FLUX COMPACTIFICATIONS, DUAL GAUGE THEORIES AND SUPERSYMMETRY BREAKING

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A dissertation submitted to the

Graduate School—New Brunswick

Rutgers, The State University of New Jersey

in partial fulfillment of the requirements

for the degree of

Doctor of Philosophy

Graduate Program in Physics and Astronomy

Written under the direction of

Professor Michael R. Douglas

and approved by

New Brunswick, New Jersey

May, 2009

ABSTRACT OF THE DISSERTATION

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The nonholomorphic sector of four dimensional theories with $\mathcal{N} = 1$ supersymmetry that arises from string compactifications is analyzed, and new models of supersymmetry breaking (both in string and field theory) are presented. The dissertation combines three complementary viewpoints.

First, space-time effects in 4d supergravity are studied from type IIB string theory compactified on warped Calabi-Yau manifolds with fluxes. The vacuum structure of supersymmetric flux compactifications is well understood and our aim is to extend this to include space-time dependence in the presence of nontrivial warping. Going beyond the static limit is required in order to compute kinetic terms and masses. We develop formalism for identifying the microscopic 10d fluctuations that give rise to fields in the low energy 4d theory, and we present a general formula for their kinetic terms. As an application, the effective theories for the universal Kähler modulus and the complex modulus of the warped deformed conifold are determined. The full effective action for warped compactifications is calculated to quadratic order, including both 4d zero modes and their light Kaluza-Klein excitations.

Next, using gauge/gravity dualities, we consider the previous results from the gauge theory side. The focus is on the warped deformed conifold, which is dual to the Klebanov-Strassler gauge theory. In the infrared it reduces to four dimensional pure super Yang-Mills, corresponding to D5 branes in the resolved conifold. The closed string analysis reveals a new term in the Kähler potential for the complex modulus, which has important effects on the low energy theory. It is suggested that this term has a natural interpretation in the dual gauge theory, in terms of the composite nature of the gaugino condensate.

Finally, new models of supersymmetry breaking are developed. From the string theory side, we analyze supersymmetry breaking by anti-self-dual flux in the deformed conifold. The theory develops a parametrically small scale of supersymmetry breaking, once warp corrections to the Kähler potential are included. In the field theory side, we construct a model of metastable supersymmetry breaking in SQCD where all the relevant parameters generated dynamically. It is argued that it is possible to balance non-perturbative effects against perturbative corrections. Furthermore, metastable vacua in SQCD with multitrace deformations are explored, with the aim of obtaining an acceptable phenomenology.

Acknowledgements

First, I would like to thank my advisor Michael Douglas for his guidance, help and support during these years. His unique way of approaching physics has opened my mind to new possibilities. I am very grateful to Profs. Tom Banks, Emanuel Diaconescu, Dan Friedan, Greg Moore, Herbert Neuberger, Joe Polchinski, Matt Strassler and Scott Thomas. Through courses and collaborations, I have learned many important lessons from them.

From my first steps in theoretical physics I have had the guidance of my M.S. advisor César Fosco; he has always been there when I needed him. I would also like to thank Diane Soyak for all her support, and Prof. Ron Ransome for his constant help through my graduate studies.

I am very grateful to Seba, Sridhar, Rouven, Kuver, Jeff, Sam, Sergio, Judy and Warren, Greg and Vale, the Mosteiro family, C.J., Alberto, Loreto and Ron, and the Rutgers Catholic Center. They have changed my life in the US. I would like to especially thank my housemates and friends Korneel, Jose, Mihai and Nachi for their support through good and bad times –and nights of work.

Finally, I wouldn't have been able to make it without the support and love from my family, Mady and Anabela.

Dedication

Para Mamá, Papá, el abuelo, la abuela y Ana.

Por enseñarme las cosas importantes de la vida.

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Chapter 1
Introduction and overview

1.1 Motivation and aims

One of the central problems in particle physics is to understand the high energy physics beyond the Standard Model (SM), and new hints are expected soon from the LHC. In this context, it becomes important to have an idea of what type of new physics is allowed. This is done by postulating high energy extensions of the SM and examining their properties. The working hypotheses in this thesis will be that there is a supersymmetric field theory extension of the SM, and that this field theory arises as a low energy limit of string theory.

Supersymmetry is a beautiful idea that leads to quantum field theories where certain parameters are protected from perturbative quantum corrections. One could suspect that field theories with such improved renormalization properties would only serve as toy models, but the surprise is that it is possible to build phenomenologically viable supersymmetric models. Some of the main successes of this approach are the stabilization of the electroweak scale, and the unification of gauge couplings in supersymmetric GUTs.

The first step in constructing a realistic supersymmetric model is to specify the amount of supersymmetry and matter content. The usual approach is to allow for four real supercharges, that is, $\mathcal{N} = 1$ in 4d. There are also models containing sectors with eight supercharges ($\mathcal{N} = 2$), and they offer some interesting alternatives to the models with $\mathcal{N} = 1$. This is also a natural scenario in string theory. The matter content consists of vector supermultiplets (gauge bosons and gauginos), chiral supermultiplets (fermions and scalars) and supergravity multiplets (graviton and gravitino).

The next step is to specify interactions leading to F- and/or D-terms that break supersymmetry spontaneously and lead to an acceptable low energy phenomenology. This is required to create mass-splittings between the observed SM fermions and their postulated superpartners. At this stage, some quantum corrections no longer cancel exactly, but rather become proportional to (powers of) the mass-splittings. The stabilization of the electroweak scale would then suggest the existence of superpartners at the TeV scale.

On the other hand, why string theory? A possible answer is that the theory appears to be unique, with no free parameters. This would lead to a unique UV completion of the SM explaining all its parameters and, at the same time, would give a consistent unification of quantum field theory and gravity. However, it has become increasingly clear that this line of thought is extremely hard to realize. The main obstacle is that it is not known what string theory is. Different limits of the theory are well understood, but the fundamental degrees of freedom are not known. However, a more pragmatic approach can be taken. Based on what is already known about string theory, it is clear that it is a consistent framework combining

- gravitational physics: supergravity, fluxes, black holes,
- gauge theories: intersecting branes, branes wrapped on cycles or at singularities,
- deep connections between both via gauge/gravity dualities (e.g. AdS/CFT).

So far, string theory seems to be the only candidate with these features, and this is a very strong motivation to use it to learn about new high energy physics. In fact, it has already led to many new ideas in gravity, supersymmetric gauge theories and phenomenology.

Moreover, string theory can reproduce a large number of effective field theories, but not every field theory can be obtained in this way. By restricting to this string landscape of theories, it is possible to find new field theory phenomena. These would be required by consistency with gravity, and would otherwise be undetected in the general framework of effective field theories.

A basic property of critical string theory is that it predicts ten space-time dimensions. There are then two ways of making contact with 4d physics. One is to localize the low energy fields on four dimensional hypersurfaces (branes). This localization is, in a sense, dynamical, and leads to beautiful geometrical reformulations of gauge theories. The other option is to compactify the six internal dimensions. A basic requirement is that this should not be put in by hand, as was originally emphasized by Cremmer and Scherk. Rather, the compactification should occur spontaneously, that is, the higher dimensional theory should have a stable vacuum of the form $\mathbb{R}^{3,1} \times X_6$. This leads to the idea of flux compactifications. Actually, these two approaches are, in many cases, dual descriptions of the same underlying string theory.

Having presented our hypotheses and motivations, let us discuss the aims of this dissertation. The first one is to reproduce space-time effects of the low energy $\mathcal{N} = 1$ theory from the point of view of string theory. What has been understood so far is how to relate the vacuum properties of 4d theory (*i.e* the extreme low energy limit) to those of the higher dimensional theory.

In contrast, the questions we will attempt to answer are of dynamical origin[1]: how do the propagating 4d fields arise from the microscopic 10d fluctuations? What are their kinetic terms and (in the susy case) the Kähler potential? Can Kaluza-Klein modes become important in the effective theory?

These questions need to be answered in order to understand most of the 4d physics, including physical masses, Yukawa couplings, etc. The emphasis will be on computing the effective action for a generic $\mathcal{N} = 1$ theory arising from string theory, but not on reproducing precise details of the SM. The analysis is based on type IIB supergravity with fluxes, because this is the simplest string theory limit that can produce viable "spontaneous compactification." Surprisingly, our analysis reveals new terms in the Kähler potential, and they can affect the effective theory even at a qualitative level. The next step is to extend the approach to include branes. This has not been solved in full generality yet, but we will be able to extract dynamical properties for certain brane configurations with the help of gauge/gravity dualities.

Once the basic properties of the $\mathcal{N} = 1$ theory are understood, the problem of supersymmetry breaking has to be addressed. Our second aim is to explore new mechanisms for supersymmetry breaking, both in string theory and directly in the 4d field theory. From the string theoretic point of view, we present a simple flux construction that can yield a parametrically small scale of supersymmetry breaking compared to the string scale. This turns out to be a direct consequence of the new terms found in the Kähler potential. From a field theoretic point of view, we will analyze supersymmetry breaking models based on metastable dynamical supersymmetry breaking. The idea that our universe may be a long-lived metastable state has opened many new avenues for model-building. Exploring some of its consequences is the subject of the last chapters in this dissertation.

This thesis focuses on the papers [1–7] published during the course of graduate studies. Their results appear as follows:

- Chapter 2 and section 4.5 are based on [1].
- Chapter 3 is based on [2].
- Sections 4.2, 4.3 and 4.4 summarize [3].
- Sections 4.5, 4.7 and 4.6 are based on [4].
- Chapter 5 is a summary of [5] and contains work in progress.

[1] The word 'dynamics' will be used in a loose sense, to mean effects involving nontrivial space-time dependence.

- Chapters 6 and 7 are based on [6] and [7], respectively.

In the rest of this chapter we will review some background material to provide the necessary tools for the rest of the dissertation.

1.2 Four dimensional theories with global supersymmetry

We start with a review of supersymmetric field theories, putting emphasis on some aspects that will be needed in the rest of the work. We follow [8] and the $\mathcal{N} = 2$ discussion is based on [9].

1.2.1 $\mathcal{N} = 1$ theories

In field theories with global $\mathcal{N} = 1$ supersymmetry, the chiral and vector superfields are

$$\begin{aligned} \Phi &= \phi + \sqrt{2}\theta\psi + \theta^2 F \\ V &= -\theta\sigma^\mu\bar{\theta}A_\mu + i\theta^2\bar{\theta}\bar{\lambda} - i\bar{\theta}^2\theta\lambda + \frac{1}{2}\theta^2\bar{\theta}^2 D\,. \end{aligned} \tag{1.1}$$

The matter content is a scalar and fermion in the case of the chiral superfield, and a gauge boson and gaugino for the vector superfield. We assume that Φ transforms in the fundamental representation of the (here arbitrary) gauge group. It is also useful to introduce the field strength

$$W_\alpha = -i\lambda_\alpha + \theta_\alpha D - \frac{1}{2}(\sigma^\mu\bar{\sigma}^\nu\theta)_\alpha F_{\mu\nu} + \theta^2(\sigma^\mu\partial_\mu\lambda)_\alpha \tag{1.2}$$

which follows from taking three superspace derivatives of V. We also recall that the holomorphic gauge coupling is defined as

$$\tau = \frac{\theta}{2\pi} + \frac{4\pi i}{g^2}\,. \tag{1.3}$$

The $\mathcal{N} = 1$ lagrangian is

$$L = \frac{1}{8\pi}\mathrm{Im}\,\mathrm{Tr}\left(\tau\int d^2\theta\, W^\alpha W_\alpha\right) + \int d^4\theta\,\Phi_i^\dagger e^{-2V}\Phi_i + \int d^2\theta\, W + c.c. \tag{1.4}$$

where W is the superpotential for the matter sector and we allow for flavors Φ_i, $i = 1, \ldots, N_f$. This lagrangian is invariant under the following supersymmetry transformations of fermionic parameter ξ:

$$\begin{aligned} \delta_\xi\phi &= \sqrt{2}\xi\psi\,,\quad \delta_\xi\psi = \sqrt{2}\xi F + i\sqrt{2}\sigma^\mu\bar{\xi}D_\mu\phi \\ \delta_\xi F &= i\sqrt{2}\bar{\xi}\bar{\sigma}^\mu D_\mu\psi + 2i\phi\bar{\xi}\bar{\lambda} \\ \delta_\xi A_\mu^a &= -i\bar{\xi}\bar{\sigma}_\mu\lambda^a + i\bar{\lambda}^a\bar{\sigma}_\mu\xi\,,\quad \delta_\xi\lambda^a = i\xi D^a + \sigma^{\mu\nu}\xi F_{\mu\nu}^a \\ \delta_\xi D^a &= -\xi\sigma^\mu D_\mu\bar{\lambda}^a - D_\mu\lambda^a\sigma^\mu\bar{\xi}\,. \end{aligned} \tag{1.5}$$

The superpotential can only receive nonperturbative corrections, while the holomorphic gauge coupling can be corrected at one loop and nonperturbatively. However, the non-holomorphic terms (like the kinetic function) are not protected against perturbative quantum corrections. Therefore, physical couplings are corrected quantum-mechanically through wavefunction renormalization.

The general lagrangian for matter fields including quantum effects is then of the form

$$L_{matter} = \int d^4\theta \, K(\Phi^\dagger, \Phi) + \int d^2\theta \, W(\Phi) + c.c.\,. \tag{1.6}$$

K is called the *Kähler potential*. The semiclassical limit corresponds to $K = \Phi^\dagger \Phi$, recovering Eq. (1.4). The effective field theory will include higher order terms as well. Expanding in components and integrating out the auxiliary field F, the Lagrangian becomes

$$\begin{aligned} L_{matter} &= -G_{i\bar{j}}\, \partial_\mu \phi^i \partial^\mu \bar{\phi}^j - iG_{i\bar{j}}\, \bar{\psi}\bar{\sigma}^\mu D_\mu \psi^i + \frac{1}{4}\, R_{i\bar{j}k\bar{l}}\, \psi^i \psi^k \bar{\psi}^j \bar{\psi}^l + \\ &- G^{i\bar{j}} \partial_i W \partial_{\bar{j}} W^* - \frac{1}{2} D_i \partial_j W\, \psi^i \psi^j - \frac{1}{2} D_{\bar{i}} \partial_{\bar{j}} W^*\, \bar{\psi}^i \bar{\psi}^j\,. \end{aligned} \tag{1.7}$$

The notation is

$$G_{i\bar{j}} = \frac{\partial^2 K}{\partial \phi^i \, \partial \phi^{*j}}\ ,\ \partial_i W = \frac{\partial W}{\partial \phi^i} \tag{1.8}$$

and $R_{i\bar{j}k\bar{l}}$ is formally the same as the curvature tensor corresponding to a "metric" $G_{i\bar{j}}$. Similarly, D_i is the covariant derivative compatible with $G_{i\bar{j}}$.

Let us interpret these results. From the kinetic term, the fields $(\Phi, \Phi^\dagger)$ can be interpreted as coordinates in a curved "target space" with metric $G_{i\bar{j}} = \partial_i \partial_{\bar{j}} K$. A complex manifold with a metric of this form is known as a Kähler manifold. In fact, the supersymmetric lagrangian also knows about the connection on this space, through the mass terms,

$$D_i \partial_j W = \partial_i \partial_j W - \Gamma^k_{ij}\, \partial_k W\,.$$

Finally, the curvature of the Kähler manifold is probed by the quartic fermionic term. It is also important to notice that Eq. (1.6) is invariant under the transformation

$$K(\phi, \phi^*) \to K(\phi, \phi^*) + f(\phi) + f^*(\phi^*)\,. \tag{1.9}$$

1.2.2 $\mathcal{N} = 2$ theories

The supermultiplets in theories with global $\mathcal{N} = 2$ supersymmetry are

- the $\mathcal{N} = 2$ vector multiplet, which contains an $\mathcal{N} = 1$ chiral multiplet $\Phi = (\phi, \psi)$ plus an $\mathcal{N} = 1$ vector multiplet $W_\alpha = (A_\mu, \lambda_\alpha)$,

- the $\mathcal{N}=2$ hypermultiplet contains two $\mathcal{N}=1$ chiral multiplets $(Q, \tilde{Q}^\dagger)$, where both are in the fundamental representation of the gauge group.

Consider the classical action for the vector multiplets. Since ψ and λ are on the same footing, there cannot be a nontrivial superpotential; also they both have the same kinetic term normalization. The action reads [9]

$$L_V = \frac{1}{8\pi} \operatorname{Im} \operatorname{Tr} \left[\tau \left(\int d^2\theta \, W_\alpha W^\alpha + 2 \int d^4\theta \, \Phi^\dagger e^{-2V} \Phi \right) \right] . \tag{1.10}$$

Integrating out the D-term produces a potential

$$V_D = -\frac{1}{2g^2} \operatorname{Tr} \left([\phi^\dagger, \phi]^2 \right) .$$

The crucial difference with the $\mathcal{N}=1$ case is that in theories with $\mathcal{N}=2$ supersymmetry, the kinetic term for matter fields is determined by the gaugino kinetic term. Therefore, the Kähler potential only receives one loop and nonperturbative corrections. All the other loop corrections cancel by holomorphy!

More concretely, the $\mathcal{N}=2$ lagrangian for a vector multiplet including quantum corrections takes the form

$$L_V = \frac{1}{8\pi} \operatorname{Im} \operatorname{Tr} \left(\int d^2\theta \frac{\partial^2 \mathcal{F}}{\partial\Phi \, \partial\Phi} \, W_\alpha W^\alpha + 2 \int d^4\theta \, \Phi^\dagger e^{-2V} \frac{\partial \mathcal{F}}{\partial \Phi} \right) . \tag{1.11}$$

Here, $\mathcal{F}$ is known as the prepotential. Supersymmetry restricts possible quantum corrections to be of the form Eq. (1.11). Expressing this lagrangian in components, we find the kinetic term metric and Kähler potential

$$G_{i\bar{j}} = \operatorname{Im} \frac{\partial^2 \mathcal{F}}{\partial \Phi^i \, \partial \Phi^j} \ , \ K(\Phi^\dagger, \Phi) = \operatorname{Im} \left(\Phi^{\dagger i} \frac{\partial \mathcal{F}}{\partial \Phi^i} \right) . \tag{1.12}$$

A target space with these geometrical properties is called a special Kähler manifold. For the case of an $SU(2)$ gauge theory, Seiberg and Witten [10] managed to compute the full prepotential, showing remarkably rich nonperturbative phenomena.

On the other hand, the action for N_f hypermultiplets $(Q_i, Q_i^\dagger)$, $i = 1, \dots, N_f$, is

$$L_H = \int d^4\theta \left(Q_i^\dagger e^{-2V} Q_i + \tilde{Q}_i e^{2V} \tilde{Q}_i^\dagger \right) + \int d^2\theta \left(\sqrt{2} \tilde{Q}_i \Phi Q_i + m_i \tilde{Q}_i Q_i \right) + c.c. \tag{1.13}$$

It has been argued in [11] that this sector of the $\mathcal{N}=2$ gauge theory does not receive quantum corrections.

1.2.3 Spontaneous supersymmetry breaking

In order to find a realistic phenomenology, supersymmetry has to be broken spontaneously. From the fermion variations in Eq. (1.5), this can happen if F or D acquire expectation values.

A simple example of supersymmetry breaking by an F-term is the O'Raifeartaigh model. It consists of three chiral superfields (X, ϕ_1, ϕ_2) with canonical Kähler potential and

$$W = X\left(\frac{1}{2}\phi_1^2 - \mu^2\right) + m\phi_1\phi_2\,. \tag{1.14}$$

The F-terms are

$$W_X = \frac{1}{2}\phi_1^2 - \mu^2\;,\; W_{\phi_1} = X\phi_1 + m\phi_2\;,\; W_{\phi_2} = m\phi_1 \tag{1.15}$$

where a subscript denotes a derivative with respect to the corresponding field. Supersymmetry is broken because W_X and W_{ϕ_2} cannot vanish simultaneously. In particular, for $|\mu|^2 < |m|^2$, the minimum is at $\phi_1 = \phi_2 = 0$, but X is arbitrary. This is our first example of a theory with a *moduli space* – a continuous degenerate set of vacua. The scale of supersymmetry breaking is measured by

$$V_{min} = |W_i|^2 = |\mu^4|\,.$$

This simple example will play an important role in the second part of this dissertation, where it will arise as the infrared limit of certain strongly coupled gauge theories.

In theories with abelian gauge groups it is possible to have D-term supersymmetry breaking by adding a Fayet-Iliopoulos (FI) term. Indeed, noticing that D transforms as a total derivative, the term

$$L_{FI} = 2\kappa \int d^4\theta\, V$$

can be added to the lagrangian Eq. (1.4) without breaking supersymmetry explicitly. This has the effect of shifting D by a constant κ. Therefore, if we start from a theory with a unique supersymmetric vacuum at $D = 0$, and then add a FI term, supersymmetry is spontaneously broken. The simplest example is super QED,

$$L = \int d^2\theta \left(\frac{1}{4}W_\alpha W^\alpha + m\Phi_1\Phi_2\right) + c.c. + \int d^4\theta \left(\Phi_1^\dagger e^{gV}\Phi_1 + \Phi_2^\dagger e^{-gV}\Phi_2 + 2\kappa V\right)\,. \tag{1.16}$$

The auxiliary fields then become

$$\begin{aligned} D &= -\kappa - \frac{g}{2}\left(|\phi_1|^2 - |\phi_2|^2\right) \\ W_1 &= m\phi_2\;,\; W_2 = m\phi_1\,. \end{aligned} \tag{1.17}$$

There is no simultaneous solution to these equations with $D = W_i = 0$, so supersymmetry is spontaneously broken.

An important property of theories with spontaneous supersymmetry breaking is that they have a Goldstino – a Nambu-Goldstone fermion associated to the breaking of supersymmetry.

This can be seen from the transformation laws (1.5). Once F or D acquire a vacuum expectation value, the fermion in the corresponding supermultiplet transforms inhomogeneously,

$$\delta\psi_I = \xi G_I + \dots$$

where ψ_I denotes ψ or λ and G_I is F or iD. This shift symmetry forbids a mass term for ψ_I.

Working with theories with $\mathcal{N}=2$ supersymmetry offers the intriguing possibility of *partial supersymmetry breaking* $\mathcal{N}=2 \to \mathcal{N}=1$, as found by [12]. This is in fact what occurs in the string compactifications analyzed in this dissertation, so let us review this in some detail. In order to understand spontaneous supersymmetry breaking for $\mathcal{N}=2$ theories, it is convenient to introduce a superfield formulation. $\mathcal{N}=2$ superspace has fermionic coordinates that we will denote (θ_1, θ_2), where each θ_I generates an $\mathcal{N}=1$ superspace. Then a superfield is a function $f(x, \theta_I, \bar{\theta}_I)$. The vector multiplet corresponds to a chiral superfield,

$$\bar{D}_1 \mathcal{A} = 0 \quad , \quad \bar{D}_2 \mathcal{A} = 0\,.$$

In components,

$$\mathcal{A} = \phi + \theta_I \psi^I + \theta_I \theta_J X^{IJ} + \frac{1}{2}\,(\epsilon^{IJ} \theta_I \sigma^{\mu\nu} \theta_J) F_{\mu\nu} + \dots \tag{1.18}$$

where $\psi^I = (\psi, \lambda)$. The antisymmetric 2×2 matrix X is an auxiliary field, satisfying the reality condition

$$X^{*IJ} = \epsilon_{IL} \epsilon_{JK} X^{LK}\,.$$

It is equivalent to the $\mathcal{N}=1$ auxiliary fields (F, D). Then Eq. (1.18) has the same content as one set of $\mathcal{N}=1$ chiral and vector superfields. The supersymmetry transformations imply that X transform as a total derivative, and

$$\delta\psi^I = i X^{IJ} \xi_J + \dots \tag{1.19}$$

We restrict to an abelian gauge group.

It is then clear that a Fayet-Iliopoulos term can be added to Eq. (1.11), preserving $\mathcal{N}=2$. Then the lagrangian becomes

$$L_V = \mathrm{Im} \int d^2\theta_1 d^2\theta_2\, \mathcal{F}(\mathcal{A}) + X^{IJ} E_{IJ} + c.c. \tag{1.20}$$

where E is a constant matrix. Choosing $\mathcal{F}$ adequately, X can acquire a nonzero vev due to the presence of the FI term, thus breaking supersymmetry spontaneously. Although it may seem that extended supersymmetry can be broken spontaneously only to $\mathcal{N}=0$, this is actually not the case. For instance, consider giving the following expectation values to X,

$$\langle X^{11} \rangle = \langle X^{12} \rangle = 0\,,\ \langle X^{22} \rangle \neq 0\,.$$

From the supersymmetry transformation Eq. (1.19), this implies that $\delta\psi = 0$ but $\delta\lambda \neq 0$. Therefore, the subgroup $\mathcal{N} = 1 \subset \mathcal{N} = 2$ corresponding to θ_1 is preserved, while the $\mathcal{N} = 1$ associated to θ_2 is broken. λ is the Goldstino. More generally, by turning on an appropriate E_{IJ}, it is possible to select which particular $\mathcal{N} = 1$ subgroup of $\mathcal{N} = 2$ will be spontaneously broken.

1.3 Four dimensional supergravity

The next step is to construct supersymmetric theories involving gravity. One approach to supersymmetrize the Einstein-Hilbert action is to promote the previous rigid supersymmetry transformations to local superspace transformations and then study the geometry of superspace. It turns out that the superspace formulation of supergravity is much more complicated than the global superspace formulation, so here we will summarize the results of [8] in component notation. The following analysis will be needed to understand how 4d supergravity is embedded in the 10d theory.

1.3.1 The supergravity action

The $\mathcal{N} = 1$ gravity multiplet has a graviton $h_{\mu\nu}$ and a spin 3/2 gravitino ψ_μ^α. The off-shell theory also has auxiliary fields $M(x)$ and $b_\mu(x)$. The equations of motion imply that M is proportional to the superpotential W of the matter coupled to gravity; b_μ equals a combination of terms containing field derivatives $\partial_\mu\phi$, fermion bilinears $\psi\sigma_\mu\bar{\psi}$, etc.

Let us focus on 4d supergravity with a matter sector given by Eq. (1.6) in the flat space limit. The complete action is given in Eq. (23.3) of [8]. The terms relevant for us are

$$L_{sugra} = -\frac{1}{2}R + \epsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\bar{\sigma}_\nu D_\rho\psi_\sigma + G_{i\bar{j}}\partial_\mu\phi^i\partial^\mu\phi^{*j} - iG_{i\bar{j}}\bar{\psi}^j\bar{\sigma}^\mu D_\mu\psi^i + V + \ldots \tag{1.21}$$

where the scalar potential is

$$V = e^K\left(G^{i\bar{j}}D_iW(D_jW)^* - 3|W|^2\right) \tag{1.22}$$

and '...' include fermionic mass terms and other trilinear and quartic interactions. The covariant derivatives are defined as

$$\begin{aligned}
D_\mu\psi^i &= (\partial_\mu + \omega_\mu)\psi^i + \Gamma^i_{jk}\partial_\mu\phi^j\psi^k - \frac{1}{4}(\partial_jK\partial_\mu\phi^j - \partial_{\bar{j}}K\partial_\mu\phi^{*j})\psi^i \\
D_\mu\psi_\nu &= (\partial_\mu + \omega_\mu)\psi_\nu + \frac{1}{4}(\partial_jK\partial_\mu\phi^j - \partial_{\bar{j}}K\partial_\mu\phi^{*j})\psi_\nu \\
D_iW &= \partial_iW + \partial_iK\,W
\end{aligned} \tag{1.23}$$

Compared to the global limit, the Kähler geometry derived from Eq. (1.21) has some important modifications related to the extra terms appearing in the covariant derivatives, Eq. (1.23). They imply that the fermions and superpotential have to transform nontrivially under the Kähler transformation Eq. (1.9). Indeed, it is possible to check that the lagrangian is now invariant under the combined Kähler-Weyl transformations

$$\begin{aligned} K(\phi,\phi^*) &\rightarrow K(\phi,\phi^*) + f(\phi) + f^*(\phi^*)\ ,\ W \rightarrow e^{-f} W\ , \\ \psi^i &\rightarrow e^{i\mathrm{Im} f/2}\psi^i\ ,\ \psi_\mu \rightarrow e^{-i\mathrm{Im} f/2}\psi_\mu\,. \end{aligned} \tag{1.24}$$

1.3.2 Supersymmetry breaking

The fermion and gravitino supersymmetry variations are of the form

$$\delta_\xi\psi^i = -\sqrt{2}e^{K/2}G^{i\bar{j}}D_{\bar{j}}W^*\,\xi(x) + \dots\ ,\ \ \delta_\xi\psi_\mu = 2D_\mu\xi(x) + ie^{K/2}W\,\sigma_\mu\bar{\xi}(x) + \dots\,. \tag{1.25}$$

As in the global case, supersymmetry is broken if $D_iW \neq 0$. The potential Eq. (1.22) is not positive definite and cannot be used to measure the supersymmetry breaking scale; for instance, it is possible to break supersymmetry and still have $V = 0$. Instead, from Eq. (1.25) the scale of supersymmetry breaking is given by $e^{K/2}G^{i\bar{j}}D_{\bar{j}}W^*$.

Once this combination acquires a nonzero expectation value, ψ^i becomes a Goldstino. The spinor mass-matrix contains a fermion-gravitino mixing of the form

$$e^{G/2}\,G_{\bar{i}}\bar{\psi}^i\bar{\sigma}^\mu\psi_\mu$$

where, following [8], we have introduced the potential

$$G := K + \log|W|^2$$

which is invariant under Kähler-Weyl transformations. Diagonalizing the fermion mass-matrix, the Goldstino is found to be

$$\eta = G_i\psi^i\,. \tag{1.26}$$

The gravitino acquires a mass

$$m_{3/2} = \frac{1}{3}e^{G/2}G^{i\bar{j}}G_iG_{\bar{j}} \tag{1.27}$$

and the corresponding eigenvector is

$$\hat{\psi}_\mu = \psi_\mu + \frac{\sqrt{2}}{3m_{3/2}}\partial_\mu\eta + i\frac{\sqrt{2}}{6}\sigma_\mu\bar{\eta}\,. \tag{1.28}$$

Therefore, the Goldstino becomes the longitudinal component of the massive gravitino. This is the super-Higgs mechanism. In particular, for vanishing cosmological constant $V = 0$, we have

$G^{i\bar{j}}G_iG_{\bar{j}} = 3$, and the gravitino mass simplifies to

$$m_{3/2} = \frac{1}{3}e^{K/2}|W|\,. \tag{1.29}$$

1.4 D-branes

We begin our analysis of the IIB superstring by first studying D-branes and, in the next section, the 10d supergravity. Introducing D-branes first helps to understand some of the features of the supergravity limit. Again, the emphasis will be on the concepts that are needed in the dissertation. We follow [13, 14].

1.4.1 D-brane action

A Dp-brane is an allowed endpoint for open strings; it carries RR charge and is BPS. Consider a Dp-brane with worldvolume coordinates ξ^a, $a = 0, \dots, p$. The embedding in 10d is specified by functions $x^M(\xi)$, $M = 0, \dots, 9$. Its action reads

$$S_{D_p} = S_{DBI} + S_{WZ} + S_{fermion} \tag{1.30}$$

where

$$S_{DBI} = -\mu_p \int d^{p+1}\xi\, e^{-\phi}\big(\mathrm{Tr}\,[-\det(G_{ab} + \mathcal{F}_{ab})]\big)^{1/2}\,,\ S_{WZ} = \mu_p \int_{D_p} \mathrm{Tr}\left(e^{\mathcal{F}} \wedge \sum_q C_q\right) \tag{1.31}$$

ϕ is the dilaton, and the gauge-invariant combination of the gauge field strength and B-field is defined as

$$\mathcal{F}_{ab} = 2\pi\alpha' F_{ab} - B_{ab}\,. \tag{1.32}$$

The fermion part can be obtained by supersymmetrizing the bosonic terms. The decoupling limit of this action gives a super Yang-Mills theory.

There are various gauge transformations. First, a shift of the B-field by an exact form induces a transformation of the gauge field A_{ab} so that $\mathcal{F}$ is left invariant:

$$\delta B_2 = d\lambda_B\ ,\ \delta A = \frac{\lambda_B}{2\pi\alpha'}\,. \tag{1.33}$$

From S_{WZ}, the internal gauge field $\mathcal{F}$ couples to RR fields C_q, inducing lower-dimensional brane charges. For example, for a D3 brane in a B-field,

$$S_{WZ} = \mu_3 \int (C_4 - B_2 \wedge C_2 + C_0\, B_2 \wedge B_2)\,.$$

Hence, a nonzero B induces D1 and D instanton charges [15]. The C_4 gauge transformation is $\delta C_4 = d\lambda_3$. Invariance of the WZ term implies that the C_2 transformation $\delta C_2 = d\lambda_C$ has to be cancelled by a corresponding change $\delta C_4 = B_2 \wedge d\lambda_C$. This implies that the gauge invariant RR field strength is

$$\tilde{F}_5 = dC_4 - C_2 \wedge H_3 \tag{1.34}$$

and not dC_4 as one might have thought. This distinction will become important in the following chapters, when we consider string compactifications with nonzero H_3 flux.

To fix ideas, consider a D5 brane wrapping an S^2, in the presence of nonzero B_2 and C_2, and set $C_0 = 0$. Dimensionally reducing on the sphere gives a 4d gauge theory. The relevant terms in the action are

$$S = -\mu_5 \int d^4x \int_{S^2} d\Omega_2\, \mathrm{e}^{-\phi} \big[-\det(G+\mathcal{F})\big]^{1/2} + i\mu_5 \int \left(\frac{1}{2}\mathcal{F}^2 \wedge C_2 + \dots\right) . \tag{1.35}$$

To compute the 4d gauge coupling we isolate the terms proportional to $F_{ab}F^{ab}$ and integrate over S^2. The result is

$$\tau_{YM} = (2\pi\alpha')^2 \mu_5 \left(\int_{S^2} C_2 - ie^{-\phi} \int_{S^2} (J - B_2)\right) . \tag{1.36}$$

J is the volume form on the sphere. This implies that $\int_{S^2} C_2$ is the 4d θ-angle, while the gauge coupling is determined by $\int_{S^2}(J - B_2)$.

The presence of a nonzero B-field leads to an interesting new possibility: we can shrink the 2-sphere to zero, keeping $\int_{S^2} B_2$ fixed. Then the gauge coupling is produced only by the internal B-field. This will be related to geometric transitions studied in chapter 4.

1.4.2 Gauge-gravity dualities

A Dp brane has a nonzero energy-momentum tensor, so it can also be analyzed via its gravitational effects. The finite brane energy means that it will backreact on the 10d space where it is placed. A simple situation corresponds to flat 10d space ($\mathbb{R}^{3,1} \times \mathbb{R}^6$) and N D3 branes extended along space-time $\mathbb{R}^{3,1}$ and localized at a point $r = 0$ of the internal space $\mathbb{R}^6$. This preserves 16 supercharges. The supergravity solution is [16]

$$\begin{aligned} ds^2 &= e^{2A(r)}\,\eta_{\mu\nu}dx^\mu dx^\nu + e^{-2A}(dr^2 + r^2 d\Omega_5^2) \\ e^\phi &= g_s\ ,\ C_4 = g_s^{-1}\, e^{4A}\, dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \end{aligned} \tag{1.37}$$

where

$$e^{-4A(r)} = 1 + \frac{R^4}{r^4}\ \ ,\ \ R^4 = 4\pi g_s N \alpha'^2\,. \tag{1.38}$$

Therefore, including the gravitational backreaction of the brane introduces a nonzero harmonic function e^{-4A} into the metric. In particular, the 4d length scale is set by e^{2A} and so it depends on the coordinate r of the internal space. This is our first example of a *warped metric*, and e^{2A} is called the warp factor. Notice that the isometries of Minkowski space are still preserved, but now the space-time is "fibered" over the internal space $\mathbb{R}^6$. One of the aims of this dissertation is to understand how the warp factor affects the 4d dynamics for geometries with 4 supercharges.

To understand better the effect of the warp factor, we take the near horizon limit of [17] which implies $R^4/r^4 \gg 1$. The background then becomes $AdS_5 \times S^5$ with $R_{AdS_5} = R_{S^5} = R$,

$$ds^2 = R^2 \left(\frac{du^2}{u^2} + u^2 \eta_{\mu\nu} dx^\mu dx^\nu \right) + R^2 d\Omega_5^2 \tag{1.39}$$

in terms of the coordinate $u = r/R^2$. Maldacena conjectured [17] that string theory on this background is dual to the decoupling limit of the worldvolume action Eq. (1.30), which in this case is four dimensional $\mathcal{N} = 4$ SYM. The $1/r^4$ dependence of the warp factor reflects the fact that the dual gauge theory is conformal. More generally, the warp factor encodes properties of the dual beta function in a geometric way [18]. In chapter 4 we will analyze the supergravity dual of a confining gauge theory.

1.5 Ten dimensional type IIB supergravity

In this section we review the basic properties of type IIB supergravity, and set the conventions for the following chapters.

1.5.1 Action and equations of motion

Consider a general 10d metric

$$ds_{10}^2 = g_{MN}(x) dx^M dx^N \,. \tag{1.40}$$

The (bosonic) type IIB supergravity action in Einstein frame is (see e.g. [19] for conventions)

$$\begin{aligned} S &= \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g_{10}}\, R - \frac{1}{4\kappa_{10}^2} \int \left(\frac{d\tau \wedge \star_{10} d\bar{\tau}}{(\mathrm{Im}\tau)^2} + \frac{1}{2} \tilde{F}_5 \wedge \star_{10} \tilde{F}_5 + \right. \\ &+ \left. \frac{1}{\mathrm{Im}\tau} \left[G_3 \wedge \star_{10} \bar{G}_3 + \frac{i}{2} C_4 \wedge G_3 \wedge \bar{G}_3 \right] \right) + S_{loc} \end{aligned} \tag{1.41}$$

where $\tilde{F}_5$ was defined in Eq. (1.34),[2] and

$$\tau := C_0 + i e^{-\phi} \;,\; G_3 := F_3 - \tau H_3 \,. \tag{1.42}$$

[2]Notice that the 4-form C_4 used in the usual supergravity definition $\tilde{F}_5 = dC_4 - \frac{1}{2} C_2 \wedge H_3 + \frac{1}{2} B_2 \wedge F_3$ does not coincide with the "physical" C_4 that couples to the D3 brane in Eq. (1.30). For this reason, we will work with the $\tilde{F}_5$ definition given in Eq. (1.34). This distinction will become important in chapter 3, when we analyze the dynamics of axions.

Also, $2\kappa_{10}^2 = (2\pi)^7 \alpha'^4 g_s^2$, where $g_s = \langle e^{-\phi} \rangle$. The term S_{loc} is the action for possible localized sources, such as branes. This action is supplemented with the self-duality condition

$$\star_{10} \tilde{F}_5 = \tilde{F}_5 \,. \tag{1.43}$$

A p-form will be normalized as

$$\omega = \frac{1}{p!}\, \omega_{M_1 \ldots M_p}\, dx^{M_1} \wedge \ldots \wedge dx^{M_p} \,.$$

The field equations were originally obtained in [20]. In terms of the energy-momentum tensor

$$T_{MN} := -\frac{2}{\sqrt{-g_{10}}}\, \frac{\delta S_{matter}}{\delta g^{MN}} \,, \tag{1.44}$$

the trace-reversed Einstein equation is

$$R_{MN} = \kappa_{10}^2 \left(T_{MN} - \frac{1}{8} g_{MN} T \right) \,. \tag{1.45}$$

The contributions from G_3 and $\tilde{F}_5$ work out to be

$$\begin{aligned} T^{(3)}_{MN} &= \frac{1}{8\kappa_{10}^2 \mathrm{Im}\tau} \left(G_{MPQ} \bar{G}_N{}^{PQ} + \bar{G}_{MPQ} G_N{}^{PQ} - \frac{1}{3} g_{MN} G_{P_1P_2P_3} \bar{G}^{P_1P_2P_3} \right) \\ T^{(5)}_{MN} &= \frac{1}{4\kappa_{10}^2}\, \frac{1}{4!}\, \tilde{F}_{MP_1\ldots P_4} \tilde{F}_N{}^{P_1\ldots P_4} \,. \end{aligned} \tag{1.46}$$

The term $g_{MN} \tilde{F}_5^2$ vanishes due to the self-duality condition, and so is absent from $T^{(5)}_{MN}$.

The equations of motion for the 3- and 5- forms are

$$\begin{aligned} d\tilde{F}_5 &= \frac{i}{2\mathrm{Im}\tau}\, G_3 \wedge \bar{G}_3 + 2\kappa_{10}^2 \rho_3^{loc} \\ d(\star_{10} G_3) &= i \tilde{F}_5 \wedge G_3 \,. \end{aligned} \tag{1.47}$$

The Bianchi identity for $\tilde{F}_5$ is

$$d\tilde{F}_5 = \frac{i}{2\mathrm{Im}\tau}\, G_3 \wedge \bar{G}_3$$

which is automatically satisfied after imposing the self-duality condition (away from the localized sources). Reciprocally, Bianchi identity plus the self-duality condition imply the equation of motion. The Bianchi identities for the 3-forms are $dF_3 = dH_3 = 0$. Finally, the equation for the axio-dilaton is, from Eq. (1.41),

$$\nabla_M \nabla^M \tau - \frac{\partial_M \tau \partial^M \tau}{i\, \mathrm{Im}\tau} = -\frac{i}{12}\, G_{MNP} \bar{G}^{MNP} \,. \tag{1.48}$$

The type IIB fermions are 2 dilatinos λ^A, $A = 1, 2$, and 2 gravitinos ψ_M^A. Both are 10d Majorana-Weyl spinors and have the same chirality. Similarly, the 32 real supercharges are grouped into 2 Majorana-Weyl generators $\varepsilon = (\varepsilon^1, \varepsilon^2)$ of the same chirality. The supersymmetry variations are

$$\begin{aligned}
\delta_\varepsilon \lambda^A &= \frac{1}{2}\Gamma^M \partial_M \phi\, \varepsilon^A - \frac{1}{24}\Gamma^M \partial_M C_0\, i\sigma_2^{AB}\varepsilon^B - \frac{e^\phi}{24}\Gamma^{MNP}\tilde{F}_{MNP}\,\sigma_1^{AB}\varepsilon^B + \\
&- \frac{1}{24}\Gamma^{MNP}H_{MNP}\,\sigma_3^{AB}\varepsilon^B \\
\delta_\varepsilon \psi_M^A &= D_M\varepsilon^A + \frac{e^\phi}{8}\,\Gamma^N\Gamma_M\partial_N C_0\, i\sigma_2^{AB}\varepsilon^B + \frac{e^\phi}{16\times 5!}\,\Gamma^{NPQRS}\,\tilde{F}_{NPQRS}\,\Gamma_M\, i\sigma_2^{AB}\varepsilon^B + \\
&- \frac{1}{8}\Gamma^{NP}H_{MNP}\,\sigma_3^{AB}\varepsilon^B + \frac{e^\phi}{48}\Gamma^{NPQ}\Gamma_M\tilde{F}_{NPQ}\,\sigma_1^{AB}\varepsilon^B
\end{aligned} \tag{1.49}$$

where, in analogy with Eq. (1.34), we have defined

$$\tilde{F}_3 := F_3 - C_0 H_3\,. \tag{1.50}$$

The fermionic variations determine the supersymmetry properties of the solution. They will allow us to understand how to embed 4d supergravity into the higher dimensional theory.

The fermionic supergravity action is rather involved; already its 4d counterpart is pretty complicated. Some of its properties will be discussed at the end of chapter 5.

1.5.2 Phenomenological restrictions on the 10d theory

Using the previous equations, let us analyze the restrictions imposed by the 4d theory on the 10d solution. First, the 10d background should preserve the isometries of the 4d space, which we take to be maximally symmetric (AdS, dS or Minkowski) with metric $\hat{g}_{\mu\nu}(x)$. The most general solution preserving these isometries is not a direct product $M_4 \times X$, but rather a *warped* product,

$$ds^2 = e^{2A(y)}\,\hat{g}_{\mu\nu}(x)dx^\mu dx^\nu + g_{ij}(y)dy^i dy^j \tag{1.51}$$

where y^i are the coordinates of the internal 6d space X. We already saw a particular case of this in the D3 brane solution Eq. (1.37). The realization that a nontrivial warp factor is allowed has led to a lot of recent progress in flux compactifications and physics of extra dimensions. The most prominent example is the model of Randall and Sundrum [21], where the electroweak hierarchy is explained geometrically by an AdS-type warp factor. We will see other examples of hierarchies generated by warping, and it will be argued that the warp factor can also be used to build models with parametrically small scales of supersymmetry breaking.

What restrictions does 4d susy place on the 10d theory? The 4d supersymmetry generator ξ has to come from decomposing the 10d generator in the form

$$\varepsilon(x,y) = \xi(x) \otimes \eta(y)$$

where $\eta(y)$ is a 6d spinor. A four dimensional supersymmetric theory then implies the existence of a globally defined (and nowhere vanishing) spinor η on the internal space X. It has to transform as a singlet under the structure group $SO(6)$,[3] which means that the structure group has to reduce to $SU(3)$. Cases where the structure group is a strict subgroup of $SU(3)$ (e.g. $SU(2)$) lead to more supersymmetries in 4d, and will not be considered here. Supersymmetry then requires the internal manifold to be a manifold of $SU(3)$ structure.

It is important to stress that, in the previous argument, we have required the existence of a set of $\mathcal{N}=1$ supercurrents. But this is not related to the question of whether supersymmetry is preserved or spontaneously broken in the vacuum. In this language, if we require supersymmetry to be preserved, the variations Eq. (1.49) have to vanish at least for some of the components of the 10d spinor ε. Hence, a 4d vacuum that preserves supersymmetry will imply certain differential constraints on ε, which will further restrict the geometry of the internal manifold. This is analyzed in detail below.

So far we have discussed the constraints imposed by the existence of a 4d theory on a space of maximal symmetry, and with a supersymmetric lagrangian. Next we ask how the various terms of the low energy effective action Eq. (1.21) are reproduced from ten dimensions. In a moment it will be argued that certain modes that deform the geometry of the internal space give rise to light four dimensional chiral superfields. The graviton corresponds to a small fluctuation of the 4d metric in Eq. (1.51). Although in this dissertation we do not focus on the fermionic sector, let us mention that the 4d gravitino arises from the 10d gravitino Ψ_M, but it includes a mixing of both Ψ_μ and Ψ_m. This will be briefly addressed in chapter 5.

We also want to be able to generate superpotential terms for the chiral superfields. Comparing the supersymmetry variations Eq. (1.25) and Eq. (1.49), a superpotential in 4d corresponds to a nonzero expectation value of the 3-form G_3. More precisely, to dimensionally reduce to four dimensions, the field strength terms in Eq. (1.49) have to be integrated over the internal manifold. Therefore, a nonzero superpotential will be produced if there are nonzero fluxes $\int G_3 \neq 0$. Fluxes are quantized as

$$\frac{1}{4\pi^2\alpha'}\int F_3\ \in \mathbb{Z}\ ,\ \ \frac{1}{4\pi^2\alpha'}\int H_3\ \in \mathbb{Z}\ . \tag{1.52}$$

An important constraint comes from integrating the $\tilde{F}_5$ Bianchi identity over the internal manifold,

$$\int_X H_3\wedge F_3 + 2\kappa_{10}^2 T_3\, Q_3^{loc} = 0\,, \tag{1.53}$$

[3]More precisely, under the cover $Spin(6)$.

so that the total $D3$ charge of a closed internal space has to vanish.

To summarize, in order to reproduce the basic properties of four dimensional supergravity, we need to consider warped flux compactifications on manifolds with $SU(3)$ structure.

1.5.3 Calabi-Yau compactifications

It is instructive to first discuss the case without fluxes. Taking a constant axio-dilaton, the supersymmetry variations Eq. (1.49) reduce to

$$D_m \eta = 0\,,$$

where η is the 6d part of the 10d supersymmetry generator. The globally defined spinor η of the $SU(3)$ structure is then covariantly constant.[4] This implies that the holonomy group (which measures how η transforms under parallel transport in closed loops) is $SU(3)$ and η is a singlet. This is called special holonomy. A manifold with $SU(3)$ holonomy (*i.e.* one covariantly constant spinor) is a Calabi-Yau (CY) manifold. The two IIB Majorana-Weyl generators have the same chirality, so they decompose as

$$\varepsilon^A = \xi^A \otimes \eta + \bar{\xi}^A \otimes \bar{\eta} \tag{1.54}$$

where ξ^A, $A = 1, 2$, are 4d Weyl spinors, and $\bar{\xi} = \xi^*$, $\bar{\eta} = \eta^*$. Therefore, a compactification on a CY without fluxes gives two 4d Weyl supersymmetries, that is, $\mathcal{N} = 2$ supersymmetry.

The internal coordinates can be split into holomorphic and antiholomorphic variables $y^i \to (z^a, \bar{z}^a)$, such that the Clifford vacuum η is annihilated by half of the 6d gamma matrices,

$$\gamma^a \eta = 0\,,\ a = 1, 2, 3\,.$$

Then we can form the spinor bilinears

$$J_{a\bar{b}} = i\eta^\dagger \gamma_{a\bar{b}} \eta\ ,\ \Omega_{abc} = i\eta^T \gamma_{abc} \eta\,. \tag{1.55}$$

Both forms are globally defined, nowhere vanishing, and closed. The $(1, 1)$ form is related to the CY metric by $g_{a\bar{b}} = -iJ_{a\bar{b}}$.

The CY has certain free parameters, corresponding to zero modes of the Ricci tensor. Since by locality the deformations can be done independently on each point x of space-time, they give rise to four dimensional massless scalars. Let us describe these modes in some detail,

[4] Recall that we are considering manifolds with strict $SU(3)$ structure.

following [22]. Consider a metric deformation δg_{ij}, and choose the gauge $\nabla^i(\delta g_{ij}) = 0$. Then the zero modes

$$R_{ij}(g + \delta g) = 0$$

separate into those of mixed type $\delta g_{a\bar{b}}$, and those of pure type δg_{ab}, $\delta g_{\bar{a}\bar{b}}$.

Kähler moduli

Deformations of mixed type are called *Kähler moduli.* Parametrizing the Kähler moduli with coordinates ρ^r, we can construct a $(1,1)$ form

$$\omega_r := i\frac{\partial g_{a\bar{b}}}{\partial \rho^r}\, dz^a \wedge dz^{\bar{b}}\,. \tag{1.56}$$

This form is harmonic because $\partial_r g_{a\bar{b}}$ is a zero mode of the Ricci tensor in transverse gauge. In other words, Kähler moduli are in one to one correspondence with $H^{(1,1)}(X,\mathbb{C})$. The dimension of this space is $h^{1,1}$.

From a four dimensional point of view, type IIB Kähler moduli form $\mathcal{N} = 2$ hypermultiplets. Each real scalar ρ^r is complemented with three more scalars that come from expanding B_2, C_2 and C_4 in harmonic 2-forms. It is also useful to represent Kähler moduli as periods of the Kähler form J. For this, expand

$$J = \rho^r e_r\ ,\ e_r \in H^2(X,\mathbb{Z})\,. \tag{1.57}$$

Then

$$\rho^r = \int_{e_r} J\ , \tag{1.58}$$

where the homology cycles satisfy $\int_{e_r} e_s = \delta^r_s$.

Complex moduli

Metric fluctuations of pure type are called *complex moduli*, and will be denoted by S^α. They correspond to deformations of the complex structure of the CY. They are in one to one correspondence with the harmonic $(2,1)$ forms

$$\chi_\alpha := -\frac{1}{2}\Omega_{abd}\,\frac{\partial g^d{}_{\bar{c}}}{\partial S^\alpha}\, dy^a \wedge dy^b \wedge dy^{\bar{c}}\ \ \in\ H^{(2,1)}(X,\mathbb{C})\,. \tag{1.59}$$

From a 4d point of view, they combine with the vectors A^α_μ in $C_4 = A^\alpha_\mu dx^\mu \wedge \chi_\alpha$ to form $\mathcal{N} = 2$ vector multiplets. The kinetic term follows from expanding $\int R_{10}$ to quadratic order in the complex structure fluctuation. The result is [22]

$$G_{\alpha\bar{\beta}} = -\frac{\int \chi_\alpha \wedge \bar{\chi}_\beta}{\int \Omega \wedge \bar{\Omega}}\,. \tag{1.60}$$

The Kähler potential can be found using the Kodaira relation

$$\frac{\partial \Omega}{\partial S^\alpha} = k_\alpha \Omega + \chi_\alpha \,, \tag{1.61}$$

which gives

$$K(S,\bar{S}) = -\log\left(i\int \Omega\wedge\bar{\Omega}\right) \,. \tag{1.62}$$

In section 1.2.2 it was argued that the vector multiplet theory is determined by a prepotential. It is interesting to understand how this arises in the context of CY compactifications. Let (A_I, B^J), $I, J = 0, \ldots, h^{(2,1)}$, be a symplectic basis for $H^3(X,\mathbb{Z})$,

$$\int_X A_I \wedge B^J = \delta_I{}^J \,.$$

The dual homology basis is denoted by (A^I, B_J). The complex moduli space can be parametrized by the A-periods

$$S^I = \int_{A_I} \Omega \,. \tag{1.63}$$

These are projective coordinates $S^I \to \lambda S^I$, because Ω is defined up to rescalings. The B-periods cannot yield independent coordinates; they turn out to be derivatives of a holomorphic function $\mathcal{F}$,

$$\int_{B_I} \Omega = \frac{\partial \mathcal{F}}{\partial S^I} \,, \tag{1.64}$$

which plays the role of the four dimensional prepotential. Then it is not hard to see that

$$e^{-K} = -2i\,\mathrm{Im}\left(S^I \frac{\partial \bar{\mathcal{F}}}{\partial \bar{S}^I}\right) \,. \tag{1.65}$$

This gives a geometrical realization of local special Kähler geometry. The field theory expression Eq. (1.12) corresponds to the global ($M_{Pl} \to \infty$) limit.

1.6 Flux compactifications with $\mathcal{N} = 1$ supersymmetry

Consider the general warped flux compactification preserving $\mathcal{N} = 1$ supersymmetry. Let us focus on the vacuum structure. The possible supersymmetric flux vacua are classified by fixing a linear relation between ε^1 and ε^2,

$$\varepsilon^1 = a\,\xi\otimes\eta + c.c.\,,\ \varepsilon^2 = b\,\xi\otimes\eta + c.c.$$

(a and b are complex parameters) and then setting to zero the supersymmetry variations Eq. (1.49). Requiring $\mathcal{N} = 1$ supersymmetry places constraints on the allowed p-forms and on the geometry of the internal space. An important consequence of having nonzero fluxes is that the internal spinor η is no longer covariantly constant in the metric compatible connection.

The extra flux terms in Eq. (1.49) can be interpreted as defining a metric with torsion, and η has to be covariantly constant in this metric. The forms J and Ω can still be defined as before, but they are not closed.

The possible type IIB flux vacua with $SU(3)$ structure are given by [23]

- "type A" solutions, with $b = 0$. They have only H_3 flux, and the internal manifold is complex non-Kähler. An example is the Maldacena-Nuñez solution [24].
- "type B" solutions, $a = \pm ib$. These solutions have imaginary self-dual G_3 and the internal manifold is conformal CY. An example is the Klebanov-Strassler solution [18]; these backgrounds were studied in detail in [25]. (GKP)
- "type C" solutions, S-dual to type A.

Understanding the low energy dynamics requires first identifying the light degrees of freedom. There is no general procedure for this yet, except in type B solutions where one starts from the low energy fields of the CY. In this dissertation we therefore restrict to such slutions, and we will analyze how fluxes and warping modify their dynamics.

1.6.1 Review of type B backgrounds

For type B solutions, the static background is a warped product of $\mathbb{R}^{3,1}$ with a six dimensional CY manifold X. Let $\tilde{g}_{mn}$ be a Ricci-flat Kähler metric on X, then the metric ansatz is

$$ds^2_{10} = e^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{-2A(y)}\tilde{g}_{mn}dy^m dy^n \,. \tag{1.66}$$

The field strengths are chosen to preserve Lorentz invariance and self-duality is imposed on the five form,

$$G_3 = \frac{1}{3!}G_{mnp}(y)dy^m dy^n dy^p = F_3 - \tau H_3 \,, \tag{1.67}$$

$$\tilde{F}_5 = \partial_m \alpha(y)(1 + \star_{10})dy^m \wedge dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \,. \tag{1.68}$$

Since we are interested in flux compactifications, we will assume that F_3 and H_3 are three-forms in nontrivial classes of $H^3(M, \mathbb{Z})$.

The warp factor is of the general form

$$e^{-4A(y)} = c + e^{-4A_0(y)} \tag{1.69}$$

where the dimensionless parameter c is related to the total volume by $V_{CY} \sim \alpha'^3\, c^{3/2}$, and A_0 is produced by matter sources (fluxes or branes). The large volume limit $c \gg e^{-4A_0}$ corresponds to a small number of fluxes or branes, where backreaction may be ignored. In the present dissertation we address the general case, which includes the strongly warped limit $c \ll e^{-4A_0}$.

Furthermore, the background satisfies certain BPS properties which are analogous to the brane metrics of AdS/CFT [25]. These are

$$\alpha = e^{4A}\ ,\ \ \frac{1}{4}(T^m_m - T^\mu_\mu)^{loc} = T_3 \rho_3^{loc}\,. \tag{1.70}$$

As a result, the equations of motion are verified automatically if the three form flux is imaginary self-dual (ISD):

$$\tilde{\star}_6 G_3 = i G_3\,. \tag{1.71}$$

Quantities with respect to $\tilde{g}_{mn}$ are denoted with tildes. Similar conditions may be written for backgrounds sourced by anti-branes, with the flux becoming IASD.

The equation obeyed by the warp factor is

$$-\tilde{\nabla}^2\, e^{-4A} = \frac{G_{mnp} \bar{G}^{\widetilde{mnp}}}{12\,\mathrm{Im}\tau} + 2\kappa_{10}^2 T_3 \rho_3^{loc}\,. \tag{1.72}$$

1.6.2 The large volume limit

As pointed out by GKP, the ISD condition fixes the complex structure moduli and leads to a constant background dilaton.

Small fluctuations around this background should be described by a 4d $\mathcal{N}=1$ supergravity effective action along the lines of section 1.3. A reasonable starting point for this is to take the $\mathcal{N}=2$ supergravity obtained by KK reduction of IIB supergravity on the CY X, and then apply an orientifold projection. This is expected to be a good description in the large volume limit, with small quantum corrections. Furthermore, we are assuming that backreaction can be ignored and thus the warp factor is essentially constant. This will be true if we take the limit $\alpha' \to 0$ while holding the number of flux units N fixed. Conversely, large N dualities or large hierarchies arise, even in the $\alpha' \to 0$ limit, when $\alpha' N$ is held fixed.

In this limit, the 4d Kähler potential for the metric moduli takes the well-known form,

$$K = -3\log(-i(\rho-\bar{\rho})) - \log\big(-i\int_X J\wedge J\wedge J\big) - \log\big(-i\int_X \Omega\wedge\bar{\Omega}\big)\,. \tag{1.73}$$

Turning on fluxes breaks spontaneously $\mathcal{N}=2 \to \mathcal{N}=1$, in the way described in section 1.2.3. Therefore, from a 4d point of view fluxes behave as FI terms for the auxiliary fields in the vector multiplets (at least at large volume). We then expect a flux-generated superpotential for complex moduli. The superpotential can be read off from the gravitino variation, as in Eq. (1.25). In our case, this comes from the 10d gravitino variation Eq. (1.49), after multiplying by η^T and integrating over the internal manifold. Recalling Eq. (1.55), this gives the Gukov-Vafa-Witten [26] type superpotential

$$W_{GVW} = \int_X G_3 \wedge \Omega \,. \tag{1.74}$$

Equivalently, dimensional reduction implies that fluxes generate a scalar potential for the complex moduli [25, 27],

$$V = -\frac{1}{2\kappa_{10}^2 \mathrm{Im}\tau} \int_M G_3 \wedge (\star_6 \bar{G}_3 + i\bar{G}_3) \tag{1.75}$$

which vanishes in the ISD case. From the scalar potential and Kähler potential we can infer Eq. (1.74). Kähler moduli only receive non-perturbative superpotential contributions [28].

Notice that the holomorphic 3-form is defined up to holomorphic scalings

$$\Omega(S) \to e^{-f(S)}\Omega(S) \tag{1.76}$$

which can depend on complex moduli S. Ω defines a line bundle over moduli space. Under this rescaling,

$$W_{GVW} \to e^{-f(S)}\, W_{GVW} \;, \quad K(S,\bar{S}) \to K(S,\bar{S}) + f(S) + f^*(S^*) \,. \tag{1.77}$$

This provides a geometric realization of the Kähler-Weyl transformations of 4d supergravity, Eq. (1.24).

1.6.3 Towards an effective description of warping

Steps towards including the effects of warping were taken in [27]. The simplest is to change the relation between the 10d Planck scale, the 4d Planck scale, and the volume of the internal manifold, to

$$\frac{M_{Pl,4}^2}{M_{Pl,10}^2} = \left(\frac{M_{Pl,10}}{2\pi}\right)^6 V_W \,,$$

where V_W is the "warped volume" of the internal manifold,

$$V_W = \int d^6y \sqrt{\tilde{g}_6}\, e^{-4A} \,. \tag{1.78}$$

Then, the potential and kinetic terms of the 4d effective action were computed by KK reduction. In particular, the kinetic terms for the moduli were obtained by varying the Einstein-Hilbert action around the metric ansatz Eq. (1.66), obtaining

$$G^W_{\alpha\bar{\beta}} = \partial_\alpha \partial_{\bar{\beta}} K_W = \int d^6y \sqrt{\tilde{g}_6}\; e^{-4A} \partial_\alpha \tilde{g}_{mn}\, \partial_{\bar{\beta}} \tilde{g}^{mn} \,. \tag{1.79}$$

Then, substituting in the variations of the 6d metric with the moduli, it was found that the warp factor corrections change Eq. (1.73) to

$$K_W \overset{?}{=} -3\log(-i(\rho - \bar{\rho})) - \log\big(-i\int_M e^{-4A} J \wedge J \wedge J\big) - \log\big(-i\int_M e^{-4A} \Omega \wedge \bar{\Omega}\big) \tag{1.80}$$

However, this analysis is somewhat oversimplified. One reason for this was that, in generalizing the KK ansatz to fields which vary in the four dimensions, one often needs to add terms which depend on derivatives of the fields. The reason for this is that, promoting a modulus u to a space-time field $u(x)$ induces extra terms

$$R_{\mu m} \sim \partial_\mu u(\dots)$$

where $(\dots)$ includes derivatives of the warp factor. The equations of motion cannot be solved consistently and the ansatz Eq. (1.66) needs to be generalized. One needs to understand how to modify the CY zero modes to account for this effect.

Another new effect is that KK modes become light in regions of strong warping, so they need to be included in the effective action. While there are a few compactifications in which a consistent truncation to a small subset of modes is possible [29], usually this is not the case, and there is no reason to believe it is so for an arbitrary Calabi-Yau compactification. Furthermore, there could be nontrivial mixings between zero modes and their KK fluctuations. This would imply that the low energy dynamics truncated to metric moduli is inconsistent. Chapters 2–5 are devoted to answering these questions.

1.7 Overview

The dissertation is organized as follows. In chapter 2 we study the problem of computing kinetic terms in warped compactifications. It is argued that a direct route to the kinetic terms is to derive them in a Hamiltonian framework. In this approach, four dimensional fields associated to internal metric variations lift to ten dimensional gauge invariant canonical momenta. We identify a gauge dependent inner product on the compactification manifold which depends explicitly on the warp factor. It is shown that kinetic terms are associated to the minimum value of this inner product over each gauge orbit.

In chapter 3 we construct the effective theory of the universal Kähler modulus in warped compactifications, as an application of the framework developed in the previous chapter. The spacetime dependent 10d solution is obtained at the linear level for both the volume modulus and its axionic partner, and nontrivial cancellations of warping effects are found in the dimensional reduction. The main result is that the Kähler potential is not corrected by warping, up to an overall shift in the background value of the volume modulus. This shift may affect nonperturbative corrections.

Chapter 4 focuses on the dynamics of complex moduli around conifold-like singularities and their dual gauge theories. We first find the gauge theory associated to a CY with general F_3

and H_3 flux. It is argued that the field theory has (p, q) 5-brane bound states and fractional gauge coupling. We use these results to prove that the number of supersymmetric flux vacua is finite. Next we focus on the nonholomorphic sector of the theory, which is affected by the warp factor. We compute the warp-modified moduli space metric and find that it leads to a new term with a power-like divergence at the conifold point. As an application, we study supersymmetry breaking by anti-self-dual flux in the deformed conifold. These results are then interpreted from the point of view of the dual gauge theory at large warping.

Chapter 5 combines the results from the previous chapters in order to obtain the effective action for type IIB warped flux compactifications up to quadratic order. The appearence of light KK modes is analyzed in detail and their contribution to the low energy theory is determined. In particular, the four dimensional potential contains a generalization of the Gukov-Vafa-Witten term, and can lead to mixings between the moduli and their KK excitations.

The last two chapters explore new mechanisms of supersymmetry breaking, based on the idea that our vacuum may be a long-lived metastable state. In chapter 6 we construct a model of metastable supersymmetry breaking where all the relevant parameters, including the supersymmetry breaking scale, are generated dynamically. A novel effect is that the metastable vacuum appears by a competition between a nonperturbative runaway direction and perturbative quantum effects.

Finally, in chapter 7, we explore metastable vacua in SQCD, in the presence of single and multitrace superpotential deformations. Our aim is to obtain a model with an acceptable phenomenology. The metastable vacua appear at one loop, have a broken R-symmetry and a magnetic gauge group that is completely Higgsed. It is argued that multitrace deformations are needed to lift hidden sector light fermions charged under the SM.

Chapter 2

Kinetic terms in warped compactifications

2.1 Introduction and summary

In this chapter, based on [1], we will develop formalism for computing the kinetic terms of 4d fields in string compactifications, particularly with warping. This is the first step towards understanding dynamical (*i.e.* space-time dependent) effects in flux compactifications.

Kaluza-Klein reduction in warped flux compactifications is very subtle, and the full 4d effective action is not known yet. One reason for this is that, in following the standard approach of substituting the Kaluza-Klein ansatz into the Lagrangian, one finds that one needs "compensator" fields [30, 31], which are difficult to solve for explicitly, and do not (at least to us) suggest any clear physical or mathematical intuition for the results.

As it turns out, a fairly direct route to the kinetic terms is to derive them in a Hamiltonian framework. The reason is that the system has constraints associated to gauge redundancies, while the physical degrees of freedom become manifest in the Hamiltonian formulation. While this does not completely eliminate the need to discuss compensators, it does provide a much clearer picture of why they arise and how to deal with them.

Perhaps the simplest way to explain the main point is to realize that the kinetic terms for metric moduli originate from a metric on the space of metrics, but the usual expression for this metric is gauge dependent. A mathematically natural [32] and physically correct [33] way to fix this ambiguity is to require that the metric fluctuations be orthogonal to gauge transformations. However, when one says "orthogonal," one has implicitly used the ten-dimensional metric, in a way which sees the warp factor. This is the point at which warping changes the usual discussion.

2.1.1 General Problem

We consider a D-dimensional theory of gravity coupled to matter, *e.g.* a supergravity. A vacuum solution is a solution of the equations of motion which at long distances "looks like" a d-dimensional space M with maximal symmetry, *i.e.* Minkowski space, AdS or dS. In general it will be a product or warped product of M with an $n = D-d$-dimensional compactification space (or internal space) X, possibly with other nonzero fields consistent with maximal symmetry (*i.e.* scalars, components of vector fields in X, etc.).

We use x^μ and y^i to denote coordinates on M and X respectively. For definiteness we will sometimes take $D = 10$ and $d = 4$, but our considerations will not depend on this.

Suppose there is a family of vacuum solutions of the D-dimensional equations of motion, with parameters u^I. Thus we can write $g_{MN}(y; u)$, $\phi(y; u)$, and so forth. To analyze the dynamics of these moduli u^I, we might try to find a family of "approximate solutions" of the equations of motion, obtained by taking the parameters to slowly vary on M [33]:

$$g_{MN}(y, u(x)). \tag{2.1}$$

The kinetic terms are then the terms in the d-dimensional effective Lagrangian of the form

$$\int d^d x \, \sqrt{g} g^{\mu\nu} \, G_{IJ}(u) \, \partial_\mu u^I \partial_\nu u^J, \tag{2.2}$$

obtained by substituting Eq. (2.1) into the D-dimensional action, integrating over X and identifying these terms.[1] Note that to compute Eq. (2.2), we need to allow "off-shell" $u(x)$ (*i.e.* $\partial^2 u \neq 0$).

However, this direct approach can become complicated. The first sign of this is that in general, the ansatz Eq. (2.1) does not solve the ten-dimensional equations of motion, even when $u(x)$ solves the four-dimensional massless field equations. One may need a more general ansatz depending on derivatives $\partial u, \partial^2 u$, etc. Further subtleties arise from gauge invariance. We will see how this happens and its consequences in examples.

2.1.2 Summary

We start in section 2.2 with the example of Yang-Mills theory, which is used to illustrate in a simple setup many of the subsequent points. Then in section 2.3 we construct a Riemannian metric on the space of metrics, with the help of the Hamiltonian of General Relativity. This metric is used in section 2.4 to construct kinetic terms arising from 10d (warped) backgrounds preserving 4d maximal symmetry. We prove that metric fluctuations should be orthogonal to gauge transformations associated to the full warped metric. This turns out to be equivalent to minimizing the value of their inner product over each gauge orbit.

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[1] The correct action may require a boundary term to cancel boundary terms in the variation, for example the Gibbons-Hawking-York term in general relativity.

In section 2.5, the previous formalism is applied to string compactifications. We first discuss the case of a Calabi-Yau manifold, where the metric for complex and Kähler moduli is recovered. The harmonic gauge choice generally considered in the literature is identified as a dynamical constraint. Next the more interesting case of conformal Calabi-Yau compactifications is analyzed; these correspond to type IIB supergravities with BPS branes and fluxes. Compensating fields are identified with Lagrange multipliers of the Hamiltonian. Their role is to set metric fluctuations into harmonic gauge with respect to the full warped metric. We find a fairly simple expression for the field space metric in terms of warped metric fluctuations. Upon rewriting this in terms of the underlying Calabi-Yau moduli we verify the expression recently found in [5].

2.2 Yang-Mills theory

We start with the simple case of a $U(1)$ field A_M with field strength F_{MN}. We suppose that there are a family of solutions of

$$D^i F_{ij} = 0$$

on X, parameterized by coordinates u^I. For example, if X is a torus, every flat connection is a solution, and the u^I might be the holonomy associated to a basis of $H^1(X, \mathbb{Z})$.

We take as the ten-dimensional action

$$S = \ldots - \frac{1}{4} \int d^4x \sqrt{g_4} \int d^6y \sqrt{g_6} g^{MN} g^{PQ} F_{MP} F_{NQ}. \tag{2.3}$$

Naively we then set $A_\mu = 0$ and write

$$F_{\mu i} = \partial_\mu A_i(y; u(x)) - \partial_i A_\mu(y; u(x)) = \frac{\partial A_i}{\partial u^I} \partial_\mu u^I,$$

and substitute this into the action, to obtain Eq. (2.2) with

$$G_{IJ} = \int d^6y \sqrt{g_6} g^{ij} \frac{\partial A_i}{\partial u^I} \frac{\partial A_j}{\partial u^J}. \tag{2.4}$$

However, on reflection, there must be a subtlety in this procedure. In defining our moduli space of solutions $A_i(y; u)$, nowhere did we specify a gauge for A_i. Two solutions which are related by gauge transformations on X,

$$\delta A_i = \partial_i \epsilon,$$

are equally good from the point of view of X. On the other hand, the expression Eq. (2.4) is not gauge invariant, so the kinetic terms will depend on which of the gauge equivalent solutions we take. Since Eq. (2.3) was gauge invariant in ten dimensions, we must have made an error.

The error was the assumption that $A_\mu = 0$ for all of these solutions. Let us look at the ten dimensional equations of motion. These can be written as

$$0 = D^\mu F_{\mu\nu} + D^i F_{i\nu}; \qquad 0 = D^\mu F_{\mu j} + D^i F_{ij}. \tag{2.5}$$

We substitute the ansatz $A_i(y; u(x))$ and require that there is no four-dimensional gauge field, $F_{\mu\nu} = 0$. This sets

$$A_\mu(x, y) = \Omega(y)\partial_\mu f(x)$$

where $\Omega(y)$ and $f(x)$ are still undetermined functions.

To find A_μ, we use the first equation of motion, which becomes $0 = \partial^i F_{i\nu}$, *i.e.*

$$\partial^i \partial_\nu A_i = \partial^i \partial_i A_\nu. \tag{2.6}$$

In general, the left hand side is nonzero, so we will have $A_\nu \neq 0$. However a simple way to make the left hand side zero is to require

$$0 = \partial^i \frac{\partial A_i}{\partial u^I}, \tag{2.7}$$

i.e. the fluctuations are taken in harmonic gauge. More generally, solving Eq. (2.6) produces an A_ν which is the parameter of the "compensating gauge transformation",

$$A_\mu(x, y) = \Omega_I(y)\partial_\mu u^I(x) \ , \ \partial^i \partial_i \Omega_I = \partial^i \ \frac{\partial A_i}{\partial u^I} \ . \tag{2.8}$$

Defining

$$\delta_I A_i := \frac{\partial A_i}{\partial u^I} - \partial_i \Omega_I \ , \tag{2.9}$$

we see that the effect of Ω is to put $\delta_I A_i$ back into harmonic gauge.

In general, it is hard to explicitly solve Eq. (2.6) for the compensator field A_ν. However, to compute the kinetic term, we do not need to do this, rather we just need to impose the condition Eq. (2.7).

2.2.1 Metric for Yang-Mills connections

One can straightforwardly generalize the above to nonabelian gauge fields. There is also a simple geometric interpretation of the final result, which leads immediately to the metric both for Yang-Mills and for gravitational configurations.

Note that Eq. (2.7) is the condition that the variation $\delta_I A$ is orthogonal in the metric Eq. (2.4) to all the gauge directions. This is a natural mathematical condition and leads to a unique definition of the metric [32].

Let $\mathcal{A}$ be the set of possible (smooth) gauge potentials on $\mathbb{R}^3$, and $\mathcal{G}$ be the group of all gauge transformations over $\mathbb{R}^3$. The four-dimensional physical configuration space is then the quotient (or orbit space) $\mathcal{C} \equiv \mathcal{A}/\mathcal{G}$.

Given a metric g_{ij} on $\mathbb{R}^3$, there is a natural metric on $T\mathcal{A}$,

$$(\dot{A}, \dot{A}) = \int d^3x \ \sqrt{g} g^{ij} \ \ \mathrm{tr} \left(\dot{A}_i(x) \dot{A}_j(x) \right) . \tag{2.10}$$

Given a path $c(t)$ in $\mathcal{C}$, we would like to define a natural Riemannian metric H on $\mathcal{C}$, which can be used in a particle action as [33]

$$S[c] = \int dt \, \frac{1}{2} \, H(\dot{c}, \dot{c}) \, . \tag{2.11}$$

Since actually one works with paths $A_i(t) \in \mathcal{A}$, the basic requirement is that $S[c]$ should be independent of the way $c(t)$ is lifted to $\mathcal{A}$. This can be accomplished by projecting the tangent vector $\dot{A}_i(t)$ on the subspace orthogonal to gauge transformations in the metric Eq. (2.10). Thus, let Π_i be this projection,

$$\Pi_i(\dot{A}) := \dot{A}_i - D_i(1/D^2) D_j \dot{A}_j \ , \ \Pi_i(D_k \lambda) = 0 \, . \tag{2.12}$$

The natural metric on $\mathcal{C}$ is then

$$H(\dot{c}, \dot{c}) = \int d^3x \, \mathrm{tr} \big(\Pi_i(\dot{A}) \, \Pi_i(\dot{A}) \big) \, . \tag{2.13}$$

From a physics point of view, $\Pi_i(\dot{A})$ is the electric field F_{0i} after eliminating A_0 by using the Gauss law. Equivalently, the projector is given by the nonabelian version of the zero mode Eq. (2.9) after solving for the compensator Ω. Substituting into the E^2 terms of the Yang-Mills action, one obtains Eq. (2.11).

There are several other formulations of the same result. One is to regard the configuration space $\mathcal{A}$ as a $\mathcal{G}$-bundle over the space of gauge orbits. The projection Eq. (2.12) then defines a preferred notion of "parallel transport" on this bundle, making Eq. (2.10) unambiguous. The metric Eq. (2.11) is then gauge invariant, in the sense that it is derived from a gauge invariant notion of parallel transport.

Another formulation is to note that, since the metric Eq. (2.10) is positive definite, evaluating it with the gauge directions projected out is the same as evaluating it on the gauge representative which *minimizes* its value.

2.2.2 Relation to Hamiltonian formulation

A slightly different way of reducing to gauge invariant variables is to go to the Hamiltonian formulation. We recall that, since the time derivatives $\partial_0 A_0$ do not appear in the action, the A_0

component of the vector potential plays the role of a Lagrange multiplier, which is conjugate to the Gauss law,

$$S = \ldots + \int A_0 D_i E^i.$$

One can then enforce the Gauss law as a constraint on the initial data (A_i, E^i), which is preserved under Hamiltonian evolution.

This is a particular example of "symplectic reduction" with respect to a symmetry group G. Starting with a phase space M with a symplectic structure $\omega(u, v)$, one identifies "moment maps" μ which are "Hamiltonians" generating the infinitesimal action of G. One can then show that the reduced phase space

$$\{x \in M : \mu(x) = 0\}/G$$

carries a symplectic structure.

In the Yang-Mills example, $G = \mathcal{G}$, and M is the direct product of the space $\mathcal{A}$ of connections $A_i(x)$ with the space of electric field strengths $E^i(x)$. It carries the symplectic structure

$$\omega(A, E) = \int d^3x \ \mathrm{tr}\ \big(A_i(x) E^i(x)\big)\,.$$

The moment maps for $\mathcal{G}$ are then $\mu = D_i E^i$. Thus, the Gauss law constraint is the natural partner of the gauge condition in this construction as well. Since the E^2 terms in the Hamiltonian are gauge invariant, they are single valued on the reduced phase space, resulting in the same metric Eq. (2.11).

Physically, we can use this formulation by considering a configuration in which the moduli u^I are linearly varying with time. The metric is then the energy density of this configuration, and the Hamiltonian framework provides a direct way to compute this. Since the phase space does not contain time-like components of vector potentials, there is no possibility for a "compensator field" A_0 to enter; rather the mixed equations of motion such as Eq. (2.5) are solved implicitly in this framework.

In general, the result of this prescription will depend on the initial choice of symplectic structure on field space. However in field theory there is usually a unique local candidate for this structure.

2.3 General relativity

In this section we consider the problem of constructing a natural Riemannian metric on the space of metrics. This will be done by using the Hamiltonian formulation of general relativity, which is well-suited for extracting the kinetic terms in a general case. At the end of the section we present a simple example where the kinetic terms are obtained via the usual Lagrangian approach, so that both perspectives may be compared.

2.3.1 Metric on the space of metrics

The problem may be formulated as follows. Consider a D-dimensional manifold equipped with a metric $g_{MN}(x)$, $M, N = 0, \ldots, D$. In many cases of interest the metric satisfies certain background equations of motion. For example, in pure Einstein gravity it is Ricci flat. However these equations depend on the theory, and thus we will not make use of them in this section.

We identify a time coordinate $t = x^0$; then Σ denotes the space-like surface $t = 0$ and h_{MN} is the pull-back of g_{MN} to Σ. Let $\mathcal{A}$ be the set of all such possible Riemannian metrics h_{MN}, and $\mathcal{G}$ the corresponding diffeomorphisms. Our aim is to identify a Riemannian metric H on $\mathcal{A}/\mathcal{G}$ and then for each path $c(t) \in \mathcal{A}/\mathcal{G}$ introduce a natural action

$$S[c] = \int dt \, \frac{1}{2} H(\dot{c}, \dot{c}) \, . \tag{2.14}$$

Following the previous discussion it will now be shown how this arises from the Hamiltonian formulation for GR [34, 35].

One starts by prescribing initial value conditions on a $D - 1$ dimensional space-like surface Σ_0, with metric h_{MN}. h_{MN} is the space-like part of g_{MN},

$$h_{MN} = g_{MN} \;\text{ for }\; M, N \neq 0 \;\; , \;\; h_{00} = 0 \, .$$

The equations of motion produce the time evolution $\Sigma_0 \rightarrow \Sigma_t$, and the physical degrees of freedom are h_{MN} and not g_{MN}. The remaining components are denoted by

$$\eta = (-g_{00})^{1/2} \;\; , \;\; \eta_M = g_{0M} \;\text{ for }\; M \neq 0 \, , \; \eta_0 = 0 \, . \tag{2.15}$$

The tensors used in the rest of the chapter have vanishing time-like components, while x^0 enters through η or time derivatives. Therefore, effectively the indices M, N will run only over the space-like directions.

The geometrical interpretation is that the time evolution $\Sigma_0 \rightarrow \Sigma_t$ given by the vector field ∂_0 can be decomposed into a normal direction plus a tangential shift η^N. The dynamics is encoded in the extrinsic curvature,

$$K_{MN} := \frac{1}{2} \eta \, (\dot{h}_{MN} - D_N \eta_M - D_M \eta_N) \, , \tag{2.16}$$

where D_N is the covariant derivative on Σ, compatible with h_{MN}. The lagrangian density takes the form

$$\mathcal{L}_G = \sqrt{-g_D} \Big({}^{(D-1)}R + K_{MN} K^{MN} - K^2 \Big) \, . \tag{2.17}$$

In terms of these variables, the canonical momentum reads

$$\pi^{MN} = \frac{\partial \mathcal{L}_G}{\partial \dot{h}_{MN}} = h^{1/2}(K^{MN} - h^{MN} K)\,, \tag{2.18}$$

from which we obtain the Hamiltonian density,

$$\begin{aligned} \mathcal{H}_G &= \pi^{MN}\dot{h}_{MN} - \mathcal{L}_G \\ &= \sqrt{h}\eta\Big(-{}^{(D-1)}R + h^{-1}\pi^{MN}\pi_{MN} - \frac{1}{D-2}h^{-1}\pi^2\Big) - 2h^{1/2}\eta_N D_M(h^{-1/2}\pi^{MN}) \end{aligned} \tag{2.19}$$

Therefore, we learn that η and η_N are Lagrange multipliers. They enforce the constraints

$${}^{(D-1)}R + h^{-1}\pi^{MN}\pi_{MN} - \frac{1}{D-2}h^{-1}\pi^2 = 0 \tag{2.20}$$

$$D_N(h^{-1/2}\pi^{NM}) = 0\,. \tag{2.21}$$

After satisfying this we can gauge fix $\eta^N = 0$. The Hamiltonian density is a sum of constraints and vanishes weakly.

The Riemannian metric on $\mathcal{A}/\mathcal{G}$ corresponds to the kinetic term from Eq. (2.19). Given a path $c_{MN}(t) \in \mathcal{A}/\mathcal{G}$ we introduce a lift $h_{MN}(t)$ to $\mathcal{A}$; to the tangent vector $\dot{h}_{MN}$ we associate the "projection" $\pi^{MN}(\dot{h})$ defined in Eq. (2.18). The metric on the space of metrics becomes

$$S[c] = \int d^D x\, \mathcal{L}_G = \int \pi^{MN}\dot{h}_{MN}\,. \tag{2.22}$$

Eliminating $\dot{h}_{MN}$ in terms of π_{MN},

$$\dot{h}_{MN} = 2h^{-1/2}\eta\left(\pi_{MN} - \frac{1}{2}h_{MN}\,\pi\right) + 2D_{(M}\eta_{N)} \tag{2.23}$$

we arrive to

$$S[c] = 2\int d^D x\,\sqrt{-g_D}\Big(h^{-1}\pi^{MN}\pi_{MN} - \frac{1}{D-2}h^{-1}\pi^2\Big) \tag{2.24}$$

The constraint Eq. (2.21) implies that $\pi^{MN}(\dot{h})$ is orthogonal to space-like gauge transformations,

$$S[\mathcal{L}_v\pi,\, \pi] = 0\,.$$

Actually, π^{MN} itself is a projector $\mathcal{A} \to \mathcal{A}/\mathcal{G}$:

$$\pi^{MN}(\mathcal{L}_v\dot{h}) = 0\,.$$

The proof is analogous to the YM case Eq. (2.12), and is based on eliminating the Lagrange multipliers η_N. On the other hand, the constraint Eq. (2.20) ensures invariance with respect to time reparametrizations.

We conclude that the Hamiltonian approach to GR yields a natural Riemannian metric Eq. (2.24) on $\mathcal{A}/\mathcal{G}$.

2.3.2 Unwarped solutions

In simple cases it is still possible to compute kinetic terms using the Lagrangian formulation, as we now discuss in an example. Consider a family of six dimensional Ricci-flat manifolds X with metric $g_{ij}(y;u)$. Examples are Calabi-Yau manifolds, with u^I parametrizing complex and Kähler moduli. The ten dimensional background is taken to be the unwarped product $M \times X$ with metric

$$ds^2 = g_{\mu\nu}(x)dx^\mu dx^\nu + g_{ij}(y;u)dy^i dy^j \,. \tag{2.25}$$

Promoting the moduli to fields $u^I(x)$ fibers X over M, but only through the implicit dependence of the moduli on the space-time coordinates. As in the Maxwell case, just replacing $u \to u(x)$ into Eq. (2.25) doesn't give a consistent D-dimensional solution. To satisfy $G_{MN} = 0$, we consider the following ansatz including a compensating field B_i:

$$ds^2 = g_{\mu\nu}(x)dx^\mu dx^\nu + 2B_{Ij}(y)\partial_\mu u^I dy^j dx^\mu + g_{ij}(y;u(x))dy^i dy^j \,. \tag{2.26}$$

In principle, an extra compensator term of the form $K_I(y)\partial_\mu\partial_\nu u^I\, dx^\mu dx^\nu$ may also be needed. This corresponds to the Lagrange multiplier η above. However, we will show that B_{Ij} is only defined modulo a total derivative term, which can be used to set $K_I = 0$.

The components of the Einstein tensor, up to two space-time derivatives, read

$$G_{\mu\nu} = (\partial_\mu\partial_\nu u^I - g_{\mu\nu}\Box u^I)\big[-\frac{1}{2}\frac{\partial g}{\partial u^I} + \nabla^j B_{Ij}\big] \tag{2.27}$$

$$G_{\mu i} = \frac{1}{2}\partial_\mu u^I \nabla^j\big(\nabla_i B_{Ij} - \nabla_j B_{Ii} + \frac{\partial g_{ij}}{\partial u^I} - g_{ij}\frac{\partial g}{\partial u^I}\big) \tag{2.28}$$

$$G_{ij} = -\frac{1}{2}\Box u^I\big[\frac{\partial g_{ij}}{\partial u^I} - \nabla_i B_j - \nabla_j B_i\big] \,, \tag{2.29}$$

where the trace part is

$$\frac{\partial g}{\partial u^I} := g^{ij}\frac{\partial g_{ij}}{\partial u^I} \,.$$

A consistent ten dimensional solution requires $G_{\mu i} = 0$, which fixes B_{Ij}, up to a total derivative $\partial_j K_I$. Then we have to require that $G_{\mu\nu} = 0$, off-shell for $u(x)$, which determines the previous function K_I:

$$\nabla^j B_{Ij} = \frac{1}{2}\frac{\partial g}{\partial u^I} \,. \tag{2.30}$$

Using Eq. (2.30) to eliminate $\partial_I g$, Eq. (2.28) can be rewritten more suggestively as

$$\nabla^i\Big[\frac{\partial g_{ij}}{\partial u^I} - \nabla_i B_j - \nabla_j B_i\Big] = 0 \,. \tag{2.31}$$

Plugging these results in the Einstein-Hilbert action, the action up to two space-time derivatives is of the form Eq. (2.2), with field space metric

$$G_{IJ}(u) = \frac{1}{4}\int d^6y\sqrt{g_6}\, g^{ij}g^{kl}\,\delta_I g_{ik}\,\delta_J g_{jl} \tag{2.32}$$

where

$$\delta_I g_{ij} := \frac{\partial g_{ij}}{\partial u^I} - \nabla_i B_j - \nabla_i B_j \, . \tag{2.33}$$

The role of the ten dimensional constraints is to set $\delta_I g_{ij}$ in the transverse traceless gauge,

$$\nabla^i \, \delta_I g_{ij} = 0 \, , \; g^{ij} \delta_I g_{ij} = 0 \, . \tag{2.34}$$

This example shows how the metric compensators repackage into a "physical" zero mode $\delta_I g_{ij}$ which is orthogonal to diffeomorphism transformations. Their effect can be simply summarized in the requirement that the zero mode has to be in the transverse traceless gauge. The upshot from this example is that harmonic gauge is not a choice, but rather a dynamical constraint.

2.4 Kinetic terms in general compactifications

The most general D-dimensional metric consistent with d-dimensional maximal symmetry is

$$ds^2 = e^{2A(y;\,u)} \, \hat{g}_{\mu\nu}(x) dx^\mu dx^\nu + g_{ij}(y;u) dy^i dy^j \, . \tag{2.35}$$

This is a warped product of a maximally symmetric space M with metric $\hat{g}_{\mu\nu}$ and an arbitrary compactification manifold X with metric g_{ij}. The internal manifold depends on parameters u^I and the aim is to find their kinetic terms. This applies to all supergravity compactifications preserving 4d maximal symmetry.

We will assume here that g_{ij} does not have exact isometries, as is the case in CY manifolds. This simplifies the analysis, since there are no gauge fields coming from the off-diagonal fluctuations $\delta g_{\mu m}$. There is a mass gap and $\delta g_{\mu m}$ are associated to massive spin 1 fields, which we choose not to excite. In a more complete treatment, one should describe how such fields combine with the graviton modes (and scalars from the internal manifold) to yield massive spin 2 degrees of freedom.

The situation is a particular case of that discussed in the previous section, where the path $c(t)$ corresponds to promoting u^I to spacetime fields. Since the 4d part $\hat{g}_{\mu\nu}$ is fixed, the metric on the space of metrics should now reduce to a metric on the parameter space $\{u^I\}$. We will not assume that g_{ij} is Ricci-flat; rather, it satisfies certain background equations of motion (for instance, including fluxes). To simplify our treatment, we will restrict to energy-momentum tensors without velocity-dependent terms.

Once the u^I are allowed to fluctuate, we have to include compensators,

$$ds^2 = e^{2A(y;\,u)} \, \left((\hat{g}_{\mu\nu}(x) + 2\partial_\mu \partial_\nu K) dx^\mu dx^\nu + 2\partial_\mu B_j \, dx^\mu dy^j \right) + g_{ij}(y;u) dy^i dy^j \, . \tag{2.36}$$

In the Lagrangian approach, the compensators are fixed by solving the equations of motion at linear order in velocities. Once this is done, the kinetic terms may be extracted from the equations which are quadratic in space-time derivatives. The K compensator corresponds to the Lagrange multiplier η in Eq. (2.15). To simplify our formulas, we fix time reparametrization invariance by setting $K = 0$.

Here the system will be analyzed from a Hamiltonian point of view; for simplicity, we take $\partial_\mu u^I = \delta^0_\mu \dot{u}^I$.[2] The kinetic term for the moduli $u^I(t)$ is obtained by plugging the corresponding time-dependent metric $\dot{h}_{MN} = \dot{u}^I(\partial h_{MN}/\partial u^I)$ in Eq. (2.24). In the linearized approximation the extrinsic curvature K_{MN} and canonical momentum π_{MN} are both proportional to $\dot{u}^I$, so we can write

$$K_{MN} = \frac{1}{2}\,(g^{tt})^{1/2}\,\dot{u}^I\,\delta_I h_{MN}\ ,\ h^{-1/2}\pi_{MN} = \frac{1}{2}\,(g^{tt})^{1/2}\,\dot{u}^I\,\delta_I \pi_{MN} \tag{2.37}$$

where $g_{tt} := |g_{00}|$. The factors of $(g^{tt})^{1/2}/2$ have been extracted for later convenience. The coefficients $\delta_I h_{MN}$ and $\delta_I \pi_{MN}$ are given by

$$\delta_I h_{MN} = \frac{\partial h_{MN}}{\partial u^I} - D_M(\eta_I)_N - D_N(\eta_I)_M \tag{2.38}$$

$$\delta_I \pi_{MN} = \delta_I h_{MN} - h_{MN}\,h^{PQ}\,\delta_I h_{PQ} \tag{2.39}$$

where we have expanded $\eta_N = \dot{u}^I\,(\eta_I)_N$.

The relation between the Lagrangian and Hamiltonian approach is that the compensators coincide with the Lagrange multipliers η_M,

$$\eta_{I\mu} = 0\ ,\ \eta_{Ij} = e^{2A}\,B_{Ij}(y)\,. \tag{2.40}$$

The advantage of the Hamiltonian formulation is that they appear explicitly as nonpropagating fields, whose only role is to impose the constraints

$$D^N\big((g^{tt})^{1/2}\delta_I \pi_{MN}\big) = 0\,, \tag{2.41}$$

which imply that the physical variations are orthogonal to gauge transformations. We remind the reader that D_N is the covariant derivative compatible with the space-like metric h_{MN}. The kinetic term derived from the Hamiltonian Eq. (2.24) reads

$$\begin{aligned} S &= \frac{1}{2}\int dt\,\dot{u}^I\,\dot{u}^J\left(\int d^{D-1}x\,\sqrt{-g_D}\,g^{tt}\left[\delta_I\pi_{MN}\,\delta_J\pi^{MN} - \frac{1}{D-2}\,\delta_I\pi\,\delta_J\pi\right]\right) \\ &= \frac{1}{2}\int dt\,\dot{u}^I\,\dot{u}^J\left(\int d^{D-1}x\,\sqrt{-g_D}\,g^{tt}\,\delta_I\pi_{MN}\,\delta_J h^{MN}\right)\,. \end{aligned} \tag{2.42}$$

[2]Recall that the difference between g_{MN} and h_{MN} is that the latter only includes space-like components.

This is the gravitational analog of the kinetic term $p\,\dot{q}$ in particle mechanics.

Let us now prove that Eq. (2.41) is equivalent to *minimizing* the inner product over each gauge orbit. Under a gauge transformation

$$\delta_I h^{MN} \to \delta_I h^{MN} - D^N v_I^M - D^M v_I^N \,,$$

the change in the inner product Eq. (2.42) is

$$-2\int d^{D-1}\sqrt{-g_D}\, g^{tt}\, v_I^M\, D^N \left[(g^{tt})^{1/2}\left(\delta_J \pi_{MN} + \mathcal{L}_v\, \delta_J \pi_{MN}\right)\right] \,. \tag{2.43}$$

Demanding that the gauge parameter minimizes this expression, we find

$$D^N \left[(g^{tt})^{1/2}\left(\delta_J \pi_{MN} + \mathcal{L}_v\, \delta_J \pi_{MN}\right)\right] = 0\,, \tag{2.44}$$

thus reproducing the prescription given in Eq. (2.41).

2.4.1 Four dimensional expression

To compactify over the internal manifold one would in principle need to know the warp factor and then extract the variation $\partial_I A$. These are complicated functions determined by the background equations of motion. But interestingly, the constraints Eq. (2.21) fix $\delta_I A$ in terms of $g^{ij}\,\delta_I g_{ij}$: from

$$0 = D^\mu(\delta_I \pi_{\mu\nu}) = -\partial_\nu\big(2\, e^{-2A}\, \delta_I e^{2A} + g^{ij}\, \delta_I g_{ij}\big)\,,$$

we obtain

$$\delta_I e^{2A} = -\frac{1}{2}\, e^{2A}\, g^{ij}\, \delta_I g_{ij}\,. \tag{2.45}$$

This implies that $\delta\pi_{\mu\nu} = 0$. Strictly speaking, there is a small subtlety in this derivation. If $u = u(t)$, then the constraint Eq. (2.41) along $M = \mu$ vanishes because the expression in parentheses is independent of x. To get a nonzero constraint, one has to allow for a more general space-time dependence $u = u(x)$ and then Eq. (2.45) follows. Once this initial value condition is found, we take the limit $|\partial_\mu u| \ll \dot{u}$. Notice that this is always allowed because our treatment is off-shell. The main motivation for taking this limit is that the space-like metric h_{MN} factorizes into 3d and 6d blocks, simplifying many of our computations.

The linearized version of the constraint Eq. (2.20) is satisfied in virtue of Eq. (2.45) and the 4d isometries. Also, the warp factor variation may be eliminated from $\delta\pi_{ij}$ yielding

$$\delta_I \pi_{ij} = \delta_I g_{ij} + \frac{1}{d-2} g_{ij}\, g^{kl} \delta_I g_{kl}\,. \tag{2.46}$$

The internal part of the constraint sets

$$D^N(e^{-A}\delta_I \pi_{Nj}) = 0\,, \tag{2.47}$$

where e^{-A} comes from $(g_{tt})^{-1/2}$, and it is important to remember that the connection is defined with respect to the full warped metric. To rewrite this in terms of 6d variables, notice that

$$D^\mu(e^{-A}\delta_I\pi_{\mu j}) = 3\, e^{-A}\, \partial^k A\; \delta_I\pi_{kj}$$

where we used the fact that $\pi_{\mu\nu} = 0$ and $h^{\mu\nu}\Gamma^k_{\mu\nu} = -3\partial^k A$. Then (2.47) becomes

$$g^{ij}\nabla_i(e^{2A}\delta_I\pi_{jk}) = 0\,. \tag{2.48}$$

With these results, the general formula for the kinetic terms is[3]

$$S_{kin} = \frac{1}{2}\int d^dx\, \sqrt{-\hat{g}_d}\, \hat{g}^{tt}\, \dot{u}^I \dot{u}^J\, G_{IJ}(u) \tag{2.49}$$

with

$$G_{IJ}(u) = \frac{1}{4}\int d^{D-d}y\sqrt{g_{D-d}}\, e^{2A}\, \delta_I g_{ij}\, \delta_J \pi^{ij}\,. \tag{2.50}$$

The warp factor dependence comes from $\sqrt{-g_d}\, g^{tt} = \sqrt{-\hat{g}_d}\, \hat{g}^{tt}\, e^{2A}$. From this expression it becomes clear that Eq. (2.48) is simply the condition that the physical variation $\delta_I\pi_{ij}$ is orthogonal to gauge transformations. The effects of the compensators are summarized in this prescription.

2.4.2 Effect of compensators

The Hamiltonian approach shows that the effect of the compensators is to make the metric fluctuations orthogonal to gauge transformations. In general it is simpler to compute the "naive" zero modes just by taking derivatives $\frac{\partial g_{ij}}{\partial u^I}$. The metric associated to these fluctuations is

$$G^0_{IJ} = \frac{1}{4}\int d^{D-d}y\sqrt{g_{D-d}}\, e^{2A}\left(\frac{\partial g_{ij}}{\partial u^I}\frac{\partial g^{ij}}{\partial u^J} - \frac{1}{D-2}\frac{\partial g}{\partial u^I}\frac{\partial g}{\partial u^J}\right), \tag{2.51}$$

which is a gauge-dependent quantity because in general $\partial_I g_{ij}$ is not orthogonal to gauge transformations.

Starting from G^0_{IJ} we can ask what is the effect of the "compensating gauge transformation"

$$\delta_I g_{ij} = \frac{\partial g_{ij}}{\partial u^I} - \nabla_i\eta_{Ij} - \nabla_j\eta_{Ii} \tag{2.52}$$

which projects down to $\mathcal{A}/\mathcal{G}$. More concretely, we are interested in analyzing $G_{IJ} - G^0_{IJ}$, which may be shown to be

$$G_{IJ} - G^0_{IJ} = \frac{1}{4}\int d^{D-d}y\, \sqrt{g_{D-d}}\, e^{2A}\, \eta_{Ij}\, \nabla_i\left(\frac{\partial\pi^{ij}}{\partial u^J}\right) + (I\leftrightarrow J)\,. \tag{2.53}$$

[3]We are ignoring the overall factor $M_{P,D}^{D-2}$; also the correct normalization of the d-dimensional Ricci term would introduce a factor of $1/Vol(X)$ in the field space metric.

Let's first derive the explicit projector analogous to the expression Eq. (2.12) for nonabelian Yang-Mills theories. From Eq. (2.47), the compensating fields satisfy the equation

$$\left(g_{ij}\, \nabla^k \nabla_k + 2\nabla_i \nabla_j + R_{ij}\right) \eta_I^j = \nabla^k \left(\frac{\partial g_{ki}}{\partial u^I}\right) \tag{2.54}$$

plus the relation Eq. (2.45) which fixes possible residual gauge transformations preserving Eq. (2.54). Defining the operator

$$\mathcal{O}_{ij} := g_{ij}\, \nabla^k \nabla_k + 2\nabla_i \nabla_j + R_{ij}\,,$$

formally the compensators are given by

$$\eta_I^i = (\mathcal{O}^{-1})^{ij}\, \nabla^k (\frac{\partial g_{kj}}{\partial u^I})\,. \tag{2.55}$$

In this way,

$$\delta_I g_{ij} = \frac{\partial g_{ij}}{\partial u^I} - \nabla_i\, (\mathcal{O}^{-1})_{jl}\, \nabla_k \left(\frac{\partial g^{kl}}{\partial u^I}\right) + (i \leftrightarrow j)\,. \tag{2.56}$$

We conclude that the effect of the compensators on the metric is

$$G_{IJ} - G^0_{IJ} = \frac{1}{2} \int d^{D-d}y\, \sqrt{g_{D-d}}\, e^{2A}\, \nabla_i \left(\frac{\partial g^{ij}}{\partial u^I}\right) \mathcal{O}^{-1}_{jl}\, \nabla_k \left(\frac{\partial g^{kl}}{\partial u^I}\right)\,. \tag{2.57}$$

This is the term responsible for minimizing the metric over each gauge orbit. A different compensator choice would imply that the gauge directions are not projected out, giving a larger result.

2.5 Application to string compactifications

The Hamiltonian derivation of the field space metric Eq. (2.50) holds quite generally. In particular supersymmetry is not assumed and the details of the matter sector (fluxes, branes, etc.) are not needed.

Of course, given supersymmetry, one can exploit its constraints. For instance, for $\mathcal{N} = 2$ supersymmetries the metric for chiral superfields may be obtained from that of the vector superpartners in the $\mathcal{N} = 2$ multiplet, which enter quadratically in the 10d action. Already for $\mathcal{N} = 1$ susy, deriving the moduli kinetic terms by dimensionally reducing the 10d action supersymmetry is a very involved task, as was shown in [5]. The main obstacle is the correct implementation of the constraints, which arise from the $(0M)$ components of Einstein equations.

On the other hand, we have shown how the kinetic terms arise more naturally from the GR Hamiltonian. In this section, some simple examples of type II compactifications will be analyzed from this point of view.

2.5.1 Calabi-Yau manifolds

To gain intuition we begin by discussing Calabi-Yau compactifications, both from the Hamiltonian and Lagrangian viewpoint. An unwarped Calabi-Yau compactification corresponds to

$$ds^2 = g_{\mu\nu}(x)dx^\mu dx^\nu + g_{ij}(y)dy^i dy^j \,, \tag{2.58}$$

where g_{ij} is a Ricci flat Kähler metric. Holomorphic coordinates are denoted by z^a, $a = 1, 2, 3$, so that the Kähler form is $J = ig_{a\bar{b}}\, dz^a \wedge d\bar{z}^b$. The metric moduli space splits into complex structure deformations $S^\alpha\, \delta_\alpha g_{ab}$, and Kähler deformations $\rho^r\, \delta_r g_{a\bar{b}}$.

The Hamiltonian analysis may be applied straightforwardly to this case. The space-time components of the constraint Eq. (2.41) imply that the metric fluctuations must be traceless, while the internal components tell us that the fluctuations are in harmonic gauge:

$$g^{ij}\, \delta_I g_{ij} = 0 \,,\ \nabla^i(\delta_I g_{ij}) = 0 \,, \tag{2.59}$$

with I running over $(\alpha,\, r)$. These conditions were a choice in the 6d approach of Candelas and de la Ossa [22], but here they emerge as constraints of the 10d Hamiltonian picture. This occurs as follows. Starting from a zero mode $\partial g_{ij}/\partial u^I$ in some arbitrary gauge, the compensators are equivalent to a diffeomorphism transformation $\partial_I g_{ij} \to \delta_I g_{ij} = \partial_I g_{ij} - \nabla_{(i}\, B_{Ij)}$ which point to point imposes the transverse-traceless constraints. The metric Eq. (2.50) gives, after reintroducing the Planck mass,

$$\begin{aligned} G_{\alpha\bar{\beta}} &= \frac{1}{4V_{CY}} \int d^6y \sqrt{g_6}\, g^{a\bar{c}} g^{b\bar{d}}\, \delta_\alpha g_{ab}\, \delta_\beta g_{\bar{c}\bar{d}} \\ G_{rs} &= \frac{1}{4V_{CY}} \int d^6y \sqrt{g_6}\, g^{a\bar{c}} g^{b\bar{d}}\, \delta_r g_{a\bar{d}}\, \delta_s g_{b\bar{c}} \,. \end{aligned} \tag{2.60}$$

Let us explain briefly how the zero modes are actually computed, because this will be necessary to understand conformal Calabi-Yau compactifications. Since Eq. (2.58) is a solution without sources, starting from a given background value g^0_{ij}, the zero modes are solutions to

$$R_{ij}(g^0 + \delta g) = 0 \,. \tag{2.61}$$

Recalling the linearized expression for the Ricci tensor [35]

$$\delta R_{ij} = -\frac{1}{2}\nabla^k\nabla_k \delta g_{ij} - \frac{1}{2}\nabla_i\nabla_j\delta g + \nabla^k\nabla_{(i}\delta g_{j)_k} \,,$$

the zero mode fluctuations satisfy

$$-\frac{1}{2}\nabla^k\nabla_k\delta g_{ij} - \frac{1}{2}\nabla_i\nabla_j\delta g + R_{k(ij)l}\delta g^{kl} + \frac{1}{2}\big(\nabla_i\nabla^k\delta g_{kj} + \nabla_j\nabla^k\delta g_{ki}\big) = 0 \,. \tag{2.62}$$

Next, imposing the gauge $\nabla^i \delta g_{ij} = 0$, the trace part can be set to zero and one is left with

$$-\frac{1}{2}\nabla^k \nabla_k \delta g_{ij} + R_{k(ij)l}\delta g^{kl} = 0\,. \tag{2.63}$$

This gauge-fixed version of $\delta R_{ij} = 0$ is the Lichnerowicz laplacian on Ricci-flat manifolds.[4] On a Kähler manifold the only nonzero components of the Riemann tensor are $R_{a\bar{b}c\bar{d}}$ up to permutations, which implies that the zero modes of mixed ($\delta g_{a\bar{b}}$) and pure (δg_{ab}) type separately verify this equation.

2.5.2 Conformal Calabi-Yau case

At the next level of complexity, we consider an internal manifold which is a conformal Calabi-Yau, with the conformal factor given by the inverse of the warp factor,

$$ds^2 = e^{2A(y)}\eta_{\mu\nu}(x)\, dx^\mu dx^\nu + e^{-2A(y)}\, \tilde{g}_{ij}(y)\, dy^i dy^j\,, \tag{2.64}$$

where $\tilde{g}_{ij}$ is the CY metric. These type IIB backgrounds preserve $\mathcal{N} = 1$ susy, and the warp factor is generated by BPS sources [25].

In terms of the unwarped fluctuations $\delta_I \tilde{g}_{ij}$, the constraint Eq. (2.45) sets

$$\delta_I A = \frac{1}{8}\, \tilde{g}^{ij}\delta_I \tilde{g}_{ij}\,; \tag{2.65}$$

this fixes the 4d gauge redundancies. Now $\delta\pi_{ij}$ given in Eq. (2.46), becomes the warped harmonic combination

$$\delta_I \pi_{ij} = e^{-2A}(\delta_I \tilde{g}_{ij} - \frac{1}{2}\tilde{g}_{ij}\, \delta_I \tilde{g})\,. \tag{2.66}$$

The constraint coming from $D_M \pi^{Mj} = 0$ sets

$$g^{ik}\nabla_i (e^{2A}\delta_I \pi_{kj}) = \tilde{g}^{ik}\tilde{\nabla}_i \big(\delta_I \tilde{g}_{kj} - \frac{1}{2}\tilde{g}_{kj}\, \delta_I \tilde{g}\big) - 4\tilde{g}^{ik}\, \partial_i A\, \delta_I \tilde{g}_{kj} = 0\,. \tag{2.67}$$

Finally, replacing Eq. (2.66) into the Hamiltonian expression Eq. (2.50), we arrive to the warped moduli space metric

$$G_{IJ}(u) = \frac{1}{4V_W}\int d^6 y \sqrt{\tilde{g}_6}\, e^{-4A}\, \tilde{g}^{ik}\tilde{g}^{jl}\, \delta_I \tilde{g}_{ij}\, \delta_J \tilde{g}_{kl}\,. \tag{2.68}$$

These results agree with those in [5], which were obtained by dimensionally reducing the action. In that approach, the compensators were gauged away; in the Hamiltonian formalism they arise as Lagrange multipliers which can always be set to zero. Furthermore, the rather

[4] If the Ricci-tensor doesn't vanish there is an extra term proportional to $R_{ik}\delta g_j{}^k$. However, the Einstein equation would also include a source piece.

complicated constraint in the r.h.s. of Eq. (2.67) has a simple interpretation in terms of the full metric with conformal and warp factors, $\nabla^i(e^{2A}\delta_I\pi_{ij})=0$. The present derivation suggests that the natural metric fluctuations are $\delta\pi_{ij}$ instead of δA and $\delta\tilde{g}_{ij}$ separately.

The presence of a nontrivial warp factor has important effects on the moduli dynamics. Eq. (2.65) implies that the fluctuations acquire a nonzero trace part proportional to $\delta_I A$; on the other hand, Eq. (2.67) imposes a gauge which is different from the harmonic condition. Therefore, although the fields u^I are the same as in the unwarped case (so that we still have complex and Kähler moduli), the internal wavefunctions that support them have changed. From Eq. (2.52), the change is by a diffeomorphism in the underlying CY,

$$\delta_I\tilde{g}_{ij}=\frac{\partial\tilde{g}_{ij}}{\partial u^I}-\tilde{\nabla}_i(e^{2A}\eta_{Ij})-\tilde{\nabla}_j(e^{2A}\eta_{Ii})\,. \tag{2.69}$$

Here $\partial\tilde{g}_{ij}/\partial u^I$ are the unwarped modes from the previous section, which are in transverse traceless gauge. The compensating fields η_{Ii} are then fixed by Eq. (2.65) and Eq. (2.67). The physical zero mode $\delta_I\tilde{g}_{ij}$ is guaranteed to satisfy $\delta\tilde{R}_{ij}=0$ separately for Kähler and complex deformations; indeed, it differs from the corresponding unwarped mode only by a gauge transformation. Notice however that the zero mode equation is no longer the Lichnerowicz laplacian which is only valid in harmonic gauge. Rather, one would have to solve the full Eq. (2.62). Of course, since we already know $\partial_I\tilde{g}_{ij}$, it is simpler to use the constraints to solve for the compensating fields.

The behavior of the compensators depends on each particular background, but from the discussion of section 2.4.2 we know that they give a nonzero contribution to the field space metric. In fact, the correct choice will minimize its value on a gauge orbit. One important consequence of this is that the metric Eq. (2.68) could mix complex and Kähler moduli. Indeed, a complex structure fluctuation acquires a nonzero mixed component $\delta_\alpha\tilde{g}_{a\bar{b}}$, while the Kähler moduli also have pure components $\delta_r\tilde{g}_{ab}$. Therefore, there can be mixed terms of the form

$$G_{\alpha r}\sim\frac{1}{V_W}\int d^6y\sqrt{\tilde{g}_6}\,e^{-4A}\left(\delta_\alpha\tilde{g}_{ab}\,\delta_r\tilde{g}^{ab}+\delta_\alpha\tilde{g}_{a\bar{b}}\,\delta_r\tilde{g}^{a\bar{b}}\right). \tag{2.70}$$

This can affect KKLT type [28] scenarios including warping, so it would be important to understand better the susy structure of the field space metric.

2.6 Conclusions

In this chapter we have argued that the natural framework to compute four dimensional kinetic terms is the Hamiltonian formulation of flux compactifications. The main outcome is that 4d fields associated to moduli are not given by simple variations of the static background.

Rather, the 10d fluctuation to which the field lifts is identified with the canonical momentum Eq. (2.39). This contains the static fluctuation (which is not gauge invariant) *plus* other pure gauge components that add up to form a 10d wavefunction which is orthogonal to gauge transformations. Once the canonical momentum is determined, the corresponding kinetic term follows from Eq. (2.42).

In situations where backreaction can be ignored, our formalism implies that metric fluctuations have to be harmonic in the CY metric. This was the "gauge" chosen in the original work of [22], although from a 10d point of view this becomes a Gauss-law type constraint. On the other hand, once the warp factor is added, the Hamiltonian constraint becomes a warped harmonic condition, Eq. (2.48). This can change qualitatively the four dimensional theory, particularly in the phenomenologically interesting limits of strong warping. The next chapters are devoted to constructing this low energy effective field theory.

Chapter 3
The universal Kähler modulus

3.1 Introduction and summary

In the supergravity limit, type IIB flux compactifications have a metric modulus that rescales the internal volume by an overall factor. This is called the universal Kähler modulus and, being insensitive to the details of the CY metric, is expected to have a simple space-time description. In this chapter, based on [2], we construct the effective theory of the universal Kähler modulus in warped compactifications using the Hamiltonian formulation described in chapter 2.

3.1.1 Beyond the Calabi-Yau case

Before starting our analysis, it is instructive to review the simpler case of a Calabi-Yau compactification without warping. We follow the discussion of [25] for IIB CY compactifications. The universal volume modulus corresponds to a simple rescaling

$$\tilde{g}_{ij} \to e^{2u}\, \tilde{g}_{ij} \tag{3.1}$$

of the internal CY metric $\tilde{g}_{ij}$. The time-dependent metric fluctuation is, at linear order,

$$ds^2 = e^{-6u(x)}\, \eta_{\mu\nu}\, dx^\mu dx^\nu + e^{2u(x)}\, \tilde{g}_{ij}(y)\, dy^i dy^j \ , \tag{3.2}$$

where the 4d Weyl factor $e^{-6u(x)}$ is needed to decouple the modulus from the graviton. This 4d rescaling defines the 4d Einstein frame and gives the Einstein-Hilbert action for the metric in 4d. The Einstein equations then reduce to the desired $\Box u = 0$ for the modulus. The 4-form field contributes an axion

$$C_4 = \frac{1}{2} a(x)\, \tilde{J} \wedge \tilde{J} + \cdots \tag{3.3}$$

($\tilde{J}$ is the fixed Kähler form associated with the fixed CY metric $\tilde{g}_{ij}$), which pairs with the volume modulus into the complex field $\rho = a + ie^{4u}$. Performing the dimensional reduction, one finds

$$K = -3\, \log\left(-i(\rho - \bar{\rho})\right) \ . \tag{3.4}$$

Backreaction from fluxes and branes (of the BPS type discussed in [25]) introduces warping to the background,

$$ds^2 = e^{2A(y)}\,\eta_{\mu\nu}\,dx^\mu dx^\nu + e^{-2A(y)}\,\tilde{g}_{ij}\,dy^i dy^j\;. \tag{3.5}$$

One could then try different ways of identifying the universal volume modulus. The simplest possibility would be to consider the same dependence as in (3.2), even in the presence of warping [27]:

$$ds^2 = e^{2A(y)}\,e^{-6u(x)}\,\eta_{\mu\nu}\,dx^\mu dx^\nu + e^{-2A(y)}\,e^{2u(x)}\,\tilde{g}_{ij}(y)\,dy^i dy^j\;. \tag{3.6}$$

This proposal does not work for a couple of reasons. Under a spacetime-independent rescaling $\tilde{g}_{ij} \to e^{2u}\tilde{g}_{ij}$, the warp factor acquires a dependence on u

$$e^{-2A}\;\to\;e^{-2u}\,e^{-2A} \tag{3.7}$$

in such a way that the full internal metric $e^{-2A}\tilde{g}_{ij}$ is actually invariant under the rescaling. Therefore, the simple rescaling of the CY metric becomes a gauge redundancy which may be set to zero by a 4d Weyl transformation. At a more technical level, Eq. (3.6) cannot solve the 10d Einstein equations, so it does not give a consistent time-dependent fluctuation.

Another possibility is suggested by the fact that the warp factor is only determined up to an overall shift,

$$e^{-4A(y)}\;\to\;e^{-4A(y)} + c\;. \tag{3.8}$$

The volume of the compact space scales as $V \sim c^{3/2}$, so it would be natural to identify this flat direction as the warped version of the universal volume modulus. One could then promote c to a spacetime field $c(x)$ by considering the metric fluctuation [31, 36, 37]

$$ds^2 = \left[c(x) + e^{-4A_0}\right]^{-1/2}\,\eta_{\mu\nu}\,dx^\mu dx^\nu + \left[c(x) + e^{-4A_0}\right]^{1/2}\,\tilde{g}_{ij}\,dy^i dy^j \tag{3.9}$$

and performing the dimensional reduction. However, this proposal does not solve the linearized equations of motion[1] either; additional components of the metric are required to satisfy all the components of the 10d Einstein equation [31]. Dimensional reduction on backgrounds for which the 10d equations of motion are not satisfied in general does not lead to good low energy effective theories, and can result in ambiguities, as noticed in previous studies [36–40].

3.1.2 Summary

Summarizing, the dynamics of the universal Kähler modulus is not understood beyond the CY case, and a more systematic approach is needed. A particularly important question is how warping effects correct the kinetic terms and Kähler potential (for $\mathcal{N} = 1$ theories).

[1]Except for special choices of $c(x)$ which appear to lead to instabilities [36, 37].

First, in sections 3.2 and 3.3, the spacetime dependent 10d solution is constructed at the linear level for both the volume modulus and its axionic partner, and nontrivial cancellations of warping effects are found in the dimensional reduction. In section 3.4 we find that the Kähler potential is in fact not corrected by warping, up to an additive shift in the background value of the modulus. This is a rather surprising outcome, because the 10d solution constructed from the Hamiltonian method is quite different from the unwarped fluctuation. It is important to notice, however, that the needed shift in the modulus would affect nonperturbative superpotentials or higher-derivative corrections that break the no-scale structure of the classical background [28, 41–43].

Most of the methods developed so far apply to moduli dynamics in the linearized approximation, namely when the fluctuations around the vacuum expectation values are infinitesimal. Understanding other effects, particularly in cosmology, beyond the 4d effective field theory, requires going beyond the linearized level. For this reason, in section 3.5 we extend our approach to the case of finite spacetime dependent fluctuations of the volume modulus. This not only should serve to eliminate remaining confusion about the relation between the 10d and 4d theories, but it is also a significant first step in developing cosmological solutions of compactified 10d supergravity. Finally, in section 3.6 we discuss the behavior of the modulus in strongly warped regions and show that there are no mixings with light Kaluza-Klein modes.

3.2 Finding the universal volume modulus

Our aim is to find the 10d solution describing a finite spacetime dependent fluctuation of the volume modulus. Now, as explained in section 3.1.1, the first problem one faces is that of defining the volume modulus in warped backgrounds. We address this issue by finding the modulus in the case of an infinitesimal fluctuation, and then showing how to integrate it to a finite variation in section 3.5.

Before proceeding, we should clarify the type of expansion being performed. One starts from a warped background of the general form

$$ds^2 = e^{2A(y;u)}\hat{g}_{\mu\nu}(x) + g_{mn}(y;u)dy^m dy^n \tag{3.10}$$

where $\hat{g}_{\mu\nu}$ is a maximally symmetric 4d metric. Then, a given modulus u is allowed to have a nontrivial spacetime dependence, acquiring a nonzero velocity $\dot{u}$ and energy $g^{tt}\,(\dot{u})^2$. The energy sources the Ricci tensor, with the result that maximal symmetry is lost; for instance, for a massless excitation we would have a pp-wave spacetime. The important point is that backreaction is proportional to the energy, and hence is quadratic in $\dot{u}$. The linearized expansion we consider here then means working at first order in moduli velocities, so that the 4d metric can still be approximated by a maximally symmetric space. In this limit, the metric fluctuations $\dot{h}_{MN} = \dot{u}^I\, \partial_I h_{MN}$ amount to a small perturbation around the background solution h_{MN} even if $\partial_I h_{MN}$ is not necessarily small. This is enough for the purposes of finding the Kähler potential.

We apply the Hamiltonian approach to find the linearized 10d wavefunctions of the universal volume modulus (in this section) and its axionic partner (in section 3.3). These results will be used in section 3.4 to compute the Kähler potential. Finally, in section 3.5 we extend our results beyond the linear approximation, finding the backreaction produced by a finite volume modulus fluctuation. We restrict to type IIB with BPS fluxes and branes, as reviewed in section 1.6.

3.2.1 Ten dimensional wavefunction

Consider an ansatz of the form

$$ds^2 = e^{2A(y;\,c)+2\Omega[c]}\, \left(\hat{g}_{\mu\nu}(x)dx^\mu dx^\nu + 2\,\partial_j B\,\partial_\mu c\, dx^\mu dy^j\right) + e^{-2A(y;\,c)}\,\tilde{g}_{ij}(y)dy^i dy^j\ , \qquad (3.11)$$

where $c(x)$ denotes the universal volume modulus. As will be seen momentarily, a compensating field proportional to a total derivative,

$$\eta_j(y) = e^{2A+2\Omega}\,\partial_j B(y)\ , \qquad (3.12)$$

solves the Hamiltonian constraints, so we have already made this identification in the ansatz. The Weyl factor is defined to bring us to 4-dimensional Einstein frame,

$$e^{2\Omega(c)} = \frac{\int d^6y\,\sqrt{\tilde{g}_6}}{\int d^6y\,\sqrt{\tilde{g}_6}\,e^{-4A(y;\,c)}} = \frac{V_{CY}}{V_W(c)}\,. \qquad (3.13)$$

Furthermore, the underlying CY metric is taken to be independent of the volume modulus because a rescaling $\tilde{g}_{ij} \to \lambda\tilde{g}_{ij}$ amounts to a 4d Weyl transformation.

At the end of the section it will be argued that $c(x)$ is actually orthogonal to the other non-universal metric zero modes $u^I(x)$. It is then consistent to set these to zero in the present discussion. Next we will show how the Hamiltonian approach determines the 10d wavefunction (3.11). The full computation is somewhat technical, so in section 3.2.2 we summarize the results.

The first step is to compute the canonical momentum (2.39) associated to the ansatz Eq. (3.11)). These are found to be

$$\begin{aligned}
\delta_c\pi_{\mu\nu} &= 2\,h_{\mu\nu}\left(4\,\frac{\partial A}{\partial c} - 2\,\frac{\partial\Omega}{\partial c} + \nabla^i\eta_i + 2\,\partial^i A\,\eta_i\right) \text{ and} \\
\delta_c\pi_{ij} &= g_{ij}\left(4\,\frac{\partial A}{\partial c} - 6\,\frac{\partial\Omega}{\partial c} + 2\,\nabla^i\eta_i + 6\,\partial^i A\,\eta_i\right) - \nabla_i\eta_j - \nabla_j\eta_i\ ,
\end{aligned} \qquad (3.14)$$

where η_i is given in (2.40).

The vanishing of the 4d momentum requires

$$4\,\frac{\partial A}{\partial c} - 2\,\frac{\partial \Omega}{\partial c} + e^{2\Omega+4A}\,\tilde{\nabla}^2 B = 0\,, \tag{3.15}$$

in terms of the derivative $\tilde{\nabla}_i$ and Laplacian compatible with $\tilde{g}_{ij}$.

The constraint $D^N\,\pi_{Nj} = 0$ requires a bit more of work. Fortunately, we can use the computation of the Ricci tensor component $R_{\mu i}$ in [31] for our purposes, recalling the relation [34]

$$R_{0i} = -D^N(h^{-1/2}\,\pi_{Ni})\,. \tag{3.16}$$

(We also need a diffeomorphism transformation to set $\eta_i = 0$ and $\eta_\mu = -e^{2A+2\Omega}\,\partial_\mu \dot{c}\,B$, which can always be done for a compensator of the form (3.12)). The constraint then sets

$$\partial_m\big(\partial_c e^{-4A(y;\,c)}\big) = 0\,. \tag{3.17}$$

This implies that the dependence of the warp factor on $c(x)$ is given by an additive shift

$$e^{-4A(y;\,c)} = e^{-4A_0(y)} + c(x)\,, \tag{3.18}$$

where $e^{-4A_0(y)}$ denotes the solution associated to the metric $\tilde{g}_{ij}$, which is independent of $c(x)$. A possible multiplicative factor is fixed using the integrated version of (3.15).

This result has an intuitive interpretation. In conformally CY flux compactifications, the background equations of motion only fix e^{-4A} up to a shift $e^{-4A} \to e^{-4A} + c$. It was noticed in [31, 36, 37] that a change in c, which is not a simple metric rescaling, also changes the internal volume, leading to the proposal that c represents the time-independent universal volume modulus. What we find here is that this shift is present in the full time-dependent case too, although the full 10d metric fluctuation has other components as well.

Finally, plugging (3.13) and (3.18) into (3.15), we obtain the differential equation that fixes the compensating field (also observed in [31]),

$$\tilde{\nabla}^2 B = -e^{-4A-2\Omega}\left(4\,\frac{\partial A}{\partial c} - 2\frac{\partial \Omega}{\partial c}\right) = -e^{-4A_0} + \frac{V_W^0}{V_{CY}}\,, \tag{3.19}$$

where $V_W^0 = \int d^6y\sqrt{\tilde{g}_6}e^{-4A_0(y)}$ is the background value of the warped volume. This equation is consistent in compact CY manifolds because the right hand side integrates to zero (which is actually the condition which fixes the factor of $e^{2\Omega}$ in (3.12)). Therefore, the 10d metric solving the Hamiltonian constraints,

$$\begin{aligned} ds^2_{10} \;\; &= \;\; \Big[e^{-4A_0(y)} + c(x)\Big]^{-1/2} e^{2\Omega[c(x)]}\,\left(\hat{g}_{\mu\nu}(x)\,dx^\mu dx^\nu + 2\,\partial_i B\,\partial_\mu c\,dy^i dx^\mu\right) + \\ &\quad + \Big[e^{-4A_0(y)} + c(x)\Big]^{1/2}\,\tilde{g}_{ij}(y)\,dy^i dy^j \end{aligned} \tag{3.20}$$

gives a consistent spacetime dependent solution representing infinitesimal fluctuations of the universal volume modulus. The last part of the 10d fluctuation is in the 4-form potential, which is proportional to e^{4A}. Intuitively, the BPS-like condition of [25] sets $C_4 = e^{4\Omega}\, e^{4A(y;\,c)}\, d^4x$, so the 4-form fluctuates along with the volume modulus. More details are given in section 3.5, where these results will be extended to finite fluctuations.

3.2.2 Summary

Briefly summarizing the main points of the previous computation, the warped universal volume modulus is not associated to a simple trace rescaling of the underlying CY metric, unlike in the unwarped case. Rather, $\tilde{g}_{ij}(y)$ stays fixed and the modulus corresponds to an additive shift

$$e^{-4A(x,y)} = e^{-4A_0(y)} + c(x) \ , \tag{3.21}$$

where $e^{-4A_0(y)}$ is the background solution with respect to $\tilde{g}_{ij}$. There is also a nonzero compensating field $\partial_i B$ determined by (3.19).

A more physical way of stating this is by noticing that in the 4d action the compensating field *only* appears through the shift [1]

$$\delta_c g_{MN} = \frac{\partial g_{MN}}{\partial c} - D_N \left(e^{2A+2\Omega}\, \partial_M B\right) - D_M \left(e^{2A+2\Omega}\, \partial_N B\right) \ , \tag{3.22}$$

where D_N is the covariant derivative with respect to the 9d spacelike metric. The physical 10d fluctuation associated to $c(x)$ then becomes

$$\delta_c g_{\mu\nu} = 2\, e^{2A+2\Omega}\, \eta_{\mu\nu} \left(\delta_c A + \frac{\partial \Omega}{\partial c}\right) \ , \ \delta_c g_{ij} = -\, e^{-2A} \left(2\, \delta_c A\, \tilde{g}_{ij} + \delta_c \tilde{g}_{ij}\right) \ , \tag{3.23}$$

where

$$\delta_c A := \frac{\partial A}{\partial c} - e^{4A+2\Omega}\, \partial^{\tilde{\imath}} A\, \partial_i B \ , \ \ \delta_c \tilde{g}_{ij} = \tilde{\nabla}_i \left[e^{4A+2\Omega} \partial_j B\right] + \tilde{\nabla}_j \left[e^{4A+2\Omega} \partial_i B\right] \ . \tag{3.24}$$

The dependence of Ω and A on $c(x)$ is given in (3.13) and (3.18). Strikingly, for non-trivial warping the universal volume modulus has an internal metric fluctuation $\delta_c g_{ij}$ which *is not pure trace*. The nontrivial dependence comes from the effect of the compensating field. Stated in gauge invariant terms, this is required so that the canonical momentum $\delta_c \pi_{MN}$ built from $\delta_c g_{MN}$ is in harmonic gauge with respect to the warped 10d metric.

Notice that in the unwarped (or large volume) limit the warp factor becomes $e^{-4A} \approx c(x) := e^{4u(x)}$, which in turn implies $e^{2\Omega} = e^{-4u(x)}$. The equation of motion for the compensator (3.19) becomes simply $\tilde{\nabla}^2 B = 0$, which is solved by $B = 0$, so we regain the usual metric for the universal volume modulus in the unwarped case (3.2).

3.2.3 Orthogonality with other modes

The metric moduli arise as independent solutions to a Sturm-Lioville problem. Different zero modes should be orthogonal to each other, and we may use this to understand how to define the universal volume modulus from the original $h^{1,1}$ moduli.

The natural inner product is given by the Hamiltonian Eq. (2.42). Consider two zero mode solutions, with canonical momenta $\delta_I \pi_{MN}$ and $\delta_J \pi_{MN}$ respectively ($I \neq J$). The orthogonality condition reads

$$G_{IJ} = \int d^{D-1}x \sqrt{-g_D}\, g^{tt} \left[\delta_I \pi_{MN}\, \delta_J \pi^{MN} - \frac{1}{D-2}\, \delta_I \pi\, \delta_J \pi\right] = 0\,, \tag{3.25}$$

where $D = 10$ in our case.

We need to compute the inner product Eq. (3.25) between the universal volume modulus and the nonuniversal metric fluctuations. Recall that the canonical momentum associated to such a fluctuation is [1]

$$\delta_I \pi_{ij} = e^{-2A} \left(\delta_I \tilde{g}_{ij} - \frac{1}{2} \tilde{g}_{ij}\, \delta_I \tilde{g} \right)\,, \tag{3.26}$$

where

$$\delta_I \tilde{g}_{ij} = \frac{\partial \tilde{g}_{ij}}{\partial u^I} - \tilde{\nabla}_i \left(e^{4A}\, B_{Ij} \right) - \tilde{\nabla}_j \left(e^{4A}\, B_{Ii} \right)\,. \tag{3.27}$$

Here B_{Ij} is the compensating field required by the time-dependent fluctuation $\partial \tilde{g}_{ij}/\partial u^I$. Unlike the case of the universal modulus, the B_{Ij} are not total derivatives; compare with Eq. (3.23) and Eq. (3.24).

Next, specialize to $I = c$, the universal volume modulus, and $J \neq c$ a nonuniversal zero mode. Using orthogonality with respect to gauge transformations and $\delta \pi_{\mu\nu} = 0$,

$$G_{cJ} = \int d^{D-1}x \sqrt{-g_D}\, g^{tt}\, \frac{\partial g^{ij}}{\partial c}\, \delta_J \pi_{ij}\,. \tag{3.28}$$

Recalling that $\partial g_{ij}/\partial c = (1/2)\, e^{4A}\, g_{ij}$, the orthogonality condition requires

$$\int d^6y\, \sqrt{\tilde{g}_6}\, \tilde{g}^{ij}\, \delta_J \tilde{g}_{ij} = 0\,, \tag{3.29}$$

which is solved by

$$\tilde{g}^{ij}\, \frac{\partial \tilde{g}_{ij}}{\partial u^J} = 0\,. \tag{3.30}$$

The compensating fields in (3.27) drop from (3.29), being total derivatives.

The nonuniversal Kähler moduli thus correspond to the $h^{1,1} - 1$ *traceless* combinations, and Eq. (3.30) defines the basis of linearly independent metric zero modes orthogonal to the universal volume modulus. It is interesting that we recover the known result from CY compactifications, although the universal mode is no longer a pure trace fluctuation of the internal metric. We should also point out that (3.30) is not a gauge condition: we can fix completely the diffeomorphism redundancies by setting the compensating fields to zero, but we would still need to impose (3.30). Rather, it tells us how to choose a particular basis in the space of solutions to the Sturm-Liouville problem of the metric zero modes. This grants that there are no kinetic mixings between the volume modulus and the other zero modes.

3.3 Axionic partner of the volume modulus

In the unwarped limit, the universal volume modulus gets complexified with the axion coming from

$$C_4 = \frac{1}{2} a(x) \tilde{J}(y) \wedge \tilde{J}(y) + \cdots \ . \tag{3.31}$$

In this section, we construct the universal axion in warped backgrounds. This will be the partner of the warped volume modulus (3.23). At the end of the section, the $h^{1,1}-1$ nonuniversal axions will be shown to be orthogonal to the universal axion, so they will be set to zero in the main part of the analysis. This is the counterpart of what happens with the universal volume modulus, as can be anticipated for supersymmetric compactifications.

The Hamiltonian formulation for antisymmetric tensors is similar to the familiar $U(1)$ Maxwell case, where the canonical momentum is the electric field,

$$E^i = \frac{\partial \mathcal{L}}{\partial \dot{A}_i} = g^{tt}\, g^{ij} \left(\partial_0 A_j - \partial_j A_0\right) \ , \tag{3.32}$$

and A_0 is a Lagrange multiplier enforcing Gauss's law $\nabla^i E_i = 0$. The shift of (3.32) by $\partial_i A_0$ is the analog of the metric fluctuation shift Eq. (2.39) by the compensating field.

The generalization to a p-form C_p is as follows. $C_{0 i_2 \ldots i_p}$ plays the role of a Lagrange multiplier, and the canonical momentum is given by the $p+1$-form

$$E := \frac{1}{(p+1)!}\, F_{0 i_1 \ldots i_p}\, dx^0 \wedge dx^{i_1} \wedge \ldots dx^{i_p}\,, \tag{3.33}$$

where $i_1, \ldots, i_p$ are spacelike indices. If there are no couplings to external fields the constraint is

$$d\left(\star_D E\right) = 0\,. \tag{3.34}$$

The Hamiltonian kinetic term is then

$$\int dt\, H_{kin} = \int E \wedge \star_D E = \frac{1}{(p+1)!} \int d^D x\, \sqrt{g_D}\, F_{0 i_1 \ldots i_p}\, F^{0 i_1 \ldots i_p}\,. \tag{3.35}$$

This is gauge invariant due to (3.34). The magnetic field contributions $F_{i_1 \ldots i_{p+1}}$ appear in the potential energy.

3.3.1 Gauge transformations and field redefinitions of C_4

The dimensional reduction of fluctuations of C_4 in 3-form flux background is slightly subtle due to its nonstandard gauge transformations. We follow the discussion of [44], which considered the case of a torus orientifold in some detail.

In terms of the 4-form that couples electrically to a D3-brane

$$S_{WZ} = \mu_3 \int C_4 \ , \tag{3.36}$$

the 5-form field strength is $\tilde{F}_5 = dC_4 - C_2 H_3$. The gauge transformations that leave $\tilde{F}_5$ invariant are

$$C_4 \to C_4 + d\chi_3 + \zeta_1^C \wedge H_3 \ , \quad C_2 \to C_2 + d\zeta_1^C \ , \quad B_2 \to B_2 + d\zeta_1^B \ . \tag{3.37}$$

In a background of nontrivial 3-form flux, the potentials B_2 and C_2 are well-defined only on coordinate patches, which must be glued together with gauge identifications $\zeta^{B,C}$. With a fixed choice of background potentials C_4, B_2, and C_2, the gauge transformations $\zeta^{B,C}$ are also fixed, so fluctuations δB_2, δC_2 must be globally defined on the internal manifold (on a torus, this means they are periodic). Hence, they have the appropriate behavior for dimensional reduction without any issue of gluing coordinate patches together.

The 4-form is slightly more complicated; the background C_4 is also defined only on patches and glued together by the gauge transformation (3.37) with H_3 the background flux. This means that the fluctuation also has a nontrivial gauge gluing $\delta C_4 \to \delta C_4 + d\chi + \zeta^C \delta H_3$. To simplify the gluing conditions, we can define $\delta C_4' = \delta C_4 - C_2 \delta B_2$ (to linear order); this is glued together by gauge transformations $\delta C_4' \to \delta C_4' + d\chi'$ with $\chi' = \chi - \lambda^C \delta B_2$, which are trivial as long as there is no quantized 5-form flux. Therefore, the 4-form potential that follows ordinary dimensional reduction is $\delta C_4'$. The field strength and complete gauge transformations work out to be

$$\delta\tilde{F}_5 \quad = \quad d\delta C_4' + \frac{ig_s}{2}\left(\delta A_2 \wedge \bar{G}_3 - \delta\bar{A}_2 \wedge G_3\right) \tag{3.38}$$

$$\delta C_4' \quad \to \quad \delta C_4' + \delta\chi' + \frac{ig_s}{2}\left(\bar{\zeta}^A \wedge G_3 - \zeta^A \wedge \bar{G}_3\right) \tag{3.39}$$

in terms of the complex potential $A_2 = C_2 - \tau B_2$, $G_3 = dA_2$. Henceforth, we drop the prime on δC_4.

3.3.2 Axion fluctuation in a warped background with flux

We now apply the previous approach to find the 10d universal axion in the presence of warping and three-form flux. There are extra subtleties arising from self-duality and the unusual gauge transformations of the 4-form potential. We start by generalizing the known form from unwarped compactifications, since the wavefunction should reduce to that form in the unwarped limit. We also find that a constant axion yields a trivial field strength, even in the presence of a fluctuating volume modulus, so the solution respects the classical axion shift symmetry. Also, recall that we are working in the limit of constant axio-dilaton.

From the discussion in the previous subsection, the globally defined 5-form and 3-form canonical momenta (3.33) are

$$\begin{aligned} \tilde{E}_5 &= d\delta C_4 + \frac{ig_s}{2}(\delta A_2 \wedge \bar{G}_3 - \delta \bar{A}_2 \wedge G_3) && (3.40) \\ E_3 &= d\delta A_2 \ , && (3.41) \end{aligned}$$

where $A_2 = C_2 - \tau B_2$. δC_4 and δA_2 denote the components of C_4 and A_2 which depend on the axion field; their explicit form will be given momentarily. The presence of the "transgression terms" in (3.40), reflects the fact that the canonical momenta are invariant under the gauge transformations,

$$\begin{aligned} \delta C_4 &\rightarrow \delta C_4 + d\chi_3 + \frac{ig_s}{2}(\bar{\zeta}_1 \wedge G_3 - \zeta_1 \wedge \bar{G}_3) \ , \\ \delta A_2 &\rightarrow \delta A_2 + d\zeta_1 \, . && (3.42) \end{aligned}$$

We expect the axion to descend from the 4-form gauge potential δC_4; however, we notice that there are two separate gauge transformations associated with δC_4, one of which arises from gauge transformations of δA_2. From the Hamiltonian perspective, gauge transformations are associated with corresponding compensators, so we expect that there should be compensators for the axion associated with *both* δC_4 and δA_2.

We take the ansatz

$$\delta C_4 = \frac{1}{2} a_0(x) \tilde{J}^2 + a_2(x) \wedge \tilde{J} - da_0 \wedge K_3 - da_2 \wedge K_1 \ , \quad \delta A_2 = da_0 \wedge \Lambda_1 \qquad (3.43)$$

(note that $\tilde{J} \wedge \tilde{J} = 2\,\tilde{\star}_6 \tilde{J}$). Here, a_0 and a_2 are spacetime 0- and 2-forms respectively, while $K_{1,3}$ and Λ_1 are forms on the internal manifold included as possible compensators. The canonical momenta (3.40-3.41) are then

$$\begin{aligned} \tilde{E}_5 &= da_0 \wedge \left(\tilde{\star}_6 \tilde{J} + dK_3 + \frac{ig_s}{2} \Lambda_1 \wedge \bar{G}_3 - \frac{ig_s}{2} \bar{\Lambda}_1 \wedge G_3 \right) + da_2 \wedge \left(\tilde{J} + dK_1 \right) && (3.44) \\ E_3 &= da_0 \wedge d\Lambda_1 \, . && (3.45) \end{aligned}$$

Notice that $\tilde{E}_5$ vanishes trivially for a constant axion a_0, so the field space metric cannot depend on the axion, as expected from the classical axion shift symmetry. The 5-form canonical momentum $\tilde{E}_5$ is self-dual, which reduces the 4d degrees of freedom to a single scalar by requiring

$da_2 \propto \hat{\star}_4 da_0$. At linear order, the proportionality constant may depend only on expectation values of moduli (at higher orders, it may also depend on fluctuations of moduli); we will see that the full wavefunction requires the choice $da_2 = e^{4\Omega}\hat{\star}_4 da_0$. In this work we only keep a_0 as an independent field, multiplying the kinetic term by 2.[2]

Imposing the constraint (3.34) for the 5-form, we find that

$$d\left[e^{4A}\left(\tilde{J} + \tilde{\star}_6\left(dK_3 + \frac{ig_s}{2}\Lambda_1 \wedge \bar{G}_3 - \frac{ig_s}{2}\bar{\Lambda}_1 \wedge G_3\right)\right)\right] = 0 \tag{3.46}$$

$$d\left[e^{-4A}\left(\tilde{\star}_6\tilde{J} + \tilde{\star}_6 dK_1\right)\right] = \frac{ig_s}{2}e^{-2\Omega}\left(\Lambda_1 \wedge \bar{G}_3 - \bar{\Lambda}_1 \wedge G_3\right) . \tag{3.47}$$

These constraints are identical to the 10d equations of motion $d(\star_{10}\tilde{F}_5) = (ig_s/2)G_3 \wedge \bar{G}_3$ evaluated for legs in the internal directions. (The factor of $e^{-2\Omega}$ on the right-hand-side of Eq. (3.47) is related to the proportionality factor in the 4d Poincaré duality between a_0 and a_2.) In this way, the Hamiltonian and Lagrangian approaches yield equivalent results, and a_0 corresponds to a massless 4d field.

For the volume modulus, the compensating field is determined by a single scalar function $\eta_i = e^{2A+2\Omega}\,\partial_i B$, and we expect the same to occur for the compensator in δC_4. The form of the compensator equation (3.46) then motivates the following ansatz,

$$e^{4A}\left[\tilde{J} + \tilde{\star}_6\left(dK_3 + \frac{ig_s}{2}\Lambda_1 \wedge \bar{G}_3 - \frac{ig_s}{2}\bar{\Lambda}_1 \wedge G_3\right)\right] = e^{2\Omega}\tilde{J} + e^{2\Omega}d\left(e^{4A}dK\right) \tag{3.48}$$

in terms of a function $K(y)$. The factor of $e^{2\Omega}$ is fixed by wedging (3.48) with $\tilde{\star}_6\tilde{J}$ and integrating over the internal space. In fact, this ansatz yields an appropriately self-dual 5-form if we take $K_1 = e^{4A}dK$, and the factor here precisely fixes the proportionality in the relation between a_0 and a_2. Replacing this ansatz in (3.47), we obtain the compensator equation for $K(y)$,

$$d\Big(\tilde{\star}_6[dA \wedge dK]\Big) + \frac{1}{8}\,de^{-4A} \wedge \tilde{J} \wedge \tilde{J} = e^{-2\Omega}\frac{ig_s}{8}\left(d\Lambda_1 \wedge \bar{G}_3 - d\bar{\Lambda}_1 \wedge G_3\right) . \tag{3.49}$$

The second constraint, associated with the A_2 gauge transformation, fixes the compensator Λ_1,

$$d(\tilde{\star}_6 d\Lambda_1) = 4i\,e^{2\Omega}e^{4A}dA \wedge dK \wedge G_3 . \tag{3.50}$$

and the 4d Poincaré duality relation (which fixes the power of $e^{2\Omega}$.

There is one other issue in this analysis. Because there is a background 5-form associated with the warp factor, the axion fluctuations can appear in the Hamiltonian equation for $\dot{\pi}_{MN}$ at linear order, through terms of the form $\delta\tilde{F}_{MP_1\ldots P_4}\,\tilde{F}_N{}^{P_1\ldots P_4}$. By examining the allowed components, we can see that the only terms that contribute are of the form

$$4\tilde{F}_\mu{}^{\nu\lambda\rho n}\delta\tilde{F}_{m\nu\lambda\rho n} + \tilde{F}_m{}^{npqr}\delta\tilde{F}_{\mu npqr} . \tag{3.51}$$

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[2]See [45] for a careful treatment of the self-dual form.

However, with the background 4-form potential proportional to the 4d volume form, self-duality of the 5-form causes this contribution to vanish for any fluctuations $\delta\tilde{F}$ with these components.

Summarizing, the gauge invariant wavefunction for the universal axion in a warped background is given by the canonical momenta

$$\tilde{E}_5 = (1+\star_{10})\left[e^{2\Omega} da_0(x)\wedge\tilde{\star}_6\left(e^{-4A}\,\tilde{J}+4\,dA\wedge dK\right)\right] \tag{3.52}$$

$$E_3 = da_0\wedge d\Lambda_1\ , \tag{3.53}$$

where K,Λ_1 satisfy the Gauss law constraints (3.49,3.50) respectively. Heuristically, the warp factor dependence arises naturally from $J\wedge J = e^{-4A}\,\tilde{J}\wedge\tilde{J}$.

In the unwarped limit, we see that the compensators become gauge trivial. First, K_1 becomes exact. Similarly Eq. (3.50) implies that Λ_1 is closed, so $\delta G_3 = 0$. The residual gauge freedom to make Λ_1 co-closed means that it must vanish (because there are no harmonic 1-forms on a CY); this same gauge transformation also forces K_3 to be closed, as required by Eq. (3.48) since $e^{2\Omega} = e^{4A} = c^{-1}$. Then it is simple to gauge away the K_1 and K_3 compensators in Eq. (3.43). As expected, we then recover the known axion wavefunction in a CY background. Also note that the compensators Λ_1 become trivial when the background 3-form flux vanishes, which we expect because δC_4 has only one gauge transformation in that case.

3.3.3 Orthogonality with nonuniversal axions

We now consider the effect of the $h^{1,1}-1$ nonuniversal axions. The story is similar to the above. For $\tilde{\rho}_r$ the independent $(1,1)$ forms in the 2nd cohomology ($\tilde{J}=\tilde{\rho}_1$), the potential now becomes

$$\delta C_4 = \sum_{r=1}^{h^{1,1}}\left[a_0^r(x)\tilde{\star}_6\tilde{\rho}_r + a_2^r(x)\wedge\tilde{\rho}_r - da_0^r\wedge dK_{3,r} - da_2^r\wedge dK_{1,r}\right]\ ,\quad \delta A_2 = da_0^r\,\Lambda_{1,r}\ . \tag{3.54}$$

Computing the canonical momentum, we obtain constraints analogous to (3.46,3.47), which along with self-duality imply

$$e^{4A}\left(\tilde{\rho}_r + \tilde{\star}_6 dK_{3,\,r} + \frac{ig_s}{2}\Lambda_{1,r}\wedge\bar{G}_3 - \frac{ig_s}{2}\bar{\Lambda}_{1,r}\wedge G_3\right) = e^{2\Omega}M_r^s(u)\ (\tilde{\rho}_s + dK_{1,\,s})\ , \tag{3.55}$$

with $M(u)$ some function of the moduli u^I, which can be diagonalized. The constraint from the 2-form gauge transformation is of a similar form as (3.50), but with a more general 1-form $K_1 \neq e^{4A}dK$ on the right hand side, because there are no harmonic 5-forms on a CY.[3]

The kinetic term mixing between the universal and nonuniversal axions is

$$\int \tilde{E}_{5,r}\wedge\star_{10}\tilde{E}_{5,1} + \frac{g_s}{2}\int E_{3,r}\wedge\star_{10}\bar{E}_{3,1} + \frac{g_s}{2}\int \bar{E}_{3,r}\wedge\star_{10}E_{3,1} \tag{3.56}$$

[3] Again, there are additional terms on T^6 or $T^2\times K3$, but they still cancel in the kinetic term.

Using the constraint equations (3.55,3.50) in a calculation similar to that presented below in section 3.4, the kinetic mixing is proportional to $(\tilde{\rho}_r, \tilde{J}) = \int \tilde{\star}\rho_r \wedge \tilde{J}$. Since this is the natural inner product on the 2nd cohomology, the universal axion is orthogonal to the other $h^{1,1}-1$ axionic excitations as long as the basis of $(1,1)$ forms is chosen to be orthogonal itself.

3.4 Kähler potential

Finally we are ready to compute the kinetic term and Kähler potential for the chiral superfield

$$\rho = a_0 + i\,c \tag{3.57}$$

which combines the universal Kähler modulus found in Eq. (3.23) with the axionic mode given in Eq. (3.52). Finding an explicit answer for the Kähler potential is in general rather involved, because the compensating fields appear explicitly in the kinetic terms. Therefore, one would have to solve the second order constraint equations (which depend on the warp factor) and then plug in the explicit solution into the kinetic terms. However, using the Hamiltonian expressions for the kinetic terms, we will find that the explicit solution to the compensating fields is actually not needed. We show that the constraint equations are enough to eliminate the compensating fields from the 4d action. In this way, we compute the explicit Kähler potential.

3.4.1 Kinetic terms

First we look at the kinetic term for $c(x)$,

$$S_{kin,\,c} = \frac{1}{\kappa_4^2} \int d^4x \sqrt{\hat{g}}\, G_{cc}\, \hat{g}^{\mu\nu}\, \partial_\mu c\, \partial_\nu c\,. \tag{3.58}$$

G_{cc} follows from replacing the canonical momentum conjugate to Eq. (3.23) in the Hamiltonian expression Eq. (2.42). A short computation reveals that

$$G_{cc} = \frac{1}{2\,V_{CY}} \int d^6y \sqrt{\tilde{g}}\, e^{4\Omega+2A} \left[e^{-2A-2\Omega}(\partial_c\Omega + \partial_c A) - e^{2A}(\partial^{\tilde{m}} A)(\partial_m B) \right] . \tag{3.59}$$

Integrating by parts to get $\tilde{\nabla}^2 B$ and replacing it by its constraint (3.15), the terms containing $\partial_c A$ cancel, and $\partial_c\Omega$ controls the kinetic term. The result is

$$G_{cc} = \frac{3}{4}\, e^{4\Omega} = \frac{3}{4} \left(\frac{V_{CY}}{c(x) V_{CY} + V_W^0} \right)^2 , \tag{3.60}$$

showing the well-known factor of 3 for the kinetic term of the universal volume modulus. It is interesting that this factor arises from nontrivial cancellations of different warping corrections, which would not occur had we neglected the compensating field contribution.

To calculate the kinetic term for the universal axion, we take the prescription for the 5-form in which we double the coefficient of the $\tilde{F}_5^2$ term in the action but consider only half the components. We will keep the terms including the scalar a_0 as opposed to a_2 (with $a_0 = a_0(t)$, this corresponds to keeping components of $\tilde{F}_5$ with time indices). Replacing the axion fluctuation (3.52) into the kinetic action, we find[4]

$$\begin{aligned} S_{kin,\,a} &= -\frac{1}{4\kappa_{10}^2}\int \left(\tilde{E}_5 \wedge \star_{10}\tilde{E}_5 + g_s\, E_3 \wedge \star_{10}\bar{E}_3\right) \\ &= -\frac{1}{4\kappa_{10}^2}\int e^{4\Omega} da_0 \wedge \hat{\star}_4 da_0 \int \Big[\left(\tilde{\star}_6(e^{-4A}\tilde{J} + 4dA\wedge dK)\right)\wedge\left(\tilde{J} + de^{4A}\wedge dK\right) \\ &\quad + e^{-2\Omega}\, g_s\, d\Lambda_1 \wedge \tilde{\star}_6 d\bar{\Lambda}_1\Big]\,. \end{aligned} \tag{3.61}$$

Note that the Chern-Simons term does not include a_0, so it does not appear. Integrating by parts and using the constraint equations (3.49,3.50) to eliminate the compensators $K(y), \Lambda_1(y)$, we arrive to

$$S_{kin,\,a} = -\frac{3}{4\kappa_4^2}\int \sqrt{-\hat{g}}e^{4\Omega}\hat{g}^{\mu\nu}\partial_\mu a_0 \partial_\nu a_0\,. \tag{3.62}$$

The factor of 3 comes from

$$\int \tilde{J}\,\wedge\,\tilde{\star}_6\tilde{J} = \frac{1}{2}\int \tilde{J}^3 = 3V_{CY}\,. \tag{3.63}$$

This reproduces precisely the field space metric of the volume modulus. As we saw with the metric volume modulus, we see that the presence of the compensators in (3.61) are crucial to obtain the correct form for the kinetic term (3.62).

3.4.2 Kähler potential and no-scale structure

The previous analysis shows that the volume modulus and universal axion can be complexified into

$$\rho(x) = a_0(x) + i\,c(x)\,. \tag{3.64}$$

In fact, since our analysis has not relied on the particular components of the 3-form flux, the volume modulus and axion form a complex scalar even in compactifications with classically broken supersymmetry. From the kinetic terms (3.58) and (3.62), we obtain

$$S_{kin} = -3\frac{1}{\kappa_4^2}\int d^4x\sqrt{\hat{g}_4}\;\frac{\hat{g}^{\mu\nu}\,\partial_\mu\rho\partial_\nu\bar{\rho}}{\left[-i(\rho-\bar{\rho}) + 2\,V_W^0/V_{CY}\right]^2} \tag{3.65}$$

This metric follows from the Kähler potential,

$$K = -3\log\left(-i(\rho-\bar{\rho}) - 2\,\frac{V_W^0}{V_{CY}}\right)\,. \tag{3.66}$$

[4]Recall that E_p is the "electric field" $F_{0i_1\ldots i_{p-1}}$.

Corrections due to warping amount to an additive constant in the Kähler potential. This proves that no-scale structure $G^{\rho\bar{\rho}}\partial_\rho K \partial_{\bar{\rho}} K = 3$ is maintained in GKP type compactifications, albeit in terms of a highly nontrivial 10d wavefunction for ρ.

The quantity (V_W^0/V_{CY}) may be interpreted as the background value for $c(x)$, so, after shifting

$$\rho \to \rho - i\,\frac{V_W^0}{V_{CY}}\,, \tag{3.67}$$

the Kähler potential is

$$K = -3\log\left[-i(\rho-\bar{\rho})\right]\,. \tag{3.68}$$

This result coincides with the unwarped expression. The correction from warping becomes important, for example, once a nonperturbative superpotential for ρ is included as in [28]. The instanton or gaugino condensation superpotential receives then an exponential correction from warping due to the shift,

$$W = A\,e^{ia\,\rho} \to A\,e^{aV_W^0/V_{CY}}\,e^{ia\rho}\,. \tag{3.69}$$

Similarly, if we consider α' corrections [41], the shift modifies the potential for the volume modulus. The modifications in both these cases deserve further study.

The fact that a series of rather subtle corrections conspire to give the very simple final result (3.66) suggests that there could be some underlying physical reason for this[5]. One way to understand this is to notice that (in the absence of contributions beyond classical supergravity) the 10d solution we have found preserves the shift symmetry $e^{-4A} \to e^{-4A} + c(x)$. This implies no-scale structure, which in turn restricts the Kähler potential to be of the general form

$$K(\rho,\bar{\rho}) = -3\log\left[-i(\rho-\bar{\rho}) + a\right] + b\,. \tag{3.70}$$

Therefore, the shift-symmetry of the full solution protects the Kähler potential from significant warping corrections.

3.5 Nonlinear solution for fluctuating volume modulus

In this section, we present a complete, nonlinear solution to the 10d supergravity field equations corresponding to a wave of the universal volume modulus. Our solutions are appropriate for compactifications of the form discussed in [25]. For ease of presentation, we will work with the covariant equations of motion.

[5] We thank S. Kachru and A. Tomasiello for discussions on this point.

The external spacetime metric in the time-dependent background takes a pp-wave form, as is appropriate for a propagating massless field. As a brief review, the pp-wave metric has the form

$$ds_4^2 = \hat{g}_{\mu\nu} dx^\mu dx^\nu = -H(x^+, \vec{x})(dx^+)^2 + dx^+ dx^- + d\vec{x}^2 \ . \tag{3.71}$$

A clear but important property of this metric is that $\hat{g}^{++} = 0$. It will be important later that most of the Christoffel symbols vanish; in particular, $\hat{\Gamma}^+_{\mu\nu} = 0$. The only nonvanishing Ricci tensor component is $\hat{R}_{++} = (1/2)\vec{\partial}^2 H$, so the Ricci scalar vanishes.

As a source, consider a massless scalar with action

$$S = -\frac{1}{2\kappa_D^2} \int d^D x \sqrt{-\hat{g}} f(\phi) \left(\partial\phi\right)^{\hat{2}} \ . \tag{3.72}$$

It is clear that any function $\phi(x^+)$ solves the scalar equation of motion, and, since $\partial\phi$ is null, the Einstein equation is (the only nontrivial component is $++$)

$$\hat{R}_{\mu\nu} = f(\phi)\partial_\mu\phi\partial_\nu\phi \ , \tag{3.73}$$

which is solved by

$$H(x^+, \vec{x}) = \frac{1}{2(D-2)} \left|\vec{x}\right|^2 f\left(\phi(x^+)\right) \left(\partial_+\phi(x^+)\right)^2 \ . \tag{3.74}$$

Since H is quadratic in the scalar velocity, we see immediately why previous attempts to solve for the volume modulus beyond linear order have failed.

3.5.1 Ten-dimensional solution

We can now present the nonlinear solution for a propagating volume modulus and verify that it solves the equations of motion. The warp factor profile in the compact dimensions remains the same as in the static case, and the compensator wavefunction is given by the linearized expression. In addition, since 3-form fluxes do not stabilize the volume modulus, we include the 3-forms quite simply, so these results apply to all GKP compactifications [25]. Throughout, we assume that 7-branes are in the orientifold limit, so that the internal space is conformally CY and the axio-dilaton is constant. We also work away from localized sources such as branes or orientifolds for simplicity; removing these assumptions is a straightforward generalization.

The 10d background corresponding to a finite fluctuation of the universal volume modulus can be written as

$$ds^2 = e^{2A(x,y)} e^{2\Omega(x)} \bar{g}_{\mu\nu}(x,y) dx^\mu dx^\nu + e^{-2A(x,y)} \tilde{g}_{ij}(y) dy^i dy^j \tag{3.75}$$

$$\tilde{F}_5 = e^{4\Omega} d^4x \wedge d\left(e^{4A}\right) + \tilde{\star} d\left(e^{-4A_0}\right) \ , \tag{3.76}$$

where we have defined the shorthand $e^{2\Omega}$ for the Einstein frame factor as in Eq. (3.13) and the warp factor as in Eq. (3.18) as well as a 4d metric

$$\bar{g}_{\mu\nu}(x,y) = \hat{g}_{\mu\nu}(x) - 2\left(\hat{\nabla}_\mu \partial_\nu c(x) + e^{2\Omega(x)}\partial_\mu c(x)\partial_\nu c(x)\right) B(y) \ . \tag{3.77}$$

Here, $\hat{g}_{\mu\nu}$ is a pp wave as defined in Eq. (3.71), and $B(y)$ is a compensator that obeys the same constraint as in the linear case Eq. (3.19). In addition, the volume modulus $c(x)$ depends only on a null direction, which we denote x^+. This means that $\hat{\nabla}_\mu\partial_\nu c = \bar{\nabla}_\mu\partial_\nu c = \partial_+^2 c$ (or for any field). In addition, since $\hat{g}_{\mu\nu}$ and $\bar{g}_{\mu\nu}$ differ from Minkowski only in the $++$ component, d^4x is the volume form for those metrics as well (conveniently written in light-cone coordinates).

The first equation of motion to check is the 5-form Bianchi identity, which is satisfied as long as A_0 is the appropriate static warp factor; with fixed background 3-form flux (and local sources), the Bianchi identity is spacetime independent. Self-duality of the 5-form then fixes the spacetime component — the external component of C_4 is just the volume form of the 4d spacetime. It is also easy to see that the axio-dilaton and 3-form equations of motion are unchanged from the static solution (up to overall factors), so they are trivially satisfied, as well.

We now proceed to the Einstein equation. The μi component is just the integrated form of Eq. (3.17), which is satisfied by the "shifted" form (3.18) assumed. The internal component is slightly more complicated because it includes sources from the 5-form and 3-forms. However, because all 4d derivatives are null and the pp wave Ricci scalar vanishes, the Einstein equation reduces to the static case, which is satisfied by assumption. This is the Poisson equation

$$\tilde{\nabla}^2 e^{-4A_0} = -\frac{g_s}{12} G_{ijk}\bar{G}^{\widetilde{ijk}} \ , \tag{3.78}$$

which also follows from the 5-form Bianchi [31].

Finally, we consider the external components of the Einstein equation. A straightforward but somewhat tedious calculation finds the Ricci tensor

$$\begin{aligned} R_{\mu\nu} \quad = \quad & \hat{R}_{\mu\nu} - 2\hat{\nabla}_\mu\partial_\nu\Omega + 4\hat{\nabla}_\mu\partial_\nu A + 2\partial_\mu\Omega\partial_\nu\Omega - 8\partial_{(\mu}\Omega\partial_{\nu)}A - 16\partial_\mu A\partial_\nu A \\ & - e^{2\Omega}e^{4A}\bar{g}_{\mu\nu}\tilde{\nabla}^2 A + e^{2\Omega}e^{4A}\left(\hat{\nabla}_\mu\partial_\nu c + e^{2\Omega}\partial_\mu c\partial_\nu c\right) B \ . \end{aligned} \tag{3.79}$$

As in calculating the other components, we have made repeated use of the fact that all spacetime derivatives lie in the x^+ direction, so contractions of them automatically vanish. The trace-reversed stress tensor (we take $R_{MN} = T_{MN}$) has external components

$$T_{\mu\nu} = -4e^{2\Omega}e^{4A}\left(\partial_i A\partial^{\tilde{\imath}} A\right)\bar{g}_{\mu\nu} - \frac{g_s}{48}e^{2\Omega}e^{4A}G_{ijk}\bar{G}^{\widetilde{ijk}}\bar{g}_{\mu\nu} \ . \tag{3.80}$$

Then the external Einstein equation simplifies with the help of Eq. (3.78) along with the relations

(3.13,3.18):

$$\hat{R}_{\mu\nu} + \hat{\nabla}_\mu \partial_\nu c \left[e^{2\Omega} - e^{4A} + e^{2\Omega} e^{4A} \tilde{\nabla}^2 B \right] + \partial_\mu c \partial_\nu c \left[-\frac{1}{2} e^{4\Omega} - e^{2\Omega} e^{4A} + e^{4\Omega} e^{4A} \tilde{\nabla}^2 B \right] = 0 \ . \tag{3.81}$$

Since we take the compensator B to obey the constraint (3.19), we end up with

$$\hat{R}_{\mu\nu} = \frac{3}{2} e^{4\Omega} \partial_\mu c \partial_\mu c \ . \tag{3.82}$$

Note that the compensator term quadratic in c is necessary to cancel all the internal space dependence in the external Einstein equation. This is just the Einstein equation (3.73) for the 4d pp wave, as we desired.

3.5.2 Comments on the nonlinear background

Let us now make a few comments about the nonlinear background.

First, compare this background to the linearized one presented earlier. The Hamiltonian approach naturally defines the compensators as metric components $g_{\mu i} \propto \partial_i B$. These can be gauged away at the cost of introducing a deformation of the internal metric. However, in the nonlinear solution, it is useful to work with coordinates in which $\tilde{g}_{ij}$ is unchanged by the fluctuation and the compensator appears in the spacetime metric. In addition, the compensator now acquires a term quadratic in the modulus velocity. Finally, since the solution singles out the lightcone coordinate x^+, we found it more convenient to work with the covariant equations of motion. Otherwise, the nonlinear background is quite similar to the linearized one, and we see that the warp factor and compensator profiles are actually identical.

The existence of this nonlinear background has several important consequences. For one, the solution provides an independent derivation of the kinetic term for the volume modulus. That is, the 10d solution satisfies the 4d Einstein equation for the pp-wave (3.73), which exactly encodes the kinetic term for the massless scalar. In fact, we see that we reproduce the field space metric (3.60), even including the famous factor of 3. This fact is a highly nontrivial consistency check of the low energy theory that we have developed.

This solution is also the first time-dependent 10d background that correctly captures the nonlinear physics of modulus motion in warped string compactifications. Since it is precisely consistent with the expected effective field theory, it should end concerns raised in [36,37] about the validity of the 4d effective theory.

Finally, it seems that this solution is likely to share a number of features with cosmological backgrounds in these compactifications; in particular, if the Kähler modulus is stabilized with a mass well below the warped KK scale, its motion will be well approximated by classical solutions. Developing cosmological backgrounds would be of relevance to models of inflation in string theory and could shed light on higher-dimensional or string physics in cosmology. Unfortunately, solving for the motion of the Kähler modulus in a cosmological background is already difficult at the 4d level, so we leave this issue as an open question.

3.6 Strongly warped limit and light KK modes

In the previous sections we have obtained the 10d solution corresponding to the universal Kähler modulus, first in the linearized approximation, and then showing how to include finite fluctuations. We also studied the 4d properties of the solution, by finding the Kähler potential and proving no-scale structure. In this section we will show how to apply our results to strongly warped throats in the compactification manifold.

Strongly warped regions are important both from a phenomenological point of view and to understand gauge/gravity dualities in string theory. Moreover, the effects from compensating fields are expected to dominate in this limit [1], so this is good place to illustrate our results. Another important dynamical effect is that at strong warping the KK mass scale is redshifted, and could become of the same order as the energy scale of the EFT for the moduli fields. This is discussed in detail in chapter 5. In the first part of the section we will find the 10d wavefunction of the volume modulus at strong warping, and illustrate its behavior for various choices of warping. Next we show will how to include light KK modes, concluding that there are no kinetic mixings with the Kähler modulus.

3.6.1 Wavefunction in the strongly warped limit

To begin with a simple example, consider an AdS warp factor $e^{-4A_0} \sim N/r^4$. Without including compensating fields, the 10d wavefunction corresponding to the volume modulus $c(x)$ scales, at small r, like

$$\delta_c g_{\mu\nu} \sim \frac{r^6}{N^{3/2}}\ , \ \ \delta_c g_{rr} \sim \frac{r^2}{N^{1/2}}\ . \tag{3.83}$$

On the other hand, including the effect of compensating fields, we obtain the qualitatively different behavior

$$\delta_c g_{\mu\nu} \sim \frac{r^2}{N^{1/2}}\ , \ \ \delta_c g_{rr} \sim \frac{N^{1/2}}{r^2}\ . \tag{3.84}$$

This illustrates the point that the correct gauge invariant 10d fluctuation may differ significantly from the naive solution.

Let us be more concrete and model the throat locally by a warped deformed conifold with metric given by the the Klebanov-Strassler solution [18],

$$\begin{aligned} ds^2 &= e^{2A_0}\eta_{\mu\nu} + e^{-2A_0}\frac{\epsilon^{4/3}}{2}K(\tau)\Big[\frac{d\tau^2+(g^5)^2}{3K^3(\tau)} \\ &\quad + \cosh^2\left(\frac{\tau}{2}\right)((g^3)^2+(g^4)^2) + \sinh^2\left(\frac{\tau}{2}\right)((g^1)^2+(g^2)^2)\Big] \end{aligned} \tag{3.85}$$

where τ is the radial coordinate along the throat. The equation for the compensator (3.19) now becomes

$$\partial_\tau\left(K^2(\tau)\cosh^2\frac{\tau}{2}\ \sinh^2\frac{\tau}{2}\ B_\tau(\tau)\right) = \left(\frac{V_W^0}{V_{CY}} - e^{-4A_0(y)}\right)\frac{\epsilon^{4/3}}{6}\cosh^2\frac{\tau}{2}\ \sinh^2\frac{\tau}{2} \tag{3.86}$$

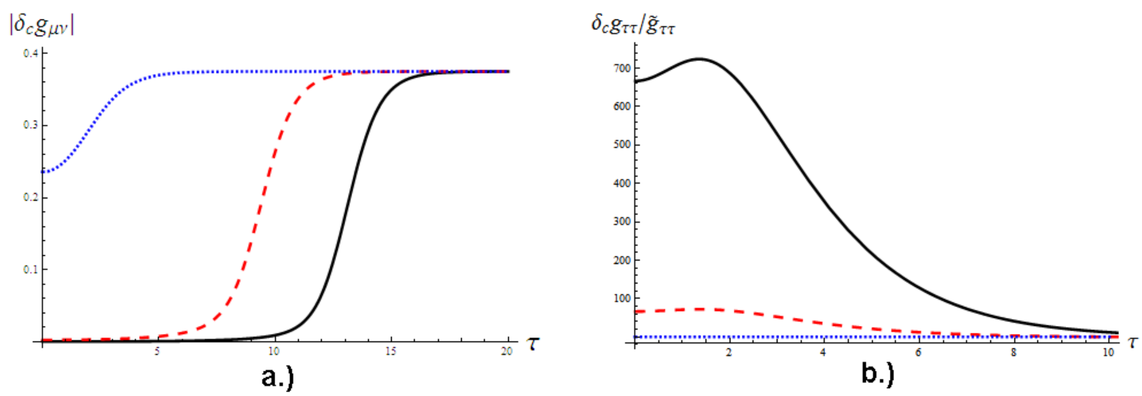

Figure 3.1: (a) The 4-dimensional wavefunction $\delta_c g_{\mu\nu}$ and (b) the internal metric wavefunction $\delta_c g_{\tau\tau}/\tilde{g}_{\tau\tau}$ in a Klebanov-Strassler warped background for various values of the warping evaluated at the tip $e^{-4A_0(0)}$: no warping $e^{-4A_0(0)} = 1$, dotted blue; weak warping $e^{-4A_0(0)} = 10^4$, dashed red; strong warping $e^{-4A_0(0)} = 10^6$, solid black. Notice that as the warping increases, the wavefunction dips deeper into the throat.

One can now solve this equation numerically for various values of the warping – the results for the wavefunctions $\delta_c g_{\mu\nu}, \delta_c g_{\tau\tau}/\tilde{g}_{\tau\tau}$ are shown in Figure 3.1. For convenience of display in Figure 3.1 we have divided out the unwarped part $\tilde{g}_{\tau\tau}$ of the metric to show that at large τ, where the warping is weak, the physical metric fluctuation asymptotes to the unfluctuated and unwarped metric, which is what we expect.

As the amount of warping increases (dashed red and solid black lines) the internal metric wavefunctions $\delta_c g_{ij}$ become more peaked in the tip region of the throat where the warping is strongest, while the 4d metric wavefunctions $\delta_c g_{\mu\nu}$ decrease to zero, as expected from our simple estimates with the AdS warp factor (3.84).

3.6.2 Inclusion of KK modes

We now address the problem of including light KK modes in the EFT of the volume modulus.[6] A general argument for the absence of kinetic mixings between zero modes and their KK excitations was given in [5]. It was based on the observation that these fluctuations are eigenvectors of a Sturm-Liouville problem, such that the orthogonality relation derived from the differential problem coincides with the Hamiltonian inner product. This then grants the absence of kinetic mixings. Since the application to p-forms may be unfamiliar, we now show that the universal axion is orthogonal to its KK excitations.

Consider then the 2-form massless and massive modes in C_4,

$$\delta C_4 = a_2(x) \wedge \tilde{J}(y) + \sum_\alpha a_2^\alpha(x) \wedge \omega_\alpha(y) \tag{3.87}$$

where ω_α are (non-closed) 2-forms, and the KK fields a_2^α are dual to spacetime scalars. The compensating fields are already absorbed into $\tilde{J}$ and ω_α. For simplicity, we are also setting the Weyl factor equal to one. There are, of course, other components, and we have not determined the complete wavefunctions for the excited KK modes, but we can see orthogonality just from these components.

Requiring that the particles have a well-defined 4d mass, $d\,(\hat{\star}_4 da_2^\alpha) = -m_\alpha^2\, \hat{\star}_4 a_2^\alpha$, we derive the eigenvector equation

$$d\,(\tilde{\star}_6 d\omega_\alpha) = m_\alpha^2\, e^{-4A}\, \tilde{\star}_6 \omega_\alpha\,. \tag{3.88}$$

The computation of the kinetic mixing between $a_2(x)$ and $a_2^\alpha(x)$ then proceeds as in Eq. (3.61):

$$\begin{aligned} \int E_5 \wedge \star_{10} E_5 \quad &\rightarrow \quad -\int_x a_2(x) \wedge d\,[\hat{\star}_4\, da_2^\alpha(x)] \int_y e^{-4A(y)}\, \tilde{J} \wedge \tilde{\star}_6 \omega_\alpha \\ &= \quad -\frac{1}{m_\alpha^2} \int_x a_2(x) \wedge d\,[\hat{\star}_4\, da_2^\alpha(x)] \int_y \tilde{J} \wedge d\,(\tilde{\star}_6 d\omega_\alpha) \end{aligned} \tag{3.89}$$

where we have used (3.88). Since $\tilde{J}$ is closed, integrating by parts the kinetic mixing vanishes.

By supersymmetry, the same holds for the universal volume modulus (since the analysis should not depend on our choice of 3-form flux, this statement holds even in classically non-supersymmetric compactifications). We conclude that light KK modes do not mix with the Kähler modulus at the level of the kinetic terms.

3.7 Conclusions

By using the Hamiltonian method, developed for warped compactifications in [1], we have computed the kinetic term and Kähler potential for the universal volume modulus and its

[6] We thank E. Silverstein for suggesting to check this.

axionic partner in IIB flux compactifications of the type studied in [25] for arbitrary warping. We found that the Kähler potential for the universal Kähler modulus takes the form

$$K(\rho,\bar{\rho}) = -3\log\left(-i(\rho-\bar{\rho}) - 2\,\frac{V_W^0}{V_{CY}}\right)\,. \tag{3.90}$$

It is rather striking that all warping corrections just amount to an additive shift $\rho \to \rho - i\,(V_W/V_{CY})$. One way to understand this result is to argue that the no-scale symmetry survives in the correct 10d warped solution. This protects the Kähler potential from further warping corrections.

It is important to emphasize that the 10d time-dependent solution that we have found is very different from the unwarped fluctuation. Therefore, the respective 4d theories are expected to be different as well, even if the Kähler potentials have the same functional dependence. In particular, once nonperturbative corrections of the form $W = A\,e^{ia\rho}$ are included, the previous seemingly innocuous shift in ρ may produce qualitative changes in the field theory. This could become important in KKLT type models [28] that rely on the existence of a strongly warped region. It would be interesting to compute the prefactor A (see [46–48]) in strongly warped backgrounds, and see how our 10d solution modifies the discussion.

In section 3.6 we showed that the warped 10d fluctuations for a time-dependent universal volume modulus are peaked at the tip of the throat, and that there are no Kähler potential mixings with light KK modes. This can be relevant for phenomenological applications in which the coupling of the universal Kähler modulus to brane and bulk fields, obtained by the 10d wavefunction overlap, is important. Also, studying further the wavefunctions of the KK modes of the universal axion could shed light on the possibility of mixing through mass terms as well as be important for studying the behavior of perturbations in strongly warped throats.

We have also shown in section 3.5 that the 10d metric fluctuations can be promoted to a fully time-dependent, warped, 10d metric for the universal volume modulus by taking into account the backreaction on the 4d space. This is a first step towards finding cosmological solutions for time-dependent Kähler moduli, which may be relevant for models of inflation.

Chapter 4

Complex structure moduli and dual gauge theories

4.1 Introduction and summary

In the previous chapter we constructed the 10d wavefunction corresponding to the (space-time dependent) universal Kähler modulus in the presence of fluxes. It was also shown that the solution preserves, at the classical level, the no-scale structure of the theory. The situation for complex deformations is quite different. As explained in section 1.6 turning on fluxes creates a nonzero superpotential

$$W = \int G_3 \wedge \Omega \tag{4.1}$$

that lifts the complex moduli.

In this chapter, based on [3, 4], we study the dynamics of complex moduli near conifold-type singularities,

$$z_1^2 + z_2^2 + z_3^2 + z_4^2 = 0 \tag{4.2}$$

The dynamics is very rich because the system admits a dual gauge theory description. In the presence of fluxes, the CY undergoes a large N geometric transition [18, 24, 49]. On the open string side of the duality, there are N D5 branes wrapping the resolved 2-cycle of the conifold. At low energy, this gives pure $SU(N)$ SYM in four dimensions, which has gaugino condensation $\langle\lambda\lambda\rangle \neq 0$. After the geometric transition, the resolved conifold is replaced by a deformed conifold with deformation parameter $S = \lambda\lambda$. The N D5 branes become N units of F_3 flux through the compact 3-cycle (A-cycle), while the gauge coupling gives nonzero H_3 through the B-cycle. Therefore, compactification on a conifold with complex modulus S and fluxes has a dual description in terms of pure SYM, where S is identified with the glueball superfield, and the flux superpotential matches precisely the Veneziano-Yankielowicz superpotential [50].

4.1.1 Summary

It turns out that in the gravity side one can turn on more general fluxes than the ones allowed in the previous dictionary (e.g. both nontrival F_3 and H_3 through the A-cycle). Therefore, in sections 4.2 and 4.3 we construct the gauge theories which are dual to configurations with more general fluxes. It will be argued that the field theories are based on (p, q) 5-brane bound states, and have fractional gauge couplings.

Next we focus on the holomorphic sector of the theory, considering the structure of supersymmetric vacua. This does not require knowing the kinetic terms. The main issue to be addressed here is whether the number of supersymmetric vacua is finite [51]. It was shown in [52] that the number of supersymmetric vacua is finite around smooth points in moduli space. The analysis may then be restricted to singularities of the moduli space where the curvature diverges. This is where the dual gauge theories come into play. In section 4.4 we argue for finiteness of flux vacua around type IIB CY singularities by proving that the Witten index of the field theory dual is finite.

In the next three sections, we focus on the non-holomorphic sector of the theory, which is affected by the warp factor. In section 4.5 we determine the effective field theory for the complex structure field S. The warp-modified moduli space metric is computed, and a new power-like divergence is found. In section 4.6 we consider supersymmetry breaking by anti-self-dual flux in the deformed conifold. We show that this leads to a parametrically small breaking scale, once warping corrections are included in the effective field theory. Finally, the dual gauge theory in the limit of strong warping is analyzed in 4.7. We find that the Kähler potential of the closed string side has a natural interpretation in terms of the composite nature of the glueball superfield.

4.2 Noncompact CY with fluxes

We begin with a description of the CY geometry. Since we are interested in analyzing a neighborhood of an ADE singularity, it is enough to consider noncompact threefolds of the form

$$P := u^2 + v^2 + F(x, y) = 0\,; \tag{4.3}$$

the nontrivial dynamics comes from the complex curve Σ: $F(x, y) = 0$. (4.3) may be thought as a decoupling limit $M_{Pl} \to \infty$ of an adequate compact variety [53], although this will not be necessary for our purposes.

For concreteness, let us consider the case of a hyperelliptic curve where we can realize singularities of the A-type:

$$F(x, y) = y^2 - W'(x)^2 - f_{n-1}(x) = 0\,. \tag{4.4}$$

$W'(x)$ is a polynomial of degree n, and will play the role of the superpotential in the gauge theory:

$$W'(x) = g_n \prod_{i=1}^{n} (x - a_i)\,. \tag{4.5}$$

$f_{n-1}(x) = \sum_{k=1}^{n} f_k x^{k-1}$ is a deformation of the singular curve $y^2 = W'(x)^2$. Its effect is to split $a_i \to (a_i^-, a_i^+)$. If all the roots of W are different then the singular curve has just ODP (conifold) singularities. We will also encounter more complicated singularities, where three or more roots coincide.

In the four dimensional effective field theory (EFT), the moduli (a_i, f_k) have a very different interpretation. Fluctuations in a_i have infinite energy and hence are non-dynamical; each arbitrary choice of a_i will give a different 4d theory so they can be interpreted as couplings. On the other hand, the f_k's are dynamical and are interpreted as scalar fields in vector multiplets. Their gauge theory meaning will become clear in section 4.3.

As shown in [54], the periods of the noncompact threefold reduce to periods of the hyperelliptic curve:

$$S_i = \int_{A_i} R(x)dx \;\;,\;\; \frac{\partial \mathcal{F}}{\partial S_i} = 2\pi i \int_{B_i} R(x)dx \tag{4.6}$$

with

$$2R(x) = W'(x) - \sqrt{W'(x)^2 + f_{n-1}(x)}\,. \tag{4.7}$$

The cycle A_i surrounds the cut $[a_i^-, a_i^+]$; B_i is the noncompact cycle dual to A_i, running from $x = a_i$ to infinity. The B-periods need to be regulated; this will be discussed shortly. Therefore all the computations can be done directly on the hyperelliptic curve $y(x)$ of genus $g = n - 1$.

When $x \to \infty$

$$R(x) \to -\frac{f_n}{2g_n x}\,. \tag{4.8}$$

This implies that R is a differential of the third kind on Σ [55]. For any value $x \in \mathbb{C}$, there are two points on the Riemann surface Σ; let $P, \tilde{P} \in \Sigma$ denote the points corresponding to $x = \infty$. Then $R(x)dx$ is a holomorphic differential only on the punctured surface $\Sigma' = \Sigma - \{P, \tilde{P}\}$.

The details of the homology of Σ and the effect of the punctures were considered in [56] and we follow their conventions. A choice of homology cycles is shown in Figure 4.1; B_j runs through the j-th cut, from $\tilde{P}$ to P. From these noncompact cycles we construct $C_i = B_i - B_n$. Besides, C_P and $C_{\tilde{P}}$ circle the punctures at P and $\tilde{P}$ respectively. The canonical symplectic basis of Σ is (A_i, C_j), $i, j = 1, \ldots, g = n-1$. In Σ, $A_1 + \ldots + A_n \equiv 0$ so A_n is not independent; however, in Σ', $A_1 + \ldots + A_n = -C_P$. This means that we can take A_n to be an independent cycle and use this to fix the values of the meromorphic differentials at infinity. A symplectic basis for $H_1(\Sigma', \mathbb{Z})$ is hence (A_i, B_j), $i, j = 1, \ldots, n$.

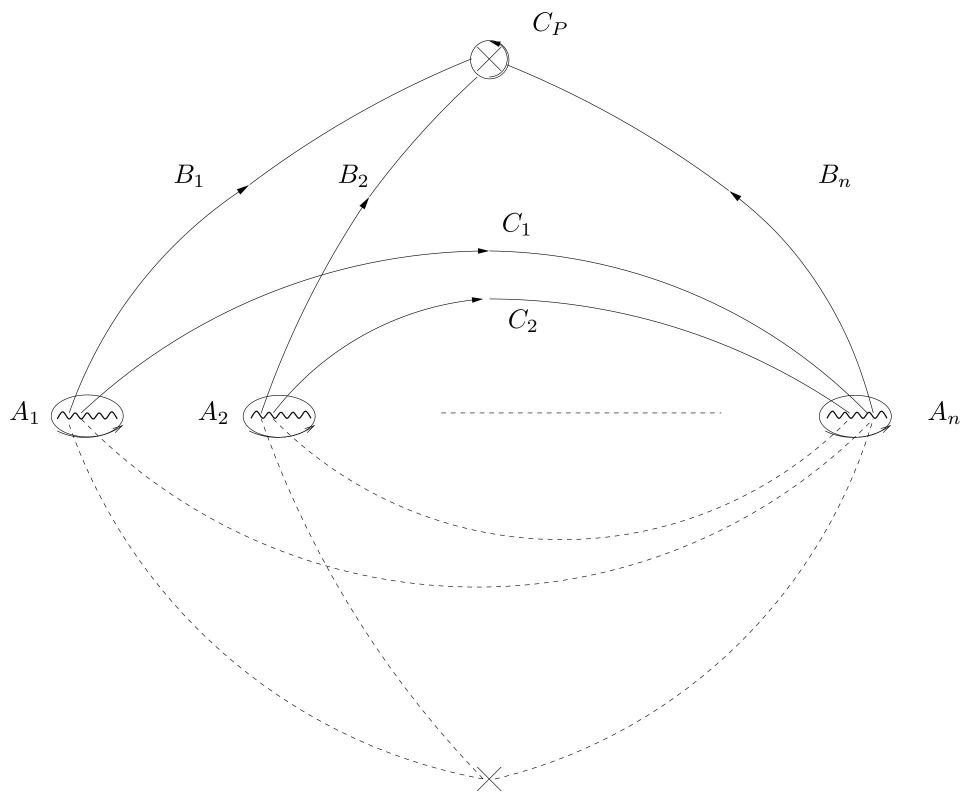

Figure 4.1: Homology elements of Σ and Σ'.

In the holomorphic decomposition $H^1(\Sigma,\mathbb{C}) = H^{1,0}(\Sigma,\mathbb{C}) + H^{0,1}(\Sigma,\mathbb{C})$, there is a unique basis of holomorphic differentials [55] $(\zeta_1,\ldots,\zeta_g)$ such that

$$\int_{A_j} \zeta_k = \delta_{jk} \ \ , \ \ \mathrm{Im}\,\Pi \geq 0 \tag{4.9}$$

where the period matrix Π is defined to be the symmetric matrix

$$\Pi_{jk} = \int_{C_j} \zeta_k \,.$$

They can be constructed as linear combinations of the differentials

$$\frac{\partial}{\partial f_k}\, y\, dx = \frac{x^{k-1}}{2y}\, dx \ \ , \ \ k = 1,\ldots,n-1\,. \tag{4.10}$$

The third kind differential

$$g_n \frac{\partial}{\partial f_n}\, y\, dx = \frac{g_n x^{n-1}}{2y}\, dx \tag{4.11}$$

has residues ± 1 at $P, \tilde{P}$ respectively. An adequate linear combination of (4.10) and (4.11) will give the unique third kind differential $\tau_{P,\,\tilde{P}}$ such that

$$\mathrm{ord}_P\, \tau_{P,\,\tilde{P}} = \mathrm{ord}_{\tilde{P}}\, \tau_{P,\,\tilde{P}} = -1\,,$$

$$\mathrm{res}_P\, \tau_{P,\,\tilde{P}} = 1\ , \ \ \mathrm{res}_{\tilde{P}}\, \tau_{P,\,\tilde{P}} = -1\,.$$

Every holomorphic differential on Σ' can be written as a linear combination of $(\zeta_1, \ldots, \zeta_g, \tau_{P\tilde{P}})$. Such differentials are meromorphic differentials on Σ with at most simple poles. A more symmetric description follows from taking A_n (instead of C_P) to be an independent cycle; hence the basis of allowed differentials will be $(\zeta_1, \ldots, \zeta_{n-1}, \zeta_n)$ where ζ_n is a superposition of ζ_i and $\tau_{P,\tilde{P}}$ fixed by $\int_{A_j} \zeta_n = \delta_{jn}$, $j = 1, \ldots, n$.

4.2.1 Geometry of the moduli space

There are different ways of parametrizing the moduli space $\mathcal{M}$ of Σ'. While from the EFT it is natural to work with the S_i, in the geometrical side it is more convenient to use the coefficients f_k of the deformation $f_{n-1}(x)$. More specifically, we parametrize $\mathcal{M}$ by combinations u_k of the f_k $(k = 1, \ldots, n)$ such that

$$\frac{\partial R}{\partial u_k} = \zeta_k \,,$$

giving directly the basis of holomorphic differentials introduced in (4.9) plus ζ_n. This is an efficient and symmetric way of taking into account the modulus from the puncture at P and will simplify our formulas.

Σ becomes singular when two branch points coincide; this leads us to define the discriminant

$$\Delta(u) := \prod_{a<b} (e_a - e_b)^2 \tag{4.12}$$

where $e_a := a_i^{\pm}$. We denote the zero locus by Σ_Δ; the moduli space is therefore

$$\mathcal{M} = \{(u_k) \in \mathbb{C}^n\} \setminus \Sigma_\Delta \,. \tag{4.13}$$

Σ_Δ is codimension one in $\mathcal{M}$ and corresponds to conifold-like singularities: around two coinciding roots we can always perform a holomorphic change of variables to rewrite the curve as

$$u^2 + v^2 + y^2 - x^2 = 0 \,.$$

Higher order Argyres-Douglas singularities [57, 58] occur when three or more roots coincide.

The moduli space is a special Kähler manifold, with metric

$$G_{i\bar{l}} = -i \int_{\Sigma'} \zeta_i \wedge \bar{\zeta}_{\bar{l}} \tag{4.14}$$

which can be derived from the Kahler potential

$$K(u, \bar{u}) = -i \int R \wedge \bar{R} \,. \tag{4.15}$$

The covariant derivative is

$$\nabla_i V^j = \partial_i V^j + \Gamma^j{}_{ik} V^k \quad , \quad \Gamma^j{}_{ik} = G^{j\bar{l}} \partial_i G_{k\bar{l}} \tag{4.16}$$

($\partial_i := \partial/\partial u^i$) and the curvature tensor is

$$R_{i\bar{j}k\bar{l}} = G_{i\bar{s}}\partial_k \Gamma^{\bar{s}}{}_{\bar{j}\bar{l}}\,. \tag{4.17}$$

A displacement in $\mathcal{M}$ deforms the complex structure of Σ, so we expect the holomorphic differentials ζ_l to mix with the antiholomorphic ones. It is easy to show that the covariant derivative of a $(1,0)$ form gives a pure $(0,1)$ form:

$$\nabla_i \zeta_j = c_{ij}{}^{\bar{k}}\bar{\zeta}_{\bar{k}}\ ,\ \ c_{ij}{}^{\bar{l}} := iG^{k\bar{l}}\int \nabla_i \zeta_j \wedge \zeta_k\,, \tag{4.18}$$

and the relation with the curvature is

$$R_{i\bar{l}j\bar{k}} = -i\, c_{ijm} c^{m}{}_{\bar{k}\bar{l}}\,. \tag{4.19}$$

4.2.2 Superpotential and fluxes

The complex moduli of the CY (X) are stabilized by Eq. (4.1). Upon integrating over the S^2 fibers given by (u,v), we obtain a superpotential on the hyperelliptic curve

$$W_{eff} = \int_{\Sigma'} T \wedge R\,. \tag{4.20}$$

The fluxes through all the compact cycles are quantized:

$$\int_{A_i} T = N_i^R - \tau N_i^{NS}\ \ ,\ \ \int_{C_i} T = c_i^R - \tau c_i^{NS}\,, \tag{4.21}$$

$N_i^R\,, N_i^{NS}\,, c_i^R\,, c_i^{NS}\ \in \mathbb{Z}$. However, the fluxes through the noncompact cycles can vary continuously and, in fact, we will argue that they have to diverge. We denote

$$-\int_{B_i} T := \beta_i^R - \tau \beta_i^{NS}\,. \tag{4.22}$$

These quantities will play the role of running gauge couplings.

Given that the B-cycles extend to infinity, and both R and T are differentials of the third kind, we need to regulate their B periods. Following [59] we introduce a cut-off at large distances $x = \Lambda_0$, replacing P and $\tilde{P}$ by Λ_0 and $\tilde{\Lambda}_0$. For the noncompact approximation to be consistent, (4.20) has to be finite in the limit $\Lambda_0 \to \infty$. We write B_i^r for the regularized version of B_i, running from $\tilde{\Lambda}_0$ to Λ_0 through the $[a_i^-, a_i^+]$ cut.

The Λ_0 dependence of $\int_{B_i^r} R$ is most easily obtained [54] by doing a monodromy around infinity $\Lambda_0^{3/2} \to \mathrm{e}^{2\pi i}\Lambda_0^{3/2}$. [1] In Σ' this corresponds to $B_i^r \to B_i^r + C_p + C_{\tilde{P}} = B_i^r - 2\sum_{i=1}^n A_i\,,$ giving

$$\int_{B_i^r} R = -\frac{1}{2\pi i}(\sum_{i=1}^{n} S_i)\log \Lambda_0^3 + \ldots \tag{4.23}$$

---

[1] The exponent is the mass dimension of x: $[x] = 3/2$, which follows from $[S] = 3$.

where ... are single valued contributions. Comparing with (4.8),

$$f_n = -4g_n \sum_{i=1}^{n} S_i \,. \tag{4.24}$$

From (4.23), we see that all the periods have the same $\log \Lambda_0^3$ dependence.

It was shown in [54] that the cutoff dependence of T is exactly the one needed to cancel the logarithmic divergence from (4.23) and yield a finite cutoff independent W_{eff}:

$$\beta_i^R - \tau\beta_i^{NS} \;=\; \frac{1}{2\pi i}(N_i^R - \tau N_i^{NS}) \log{(\Lambda_0/\Lambda_i)^3} \,. \tag{4.25}$$

The β_i where defined in (4.22) and Λ_i are a set of finite energy scales. Therefore (4.25) may be interpreted as a *geometric renormalization* of certain bare coupling constants (β_i^R, β_i^{NS}). This is the geometric analog of the RG running of the gauge couplings (see sections 4.3 and 4.4).

4.3 Gauge theory duals

Let us first quickly review the duality discovered by Vafa and collaborators, which corresponds to the flux subspace $N_i^{NS} = 0$, $\beta_n^R = 0$ and $\beta_i^{NS} = \beta_n^{NS}$ for all $i = 1, \ldots, n-1$. The large N duality between open/closed topological strings was derived in [60]. The role of the holomorphic matrix model and the relation to $\mathcal{N} = 1$ SYM was considered in [49,54,61]. On the other hand, in [62] the DV relation was derived purely from the field theory side, using the chiral ring relations and the Konishi anomaly.

Close to the semiclassical limit $|a_i^+ - a_i^-| \ll a_i$, $S_i \to 0$, the geometry (4.4) corresponds to a product of n independent deformed conifolds. They are cones over $S^3 \times S^2$ and, while the S^2s are collapsed to zero, the S^3s have finite size as measured by $S_i \neq 0$. In the geometric transition the n 3-spheres A_i are collapsed and we blow-up the conifolds at $x = a_i$ by introducing n $\mathbb{P}^1$'s. Then the RR fluxes N_i^R will disappear and, instead, we will have N_i^R D5 branes wrapping the corresponding $\mathbb{P}^1$s. The DV correspondence states that the large $N^R := \sum_{i=1}^n N_i^R$ limit of the closed string theory on the deformed threefold is equivalent to the open string theory on the resolved threefold, with the previous relation between RR fluxes and D5 branes.

$W(x)$ plays the role of a tree-level superpotential for the chiral superfield Φ in the $\mathcal{N} = 2$ vector multiplet of a pure $U(N^R)$ SYM; this potential breaks $\mathcal{N} = 2$ to $\mathcal{N} = 1$. Classically, the number of vacua is given by the number of ways of choosing N_i^R eigenvalues of Φ equal to a_i, with $\sum_i N_i^R = N^R$. This breaks $U(N^R) \to \prod_i U(N_i^R)$. β_n^{NS} is the bare gauge coupling of $U(N^R)$, while c_i^R are relative changes in the θ-angles of the $U(N_i^R)$ factors [62]. Furthermore, the complex moduli measure gaugino condensation

$$S_i = -\frac{1}{32\pi^2}\langle \mathrm{Tr}\, W_\alpha W^\alpha P_i \rangle \tag{4.26}$$

(P_i projects onto $\Phi = a_i$).

4.3.1 Dualities and geometric transition

We return now to the general flux configuration (N_i^R, N_i^{NS}), $(\beta_i^R, \beta_i^{NS})$. Denote $N^R := \sum_{i=1}^n N_i^R$, $N^{NS} := \sum_{i=1}^n N_i^{NS}$ and $r = \gcd(N^R, N^{NS})$, i.e., $N^R = n_R r$ and $N^{NS} = n_{NS} r$ with n_R and n_{NS} relatively prime.

Consider first the effect of the geometric transition around the semiclassical regime. In the open string side we end with N_i^R D5-branes and N_i^{NS} NS5-branes wrapping the i-th $\mathbb{P}^1$. The β_i do not have a brane analogue since the B-cycles remain 3-cycles; their meaning will become clear later. Our aim is to find a gauge theory interpretation for these n (N_i^R, N_i^{NS}) 5-brane states. The basic requirement is that the infrared limit of this configuration shall be given by composite fields S_i with an effective superpotential

$$W_{eff} = \sum_{i=1}^n (N_i^R - \tau N_i^{NS}) \frac{\partial \mathcal{F}}{\partial S_i} - 2\pi i \sum_{i=1}^n (\beta_i^R - \tau \beta_i^{NS}) S_i \, ; \tag{4.27}$$

we omitted a $(-1/2\pi i)$ factor as compared to (4.20).

We expect each (N_i^R, N_i^{NS}) 5-brane to decay to r_i copies of an (n_i^R, n_i^{NS}) bound state [14]; here $N_i^R = n_i^R r_i$, $N_i^{NS} = n_i^{NS} r_i$ with n_i^R and n_i^{NS} coprime. However, the generic point in flux space will give n different types of bound states and it is hard to see how this may come from a unique UV gauge theory. Instead, the straightforward way of getting a gauge theory is if on each $\mathbb{P}^1$ we have *the same* type of bound state. Combining this with the requirement that the sum of fluxes $(N^R = n_R r, N^{NS} = n_{NS} r)$ remains constant implies that we will have r copies of the bound state of type (n_R, n_{NS}) distributed over all the different $\mathbb{P}^1$s.

The physical mechanism that may be responsible for this is already known, namely, eigenvalue tunnelling in matrix models. Consider what happens when we tune the couplings a_k from (4.5) so that the n cuts come very close together: $y^2 = x^{2n} + \epsilon$, $\epsilon \to 0$. In this limit, the process of eigenvalue tunnelling between different cuts becomes relevant; this will result in RR flux transfer until we end with the same (n_R, n_{NS}) bound states in all the cuts. The tunnelling is explained by $D5$ branes wrapped around an S^3 interpolating between two S^2s in the resolved geometry [61]. This object is a domain wall from the EFT point of view, with tension $\partial \mathcal{F}/\partial S_i - \partial \mathcal{F}/\partial S_j$. After the tunnelling has taken place, we can tune back the couplings to their initial values.

We will now start to argue that the previous gauge theory is indeed the dual to our gravity configuration. The key elements entering into the argument are S-duality (decay to bound states) and moving the A_i cycles around, which is associated to an $Sp(2n-2, \mathbb{Z})$ symmetry transformation. We work in the deformation side. Denote the deformed threefold defined in (4.3) and (4.4) by X_d; the limit $f_{n-1}(x) = 0$ is a singular CY X_s with (generically) conifold degenerations.

Recall that S-duality acts by $SL(2, \mathbb{Z})$ transformations

$$\begin{pmatrix} F_3 \\ H_3 \end{pmatrix} \to \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} F_3 \\ H_3 \end{pmatrix} \ , \ \tau \to \frac{a\tau + b}{c\tau + d} \ , \ ad - bc = 1\,. \tag{4.28}$$

This doesn't change the geometry of the hyperelliptic curve (off-shell). On the other hand, the curve (4.4) has a symmetry group $Sp(2n-2, \mathbb{Z})$ of matrices mixing the canonical cycles $(A_i\,,\,C_j)$. These transformations are generated by all the possible interchanges of the roots $a_i^{\pm}$. The generators are [63]

$$J = \begin{pmatrix} 0 & \mathbb{I} \\ -\mathbb{I} & 0 \end{pmatrix} \ , \ \mathcal{A} = \begin{pmatrix} (A^t)^{-1} & 0 \\ 0 & A \end{pmatrix} \ , \ \mathcal{B} = \begin{pmatrix} \mathbb{I} & 0 \\ B & \mathbb{I} \end{pmatrix} . \tag{4.29}$$

$A \in GL(n-1,\, \mathbb{Z})$ and B is a symmetric matrix with integer coefficients. Note that $A_1 + \ldots + A_n = -C_P$ is invariant under $Sp(2n-2, \mathbb{Z})$ because the loop around infinity doesn't change under monodromies of the roots.

The first step is to use S duality to set the total NS flux $N^{NS} = 0$ and hence $N^R = r$. The transformation doing this is

$$\begin{pmatrix} n_R r \\ n_{NS} r \end{pmatrix} \to \begin{pmatrix} a & -b \\ -n_{NS} & n_R \end{pmatrix} \begin{pmatrix} n_R r \\ n_{NS} r \end{pmatrix} = \begin{pmatrix} r \\ 0 \end{pmatrix} \tag{4.30}$$

for some integers (a, b) solving $an_R - bn_{NS} = 1$. We denote with tildes the transformed quantities after S duality.

Next we set $\tilde{N}_i^{NS} = 0$, $i = 1, \ldots, n-1$ with $Sp(2n-2, \mathbb{Z})$ transformations. This is done with the 'diagonal' $SL(2, \mathbb{Z})_i \subset Sp(2n-2, \mathbb{Z})$ which mix the A_i and C_i cycles only:

$$\begin{pmatrix} \tilde{N}_i^{NS} \\ \tilde{c}_i^{NS} \end{pmatrix} \to \begin{pmatrix} a_i & b_i \\ c_i & d_i \end{pmatrix} \begin{pmatrix} \tilde{N}_i^{NS} \\ \tilde{c}_i^{NS} \end{pmatrix} = \begin{pmatrix} 0 \\ \tilde{c}_i'^{NS} \end{pmatrix} . \tag{4.31}$$

Primes refer to the transformed cycles. Symplectic transformations act in a complicated way on A_n; however, since we already fixed $N^{NS} = 0$ and $A_1 + \ldots + A_n$ is a symplectic invariant, we deduce that the combined application of (4.30) and (4.31) fixes *all* $\tilde{N}_i'^{NS} = 0$, $i = 1, \ldots, n$.

Summarizing, we have showed how $S \otimes Sp(2n-2, \mathbb{Z})$ may be used to set all the NS fluxes through the A cycles to zero. The transformed axio-dilaton is $\tilde{\tau} = (a\tau - b)/(-n_{NS}\tau + n_R)$; the transformation of β_i will be analyzed shortly. We end with r copies of *the same* 5-brane bound state (n_R, n_{NS}), wrapping the n $\mathbb{P}^1$ s. The gauge theory is then $U(r) \to \prod_i U(\tilde{N}_i'^R)$ where $\tilde{N}_i'^R$

is the number of (n_R, n_{NS}) bound states on the i-th $\mathbb{P}^1$. This is in agreement with our previous bound state reasoning in terms of eigenvalue tunnelling. The 3-cycles B_i don't collapse in the geometric transition, so in the open string side we still have the fluxes $(\beta_i^R, \beta_i^{NS})$.

4.3.2 Properties of the gauge theory

As a test of the correspondence, it has to be checked that the effective superpotentials in both sides agree. Starting from the closed string side, consider how the effective flux superpotential (4.27) transforms under the $S \otimes Sp(2n-2, \mathbb{Z})$ transformation given by (4.30) and (4.31):

$$\tilde{W}'_{eff} = \sum_{i=1}^{n} \tilde{N}_i'^{R} \frac{\partial \mathcal{F}}{\partial S'_i} - 2\pi i \sum_{i=1}^{n} \left(\frac{\beta_i'^{R} - \tau \beta_i'^{NS}}{n_R - \tau n_{NS}} \right) S'_i \,.$$

We made explicit the S duality transformation in the second term to exhibit the fractional dependence on $(n_R - \tau n_{NS})$; apart from this, $(\tilde{N}_i'^{R},\, \beta_i'^{R},\, \beta_i'^{NS})$ are all integers. Rename $\tilde{N}_i'^{R} \to N_i$ and drop all the primes:

$$W_{eff} = \sum_{i=1}^{n} N_i \frac{\partial \mathcal{F}}{\partial S_i} - 2\pi i \sum_{i=1}^{n} \left(\frac{\beta_i^{R} - \tau \beta_i^{NS}}{n_R - \tau n_{NS}} \right) S_i \,. \tag{4.32}$$

Here $(N_i,\, \beta_i^R,\, \beta_i^{NS})$ are arbitrary integers and shouldn't be confused with the original parameters appearing in (4.27).

Let us spell out the holomorphic properties of the gauge theory. Six dimensional gauge theories based on (p, q) 5-branes were studied for example in [64]. The situation here is more complicated, because the bound states are wrapping $\mathbb{P}^1$ s, and there is $(\beta_i^R, \beta_i^{NS})$ flux through such cycles.

Given that we have the same bound states (n_R, n_{NS}) in every $\mathbb{P}^1$, it is enough to study a single bound state wrapping a $\mathbb{P}^1$ and extending in four space-time dimensions. Since n_R and n_{NS} are relatively prime, the S-duality transformation (4.30) maps the bound state to a single D5 brane. We denote with tildes the variables after the transformation. The DBI action is [14]

$$S = S_{kin} + S_{CS}$$

$$S_{kin} = -\mu_5 \int d^4x \int_{S^2} d\Omega_2\, \mathrm{e}^{-\tilde{\Phi}} \big[-\det(\tilde{G} + \tilde{B} + F) \big]^{1/2}$$

$$S_{CS} = i\mu_5 \int \big[\tilde{C}_6 + (\tilde{B} + F) \wedge \tilde{C}_4 + \frac{1}{2} (\tilde{B} + F)^2 \wedge \tilde{C}_2 + \frac{1}{6} (\tilde{B} + F)^3\, \tilde{C}_0 \big] \,. \tag{4.33}$$

$F := 2\pi\alpha' F_{ab}$ denotes the $U(1)$ gauge field on the D-brane. Near the geometric transition point, where the S^2 shrinks, the holomorphic gauge coupling is given by

$$\tilde{\tau}_{YM} = (2\pi\alpha')^2 \mu_5 \Big(\int_{S^2} \tilde{C}_2 - (\tilde{C}_0 + i\mathrm{e}^{-\tilde{\Phi}}) \int_{S^2} \tilde{B}_2 \Big) \,. \tag{4.34}$$

The action for the (n_R, n_{NS}) bound state and the properties of its gauge theory follow from (4.33) and S-duality:

$$\tilde{\tau} = \tilde{C}_0 + i\mathrm{e}^{-\tilde{\Phi}} = \frac{a\tau - b}{-n_{NS}\tau + n_R} ,$$

$$\tilde{C}_2 = aC_2 - bB_2 \ , \ \ \tilde{B}_2 = -n_{NS}C_2 + n_R B_2$$

$$\tilde{G}_{ab} = |n_R - n_{NS}\tau|\, G_{ab} \ , \ \ \tilde{C}_4 = C_4$$

$$\tilde{B}_6 - \tilde{\tau}\tilde{C}_6 = \frac{B_6 - \tau C_6}{n_R - \tau n_{NS}} \, . \tag{4.35}$$

Noting that

$$\int_{S^2} (C_2 - \tau B_2) = \beta^R - \tau\beta^{NS} \, ,$$

the gauge coupling becomes

$$\tilde{\tau}_{YM} = \frac{\beta^R - \tau\beta^{NS}}{n_R - \tau n_{NS}} \, , \tag{4.36}$$

where we set $(2\pi\alpha')^2\mu_5 = 1$. This coincides exactly with the fractional holomorphic coupling derived from the flux side, eq. (4.32). Furthermore, once we map the system of (p, q) 5-branes to D5 branes, the arguments of [62] may be applied to this $\mathcal{N} = 1$ SYM theory to deduce that the effective superpotential has precisely the form given in (4.32). Generalizing to the case of n $\mathbb{P}^1$s, the gauge theory is $U(r) \to \prod_i U(N_i)$, $\sum_i N_i = r$, and each $U(N_i)$ has a holomorphic coupling

$$\tau_i := \frac{\beta_i^R - \tau\beta_i^{NS}}{n_R - n_{NS}\tau} \, . \tag{4.37}$$

From our previous construction, it is clear that we didn't fix all the symplectic symmetries. In particular, we can still perform monodromies $S_i \to \mathrm{e}^{2\pi i}\, S_i$ corresponding to $B_i \to B_i + A_i$. This implies that τ_i is defined only modulo N_i or, equivalently,

$$\beta_i^R = 0, \ldots, n_R N_i - 1 \, ; \ \ \beta_i^{NS} = 0, \ldots, n_{NS} N_i - 1 \, . \tag{4.38}$$

We thus see that the information in the original brane system is not lost after the S-duality $(N^R, N^{NS}) \to (r, 0)$, but rather it is encoded in the holomorphic gauge couplings of the new theory.

It is worth noting that the holomorphic couplings τ_i, besides being fractional, they are also independent since we can choose arbitrary integers β_i. Equivalently from (4.25), each $U(N_i)$ factor has an independent physical scale Λ_i. This situation is natural from the DBI action, but it cannot arise as the IR limit of the usual $\mathcal{N} = 2$ $U(r)$ SYM broken to $\mathcal{N} = 1$ by the tree level superpotential $W(\Phi)$. Let us exhibit a simple generalization that may account for independent

τ_i s. Coming from string theory, we won't require this UV gauge theory to be renormalizable, so we look for a modified kinetic term

$$\mathcal{L}_{kin} \sim \int d^2\theta \, \mathrm{Tr}\left(W^\alpha W_\alpha \, f(\Phi)\right). \tag{4.39}$$

If $W(\Phi) = 0$, the gauge group is not broken and $f(\Phi_{class}) = \tau_{YM}$ should give a unique gauge coupling. On the other hand, when we turn on the superpotential, the basic property of $f(\Phi)$ is that it should be equal to τ_i on the subspace $\Phi = a_i$. The matrix function that does this is simply constructed from the idempotents of the classical chiral ring:

$$E_i(\Phi) = \frac{\prod_{j\neq i}(\Phi - a_j \mathbb{I})}{\prod_{j\neq i}(a_i - a_j)}, \tag{4.40}$$

which satisfy $E_i(a_j) = \delta_{ij}$. Then we may define

$$f(\Phi) := \sum_{i=1}^{n} \tau_i E_i(\Phi). \tag{4.41}$$

The nonrenormalizable gauge theory (4.39) with this choice of $f(\Phi)$ gives independent gauge couplings in the infrared.

To summarize, using $S \otimes Sp(2n-2, \mathbb{Z})$ in this section we mapped a general flux configuration to a gauge theory, after the geometric transition. All the flux parameters have a natural gauge interpretation; in particular the fluxes $(\beta_i^R, \beta_i^{NS})$ through the 3-cycles, which don't collapse after the transition, don't contribute brane degrees of freedom. They combine in a nontrivial way to determine the holomorphic gauge couplings of the different gauge factors.

4.4 Finiteness of flux vacua from geometric transitions

Using the previous gauge theory duals, in this section we prove that the number of supersymmetric vacua

$$D_i \left(\int G_3 \wedge \Omega \right) = 0 \tag{4.42}$$

is finite. In particular, in the limit $M_{Pl} \to \infty$, supersymmetric solutions are given by $\partial_i W_{eff} = 0$, where $\partial_i := \partial/\partial u_i$. As explained before, this limit corresponds to taking into account only a neighborhood of the singularity, so that supergravity effects are negligible.

4.4.1 Counting vacua in the presence of singularities

Solutions to these equations may be viewed in two equivalent ways. If we want to stabilize at a particular point in the moduli space, $\partial_i W_{eff} = 0$ is an on-shell condition that restricts the possible values of the fluxes to a subspace. Indeed, since $\partial_i R$ gives by construction a basis of $H^{1,0}(\Sigma')$,

$$\partial_i W_{eff} = \int T \wedge \partial_i R = 0 \ \ , \ \ i = 1, \dots, n$$

implies that

$$T \ = \ (N^R - \tau N^{NS})\, \tau_{P,\,\tilde{P}} + \sum_{i=1}^{g} (N_i^R - \tau N_i^{NS})\, \zeta_i \ = \ \sum_{i=1}^{n} (N_i^R - \tau N_i^{NS})\, \zeta_i \, . \tag{4.43}$$

On the other hand, a holomorphic differential is uniquely specified by giving its A-periods. Indeed, the B-periods are then functions of the period matrix:

$$\int_{B_j} T = \sum_{i=1}^{n} (N_i^R - \tau N_i^{NS}) \int_{B_j} \zeta_i \, . \tag{4.44}$$

The other possible point of view is that we can turn on arbitrary fluxes through all the cycles; this will lift almost all the degeneracy of the $\mathcal{N} = 2$ supersymmetric moduli space, leaving only some number of $\mathcal{N} = 1$ supersymmetric vacua. Therefore, if we specify arbitrarily both the A and B fluxes, (4.44) stabilizes the complex moduli of the curve:

$$\beta_j^R - \tau \beta_j^{NS} = - \sum_{i=1}^{n} (N_i^R - \tau N_i^{NS}) \int_{B_i} \zeta_j \, . \tag{4.45}$$

The ingredient that makes the number of vacua finite in compact Calabi-Yau manifolds is the tadpole cancellation condition [52]. There is no such constraint in the noncompact case, since the flux can go off to infinity. However, the fluxes cannot be arbitrarily large, because once their associated energy is of order M_{Pl}, the noncompact approximation breaks down: our local variety will be mixed with far away cycles in the CY. Therefore, in counting the total number of vacua, we have to impose by hand a tadpole condition. By analogy with the compact case [25], we require that

$$\frac{i}{2\mathrm{Im}\tau} \int_{\Sigma'} T \wedge \bar{T} \ = L \, . \tag{4.46}$$

Using the on-shell formula (4.43) and recalling (4.14), the tadpole condition becomes

$$0 \leq L = \frac{1}{2\mathrm{Im}\tau}\, G_{i\bar{l}} U^i \bar{U}^l \leq L_* \tag{4.47}$$

where $U^i := N_i^R - \tau N_i^{NS}$. L_* is the maximum value of L, fixed by data of the compact CY that we choose to embed (4.4).

From (4.47), the counting of supersymmetric vacua may be rephrased in terms of the geometry of Σ: over each point (u^k) in moduli space we have a 'solid sphere' $U^i(u)$, with volume L_*. Each of these allowed points determines a point in flux space; the number of such points will give the number of supersymmetric vacua. Furthermore, (4.47) shows why degeneration limits may produce an infinite number of vacua: if $G_{i\bar{l}}$ develops a null direction, the tadpole condition will not bound the number of flux points. In other words, from this analysis it is not clear how configurations where one flux goes to infinity and another goes to minus infinity, in a correlated way such that $L \geq 0$ stays finite, will be ruled out. The gauge theory analysis will shed light on this point.

Finally, even with the tadpole condition, the number of solutions to the equations of motion (4.45) with continuous fluxes β_i will be infinite. Fortunately, there is a simple way out of this problem. Recall that the noncompact hyperelliptic curve should be considered as part of a compact CY. Instead of parametrizing the fluxes with arbitrary energy scales Λ_i, we take them to be integers. Then (4.25) will *fix* the energy scales at particular values, depending on the fluxes. This approach was also taken in [25] to study the consequences of the Klebanov-Strassler solution [18] and leads to the usual exponentially large hierarchies of energy scales, as we show later.

Now we have all the elements to count vacua on complex curves with punctures; the derivation of the formula for the density of vacua continues as in [52]: the number of supersymmetric vacua is given by

$$N_{vac}(L \leq L_*) = \int_0^\infty dL\, \theta(L_* - L) \sum_{N_R, N_{NS}} \delta(L - \frac{1}{2\mathrm{Im}\tau}\, G_{i\bar{l}} U^i \bar{U}^l) \times$$

$$\times \int (\prod_{i=1}^{n} d^2 u^i)\, \delta(\partial W) \tag{4.48}$$

with

$$\delta(\partial W) := \prod_l \delta(\partial_l W)\, \delta(\partial_{\bar{l}} W^*)\, |\det \partial^2 W\,| \,.$$

Here,

$$\partial^2 W := \begin{pmatrix} \partial_l \partial_n W & \partial_l \partial_{\bar{n}} W^* \\ \partial_{\bar{l}} \partial_n W & \partial_{\bar{l}} \partial_{\bar{n}} W^* \end{pmatrix} . \tag{4.49}$$

Because of $\delta(\partial W)$, we can replace $\partial_l \to \nabla_l$ in (4.49).

The main simplification in the noncompact case is that, since $\nabla_l \bar{\zeta}_{\bar{n}} = 0$, $\nabla_l \partial_{\bar{n}} W^* = 0$, and then

$$|\det \partial^2 W\,| = \det \partial^2 W = |\det \nabla_l \partial_n W\,|^2 \,. \tag{4.50}$$

Therefore *the number of supersymmetric vacua coincides with the supersymmetry index*, which is topological and, as we shall see, much easier to compute. On the contrary, in the compact case, when gravity is not decoupled, the supersymmetric index gives just a lower bound to the number of vacua.

The final result is

$$N^{C}_{vac}(L_*) = \frac{(2\pi L_*)^{2n}}{\pi^n (2n)!} \int_{\mathcal{M}} \det(-R) \tag{4.51}$$

where $\det R := \det_{\bar{s}\bar{r}}\left(R^{\bar{s}}{}_{\bar{r}k\bar{l}}\, du^k \wedge d\bar{u}^l\right)$. As expected, this coincides with [52] when $M_{Pl} \to \infty$. The index C is introduced for clarity reasons, to mean that this is the result from the closed string side.

4.4.2 Proof of the finiteness of N_{vac}

We begin by showing that the number of supersymmetric gauge vacua, i.e., solutions to $\partial W_{eff}/\partial S_i$ from equation (4.32), is finite. As discussed before, this is based on the tadpole constraint

$$L = \sum_{i=1}^{n} N_i \tilde{\beta}_i^{NS}\,. \tag{4.52}$$

Here $\tilde{\beta}_i^{NS} = (n_R \beta_i^{NS} - n_{NS}\beta_i^{R})$; also recall that $N_i := \tilde{N}_i^{\prime R}$, $\beta_i^R := \beta_i^{\prime R}$, $\beta_i^{NS} := \beta_i^{\prime NS}$.

We have to sum over all choices of fluxes satisfying (4.52). Here we run into the main obstacle. The reason why this could in principle diverge is that there may be flux configurations such that two terms in L grow in a correlated way to plus and minus infinity respectively, but keeping L finite and positive. This would give an infinite number of allowed flux points (and hence supersymmetric vacua).

This is the point where having a gauge theory based on the geometry (4.4) proves useful. In the gauge theory, W_{eff} is holomorphic in the couplings a_k, so the number of solutions to the equations $\partial W_{eff}/\partial f_i = 0$ is invariant under smooth changes of the parameters, being protected by holomorphy.[2] An equivalent statement is that the number of vacua coincides with the dimension of the chiral ring of the theory, and such a quantity is independent of the gauge couplings. This topological behavior was already encountered in the gravity side, when we showed (section 4.4.1) that the number of supersymmetric vacua coincides with the supersymmetric index.

We now argue, from a variation of the a_k, that each term in L is in fact positive even around singularities. The discriminant locus consists of generic conifold points and higher codimension AD singularities. The later cannot be neglected because they have a higher 'weight' in the counting of degrees of freedom, as measured by $\det(R)$.

Consider a point in moduli space $\mathcal{M}$ corresponding to the semiclassical limit. This is just

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[2] Since off-shell the f_i don't depend on a_k, it is more convenient to take derivatives w.r.t. f_i and not S_i.

the origin $S_i \to 0$ of $\mathcal{M}$. In this case the geometry is a product of independent conifold-like configurations. The effective superpotential follows from (4.32) using monodromy arguments [54]:

$$W_{eff} = \sum_{i=1}^{n} N_i S_i \big(\log(\frac{\Lambda_0^3}{S_i}) + 1\big) - 2\pi i \sum_{i=1}^{n} \left(\frac{\beta_i^R - \tau \beta_i^{NS}}{n_R - \tau n_{NS}} \right) S_i \,. \tag{4.53}$$

Denoting $\theta_i/2\pi := \mathrm{Re}(\tau_i)$ and $1/g_i^2 := \mathrm{Im}(\tau_i)$, the supersymmetric vacua may be written as

$$S_i = \exp(-i\theta_i/N_i) \exp(-2\pi/g_i^2 N_i)\, \Lambda_0^3 = \exp(-i\theta_i/N_i)\, \Lambda_i^3 \,. \tag{4.54}$$

Then counting vacua in the neighborhood of the conifold limit implies summing over fluxes giving $0 \le |S_i| \le ({\Lambda_i}^f)^3$. [3] Clearly this requires $\mathrm{sign}(n_R \beta_i^{NS} - n_{NS} \beta_i^R) = \mathrm{sign}(N_i^R)$, to avoid vacua exponentially far away from the origin. We therefore see that the number of vacua around the semiclassical point is finite because each term in L is *separately* positive. Without loss of generality, we can just take all the fluxes to be positive.

The holomorphic dependence of W_{eff} on a_k implies that this is true for the whole moduli space. Indeed, every point in moduli space can be connected to the semiclassical limit by such a variation of couplings. Of course, strongly coupled limits may have quite complicated superpotentials, but we are interested in the number of vacua, which is a topological invariant.

For concreteness, we show this for $n = 2$. The hyperelliptic curve is

$$y^2 = (x^2 + g_1 x + g_0)^2 + f_2 x + f_1 \,. \tag{4.55}$$

We only need to worry about singularities in $\mathcal{M}$ since it is known that N_{vac} is finite around smooth points. There are two types; the codimension one singularities are conifolds, and correspond to the semiclassical regime where we showed the finiteness of N_{vac}. There is also a codimension two A_2 singularity. It corresponds to the singular limit of y:

$$y^2 = (x^3 - \delta u x - \delta v)(x - 1) \;\; ; \;\; \delta u \,,\;\; \delta v \to 0 \,. \tag{4.56}$$

Three roots coincide at $x = 0$ giving two vanishing intersecting cycles, while the last one is fixed at $x = 1$. Comparing to (4.55), we find the 'double scaling' limit

$$f_1 = \delta v - (\frac{1}{8} + \frac{\delta u}{2})^2 \;,\;\; f_2 = -\frac{1}{8} + \frac{\delta u}{2} - \delta v \,, \tag{4.57}$$

and, for the couplings,

$$g_1 = -\frac{1}{2} \;\;,\;\; g_0 = -(\frac{1}{8} + \frac{\delta u}{2}) \,. \tag{4.58}$$

To connect this to the semiclassical point, vary the couplings g_i from their previous double-scaled values to $g_i \gg f_i$, while keeping the f_i fixed at (4.57). Clearly, at the new point in $\mathcal{M}$ the semiclassical approximation is valid. This process is depicted in Figure 4.2.

[3] $({\Lambda_i}^f)^3$ is some final energy scale associated to $U(N_i)$.

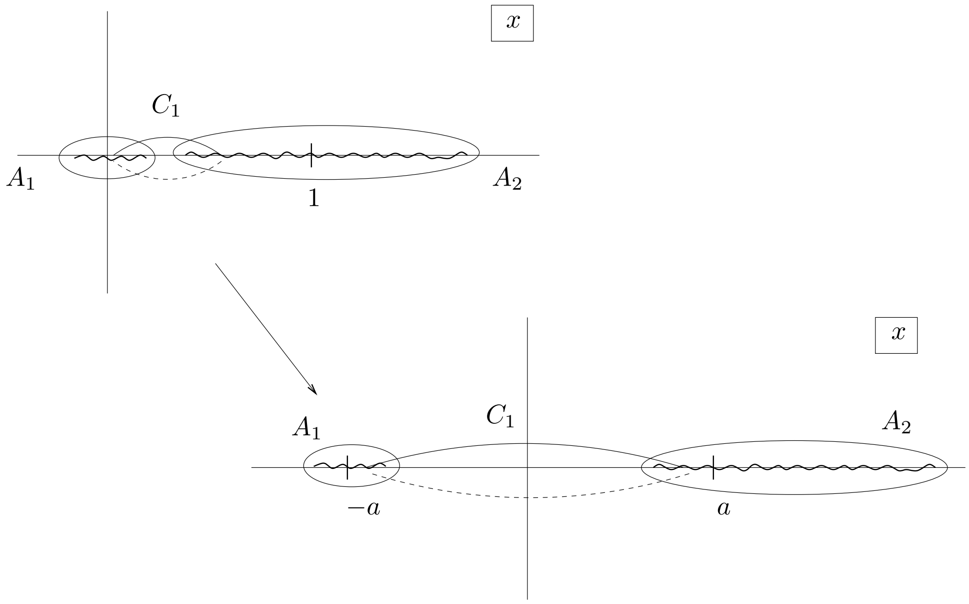

Figure 4.2: Holomorphic change of couplings that connects the AD point and the semiclassical limit.

Therefore we have shown that any point in $\mathcal{M}$ can be connected to the conifold limit by a smooth variation of the a_k. In other words, the gauge theory tells us how to do, on every point in moduli space, a change of variables $S_i(a_k) \to S_i(\tilde{a}_k)$ such that: (i) each term in L is explicitly positive and (ii) the number of supersymmetric vacua doesn't change. Furthermore, since we can work in a regime $f_i \to 0$ by tuning $a_i \gg f_i$, we can always do power-series expansions and hence the change of variables is continuous. This maps compact regions to compact regions, assuring that the number of vacua doesn't diverge.

The meaning of this transformation becomes transparent if we consider the chiral ring. It is generated by idempotents and nilpotents [65]. If we move around the moduli space S_i by changing the couplings until we encounter a singularity, the result on the chiral ring is that some idempotents become nilpotents. The total number of generators is conserved in the process.

4.4.3 Formula for $N_{vac}(L_*)$

In order to compare with the gravity side result (4.51), we next compute the number of supersymmetric gauge vacua around an arbitrary point in $\mathcal{M}$. As argued before, holomorphy implies that we can as well compute it around the semiclassical limit.

Because of the monodromies leading to (4.38), at fixed N_i, the number of vacua is

$$N_{vac}(\{N_i\}) = (n_R n_{NS})^n \prod_{i=1}^{n} N_i^2\,; \tag{4.59}$$

the N_i satisfy $\sum_i N_i = r$. This is quite different to the result from a standard $N=1$ SYM, $\prod_i N_i$. Eq. (4.51) includes an integration over a region in moduli space. We need to specify the analogous condition in the gauge side. It is associated to the RG flow of the gauge theory from the cutoff Λ_0 up to some IR energy scale Λ^f. For concreteness, we compute N_{vac} for the simplest case, namely when each $U(N_i)$ flows up to a scale ${\Lambda_i}^f$. In other words, we assume that we are integrating on disks $0 \le |S_i| \le ({\Lambda_i}^f)^3$.

The renormalization of gauge couplings (4.25) applied to the case (4.32) gives

$$\frac{\tilde{\beta}_i^{NS}}{n_R^2 + n_{NS}^2} = \frac{1}{2\pi} N_i \log\Big(\frac{\Lambda_0}{\Lambda_i}\Big)^3 . \tag{4.60}$$

Here we set, for simplicity, $C_0 = 0$, $g_s = 1$. This is possible because in the noncompact model the axio-dilaton is fixed and behaves as a coupling; therefore N_{vac} cannot depend on it. Since we are summing the degrees of freedom with $0 \le \Lambda_i \le {\Lambda_i}^f$, (4.60) implies

$$\tilde{\beta}_i^{NS} \ge \frac{1}{2\pi}(n_R^2 + n_{NS}^2) N_i \log\Big(\frac{\Lambda_0}{{\Lambda_i}^f}\Big)^3 . \tag{4.61}$$

Replacing in the gauge tadpole condition (4.52),

$$(n_R^2 + n_{NS}^2) \sum_{i=1}^{n} N_i^2 \log\Big(\frac{\Lambda_0}{{\Lambda_i}^f}\Big)^3 \le 2\pi\, L\, . \tag{4.62}$$

Once we fix arbitrary (N_i), the dual fluxes $(\tilde{\beta}_i^{NS})$ are integers satisfying the diophantine equation (4.52). This has solutions iff $\gcd(N_i)|L$; the number of integer solutions is of course infinite, but we argued that $\mathrm{sign}(N_i) = \mathrm{sign}(\tilde{\beta}_i^{NS})$. So we take the fluxes to be positive, and multiply the number of vacua by 2^n. The number of positive solutions to the tadpole constraint will be denoted by $b_+(\{N_i\})$. For large L, this number is typically of order 1.

Combining all the previous elements, the total number of supersymmetric vacua is

$$N_{vac}(L_*; \Lambda^f) = 2^n \sum_{L=0}^{L_*} \sum_{n_R,\, n_{NS}\, coprime} (n_R n_{NS})^n \sum_{\{N_i\}:\, gcd(N_i)\,|\,L} [\prod_{i=1}^n N_i^2] \times$$
$$\times\, b_+(\{N_i\}) \cdot T(N_i; n_R,\, n_{NS})\, . \tag{4.63}$$

The notation here is the following. The sum on (n_R, n_{NS}) is over coprime integers. The sum on (N_i) should be done over inequivalent fluxes with respect to the residual symplectic transformations; indeed, some generators in (4.29) were not fixed by the mapping to the region $(N_i^R, N_i^{NS}) \to (N_i^R, 0)$. Also, recall that $b_+(\{N_i\})$ is the number of positive solutions to the diophantine equation (4.52); for large L_*, it will give subleading contributions so, to a good approximation, we may set $b_+ \sim 1$. Lastly, $T(N_i; n_R,\, n_{NS})$ specifies the region in flux space over which we are summing vacua. For instance, if we integrate on disks of radius $({\Lambda_i}^f)^3$, (4.62) gives the Heaviside function

$$T(N_i; n_R,\, n_{NS}) = \Theta\Big(2\pi\, L - (n_R^2 + n_{NS}^2) \sum_{i=1}^{n} \log\big(\frac{\Lambda_0}{\Lambda_i^{\ f}}\big)^3\, N_i^2\Big)\,. \tag{4.64}$$

Finally, in [3] the formulas (4.51) and (4.63) for N_{vac} in the gravity and gauge side were compared for the conifold and Argyres-Douglas degenerations. It was found that both results are in agreement.

4.5 Including warp effects: the warped deformed conifold

So far we have discussed the holomorphic sector of the theory, which is not corrected by warping. We now turn our attention to the computation of kinetic terms which, as shown in chapter 2, are affected by backreaction from fluxes. The warp factor enters explicitly in the expression Eq. (2.50),

$$G_{IJ}(u) = \frac{1}{4V_W} \int d^6y \sqrt{g_6}\, e^{2A}\, \delta_I g_{ij}\, \delta_J \pi^{ij}\,. \tag{4.65}$$

In the rest of the chapter we will analyze the effects of warping in the deformed conifold, where the background metric is known explicitly [18]. This rather difficult problem will be discussed from different angles: a 10d point of view, a low energy 4d description, and effects from the dual gauge theory.

4.5.1 The Klebanov-Strassler background

Let us begin by describing the background solution, given by Klebanov and Strassler [18], corresponding to

$$u^2 + v^2 + y^2 - x^2 + S = 0\,. \tag{4.66}$$

Its holomorphic properties are a particular case of the situation discussed in section 4.2. There are only two nontrivial 3-cycles, (A, B), $A \cap B = 1$; for $S \to 0$, $A \to 0$ and B is noncompact. The A cycle is an S^2 fibration ($u,\, v\ \in \mathbb{R}$) over the cut $x \in (-\sqrt{S},\, +\sqrt{S})$ of the hyperelliptic curve

$$F(x, y) = y^2 - x^2 + S = 0\,. \tag{4.67}$$

The noncompact B-cycle extends between $y = \pm\infty$ and runs through the previous cut. We introduce a geometrical cutoff Λ_0 such that the points at infinity become $\pm\Lambda_0$. From the usual monodromy arguments, the periods are

$$\int_A \Omega = S\ ,\ \ \int_B \Omega = \frac{\partial \mathcal{F}}{\partial S} = \Pi_0 + \frac{1}{2\pi i}\, S \log \frac{\Lambda_0^3}{S} + \ldots \tag{4.68}$$

where $\mathcal{F}$ is the prepotential of the geometry, Π_0 is a constant, and $\ldots$ are analytic subleading contributions.

We warm up with the singular conifold, with $S=0$. It has

$$ds_6^2 = dr^2 + r^2\, ds^2_{T^{1,1}}\ ,\ \ ds^2_{T^{1,1}} = \frac{1}{9}(g^5)^2 + \frac{1}{6}\sum_{i=1}^{4}(g^i)^2\,. \tag{4.69}$$

The basis of one forms g^i was introduced in [18,66]; they arise from the angular variables of the base $S^2\times S^3$. In terms of

$$\omega_2 := \frac{1}{2}(g^1\wedge g^2 + g^3\wedge g^4)\ ,\ \ \omega_3 := \frac{1}{2}g^5\wedge(g^1\wedge g^2 + g^3\wedge g^4)\,, \tag{4.70}$$

the $(2,1)$ form reads

$$\chi_S = \omega_3 - 2i\frac{dr}{r}\wedge\omega_2\,. \tag{4.71}$$

Notice that this form is 'localized' at small r.

The actual solution of interest is the deformed conifold. In the basis (τ, g^i) of [18] the metric is diagonal

$$ds_6^2 = \frac{1}{2}|S|^{2/3}K(\tau)\Big[\frac{d\tau^2+(g^5)^2}{3K^3(\tau)} + \cosh^2\frac{\tau}{2}\left((g^3)^2+(g^4)^2\right) + \sinh^2\frac{\tau}{2}\left((g^1)^2+(g^2)^2\right)\Big] \tag{4.72}$$

where

$$K(\tau) := \frac{\left(\sinh(2\tau)-2\tau\right)^{1/3}}{2^{1/3}\sinh\tau}\,.$$

Note that all the moduli dependence is contained in the single prefactor $|S|^{2/3}$. For large τ, the relation with the conical radius is

$$r^2 = \frac{3}{2^{5/3}}|S|^{2/3}\,e^{2\tau/3}\,. \tag{4.73}$$

The $(2,1)$ form is now more complicated:

$$\chi_S = g^5\wedge g^3\wedge g^4 + d\big[F(\tau)(g^1\wedge g^3 + g^2\wedge g^4)\big] - i\,d\big[f(\tau)g^1\wedge g^2 + k(\tau)g^3\wedge g^4\big]\,, \tag{4.74}$$

where the functions F, f and k were computed in [18]:

$$F(\tau) = \frac{\sinh\tau-\tau}{2\sinh\tau}\ ,\ \ f(\tau) = \frac{\tau\coth\tau-1}{2\sinh\tau}(\cosh\tau-1)\ ,$$
$$k(\tau) = \frac{\tau\coth\tau-1}{2\sinh\tau}(\cosh\tau+1)\,. \tag{4.75}$$

As the next step we turn on the following quantized fluxes:

$$\int_A F_3 = N\ ,\ \ \int_B H_3 = -\beta^{NS}\,. \tag{4.76}$$

The configuration might be seen either as the gravity side of the Dijkgraaf-Vafa duality, or as the end of the duality cascade of Klebanov-Strassler. Using (4.68) and (4.76), the flux superpotential for the conifold reads

$$W = \frac{N}{2\pi i} S\big(\log\frac{\Lambda_0^3}{S} + 1\big) - \frac{i}{g_s}\beta^{NS} S\,. \tag{4.77}$$

For $N > 0$, $\beta^{NS} > 0$, solving $\partial_S W = 0$, gives the supersymmetric vacuum Eq. (4.78),

$$S_{min} = e^{-2\pi\beta^{NS}/g_s N}\,\Lambda_0^3\,, \tag{4.78}$$

The backreaction on the geometry warps the CY,

$$ds^2 = e^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{-2A(y)}\tilde{g}_{ij}dy^i dy^j\,, \tag{4.79}$$

where e^{-4A} may be written as [18]

$$e^{-4A(\tau)} = 2^{2/3}\,\frac{(g_s N\alpha')^2}{|S|^{4/3}}\,I(\tau) \tag{4.80}$$

and $I(\tau)$ is an integral expression defined in [18].

4.5.2 Effective action for the complex structure deformation

We need to evaluate the extrinsic curvature $\delta_S h_{ij}$ and canonical momentum $\delta_S\pi_{ij}$ associated to fluctuations of the complex modulus, and then replace this into Eq. (4.65). The relevant formulas are given in (2.38), (2.39), (2.46) and (2.48).

Since $\partial_S g_{ij} = 0$, we have (suppressing the subindex 'S' in η_{Si})

$$\delta_S g_{ij} = -\nabla_i\eta_j - \nabla_j\eta_i\,.$$

Hence the internal metric fluctuation is produced solely by the compensating field! This contribution is nonzero because a time-dependent fluctuation in S does modify the 4d piece of the metric, and this requires non-vanishing compensators. Thus the KS solution is very good for illustrating the effects of compensators, since $G_{S\bar{S}}$ would vanish if they were not taken into account.[4]

Plugging this metric fluctuation into Eq. (4.65), the integrand becomes a total derivative. Integrating over τ gives

$$G_{S\bar{S}} = -\frac{1}{2V_W}\,\Big(\int\prod_i g^i\Big)\,\sqrt{g_6}\,e^{2A}\,\eta_i\,\delta_S\pi^{i\tau}\Big|_{\tau=0}^{\tau=\tau_\Lambda}\,. \tag{4.81}$$

[4]Since we are working at strong warping, we are ignoring the constant term in e^{-4A}. This term can be added easily, and gives the usual metric $G_{S\bar{S}} \sim \log|S|$.

Solving the compensator equations explicitly is a very involved task. Indeed, Eq. (2.45) and Eq. (2.48) gives a system of six coupled second order PDEs, with coefficients that contain various combinations of (hyperbolic) trigonometric functions, plus $I(\tau)$ which only has an integral expression. Now, the problem is simplified by the fact that in order to evaluate Eq. (4.81) only the solutions close to the boundaries are needed. The approach is then to expand the KS solution near each boundary, and find the solutions separately in each region after making simplifying ansatze for the compensators taking into account the isometries of the background.

Still the problem turns out to be too complicated to allow for an intuitive understanding of the underlying physics. Instead, we will consider the so-called *hard-wall* approximation, where the regular background is replaced by an AdS space with a cut-off at $r = |S|^{1/3}$ plus boundary conditions to match the known KS values. The warp factor is taken to be

$$e^{-4A(r)} = \frac{a_0 (g_s N \alpha')^2}{r^4} \tag{4.82}$$

where $a_0 = 2^{2/3}\, I(0)$ is chosen so that at $r = |S|^{1/3}$ this agrees with the KS warp factor at $\tau = 0$. Similarly, the 10d metric will be approximated by

$$ds^2_{10} = e^{2A(r)}\, \eta_{\mu\nu}\, dx^\mu dx^\nu + e^{-2A(r)} \big(dr^2 + r^2\, ds^2_{T^{1,1}}\big)\,. \tag{4.83}$$

In the hard-wall approximation there is one IR boundary at $r = |S|^{1/3}$ and the space has a UV cutoff at $r = \Lambda_0$. However, due to the fall-off of the metric fluctuations at large r, only the IR boundary turns out to contribute to the field space metric. Therefore we only need to solve for the compensators around the tip of the conifold.

Before proceeding, let us pause and ask about the validity of this approximation. The work of [67] performed a detailed numerical analysis of the mass spectrum in the full KS solution without any approximation in the background. Their results were compared to the ones obtained in the hard-wall approximation and it is found that, although the precise numerical coefficients don't agree, both spectra have the same dependence on the parameters of the problem. Since the masses depend directly on the kinetic term metric, the hard-wall method gives the correct dependence on $g_s N \alpha'$ and S, while more work would be required to get the numerical coefficients right.

From Eq. (2.45) and Eq. (2.48), the constraint equations that need to be solved are

$$\begin{aligned} &g^{ij}\, \nabla_i\, \eta_j + 2\, g^{ij}\, \partial_i A\, \eta_j = 2\, \partial_S A \\ &g^{ij}\, \nabla_i (\delta_S \pi_{jk}) + 2\, g^{ij}\, \partial_i A\, \delta_S \pi_{jk} = 0 \end{aligned} \tag{4.84}$$

with

$$\delta_S \pi_{ij} = -\nabla_i \eta_j - \nabla_j \eta_i - g_{ij}\, (g^{kl}\, \nabla_k \eta_l)\,.$$

The covariant derivatives here are with respect to the warped 6d metric g_{ij}.

Due to the $SU(2) \times SU(2)$ symmetry, the angular components of the compensators may be rotated to point in the ψ direction. A radial compensator is of course needed due to the source term produced by $\partial_r A$. Then from Eq. (4.84) we learn that η_r and η_ψ only depend on the radial direction. Notice that at least two nonzero components are needed to be able to construct a metric fluctuation orthogonal to gauge transformations. Summarizing, our ansatz for the compensating field is

$$\eta_i(y) = \big(\eta_r(r),\, \eta_\psi(r), 0, 0, 0, 0\big) \tag{4.85}$$

where the last 4 components refer to the coordinates (θ_i, ϕ_i).

This is admittedly not the most general ansatz; one could find others with less symmetry. However, since the kinetic term coefficient is the integral of a positive definite quantity, it seems very implausible to us that a solution with less symmetry could lead to a smaller result.

Granting Eq. (4.85), the system Eq. (4.84) then becomes one second order equation for η_ψ and two equations (one first order and one second order) for η_r. Concentrating on η_r first, the general solution to the first order equation is

$$\eta_r(r) = \sqrt{a_0}\,\frac{(g_s N \alpha')}{|S|}\,\frac{1}{r} + \frac{c_1}{r^3}$$

Plugging this into the second order constraint sets $c_1 = 0$. The role of this compensator is to cancel the contribution of the nontrivial warp factor; it may be checked that η_r is covariantly constant, $\nabla_r \eta_r = 0$. This then implies that

$$g^{kl}\nabla_k \eta_l = 0\;,\;\; \delta_S \pi_{rr} = 0\,.$$

Due to these properties, η_r drops out from the second order equation for η_ψ, and the solution around $r \approx |S|^{1/3}$ is

$$\eta_\psi(r) = \frac{b_1}{r}\,.$$

The constant b_1 is fixed by matching $||\delta_S \pi_{\psi r}||^2$ at $r = |S|^{1/3}$ to $||\chi_S||^2$ at $\tau = 0$, ensuring that the metric fluctuations are normalized in the same way. This boundary condition is required because the IR cutoff $r = |S|^{1/3}$ is imposed by hand. The result is

$$\eta_\psi(r) \approx k\,\frac{(g_s N \alpha')}{|S|^{2/3}}\,\frac{1}{r}\,,$$

where from now on we will absorb the dimensionless order one constants into k. The dependence on $(g_s N\alpha')$ and $|S|^{2/3}$ can also be understood as follows. Since $\delta g_{\psi r} = e^{-2A}\,\delta\tilde{g}_{\psi r}$ and $\delta\tilde{g}$ is independent of fluxes, the warped metric fluctuation has to be proportional to $(g_s N\alpha')$. Then $|S|^{-2/3}$ follows from dimensional analysis.

Putting these results together, the compensating field in the hard-wall approximation is

$$\eta_i(y) = \big(\sqrt{a_0}\,\frac{(g_s N\alpha')}{|S|}\,\frac{1}{r},\; k\,\frac{(g_s N\alpha')}{|S|^{2/3}}\,\frac{1}{r}, 0, 0, 0, 0\big)\,. \tag{4.86}$$

With these components, the only nonvanishing metric fluctuation is

$$\delta_S \pi_{\psi r} = -k\,\frac{(g_s N\alpha')}{|S|^{2/3}}\,\frac{1}{r^2}\,. \tag{4.87}$$

Naively, one might find it peculiar that the metric variation is an off-diagonal component, not present in the original Klebanov-Strassler metric Eq. (4.72). But, as we commented, the 6d part of the Klebanov-Strassler metric is actually independent of S, and the variation is pure gauge. Nevertheless it must be non-zero to satisfy the orthogonality condition.

Finally, replacing Eq. (4.86) into the expression Eq. (4.81), the result is

$$G_{S\bar{S}} = \frac{1}{V_W}\,\left(c\log\frac{\Lambda_0^3}{|S|} + k\frac{(g_s N\alpha')^2}{|S|^{4/3}}\right)\,, \tag{4.88}$$

where we have combined all the order one numerical constants into c and k. The first term in Eq. (4.88) is the usual unwarped result, determined by special geometry and interpreted as integrating out BPS $D3$ branes wrapping the A cycle. The second term is the new contribution; such a term could not appear in $\mathcal{N}=2$ compactification, both on mathematical grounds [68] and because loop effects of massless particles cannot lead to this type of power-like divergence. However it is a natural consequence of warping in $\mathcal{N}=1$, and also has a suggestive interpretation in the dual gauge theory, as we discuss later.

Notice that at small enough $|S|$, the second term will dominate. Since it is singular at $S=0$, one should ask whether it is valid in this regime. We will examine the consistency condition in supergravity in subsection 4.5.3, concluding that for $g_s N >> 1$ (the standard supergravity regime) this is valid all the way down to $S=0$.

The expression for the vacuum energy from dimensional reduction is [27]

$$V = \frac{1}{2\kappa_4^2}\,\frac{1}{V_W \mathrm{Im}\,\tau(\mathrm{Im}\,\rho)^3}\,\frac{1}{\parallel\Omega\parallel^2 V_W}\,G^{S\bar{S}}|\partial_S W|^2\,. \tag{4.89}$$

Using the known expression for $G_{S\bar{S}}$, we obtain

$$V = \frac{1}{2\kappa_{10}^2}\,\frac{g_s}{(\mathrm{Im}\,\rho)^3}\,\Big[c\log\frac{\Lambda_0^3}{|S|} + k\,\frac{(\alpha' g_s N)^2}{|S|^{4/3}}\Big]^{-1}\Big|\frac{N}{2\pi i}\log\frac{\Lambda_0^3}{S} + i\frac{\beta^{NS}}{g_S}\Big|^2\,. \tag{4.90}$$

To avoid cluttering, we have absorbed numerical factors like $64\pi^3$ into the constants c and k, although c still denotes the universal Kahler modulus $c \sim V_W^{2/3}$.

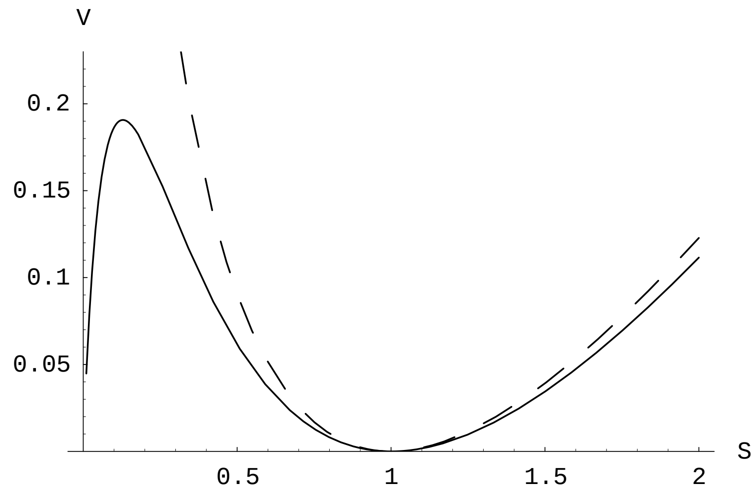

Figure 4.3: Behavior of the potential (4.90) for the supersymmetric $N > 0$ case, with (full line) and without (dashed line) warping effects. The point $S = 1$ is the supersymmetric vacuum.

4.5.3 A closer look into warping effects

Near the mouth of the throat, where warping is small, the usual intuition from special geometry and the deformed conifold is valid. In particular, in the limit $S \to 0$, the S^3 collapses, its radius being controlled by the factor $|S|^{2/3}$ in the metric (4.72).

We may, however, tune the fluxes to get *strong warping* $e^{-4A} \gg c$. As we now discuss, this changes radically the picture. To begin with, consider a sample potential with and without warping, plotted in Figure 4.3. We have taken order one parameters so that the various regimes are easily visible on the same plot.

The dashed curve is the potential without the $S^{-4/3}$ correction to the metric. It has a minimum given by Eq. (4.78), where it vanishes, while it goes to infinity at $|S| \to 0$ and at $|S| \to \Lambda_0^3$. One might have expected that for S small, the system should become unstable and undergo a geometric transition.

On the other hand, the behavior of the potential Eq. (4.90) including the warped metric is quite different. At

$$c \log \frac{\Lambda_0^3}{|S|} \sim \frac{(\alpha' g_s N)^2}{|S|^{4/3}}$$

the $S^{-4/3}$ starts to dominate; a maximum value is attained and after that the system starts to roll down to $S = 0$!

The reason why this effect was not detected before is that fluxes break $\mathcal{N} = 2$ softly, so at string tree level the Kahler metric is still given by special geometry. However, if we want to analyze the geometric transition in more detail, we have to consider what happens for $S \to 0$.

In this case, the g_s correction of (4.90) is important, showing that the system becomes unstable.

Clearly, the supergravity solution is singular at $S=0$. For which range of small (but finite) S can we trust the supergravity analysis? To answer this we need to study the curvature of the background. We consider the 'near horizon' limit $\tau \to 0$, where the largest curvatures may be generated; strong warping implies the boundary condition $e^{-4A(\tau)} \to 0$ as $\tau \to \infty$, which is exactly the KS end of the cascade. In this case, the metric for the warped-deformed conifold

$$ds_{10}^2 = e^{2A(\tau)} \eta_{\mu\nu} dx^\mu dx^\nu + e^{-2A(\tau)}\, ds_6^2$$

with ds_6^2 given in (4.72), becomes

$$ds_{10}^2 \approx \frac{1}{2^{1/3} a_0^{1/2}} \frac{|S|^{2/3}}{\alpha' g_s N}\, \eta_{\mu\nu} dx^\mu dx^\nu + \frac{a_0^{1/2}}{6^{1/3}}\, \alpha' g_s N \left[\frac{d\tau^2}{2} + d\Omega_2^2 + d\Omega_3^2\right]. \tag{4.91}$$

Here we used the fact that for $\tau \to 0$, the function $I(\tau)$ introduced in (4.80) behaves as $I(\tau \to 0) \to a_0 \sim 0.7180$ [18]. Furthermore, we included explicitly the S^2 and S^3 at the base of the cone:

$$d\Omega_2^2 = \frac{\tau^2}{2}\left((g^1)^2 + (g^2)^2\right),\; d\Omega_3^2 = \frac{1}{2}(g^5)^2 + (g^3)^2 + (g^4)^2\,. \tag{4.92}$$

The S^3 has finite radius, while the S^2 collapses, as expected.

The fact that the S dependence cancels out in $e^{-2A(\tau)}\, ds_6^2$ is quite striking; this was already derived in [18], but we would like to point out some of its consequences. In the strong warping limit, we see that the volume of the S^3 is not proportional to S; in particular this 3-cycle *does not* vanish when $S \to 0$! The order of limits matters and we cannot recover the (strongly) warped deformed conifold by taking $S \to 0$ in the deformed conifold and then introducing the warp factor for the singular conifold. The modulus S no longer parametrizes the size of a cycle in the warped deformed geometry. Note, however, that not all the dependence on S of the six-dimensional geometry has disappeared. Indeed, unlike r, τ is a dimensionless coordinate; cutting off the conifold at some finite τ_Λ requires both scales Λ_0 and S,

$$\tau_\Lambda = \frac{3}{2}\log\frac{2^{5/3}}{3} + \log\frac{\Lambda_0^3}{|S|^2}\,. \tag{4.93}$$

Hence, as $S \to 0$, the throat becomes infinite (even at fixed Λ_0). Of course, once e^{-4A} is small enough, the bulk effects become relevant, cutting off the geometry; but still, this behavior is very different to the deformed case without warping.

The analysis of the curvature tensor of (4.91) is straightforward. A crucial point is that the only dependence on S is through $\eta_{\mu\nu} dx^\mu dx^\nu$; since the curvature does not depend on x^μ, defining orthonormal Minkowski coordinates

$$\tilde{x}^\mu := \Big(\frac{1}{2^{1/3} a_0^{1/2}} \frac{|S|^{2/3}}{\alpha' g_s N}\Big)^{1/2} x^\mu\,, \tag{4.94}$$

none of the components of R^M_{NRS} will depend on S. An explicit computation to order τ^2 gives the scalar curvature

$$R = -\frac{6^{1/3}}{5\sqrt{a_0}}\frac{1}{\alpha' g_s N}\left[3(1+20k) - (6+9k+880k^2)\tau^2\right] + \mathcal{O}(\tau^3) \tag{4.95}$$

where $I(\tau) \sim a_0(1+k\tau^2)$, k being an order one constant. Therefore, unlike the unwarped case, we can trust the supergravity approach as long as $g_s N \beta^{NS} \gg 1$, even if $S \to 0$ (but finite). Incidentally, (4.94) implies that the time x^0 necessary to roll down to $S=0$ tends to infinity, at fixed orthonormal time $\tilde{x}^0$.

If the modulus S doesn't have now a geometric interpretation, what is its meaning? As explained by KS, (4.91) is the 'supergravity version' of confinement. Since the prefactor multiplying $\eta_{\mu\nu}dx^\mu dx^\nu$ is finite for $\tau = 0$, Wilson loops will have an area law. Furthermore, we see that the theory generates dynamically a confinement scale

$$M^2_{conf} \sim \frac{|S|^{2/3}}{\alpha' g_s N} \tag{4.96}$$

controlled by S. From this point of view, the previous noncommutativity of limits is expected: the warped singular conifold cannot reproduce these nonperturbative effects.

4.6 Warping and supersymmetry breaking

In this section, we analyze supersymmetry breaking by anti-self dual flux in the deformed conifold. This theory has been argued to be a dual realization of susy breaking by antibranes [69]. As such, one might expect it to lead to a hierarchically small breaking scale, but only if the warp factor is taken into account. We verify this by using the warp-modified moduli space metric computed in the previous sections.

Let us first discuss some details of the supergravity configuration, that will be relevant to understanding the mechanism of susy breaking. A simple embedding of the conifold in a compact Calabi-Yau orientifold was constructed in [25]. However the details of the embedding do not matter for our discussion, so we consider the following simplified model. We 'zoom in' to a local neighborhood of a compact CY X, containing the deformed conifold (4.66). The only information that we keep from the rest of X is that there is an orientifold projection, breaking $\mathcal{N}=2 \to \mathcal{N}=1$, and preserving the chiral supermultiplet with scalar component S. While a complete discussion requires solving the D3 tadpole condition, which usually requires adding wandering $D3$ branes, we keep these away from the conifold, so that they don't enter these results.

In the presence of the 4-form C_4, compactifying on the conifold contributes an $\mathcal{N}=2$ 4d vector multiplet $\mathcal{A}=(S,\psi,\lambda,A_\mu)$. The orientifold action is [70]

$$\mathcal{O}=(-)^{F_L}\Omega_p\sigma^*\ ,\ \sigma^*\Omega=-\Omega\,;$$

Ω_p is the worldsheet parity, F_L is the left moving 4d fermion number and σ^* is the holomorphic involution (acting on forms). This will produce $O3/O7$ planes. Orientifolding splits the $\mathcal{N}=2$ vector multiplet into an $\mathcal{N}=1$ chiral multiplet (S,ψ) and a vector multiplet (λ,A_μ). Since we want to keep S as a low energy 4d field, we take the action of the involution to be $\sigma^*\chi_S=-\chi_S$. In this way the vector multiplet is projected out and we are left with only (S,ψ).

As explained in section 1.2.3, in an $\mathcal{N}=2$ formalism, the fluxes (4.76) may be seen as FI terms for the auxiliary components of the superfield $\mathcal{A}$. Based on this identification, it was noted in [69] that for $N>0$ (we always take $\beta^{NS}>0$), the supersymmetry variations are

$$\delta_\epsilon\psi=0\ ,\ \ \delta_\epsilon\lambda=i\epsilon\,\frac{1}{\mathrm{Im}\,\partial_S^2\mathcal{F}}\,\big(\frac{i}{g_s}\beta^{NS}+N\overline{\partial_S^2\mathcal{F}}\big)\,.$$

Therefore positive flux respects the same supersymmetry as the orientifold; we still have an $\mathcal{N}=1$ theory because λ is projected out from the spectrum, so $\delta\lambda\neq0$ is not seen. This type of flux dual was used in the discussion of supersymmetry breaking by anti-D3 branes in [71].

4.6.1 Supersymmetry breaking without warping

We now consider the effect of misaligning the supersymmetry preserved by the $O7$ and the one preserved in the conifold, by turning on negative flux $N<0$. Corrections from the warp factor will be ignored; this corresponds to the limit $\alpha'\to0$ and N fixed.

In the case $N<0$ [69] showed that $\delta\lambda=0$ but

$$\delta_\epsilon\psi=i\epsilon\,\frac{1}{\mathrm{Im}\,\partial_S^2\mathcal{F}}\,\big(\frac{i}{g_s}\beta^{NS}+N\partial_S^2\mathcal{F}\big)\,. \tag{4.97}$$

and hence this flux configuration breaks $\mathcal{N}=1\to\mathcal{N}=0$ spontaneously.

For $N<0$, $\beta^{NS}>0$, (4.78) would give a result $S\gg\Lambda_0^3$! It was argued in [69] that the physical vacuum is instead the minimum of the scalar potential Eq. (4.90). Ignoring warp corrections to the field space metric, this is located at

$$S_{N<0}=e^{-2\pi i\beta^R/N}\,e^{-2\pi|\beta^{NS}/g_sN|}\,\Lambda_0^3\,. \tag{4.98}$$

On this vacuum,

$$\partial_S W_{N<0}=2i\frac{\beta^{NS}}{g_s}\neq0 \tag{4.99}$$

and hence supersymmetry is broken by an explicit non-zero F-term $\partial_S W$.

In principle there are various ways one could define the scale of supersymmetry breaking, and in [69] definitions involving the mass splittings among supermultiplets were studied. However, in a spontaneously broken $\mathcal{N}=1$ supergravity theory, the standard definition is the norm of the F terms, or equivalently the scale determined by the F term contribution to the scalar potential

$$M_{susy}^4 = V = e^K \left(G^{i\bar{i}} D_i W D_{\bar{i}} W^*\right).$$

In a realistic compactification with near-zero cosmological constant, this scale will also determine the gravitino mass, as $m_{3/2} = M_{susy}^2/\sqrt{3}M_{Planck}$. How exactly it enters into observable susy breaking depends on the mediation mechanism, but very generally one expects soft terms of order M_{susy}^2/M_{Planck} from gravitational couplings and gravitino loop effects. Thus one generally requires $M_{susy} < 10^{11}$GeV (the intermediate scale) for a model which naturally solves the hierarchy problem, and this is the operational definition of a low scale of susy breaking.

Plugging the value of the nonsupersymmetric vacuum into the potential, and using $G_{S\bar{S}} \sim c \log \frac{\Lambda_0^3}{|S|}$, we find and thus

$$V \sim N \, \frac{\beta^{NS}}{g_s}. \tag{4.100}$$

Since we are working in conventions in which α' is order one, the upshot is that $\mathcal{N}=1$ supersymmetry is broken at a high scale. This can be confirmed by a $d=10$ computation of the mixing between the gravitino and the Goldstino, here the fermionic component of S. The essential content of this computation is already present in the supersymmetry variation Eq. (4.97).

Since the energy Eq. (4.100) is the expected tension of N anti D5-branes, in retrospect this result should not be very surprising. However it raises the question of whether and how it would be changed by including the warp factor.

4.6.2 Breaking supersymmetry at strong warping

We expect that a parametrically small scale of supersymmetry breaking can be generated in a regime where the warp factor dominates. From Eq. (4.88), this corresponds to

$$c \log \frac{\Lambda_0^3}{|S|} \ll \frac{(\alpha' g_s N)^2}{|S|^{4/3}}\,. \tag{4.101}$$

which may be attained by an adequate choice of fluxes $\beta^{NS} \gg g_s N$. In this case, $G^{S\bar{S}}$ scales like $|S|^{4/3}$ and this introduces an exponentially small factor in front of the F-term. The vacuum energy then becomes

$$M_{susy}^4 := V_{N<0} = \frac{k}{\kappa_4^2}\,\frac{1}{V_W(\mathrm{Im}\,\rho)^3 g_s}\,\Big|\frac{\beta^{NS}}{g_s N}\Big|^2 \exp\Big(-\frac{8\pi}{3}\,\frac{\beta^{NS}}{g_s|N|}\Big)\,\Lambda_0^4\,. \tag{4.102}$$

This has the desired exponential suppression in the semiclassical limit $\beta^{NS}/g_sN \gg 1$. Note that in the limit $V_W \to \infty$, the orientifold will be far away from the throat and $V_{N<0} \to 0$, which agrees with the idea that the system is locally supersymmetric.

From the point of view of the potential (4.90), the prescription of [69] for the physical vacuum (4.98) puts us in an unstable point, rolling directly to $S = 0$! One option would be that the present description, in terms of a single field S is not valid in strongly warped regimes. Indeed, in the known holographic descriptions of confined pure SYM [18, 19, 24], the masses of KK modes (from dimensional reduction on the conifold) are comparable to the glueball mass. Including these fields is not a simple task, requiring, in particular, a better understanding of the Green's functions on the deformed conifold.

Here we briefly discuss another option, namely that the breaking of the no-scale structure (due to the absence of supersymmetry) may stabilize the vacuum. Indeed, as the analysis of [72] suggests, metastable vacua in general require two scales, one generated by the gauge theory, and another coming from UV effects (in their case, the small mass m for quarks). Our discussion so far has no analog of this second scale.

To begin a full discussion, one would have to incorporate the various ingredients of moduli stabilization discussed in [28], including stabilization of the dilaton and the complex structure moduli other than S, and breaking of no-scale structure and stabilization of Kähler moduli due to stringy and quantum corrections which depend on the these moduli. We now assume that this has been done in some way which does not affect the physics in the throat, and discuss the remaining physics in the throat after integrating these modes out, using the supergravity potential

$$V = \kappa_4^2 e^K \Big[G^{S\bar{S}} |D_S W|^2 - 3|W|^2 \Big] \tag{4.103}$$

Actually this expression would only be exact in a limit in which the other moduli were infinitely massive; otherwise it will receive corrections from cross-coupling between the other moduli and S. However, one can easily state conditions under which these effects will not qualitatively affect the results, so we neglect this.

Now, taking the anti-self-dual flux configuration, an important point is that the dual period $\partial\mathcal{F}/\partial S$ does not vanish in the limit $S \to 0$. Writing

$$\int_B \Omega = \frac{S}{2\pi i} \log \frac{\Lambda_0^3}{S} + \Pi_0 \,,$$

then

$$W = \frac{N}{2\pi i} S \big(\log \frac{\Lambda_0^3}{S} + 1 \big) + N\Pi_0 + \frac{i}{g_s} \beta^{NS} S \tag{4.104}$$

and the condition for having a minimum at small S is

$$S^{1/3}\log\frac{\Lambda_0}{S^{1/3}} \approx \frac{2\pi c'\,\Pi_0}{\int e^{-4A}\Omega\wedge\overline{\Omega}}\,(\alpha' g_s N)^2\,. \tag{4.105}$$

Typically,

$$\frac{2\pi\Pi_0}{\int e^{-4A}\Omega\wedge\overline{\Omega}} \sim V_W^{-1/2}$$

so by choosing a bulk volume $V_W^{1/2} \gg \alpha'^2 g_s N\beta^{NS}$, the modulus is stabilized at a parametrically small (though no longer exponentially small) scale. The vacuum energy here is of the order

$$V_{min} \approx \frac{1}{2\kappa_{10}^2}\frac{g_s}{\mathrm{Im}\rho^3}\,\frac{|\Pi_0|^2}{\int e^{-4A}\Omega\wedge\overline{\Omega}}\,.$$

Since the natural scale of the potential away from the $S \to 0$ limit is set by β^{NS}/g_sN, the height and breath of the barrier separating this minimum from the true vacuum scale in the same way. It's worth mentioning at this point that in the GKP conifold setup the parameter choices leading to a controllable hierarchy are $1 << g_sN$, so that the supergravity approximation is reliable at the tip, and $1 << \beta^{NS}/g_sN$, which in the supersymmetric GKP setup sets the scale of the hierarchy. These are precisely the same relations which yield a reliable metastable vacuum here.

Finally, once one has found a stable vacuum from the point of view of the $\mathcal{N}=1$ effective Lagrangian, one needs to ask whether other effects could destabilize it, in particular whether a KK mode which was dropped in deriving the Lagrangian could go tachyonic. The basic answer to this question is that, since there is a limit in which the throat solution would have been $\mathcal{N}=1$ supersymmetric had not that supersymmetry been projected out by the orientifolding, it will satisfy the constraints of this $\mathcal{N}=1$ supersymmetry, up to small corrections. Thus, one can restrict attention to the light modes in the $\mathcal{N}=1$ effective Lagrangian, and see whether the new couplings introduced at this point destabilize any of them; massive KK modes will be stable since the original KS solution was stable.

4.7 The dual gauge theory at strong warping

The discussion we just gave should be valid for $g_sN >> 1$. For small g_sN, we would expect a description in terms of the gauge theory on the wrapped anti-D5 branes to be more appropriate. We do not know how such a description would work in detail, but we can make the following comments on the problem.

Let us start by considering the embedding of the conifold with anti-self-dual flux into an $\mathcal{N}=2$ compactification. There, the gauge theory under discussion is *the same* as the gauge theory usually invoked in this duality, namely the $U(N_1) \times U(N_2)$ supersymmetric gauge theory of [18] in the UV, undergoing a "cascade" down to pure $U(N)$ super Yang-Mills theory. This theory has N supersymmetric vacua, and we recover the standard discussion, with the sole change being the sign of the RR fluxes and the identification of the unbroken $\mathcal{N} = 1$ subalgebra in $\mathcal{N} = 2$.

As we saw in the supergravity analysis, it seems very plausible that the essential phenomenon is a misalignment of the $\mathcal{N} = 1$ supersymmetries preserved by the bulk and by the antibrane. To describe this in gauge theory terms, we might try to identify the action of bulk $\mathcal{N} = 2$ supersymmetry on the gauge theory, and the $\mathcal{N} = 2$ stress tensor multiplet, which would couple to the $d = 4$, $\mathcal{N} = 2$ supergravity obtained by KK reduction. The difference between the D5 and anti-D5 theories then arises when we do the orientifold projection, obtaining a $d = 4$, $\mathcal{N} = 1$ supergravity. Whereas for the D5 theory, we couple the $\mathcal{N} = 1$ stress tensor multiplet to $\mathcal{N} = 1$ supergravity, for the anti D5-brane we would instead couple to the *broken* $\mathcal{N} = 1$ subalgebra of $\mathcal{N} = 2$.

This idea is simple to realize in the case of branes embedded in flat space. Consider for example the world-volume theory of N D3-branes; it is $\mathcal{N} = 4$ super Yang-Mills with 16 linearly realized supersymmetries. It also has 16 nonlinearly realized supersymmetries, the constant shifts of the diagonal components of the gauginos. An analog of the theory under discussion is obtained by truncating this to a linearly realized $\mathcal{N} = 1$ and a nonlinearly realized $\mathcal{N} = 1$. Thus, the antibrane couples to the $\mathcal{N} = 1$ gravitino, not through the standard supercurrent, but through the gaugino.

This leads to spontaneous supersymmetry breaking, at a scale controlled by the antibrane tension. However, it is not obvious how strong coupling effects could lower this scale. Naively, since the supersymmetry breaking is all in the coupling to the $U(1)$ sector, the nonabelian Yang-Mills sector does not seem to play any role. However, the sectors could be coupled by higher dimension operators, so this conclusion is probably too quick.

According to the usual discussions of the AdS/CFT correspondence, the $\mathcal{N} = 2$ supersymmetry of the underlying string background, is reflected in the $\mathcal{N} = 1$ superconformal symmetry of the gauge theory. Thus, the idea would be to couple the gravitino of $\mathcal{N} = 1$ supergravity, not to the standard supercurrent, but to the superconformal current of the gauge theory.

Unfortunately, this idea is not consistent as it stands, as the superconformal symmetry in these gauge theories is explicitly broken by quantum effects (the beta function is non-zero) and we cannot gauge an explicitly broken symmetry. Nevertheless it might be correct if a suitable compensator field is present in the bulk theory.

4.7.1 Effective potential

Granting that there is a microscopic definition of the theory as a gauge theory coupled to $\mathcal{N} = 1$ supergravity, we next ask whether the effective potential we have derived and justified at $g_s N >> 1$, should be expected to give a good qualitative description for small $g_s N$. As we commented earlier, even in the supersymmetric vacua the precise interpretation of this type of effective action is not entirely clear, so we limit ourselves to questions about vacuum energy, supersymmetry breaking and stability.

We begin with the $|S|^{2/3}$ term in the Kähler potential. It is amusing and perhaps significant that such a term was already suggested in the pioneering work of Veneziano and Yankielowicz on pure $SU(N)$ SYM [50]. The argument there was that the gaugino bilinear S, being a composite field, does not have the canonical dimension of a scalar field. At weak coupling, its dimension should be close to that of a fermion bilinear in free field theory, namely $[S] = 2[\lambda] = 3$. On the other hand, a $d^4\theta$ kinetic term should have dimension 2. If we are not allowed any dimensionful constants, this forces

$$K(\mathcal{S}\bar{\mathcal{S}}) = \alpha(\bar{\mathcal{S}}\mathcal{S})^{1/3} \tag{4.106}$$

for some numerical constant α. This precisely matches the new term coming from warping in Eq. (4.88)!

Unfortunately, for $S \to 0$ the gauge theory is strongly coupled, and it is not known how to compute the Kähler potential in this regime. On general grounds one would expect corrections controlled by the dynamical scale Λ. While it is true that Λ does not appear explicitly in the superpotential, only emerging upon solving for the vacuum, there is no obvious reason that the Kähler potential should work the same way. Thus at this point we can not say we have strong evidence for such a term at weak coupling, although it is certainly a very suggestive coincidence.

In any case, if we accept that the theory breaks supersymmetry at the dynamical scale, the claim that the metric $G_{S\bar{S}} \sim |S|^{-\alpha}$ for some $\alpha > 0$ would seem to be a very natural way to describe this in an $\mathcal{N} = 1$ effective Lagrangian. It might not be inevitable, as one can also imagine inverse powers of Λ playing this role. However, this would violate the general principle that nonperturbative effects should vanish in the weak coupling limit $\Lambda \to 0$, so it seems a reasonable hypothesis that such effects are not present.

But, as we saw in our explicit example, any structure in which the vacuum energy is warped down by a power of S, leads directly to a potential with a zero energy minimum at $S = 0$. We discussed how in a string theory compactification this might be prevented by bulk effects. But in the gauge theory limit, such effects would presumably be absent, so the result would be a theory which rolls down to $S = 0$.

Could there be another supersymmetric vacuum at $S = 0$? A suggestion that super Yang-Mills theory has additional vacua at $S = 0$ was made in [73], however at present this is not believed to be the case.

One straightforward way to reconcile these claims is if the effects we are discussing, in particular the correction to the Kähler potential and the corresponding lowering of the supersymmetry breaking scale, are not present at small $g_s N$. Now some brane-antibrane realizations of supersymmetry breaking, for example [74, 75], lead to a non-trivial phase structure, and it might be the case here. However, in these realizations, the supersymmetry breaking vacuum exists at weak coupling, and disappears at strong coupling, so the opposite claim might be surprising.

It also seems possible to us that while this effective field theory is qualitatively valid, the configuration rolling down to $S = 0$ is not a conventional vacuum. This is true in the supergravity limit, as the value of S controls the warp factor in $d = 4$, so that $S = 0$ cannot be realized. One can still imagine solutions in which S rolls to zero, but these are essentially dynamical. In particular, since the warp factor multiplies the g_{00} component of the metric, the time evolution is very different than the flat space evolution in such a potential. As we explained in the discussion below Eq. (4.95), this suggests that the minimum $S = 0$ is not reached in finite physical time.

Chapter 5
Four dimensional effective theory

5.1 Introduction and summary

In this final chapter on dynamical aspects of flux compactifications, we analyze the four dimensional effective action including both 4d zero modes and their KK excitations (which become light at large warping). This is based on [1, 5].

Understanding the effective action with warp effects is important both for theoretical issues (e.g. gauge/string dualities) and for making more precise four dimensional predictions. The original motivation for warped geometries comes from considering stacks of large numbers of branes, and reveals deep dualities between supergravities and gauge theories, as in the AdS/CFT correspondence [17]. The example discussed in chapter 4 involves the gauge theory of D5-branes wrapping an isolated two-cycle of the conifold.

Warped geometries also play an important role in supersymmetry breaking scenarios from string theory. For instance, placing antibranes at the end of the conifold, [71] found a supergravity dual of a nonsupersymmetric field theory. Another approach, based on anti-self dual flux in the conifold, was described on chapter 4. There have also been recent developments in metastable vacua, following the work of [72]. Overall, these works suggest that strongly warped supergravities which break supersymmetry are dual to dynamical supersymmetry breaking in gauge theories. Models with metastable supersymmetry breaking are explored in chapters 6 and 7.

To begin with some general comments, the warp factor is of the general form

$$e^{-4A(y)} = c + e^{-4A_0(y)} \tag{5.1}$$

where the dimensionless parameter c is the static part of the universal Kähler modulus, and A_0 is produced by matter sources (fluxes or branes). The large volume limit $c \gg e^{-4A_0}$ corresponds to a small number of fluxes or branes, where backreaction may be ignored. Here we address the general case, which includes the strongly warped limit $c \ll e^{-4A_0}$. The warp factor is not holomorphic and thus one expects it not to affect holomorphic quantities such as the superpotential and gauge kinetic terms. However, we have shown that in general it will affect non-holomorphic quantities such as the Kähler potential.

5.1.1 Summary

The effective theory for warped compactifications is developed as follows. We begin in section 5.2 by analyzing the physics of KK modes at strong warping. Considering the case of a scalar field in a warped geometry, we argue that new light KK modes are generated, corresponding to "bound states" around warped throats. This can be interpreted as a 6d Coulomb problem. We also discuss KK modes for internal metric fluctuations in the KS geometry, both from the 10d and 4d perspectives, and relate our findings to glueballs in pure SYM.

In section 5.3, the theory is analyzed from a ten dimensional perspective. We identify the 10d fluctuations and compute their equations of motion. In subsection 5.3.1 we define, for each type of fluctuation, a basis of "warped" internal wavefunctions which will be used throughout the work. A general formula for the effective action is provided in Eq. (5.35).

Section 5.4 presents the four dimensional kinetic terms for the different fluctuations. It is argued that there are no terms with two space-time derivatives mixing the metric moduli with any of the other light modes, in the basis of "warped" KK modes defined in subsection 5.3.1. Therefore, even in the strongly warped limit it is consistent to study the propagators associated to such moduli independently of the other fields.

In section 5.5 we study the geometrical KK masses, and mass terms induced by fluxes. It is shown that KK mass terms do not mix the metric moduli with KK excitations, while we do find quadratic couplings between massive graviton and internal metric modes. The computation of the flux potential for metric fluctuations, including KK modes, is given in Eq. (5.66). This, is a warped generalization of the Gukov-Vafa-Witten potential and it exhibits possible mixings between the moduli and their KK tower.

In section 5.6 we end the chapter with comments on future directions.

5.2 Kaluza-Klein modes at strong warping

In this section we analyze the physics of KK modes in strongly warped limits.

5.2.1 Scalar field case

For concreteness, let us work in detail the case of a ten-dimensional scalar field; the other modes follow a similar pattern. Consider the action with a possible nontrivial potential,

$$S = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{g} \left(g^{MN} \partial_M \phi \, \partial_N \phi + V(\phi) \right) . \tag{5.2}$$

Using the ansatz

$$\phi(x, y) = \sum_i \varphi_i(x) Y_i(y)$$

the dimensionally reduced action becomes

$$S = -\frac{1}{2\kappa_{10}^2} \int d^4x \Big[(Y_i, e^{-4A} Y_j) \, \varphi_i \Box \varphi_j + (Y_i, \tilde{\nabla}^2 Y_j) \, \varphi_i \varphi_j + V(\varphi) \Big] \tag{5.3}$$

where we have introduced the natural inner product on the Calabi-Yau manifold,

$$(f, g) := \int d^6y \sqrt{\tilde{g}_6} \, f(y) g(y) \, . \tag{5.4}$$

Both operators e^{-4A} and $\tilde{\nabla}^2$ are self-adjoint with respect to this product, so that we have a well-defined action.

A preferred basis for $Y_i(y)$ would be the one in which both the field space metric and mass matrix are simultaneously diagonalized, if possible. In our case, such functions are given as the eigenvectors of the following differential problem:

$$\tilde{\nabla}^2 \, Y_i(y) = e^{-4A(y)} \, \lambda_i^2 \, Y_i(y) \, . \tag{5.5}$$

Since this is a well-defined Sturm-Liouville problem, non-degenerate eigenvectors will be orthogonal and we can orthogonalize degenerate eigenvectors. Thus we can choose a basis in which

$$\frac{1}{V_W} \int d^6y \sqrt{\tilde{g}_6} \, e^{-4A} \, Y_i(y) Y_j(y) = G \, \delta_{ij} \, . \tag{5.6}$$

Then the action acquires the desired diagonal form

$$S = -\frac{1}{2\kappa_4^2} \int d^4x \Big[G \, \varphi_i (\Box + \lambda_i^2) \varphi_i + V(\varphi) \Big] \, . \tag{5.7}$$

One arrives to the same results by requiring that the 4d scalar has a well-defined mass, $\eta^{\mu\nu} \partial_\mu \partial_\nu \varphi_i = -\lambda_i^2 \varphi_i$. These are the mass eigenstates in the limit $V \to 0$. Comparing Eq. (5.5) with the usual unwarped KK modes,

$$\tilde{\nabla}^2 \, \mathcal{Y}_i(y) = \nu_i^2 \, \mathcal{Y}_i(y) \, , \tag{5.8}$$

we conclude that a nontrivial warp factor modifies the eigenvalue problem for KK modes, while the unwarped eigenfunctions $\mathcal{Y}_i$ do not have a 4d interpretation as single-particle excitations.

To gain more intuition on the behavior of KK modes at strong warping, let us rewrite Eq. (5.5) as

$$\left(\frac{1}{\lambda_i^2}\tilde{\nabla}^2 - e^{-4A(y)}\right) Y_i(y) = 0\,. \tag{5.9}$$

From Eq. (1.72), the warp factor is the analog of the Coulomb potential, but in a 6d curved space. Then Eq. (5.9) can be interpreted as a Schrödinger equation for the wavefunction Y_i with a Coulomb potential determined by the warp factor [76,77]. In analogy with the hydrogen atom, there will be light warped KK modes corresponding to bound states in such potential, while the unwarped modes are associated to states whose interactions are warp factor insensitive in a box of size V_W.

5.2.2 KK modes in the deformed conifold

The light KK modes have support on regions of strong warping, so their main features can be understood from a local analysis of the geometry. Consider a warped throat modeled by the warped deformed conifold, as described in chapter 4. Eq. (5.5) can be solved numerically; see for instance [67] and references therein. In particular, it is found that the mass scale is of order of the local scale Λ of the throat, namely (see Eq. (4.78))

$$\Lambda^3 = S_{min} = e^{-2\pi\beta^{NS}/g_s N}\,\Lambda_0^3\,, \tag{5.10}$$

where N is the F_3 flux in the A-cycle, while β^{NS} is the H_3 flux through the noncompact B-cycle.

It is also useful to understand this result from the point of view of the effective theory. In the KS solution, the field S parametrizing the size of the S^3 can be interpreted as the lowest mode of a KK tower, and using the results from chapter 4, we can compute its mass. After changing variables to canonically normalized fields, the physical mass is

$$|m_S| \sim G^{S\bar{S}}\left|\partial_S^2 W(\Lambda^3)\right|\,. \tag{5.11}$$

From this expression, it is clear that the factor responsible for generating a parametrically small mass is the kinetic metric $G_{S\bar{S}}$. Using the result $G_{S\bar{S}} \sim |S|^{-4/3}$ from Eq. (4.88), we obtain

$$|m_S| \sim \frac{\Lambda}{g_s^2 N} \tag{5.12}$$

so the local scale Λ sets the value of the mass, in agreement with the numerical approach. The masses are expressed in α' units. On the other hand, if warp corrections to the Kähler potential are ignored, the metric is $G_{S\bar{S}} = c\log(\Lambda_0^3/|S|)$, and then the mass becomes

$$|m_S| \sim \frac{g_s N^2}{\beta^{NS}\Lambda^3} \tag{5.13}$$

with a qualitatively different limit for $\Lambda \to 0$.

In the dual gauge theory, S is interpreted as the glueball composite $\lambda\lambda$. Similarly, higher modes in the KK tower correspond to various glueball excitations of pure SYM. Since the theory has a mass gap Λ, the lowest excitations have masses of order Λ. From this point of view, the denominator in Eq. (5.12) could be interpreted as an effect appearing at large 't Hooft coupling. It would be interesting to continue along these lines and try to understand the gauge theory for other glueballs, by computing warp corrections to the KK mode effective action.

5.3 From 10 to 4: warped fluctuations and effective action

In this section, the general procedure for obtaining the effective action for warped flux compactifications will be described. This combines the Hamiltonian approach developed in chapters 2–4 with the results of the previous section on light KK modes.

5.3.1 10d perspective for fluctuations in IIB supergravity

From a ten-dimensional point of view, the dynamics follows by considering infinitesimal fluctuations around the background studied in section 1.6. Moduli[1] are promoted to spacetime fields, and the corresponding equations of motion have to be solved. The zero mode sector includes the complex and Kähler moduli $u^I = (\rho^r, S^\alpha)$, the 4d graviton $h_{\mu\nu}(x)$, the axio-dilaton $\tau_0(x)$ (both are constant on the internal manifold), and the various massless p-form fields coming from decomposing (C_2, B_2, C_4) into harmonic forms. For each of them we have to include the corresponding tower of KK excitations.

Following the prescription of chapter 2, we work in the gauge where the compensators vanish, and then impose the Hamiltonian constraints on the physical fluctuations. In the presence of dynamical moduli, the metric fluctuations have the form

$$\delta(ds^2) = \delta g_{\mu\nu} dx^\mu dx^\nu + \delta g_{mn} dy^m dy^n \,. \tag{5.14}$$

where

$$\delta g_{\mu\nu}(x,y) = e^{2A(y)}[2\,\delta A(x,y)\eta_{\mu\nu} + \delta_K g_{\mu\nu}(x,y)]\,, \tag{5.15}$$

$$\delta g_{mn}(x,y) = e^{-2A(y)}[-2\,\delta A(x,y)\tilde{g}_{mn}(y) + \delta\tilde{g}_{mn}(y,y)]\,. \tag{5.16}$$

[1]We will loosely refer to complex structure deformations as complex moduli, even though they are lifted by fluxes.

Here $\delta_K g_{\mu\nu}$ are 4d graviton KK modes (which are not necessarily transverse-traceless), while $\delta\tilde{g}_{mn}$ encode the metric moduli u^I and their KK modes. Since the warp factor depends on the moduli, a fluctuation $\delta\tilde{g}_{mn}$ induces in turn a variation $\delta A \neq 0$.

To isolate the zero modes from their KK partners, we expand the metric fields in a basis of eigenmodes for the internal manifold:

$$\delta_K g_{\mu\nu}(x,y) = \sum_{I_1} h^{I_1}_{\mu\nu}(x) Y^{I_1}(y)\,. \tag{5.17}$$

$$\delta\tilde{g}_{mn}(x,y) = \sum_{I_2} u^{I_2}(x)\, Y^{I_2}_{mn}(y)\,, \tag{5.18}$$

The multi-index I_i, $i = 1, 2$, runs over the different types of metric fluctuations and, for each type of fluctuation, over the 4d KK tower. $I_i = 0$ gives the zero mode of the appropriate Laplacian on the internal manifold.

We certainly have the freedom to choose different complete bases of internal wavefunctions; this amounts to making field redefinitions in the four-dimensional theory. Different choices represent the extra dimensions in rather different ways and our aim is to find the one that yields the simplest description. Following section 5.2, we choose the Fourier mode expansion describing the light KK modes,

$$\tilde{\nabla}^2\, Y_i(y) = e^{-4A(y)}\, \lambda_i^2\, Y_i(y)\,. \tag{5.19}$$

In the following, this is made explicit in each sector.

Metric fluctuations

We will take the eigenmodes (5.17), (5.18) to be solutions to the respective eigenvalue equations,

$$\tilde{\nabla}^2\, Y^{I_1} = e^{-4A(y)} \lambda^2_{I_1}\, Y^{I_1}\,, \tag{5.20}$$

$$\frac{1}{2}\tilde{\Delta}_L\, Y^{I_2}_{mn}(y) = \delta\tilde{G}_{mn} = e^{-4A(y)} \lambda^2_{I_2}\, Y^{I_2}_{mn}(y)\,; \tag{5.21}$$

here $\tilde{\Delta}_L$ is the Lichnerowicz laplacian for $\tilde{g}_{mn}$. The eigenmode expansions (5.21, 5.20) lead to orthogonality relations between different modes

$$\frac{1}{2V_W}\int d^6y\sqrt{\tilde{g}_6}e^{-4A} Y^{I_1}\, Y^{J_1} = \mathcal{M}^{kk}\delta_{I_1,J_1}\,. \tag{5.22}$$

$$\frac{1}{V_W}\int d^6y\sqrt{\tilde{g}_6}\, e^{-4A(y)}\, Y^{I_2}_{mn}(y)\bar{Y}^{J_2\,\widetilde{mn}}(y) = G^{(u)}\,\delta_{I_2,J_2}\,. \tag{5.23}$$

Indices with tildes are raised with $\tilde{g}^{mn}$.

Dilaton fluctuations

Fluctuations of the dilaton are of the form $\tau = \tau_0 + \delta\tau(x,y)$, where $\mathrm{Im}\,\tau_0 = g_s^{-1} \gg 1$ and is constant in the internal space. Since the axio-dilaton is a scalar from the six-dimensional point of view, its expansion in KK modes is the same as for the graviton:

$$\delta\tau = \delta\tau_0(x) + \delta_K\tau(x,y) = \delta\tau_0(x) + \sum_{I_1} t^{I_1}(x) Y^{I_1}(y)\,. \tag{5.24}$$

The orthogonality relation for the dilaton KK modes then reads,

$$-\frac{1}{4(\mathrm{Im}\tau_0)^2\, V_W}\int d^6y\sqrt{\tilde{g}_6}\, e^{-4A}\, Y^{I_1}(y)Y^{J_1}(y) \equiv \mathcal{M}^{kk}_{\tau}\,\delta_{I_1 J_1}\,. \tag{5.25}$$

p-form fluctuations

The various antisymmetric tensors have an expansion of the form

$$\begin{aligned} &F_3 \to F_3 + d\,\delta C_2\;,\; H_3 \to H_3 + d\,\delta B_2\;,\\ &F_5 \to F_5 + d\,(\delta\alpha)\wedge d^4x + d\,\delta C_4\,. \end{aligned} \tag{5.26}$$

(F_3, H_3, F_5) denote the background GKP values, while the derivative variations include the zero modes (harmonic forms) and their KK excitations. Note that because of the background BPS relation $\alpha = e^{4A}$, fluctuations of the 4-form are induced by fluctuations of the warp factor (which are in turn induced by fluctuations of moduli), e.g. $\delta\alpha = \delta e^{4A}$.

In principle, a nontrivial warp factor could induce mixings between the four dimensional scalars coming from these p-forms and the metric moduli. However, we will argue in section 5.5.2 that this is not the case; for this reason, we will not perform a full analysis of this sector. Also, the dimensional reduction of the universal axion from C_4 was discussed in chapter 3.

5.3.2 10d fluctuated equations and effective action

The linearized equations for the fluctuations are

$$\delta G_{MN} = \kappa_{10}^2\delta T_{MN}\;,\; d\delta F_5 = \delta(H_3\wedge F_3) + 2\kappa_{10}^2\delta(T_3\rho_3^{loc})\,, \tag{5.27}$$

$$d\,\delta\big[e^{4A}(\star_6 G_3 - iG_3)\big] = 0\,,\;\delta(\star_{10}F_5) = \delta F_5\,, \tag{5.28}$$

$$\nabla_M\nabla^M\delta_K\tau = -\frac{i}{12}\delta(G_3\cdot G_3)\,, \tag{5.29}$$

using the usual formula

$$\delta R_{MN} = -\frac{1}{2}\nabla^P\nabla_P\delta g_{MN} - \frac{1}{2}\nabla_M\nabla_N g^{PQ}\delta g_{PQ} + \frac{1}{2}\nabla^P\nabla_M\delta g_{NP} + \frac{1}{2}\nabla^P\nabla_N\delta g_{MP}\,. \tag{5.30}$$

Some of these equations contain at most first order space-time derivatives, and these are precisely the Hamiltonian constraints. The initial value constraints in general relativity are

$G_{0M} = T_{0M}$; assuming that the stress-energy tensor does not have velocity dependent terms, it is found that (see e.g. [35])

$$\begin{aligned} T_{00} &= G_{00} = \frac{1}{2}\left({}^{(9)}R - h^{-1}\pi^{MN}\pi_{MN} + \frac{1}{2}h^{-1}\pi^2\right) \\ 0 &= G_{0M} = D_N\left(h^{-1/2}\pi^N_{\ M}\right) \quad , \quad M \neq 0\,. \end{aligned} \tag{5.31}$$

Here, h is the 9d (space-like) metric, ${}^{(9)}R$ is its scalar curvature, and the canonical momentum was defined in Eq. (2.18). Evaluating the second constraint for $M = \mu$ sets $\pi_{\mu\nu} = 0$ or, in 4d terms,

$$\delta A = \frac{1}{8}\delta\tilde{g}\,. \tag{5.32}$$

Computationally, this is a very useful relation, allowing us to replace the potentially complicated fluctuation δA by $\delta\tilde{g}$. By Lorentz invariance, the linearized version of the first constraint is then satisfied. The second equation along $M = m$ is the 6d constraint Eq. (2.67), which we reproduce for convenience,

$$\tilde{\nabla}^n\left(\delta\tilde{g}_{mn} - \frac{1}{2}\tilde{g}_{mn}\,\delta\tilde{g}\right) = 4(\partial^{\tilde{n}}A)\,\delta\tilde{g}_{mn} \tag{5.33}$$

The p-form equations of motion also contain initial value constraints, which are the generalization of Gauss' law in electrodynamics. These were discussed in chapter 3.

As the final step in our general discussion, we wish to understand the dynamics from a four dimensional action principle, by first compactifying the supergravity action on a background satisfying the ten-dimensional equations of motion, and then integrating over the internal coordinates, along the lines of [22].

There are some known subtleties in doing this. First, the type IIB supergravity action is ill-defined due to the self-duality of the five form. The procedure which will be followed here is to project out half of the 4-dimensional degrees of freedom of the five form and double the coefficients of the $\tilde{F}_5^2$ and Chern-Simons term. Second, the Gibbons-Hawking-York term [78,79] must be included in the dimensional reduction to cancel certain total derivative terms of the variation of the gravitational action.

The dimensionally reduced effective action is then obtained by expanding

$$S = \int_M d^{10}x\sqrt{g}\,(g^{MN}\,R_{MN} + \mathcal{L}_{matter}) + 2\int_{\partial M} d^9x\,\sqrt{h}\,K \tag{5.34}$$

to second order in fluctuations. After some algebra, the result is,

$$S_{eff} = \frac{1}{4\kappa_{10}^2}\int d^{10}x\,\sqrt{g}\left[-(\delta g)^{MN}\left(\delta G_{MN} - \delta T_{MN}\right) + \delta^2\mathcal{L}_{matter}\right] + \mathcal{O}(\delta g^3)\,, \tag{5.35}$$

where $\delta^2\mathcal{L}_{matter}$ represents second order fluctuations of the flux and dilaton terms in the effective action with respect to the flux and dilaton fields. This reproduces the correct ten dimensional equations of motion. To perform the dimensional reduction, one expands the fluctuations in terms of internal eigenmodes and integrates over the compactification space, while imposing the constraint equations (5.32, 5.33) derived in the previous subsection.

Formally, in this process we are including all the (infinite tower of) KK modes, to quadratic order. The details of each particular region of strong warping will then determine a truncation of the KK tower to keep only the lightest modes. Higher order interactions may also be analyzed by further expanding, for instance, the Einstein-Hilbert term. The existence of trilinear couplings between moduli and KK modes may have interesting consequences.

5.4 Warped kinetic terms

This section is devoted to the analysis of the terms in the effective action that contain space-time derivatives. The main result will be that the kinetic terms for all sectors decouple and are diagonal in the KK mode expansion,

$$\mathcal{L}_{kin} = G^{(u)} \sum_{I_2} u^{I_2} \, \Box \bar{u}^{I_2} + \mathcal{M}_h^{kk} \sum_{I_1} E^{\mu\nu}_{I_1} \, h^{I_1}_{\mu\nu} + \mathcal{M}_\tau^{kk} \sum_{I_1} t^{I_1} \, \Box \bar{t}^{I_1} \, .$$

Here, $E_{\mu\nu}$ is the linearized four dimensional Einstein tensor and I_i runs over the different moduli and their KK towers. This implies that (even with a nontrivial warp factor) the propagators for the moduli and for the light KK modes do not mix. The diagonality of the kinetic terms implies that the Kähler potential is also diagonal to quadratic order in the fluctuations of the moduli and the dilaton (and their KK modes),

$$K = G^{(u)} \sum_{I_2} u^{I_2} \bar{u}^{I_2} + \mathcal{M}_\tau^{kk} \sum_{I_1} t^{I_1} \bar{t}^{I_1} \, . \tag{5.36}$$

5.4.1 Axio-dilaton and p-form modes

As a warm-up, we first consider the kinetic terms for the axio-dilaton and the p-forms. Here the dimensional reduction is simple but nonetheless illustrates some of the features that will appear in the more involved analysis of metric fluctuations.

The axio-dilaton is a ten-dimensional scalar field with a nonlinear metric. Expanding around the background value τ_0 according to (5.24), the kinetic term turns out to be

$$\mathcal{L}^{(\tau)}_{kin} = \sum_{I_1, J_1} G^{(\tau)}_{I_1 J_1} t^{I_1}(x) \Box \bar{t}^{J_1}(x) \tag{5.37}$$

where the warped metric is

$$G^{(\tau)}_{I_1 J_1} = \frac{1}{4 (\mathrm{Im}\, \tau_0)^2 \, V_W} \int d^6 y \sqrt{\tilde{g}_6} \, e^{-4A(y)} Y_{I_1}(y) Y_{J_1}(y) \, . \tag{5.38}$$

The dilaton metric (5.38) is proportional to the orthogonality relation on the internal space (5.25), so that the Kähler potential, to quadratic order, becomes (taking $\mathcal{M}_\tau^{kk} \equiv G^{(\tau)}_{I_1 I_1}$)

$$K^{(\tau)} = \mathcal{M}_\tau^{kk} \sum_{I_1} t^{I_1}(x) \bar{t}^{I_1}(x)\,. \tag{5.39}$$

The conclusion is that, at this order, there is no mixing between the various dilaton KK modes in the kinetic term. Had we used the unwarped KK expansion of Eq. (5.8), the kinetic term would have exhibited complicated mixings.

Next we discuss a rather different behavior, arising from the one-form KK modes of C_4:

$$\delta C_4 = \sum_I V^I_\mu(x) dx^\mu \wedge \chi^I(y)\,,$$

where $\chi^I(y) = \frac{1}{3!}\chi^I_{mnp}\, dy^{mnp}$. This case is relevant for D-term supersymmetry breaking [80,81]. Replacing this in the $\tilde{F}_5^2$ term of the IIB action, we find

$$\mathcal{L}^{(V)}_{kin} = G^{(V)}_{IJ}\, F^I \wedge \star_4 F^J\,. \tag{5.40}$$

The metric appearing here is

$$G^{(V)}_{IJ} = \sum_{I,J} \frac{1}{4\, V_W} \int_M \chi_I(y) \wedge \star_6 \chi_J(y)\,, \tag{5.41}$$

Therefore, the field space metric coincides with the unwarped one, with the result that the corresponding D-term cannot be made parametrically small by the large hierarchy of the throat. The field space metric $G^{(V)}$ for the massless mode can be shown to coincide with $\mathrm{Im}\, \partial^2 \mathcal{F}$, where $\mathcal{F}$ is the Calabi-Yau prepotential.

5.4.2 Graviton fluctuations

After having gained some intuition with the previous simpler sectors, we will now consider the metric fluctuations,

$$\delta(ds^2) = e^{2A}\big[2\delta A\, \eta_{\mu\nu} + \delta_K g_{\mu\nu}\big] dx^\mu dx^\nu + e^{-2A}\big[-2\delta A\, \tilde{g}_{mn} + \delta \tilde{g}_{mn}\big] dy^m dy^n\,. \tag{5.42}$$

The kinetic term for this sector follows from dimensionally reducing the Einstein-Hilbert part of the supergravity action, according to the prescription Eq. (5.35).

The variation of the warp factor makes this computation highly nontrivial in at least two aspects. First, from the space-time variation we expect mixings between the trace part of the graviton mode[2] and the moduli through δA, $\delta \tilde{g}$. Further, the internal metric variation is no

[2]Note that we have not chosen the standard transverse traceless gauge for the graviton, which is in general not consistent with the gauge where the compensators vanish.

longer proportional to $\delta\tilde{g}_{mn}$ (since it includes δA fluctuations) so the relation between a complex modulus of the underlying Calabi-Yau ($\delta\tilde{G}_{mn}=0$) and a zero mode of the full warped metric ($\delta G_{MN}=0$) may be very involved.

Our first result will be to show that, in spite of the possible couplings suggested by (5.42), there are no space-time derivative mixings between $\delta_K g$ and $\delta\tilde{g}_{mn}$. The simplest way of understanding this is by doing a conformal transformation, and for this it is actually better to work with the metric containing the fluctuations to all orders:

$$ds^2 = e^{2A(x,y)}\, g_{\mu\nu}(x,y)\, dx^\mu dx^\nu + e^{-2A(x,y)}\, \tilde{g}_{mn}(x,y)\, dy^m dy^n\,. \tag{5.43}$$

The x dependence in A and $\tilde{g}_{mn}$ comes from promoting the moduli to space-time dependent fields and from their KK modes. The graviton is associated to the fluctuating metric $g_{\mu\nu}(x,y)$ and does not induce warp factor fluctuations.

Consider the conformal transformation

$$ds^2 = e^{2A(x,y)}\, d\hat{s}^2\;,\;\hat{g}_{\mu\nu}=g_{\mu\nu}\;,\;\hat{g}_{mn}=e^{-4A}\,\tilde{g}_{mn}\,, \tag{5.44}$$

which leads to a change in the Ricci scalar [35]

$$R \to e^{-2A}\big(\hat{R} + 9\times 8\,\hat{g}^{LM}\partial_L A\,\partial_M A\big)\,, \tag{5.45}$$

after an integration by parts. The conformally rescaled metric $\hat{g}_{MN}$ is block-diagonal (in terms of its coordinate dependence) and decouples the graviton and internal metric fluctuations. Dimensionally reducing $\hat{R}$ on this ansatz for $\hat{g}_{MN}$ leads to kinetic terms for the graviton and internal metric fluctuations without off-diagonal mixings. Further, the spacetime derivative contribution of the extra term in (5.45) does not contain graviton pieces (at quadratic order). This proves that there are no space-time derivative mixings. The same result is derived in [5] by performing the computation in the original unrescaled metric.

After having established this, it is straightforward to compute the kinetic term for the graviton modes, since we only need to consider a metric perturbation

$$\delta(ds^2) = e^{2A(y)}\,\delta_K g_{\mu\nu}(x,y) dx^\mu dx^\nu\,. \tag{5.46}$$

The result is a warped version of the linearized Einstein-Hilbert action around a flat background [35],

$$\begin{aligned} S^{(h)}_{kin} &= \frac{1}{2\kappa_4^2 V_W}\int d^4x\int d^6y\sqrt{\tilde{g}_6}e^{-4A}\delta_K g^{\mu\nu}\,\delta_K G^{(4)}_{\mu\nu} \\ &= \frac{1}{2\kappa_4^2 V_W}\int d^4x\int d^6y\sqrt{\tilde{g}_6}e^{-4A}\Big[-\frac{1}{2}(\delta_K g^{\mu\nu}\,\Box\delta_K g_{\mu\nu}-\delta_K g\,\Box\delta_K g)+ \\ &+\;\delta_K g^{\mu\nu}(\partial^\sigma\partial_{(\mu}\delta_K g_{\nu)\sigma}-\partial_\mu\partial_\nu\delta_K g)\Big]\,. \end{aligned} \tag{5.47}$$

Indices are raised with $\eta^{\mu\nu}$, and $\delta_K g := \eta^{\mu\nu}\delta_K g_{\mu\nu}$.

Expanding $\delta_K g_{\mu\nu}$ in the internal fluctuations of (5.17), the field space metric is seen to be

$$G^{(h)}_{I_1 J_1} = \frac{1}{4\,V_W} \int d^6y\sqrt{\tilde{g}_6}\, e^{-4A(y)} Y_{I_1}(y) Y_{J_1}(y)\,, \tag{5.48}$$

which, from the orthogonality relation (5.22), is proportional to the identity matrix. We thus obtain a warped generalization of the usual gravity lagrangian (again taking $\mathcal{M}^{kk}$ to be the diagonal part of the metric (5.48))

$$\mathcal{L}^{(h)}_{kin} = \mathcal{M}^{kk} \sum_{I_1} E^{I_1}_{\mu\nu}(x)\, h^{\mu\nu}_{I_1}(x) \tag{5.49}$$

in terms of the linearized Einstein tensor

$$E^{I_1}_{\mu\nu}(x) := \frac{1}{2}\big(\Box h^{I_1}_{\mu\nu} - \eta_{\mu\nu}\Box h^{I_1} + \partial_\mu\partial_\nu h^{I_1} - \partial_\mu\partial^\lambda h^{I_1}_{\lambda\nu} - \partial_\nu\partial^\lambda h^{I_1}_{\lambda\mu} + \eta_{\mu\nu}\partial^\lambda\partial^\rho h^{I_1}_{\lambda\rho}\big)\,. \tag{5.50}$$

Since the four dimensional theory has $\mathcal{N} = 1$ supersymmetry, this is the bosonic part of a D-term. In terms of the real vector superfields H_μ (which contains the graviton and gravitino) and E_μ (whose $\bar{\theta}\theta$ component is the Einstein tensor), this D-term is

$$\mathcal{L}^{(h)}_{D} = M^{kk} \sum_{I_1} \eta^{\mu\nu} E^{I_1}_{\mu}\, H^{I_1}_{\nu}\,, \tag{5.51}$$

where we follow the notations of [82].

5.4.3 Kinetic terms for internal metric fluctuations

Consider now metric perturbations

$$\delta(ds^2) = 2e^{2A}\,\delta A\,\eta^{\mu\nu}dx^\mu dx^\nu + e^{-2A}\big(-2\delta A\,\tilde{g}_{mn} + \delta\tilde{g}_{mn}\big)dy^m dy^n\,, \tag{5.52}$$

induced by fluctuations $\delta\tilde{g}_{mn}$ of complex and Kähler structure deformations *plus* their KK modes. The zero mode sector was analyzed in chapter 2, where we computed the kinetic term Eq. (2.68). The full KK tower Eq. (5.16) can be taken into account in the same way, and we find

$$S^{(u)}_{kin} = \frac{1}{8\kappa^2_{10}} \int d^4x \int d^6y\,\sqrt{\tilde{g}_6}\,e^{-4A}\,\delta\tilde{g}^{mn}\Box\,\delta\tilde{g}_{mn} = \frac{1}{2\kappa_4^2}\int d^4x\, G^{(u)}_{I_2\bar{J}_2}\, u^{I_2}\,\Box\bar{u}^{J_2} \tag{5.53}$$

where we have used the expansion of Eq. (5.18) to write this in terms of a field space metric,

$$G^{(u)}_{I_2\bar{J}_2} = \frac{1}{4V_W}\int d^6y\,\sqrt{\tilde{g}_6}\,e^{-4A}\,Y_{I_2,\,mn}(y)\bar{Y}^{\widetilde{mn}}_{J_2}(y)\,. \tag{5.54}$$

Geometrically, $\delta\tilde{g}_{mn}$ is the extrinsic curvature specifying how the internal manifold is fibered over the space of fluctuations u^I. They are constrained by Eq. (5.33).

Again, the metric (5.54) is proportional to the orthogonality relation (5.23), so that the kinetic term is diagonal in the different KK levels,

$$\mathcal{L}^{(u)}_{kin} = G^{(u)} \sum_{I_2} u^{I_2} \Box \bar{u}^{I_2} \,. \tag{5.55}$$

This is one of the main results of our work [5]. If one is only interested in the propagator of the metric moduli, then even at strong warping it is consistent to truncate the analysis to the zero KK level. As was pointed out by [31], a warp factor does induce mixings between the moduli and the *unwarped* KK modes, given in Eq. (5.8). Such modes don't represent light four dimensional excitations. Rather, these are given by the warped eigenvectors of Eq. (5.21), in terms of which there is no kinetic mixing.

One intriguing consequence of Eq. (5.54) is that it could allow for a nontrivial mixing between complex and Kähler moduli. This would happen if, for instance, solving Eq. (5.33) requires a complex structure deformation to acquire $(1,1)$ pieces Such terms would have a nontrivial overlap with the Kähler moduli wavefunction. While the explicit mixings are not known yet, a possible explanation is that in the presence of warping the true holomorphic coordinates become combinations of the CY complex and Kähler deformations.

5.5 Geometric masses and flux-induced interactions

To complete our analysis of the effective action to quadratic order, we need to compute the mass terms for the low energy fields. These arise from geometric effects (KK masses) and/or from the background flux. Let us consider the KK mass matrix first.

5.5.1 Kaluza-Klein masses

The simplest case corresponds to the dilaton. After expanding around τ_0, the mass matrix reads

$$M^{(\tau)2}_{I_1 J_1} = \frac{1}{4 \mathrm{Im}\, \tau_0 \, V_W} \int d^6 y \sqrt{\tilde{g}_6}\, Y_{I_1}(y)\, \tilde{\nabla}^2 Y_{J_1}(y) = \mathcal{M}^{kk}_{\tau}\, \delta_{I_1 J_1} \,. \tag{5.56}$$

Therefore,

$$\mathcal{L}^{(\tau)} = \mathcal{M}^{kk}_{\tau} \sum_{I_1} t^{I_1} (\Box + \lambda^2_{I_1}) \bar{t}^{I_1} \,. \tag{5.57}$$

As an example of KK masses for p-forms, we can write down the mass term for the vector coming from C_4,

$$M^{(V)2}_{IJ}\, V^I \wedge \star_4 V^J \tag{5.58}$$

where

$$M^{(V)2}_{IJ} = \frac{1}{4\, V_W} \int_M e^{4A}\, d\chi_I(y) \wedge \tilde{\star}_6 d\, \chi_J(y) \,. \tag{5.59}$$

Notice that, while the field space metric for the vector V_μ is unwarped, the warp factor enters into the mass matrix. Therefore this sector also exhibits light bound states, much as in the scalar field discussion.

The KK masses for the metric fluctuations follow from the effective action

$$S_{eff} = -\frac{1}{4\kappa_{10}^2} \int d^{10}x \sqrt{g} \left[(\delta g)^{\mu\nu} \delta G_{\mu\nu} + (\delta g)^{mn} \delta G_{mn} \right] \tag{5.60}$$

if we consider the metric fluctuations (5.42), but with variations being space-time independent. The conformal rescaling used to explain why there are no spacetime derivative mixings between the graviton and internal metric modes does not rule out mass mixings of the form $\delta_K g\, \delta\tilde{g}$. Therefore we need to consider both types of fluctuations simultaneously.

The full computation is explained in [5], and here we summarize the results. After making use of the constraints in (5.32) and (5.33), the mass terms simplify to

$$S_{mass} = \frac{1}{4\kappa_{10}^2} \int d^{10}x \sqrt{\tilde{g}_6} \Big[\frac{1}{2} \delta_K g_{\mu\nu} \tilde{\nabla}^2 (\delta_K g^{\mu\nu} - \eta^{\mu\nu} \delta_K g) - \delta\tilde{g}^{mn} \delta\tilde{G}_{mn} + \frac{1}{2} \delta_K g \delta\tilde{R} \Big]\,. \tag{5.61}$$

The first two terms give rise to geometric KK masses for the graviton and internal metric, while the last one mixes these massive sectors. We conclude that there are no mixing with the metric moduli, which satisfy $\delta\tilde{G}_{mn} = 0$. It is easily seen that $\delta\tilde{G}_{mn} = 0$ implies $\delta\tilde{R} = 0$ for a background unwarped metric which is Ricci-flat, i.e., $\tilde{R} = 0$.

Eq. (5.61) shows massive gravitons coupled to KK modes from the internal metric. This has a natural interpretation as a Higgs-type mechanism triggered by the spontaneous breaking of ten dimensional diffeomorphism invariance ($\langle g_{\mu\nu} \rangle$ and $\langle g_{mn} \rangle$ are nonzero). For instance, in the original Kaluza-Klein compactification on $\mathbb{R}^{(3,1)} \times S^1$, the infinite tower of massive spin 2 fields comes from combining the 4d gravitons plus Goldstone modes of spin 0 (from g_{55}) and spin 1 (from $g_{\mu 5}$). As in the gauge theory case, it should be possible to represent the massive states by a gauge invariant field combining the states of helicity 0, 1 and 2. This was done for the S^1 case in [83, 84]. It would be very interesting to extend that analysis to the warped compactifications discussed here, and also to provide an explicitly supersymmetric construction [85].

5.5.2 No mixing with p-form modes

Before starting our analysis of the flux potential, we show here that there are no mass mixings between the complex moduli S and KK modes coming from (B_2, C_2, C_4). After dimensional reduction, these 10d forms give 4d forms of various ranks. The first point to note is that, due to Lorentz invariance, the scalar field S can only mix with the zero forms; hence we restrict our attention to them.

First consider possible mixings coming from C_4 and the self-dual term. To account for self-duality, we set $\tilde{F}_5 = dC_4$ and multiply by two the terms where C_4 appears. After eliminating half of the degrees of freedom, the remaining KK modes from C_4 which contribute to $\tilde{F}_5^2$ term are either 1 and 2-forms in space-time, which cannot lead to mixing with moduli by Lorentz invariance, or 0-forms. Explicit computation shows that the scalar coming from C_4 does not lead to mixing.

Next, the bilinear terms involving S and the zero forms from (B_2, C_2) come from combining the $|G_3|^2$ and CS terms, yielding the usual term

$$S_{mix} = -\frac{1}{4\kappa_{10}^2 \operatorname{Im}\tau} \int G_3 \wedge \left(\star_{10} \bar{G}_3 + iC_4 \wedge \bar{G}_3 \right) . \tag{5.62}$$

Here we assume a constant dilaton background; mixings with the dilaton KK modes will be analyzed momentarily. Expanding in a complete basis of internal two forms $\omega_A(y)$, the KK mode contribution to the 3-form reads

$$\delta G_3^{KK} = d\Big([c_A(x) - \tau b_A(x)]\, \omega_A(y) \Big) ,$$

where a sum over A is implicit. If $\omega_A \in H^2(M)$, we recover the usual four dimensional zero modes which do not mix with S. Here we are interested in the massive modes, for which ω_A is not closed. Replacing in (5.62) and expanding to quadratic order in the fields, we have (note that there are no quadratic terms with spacetime derivatives),

$$S_{mix} = -\frac{1}{4\kappa_{10}^2 \operatorname{Im}\tau} \int d^4x\, (c_A - \tau b_A)\bar{S} \int_M d\omega_A(x) \wedge \partial_{\bar{S}} \Big(e^{4A} [\star_6 \bar{G}_3 + i\bar{G}_3] \Big) . \tag{5.63}$$

Under a complex moduli fluctuation, the G_3 equation of motion implies that $\bar{\Lambda} = e^{4A}[\tilde{\star}_6 \bar{G}_3 + i\bar{G}_3]$ is closed, so $S_{mix} = 0$ after integrating by parts.[3]

5.5.3 Flux-induced mass terms

In truncations to the zero mode sector, the main role of these quantized fluxes is to lift the complex moduli, via the Gukov-Vafa-Witten superpotential. It is well understood how this contribution arises in unwarped scenarios but, as expected, the presence of warping introduces many new subtleties and potential mixings.

The flux-induced masses for metric moduli and KK modes follow from Eq. (5.35). This involves computing the fluctuated energy momentum tensors for G_3 and $\tilde{F}_5$, and then contracting with the fluctuated metric. Also, recalling that we are working in backgrounds satisfying

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[3] In particular, $\bar{\Lambda}$ is a linear combination of Ω and $\bar{\chi}_S$.

$e^{4A} = \alpha$, one gets extra pieces coming from the gravitational part δG_{MN}, which depend on the warp factor. Furthermore, the equation of motion for α has to be imposed as a constraint, and this introduces flux dependence.

It turns out that the computation may be done including the moduli (the relevant ones here are the complex moduli and axio-dilaton) and their KK modes, in a symmetric way; we refer to [5] for more details. The flux induced mass terms including moduli and KK modes are

$$\begin{aligned} S_{flux} &= -\frac{1}{2\kappa_{10}^2}\int d^4x \int d^6y\, \sqrt{\tilde{g}_6}\; e^{-2A}\left\{|\delta_K\tau|^2\partial_\tau\partial_{\bar{\tau}}(\frac{G_3\cdot\bar{G}_3}{24\mathrm{Im}\tau}) + \right.\\ &+ \left. \delta\tilde{g}_{\tilde{m}}^{\tilde{n}}\delta\left[\frac{1}{8\mathrm{Im}\tau}\left(G_{npq}\,\bar{G}^{mpq} - \frac{1}{6}\delta_n^m\,|G_3|^2\right)\right]\right\} \end{aligned} \tag{5.64}$$

where the variation 'δ' in the last line includes both the axio-dilaton and internal metric fluctuations. To make the result more compact, indices with tildes are raised with $\tilde{g}_{mn}$, while the ones without tildes are raised with $g_{mn} = e^{-2A}\tilde{g}_{mn}$.

Restricting to the zero mode sector, this result shows the usual lifting of the moduli by fluxes. However, we would like to stress that we are including KK modes as well, as can be seen by inserting the mode expansion (5.18) into (5.64). One very interesting consequence of Eq. (5.64) is that the flux contribution may mix the zero modes with the massive fluctuations. It is very important to understand such mixings, since so far all the other terms in our effective action do not exhibit this effect (at least to quadratic order).

Unfortunately, S_{flux} presents a rather complicated structure and it seems that statements about mixings will depend strongly on the particular background, with the corresponding flux choice and form of $\delta\tilde{g}_{mn}$. Nevertheless, we now describe an alternative approach for finding S_{flux} which may be better suited for answering these sorts of questions. The method is based on two observations: first, to compute the potential it is enough to consider space-time independent fluctuations. Also, the expression as a power series (5.35) is only necessary to identify the 'geometrical' KK masses. In order to find the flux potential such terms may be set to zero, and an appropriate use of the 10d equations of motion gives us an answer to all orders in the fluctuations.

This is in fact the spirit of the original GKP derivation [25] or the more detailed approach of [27]. However, for a nontrivial warp factor some terms would be missing in their derivation, and we also want to include KK modes. The terms contributing to the potential are given in Eq. (1.41). First, the Ricci scalar part has the form

$$\int d^{10}x\sqrt{-g}R = \int d^4x \int d^6y\,\sqrt{g_6}\,\left[-8e^{4A}(\nabla A)^2 + \ldots\right]\,, \tag{5.65}$$

where the dots refer to terms induced by the KK modes, which are related to their geometric masses and do not depend on moduli. The flux dependence here comes from the equation of motion

Next, the G_3 term is already in the desired form. Finally, after integrating by parts and using the Bianchi identity for the 5-form, the $\tilde{F}_5^2$ and CS terms give

$$\int \left(\tilde{F}_5 \wedge \star_{10} \tilde{F}_5 + \frac{C_4 \wedge G_3 \wedge \bar{G}_3}{\mathrm{Im}\tau} \right) \to i \int d^4x \int \frac{e^{4A(y)}}{\mathrm{Im}\,\tau} \, G_3 \wedge \bar{G}_3 \,.$$

Combining these contributions, we arrive to

$$S_{flux} = -\frac{1}{4\kappa_{10}^2} \int d^4x \int \frac{e^{4A}}{\mathrm{Im}\,\tau} \, G_3 \wedge \left(\tilde{\star}_6 \bar{G}_3 + i \bar{G}_3 \right) . \tag{5.66}$$

As a check, the second order variation of this expression reproduces our previous result Eq. (5.64).

Summarizing, Eq. (5.66) gives the full flux potential for the metric fluctuations including KK modes. This has the same form as the potential including only complex moduli. There are, however, two differences. This expression is valid including axio-dilaton fluctuations, while the original derivation set τ to a constant. Further, the massive metric modes are encoded in the Hodge star. One cannot use the method of [27] to obtain the GVW superpotential from here, since for arbitrary massive fluctuations we do not know which are the 3-forms with definite self-duality properties under $\tilde{\star}_6$. It would be interesting to compute Eq. (5.66) for the Klebanov-Strassler geometry, where a nontrivial mixing between a complex modulus and their KK excitations would mean new terms in the Veneziano-Yankielowicz superpotential [50] coupling the gaugino condensate to other glueballs.

5.6 Future directions

We have computed the effective action to quadratic order, but we do not have a formulation in terms of $\mathcal{N} = 1$ superfields. This requires integrating the kinetic metrics of section 5.4 to find the Kähler potential, and also finding the generalization of the GVW superpotential that reproduces Eq. (5.66). Understanding the $\mathcal{N} = 1$ structure of string compactifications would be very valuable, both for theoretical reasons (e.g. better control over supergravity limits and gauge/gravity dualities) and for phenomenological applications.

In this last part of our analysis of dynamics of flux compactifications, we describe some future directions of research.

5.6.1 Towards the Kähler potential

Finding the Kähler potential for compactifications with 4 supercharges is still an open problem. Actually, this is common to other systems with $\mathcal{N} = 1$. Another example is the Kähler potential on the moduli space of hermitean Yang-Mills connections on heterotic compactifications [86].

In our case, the main difficulties stem, as expected, from the warp factor. It appears in the general formula for kinetic terms Eq. (2.50) and also in the constraints that physical fluctuations have to obey in order to be orthogonal to gauge transformations – Eqs. (2.45) and (2.48). The warp factor is determined from the rather complicated background equation XXX; the solution e^{-4A} will depend both on the sources which are present and on the CY metric. Because of this, it is hard to make progress in evaluating more explicitly the kinetic terms. One needs methods going beyond holomorphy and algebraic geometry.

Nevertheless, in this dissertation we have succeeded in evaluating the Kähler potential for two low energy modes of the theory, namely the universal Kähler modulus and the S field in the deformed conifold. So we will try to extract some general lessons from these results.

In chapter 3, we found the Kähler potential for the universal Kähler modulus,

$$K = -3 \log \left(-i(\rho - \bar{\rho}) - 2\, \frac{V_W^0}{V_{CY}} \right) . \tag{5.67}$$

This required constructing the full 10d (off-shell) solution. However, an interesting feature is that the explicit form of the compensating field B appearing in Eq. (3.23) was not needed to arrive to Eq. (5.67). It could be that this is a special feature of the universal modulus which is, to some extent, insensitive to internal fluctuations of the CY. But it would be worth exploring if for other moduli there exists a change of variables that explicitly decouples compensating fields from the kinetic term.

This Kähler potential can also be rewritten more suggestively in terms of the warped volume as

$$K = -3 \log \frac{V_W(\rho)}{V_{CY}} . \tag{5.68}$$

It is then tempting to hypothesize that this expression captures the Kähler potential for nonuniversal moduli as well, in analogy with the CY case. Then one would have

$$K = -\log \int e^{-4A}\, \tilde{J} \wedge \tilde{J} \wedge \tilde{J} , \tag{5.69}$$

where $\tilde{J} = \rho_r\, \omega^r$ is the harmonic Kähler form of the underlying CY, $\omega^r \in H^{(1,1)}(CY, \mathbb{C})$. This expression was first suggested by [27], but it suffers from some problems. For instance, the Kähler metric would be (ignoring variations of the warp factor),

$$G_{rs} = \frac{1}{2V_W} \int e^{-4A}\, \omega_r \wedge \tilde{\star}_6 \omega_s . \tag{5.70}$$

The ω_r are harmonic in the CY metric, but this does not agree with the kinetic term derived from the Hamiltonian approach, which implies that the gauge invariant forms satisfy a warped harmonic condition. Still, it may be that the answer is not far away. Computations in progress suggest that the Kähler potential for Kähler moduli could instead take the form

$$K = -\log \int e^{-4A}\, \tilde{J}_W \wedge \tilde{\star}_6 \tilde{J}_W \,, \tag{5.71}$$

in terms of a "warped" Kähler form $\tilde{J}_W := \tilde{J} + dK$. K is defined by requiring that $\tilde{J}_W$ be orthogonal to diffeomorphisms of the warped geometry.

On the other hand, in chapter 4, the Kähler metric for the complex modulus S in the warped deformed conifold was computed. Eq. (4.88) can be integrated and yields the Kähler potential

$$K(S,\bar{S}) = c\,|S|^2 \,\log\frac{\Lambda_0^3}{|S|} + k\,(\alpha' g_s N)^2\,|S|^{2/3} \tag{5.72}$$

where c and k are numerical constants. This example illustrates how warping effects can modify the low energy theory. The second term in Eq. (5.72) is a purely $\mathcal{N}=1$ contribution produced by the warp factor; it gives an extra term $G_{S\bar{S}} \sim |S|^{-4/3}$ that dominates in the semiclassical limit $S \to 0$.

In analogy with Eq. (5.68), one could try to reproduce Eq. (5.72) in a compact internal manifold, by postulating a Kähler potential of the form [27]

$$K = -\log \int e^{-4A}\,\Omega \wedge \bar{\Omega}\,. \tag{5.73}$$

However, this suffers from the same problems than the Kähler moduli proposal: the corresponding field space metric

$$G_{S\bar{S}} = -\frac{\int e^{-4A}\,\chi_S \wedge \bar{\chi}_S}{\int e^{-4A}\,\Omega \wedge \overline{\Omega}}\,. \tag{5.74}$$

is not orthogonal to gauge transformations since χ_S is harmonic with respect to the unwarped metric, while the physical fluctuations should be harmonic with respect to the full 10d metric.

Nevertheless, it was shown in [4] that Eq. (5.74) does reproduce the correct power-like scaling found in Eq. (5.72). One could then try to modify Eq. (5.74) by adding appropriate exact pieces to the 3-forms, to make it gauge invariant. In fact, this can be done explicitly for the conifold. Recalling the results of section 4.5.2 in chapter 4, we see that the effect of the η_r compensator is simply to set

$$\delta_S A = 0\ ,\ \delta_S g = 0\,.$$

In terms of the physical fluctuations, the warp factor becomes independent of S and the metric fluctuation is traceless. In fact, both are equivalent by the 4d constraint Eq. (5.32). Then the other constraint (Eq. (5.33)) may be rewritten as

$$\tilde{\nabla}^i\big(e^{-4A}\,\delta_S\tilde{g}_{ij}\big) = 0 \tag{5.75}$$

which is a warped generalization of the harmonic gauge. The associated 3-form

$$\chi_S = \tilde{g}^{ln}\,\Omega_{ijl}\,\delta_S\tilde{g}_{nk}\,dy^i dy^j dy^k \tag{5.76}$$

then satisfies

$$d\,\tilde{\star}_6(e^{-4A}\,\chi_S) = 0\,. \tag{5.77}$$

In other words, the effect of the compensating field η_ψ (see Eq. (4.85)) is to shift the original harmonic $(2,1)$ form by an exact piece so that the "physical" χ_S satisfies Eq. (5.77).

With this constraint, the field space metric reads

$$G_{S\bar{S}} = -\frac{\int e^{-4A}\,\chi_S\wedge\tilde{\star}_6\,\bar{\chi}_S}{\int e^{-4A}\,\Omega\wedge\overline{\Omega}}\,. \tag{5.78}$$

The Hodge star is needed because χ_S is no longer harmonic. The next step would be to prove that Eq. (5.77) is also valid in more general compactifications which admit a covariantly constant spinor. From here, one has to rework the moduli space geometry along the lines of [22].

5.6.2 Fermions in supergravity

An important point that could be explored is the dimensional reduction of the 10d fermionic sector in warped backgrounds. This is necessary if we want to determine the dynamics of the 4d gravitino and/or interactions with the Standard Model fermions. Also, analyzing the fermionic partners of the internal metric fluctuations can provide an alternative approach to computing the Kähler potential.

The first step is to relate the 4d and 10d gravitinos. The 10d fermionic action is [20, 27]

$$S_f = \frac{i}{\kappa_{10}^2}\int\left[\bar{\Psi}_M\Gamma^{MNP}\left(D_N\Psi_P - \frac{i}{2}Q_N\Psi_P - R_P\Psi_N\right) - \frac{1}{2}\left(\bar{\Psi}_M\Gamma^{MNP}S_P\bar{\Psi}^*_N - c.c.\right) + \ldots\right] \tag{5.79}$$

Here,

$$R_M = -\frac{i}{16\times 5!}\,\Gamma^{M_1\ldots M_5}\tilde{F}_{M_1\ldots M_5}\,\Gamma_M\ ,\ S_M = \frac{1}{96}\,(\mathrm{Im}\tau)^{-1/2}\left(\Gamma_M^{\ NPQ}G_{NPQ} - 9\Gamma^{NP}G_{MNP}\right) \tag{5.80}$$

and

$$Q_N = \frac{\mathrm{Im}(B\partial_M B^*)}{1-|B|^2}\ ,\ B = \frac{1+i\tau}{1-i\tau}\,. \tag{5.81}$$

Furthermore, '...' includes the dilatino terms plus higher order interactions.

In general, it is assumed that the 4d gravitino ψ_μ is embedded in the 10d field as

$$\Psi_\mu = \psi_\mu \otimes e^{A/2}\eta$$

where η is the covariantly constant CY spinor. However, this is not correct because Ψ_μ does not have diagonal kinetic terms. The 10d matrix Γ^{MNP} is not block-diagonal in the 4 and 6 dimensions, and this induces kinetic mixings between Ψ_μ and Ψ_m. This was noticed in [87].

Therefore, already the 4d gravitino exhibits some subtleties that have to be analyzed. A short computation reveals that the fields with diagonal kinetic terms are

$$\hat{\Psi}_\mu = \Psi_\mu + \frac{1}{2}\Gamma_\mu(\Gamma^m \Psi_m) \ \ , \ \ \hat{\Psi}_m = \Psi_m - \frac{1}{2}\Gamma_m(\Gamma^n \Psi_n)\,. \tag{5.82}$$

The 4d gravitino follows from the dimensional reduction of $\hat{\Psi}_\mu$. It would be interesting to work out its effective action explicitly – results on the holomorphic sector can be found in [87]. The presence of $\Gamma^n\Psi_n$ in $\hat{\Psi}_\mu$ could lead to new interaction terms relevant for supersymmetry breaking models.

Let us also make some preliminary comments on the internal gravitinos $\hat{\Psi}_m$. Their dimensional reduction gives the superpartners of the metric moduli and scalars arising from p-forms. It is not hard to check that Ψ_0 is an auxiliary field which, in the absence of matter, imposes the constraint

$$\Gamma^{0MN}\, D_M \Psi_N = 0\,.$$

Therefore, this sector should also be analyzed with the Hamiltonian approach, along the lines of chapter 2. Unfortunately not much is known about the Hamiltonian approach in supergravity; some references are [88, 89]. Also, the analysis should take into account the mass-mixings between $\hat{\Psi}_m$ and the dilatino. Notice that the fermionic fields are self-conjugate, and their kinetic terms are already in canonical form. It may be possible to use them to extract the Kähler potential, and we hope to come back to this point in the future.

5.6.3 Relations to generalized geometry

Finally, along the lines of the previous subsection, it is worth exploring approaches where the dimensional reduction keeps $\mathcal{N}=1$ supersymmetry manifest. The focus of this thesis has been on the bosonic sector of type IIB supergravity. In order to continue simplifying the Kähler metric and Hamiltonian constraints, one has to use the fact that the background preserves four supercharges.

In the adiabatic limit, where space-time dependence can be ignored, the most efficient way to encode the supersymmetric properties seems to be with generalized geometry [90, 91]. One could try to extend this formalism to allow for space-time dependence. Another point of view is currently being developed by [87, 92, 93]. In this approach, the 10d supergravity is formally rewritten as a theory with 4 supercharges by decomposing the structure group $Spin(1,9) \rightarrow Spin(1,3) \times Spin(6)$. No Kaluza-Klein reduction is made, and instead all the 10d degrees of freedom are kept. This allows to derive various structures analog to the $\mathcal{N}=1$ compactified theory. As a next step, one would have to find a recipe to extract the low energy degrees of freedom, and also impose the Hamiltonian constraints described above.

Chapter 6

Metastable supersymmetry breaking near points of enhanced symmetry

The last two chapters of this dissertation we shift emphasis from string theory to field theory, exploring new mechanisms of supersymmetry breaking directly in supersymmetric field theories.

6.1 Introduction and summary

The idea that our universe may be in a long-lived metastable state in which supersymmetry is broken has recently led to an increased interest in developing models of supersymmetry breaking. This has opened many new possibilities in constructing field theory and string theory models. In the last two chapters of this thesis, we explore new models of metastable supersymmetry breaking. Our aim is two-fold: to find models where the relevant parameters are generated dynamically, and to obtain an acceptable phenomenology. In this chapter, based on the work [6], we focus on the first aspect, while a model with realistic phenomenology is analyzed in chapter 7.

Relaxing the requirement that supersymmetry breaking occurs in the true vacuum (see e.g. [94–97]) can help overcome many of the constraints of dynamical supersymmetry breaking with no supersymmetric vacua [98]. Recently, Intriligator, Seiberg and Shih (ISS) [72] have shown that metastable dynamical supersymmetry breaking is rather generic and easy to achieve. They found that metastable vacua occur in supersymmetric QCD (SQCD), in the free magnetic range, when the quarks have small masses,

$$W = \mathrm{tr}\big(m\tilde{Q}Q\big)\,. \tag{6.1}$$

This has opened many new avenues for model building and gauge mediation; see [6, 99–114] for some examples of recent work, and [115] for a review and a more complete list of references. The ISS construction still contains relevant couplings in the form of masses for the quarks though, and the search for models with all the relevant parameters generated dynamically has proven difficult.

One lesson from ISS is that certain properties of moduli spaces can hint at the existence of metastable vacua. In their case, it was the existence of supersymmetric vacua coming in from infinity that signaled an approximate R-symmetry. Here we will point out that one should also look for another feature, namely, enhanced symmetry points, which are defined by the appearance of massless particles. We claim that if the moduli space has certain coincident enhanced symmetry points, metastable vacua with all the relevant couplings arising by dimensional transmutation may be obtained.

Let us motivate this claim. In order to generate relevant couplings dynamically, a gauge sector is required, which gives nonperturbative contributions to the superpotential. However, in general this leads to a runaway behavior. We will show that starting with two gauge sectors, the runaway may now be stabilized by one loop effects from the additional gauge sector, but only around enhanced symmetry points where quantum corrections are large enough. Such runaways which are stabilized by perturbative quantum corrections will be called 'pseudo-runaways'. Surprisingly, the gauge theories where this occurs turn out to be generic.

The model considered here consists of two SQCD sectors, each with independent rank and number of flavors, coupled by a singlet. It involves only marginal operators with all scales generated dynamically. At the origin of moduli space, the singlet vanishes and the quarks of both sectors become massless simultaneously. There are thus two coincident enhanced symmetry points at the origin. While one of the SQCD sectors is in the electric range and produces a runaway, the other has a magnetic dual description as an O'Raifeartaigh-like model. Near the enhanced symmetry point, the Coleman-Weinberg corrections stabilize the nonperturbative instability producing a long-lived metastable vacuum. A feature of our model is that it may be possible to gauge parts of its large global symmetry to obtain renormalizable, natural models of direct gauge mediated supersymmetry breaking with a singlet. R-symmetry is broken both spontaneously and explicitly in our model.

The chapter is organized as follows. In Section 6.2, our model is introduced and its supersymmetric vacua are studied. In Section 6.3, we analyze in detail the non-supersymmetric vacua and argue that they are parametrically long-lived. In Section 6.4, we give a detailed analysis of the particle spectrum and the R-symmetry properties. In Section 6.5, we argue that such metastable vacua may be generic near points of enhanced symmetry in the landscape of effective field theories. In Section 6.6, we give our conclusions.

6.2 The Model and its supersymmetric vacua

We consider models with two supersymmetric QCD (SQCD) sectors characterized by (N_c, N_f, Λ) and (N_c', N_f', Λ'), respectively, that are coupled to the same singlet field Φ. The field Φ provides the mass of the quarks in both sectors. In Section 2.1, the general properties of such models will be discussed and their global symmetries analyzed. In Section 2.2, we analyze the supersymmetric vacua. Section 2.3 will discuss for which range of the parameters (N_c, N_f, Λ) and (N_c', N_f', Λ') metastable vacua will be shown to exist. The upshot will be that one sector has to be taken in the electric range and the other sector in the free magnetic range.

6.2.1 Description of the Model

The matter content of the models considered here consists of two copies of supersymmetric QCD, each with independent rank and number of flavors, and a single gauge singlet chiral superfield:

$$\begin{array}{cccc} & SU(N_c) & SU(N_c') & \\ Q_i & \square & 1 & i = 1, \ldots, N_f \\ \overline{Q}_i & \overline{\square} & 1 & \\ P_{i'} & 1 & \square & i' = 1, \ldots, N_f' \\ \overline{P}_{i'} & 1 & \overline{\square} & \\ \Phi & 1 & 1 & \end{array} \tag{6.2}$$

The most general tree-level superpotential with only relevant or marginal terms in four dimensions for the matter content (6.2) with N_c, $N_c' \geq 4$ is

$$W = (\lambda_{ij}\Phi + \xi_{ij})Q_i\overline{Q}_j + (\lambda'_{i'j'}\Phi + \xi'_{i'j'})P_{i'}\overline{P}_{j'} + w(\Phi)\,, \tag{6.3}$$

where $w(\Phi)$ is a cubic polynomial in Φ. Remarkably, we shall find metastable vacua even in the simplest case of $w(\Phi) = 0$, which we assume from now on. The general situation is discussed in Section 5 (in [116], the case $w(\Phi) = \kappa\Phi^3$ was used to stabilize Φ supersymmetrically).

At the classical level, the superpotential with $w(\Phi) = 0$ has an $U(1)_R \times U(1)_V \times U(1)'_V$

global symmetry under which the fields transform as

$$
\begin{array}{cccc}
 & U(1)_R & U(1)_V & U(1)'_V \\
Q_i & +1 & +1 & 0 \\
\overline{Q}_i & +1 & -1 & 0 \\
P_{i'} & +1 & 0 & +1 \\
\overline{P}_{i'} & +1 & 0 & -1 \\
\Phi & 0 & 0 & 0 \\
\Lambda^{3N_c-N_f} & 2N_c & 0 & 0 \\
\Lambda'^{3N'_c-N'_f} & 2N'_c & 0 & 0
\end{array}
\tag{6.4}
$$

where the normalizations of the $U(1)_V \times U(1)'_V$ charges are arbitrary. In the quantum theory the $U(1)_R$ symmetry is anomalous with respect to the $SU(N_c)$ and $SU(N'_c)$ gauge dynamics. The theta angles θ and θ' transform inhomogeneously under $U(1)_R$, and the holomorphic dynamical scale,

$$
(\Lambda/\mu)^{3N_c-N_f} = e^{-8\pi^2/g^2(\mu)+i\theta}\,, \tag{6.5}
$$

and likewise for $\Lambda'^{3N'_c-N'_f}$, transform with charges given in (6.4). The $U(1)_R$ symmetry is broken explicitly by the anomalies to the anomaly free discrete subgroups $Z_{2N_c} \subset U(1)_R$ and $Z_{2N'_c} \subset U(1)_R$, respectively. The largest simultaneous subgroup of both Z_{2N_c} and $Z_{2N'_c}$ which is left invariant by the superpotential (6.3) which couples the two gauge sectors through Φ interactions is $Z_{\mathrm{GCD}(2N_c,2N'_c)} \subset U(1)_R$, where $\mathrm{GCD}(2N_c, 2N'_c)$ is the greatest common divisor of $2N_c$ and $2N'_c$.

In the $SU(N_f)_V \times SU(N'_f)_V$ global symmetry limit the superpotential (6.3) (with $w(\Phi) = 0$) reduces to

$$
W = (\lambda\Phi + \xi)\mathrm{tr}(Q\overline{Q}) + (\lambda'\Phi + \xi')\mathrm{tr}(P\overline{P})\,. \tag{6.6}
$$

This superpotential has the same $U(1)_R \times U(1)_V \times U(1)'_V$ global symmetry as (6.3), as well as a $Z_2 \times Z_2$ conjugation symmetry under which $Q_i \leftrightarrow \overline{Q}_i$ and $P_i \leftrightarrow \overline{P}_i$, respectively. The form of the superpotential (6.6) may be enforced for any N_c and N'_c by weakly gauging the $SU(N_f)_V \times SU(N'_f)_V$ symmetry. One of the masses, ξ or ξ', may always be absorbed into a shift of Φ. For $\xi = \xi'$ both masses may simultaneously be absorbed into a shift of Φ, and the tree level superpotential in this case reduces to

$$
W = \lambda\Phi\ \mathrm{tr}(Q\overline{Q}) + \lambda'\Phi\ \mathrm{tr}(P\overline{P})\,. \tag{6.7}
$$

This form agrees with the naturalness requirement that there be no relevant couplings. $\Phi = 0$ is an enhanced symmetry point for both sectors, where the respective quarks become massless. The case $\xi \neq \xi'$ is analyzed in Section 5.

At the classical level this superpotential has an $U(1)_R \times U(1)_A \times U(1)_V \times U(1)'_V$ global symmetry

$$
\begin{array}{ccccc}
 & U(1)_R & U(1)_A & U(1)_V & U(1)'_V \\
Q_i & +1 & -\frac{1}{2} & +1 & 0 \\
\overline{Q}_i & +1 & -\frac{1}{2} & -1 & 0 \\
P_{i'} & +1 & -\frac{1}{2} & 0 & +1 \\
\overline{P}_{i'} & +1 & -\frac{1}{2} & 0 & -1 \\
\Phi & 0 & +1 & 0 & 0 \\
\Lambda^{3N_c-N_f} & 2N_c & -N_f & 0 & 0 \\
\Lambda'^{3N'_c-N'_f} & 2N'_c & -N'_f & 0 & 0
\end{array}
\tag{6.8}
$$

where the normalizations of the $U(1)_A \times U(1)_V \times U(1)'_V$ charges are arbitrary. The $U(1)_R$ charges are only defined up to an addition of an arbitrary multiple of the $U(1)_A$ charges. In the quantum theory both the $U(1)_R$ and $U(1)_A$ symmetries are anomalous. With the classical charge assignments (6.8) the $U(1)_R$ symmetry is broken explicitly by the $SU(N_c)$ and $SU(N'_c)$ gauge dynamics to the anomaly free discrete subgroup $Z_{\mathrm{GCD}(2N_c,2N'_c)} \subset U(1)_R$ as described above. Likewise, the $U(1)_A$ symmetry is broken explicitly by $SU(N_c)$ and $SU(N'_c)$ gauge dynamics to anomaly free discrete subgroups $Z_{N_f} \subset U(1)_A$ and $Z_{N'_f} \subset U(1)_A$, respectively. The largest simultaneous subgroup of both Z_{N_f} and $Z_{N'_f}$ which is left invariant by the superpotential (6.7) is $Z_{\mathrm{GCD}(N_f,N'_f)} \subset U(1)_A$. The form of the potential (6.7) may be enforced by gauging the non-anomalous discrete $Z_{\mathrm{GCD}(N_f,N'_f)}$ symmetry if it is non-trivial, along with weakly gauging the $SU(N_f)_V \times SU(N'_f)_V$ symmetry. This forbids the presence of a polynomial dependence $w(\Phi)$.

The marginal tree-level superpotential (6.7) is, up to irrelevant terms, of rather generic form within many UV completions of theories with moduli dependent masses. It requires only that the masses of the flavors of both gauge groups are moduli dependent functions, and that all flavors become massless at a single point in moduli space, here defined to be $\Phi = 0$. Importantly for the discussion of metastable dynamical supersymmetry breaking below, the superpotential (6.7) contains only marginal terms, so that any relevant mass scales must arise from dimensional transmutation. Generalizations to other gauge groups and matter contents in vector-like representations with the superpotential (6.7) are straightforward.

The classical moduli space for the theory (6.2) with superpotential (6.7) depends on the gauge group ranks and number of flavors. For $\lambda = \lambda' = 0$ the moduli space is parameterized by Φ, meson invariants $M_{ij} = Q_i\overline{Q}_j$ and $M'_{i'j'} = P_{i'}\bar{P}_{j'}$ and for $N_f \geq N_c$ and/or $N'_f \geq N'_c$

baryon and anti-baryon invariants $B_{i_1 i_2 \ldots i_{N_c}} = Q_{[i_1} Q_{i_2} \cdots Q_{i_{N_c}]}$, $\overline{B}_{i_1 i_2 \ldots i_{N_c}} = \overline{Q}_{[i_1} \overline{Q}_{i_2} \cdots \overline{Q}_{i_{N_c}]}$, and/or $B'_{i_1 i_2 \ldots i_{N'_c}} = P_{[i_1} P_{i_2} \cdots P_{i_{N'_c}]}$, $\overline{B}'_{i_1 i_2 \ldots i_{N'_c}} = \bar{P}_{[i_1} \bar{P}_{i_2} \cdots \bar{P}_{i_{N'_c}]}$ respectively. For $\lambda, \lambda' \neq 0$ the superpotential (6.7) lifts all the moduli parameterized by the mesons. The remaining moduli space has a branch parameterized by Φ. For $\Phi \neq 0$ the flavors are massive and the baryon and anti-baryon directions are lifted along this branch. For $N_f \geq N_c$ and/or $N'_f \geq N'_c$ there is a second branch of the moduli space parameterized by the baryons and anti-baryons with $\Phi = 0$. The two branches touch at the point where all the moduli vanish.

6.2.2 Supersymmetric Vacua

The classical moduli space of vacua is lifted by nonperturbative effects in the quantum theory. Since the metastable supersymmetry breaking vacua discussed below arise for $\Phi \neq 0$, only this branch of the moduli space will be considered in detail. On this branch, holomorphy, symmetries, and limits fix the exact superpotential written in terms of invariants, to be

$$
\begin{aligned}
W \quad &= \quad \lambda \Phi \ \mathrm{Tr} M + (N_c - N_f) \left[\frac{\Lambda^{3N_c - N_f}}{\det\, M} \right]^{1/(N_c - N_f)} \\
&\quad + \lambda' \Phi \ \mathrm{Tr} M' + (N'_c - N'_f) \left[\frac{\Lambda'^{3N'_c - N'_f}}{\det\, M'} \right]^{1/(N'_c - N'_f)}
\end{aligned} \tag{6.9}
$$

For gauge sectors in the free magnetic range, the nonperturbative contribution refers to the Seiberg dual. Since the meson invariants are lifted on this branch, they may be eliminated by equations of motion, $\partial W / \partial M_{ij} = 0$ and $\partial W / \partial M'_{i'j'} = 0$, to give the exact superpotential in terms of the classical modulus Φ

$$
W = N_c \left[(\lambda \Phi)^{N_f} \Lambda^{3N_c - N_f} \right]^{1/N_c} + N'_c \left[(\lambda' \Phi)^{N'_f} \Lambda'^{3N'_c - N'_f} \right]^{1/N'_c} . \tag{6.10}
$$

The supersymmetric minima are given by stationary points of the superpotential, $\partial W / \partial \Phi = 0$, for which

$$
N_f \left[(\lambda \Phi)^{N_f} \Lambda^{3N_c - N_f} \right]^{1/N_c} + N'_f \left[(\lambda' \Phi)^{N'_f} \Lambda'^{3N'_c - N'_f} \right]^{1/N'_c} = 0 \, . \tag{6.11}
$$

Physically distinct supersymmetric vacua are distinguished by the expectation value of the superpotential.

6.2.3 Parameter ranges for the gauge sectors

Under mild assumptions we thus end up considering two SQCD sectors, characterized by (N_c, N_f, Λ) and (N'_c, N'_f, Λ'), respectively, and superpotential couplings (6.7). Different choices may be considered here; to restrict them, it is important to note that calculable quantum corrections can be generated in two different limits.

For $\lambda_i\Phi \gg \Lambda_i$, with $\Lambda_i = \Lambda$ or Λ', the corresponding gauge group is weakly coupled and hence generates small calculable corrections to the Kähler potential. Integrating out the massive quarks, for energies below Φ, leads to gaugino condensation, which gives nonperturbative contributions as in (6.10).

On the other hand, for $\lambda_i\Phi \ll \Lambda_i$, the corresponding gauge sector becomes strongly coupled. The calculable case corresponds to having the gauge theory in the free magnetic range. For concreteness, we choose this sector to be $SU(N_c)$ (the unprimed sector), so that $N_c+1 \leq N_f < \frac{3}{2}N_c$.

For the (N_c', N_f', Λ') (primed) sector, the interesting case arises for $N_f' < N_c'$ and $\lambda'\Phi \gg \Lambda'$. Although the classical superpotential pushes Φ to zero, the primed dynamics generates a nonperturbative term which makes the potential energy diverge as $\Phi \to 0$, in agreement with the fact that $\Phi = 0$ corresponds to an enhanced symmetry point where P and $\bar{P}$ become massless. Balancing the primed and unprimed contributions leads to a runaway direction in moduli space which will be lifted by one loop corrections. This stabilizes Φ at a nonzero value. Calculability demands working in the energy range $E \gg \Lambda'$ and $E \ll \Lambda$ so the dynamically generated scales must satisfy $\Lambda' \ll \Lambda$.

The semiclassical limit corresponds to energies $E \gg \Lambda, \Lambda'$, where both sectors are weakly coupled. Since $\Lambda' \ll \Lambda$, $SU(N_c)$ confines first when flowing to the IR. For $\Lambda' \ll E \ll \Lambda$, the primed sector is weakly interacting while the unprimed sector has a dual weakly coupled description [117] in terms of the magnetic gauge group $SU(\tilde{N}_c)$ with $\tilde{N}_c = N_f - N_c$, N_f^2 singlets M_{ij}, and N_f magnetic quarks $(q_i, \tilde{q}_j)$. In terms of this description, the full non-perturbative superpotential reads

$$\begin{aligned} W &= m\Phi \mathrm{tr} M + h \mathrm{tr} q M \tilde{q} + \lambda' \Phi \mathrm{tr} P\bar{P} + (N_c' - N_f') \left(\frac{\Lambda'^{3N_c' - N_f'}}{\det P\bar{P}} \right)^{1/(N_c' - N_f')} \\ &\quad + (N_f - N_c) \left(\frac{\det M}{\tilde{\Lambda}^{3N_c - 2N_f}} \right)^{1/(N_f - N_c)} . \end{aligned} \tag{6.12}$$

Hereafter, $M_{ij} = Q_i\bar{Q}_j/\Lambda$, and $m := \lambda\Lambda$. The magnetic sector has a Landau pole at $\tilde{\Lambda} = \Lambda$.

In this description, the meson M and the primed quarks $(P,\ \bar{P})$ become massless at $\Phi = 0$. $M = 0$ is also an enhanced symmetry point since here the magnetic quarks $(q,\ \tilde{q})$ become massless.

6.3 Metastability near enhanced symmetry points

In this section, metastable vacua near the origin of moduli space will be shown to exist for the theory with superpotential (6.12). In Section 3.1, we analyze the branches of the moduli space and determine where Coleman-Weinberg effects may lift the runaway. Next, in 3.2, we focus on the region containing metastable vacua. In 3.3, we argue that other quantum corrections are under control and do not affect the stability of these vacua. Finally, in Section 3.4 the metastable vacua are shown to be parametrically long-lived.

6.3.1 Exploring the moduli space

Starting from the superpotential (6.12), the discussion is simplified by taking the limit $\tilde{\Lambda} \to \infty$, while keeping m fixed. The nonperturbative $\det M$ term is only relevant for generating supersymmetric vacua, as discussed in (6.10), and not important for the details of the metastable vacua that will arise near $M = 0$. Thus, for $M/\tilde{\Lambda} \to 0$ and $\Phi/\tilde{\Lambda} \to 0$, it is enough to consider the superpotential

$$W = m\Phi \text{ tr } M + h \text{ tr } qM\tilde{q} + \lambda'\Phi \,\mathrm{tr}\, P\bar{P} + (N'_c - N'_f) \left(\frac{\Lambda'^{3N'_c - N'_f}}{\det P\bar{P}} \right)^{1/(N'_c - N'_f)} . \tag{6.13}$$

In this limit all the fields are canonically normalized and the classical potential is

$$V = V_D + V'_D + \sum_a |W_a|^2 \tag{6.14}$$

where $W_a = \partial_a W$, and a runs over all the fields. V_D and V'_D are the usual D-term contributions from $SU(\tilde{N}_c)$ and $SU(N'_c)$. Since both gauge sectors are weakly coupled, it is enough to consider the F-terms on the D-flat moduli space, parametrized by the chiral ring. This restriction has no impact on the analysis of the metastable vacua.

Let us study the regime $P\bar{P} \to \infty$. Then nonperturbative effects from $SU(N'_c)$ may be neglected, and the classical superpotential

$$W_{cl} = m\Phi \text{ tr } M + h \text{ tr } qM\tilde{q} + \lambda'\Phi \,\mathrm{tr}\, P\bar{P} \tag{6.15}$$

is recovered. Setting

$$W_{M_{ij}} = m\Phi\delta_{ij} + hq_i\tilde{q}_j = 0\,, \tag{6.16}$$

we obtain $\Phi = 0$ and $hq\tilde{q} = 0$. This implies $W_{\mathrm{tr}P\bar{P}} = W_q = 0$. The locus $W_\Phi = 0$ then defines a classical moduli space of supersymmetric vacua.

Let us keep $P\bar{P}$ large, but include the non-perturbative effects from $SU(N'_c)$. Then $W_{\mathrm{tr}P\bar{P}} = 0$ sets $P\bar{P} \to \infty$ and $W_\Phi = 0$ implies $M \to \infty$. Therefore the model does not have a stable vacuum in the limit $\tilde{\Lambda} \to \infty$. As discussed above, for $\tilde{\Lambda}$ finite and M large enough, the nonperturbative $\det M$ term introduces supersymmetric vacua as in (6.10).

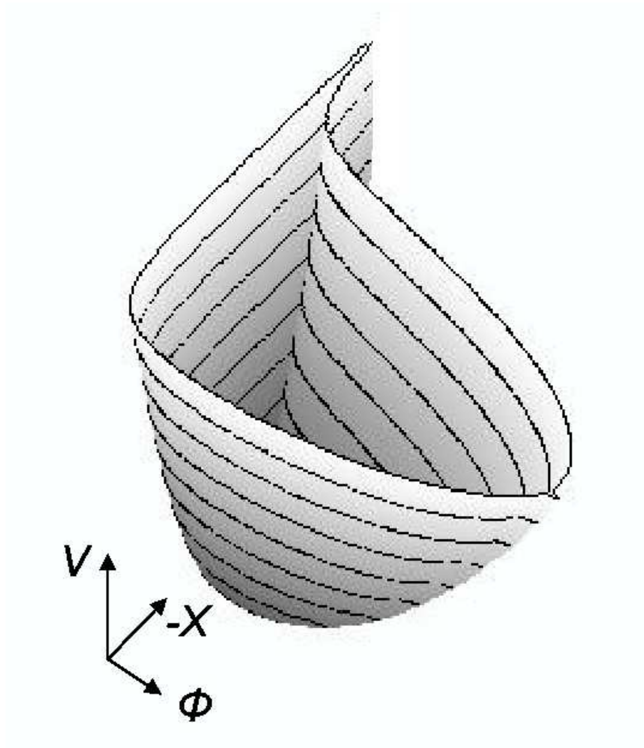

Figure 6.1: A plot showing the global shape of the potential. M has been expanded around zero as in equation (3.8). Note the runaway in the direction $X \to -\infty$ and $\phi \to 0$. The singularity at $\phi = 0$ and the "drain" $W_\phi = 0$ are clearly visible. Also visible is the Coleman-Weinberg channel near $X = 0$ and ϕ large, discussed later.

All the F-terms are small in the limit $M \to \infty$, $\Phi \to 0$, which thus corresponds to $M_F^2 \gg |F|$. The one-loop corrections give logarithmic dependences on the fields (Φ, M) and these cannot stop the power-law runaway behavior.

Thus we are led to consider the region near the enhanced symmetry point $M = 0$. As we shall see below, this still has a runaway. Crucially, it turns out that one-loop corrections stop this runaway (this novel effect is characterized as a "pseudo-runaway"). The reason for this is that the Coleman-Weinberg formula [118]

$$V_{CW} = \frac{1}{64\pi^2}\,\mathrm{Str}\, M^4 \,\ln\, M^2 \tag{6.17}$$

will have polynomial (instead of logarithmic) dependence. This will be explained next.

A global plot of the potential is provided in Fig. 6.1, where M has been expanded around zero as below in equation (3.8). In the graphic, the 'drain' towards the supersymmetric vacuum corresponds to the curve $W_\Phi = 0$.

6.3.2 Metastability Along the Pseudo-Runaway Direction

In the region $\Phi \neq 0$, $(P, \bar{P})$ may be integrated out by equations of motion provided that $\Lambda' \ll \lambda'\Phi$. This is a good description if we are not exactly at the origin but near it, as given by $\Phi/\tilde{\Lambda} \ll 1$. Taking, as before, $\tilde{\Lambda} \to \infty$ and m fixed, the superpotential reads

$$W = m\Phi\,\mathrm{tr}\,M + h\,\mathrm{tr}\,qM\tilde{q} + N_c'\big[\lambda'^{N_f'}\,\Lambda'^{3N_c'-N_f'}\,\Phi^{N_f'}\big]^{1/N_c'} . \tag{6.18}$$

This description corresponds to an O'Raifeartaigh-type model in terms of magnetic variables but with no flat directions.

Given that $\phi = \langle\Phi\rangle \neq 0$, we will expand around the point of maximal symmetry

$$q = \begin{pmatrix} q_0 & 0 \end{pmatrix} \ , \ \tilde{q} = \begin{pmatrix} \tilde{q}_0 \\ 0 \end{pmatrix} \ , \ M = \begin{pmatrix} 0 & 0 \\ 0 & 0 + X \cdot I_{N_c \times N_c} \end{pmatrix} . \tag{6.19}$$

Here q_0 and $\tilde{q}_0$ are $\tilde{N}_c \times \tilde{N}_c$ matrices satisfying

$$hq_{0i}\tilde{q}_{0j} = -m\phi\,\delta_{ij} \ , \ i,j = \tilde{N}_c + 1, \ldots N_f \ , \tag{6.20}$$

and the nonzero block matrix in M has been taken to be proportional to the identity; indeed, only $\mathrm{tr}\,M$ appears in the potential. This minimizes W_M and sets $W_q = W_{\tilde{q}} = 0$. The spectrum of fluctuations around (6.19) is studied in detail in Section 4, where it is shown that the lightest degrees of freedom correspond to (ϕ, X) with mass given by m. The effective potential derived from (6.18) is

$$V(\phi, X) = N_c m^2|\phi|^2 + \left| mN_cX + N_f'\lambda'^{N_f'/N_c'}\left(\frac{\Lambda'^{3N_c'-N_f'}}{\phi^{N_c'-N_f'}}\right)^{1/N_c'}\right|^2 + V_{CW}(\phi, X) \, , \tag{6.21}$$

where the second term comes from W_ϕ. The Coleman-Weinberg contribution will be discussed shortly.

As a starting point, set $X = 0$ and $V_{CW} \to 0$. Minimizing $V(\phi, X = 0)$ gives

$$|\phi_0|^{(2N_c'-N_f')/N_c'} = \sqrt{\frac{N_c' - N_f'}{N_c N_c'}}\; N_f'\,\frac{\lambda'^{N_f'/N_c'}}{m}\Lambda'^{(3N_c'-N_f')/N_c'} \, , \tag{6.22}$$

and since $W_{\phi\phi} \sim m$, $V(\phi_0 + \delta\phi, X = 0)$ corresponds to a parabola of curvature m. The nonperturbative term only affects ϕ_0 but not the curvature m; this will be important in the discussion of subsection 3.4.

Next, allowing X to fluctuate (but still keeping $V_{CW} \to 0$), $V(\phi_0, X)$ gives a parabola centered at

$$X_{W_\phi=0} = -\sqrt{\frac{N_c'}{N_c(N_c' - N_f')}}|\phi_0| \tag{6.23}$$

and curvature m. In other words, $X = 0$ is on the side of a hill of curvature m and height $V(\phi_0, 0) \sim m^2|\phi_0|^2$.

To create a minimum near $X = 0$, V_{CW} should contain a term $m_{CW}^2|X|^2$, with $m_{CW} \gg m$; this would overwhelm the classical curvature. As explained in Section 4, the massive degrees of freedom giving the dominant contribution to V_{CW} come from integrating out the massive fluctuations along q_0 and $\tilde{q}_0$. The result is

$$V_{CW} = N_c b h^3 m|\phi||X|^2 + \ldots \tag{6.24}$$

with $b = (\log 4 - 1)/8\pi^2\tilde{N}_c$ [72], and '...' represent contributions that are unimportant for the present discussion. In this computation, X and ϕ are taken as background fields. It is crucial to notice that the quadratic X dependence appears because $X = 0$ is an enhanced symmetry point.

In order to be able to produce a local minimum, the marginal parameters (λ, λ') will have to be tuned to satisfy

$$\epsilon \equiv \frac{m^2}{m_{CW}^2} = \frac{m}{bh^3|\phi|} \ll 1\,. \tag{6.25}$$

In this approximation, the value of ϕ at the minimum is still given by (6.22); also, X is stabilized at the nonzero value

$$X_0 = -e^{-i\frac{N_c'-N_f'}{N_c'}\alpha_\phi}\,\frac{N_f'}{bh^3}\lambda'^{N_f'/N_c'}\left(\frac{\Lambda'^{3N_c'-N_f'}}{|\phi_0|^{2N_c'-N_f'}}\right)^{1/N_c'}\,. \tag{6.26}$$

The phases of ϕ and X are thus related by

$$\alpha_X + \frac{N_c' - N_f'}{N_c'}\alpha_\phi = \pi\,. \tag{6.27}$$

Inserting (6.22) into (6.26) gives

$$|X_0| = \sqrt{\frac{N_c N_c'}{N_c' - N_f'}}\,\frac{m}{bh^3}\,. \tag{6.28}$$

At the minimum, (6.25) gives

$$(m/\Lambda')^{3N_c'-N_f'} \ll (bh^3)^{(2N_c'-N_f')/N_c'}\lambda'^{N_f'} \tag{6.29}$$

so the Yukawa coupling λ in $m = \lambda\Lambda$ must be taken small for the analysis to be self-consistent. The calculability condition $\Lambda' \ll \lambda'\Phi$ follows as a consequence of this. At the minimum, $X_0 \ll \phi_0$. The F-terms are given by

$$W_\phi \approx \sqrt{\frac{N_c N_c'}{N_c' - N_f'}}m\phi_0 \sim W_X\,. \tag{6.30}$$

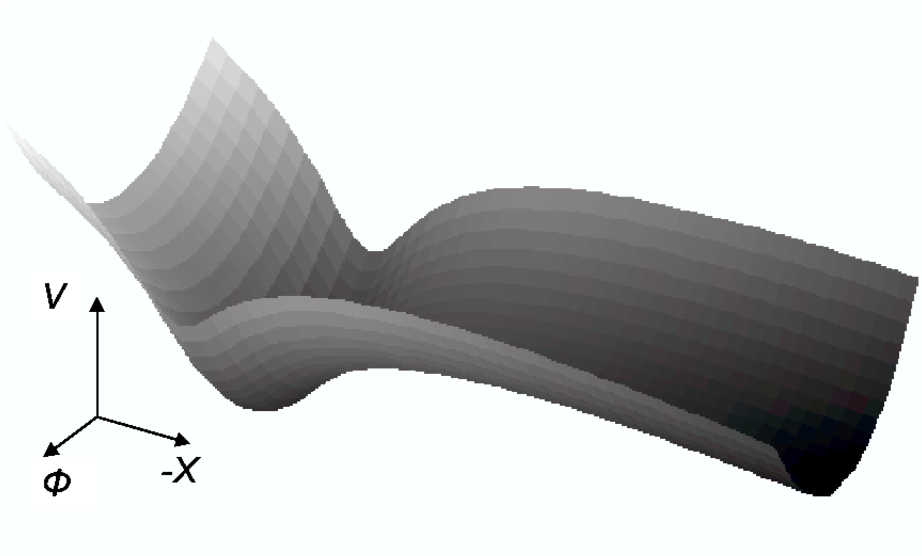

Figure 6.2: A plot showing the shape of the potential, including the one-loop Coleman-Weinberg corrections, near the metastable minimum. In the ϕ-direction the potential is a parabola, whereas in the X-direction it is a side of a hill with a minimum created due to quantum corrections.

and from (6.22) the scale of supersymmetry breaking is thus controlled by the dynamical scales of both gauge sectors. In the next subsection, the vacuum will be shown to be long-lived if (6.25) is satisfied.

Thus the model has a metastable vacuum near the origin, created by a combination of quantum corrections and nonperturbative gauge effects. The pseudo-runaway towards $X = X_{W_\phi=0}$ has been lifted by the Coleman-Weinberg contribution, as anticipated. This is the origin of the $1/b$ dependence in (6.28). The local minimum is depicted in Fig. 6.2.

6.3.3 Stability under other quantum corrections

The metastable vacuum appears from a competing effect between a runaway behavior in the primed sector and one loop corrections for the meson field X. One is naturally led to ask if, under these circumstances, other quantum effects are under control. These include higher loop terms from the massive particles producing V_{CW} as well as perturbative g' corrections.

Let us first study higher loop contributions from the massive fields in $(q, \tilde{q})$. They can correct the potential by additive terms of the form X^n, $n > 2$; these are automatically subleading, because $|X_0|^2 \ll m|\phi_0|$. They can also produce higher ϕ powers. However, such quantum corrections can only depend on the combination $m\phi$, and thus will be suppressed by powers of the UV cutoff Λ_0. For instance, a quartic term would appear as $(m\phi)^4/\Lambda_0^4$. We conclude that all these effects are subleading to (6.24).

Furthermore, since nonperturbative effects from $SU(N_c')$ were used, we should make sure that perturbative g' effects are not important. First note that the nonperturbative term in (6.21) is of the same order as the classical height of the potential $m^2|\phi|^2$ (see eq. (6.30)). It thus suffices to show that g' perturbative corrections to this height are subleading. A simple argument for this is as follows. Loops generate typical quartic terms in the Kähler potential

$$\delta K = \frac{\alpha}{\Lambda_0^2}(\Phi^*\Phi)^2 \tag{6.31}$$

which change the scalar potential by

$$\left[\frac{\alpha}{\Lambda_0^2}\,|\phi|^2\right](m^2|\phi|^2)\,. \tag{6.32}$$

The prefactor is parametrically small, making these contributions negligible.

6.3.4 Tunneling Out of the Metastable Vacuum

This section will show that the metastable non-supersymmetric vacuum can be made parametrically long-lived by taking the parameter $\epsilon \equiv \frac{m}{bh^3|\phi_0|}$ sufficiently small. The lifetime of the metastable vacuum may be estimated using semiclassical techniques and is proportional to the exponential of the bounce action, e^B [118].

First, the direction of tunneling in field space needs to be determined. Recall that the metastable vacuum in the $(|\phi|, X)$ space lies at

$$|\phi_0|^{\frac{2N_c'-N_f'}{N_c'}} = \sqrt{\frac{N_c'-N_f'}{N_cN_c'}}\;N_f'\,\frac{\lambda'^{\frac{N_f'}{N_c'}}}{m}\Lambda'^{\frac{3N_c'-N_f'}{N_c'}}\,,\quad X_0 = -\sqrt{\frac{N_cN_c'}{N_c'-N_f'}}\,\frac{m}{bh^3}\,. \tag{6.33}$$

(The phase of ϕ, not of qualitative importance for the present discussion, has been chosen to be zero. This fixes X to be real - see equation (6.27).) For fixed X the potential has a minimum at $|\phi| = |\phi_0|$; while quantum corrections may change this value by an order one number, corrections to the curvature of the potential in the $|\phi|$ direction are negligible. This curvature is positive, and thus the potential increases as $|\phi|$ moves away from $|\phi_0|$. The field therefore does not tunnel in the $|\phi|$ direction (see (6.2)). Along the X direction, however, the potential without quantum corrections near the enhanced symmetry point is like the side of a hill. For fixed $|\phi| = |\phi_0|$, the potential decreases in the negative X direction, and the classical curvature at $X = 0$ is m.

Quantum corrections are qualitatively important when $|X|$ is sufficiently small. For $|X|^2 \ll |W_X|$, their size grows quadratically as a function of X and they are sufficient to change the slope of the classical potential enough to introduce a minimum. For $|X|^2 \simeq |W_X|$, the growth of the quantum corrections is only logarithmic, and the slope of the classical potential again starts to dominate. Hence, the total potential has a peak that parametrically may be estimated to lie near

$$X_{\rm peak} \simeq -\sqrt{|W_X|} = -\sqrt{N_c m |\phi_0|}. \tag{6.34}$$

For $|X| > |X_{\rm peak}|$, the potential decreases as X becomes more negative until X reaches the 'drain' $W_\phi = 0$,

$$X_{W_\phi=0} = -\sqrt{\frac{N_c'}{N_c(N_c'-N_f')}}|\phi_0|. \tag{6.35}$$

The direction in field space to tunnel out of the false vacuum is towards negative X with fixed $|\phi| = |\phi_0|$. It thus suffices to consider the tunneling in the one-dimensional potential, $V(X) \equiv V(|\phi_0|, X)$. Note that parametrically $|X_0| \ll |X_{\rm peak}| \ll |X_{W_\phi=0}|$ as $\epsilon \to 0$.

For negative X, using equations (6.21) and (6.33), the one-dimensional potential may be written as

$$V(X) = \left(\frac{2N_c' - N_f'}{N_c' - N_f'}\right) N_c\, m^2\, |\phi_0|^2 + N_c^2\, bh^3\, m^2\, |\phi_0|^2\, f\left(\frac{-|X|}{bh^3|\phi_0|}\right). \tag{6.36}$$

In the region $|X| \ll |X_{\rm peak}|$, the function $f(x)$ is dominated by quantum corrections and may be approximated by

$$f(x) \simeq \frac{bh^3}{N_c\,\epsilon}\, x^2\,, \tag{6.37}$$

where a constant piece coming from the quantum corrections, again not important for the calculation of the bounce action, has been neglected. On the other hand, in the region $|X_{\rm peak}| \ll |X| \ll |X_{W_\phi=0}|$, the constant slope of the classical potential dominates. The potential in this region may be approximated by the classical potential plus a constant contribution from the quantum corrections whose size is roughly given by the height of the potential barrier. The height of the potential barrier is, from (6.37), of order $f(X_{\rm peak}/bh^3|\phi_0|) = 1$, and it is thus loop-suppressed compared to the overall magnitude of the potential near the metastable minimum. The potential in this region will be parametrized by a straight line

$$f(x) \simeq 1 - 2\sqrt{\frac{N_c'}{N_c(N_c'-N_f')}}\,(x - x_{\rm peak}). \tag{6.38}$$

In order to estimate the bounce action it is not appropriate to use the thin-wall approximation [118]. Instead, the potential may be modeled as a triangular barrier [119]. Using the results of [119], the value to which the field tunnels to is

$$\tilde{X} \sim -\,b\,h^3|\phi_0|. \tag{6.39}$$

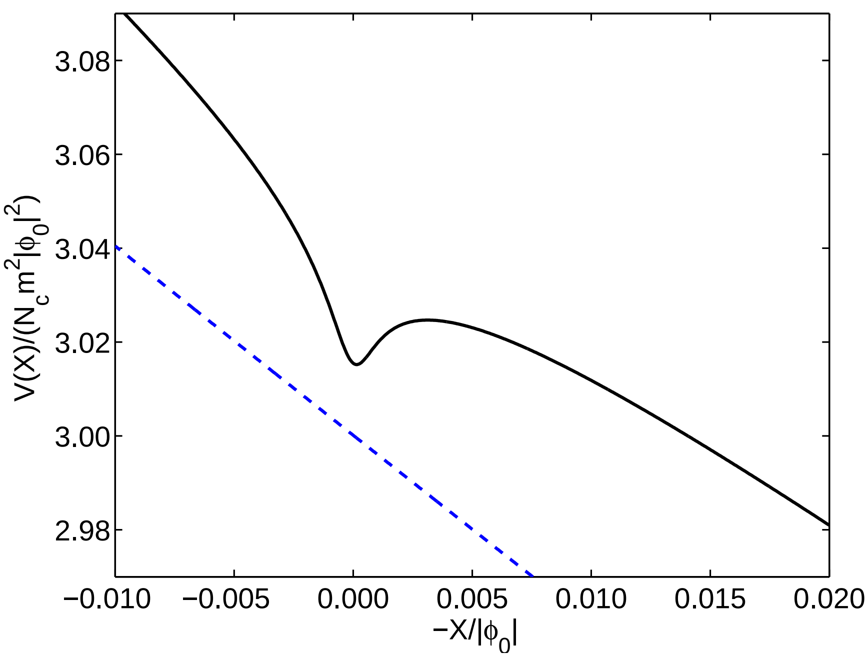

Figure 6.3: A plot of the classical potential (dashed line) and the total potential including one-loop corrections (solid line) for fixed $|\phi| = |\phi_0|$, where $|\phi_0|$ is the position of the metastable minimum in the ϕ-direction, defined in (6.33). In the figure, $N_f = 3$, $N_c = 2$, $N_f' = 1$ and $N_c' = 2$. The values were scaled so that the position of the "drain", $W_\phi = 0$, equals 1 on both axes. In these units, the position of the metastable minimum is on the order of 10^{-4}.

Note that parametrically $|X_0| \ll |X_{\rm peak}| \ll |\tilde{X}|$ as $\epsilon \to 0$, and that $|\tilde{X}|$ is loop-suppressed compared to $|X_{W_\phi=0}|$. The bounce action scales as

$$B \sim \frac{\tilde{X}^4}{V(X_{peak}) - V(X_0)} \sim b\, h^3\, \frac{1}{\epsilon^2}. \tag{6.40}$$

Therefore $B \to \infty$ as $\epsilon \to 0$, and the metastable vacuum is parametrically long-lived.

The total potential $V(X)$, including the full one-loop Coleman-Weinberg potential computed numerically with the help of [120], is shown in Fig. 6.3. The program of [120] also allowed us to check numerically the previous tunneling properties.

6.4 Particle spectrum and R-symmetry

In this section, we discuss in more detail the particle spectrum of the model and comment on the R-symmetry properties.

The fluctuations of the fields around the metastable minimum may be parametrized following

	Fermions			Bosons		
	Weyl mult.	mass^2	$U(N_f-1)$	Real mult.	mass^2	$U(N_f-1)$
ϕ, tr$X$	2	$\mathcal{O}(m^2)$	1_0	1	0	1_0
				3	$\mathcal{O}(m^2)$	1_0
$X_{ij}-\mathrm{tr}X$	$(N_f-1)^2-1$	0	Adj$_0$	$2((N_f-1)^2-1)$	0	Adj$_0$
Y, χ $\tilde{\chi}$	1	0	1_0	1	0_{GB}	1_0
				1	0_{NCGB}	1_0
	2	$\mathcal{O}(hm\lvert\phi_0\rvert)$	1_0	4	$\mathcal{O}(hm\lvert\phi_0\rvert)$	1_0
Z,$\tilde{Z}$, ρ, $\tilde{\rho}$	$2(N_f-1)$	$\mathcal{O}(hm\lvert\phi_0\rvert)$	$\Box_1+\overline{\Box}_{-1}$	$2(N_f-1)$	0_{GB}	$\Box_1$
				$2(N_f-1)$	$\mathcal{O}(hm\lvert\phi_0\rvert)$	$\overline{\Box}_{-1}$
	$2(N_f-1)$	$\mathcal{O}(hm\lvert\phi_0\rvert)$	$\Box_1+\overline{\Box}_{-1}$	$2(N_f-1)$	$\mathcal{O}(hm\lvert\phi_0\rvert)$	$(\Box_1+$
				$2(N_f-1)$	$\mathcal{O}(hm\lvert\phi_0\rvert)$	$\overline{\Box}_{-1})$

Figure 6.4: Table showing the classical mass spectrum, grouped in sectors of $\mathrm{Str}\, M^2 = 0$ for $N_f = N_c + 1$. The $O(m^2)$ fields in $(\phi, \mathrm{tr}\, X)$ are not degenerate. Although supersymmetry is spontaneoulsy broken, there is no Goldstino at the classical level.

ISS,

$$\phi = \phi_0 + \delta\phi\ ,\ M = \begin{pmatrix} Y_{\tilde{N}_c\times\tilde{N}_c} & Z^T_{\tilde{N}_c\times(N_f-\tilde{N}_c)} \\ \tilde{Z}_{(N_f-\tilde{N}_c)\times\tilde{N}_c} & X_0 + X_{(N_f-\tilde{N}_c)\times(N_f-\tilde{N}_c)} \end{pmatrix} \tag{6.41}$$

$$q = \begin{pmatrix} q_0 + \chi_{\tilde{N}_c\times\tilde{N}_c} \\ \rho_{(N_f-\tilde{N}_c)\times\tilde{N}_c} \end{pmatrix}\ ,\ \tilde{q} = \begin{pmatrix} \tilde{q}_0 + \tilde{\chi}_{\tilde{N}_c\times\tilde{N}_c} \\ \tilde{\rho}_{(N_f-\tilde{N}_c)\times\tilde{N}_c} \end{pmatrix}\ , \tag{6.42}$$

where $q_0\tilde{q}_0 := -m\phi_0/h$. All fields are complex; ϕ_0 and X_0 are the values at the metastable minimum.

The relevant mass scales are

$$M^2 = 0,\ m^2,\ m^2_{CW} = bh^3 m|\phi_0|,\ hm|\phi_0|\,. \tag{6.43}$$

The particles may be divided into three 'sectors' with small mixing amongst themselves. Up to quadratic order, the superpotential is

$$\begin{aligned} W \quad = \quad & W_{\phi\phi}\delta\phi\delta\phi + mN_c\delta\phi(X_0 + X) + m\delta\phi\sum_{\alpha=1}^{\tilde{N}_c} Y_{\alpha\alpha} \\ & + mN_c\phi_0(X_0+X) + h\sum_{f=1}^{N_c}\left[q_0(\tilde{\rho}Z^T)_{ff} + \tilde{q}_0(\rho\tilde{Z}^T)_{ff} + X_0(\rho\tilde{\rho}^T)_{ff}\right] \\ & + h\sum_{\alpha=1}^{\tilde{N}_c}\left[q_0(\tilde{\chi}Y)_{\alpha\alpha} + \tilde{q}_0(\chi Y)_{\alpha\alpha}\right]. \end{aligned} \tag{6.44}$$

The first line is related to the new dynamical field $\delta\phi$; unlike ISS, now X is not a pseudo-flat direction. The second and third lines are as in ISS.

Consider the case $N_f = N_c + 1$; the spectrum of classical masses is shown in Fig. 6.4, and the spectrum of the masses including one-loop CW corrections is shown in Fig. 6.5. The fields are grouped in sectors of $\mathrm{STr}M^2 = 0$.

	Fermions			Bosons		
	Weyl mult.	mass^2	$U(N_f-1)$	Real mult.	mass^2	$U(N_f-1)$
ϕ, tr$X$	1	0	1_0	1	0	1_0
	1	$\mathcal{O}(m^2)$	1_0	1	$\mathcal{O}(m^2)$	1_0
				2	$\mathcal{O}(m^2_{\mathrm{CW}})$	1_0
$X_{ij}-\mathrm{tr}X$	$(N_f-1)^2-1$	0	Adj$_0$	$2((N_f-1)^2-1)$	$\mathcal{O}(m^2_{\mathrm{CW}})$	Adj$_0$
Y, χ $\tilde{\chi}$	1	0	1_0	1	0_{GB}	1_0
				1	$\mathcal{O}(m^2_{\mathrm{CW}})$	1_0
	2	$\mathcal{O}(hm\lvert\phi_0\rvert)$	1_0	4	$\mathcal{O}(hm\lvert\phi_0\rvert)$	1_0
Z,$\tilde{Z}$, ρ, $\tilde{\rho}$	$2(N_f-1)$	$\mathcal{O}(hm\lvert\phi_0\rvert)$	$\square_1+\overline{\square}_{-1}$	$2(N_f-1)$	0_{GB}	$\square_1$
				$2(N_f-1)$	$\mathcal{O}(hm\lvert\phi_0\rvert)$	$\overline{\square}_{-1}$
	$2(N_f-1)$	$\mathcal{O}(hm\lvert\phi_0\rvert)$	$\square_1+\overline{\square}_{-1}$	$2(N_f-1)$	$\mathcal{O}(hm\lvert\phi_0\rvert)$	$(\square_1+$
				$2(N_f-1)$	$\mathcal{O}(hm\lvert\phi_0\rvert)$	$\overline{\square}_{-1})$

Figure 6.5: Table showing the mass spectrum, including one-loop corrections, grouped in sectors of $\mathrm{Str}\, M^2 = 0$ for $N_f = N_c + 1$. Notice the appearance of the Goldstino in the $(\phi, \mathrm{tr}\, X)$ sector. The $O(m^2)$ fields in $(\phi$, $\mathrm{tr}\, X)$ are not degenerate; here $m^2_{CW} = bh^3 m|\phi_0|$.

	Fermions				Bosons			
	Weyl mult.	mass^2	$U(N_f-\tilde{N}_c)$	$SU(\tilde{N}_c)_D$	Real mult.	mass^2	$U(N_f-\tilde{N}_c)$	$SU(\tilde{N}_c)_D$
ϕ, tr$X$	2	$\mathcal{O}(m^2)$	1_0	1	1	0	1_0	1
					3	$\mathcal{O}(m^2)$	1_0	1
$X_{ij}-\mathrm{tr}X$	$(N_f-\tilde{N}_c)^2-1$	0	Adj$_0$	1	$2((N_f-\tilde{N}_c)^2-1)$	0	Adj$_0$	1
Y, χ $\tilde{\chi}$	$\tilde{N}_c^2$	0	1_0	Adj	$\tilde{N}_c^2$	0_{GB}	1_0	Adj
					$\tilde{N}_c^2$	0_{NCGB}	1_0	Adj
	$2\tilde{N}_c^2$	$\mathcal{O}(hm\lvert\phi_0\rvert)$	1_0	Adj	$4\tilde{N}_c^2$	$\mathcal{O}(hm\lvert\phi_0\rvert)$	1_0	Adj
Z,$\tilde{Z}$, ρ, $\tilde{\rho}$	$2\tilde{N}_c(N_f-\tilde{N}_c)$	$\mathcal{O}(hm\lvert\phi_0\rvert)$	$\square_1+\overline{\square}_{-1}$	$\square+\overline{\square}$	$2\tilde{N}_c(N_f-\tilde{N}_c)$	0_{GB}	$\square_1$	$\overline{\square}$
					$2\tilde{N}_c(N_f-\tilde{N}_c)$	$\mathcal{O}(hm\lvert\phi_0\rvert)$	$\overline{\square}_{-1}$	$\square$
	$2\tilde{N}_c(N_f-\tilde{N}_c)$	$\mathcal{O}(hm\lvert\phi_0\rvert)$	$\square_1+\overline{\square}_{-1}$	$\square+\overline{\square}$	$2\tilde{N}_c(N_f-\tilde{N}_c)$	$\mathcal{O}(hm\lvert\phi_0\rvert)$	$(\square_1+$	$(\overline{\square}+$
					$2\tilde{N}_c(N_f-\tilde{N}_c)$	$\mathcal{O}(hm\lvert\phi_0\rvert)$	$\overline{\square}_{-1})$	$\square)$

Figure 6.6: Table showing the classical mass spectrum, grouped in sectors of $\mathrm{Str}\, m^2 = 0$, for $N_f > N_c + 1$. After gauging $SU(\tilde{N}_c)$, the traceless goldstone bosons from $(\chi, \bar{\chi})$ are eaten, giving a mass $m_W^2 = g^2 m|\phi_0|/h$ to the gauge bosons. Further, from $V_D = 0$, the noncompact goldstones also acquire a mass m_W^2. Including CW corrections, $\mathrm{tr}\, X$ acquires mass m^2_{CW} and one of the fermions becomes massless.

The fields $(Y, \chi, \tilde{\chi})$ form three chiral superfields, with supersymmetric masses, and hence do not contribute when integrated out at one loop. The Coleman-Weinberg potential is generated by the fields $(Z, \tilde{Z}, \rho, \tilde{\rho})$, which are the heaviest in the spectrum. Including such quantum corrections, $\mathrm{tr}\, X$ acquires a mass m^2_{CW}, while the mass of ϕ is not modified. Interestingly, at the classical level there is no massless Goldstino, since the expansion is not around a critical point of the classical potential. Including quantum corrections, one of the massive fermions in the $(\phi, \mathrm{tr}\, X)$-sector becomes massless, as may be seen in Fig. 6.5. A similar situation, in the opposite limit of small supersymmetry breaking, has been discussed recently in [115].

The case $\tilde{N}_c = N_f - N_c > 1$ can be similarly analyzed, and is shown in Fig. 6.6.

The Standard Model gauge group can be embedded inside the global symmetry group of this model. In this way, renormalizable models of direct gauge mediated supersymmetry breaking may be constructed.

6.4.1 Breaking the R-symmetry

To have gaugino masses, any R-symmetry must be broken, explicitly and/or spontaneously [72], [115]. The low energy superpotential 6.18 has the following $U(1)_R$ symmetry:

$$R_\phi = 2\frac{N_c'}{N_f'}\ ,\ R_X = 2\frac{N_f' - N_c'}{N_f'}\ ,\ R_q = R_{\tilde{q}} = \frac{N_c'}{N_f'}\,. \tag{6.45}$$

Since the VEV's of these fields are nonzero in the metastable vacuum, the R-symmetry is spontaneously broken, and there is an R-axion a. In terms of the phase of the i-th field, the axion is

$$\phi_i = \frac{1}{\sqrt{2}}\frac{f_R}{R_i}\,e^{iR_i(a/f_R)}\,, \tag{6.46}$$

where the decay constant f_R is defined as

$$f_R = \Big[\sum_i \big(\sqrt{2}R_i|\langle\phi_i\rangle|\big)^2\Big]^{1/2} \tag{6.47}$$

and R_i is the R-charge of ϕ_i. In [121] it was pointed out that if R-symmetry is broken spontaneously in an O' Raifeartaigh model, then the theory should contain a field with R-charge different than 0 or 2. This is also the case in the present situation, although our model does not contain the linear O' Raifeartaigh term.

For finite $\tilde{\Lambda}$, the det X contributions need to be taken into account, and the $U(1)_R$ symmetry becomes anomalous. Adding this term induces a tadpole for Y, which now acquires an expectation value of order

$$Y \sim \left[\frac{X_0}{\tilde{\Lambda}}\right]^{\frac{3N_c-2N_f}{N_f-N_c}} X_0\,, \tag{6.48}$$

so that $|Y| \ll |X_0|$. Then the mass of the R-axion follows from

$$|W_X|^2 \sim \left|m\phi + cX_0^2\left[\frac{X_0}{\tilde{\Lambda}}\right]^{2\,\frac{3N_c-2N_f}{N_f-N_c}}\right|^2. \tag{6.49}$$

Deriving twice the cross-term, which is proportional to $\cos(a/f)$, yields the axion mass

$$m_a^2 \sim m^2\left(\left[\frac{\lambda}{bh^3}\right]^{2\,\frac{3N_c-2N_f}{N_f-N_c}}\frac{\epsilon}{bh^3}\right) \ll m^2\,, \tag{6.50}$$

where λ is the Yukawa coupling appearing in $m = \lambda\Lambda$. Thus, R-symmetry is both spontaneously and explicitly broken.

6.5 Metastability near generic points of enhanced symmetry

In this section, the existence and genericity of metastable vacua near enhanced symmetry points is explored. Statistical analyses of the supersymmetry breaking scale up to date have not taken into account loop quantum effects as these corrections are hard to evaluate on an ensemble of field theories. However, metastable vacua introduced by the Coleman-Weinberg potential, with all the relevant parameters generated dynamically, may change such results. Before considering the general case, let us analyze (6.6).

6.5.1 Non-coincident enhanced symmetry points

Consider two gauge sectors as in (6.6), with enhanced symmetry points at $\Phi = 0$ and $\Phi = \xi$, respectively. The free magnetic sector is taken to be massless at $\Phi = 0$; integrating over the other primed sector gives

$$W = m\Phi \,\mathrm{tr}\, M + h\,\mathrm{tr}\, qM\tilde{q} + N_c'\big[\lambda'^{N_f'}\,\Lambda'^{3N_c'-N_f'}\,(\Phi+\xi)^{N_f'}\big]^{1/N_c'}\,. \tag{6.51}$$

Since metastable vacua were shown to exist for $\xi = 0$, here the discussion is restricted to the limit of ξ much bigger than all the energy scales in the problem. This is consistent with the fact that naturalness demands any relevant coupling to be of order the UV cutoff.

Introducing the notation

$$\alpha = N_f'/N_c'\ ,\ K = N_c'\lambda'^{N_f'/N_c'}\,\Lambda'^{(3N_c'-N_f')/N_c'}\,, \tag{6.52}$$

the equations of motion for ϕ and X give

$$N_c m^2\phi = \alpha^2(1-\alpha)\frac{K^2}{\xi^{3-2\alpha}}\,. \tag{6.53}$$

$$|X| = \frac{N_c}{\alpha(1-\alpha)}\,\frac{m^2\xi^{2-\alpha}}{K}\,. \tag{6.54}$$

Without fine-tuning m or K, X tends to be driven away from the origin as ξ increases. The fine-tuning may be seen, for instance, from the requirement $m_{CW} \gg m$, which implies

$$m^3 \ll bh^3\frac{K^2}{\xi^{3-2\alpha}}\,. \tag{6.55}$$

Although this resembles the calculability condition (6.29), now there are powers of the large scale ξ in the denominator. For ξ of order the UV cutoff, this represents a big fine-tuning, either on the coefficient K or on the small mass parameter m.

The conclusion is that, while metastable vacua can occur for far away enhanced symmetry points, this situation is not generic and requires fine-tuning. This is to be expected, once relevant parameters are allowed to appear in the superpotential.

6.5.2 General Analysis

A generic structure in the landscape of effective field theories corresponds to a gauge theory with vector-like matter and mass given by a singlet, whose dynamics is related to another sector. The superpotential may be written as

$$W = f(\Phi) + \lambda\,\Phi \mathrm{tr}(Q\bar{Q})\,. \tag{6.56}$$

Here, $(Q, \bar{Q})$ are N_f quarks in $SU(N_c)$ SQCD; $f(\Phi)$ may be generated, for instance, from a flux superpotential, by nonrenormalizable interactions [101], or, as in the case studied in this work, by another gauge sector. Next, it is required that the SQCD sector be in the free magnetic range; this is still a generic situation. The dual magnetic description is weakly coupled near the enhanced symmetry point $\Phi = 0$, where the superpotential reads

$$W = f(\Phi) + m\Phi\,\mathrm{tr}\;M + h\,\mathrm{tr}\;qM\tilde{q}\,. \tag{6.57}$$

The question that will be addressed here is: what restrictions need to be imposed on $f(\Phi)$, so that the one loop potential V_{CW} can create a metastable vacuum near $M = 0$? Since we are interested in the novel effect of pseudo-runaway directions we will demand $f'(\Phi) \neq 0$. The case $f'(\Phi) = 0$ is standard in such analyses, see e.g. [116].

As discussed in Section 3, this is possible only if

$$m_{CW}^2 := N_c b h^3 m|\phi| \gg m^2 \tag{6.58}$$

where ϕ denotes the expectation value of Φ at the metastable vacuum. Further, one needs to impose that

$$h^2|X|^2 \ll m|\phi| \tag{6.59}$$

in order for the Taylor expansion of V_{CW} around $X = 0$ to converge. Evaluating the potential as in (6.21),

$$V = N_c m^2|\phi|^2 + \left|f'(\phi) + mN_c\,X\right|^2 + m_{CW}^2|X|^2\,. \tag{6.60}$$

The rank condition, an essential ingredient in the discussion, just follows from having SQCD in the free magnetic range. This fixes the first term, which comes from W_M, and the block structure of the matrix M; X was defined in (6.19).

Extremizing $V(\phi, X = 0)$ leads to

$$N_c m^2\phi = -f'(\phi)\,f''(\phi)^*\,. \tag{6.61}$$

On the other hand, minimization with respect to X in the approximation $m_{CW}^2 \gg m^2$, gives the metastable vacuum

$$m_{CW}^2\,X = -N_c m f'(\phi)\,. \tag{6.62}$$

Notice that $m_{CW}^2 \gg m^2$ makes this value parametrically smaller than the position of the 'drain' $f'(\phi) + mN_c X = 0$. This ensures the stability of the nonsupersymmetric vacuum. Replacing (6.61) in (6.62) (with $m_{CW}^2 = N_c bh^3|\phi|$) yields

$$|X| = \frac{N_c m^2}{bh^3} \frac{1}{|f''(\phi)|} \,. \tag{6.63}$$

It is possible to combine the conditions (6.58) and (6.59) with the values at the metastable vacuum (6.61), (6.63), to derive constraints on $f(\phi)$: (6.58) now reads

$$\frac{|f'(\phi)f''(\phi)|}{m^3} \gg \frac{1}{bh^3} \,, \tag{6.64}$$

while (6.59) gives

$$h^2|f'(\phi)|^2 \ll m(bh^3)^2|\phi|^3 \,. \tag{6.65}$$

Summarizing, the necessary conditions for metastable vacua near $X = 0$ to exist are (6.64) and (6.65). As illustrated in the previous subsection, they require fine-tuning the coefficients of $f(\phi)$, except in the case of coincident enhanced symmetry points, where there are no relevant scales.

6.6 Conclusions

We have constructed a model with long-lived metastable vacua in which all the relevant parameters, including the supersymmetry breaking scale, are generated dynamically by dimensional transmutation. The model consists of two $N = 1$ supersymmetric QCD sectors with flavors whose respective masses are controlled by the same singlet field. One of the gauge sectors is in the free magnetic range while the other is in the electric range. The metastable vacua are produced near a point of enhanced symmetry by a combination of nonperturbative gauge effects and, crucially, perturbative effects coming from the one-loop Coleman-Weinberg potential.

The model has the following desirable features: an explicitly and spontaneously broken R-symmetry, a singlet, a large global symmetry, naturalness and renormalizability.

There are two points that have to be stressed. First, a salient feature of the model is the existence of pseudo-runaway directions. They correspond to a runaway behavior that is lifted by one loop quantum corrections. This has not been observed before, the closest analog corresponding for example to the pseudo-moduli of [72]. It is quite plausible that this phenomenon appears in other models as well. The criterion is that the height of the potential has to be parametrically larger than the curvature, as quantified in Section 3. The strength of the quadratic Coleman-Weinberg corrections is set by this height, thus introducing a local minimum of high curvature in the (otherwise) runaway potential.

In dynamical supersymmetry breaking models [122–127], nonsupersymmetric vacua generally arise due to competing effects between a nonperturbative runaway and a classical term in the superpotential, as in the (3,2) model [128]. Our analysis shows that it is possible to stabilize such runaways even without tree-level terms, provided that one is close to certain enhanced symmetry points.

The second feature worth emphasizing is the connection between enhanced symmetry points in gauge theory moduli spaces and metastable dynamical supersymmetry breaking. There are reasons to believe that such vacua are generic. At the field theory level this is associated to the fact that a nonzero Witten index [129] may still allow an approximate R-symmetry [130]. While dynamical ISS models are not hard to construct, in general these mechanisms involve discrete R-symmetries [101]. This is very suppressed in the landscape of string vacua, corresponding to a high codimension locus in the flux lattice [131]. On the other hand, the construction presented here does not suffer from the previous difficulty. Therefore, it would be interesting to study how statistical estimates of the scale of supersymmetry breaking change, once the model is embedded in string theory.

Chapter 7

Metastable supersymmetry breaking and multitrace deformations of SQCD

7.1 Introduction and summary

Based on [7], in this chapter we explore metastable vacua in supersymmetric QCD in the presence of single and multitrace deformations of the superpotential, with the aim of obtaining an acceptable phenomenology.

It is not possible to build a phenomenologically viable model of gauge mediation using directly the ISS superpotential (6.1). This is due to an unbroken R-symmetry that forbids non-zero gaugino masses. A natural question is then how the phenomenology changes when the superpotential is a more general polynomial in $\tilde{Q}Q$. While this has been considered before for some particular superpotential deformations (see e.g. [104, 107, 108, 112, 113]), a more detailed account of the space of metastable vacua and the low energy phenomenology is needed. For instance, the light fermions of the model have not been fully explored. The aim of this work is to analyze the IR properties of the theory and its phenomenology in the presence of a generic $U(N_f)$-preserving polynomial superpotential

$$W = m\,\mathrm{tr}(Q\tilde{Q}) + \frac{1}{2\Lambda_0}\,\mathrm{tr}\left[(Q\tilde{Q})^2\right] + \frac{1}{2\Lambda_0}\,\gamma\big[\mathrm{tr}\,(Q\tilde{Q})\big]^2 + \ldots\ , \tag{7.1}$$

where $\Lambda_0 \gg \Lambda$ is some large UV scale, γ is an order one coefficient, and '…' are sextic and higher dimensional operators.

Deforming (6.1) by a generic polynomial in $\tilde{Q}Q$ breaks R-symmetry explicitly at tree level, and additional supersymmetric vacua are introduced [132]. The supersymmetric vacua for a single trace superpotential were analyzed in detail in [108], where it was found that the magnetic theory has classical supersymmetric vacua with various possible unbroken subgroups of the magnetic gauge group. This should be contrasted with the case of ISS, Eq. (6.1), where the magnetic gauge group is completely Higgsed and supersymmetry is broken classically by the rank condition.

After taking into account one loop quantum corrections in the magnetic theory, one finds the deformed theory also has metastable vacua at low energies [108]. The dynamical reason for this is that the deformations to the magnetic superpotential come from irrelevant operators in the electric theory, which are parametrically suppressed. Therefore, we end up with a controllable deformation of the ISS construction in the IR. These vacua break R-symmetry spontaneously, and in phenomenologically interesting regions of parameter space the spontaneous breaking is much larger than the explicit breaking.

Since supersymmetric vacua allow for unbroken magnetic gauge groups, one might expect the same to occur for metastable vacua. However, the metastable vacua in the theories we explore below have a completely broken magnetic gauge group; vacua with unbroken subgroups of the magnetic gauge group do not occur. This is in some disagreement with [108] and it would be interesting to see how this effect appears in the brane constructions of metastable vacua [75].

Next we will analyze the phenomenological properties of the spectrum, with particular attention to the light fermions, including the Standard Model gauginos and a multiplet of fermions from the "meson" superfield $M = \tilde{Q}Q$. If the superpotential contains only single traces of powers of M, the singlet and adjoint parts of the meson superfield $M = \tilde{Q}Q$ have the same one loop effective action. The singlet fermion is the Goldstino, and must be massless at one loop through a cancellation of its nonzero tree level mass against a one loop correction. The adjoint fermions (or more precisely, a certain subset thereof) have the same tree and one loop effective action, and so their masses arise only at two loops (and/or through equally small mixing effects.) Consequently their masses are small compared with those of the Standard Model gauginos, which arise at one loop.

In this paper we will be considering the case where the embedding of the Standard Model gauge group into the $U(N_f)$ flavor group endows these fermions with Standard Model quantum numbers. With such light masses, these fermions would already have been observed, and so these models would be phenomenologically unacceptable.

We are therefore led to consider a multitrace deformation of the superpotential; in particular, we must take $\gamma \neq 0$ in Eq. (7.1). Then the cancellation between the tree level and one loop masses for the Goldstino fails for the adjoint fermions, leaving them with masses proportional to γ. The phenomenology of direct gauge-mediated models based on this theory is quite rich, since the adjoint fermions may be lighter or heavier than the Standard Model gauginos, depending on γ. Mixing between these fermions and the gauginos is negligibly tiny, due to a charge-conjugation symmetry in (7.1).

The various sections are arranged as follows. In Section 7.2, we discuss the moduli space of SQCD with the superpotential Eq. (7.1), keeping only terms up to quartic order in the electric fields. In Section 7.3, we review SQCD without deformations (ISS), with emphasis on the spectrum and associated phenomenological issues. In Section 7.4, we study single trace deformations of the ISS superpotential, that is, the case $\gamma = 0$. We show that all metastable vacua have a magnetic gauge group that is completely Higgsed, and we discuss the spectrum, showing it is unacceptable for phenomenology. Next, in Section 7.5 we consider $\gamma \neq 0$, describing the spectrum in detail. Finally, Section 7.6 contains a brief overview of the phenomenology of and constraints on such models.

7.2 SQCD with a multitrace superpotential

In this section, we analyze the symmetries and supersymmetric vacua of SQCD in the presence of a generic $U(N_f)$-preserving polynomial superpotential.

Supersymmetric QCD with gauge group $SU(N_c)$ and N_f flavors $(Q_i, \tilde{Q}_j)$ with equal masses m has a global symmetry group

$$SU(N_f)_V \times U(1)_V \tag{7.2}$$

under which $(Q_i,\, \tilde{Q}_i)$ transform as $(\square_{+1},\, \bar{\square}_{-1})$. There is also a discrete $\mathbb{Z}_2$ charge conjugation symmetry $Q_i \leftrightarrow \tilde{Q}_i$. For phenomenological applications we will later weakly gauge a subgroup of $SU(N_f)_V$ and identify it with the Standard Model gauge groups. We will also gauge $U(1)_V$ to remove a Nambu-Goldstone boson.

The most general quartic superpotential preserving this symmetry is of the form

$$W = m\,\mathrm{tr}(Q\tilde{Q}) + \frac{1}{2\Lambda_0}\,\mathrm{tr}\left[(Q\tilde{Q})^2\right] + \frac{1}{2\Lambda_0}\,\gamma\Big[\mathrm{tr}\,(Q\tilde{Q})\Big]^2\,. \tag{7.3}$$

We will typically consider $\Lambda_0 \gg \Lambda \gg m$, and take γ to be of order one or smaller. We will not consider sextic or higher operators, since they are suppressed by higher powers of Λ_0 and would not affect our discussion. The nonrenormalizable superpotential (7.3) could be generated from a renormalizable theory, for example by integrating out fields with masses $\sim \Lambda_0$ that couple to $Q\tilde{Q}$.

Let us consider the theory in various limits. First, for $W = 0$ there is a moduli space of vacua parameterized by mesons and baryons modulo classical constraints. The global symmetry is enhanced to $SU(N_f)_L \times SU(N_f)_R \times U(1)_V$, and there is a non-anomalous $U(1)_R$ symmetry as well as an anomalous $U(1)_A$ axial current.

For $m/\Lambda \neq 0$ but $\Lambda_0 \to \infty$, the superpotential is renormalizable, and the theory has an exact classical $U(1)_R$ symmetry which is anomalous at the quantum level.[1] The non-anomalous

[1]There is also an approximate non-anomalous R-symmetry "$U(1)_{R'}$" which is restored as $m \to 0$, but we will

symmetries of the model are

	$SU(N_f)_V$	$U(1)_R$	$U(1)_V$
Q_i	$\square$	$+1$	$+1$
$\tilde{Q}_i$	$\bar{\square}$	$+1$	-1
$\Lambda^{3N_c-N_f}$	0	$2N_c$	0

plus the $\mathbb{Z}_2$ charge conjugation. The F-term relations lift the moduli space and the only vacuum is at the origin.

On the other hand, for $m \neq 0$ and Λ_0 large but finite, all R-symmetries are explicitly broken at the classical level. New discrete supersymmetric vacua appear in the regime

$$\tilde{Q}\,Q \sim m\Lambda_0\,.$$

7.2.1 Magnetic dual

Below the scale Λ, the theory is described by an effective theory, called the "dual magnetic theory", with gauge group $SU(\tilde{N}_c)$, singlet mesons Φ_{ij}, and N_f fundamental flavors $(q_i, \tilde{q}_j)$; we define $\tilde{N}_c \equiv N_f - N_c$. The theory has a positive beta function and is weakly-coupled in the infrared. After an appropriate change of variables, the classical tree level superpotential reads

$$W = h\,\mathrm{tr}(q\Phi\tilde{q}) - h\mu^2\,\mathrm{tr}\,\Phi + +\frac{1}{2}h^2\mu_\phi\Big(\mathrm{tr}\,\Phi^2 + \gamma(\mathrm{tr}\,\Phi)^2\Big)\,. \tag{7.4}$$

where the first trace is over magnetic color and the remaining traces are over flavor indices. The relation with the electric variables is (roughly)

$$\Lambda\Phi \sim \tilde{Q}Q,\ h\,\mu^2 \sim \Lambda m\,,\ h^2\,\mu_\phi \sim \frac{\Lambda^2}{\Lambda_0}\,.$$

More details may be found in [72].

As in ISS, we restrict to small quark masses $m \ll \Lambda$. We will also restrict ourselves to the range

$$\Lambda_0 \gg \sqrt{\frac{\Lambda}{m}}\,\Lambda\,, \tag{7.5}$$

which guarantees that $h\mu_\phi \ll \mu$. This will be needed to have long-lived metastable vacua. There are nonperturbative corrections to the superpotential (7.4), but they are all small enough not to affect our calculations given (7.5).

Also, these conditions ensure that the symmetries of the model at the scale Λ are approximately $SU(N_f)_L \times SU(N_f)_R \times U(1)_V \times U(1)_{R'}$, broken to $SU(N_f)_V \times U(1)_V$ only by effects of

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not need to consider this symmetry.

order m/Λ and Λ/Λ_0. Therefore, to an excellent approximation, both the superpotential *and* the Kähler potential satisfy the larger symmetry group, under which the trace and traceless parts of Φ_{ij} transform as a single irreducible multiplet. We will work only to leading non-vanishing order in the symmetry-breaking effects from non-zero m and non-infinite Λ_0.

Furthermore, the discrete $\mathbb{Z}_2$ charge-conjugation symmetry of the electric theory appears as the transformation

$$\Phi \to \Phi^T\,,\ q_i \leftrightarrow \tilde{q}_i\ . \tag{7.6}$$

This transformation plays an important role in the phenomenology of gauge mediation models based on (7.4), and indeed in other ISS-related models (see e.g. [133]).

As in the electric theory, the R-symmetry is explicitly broken, and we expect new supersymmetric vacua parametrically at μ^2/μ_ϕ. Indeed, the solutions to the F-term constraints

$$\begin{aligned} \big(-h\mu^2 + h^2\mu_\phi\gamma\,\mathrm{tr}\,\Phi\big)\,I_{N_f\times N_f} + h^2\mu_\phi\,\Phi + h\,\tilde{q}q &= 0 \\ q\Phi = \Phi\tilde{q} &= 0\,, \end{aligned} \tag{7.7}$$

are

$$\langle h\,\Phi\rangle = \frac{1}{1+(N_f-k)\gamma}\,\frac{\mu^2}{\mu_\phi}\begin{pmatrix} 0_{k\times k} & 0_{k\times(N_f-k)} \\ 0_{(N_f-k)\times k} & I_{(N_f-k)\times(N_f-k)} \end{pmatrix} \tag{7.8}$$

and

$$\langle \tilde{q}q\rangle = \frac{1}{1+(N_f-k)\gamma}\,\mu^2\begin{pmatrix} I_{k\times k} & 0_{k\times(N_f-k)} \\ 0_{(N_f-k)\times k} & 0_{(N_f-k)\times(N_f-k)} \end{pmatrix} \tag{7.9}$$

with $k = 1,\ldots,N_f-N_c$. (Here I represents the identity matrix, and a subscript $r\times s$ indicates a block matrix of the corresponding size.) The appearance of the extra parameter k classifying different classical vacua has been observed for $\gamma = 0$ by [108]. In particular, for $k < N_f - N_c$ there is an unbroken magnetic gauge group $SU(N_f - N_c - k)$.

7.3 Metastable DSB in the R-symmetric limit

In the next three sections, we will analyze the IR dynamics of (7.4) in three steps. First, we review the ISS model [72], the R-symmetric limit $\mu_\phi = 0$, which corresponds to an electric SQCD with massive flavors and no irrelevant operators. We will highlight the spectrum and associated phenomenological problems. In Section 7.4, we show how these problems are not entirely solved by making μ_ϕ non-zero but leaving $\gamma = 0$. Finally, in Section 7.5, we show how the theory with $\gamma \neq 0$ resolves the remaining problems.

7.3.1 The model and its spectrum

The ISS model considers massive SQCD near the origin in field space in the free magnetic range $N_c + 1 \le N_f < \frac{3}{2} N_c$, where the theory has a dual magnetic description with superpotential

$$W = -h\mu^2 \,\mathrm{tr}\, \Phi + h\mathrm{tr}(q \Phi \tilde{q})\,. \tag{7.10}$$

At the classical level the theory breaks supersymmetry by the rank condition. We parametrize the fields by

$$\Phi = \begin{pmatrix} Y_{\tilde{N}_c \times \tilde{N}_c} & Z^T_{\tilde{N}_c \times N_c} \\ \tilde{Z}_{N_c \times \tilde{N}_c} & X_{N_c \times N_c} \end{pmatrix} \tag{7.11}$$

$$q^T = \begin{pmatrix} \chi_{\tilde{N}_c \times \tilde{N}_c} \\ \rho_{N_c \times \tilde{N}_c} \end{pmatrix} \;,\; \tilde{q} = \begin{pmatrix} \tilde{\chi}_{\tilde{N}_c \times \tilde{N}_c} \\ \tilde{\rho}_{N_c \times \tilde{N}_c} \end{pmatrix} \;, \tag{7.12}$$

where $\tilde{N}_c = N_f - N_c$ is the rank of the magnetic gauge group. The classical moduli space of vacua is parametrized by $\langle \chi \tilde{\chi} \rangle = \mu^2 \, I_{\tilde{N}_c \times \tilde{N}_c}$ and $\langle X \rangle$. The other fields have vanishing expectation values. In the rest of the paper we will restrict to metastable vacua with maximal unbroken global symmetry, by choosing the ansatz

$$\langle X \rangle = X_0 \, I_{N_c \times N_c} \;,\; \langle \chi \rangle = q_0 \, I_{\tilde{N}_c \times \tilde{N}_c} \;,\; \langle \tilde{\chi} \rangle = \tilde{q}_0 \, I_{\tilde{N}_c \times \tilde{N}_c} \,. \tag{7.13}$$

It will be checked that this is a self-consistent choice.

The vev for $\chi\tilde{\chi}$ breaks the gauge group $SU(\tilde{N}_c)_G$ completely, with the breaking pattern

$$SU(\tilde{N}_c)_G \times SU(N_f)_V \times U(1)_V \to SU(\tilde{N}_c)_V \times SU(N_c) \times U(1)'\,. \tag{7.14}$$

(Here all groups except $SU(\tilde{N}_c)_G$ are global; we remind the reader that $\tilde{N}_c = N_f - N_c$). The reduction of the global symmetry group leads to $2N_c\tilde{N}_c+1$ Nambu-Goldstone modes. The fields $(\rho, \tilde{\rho}, Z, \tilde{Z})$ are charged under $U(1)'$, which plays the role of a messenger number symmetry. See [72] for a more detailed discussion.

The flat directions X are not protected by holomorphy or symmetries and, as we shall review shortly, become massive at one loop. (A field with these properties is called a "pseudo-modulus" [72].) In particular, X is stabilized at the origin. Near the origin of moduli space the rank condition imposes

$$|F_X| = |h\mu^2|\,, \tag{7.15}$$

and the scale of supersymmetry breaking is

$$V_{min} = N_c \, |h^2 \mu^4|\,. \tag{7.16}$$

To analyze the spectrum of the theory, it is convenient to rewrite the superpotential in terms of the component fields,

$$\begin{aligned} W &= -h\mu^2 \operatorname{tr} X + h \operatorname{tr} \begin{pmatrix} \rho & Z \end{pmatrix} \begin{pmatrix} X & \mu \\ \mu & 0 \end{pmatrix} \begin{pmatrix} \tilde\rho \\ \tilde Z \end{pmatrix} \\ & \quad + h\mu \operatorname{tr}\big[Y(\chi+\tilde\chi)\big] + h \operatorname{tr}\big(\chi Y \tilde\chi + \rho \tilde Z \tilde\chi + \chi Z \tilde\rho\big)\,. \end{aligned} \tag{7.17}$$

The spectrum consists of three sectors, each consisting of fields satisfying $\operatorname{Str} M^2 = 0$.

(1) *The* (ρ, Z) *sector*: Treating X as a background superfield, the (ρ, Z) supersymmetric mass matrix is

$$M_f = \begin{pmatrix} hX & h\mu \\ h\mu & 0 \end{pmatrix} \tag{7.18}$$

while the bosonic matrix is computed, as usual, including off-diagonal blocks with F-terms.

There are $2N_c\tilde N_c$ Dirac fermions that come from $(\psi_\rho,\, \psi_Z)$ and $(\psi_{\tilde\rho},\, \psi_{\tilde Z})$. Near the origin of field space, their masses are of order $h\mu$, from (7.18). The scalars combine into $4N_c\tilde N_c$ complex fields, which are linear combinations of $(\rho, Z, \tilde\rho^*, \tilde Z^*)$. There are $N_c\tilde N_c$ complex Nambu-Goldstone bosons from the combinations $\operatorname{Re}(\rho+\tilde\rho)$ and $\operatorname{Im}(\rho-\tilde\rho)$. The $3N_c\tilde N_c$ remaining complex scalars have splittings of order, and centered around, $h\mu$. The numerical coefficients adjust to preserve $\operatorname{Str} M^2 = 0$.

This sector will play the role of the messenger sector in gauge mediation applications. Once a subgroup of the flavor symmetry is identified with the Standard Model, and gauged with couplings g_{SM}, the Nambu-Goldstone modes will acquire a one loop mass of order $g_{SM}\mu/(4\pi)$. (In particular, we will study the case where $SU(N_c)$ is gauged — see Eq.(7.14).) The lightest state will be stable in the full theory, since the messenger sector is protected by the non-anomalous $U(1)'$ messenger number.

(2) *The* (Y, χ) *sector*: Fermions from $Y, (\chi+\tilde\chi)$ form $\tilde N_c^2$ Dirac fermions with mass $\sim h\mu$. The traceless part[2] of the chiral superfield $(\chi-\tilde\chi)$, which contains the NG bosons $\operatorname{Im}(\chi'-\tilde\chi')$, is eaten by the superHiggs mechanism when the magnetic group is gauged.

The field $\operatorname{Im}\operatorname{tr}(\chi-\tilde\chi)$ is a NG boson associated to the breaking of $U(1)_V$. The field $\operatorname{Re}\operatorname{tr}(\chi-\tilde\chi)$ corresponds to a pseudo-modulus, which will be lifted at one loop. The fermion from $\operatorname{tr}(\chi-\tilde\chi)$ is massless. This sector has a supersymmetric spectrum at tree level.

The massless fields from $\operatorname{tr}(\chi-\tilde\chi)$ would be phenomenologically forbidden. This forces us to gauge $U(1)_V$, so that the superfield $\operatorname{tr}(\chi-\tilde\chi)$ is eaten by the $U(1)_V$ gauge boson and at tree level acquires a mass of order $g_V\mu$.

[2]We denote traceless fields with primes; for instance X' is the traceless part of X.

(3) *The X sector*: X is a flat direction, with massless fermionic partner at tree level. In particular, $\psi_{\mathrm{tr}\,X}$ is the Goldstino.

One loop contributions from heavy particles lift the pseudo-moduli. The fields $(Y, \chi, \tilde{\chi})$ do not couple at tree level to the supersymmetry breaking sector, so they do not contribute to the one loop effective potential for the pseudo-moduli. Because we are in the regime where $|F_X| = |h\mu^2|$ is of order the square of the messenger masses, the effect of integrating out the messengers does not have a simple expression in superspace, and it is more convenient to work directly with nonsupersymmetric expressions. The bosonic action is given by the usual Coleman-Weinberg formula [118]

$$V_{CW} = \frac{1}{64\pi^2}\,\mathrm{STr}\, M^4 \log\frac{M^2}{\Lambda^2}\,. \tag{7.19}$$

Near the origin of moduli space $X \ll \mu$, the potential is approximated by [72]

$$V_{CW} \approx \frac{a}{2}\,|h^4\mu^2|\,\mathrm{tr}\left(\mathrm{Re}\,\frac{1}{\sqrt{2}}[\chi - \tilde{\chi}]\right)^2 + b|h^4\mu^2|\,\mathrm{tr}\,(X^\dagger X) \tag{7.20}$$

with

$$a = \frac{\log 4 - 1}{8\pi^2}\,N_c\,, \quad b = \frac{\log 4 - 1}{8\pi^2}\,\tilde{N}_c\,. \tag{7.21}$$

Therefore, in the ISS model the pseudo-moduli are consistently stabilized at the origin and R-symmetry is preserved. In this approximation, the one loop mass of the bosonic field X is given by

$$m^2_{CW} = b|h^4\mu^2| = \frac{\log 4 - 1}{8\pi^2}\,\tilde{N}_c\,|h^4\mu^2|\,. \tag{7.22}$$

7.3.2 Phenomenological problems

One could try to use the ISS construction as the supersymmetry breaking sector in models of direct gauge mediation. However, since R-symmetry is preserved in the metastable vacuum, Majorana masses for the Standard Model gauginos are forbidden. The same applies to the fermions ψ_X and $\psi_{\chi-\tilde{\chi}}$, which may have SM quantum numbers after embedding the SM gauge group into the flavor symmetry group of the model. For these reasons, this model does not give an acceptable phenomenology.

There are various ways of improving this situation (see, for instance, [116, 134, 135]). One very interesting proposal [133, 136] is that the gauginos could come from Dirac fermions, whose mass is not constrained to vanish by an unbroken R-symmetry. This idea was applied to the ISS model in [137], by adding new fields and interactions to the superpotential. Dirac masses appear from one loop diagrams mixing the MSSM Weyl gauginos with the new Weyl fermions.

One problem with this approach is that doubling the number of fields (in order to have Dirac fermions) creates a Landau pole close to the messenger scale. In this case, corrections from the microscopic theory may become important.

Another possibility is to deform the superpotential by higher powers of the meson superfield, explicitly breaking the R-symmetry at tree level [104, 107, 108]. We consider this possibility in detail below.

7.4 Single trace deformation

We begin by considering the superpotential Eq. (7.4) with $\gamma = 0$, that is, with only a single trace perturbation:

$$W = -h\mu^2 \operatorname{tr}\Phi + h\operatorname{tr}(q\Phi\tilde{q}) + \frac{1}{2}h^2\mu_\phi \operatorname{tr}(\Phi^2)\,. \tag{7.23}$$

This model was discussed in [108], where it was suggested that new metastable vacua, with unbroken magnetic group, appear around $X \sim \mu$. However, this region of parameter space is subtle, because higher order corrections to (7.20) become important. We will have two new things to say about this model.

(1) By considering the full logarithmic one loop potential (7.19), it is possible to show that the metastable vacua with unbroken magnetic gauge group are actually unstable. Thus, one is led to study only the ISS-like vacuum where the magnetic gauge group is completely Higgsed.

(2) Gauginos indeed become massive at one loop in this model, as expected from the R-symmetry breaking. However (ignoring some subtleties which we will discuss later) the adjoint fermions $\psi_{X'}$ become massive only at two loops, because diagrammatic cancellations that make the Goldstino $\psi_{\operatorname{tr}X}$ massless at one loop also force the adjoint fermions $\psi_{X'}$ to be massless at this order. This provides the main motivation for studying non-zero γ below.

7.4.1 Metastable supersymmetry breaking

The classical supersymmetric vacua are obtained by setting $\gamma = 0$ in (7.8) and (7.9). In order to analyze the effect of the deformation on the ISS metastable vacuum, the cases $k = N_f - N_c$ and $k < N_f - N_c$ have to be distinguished.

Case $k = N_f - N_c$

This is the analog of the ISS construction, with no unbroken gauge group. The fields are parameterized as in Eqs. (7.11) and (7.12). We will now review why a metastable vacuum appears at a distance of order μ_ϕ/b away from the origin [108].

	Fermions				Bosons			
	Weyl mult.	mass	$U(N_c)$	$SU(\tilde{N}_c)_D$	Real mult.	mass	$U(N_c)$	$SU(\tilde{N}_c)_D$
$\mathrm{tr}X$	1	$h^2\mu_\phi$	1_0	1	2	$h^2\mu_\phi$	1_0	1
X'	N_c^2-1	$h^2\mu_\phi$	Adj_0	1	$2(N_c^2-1)$	$h^2\mu_\phi$	Adj_0	1
Y, χ, $\tilde{\chi}$	$\tilde{N}_c^2$ $\tilde{N}_c^2$ $\tilde{N}_c^2-1$ 1	$\mathcal{O}(h\mu)$ $\mathcal{O}(h\mu)$ $g_{\mathrm{mag}}\mu$ 0	1_0 1_0 1_0 1_0	Adj Adj Adj 1	$2\tilde{N}_c^2$ $2\tilde{N}_c^2$ $2(\tilde{N}_c^2-1)$ 1 1	$\mathcal{O}(h\mu)$ $\mathcal{O}(h\mu)$ $g_{\mathrm{mag}}\mu$ 0_{NGB} 0	1_0 1_0 1_0 1_0 1_0	Adj Adj Adj 1 1
Z,$\tilde{Z}$, ρ, $\tilde{\rho}$	$2N_c\tilde{N}_c$ $2N_c\tilde{N}_c$	$\mathcal{O}(h\mu)$ $\mathcal{O}(h\mu)$	$\square_1+\overline{\square}_{-1}$ $\square_1+\overline{\square}_{-1}$	$\square+\overline{\square}$ $\square+\overline{\square}$	$2N_c\tilde{N}_c$ $2N_c\tilde{N}_c$ $2N_c\tilde{N}_c$ $2N_c\tilde{N}_c$	0_{NGB} $\mathcal{O}(h\mu)$ $\mathcal{O}(h\mu)$ $\mathcal{O}(h\mu)$	$\square_1$ $\overline{\square}_{-1}$ $(\square_1+$ $\overline{\square}_{-1})$	$\overline{\square}$ $\square$ $(\overline{\square}+$ $\square)$

Figure 7.1: The classical mass spectrum, grouped in sectors with $\mathrm{Str}\, M^2 = 0$. Since supersymmetry is spontaneously broken only after including one loop effects, there is no Goldstino at tree level. g_{mag} is the magnetic gauge coupling. A subscript “NGB” indicates the particle is massless because it is a Nambu-Goldstone boson. Subscripts in the third column indicate the charge under the $U(1)$ subgroup. Note this table gives the spectrum before the Standard Model gauge group is gauged.

As a starting point, set $V_{CW} \to 0$. Due to the classical deformation, X is no longer a flat direction, unlike the ISS case. Rather, the origin $X_0 \sim 0$ is at the side of a paraboloid of classical curvature $|h^2\mu_\phi|^2$. In other words, the origin is unstable against classical flow of X_0 toward the supersymmetric vacua discussed before. The tree level spectrum near the origin is shown in Figure 7.1.

In order to create a local minimum, the quantum contribution $V_{CW} \sim m_{CW}\,|X_0|^2$ should overwhelm the curvature of the classical potential, i.e., $m_{CW} \gg |h^2\,\mu_\phi|$. This rather interesting effect, where a one loop contribution stabilizes a classical runaway direction, was analyzed in [6]. Here, the stabilization of X_0 can occur naturally, since μ_ϕ, arising from a nonrenormalizable operator in the microscopic theory, is parametrically small. The condition that the one loop potential introduces a supersymmetry breaking minimum,

$$\epsilon \equiv \frac{m_{cl}^2}{m_{CW}^2} \approx \left|\frac{\mu_\phi^2}{b\mu^2}\right| \ll 1\,, \tag{7.24}$$

is naturally satisfied.

The potentials at tree level and at one loop, as a function of X_0, are shown in Figure 7.2. As seen from the figure, the tree level potential (lower magenta curve), which is obtained from the

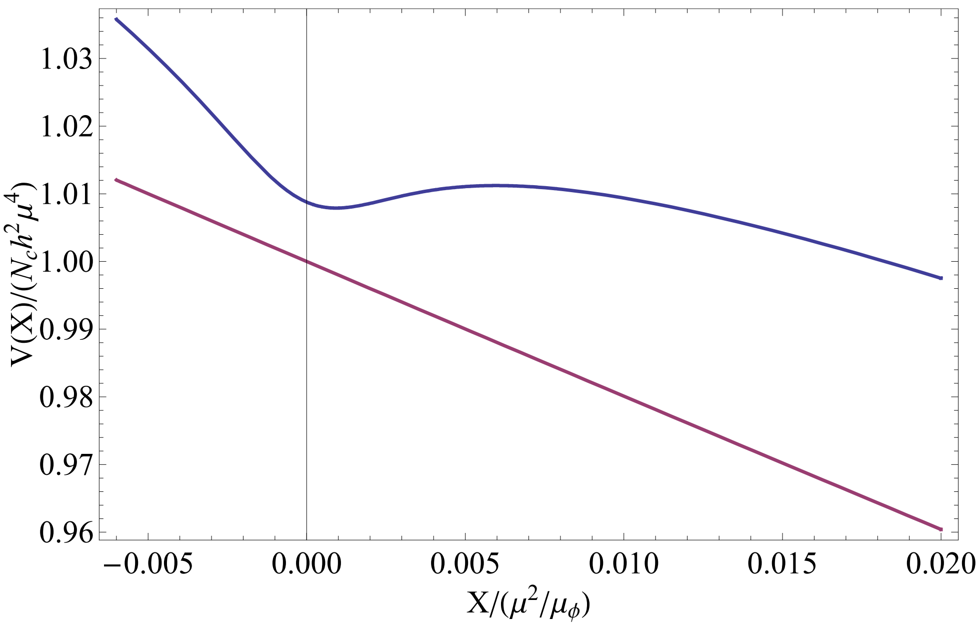

Figure 7.2: Metastable vacuum near $X \sim 0$, for a single trace quadratic deformation of the superpotential (i.e. $\gamma = 0$). All parameters have been chosen to be real. The bottom (magenta) line is the tree level potential, while the top (blue) line shows the tree level potential plus one loop Coleman-Weinberg corrections. The X-axis has been normalized such that the position of the tree level supersymmetric vacuum lies at $X/(\mu^2/\mu_\phi) = 1$. Notice how the one loop corrections create a (metastable) minimum near the origin.

superpotential in (7.23), has no supersymmetry breaking minimum. A metastable minimum is created near the origin once the one loop quantum corrections in the form of V_{CW} are included (upper blue curve).

As a result of the competition between the classical and quantum contributions, a metastable vacuum is created at

$$hX_0 \approx \frac{\mu^2 \mu_\phi^*}{b|\mu|^2 + |\mu_\phi|^2}\ ,\ q_0 \tilde{q}_0 = \mu^2\ ; \tag{7.25}$$

see Eq. (7.13) for the notation. As expected, X_0 is proportional to the explicit R-symmetry breaking parameter μ_ϕ. However, it is larger than this by the inverse loop factor $1/b$. This follows from the fact that the minimum appears from balancing a tree level linear term of order $\mu^2\,\mu_\phi$ against a one loop quadratic term of order $b\mu^2$.

The pattern of symmetry breaking in this vacuum is

$$SU(\tilde{N}_c)_G \times SU(N_f)_V \times U(1)_V \to SU(\tilde{N}_c)_V \times SU(N_c) \times U(1)'\,, \tag{7.26}$$

where only the messengers transform under $U(1)'$. Unlike the ISS construction, here $X_0 \neq 0$, so that the R-symmetry is both explicitly and spontaneously broken, *with the latter dominating* since $|hX_0| \gg |\mu_\phi|$.

Case $k < N_f - N_c$

The possibility of metastable vacua with $k < N_f - N_c$ is very interesting; coupling this to the MSSM, it would imply unbroken gauge groups in the hidden sector. Properties of such configurations were discussed in [108]. Unfortunately, we will now show that there are generically no metastable vacua in this regime.

Such vacua should be of the form

$$\Phi = \begin{pmatrix} 0 & 0 \\ 0 & X_{(N_f-k)\times(N_f-k)} \end{pmatrix} \ , \ \tilde{q}q = \begin{pmatrix} \mu^2 I_{k\times k} & 0 \\ 0 & 0 \end{pmatrix} \ . \tag{7.27}$$

The parametrization of the fluctuations is slightly more involved,

$$\Phi = \begin{pmatrix} Y_{k\times k} & Z_{k\times(N_f-k)} \\ \tilde{Z}_{(N_f-k)\times k} & X_{(N_f-k)\times(N_f-k)} \end{pmatrix} \ , \ q = \begin{pmatrix} V_{k\times k} & T_{k\times(\tilde{N}_c-k)} \\ P_{(N_f-k)\times k} & \varphi_{(N_f-k)\times(\tilde{N}_c-k)} \end{pmatrix} \tag{7.28}$$

and similarly for $\tilde{q}$. As in the case $k = N_f - N_c$, the expectation values are chosen to be of the form

$$\langle X\rangle = X_0\, I_{(N_f-k)\times(N_f-k)} \ , \ \langle V\rangle = q_0\, I_{k\times k} \ , \ \langle\tilde{V}\rangle = \tilde{q}_0\, I_{k\times k} \ .$$

The new fields $(\varphi,\, \tilde{\varphi})$ and $(T,\, \tilde{T})$ do not exist for $k = N_f - N_c$. They are fundamental flavors of the unbroken magnetic group $SU(N_f - N_c - k)$.

As was found in [108], positivity of the bosonic mass matrix of $(\varphi, \tilde{\varphi})$ implies

$$|X_0|^2 \geq |\mu^2 - h\mu_\phi X_0| \, .$$

This places us in the regime $X_0 \gtrsim \mu$. In this regime, the quadratic approximation (7.20) to the Coleman-Weinberg potential is no longer valid. For $X_0/\mu \sim 1$, all the higher order terms in V_{CW} give contributions comparable to (7.20). In other words, it is necessary to use the full expression appearing in Eq. (7.19).

Therefore, to establish the existence of such vacua, a detailed analysis of V_{CW} is required. As shown in [7], all such vacua are unstable once the full form of V_{CW} is included. The intuitive reason for this is that at large X_0 the logarithmic growth of V_{CW} cannot overwhelm the quadratic terms in the classical potential. A similar behavior was found in [6].

The plot of $V_{tree}+V_{CW}$ for this case is almost the same as that of Figure 7.2. For sufficiently large $|X_0/\mu| > 1$, the classical falling potential dominates the logarithmic rise of the V_{CW}, and no critical points are found until the supersymmetric vacuum is reached.

Summarizing, metastable states occur only for $k = N_f - N_c$. The fields have expectation values Eq. (7.25), breaking the magnetic gauge group completely at the scale μ.

7.4.2 Light fermions

We therefore return to the one remaining vacuum, the ISS-like case with $k = N_f - N_c$. From the previous analysis, the bosons from X and the traceless part of $\chi - \tilde{\chi}$ acquire masses of order m_{CW}. The aim of this section is to compute the fermion masses at one loop, and show that $\psi_{X_{ij}}$ remains massless at this order, contrary to naive expectations from R-symmetry breaking.

First we explore one loop effects involving the Goldstino $\psi_{\mathrm{tr}\,X}$. At tree level it has a nonvanishing mass $h^2\mu_\phi$. We are not expanding around a critical point of the classical potential, but rather one of the full one loop potential, and therefore the Goldstino should become massless only once one loop effects are included. This implies that the one loop diagram has to give

$$m_{\psi_{\mathrm{tr}\,X}}^{1-loop} \approx -h^2\mu_\phi\,, \tag{7.29}$$

such that $m_{\psi_{\mathrm{tr}\,X}}^{tree} + m_{\psi_{\mathrm{tr}\,X}}^{1-loop} \approx 0$. Indeed, the explicit evaluation of the one loop diagram in [7] corroborates (7.29). These results are approximate because we are neglecting (subleading) mixings with other singlet fermions; see below and [7] for more details.

At a first glance it is surprising that the one loop contribution can be equal to the tree level one. This is so because the one loop diagram is of order

$$\frac{h^2}{16\pi^2}\,h\,X_0\,.$$

However, since $hX_0 \sim \mu_\phi/b$, with b defined in Eq. (7.21), we obtain the result (7.29). This is another manifestation of the pseudo-runaway behavior discussed in the previous section.

Next, notice that within the classical superpotential (7.23), X_{ij} only appears in single traces. On the other hand, the one loop contribution is a single trace of a function of X_{ij}, because it comes from exponentiating bosonic and fermionic determinants (denoted by Δ) arising from messengers in the fundamental representation of $SU(N_c)$. Therefore, the full one loop effective action

$$S_{eff}(X, \psi_X) = S_{tree} + \mathrm{Tr}\left(\log\Delta\right)$$

can be written as a single trace of products of X_{ij} and its superpartner. This means that *the tree level plus one loop* contribution to the masses of the X fields must be of the form $\mathrm{Tr}(X^\dagger X)$, and therefore the singlet and adjoint parts of X get identical masses through one loop. The same is true for the fermionic partners of X: at one loop the masses of the singlet $\psi_{\mathrm{tr}\,X}$ and the adjoint $\psi_{X'}$ are the same. Diagrammatically, there is a cancellation between the tree level Weyl mass and the one loop correction.

We note two small subtleties. First, we have assumed here that the kinetic terms for the singlet and adjoint parts of X have the same normalization. This is true to a very good approximation. We assumed $m \ll \Lambda \ll \Lambda_0$, which ensured that the high-energy theory's approximate $SU(N_f) \times SU(N_f)$ symmetry is only weakly broken to $SU(N_f)_V$ at the scale Λ. Under this larger symmetry, the singlet and adjoint transform as a single irreducible representation, assuring equally normalized kinetic terms, up to negligible order(μ/Λ) corrections.

Second, and irreducibly, the Goldstino is not quite $\psi_{\mathrm{tr}\, X}$. As discussed in more detail in [7], it mixes slightly with the fields $\psi_{\mathrm{tr}\, Y}$ and $\psi_{\mathrm{tr}\,(\chi+\tilde{\chi})}$, with mixing angles of order a one loop factor, $\sim 1/16\pi^2$ and $\sim X_0/(16\pi^2\mu)$, respectively. Consequently the tree level and one loop ψ_X masses fail to cancel precisely, though by an amount that is one further loop-order suppressed. Thus our statement that the ψ_X masses vanish at one loop is effectively correct.

7.4.3 Phenomenology of the $\gamma = 0$ model

After gauging a subgroup of the flavor group $SU(N_c)$ — see Eq. (7.14) — and identifying it with the Standard Model gauge group, the adjoint fermions $\psi_{X'}$ will carry Standard Model gauge charges. The fact that they are approximately massless at one loop is unacceptable phenomenologically. They do become massive at two loop order, through the above-mentioned mixings, and through explicit two loop diagrams. For example, Standard Model gauge bosons, which do not impact the singlet $\psi_{\mathrm{tr}\, X}$, generate for the other fields a two loop mass of order

$$m_{\psi_{X'}} \ \sim \ g^2 \, \frac{X_0}{(16\pi^2)^2} \ \sim \ g^2 \, \frac{\mu_\phi}{16\pi^2} \ . \tag{7.30}$$

But the Standard Model gauginos have a one loop mass of order $X_0/16\pi^2 \sim \mu_\phi$. Importantly, the charge conjugation symmetry discussed in Section 7.2 forbids significant mixing between λ and ψ_X, so the masses for the $\psi_{X'}$ fields cannot be raised through mixing effects. *Consequently, requiring the gauginos are at a scale $\sim$ 1 TeV implies the $\psi_{X'}$ would be so light that they would have already been observed.*

7.5 The deformation with $\gamma \neq 0$

Clearly the root of this phenomenological problem lies in treating $\psi_{X'}$ and the Goldstino $\psi_{\mathrm{tr}\, X}$ on the same footing in the tree level superpotential. A solution is to allow non-zero γ,

$$W = h\mathrm{tr}(q\Phi\tilde{q}) - h\mu^2\, \mathrm{tr}\, \Phi + \frac{1}{2}h^2\mu_\phi\Big(\mathrm{tr}\,(\Phi^2) + \gamma(\mathrm{tr}\,\Phi)^2\Big)\,. \tag{7.31}$$

such that the two have different tree level masses. Then the total one loop mass for $\psi_{X'}$ becomes proportional to $\gamma\mu_\phi$.

The motivation for considering non-zero γ

$$W = -h\mu^2 \operatorname{tr}\Phi + h\operatorname{tr}(q\Phi\tilde{q}) + \frac{1}{2}h^2\mu_\phi\Big(\operatorname{tr}(\Phi^2) + \gamma(\operatorname{tr}\Phi)^2\Big)\,, \tag{7.32}$$

extends beyond phenomenological utility. No symmetry enforces $\gamma = 0$ once μ_ϕ or even μ are non-zero, so it is quite natural for γ to be nonzero.[3]

Let us now analyze the metastable vacua of the theory. For $h\mu_\phi \ll \mu$ (and for $|\gamma|$ roughly of order 1), the Coleman-Weinberg potential is approximately as in ISS. The only stable local minimum occurs for $k = N_f - N_c$. The multitrace deformation adds a term proportional to the identity matrix to W_Φ, so we obtain

$$q_0\tilde{q}_0 = \mu^2 - h\mu_\phi\, N_c\, \gamma\, X_0\,. \tag{7.33}$$

$$hX_0 \approx \frac{\mu^2\mu_\phi^*(1+N_c\gamma^*)}{b|\mu^2| + |\mu_\phi|^2 + f(\gamma,\gamma^*)} \tag{7.34}$$

with

$$f(\gamma,\gamma^*) = |\mu_\phi|^2\left[N_c\,(\gamma+\gamma^*) + N_c^2\,|\gamma|^2\right].$$

In the limit $h\mu_\phi \ll \mu$, the effect of γ is qualitatively unimportant:

$$hX_0 \approx \frac{\mu^2\mu_\phi^*(1+N_c\gamma^*)}{b|\mu|^2}\,,\; q_0\tilde{q}_0 \approx \mu^2\,, \tag{7.35}$$

so that $|hX_0| \gg |\mu_\phi|$. While $\gamma \neq 0$ does not alter the qualitative features of the vacuum, it is important, when computing the spectrum, that the precise values (7.33) and (7.34) be used.

7.5.1 Spectrum

We now analyze the spectrum in the metastable vacuum. As in Section 7.4, the Goldstino is not massless at tree level. Some of the one loop diagrams exactly cancel the tree level contributions and for this reason we discuss directly the tree level plus one loop results.

We first consider the fermions of the pseudo-modulus X. The singlet fermion (the Goldstino) is massless at one loop. For the adjoint fermions, the tree level mass $h^2\mu_\phi$ is partially canceled against the one loop contribution, and the full mass is of order

$$m_{\psi_{X'}} \approx h^2\mu_\phi\, N_c\gamma\,. \tag{7.36}$$

[3]Considering the preserved symmetries, one might wonder why the coefficients of $q\Phi\tilde{q}$ should be taken precisely equal. The point is that the physical couplings are constrained by the approximate $SU(N_f)_L \times SU(N_f)_R$ in the electric theory, which is still valid at and just below the scale Λ. In other words, the $\mu \to 0$ and $\mu_\phi \to 0$ limit implies equal couplings. Nothing comparable favors $\gamma = 0$.

Of course this vanishes in the limit $\gamma \to 0$, as required from Section 7.4.

Interestingly, we will see in Section 7.6 that the Majorana gaugino masses are proportional to $(1 + N_c\gamma)$. By changing the dimensionless parameter γ, the adjoint fermions may thus be made lighter or heavier than the gauginos. This allows a variety of spectra with different phenomenological signatures, see Section 7.6.

As for the bosons of X, both the adjoint and one component of the singlet acquire one loop masses of order m_{CW}; see Eq. (7.22). The other part of the singlet, $\mathrm{Arg}(X)$, is a massive R-axion. This is because X has a large nonzero expectation value $X_0 \sim 16\pi^2\mu_\phi \gg \mu_\phi$, which spontaneously breaks the approximate $U(1)_R$ symmetry at a scale much larger than any explicit breaking. The mass of the R-axion is given by

$$m_a^2 = \frac{2\sqrt{N_c}}{N_c|X_0|}\,\mathrm{Re}\left[h\mu^2\,(h^2\mu_\phi)^*\right] \sim b|h^4\,\mu^2|\,. \tag{7.37}$$

This is of the same order as the one loop mass m_{CW}, Eq. (7.22).

Finally, the $(Y, \chi, \tilde{\chi})$ and $(Z, \tilde{Z}, \rho, \tilde{\rho})$ sectors are as in Section 7.3.1. We remind the reader that we have gauged the $U(1)_V$ symmetry, and g_V denotes its gauge coupling. The (otherwise massless) fields from $\mathrm{tr}(\chi - \tilde{\chi})$ acquire masses of order $g_V\mu$, as shown in the table. Furthermore, the NG bosons from (ρ, $\tilde{\rho}$, Z, $\tilde{Z}$) acquire a one loop mass of order $g_{SM}\mu/4\pi$ once the Standard Model is gauged, as a subgroup of the flavor symmetry group. The lightest of these is stable due to the unbroken messenger number $U(1)'$ from Eq. (7.26).

7.5.2 Lifetime of the metastable vacuum

Here we check that the metastable non-supersymmetric vacuum can be sufficiently long-lived. This vacuum can decay to the ISS-like supersymmetric vacuum with $k = N_f - N_c$, or to the supersymmetric vacua with $k < N_f - N_c$ (see Section 7.2.1). The decay to the vacua with $k < N_f - N_c$ requires changing the expectation value of (some of the elements of) $q\tilde{q}$, from $h\mu^2$ to 0. This is strongly suppressed by the quartic potential term $V = \ldots + |hq\tilde{q}|^2$. The dominant decay channel will be to the supersymmetric vacuum with $k = N_f - N_c$, which we now analyze.

The lifetime of the vacuum may be estimated using semiclassical techniques and is proportional to the exponential of the bounce action, e^B [138]. We will see that the tunneling takes place in the direction of $\mathrm{tr}\,X$, in a region where $q\tilde{q} \approx \mu^2$ is almost constant. The potential as a function of $\mathrm{tr}\,X$, including the one loop quantum corrections from the Coleman-Weinberg potential, was analyzed in detail in [7]; we summarize its behavior in Figure 7.2. It may be modeled as a triangular barrier, and the bounce action may be estimated using the results in [119].

	Fermions				Bosons			
	Weyl mult.	mass	$U(N_c)$	$SU(\tilde{N}_c)_D$	Real mult.	mass	$U(N_c)$	$SU(\tilde{N}_c)_D$
$\mathrm{tr}X$	1	0	1_0	1	1 1	$\mathcal{O}(m_{CW})$ $\mathcal{O}(\sqrt{b}h^2\mu)$	1_0 1_0	1 1
X'	N_c^2-1	$h^2\mu_\phi N_c\gamma$	Adj_0	1	$2(N_c^2-1)$	$\mathcal{O}(m_{CW})$	Adj_0	1
Y, χ, $\tilde{\chi}$	$\tilde{N}_c^2$ $\tilde{N}_c^2$ $\tilde{N}_c^2-1$ 1	$\mathcal{O}(h\mu)$ $\mathcal{O}(h\mu)$ $g_{\mathrm{mag}}\mu$ $g_V\mu$	1_0 1_0 1_0 1_0	Adj Adj Adj 1	$2\tilde{N}_c^2$ $2\tilde{N}_c^2$ $2(\tilde{N}_c^2-1)$ 2	$\mathcal{O}(h\mu)$ $\mathcal{O}(h\mu)$ $g_{\mathrm{mag}}\mu$ $g_V\mu$	1_0 1_0 1_0 1_0	Adj Adj Adj 1
Z,$\tilde{Z}$, ρ, $\tilde{\rho}$	$2N_c\tilde{N}_c$ $2N_c\tilde{N}_c$	$\mathcal{O}(h\mu)$ $\mathcal{O}(h\mu)$	$\square_1+\overline{\square}_{-1}$ $\square_1+\overline{\square}_{-1}$	$\square+\overline{\square}$ $\square+\overline{\square}$	$2N_c\tilde{N}_c$ $2N_c\tilde{N}_c$ $2N_c\tilde{N}_c$ $2N_c\tilde{N}_c$	0_{NGB} $\mathcal{O}(h\mu)$ $\mathcal{O}(h\mu)$ $\mathcal{O}(h\mu)$	$\square_1$ $\overline{\square}_{-1}$ $(\square_1+$ $\overline{\square}_{-1})$	$\overline{\square}$ $\square$ $(\overline{\square}+$ $\square)$

Figure 7.3: The mass spectrum, including one loop corrections (but without Standard Model gauge interactions), grouped in sectors with Str $M^2=0$. Notice the appearance of the Goldstino in the tr (X) sector. The details of the spectrum are described further in the text. Notation is as in Figure 1.

We will see in the next section that, in order to have large enough gaugino masses but a low SUSY-breaking scale and low sfermion masses, the ratio μ_ϕ/μ cannot be made too small. Nonetheless, it is useful to first analyze the bounce action in the limit $\mu_\phi \ll \mu$, where it is clear the vacuum is parametrically stable.

The dimensionful parameters controlling the shape of the potential are μ and μ_ϕ. We assume h, γ, N_f, and N_c are all of order 1. The SUSY vacua are parametrically far away from the metastable vacua in the limit

$$\epsilon \equiv \left|\frac{\mu_\phi^2}{b\mu^2}\right| \ll 1 \; . \tag{7.38}$$

In this limit, the calculation of the bounce action is very similar to that done in [6], as long as only tr X varies. Let us assume $q\tilde{q}$ is essentially constant.

The metastable SUSY-breaking vacuum lies at $X_0 \sim \mu_\phi/b$, the peak of the potential is near $X_{\mathrm{peak}} \sim b\mu^2/\mu_\phi$, and the SUSY vacuum is at $X_{\mathrm{susy}} \sim \mu^2/\mu_\phi$, where phases and $\mathcal{O}(1)$ numbers have been ignored. Moreover, the potential difference between the peak and the metastable SUSY-breaking minimum is roughly $V(X_{\mathrm{peak}}) - V(X_0) \sim b\,\mu^4$, much smaller than $V(X_0) - V(X_{\mathrm{susy}}) \sim \mu^4$. The results of [119] then show that the field tunnels not to the SUSY vacuum directly but rather to $X_{\mathrm{tunnel}} \gtrsim X_{\mathrm{peak}}$. For this value of X_{tunnel}, Eq. (7.7) implies

$q\tilde{q} \approx \mu^2$, and thus $q\tilde{q}$ indeed stays approximately constant in the tunneling region. This confirms that the results in [119] apply.

In the limit $\epsilon \ll 1$, the bounce action scales parametrically as

$$B \sim \frac{(X_{\text{tunnel}})^4}{V(X_{\text{peak}}) - V(X_0)} \sim b\,\frac{1}{\epsilon^2}, \tag{7.39}$$

where we have neglected some numerical factors, see [119]. Thus, $B \to \infty$ as $\epsilon \to 0$, and the metastable vacuum can be made parametrically long-lived.

In Section 7.6, we will see that in order to obtain sfermion masses that are roughly of the same size as gaugino masses, we need to take $\mu_\phi \sim b\mu$ (and thus $\epsilon \sim b$.) In this regime X_0, X_{peak} and X_{tunnel} are all parametrically of order bX_{SUSY}. A numerical study is required to determine the existence and lifetime of the metastable vacuum. Taking the gaugino masses to lie at their experimental lower bound, of order 100 GeV, we find that the existence of a metastable vacuum sets a lower bound on the sfermion masses — typically a few TeV for the squarks and at least a few hundred GeV for the right-handed sleptons. Once such a metastable vacuum is obtained, it is easy to make the bounce action larger than the required 400 by a small increase (of order 5%) in the sfermion masses.

7.6 Comments on the phenomenology

This section briefly discusses some of the phenomenology associated with the multitrace deformation of the ISS model, equation (7.31).

The ISS-like supersymmetry breaking models are interesting from a phenomenological point of view due to the presence of the large global symmetry group

$$SU(\tilde{N}_c)_V \times SU(N_c) \times U(1)'\,. \tag{7.40}$$

A model of direct gauge mediation can be built by weakly gauging a subgroup of (7.40) and identifying it with the Standard Model (SM) gauge group. The fields ρ, Z, $\tilde{\rho}$, and $\tilde{Z}$ in (7.11) and (7.12) act as messengers that mediate the supersymmetry breaking effects to the visible sector. Loops involving these messengers can give non-zero masses to the scalar superpartners of the SM fermions and, provided there is no unbroken R-symmetry, non-zero Majorana fermion masses to the gauginos.

In this section, we will consider gauging the $SU(3) \times SU(2) \times U(1)$ subgroup of $SU(N_c)$ for $N_c = 5$ in the $\gamma \neq 0$ model, and identifying it with the SM gauge group.

Under the SM gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$, the adjoint field X' decomposes as

$$X' = X_{\mathbf{24}} = X_{(\mathbf{8},\mathbf{1})_0} \oplus X_{(\mathbf{1},\mathbf{3})_0} \oplus X_{(\mathbf{3},\mathbf{2})_{-5/6}} \oplus X_{(\bar{\mathbf{3}},\mathbf{2})_{5/6}} \oplus X_{(\mathbf{1},\mathbf{1})_0}\,. \tag{7.41}$$

The fermions from the superfields $X_{(\mathbf{8},\mathbf{1})_0}$, $X_{(\mathbf{1},\mathbf{3})_0}$, and $X_{(\mathbf{1},\mathbf{1})_0}$ carry the same gauge charges as the gluino, wino, and bino, respectively, and the first two could be directly produced at colliders.[4] Also, there are new light fermions from the superfields $X_{(\mathbf{3},\mathbf{2})_{-5/6}}$ and $X_{(\bar{\mathbf{3}},\mathbf{2})_{5/6}}$; these are stable unless given new interactions, and require a special discussion below.

7.6.1 Phenomenology of $\psi_{X'}$ and λ

A very important property of the model is that *the gauginos and the adjoint $\psi_{X'}$ do not mix.* This is due to the fact that λ and $\psi_{X'}$ have charge conjugation transformations that differ by a sign,

$$C(\psi_{X'_{ij}}) = \psi_{X'_{ji}}\ ,\ C(\lambda_{ij}) = -\lambda_{ji}\,. \tag{7.42}$$

This discrete symmetry forbids any mixing at low orders between the two sets of fermions. More precisely, C-violation in the SM allows λ and $\psi_{X'}$ to mix, but this occurs only at three loops and is thus negligibly small.

Let us estimate the gaugino and $\psi_{X'}$ masses. As discussed in Section 7.5.1, the metastable vacuum has an approximate R-symmetry that is spontaneously broken through the non-zero vev $X_0 \sim (1+N_c\gamma)\mu_\phi/b$, where $b \sim 1/(16\pi^2)$ is a loop factor (7.21). Therefore, gauginos obtain a one loop mass of order

$$m_\lambda \sim \frac{g^2}{16\pi^2}\, X_0 \sim g^2\,(1+N_c\gamma)\mu_\phi\,. \tag{7.43}$$

Neglecting $\mathcal{O}(1)$ numbers and factors of the gauge coupling g, an interesting phenomenology is obtained for

$$m_\lambda \sim \mathcal{O}\,(1 \text{ TeV})\,, \tag{7.44}$$

i.e. for

$$\mu_\phi \sim \mathcal{O}(1 \text{ TeV}). \tag{7.45}$$

The $\psi_{X'}$ also obtain a mass at one loop, which, using equation (7.36), is of order

$$m_{\psi_{X'}} \sim h^2\,\mu_\phi\,N_c\,\gamma \sim \gamma\,\times\,\mathcal{O}\,(1 \text{ TeV})\,, \tag{7.46}$$

neglecting factors of h and g and other $\mathcal{O}(1)$ numbers. By adjusting γ, $\psi_{X'}$ can be made heavier or lighter than λ, leading to very different collider signatures as we will discuss next.

The $\psi_{X'}$ do not mix with the Standard Model gauginos at a level that determines their decays. Instead, if they are heavy enough, they can decay (promptly) into a gaugino and a

[4]The X bosons in (7.41) get a mass of order $\sqrt{b}h^2\mu \sim \mathcal{O}(10 \text{ TeV})$ from the Coleman-Weinberg potential and are thus rather heavy. If produced in the early Universe, they would have decayed promptly into ψ_X and a gaugino, excepting gauge singlets which would decay a bit more slowly through higher dimension operators.

gauge boson through the dimension five operator $\psi_{X'}\sigma^{\mu\nu}\lambda F_{\mu\nu}$:

$$\psi_{X'} \to \lambda + \text{gauge boson}\,. \tag{7.47}$$

The gauginos can decay through all the usual supersymmetric decay modes, and/or through the standard coupling of each gaugino to a gauge boson and Goldstino:

$$\lambda \to \psi_{\mathrm{tr}\,X} + \text{gauge boson} \tag{7.48}$$

If instead the $\psi_{X'}$ are lighter than the gauginos, then the gauginos will decay into the $\psi_{X'}$ plus a gauge boson via the above-mentioned operator. The $\psi_{X'}$ decays to a gauge boson and an off-shell gaugino.

From (7.41), we see that there are *new* $(\mathbf{3},\mathbf{2})$ *fermions*, with charges $(\mathbf{3},\mathbf{2})_{-5/6}$ and $(\bar{\mathbf{3}},\mathbf{2})_{5/6}$. By binding to quarks, these form hadrons, some of which are charged. The lightest of these novel hadrons, whether charged or neutral, would be stable in the model as described so far. But this would be ruled out, since these hadrons would have been created in the early Universe, violating the bounds on the existence of heavy stable particles [139, 140]. These fermions must thus be made to decay through additional baryon-number violating operators in the superpotential and/or the Kähler potential. It is possible to show that additional dimension five Kähler potential terms, coupling the adjoint X' to SM quarks and leptons, can allow the $(\mathbf{3},\mathbf{2})$ fermions to decay without affecting Big-Bang Nucleosynthesis or violating current bounds on proton decay.

7.6.2 Sfermion masses, the SUSY-breaking scale and a light gravitino

Since the supersymmetry breaking scale is $|\sqrt{F}| = |\sqrt{h}\mu|$ and the mass scale of the messengers is of the same order, the soft scalar masses are roughly given by

$$m_S \sim \frac{g^2}{16\pi^2}\,\mu\,. \tag{7.49}$$

Comparing this to (7.43), the sfermions and gauginos have similar masses if

$$\mu_\phi \sim \mu/(16\pi^2). \tag{7.50}$$

We recall that the existence and longevity of the metastable vacuum requires $\mu_\phi \ll \mu$, see Section 7.5.2.

More concretely, there is an interesting parameter region characterized by (7.50) and a low supersymmetry breaking scale

$$\sqrt{F} \approx \mu \sim \mathcal{O}\,(100 - 200\ \text{TeV})\,. \tag{7.51}$$

In this case, one can show that the heaviest sfermions (squarks) have masses of a few TeV, the lightest sfermions (right-handed sleptons) haves masses of a few hundred GeV, the gaugino masses are of order several hundred GeV, and there is a large enough lifetime for the metastable vacuum. The gravitino mass is

$$m_{3/2} \sim \frac{F}{\sqrt{3}M_{\rm Pl}} \sim \mathcal{O}(1\text{–}10 \text{ eV}) \,, \tag{7.52}$$

where $M_{\rm Pl} \simeq 2.4 \times 10^{18}$ GeV is the reduced Planck mass. Such a light gravitino does not violate any cosmological or astrophysical constraints [141].

7.6.3 Further comments on the spectrum

As discussed in Section 7.5, the messenger sector (ρ, $\tilde{\rho}$, Z, $\tilde{Z}$) contains $2N_c\tilde{N}_c$ real NG bosons, all of which become massive at one loop after weakly gauging the flavor symmetry. In the parameter range (7.51), this mass is of order of several TeV. The $U(1)'$ messenger number in (7.26) forbids the decay of the lightest of these messenger particles, which is thus stable. If the lightest messenger is neutral and weakly interacting and has an appreciable relic density, it would have a tree-level coupling to nuclei via Z-boson exchange and would have been seen at a dark matter direct detection experiment [142]- [143]. If the stable state is charged and/or colored, the experimental constraints are even stronger [139,140]. Thus experimental constraints rule out the possibility that the lightest messenger is dark matter.

We also note that the SM gauge couplings have a Landau pole well below the GUT scale, due to the presence of extra matter charged under the SM gauge group. As one runs up to the high scale, the $SU(3)_C$ gauge coupling blows up first at about 10^9 (10^7) GeV for $\tilde{N}_c = 1$ (3), so that new physics has to enter at or below this scale. Larger values of $\tilde{N}_c$ lower this scale to the point that it affects our discussion materially. See [144] for a recent discussion of the Landau pole problem in ISS-like SUSY-breaking models.

7.6.4 Illustrative choices of parameters

Preliminarily, it appears possible to satisfy simultaneously all of the conditions considered above. For example, for $\tilde{N}_c = 1$,[5] the parameters of the electric theory Eq. (7.3) that are consistent with (7.50) and (7.51) are m of order 0.01–10 TeV, $\Lambda \sim 10^{3-5}$ TeV, and $\Lambda_0 \sim 10^{6-9}$ TeV. With these choices, the models appear to have no insuperable problem below the scale of the Landau pole.

[5] In this case, the magnetic gauge group is trivial and, after a field redefinition, the superpotential is given by (7.4) plus $\det\Phi/\Lambda^{N_c-2}$. For $N_c > 2$ this term is negligible near the origin, so our analysis is self-consistent.

On the other hand, for $\tilde{N}_c \geq 3$, Λ has to be below 10^3 TeV, and the ratio m/Λ is not parametrically small. In this case, the corrections from the microscopic theory are not guaranteed to be small, and the violations of the approximate symmetries may be large. In particular, the cancellations described in section 7.4.2 may be imperfect, requiring a more elaborate analysis. However, the argument for nonzero γ still holds, and its effects can still dominate, in which case the phenomenology outlined here will be largely unchanged.

7.6.5 Summary

While these models are not yet entirely plausible, they represent an advance over the models with $SU(N_c)$ gauged and $\gamma = 0$, which as we showed are excluded by the presence of overly-light charged and colored fermions. We have demonstrated that with $\gamma \neq 0$, it is possible to obtain models with a long-lived metastable vacuum, a spectrum with all standard model superpartners in the TeV range, and with no obvious unresolvable conflict with any experiment.

The minimal versions of these models have new TeV-scale fermions in the adjoint representations of the Standard Model gauge group that do not mix with standard model gauginos. They also have squarks and sleptons significantly heavier than the gauginos, and exotic stable hadrons which must be made to decay through additional interactions. They also suffer from the ubiquitous intermediate-scale Landau pole for standard model gauge couplings.

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Chapter 8

Curriculum Vitae
Gonzalo Torroba

Education

1998 – 2000	**National University of La Pampa**, Argentina *Beginning of B. S. in Physics*
2000 – 2004	**Balseiro Institute**, Bariloche, Argentina **Degree**: *B. S. and M. S. in Physics*
2004 – 2009	**Rutgers University**, Department of Physics, New Jersey, USA **Degree**: *PhD in Physics*

Academic experience and positions held

- **Balseiro Institute Fellowship** (2000 – 2004)
- **Teaching Assistant**, Department of Physics and Astronomy, Rutgers (2004 – 2006)
- **Graduate Assistant**, Department of Physics and New High Energy Theory Center, Rutgers (2006 – present)
- **Kavli Institute for Theoretical Physics**, graduate fellow (Fall 2008)

Publications

1. R. Essig, J. F. Fortin, K. Sinha, G. Torroba and M. J. Strassler, *"Metastable supersymmetry breaking and multitrace deformations of SQCD,"*, JHEP 0903:043,2009, arXiv:0812.3213 [hep-th].
2. M. R. Douglas, A. Frey, G. Torroba and B. Underwood, *"The universal Kähler modulus in warped compactifications,"*, JHEP 0901:036,2009, arXiv:0810.5768 [hep-th].
3. M. R. Douglas and G. Torroba, *"Kinetic terms in warped compactifications,"* arXiv:0805.3700 [hep-th].

4. G. Shiu, G. Torroba, B. Underwood and M. R. Douglas, *"Dynamics of Warped Flux Compactifications,"* JHEP **0806**, 024 (2008), arXiv:0803.3068 [hep-th].

5. R. Essig, K. Sinha and G. Torroba, *"Meta-Stable Dynamical Supersymmetry Breaking Near Points of Enhanced Symmetry,"* JHEP **0709**, 032 (2007), arXiv:0707.0007 [hep-th].

6. M. R. Douglas, J. Shelton and G. Torroba, *"Warping and supersymmetry breaking,"* arXiv:0704.4001 [hep-th].

7. C. D. Fosco, G. Torroba and H. Neuberger, *"A simple derivation of the Overlap Dirac Operator,"* Phys. Lett. B **650**, 428 (2007), arXiv:0704.2433 [hep-lat].

8. G. Torroba, *"Finiteness of flux vacua from geometric transitions,"* JHEP **0702**, 061 (2007), arXiv:hep-th/0611002.

9. C. D. Fosco and G. Torroba, *"Zero modes in non local domain walls,"* Phys. Lett. B **620**, 174 (2005), arXiv:hep-th/0505002.

10. C. D. Fosco and G. Torroba, *"Noncommutative theories and general coordinate transformations,"* Phys. Rev. D **71**, 065012 (2005), arXiv:hep-th/0409240.

11. D. H. Correa, C. D. Fosco, F. A. Schaposnik and G. Torroba, *"On coordinate transformations in planar noncommutative theories,"* JHEP **0409**, 064 (2004), arXiv:hep-th/0407220.

12. C. D. Fosco and G. Torroba, *"Planar field theories with space-dependent noncommutativity,"* J. Phys. A **38**, 3695 (2005), arXiv:hep-th/0405090.