PERCEPTUAL DECISIONS UNDER RISK IN A
MOTION EXTRAPOLATION PARADIGM

by

SHALIN SHAH

A thesis submitted to the
Graduate School - New Brunswick
Rutgers, The State University of New Jersey
in partial fulfilment of the requirements
for the degree of
Master of Science
Graduate Program in Psychology
written under the direction of
Manish Singh
and approved by

____________________________
____________________________
____________________________

New Brunswick, New Jersey
January, 2007
Classical work on decision-making has demonstrated systematic deviations from normative behavior. Recent experiments on movement planning, however, have shown that subjects can be indistinguishable from optimality in their visuo-motor decisions (Trommershaeuser et al., 2003a, 2003b). Unlike the classical case, the uncertainty in the visuo-motor case is internal to the subjects; it arises from their motor variability—variability in executing a motor action.

We asked whether observers can also combine the intrinsic variability in their perceptual representation with externally-specified reward-and-penalty structure to make optimal decisions. We extended a paradigm used to study the visual extrapolation of static contour geometry (Singh & Fulvio, 2005), to examine observers’ decisions under risk in extrapolating curved motion trajectories.

Observers viewed a dot moving along a parabolic trajectory disappear behind the straight-edge of a half-disk occluder. Their task was to “catch” the dot from the opposite curved side, by adjusting the angular position of an arc or “mitt”. In the risky conditions, a double-mitt was used, comprising a green reward region and a red penalty region, with partial overlap. Observers’ gains/losses were determined by the part of the double-mitt
that “caught” the dot. Variables manipulated were: trajectory curvature, penalty value, and mitt overlap.

Observers’ performance in the baseline condition was used to estimate individual bias and variability. Based on these, predictions of optimal shift and optimal score were computed, using maximization of expected gain. We found that observed shifts were well predicted by optimal shifts. Moreover, observer efficiency (observed/optimal score) was high (80% - 114%). The results indicate that observers are implicitly aware of the intrinsic variability in their perceptual representation, and can combine it with externally-specified reward-and-penalty structure to make near-optimal decisions.
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abstract</td>
<td>ii</td>
</tr>
<tr>
<td>Introduction</td>
<td>1</td>
</tr>
<tr>
<td>Methods</td>
<td>16</td>
</tr>
<tr>
<td>Results</td>
<td>21</td>
</tr>
<tr>
<td>Discussion</td>
<td>30</td>
</tr>
<tr>
<td>Figures</td>
<td>37</td>
</tr>
<tr>
<td>References</td>
<td>49</td>
</tr>
</tbody>
</table>
INTRODUCTION

Making choices and decisions is vital to our day-to-day life. People make several important choices in their lifetimes—choosing a career, selecting a mate, choosing a place to live or relocate, and so on. Moreover, people have several options to choose from when they make any particular decision. It is in their best interest to make a choice that would optimize their general well-being. Economists and rationalists have claimed that, when making a decision, people should choose options that maximize their Expected Gain.¹

As a simple example, suppose one is faced with a choice between two lotteries. Lottery A has a 70% chance of winning $900 (and 30% chance of winning nothing), and Lottery B has an 80% chance of winning $600 (and 20% chance of winning nothing). The Expected Gain of Lottery A is then given by .70 x $900 + .30 x $0 = $630. The Expected Gain of Lottery B is given by .80 x $600 + .20 x $0 = $480. Hence, Lottery A has the higher Expected Gain, and it is therefore more lucrative to choose Lottery A. Expected Gain provides a normative prediction, i.e., the model describes what people should do in such a situation (namely, choose the lottery with the higher Expected Gain).

¹ It should be noted that gain does not necessarily refer to monetary gains and losses, but also reflect, e.g., pleasurable or painful experiences, and biomechanical costs.
Normative models and theories generally describe what people ought to do in order to act rationally. On the other hand, descriptive theories describe what people actually do in different situations. Over the years, psychologists have demonstrated that people deviate systematically from these normative models; many of these studies are reviewed below. There have been serious debates in the literature about whether a proposed normative model or rule is a good description of subjects' rationality; these debates thus highlight the question of what is ultimately “rational”.

**Cognitive decision-making**

Studies in cognitive decision-making have shown that humans exhibit systematic deviations from normative decision theory (Bazerman, 2006; Kahneman and Tversky, 1973; 1979; Shafir, 2000; Tversky & Kahneman, 1982). In one of the seminal studies in the field of decision-making, Kahneman and Tversky (1979) demonstrated the now well-documented phenomenon of risk-aversion (first noted by Bernoulli, 1738). For example, subjects were found to prefer Lottery A (a sure gain of $800) over Lottery B (85% chance of winning $1000; 15% chance of winning nothing), even though the expected gain of Lottery B ($850) is higher than that of Lottery A ($800). In general, subjects were found to prefer a sure (i.e., riskless) outcome with a lower payoff to an outcome that had a higher payoff but
was uncertain and risky. Moreover, Kahneman and Tversky (1979) also showed that subjects were risk seeking for gains and risk averse for losses for low values of probabilities, and they were risk averse for gains and risk seeking for losses of high probability values. They proposed that a non-linear transformation of the probability scale may be responsible for the non-normative behavior of humans in these studies. Specifically, they suggested that humans tend to underweight moderate to high probability values, and overweight low probability values. Similarly, Tversky & Fox (1995) found that when subjects were not explicitly aware of the probability values, they behaved in a way that deviated from the rational model. For example, observers were found to exaggerate the impact of an event as it changed from impossible to possible, or when it changed from possible to certain, as compared with when it became more or less possible (by an equal amount).

In explaining why people often make decisions or choices that are not aligned with a rational model, researchers have proposed that subjects use various heuristics in making decisions rather than perform computations consistent with a normative model. Some have suggested that the limited capacity of the brain as an information-processing system makes the use of heuristics very important in making quick judgments (Thaler, 1992). Although these heuristics can sometimes lead to
normatively correct choices, they also generate various deviations (or “errors”) from normative behavior.

A prominent such heuristic is the representativeness heuristic (Kahneman & Tversky, 1973; Tversky & Kahneman, 1982), in which subjects make their judgments simply based on how representative, or similar, an item or person is to various categories, while ignoring the baseline frequency of occurrence of those categories. This heuristic is closely related to the phenomenon of base-rate neglect. This refers to subjects’ failure to use relevant information available to them, in the form of baseline frequencies, in making their decisions. As an example, given some personality description of a student, subjects were asked to judge the likelihood of this student being in one of nine graduate programs. The subjects were also aware of the base rate, i.e., the baseline (or “prior”) probabilities, of a student being in each of the nine graduate programs. Kahneman and Tversky (1973) found that subjects tend to ignore the base rates in making their judgment. As a result, their choice is based on the personality description alone. Some researchers have suggested that other fallacies like the attribution error (i.e., attributing an event or behavior to someone based on personality information, while ignoring situational factors; Jones & Harris, 1967) can also be partly explained in terms of base-rate neglect (Nisbett, Borgida, Crandall, & Reed, 1976).
Deviations from normative behavior also occur as a result of framing. A frame consists of a mental model for a decision problem that individuals use to solve the problem (Johnson-Laird, 1983). It includes information about the problem, as well as the context. Normative theories assume that the solution to such a problem should be independent of the context. Hence, as long as the problem contains the same information, the solution to that problem should not change. However, a number of studies have demonstrated that this is not the case with human subjects.

Kahneman and Tversky (1979) were one of the first to observe that subjects will tend to hold on to whatever frame of the problem they were initially presented with, and not transform the problem in a way that would make it more conducive to normative analysis. For example, Tversky and Kahneman (1981) were able to demonstrate violations of the invariance principle (namely, that the solution to a problem should not vary with how it is framed). In this study, they presented subjects with two versions of the same decision problem—one framed as a gain, the other as a loss. Normatively, subjects should make the same choice irrespective of what version they were presented with (the invariance principle). However, the experimenters found that subjects tend to be risk seeking when presented with a version framed as a loss, and conversely, they tend to be risk-aversive when the problem is framed as a gain. This causes them to make different choices in the two cases.
Based on the above studies, Kahneman and Tversky (1979) proposed that “losses loom larger than gains”—a tendency known as loss aversion. This tendency manifests itself in other observed phenomena like the endowment effect, namely, placing a higher value in an object that one owns. For example, Thaler (1980) conducted a study in which subjects were queried about how much they would be willing to give up their special “reserve” wine bottle for. The results showed that people often demand much more to give up an object they own, relative to what they would pay to acquire it. One explanation of this phenomenon is that giving up an object that one owns is interpreted as a loss, and the subjective or ‘mental’ value associated with that loss is much greater than the corresponding gain value associated with acquiring the same object.

Loss aversion also partly expresses itself in the ‘Sunk-Cost’ effect. Arkes and Blumer (1985) performed several studies in which they demonstrated that subjects tend to continue to endeavor in some enterprise, just because of prior investment in it, even though it may not be worth it, normatively speaking. For example, when college students were asked to choose between a $50 ski trip that would be more enjoyable, as opposed to a $100 ski trip which would be less enjoyable (both of which they had mistakenly paid for), more students chose the more expensive (but less enjoyable) trip because of their higher prior investment in it.
The work on heuristics and biases summarized above has often been taken to argue that the human brains may not be suited to thinking in purely probabilistic and statistical terms (e.g., Piattelli-Palmarini, 1994). However, it is important to note that in the studies by Kahneman and colleagues, the probabilistic information was presented to subjects in an explicit symbolic—verbal or numerical—form (“decision by description”). In real life, though, humans and animals generally acquire probabilistic information about pay-offs implicitly, based on experience and interaction with their environment (e.g., Gigerenzer & Hoffrage, 1995; Weber, Shafir, & Blais, 2004).

In recent years, some studies examining the behavior of human subjects have been carried out under the paradigm of experiential decisions, where probabilistic pay-offs are experienced interactively, rather than described symbolically as in the classical paper-and-pen decision-making tasks. The results of such studies show the subjects’ performance is closer to the normative predictions based on maximization of Expected Gain. For example, Barron & Erev (2003) conducted a study in which the subjects were not overtly supplied with the pay-off distributions for the different outcomes. Rather they were presented with 2 virtual buttons on the computer screen, and were instructed to make binary choices that resulted in pay-offs from underlying distributions. Thus the probabilities of
the outcomes were learned through repeated choices and experiencing their outcomes. Barron & Erev (2003) found a substantial reduction in non-normative behavior resulting from risk aversion. In contrast to Kahneman & Tversky's findings, a majority of subjects in their study chose the risky option with a higher expected value (e.g., $4 with probability 0.8; $0 otherwise) over a certain option with lower expected value ($3 for sure) (see also Hertwig, Barron, Weber, & Erev, 2004). Although the behavior of their subjects was not perfectly normative (e.g., loss aversion was still observed, and there was an underweighting of rare events due to recency effects), in general, their subjects were closer to maximizing expected gains than in classical decision making tasks.

These and other similar studies with different variations (Hertwig et al., 2004; Weber et al., 2004) together indicate that when information about external uncertainty and pay-offs is obtained through direct experience and feedback, subjects' behavior is more likely to be aligned with a rational model than when they are provided this information in summarized descriptive form.

**Motor decision making**

A common element in the pay-off distributions in the cognitive decision-making studies reviewed above is that this uncertainty is external to the
observer, hence the source of the risky choices is external. These studies do not consider any form of uncertainty arising from within the individual (e.g., in the representational state of an observer, or in the execution of a motor action).

In recent studies, Trommershäuser, Landy, & Maloney (2003a, 2003b) have conducted a series of experiments investigating whether observers take into account their own implicit motor uncertainty (or variance) when making visuo-motor decisions. Contrary to the research on decision-making in the cognitive domain, Trommershäuser et al. have found that observers are near optimal in the visuo-motor domain.

Trommershäuser et al. (2003a, 2003b) used a touch-screen monitor for their studies. Their stimuli consisted of the following (see Figure 1):

a) A red penalty circle. The penalty value associated with landing within this circle was either 100 or 500 points.

b) A green reward circle. The reward value associated with landing within this circle was fixed at 100 points.

The participants’ task was to hit the green circle with their index finger (starting with the finger placed on the space bar of the keyboard), and to accumulate as many points as possible over the course of the experiment. They were penalized if they did not respond (i.e., hit the screen) within 700
milliseconds. At the end of each trial, observers were shown the number of points made on that trial.

The variables manipulated in their study were:

1. The penalty value associated with the red circle (either 100 or 500). The value remained fixed within an experimental session, but alternated across sessions.

2. Degree of overlap between the penalty and reward circles. The green reward region was displaced horizontally relative to the penalty region either to the left or the right. There were 3 possible magnitudes for the displacement on either side, yielding a total of 6 different “overlap” values (see Figure 1).

In the absence of any penalty region (the red circle), observers would be expected to simply aim for the center of the green circle. However, with a red circle present (and partly overlapping the green circle), such a strategy becomes suboptimal because the observer’s finger would land within the red circle on some proportion of the trials (due to the observer’s intrinsic motor variability). In such a case, the optimal strategy would be to move the target location further away from the red circle. The exact magnitude of the optimal shift would depend on (i) the degree of overlap between the red and green circles (the higher the overlap, the greater the predicted shift), (ii) the penalty value associated with the red circle (the higher the
penalty, the greater the predicted shift), and (iii) the observer’s own internal motor variability (the larger the variability, the greater the predicted shift). It should be noted that this last prediction assumes that observers are in fact aware of their own motor variability. Trommershäuser et al. (2003a, 2003b) independently measured each observer’s motor variability, and used that variability to derive the predicted optimal shifts, using the maximization-of-expected-gain (MEG) model. For example, the MEG model would predict that observers with higher intrinsic variance should shift their target point further away from the penalty circle.

To make this more precise, consider a fixed stimulus (i.e., fixed overlap between the red and green circles, and fixed reward and penalty values associated with the two circles). We assume, following Trommershäuser et al., that when an observer aims for a given target location \((x, y)\) on the screen, the actual landing position \((x', y')\) is determined by a 2D Gaussian distribution centered on \((x, y)\), with some isotropic variance \(\sigma^2\) that can differ across observers. Each possible landing position is thus an “outcome,” whose probability is determined by (i) the point \((x, y)\) that the observer is aiming for, and (ii) the observer’s motor variability \(\sigma\).

The Expected Gain associated with each target location \((x, y)\) is given by:

\[
EG(x, y | \sigma) = \text{Gain(green circle)} \cdot p(\text{green circle} | x, y, \sigma) \\
+ \text{Gain(red circle)} \cdot p(\text{red circle} | x, y, \sigma) \quad (1)
\]
where the “gain” associated with the red circle is either -100 or -500; and 
\( p(\text{red circle} \mid \bar{x}, \bar{y}, \sigma) \) is determined by integrating the volume of the 
bivariate Gaussian distribution \( N(\bar{x}, \bar{y}, \sigma) \) that falls within the bounds of 
the red circle. Computing the expected gain using Equation (1) for each 
possible target location \((\bar{x}, \bar{y})\) on the screen, one obtains a 2-dimensional 
Expected-Gain surface. Figure 2 shows the gain surface for the stimulus 
setup shown in Figure 1. The precise shape of this gain surface depends 
on the individual variance of the observer, the overlap between the red 
and green circles, and the penalty value associated with the red circle. 
The target location that maximizes the expected gain (the peak of the gain 
surface) determines the optimal shift. The height of the surface at this 
location gives the optimal score. These two values—optimal shift and 
optimal score—provide two normative measures against which observers’ 
behavior (their actual shift and points scored) can be compared, in testing 
for optimality.

As noted above, one would expect observers’ shift from the center of the 
green circle to increase with the penalty value associated the red circle, 
and also to increase with the degree of overlap between the two circles. 
Both of these predictions were confirmed in Trommershäuser et al. ’s 
(2003a, 2003b) experiments. Observers’ shift values were well predicted 
by the optimal shifts in the corresponding conditions. Moreover, each
observer’s total score was very close to the optimal score for that observer (i.e., based on his/her individual variance). As a result, the total efficiency value for observers (ratio of total obtained score to total optimal score) was very high—ranging from 97% and 108%. Finally, observers with higher intrinsic variance scored lower overall than those with a lower variance, consistent with the optimality analysis. Such observers also shifted away more on average than observers with lower variance, indicating that observers were aware of their variability in motor execution.

Trommershäuser et al. (2003a, 2003b) were thus able to show that observers are implicitly aware of their internal uncertainty, and can adapt their behavior in a given task to maximize their expected gain. It should be noted, however, the implicit uncertainty in their experiments arose from motor variability, i.e., variability in executing a motor action. An intriguing question is whether observers are also implicitly aware of the intrinsic variability in their perceptual representation, and whether they can use this variability, along with the reward and penalty structure in a given perceptual task, in making optimal perceptual decisions.

Perception and perceptual decisions

In contrast to the Trommershäuser et al.’s study where the intrinsic variability arose from the execution of a motion action, the current study is
concerned with variability in perceptual representation itself. In particular, the question it addresses is: *Can observers combine their own intrinsic perceptual variability, and an externally specified reward and penalty structure, to make optimal perceptual decisions?*

To address this question, we adapted a perceptual task used by Singh and Fulvio (2005; in press), and added a reward and penalty structure to it. Figure 3(a) shows the stimulus used in their study. The stimulus consisted of a half-disk occluder and an inducing contour that disappeared behind the straight edge of the occluder. A short line probe was placed around the curved edge of the occluder. On each trial, the observer iteratively adjusted two parameters in order to optimize the percept of extrapolation: (a) the angular position of the probe along the curved edge, and (b) the orientation of the probe (see Figure 3b). The study manipulated three variables: shape of the inducing contour (linear, circular, or parabolic), curvature of the inducing contour (4 values for each non-linear shape), and radial distance from the point of occlusion (6 values, ranging from 0.71 – 4.1 degrees of visual angle). Their study thus mapped out observers’ visually extrapolated contours, in terms of angular position and orientation measured at multiple distances from the point of occlusion.
For the purpose of the current study, the most relevant results from Singh & Fulvio (2005) were the following. The visual system takes into account the curvature of the inducing contour in extrapolating contours; there was a linear increase in the curvature of the extrapolated contour with increase in inducer curvature. However, there was an associated ‘cost of curvature,’ in that increase in contour curvature led to a systematic decrease in the precision with which the extrapolated contour was represented. The shape of visually extrapolated contours was characterized by a monotonic decrease in curvature, irrespective of the nature of the inducing contour (see also Singh & Fulvio, in press). In particular, it did not matter whether the inducing contour was circular or parabolic—a parabolic model always explained the extrapolation data better than a circular-arc model.

In modeling these results, Singh & Fulvio (2005) formulated a Bayesian model, involving the interaction between two shape constraints. The prior in their model captured a default preference for minimizing curvature (a tendency toward straightness), and was expressed as a probability distribution on curvature, centered on 0. The likelihood captured the tendency to continue the estimated curvature of the inducer (a tendency toward co-circularity); it was expressed as a distribution centered on the estimated inducer curvature at the point of occlusion. The model assumes a Weber-like dependence of—i.e., a linear increase in—standard deviation
in extrapolation curvature with distance from the point of occlusion. Near the point of occlusion, the likelihood dominates the prior because of its high reliability (low variance), and hence the extrapolated contour is maximally curved there. As the distance from the point of occlusion increases, the decrease in the reliability of the likelihood leads to a systematic shift in relative weights from the likelihood to the prior (i.e., the prior starts to dominate the likelihood), hence causing a systematic decay in curvature.

In the current study, we adapted the stimulus setup used by Singh and Fulvio (2005). We modified their task into a motion-extrapolation task, i.e., involving the extrapolation of the curved motion trajectory of a moving dot, and we added a reward/penalty structure to it.

**METHODS**

The experimental paradigm used by Singh & Fulvio (2005) is well suited for our study because it can be readily adapted to include a reward/penalty structure. To create a more natural task involving rewards and penalties, their task was modified from extrapolation of a static contour to extrapolation of a dot moving along a curved trajectory (i.e., the contour now defines the trajectory of a moving dot). Hence, the current experiment becomes one in which observers’ task is to ‘catch’ a moving
dot from the opposite side of the occluder, in a way that maximizes their gain. One relevant result from the Singh & Fulvio (2005) study is that variability in extrapolation settings increases with inducer curvature. An intriguing question, assuming the same is true in extrapolating the curved trajectory of a moving dot, is whether observers can take their variability into account, and combine it with an externally specified reward-and-penalty structure, to maximize their gains in a perceptual decision-making task. This is the basic question that the current study investigates.

**Observers**

6 observers participated in the study. All were affiliated with Rutgers University, and were naïve about the purposes of the experiment.

**Stimuli**

The stimuli consisted of 3 components:

a) A blue half-disk occluder of radius 4.06 deg of visual angle, with its straight edge vertical.

b) A dot (6 pixels in diameter) moving along a parabolic trajectory (with one of two levels of curvature). The length of the visible component of the trajectory was 4.56 degrees of visual angle. The speed of the moving dot was 2.82 degrees/sec.

c) A “double mitt” consisting of two partly-overlapping curved segments (or “mitts”), each subtending 20 degrees of polar angle (see
Figure 4). One of these was assigned to be the reward region (green in color) and the other was assigned to be the penalty region (red in color).

**Design**

The following variables were manipulated:

i. *Curvature* of the parabolic path, measured at the point where the dot disappeared behind the occluder (the vertex of the parabola). The two values of curvature used were: 0.1185 and 0.237 deg$^{-1}$.

ii. The magnitude of the *penalty* associated with the red portion of the double mitt. The two penalty values used were: 200 and 500. The penalty was manipulated between experimental sessions; it remained constant within a given session.

iii. The *overlap* between the red and the green portions of the double-mitt. The magnitude of the overlap was either one-half or one-quarter of the length of each separate mitt (hence, either 5° or 10° of polar angle). In addition, the green portion of the mitt either had a higher polar angle than the red (designated as positive overlap), or a lower polar angle (designated as negative overlap); see Figure 4. Hence, four overlap conditions were possible: +0.25, +0.50, -0.25, and -0.50.
Procedure

Observers’ task was to “catch” the moving dot with the green, reward, portion of the double-mitt. They first viewed the dot moving along a parabolic trajectory and disappear behind the straight edge of the occluder (at its mid-point). The orientation of the parabolic trajectory (measured at the point of occlusion) was randomly set between $\pm (15^\circ - 45^\circ)$. Thereafter, observers adjusted the angular position of the double-mitt, along the curved edge of the occluder, in order to maximize their chances of catching the dot with the green mitt, while avoiding the red mitt. The observers’ setting of the double-mitt was coded in terms of the polar angle of the center of the green mitt. Once they had made their setting, they were shown the dot emerge from the other side of the occluder (i.e., moving along the extension of its parabolic path). The observers could thus see where the moving dot would have passed relative to their placement of their double mitt. Moreover, the reward or penalty value incurred by their placement was displayed on the screen. Rewarded points were displayed in green with a positive sign, and penalized points were displayed in red with a negative sign.

It should be noted that the task was not set up as a real-time or “active” catching task; once the dot disappeared behind the occluder, the observers did not see it emerge from the other side until after they had made their setting. No time constraints were imposed on the observers in
making their setting. In fact, they were given the option of watching the motion sequence multiple times if they so wished. This was done because our interest was not in the motor variability that would arise from a real-time catching task, but rather in the variability in the perceptual representation of the visually-extrapolated trajectory.

If the dot passed through the green portion of the double mitt, observers received a reward of 200 points. If it passed through the red portion, they incurred a penalty of either 200 or 500 points, depending on the experimental session. Passing through the overlap region earned them both the reward and the penalty associated with those regions (0 or -300, depending on the experimental session). Missing the double-mitt altogether incurred no penalty and no reward. The observers’ goal was to make as many points as possible. Observers were awarded 50c for every 1000 points made, in addition to the payment of $14 per hour for participating in the experiment.

Each subject performed adjustments in 6 experimental sessions (preceded by 1 practice session). The 6 experimental sessions included 2 baseline sessions and 4 risky sessions:

a. Baseline Sessions:
The first and last sessions comprised the baseline condition. The baseline condition lacked the penalty portion of the double-mitt and hence consisted of just a single green mitt. The baseline condition was used to estimate the individual bias and variability of each observer. The baseline sessions consisted of 48 trials each (2 curvatures x 24 repetitions).

b. Risky Sessions:
There were 4 risky sessions (sessions 2 through 5) that contained both the reward (green) and penalty (red) mits. Two sessions had a penalty of 200 associated with the red mitt, and two had a penalty of 500. Their order was counterbalanced. Each session consisted of 72 trials (2 curvatures X 4 mitt overlaps X 9 repetitions).

RESULTS

To evaluate if the observers performed optimally (taking into account the extrinsic overlap and penalty, as well as their intrinsic variability), we used two measures:

a. **Shift** in the angular setting in the risky condition relative to the baseline condition. Observers' shift was compared against the optimal shift (i.e., the shift predicted by the maximization-of-expected-gain model).
b. \textit{Score} or the total number of points made. This was again
compared against the total optimal score, based on the maximization-of-
expected-gain model.

It should be emphasized that both predicted optimal shift and optimal
score depend on the observers' intrinsic variance. Hence, the optimal shift
and optimal score values were computed individually for each observer,
based on their own variance estimated from the baseline condition.

\textbf{Baseline condition:}

Observers' settings in the baseline condition were used to estimate their
intrinsic bias and variability. Settings were collapsed over the two baseline
sessions as preliminary analyses revealed no systematic differences
between them. Figure 5 shows the polar histograms for all observers, for
both curvature levels. The curves denote the true extensions of the
parabolic trajectories.

The bias was defined in terms of the mean of an observer's settings for
each curvature level. It served as the baseline for an observer, relative to
which the "shift" was measured in the risky conditions.
The variability was defined in terms of the standard deviation of an observer’s settings for each curvature level. It was used to compute the optimal shift and optimal score for each observer (see below).²

² We initially fit skew-normal distributions (see, e.g., Azzalini & Capitanio, 2003) to the baseline histograms, to capture any possible skew. However, in most (9/12) cases, the estimated skew was not statistically different from 0. Hence, we treat these histograms as arising from Gaussian distributions that can be summarized simply by their mean and standard deviation.
Observed versus Optimal Shift

Given an observer’s intrinsic variance (estimated from the baseline condition), the optimal shift for a given risky condition is defined as the setting, measured relative to the baseline, that maximizes the observer’s expected gain. Figure 6 clarifies the notion of optimal shift. The Gaussian distribution in this figure depicts an observer’s hypothetical perceptual estimate of extrapolation, i.e., the distribution of perceived exit locations for a given motion trajectory. If the double-mitt is placed too close to the mean of the distribution (relative to its variance), as in Figure 6(a), the probability of the dot passing through the red portion of the double-mitt becomes large, so the observer stands to lose points. Conversely, if the double-mitt is placed too far from the mean of this distribution, as in Figure 6(b), the probability of the dot passing through the green portion of the double-mitt is extremely low; so the observer is unlikely to make any points whatsoever. The optimal setting of the double-mitt would thus be some intermediate position (see Figure 6(c)) that maximizes the probability of the dot passing through the green mitt (the mass of the distribution falling within the green mitt), while minimizing the probability of the dot passing through the red mitt (the mass of the distribution falling within the red mitt).

To express this quantitatively, let $\theta^*$ be the mean of the Gaussian distribution, and $\theta$ be the setting of the double-mitt (as before, coded in
terms of the polar angle of the center of the green mitt; see Figure 7(a)).

Then, the expected gain for a given shift value \(\theta - \theta^*\) is given by:

\[
EG(\theta - \theta^* | \sigma) = \text{Gain(green mit)} \cdot p(\text{green mit} | \theta - \theta^*, \sigma)
+ \text{Gain(red mit)} \cdot p(\text{red mit} | \theta - \theta^*, \sigma)
\]

(2)

where rewards are coded as positive gains, and penalties as negative gains. The term \(p(\text{green mit} | \theta - \theta^*, \sigma)\) corresponds to the probability that the dot will pass through the green portion of the mitt, and is determined by integrating the area under the Gaussian distribution around \(\theta^*\) that falls within the green mitt. Similarly, \(p(\text{red mit} | \theta - \theta^*, \sigma)\) is given by the area under the Gaussian distribution that falls within the red mitt. Because of the presence of overlap between the two portions of the double-mitt, these two possibilities are not mutually exclusive.

By using Equation (2) to compute the expected gain for all possible values of shift \(\theta - \theta^*\), one obtains the expected gain profile \(EG(\theta - \theta^*)\) (see Figure 7(b)). The optimal shift is now obtained by determining the shift value that maximizes the expected-gain function (Figure 7(b)). Hence, given an observer's bias and intrinsic variability (i.e., mean and standard deviation, respectively, in the baseline condition), the optimal shift can be computed.
for each risky condition (i.e., each combination of penalty value and overlap).

Figure 8 plots the observed-shift values versus the optimal-shift values, for all observers across all conditions (penalty = -200/-500, overlap = -/+ .5, -/+ .25, curvature = .1187/.237 deg⁻¹). The data points lie close to the identity line suggesting that the observed shifts are generally well predicted by the optimal shifts. In order to assess this quantitatively, we fit regression lines to the data for each penalty condition (penalty = -200; penalty = -500).

We first performed a test to estimate if the slope of the line was significantly different from 0. A slope of 0 would indicate that the observers are not shifting the angular position of the double-mitt in accordance with optimal behavior. Not surprisingly, in all 12 cases (6 subjects for each penalty condition), the null hypothesis of 0 slope was rejected (at the .05 level, with a Bonferroni correction for multiple tests). Secondly, we tested if the slope was significantly different from 1. A slope of 1 indicates that the observers are shifting perfectly in accordance with the shifts derived from the MEG model. A slope greater than 1 indicates that an observer shifts too much—is overly “afraid”—relative to the MEG model. A slope less than 1 indicates that the observer did not shift sufficiently. In 8 of the 12 tests, we could not reject the null hypothesis of a slope of 1 at the .05 level (with
a Bonferroni correction). Of the remaining 4 cases, one deviation from a slope of 1 occurred in the 200 penalty condition, and three occurred in the penalty 500 conditions. All except one involved a slope smaller than 1, i.e., an underestimation of shift. Overall, the tests indicate that observers generally responded in accordance with the optimal model.

The plots in Figure 8 also suggest a systematic influence of penalty and overlap on observers’ shift values, with the higher penalty conditions (red symbols), for example, generating greater shifts that the lower penalty conditions (blue symbols). To investigate the influence of overlap and penalty, we plotted observed shifts as a function of mitt separation (= reward-mitt center – penalty-mitt center; see Figure 9). Mitt separation is inversely related to overlap: an overlap of -.25/+.25 corresponds to a mitt separation of -15/+15 degrees, and an overlap of -0.5/+0.5 corresponds to a separation of -10/+10 degrees. Also shown in Figure 9 are the optimal-shift curves. The red and blue curves correspond to the high and low penalty conditions respectively, whereas the solid and the dashed curves correspond to high and low curvatures, respectively. These curves were derived by computing optimal shifts for values of mitt-separation ranging from -20 to 20 degrees (see Figure 9). The observer’s data points are similarly coded. The red and blue points denote high and low penalty conditions respectively. The star and the open circle denote high and low curvature conditions respectively.
As is evident from Figure 9, as the magnitude of mitt separation decreases (i.e., as overlap increases), the predicted magnitude of the optimal shift increases. Moreover, the high-penalty conditions (red curves) generate a higher predicted magnitude of optimal shift than the corresponding low-penalty conditions (blue curves). Figure 9 shows that observers’ shifts exhibit these same dependencies on mitt separation (hence overlap) and penalty values: the observed shift magnitudes increase with decreasing mitt separation (i.e., increasing overlap), and increase with increasing penalty. These data thus indicate that observers take into account the extrinsic reward and penalty structure (overlap and penalty value) imposed by the experiment in performing the task.

An assessment of optimal behavior must consider not only whether observers take into account extrinsic costs, but also if they take into account their own intrinsic variance. The manipulated variable that affects observers’ variance in our task is the curvature of the parabolic path—with higher curvature expected to generate higher variance. However, the observed difference in variance between the two curvature conditions is quite small within each observer (see Figure 10). (Note that, consistent with this, the optimal curves corresponding to the two curvature levels in Figure 9 are very close to one another, within each observer’s plot.) Thus, to investigate the influence of observers’ intrinsic variance on their shifts, we plotted observed shift magnitudes as a function of observers’ intrinsic
variability (their standard deviations in the baseline condition) across all observers (Figure 10). Although there is a great deal of variance in observed shifts for each standard deviation value (arising from the penalty and overlap conditions), there is nevertheless a statistically reliable influence of intrinsic variability (SD) on observed shifts (slope = 0.455, p < 0.05). Observers with higher intrinsic variability tend to shift the double-mitt further away the mean of their perceptual estimate than observers with lower intrinsic variability. Observers thus do take into account their intrinsic perceptual variability in making perceptual decisions.

**Observed versus Optimal Score**

The second measure we employ to assess observers’ performance is their score. As before, we compare observers’ scores in various conditions to the corresponding optimal scores. The *optimal score* is the total score predicted by the maximization-of-expected-gain model (hence by shifting the mitt through the optimal shift value; see above) for each trial of the experiment. It is therefore determined by a combination of the penalty and overlap conditions, as well as each observer’s individual intrinsic variance.

Figure 11 plots observers’ total scores against the corresponding optimal scores across the different conditions of penalty, overlap, and curvature (penalty= -200/-500, overlap = -/+ .5, -/+ .25, curvature = .1185/.237
deg$^1$). Data points lie close to, or just below, the identity line for all observers, again suggesting near-optimal performance. To quantify how well observers did in terms of their total score, we defined each observer’s overall *efficiency* in terms of the ratio of their total score to the total optimal score. The efficiency scores ranged from 80% to 114%, which indicates near-optimal behavior$^3$ (see Figure 12).

**DISCUSSION**

Decision-making has been studied extensively in the cognitive domain by a large number of investigators, starting from the work of Kahneman and Tversky (1973, 1979). Most work in this domain demonstrates that humans deviate systematically from normative or rational theories of decision-making. The extensive research in this area has led to the identification of several heuristics and biases that humans employ in making choices. This approach is consistent with Simon’s (1957) notion of ‘bounded rationality,’ according to which the capacity of the brain as an information-processing system is limited; hence using heuristics provides a way to make quick decisions in real time. A consequence of these heuristics is that they can lead to deviations from rational theories, and errors in judgment. Others have argued that the human brain may not be suited to deal with information in purely probabilistic terms (e.g., Piattelli-

---

$^3$ It should be noted that the "optimal" score is the *expected score* based on using an optimal strategy (i.e., shifting by the amount predicted by the MEG model). It should not be confused with the highest score that an observer can make.
Palmarini, 1994). These issues have led to an ongoing debate between normative/prescriptive theories and descriptive theories of human reasoning and decision-making.

More recent experiments in the cognitive domain have revealed an interesting insight: Subjects’ performance is closer to the normative prediction when the probability information is conveyed to them implicitly through experience and feedback (e.g., if subjects are allowed to sample repeatedly from pay-off distributions), rather than given explicitly in a descriptive form, as in classical paper-and-pencil decision-making tasks. Thus, when information about external uncertainty is obtained through interactive experience, people are able to learn the pay-off distributions and use them in making decisions (Barron & Erev, 2003; Hertwig et al., 2004; Weber et al., 2004).

Recent investigations by Trommershäuser et al. (2003a, 2003b) have extended the study of decision-making to the visuo-motor domain. Unlike the cognitive domain, the uncertainties in this domain are internal to the observer, arising from observers’ own motor variability: when an observer aims for a point $(x, y)$ on the screen, the finger’s actual landing position $(\bar{x}, \bar{y})$ is determined by a bivariate Gaussian distribution centered around $(\bar{x}, \bar{y})$. The studies by Trommershäuser et al. (2003a, 2003b) demonstrated that observers’ performance is essentially indistinguishable
from optimal in a visually-guided motor task involving reward and penalties. These studies also showed that observers are implicitly aware of their intrinsic motor variability, and can combine it with external rewards and penalties in making optimal decisions.

Although the uncertainty in Trommershäuser et al.’s (2003a, 2003b) experiments is internal to the observer, the variability arises from the execution of a motor movement. The current study considered the context of perceptual decisions, in which the uncertainty or variability is present in the perceptual representation itself. The results showed that perceptual decisions in this task are well predicted by the maximization-of-expected-gain model. For example, observers shifted their settings further away from baseline for higher penalties than for lower penalty values, and similarly for higher overlap than lower overlap. Furthermore, observers with higher baseline variability shifted their settings to a further extent than those with lower variability. Thus, in the context of this motion-extrapolation paradigm, observers do seem to be aware of their variability in perceptual representation, and are able to combine it with the externally imposed reward-and-penalty structure in making near-optimal perceptual decisions.

One interpretation of the body of work on decision-making is that there is a basic dichotomy between the cognitive domain on the one hand and the
motor and perceptual domains on the other. Observers systematically deviate from normative behavior when making decisions in the cognitive domain, but they do not in the motor and perceptual domains. However, an alternative dichotomy is based on how subjects are provided the probabilistic information: implicitly or explicitly. Their performance is near-optimal when the probability information is learned through experience, but deviates systematically from optimality when it is provided in summary descriptive form. As noted above, decision-making even in the cognitive domain becomes more aligned with normative models when subjects learn the pay-off distributions through interactive experience. Moreover, a very recent study by Maloney, Trommershäuser, & Landy (in press) has shown that performance even in the visuo-motor domain becomes sub-optimal when the probabilistic information is provided to subjects in explicit form (e.g., subjects are told that hitting a certain region on the screen is associated with a 50% chance of gaining 100 points).

The above considerations suggest that the primary factor that determines whether or not decision-making behavior will be optimal is likely to be how the probabilistic information is conveyed: whether it is acquired by subjects through experience and feedback, or given descriptively. In the cognitive and motor domains, it is easy to see how probabilistic information can be acquired through experience. In the motor domain, for instance, repeated executions of a motor action provide direct feedback
concerning the variability in the landing position relative to the target location. However, there are interesting open questions concerning how people come to gain implicit knowledge about the variability in their own perceptual representations. It would be interesting in future work to investigate (i) how well our findings in the motion-extrapolation domain generalize to other perceptual domains, and (ii) how well subjects are able to combine their implicit knowledge of variability in their perceptual representation with externally given probabilistic information.
Figure 1. A depiction of the stimuli used by Trommershäuser et al. (2003). The stimuli consisted of a red penalty circle (solid circle, in the center) and green reward circles (dashed circles) at different locations relative to the reward circle.
Figure 2. An example of the expected gain profile from Trommershäuser et al. (2003). The z-axis depicts the optimal number of points per trial, and the x and y axes depict the location relative to the center of red circle (which lies at \((x,y) = (0,0)\)). The point on the x-axis where the height of the profile is maximum depicts the optimal shift (the location of the green circle was manipulated only in the x-direction).
Figure 3. A depiction of the stimuli used by Singh and Fulvio (2005). (a) The stimuli consist of a half-disc occluder and a contour that disappears behind the straight edge of the occluder. The curved side of the occluder has a probe which is used to make two measurements – position and orientation – that optimize the extrapolation of the inducing contour. (b) Observer settings were measured in terms of polar angle $\theta$ and orientation angle $\phi$, relative to the tangent direction of the inducing contour at the point of occlusion.
Figure 4. Illustration of the sign of overlap: Positive overlap is defined by the green mit positioned higher (i.e., with a larger polar angle) than the red mit; and negative overlap is defined by the green mit being positioned lower than the red mit.
Figure 5. Polar histograms of observers’ settings in the baseline condition. The figures on the top correspond to the low-curvature condition, and the ones at the bottom correspond to the high-curvature condition. The dashed curves are the true extensions of the parabolic paths.
Figure 6. Illustration of the notion of optimal shift. The Gaussian distribution (in blue) depicts the hypothetical perceptual estimate of the observer. In (a), the double-mit is too close to the mean of the perceptual estimate, given the observer’s variance: the moving dot will be caught by the red (penalty) portion of the mit in a fair proportion of the trials. In (b), the double-mit is too far. The observer will not incur any penalty (the red portion doesn’t fall within the bulk of the distribution), but will not make any points either. In (c), the double-mit is more appropriately placed to maximize points gained, and minimize losses.
Figure 7. Computation of the expected-gain profile. (a) $\theta^*$ corresponds to the mean of the observer’s perceptual estimate. $\theta$ denotes all possible angular positions, relative to $\theta^*$, that the double-mit can take. For each possible shift $\theta$, Equation (2) is used to determine the expected gain for that position of the double mit. (b) The expected-gain profile shown in polar coordinates, as a function of relative position $\theta$ of the double-mit.
Figure 8. Plots of observed shift (on the y-axis) versus predicted shift (on the x-axis) for all observers. The square symbols correspond to the $\frac{1}{2}$ overlap condition, and the triangles to the $\frac{1}{4}$ overlap condition. Red symbols indicate higher penalty (500) whereas blue indicates lower penalty (200). The diagonal black line indicates perfect correspondence between the predicted and the observed shift. Note that most of the points lie close to this diagonal, thereby indicating optimal behavior by the observers.
Figure 9. Plots of observed shift versus mit separation (= center of green mit – center of red mit). The blue and the red colors correspond to low and high penalties respectively. The dashed and the solid lines indicate optimal curves for low and high curvatures respectively. Solid and open circles correspond to low and high curvature respectively. The graphs show that, consistent with the optimal prediction, there is an increase in the observed shift with a decrease in mit-separation (i.e., increase in overlap). Moreover, as predicted by the optimal model, higher penalty leads to a greater shift than lower penalty for the same curvature level.
Figure 10. Plot of observed shift (on the y-axis) versus setting variability (SDs) in the baseline condition (on the x-axis across) across all observers. The circle symbols indicate low curvature and the stars indicate high curvature. The different colors represent each of the 6 different observers. The black line is the regression line with a slope of .455 (m= .455, p < .05). The linear fit indicates that there is a statistically reliable influence of internal observer variance on the observed shifts.
Figure 11. Plots of observed scores (on the y-axis) verses optimal scores (on the x-axis) for all observers. The square and the triangle symbols correspond to $\frac{1}{2}$ overlap and $\frac{1}{4}$ overlap respectively. The red and blue symbols indicate higher and lower penalty respectively. The black diagonal line denotes optimal performance. Most data points lie near, or just below, the optimal line.
Figure 12. A histogram plot of the efficiency scores (ratio of total observer score to total optimal score) for all observers. The range of the values is from 80% to 114%, indicating near-optimal performance.
REFERENCES


