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Abstract of the Dissertation<br>Fostering Effective Mathematics Teaching: Professional Coaching and Teachers' Instructional Practices and Beliefs<br>By James A. Neuberger<br>\section*{Dissertation Chair: Carolyn A. Maher, Ed. D.}

Two decades ago the National Council of Teachers of Mathematics challenged the mathematics education community to promulgate a comprehensive set of learning goals for K-12 students that would guide mathematics curriculum, teaching, and assessment for the future. One consequence was an emphasis on professional development of teachers. Accordingly, in 2003, New York City's public schools started a math coaching program, whereby math education experts worked closely with math teachers for an extended period of time in the teachers' schools. This program became an opportunity for important research regarding the effectiveness of coaching

This study describes the collaboration between one coach and one teacher in the implementation of the coaching system. The researcher observed and videotaped a lesson and the subsequent debriefing between the teacher and coach; and interviewed the teacher, coach, and principal.

The benefit to the classroom teacher was supported by analysis of the data. The teacher reported that, for the first time, math was "fun," she was more confident, and more class time was devoted to mathematics. The teacher paid closer attention to student work, reflected on her own practice, grouped students more beneficially,
encouraged them to interact, and to make their thinking public. She did not view answers as just right or wrong, but rather as part of a process of making sense of ideas.

The data suggest:

1. The teacher reported that some of her beliefs about math teaching had changed due to the coaching process.
2. Teacher practices mirrored teacher beliefs. There are signs that the coaching is influencing the teacher's practice.
3. The coach helped the teacher learn mathematics and pay attention to the math learning of her students.
4. The teacher is in a state of transition in many of her emerging beliefs, suggesting that some of them are fragile.

While results of the study are promising, further research is recommended to examine long term effects of coaching with more teachers and coaches over several cycles.

## Acknowledgements

The completion of this dissertation is something of a miracle. I had many obstacles to overcome, some self-imposed. One and a half years ago, I told my advisor, Carolyn Maher, that I was finally giving in to the inevitable and dropping out of the program. She would have none of it. And she was right. What I have accomplished means a great deal to me. I thank her for helping to bring that fulfillment to me and for her essential support and enthusiasm.

My late mother, Marie Salant Neuberger, had an insatiable mind and would never settle for the obvious or popular answer. She also had impeccable ideals and integrity. I would like to think that she planted some seeds in me that continue to germinate. My remarkable father, Roy R. Neuberger, who is one hundred and six years old, has always encouraged me to follow my passion. I love and owe a great debt to him.

I never liked school, nor was I good at it until, later in life, I studied at Bank Street College of Education. Bank Street was a wonderful place for me, and elicited skills and insights that were long buried. When I got there I was already taken with the idea of helping math teachers, but Hal Melnick was instrumental in guiding me to develop my ideas.

Alice Alston contributed invaluable insights. Charlene Marchese connected me to the Melville School and helped in many ways. Of course, without the cooperation of the people who go by the names Angela, Matt, and Jane at the school, this study would have been impossible. Over the years, others have assisted me in countless ways, including Liz Uptegrove, Ralph Pantozzi, and the indispensable Marjory Palius.

Lastly I want to express my profound love and gratitude to my wife, Helen Stambler Neuberger, who was always encouraging despite my resistance (and is a terrific editor); and to two old and very dear friends, the late Denis Berger, who was exuberant, insistent and persistent in his unique way; and the brilliant, supportive Mark Jacobs.

## Dedication

I would like to dedicate this dissertation to the late Robert B. Davis, a superb and beloved person. He was extremely smart, but always modest. He was seemingly curious about everything. He was very generous to me. I adored him.

I first considered coming to Rutgers after reading an article written by Bob (and Carolyn Maher). Bob was brilliant, but he always wanted to hear what I was thinking. That was his way. He died suddenly and much too soon, after I had known him for only three years. But his sense of curiosity, childlike in many ways, animates this program at Rutgers. I am grateful that I was able to spend as much time with him as I did, for he taught me much. For carrying on his spirit, Carolyn deserves special kudos.

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## Chapter 1 Introduction

Schoolchildren have an inadequate knowledge of mathematics in this country. The evidence for that statement is varied, from widespread self-reports of ignorance and anxiety about the subject, to the results on international tests (see, e.g., McKnight, et al., 1987) to reports on the unpreparedness of our workforce (Everybody Counts, 1989).

Many feel we can and must do much better (A Nation at Risk, 1983), especially with regard to teaching (Before It's Too Late, 2000).

In the last few decades we have learned a great deal about how students learn (see, e.g., Steencken and Maher, 2003) and about effective teaching practices (e.g., Hiebert, et al., 1997). Furthermore, we have articulated policy documents so that these pedagogical ideas would be incorporated into practice (e.g., Curriculum and Evaluation Standards for School Mathematics, 1989).

And yet, something is clearly still amiss. For the most part, knowledge about learning and teaching has not made its way into the classroom (Ball, 2001). Thus the problem becomes one of translating the theoretical changes to a deeper, more resistant, practical level. Can this be done? If so, how?

In the fall of 2003, the New York City public schools started implementing a program to support mathematics teachers deeply and on an ongoing basis. (A similar program for reading and writing was also implemented.) The schools instituted a system of universal coaching, whereby all teachers of math had an opportunity to work with coaches on a one-to-one basis to help them improve
their teaching. Relationships between coaches and schools were intended to last for an extended period, perhaps two to three years, but the system varied from region to region. ${ }^{1}$

Coaching is a particular type of professional development strategy whereby expert coaches "provide one-on-one learning opportunities for teachers focused on improving teaching practice ... [Coaching practices] incorporate a traditional supervisory model focused on classroom observations and use a preconference-observation-postconference cycle" (Loucks-Horsley, Hewson, Love, and Stiles, 1998, p. 126). The particular model for coaching used by the coach studied here is set forth in the book Content-Focused Coaching:

Transforming Mathematics Lessons (West and Staub, 2003).

This study examined this coaching and teaching system in one of the ten regions in New York City, focusing on one coach and one teacher. Its broad goal, of course, is to effectuate improvement in math teaching and learning. When effective methods for improving teaching are found, they must be documented and made available for the wider community. The narrower goal was and is to see what relationship there is, if any, between coaching and teaching. More specifically, what coaches and teachers can do to help teachers teach more effectively?

[^0]Subsequently, the coaching system was removed as a mandate for each school and decisions as to what type of professional development would be used in a school was largely left to the discretion of the principal. However, it has been reported that coaching is still widely employed in both mathematics and reading and writing.

### 1.1. Region 9

At the time of this study Region 9 was one of ten regions in the public school system, composed of approximately 180 schools (70 elementary, 40 middle, and 70 high schools) in Manhattan and the Bronx. To implement the coaching system there were 88 coaches, composed of three levels: five Regional Instructional Supervisors (RISs), ten lead coaches and 73 coaches. In turn each coach was assigned to two schools. Some high school assistant principals were also utilized as coaches. ${ }^{2}$

### 1.2. Statement of the Problem

In the year 2003, virtually all of the components of a high quality mathematics education system---policy documents, textbooks, manipulative materials, pedagogical ideas, assessment tools---were available, yet the system was not functioning as it was envisioned. It was becoming increasingly clear that these components, having been developed primarily by experts in the

[^1]mathematics education community, had not been internalized in a way that math teachers could fully utilize them..

Thus the problem was: How to transform high quality mathematics instruction from policy into practice? With the advent of the coaching regimen in New York City, an opportunity was presented to contribute to the solution of the problem.

The problem of low quality teaching was widespread and pervasive. This research was small-scaled, looking only at a case of coaching and teaching in a large, urban school system. Yet, by shining a light on the coaching and teaching process, it carries important potential lessons. If it can be shown that certain coaching practices help teachers improve their instructional practices, then it will serve as proof of the existence of a coaching-teaching link; it can pinpoint specific coaching practices that help teachers improve, and preserve them for others to emulate.

## Chapter 2 Research questions

This study addressed the following research questions.

1. What relationship, if any, is there between the teacher's beliefs and practices and the coaching process?
2. Which teacher beliefs and practices, if any, appeared to change as a consequence of the coaching?

## Chapter 3 Review of the Literature

### 3.1 Education Woes

Problems with our educational system are not new. They exist on many
levels. One problem concerns a lack of attention to children's' needs. Forty-two
years ago, John Holt observed in How Children Learn (1967):
Children have a style of learning that fits their condition, and which they use naturally and well until we train them out of it. We like to say that we send children to school to teach them to think. What we do, all too often, is to teach them to think badly, to give up a natural and powerful way of thinking in favor of a method that does not work well for them and that we rarely use ourselves (vii).

Holt went on to recognize the need to figure out how children learn, a
process that has made great strides in the intervening years, but is still ongoing:
When we better understand the ways, conditions, and spirit in which children do their best learning, and are able to make school into a place where they can use and improve the style of thinking and learning natural to them, we may be able to prevent much of the failure. School may then become a place in which all children grow, not just in size, not even in knowledge, but in curiosity, courage, confidence, independence, resourcefulness, resilience, patience, competence, and understanding (viii).

Twenty-six years ago a national spotlight was placed on the unfortunate state of our educational system and the need for reform by the report $A$ Nation at Risk (1983) and the famous words:

If an unfriendly foreign power had attempted to impose on America the mediocre educational performance that exists today, we might well have viewed it as an act of war. As it stands, we have allowed this to happen to ourselves (p.5).

### 3.2 Mathematics Reform and the NCTM Standards

Problems are particularly acute in mathematics. In 1989 the National Resource Council bemoaned the state of "underachievement" in mathematics education (Everybody Counts, p. 1). In 2000, the National Commission on Mathematics and Science Teaching for the $21^{\text {st }}$ Century reported: "In an age now driven by the relentless necessity of scientific and technological advance, the preparation our students receive in mathematics and science is, in a word, unacceptable (Before It's Too Late, p. 10)."

Results on international tests in the 1960s (Husen, 1967), the 1980s (McKnight et al., 1987), and the 1990s (Pursuing Excellence, 1996) served to reinforce and quantify this sense of failure. In 1989 the mathematics education community responded to this need with Curriculum and Evaluation Standards for School Mathematics (National Council of Teachers of Mathematics) setting forth a vision of K-12 mathematics and a blueprint for reform. That vision is described in the Introduction to a recent book, Beyond Classical Pedagogy (2001):
[The NCTM Standards] and the research that lay behind it created a vision of mathematics instruction that took seriously the fact that children construct their mathematical knowledge. In this view, teaching would no longer be a matter of viewing students' minds as blank slates and getting them to internalize the correct mathematics. Rather, the work of teaching would consist of developing instructional contexts in which students could move from their own, intuitive, mathematical understandings to those of conventional mathematics. The goal of instruction also changed. Rather than aiming for content mastery alone, the goal now included having students see mathematics as a sense-making activity (Nelson, Warfield, and Wood, 2001, pp. 6-7).

With the publication of the Standards, the thrust of the mathematics education community was largely channeled into writing or revising textbooks and assessments and drafting state and local policies, all to comport with the Standards. Pedagogical guides and courses were offered to teachers.

The Standards movement caused a number of reactions. It was endorsed by "virtually every professional mathematical science organization in the U.S" (Counting on You, 1991, p. 11). There was much excitement in many segments of the math education community and several notable successes (see, e.g., Fennema and Nelson, 1997; Fosnot, Dolk, and van den Heuvel, 2001). Many commissions or other bodies were formed and reports or books published.

And there was a backlash, especially from people who thought the Standards gave short shrift to "traditional instruction," "correct answers," and to the importance of memorizing basic facts (e.g., Addington and Roitman, 1996). Textbook publishers responded by writings books emphasizing traditional mathematics approaches, e.g., Saxon Math. Parents increasingly became concerned about whether their children were learning their math facts well enough and increasingly enrolled in after-school programs like Kumon Math. And advocacy groups of educators and others formed organizations to oppose or influence the reforms, e.g., Mathematically Correct.

Yet, it might prudently be asked, what was the impact of the Standards on the classroom? Was the theory truly being practiced? Indeed researchers found
that the Standards were very difficult to implement and so not being observed in the vast majority of classrooms. Ball (2001). There were several reasons why.

One putative reason is that teachers lacked sufficient knowledge of mathematics (Ma, 1999). Another was that teachers' deeply held beliefs and attitudes undermine their good intentions (Ball, 1990; Cohen, 1990). Furthermore, jurisdictions expected teachers to change, but failed to support teachers with sufficient training or resources (Cohen \& Ball, 2001).

For example, as Schorr, Firestone, and Monfils (2003) found, a state instituted a $4^{\text {th }}$ grade math test to challenge teachers to make fundamental revisions in their teaching. While teachers reported they had made changes, observers suggested the changes were minimal and adopted without having to alter their basic beliefs or practices.

Ball, Hill, and Bass (2005) felt that the "research and experience consistently reveal" that, in the process of reform, not enough attention was given to teachers (p. 14.) Ball (2001) summed up the minimal impact of the Standards on instruction:

Most evidence suggests that mathematics teaching that focuses on understanding as well as skill, that involves children in significant mathematical reasoning, and that strives for high standards of progress and accomplishment, is not commonplace. In most classrooms, skill practice still dominates. Teachers explain and children follow. Explanations are based on rules and steps more often than on mathematical structure and reasoning. And student learning continues to be low, distressingly stratified by race, gender, and class (pp. 11-12).

Recognizing an additional need, supporters of the Standards made some refinements. In 2000 NCTM published Principles and Standards for School

Mathematics (PSSM), a book that clarified some issues and streamlined the Standards. In PSSM, NCTM made clear that it had never intended to gloss over the importance of basic facts; instead it had wanted to stress its view that passive memorization is not sufficient, that students need to become actively engaged in thoughtful processes.

The other refinement was a much greater emphasis on the training of teachers, in terms of both research and implementation. The irony was clear: the academic community had said that students needed to make sense of their mathematical ideas in order to understand them, yet it had not given the same opportunities or respect to math teachers.

### 3.3 Professional Development

In 2000 the National Commission on Mathematics and Science Teaching for the 21st Century reported its number one goal was to "Establish an ongoing system to improve the quality of mathematics and science teaching in grades K12" (Before It's Too Late, 2000, p. 24). The report went on to explain:

If high-quality teaching is the leverage point for improving mathematics and science education, and if professional development is a prerequisite for a well-qualified and effective teaching force, then teachers need a focused support system and enough time to grow as professionals (p.24).

Researchers have heeded the call. "The cry for professional development has now become a roar" (Fosnot and Dolk, 2002, p. 146). Loucks-Horsley, et al. (1998) examined the issue of professional development in mathematics from a global perspective. They proposed a set of principles
that were common to effective professional development programs, including, in simplified form:

1. Defining an image of effective learning and teaching, including an emphasis on inquiry-based learning.
2. Providing opportunities for teachers to build their knowledge and skills, including knowledge of mathematics and pedagogical content.
3. Providing models of strategies teachers will use.
4. Building a learning community and promoting ongoing selfassessment.
5. And supporting teachers to become leaders (p. 36-37).

Loucks-Horsley, et al. went on to posit fifteen strategic approaches to professional development and their characteristics. One approach was the workshop. They made clear that, while the workshop had hitherto reigned as the dominant mode of professional learning, it was to be viewed now as only one strategy on a lengthy menu, since it may be more effective at helping a teacher build knowledge than at building a learning community or encouraging reflection. Examples of other approaches given were study groups, curriculum implementation, and professional networks.

Another approach expounded in Loucks-Horsley, coaching and mentoring, is the subject of Content-Focused Coaching: Transforming Mathematics Lessons by West and Staub (2003) and of this study.

Content-Focused Coaching sets forth a program whereby a coach meets with a teacher one-on-one on an ongoing basis to plan, observe and reflect on actual math lessons. The transformation in instruction the authors contemplate "reflects a profound change in the definition of teaching---from teaching as mechanically implementing curriculum to teaching as mindfully making use of curriculum" (p. 5). According to Content-Focused Coaching, two areas the coach would be looking at are how teachers design lessons and what are their needs, in terms of knowledge, beliefs, and skills.

With regard to lesson design, the book (p. 11) sets forth some "core issues" that the teacher should be thinking about when planning lessons. Among these are: What is the mathematics in the lesson? What are students' prior knowledge and potential difficulties? And how does the lesson help students reach the lesson goals?

Regarding teachers' needs, West and Staub state, in a section entitled "Diagnosing Teachers' Needs," that coaches need to elicit "what teachers know and can do and what they need to know and be able to do ... [and] the gap between the two" (p. 19).

Specifically coaches will assess the following teachers' needs (p. 19): content knowledge and disposition toward mathematics; pedagogical knowledge and underlying beliefs about learning; pedagogical content knowledge; diagnosis of children's thinking and assessing prior knowledge; and habits of planning and engagement with curriculum materials.

The coaching method described in Content-Focused Coaching is grounded in an actual lesson, planned, taught, and reflected on by the teacher. As such it borrows from lesson study, a system whereby a teacher strives to craft a lesson with help from the whole school community and indeed a sincere outreach to a larger community. Lesson study is very popular in Japan and is receiving increased attention in the United States (Lewis, 1995; see also, Stigler and Hiebert, 1999). Yet, it is not one of the strategies set forth in Loucks-Horsley, et al.

### 3.4 Instructional Practices

Over the years researchers have examined instructional practices in classrooms to see what insights they could acquire into the processes of learning and teaching. In 1967 Holt told an anecdote (pp. 147-148) of his experience of taking over a class of young children for a week from the regular teacher. It had been her habit to put up some addition sentences every day on the board for the class to complete. The sums were always less than twenty, usually less than ten. One day Holt forgot to put up any problems and the children asked him if they could do so. He said "yes." At first they wrote sentences of the same order; but soon they started writing ones with larger addends. These challenging problems stretched the students' thinking and led directly to new learning and long conversations that Holt felt certain would never have happened without his fortuitous forgetfulness.

Recently more systematic analysis has taken place. Stigler \& Hiebert (1999) compared mathematics teaching in the United States to that in Japan and Germany. They quoted from an unnamed mathematics educator who was a participant in their project:

In Japanese lessons, there is the mathematics on one hand, and the students on the other. The students engage with the mathematics, and the teacher mediates the relationship between the two. In Germany, there is the mathematics as well, but the teacher owns the mathematics and parcels it out to students as he sees fit, giving facts and explanation at just the right time. In U.S. lessons, there are the students and there is the teacher. I have trouble finding the mathematics; I just see interactions between students and teachers (pp. 25-26).

Stigler and Hiebert remarked that, "although perhaps oversimplified," this description "captured an important aspect" of American math teaching. Could the aspect they were alluding to be similar to what Holt felt in his anecdote: students were not being challenged by the mathematics?

Hiebert, et al. (1997) also emphasized how important it was that students be provoked and expected to think:

In problem-centered learning classrooms, students are presented with computation problems that are meaningful and interesting to them, but which they cannot solve with ease using routinized procedures or drilled responses. The teacher does not demonstrate a solution method, nor does the teacher indicate a preferred method, yet she or he expects every student to attempt to solve it. Students' own invented methods are expected and encouraged, It is expected of students to discuss, critique, explain, and when necessary justify their interpretations and solutions (p. 115).

This style of teaching necessarily places greater emphasis on the quality of discourse in the classroom. Stein, Smith, Henningsen, \& Silver (2000), in analyzing mathematical tasks, paid special attention to different levels of cognitive demand placed on learners, i.e., ranging from the low demand needed to recall a previously learned fact to the high demand of exploring mathematical concepts or processes. They went on to observe how lessons evolved, often declining in cognitive demand as they moved from an introductory to an implementation phase.

Perkins (1992) talked about how the "understanding" that most students attain is very fragile and superficial, even after considerable instruction in a subject. In 1993 he proposed that schools go about teaching in a completely different way.
[U]nderstanding a topic of study is a matter of being able to perform in a variety of thought-demanding ways with the topic, for instance to: explain, muster evidence, find examples, generalize, apply concepts, analogize, represent in a new way, and so on.

Skemp (1987) realized that people have two different meanings in mind when they use the word understanding. Relational understanding occurs when a person has a constellation of knowledge that constitutes both what to do and why. Instrumental understanding is more isolated, constituting rules without reasons or what, but not why. Schools must strive for relational understanding.

One path to relational understanding relies on the exploration of ideas. Martino \& Maher (1999) talk of the importance of justification and generalization and of the crucial role that teacher questioning plays in promoting them.

Generations ago John Dewey (1922) recognized the importance of debate and challenge:

Conflict is the gadfly of thought. It stirs us to observation and memory. It instigates invention. It shocks us out of sheep-like passivity, and sets us at noting and contriving ... conflict is a sine qua non of reflection and ingenuity (p. 275).

Reflecting on one's practice as a strategy for developing teacher thoughtfulness is gaining increased attention. Schoenfeld \& Kilpatrick (2008) cite Dewey as one of the first to recommend reflection in education, not just for perspective teachers, but for teachers throughout their careers. Dewey (1933) defined reflective thinking as:

Active, persistent, and careful consideration of any belief or supposed form of knowledge in the light of the grounds that support it and the further conclusion to which it tends constitutes reflective thought (p.9).

Mewborn (1996) elaborates: "In an educational context ... reflecting on teaching practice can assist teachers in making intelligent choices about future teaching actions (p. 14)." In a similar vein, Hiebert, Morris, Berk, \& Jansen (2007) propose a framework to help teachers learn from own teaching.

As another strategy that gained increased attention in the 1960s, manipulative materials are rather widely used, especially in elementary classrooms. Yet their use is not as common as some assume. (Adding it up, p. 45.) The lesson observed in the study described herein was one where students actually made materials that were to be used in future lessons. The NCTM Standards advocated the increased use of manipulatives as useful concrete aids to math learning. On the other hand, many commentators warn against them
being used passively by students. Ball (1992) warned: "My main concern about the enormous faith in the power of manipulatives, in their almost magical ability to enlighten, is that it will be misled into thinking that mathematical knowledge will automatically arise from their use (p. 18)."

Nührenborger \& Steinbring (2008) voice a similar concern:
A manipulative material, in order to reap that potential must be utilized at several levels by students. It should be used as a tool certainly, but its significance and structural character should also be pushed. They emphasize how difficult this goals can be for a teacher (p.158).

### 3.5 Knowledge Base for Teaching

What do math teachers need to know in order to teach effectively? The answer to this question has for some time been undergoing change, as researchers have achieved various insights.

To investigate what preparation teachers had for teaching, Shulman (1986) compared teacher-entry examinations from two different eras. A review of teacher certification materials from the 1870s showed a high level of breadth and depth was required. By contrast in the 1980s exams typically contained almost no mathematics and much pedagogy. Shulman judged that while in fact, both content and pedagogy are necessary for teaching, a third kind of knowledge is also needed: curricular. He described curricular knowledge as the practical repertory on which teachers draw to present content or to remediate or evaluate students' ideas; he named it "pedagogical content knowledge."

Thus pedagogical content knowledge includes familiarity with mathematics textbooks and other writings, as well as with tools, such as calculators, computers, and manipulable materials, e.g., Unifix cubes, Geoboards. Effective teachers need to know when and how to use these resources and materials to help students develop understanding of mathematical ideas. Thus they should know when and how to introduce challenging and appropriate content, foster student exploration, elicit understandings and misunderstandings, and assess learning.

Ball (2000) was also concerned with pedagogical content knowledge. She noted how subject matter and pedagogy had been historically divided into separate realms and in many ways still were; the division was reflected, for example, in the chasm common at many universities between university math and math education departments. The burden of integrating the two disciplines was often left to K-12 teachers. Ball invited researchers to help teachers understand this knowledge that matters for teaching.

Teachers' beliefs can certainly have a profound effect on their practice. Forgasz and Leder (2008) "trace the growing research in teachers' beliefs and describe the multiple ways the term belief is used (p.173)." They warn that researchers should not ignore the role of beliefs.

Raymond (1997) found that a teacher could be greatly influenced in her teaching by her own childhood experiences as a student even years later.

A similar claim could be made about attitudes. Why is there is so little attention to teacher attitudes in the research? This researcher maintains that teachers' attitudes are not the same as beliefs, but can have a profound effect on beliefs and practices.

## Chapter 4 Methodology

This study was conducted as a case study of a coach, Matt, and a teacher, Angela, working together on an ongoing basis in a New York City public elementary school. It was intended to examine coaching and teaching as a system in schools where the Content-Focused Coaching model could be implemented.

Professional or expert math coaches, at least in the New York City program, were generally mathematics teachers who had decided they wanted to become mentors or other types of administrators. Often they got master degrees in math leadership.
"Expert coaching" is a particular type of professional development strategy whereby expert coaches "provide one-on-one learning opportunities for teachers focused on improving teaching practice ... [Coaching practices] incorporate a traditional supervisory model focused on classroom observations and use a preconference-observation-postconference cycle" (Loucks-Horsley, Hewson, Love, and Stiles, 1998, p. 126). The particular model for coaching used in this study is set forth in the book Content-Focused Coaching: Transforming Mathematics Lessons (West and Staub, 2003).

Several types of data were collected: videotapes of two sessions, a lesson and a debriefing session, and audiotapes of five interviews (two with the teacher and coach, and one with the school principal). In chronological order, the following cycle of data were collected. (All sessions were observed by the researcher and he conducted all interviews. Two sessions, the lesson and the
debriefing, were videotaped. Transcripts of all sessions are attached hereto in the Appendix, but not in chronological order.)

- April 28th, 2004. The principal of the school was interviewed.
- March $3^{\text {rd }}$. A lesson was taught by Angela and observed by the coach.
- March $3^{\text {rd }}$. The coach was interviewed for the first time.
- March $4^{\text {th }}$. The teacher and coach met to debrief the previous day's lesson
- March $10^{\text {th }}$. The teacher was interviewed for the first time. ${ }^{1}$
- June $17^{\text {th }}$. The coach was interviewed for the second time. ${ }^{2}$
- June $28^{\text {th }}$ The teacher was interviewed for the second time.

The researcher spent several hours talking with the teacher and observing classes in the teacher's classroom before the cycle of study began in order to establish rapport.

After data were collected, they were analyzed according to some of the ideas described in Powell, Francisco, \& Maher (2003). All data were transcribed and narratives of the lesson and debriefing session were developed. Some of the "ground rules" observed in the study are set forth now.

[^2]- Camera. When the lesson was videotaped, one camera was used and it was largely trained on the teacher, and not on students.
- Interviews with teacher. In the first interview with the teacher, the researcher attempted to elicit the teacher's training and experience, beliefs and attitudes about math and teaching, books and materials used, expectations of the coaching, perceived strengths and weaknesses, and problems or issues that she wanted help with. In the second interview, the teacher's reaction to the lesson and to the coaching process were be probed, specifically, any surprises or practices that might have been influenced by the coaching process.
- Interviews with coach. The coach was interviewed as to his experiences, attitudes and beliefs about math, expectations of the coaching, assessment of the teacher's strengths and weaknesses, problems or issues that he wanted to help the teacher with, reactions to the lesson and to the coaching process, and any perceived effects of the coaching on the teacher's practice. [Questions from a doctoral dissertation (Landis, 1990) were utilized in the interviews and are set forth in the Appendix.] All of the transcribers, i.e., of the interviews, lesson, debriefing, were instructed by the researcher to pick up nuances in speech, e.g., hesitations, pauses, repetitions, etc. in order to convey a more
accurate sense of the speakers patterns, thoughtfulness, sense of certainty, etc.
- Analysis of lesson. In examining the teacher's instructional practices, the study looked specifically for many of the same things the coach was looking for, e.g., how tasks were introduced, the way questions were asked and answered, with particular regard to how inquiry, discourse, and reflections were engendered, and how student ideas were elicited.
- Analysis of coach's process. The research looked at what coaches did to effect teacher development. What did the coach ask or tell the teacher? What kinds of things did he emphasize?
- Finally evidence of teacher development was sought, through direct observation, or indirectly from the teacher or coach referring to modifications or changes in beliefs, attitudes, practices, or knowledge.

It is well to keep in mind that this research used a case study approach, in which a broad proposition was investigated by looking at a single case. Obviously the vast majority of coach-teacher sessions or classroom lessons were not observed. A judgment is made on the extent to which the case does or does not represent the broader picture and the meaning of that will be argued. Whether or not this case is deemed to be typical it should give considerable insight into the relationship between coaching and teaching.

The theoretical model for this study is ethnography, where the observer attempted, as far as is practicable, not to affect the settings (i.e., the lessons and the coaching sessions) so that he viewed them as if he had not been present. If called upon to interact with the participants, the researcher attempted not unduly to influence the events he observed. Thus the observer was trying to answer two basic questions in relation to the research questions: What was happening? What did it mean?

## Chapter 5 Results

This chapter focuses on what happened during the course of the study. These results are set forth objectively. Each data set, i.e., the interviews, the lesson, and the debriefing, is summarized separately. If more specificity is desired, the transcripts of each are set forth in the Appendix. A number of photographs are copied from the lesson and the debriefing and are reproduced in those transcripts. Some photos are also reproduced in this chapter. (References to line numbers in transcripts, which are located in the Appendix, will be indicated by a code: TI\#1 refers to the first teacher interview and will be followed by line numbers; TI\#2 refers to the second teacher interview; codes referring to the coach and principal interviews will be: CI\#1, CI\#2, and PI; L will refer to the lesson and D to the debriefing.)

### 5.1. Summary of Interview with School Principal

Having learned that Jane, the principal of the Melville School, was interested and involved with the math coaching regimen in the school, the researcher determined that he should interview her, which was done on 4/28/04. This summary of the interview is divided into 3 sections: the history of the school, the school now, and the math program and the coaching process.

### 5.1.1. The history of the Melville School.

The Melville School was started almost 20 years ago by a director, 3 teachers, and supportive parents. At the time, there was one progressive school
in the neighborhood and demand for a second one. Originally, it was designated by the region as a "gifted and talented" school, but when it started the teachers and director wanted it to be "open to everybody," and so it soon became open. It started as pre-K to $1^{\text {st }}$ grade, with 3 classroom teachers and a cluster teacher, based on a "progressive model of education ... group-based, core curriculum, and kids engaged in real, important, authentic work." Jane was a pre-K/K teacher. [PI, 6-16.]

A grade was added each year until it reached its present configuration, pre-K to $6^{\text {th }}$ grade. From the beginning the classes had mixed age groupings, students did narrative writing and portfolios were kept for each student. The core curriculum was based on social studies or science. "At every point every year we talk about what various things mean, how we report to parents, how we have family conferences, what should go in a portfolio, but the foundation of our practice is descriptive work, looking at the individual children and seeing what comes next for those kids, and also, how to make something out of that for the whole group." [PI, 24-33.]

### 5.1.2. The Melville School now.

The Melville School in 2004 had 242 students in 11 mixed-age classrooms (3 pre-K/K, $31^{\text {st }} / 2^{\text {nd }}, 33^{\text {rd }} / 4^{\text {th }}$, and $25^{\text {th }} / 6^{\text {th }}$ ). All kids had music and art once a week; upper grades had theater where they wrote and acted out their own plays. There were 32 or 33 adults who worked in the school. Slightly more than a
quarter of the kids were Latino, the same with white, slightly less than a quarter were black, and 17\% Asian. [PI, 60-66, 87-89.]

A "key" point is that
social and academic work first go hand in hand ... and we look at what kind of adults we want the kid to be and our practice follows from that. That we want them to be full participants in a democratic society, to be able to critique what's going on, and work productively toward that. To be able to work as individuals" ... as members of a group, because you can't always be doing your own work, you have to work with other people as an adult, and we want kids to be engaged .. do good things. Be fulfilled people. Those goals are "explicitly stated in the mission statement in our parent handbook. [PI, 37-46.]

### 5.1.3. The math program and the coaching process.

When Jane, who had left the school for several years, returned to become its principal, the math program "wasn't coherent," so the "first thing I did when I came back as director was to ...build coherence, so I introduced her [she is probably referring to Constance Kamii, but she also referred to Marilyn Burns and Math Your Way] to the staff and we started having meetings about different math strands and how to build that ..." [PI, 102-109.]

Jane saw her role as providing for good professional development, by bringing in educators, by having teachers and parents go to classes, and by working with teachers to "build curriculum, making sure that teachers have materials need really try to look at kids' mathematical thinking and understand it better ... observe lessons and working with teachers on pedagogy ..." [PI, 121125.]

Jane liked to attend meetings between math coaches and teachers, "partly because I like to learn, partly because it's really important and that signifies that it's really important ... Also I don't see myself in opposition to the teachers, but working alongside the teachers ... " She attended all the coaching meetings for the first couple of months because she wanted "the process to be set up well ... To protect the teachers and ... the coach," but now attends them less often. [PI, 153-164.]

She also met with the coach. "The coach can really elevate the practice or he can ruin (she laughs) the school, either one. ... I certainly didn't want it to be a waste of anybody's time." [PI,173-176.]

In the past the professional development had not been consistent, but now it was more so, although "for different grade levels it's been a different experience." "I think ultimately it will be better for the school to have the coaching." [PI, 184-188.]

As for specifics, Jane noticed a literacy lesson where she thought that the teacher got an idea from the coach. She has noticed " some grade levels are much more meticulous planning their curriculum," and how the classes repeat patterns from year to year but on a different level. [PI, 189-196.]

Jane considered herself a "baaaad mathematician but a very good math teacher." She is much better at estimating numbers than she used to be. In second grade she didn't understand math; that continued in middle school, so "I
memorized the algorithms and did fine but not really understanding what it was I was doing." [PI, 205-223.]

### 5.2. Summary of Teacher Interviews

The teacher, Angela, taught a mixed $3^{\text {rd }}$ and $4^{\text {th }}$ grade class at Melville. She was interviewed two times, once on $5 / 10$ and once on $6 / 28 / 04$, about a week after her class was observed. This summary focuses on three aspects: first, what her background is; second, what she wanted help with from the coach; and third, what were her impressions of coaching and its effect.

### 5.2.1. The teacher's background.

As a student in school, Angela loved art and felt herself to be a "very bad, very poor math student." [TI\#1, 5-9, 11.] She was in a remedial or slow math class, that was taught by the only good math teacher she had (until education graduate school) [TI\#1, 12-14]. She was good, because she was nice, kind, caring, warm, and laid math out very clearly. [TI\#1, 19-21.] Another math teacher, on the other hand, was a "really angry, mean, drunk guy ... and would come in with a hangover every day and just scream at everybody." Her experience was that, "if you didn't get it ... that was the end of it." [TI\#1, 29-30.]

In graduate school math methods, basically how to teach math to kids, it was "very fun," "lots of hands-on work." [TI\#1, 31-41.]

Angela talked about her experience with children. She baby-sat a lot as a young person and just loved children. For one day she helped a friend of hers, an artist, in a school she was working in, and just "loved being with the kids." Thus
teaching " sort of found me in a weird way." Angela has children of her own. [TI\#1, 48-53, 72-73.]

She "wants kids to think of math as fun, because it is for me now, and it never was." [TI\#1, 95-96.]

As an undergraduate, Angela took no math classes. [TI\#1, 37.] She then did "pre-engineering work ... where I had to take a lot more math." She found it "very hard and very painful." [TI\#1, 14-15.]

She observed that her "Dad is an incredible mathematician, so is my brother, so it was really an odd thing that I didn't ..." This thought was not concluded, but one can infer that Angela wonders why she was not better at math or enjoyed it more. [TI\#1, 16-17.]

While Angela was getting her master's degree in education, she started substitute teaching at the Melville School, where she now teaches. "Never knew schools like this existed. I had done my student teaching placements in very traditional schools and I completely fell in love with the school. I thought, 'This is how kids have to learn.' " [T1\#1, 59-63.]

She started teaching first and second grade for two or three years, and then taught a combined 2/3 class for a year. Then she started teaching a combined 3rd and 4th grade class, and she's been doing that for five or six years. So she's been at Melville for 10 or 11 years. She also took three years off, two when she had her children. [TI\#1, 63-69.]

Previously Angela said she loved art in school and that it was her favorite subject. "I like to do things with my hands, I like to make stuff, and that helps me learn..." [TI\#2, 302-303.]

### 5.2.2. What the teacher wanted help with from the coach.

Angela felt that her weakness generally was in mathematical knowledge, more than in pedagogical knowledge, as she felt she was an experienced teacher. [TI\#1, 152-3.]

She also expressed admiration for the mathematical prowess of others, including her father, brother, and the other teachers in her grade [T1, 16-19, 114116.].

According to Angela, the coach emphasized with her that she look more closely at student work; she finds that this focus has spread into her other subjects and has been "really nice." [TI\#1, 154-6.] As part of this process, she has learned how to place students into groups in a way that is more meaningful than previously. [TI\#1, 161-5.]

One thing has changed in particular in her practice: "I try to do math all the time." [TI\#1, 169.]

Also, she said that she tried to insure that students share their work with each other, make it public.

I feel like it's important to get kids to share their thinking, and to make that public, which I think Matt helped me do. Um... Because it's shown me that that's how they're making sense of what they're doing. [TI\#1, 170-3.]

Angela reported that one practice that she often asked the coach to do for her was the wrap-up, and she found it "really helpful" that he has done that. "... I feel like that's the place where I could push the kids' thinking more and I feel like that's where I really need his help ,,," [T<br>\#1,147-149.]

In the second teacher interview, she said she lamented that what she modeled for the second lesson ${ }^{1}$ ended up being what almost all the students did and she wanted much more variation. [TI\#2, 231-4.] If she did this exercise again, she would model several ways. [TI\#2, 235.]

She also talked about the first lesson. She thought that lesson had gone really well, that students had followed instructions well, that they had been clear. [TI\#2, 265-73.] She recalled that the fraction cards fashioned in that lesson had been used in subsequent lessons. [TI\#2, 275.] She said she would not teach it differently if she did it again. [TI\#2, 277-8.]

### 5.2.3. The teacher's impressions of coaching and its effect.

Matt started working with Angela in February, 2004, about two months before the study began. (Before then he had been working at Melville, but with younger grades.) [TI\#1, 120, 122.] Even before then, she had formed a collegial bond with her two grade-level colleagues, whom she felt to be strong math teachers, and felt free to ask them questions. [TI\#1, 100-116.]

She reported feeling the coach was "really helpful," and that she had "really gotten a lot out of it." She considered some of his suggestions to be "just

[^3]amazing." [TI\#1, 135-140.] In particular, he helped her to look more closely at students' work, to find more meaningful ways of putting students into groups, and to emphasize that students make their thinking public. [TI\#1, 155-156, 161-165, 170-175.] She appreciated the "immediate feedback" of the coaching process, compared to other forms of professional development." [TI\#1, 186-194.]

Angela mentions that she is now willing to try her own ideas, as if by intuition or instinct and she credits Matt for this. [TI\#1, 145-146.] And, for the first time in her life she thinks math is fun and she holds this as a goal for her students: "I want kids to think of math as fun, because it is for me now, and it never was." [TI\#1, 95-96.]

### 5.3. Summary of Coach Interviews

The coach, Matt, was interviewed two times, on $5 / 3 / 04$ and on $6 / 17 / 04$, on both occasions right after lessons. ${ }^{2}$ This summary focuses on three aspects: first, what his background was; second what he wanted to and did work on in his coaching sessions with Angela; and thirdly, he talked more generally about the Melville School and the coaching process.]

### 5.3.1. The coach's background.

Matt, the coach, always loved math. "I always expected to make sense of math, and I gave myself permission from a very early age to expect it to make sense and when teachers were talking about math in ways that didn't make sense, I kind of went the extra step to, uh, try to figure out ..." [CI\#1, 7-15.] In

[^4]fourth grade, he tutored "other kids in the class cause I felt like adults have this one language for talking about math and that I was able to really easily translate between what the adults were saying and what, what it really meant." [CI\#1, 1619.]

Matt remembered an incident from 3rd grade where he was excited about his solution to a problem, perhaps 36 times 4 . He wrote 36 four times and added it up. Thrilled with his discovery, he told the teacher; she said that was not how we do it here. So, he felt she had been very rigid. Generally he was not encouraged by his elementary school teachers. As to the source for his feeling of enjoyment and empowerment, he points to his mother. "She was always about making sense and giving yourself permission to make sense about following the rules per se." [CI\#1, 23-34.]

Matt's mother was an artist, a rebel. She left the United States for Mexico when she was 18 , after getting kicked out of college for smoking pot. She got her bachelors and masters degrees in fine arts in Mexico. "She's always been a really independent spirit, creative, honest, hard-working ..." She had 2 children, got divorced and moved back to the U.S. with her children. So Matt was born in Mexico and moved to the U.S. at the age of 4. [CI\#1, 37-45, 174-175.]

Matt's mother was not especially good in math - he remembers fighting with her about his homework - and didn't especially encourage him to reason about it. Thus, while Matt's mother didn't encourage him to reason about math in
particular, she "encouraged reasoning about life in general." [CI\#1, 46-53, 51, 5657.]

Matt always knew he was a good math student and that feeling was reinforced by his good results on standardized tests. He calls that "kind of crazy" because apparently he does not "feel that way" about tests now. [CI\#1, 63-70.]

An important experience for Matt in middle school was joining a math team, which meant that he spent time with six or eight other students working on problems - "an alternative curriculum." (He did not like his "boring" math teacher.) He loved the:
workshop-y feel in the sense that everybody would go off and work on their own and then we'd come back together and share how we did it and invariably someone else's way of thinking about it was different from mine and would make me think about it in a different way ... (I)t really brought out the creativity behind mathematical thinking ... a really important experience for me." [CI\#1, 83-89.]

Matt studied economics in college. He felt he had an "intuitive grasp of the relationships beyond the various formulas." He did not take much advanced math. In graduate school he studied a lot of math education. He also took a summer institute. He got a masters degree in technology education, then a second masters in math leadership. [CI\#1, 108, 112, 131-145, 147-157.]

Matt was a 4th grade teacher for 3 years at a large, overcrowded public school in a school with mostly bilingual (Spanish-speaking) kids in Washington Heights. He spoke Spanish, having lived in Mexico until he was 4, but had to
study it again in high school and college. He taught 5th grade at the same school for a year. [CI\#1, 170-177.]

This was a SURR school (Schools Under Registration Review) ${ }^{3}$, which enabled him to participate in "Math in the City" at a local college. The practices he was learning there "started to clash with the pacing calendars and rigid curriculum mandates" at his school. He calls it "mindless work" where "test prep was a curriculum area ... valued above science, above social studies, sometimes above math." Accordingly, he "started to reshape my practice, but I clash too much with administration." While chairing a math/science restructuring committee, "my efforts to try and organize other people around thoughtful practice weren't, I wouldn't say they were badly received, but they weren't supported, they were permitted, but not supported." [CI\#1, 177-193.]

Leaving that school, Matt taught 3 years at a small progressive public school in East Harlem. He also participated in an "institute on descriptive inquiry" and some reading and writing projects at Teachers College. [CI\#1, 195-201.]

He then was invited to be a coach in District 3, where he worked in anywhere from 3-5 schools for 4 years as a math staff developer. [CI\#1, 205216.]

[^5]
### 5.3.2. What the coach wanted to work on in coaching generally and with this teacher.

The coach, Matt, with all his experience in schools and with teaching, believed that teaching was "really hard" and wanted to make "life a little better" for teachers, so that they could make the "best decisions they can." [CI\#1, 253263.]

Matt remembered a workshop for teachers and coaches where several teachers from the same school attended together. He thought it was really good that they came "in teams," so they could implement ideas together and develop a community in their schools. This approach also encouraged teachers to look at the "big ideas" in the math units. [CI\#1, 343-352.]

Matt identified Angela's biggest concern as planning and sequencing. Thus in their debriefing sessions, he tried to lay out a few weeks of lessons. [CI\#2, 394-397.]

He identified an issue that he wanted Angela to work on: managing groups during class. Previously he had noticed that she tended to call on a few kids for answers, while the needs of the majority of kids were not met. In this he wanted her to develop more of a "community" in the class. [CI\#2, 397-404.] Related closely to this, he wanted the teacher to think less in terms of answers as either right or wrong, but as part of a thinking process. [CI\#2, 405-409.]

He also wanted to help the teacher become less self-conscious and to develop more confidence. [CI\#2, 414-416.]

Turning to the question of whether Matt had noticed any changes in his work in Angela's class, he said "definitely." [CI\#2, 420.] For example, he cited a management mistake she had made in that day's class: she had distributed materials to individuals herself, thereby inhibiting her ability to observe children's behavior. However, she had noticed her own mistake before he pointed it out; this self-awareness impressed him. [CI\#2, 425-429.]

Moving on to discuss class wrap-up, an issue the teacher had cited, the coach said the teacher still wanted him to do it, and he indeed did it, but that he asked her to "imagine what it would look like." She responded that she would want students to explain and justify their thinking. [CI\#2, 431-440.]

As to Angela's issue of planning, Matt felt that she had made progress too. When he first started working with her, she had activities "to keep them busy," but "she wasn't able to articulate what her mathematical goals were." Now, she is much better able to articulate the goals. [CI\#2, 458-468.]

Matt had other concerns in his observation of the 6/17 lesson (not included in this study.) He noticed that almost half of the students had made a quilt design that looked just like the one the teacher had modeled for them. He wanted more divergence. [CI\#2, 481-483.] Addditionally, he noticed that students were not explaining their work. Therefore when he did the wrap-up he focused on reasoning. [CI\#2, 483-488, 496-499.]

Matt was asked whether he thought that his coaching had had an impact on Angela's teaching, particularly in the 6/17 class. He said, "I think so," but went
on to say how complicated an issue that is, and to his belief that progress could better be seen over a longer time period. The improvements he saw immediately he viewed as shallower and not so meaningful. [CI\#2, 500-517.]

In comparing different models of professional development, he viewed other types of professional development models as focusing more on the short term than coaching, and "that's not always helpful to teachers." [CI\#2, 521-524.]

As an example of just that point, he talked about the issue of Angela's need to develop confidence.

Like I feel like confidence was a big issue for Angela, and I couldn't have helped her develop her confidence if I were too aggressive, you know? And I feel like I cover a lot of ground with her. If I were any more aggressive I think I would scare her off ... [CI\#2, 524-527.]

### 5.3.3. The school and coaching

Matt talked about the Melville School and its principal, Jane. Among other things, he valued the schools role in helping to build a mathematical community [PI, 564]. He noted, "nine out every ten days I'm not working with that group of teachers," and emphasized therefore how important it was that teachers view each other as valuable resources." He cited this sense of community among the teachers as "probably the single most important thing that I think has shifted this year." [CI\#2, 569-575.] In contrast, Matt said that in many years of working in schools, there were many exemplary aspects at Melville. [CI\#2, 587-604.] Matt
expected to coach at Melville for two to three years, although he did not state that in his interviews. ${ }^{4}$

Finally, it was noted that Matt coached at two different schools that year. He was asked to compare them. The Melville School, he said, had "stability" in its staff and was organized in grade teams, which helped to foster a community. [CI\#2, 612-616.] The other school has "opposite characteristics." For example, he worked there more with individual teachers than with groups. [CI\#2, 616-618, 632-634.] Interestingly, the other school agreed to form "partnerships" between teachers for the following year. [CI\#2, 635-642.]

### 5.4. The Lesson

On May $3^{\text {rd }}, 2004$, the teacher, Angela, taught a class to her $3^{\text {rd }}$ and $4^{\text {th }}$ graders that was captured on videotape and observed by the coach, Matt, and the researcher. The goal of the class was for all students to make a set of fraction cards out of colored paper and subsequently to be used in various activities. This lesson was based on a unit from "Investigation in Date, Number, and Space" (see teacher page "How to Make Fraction Cards" in Appendix I). ${ }^{5}$

[^6]Several themes can be gleaned after studying the video and transcript. The details and evidence for these themes will be set forth in the sections that follow.

First, students were active for the bulk of the class, moving around, using materials and tools to fashion products. Second, in the course of this activity, students were sometimes thoughtful, reasoning about the process they were undertaking and even explaining it. But sometimes they acted in a rote manner, following the teacher's directions without apparent engagement. Third, a sense of community was established in the classroom, with students often talking to, listening to, and helping each other. (Note: When reference is made to, or a quotation taken from the lesson, the line numbers are preceded by 'L,' e.g., "[L, 152-155.]" This code refers to those lines from the lesson transcript, which is located in Appendix F. Angela and Matt are referred to, either as $A$ and $M$, or as teacher and coach, T and C, respectively.)

### 5.4.1. Students were active during part of the lesson.

The class lasted 45 minutes. There was a two-minute introduction and a nine-minute wrap-up; in between, for 35 minutes, students were seated at their desks (but were permitted to stand and walk around). During this long interval, there were periods when Angela was speaking to the whole class, e.g., when she was assigning a new task, periods when she was speaking to individual students or the coach, and periods when she was observing. Meanwhile students were seated at their clustered desks. They would listen to the teacher's instructions,
carry out the given task, and talk to their neighbors. Thus the students were both mental and physical; mental in carrying out instructions, physical in folding, creasing, and cutting pieces of paper.

In the course of the lesson each student was given the materials to make a set of five fraction cards to represent, in this order, halves, fourths, eighths, thirds, and sixths. Students started with five sheets of colored paper, approximately $81 / 2^{\prime \prime} \times 11^{\prime \prime}$. They would then fold them according to the teacher's instructions, put a crease or creases in them, and cut them with scissors. With eighths, for example, at the end of the process there would be eight pieces for each student. Students would then mark each piece so as to show the appropriate fraction name and their initials.

The evidence of activity on the part of students is not so much measured in their words, nor accordingly in the transcript, but in the video, or in the descriptions made by the researcher (and shown in brackets in the transcript). S Students are shown folding, creasing and cutting paper and writing fractional names on the pieces. [See Figure 1, below, for example. L288 in transcript.]


Figure 1: Student folding, creasing, and cutting paper.

Sometimes student work is described by the researcher: "[T is writing on paper, holding it up. Each folded portion says " $1 / 4$ " on one side and "fourth" on the other.]" or "[T holds up a new piece of blue paper, then folds it. She folds paper again.]" [L, 95-96, 81-82.]

### 5.4.2. Teacher often directive in describing tasks.

During the lesson students had five tasks to perform: to make cards from five pieces of paper to represent fractions. What path did they follow in carrying out their tasks? Were they thoughtful or just following directions? Were they all doing the same thing? Were they working as individuals or were they getting ideas from each other? Were they sharing ideas with the whole class? In fact, some of all of the above behaviors occurred; examples will be shown below.

As soon as the students were given the materials and were seated at their desks, the teacher started (at L34) giving them specific and detailed instructions, which entailed emulating just what she modeled for them.

Okay. All you need right now is one piece of paper in your hands. No scissors. Nothing but the paper. So if you're holding the scissors in your hand right now, you need to put it down.... All right. The first piece of paper we're going to cut in half. So we're going to very carefully ... can you watch first? Can everybody watch first? Can you guys see over here? Table 1? All right. Make sure that your two sides of the paper match up. So that you get a perfect half. And then what I'd like to do ... is I put it on the table, and I use the side of my thumb, and I make a nice crease. Okay? Get it as close as you can. And then that's going to make a guideline for you. Because then you're just going to go ahead and cut it. All right? So once you feel like you've got two really nice, even halves, you're going to go ahead and very carefully cut it. [L, 34-36, 38-45.]

When a student asked "Can we do it like any shape?,"she replied "No, you may not. You may do it like I just showed you." [L, 47-48.]

Next she instructed students as to what to write on the front and back of their newly-cut halves:

As soon as you're finished cutting, you're going to take your pencil, and you're going to write one over two on both sides. ... And then, on the back of those, what do you think you're going to do? ... You're going to actually write the word for what the piece is. So this is what? ...These are two halves. And then very lightly, somewhere on the back, where the words are, put your initials. [L, 49-51, 62-63, 65-66.]

The second task, making cards for fourths, was started out in the same way, but then the teacher asked students to imagine or predict how they should proceed.

T: The next piece...Is everybody ready to do the next piece?... You're going to fold it in half. So do what you just did, exactly the same. Do that all over again. So do the same thing all over again. And then, I'm going to fold this piece in half again. Raise your hand if you think you know what's going to happen. Just four people? Just five people? I folded it in half. Then I folded it in half again. So, Donald, what do you think's going to happen?
Student: Four. There's going to be four. [L, 75, 77-81.]

In the third task (starting at L122), students folded the paper per strict instructions from the teacher, but the fraction they were representing, eighths, was not named by the teacher. "Then fold it in two quarters. And now if I fold it again, which gets a little trickier because the paper's getting thick, we have to do this really carefully." [L, 124-125.]

When a student said "Now there's eight," [L127] the teacher asked "Why do you think it's an eighth, Jonathan?" [L130.] The student responded "I don't know. I just do." [L131.] The teacher probed him: "What were you doing that made you think it was an eighth?" [L133.] After an inaudible comment by another student, the teacher said "Barbara said it's like doing 4+4. We're doubling it. Well let's see. Open it up, and see how many ... How many pieces do you have?" [L, 142-143.] Then the teacher instructed students to write "one-eighth" on the front of all eight pieces and "eighth" on the back [L, 145-147].

In this episode three different terms were used for the fractional part: eight, eighth, and one-eighth.

While students were working on task 3 , the teacher raised the potential of divergent pathways for paper folding in a conversation with the coach: "There were two different ways to do it. [Inaudible.] Have them do it two different ways. ... Cause they can fold it the long skinny way, or they can fold it ..." The coach encouraged her by saying "That's a good idea." [L, 154-156.] He also pointed out to her that "you were leading them." [L, 165.] Interesingly perhaps, the teacher seems to exert less control and the divergence emerges more clearly in the next tasks, when students make thirds and sixths.

The first three tasks involved folding paper into halves. Tasks 4 and 5 (starting at L203 and L250, respectively), comprised folding the paper into thirds and the teacher warned that task 3 is tricky, but gave fewer explicit directions:

T: For this piece, we're going to make thirds. Larry, please don't fold it yet, because this is really tricky. And that's going to confuse your cut.

Everybody needs to look up here. To make thirds, it's sort of tricky. You need to ... not make a crease yet. You need to sort of scooch it around like a cylinder. And then when you feel like it's even and I can't tell you an easier way to do it, other than you just have to sort of eyeball it, and see when your edges ... And then you can make your crease, once you feel like you've got three equal sides. [ $T$ folded a piece of blue paper into thirds and started to "scooch" it without creasing it. See Figure 2, below, L 323.] [L, 204-213.]


Figure 2: Teacher folding paper into thirds.

In task 5, when students made sixths, the teacher instructed students to fold their papers in thirds as in task 4, and then to fold the resulting thirds in half. [L, 252-255.]

About 23 minutes into the class, after the teacher introduced the fifth and final task, her role in the lesson appeared to change. She largely pulled back from giving directions and started to walk around the classroom, look at the work of students, and occasionally talk to students or the coach.

The lesson revealed the extent to which the teacher dominated the class conversation. In the 20 minutes when students were working independently of her, until the point when the wrap-up began, she spoke more than $90 \%$ of the words, 1442 words to 142 words for students. Her words were also clearer or
louder, as judged by this fact: more of the inaudible words or phrases (7) were spoken by students than by her (2). ${ }^{6}$

After the students worked independently and made their fraction cards, there was a wrap-up session, where students were gathered together in the front of the room, The wrap-up session was led by the coach and he utilized a white board, while the teacher watched. Several times he encouraged students to experiment and to figure things out by using their own sensory abilities.

In trying to fold papers into sixths one student had made a shape where the sides did not line up evenly and looked very different from the others. The coach showed the class a folded green paper (see Figure 3, below), and simultaneously asked "So do you think that these two shapes that he ends up with are even?" [L, 520.]


Figure 3: Coach's recreation of Zolu'a green paper folded without right angles.

After a converstation, the coach asked another question and got an immediate answer:

C: How could I find out if these actually came out to be the same size? Yup. [Students raising hands]
S: Use the one sixth you have, and measure each of them. But you'll be having to flip them over. [L. 534-537.]

[^7]After another student does a similar folding with a blue paper, the paper is unfolded and one of the areas is cut out and shown (at L, 547; see Figure 4, below).


Figure 4: Colleen's blue cut-out of a piece without right angles.

Then the coach probes further and gets an answer:

C: How can we call ... these sixths, even though they don't look like any of the other shapes that we had? Why can we call them sixths? Just two people know? Four people? Five? Why can we call these sixths? Yup? S: Because they're all the same size. You just have to turn them over. [L, 565-568.]

Then the little shape (Figure 4) is matched up against the unfolded green paper (at L, 539) and then flipped over (at L, 542). (See Figures 5 and 6, below.)


Figure 5: The blue piece shown to conform to a folded area of green paper.


Figure 6: The blue piece "flipped" to conform to another area.

The coach emphasized the question, even writing it on the whiteboard "How do we know we've got sixths?" He made a drawing (see Figure 7, below), and asked the question "Tell me what you think of this drawing" and [L, 582-583, 588-589.]. Referring to Figure 7, he asked: "Can I call these sixths?" [L, 594.]

Figure 7: Coach's drawing of sixths from whiteboard.

Many students responded "Yes. [L, 598.]"
Predictably students made discoveries and manipulated materials in new ways. Here are some examples of students' comments. "Use the one sixth you have, and measure each of them. But you'll be having to flip them over." [L, 536537.] "You put the two corners together." [L559.] "But they're overlapping." [L545.] "Because they're all the same size. You just have to turn them over." [L568.] "You just have to turn it over, like one-half you have to turn over... To make them look the same." [L, 574-576.]

The teacher probed the thinking of students, e.g., saying "Why choose that way to fold it? I never would have thought of that way. Zolu, what made you do that?" [L,555-556.]

In sum, this lesson shows the teacher sometimes encouraging students to reason and sometimes discouraging them.

### 5.4.3. The class showed a feeling of community.

There was an unmistakable sense that the class was a community in which students and adults were meant to help and care for each other. Student ideas were elicited and shared with the whole class. Students were invited to
help each other. Informal and friendly language was used. Examples abound and will be shown below.

### 5.4.3.1. Students share their thinking

On a number of occasions the teacher asked a student to share what she or he had said, done, or was thinking about with the rest of the class. For example, the teacher said to a student: "Kara just had a thought about this. Can you share with everybody, because I think that was a really good thought. [L, 188-189.]

And "Colleen, could you explain what you were thinking?" [L, 312.] During wrap-up the teacher probed a student's thinking and asked her to make it public:

T: Can I ask Colleen what she was thinking when she, or Zolu, what they were thinking? Why, why choose that way to fold it? I never would have thought of that way. Zolu, what made you do that?

Colleen: Put the two corners together. [L, 554-556, 559]

One time a student said that another student had "cheated" by observing and emulating a neighbor. "He saw. He cheated. Immediately the teacher scolded the accuser: "Shhh. Shari, that's not kind." [L, 134-135.] Apparently the teacher was establishing a culture in the classroom: It is okay to observe and learn from other students and treat each other nicely.

### 5.4.3.2. Contrasts between students made public

At one point, the teacher noticed that two students had folder their papers in different ways, so she brought it to the class' attention: "All right. Something
happened. Kara got this, for sixths. [T holds up green "skinny" folded paper. See Figure 8, below.] And Zora got this for sixths. [T shows a "fatter" green paper, indeed almost a square. See figure 9 below.] What happened? Can somebody explain what happened? They can't possibly be the same thing. Shhh. How did this happen? Colleen? How did this happen?" [L, 300-306.]


Figure 8: Kara's "skinny" green folded paper.


Figure 9: Zora's "fatter" green folded paper.

### 5.4.3.3. Teaching aide took active role.

The teaching aide also participated, helping a student when she got confused. Addressing the student after the student became confused, she said: "Joanie, remember what we did at the beginning, instead of folding it like this, you folded it like this, right? ...You remember doing that? [The student nods her head affirmatively.]" [L, 360-367.]

### 5.4.3.4. Students encouraged to help each other.

Students were also encouraged, when they had finished their own work, to help their neighbors. "If there's somebody who's finished at your table, could you help somebody who's not finished cut out their pieces?" [L, 180-181.] This
invitation was repeated: "If you feel like you figured out how to do it, can you help somebody at your table make your thirds?" [L, 213-214.]

There were no explicit example of a student helping another student, either observed by the researcher or picked up by the mocrophone, but students did talk to each other and their conversations may have constituted "help."

### 5.4.3.5 Teacher used management techniques

There were several times where the teacher exercised various management techniques, typically when she was giving instructions, when the class was making transitions, or when the room was noisy. For two examples:

Okay. When you're ready, could you look up at me, please? Larry's ready? Tammi's ready. Oren's ready. Put it in your baggie. Excellent, Colleen. Okay. The next piece...Is everybody ready to do the next piece? [L, 73-75.]

And:

Okay. Once you've finished ... Can everybody stop, and look up here? Can you stop what you're doing for a minute and look up here? Thank you, Lori. Thanks, Daryl. You need to make sure that all of your pieces are inside your baggies, and your baggies are zipped up. And then you can come to the rug. [L447-450.]

To quiet the class the teacher used a "rain stick" on several occasions. Indeed she poured it three times in the second minute of class. [See L, 20, 22, 24.]

### 5.5. Teacher-Coach Debriefing

On May $3^{\text {rd }}, 2004$, Angela taught a lesson to her $3^{\text {rd }} / 4^{\text {th }}$ grade class observed by the coach, Matt, and several others. The following day, Angela and

Matt met to debrief about the class. (Subsequently there was a second lesson and debriefing, also videotaped and observed by the researcher, but not analyzed.)

After looking at the video and reading the transcript of the session, several conclusions about it can be made. As intended, the debriefing was grounded in the lesson that preceded it. Second, the teacher and coach were continually asking themselves and each other how the lesson could have been improved upon; in other words, they strove to craft the lesson. Thirdly, the teacher used the coach as a sounding board, wanting to learn some aspect of mathematics or math pedagogy. Finally, both the teacher and the coach managed to summon a great deal of enthusiasm and ideas, seemingly enjoying the process they were going through; their positive spirit was palpable.

These four themes will be spelled out below in greater detail and supporting evidence will be shown. (In the debriefing transcript [D], the teacher and coach are referred to as Angela and Matt, or $A$ and $M$, respectively.)

### 5.5.1. Debriefing linked with lesson

The debriefing was specifically intended to be practical, grounded in the lesson taught the day before by the teacher and observed by the coach. As such it took some of its rationale from the book "Content-focused coaching," and from several professional models, particularly lesson study, insofar as lesson study also emphasizes a cycle of a teacher planning a lesson, teaching it, and then discussing it, all with help from others.

Was the lesson talked about in the debriefing session? Did this actually happen?

In fact, the entire debriefing session was linked to the lesson. The previous day's lesson had ended with the coach leaving the room before he had been able to complete a wrap-up exercise and the teacher thus taking over for him. So the coach began the debriefing session by asking the teacher what had happened after he had left the classroom. In response, the teacher showed the coach a diagram she had made copying a student's drawing of an array and she paraphrased the student explaining her work. [D, 48-52.]

Thus the teacher was aware of the coach's need to know what had happened when he was absent. The rest of the session was conducted similarly, with coach and teacher looking for material for their dialogue from students' ideas and representations and reflecting on their own.

### 5.5.1.1. Looking closely at student work.

In addition to observing individual students in class, the teacher made extensive notes of student work, including at times, their words and representations. This enabled her to refer with specificity to their ideas in the debriefing session.

Thus the teacher often referred to ideas students had expressed during the lesson or representations made by students [D, 68-74, 93-97, 315-323].

For example, at one point the teacher talked about an idea of some students, along with a representation of it, and contrasted it with an idea of other students and its accompanying drawing.

And then there were kids did it this way [A points to a drawing in her notebook of a rectangle containing 1 horizontal line and 2 vertical ones, thus making 6 roughly equal squarish sub-rectangles. See Figure 10.] and there were kids did it this way -- [A points to another drawing of a rectangle with 5 vertical lines, thus making 6 roughly equal "slim" rectangles. See Figure 11.] [D, 93-98.]


Figure 10: Teacher's copy of a student drawing of rectangle with squarish sixths.


Figure 11: Teacher's copy of a student drawing of rectangle with skinny sixths.

In one instance, the teacher reads a student explanation of a concept, leading the coach to probe further. Finally, the coach and teacher ponder whether or not the student really understood what she said she understood.

A: She says like, "These are even and those are even, and both sides are even, so they're each a quarter of the whole." Then she does the math.

So that's a totally different way of looking at it. ... See now ... I don't know what she's ... what looks the same. They look the same and I took a ruler and measured it. [The camera pans down and zooms in on Angela holding and pointing to the paper and then pans further up the table to show a paper with diagrams on the table. The printed workbook page says: "Different - Shaped Pieces - Here is a picture of a brownie cut in four pieces. Some people think these are not fair shares. Write what you believe." Then the camera moves back to the papers in front of Angela as she goes through them. See Figure 12 below.]
M So what do you think? Does that--
A I don't think she understands what she is doing.
M That doesn't clearly convince me.
A No.
M But this is good we're getting a --, like, so "What are the ways that kids can come at thinking about this?" [D, 299-313].


Figure 12: Drawing from workbook of rectangle cut into four pieces.

Another time the teacher reads a student explanation about shape,
leading the teacher to formulate a thoughtful line of inquiry.
A Okay, so Oliver says, "It is obvious a rectangle and a triangle, big difference."
M Right.
A So he is looking at the shape. [The camera pans to and zooms in on the paper in Angela's hand, showing the diagram shown in the quote directly above.]
M So he's looking at the shape and he's saying they're not congruent. So the problem here is that they're not congruent. And when they aren't congruent, what can we do?
A Right.
M Oh, we can cut them to make them congruent? [Matt draws an arrow on the diagram. See Figure 13.] [L, 315 - 322.]


Figure 13: Coach's drawing of a shape being "moved" to show congruence.

From these examples it is clear that the teacher and coach were not engaging in a theoretical or abstract exercise. They had specific student ideas to struggle with and their conclusions would potentially be put to use in the future.

### 5.5.1.2. Benefits of close attention.

The teacher prepared for the debriefing session by taking copious notes, which included copies of drawings made by students (many of the teacher's drawings are included in this narrative), words spoken by students (also included herein), and her own analysis of student thinking. Her notes were impressive enough, that when the coach saw them, he said "Oh wow!" [D, 88.] The notebook was not captured on camera.

Having a record of students' words and drawings, the teacher was able to gain insight into what students were thinking. For example, at one point, the teacher felt that two students had made "wacky" drawings when the task was one of making sixths:

A: So this is -- you know, I just sort took these notes about ... [A opens a notebook, but it is not shown on camera.] ... kids that -- sort of saw -- like things. This was such a wacky way of making the sixth, and yet Cleo and Sara both like gravitated towards that, and both, and I had no problem with it. [D, 87-91.]

Angela showed Matt her copy of Sara's diagram showing a rectangle with her starting to make a grid. She had made seven vertical lines and then was making horizontal ones. (See Figure 13A below. L, 65.)


Figure 14: Teacher's copy of student's rectangular array.

Cleo's drawing (see Figure 14, L., 71), as copied below by the teacher, was related to Sara's in that it was an array. But it was a large rectangle divided into only 4 thin smaller rectangles. The sub-rectangle to the right in the drawing appeared larger than the others and she divided that one into two parts, thereby, perhaps, attempting to make a representation of sixths.


Figure 15: Teacher's copy of student's rectangular array.

Referring to Sara, the teacher stated: "She just used the area," recalling how the class had worked with arrays earlier.

But the drawings were shared with the class and made an immediate
impact. Angela recalls how Sara and Cleo started counting and the rest of the class took notice. The coach was also impressed.

A: And she said, "I'm just counting the boxes. There are 16, I know $2 \times 8$ is 16 , there are 2 rows of 8 ." So she was totally going back to the work that we did with arrays. ... And then she said, and $4 \times 4$, here we have 4 rows of 4 is 16 , so it's the same number of boxes. ... And once she started to count the boxes like hands just shot up in the air, like oh, oh, oh. Because kids totally saw that was a way to prove that they were the same. That was great.

M: So, that's awesome, so a lot of kids connected to that idea of using the area to prove that they are the same amount even though they are not the same shape, yeah? [L, 73-83.]

Angela then goes on to contrast that representation with ones made by other students

A: And then there were kids did it this way [A points to a drawing in her notebook of a rectangle containing 1 horizontal line and 2 vertical ones, thus making 6 roughly equal squarish sub-rectangles. [See figure 10, above.] and there were kids did it this way -- [A points to another drawing, Figure 11, above, of a rectangle with 5 vertical lines, thus making 6 roughly equal "slim" rectangles. The camera zooms in on the papers on the table and specifically focuses on the diagrams in A's notes.] [D, 9399.]

Of course this type of close analysis enables both the teacher and coach to compare different ways of thinking or drawing. For instance, the coach looked at the two contrasting representations the teacher showed him in the examples above and said:

M: Now, I would say that this [He points to a diagram in A's notes of a rectangle divided into sixths with 2 vertical lines and a zig-zag, somewhat horizontal, line. See Figure 15, below.] was easier actually to see than what Sara did in this one [Figure 13A, above], because here [Again M points to zig-zag drawing.] the shapes are congruent. ... So you got this idea of congruence, but then you got
this idea of equivalence; and it doesn't have to be congruent to be equivalent; and that's where Sara's area computation helped. [D, 101-109.]


Figure 16: Teacher's copy of a student's drawing of a rectangle divided by zig-zag lines.

At other times, the teacher recorded words the student had used to interpret the drawing that the student had made, below, e.g., "These are even and those are even, and both sides are even, so they're each a quarter of the whole. [D, 299-300.]" [See Figure 10, above.]

This type of close attention to detail enables the teacher to make connections between student ideas and previous work they had done, as we saw with the arrays, whether from a recent lesson [D68-74], one from earlier in the year [D, 606-607], or even an earlier year, as shown below where the teacher quoted a student.

The teacher and coach were able to focus on common error patterns both had seen. The teacher noticed how students had trouble folding strips of paper to represent thirds:

A: I have to say like when we did it, James did this thing where he just couldn't figure out how to do thirds and so he did quarters this way and then he realized all I needed was three and he just literally threw it away hoping I wouldn't notice. [The camera pans down and then zooms in on Angela's hands as she is drawing a diagram on the paper. See Figure 16, below.] [D, 215-217.]


Figure 17: Teacher's copy of a student's drawing of a rectangle divided into fourths with one fourth "crossed out."

The coach responded with a similar folding problem he had often noticed:
M : Another thing I see a lot when kids are trying to make thirds, is especially third graders the first time they're dealing with this stuff, is they will cut it in half 'cause everything starts with cut it in half. And then they will cut half in half. And so then they will say there are three parts, right? [Matt made a drawing. See Figure 17, below.] [D, 222-225.]


Figure 18: Coach's drawing of a rectangle divided into three parts.

### 5.5.2. Crafting the lesson

Another theme apparent in the debriefing session was that, starting from the lesson itself, the coach and teacher examined the effectiveness of several teaching episodes and made judgments about them. Presumably, the main goal was to explore teaching as a craft, to improve the lesson, or a similar one, the next time it might be used. The theme of crafting the lesson was related to the previous theme, looking closely at student work, but it is slightly broader. It also looked closely at the teacher's words and actions, tasks and materials used, and at the lesson in the context of what came, or should have come, before and after
the lesson. Crafting lessons is the focus of "Lesson Study," a professional development technique, practiced widely in Japan.

In this debriefing session, many aspects of the lesson, and lessons generally, were discussed. One aspect was rather theoretical, the goal of math class was declared to be making sense of the material. More practical were the means taken to achieve the goal, including tasks, strategies; and other pedagogical considerations. Finally, the effectiveness of the lesson was assessed.

### 5.5.2.1. Sense-making as a major goal.

At various times in the debriefing session the coach stressed the importance he placed on having students reason or make sense of what they were doing, not on accepting what the teacher or book was saying without really understanding it. And most significantly, the teacher appeared to buy into this notion.

For example when the class was discussing equivalent fractions, there arose a debate among some students who saw two shapes as looking different and therefore concluding that they could not be equivalent and others who counted the number of little squares in the two shapes, found they were equal, and concluded the shapes were equivalent. The coach argued in favor of risking polarizing the class so that they would then be encouraged to search for convincing evidence to support their judgments. One should notice there is no
talk of the coach or teacher giving students the answer in order to resolve any conflict. He says:

M: So, I would push to have this [introducing an exercise that promotes argument among students] become part of the class, like ...vocabu...it's not vocabulary, but sense-making, or the logic of equivalency, that these are equivalent pieces, because ... So, it's interesting, because in our meeting earlier, I said that maybe this isn't equivalency as we typically think about it. Typically we think of equivalency as being like different-looking numbers in the fractions, but I think it could also relate to different-looking shapes that are the same amount, just the way $2 / 4$ is equivalent to a $1 / 2$, triangle $1 / 4$ is equivalent to rectangle 1/4. So, I guess in a way it's an early concept of equivalency. [D, 344-350.]

The coach here was also modeling the reasoning process for the teacher. As he talked about what students did, he himself was making sense of what they meant.

The teacher did not just go along with the coach as to the importance of these instances, but was very excited about the potential for thought and reasoning among students. [See, e.g., D, 339-341, 364-376, 640-643.] That potential was what she apparently was referring to when she extolled the value of students' starting out a unit with their everyday use of fractions in the real world and then building to a more formal language. [D, 174-179.]

### 5.5.2.2. Ways to promote sense-making

The job of selecting tasks was critical. The coach explicitly raised the issue of what considerations a teacher might think about in choosing a task in order to promote reasoning:

What kind of context are you going to choose, as the teacher to work from, so that ... so, you're choosing a context that jives with
their kind of natural sense-making or intuitive sense-making about fractions? [D, 189-191.]

The teacher liked exercises that started with students' informal knowledge.
She quoted from a book [there is some evidence that the book was one written
by Marilyn Burns; see lines D, 174-179 and D, 409-413] she liked:
A: One of the nice things she talks about in this book too is, which I feel like, maybe we could go back to do this, as part of this conversation, like "Where are all those places in the world where you hear, or you use fractions?" and you know, "Well do you have half-a-dollar?" or "I'll be there in three-quarters of an hour." or whatever it is. [D, 174-179.]

The teacher goes on to explain that she liked the unit because it was
"accessible," "visual" and "concrete" to students [D, 197-199.]
From an informal or intuitive grasp initially, students could go on to think more formally or abstractly. The coach saw this happening with a student:

M: So, this kid is, Ivan is really using two strategies: one is, "Can you make them congruent?" But the other strategy is using kind of reasoning about, well, if I cut it in half, and then I cut each of those in half again, so it's half of a half. Do you know what I mean? ... She says like, "These are even and those are even, and both sides are even, so they're each a quarter of the whole." Then she does the math. So that's a totally different way of looking at it. [D, 289301.]

The coach also placed an emphasis on activities that, for example, asked students to notice patterns. He talked at one point about ordering of fractions and wrote down several in order from small to large: $1 / 8,1 / 6,1 / 4,1 / 3$, and $1 / 2$, along with roughly proportional rectangles. He said:

M : And so, one of the big things that comes out of the ordering work, is that, if you have $1 / 8$, then $1 / 6,1 / 4,1 / 3$ and $1 / 2$, that they, then the idea is, "What patterns are we seeing here? Oh, the smallest piece has the biggest number on the bottom. [ M makes a
diagram. See Figure 18, below.] ... The biggest piece has the smallest number on the bottom. Why do you think that is?" So you start with the unit fractions, and so that's a really important conversation to have, because it gets into the idea of, what does the numerator mean, what does the denominator mean? [D, 437444.]

Figure 19: Coach's drawing of 5 rectangles with corresponding fractions.

The coach can also be cautionary. When the teacher proposed an activity about groups, where students figured out "how many red tiles out of all of the tiles are there?" [D, 469], the coach demurred, thinking that could confuse students who are primarily dealing with comparing fractions. He says "I wouldn't mix it in right now while you're doing this though, right? [D, 471.] Instead he suggests an activity making number sentences using their fraction cards. [D, 474-476.]

Thoughtfulness was promoted throughout. The coach talks about how sense-making can be promoted by allowing different ways of thinking and multiple entry points into a problem.

M : And that's one of the beautiful things about it, is that there's context, which supports sense-making. And then from that you attach symbols to it. And the symbols are really representing what you're manipulating, what you're making, and it's just kind of a shorthand notation. So rather than starting from manipulating symbols, you're starting from, "How do I find a shorthand way for communicating my amounts?" Yeah, that would be really cool. And I think for 4th graders that find this kind of easy, to say, "Well, on the front I'm going to give you three with pictures, and on the back I'm going to give you just the numbers. You can draw a picture if you need to, but if you can find some ways of thinking
about this without having to draw a picture, then try doing that to challenge yourself." [D, 611-619.]

At another time the coach makes a similar judgment:
M : So, this kid is, Ivan is really using two strategies: one is, "Can you make them congruent?" But the other strategy is using kind of reasoning about, well, if I cut it in half, and then I cut each of those in half again, so it's half of a half. Do you know what I mean? [D, 289-292.]

### 5.5.2.3. Effectiveness of the lesson

After examining an exercise, the coach asks a question: "Now, the tricky thing is, I think that's really wonderful problem-solving. The question is: Is that an efficient way to get kids to be able to think about equivalence?" [D, 585-586.]

There were numerous examples of the teacher and coach admiring the quality of tasks introduced and of students' thoughtfulness in the lesson. Many are cited above. [E.g., D, 80-81, 192-193, 590-596.]

### 5.5.3. Teacher asks for help from coach

During the course of the debriefing session the teacher apparently learned several things, some of which could be characterized as large or conceptual ideas and others as smaller, often fostered by specific teacher questions.

A number of times the teacher asked specific questions of the coach, e.g., about how to frame a particular exercise [D, 364-378], for an example of a more challenging question [D, 499-504], or simply for an explanation of the name "mixed fraction" [D, 510-522].

For example, in the framing situation, the teacher and coach were discussing how best to orchestrate a conversation about equivalency when some students counted the areas of two shapes and saw equality and others saw two different shapes and asserted they could not be equal. The teacher started a dialogue with the coach in which he emphasized that the teacher not be directive:

T: So, I wonder, do I say to them, "I know that they're the same. How can we prove it?" Or ...
C: No. I would start with, "I was looking at the work, and Oliver looked at this, and he said it's obvious they're not the same shape.
T : And he is the only one that thinks that.
$C$ : Fine.
T: Do I even say "Oliver is the only one who disagrees with us," or no?
C: Well, I think...
T: I mean just to make it an even ...
C: I wouldn't necessarily...
T: Okay, all right.
C; I think it adds dramatic flair to it, maybe, but I think ... no, I think it's better to just say, "Here's what one student said," and it's weighty enough that [inaudible] it's Oliver, right?
T: And everybody else disagrees with him, too. They're not going to know necessarily that they do, but....
C: I mean you could say that, so...
T: No, I hear you. I hear you. I don't need to...
C: So, how can we either convince him or realize that he's correct? Does anyone have an argument that could convince him? So, you want to look ahead a bit, yeah? [D, 364-382.]

In their conversation about mixed or "improper" fractions, Matt says there is nothing really "improper" about them, so he prefers other names, e.g., mixed numbers, numbers greater than one [D, 513-522.] He went on to advise her of a good follow-up activity, making equivalent number sentences. He made diagrams for two ways of doing this. [See Figures 19 and 20, below, at D, 532 and 537, respectively.]


Figure 20: Coach's idea for making equivalent fractions.


Figure 21: Coach's second idea for showing equivalent fractions.

Matt suggests a different idea for introducing fractions with unusual numerators and denominators, using the idea of dividing imaginary brownies among students. ${ }^{7}$ The first example is five brownies divided among four children, then seven brownies among four children (see Figures 21 and 22, respectively. [D, 550 and 573.]


Figure 22: Coach's drawing for dividing "brownies."

[^8]

Figure 23: Coach's drawing for apportioning brownies.

This activity also leads to more ways to operate with fractions, such as when each student gets a brownie, plus a half, plus a quarter. [Matt's diagram is Figure 23, below, D, 481.]


Figure 24: Coach's picture for computing brownie shares.

## Chapter 6 Reflection on lesson in light of interviews

Looking at the lesson in light of the teacher and coach interviews (which are included in the appendix) is revealing in one major way: $A$ researcher cannot say with certainty whether or not many of the changes in practice that they wanted to happen actually happened; but a few things did.

The interviews emphasized two related aspects of the lesson: improving teacher practices; and improving the teacher's attitudes and beliefs. (In this section the references to line numbers refer to the numbers in the interviews located in the appendix. References, e.g., to the second teacher interview would start with TI \#2; CI\#1 refers to the first coach interview.

There were many important aspects of Angela's practice or attitude and beliefs that were not explicitly talked about in the interviews and they are not discussed below.

### 6.1. Improving Teacher Practice

Six goals for Angela were expressed in the interviews, by the teacher, the coach, or both: Angela becoming part of a team; observing students; modeling without dominating; encouraging students to think for themselves at the same time they communicated with each other and the class; focusing more on process and less on right/wrong answers; and wrapping-up better.

### 6.1.1. Individual teacher becoming part of a team.

The teacher, the coach, and the principal were all drawn to the Melville School by its spirit of collaboration. They each expressed it in slightly different ways.

The teacher was first drawn to Melville by its non-traditional atmosphere and she completely fell in love with it before she ever taught there. Among the things that attracted her was a communal spirit where teachers formed real bonds with each other and discussed curricular issues as a group. [TI\#1, 59-63, 100-116.]

The coach extolled the sense of community in the school. He noted rather selflessly that he would not be present with the teacher most of the time, so he wanted her to build confidence in herself and be part of a group of people who could rely on each other for support. He said:
[ $N$ ]ine out of every ten days I'm not working with that group of teachers ...They went to each other with ideas, with questions, with materials, with resources. And I think that happened to some extent before, but it happened at a much deeper level where they really were connected to each other's work because of that storytelling, and during our team meetings learning landscapes that we were negotiating and building together ... That is probably the single most important thing that I think has shifted this year. [CI\#2, 569-575.]

Matt also remembered a positive team experience he had once experienced where he had attended:
a full day workshop ... The good, the really smart idea ... was that teachers came in teams, so they had someone to talk about the school, but they also came ... with their coaches ... so that they came, went back to school with those teachers and helped
them implement some of these ideas. It wasn't ... that you just take this one idea and you go implement it, it was really more about ... we did some adult math to engage people in a math experience and think about themselves as learners, to help them formulate thinking about learners in general, and, but then we got into looking at parts of the curriculum and what the math was behind a unit, like what the big mathematical ideas were behind this one unit ... [CI\#1, 343-352.]

The principal talked about how she envisioned the way the young people would be able to function as adults down the road; she wanted them to be both good individuals, able to make good decisions, and good players on a team, able to build good working relationships with others. [PI, 36-46.]

The coach wanted this school-wide collaborative feeling to imbue the math classroom, and he felt Angela needed to work on that. He specifically wanted her to build her classroom into a "community."

### 6.1.2. Observing students

In their interviews both the teacher and the coach talked about how the teacher had made an error of judgment in the way she had distributed materials at the outset of the lesson. The "error" is described below. [The incident in question occurred in the second lesson and was observed by the researcher, but not analyzed in this study, and is not included in the Appendix.] The coach said:
(T)oday for example, she made one big management mistake ... It's the second to the last week of school ... I wouldn't say that it's an atypical kind of mistake. Like she was giving out the papers individually to kids that they needed to cut up to make the fractions, and that took all of her energy and all of her focus away from observing children and their thinking, and put it all on doling out paper. And ... you know, she had to, like, search for
the colors, and how many, and there's too much of that kind of interaction and not enough opportunity for her to observe what is going on. [CI\#2, 420-426.]

The coach stated that he did not cite this 'mistake' to chastise the teacher. Quite to the contrary, he mentioned it to praise her, for she had immediately recognized her mistake, regretted making it, would do it differently next time, and mentioned it to him in an aside. [CI\#2, 427-429.] Indeed when the coach was asked whether there were changes in the teacher's practice he said "definitely" and cited this moment. [CI\#2, 417-420.]

For her part the teacher also regretted this move and contended she would handle it differently in the future. After immediately citing this a "very hard" management moment, she said:

So, you know, I would've done a bunch of things differently. You know, if we do it again...We had talked with Matt afterwards about putting just a couple of each color on a table so that kids could trade with each other and share a whole square, and then they would realize that if you only wanted a half, then somebody else could have another half of the paper, so there would be more going on within the tables and within the groups. [TI\#2, 210-218.]

### 6.1.3. Modeling without dominating .

The incident described above concerning the teacher handing out materials also illustrates another behavior that many teachers apparently have difficulty with: giving up control. Both teacher and coach were bothered by the uniformity of the products that the students produced and it is only
logical that uniformity increases the more tightly the teacher controls the situation.

In his interview the coach mentioned something that "caught his attention was that half the kids almost made their ... fraction quilt square look just like the teacher's, and so that was a concern to me [CI\#2, 487-488.]"

For her part the teacher lamented the fact that she had modeled only one pattern and that almost the students had followed her example. [TI\#2, 231-234.] Indeed the teacher vowed that, were she to teach this unit again, she would model several patterns, in order to promote divergence and thoughfulness. [TI\#2, 231.]

Thus both became aware of the need for the teacher to learn to model activities while still allowing students to be somewhat independent. So the question is: If, after seeing student work, the teacher laments the lack of divergence, why does the teacher persist in exerting such control? One can only speculate that such behavior is deeply rooted.

And yet we see the teacher at certain times becoming very directive in her instructions to students in how to fold paper for the fractions strips. [L, 4849, 52-59, 218-227.] One thinks that the balance between telling students what to do and allowing them to fail or struggle might be an ongoing issue.

### 6.1.4. Encouraging students to think for themselves and communicate with each other and the class.

One of the issues for both teacher and coach is how to achieve a delicate balance in the classroom between the teacher giving students enough information and direction to solve the problems correctly, on one hand, and giving them enough freedom to discover the meaning of their ideas on their own, with the possibility inherent in that freedom that some students will really struggle. In answering that question, let us go back and review the rhythm of the lesson.

After the launch of the lesson, about two minutes, the class was broken into groups. Each student brought his or her materials to a desk that was clustered together into 'tables' of 4,5 , or 6 students.

The great bulk of class time ( 35 out of 45 minutes) was spent when students worked at their own tables.

The lesson ended when the whole class came to a meeting where the coach sat on a chair in front of them and used a whiteboard to aid in wrapping up some of the activities or thinking of the class. That part of the class lasted nine minutes.

Matt, in his interview, stressed the importance of Angela learning to use the grouping of students more effectively. [CI\#2, 401-402.] She appreciated this focus:

It's been really good for me to be able to group kids in a manageable way because ... there's a really broad spectrum of abilities right now mathematically in my classroom. And so it's
been really great to have him show me how to organize, like, what groups I can put kids into so that they're not seemingly all over the place. Um...That's been really helpful. [TI\#1, 154-156.]

Interestingly, nowhere in the interviews or in the class itself, is any technique for grouping students discussed, so it cannot be said whether or not the groups were effective.

Apparently a crucial part of the flow of the class was that student work and ideas be made public to the whole class. Both teacher and coach emphasized this point in interviews. The teacher said:

I feel like it's important to get kids to share their thinking, and to make that public, which I think Matt's helped me do. ... Because it's shown me that that's how they're making sense of what they're doing. Like if they're able to articulate it, and then I value it by putting it up there publicly, um...It's just given me a window into how they're making sense of it, and then that in turn sometimes shows me the direction that I need to go in. [TI\#1, 168-175.]

There were indeed instances in the lesson where students talked to each other and instances where their ideas were made public. (See Section, 4.4.3, above, for examples.) What was not at all clear was how much they reasoned and explained. The evidence in that section was indeed ambivalent: sometimes they were encouraged to think and sometimes they were encouraged not to think. Nor can any comparison be made with their thoughtfulness in other classes.

Both the teacher and coach talked in their interviews about wanting to engender certain practices, attitudes, and beliefs in students. For example, the coach noticed that students were generally not explaining their work. So
he made a point in his wrap-up of getting students to think about reasoning. And so, "I chose to make the wrap-up with the kids about modeling: what it sounds like, and looks like when you're reasoning about the fractional pieces." [TI\#2, 497-499.]

The coach also talked about having students think in different ways about a problem. He wanted divergent thinking. (He referred to a moment in the second class that is not analyzed herein.) [TI\#2, 481-483.]

Thus the lesson involved a number of different types of interactions. Instructions from the teacher, the potential for independent thinking by individual students, discussion among students, students talking to the whole class, and a whole class discussion, but this time with the class gathered together in a group. This is working towards a classroom where the teacher and the students might be feeling more comfortable with the subject matter and, on the other hand, they are also becoming more adept at figuring things out for themselves.

### 6.1.5. Focusing more on process and less on right/wrong answers.

The coach wanted the teacher to effect more of a communal feeling in the room. He cited several reasons for this. One was her tendency to favor some students over others. He said:

When I first started working with her she had, maybe, four or five go-to kids that would always answer all the questions ... She relied heavily on them while kind of disciplining the rest of the group. ... And so one of the big things that I was thinking about
was: How can I make this feel like more of a math community? How can I bring more voices into it? [CI\#2, 400-404.]

He also wanted her to focus more on a spirit of inquiry. He felt that the teacher focused too much attention on the best or most aggressive students, thereby not meeting the needs of the majority of the class. He said:

Oh, and the other thing that she did was, she would, um...When kids gave answers, they'd either be right or wrong, and she'd kind of speed through the wrong ones and spend a little bit more time on the right ones. But it felt more about, like it was about your answer, not about your thinking process. [CI\#2, 404-408.]

### 6.1.6. Wrapping-up more effectively

As to the wrap-up at the end of the lesson, the teacher said she was unsure of herself in this role and wanted the coach to do it for her. She said:

I ask him a lot of times to do the wrap-up because I feel like that's the place where I could push the kids' thinking more and I feel like that's where I really need his help, and, um...So that's been really helpful too." [TI\#1, 147149.]

And so the coach led the wrap-up. As for his part the coach said he acceded to the teacher's request, but that he wanted the teacher to "imagine what it would look like." She responded that she would want students to explain and justify their thinking. He wanted her to push their mathematical thinking more. [CI\#2, 431-434, 435-436, 439-440, 444-446.]

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The researcher felt that progress had definitely been made in these areas, as was noted in Section 4.4.4, above.

The teacher stated that the coach wants her to look more closely at students' work and she added, apparently proudly, that this concept has spread into other subject areas she teaches. She felt this was "really nice." [T1\#1, 154-156.]

### 6.2. Improving the teacher's self-confidence

Angela thought of herself as "a very bad, very poor math student."
[TI\#1, 11.] This negative belief from childhood still persisted into her selfimage as a math teacher [TI\#1, 150-152].

The coach recognized several things about Angela's attitude. He wanted to help her improve this self-image in terms of self-confidence and decision-making. He said: "She's, she can be a little self-conscious ... Another big thing is helping to boost her confidence and her, you know, the framework for making decisions so she would be more confident." [CI\#2, 412-416.]

But he also wanted to be gentle with her, not making her self-
conscious. [CI\#2,, 412-414.]
With regard to her teaching Matt also had some insights and goals for her: to make Angela the best teacher she could be.

I think about what my life was like in the classroom and it was hard, it was really hard, um, and so I think that my first priority is how can I make life a little bit better for teachers, a little bit easier, and sometimes that means helping to make some materials and make copies or whatever, but it really means, to me it means, helping them develop the teaching tools that make the work not so scary, but even if it's hard work, even if it's exhausting, they have, they have an idea of what (long pause) what good teaching looks like and what it's about, and how, and that they, that they see that they have access to good teaching, they step by step are making, making their way towards the best teaching possible and understanding why it's, what's good about it and why it's good and how to make the best decisions they can so they can feel really proud of the work they do with kids. [LI\#1, 253-263.]

Although it is hard to state with much confidence, based on one observation, that the teacher's attitudes or beliefs have developed on account of the coaching process, the teacher reported "I do try to do math all the time. It's something that's a priority ... " [TI\#1, 169-170.]

## Chapter 7 Findings and Conclusions

Chapter 5 described what was happening in the lesson, the debriefing, and the interviews. In this chapter, attention will be given to interpretation, the meaning of what happened. Analysis of the data have produced findings, which are stated at the outset. Then the data will be examined by looking at each data source separately, looking at them altogether, and finally responding to the research questions in an effort to discern their significance, if any. In this chapter, fewer references will be made to line numbers in any transcript, as the points made will be more general.

Section 7.2 will focus on the data sets, i.e., the interviews, the lesson, and the debriefing quickly, to point out their most significant features. Overall conclusions about the data will be rendered in Section 7.3, tethered less to each set as a separate entity, and more to an impressionistic sense of the whole. Section 7.3 sets forth the research questions that animated the study and attempts to answer them.

### 7.1. Findings

### 7.1.1. The teacher reports that some of her beliefs about math teaching have changed due to the coaching process.

The teacher talks about how her beliefs have undergone change as a result of the coach and the coaching process. Among the changes, or emerging changes, are: that students should make sense about their mathematical ideas; that the classroom should function as a community of people who try to help
each other; that math teaching is within her capability; that math is a priority in her classroom; and that the teacher should reflect on her own practice. Her selfconfidence has also increased.

Some of these beliefs are interrelated. For example, in making sense of one's ideas, a student should view other students and the teacher as helpful resources and that, of course, implies a working community. So too, for a teacher to help a student, she has to know what the student's ideas are, so the teacher has to ask questions to elicit the student's thinking. Implicit in this type of learning is also the idea that mathematical reasoning is a complex process; therefore students' incorrect ideas should not be viewed as wrong, but as part of a pathway to understanding.

The coach stated that he had observed many of the same changes in beliefs.

### 7.1.2. Teacher practices mirror teacher beliefs. There are signs that the coaching is influencing the teacher's practice.

Among the practices observed or discussed were the close collaboration between the coach and teacher, including the way they conferred during class and met to plan and debrief about lessons. The teacher started paying closer attention to what students were saying, writing, and doing and also reflected deeply on her own practices. The teacher started paying more attention to the way students were grouped, encouraged them to interact, and to talk to the whole class about their ideas. She called on more children, not just the ones who
knew the answers. Answers were not viewed as just right or wrong, but as part of a process of making sense of student ideas. A communal spirit was fostered.

Also the teacher reports that she spends more time on math, that indeed it is now a priority for her, She also has more fun and

Moreover, the link was unmistakable between various observed teacher practices and corresponding beliefs. These links are made explicit in Chapter 7 when each practice is correlated to an antecedent belief.

For example, the practice of students interacting with each other stems from two beliefs: that students should make sense of what their ideas are and that the classroom is a community. Many other examples where beliefs appear to be antecedent to practices appear above. They are also shown in Figure 24, the Map of the Teacher's Evolution.

### 7.1.3. The coaching is helping the teacher to learn mathematics and to pay attention to the math learning of her students.

There are many instances where the teacher is learning both mathematical and pedagogical content from the coaching. Examples come from both the lesson the debriefing.

Regarding mathematical knowledge, the teacher and coach discuss the distinction between congruence and equivalence, and different ways to prove equivalence, including a concrete model (i.e., constructing and counting boxes) and an inductive or Intuitive model (e.g., if two areas are equal, then half of one is equal to half of the other).

The teacher has these conversations with the coach and also with students during the lesson. As such they reflect both her content knowledge and the knowledge of her students.

As to pedagogical content knowledge, the teacher (at some points) encourages students to fold paper in different ways to illustrate different models for fractions and to link with more abstract ideas. Much of the learning discussed in other findings, e.g., having student discuss their thinking, the teacher reflecting on her own ideas, would certainly be thought of as pedagogical content knowledge.

### 7.1.4. The teacher is in a state of transition in many of her emerging beliefs, so that many of them are fragile.

As with any person who is in a process of learning or transition, many beliefs are still emerging or evolving. They are not necessarily stable.

One example is her belief that students should make sense of their mathematical ideas. At times, she allows them leeway to devise divergent solutions and to make mistakes. But at other times, she gives explicit, perhaps rigid, directions, apparently wanting students to "obey" her and get the right answer rather than to figure it out for themselves, thereby risking errors.

### 7.2. A summary of each data set

The five data sets (interviews with the teacher, the coach, and the school principal, then the two longer sessions, the class lesson, and the debriefing session) will be synthesized, with attention to their salient points.

### 7.2.1. Teacher interviews.

Angela, the teacher, initially expressed negative feelings about mathematics, recalling her own bad teachers in elementary school. She considered herself an "art person". The negative feelings carried over to her teaching in that she appeared to have developed empathy for her struggling students, thus identifying with their endeavors, and wanted them not to be intimidated by math or math class. Yet she reported that, for the first time, math was fun and expressed hope her students would feel that way. She also said that she spent more class time doing math than before.

Angela brought a very positive attitude into her relationship with the coach, likely in an effort to overcome some of the negative ones felt inside her. Indeed, she appeared to have internalized some of the values or beliefs that the coach had, to wit: an emphasis on having students make sense of what they are doing, rather than just memorizing what she tells them to. She seemed to be proud of the progress she has made.

But Angela continued to exhibit some of her persistent insecurities about math when she described some of the areas she wanted help from the coach. She expressed positive views about the assistance from the coach and the coaching process, indicating that it has helped her feel more comfortable about teaching it, and that has even carried over into her teaching of other subjects.

In sum, the coaching has apparently had a very positive effect on her overall attitude towards mathematics.

### 7.2.2. Coach interviews.

From early on Matt, the math coach assigned to the Melville School, had very strong ideas about math: that he was very good at it, that he expected to make sense of it, and that his childhood teachers were fallible and could actually get in the way of his learning.

As a former math teacher, he had great empathy for teachers, and in the coaching process he respected the teachers' efforts and did not wish to impose his values on an unready or unwilling teacher, thereby risking a negative reaction. Yet, notwithstanding this belief, he worked very hard to form bonds with the teacher that would enable the teacher to come to share many of his beliefs, i.e., to study students closely in order to understand their thinking, and to emphasize process, rather than right/wrong answers.

Matt was able to form a relationship with Angela in which each felt comfortable with the other, and as a result were able to express needs and questions. He indicated that he believed his work with her had had a definite effect on her teaching, not so much on individual practices, but on longer term beliefs and attitudes.

Matt emphasized that his purpose in coaching generally was to instill better long-term beliefs, attitudes, and practices, in contrast to other professional development methods, that he felt strove for immediate impact. One coaching impact he pointed to was that teachers were talking to each other more about their lessons when he wasn't present, and that he was especially pleased with this outcome.

Another observation about Matt is appropriate: he was very thoughtful and not glib in his answers. The best example of this was when he was asked whether he thought the coaching had had an impact of Angela's teaching and he responded "I think so," and gave an instance, but went on to explain how complex an issue that was and that a longer time period was necessary to assess the impact the true impact.

### 7.2.3. Principal interview.

Jane, the principal, was vitally interested in building coherence into the school-wide math curriculum. She built a rapport with teachers, parents, and the math coach and occasionally attended their meetings. She saw herself as on the side of teachers. She changed schedules so that teachers and coaches could meet and teachers could meet with each other.

She thought of herself as not good in math, but was not discouraged by that from taking risks to improve the program. She saw coaching and other forms of professional development as important. She was impressed with the job Matt was doing with her teachers. She seemed completely sincere in her wish to improve the teaching and learning.

### 7.2.4. Lesson.

The lesson started with a class meeting, then went to a long period of individual work, and ended with another class meeting. The initial meeting is very fast, lasting about one minute, only long enough for the teacher to hand out packets to students and disburse them to their individual desks. At their own
desks and clustered in groups of four to six, students are given them their initial task by the teacher which they work on first, individually, and then talking to their neighbors, which is encouraged by the teacher. They are given four other tasks and work on them for 36 minutes. The teacher walked around the class and occasionally talked to the whole class or asked students to talk to the class. She also talked several times to the coach. The wrap-up at the end of class lasted eight minutes and was conducted by the coach.

If we look more analytically at the lesson, we see a different picture of the teacher. For many years the teacher was carrying on an interior battle. She had come to believe that students should be encouraged to reason about math ideas and figure things out for themselves. So she gave students opportunities to make sense of their work. Much of the thinking and paper folding in the first lesson shows a variety of thought and reasoning.

On the other hand, she also held somewhat contradictory beliefs that carried her in a different direction. Many of these she likely retained from her own math experiences, long-held and largely negative. Among these were that students should be given very clear directions as to how to solve problems or otherwise they would be confused and unable to proceed. Thus she wanted to take away their discretion and divergent ideas. This was illustrated in the second lesson, ${ }^{1}$ when she gave directive instructions, and the students produced cardboard quilt patterns that were almost all the same. (The teacher immediately noticed the lack of variation and rued her own directiveness.)

[^9]That two contrary beliefs could coexist initially seems curious and rather significant. Yet, upon further consideration, how could it not be so? Here was a person who held a set of strong beliefs for several decades. Now she was undergoing a completely different experience and the new beliefs that went with them. An underlying ambivalence seems totally appropriate to this situation.

Moving on to another observation, the prevailing ethos in the classroom was apparent: it was a community, not just a set of individuals. This spirit was accomplished by a set of practices and admonitions. Students were encouraged to talk and work with each other and asked to share ideas with the class.

The teacher and the coach had a close, positive relationship. They conferred during class; after these private conferences, the teacher often immediately did something different, leading to speculation that the coach had suggested a practice that she was now carrying out.

### 7.2.5. Debriefing.

The debriefing session was impressive for its positive spirit. It lasted 52 minutes. The seriousness with which both teacher and coach took their responsibilities was noteworthy, as was the energy and persistence of each, as indicated by the near absence of "down-time," or time limited to extraneous or frivolous matters. Also remarkable was the give and take between the two; they each initiated questions and ideas and responded to the other. They were a team.

To illustrate, several of the positive interactions between the teacher and the coach are shown. The teacher looked closely at students' work and discussed it with the coach. From that basis, sometimes she questioned the coach about students' understandings. She was able to do this partly because she had taken notes of what they had said and copied many of their representations into her notebook. Indeed, the coach was very impressed with her note-taking.

Both brought a critical quality to their conversation. For his part, the coach had suggestions to make to the teacher about follow-up questions or activities. The teacher looked back upon her words to and interactions with students, questioning how she might have done better. To the researcher, it was an exemplary session.

### 7.3. Looking at the Data Overall

Before we even get to the lesson, debriefing or interviews, there is one overarching structure: the school itself. The Melville School provided certain fertile conditions that contributed greatly to the potential success of the coaching system. Among these were a supportive principal, collaborative teachers, and a school-wide disposition to utilize systems and practices likely to promote student learning.

There are many logistical aspects to a teacher's job, including time and space management. Melville had a history of trying to accommodate teachers' needs. The principal and the teacher remarked upon this in their interviews. Thus
time and coverage were provided when grade level teachers needed to meet with each other or with the coach. In addition the coach was made to feel as if he was an integral part of the school community.

Yet, aside from logistics, another important message was conveyed by the school atmosphere and the principal's attitude: the development of teachers' mathematical thinking was very important work! This was a pervasive psychological aspect of the school's ethos. A tone was set for all teachers.

We should not minimize Angela's attitude. She was very open to inquiry, she wanted help. Perhaps we can conjecture that this is a necessary condition for success in a coach-teacher relationship.

The debriefing session stood out from all the others; it was truly impressive. The teacher was really digging into tough issues, and serious about helping herself become a better mathematics teacher. The coach was helping her accomplish this. She was not afraid to reveal her own insecurities about math. Her notes incorporating students' thinking and representations was notable, enabling her to examine her students' reasoning even more closely and to explore her observations with the coach.

The attitude of the coach towards the teacher can be summed up in this way: he treated her as if she were a professional, capable of making skillful judgments and decisions about her teaching, and someone to be respected. His confidence in her was clearly reciprocated.

The lesson was more problematic than the debriefing. One might think of the lesson as more practical and the debriefing as theoretical. The lesson was much less subject to control, involving as it always does, many more unpredictable factors, i.e., the interactions of many students, and the introduction of subject matter that would tend to have varying effects on different participants.

In addition, any lesson would tend to raise issues for the teacher that might be much more challenging than the debriefing session would. Her beliefs and attitudes about math and math learning, long-held and largely very negative, would have tended to clash with much of what she had seen in the coaching process and were coming to emerge in her beliefs. It is hard to imagine that the latent and deeply-felt ideas would not impact negatively on her teaching.

Yet the lesson did incorporate many of the theoretical ideas the teacher and coach wanted to incorporate, and had talked about in their interviews and debriefing. Among these ideas were creating a community of learners sharing ideas with each other, and not in competition; trying to include more of the class in the learning process; and emphasizing reasoning about ideas, not just whether answers were right or wrong.

### 7.4. Responses to Research Questions

### 7.4.1. Research Questions.

These questions are posed in Chapter 2, above, and rewritten here for convenience.

1. What relationship, if any, is there between the teacher's beliefs, and practices and the coaching process?
2. Which teacher beliefs, and practices, if any, appeared to change as a consequence of the coaching?

### 7.4.2. Responses to Research Question 1.

Research Question 1 deals with whether the math coaching had any affect on math teaching. Question 2 delves into the specifics of what, if any, those effects are.

The teacher and coach, in their interviews and in the debriefing session, frequently cited areas they wanted or needed to change in the teacher's pedagogical content knowledge, beliefs, or practices. Many of the topics they talked about did indeed arise in the lesson and it can be presumed that many of these occurrences were influenced by what was said or done in the coaching process. Much of the evidence for changes depends on the word of participants, in their interviews.

The teacher appeared to exhibit large changes in her beliefs about math and math teaching, especially as seen in her enjoyment of the debriefing session. This conclusion is reinforced by her interview. In general it can be stated that the teacher developed negative feelings about math as a child and that at the time of this study, her attitudes are largely positive. It is also clear that the positive changes began before the coach came into the school, but the coaching has given great impetus and focus to the changes. The teacher also displayed
ambivalent attitudes in the lesson, to wit: on occasion she seemed to want to bring a new, emerging attitude to class, but was influenced by her deeply held earlier views.

Angela stated, apparently with great pride, that math is now "fun" for her, that it never had been previously, and that she wants her students to feel the same way. These are obvious positive trends and seem likely to lead to other beneficial effects.

The teacher's beliefs about mathematics and its teaching had undergone substantial changes, largely due to the coaching process. This conclusion is based largely on teacher self-reports, both from the interview and the debriefing session, bolstered by evidence from the lesson and statements of the coach. Of course, the depth and endurance of the changes cannot be known, but their existence at a moment in time is patent. It should also be noted that beliefs and practices have a relationship. If a change in belief does not lead to a change in practice, what purpose does it serve, beyond the edification of one person? The coaching regimen is, after all, intended to effect improvements in teachers' classroom practices and ultimately to effect students' understandings.

Other questions follow: Can sustained changes in a teacher's beliefs fail to change her practices? Can she teach the same way she has for years when basic assumptions are different? It is submitted that practices must change assuming beliefs are truly changed - although proving this point is admittedly not simple and cannot be achieved in a short-term study such as this one.

From the way this study was conducted it is difficult to trace causality, i.e., one cannot aver that something that happened in the coaching process was the proximate cause of something that happened in the lesson, for either of two reasons. First, a new practice might have happened anyway. Secondly, the time frame here was too short for change to be observed.

What can be said, however, is that this coaching relationship was successful. practices and beliefs, that the coach and/or the teacher wanted to work on, were worked. Moreover, changes in the teacher's beliefs did occur, and they clearly related to the likely changes in practices. These changes will be spelled out in some detail below.

This study comprised a quick look at a case of one coach and one teacher working together to improve the teacher's mathematics teaching practices through the professional development model known variously as coaching or mentoring. It is called "coaching" in this study. The evidence adduced established to a high degree of confidence to the researcher that the teacher changed deeply, and has started on a process of continuing change, that will result in her becoming a more thoughtful and effective math teacher who is likely to help her students to develop a more profound understanding of their mathematical ideas.

That one coaching relationship was successful says only that coaching can be successful form of professional development. It is proof by existence. Under other circumstances, coaching could be less successful. That's why more
research is necessary. Suggestions for further research are recommended in the following chapter.

### 7.4.3. Responses to research Question 2

In this section, some of the specifics of the changes will be addressed. A diagram entitled "Map of Teacher’s Evolution" appears in Figure 25 below. It represents a description of the interpreted relationships among teacher beliefs and practices as they were observed in this study. It gives a picture of the evolution of Angela as a mathematics teacher during the course of this study. Four sections follow.

## BELIEFS



Figure 25: Map of Teacher's Evolution

### 7.4.3.1. Beliefs: Many of the teacher's beliefs underwent change, most likely due to coaching

Many of the teacher's beliefs underwent reflection and change, apparently ue to the coaching process. Beliefs, along with their antecedents and/or succeeding developments, will be shown below:

Affective beliefs. Angela stated in her first interview that math had become a priority; she even said she thought it was "fun." [TI1, 95-96, 169-170.] Largely because of this new belief, she was now spending more class time on math. [TI1, 166-170.]

In her interviews Angela described herself as an 'art' person who had negative feelings about math from her earliest days in school. She had one math teacher she liked. All the others she did not like, especially one who terrified her.

Her feelings about math started to turn more positive when, years later, in graduate school of education she had a math teacher who made class 'fun' and when she got to the Melville School, where there was a collegial atmosphere and she formed a bond with her two grade-level colleagues, both strong math teachers, and consulted often with them.

When the coach, Matt, arrived at Melville earlier that year, her attitudes underwent further changes. From Angela's interviews, she expressed a feeling that she now had someone by her side, or sitting on her shoulder, for her to utilize as a resource and to answer her questions. She described some of the suggestions he made to her during class as 'amazing.' While the question remains of whether this kind of assistance would enable her to become a more
confident teacher independent of him, it clearly helped her form a paradigm of sense-making, and a model of the classroom as a supportive community.

She also added in her interviews that she was spending more class time on math than she had previously. She added that math was now "fun" for her and a priority in her classroom. She also said the coach had made a difference in her teaching in other subjects.

Aside from the interviews, there were other indications of the positive change in orientation. She appeared to truly enjoy the debriefing session. Although the session was long - 52 minutes - and came at the end of a work day near the last day of the school year, in a hot non-air-conditioned room, she was a bundle of enthusiasm, energy, and ideas. These attitudes suggest a person engaged and stimulated by a process.

Students should make sense of their ideas. Angela apparently was coming to a belief that mathematical solutions should be arrived at through a thoughtful, rational approach. The coach, in one of his interviews, spoke very forcefully about his own belief as a child in making sense of his ideas and even challenging the teacher for help when he did not understand something. In the debriefing session, he stressed to the teacher the importance of creating an atmosphere whereby student's ideas are elicited, examined, and made sense of such that the ideas become more meaningful and ultimately understood. In the lesson the teacher asked students to make their ideas public, to debate with each other when their ideas diverged, and to attempt to make sense of the differences. In
the debriefing, she also talked about the importance of making sense of their ideas, apparently adopting this value as her own.

Angela also spoke about "fun" in her interview, as a goal both for her and for the students. She also made clear that fun had never been a feeling that she had previously associated with math or math class. She also explicitly linked having fun with achieving understanding. Thus she saw a connection between the emotional and the intellectual.

The classroom began to feel more like a community In one of his interviews, the coach expressed his belief that the teacher called on too few children in class, while the needs of the rest of the class were not met. So he wanted her to come to view the classroom as more of a community and he developed several strategies for carrying this ideal forward.

One idea was that the groupings in the classroom be rationalized, presumably so that needy students would be placed with supportive ones. Another was that students should be encouraged to interact, i.e., to talk to each other, to listen, and to help. Another was that students would make their solutions public; often the teacher asked students to tell the class what he or she had done or was thinking; and the rest of the class was expected to listen and respond. All of these practices were indeed observed in the lesson. And the teacher also talked about her evolving sense of the classroom as a community in the debriefing session.

The practices observed in the lesson and espoused by the teacher in the debriefing session seem a far cry from the coach's description of earlier classes. This presents strong evidence of her evolution.

The teacher reflected on her own practice. One of the premises of the coaching regimen is for teachers to reflect on their own practices. While one could assume that some participating teachers might resist this injunction, Angela clearly accepted it.

In the debriefing session in particular, the teacher was very engaged in the act of self-reflection. Her serious attitude and impressive energy betoken her keen interest in examining what had transpired in the lesson. So too do her copious notes of student words and representations indicate her commitment to figuring out their thinking as well as her own reactions to it.

The teacher was coming to feel that mathematics teaching was within her capability. The teacher talked at some length in one of her interviews about her lifelong alienation from mathematics. Many of the positive changes that ensued are discussed above. Through them she was able to use herself as a source of teaching ideas. She summed up her evolution when she said: "I want kids to think of math as fun, because it is for me now, and it never was." [TI1, 95-96.]

### 7.4.3.2. Practices: Teacher practices mirror changes in beliefs

Of course many teacher practices were seen, but it seems noteworthy that, while they may or may not have been changes from previous practices, or have been caused by the coaching, they can be linked to changes in the
teachers beliefs discussed above, and those changes can be said to have been caused by the coaching. This observation makes it much more likely that the practices had indeed changed, and that the changes occurred as a consequence of the coaching.

Most of the practices discussed in this section were observed during the lesson, but some were discerned during the debriefing session or interviews. Those practices that were observed during the lesson will be described first, followed by those from the debriefing session or in one of the interviews. In each case a practice will be linked to one or more antecedents, either attitudes or beliefs.

First, students were encouraged to interact with each other and with whole class. At the beginning of the lesson, students were placed in groups of four, five or six students. One of the hallmarks of the lesson was that the teacher encouraged students to interact with members of their group, including that they talk to, listen to, and help each other. The extent to which they did so is harder to discern, in that only a few such interactions were observed.

The teacher also asked students to share their ideas with the whole class. There were several times during the lesson that the teacher, noticing the work of a particular student, halted the work of the class and asked the student to say what he or she had done or was thinking.

Similarly, seeing the classroom as a community depends, not only on students talking, but caring about and helping each other, and adopting other
supportive roles, like not making fun of each other or leaving some on the outside of conversations. Each of these behaviors were stressed by the teacher.

These interactions between and among students can be seen as initiating the process of reasoning about ideas.

Two of Angela's emerging beliefs can be viewed as clear antecedents for these various practice of interaction: her notion that students should make sense of their ideas and her wish to make the classroom see itself as a community.

Students reason about ideas. After giving students a task, the teacher often, but not always, encouraged them to be thoughtful in the process of solving it. This reasoning process has at least two antecedents in the teacher's emerging beliefs, the same as the ones for interaction: that students should make sense of their thoughts and that the classroom should be a community.

Making sense of, or reasoning about, ideas, can take place inside a person's head, with another person, or in a large group. Whether an experience is personal or shared, a student expresses ideas, or becomes aware of another student's ideas, and then ruminates and evaluates his or her own ideas or the other person's, and argues about the their merits.

One of the elements that is crucial to enable a students to make sense of math is that they be given appropriate tasks to reason about. The task should be meaningful or fun to the students, not something too abstract or theoretical. The meaningfulness may be enhanced by manipulative objects, games, or real-life examples.

A belief was encouraged that a good task should not be so easy as to be readily solvable without thought, nor too hard as to frustrate students. It should encourage students to be thoughtful and allow them to think in ways that are natural to them and are likely to often differ from each other. It should also allow multiple entry points, i.e., students who can do a difficult or multi-step problem without assistance either from the teacher or from concrete materials or visual aids should be able to work on their own, while students who need assistance should be able to get it. Tasks should also incorporate a review of previously learned material.

Through these processes students start to learn to judge what makes thinking strong or weak, an explanation clear or murky, or an argument persuasive or not. It allows students to formulate questions or comments. These are, after all, the building blocks for thinking and reasoning.

One of the elements unleashed by this sharing process, is that students inevitably have different ideas from each other; their thinking is divergent. This divergence will introduce complications and confusions to the lesson and, in certain ways, will make the teacher's job harder.

Angela encouraged her students to think divergently because she had come to believe that this type of thinking reflected their true understandings, coming from within them. This belief carried with it a conviction that students would thereby gain a more meaningful and durable grasp of the ideas in the long run and was well worth making the teacher's job harder in the short run.

Yet, as mentioned above, she projected a certain ambivalence for, at times, she did not encourage her students to think divergently: she told them what to do and therefore, to some extent, what to think. At these times, the researcher believes, she was reverting to the beliefs of her childhood, to a time when she did not want to think, she wanted to be told what to do, to make things simple and correct.

The teacher asked the coach to do the wrap-up. There are at least two antecedents for the importance the teacher places on the wrap-up and her wish to have him do it. One is the teacher's belief that students should be making sense of the lesson. Another is the belief in setting a context for the learning. Not only should the wrap-up reinforce ideas from the just-completed lesson, but it can provoke questions or set a stage for the next lesson. So the lesson becomes part of a sequence, not just an isolated moment.

In the first interview, Angela stated that one of the most helpful things the coach did for her was when he did the wrap-up. The reason she wanted him to wrap-up was "... because I feel like that's the place where I could push the kids' thinking more and I feel that's where I really need his help." [TI1, 147-149.]

On the issue of wrap-up, the coach said that while he obliged and modeled it for her, he wanted her to "imagine" what she would do, so in time she would do it for herself. [CI2, 430-435.]

Both the teacher [D, 397-409, 473-477] and the coach [CI2, 393-395, 455471] were also concerned with issues of planning and sequencing. These concerns certainly impacted the wrap-up.

Taking a broad look we see Angela was very serious about debriefing. Her seriousness and energy can be seen as having antecedents in at least two beliefs. One is her emerging belief that she is competent as a math teacher and improving, that the subject is within her capability. Another is that she should reflect on her own practice as a math teacher.

The coaching regimen and academic research emphasize the importance of reflection. The idea of a debriefing session focusing on a particular lesson is an invitation to reflect on what has taken place, including the words and actions of the teacher. Also, the planning session among the grade level teachers (which is not analyzed in this study) is part of a cycle where self-reflection is an integral element.

Furthermore, both the teacher and the coach clearly report progress that Angela has made and attribute it largely due to the coaching, and say so [TI1, 135-140; Cl2, 393-397; D, 455-471.]. So this is great feedback and clear incentive for her to keep improving and the debriefing is unquestionably an important piece of that process.

One indicator of her investment in the debriefing and in bettering herself was the record of student work, including words they had used and representations they had made. Antecedents for the practice were in her beliefs
that she should examine students' work closely and that she should reflect on her own practice.

One way of looking closely at students' work is by making a record of it. She could also have collected it from students, but she did not do that. But her notes, which the coach thought were "awesome," did enable her, along with the coach, to examine ideas with a powerful lens and a good amount of time.

These notes also permitted the teacher to reflect on her own practice, either by utilizing them in the debriefing session, alongside the coach, or perhaps even recalling what she had said in class, or using them to plan or make examples or comparisons for a future lesson.

### 7.4.3.3. Knowledge: The teacher exhibited growth in mathematical and pedagogical content knowledge.

There were several conversations in the lesson and debriefing regarding the way equivalency is determined in fractions. In the debriefing session, the teacher tells the coach what had happened after the coach had left the previous days' wrap-up session. The excitement in her voice was evident [D, 50-109]. Students made very different representations to show sixths, showing thinking that varied from an array model (where a student, using gridded paper, divided a rectangle into boxes and counted them to show equality [D, 65]), to others with congruent rectangles, where the teacher had made drawings based either on students' drawings or paper folding showing rectangles with squarish sixths [D, 96] or skinny sixths [D, 100], or even an imaginative zig-zag pattern [D, 105].

In the wrap-up, the coach provoked a class conversation by focusing on a student who had folded a piece of paper into a shape that had no right angles [L, 474-547]. Students had to "flip" a template to show equivalency. Also at the end of the wrap-up the coach had made a non-standard drawing (see Figure 7) and asked the question: "Can I call these sixths?" [L, 596].

In another instance the teacher made a drawing taken from an earlier class assignment where the issue was whether a triangle could be equivalent in size to a rectangle and how it could be proved (see Figure 12.). The teacher and coach, as well as the class, had a conversation about the relationship between congruence and equivalence [D, 241-280].

In all of these situations the teacher appeared to gain knowledge, both mathematical and pedagogical.

The teacher gained some mathematical knowledge when she asked the coach about mixed fractions [D, 518-520].

In response to a direct question, Angela stated in her first interview that she felt that the coaching was really helping more with mathematical ideas than with pedagogical ideas [TI1, 150-154]. But this response might have merely reflected her perspective that she was an experienced teacher, having had 10 or 11 years in the classroom [TI1, 71], and, given her longstanding negative feelings about math, that a boost in that area was more important than pedagogy. When one looks at the first coach interview, however, one gets a different perspective: many of the topics he wanted to work with her on could be categorized as
pedagogical content knowledge, e.g., planning and sequencing of lessons, animating her to think of the class as more of a community, and helping her to gain more confidence in herself. [See section 7.2.2 in the summary of the coach's interviews.]

Indeed the coach's perspective on the intent of the coaching seems to be bolstered when one examines, Content-Focused Coaching, the book that all the coaches were given and which served as their primary manual. Mathematical knowledge is only one of the teacher needs that coaches are expected to assess:

Specifically coaches will assess the following teachers' needs: content knowledge and disposition toward mathematics; pedagogical knowledge and underlying beliefs about learning; pedagogical content knowledge; diagnosis of children's thinking and assessing prior knowledge; and habits of planning and engagement with curriculum materials (West \& Staub, 2003, p. 19).

Angela stated in one of her interviews that she was spending more class time on math than she had before. This practice seems to flow from a belief that math has become a priority for her. [TI1, 168-170], and before that from new attitudes that she likes math more than she used to - she even said math is "fun"- and that her self-confidence in teaching it is growing.

Angela's lesson planning improved. The coach, when asked what he was focusing on with Angela, said in his second interview that the problem she had identified as "most pressing" was planning and sequencing of lessons. [Cl2, 393397.] Curiously, in her interviews, the teacher did not mention this as an area of particular concern. But she is concerned with sequencing as can be determined by the number of occasions where she brings it up. She talked about planning as
it pertained to wrap-up [TI1, 146-149] and to moving from informal to formal knowledge [D, 180-185]. She asked the coach what he thought about introducing one type of fraction when the class was working on another; he thought that would be a mistake. [D, 475-478.] She also talked about the importance she places on planning when she talked with her colleagues [TI1, 106-109.] So it is clear that Angela has a strong belief in the importance of sequencing that has served as a catalyst to her learning more about it. Matt expresses a strong opinion that Angela's thinking about planning has improved significantly since he started working with her.

I thought it was really impressive that she was able to show the progression of lessons that she was following, and as I was looking at that progression it made a lot more sense than previous progressions that she had used. I don't know if she had planned those out by herself, but whether she's planning it by herself or not, um...What it says to me is that she's aware of it, and she's...Whether it's coming from her or not, she's got the resources to have coherent, cohesive, uh, sequences of lessons. [Cl2, 465471.]

## Chapter 8 A Critical Look at the Study: Limitations, Suggestions for Further Research, Implications

For research to be respected, convincing, and useful, its researcher must look at the work critically. Limitations must be pointed out. Suggestions for directions in future study should be made. And the wider implications of the research must be acknowledged.

### 8.1. Limitations of this study

The data collected in this study were collected by one researcher over a period of just over two months and concentrated on one coach and one teacher. The limitations imposed by the small span of time and number of participants is clear. The small span of time meant that only one cycle of coaching was examined.

Thus there could be no direct observation of changes in teacher practices, unless they happened to take place during the course of a lesson. (They did not.) Any judgments on changes in teacher behavior depended primarily on statements made in the interviews or in the debriefing session.

Furthermore, only one camera was used during the taped sessions and the operator was instructed to focus primarily on the teacher. Thus students were rarely observed during the lesson. In addition the protocol called for students to be excluded from being studied directly.

### 8.2. Suggestions for further research

In 2008 the National Mathematics Advisory Panel of the U.S. Department of Education recommended further research into the efficacy of professional development for teachers of mathematics at every level. And this study itself gives rise to several ideas for further study. Four major areas are ripe for examination; they are set forth below.

### 8.2.1. A larger and longer study.

Presumably the primary goal of any professional development intervention is to change the teaching practices of teachers and thereby to improve students' learning. In order to accomplish these goals a research project would be better served by spending enough time observing teachers to observe whether they are changing their practices, determining in what ways they are or are not, and ascertaining whether coaching might be a major cause. In this process, the techniques of several coaches, and a comparison of their methods would be invaluable. And, of course, different teachers in different settings would be necessary to indicate whether the process is potentially useful on a wide basis. Along with the size and length of the study would go other amenities, i.e., more observers, more cameras.

One amenity that might be especially influential would be showing the videos to the teachers. Maher (2008) talks about the value of videotapes as a teaching tool, that the process is particularly fruitful in getting teachers to reflect on past practices.

This study lasted approximately three months. The belifes, practices, and other behaviors pf the teacher were observed or stated in interviews only briefly. Angela was a person in transition, thus it is not possible to say in such a 'glimpse" how durable her conduct was. A longer study is necessary in order the test her fragility.

### 8.2.2. The effect of coaching on students.

This study did not have as a goal the study of students when their teachers were in a coaching situation, yet such an examination would be invaluable. As stated above, improvement in student learning is the ultimate goal of a coaching system or any professional development. The type of system is only a pathway to that goal.

If changes in teacher practice can be found, it would be very useful to correlate them with changes in students learning, including student behavior, writings, interviews, and test scores. Concomitantly it would be useful to examine what kinds of student learning are most susceptible to change and what kinds most resistant.

### 8.2.3. The cost of coaching.

Coaching is clearly a very labor-intensive type of professional development, depending as it does on a great deal of one-on-one time between a teacher and a coach. Is it worth the expenditures of time and money? Are there more cost-effective systems? Or might coaching be able to overcome arguments of high cost by evidence of its profound and enduring impact?

Communities, administrators (including school boards), parents, and opinion leaders, including politicians, are crucial elements in the success of such programs. A key question must be dealt with: Would the community support a system that would likely be costly, even though it has the potential for profound change?

### 8.2.4 Coaching compared to other forms of professional development.

How does coaching compare to other forms of professional development, in terms of cost and effectiveness? Are some forms better in some settings or cultures than other forms? All of these questions are worth asking and answering.

Perhaps a hybrid form of coaching could be developed that preserves the essential points of coaching and makes it more practical for schools and an economy that are very attuned to issues of money and time. Might the researcher suggest that the essential parts of coaching have to do with the intense relationship between an expert and a teacher, not on a fleeting basis, but one that has the opportunity to build up trust and mutual respect. Perhaps that relationship needs to be fostered also with other supportive people in the school community who can serve as a support team when the coach is not present. In this way the teacher can become reflective, independent, and resourceful, incorporating aspects of other professional approaches. Perhaps even such a form of professional development already exists, but has not been documented or publicized.

### 8.3. Implications

This study looked at a single case of a teacher and coach working together in a particular setting. How applicable is it to other settings? Do the major participants - the teacher and the coach - and the various settings - the classroom and the school - represent typical people and places, or are they idiosyncratic in some way? These questions cannot be answered by this study. We cannot know whether this one case is representative of any other setting.

Yet, it is the strong belief of this researcher that, if mathematics learning is going to improve in the United States, it will not primarily be due to better theories of learning, better textbooks, better tests, or better test scores, although all these may play a role. It will be due primarily to better teaching.

Good math teaching is indispensable to math learning. This country has never had enough good math teachers, but we finally seem to be asking why that is and trying to do something serious about it. The New York City coaching regimen that this study looked at, was but one example of that seriousness. While the program in its original form ended, the emphasis placed on effective professional development of teachers has not and indeed has continued to burgeon.

So what can be said about coaching, which is after all, only one form of professional development? The findings in this study, limited as they are, make clear that math coaching can be a positive influence on one teacher's classroom behavior by changing her attitudes and beliefs. Of course, it helps tremendously,
perhaps it is necessary, to have certain circumstances in place, as in the school studied here, namely a willing, enthusiastic, refective teacher, a supportive principal and environment, and a respectful, knowledgeable, dedicated coach. But it is undeniable that this teacher went through tremendous changes in attitudes and beliefs and that more changes in practice will accompany them over time. That, in and of itself, has significance.

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## Appendix A

## Transcript of First Teacher Interview (TI\#1), 5/10/04

Angela, a teacher at the Melville School who taught a combined $3^{\text {rd }}$ and $4^{\text {th }}$ grade class was interviewed twice. Below is the transcript of the first interview, recorded on 5/10/04. References to this transcript in the body of the dissertation start with the letters T1\#1, ending with line numbers.

Researcher (R): I am at Melville School interviewing Angela. I have some questions in three basic, um, areas: Your background as a student, the teaching experience, and the coaching process. First question: if you had to pick one subject as your favorite subject in school, what would it be? And why? Teacher (T): Um...Do I get to say art?

R: Sure!

T: That was by far my favorite subject in school because I love it and it's something I've always done, and I have an affinity for it and it comes easy me. So I would say that that was...That that still is my favorite thing to do.

R: How did you feel about math?
T: Um...I was a very bad, very poor math student. Um...All through school I, when I was in high school I was in what was considered the remedial, or slow, math class. Um...I actually had a wonderful teacher so I started to enjoy it more because the teacher was so good. But math was always really hard for me. Um...I ended up doing pre-engineering work when I graduated college, where I had to take a lot more math. Um...And it was very hard and very painful. Uh...My
dad is an incredible mathematician, so is my brother, so it was really an odd thing that I didn't, didn't...

R: What made that math teacher good? The remedial math teacher?
T: She was very clear, she was very nice. She was a kind person. I don't know...She was just a very nice, warm, caring person who just laid it out very clearly. So it was easy for me to understand.

R: Did you have any other math teachers that you would consider good math teachers?

T: No.
R: How about a particularly poor math teacher?
T: Yes.
R: What made him or her poor?
T: Um...I remember my...In high school before I got into the ninth teacher's math class, um, he was a really angry, mean, drunk guy. And he would just come in with a hangover every day and just scream at everybody. So that was my experience in math and, you know, if you didn't get it you didn't get it, and that was, that was the end of it.

R: Well, uh...I have a question about what math courses did you take in college or after college. And you've partly answered that, but do you have anything to add?

T: Uh...when I was thinking about going into engineering I took some pre-calculus type classes...Um...

R: In college?

First Teacher Interview

T: Yeah, I guess it would be...It wasn't graduate work, I had already finished my Bachelor's, and in college undergrad I didn't take any math. I didn't have to and so I didn't take it. So once I got my BA I then went back and took a couple of precalculus classes. Um...And then in graduate school for education I took math methods - you know, how to teach math to kids. Um...Which was very fun, but that was sort of at my math level at the time, so, you know...

R: So that would be another example...Was that a good teacher?
T: Yes, it was very fun. Very fun, and we did a lot of hands-on work, um...
R: Where, where was...?
T: At Hunter.
R: Now l'm going to ask you about your experience as a teacher. How did you come to choose teaching as your career?

T: Um...I don't know. I had helped a friend of mine who is an artist...She was working in a school and needed me to help her one day. And, um...I loved it. I just loved being with the kids. I always babysat as a, as a young person, and I really enjoyed it, and I had a real affinity towards the kids, it was really just very fun. And I was painting at the time and waiting tables, and I thought, "Oh, I'll try this." And, uh...I just loved it. I really loved it. And it was sort of, I don't know. It sort of found me in a weird way.

R: The sort of teaching positions that you've had - how many years have you taught, what grade levels have you taught?

T: Um...I started working with kids in pre-school, like three and four year olds, teaching art to the pre-schoolers.

R: In what setting?
T: In a private pre-school. Um...And then I started substitute teaching throughout the city while I was getting my Master's, and I actually started subbing here, at Melville School, while I was, you know, working my way through grad school, and um...Never knew schools like this existed. I had done my student teaching placements in very traditional schools and I completely fell in love with the school. I thought, "This is how kids have to learn." And so pursued getting a job here, so I was very, very fortunate that this was my first full-time teaching job. I started teaching first and second grade, and I did that for I think two or three years. Then we had, like, a weird, uh, clump of kids in the second grade or third grade one year, and so we had to reconfigure our grade. And so I spent one year teaching second and third together, um, and then moved to third and fourth. And I've been there...I'm going to say this is sixth, or fifth, year or so as a fourth teacher (?).

R: So you've been here for eight or nine...
T: I took off...This would've been, my I think my tenth or eleventh year. I'd have to do the numbers. But I took off three years altogether when I had my children. So I took off a year, and then two when I had my kids.

R: And the sort of students you've taught?
T: Um...They've changed a lot. Uh...When the school was first starting out, um...We had kids I would say were mostly coming from educated, middle-class, sort of alternative families who were looking for a different way to...For their kids to be educated. Who were really behind the progressive school movement.

Um...And kids were pretty well behaved I would say, and came from pretty intact families, um...And that's changed a lot over the years. Um...We're getting a much more diverse, I think much more diverse, population now. We get families who don't necessarily understand what Progressive Ed. is, but sort of like the feel of the school, but l'm not sure are completely behind it philosophically. So we have a clash there a lot of times, um...I find that we're getting a lot more kids with learning disabilities than we ever used to. Uh...Kids with attention issues, uh...A lot of behavior stuff that wasn't necessarily there in my first few years. I find that it's much tougher than it used to be.

R: And do you think that has to do with your population of kids?
T: I don't...You know, it's hard to say. I mean, I think some of it's cultural, I think kids are being asked to grow up a lot faster than they were...I don't know. I mean, I couldn't really begin to say why that is.

R: What do you think is important in teaching math? Please explain. T: Oh...First of all, that kids aren't afraid of it. That they're not intimidated by it. Um...I think it gets a really bad rap, um, or rep. That it become something that kids can approach from wherever their understanding is, that whichever way they may need to make sense of numbers and space and problem solving - that that's OK. Um...That it should be fun. I want kids to think of math as fun, because it is for me now, and it never was. Um...I want kids to realize that they can all do it, that you're not necessarily stupid if it doesn't make sense to you on some level, that you just have to figure out another way to approach it. Um...

R: Uh...I know the answer to this is 'yes,' but could you explain it: Do you meet with other teachers to discuss math issues?

T: Yes. All the time. I mean, I do feel like because math has never been my strong suit, uh, and we are a school that collaborates a lot, um...That's the one thing that I do go to my colleagues for probably more than other things. You know, I don't feel as, uh, inadequate in other areas as I do in math. And so...And I happen to work with two really strong math teachers. Both Susan and Barbara are really great at teaching math, and so I go to them a lot and I ask for their help a lot, and just laying things out in a clear way, and, um, sequencing of, of different units, you know. I go to them for help with that. How to sort of lay it out in a way that makes sense sequentially.

R: So you have a strong rapport with them?
T: Yes.

R: And does that precede a coach, uh, Matt being here?
T: Yes, yes.

R: Um...
T: I think because we didn't have a coach before, we were all we had. So we were like, we got used to sort of relying on each other, and that's just the way it is.

R: OK, now I'm going to ask you some questions about your experience with the coaching process. How many coaching cycles - observation and postconference is kind of a cycle - have you had?

T: I guess we started working with Matt around, um...I'm going to say February.

First Teacher Interview

R: Oh! You didn't work with him in the fall?

T: No, we didn't. He was just working with the lower grades. Um...So we've only been working with him since February, and I guess he comes every two weeks, so whatever that ends up being. And we've met with him, um...Once a week individually I meet with him, and then we meet as a grade, so I guess that would be two times, every other week, that he comes.

R: And when you say "meet with him individually" you mean...?
T: I have my own little private session with Matt.
R: And then he observes your class, or...?
T : No, what we do is he will come spend time in my room. Either he'll watch me do something or I'll watch him do something, and then we meet after, at the end of that day to talk about it. And usually we try to put in a little bit of time to try to think about what we're going to do the next time.

R: So how would you say the coaching has gone?
T: It's been really nice; it's been really helpful. l've really gotten a lot out of it, and it's just nice having somebody I know I can touch base with, and um...Who is going to help. I mean I really do feel like he's a help. Um...My first day when he was in the class I felt like I had this little person on my shoulder, you know, sort of coaching me like, "Yeah! That's good" and "Why don't you try this?" and "Why don't you ask this kind of question?" And it was like he was, you know, this little shadow, and it was just amazing.

R: Did he tell you things, or was it...?

T : Yeah, yeah. As I was sort of in the middle of doing this lesson, he was just there sort of making it better for me, you know? It was really...

R: Does anything stand out as particularly helpful or unhelpful?
T : Well, that, that kind of thing is really helpful. When he lets me sort of go, go for it, and do the lesson, and he can be there sort of to catch me. That's been really nice. Um...And I also, he...I ask him a lot of times to do the wrap-up because I feel like that's the place where I could push the kids' thinking more and I feel like that's where I really need his help, and, um...So that's been really helpful too. R: Now, uh...What kinds of things...Would you say he helps you with mathematical ideas or with pedagogical ideas? Or do you have...could you...? T: I would say it's mostly mathematical ideas. Because I do feel like I've been teaching a long time, and so the sort of other side of it, I don't feel like we need to spend that much time on. Although in the way that he gets me to look at kids' work, uh, spreads out into other areas of my teaching. Um...He's helped me sort of organize my thinking around how I look at kids' work, which has been really nice.

R: Have you learned anything in particular about math, math teaching or about yourself as a math teacher?

T: Well...I think, um...
R: Well I guess...
T: Yeah, how to, how to look at their work and how to sort of, um...It's been really good for me to be able to group kids in a manageable way because we have a real, um...There's a really broad spectrum of abilities right now mathematically in
my classroom. And so it's been really great to have him show me how to organize, like, what groups I can put kids into so that they're not seemingly all over the place. Um...That's been really helpful.

R: Um...Have you reinforced, modified, or changed any of your beliefs about math or math teaching as a result of coaching? Please explain.

T: Well...I feel like I do it in a more consistent and, um, I feel like I'm more consistent about my teaching. That, you know, I do try to do math all the time. It's something that's a priority, um... I feel like it's important to get kids to share their thinking, and to make that public, which I think Matt helped me to do.

Um...Because it's shown me that that's how they're making sense of what they're doing. Like if they're able to articulate it, and then I value it by putting it up there publicly, um...It's just given me a window into how they're making sense of it, and then that in turn sometimes shows me the direction that I need to go in. R: Is, uh...Well...OK. The question as written is: With regard to the teachercoaching cycle that's about to happen, is there anything in particular you're working on? But in fact, you seem to be...Of course after I watched you in the class, so is there anything in the cycle you just had that you were working on in particular?

T: Well, we're actually right in the middle of, of a unit on fractions. And I think you saw us in that brainstorming session. But we're still in the middle of it.

R: Well, actually the day I, uh, observed you were folding fraction cards.

## First Teacher Interview

T: Right. And Matt and I have met to sort of talk about the direction that it can go in, and the kinds of things that I want to make sure I hit on. And so we're doing that right now, we're sort of in the middle of it now.

R: How do you think coaching compares with other forms of professional development?

T: I prefer it...
R: Why?
T: Because there's immediate feedback, and it's, it's a constant. It's not like you go to a workshop and then, you know, you take notes on it and then hopefully you open your notebook and you try some of the stuff. This is something that just keeps coming back at you, you know? And you get a chance to try it out, and you either succeed or fail, and you talk about it and you try it again. Um...You know, it's ongoing, which makes it much more acceptable and, you know I feel like I try stuff more readily.

R: OK, that's all the questions on those topics. Thank you very much. [END OF FIRST INTERVIEW]

## Appendix B

## Transcript of Second Teacher Interview (TI\#2), 6/28/04

Angela, a teacher at the Melville School, was interviewed two times, once before teaching her lesson and once after, over the telephone on $6 / 28 / 04$. (Line numbering is continued from the numbering from the first teacher interview.) References to this transcript in the body of the dissertation start with the letters TI\#2, ending with line numbers.
[Second interview begins.]

R: It's Joe (the researcher [R]). What is today? It's June $28^{\text {th }}$ and I'm on the phone with Angela (T), and let me just hear...let me just play this back and see if you can hear it.

T: OK.
(Pause.)

R: All right. I have some questions for you which basically are a continuation of the previous interview we had. Um...First of all I'm going to ask you questions about a math lesson that was ... of yours, that was, uh, recently taught. And that's the lesson about the quilts.

T: Uh-huh.

R: Um...How did the lesson go, would you say? And please explain.
T: Um...I think that ultimately the kids understood what it was we were asking them to do. Uh...They understood equivalencies, which is what we were getting at, um...But the management of it was very hard, and if you remember there was a lot of sort of handing out of paper, um...Kids choosing colors that they wanted to use - and it was really hard to manage that. So, you know, I would've done a bunch of things differently. You know, if
we do it again...We had talked with Matt afterwards about putting just a couple of each color on a table so that kids could trade with each other and share a whole square, and then they would realize that if you only wanted a half, then somebody else could have another half of the paper, so there would be more going on within the tables and within the groups. Um...But you know it was OK. It was also very hot, if you remember, and it was, you know, probably our last math lesson of the year, so it was sort of hard to keep their interest.

R: Right. And you mentioned that this was really not the best time to have an observation. Could you explain why you feel that way?

T: Um...Because at the end of the year everybody's sort of done (speaks to child for a moment) ...people are done. The kids are done having to focus and pay attention; teachers are sort of tired and tried...And um...I think it's just very hard at the end of the year. And, um...It's also, the weather is hot and it's an old building and it's not very comfortable to be there and so it just adds to make it difficult.

R: Now aside from what you just mentioned about doing...About handing out the materials differently, uh...Are there other things that you would do differently in the class? Or did you...was the teaching basically the same?

T: You know, I think I might've modeled a little...a couple of different ways. What ended up happening is that the one that I modeled for the class ended up being the one that every - almost every kid - did, except for two. You know, two girls chose to do a different pattern, um...But everybody else in some form or another presented the one that I modeled. Um...And so I think if I were going to do it again I would've showed them a
bunch of different possibilities, um...So that there would've been a variety in their responses.

R: Have you taught a lesson like this in the past?
T: Um...Not exactly. I've done, uh, fraction quilts where, uh, we repeated the same pattern over and over again and then just put that into a whole quilt...Um...But I hadn't done this particular one before.

R: Uh...Was there something the coach said or did before the lesson that influenced you in the lesson?

T: No...Not that particular one. You know, we had, I guess, you know, one of our fractions meetings, uh, that my two colleagues and I had come up with. You know, all the things with Matt that we wanted kids to take away, uh, through the study. And one of the things was this understanding of equivalencies and adding fractions - like and unlike fractions. And so I guess because Matt sort of helped us, uh, articulate that, um...That's where this lesson came from. And trying to make sure that kids came away with the stuff we'd wanted them to come away with.

R: This is...So the coach's influence was a little more, uh, careful than it was specifically this lesson...

T : Yes, yes.
R ... It was just equivalencies being an important thing that you wanted...Thought should be emphasized.

T: Yes.

R: OK. Um...Now that's all the questions I have on, um, uh, last week's lesson. Um...I don't know if you remember, in the previous lesson that was filmed, uh...

T: What was that?
R: Uh...It had to do with...
T: That's when we made the fraction cards?
R: Exactly.
T: Yeah.
R: Yeah. Uh...I don't know if you have any memory, but let me ask you how you, if you remember, how did that lesson go?

T: Well, I think that lesson I thought went really well. It was very, very clear and very concrete, and um...All of the kids were doing exactly the same thing at exactly the same time along with me. So, you know, it was pretty hard for them to mess up. You know, we would take one, one fraction piece at a time, and so I felt like, you know...I wanted them basically to come away with a set of fraction cards so that we could use them to play some games. So...So everyone was successful, everyone was able to do that, so I thought that that went really well.

R: So were the...Were the fraction cards used subsequent to that lesson?
T: Yeah, there were a couple of games that we learned that, uh, allowed them to use the cards.

R: Um...OK, now would you basically...If you had to do that lesson again, would you do it basically the same way?

T: Yeah, I would. And I've done that exact lesson a couple of times already in previous years, so you know, from experience I knew that that was something that would work.

R: Did you do it any different this year than in the previous times?
T: No, not really.
R: Um...So that was not a lesson that was, uh, was uh, say affected by coaching? Uh...
T: No...
R: Uh...Is that...?
T: No, but, um...You know after our brainstorming session with Matt about what fractions work we wanted to do, um...It became clear that that was something that I wanted to use - that lesson and those, the games, because again it sort of satisfied those goals that we had.

R: Uh, OK. But, but, but, do your, uh...Nothing in particular was especially different in this lesson?

T: No.
R: Um...And again, does that mean...Was, was there anything that the coach said or did before that lesson that influenced you in particular that you can remember?

T: No, I don't think so.
R: Now, in our interview you mentioned that you were very interested, uh, in arts. And, uh...I, I can't help but notice in the two lessons that, that I observed, uh, that you were teaching were kind of, uh, creative or artistic. Uh...Now were kids using materials and uh, and uh, tools, uh...Is this something that has been the same all along, or is there something different this year with the coaching or...? Could you talk about that?

T: Well, you know, I guess I never really looked at it through that lens before, um...I guess, you know, I, I like to do things with my hands, I like to make stuff, and that helps me learn - when I have to make something. Um...And this year I had kids who really
needed the work to be broken down and very concrete, and so I think that doing those kinds of lessons with them, where they have to make something and construct - actually construct something - um...Made it much clearer for them. Um...You know I never really thought of it in those terms, that it was like, a creative thing. It was more just doing something with your hands and it being concrete. I don't know if that answers the question...

R: Right, right. Um...OK, well I think that's all the questions I have. Thank you, thank you very much.
A: Well good luck to you!
[END OF TAPE,

## Appendix C

## Transcript of First Coach Interview (Cl\#1), 5/3/04


#### Abstract

Matt, math coach at Melville School, was interviewed on two dates, once early and once late in the study. This is the transcript for the first interview, recorded on $5 / 3 / 04$. References to this transcript in the body of the dissertation start with the letters CI\#1, ending with line numbers.


Researcher (R): Ok, uh, this is the researcher, it's, um Monday, May $3^{\text {rd }}, 2 \mathrm{pm}$ and I am interviewing Matt, math coach at the Melville School. Alright Matt, I have a few questions for you, um, and let's start out. The questions are in the following categories: first I'll talk more about your background, and then I'll talk about the coaching process, and um, and the lessons observed today. Ok, concerning your background, how did you come to be interested in math? What did you like (or dislike) about math?

Coach (C): Um, I always liked math, actually, as a, as a young child, I don't know maybe it was just the way they illustrated the math, uh, workbooks, but I, I preferred using a, I preferred working in a math workbook to coloring a coloring book, and uh, uh I remember then someone once, you know, asking if I were smart I would know what nine times nine was and I remember wanting to be smart, and uh, and from there, I don't know, I just, I guess that's, that's not really the only thing... I mean I really liked, uh ... I always expected to make sense of math, and I gave myself permission from a very early age to expect it to make sense and when teachers were talking about math in ways that didn't make sense, I kind of went the extra step to, uh, try to figure out what math, um, so that actually got me into tutoring when I was like in fourth grade. I would tutor other kids
in the class cause I felt like adults have this one language for talking about math and that I was able to really easily translate between what the adults were saying and what, what it really meant.

R: And you, you've said attitude of, uh, feeling empowered in math, did that come from within you or were you encouraged by someone or something happened, something any of your teachers had said.

C: I wouldn't say I was encouraged by my teachers. In fact I'd say I was discouraged by my teachers. There were, I remember in third grade, uh ... We were just starting multiplication after a while of division, of, sorry, of addition, and um, we had a problem like 36 times 4, for example, and I remember distinctly, uh, writing 36 four times and adding it up to figure it out, and the teacher explained it a different way, and I raised my hand excitedly and told the teacher uh, hey look, I figured out another way to do it and I got the same answer, and I was thrilled with my discovery, and uh, she came over, looked at my work and said, that's not how we do it here. So, I was not encouraged by teachers in elementary school around math, but um, and I don't know that I was explicitly encouraged to be into math by my, by my mother, but she was always about making sense and giving yourself permission to make sense above following the rules, per se. R: That sounds unusual to me, and I'm wondering, uh, if you could pinpoint anything in your mother's makeup that would account for that, uh, attitude.

C: She, she's an artist, she's a rebel, uh, she left for Mexico when she was eighteen-years-old.

R: From the...

C: From the United States. Um, uh, got kicked out of college for smoking pot, I think, (laughs) and uh, went to Mexico and uh, got her bachelors and masters in fine arts, painted with Siquieros, came back, uh, yeah, and uh, had two kids, marriage didn't work out, got divorced, came back, she's always been a really independent spirit, creative, honest, hardworking but in a very a, in a way that doesn't give in to status quo and just following, you know.

R: But that ... You also sometimes hear that artists don't feel comfortable about mathematics, and uh ... Did she feel comfortable?

C: No she didn't, she didn't. She actually ... uh ... She tried to help me with my homework and it was always a fight and she'd say well, this is how you do it and I'd say, no that's not how we do it in school or, and she'd say well, you've got ways of doing things that I don't even understand. Eventually, we would fight a lot about math homework, but I think eventually she just started to kind of trust what I was, what I was doing (laughing), and I think one time she even said, uh oh, um, I learn from you, you know, so it was.

R: Well now the part about, uh, encouraging you to reason about the math, that was, where did that come from, do you think?

C: It wasn't ... I don't think she specifically encouraged me to reason about math, she just encouraged reasoning about life in general, she was, she was always, uh, really, I don't know, she, I think, because uh, she was my only parent, we spent a lot of time together and so I think, I watched her negotiate life and make sense of things and life, you know, and so I just, I took that as a (pause) I took that as a, as given that that's what
you're supposed to do, you're supposed to make sense of this world, not just follow blindly.

R: Do you remember how you felt about yourself as a math student?
C: I always knew I was a good math student, um, I meant I didn't think I was all that exceptional, I, you know I felt like I did just fine uh, in elementary school, uh, you know honestly I think standardized tests reinforced my feelings about being good at math more than anything else, which is kind of crazy because uh, I don't feel, I don't feel that way about them now necessarily, although I know that I've seen kids who really want that kind of, uh, reinforcement, you know, want to be told that they're doing well by some overt measure and uh, I remember I would score in the $99^{\text {th }}$ percentile on the standardized tests and uh, and I'd ask, so what does that mean, you know I didn't know what percentiles were, and they basically explained to me, well if 100 people take this test, you'll do better than 99 of them, and so, so it was like, that, that made me feel good, but really, I don't know I think, I think that the tests, again were about giving yourself permission to make sense because honestly, I remember coming across lots of problems on the tests and thinking, we didn't do this in class, this doesn't look like anything I've ever seen before, and saying, saying oh, well let me see, what do I think they're trying to get at, what do I think they want from me, and I don't know, I always did well, and I think I actually carry that into my teaching too. I expect people to, I don't, I don't believe that just following the curriculum is what, you know, mindlessly is what gives you the ability to do well on tests. I think you have to really give yourself permission to make sense of what's in front of you.

R: What mathematics courses did you take in college or after college?

C: Oh, sorry, one other thing. In middle school I did, realizing that I, that I liked math, I think that's when I realized, and I actually ended up joining a math team in my middle school, there was like, I didn't really like my math teacher, I thought he was boring, but um, but I did, but there was another math teacher who was coaching this uh, math team and we would kind of get together, there were maybe six or eight of us who would get together and work out math problems, and it was kind of an alternative curriculum, uh, that we would work on to prepare for these math meets. And um...

R : Like the Olympiad or something?
C: Yeah, yeah, and I, I loved, you know the thing that I loved the most about that was that it was kind of a workshopy feel in the sense that everybody would go off and work on their own and then we'd come back together and share how we did it and invariably someone else's way of thinking about it was different from mine and would make me think about it in a different way and so I would look at that and say, oh, I never thought of approaching this that way, you know, and I, I was able, it really brought out the creativity behind mathematical thinking and the art of, you know, thinking mathematically, it was a really, I'd say that was a really important experience for me.

R: Great.

C: So but you wanted to know...
R : What math courses in college or after college.
C: You know in high school I had taken, um, (pause) because I did so well on standardized tests, and middle school I was on an accelerated track, took algebra in ninth, in eighth grade, um and passed the Regents, and then in high school I took calculus and I did really well on the first semester, and on the second semester I had senioritis and you
know, it was more about just fooling around and having a good time, and um, so I didn't test out of doing, um, out of taking calculus again, and in a way it was a good thing because I took it again in college and uh, I really enjoyed it, but I studied economics in college, so there was some math there, but um, and what I, what I, the strength that I brought with me to the economics were, was, really striving to understand what each part of each formula had to do with anything, rather than just memorizing the formula and just applying it to situations. Um, and so I felt like I had a really good, uh, intuitive grasp of the relationships behind the various formulas, but um, I didn't take a ton of, I didn't take a lot of advanced math, I did, I took statistics as well, but um, I don't, I didn't take much beyond that. Um, in graduate school at bank, I mean I also studied a lot of math education, um, I went to, uh, Math in the Cities course where, um...

R: What, what was that?
C: When?
R : What is that?
C: Oh what is that. Um, Math in the City is a grant fund, is an NSF grant-funded, uh, collaboration between some ebullient (?) scholars from the Freudenthal Institute in the Netherlands who, who have developed a realistic approach to mathematics education and Kathy Fosnot here at City College in New York, um, who has a solid background in constructive, a PHD study in constructivism and she'd also done some work with Debra Shifter, up at, I think Holyoke, where I think...

R: Mount Holyoke College?
C: Yeah, I think they were doing summer math for teachers looking at in-service work, that would support teachers' ability to teach math. Um...

R: And how long was that, how long were you in Math in the City project?
C: It, you know I started there, um, my second year of teaching, the second summer after I began teaching, um, and, I took, I took the summer institute which is a two week institute designed to kind of help people shift their paradigm and begin looking at, uh, children's work and children's thinking much more closely and beginning to think about what learning lines could look like and then I took it a second time as part of a bigger project which Kathy Fisner had embarked on where there was additional follow-up coursework during the year as well as, uh, mentoring by faculty from the project, um, so that was for another, I think three years, and then I also, as part of that first group, began to, uh, co-teach courses, the same courses that I had taken, summer institutes and yearlong courses.

R : And did you get a degree from that?
C: I used it as part of a degree from City College, yeah.
R: Masters in?
C: A masters, it was in, uh, technology education, it was coupled with a lot of inquiry science from the workshop center, other inquiry science from a NASA project, a lot of technology, uh, stuff, internet and uh, fairly early use of internet and networking.
R: And have you had any other math education courses?
C: Um, I also studied at Bank Street, I got a second masters there, so there was Math for Teachers, which was a really broad, general overview, I actually, I felt like the formats developed, that Math in the City helped me develop, were much deeper, more fundamental, like, uh, frameworks for making classroom decisions, and I felt like, um, Math for Teachers was more of a survey, you know kind of an overview of different, it
was like a sampler plate of different kinds of alternative math practices, but it wasn't as, uh, framework-building as Math in the City, but that was a lot more coursework at Math in the City, and then we also took, uh, a series of three integrated math courses at Bank Street.

R: Did you go through a degree program at Bank Street? What was that?
C: It was the Math Leadership program.
R: Oh.
C: Yeah.
R:And that was a, that's a summer program, could you explain that a little bit?
C: Three derives (?) plus some coursework during the year, um, and it's I think 45, 48 credits, um, heavily focused on math, um, and math teaching, uh, there's a curriculum course with Anna (?) on designing curriculum, a professional development course where you're looking at various models of professional development, there's a, two, one course on designing small schools, small democratic schools, one course on, uh, on high spective (?) leadership and principle and, um, so you graduation with a masters in education, but if you take six additional credits, which I took, you also get a um, an administrative, you have the coursework to apply for a mid-administrative credential. R: Now, what, what grades have you taught, and how many years C: I was a, I was a fourth grade teacher for three years at a large, uh, overcrowded, uh public school, PS 115 in Washington Heights, mostly uh, bilingual kids, immigrant Dominican families.

R: Do you speak, uh, another language?

C: I speak Spanish, I was born in Mexico, um, but, I had to study Spanish, I came here when I was four, so I had to study all over again in high school and college, but um, and then I was a fifth grade teacher at that same school for one year, and they actually got me to participate in Math in the City because it was a SURR school, um ...

R:That's Schools Under...?
C: Registration Review.
R: Under Registration Review.
C: They were, they had tried before kind of corrective action, sort of, and um, and so they had opportunities to go, we had opportunities to various district-initiated coursework, so that's how, that's why I got to work at, participate at the workshop center and with Math in the City, um, and the, so I was there for fours, but when, it, when the practices I was learning started to clash with the pacing calendars and the rigid curriculum mandates, you know, be on this page on this day, um, mindless work, a lot of it, and where test prep was a curriculum area, it was, valued above science, above social studies, sometimes about math, um, I started to reshape my practice there, but I clashed too much with the administration, I mean we got along just fine, but they, they really would have preferred that I just get on board and organizing efforts, I was chairing in a math science restructuring committee there, and my efforts to try and organize other people around thoughtful practice weren't, I wouldn't say they were badly received, but they weren't supported, they were permitted, but not supported, occasionally they were clashes and I just, I realized I needed to go somewhere where they were going to allow me to exercise these new ideas and even support them, so then I taught for 3 more years at River East Elementary School, it's a small progressive school in East Harlem, um, and
there I was given pretty much free rein and I got to learn a lot about progressive schooling, um, it got me to participate at Prospect Institute, and um, which is an institute on descriptive inquiry, um, and it got me to go to Teachers College to learn about the writing project, reading/writing project, and it allowed me to continue my relationship with City College, with Math in the City. That's actually how I met Sam Morris the principal at River East was through Math in the City, he had been, he as the principal was attending with some of his teachers, so, so I felt like I was maintaining some of my connections but I was going to a place that would allow me to practice the pedagogy of those, uh, other alternative ways of teaching, so, and then, and then I had an opportunity to join a coaching staff at, in District 3, it was someone else who had been in Math in the City with me, Terry Kay, she uh, she invited me to come join her in District 3 because she, I felt like we were like-minded enough that we could really make some, find a place to get some of the ideas from Math in the City to take root, um, and so, so I couldn't pass up that opportunity. I loved teaching math and I was actually teaching one of the Math in the City classes out of my classroom, so people from District 4 would come to my classroom one day a week and sit and patina (?), and I other, other co-teachers from Math in the City and one or two other people who came sometimes, uh, would co-teach this class in my, out of my classroom. It was really great.

R: So what would you do in District 3?
C: Oh, in District 3 I was a math staff developer, so I worked in, anywhere from 3-5
public schools for (pause) 4 years, um, and, some of my school were large, like 870 kids, five, six teachers on a grade, some of my schools were small, um, maybe 250 kids, uh, some of them were, had gifted and talented programs and kids who were scoring off the
charts on their tests, and some of the schools were, uh, actually shut down and redesigned, so I worked at a wide range of places and got to see really the full spectrum of, um, of (pause) school organization, children's needs, uh, teaching styles and teaching abilities, um, and worked with, worked with all of it and it was really an incredible learning experience. I mean, I felt like I contributed a lot. I think my biggest contribution there was the idea that I could go into any classroom in any school and do math work with people, you know, and kids would get excited, teachers would get excited, and it felt like (pause) it was an up, it let people know that um, it's, that no matter what label people put on you, your school, your kids, good stuff can happen, and you just have to have an understanding of what makes it, what makes teaching good and good teaching can happen, and good learning can happen anywhere.

R: Have you had any other math experiences that prepared you to be a coach?
C: Um, I would say that, that I learned a lot, I mean, the things that prepared me most was the co-teaching from Math in the City, um, I think Karen Fisher, part of, at one point she realized that her initial intention was to develop teachers' knowledge and teachers' teaching ability in math, but she realized that she was at the same time also producing leaders in math, and um, and so she deliberately built into her model the development of, of leaders and leadership and she was, she invited us and involved us in, um, co-teaching classes, um, and so she gave me the opportunity, but then working along side, um, Lucy West, you know in the early days when she had just finished her own coursework at Math in the City, um, so we were first playing with these ideas together and I think she had done a lot of work with Marilyn Burns before that, so, um, but also Myron, uh, Dahl from the Freudentha; Institute and Willard from the Freudenthal Institute, they were
incredible, incredible teachers, and their, you know, being from another country, from another culture, they brought something to the work that was really unfamiliar, but at the same time, uh, gut-wrenchingly resonating, well I don't know if that's the right, anyway it was really familiar the same time, they, it was so compelling and clearly legitimate, you could see it on their faces, on the faces of the children they were working with, even though they were speaking Dutch, you know there were connections being made, there was this mutual respect and deep understanding of what teaching and learning is all about, and so, I learned a lot from, from them as well.

R : In general, what are your hopes and expectations for the coaching process?
C: Um, I think about was my life was like in the classroom and it was hard, it was really hard, um, and so I think that my first priority is how can I make life a little bit better for teachers, a little bit easier, and sometimes that means helping to make some materials and make copies or whatever, but it really means, to me it means, helping them develop the teaching tools that make the work not so scary, but even if it's hard work, even if it's exhausting, they have, they have an idea of what (long pause) what good teaching looks like and what it's about, and how, and that they, that they see that they have access to good teaching, they step by step are making, making their way towards the best teaching possible and understanding why it's, what's good about it and why it's good and how to make the best decisions they can so they can feel really proud of the work they do with kids.

R: How many teachers are you coaching now?
C: Um, when I was in District 3 I was coaching probably about 30-35 teachers in a year, not all at once always, but, uh, in a year I would work with anywhere from 30 to 35 .

Now I'm working with, um, uh, 20 classroom teachers and 2 core, 3 sets teachers, and I was also mentoring 2 new coaches, so via them I was working with another (pause) 20 teachers, but really that, I, I stopped that in February, but I was, you know, I wasn't directly responsible for those 20 teachers. And if you count workshops, you know, it's a lot more.

R : But in terms of strict coaching?
C: 20 teachers.
R : How is that going?
C: Um (small laugh), I think it's going really well, I think, I'm learning a lot this year, uh, I'm, I'm (pause)...

R: Well, let me clarify, that, those teachers are in two schools.
C: Two schools. So there are 11 at School 1 and there are 9 at the East Village community school. I deliberately chose to work in two small progressive schools, um, because, I have a history with progressive schools and I know what some of the pitfalls are and if things get, get, people on the outside perceive to make, to be problematic about progressive schools, um and so partly I want to help them be more clearly compelling to people on the outside, um, because I think there are some really important and special practices that go on at these schools that need to be appreciated, um, and so I want to help give them the tools for being able to, uh, to practice thoughtful teaching. Um, at the same, at the same, (pause) uh at the same time there's a lot that goes on in these schools that I feel I can learn from too, um, so I feel like it's a really mutual exchange, I mean I'm, I'm happy working anywhere, I love to teach, I love working with teachers, so I would have been happy anywhere, I've worked in all kinds of schools in District 3, um,
and was happy in all of them, but this was a deliberate move to try to, kind of accomplish something over the next few years, um, and, uh, so I, in my mind it's a multi-year commitment, um, and (pause) and I'm getting to try things with, with the groups of teachers that I work with that I hadn't gotten to try before, so I feel like I'm taking my work to a new level while helping them take their work to a new level, it's a really, uh, it's a great mutual exchange I think.

R: Do you meet with other coaches or lead coaches, and please explain that process. C: I do I meet with, um, I meet with both other coaches and lead coaches and risses uh,...

R: Well are you, do you, are you called a coach or a lead coach?
C: I, I'm a lead coach, I mean basically we're all called coaches, but um, yeah I am a lead coach that, so the extra responsibilities of lead coaching was, uh, were mentoring to 2 new coaches, um, and taking the lead on planning, uh, and delivering workshops, regional, region-wide workshops.

Someone else enters room: Hi guys.
C: Hey, could you pause it for a minute.
[Tape recorder turned off, then turned back on.]
R: Ok, continue, uh, the interview with Matt. You may have answered this, either all or in part, but let me, let me ask you, but you were talking about your meetings with other coaches and uh, this is, uh, along the same idea. Have you been having workshops or other professional development experiences, and how have they been?

C: Um, this year we've, we, as lead coaches, well, one of the things that, that guides how they're trying to develop the coaches is um, one is an apprenticeship model, so whether
it's us, whether it's lead coaches, well, one thing is, never do anything by yourself, so we do things in teams quite a bit, um, and so we always have the benefit of each other's thinking to help us through things, and we also have the risses, the, typically more experiences others, uh, participating on some of the planning, and then we're also supporting new coaches as they, and they, they get filtered into the planning with, with us, so when we were planning, um, curriculum workshops to introduce people to the new curriculum, uh, we, uh, a group of lead coaches met maybe once, then, then we also worked with, uh, newer coaches, um, several times to plan a workshop, um, and then, uh, and the risses co-planned parts of that with us, and then uh, and then we delivered the workshop multiple times, so we each played in, in giving the workshop, uh, transferring the leadership from, of the workshops, from the most experienced people at first, and then moving, adding more responsibility to less experienced people as we went through it so that's one form of support.

R: Now when you say curriculum, you're talking about the, the...
C: Everyday Math.
R: Everyday Math, and so the workshops were, uh, the coaches teaching other coaches for the most part.

C: Well the plan of the workshop was coaches working with and somewhat teaching other coaches about giving workshops for teachers, and so we were, we actually did the workshops with teachers, um, and uh, it was, yeah, it was, I think there were several hundred teachers that...

R: Was that, was that done, uh, when was that done and how often, well let's say, you, how many teachers might be in a workshop let's say?

C: There were maybe 30 teachers in a workshop, and there were 2 , so it was broken down by grade and location, and so, for example, for uh, second grade, there were 2 locations and there were maybe 30 teachers per location and then we gave that workshop, I think, I'm thinking maybe twice, for some reason it feels like a lot more, but, and then we also attended each...

R: Let me, if I'm a teacher, second grade teacher in one of those, do I take, how much of the workshop did I have, did I have it one day or?

C: Oh, ok, so uh, it was a full day workshop, but it was one full day workshop, and, um, the good, the really smart idea, uh, was that teachers came in teams, so they had someone to talk about the school, but they also came with, with their coaches, so any coach that was not giving the workshop who had teachers attending the workshop came with their teachers, so that they came, went back to school with those teachers and helped them implement some of these ideas. It wasn't exactly, you know, ideas that, that you just take this one idea and you go implement it, it was really more about, um, we did, we did some adult math to engage people in a math experience and think about themselves as learners, to help them formulate thinking about learners in general, and, but then we got into looking at parts of the curriculum and what the math was behind a unit, like what the big mathematical ideas were behind this one unit, and then, we did a fishbowl where we modeled a plan in process, because one of the things that we're concerned about as a region is that, um, people, people's planning, um, that there are opportunities in the planning that people just aren't familiar with capitalizing on, like, particularly at the beginning with what is the math you're going for and designing the lesson around that, um, you have time? And so we fi, we did a a fishbowl of a planning session where we
modeled what it looks like as you go through the lesson and question various parts of the lesson, everything from, you know, how do these materials support children, maybe we want to make different decisions, to how these questions, to how do the, how do the grouping structures and the transitions and the timing and, everything, just really getting deeply into planning a lesson and then, they planned together, which I think is another strength of these workshops, um, the teachers planned together, with each other, and with their coach, right there during the workshop, a continuation off of what we had been looking at in that unit, and then, uh, and then the coach was there to follow up with them back at the schools to observe, co-teach, model, continue the planning process which we had modeled there, um, so I thought it was a really powerful series of workshops, and it actually really supports what I do here, as much as I think that this curriculum is problematic, um, I think that the way that we support people in using it and taking ownership of it, and making sense of it, made for, made for a really positive opportunity, uh, with, I think, not the, possibly problematic materials.

R: The, uh curriculum you're using is?
C: Everyday Math.
R: Everyday Math. In all grades here?
C: No, it was mandated for the city, uh, K-2, and um, next year, the city had originally planned to mandate it...
(End of first coach interview, taped on 5/3/04.)

## Appendix D

## Transcript of Second Coach Interview (CI\#2), 6/17/04


#### Abstract

Matt, math coach at Melville School, was interviewed twice, once at the beginning of the study, once towards the end. This is a transcript of the second interview. (Line numbering is continued from the numbering from First Coach Interview, above.) References to this transcript in the body of the dissertation start with the letters CI\#2, ending with line numbers.


$R$ : This is the researcher. It's June $18^{\text {th }} \ldots$ June $17^{\text {th }}$. I'm interviewing Matt, the coach at Melville School 1. Matt, I asked you questions earlier about your background as a coach, now I'm going to ask you some questions about the coaching process and the lesson you observed. Um...How many coaching cycles, uh, we try to define as pre-conference, observation, and post conference - how many coaching cycles have you had?

Coach (C): Have I had with a particular teacher, or...?
R: Well...Let's say with Angela.
C: Hmm...I...
R: Could you give an estimate?
C: Yeah. I think I may have had about, uh, six with her. Probably about six. I didn't start working with her until February. Or maybe mid-January or February.

R: And how about with Susan?
C: Also about six or seven.
R: Are there any particular issues or problems that you've been working on with Angela? C: Um...I mean we, her...The problem that she identified, the issue that she identified as the most pressing for her was planning, and knowing what to do, what to do next.

Um...So, as part of our debriefing we always kind of lay out a few weeks of lessons, uh, following the one that we just observed. So planning is one big thing. I think one of the issues that I've identified that she may not have identified, uh, although I'm sure she's aware of it is...I also think, um...How she managed the group is a question for me. Um...How to get more...When I first started working with her she had, maybe, four or five go-to kids that would always answer all the questions and, um...She relied heavily on them while kind of disciplining the rest of the group. And um...And so one of the big things that I was thinking about was: How can I make this feel like more of a math community? How can I bring more voices into it? How can I make it...Oh, and the other thing that she did was, she would, um...When kids gave answers, they'd either be right or wrong, and she'd kind of speed (? I don't know what this word is: time code is about 137 on the tape) through the wrong ones and spend a little bit more time on the right ones. But it felt more about, like it was about your answer, not about your thinking process. And so I tried really hard to model and talk about ways to shift that. I never specifically said to her, "What I'd like to work on with you is your, you know, your..." Well, I guess there were times when I said specifically, "How can we get more kids into the conversation?" I tried not to pose it as, like, "This is a problem for you" but more like, "How can we get this going on, or that going on?" Because I didn't want to make her self-conscious. She's, she can be a little self-conscious, but um...I feel like I've...Another big thing is helping to boost her confidence and her, you know, the framework for making decisions so she would be more confident.

R: Have you noticed any change, uh, in that area?
C: What...

R: The area of, uh, getting more kids into the community?
C: Definitely. I mean, today for example, she made one big management mistake that is...It's the second to the last week of school...I wouldn't say that it's an atypical kind of mistake. Like she was giving out the papers individually to kids that they needed to cut up to make the fractions, and that took all of her energy and all of her focus away from observing children and their thinking, and put it all on doling out paper. And...You know, she had to, like, search for the colors, and how many, and there's too much of that kind of interaction and not enough opportunity for her to observe what is going on, um...But, at the same time, she caught that mistake right away. She said immediately to me, uh, probably off camera, uh "You know what? If I were going to do this again, I wouldn't...I would've had the papers pre-sorted." And even in our debriefing she even elaborated on that. But the other thing that she did is that she talked about, um...When I asked her, "Well, if we were going to have a wrap-up, what do you think it would look like?" Of course she wanted me to do it because I feel, I think...She still feels like she wants to see things modeled. What I don't...I keep the accountability on her by saying, "Alright, think through what you imagine it would look like. I'm happy to do it, but you need to think about what it would look like." And she said that she would have them, uh, explain and kind of justify why, uh, why they were picking the sections that they...Why, why they made the shapes, the sizes they did and how, how they knew that corresponded to different fractions. Um...I guess that doesn't really show how she would involve more voices, but um...Yeah, I guess...I don't know. I feel like she, I feel like she's moved a little bit on that, but that wasn't a good example.

R: Well, I think there was a point where one kid made an explanation and other kids were to, uh, interpret or, or repeat it. That, maybe that's a...

C: Yeah...And that, I mean you could see in my debriefing, I really deliberately am trying...You know, sometimes that's a priority above the specific mathematical content that I want to go for, like I think I could've pushed them a little bit further mathematically, I probably could've, you know from my initial assessment, walking around the class, I probably could've skipped a little sooner to, uh, Laura's quilt square...or to Celine's quilt square, but I decided not to because for me it was...Even above that was building mathematical community and modeling how that happens. And so Megan, who's very shaky on her own, it was huge that she was willing to...She volunteered to share, you know, which is, she rarely does, um...Colette, who doesn't speak that often, I made sure she spoke. And I balanced it, I was trying to model balancing between kids that are more solid and more shaky, and how, how that can interplay a little bit.

R: How has, um...You also mentioned that you wanted to work with Angela on planning. How is that going?

C: Well, it's a little bit hard to gauge, just because, um...Just because it's been a month since I was in her classroom last, but um...But what I will say is that when I first started working with her, the mathematical goals weren't...were really not obvious of what she was doing, and they were...It was more like she had activity plans to keep them kind of busy, and it was related to the mathematics, but she wasn't able to articulate what her mathematical goals were. Whereas today she was much clearer about what is it that she was hoping they got from this, and, uh, she said... She was saying that she wanted to, to
really start talking about and using the relationships between the different size fractions, um...That a fourth is a half of a half, and so on. Um...She, she also...I thought it was really impressive that she was able to show the progression of lessons that she was following, and as I was looking at that progression it made a lot more sense than previous progressions that she had used. I don't know if she had planned those out by herself, but whether she's planning it by herself or not, um...What it says to me is that she's aware of it, and she's...Whether it's coming from her or not, she's got the resources to have coherent, cohesive, uh, sequences of lessons.

R: Uh, did you see or hear anything in the lesson today that particularly caught your attention? What were they?

C: Well, I mean I think the one biggest thing that was distracting me was the management of materials. Um...But I decided not to make that the issue that I talked about with her in the debriefing because, um...She had already mentioned something to me, and then she did raise it on her own. And so I didn't want to...It's such a big distraction that I really...And it was so obvious, too, that I tried to really not let myself get distracted. Like, I was tempted to help pass out the papers, but then I decided, well, if I do that I'm not going to get to see what kids are thinking, and I...So that was the first thing that caught my attention, um...Another thing about the less, about the lesson that caught my attention was that half the kids almost made their, uh, fraction quilt square look just like the teacher's, and so that was a concern to me. A third thing that caught my attention was that they weren't writing explanations, uh, about how they knew, even though there was a big space there for it. They weren't explaining how they knew that the quilt square...That the shapes they made were the fractions that they thought they were.

Um...They weren't even always recording what fraction they had made blue, or what fraction was pink. Um, yeah...

R: Was that why...You asked that in the wrap-up, in the, uh, end of the lesson you asked them to explain it...Is that why you asked that?

C: Exactly. Because I, I felt like during the work time I realized I wasn't able to get information about what they know about the relationships between the fractions by looking at their papers. So I interviewed a lot of kids. I just, I particularly chose kids that I knew struggle a little bit with math, and um...But I also checked in with kids that don't struggle as...That don't really struggle, to hear what kind of sense are they making of it. Um...And I was satisfied that there is a common vocabulary in the class, but that not everyone has it. And so, so I did...I chose to make the, the wrap-up with the kids about modeling: what it sounds like, and looks like when you're reasoning about the fractional pieces. Yeah, no...That was, that was based on that.

R: Do you think that you as a coach had any impact on the teacher's practice in today's lesson?

C: I think so. I mean, I think...I think that it's a really complicated dynamic, you know? Being a teacher, you've got a million things going on. Um...You've got, you know, balancing the incredible demands of school, and, and life. Uh...You've got all the different subjects to teach, reports to write, management issues - this new kid who came into her class a month ago. So there's an incredible amount. Um...But I see it as kind of a long-term commitment, and so I feel like today we were, we were building foundations you know, that are, that are going to serve her in her...throughout her teaching life. So I mean I think it'll have a direct and immediate im--...Not that she's going to be teaching
more math next week, but I think that if she were to be, it would have a direct and immediate impact on some of the things that we talked about: managing the materials, uh...Being thoughtful about wrap-ups, being thoughtful about planning sequences, um...But I don't expect that she'd do it the same way, or to the extent that I do. But, but yeah, I think it definitely has an impact. And I think over the long run it...it'll make a big difference. But I would argue that you have to be in a coaching relationship for more than just, you know, six or seven sessions, for more than just one year. I think it's really helpful throughout your career.

R: Well, you will be with them next year also?
C: Yes.
R: With the same teachers...Let me ask you, um...
C: Sorry, one other thing: That informs my decision-making as well. Like, it...I think that people who are in for a shorter term commitment don't focus on the long-term growth of the teacher, they just try to make as much of an impact in as short a time as possible, and that's not always helpful to teachers. Like I feel like confidence was a big issue for Angela, and I couldn't have helped her develop her confidence if I were too aggressive, you know? And I feel like I cover a lot of ground with her. If I were any more aggressive I think I would scare her off, so...All right, sorry.

R: Um...I can't help but thinking that this school is one that, um...That, that makes it easier to have a thoughtful math process and program going on. And I'm just...You've only been in the school for this year. I'm just wondering if you have nay thoughts on, on the impact that you've made, or, or on the uh, atmosphere before you got here - which
you don't know anything about directly, but - uh...Just some thoughts on the school generally?

C: You know, I'd be interested to hear also what teachers have to say about that, you know, how they felt like this year has impacted on the school. But my feeling is that, I mean I did a lot of talking to teachers and trying to assess the situation as I was getting started with the work, and uh...One of the things that stood out to me is that teachers...There's, there's a strong thematic approach here, where they integrate curriculum around themes, and so they do a fabulous job of...And it's also a strong focus on inquiry, and they do a fabulous job of, uh, doing that around literacy, around social studies, around science, and they really wanted to do that with math. The problem is, I don't think that that's the most effective way to teach math. And so, um...So I think one issue was, how do I - without making them abandon their beliefs - support them in, in giving math the attention it needs and the focus that it needs to make connections. I think you maximize opportunities for kids to make connections when you stick with an idea in math for, for a certain amount of time and then you follow up with related ideas, so that you're maximizing the opportunities to make mathematical connections. Um...And so I think one thing that I helped them do is prioritize that kind of attention to math. Another thing that I think I helped them do is, um...Is to articulate their firmly held beliefs, goals, ideas about teaching math. So part of it was uncovering where they're already coming from, where they're clear and where they're not clear, and part of it was trying to push them a little bit in another direction. The other, the other thing that I think is a huge...has made a really big impact is that people weren't really...People didn't have any framework, protocol, or, like, ritual of coming together around mathematical ideas, and
so this year we've spent every, every, every time that I spend a day with a group of teachers, I do it on one grade, grade team at a time...Because every time I was here I met with just one grade on a day, and we would...And I give a lot of credit to Jane, the principal, for this, she was able to help arrange the schedules so that I would also, in addition to working in their classrooms with them and having debriefing times with them, I also had time to meet with them at lunchtime and we would meet for about an hour to do planning. And I think, while I think that the planning sessions were productive and useful individually, I think that collectively, over the course of the year, working with your colleagues and having mathematical conversations and planning and negotiating in this group form, in this...really helped to build mathematical community. And, and, so it helped them get a sense of where everyone was, what the school's philosophy is, and gave them a chance to re-negotiate that philosophy, uh, on a regular basis. But the other thing that happened is that they were able to then follow up with each other. They started to see each other as resources and so, while I was not, on the days that I wasn't here - so this is nine out every ten days I'm not working with that group of teachers - um...They went to each other with ideas, with questions, with materials, with resources. And I think that happened to some extent before, but it happened at a much deeper level where they really were connected to each other's work because of that storytelling, and during our team meetings learning landscapes that we were negotiating and building together, um...So that made it, I think that...That is probably the single most important thing that I think has shifted this year.

R: How do you know that happened?

C: Um, teachers approached me in the hall, you know, uh, telling me the things they had come up with, uh, while I was away, um...At our mee--, at our, uh, bi-weekly meetings they would refer to conversations they had at other times. Um...I would see it in the planning, you know, "Oh! You're doing the same activity as this other teacher - Oh yeah, we shared, we shared this idea," you know.

R: Actually, I, I, witnessed a...The first or second day I was at school, I witnessed a teacher coming up to you and asking you a question in the hallway, which seemed to me a very thoughtful question and was obviously on her mind before she saw you. And the, and your answer had to do with something that was about to happen that day or the next day, so there was clearly a process going on with teachers' minds.

C: And I think in a...considering something you said before about, well, at this school it seems like things are really organized around being successful, um...I, I agree with that. I mean I think, I think the things that I do here are things that I realized would be helpful in my previous settings. I've been doing this...This is my fifth year doing this, and in some of the schools, I'd worked there for four... in some other schools for up to four years. Um...Some typically successful schools, and some of...Some really failing schools, some of the toughest schools in the city. And I think that there's some universal, uh, good in the kinds of things that we're doing here that actually I had tried to set up at other schools. And when I was successful at getting those things - for example, grade...Regular grade-team meetings at PS [number omitted] on, uh, just south of Harlem, um...There, there's, there's a fifth grade team that was really resistant to any kind of new practices, to talking to each other about math, to working with a coach, to letting anyone else see their teaching. And it was really hard to get grade-team meetings going, but once we did
people opened up in ways that were unimaginable, you know, that in the five previous years, no one had ever really seen. And so, you know, the same kind of cultural shift, the same kind of focus on content and attention to children's thinking did happen at PS [number withheld] uh, with the fifth grade team who is considered the toughest nut in the...you know, to crack, in the group.

R: You're coaching, right now, at two different schools, uh...Could you speak a little bit about the similarities and differences?

C: Um...Between the two schools? Well, I feel like [School 1] has a much more stable, consistent staff, um...They've got probably four teachers here that are founding teachers they've been here since the founding of the school fourteen years ago. Um, the principal is one of the founding teachers, um...There...Everybody on, everybody on the staff is planning on returning next year, um...And most of the staff has, uh, has...I think there's only one person that's new this year. Um...So there's an incredible stability that [School 1] has, and kind of a consistency of routine and ritual. Um...The other thing that they have is that they're organized in terms of grade-teams. There's three, three teachers on a grade-team, and their schedule is designed to provide opportunities for people to function as...for teachers to function as grade-teams, not to be independent. I'd say those are the biggest differences between the schools, so [name omitted] School has kind of the opposite of each of those characteristics. Um...They, they only have, they have three teachers on the pre-K/K, three on $1 / 2$, but then a $3 \mathrm{~s} / 4 \mathrm{~s}$ teacher by, by herself, a $4 \mathrm{~s} / 5 \mathrm{~s}$ teacher by herself, and a $5 \mathrm{~s} / 6 \mathrm{~s}$ teacher by himself, um, so on the upper grades there's nobody to really collaborate with. And on the lower grades, because they only have two prep teachers, they can never free up all three people at the same time. And so they've
chosen other priorities in designing their schedule, and as a result there is, there are almost no opportunities for collaboration among them. So that, that really impedes the progress. Um...So two things that I did to support that, um...One is I gave up some of the bi-weekly professional development time in the afternoons for them to be able to meet as grade-teams. So I would do out of every four sessions, one session was all mine - I could, I could guide them through some kind of workshop-y thing. The following three sessions I would only do maybe a twenty-minute mini-lesson, and then let them go off to work with their grade teams, and then they'd come back and report a little bit about what went on in their grade-team meetings. And sometimes I would give them a theme to...you know, like based on what I'd been hearing from them in the classrooms. But, so what ended up happening was that I was working with nine individual cases, and each one was its own separate case, whereas here at [School 1], each case informs the others. And I was able to get them to work together. The other big thing that's going to change at [name omitted] School is that they've agreed next year to re-organize the school so that they've got two teachers on a grade-band (?), all the way up through the grades. So two pre-K/Ks, two $1 \mathrm{~s} / 2 \mathrm{~s}$, two $3 \mathrm{~s} / 4 \mathrm{~s}$, two $5 \mathrm{~s} / 6 \mathrm{~s}$. So everyone is going to have one grade-team partner, um...And I think...And so this way, they'll be able to cover both people at the same time, and everyone will have a partner. Now their big question is, well, just because you have partnerships doesn't mean that you have partnerships, you know what I mean? In the sense that you still then have to negotiate how does that dynamic work? And here at [School 1], people cried in the first couple of sessions. People got on each other's nerves, they got intimidated, they got nervous. For whatever reason, it was not easy to start and it didn't happen by magic. We...it took a lot of hard work to get to where we are,
but I think that you can't get there if you don't make the commitment in the first place. And so it's always going to be difficult to get off the ground, but I think, um...You've got to make that commitment. So I'm really proud of the principal for recognizing her... Jane, Melville School, for recognizing her school's needs and organizing around fulfilling those.

R: Well, was that...Is that re-organization being done, uh, because of your urging or the urging of you and others? Were there other people who want that re-organization?

C: Um...I'd say that I was the biggest proponent of it. Um...I, I think that, um...I collaborated a little bit with the local instructional supervisor, but...I think the local instructional supervisor was willing...was, was...May have been willing to accept other possibilities. I think one big question that they had there was, um...Well if...Do...I think the two choices were grade teams or, uh, or bridge classes - so having, like, a pre-K/K, a K/1, a $1 / 2$, a $2 / 3$ - where they overlap, so that...Because they were worried that, "Well, what about our really sharp, uh, second graders? Do we really want them in a $1 / 2$, or would they be better served in a $2 / 3$ with kids that are ahead of them?" And I think that no matter what you are always going to have a range of kids in your class, but what they need now, more than anything else, is anything that contributes to professional community and stability, not something else where...Because they have talented individuals, but relying on the individual talents of each of those people is, I think, inferior to designing structures that support the community of those talented individuals, so that everybody's efforts get magnified, everybody's efforts support everyone else's efforts.

R: Right. Thank you very much. [[END OF TAPE]

## Appendix E

## Transcript of Principal Interview (PI), 4/28/04


#### Abstract

Jane, the principal of the Melville School, was interviewed by the researcher on $4 / 28 / 04$. This is the transcript of that interview. References to this transcript in the body of the dissertation start with the letters PI, ending with line numbers.


Reseacher (R): Hi, this is Joe. April $28^{\text {th }}$. I'm at the Melville School with the principal Jane. Um, I have, I asked Jane if she would answer a few questions for me since I was, from our first conversation a couple weeks ago it was clear that she was involved with the math program here. First, um, you told me a little bit before about the history of the Melville School. Could you just tell, give me a little background about the school.

Principal (P): Uh huh, it was started, uh, thirteen years ago by director, three teachers, supportive parents. The neighborhood at the time had one progressive school and there was demand for more and this was the second school that was started. Actually, it was started as a gifted and talented school, it was called the Neighborhood Enrichment School, by the region, by as soon as the teachers, the director got here, we decided that we wanted it, um, to be open to everybody so we did away with that idea and changed it to just the Melville School. It was started pre-K through first grade, um, with three teachers and, three classroom teachers, and a cluster teacher.

R : And what, what were you, what was your role?
P: I was a pre-K/K teacher, and it was based on a progressive, a, model of education, um, in group based, core curriculum, and kids engaging in real, important, authentic work. And, um Dena who was one of the pre-K/K teachers, had had experience, um, with Elaine Weber at City

College workshop center. Dena was a traditional teacher, um, who wanted to change her practice, she taught pre-K, and, um, I had had kind of an odd background, um, but was really interested in the City and Country School and was, was looking at that as the basis for my practice.

R: And how did the school evolve after that, was there anything you could say about what happened between its founding and now?

P: Yeah, each year we added on a grade to about our present configuration and we always had the idea that we were going to have mixed age grouping, and um, we developed our, um, practice and kind of our organization as we went, so, we knew that we were going to keep portfolios for the kids, that we were going to write narrative reports, but, um, we were going to have a social studies or science-based core curriculum. At every point every year we talk about, um, what various things means, and we find, um, how we report to parents, or how we have family conferences, or what should go in a portfolio, but the foundation of our practice is descriptive work, looking at the individual children and seeing what comes next for those kids, and also, how to make something out of that for the whole group.

R: Could you describe the present school?
P: Mm hm, um the...
R: First of all...
P: Social and academic work first go hand in hand, that is really, um, key, and that we look at what kind of adults we want the kid to be and our practice follows from that. That we want them to be full participants in a democratic society, to be able to critique what's going on, and work productively toward that. To be able to work as individuals, to be able to work as, um, part, you know, members of a group, because you can't always be doing your own work, you have to work
with other people as an adult, and we want kids to be able to be engaged, and um, you know, do good things, as well as do, uh, be, be uh, fulfilled people.

R : Is that, are those goals expressed in any way to the, uh um, explicitly?
P: Yes, when people come on tours, um, to place their kids in this school, that's stated explicitly in, uh, our mission statement, in our parent handbook, that's explicit a practice is set up with mixed age grouping because that supports social growth, as well as academic growth. Um, kids take on, um responsibility for each other in, um, situations such as reading buddies or math buddies. Um, kids will do community projects often within the classroom. If we're studying, um, a topic such as, um (pause), you know slavery, um, in colonial times, or if we're studying the, the silk route, um, whatever it is, uh, the concepts that are being studied, it's how does that apply to now, so what does that mean in terms of now, and um, really, using history and social studies to understand, um, our contemporary times. Um, same thing with math, next year when there are the elections I assume, that we'll be doing a really big, um, (pause) uh, study in, um, mode, medium, you know, that graphing, um, poling, um, what graphs tell you, how you read graphs, um, how you read opinion polls (small laugh), because you can say "only $40 \%$ of the people blah, blah, blah", or you can say " $40 \%$ of the people blah, blah, blah" so we want kids to apply what they know to their lives.

R: Uh, how many students are in the school, how many teachers?
P: We have 242 kids, we've got 11 classrooms, we have, um, 3 cluster teachers, um, science technology, a Spanish teacher and a physic, conflict resolution teacher. Then we have, um, power professionals for the pre-K/Ks, and some kids who have their own. We have probably about, um, we have occupational therapists who are part time, social worker, um, sets teachers
who are learning specialists, so altogether we probably have about, um, 32,33 adults who work in different ways with children and each other.

R: Only a, the ages, the, what classes do you...?
P: Um, they're four-year-olds to sixth graders.
R: And, um, how, how many classes per grade?
P: Um, we have 3 pre-K/Ks, 3 first/second grades, 3 third/fourth grades, and 2 fifth/sixth grades. R: Ok. (pause)

P: And all kids have music once a week, um, art once a week, um, with a visu..., with an artist or a musician, and our upper grades also have, um, theater where they work with a, um, playwright to write their own play and act it out because we feel that kids can express what they learned in a variety of ways, and, um, that those ways are as important as, as any other way. And um, that, math or technology or reading or writing are skills to ex..., to communicate, but the, the meat of what you want to communicate is in, you know, science or social studies, so we'll, um, use those skills for doing other things, for investigating and researchers, and we look at what is being a researcher mean when you're four-years-old, and what does a researcher mean when you're twelve-years-old, and, um, give kids the opportunities to be researchers and investigators and artists and look at life, um, through those lenses and try and understand their own lives with, through those lenses.

R: (pause) Could you describe the population of, uh, students?
P: Mm hm, we have just about half boys and girls (swallows), a little more, um, boys than girls, and we've got, um, probably about, um, a little over a quarter of our kids are, um, Latino, a little bit, over a quarter of our kids are white, um, a little under a quarter are, um, black, and, um, probably about $17 \%$ are Asian, but that's, um, we have kids from every religious group, um,
varied economic groups, um, I think, you know, um, this is, um, the Lower East Side, so we have a lot of blended families, bi-cultural families, um, families that are mixed in a lot of ways, um, we have families from all different countries: Europe, um, Spanish speaking countries, um...

R: Recent, recent immigrants?
P: Uh, some recent immigrants, um, not a huge amount, um, but some and, in, um, from a variety of countries.

R: Now let's turn to the, um, math program. You, you indicated (pause), uh, in your, to me, before that you're interested in the math program, what is your involvement with, um, with math?

P: Um, well, when I came to this school, um, I had been a pre-K/K teacher, and left the school for a couple of years, um, to be a staff developer in District 1, then District 10 in the Bronx in, um, early childhood literacy, then in, um, then in, um, school reform and early childhood work, and when I came back the school had kind of had mish mosh, um, uh math programs, and, uh, the pre-K/K was doing, um, some Marilyn Burns work, and some Math Your Way, and we were looking at Constance Kamii, the first and second grade had their own curriculum, they had to buy some, the third grade had to buy some (inaudible) curriculum, it didn't, um, it wasn't coherent, so the first thing I did when I came back as director was to, um, try and, um, build coherence so, I introduced her, um, to the teachers, to the staff, and we started having meetings about, um, different math strands and, and how to build that, so I've been involved in building curriculum.

R: I'm, I'm going to have to interrupt now, uh, we'll, let's continue this later.
R: This is a resumption of my interview with Jane, the principal, on April $28^{\text {th }}$, um, I, we were, you were answering my question about your involvement when the math program when I had to
interrupt the interview. Uh, (pause), do, (pause) could, could you continue, that, to, um describe your involvement with the math program?

P: Mm hm. So it's like any other subject area here where I work with teachers to build curriculum, and, um, arrange opportunities for teachers and for parents to have really good professional development. Um, before we had the coaches this year, we worked a lot with Marla Dumont and I'd arrange for teachers to go to City College, um, to the, um, you know, City College, um, uh, math stuff, and also, um, when we knew we were going to get a math coach, then I, um advertised and looked around and tried to hire somebody who I thought would be really good, so, um, my involvement is, like, providing for good professional development, um, working with teachers to build curriculum, making sure that teachers have materials they need, really try to look at kids' mathematical thinking and understand it better, and, um, you know, observing lessons and working with teachers on, um, pedagogy, and, um, working with teachers on projects, um, that support kids' math work and, um, use math skills and...

R: So, so...
P: ...stuff like that.
R: ...so, let's um...

P: Working with parents, um, you know, um to help them understand math curriculum.
R : So those are things you do? Or they...?
P: Mm hm.
R: You described two kinds of professional development, at least two kinds, uh, one where teachers go to different programs or workshops I guess, and one where you have someone come into the school, uh, and I (pause) uh, now, uh, the system now is more geared to someone
coming into the school, someone part of the school, the coach. How would you compare that, uh, the coaching with the workshops instead? Would, would you...

P: I think you need both, um, in an optimum way, you always need to go out and look at something in depth, and refresh yourself and come back, and um, you also always need somebody to be with you in your room over time, and, um, help you with that too, so I would say one or the other, um, isn't good but both together are really good.

R: Well, why do you, why does it help to have someone come back to see you in your room in this way?

P: Um (pause) going to a workshop and then coming back, there, there are some issues about how to implement that, and, um, how to really conceptualize it and how to begin in that, um, it's better for people to be able to have somebody to consult with about their own practice, their own kids, um, their own issues and to be able to help you with that. It's, it's like having, you know, um, a master carpenter if you're a carpenter, I mean, you can always, um, you know it's learning from somebody, same way we don't have kids, you know, just be on a computer, it's that human interaction and taking, being able to advance your own needs.

R: Um (pause), when, you, you also (pause) uh, spent time, when, when the math coach meets with other coaches and...

P: With teachers.
R: With with, um, sorry, with, other, with teachers. Um, (pause) why do you do that, what, why do you do that?

P: Um, partly I like to learn, partly because it's really important and that signifies that it's really important, um, when I go to meetings, that it's not trivial, and um, also, I don't see myself in opposition to the teachers, but working along side the teachers in a different role, and, um, the
first couple of months of school I went to all the coaching meetings because I wanted the coaching, um, process to be set up well, cause I kind of wanted the teachers, um, to protect the teachers and kind of protect the coach (laughs) at the same time, like, make sure that people could work out a good working relationship, and that nobody felt abused or trampled on, or, um, (short pause) unlistened to, or that it, it was going to be a mutually beneficial relationship for both the coach and for the teachers.

R: Now do you, um, meet with the coach, uh, do, do you (pause) do you have a relationship with the coach...

P: Mm hm..
R: Do you talk about, uh, issues?
P: Yeah, I, um, I have goals for the school, goals for individual teachers, um, goals for kids, and, um, the coach and I talk about that. The coach is really, um, I think, the school leader's kind of right arm into the classroom, um, and can, um, and is the teachers' um, you know, um, I don't know ... extra brain or something, that, um, the coach can really elevate the practice or he can ruin (laughs) the school, either one. Or it can be just one more blah thing to have to do and I didn't want it to be a waste of anybody's time, I didn't, certainly didn't want it to waste the school.

R: Do you feel that, um, that, this, uh, any change that has happened since the coach has come into the school? And if so, could you describe it?

P: Um, (short pause) I think it's worked out, um, (pause) I think, you know, we, we've had, we haven't had this consistent coaching in this school, we've had people come in, we, Marla Dumont came in and worked with the teachers a lot, um, I would say it's, it's kind of a doubleedged thing in the sense that the coach is, it's part of a whole larger thing, I think, ok, if, if you're
looking at Matt himself coming in as, as a math coach, a math professional developer working with grade levels, I think for different grade levels it's been a different experience, and I think that ultimately, um, it is beneficial. I think it's, um, everything's tit for tat too, I think, you know sometimes, um, you let one thing go because you're concentrating on something else, and, um, I think ultimately it will be better for the school to have the coaching.

R : Uh, have you noticed any difference yet?
P: Um...
R: From the coaching process?
P: I think, yeah, I think what, what has been, um, beneficial is that, um, I think, um, yeah I was, I was with a teacher yesterday, actually, and he was working with a class in a mini lesson and he did something that I thought, "Oh, he got that from Matt." It was a literacy lesson, but I think, um, what he got from Matt was good, and, um, I've seen that some of the grade levels are much more meticulous, um, planning their curriculum and, um, I think that that was something I was hoping that Matt would do this year was, um, really look at how we, um, if I had told him, you know, um, to really look at if we're going to do repeating pattern, pre- $\mathrm{K} / \mathrm{K}$, and then do it in first/second grade, how do you they take it, how do they know what happened before, and how do they take it to a different level. And kind of how to deepen the math.

R: So that is on, on your mind and on teachers' minds, uh, sort of, where does something come from and where is it going to, and where is it now, and where does it come from and where is it going. Um, now, would you describe your math in school yourself, are you...?

P: Right, I'm a baaaad mathematician (laughs) but a very good math teacher! Right I've learned a lot from, I've learned a lot from teaching math in pre-K/K, I can estimate distance much more, I can estimate quantity much better, and volume, and, um...

R: So you're saying...
P: ... and I really have to sweat if the kids, um, are going to, um, be, going to challenge me to play a logic game with them and in sixth grade I've really got to bone up, but I did notice that when I went to a parent workshop that Marla was leading and I got the problem when the other people didn't because they were stuck on their algorithms and I was much more in the elementary school mode of trying different techniques to solve the problem, so I'm proud of myself that way.

R: So would you say, uh, had a belief, for, since you were, let's say a kid, or a, that you were not good in math?

P: Um, I knew I was not good in math I wasn't that interested in math, um, I was a kid who, um, stuck, um, a novel inside the math book in second grade and also knew that I wasn't understanding whatever it is that they were trying to do and expressed that, um, to teachers, um, in middle school that there was something I wasn't getting and I didn't know what it was but they didn't either, so, um, it just kind of continued, so I memorized the algorithms and would, you know, do fine in math but not really, um, understand what it was I was doing. R: Well let's um, uh, that's all the questions I have for now, so thank you very much. P: Ok.
[End of Transcript]

## Appendix F

## Transcript of Angela's Lesson at Melville School (L), 5/3/04

Angela, the teacher, taught a lesson at Melville on $5 / 3 / 04$, observed by the coach, the researcher, and videotaped by a professional cameraman. In the transcript below, the left-hand column indicates the line number in the transcript and the second column shows the time for each speaker in minutes, seconds and tenths of seconds. The third column identifies the speaker. The fourth column contains spoken words and/or description. Descriptions are made by the researcher and enclosed in brackets. Names of all participants are changed to conceal their identities. Angela, the teacher is identified as T , the coach as C , the teacher aide as TA. If a student speaks more than one time in a row, or in a short sequence, then he or she is designated by a name, if known, or $S$ and a number, e.g., S1, S2, etc. If a student speaks only once in a short sequence, then he or she is designated only by $S$; thus $S$ stands for many different students. If several students talk simultaneously, saying similar words, they are designated Ss. References to this transcript in the body of the dissertation start with the letter $L$ for lesson, ending with line numbers.
[The lesson is in progress. The teacher is sitting in a chair next to a large easel, with students sitting on the floor around her. She holds a yellow folder.]

00:00 Angela - Teacher [T] ...keeping up with us. So if you start getting distracted, you're going to fall behind and you're going to get lost. Okay? So in about two seconds, I'm going to send everybody off to their tables. Some of you, I started to put things down at your seats. The rest of you, I have them here. What you're getting is a baggie with your name on it. [Holds up yellow folder] It should have five pieces of
paper of the same color in it. And we're going to make a set of fraction cards out of this. Okay? We're going to be folding really carefully. And cutting really carefully five different fractional pieces. Okay? So what I need to do, is when I give this to you, you're going to go quickly sit down. Kyla [teacher aide (TA)] is going to bring around scissors, so you don't need to do anything, except go to your tables. Okay?

00:53 Student [S] [Inaudible.]

00:54 T Ira, I'm going to tell everybody everything you need to do. Here you go Linda. Okay, quickly. [ $T$ is handing out the folders. As $T$ calls out each name, one student stands, takes the folder from her, and walks out of the camera frame. There is an increasing amount of talk.] That's all. Katie? Matt? Zoe? Kara? Jeanie? Tom? Okay, if you're sitting here, then that means that your stuff is at your desk. Let's see, why don't you sit at Zoe's table? Okay, that's one. [T 'pours' rain stick, making a loud noise.] 01:53 S I just didn't understand what you said so can I ask you questions?

01:57 T I'm going to show everybody. That's once. [T pours rain stick again.] That's twice. Sally, could you move your chair in, so she can get by? This is the third time. [Pours rain stick a third time.] All you need right now is to look at me. You don't need anything in your hands. You don't need pencils. Okay. What you're going to do, right this minute, is: You're going to take out all five pieces and make a neat pile in front of you. Take out all five pieces. Make a neat pile on the table in front of you. And now take out one sheet. Shh.

02:48 S Sixpieces.

02:49 T Shhh. If there's talking, Donna, then people can't hear me and you really
need to listen now

2:58 S I'm not even talking.

02:59 T [Teacher standing among students, holding blue sheet of paper] Good. I'm glad. Shhh. It's okay. Okay. All you need right now is one piece of paper in your hands. No scissors. Nothing but the paper. So if you're holding the scissors in your hand right now, you need to put it down.

03:17 S Okay.

03:18 T All right. The first piece of paper we're going to cut in half. So we're going to very carefully ... can you watch first? Can everybody watch first? Can you guys see over here? Table 1? All right. Make sure that your two sides of the paper match up. So that you get a perfect half. And then what I'd like to do, is I put it on the table, and I use the side of my thumb, and I make a nice crease. Okay? Get it as close as you can. And then that's going to make a guideline for you. Because then you're just going to go ahead and cut it. All right? So once you feel like you've got two really nice, even halves, you're going to go ahead and very carefully cut it. [Moves to front of room. Folds paper in half]

04:20 S Can we do it like any shape?

04:23 T No, you may not. You may do it like I just showed you. Ira, I started the lesson off by saying that you really need to follow along and keep up with us. As soon as you're finished cutting, you're going to take your pencil, and you're going to write one over two on both sides. ... Can you look up here for one second? Can you see that? I'll do it in marker. [T writes on blue paper and holds it up.]

05:02 S Got it. [Students are working quietly at their desks.]

05:21 T So on both sides on each, on each half it's going to look like this. [holds up blue paper, says $1 / 2$ on each side of paper]

05:27 S [Inaudible.]

05:29 T Excellent. And then, on the back of those, what do you think you're going to do?

05:30 S Put your initials.

05:36 T No.

05:37 S [Inaudible]. [Many voices]

05:43 T You're going to actually write the word for what the piece is. So this is what?

05:48 SS Half.

05:48 T These are two halves. And then very lightly, somewhere on the back, where the words are, put your initials. So I'm going to just put my initials very lightly in the corner. So on one side of you have two pieces that look like this. Can everybody look up here to just double-check? You have two pieces that look like this? Mark? And then on the back, it'll have the actual wording: half, half, and your initials should be on each of those fractions. Each of those pieces. And then take those two halves, and you're going to stick them in your baggie.[Writes on back of paper][Many voices] 06:30 T H-A-L-F. Here, see?[Many voices] 06:45 T Okay. When you're ready, could you look up at me, please? Larry's
ready? Tammi's ready. Oren's ready. Put it in your baggie. Excellent, Colleen. Okay. The next piece...Is everybody ready to do the next piece?

07:17 S Yeah.
07:24 T You're going to fold it in half. So do what you just did, exactly the same. Do that all over again. So do the same thing al.I over again. And then, I'm going to fold this piece in half again. Raise your hand if you think you know what's going to happen. Just four people? Just five people? I folded it in half. Then I folded it in half again. So, Donald, what do you think's going to happen? [ $T$ holds up a new piece of blue paper, then folds it. She folds paper again.]

08:03 S Four. There's going to be four.
08:05 T I'm going to have four pieces. So what am I going to call each one of these? [Holds paper up showing $1 / 4$ on each side.]

## 08:09 S One-Fourth.

08:11 T Fourths. These are each fourths. Okay. So, again, before you cut it out, I want you to write one over four on top of each fourth. So it's going to look like this. I like how Darla looked up just to check to make sure. Jan, should you look up, just to make sure you know what you're going to do?[Writing on paper]

08:40 S Do you write fourths on the back?
08:41 T And then, I'm going to write fourth F-O-U-R-T-H. And what else am I going to put on the back?

08:50 S Your initials?

08:51 T Excellent. The back will look like this. [T is writing on paper, holding it up. Each folded portion says " $1 / 4$ " on one side and "fourth" on the other.]

09:05 S Can Isee?

09:06 T Yup. Okay? Just like "four" with a "th" at the end, Luke, over here. Please? Back. Front. And then what's missing on my...?

09:21 S Initials.

09:21 T My initials. Okay. So make sure you put them on every single part. And then you can cut them out.

09:28 S Can you write like "one fourth" on it?

09:31 T You can.

09:32 S Okay.
09:36 S How do you spell "fourth"?

09:37 T But each one of these pieces we say is "a fourth". Okay? Could you keep up Carla, cause we're gonna move on. And then you can very carefully ... it's really important you do this carefully on the lines ... because we'll be using these to play some games later. And it's really important that the lines be straight, and that all of your pieces are even. [Many voices]

10:12 T If they're a teeny little bit off, that's okay, Dara. But really try to make that's fine. That's fine. And then what are you going to do one you get all four pieces cut?

10:20 S Put it in your baggie?

10:24 T Put it in your baggie. Okay?[Many voices][Teacher folds more blue paper.

Camera pans to a student at a circular table folding brightly colored paper and tearing it.]

11:39 S Look at me, the scissor man! [Boy holding scissors up to the camera.]

11:40 T Okay? Put them inside your bags so that they don't get mixed up with somebody else's. [Camera back on teacher][Many voices]

11:52 T How's Table 1 doing? You guys done? Table 2? You guys done? Okay. I'm going to move on. So you're - take out your next piece. And you're going to fold it in half. Iris and Oren? Fold it in half [Folding more blue paper][Many voices]

12:19 T Then fold it in two quarters. And now if I fold it again, which gets a little trickier because the paper's getting thick we have to do this really carefully.[Many voices]

12:37 S1 Now there's eight.
12:38 T Who said that?

12:40 SS Jonathan [S1].
12:41 T Why do you think it's an eighth, Jonathan?
12:42 S1 I don't know. I just do.

12:43 S I know.

12:45 T What were you doing that made you think it was an eighth?
12:47 S He saw. He cheated.

12:49 T Shhh. Shari, that's not kind.

12:53 S1 Um...because I did it in the other thing [Inaudible] over and over.
[Inaudible] fold it like that [inaudible].

13:05 T What's going to happen? If we have fourths, and then we fold it again[multiple voices, students raising hands]

13:13 T Barbara?

13:14 S [Inaudible].
13:16 T Barbara said it's like doing 4+4. We're doubling it. Well let's see. Open it up, and see how many ... How many pieces do you have?

## 13:24 SS Eight.

13:27 T So, you're going to write ... shhh ... "one eighth" on all eight pieces. [Many voices.]

13:36 T And then you're going to write "Eighth" on the back, that looks like this. And then your initials. And cut it out. And then put it in the baggie. Okay? [holds up blue paper][Many voices]

13:58 S Angela, how do you spell "eighth"?
14:02 T E-I-G-H-T-H. [Camera pans around classroom, showing large class with students working at round tables with multicolored paper. Zooms in on some students' work. Teacher talking to coach, aside.]

14:48 T [inaudible] There were two different ways to do it. [inaudible] Have them do it two different ways.

14:58 C That's a good idea.
14:59 T Cause they can fold it the long skinny way, or they can fold it.

15:02 C Right, so you would take them through the first part, making thirds. 15:04 T And then...

15:05 C And then say, you know what? But I, what would happen if we cut out our thirds [and firsts?] How could we make sixths [inaudible] people think of more than one way to do I don't know. I mean, I think you still want to lead it in a discussion. You don't want to just open it up and let them do whatever they decide.

15:25 T Right. And point out the two different ways.
15:26 C Just because the whole rest of the time, you were leading them.

15:34 T [T raises her voice, apparently addressing the class again.]Okay. Shari, are you done?

15:36 S Almost.[Many voices]
15:37 T So you need to be keeping on with your work and not talking. Okay, um, okay, hurry up.

15:50 T [To Teacher's Aide (TA)] So Kyla, you may need to help some of them.
15:55 TA Yeah, that's what I was showing them to do last time. To just fold like that, and then [inaudible]. Okay.

16:02 T Okay. Three. Two. Shhh. One. Table 1, can I have your attention? Table 2 are you done? Table 3, how you doing? Okay? Jonathan, Fabio. Shhh. For the next one that we're going to make together, you really need to look up here. Because this is the trickiest one there is to make. [ T is addressing class][C walks over to T at front of classroom and says something to her.]

16:42 C [Inaudible.]

16:47 T If there's somebody who's finished at your table, could you help somebody who's not finished cut out their pieces? [Teacher moves to a table to help student cut orange paper] [Many voices]

17:11 T I'm sorry. Were you going to use this?
17:14 S No, I was trying to help him.

17:14 T I thought you were done. [Many voices]

17:21 T [Speaking now to the whole class.] Okay. Shhh. [Many voices]
17:27 T Kara, can you wait a second? Because l'd like everybody to hear that. Shhh. Kara just had a thought about this. Can you share with everybody, because I think that was a really good thought.

17:41 S [Inaudible]

17:42 T Did everybody hear what she said?

17:43 SS Yes. Multiplying by 2.

17:45 T So what are you multiplying by 2? What are you multiplying by 2, Kara? 1 S [Inaudible.]

17:50 T I'm sorry. Oren, I can't hear Kara. She's sitting right behind you.

17:55 S [Inaudible] six.

17:59 T So what do you do what do you do to the 2 ? If you multiply it by 2 , what do you get?

18:08 S: Two. I mean four.

18:09 T You get fourths. And then if you multiply the fourth by 2, what do you get?
18:16 S Eighths. Oh, I get it.
18:18 S By 2.
18:20 T Okay. Last piece that you're going to do with me and then you're going to do a piece on your own. For this piece, we're going to make thirds. Larry, please don't fold it yet, because this is really tricky. And that's going to confuse your cut. Everybody needs to look up here. To make thirds, it's sort of tricky. You need to, you need to not make a crease yet. You need to sort of scooch it around like a cylinder. And then when you feel like it's even and I can't tell you an easier way to do it, other than you just have to sort of eyeball it, and see when your edges ... I'll come around. [T is holding up blue sheet of paper.]

19:11 S Are we going to do the fraction game again?
19:16 T And then you can make your crease, once you feel like you've got three equal sides. Excellent, Kara. If you feel like you figured out how to do it, can you help somebody at your table make your thirds? [Holds up creased paper] [Many voices] [Camera pans around classroom, students working on papers]

19:44 T And then raise your hand if you think you know what you're going to do. Larry, what am I going to do now?

19:50 S Cut one third.
19:51 T One third on each section.

19:54 S And then you write thirds on the back.

19:56 T Third on the back. And?

19:58 S Your initials.
19:59 T And your initials. Excellent. So let's do that, when we're [Many voices][T leaning on desk writing on blue paper]

20:11 T I'll help you Chris. Thirds on one side. Look up here for a sec. Look up here for a second. Just to double-check yourselves. Laura, you're not looking up here. Table 1. Table 2. One-third. [T holds up a blue sheet to the class, showing " $1 / 3$ " on one side and "Thirds" on the other.]

20:28 S I already did it.
20:29 T Excellent. And then on the back, you're going to write "third" on each piece, and your initials. And then, cut it out and put it in your baggie. [Many voices] [Shots of students working on papers, cutting.]

20:45 S Thirds, okay, I'm going to be done now. Ta da! Done. Easy, easy. [Many voices heard.]

21:05 S [Inaudible] doggie dog. [Camera focuses on another table, boys making faces, playing around.

21:08 S This is my name.
21:10 S Bye bye, Mommy. I love you Daddy.
21:14 S My eighth isn't cut exactly. [Camera back on teacher going around classroom to help students.]

21:17 S Are we going to do sixteens [or does he say 'sixteenths"?] by ourselves?

Can I close my bag up?

21:22 T No because you're going to need to put more in there. Are you done? 21:25 S With that one.

21:28 T Hold on. Hold on. Hold on. [Many voices]

21:45 T It's not a 3, it's a third.[Many voices]

21:55 S You're supposed to put third, instead of 3.

21:57 T With this next one, you're on your own.

21:58 S Yay![Many voices]
22:03 T Okay. You are going to make yourselves Can everybody look up here? Jonathan? Mark? Daria? Table 1. Jonathan. Matt. Daria. Jonathan. Matt. Thank you. Okay. The next one you're going to do, is you're going to make another third. Lara. You're going to make another third, just like you did. And then, you're going to fold it in half. And I'm not going to tell you which way to do that. And what are you going to get once we fold a third in half? Julie? [Teacher is back at front of class, holding blue paper.]

22:44 S A sixth.

22:44 T A sixth. Okay. So you're going to make sixths next. And you're going to do that by folding your thirds in half, in some way. Just make one half of a third. Okay? [Many voices] [Teacher walking around helping students.]

23:14 T Kara, were you sick last week? You were sick? Are you feeling better? Did you have [inaudible]?

23:20 S No, [inaudible].

23:24 S I did that already, I did that last year.

23:29 T Because Miriam said that she thought that's what you were doing. [inaudible] Now you've got to fold that in half. [Camera focusing on different tables, students working] [Many voices]

23:42 S [Boy is folding pink paper.] All these I have to write sixths. 23:50 [Camera focuses on another boy at same table folding pink paper and writing with pencil.] [Many voices.]

24:02 T [Camera now shows another boy, Zora (S1 in blue shirt), who is cutting green paper along folds; T addresses him.] I want to use yours as an example once you get it cut out. [But it is not clear at this point why she wants to use his work as an example.] [T calls to another student.] Shhh. Neil, you need to stop talking and do more work. [Many voices.] [T and C talking aside. Their words are inaudible. C talks, T nods, as if agreeing. Camera focuses on class. T goes back to walking around helping students with work. Camera remains focused on boy, S2 cutting green paper. T comes to S 2 and folds green piece of paper, apparently in thirds.]

24:47 T [To S2] And it would help if you watched as I was doing it, so that you could learn to do it yourself. So here are your thirds. Right? [T folds paper in thirds, widthwise and hands it to S2. He then folds it in half so as to make 'chunky' sixths.] [Many voices.]

25:01 T Are you done? Nice, very nice. [Voices quiet.]

25:14 T [Camera shows S 2 in foreground. T can be seen in background talking to other students.] Stop. Stop talking and cut. More cutting. Less talking.

25:19 S Less talking and more cutting. [Many voices] [S2 carefully folds and cuts green paper. See Figure 1 below.]


Figure 1

26:18 S [Camera now shows new shot of boy with pink paper, talking to teacher.] And then like this, same way. Look. For one third I did this, and I did this. And then you have one third, right? And then I folded it. [Folds appear to be the same as at lines 2946.]

26:36 T How come your pieces came out this shape?[Camera does not show what shape $T$ is referring to.]

26:39 S Because. One two three, one two three.

26:47 T Very interesting. Because normally if you do what you just did, they come out that shape. Or, they come out this shape. Beautiful. [Teacher gets up and walks to front of room, by computer. Holds up rain stick to signal quiet.] [Many voices.]

27:20 T That's one. I need everybody to look up here. Table 1. Table 2. Shhh.

Thank you, Colleen. Thank you, Barbara and Kara. All right. Something happened. Kara got this, for sixths. [T holds up green "skinny" folded paper. See Figure 3, below.]


Figure 3
And Zora [S1] got this for sixths. [T shows a "fatter" green paper, indeed almost a square. See Figure 3.]


Figure 4
What happened? Can somebody explain what happened? They can't possibly be the same thing. Shhh. How did this happen? Colleen? How did this happen?

28:00 S They cut it different ways. They did half, we did half of these.
28:06 T What do you mean?
28:07 S I know what. I know what.
28:08 T You know what? If there's three people talking at once, it makes it really hard for us to hear. Colleen, could you explain what you were thinking?

28:16 S [Camera shows in the distance a girl in red shirt.] One person did it that way. Another did it that way.

28:20 T So when Zora ... Can everybody look up here? I'm going to give you time to finish, Jonathan. Jonathan, I'm still waiting for you. Matt, I'm still waiting for you. So one way to do this is ... Here's our thirds, right? [T holds up blue paper, folded into thirds.] Everybody got to this point. And then what did Kara do to get her, her skinny
sixths? What did she do, Collette?

28:56 S She cut it down the middle like this.

28:58 T So.

28:59 S That way. [T folds down middle.]


Figure 2

28:59 T She folded it.

29:02 S Yeah.

29:04 T Like this. So she's getting a half of a third, right? And mine aren't that even, but does that work? Are those six pieces? Let's see. 1, 2, 3, 4, 5. [T counts the folded sections.]

29:18 S Yeah, yeah.

29:20 T And then, let's see. Zora looked at it in a different way. He folded it in half. He made his thirds. And then what did he do? What did you do, Zora, to get your sixths, that look totally different? [Zora comes to front, takes blue paper. See figure below.]


29:44 S1 This way. I fold it this way.

29:44 T Ah. Aaah.

29:46: S1 Because I figured that since that's the way we..

29:49 T Shh...I'm sorry, Zora. I can't hear because Oren is talking now. Could you say it again?

29:53 S1 Because I figured since that's the way we've been ... we've ... we did it ... we've been doing it today, that's probably the way you were talking about. Because you said you weren't going to tell us the way we we're going to fold it, but I thought it was going to be like the way we've been doing.

30:09 T Which was doing what? What were we doing to all of the pieces?

30:12 S1 Folding in like proper halves?
30:14 T We were making halves of the original fraction that we started with. Raise your hand if you got a skinny sixth. Raise your hand if you got a skinny sixth. 1, 2, 3, 4 people got skinnies? And raise your hand if you got the short fat ones. [T doesn't say how many got fat sixths, but within camera shot, 5 children (out of 7 who can be seen) are raising their hands.]

30:39 S No, no, no. I got this one!

30:40 T There...There was a third way.

30:44 S Loren's way.

30:44 T Loren and Joanie got a third way.
30:47 S Tattoo it.

30:48 T How did you do yours, Joanie [S2 in red shirt]?[T is looking at S2, but
camera is not yet focusing on her.]

30:50 S2 No, no. Can I do it, can I do it?

30:52 T Craig, can you sit down please, that's not helpful.

30:56 S2 I went like this.[Girl folding paper]

30:58 T So you started with your thirds, right? Like this? And then, you fold the sheet the long way. [Camera now zooms in on S2. Girl is folding yellow paper]

31:31 S2 Yeah. This way.

31:32 T But that's the same way as ... no, you did it a different way. [Many voices]

31:39 TA [TA speaks to S2, while folding paper.] Joanie, remember what we did at the beginning, instead of folding it like this, you folded it like this, right?

31:47 T And then you folded it in half that way. [TA leaning over table, showing S2.]

31:49 TA You remember doing that? [S2 nods her head affirmatively.] 31:51 T Yes. 31:52 S Oh, so like this.

31:55 T So the third way was, you made your thirds this way. Jonathan, which way? Which way did you make your thirds, then? No, Jonathan. I needed you to be looking up here. That's why I did that. Can you look up here, please? Can you sit on your bottom and look up here?

32:15 S What are we going to do with this?

32:16 T You can make your third the long, skinny way.

32:20 S The long, skinny way.

32:23 T And then, if you fold that in half, you get a third a third way of making yet a different way of making your, your sixths. [ $T$ folds blue paper in 'skinny' thirds and then folds them in half so as not to make them skinnier. See figure below.] Okay, and then your sixths end up this way.


32:37 S That's the same as Kara's way, though.

32:39 T No, Kara's were long and skinny. And Zora's were...

32:45 S1 [S1 stands.] Square.

32:45 T Square and fat.

32:47 S And that's short and skinny.

32:48 T And this is sort of in between both.

32:50 S Yeah.[Many voices]

32:50 T Zoe, shhh.[Many voices]

33:05 T Kara? Please.

33:06 S It won't be exactly but you fold it like this diagonally and like this. [Camera focuses on boy folding yellow paper.]

33:15 S You're even. You're even.

33:16 S I know.

33:17 T So if all six pieces aren't even, are they each a sixth then, if they're not even pieces? If they're not equal pieces?

S [Inaudible.]

33:29 T What's that? If you want, take your last piece and see if you can come up with another way of making sixths. That we haven't shown. See if you can figure out a way to do it. [Multiple voices heard.] [Teacher picks up blue paper, starts folding]

T I think there's one more way. I think there's one more way. You do thirds [Girl walks over]

33:56 S You do thirds. And then you put the two diag...the two corners together. [S folds so one corner aligns with diagonal corner]

33:59 T Yeah, what if you cut your thirds. Zora, can I have your scissors for a minute? Zora, can I can have your scissors for a minute?

34:04 S They go like this. [Many voices]

34:08 S Angela, do you have to?
34:09 T Yeah. See if you can figure it out on the diagonal.

34:13 S Yeah. Look. They are even.

34:19 T AideWhat's up, buddy? Did you try to make it a new way?

34:24 T So do you want to [inaudible] [Talking with C]

34:28 C Um, yeah, yeah. That would be okay. So this is really, this conversation is going really nicely. Um...Yeah. We can since they're working with sixths, the big math
piece that's going right now is, is are they equal pieces? Are they six pieces, right? And I heard it come out in little bits and pieces, and so one thing we can do, is we can up with some examples to challenge their ideas about this.

35:00 T Um hm. Put your stuff in your baggie. [to student]
35:03 C I can draw this up.

35:05 T Mm hmm.

35:06 C And then I can draw one that doesn't work out. Where I can say, oh, I saw some people doing this. Wait. Let me think. Like, if I make six equal pieces, are these sixths?

35:23 T Mm hmm

35:24 C No, because there's a seventh piece, so okay, let me cut if off. Now are they sixths? Well, there are six of them each. [inaudible]

35:30 T Jonathan did that when we first making quarters and we had to make...

35:37 S [Girl approaches T and C.] I made a twelfth. [She holds up green paper that apparently makes diagonal folds.]

35:38 T Wow! That's beautiful. Good for you. That's excellent. [Many voices] 35:48 T We were trying to make thirds and he didn't get how you folded in thirds. And so he made quarters, because that was easy. And then he just threw one away. So we shared that. You know, we shared how, you know, that doesn't work, that's sort of cheating.

36:05 C Oh, okay.

36:06 T So. We just sort have had that conversation.
36:07 C [Overlapping] So you guys have already started to talk about that. 36:08 T Yeah.

36:09 C But you just had that one or....?
36:11 T No. Because they were doing a TERC thing [TERC is the name of a math program.]. You know, the...

36:13 C Oh, they were.
36:14 T --sharing brownies. So ...

36:15 C Ahh. Gotcha. Gotcha. Okay.
36:17 T [Addressing the class] Okay. Once you've finished Can everybody stop, and look up here? Can you stop what you're doing for a minute and look up here? Thank you, Lori. Thanks, Daryl. You need to make sure that all of your pieces are inside your baggies, and your baggies are zipped up. And then you can come to the rug. [Many voices]

36:39 T You shouldn't leave this. [Camera pans around room to students at desks putting papers away. C sitting by easel talking to students, then writing on easel. He is about to lead a sharing session. Teacher walks around room.][Many voices. A boy (S1) pours rain stick twice.]

36:47 C And so I want to start

36:55 T You weren't supposed to bring anything with you. Just leave it on your table. Okay. [Multiple voices and children assembling, sitting on floor near teacher and
$\mathrm{C}, \mathrm{C}$ writing on easel. He draws 2 "horizontal" rectangles, one below the other on a large tablet.]

37:00 T Leave everything at your tables, please. Leave it at your table. Lori, we are waiting for you. Joanie, waiting for you. Shhh. [T is sitting with students.]

37:22 C First thing I want to do, is take a look at something that Zora had started to talk about. [C holds a green paper already folded, apparently in thirds.]

37:30 S Zolu.
37:31 C Zolu, sorry. That Zolu had started to talk about. He I'm waiting for everybody to be looking up here, though.[Many voices]

37:44 C Can I have everybody facing the front here?
37:45 T Mark, you're facing the wrong way, buddy. Your body should be facing Maura. Thank you. Excellent.

37:50 C Only got five minutes. And we have a lot of ground to cover. So. The first thing Zora did, what would you say...

37:57 SS Zolu.
37:59 C Oh, man. I'm having a hard time getting this right. Okay, the first thing that Zolu did, what would you describe, the first thing he did? As?

38:07 S Thirds.

38:08 C He made thirds. Right? Okay. So he made thirds. Then, the next thing he did and I can still see his crease here was he folded it like this. [C folds paper with diagonal aligned with opposite diagonal. See Figure 3 below.] What would you describe
what do you think he was going for? What do you think he was try...


38:29 S I don't know.

38:29 T We're not calling out answers.

38:29 C What do you think he was trying to do here? What was his intention? Any idea? It's kind of a strange-looking shape. What do you Loren, I need your attention. What do you think he was trying to do here? I know you know. Because we're going to show yours in a second. But...

38:46 S Oh!

38:47 C But any idea? What do you think he was trying to do here?

38:50 S I know, I know.

38:52 C No idea?

38:54 S I know.

38:55 T Want to take a guess?

38:56 S I know. I know.

38:59 T Zora, we know you know. Because you did the same thing.

39:01 S But I know it.

39:02 C You want to take a guess? What do you think he was trying to do?
39:05 S I know.

39:06 C Okay. What do you think he was trying to do?

39:08 S I think he was trying to make a sixth and then [inaudible].

39:11 C He was trying to make sixths. But why did why do you think he did this?
What do you think - you're calling out. I'm looking for someone that's not calling out.
Why So he's trying to make sixths. Why do you think he folded it like this? Trying to make sixths? Yeah?

39:28 S He was trying to make a hexagon.
39:31 C And is that an idea for making sixths?

39:33 S No.

39:34 SS Yes.

39:36 C A hexagon has six sides, but ...

39:38 S Oh, yeah.

39:40 C But I'm not sure if I'm not sure how what the connection is to this.

39:43 S Ohh. I know, I know.

39:43 C Do you want to ...
39:44 S Sixths.

39:46 C Okay. Last thought?

39:48 S Because the first way he does it, he has to fold it evenly.

39:54 C Lori, make sure you hear.

39:55 S But, so he already did it that way, so he's probably trying to do it because
there's not another way to do it.

40:02 C So do you think that these two shapes that he ends up with are even?
[The shapes are trapezoids, but as they are not "parallel" they do not quite look congruent or equal.]

40:05 SS Yes.

40:06 S No!

40:07 S I don't know.

40:08 C So he's trying to fold it evenly.

40:09 S Yes.

40:10 S Nobody's [inaudible]
40:11 C Not You don't think it is going to end up even. Can you see the crease in here?

40:14 S Yes it is.

40:16 C Well, let's Hold on. So now we've got this interesting pattern that Zora showed us before. And Colleen actually made a very similar pattern, and she cut it out. How could I find out if these actually came out to be the same size? Yup. [Students raising hands.]

40:36 S Use the one sixth you have, and measure each of them. But you'll be having to flip them over.

40:39 C I'll have to flip it over. So here's ... here's one.


And now, if I flip it Oh! Then it fills this other shape. So would you say that they came out to be equal?


40:53 SS Yes.

40:54 C Yeah.

40:55 S But they're overlapping.

40:57 C Now if I fold it back up, I could also take this cut-out one.

[Folded up, all the corners do not meet, but the cut-out shape is seen to be congruent to both shapes.]

41:01 T Turn your body.

41:02 C And it covers this shape. And it covers that shape.[Many voices]

41:08 C So they did all come out they did all come out even? That was a really creative approach to sixths. I want to take...

41:14 T Can I ask Colleen what she was thinking when she, or Zolu, what they were thinking? Why, why choose that way to fold it? I never would have thought of that
way. Zolu, what made you do that?

41:27 S Because there was [inaudible].

41:31 T Were you trying to [inaudible]. [Many voices]

41:33 S You put the two corners together.

41:36 C Put the two corners together.

41:38 S You can fold eighths

41:40 C Okay. Here's what I need. I need all hands down for one second. I want to I have about one minute. And I really want to take us through something. Uh...What was it that they needed to have, to know that these are going to ... to know that these are going to be sixths? How can we call why can we call these sixths, even though they don't look like any of the other shapes that we had? Why can we call them sixths? Just two people know? Four people? Five? Why can we call these sixths? Yup?

42:16 S Because they're all the same size. You just have to turn them over.

42:18 C Yeah, so one thing about them. Hold on. All the same size. And they're kind of the same shape, too, right? [C writes on easel ("all same size (and same shape)"]

42:28 S Yeah.

42:29 C And same shape.

42:33 S You just have to turn it over, like one-half you have to turn over.

42:35 C Okay.
42:36 S To make them look the same.

42:38 C Okay. What else lets us know that these are sixths? What else lets us know that these are sixths? Yes?

42:47 S Because it takes six of those shapes to make a whole.

42:53 C Ahhh. It takes is that what you were going to say, Colleen? It takes six of them [C writes this on tablet: "It takes 6 of them to make a whole."] it takes six of them to make a whole. Okay. So using those two things for -- How do we know we've got sixths? [He writes at the top "How do we know we've got sixths?"] Oh, l'm writing too small. Okay. So using these two things for knowing how we that we've got sixths, I'm going to draw a couple, and see, do you think that they're sixths? You have one quick thing to say?

43:36 S I can make eighths.
43:38 C Okay. Right now we're working with sixths. Hold on. So take a look. Tell me what you think of this drawing. Ready? I don't see everybody looking.

43:50 T All eyes on [inaudible]
43:51 C Okay. [C drawing lines in a box on the easel.]
44:17 C Uh. Oh, oh.
44:18 S There's two.

44:18 C Oh right. Sorry. Okay. Can I call these sixths? [C points to completed drawing on shiteboard. See Figure 7 below.]


44:26 S Uh Huh.

44:27 SS Yes.

44:27 C Give me a thumbs-up when you're done thinking about it. I don't want to hear any sounds until you're done thinking about it. You know what? I have to leave.

44:33 T Okay, I'm going to send to you what they think.

44:35 C Thank you. All right. Thank you.
[End of Recording]

## Appendix G

## Transcript of Teacher Debriefing with Coach (D), 5-4-04

In the transcript below, Angela, a $3^{\text {rd }} / 4$ th Grade teacher at the Melville School meets with her math coach, Matt, for a debriefing of the lesson she had recently taught and he observed. Names of the school and all participants are changed to conceal their identities.

The left-hand column below indicates the line number in the transcript. The second column shows the time for each speaker in minutes and seconds. The third column identifies the speaker. The fourth column contains spoken words and/or description made by the researcher. Descriptions and occasional interjections are enclosed in brackets. References to this transcript in the body of the dissertation start with the letters D for debriefing, ending with line numbers.

00:12 Matt (M) [Angela, a teacher, and Matt, a math coach, are seated next to each at a table. There are papers on the table. The conversation picks up in the middle of a sentence.] ... that you were looking through books, even though the other teachers were just searching their memories, but I think whatever works best for us I didn't mean it to be one way or the other. I think it would be good if we all, one thing I was thinking about for next time is if people, well I guess I said it, bring some kind of --

00:37 Angela (A) I think all that stuff comes out of these books. Are you agree -- year and year of interpreter and assess --

00:40 M Yeah, oh totally, it totally does. Years of using [inaudible].
00:47 A I don't think we are reinventing the wheel.
00:50 M No, not at all! That's not our goal.
01:05 A Well I thought, I don't know what you wanted to do, but I thought maybe we can just sort of ... map out a couple of days on our schedule. I think my schedule with you is all
messed up because I'm off a week.

01:17 M All right, let's look at the schedule.
01:20 A So you are not here next week even though I had you down for next week? [Angela and Matt are reviewing their calendars and schedules].

01:23 M I am here on -- the 17th; but time -- so that's a day we will work together.
01:40 A Right, and we're going to have; I guess this doesn't affect us. I don't know if Jim talked to you but we are going to have this music performance and dress rehearsal in the afternoon.

01:50 M Right, that doesn't affect us, right?
01:53 A doesn't affect you and I.
01:57 M OK, good, so in the afternoon there's a dress rehearsal?
02:05 A It will be starting at $1: 15$, so I don't know if you are working with lower grades, but they won't be there.

02:11 M OK, and the whole school is attending? [The camera moves in on Angela.]
02:14 A Uh-huh.

02:18 M That's fine, I mean what I'll probably end up, Oh my God it's going to be a long time before I see you guys again, before we have a planning meeting again.

02:27 A Why, we don't meet on that Tuesday?
02:29 M No, I am not here the 18th, 19th, the 20th. I am at the -- I have to score the 4th grade math tests study. [The camera pans over to Matt and then pulls out to show them both.] 02:42 A I wonder if we should try to figure out a way to do our planning meeting on this one-day though. Because we all go to town --

02:48 M I'll ask the other teachers.

02:51 A So I'm going to put down --.
03:00 M OK, so we're still on for 10:40am, right?
03:07 A Uh-huh.
03:11 A I think you are not here on the 18th.
03:13 M Right, so I try to set up a planning meeting for all four of us.
03:24 M Wait a minute you guys aren't free until 12:20, right? That's lunch.
03:30 A Uh-huh, but I'm thinking that maybe either we could do a lunch meeting, an official lunch meeting, or choose to get coverage for us for the period before.

03:46 M I see.
03:52 A I'll ask her too.
03:57 M All right. Do you want to just catch me up a little bit on what happened after I left the other day?

04:07 A Sure, We just continued the conversation about how they knew the pieces were equal. 'cause you had done on the grid paper, those was really great, I thought it was really great. You had drawn a rectangle and you had done this. [The camera moves in on Angela and then pans down to show her drawing a diagram on paper. With a thick Sharpie pen, she draws a large - maybe $6 \times 4$ - horizontal rectangle with about 10 vertical lines, 3 of them darker than the others.]

04:37 A But anyway there were, for each part you drew and I think we were talking about fourths? Let's say we were talking about fourths.

04:46 M I think I made -- it was going to be sixths.
04:52 A Let's just for argument sake,

04:54 M Okey, That's fine.

04:55 A 'cause it's the same idea. Lets say it was fourths, and you used the gridded paper, so it was like -- [The camera shows Angela drawing 7 horizontal lines - there is already 1 vertical line - in the leftmost sub-rectangle, and then she counts. The drawing below, Figure 13 A , is apparently meant to simulate grid paper.]


So for each quarter there were six, Sara noticed there were sixteen boxes.
05:18 M So she just used the area.

05:19 A So she just used the area. So for the last part, you had done something like that ... [The camera shows Angela drawing another rectangle below the first one, in which there are 3 vertical lines and, in the rightmost sub-rectangle, a horizontal line. See Figure 14, below.]


Figure 14

05:31 M That's awesome.

05:33 A And she said, I'm just counting the boxes. There are 16 , I know 2 X 8 is 16 , there are 2 rows of 8 . So she was totally going back to the work that we did with arrays.

05:47 M Right.

05:50 A And then she said, and $4 \times 4$, here we have 4 rows of 4 is 16 , so it's the same number of boxes. [The camera pulls back from the table to show both Angela and Matt. The camera pans down to show Angela writing in the diagram on the paper. The camera zooms in on Angela's hands. The camera pans back up to Angela.]

06:00 A And once she started to count the boxes like hands just shot up in the air, like oh, oh, oh. Because kids totally saw that was a way to prove that they were the same. That was great. 06:12 M So, that's awesome, so a lot of kids connected to that idea of using the area to prove that they are the same amount even though they are not the same shape, yeah? 06:22 M Did you record that in anyway? [The camera pans out to show both Angela and Matt.]

06:25 A I took notes to myself afterwards ... of our staff meeting. So I'm going to put that in my letter. So this is -- you know, I just sort took these notes about ... [A opens a notebook.] M [Looking at A's notebook] Oh Wow!

A ... kids that -- sort of saw -- like things. This was such a wacky way of making the sixth, and yet Cleo and Sara both like gravitated towards that, and both, and I had no problem with it.

06:54 M . Yeah.
06:56 A And then there were kids did it this way [A points to a drawing in her notebook of a rectangle containing 1 horizontal line and 2 vertical ones, thus making 6 roughly equal squarish sub-rectangles. See Figure 10 below.]

and there were kids did it this way -- [A points to another drawing of a rectangle with 5 vertical lines, thus making 6 roughly equal "slim" rectangles. The camera zooms in on the papers on the table and specifically focuses on the diagrams in A's notes. See Figure 11 below.]


Figure 11
07:01 M Now, I would say that this [He points to a diagram in A's notes of a rectangle divided into sixths with 2 vertical lines and a zig-zag, somewhat horizontal, line.] was easier actually to see than what Sara did in this one, [ M points to a diagram just off camera.] because here [Again M points to zig-zag drawing. See Figure 15 below.]


Figure 15
the shapes are congruent.

## 07:10 A Right, Right.

07:11 M So you got this idea of congruence, but then you got this idea of equivalence; and it doesn't have to be congruent to be equivalent; and that's where Sara's area computation helped. [Matt and Angela are pointing to the diagrams on the paper.]

07:30 M One of the things that I was trying deliberately to do, and I did a little of a sloppy job of it, but it was my intention, was to kind of on that same chart where we were about to do these examples, record, so what is it we know so far? Just kind of like reviewing--

07:48 A So we were starting to do that, but they have to be, well, we went back to that because you said they had to be the same shape; and so --

07:59 M Well, I didn't say they have to be. What I said was ...
08:01 A Well, you wrote, "same shape" for something --

08:03 M Right.

08:04 A and so when we started to do this, [A points to drawings in her notebook.] we pointed it to that. Well, it's not the same shape but we are agreeing that it is the same -- value or 08:12 M So having the shape is one way to know ... is one way of knowing, but it doesn't mean that it has to be the same shape.

08:20 M So I think what I wrote on top is "How do we know if it is ... " something like that, yeah?

08:25 A "if it is equal?" [The camera pans down to Angela writing this on the papers.]
08:33 A So we could just sort of list the different ways.
08:35 M Yeah. I think like, a regular reviewing of ... so "What are the criteria that we know this has to kind of submit to?" That's adult language for it, but "What are the things we know about how we can tell when it is a fraction?" [The camera pans up to Matt speaking.] So like, in some of the initial things, it can be just oral, but at the same time I wanted it written so as we went through, I'll $\ldots$. [The camera pans out to show Matt and Angela.]

09:02 A I was thinking this would be really nice to have something like this hanging up. And quite honestly the only reason I haven't done it was because I knew I would have to take it down for this week. So I'm not going to put anything new up there now.

09:12 M Sure, sure.

09:14 A But I thought this would be a really nice thing to just show you know we can look at a whole and its parts in all of these different ways, and -- [The camera pans down to the table where Angela is pointing to specific diagrams on the paper. The camera continues to slowly pan down the page showing the various diagrams.]

09:23 M So the question is--
09:24 A Just assigning kids' names to them. I think that is really nice too.
09:30 M So assigning names to --
09:32 A Just the way they looked at it. How Emil saw six this way, Cara saw six this way. [Matt points to the diagrams on the paper and then the camera pans out to show both Matt and Angela.]

09:40 M So the goal of your chart would be that there are different ways that we can make a sixth, yeah? and so I think that is great to have examples, like I think that's really useful to have illustrations of it. I would also say that having something like this [M points to A's notes] next to it--

10:00 A Right, and what's the same about all of these things? Well, they are, you know, the same size, shape, or whatever, whatever that bulleted list that we come up with.

10:10 M Exactly, and it can be revisable, it can be ongoing. You know, that it gets developed; it doesn't have to be all figured out before you put any of it up. But I know I found that really helpful when I was taking classes with Beatrice DeSica. Do you know who she is? She's at Bank Street. She was teaching a calculus course; and like at the beginning of every class, she'd kind of review with us what we knew; but kind of by asking us questions. So a question is, so, " How do we know?" You know this was a review question really for them at that point. It wasn't the first time they were thinking about that.

10:51 A Right. And the truth is that for ... hopefully for a lot of the fourth graders this should be review for them. For some of the third graders, maybe they have been taking some of this already a little. [The camera pans down to focus on the table and Angela's hand gestures and then moves back up to show both Angela and Matt.]

11:03 M So there is not that they -- that -- [The camera shows that Matt gentlely rub his forehead with left hand and Angela sit silently.]

11:16 M So I mean I just that -- so, I think that for one, it's the reviewing of it; like hearing it again and again as the common language. And the other thing is that then, those are right there for you to refer to, as their reasoning. So when a kid makes an argument about why something is or isn't a fraction, you can say, so "Does that follow this idea"? "No?" "Does that follow this idea?" "Oh, you have a new idea!" [The camera zooms in on Angela who is nodding her head.]

11:42 A Right -- "Should we add that?" Or,"Is that always true?"
11:44 M Exactly, exactly. It helps make it public; and it creates a reference that they can come back to.

11:55 A One of the nice things she talks about in this book [A appears to point to a book, but it is out of camera range. Speculation: she is alluding to a Marilyn Burns' book, which she refers to below at 29:03.] too is, which I feel like, maybe we could go back to do this, as part of this conversation, like "Where are all those places in the world where you hear, or you use fractions?" and you know, "Well do you have half-a-dollar?" or "I'll be there in three-quarters of an hour." or whatever it is.

12:20 A So that -- it just sort empowers them with a little more information. And they know, "Oh wait, I do know a lot about this already." and "I've been using these quite easily already." En-- I think that would be a really nice place to start this conversation, just with you.
[The camera pans over to Matt.]
12:36 M I agree. That's actually something we talked about last time when we started our multiplication/division map. We didn't do it this time. I made a note actually as I was typing it up ...

A Right.

12:50 M ... to raise that. That informs, it's not just for them to tap into; but also informs, what kind of context are you going to choose, as the teacher to work from, so that ... so, you're choosing context that jives with their kind of natural sense-making or intuitive sense-making about fractions.

13:17 M Maybe like, they could start by for homework, like looking for, "Where do you see fractions? Cut it out if you see it; write next to it where you found it." [The camera pans out to show Angela and Matt and then to Angela copying the words he has just said.]

13:48 M It's a good thing for us to think about it as well. Because then, we can make decisions about where, which contexts we are going to use.

13:58 A It's funny like, I always find this unit to be really accessible for most kids. I don't know what it is. Maybe because it is really visual, for at least the beginning parts of it. It is so visual, and it is so concrete. And I think that the way those measurements came were really successful too, is that they were physically moving around pieces of paper. And, I don't know, something about that. [The camera pans out showing Angela from her right side and then swings around to show both Angela and Matt from the right side of Angela.]

14:28 M I think, that there's the visual and the concrete. I think about it in the terms of modeling. It creates a model of the relationships, so they're using the visual and the concrete. I don't think any old visual and any old concrete works. But these are particularly chosen, because
they lend themselves to representing the relationships in it. The other thing I think helps is that there's context around it. Like, it's not like you're saying, "OK everybody I want you to draw a rectangle and cut it into two equal parts." you know. Like one of the things about the brownie activity for example is ... because it's food, you're less likely to get kids that ... who do the thing where they tried to cut it into half; but it wasn't quite half; so then they cut this off. [M makes a drawing of a rectangular "brownie," with 1 vertical line in the center and "cuts off" the right side.] Because it's a brownie. You're not wasting any brownie. Brownies are sacred, you know. [The camera pans down and then zooms in on Matt's hands as he is drawing a diagram on the paper. The camera pulls back to show both Angela and Matt.]

15:34 A I have to say like when we did it, James did this thing where he just couldn't figure out how to do thirds and so he did quarters this way and then he realized all I needed was three and he just literally threw it away hoping I wouldn't notice. [The camera pans down and then zooms in on Angela's hands as she is drawing a diagram on the paper.]


Figure 16
15:49 M Another thing I see a lot when --
15:51 A I think you were here that day, when he did that.
15:55 M Another thing I see a lot when kids are trying to make thirds, is especially third graders the first time they're dealing with this stuff, is they will cut it in half 'cause everything starts with cut it in half. And then they will cut half in half. And so then they will say there are three parts, right? [M makes a diagram. See Figure 17 below.]


Figure 17

16:12 A Right.

16:13 $\mathrm{N} \quad$ And so it's a nice opportunity to open up the question of, "So do you agree that these are thirds?" "Well, if you don't agree, why not?" Well, because thirds means one of three equal pieces and they're not equal pieces. And then someone else might also make the argument that, "Well, that piece is actually a fourth, so I know it can't be a third." [M draws a diagram. See figure below.]


Figure

This piece is actually a half, which is a more sophisticated argument probably. [ M writes " $1 / 2$ " in the right part of the earlier diagram. See Figure below.]


It's not the one that's going to be the first entry for a kid. [The camera shifts over to show Matt drawing a diagram on the paper. The camera pulls back to show both Matt and Angela. Again, the camera zooms in to show Matt pointing to the diagrams and then back again to show them both.]

16:46 A Well, for homework one night they had this problem where, you know it's like this -- and then there was that -- and they said, you know--

16:55 M They already had that for homework? [The camera zooms in and down to show Angela pointing to and marking a diagram on the paper. See Figure 12 below.]


Figure 12
16:56 A Yeah.
16:57 M Okay, how did that go?
17:01 A For the most part you know, kids who were comfortable with fractions and - we had done the brownie shares in class that day - they got that. It came out of this whole piece. They got that. It doesn't matter which way you cut them up, as long as they were four pieces that make up the whole again. They were all considered, they each considered a fourth. Except interestingly Oliver, he says, "No, they are not equal", and that really surprised me. [The camera shows Angela looking for something in her folder.]

17:35 A I don't want to waste your time looking for it, but he was the one kid who didn't buy it. [The camera shows Angela keep flipping pages of her documents.]

17:46 A Like Marcey said, "We made these different halves. We made halves that looked like these different configurations. So we made the diagonal halves. We made, cut in half, this way halves,--" So she's relating to the -- I should say --. [The camera pulls up and out to show both Angela and Matt. The camera pans down to show Angela going through papers and pointing to the diagram on the sheet. (See Fiure below.) Angela continues to sift through the pages.]


Figure
18:03 M And they did that for homework?
18:05 A They took this home for homework.
18:06 M Did you guys get to talk about it?
18:09 A No, because we haven't had time to do any of this. [The camera pans out to show Matt.]

18:11 M No, not right now. --This task is a -- looming overhead.
18:15 A Yeah, it sort of stinks. 'Cause we are just starting this, and I don't really feel like ... I should just have waited.

18:31 A Yes, so she sees that each piece is a quarter of the whole, right? [The camera pans over to a close up of Angela. The camera pans down to the papers and zooms in on the diagrams.]

18:45 M I see another strategy on her page too. She did this-- She said, "Oh I can cut this off and stick it here, and then I have the same shape of pieces." [The camera pulls back to show close-ups of Angela and then Matt and then zooms back down to the table as Matt draws a diagram. See Figure 13 below.]


Figure 13
19:00 A Yeah, eventually I wanted to actually do it for them.
19:04 M So one strategy for figuring this out, is "cut and make congruent."
19:10 A Yes, he's doing the same thing. Ivan.
19:16 M Trying to make this that.

19:17 A [A is reading from Ivan's paper.] "Two squares make one big square, and the two triangles make one big square. Both big squares are the same size and one big square is half of the brownie. Add them, and if, this -- put the whole square brownie on top of the one triangle too, ends to the square will stick out, and both ends fit in the faces -- the square." [Matt makes some notes on the paper. The camera pulls out to show the piece of paper with diagrams Angela is holding. The camera moves up to show both Matt and Angela.]

19:40 M So, this kid is, Ivan is really using two strategies: one is, [M adopts Ivan's voice] "Can you make them congruent?" But the other strategy he is using kind of reasoning about, "Well, if I cut it in half, and then I cut each of those in half again, so it's half of a half." Do you know what I mean? [The camera pans down and zooms in on Matt drawing two lines from the Angela's diagram, and writes "cut and make congruent," the other " $1 / 2$ of $1 / 2$ ".] 19:58 A Right.

19:58 M That, like the reasoning that if I cut it in half, these two I know are equal. Then if I cut this in half, and I cut this in half, even though I cut them in half differently, I know half of a half is the same. [The camera pulls out to show Angela and Matt.]

20:12 A [She nods her head as she says:] Right. [Angela looks down, apparently reading
from an unnamed girl's work:] She says like, "These are even and those are even, and both sides are even, so they're each a quarter of the whole." Then she does the math. So that's a totally different way of looking at it. See now ... I don't know what she's ... what looks the same. They look the same and I took a ruler and measured it. [The camera pans down and zooms in on Angela holding and pointing to the paper and then pans further up the table to show a paper with diagrams on the table. The printed workbook page says: "Different - Shaped Pieces - Here is a picture of a brownie cut in four pieces. Some people think these are not fair shares. Write what you believe." Then the camera moves back to the papers in front of Angela as she goes through them.]

20:40 M So what do you think? Does that--
20:42 A I don't think she understands what she is doing.
20:43 M That doesn't clearly convince me.
20:44 A No.
20:45 M But this is good we're getting a --, like, so "What are the ways that kids can come at thinking about this?" [The camera pulls back to show Angela and Matt as Angela flips through papers.]

21:05 A Okay, so Oliver says, "It is obvious a rectangle and a triangle, big difference." 21:12 M Right.

21:12 A So he is looking at the shape. [The camera pans to and zooms in on the paper in Angela's hand, showing the diagram described above.]

21:16 M So he's looking at the shape and he's saying they're not congruent. So the problem here is that they're not congruent. And when they aren't congruent, what can we do?

12:30 A Right

12:31 M "Oh, we can cut them to make them congruent?" or "We can use some other kind of reasoning." [The camera zooms in on Matt's hands as he draws diagrams and writes notes. The camera pulls back to show both Angela and Matt.]

21:36 A Like Sara's.
21:38 M So it's not enough to just see that they're not the same shape. And that's ... I mean I'm surprised in a way. But I'm also not surprised. You know what I mean? That's a clearly confusing aspect of it. Honestly, in terms of the priority of things, I don't think it is the most important thing in the world. But I think that one of the things to talk about, maybe --, I don't know if you'll have time to talk about it with them, but if you were going to, I would look at both of ... I would start maybe with Oliver and says he says, "They're not the same shape." I agree. I don't see the same shape here. So, we need some really convincing argument that's going to make it make sense why people might think they might be equal. [The camera zooms in on Matt.]

22:33 A And what are some ways?
22:33 M And having [inaudible] Oliver that is, that he doesn't agree, is going to really polarize the class, and that's going to make them, I think, really push for, so, "Is this convincing evidence?" [The camera pulls out to show both Angela and Matt.]

22:47 A And I wonder because we did this before we looked at the area model. I wonder if there won't be some attempts to sort of figure out, ok so what's the area then? So, is that going to help us to get to it?

M Right.
A Like, what if we did this on a piece of graph paper?
23:03 M So, I would push to have this become part of the class, like ...vocabu...it's not
vocabulary, but sense-making, or the logic of equivalency, that these are equivalent pieces, because ... So, it's interesting, because in our meeting earlier, I said that maybe this isn't equivalency as we typically think about it. Typically we think of equivalency as being like different-looking numbers in the fractions, but I think it could also relate to different-looking shapes that are the same amount, just the way $2 / 4$ is equivalent to a $1 / 2$, triangle $1 / 4$ is equivalent to rectangle $1 / 4$. So, I guess in a way it's an early concept of equivalency. Because... I feel like it takes a lot to get a kid to buy into the idea that even though this doesn't look the same, it's the same. And the same issue applies with this ... even though this doesn't look the same, I've got two little pieces and one big piece here. How are you telling me that that's the same? The idea that you could put those two together and then take this piece and put it on top, and you've got the same shape. [The camera zooms in on paper with notes and diagrams. The camera pulls back to show Matt and Angela. The camera zooms in on paper with notes and diagrams and pans to Angela making notes.]

24:38 A Right. Because we're saying before, that one of the ways that we... the one that looked like this ... is that we could just quite simply lay one on top of the other. Oh, look, if we turn these around, they're exactly the same ...[The camera shows Angela's hand gestures as she speaks.]

24:57 M Does an easy congruence argument ... [The camera pulls back to show a close-up of Angela and Matt.]

24:59 A So, we can't do that with them. So, I wonder, do I say to them, "I know that they're the same. How can we prove it?" Or ...

25:06 M No. I would start with, "I was looking at the work, and Oliver looked at this, and he said it's obvious they're not the same shape.

25:16 A And he is the only one that thinks that.
25:20 M Fine.
25:21 A Do I even say "Oliver is the only one who disagrees with us," or no?
25:28 M Well, I think...

25:29 A I mean just to make it an even ...
25:30 M I wouldn't necessarily...

25:32 A Okay, all right.
25:34 M I think it adds dramatic flair to it, maybe, but I think ... no, I think it's better to just say, "Here's what one student said," and it's weighty enough that [inaudible] it's Oliver, right?

25:49 A And everybody else disagrees with him, too. They're not going to know necessarily that they do, but.... [The camera pans down to text on paper in front of Angela.]

25:55 M I mean you could say that, so...
25:57 A No, I hear you. I hear you. I don't need to...
26:03 M So, how can we either convince him or realize that he's correct? Does anyone have an argument that could convince him? So, you want to look ahead a bit, yeah? [The camera pans over to Matt's hand as he touches the pen on the table.]

26:23 A We can. I realize...we can, but I don't have a whole lot ... [The camera moves up to a close-up of Angela.]

26:33 M So, how long should we spend, just so I...so we know...
26:36 A Another 15? [The camera pulls back to show both Angela and Matt.]
26:38 M That's...fine.
26:44 A I'm thinking like, by Friday, I'm hoping on Friday I'll get to do this as much as we can . I've been going ...you know, our schedule has been pretty tight in the next two days, so I
don't really even think we have the physical time in the day. But I'm thinking, on Friday I wanted to have this conversation with them, that just sort of reviewing and where have you seen this in the world? And what can we say so far? And then maybe from there we can look at this. 27:18 M That sounds good. [Pause.]

27:34 A And then, honestly, I was going to review with you the... follow in the sequence of how TERC lays it out in the 3rd grade book, because there's a bunch of games, and there's the pattern block wave, looking at equivalence, and.... I also wanted to look at the order of stuff, from smalls to largest. Because we were talking about looking at the number line and placing stuff in the number line, too. I thought that could easily get thrown in there. If you have a more sensible way to sequence this ...[The camera pans down to Angela making notes in her calendar and then pulls back to show both Angela and Matt.]

28:19 M No. I think for the most part the 3rd grade book ... it's a $3 / 4$ book, and so it's mostly fine. I think that there are a couple of things from the 4th grade book that would be good to include. [The camera zooms in to show the notes Matt has written and as he continues to write.]

28:38 A Which ones?
28:38 M Like, they go a little bit further into ...into comparing fractions. They do that activity, where you sort them into equal to zero, between zero and a half,

29:01 A Well, like Beatrice was talking..

29:03 M like Beatrice was talking about. So that's in Marilyn Burns' book, it's also in the 4th grade Investigations. They do a lot of work at the beginning on different shapes and equal pieces ... that's the name of the book ... and so they divide Geoboards up into halves, fourths. I don't think you need to spend a lot of time on that. [The camera pulls out to a close-up
of Matt. The camera pulls out a little to show a close-up of both Matt and Angela and then pans down to show the papers on the table.]

29:26 A Honestly, too, I'm just thinking management-wise with the level of immaturity in here, rubber bands and geo boards just doesn't seem ... [The camera zooms in on the notes on the papers in front of Angela.]

29:34 M Well, they do it on dot paper. They don't have to do it on Geoboards. So they could just do the drawings on dot paper. I mean, you might want to choose ...

29:44 A That might be nice, to sort of throw in there like ...
29:47 M But I wouldn't spend too many days on it. Like, maybe you could do halves and fourths in one day. But so ... and then they go a little bit further into comparing fractions, and then they get into decimals a bit more. The connection to decimals and percents a bit more at the end of the 4th grade book than they do in the 3rd grade. So I would definitely think that a goal is to get a little bit into the connections between decimals and percents. So, coming back for a minute, you guys have made this set of fraction cards. I know that Brenda and Susan [the other grade level teachers] had chosen to do more of the fraction strips. I think each have their merits, and we could talk about that, but I don't think it's the best use of our few minutes left. So, the things that they say in the TERC book, to do with those ,... one is...one is ordering them on a number line, right? [Angela makes some notes. The camera pulls back to show both Angela and Matt .]

31:11 A Then there's this really nice discussion where there's a piece that's like this ... sort of figure out what ... [A makes a diagram. See Figure below.]


Figure
31:23 M I think you're figuring out what the whole is based on the missing part, right?
And that goes along with the ordering. And so, one of the big things that comes out of the ordering work, is that [M starts making a diagram.] ...

if you have $1 / 8$, then $1 / 6,1 / 4,1 / 3$ and $1 / 2$, that they, then the idea is, "What patterns are we seeing here? Oh, the smallest piece has the biggest number on the bottom. The biggest piece has the smallest number on the bottom. Why do you think that is?" So you start with the unit fractions, and so that's really nice ... a really important conversation to have, because it gets into the idea of, what does the numerator mean, what does the denominator mean?"Why do you think ..."

First, they have to notice the pattern, but then the "why" is the really important thing, because when the number on the bottom is bigger, we're cutting it into more pieces.

32:41 A Right.
32:42 M I think that's a really important aspect of that. And then I don't know if they say
to do this in the ... Teacher's Guide, but maybe ... They've got all unit fractions as it is now. So, it's all unit fractions, like $1 / 4,1 / 8$. And what you've got there is like what's missing from a unit fraction. So, it could be interesting to have some cards that are like $2 / 4,2 / 3 \ldots$ [The camera pans down to Angela pointing to her papers and drawing diagrams. The camera zooms in on Angela's hand and the diagrams on the paper. The camera shifts to a close-up of Matt writing on paper and pointing to notes he's written. The camera pulls up and back to show a view of Angela and Matt from the side and then back to Angela writing notes on paper. The camera pulls back to show Matt from behind Angela.]

33:26 A Right. And where do those fit?
33:29 M Yeah. And where do those fit? And so, moving those around. But I don't know how you'd make those. And I don't know that you want to spend the time to make them right now.

33:38 A There are some games, too, where you have to make number sentences, and you have to come up with some equivalence too, based on the fraction cards that you made. [The camera pans to show the table and zooms in on the notes and Angela and Matt's hands.]

33:49 M That's, I'd say that that's different from this [It is unclear what M refers to by that and this], but that's another really good thing to do.

33:53 A Well, I was thinking too, as you were talking that, I haven't plugged in any parts of the group stuff, you know like how many red tiles out of all of the tiles are there? So, I feel like that needs to happen...for maybe a day or two

34:08 M Right. I wouldn't mix it in right now while you're doing this though, right?
34:11 A No. But it got to something that's completely, in my mind ... maybe I'm wrong ...
it's completely separate from looking at fractions this way.
34:18 M I think so. I think there's looking at fractions as area of one contiguous shape, and looking at fractions as part of a group. So, the other thing to do with the fraction cards is to do a page of ... so, just what are some number sentences you could make using your fraction cards? And so, they'll say things like, "Oh, $1 / 4$ plus $1 / 4$ equals $2 / 4.1 / 4$ plus $1 / 4$ equals $1 / 2$. " [M writes as he speaks. The camera focuses on the notes on the paper and then pulls back to show their hands as they write notes.]

34:57 A That's one of the exercise...one of the investigations they have to do that. They have to come up with the whole.

35:03 M So they come up with as many number sentences as they can. You may want to constrain it, like, "Okay, find all the number sentences you can come up with for making 1/2. Find all the number sentences you can come up with for making one whole." You know what I mean? So like, at first constraining it so that when you come together, you can have a more focused conversation about some of the things that they noticed. [The camera pulls back to show part of Angela and Matt hands over the papers. The camera shifts to show both Angela and Matt.]

35:37 A Right.
35:38 M And with the half, I think if you start with $1 / 2$, that could be nice because one thing to notice ... there are some really important things to notice about when something is equivalent to $1 / 2$. So I would start with that first. Like, "Oh, $3 / 6 \ldots 1 / 6$ plus $1 / 6$ plus $1 / 6$ makes 1/2." "Okay." "Is there another way to say that?" "Oh yeah, $3 / 6$ equals $1 / 2$. . So, if they just said the first part, you would need to ask that question, because an important thing to notice is, "Oh when the denominator is sixths, I know half of 6 is 3 , so $3 / 6$ is going to be $1 / 2$." "Yeah,
does that make sense? Why does that make sense?" "Oh, because you need six of them to make the whole, so half of them is ... just ..." 36:22 A Right.

36:24 M And someone else might say, "Oh, the numerator is half the denominator." So there's a lot of discussion that could come out of that. And you could, on the back of that sheet, have a different one or a more challenging one, but maybe everybody talks about the front side, and for your kids that are moving through it more quickly, they can do a more challenging one on the back, or maybe more open.

36:47 A What would be a more challenging one?
36:49 M I think ... well, you could ... one possibility would be ... I'm playing with either making a mixed number, a number greater than 1 , or it could just be more open, like, "Find any combinations that you want to find ... write any number sentences you want." And then see where their kind of preferences lead them. But what do you think? Do you have a thought about that?

37:39 A I think if they had to do ... what do you call ...do you call them mixed fractions ... what do you call those, where it's $1-3 / 5$ or something? What do you call that? [The camera pans to Angela and zooms in a bit.]

37:52 M I think they're called mixed numbers, and it's still okay to call them. You can also call them "amounts greater than 1." Like, it's not so cool to call them "improper fractions," for example, like $12 / 3$, because there's nothing improper about it. Fractions greater than $1 \ldots$ that gets a little bit tongue-twistery. But, I think it's more accurate. [The camera shifts a bit to include Matt .]

38:22 A We used to call them "improper fractions."

38:24 M Everybody called it "improper fractions," right? But you can see how that's a little bit problematic for a kid just starting to make sense of it. Because then they miss the idea that there are equivalencies, because "Oh, it's improper, so I can't [inaudible -- two voices] to anything."

38:46 M So, kind of like you guys did with all the measurement stuff, I definitely like the idea of making number sentences out of what you've got. Like every time you have a new model for fractions, using that to help you make number sentences, and to find equivalences. So this starts the conversation of operations. This starts the conversation of equivalence. This starts the conversation of adding unlike denominators, because they're going to find some equivalence with unlike denominators, too. Like $1 / 3$ and $1 / 6$ makes $1 / 2$. But it's just the beginning. One thing you might want to put up is an equivalency chart, like every time we find ... it's really ... [The camera pans down to the papers and zooms in on Matt pointing to the notes. Then the camera slowly pans to Angela's hand on the paper with notes.]


Figure 19
39:38 A You can't do it until after the [inaudible].
39:40 M It's really done in the 5th grade units, but I think it's good throughout where it's just a chart that says, " $1 / 2=\ldots$ " and any time you find something that's equivalent to $1 / 2 \ldots$.. " $1 / 3=$ ..." and anytime you find something, and so on. " $1 / 4$... " [Matt makes a diagram.]


Figure 20

Just so that they have it kind of as a reference. And it's also, every time something new comes up in your work, it's a reason to come back to that chart, so it just keeps it on their mind, the idea of equivalence. And then you were thinking about the fraction cookie stuff, and so that gets into thirds, halves and sixths.

40:24 A And then, that also addresses like operations of unlike denominators, because you're using like $1 / 3$ and $1 / 2$. You're using pattern blocks. [The camera pulls back to show both Matt and Angela.]

40:40 M So it's another area model, and it gives you another opportunity to come back to this writing number sentences. So it doesn't all have to come out of these fraction cards, is one important aspect of it. And then for ordering and reasoning ... Well, in between that, I know there's something where they have like... 5 brownies and 4 kids, or something. [ M draws a diagram. See Figure 21 below.]


Figure 21

So, what do they each get? Or 7 brownies, 4 kids? [The camera pans to table with Angela taking notes and Matt adds 2 square "brownies" to his previous diagram. The camera then
focuses on the diagrams on the paper. The camera pulls back to focus on Matt and then includes Angela. The camera pans down and zooms in on the diagrams on the paper and Matt writing. See Figure below.]


Figure
41:32 A So they each get ...
41:35 M What does each person get, right? And they word the problems always on purpose to be, kids ... brownies ... but really, ultimately, it becomes "Oh, you're dividing 7 brownies among 4 kids," which is $7 / 4$, which ends up being your answer. I don't know that... I don't think I would want to rush, no, I don't want to rush 3rd graders into seeing this pattern. I wouldn't necessarily want to rush 4th graders, either, into seeing that. But what I ...[The camera pulls back to show Angela and Matt.]

42:16 A But what's an equivalent to 7/4? Like, how many whole brownies can you make from 7/4? I think that's worthwhile.

42:29 M Well, that can come ... so I wouldn't rush them to the shortcut of saying, "Oh, just flip it around and turn it into a fraction," or, "Flip it around and divide." But at first, kids do very concrete representations of it, and so, one thing becomes ... well, I'm thinking particularly for the 3rd graders, but it's good for the 4th graders, too, I think ... the argument of, "Someone got this as their answer, got a whole for each person plus $1 / 2$ for each person, plus $1 / 4$." So, someone got a
whole plus $1 / 2$ plus $1 / 4$. [ M adds to prior diagram.] And someone else got $7 / 4 \ldots 1 / 4$ plus $1 / 4$ plus $1 / 4$ plus $1 / 4$ plus $1 / 4$ plus $1 / 4$ plus $1 / 4$. [ $M$ draws 7 small squares. See Figure 22 below.]


Figure 22
Because they divided all of these into fourths. You know what I mean? Who's better off? Which is the better deal? Which one would you rather have? Now, it's possible that they think, "I don't want all those little pieces to get lost," but, so then the question could be, "Who's fuller? Who gets to eat more brownie?" And you might be, I don't know, you might be surprised that some kids could end up thinking, "Oh, one of those is more than the other." So coming back and arguing or discussing about why are these equivalent? [The camera pans to notes on table and Matt writing diagrams. The camera zooms in on the diagram. See Figure 23 below.]


Figure 23
[The camera pulls back to a close-up of Matt's hands and then pulls back to show both Angela and Matt. Matt makes a diagram. See Figure below.]


Figure

44:07 M Now, the tricky thing is, I think that's really wonderful problem-solving. The question is, is that an efficient way to get kids to be able to think about equivalence? It's not the
typical way that you use equivalence. Is it a good way to get kids thinking of ... One thing that it definitely does is, it generates a reason to have to add unlike denominators for some kids. And so, one question could be like...let's say most...

44:42 A I see it as like, when they're forced to look at this, then they're forced to realize, "Well, I have to break a half into quarters and I have to break the whole into quarters," and it forces them to try to figure out a way to resolve the differences that are here, to make it something that they can work with, which is, $1 / 4$ plus $1 / 4 \ldots$... So, I feel like this is a really good exercise. [The camera zooms in on diagrams on paper.]

## 45:09 M Right.

45:09 A How do we get this to look like that? Maybe that's the other way to [inaudible].
45:13 M And actually, neither of them are completely ... Like, you could do this, you could say, "Here's one, and I know that two of these make a half, and this makes a quarter." [Using his previous drawing, $M$ draws a circle around 4 of the $1 / 4 \mathrm{~s}$, another circle around 2 of them, and points to the other $1 / 4$.] Or you could say, "This is 1 and this is $3 / 4$. So I have $13 / 4$." So you could just as easily turn this into this, or find some kind of middle ground that they both meet at. Or you could break this up. So, that all three of those ways would be great. So, it does generate that. So, I don't know how much time you feel like you have, but I like these. I think it's a great investigation. It's a great opportunity to have to do some sense-making around fractions. [Matt pointing to and writing diagrams on paper. The camera pulls back to include both Matt and Angela.]

46:06 A These problems also like make ... how they start solving these makes me think exactly about the milk carton problems that they did earlier in the year.
M Yeah, yeah.

A ... That experience of having to take something and share it.
46:25 M And that's one of the beautiful things about it, is that there's context, which supports sense-making. And then from that you attach symbols to it. And the symbols are really representing what you're manipulating, what you're making, and it's just kind of a shorthand notation. So rather than starting from manipulating symbols, you're starting from, "How do I find a shorthand way for communicating my amounts?" Yeah, that would be really cool. And I think for 4th graders that find this kind of easy, to say, "Well, on the front I'm going to give you three with pictures, and on the back I'm going to give you just the numbers. You can draw a picture if you need to, but if you can find some ways of thinking about this without having to draw a picture, then try doing that to challenge yourself." And that could be cool. And then they'll probably end up seeing this pattern. [The camera zooms in on diagrams on paper and then pulls back to include both Matt and Angela.]

47:32 A Right.
47:35 M So, if you were to do it for two days, that would be ... And then you could revisit this after you do some comparing fractions, after you get into comparing and ordering, and they have this either in the 3rd or 4th grade book. I think it's in the 3rd grade book. After they've done some comparing, there's ... "At one table there are four kids and seven brownies, and at another table there's three kids and five brownies. Which table gets the better deal? Which table would you rather be a part of?" And one nice thing about this is that it gets into comparing fractions ... and this can be the last thing we talk about ... One of the nice things that this does is, it promotes the idea of ... this idea of like same ... when the numerator ... So, this is a bad example. Four kids and seven brownies, four kids and five brownies. So, the idea is, of course, if it's four kids sharing, you want to be at the table with more stuff. If it's the same number of
kids at both tables, you want to be at the table with the most stuff. And if it were the other way around, if it were seven kids, four brownies, and five kids, four brownies, then if the number of brownies is the same, you want to be at the table with the fewer ...[The camera zooms in on Matt making notes on the paper.]

49:18 A Fill their stomach.
49:19 M Exactly. That connects really nicely to the reasoning behind comparing 7/4 to 7/5 to $5 / 4$, and $4 / 7$ to $5 / 7$, that if you're comparing these you're saying, "Oh, it's the same size pieces, but I have more of them." They're very related, but it's not as clear...

49:51 A If they order these they can see that these pieces are bigger and these pieces are smaller, and what does that do to the number of pieces that are smaller and the number of pieces that are ... You know what I'm saying? They've already hopefully realized that on the number line of fractions, that $1 / 7$ is going to be a lot smaller piece than $1 / 4$.

50:15 M And that's when it gets tricky, when you're comparing, let's say $3 / 4$ to $4 / 7$. How do you compare those? "Okay, these are smaller ..." Make it $2 / 4$ and 4/7. These are smaller pieces, [pointing to the 4 of 2/4] and these are bigger pieces [pointing to the 7 of the $4 / 7$ ]. But I have a lot of the small pieces [pointing to the 2 of $2 / 4$ ] and a few of the big pieces [pointing to the 4 of 4/7]. It's close, it's very close.

50:47 A Right.
50:48 M So, another strategy ... the same denominator, there's the same ... Oops, I messed something up. Well, anyway, there's the same denominator strategy, there's having the same numerator. So, this should have been $4 / 7$ and $4 / 5$. There's having the same denominator, there's having the same numerator, but when they're not the same denominator or the same numerator, that's when something like, "Oh, this equals $1 / 2$ and this is ... for 7 I would need $31 / 2$ of them ...

It's more than half." I don't know that's enough, it's greater than a half ...[The camera pulls out to show both Matt and Angela. The camera zooms back into Matt making notes on the paper.]

51:41 A So that probably is more like that, closer to 1 , which one is closer to $1 \ldots$
51:47 M Yeah, yeah, exactly.
51:51 A Alrighty.
51:52 M You have some ideas of where you're going to go?
51:55 A I have enough data. [The camera pulls out to show Matt and Angela.]
52:03 M I'll see you on that Monday, in 2 weeks, on the 17th.
52:07 A Right before we go to camp, so they should be nice and crazed. Want to come to camp with us?

52:15 M I wish I could. I'm going to have to be grading the 4th grade math tests. 52:20 [End of File]

## Appendix H

## Interview Questions

(Questions taken largely from Landis, 1990)

## Pre-Lesson Teacher Interview

## A. School Background

I want to ask you some questions about your background as a student.

1. If you had to pick one subject as your favorite subject in school, what would it be? Why?
2. (If answer to \#1 was not math:) How did you feel about math?
3. Do you remember a particularly good math teacher? What made him or her good?
4. How about a particularly poor math teacher? What made him or her poor?
5. Did you major or minor in math? What mathematics courses did you take in college or after college?

## B. Teaching Experience

Now l'm going to ask about your experience as a teacher.
6. How did you come to choose teaching as a career?
7. Describe the teaching positions you have had. How many years have you taught? What grade levels have you taught?
8. Describe the students you have taught.
9. What do you think is important in teaching math? Please explain.
10. Do you meet with other teachers to discuss math issues? Please explain.

## C. Coaching Process

Now l'm going to ask you about your experience with the coaching process.
11. How many coaching cycles --- preconference-observation-postconference --have you had?
12. How have they gone?
13. Does anything stand out as particularly helpful or unhelpful?
14. Have you learned anything in particular about math, math teaching, or about yourself as a math teacher?
15. Have you reinforced, modified or changed any of your beliefs about math or math teaching as a result of coaching? Please explain.
16. With regard to the coaching cycle that is about to happen, is there anything in particular you are working on?
17. How do you think coaching compares with other forms of professional development? Please explain.

## Post-Lesson Teacher Interview

## D. Observed Lesson

I want to ask you some questions about the math lesson you just taught.
18. How did the lesson go? Please explain.
19. Would you do it basically the same way again? Please explain.
20. Have you taught a lesson like this in the past?
21. (If yes to \#20) Did you do anything differently this time?
22. (If yes to \#21) Can you describe what was different? Why did you do things differently?
23. Was there something the coach said or did that influenced you? What?

## Coach Interviews

## E. Background of Coach

I want to ask you some questions about your background.
24. How did you come to be interested in math? What did you like (or dislike) about math?
25. Do you remember how you felt about yourself as a math student? Please explain.
26. What mathematics courses did you take in college or after college?
27. Have you taught math? What grades? How many years?
28. Aside from teaching, what math jobs, if any, have you had?
29. Have you had any other math experiences that prepared you to be a coach?
30. In general, what are your hopes and expectations for the coaching process?
31. How many teachers are you coaching now?
32. How is that going?
33. Do you meet with other coaches or lead coaches? Please explain.
34. Have you been having workshops or other professional development experiences? How have they been?

## F. Coaching Process and Observed Lesson

Now I want to ask you some questions about the coaching process (with this teacher).
35. How many coaching cycles --- preconference-observation-postconference --have you had?
36. Are there any particular issues or problems that you have been working on? Please explain.
37. How has that gone?
38. Did you see or hear anything in this lesson that particularly caught your attention? What?
39. Do you think you had any impact on the teacher's practice? Please explain.

## APPENDIX I

## Page from TERC workbook

## HOW TO MAKE FRACTION CARDS

Materials: 5 sheets of paper (all the same color), a pen or crayon, scissors

## What to Do

Fold and label sheets as shown. Write each fraction on one side only. Mark the fold lines. Cut on the lines.
Halves Fold the sheet in the middle.
Thirds Fold the sheet in 3 equal pieces.
Fourths Fold the sheet in half one way, then in half the other way, to make 4 equal pieces.
Sixths Fold the sheet the way you did to make thirds. Cut apart the thirds. Fold two of the thirds
 in half and cut to make long, skinny sixths. Cut the other third in half the other way to make chunky sixths.
Eighths Fold the sheet into fourths. Open the paper to see the folds. Fold each fourth in half.
Store your Fraction Cards in a plastic bag or envelope. Save these directions with them. You may want to make another set for playing games.

## Ideas to Try at Home

Turn over the cards to hide the labels. Order the cards from smallest to largest. Then turn them over. Look at the number pattern. What do you see?
Find different combinations to make one whole. Put
 the Fraction Cards on top of a whole piece of paper to keep track. Keep a list of your combinations. For example:

$$
\frac{1}{2}+\frac{1}{3}+\frac{1}{6}=1 \quad \frac{1}{3}+\frac{1}{3}+\frac{1}{3}=1
$$


[^0]:    ${ }^{1}$ The coaching system was changed in 2006 or 2007 and decisions whether to have a coach was largely left to schools, often to its principal. In fact many, probably most, schools still have math coaches; the nature and organization of the coaching has undergone some major changes. (S. Werner, a mathematics coach, personal communication, 10/18/09.)

[^1]:    ${ }^{2}$ The regions were abolished in 2007.

[^2]:    ${ }^{1}$ On May $4^{\text {th }}$ and June $1^{\text {st }}$, several of the $3{ }^{\text {rd }} / 4^{\text {th }}$ grade teachers, including Angela, met with Matt to discuss plans for future lessons. The sessions were observed by the researcher and videotaped, but were not analyzed.
    ${ }^{2}$ On June $17^{\text {th }}$, Angela taught a lesson on designing cardboard quilts. The lesson was observed by the researcher and videotaped, but not analyzed.

[^3]:    ${ }^{1}$ This refers to the lesson, taught on 6/17/04, where students made quilt patterns. That lesson is not analyzed in this study.

[^4]:    ${ }^{2}$ To repeat, the first lesson is analyzed herein, the second lesson is not.

[^5]:    ${ }^{3}$ This status means that a school has fallen below far below standards and is in danger of being closed.

[^6]:    ${ }^{4}$ Matt was math coach at Melville for two full years starting in 2003 and part of another. (Personal communication, 8/25/09.)
    ${ }^{5}$ Angela also taught a lesson observed by the coach and researcher on 6/17/04 and referred to by both the teacher and coach in their interviews. The primary activity of the class was to cut out and color paper "quilts" with different patterns. The lesson was not analyzed in this study.

[^7]:    ${ }^{6}$ It should be pointed out that the cameraman, and thus the microphone, were instructed to focus on the teacher, and thus the statistics are bound to be somewhat skewed.

[^8]:    ${ }^{7}$ This example is likely to come from the TERC Investigations books.

[^9]:    ${ }^{1}$ The second lesson was not analyzed or included in the Appendix.

