ESSAYS ON BEHAVIORAL FINANCE AND MARKET MICROSTRUCTURE

by

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ABSTRACT OF THE DISSERTATION

Essays on Behavioral Finance and Market Microstructure

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This dissertation is comprised of three essays that study behavioral finance and market microstructure.

The first essay models a game of individual day traders' interactions in a stock trading chat room and empirically tests the model's conclusions. Trading behaviors are analyzed in an Internet chat room with free entry but secure identity, and traders' interactions are modeled as a dynamic game with informed traders, momentum traders, arbitragers and noise traders. Three empirical predictions are generated in the model's equilibrium. The unique data set consists of stock trading chat room posts of more than 1,000 individual semi-professional day traders and their interactions and transactions are investigated in a time series. All the three predictions from the model's equilibrium are affirmed by empirical tests.

The second essay assesses the effects of the entire limit order book and analyzes the market impacts of the quotes in the Shanghai and Shenzhen Stock Exchange in China, where the stock market has a pure order-driven trading mechanism without market makers. Firstly, in the empirical modeling the limit order books, the structural vector autoregressive

model of Hasbrouck (1991) is used and extended to incorporate more information beyond the inside quotes. Secondly, the market impact of stocks is also analyzed cross sectionally with market capitalization, tick frequency, turnover, average price, etc. Finally, the market impacts and order imbalance of small trades are distinguished. Small trades, usually linked with individual investors, have proportionally small market impact. Besides, the volume-weighted daily order imbalances of small trades and next-day's and contemporaneous daily returns are negatively related with each other. This is in accordance with the `pain theory' of the individual traders.

The third essay investigates microstructure characteristics of the Credit Default Swap (CDS) market. During the sample period, April 2006 -- March 2008, CDS are traded on the over-the-counter (OTC) market, through brokers' voice-based or electronic-based systems. The study analyzes CDS spread, trade-to-quote ratio, bid-ask spread, the frequency that the orders fall between the quotes, and the relationship between the order imbalance and the daily change of CDS spread.

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I leave my last thanks to my parents, Jianrong Lu and Longying Fang, and my husband, Zhenyu Wu. Their love and care support me to complete my degree.

Dedication

To My Parents and Husband

Table of Contents

Lists of Tables

List of Figures

Chapter 1

Introduction

1.1 Strategic Interaction in A Stock Trading Chat Room

The studies on individual traders are initiated by Odean (1999). His paper documented the poor returns in a sample of more than 35,000 individual traders' accounts. He attributes the underperformance to both overtrading and the disposition effect, the tendency to sell winners and hold losers. Since then, a literature on individual traders has been established, mainly focusing on their trading performance and their psychological bias. Some recent papers, including Coval, Hirshleifer, and Shumway (2005) and Niccolosi, Peng, and Zhu (2003), have suggested that traders might gain experience that improves their performance over time. Mizrach and Weerts (2007) show that skills may be stock specific. Antweiler and Frank (2004) study Internet bulletin board posts, but these are not observed in real time. As far as we know, this essay is the first one to study the real-time interactions between individual traders and also the first one to investigate the online stock trading chatroom.

This essay takes advantage of a unique data set of the chat room posts of more than 1,000 individual traders and analyzes their interaction and transactions in time series. Several basic questions are studied: Who communicates the most? When do they communicate? And why?

Firstly, the individual day traders' trading decision making process without communication is studied. The process is modeled as a dynamic game. A Bayesian Nash Equilibrium is found for the game, and it is a symmetric equilibrium. Secondly, the model is extended to consider the case with communication. A symmetric equilibrium is also found for this game. Thirdly, the equilibrium establishes three empirical predictions: (1) More profitable traders post more fundamental analysis and less profitable traders post more non-fundamental analysis, (2) The less profitable a trader is,

the more frequently she follows by others, and (3) The more profitable a trader is, the more frequently she is followed by others. Finally, all the three empirical implications from the model's equilibrium are confirmed by the data set.

This essay's conclusions are not limited to the individual traders, but also shed light on what happens in Wall Street. Actually, most individual traders in the data set are semi-professionals. The average value of their active trading portfolio is about 1/4 million dollars and they always stay in the chatroom doing trading and posting during the trading hours. In fact, this kind of semi-professional traders' trades account for about 25% of the Nasdaq's total trading volumes. Thus, it is not surprising that this type of traders' interactions is just a small version of Wall Street: building positions before releasing information (see e.g. Mizrach (2005)).

1.2 An Empirical Analysis of the Shanghai and Shenzhen Limit Order Books

The Shanghai Stock Exchange and the Shenzhen Stock Exchange are the only two stock exchanges in mainland China. Both markets have seen impressive growth since they were founded in 1990. By December 2007, Shanghai Stock Exchange's market capitalization ranked sixth worldwide and Shenzhen ranked 20th. Their combined market capitalization of $32,714$ billion RMB ($4,474$ billion USD) was the second largest globally after the United States.

There is a limited literature about the microstructure of the Chinese stock market, and only a few papers analyze intraday limit order book information. Xu (2000) discussed the trading mechanism of Chinese stock market but the paper's quantitative study focused on stocks's daily returns. As to limit order book, Shenoy and Zhang (2007) studied the relationship between daily order imbalance from limit order book and daily stock returns. Bailey, Cai, Cheung and Wang (2006) separated the order imbalance from individual, institutional and proprietary investors and investigated

the various influences of different traders. As far as we know, this essay is the first one to apply vector autoregressive models into analyzing the intraday quotes and limit order book in Chinese stock market.

This essay studies the market impact of the entire limit order book of the Shanghai Stock Exchange and the Shenzhen Stock Exchange. Firstly, the structural vector autoregressive model of Hasbrouck (1991) is extended to incorporate more information beyond the inside quotes, such as returns, tick directions and the differences of 5 bid and ask quantities. And the market impact is calculated from both Hasbrouck model and the extended model. Secondly, the market impact of stocks is also analyzed cross sectionally with market capitalization, tick frequency, turnover, average price, etc. Finally, small trades' information effect and daily order imbalances are distinguished. Small traders have lower market impacts than other trades. And their volume-weighted daily order imbalances and next-day's and contemporaneous daily returns are negatively related with each other. This is in accordance with the 'pain theory' of the individual traders.

1.3 An Empirical Microstructure Study of the CDS Over-the-counter (OTC) Market

As a financial derivative, Credit Default Swaps (CDS) provide insurance against a default by a particular company or sovereign entity. The CDS buyer makes periodic payments to the seller and in return obtains the right to sell a bond issued by the reference entity for its face value if a credit event occurs. Before March 2009, Credit default Swaps are traded over the counter (OTC). Major dealers are banks and financial institutions with good credit ratings. Between major dealers, there are interdealer brokerage companies offer both voice brokerage and electronic brokerage. Trades are facilitated through phone systems or electronic platforms. The contracts and trades follow the standards set by the International Swaps and Derivatives Asso-

ciation (ISDA). The CDS OTC market is one of the most rapidly expanding derivative markets. The notional outstanding amount of CDS has reach its peak, about 45 trillion dollars, in 2007. However, as the financial crisis exploded, the notional market value of CDS has fallen 38% to 38.6 trillion dollars in 2008.

There is an established literature about the CDS market but most of them focus on CDS pricing and only a few papers correlate with this one in the microstructure study. Gunduz, Ludecke and Uhrig-Homburg (2007) described the hybrid structure of interdealer brokers and compared the liquidity from the two trading systems, voice brokerage and electronic brokerage. Archary and Johnson (2007) studied the informed trading in CDS market and stock market. Some papers on CDS liquidity risk pricing also included a little microstructure studies. Tang and Yan (2007) constructed liquidity proxies to test whether liquidity risk is priced in CDS market. Chen, Fabozzi and Sverdlove (2008) showed the large bid-ask spread can profoundly affect the estimation of credit risk and liquidity risk.

This essay is an empirical microstructure study of the CDS over-the-counter (OTC) market. The sample period is from April 2006 to March 2008. During that period, CDS trading occurs in OTC market and relies heavily on the interdealer broker system. The first part describes and explains the trend and spikes of CDS spreads during the sample period, including the beginning of the financial crisis. The second part analyzes the trade-to-quote ratio, which indicates how easily to find a trading counter-party. The third part focuses on the bid-ask spread, which is one of the most widely used liquidity measures. The fourth part shows the frequency that the orders fall between the quotes. The fifth part studies the relationship between the daily order imbalance and the next-day's or contemporaneous daily changes of CDS spreads.

Chapter 2
Strategic Interaction in A Stock Trading Chat Room

2.1 Introduction

Wall Street has a lot in common with Madison Avenue. There is a great deal of information disseminated to influence portfolio selection. There are numerous communications among professionals and here comes out the questions: will trader A take positions in a stock after trader B says he has loaded up? When will trader B tell the truth and when will he lie? There is no effective way to study the real time effects of such informal communications among professional investors. However, when stock trading chat rooms come out, we now can study similar interactions among individual semi-professional traders. This essay studies the influence of communications among individual day traders on their trading decisions. And we takes advantage of a unique data set of the chat room posts of more than 1,000 individual day traders and studied their interaction and transactions in time series.

There are two advantages that individual day traders are good objectives to study the effect of informal communications in trading decision making: (1) unlike professionals, they do not have any trading rules or trading guidelines forced on them, which make their trades more personal-decision driven; (2) unlike professionals, they do not have enough capital to verify others' news/rumors/ideas by testing market liquidity, which makes the influence of the real-time interaction on their trading decisions more easily to study.

There is now an established literature on the performance of individual traders. Odean (1999) initiated the studies on individual traders and documented poor returns in a sample of more than 35,000 households. He attributes the underperformance to both overtrading and the disposition effect, the tendency to sell winners and hold losers.

Some recent papers, including Coval, Hirshleifer, and Shumway (2005) and Niccolosi, Peng, and Zhu (2003), have suggested that traders might gain experience that improves their performance over time. Mizrach and Weerts (2007) show that skills may be stock specific. As far as we know, the literature has not looked at the real-time interactions between individual traders, perhaps because of data limitations.

This essay models individual day traders' interactions as a dynamic game and studies several basic questions: Who communicates the most? When do they communicate? And why? The model establishes three empirical predictions: (1) More profitable traders post more fundamental analysis and less profitable traders post more non-fundamental analysis, (2) The less profitable a trader is, the more frequently she follows by others, and (3) The more profitable a trader is, the more frequently she is followed by others.

We typically don't observe the message traffic between traders and their brokers. And we also don't see trading decisions linked directly to their posts. Antweiler and Frank (2004) study Internet bulletin board posts, but these are not observed in real time.

This paper takes advantage of a unique data set of the chat room posts of more than 1,000 individual traders, with which we confirm the three main empirical predictions of our model.

The essay is organized as follows: Section 2.2 describes the equilibrium if traders cannot communicate; Section 2.3 describes the equilibrium with communication and the empirical implications; Section 2.4 introduces the data; Section 2.5 presents our empirical results; Section 2.6 concludes and speculates about the generalizability of the results.

2.2 Model

This model describes how traders with different information levels trade in the

market and how they move the stock price according to their expectations.

There is a risky asset V with initial value v_0. Information is released at time $t = \Gamma$ which changes the risky asset's value to v. The value of v depends on the state of the world, which takes two values from the set $\omega = \Omega = \{\omega^-, \omega^+\}$. $v = v_0 + \widehat{v}$ in state ω^+ and $v = v_0 - \widehat{v}$ in state ω^-. The prior probability of each state $\{\omega^+, \omega^-\}$ is $\{\frac{1}{2}, \frac{1}{2}\}$. We divide $[0, \Gamma]$ into 2 periods and v is revealed as information is released at period 2.

There are four kinds of traders in the market: QQ_I informed traders S_I, QQ_M momentum traders S_M, noise traders and arbitragers, where QQ_I and QQ_M are positive integers denoting numbers of traders. Noise traders trade for liquidity reasons and simply add noise into prices. Arbitragers trade the asset elastically to keep price at the fundamental value once it is revealed. We study two kinds of traders' optimal strategy: informed traders S_I, and momentum traders S_M, who receive signals and trade for profits according to all information they can get, and we also take into account the noise term in the price which comes from noise traders' behaviors and arbitrage factor after information is released.

Each trader i, either S_I or S_M, receives a signal at period 0. Informed traders S_I are more skillful traders. In the model, they have perfect information about the state. Momentum traders S_M are less skillful traders and they have no information about the state. Arbitrager enter the market only when information is released and the asset's true value is revealed to everyone. Table 2.1 shows the signals traders receive in different states.

[Insert Table 2.1 Here]

At period s, trader i's action is denoted as $a_s^i \in \Lambda_1 = \{-1, 0, 1\}$, where $\{-1, 0, 1\}$ is the action set, 1 means buying 1 unit (long position) of risky asset, -1 means selling 1 unit (short position), and 0 means no order submitted. We assume that any trader can be at most one unit long or short, $-1 \leq \sum_t a_t^i \leq 1$ for $t = 0, 1, 2$. We also assume

that S_I and S_M all hold position 0 at the beginning, period 0, and that traders must be flat at the end of the game, $\sum_{t=0}^{2} a_t^i = 0$. These imply the actions in the first two periods

$$a^i = \left(a_0^i, a_1^i\right) \in \{(1,0), (1,-1), (0,1), (0,0), (0,-1), (-1,1), (-1,0)\}.$$

For any informed trader S_I^i, a strategy is a pair $\sigma^{i,I} = (\sigma_0^{i,I}, \sigma_1^{i,I})$ where $\sigma_0^{i,I} : \Omega \to \Lambda_1$ and $\sigma_1^{i,I} : \Omega \times \Lambda_1 \times R \to \Lambda_1$. For any momentum traders S_M^j, a strategy is a pair $(\sigma_0^{j,M}, \sigma_1^{j,M})$ where $\sigma_0^{j,M} \in \Lambda_1$ and $\sigma_1^{j,M} : \Lambda_1 \times R \to \Lambda_1$.

At each period, all orders are submitted to a market maker, and each unit of order flow has the same market impact λ on next period's price that is very small compared with P_t, v_0 and $\widehat{v}$. The noise in the market clearing price is determined by noise buyers and sellers, making ε_t uniformly distributed on $[-\lambda QQ_N, \lambda QQ_N]$.

At period 0, asset value is v_0 and market price is $P_0 = v_0$. Traders receive their signals and submit their orders a_0^i to maximize their expected payoffs. At period 1, asset price reflects the trades from the initial period,

$$P_1 = P_0 + \lambda \left(\sum_i a_0^i\right) + \varepsilon_1.$$

Denote p_0^i as the execution price for trader i's order submitted at period 0. We assume that trader i's order submitted at period 0 has equivalent chances to be filled between the price P_0 and the price P_1. This implies $E\left[p_0^i\right] = \frac{P_0 + P_1}{2}$.

Traders observe the price P_1 and may submit their orders a_1^i according to their updated expectations. We denote their execution price as p_1^i and note that $E\left[p_1^i\right] = \frac{P_1 + P_2}{2}$ as we assume that trader i's order submitted at period 1 has equivalent chances to be filled between the price P_1 and the price P_2.

At period 2, information will be released and the final payoffs are determined. Asset price reflect the information $v(\omega)$, where $v(\omega^+) = v_0 + \widehat{v}$ and $v(\omega^-) = v_0 - \widehat{v}$.

Traders submit orders to clear their positions $a_2^i = -(a_0^i + a_1^i)$. Arbitragers will get into the market after information release and move the orders a_2^i's execution price closer to $v(\omega)$ if they can make gains within their transactions costs c. Asset price reflect the true value and arbitrage effect. And the orders submitted at period 2 can be filled at the price p_2 due to the arbitrage activities.

$$p_2(\omega, P_2) = \begin{cases} v(\omega) - c & if \ P_2 \leq v(\omega) - c \\ P_2 & if \ v(\omega) - c < P_2 < v(\omega) + c \\ v(\omega) + c & if \ P_2 \geq v(\omega) + c \end{cases},$$

where $P_2 = P_1 + \lambda \left(\sum_i a_1^i\right) + \varepsilon_2$, $v(\omega^+) = v_0 + \widehat{v}$ and $v(\omega^-) = v_0 - \widehat{v}$.

The end of game payoff of trader i is

$$\pi^i = \left[\sum_{s=0}^1 \left(-a_s^i p_s^i\right)\right] - a_2^i p_2,$$

where $i \in \left\{S_I^1, \ldots, S_I^{QQ_I}, S_M^1, \ldots, S_M^{QQ_M}\right\}$.

To simplify the problem, assume without loss of generality, $P_0 = v_0 = 0$. The initial period expected payoffs for informed traders are,

$$E_0\left[\pi^{i,I}|\omega\right] = E\left[-a_0^I \cdot p_0^i - \sigma_1^I \cdot p_1^i + \left(a_0^I + \sigma_1^I\right) \cdot p_2|\omega\right].$$

Momentum traders cannot condition on the signal, so their expected payoff is,

$$E_0\left[\pi^{i,M}\right] = E\left[-a_0^M \cdot p_0^i - \sigma_1^M \cdot p_1^i + \left(a_0^M + \sigma_1^M\right) \cdot p_2\right].$$

In making their final strategic choices before the game ends, the expected payoff for informed traders are,

$$E_1\left[\pi^{i,I}|\omega, P_1, a_0^I\right] = -a_1^I \cdot E\left[p_1^i|\omega, P_1, a_0^I\right] + \left(a_0^I + a_1^I\right) \cdot E\left[p_2|\omega, P_1, a_0^I\right].$$

And for momentum traders,

$$E_1\left[\pi^{i,M}|P_1,a_0^M\right] = -a_1^M \cdot E\left[p_1^i|P_1,a_0^M\right] + \left(a_0^M + a_1^M\right) \cdot E\left[p_2|P_1,a_0^M\right].$$

DEFINITION.

A *symmetric equilibrium without communication* is a strategy profile $\forall i \in I, M$, $\sigma^{i.I} = \sigma^I = (\sigma_0^I, \sigma_1^I)$ and $\sigma^{i.M} = \sigma^M = (\sigma_0^M, \sigma_1^M)$, satisfying:

1. $\sigma^I = (\sigma_0^I, \sigma_1^I)$, where $\sigma_0^I : \Omega \to \Lambda_1$ and $\sigma_1^I : \Omega \times \Lambda_1 \times R \to \Lambda_1$,

2. $\sigma^M = (\sigma_0^M, \sigma_1^M)$, where $\sigma_0^M \in \Lambda_1$ and $\sigma_1^M : \Lambda_1 \times R \to \Lambda_1$.

3. Given ω,

$$\sigma_0^I(\omega) \in \underset{a_0^I \in \Lambda_1}{\arg\max}\left\{E_0\left[\pi^I|\omega\right]\right\}.$$

4.

$$\sigma_0^M \in \underset{a_0^M \in \Lambda_1}{\arg\max}\left\{E_0\left[\pi^M\right]\right\}.$$

5. Given $\left(\omega, P_1, a_0^I\right)$,

$$\sigma_1^I\left(\omega, P_1, a_0^I\right) \in \underset{a_1^I \in \Lambda_1,\ -1 \leq a_0^I + a_1^I \leq 1}{\arg\max}\left\{E_1\left[\pi^I|\omega, P_1, a_0^I\right]\right\}.$$

6. Given $\left(P_1, a_0^M\right)$,

$$\sigma_1^M\left(P_1, a_0^M\right) \in \underset{a_1^M \in \Lambda_1,\ -1 \leq a_0^M + a_1^M \leq 1}{\arg\max}\left\{E_1\left[\pi^M|P_1, a_0^M\right]\right\}.$$

We begin to describe the best response strategies in the next section.

2.3 Equilibrium without Communication

2.3.1 Optimal Trading Strategies

We describe the optimal trading strategies of the informed and momentum traders in Proposition 1. We provide the proof in Section 2.7 Proof A.

PROPOSITION 1: *Suppose that the number of traders satisfies (A.1) and the market impact condition satisfies (A.2), then the following strategy pairs are a symmetric equilibrium to the trading game without communication.*

On the Equilibrium path:

1. *Informed traders* S_I

$$\sigma_0^I(\omega^+) = 1, \; \sigma_0^I(\omega^-) = -1,$$

$$\sigma_1^I\left(\omega^+, a_0^I, P_1\right) = 0 \text{ if } P_1 \in [-P^*, \overline{P}], a_0^I = 1,$$

$$\sigma_1^I\left(\omega^-, a_0^I, P_1\right) = 0 \text{ if } P_1 \in [-\overline{P}, P^*], a_0^I = -1,$$

where $P^* = \lambda\left(QQ_N - QQ_I\right), \overline{P} = \lambda\left(QQ_I + QQ_N\right).$

2. *Momentum traders* S_M:

$$\sigma_0^M = 0,$$

$$
\begin{aligned}
\sigma_1^M\left(a_0^M, P_1\right) &= 1 \text{ if } P_1 \in \,]P^*, \overline{P}], a_0^M = 0, \\
&= 0 \text{ with } prob \; 1 - q^+ \text{ and } -1 \text{ with } prob \; q^+ \text{ if } P_1 \in \,]\frac{\lambda}{2}, P^*], a_0^M = 0, \\
&= 0 \text{ if } P_1 \in [-\frac{\lambda}{2}, \frac{\lambda}{2}], a_0^M = 0, \\
&= 0 \text{ with } prob \; 1 - q^- \text{ and } 1 \text{ with } prob \; q^- \text{ if } P_1 \in [-P^*, -\frac{\lambda}{2}[, a_0^M = 0, \\
&= -1 \text{ if } P_1 \in [-\overline{P}, -P^*[, a_0^M = 0,
\end{aligned}
$$

where $q^+ = \frac{2P_1 - \lambda}{\lambda(QQ_M - 1)} > 0$ if $P_1 \in \,]\frac{\lambda}{2}, P^*]$ and $q^- = \frac{-2P_1 - \lambda}{\lambda(QQ_M - 1)} > 0$ if $P_1 \in [-P^*, -\frac{\lambda}{2}[.$

Off the Equilibrium path:

1.' *Informed traders*:

$$\sigma_1^I\left(\omega^+, a_0^I, P_1\right) = 1 \text{ if } P_1 \in [-\lambda - P^*, -\lambda + \overline{P}], a_0^I = 0,$$

$$\sigma_1^I\left(\omega^+, a_0^I, P_1\right) = 1 \text{ if } P_1 \in [-2\lambda - P^*, -2\lambda + \overline{P}], a_0^I = -1,$$

$$\sigma_1^I\left(\omega^-, a_0^I, P_1\right) = -1 \text{ if } P_1 \in [\lambda - \overline{P}, \lambda + P^*], a_0^I = 0,$$

$$\sigma_1^I\left(\omega^-, a_0^I, P_1\right) = -1 \text{ if } P_1 \in [2\lambda - \overline{P}, 2\lambda + P^*], a_0^I = 1.$$

2.' *Momentum traders*

$$\sigma_1^M\left(a_0^M, P_1\right) = 0 \text{ if } P_1 \in]P^* + \lambda, \overline{P} + \lambda], a_0^M = 1,$$

$$= -1 \text{ if } P_1 \in [P^*, P^* + \lambda], a_0^M = 1,$$

$$= 0 \text{ if } P_1 \in [-P^*, P^*[, a_0^M = 1,$$

$$= -1 \text{ if } P_1 \in [\lambda - \overline{P}, -P^*[, a_0^M = 1.$$

$$\sigma_1^M\left(a_0^M, P_1\right) = 1 \text{ if } P_1 \in]P^*, \overline{P} - \lambda], a_0^M = -1,$$

$$= 0 \text{ if } P_1 \in [-P^*, P^*], a_0^M = -1,$$

$$= 1 \text{ if } P_1 \in [-\lambda - P^*, -P^*[, a_0^M = -1,$$

$$= 0 \text{ if } P_1 \in [-\lambda - \overline{P}, -\lambda - P^*[, a_0^M = -1.$$

Proof: See Section 2.7 Proof A.

Since the informed traders S_I receive perfect information about $\tilde{v}$, their optimal strategy is to trade on their private signals immediately. They long (short) at period 0 in state ω^+ (ω^-). And they hold their positions at period 1.

The momentum traders S_M rely on the price path to make trading decisions. They

never trade at the very beginning, period 0, because they only have uninformative signals. They infer informed traders S_I's actions from the price path and make their trading decisions based on this. If the price at period 1 passes the threshold P^* ($-P^*$), then they optimally follow informed traders to enter the market to long (short). If the price P_1 does not pass the threshold, their optimal strategy is mixed.

2.3.2 Leading example

We consider a case in which there are more momentum traders than informed traders and the noise in the price path is large enough that momentum traders cannot attain perfect information about informed traders' action from the price path. Besides, each trader's market impact is small, compared with the value change caused by information release. Furthermore, the informed traders do not have enough capital to move price to the asset's underlying value.

Suppose that $P_0 = 10$ and $\widehat{v} = 10$, and the arbitragers cost c is 5. $QQ_I = 200, QQ_N = 400$, $QQ_M = 400$ and $\lambda = 0.01$. Thus, $P^* = 2$, $\overline{P} = 6$ and $\widehat{v} - c = 5, \widehat{v} + c = 15$.

In the event of a positive (negative) state, we know that all of the informed traders will trade long (short), with market impact of 2 (-2). The noise traders add noise $[-4, 4]$ into the price. We can describe the price path using Table 2.2 and Table 2.3.

[Insert Table 2.2 Here]

[Insert Table 2.3 Here]

The momentum trader's strategy at period 1 is shown in Table 2.4.

[Insert Table 2.4 Here]

The price distribution at period 2 is shown in Table 2.5 and Table 2.6.

[Insert Table 2.5 Here]

[Insert Table 2.6 Here]

In this situation, the momentum traders' payoff is:

$$E_0 \left[\pi^M | a_0^M = 0 \right] = \frac{\lambda QQ_I \cdot QQ_M}{2QQ_N} = 1$$

And the informed traders' payoff is:

$$E_0 \left[\pi^I | a_0^I = 1, \omega^+ \right]$$
$$= (\widehat{v} - c) \cdot \left(1 - \frac{QQ_I}{QQ_N} \right) + \lambda QQ_I \cdot \left(\frac{QQ_M}{QQ_N} + \frac{1}{2} \right)$$
$$= 5.5$$

Given the two assumptions (A.1) and (A.2) are satisfied, the momentum traders S_M's payoff $E_0 \left[\pi^M \right]$ increase as QQ_I, QQ_M, λ increase or QQ_N decreases; the informed traders S_I's payoff $E_0 \left[\pi^I \right]$ increase as QQ_M, λ, $(\widehat{v} - c)$ increase. If $\lambda QQ_M < \widehat{v} - c$, $E_0 \left[\pi^I \right]$ increases as QQ_N increases; otherwise, $E_0 \left[\pi^I \right]$ increases as QQ_N decreases. If $\lambda \left(QQ_M + \frac{QQ_N}{2} \right) > \widehat{v} - c$, $E_0 \left[\pi^I \right]$ increases as QQ_I increases; otherwise, $E_0 \left[\pi^I \right]$ increases as QQ_I decreases.

These conclusions hold as long as the two assumptions (A.1) and (A.2) are satisfied.

2.4 Equilibrium with Communication

2.4.1 Optimal Trading Strategies

Now, suppose informed and momentum traders can choose to communicate with other traders. And suppose each trader has perfect information about all posters' types. The action space is two-dimensional, including trader i's trades and posts. At period s, trader i's action is denoted as $a_s^i \in \Lambda_1 = \{-1, 0, 1\}$, $b_s^i \in \Lambda_2 = \{l, s, n\}$, where $-1, 0, 1$ are defined as previous part, and l means posting long positions, s

means posting short positions, and n means not to post at all. Trader i's strategy in periods $s = 0, 1$ can be denoted as $\{\sigma_0^i, \gamma_0^i, \sigma_1^i, \gamma_1^i\}$.

DEFINITION.

A *symmetric equilibrium with communication* is a strategy profile $\forall i \in I, M$, $\sigma^{i.I} = \sigma^I = (\sigma_0^I, \sigma_1^I)$, $\gamma^{i.I} = \gamma^I = (\gamma_0^I, \gamma_1^I)$ and $\sigma^{i.M} = \sigma^M = (\sigma_0^M, \sigma_1^M), \gamma^{i.M} = \gamma^M = (\gamma_0^M, \gamma_1^M)$, satisfying, besides the conditions 1-4 in the non-communication equilibrium:

1'. $\gamma^I = (\gamma_0^I, \gamma_1^I)$, where $\gamma_0^I : \Omega \times \Lambda_1 \to \Lambda_2$, $\gamma_1^I \in \Lambda_2$;

2'. $\gamma^M = (\gamma_0^M, \gamma_1^M)$ where $\gamma_0^M \in \Lambda_2$, $\gamma_1^M \in \Lambda_2$;

3'. Given $\left(\omega, a_0^I\right)$,

$$\gamma_0^I\left(\omega, a_0^I\right) \in \underset{b_1^I \in \Lambda_2}{\arg\max} \left\{E_0\left[\pi^I \middle| \omega, a_0^I\right]\right\};$$

4'. Given a_0^M,

$$\gamma_0^M\left(a_0^M\right) \in \underset{b_1^M \in \Lambda_2}{\arg\max} \left\{E_0\left[\pi^M \middle| a_0^M\right]\right\}.$$

5'. Given $\left(\omega, P_1, a_0^I, b_0^I, b_0^M\right)$,

$$\sigma_1^I\left(\omega, P_1, a_0^I, b_0^I, b_0^M\right) \in \underset{a_1^I \in \Lambda_1, \, -1 \leq a_0^I + a_1^I \leq 1}{\arg\max} \left\{E_1\left[\pi^I \middle| \omega, P_1, a_0^I, b_0^I, b_0^M\right]\right\}.$$

6'. Given $\left(P_1, a_0^M, b_0^I, b_0^M\right)$,

$$\sigma_1^M\left(P_1, a_0^M, b_0^I, b_0^M\right) \in \underset{a_1^M \in \Lambda_1, \, -1 \leq a_0^M + a_1^M \leq 1}{\arg\max} \left\{E_1\left[\pi^M \middle| P_1, a_0^M, b_0^I, b_0^M\right]\right\}.$$

We describe the optimal trading strategies of the informed and momentum traders in Proposition 2. We provide the proof in Section 2.8 Proof B.

PROPOSITION 2: *Suppose that the number of traders satisfies (A.1) and the market impact condition satisfies (A.2), then the following strategy pairs are a symmetric*

equilibrium to the trading game without communication.

`On the Equilibrium Path:`

1. *Informed traders* S_I has the same trading strategy as without communication. As to the posting strategy, informed traders S_I at $t = 0, 1$:

$$\gamma_0^I(\omega^+, a_0^I) = l, \text{if } a_0^I = 1,$$
$$\gamma_0^I(\omega^-, a_0^I) = s, \text{if } a_0^I = -1,$$

$$\gamma_1^I\left(a_0^I\right) = n, \text{if } a_0^I = 0.$$

2. *Momentum traders* S_M:

$$\sigma_0^M = 0, \gamma_0^M = n$$

$$\gamma_1^M\left(a_0^M\right) = l, \text{if } a_0^M = 1,$$
$$\gamma_1^M\left(a_0^M\right) = s, \text{if } a_0^M = -1.$$

`Off the Equilibrium Path:`

On the off-equilibrium path, S_M can only update their beliefs from the price path and trade as if without communication when there is no post from S_I or S_I's posts conflict, i.e. $b_1^I = mix\{l, n, s\}$.

1'. Informed traders S_I have the same trading strategy as without communication.

As to the posting strategy, informed traders S_I:

$$
\begin{aligned}
\gamma_0^I(\omega^+, a_0^I) &= s, \text{if } a_0^I = -1, \\
\gamma_0^I(\omega^+, a_0^I) &= s \text{ or } n, \text{if } a_0^I = 0, \\
\gamma_0^I(\omega^-, a_0^I) &= l, \text{if } a_0^I = 1, \\
\gamma_0^I(\omega^-, a_0^I) &= l \text{ or } n, \text{if } a_0^I = 0,
\end{aligned}
$$

$$
\begin{aligned}
\gamma_1^I\left(a_1^I\right) &= l, \text{if } a_1^I = 1, \\
\gamma_1^I\left(a_1^I\right) &= s, \text{if } a_1^I = -1.
\end{aligned}
$$

2'. Momentum traders S_M:

$$
\begin{aligned}
\sigma_1^M\left(P_1, a_0^M, b_0^I, b_0^M\right) &= 0, \text{if } a_0^M = 1, b_0^I = l, \\
\sigma_1^M\left(P_1, a_0^M, b_0^I, b_0^M\right) &= 1, \text{if } a_0^M = -1, b_0^I = l, \\
\sigma_1^M\left(P_1, a_0^M, b_0^I, b_0^M\right) &= 0, \text{if } a_0^M = -1, b_0^I = s, \\
\sigma_1^M\left(P_1, a_0^M, b_0^I, b_0^M\right) &= -1, \text{if } a_0^M = 1, b_0^I = s.
\end{aligned}
$$

Proof: See Section 2.7 Proof B.

The informed traders S_I still receive perfect information about $\tilde{v}$ and their optimal strategy is to long (short) at period 0 in state ω^+ (ω^-). After building their positions, S_I post truthfully to attract followers, which helps the asset realize the true value. And they hold their positions at period 1.

The momentum traders S_M do not post and trade at period 0 because they do not have any information in this period. At period 1, S_M with communication face similar situations as S_I: with S_I's l posts, S_M can indicate state ω^+; and with S_I's s posts, S_M can indicate state ω^+, because S_I has no incentive to cheat or hide. Thus,

the momentum traders optimally follow the informed traders' posts.

2.4.2 Empirical Implications

This part summarizes the observable implications in the equilibrium of the model. We have three hypothesis indicated from the equilibrium:

HYPOTHESIS 1. *Who posts: More profitable traders post more fundamental analysis and less profitable traders post more non-fundamental analysis.*

Traders' profit levels shows their informativeness. When observing the data, we should see that informed traders get information from their own fundamental analysis and attain high profit record while momentum traders pay attention to public price path and attain low profits.

HYPOTHESIS 2. *Who follows others: The less profitable a trader is, the more frequently she follows others.*

We use a "following" trade to denote a trade which have a previous trade traded on the same direction and posted by another trader within five minutes. Based on this definition, in the equilibrium, S_I seldom follow others while S_M frequently follow others in stock picking. Thus, when observing the data, we should see that a trader's profitability is negatively related with her following frequency.

HYPOTHESIS 3. *Who is followed: The more profitable a trader is, the more frequently she is followed by others.*

We define the trade followed by a "following" trade as a "being followed" trade. In the equilibrium, S_I are followed with higher probability than S_M. Thus, when observing the data, we should see that a trader's profitability is positively related with the number of her "being followed" trades.

2.5 Data and Empirical Tests

2.5.1 Data

We collected the posts from the Active Trader Financial Chatroom at sporadic intervals over a four year period from 2000 to 2003. Our sample period is the most active trading month October 2000. The logs contain several interruptions when the chat client froze or when the author neglected to capture the feed. In October 2000, we have 14 trading days of information. Posts are time stamped to the minute. Trader identities are in <.>

The posts contain information about fundamental and technical analysis, trades, and some irrelevant information. Here is a sample chat log from 11:48 to 11:53 Eastern time on October 30, 2000.

<UofMichigan> CSCO chart support 37, can't believe we will see that

<Tommy> CSCO wants low 40's

<Fleance> CSCO selling 46

double_odds buys COVD 5 3/16

<UofMichigan> CSCO PE not looking that bad

<getnby> sells CSCO

<aim> INTC going down with CSCO

<Sodo> CSCO 46

Matrix in CSCO

<Fleance> CSCO 800,000 shares traded last min

WallStArb buys CSCO 46 1/16

buyinlow in csco

<tradem> adding csco

<DMS> buys ITRU on NEWS

double_odds sells INDG +1/2

<[MrB]> added CSCO here

<Amokk> CSCO bounce

<ghe> buys INTC

WallStArb places 46 1/8 stop on CSCO

Matrix sells some CSCO

Targetman Buys NAS-FUTURES @ 3102

Matrix buys YHOO 52

<Commonman> $35.70/share BOUT? at what PRM price?

Targetman Buys SP-FUTURES @ 1393.50

<scalper> smart move Wally

<HITTHEBID> naz looks overdone

<phishy> bvsn stoch upcross + spoos candle bottom

Targetman Buys CSCO @ 46 3/8

<Bill1> adds xxia 18 3/4

We summarize the type of posts, number of posters and frequency in Table 2.7.

[Insert Table 2.7 Here]

Although day traders trade mostly on non-fundamental analysis, those traders did post and use fundamental information in making trading decisions. They analyzed typical fundamental indicators, stock valuation, company financial status, CEO performances and product innovations. A typical fundamental post in the example log is "[11:50] <UofMichigan> CSCO PE not looking that bad," which refers to the price earnings ratio.

Most posts about stock trading are non-fundamental posts, including technical analysis and price statements mentioning the new updates on the price path. A typical technical analysis is "[11:48] <UofMichigan> CSCO chart support 37" or "[11:53] <phishy> bvsn stock upcross + spoos candle bottom"; A typical statement about price direction is "[11:50] <aim> INTC going down with CSCO", which is

simply repeating the price path, which is public information.

Traders also post their trades, which gives us the information about their real skills. A typical trade post is "[11:53] Targetman Buys CSCO @ 46 3/8", in which the trader <Targetman> bought CSCO at the price he showed. We do not rely on the trader's posted price and profit information, but instead verify this from transactions records.

There are posts irrelevant with stock trading, such as "[11:53] <scalper> smart move Wally" in the sample chat log. However, since there are chatroom administrators who keep the room focus on stock trading within trading hours, most totally irrelevant posts appear after trading hours.

We also summarize the trading activity for October 2000 in Table 2.7.

Traders use a wide variety of slang for their trades. We used various forms of the keywords, including their abbreviations and misspelled variants, to indicate buying activity: Accumulate; Add; Back; Buy; Cover; Enter; Get; Grab; In; Into; Load; Long; Nibble; Nip; Pick; Poke; Reload; Take; and Try. Keywords for selling were: Dump; Out; Scalp; Sell; Short; Stop; and Purge.

We cannot match open and closing trades for about 70% of the posts. We assume that all open positions whether long or short are closed at the end of the day. We do not consider after hours trades.

To compute dollar profit and losses for each trader, we make transaction cost assumptions for position size assumptions. For position size A, we assume a $20 commission. This is a $0.02 per share commission on the 1,000 share round trip. Numerous brokers offer commissions in this range. For position size B, we assume a $0.005 per share commission and a 50 basis point slippage. These reflect the lower commissions typically paid on larger lot sizes, and some market impact on the larger trades. We find that none of the position or transaction costs assumptions has a qualitative impact on our profit estimates.

We examine profits for all trades. The first profit measure is the aggregate difference between selling and buying prices so the reader can gauge the effect of the transactions costs. The second measure A uses the low cost estimate with flat commissions. The second measure B has higher transactions costs, but sometimes benefits from the larger lot sizes.

In our sample period, more than 50% of traders are profitable under A while 47.48% of the traders are profitable under B. These are much higher ratios of profitable traders found in other studies of retail investors or day traders. This is why we feel comfortable regarding some semi-professional and professional traders as informed traders. The experts in our chat room are "Activetraders" for a good reason; trading, for them, is a profitable activity.

Our traders make money trading both long and short. When we break apart profits short versus long, we find that 74.7% of profits are made trading long and 25.3% short. Trades are equally likely to be profitable long versus short, 53.97% long compared to 56.07% short. The marginal profit per trade is substantially higher on the short side than the long, $210.84 per trade short versus $110.87 long in the pooled sample. Short traders are also more skillful overall. Over the four years, 51.55% of traders who never short are profitable under assumption A, compared with 62.21% for traders who trade both short and long.

For the remainder of this section, we will utilize the more conservative profit assumptions A.

2.5.2 Empirical Results

We summarize the empirical results in Table 2.8.

[Insert Table 2.8 Here]

RESULT 1. *Who posts: More profitable traders post more fundamental analysis and less profitable traders post more non-fundamental analysis.*

Our first test of the model is about posting frequency by trader j for the four types of posts: (1) fundamental posts, FP_j;(2) non-fundamental posts, NFP_j;(3) trade posts, TRP_j; (4) irrelevant posts, IRR_j. Trader j's total posts are

$$NP_j = FP_j + NFP_j + TRP_j + IRR_j.$$

H1 tests the posting frequency of trades, FP_j/NP_j and NFP_j/NP_j.

We calculate our standard profit measure, the profit per trade of trader j

$$\pi_j = \frac{\sum_{t=1}^{Tr_j} \pi_{j,t}}{\sum_{t=1}^{Tr_j} Tr_{j,t}}.$$

We regress the fundamental posting frequency, FP_j/NP_j, on profits per trade π_j on

$$FP_j/NP_j = \alpha_{1a} + \beta_{1a}\pi_j \ ,$$

and regress the technical posting frequency, NFP_j/NP_j, on profits per trade π_j on

$$NFP_j/NP_j = \alpha_{2a} + \beta_{2a}\pi_j \ .$$

We find that β_{1a} is significantly positive and β_{1b} is significantly negative, consistent with Hypothesis 1.

The higher profit record a trader has, the more frequently he posts fundamental analysis, and the less frequently he posts non-fundamental analysis .

RESULT 2. *Who follows others: The less profitable a trader is, the more frequently she follows others.*

We first test hypothesis H2a: The less profitable a trader is, the more frequently she follows others. We partition trade profits into following and non-following, $\pi_j = \pi_j^{(f)} + \pi_j^{(nf)}$, using profits obtained while not following as a measure. We regress the

following rate, $F_j = TR_j^{(f)}/(TR_j^{(f)} + TR_j^{(nf)})$, on profits per trade π_j

$$F_j = \alpha_{2a} + \beta_{2a}\pi_j \ .$$

We find that β_{2a} is significantly less than zero, consistent with the hypothesis.

We next test hypothesis H2b: Do less profitable traders benefit more from following? We consider trades where a less profitable trader $\pi_j < 0$ follows a high profit trader, $\pi_j > 0$. We partition trade profits into following and non-following, $\pi_j = \pi_j^{(f)} + \pi_j^{(nf)}$ and regress total profits on the difference,

$$\pi_j^{(f)} - \pi_j^{(nf)} = \alpha_{2b} + \beta_{2b}\pi_j \ .$$

We find that $\beta_{2b} < 0$.

$\beta_{2a} < 0$ and $\beta_{2b} < 0$ shows traders' profitability are negatively related with their following frequency and their profits from following. The more profitable a trader is, the less frequently she follows others.

RESULT 3. *Who is followed: The more profitable a trader is, the more frequently she is followed by others.*

Hypothesis 3 asks whether more profitable traders have more followers? Define trader j's total trades and her trades followed by traders other than j as Tr_j and $Tr_{-j}^{(f)}$, and define the being followed rate,

$$F_{-j} = Tr_{-j}^{(f)}/Tr_j.$$

We then regress the profit level on the "being followed" rate

$$F_{-j} = \alpha_3 + \beta_3\pi_j \ ,$$

and find that $\beta_3 > 0$, indicating strong support of the hypothesis.

$\beta_3 > 0$ shows traders' profit records are positively related with their being-followed rate. The more profitable a trader is, the more frequently she is followed by others.

2.6 Conclusions

This essay studies individual day traders and their communications. An interaction game is built up to explain individual traders' strategic behaviors in an internet stock trading chat room. In the equilibrium without communication, the informed traders enter the market at the very beginning because they have perfect information while the momentum traders rely on the price path to make trading decisions and optimally follow informed traders to enter the market if the price at period 1 passes the threshold. In the equilibrium with communication, informed traders S_I still trade on their private signals immediately and they post truthfully to attract followers, which helps the asset realize the true value, while the momentum traders S_M optimally follow informed traders' posts. In sum, we model how communications influence traders' trading decisions and explain how the chat room is beneficial to all participants, even the informed traders.

We motivate three empirical results from the model's equilibrium: (1) More profitable traders post more fundamental analysis and less profitable traders post more non-fundamental analysis; (2) The less profitable a trader is, the more frequently she follows others; and (3) The more profitable a trader is, the more frequently she is followed by others. And we do find out that traders have some knowledge of who the profitable traders are and follow more often the most profitable traders, instead of the most active ones.

It is interesting to speculate whether Wall Street is just a large version of the chatroom. For example, large financial institutions are doing two things which skillful traders did in this chat room: (1) building positions before releasing information (see

e.g. Mizrach (2005)); and (2) taking advantage of reputation as was disclosed in Elliot Spitzer's investigations in 2002.

2.7 Proof A. Proof of Proposition 1

We will present the proof in four parts.

To begin, recall $P^* = \lambda \left(QQ_N - QQ_I \right)$, $\overline{P} = \lambda \left(QQ_I + QQ_N \right)$.

Suppose the number of traders satisfies:

(A.1) $QQ_I \geq 2, QQ_N - QQ_I \geq \frac{QQ_M}{4} \geq 3$

and the market impact satisfies:

(A.2) $2\lambda \left(QQ_N - QQ_I \right) < \widehat{v} - c < \lambda \left(QQ_N + QQ_M - QQ_I - 1 \right)$,

and $\lambda \left(QQ_I + QQ_M + QQ_N \right) < \widehat{v} + c.$

Part 1: For a fixed I player who chooses a_0^I and given that all other I players and all M players use the equilibrium strategies defined above, it follows that the random variable

$$
\begin{aligned}
P_1 &= \lambda \left[a_0^I + (QQ_I - 1)\sigma_0^I(\omega) + (QQ_M)\sigma_0^M \right] + \varepsilon_1 \\
&= \lambda \left[a_0^I + (QQ_I - 1)\sigma_0^I(\omega) \right] + \varepsilon_1
\end{aligned}
$$

Therefore, in state ω^+, P_1 is uniformly distributed on the interval

$$
J^{\omega^+}(a_0^I) = [\lambda(a_0^I - 1) - P^*, \lambda(a_0^I - 1) + \overline{P}]
$$

and in state ω^-, P_1 is uniformly distributed on the interval

$$
J^{\omega^-}(a_0^I) = [\lambda(a_0^I - 1) - \overline{P}, \lambda(a_0^I - 1) + P^*]
$$

We claim that σ_1^I is a best response for type I traders in period 1, i.e., we claim that,

for each a_0^I, ω, and $P_1 \in J^\omega(a_0^I)$,

$$\sigma_1^I\left(\omega, P_1, a_0^I\right) \in \underset{a_1^I \in \Lambda_1,\ -1 \leq a_0^I + a_1^I \leq 1}{\arg\max} \left\{ E_1\left[\pi^I | \omega, P_1, a_0^I\right] \right\}$$

$$\begin{aligned} & E_1\left[\pi^I | \omega, P_1, a_0^I\right] \\ = \ & -a_1^I \cdot \left(P_1 + \frac{\lambda a_1^I + \lambda\left(QQ_I - 1\right)\sigma_1^I + \lambda QQ_M \cdot \sigma_1^M}{2}\right) \\ & + \left(a_0^I + a_1^I\right) \cdot E\left[p_2 | \omega, P_1, a_0^I\right] \end{aligned}$$

Case 1.1: ω^+, $a_0^I = 1$, $P_1 \in J^{\omega^+}(1) = [-P^*, \overline{P}]$:

Case 1.1.1: $P_1 \in]P^*, \overline{P}]$:

Since $-1 \leq a_0^I + a_1^I \leq 1$ and $a_0^I = 1$, it follows that $a_1^I \in \{-1, 0\}$.

Using (A.2), we conclude that $\widehat{v} - c < P_2 < \widehat{v} + c$.

$$\begin{aligned} a_1^I \ = \ & 0 \Rightarrow \\ & E_1\left[\pi^I | \omega, P_1, a_0^I\right] \\ = \ & -a_1^I \cdot \left(P_1 + \frac{\lambda a_1^I + \lambda\left(QQ_I - 1\right)\sigma_1^I + \lambda QQ_M \cdot \sigma_1^M}{2}\right) \\ & + \left(a_0^I + a_1^I\right) \cdot E\left[p_2 | \omega, P_1, a_0^I\right] \\ = \ & 0 \cdot \left(P_1 + \frac{\lambda \cdot 0 + \lambda\left(QQ_I - 1\right) \cdot 0 + \lambda QQ_M \cdot 1}{2}\right) \\ & + (1 + 0) \cdot \left(P_1 + \lambda QQ_M\right) \\ = \ & P_1 + \lambda QQ_M \end{aligned}$$

$$a_1^I \quad = \quad -1$$

$$\Rightarrow \quad E_1\left[\pi^I|\omega, P_1, a_0^I\right]$$

$$= \quad -a_1^I \cdot \left(P_1 + \frac{\lambda a_1^I + \lambda\left(QQ_I - 1\right)\sigma_1^I + \lambda QQ_M \cdot \sigma_1^M}{2}\right)$$

$$\quad + \left(a_0^I + a_1^I\right) \cdot E\left[p_2|\omega, P_1, a_0^I\right]$$

$$= \quad +1 \cdot \left(P_1 + \frac{\lambda \cdot (-1) + \lambda\left(QQ_I - 1\right) \cdot 0 + \lambda QQ_M \cdot 1}{2}\right)$$

$$\quad + (1 - 1) \cdot (P_1 + \lambda QQ_M)$$

$$= \quad P_1 + \lambda\frac{QQ_M - 1}{2}$$

$$< \quad P_1 + \lambda QQ_M$$

Therefore,

$$\sigma_1^I\left(\omega^+, P_1, a_0^I\right) = 0 \in \underset{a_1^I \in \Lambda_1, \ -1 \leq a_0^I + a_1^I \leq 1}{\arg\max} \left\{E_1\left[\pi^I|\omega, P_1, a_0^I\right]\right\}.$$

Case 1.1.2: $P_1 \in]\frac{\lambda}{2}, P^*]$:

Since $-1 \leq a_0^I + a_1^I \leq 1$ and $a_0^I = 1$, it follows that $a_1^I \in \{-1, 0\}$.

Using (A.2), we conclude that $p_2 = \widehat{v} - c$.

$$a_1^I = 0 \Rightarrow$$

$$E_1\left[\pi^I|\omega, P_1, a_0^I\right]$$

$$= -a_1^I \cdot \left(P_1 + \frac{\lambda a_1^I + \lambda\left(QQ_I - 1\right)\sigma_1^I + \lambda QQ_M \cdot \sigma_1^M}{2}\right)$$

$$+ \left(a_0^I + a_1^I\right) \cdot E\left[p_2|\omega, P_1, a_0^I\right]$$

$$= 0 \cdot \left(P_1 + \frac{\lambda \cdot 0 + \lambda\left(QQ_I - 1\right)\cdot 0 - \lambda QQ_M \cdot q^+}{2}\right)$$

$$+ \left(1 + 0\right) \cdot \left(\widehat{v} - c\right)$$

$$= \widehat{v} - c$$

$$a_1^I = -1 \Rightarrow$$

$$E_1\left[\pi^I|\omega, P_1, a_0^I\right]$$

$$= -a_1^I \cdot \left(P_1 + \frac{\lambda a_1^I + \lambda\left(QQ_I - 1\right)\sigma_1^I + \lambda QQ_M \cdot \sigma_1^M}{2}\right)$$

$$+ \left(a_0^I + a_1^I\right) \cdot E\left[p_2|\omega, P_1, a_0^I\right]$$

$$= +1 \cdot \left(P_1 + \frac{\lambda \cdot (-1) + \lambda\left(QQ_I - 1\right)\cdot 0 - \lambda QQ_M \cdot q^+}{2}\right)$$

$$+ \left(1 - 1\right) \cdot \left(\widehat{v} - c\right)$$

$$= P_1 - \lambda\frac{QQ_M \cdot q^+ + 1}{2}$$

$$< 0$$

Since $\widehat{v} - c > 0$ by the assumptions (A.1) and (A.2), we conclude that

$$\sigma_1^I\left(\omega^+, P_1, a_0^I\right) = 0 \in \underset{a_1^I \in \Lambda_1,\ -1 \leq a_0^I + a_1^I \leq 1}{\arg\max}\left\{E_1\left[\pi^I|\omega, P_1, a_0^I\right]\right\}.$$

Case 1.1.3: $P_1 \in [-\frac{\lambda}{2}, \frac{\lambda}{2}]$:

Since $-1 \le a_0^I + a_1^I \le 1$ and $a_0^I = 1$, it follows that $a_1^I \in \{-1, 0\}$.

Using (A.2), we conclude that $p_2 = \widehat{v} - c$.

$$
\begin{aligned}
a_1^I &= 0 \Rightarrow \\[2mm]
&\quad E_1\left[\pi^I | \omega, P_1, a_0^I\right] \\[1mm]
&= -a_1^I \cdot \left(P_1 + \frac{\lambda a_1^I + \lambda\left(QQ_I - 1\right)\sigma_1^I + \lambda QQ_M \cdot \sigma_1^M}{2}\right) \\[1mm]
&\quad + \left(a_0^I + a_1^I\right) \cdot E\left[p_2 | \omega, P_1, a_0^I\right] \\[1mm]
&= 0 \cdot \left(P_1 + \frac{\lambda \cdot 0 + \lambda\left(QQ_I - 1\right)\cdot 0 - \lambda QQ_M \cdot 0}{2}\right) \\[1mm]
&\quad + (1 + 0) \cdot (\widehat{v} - c) \\[1mm]
&= \widehat{v} - c
\end{aligned}
$$

$$
\begin{aligned}
a_1^I &= -1 \Rightarrow \\[2mm]
&\quad E_1\left[\pi^I | \omega, P_1, a_0^I\right] \\[1mm]
&= -a_1^I \cdot \left(P_1 + \frac{\lambda a_1^I + \lambda\left(QQ_I - 1\right)\sigma_1^I + \lambda QQ_M \cdot \sigma_1^M}{2}\right) \\[1mm]
&\quad + \left(a_0^I + a_1^I\right) \cdot E\left[p_2 | \omega, P_1, a_0^I\right] \\[1mm]
&= +1 \cdot \left(P_1 + \frac{\lambda \cdot (-1) + \lambda\left(QQ_I - 1\right)\cdot 0 - \lambda QQ_M \cdot 0}{2}\right) \\[1mm]
&\quad + (1 - 1) \cdot (\widehat{v} - c) \\[1mm]
&= P_1 - \frac{\lambda}{2} \\[1mm]
&< 0
\end{aligned}
$$

Since $\widehat{v} - c > 0$ by the assumptions (A.1) and (A.2), we conclude that

$$
\sigma_1^I\left(\omega^+, P_1, a_0^I\right) = 0 \in \underset{a_1^I \in \Lambda_1,\ -1 \le a_0^I + a_1^I \le 1}{\arg\max}\left\{E_1\left[\pi^I | \omega, P_1, a_0^I\right]\right\}.
$$

Case 1.1.4: $P_1 \in [-P^*, -\frac{\lambda}{2}[$:

Since $-1 \leq a_0^I + a_1^I \leq 1$ and $a_0^I = 1$, it follows that $a_1^I \in \{-1, 0\}$.

Using (A.2), we conclude that $p_2 = \widehat{v} - c$.

$$
\begin{aligned}
a_1^I &= 0 \Rightarrow \\
&E_1\left[\pi^I | \omega, P_1, a_0^I\right] \\
&= -a_1^I \cdot \left(P_1 + \frac{\lambda a_1^I + \lambda\left(QQ_I - 1\right)\sigma_1^I + \lambda QQ_M \cdot \sigma_1^M}{2}\right) \\
&\quad + \left(a_0^I + a_1^I\right) \cdot E\left[p_2 | \omega, P_1, a_0^I\right] \\
&= 0 \cdot \left(P_1 + \frac{\lambda QQ_M \cdot q^-}{2}\right) + (0 + 1) \cdot (\widehat{v} - c) \\
&= \widehat{v} - c
\end{aligned}
$$

$$
\begin{aligned}
a_1^I &= -1 \Rightarrow \\
&E_1\left[\pi^I | \omega, P_1, a_0^I\right] \\
&= -a_1^I \cdot \left(P_1 + \frac{\lambda a_1^I + \lambda\left(QQ_I - 1\right)\sigma_1^I + \lambda QQ_M \cdot \sigma_1^M}{2}\right) \\
&\quad + \left(a_0^I + a_1^I\right) \cdot E\left[p_2 | \omega, P_1, a_0^I\right] \\
&= +1 \cdot \left(P_1 + \frac{\lambda \cdot (-1) + \lambda\left(QQ_I - 1\right) \cdot 0 + \lambda QQ_M \cdot q^-}{2}\right) \\
&\quad + (1 - 1) \cdot (\widehat{v} - c) \\
&= P_1 + \frac{\lambda}{2}\left(QQ_M \cdot q^- - 1\right) \\
&< -\lambda \\
&< 0
\end{aligned}
$$

Since $\widehat{v} - c > 0$ by the assumptions (A.1) and (A.2), we conclude that

$$
\sigma_1^I\left(\omega^+, P_1, a_0^I\right) = 0 \in \underset{a_1^I \in \Lambda_1, \ -1 \leq a_0^I + a_1^I \leq 1}{\arg\max}\left\{E_1\left[\pi^I | \omega, P_1, a_0^I\right]\right\}.
$$

Case 1.2: $\omega^+, a_0^I = 0, P_1 \in J^{\omega^+}(0) = [-\lambda - P^*, -\lambda + \overline{P}]$:

Case 1.2.1: $P_1 \in]P^*, -\lambda + \overline{P}]$:

Since $-1 \leq a_1^I \leq 1$ and $a_0^I = 0$, it follows that $a_1^I \in \{-1, 0, 1\}$.

Using (A2), we conclude that $\widehat{v} - c < P_2 < \widehat{v} + c$.

$$
\begin{aligned}
a_1^I &= 1 \Rightarrow \\
& E_1 \left[\pi^I | \omega, P_1, a_0^I \right] \\
&= -a_1^I \cdot \left(P_1 + \frac{\lambda a_1^I + \lambda (QQ_I - 1) \sigma_1^I + \lambda QQ_M \cdot \sigma_1^M}{2} \right) \\
&\quad + \left(a_0^I + a_1^I \right) \cdot E \left[p_2 | \omega, P_1, a_0^I \right] \\
&= -1 \cdot \left(P_1 + \frac{\lambda \cdot 1 + \lambda (QQ_I - 1) \cdot 0 + \lambda QQ_M \cdot 1}{2} \right) \\
&\quad + (1 + 0) \cdot (P_1 + \lambda QQ_M) \\
&= -\left(P_1 + \lambda \frac{QQ_M + 1}{2} \right) + 1 \cdot (P_1 + \lambda QQ_M) \\
&= \lambda \frac{QQ_M - 1}{2}
\end{aligned}
$$

$$
a_1^I = 0 \Rightarrow E_1 \left[\pi^I | \omega, P_1, a_0^I \right] = 0
$$

$$
\begin{aligned}
a_1^I &= -1 \Rightarrow \\
E_1 \left[\pi^I | \omega, P_1, a_0^I \right] &= -(-1) \cdot \left(P_1 + \lambda \frac{QQ_M - 1}{2} \right) \\
&\quad + (0 - 1) \cdot (P_1 + \lambda QQ_M) \\
&= -\lambda \frac{QQ_M + 1}{2}
\end{aligned}
$$

Therefore,

$$\sigma_1^I\left(\omega^+, P_1, 0\right) = 1 \in \underset{a_1^I \in \Lambda_1,\ -1 \leq a_0^I + a_1^I \leq 1}{\arg\max} \left\{ E_1 \left[\pi^I | \omega, P_1, a_0^I \right] \right\}.$$

Case 1.2.2: $P_1 \in]\frac{\lambda}{2}, P^*]$:

Since $-1 \leq a_1^I \leq 1$ and $a_0^I = 0$, it follows that $a_1^I \in \{-1, 0, 1\}$.

Using (A.2), we conclude that $p_2 = \widehat{v} - c$.

$$
\begin{aligned}
a_1^I &= 1 \Rightarrow \\
&\quad E_1 \left[\pi^I | \omega, P_1, a_0^I \right] \\
&= -a_1^I \cdot \left(P_1 + \frac{\lambda a_1^I + \lambda \left(QQ_I - 1\right) \sigma_1^I + \lambda QQ_M \cdot \sigma_1^M}{2} \right) \\
&\quad + \left(a_0^I + a_1^I \right) \cdot E \left[p_2 | \omega, P_1, a_0^I \right] \\
&= (-1) \cdot \left(P_1 + \frac{\lambda \cdot 1 + \lambda \left(QQ_I - 1\right) \cdot 0 - \lambda QQ_M \cdot q^+}{2} \right) \\
&\quad + (1 + 0) \cdot \left(\widehat{v} - c \right) \\
&= -\left(P_1 + \lambda \frac{-q^+ QQ_M + 1}{2} \right) \\
&\quad + \left(\widehat{v} - c \right) \\
&> \left(\widehat{v} - c \right) - \lambda \\
&> 0
\end{aligned}
$$

$$a_1^I = 0 \Rightarrow E_1 \left[\pi^I | \omega, P_1, a_0^I \right] = 0$$

$$a_1^I = -1 \Rightarrow$$

$$E_1 \left[\pi^I | \omega, P_1, a_0^I \right]$$

$$= -(-1) \cdot \left(P_1 + \lambda \frac{-q^+ \cdot QQ_M - 1}{2} \right) + (0 - 1) \cdot (\widehat{v} - c)$$

$$= \left(P_1 + \lambda \frac{-q^+ \cdot QQ_M - 1}{2} \right) - (\widehat{v} - c)$$

$$< -(\widehat{v} - c)$$

$$< 0$$

Since $\widehat{v} - c > \lambda > 0$ by the assumptions (A.1) and (A.2), we conclude that,

$$\sigma_1^I \left(\omega^+, P_1, 0 \right) = 1 \in \operatorname*{arg\,max}_{a_1^I \in \Lambda_1,\ -1 \leq a_0^I + a_1^I \leq 1} \left\{ E_1 \left[\pi^I | \omega, P_1, a_0^I \right] \right\}.$$

Case 1.2.3: $P_1 \in [-\frac{\lambda}{2}, \frac{\lambda}{2}]$:

Since $-1 \leq a_1^I \leq 1$ and $a_0^I = 0$, it follows that $a_1^I \in \{-1, 0, 1\}$.

Using (A.2), we conclude that $p_2 = \widehat{v} - c$.

$$a_1^I = 1 \Rightarrow$$

$$E_1\left[\pi^I|\omega, P_1, a_0^I\right]$$

$$= -a_1^I \cdot \left(P_1 + \frac{\lambda a_1^I + \lambda\left(QQ_I - 1\right)\sigma_1^I + \lambda QQ_M \cdot \sigma_1^M}{2}\right)$$

$$+ \left(a_0^I + a_1^I\right) \cdot E\left[p_2|\omega, P_1, a_0^I\right]$$

$$= (-1) \cdot \left(P_1 + \frac{\lambda \cdot 1 + \lambda\left(QQ_I - 1\right) \cdot 0 + \lambda QQ_M \cdot 0}{2}\right)$$

$$+ (1 + 0) \cdot (\widehat{v} - c)$$

$$= (\widehat{v} - c) - \left(P_1 + \frac{\lambda}{2}\right)$$

$$> (\widehat{v} - c) - \lambda$$

$$> 0$$

$$a_1^I = 0 \Rightarrow E_1\left[\pi^I|\omega, P_1, a_0^I\right] = 0$$

$$a_1^I = -1 \Rightarrow$$

$$E_1\left[\pi^I|\omega, P_1, a_0^I\right]$$

$$= -(-1) \cdot \left(P_1 - \frac{\lambda}{2}\right) + (0 - 1) \cdot (\widehat{v} - c)$$

$$= \left(P_1 + \frac{\lambda}{2}\right) - (\widehat{v} - c)$$

$$< -(\widehat{v} - c) + \lambda < 0$$

Since $\widehat{v} - c > \lambda > 0$ by the assumptions (A.1) and (A.2), we conclude that,

$$\sigma_1^I\left(\omega^+, P_1, 0\right) = 1 \in \underset{a_1^I \in \Lambda_1,\ -1 \leq a_0^I + a_1^I \leq 1}{\arg\max}\left\{E_1\left[\pi^I|\omega, P_1, a_0^I\right]\right\}.$$

Case 1.2.4: $P_1 \in [-P^*, -\frac{\lambda}{2}[$:

Since $-1 \leq a_1^I \leq 1$ and $a_0^I = 0$, it follows that $a_1^I \in \{-1, 0, 1\}$.

Using (A.2), we conclude that $p_2 = \widehat{v} - c$.

$$
\begin{aligned}
a_1^I &= 1 \Rightarrow \\
& E_1\left[\pi^I|\omega, P_1, a_0^I\right] \\
&= -a_1^I \cdot \left(P_1 + \frac{\lambda a_1^I + \lambda\left(QQ_I - 1\right)\sigma_1^I + \lambda QQ_M \cdot \sigma_1^M}{2}\right) \\
&\quad + \left(a_0^I + a_1^I\right) \cdot E\left[p_2|\omega, P_1, a_0^I\right] \\
&= (-1) \cdot \left(P_1 + \frac{\lambda \cdot 1 + \lambda\left(QQ_I - 1\right) \cdot 0 + \lambda QQ_M \cdot q^-}{2}\right) \\
&\quad + (1 + 0) \cdot (\widehat{v} - c) \\
&= -\left(P_1 + \lambda\frac{q^- QQ_M + 1}{2}\right) + (\widehat{v} - c) \\
&> (\widehat{v} - c) - \frac{\lambda}{2} \\
&> 0
\end{aligned}
$$

$$
a_1^I = 0 \Rightarrow E_1\left[\pi^I|\omega, P_1, a_0^I\right] = 0
$$

$$
\begin{aligned}
a_1^I &= -1 \Rightarrow \\
E_1\left[\pi^I|\omega, P_1, a_0^I\right] &= \left(P_1 + \lambda\frac{q^- QQ_M - 1}{2}\right) - (\widehat{v} - c) \\
&< -(\widehat{v} - c) - \frac{\lambda}{2} \\
&< 0
\end{aligned}
$$

Since $\widehat{v} - c > \frac{\lambda}{2}$ by the assumptions (A.1) and (A.2), we conclude that,

$$\sigma_1^I\left(\omega^+, P_1, 0\right) = 1 \in \underset{a_1^I \in \Lambda_1, \ -1 \leq a_0^I + a_1^I \leq 1}{\arg\max} \left\{ E_1\left[\pi^I | \omega, P_1, a_0^I\right] \right\}.$$

Case 1.2.5: $P_1 \in [-\lambda - P^*, -P^*[$:

Since $-1 \leq a_1^I \leq 1$ and $a_0^I = 0$, it follows that $a_1^I \in \{-1, 0, 1\}$.

Using (A.2), we conclude that $p_2 = \widehat{v} - c$.

$$
\begin{aligned}
a_1^I &= 1 \Rightarrow \\[1em]
& E_1\left[\pi^I | \omega, P_1, a_0^I\right] \\
&= -a_1^I \cdot \left(P_1 + \frac{\lambda a_1^I + \lambda\left(QQ_I - 1\right)\sigma_1^I + \lambda QQ_M \cdot \sigma_1^M}{2} \right) \\
& \quad + \left(a_0^I + a_1^I\right) \cdot E\left[p_2 | \omega, P_1, a_0^I\right] \\
&= (-1) \cdot \left(P_1 + \frac{\lambda \cdot 1 + \lambda\left(QQ_I - 1\right) \cdot 0 - \lambda QQ_M \cdot 1}{2} \right) \\
& \quad + (1 + 0) \cdot (\widehat{v} - c) \\
&= (\widehat{v} - c) - \left(P_1 - \lambda\frac{QQ_M - 1}{2} \right) \\
&> 0
\end{aligned}
$$

$$a_1^I = 0 \Rightarrow E_1\left[\pi^I | \omega, P_1, a_0^I\right] = 0$$

$$
\begin{aligned}
a_1^I &= -1 \Rightarrow \\
E_1\left[\pi^I | \omega, P_1, a_0^I\right] &= -(\widehat{v} - c) + \left(P_1 - \lambda\frac{QQ_M + 1}{2} \right) \\
&< 0
\end{aligned}
$$

Therefore, we conclude that,

$$\sigma_1^I\left(\omega^+, P_1, 0\right) = 1 \in \underset{a_1^I \in \Lambda_1,\ -1 \leq a_0^I + a_1^I \leq 1}{\arg\max} \left\{ E_1\left[\pi^I | \omega, P_1, a_0^I\right] \right\}.$$

Case 1.3: ω^+, $a_0^I = -1$, $P_1 \in J^{\omega^+}(-1) = [-2\lambda + P^*, -2\lambda + \overline{P}]$:

Case 1.3.1: $P_1 \in]P^*, -2\lambda + \overline{P}]$:

Since $-1 \leq a_0^I + a_1^I \leq 1$ and $a_0^I = -1$, it follows that $a_1^I \in \{1, 0\}$.

Using (A.2), we conclude that $\widehat{v} - c < P_2 < \widehat{v} + c$.

$$a_1^I = 0 \Rightarrow E_1\left[\pi^I | \omega, P_1, a_0^I\right] = -\left(P_1 + \lambda Q Q_M\right)$$

$$a_1^I = 1 \Rightarrow E_1\left[\pi^I | \omega, P_1, a_0^I\right] = -\left(P_1 + \lambda \frac{Q Q_M + 1}{2}\right)$$

Therefore,

$$\sigma_1^I\left(\omega^+, P_1, -1\right) = 1 \in \underset{a_1^I \in \Lambda_1,\ -1 \leq a_0^I + a_1^I \leq 1}{\arg\max} \left\{ E_1\left[\pi^I | \omega, P_1, a_0^I\right] \right\}.$$

Case 1.3.2: $P_1 \in]\frac{\lambda}{2}, P^*]$:

Since $-1 \leq a_0^I + a_1^I \leq 1$ and $a_0^I = -1$, it follows that $a_1^I \in \{1, 0\}$.

Using (A.2), we conclude that $p_2 = \widehat{v} - c$.

$$a_1^I = 0 \Rightarrow E_1\left[\pi^I | \omega, P_1, a_0^I\right] = -\left(\widehat{v} - c\right)$$

$$
\begin{aligned}
a_1^I &= 1 \Rightarrow \\
E_1\left[\pi^I | \omega, P_1, a_0^I\right] &= \frac{q^+ Q Q_M - 1}{2}\lambda - P_1 > -\lambda
\end{aligned}
$$

Since $\widehat{v} - c > \lambda$ by the assumptions (A.1) and (A.2), we conclude that,

$$\sigma_1^I\left(\omega^+, P_1, -1\right) = 1 \in \underset{a_1^I \in \Lambda_1, \ -1 \leq a_0^I + a_1^I \leq 1}{\arg\max} \left\{E_1\left[\pi^I | \omega, P_1, a_0^I\right]\right\}.$$

Case 1.3.3: $P_1 \in [-\frac{\lambda}{2}, \frac{\lambda}{2}]$:

Since $-1 \leq a_0^I + a_1^I \leq 1$ and $a_0^I = -1$,it follows that $a_1^I \in \{1, 0\}$.

Using (A.2), we conclude that $p_2 = \widehat{v} - c$.

$$a_1^I = 0 \Rightarrow E_1\left[\pi^I | \omega, P_1, a_0^I\right] = -\left(\widehat{v} - c\right)$$

$$a_1^I = 1 \Rightarrow E_1\left[\pi^I | \omega, P_1, a_0^I\right] = -\frac{\lambda}{2} - P_1 > -\lambda$$

Since $\widehat{v} - c > \lambda$ by the assumptions (A.1) and (A.2), we conclude that,

$$\sigma_1^I\left(\omega^+, P_1, -1\right) = 1 \in \underset{a_1^I \in \Lambda_1, \ -1 \leq a_0^I + a_1^I \leq 1}{\arg\max} \left\{E_1\left[\pi^I | \omega, P_1, a_0^I\right]\right\}.$$

Case 1.3.4: $P_1 \in [-P^*, -\frac{\lambda}{2}[$:

Since $-1 \leq a_0^I + a_1^I \leq 1$ and $a_0^I = -1$,it follows that $a_1^I \in \{1, 0\}$.

Using (A.2), we conclude that $p_2 = \widehat{v} - c$.

$$a_1^I = 0 \Rightarrow E_1\left[\pi^I | \omega, P_1, a_0^I\right] = -\left(\widehat{v} - c\right)$$

$$
\begin{aligned}
a_1^I &= 1 \Rightarrow \\
E_1\left[\pi^I | \omega, P_1, a_0^I\right] &= -\frac{q^- Q Q_M + 1}{2}\lambda - P_1 > -\frac{\lambda}{2}
\end{aligned}
$$

Since $\widehat{v} - c > \frac{\lambda}{2}$ by the assumptions (A.1) and (A.2), we conclude that,

$$\sigma_1^I\left(\omega^+, P_1, -1\right) = 1 \in \underset{a_1^I \in \Lambda_1,\ -1 \le a_0^I + a_1^I \le 1}{\arg\max} \left\{E_1\left[\pi^I | \omega, P_1, a_0^I\right]\right\}.$$

Case 1.3.5: $P_1 \in [-2\lambda - P^*, -P^*[$:

Since $-1 \le a_0^I + a_1^I \le 1$ and $a_0^I = -1$, it follows that $a_1^I \in \{1, 0\}$.

Using (A.2), we conclude that $p_2 = \widehat{v} - c$.

$$a_1^I = 0 \Rightarrow E_1\left[\pi^I | \omega, P_1, a_0^I\right] = -\left(\widehat{v} - c\right) < 0$$

$$a_1^I = 1 \Rightarrow$$
$$E_1\left[\pi^I | \omega, P_1, a_0^I\right] = \lambda \frac{QQ_M - 1}{2} - P_1 > 0$$

Since $\lambda \frac{QQ_M - 1}{2} - P_1 > 0 > -\left(\widehat{v} - c\right)$ by the assumptions (A.1) and (A.2), we conclude that,

$$\sigma_1^I\left(\omega^+, P_1, -1\right) = 1 \in \underset{a_1^I \in \Lambda_1,\ -1 \le a_0^I + a_1^I \le 1}{\arg\max} \left\{E_1\left[\pi^I | \omega, P_1, a_0^I\right]\right\}.$$

Case 1.4: $\omega^-, a_0^I = -1, P_1 \in J^{\omega^-}(-1) = [-\overline{P}, P^*]$:

This proof is exactly the symmetric case to **Case 1.1**,

$$\sigma_1^I\left(\omega^-, P_1, -1\right) = 0 \in \underset{a_1^I \in \Lambda_1,\ -1 \le a_0^I + a_1^I \le 1}{\arg\max} \left\{E_1\left[\pi^I | \omega, P_1, a_0^I\right]\right\}.$$

Case 1.5: $\omega^-, a_0^I = 0, P_1 \in J^{\omega^-}(0) = [-\lambda - \overline{P}, -\lambda + P^*]$:

This proof is exactly the symmetric case to **Case 1.2**,

$$\sigma_1^I\left(\omega^-, P_1, 0\right) = -1 \in \underset{a_1^I \in \Lambda_1,\ -1 \le a_0^I + a_1^I \le 1}{\arg\max} \left\{E_1\left[\pi^I | \omega, P_1, a_0^I\right]\right\}.$$

Case 1.6: ω^-, $a_0^I = 1$, $P_1 \in J^{\omega^-}(1) = [-2\lambda - \overline{P}, -2\lambda + P^*]$:

This proof is exactly the symmetric case to **Case 1.3**,

$$\sigma_1^I\left(\omega^-, P_1, 1\right) = -1 \in \underset{a_1^I \in \Lambda_1,\ -1 \leq a_0^I + a_1^I \leq 1}{\arg\max} \left\{E_1\left[\pi^I | \omega, P_1, a_0^I\right]\right\}.$$

Part 2: For a fixed I player who chooses a_0^I and given that all other I players and all M players use the equilibrium strategies defined above, it follows that the random variable

$$
\begin{aligned}
P_1 &= \lambda\left[a_0^I + (QQ_I - 1)\sigma_0^I(\omega) + (QQ_M)\sigma_0^M\right] + \varepsilon_1 \\
&= \lambda\left[a_0^I + (QQ_I - 1)\sigma_0^I(\omega)\right] + \varepsilon_1.
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
&E\left[P_1 | \omega^+, a_0^I\right] \\
&= E[\lambda\left[a_0^I + (QQ_I - 1)\sigma_0^I(\omega) | \omega^+, a_0^I\right] + E[\varepsilon_1 | \omega^+, a_0^I] \\
&= \lambda\left[a_0^I + (QQ_I - 1)\sigma_0^I(\omega^+)\right] + 0 \\
&= \lambda\left[a_0^I + (QQ_I - 1)\right].
\end{aligned}
$$

and

$$
\begin{aligned}
&E\left[P_1 | \omega^-, a_0^I\right] \\
&= E[\lambda\left[a_0^I + (QQ_I - 1)\sigma_0^I(\omega) | \omega^-, a_0^I\right] + E[\varepsilon_1 | \omega^-, a_0^I] \\
&= \lambda\left[a_0^I + (QQ_I - 1)\sigma_0^I(\omega^-)\right] + 0 \\
&= \lambda\left[a_0^I - (QQ_I - 1)\right].
\end{aligned}
$$

We claim that σ_0^I is a best response for type I traders in period 1, i.e., we claim that, for each ω,

$$\sigma_0^I(\omega) \in \underset{a_0^I \in \Lambda_1}{\arg\max} E_0\left[\pi^I|\omega\right]$$

, where

$$E_0\left[\pi^I|\omega\right] = E\left[\begin{array}{c} -a_0^I \cdot \left(\frac{\lambda a_0^I + \lambda(QQ_I - 1)\sigma_0^I + \lambda QQ_M \cdot \sigma_0^M}{2}\right) \\ -\sigma_1^I \cdot \left(P_1 + \frac{\lambda QQ_M \cdot \sigma_1^M + \lambda QQ_I \cdot \sigma_1^I}{2}\right) \\ + \left(a_0^I + \sigma_1^I\right) \cdot p_2 \end{array}\middle| \omega\right]$$

Case 2.1: In state ω^+,

$$E_0\left[\pi^I|\omega^+\right] = E\left[\begin{array}{c} -a_0^I \cdot \left(\frac{\lambda a_0^I + \lambda(QQ_I - 1)\sigma_0^I + \lambda QQ_M \cdot \sigma_0^M}{2}\right) \\ -\sigma_1^I \cdot \left(P_1 + \frac{\lambda QQ_M \cdot \sigma_1^M + \lambda QQ_I \cdot \sigma_1^I}{2}\right) \\ + \left(a_0^I + \sigma_1^I\right) \cdot p_2 \end{array}\middle| \omega\right]$$

Consequently,

$$
\begin{aligned}
a_0^I &= 1 \Rightarrow \sigma_1^I\left(\omega^+, a_0^I, P_1\right) = 0 \\
&\Rightarrow E_0\left[\pi^I \middle| \omega^+\right] = E\left[\left.\begin{array}{c} -1 \cdot \left(\frac{\lambda + \lambda(QQ_I - 1)}{2}\right) \\ +0 + (1+0) \cdot p_2 \end{array} \middle| \omega^+\right.\right] \\
&= E\left[-\left(\frac{QQ_I \lambda}{2}\right) + p_2 \middle| \omega^+\right] \\
&= -\left(\frac{QQ_I \lambda}{2}\right) + E\left[p_2 \middle| \omega^+\right] \\
&= -\left(\frac{QQ_I \lambda}{2}\right) + \left\{\begin{array}{c} \int_{P^*}^{\overline{P}} (x + \lambda QQ_M) \frac{dx}{2\lambda QQ_N} \\ +\int_{-P^*}^{P^*} (\widehat{v} - c) \frac{dx}{2\lambda QQ_N} \end{array}\right\} \\
&= (\widehat{v} - c) \cdot \left(1 - \frac{QQ_I}{QQ_N}\right) + \lambda QQ_I \cdot \left(\frac{QQ_M}{QQ_N} + \frac{1}{2}\right) \\
&\triangleq A^1
\end{aligned}
$$

$$
\begin{aligned}
a_0^I &= 0 \Rightarrow \sigma_1^I\left(\omega^+, a_0^I, P_1\right) = 1 \\
&\Rightarrow E_0\left[\pi^I|\omega^+\right] = E\left[p_2 - P_1 - \left(\frac{QQ_M \cdot \sigma_1^M + 1}{2}\lambda\right)\bigg|\omega^+\right] \\
&= \int_{P^*}^{\overline{P}-\lambda}\left[x + \lambda QQ_M - x - \lambda\frac{QQ_M+1}{2}\right]\frac{dx}{2\lambda QQ_N} \\
&\quad + \int_{\frac{\lambda}{2}}^{P^*}\left[(\widehat{v}-c) - x - \left(\frac{-q^+QQ_M+1}{2}\lambda\right)\right]\frac{dx}{2\lambda QQ_N} \\
&\quad + \int_{-\frac{\lambda}{2}}^{\frac{\lambda}{2}}\left[(\widehat{v}-c) - x - \frac{\lambda}{2}\right]\frac{dx}{2\lambda QQ_N} \\
&\quad + \int_{-P^*}^{0}\left[(\widehat{v}-c) - x - \left(\frac{q^-QQ_M+1}{2}\lambda\right)\right]\frac{dx}{2\lambda QQ_N} \\
&\quad + \int_{-P^*-\lambda}^{-P^*}\left[(\widehat{v}-c) - x - \left(\frac{-QQ_M+1}{2}\lambda\right)\right]\frac{dx}{2\lambda QQ_N} \\
&= (\widehat{v}-c)\cdot\left(1 - \frac{QQ_I}{QQ_N} + \frac{1}{2QQ_N}\right) \\
&\quad + \frac{QQ_M-1}{2QQ_N}\lambda\left(QQ_I - \frac{1}{2}\right) \\
&\triangleq A^0
\end{aligned}
$$

$$
\begin{aligned}
&A^1 - A^0 \\
&= (\widehat{v}-c)\cdot\left(-\frac{1}{2QQ_N}\right) + \lambda QQ_I\cdot\left(\frac{QQ_M+1}{2QQ_N} + \frac{1}{2}\right) + \frac{QQ_M-1}{4QQ_N}\lambda \\
&> \frac{1}{2QQ_N}\left\{\lambda QQ_I\cdot(QQ_N + QQ_M + 1) + \lambda\frac{QQ_M-1}{2} - (\widehat{v}-c)\right\} \\
&> {}^1\frac{1}{2QQ_N}\left\{\lambda(QQ_N + QQ_M + 1) + \lambda\frac{QQ_M-1}{2} - (\widehat{v}-c)\right\} \\
&> {}^2 0
\end{aligned}
$$

where $>^1$ follows from (A.1) and $>^2$ follows from (A.2).

$$
\begin{aligned}
a_0^I &= -1 \Rightarrow \sigma_1^I\left(\omega^+, a_0^I, P_1\right) = 1 \\
&\Rightarrow E_0\left[\pi^I | \omega^+\right] \\
&= E\left[\left(\frac{QQ_I - 2}{2}\lambda\right) - P_1 - \left(\frac{QQ_M \cdot \sigma_1^M + 1}{2}\lambda\right)\Big|\omega\right] \\
&= \lambda\left(\frac{QQ_I}{2} - 1\right) + \int_{P^*}^{\overline{P} - 2\lambda}\left[-x + \frac{QQ_M - 1}{2}\lambda\right]\frac{dx}{2\lambda QQ_N} \\
&\quad + \int_{\frac{\lambda}{2}}^{P^*}\left[-x + \frac{-q^+ QQ_M - 1}{2}\lambda\right]\frac{dx}{2\lambda QQ_N} \\
&\quad + \int_{-\frac{\lambda}{2}}^{\frac{\lambda}{2}}\left[-x - \frac{\lambda}{2}\right]\frac{dx}{2\lambda QQ_N} \\
&\quad + \int_{-P^*}^{-\frac{\lambda}{2}}\left[-x + \frac{q^- QQ_M - 1}{2}\lambda\right]\frac{dx}{2\lambda QQ_N} \\
&\quad + \int_{-P^* - 2\lambda}^{-P^*}\left[-x + \frac{-QQ_M - 1}{2}\lambda\right]\frac{dx}{2\lambda QQ_N} \\
&= -\frac{\lambda}{2}\left(QQ_I - 1\right) - \frac{QQ_M}{QQ_N}\left(\frac{\lambda QQ_I}{2} + \lambda\right) \\
&<^1 0
\end{aligned}
$$

where $<^1$ follows from (A.1), and $A^1 > 0$ follows from (A.1).

Therefore, we conclude that

$$
\sigma_0^I\left(\omega^+\right) = 1 \in \underset{a_0^I \in \Lambda_1}{\arg\max} E_0\left[\pi^I | \omega^+\right].
$$

Case 2.2: In state ω^-,

This proof is exactly the symmetric case to **Case 2.1**,

$$
\sigma_0^I\left(\omega^-\right) = -1 \in \underset{a_0^I \in \Lambda_1}{\arg\max} E_0\left[\pi^I | \omega^-\right].
$$

Part 3: For a fixed M trader who chooses a_0^M and given that all I players and

all other M players use the equilibrium strategies defined above, it follows that

$$P_1 = \lambda \left[QQ_I \sigma_0^I(\omega) + a_0^M \right] + \varepsilon_1.$$

Conditional on $\omega = \omega^+$, the random variable P_1 is uniformly distributed on the interval

$$K^+(a_0^M) = [\lambda a_0^M - P^*, \lambda a_0^M + \overline{P}]$$

and conditional on $\omega = \omega^-$, the random variable P_1 is uniformly distributed on the interval

$$K^-(a_0^M) = [\lambda a_0^M - \overline{P}, \lambda a_0^M + P^*].$$

We claim that σ_1^M is a best response for type I traders in period 1, i.e., we claim that, for each a_0^M and $P_1 \in K^+(a_0^M) \cup K^-(a_0^M)$,

$$\sigma_1^M \left(P_1, a_0^M \right) \in \underset{a_1^M \in \Lambda_1, \ -1 \leq a_0^M + a_1^M \leq 1}{\arg\max} \{ E_1 \left[\pi^M | P_1, a_0^M \right] \},$$

where

$$
\begin{aligned}
& E_1 \left[\pi^M | P_1, a_0^M \right] \\
={} & -a_1^M \cdot \left(P_1 + \frac{\lambda a_1^M + \lambda \left(QQ_M - 1 \right) \sigma_1^M + \lambda QQ_I \cdot \sigma_1^I}{2} \right) \\
& + \left(a_0^M + a_1^M \right) \cdot E \left[p_2 | P_1, a_0^M \right].
\end{aligned}
$$

Case 3.1: Suppose that $a_0^M = 0$. Since $K^+(0) = [-P^*, \overline{P}]$ and $K^-(0) = [-\overline{P}, P^*]$, we consider realizations of P_1 lying in the intervals $[-\overline{P}, -P^*[, [-P^*, 0[, [0, P^*]$ and $]P^*, \overline{P}]$.

Case 3.1.1: $P_1 \in]P^*, \overline{P}]$:

In this case, $\Pr\left(\omega^+ | P_1, a_0^M\right) = 1$.

Since $-1 \leq a_0^I + a_1^I \leq 1$ and $a_0^M = 0$,it follows that $a_1^I \in \{-1, 0, 1\}$.

Using (A.2), we conclude that $\widehat{v} - c < P_2 < \widehat{v} + c$.

$$
\begin{aligned}
& E_1\left[\pi^M | P_1, a_0^M\right] \\
= \ & -a_1^M \cdot \left(P_1 + \frac{\lambda a_1^M + \lambda\left(QQ_M - 1\right)\sigma_1^M + \lambda QQ_I \cdot \sigma_1^I}{2}\right) \\
& + \left(a_0^M + a_1^M\right) \cdot E\left[p_2 | P_1, a_0^M\right] \\
= \ & -a_1^M \cdot \left(P_1 + \frac{\lambda a_1^M + \lambda\left(QQ_M - 1\right) \cdot 1 + \lambda QQ_I \cdot 0}{2}\right) \\
& + \left(0 + a_1^M\right) \cdot \left(P_1 + \lambda a_1^M + \lambda\left(QQ_M - 1\right) \cdot 1 + \lambda QQ_I \cdot 0\right) \\
= \ & a_1^M \left(\lambda \frac{a_1^M + \left(QQ_M - 1\right)}{2}\right)
\end{aligned}
$$

Therefore, we conclude that

$$
\sigma_1^M\left(P_1, a_0^M\right) = 1 \in \underset{a_1^m \in \{1, 0, -1\}}{\arg\max}\left\{E_1\left[\pi^M | P_1, a_0^M\right]\right\}
$$

Case 3.1.2: $P_1 \in]\frac{\lambda}{2}, P^*]$:

In this case,

$$
\begin{aligned}
& \Pr\left(\omega^+ | P_1, a_0^M\right) \\
= \ & \frac{f\left(P_1 | \omega^+\right) \cdot \Pr\left(\omega^+\right)}{f\left(P_1 | \omega^+\right) \cdot \Pr\left(\omega^+\right) + f\left(P_1 | \omega^-\right) \cdot \Pr\left(\omega^-\right)} \\
= \ & \frac{1}{2}
\end{aligned}
$$

Since $-1 \leq a_0^I + a_1^I \leq 1$ and $a_0^M = 0$,it follows that $a_1^I \in \{-1, 0, 1\}$.

Using (A.2), we conclude that $p_2 = \widehat{v} - c$ in state ω^+ and $p_2 = -\left(\widehat{v} - c\right)$ in state

ω^-, so that $E\left[p_2|P_1, a_0^M\right] = 0$ and

$$E_1\left[\pi^M|P_1, a_0^M\right]$$

$$= -a_1^M \cdot \left(P_1 + \lambda\frac{a_1^M + (QQ_M - 1)\,\sigma_1^M + QQ_I \cdot \sigma_1^I}{2}\right)$$

$$+ \left(a_0^M + a_1^M\right) \cdot E\left[p_2|P_1, a_0^M\right],$$

$$= -a_1^M \cdot \left(P_1 + \frac{a_1^M - q^+\,(QQ_M - 1)}{2}\lambda\right).$$

$$a_1^M = -1 \Rightarrow$$

$$E_1\left[\pi^M|P_1, a_0^M\right] = P_1 - \frac{\lambda}{2} - \frac{q^+\,(QQ_M - 1)}{2}\lambda$$

$$= 0,$$

$$a_1^M = 0 \Rightarrow E\left[\pi_1^M|P_1, a_0^I\right] = 0,$$

$$a_1^M = 1 \Rightarrow$$

$$E_1\left[\pi^M|P_1, a_0^M\right] = -P_1 - \frac{\lambda}{2} + \frac{q^+\,(QQ_M - 1)}{2}\lambda$$

$$= -\lambda,$$

where $q^+ = \frac{2P_1 - \lambda}{\lambda(QQ_M - 1)} > 0$.

Therefore,

$$\sigma_1^M\left(P_1, a_0^M\right) = \left\{0 \text{ with } prob\ 1 - q^+ \text{ and } -1 \text{ with } prob\ q^+\right\}$$

$$\in \underset{a_1^m \in \{1,0,-1\}}{\arg\max}\left\{E_1\left[\pi^M|P_1, a_0^M\right]\right\}$$

Case 3.1.3: $P_1 \in \left[-\frac{\lambda}{2}, \frac{\lambda}{2}\right]$:

In this case,

$$
\begin{aligned}
&\Pr\left(\omega^+|P_1, a_0^M\right) \\
&= \frac{f\left(P_1|\omega^+\right) \cdot \Pr\left(\omega^+\right)}{f\left(P_1|\omega^+\right) \cdot \Pr\left(\omega^+\right) + f\left(P_1|\omega^-\right) \cdot \Pr\left(\omega^-\right)} \\
&= \frac{1}{2}
\end{aligned}
$$

Since $-1 \le a_0^I + a_1^I \le 1$ and $a_0^M = 0$, it follows that $a_1^I \in \{-1, 0, 1\}$.

Using (A.2), we conclude that $p_2 = \widehat{v} - c$ in state ω^+ and $p_2 = -\left(\widehat{v} - c\right)$ in state ω^-, so that $E\left[p_2|P_1, a_0^M\right] = 0$ and

$$
\begin{aligned}
&E_1\left[\pi^M|P_1, a_0^M\right] \\
&= -a_1^M \cdot \left(P_1 + \lambda \frac{a_1^M + (QQ_M - 1)\sigma_1^M + QQ_I \cdot \sigma_1^I}{2}\right) \\
&\quad + \left(a_0^M + a_1^M\right) \cdot E\left[p_2|P_1, a_0^M\right] \\
&= -a_1^M \cdot \left(P_1 + \frac{a_1^M}{2}\lambda\right)
\end{aligned}
$$

$$
a_1^M = -1 \Rightarrow E_1\left[\pi^M|P_1, a_0^M\right] = P_1 - \frac{\lambda}{2} < 0
$$

$$
a_1^M = 0 \Rightarrow E\left[\pi_1^M|P_1, a_0^I\right] = 0
$$

$$
a_1^M = 1 \Rightarrow E_1\left[\pi^M|P_1, a_0^M\right] = -P_1 - \frac{\lambda}{2} < 0
$$

Therefore,

$$\sigma_1^M\left(P_1, a_0^M\right) = 0 \in \underset{a_1^m \in \{1,0,-1\}}{\arg\max} \left\{ E_1\left[\pi^M | P_1, a_0^M\right] \right\}.$$

Case 3.1.4: $P_1 \in [-P^*, -\frac{\lambda}{2}[$:

This proof is exactly the symmetric case to **Case 3.1.2**,

$$\sigma_1^M\left(P_1, a_0^M\right) = \left\{ 0 \text{ with } prob \ 1 - q^- \text{ and } 1 \text{ with } prob \ q^- \right\}$$

$$\in \underset{a_1^m \in \{1,0,-1\}}{\arg\max} \left\{ E_1\left[\pi^M | P_1, a_0^M\right] \right\},$$

where $q^- = \frac{-2P_1 - \lambda}{\lambda(QQ_M - 1)} > 0$.

Case 3.1.5: $P_1 \in [-\overline{P}, -P^*[$:

This proof is exactly the symmetric case to **Case 3.1.1**,

$$\sigma_1^M\left(P_1, a_0^M\right) = -1 \in \underset{a_1^m \in \{1,0,-1\}}{\arg\max} \left\{ E_1\left[\pi^M | P_1, a_0^M\right] \right\}$$

Case 3.2: Suppose that $a_0^M = 1$. Since $K^+(1) = [\lambda - P^*, \lambda + \overline{P}]$ and $K^-(1) = [\lambda - \overline{P}, \lambda + P^*]$, we consider realizations of P_1 lying in the intervals $[\lambda - \overline{P}, -P^*[, [-P^*, \lambda - P^*[, [\lambda - P^*, -\frac{\lambda}{2}[, [-\frac{\lambda}{2}, \frac{\lambda}{2}],]\frac{\lambda}{2}, P^*],]P^*, P^* + \lambda]$ and $]P^* + \lambda, \overline{P} + \lambda]$.

Case 3.2.1: $P_1 \in]P^* + \lambda, \overline{P} + \lambda]$:

In this case, $\Pr\left(\omega^+ | P_1, a_0^M\right) = 1$.

Since $-1 \le a_0^I + a_1^I \le 1$ and $a_0^M = 1$,it follows that $a_1^I \in \{-1, 0\}$.

Using (A.2), we conclude that $\widehat{v} - c < P_2 < \widehat{v} + c$.

$$E_1\left[\pi^M|P_1,a_0^M\right]$$

$$= -a_1^M\cdot\left(P_1+\frac{\lambda a_1^M+\lambda\left(QQ_M-1\right)\sigma_1^M+\lambda QQ_I\cdot\sigma_1^I}{2}\right)$$

$$\quad+\left(a_0^M+a_1^M\right)\cdot E\left[p_2|P_1,a_0^M\right]$$

$$= -a_1^M\cdot\left(P_1+\frac{\lambda a_1^M+\lambda\left(QQ_M-1\right)\cdot 1+\lambda QQ_I\cdot 0}{2}\right)$$

$$\quad+\left(0+a_1^M\right)\cdot\left(P_1+\lambda a_1^M+\lambda\left(QQ_M-1\right)\cdot 1+\lambda QQ_I\cdot 0\right)$$

$$= a_1^M\left(\lambda\frac{a_1^M+\left(QQ_M-1\right)}{2}\right)$$

Therefore, we conclude that

$$\sigma_1^M\left(P_1,a_0^M\right)=0\in\operatorname*{arg\,max}_{a_1^m\in\{-1,0\}}\left\{E_1\left[\pi^M|P_1,a_0^M\right]\right\}$$

Case 3.2.2: $P_1\in]P^*,P^*+\lambda]$:

In this case,

$$\Pr\left(\omega^+|P_1,a_0^M\right)=\frac{f\left(P_1|\omega^+\right)\cdot\Pr\left(\omega^+\right)}{f\left(P_1|\omega^+\right)\cdot\Pr\left(\omega^+\right)+f\left(P_1|\omega^-\right)\cdot\Pr\left(\omega^-\right)}=\frac{1}{2}$$

Since $-1\le a_0^I+a_1^I\le 1$ and $a_0^M=1$,it follows that $a_1^I\in\{-1,0\}$.

Using (A.2), we conclude that $\widehat{v}+c<P_2<\widehat{v}-c$ in state ω^+ and $p_2=-\left(\widehat{v}-c\right)$ in state ω^-,so that

$$E_1\left[\pi^M|P_1,a_0^M\right]$$

$$= -a_1^M\cdot\left(P_1+\lambda\frac{a_1^M+(QQ_M-1)\,\sigma_1^M+QQ_I\cdot\sigma_1^I}{2}\right)$$

$$+\left(a_0^M+a_1^M\right)\cdot E\left[p_2|P_1,a_0^M\right]$$

$$= -a_1^M\cdot\left(P_1+\frac{a_1^M+(QQ_M-1)}{2}\lambda\right)$$

$$+\left(1+a_1^M\right)\cdot\left\{\frac{1}{2}\left[P_1+\left(QQ_M-1+a_1^M\right)\lambda\right]-\frac{1}{2}\left(\widehat{v}-c\right)\right\}$$

$$= \left\{\frac{1}{2}\left[P_1+\left(QQ_M-1+a_1^M\right)\lambda\right]-\frac{\widehat{v}-c}{2}\right\}$$

$$-a_1^M\cdot\left(\frac{P_1}{2}+\frac{a_1^M+(QQ_M-1)}{2}\lambda+\frac{\widehat{v}-c}{2}\right)$$

Therefore, we conclude that

$$\sigma_1^M\left(P_1,a_0^M\right)=-1\in\operatorname*{arg\,max}_{a_1^m\in\{-1,0\}}\left\{E_1\left[\pi^M|P_1,a_0^M\right]\right\}$$

Case 3.2.3: $P_1\in]\frac{\lambda}{2},P^*]$:

In this case,

$$\Pr\left(\omega^+|P_1,a_0^M\right)$$

$$= \frac{f\left(P_1|\omega^+\right)\cdot\Pr\left(\omega^+\right)}{f\left(P_1|\omega^+\right)\cdot\Pr\left(\omega^+\right)+f\left(P_1|\omega^-\right)\cdot\Pr\left(\omega^-\right)}$$

$$= \frac{1}{2}$$

Since $-1\le a_0^I+a_1^I\le 1$ and $a_0^M=1$,it follows that $a_1^I\in\{-1,0\}$.

Using (A.2), we conclude that $p_2=\widehat{v}-c$ in state ω^+ and $p_2=-\left(\widehat{v}-c\right)$ in state ω^-,so that $E\left[p_2|P_1,a_0^M\right]=0$ and

$$E_1\left[\pi^M|P_1,a_0^M\right]$$
$$= -a_1^M\cdot\left(P_1+\lambda\frac{a_1^M+(QQ_M-1)\,\sigma_1^M+QQ_I\cdot\sigma_1^I}{2}\right)$$
$$+\left(a_0^M+a_1^M\right)\cdot E\left[p_2|P_1,a_0^M\right]$$
$$= -a_1^M\cdot\left(P_1+\frac{a_1^M-q^+\,(QQ_M-1)}{2}\lambda\right)$$

$$a_1^M = -1\Rightarrow$$
$$E_1\left[\pi^M|P_1,a_0^M\right] = P_1-\frac{\lambda}{2}-\frac{q^+\,(QQ_M-1)}{2}\lambda=0$$

$$a_1^M=0\Rightarrow E_1\left[\pi^M|P_1,a_0^M\right]=0,$$

where $q^+=\frac{2P_1-\lambda}{\lambda(QQ_M-1)}$.

Therefore, we conclude that

$$\sigma_1^M\left(P_1,a_0^M\right)=0\in\underset{a_1^m\in\{-1,0\}}{\arg\max}\left\{E_1\left[\pi^M|P_1,a_0^M\right]\right\}$$

Case 3.2.4: $P_1\in[-\frac{\lambda}{2},\frac{\lambda}{2}]:$

In this case,

$$\Pr\left(\omega^+|P_1,a_0^M\right)$$
$$= \frac{f\left(P_1|\omega^+\right)\cdot\Pr\left(\omega^+\right)}{f\left(P_1|\omega^+\right)\cdot\Pr\left(\omega^+\right)+f\left(P_1|\omega^-\right)\cdot\Pr\left(\omega^-\right)}$$
$$= \frac{1}{2}$$

Since $-1 \leq a_0^I + a_1^I \leq 1$ and $a_0^M = 1$, it follows that $a_1^I \in \{-1, 0\}$.

Using (A.2), we conclude that $p_2 = \widehat{v} - c$ in state ω^+ and $p_2 = -(\widehat{v} - c)$ in state ω^-, so that $E\left[p_2 | P_1, a_0^M\right] = 0$ and

$$
\begin{aligned}
& E_1\left[\pi^M | P_1, a_0^M\right] \\
= \ & -a_1^M \cdot \left(P_1 + \lambda \frac{a_1^M + (QQ_M - 1)\sigma_1^M + QQ_I \cdot \sigma_1^I}{2}\right) \\
& + \left(a_0^M + a_1^M\right) \cdot E\left[p_2 | P_1, a_0^M\right] \\
= \ & -a_1^M \cdot \left(P_1 + \frac{a_1^M}{2}\lambda\right)
\end{aligned}
$$

$$
a_1^M = -1 \Rightarrow E_1\left[\pi^M | P_1, a_0^M\right] = P_1 - \frac{\lambda}{2} < 0
$$

$$
a_1^M = 0 \Rightarrow E_1\left[\pi^M | P_1, a_0^M\right] = 0
$$

Therefore, we conclude that

$$
\sigma_1^M\left(P_1, a_0^M\right) = 0 \in \arg\max_{a_1^m \in \{-1, 0\}} \left\{E_1\left[\pi^M | P_1, a_0^M\right]\right\}.
$$

Case 3.2.5: $P_1 \in [\lambda - P^*, -\frac{\lambda}{2}[$:

In this case,

$$
\begin{aligned}
& \Pr\left(\omega^+ | P_1, a_0^M\right) \\
= \ & \frac{f\left(P_1 | \omega^+\right) \cdot \Pr\left(\omega^+\right)}{f\left(P_1 | \omega^+\right) \cdot \Pr\left(\omega^+\right) + f\left(P_1 | \omega^-\right) \cdot \Pr\left(\omega^-\right)} \\
= \ & \frac{1}{2}
\end{aligned}
$$

Since $-1 \leq a_0^I + a_1^I \leq 1$ and $a_0^M = 1$, it follows that $a_1^I \in \{-1, 0\}$.

Using (A.2), we conclude that $p_2 = \hat{v} - c$ in state ω^+ and $p_2 = -(\hat{v} - c)$ in state ω^-, so that $E\left[p_2 | P_1, a_0^M\right] = 0$ and

$$
\begin{aligned}
&E_1\left[\pi^M | P_1, a_0^M\right] \\
&= -a_1^M \cdot \left(P_1 + \lambda \frac{a_1^M + (QQ_M - 1)\,\sigma_1^M + QQ_I \cdot \sigma_1^I}{2}\right) \\
&\quad + \left(a_0^M + a_1^M\right) \cdot E\left[p_2 | P_1, a_0^M\right] \\
&= -a_1^M \cdot \left(P_1 + \frac{a_1^M + q^-\,(QQ_M - 1)}{2}\lambda\right)
\end{aligned}
$$

$$
\begin{aligned}
a_1^M &= -1 \Rightarrow \\
E_1\left[\pi^M | P_1, a_0^M\right] &= P_1 - \frac{\lambda}{2} + \frac{q^-\,(QQ_M - 1)}{2}\lambda \\
&= -\lambda \\
&< 0
\end{aligned}
$$

$$
a_1^M = 0 \Rightarrow E_1\left[\pi^M | P_1, a_0^M\right] = 0
$$

Therefore, we conclude that

$$
\sigma_1^M\left(P_1, a_0^M\right) = 0 \in \arg\max_{a_1^m \in \{-1,0\}} \left\{E_1\left[\pi^M | P_1, a_0^M\right]\right\}
$$

Case 3.2.6: $P_1 \in [-P^*, \lambda - P^*[$:

In this case, $\Pr\left(\omega^- | P_1, a_0^M\right) = 1$.

Since $-1 \leq a_0^I + a_1^I \leq 1$ and $a_0^M = 1$, it follows that $a_1^I \in \{-1, 0\}$.

Using (A.2), we conclude that $p_2 = -(\hat{v} - c)$.

$$E_1\left[\pi^M|P_1, a_0^M\right]$$

$$= -a_1^M \cdot \left(P_1 + \frac{\lambda a_1^M + \lambda\left(QQ_M - 1\right)\sigma_1^M + \lambda QQ_I \cdot \sigma_1^I}{2}\right)$$

$$+ \left(a_0^M + a_1^M\right) \cdot E\left[p_2|P_1, a_0^M\right]$$

$$= -a_1^M \cdot \left(P_1 + \frac{a_1^M + q^-\left(QQ_M - 1\right) \cdot 1 + QQ_I \cdot 0}{2}\lambda\right)$$

$$+ \left(1 + a_1^M\right) \cdot \left[-\left(\widehat{v} - c\right)\right]$$

$$= -\left(\widehat{v} - c\right) -$$

$$a_1^M \cdot \left[\left(\widehat{v} - c\right) + \left(P_1 + \frac{a_1^M + q^-\left(QQ_M - 1\right)}{2}\lambda\right)\right]$$

$$a_1^M = -1 \Rightarrow$$

$$E_1\left[\pi^M|P_1, a_0^M\right] = P_1 - \frac{\lambda}{2} + \frac{q^-\left(QQ_M - 1\right)}{2}\lambda$$

$$= -\lambda$$

$$a_1^M = 0 \Rightarrow E_1\left[\pi^M|P_1, a_0^M\right] = -\left(\widehat{v} - c\right)$$

Since $\widehat{v} - c > \lambda$ by the assumptions (A.1) and (A.2),, we conclude that

$$\sigma_1^M\left(P_1, a_0^M\right) = -1 \in \underset{a_1^m \in \{-1, 0\}}{\arg\max}\left\{E_1\left[\pi^M|P_1, a_0^M\right]\right\}$$

Case 3.2.7: $P_1 \in [\lambda - \overline{P}, -P^*[$:

In this case, $\Pr\left(\omega^-|P_1, a_0^M\right) = 1$.

Since $-1 \le a_0^I + a_1^I \le 1$ and $a_0^M = 1$, it follows that $a_1^I \in \{-1, 0\}$.

Using (A.2), we conclude that $-\left(\widehat{v} + c\right) < P_2 < -\left(\widehat{v} - c\right)$, so that

$$E_1\left[\pi^M|P_1,a_0^M\right]$$

$$= -a_1^M\cdot\left(P_1+\lambda\frac{a_1^M+(QQ_M-1)\,\sigma_1^M+QQ_I\cdot\sigma_1^I}{2}\right)$$

$$+\left(a_0^M+a_1^M\right)\cdot E\left[p_2|P_1,a_0^M\right]$$

$$= -a_1^M\cdot\left(P_1+\frac{a_1^M-(QQ_M-1)}{2}\lambda\right)$$

$$+\left(1+a_1^M\right)\cdot\left(P_1+a_1^M\lambda-(QQ_M-1)\,\lambda\right)$$

$$a_1^M=-1\Rightarrow E_1\left[\pi^M|P_1,a_0^M\right]=P_1-\lambda\frac{QQ_M}{2}$$

$$a_1^M \;=\; 0\Rightarrow$$

$$E_1\left[\pi^M|P_1,a_0^M\right] \;=\; P_1-\lambda\,(QQ_M-1)$$

$$<\; {}^1P_1-\lambda\frac{QQ_M}{2}$$

where $<^1$ follows from (A.1).

Therefore, we conclude that

$$\sigma_1^M\left(P_1,a_0^M\right)=-1\in\underset{a_1^m\in\{-1,0\}}{\arg\max}\left\{E_1\left[\pi^M|P_1,a_0^M\right]\right\}.$$

Case 3.3: Suppose that $a_0^M=-1$. Since $K^+(-1)=[-\lambda-P^*,-\lambda+\overline{P}]$ and $K^-(-1)=[-\lambda-\overline{P},-\lambda+P^*]$, we consider realizations of P_1 lying in the intervals $[-\lambda-\overline{P},-\lambda-P^*[,[-\lambda-P^*,-P^*[,[-P^*,-\frac{\lambda}{2}[,[-\frac{\lambda}{2},\frac{\lambda}{2}],]\frac{\lambda}{2},P^*-\lambda],[P^*-\lambda,P^*]$ and $]P^*,\overline{P}-\lambda]$.

This proof is exactly the symmetric case to **Case 3.2**.

Part 4: For a fixed M trader who chooses a_0^M and given that all I players and all other M players use the equilibrium strategies defined above, it follows that

$$P_1 = \lambda \left[QQ_I \sigma_0^I(\omega) + a_0^M \right] + \varepsilon_1.$$

As in Part 3, conditional on $\omega = \omega^+$, the random variable P_1 is uniformly distributed on the interval

$$K^+(a_0^I) = [\lambda a_0^M - P^*, \lambda a_0^M + \overline{P}]$$

and conditional on $\omega = \omega^-$, the rv P_1 is uniformly distributed on the interval,

$$K^-(a_0^I) = [\lambda a_0^M - \overline{P}, \lambda a_0^M + P^*].$$

We claim that σ_0^M is a best response for type I traders in period 1, i.e., we claim that

$$\sigma_0^M = 0 \in \arg\max_{a_0^M \in \Lambda_1} \left\{ E_0 \left[\pi^M \right] \right\},$$

where

$$E_0 \left[\pi^M \right] = E \begin{bmatrix} -a_0^M \cdot \left(\frac{\lambda a_0^M + \lambda(QQ_M - 1)\sigma_0^M + \lambda QQ_I \cdot \sigma_0^I}{2} \right) \\ -\sigma_1^M \cdot \left(P_1 + \frac{\lambda QQ_M \cdot \sigma_1^M + \lambda QQ_I \cdot \sigma_1^I}{2} \right) \\ + \left(a_0^M + \sigma_1^M \right) \cdot p_2 \end{bmatrix}.$$

If $a_0^M = 0$, then

$$
\begin{aligned}
& E_0\left[\pi^M\right] \\
= & \; E\left[\begin{array}{c} 0 \cdot \left(\frac{\lambda QQ_I \cdot \sigma_0^I}{2}\right) - \sigma_1^M \cdot \left(P_1 + \frac{\lambda QQ_M \cdot \sigma_1^M}{2}\right) \\ + \left(0 + \sigma_1^M\right) \cdot p_2 \end{array}\right] \\
= & \; \Pr\left(\omega^+\right)\left\{\begin{array}{c} \int_{P^*}^{\overline{P}} \left(\frac{\lambda QQ_M}{2}\right) \frac{dx}{2\lambda QQ_N} + \int_{-\frac{\lambda}{2}}^{\frac{\lambda}{2}} 0 \frac{dx}{2\lambda QQ_N} \\ + \int_{\frac{\lambda}{2}}^{P^*} q^+ \cdot \left[x + \frac{-1 - q^+ \cdot (QQ_M - 1)}{2}\lambda - \left(\widehat{v} - c\right)\right] \frac{dx}{2\lambda QQ_N} \\ + \int_{-P^*}^{-\frac{\lambda}{2}} q^- \cdot \left[-x - \frac{1 + q^- \cdot (QQ_M - 1)}{2}\lambda + \left(\widehat{v} - c\right)\right] \frac{dx}{2\lambda QQ_N} \end{array}\right\} \\
& + \Pr\left(\omega^-\right)\left\{\begin{array}{c} \int_{-\overline{P}}^{-P^*} \left(\frac{\lambda QQ_M}{2}\right) \frac{dx}{2\lambda QQ_N} + \int_{-\frac{\lambda}{2}}^{\frac{\lambda}{2}} 0 \frac{dx}{2\lambda QQ_N} \\ + \int_{-P^*}^{-\frac{\lambda}{2}} q^- \cdot \left[-x - \frac{1 + q^- \cdot (QQ_M - 1)}{2}\lambda - \left(\widehat{v} - c\right)\right] \frac{dx}{2\lambda QQ_N} \\ + \int_{\frac{\lambda}{2}}^{P^*} q^+ \cdot \left[x + \frac{-1 - q^+ \cdot (QQ_M - 1)}{2}\lambda + \left(\widehat{v} - c\right)\right] \frac{dx}{2\lambda QQ_N} \end{array}\right\} \\
= & \; \frac{\lambda QQ_I \cdot QQ_M}{2QQ_N} \\
\triangleq & \; B^0
\end{aligned}
$$

If $a_0^M = 1$, then

$$E_0\left[\pi^M\right]$$

$$= E\left[\begin{array}{c} -1\cdot\left(\frac{\lambda+\lambda QQ_I\cdot\sigma_0^I}{2}\right) \\ -\sigma_1^M\cdot\left(P_1+\frac{\lambda QQ_M\cdot\sigma_1^M}{2}\right)+\left(1+\sigma_1^M\right)\cdot p_2 \end{array}\right]$$

$$= \Pr\left(\omega^+\right)\left\{\begin{array}{c} \int_{P^*+\lambda}^{\overline{P}+\lambda}\left[-\frac{\lambda(QQ_I+1)}{2}+x+\lambda\left(QQ_M-1\right)\right]\frac{dx}{2\lambda QQ_N} \\ +\int_{P^*}^{P^*+\lambda}\left[-\frac{\lambda(QQ_I+1)}{2}+x+\frac{\lambda(QQ_M-2)}{2}\right]\frac{dx}{2\lambda QQ_N} \\ \int_{\lambda-P^*}^{P^*}\left[\left(\widehat{v}-c\right)-\frac{\lambda(QQ_I+1)}{2}\right]\frac{dx}{2\lambda QQ_N} \end{array}\right\}$$

$$+\Pr\left(\omega^-\right)\left\{\begin{array}{c} \int_{P^*}^{P^*+\lambda}\left[-\frac{\lambda(-QQ_I+1)}{2}+x+\frac{\lambda(QQ_M-2)}{2}\right]\frac{dx}{2\lambda QQ_N} \\ +\int_{-P^*}^{P^*}\left[-\frac{\lambda(-QQ_I+1)}{2}-\left(\widehat{v}-c\right)\right]\frac{dx}{2\lambda QQ_N} \\ +\int_{-\overline{P}+\lambda}^{-P^*}\left[-\frac{\lambda(-QQ_I+1)}{2}+x-\frac{\lambda QQ_M}{2}\right]\frac{dx}{2\lambda QQ_N} \end{array}\right\}$$

$$= \frac{1}{4QQ_N}\left\{\begin{array}{c} -\left(\widehat{v}-c\right)+ \\ \lambda\left[QQ_I\cdot QQ_M-QQ_I+QQ_N+\frac{3}{2}QQ_M-\frac{3}{2}\right] \end{array}\right\}$$

$$\triangleq B^1$$

If $a_0^M=-1$, then

$$E_0\left[\pi^M\right]$$

$$= E\left[\begin{array}{c} 1\cdot\left(\frac{-\lambda+\lambda QQ_I\cdot\sigma_0^I}{2}\right) \\ -\sigma_1^M\cdot\left(P_1+\frac{\lambda QQ_M\cdot\sigma_1^M}{2}\right)+\left(-1+\sigma_1^M\right)\cdot p_2 \end{array}\right]$$

$$= \frac{1}{4QQ_N}\left\{\begin{array}{c} -\left(\widehat{v}-c\right)+ \\ \lambda\left[QQ_I\cdot QQ_M-QQ_I+QQ_N+\frac{3}{2}QQ_M-\frac{3}{2}\right] \end{array}\right\}$$

$$= B^1$$

Next, note that

$$B^0 - B^1$$

$$= \frac{1}{4QQ_N} \left\{ (\widehat{v} - c) + \lambda \begin{bmatrix} QQ_M \cdot QQ_I - QQ_N \\ +QQ_I - \frac{3}{2}QQ_M + \frac{3}{2} \end{bmatrix} \right\}$$

$$= \frac{1}{4QQ_N} \left\{ \begin{array}{c} \left[(\widehat{v} - c) - \lambda \left(QQ_N - QQ_I - \frac{3}{2} \right) \right] \\ +\lambda QQ_M \left[QQ_I - \frac{3}{2} \right] \end{array} \right\}$$

$$\geq^1 \frac{1}{4QQ_N} \left\{ (\widehat{v} - c) - \lambda \left(QQ_N - QQ_I - \frac{3}{2} \right) \right\}$$

$$\geq^2 0$$

where $\geq^1$ follows from (A.1) and $\geq^2$ follows from (A.2).

Therefore, we have shown that

$$\sigma_0^M = 0 \in \underset{a_0^M \in \Lambda_1}{\arg\max} \left\{ E_0 \left[\pi^M \right] \right\}. \blacksquare$$

2.8 Proof B. Proof of Proposition 2

We will present the proof in three parts.

To begin, recall $\overline{P} = \lambda \left(QQ_I + QQ_N \right)$ and $P^* = \lambda \left(QQ_N - QQ_I \right)$.

Consider an equilibrium where, on the off-equilibrium path, S_M update their beliefs from the price path and trade as if without communication when there is no post from S_I or S_I's posts conflict, i.e. $b_1^I = mix\{l, n, s\}$.

Part 1: For a fixed I player, the best trading strategy is the same as without communication case. And the proof is also similar.

As to the posting strategy, we claim that $\gamma_0^I(\omega)$ is a best response for type I traders in period 1, i.e., we claim that

$$\gamma_0^I \left(\omega, a_0^I \right) \in \arg\max_{b_0^I \in \Lambda_2} \left\{ E_0 \left[\pi^I \middle| \omega, a_0^I \right] \right\},$$

where

$$E_0 \left[\pi^I \middle| \omega, a_0^I \right]$$
$$= E \left[\begin{array}{c} -\sigma_1^I \cdot \left(P_1 + \frac{\lambda QQ_I \cdot \sigma_1^I + \lambda QQ_M \cdot \sigma_1^M}{2} \right) \\ + \left(a_0^I + \sigma_1^I \right) \cdot p_2 \end{array} \middle| \omega, a_0^I \right]$$

Case 1.1: In state ω^+,

Case 1.1.1 $a_0^I = 1 \Rightarrow \sigma_1^I = 0$

$$b_0^I = l \Rightarrow$$

$$
\begin{aligned}
E_0 \left[\pi^I \middle| \omega^+, a_0^I \right] \\
= E \left[0 \cdot \left(P_1 + \frac{\lambda QQ_M \cdot \sigma_1^M}{2} \right) + (1 + 0) \cdot p_2 \middle| \omega, a_0^I \right] \\
= E \left[p_2 \middle| \omega, a_0^I \right] = \lambda \left(QQ_I + QQ_M \right)
\end{aligned}
$$

$$b_0^I = s \Rightarrow$$

$$E_0 \left[\pi^I \middle| \omega^+, a_0^I \right] = \left(1 - \frac{QQ_I}{QQ_N} \right) (\widehat{v} - c) + \frac{QQ_I}{QQ_N} \lambda \left(QQ_I + QQ_M \right)$$

$$b_0^I = n \Rightarrow$$

$$E_0 \left[\pi^I \middle| \omega^+, a_0^I \right] = \left(1 - \frac{QQ_I}{QQ_N} \right) (\widehat{v} - c) + \frac{QQ_I}{QQ_N} \lambda \left(QQ_I + QQ_M \right)$$

Since $\widehat{v} - c < \lambda \left(QQ_I + QQ_M \right)$ by the assumption (A.2), we conclude that

$$\gamma_0^I \left(\omega, a_0^I \right) = l \in \underset{b_0^I \in \Lambda_2}{\arg \max} \left\{ E_0 \left[\pi^I \middle| \omega^+, a_0^I \right] \right\}.$$

Case 1.1.2 $a_0^I = -1 \Rightarrow \sigma_1^I = 1$

$$b_0^I = l \Rightarrow$$

$$E_0\left[\pi^I \,\middle|\, \omega^+, a_0^I\right]$$

$$= E\left[-1 \cdot \left(P_1 + \frac{\lambda\left(QQ_M + 1\right)}{2}\right) + (-1 + 1) \cdot p_2 \,\middle|\, \omega, a_0^I\right]$$

$$= -\lambda\left(QQ_I + \frac{QQ_M - 3}{2}\right)$$

$$\triangleq C^l$$

$$b_0^I = s \Rightarrow$$

$$E_0\left[\pi^I \,\middle|\, \omega^+, a_0^I\right]$$

$$= E\left[-1 \cdot \left(P_1 + \frac{\lambda + \lambda QQ_M \cdot \sigma_1^M}{2}\right) + (-1 + 1) \cdot p_2 \,\middle|\, \omega, a_0^I\right]$$

$$= -\left\{ \begin{array}{c} \int_{P^*}^{\overline{P}-2\lambda} \left(x + \frac{\lambda(QQ_M+1)}{2}\right) \frac{dx}{2\lambda QQ_N} \\[2mm] + \int_{\frac{\lambda}{2}}^{P^*} \left(x + \frac{\lambda - q^+ \cdot \lambda QQ_M}{2}\right) \frac{dx}{2\lambda QQ_N} \\[2mm] + \int_{-\frac{\lambda}{2}}^{\frac{\lambda}{2}} \left(x + \frac{\lambda}{2}\right) \frac{dx}{2\lambda QQ_N} \\[2mm] + \int_{-P^*}^{-\frac{\lambda}{2}} \left(x + \frac{\lambda + q^- \cdot \lambda QQ_M}{2}\right) \frac{dx}{2\lambda QQ_N} \\[2mm] + \int_{-P^*-2\lambda}^{-P^*} \left(x + \frac{\lambda - \lambda QQ_M}{2}\right) \frac{dx}{2\lambda QQ_N} \end{array} \right\}$$

$$= -\lambda\left\{ QQ_I - \frac{1}{2} + \frac{QQ_M\left(QQ_I - 2\right)}{2QQ_N} \right\}$$

$$\triangleq C^s$$

$$b_0^I = n \Rightarrow E_0\left[\pi^I \,\middle|\, \omega^+, a_0^I\right] = C^s$$

Since

$$C^s - C^l$$
$$= \lambda \left\{ \left(QQ_I + \frac{QQ_M - 3}{2} \right) - \left[QQ_I - \frac{1}{2} + \frac{QQ_M (QQ_I - 2)}{2QQ_N} \right] \right\}$$
$$= \lambda \left\{ \frac{QQ_M}{2} \left(1 - \frac{QQ_I - 2}{2QQ_N} \right) - 1 \right\}$$
$$>\ ^1\lambda \left\{ \frac{QQ_M}{4} - 1 \right\}$$
$$>\ ^2 0$$

where $>^1$ follows from (A.1) and $>^2$ follows from (A.1).

Therefore, we conclude that

$$\gamma_0^I \left(\omega, a_0^I \right) = s \text{ or } n \in \arg\max_{b_0^I \in \Lambda_2} \left\{ E_0 \left[\pi^I \middle| \omega^+, a_0^I \right] \right\}.$$

Case 1.1.3 $a_0^I = 0 \Rightarrow \sigma_1^I = 1$

$$b_0^I = l \Rightarrow$$

$$E_0 \left[\pi^I \middle| \omega^+, a_0^I \right]$$
$$= E \left[\begin{array}{c} -1 \cdot \left(P_1 + \frac{\lambda(QQ_M+1)}{2} \right) \\ + (1+0) \cdot p_2 \end{array} \middle| \omega, a_0^I \right]$$
$$= E \left[p_2 - P_1 - \frac{\lambda (QQ_M + 1)}{2} \middle| \omega, a_0^I \right]$$
$$= \frac{\lambda (QQ_M + 1)}{2}$$
$$\triangleq D^l$$

$$b_0^I \quad = \quad s \Rightarrow$$

$$E_0\left[\pi^I\middle|\omega^+, a_0^I\right] \quad = \quad E\left[p_2 - P_1 - \frac{\lambda\left(QQ_M \cdot \sigma_1^M + 1\right)}{2}\middle|\omega, a_0^I\right]$$

$$= \quad \left\{ \begin{array}{c} \displaystyle\int_{P^*}^{\overline{P}-\lambda} \left[\frac{\lambda(QQ_M+1)}{2}\right]\frac{dx}{2\lambda QQ_N} \\[2ex] \displaystyle + \int_{\frac{\lambda}{2}}^{P^*} \left[(\widehat{v}-c) - \left(x + \frac{\lambda - q^+ \cdot \lambda QQ_M}{2}\right)\right]\frac{dx}{2\lambda QQ_N} \\[2ex] \displaystyle + \int_{-\frac{\lambda}{2}}^{\frac{\lambda}{2}} \left[(\widehat{v}-c) - \left(x + \frac{\lambda}{2}\right)\right]\frac{dx}{2\lambda QQ_N} \\[2ex] \displaystyle + \int_{-P^*}^{-\frac{\lambda}{2}} \left[(\widehat{v}-c) - \left(x + \frac{\lambda + q^- \cdot \lambda QQ_M}{2}\right)\right]\frac{dx}{2\lambda QQ_N} \\[2ex] \displaystyle + \int_{-P^*-\lambda}^{-P^*} \left[(\widehat{v}-c) - \left(x + \frac{\lambda - \lambda QQ_M}{2}\right)\right]\frac{dx}{2\lambda QQ_N} \end{array} \right\}$$

$$= \quad \frac{1}{2QQ_N}\left\{ \begin{array}{c} \lambda QQ_M \cdot QQ_I + \lambda\left(QQ_I - \frac{1}{2}\right) \\[1ex] + (2QQ_N - 2QQ_I + 1)\left(\widehat{v}-c\right) \end{array} \right\}$$

$$\triangleq \quad D^s$$

$$b_0^I = n \Rightarrow E_0\left[\pi^I\middle|\omega^+, a_0^I\right] = \frac{\lambda\left(QQ_M + 1\right)}{2} = D^s$$

Since

$$D^s - D^l$$

$$= \quad \frac{1}{2QQ_N}\left\{ \begin{array}{c} \lambda QQ_M \cdot (QQ_I - QQ_N) \\[1ex] + (2QQ_N - 2QQ_I + 1)\left(\widehat{v} - c - \frac{\lambda}{2}\right) \end{array} \right\},$$

$$> \quad {}^1\frac{1}{2QQ_N}\left\{2\left(QQ_N - QQ_I\right)\left(\widehat{v} - c - \frac{\lambda}{2} - \frac{\lambda QQ_M}{2}\right)\right\}$$

$$> \quad {}^2 0$$

where $>^1$ follows from (A.2) and $>^2$ follows from (A.1).

We conclude that

$$\gamma_0^I\left(\omega, a_0^I\right) = s \text{ or } n \in \arg\max_{b_0^I \in \Lambda_2}\left\{E_0\left[\pi^I\middle|\omega^+, a_0^I\right]\right\}.$$

Case 1.2: In state ω^-,

Case 1.2.1 $a_0^I = -1 \Rightarrow \sigma_1^I = 0$

This proof is exactly the symmetric case to **Case 1.1.1.**

$$\gamma_0^I\left(\omega, a_0^I\right) = s \in \arg\max_{b_0^I \in \Lambda_2}\left\{E_0\left[\pi^I\middle|\omega, a_0^I\right]\right\}.$$

Case 1.2.2 $a_0^I = 1 \Rightarrow \sigma_1^I = -1$

This proof is exactly the symmetric case to **Case 1.1.2.**

$$\gamma_0^I\left(\omega, a_0^I\right) = l \text{ or } n \in \arg\max_{b_0^I \in \Lambda_2}\left\{E_0\left[\pi^I\middle|\omega, a_0^I\right]\right\}.$$

Case 1.2.3 $a_0^I = 0 \Rightarrow \sigma_1^I = -1$

This proof is exactly the symmetric case to **Case 1.1.3.**

$$\gamma_0^I\left(\omega, a_0^I\right) = l \text{ or } n \in \arg\max_{b_0^I \in \Lambda_2}\left\{E_0\left[\pi^I\middle|\omega, a_0^I\right]\right\}.$$

Part 2: For a fixed M trader who chooses a_0^M and given that all I players and all other M players use the equilibrium strategies defined above, it follows that

$$P_1 = \lambda\left[QQ_I\sigma_0^I(\omega) + a_0^M\right] + \varepsilon_1.$$

Conditional on $\omega = \omega^+$, the random variable P_1 is uniformly distributed on the interval

$$K^+(a_0^M) = [\lambda a_0^M - P^*, \lambda a_0^M + \overline{P}]$$

and conditional on $\omega = \omega^-$, the random variable P_1 is uniformly distributed on the interval

$$K^-(a_0^M) = [\lambda a_0^M - \overline{P}, \lambda a_0^M + P^*].$$

We claim that σ_1^M is a best response for type I traders in period 1, i.e., we claim that, for each a_0^M and $P_1 \in K^+(a_0^M) \cup K^-(a_0^M)$,

$$\sigma_1^M\left(P_1, a_0^M, b_0^I, b_0^M\right) \in \underset{a_1^M \in \Lambda_1, \ -1 \le a_0^M + a_1^M \le 1}{\arg\max} \{E_1\left[\pi^M | P_1, a_0^M, b_0^I, b_0^M\right]\},$$

where

$$
\begin{aligned}
&E_1\left[\pi^M | P_1, a_0^M, b_0^I, b_0^M\right] \\
&= -a_1^M \cdot \left(P_1 + \frac{\lambda a_1^M + \lambda\left(QQ_M - 1\right) \cdot \sigma_1^M + \lambda QQ_I \cdot \sigma_1^I}{2}\right) \\
&\quad + \left(a_0^M + a_1^M\right) \cdot E\left[p_2 | P_1, a_0^M, b_0^I, b_0^M\right].
\end{aligned}
$$

Case 2.1: Suppose that $a_0^M = 0$.

Case 2.1.1: $b_0^I = l$:

In this case, $\Pr\left(\omega^+ | P_1, a_0^M\right) = 1$ so that

$$
\begin{aligned}
&E_1\left[\pi^M | P_1, a_0^M, b_0^I, b_0^M\right] \\
&= -a_1^M \cdot \left(P_1 + \frac{\lambda a_1^M + \lambda\left(QQ_M - 1\right)}{2}\right) \\
&\quad + \left(0 + a_1^M\right) \cdot E\left[p_2 | P_1, a_0^M, b_0^I, b_0^M\right], \\
&= a_1^M \frac{\lambda a_1^M + \lambda\left(QQ_M - 1\right)}{2}.
\end{aligned}
$$

We conclude that

$$\sigma_1^M \left(P_1, a_0^M, b_0^I, b_0^M \right) = 1 \in \underset{a_1^I \in \Lambda_1, \ -1 \le a_0^I + a_1^I \le 1}{\arg\max} \left\{ E_1 \left[\pi^M | P_1, a_0^M, b_0^I, b_0^M \right] \right\}$$

Case 2.1.2: $b_0^I = s$:

In this case, $\Pr \left(\omega^- | P_1, a_0^M \right) = 1$.

This proof is exactly the symmetric case to **Case 2.1.1.**

$$\sigma_1^M \left(P_1, a_0^M, b_0^I, b_0^M \right) = -1 \in \underset{a_1^I \in \Lambda_1, \ -1 \le a_0^I + a_1^I \le 1}{\arg\max} \left\{ E_1 \left[\pi^M | P_1, a_0^M, b_0^I, b_0^M \right] \right\}$$

Case 2.2: Suppose that $a_0^M = 1$.

Case 2.2.1: $b_0^I = l$:

In this case, $\Pr \left(\omega^+ | P_1, a_0^M \right) = 1$ so that

$$
\begin{aligned}
& E_1 \left[\pi^M | P_1, a_0^M, b_0^I, b_0^M \right] \\
= {} & -a_1^M \cdot \left(P_1 + \frac{\lambda a_1^M + \lambda \left(QQ_M - 1 \right)}{2} \right) \\
& + \left(1 + a_1^M \right) \cdot E \left[p_2 | P_1, a_0^M, b_0^I, b_0^M \right] \\
= {} & P_1 + \lambda \left(QQ_M - 1 \right) \\
& + a_1^M \cdot \frac{\lambda a_1^M + \lambda \left(QQ_M + 1 \right)}{2}
\end{aligned}
$$

Since $-1 \le a_0^M + a_1^M \le 1$ and $a_0^M = 1$, it follows that $a_1^M \in \{-1, 0\}$. We conclude that

$$\sigma_1^M \left(P_1, a_0^M, b_0^I, b_0^M \right) = 0 \in \underset{a_1^I \in \Lambda_1, \ -1 \le a_0^I + a_1^I \le 1}{\arg\max} \left\{ E_1 \left[\pi^M | P_1, a_0^M, b_0^I, b_0^M \right] \right\}$$

Case 2.2.2: $b_0^I = s$:

In this case, $\Pr\left(\omega^-|P_1, a_0^M\right) = 1$ so that

$$
\begin{aligned}
& E_1\left[\pi^M|P_1, a_0^M, b_0^I, b_0^M\right] \\
= {} & -a_1^M \cdot \left(P_1 + \frac{\lambda a_1^M - \lambda\left(QQ_M - 1\right)}{2}\right) \\
& + \left(1 + a_1^M\right) \cdot E\left[p_2|P_1, a_0^M, b_0^I, b_0^M\right] \\
= {} & P_1 - \lambda\left(QQ_M - 1\right) + a_1^M \cdot \frac{\lambda a_1^M - \lambda\left(QQ_M - 3\right)}{2}
\end{aligned}
$$

Since $-1 \le a_0^M + a_1^M \le 1$ and $a_0^M = 1$, it follows that $a_1^M \in \{-1, 0\}$. Therefore, we conclude that

$$
\sigma_1^M\left(P_1, a_0^M, b_0^I, b_0^M\right) = -1 \in \underset{a_1^I \in \Lambda_1,\ -1 \le a_0^I + a_1^I \le 1}{\arg\max} \left\{E_1\left[\pi^M|P_1, a_0^M, b_0^I, b_0^M\right]\right\}
$$

Case 2.3: Suppose that $a_0^M = -1$.

Case 2.3.1: $b_0^I = l$:

In this case, $\Pr\left(\omega^+|P_1, a_0^M\right) = 1$.

This proof is exactly the symmetric case to **Case 2.2.2.**

$$
\sigma_1^M\left(P_1, a_0^M, b_0^I, b_0^M\right) = -1 \in \underset{a_1^I \in \Lambda_1,\ -1 \le a_0^I + a_1^I \le 1}{\arg\max} \left\{E_1\left[\pi^M|P_1, a_0^M, b_0^I, b_0^M\right]\right\}
$$

Case 2.3.2: $b_0^I = s$: In this case, $\Pr\left(\omega^-|P_1, a_0^M\right) = 1$.

This proof is exactly the symmetric case to **Case 2.2.1.**

$$
\sigma_1^M\left(P_1, a_0^M, b_0^I, b_0^M\right) = 0 \in \underset{a_1^I \in \Lambda_1,\ -1 \le a_0^I + a_1^I \le 1}{\arg\max} \left\{E_1\left[\pi^M|P_1, a_0^M, b_0^I, b_0^M\right]\right\}
$$

Part 3: For a fixed M trader who chooses a_0^M and given that all I players and

all other M players use the equilibrium strategies defined above, it follows that

$$P_1 = \lambda \left[QQ_I \sigma_0^I(\omega) + a_0^M \right] + \varepsilon_1.$$

As in Part 2, conditional on $\omega = \omega^+$, the rv P_1 is uniformly distributed on the interval

$$K^+(a_0^I) = [\lambda a_0^M - P^*, \lambda a_0^M + \overline{P}]$$

and conditional on $\omega = \omega^-$, the rv P_1 is uniformly distributed on the interval

$$K^-(a_0^I) = [\lambda a_0^M - \overline{P}, \lambda a_0^M + P^*].$$

We claim that σ_0^M is a best response for type I traders in period 1, i.e., we claim that

$$\sigma_0^M = 0 \in \arg\max_{a_0^M \in \Lambda_1} \left\{ E_0 \left[\pi^M \right] \right\},$$

where

$$E_0 \left[\pi^M \right] = E \left[\begin{array}{c} -a_0^M \cdot \left(\frac{\lambda a_0^M + \lambda(QQ_M - 1)\sigma_0^M + \lambda QQ_I \cdot \sigma_0^I}{2} \right) \\ -\sigma_1^M \cdot \left(P_1 + \frac{\lambda QQ_M \cdot \sigma_1^M + \lambda QQ_I \cdot \sigma_1^I}{2} \right) \\ + \left(a_0^M + \sigma_1^M \right) \cdot p_2 \end{array} \right].$$

Since each trader has perfect information about posters' types, S_M are indifferent with lying or telling the truth. We suppose, under such a situation, traders choose to tell the truth, i.e. S_M post their action σ_0^M.

If $a_0^M = 0$, then

$$E_0\left[\pi^M\right]$$

$$= E\begin{bmatrix} 0 \cdot \left(\frac{\lambda QQ_I \cdot \sigma_0^I}{2}\right) \\ -\sigma_1^M \cdot \left(P_1 + \frac{\lambda QQ_M \cdot \sigma_1^M}{2}\right) \\ + \left(0 + \sigma_1^M\right) \cdot p_2 \end{bmatrix},$$

$$= \Pr\left(\omega^+\right)\left\{\frac{\lambda QQ_M}{2}\right\} + \Pr\left(\omega^-\right)\left\{\frac{\lambda QQ_M}{2}\right\},$$

$$= \frac{\lambda QQ_M}{2}.$$

If $a_0^M = 1$, then

$$E_0\left[\pi^M\right] = E\begin{bmatrix} -1 \cdot \left(\frac{\lambda a_0^M + \lambda QQ_I \cdot \sigma_0^I}{2}\right) \\ -\sigma_1^M \cdot \left(P_1 + \frac{\lambda \cdot \sigma_1^M + \lambda(QQ_M - 1) \cdot \sigma_1^M}{2}\right) \\ + \left(1 + \sigma_1^M\right) \cdot p_2 \end{bmatrix},$$

$$= \Pr\left(\omega^+\right)\left\{\begin{array}{c} -1 \cdot \frac{QQ_I + 1}{2}\lambda + \\ (0 + 1) \cdot \left[E\left[P_1 | \omega^+\right] + \lambda\left(QQ_M - 1\right)\right] \end{array}\right\}$$

$$+ \Pr\left(\omega^-\right)\left\{\begin{array}{c} -1 \cdot \frac{-QQ_I + 1}{2}\lambda - \\ (-1) \cdot \left[E\left[P_1 | \omega^-\right] - \frac{\lambda(QQ_M - 2)}{2}\right] \end{array}\right\}$$

$$= \frac{\lambda QQ_M}{4} - \frac{\lambda}{2}.$$

If $a_0^M = -1$, then

$$E_0\left[\pi^M\right] = E\begin{bmatrix} -\left(-1\right)\cdot\left(\frac{\lambda a_0^M+\lambda QQ_I\cdot\sigma_0^I}{2}\right) \\ -\sigma_1^M\cdot\left(P_1+\frac{\lambda\cdot\sigma_1^M+\lambda(QQ_M-1)\cdot\sigma_1^M}{2}\right) \\ +\left(-1+\sigma_1^M\right)\cdot p_2 \end{bmatrix},$$

$$= \frac{\lambda QQ_M}{4}-\frac{\lambda}{2}.$$

We have therefore shown that $\sigma_0^M = 0 \in \underset{a_0^M\in\Lambda_1}{\arg\max}\left\{E_0\left[\pi^M\right]\right\}.\blacksquare$

2.9 Tables of Chapter 2

Table 2.1

Signals and States

Trader i	state ω^+	state ω^-
Informed Traders S_I	ω^+	ω^-
Momentum Traders S_M	no signal	no signal

Table 2.2

the Price Path in state ω^+ for the leading example

ω^+	Price	S_M's strategy	S_I's strategy
Period 0	$P_0 = 10$	$\sigma_0^M = 0$	$\sigma_0^I = 1$
Period 1	$P_1 \in [8, 16]$	$P_1 \in \,]12, 16] \Rightarrow \sigma_1^M = 1$ $P_1 \in \,]10.005, 12] \Rightarrow \sigma_1^M = \begin{cases} 0 \text{ w. prob } 1 - q^+ \\ -1 \text{ w. prob } q^+ \end{cases}$ $P_1 \in [9.995, 10.005] \Rightarrow \sigma_1^M = 0$ $P_1 \in [8, 9.995[\Rightarrow \sigma_1^M = \begin{cases} 0 \text{ w. prob } 1 - q^- \\ 1 \text{ w. prob } q^- \end{cases}$	$\sigma_1^I = 0$

Table 2.3

the Price Path in state ω^- for the leading example

ω^-	Price	S_M's strategy	S_I's strategy
Period 0	$P_0 = 10$	$\sigma_0^M = 0$	$\sigma_0^I = -1$
Period 1	$P_1 \in [4, 12]$	$P_1 \in [4, 8[\Rightarrow \sigma_1^M = -1$ $P_1 \in [8, 9.995[\Rightarrow \sigma_1^M = \begin{cases} 0 \text{ with } prob\ 1 - q^- \\ 1 \text{ with } prob\ q^- \end{cases}$ $P_1 \in [9.995, 10.005] \Rightarrow \sigma_1^M = 0$ $P_1 \in]10.005, 12] \Rightarrow \sigma_1^M = \begin{cases} 0 \text{ w. } prob\ 1 - q^+ \\ -1 \text{ w. } prob\ q^+ \end{cases}$	$\sigma_1^I = 0$

Table 2.4

the Momentum Trader's Strategy at period 1 for the leading example

State	P_1	σ_1^M
$Pr\left(\omega^+\right) = 1$	$P_1 \in [4, 8[$	$\sigma_1^M = 1$
$Pr\left(\omega^+\right) = \frac{1}{2}$	$P_1 \in [8, 9.995[$	$\sigma_1^M = 0$ with $prob$ $1 - q^+$ and -1 with $prob$ q^+
$Pr\left(\omega^+\right) = \frac{1}{2}$	$P_1 \in [9.995, 10.005]$	$\sigma_1^M = 0$
$Pr\left(\omega^+\right) = \frac{1}{2}$	$P_1 \in]10.005, 12]$	$\sigma_1^M = 0$ with $prob$ $1 - q^-$ and 1 with $prob$ q^-
$Pr\left(\omega^+\right) = 0$	$P_1 \in [12, 16]$	$\sigma_1^M = -1$

Table 2.5

the Price Distribution at period 2 in state ω^+ for the leading example

ω^+	P_1	p_2
	$P_1 \in \,]12, 16]$	$p_2 \in \,]15, 20]$
	$P_1 \in [8, 12]$	$p_2 \in 15$

Table 2.6

the Price Distribution at period 2 in state ω^- for the leading example

ω^-	P_1	p_2
	$P_1 \in [8, 12]$	$p_2 = 5$
	$P_1 \in [4, 8[$	$p_2 \in [0, 5[$

Table 2.7

Summary of Posts and Trades

Year	2000
Number of Posts	$77,712$
Number of Trades	$3,658$
Number of Posters	2184
Overall Profits	$349,578.10
Profit per trade	$135.06
% Profitable	52.82%

Table 2.8

Empirical Tests

Hypothesis	Dep. Var.	π_j	R^2
H1 A	FP_j	0.011 (2.15)	8.2%
H1 B	NFP_j	-0.075 (−2.02)	5.8%
H2 a	F_j	-0.974 (−2.27)	9.2%
H2 b	$\pi_j^{(f)} - \pi_j^{(nf)}$	-2.152 (−5.60)	38.1%
H3	F_{-j}	0.062 (2.73)	17.9%

* In H1A, we limit the sample to traders with more than 1 trade, more than 10 posts and more than 1 fundamental post, and also exclude one ID who specialized in releasing news. In H1B, we limit the sample to traders with more than 1 trade, more than 10 posts and more than 1 non-fundamental post. In H2, we include only those who follow more than 1 time. In H3, we limit the sample to traders who are followed by others more than once but not always been followed.

Chapter 3

An Empirical Analysis of the Shanghai and Shenzhen Limit Order Books

3.1 Introduction

There are two stock exchanges in mainland China. The Shanghai Stock Exchange was founded on November 26, 1990 and trading began on December 19,1990. The Shenzhen Stock Exchange started stock trading on December 1, 1990. After the first year of trading, the market capitalization, including all shares in Shanghai Stock Exchange and Shenzhen Stock Exchange, was only about three billion Renminbi (RMB). Shanghai had only eight listings, and had a daily average turnover of only 18 million RMB.

Since these modest beginnings, both markets have seen impressive growth. By December 2007, Shanghai Stock Exchange's market capitalization ranked sixth worldwide and Shenzhen ranked 20th. Their combined market capitalization of $32,714$ billion RMB ($4,474$ billion USD) was the second largest globally after the United States. There are more than $1,600$ listings on the two markets, and combined daily average trading volume exceed 100 billion RMB.

After peaking in 2007-8, the markets have fallen by more than half. The Shanghai Stock Exchange Composite Index (SSEC), which once reached $6,092$ in October 2007, retreated to $1,821$ at the end of 2008. The Shenzhen Composite Index closed 2008 at 553.02, after peaking at $1,576.501$ on January 15, 2008. The combined loss in market value in 2008 was over $20,000$ RMB billion, a loss of almost 63%.The basic statistics of the Shanghai Stock Exchange and the Shenzhen Stock Exchange are are summarized in Table 3.1.

[Insert Table 3.1 Here]

The trading mechanism of the stock market in mainland China is similar to that of the Hong Kong or Tokyo Stock Exchanges. Both Shanghai and Shenzhen run a pure order-driven trading mechanism on electronic systems without official market makers. Trading is conducted from Monday to Friday, except holidays. For each trading day, there is a morning session and afternoon session. The morning session includes one pre-trading auction 9:15-9:25 AM and one continuous trading period 9:30-11:30 AM. The afternoon session includes only one continuous trading period 13:00-15:00. Only limit orders and market orders are allowed in both exchanges and orders are filled following price, time and size priority. The limit of price change for each trading day is $\pm 10\%$ of the previous closing price, beyond which, trading will be halted for the rest of the day. The quantity of stock purchased must be in round lots of 100 while there is no requirement on the quantity of sales.

There are three types of shares in the market, A shares that are denominated in Renminbi, H shares that are denominated in Hong Kong Dollar (HKD) and B shares that are dominated by U.S. Dollar (USD). H shares are only traded in Shenzhen Stock Exchange while B shares are only traded in Shanghai Stock Exchange. A shares are traded in both exchanges. Domestic investors can trade all 3 types of shares while the foreign investors only have access to B shares and H shares. The minimum tick size for A shares, B shares and H shares are 0.01RMB, 0.001USD and 0.01HKD, respectively.

The comparison of the Shanghai Stock Exchange and the Shenzhen Stock Exchange are with other stock exchanges are summarized in Table 3.2.

[Insert Table 3.2 Here]

There is a limited literature about the microstructure of the Chinese stock market, and only a few papers analyze intraday limit order book information. Xu (2000) discussed the trading mechanism of Chinese stock market but the paper's quantitative study focused on stocks's daily returns. As to limit order book, Shenoy and Zhang

(2007) studied the relationship between daily order imbalance from limit order book and daily stock returns. Bailey, Cai, Cheung and Wang (2006) separated the order imbalance from individual, institutional and proprietary investors and investigated the various influences of different traders. As far as we know, this essay is the first one to apply vector autoregressive model into analyzing the intraday quotes and limit order book in Chinese stock market.

The essay is organized as follows: Section 3.2 introduces the data and basic statistics. Section 3.3 specifies the baseline Hasbrouck model and reports the market impact of quotes and trades on stock prices. Section 3.4 extends Hasbrouck model to incorporate other information on limit order book and assess the market impact of one buy order in our limit order book model. Section 3.5 studies the relationship between market impacts and microstructure characteristics. Section 3.6 pays particular attention to small orders' market impacts. Section 3.7 concludes.

3.2 Data

We have the limit order book information on $1,652$ Chinese stocks for the month of June 2007, including all A shares, B shares and H shares traded on Shanghai Stock Exchange and Shenzhen Stock Exchange during the sample period. In this limit order book, we have trade-driven data with 5 bids and 5 asks with quantities, with updates no faster than every second. The trades are not combined with each other even if they happened on the same price at the same time. We report summary statistics on the three share classes in Table 3.3.

[Insert Table 3.3 Here]

A shares' median price in our data set is 12.26 RMB, while the median prices of B shares and H shares are 0.998 USD (about 6.78 RMB) and 6.65 HKD (about 5.86 RMB), respectively. As to market cap, the median market cap of A share is $1,964$ RMB (mn), higher than that of B shares, 201 USD (mn) or about $1,367$ RMB (mn),

and that of H shares, 999 HKD (mn), or about 879 RMB (mn). A shares have much higher turnover 0.0537 than H shares and B shares, whose turnover rate are both around 0.0202. This is in accordance with the common understanding that A shares are traded much more actively than B shares and H shares.

3.3 Hasbrouck Model

Hasbrouck's vector autoregressive model (1991) is regarded as the standard model in analyzing intraday quotes and trades of a limit order book. According to Hasbrouck's theory, the ultimate price impact of a trade can meaningfully measure the trade's information effect.

We begin our empirical modeling of Chinese stock market's limit order book using of Hasbrouck's model. Let r_t be the percentage change in the midpoint of the bid-ask spread, $\log((p_t^b + p_t^a)/2) - \log((p_{t-1}^b + p_{t-1}^a)/2)$. Let x_t denote the sequence of signed trades, where trade initiation is determined by the distance from the the bid-ask midpoint. A transaction is considered to be a buy (sell) and is signed $+1$ (-1) if it is initiated by a buy(sell) order. The quote revision model is specified as

$$r_t = a_{r,0} + \sum_{i=1}^{M} a_{r,i} r_{t-i} + \sum_{i=0}^{M} b_{r,i} x_{t-i} + \varepsilon_{r,t},$$

$$x_t = a_{x,0} + \sum_{i=1}^{M} a_{x,i} r_{t-i} + \sum_{i=1}^{M} b_{x,i} x_{t-i} + \varepsilon_{x,t}.$$

where M is the average length in ticks corresponding to roughly 3 minutes. Market impact, which indicates the trade's information effect, is determined by the arrival of a buy order to the market,

$$\partial r_{t+s}/\partial x_t.$$

We apply the model to our data set and limit our sample to stocks that trade

at least $1,000,000$ shares in the trading month. The market impact of a trade is summarized across different share classes and market caps in Table 3.4.

[Insert Table 3.4 Here]

Based on Hasbrouck's model, the median market impact $5 \times M$ periods ahead is 0.1367% on price. This means, on average, a buy trade increases the quote midpoint of the stock by 0.1367% after $5 \times M$ periods.

A shares' median market impact is 0.1374%. Since A shares include much more stocks than B shares and H shares, we should consider A shares as a large sample whose market impact range $(0.0006\%, 3.24\%)$ contains B shares' $(0.006\%, 0.5\%)$ and H shares' $(0.036\%, 1.2\%)$. Thus, we cannot simply compare A shares with B shares or H shares.

B shares has lower median market impact 0.0993% than H shares' 0.1594%, indicating that the average trade's price impact in B shares is lower than that in H shares. The reason will be explained in Section 3.5.

3.4 An Empirical Model of the Limit Order Book

In this section, we extend the VAR model as in Mizrach (2008) to incorporate more details in the limit order book, beyond the inside quote and apply the model to our data set.

Let $p_{k,t}^b$ be the bid on the tier k of the quote montage at time t, and let $p_{k,t}^a$ be the corresponding quote on the tier k of the ask. The posted depths of each participant are denoted by $q_{k,t}^b$ and $q_{k,t}^a$. Now we incorporate the entire book of quotes and depths into an extended specification for the VAR,

$$r_t = a_{r,0} + \sum_{i=1}^{M} a_{r,i} r_{t-i} + \sum_{i=0}^{M} b_{r,i} x_{t-i} + \sum_{i=1}^{M} \sum_{k=1}^{5} \beta_{r,k} (q_{k,t-i}^b - q_{k,t-i}^a) + \varepsilon_{r,t},$$

$$x_t = a_{x,0} + \sum_{i=1}^{M} a_{x,i} r_{t-i} + \sum_{i=1}^{M} b_{x,i} x_{t-i} + \sum_{i=1}^{M} \sum_{k=1}^{5} \beta_{x,k}(q_{k,t-i}^b - q_{k,t-i}^a) + \varepsilon_{x,t},$$

$$q_{k,t}^b - q_{k,t}^a = a_{i,0} + \sum_{i=1}^{M} a_{n,i} r_{t-i} + \sum_{i=1}^{M} b_{n,i} x_{t-i} + \sum_{i=1}^{M} \sum_{k=1}^{5} \beta_{1,i}(q_{k,t-i}^b - q_{k,t-i}^a) + \varepsilon_{q,k,t},$$

$$k = 1, ..., 5.$$

where M is the average length in ticks corresponding to roughly 3 minutes.

The three variable VAR is now given as above. While there are about $7 \times M$ parameters in each equation, the large data sample makes the estimation feasible.

We then use this system to examine the effects over the next $5 \times M$ periods of a net one unit buy, $x_t = 1$. We still limit our sample to stocks that trade at least $1,000,000$ shares in the trading month. The estimates are summarized in Table 3.5.

[Insert Table 3.5 Here]

In the extended model, the median market impact $5 \times M$ periods ahead is 0.1021% on price, less than that of Hasbrouck's model, but the $5\% - 95\%$ range of market impact, $0.0086\% - 0.4343\%$, is larger than that of Hasbrouck model, $0.0098\% - 0.4192\%$. A shares' median market impact is 0.1000%. We still have B shares' median market impact 0.0887% lower than H shares' 0.1531%. We will try to put these results into perspective in the next section.

3.5 Cross Section Estimation of Market Impact

Hasbrouck (1991) stated that information asymmetries are larger for smaller companies. Mizrach (2008) empirically checked the cross-sectional market impacts on the Nasdaq and found them to be positively related with average price, tick frequency,

number of market makers and negatively related with market capitalization.

As for the Chinese markets, we investigate cross-sectional market impacts first for the A shares and summarize the results in Table 3.6.

[Insert Table 3.6 Here]

The cross-sectional market impact fits the following relationship with the average price, turnover, market cap and tick frequency:

Average price has an insignificant influence in this case, and we omitted it from the final specification. For all A shares, the market impacts are positively related with turnover and market cap while negatively related with tick frequencies within the sample period. Those A share stocks, which have large market cap, high turnover and traded less often, attain higher market impact from transaction.

If we consider A shares, B shares and H shares altogether, market cap becomes insignificant. The market impacts are only positively related with turnover and negatively related with tick frequencies within the sample period. Those stocks with high turnover and traded less often attain higher market impact from transaction. The median number of ticks for B shares is $14,446$ and for H shares, $11,687$. Compared with B shares, H shares have the same turnover but lower tick frequency. Thus H shares' median market impact is larger than B shares, consistent with our findings in Section 3.3 and 3.4.

3.6 Small Trades

In Hasbrouck's empirical tests, all trade sizes are constrained to have a similar price impact. In this section, we distinguish the information effects and order imbalances of small trades.

3.6.1 Market impact

Ng and Wu (2007) analyzed Chinese individual and institutional investors' trading

behaviors from brokerage accounts. According to their survey in 2000-2001 period, the average trading sizes of small individual accounts, middle individual accounts, wealthy individual accounts and institutional accounts are about $650, 2150, 16800$ and 111800 shares, respectively. Thus, we classify trades with size less than 650 shares as small trades and others as average trades. And we report the results for Hasbrouck's model in the left side of Table 3.7.

[Insert Table 3.7 Here]

The median market impact of small trades is 0.0234% , while the median market impact of average trades is much larger, 0.1026%.

This conclusion is robust in our empirical models with other limit order book information which appears in the right side of Table 3.7. The median market impact of small and average trades are 0.0445% and 0.1151%, respectively.

3.6.2 Order Imbalance

To investigate the small market impact of small trades, we also check the relationship between daily order imbalance of small trades and contemporaneous daily return. In Table 3.8, we show that the volume-weighted daily order imbalances of small trades and next-day's and contemporaneous daily returns are negatively related with each other.

[Insert Table 3.8 Here]

According to Hasbrouck's analysis, the market impact of a trade is a function of how informed the trader is. Since most small trades are from individual investors, it is reasonable to assume that the small trades are less informed and have less market impact.

There is an established literatures on retail investors' poor trading performance. Hvidkjaer (2008) found that small trades are negatively related with a stocks' future

performance. Stocks with intensive sell-initiated small trade volume outperform those with intensive buy-initiated small trade volume, from one month to two years later. And Barber, Lee, Liu and Odean (2008) also showed that, in Taiwan's stock market, individual traders' losses are equivalent to 2.2% of Taiwan's GDP. Our empirical findings actually show that small trades, which are mostly conducted by retail investors, may be a magnet for informed traders and result in less market impacts.

3.7 Conclusions

This essay investigates the microstructure of the Chinese stock markets and focuses on limit order book information. Firstly, the Shanghai and Shenzhen Stock Exchange's trading mechanism are compared with other markets' microstructures. Secondly, Hasbrouck's vector autoregressive model is applied and extended to incorporate more limit order book information. Market impacts are studied from both Hasbrouck's model and the extended VAR model. Thirdly, the cross sectional relationship, between market impact with market capitalization, tick frequencies, and turnover, is analyzed. Finally, the market impacts of small trades are distinguished. The small trades have a proportionally smaller market impact than average trades. And the volume-weighted daily order imbalances of small trades and next-day's and contemporaneous daily returns are negatively related with each other.

There is additional work needed on the properties of the limit order book, such as liquidity, depth, and clustering. A direct comparison of price impacts in mainland China to Hong Kong and Tokyo, for stocks of similar size and liquidity, would also provide a useful quantitative perspective.

3.8 Tables of Chapter 3

<div align="center">

Table 3.1

Market Statistics for Shanghai and Shenzhen

</div>

	Dec. 2007	Dec. 2008
Market cap:(RMB bn)	$32,714$	$12,136$
Shanghai	$26,984$	$9,725$
Shenzhen	$5,730$	$2,411$
Daily ave. trading volume (RMB bn):	147.489	101.473
Shanghai	99.480	68.070
Shenzhen	48.009	33.403
Number of listings	1530	$1,604$
Shanghai	860	864
Shenzhen	670	740

* Source: World Federation of Exchanges
(http://www.world-exchanges.org/statistics).
* Capitalization and daily average trading volume are in billions of Renminbi
(RMB bn).

Table 3.2

Comparison of Microstructures

Microstructure Characteristics	Shanghai Stock Exchange Shenzhen Stock Exchange	NYSE	NASDAQ
Market Type	Order-driven	Hybrid	Hybrid
Floor Trading	No	Yes	No
Market makers	No	Yes	Yes
Open Hours	9:30AM-11:30AM &13PM-15PM	9:30AM-16PM	9:30AM-16PM
Pre-trading Period or Opening Session	9:15-9:25AM	4AM-9:30AM	7AM-9:30AM
After hours trading	No	16PM-20PM	16PM-20PM
Market Order	Yes	Yes	Yes
Limit Order	Yes	Yes	Yes
Stop Limit Order	No	Yes	Yes
Fill-or-kill Order	No	Yes	Yes
Call auction used?	Yes	Yes	No
Call auction at opening?	Yes	Yes	No
Call auction at closing?	No	No	No
Call Auction Design	Price/Time	Price/Time	N/A
Intraday trading mechanism	Continuous Auction	Continuous Auction	Continuous Auction
Priority	Price/Time/Size	Price/Time	Price/Time/Size or Price/Size/Time or Price/Time/Access Fee
Tick size	A shares: 0.01RMB B shares: 0.001USD H shares: 0.01HKD	0.01 USD	0.01 USD

Table 3.2

Comparison of Microstructures (cnts1)

Microstructure Characteristics	Shanghai & Shenzhen	Tokyo Stock Exchange
Market Type	Order-driven	Order-driven
Floor Trading	No	No
Market makers	No	No
Open Hours	9:30AM-11:30AM &13PM-15PM	9AM-11AM & 12:30PM-15PM
Pre-trading Period or Opening Session	9:15-9:25AM	No
After hours trading	No	No
Market Order	Yes	Yes
Limit Order	Yes	Yes
Stop Limit Order	No	No
Fill-or-kill Order	No	No
Call auction used?	Yes	Yes
Call auction at opening?	Yes	Yes
Call auction at closing?	No	Yes
Call Auction Design	Price/Time	Price/No time priority
Intraday trading mechanism	Continuous Auction	Continuous Auction
Priority	Price/Time/Size	Price/Time
Tick size	A shares: 0.01RMB B shares: 0.001USD H shares: 0.01HKD	$\leq$ 2k JPY: 1 JPY 2k-3k JPY: 5 JPY 3k-30k JPY: 10 JPY 30k-50kJPY: 50 JPY 50k-500k JPY: 100 JPY 500k - 1M JPY: 1k JPY 1M - 20M JPY: 10k JPY 20M - 30M JPY: 50k JPY > 30M JPY: 100k JPY

Table 3.2

Comparison of Microstructures (cnts2)

Microstructure Characteristics	Shanghai & Shenzhen	Hongkong Stock Exchange
Market Type	Order-driven	Order-driven
Floor Trading	No	No
Market makers	No	Yes
Open Hours	9:30AM-11:30AM &13PM-15PM	10AM-12:30PM & 2:30PM-14:30pm & 14:30PM-16PM
Pre-trading Period or Opening Session	9:15-9:25AM	9:30AM-10AM
After hours trading	No	16PM-16:10PM
Market Order	Yes	No
Limit Order	Yes	Yes
Stop Limit Order	No	No
Fill-or-kill Order	No	Yes
Call auction used?	Yes	Yes
Call auction at opening?	Yes	Yes
Call auction at closing?	No	No
Call Auction Design	Price/Time	Order type/Price/Time
Intraday trading mechanism	Continuous Auction	Continuous Auction
Priority	Price/Time/Size	Price/Time
Tick size	A shares: 0.01RMB B shares: 0.001USD H shares: 0.01HKD	$\leq$0.25 HKD: 0.001 HKD 0.25-0.5 HKD: 0.005 HKD 0.5-2 HKD: 0.01 HKD 2-5 HKD: 0.025 HKD 5-30 HKD: 0.05 HKD 30-50 HKD: 0.1 HKD 50-100 HKD: 0.25 HKD 100-200 HKD: 0.5 HKD 200-1k HKD: 1 HKD 1k-9995 HKD: 2.5 HKD

Table 3.3

Statistics on Share Classes

A shares (RMB)	Median	5%	95%
Price	12.26	6.75	40.49
Market Cap (mn)	1,964	525	15,656
Shares Outstanding (mn)	146	33	832
Turnover	0.0537	0.0138	0.0929

B shares (USD)	Median	5%	95%
Price	0.998	0.547	2.213
Market Cap (mn)	201	63	845
Shares Outstanding (mn)	176	59	519
Turnover	0.0202	0.0078	0.0348

H shares (HKD)	Median	5%	95%
Price	6.65	3.30	31.57
Market Cap (mn)	999	260	6,629
Shares Outstanding (mn)	133	57	736
Turnover	0.0202	0.0050	0.0442

Table 3.4

Hasbrouck Model Market Impact Estimates

	Median	5%	95%
A.B.H: overall	0.1367%	0.0098%	0.4192%
A: overall	0.1374%	0.0094%	0.4099%
A: Small Cap	0.1267%	0.0085%	0.3686%
A: Mid Cap	0.1559%	0.0125%	0.3903%
A: Large Cap	0.0993%	0.0079%	0.4489%
B: overall	0.0993%	0.0155%	0.3752%
H: overall	0.1594%	0.0609%	0.5828%

* Small Cap, < 1B RMB, 247 stocks; Middle Cap, 1B~4B RMB, 757 stocks; Large Cap, 4B RMB,345 stocks

Table 3.5

Order Book Model Market Impact Estimates

	Median	5%	95%
A.B.H: overall	0.1021%	0.0086%	0.4343%
A: overall	0.1000%	0.0085%	0.4299%
A: Small Cap	0.0986%	0.0093%	0.3612%
A: Mid Cap	0.1071%	0.0083%	0.4181%
A: Large Cap	0.0869%	0.0081%	0.4830%
B: overall	0.0887%	0.0201%	0.4723%
H: overall	0.1531%	0.0254%	0.6131%

* Small Cap, < 1B RMB, 247 stocks; Middle Cap, 1B~4B RMB, 757 stocks; Large Cap, 4B RMB,345 stocks

Table 3.6

Cross Sectional Market Impact Estimates

Dep. Var.	Constant	Ticks	Turnover	Market Cap	$\overline{R}^2$
A: overall	8.40×10^{-4} (4.73)	-2.33×10^{-8} (−4.62)	0.0250 (14.95)	4.37×10^{-15} (2.02)	0.1506
A: Small Cap	2.10×10^{-3} (3.36)	-1.90×10^{-7} (−7.74)	0.0354 (12.41)	1.16×10^{-12} (1.34)	0.4725
A: Mid Cap	8.60×10^{-4} (2.88)	-7.00×10^{-8} (−6.54)	0.0270 (6.92)	5.76×10^{-13} (5.44)	0.0737
A: Large Cap	8.96×10^{-4} (2.93)	-1.70×10^{-8} (−2.12)	0.0290 (9.79)	7.77×10^{-15} (1.24)	0.2297
A.B.H: overall	0.001 (6.70)	-2.56×10^{-8} (−5.65)	0.0240 (15.09)		0.1443

Table 3.7

Market Impacts by Trade Size

	Hasbrouck Model			Order Book Model		
	Median	5%	95%	Median	5%	95%
Small Orders	0.0234%	−0.2587%	0.3826%	0.0445%	−0.2407%	0.3947%
Ave. Orders	0.1026%	−0.1598%	0.4952%	0.1151%	−0.1499%	0.4801%

Table 3.8

Impact of Trade Size on Returns

	r_t	r_{t+1}
Small Orders (<650)		
Volume Weighted OIB	-1.340×10^{-6} (-83.39)	-5.525×10^{-7} (-30.48)
Ave. Orders (>650)		
Volume Weighted OIB	1.401×10^{-9} (14.04)	3.902×10^{-10} (3.90)

Chapter 4

An Empirical Microstructure Study of the CDS Over-the-counter Market

4.1 Introduction

Credit Default Swap CDS market is one of the most rapidly expanding derivative markets since CDS was formally invented by J.P.Morgan in 1997. The notional outstanding amount of CDS has reach its peak, about 45 trillion dollars, in 2007. However, as the financial crisis exploded, the notional market value of CDS has fallen 38% to 38.6 trillion dollars in 2008.

As a financial derivative, Credit Default Swaps (CDS) provide insurance against a default by a particular company or sovereign entity. The company or the sovereign entity is known as the reference entity and a default is known as a credit event. The CDS buyer makes periodic payments to the seller and in return obtains the right to sell a bond issued by the reference entity for its face value if a credit event occurs. Thus, Credit Default Swaps (CDS) offers a way to hedge the credit risk of holding various corporate bonds, which are hard to short. And it can also be used to bet for or against the likelihood that credit events happen to particular companies or portfolios.

Before March 2009, Credit default Swaps are traded over the counter. Major dealers are banks and financial institutions with good credit ratings. Between major dealers, there are interdealer brokerage companies offering voice brokerage and electronic brokerage. Trades are facilitated through phone systems or electronic platforms. The contracts and trades follow the standards set by the International Swaps and Derivatives Association (ISDA).

Due to the system risk shown in the recent financial crisis, more transparency and regulations are required in CDS market. On March 9, 2009, Intercontinental

Exchange (ICE) established 'ICE Trust' to serve as a central clearing facility for North American CDS indexes. As a New York trust company and member of the Federal Reserve System, 'ICE Trust' is subject to direct regulation and supervision by the Federal Reserve and the New York State Banking Department. And on July 31, 2009 , ICE launched 'ICE Clear Europe' to clear European CDS transactions. 'ICE Clear Europe' is supervised by the Financial Services Authority in accordance with the Financial Services and Markets Act (FSMA). 'The CDS Big Bang' published by Markit Group Limited provided some details about the changes of the trading mechanism.

The sample period of our data set is from April 2006 to March 2008. During that period, CDS trading occurs in the over-the-counter (OTC) market and relies heavily on the interdealer broker system. This study focuses on the microstructure characteristics of CDS market, especially the bid-ask spread and the order imbalance.

There is an established literature about the CDS market but most of them focus on CDS pricing and only a few papers correlate with this one in the microstructure study. Gunduz, Ludecke and Uhrig-Homburg (2007) described the hybrid structure of interdealer brokers and compared the liquidity from the two trading systems, voice brokerage and electronic brokerage. Archary and Johnson (2007) studied the informed trading in CDS market and stock market. Some papers on CDS liquidity risk pricing also include a little microstructure studies. Chen, Cheng and Wu (2005) showed the different liquidity levels drive different CDS spreads. Tang and Yan (2007) constructed liquidity proxies to test whether liquidity risk is priced in CDS market. Chen, Fabozzi and Sverdlove (2008) showed the large bid-ask spread can profoundly affect the estimation of credit risk and liquidity risk. Buhler and Trapp (2008) addressed the liquidity premia in CDS pricing. Han and Zhou (2008) discussed the relationshp between the nondefault component of CDS spreads and illiquidity. Dunbar (2008) developed a three-factor model including a liquidity proxy. Brunnermeier

and Pederson (2008) modeled the situation where market liquidity and funding liquidity mutually reinforce and lead to market liquidity dry-up. Some studies focus on the Sovereign CDS, such as Pan and Singleton (2007), Blanco, Brennan and Marsh (2003), Remolona, Scatigna and Wu (2008), Longstaff, Pan, Pedersen and Singleton(2008). This essay discusses the market microstructure of the Sovereign CDS separately as one group. The literature in pricing liquidity risk of corporate bonds also helped ignite this essay. Jong and Driessen (2007) addressed the liquidity risk premia in corporate bonds. Downing and Covitz (2007) discussed the liquidity risk in the commercial paper yield spreads. Besides, Downing, Underwood and Xing (2007) conducted an interday empirical analysis on informational efficiency of bonds.

The essay is organized as follows: Section 4.2 introduces the data set and the basic statistics. Section 4.3 describes the CDS spreads. Section 4.4 analyzes the trade-to-quote ratio. Section 4.5 investigates the bid-ask spread. Section 4.6 shows the frequency that the trades fall between the quotes. Section 4.7 studies the relationship between the order imbalance and the daily change of CDS spread. Section 4.8 concludes.

4.2 Data

We have the intraday CDS trading records, about $1,555,000$ quotes and $135,500$ trades, from April, 2006 to March, 2008, collected by the interdealer broker GFI Group Inc., a top international broker in credit derivative market. The data is quote-driven. And we have 1 bid, 1 ask and trade price recorded whenever updated. During the sample period, CDS is traded in the OTC market relying heavily on the voice and electronic brokerage system. The records in our data set are stored from both GFI's brokerage desks and its electronic trading platform CreditMatch. Our data set have about 2700 reference entities, including both CDS Indexes and various individual bonds, such as sovereign bonds, municipal bonds, bank and corporate bonds and etc.

The reference entities are from the various regions including North America, Europe, Asia, Australia, South America and Africa, and the contracts are denominated in various currencies including USD, EUR, JPY, GBP and AUD.

We divide the reference entities into the four categories: (a) CDS Indexes, (b) Governmental bonds, including sovereign, municipal and nation-owned corporate, (c) Corporate bonds in Financial Industry, including financial services, fund managers, insurance, real estate and security brokers, and (d) Corporate bonds in other industries. We report summary statistics on the four categories in Table 4.1 and Table 4.2.

[Insert Table 4.1 Here]

[Insert Table 4.2 Here]

Our data set has 1554732 quotes, including 135544 trades, 1192839 bids and 1073450 asks. As shown in Table 1, there are more bids than asks in our data set. Bids are 10.64% more than asks in the 'Index' category, 5.39% in 'Governments', 15.09% in 'Financial', 10.71% in 'Industrial' and 11.12% in the full set. The asymmetry shows, during the sample period, there are generally more buyers than sellers among all the dealers. One possible reason is that more buyers are informed traders during the sample period. Thus, the brokerage offers liquidity for the informed traders, which is more likely to be a buyer.

The distribution of quotes and trades are not so concentrated. Considering Herfindahl Index $= \sum_{i=1}^{N} s_i^2$, HHI is only 0.00556 for the trades and 0.00286 for the quotes.

Reference entity is the company or the sovereign entity which a CDS contract provide insurance on. Table 4.2 shows the regions and currencies of the reference entities in our data set. There are totally 2695 reference entities in our data set. 39% of the reference entities are in Europe and 35% are in North America. For 'Index'

CDS, 81% reference entities of the 'Index' CDS denominated in US dollar are in North America. All the reference entities of the 'Index' CDS denominated in Euro are in Europe. And all the reference entities of the 'Index' CDS denominated in Japanese Yen are in Asia. For the CDS contracts of governmental bonds, 97.5% reference entities of the 'Sovereign' CDS are denominated in US dollar and 95% reference entities of the Municipal CDS are in Asia and denominated in Japanese Yen. For all the Corporate CDS, 66.7% reference entities of the Corporate CDS denominated in US dollar are in North America, 96.4% reference entities of the Corporate CDS denominated in Euro are in Europe, and 99% reference entities of the Corporate CDS denominated in Japanese Yen are in Asia. Thus, in the following parts, our discussion focuses on the categories 'Index, USD, North American', 'Index, EUR, Europe', 'Index, JPY, Asia', 'Sovereign, USD', 'Financial or Industrial, USD, North American', 'Financial or Industrial, EUR, Europe', 'Financial or Industrial, JPY, Asia'.

4.3 Spread

CDS spread is the rate of the payments made per year by the buyer. In this essay, we use Spread for the CDS spread and BAS for the bid-ask spread which will be defined later. Usually, CDS spread is denoted by the percentage of the CDS's nominal value and the unit of CDS Spread is basis point (bps), 1 bps = 0.01%. For example, on Jan 21, 2008, BHP's CDS spread on 5 year bonds was 105.0 bps with no upfront fee. Considering the common nominal value of CDS contracts is 10 Million, in this example, the buyer of the CDS need to pay 105,000 US dollar per year to the seller for the protection on BHP's 5 year bonds.

[Insert Table 4.3 Here]

[Insert Figure 4.1 - 4.12 Here]

Table 4.3 summarizes the CDS Spreads in our data set and Figure 4.1 - 4.12 shows the trend of the CDS Spreads over time. All the spreads in the table and figures come from the recorded trade prices.

During the sample period, in North America, the Financial CDS have lower median spread than the Corporate CDS. However, after the beginning of the financial crisis, the late summer of 2007, the Financial CDS's spreads have increased very sharply and the Corporate CDS's spread have not had much difference until December 2007. The possible reasons for this break point are the closure of Lehman's subprime lender in August, the Countrywide's trouble announcement in August, and also the first bank run of this financial crisis in September (Northern Rock, a medium British bank), which caused the first TED spread spike. This probably means that the market believed the Financial industries had much better credit than the other industries until the financial crisis hit the banks and sharply raised the market's expectations on the bond risks of the Financial industries'. In European and Asia, the similar thing happened: the Financial CDS's spreads have sharply increased since August - September 2007 while the Corporate CDS's spreads have not had any significantly rise until December 2007 when the credit market deteriorated with the various rumors such as Countrywide's bankruptcy, and the TED spread spiked for the second time, and the Federal Reserve established the Term Auction Facility (TAF) program and united the Bank of Canada, the Bank of England, the European Central Bank, and the Swiss National Bank to ease the stress on the short-term funding markets.

The CDS Indexes in North America and Asia have not been much worse until December 2007, and the CDS Indexes in Europe have had much more volatile spreads and began its up trend since May 2007, when HSBC announced it was pulled down by its two US-based unit, one of which is the lender, Household International.

As to the governmental bonds, during the sample period, the various Sovereign CDS denominated in US dollar seemed remain in the normal state, and in the mean-

time, the Japanese Municipal CDS's spreads had a clearly up trend since the beginning of the financial crisis.

4.4 Trade-to-Quote Ratio

We use the trade-to-quote ratio to show how easily to find a trading counter-party. A higher trade-to-quote ratio means it's more rapidly to complete a trade.

Table 4.4 summarizes each reference entity's trade-to-quote ratio. The median trade-to quote ratio is 0.0336. Thus, on average, for every 30 quotes, there is one trade.

The U.S. Corporate CDS denominated in US dollar, have the highest median trade-to-quote ratio, especially those in financial industries. This indicates the U.S. Financial CDS denominated in US dollar are the most liquid in the CDS markets. In the U.S. Financial CDS market, there is one trade for every 2.5 quotes. And in other U.S. Corporate CDS market, there is one trade for every 8.4 quotes. These two median trade-to-quote ratios, 0.4074 for 'US Financial' and 0.1190 for 'US Industrial', are much higher than the median value of the whole data set.

Within all Corporate CDS, the financial industries have relatively higher trade-to-quote ratio than the other industries. It probably means the financial CDS are more liquid. However, the Asian financial CDS denominated in US dollar have a very low trade-to-quote ratio, 0.0099.Besides, the European Corporate CDS denominated in Euro, has higher trade-to-quote ratio than the Asian Corporate CDS denominated in Japanese Yen.

The government related CDS has the lowest trade-to-quote ratio, 0.0179, in the four categories. However, the Sovereign CDS denominated in US dollar has a median trade-to-quote ratio 0.0468, which is higher than the median value of the whole data set.

In the CDS Index market, there is one trade for every 35 quotes. The U.S. CDS

Indexes denominated in US dollar have the highest median trade-to-quote ratio in this category, 0.0871, and the Asian CDS Indexes denominated in Japanese Yen have the lowest trade-to-quote ratio in this category.

[Insert Table 4.4 Here]

4.5 Bid-ask Spread

Bid-ask spread is the difference between the immediate bid price and the immediate ask price, and it is widely used to measure the liquidity of the market, $BAS = ask - bid$. Since CDS contracts have different nominal prices, the percentage bid-ask spread are also calculated, $\%BAS = \frac{ask-bid}{(ask+bid)/2}$.

In this essay, we use BAS to denote the bid-ask spread and %BAS to denote the percentage bid-ask spread. Table 4.5 summarizes the bid-ask spreads and Table 4.6 summarizes the percentage bid-ask spreads for each CDS category.

[Insert Table 4.5 Here]

[Insert Table 4.6 Here]

Table 4.5 shows the U.S. Financial CDS denominated in U.S. dollar have relatively low BAS while Table 4.6 shows their %BAS is not so low. Considering those U.S. financial CDS have the highest trade-to-quote ratio, this means, within all CDS, it is the most easily to find a trading counter-party for the U.S. financial CDS denominated in U.S. dollar, and the BAS costs paid in trading those CDS are also low. Both indicate those U.S. financial CDS are more liquid than others. However, since those CDS have low average spreads, the %BAS costs are relatively high.

In Table 4.4 - 4.6, the Asian Financial CDS denominated with U.S. dollar have low trade-to-quote ratio, high BAS and also high %BAS, and thus, are relatively illiquid.

For the Corporate CDS outside the financial industries, the U.S. Corporate CDS denominated in US dollar have higher median BAS and lower median %BAS than

the Asian Corporate CDS denominated in Japanese Yen and the European Corporate CDS denominated in Euro, due to those U.S. Corporate CDS's much higher spreads.

Within all CDS Indexes, the U.S. CDS Indexes denominated in US dollar have higher trade-to-quote ratios, higher median BAS and higher median %BAS than the Asian CDS Indexes denominated in Japanese Yen and the European CDS Indexes denominated in Euro. This means, although those U.S. CDS Indexes are more frequently traded than those Asian or European CDS Indexes, the BAS costs and the %BAS costs in trading those U.S. CDS Indexes are also higher.

The Sovereign CDS denominated in US dollar have high trade-to-quote ratio, low BAS and the lowest %BAS in all the CDS, and thus, are very liquid. The Municipal CDS denominated in Japanese Yen have few trades, high BAS and the highest %BAS in all the CDS, and thus, are very illiquid.

We also calculate the average bid-ask spread within each month. Figure 4.13 - 4.42 shows the monthly average BAS and %BAS. And we can compare these figures with Figure 4.1 - 4.12.

[Insert Figure 4.13 - 4.42 Here]

For the CDS Indexes, we can easily find in Figure 4.13 - 420 the two BAS spikes, one in the late summer of 2007 and the other at the beginning of 2008. This is consistent with the two spread spikes in Figure 4.1 - 4.3 and the expansion of the financial crisis. When calculating the %BAS, the two spikes are largely smoothed. The %BAS over months shows a steady down trend. The only exception is the European CDS Indexes denominated in Euro. The spike in the late summer of 2007 is still obvious.

Figure 4.23 - 4.24 shows the Sovereign CDS denominated in US dollar have not been significantly impacted by the financial crisis during the sample period. And this is also in consistency with the narrow range of the spreads in Figure 4.4. Besides, the %BAS of those Sovereign CDS have also remained in a narrow range, instead of

a down trend, during the sample period.

Figure 4.25 - 4.34 shows the Financial CDS's BAS have begun to climb up steadily since the late summer of 2007 which is the beginning of this financial crisis. Please note that the Financial CDS's spreads had similar upward slopes in Figure 4.6 - 4.9. Moreover, those CDS's %BAS have been in a range during the sample period. The only exception is the U.S. Financial CDS denominated in USD. Their BAS and %BAS have been much more volatile after the financial crisis.

As to the Corporate CDS outside financial industries, the CDS BAS over months in Figure 4.35 - 4.42 show us more than what the CDS spreads do in Figure 4.10 - 4.12. In Figure 4.35 - 4.42, the Industrial CDS have their BAS gradually increased since the late summer of 2007, the beginning of the financial crisis, though in Figure 4.10 - 4.12, their spreads have not reacted to the crises until the beginning of 2008. Furthermore, the Industrial CDS's %BAS remained in a range during the sample period. The only exception is the increasing %BAS of the Asian Industrial CDS denominated in Japanese Yen.

4.6 Trades

This section discusses whether a trade is completed within quotes provided. We checked (1) whether a trade is conducted with bid or ask provided by the brokerage, and (2) whether the trade falls into quotes.

Biasis, Hillion and Spatt (1995) showed order placement is concentrated at or within the quotes in stock market. Bollen and Christie (2007) discussed the clustering characteristics in the pink sheet market. Since CDS are traded on OTC market, it is not surprising that only 53% trade have both bids and asks provided, and less than half, only 45.3% of trades are completed at or within the bids and asks. Particularly, the Municipal CDS denominated with Japanese Yen have only 9.09% trades with both bids and asks, probably due to its illiquid markets.

Table 4.7 summarizes all the cases. For the data set, within the 261596 trades, 19.86% trades have neither bid nor ask provided, 9.42% trades have only ask provided, 17.72% trades have only bid provided, and 53% trades have both bids and asks provided, including 3.20% traded below bid, 11.70% traded on bid, 23.06% traded between bid and ask, 10.54% traded on bid, and 4.50% traded above ask.

[Insert Table 4.7 Here]

4.7 Order Imbalance

In this section, we calculate the daily order imbalance for each CDS contracts and also check the relationship between the daily order imbalance and the daily spread changes.

Table 4.8 summarizes the daily order imbalance per CDS contract, and Table 4.9 shows the regression of the next-day's or contemporaneous daily spread changes on the daily order imbalances (without volume).

[Insert Table 4.8 Here]

Table 4.8 shows that the CDS Indexes and the Financial CDS have positive median order imbalance (per reference entity), while the Sovereign and Industrial CDS have negative median order imbalance (per reference entity). This probably indicates that the CDS Indexes and the Financial CDS are more liquid. As we know, a liquid market attracts more informed trades. Considering the buyers are more likely to informed traders in CDS market, especially during the financial crisis period, the more liquid markets should have more buyers.

[Insert Table 4.9 Here]

In Table 4.9, the CDS spreads obviously increase as more buyer-initiated trades and decrease as more seller-initiated trades. On average, CDS spread increases by 0.0192 bps for each buyer-initiated trade and decreases by 0.0054 bps for each seller-

initiated trade. This asymmetry also indicates that the market did expect the buyers are more likely to be informed traders. There is an established literature on the relationship between next-day's returns and daily order imbalances in stock market. We can also find such a significant relationship in the Financial CDS market.

We also show the monthly OIB per reference entity in Figure 4.43 - 4.61. Since the financial crisis expanded, the order imbalance tended to the positive side. There was an increasingly number of buyer-initiated trades as the financial crisis raise the market's expectation on risk. However, there is no obvious pattern for the relationship between monthly OIB and monthly average spreads.

[Insert Figure 4.43 - 4.61 Here]

4.8 Conclusions

This essay studies the microstructure of the CDS OTC markets and focuses on the trade-to-quote ratio, the bid-ask spread, and the order imbalance. Firstly, we analyze the influence of the financial crisis on the CDS spreads. Secondly, we study the trade-to-quote ratio, the bid-ask spread, and other microstructure characteristics and check their implications on the market liquidity. Finally, we focus on the daily order imbalance and its relationship with the daily changes of the CDS spreads.

As we mentioned, ICE have established the 'ICE Trust' and 'ICE Clear Europe' to start the CDS's exchange trading system. A comparison of CDS OTC market to the exchange trading would provide a meaningful perspective. And there is also additional work needed on the microstructure studies about CDS's OTC market, such as hidden liquidity and the hybrid brokerage system.

4.9 Tables of Chapter 4

Table 4.1

Summary of Observations

	# of Quotes	# of Trades		# of Bids		# of Asks	
Index	296571	34449	11.62%	228071	76.90%	206132	69.51%
Gov.	86450	14062	16.27%	52445	60.67%	49764	57.56%
Financial	252859	21864	8.65%	189440	74.92%	164607	65.10%
Industrial	918852	65169	7.09%	722883	78.67%	652947	71.06%
Full set	1554732	135544	8.72%	1192839	76.72%	1073450	69.04%

* Index: bond index or CDS index.

* Governmental: Sovereign and local governments, state-owned firms.

* Financial: Banks, Financial Services, Fund Managers, Insurance, Real Estate, and Security Brokers.

* Industrial: All other industries.

Table 4.2

Summary of Reference Entities (# of Reference Entities)

		N. America	European	Asia	Australia	S. America	Africa	Total
Index	USD	258	0	37	14	10	0	319
	EUR	0	254	0	0	0	0	254*
	JPY	0	0	44	0	0	0	44
	GBP	0	3	0	0	0	0	3*
	Full set	258	255	81	14	10	0	618
Gov.	USD	12	34	24	2	8	4	84
	EUR	1	8	2	0	1	1	13
	JPY	0	0	57	0	0	0	57
	Full set	12*	35*	80	2	8*	4*	141*
Sovereign	USD	10	33	21	2	8	4	78
	EUR	1	7	2	0	1	1	12
	JPY	0	0	2	0	0	0	2
	Full set	10	33*	23*	2	8*	4*	80
Municipal	USD	2	1	0	0	0	0	3
	JPY	0	0	53	0	0	0	53
	Full set	2	1	53	0	0	0	56

* Some reference entities have CDS denominated in more than one currencies.

Table 4.2 (cnts)

Summary of Reference Entities (# of Reference Entities)

		N. America	European	Asia	Australia	S. America	Africa	Total
Financial	USD	107	29	86	22	1	1	246
	EUR	1	144	8	0	0	0	153
	JPY	1	0	75	0	0	0	76
	Full set	108*	159*	145*	22	1	1	436
Industrial	USD	563	51	100	38	4	2	758
	EUR	2	500	10	1	0	2	515
	JPY	2	0	212	0	0	0	214
	GBP	0	58	1	0	0	0	59
	AUD	0	0	0	1	0	0	1
	Full set	565*	593*	296*	38	4	4	1500

* Some reference entities have CDS denominated in more than one currencies.

Table 4.3

Summary of CDS Spread in trades

	Currency	Region	Mean	Min	5%	25%	Median	75%	95%	Max
Index	USD	N.America	309.51	2.25	28.93	100.38	282	500	597	1435
	EUR	Europe	168.82	0.25	23.25	45.25	98	263	500	1400
	JPY	Asia	69.47	13.50	18.13	28.75	43.50	95.81	189.05	245
Sovereign	USD	Full set	162.29	3	40	99	158	204	305	1500
Financial	USD	N.America	138.69	2.80	13.50	34	76	175	500	1350
	USD	Asia	302.09	6.50	21	74.75	240	485.75	745	1200
	EUR	Europe	61.86	2	9	22	41.50	68	180.40	1150
	JPY	Asia	84.72	4	12.46	38.25	61.50	90	286.60	535
Industrial	USD	N. America	260.32	4	23	75.63	195	410	675	1200
	EUR	Europe	79.50	2	20	35	51	85	240	1125
	JPY	Asia	46.40	2.25	8	16.50	26	42	200	630

Table 4.4

Trade-to-Quote Ratio per Reference Entity (T2Q)

	Currency	Region	# Quotes	# Trades	# Firms	Median T2Q per reference entity	25%	75%
Index	USD	N.America	43803	6538	258	0.0871	0.0098	0.2760
	EUR	Europe	200131	21920	254	0.0196	0	0.0541
	JPY	Asia	21134	1820	44	0	0	0.0493
	Full set	Full set	296571	34449	618	0.0286	0	0.111
Sovereign	USD	Full set	83678	13974	78	0.0468	0.0210	0.2932
Gov.	Full set	Full set	86450	14062	141	0.0179	0	0.1323
Financial	USD	N.America	18874	10830	107	0.4074	0	0.6667
	USD	Asia	23129	1728	86	0.0099	0	0.0460
	EUR	Europe	160021	6233	144	0.0322	0.0197	0.0543
	JPY	Asia	27874	1438	75	0.0392	0.0360	0.0490
	Full set	Full set	252859	21864	463	0.0320	0	0.1929
Industrial	USD	N. America	107661	16822	563	0.1190	0.0448	0.1983
	EUR	Europe	653024	41233	500	0.0215	0	0.0493
	JPY	Asia	84297	3472	212	0.0319	0	0.0464
	Full Set	Full set	918852	65169	1500	0.0367	0	0.0988

Table 4.5

Bid-ask Spread (BAS)

	Currency	Region	# Obs	Median	Min	5%	25%	75%	95%	Max
Index	USD	N. America	22791	4	0.01	0.25	2	7	19	200
	EUR	Europe	132705	2	0.01	0.25	0.75	8	12	200
	JPY	Asia	12884	1.25	0.05	0.25	0.75	3	13	160
Sovereign	USD	Full set	39267	5	0.125	1	3	5	20	1400
Municipal	JPY	Asia	1296	7	2.5	4	6	8	15	30
Financial	USD	N.America	4392	3	0.25	1	2	5	15	100
	USD	Asia	10449	8	0.5	2	4	16.5	50	500
	EUR	Europe	101862	3	0.5	1	2	8	25	800
	JPY	Asia	15236	7	0.5	2	4	18	55	820
Industrial	USD	N. America	55622	10	0.5	3	5	15	40	555
	EUR	Europe	411618	4	0.5	1	3	9	30	550
	JPY	Asia	43600	5	0.3	1.5	3	10	65	930

* only consider non-zero bids and asks.

Table 4.6

Percentage Bid-ask Spread (%BAS)

	Currency	Region	#Obs.	Median	Min	5%	25%	75%	95%	Max
Index	USD	N.America	22791	6.45%	0.01%	0.24%	2.23%	17.24%	54.55%	171.43%
	EUR	Europe	132705	1.63%	0.01%	0.45%	0.97%	3.32%	14.93%	166.67%
	JPY	Asia	12884	3.39%	0.27%	0.85%	1.95%	6.14%	21.14%	142.86%
Sovereign	USD	Full set	39267	4.76%	0.21%	1.38%	2.90%	5.31%	25.00%	180.95%
Municipal	JPY	Asia	1296	90.91%	20.69%	44.44%	66.67%	94.74%	120.00%	146.67%
Financial	USD	N.America	4392	11.11%	0.38%	4.08%	7.79%	15.39%	26.09%	85.71%
	USD	Asia	10449	19.51%	0.83%	6.25%	13.33%	28.57%	50.00%	120.00%
	EUR	Europe	101862	15.39%	0.48%	4.38%	9.52%	22.22%	40.00%	160.66%
	JPY	Asia	15236	13.70%	0.48%	3.77%	8.06%	25.00%	55.44%	169.81%
Industrial	USD	N.Amreica	55622	8.23%	0.19%	1.71%	4.26%	14.71%	30.77%	157.90%
	EUR	Europe	411618	8.45%	0.61%	2.88%	5.55%	13.35%	25.60%	191.11%
	JPY	Asia	43600	21.28%	0.31%	5.13%	12.25%	36.36%	76.92%	173.83%

* only consider non-zero bids and asks

Table 4.7

Trades within bid/ask or not?

				wo.Bid or Ask, Trade vs. (bid,ask)					
	Currency	Region	#Trades	wo.Bid		wo.Ask		wo.B&A	
Index	USD	N.America	6538	2278	34.84%	488	7.46%	530	8.11%
	EUR	Europe	21920	1345	6.14%	3057	13.95%	3004	13.70%
	JPY	Asia	1820	116	6.37%	277	15.22%	274	15.05%
	Full set	Full Set	34449	3789	11.00%	5082	14.75%	5302	15.39%
Sovereign	USD	Full set	13974	1418	10.15%	4260	30.49%	3801	27.20%
Municipal	JPY	Asia	44	7	15.91%	33	75.00%	0	0
Gov.	Full set	Full set	14062	1454	10.34%	4299	30.57%	3806	27.07%
Financial	USD	N.America	10830	3413	31.51%	1417	13.08%	4102	37.88%
	USD	Asia	1728	975	56.42%	210	12.15%	236	13.66%
	EUR	Europe	6233	98	1.57%	1255	20.13%	1484	23.81%
	JPY	Asia	1438	10	0.70%	265	18.43%	368	25.59%
	Full set	Full set	21864	4899	22.41%	3609	16.51%	6448	29.49%
Industrial	USD	N. America	16822	1082	6.43%	2714	16.13%	3362	19.99%
	EUR	Europe	41233	1041	2.52%	7285	17.67%	7010	17.00%
	JPY	Asia	3472	66	1.90%	662	19.07%	657	18.92%
	Full set	Full set	65169	2647	4.06%	11435	17.55%	11582	17.77%
Full set	Full set	Full set	261596	24638	9.42%	46348	17.72%	51966	19.86%

Table 4.7(cnts)

Trades within bid/ask or not?

	Curr.	Region	w.Bid & Ask, Trade vs. (bid,ask)									
			<bid		=bid		$\in$(bid,ask)		=ask		>ask	
Ind.	USD	N.Am.	644	9.85%	230	3.52%	1554	23.77%	196	3.00%	618	9.45%
	EUR	Eur.	527	2.40%	3282	14.97%	6372	29.07%	3252	14.84%	1081	4.93%
	JPY	Asia	44	2.42%	231	12.69%	474	26.04%	299	16.43%	105	5.77%
	All	All	1257	3.65%	4202	12.20%	8718	25.31%	4260	12.37%	1839	5.34%
Sov.	USD	All	706	5.05%	1133	8.11%	980	7.01%	1036	7.41%	640	4.58%
Mun.	JPY	Asia	0	0	2	4.55%	2	4.55%	0	0	0	0
Gov.	All	All	706	5.02%	1135	8.07%	983	6.99%	1036	7.37%	643	4.57%
Fin.	USD	N.Am.	391	3.61%	154	1.42%	501	4.63%	114	1.05%	738	6.81%
	USD	Asia	12	0.69%	65	3.76%	154	8.91%	67	3.88%	9	0.52%
	EUR	Eur.	197	3.16%	745	11.95%	1687	27.07%	621	9.96%	146	2.34%
	JPY	Asia	19	1.32%	147	10.22%	472	32.82%	143	9.94%	14	0.97%
	All	All	640	2.93%	1223	5.59%	2997	13.71%	1067	4.88%	981	4.49%
Ind.	USD	N. Am.	1090	6.48%	1579	9.39%	4191	24.91%	1164	6.92%	1640	9.75%
	EUR	Eur.	458	1.11%	6844	16.60%	11898	28.86%	6061	14.70%	636	1.54%
	JPY	Asia	31	0.89%	402	11.58%	1208	34.79%	344	9.91%	102	2.94%
	All	All	1646	2.53%	9231	14.16%	18142	27.84%	7916	12.15%	2570	3.94%
All	All	All	8368	3.20%	30605	11.70%	60333	23.06%	27576	10.54%	11762	4.50%

Table 4.8

Entire-period OIB for each Reference Entities (without Volume)

	Currency	Region	Mean	Min	5%	25%	Median	75%	95%	Max
Index	USD	N.America	-0.30	-100	-11.4	-2	0	2	10.4	45
	EUR	Europe	3.46	-124	10	-2	0	2	37.95	149
	JPY	Asia	8.31	-14	-8	-2	0	2	57.4	94
	Full Set	Full Set	1.93	-124	-12	-2	0	2	21	149
Sovereign	USD	Full Set	-3.29	-144	-15.9	-3	0	1.25	14.45	57
Gov.	Full set	Full Set	-3.07	-144	-15.5	-2.25	0	1	14.25	57
Financial	USD	N.America	5.71	-32	-18.65	-2.25	0	1.25	20.25	329
	EUR	Europe	-2.19	-56	-14.1	-5	-1	2	8	16
	JPY	Asia	-0.52	-39	-13	-1.5	1	4.5	8.8	11
	Full set	Full Set	0.605	-56	-15	-3	0	2	11	329
Industrial	USD	N America	-0.0825	-91	-10	-3	0	2	12.05	33
	EUR	Europe	-8.34	-135	-51.55	-11	-2	1	11.65	34
	JPY	Asia	0.098	-11	-6	-2	0	2	6	10
	Full Set	Full Set	-2.468	-135	-19	-4	-1	2	10.25	72

* only consider all trades with non-zero bids and asks.

Table 4.9 OIB vs. CDS Spread Changes

(Dependent. Var.: Spread Changes)

	OIB_t	OIB_{t-1}
Governmental	$3.24E-3$ (4.75)	$1.15E-4$ (0.16)
Financial	$1.12E-2$ (11.00)	$5.86E-3$ (5.63)
Industrial	$7.29E-3$ (13.99)	$1.25E-4$ (0.25)

* not consider the sub groups' regressions because of the limited sample size.
* trim all contracts without OIB or with zero OIB.

4.10 Figures of Chapter 4

Figure 4.1. Monthly Ave. CDS Spread (Index,USD,N. America)
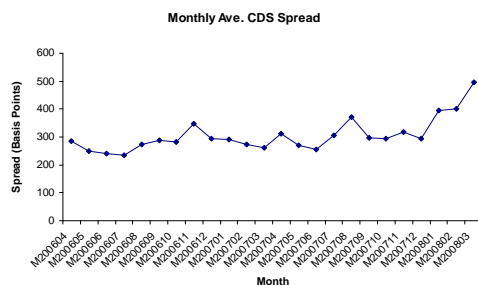
Figure 4.2. Monthly Ave. CDS Spread (Index,EUR,Europe)

Monthly Ave. CDS Spread

Monthly Ave. CDS Spread
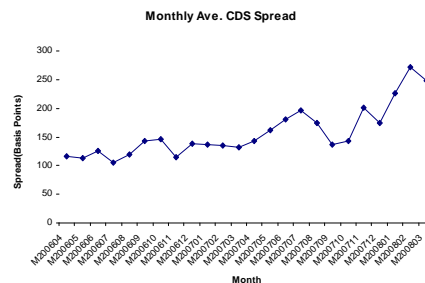
Figure 4.3. Monthly Ave. CDS Spread (Index,JPY,Asia)
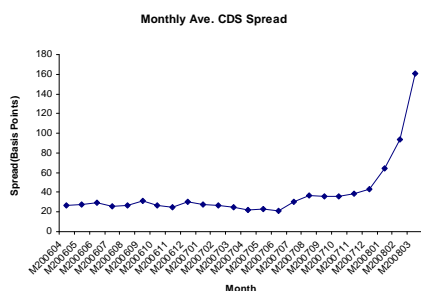
Figure 4.4. Monthly Ave. CDS Spread (Sovereign,USD)

Monthly Ave. CDS Spread

Monthly Ave. CDS Spread
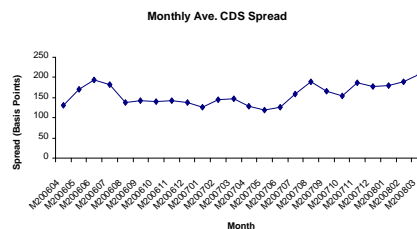
Figure 4.5. Monthly Ave. CDS Spread (Municipal,JPY)
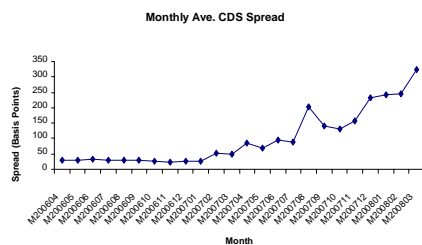
Figure 4.6. Monthly Ave. CDS Spread (Financial,USD,N.America)

Monthly Ave. CDS Spread

Monthly Ave. CDS Spread
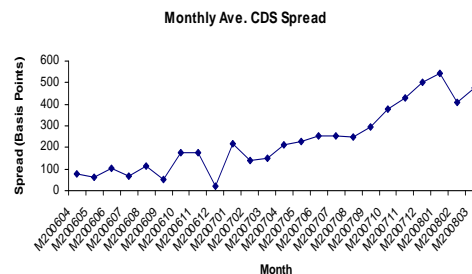
Figure 4.7. Monthly Ave. CDS Spread (Financial,USD,Asia)

Monthly Ave. CDS Spread
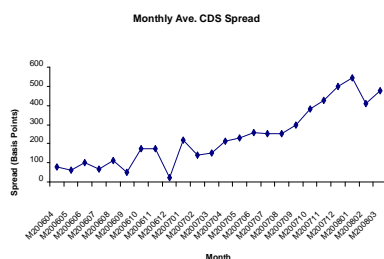
Figure 4.8. Monthly Ave. CDS Spread (Financial,EUR, Europe)

Monthly Ave. CDS Spread
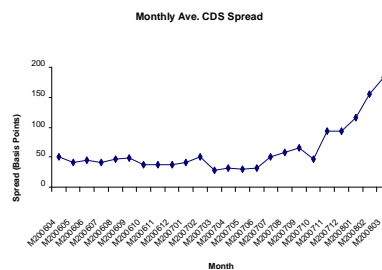
Figure 9. Monthly Ave.CDS Spread (Financial,JPY,Asia)
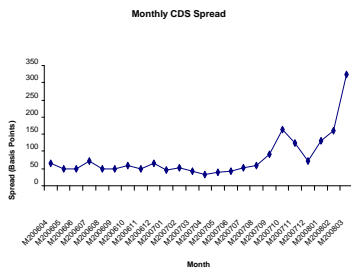
Monthly CDS Spread

Figure 10. Monthly Ave. CDS Spread (Industrial,USD,N.America)

Monthly Ave. CDS Spread
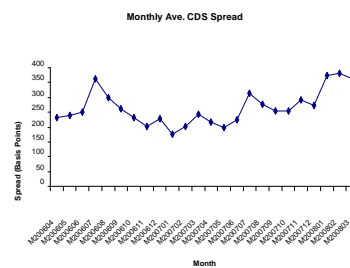
Figure 11. Monthly Ave. CDS Spread (Industrial,EUR,Europe)

Monthly Ave. CDS Spread
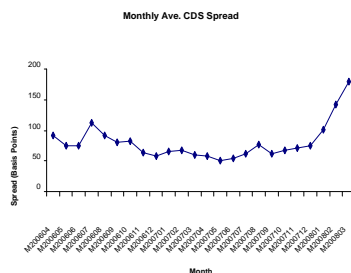
Figure 12. Monthly Ave. CDS Spread (Industrial,JPY,Asia)

Montly Ave. CDS Spread
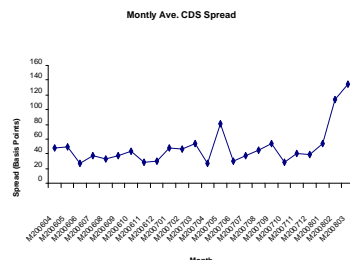
Figure 4.13. Monthly Ave. BAS (Index)
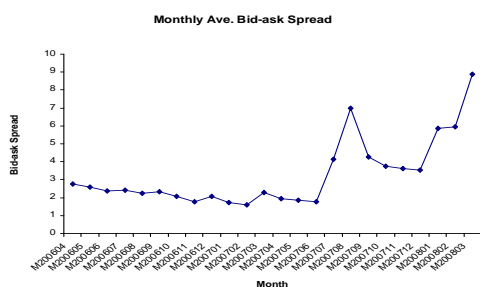
Figure 4.14. Monthly Ave. %BAS (Index)
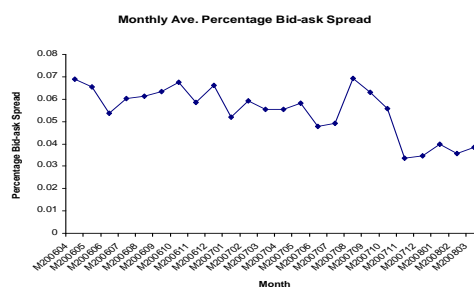
Figure 4.15. Monthly Ave. BAS (Index,USD,North America)
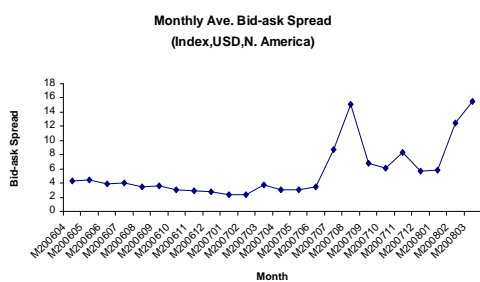
Figure 4.16. Monthly Ave. %BAS (Index,USD,North America)
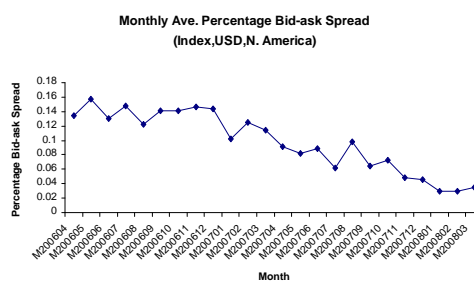
Figure 4.17. Monthly Ave. BAS (Index,EUR,Europe)
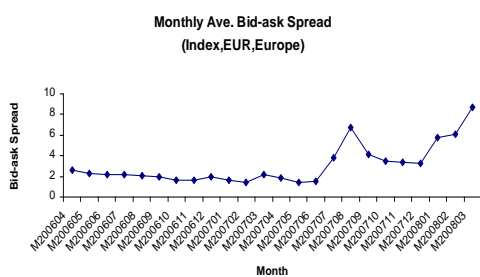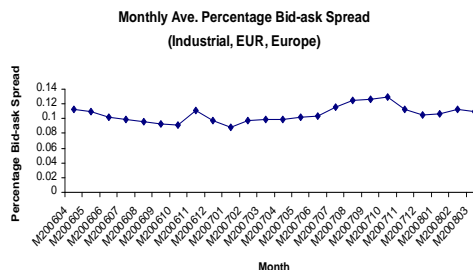

Figure 4.18. Monthly Ave. %BAS (Index,EUR,Europe)
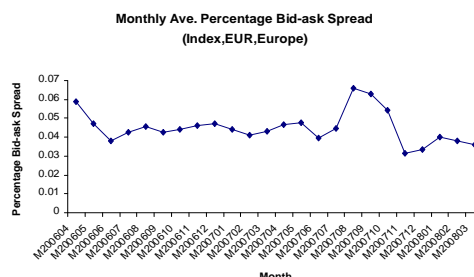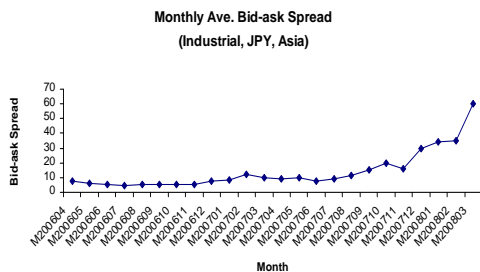

Figure 4.19. Monthly Ave. BAS (Index,JPY,Asia)
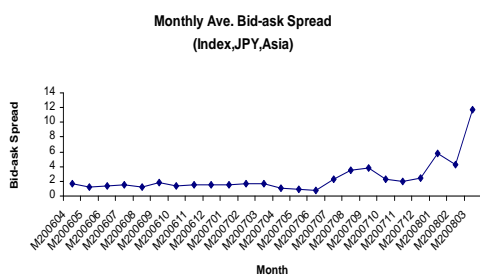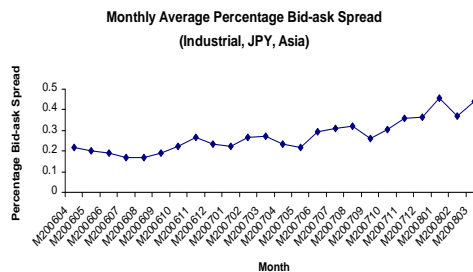

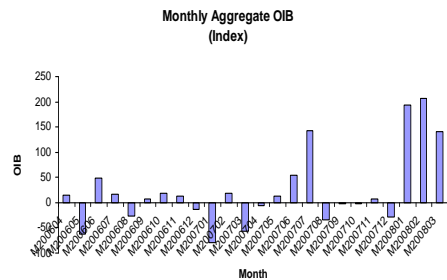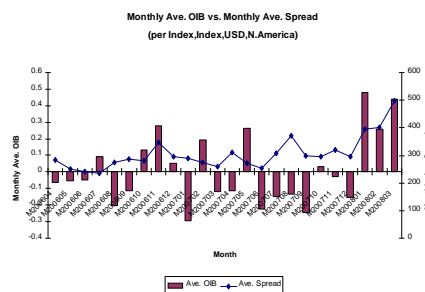Figure 4.20. Monthly Ave. %BAS (Index,JPY,Asia)

Figure 4.21. Monthly Ave. BAS

(Governmental)

Monthly Ave.Bid-ask Spread

Figure 4.22. Monthly Ave. %BAS

(Governmental)

Monthly Ave. Percentage Bid-ask Spread

Figure 4.23. Monthly Ave. BAS

(Sovereign,USD)

Monthly Ave.Bid-ask Spread
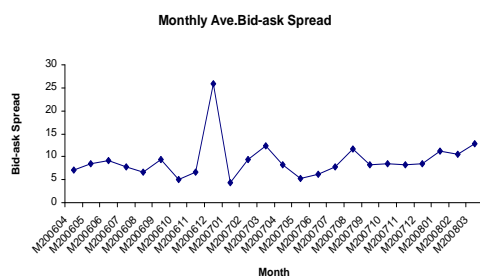
Figure 4.24. Monthly Ave. %BAS

(Sovereign,USD)

Monthly Ave. Percentage Bid-ask Spread
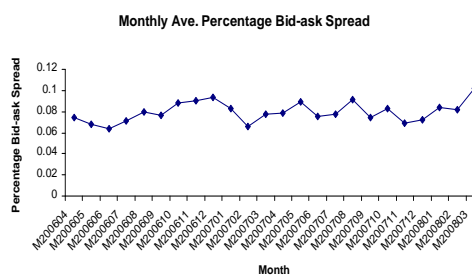
Figure 4.25. Monthly Ave. BAS

(Financial)

Monthly Ave. Bid-ask Spread
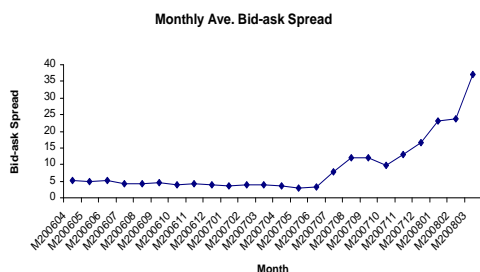
Figure 4.26. Monthly Ave. %BAS

(Financial)

Monthly Ave. Percentage Bid-ask Spread
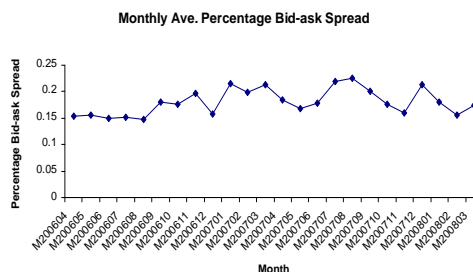
Figure 4.27. Monthly Ave. BAS

(Financial,USD,N. America)

Monthly Ave. Bid-ask Spread
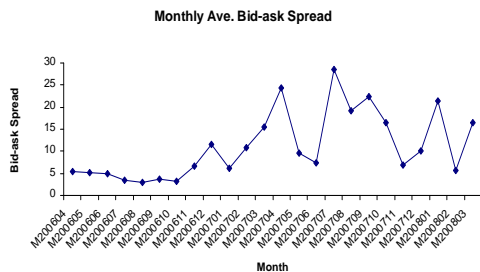
Figure 4.28. Monthly Ave. %BAS

(Financial,USD,N. America)

Monthly Ave. Percentage Bid-ask Spread
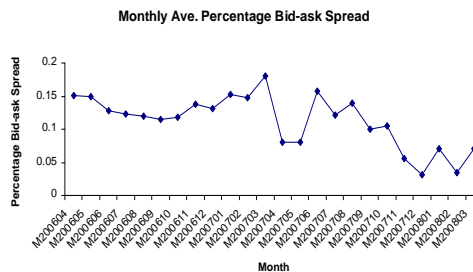
Figure 4.29. Monthly Ave. BAS

(Financial, USD, Asia)

Monthly Ave. Bid-ask Spread
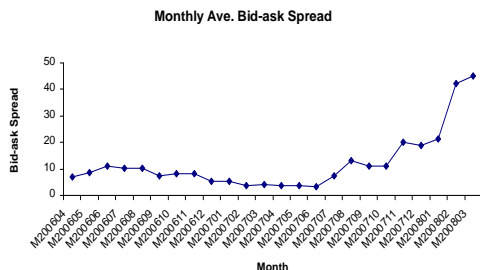
Figure 4.30. Monthly Ave. %BAS

(Financial, USD, Asia)

Monthly Ave. Percentage Bid-ask Spread
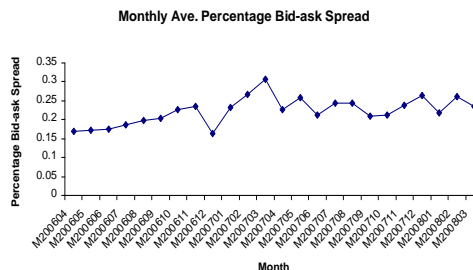
Figure 4.31. Monthly Ave. BAS

(Financial,EUR,Europe)

Monthly Ave. Bid-ask Spread

Figure 4.32. Monthly Ave. %BAS

(Financial,EUR,Europe)

Monthly Ave. Percentage Bid-ask Spread

Figure 4.33. Monthly Ave. BAS

(Financial,JPY,Asia)

Monthly Ave. Bid-ask Spread

Figure 4.34. Monthly Ave. %BAS

(Financial,JPY,Asia)

Monthly Ave. Percentage Bid-ask Spread
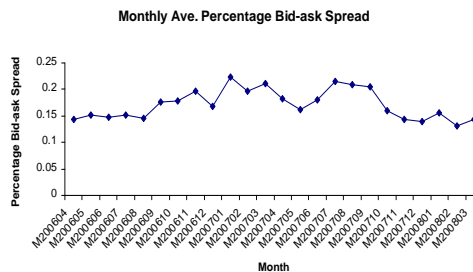
Figure 4.35. Monthly Ave. BAS
(Industrial)

Monthly Ave. Bid-ask Spread
(Industrial)

Figure 4.36. Monthly Ave. %BAS
(Industrial)

Monthly Ave. Percentage Bid-ask Spread
(Industrial)

Figure 4.37. Monthly Ave. BAS
(Industrial,USD,North America)

Monthly Ave. Bid-ask Spread
(Industrial, USD, N.Amreica)

Figure 4.38. Monthly Ave. %BAS
(Industrial,USD,North America)

Monthly Average Percentage Bid-ask Spread
(Industrial, USD, N.Amreica)
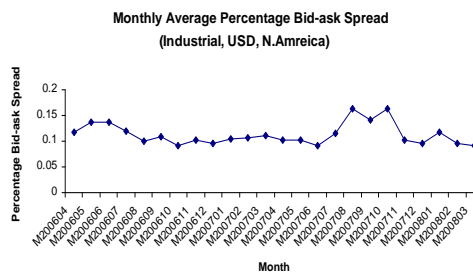
Figure 4.39. Monthly Ave. BAS
(Industrial,EUR,Europe)

Monthly Ave. Bid-ask Spread
(Industrial, EUR, Europe)
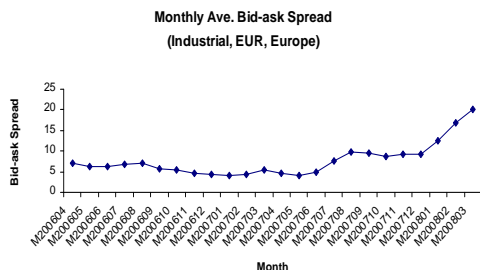
Figure 4.40. Monthly Ave. %BAS
(Industrial,EUR,Europe)

Monthly Ave. Percentage Bid-ask Spread
(Industrial, EUR, Europe)

Figure 4.41. Monthly Ave. BAS
(Industrial,JPY,Asia)

Monthly Ave. Bid-ask Spread
(Industrial, JPY, Asia)

Figure 4.42. Monthly Ave. %BAS
(Industrial,JPY,Asia)

Monthly Average Percentage Bid-ask Spread
(Industrial, JPY, Asia)

Figure 4.43. Monthly Ave. OIB

(Index)

Monthly Ave. OIB per Index
(Index)

Figure 4.45. Monthly Ave. OIB

(Index,USD,N.America)

Monthly Ave. OIB vs. Monthly Ave. Spread
(per Index,Index,USD,N.America)

Figure 4.44. Monthly Aggregate OIB

(Index)

Monthly Aggregate OIB
(Index)

Figure 4.46. Monthly Ave. OIB

(Index,EUR,Europe)

Monthly Ave. OIB vs. Monthly Ave. Spread
(per Index,Index,EUR,Europe)
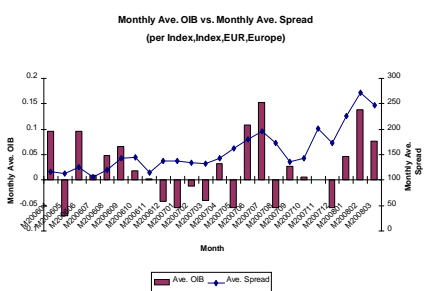
Figure 4.47. Monthly Ave. OIB

(Index,JPY,Asia)

Monthly Ave. OIB vs. Monthly Ave. Spread
(per Index,Index,JPY,Asia)
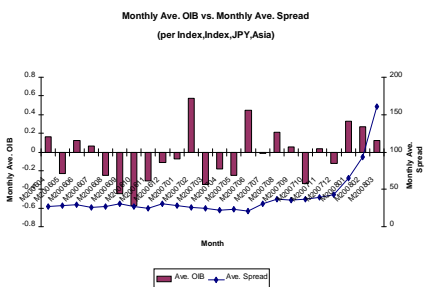
Figure 4.48. Monthly Ave. OIB

(Governmental)

Monthly Ave. OIB
(Governments)
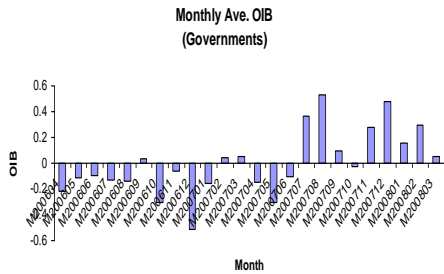
Figure 4.50. Monthly Ave. OIB

(Sovereign,USD)

Monthly Ave. OIB vs. Monthly Ave. CDS Spread
(per Reference Entity,Sovereign,USD)
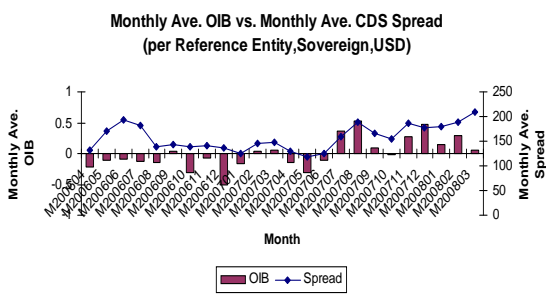
Figure 4.49. Monthly Aggregate OIB

(Governmental)

Monthly Aggregate OIB (Governments)
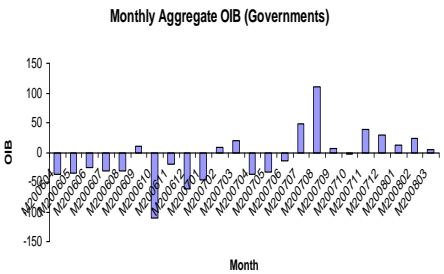
Figure 4.51. Monthly Ave. OIB

(Financial)
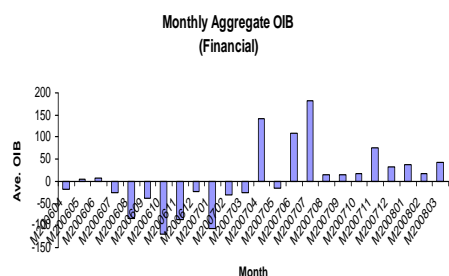
Figure 4.52. Monthly Aggregate OIB

(Financial)
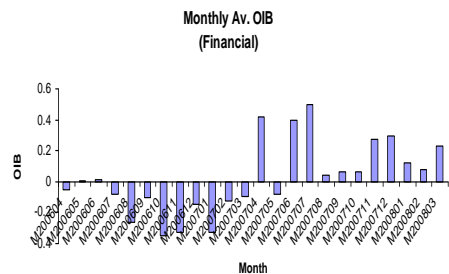
Figure 4.53. Monthly Ave. OIB

(Financial,USD,N.America)
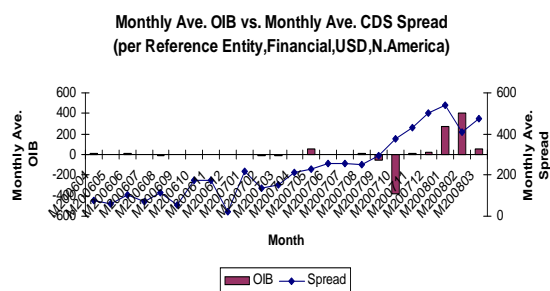
Figure 4.54. Monthly Ave. OIB

(Financial,USD,Asia)
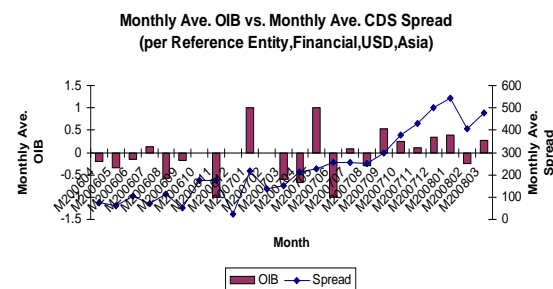
Figure 4.55. Monthly Ave. OIB

(Financial,EUR,Europe)
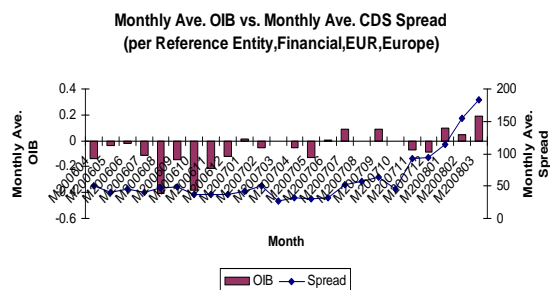
Figure 4.56. Monthly Ave. OIB

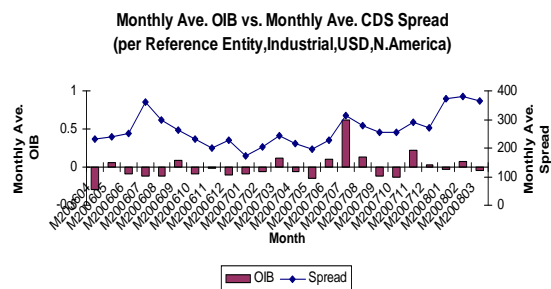(Financial,JPY,Asia)
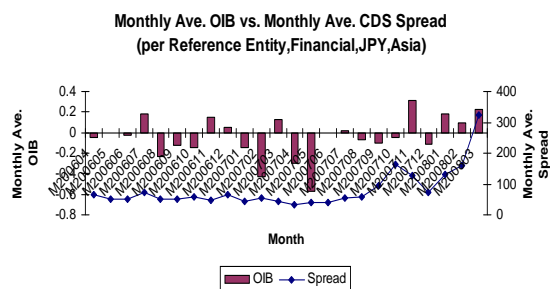
Figure 4.57. Monthly Ave. OIB

(Index)

Figure 4.58. Monthly Aggregate OIB

(Index)

Figure 4.59. Monthly Ave. OIB
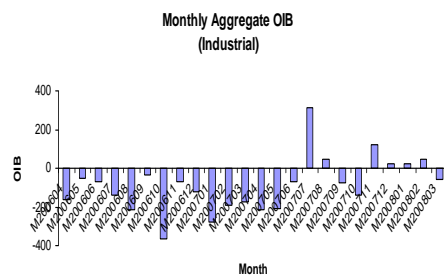
(Index,USD,N.America)

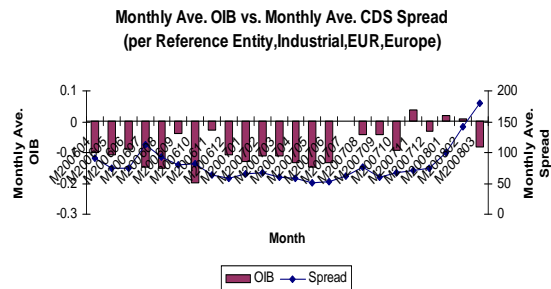Figure 4.60. Monthly Ave. OIB
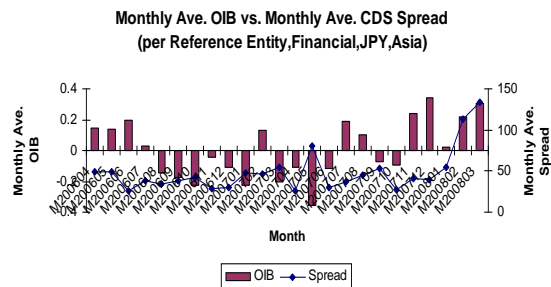
(Index,EUR,Europe)

Figure 4.61. Monthly Ave. OIB

(Index,JPY,Asia)

Chapter 5

Conclusion and Summary

This dissertation consists of three essays on behavioral finance and market microstructure.

In Chapter 2, I focus on individual day traders' trading behaviors, and model their interactions in an Internet stock trading chatroom. A symmetric Bayesian Nash Equilibrium exists for either the game without communication or the game with communication. This model generates three empirical predictions, which are confirmed by the data set consisting of stock trading chat room posts of more than 1,000 individual semi-professional day traders. This essay studies several basic questions: Who communicates the most? What do they communicate? And why? As to these questions, the model indicates: (1) both informed and momentum traders can benefit from entering into the chatroom and communicating with others, (2) the informed traders post more fundamental analysis while the momentum traders post more non-fundamental analysis, and (3) the traders' optimal strategy is to follow the informed traders. On the other side, the empirical results demonstrate: (1) there are both high profitable and low profitable traders in the chatroom, (2) the high profitable traders, who are more likely to be informed traders, post more fundamental analysis, while the low profitable ones, who are more likely to be momentum traders, post more technical analysis, and (3) the traders did follow the high profitable traders much more frequently and the low profitable traders are more likely to follow.

In Chapter 3, I analyze the effects of the entire limit order book in Shanghai Stock Exchange and the Shenzhen Stock Exchange, which are the only two stock exchanges in mainland China. The two exchanges have a pure order-driven trading mechanism without market makers. And the three types of shares in the market: A share, B share and H share are all studied. To analyze the limit order books, the structural

vector autoregressive model of Hasbrouck (1991) is extended to incorporate more information beyond the inside quotes. Both Hasbrouck's model and the extended model are applied in assessing the market impact of quotes. One meaningful finding is that the A shares' market impacts are to be positively related with turnover and market cap while negatively related with tick frequency. Another interesting finding is small trades, usually linked with individual investors, have proportionally smaller market impact, and the positive daily order imbalances of small trades shadow the next-day's returns.

In Chapter 4, I study the Credit Default Swap (CDS) market microstructure. Before March 2009, CDS are traded on the over-the-counter (OTC) market, through brokers' voice-based or electronic-based systems. This empirical analysis includes the CDS spread, the trade-to-quote ratio, the bid-ask spread, the frequency that the trades fall into the quotes, and the relationship between the daily order imbalance and the daily changes of the CDS spread.

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