready to move forward.

  Carl Leads the Class. Over the next ten minutes, students use the TI-89 calculator to calculate the area under the curve using different numbers of rectangles assigned by Dr. G. Members of the class put values into a table on the board that shows the area for an increasing number of rectangles. When the table is completed, Wendy evaluates F(3)-F(1). After that, a homework assignment is given to write the summation as an area function for an infinite number of rectangles. Carl is called to the board to begin to write the expression for the infinite number of rectangles. Carl writes

$\lim\limits_{n\to\infty}\sum\limits_{i=1}^{n}\dfrac{1}{n}[$. The first section of transcript is an example of the inviting theme and

supporting theme occurring simultaneously.

Inviting and Supporting Themes

51:39	Dr. G	T(d)	Question Kristen.
51:40	Kristen	S(ta)	Shouldn't it be 2 over n cause that is how you did everything else?
51:46	Carl		(inaudible) [calls on a student to answer]
51:53	Matthew	S(ta)	It should be. That first expression should be n over 2, which that would be. So that top number which is n is four. So the number in the denominator is four.
52:03	Kristen		But that.

| 52:05 | Matthew | S(qs) | What? |

| 52:06 | Dr. G | T(r) | Kristen, let us hear your argument again please. |

| 52:10 | Kristen | S(pb) | Okay, it should be 2 over n. because like to get one half, you do two over four. Cause it was like you have four triangles and the area two. Two. So it was two over four which is one half and you had two over ten which is one fifth. And so on. So wouldn't it be two over n. |

Shift to Supporting Theme

| 52:35 | Dr. G | T(con-t), T(d) | Any disagreement? Everyone understand that. What'd she say Chris? |

| 52:47 | Chris | S(c) | She said that since those four triangles and those were split into two parts. I mean four rectangles. It was two over n, which was reduced to one half and that's how we got one half. |

| 53:06 | Dr. G | T(con-t) | Is that what you said Kristen? |

| 53:09 | Kristen | S(ans) | Uh, not really. |

| 53:10 | Dr. G | T(r) | Try it again, Kristen. And then we'll back it up and try and have him run it through again. |

| 53:16 | Kristen | S(pb) | Okay, I don't know how. I'm sure how to explain it. Yeah, You had okay. You had like two, one to three, three minus one is two. So, you have like two spots and |

you're doing it for four rectangles. So you did two over four is equal to one half and then you did two over ten is one fifth and so on. So to get it for n, you get two over n. I think that's what I said.

| 53:48 | Dr. G | T(d) | So, now you try and say it Chris. |

| 53:51 | Chris | S(c) | Yeah, fine. Uh, Since we're going from one to three, the top part is two. and since there's four rectangles, the bottom part is four. |

54:01	Dr. G	T(con-s),	It's the length of the interval divided by the number of rectangles. [points to the equation written on the board] It's the length of the interval divided by the number of rectangles. Now Kristen, that's probably the best piece of thinking I heard you do in six months. That was excellent. No one else picked up on that. That was excellent. And by the way, it is those kinds of nuances that are going to kill you. There is a tremendous amount of technical stuff throughout these kinds of problems and you've got to be alert to every piece of it. Excellent job, Kristen.
		T(con-t),	Now any problems with what Carl wrote?
		T(p)	Put equals 60 because that's what you're going to try to prove. Now Carl take the projector, the calculator that projects and graph the derivative function for me

The questioning in this part of the conversation is in the inviting and supporting theme. Dr. G invites Kristen to share her thinking with a discoursive question (51:39). Carl, Matthew and Kristen discuss her reasoning and Dr. G supports the student conversation by asking a retracing question (52:06) so Kristen can explain her thinking again. Kristen's now justifies her thinking (52:10) rather than just sharing her thoughts as in her previous utterance (51:53). Dr. G invites any disagreement and asks Chris for his thinking about Kristen's utterance (52:35). Once Chris contributes to the conversation, Dr. G's questions are supporting the discussion between Chris and Kristen. Dr. G asks a confirmation question to see if Kristen agrees with Chris' utterance (53:06), and then asks a retracing question (53:10) so Kristen can explain her reasoning again. Kristen's justification response is more sophisticated than her previous justification response. Dr. G asks Chris to contribute to the conversation (53:48) and finally makes a confirmation utterance (54:01) about the reasoning discussed by the two students to summarize for the whole class. The use of questions within both themes elicited the reasoning of Kristen and engaged other students in the conversation.

Kenilworth Setting. The conversation selected from the Kenilworth data comes from the final day of data collection. The students worked on the following problem[4]:

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[4] Robert Speiser of Brigham Young University developed the Placenticeras task for an honors calculus course. He participated in the summer institute as a researcher and presented the problem to the students.

To: Rutgers-Kenilworth Summer Institute
From: R. Speiser and C. Walter
Subject: Placenticeras
Date: July 7, 1999

This task is about a spiral shell. The shell here is a fossil Placenticeras, an ammonite which fell to the bottom of a shallow sea 170 million years ago near what is now Glendive, Montana, and was buried in a mudslide.

Several people can join forces to build a solution together.

1. In your photo, locate the center of the spiral, and then draw a ray from this center, pointing in any direction you like. Having chosen a center and a ray, we can now use polar coordinates to describe the spiral of the shell.

2. Make a table of r as a function of θ, for the spiral of the shell, using the photo and a metric ruler. (Let's measure distances in centimeters and angles in radians.) Based on the information you have gathered, what can you say about r, as a function of θ?
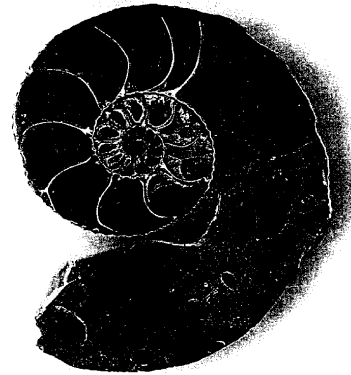
Figure 1. Copy of Placentaceras task given to students.

Before this conversation the students have collected data about shell by performing measurements on the copy of the shell. Using this data, the students developed a table that contains all of their measurements and equations that model the shell. The students have two equations that model the shell in different ways, the first in the x-y coordinate plane and the second in the polar plane. The conversation selected discusses why their equations produce different representations and what the students' equations model about the shell. This conversation's divisions are based on the different topics discussed during the interaction with Dr. A.

The transcript presented contains four parts based upon the topic discussed during the interactions. For part one, discussing the equations, the transcript shows the initiating theme. For part two, meaning of the coordinates, the transcript shows the following themes: initiating, revisiting, inviting and revisiting. For part three, shape of the graph, the transcript shows the revisiting and inviting themes together followed by the inviting

theme twice. For part four, meaning of the model, the transcript shows the initiating themes followed by the revisiting theme twice.

Discussing the Equations. Dr. A has been listening to a conversation between the students about why one graph is spiraled and the other is not.

Initiating Theme

00:39:59:19	Dr. A	T(r)	Can you explain to me (inaudible) the two equations you're working with?
00:40:05:07	Robert	S(pb)	No, we were just saying like. To explain like why this becomes that. That' s what we're, that's what we're trying to explain. Um, because this is like that, kind of like unraveled, un-spiraled. And like all the points would be the same. Like if I graph the points in polar it would be that.
00:40:24:09	Robert	S(pb)	If I graph it in a function, it would be like line. If you get an equation that goes through the points, that when we like spiral it or put it in polar, it will go through the points too. It will go through the points when they're circular if you just put it, if you get it to go through when it's a function. We just saw it like this, this kind of like that unraveled, like if you just took the spiral out and made it into a straight line. And then we um, and then we just kind of respiraled it.

00:40:55:29 Dr. A   T(c)   Can you, could you use a pen and just give me a quick,

easy explanation on the paper of what you just said,

maybe not use all the points, but do something that helps

me to understand it?

00:41:08:14 Robert   Um.

00:41:12:26 Robert   S(pb)   Say we had three points that like here, here, and here.

00:41:17:06 Dr. A   T(con-s) Okay.

00:41:17:28 Robert   S(pb)   And let's say like, this was point five, I don't know, like

one point two and like two. I (inaudible). [Robert draws

a spiral with three points with values of .5, 1.2, 2.]

00:41:27:08 Dr. A   T(c)   Okay, okay, what do those numbers represent?

00:41:29:29 Robert & Sherly S(c) The distance away from the

00:41:31:21 Sherly   S(c)   center.

00:41:32:09 Robert   S(con)   Yeah.

00:41:33:11 Dr. A   T(con-t) Okay, and so you've done the spiral and you've said that.

Let me see what you've done. And so you had a point

over here and maybe a point over here. And a point over

here or whatever. And so the distance from the center is

your, is your number, is your radius.

00:41:52:06 Robert   S(ans)   Yeah and then we figured

00:41:52:20 Dr. A   T(con-s) Okay.

00:41:53:14 Robert S(pb)   I don't know, I guess like then if you draw it on one of

these. [Creates an axis with the distance around the spiral

on the x-axis, and the number of points on the y-axis.]

00:41:57:24 Dr. A   Mm hmm.

00:41:57:24 Robert S(pb)   I guess this would be the point number, like 1, 2, 3. Like

this would be the first point the second point the third

point.

00:42:04:29 Dr. A   Mm hmm.

00:42:05:18 Robert S(pb)   And then you go over like, here I'll make this .5, 1.5, 2.

So you go over to .5 and you go up one

00:42:07:23 Sherly   (Inaudible)

00:42:13:02 Dr. A   Uh, huh.

00:42:13:15 Robert S(pb)   And then you put a line here. And you go to 1.2, up to 2,

and 2 up to 3. And you want to find an equation. [graphs

each point.]

     This part of the conversation is an example of questions in the initiating theme.

Dr. A begins her interaction with the group by asking a retracing question (39:59:19) of

Robert. By asking Robert to explain the equations to her after explaining it within the

group, Dr. A places Robert's thinking about the two equations as the main reasoning for

the group's work. Robert's proof building response tries to justify why both equations

create the same graph depending on if the points are polar or not. Dr. A further engages

Robert in conversation by asking a clarification question (40:55:29) about his explanation

using an example. Robert continues to verbalize a justification for his reasoning by

attempting to draw an example graph on a few points. Both questions by Dr. A initiate

Robert into a public discourse by expressing his reasoning about the two equations and

the graphs they produce.

     Meaning of the Coordinates. After Robert gives an example of plotting points to

find an equation, Dr. A changes the topic by asking about the meaning of the coordinates

at a point.

Initiating Theme

00:42:19:24	Dr. A	T(c)	Okay, I'm pretty dumb about this. I need to know which numbers, what is, what are you're two coordinates at those points?
00:42:27:28	Robert	S(pb)	Oh yeah right. This is distance around spiral. [Labels the horizontal axis 'distance around spiral'.]
00:42:37:04	Dr. A	T(f)	Okay, and so you're taking this point up here? Where the distance was
00:42:42:20	Robert	S(ans)	Yeah, but see we
00:42:43:00	Dr. A	T(con-t)	0.5, or something like that.
00:42:44:10	Robert	S(ans)	Yeah, we put .5
00:42:46:06	Dr. A	T(c)	And you're taking it over here?
00:42:48:14	Robert	S(ta)	Yeah, I, but this is like .5, this is 1 this is 1.5. Like it's going by one-half, and this is 1, 2, 3. And this is like [labels the tick marks on the horizontal axis.]
00:42:58:13	Dr. A	T(j)	Okay, how did you decide how far up for them to go?

00:43:01:20   Robert   S(pb)   Alright, so this is. This is like the first point, the second

point, the third point.

00:43:06:13   Dr. A   T(c)   I don't get it.

00:43:08:10   Robert   Alright, um.

Shift to Revisiting Theme

00:43:09:13   Dr. A   T(r)   I thought that generally when you plotted a point you had

two coordinates and they both meant something.

00:43:16:20   Robert   S(ta)   Yeah, that's why I don't know what this one means. Like

I actually I thought like one number it would be...

00:43:22:28   Robert   S(ta)   But it might have something the angle, I don't know. It

might be like how much it turned or something. But um,

I don't know that part yet.

00:43:35:10   Dr. A   T(con-t)   But you got those points to plot on the scatter plot?

00:43:40:11   Robert   S(con)   Yeah.

00:43:40:22   Dr. A   T(r)   What were you plotting and what were the two

coordinates then?

00:43:44:09   Robert   S(pb)   Ah, c2, so that would be like the degree here. [points to

axes he drew.] Like the amount or something. And I

guess like, compared to one circle, so if like you go 90

degrees around, this would be like 1/4, I guess, and then

if you go like half way around the circle this would be

1/2. And then um 3/4's, if you go like 3/4's around the
circle.

Shift to Inviting Theme

00:44:05:24	Dr. A	T(r)	Okay, I can't, Victor said something a minute ago about which was the x and which was the y? Which is the
00:44:12:04	Sherly	S(ans)	C2 is the x, and c1 was the y.
00:44:15:10	Victor	S(con)	The degrees is c1, um wait yeah. [Victor is looking down at his calculator in his lap.]
00:44:20:11	Sherly	S(con)	C2 was the x and c1 was the y.
00:44:22:08	Dr. A	T(c)	Okay, c2?
00:44:23:08	Sherly	S(ans)	Was the x.
00:44:24:08	Victor	S(con)	Yeah and c2
00:44:24:21	Dr. A	T(con-t)	Is the x.
00:44:25:28	Victor	S(con)	Uh huh.
00:44:26:08	Dr. A	T(c)	And what is c2?
00:44:27:29	Victor	S(ans)	That's, they're radians.
00:44:30:23	Sherly	S(con)	Yeah.
00:44:32:27	Dr. A	T(c)	Is the number, is what you were just talking about? And c1 is the y. And what was c1?
00:44:41:17	Sherly		Um.
00:44:42:17	Victor	S(ans)	Distance.
00:44:43:15	Dr. A	T(con-s)	Is the distance.

00:44:44:16 Robert S(pb) Oh, so it's like, if this went to like, if this was over here, and then this was over here.

Shift to Revisiting Theme

00:44:50:18 Dr. A T(r) Can you do it again?

00:44:51:23 Robert S(ans) Alright. [creates a new coordinate plane with the distance as the y-axis and the degrees of the points as the x-axis.]

00:44:51:23 Dr. A T(j) So I can see what you mean.

00:44:55:18 Robert S(ans) Distance.

00:45:03:04 Robert S(ta) And then I guess this would be degrees around or radians around.

00:45:08:13 Dr. A Mm hmm

00:45:11:16 Dr. A T(c) And so it would be like pi/2

00:45:15:17 Robert S(con) Yeah.

00:45:16:12 Dr. A T(con) and pi? Is that right?

00:45:18:23 Robert S(seek) Yeah, because we are going to be in radians right? (whispers so pi/2).

00:45:21:09 Dr. A T(f) I think, I don't know.

00:45:22:17 Robert S(ans) Yeah, 3pi/2, 2pi. [labels the tick marks on x-axis.]

00:45:28:04 Dr. A T(r) Okay, now show me how a point would (inaudible).

00:45:29:11 Robert S(pb) Oh, (inaudible), just pretend it's like taken every ninety degrees. Can we just like pretend that?

00:45:34:16	Dr. A	T(con)	That's fine with me.
00:45:35:05	Robert	S(pb)	Alright, so then we go over to ninety degrees or pi/2
00:45:38:20	Dr. A		Uh huh.
00:45:39:15	Robert	S(pb)	and then go up to .5 and put a point
00:45:42:11	Dr. A		Uh.
00:45:42:19	Robert	S(pb)	and then go over like ninety degrees more
00:45:45:15	Dr. A		Mm hmm.
00:45:46:09	Robert	S(pb)	and go to 1.2, and put a point. And then go to ninety degrees more and go to 2pi.
00:45:50:13	Dr. A		Mm hmm.
00:45:51:14	Robert	S(pb)	(Inaudible)
00:45:55:06	Robert	S(pb)	And we want to find an equation that goes through all of these. [referring to the points he just plotted.]

By changing topics through a clarification question (42:19:24) about the meaning of the points Robert plotted, Dr. A is questioning within the initiating theme. Robert's thinking about the coordinates is now the focus of the conversation. Dr. A continues the conversation by asking Robert to clarify (42:46:06) and justify (42:58:13) his thinking.

Following Robert's attempt to justify the meaning of the coordinates, Dr. A's questions move to the revisiting theme. Dr. A uses retracing questions (43:09:13, 43:40:22) to further engage Robert in conversation. These questions lead Robert to justify the meaning by mentioning one value, c2, of the coordinate represents a degree measure. At 44:05:24, Dr. A asks a retracing question to bridge between the revisiting and inviting theme. The question refers to something Victor previously stated, but also

invites Victor to comment about the meaning of the coordinates. Victor's input into the conversation clarifies c2 is radian degrees and c1 is distance. Dr. A continues the conversation with Robert using Victor's contributions by asking a retracing question (44:50:18), which moves her questioning back into the revisiting theme. Robert's responses demonstrate a clearer understanding of each axis on his new coordinate plane. Dr. A asks a retracing question (45:28:04) to have him show to plot a point. Robert's proof building responses (45:35:05 to 45:55:06) justify what each point means and that the equation would go through all the plotted points.

Shape of the Graph. After Robert explains the plotted points in the x-y coordinate plane and that their equation would go through all of the points, Dr. A changes the topic to discussing the shape of the graph. The first section of transcript is an example of the revisiting theme and initiating theme occurring simultaneously.

Revisiting and Inviting Themes

00:45:59:22 Dr. A   T(r)   And it's not going to be a spiral?

00:46:02:03 Robert   S(pb)   No, cause we're in um x and y. And you can't make a
                                    spiral with that because what we did yesterday with the
                                    vertical line test. Or Friday.

00:46:10:02 Sherly   S(qs)   (Inaudible) makes a line that goes through all of them,
                                    right?

00:46:12:10 Robert   S(ans)   Yeah.

00:46:13:09 Sherly   (Inaudible).

00:46:13:09 Dr. A   T(con-t)   So, it's not a spiral then?

00:46:15:10 Robert   S(con)   No.

00:46:16:00 Sherly   S(c)   Unless you go to polar.

00:46:17:01 Dr. A   T(c)   Because you unraveled it?

00:46:19:13 Sherly   S(con)   Yeah.

00:46:20:03 Dr. A   T(con-t) So, it's not a spiral?

00:46:22:05 Robert   S(ans)   Yeah

00:46:24:01 Dr. A   Hmm.

00:46:25:03 Robert   S(ans)   Because like, I don't know why, but

00:46:29:03 Sherly   S(pb)   Cause like x, y like the radius is like the point, and in

polar (inaudible) like circle, or circular. And so,

00:46:37:03 Dr. A   Mm hmm.

00:46:38:18 Robert   S(ta)   And then we just figure if we find an equation that goes

through this, like this. [draws on his paper by connecting

the points with a curve]

00:46:44:04 Robert   S(ta)   That like when we go to here that equation will be this.

[moves his pen to the spiral he drew with .5, 1, 1.5

marked on it when first beginning to explain to Dr. A.]

00:46:47:25 Dr. A   T(j)   Why?

00:46:49:19 Robert   S(pb)   Uh, because it goes through all the same points and

they're both like counting on the same thing. Like, this

depends on the distance around the circle and the degrees

around, like where it goes to the point. And so does this,

and they both are the same points, so we just figured it

goes through one it's got to go through the other.

00:47:08:23 Dr. A		Mm hmm.

00:47:09:17 Robert S(pb) Cause I was trying to say like, ah let's see, but this kind of looks like this unspiraled, like on the calculator. Like if you just took it out, and kept the points where they were and just like kept it going through, it would look like this sorta and then...that's it.

00:47:26:21 Dr. A T(c) Okay, but then you have one equation or you have two equations that makes those two different things?

00:47:31:07 Sherly S(c) No, it's one, but it's like in a different format

00:47:34:01 Robert S(con) Yeah, one it works for both.

00:47:35:02 Sherly S(pb) One's the polar coordinate which makes it spiral and the other one's in xy which makes it show

00:47:38:05 Dr. A Oh,

00:47:38:25 Dr. A Okay, how do you

00:47:38:25 Robert S(c) except the x is replaced with theta in polar. That's it though.

00:47:43:09 Dr. A T(f) Oh, so when you're throw theta in there instead of an x

00:47:45:29 Robert S(pb), S(qs) It acts as x, because doesn't polar like go on degrees or something?

00:47:49:08 Sherly S(con) Mm hmm.

00:47:50:16 Robert Instead of

00:47:51:18 Dr. A T(c) So it circles it around.

00:47:53:03 Robert S(con) Yeah.

Shift to Inviting Theme

00:47:55:12	Dr. A	T(d)	Angela does that make sense to you at all? or not?
00:47:59:04	Angela	S(ans)	Sort of. I like get lost with all this stuff. I hate this.
00:48:02:24	Dr. A	T(f)	What, what are the, what is the sort of question that throws you?
00:48:08:22	Angela	S(ans)	Like, I get like little bits and pieces of what he's explaining, but I don't really get all of it.
00:48:14:04	Sherly	S(qs)	What don't you understand?
00:48:16:29	Angela	S(ans)	I don't know.
00:48:18:24	Dr. A	T(con-t)	I think I hear her say, she doesn't even know what she's asking. Um what, what are you asking...in this?
00:48:28:11	Angela	S(ans)	I don't like have a specific question. I just don't like understand the whole, like everything you just explained. Like why that, the whole thing, like what you were saying like why it's like the spiral unraveled or something like that. Like I don't even know how to explain, the points and just trying to follow and I just didn't.
00:48:53:27	Robert	S(ans)	It's kind of hard to explain.
00:48:54:10	Sherly	S(qs)	Which one are you on?
00:48:56:00	Angela	S(a)	Huh?
00:48:57:07	Sherly	S(a)	No, I was talking to Michelle.
00:48:59:08	Angela	S(qs)	Do you get this? [to Ashley]
00:48:59:08	Ashley	S(qs)	Do you understand? [to Michelle]

00:49:01:09 Michelle S(ans)   What he just said. Kinda.

Continuation of Inviting Theme (Questioning leads to student conversation)

00:49:06:18 Dr. A   T(d), T(r)   Could you, could you try Michelle, to explain.

Cause every time one of you explains it, it helps me a

little more. This is really just as foggy for me, Angela, as

it is for you. I am even further away than you, from this

stuff, because I don't understand the calculator either. So

Michelle could you try it again.

00:49:26:18 Michelle S(pb)   Okay, um. Ooh. Alright if you took, let me draw a piece

of the spiral, and you picked like certain points, whatever

ones they were. Right? Robert?

00:49:41:02 Robert   Yeah.

00:49:41:21 Michelle   Okay (laugh), just checking. And like um, you're doing

the ninety degree intervals, which is like the pi over two,

you know what I mean, and like you graph them...

00:50:00:10 Michelle   Okay, then you're saying that r would be the radians? Is

that what you're saying?

00:50:04:15 Robert   Yeah.

00:50:05:27 Michelle   Well or whatever, then you decided that, I guess.

00:50:09:05 Robert   No.

00:50:09:17 Michelle   (inaudible).

00:50:11:00 Robert   Yeah, because we put in scatter plot, we put c2 first

00:50:14:10	Michelle		Oh, you switched 'em?
00:50:15:07	Robert		Yeah and then c2 is the radians or whatever, like distance. Radians. Radians.
00:50:21:00	Michelle		Okay, so this is like the c2 thing. This one's like the c1.
00:50:25:12	Dr. A	T(c)	Okay, what's c2?
00:50:26:23	Michelle	S(c)	C2 is like um, the interval around the circle like the radian thing.
00:50:31:27	Robert		Yeah, like
00:50:32:07	Michelle		Right?
00:50:32:25	Robert		how far you turn
00:50:34:06	Michelle		That's it.
00:50:35:00	Robert		from like the beginning.
00:50:35:29	Sherly		And we have like (inaudible).
00:50:39:03	Michelle		And then this one's um, pi over two right?
00:50:42:26	Robert		Yeah.
00:50:43:10	Michelle		This one's um
00:50:43:27	Sherly		Pi
00:50:44:28	Michelle		Pi. See I don't remember these. Three pi over two.
00:50:47:07	Sherly		Mm hmm. Two Pi
00:50:48:25	Michelle		this is two Pi. That's like the intervals we went around the circle.
00:50:53:03	Sherly		Mm hmm.
00:50:53:03	Michelle		Right? And that's the c2?

00:50:54:18 Dr. A   T(con-t) If it were a circle rather than a spiral. Yeah.

00:50:57:08 Michelle S(con) Yeah. This one's like pi over two, which is like ninety degrees. This one's like pi which is like one-eighty. That's three pi over two, which is like um, two seventy (laugh). This is like pi, which is 360.

00:51:11:03 Michelle And then, um this is like the actual distance from the circ, like center thing. Right? Like this would be like .5 and this would be like...what?

00:51:22:21 Robert Just say one.

00:51:23:21 Michelle One. And this one's like two. So you have...

00:51:29:01 Sherly I have a question though. Is it like distance from like a straight line or is it like

00:51:33:11 Robert No, it's distance like around the thing.

00:51:35:00 Sherly (Inaudible) just the spiral?

00:51:36:03 Robert Yeah.

00:51:36:22 Michelle Which is the distance from like center to that actual point?

00:51:38:28 Robert No, it's like 2 would be the distance from the beginning

00:51:42:17 Sherly Going all the way around it.

00:51:43:12 Robert all the way around to two. [points with his pen to the spiral and traces in a curve following the shape of the spiral.]

00:51:44:24 Michelle Oh, like

00:51:46:06 Dr. A   Oh.

00:51:47:24 Robert   I think that's what it means.

00:51:48:19 Dr. A   T(con-t) Is that true?

00:51:50:12 Michelle S(ta)   So like from this

00:51:50:22 Robert   S(ans)   That's what we're doing.

00:51:51:21 Michelle S(ta)   point we're saying that we're going all the way around

                                               and reaching 2. That's like 2 units.

00:51:59:23 Robert   S(con)   Yeah, like two centimeters, I guess we're using.

00:52:02:23 Michelle S(ta)   Oh, got that people? Got it? So then like if you just

                                               graph it, like the point five was at ninety.

Dr. A's first question, which transitions the conversation to discussing the shape of the graph, is a retracing question (45:59:22) that fits in the initiating and revisiting theme. The question revisits the shape of the graph that Robert alludes to in his first utterances (40:05:07 and 40:24:09) and initiates Robert's thinking about the shape of the graph based on his understanding of the coordinates. Robert's proof building response justifies that it does not make a spiral because the points are graphed in the Cartesian plane. Dr. A continues Robert's engagement in the conversation through confirmation questions (46:13:09, 46:20:03) and a clarification question (46:17:01). Sherly is also engaged in the conversation, but Robert continues to share his thinking (46:38:18, 46:44:04) and justify his reasoning (46:49:19, 47:09:17).

After listening to Robert's explanation with Sherly's help, Dr. A promotes further conversation by asking a question in the inviting theme. Dr. A asks a discoursive question (47:55:12) to Angela about her understanding of Robert's ideas. Angela's

response that her understanding is not completely clear, leads to another inviting question

when Michelle states she has some understanding (49:01:09). Dr. A's second discoursive

questions invites Michelle to share her thinking. The second inviting question leads to a

student conversation where Dr. A only asks two questions during a two and a half minute

time period. By inviting other students to comment on Robert's public thinking about the

equation, coordinates and shape of the graph, the students have shared their reasoning

and engaged in a mathematical conversation with little intervention from the teacher.

Meaning of the Model. After Michelle's explanation of the graph, Dr. A changes

the topic to discuss what their equation represents about the spiral. In previous days, the

students developed an equation that determined the arc length of the spiral by calculating

an average change in length per degree. Dr. A questions about what the distance

represents in this model.

Initiating Theme

00:52:12:03	Dr. A	T(c)	Hey Robert. I need to really clarify that in my head. In your data set
00:52:17:02	Robert	S(ans)	Yeah (inaudible). It doesn't make sense.
00:52:18:23	Dr. A	T(c)	isn't it these numbers? [pulls out a piece of paper from the middle of the table.]
00:52:22:15	Robert	S(ans)	I don't know I didn't put the numbers in.
00:52:24:27	Dr. A	T(d)	Victor.
00:52:26:14	Victor	S(a)	What?
00:52:27:12	Dr. A	T(c)	In your c1, which is the distance. Isn't c1 the distance?
00:52:32:19	Victor	S(con)	Right, right.

00:52:33:18 Dr. A   T(c)   Okay, when you put it in, I still need clarification as to whether it is the distance from the origin to the thing. Which is called a radius.

00:52:44:18 Dr. A   T(c)   Or is it the distance now, around the spiral. I know you all worked with both of those data sets. Which one are you using?

00:52:52:24 Victor   S(ans)   Oh, man.

00:52:54:05 Michelle S(ans)   I used

Shift to Revisiting Theme

00:52:55:00 Dr. A   T(r)   Because Angela and Sherly, I think and Ashley, whoever it was. You all were very careful and good at measuring those distances with the rubber band.

00:53:03:25 Dr. A   T(r)   Do you remember?

00:53:05:08 Angela   S(con)   Mm hmm.

00:53:06:03 Dr. A   T(c)   Okay, I need to know which data set you're working on right now.

00:53:13:08 Victor   Um

00:53:14:05 Angela   S(qs)   Isn't it like around?

00:53:15:00 Victor   S(a)   Ask that question again.

00:53:16:12 Dr. A   T(c)   Okay, I think and I just heard Robert say also something that I need clarifying, that you have in your notes. Where's the data set for the distances on the rubber band?

00:53:32:03 Angela   Um

00:53:33:22 Robert   S(ta)   Actually I think it's the radius,

00:53:34:07 Angela   S(ans)   Wait I just remem um, yeah

00:53:35:28 Robert   S(ta)   because that's what we're comparing.

00:53:35:28 Angela   I just

00:53:36:29 Dr. A   T(con-t) Didn't you write them down?

00:53:36:29 Sherly   S(ans)   (Inaudible) 47.5.

00:53:38:00 Angela   S(ans)   I just measured from the beginning to the end.

00:53:39:19 Dr. A   T(r), T(con-t)   You only had one long thing didn't you?

00:53:43:00 Sherly   S(qs)   It was 47.5 isn't it?

00:53:45:14 Victor   S(con)   Yeah (inaudible).

00:53:46:08 Dr. A   T(c)   It was 46.5 or 47.5 or something like that?

00:53:49:29 Several Students S(con)   Mm hmm.

00:53:50:10 Dr. A   T(c)   Yeah. Okay and so, and so you're saying what Robert?

00:53:54:19 Robert   S(ta)   No, I was wrong. I think now it's just like the point to

                                          here.

00:53:57:29 Dr. A   Oh.

00:53:57:29 Robert   S(pb)   I at first thought it was this but then I remember that we

                                          only measured the whole thing.

00:54:02:19 Dr. A   T(c)   It's forty (inaudible) or something or other?

00:54:04:04 Michelle S(qs)   So it's like from the origin to the actual point?

00:54:05:12 Robert   S(con)   Yeah, 47. [to Dr. A]

00:54:07:12 Robert   S(con)   Yeah. You were right first. [to Michelle]

00:54:07:12 Dr. A T(con) Yeah.

00:54:09:16 Michelle Sorry Bob. I told you.

00:54:13:03 Dr. A T(r) Because then you, then you divided it by the total number of angles?

00:54:19:04 Robert Um,

00:54:19:04 Dr. A T(r) The total, the total number of degrees around,

00:54:21:03 Robert S(con) Yeah.

00:54:21:03 Dr. A T(r) and around and around and around which was like 7 or 6 pi. And I asked,

00:54:27:07 Robert S(con) Yeah.

00:54:27:07 Dr. A T(c) and were able to figure an approximate distance for everything?

00:54:31:09 Robert S(ans) Yeah, it's like point three or something or .03.

00:54:33:00 Dr. A T(con-s) Yeah, yeah, yeah

00:54:37:05 Dr. A T(c) But is that what you're working with now?

00:54:39:19 Robert S(ans) Um, nah I just think we are just working with the original things we got here.

00:54:45:01 Dr. A T(c) Which is?

00:54:46:10 Robert S(ta) Distance from the center to the point. Like the radius. Yeah I remember. Isn't that the problem?

00:54:50:01 Dr. A T(r) Yeah, I think one of the really interesting things about you guys working on Friday, was that you were really

			working with these two different, very different ideas that are both really really good,
00:55:04:00	Dr. A	T(r)	but um, but this notion, I remember, at the end we were plugging things in and you were coming out and you were able, amazingly, could tell me a,
00:55:15:03	Dr. A	T(r)	if it was 46 all the way around when you got the average rate of change per degree, you could tell me how far around, along, around the spiral. Do you remember that?
00:55:24:20	Angela	S(ans)	Yeah. It's the one we used.
00:55:28:04	Dr. A	T(con-t)	Yeah, yeah, yeah. Which to me is really impressive. But I think I agree that that's not the same thing, that you're talking about now, in terms of
00:55:36:02	Robert	S(con)	Yeah it's different.
00:55:37:05	Angela	S(a)	For this one we used that. [shows what is on her calculator.]
00:55:43:17	Dr. A	T(r)	Yeah, yeah, that was when you found an approximate average distance per degree that was going around the spiral.
00:55:52:13	Angela		Mm hmm.
00:55:52:25	Dr. A	T(r)	Do you remember?
00:55:53:20	Angela	S(con)	Yeah, plugging in the angles and then
00:55:56:20	Dr. A	T(f)	And you'd plug in an angle
00:55:57:20	Angela	S(con)	it would tell you how far around.

00:55:59:20	Dr. A	T(f)	and it would spit out a distance around. Sort of an approximate distance.
00:56:04:05	Sherly	S(con)	Yeah.
00:56:05:20	Dr. A	T(con)	Yeah and I guess what I was trying to, to make sure is, or clarify as to whether that, you know, was...was what you're doing now. And it doesn't sound like it.
00:56:20:22	Robert	S(ans)	No, I don't think so.
00:56:21:28	Victor	S(ans)	Ah, no.

Shift to Revisiting Theme (Different topic than previous revisiting theme section)

00:56:23:11	Dr. A	T(r)	Okay, but so now back to the question that I was, that Michelle you were helping me understand which is that, that you'd come up higher, and what would you do?
00:56:37:20	Michelle		Um, like
00:56:38:01	Dr. A	T(p)	I want a, I want you to plot me a point.
00:56:40:24	Michelle	S(ans)	Oh, like...the one. [referring to the graph on the paper.]
00:56:45:02	Robert		Yeah, you said that's 2pi right?
00:56:48:20	Michelle		Okay, so if you write pi, I guess it's kind of there. Like that and then [plotting points on her graph on her paper.]
00:56:57:13	Robert		You said the other one was 2 or something.
00:56:59:20	Michelle		So, say the other one was 2
00:57:01:20	Robert		2Pi.
00:57:03:25	Michelle		It has to be 2pi?

00:57:05:04	Dr. A		What's the, what's the
00:57:05:15	Robert		That's what you said.
00:57:06:22	Michelle		Well, I just took numbers. It don't matter does it?
00:57:10:07	Robert		No.
00:57:11:01	Dr. A	T(c)	Okay so the first point was...
00:57:13:14	Michelle	S(ans)	Well like the first point
00:57:14:26	Dr. A	T(f)	Would have been this one.
00:57:15:21	Michelle	S(ans)	would be this .5 thing. Which is like
00:57:20:00	Dr. A	T(j)	Could it be here?
00:57:22:10	Michelle	S(ta)	Yeah, can't it be anywhere? Well, wait I don't
00:57:25:04	Robert		Well, what you're saying that's like
00:57:26:27	Michelle		Wait
00:57:28:24	Dr. A	T(c)	I thought it has to be further than ninety degrees.
00:57:30:26	Robert	S(ans)	Yeah, she's just saying like, pretend it's ninety degrees around.
00:57:33:26	Dr. A	T(con-t)	Pretend what's ninety degrees?
00:57:35:04	Robert	S(ans)	That .5 is ninety degrees around the thing.
00:57:39:19	Dr. A		Oh.
00:57:41:08	Robert	S(pb)	Like if you were starting here. Just saying like pretend turn ninety degrees, and now you're here. And that's where the .5 is.
00:57:49:28	Robert	S(pb)	That's not ninety degrees here, or where ever, here.

00:57:54:02	Dr. A	T(j), T(r)	How are you gonna be able tell when you've turned ninety degrees? Didn't you all have that your very first day?
00:58:01:29	Robert	S(ans), S(qs)	Yeah, mixed right angle, right?
00:58:03:10	Sherly	S(a)	Hmm?
00:58:03:10	Michelle	S(con)	Oh, yeah.
00:58:04:10	Dr. A	T(c)	With your overlay, didn't you have that?
00:58:06:26	Sherly	S(ans)	We just used a ruler.
00:58:08:08	Dr. A	T(con-t)	Oh, you used a ruler, or something?
00:58:10:18	Sherly	S(con)	Mm hmm.
00:58:11:00	Dr. A	T(con-t)	Yeah. Okay and so then when you turned ninety degrees, you had, you measured the radius, is that right?
00:58:21:00	Michelle	S(con)	Mm hmm, right?
00:58:22:09	Dr. A	T(con-s)	Uh huh, I think.
00:58:22:26	Robert	S(con)	Yeah.
00:58:23:00	Michelle	S(ta)	Wait, so like it went like this was ninety degrees, but it's really not
00:58:26:08	Dr. A		Uh huh.
00:58:27:12	Michelle	S(ta)	like it had like turned to the point where like, we were like .5 centimeters away from the origin
00:58:34:00	Dr. A		Mm hmm, mm hmm.
00:58:35:04	Michelle	S(ta)	And then like when you went to like, when you went around 180 degrees which is like pi radians

00:58:41:21 Dr. A   Mm hmm.

00:58:42:21 Michelle S(ta)   you'd be like 1 centimeter away from the origin.

00:58:45:26 Dr. A   T(con-s) Okay.

00:58:46:03 Michelle S(ta)   And it just gives you

00:58:48:16 Dr. A   Mm hmm, mm hmm.

00:58:53:19 Dr. A   T(r)   And so, I think Angela was asking the question, the same

                                        question I had before as to why it's not a spiral anymore.

00:59:06:03 Victor   S(a)   Why it's not a spiral?

00:59:07:14 Dr. A   T(r)   Yeah. When you connect those things, why isn't it a

                                        spiral anymore?

00:59:12:00 Robert   S(pb)   Because for this one like, it's not a function. Like if you

                                        put a number in you can only get like 1 output. But for

                                        the other one, like you, same thing, but except it doesn't

                                        go like x and y, it like, in circles.

Dr. A initiates Robert's thinking about the data set used to create the scatter plot by asking a clarification question (52:12:03). When Robert says he does not know, Dr. A invites Victor into the conversation by asking a discoursive question (52:24:27) followed by a clarification question (52:27:12) about the data. Dr. A continues the conversation to elicit student thinking by asking questions in the revisiting theme.

The first group of revisiting questions refer back to the work students did measuring lengths of the spiral (52:55:00 to 56:21:28). During this part of the conversation, Dr. A asks different types of questions to elicit students' previous thinking about the distance around the spiral. Dr. A tries to connect the previous idea to the

current model by asking a clarification question (54:37:05) to which Robert's thinking aloud response is that the ideas are different. Dr. A completes this group of questioning with two different confirmation utterances (55:28:04 and 56:05:20) that the ideas are different.

The second group of revisiting questions elicits students' reasoning about the explanations given in earlier parts of this conversation (56:23:11 to 59:12:00). Dr. A discusses with several students how the points they were explaining to her earlier represent a distance from the origin. Michelle's understanding of the points plotted in this new model is revealed in her thinking aloud responses (58:23:00 to 58:42:21) as she states the point at pi is one centimeter away from the origin. After Michelle's reasoning is made clear, Dr. A revisits the shape of the graph and why it is not a spiral anymore (58:53:19). Robert gives a proof building response, but Dr. A steps away from the group as they continue to discuss the idea.

Relationship between Questions and Themes.

When coding for themes in the remainder of the data, the Old Bridge setting accounted for 16 instances of initiating, 19 instances of inviting, and 7 instances of supporting. In the Kenilworth data, there were 14 instances of initiating, 6 instances of inviting and 9 instances of revisiting. The occurrences happened over varying length time periods and at times, as shown in the vignettes, questions overlapped themes. The questioning themes also occurred in different orders so it can not be concluded that one theme began each part of the conversation. Another pattern evident in the Old Bridge setting was that as students worked on different problems over the three day period, the

questioning themes moved from mostly initiating and inviting during the first problem to more inviting and supporting questioning by the fourth problem. Despite the variety of when and how the themes emerged throughout the data, the teachers' questioning elicited student thinking. In the Old Bridge setting, students were successful in discussing and solving four different area problems over the three days of data collection. In the Kenilworth setting, students were successful in developing two different models for the growth of the shell, and possibly a third that determined the arc length before being abandoned for the models discussed in the vignette. Therefore, I examine how the questions teachers asked, as described in chapter four, support each questioning theme.

Questions and Initiating Theme. The initiating theme plays an important role in discussion because it provides the participants with a chance to hear another's thinking. Whether as part of a whole class discussion or intervening with a group solving a problem, the teacher can use questions to make student ideas public. Questions asked within the initiating theme can also begin the process of engaging students in mathematical conversation by hearing one student's ideas. The most direct questions to ask students to share their thinking are explanation questions. For example, Dr. G used explanation questions when students went to the board and began to write mathematical inscriptions on the board. The explanation questions asked students to verbalize the thinking behind their inscriptions and therefore place their thinking in the public domain for further questions by either the teacher or other students.

To draw out further student thinking, a teacher can use clarification or justification questions. Clarification questions can ask a student to speak using more mathematical language in their explanation. Justification questions can ask students to

provide a mathematical argument for their thinking. Both of these questions provide the other participants with more information to consider in the conversation. Procedural questions that direct students to perform additional mathematical tasks in order to give the student additional processes to explain. When questions no longer elicit further thinking, the teacher can use a questions in a different theme to continue the conversation.

Questions and Inviting Theme. The inviting theme plays an important role in discussion because it provides other participants with an opportunity to share their thinking. When the student giving the initial explanation can no longer provide the group with information, engaging other students in mathematical conversation perpetuates the discussion. Another purpose of the inviting theme in a discussion is to allow other students to present alternative ideas to the current thinking or justify why another's ideas are correct. Discoursive questions are the most direct way for a teacher to ask other students to engage in mathematical conversation. Both teachers used discoursive questions to address students specifically and generally for a contribution to the conversation. In the whole class setting, Dr. G also used discoursive questions as a way to recognize a student who raised their hand so their thinking can be heard.

Confirmation questions where the teachers asks for agreement are another way teachers can engage students in mathematical conversation. When a teacher asks if the listeners agree with the public thinking, the listener can acknowledge agreement and provide support for that agreement or the listener can share their thinking about their disagreement and provide a different approach to thinking about the mathematical ideas. Teachers can also use procedural questions to ask other students to join the conversation

via performing some mathematical task. Dr. G included other students by asking them to go to the board and demonstrate a specific part of an inscription or work with the student at the board to complete some mathematical task. Once another student is engaged in the work being discussed, they can be questioned further to elicit their thinking as a way to invite another voice into the conversation.

Questions and Supporting Theme. When students are engaged in mathematical conversation with other students, the teacher takes on a secondary role. Rather than using questions to elicit ideas and promote further discussion, the teacher must now determine if the conversation taking place is productive for reaching the established mathematical goals. Using questions to support student-to-student conversation is challenging because the teacher must balance between letting students present their ideas, which may be unproductive or mathematically incorrect and ensuring that particular conclusions are ultimately reached. Following questions, which are based on student ideas, are an approach to supporting the conversation that will help the teacher balance their role. With these questions, the teacher is still building on students' ideas, but steering them in a desired direction. The key for the teacher to ask appropriate following questions is to listen to the student ideas.

Several other questions can be asked while students talk to each other in order to assist their conversation. Clarification questions can help the students provide clearer information to each other. If asked to a student other than the one making the initial statement, a clarification or explanation question can assist in helping other students understand each other's utterances. Justification questions can ask students prove their thinking or explain the validity of another student's thinking. Suggestion or procedural

questions can give students a method for clearing up a misunderstanding or inability to discuss their thinking further. Finally, confirmation questions that express agreement with the student can inform the students that their ideas are valid and they should continue sharing them with each other.

Questions and Revisiting Theme. When students are developing ideas through discussion, their understanding will occur over a period of time. When a teacher backs away from a group of students engaged in mathematical activity, such as problem-solving, modeling data or developing formulas, students continue to share their thinking with each other. In the Kenilworth setting, Dr. A continually moves in and out of the conversation with the students. She uses questions in the revisiting theme to elicit student thinking about their equation models. Rather than draw conclusions for the students about their work, Dr. A revisits what the group's model represents, the meaning of the data collected, and the group's first model about average rate of change per degree. By revisiting these ideas, students gain an understanding of what their equations model about the shell.

The most direct way to revisit ideas is by asking retracing questions, which refer to ideas previously discussed by the students. These questions work in a group setting because they can elicit student thinking from when the teacher was not present in the conversation. Retracing questions can also refer students back to important ideas that can help them solve a problem or help students determine an error in their thinking. In a large class setting, retracing questions can also be used to help students connect ideas between problems. Dr. G. used retracing questions to refer to parts of previous problems and discuss differences between the results of the students. By revisiting previous

student conclusions, the teacher can challenge student conceptions and cause them to develop a deeper understanding by having to reformulate their arguments to the current problem.

Summary

This research set out to examine how teacher questions can engage students in mathematical conversation. With that goal in mind, two research questions framed the structure and analysis of this study. The first question sought to determine the types of questions two mathematics teachers asked in the student-centered settings. The research identified 11 questions asked, of which 10 were common to both research settings. The analysis also determined the frequency of each question and they are listed from most to least frequent: confirmation– asking for student agreement, confirmation – agreeing with student statements, clarification, procedural, following, retracing, discoursive, justification, explanation, suggestion, and repeat. These questions were used by the teachers throughout the data to engage students in a mathematical conversation about solving area problems using calculus and a modeling problem.

The second question of this research sought to determine the extent and in what ways these teachers' questions engaged students in mathematical conversation. Specifically, examining how the teachers elicited students' mathematical reasoning. The analysis to answer this question was two-fold. First, I identified and determined the frequency of student responses to the teacher questions. The research identified nine student responses. The responses from most to least frequent were: answer, thinking

aloud, confirmation, proof building, attunement, questions student, seeking, clarification, and does not contribute.

The second part of the analysis was to develop an understanding of how specific types of questions could engage students to share their reasoning through conversation. The first conclusion from this analysis was that teacher questioning occurred within themes depending upon the discussion occurring among the participants. In order to have students share their thinking, the teachers used questions to initiate student thinking as public knowledge, invite other students to share their thinking, revisit previous student ideas, and support student-to-student conversation. Three types of questions emerged as focal points for engaging students in conversation throughout different parts of a discussion: explanation questions for the initiating theme, discoursive questions for the inviting, and retracing questions for revisiting theme. Other questions served the purpose of developing discussion, but explanation, discoursive and retracing questions were most direct method for a teacher to question within each theme. The other identified questions in this study served as supplemental ways to help teachers continue the conversation through eliciting further thinking from the students. The most evident application of multiple question types within a theme was the supporting theme using several questions to assist student conversation rather than one focal question.

The second conclusion from this analysis was that teachers moved between themes and questions to create a mathematical conversation where students shared their reasoning. In order to engage students in mathematical conversation, teachers can use the questions identified in this study with different intents. The first intent is to initiate student thinking into a public discussion. Using questions to initiate student thinking into

the public discourse is an essential part of discussion because student thinking provides a foundation for further discussion. The most direct way to elicit student thinking is to ask explanation questions. Once a student explains their thinking, the teacher can ask clarification, justification or following questions to elicit more detail from the student about their ideas. In order to develop a more robust mathematical conversation, teachers can invite other students to contribute to the discussion. When other students contributed, they brought their own thinking to the public discourse. Discoursive questions by definition provide a method for teachers to elicit other students' thinking. Additionally asking students for agreement about the discussion, using confirmation questions, can allow students more opportunities to engage in the conversation. These two themes of questioning allow ideas to flow throughout the classroom and encourage discussion about mathematics.

The other two themes present in the study, which were setting specific, mimic common practices in mathematics classroom. The supporting theme occurred in a whole class discussion, where the structure allows for students to freely engage with each other. The revisiting theme occurred while students worked in groups and the teacher intervened throughout their work. These two methods of student organization place the teacher in different roles within the classroom. During a whole class discussion, the teacher can drive the conversation through their questioning. However, when students begin to engage in their own conversations, the teacher should support this part of the conversation by using various questions that assist students in clearly stating their ideas within the mathematical community. Supporting questions provide oversight of the student talk to ensure student utterances are clear and express valid mathematical ideas.

Questions that can help accomplish this support are clarification, justification, suggestion or confirmation questions.

When students are developing mathematics through group discussion, the notion of revisiting their thinking can be an important part of engaging students in conversation. If the students are working in groups, particularly on open-ended activities as in this research, the teacher becomes passive in the conversation since they may not be able to observe and listen to the students' entire process. By asking questions that refer to previous ideas, the teacher can hear how students have progressed in their thinking since the last intervention. Using other question types can elicit further thinking about specific aspects of student thinking. Retracing questions can also provide the teacher with a way to have students develop a more sophisticated understanding of their work over time.

This research shows that the two teachers studied used questions in a way that made students' thinking public and invited several participants to contribute their ideas to the conversation. Once these teachers established a public conversation based upon student ideas and reasoning, the teachers used questions to support student thinking and revisit their ideas in order to make sure the conversation promotes valid mathematical thinking.

Implications and conclusions

The specific questions and questioning themes identified in this study can inform mathematics teachers of the characteristics involved in the classroom discussions of the observed teachers. Since the questions are related to student responses, a teacher may be able to better understand what types of student feedback are likely when a specific type of question is asked. Although no specific question types *always* had students describe their thinking or justify their reasoning, discoursive and justification questions led students to do so more than other question types. Further, the themes of questioning described in the study's discussions provide some insight into how questions could be used to engage students in mathematical conversation.

Relation to Research

As Lampert, Rittenhouse, and Crumbaugh (1996) noted, the NCTM did not give teachers specific guidance about creating productive discussion in the mathematics classroom. Research provides ideas about the roles or tasks teachers take on during class discussions (Staples & Colonis, 2007; Truxaw & Franco, 2007). This study supports the arguments of other researchers by finding similar references within the analysis. Steele (2003), Martino and Maher (1999), Brent Davis (1997), van Zee and Minstrell (1997) and Dann, Pantozzi and Steencken (1995) encourage teachers to ask questions that probe, extend or elaborate upon previously articulated student thinking or ask them to justify their ideas. Another example is Herbst (2002) referring to suggestion as a way for teachers to assist students and this study identified suggestion questions as a question

where the teacher provides information to the student. This study identified suggestion,
following, clarification, and justification questions as examples of questions that coincide
with the researchers' work.

The questioning themes identified in this study can also be connected to the
research. Manouchehri and St. John (2006) argue that questions revealing student
thinking can foster discussion and the teachers in this study used questions to initiate
students into conversation. The goal of the initiating theme is to make student thinking
public and the teachers used various questions, such as explanation questions, to
accomplish this goal. White (2003) commented on the positive influence of student
mathematical thinking when several students were involved in the discussion. The
inviting questioning theme, used by both teachers, elicited thinking from other students
and allowed them to share their ideas. It also created the opportunity to make the original
student's reasoning the object of debate, a process that Weber, Maher, Powell, and Lee
(2008) argue creates learning opportunities for students to engage in more sophisticated
reasoning. When multiple students shared their thinking, they were better able to reach
conclusions about the problems they were solving.

The supporting theme occurred when students engaged in conversations among
themselves rather than in a dialogue with the teacher. Blanton and Stylianou (2003)
studied how a teacher focused on giving students the responsibility of constructing
mathematical knowledge. Questions in the supporting theme allowed students to be
responsible for their ideas while the teacher ensured they understood each other and
making mathematically valid arguments. This move allowed the teacher to steer the
discussion in directions that he or she feels is appropriate while still having the ideas in

the discussion be generated by the students. The revisiting theme accomplishes a similar notion when teachers asked questions that had students return to their previous ideas.

The existing research extends the previous research in three ways. First, much of the previous research categorized questions by the types of responses students were *expected* to give, while this research analyzed the correlation between what teachers asked and how students *actually* responded. The second is in the level of detail of the question coding. The results of this study provide question types that were more specific than the categories suggested by previous researchers. The third is in how the questions were used to accomplish a broader goal—using question types in tandem to create mathematical conversations that were student-centered and mathematically productive.

Contributions

Identifying questions that engage students in mathematical conversation is a small, but useful, contribution to the vast field of knowledge about discussion and questions. Being aware of these questions may allow teachers to use them in their own classrooms to gain insight into student reasoning by having students share their thinking publicly. The teacher then has the responsibility to use student ideas to help them make connections, argue with proper mathematical language, and understand mathematical concepts.

Identifying student responses within an environment of established conversational norms contributes information about how students reply to questions. A teacher wishing to assess their students' understanding would want students to give thinking aloud or proof building responses. However, students engaged in conversation respond in many

ways. Having knowledge of the different student responses during a conversation allows the teacher to have expectations of student replies. For example, if students respond to teacher questions with questioning student and seeking responses, the teacher gains information about student understanding. While this understanding may not be explicit, a seeking response may mean the student needs more information to be able to express their understanding within established classroom norms. Similarly, an attunement response may mean a student is still processing earlier information into their own framework of understanding. Each type of student response within a conversation may provide the teacher with feedback about how to continue to engage students in mathematical conversation.

Examining a specific type of student response, the presence of so many answer questions within environments where teachers had established conversational norms is an interesting result. The ideal of students engaged in conversation where they share thinking and construct mathematical concepts within a community of talk may be unrealistic. With so many short, closed recall responses by students, the dynamics of mathematical conversation are shown to vary between long responses where student ideas are revealed and short responses where thinking may not be revealed at all. This may be viewed as a disappointing result because mathematics teachers hoping to include conversation as a centerpiece of their classroom may see answer responses as a failure to accomplish rich discussion. However, one can not assume that a student does not have knowledge to answer a questions because they do not participate in an elaborate response. Rather it may be possible the student gives an answer response because they do not know

how to give a good justification or that they were supposed to provide a reason with their answer. Further questions can help determine why a student gave an answer response.

The relationship between teacher questions and student responses did provide direct insight into questions that provoked particular responses. The most frequent responses to specific teacher questions provided a dichotomy in many cases. The frequency of the answer response contributed to that result, but there is one relationship that may help teachers engage students in mathematical conversation. When teachers asked discoursive questions, the most frequent response was a thinking aloud response. It should also be noted that nearly 50% of responses to discoursive questions were thinking aloud and proof building responses. The implication of this result is that when teachers invite other students to contribute to the conversation the student will most likely share or justify their thinking. It seems likely that when one student has already shared their thinking, a new student would be encouraged to share their own thinking to either explain why they agreed with the first student or to justify why their response was different. If students respond in this manner, the teacher can promote conversation in the classroom by having students listen to each other's thinking, convince each other their thinking, and providing feedback.

A final contribution of this research is a primitive model of teaching for how to engage students in conversation and elicit their thinking. The idea of a teaching model is similar to a lesson plan structure, where teachers prepare a warm-up problem, followed by instruction about a topic, and ending with guided practice. This research suggests how teachers should use questions to engage students in mathematical conversation. When teachers plan their lessons with the idea that instruction should include discussion about

mathematics, the initiating and inviting themes of questions suggest a possible foundation

for how such a discussion can occur. When a teacher asks questions within the initiating

theme, the results of this study suggest that students' thinking becomes public. By also

asking questions within the inviting themes, the results of this study suggest that several

participants may contribute their ideas to the conversation. If teacher questioning has

these purposes in mind, teachers can accomplish a similar conversation demonstrated by

the two teachers in this study. Once teachers have established a public conversation

based upon student ideas and reasoning, this research suggests how the supporting and

revisiting themes can promote valid mathematical thinking.

Implications

The are two implications about the teaching of mathematics that can be drawn

from this research. The first implication is for teachers and refers to the manner in which

they interact with students. Gaining insight into student understanding is an important

part of teaching. Typically, teachers use written assessments to collect information about

student knowledge. This snapshot of student understanding does not provide teachers

with insight into the development of student understanding and slights the inherent

fluidity of the process of coming to understand mathematics. Conversation is one way

teachers can acquire information about student understanding. However, as demonstrated

in this study, the nature of engaging students in conversation is a complex process.

Simply, asking the right questions, such as explanation and justification, does not mean

students will share their thinking. Therefore, in order for teachers to become adept at

promoting conversation in their classrooms, knowing the research and questions to ask is

only a small part of the process. Teachers need to experience these classrooms first hand and discuss their observations with colleagues.

The second implication of this study is for mathematics educators and administrators and refers to how to support teachers to become adept at conversation in the classroom. From a pre-service perspective, mathematics educators should afford pre-service teachers with opportunities to practice creating conversation. For example, pre-service teachers could examine video of classroom conversation and analyze the teacher's questions and actions and how the students responded. Another way to expose pre-service teachers to conversation-based classroom requires the help of school administrators. Pre-service teachers should observe and complete their teaching field experience under the supervisor of teachers who know the benefits of and practice conversation in their classroom. If future teachers are not exposed to the classroom promoted in the literature and supported by administrators once they are in the field, their opportunity to create student-centered and conversation-based classroom will be hindered. As shown in this study, eliciting student thinking through conversation is a complex, dynamic and challenging task for even experienced mathematics teachers. Therefore, the support system for future teachers must become adept at helping them achieve the type of classroom promoted by so many in the mathematics education profession.

Future Research Questions

I suggest two possible avenues for future research based upon this study. The first is to examine a teacher-centered classroom and describe the teacher questions and

students responses. The types of questions and responses from such a study would allow for a comparison of the information found in this study. A possible research question could be: What types of questions, if any, in a teacher-centered classroom elicit student thinking? One may expect an overlap of some questions and responses in a student-centered and a teacher-centered classroom, such as procedural questions or answer responses. However, by also searching for themes of questioning in a teacher-centered classroom, the use of questions by the teacher can provide similarities and differences to this study with regards to eliciting student thinking. One potentially very interesting finding would be that the same questions in student-centered and teacher-centered classrooms led to different types of student responses. If so, this could provide some insight into how different classroom norms and environments directly influence students' participation and reasoning.

The second area for future research is to examine the role of questioning and discussion in determining the classroom environment. The two research settings for this study were purposefully selected in order to describe teacher questions, student responses and the relationship between questions and responses occurring in student-centered classrooms. Since the teachers already established the environments before collecting data, a possible study would examine the use of these questions from the beginning of the year to determine if a discussion filled environment occurs. A possible research questions could be: Do teacher questions, as identified in this study and according to the themes, promote an environment of discussion? By examining how questions contributed to the classroom environment, the study would determine if non-questioning factors, such as students writing at the board or working in groups, is sufficient to engage

students in mathematical conversation or if teacher questions play an important role in creating the conversation.

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Appendix A: Rejected Codes

This appendix elaborates on the coding process by explaining the reasoning that certain teacher and student codes were not included in the final list.

Rejected Teacher Codes

As mentioned in chapter 4, the *new question* coding was changed to be coded with *suggestion questions*. Two other codes were rejected and the questions receiving the codes were changed to various codes. One eliminated question is a *connection question*, which asks the student to draw upon previous similar (isomorphic) ideas for use in the present or extend the present idea to a different situation (extension). Originally less than ten questions were coded as connection questions so each question was revisited to determine if the question fit better under another category. Some questions were actually *procedural questions* that spoke about other problems and others were *suggestion questions* to give students something to think about rather than draw connections. An example of a connection question relabeled as *procedural question* is shown below. The question is *procedural* because Dr. G is telling the student to perform a mathematical task.

| 24:34 | Dr. G | T(ct) | Now run this graph into your calculator and set in exact mode. Uh, I take that back. Don't do the graph first. Do the integration process first under calculus menu and set it in exact mode. Go ahead Mikey. In your calculators, under the calculus menu, in exact mode. See what it comes up with. |

The other eliminated question is an *information question*, which asks the student for mathematical information that is based on recall of known information that is not a result of the current classroom discourse. Once again a minimal amount of questions received this code and they were changed to other categories. As an example, the following question is an *explanation question* since Dr. A is asking the student to explain an answer for the first time.

58:25:18	Dr. A	T(i)	So that's not far off. What would be this one? What would ,what would be the number of degrees for this one. For the, for the point before it

Rejected Student Codes

One student response code is not included in the final list of codes. The code is a natural response to a teacher asking a *repeat question*, but rarely did students repeat exactly the same phrase when asked to repeat their statement. Only two exact repeats were found in the data. The *repeat response* is an identical statement to a previously made statement and is most likely given because the original statement was not heard. The two incidences are:

28:58	Ronak	S(ta)	Charlie. Don't multiply the whole thing by n squared.
29:05	Charlie	S(qs)	What?
29:06	Ronak	S(r)	Don't multiply the whole thing by n squared.

01:14:21	Mike	S(seek)	Can we get it out the way?
01:14:24	Dr. G	T(rep)	What?

01:14:25   Mike   S(r)   Can we get it out the way?

The first S(r) is an S(ta) since the student is expressing mathematical thought without justification and the second is an S(seek) since the student is checking with teacher about removing the calculator graph displayed on the board. Most times when the students were asked a *repeat question*, there was an *attunement* or *clarification* response rather than a verbatim restatement. Therefore, this code was removed from the list.

Appendix B: Kenilworth Transcript with Codes

July 8th   AM Session

The speaker begins with a brief introduction of the Placentaceras. The question posed to the students is "How did it grow?". Students begin a whole group discussion about their opinions to the question.

After 12 minutes of a whole group discussion, the students continue the discussion in groups at their table. The students at table three continue their discussion of how they think the animal grew. After 5 minutes, Mrs. W engages the table in conversation about what they are discussing about the growth for about 2 minutes. They discuss the direction the animal is growing in the shell. Approximately, 21 minutes after the start of the problem, Bob joins table three's discussion for about 5 minutes. They discuss if the animal was born at full size, what the shell is made of and how they think the chambers grew.

After approximately 30 minutes of discussion, the entire group is brought back together to share their ideas. The discussion lasts for 30 minutes before students begin to work in their own group.

The students at Table 3 begin their mathematical work by recalling how to do polar conversions. Angela begins to draw rays and right triangles. There are marked

measurements on each ray. Four minutes after the students have started, Mrs. W engages

the group in conversation.

01:08:25:14	Mrs. W	T(c)	I'm not sure what you're doing, because when you started just a few minutes ago
01:08:30:20	Robert		Uh huh.
01:08:31:13	Mrs. W	T(p)	Uh, I don't know where you picked up from. Could you maybe put your transparency out in the middle of the table?
01:08:38:15	Angela		Uh huh.
01:08:40:13	Mrs. W	T(r)	uh, so that I can see it and we can get a good look, uh, with the camera? And could you kind of recap what brought you to this point?
01:08:53:20	Angela		
01:08:57:22	Mrs. W	T(d)	Now when I say "you" I mean everybody contribute to what brought you to the point that you're at.
01:09:03:03	Sherly		Let's start.
01:09:09:28	Angela	S(a)	Me? Alright.
01:09:12:28	Angela	S(ta)	We traced the spiral that we...(inaudible) first made the ray
01:09:19:25	Angela	S(ta)	Alright, and then we measured how far, well, we traced the spiral too,

01:09:24:28	Angela	S(ta)	We measured, like, wherever the spiral intersects the ray, we measured how far from the center it was.
01:09:30:28	Angela	S(ta)	We have our measure in centimeters. So we do the second line, and we had to figure out what that was
01:09:37:14	Angela	S(ta)	so we made that a right triangle and used a ruler so it might not be exactly accurate, but, um,
01:09:45:28	Angela	S(ta)	you know, like, we used cosine to figure out what the angle was and then we changed it into radians.
01:09:51:19	Sherly	S(ta)	We traced it (inaudible)
01:09:52:25	Angela	S(a)	What?
01:09:53:11	Sherly	S(qs)	we didn't do all those yet, did we?
01:09:55:05	Angela	S(ans)	No, we did that for the first one.
01:09:56:29	Angela	S(ta)	Now we just, um, we just traced it on a transparency so that we'd be able to present it, or whatever and show it better.
01:10:06:27	Angela	S(ta)	And um, we just, so far we just kinda made the line, well, we just, you know, did that and we measured how far the points were for this other one
01:10:17:03	Angela	S(ta)	and so we figured out what theta was
01:10:19:11	Angela	S(ta)	and we kind of wrote it in the color that we have it up there

01:10:23:20　Angela　S(ta)　We're just doing that for the second line now, we

measured the distances, and now we're just going to

figure out what, um, the angle is.

The students continue to discuss if they have all the measurements, then continue to draw rays and calculate measurements using inverse cosine. Angela continues to draw rays to find measurements for the spiral while other students perform calculations.

01:35:34:26　Mrs. W　T(d), T(e)　Robert do you know what she's doing right now? Could you explain to me like what?

35:49:00　Robert　S(ta)　Draw lines, like this thing. Just drawing lines. She's just

36:02:00　Angela　　Here let me get them all even.

36:10:00　Robert　S(ta)　Well we started by drawing a line up, in the center up,

and then we took lines all over. We started going out

this way. (pointing to the left) and then we measured

the length.

36:20:00　Mrs. W　T(c)　Going which way?

36:22:00　Robert　S(c)　Well we started here and then we drew a line going

diagonal this way.

36:25:00　Mrs. W　T(con)　So you started on this red line going up and then this

was your second line

36:31:00　Robert　S(ans)　Yeah, and then we measured it

36:34:00　Mrs. W　T(c)　Measured what?

36:36:00	Robert	S(c)	Measured the all the, all the distances
36:37:00	Sherly	S(ans)	The distances
36:38:00	Angela	S(c)	Like where the spiral intersects
36:42:00	Mrs. W	T(e)	Okay and you've recorded those where?
36:43:00	Robert	S(ans)	There (points to transparency) and the colored lines are the color it's written in
36:47:00	Mrs. W	T(con)	Okay, Okay. So it's color coded. Okay.
36:51:00	Robert	S(ta)	Uh, and then uh we just used cosine to figure out the angle. Like we took all the measurements and divided them and then used cosine and all that and then put them into radians too; converted them to radians.
37:04:00	Ashley	S(ans)	We made a ninety degree angle
37:07:00	Robert	S(c)	Yeah we made a 90 degree angle connecting these two lines
37:10:00	Mrs. W	T(c)	Where's the 90 degree angle?
37:12:00	Robert	S(c)	See the dotted line
37:13:00	Mrs. W	T(con)	The dotted line
37:15:00	Robert	S(ta)	Yeah, and then we just recorded it in degrees and radians for all our measurements and we just kept drawing lines.
37:22:00	Mrs. W	T(c)	And so when you said you used, you drew a right triangle and you used the right angle and you. What did you do with that?

37:31:00	Robert	S(c)	Uh, well then after we drew the right angle
37:33:00	Sherly	S(ans)	We used cosine
37:34:00	Robert	S(c)	We used the right triangle to use cosine
34:38:00	Sherly	S(ans)	Adjacent over hypotenuse.
37:39:00	Mrs. W	T(c)	Okay, you used cosine to get what?
37:41:00	Sherly	S(c)	The theta.
37:45:00	Angela	S(con), S(c)	The theta thing. In degrees and then we converted to radians
37:50:00	Mrs. W	T(e)	Okay and then you did a regression with additional rays. Can you show me how that went?
37:55:00	Ashley	S(ta)	(Angela places a transparency on top of their previous work) That is the next line and those are the measurements for the black line. Like you see we put our 4, 3, 2,1 those are our measurements.
38:03:00	Mrs. W	T(con)	Okay. Okay.
38:04:00	Ashley	S(ta)	And then we went from there and just did the same thing.
38:08:00	Mrs. W	T(con)	Okay
38:09	Robert	S(pb)	Um and the measurements are from the line to the original ray not from like this to this. Its from this to this. Like we just drew the right angle connecting these two because it would be easier. And then we added the angles

38:18	Angela	S(pb)	Yeah, to figure out that theta, like we just did it like that because it would be kind of retarded to you know go.
38:27:00	Mrs. W	T(con)	So you drew your right angle using this line that you drew and this line that you drew. You drew the right angle between those two.
38:35:00	Angela	S(c)	Yeah, and then we just added the two angles to find out what this whole angle was
38:40:00	Mrs. W	T(con)	Oh, okay.
38:42:00	Robert	S(c)	So this 84.4 is really like this angle the whole angle, not just this small one here.
38:47:00	Sherly	S(con)	The total yeah
38:50:00	Angela	S(ans)	And then the next one. (places another transparency over their work)
38:56:00	Sherly	S(c)	The total is together. No because I thought it was this one
39:04:00	Angela	S(pb)	I'll explain that's all. Just come over later (to Sherly). It's our next line. We did basically the same thing, but we knew it was just kind of a straight line. So we didn't actually have to figure out anything. Once we just used 180. Like we know how many radians is in a straight line, you know with the whole circle thing. The unit circle. So that is the right angle instead. (places another transparency on their work) Then that's the last one.

The orange up there.   And we did the same thing. Except we figured out theta by doing this, because you can make a right triangle out of, from this line and this line cause it's over 90 degrees. So um we just figured out this angle and then subtracted that from 360 because there's 360 in a circle. And then you know we figured out that whole angle by doing that and then changing it to radians.

| 40:16:00 | Mrs. W | | Thank you |

| 40:18:00 | Robert | S(ta) | When we added up we got 6.28 radians total and that's how many are in a circle so. |

The students continue their discussion amongst each other by referring to the spiral, rays and measurements.

| July 8th | PM Session | | |

At the start of this session, the students are using graphing calculators and working with the shell and their measurements. They ask if they can say anything about r as a function of theta. Victor comes to the table and says that another group is taking radii every 90 degrees to get the change and found an average.

| 03:14 | Dr. W | T(d) | You mind if I join you for a few minutes. |

03:16	Robert	S(con)	Sure

03:18	Dr. W	T(e)	I've been looking around other places and I haven't been able to see what you guys are really doing, but what you have here looks very interesting and I wondered if you could maybe you could kind of walk, walk by me what you've been doing. You know what you've got it's very interesting what's developed here.

03:45	Angela	S(ta)	We started off with we traced the spiral and then we drew the like you know the axis thingy and then um we measured the distance well from the center of these points where it you know intersects.

04:02	Dr. W	T(con)	okay

04:03	Angela	S(ta)	we wrote it down there that's why it's letter because that's the original one

04:06	Dr. W	T(con)	okay

04:07	Angela	S(ta)	then that's like zero degrees, zero radians or something like that.

04:14	Dr. W	T(c)	That's this (points to red axis on transparency)

04:16	Angela	S(ta)	That's the original in there ,which we have on the chart. Then we drew the first line and we had to figure out the angle. So we made a right triangle using a ruler like that

04:34	Dr. W	T(f)	So you just drew that line in

04:36	Angela	S(ans)	anywhere

04:37	Dr. W	T(f)	somewhere that looked interesting
04:38	Angela	S(ta)	Yeah where ever we felt like it. And then we measured the different distances and we figured out the angle in degrees using inverse cosine. And then we converted to radians and we have the measurements right up there (points to corner of transparency)
04:53	Dr. W	T(f), T(c)	So how did you use the inverse cosine here?
04:57	Angela	S(c)	We did the adjacent divided by the hypotenuse and then inverse cosine
05:04	Dr. W	T(con)	So you measured a couple of line segments. Is that true?
005:10	Angela	S(con)	Um hum
05:12	Dr. W	T(c)	So which line segments?
05:16	Angela	S(c), S(pb)	Well for the angle we did this to there. (pointing to transparency) and there to there. That's the hypotenuse. I mean we could have use um, what is that tangent, no? opposite over adjacent. yeah, tangent like for measuring that, we just did cosine. We could have used anything actually. Um, so we figured out the angle and converted it to radians and we have it right up there and there too(pointing to transparency) . Then we measured out the segments, you know where the points are. How

far away from the center and that line. Then we did it
on this one too (takes out second transparency)

06:01	Dr. W	T(con)	Oh I see, this is nice

06:03	Angela	S(ta)	And we kept just doing it and you know all the measurements are there. And we did the same thing except we measured the angle from here to here (new angle) and added it to this angle (old angle) and then converted it to radians.

06:15	Dr. W	T(c)	This angle looks very much like this one. Is that on purpose?

06:21	Angela	S(pb)	No, it's just how, we just felt like drawing lines where ever. I was like is here okay? And

06:26	Dr. W	T(con)	Sure go ahead

06:36	Dr. W		(Angela gets another transparency) Oh yet again

06:39	Angela	S(ta)	Then we just drew the straight line right across and we know it's 180 degrees or pi radians so we didn't have to bother with the all that adding angles

06:47	Dr. W	T(f)	So the calculations there were real easy.

06:49	Angela	S(pb)	yeah and we just measured the distances and we sort of lost one of the points because if the spiral had continued then we would have four points but like since it didn't we only had three. We recorded those then we did it one more time. In this one we measured this angle(the

angle between the original axis and new line) cause you can't make a right triangle out of that. So we measured that angle like with that and then we subtracted that from 360 degrees because we were interested in that angle (larger angle from original axis and new line) not really that one. Then we converted this angle in degrees to radians which is right where's the orange, up there. and then

07:30	Dr. W	T(c)	So let's see now the angle you the angle you told me is where?
07:37	Angela	S(c)	This one. (points to large angle around spiral) Like we found out what this angle was and then subtracted that from 360 because there's 360
07:42	Dr. W	T(con)	I see
07:44	Angela	S(c), S(pb)	in the circle. So then we had that angle and then we converted to radians and measured. And we also lost one of the points because it didn't go all the way around.
07:54	Dr. W	T(con)	Okay. Okay
07:55	Angela	S(pb)	And so far we've gotten to…and all of the angles add up to 6.28 radians so we know that we didn't mess up with the angles. Cause that's how many there are.

08:04	Dr. W	T(c)	Oh I see. So…so these are then what I see down there. Is that right. (pointing to colored measurements.)
08:18	Angela	S(ans)	Yes. We just recorded
08:20	Dr. W		You color coded. Very nice, I can follow this.
08:25	Angela		Yep perfectionist.
08:27	Dr. W	T(c)	You didn't, you choose not to use any angles other than this one down in here. Is that true?
08:32	Sherly	S(pb)	Yeah because we didn't have a lot of; we didn't have that many rays. Over here we had four. So we just used this one and this one. Cause those are not, we don't have to put another one here. (point to bottom of transparency where there is only one ray)
08:43	Dr. W	T(j)	So you're feeling this is giving you a significant number of data points that maybe you can work with.
08:51	Angela	S(con)	Yeah, that's what he's doing right now.
08:55	Dr. W	T(d)	What's Victor working on now or what are you working on now?
08:59	Angela	S(ta)	He's trying to see the like rate of change per like each angle and the points. Right
09:06	Victor	S(ta)	Right what we're going to do is take uh radian 4 the last radian which is 6.9, 6.5,5.4 and then what we're going to do is, um, find the average and then divide by 90 degrees and get a rate of change per 90, per degree, and

then. So we can get an actual distance around this. (points to shell)

09:28	Dr. W	T(r)	Okay now, you said 90 degrees Victor and I'm a little, I'm wondering where that came from.
09:36	Victor	S(pb)	That came from because everything. When we figured out theta and stuff like that
09:40	Dr. W	T(con)	uh huh
09:41	Victor	S(qs)	We used 90 degree angles right? Yeah right
09:43	Sherly	S(con)	Yeah. Um huh.
09:46	Victor	S(ans)	We used all 90 degree angles
09:48	Sherly	S(pb)	We made them all 90 degree angles so we could figure them out.
09:51	Dr. W	T(con)	Okay so you're looking at these 90 degree angles.
09:53	Victor	S(con)	Yeah the measurement of the red
09:58	Dr. W	T(f)	Okay, I see. Now are you anticipating something from these calculations
00:10:18:05	Victor	S(ta)	More or less. Because once we get the rate of change, average rate of change per whole degree we can multiply that by, probably two pi radians, which is equal to three-hundred sixty degrees, and we can get, you know, um, we can get the distance around the whole spiral. You getting, understanding what I'm saying?

00:10:46:22	Dr. W	T(f)	I'm working on it okay. I'm not quite there yet.
00:10:49:21	Victor	S(c)	Okay. We have um. Wait, hold on. Anyway, it says, the whole thing goes three-hundred sixty degrees, right?
00:11:04:14	Dr. W	T(c)	Once around, is that what you're saying?
00:11:09:21	Victor	S(ans)	Once around.
00:11:12:05	Victor	S(dnc)	I'll get back to you on that.
00:11:14:15	Dr. W	T(con)	Okay, you certainly can do that.
00:11:22:19	Angela	S(seek)	That's all we have so far. Are we on the right track?

During the rest of the day, Victor is working on calculating the average. Each table presents their work so far. After the presentation, Victor's work leads the group as he tries to use the measurements from the transparencies to calculate an average. He states he is trying to find a distance for the shell to the group and he measures the spiral several times. The rest of the students become involved in finding a distance when Victor leaves the table to speak with table 2. He returns and the group questions again what Victor is trying to determine. Sherly shares that he is trying to find the whole distance around the shell. Victor announces he has 1207 degrees and 31.8001 is the distance around the outline of the shell in centimeters. He then says that they still do not have a function. Victor again discusses his work with students at other tables as the day comes to an end.

July 9th   AM Session

Students begin by trying to figure out what they accomplished the day before. Angela

asks if they can graph the points and Victor says that is what he was trying to do. He says

he put .02356251, 1207 degrees and was trying to figure out the length referring to a

string being pulled out to 31 centimeter for the measurement of the spiral. The rest of the

students begin to try and plot the points to get a function that resembles the spiral.

11:37:27	Dr. A	T(d)	Tell me what you're doing?
11:39:10	Angela	S(ta)	Putting in the points, to see if it will graph it. I really don't know what I'm doing, honestly.
11:47:00	Mrs. W	T(con)	Is that what you were trying to do Ashley?
11:47:17	Dr. A	T(c)	But, c2 is which? Is the
11:51:10	Angela	S(ans)	Um, it ends up being (inaudible). That's wrong.
11:55:24	Dr. A	T(c)	No, I mean, it's this stuff isn't it?
11:57:21	Angela	S(con)	Yeah.
11:57:21	Victor		(Inaudible).
11:59:22	Dr. A	T(c)	And those were?
12:01:04	Sherly		Oh.
12:03:29	Dr. A	T(r)	Help me to remember what your things stand for?
12:06:28	Angela	S(ans)	It's the distance from the center at the different, um angles.
12:06:28	Ashley	S(qs)	Victor, what are you doing?
12:09:15	Victor	S(ta)	I'm trying to see if I can, I want to see this, so I'm trying to write this stuff on here. (Inaudible).

12:12:21   Dr. A   T(con)   Uh, huh.

12:19:06   Angela   S(qs)   What do we do now?

The students examine the graph of their function and discuss what they should compare it to.

13:34:27   Mrs. W   T(d)   Angela, Ashley. Robert, why don't you share with her, what you have?

13:41:18   Robert   S(ta)   Oh, I just took this number here, and timesed it by theta and put it in the graph.

13:49:23   Mrs. W   T(e)   Show her what you got.

13:53:16   Robert   S(ans)   It's kind of nothing like it, but

13:55:23   Sherly   I can't see. I still can't see. (laugh).

14:00:17   Angela   Sherly's blind.

14:01:25   Sherly   S(qs)   No, he's holding it. Okay. So what does that mean? No, wait can I see?

14:07:28   Ashley   S(ans)   That's a spiral.

14:09:21   Dr. A   T(r)   How did you. Robert, can you explain again, how that worked?

14:14:02   Robert   S(pb)   Oh, well, I just took the degrees here, and divide, or no wait we took the total length of it, and divided by degrees and just put this in there.

The students immediately return to their conversation about how to graph the function in the calculator. They ignore Dr. A's next attempt to ask about the equation they are working on.

16:31:09	Dr. A	T(d)	Do you understand where the degrees came from?
16:39:18	Victor	S(pb)	What we did was we took, what we kind of did yesterday, like start here, thats
16:49:23	Victor	S(pb)	2, 3, wait hold up.
17:01:26	Victor	S(pb)	So, 1, 2, 3, and then.
17:11:09	Victor	S(pb)	That's a half a radian and that's our other part(inaudible word). Wait. 1 radian
17:21:26	Victor	S(pb)	That's like 4 radians or 5.
17:24:02	Dr. A	T(c)	And that's for a hundred, one thousand two-hundred and seven. ?
17:28:02	Victor	S(con)	Yeah, pretty much, and then we had all the points.
17:34:05	Victor	S(pb)	Um, and we separate each point by 90 degrees.
17:40:09	Victor	S(pb)	like it's apart by 90 degrees and that's the thing we just measured out. Like here's our origin. Our first point is right here. Then our second point on the, is right here. And it's like 90 degrees from that. The third point is right here and it goes up by 90 degrees. Another 90 degrees. We just (?) up
18:07:29	Dr. A	T(c)	What, you mean you measured them?

18:09:10	Victor	S(c)	From the origin to the point, once we got them all plotted.
18:13:14	Dr. A	T(c)	That's what these numbers are?
18:15:10	Victor	S(con)	Exactly.
18:16:12	Dr. A	T(c)	And so it's like from the origin out to R7 is this?
18:21:14	Victor	S(ta)	45.7 centimeters. We added them all up and then we divided by um, 90, oh we added them all up
18:31:01	Dr. A	T(f)	You added all those up, and what did you get?
18:32:28	Victor	S(ans)	We got the average.
18:35:01	Dr. A	T(c)	No, you added them all up and got a number.
18:38:02	Victor	S(ta)	We got a number and then we divided that by 13 and got the average. And then we divided that by 90 degrees and got the rate of change, per degree.
18:49:15	Dr. A	T(j)	You need to start. I don't understand how's that.
19:01:00	Mrs. W	T(con)	I, I understand how you mean
19:02:00	Victor	S(con)	You understand the box points.
19:03:18	Mrs. W	T(r)	I understand where these points came from. I'm cool with that. The thing that I, that then I get lost is...I understand you added them up.
19:13:23	Victor	S(con)	Okay, what we did was we added them up, right.
19:16:24	Mrs. W	T(j)	Why? Why?
19:18:00	Victor	S(pb)	So we can get an average, we are trying to find an average.

19:20:09	Mrs. W	T(c)	Average what?
19:21:15	Victor	S(ans)	Average distance per 90 degrees.
19:25:09	Dr. A	T(c)	Distance of what?
19:26:15	Victor	S(ans)	Of this whole thing.
19:29:17	Mrs. W	T(con)	So, you're taking the distance from the origin to the points, and added all those distances up,
19:35:03	Victor	S(ans)	And divided by
19:35:03	Mrs. W	T(con)	and so you're trying to find an average
19:37:08	Victor	S(ans)	average.
19:37:17	Mrs. W	T(con)	of those distances?
19:39:04	Victor	S(con)	Yes. Average of (inaudible).
19:42:06	Victor	S(con)	Yes, an average.
19:42:20	Mrs. W	T(e)	That's what you did, but so you added up all the lengths, to get the, what number?
19:48:00	Victor	S(ta)	Average. Uh, I didn't write it down. But then we found out the average. Which was all these added up,
19:54:23	Mrs. W	T(con)	Uh huh.
19:54:23	Victor	S(ta)	divided by 13. We got 13 points. And we got the, that's, so 2.
20:00:08	Dr. A	T(con)	That's the 2.36.
20:01:04	Victor	S(ta)	2, 9, 2, 3. That's the average the, that's the average distance for every 90 degrees, of change from like

20:11:01	Victor	S(ta)	From this point to that point the average is about 90 degrees.
20:14:20	Mrs. W	T(con)	And when, okay. So you're saying the average
20:17:29	Victor	S(ans)	From point to point.
20:19:23	Mrs. W	T(con)	of every 90 degrees, it's because each of those points are 90 degrees
20:24:27	Victor	S(ans)	Apart.
20:25:08	Mrs. W	T(con)	Apart.
20:26:29	Victor	S(con)	Okay.
20:27:06	Dr. A	T(j)	But it's the average distance, what does the number represent?
20:33:04	Victor	S(con)	What number?
20:34:00	Dr. A	T(r)	This number when you got it, dividing.
20:37:16	Victor	S(pb)	Well, I divided by 90 degrees
20:39:08	Dr. A	T(r)	No, no, no, no, no, before that, the 2.36, you're saying it's the average what?
20:46:14	Victor	S(c)	The average distance for every 90 degrees.
20:54:04	Dr. A	T(c)	The average distance from where to where?
20:58:15	Victor	S(c)	The whole thing. (laugh). Okay, it's the average of, it's...
21:07:00	Mrs. W	T(d)	Okay, now, when he finishes explaining this again, will you rephrase what he said so that
21:13:04	Sherly	S(ans)	We can understand.

21:14:00	Mrs. W	T(d)	Yeah, I mean, not that we're not understanding what he's saying, as much as we're really trying to understand every little piece of how it all fits together.
21:15:14	Sherly		I'm sorry.
21:27:02	Sherly		Mm hmm.
21:27:02	Mrs. W	T(d)	And if you could help us, after he resta(sic), he's being very patient in restating things. You're doing great, I appreciate it.
21:34:28	Dr. A		And we're generally (inaudible word)
21:36:26	Mrs. W	T(d)	I know, I know, I'm really trying to understand, so if you would restate it and then if you would sort of help.
21:44:08	Robert	S(ta)	You just take all these numbers and add them all up and you just divide by this, you get the same numbers as so. Like I guess he's just, just doing it a different way.
21:55:04	Dr. A	T(c)	He gets the same number as what?
21:56:29	Robert	S(ans)	This .026, and I got .02632 and then continue on.
22:05:22	Mrs. W	T(d)	So where you understand that .026325, could you each respond somehow and?
22:15:16	Sherly	S(dnc)	I can't see, what happened?
22:15:26	Robert	S(ans)	The length of one degree around the thing.
22:18:06	Mrs. W	T(f)	The length of one degree?
22:19:16	Robert	S(c)	Around the thing in centimeters, like
22:22:13	Mrs. W	T(f)	The length of one degree. (whispered)

22:24:15	Robert	S(c)	Like if you turn one degree, that's how far you go around the spiral.
22:24:22	Ashley	S(con)	At each degree.
22:24:22	Angela	S(c)	Like here, like the length of this spiral, in one degree.
22:31:27	Robert	S(c)	Yeah, if you turn one degree, then you are going to .026 centimeters.
22:34:22	Angela	S(c)	Average.
22:35:10	Ashley	S(c)	Centimeters.
22:36:00	Mrs. W	T(c)	Okay, so turning one degree means doing what? Like
22:39:20	Angela	S(ans)	Moving
22:40:15	Sherly	S(ans)	.02.
22:41:00	Angela	S(ans)	Yeah. Around the spiral.
22:44:08	Mrs. W	T(c)	So, every time I move one degree more
22:48:11	Dr. A	T(p)	Can you put this behind it, I can't see.
22:52:13	Dr. A		Okay now.
22:58:05	Mrs. W	T(j)	Everytime I turn one degree more, what do I know?
23:01:07	Sherly	S(ans)	You're moving about .026375.
23:06:16	Angela	S(c)	Yeah, centimeters around so
23:07:17	Ashley	S(pb)	That's what we we're trying to figure out. That's why he added everything together, and then took the average from 90 degrees.
23:17:15	Dr. A	T(r)	I need to be stupid again. I need to know what those numbers that you measured, what did you measure?

23:25:27	Robert	S(ans)	The length, we went every 90 degrees
23:28:07	Sherly	S(con)	Yeah.
23:28:18	Robert	S(ta)	and put a point there. And then measured the distance between the point before and this new point. And if you just gonna put a.
23:37:09	Sherly	S(qs)	Didn't we measure from here to here?
23:39:04	Victor	S(con)	Yeah, from the origin. To the origin.
23:39:14	Robert	S(con)	Yeah.
23:40:28	Dr. A	T(j)	Is it the same thing?
23:44:17	Victor	S(pb)	No, I think, cause um, if this curves in, it would have been difficult to just measure from point to point. We would have needed some string or something, so
23:51:27	Angela	S(con)	Yeah.
23:56:04	Dr. A	T(con)	Okay, and so you're saying that you measured from the origin to the point.
24:02:23	Victor	S(pb)	And plus it was keeping it consistent with what we were doing before, because we were using right angles to find theta and just like, that so that's why I picked that way.
24:13:23	Robert	S(pb)	Even if you just did um, put it like every 180, and then divide it by 180 and probably get the same thing. We just chose 90 because (inaudible). So it's like you choose whatever yeah.

24:26:02	Dr. A	T(con)	But you're saying then that these numbers here, in each case, are the distance from the origin to the point?
24:32:24	Sherly	S(con)	Mm hmm.
24:33:21	Dr. A	T(d), T(con)	And all of you agree that?
24:35:23	Sherly	S(con)	Yeah.
24:36:03	Dr. A	T(r)	Then added them up, and that number is what?
24:42:29	Ashley	S(ans)	And divided by 90 degrees.
24:44:19	Robert	S(ans)	This one right here.
24:45:07	Victor	S(ta)	I didn't write it down. That's, that's the distance(in response to Robert pointing to Victor's paper)
24:47:21	Dr. A	T(c)	That doesn't matter. But you added it up and, but you didn't divide it by 90 degrees first.
24:52:26	Victor	S(pb)	No, we divided by 13 so we could find an average.
24:55:19	Sherly	S(con)	Yeah, cause there is different points.
24:58:08	Dr. A	T(c)	So when you found that average, that average was
25:04:06	Ashley	S(ans)	Divided by 90.
25:05:04	Victor	S(pb)	Is the average distance it travels per 90 degrees. Like for instance, from right here. From right here, from R2 to R5, it's 7 centimeters.
25:24:22	Dr. A	T(f)	R, like R
25:26:13	Victor	S(pb)	R12 and R13 it oh no.
25:28:00	Dr. A	T(f)	From R12 to R13

25:28:12	Victor	S(pb)	And R13, from here to here it's about 13, I mean 7 centimeters. And from the origin to R12 is about 5 centimeters.
25:39:05	Dr. A	T(con)	Uh huh.
25:43:21	Victor	S(pb)	So, you just find the average distance of those to points and you get about the average distance from here to, the average distance if you had say a point in the middle, so you add 7 + 5, and you get 12 divided by 2 average of those two points. See?
26:02:04	Dr. A	T(c)	I understand, but if you added, if you added the 5 and the 7, you get 12, and the average is 6. What does that 6 mean?
26:15:00	Sherly	S(ta)	Take average distance of it and then you divide it by
26:17:19	Dr. A	T(c)	The distance from where to where?
26:18:21	Sherly	S(pb)	(Inaudible) per degree. That's why we divided by 90 degrees.
26:22:21	Dr. A	T(c)	This is 6, what's the. I don't get it
26:25:25	Angela	S(qs)	The average distance in 90 degrees, right?
26:29:12	Victor	S(con)	The averages traveled for that 90 degrees.
26:31:22	Angela	S(con)	Yeah.
26:33:24	Mrs. W	T(c)	Traveled where?
26:36:06	Angela	S(ans)	On the spiral.

26:39:01	Robert	S(qs)	But in this case we went just between R12 and R13, right?
26:43:05	Sherly	S(con)	Yeah.
26:43:05	Robert	S(pb)	because those are the only two numbers. So it would be in between those two points.
26:46:08	Dr. A	T(con)	So you mean from here around the corner to here,
26:50:14	Sherly	S(con)	Mm hmm.
26:50:28	Dr. A	T(j)	should be 6 centimeters, because
26:54:03	Victor	S(c)	On average.
26:54:03	Dr. A	T(j)	because it's the average of this distance and this distance?
26:59:10	Victor	S(ans)	Just about.
27:02:08	Mrs. W	T(d)	Okay, can I ask that. I think it's the same question you have.
27:07:17	Dr. A	T(con)	Yeah.
27:10:22	Mrs. W	T(s)	If I travel from, now if, does it matter which direction I go?
27:18:09	Victor	S(ans)	It does, I think so.
27:20:15	Mrs. W	T(con)	It does?
27:22:25	Victor	S(ans)	I think you have to go clockwise.
27:26:07	Mrs. W	T(f)	Clockwise, which would be which way?
27:27:02	Victor	S(ans)	Cause that's how
27:29:08	Ashley	S(con)	Yeah.

27:30:07	Mrs. W	T(con)	Like that.
27:31:04	Dr. A	T(con)	You went to the outside of the spiral
27:33:03	Mrs. W	T(con)	Like that. Like, so for example if I went from R10 to R11, and I traveled this path.
27:45:02	Victor	S(con)	Mm hmm.
27:45:28	Mrs. W	T(r)	The average, the average, I'm not understanding the average of this length, and then if I continued and traveled from R11 to R12, the average of that length, is 6?
28:06:20	Victor	S(pb)	No. You have to take 3, 4, and then 5. 3, 4.5, and 5, add those up and the divide by 3 and then you went 3 different points
28:21:16	Robert	S(ans)	It's like four something.
28:22:27	Victor	S(ans)	Seems to be that, like four something
28:25:09	Mrs. W	T(f), T(c)	So then that must mean that, that this, if I am hearing you right, are you saying that this, where would I get the length for this? Now we are talking about he average lengths, but where would I get the length for this? To consider it's average, if I compared it with other lengths?
28:48:15	Ashley	S(ans)	You add 3 and the 4.5, and then you divide by 2.
28:55:04	Dr. A	T(f)	You add the 3 and the 4.5, and you do what?
28:59:29	Ashley	S(c)	And you divide by 2.

29:01:16	Dr. A	T(c)	And you get?
29:02:14	Ashley	S(ans)	The average.
29:03:14	Dr. A	T(c)	Yeah, it would be?
29:05:20	Robert	S(ans)	3.75
29:09:06	Dr. A	T(j)	3.75? And you're telling me that that is the distance from here to here? Can you prove it?
29:17:03	Victor	S(dnc)	Um.
29:18:11	Dr. A		But don't stretch it.
29:25:21	Dr. A	T(p)	You can mark that with pen. Remember, don't stretch it.
29:35:13	Victor	S(con)	From here to here?
29:39:05	Angela		(Inaudible).
29:39:05	Dr. A	T(f)	Isn't that what you said. You said it was supposed to be 3.75.
29:42:18	Robert	S(ans)	3.75.
29:56:19	Sherly	S(ans)	What? It's off average. (laugh).
29:58:13	Angela	S(ans)	5.6.
30:00:08	Victor		(Inaudible).
30:01:10	Dr. A	T(e)	Is it, no I'm just. What was it?
30:03:21	Victor	S(con)	I saying that you were right, go ahead.
30:06:08	Dr. A	T(e)	No, no, no. What was it?
30:08:23	Victor	S(ans)	5. something.
30:09:07	Angela	S(ans)	5.6.

| 30:13:25 | Dr. A | T(f) | What about the one on the other side? |
| 30:23:12 | Sherly | S(dnc) | It's okay Victor. |

The students recalculate their averages between points and try to determine why their values are not working. They remeasure some values and Sherly suggested that should be looking at circles because they are assuming their measurements work for circles. Victor does some more calculations and gets 47.34172 for the distance by adding up the measurements and dividing by 90. On the previous attempt he divided by 13 then by 90.

40:43:29	Dr. A		So did you (inaudible)
40:45:28	Sherly	S(ans)	Yeah we were wrong, but then we did something different and we got a number that was closer
40:49:15	Dr. A	T(c)	Oh really, what do you mean?
40:52:02	Sherly	S(a)	Huh
40:54:22	Dr. A		You're tired
40:55:23	Angela	S(ta)	Instead of adding up and then dividing by 13, we just divided by 90 after we added them all up and got.
41:03:01	Dr. A	T(j)	Why?
41:04:01	Ashley	S(ta)	And some of the measurements were off because we got a new distance.
41:10:06	Angela	S(pb)	Cause we um, did the whole thing with a rubber band.
41:13:08	Dr. A	T(f)	Yeah and got a new data (inaudible)

41:16:23	Sherly	S(ta)	No, no, just a different number here, for the total distances.
41:17:18	Angela	S(ans)	No, we just
41:20:03	Dr. A	T(c)	Oh, Oh. So what is your number?
41:27:18	Sherly	S(a)	Of what
41:27:29	Angela	S(ans)	It should be on his calculator
41:29:12	Dr. A	T(c)	What's the new distance
41:31:14	Sherly	S(ans)	47.3412
41:36:03	Dr. A	T(c)	And it's just the string all the way around the outside.
41:37:28	Sherly	S(ans)	No that was actually when we used the calculator
41:39:06	Angela	S(con)	Yeah
41:39:21	Sherly	S(ta)	The one with the string was about 49 but it was because, yeah (inaudible)
41:41:20	Angela	S(pb)	it was because we, like you know. it's not going to be perfect because it wasn't exactly on maybe.
41:50:10	Dr. A	T(d)	Tell me what you did?
41:52:09	Victor	S(a)	What are you talking about?
41:53:06	Dr. A	T(f)	Well I was just coming to catch up with this group.
41:57:12	Victor	S(con)	Oh
41:57:14	Dr. A	T(f)	See if you measured around.
41:59:18	Victor	S(ta)	No I just took all the measurements that I had before and I added them up and got 35.3 and then divide that by 90. Had a new

42:07:22	Dr. A	T(j)	Why?
42:08:21	Victor	S(ans)	Instead of 13
42:10:00	Dr. A	T(j)	Why?
42:11:11	Sherly	S(ans)	Just to see
42:11:21	Victor	S(pb)	My thought was that, maybe it's just like the average of 90 degrees. Not the average and then the average again. You never take two averages of something. So I just took the average over every 90 degrees and got .3922 centimeters change. And then multiply that by total degrees, which was1027.5 and got 47.
42:38:22	Dr. A	T(r)	You have, you have to catch me up on that. You added up all these radiuses. R1 through R13.
42:50:06	Victor	S(ans)	35.3
42:51:27	Dr. A	T(con)	35.3
42:53:12	Victor	S(con)	exactly
42:53:26	Dr. A	T(r)	That was the number that was missing awhile ago. why did you divide it by 90?
43:01:04	Victor	S(pb)	let me think, um. let me see now. I lost the thought here. Okay. What I started to think was that maybe it's not. Cause what we did was take two averages, and I never heard in mathematics of taking two averages. Like take one average and then take another average of

the one you have before. So we just took, instead of just

taking the average.

43:27:19	Dr. A	T(r)	Can, can you go back to the first time? I'm not sure. I wanna know what those two averages are. You said the first thing you did was to add up what you considered to be all those distances.
43:40:11	Victor	S(con)	Uh, hmm
43:40:22	Dr. A	T(con)	Is that right?
43:41:10	Victor	S(con)	Right
43:42:01	Dr. A	T(con)	You added the distances?
43:44:20	Victor	S(con)	hm, hmm
43:45:24	Dr. A	T(con)	Okay
43:46:23	Mrs. W	T(c)	which distances?
43:48:06	Dr. A	T(c)	And it was the distance from
43:49:02	Sherly	S(ans)	Distance from the origin to each point.
43:51:29	Mrs. W	T(r)	that's, when we say distances, we're talking about the origin out to each point. You added those distances. We're like recapping here.
43:59:04	Sherly	S(con)	uh,hmm
43:59:06	Mrs. W	T(con)	Okay
43:59:23	Dr. A	T(con)	From the origins to the point. Okay. And, and then you divided by 13
44:12:27	Sherly	S(c)	No that was last time.

44:14:10	Dr. A	T(r)	No, no, no. I am still having to know what last time was
44:16:20	Sherly	S(con)	Oh okay
44:16:28	Dr. A	T(r)	And you divided by 13.
44:20:03	Sherly	S(ans)	divided by how many points we had
44:25:08	Dr. A	T(c)	But from where
44:26:00	Victor	S(ans)	per 90 degrees
44:27:10	Sherly	S(c)	But no, you didn't divide by 90 yet.
44:28:06	Dr. A	T(r)	No, Oh, no, it was (inaudible) . You were right. But I want to know what you thought that distance was representing?
44:37:15	Victor	S(ta)	I thought that distance was representing the average change from point to point around.
44:41:12	Dr. A	T(f)	around the outside.
44:43:23	Victor	S(ans)	The average
44:44:17	Angela	S(qs)	did you say that was the number?
44:47:11	Dr. A	T(con)	Okay, The average distance around the outside. Okay
44:47:11	Angela	S(ans)	I got 47.5 when I measured it again.
44:50:23	Dr. A	T(con)	Okay
44:52:02	Angela	S(ta)	I just measured it, like, perfectly without stretching it and got 47.5
44:57:19	Victor	S(qs)	That's the number we got right?
44:58:22	Angela	S(con)	uh, hmm
44:58:24	Victor	S(con)	Oh okay

44:59:09	Dr. A	T(r)	Okay, now Victor. You were saying that number you got when you were dividing by 13. you thought approximate the average distance around the outside.
45:11:04	Victor	S(c)	from point to point
45:12:09	Dr. A	T(r)	Yeah I know. from point to point, per 90 degrees around the outside. Okay. And so, and so that was kind of wrong.
45:22:21	Victor	S(con)	wrong(inaudible)
45:24:01	Dr. A	T(f)	Yeah. And so then I thought you were then going to try to find out what that really implies. And you're not going to do that.
45:34:13	Victor	S(pb)	No, see. Okay, that's one now. See this now right. It was proved that the second method was correct. Why? Cause we took. You said the average was from point to point, right? The average from point to point and added them all up.
45:54:07	Angela	S(c)	I measured around the whole thing.
45:55:23	Dr. A	T(f)	with the string.
45:56:16	Victor	S(ans)	got 47.5
45:59:04	Angela	S(ans)	47.5
45:59:18	Dr. A	T(con)	You got 47.5
45:59:18	Victor	S(ans)	and that is exactly what we had the second time.
46:03:05	Dr. A	T(con)	You got 47.5 point and that was

46:05:09	Victor	S(ans)	the second time.
46:05:09	Angela	S(ans)	All the way
46:05:22	Dr. A	T(con)	All the way around with the string.
46:07:10	Angela	S(con)	Yeah
46:07:10	Victor	S(con)	On the dot.
46:08:23	Dr. A	T(con)	with the string.
46:10:05	Sherly	S(c)	no we were off by .2
46:12:10	Angela	S(ans)	That's good that could be my fault.
46:12:13	Dr. A	T(f)	With the string and then, and then what did you do?
46:15:26	Sherly	S(ans)	And then the number matched up.
46:17:28	Angela	S(pb)	Yeah it matches up with the number he got when he added them all and divided by 90
46:21:19	Victor	S(c)	Not this number. Don't look at this. That's garbage.
46:24:11	Dr. A	T(con)	Okay, so now
46:25:24	Victor	S(c)	this number
46:27:06	Dr. A	T(c)	Which is?
46:28:02	Victor	S(pb)	This is the new rate of change. This is when we added all the distances. All these up.
46:31:16	Dr. A	T(con)	Yeah
46:32:08	Victor	S(pb)	and 35.3. divided by 90 degrees. Instead of by 13 and then by 90, we just divided by 90. And I got 3.92 repeating.

46:41:23	Dr. A	T(f)	Okay, you added them all up and you got 35.3 and then you divided by.
46:49:21	Victor	S(ans)	90
46:50:18	Dr. A	T(f)	By 90, and you got.
46:53:12	Victor	S(dnc)	uh.
46:53:12	Dr. A	T(f)	0.399
46:56:01	Victor	S(ans)	32 repeating.
46:58:24	Dr. A	T(con)	392
47:01:17	Victor	S(ans)	repeating, and then we multiplied that.
47:02:29	Dr. A	T(con)	Okay, so you got 392 repeating.
47:07:03	Sherly		
47:07:14	Victor	S(ans)	That was our rate.
47:09:06	Dr. A	T(con)	Okay.
47:09:17	Sherly	S(ans)	I knew you had a question
47:10:07	Victor	S(ans)	then we multiplied by, oh.
47:14:24	Sherly	S(pb)	He did the same thing he did before, but rather than divide by 13, he just divided by 90
47:18:15	Victor		This is not good.
47:19:04	Angela	S(qs)	Why, What's not good?
47:21:04	Victor	S(ans)	I multiplied by the wrong number.
47:23:28	Angela	S(ans)	Multiply again.
47:25:25	Victor	S(ans)	I multiplied by 120.7.
47:30:23	Angela	S(qs)	What were you supposed to multiply by?

47:31:06	Victor	S(ans)	Oh man.1207
47:32:15	Sherly	S(pb)	The new rate of change that he got when he divided by 90.
47:34:08	Dr. A	T(d)	Okay, Angela, what did you do with you 47.3417
47:40:06	Angela	S(ans)	I didn't do anything with it. I was just um,
47:41:20	Sherly	S(ans)	figuring out numbers.
47:42:22	Angela	S(ans)	47.5. I got.
47:45:22	Mrs. W	T(c)	So there's 47.5 and 47.3.
47:49:19	Sherly	S(c)	No, see 47.3 was our number, what we screwed up when we multiplied by the wrong number. So let's just cancel that
47:56:29	Victor	S(dnc)	Man, this is garbage.
47:56:29	Mrs. W	T(c)	So what's the right number?
47:58:13	Sherly	S(ans)	The actual distance that she measured was 47.5
48:02:12	Angela	S(con)	yeah.
48:02:28	Sherly	S(c)	with the rubber band.
48:04:12	Angela	S(c)	all around the spiral
48:05:06	Sherly	S(c)	Like physically doing it.
48:07:26	Mrs. W	T(p)	Now when you say all the way around, show me.
48:10:11	Angela	S(pb)	Starting
48:11:07	Sherly	S(pb)	this point here.
48:12:12	Angela	S(pb)	to going all the way around to there.
48:15:23	Mrs. W	T(c)	Okay, okay. And that's what number.

48:18:13	Angela	S(ans)	47.5

48:23:06   Dr. A   T(d)   Okay, can I ask you one other question. Victor, come back. Forget this. If she said 47.5 is the distance all the way around.

48:33:28   Victor   S(con)   uh, hmm

48:34:01   Dr. A   T(r)   What does that correspond to with what you were doing earlier, I mean what, in terms of what you were looking for?

48:42:24   Victor   S(a)   What we were looking for

48:43:14   Sherly   S(ta)   We were trying to get the distance (inaudible) with his

48:44:27   Victor   S(ans)   mathematically.

48:47:02   Sherly   S(ta)   but the numbers didn't prove right.

48:49:24   Victor   S(pb)   Cause we figured if we could get one thing. If we could get like, the thought. Okay. Well a spiral consists of r is equal to a theta, right. So we figured if we could get some.

49:02:28   Mrs. W   T(c)   Wait, what?

49:04:21   Victor   S(pb)   This is the equation, right, that you get in calculus or whatever. R is equal to a theta, right. So that's how you get some Archimedes spiral or something like that. So what I was trying to figure out some kind rate of change that will affect how big the spiral is. And that's

the problem I'm having getting here so I can it's

function. See, I'm trying to find a good *a*.

49:41:21	Sherly	S(qs)	What was the rate of change?
49:44:05	Victor	S(a)	rate of change.
49:45:05	Dr. A	T(r)	Tell me again what *a* is?
49:47:29	Victor	S(ans)	I don't even know what *a* is yet.
49:49:18	Dr. A	T(c)	No, no, no, but in your formula
49:49:23	Sherly	S(ans)	It's the rate of change.
49:52:15	Dr. A	T(con)	It's the rate of change.
49:53:15	Victor	S(con)	Right just about.
49:55:15	Dr. A	T(con)	It's the rate of change,
49:56:26	Victor	S(con)	uh, huh
49:57:00	Dr. A	T(c)	per degree, is that what you are saying.
49:59:13	Victor	S(con)	Exactly, uh huh
50:00:10	Dr. A	T(con)	Okay, rate of change, per degree.
50:03:15	Victor	S(con)	Right
50:04:27	Dr. A	T(r)	Okay and you were telling me that if you thought you knew the distance around the spiral and you knew the number of degrees. You told me a while ago you know the number of degrees.
50:17:28	Sherly		(inaudible)
50:18:23	Dr. A	T(r)	That all the way around. How many? I mean what is the. How many. You were counting them. I saw you.

50:47:08	Dr. A	T(c)	Is it this many?
50:52:24	Sherly	S(ans)	No because that was when you divided by 13. Now we divided by 90.
50:55:17	Dr. A	T(r)	No, it was the number of the number of degrees.
51:01:19	Sherly	S(ans)	1, 2, 0, 7
51:02:01	Dr. A	T(con)	1, 2, 0, 7 point 8.
51:05:06	Sherly	S(c)	But that was when he had divided by 13, then divided by 90 with a different rate of change. Because he just divided by 90. And he times, what did you do, times this number by that number and it came out.
51:20:03	Dr. A	T(s)	Okay. How many degrees. I guess you need to think about. How many degrees does it travel around that spiral. Or does it matter?
51:33:08	Victor	S(ans)	It does
51:35:13	Dr. A	T(f)	Because you now know the length of the , you know the length of the spiral.
51:42:15	Robert	S(ta)	Isn't that what we had there, 1207.
51:44:07	Victor	S(ta)	Well, (inaudible)right. What I just did was take 47.5, hold on.
51:46:12	Sherly	S(qs)	How is this the same number?
51:49:09	Angela	S(ta)	Yeah isn't that the same number because.
51:53:22	Sherly	S(qs)	Victor
51:55:07	Victor	S(a)	Huh

51:55:21	Sherly	S(qs)	How is this the same number?
51:59:26	Victor	S(a)	What?
52:00:17	Sherly	S(qs)	How are this and this the same number?
52:02:27	Angela	S(ans)	Cause they're the same thing.
52:03:14	Sherly	S(a)	This is the total distance.
52:05:25	Victor	S(con)	I thought it was.

Students discuss if the have found total distance or total degrees. They are interrupted by a group presenting to the whole group. The AM session ends with another student coming over and talking with the group about matching the graph to the spiral.

July 9th   PM Session

The students begin a discussion about how a negative affects the graph. They also discuss who is going to present and what they are going to present. Angela then asks for the equations from Victor and Robert so everyone can have it and she and Ashley can understand what is going on.

11:19:24	Dr. A	T(r)	Is this what you all started with? You had this from the very beginning. That or is that the problem statement.
11:29:06	Sherly	S(ans)	From the beginning.
11:29:26	Dr. A	T(con)	From the beginning.
12:00:03	Angela		You're not in polar

12:02:13	Ashley		I know that.
12:05:22	Angela		That's not good. Alright
12:16:20	Sherly	S(qs)	How'd you get to flip the other side, flip over like y?
12:19:11	Victor	S(a)	Huh
12:20:02	Sherly	S(qs)	How do you get it to flip over the y axis?
12:22:23	Victor	S(ans)	figure it. A negative.
12:25:23	Sherly	S(con)	I did that.
12:25:23	Victor	S(ans)	Oh, I don't know
12:36:08	Dr. A	T(f)	What happens when you do a negative?
12:39:08	Sherly	S(ans)	It like flips it over. Watch
12:41:28	Victor	S(ans)	Flips it over the x axis.
12:43:06	Dr. A	T(c)	upside down.
12:46:09	Sherly	S(qs)	This is. whoops. How do you get that thing to come below?
12:58:12	Dr. A	T(e)	How do you flip?
13:33:11	Sherly	S(ans)	See this one stops here.
13:37:06	Dr. A	T(con)	Okay it starts. (inaudible)
13:41:12	Sherly	S(pb)	No I put two on each (unclear)
13:43:29	Dr. A	T(r)	Okay go back again. I wanna see what the first one was.
13:45:25	Sherly	S(con)	First one. The first one was. [Show calculator screen]
14:04:27	Dr. A	T(f)	And the other one
14:12:20	Dr. A	T(c)	So it flips this way.
14:14:21	Sherly	S(ans)	It kinds of flips over. It kind of flips over both axes.

14:20:05	Dr. A	T(f)	Okay this one went this way (other students talking about movies over voices of Dr. A and Sherly) And this is the... Now lets...
14:36:17	Sherly		(inaudible)
14:38:22	Dr. A	T(con)	So it goes it that way.
14:40:28	Sherly	S(con)	uhumm
14:43:26	Dr. A	T(con)	And then it flipped over this axis.
14:45:25	Sherly	S(ta)	Well kind of both axes. Cause if you go this way. It'll go like a (unclear, other students talking over her about movies)
14:55:03	Dr. A	T(con)	Oh I see. So if you flipped it this way.
14:58:18	Sherly	S(con)	Yeah and then over.
14:59:16	Dr. A	T(con)	And this was it's like a (unclear word)
15:01:07	Sherly	S(con)	Yeah.
15:07:00	Dr. A	T(f)	Is that what you wanted to do?
15:09:07	Sherly	S(ta)	Well because the picture like ends here but not really. It kind of
15:13:17	Dr. A	T(f)	Where's the picture?
15:20:15	Sherly	S(ans)	It's just that it goes like that.
15:26:05	Victor	S(ans)	We can make it look any way we want it to be.
15:27:06	Dr. A	T(con)	Oh I see. So
15:30:06	Sherly	S(ans)	But we don't care.

15:31:00	Dr. A	T(c)	No, I'm just this really might be. So what you're saying is you want it to go this. You want this one to flip to this.
15:39:12	Sherly	S(c)	that way.
15:42:10	Dr. A	T(s)	Ok. If you were, If you were a line. You're gonna want that. Is that right?
15:53:15	Sherly	S(con)	Uh hm
15:55:01	Dr. A	T(con), T(e)	Is is. Does that makes sense? If it were a line and you just flipped it over here. You wanted that. How did you get that?
16:14:17	Sherly	S(pb)	No, isn't. That's not the same thing as a negative cause like it's kind of going down and over.
16:18:27	Dr. A	T(s)	Wha, what's changing? Let's let's do an easier one. If it's. If it's this line. That's y is equal to x. and this one. Is y equals negative x. Okay but, but it doesn't always work. If it's this one which is y equals two x and you go this way. Is it like (inaudible) And so if you are moving, that's this. Is that right?
17:25:06	Sherly	S(con), S(ans)	uh hmm. No the other half is this.
17:35:28	Dr. A	T(c)	And what you want is this?
17:55:06	Dr. A	T(c)	That's really interesting. So what you're wanting to do is turn it down.

18:04:07	Sherly	S(pb)	Oh but see it's easier with this one cause this one you just have flip it over. Cause you have it like this. Oh no.
18:14:21	Dr. A	T(con)	That's this.
18:17:05	Sherly	S(ta)	Yeah, well two things to figure out. Cause on the calculator it's like that and we want it to go like. I don't know.
18:51:08	Sherly	S(qs)	Bob what are you working over there? Bob. What are you working on over there.
18:56:16	Robert	S(a)	Huh
18:58:19	Dr. A	T(d)	Were you trying to do the same thing that she was trying?
19:01:21	Victor	S(ans)	sort of, kind of, yeah.
19:03:04	Dr. A	T(c)	trying to flip it.
19:05:00	Victor	S(con)	um hmm.
19:05:25	Dr. A	T(c)	and turn it. What was your (unclear). What was your solution?
19:14:16	Victor	S(ans)	We didn't come up with no solution.
19:15:29	Dr. A	T(c)	You just came up. You just had the when it from positive to negative.
19:18:10	Victor	S(ta)	I just looked at the way. It doesn't really matter cause I looked at the transparency. I can flip it anyway I want to.

19:22:12	Dr. A	T(s)	What would it. This is, this just a curious thing. What if it was one over alpha. What would the picture look like?
19:29:23	Sherly	S(ans)	Nope it doesn't work
19:31:14	Dr. A	T(con)	One over alpha
19:32:24	Sherly	S(pb)	No, we cause we just tried it now. Cause it gave me that line. Oh wait this line. It gave me like a straight line in that direction. Oh, see that's the second one, the one over.
19:46:04	Dr. A	T(con)	Yeah, this is the one over. No
19:48:18	Sherly	S(con)	This line right here.
19:50:01	Dr. A	T(con)	Oh the straight line there.
19:51:23	Sherly	S(con)	Yeah
20:49:00	Dr. A	T(s)	What if you did one minus the equivalent of them
20:51:18	Sherly	S(con)	One minus it
20:52:18	Dr. A	T(con)	uh hmm
21:12:13	Dr. A	T(c)	Is that what that was?
21:15:09	Sherly	S(pb)	Yeah, I don't (coughing) like on the direction. Cause it's just a different rate then. and the spiral would be the right size
21:28:19	Dr. A	T(con)	uh hmm really
21:31:14	Sherly	S(seek)	Right?

21:31:22	Dr. A	T(r)	I don't know if that's what the answer is. What was the original one?
21:34:11	Sherly	S(a)	What original one. that one
21:35:18	Dr. A	T(con)	Yeah
21:48:13	Dr. A	T(r)	Okay and what's one minus them
21:49:19	Sherly	S(pb)	One of those I guess it's changing the size of it cause there's two on top of each other.
21:53:12	Dr. A	T(c)	Is it one of them is the thing itself and the other one is one minus the thing itself and they are right on top of each other?
22:00:29	Sherly	S(ta)	Well I think not completely. There's like a little.
22:09:08	Sherly	S(ans)	I don't think that
22:11:01	Dr. A		(unclear)
22:12:05	Shelly	S(qs)	Do you guys have an equation?

The students discuss amongst themselves about the mode of the calculator and then complain about being tired of this problem.

23:16:16	Dr. A	T(f)	What is, what's your issue, Victor?
23:19:24	Sherly	S(ans)	That we're not solving anything.
23:22:12	Victor	S(ans)	We're not solving anything
23:26:12	Dr. A	T(c)	What would it mean to solve for something?
23:29:04	Sherly	S(ans)	Cause you're asking for what r. r as a function of theta

23:34:14	Dr. A	T(r)	Make a table of r as a function of theta.
23:37:24	Dr. A	T(con)	I thought you just did that.
23:39:07	Sherly	S(ans)	No we didn't'
23:41:25	Dr. A	T(con)	You made the table of r.
23:43:11	Victor	S(con)	Yep. Our table is kind of funny
23:46:24	Sherly	S(ans)	Or we couldn't make an equation though
23:48:15	Dr. A	T(r)	What can you say about r as a function of theta?
23:53:28	Sherly	S(ans)	Well we made a table sort of
23:55:04	Victor		Hold up
23:55:12	Sherly	S(ans)	But we don't have an equation
23:57:22	Victor	S(ans)	Our table's kind of messed up.
23:57:22	Dr. A	T(r)	You've been working with an equation what does it do?
23:59:21	Sherly	S(ta)	But then we just changed it to radians and that sort of.
24:02:14	Victor	S(dnc)	Hold on. I'll tell you. I think that.
24:05:04	Dr. A	T(c)	The equation you've been working with is.
24:07:20	Sherly	S(ans)	was in degrees
24:08:23	Dr. A	T(c)	y is equal to?
24:11:05	Sherly	S(ans)	point
24:13:17	Dr. A	T(c)	is alpha was equal to point.
24:16:14	Sherly	S(ans)	zero, three, nine, three, three, seven times.
24:22:08	Dr. A	T(f)	three
24:23:01	Sherly	S(ans)	zero, three, nine, three, three, seven.

24:27:14	Dr. A	T(r)	zero, three, nine, three, three, seven. And that was what you got when you divided 47.5
24:35:06	Sherly	S(ans)	divided by one two zero seven, point eight.
24:40:27	Dr. A	T(f)	one thousand two
24:44:00	Sherly	S(ans)	two hundred seven point eight.
24:44:13	Dr. A	T(c)	seven point eight degrees. which was the total number of degrees.
24:48:11	Sherly	S(con)	Uh huh
24:50:07	Dr. A	T(c)	in degrees.
24:56:11	Sherly	S(seek)	What if we focus them on radians?
24:58:08	Dr. A	T(con)	What if you did it in radians? Is that what you just said?
25:00:21	Sherly	S(con)	Yeah
25:01:20	Dr. A	T(f)	What would it, what would be the total number of radians with them, before you can put it into twelve oh seven
25:06:29	Sherly	S(qs)	How do you do radians again. divide by 180
25:11:23	Dr. A	T(r)	What is the total number of radians?
25:12:22	Victor	S(con)	Um yeah.
25:18:14	Sherly	S(ans)	six point seven one. And that's that.
25:21:21	Victor	S(con)	And that's that.
25:21:21	Sherly	S(ans)	That's 6.715 radians.
25:25:06	Dr. A	T(r)	six point seven one five. And so you could have divided 47.5 by 6 point. What are you going to use.

25:33:11	Victor	S(ta)	What I need is to get the number of radians per point. How many radians is that?
25:40:02	Dr. A	T(c)	But does it matter.
25:41:13	Victor	S(ans)	That's half a radian.
25:42:26	Dr. A	T(s)	You divided by this. What if you divided by the thing before you converted it.
25:47:12	Victor	S(dnc)	I know, oh. Oh no.
25:48:16	Sherly	S(qs)	Where's pi?
25:51:04	Robert	S(ans)	Carrot.
25:52:19	Sherly	S(a)	Oh is it.
25:53:01	Robert	S(ans)	Under clear
25:54:12	Sherly	S(con)	Thanks
25:57:10	Dr. A	T(c)	So, what is it?
25:59:16	Sherly	S(ans)	22 point. 2393
26:02:23	Dr. A	T(con)	twenty point two three
26:04:11	Sherly	S(ans)	nine three.
26:04:11	Dr. A	T(con)	nine three.
26:05:08	Sherly	S(con)	22.23
26:06:23	Dr. A	T(c)	Okay then, what did you do. You divided that into 47.5. Is that right?
26:13:27	Sherly	S(a)	What? Wait I am sorry
26:15:09	Dr. A	T(con)	Okay, this corresponds to this.
26:18:29	Sherly	S(con)	yes

26:20:14	Dr. A	T(c)	Okay then didn't to get your a. times theta.
26:25:09	Sherly	S(con)	Uh huh
26:26:06	Dr. A	T(c)	didn't you divide this by this.
26:29:21	Sherly	S(ans)	I divided 47.5 divided by 6.715
26:35:10	Dr. A	T(c)	Okay you did it, and what did you get?
26:36:26	Sherly	S(ans)	22.2393
26:39:08	Dr. A	T(con)	Oh that's where you got that.
26:40:10	Sherly	S(con)	Yeah.
26:40:14	Dr. A	T(c)	Okay, so then.
26:42:02	Sherly	S(ans)	I had to put that number.
26:51:04	Dr. A	T(c)	times theta
26:51:28	Sherly	S(con)	Yeah
27:29:18	Sherly	S(ans)	Um, that didn't look right.
27:58:22	Sherly	S(ta)	Probably change the theta back to zero degrees
28:02:25	Sherly	S(ta)	or take the window. Oh I see.
28:36:27	Sherly	S(seek)	What do I change theta max to. Just a number.
29:51:03	Dr. A	T(s)	Is it (inaudible) to keep that picture on yours and to get the other picture on (inaudible)
29:58:03	Sherly	S(qs)	Angela, do you (inaudible)
30:00:14	Angela	S(ans)	This one
30:44:11	Dr. A	T(c)	This is the right answer.
30:46:11	Sherly	S(con)	the what, yeah?

30:47:08	Dr. A	T(c)	This is the right answer. Um, hmm and this is the biggest radius
30:53:23	Sherly	S(ta)	Oh you know what. They're different
30:56:28	Angela	S(qs)	Why is it different?
30:57:29	Sherly	S(pb)	Cause look how mine starts. It starts here, and it's going this way and yours is going this way. Oh, it's the same way, never mind.
31:09:14	Dr. A	T(s)	Okay, your scale. Was. used to scale it with and her is scale is
31:18:20	Victor	S(ans)	I think it I got it.
31:19:11	Sherly	S(con)	What?

Robert continues to help Ashley and Angela perform a cubic regression. Victor confirms the answer and explains what he did to the group.

34:34:07	Dr. A	T(r)	Can I ask one more time how you got this number?
34:37:01	Sherly	S(pb)	I don't know. Okay, Um. Oh yeah, cause this number this is degrees and in radians it's 6.71. So instead of dividing total distance of 47.5 in by degrees, I did it by radians.
34:56:20	Dr. A	T(j)	I understand that. yes, that's what I thought you were saying, but. Can you show me?
35:05:21	Sherly	S(ans)	Well it's already there. Do I have to do it over again?

35:07:03	Dr. A	T(f)	No, not if you talk me through it. Okay, So 6 point 71.
35:17:19	Sherly	S(ans)	That's how it just came up.
35:20:06	Dr. A	T(c)	But is the pi in the number.
35:21:20	Sherly	S(ans)	Well yeah, cause look it's right there.
35:24:14	Dr. A	T(c)	so its 47.5 times 6 point
35:26:16	Sherly	S(ans)	No divide by
35:29:10	Dr. A	T(c)	71. hmm.
35:33:25	Sherly	S(c)	See, that's what was in here.
35:36:09	Dr. A	T(j)	My only question is. Why is that (whispering low) to know that you're dividing by it.
35:41:26	Sherly	S(pb)	Because I typed it in and that's how it came out.
35:43:27	Dr. A	T(f)	Okay. I see. So. Can you just on the calculator show me what 6.71 times pi is.
35:56:24	Sherly	S(ans)	Maybe cause I don't if it sometimes, it doesn't sometime show up.
36:02:29	Victor	S(ans)	A couple times that decimal will show up.
36:04:24	Sherly	S(con)	It will
36:06:20	Victor	S(ans)	Like 2.0
36:07:04	Sherly	S(con)	Okay
36:12:13	Dr. A	T(j)	So it's 21 point 'oh' eight 'oh' one. Okay now can you show me
36:19:05	Sherly	S(ans)	Divide 47.5 by that
36:19:25	Dr. A	T(p)	47.5 divided by 21.0801

36:34:24	Sherly	S(ta)	Oh wait, I am off a decimal point. Oh, I am off a decimal point or something.
36:42:14	Dr. A	T(s)	It shouldn't matter
36:43:24	Sherly		Does it (inaudible)
36:47:12	Dr. A		(inaudible) point 33. It's not this.
36:52:13	Sherly		Oh, I don't know.
36:54:16	Dr. A	T(p)	But it's because you're not dividing by it. Divide 47.5 by 6.71. Okay. And multiply the answer times pi. What do you get?
37:17:17	Sherly	S(ans)	22.2
37:19:14	Dr. A	T(s)	yeah see you're not dividing. Your pi doesn't (?) the denominator. You understand what I am saying.
37:24:14	Sherly	S(con)	Oh
37:25:23	Dr. A	T(s)	It's all in the wrong (?). So the number that you should be using isn't this one. It's that other one.
37:34:07	Sherly	S(con)	What one?
37:35:27	Dr. A	T(s)	It was this one.
37:37:15	Sherly	S(con)	Oh, yeah.
37:38:23	Dr. A	T(con)	Do you understand what I am saying?
37:40:00	Sherly	S(con)	Okay
37:41:11	Dr. A	T(p)	Now can you, can you, can you graph it. This one became. Is that what your A is?
37:50:10	Sherly	S(a)	My A.

37:51:14	Dr. A	T(con)	Uh, huh.
37:51:28	Sherly	S(seek)	should it be that then?
37:53:07	Dr. A	T(con)	Uh huh.
37:53:16	Sherly	S(con)	Okay
37:54:01	Dr. A	T(con)	I mean let's see what happens. Do you understand what.
37:56:19	Sherly	S(con)	Yeah
37:57:01	Dr. A	T(con)	what we're trying to do.
37:57:21	Sherly	S(con)	uh, huh. Hold on.
38:00:06	Dr. A	T(p)	Put it in for another function.
38:02:11	Sherly	S(con)	Okay, I just wanna
38:08:07	Dr. A	T(p)	It's 21.08. Wait a sec.
38:15:21	Sherly	S(ans)	No I didn't put 7
38:18:27	Dr. A	T(c)	21.08 and do 47.5 is that what you're saying.
38:25:15	Sherly	S(con)	Yeah
38:26:07	Dr. A	T(con)	Okay
38:54:19	Dr. A	T(s)	It was 2.253
38:55:26	Sherly	S(ans)	Something like that. it just totally didn't work
39:09:07	Dr. A	T(s)	I think your window, your window's up too much.
39:46:12	Dr. A	T(s)	Can you get in between those two things?
39:48:06	Sherly	S(con)	Oh
39:51:10	Dr. A	T(con)	You know what I mean.
39:52:03	Sherly	S(con)	Yeah
40:29:08	Dr. A	T(j)	This is the same thing?

40:30:29	Sherly	S(a)	The what?
40:31:21	Dr. A	T(c)	This is, how does this compare to yours?
40:35:05	Sherly	S(ans)	The same.
40:35:17	Dr. A	T(con)	Um, hmm. It's the same. Think you did enough, to make you do a little work.
40:41:26	Sherly	S(con)	That's okay.
40:42:23	Dr. A	T(c)	Don't you think that's more accurate?
40:44:15	Sherly	S(ans)	Yeah, yeah. I didn't realize that.
40:46:18	Dr. A	T(con)	Yeah

Angela and Sherly continue to question Robert, but quickly go off topic. Benny comes from Table 2 and Dr. A tries to interact with them, but Victor asks for Benny's help on how to do a regression on the calculator.

48:07:20	Dr. A	T(d)	Tell me what you think you found.
48:10:14	Angela	S(ans)	I don't think I found anything. I seriously I don't like did all that. The only thing I understood is what we did in the very beginning and it's like proven to be useless. So I have no clue about anything anymore.
48:21:21	Dr. A	T(d)	Tell me what you, what you know. You have this table. This table.
48:30:21	Sherly		Oh, they moved it away.
48:32:11	Angela	S(ans)	That table. I think that table.

48:32:22	Dr. A	T(con)	these numbers, did you, your group. This table. That's this table.
48:48:26	Angela	S(con)	Okay.
48:49:24	Dr. A	T(r)	For the, for your thirteen
48:54:05	Angela	S(ans)	um, hmm. It's just how far away from the middle it is, the origin.
48:59:02	Dr. A	T(r)	Um, hmm. What that one of the things you were supposed to do. Make a table
49:05:23	Angela	S(a)	You want me to.
49:06:20	Dr. A	T(con)	I mean, no, you just did and it's right there.
49:08:14	Angela	S(con)	Yeah okay,
49:10:26	Dr. A	T(f)	So you have this table.
49:13:26	Angela	S(dnc)	um, hmm. I don't understand this problem. Everytime people explain it to me I feel stupid because I don't get it.
49:23:04	Dr. A	T(c)	Yeah, and so what you're saying is?
49:30:26	Angela	S(dnc)	I have like no clue where we're going with this.

Students say they don't understand what is going on and that they don't have an answer after 8 hours of working on the problem. Dr. A tries to interact with what students don't understand, but she is ignored. Dr. W also stops by and the students want to know when the next problem is going to start.

54:50:13	Dr. A	T(d)	Hey Victor
54:51:01	Victor	S(a)	Huh
54:51:15	Dr. A	T(d)	Let me ask you this question. If we're saying, you told me the radius, which is the distance out to a point, is equal to that number that you came up with which was .039337
54:56:16	Victor	S(con)	mm, hmm.
54:57:20	Dr. A	T(con)	Is that right?
54:58:17	Victor	S(ans)	I guess that, that's right for what I said, not right like correct
55:04:00	Dr. A	T(f)	Oh, no. I'm, I'm just saying.
55:06:03	Victor	S(con)	Okay
55:06:21	Dr. A	T(c)	that's what you got and that's what you, that's what you based your spiral on.
55:09:21	Victor	S(con)	mm, hmm.
55:11:03	Dr. A	T(c)	times theta.
55:13:14	Victor	S(con)	right.
55:14:18	Dr. A	T(c)	And theta for this number was in degrees or in radians?
55:20:14	Victor	S(ans)	Degrees
55:21:14	Dr. A	T(s)	Okay, and so then shouldn't you be able to plug in a number of degrees and get one of your points on the spiral?
55:37:04	Victor	S(a)	Say that again now.

55:39:08	Dr. A	T(r)	If this is an equation that says r is equal to .039337 times theta. What happens if you plug in a number of degrees? And multiply it by .039337. What should you get?
56:01:20	Victor	S(seek)	A number of radius I guess. The number of the the radius right?
56:06:02	Dr. A	T(c)	The, the length of a radius? Is that what you should get?
56:10:21	Victor	S(con)	Uh huh
56:11:18	Dr. A	T(con)	But now, isn't that what these things are?
56:15:10	Victor	S(con)	Right
56:16:25	Dr. A	T(r)	Well shouldn't she be able to check by going backwards or not?
56:20:21	Victor	S(ta)	She should, but then again, well she should, but I think, something. I think that's just an average and this a whole bunch of distortion. But I don't know, you can go check.
56:32:23	Dr. A	T(f)	I don't know, that's what I was saying.
56:35:28	Victor	S(ans)	Well why don't you, let's go check Ms. Alston. Let's just prove me wrong here. Let's go
56:39:27	Dr. A	T(f)	No, no, no, no, no.
56:41:02	Victor	S(ans)	No, you're about to do it. Come on, let's go. Um, which one is it now. Um, some what degree.
56:50:27	Dr. A	T(e)	What's the degree to get this guy out here?

56:57:29	Victor	S(ta)	Uh, 6.55 radians. Which is about, 6.5 um, 6.5 times 180. 1107
57:20:02	Dr. A	T(con)	mm,hmm
57:21:06	Victor	S(ans)	times
57:21:09	Dr. A	T(r)	which is almost your total, wasn't it?
57:23:00	Victor	S(ta)	.0, hmm, .0 what, three
57:30:08	Dr. A		9337
57:38:22	Victor	S(ta)	No, you don't get the radius. You don't get the radius. Oh dang.
57:43:21	Dr. A	T(f)	No I just
57:44:06	Victor	S(ta)	You don't get the radius, you get the length.
57:46:07	Dr. A	T(c)	What do you get? do you get the length?
57:50:04	Angela	S(qs)	Is it length from like, length like this, that's what you get?
57:55:21	Victor	S(con)	Yeah
57:56:04	Dr. A	T(c)	So is that what you are getting?
57:58:02	Victor	S(a)	That's what I mean
57:58:24	Dr. A	T(r)	Oh, okay and so when we did it for, do it for 360. and then what, what.
58:05:07	Victor	S(a)	Huh?
58:07:14	Dr. A	T(r)	If for 360 degrees. Uh what did you get? What number did you get when you did that last week?
58:14:26	Victor	S(a)	Huh?

58:14:29	Dr. A	T(r)	When you tried that last time, what number did you get?
58:17:08	Victor	S(ans)	46
58:21:29	Angela	S(ans)	the whole thing's 47.5
58:25:18	Dr. A	T(e)	So that's not far off. What would be this one? What would ,what would be the number of degrees for this one. For the, for the point before it
58:43:16	Victor	S(ans)	let me see now, 6.
58:46:20	Dr. A	T(c)	For r12, what's the number of degrees? Wasn't it.
58:55:23	Victor	S(ans)	1080
58:57:29	Dr. A	T(con)	Wasn't it, wasn't it six pi?
59:03:02	Victor	S(con)	Six pi, so it's 1080
59:04:09	Dr. A	T(j)	Okay, so what does it tell you?
59:15:23	Victor	S(ta)	42.48, that's just the distance from here to here.
59:20:01	Dr. A	T(j)	But it really works.
59:21:06	Victor	S(con)	Yeah.
59:38:12	Dr. A	T(c)	so, so what, what is the thing over here? What is thing thing over here. I mean what it, what's the name of it is.
59:58:24	Victor	S(ans)	I don't know. um, (inaudible) of the spiral.
01:00:03:09	Angela	S(ans)	Some kind of arc length type deal. I don't know.
01:00:08:16	Dr. A	T(con)	Well arc, arc, whatever. Arc length, Isn't that was you call that?

01:00:12:10	Victor	S(ans)	Arc something.
01:00:16:15	Dr. A	T(f)	I don't know I mean this is all new to me. (inaudible)
01:00:25:04	Sherly	S(seek)	Just give a hint.
01:00:26:21	Dr. A	T(s)	Okay so arc length is the (inaudible)
01:00:27:10	Victor	S(seek)	No I mean I did. Well I did everything we were supposed to do. I plot. Where is it?
01:00:33:15	Dr. A	T(con)	I think you've done great.
01:00:34:24	Sherly	S(ans)	Well we're annoyed cause we can't get an answer.
01:00:39:22	Dr. A	T(c)	I don't understand what the answer is.
01:00:43:13	Sherly	S(c)	Stupid answers.
01:00:43:14	Dr. A	T(r)	No, no, no, no. I don't want 3. I don't understand what the question is that you're looking for answer to.
01:00:52:20	Benny	S(ans)	I know what the question is.
01:00:54:06	Sherly	S(a)	The one on the paper.
01:00:55:04	Robert	S(ans)	How is r a function of theta.
01:00:56:07	Dr. A	T(c)	Which is?
01:00:57:15	Sherly	S(ans)	R is a function of theta.
01:00:59:14	Angela	S(qs)	isn't that the equation we have.
01:01:02:28	Sherly	S(a)	But is that right, cause it's not like
01:01:05:08	Benny	S(qs)	What equation do you have
01:01:07:13	Angela	S(ans)	r equals a.

The students continue to discuss what they have to answer the question. Bob then says

groups want to present as the conversation ends.

01:03:22:29	Dr. A		(inaudible) You've developed
01:03:36:11	Victor	S(ans)	A function for the wrong thing.
01:03:38:15	Dr. A	T(j)	What you mean for the wrong thing?
01:03:39:29	Sherly	S(ans)	Damn it, we're saying this and that's the end of the story
01:03:42:28	Dr. A	T(c)	What is it that you've developed a function for?
01:03:46:02	Victor	S(ans)	Arc length or something
01:03:47:14	Dr. A	T(f)	And that's what Angela and I were guessing. But I think it's very, very important. But you've also developed it in two different ways.

Tables present to end the day.

July 12th   AM Session

The day starts with Bob Speiser welcoming everyone back and telling them to continue to

discuss their work and that there will be some presentations today.

| 00:02:02:14 | Mrs. W | | Sherly, Ashley, Victor, Robert. Michelle was gone for a couple of days, and |

00:02:10:07	Mrs. W	T(r)	so do you think it might be nice if they could sort of back-track through what they had done while you were gone?
00:02:17:16	Michelle	S(con)	Uh huh.
00:02:17:16	Mrs. W	T(r)	And help get you up to, to speed on where they left off and then uh,
00:02:17:25	Angela		Go head Rob.
00:02:22:10	Sherly	S(qs)	Where's your folder with all the um, transparencies?
00:02:21:16	Mrs. W	T(r)	and then, um when you all feel like you've gotten back to where everybody's on the same page then maybe it would be well to do the presentation, and that will get everybody back thinking about the problem. So

The group spends about 9 minutes explaining their work to Michelle including the regression equation the calculator gave them.

00:10:03:21	Dr. A	T(r)	And say again, you all. What were the. You had c2?
00:10:09:04	Victor	S(ans)	c2 for x. And c1 for the
00:10:12:08	Dr. A	T(f)	For the, for the x?
00:10:13:04	Victor	S(con)	Mm hmm.
00:10:14:02	Dr. A	T(con)	And c1 for the y?
00:10:16:10	Victor	S(con)	The y.
00:10:18:09	Dr. A	T(f)	But

00:10:19:01   Victor   S(pb)   We had to be rushed before, that's why we were kept

getting messed up.

00:10:21:20   Dr. A   T(c)   But in what, what c2? What were the, what were your

entries for c2?

00:10:25:05   Victor   S(c)   c2 was, c2 was um, degrees, radians. Yeah, something

like that.

00:10:30:14   Dr. A   T(con) Okay, so these were your, your degrees in radians?

00:10:34:11   Victor   S(con), S(c)   Uh, huh. And then c1 was our actual

measurements.

00:10:36:02   Michelle S(qs)   Did you copy down this equation?

00:10:39:28   Sherly   S(ans)   I don't know, (inaudible).

00:10:40:07   Victor   S(ans)   Yeah, I did.

00:10:40:08   Sherly   S(con)   Okay.

00:10:42:22   Dr. A   T(c)   And c1 was your...

00:10:45:08   Victor   S(c)   And then all we, and then c1 was our measurements

that we actually measured off the thirteen points.

00:10:49:08   Dr. A   T(con) Mm hmm.

00:10:51:08   Victor   S(pb)   And then instead of using x, we just used x of theta, and

then we put it in our program, and that's how we got it.

00:11:03:13   Dr. A   T(j)   And so when you got that what do you have?

00:11:06:12   Victor   S(ans)   We got a...

00:11:06:12	Michelle	S(ta)	Wait, I lost the equation. I just pushed some button. I didn't lose it. I didn't lose it. I just like went off the screen, I don't know how to get out.
00:11:14:08	Ashley	S(ans)	Second, second quit.
00:11:16:27	Michelle	S(a)	Second quit? Okay there's a bunch of numbers that I don't get. I'm checking it.
00:11:26:12	Michelle	S(ans)	I don't know how to work the calculator.
00:11:27:29	Victor	S(ans)	Let me see.
00:11:31:24	Victor	S(a)	(inaudible) you want the equation again?
00:11:32:13	Mrs. W	T(s)	So do all of you have the information in your calculators? Do you all have c1 and c2?
00:11:38:02	Sherly	S(ans)	No.
00:11:39:15	Mrs. W	T(con)	You don't, you don't all have the c1 and c2?
00:11:40:17	Victor	S(ans)	And this is the equation that, that group came up with, um the first group.
00:11:41:26	Sherly	S(ans)	No.
00:11:44:24	Mrs. W	T(s)	With the link and the (inaudible) and you can get it.
00:11:45:16	Victor	S(ta)	But we don't really get that equation. They were just playing around and kinda, you know came across that number.
00:11:53:14	Sherly	S(con)	Oh okay, okay.
00:11:53:29	Michelle	S(con)	Okay.

00:11:54:12	Robert	S(ta)	You know because they, they like plotted a function or something. And then they figured out how to go through all the points. So then, they made this equation that went through all the points.
00:12:02:22	Robert	S(ta)	Like they plotted all those points, and this, um kind of went through all of them and when they graphed it in polar it kind of went spiral.
00:12:11:09	Mrs. W	T(con)	Have you done that yet?
00:12:13:05	Robert	S(a), S(ans)	What? No, I don't think so.
00:12:15:23	Victor	S(qs)	Did what?
00:12:17:06	Robert	S(ans)	A, what they did like plot all the points and then drew a line that go through all the points.
00:12:22:11	Victor	S(ans)	Oh yeah, we did that.
00:12:23:10	Robert	S(a)	We did?
00:12:24:12	Victor	S(con)	Mm hmm.
00:12:25:29	Mrs. W	T(e)	What did you see?
00:12:27:29	Robert	S(ans)	I don't remember.
00:12:28:19	Victor	S(ta)	Um, we just boxed em. We put em. Our thing was a box, and we saw a whole bunch of them like together and then some were just spread out. I could do it again, but
00:12:39:09	Mrs. W	T(f)	I'd like to see it.
00:12:40:09	Sherly	S(qs)	What did we call those points?

00:12:42:10	Victor	S(a)	Points?
00:12:43:03	Sherly	S(a)	We called it points right?
00:12:44:16	Victor	S(ans)	No I, I don't know. I think
00:12:46:11	Sherly		No, it was something different.
00:12:49:19	Victor		Oh, I called it x.
00:12:51:13	Sherly		It was x?
00:12:51:25	Victor		Yeah.
00:12:52:14	Sherly		Okay.
00:12:54:13	Sherly		Oh wait. Oops.
00:12:55:05	Robert		Is this yours or his?
00:12:56:13	Michelle		I think it's his.
00:12:56:28	Sherly		Oops. (inaudible)
00:12:58:22	Angela		Tell me one.
00:12:59:15	Sherly		Hold on. Kay.
00:13:01:14	Victor		Ohh. (laugh) (inaudible)
00:13:01:14	Dr. A	T(r)	Robert, do you have the data in your calculator, that you could show me what you were saying that, that they first did?
00:13:04:18	Robert	S(ans)	I don't think so, cause we doing something different, but um
00:13:21:15	Robert	S(ta)	Yeah I have this, but I don't know if it's right. Cause we were messing around. So, I don't think this is right.
00:13:31:15	Dr. A	T(r)	But your c1 are the angles?

00:13:36:22	Robert	S(ans)	Yeah, I think so.
00:13:37:28	Dr. A	T(c)	And your c2 are the
00:13:39:12	Angela	S(ans)	(Inaudible) right, yeah?
00:13:40:16	Sherly	S(con)	Mm hmm.
00:13:40:24	Victor	S(con)	Mmm, yeah c2 is the angle.
00:13:43:10	Robert	S(con)	Yeah, and then this is the distance.
00:13:45:02	Victor	S(con)	And then c1 is the distance.
00:13:48:09	Robert	S(pb)	I don't think it's from this. I think it's just from trying equations and then these numbers (inaudible).
00:14:00:00	Mrs. W	T(d)	Okay, I'm a little confused cuz I'm not sure what everyone one is doing.
00:14:04:29	Victor	S(ans)	Just kind of like, I mean, making sure that the data is in everybody's um calculators.
00:14:09:05	Mrs. W	T(con)	So you're sharing the data.
00:14:10:16	Victor	S(con)	That's right
00:14:11:02	Mrs. W	T(r)	Okay, and then Robert you did something. What did you do? Because I don't think on this side of the table we know what you got.
00:14:16:26	Robert	S(ans)	I didn't do anything. I don't know.
00:14:18:24	Mrs. W	T(c)	What was on your calculator?
00:14:20:14	Robert	S(ans)	Oh, nothing I was just trying the equation. She wanted to see if I had it, like what the points were.
00:14:25:09	Mrs. W	T(con)	Did you get the points?

00:14:26:17   Robert   S(con) No. I have to re-copy them down. Because I had them

in here, but then

00:14:29:01   Mrs. W   T(f)   Oh, so you need the data too?

00:14:31:02   Robert   S(ans) Yeah.

00:14:31:19   Mrs. W   T(con) Okay. So, everybody needs to have the data. Michelle

do you have the data?

00:14:35:14   Michelle   S(ans) No.

00:14:36:01   Mrs. W   T(con) Okay.

00:14:36:13   Michelle   S(ans) I don't think I have anything.

00:14:38:00   Mrs. W   T(con) Okay.

00:14:40:01   Michelle   S(ans) I'm way lost.

00:14:41:18   Mrs. W   T(d)   Ashley, do you have it?

The students transfer the data to each other's calculator. Robert and Victor explain to the

other group members how to find the regression equation. The group tries to get the

graph of the equation.

00:20:49:04   Mrs. W   T(j)   Now, I'm not convinced.

00:20:50:26   Victor   S(a)   You're not convinced about what?

00:20:50:26   Dr. A   T(j)   I'm not either, at all.

00:20:52:11   Mrs. W   T(j)   I am not convinced that what

00:20:53:10   Victor   S(con) You want to see the points right? I see, okay.

00:20:57:22   Robert   S(qs)   Is it supposed to be like this?

00:21:00:20	Victor	S(ans)	Yeah.
00:21:12:22	Dr. A	T(e)	Can you explain what you've done there?
00:21:16:29	Robert	S(ans)	I just put scatter plot.
00:21:19:03	Dr. A	T(c)	You did a scatter plot of what?
00:21:21:13	Robert	S(ta)	Of um, the points. And then we just used the equation that he had to um, then we put in y equals and then we just graphed the points.
00:21:41:15	Robert	S(ta)	And then
00:21:42:14	Mrs. W	T(con)	I'd like to make sure that the whole group is on the same page with what's going on and uhh the question is.
00:21:56:16	Mrs. W	T(con)	What I've seen so far is that you all have the points, right?
00:22:01:14	Mrs. W	T(con)	You all have the points. I 'm just trying to get clear on where we are. And I want to make sure that everybody is in the same place, because I feel like some people are getting left out of the conversation. So, when you get to a good spot Angela, we'll
00:22:18:15	Angela		Mm hmm.
00:22:19:29	Mrs. W	T(s)	We'll proceed. You tell me when.
00:22:25:23	Mrs. W	T(d)	What you doin?
00:22:27:26	Angela	S(ans)	Trying to get the graph.
00:22:31:08	Mrs. W	T(e)	What are you trying to get a graph of?
00:22:33:03	Angela	S(ans)	The points.

00:22:35:15	Mrs. W	T(c)	Are you trying to get a graph like Robert has?
00:22:37:16	Angela	S(con)	Yeah.
00:22:40:26	Robert	S(ans)	She's putting what um, we used over there.
00:22:44:06	Mrs. W	T(con)	Okay.
00:22:44:29	Angela	S(qs)	What do you mean?
00:22:45:22	Mrs. W	T(con)	Okay.
00:22:46:12	Robert	S(ans)	The one that he got from the thing.
00:22:48:02	Ashley	S(ans)	Let me see.
00:22:48:14	Robert	S(con)	Yeah.
00:22:52:06	Dr. A	T(r)	Okay, and so what, you did the scatter plot first?
00:22:55:08	Robert	S(ta)	Oh, yeah and then they got the points and then like I gotta a window so you can see the whole thing.
00:23:00:07	Dr. A	T(con)	Yeah.
00:23:00:18	Robert	S(pb)	Cause at first the window was like, it only went up like this high and then this high.
00:23:04:17	Robert	S(pb)	But it went like this far off the screen, so I had to adjust it.
00:23:08:14	Dr. A	T(con)	So, that the window had room for all the points. Is that what you're saying?
00:23:11:09	Robert	S(con)	Yeah, and then um, I just used that equation, and put it in (inaudible).
00:23:17:07	Dr. A	T(c)	You already had the equation from
00:23:19:21	Robert		Yeah, (inaudible) um.

00:23:19:21   Ashley   Oh, (inaudible) you don't have to polar?

00:23:23:08   Dr. A   T(d)   Ashley.

00:23:24:09   Ashley   S(ta) Cause she just told me to change it to polar.   My thing

is not in polar. You don't have to be in polar?

00:23:25:18   Angela   Yeah, that's what I thought, (inaudible). She is.

00:23:30:00   Robert   No, it was like x or something, that we used to put it in.

And after that

00:23:38:18   Robert   it just went through all the points, so I guess that's like

saying that it's the spiral (inaudible).

00:23:42:27   Sherly   I'm not getting all the points.

00:23:42:27   Robert   And then like went through all the (inaudible), like

nicked the edges or something, but

00:23:46:15   Victor   We're close enough.

00:23:47:09   Dr. A   T(j)   But it looked pretty good to you?

00:23:49:08   Robert   S(con) Yeah.

The students begin to check with each other that they all have the same graph. They

discuss and compare the graph and the window they have. Dr. A and Mrs. W

intermittently question the students to see if they have all gotten to the same place with

the graph. Most of their questions are ignored by the students

00:32:21:02   Dr. A   T(r)   So, what's that data?

00:32:22:09   Sherly   S(ans) I don't know.

00:32:23:03	Michelle	S(ans)	Well, he typed it in on mine.
00:32:24:13	Dr. A	T(e)	No, no, no. What are you asking it to do?
00:32:29:02	Ashley	S(dnc)	It's freezing cold.
00:32:29:15	Dr. A	T(e)	You're asking it to. I don't understand. What do you mean you're asking it to do a spiral?
00:32:33:28	Sherly	S(ans)	Like his picture. It's not coming out.
00:32:42:05	Dr. A	T(c)	This comes from what in the data?
00:32:45:15	Victor	S(ta)	Hey, that was in degrees before. That comes from our equation, but we came out with data that we entered into the table. Yeah.
00:32:54:11	Dr. A	T(con)	The data that you did on Friday?
00:32:57:26	Victor	S(ans)	Right.
00:33:00:08	Victor	S(ta)	These are the measurements that across the (inaudible) points and then the degrees is um
00:33:06:03	Victor	S(ta)	a half, 1, one and a half and (inaudible) half pi.
00:33:10:13	Victor	S(ta)	Like a half of pi,
00:33:11:23	Dr. A	T(con)	Yeah.
00:33:13:04	Victor	S(ta)	then pi, 1 1/2 pi's,
00:33:14:16	Dr. A	T(con)	Yeah, so pi/2, pi and 3pi/2, it's (inaudible)
00:33:37:00	Dr. A	T(j)	Okay, and and and so um, that came from.
00:33:44:23	Dr. A	T(con)	That's your graph.
00:33:46:07			
00:33:50:17	Dr. A	T(c)	But it's not that equation.

00:33:52:02   Victor   S(ans)   That's that equation right there.

00:33:53:16   Dr. A   T(con)   Oh, it's that equation.

00:33:55:20   Victor   S(a)   And how did we get that equation?

00:33:57:01   Dr. A   T(con)   Yeah.

00:34:00:08   Victor   S(ans)   The quadratic regression thing that we learned on Friday.

00:34:04:12   Dr. A   T(con)   Okay. So you went down to the quadratic regression

00:34:08:13   Dr. A   T(c)   and you had put in your points, not their points, but your points.

00:34:15:10   Dr. A   T(c)   And it spit out this.

00:34:17:14   Victor   S(ans)   Exactly.

00:34:18:18   Dr. A   T(f)   And so when did you

00:34:19:26   Victor   S(pb)   We had no problem coming up with that. It was just that we were putting, for x we put c1. Our information on c1, our distance. When we should have put our theta. That was the only problem that we were having all this time basically, yeah.

00:34:34:09   Dr. A   T(con)   So that you had your two, your variables backwards.

00:34:37:00   Victor   S(pb)   Exactly. And that's why we were not coming up with the graph.

00:34:40:18   Victor   S(pb)   That's why we thought what we were doing until they said c2 and c1. And we thought "Okay then...

00:34:47:09   Dr. A   T(d)   Who said that? (Inaudible)?

00:34:48:15	Victor	S(ans)	Yeah, (inaudible).
00:34:53:18	Victor	S(ans)	(inaudible) then we put c two and I was like (inaudible)
00:35:18:29	Dr. A	T(r)	So, what does this have to do with the scatter plot and and (inaudible)?
00:35:25:08	Victor	S(ans)	Mmm, about a (inaudible)
00:35:31:04	Victor	S(ans)	And say that all our points, um (inaudible), and a
00:35:37:16	Victor	S(seek)	Those are all our points right?
00:35:39:12	Dr. A	T(con)	Mm hmm.
00:35:41:01	Victor	S(ans)	And they should come across about

Robert explains to Michelle what the graph is trying to model. Robert explains that his graph is a function graph, not a polar graph, so it doesn't spiral. They discuss who would explain this to the whole group. Sherly tries to explain to Michelle also that they need the graph to go through the points on the scatter plot to make the spiral be the same even though the spiral is "unraveled".

| 00:39:59:19 | Dr. A | T(r) | Can you explain to me (inaudible) the two equations you're working with? |
| 00:40:05:07 | Robert | S(pb) | No, we were just saying like. To explain like why this becomes that. That' s what we're, that's what we're trying to explain. Um, because this is like that, kind of like unraveled, un-spiraled. And like all the points |

would be the same. Like if I graph the points in polar it would be that.

00:40:24:09 Robert S(pb) If I graph it in a function, it would be like line. If you get an equation that goes through the points, that when we like spiral it or put it in polar, it will go through the points too. It will go through the points when they're circular if you just put it, if you get it to go through when it's a function. We just saw it like this, this kind of like that unraveled, like if you just took the spiral out and made it into a straight line. And then we um, and then we just kind of respiraled it.

00:40:55:29 Dr. A T(c) Can you, could you use a pen and just give me a quick, easy explanation on the paper of what you just said, maybe not use all the points, but do something that helps me to understand it?

00:41:08:14 Robert Um.

00:41:12:26 Robert S(pb) Say we had three points that like here, here, and here.

00:41:17:06 Dr. A T(con) Okay.

00:41:17:28 Robert S(pb) And let's say like, this was point five, I don't know, like one point two and like two. I (inaudible).

00:41:27:08 Dr. A T(c) Okay, okay, what do those numbers represent?

00:41:29:29 Robert & Sherly S(c) The distance away from the

00:41:31:21 Sherly S(c) center.

| 00:41:32:09 | Robert | S(con) | Yeah. |

00:41:33:11 Dr. A T(con) Okay, and so you've done the spiral and you've said that. Let me see what you've done. And so you had a point over here and maybe a point over here. And a point over here or whatever. And so the distance from the center is your, is your number, is your radius.

00:41:52:06 Robert S(ans) Yeah and then we figured

00:41:52:20 Dr. A T(con) Okay.

00:41:53:14 Robert S(pb) I don't know, I guess like then if you draw it on one of these.

00:41:57:24 Dr. A Mm hmm.

00:41:57:24 Robert S(pb) I guess this would be the point number, like 1, 2, 3. Like this would be the first point the second point the third point.

00:42:04:29 Dr. A Mm hmm.

00:42:05:18 Robert S(pb) And then you go over like, here I'll make this .5, 1.5, 2. So you go over to .5 and you go up one

00:42:07:23 Sherly (Inaudible)

00:42:13:02 Dr. A Uh, huh.

00:42:13:15 Robert S(pb) And then you put a line here. And you go to 1.2, up to 2, and 2 up to 3. And you want to find an equation.

00:42:19:24	Dr. A	T(c)	Okay, I'm pretty dumb about this. I need to know which numbers, what is, what are you're two coordinates at those points?
00:42:27:28	Robert	S(pb)	Oh yeah right. This is distance around spiral.
00:42:37:04	Dr. A	T(f)	Okay, and so you're taking this point up here? Where the distance was
00:42:42:20	Robert	S(ans)	Yeah, but see we
00:42:43:00	Dr. A	T(con)	0.5, or something like that.
00:42:44:10	Robert	S(ans)	Yeah, we put .5
00:42:46:06	Dr. A	T(c)	And you're taking it over here?
00:42:48:14	Robert	S(ta)	Yeah, I, but this is like .5, this is 1 this is 1.5. Like it's going by one-half, and this is 1, 2, 3. And this is like
00:42:58:13	Dr. A	T(j)	Okay, how did you decide how far up for them to go?
00:43:01:20	Robert	S(pb)	Alright, so this is. This is like the first point, the second point, the third point.
00:43:06:13	Dr. A	T(c)	I don't get it.
00:43:08:10	Robert		Alright, um.
00:43:09:13	Dr. A	T(r)	I thought that generally when you plotted a point you had two coordinates and they both meant something.
00:43:16:20	Robert	S(ta)	Yeah, that's why I don't know what this one means. Like I actually I thought like one number it would be...

00:43:22:28	Robert	S(ta)	But it might have something the angle, I don't know. It might be like how much it turned or something. But um, I don't know that part yet.
00:43:35:10	Dr. A	T(con)	But you got those points to plot on the scatter plot?
00:43:40:11	Robert	S(con)	Yeah.
00:43:40:22	Dr. A	T(r)	What were you plotting and what were the two coordinates then?
00:43:44:09	Robert	S(pb)	Ah, c2, so that would be like the degree here. Like the amount or something. And I guess like, compared to one circle, so if like you go 90 degrees around, this would be like 1/4, I guess, and then if you go like half way around the circle this would be 1/2. And then um 3/4's, if you go like 3/4's around the circle.
00:44:05:24	Dr. A	T(r)	Okay, I can't, Victor said something a minute ago about which was the x and which was the y? Which is the
00:44:12:04	Sherly	S(ans)	C2 is the x, and c1 was the y.
00:44:15:10	Victor	S(con)	The degrees is c1, um wait yeah.
00:44:20:11	Sherly	S(con)	C2 was the x and c1 was the y.
00:44:22:08	Dr. A	T(c)	Okay, c2?
00:44:23:08	Sherly	S(ans)	Was the x.
00:44:24:08	Victor	S(con)	Yeah and c2
00:44:24:21	Dr. A	T(con)	Is the x.
00:44:25:28	Victor	S(con)	Uh huh.

00:44:26:08	Dr. A	T(c)	And what is c2?
00:44:27:29	Victor	S(ans)	That's, they're radians.
00:44:30:23	Sherly	S(con)	Yeah.
00:44:32:27	Dr. A	T(c)	Is the number, is what you were just talking about?
			And c1 is the y. And what was c1?
00:44:41:17	Sherly		Um.
00:44:42:17	Victor	S(ans)	Distance.
00:44:43:15	Dr. A	T(con)	Is the distance.
00:44:44:16	Robert	S(pb)	Oh, so it's like, if this went to like, if this was over here,
			and then this was over here.
00:44:50:18	Dr. A	T(r)	Can you do it again?
00:44:51:23	Robert	S(ans)	Alright.
00:44:51:23	Dr. A	T(j)	So I can see what you mean.
00:44:55:18	Robert	S(ans)	Distance.
00:45:03:04	Robert	S(ta)	And then I guess this would be degrees around or
			radians around.
00:45:08:13	Dr. A		Mm hmm
00:45:11:16	Dr. A	T(c)	And so it would be like pi/2
00:45:15:17	Robert	S(con)	Yeah.
00:45:16:12	Dr. A	T(con)	and pi? Is that right?
00:45:18:23	Robert	S(seek)	Yeah, because we are going to be in radians right?
			(whispers so pi/2).
00:45:21:09	Dr. A	T(f)	I think, I don't know.

00:45:22:17	Robert	S(ans)	Yeah, 3pi/2, 2pi.
00:45:28:04	Dr. A	T(r)	Okay, now show me how a point would (inaudible).
00:45:29:11	Robert	S(pb)	Oh, (inaudible), just pretend it's like taken every ninety degrees. Can we just like pretend that?
00:45:34:16	Dr. A	T(con)	That's fine with me.
00:45:35:05	Robert	S(pb)	Alright, so then we go over to ninety degrees or pi/2
00:45:38:20	Dr. A		Uh huh.
00:45:39:15	Robert	S(pb)	and then go up to .5 and put a point
00:45:42:11	Dr. A		Uh.
00:45:42:19	Robert	S(pb)	and then go over like ninety degrees more
00:45:45:15	Dr. A		Mm hmm.
00:45:46:09	Robert	S(pb)	and go to 1.2, and put a point. And then go to ninety degrees more and go to 2pi.
00:45:50:13	Dr. A		Mm hmm.
00:45:51:14	Robert	S(pb)	(Inaudible)
00:45:55:06	Robert	S(pb)	And we want to find an equation that goes through all of these.
00:45:59:22	Dr. A	T(r)	And it's not going to be a spiral?
00:46:02:03	Robert	S(pb)	No, cause we're in um x and y. And you can't make a spiral with that because what we did yesterday with the vertical line test. Or Friday.
00:46:10:02	Sherly	S(qs)	(Inaudible) makes a line that goes through all of them, right?

00:46:12:10	Robert	S(ans)	Yeah.
00:46:13:09	Sherly		(Inaudible).
00:46:13:09	Dr. A	T(con)	So, it's not a spiral then?
00:46:15:10	Robert	S(con)	No.
00:46:16:00	Sherly	S(c)	Unless you go to polar.
00:46:17:01	Dr. A	T(c)	Because you unraveled it?
00:46:19:13	Sherly	S(con)	Yeah.
00:46:20:03	Dr. A	T(con)	So, it's not a spiral?
00:46:22:05	Robert	S(ans)	Yeah
00:46:24:01	Dr. A		Hmm.
00:46:25:03	Robert	S(ans)	Because like, I don't know why, but
00:46:29:03	Sherly	S(pb)	Cause like x, y like the radius is like the point, and in polar (inaudible) like circle, or circular. And so,
00:46:37:03	Dr. A		Mm hmm.
00:46:38:18	Robert	S(ta)	And then we just figure if we find an equation that goes through this, like this.
00:46:44:04	Robert	S(ta)	That like when we go to here that equation will be this.
00:46:47:25	Dr. A	T(j)	Why?
00:46:49:19	Robert	S(pb)	Uh, because it goes through all the same points and they're both like counting on the same thing. Like, this depends on the distance around the circle and the degrees around, like where it goes to the point. And so does this, and they both are the same points, so we just

figured it goes through one it's got to go through the other.

00:47:08:23	Dr. A		Mm hmm.
00:47:09:17	Robert	S(pb)	Cause I was trying to say like, ah let's see, but this kind of looks like this unspiraled, like on the calculator. Like if you just took it out, and kept the points where they were and just like kept it going through, it would look like this sorta and then...that's it.
00:47:26:21	Dr. A	T(c)	Okay, but then you have one equation or you have two equations that makes those two different things?
00:47:31:07	Sherly	S(c)	No, it's one, but it's like in a different format
00:47:34:01	Robert	S(con)	Yeah, one it works for both.
00:47:35:02	Sherly	S(pb)	One's the polar coordinate which makes it spiral and the other one's in xy which makes it show
00:47:38:05	Dr. A		Oh,
00:47:38:25	Dr. A		Okay, how do you
00:47:38:25	Robert	S(c)	except the x is replaced with theta in polar. That's it though.
00:47:43:09	Dr. A	T(f)	Oh, so when you're throw theta in there instead of an x
00:47:45:29	Robert	S(pb), S(qs)	It acts as x, because doesn't polar like go on degrees or something?
00:47:49:08	Sherly	S(con)	Mm hmm.
00:47:50:16	Robert		Instead of

00:47:51:18	Dr. A	T(c)	So it circles it around.
00:47:53:03	Robert	S(con)	Yeah.
00:47:55:12	Dr. A	T(d)	Angela does that make sense to you at all? or not?
00:47:59:04	Angela	S(ans)	Sort of. I like get lost with all this stuff. I hate this.
00:48:02:24	Dr. A	T(f)	What, what are the, what is the sort of question that throws you?
00:48:08:22	Angela	S(ans)	Like, I get like little bits and pieces of what he's explaining, but I don't really get all of it.
00:48:14:04	Sherly	S(qs)	What don't you understand?
00:48:16:29	Angela	S(ans)	I don't know.
00:48:18:24	Dr. A	T(con)	I think I hear her say, she doesn't even know what she's asking. Um what, what are you asking...in this?
00:48:28:11	Angela	S(ans)	I don't like have a specific question. I just don't like understand the whole, like everything you just explained. Like why that, the whole thing, like what you were saying like why it's like the spiral unraveled or something like that. Like I don't even know how to explain, the points and just trying to follow and I just didn't.
00:48:53:27	Robert	S(ans)	It's kind of hard to explain.
00:48:54:10	Sherly	S(qs)	Which one are you on?
00:48:56:00	Angela	S(a)	Huh?
00:48:57:07	Sherly	S(a)	No, I was talking to Michelle.

00:48:59:08 Angela   S(qs)   Do you get this?

00:48:59:08 Ashley   S(qs)   Do you understand?

00:49:01:09 Michelle S(ans)   What he just said. Kinda.

00:49:06:18 Dr. A   T(d), T(r)   Could you, could you try Michelle, to explain.

Cause every time one of you explains it, it helps me a

little more. This is really just as foggy for me, Angela,

as it is for you. I am even further away than you, from

this stuff, because I don't understand the calculator

either. So Michelle could you try it again.

00:49:26:18 Michelle S(pb)   Okay, um. Ooh. Alright if you took, let me draw a

piece of the spiral, and you picked like certain points,

whatever ones they were. Right? Robert?

00:49:41:02 Robert   Yeah.

00:49:41:21 Michelle   Okay (laugh), just checking. And like um, you're

doing the ninety degree intervals, which is like the pi

over two, you know what I mean, and like you graph

them...

00:50:00:10 Michelle   Okay, then you're saying that r would be the radians? Is

that what you're saying?

00:50:04:15 Robert   Yeah.

00:50:05:27 Michelle   Well or whatever, then you decided that, I guess.

00:50:09:05 Robert   No.

00:50:09:17 Michelle   (inaudible).

00:50:11:00	Robert		Yeah, because we put in scatter plot, we put c2 first
00:50:14:10	Michelle		Oh, you switched 'em?
00:50:15:07	Robert		Yeah and then c2 is the radians or whatever, like distance. Radians. Radians.
00:50:21:00	Michelle		Okay, so this is like the c2 thing. This one's like the c1.
00:50:25:12	Dr. A	T(c)	Okay, what's c2?
00:50:26:23	Michelle	S(c)	C2 is like um, the interval around the circle like the radian thing.
00:50:31:27	Robert		Yeah, like
00:50:32:07	Michelle		Right?
00:50:32:25	Robert		how far you turn
00:50:34:06	Michelle		That's it.
00:50:35:00	Robert		from like the beginning.
00:50:35:29	Sherly		And we have like (inaudible).
00:50:39:03	Michelle		And then this one's um, pi over two right?
00:50:42:26	Robert		Yeah.
00:50:43:10	Michelle		This one's um
00:50:43:27	Sherly		Pi
00:50:44:28	Michelle		Pi. See I don't remember these. Three pi over two.
00:50:47:07	Sherly		Mm hmm. Two Pi
00:50:48:25	Michelle		this is two Pi. That's like the intervals we went around the circle.
00:50:53:03	Sherly		Mm hmm.

00:50:53:03	Michelle		Right? And that's the c2?
00:50:54:18	Dr. A	T(con)	If it were a circle rather than a spiral. Yeah.
00:50:57:08	Michelle	S(con)	Yeah. This one's like pi over two, which is like ninety degrees. This one's like pi which is like one-eighty. That's three pi over two, which is like um, two seventy (laugh). This is like pi, which is 360.
00:51:11:03	Michelle		And then, um this is like the actual distance from the circ, like center thing. Right? Like this would be like .5 and this would be like...what?
00:51:22:21	Robert		Just say one.
00:51:23:21	Michelle		One. And this one's like two. So you have...
00:51:29:01	Sherly		I have a question though. Is it like distance from like a straight line or is it like
00:51:33:11	Robert		No, it's distance like around the thing.
00:51:35:00	Sherly		(Inaudible) just the spiral?
00:51:36:03	Robert		Yeah.
00:51:36:22	Michelle		Which is the distance from like center to that actual point?
00:51:38:28	Robert		No, it's like 2 would be the distance from the beginning
00:51:42:17	Sherly		Going all the way around it.
00:51:43:12	Robert		all the way around to two.
00:51:44:24	Michelle		Oh, like
00:51:46:06	Dr. A		Oh.

00:51:47:24	Robert		I think that's what it means.
00:51:48:19	Dr. A	T(con)	Is that true?
00:51:50:12	Michelle	S(ta)	So like from this
00:51:50:22	Robert	S(ans)	That's what we're doing.
00:51:51:21	Michelle	S(ta)	point we're saying that we're going all the way around and reaching 2. That's like 2 units.
00:51:59:23	Robert	S(con)	Yeah, like two centimeters, I guess we're using.
00:52:02:23	Michelle	S(ta)	Oh, got that people? Got it? So then like if you just graph it, like the point five was at ninety.
00:52:12:03	Dr. A	T(c)	Hey Robert. I need to really clarify that in my head. In your data set
00:52:17:02	Robert	S(ans)	Yeah (inaudible). It doesn't make sense.
00:52:18:23	Dr. A	T(c)	isn't it these numbers?
00:52:22:15	Robert	S(ans)	I don't know I didn't put the numbers in.
00:52:24:27	Dr. A	T(d)	Victor.
00:52:26:14	Victor	S(a)	What?
00:52:27:12	Dr. A	T(c)	In your c1, which is the distance. Isn't c1 the distance?
00:52:32:19	Victor	S(con)	Right, right.
00:52:33:18	Dr. A	T(c)	Okay, when you put it in, I still need clarification as to whether it is the distance from the origin to the thing. Which is called a radius.

00:52:44:18	Dr. A	T(c)	Or is it the distance now, around the spiral. I know you all worked with both of those data sets. Which one are you using?
00:52:52:24	Victor	S(ans)	Oh, man.
00:52:54:05	Michelle	S(ans)	I used
00:52:55:00	Dr. A	T(r)	Because Angela and Sherly, I think and Ashley, whoever it was. You all were very careful and good at measuring those distances with the rubberband.
00:53:03:25	Dr. A	T(r)	Do you remember?
00:53:05:08	Angela	S(con)	Mm hmm.
00:53:06:03	Dr. A	T(c)	Okay, I need to know which data set you're working on right now.
00:53:13:08	Victor		Um
00:53:14:05	Angela	S(qs)	Isn't it like around?
00:53:15:00	Victor	S(a)	Ask that question again.
00:53:16:12	Dr. A	T(c)	Okay, I think and I just heard Robert say also something that I need clarifying, that you have in your notes. Where's the data set for the distances on the rubberband?
00:53:32:03	Angela		Um
00:53:33:22	Robert	S(ta)	Actually I think it's the radius,
00:53:34:07	Angela	S(ans)	Wait I just remem um, yeah
00:53:35:28	Robert	S(ta)	because that's what we're comparing.

00:53:35:28	Angela		I just
00:53:36:29	Dr. A	T(con)	Didn't you write them down??
00:53:36:29	Sherly	S(ans)	(Inaudible) 47.5.
00:53:38:00	Angela	S(ans)	I just measured from the beginning to the end.
00:53:39:19	Dr. A	T(r), T(con)	You only had one long thing didn't you?
00:53:43:00	Sherly	S(qs)	It was 47.5 isn't it?
00:53:45:14	Victor	S(con)	Yeah (inaudible).
00:53:46:08	Dr. A	T(c)	It was 46.5 or 47.5 or something like that?
00:53:49:29	Several Students	S(con)	Mm hmm.
00:53:50:10	Dr. A	T(c)	Yeah. Okay and so, and so you're saying what Robert?
00:53:54:19	Robert	S(ta)	No, I was wrong. I think now it's just like the point to here.
00:53:57:29	Dr. A		Oh.
00:53:57:29	Robert	S(pb)	I at first thought it was this but then I remember that we only measured the whole thing.
00:54:02:19	Dr. A	T(c)	It's forty (inaudible) or something or other?
00:54:04:04	Michelle	S(qs)	So it's like from the origin to the actual point?
00:54:05:12	Robert	S(con)	Yeah, 47.
00:54:07:12	Robert	S(con)	Yeah. You were right first.
00:54:07:12	Dr. A	T(con)	Yeah.
00:54:09:16	Michelle		Sorry Bob. I told you.
00:54:13:03	Dr. A	T(r)	Because then you, then you divided it by the total number of angles?

00:54:19:04	Robert		Um,
00:54:19:04	Dr. A	T(r)	The total, the total number of degrees around,
00:54:21:03	Robert	S(con)	Yeah.
00:54:21:03	Dr. A	T(r)	and around and around and around which was like 7 or 6 pi. And I asked,
00:54:27:07	Robert	S(con)	Yeah.
00:54:27:07	Dr. A	T(c)	and were able to figure an approximate distance for everything?
00:54:31:09	Robert	S(ans)	Yeah, it's like point three or something or .03.
00:54:33:00	Dr. A	T(con)	Yeah, yeah, yeah
00:54:37:05	Dr. A	T(c)	But is that what you're working with now?
00:54:39:19	Robert	S(ans)	Um, nah I just think we are just working with the original things we got here.
00:54:45:01	Dr. A	T(c)	Which is?
00:54:46:10	Robert	S(ta)	Distance from the center to the point. Like the radius. Yeah I remember. Isn't that the problem?
00:54:50:01	Dr. A	T(r)	Yeah, I think one of the really interesting things about you guys working on Friday, was that you were really working with these two different, very different ideas that are both really really good,
00:55:04:00	Dr. A	T(r)	but um, but this notion, I remember, at the end we were plugging things in and you were coming out and you were able, amazingly, could tell me a,

00:55:15:03	Dr. A	T(r)	if it was 46 all the way around when you got the average rate of change per degree, you could tell me how far around, along, around the spiral. Do you remember that?
00:55:24:20	Angela	S(ans)	Yeah. It's the one we used.
00:55:28:04	Dr. A	T(con)	Yeah, yeah, yeah. Which to me is really impressive. But I think I agree that that's not the same thing that you're talking about now, in terms of
00:55:36:02	Robert	S(con)	Yeah it's different.
00:55:37:05	Angela	S(a)	For this one we used that.
00:55:43:17	Dr. A	T(r)	Yeah, yeah, that was when you found an approximate average distance per degree that was going around the spiral.
00:55:52:13	Angela		Mm hmm.
00:55:52:25	Dr. A	T(r)	Do you remember?
00:55:53:20	Angela	S(con)	Yeah, plugging in the angles and then
00:55:56:20	Dr. A	T(f)	And you'd plug in an angle
00:55:57:20	Angela	S(con)	it would tell you how far around.
00:55:59:20	Dr. A	T(f)	and it would spit out a distance around. Sort of an approximate distance.
00:56:04:05	Sherly	S(con)	Yeah.

00:56:05:20	Dr. A	T(con)	Yeah and I guess what I was trying to, to make sure is, or clarify as to whether that, you know, was...was what you're doing now. And it doesn't sound like it.
00:56:20:22	Robert	S(ans)	No, I don't think so.
00:56:21:28	Victor	S(ans)	Ah, no.
00:56:23:11	Dr. A	T(r)	Okay, but so now back to the question that I was, that Michelle you were helping me understand which is that, that you'd come up higher, and what would you do?
00:56:37:20	Michelle		Um, like
00:56:38:01	Dr. A	T(p)	I want a, I want you to plot me a point.
00:56:40:24	Michelle	S(ans)	Oh, like...the one.
00:56:45:02	Robert		Yeah, you said that's 2pi right?
00:56:48:20	Michelle		Okay, so if you write pi, I guess it's kind of there. Like that and then
00:56:57:13	Robert		You said the other one was 2 or something.
00:56:59:20	Michelle		So, say the other one was 2
00:57:01:20	Robert		2Pi.
00:57:03:25	Michelle		It has to be 2pi?
00:57:05:04	Dr. A		What's the, what's the
00:57:05:15	Robert		That's what you said.
00:57:06:22	Michelle		Well, I just took numbers. It don't matter does it?
00:57:10:07	Robert		No.
00:57:11:01	Dr. A	T(c)	Okay so the first point was...

00:57:13:14 Michelle S(ans) Well like the first point

00:57:14:26 Dr. A T(f) Would have been this one.

00:57:15:21 Michelle S(ans) would be this .5 thing. Which is like

00:57:20:00 Dr. A T(j) Could it be here?

00:57:22:10 Michelle S(ta) Yeah, can't it be anywhere? Well, wait I don't

00:57:25:04 Robert Well, what you're saying that's like

00:57:26:27 Michelle Wait

00:57:28:24 Dr. A T(c) I thought it has to be further than ninety degrees.

00:57:30:26 Robert S(ans) Yeah, she's just saying like, pretend it's ninety degrees

around.

00:57:33:26 Dr. A T(con) Pretend what's ninety degrees?

00:57:35:04 Robert S(ans) That .5 is ninety degrees around the thing.

00:57:39:19 Dr. A Oh.

00:57:41:08 Robert S(pb) Like if you were starting here. Just saying like pretend

turn ninety degrees, and now you're here. And that's

where the .5 is.

00:57:49:28 Robert S(pb) That's not ninety degrees here, or where ever, here.

00:57:54:02 Dr. A T(j), T(r) How are you gonna be able tell when you've turned

ninety degrees? Didn't you all have that your very first

day?

00:58:01:29 Robert S(ans), S(qs) Yeah, mixed right angle, right?

00:58:03:10 Sherly S(a) Hmm?

00:58:03:10 Michelle S(con) Oh, yeah.

00:58:04:10	Dr. A	T(c)	With your overlay, didn't you have that?
00:58:06:26	Sherly	S(ans)	We just used a ruler.
00:58:08:08	Dr. A	T(con)	Oh, you used a ruler, or something?
00:58:10:18	Sherly		Mm hmm.
00:58:11:00	Dr. A	T(con)	Yeah. Okay and so then when you turned ninety degrees, you had, you measured the radius, is that right?
00:58:21:00	Michelle	S(con)	Mm hmm, right?
00:58:22:09	Dr. A	T(con)	Uh huh, I think.
00:58:22:26	Robert	S(con)	Yeah.
00:58:23:00	Michelle	S(ta)	Wait, so like it went like this was ninety degrees, but it's really not
00:58:26:08	Dr. A		Uh huh.
00:58:27:12	Michelle	S(ta)	like it had like turned to the point where like, we were like .5 centimeters away from the origin
00:58:34:00	Dr. A		Mm hmm, mm hmm.
00:58:35:04	Michelle	S(ta)	And then like when you went to like, when you went around 180 degrees which is like pi radians
00:58:41:21	Dr. A		Mm hmm.
00:58:42:21	Michelle	S(ta)	you'd be like 1 centimeter away from the origin.
00:58:45:26	Dr. A	T(con)	Okay.
00:58:46:03	Michelle	S(ta)	And it just gives you
00:58:48:16	Dr. A		Mm hmm, mm hmm.

00:58:53:19	Dr. A	T(r)	And so, I think Angela was asking the question, the same question I had before as to why it's not a spiral anymore.
00:59:06:03	Victor	S(a)	Why it's not a spiral?
00:59:07:14	Dr. A	T(r)	Yeah. When you connect those things, why isn't it a spiral anymore?
00:59:12:00	Robert	S(pb)	Because for this one like, it's not a function. Like if you put a number in you can only get like 1 output. But for the other one, like you, same thing, but except it doesn't go like x and y, it like, in circles.

The students continue to discuss why the graph is not a spiral. Angela asks which equation they will present. Victor is still working on his calculator to get a spiral graph. Robert shows how changing the mode on the calculator and using theta instead of x makes a spiral. After that, the students again discussion who will present.

01:05:04:03	Dr. A	T(c)	So this is the polar graph?
01:05:05:28	Robert	S(ta)	(Inaudible), you know how, (inaudible) changes by 360,
01:05:12:28	Robert	S(ta)	I don't know, just x. Um, I was going to say like if you just graph polar by itself, you get something like this.
01:05:20:00	Robert	S(ta)	I didn't put (inaudible) make it smaller, cause it was too big...
01:05:22:27	Dr. A	T(c)	How did you get polar by itself?

01:05:24:12	Robert	S(pb)	I just graphed theta, and that's like the function you use or whatever, that's like x, an xy graph. I just graphed it and it just comes out like this.
01:05:34:04	Robert	S(pb)	Like it makes one full circle, and it goes over a certain amount.
01:05:39:01	Dr. A	T(con)	This has nothing to fit your data? This is just a random graph.
01:05:41:20	Robert	S(ta)	No, I was trying to. Yeah, I was trying to explain like why one spirals and one doesn't.
01:05:49:03	Dr. A	T(f)	Oh, this is a thing y to theta?
01:05:52:11	Robert	S(ans)	Yeah, just theta, nothing else.
01:05:54:21	Dr. A	T(f)	And then, if you graph y to x?
01:05:57:07	Robert	S(ta)	Yeah, and I just wanted a function, but then now (inaudible) just graph y to x and you get this.
01:06:01:21	Dr. A		Yeah.
01:06:02:12	Robert	S(ta)	And it shows that like if you just graph regular numbers, it's not going to like spiral, but if you graph the other one, it spirals.
01:06:08:27	Dr. A		Yeah.
01:06:10:06	Michelle	S(ans)	It's when variables like degrees, or whatever.
01:06:13:00	Dr. A		Mm hmm.
01:06:13:00	Robert	S(con)	Yeah, and then we just did (inaudible).
01:06:15:23	Dr. A	T(con)	There's just two different kinds of coordinatizing.

01:06:19:29	Michelle	S(ans)	It's like two different ways to represent. Right?
01:06:24:10	Robert	S(ta)	Yeah. And then if you want like 720, then it turns around twice, but I don't know like
01:06:30:24	Dr. A	T(p)	Why don't you, show me what y verses 2 theta would be.
01:06:35:29	Robert	S(con)	Alright.
01:06:37:05	Dr. A	T(e)	What's (inaudible) your graph?
01:06:39:04	Robert	S(ta)	Ah, I don't think it will be 2 theta will be too big, because like, I put like 1 theta and it was like huge. So, I had to put a .01 in front of it to make it fit.
01:06:44:29	Dr. A	T(p)	Oh, well then do point oh two..
01:06:54:13	Robert	S(ta)	You know, it kind of starts, it kind of goes out a little farther.
01:06:59:01	Robert	S(ta)	And then it kind of starts a little (inaudible), but it still rotates around twice.
01:07:04:04	Dr. A		(Inaudible).
01:07:06:03	Robert	S(ta)	Ah, yeah. But this one is kind of like wider. I can make it just rotate around once if you want to, but
01:07:14:05	Dr. A	T(con)	Okay. So, it just changes the sizes.
01:07:18:02	Robert	S(pb)	Yeah. I guess when you make the number bigger, it makes it wider. And like how you make the number bigger with the x it makes it steeper.
01:07:22:08	Victor		Wo, hello.

01:07:26:05	Mrs. W	T(d)	What happened?
01:07:28:19	Mrs. W		Just shocked?
01:07:31:22	Victor	S(ans)	Something like that. Yeah, go l.
01:07:33:25	Dr. A	T(f)	What did you find?
01:07:34:27	Victor	S(dnc)	I didn't find anything. Who said I found anything, man? Just go away. Move away from me. Go ahead cut that out.
01:07:43:10	Dr. A		I'll go away. (laugh).

The students discuss who is going to present. Mrs. W and Dr. A help them decide who will talk about the different pieces of their work. The group then discuss their presentation content plans and begin to prepare materials. Robert and Michelle discuss how to graph the spiral when Dr. A questions them.

01:16:36:08	Dr. A	T(s)	Would it help if you actually maybe with another color, drew lines? (Inaudible).
01:16:48:13	Dr. A	T(f)	Or show me. Even, even do you have a blank one?
01:16:51:17	Michelle	S(ans)	No.
01:16:53:26	Dr. A	T(j)	(Inaudible) that's so pretty, and he made (inaudible). Show me what you mean when you say this (inaudible) adds up into the 90 degrees spiral. What are you talking about?

01:17:03:25   Michelle   S(ta)   So like, I thought it was like, this was like an axis

thingy.

01:17:08:29   Dr. A   T(j)   Can you draw it in? That's the reason I was saying, so

cause you could even (inaudible).   Okay.

01:17:12:28   Michelle   S(dnc)   Ooh. It looks like Christmas.

01:17:17:18   Dr. A   T(r)   So there's this axis thing.

01:17:20:07   Michelle   S(pb)   Like this is what I thought it was, was like from here to

here, like this was like a 90 degree thing, and I'm like

from like, oh.

01:17:28:03   Dr. A   T(con)   Which would be the 90?

01:17:28:28   Michelle   S(ans)   From like here, this line here. I'm just picking.

01:17:33:10   Dr. A   T(con)   Well, for just the first point.

01:17:35:06   Robert   S(ta)   But then you could also say this as 90, but it's not.

Cause it's like (inaudible).

01:17:39:29   Michelle   S(qs)   Yeah, but you're going around in a circle, so isn't it

going to repeat?

01:17:43:13   Dr. A   T(c)   Okay, are you starting?

01:17:51:14   Dr. A   T(e)   Explain to me where you starting, Michelle.

01:17:53:27   Michelle   S(ans)   Here.

01:17:55:24   Dr. A   T(c)   Where on the spiral, have you   gone 90 degrees?

01:18:01:14   Michelle   S(ans)   I don't know.

01:18:03:00   Dr. A   T(c)   Where does this. Where does this spiral go?

01:18:04:06   Robert   S(ans)   No, that's 180.

01:18:07:08   Michelle   S(ans)   This way.

01:18:08:05   Dr. A   T(con)   Uh, huh.

01:18:09:10   Michelle   S(seek)   But like how do you know how far like when you

reach 90 degrees?

01:18:12:05   Dr. A   T(j)   How do you know?

01:18:13:20   Michelle   S(seek)   I'm prob, it's probably really obvious isn't it?

01:18:16:15   Dr. A   T(f)   What do you (inaudible)?

01:18:16:20   Michelle   S(dnc)   Everybody's probably laughing at me.

01:18:18:27   Robert   S(ans)   We just guessed.

01:18:18:27   Dr. A   T(d)   It seems to me everybody's having this question. When

you, if you're starting here and going around, when

have you traveled 90 degrees?

01:18:27:03   Angela   S(ta)   When you cross the next line. When it intersects the

little axis thing.

01:18:32:14   Dr. A   T(j)   Show me Angela, what do you mean?

01:18:34:13   Angela   S(pb)   Like if you start here,

01:18:36:16   Dr. A   T(con)   At the first point.

01:18:36:16   Angela   S(pb)   and there's 90 degrees.

01:18:38:04   Dr. A   T(con)   Right there? Alright.

01:18:38:21   Sherly   (Inaudible).

01:18:39:26   Michelle   S(ta)   Oh, so this would be like .5, right. And then like this

one would be like

01:18:46:04   Robert   S(ans)   1

01:18:46:19	Michelle	S(ta)	1. And then like this one would be like 1.5, whatever.
01:18:49:00	Angela	S(ans)	1.5
01:18:50:10	Dr. A		Or whatever,
01:18:51:00	Angela		etc.
01:18:52:00	Michelle	S(ans)	And then the other's would be like 2.
01:18:54:02	Angela	S(ans)	2
01:18:56:00	Michelle	S(ans)	Oh, but isn't it won't repeat again.
01:18:58:20	Dr. A	T(c)	Okay, but now. And it keeps going for awhile, doesn't it? Okay, but now this was, this was .5 what?
01:19:09:09	Robert	S(ans)	Distance radius.
01:19:10:15	Dr. A	T(con)	The distance was .5?
01:19:12:02	Robert	S(con)	Yeah.
01:19:12:15	Dr. A	T(j)	So, help me with this as compared with that thing up there.
01:19:16:24	Michelle	S(ans)	Okay. This is like 90 degrees, which is like pi/2 radians.
01:19:21:22	Sherly	S(dnc)	Michelle. (Inaudible). No just turn it.
01:19:27:05	Michelle		Where do you want me to put it?
01:19:28:14	Sherly		Turn your paper so the camera.
01:19:30:23	Ashley		Or move a little closer to Bobby.
01:19:30:23	Michelle		Here?
01:19:32:06	Sherly		Yeah.
01:19:34:12	Michelle		Okay. So, okay, um

01:19:37:04	Sherly		She's all like
01:19:38:01	Michelle		What happened to the spiral it moved?
01:19:39:24	Dr. A	T(c)	You just moved it.
01:19:41:23	Michelle	S(ans)	Okay. So like the .5 is like that distance.
01:19:46:13	Dr. A		Mm hmm.
01:19:47:18	Michelle	S(ta)	And then like the 1 would be like, from the origin to that point. And 1.5 would be from the origin to that point. So on and so
01:19:57:04	Dr. A	T(j)	Okay, that's fine, but then how do you it get into this form up here?
01:20:01:09	Michelle	S(pb)	Okay, cause this, from like the origin to this like .5 point. That you traveled the 90 degrees, which is pi/2 radians.
01:20:11:07	Dr. A	T(con)	Oh, okay.
01:20:12:25	Michelle	S(ta)	So like you just like the x axis is like the degrees, the radians that you traveled or whatever.
01:20:22:13	Michelle	S(ta)	And like at that point, the .5 is how far away you are from the origin.
01:20:29:00	Dr. A	T(con)	Oh. Okay, okay.
01:20:33:15	Robert	S(ta)	And then this is 5pi/2, not just pi/2, right?
01:20:38:24	Michelle	S(con)	Oh, cause yeah.
01:20:41:16	Angela		Can you (inaudible).
01:20:41:16	Dr. A	T(c)	Which one is 5pi/2?

01:20:42:17   Robert   S(ans)   Actually (inaudible).

01:20:43:08   Michelle   S(seek) This one is 2pi right?

01:20:46:00   Dr. A   T(c)   This one right here is (inaudible). This one down here

is what?

01:20:48:27   Michelle   S(seek) This one is um, pi, pi/2. right?

01:20:51:12   Dr. A   T(con)   Mm hmm.

01:20:56:06   Robert   S(qs)   I got a question real quick.

01:20:58:03   Robert   S(qs)   If this is going to be 5pi/2 like

01:21:01:14   Dr. A   T(c)   Which one is 5pi/2?

01:21:02:28   Robert   S(ans)   This one. Isn't that like the next one but wouldn't that

um

The students continue to have side discussions about what they are presenting and how to explain their work. The data is incomplete because many statements can not be heard and a complete conversation can not be documented. Dr. A continues to questions students about the function graph and if it will spiral. Robert says it will not spiral back. Bob Speiser comes over and the group presents their work to the whole group.

Appendix C: Old Bridge Transcript with Codes

OBHS_17Apr02_1a&b

Tape starts with Dr. G explaining how to enter a product into the TI-89 Calculator from a
homework problem.

00:52	Dr G.	T(e)	Now. What's the function Andrew, for which you were to find the area under the curve?
01:15	Andrew	S(a)	Which problem?
01:16	Dr. G	T(f)	The one you worked on last night.
01:20	Andrew	S(ans)	x squared plus one.
01:35	Dr. G	T(e)	And what was the interval? Andrew?
01:41	Andrew	S(ans)	Zero to one, no, I'm sorry, one to two.
01:53	Dr. G	T(p)	Now, Kristen, the first thing I would see is a reasonable sketch of this function and all you need to do is give me two relevant points of interest. On a reasonable coordinate plane
02:44	Dr. G	T(r)	Kristen, do you recall the instructions I gave you when you, before you went up there.
02:50	Kristen	S(ans)	Yeah
02:52	Dr. G	T(c)	What were they?
02:53	Kristen	S(ans)	Draw a sketch. Reasonable
02:55	Dr. G	T(p)	All you have to do is give me two points of interest

03:01	Kristen	S(a)	Oh, you don't want a graph
03:03	Dr. G	T(r)	Two points. You've already given me an infinite number of points
03:23	Kristen	S(seek)	I did it wrong
03:27	Dr. G	T(d)	Amanda, clue?
03:28	Amanda	S(ans)	He just wants you to put a point at 1 (Inaudible)
03:53	Kristen		Oh
04:19	Dr. G	T(s)	Kristen, is there any reason for us to believe that this function is not continuous between these two points
04:31	Kristen	S(ans)	um, no
04:36	Dr. G	T(s)	Alright. Now. Where you going? Is the function increasing or decreasing between those two points?
	Dr. G	T(j)	How do you know?
04:49	Kristen	S(pb)	Um, Increasing because when you put x values in, you get higher y's. I don't know.
05:02	Dr. G	T(rep)	Say that again
05:04	Kristen	S(pb)	I said increasing because when you put x values in you get higher y, but I think that was wrong.
05:10	Dr. G	T(r)	Over the past few weeks, what did we practice that gave a serious clue as to whether or not the function would be increasing or decreasing?
05:31	Dr. G	T(d)	Jason, clue
05:33	Jason	S(ans)	The derivative

05:34	Kristen	S(a)	I can't hear him
05:34	Jason	S(ans)	Derivative
05:35	Dr. G	T(con-t)	You hear him now?
05:37	Kristen	S(ans)	Yeah
05:38	Dr. G	T(c)	What'd he say?
05:39	Kristen	S(ans)	Derivative
05:39	Dr. G	T(con-t)	Are we both okay?
05:41	Kristen		(nods yes)
05:45	Dr. G	T(s)	What is the derivative of this function?
04:49	Kristen	S(a)	What, what are you trying to say? Can you speak up please
05:51	Dr. G	T(f)	What is the derivative of this function?
05:56	Kristen	S(ta)	2x
05:57	Dr. G	T(p)	Write it please
06:01	Kristen	S(seek)	Do I have to show you how I got it? Or do I ?
06:03	Dr. G	T(con-s)	I think we understand how you got it
06:12	Dr. G	T(s)	Now over the interval 1 to 2 inclusive. Two x, is it positive or negative?
06:24	Kristen	S(a)	What was the question? Sorry
06:29	Dr. G	T(d)	Jason repeat the question
06:31	Jason	S(ans)	Is the derivative positive or negative?
06:33	Dr. G	T(f)	Over the interval
06:34	Jason	S(ans)	from one to two

06:37	Kristen	S(ans)	positive
06:38	Dr. G	T(j)	How do you know that?
06:51	Kristen	S(dnc)	Um, I have no clue. Just explain it to me.
06:53	Dr. G	T(d)	Eric, how do you know it's positive or how do you know it's negative?
06:58	Eric	S(pb)	Ah, I think because if you solve for x. and uh, I don't know it's positive
07:08	Dr. G	T(d)	Becca
07:09	Becca	S(pb)	Because the derivative is the slope, so it's like going up two and over one
07:20	Dr. G	T(d)	Matthew
07:21	Matthew	S(pb)	If you fill in any number between the interval between one and two, the answer is positive because graph is
07:31	Dr. G	T(f), T(p)	Graph y equals 2x between 1 and 2. Rather, I am going to change that slightly. Graph y equals 2 x over the interval 1 to 2 inclusive.
07:55	Dr. G	T(p)	On a new graph.
08:21	Dr. G	T(r)	Now, is the derivative always positive on that interval?
08:38	Kristen		Um
08:39	Dr. G	T(d)	Time out. Repeat the question Brittany
08:44	Brittany	S(ans)	Is that always positive in the interval?
08:47	Dr. G	T(c)	No pronouns
08:51	Brittany	S(c)	Is the derivative always positive on that interval?

08:53	Dr. G	T(d)	Kristen
08:55	Kristen	S(ans)	Yes
08:55	Dr. G	T(j)	How so? How is it we know this? How did you make that decision?
09:02	Dr. G	T(j)	By the look on your face that was a fifty-fifty guess type thing?
09:10	Kristen	S(dnc)	I don't, I don't, I have no clue
09:12	Dr. G	T(d),T(c)	Alright. Rachel, explain what we mean by the derivative being positive over that interval.
09:18	Rachel	S(pb)	Is it because of all the number that you put in for are going to be positive for x and when you multiply by 2, it is going to be positive
09:29	Kristen	S(a)	What?
09:30	Dr. G	T(rep)	Louder
09:31	Rachel	S(pb)	All the numbers you're putting in for x are positive. And when you multiply a positive by a positive you are going to get a positive.
09:41	Kristen	S(con)	okay
09:43	Dr. G	T(d)	Anyone want to say it another way?
09:47	Dr. G	T(f)	Is your graph of the derivative above or below the x axis?
09:53	Kristen	S(ans)	Above
09:53	Dr G	T(j)	Which means it's?
09:54	Kristen	S(ans)	Positive

09:58	Dr. G	T(r)	Which means our function is increasing or decreasing?
10:00	Kristen	S(ans)	Increasing
10:01	Dr. G	T(s)	Now, we have one more thing to talk about. We know it's increasing right. So, suppose I do this (draws a concave up function). And Rachel says, no it goes like this.(draws as concave down function) Do we have any way to know?
10:30	Kristen	S(dnc)	I am sure that's right, but I don't know
10:34	Dr. G	T(d)	Mikey
10:34	Mike	S(ans)	Second derivative
10:38	Dr. G	T(c)	What's the second derivative?
10:44	Kristen	S(ta)	Two
10:50	Dr. G	T(f)	So, now graph the second derivative over that interval
10:56	Kristen	S(seek)	On the same?
10:56	Dr. G	T(p)	Yep, and Chance is going to pick up for you from here. So why don't you rest for a minute
11:00	Dr. G	T(p)	New graph
11:01	Kristen		Huh, Ahh
11:03	Dr. G	T(p)	No no, Chance is going to pick it up for. Kristen, you can rest for a minute
11:05	Kristen	S(a)	Oh, I'm done
11:06	Dr. G		Yes (Chance gets up and goes to the board.)

11:09	Dr. G	T(p)	New graph. This time the second derivative. Same interval. (Chance draws a coordinate axis and the line y = 2.)
11:28	Dr. G	T(f)	Very good. Now is the second derivative positive or negative over this interval?
11:35	Chance	S(pb)	Well it has the slope of 0. So. So. I mean, I know it's positive, like about the x axis but
11:43	Dr. G	T(f)	Okay, Alright. Since it's positive. We know this because the graph exists totally above the x axis
11:51	Chance	S(con)	Right
11:51	Dr. G	T(f)	that tells us what about the original parent function?
11:57	Chance	S(ta)	That it was positive also
12:03	Dr. G	T(d)	Kelly
12:05	Kelly	S(ta)	It was concave up
12:06	Dr. G	T(con-s)	Concave up
12:08	Chance	S(seek)	Up. You want me to draw that?
12:10	Dr. G	T(con-s)	Yep.
12:13	Dr. G	T(s)	Alright. Now that is not the way we used to graph something. But that is the way I would like you to begin to think about some of these simpler graphs. It is a chance for you to review some of the basic calculus you've been studying. Alright, Chance. We are going to find the area where?

12:37	Chance	T(seek)	Oh, Between the 1 and 2. You want me to draw the rectangles?
12:42	Dr. G	T(r)	We are going to find the area of what region?
12:50	Chance	S(ans)	Oh, below the, below the curve
12:53	Dr. G	T(p)	Lightly shade the area.
13:04	Dr. G	T(p)	And I need two more boundary lines
13:07	Chance	S(con)	Two more boundary lines?
13:08	Dr. G		Uh huh
13:10	Chance	S(seek)	What do you mean by that? The dots that come down like that.
13:14	Dr. G	T(f)	How about just the line?
13:20	Chance	S(dnc)	I don't know
13:21	Dr. G	T(f)	How about a line perpendicular to something someplace?
13:27	Chance	S(seek)	That's perpendicular to this
13:28	Student	S(ans)	So make it there
13:30	Chance	S(con)	Okay
13:31	Dr. G		Ah, Ah, Ah
13:32	Chance		Uh oh
13:33	Dr. G	T(p)	Just what you need. From the point down. No sense in complicating things
13:35	Chance	S(con)	Oh Okay, I get it.
13:36	Dr. G	T(p)	One more
13:37	Chance	S(con)	Okay

13:39	Dr. G	T(p)	Alright, now Chance. We're going to estimate the area using four rectangles
13:46	Chance	S(con)	Okay
13:48	Dr. G	T(p)	You can go ahead and draw them if you want. Such that when I look at the first rectangle I am going to measure it's height on the right hand side. Go ahead.
14:29:16	Dr. G	T(e)	Now talk to me about the area of the first of these rectangles. You can erase those two graphs now because we are going to need a lot of board space.
14:58:02	Chance	S(seek)	Plot the first one.
14:59:01	Dr. G	T(con-s)	Yep
14:59:22	Chance	S(pb)	Okay well the width is one-fourth cause there's four rectangles. and then. So each of them have a width of one-fourth and then, um. The height is two.
15:13:20	Dr. G	T(j)	How do we know this?
15:15:19	Chance	S(ta)	Well actually that might not even be true. That's not.
15:17:22	Dr. G	T(con-s)	Okay, so you lied to me.
15:20:05	Chance	S(con)	Yes I was lying
15:21:05	Dr. G		Not Nice
15:21:29	Chance		Sorry
15:22:23	Dr. G		Alright
15:23:20	Chance	S(seek)	Okay, um, what else do you want me to say?
15:28:12	Dr. G	T(r)	How do you find the area of that first rectangle you drew?

15:34:07 Chance	S(seek)	Okay, um, using like summation.
15:38:15 Dr. G	T(f)	Using
15:40:00 Chance	S(ta)	Oh, okay, okay. Um, Okay, well you know that the width. Okay. Area equals length times width so um, Wait you know the width is one-fourth and then. Your starting point, I mean not your starting point, but like.
Chance	S(seek)	How do I have to do that?
16:02:28 Dr. G	T(con-s)	That's okay for you to say
16:05:11 Chance	S(ta)	Okay so the starting point is one. So you do one plus the. Oh God. Okay wait. Hold on. I am trying to remember, um. Length equals. I can't remember. I can't remember what you add.
16:29:10 Dr. G	T(d)	Maureen, can you give her a clue.
16:34:05 Maureen	S(ans)	Like, one times one-fourth
16:38:16 Chance	S(con)	So wait, one plus one times one-fourth.
16:40:16 Maureen	S(con)	Yeah
16:41:04 Dr. G	T(j)	Now why is that Chance?
16:42:19 Chance	S(dnc)	I don't know I was about to ask her the same question.
16:44:14 Dr. G	T(d)	uh huh, Maureen.
16:46:16 Maureen	S(pb)	Cause you go the difference, um
16:49:26 Chance	S(ta)	Oh yeah duh, I am having memory loss. Okay.

17:00:09 Dr. G	T(e)	Now. That, slide over. This piece right here. Tell me what that represents in terms of this graph. (pointing to 1+1(1/4))
17:13:01 Chance	S(ans)	the length
17:20:25 Dr. G	T(d)	Charlie, what do you think this represents? (points to 1+ 1(1/4) again)
17:26:17 Charlie	S(ans)	The area of the first rectangle
17:31:01 Dr. G	T(d)	Anita
17:33:12 Anita	S(dnc)	Uh.
17:33:23 Dr. G	T(con-s)	Now so far we got the height of the first rectangle, the area of the first rectangle.
17:40:21 Anita	S(ans)	The width.
17:41:29 Dr. G	T(d)	The width of the first rectangle. Now Maureen wants to take it back and start over.
17:47:00 Maureen	S(ans)	A value of x
17:48:03 Dr. G	T(con-s)	A value of x is all it represents. A particular value of x. Where is that particular value of x,
Dr. G	T(d)	Chance?
17:57:24 Chance	S(ans)	Um, this. (Points to entire right side of first rectangle.)
18:02:22 Dr. G	T(d)	Maureen.
18:03:21 Maureen	S(ans)	It's at the bottom.
18:05:14 Chance		Oh yeah, cause you wanna.

18:06:24 Dr. G	T(e)	Point to that place on the x axis. Okay. Now notice how easy it was to mix up the words. Area, height, value of x, you can't just use these words interchangeably. You gotta think about the words you're gonna use. So, Chance, if we take that value of x, which you said was right here (points to place on x axis Chance just pointed to) and I wanna know how high this rectangle is, what do I do with that value of x to know how high this is?
18:49:01 Chance	S(ta)	You add it the the x, x variable.
18:51:25 Dr. G	T(d)	Andrew, what do I do (Interrupted by someone coming into class. Responds to the person) Andrew, what do I do?
19:05:04 Andrew	S(ta)	Uh. Multiply by the one-fourth. Multiply it by the one-fourth.
19:19:29 Dr. G	T(j)	Why?
19:21:20 Andrew	S(pb)	Cause one-fourth is supposed to represent the width
19:23:23 Dr. G	T(rep)	Louder.
19:25:11 Andrew	S(c)	One-fourth represents the width of the rectangle.
19:28:07 Dr. G	T(f)	So, if I multiply it by one-fourth and the one-fourth is the width, you're telling me that is the.
19:39:15 Andrew	S(ans)	the height.
19:41:28 Maureen		(inaudible)
19:43:05 Dr. G	T(rep)	I don't think he heard you.

19:45:22 Maureen	S(ans)	It's a value of x.
19:47:21 Dr. G	T(f), T(d)	It's a value of x that does what Andrew.
19:59:17 Dr. G	T(d)	That does what, Nick
20:01:22 Nick	S(ans)	Moves quarterly.
20:04:18 Dr. G	T(rep)	Louder.
20:04:22 Nick	S(c)	Moves quarterly to the right.
20:09:13 Dr. G	T(con-s)	It's a value of x that moves quarterly to the right. There's some truth in that.
Dr. G	T(d)	Michael
20:17:22 Michael	S(dnc)	I don't know what it does.
20:20:02 Dr. G	T(d)	Richi
20:21:26 Richi	S(ans)	You have to put it into the function.
20:23:21 Dr. G	T(c)	To do what Richi.
20:28:01 Richi	S(ans)	You put it into the function $x^2 + 1$
20:30:02 Chance	S(con)	Oh okay.
20:30:17 Dr. G	T(c)	Because it is that function which produces
20:33:28 Chance	S(ans)	the value
20:35:02 Dr. G	T(f)	the y value here, which is the same as the height of the rectangle. So.
20:40:27 Chance	S(con)	Okay
20:41:08 Dr. G	T(p)	Whoop, whoop, whoop, whoop. Just up there.
20:44:12 Chance	S(seek)	What do you mean, just like.
20:49:14 Dr. G	T(p)	We're gonna take this off for a second.

20:51:20 Chance		um hmm.
20:52:02 Dr. G	T(p)	This is going to be the area of the first rectangle.
20:55:27 Chance	S(con)	Okay.
20:56:08 Dr. G	T(r)	So what do you have to do yet?
20:59:12 Chance	S(seek)	Well you want to plug it into the x right?
21:01:22 Dr. G	T(con-s)	Uh huh.
21:03:03 Chance	S(seek)	Okay, this all done.
21:04:03 Dr. G	T(p)	Unt, uh. Same line, just edit.
21:11:22 Chance		Oh.
21:14:19 Dr. G	T(d)	Aright Andrew, I want to you look at that very carefully and tell me if you think you like that answer.
21:31:08 Andrew	S(ans)	Yeah, I think so
21:32:15 Dr. G	T(d)	Mikey.
21:33:03 Michael	S(ta)	No, there's gotta be a parentheses around the point, around the outside one.
21:37:25 Dr. G		Wait she's getting nervous up there.
21:40:02 Chance		No, I wasn't getting nervous. I just.
21:42:17 Dr. G	T(c)	So, Mikey what do you want?
21:44:05 Michael	S(ta)	There's gotta be a parentheses after the one-fourth on the left and at the. No. After
21:50:06 Chance		Oh.
21:50:17 Michael	S(ta)	and at the end of the one.
21:52:25 Chance		At the.

21:53:17 Michael		At the end of the.
21:54:26 Chance		two.
21:55:24 Michael		No at the end.
21:57:05 Chance		Oh, Okay.
21:58:09 Dr. G	T(c)	Now, Chance, using your hands as parenthesis. Okay. Give me a value of x that I use as input for the function (Student points to 1+1(1/4)). Good. Give me the height as output of the function.
22:21:05 Chance	S(ans)	Um, this. (Points to $((1+(1/4))^2 + 1)$
22:24:15 Dr. G	T(c)	And give me the width, which is constant. (Points to 1/4). Very nicely done Chance, put a plus sign after that. Amanda you got the second rectangle. There's only one Amanda left. Yeah the other one snuck out.
23:25:19 Dr. G	T(e)	[Student writes on the board $1/4((1+2(1/4))^2 + 1)$] Alright talk to me Amanda.
23:28:14 Amanda	S(a)	About what.
23:29:29 Dr. G	T(f)	What you wrote.
23:31:08 Amanda	S(ta)	Alright, this is the width [points to 1/4], that's the starting point [points to first 1 in the parenthesis], this is two widths away from the starting point [points to the 2].
23:39:06 Dr. G	T(c)	Show me where that particular value of x lies on the graph [Student points to the x axis where the right side of the second rectangle is]. Good. Now we're going to do

the hand thing. Your hands are parenthesis. Are you ready?

23:49:29 Amanda   S(ans)   Yeah.

23:50:21 Dr. G   T(c)   The constant width[points to 1/4] The value of x we use as input into the function.[points to (1+2(1/4))]. The output of the function equivalent to the height of the second rectangle.[points to ((1+2(1/4))^2+1)] Very nicely done. Finish that whole line while you're up there Amanda. [Student writes: 1/4((1 + 3(1/4))^2)+1) + 1/4((1+4(1/4)^2)+1)]

24:53   Dr. G   T(s)   Alright Amanda. Assuming you did all that already. Would our estimation of the area be too small or too large?

         Dr. G   T(j)   And Why?

25:06   Amanda   S(ans)   Um, too large

25:10   Dr G   T(j)   Too Large. Tell me why?

25:15   Amanda   S(pb)   Because these are higher. [points to rectangle on graph.]

25:19   Dr. G   T(s)   So, we're going to go ahead and do these four and get an idea, a rough idea, for what the area is. Then we'll rework the same situation for an infinitely large number of rectangles. So, thank you Amanda. Mikey, summarize that line, will you?

25:43   Mikey   S(a)   On the board?

25:56	Dr. G	T(r)	[Student writes =]. Whoop. Well I might have spoken too soon. I'll wait. Keep going. [Student writes a parenthesis.] No, that's a definite whoop. Summarize that line is the instruction, right?
26:12	Mikey	S(con)	Yep
26:13	Dr. G	T(p)	But give me an alternative summary. I don't like that summary. Give me an alternative summary.
26:18	Mikey	S(con)	Oh, Sum.
26:26	Dr. G.	T(con-s)	There you go Mikey.
26:35	Dr. G	T(p)	Now please, when you make these i's as trivial as this is going to sound, Put some curve to them, and put a nice dark dot on top of the i because in an earlier group this morning, on three different occasions, we tripped over it as a one, and it caused problems.
27:01	Mikey	S(seek)	What do I do? Do I just put the equation in now?
27:04	Dr. G	T(e)	I don't know. You'll have to explain how this works. I forgot.
27:08	Mikey	S(con)	Yeah, me too.
27:29	Mikey	S(qs)	Someone want to help me out here. Yeah Kelly
27:33	Kelly	S(ta)	If you look at all four terms on top, there's only one thing that changes.
27:41	Dr. G	T(f)	Hold it right there. Now Kristen, we need a helper. Walk up there. Kelly's going to say it again loud and clear and

you're going to point to the things she's talking about.

Kelly loud and clear.

27:52	Kelly	S(c)	In each term in the line above the sigma thing there is only on number that changes in each of them.
27:59	Dr. G	T(p)	Now point to them Kristen.
28:05	Dr. G	T(r)	Now what does that have to do with what you're trying to write Mikey?
28:11	Mikey	S(ans)	That'd be i.
28:12	Dr. G	T(con-s)	Good. Excellent
28:14	Mikey	S(seek)	Just substitute i.
28:16	Dr. G.		I don't know. Thank you Kristen
28:37	Mikey		No, duh
28:44	Dr. G	T(d)	Okay. Comments, critique. Rebecca
28:49	Rebecca	S(ta)	You can simplify that further because two plus i times parenthesis squared. You have two one's on either end and they can be put together
28:59	Dr. G	T(d)	Okay, now before we begin to do what Rebecca is suggesting are there any comments on what we see right now? Vinny.
29:06	Ryan		I'm Ryan
29:07	Dr. G		I'm sorry. Ryan

29:08	Ryan	S(pb)	I think just before you do what Rebecca said we should multiply the binomial first because you do exponents before you add and that's outside the parenthesis.
29:16	Dr. G	T(d)	What'd he say, Mikey?
29:18	Mikey	S(c)	Multiply the binomial first
29:21	Ryan	S(c)	Square it.
29:22	Dr. G	T(d)	Okay, now let's see what Amy wants.
29:24	Amy	S(pb)	I don't think you can do that because you have the parentheses, you're squaring the parenthesis, so you have to foil it first.
29:30	Ryan	S(con)	Well that's yeah,
29:33	Several Students	S(con)	What we just said
29:34	Amy		Oh, Sorry
29:37	Dr. G		I'm glad you apologized because you really riled them up there for a minute.
29:40	Amy		I thought Rebecca said to add the one plus one.
29:43	Ryan		No, that's what I just said
29:44	Amy		Oh, I'm sorry.
29:45	Dr. G		Okay. Here we go. Alright Mikey
29:47	Mikey	S(seek)	Can I start?
29:48	Dr. G	T(con-s)	Yep
29:51	Mikey	S(seek)	Can I make some room somewhere?

29:54	Dr. G	T(p)	Uh, if you want to do that part as an aside stick it up top there some place. Erase that stuff that I wrote.
30:05	Dr. G	T(p)	Now he's just going to do the multiplication as an aside and then he'll insert.
30:56	Dr. G	T(con-t)	[Student writes: $(i(1/4))^2 +2(i(1/4)+1]$ Gretchen? Everybody alright?
	Dr. G	T(p)	Keep going. Simplify it first. Up there.
31:07	Mikey	S(seek)	Where?
32:57	Dr. G	T(con-t)	Everybody alright?
33:01			Several students mumble
33:05	Dr. G	T(d)	Molnar. Talk to him
33:10	Molnar	S(ta)	It says i squared times one over six, it should be one over 16.
33:19	Mikey	S(qs)	Would it be easier to write it the other way?
33:25	Molnar	S(c)	If you want to write it another way, you can just put i squared over 16.
33:29	Dr. G	T(d)	Wait a minute, Rachel
33:31	Rachel	S(ta)	It says at the end one half i shouldn't there be another one at the end
33:37	Student	S(ans)	There's a plus sign
33:39	Rachel	S(qs)	Isn't it supposed to be plus one?
33:41	Student	S(ans)	No that was part of the (inaudible)
33:45	Kristen	S(con)	You mean there

33:46	Dr. G	T(p)	Walk up here Rachel. Show him. [Rachel goes to the board and points at the plus sign from Mikey's line above and then points to the line he has rewritten at the bottom of the board.]
33:59	Dr. G	T(con-t)	Alright. Now everybody alright. Ricci next line.
34:06	Ricci	S(ans)	Uh, you could simplify like the
34:10	Dr. G	T(p)	Like, uh, walk up there and write the next line. Thank you Mikey.
35:12	Dr. G	T(e)	Can you do anything with the expression to further simplify it on that line?
35:21	Ricci	S(seek)	Do I even need this parenthesis (points to last parenthesis in expression) Can I just combine those two? (point to the one's at the end of the expression)
35:25	Dr. G	T(c)	What do you think?
35:27	Ricci	S(ans)	Yes
35:28	Dr. G	T(j)	Why? What purpose do they serve?
35:33	Ricci	S(pb)	I started thinking of it as numbers
35:35	Dr. G	T(rep)	Louder
35:36	Ricci	S(pb)	I started thinking of them as numbers not as like having purposes. So, I forgot what
35:41	Dr. G	T(con-t)	Let me get this straight. You're thinking of the parenthesis as a number that has no purpose.

35:50	Ricci	S(ta)	Like instead of a like, like a length and such. Can I just combine them?
35:57	Eric	S(ans)	I think you can
35:58	Student	S(ans)	Just don't call them numbers
36:00	Dr. G	T(c)	The parenthesis that begins. The first parenthesis after sigma. Point to it. Now. Where is the matching parenthesis for that?
36:15	Ricci	S(seek)	Oh interesting. Can I just combine those two? [Erases last) and then 1) + 1 and writes 2.]
36:23	Dr. G	T(f)	You didn't answer my question. You continue to erase stuff (inaudible, student laughing)
36:31	Dr. G	T(p)	Alright now, one more line
36:36	Ricci	S(ans)	Um, that's not going to happen. Alright, um
36:48	Dr. G	T(s)	Suppose I said I want you to factor the fraction 1/16th and write this common factor to the left of sigma.
37:06	Ricci	S(con)	Wait. You're saying 1/16th what?
37:08	Dr. G	T(e)	I want to get a common factor as the fraction 1/16th and somehow move it to the left of the sigma notation. How does that happen?
38:11	Dr. G	T(e)	Now explain how that happened Ricci
38:14	Ricci	S(ta)	Uh, I did 1/16th times 2. Well, not actually, I did. I figured if 1/16th was out here to get 1/2, it'd have to be 1/32

38:33	Dr. G	T(c)	Show me how that worked?
38:56	Ricci	S(seek)	Like that
38:59	Dr. G	T(c)	Well, I am still waiting for you to show me how it works
39:04	Ricci	S(ta)	Oh. Okay. Oh, that's not it at all. Um, Someone
39:16	Dr. G	T(c)	someone what?
39:19	Ricci	S(dnc)	I hate simple math. Man, it's hot in here.
39:34	Dr. G	T(con-s)	Yes.
39:39	Ricci	S(ans)	That's the one for this
39:41	Dr. G	T(d), T(con-t)	Alright now. Uh, Rajeesh. You understand?
39:51	Rajeesh	S(qs)	I am wondering what?
39:52	Dr. G	T(c)	You're wondering what he's doing right? He trying to factor 1/16th from 1/2. How's that work?
40:02	Rajeesh	S(ans)	Take 1/16th and um factor
40:07	Ricci	S(qs)	What? I can't hear what you're saying?
40:15	Rajeesh	S(ans)	Using that equation, the 1/16th
40:17	Ricci	S(ans)	Yeah
40:18	Rajeesh	S(ans)	on the last line
40:24	Ricci	S(qs)	Where?
40:26	Rajeesh	S(ans)	No
40:28	Ricci	S(qs)	Here?
40:29	Rajeesh	S(ans)	Take a common factor out
40:32	Ricci	S(qs)	Common factor of what 4 and 1/4th?
40:36	Rajeesh	S(ans)	Multiply the one over two by that.

40:44	Dr G	T(p)	Don't give this away. Let these two work for a minute
40:48	Ricci	S(qs)	I still can't hear him. Multiply the something?
40:53	Rajeesh	S(ans)	Multiply 16 times 1 over 2 that side.
40:58	Ricci	S(a)	Multiply 16, times 16
41:02	Dr. G	T(p)	Don't give this away ladies and gentlemen.
41:07	Ricci	S(ans)	Eight.
41:09	Dr. G	T(p)	Don't look at me. He's talking to you.
41:10	Rajeesh	S(seek)	Can I go up there?
41:12	Dr. G	T(con-s)	Yes.
41:13	Ricci		Sorry.
41:17	Dr. G	T(e)	Rajeesh. By the way. When your done with whatever you're going to do up there. He still has to explain this to us.
41:23	Ricci	S(ans)	Yeah
41:24	Rajeesh	S(ta)	Okay. Take 16 out. (Inaudible.)
41:33	Ricci	S(ans)	Oh, I was thinking about that. I wasn't sure.
41:40	Rajeesh	S(ta)	over here. 32i. 32i
41:46	Ricci	S(qs)	Why is it 32i?
42:48	Rajeesh	S(pb)	You multiply by 16
42:50	Ricci/Rajeesh		[Whispering to each other]
42:08	Dr. G	T(e)	Now how did that happen?
42:09	Ricci	S(dnc)	Huh
42:11	Dr. G	T(e)	How did that happen?

42:12	Ricci	S(pb)	I multiplied by 16 because that's what I just took out. [Writes + 32)] Basically, he told me that because I'm taking out the sixteenth and I already have the fourth out that has to be come one over 64. And then I have the uh 16 here so that 1/2 times 16 is 8 and uh 16 times 2.
42:38	Dr. G	T(c)	So, what was wrong with that equation you wrote over there?
42:42	Ricci	S(ans)	Oh, I don't know. Nothing
42:48	Dr. G	T(d)	Question, Chris
42:51	Chris	S(ta)	If your multiplying by 16 on the right side shouldn't you multiply..
42:56	Ricci	S(pb)	Well, it's not an equation, but it's still in the thing. We already took one fourth out here.
43:03	Chris	S(con)	Yeah
43:04	Ricci	S(pb)	So now to get to the 64 all we're doing is taking the one-sixteenth out.
43:08	Chris	S(qs)	I understand that. By multiplying both sides by 1/16th right?
43:12	Dr. G	T(c)	Both sides of what Chris?
43:21	Ricci	S(ans)	It's not an equation.
43:24	Chris		Oh.
43:25	Dr. G	T(p)	Ricci, for one second, split the 1/64th into the 1/4th times 1/16th.

43:32	Ricci	S(a)	Say that again.
43:34	Dr. G	T(p)	Split the 1/64th into 1/4th times 1/16th.
43:38	Ricci		Oh.
43:50	Dr. G	T(p)	Now. Point for me. 1/16th on the left. Times one on the right.
44:00	Ricci	S(a)	What?
44:01	Dr. G	T(con-s)	Coefficient of i squared is 1/16th. Correct, Chris?
44:06	Chris	S(ans)	Oh, I didn't see that.
44:08	Dr. G	T(con-t)	Second one would be 8/16th or 1/2. The third one would be 32/16th or 2. Okay. Is everybody alright with this? Alright. Thank you. Ryan next line.
44:23	Ryan	S(seek)	Next line. Isn't that simplified?
44:29	Ricci	S(a)	Is that me still?
44:32	Dr. G	T(p)	Ryan, next line.
44:33	Ryan	S(seek)	This next line isn't that simplified?
44:35	Dr. G	T(s)	No, we're just getting started.
44:40	Ryan	S(qs)	Oh. [Student goes to board]. Hey buddy, help me out? [to student]
44:46	Dr. G	T(e)	Oh, How about. Do you have any ideas?
44:51	Ryan	S(dnc)	Not at this point.
44:52	Dr. G	T(d)	Anybody have any ideas? Chris?

00:16	Chris	S(ta)	Well, I am assuming one of them can be removed. Is there any way we can take the number 32 out? I'm not sure.
00:31	Dr. G	T(s)	Suppose, Suppose. Instead of using his one sigma, we rewrite this using three sigmas.
00:41	Ryan	S(a)	You'll have to explain that.
00:46	Dr. G	T(p)	Suppose. [walks to board] We deal with this, then we say plus this.
00:57	Ryan	S(con)	Ok.
00:57	Dr. G	T(p)	and you put, [points to sigma] and then [points to last term].
01:06	Ryan	S(seek)	So, So it would just be like.
01:16	Kelly		Ryan
01:17	Dr. G	T(d)	Wait a minute. Kelly
01:18	Kelly		Can you write a bit darker?
01:19	Ryan		Darker.
01:43	Dr. G	T(p)	uh, uh, Times 8. Put the 8 out there too.
01:49	Ryan	S(con)	Uh, where?
01:50	Dr. G	T(p)	With the 1/64th.
01:52	Ryan	S(seek)	Here?
01:54	Dr. G	T(con-s)	Yep.
01:55	Ryan	S(con)	oh, okay.

01:56	Dr. G	T(d)	Otherwise known as and then you have a question from Chance.
01:56	Ryan	S(ta)	Otherwise known as [Student writes 1/8 sigma i]
02:15	Ryan	S(qs)	Okay, Chance you have question
02:16	Chance	S(qs)	Yeah, um, what your doing know. What's the point of this?
02:20	Ryan	S(ta)	What's the? I don't know what the point is. I would think that would be simpler, but
02:24	Chance	S(ans)	Wait, so like
02:26	Dr. G	T(s)	Oh, Okay. Hang on a second. This version is no calculator what so ever.
02:32	Chance	S(qs)	Okay so. How did you know to bring the 1/8th. Multiply like. Like what did you. Multiply by the reciprocal of eight.
02:42	Ryan	S(a)	What this?
02:43	Chance	S(c)	Yeah like how you have 1/8th
02:45	Ryan	S(pb)	I just brought the 8 out. Put 8 over 64. which is one eighth.
02:47	Chance	S(qs)	Why?
02:49	Ryan	S(ans)	Cause that's what he told me to do.
02:49	Chance	S(con)	Oh Okay
02:50	Ryan	S(pb)	Cause you can bring the eight over because it's a common factor.

02:53	Chance	S(con)	Oh Okay
02:53	Dr. G	T(d), T(j)	Does anybody know the why of that? Stephanie
02:56	Stephanie	S(pb)	Um, I think because 8 is a constant, but I'm not sure.
02:58	Ryan	S(pb)	Cause it's a common factor in all the summations. All the terms. It's uh, it's the common factor
03:11	Dr. G	T(con-t)	No one's recognizing this?
03:13	Amy	S(con)	What was the question?
03:15	Eric	S(ans)	How do you take the 8 out of?
03:16	Dr G	T(d)	Kelly
03:17	Kelly	S(dnc)	Um, I thought I had something, but I don't. Sorry
03:20	Dr. G	T(d)	Look at this one. Look at this one. Ronak.
03:24	Ronak	S(pb)	Well the i squared is from, is from the equation that we had over there. And we're basically separating them. You know how we had 1/ 64th. So we're just multiplying 1/8th over it.
03:39	Ryan	S(qs)	Shouldn't that be 8?
03:40	Stephanie	S(qs)	Are we getting all of the i's together?
03:42	Ryan	S(ans)	No
03:44	Dr. G	T(d)	Jason, where am I going with this?
03:50	Jason	S(ta)	Wait, isn't this induction or something?
03:53	Dr. G	T(d)	Rachel, where am I going with this?
03:54	Rachel	S(ta)	Is it cause like after you simplify it and eventually in the line you can just plug in one for i, two for i, and three

04:02	Dr. G	T(con-s)	You could.
04:05	Yla	S(ta)	Aren't you just distributing the one across the equation for us then we won't have to use our calculator to plug in the one, two, three and so on wherever i is
04:19	Dr. G	T(r)	Alright. We're going to finish the line and then restate the question. Do this for me (To student at board). And this time before we pose the question, why do we write it like this, think about the fact that suppose I had said we are going to use a 100 rectangles instead of 4. We want to look at this arithmetic, if you will, for a larger number of rectangles than 4.(Student finishes writing on the board) Now look at those and what are they?
05:02	Student	S(ta)	I was just going to say if you put a large number in for i, it going to be 8 i.
05:11	Dr. G	T(c)	So you're still convinced that we're going to do this as 1 squared, plus two squared, plus three. How are we going to do this?
05:19	Student	S(ans)	We still (inaudible)
05:20	Dr. G	T(rep)	Louder
05:21	Student	S(ans)	We still need the one over four
05:25	Dr. G	T(d)	October, What do we do?
05:29	October	S(ans)	Pascal's Triangle
05:33	Dr. G	T(d)	Rebecca

05:34	Rebecca	S(ta)	We use like those series things to find the n terms
05:38	Dr. G		You have formulas for all of this and you proved them by induction a few months ago.
05:43	Eric	S(ans)	I said there'd be induction.
05:44	Dr. G	T(f)	So that if this is a very large number you don't have to do one squared, plus two squared, plus three squared, up to one hundred squared. You don't have to do any of that. Now where are those formulas summarized?
06:01	Ricci	S(ans)	The chapter one of our handout
06:05	Dr. G	T(f)	In the calculus text, where are they summarized?
06:07	Students	S(ans)	Page 270
06:10	Dr. G	T(p)	Alright Ryan page 270 they say
06:20	Dr. G	T(d)	Ronak?
06:23	Ronak	S(ta)	Um, you know how you put 1/8 and 1/2 is that before you made 16 the common denominator? Is that how got it?
06:31	Ryan	S(a)	Wait, come again.
06:32	Dr. G	T(rep)	I am not sure I heard that either
06:36	Ronak	S(ans)	You now you have 1/8th and 1/2.
06:37	Ryan	S(ans)	Yeah
06:38	Ronak	S(qs)	Well I don't really know how you got that?
06:40	Dr. G	T(p)	Ladies and gentlemen. You don't have to shuffle around. It's not going to be that big a deal.

06:44	Ryan	S(pb)	Where I got that? Cause what I did was. Since 8 is a common factor in all the terms of the summation, you can just bring it out. And 8 over 64 is 1/8th.
06:54	Ronak	S(con)	Oh so, okay. So you're actually multiplying 8 times 1 over 64.
06:58	Ryan	S(con)	Yeah.
06:59	Ronak	S(con)	Okay.
07:03	Student		Ryan.
07:03	Ryan		Yeah.
07:04	Student	S(qs)	8 is the common factor of what?
07:06	Ryan	S(pb)	Is the, cause 8 plus one, um 8 i, because you know the summation would be 8i plus 8i plus 8i.
07:20	Student		Oh Okay.
07:21	Ryan	S(pb)	There's i's. it's a common factor in all the terms.
08:20	Dr. G	T(p)	All we're gonna do is substitute 4 in.
09:00	Dr. G		Be reminded that we did prove these a couple months ago and you are expected to know how to do that today.
09:40	Dr. G	T(d)	Question Chris
09:41	Chris	S(ans)	Ryan, it would be 1/2 times 4.
09:42	Ryan	S(ans)	Yeah.
09:45	Chris	S(ans)	You know
09:46	Ryan	S(ans)	Oh, yeah, I get it
09:47	Chris	S(ans)	1/2 times 4

09:54	Dr. G	T(p)	Alright, let's make it simpler.
10:43	Dr. G		Please be reminded that the thing I want you to be thinking about here is you are going back 20 or so years. You had to do this work without access to a simple calculator. This strategy allows you to simplify the arithmetic and actually perform the calculation. This certainly not the way we want to do it as a matter of routine. Keep going Ryan.
11:16	Ryan	S(seek)	Can I erase this?
11:18	Dr. G	T(con-s)	Yeah
11:28	Ryan		Oh boy
11:44	Dr. G	T(p)	Reduce it and keep going or something
11:46	Ryan		Yeah
11:46	Dr G.	T(p)	Try and get down to at least something reasonable
12:11	Ryan	S(seek)	Is that right?
12:13	Dr. G	T(p)	Keep going
12:19	Ryan	S(ta)	(to himself) 4, that's 2 plus a three
13:09	Dr. G	T(p)	One more
13:30	Dr. G	T(con-t)	Okay. Any questions? Thank you Ryan. Now we have a very rough estimate. Three and some ridiculous fraction. Now you know from your calculator work that the perfect, rather near perfect answer is three and one-third. So that's not bad, but now we're going to an infinitely

large number of rectangles, infinitely large number of rectangles. Instead of four, how many rectangles, Andrew?

14:11	Andrew	S(ans)	Uh, the next one. 10
14:20	Dr G.		I think that was a called strike three
	Students		Comments making fun of Andrew
14:26	Dr G	T(d)	Eric
14:27	Eric	S(ans)	An infinite
14:28	Dr. G	T(f)	An infinitely large number of rectangles.
14:30	Student		Andrew you're letting Eric
14:31	Eric		I wasn't paying attention
14:37	Dr. G		Charlie.
14:50	Dr. G	T(s)	This is a critical piece, but we want to change it to an infinitely large number. (pointing to previous work). So come up to here and start over.
15:01	Charlie	S(seek)	From here.
15:02	Dr. G	T(p)	That's something you'll probably want to look at so I wouldn't erase that just yet.
16:01	Dr. G	T(d)	Alright, now time out. Chance
16:03	Chance	S(ta)	Yeah okay. I just want to look back on that for a second. Okay. How come when I did it when I was simplifying I got like 13. Can you do it with 202 + 40/3 +16/32?
16:14	Ryan	S(ans)	118/32

16:16	Chance	S(ans)	No, 71 over 32, but isn't it.
16:17	Student	S(ans)	It's 64 over 32.
16:22	Chance	S(ans)	Oh duh, sorry.
16:23	Dr. G	T(c)	Okay. Now. Look at the language that Charlie is using. Since I said an infinitely large number of rectangles he simply inserted infinity for the number of rectangles. Now intuitively that seems to make very good sense, but formally we don't like the language that way. How do we write it Charlie?
16:53	Charlie	S(ans)	Limit
16:54	Dr. G	T(con-s)	Good
17:02	Dr. G	T(c)	Now be careful. As what approaches infinity? There you. Ah, as what approaches infinity? We didn't use an x there, what did we use? only because it corresponds to the formula. What did we use?
17:18	Charlie	S(ans)	i
17:22	Dr. G	T(d)	Carl, What'd we use?
17:23	Carl	S(ans)	n.
17:24	Dr G	T(con-s)	n. Because that is what we used in those formulas. It makes an easy connection.
	Dr G	T(d)	Yla Question?
17:31	Yla	S(ta)	Um, is it (inaudible) or was it from (inaudible)
17:36	Dr. G		I don't know, he erased it.

17:39	Charlie	S(a)	What did I do?
17:40	Yla	S(ta)	I thought that. You didn't put the parenthesis around the first one and after the fraction right before the square.
17:50	Dr. G	T(p)	It's a moot point unless you put it back.
17:52	Charlie		Alright.
17:54	Dr. G	T(p)	Put it back.
17:55	Charlie	S(a)	I got to write it over
17:57	Dr. G	T(con-s)	Yes
17:59	Charlie	S(ans)	I don't even know what I wrote.
18:28	Dr. G	T(con-t)	Now did he get it this time Yla or not?
18:32	Yla	S(ans)	No.
18:35	Eric	S(ans)	You got too many.
18:41	Yla	S(ta)	You have too many and you still didn't do that infinity properly.
18:49	Charlie	S(qs)	What?
18:54	Yla	S(ans)	Put one after the square.
18:56	Student	S(ans)	This is wrong anyway.
18:57	Charlie	S(con)	After it.
18:58	Yla	S(con)	Yeah.
19:00	Dr. G	T(f)	Now, what's a matter with it, Yla?
19:04	Eric	S(ta)	I don't know what the hell he did. Students saying "He still has too many parenthesis." and other students responding, "Yes and No"

19:10	Eric	S(ta)	You still got too many parenthesis
19:10	Students	S(ans)	No he doesn't
19:11	Eric	S(ans)	Yeah he does
19:12	Students	S(ans)	No he doesn't
19:12	Eric	S(ans)	Yeah he does
19:13	Students	S(ans)	No
19:13	Eric	S(ans)	Yeah he does
19:15	Dr. G	T(p)	Eric, ease up. Now, what's the problem. Go ahead
19:18	Charlie	S(ans)	Yeah I do
19:19	Yla	S(qs)	Charlie, is that a one or is that a parenthesis?
19:23	Eric	S(ta)	That's what I was about to go up there and point
19:23	Charlie	S(ans)	That's a parenthesis
19:27	Eric		Now I can sit down and say I was right
19:30	Michelle		Oh my God
19:32	Dr. G	T(s)	Now be reminded that that is not the language I want you to use. No matter how you perfect the parenthesis. That's not the issue here. So leave that there as a reminder and in your notes you might want to say that is not the correct language. Yes.
19:56	Rebecca	S(ta)	So uh, infinity should be n
19:59	Dr. G	T(rep)	Louder
20:00	Rebecca	S(seek)	Should the infinity be n
20:03	Charlie	S(ans)	Yeah right here

20:04	Dr. G	T(p)	That's what he's doing on the next line
20:05	Rebecca	S(con)	Okay
20:25	Dr. G	T(con-t)	Everybody alright.
	Dr. G	T(d)	Now you begin the process again and a couple of you have already noted. Wait a minute we have a couple of questions.
20:36	Eric	S(ta)	Uh, is the squared supposed to be inside or outside that parenthesis
20:41	Charlie	S(con), S(qs)	Didn't you guys say this was right?
20:43	Rebecca	S(ans)	It's wrong
20:46	Several students	S(ans)	It's wrong
20:48	Eric	S(ta)	Look down there you can see where the two is, where the square is. Look down. See where the square is.
21:01	Dr. G	T(p)	Charlie you're going to have to fix the one above.
21:06	Charlie	S(ans)	I hate parenthesis.
21:08	Dr. G	T(f)	Now, what do you hate Charlie?
21:10	Charlie	S(ans)	Nothing.
21:16	Kristen	S(ans)	You're missing parenthesis at the end.
21:21	Dr. G	T(con-t)	Now, everybody content? Alright. What you're going to see is a rather vivid comparison here. The process is going to look very much like what you just did until you get to the end. So, let's begin.

23:01	Dr. G	T(r)	Now since we have. You keep right on going Charlie. Since we have increased the number of rectangles to infinitely large number of rectangles. What should the answer come out to be this time Andrew?
23:25	Andrew	S(a)	Say that one more time, I was trying to get this down.
23:29	Dr. G	T(r)	Since we have increased the number of rectangles to an infinitely large number of rectangles. We're not going to get three and that ridiculous fraction we had a minute ago. We're going to get three and what?
23:46	Andrew	S(ans)	one-third.
23:47	Dr. G	T(j)	one-third. How do you know this?
23:56	Dr. G	T(j)	How do you know it should be three and one-third?
24:16	Dr. G	T(d)	Matthew.
24:17	Matthew	S(pb)	I was going to say, we did it on the calculator yesterday and it came through whatever the process and that's what it was.
24:26	Dr. G	T(d)	There's one piece of evidence. The calculator told you it should. Now supposed you didn't have that little trick. How would you come up with three and one-third. Rachel?
24:40	Rachel	S(pb)	When we made that graph yesterday with the x and y coordinates. It like started to go down to the number that we're looking for and then it kind of stays.

24:47	Dr. G	T(d)	What'd she say Andrew?
24:49	Andrew	S(dnc)	[Shrugs shoulders]
24:50	Dr. G	T(rep)	Yeah, I didn't hear it either. I want you to say it loud enough that Andrew can hear it.
24:55	Rachel	S(pb)	When we did it yesterday with the other equation and we did like the x and y axis and like we put an x and got y it started to go down and down until it was like the same number each time.
25:06	Dr. G	T(f)	So if we did for ten rectangles, fifty rectangles, one-hundred rectangles, five-hundred rectangles, etcetera and you looked at the estimated area each time.
	Dr. G	T(d)	The limit of that estimation would be what Andrew?
25:25	Andrew	S(ans)	Three and one-third.
25:26	Dr. G	T(con-s)	Three and one-third. Therefore in this abstract this limit better come out to be three and one-third.
25:44	Charlie	S(seek)	I think I'm stuck.
25:45	Dr. G	T(d)	Alright, Charlie's stuck. Ronak.
25:48	Ronak	S(ta)	Um, Take a common denominator like we did on the other one. Take a common denominator of n squared.
25:57	Charlie	S(con)	Alright. So we're taking that out.
25:59	Ronak	S(pb)	No, just like, first make a common denominator of n squared.

26:03	Dr. G	T(c), T(d)	Time out. Before you do that. Technical error. Anybody see it. Nick.
26:11	Nick	S(ans)	Parenthesis.
26:12	Dr. G	T(con-s)	Parenthesis. Charlie's favorite things.
26:19	Charlie	S(qs)	What did you say Ronak?
26:21	Ronak	S(ans)	Common denominator.
26:24	Charlie	S(con), S(qs)	You mean this?
26:25	Ronak	S(ta)	No the other side. Yeah that. Get the common denominator for all the terms.
26:38	Charlie	S(ans)	You're confusing me.
26:40	Ronak	S(ta)	Alright, uh. Like you see where the two is there.
26:44	Charlie	S(con)	Yeah
26:44	Ronak	S(ta)	Well you have the two. You can multiple the two by n squared over n squared and you'll have a common denominator of n squared.
26:58	Dr. G	T(d)	Question, Stephanie.
27:00	Stephanie	S(ta)	Um, I don't know if it would be easier, but yesterday we substituted the numbers, the summation formula numbers for like n times the quantity n plus one and then whatever over six. We put that in for n after we got to a certain point, and I think it would be easier to put those in now and then simplify it than to do the whole common

denominator thing. Because then you'll have n's all over

the place.

27:37	Ronak	S(ta)	But then after you get a common denominator you can move it to the outside.
27:42	Stephanie	S(ans)	I don't know it was just a thought.
27:45	Dr. G	T(f)	Charlie, you got the chalk. You decide.
27:51	Charlie	S(ans)	I think I'm going to go with Ronak.
28:02	Dr. G	T(d)	So, Charlie decided to do. Okay I got it. And Wendy, Kelly. Sorry. Kelly
28:12	Kelly	S(ta)	The first term you always have parenthesis.
28:17	Charlie	S(con)	Right here.
28:19	Kelly	S(ans)	Okay. Oh you said that. I take that back and sincerely apologize.
28:22	Dr. G		She takes and sincerely apologizes.
28:26	Kelly		Yeah, no. I was being Wendy.
28:28	Students		Oooh.
28:31	Eric		Best friends. Fight.
28:31	Kelly		I was just kidding.
28:58	Ronak	S(ta)	Charlie. Don't multiply the whole thing by n squared.
29:05	Charlie	S(qs)	What?
29:06	Ronak	S(ta)	Don't multiply the whole thing by n squared.
29:08	Charlie	S(qs)	What do you want me to do just the first one?
29:11	Ronak	S(con)	Yeah, cause yeah.

29:13	Charlie	S(con)	Alright.
29:14	Ronak	S(ta)	And the second one you have to multiply by n.
29:45	Dr. G	T(d)	Rachel.
29:46	Rachel	S(a)	He's trying to get n squared out right. That's what he's trying to do.
29:51	Ronak	S(con)	Yeah.
29:52	Rachel	S(qs)	So why he do. Nevermind. So why is there an n squared on the second one?
30:03	Dr. G	T(c)	What's the question Charlie?
30:04	Charlie	S(dnc)	I don't know.
30:05	Rachel	S(dnc)	Nevermind.
30:05	Dr. G	T(d)	Rachel again.
30:06	Rachel	S(dnc)	Nevermind.
30:07	Dr. G	T(p)	Alright. Charlie, doesn't the third one already have a denominator of n squared. Now we need another set of parenthesis. [Teacher assigns homework problems and bell rings for end of class.]

OBHS_18Apr02_1a&b

| 00:00 | Dr G | | $F(x) = 3x^2 + 2x$ and gives directions to graph the derivative of the function over the interval [1, 3] without using a calculator. Students are instructed to use the first |

and second derivative in order to make the graph.

Students work independently on this task.

06:05	Dr. G	T(p)	Alright Michael. I want you to start with two points that belong to the function we are, the function lower case f, two points.
06:23	Michael	S(a)	the function you want is?
06:25	Dr. G		Hmm
06:26	Michael	S(ans)	Um, I not sure what the parent function is
06:30	Dr. G	T(p)	The parent function is upper case F, the derivative is lower case f. I need two points that belong to the derivative function because that's where we're concerned about area under that curve.
06:52	Michael	S(seek)	You want me to go up there?
06:52	Dr. G	T(con-s)	Yep
07:34	Michael	S(seek)	Is that what you want?
07:41	Dr. G	T(d)	Yep, that's what I want, but now I realize something else. Eric, question.
07:51	Eric	S(ta)	Um, you should probably do uh point number three also and then that's another point because that's where you're graphing over one to three.
08:04	Dr. G	T(con-s)	That's a good idea.
08:13	Dr. G	T(d)	Question.

08:13	Chris	S(ta)	Um, isn't this going to be a linear function because well, are we going to estimate the area the usual way?
08:25	Dr. G	T(c)	I don't understand the question.
08:28	Chris	S(c)	Well the function is a linear function right, the derivative?
08:32	Dr. G	T(con-s)	The derivative is a linear function, I agree.
08:34	Chris	S(ta)	Okay, then uh, like, perhaps I'm jumping too far ahead, but wouldn't it be simpler if we just used the area of a triangle and added the area of a rectangle on to it.
08:48	Dr. G	T(f)	Could be. Let's draw the graph.
09:46	Dr. G	T(r)	Alright now. We are attempting to find the area under that curve. Now, what is it you said about that region, Chris?
10:06	Chris	S(c)	Um, you could simply take the area and split it up into two separate parts. One a right triangle.
10:19	Dr. G	T(f)	Go show us this stuff. Thank you, Mike
10:23	Eric	S(ta)	This is a good way to find the area beforehand and then we can do all that other crap.
10:29	Chris	S(ta)	Yeah, exactly, yeah. We could split this area up into two separate parts. A right triangle, represented her and a rectangle represented here.
	Chris	S(seek)	Okay, so um. So, is there something specific you want me to explain?

11:03	Dr. G	T(r)	I would like to know the area under the curve.
11:06	Chris	S(con)	Okay.
11:07	Dr. G	T(e)	That is specifically what I would like you to explain.
11:11	Chris	S(pb)	Alright, so the, the rectangle, the area. We have to find the areas of the triangle and the rectangle. The area of the rectangle will be derived from the width of the rectangle times the height of the rectangle. Okay? So, we know the rectangle is, has a width of two and we know the height of the rectangle is four.
11:18	Student	S(ans)	Eight
11:49	Chris	S(pb)	Eight. I didn't see the scale. Eight. And so, the uh. The area of the rectangle equals the length, is two, and that's times by the height of the rectangle, which is 8. So the area of the rectangle would then be 16. Then we need to find the area of the triangle. Uh, the, the formula for the area of a triangle is one-half b, h. The base of the uh triangle has a width of two, which is the same thing as the rectangle. So in the formula I'm going to put a 2 here. And as for the height, we know that since this point here is 20, I'm assuming, and this is eight. Well that would mean that the height would be twenty minus eight or twelve. So the height is twelve and the resulting area would be one-half times twelve times two, which is

twelve. And then the final step on the area of the shaded region would be to add the areas of the rectangle and the triangle. (Writes 12 + 16). Which is twenty-eight.

13:42	Dr. G	T(con-t)	Questions so far. Thank you Chris.
	Dr. G	T(p)	Now Charlie, this Charlie. I want you to evaluate something for me.
			Charlie writes $3(3)^2-2(3)-3(1)^2+2(1)$
14:37	Dr. G	T(c)	Charlie. I want you to evaluate a little better.
14:44	Charlie	S(seek)	You mean for here.
14:48	Dr. G	T(p)	Charlie. I'd like you to write line two with some improvement.
14:54	Charlie	S(con)	Okay.
14:59	Dr. G	T(p)	Don't erase anything.
15:02	Charlie		That's not line two.
15:05	Dr. G	T(p)	Charlie, don't go to line three yet. Ah, Ah don't say anything, let him think.
15:32	Charlie	S(ans)	Parenthesis. I don't know.
15:35	Dr. G	T(d)	Kristen, help the boy. He's starting to sweat up there.
15:42	Kristen	S(ta)	Um, he's sweating. I'm sweating. Parenthesis around the F(3) part and the F(1) part.
15:59	Dr. G	T(f)	Okay, she wants them around both part. We'll play along.
16:03	Charlie	S(con)	I was going to do that

16:05	Dr. G	T(f)	I thought that's what you were going to do. I apologize for stepping in there so quick.
	Dr. G	T(r)	Now what was the one that created the error though Charlie
16:14	Charlie	S(con)	That's a good question
16:16	Kristen	S(ans)	Oh cause.
16:17	Chris		Yes. Go.
16:18	Kristen	S(pb)	It was the second part because if you do that without the ones you would add those two and then you would take that and subtract it before you got to the end. You really had to only put it on the end.
16:31	Dr. G	T(f)	So now what pair was absolutely critical. Charlie. The first pair or the second pair?
16:40	Charlie	S(ans)	The second pair
16:40	Dr. G	T(con-s)	The second pair. Alright Charlie go ahead
17:09	Dr. G	T(j)	Now what do you find interesting about that?
17:11	Charlie	S(ans)	That and that are the same
17:14	Dr. G	T(c)	Yeah, now Charlie all you have to do is explain to me how the hell that happened
17:24	Charlie	S(con)	That's a good question.
17:25	Dr. G	T(con-s)	That's a good question. It certainly is. And that's what I want you guys to think about over the course of the next week or so. How that happened because that's going to

happen a lot and sooner or later one of you might know why. But I'm not telling. Alright now. Thank you Charlie.

	Dr. G	T(r)	We're gonna going back to the beginning for a minute and I'm going to change a few things like.
18:37	Dr. G	T(d)	So. Andrew, you're up. Now you can erase everything but my question.
19:57	Dr. G	T(e)	Now a little verbalization, Andrew would help.
20:01	Andrew	S(ans)	Alright, the derivative of the parent function.
20:02	Dr. G		Remember when your mother watches this on public access channel. She wants to hear you talk.
20:10	Andrew	S(ta)	Alright. The derivative of the parent function would be $6x^2 + 2x$, which is the power rule. And then to draw the graph, just plug in one, two and three to get the points.
20:23	Dr. G	T(p)	Let's just use one and three. Just the endpoints.
20:26	Andrew	S(con)	okay.
21:18	Dr. G	T(p)	Now the graph can be rough. Doesn't have to be finally tuned with lots of tick marks. Okay. Just start with those two points and Amy will take it from there.
21:57	Andrew	S(seek)	Should I put any tick points or just?
22:10	Dr. G	T(c)	Alright. Amy. Now what are you about to do Amy?
22:16	Amy	S(ans)	I am going to start the rectangles
22:18	Dr. G		No.

22:20	Amy	S(a)	Why?
22:21	Dr. G	T(f)	Oh, I don't want to Amy.
22:23	Amy		Geez.
22:24	Dr. G	T(f)	What else could you do?
22:26	Amy	S(ta)	Oh Uh, I have to do the thing to figure out the increases, concave up.
22:33	Dr. G	T(e)	Very good. Talk to us Amy.
22:36	Amy	S(pb)	Okay, um. So you have to find the area of the derivative of the parent function. So to find out if it increases you have to find the derivative of this. So, the derivative of the derivative of the parent function would be $12x + 2$. And then I should graph it. And then when it's one it's going to be, when it's one it's going to be 14. And then when its three it going to be, 36 times, 38. And then we know that this function is linear because it's to the power of one and it's above the x axis so you know that it's increasing.
23:54	Dr. G	T(c)	Whoop, whoop, whoop, whoop, whoop. What did you say?
23:58	Amy	S(ta)	Wait, you know that it's above the x axis
24:02	Dr. G	T(c)	Wait. Time out. Remember this rule we have about pronouns. That they stink because we don't know what they mean half the time.

24:08	Amy	S(c), S(pb)	Okay, you have these two points that, of the, of lower case f(x) and you have to figure out if the graph increases and the concavity of it. So you have to find the first derivative of the derivative to find out if this line, this linear function, is above the x axis or below the x axis and if it's above the x axis then you know that this graph is increasing.
24:34	Dr. G	T(d), T(c)	What'd she say Mikey?
24:39	Mikey	S(c)	The derivative above the x axis increases.
24:43	Dr. G	T(rep)	A little louder Mikey, I almost caught it.
24:45	Mikey	S(c)	The derivative of the derivative is above the x axis it's increasing.
24:49	Dr. G	T(c)	You uses this word it's and that's where I lost you. Try again.
24:53	Mikey	S(c)	The original graph. The first derivative of the parent function.
24:57	Dr. G	T(rep)	So say it all again.
25:00	Mikey	S(c)	Okay. If the first derivative of the derivative of parent function is above the x axis then the derivative of the parent function is increasing.
25:10	Dr. G	T(con-s)	That's not bad. Go ahead.
25:12	Amy	S(pb)	So, this line is above the x axis so you know that it's increasing

25:18	Dr. G	T(c)	What's increasing?
25:22	Amy	S(c)	The derivative is increasing. The derivative of the parent function is increasing.
	Amy	S(ta)	So then now you have find the concavity of it. So now you have to find the derivative of the derivative of the derivative of the parent function.
25:40	Eric	S(ans)	It's called the second derivative
25:45	Dr. G		Now Amy, the next time you say something like that please look at the camera, because nobody would believe this. Alright.
25:52	Amy	S(ta)	Okay. So, I'm finding the second derivative. So then I use the thing. And then
25:59	Dr. G	T(c)	Uh, the thing.
26:01	Amy	S(c)	I use the power rule.
26:03	Dr. G	T(con-s)	Okay.
26:04	Amy	S(pb)	And so it's just 12. So. (Draws the first quadrant of the coordinate plane.) So then like. The domain is restricted to one and three so you just need it at 12 between 1 and 3. And now, this graph it's above the x axis again, so now you know the concavity of this because it's increasing so you know that it concaves it up. So now that we've determined that it increases that this, the lowercase f(x) increases and it concaves up. So you can draw the line.

26:59	Dr. G	T(con-s)	Alright. Very Good
27:01	Chris	S(qs)	Amy can I ask a question?
27:03	Amy		Yeah
27:04	Chris	S(a)	Because the uh second derivative of the original derivative is above the x axis, it means that the first derivative is concave up.
27:16	Amy	S(ta)	The derivative, yeah. The derivative of the parent function is concave up. Which is this
27:19	Chris		Alright
27:20	Dr. G	T(con-t)	Alright so now we have a reasonable analysis of what the graph looks like over the interval for which we're interested.
27:31	Amy	S(ans)	Correct.
27:32	Dr. G	T(f)	Now what are we gonna do?
27:33	Amy	S(ans)	Rectangles
27:35	Dr. G		Alright, at that point. Wendy. [interrupted by announcement on public address system]
27:49	Dr. G	T(p)	Now Wendy. Um, let's start with four
27:54	Wendy	S(con)	Okay.
27:55	Dr. G	T(p)	and then we'll build our way up. I'm going to assign certain groups other numbers of rectangles and then you can create some sort of chart for us. Alright
28:05	Wendy	S(con)	Alright.

28:07	Dr. G	T(p)	So Wendy is going to do four and then we'll break this up into some teamwork.
28:45	Dr. G	T(p)	So you go ahead and do four and we'll fill this in later
28:49	Wendy		Um.
28:50	Dr. G	T(e)	Now Wendy I'm really really interested in hearing you talk about how you put this together. With sigma notation.
29:01	Wendy	S(seek)	Wait should I do the one-fourth?
29:05	Dr. G	T(p)	Let's see if we can go straight to sigma and see if everybody buys it.
29:10	Wendy	S(seek)	Just write it.
29:11	Dr. G	T(p)	Straight to sigma.
29:57	Dr. G	T(d)	Time out. Now talk to us.
30:02	Wendy	S(ta)	Um. Since this is divided into four rectangles so to get from. To figure out the area of the first rectangle is going to be one-fourth times um. This is the starting value which is one. Can I answer a question?
30:25	Dr. G	T(f)	Sure can.
30:26	Wendy		Mike.
30:27	Mike	S(pb)	Yeah, you should only divide it into one half cause even though it's four rectangles it's over a space of two. So it would be one-half instead of one-fourth
30:36	Wendy	S(qs)	Wait why is?

30:37	Dr. G	T(c)	Wendy, What did he say?
30:38	Wendy		I can't hear him.
30:39	Dr. G		I couldn't either.
30:39	Mike	S(c)	You would divide it into one-half instead of one-fourth.
30:42	Wendy	S(qs)	Why?
30:42	Mike	S(pb)	Cause even though you are dividing it into four rectangles it's over an interval of two from one to three. So. So each distance would be one-half.
31:04	Wendy	S(a)	I'm not sure I understand
31:06	Dr. G	T(p)	Okay. Put your hands around the width of the first rectangle.
31:13	Wendy	S(seek)	This.
31:14	Dr. G	T(r)	Yeah, now Mikey how wide is that?
31:17	Mike	S(ans)	one-half.
31:19	Dr. G	T(e)	How did he get that?
31:20	Wendy	S(ans)	By saying it's one half of this interval.
31:22	Mike	S(c)	No, it's one half with the first rectangle. Yeah.
31:28	Dr. G	T(c)	Now Mikey, try to explain how you arrived at that conclusion.
31:31	Mike	S(pb)	Alright see from one to two.
31:33	Wendy	S(con)	Uh huh.
31:33	Mike	S(pb)	Halfway between that. Yeah. So from one to that point is one- half.

31:39	Wendy	S(con)	Uh huh. Then you still start from one.
31:41	Mike	S(con)	Yeah.
31:43	Wendy	S(a)	You still start from one right.
31:45	Dr. G	T(f)	Now wait a minute. Show me the second rectangle.
31:50	Wendy	S(ans)	Here.
31:51	Dr. G	T(f)	How wide is that one?
31:52	Wendy	S(ans)	one-half
31:53	Dr. G	T(con-s)	They're all one half.
31:53	Wendy		Okay
31:54	Dr. G	T(p)	Now show me where you put that one-half.
31:58	Wendy	S(ans)	Here.
32:00	Dr. G	T(f)	Where else?
32:01	Wendy	S(ans)	Here.
32:03	Dr. G	T(j)	Why there?
32:09	Wendy	S(dnc)	I don't know. Why?
32:12	Dr. G	T(r)	Back to the first rectangle. Show me the height. Now where are we going to determine the height. On the right hand side correct?
32:24	Wendy	S(seek)	Right here?
32:25	Dr. G	T(f)	So how high is that?
32:43	Wendy	S(seek)	Is it one plus one times one times twelve?
32:49	Dr. G	T(d)	Suppose I said. It's this high when x equals what Ronak.
33:02	Ronak	S(a)	Wait. Can you say that one more time?

33:04	Dr. G	T(f)	It's this high when x equals?
33:11	Ronak	S(ta)	x equals the. When x equals one and a half.
33:21	Dr. G	T(r)	Now you want to try it again?
33:22	Wendy	S(seek)	Am I supposed to change this number?
33:36	Dr. G	T(r)	Okay. Now now. Time out. Right now what you're trying to do is edit a number and you're not looking at the big picture. So let's take all this out here [erases Wendy's work]. Every one of those rectangles is how wide?
33:45	Wendy	S(ans)	one -half.
33:45	Dr. G	T(f)	One-half. So every length or height is going to be multiplied by the same factor of?
33:55	Wendy	S(ans)	One -half.
33:56	Dr. G	T(c)	One-half. So we could write the one-half one time if we want because it's a factor of all these areas. So. Where do you want to write it?
34:09	Wendy	S(ans)	All the way at the beginning.
34:11	Dr. G	T(con-s)	Fine by me.
34:18	Wendy	S(seek)	I just multiply.
34:19	Dr. G	T(r)	Close. Because remember that's a common factor right.
34:21	Wendy	S(con)	Okay.
34:22	Dr. G	T(r)	Now hang on a second. Now here's the tricky part. Let's concentrate just on the first rectangle. What value of x are you going to use in the function as input?

34:41	Wendy	S(ans)	One.
34:43	Dr. G	T(f)	Plus.
34:45	Wendy	S(seek)	Should I write it?
34:46	Dr. G	T(r)	Go ahead. Now how many widths do you have walk to get to that value of x that's interesting?
34:55	Wendy	S(ans)	One.
34:57	Dr. G	T(c)	Where is that one already accounted for?
35:04	Wendy	S(ans)	Here. This one.
35:09	Dr. G	T(c)	Where is that one already addressed?
35:17	Wendy	S(a)	What do you mean?
35:24	Dr. G	T(d)	Yla, where is that one already addressed?
35:27	Yla	S(ta)	In the summation.
35:29	Dr. G	T(con-t)	In the summation counting numbers. Okay?
35:31	Wendy	S(con)	Okay.
35:44	Dr. G	T(c)	Time out. Pretend for a minute. You know that value of x. What do you do with it?
35:59	Wendy	S(ta)	Plug it in.
36:01	Dr. G	T(con-t)	Is that what you're doing?
36:05	Wendy	S(ans)	No.
36:07	Dr. G	T(r)	What's the function start with?
36:08	Wendy	S(ans)	Two.
36:09	Dr. G	T(c)	So is that what you're doing?
36:11	Wendy		Oh.

36:12	Dr. G	T(c)	Didn't that come a little late? Yeah. Now what's the first thing you see in that function?
36:26	Wendy	S(ans)	Two.
36:26	Dr. G	T(con-s)	Two.
36:28	Anthony		Wait.
36:33	Dr. G	T(d)	Carl.
36:35	Carl	S(ta)	Shouldn't you. Aren't we using the derivative of the function not the parent function. So shouldn't it be 6 times.
36:45	Dr. G	T(c)	[to Amy]We're using the wrong one.
	Dr. G		inaudible conversation with Wendy.
37:20	Dr. G	T(r)	Alright now. What does the function start with?
37:21	Wendy	S(ans)	Six.
37:23	Dr. G	T(c)	Okay. Now we need a value of x. How do we get that value of x?
37:37	Wendy	S(ans)	One plus one-fourth. Is it one-fourth or is it one-half?
37:44	Dr. G	T(con-s)	One-half. It's always one-half.
37:46	Wendy		Okay.
37:58	Dr. G	T(r)	How many one-halves? [Inaudible conversation with Wendy as she finishes writing the expression.]
38:36	Dr. G	T(d)	Now comments. Anthony. Question.
38:40	Anthony	S(ans)	You're missing one parenthesis.

38:47	Chance	S(ta)	I still don't know where the one-half comes from. Like I'm totally confused about that. Like. Cause all this time I thought it was four like there's four intervals so it would be one-fourth.
38:58	Wendy	S(pb)	Because this point is one.
39:00	Chance	S(con)	Right.
39:01	Wendy	S(pb)	and one-half.
39:02	Chance	S(con)	right.
39:03	Wendy	S(pb)	So the width is going to be one-half. One to one-half.
39:08	Chance	S(con)	Oh. Okay. Okay. So you have to look at the.
39:10	Wendy		Amy.
39:11	Amy	S(ta)	I think, but I'm not sure. But I think you need parenthesis around the whole thing cause. I think. I don't know if it's necessary.
39:18	Dr. G	T(c)	What whole thing Amy?
39:20	Wendy		She wants the whole
39:20	Amy	S(ans)	The whole expression.
39:21	Dr. G	T(s)	If it was a plus sign in there some place yes. That's a product so we're okay.
39:26	Kristen	S(c)	You know how he talked to you and was like okay one-half is the common factor and the six is where you started.
39:30	Wendy	S(con)	Yeah

39:32	Kristen	S(a)	What is the one after that. Like, what does that stand for?
39:34	Wendy	S(ans)	You start at one.
39:36	Kristen	S(con)	Okay.
39:38	Dr. G	T(con-t)	Alright. Now. Jason are you ready.
39:41	Jason	S(con)	Yeah
39:42	Dr. G	T(p)	Let's get that in the calculator at least one time and let's see what happens.
40:00			Students use the TI-89 to calculate area under the curve using different numbers of rectangles assigned by Dr. G. Class puts values into a table on the board.
50:00			Class completes table and Wendy evaluates F(3)-F(1). A homework assignment is given to write the summation as an area function for an infinite number of rectangles.
50:17			Carl is called to the board to begin to write the expression for the infinite number of rectangles.
51:39:07	Dr. G	T(d)	Question Kristen.
51:40:20	Kristen	S(ta)	Shouldn't it be 2 over n cause that is how you did everything else?
51:46:09	Carl		(inaudible) [calls on a student to answer]

51:53:01 Matthew	S(ta)	It should be. That first expression should be n over 2, which that would be. So that top number which is n is four. So the number in the denominator is four.
52:03:25 Kristen		But that.
52:05:02 Matthew	S(qs)	What?
52:06:00 Dr. G	T(r)	Kristen, let us hear your argument again please.
52:10:22 Kristen	S(pb)	Okay, it should be 2 over n. because like to get one half, you do two over four. Cause it was like you have four triangles and the area two. Two. So it was two over four which is one half and you had two over ten which is one fifth. And so on. So wouldn't it be two over n.
52:35:16 Dr. G	T(con-t)	Any disagreement? Everyone understand that. ?
Dr. G	T(d)	What'd she say Chris
52:47:19 Chris	S(c)	She said that since those four triangles and those were split into two parts. I mean four rectangles. It was two over n, which was reduced to one half and that's how we got one half.
53:06:26 Dr. G	T(con-t)	Is that what you said Kristen?
53:09:27 Kristen	S(ans)	Uh, not really.
53:10:24 Dr. G	T(r)	Try it again, Kristen. And then we'll back it up and try and have him run it through again.
53:16:18 Kristen	S(pb)	Okay, I don't know how. I'm sure how to explain it. Yeah, You had okay. You had like two, one to three, three

minus one is two. So, you have like two spots and you're doing it for four rectangles. So you did two over four is equal to one half and then you did two over ten is one fifth and so on. So to get it for n, you get two over n. I think that's what I said.

53:48:01	Dr. G	T(d)	So, now you try and say it Chris.
53:51:07	Chris	S(c)	Yeah, fine. Uh, Since we're going from one to three, the top part is two. and since there's four rectangles, the bottom part is four.
54:01:06	Dr. G	T(con-s)	It's the length of the interval divided by the number of rectangles. [points to the equation written on the board] It's the length of the interval divided by the number of rectangles. Now Kristen, that's probably the best piece of thinking I heard you do in six months. That was excellent. No one else picked up on that. That was excellent. And by the way, it is those kinds of nuances that are going to kill you. There is a tremendous amount of technical stuff throughout these kinds of problems and you've got to be alert to every piece of it. Excellent job, Kristen.
	Dr. G	T(con-t)	Now any problems with what Carl wrote.

Dr. G	T(p)	Put equals 60 because that's what you're going to try to prove. Now Carl take the projector, the calculator that projects and graph the derivative function for me.
55:00:00		Class sets equation equal to 60. A graph of the derivative function is put on the calculator and the area under the curve for the interval [1,3] is found using the calculator. Teacher draws student attention to the integral notation on the calculator and introduces paper and pencil integral language formally, graphically and using F(x) notation.
01:02:30		Teacher explains how to use the TI-89 to calculate the integral of their given derivative function. Teacher also mentions that the constant C is not given by the calculator when it determines the integral. A new problem is introduced giving the derivative function and determining the parent function: Area under the curve $f(x) = 2x^2 + 3x$ over [0,1]. A student then goes to the board to calculate the parent by reversing the power rule for derivatives.
01:10:00		Student then evaluates the integral for the given interval using integral language. Another student evaluates the integral using the overhead calculator. Teacher asks another student to set up the limit equation which they will determine for homework.

01:12:22 Dr. G	T(p)	Alright. Now it would be a good idea to set up the limit expression before we get out of here because that's one that probably the most difficult to set up. Mike it's all yours
01:13:04 Dr. G	T(e)	I want you to talk to us as you go.
01:13:26 Mike	S(seek)	So would I start with a sigma right here.
01:13:30 Dr. G	T(p)	Why don't you start with a rough graph?
01:13:34 Mike	S(a)	So I'm making the graphs.
01:13:35 Dr. G	T(con-s)	Yep.
01:13:36 Mike	S(con)	Okay.
01:13:37 Dr. G	T(p)	Jason, do a graph for him real quick and we'll just project it for him to use.
01:14:01 Dr. G		There it is.
01:14:11 Mike	S(a)	Wait. So what do you want me to do with the graph?
01:14:18 Dr. G	T(f)	I don't care. Anything you like? You can turn it off if you like
01:14:21 Mike	S(seek)	Can we get it out the way?
01:14:24 Dr. G	T(rep)	What?
01:14:25 Mike	S(seek)	Can we get it out the way?
01:14:26 Dr. G	T(p)	Well turn it off
01:14:43:04 Mike	S(a)	So just, uh, write the equation.

01:14:48:29	Dr. G	T(r)	Now, why one over n. Awhile ago it was two over n.
01:14:55:00	Mike	S(pb)	Because the uh, zero to one, not one to three.
01:14:57:00	Dr. G	T(con-s)	Oh, okay.
01:15:09	Mike		So now.
01:15:41	Ricci	S(qs)	i equals one?
01:15:44	Dr. G	T(e)	Time out. Explain that.
01:15:47	Mike	S(ans)	Alright. i is the counting number.
01:15:52	Dr. G	T(f)	Yeah. What else?
01:15:56	Mike	S(ta)	And uh. This is the uh length, and this is the height. Well I'm not finished writing the height yet.
01:16:04	Dr. G	T(f)	Okay. So there's. Well up there it says two-thirds x cubed.
01:16:05	Mike	S(a)	Where?
01:16:06	Dr. G	T(p)	Right above it.
01:16:14	Mike	S(ans)	That's the parent function.
01:16:16	Dr. G	T(con-t)	Oh, did I get confused?
01:16:17	Mike	S(con)	Yeah.
01:16:18	Dr. G	T(con-s)	Okay.
01:16:49	Dr. G	T(con-t)	Now is everybody happy with the statement. And that's supposed to equal.
01:16:56	Several students	S(ans)	The answer. What's the answer
01:16:58	Dr. G	T(con-t)	Thirteen-thirds was it?

01:17:01 Students   S(ans)   Thirteen-sixths.

01:17:02 Dr. G   T(con-s) Thirteen-sixths. Equals thirteen-sixths. [Mike writes =

13/6].

Dr. G   T(p)   Now those are the two things I want you to prove tonight.

And it would be a good idea if you had somebody to

compare notes with halfway through. These get very

tedious. Good job Mike.

Class dismissed.

OBHS_19Apr02_1a&b

00:00   Teacher writes homework assignment for the day on the

board as the class gets settled. Explains to students that

these are problems they should practice in order to

become efficient at solving calculus problems.

02:14   Dr. G   T(s)   Now, we're going to start with one of them in particular

from the second set, which is this situation right up here.

(Integral 1 to 2 of sq rt 2/x dx). First of all how do you

read that?

03:02   Dr. G   T(r)   How do you read that?

Students talk to each other in small groups

03:20   Dr. G   T(d)   Okay, Rebecca.

03:22	Rebecca	S(ta)	The derivative of two divided x over the interval one comma two.
03:28	Dr. G	T(rep)	One more time.
03:29	Rebecca	S(c)	The derivative of the square root of two divided x over the interval one to two.
03:37	Dr. G	T(con-t)	Agreed?
03:41	Dr. G	T(d)	[Carl if you would please shut the door.] Stephanie.
03:45	Stephanie	S(ta)	The integral of the square root of two over x with respect to the independent variable x over the domain of one to two inclusive.
03:56	Dr. G	T(c)	One more time, in about half the words.
03:58	Stephanie		Okay, um.
04:01	Dr. G	T(f)	Wait before you do that. What was the key difference between the two responses?
04:13	Rebecca	S(ans)	She stated that the interval first.
04:13	Another student	S(ans)	The independent variable.
04:17	Dr. G	T(r)	And you said.
04:18	Rebecca	S(ans)	The variable last. The derivative first.
04:20	Dr. G	T(r)	Oh interval. I missed the word interval. Alright the two ladies are going to say it again. I want you to pay attention to a key word in each response. Rebecca.
04:29	Rebecca	S(ta)	The derivative of the square root of two.
04:32	Dr. G	T(d)	Stop. Stephanie.

04:33	Stephanie	S(ans)	The integral of the.
04:36	Dr. G	T(r)	Stop. Difference.
04:40	Amy	S(ans)	Integral.
04:41	Dr. G		Versus.
04:42	Amy	S(ans)	Derivative.
04:43	Dr. G	T(c)	Derivative. Integral versus derivative. You can almost think of that as opposites. One undoes the other, so to speak. Alright. Now the one we have here is ?
05:04	Dr. G		(Goes to problem written on the board.) That is a derivative function
05:08	Rebecca		Right
05:09	Dr. G	T(r)	Of some other function. This is called the integral of that derivative function. But we are going to integral this function. Meaning we're going to ask the question, Where did it come from? What parent function generated that derivative? And then we're going to think about it over the interval one to two. Alright. Now I want to construct a graph of that expression because when you look at this what geometric word should come to mind immediately? (points to integral on board)
06:32	Dr. G	T(r)	When you look at this. When you're looking at some sort of algebraic calculus notation, abbreviation, syntax, for what geometric term? Concept?

06:49	Amy	S(ans)	Hyperbola.
06:55	Dr. G	T(e)	I wasn't expecting that. Where did you get hyperbola? You graphed something in your calculator and it looked hyperbolic? Right.
07:03	Amy	S(con)	Sure.
07:04	Dr. G	T(d)	Yeah, I thought so. You got to take it to another level. Chris
07:10	Chris	S(ans)	Area
07:11	Dr. G	T(con-s)	Area. Very good Chris. Very good. Class laughs
07:17	Dr. G	T(r)	Alright now. I want to show that graph. Okay. How do we begin? By putting away the calculator. Some of you already cheated and know what it looks like. Now I want to start by finding one of the end points to this curve. Alright. So, Yla come on down and find an end point for me.
08:34	Yla	S(seek)	That an end point?
08:35	Dr. G	T(e)	What are we going to do with that?
08:37	Yla	S(ta)	Well that the value of. When you put one into the function you get square root of two.
08:49	Dr. G	T(r)	Okay. What are we going to do with that?
08:51	Yla	S(ans)	Put it on a graph.
08:52	Dr. G	T(con-s)	Alright.

08:55	Chance	S(qs)	Yla. How'd you plug one into it?
08:57	Yla	S(pb)	Because we're looking at between one and two. So just take one.
	Yla	S(seek)	How am I going to graph the square root of two?
09:17	Dr. G	T(f)	Pretend we don't know. Can you estimate it for me?
09:29	Dr. G	T(f)	It's between what and what?
09:33	Yla	S(ans)	Between one and two. One and two.
09:36	Dr. G	T(e)	How is it you know this?
09:38	Yla	S(pb)	Because one squared is one and two squared is four.
09:45	Dr. G	T(con-s)	Somewhere between one and two. That's good enough for all we know.
09:51	Yla	S(seek)	So. (makes tick marks on coordinate graph) Should I just make a point anywhere here?
10:23	Dr. G	T(p)	Now that really bothers some of you. And somehow we got to get you beyond that. The fact that she just put that dot there without really knowing specifically exactly where it goes really bothers some of you. You got to get beyond that. You got to let it go. Okay. Now try the other one.
11:01	Yla		Right
11:13	Dr. G	T(f)	Okay, now that you know something about the graph. Do you think we have it designed very well?
11:25	Yla	S(ans)	No

11:27	Dr. G	T(d)	Possibly not, but let's hold on that for a second okay. Tracy, if she had to decide what the curve does. How the curve behaves between those two points. What would offer a good clue?
11:47	Tracy	S(ans)	The first derivative and the second derivative
11:48	Dr. G	T(p)	So you go up and take the first derivative. Thank you Yla
11:58	Dr. G	T(d)	Question Chris.
12:00	Chris	S(con)	I am just confirming. That function itself up there. The square root of two over x that itself is the derivative of some parent function.
12:08	Dr. G	T(f)	Very good Chris. And we are going to find the area under the curve represented by that derivative
12:22	Tracy	S(seek)	How would you. How would you would express like. You know you have f of x, you put with a line and then x. How would you do that?
12:30	Dr. G	T(f)	Is this lower case or upper case F?
12:34	Tracy	S(dnc)	I don't know which this is?
12:37	Dr. G	T(rep)	I can not hear you.
12:38	Tracy	S(dnc)	Did you do this yesterday? I don't know I wasn't here.
12:44	Dr. G	T(f)	You're going to find the derivative of what?
12:46	Tracy	S(ans)	Of this (points to square root of 2/x)
12:51	Dr. G	T(con-s)	Okay, so that has nothing to do with whether or not your were here yesterday

12:55	Tracy	S(seek)	So that's like a lower f.
12:58	Dr. G	T(con-s)	Okay
13:01	Tracy	S(seek)	But then what's upper F?
13:02	Dr. G	T(s)	Where that came from. The parent
13:03	Tracy	S(ans)	Oh, that's already a derivative.
13:04	Dr. G	T(con-s)	That's already a derivative.
13:04	Tracy	S(seek)	So this. Would this be called a second derivative?
13:06	Dr. G	T(s)	No this would a be a first.
13:07	Tracy	S(seek)	The first derivative of that
13:07	Dr. G	T(s)	derivative of that derivative, but it would be second derivative of the parent function
13:09	Tracy		Parent.
13:12	Dr. G	T(p)	But we don't care so let's stay with the first derivative.
13:16	Tracy	S(seek)	So what do I do?
13:17	Student	S(ans)	Little f with two.
13:20	Students	S(ans)	one one one.
13:23	Tracy	S(a)	off.
13:23	Rajeesh		Groninger
13:24	Dr. G		Yeah.
13:25	Rajeesh	S(ans)	No pronouns
13:25	Dr. G	T(d)	No pronouns. Yla?

	Yla		Student speaks to Dr. G inaudible. Several students speak at once. Some to Tracy at the board and some in response to Dr. G's comment to Yla.
13:38	Tracy	S(seek)	Is this?
13:42	Dr. G	T(f)	Now, what's a matter?
13:43	Tracy	S(seek)	I don't know. Now what do I do?
13:46	Dr. G	T(p)	Okay. Right above that
13:48	Tracy	S(seek)	How do you take the derivative?
13:49	Dr. G	T(p)	Right above that. Write the function lower case f.
13:59	Dr. G	T(c)	Okay, Now what do you want to know. (Tracy points to board) What's that mean?
14:05	Tracy	S(seek)	How to do it?
14:06	Dr. G		I don't know.
14:08	Student A	S(ta)	You make it equal to, um, two over x raised to one-half. Then you can do the chain rule
14:21	Tracy	S(a)	Alright, this would be up here.
14:42	Tracy	S(qs)	Is this wrong? Is this supposed to be the derivative of two x here?
14:45	Carl	S(ans)	No that's fine
14:45	Tracy	S(a)	That's fine.
14:55	Tracy	S(ta)	And now the derivative of that(points to 2 over x). Do you do the whole quotient rule thing again. Should I do that on the side? Well is it like easy cause now I'm just

			not [talking with student in front row. Student just nods yes or no]
15:24	Tracy	S(seek)	Would they both be like? Well
15:26	Dr. G	T(e)	A little more verbalization so we can hear all the way back here.
15:37	Dr. G	T(f)	She ignores us. What's up with that?
15:39	Tracy	S(ta)	What is the derivatives of two and x. I am missing something. What is this?
15:48	Kristen	S(ans)	The derivative of x is two.
15:52	Tracy	S(ta)	Oh yeah because of the coefficient. Two minus nothing.
16:09	Kristen	S(ans)	equals.
16:10	Dr. G	T(e)	Tracy. Talk to us real nice and loud.
16:13	Tracy	S(ans)	I was talking with Kristen, that's why.
16:16	Dr. G		Well maybe we can help you.
16:19	Kristen	S(c)	It's just two.
16:20	Tracy	S(a)	Like that.
16:23	Dr. G	T(c)	Now you're going to have to explain that.
16:25	Tracy	S(dnc)	Why?
16:27	Eric	S(ans)	Cause it's wrong.
16:28	Tracy	S(a)	It's wrong.
			Students joke with Tracy because she is being recorded
16:59	Eric	S(ta)	Do the quotient rule. It makes it really easy. Just do the quotient rule.

17:09	Tracy	S(seek)	Is that it?
17:10	Dr. G	T(c)	There you go. Now talk to us about what that means.
17:13	Tracy	S(dnc)	Why? You know what it means. I'm the only one who doesn't know it.
17:18	Eric		Oh, we got a rebel.
17:22	Tracy	S(a)	Is that right? (turns to Kristen for approval. Kristen nods)
17:42	Tracy	S(qs)	Is that right?
17:43	Eric	S(ans)	Yes.
17:51	Dr. G	T(con-t)	Don't erase that for one second until everyone buys in. Everybody alright? One more line Tracy.
18:12	Tracy	S(ta)	Should I leave that to the side. You mean like simplify it. Should I do that? (taps terms on the board). What do you do? Amy?
18:28	Amy	S(ta)	You know that it's a negative exponent so that means you flip the thing and then you raise it to the one-half and square root.
18:38	Tracy	S(a)	So it would be the square root of x and the square root of two.
18:57	Tracy	S(ta)	Should I square them? Wait can you do that?
19:22	Tracy	S(ta)	Would that be wrong? (no response from students. Tracy keeps simplifying)
20:00	Several students		one-half

20:15	Tracy	S(ta)	Would it just be over four. How do you multiply fractions?
20:18	Kristen	S(ans)	Multiply across
20:43	Tracy	S(qs)	Can I just put this together? [Students nod.]
21:03	Dr. G	T(d)	Okay. Now. Comments. Corrections. Additions. Questions
21:10	Carl	S(ta)	Uh, I forget the rule, but is the square of two times the square root of x the square root of two x.
21:37	Tracy	S(ta)	Can you take this out of the bottom?
21:45	Dr. G	T(j)	Thank you Tracy. Now. Matthew why do you do that?
21:55	Matthew	S(a)	Why did we find the derivative?
21:57	Dr. G	T(con-s)	Yeah.
21:59	Matthew	S(pb)	Because we want to graph it to see if the function increases or decreases.
22:05	Dr. G	T(rep)	Can't hear you.
22:06	Matthew	S(ta)	We want to graph what the derivative is to see if the function increases or decreases.
22:14	Dr. G	T(f)	Suppose I don't want to do that. Give me an option. Give me another option. I don't want to do the graph. Too tired.
22:24	Matthew	S(ans)	You can plug in numbers.
22:27	Dr. G	T(p)	I don't want to do that either.
22:34	Matthew	S(ans)	You can tell by looking at it.
22:36	Dr. G	T(f)	What can you tell from looking at this?

22:39	Matthew	S(ans)	The negative sign.
22:43	Dr. G	T(p)	Come up here. Talk to us. I don't want to plug in numbers, I don't want to graph it. We want an easy way out. Talk to us.
23:02	Matthew	S(ta)	Since we know that the domain we're working with is from one to two and they are both positive numbers and if you know that a positive times a positive number is positive. So and a negative number times a positive number is negative. So since these are positive then this would be. This part would be positive. And then
23:27	Dr. G	T(c)	What part would we positive?
23:29	Matthew	S(ans)	Like right here.
23:30	Dr. G	T(j)	Why?
23:32	Matthew	S(pb)	Because a positive times a positive is a positive
23:34	Dr. G	T(j)	Why does x have to be positive?
23:37	Matthew	S(pb)	Because x is in the domain one to two.
23:40	Dr. G	T(con-s)	Alright.
23:41	Matthew	S(pb)	And then same thing would be here. And then since it's multiplied by a negative you know that the final answer would be negative. So then it decreases.
23:52	Dr. G	T(c)	So what decreases?
23:54	Matthew	S(ans)	The function decreases
23:55	Dr. G	T(c)	Which one?

23:58	Matthew	S(ans)	The original
24:00	Dr. G	T(f)	Okay. Now we know it decreases over the entire interval because that derivative is always going to be negative. Okay. Now what else do we want to know, if possible?
24:16	Matthew	S(a)	From this?
24:17	Dr. G	T(f)	From anything about graphing.
24:20	Matthew	S(ta)	We want to know if it concaves up or concaves down.
24:23	Dr. G	T(c)	Alright, how so?
24:26	Matthew	S(ans)	You have to find the second derivative.
24:27	Dr. G	T(p)	Alright. Go ahead
24:30	Matthew	S(con)	Okay.
24:52	Chance	S(ans)	Matthew. You need an x.
25:05	Matthew	S(qs)	Wait before I start is quotient rule the best way. Anybody agree or disagree?
25:11	Student	S(ans)	Yeah
25:12	Matthew	S(con)	Alright
25:14	Dr. G	T(f)	Timeout. Can we manipulate that to make it more friendly before we try to the quotient rule?
25:32	Matthew	S(ta)	I'm not sure at the order, but can you just take the two out and divide it in the four? You can do that. Alright. Should I just write under here.

25:55	Dr. G	T(r)	And my question still stands. Can we manipulate that to make it more friendly for finding it's derivative before we would attempt the quotient rule.
26:05	Matthew		Chance
26:07	Chance	S(ta)	Well actually this might even be harder, but I was thinking like negative two x to the one-half that's still just as bad right. Nevermind.
26:20	Student	S(a)	Wait. What did she say?
26:21	Dr. G	T(p)	Louder.
26:23	Chance	S(c)	I said like I think this might be even like harder to like work with but I said make it like negative two x to the one-half, but I don't know.
26:33	Matthew	S(qs)	Maybe. Um, Where's Rohan? Rohan.
26:40	Rohan	S(ta)	Um, like maybe if you could multiply the numerator and denominator by x, square root of x. Multiply the top and bottom by the square root of x.
26:54	Matthew	S(pb)	No because then the bottom, like x squared times square root of x then you'd have square root of x on the bottom, you couldn't simplify that anymore.
27:02	Carl	S(ans)	No it would be x
27:05	Rohan	S(ans)	Wouldn't it be two x.
27:08	Matthew	S(pb)	No because the square root of two times two for example. That's that square root of two. Eric

27:18	Eric	S(ta)	Would it make it any easier to just move the negative to the bottom. (Class laughs in response.)
27:24	Matthew		Amy
27:25	Amy	S(ta)	I don't know if it's easier, but if you change the square root of 2x to 2x to the one-half.
27:28			Several students start saying that Chance said that earlier
27:32	Amy	S(a)	Is that what she just just said.
27:33			Several students respond affirmatively
27:36	Dr. G		Chance she tried to steal your idea.
27:38	Amy		I'm sorry.
27:40	Chance		She's adding a further step.
27:44	Rebecca	S(ta)	Kind of but not really. If you have like, just picture it as negative two x to the one-half, then you can subtract the exponents and the two x's cancel out right? And then it's just negative one.
28:00	Matthew	S(ans)	I think that makes sense, but I am not sure.
28:04	Chance	S(ta)	That would be the simplest because then your exponents are gone.
28:05	Matthew	S(ans)	Cause then.
28:06	Rebecca	S(ans)	It's like what one-half minus four over two.
28:10	Chance & Matthew	S(ans)	four over two
28:11	Matthew		Oh.
28:12	Rebecca	S(ans)	Simple. You got to have a common denominator.

28:13	Chance		Oh Okay.
28:15	Matthew	S(a)	Yeah. Alright. Anybody agree or disagree. Should I try that?
28:18	Chance	S(con)	Agree let's try it.
28:20	Amy	S(a)	Wait. I didn't hear it.
28:21	Matthew	S(c)	She said that if you turn this into two x to the one-half because of the rules of algebra or whatever you can subtract one-half from two, I think, one-half from two. Okay.
28:42	Dr. G	T(e)	Once you set this up I want you to turn around and explain what happened?
28:46	Matthew		Okay
28:56	Matthew	S(ta)	Alright for now I just changed that to one-half just manipulated it. I didn't do anything to dramatically change it. Oh I see.
29:12	Dr. G	T(f)	So is that to suggest that next will be very dramatic?
29:16	Matthew	S(ans)	A little more dramatic.
29:18	Dr. G		A little more dramatic
29:20	Ricci	S(qs)	I have a question
29:21	Matthew	S(ans)	Yes
29:22	Ricci	S(ta)	Isn't only x squared?
29:26	Matthew	S(ans)	Oh.
29:27	Chance	S(con)	Yeah that's right.

29:31	Matthew	S(ta)	Will it still work if you do that. If it's only x squared then
29:33	Ricci	S(ans)	The x, the x to the one-half.
29:39	Matthew	S(ta)	Well maybe you can do this. Just x to the one-half, two to the one-half.
29:45	Yla		Matt.
29:46	Matthew		One sec. Yla.
29:56	Dr. G	T(con-t)	Go ahead. You sure? You done. Alright. Okay. So continue. Several students are discussing with each other the algebra steps.
30:22	Matthew	S(ta)	Wait. What do you do? Do you.
30:24	Rebecca	S(ans)	Well you have
30:25	Matthew	S(qs)	Which one do you subtract from?
30:27	Rebecca	S(ans)	You subtract the top from the bottom.
30:29	Matthew	S(qs)	And then put the x equals to the top?
30:32	Amy		No.
30:34	Rebecca	S(ans)	It's like one-half minus four over two.
30:51	Dr. G	T(c)	Alright so now Matt. Talk to us. What's happening here?
30:55	Matthew	S(ta)	Alright. We're. We're moving the exponent down to, wait, I'm not sure. Alright. I think I should do one more step to make it a little clearer. I think. So I'm just going to chance my parenthesis.

31:26	Matthew	S(ta)	Okay here I subtracted one-half. From one-half I subtracted two to bring the exponent to the top. Is that understood?
31:37	Dr. G	T(c)	How is it that you choose only to do it with the x rather than with the base of two as well?
31:46	Matthew	S(ans)	I didn't realize that I can do both
31:49	Dr. G	T(con-s)	Okay
31:51	Matthew	S(ans)	I'm just going to put that there so I remember. Okay
32:03	Dr. G	T(con-t)	Now what do we have there. Negative two to the negative one-half power times x to the negative three-halves power. Everybody agree with this?
32:14	Chris	S(ans)	No
32:18	Dr. G	T(c)	Alright now you got to talk to them because they're not with you.
32:20	Matthew		Chris
32:21	Chris	S(ta)	Alright when you move the x squared up. You did what? You
32:27	Matthew	S(ta)	You subtracted. You have this and you subtracted two from it.
32:35	Chris	S(con)	okay.
32:40	Dr. G	T(con-t)	Is everybody alright?
32:41	Eric	S(ans)	No
32:42	Dr. G	T(d)	Eric. Talk to him.

32:43	Eric	S(qs)	I don't understand. I don't understand any of this. I don't know what you did.
32:45	Matthew	S(seek)	Alright. Should I show it on the side?
32:46	Dr. G	T(p)	Hang on a second.
			Class converses as Dr. G works with Eric on two exponent rules. Class is interrupted by an announcement. A student checks what Eric has written on the board. Eric fills in another rule.
34:25	Dr. G	T(con-t)	Now Eric do those manipulative rules make sense to you?
34:29	Eric	S(ans)	Uh. Yeah, I think. Yeah.
34:37	Dr. G	T(d)	Question Yla.
34:38	Yla	S(qs)	I don't get what happened to the two?
34:41	Dr. G	T(f)	Talk about the two
34:44	Matthew	S(pb)	Alright. It was just the same thing, because, but instead of a variable, it was just. Two is kind of like the x. So it's two to the one and that's two to the one-half. So then it would be two to the one-half over two to the one and then you do one-half minus one. Do you understand that?
35:06	Yla	S(con)	Yeah.
35:09	Dr. G	T(r)	Now what rule are you going to contemplate for your derivative?
35:16	Matthew	S(ans)	Power Rule.
35:19	Dr. G	T(con-s)	Alright.

35:23	Matthew	S(ta)	I'm just a little confused. You can't do anything right now. You don't use. You just turn that into something more easier to see right. Cause you don't do the power.
35:38	Chance	S(a)	What? Two to the negative one-half.
35:39	Matthew	S(seek)	Cause it's a constant right?
35:41	Dr. G	T(f), T(c)	What's a constant? Where is your constant? Tell me what your constant is.
35:48	Matthew	S(ans)	Negative two to the negative one-half.
35:49	Dr. G	T(con-s)	Correct
35:51	Matthew	S(seek)	Where should I do this? Here? Okay I'm going to rewrite it. What should I? Should I leave it at that?
36:06	Dr. G	T(con-s)	For now I would.
36:07	Matthew	S(con)	Okay.
36:31	Dr. G	T(r)	Okay. Thank you Matt. Jason. Now before you continue Jason. I want to remind you how it is we used this derivative. That was Matt just looked at this expression is always going to have what kind of value?
36:56	Jason	S(ans)	Negative.
36:57	Dr. G	T(p)	A negative value. We didn't have to graph it all we had know what that and how we knew that. So, when you proceed. Whenever you're ready to discuss something like that you can stop manipulating that. Okay.
37:24	Jason	S(ta)	That's the formula for the product rule.

37:26	Dr. G	T(d)	Alright hand on a second. Matt take a look at what he just did. Counsel the boy.
37:33	Matthew	S(a)	What is he trying to find out right now?
37:36	Dr. G	T(f)	Ask him.
37:38	Matthew	S(qs)	What are you trying to find out right now?
37:39	Dr. G	T(d)	Jason.
37:43	Jason	S(ans)	Yeah.
37:44	Dr. G		Your partner is way back here.
37:44	Jason		Matt.
37:46	Matthew	S(qs)	Why are you finding another derivative?
37:53	Jason		Yeah Chris
37:54	Dr. G	T(p)	Wait, wait wait, wait, wait. I would like to hear a dialogue between these two.
38:08	Jason		Okay Matt.
38:10	Matthew	S(qs)	What are you trying to find right now?
38:16	Dr. G	T(f)	So you're not trying to find anything right now?
38:18	Jason		Um.
38:22	Dr. G	T(c)	Alright. What's the overall question we're involved in right now?
38:32	Jason	S(ans)	Trying to find out if it's negative.
38:38	Dr. G	T(f)	We know this one is negative. Which means if I draw the curve between this point and this point it should be

decreasing over the entire domain. Now what's your question?

38:52	Jason	S(ta)	Find if this is negative. Positive or negative.
38:55	Dr. G	T(c)	Okay. Which one though? This is still the original. So what's this compared to this?
39:06	Jason	S(ans)	The derivative of that.
39:09	Dr. G	T(con-t)	This is the derivative of this correct. Now you're going to manipulate this until you can decide if it's positive or negative. Someway. Somehow. Okay? Alright.
40:09	Chris		Jason
40:10	Jason		Yeah
40:11	Chris		Are we not (inaudible) to (inaudible) yet?
40:14	Jason		(Inaudible)
40:16	Chris		Oh I didn't see that. Sorry
40:30	Jason	S(seek)	Is that right?
40:32	Dr. G	T(d)	So we have. Negative one over the square of two times negative three-..then we went to. Okay. Now are you able to. Question Chris?
40:48	Chris	S(a)	I'm still confused.
40:51	Dr. G	T(c)	Jason. He needs to be understanding how you did this derivative or how Matt did this derivative.
41:01	Jason	S(ans)	I didn't find it. I'm just simplifying.
41:04	Dr. G	T(r)	I know. Where did Matt get that derivative?

41:10	Jason	S(a)	This derivative.
41:11	Dr. G	T(con-s)	That's the one.
41:20	Jason	S(ans)	He just simplified
41:22	Dr. G	T(c)	He did what?
41:25	Jason	S(ans)	He simplified what he had here.
41:30	Dr. G	T(r)	Jason the question is. How did we go from this line (draws an arrow between two lines) What happened?
42:04	Jason	S(ans)	He used the chain rule.
42:05	Dr. G	T(rep)	Pardon.
42:06	Jason	S(ans)	He used the chain rule.
42:08	Dr. G	T(j)	Well explain it. How did it work?
42:11	Jason	S(pb)	Um, He took this, the first derivative, and then he did the chain rule and got that?
42:21	Dr. G	T(j)	Yeah but how does the chain rule work Jason?
42:25	Jason		You take.
42:28	Chris		Hey Jason
42:30	Jason		Chris.
42:31	Chris	S(ans)	He used the power rule.
42:35	Eric	S(con)	Yeah that's the power rule.
42:37	Jason	S(con)	Oh yeah.
42:44	Dr. G	T(f)	Now what rule are you going to use Jason?
42:46	Jason	S(ans)	Power Rule
42:48	Dr. G	T(c)	Alright so explain the power rule to me?

42:52	Jason	S(ans)	There's a constant.
42:57	Dr. G		I'm listening.
43:00	Jason	S(ans)	There's a constant. You have like something here.
43:08	Dr. G	T(rep)	Can't hear you.
43:09	Jason	S(c)	And then you have like something in there.
43:11	Dr. G	T(con-s)	Something in there. Alright.
43:12	Jason	S(ans)	Now wait. Alright. Something to the power of.
43:18	Dr. G	T(r)	Why don't we just use your problem?
43:22	Jason	S(seek)	Alright. What do you mean? What do you mean?
43:31	Dr. G	T(d)	Matt. Go save him. Get up there and tell him what you did and he'll turn around and tell me.
43:48	Matthew	S(pb)	This is the constant. So you can do this, but then you multiply the power and subtract one.
43:57	Jason		Oh yeah. Yeah.
43:58	Matthew		No erase what.
43:59	Jason		This part
44:00	Matthew		No that's, that's . Use that for how you got from here to here. Do you understand now?
44:08	Jason		Yes. Multiply this.
44:12	Matthew		Use this.
44:13	Jason		Use the constant.
44:16	Dr. G	T(p)	Ready Jason. Okay now you got to turn the volume up.
44:21	Jason	S(ans)	Use the constant .

44:21	Dr. G	T(p)	It's hard to hear you back here.
44:22	Jason	S(ta)	Use the constant which is negative one-half. Negative two to the negative one-half power. Multiply that by the exponent that x had. And then he uh. Negative two.
44:40	Matthew	S(ans)	Subtract one from the x's
44:43	Jason	S(ta)	Oh yeah he multiplied this (points to coefficient) by the power and then subtracted one from the exponent.
44:49	Dr. G	T(r)	Okay. Now then you began to manipulate that. Now eventually you want to be able look at your derivative and decide is it always positive or is it always negative in this domain. Now what do you think are you ready to make that decision?
45:11	Jason	S(ans)	No.
45:12	Dr. G	T(rep)	Pardon me.
45:14	Jason	S(ans)	Yes, Yes.
45:15	Dr. G	T(e)	Explain to me how you know whether it's always positive or always negative.
45:23	Jason	S(pb)	If it was a negative number, like take negative one raised to the fifth power, it would still be negative. Yla?
45:30	Yla	S(ta)	You have negative one times x squared times negative three
45:40	Jason	S(a)	Wait. This part is positive then.

45:44	Yla	S(pb)	No since you have two negatives there a negative times a negative always equals a positive. So you know it's positive.
45:55	Jason	S(qs)	Yeah but. How does
45:58	Dr. G	T(p)	Time out. Time out. Time out. Now Jason and Yla. Let, the rest of you let them finish this conversation. Jason turn up the volume a little bit.
46:09	Jason	S(c)	I was just showing how the denominator of this part would have to be a positive.
46:15	Yla	S(pb)	But negative values of x isn't in the question because the domain is between one and two.
46:23	Jason	S(ta)	Oh yeah. That's true. So that would be a negative times a negative.
46:29	Dr. G	T(rep)	Louder.
46:30	Jason	S(c)	It would be a negative times a negative and that would be positive.
46:32	Dr. G	T(con-s)	So that thing is always going to be positive because x is always positive and there's nothing there to change that. So now you come over and draw the function.
46:50	Jason	S(a)	Draw the derivative.
46:55	Dr. G	T(d)	Which uh. Rebecca. Walk up there and point to the one he's supposed to draw.

47:03	Rebecca	S(seek)	Which one he's supposed to draw? What do you mean
			he's supposed to draw?
47:05	Dr. G		Oh my God. Oh my God.
47:07	Rebecca	S(pb)	Well since this is, this is positive right so he can connect
			the dots here up or down. Cause it concaves up now right.
47:15	Dr. G	T(r)	Time out. Time out. Time out. The last instruction was
			point to the function we're trying to draw.
47:22	Rebecca	S(ans)	That one.
47:27	Dr. G	T(r)	What do we know about it?
47:29	Rebecca	S(ans)	That it's decreasing
47:32	Dr. G	T(f)	And?
47:33	Rebecca	S(ans)	And it concaves up
47:34	Dr. G	T(c)	Over?
47:36	Rebecca	S(ans)	one to two
47:37	Dr. G	T(p)	A small interval one to two. Go ahead Jason. Alright
			now. Complete the drawing. Complete the graphic of
			what that expression means.
47:50	Jason	S(a)	Wait what?
47:52	Dr. G	T(p)	Complete the graphic, the drawing, the visual of what this
			means
48:00	Chance		I have a question
48:01	Dr. G		Timeout.

48:03	Chance	S(ta)	Okay. This is random, but like when you like you know like how we knew that it was decreasing the graph. Well either way it would look like it was decreasing right. Cause when you draw it, the point.
48:14	Jason	S(ans)	It could look this this though
48:17	Chance	S(con)	Okay
48:18	Dr. G	T(p)	Okay. Complete the drawing.
48:26	Jason	S(a)	I don't get what you want.
48:28	Dr. G	T(p)	Rachel walk up there and draw it for me and explain it to him.
48:49	Dr. G	T(s)	Alright Jason, very good. Thank you. Now we're about the find the area of that bounded decreasing shaded region. Now if we knew the parent function, that would be a rather easy task. If we knew the parent function, all we would have to say is: This area (Writes $F(x)|2,1$) And we'd be done. Now we can do that or we can divide that region into an infinite number of geometric figures for which we are familiar, rectangles, and do it the hard way. Your choice. Mike, your choice.
50:06	Mike	S(ans)	Alright. Let's do it that way.
50:07	Dr. G	T(p)	Go ahead. Several students start to discuss what Mike means by "do it that way"

00:03	Dr. G	T(f)	Mikey wants to do it that way he says
00:05	Eric	S(a)	yeah which way is he talking about.
00:06	Dr. G	T(con-s)	I don't know, but he's going to do it that way
			Students continue to comment about which way Mike
			means
00:22	Dr. G	T(d)	Rebecca
00:23	Rebecca	S(ta)	Why can't we do it the way we did with F of. Oh wait, we
			don't know what F is
00:33	Dr. G	T(r)	The task is for you to think about how to find the parent
			function without looking in your calculator. Don't cheat.
			Eventually you'll do that routinely. But now I want you to
			think about the reverse process here. How do you go from
			the derivative back to the parent function and the only
			clue you have so far is reversing the power rule?
01:06	Eric	S(con)	Yeah that's pretty much the only thing we have here.
01:34	Dr. G	T(p)	Anita. You're up. I want you to lead this group through
			trying to figure out how to reverse the power rule on this.
01:52	Anita	S(qs)	I don't even know how to reverse it. Who knows?
			Stephanie, you know.
01:59	Stephanie	S(ta)	Well I know how to. You have to add one to the. Well
			you can make it two over x to the one-half.
02:06	Anita	S(seek)	Where can I erase?
02:08	Dr. G	T(p)	Anything you like.

02:09	Anita	S(con)	Okay.
02:11	Dr. G	T(p)	Except the graph. Leave the graph.
02:13	Anita	S(con)	Alright.
02:25	Anita	S(a)	What did you say Stephanie?
02:27	Stephanie	S(ans)	the first (inaudible)
02:28	Anita	S(con)	Okay. Oops Sorry. Okay
02:36	Stephanie	S(ta)	Equals to two over x to the one-half, quantity to the one-half
02:41	Anita	S(con)	Alright
02:45	Stephanie	S(ta)	And then you're supposed to add one to the exponent
02:49	Anita	S(con)	Okay
02:58	Dr. G	T(s)	Time out. What you now have is a false statement.
03:09	Stephanie	S(con)	Yeah don't write it like that.
03:10	Anita	S(con)	Okay.
03:13	Stephanie	S(ans)	Erase the plus one too. And erase the equals
03:23	Anita	S(a)	I'm going from the bottom.
03:24	Stephanie	S(con)	Yeah.
03:25	Anita		Chance
03:28	Chance	S(qs)	Is this like rule with the function thing or?
03:30	Dr. G	T(con-t)	Time out. I'll help you with that. Right now this is still a derivative right?
03:36	Anita	S(con)	Right

03:37	Dr. G	T(f)	So we're still in our lower case. Where are you going to take it?
03:45	Anita	S(ans)	Underneath
03:47	Dr. G	T(c)	This is going from where to where. Lowercase to
03:49	Anita	S(ans)	To big f.
03:50	Dr. G	T(con-s)	Alright.
04:06	Anita		Like that.
04:09	Stephanie	S(a)	What does it say?
04:11	Anita	S(ans)	Three over two
04:12	Stephanie	S(ans)	Okay, but you have to like. Erase it and put a little farther away to the equal sign because
04:18	Anita	S(con)	Okay.
04:19	Stephanie	S(ta)	Well actually no, you can do it another line. But then you have to divide the whole expression by the exponent.
04:27	Anita	S(a)	Divide by three over two
04:28	Stephanie	S(con)	Yes
04:30	Dr. G	T(p)	Woop, woop. Finish that sentence
04:32	Stephanie	S(con)	yeah in that line
04:38	Anita		Show it. And now I can
04:43	Dr. G	T(d)	Almost. Chris
04:47	Chris	S(qs)	Can we multiply by the reciprocal here.
04:50	Anita	S(con)	Yeah
04:51	Dr. G	T(p)	Excellent. But finish this line which is almost done.

04:55	Chris		Okay, um.
05:08	Anita		Amy
05:11	Amy	S(ta)	Um, it'll help if you. You know how you have the inside the parenthesis is two over x. If you move the two on the outside and do one over x.
05:18	Dr. G	T(p)	Not yet. Not yet
05:20	Tracy	S(ans)	Plus C
05:22	Dr. G	T(con-t)	There you go. Remember we are generating a family of potential parent functions. Okay?
05:27	Chance	S(qs)	What's C?
05:29	Anita	S(ans)	A constant
05:33	Dr. G	T(p)	Now. I'm going to let you go one more line anticipating a little simplifying
05:39	Anita	S(seek)	Okay. What does all this simplify to?
05:47	Dr. G	T(rep)	Pardon
05:47	Stephanie		When you.
05:48	Anita		Yes, Stephanie.
05:49	Stephanie	S(ta)	When you divide by a fraction.
05:51	Anita	S(ta)	That's like multiplying by reciprocal. Right. So it would be like two over x times two over three. Like the two x times the two
06:01	Stephanie	S(ta)	two-thirds times the whole quantity.
06:06	Anita	S(a)	Times all of this

06:07	Stephanie	S(con)	Yes
06:10	Anita	S(a)	Which is
06:12	Stephanie	S(ans)	Two-thirds. Just write two-thirds.
06:14	Anita	S(con)	Okay
06:23	Dr. G	T(f)	Woah, woah, woah. How many parenthesis are you going to use here?
06:25	Anita	S(ans)	Well it's like here
06:27	Dr. G	T(p)	I see what you're doing. Go ahead.
06:30	Dr. G	T(d)	Alright. Now hold on a second. Um. Questions so far before I walk into this.
06:41	Anita		Eric
06:42	Eric	S(qs)	Do we need one of those sets of parenthesis. The outer set.
06:45	Anita	S(pb)	Well it's just these should be multiplied, but you just put a dot.
06:52	Eric		I was just saying. I don't know.
			Several students discuss the parenthesis while Dr. G points something out to Anita on the board.
07:05	Dr. G	T(s)	Now. Here's the part that you cannot forget to think about. This is supposed a potential function for which the derivative is known. This is supposed to be a potential function that when we take the derivative of this function, we will get this. So thank you Anita. Vinny come on

down here. (writes F(x) on board) To Vinny: Bring this up here and take the derivative.

08:47	Vinny	S(seek)	Should I keep little f of x
09:17	Dr. G	T(d)	Timeout. Chris.
09:18	Chris	S(qs)	Can you write capital F?
09:21	Vinny	S(a)	Capital F
09:23	Chris	S(con)	Yeah
09:24	Vinny	S(a)	For this
09:25	Chris	S(con)	Yeah. Right
09:27	Vinny	S(pb)	It's little f because it's eventually going to come to this
09:31	Chris	S(con)	Right
09:32	Vinny		So
09:35	Chris	S(a)	You're making your way to that
09:36	Vinny	S(con)	Yeah
09:38	Chris	S(a)	So. Yeah. You're continuing the top right.
09:42	Vinny	S(ans)	No, I'm doing the derivative
09:47	Dr. G		Time out.
09:48	Vinny		Amy
09:50	Amy	S(qs)	Why are you doing the derivative of it?
09:51	Vinny	S(ans)	To prove that it's correct.
09:54	Amy	S(a)	That what's correct
09:55	Vinny	S(ans)	That this is the parent function.

09:58	Dr. G	T(d)	Time out. We're going to wait to for more critique of line 2.
10:08	Vinny		Uh.
10:11	Dr. G	T(d)	Saran
10:12	Saran	S(ans)	You have to find the derivative of a quotient.
10:14	Dr. G	T(con-s)	Excellent. Now every now and then one of you comes off with an observation that is very astute.
10:18	Vinny	S(con)	Oh yeah
10:18	Dr. G	T(con-t)	Okay? That was the critical piece this afternoon. Okay?
10:38	Vinny	S(seek)	Can I erase all this?
10:39	Dr. G	T(con-s)	Yep
11:12	Dr. G		Now if you need help with this part Tracy is an expert
11:15	Tracy	S(ta)	We did it already so why don't you just. It's negative two over x.
11:22	Rebecca	S(ans)	It's not negative two over x
11:26	Tracy	S(ans)	We did it already.
12:19	Dr. G	T(r)	Okay. Now. Is the derivative of this function the one we started with?
12:29	Students	S(ans)	No
12:32	Dr. G	T(e)	Now. What went wrong? When you reverse the power process where was the error? Nick
12:43	Nick	S(ta)	The two over three in that function should be separate parenthesis.

12:48	Eric	S(ans)	No.
12:52	Dr. G	T(con-s)	This is okay outside the parenthesis because to divide by this fraction is to multiply everything in the numerator by it's reciprocal.
13:04	Eric	S(ta)	It's from the chunk in the middle, the two over x.
13:08	Dr. G	T(d)	The power rule goes with the single variable x, so to speak. Here we have a function of x. So you were not reversing the power rule as you thought you were because this a function of x not just the independent variable x. So Amy, what's the point here.
13:33	Amy	S(ta)	If you pulled it out and just did one over x would that
13:37	Dr. G	T(f)	That's still a function of x. Until you have x as simply the independent variable x not a function there of, you can not reverse the power rule.
13:49	Eric	S(qs)	How do you reverse the quotient rule?
13:51	Amy	S(ta)	But you're saying that the problem is one over x now.
13:53	Dr. G		That's a function of x
13:56	Amy	S(ta)	So instead of, why can't you just make the exponent negative?
14:00	Dr. G	T(j)	Why don't we do that?
14:02	Eric		Let's go for it. Vinny
14:04	Vinny	S(a)	Wait, what am I going to ?

14:06	Dr. G	T(r)	You got to back up and manipulate lower case f in a different way
14:15	Vinny	S(con)	Alright
14:16	Dr. G		That's not going to fly
	Several students		Students comment on whether or not Dr. G knew there was a mistake. Eric states that he has done this all year to them.
14:44	Dr. G	T(p)	So Vinny. Point to the first line under lower case f.
14:48	Vinny	S(a)	First line under it.
14:49	Dr. G	T(p)	Under it. Erase from that line on through. It's all useless.
15:05	Vinny	S(seek)	Should I erase that too?
15:06	Dr. G	T(con-s)	Yep
15:25	Dr. G	T(r), T(d)	Alright so how we going to do this? Chris
15:30	Chris	S(dnc)	Um, Nevermind
15:38	Dr. G	T(d), T(r)	Vinny what do you think? Is there another way to write that more friendly to our cause?
15:45	Vinny	S(ans)	Probably, but I don't know
15:49	Dr. G	T(d)	Stephanie
15:50	Stephanie	S(ta)	Um, the square root of two times x to the negative one-half.
15:58	Vinny	S(con)	Okay.
16:15	Dr. G	T(p)	Now you can reverse the power rule.
16:26	Dr. G		Ah, ah, ah.

16:30	Vinny	S(a)	Should it be. [writes f(x) = sqrt 2 x^(-1/2)]
17:15	Dr. G	T(d)	How do you reverse it Stephanie?
17:17	Stephanie	S(ta)	Uh, What. Oh. Um, The square root of two is like a constant so you don't really have to take that into account when you do the whole add one and divide by the exponent thing. So you can just leave it as the square of root of two.
17:34	Vinny	S(con)	Okay so
17:34	Stephanie	S(con)	I'm, I'm pretty sure this is correct.
17:36	Dr. G	T(con-s)	Correct. You're doing good.
17:36	Stephanie	S(ta)	The square root of two times x to the one-half plus one, which is three-halves. So just change the exponent to three-halves.
17:48	Vinny	S(qs)	This is to the negative one-half so do I just go one-half
17:50	Stephanie	S(ta)	Oh so yeah. Yeah okay. And then divide by one-half
17:58	Vinny	S(con)	Uh, Okay [Writes / under whole expression, then 1/2]
18:05	Stephanie	S(ta)	But the just the x by the one-half not the whole line.
18:08	Dr. G	T(c)	What's that mean just the x?
18:10	Stephanie	S(c)	Just the variable not the constant.
18:13	Dr. G	T(f)	But since it's a multiplication problem does it matter?
18:17	Stephanie	S(ans)	No, no because
18:40	Dr. G	T(con-t)	Now. Questions?
18:47	Student	S(ta)	Is there supposed to be a constant?

18:56	Dr. G	T(p)	Technically that would have been written on the previous line too. Okay. Thank you Vinny. Now Eric go find the derivative of upper case f.
19:19	Eric	S(seek)	Uh, Should I change this back to, uh, two x to the one-half, to the one-half power
19:32	Dr. G	T(d)	Question Amy?
19:33	Amy	S(ta)	Is the x supposed to be under the square root or no?
19:35	Students	S(ans)	yes
19:36	Amy	S(qs)	Why?
19:39	Students		Cause it's x to
19:52	Dr. G		Timeout.
19:54	Chris	S(ta)	You know the quickest way to do it. Do two times the square root of two times x to the one-half times. It's the same thing. It's quicker
20:01	Eric	S(a)	Wait, what do I want?
20:04	Chris	S(c)	Two, two times the square root of two times x to the one-half. Just write it that way it's quicker
20:27	Dr. G	T(c)	Now we need some clarification for line two. Chris.
20:31	Chris	S(ans)	Yeah. No keep going do one-half.
20:35	Dr. G		Wait, wait, wait
20:37	Student	S(ans)	Plus C
20:46	Eric	S(ta)	Won't the C equal zero. Oh wait this is still the parent function. Alright. Keep going. Okay.

21:31	Dr. G	T(p)	Now. If you leave it that way Eric, you're leaving the reader with a decision to make and you don't want to do that.
21:37	Eric	S(con)	Oh, Right.
21:44	Dr. G	T(r)	Okay. Now you have validated that you have indeed the correct parent function. Given that Mikey, go finish the area. Go finish calculating the area.
22:03	Mike	S(seek)	The interval
22:04	Dr. G	T(rep)	What's that?
22:05	Mike	S(seek)	What's the interval?
22:08	Chris(2)	S(ans)	From one to two
22:09	Mike	S(con)	Oh yeah
22:24	Dr. G	T(p)	Now what I want you to do when you leave here. I want you to get in the calculators and look at finding this area two different ways with the calculator and I want you to finish it as well using the limit process for an infinite number of rectangles.
23:01	Dr. G	T(con-t)	Questions?
	Dr. G	T(e)	Now explain that piece right there
23:12	Mike	S(pb)	Yeah, you just take the larger number of the distance that we
23:20	Dr. G	T(con-s)	Okay
23:20	Mike	S(pb)	and subtract it from the smaller number

23:23	Dr. G	T(con-s)	Alright
23:24	Mike	S(seek)	Uh, what do I do with c, just leave it out?
23:27	Eric	S(a)	What are you talking about?
23:29	Dr. G	T(f)	Put it in once for the moment and see what happens
23:39	Dr. G	T(p)	Keep going
23:40	Mike	S(seek)	Why am I wasting time?
23:42	Dr. G	T(p)	Keep going
23:48	Student	S(pb)	Cause it's going to be plus c and minus
23:51	Mike	S(con)	Oh. I see.
24:34	Dr. G	T(p)	Now run this graph into your calculator and set in exact mode. Uh, I take that back. Don't do the graph first. Do the integration process first under calculus menu and set it in exact mode. Go ahead Mikey. In your calculators, under the calculus menu, in exact mode. See what it comes up with. Students commenting to each other that they get the same thing
25:21	Dr. G	T(p)	Now Chris let's begin to set up this limit form which has to come out to four minus two root two. So go start. This is your primary job for the weekend. Make this limit question actually equal four minus two root two.
25:42	Chris	S(a)	So find the limit of the derivative.

25:45	Dr. G	T(r)	No, find the limit for the sum of the areas of an infinitely large number of rectangles?
26:02	Eric	S(qs)	What is that?
26:03	Student	S(ans)	Limit
26:03	Eric	S(qs)	I know, but what's the thing.
26:04	Dr. G	T(s)	An n. Eric you should know that
26:08	Eric	S(con)	No, I know it's an n, but it looks. Alright
26:26	Dr. G	T(d)	Comment Chance.
26:29	Chance	S(a)	Me Comment
26:30	Dr. G	T(con-s)	Yeah
26:31	Chance	S(dnc)	I don't have a comment
26:32	Dr. G	T(rep)	Yeah you do.
26:33	Chance	S(a)	Really.
26:34	Dr. G	T(d)	Alright Charlie.
26:34	Chance	S(ta)	n. n. Oh I know. N on top of the summation thing instead of writing infinity. N
26:42	Chris	S(con)	Oh, I understand now.
26:53	Dr. G	T(r)	Now how wide is one of these?
26:55	Chris	S(ans)	One-fourth
16:56	Dr. G	T(c)	Say what?
26:58	Chris	S(ans)	One-fourth
26:59	Dr. G	T(c)	Say what?
27:04	Chris	S(ans)	One-fourth

27:05	Dr. G	T(c)	Say what?
27:07	Chance	S(ans)	No it's not
27:35	Dr. G	T(p)	Now. You know. So you kind of know where this is going that radical is probably going to have to be enlarged.
27:43	Chris		Yeah
27:50	Dr. G	T(p)	You got a lot of stuff to put in that denominator and you got to make it very clear it's all in that denominator
28:26	Dr. G	T(p)	The bracket is probably unnecessary.
28:29	Chris		Correct.
28:31	Dr. G	T(con-t)	Now anybody have a problem with this?
	Dr. G	T(d)	I have a strong sense you're definitely going to have a problem when you sit down and do it, but how about the first line. Amy
28:48	Amy	S(ta)	Is the parenthesis supposed to be one?
28:52	Dr. G	T(rep)	Pardon
28:53	Amy	S(ta)	Is the parenthesis. Instead of parenthesis is it supposed to be one. Cause we're only
28:57	Dr. G	T(c)	I don't even any see parenthesis
28:58	Amy	S(ans)	Right there in the denominator
29:02	Dr. G	T(con-s)	Oh that one
29:03	Amy	S(seek)	Should it be one?
29:03	Dr. G	T(j)	Why?

29:04	Amy	S(ta)	Cause there. I don't know, cause we're only moving one block over.
29:09	Dr. G	T(d)	Can you answer that question Chris?
29:11	Chris	S(pb)	You're going, like the I, it's a number, so you're going by one over n, that infinite number. You're going from one to say four. You know what I mean. To like the four. You know what I mean. Chris.
29:30	Chris(2)	S(pb)	Um, in the parenthesis it's the, uh, distance over the amount of rectangles and n is the amount of rectangles and the distance is two minus one equals one.
29:45	Dr. G		He walked across one width, then he'll walk across two widths, three widths, four widths. The i represents the count. The one over n represents the width and to get the width it's the length of the interval, as Chris (2) just said, divided by the number of rectangles. Now the critical part is what to do with that radical ladies and gentlemen.
			Bell rings and class is dismissed

Curriculum Vitae

1.   <u>Academic Degrees</u>

Ph.D. Rutgers University   2009   Mathematics Education
M.S.   Rutgers University   2006   Mathematics
Ed.M. Rutgers University   1999   Mathematics Education
BA   Drew University   1997   Mathematics

2.   <u>Professional Experience</u>

T3 Regional Instructor, Texas Instruments, Dallas, TX

Professional development of teachers using TI technology. Presentation at conferences, development and delivery of workshops, and continued personal training on new technology.

Supervisor of Mathematics, 6- 12, Ridgewood Public Schools, Ridgewood, NJ

Oversight of 6-12 mathematics program, including implementation of Connected Mathematics program in grades 6 – 8, observation and professional development, determination of department budget, and other supervisor committees.

3.   <u>Publications</u>

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