Risk-Adjusted Information Content in Option Prices

By Durga Prasad Panda

A dissertation submitted to the

Graduate School –Newark

Rutgers, The State University of New Jersey

in partial fulfillment of requirements

for the degree of

Doctor of Philosophy

Ph. D. Program in Management

Written under the direction of

Dr. Ren-Raw Chen

and approved by

Newark, New Jersey

May, 2010

Abstract

Risk-Adjusted Information Content in Option Prices

By Durga Prasad Panda

Dissertation Director:
Dr. Ren-Raw Chen

There are many measures to price an option. This dissertation investigates a risk-adjusted measure to price the option with an alternative numeraire that retains the expected return of the underlying in the pricing equation. This model is consistent with the Black-Scholes model when their assumptions are imposed and is consistent with the standard capital asset pricing model. Unlike many asset pricing models that rely on historical data, we provide a forward-looking approach for extracting the ex ante return distribution parameters of the underlying from option prices.

Using this framework and observing the market prices of options, we jointly extract implied return and implied volatility of the underlying assets for different days-to-maturity using a grid search method of global optima. Our approach does not use a preference structure or information about the market such as the market risk premium to estimate the expected return of the underlying asset. We find that when there are not many near-the-money traded options available our approach provides a better solution

to forecast future volatility than the Black-Scholes implied volatility. Further, our results show that option prices reflect a higher expectation of stock return in the short-term, but a lower expectation of stock return in the long-term that is robust to many alternative tests.

We further find that ex ante expected returns have a positive and significant cross-sectional relation with ex ante betas even in the presence of firm size, book-to-market, and momentum. The cross-sectional regression estimate of ex ante market risk premium has a statistical significance as well as an economic significance in that it contains significant forward-looking information on future macroeconomic conditions. Furthermore, in an ex ante world, firm size is still negatively significant, but book-to-market is also negatively significant, which is the opposite of the ex post results.

Our risk-adjusted approach provides a framework for extraction of ex ante information from option prices with alternative assumptions of stochastic processes. In this vein, we provide a risk-adjusted stochastic volatility pricing model and discuss its estimation process.

Acknowledgements

My deepest indebtedness goes to my advisor Dr. Ren-Raw Chen for introducing me to financial mathematics during the earlier part of my study that has been the building block of knowledge throughout this research and will be invaluable to me beyond this program; for introducing me to financial research which lead to this thesis; for joint formulation of two papers out of the risk-adjusted framework; for countless hours of his time for guidance and discussions on current and future research. I wish to dedicate this thesis to him.

I am also deeply indebted to Dr. Dongcheol Kim for many hours of valuable discussions on new research possibilities, exploration on extraction of ex ante beta, many new findings using ex ante information, and joint formulation of two papers based on this research; to Dr. Kose John for being a valuable source of inspiration for me to think in different ways to recognize the important aspects of a given research; to Dr. Cheng-Few Lee for many helpful discussions during the research process; to Dr. Oded Palmon for the feedback on the earlier draft of this dissertation, and suggestions for many robustness tests throughout this process.

I am also grateful to Dr. Ivan Brick for many valuable suggestions about teaching and research. My sincere gratitude goes to Dr. Glenn Shafer for many helpful discussions during my research. I am grateful to Dr. Shari Gifford, Dr. Darius Palia, Dr. Robert Patrick, Dr. Abraham Ravid, Dr. Tavy Ronen, Dr. Yangru Wu, and Dr. Feng

Zhao for the knowledge I gained during my Ph. D. study. I thank Rutgers University for providing me the Teaching Assistantship and Fellowship during this process. I also wish to thank my classmates for many cordial discussions during the research process. Finally, I must thank my wife, Minati, and my son, Om for their patience and support during my Ph.D. study and research. Without their support this would not have been possible.

Table of Contents

Chapter-1

Introduction

The long history of the theory of option pricing began when the French mathematician Louis Bachelier in 1900 deduced a formula based on stock price that follows a zero drift Brownian motion. Many year after Bachelier, the celebrated Black and Scholes (1973) paper provided a pricing model for European options assuming a positive drift Brownian for the stock price that is closer (than zero drift) to the historical price movement of stocks. In this setting, they show that the option can be priced by forming a continuous hedging portfolio of the stock, and the option so that at any instant of time the portfolio thus formed is riskless; which intuitively implies, in this approach, the drift factor and the Weiner component cancel out from the pricing equation. Thus, Black-Scholes pricing formula does not depend on risk preference of the representative investor. Although the pricing formula can be obtained with a specific utility framework as shown in Rubenstein (1976), it is not necessary to go through a utility route to achieve this option pricing formula. In addition, one of the main objectives of the Black-Scholes option pricing formula is to obtain a valuation method that will be a function of parameters, which are mostly known at the time of pricing. From this perspective, we see the option can be priced by knowing the interest rate, stock price, strike price, time-to-maturity, and the stock return volatility. All these parameters except the stock return volatility is known with certainty at the time of pricing a European option.[1]

In fact the Black-Scholes model uses the so called traditional risk-neutral measure to price the option in which the money market account is the numeraire.[2] It turns out that by using this measure we can price the option with least number of unknowns. Even though the traditional risk-neutral model provides a parsimonious measure for pricing the option, it is not the only measure that can price the option. For example, we can use the zero coupon bond price as another numeraire and formulate another option pricing equation. Therefore, there are many measures to price the same option. However other measures may contain more unknowns; therefore may not be the measures of choice when it comes to pricing. Nonetheless, irrespective of the measures we use, the price of the option should be the same.

Our objective in this research is not to price the option. Therefore we are not looking for a measure that is parsimonious; rather we are looking for a measure that contains the parameters we seek to estimate, such as the expected return of the stock. In this vein, we pursue a discrete time physical measure approach in which every asset grows by their corresponding risk-adjusted growth rate. Therefore, our approach retains the expected growth rate of the stock. Unlike the Black-Scholes model, the advantage of our risk-adjusted model is that it does not require a continuous rebalancing assumption. However the disadvantage of our model is that it has many unknowns, whereas the

---

[1] A European option is an option that can be exercised at the time of maturity as opposed to an American option that can be exercised at any time until maturity.

[2] Black-Scholes assume the short-term interest rate is known and constant.

Black-Scholes equation has only one unknown namely the volatility of the stock return. In fact our approach can be thought of as a generic model of which Black-Scholes is a special case. For example when investment horizon is infinitesimal or continuous rebalancing is assumed, our model will collapse to the Black-Schoels model.[3] Therefore, with the assumptions of Black-Scholes, our approach is consistent with their model. Furthermore, using a discrete time approach we make our model consistent with the standard capital asset pricing model (CAPM). This means that the expected return we extract from this model could be used to test this version of the CAPM. In fact we can think of our research as a framework, where with different assumptions of stochastic processes along with risk-adjusted numeraire we could extract additional information from option prices. For example, we could have a risk-adjusted pricing equation with stochastic volatility that extracts additional ex ante information from option prices.

In this dissertation, we derive the risk-adjusted pricing formula and work on the following branches of research:

1) Extraction of risk-adjusted expected return and volatility from market observed option prices and robustness test of the term structure of expected return.

2) Comparison of information content of risk-adjusted implied volatility and risk-neutral implied volatility to forecast future volatility.

3) Study of cost of equity using the risk-adjusted expected return.

---

[3] Therefore, if market prices of options truly reflect these Black-Scholes assumptions then we will not be able to extract the expected stock return from these prices.

4) Use of risk-adjusted expected return to test the standard CAPM and the investigation of its relationship with macroeconomic variables.

Findings of this Dissertation

Using the risk-adjusted model on OptionMetrics month-end data for the period of January 1996-April 2004 we jointly estimate the ex ante expected stock return and volatility based on a grid search method to look for the global optima. We estimate these parameters separately for S&P500 index options and all stock options. Our approach estimates different implied expected return for different time horizon based on days-to-maturity of the option.

There are three advantages of our approach of estimation of implied expected return. First, the expected return of a stock can be computed without using any information of the market portfolio such as the market risk premium. This implies we do not have to define what the 'market' consists of, and we do not have to estimate the risk premium of the market, which is required in traditional asset pricing models to estimate the expected return. Second, our approach extracts implied stock return based on forward looking options data unlike the Fama and French model, and the CAPM that rely on historical information. Third, we do not use a preference structure to arrive at our results.[4]

---

[4] As we know, although Black-Scholes does not use a preference structure, it is consistent with CPRA utility function as shown by Rubenstein (1976). Similarly, even though we do not use a preference structure, our approach is consistent with the quadratic utility structure.

Using S&P 500 Index options, we discover the following four results. First, our result shows that investors have higher expectations of stock returns in the short-term, but lower expectations in the long-term. This term structure finding is robust to many alternative tests. Second, the term structure of volatility using our model is much flatter than the term structure using the Black-Scholes model. Third, the empirical investigation shows that a combination of our implied expected return and implied volatility with Black-Scholes implied standard deviation provides a better model, than Black-Scholes implied standard deviation alone to forecast future volatility of stocks for any combination of moneyness and maturity. Finally, the implied volatility of our model can predict much better future realized volatility than the implied volatility of the Black-Scholes model, more so for short maturities of 90-days or less. In general, our risk-adjusted approach provides a better measure (than Black-Scholes implied volatility) that captures moneyness biases even without adjusting for stochastic volatility. Therefore, if we are concerned about the smile while forecasting future volatility using all options data for a stock, then our approach provides a better solution so that we do not need any adjustment for moneyness bias. This implies, when there are not many near-the-money traded options available, our approach provides a better alternative to forecast future volatility.

Using all stock options data we estimate the ex ante expected return for individual stocks. We use this expected return to compute the cost of equity for different industry groups. Unlike the CAPM and Fama and French costs of equity estimates, our approach doesn't need the unobservable market risk premium. We find the option implied expected returns are more stable over time than the Fama and French estimates. In fact Fama and French cost of equity estimates in some cases become negative, which is not the case using our model. Furthermore, our result shows even using all stock options the downward sloping term structure of expected return is maintained.

We also examine the cross-sectional relations between ex ante expected returns from our risk-adjusted model and ex ante betas. We find that ex ante expected returns have a positive and significant cross-sectional relation with ex ante betas in all investment horizons considered. This significant relation is maintained regardless of the inclusion of firm size, book-to-market, and momentum. The cross-sectional regression estimate of ex ante market risk premium has a statistical significance as well as an economic significance in that it contains significant forward-looking information on future macroeconomic conditions. Further, we find that ex ante betas have significant explanatory power for realized ex post returns. A significant relation between ex ante forward returns and forward betas is also found. Other interesting findings are that, in an ex ante world, firm size is still negatively significant, but book-to-market is also negatively significant, which is the opposite of the ex post results; also, investors' ex ante expectation on returns is not predicated on past stock performance.

Chapter-2

Related Literature

The area of this dissertation touches a broad spectrum of research from derivatives to asset pricing. However, in this chapter we discuss the literature that is immediately related to the risk-adjusted option pricing model and its empirical findings. Broadly speaking there are three areas of research that branch out of Black-Scholes (1973) option pricing model. First area of research is the study of option properties and market efficiency. Second group of research is on extending the Black-Scholes model to include additional features such as stochastic volatility, and jumps. Third area of research is on extraction of information using observed market prices of options. In the following paragraphs we discuss the first two areas in brief and the third area in detail, since our findings are related more to the third area of research. We also discuss related literature that extract ex ante expected return from other sources. For completeness, the last section of this chapter reviews some of the option pricing models using various utility structures.

The first group of research is the extensive study of Black-Scholes model to examine the properties of American and European option prices. For example, Merton (1973) shows the pricing relationship of different contingent claims on any stock based on the weak assumption that investors prefer more to less. Even though this assumption may not give the option price in exact form, it helps in formulating tight bounds and relationships across various options of that stock without any distributional assumptions. Since Merton's paper, many researchers have expanded the literature to understand the pricing relationship of different contingent claims and related market efficiency[1].

The second group of research is based on expanding the model of Black-Scholes to more generalized equations. For example, Merton (1976a) extends the option pricing model to have both continuous time Wiener and noncontinuous jumps in the stock price dynamics. With this setting, Merton shows that if investors use Black-Scholes formula when the true process contains jumps, that will introduce significant error in the option pricing. In this line of research papers by Cox and Ross (1976a and 1976b), Cox, Ross and Rubenstein (1979), Scott (1987), Hull and White (1987), Wiggins (1987), Stein and Stein (1991), and Heston (1993a and 1993b) provide extensions to the Black-Scholes model to have jumps and stochastic volatilities so that the models are close to the reality of observed option prices. We discuss the stochastic volatility models in more detail in chapter 6.

The third line of research is to view the option pricing models not as a pricing mechanism but as a method to extract the properties of the underlying asset return by using observed option prices in the market. Our current work on estimation of implied expected return, beta, and volatility is aligned with this line of research. The existing research in this line can be divided into three sub-groups. We discuss these in details in the following paragraphs.

[1] Following is a partial list of papers in this area of research:
Ross (1976), Jarrow (1980), Whaley (1986), and Hentschel (2003).

2.1 Implied Volatility from Option Prices

The first sub-group of research is based on extracting implied volatility from option prices and examining its properties for different values of option maturity and moneyness[2]. We briefly discuss some of these papers in this section. For example the papers that find Black-Scholes implied volatility (ISD) is a better measure for forecasting volatility are given by Latane and Rendleman (1976) (LR), and Chiras and Manaster (1978) (CM). The paper by Latane and Rendleman (1976) (LR) computes the implied standard deviation using the Black-Scholes (B-S) model. To adjust for sensitivity of option prices to implied volatility they compute the weighted average implied standard deviation (WISD) in which the implied standard deviation on all options on a given underlying stock are weighted by the partial derivatives of price of option in B-S equation with respect to each implied standard deviation. Then they compare the two methods to compute volatility, namely, the historical method, and the WISD method. They use continuous hedging of a portfolio of stock and its option for over-priced and under-priced options. The pricing and hedging are based on different combinations of computing volatilities by historical method and WISD. They argue with an approximate continuous hedging the portfolio should earn close to risk-free rate with lowest standard deviation. In their experiment they show that the portfolio return where historical volatility is used in the hedge weight computation has the highest standard deviation thus are far from being risk-less compared to the portfolio return where WISD is used. They also show that the mean return of the portfolio formed on

[2] In this sub-group of research the papers are by Latane and Rendleman (1976) (LR), Chiras and Manaster (1978) (CM), Beckers (1981), Day and Lewis (1992), Canina and Figlewski (1993) (CF), Christensen and Prabhala (1998), Lamoureux and Lastrapes (1993), Blair, Poon and Taylor (2001). Granger and Poon (2005) provides a comparison of different methods of forecasting volatility.

the basis of WISD is significant, and is in the expected direction and thus it is a better estimate of market volatility. Chiras and Manaster (1978) (CM) argue that the weighted average in LR is not truly a weighted average since the sum of weights is less than one. They compute the weighted average of ISD by price elasticity of the option with respect to its implied standard deviation, which they argue is a better method to compute the WISD. With empirical experimentation, they show that WISD does a better job than the historical volatility in predicting the realized volatility. Papers that do not support the hypothesis that the information content of implied volatility is superior are given by Day and Lewis (1992), and Canina and Figulewski (1993). Day and Lewis (1992) argue that the implied volatility is biased and inefficient since in their research, past volatility contained predictive information beyond the information content of implied volatility. One of the most interesting researches not supporting the ISD is given by Canina and Figulewski (1993) (CF). Using binomial model of option pricing that adjusts for dividends they argue implied volatility is not as better a predictor of realized volatility as the prior research suggested. Most importantly, Canina and Figulewski show that the implied volatility is not same for different maturity options, thus we cannot combine them to compute a WISD, since implied volatilities for different maturities may be influenced by systematic factors rather than the noise in the data. To take into account the possible systematic effects of time to maturity and moneyness they formed different groups based on these two factors and analyze each group separately. They show neither the implied volatility nor the realized volatility is an appropriate volatility forecast. Thus, they suggest, a better way might be to incorporate all sources of information rather than use only implied volatility to forecast realized volatility.

However, there are no papers, which show how we can combine different information to have a better volatility forecast. CF finding questions the B-S model in the following way. As shown in previous literature B-S can be thought of as a pricing model that prices the future volatility, However CF findings of no significant relationship between option's prices through implied volatility with the realized volatility refutes this belief within the rational expectations setting. In subsequent research, Christensen and Prabhala (1998) (CP) show that implied volatility is a better forecast of future volatility than previously reported. Christensen and Prabhala use monthly observations to avoid data overlaps and adjust for regime shift around the market crash of October 1987 that was not taken into account in Canina and Figulewski. Christensen and Prabhala also show past volatility has no incremental explanatory power over implied volatility in their test which is in contrast to Canina and Figulewski findings. They argue that the reason for this could be in extreme overlap in CF data that might have caused biased estimates as opposed to the nonoverlapping data in their experiment. Findings in Christensen and Prabhala research supports the idea that B-S model can be better used as a volatility forecaster than previously thought. Recent survey by Granger and Poon(2005) categorizes the future volatility forecast into four methods namely: historical volatility method, ARCH and GARCH models, stochastic volatility models, and implied volatility method. They rank these methods based on past literature. Their overall ranking suggests that B-S implied volatility provides the best forecast, followed by historical volatility and GARCH roughly with equal performance. Despite the added flexibility and complexity of stochastic volatility models, they find no clear evidence that it provides a superior volatility forecast. Our research is closer to the B-S framework. However, we use a risk-adjusted discrete time pricing method that retains not only the implied volatility but also the implied return in the equation. Using near-the-money options and computing different implied volatilities and returns for different days-to-maturity we avoid the systematic effects of moneyness and maturity described in Canina and Figulewski literature.

2.2 Implied Beta from Option Prices

The second sub-group of research is in the area of extracting implied beta from option prices. Papers in this area include Siegel (1995), and Christoffersen, Jacob and Vainberg (2006). Segiel (1995) proposes a new 'exchange option', the price of which is based on number of units of a specific stock, that can be exchanged for one unit of an index. Thus, he argues the price of this exchange option can reveal the implied beta of the stock. More recently, Christoffersen, Jacob and Vainberg (2006) show that implied beta can be extracted from option prices without using this new derivative. The beta in their model is computed using forward-looking variance and skewness. Using methods from previous literature, they retrieve the underlying distributions for index options and stock options from cross-section of option prices. Then they use traditional one-factor model and express the forward-looking beta as a function of the skewness and variance of the underlying distribution. They show these forward-looking betas perform well compared to historical betas in many cases. However, the main limitation in their approach could be the extraction of market betas from skewness. As shown in past literature market beta obtains when the stock returns are multivariate normal or preference is quadratic. The use of skewness to compute beta is at odds with multivariate normal assumption of the CAPM. On the other hand, our method to compute beta uses a time series estimate based on ex ante information set of market and stock expected returns.

2.3 Implied Expected Stock Return from Different Sources

2.3.1 Implied Expected Stock Return from Option Prices

The third sub-group of research is based on extraction of implied expected stock return (or implied return) from option prices. Option pricing models of Sprenekle (1961), Ayres (1963), and Boness (1964) had implicitly or explicitly assumed that the investors buy and hold the options until maturity to extract the option implied returns, which then could be linked to the stock implied return. However, none of these models provides an adequate theoretical structure to determine the implied return values. The Black-Scholes (1973), models the option price by taking advantage of the interesting feature that a certain portfolio of the stock and the option can cancel out the unknowns namely the implied stock return and the implied option return in continuous time. Thus if our objective is to value the option then we remain in this risk-neutral framework so that implied returns are not required in the pricing formula. However if our objective is to extract implied return given the market price of options we form the corresponding risk-adjusted valuation model that will retain the expected returns in the pricing models. Comparison between risk-neutral and risk-adjusted model of option pricing was given by Galai (1978), in which the author shows that if we use the risk-adjusted model then it will retain the stock implied return in the pricing equation. Our approach parallels this approach. However, there are at least three differences between our approach and his approach. First, he compares the properties of implied option return derived from risk-adjusted model with the risk-neutral model, whereas we derive a relationship between the implied stock return and option return in discrete time and then link that with the risk-adjusted model. Second, we derive a discrete time version of equations for covariance of option return and stock return, and variance of stock return. Finally, we compute the implied stock return and volatility from observed market price of options whereas his paper takes a range of implied stock returns as given and then uses the equation to compute a range of implied option returns.

Another paper that studies the properties of the risk-neutral valuation is given by Heston (1993b). In this research, the author suggests a generic framework under which the prior option pricing models do not depend on risk aversion parameters. For example, in diffusion models (Black and Scholes (1973)) option prices are independent of the stock drift; in Poisson models (Cox and Ross (1976a)) option prices are independent of the Poisson intensity, and in binomial models (Cox, Ross, and Rubinstein (1979)), option prices are independent of the jump probabilities. To derive these models we need the assumption about market completeness so that continuous time hedging is possible. Another alternative is to have a certain preference structure so that option pricing will not depend on risk aversion parameter even if we do not have continuous hedging. By this approach, Heston generalizes Rubinstein (1976) preference structure and by combining that with log-normal spot asset prices, he obtains the B-S pricing formula free of risk aversion parameters. Heston also shows a log-gamma formula, which depends on, mean return parameter but is independent of volatility, the scale parameter. If this distribution holds then option prices will be insensitive to sigma which contrasts with many findings that implied volatility has useful information to explain realized volatility. In contrast to these papers we follow a discrete time risk-adjusted approach with geometric Brownian price process, so that the implied return and implied volatility parameters are retained in our pricing equation.

Another paper related to implied stock return is given by McNulty et al. (2002). They use a heuristic approach to compute the 'real cost of equity capital'. Their findings of higher implied return in the short-term and lower implied return in the long-term matches with our finding; however, their approach lacks the theoretical support. Another recent paper, which computes the stock implied return from option prices, is by Camara, Chung, and Wang (2007). There are two aspects of their approach. First, they assume a specific utility structure such that the marginal utility of wealth of the representative investor is:

$$U'(W_T) = W_T^{\alpha} + \beta$$

where α and β are risk preference parameters. Based on this utility structure they show their option pricing equation contains implied stock return as one of the parameters to be estimated.[3] Second, their approach requires an intermediate parameter that needs to be computed using options of all companies, before they compute the implied return of any individual firm. In contrast to the above papers, we follow a discrete time approach

[3] Unlike the Camara, Chung, and Wang (2007) paper, our approach is consistent with the standard CAPM and thus consistent with a specific utility structure of hyperbolic absolute risk aversion (HARA) preference family namely the quadratic utility structure.

that has the advantage of being consistent with single period standard CAPM. Using our approach, the expected return of a stock can be computed without using any information of option of all companies or of the market portfolio such as the market risk premium. This implies we do not have to define what the 'market' consists of, and we do not have to estimate the risk premium of the market, which is required in traditional asset pricing models to estimate the expected return.

2.3.2 Other Sources of Expected Stock Return

Recent research explores different sources to extract ex ante stock return. In this section we will briefly discuss some of those studies. Campello, Chen, and Zhang (2008) use corporate bond yields to estimate expected equity returns. They argue that, since forward-looking bond yields are reflected in bond prices, this provides a natural selection of data source for ex ante information. However, their approach is not entirely based on ex ante information. For example, they use existing default information to gauge expected default losses that is required to back out the systematic component of yield spread. Further, to estimate the extent of empirical relationship between the bond yield and stock expected return using the elasticity of equity value with respect to the bond value, they again use historical data. Therefore, the 'links' the process of extraction of expected stock return goes through relies on ex post information at many intermediate steps. Fama and French (2002), also use the historical average dividend growth as the expected rate of capital gain and measure the equity premium as the sum of the expected rate of capital gain and the average dividend yield. However, use of ex post return in any form for test of ex ante models is a questionable assumption.[4] In fact as Sharpe (1978) pointed out:

> "All the econometric sophistication in the world will not completely solve the basic problem associated with the use of ex post data to test theories dealing with ex ante prediction, however. The Capital Asset Pricing Model deals with predictions concerning a future period [...]. It does not assume that the predictions or the implied relationships among them are stable over time. Nor does it assume that actual results will accord with such predictions, either period-by-period or, in any simple sense, 'on average'." (p. 920)

Unlike these approaches our model relies on option prices which is a direct source of ex ante expected return for the underlying stock.

Another group of literature relies on accounting information to estimate the expected returns. Using Value Line forecasts of dividends and target prices, Botosan and Plumlee (2005) obtain estimates of firm cost of capital and ask whether these estimates are correlated with firm characteristics. They find a positive relation between market beta and cost of equity. However, they generally find no association between market capitalization and Value Line estimates of the cost of equity. In a similar vein Brav, Lehavy, and Michaely (2005) use the Value Line forecasts and First Call analyst's expectations, and argue that researchers and practitioners use this database of earning and growth forecast as a proxy for expectation of these variables. Thus they argue, this source of information is superior to using the realized return for asset pricing

---

[4] Pastor, Sinha, and Swaminathan (2008) use simulations to show that, except for very long time windows, realized returns do not converge to expected returns and often yield wrong inferences. Moreover using realized returns as a proxy for expected returns, the evidence is mixed. Early tests, such as Fama and MacBeth (1973) find that firms' betas are positively related to their realized returns. Using later data and monthly return intervals, Fama and French (1992, 1993) and others do not find a significant relation. However, when annual return intervals are used (Kothari, Shanken. and Sloan, 1995) find that beta is significantly related to average realized returns.

tests. Using these data sources they find that market beta is positively associated with expected returns. Furthermore, using Value Line expectations, they do not find evidence that high book-to-market stocks have higher expected returns than low book-to-market stocks. When they use the analysts expected returns from First Call, they find that the coefficient on book-to-market is negative and significant. These results challenge the notion that the market perceives high book-to-market stocks as riskier and therefore they command higher expected returns. In fact Brav, Lehavy, and Michaely (2005) finding is consistent with our finding that there is no evidence of high book-to-market stock being riskier than low book-to-market stocks. [5] However, their approach has strong assumptions regarding the future evolution of accounting variables. For example they assume that dividends will continue to grow at the same historical rate, in the following four years. Furthermore, their paper and Botosan and Plumlee (2005) use indirect measures for expected stock returns such as the analyst's price targets by Value Line and expected returns from First Call.

To overcome the shortcomings of the above mentioned measures, we use option prices to extract information regarding ex ante expected returns and market beta of the underlying asset. Since option prices reflect investor expectations for future stock price movements, option data are an excellent information source for ex ante parameters.

---

[5] This finding is consistent with Shefrin and Statman (2003), who use an ordinal ranking of recommendations as their proxy for expected returns and relate them to firm characteristics such as book-to-market and market capitalization. They find that stocks with buy recommendations are more likely to be low book-to-market stocks. They interpret this finding as an indication of higher expected return for those types of stocks, which is consistent with our findings.
Furthermore, Lakonishok, Shleifer, and Vishny (1994) find that there is no eveidence of high book-to-market being fundamentally risker. To be fundamentally riskier, high book-to-market (value) stocks must underperform low book-to-market (glamour) stocks with some frequency, and particularly in the states of the world when the marginal utility of wealth is high. They find little, if any, support for the view that value strategies are fundamentally riskier.

Unlike the information content in bond prices which provides an indirect relationship with model assumptions, our approach is a direct source of ex ante expected stock return. Our risk-adjusted approach jointly extracts implied mean return and implied volatility of the underlying asset from forward-looking option prices. We use this implied mean return as a proxy for ex ante expected return.

2.4 Brief Review of Utility Based Option Pricing

Rubinstein (1976) and Brennan (1979) use specific utility structures to price the options in discrete time. Rubinstein (1976) obtains the Black-Scholes model with constant proportional risk aversion (CPRA) preferences. He also assumes that aggregate consumption and the underlying asset are bivariate lognormally distributed. Brennan (1979) derives a risk-neutral valuation relation assuming a representative agent who has a negative exponential utility function, and a bivariate normal distribution for aggregate wealth and the underlying asset.[6] Using Rubinstein (1976) approach with a general pricing distribution and discrete trading, Perrakis and Ryan (1984) show the upper and lower bound for call options based on a utility structure such that the normalized conditional expected marginal utility for consumption is non-increasing in the price change of the stock. Bates (1991) show that the high price for 'crash insurance' during 1987 cannot be explained by standard option pricing models with positively skewed distributions, such as Black-Scholes, constant elasticity of variance, or GARCH; instead a

[6] Brennan (1976) and Rubinstein (1976) have the following additional common assumptions: (i) the single-price law of markets, (ii) non-satiation, (iii) perfect, competitive, and Pareto-efficient financial markets, (iv) rational time-additive tastes, and (v) weak aggregation, or the existence of an average investor.

jump diffusion process with time-separable power utility function explains this crash, when the jump risk is systematic and nondiversifiable.

Levy (1985) shows upper and lower bound for call options with less restrictive assumption on the utility structure using a discrete time model. Levy argues, on the one hand, Brennan (1979), assuming some specific stock value distributions and investor utility functions, derives a relative pricing relationship between stock and the option. On the other hand, Merton (1973), imposing no restrictions on the stock price behavior and the investors' characteristics, obtained upper and lower bounds on the option value relative to the stock value. Knowing these two extreme cases the upper and lower bounds can be further improved by assuming simply a concave utility function. Levy shows that the bounds are much tighter than Merton bounds with this simple assumption.

Camara (2003) generalizes Brennan-Rubinstein approach to show a new range of preferences and distributions of wealth pairs under which the Black-Scholes model holds. The author shows Black-Scholes model might be obtained, when the underlying asset has a lognormal distribution, with any of the following risk preferences and wealth distribution pairs: (i) The utility function is an extended power displaying DARA and aggregate wealth has a displaced lognormal distribution. (ii) The utility function is a negative exponential displaying CARA and aggregate wealth has a normal distribution.

(iii) The utility function is a cubic one displaying IARA and aggregate wealth has a negatively skew lognormal distribution.

Vanden (2006) analyzes asset pricing with nonnegative wealth constraints. In the presence of these constraints, using exponential, power, and quadratic utility functions, Vanden shows that options on the market portfolio are nonredundant securities and the economy's pricing kernel depends on both the market's return and the option's returns. This leads to a pricing model in which the expected excess return on any risky asset is linearly related to the expected excess return on the market portfolio and to the expected excess returns on the nonredundant options. The empirical results indicate that the inclusion of the option returns can improve the CAPM and this improvement is significant for nonsmall stocks.

Chapter – 3

Risk-Adjusted Information from Option Prices[1]

As option's payoff depends upon future stock price, option prices contain important information of their underlying stocks. For a bullish stock, the price of the call goes up and the put goes down. However, using the Black-Scholes model, we can only retrieve the volatility information, as risk preference disappears from the pricing model. In this paper, we price options with the physical measure where we can jointly estimate the expected return (μ) and implied volatility of the underlying stock from market prices of options.

Pricing measures are not unique. Yet the law of one price (or known as no arbitrage) guarantees all pricing measures lead to a unique option price. As a result, there exists a pricing measure where μ is present and the same option price is obtained. In this paper, we choose the physical measure to price options so that we can jointly estimate the expected return and implied volatility of the underlying stock. The use of the physical measure in pricing assets has been the standard methodology in microeconomic theories. In fact, the earlier literature (such as Sprenkle (1961) and Samuelson (1965)) in option pricing used the physical measure to price options. Our contribution is to extend those models and further derive the closed form solution to the

---

[1] This chapter is a superset of a joint paper with my dissertation committee members Dr. Ren-Raw Chen (advisor), and Dr. Dongcheol Kim. We wish to thank Dr. Kose John, Dr. C.F. Lee, Dr. Oded Palmon, and the 2009 Financial Management Association Meetings participants for their helpful comments and suggestions. We thank the Whitcomb Financial Center for data assistance.

expected return of the option as a function of the expected return of the stock.

Black and Scholes (1973) show that if the market is complete,[2] then the expected return of the stock should disappear from the valuation of the option as dynamic hedging (or known as continuous rebalancing, price by no arbitrage, or risk neutral pricing) should effectively remove the dependence of the option price on the stock return. This is true, however, only if the market is truly complete in reality. In other words, if the reality were exactly described by the Black-Scholes model, it is impossible to theoretically solve for both the expected return and implied volatility of the stock . However, it has been empirically shown that the Black-Scholes model cannot explain all option prices (known as the volatility smile and volatility term structure). As a result, we can solve for these two parameters simultaneously under our model.

Except for the expected return parameter, the physical pricing measure adopted by our model assumes the same assumptions of the Black-Scholes model. In particular, we assume the same stock price process as the Black-Scholes model does. This design is to assure that we have a closed form solution to our model. In theory, we could relax as many assumptions by the Black-Scholes model as possible and build a model that can explain every traded option price in the market place. However, in doing so, we shall lose the closed form solution and furthermore once we have as many parameters as the number of the traded options, the model can no longer “price” any option as all option prices are used to calculate parameters. As a result, we need to seek balance between over-parameterization (having same number of parameters as option prices),

[2] This is complete market in the dynamic sense, as later described carefully by Duffie and Huang (1985).

under-parameterization (such as the Black-Scholes model), and computation feasibility (maintaining closed form solution). As we shall show in our empirical study, with two parameters (expected return and implied volatility), we find that we can predict realized volatility much better than the Black-Scholes model.

Option pricing models of Sprenkle (1961), Ayres (1963), Boness (1964), and Samuelson (1965) employed the physical measure and implicitly or explicitly assumed some form of risk-adjusted model such that the investors buy and hold the options until maturity to extract the option implied return, which then could be linked to the stock return. However, none of these models provides an adequate theoretical structure to determine the implied return values.[3] Under the risk neutral pricing measure, Heston (1993b) shows that, under a log-gamma dynamic assumption for the stock price, the expected stock return will show up in the pricing formula and yet the volatility disappears. Hence, his model is not capable of jointly determining both the expected return and volatility of the stock price. Nonetheless, Heston's paper shows the possibility of retaining the expected return parameter in the model with suitable adjustments to the pricing equation.

Using the S&P 500 index call options, we estimate expected stock return and implied volatility with our model. We use options with various strikes at a given day and compute expected return and volatility for each time to maturity. As a result, we obtain jointly the term structure of expected return and the term structure of implied volatility of the stock. We find a downward sloping term structure of expected return that is consistent with existing studies to be reviewed in details later in the empirical

[3] Galai (1978) later showed that the Boness model and the Black-Scholes model are consistent.

section. We find that implied volatility carries more information in predicting realized volatility of the stock than the term structure of Black-Scholes implied volatility.

The reminder of this chapter is organized as follows. Section 3.1 presents the risk-adjusted discrete time model that retains the stock expected return in the option pricing equation. Section 3.2 presents the data and estimation methodology. Section 3.3 discusses the empirical results of our estimation. Section 3.4 provides the concluding remarks.

3.1 The Model

It is well known that the Black-Scholes model can be used to compute implied volatility and not implied expected return of the underlying stock due to the fact that no-arbitrage argument renders a preference-free model and hence contains no such parameter. In this sub-section, we demonstrate that such parameter can be re-discovered via an "equilibrium" pricing approach similar to Samuelson (1965) and Sprenkle (1961). Let the stock price follow the usual log normal process under the physical measure:

(1) $$\frac{dS}{S} = \mu dt + \sigma dW$$

where the annualized instantaneous expected return is μ and the volatility is σ. The classical economic valuation theory states that any price today must be a properly discounted future payoff:

(2) $$C_t = E_t[M_{t,T} C_T]$$

where $M_{t,T}$ is the pricing kernel, also known as the marginal rate of substitution, between time t and time T.

Continuous rebalancing, which constitutes a dynamically complete market, guarantees the existence of the risk neutral pricing measure where the risk premium is removed from the expectation and hence the discount rate is the risk-free rate as follows:[4]

$$
\begin{aligned}
C_t &= E_t[M_{t,T}C_T] \\
&= E_t[M_{t,T}]E_t^Q[C_T] \\
&= \begin{cases} e^{-r(T-t)}E_t^Q[C_T] & \text{if interest rate is constant} \\ P_{t,T}E_t^{F(T)}[C_T] & \text{if interest rate follows a random process} \end{cases}
\end{aligned}
\tag{3}
$$

where Q represents the risk neutral measure and $F(T)$ represents the T-maturity forward measure and, $P_{t,T}$ is the risk free zero coupon bond price of \$1 paid at time T. Or alternatively, one can find a more familiar pricing measure where the expected payoff is discounted at a properly risk-adjusted discount rate as follows:

$$
\begin{aligned}
C_t &= E_t[M_{t,T}C_T] \\
&= E_t^C[M_{t,T}]E_t[C_T] \\
&= e^{-k(T-t)}E_t[C_T]
\end{aligned}
\tag{4}
$$

where *C* represents the measure where the option price serves as a numeraire, and *k* is the annualized expected instantaneous return on this option in the physical world. We then assume that the *C*-measure expectation of the pricing kernel takes a form of continuous discounting. Now, we can derive our option pricing formula as:

---

[4] See Duffie and Huang (1985) for this result.

$$
\begin{aligned}
C_t &= e^{-k(T-t)} E_t[\max\{S_T - K, 0\}] \\
&= e^{-k(T-t)} \left[\int_K^{\infty} S_T \phi(S_T) dS_T - K \int_K^{\infty} \phi(S_T) dS_T \right] \qquad (5) \\
&= e^{(\mu - k)(T-t)} S_t N(h_1) - e^{-k(T-t)} K N(h_2)
\end{aligned}
$$

where t and T are the current time and maturity time of the option, and K is the strike price of the option and

$$
\begin{aligned}
h_1 &= \frac{\ln S - \ln K + \ \ \mu + ½\sigma^2 \ \ (T-t)}{\sigma\sqrt{T-t}} \\
h_2 &= h_1 - \sigma\sqrt{T-t}
\end{aligned}
$$

To derive a pricing formula that contains μ, we need the following propositions. These propositions describe how implied return and volatility can be simultaneously estimated from option prices.

Proposition 1. Assume stock price S follows a geometric Brownian motion with an annualized expected instantaneous return of μ and volatility of σ. Let a call option on the stock at any point in time t is given by $C(S,t)$ that matures at time T. Let *k* is the annualized expected instantaneous return on this option. Then for a small interval of time Δt, the relationship between μ and *k* can be given by:

$$k = r + \beta(\mu - r) \qquad (6)$$

where

$$\beta = \frac{\mathrm{cov}(r_C, r_S)}{\mathrm{var}(r_S)} \qquad (7)$$

and $r_S = \Delta S / S$ and $r_C = \Delta C / C$ are two random variables representing the stock return and call option return respectively during the period Δt. And, r is the annualized constant risk-free rate for the period of the option. Proposition 1 can be proved without assuming the CAPM.

Proof. See Appendix 3.A.1.1.[5]

Equation (6) holds for a small interval of time Δt. We assume the distributions of stock return, r_S and option return, r_C are stationary over the period of the option. This implies the annualized instantaneous expected return and variance over a small interval of time and the annualized instantaneous expected return and variance over the discrete time (from time t and time T) will be same. This also implies β is constant over this period, which means the linear relationship between k and μ as in equation (6) is valid over the life of the option from current time t to maturity time T.[6] Since our approach will be pricing the option in a discrete setting, we approximate the β over the discrete time from t to T as:

(7a) $$\beta_{t,T} = \frac{\operatorname{cov}\left(\dfrac{C_T}{C_t}, \dfrac{S_T}{S_t}\right)}{\operatorname{var}\left(\dfrac{S_T}{S_t}\right)} = \frac{S_t}{C_t}\frac{\operatorname{cov}\ C_T, S_T}{\operatorname{var}(S_T)}$$

and with the assumption of stationarity as above, equation (6) holds for the life of the option as:

(6a) $$k_{t,T} = r_{t,T} + \beta_{t,T,K}(\mu_{t,T} - r_{t,T})$$

[5] Appendix 3.A.2.1 provides a similar derivation for put options.

[6] This can be easily seen by integrating both side of equation (6) from *t* to *T*.

Equation (6) with equation (7), in continuous time, and equation (6a) with equation (7a) in discrete time can also be proved using the CAPM. However, for these two equations to hold it is not necessary that the CAPM should hold. The assumptions of the CAPM are much stronger so that all return distributions are stationary, however here we need only the stationarity of the stock and the option return to obtain these two equations. Hence stationarity assumption of r_S and r_C is a weaker assumption than what is needed for CAPM. Further Galai (1978) shows many similarities between the continuous time and discrete time properties of r_C that support our assumption of stationarity of distribution.

Equation (5) is obtained based on the assumption that the expected return of the option k, expected return of the stock μ and volatility σ are constants. We approximate β by $\beta_{t,T}$ based on the discrete time period of the option from *t* to *T* as explained above. Furthermore, we assume that the stock price follows a geometric Brownian motion. In discrete time, equation (5) can be written as:

(5a) $$C_{t,T,K} = e^{(\mu_{t,T} - k_{t,T})(T-t)} S_t N(h_1) - e^{-k_{t,T}(T-t)} KN(h_2)$$

where t and T are the current time and maturity time of the option, and K is the strike price of the option and

$$h_1 = \frac{\ln S_t - \ln K + \left(\mu_{t,T} + \tfrac{1}{2}\sigma_{t,T}^2\right)(T-t)}{\sigma_{t,T}\sqrt{T-t}}$$
$$h_2 = h_1 - \sigma_{t,T}\sqrt{T-t}$$

Combining equations (7a) and (5a), we arrive at the following proposition.

Proposition 2: The $\beta_{t,T,K}$, based on the life of the option can be written as:

$$\beta_{t,T,K} = \frac{S_t\left[e^{\sigma_{t,T}^{2}(T-t)}N(h_3) - \frac{K}{S_t}e^{-\mu_{t,T}(T-t)} \quad N(h_1) - N(h_2) \quad - N(h_1)\right]}{C_t\left(e^{\sigma_{t,T}^{2}(T-t)} - 1\right)} \tag{8}$$

where

$$h_3 = \frac{\ln S_t - \ln K + \quad \mu_{t,T} + 1\frac{1}{2}\sigma_{t,T}^{2} \quad (T-t)}{\sigma_{t,T}\sqrt{T-t}}$$

Proof: See Appendix 3.A.1.2.[7]

It should be noted that we do not use the distributional properties of the market return r_M to obtain (8). Using (8), (5a), and (6a) we can solve for the call price $C_{t,T,K}$ explicitly in terms of the known values: stock price (S_t), strike price (K), risk free rate (r), time-to-maturity ($T-t$), and two important unknown parameters: expected stock return $\mu_{t,T}$ and volatility $\sigma_{t,T}$.[8] If we observe the values of two or more call options, with same time-to-maturity with different strike prices, we can then simultaneously

[7] Appendix 3.A.2.2 provides the corresponding derivation for put options.

[8] $\mu_{t,T}$ and $\sigma_{t,T}$ represent expected stock return (μ) and implied volatility of the stock (σ) respectively for a specific time period, where *t* is the date of observation of option prices, and *T* is the maturity date of the options.

solve for $\mu_{t,T}$ and $\sigma_{t,T}$.[9]

It should be noted that unlike the Black-Scholes model, the advantage of the risk-adjusted (physical measure) pricing equation is that it does not require a continuous rebalancing assumption. However the disadvantage of the physical measure approach is that it has many unknowns, whereas the Black-Scholes equation has only one unknown namely the volatility of the stock return. It can be easily shown that with the assumption of continuous rebalancing (or instantaneous holding period) the risk-adjusted model will collapse to the Black-Scholes model so that the pricing equation will not contain the expected stock return. Therefore, our model is consistent with Black-Scholes model when their assumptions hold. Furthermore our model is applicable in discrete time and is consistent with the standard CAPM.

3.2 Data and Estimation Methodology Using S&P 500 Index Options

3.2.1 Data

To extract implied expected return from option prices we use the end-of-day OptionMetrics data of options on S&P 500 (SPX) for the last business day of every month during January 1996 – April 2006. This data file contains the end-of-day stock CUSIP, strike price, offer, bid, volume, open interest, days-to-maturity, and Black-Scholes implied volatility for each option. From this dataset, we exclude all put options and options with zero trading volume. We also exclude single option records for a

---

[9] With prices for options with more than two strike prices, we can find values for $\mu_{t,T}$ and $\sigma_{t,T}$ that produce option prices closest to the observed prices in the least squares sense. A similar least-squares methodology was used by Melick and Thomas (1997).

particular trade date and days-to-maturity.[10]

We obtain daily levels of the index and returns from CRSP. We need the returns for realized volatility computation. To match the CRSP records with option records, we use the trade date and CUSIP of the index. In our data all S&P500 records have a common CUSIP. Merging CRSP and option data by trade date and CUSIP can be used for any stock option in general.

For the interest rates, we use the St. Louis Fed's 3-months, 6-months, 1-year, 2-year, 3-year, and 5-year Treasury Constant Maturity Rates. Assuming a step-function of interest rates, we match the days-to-maturity in the option record with its corresponding constant maturity rate. For example if the days-to-maturity of the option is less than or equal to 3-months we use 3-months rates, and if the days-to-maturity is between 3-months and 6-months, we use the 6-months rate and so on.

In this paper, the results are based on the last business day observations for each calendar month. This results in 124 months, 791 different trade date and maturities combinations (on average 6.38 maturities per month), and a total of 7865 options (9.94 different moneyness levels per trade date and maturity combination). Taking any other day of the month produces similar results. For example, we verified our results by taking first working day, second Thursday, and third Friday of every month. The results are similar. Table I shows the summary statistics of all moneyness S&P500 index call

[10] We need at least two option records for a specific trade date and days-to-maturity to compute $\mu_{t,T}$ and $\sigma_{t,T}$.

option input data that are used to compute $\mu_{t,T}$ and $\sigma_{t,T}$.[11]

3.2.2 Estimation of Implied Expected Stock Return and Implied Volatility

We jointly estimate the implied expected stock return ($\mu_{t,T}$) and implied volatility ($\sigma_{t,T}$) using the risk-adjusted option pricing model described in previous section. For a given trade date for S&P500 index, we have many call options with same days-to-maturity. We use all these options records to compute implied stock return and implied volatility by a method of grid search to look for the global optima that minimizes the square error. A square error is defined as the square of the difference between the market observed option price and right hand side of the equation used to compute the option price based on the observed values.[12] Since we are searching for the entire spectrum for the global optima we need to specify search intervals without which we would not be able to implement the search.[13] We use the implied expected return ($\mu_{t,T}$) search range from 0.0% to 200.00%, and implied volatility ($\sigma_{t,T}$) search range from 0.0% to 100.00% for the grid search. To compute implied expected returns we need two or more records with same key value of trade date, CUSIP, and days-to-maturity. Thus, all the single records for a key value cannot be used to compute implied return and are discarded. By this

[11] The option data also contain Black-Scholes implied volatilities adjusted for stock dividends. Using this information along with the interest rates, we can reverse compute the corresponding European option price. If the European option price thus computed is higher than the bid and ask midpoint price, then we take the bid and ask midpoint price, else we take the European price as the option price to compute $\mu_{t,T}$ and $\sigma_{t,T}$ .S&P 500 options are European style and the prices should reflect as such. However, minor differences exist between the reported closing prices and the prices reversely computed from end of day implied volatilities.

[12] The observed values used on the right hand side of the equation are stock price, strike price, option price, days to maturity, and interest rate.

[13] Theoretically an interval of $-\infty$ to $+\infty$ is the full search interval for both $\mu_{t,T}$ and $\sigma_{t,T}$. However, we would not be able to practically implement such a search for global optima given limited processing power of resources. Therefore, we choose the upper and lower bound based on the most feasible interval possible from prior experience.

method, we extract the market implied return and market volatility for different days-to-maturity based on S&P500 index option prices, and corresponding S&P500 index levels.

3.3 Results

Using the S&P 500 monthly index option prices from January 1996 till April 2006, we estimate expected stock return ($\mu_{t,T}$) and volatility ($\sigma_{t,T}$) with our model. We use options with various strikes at a given day and compute $\mu_{t,T}$ and $\sigma_{t,T}$ for each time to maturity. As a result, we obtain jointly the term structure of $\mu_{t,T}$ and the term structure of $\sigma_{t,T}$.

Using the S&P 500 index call options of all moneyness,[14] we find the following:

- A downward sloping term structure of $\mu_{t,T}$ that is consistent with existing studies to be reviewed in details later in section 3.3.1,
- Much flatter term structure for $\sigma_{t,T}$ than the Black-Scholes model,
- $\sigma_{t,T}$ carries more information in predicting realized volatility than the Black-Scholes implied volatility (i.e., average implied standard deviation, or $\overline{\sigma}_{t,T}^{BS}$) based on near term options maturing in 90-days or less.[15]

[14] We also perform combined call and put option testing. The results are presented in robustness test section.

[15] Average implied standard deviation is the arithmetic average of Black-Scholes implied standard deviation of all options with different strike prices that are used to estimate $\mu_{t,T}$ and $\sigma_{t,T}$.

- A combination of our implied expected return ($\mu_{t,T}$) and implied volatility ($\sigma_{t,T}$) with $\overline{\sigma}_{t,T}^{BS}$ provides a better model, than using $\overline{\sigma}_{t,T}^{BS}$ alone to forecast future volatility for any maturity and moneyness combination.

3.3.1 The Term Structure of $\mu_{t,T}$

Table II shows the descriptive statistics of implied expected return ($\mu_{t,T}$) and implied volatility ($\sigma_{t,T}$) using all moneyness S&P 500 index call options. To analyze the results we classify the data into different days-to-maturity groups. Thus the options whose days-to-maturity is less than or equal to 90 days are classified into '<=90' group. The options whose days-to-maturity is greater than 90 days are classfied into '>90' group. Figure I shows $\mu_{t,T}$ and $\sigma_{t,T}$ graphs for S&P500 index call options of all moneyness. In these table and graph, we see a term structure of $\mu_{t,T}$. For example in Table II for '<=90' days-to-maturity $\mu_{t,T}$ is 19.5%, whereas for '>90' days to-maturity it is 9.41.[16]

The term structure of $\mu_{t,T}$ implies the expected return is impacted by the time horizon of investment. McNulty et al. (2002) study the 'real cost of equity capital' using option prices. They find high expected returns in the short term and low expected returns in the long term, which is consistent with our finding. They argue that the marginal risk of an investment (the additional risk the company takes on per unit time) declines as a function of square root of time. The falling marginal risk should be

---

[16] We also see the term structure when we group the data into 30, 60, 90, and so on days-to-maturity groups.

reflected in the annual discount rate.[17] Our term structure of $\mu_{t,T}$ is consistent with this explanation. However, unlike our approach, their approach is heuristic and lacks the theoretical foundation. Recently, Camara et al. (2007) compute the cost of equity from option prices using a specific utility function and arrive at the same downward sloping term structure of expected stock returns as did by McNulty et al. Their approach requires an intermediate parameter that needs to be computed using options of all firms before they compute the implied expected return of any individual firm. In contrast to their approach, we do not assume any explicit utility function.[18]

The data points for the term structure graphs (Figure I) are generated by non-parametric spline interpolation using the neighborhood data points. Our approach can be used to estimate the cost of equity for any time horizon of investment.[19] One of the advantages of our approach is that the expected return of a stock can be computed without using any information of the market portfolio such as the market risk premium. This implies one does not have to define what the 'market' consists of, and one does not have to estimate the risk premium of the market, which is required in traditional asset pricing models, to estimate the expected return.

To validate the robustness of our finding, we examine the influence of market friction proxies such as the option open interest, volume, and bid-ask spread on the term structure of implied expected return. We control for time to expiration bias, moneyness

---

[17] This is explained in McNulty et al. (2002).

[18] Note that our model is consistent with the Black-Scholes and assumes normality of stock returns. As a result, our model is implicitly consistent with the quadratic utility function.

[19] Our approach can be used to estimate cost of equity for different industry portfolios. We do similar experiments and show the term structure of expected return persists for these industry portfolios.

bias, and volatility bias in this regression.[20] Our results show, the market friction proxies do not explain this term structure. We also find the term structure of expected return remains for deep-in and deep-out of the money call options. Furthermore, this term structure also persists for combined call and put options (discussed in robustness section).

3.3.2 Comparison of Term Structure of $\sigma_{t,T}$ and Black-Scholes Volatility

Our model also demonstrates a flatter (less variation) term structure of $\sigma_{t,T}$.[21] From Figure I, we can eyeball the two volatility term structures from $\sigma_{t,T}$ of our model and $\bar{\sigma}_{t,T}^{BS}$ of the Black-Scholes model that the term structure of $\sigma_{t,T}$ is much flatter than the term structure of $\bar{\sigma}_{t,T}^{BS}$. While it is not easy to compare the two term structures statistically, we can compute the relative variation of the two term structures from Table II. For all maturities, the mean and variation (standard deviation) of $\sigma_{t,T}$ are 0.2139 and 0.0705 respectively; and of $\bar{\sigma}_{t,T}^{BS}$ are 0.1968 and 0.0785 respectively. Hence, the relative variation, defined as standard deviation divided by the mean, is 0.3296 for our model and 0.3989 for the Black-Scholes model.[22] This demonstrates that the $\sigma_{t,T}$ of our model presents a “flatter” term structure than the $\bar{\sigma}_{t,T}^{BS}$ of the Black-Scholes model.

[20] Papers by Chiras and Manaster (1978), Macbeth and Merville (1980), Rubenstein (1985), and Canina and Figlewski (1993) find these biases. Longstaff (1995) has similar controls for these biases.

[21] By term structure of $\sigma_{t,T}$, we mean the value of $\sigma_{t,T}$ for different days to maturity of T, for same observation date, t.

[22] Table III provides a detail comparison of $\sigma_{t,T}$ and $\bar{\sigma}_{t,T}^{BS}$.

When we divide the sample into short term (<=90 days) and long term (>90 days), we find that our model performs better than the Black-Scholes model for the short term options – 0.3481 versus 0.4310; yet worse for the long term options – 0.3143 versus 0.2992. This demonstrates that the term structure of the Black-Scholes $\overline{\sigma}_{t,T}^{BS}$ dissipate off, for higher days to maturity options.

To have a detail comparison of the characteristics of $\sigma_{t,T}$ of our model and the implied volatility (i.e. implied standard deviation, or $\overline{\sigma}_{t,T}^{BS}$) of the Black-Scholes model, we estimate various attributes of comparison as shown in Table III. $\sigma_{t,T}$ is jointly estimated with $\mu_{t,T}$ using multiple option records as described in section 3.2.1 and 3.2.2. To compute the values in this table, first, we estimate the mean and standard deviation of $\sigma_{t,T}$ and Black-Scholes implied volatility ($\overline{\sigma}_{t,T}^{BS}$) for each year and days-to-maturity based on our entire dataset. Then we compute the difference of these means and standard deviations of $\sigma_{t,T}$ and $\overline{\sigma}_{t,T}^{BS}$ for each year and days-to-maturity.[23] Panel A of Table III provides the summary statistics of the difference of the means for different days-to-maturity groups. Panel B provides the summary statistics of the difference of the standard deviations for different days-to-maturity groups. As we see in Panel A, the *t*-statistics is significant for all maturity groups. Similarly in Panel B the *t*-statistics is significant for both ‘<=90’ days-to-maturity and ‘all maturities’ groups and they are negative. This shows the standard deviation is lower for sigma than $\overline{\sigma}_{t,T}^{BS}$. Panel C shows the summary statistics of the difference of coefficient of variation (CV) of $\sigma_{t,T}$ and $\overline{\sigma}_{t,T}^{BS}$

[23] Difference of the means is computed as the mean of $\sigma_{t,T}$ minus the mean of $\overline{\sigma}_{t,T}^{BS}$. Similarly we compute difference of standard deviation and difference of coefficient of variation.

for different days-to-maturity groups. Here we see the CV of $\sigma_{t,T}$ and $\overline{\sigma}_{t,T}^{BS}$ are statistically different. Similar to Table II we see CV of $\sigma_{t,T}$ are lower compared to CV of $\overline{\sigma}_{t,T}^{BS}$ and thus $\sigma_{t,T}$ is 'flatter' than $\overline{\sigma}_{t,T}^{BS}$. Overall, Table III shows that $\sigma_{t,T}$ has lower standard deviation, lower CV, and higher mean compared to $\overline{\sigma}_{t,T}^{BS}$. This implies that $\sigma_{t,T}$ of our risk-adjusted model might have additional information beyond $\overline{\sigma}_{t,T}^{BS}$ that might be valuable to estimate the characteristics of the underlying stock.

3.3.3 Volatility Forecast

In this section we analyze whether the $\mu_{t,T}$ and $\sigma_{t,T}$ pair of our model carries more information than $\overline{\sigma}_{t,T}^{BS}$ of the Black-Scholes model to forecast realized volatility. We find that $\sigma_{t,T}$ alone can predict the future realized volatility significantly better than the Black-Scholes $\overline{\sigma}_{t,T}^{BS}$ when we use options of all moneyness. More interestingly, we find that when $\mu_{t,T}$, $\sigma_{t,T}$ and $\overline{\sigma}_{t,T}^{BS}$ are all used in the prediction, the result is significantly better than either $\sigma_{t,T}$ or $\overline{\sigma}_{t,T}^{BS}$ alone. These results are stronger for near term options. First, when we use all moneyness, $\sigma_{t,T}$ and its second order term does better than $\overline{\sigma}_{t,T}^{BS}$ and its second order term for both the days-to-maturity groups namely '<=90' and '>90', based on adjusted R-square. Second, for near term options, the coefficients of $\sigma_{t,T}$, and the second-order term are significant even in the presence of $\overline{\sigma}_{t,T}^{BS}$. Furthermore, a likelihood ratio test rejects the null hypothesis that the restricted model with $\overline{\sigma}_{t,T}^{BS}$ and its second-order term is better than the unrestricted model with all the three variables and their second-order terms for all near and far maturity groups, and for any moneyness level. [24]

A vast body of literature exists on the volatility forecasting front, that investigates the forecasting capability of implied volatility from option prices.[25] In a recent comparison study, Granger and Poon (2005) finds that the Black-Scholes (1973) implied volatility provides a more accurate forecast of realized volatilities. In their paper, they show the outcomes of 66 previous studies in this area that uses different methods to forecast the realized volatility. These methods are historical volatility, ARCH, GARCH, Black-Scholes (1973) implied volatility, and stochastic volatility (SV).[26] Based on their ranking they suggest that Black-Scholes (1973) implied volatility provides the best forecast of future volatility. Despite the added flexibility of SV models, authors find no clear evidence that they provide superior volatility forecasts. Furthermore, they find Black-Scholes (1973) implied volatility dominates over time-series models because the market option prices fully incorporate current information and future volatility expectations. Therefore, we choose Black-Scholes implied volatility ($\overline{\sigma}_{t,T}^{BS}$) as the benchmark, and compare the information content of our

---

[24] As we show in Table V, we take all moneyness or near-the-money options; we take '<=90' days and '>90' days-to-maturity groups. In all these cases we reject the restricted model that uses only Black-Scholes implied volatility and its second order term to predict the realized volatility.

[25] Papers are by Latane and Rendleman (1976) (LR), Chiras and Manaster (1978) (CM), Beckers (1981), Day and Lewis (1992), Canina and Figlewski (1993) (CF), Christensen and Prabhala (1998), Lamoureux and Lastrapes (1993), Blair et al. (2001). Granger and Poon (2005) provides a comparison of different methods of forecasting volatility.

[26] Option pricing models by Merton (1976a), Cox and Ross (1976a), Hull and White (1987), Scott (1987), and Heston (1993a) extend basic Black-Scholes (1973) model to incorporate stochastic volatility and jumps.

implied expected return ($\mu_{t,T}$) and implied volatility ($\sigma_{t,T}$) with the $\bar{\sigma}_{t,T}^{BS}$. To understand the forecastability of realized volatility using $\mu_{t,T}$ and $\sigma_{t,T}$ and $\bar{\sigma}_{t,T}^{BS}$ we plot these time series values in Figure II and Figure III for '<=90' days-to-maturity and '>90' days-to-maturity groups respectively for S&P500 index options using all moneyness.

3.3.3.1 Information Content of the Nested Model

The comparison of information content of $\bar{\sigma}_{t,T}^{BS}$ over a model of $\mu_{t,T}$ and $\sigma_{t,}$ and $\bar{\sigma}_{t,T}^{BS}$ can be evaluated using the following regressions:

(R1) $\sigma_{t,T}^{RE} = \alpha_{10} + \alpha_{11}\bar{\sigma}_{t,T}^{BS} + \alpha_{12}\bar{\sigma}_{t,T}^{BS^2} + \omega_{1t,T}$

(R2) $\sigma_{t,T}^{RE} = \alpha_{20} + \alpha_{21}\sigma_{t,T} + \alpha_{22}\sigma_{t,T}^2 + \omega_{2t,T}$

(R3) $\sigma_{t,T}^{RE} = \alpha_{40} + \alpha_{41}\bar{\sigma}_{t,T}^{BS} + \alpha_{42}\bar{\sigma}_{t,T}^{BS^2} + \alpha_{43}\mu_{t,T} + \alpha_{44}\mu_{t,T}^2 + \alpha_{45}\sigma_{t,T} + \alpha_{46}\sigma_{t,T}^2 + \omega_{4t,T}$

Past literature typically uses equation (R1) without the second-order term. In our investigation we include the second-order terms[27] to capture the higher order effects to explain the annualized 'realized' volatility ($\sigma_{t,T}^{RE}$), where *t* is the date of observation of option prices for a given stock, and *T* is the maturity date. To compute the $\bar{\sigma}_{t,T}^{BS}$, we use the dividend adjusted Black-Scholes implied volatilities given in the OptionMetrics

[27] We test the validity of the restricted model without the square term. Based on the likelihood ratio test our results in most cases reject the restricted model. Therefore, we take the variables ($\mu_{t,T}$, $\sigma_{t,T}$, or Black-Scholes implied standard deviation) with the square terms.

data file. $\overline{\sigma}_{t,T}^{BS}$ is the average of these implied volatilities of all options that are used to estimate the $\mu_{t,T}$ and $\sigma_{t,T}$ pair.[28] To compute $\sigma_{t,T}^{RE}$, first, we compute daily 'realized' volatility based on ex post daily returns of the underlying asset for the remaining life of the option and then multiply by $\sqrt{252}$:

$$\sigma_{t,T}^{RE} = \sqrt{\frac{252}{\tau - 1}\sum_{i=1}^{\tau}\left(u_i - \overline{u}_i\right)^2}$$

where τ is the remaining life (in working days) of the option; $u_i = \ln(1 + r_i)$; r_i is the daily return of the underlying asset for day *i* in CRSP database; $\overline{u}_i$ is the mean of the u_i series.[29] Table II shows the summary statistics of 'realized' volatilities ($\sigma_{t,T}^{RE}$) of S&P500 index options for different day-to-maturity groups of options.

Andersen et al. (2001) show that the conventional squared returns produce inaccurate forecast if daily returns are used. The inaccuracy is a result of noise in these returns. They further show that impact of noise component is reduced if high-frequency returns are used (e.g., 5-minute returns). However, a relatively recent study by Aït-Sahalia, Mykland, and Zhang (2005) demonstrate that more data does not necessarily lead to a better estimate of realized volatility in the presence of market microstructure noise. They show that the optimal sampling frequency is jointly determined by the magnitude of market microstructure noise and the horizon of realized volatility. For a given level of noise, the realized volatility for a longer horizon (e.g., one month or

[28] $\mu_{t,T}$, $\sigma_{t,T}$ represent μ and σ respectively for a specific time period, where *t* is the date of observation of option prices, and *T* is the maturity date of the options.

[29] Hull (2002) uses a similar procedure to compute realized volatilities.

more) should be estimated with less frequent sampling than the realized volatility for a shorter horizon (e.g., one day). Since our experiments are mostly for more than one month time horizon, the optimum data frequency should neither be 5-minutes nor be the daily returns. In the absence of high-frequency data, to the extent the optimum frequency is closer the daily return our measure based on this frequency should closely represent the realized returns.[30]

Using the above regression models, (R1) ~ (R3), we can test three hypotheses. First, we can test if $\sigma_{t,T}$ predicts better than $\overline{\sigma}_{t,T}^{BS}$. Second, we can verify if the coefficients of $\mu_{t,T}$ and $\sigma_{t,T}$ are significant even in the presence of $\overline{\sigma}_{t,T}^{BS}$. Third, we can test the hypothesis H_0: $\alpha_{43} = \alpha_{44} = \alpha_{45} = \alpha_{46} = 0$. If we reject this null hypothesis then we can argue that $\mu_{t,T}$ and $\sigma_{t,T}$ have significant contribution in forecasting the future volatility using the model as given in equation (R3).

The regression results are shown in Table IV. We have separate regressions for different maturity groups. As before, if days-to-maturity is less than or equal to 90 days then the observations are in '<=90' days-to-maturity group. If days-to-maturity is greater than 90 days then the observations are in '>90' days-to-maturity group. We estimate these regressions using the generalized method of moments. Using OLS may not be appropriate for our data in the presence of nonspherical disturbances.

Panel A of Table IV shows the regression results using all moneyness of

[30] Therefore we use 'realized' volatility, 'ex post' volatility, and 'historical' volatility interchangeably.

S&P500 index call options.[31] As shown in this panel the coefficients of $\overline{\sigma}_{t,T}^{BS}$, $\sigma_{t,T}$ and $\sigma_{t,T}^2$ are significant using models (R1) and (R2) respectively. However, the adjusted R-square is higher for the equation containing $\sigma_{t,T}$ and $\sigma_{t,T}^2$ for every maturity group. This shows, when we take all options $\sigma_{t,T}$ provides a better forecast of realized volatility of the stock than the $\overline{\sigma}_{t,T}^{BS}$. To investigate the performance of $\sigma_{t,T}$ further we have similar regressions in Panel B and Panel C of Table IV. As we see in Panel B, for stock price/strike price between 0.95 and 1.05 the adjusted R-squares are not higher for the equations containing $\sigma_{t,T}$ and $\sigma_{t,T}^2$. However, the adjusted R-squares are higher for the equations containing $\sigma_{t,T}$ and $\sigma_{t,T}^2$ using far-the-money options.[32] This shows $\sigma_{t,T}$ provides a better representation of ex ante volatility than $\overline{\sigma}_{t,T}^{BS}$ using the information in far-from-the-money options. Even though $\overline{\sigma}_{t,T}^{BS}$ does better when we take only near-the-money options, it is unable to provide a single implied volatility that we can use for options of all moneyness. On the other hand $\sigma_{t,T}$ provides a better measure of ex ante volatility that can be used for options of all moneyness.

How does equation (R1) compare with equation (R3) in explaining the realized volatility? To address this question first we see for all panels using near-the-money, all moneyness, and far-the-money options, the adjusted R-square is higher for the unrestricted regression (R3) as shown in Table IV. For example, in Panel A for '<=90' days-to-maturity group the adjusted R-square for the unrestricted model (R3) is 46.29%

[31] In all our samples, we do not include options that have zero trading volume.

[32] Options are defined to be far-the-money if the stock price divided by strike price is either higher than 1.05 or lower than 0.95.

and for the restricted model (R1) it is 41.87%. This shows that equation (R3) provides a better model such that it has a higher adjusted R-square for near-the-money, far-the-money, and options of all moneyness. Second, for all maturities the coefficients of $\sigma_{t,T}$ and $\sigma_{t,T}^2$ are significant for all Panels of Table IV in the unrestricted equation (R3). However that is not the case with $\overline{\sigma}_{t,T}^{BS}$. For example in Panel A and Panel C the coefficients of $\overline{\sigma}_{t,T}^{BS}$ are not significant.

Finally, we use the likelihood ratio to test the hypothesis H_0: $\alpha_{43} = \alpha_{44} = \alpha_{45} = \alpha_{46} = 0$. The likelihood ratios are significant in our experiment for all panels of Table IV. Therefore, we reject the restricted model as given in equation (R1) for all maturity groups shown in this table. This result indicates that the inclusion of $\mu_{t,T}$ and $\sigma_{t,T}$, and their second-order terms provides a better model than simply using Black-Scholes implied volatility to forecast the realized volatility for all near and far maturity groups, and for any moneyness level.

3.3.3.2 Information Content of Non-Nested Models

In this subsection we compare the non-nested models that have only the risk-adjusted variables ($\mu_{t,T}$ and $\sigma_{t,T}$, and the square terms) or the $\overline{\sigma}_{t,T}^{BS}$ variable (and its square term) to forecast realized volatility. We use two different variations of *J*-test that are popularly used in the literature.

The non-nested models that we use to forecast realized volatility can be given by the following regressions:

(R4) $\sigma_{t,T}^{RE} = \alpha_{10} + \alpha_{11}\bar{\sigma}_{t,T}^{BS} + \alpha_{12}\bar{\sigma}_{t,T}^{BS^2} + \omega_{1t,T}$

(R5) $\sigma_{t,T}^{RE} = \alpha_{20} + \alpha_{21}\sigma_{t,T} + \alpha_{22}\sigma_{t,T}^2 + \omega_{2t,T}$

(R6) $\sigma_{t,T}^{RE} = \alpha_{30} + \alpha_{31}\sigma_{t,T} + \alpha_{32}\sigma_{t,T}^2 + \alpha_{33}\mu_{t,T} + \alpha_{34}\mu_{t,T}^2 + \omega_{3t,T}$

To compare (R5) or (R6) with (R4) we take the fitted values of $\sigma_{t,T}^{RE}$ from these equations and use the following *J*-test regressions:

(R7) $\sigma_{t,T}^{RE} = \phi_1[\alpha_{20} + \alpha_{21}\sigma_{t,T} + \alpha_{22}\sigma_{t,T}^2] + (1-\phi_1)[\sigma_{t,T}^{RE} - \omega_{1t,T}] + e_{t,T}$

(R8) $\sigma_{t,T}^{RE} = \phi_1[\alpha_{30} + \alpha_{31}\sigma_{t,T} + \alpha_{32}\sigma_{t,T}^2 + \alpha_{33}\mu_{t,T} + \alpha_{34}\mu_{t,T}^2] + (1-\phi_1)[\sigma_{t,T}^{RE} - \omega_{1t,T}] + e_{t,T}$

(R9) $\sigma_{t,T}^{RE} = \phi_2[\alpha_{10} + \alpha_{11}\bar{\sigma}_{t,T}^{BS} + \alpha_{12}\bar{\sigma}_{t,T}^{BS^2}] + (1-\phi_2)[\sigma_{t,T}^{RE} - \omega_{2t,T}] + e_{t,T}$

Using (R7) and (R9) we can test whether the Black-Scholes implied standard deviation offers any incremental information over risk-adjusted implied volatility. If the Black-Scholes model does not have any incremental information, then ϕ_1 should be close to 1 and significant, and ϕ_2 should be insignificant.[33] To find whether ϕ_1 is in fact 1, we

[33] Our discussions compare (R5) with (R4). However, we can also compare (R6) with (R4) to find if Black-Scholes implied standard deviation offers any incremental information over risk-adjusted $\sigma_{t,T}$ and $\mu_{t,T}$. In that case we use (R8) instead of (R7) and (R9) is given by:

test the null hypothesis of H_0 : ϕ_1=1. Since our null hypothesis is the result intended, in this test, to minimize the Type II error *p*-value should be higher.[34] The left side of Table V shows the results of this comparison. As we see from left side of Panel A using all moneyness, ϕ_2 is insignificant for '<=90' days-to-maturity group. Also, ϕ_1 is significant and we fail to reject the null hypothesis that ϕ_1=1 for this maturity group. This show that for '<=90' days-to-maturity group Black-Scholes implied standard deviation provide no incremental information over our implied volatility. However for '>90' days-to-maturity group we cannot say that the Black-Scholes implied standard deviation provide no incremental information over the risk-adjusted $\sigma_{t,T}$. Results are similar when we take both $\mu_{t,T}$ and $\sigma_{t,T}$ to compare with the Black-Scholes implied standard deviation. Even for the near-the-money options (Panel B) for '<=90' days-to-maturity group, ϕ_2 is insignificant, and we fail to reject the null hypothesis that ϕ_1=1. This indicates that even when we do not have a volatility smile the risk-adjusted $\sigma_{t,T}$ performs marginally better than $\bar{\sigma}^{BS}_{t,T}$. Furthermore, as we see from Panel C, of Table V, consistent with the prior literature, when we have many far-from-the-money options, $\bar{\sigma}^{BS}_{t,T}$ does not provide any incremental information. These results suggest, to forecast volatility for shorter maturity of 90-days or less, the risk-adjusted $\sigma_{t,T}$ provides a better alternative over the $\bar{\sigma}^{BS}_{t,T}$ for any moneyness level. Furthermore, if we have many far-from-the-money options, then $\sigma_{t,T}$ is a better choice irrespective of days-to-maturity.

(R9) $\sigma^{RE}_{t,T} = \phi_2[\alpha_{10} + \alpha_{11}\bar{\sigma}^{BS}_{t,T} + \alpha_{12}\bar{\sigma}^{BS^2}_{t,T}] + (1-\phi_2)[\sigma^{RE}_{t,T} - \omega_{3t,T}] + e_{t,T}$
We show the results for both (R5), (R4) comparison, and (R6), (R4) comparison in Table VI.

[34] We take 5% significance level as the cutoff point, approximately in the middle of 10% and 1%.

We also test another variation[35] of the above *J*-test using the following regressions:

$$\text{(R10)}\quad \sigma^{RE}_{t,T} = (1-\psi_2)[\alpha_{20} + \alpha_{21}\sigma_{t,T} + \alpha_{22}\sigma^2_{t,T}] + \psi_2[\sigma^{RE}_{t,T} - \omega_{1t,T}] + e_{t,T}$$

$$\text{(R11)}\quad \sigma^{RE}_{t,T} = (1-\psi_2)[\alpha_{30} + \alpha_{31}\sigma_{t,T} + \alpha_{32}\sigma^2_{t,T} + \alpha_{33}\mu_{t,T} + \alpha_{34}\mu^2_{t,T}] + \psi_2[\sigma^{RE}_{t,T} - \omega_{1t,T}] + e_{t,T}$$

$$\text{(R12)}\quad \sigma^{RE}_{t,T} = (1-\psi_1)[\alpha_{10} + \alpha_{11}\bar{\sigma}^{BS}_{t,T} + \alpha_{12}\bar{\sigma}^{BS^2}_{t,T}] + \psi_1[\sigma^{RE}_{t,T} - \omega_{2t,T}] + e_{t,T}$$

For Black-Scholes implied volatility not to have any incremental contribution to forecast realized volatility, ψ_2 should be insignificant in (R10) and ψ_1 should be significant and closer to 1 in (R12).[36] Similar to the prior *J*-test, we test the null hypothesis that $H_0: \psi_1 = 1$. The results are given on the right side of Table V. The results using this alternative *J*-test are mostly similar to the prior *J*-test. Consistent with the prior *J*-test, when we take any moneyness for near term options (90-days or less), our results show Black-Scholes implied standard deviation does not contain incremental information beyond the risk-adjusted $\sigma_{t,T}$ (or $\mu_{t,T}$ and $\sigma_{t,T}$). However, for far term options (more than 90-days), we cannot argue that $\sigma_{t,T}$ (or $\mu_{t,T}$ and $\sigma_{t,}$) alone is sufficient to forecast realized volatility. Nonetheless, in this case we can still use the unrestricted regression using all the three variables which provide a better model for all

---

[35] Davidson and MacKinnon (1981).

[36] If we use risk-adjusted $\sigma_{t,T}$ and $\mu_{t,T}$. instead of just $\sigma_{t,T}$, then we use (R11) instead of (R10) and (R12) will be given by:

$$\text{(R12)}\quad \sigma^{RE}_{t,T} = (1-\psi_1)[\alpha_{10} + \alpha_{11}\bar{\sigma}^{BS}_{t,T} + \alpha_{12}\bar{\sigma}^{BS^2}_{t,T}] + \psi_1[\sigma^{RE}_{t,T} - \omega_{3t,T}] + e_{t,T}$$

near and far maturity groups, and for any moneyness level as we find in Table IV. In general, our risk-adjusted approach provides a better measure (than $\overline{\sigma}_{t,T}^{BS}$) that captures moneyness biases even without adjusting for stochastic volatility. Our results are stronger in forecasting the short term volatility for 90-days or less. Therefore, if we are concerned about the smile while forecasting realized volatility using all options data, then our approach provides a better solution than $\overline{\sigma}_{t,T}^{BS}$ so that we do not need any adjustment for moneyness bias.

3.3.4 Measurement Error and Robustness Checks

Option spread and option volume could be one possible reason for the term structure of $\mu_{t,T}$.[37] As we see in Table I, spread and option volume are lower for higher days-to-maturity.[38] This experiment is also motivated by the findings of Longstaff (1995). Using S&P100 index options and Black-Scholes (1973) risk-neutral valuation Longstaff shows that the implied cost of the index is significantly higher in the option market than in the stock market. The author also shows the percentage pricing difference between the implied and actual index is directly related to the measures of transaction costs and liquidity such as the option spread, volume, and open interest. To examine the possible influence of these market friction proxies on the term structure of $\mu_{t,T}$, we regress $\mu_{t,T}$ on transaction cost proxy that is given by the average spread, and liquidity measures that

[37] Term structure of $\mu_{t,T}$ is the value of $\mu_{t,T}$ for different option maturity date of T, for a given option pricing date of t.

[38] When we take finer groups, such as 30, 60, 90 days-to-maturity groups we clearly see the average volume and spread decrease with days-to-maturity.

are given by average volume and total open interest. We also control for other finding of pricing biases of Black-Scholes model. These findings include Chiras and Manaster (1978), Macbeth and Merville (1980), Rubenstein (1985), and Canina and Figlewski (1993). These studies find three types of pricing bias in Black-Scholes model namely a time to expiration bias, a moneyness bias, and a volatility bias. To control for these biases we include the time to expiration, moneyness (stock price/strike price), and current and first two lagged values of absolute daily returns. To control for volatility bias, we use current and first two lagged values of absolute daily returns instead of implied volatility $\sigma_{t,T}$ since this parameter is jointly estimated with $\mu_{t,T}$, which can induce spurious correlation. Further, we use number of calls to compute $\mu_{t,T}$ and $\sigma_{t,T}$ as a measure of trading activity, current and lagged daily returns as a measure of path-dependent effects (Leland (1985)). The results are shown in Table VI. The regression results provide mixed evidence that term structure of $\mu_{t,T}$ is related to the market friction proxies namely spread, volume, and open interest. For example, for '>90' days-to-maturity group the coefficient of average spread and total open interest are 0.0989 and -1.04E-07 respectively and are significant, whereas average volume is not significant. Similarly, for '<=90' days-to-maturity group only total open interest is significant. Interestingly coefficient of total open interest is negative and significant for all maturity groups. However, in the data, total open interest does not increase (as the days to maturity increases) to support the declining term structure of $\mu_{t,T}$.[39] As we see average spread is not significant for '<=90' days to maturity groups, that means spread cannot explain the sharp term structure of $\mu_{t,T}$ especially for the lower days-to-maturity group

[39] Open interest is mostly lower for higher days to maturity.

as seen in Figure I,. Therefore, our evidence shows that friction proxies are not the cause of the term structure of $\mu_{t,T}$.[40]

Our modified risk-adjusted approach can be questionable in a framework with stochastic volatility and jumps, which means we may not be using the exact model of option pricing. Many of the past literature for example Merton (1976a), Cox and Ross (1976a), Hull and White (1987), Scott (1987), and Heston (1993a) extend basic Black-Scholes (B-S) model to incorporate jumps and stochastic volatility. However, the risk-adjusted formulas we use do not have these adjustments and assumes a lognormal diffusion process. This can create errors-in-variable problem in implied return and implied volatility computation. To minimize the effect of errors-in-variable bias, we alternatively take options, which are only near-the-money (stock price divided by strike price is between 0.95 and 1.05).[41] We still see a strong term structure of $\mu_{t,T}$ in this case. Moreover, we do not take options that do not have any trading in a given day. We also separately estimate $\mu_{t,T}$ and $\sigma_{t,T}$ for deep-in-the-money call options where stock price divided by strike price is greater than 1.20, and deep-out-of-the-money call options where stock price divided by strike price is less than 0.90. In both cases, we still get the term structure of $\mu_{t,T}$. Measurement error may be systematically affected by time-to-maturity (Canina and Figlewski (1993)). To mitigate these errors, options with same days-to-maturity are used to compute implied expected return and implied volatility. It may also be possible to have systematic bias in our computation due to other factors

---

[40] Table IV is based on all moneyness of S&P500 index options. When we take only near-the-money (stock price divided by strike price is between 0.95 and 1.05) the evidence of friction proxies on $\mu_{t,T}$ are much weaker; however, we still see a very strong term structure of $\mu_{t,T}$ even in this case.

[41] The term structure of $\mu_{t,T}$ using near-the-money is also downward sloping.

such as the market friction (Longstaff (1995)) proxies. To examine this possibility, we regress $\mu_{t,T}$ on these proxies to show in the previous paragraph that they do not explain the term structure of $\mu_{t,T}$.

Furthermore, our procedure might have problems of computing European option prices from OptionMetrics implied volatility and using that to compute our implied return and implied volatility. As a part of our robustness check, we show even if we use different methods to compute option prices, the term structure of implied expected return remains in our result. For example, in our main result we compute the European price using the OptionMetrics implied volatility adjusted for dividends. If this price is higher than the bid-ask midpoint then we take the bid-ask midpoint, else we take the European price as the option price for $\mu_{t,T}$ and $\sigma_{t,T}$ estimation. In our robustness check, we compute $\mu_{t,T}$ and $\sigma_{t,T}$ first by taking the European price, and then by taking the bid-ask midpoint price as the option price and we get clear term structures of implied expected return in both cases.

As we discussed before, the term structure of implied expected return ($\mu_{t,T}$) is robust to various tests using call options. However it would be interesting to find out if the term structure persists using both call and put options. For this experiment, we take a set of 'balanced' call and put options. Balanced options means we take only the options that have both call and put with same strike price. If any call (put) does not have a corresponding put (call) with same strike price we do not take that option. Since for a bullish stock, the price of the call goes up; that might be the cause of the term structure of $\mu_{t,T}$ using only call options. Similarly, taking just the put options might reflect only specific set of investor needs. [42] From this argument it is clear that if we take all the calls and puts for a given maturity we might have either more number of calls or more number of puts, and thus our inference might be dominated by a specific type of option. Therefore, to make sure we have same number of calls and puts, we take a 'balanced' options approach to estimate the implied expected return ($\mu_{t,T}$) jointly with implied volatility ($\sigma_{t,T}$). The input data summary statistics for these observations are in Table VII and the $\mu_{t,T}$ and $\sigma_{t,T}$ results are in Table VIII. As we see in Table VII the total number of observations used is 6242. This compares with 7865 number of observations in Table I where we use only call options. Number of options in the balanced dataset will be lower if we do not have a corresponding put option with the same strike price. Alternatively if for some maturities we had rejected the call options because we did not have at least two options, those records might not get rejected when we take both call and put options, thus increasing the number of observations. Therefore, taking a balanced set does not imply that the total number of observation will increase or decrease compared to taking only the call options. As we see from Table VIII, $\mu_{t,T}$ for less than 90 days group is 15.53% whereas for more than 90 days group is 9.83%.[43] This compares with corresponding $\mu_{t,T}$ value of 19.50% and 9.41% when we take only call options. Figure IV also shows a similar term structure. This graph is sharp near zero days to maturity (only for the recent year) due to the extrapolation effect of the spline algorithm. Nonetheless our experiment shows that the term structure of $\mu_{t,T}$ still persists when we use balanced call and put options.

---

[42] Buying a put does not have the same payoff as writing a call. So the investor needs to choose a suitable option (call or put) and suitable side (buy or sell) of the trade for the investment need.

[43] We also see this term structure when we break into smaller interval groups of days-to-maturity.

3.4 Possible Explanations of the Term Structure of Expected Return

As we show in this chapter: 1) there is a term structure of stock expected return in option prices; 2) this term structure is robust to near-the-money, far-the-money, and all moneyness. It is also robust to all stock options (shown in chapter 4) and S&P500 index options. Further, it is robust to the bid and ask midpoint price and European option price. Therefore the next phase of natural exploration is why the term structure is there in the option price. Following are few possible explanations for this term structure for future investigation.

First, the term structure of expected return could be model dependent. This means the geometric Brownian with constant volatility assumption might be little restrictive to describe the evolution of the price process that might be resulting this term structure. Therefore as a future extension of our research we suggest a stochastic volatility risk adjusted model in chapter 6. Nonetheless, even in the presence of this term structure, we show in chapter 5 that ex ante expected return has the properties so that it satisfies the tradition CAPM and has information about future macroeconomic factors. Using stochastic volatility should possibly further improve the information content of this ex ante expected return. The second possible story could be the urgency to rebalance and cost of liquidity. Imagine two options on the same stock: one that matures in one month and the second that matures in six months. In the absence of any transaction cost, the more we rebalance the more we are close to the Black-Scholes price with lower standard error.[44] . Let us assume we need to rebalance around *n* times during the life of the option to have a specific level of standard error.[45] So the liquidity cost (in terms of immediacy of availability) of obtaining *n* opportunities in a short period of one month is higher than in a long period of six months. Including this cost in the option price lowers the price of the option and raises the expected return of the option (and thus raises the expected return of the stock) in the short term. The above discussion is based on a flat volatility term structure. In the presence of a downward sloping term structure of volatility, this reasoning even becomes stronger. Third possibility is related to a possible extension of Leland (1985). Leland's paper has developed a technique for replicating option returns in the presence of transactions costs. The strategy depends upon the level of transactions costs and the time period between portfolio revisions, in addition to the standard variables of option pricing. However, our finding might imply a correlation between the transaction cost and the time period between revisions. Therefore, Leland's transaction cost option pricing could possibly be extended to address this term structure of expected return.

3.5 Conclusion

This dissertation uses a risk-adjusted method for joint estimation of implied expected stock return and volatility from market observed option prices. We find that investors in option markets have a higher expectation of stock return in the short-term, but a lower

[44] This can be seen using MonteCarlo simulation.

[45]We assume *n* is a function of asset characteristics, more specifically the volatility of the stock. So if volatility term structure is flat then we will need same number of rebalancing, *n* for short- and long-term options for a given standard error. Also, keeping all parameters same, if we change the volatility to obtain the price of the option, using MonteCarlo simulation, we can easily see, that the standard error of option price is higher when we the volatility is higher.

expectation of stock return in the long-term. This term structure of expected stock return also remains for deep-in and deep-out of the money call options. We also find that the market friction proxies such as volume, open interest and bid-ask spread do not explain this term structure. It also persists for combined call and put options. This term structure finding supports McNulty et al. (2002) explanation where the authors argue that shorter horizon investments should be discounted at a higher rate. However, they use a heuristic approach without a theoretical setting to arrive at these results. On the other hand, our research provides the necessary theoretical support for this finding. Using all moneyness options, we further find that the term structure of our volatility is 'flatter' than the term structure of Black-Scholes implied standard deviation. We also find that the implied volatility ($\sigma_{t,T}$) provides a better model than Black-Scholes implied standard deviation ($\overline{\sigma}_{t,T}^{BS}$) to forecast realized volatility for maturities of 90-days or less for any moneyness level. In general, our risk-adjusted approach provides a better measure (than $\overline{\sigma}_{t,T}^{BS}$) that captures moneyness biases even without adjusting for stochastic volatility. Therefore, if we are concerned about the smile while forecasting realized volatility using all options data, then our approach provides a better solution than $\overline{\sigma}_{t,T}^{BS}$ so that we do not need any adjustment for moneyness bias. In addition, we find that a combination of our implied expected return ($\mu_{t,T}$) and implied volatility ($\sigma_{t,T}$) with $\overline{\sigma}_{t,T}^{BS}$ provides a better model, than using $\overline{\sigma}_{t,T}^{BS}$ alone to forecast future volatility for all near and far maturity groups, and for any moneyness level.

These findings may provide a starting point for further research. For example, our approach may be used to estimate the cost of equity for different industry portfolios.

Especially estimates of expected return for one-year or more will have lower standard error, which is a necessary condition for this to be useful as an estimate of cost of equity. Using this approach, we can compute the expected return of any individual stock without using any information of the market portfolio such as the market risk premium. Moreover, our results can be deduced without assuming a utility structure for the representative agent. Furthering the research, we plan to investigate whether the term structure persists using other approaches. Nonetheless, better forecasting capability of future volatility using our sigma and expected return might suggest additional investigation of information content in these findings.

3.A Appendix

3.A.1 Risk-Adjusted Formulas for Call Options

3.A.1.1 Proof of Proposition 1:

We prove the proposition without assuming the CAPM. Let the price change for the stock and option during a small interval of time Δt are ΔS and ΔC respectively. Without loss of generality, we assume *t* as the current time. Let the current stock and option prices are S_t and C_t respectively. This implies:

(A1) $$\begin{aligned} &\frac{\Delta S}{S_t} = r_S \\ &\frac{\Delta C}{C_t} = r_C \\ &E[r_S] = \mu \Delta t \\ &E[r_C] = k \Delta t \end{aligned}$$

When Δt is a small interval of time, then Δt tends to dt, ΔS tends to dS, and ΔC tends to dC.

Since stock price *S* follows a geometric Brownian, the change in the price of the stock ΔS during the small interval of time Δt is:

(A2) $$dS = \mu S_t dt + \sigma S_t dW$$

where dW is the Wiener differential. Then, following Ito's Lemma, option price change is given by:

(A3)
$$\begin{aligned} dC &= \frac{\partial C}{\partial S} dS + \left(\frac{1}{2} \frac{\partial^2 C}{\partial S^2} \sigma^2 S_t^2 + \frac{\partial C}{\partial t} \right) dt \\ &= \frac{\partial C}{\partial S} dS + \left(rC_t - \frac{\partial C}{\partial S} rS_t \right) dt \end{aligned}$$

where the second line of (A3) is derived from the Black-Scholes PDE (partial differential equation). From (A3), we can then compute the covariance between the option return and the stock return as follows:

(A4)
$$\begin{aligned} \operatorname{cov}\left[\frac{dC}{C_t}, \frac{dS}{S_t} \right] &= \frac{1}{C_t S_t} \operatorname{cov}[dC, dS] \\ &= \frac{1}{C_t S_t} \frac{\partial C}{\partial S} \operatorname{var}[dS] \\ &= \frac{S_t}{C_t} \frac{\partial C}{\partial S} \operatorname{var}\left[\frac{dS}{S_t} \right] \end{aligned}$$

Then it follows that:

(A5)
$$\begin{aligned} \frac{S_t}{C_t} \frac{\partial C}{\partial S} &= \frac{\operatorname{cov}\left[\frac{dC}{C_t}, \frac{dS}{S_t} \right]}{\operatorname{var}\left[\frac{dS}{S_t} \right]} \\ &= \beta \end{aligned}$$

Finally, taking the expectation of (A3), we obtain:

(A6)
$$kdt = \beta \mu dt + r(1 - \beta) dt$$

Q.E.D.

Further, we note that, if we take covariance of both sides of (A3) with respect to the market return r_M , then we will obtain the following:

(A7)
$$k = r + \beta(\mu - r)$$

where

$$\beta = \frac{\beta_C}{\beta_S}$$

$$\beta_C = \frac{\operatorname{cov}(r_C, r_M)}{\operatorname{var}(r_M)}$$

$$\beta_S = \frac{\operatorname{cov}(r_S, r_M)}{\operatorname{var}(r_M)}$$

This implies:

(A8) $$\begin{aligned}\beta &= \frac{\operatorname{cov}(r_C, r_S)}{\operatorname{var}(r_S)} \\ &= \frac{\operatorname{cov}(r_C, r_M)}{\operatorname{cov}(r_S, r_M)}\end{aligned}$$

3.A.1.2. Proof of Proposition 2:

For readability we drop the subscript *t,T* for μ, σ, and *k* during this proof. From (5a), we can compute the expected value of the call payoff using the risk-adjusted measure as:

(A9) $$\begin{aligned}E[C_T] &= e^{k(T-t)}C_t \\ &= S_t e^{\mu(T-t)} N(h_1) - KN(h_2)\end{aligned}$$

From the known result of the moment generating function of a Gaussian variable, we have:

$$\text{(A10)}\qquad \begin{aligned} \text{var}\ S_T &= E[S_T^2] - \ E[S_T]\ ^2 \\ &= S_t^2 e^{(2\mu+\sigma^2)(T-t)} - S_t^2 e^{2\mu(T-t)} \\ &= S_t^2 e^{2\mu(T-t)}\ \ e^{\sigma^2(T-t)} - 1 \end{aligned}$$

and

$$\text{(A11)}\qquad \begin{aligned} E[S_T C_T] &= \int_0^\infty S_T \max\{S_T - K, 0\}\phi(S_T)dS_T \\ &= \int_K^\infty S_T^2 \phi(S_T)dS_T - K\int_K^\infty S_T\phi(S_T)dS_T \\ &= S_t^2 e^{(2\mu+\sigma^2)(T-t)} N(h_3) - KS_t e^{\mu(T-t)} N(h_1) \end{aligned}$$

where

$$h_3 = \frac{\ln S - \ln K + \ \ \mu + \frac{3}{2}\sigma^2 \ \ (T-t)}{\sigma\sqrt{T-t}}$$

Hence, the covariance term in (7a) can be computed as:

$$\text{(A12)}\qquad \begin{aligned} \text{cov}\ S_T, C_T &= E[S_T C_T] - E[S_T]E[C_T] \\ &= S_t^2 e^{(2\mu+\sigma^2)(T-t)} N(h_3) - KS_t e^{\mu(T-t)} N(h_1) - S_t e^{\mu(T-t)} \left[S_t e^{\mu(T-t)} N(h_1) - KN(h_2) \right] \\ &= S_t^2 e^{2\mu(T-t)} \left[e^{\sigma^2(T-t)} N(h_3) - \frac{K}{S_t} e^{-\mu(T-t)} \ \ N(h_1) - N(h_2) \ \ - N(h_1) \right] \end{aligned}$$

Finally, combining equations (7a), (A10), and (A12) we have:

$$\text{(A13)}\qquad \beta = \frac{S_t \left[e^{\sigma^2(T-t)} N(h_3) - \frac{K}{S_t} e^{-\mu(T-t)} \ \ N(h_1) - N(h_2) \ \ - N(h_1) \right]}{C_t \ \ e^{\sigma^2(T-t)} - 1}$$

With the subscripts *t*,*T* attached to the parameters, equation (A13) can be written as equation (8) of Proposition (2).:

Q.E.D.

3.A.2 Risk-Adjusted Formulas for Put Options

3.A.2.1 Proposition 1 for put options:

Assume stock price S follows a geometric Brownian motion with an annualized expected instantaneous return of μ and volatility of σ. Let a put option on the stock at any point in time t is given by $P(S,t)$ that matures at time T. Let k is the annualized expected instantaneous return on this option. Then for a small interval of time Δt, the relationship between μ and k can be given by:

(A14) $k = r + \beta(\mu - r)$

where

(A15) $\beta = \dfrac{\mathrm{cov}(r_P, r_S)}{\mathrm{var}(r_S)}$

and $r_S = \Delta S \,/\, S$ and $r_P = \Delta P \,/\, P$ are two random variables representing the stock return and put option return respectively during the period Δt. And, r is the annualized constant risk-free rate for the period of the option.

Proof: The proof is similar to proposition 1.

As in proposition 1, we have:

$$\text{(A16)}\quad \begin{aligned} &\frac{\Delta S}{S} = r_S \\ &\frac{\Delta P}{P} = r_P \\ &E[r_S] = \mu \Delta t \\ &E[r_P] = k \Delta t \end{aligned}$$

When Δt is a small interval of time, then Δt approaches dt, ΔS approaches dS, and ΔP approaches dP.

Since stock price *S* follows a geometric Brownian, the change in the price of the stock ΔS during the small interval of time Δt is:

$$\text{(A17)}\quad dS = \mu S_t dt + \sigma S_t dW$$

where dW is the Wiener differential. Then, following Ito's Lemma, option price change is given by:

$$\text{(A18)}\quad \begin{aligned} dP &= \frac{\partial P}{\partial S} dS + \left(\frac{1}{2} \frac{\partial^2 P}{\partial S^2} \sigma^2 S_t^2 + \frac{\partial P}{\partial t} \right) dt \\ &= \frac{\partial P}{\partial S} dS + \left(r P_t - \frac{\partial P}{\partial S} r S_t \right) dt \end{aligned}$$

where the second line of (A18) is derived from the Black-Scholes PDE (partial differential equation). From (A18), we can then compute the covariance between the option return and the stock return as follows:

$$\text{(A19)}\quad \begin{aligned} \operatorname{cov}\left[\frac{dP}{P_t}, \frac{dS}{S_t} \right] &= \frac{1}{P_t S_t} \operatorname{cov}[dP, dS] \\ &= \frac{1}{P_t S_t} \frac{\partial P}{\partial S} \operatorname{var}[dS] \\ &= \frac{S_t}{P_t} \frac{\partial P}{\partial S} \operatorname{var}\left[\frac{dS}{S_t} \right] \end{aligned}$$

Then it follows that:

(A20) $$\frac{S_t}{P_t}\frac{\partial P}{\partial S} = \frac{\text{cov}\left[\frac{dP}{P_t}, \frac{dS}{S_t}\right]}{\text{var}\left[\frac{dS}{S_t}\right]} \\ = \beta$$

Finally, taking the expectation of (A18), we obtain:

(A21) $$kdt = \beta\mu dt + r(1-\beta)dt$$

Q.E.D.

Without the subscripts of t,T we write β over the life of the put options as:

(A22) $$\beta = \frac{\text{cov}\left(\frac{P_T}{P_t}, \frac{S_T}{S_t}\right)}{\text{var}\left(\frac{S_T}{S_t}\right)} = \frac{S_t}{P_t}\frac{\text{cov}\ \ P_T, S_T}{\text{var}(S_T)}$$

The put option risk-adjusted pricing equation is:

(A23) $$\begin{aligned} P_t &= e^{-k(T-t)}E_t[\max\{K - S_T, 0\}] \\ &= e^{-k(T-t)}\left[K\int_0^K \phi(S_T)dS_T - \int_0^K S_T\phi(S_T)dS_T\right] \\ &= e^{-k(T-t)}KN(-h_2) - e^{(\mu-k)(T-t)}S_tN(-h_1) \end{aligned}$$

where t and T are the current time and maturity time of the option, and K is the strike price of the option and

$$\begin{aligned} h_1 &= \frac{\ln S - \ln K + \ \ \mu + ½\sigma^2 \ \ (T-t)}{\sigma\sqrt{T-t}} \\ h_2 &= h_1 - \sigma\sqrt{T-t} \end{aligned}$$

3.A.2.2 Proposition 2 for put options:

The β, based on the life of the put option can be written as:

$$\text{(A24)} \quad \beta = \frac{S_t}{P_t} \frac{-\left[e^{\sigma^2(T-t)}N(-h_3) - \frac{K}{S_t}e^{-\mu(T-t)}\{N(-h_1) - N(-h_2)\} - N(-h_1)\right]}{e^{\sigma^2(T-t)} - 1}$$

Proof:

The expected value of the put payoff using the risk-adjusted measure is:

$$\text{(A25)} \quad E[P_T] = KN(-h_2) - e^{\mu(T-t)}S_t N(-h_1)$$

From the known result of the moment generating function of a Gaussian variable, we have:

$$\text{(A26)} \quad \begin{aligned} \mathrm{var}\ S_T &= E[S_T^2] - \ E[S_T]\ ^2 \\ &= S_t^2 e^{(2\mu+\sigma^2)(T-t)} - S_t^2 e^{2\mu(T-t)} \\ &= S_t^2 e^{2\mu(T-t)} \ \ e^{\sigma^2(T-t)} - 1 \end{aligned}$$

and

$$\text{(A27)} \quad \begin{aligned} E[S_T P_T] &= \int_0^\infty S_T \max\{K - S_T, 0\}\phi(S_T)dS_T \\ &= K\int_0^K S_T\phi(S_T)dS_T - \int_0^K S_T^2\phi(S_T)dS_T \\ &= KS_t e^{\mu(T-t)}N(-h_1) - S_t^2 e^{(2\mu+\sigma^2)(T-t)}N(-h_3) \end{aligned}$$

where

$$h_3 = \frac{\ln S - \ln K + \ \ \mu + \frac{3}{2}\sigma^2 \ \ (T-t)}{\sigma\sqrt{T-t}}$$

Hence, the covariance term in (A22) can be computed as:

(A28)

$$\begin{aligned}
\text{cov } S_T, P_T &= E[S_T P_T] - E[S_T]E[P_T] \\
&= [KS_t e^{\mu(T-t)} N(-h_1) - S_t^2 e^{(2\mu+\sigma^2)(T-t)} N(-h_3)] - S_t e^{\mu(T-t)}[KN(-h_2) - S_t e^{\mu(T-t)} N(-h_1)] \\
&= KS_t e^{\mu(T-t)}[N(-h_1) - N(-h_2)] - S_t^2 e^{2\mu(T-t)}[e^{\sigma^2(T-t)} N(-h_3) - N(-h_1)] \\
&= S_t^2 e^{2\mu(T-t)} \left[\frac{K}{S_t} e^{-\mu(T-t)} \{N(-h_1) - N(-h_2)\} - e^{\sigma^2(T-t)} N(-h_3) + N(-h_1) \right]
\end{aligned}$$

Finally, combining equations (A22), (A26), and (A28) we have:

(A29) $$\beta = \frac{S_t}{P_t} \frac{-\left[e^{\sigma^2(T-t)} N(-h_3) - \dfrac{K}{S_t} e^{-\mu(T-t)} \{N(-h_1) - N(-h_2)\} - N(-h_1) \right]}{e^{\sigma^2(T-t)} - 1}$$

Q.E.D.

Tables

Table I: Input Data Summary Statistics of S&P500 Index Options

This table presents the summary statistics of all moneyness month-end S&P 500 index call options having positive trading volume based on the month-end observations for the period of January 1996-April 2006. Days-to-maturity groups are formed based on option days-to-maturity. For example, if days to maturity is less than or equal to 90 days then the observation is in '<=90' days-to-maturity group. If days to maturity is greater than 90 days it is in '> 90' days-to-maturity group. Moneyness we define as the stock price divided by the strike price. For S&P500, stock price is the level of the index. Avg. volume is the average of volume of call options used for a $\mu_{t,T}$ and $\sigma_{t,T}$ pair estimate. Avg. spread is the average of spread of call options used for a $\mu_{t,T}$ and $\sigma_{t,T}$ pair estimate. Spread is defined as (offer - bid)/call price. Call price is the midpoint of bid and offer or the European option price whichever is lower. European option price is computed from Black-Scholes implied volatility in the data. Number of calls used is the number of option records that are used to compute a $\mu_{t,T}$ and $\sigma_{t,T}$ pair.

Days-to-maturity groups	**<= 90 Days**	**> 90 Days**	**All Maturities**
Number of observations	5602	2263	7865
Days-to-maturity Mean	48.9749	321.2702	198.0316
Avg. moneyness Mean	0.9818	0.953	0.966
Std. Dev.	0.0298	0.0709	0.0579
Min	0.8757	0.6242	0.6242
Max	1.1446	1.3697	1.3697
Median	0.9832	0.954	0.9714
Number of calls used Mean	15.648	5.2263	9.9431
Std. Dev.	7.9866	3.1247	7.8171
Min	2	2	2
Max	42	26	42
Median	15	4	7
Avg. spread Mean	0.1365	0.035	0.0809
Std. Dev.	0.1347	0.0418	0.1082
Min	0.0065	0.0008	0.0008
Max	1.0806	0.3773	1.0806
Median	0.1016	0.0229	0.0427
Avg. volume Mean	603.0293	350.6503	464.8749
Std. Dev.	526.9405	660.0032	616.0207
Min	3.5	1	1
Max	4385.5	8186.75	8186.75
Median	472.8111	191.0833	301
Total open interest Mean	143911.3017	44156.9307	89304.9267
Std. Dev.	180980.9924	50192.6798	136556.5071
Min	0	0	0
Max	1336404	280941	1336404
Median	88911.5	23619	41591

Table II: Implied and Realized Summary Statistics Using S&P500 Index Options

The sample consists of all moneyness month-end S&P 500 index call options based on the month-end observations for the period of January 1996-April 2006. Days to maturity groups are formed based on option days-to-maturities. For example, if days to maturity is less than or equal to 90 days then the observation is in '<=90' days-to-maturity group. If days to maturity is greater than 90 days it is in '> 90' days-to-maturity group. We use all the call options on the same CUSIP, days-to-maturity, and trade date to compute the implied expected return and implied volatility by a grid search method that minimizes the square of difference between the observed and computed option price. Realized volatility is computed based on actual return of the index from trade date to maturity date of the option. Implied standard deviation ($\overline{\sigma}_{t,T}^{BS}$) is the Black-Scholes implied volatility. Results are shown in decimals.

Days-to-maturity groups	**<= 90 Days**	**> 90 Days**	**All Maturities**
Implied expected return $\mu_{t,T}$			
Mean	0.195	0.0941	0.1397
Std. Dev.	0.0876	0.0379	0.0823
Min	0.0745	0	0
Max	0.5887	0.2428	0.5887
Median	0.173	0.0897	0.1216
Implied volatility $\sigma_{t,T}$			
Mean	0.2146	0.2132	0.2139
Std. Dev.	0.0747	0.0670	0.0705
Min	0.0788	0.1017	0.0788
Max	0.464	0.4611	0.464
Median	0.2068	0.2073	0.2071
Implied standard deviation ($\overline{\sigma}_{t,T}^{BS}$)			
Mean	0.1979	0.1942	0.1968
Std. Dev.	0.0853	0.0581	0.0785
Min	0.0738	0.0898	0.0738
Max	1.7805	1.1215	1.7805
Median	0.1854	0.1889	0.1865
Realized volatility			
Mean	0.1678	0.1712	0.1697
Std. Dev.	0.0697	0.0544	0.0618
Min	0.0632	0.0882	0.0632
Max	0.4324	0.3255	0.4324
Median	0.1553	0.181	0.166

Table III: Comparison of Sigma and Black-Scholes Implied Volatility

This table presents the summary statistics of comparison of our sigma ($\sigma_{t,T}$) estimates and Black-Scholes implied volatility ($\bar{\sigma}_{t,T}^{BS}$) for different days-to-maturity groups based on all moneyness S&P500 Index call options for the period of January 1996-April 2006. Days to maturity groups are formed based on option days-to-maturities. For example, if days to maturity is less than or equal to 90 days then the observation is in '<=90' days-to-maturity group. If days to maturity is greater than 90 days it is in '> 90' days-to-maturity group. For this table, first, we compute mean and standard deviation of sigma and $\bar{\sigma}_{t,T}^{BS}$ for each year and days-to-maturity. Panel A presents the test of difference between mean level of sigma and $\bar{\sigma}_{t,T}^{BS}$ for different maturity groups. Panel B presents the test of difference between standard deviation level of sigma and $\bar{\sigma}_{t,T}^{BS}$ for different maturity groups. For Panel C, we compute the coefficient of variation (CV) of sigma and $\bar{\sigma}_{t,T}^{BS}$ as corresponding standard deviation divided by the mean for each year and days-to-maturity. Then we take the difference of CV of sigma and $\bar{\sigma}_{t,T}^{BS}$ for each year and days-to-maturity. The *t*-statistics shows whether these differences are significant for different days-to-maturity groups. ** and * represent the *p*-values of less than 0.01, and between 0.01 and 0.05 respectively.

Days-to-maturity groups	**<= 90 Days**	**> 90 Days**	**All Maturities**
Panel A: Test of difference between level of Sigma and $\bar{\sigma}_{t,T}^{BS}$			
Difference:			
Mean	0.0192	0.0201	0.0153
Standard Deviation	0.0232	0.0274	0.0176
t-statistics	10.9284**	13.3524**	2.88**
Panel B: Test of difference between standard deviation of Sigma and $\bar{\sigma}_{t,T}^{BS}$			
Difference:			
Mean	-0.0071	-0.0004	-0.0203
Standard Deviation	0.0313	0.0207	0.0115
t-statistics	-2.2813*	-0.1710	-5.8584**
Panel C: Test of difference between coefficient of variation of Sigma and $\bar{\sigma}_{t,T}^{BS}$			
Difference:			
Mean	-0.0565	-0.0200	-0.1232
Standard Deviation	0.1371	0.0870	0.0600
t-statistics	-4.1222**	-2.0475*	-6.8079**

Table IV: Information Content of the Nested Model Using Mu, Sigma, and Black-Scholes Implied Volatility

This table presents the generalized method of moments regression for forecast of realized volatility using $\mu_{t,T}$, $\sigma_{t,T}$, and Black-Scholes implied volatility ($\bar{\sigma}_{t,T}^{BS}$) for different maturity groups for the period of January 1996-April 2006. Days to maturity groups are formed based on option days-to-maturities. For example, if days to maturity is less than or equal to 90 days then the observation is in '<=90' days-to-maturity group. If days to maturity is greater than 90 days it is in '> 90' days-to-maturity group.Values in parenthesis are *t*-statistics. Dependent variable is realized volatility of the index for the period of the option. $\bar{\sigma}_{t,T}^{BS}$ is Black-Scholes implied volatility, $\mu_{t,T}$ and $\sigma_{t,T}$ are the estimated values from our model. For S&P500 index (SPX), stock price is the level of the index. LR is the likelihood ratio to test whether the restricted regressions are valid. ** and * represent the *p*-values of less than 0.01, and between 0.01 and 0.05 respectively.

Panel A. SPX Call Options Using All Moneyness

Intercept	$\sigma_{t,T}$	$\sigma_{t,T}^2$	$\mu_{t,T}$	$\mu_{t,T}^2$	$\bar{\sigma}_{t,T}^{BS}$	$\bar{\sigma}_{t,T}^{BS^2}$	Adj. R^2	LR
Days-To-Maturity of Less Than or Equal To 90 Days								
-0.0299(-1.17)	1.2336**(4.82)	-1.2986*(-2.28)					0.4379	20.3474**
-0.0236(-0.79)					1.1905**(3.68)	-0.9544(-1.18)	0.4187	32.4164**
0.0197(0.68)	1.9075**(4.67)	-2.9228**(-3.77)	0.0526(0.36)	-0.2457(-0.95)	-1.3344*(-2.49)	3.6476**(3.23)	0.4629	
Days-To-Maturity of Greater Than 90 Days								
-0.0681**(-4.45)	1.7558**(12.17)	-2.7056**(-8.89)					0.4434	13.9759**
-0.1891**(-6.84)					3.0648**(10.11)	-5.8532**(-7.67)	0.4328	22.1500**
-0.1912**(-4.42)	0.3136(0.81)	-0.2417(-0.35)	0.627*(2.08)	-2.4604(-1.65)	2.4852**(3.5)	-5.2144**(-3.19)	0.456	
All Maturities								
-0.0461**(-2.89)	1.468**(9.25)	-1.9368**(-5.37)					0.4282	20.6095**
-0.0663**(-2.6)					1.6909**(5.96)	-2.2575**(-3.05)	0.4048	52.3657**
-0.0073(-0.28)	1.8209**(6.83)	-2.9905**(-6.38)	-0.0092(-0.11)	-0.0526(-0.3)	-0.8351(-1.85)	2.5825*(2.49)	0.4401	

Table IV. Continued

Panel B. SPX Call Options Using StockPrice/StrikePrice Between 0.95 and 1.05 (Near the Money)

Intercept	$\sigma_{t,T}$	$\sigma^2_{t,T}$	$\mu_{t,T}$	$\mu^2_{t,T}$	$\bar{\sigma}^{BS}_{t,T}$	$\bar{\sigma}^{BS^2}_{t,T}$	Adj. R^2	LR
Days-To-Maturity of Less Than or Equal To 90 Days								
0(0)	0.9077**(4.55)	-0.6131(-1.48)					0.4812	21.5486**
-0.0114(-0.43)					1.0455**(3.64)	-0.5166(-0.73)	0.4853	18.6622**
0.0547(1.89)	2.1375**(3.76)	-3.8914**(-3.46)	-0.1606(-1.27)	0.2987(1.62)	-1.8916*(-2.14)	5.8757**(2.77)	0.5055	
Days-To-Maturity of Greater Than 90 Days								
0.0041(0.38)	0.9171**(12.63)	-0.7982**(-7.27)					0.4759	36.3098**
-0.193**(-5.73)					2.9321**(8.03)	-5.2307**(-5.88)	0.5191	12.4349*
-0.2158**(-5.78)	-0.1912(-0.69)	0.1589(0.72)	0.7401*(2.11)	-1.9228(-1.61)	2.918**(6.76)	-5.1362**(-6.07)	0.5333	
All Maturities								
-0.0053(-0.63)	0.9771**(16.1)	-0.829**(-9.58)					0.4765	16.1417**
-0.0501(-1.94)					1.447**(5.04)	-1.5308*(-2.08)	0.4785	13.6985**
-0.0389(-1.49)	0.53*(2.55)	-0.4676**(-2.62)	-0.0485(-0.57)	0.2009(1.42)	0.8354*(2.15)	-1.1626(-1.45)	0.4863	

Panel C. SPX Call Options Using StockPrice/StrikePrice of Less Than 0.95 or Greater Than 1.05 (Far from the Money)

Intercept	$\sigma_{t,T}$	$\sigma^2_{t,T}$	$\mu_{t,T}$	$\mu^2_{t,T}$	$\bar{\sigma}^{BS}_{t,T}$	$\bar{\sigma}^{BS^2}_{t,T}$	Adj. R^2	LR
Days-To-Maturity of Less Than or Equal To 90 Days								
-0.001(-0.08)	1.0099**(10.64)	-0.9703**(-8.19)					0.3331	11.6945*
-0.0146(-0.52)					1.2185**(4.17)	-1.3647(-1.96)	0.2742	39.3134**
0.0246(0.91)	1.2148**(6.72)	-1.1504**(-6.76)	-0.0812(-0.75)	-0.1393(-1.12)	-0.4097(-1.16)	0.9081(1.23)	0.3485	
Days-To-Maturity of Greater Than 90 Days								
-0.0172(-1.12)	1.3344**(10.01)	-1.8246**(-6.63)					0.3912	9.8708*
-0.1313**(-5.71)					2.6113**(10.18)	-4.9591**(-7.61)	0.3808	15.2859**
-0.1274**(-3.7)	0.3576(0.9)	-0.1559(-0.21)	0.4193(1.58)	-1.4697(-1.16)	2.0244**(2.95)	-4.5422**(-2.86)	0.4021	

All Maturities								
0.0063(0.5)	1.0288**(10.17)	-1.0906**(-5.84)					0.3383	12.8482*
-0.0423(-1.96)					1.5663**(6.68)	-2.2206**(-3.69)	0.2972	51.9050**
0.0037(0.17)	1.0571**(6.79)	-1.0746**(-5.44)	-0.0591(-0.76)	-0.1118(-0.97)	0.0204(0.06)	0.0497(0.07)	0.3473	

Table V: Comparison of Non-Nested Models Using Mu, Sigma, or Black-Scholes Implied Volatility

This table presents the generalized method of moments regression to compare non-nested model of Black-Scholes volatility (and the square term) with the $\sigma_{t,T}$ (and the square term) or $\mu_{t,T}$ and $\sigma_{t,T}$ (and the square terms) using month end data for different maturity groups for the period of January 1996-April 2006. The left and right side panel provide two different versions of the *J*-test as described in section 3.3.3.2. In the first version (the left side panel) of the *J*-test regression, $(1-\emptyset_1)$ is the coefficient of the fitted value from the Black-Scholes implied standard deviation (and the square term) non-nested equation $(1-\emptyset_2)$ is the coefficient of the fitted value from $\sigma_{t,T}$ and the square term (or $\mu_{t,T}$ and $\sigma_{t,T}$ and the square terms) non-nested equation. In the second version (right side panel) of the *J*-test regression, ψ_1 is the coefficient of the fitted value from $\sigma_{t,T}$ and the square term (or $\mu_{t,T}$ and $\sigma_{t,T}$ and the square terms) non-nested equation; ψ_2 is the coefficient of the fitted value from Black-Scholes implied standard deviation and the square term non-nested equation. Days to maturity groups are formed based on option days-to-maturities. For example, if days to maturity is less than or equal to 90 days then the observation is in '<=90' days-to-maturity group. If days to maturity is greater than 90 days it is in '> 90' days-to-maturity group.Values in parenthesis are *p*-values. Dependent variable is realized volatility of the index for the period of the option.χ^2 is based on the likelihood ratio tests.

		ϕ_1	$H_0: \phi_1=1$	ϕ_2	ψ_1	$H_0: \psi_1=1$	ψ_2
SPX Calls	Days to maturity	Coeff.(*p*-val)	LR χ^2 (*p*-val)	Coeff.(*p*-val)	Coeff.(*p*-val)	LR χ^2 (*p*-val)	Coeff.(*p*-val)
Panel A. using all moneyness							
Sigma, and square term	<= 90 days	0.6797(0.004)	1.87(0.172)	0.1218(0.558)	0.8781(0.000)	2.82(0.093)	0.3202(0.172)
	> 90 days	0.6044(0.002)	4.03(0.044)	0.3594(0.067)	0.6405(0.001)	4.60(0.032)	0.3955(0.045)
	All Maturities	0.7496(0.000)	2.48(0.115)	0.1313(0.366)	0.8686(0.000)	2.43(0.118)	0.2503(0.115)
Sigma, Mu, and square terms	<= 90 days	0.5459(0.036)	3.03(0.081)	0.1429(0.390)	0.8570(0.000)	3.00(0.083)	0.4540(0.082)
	> 90 days	0.3288(0.115)	10.37(0.001)	0.3639(0.043)	0.6360(0.000)	6.00(0.014)	0.6711(0.001)
	All Maturities	0.7364(0.000)	2.64(0.104)	0.1274(0.367)	0.8725(0.000)	2.47(0.116)	0.2635(0.104)

Table V. Continued

SPX Calls	Days to maturity	ϕ_1 Coeff.(*p*-val)	$H_0: \phi_1=1$ LR χ^2 (*p*-val)	ϕ_2 Coeff.(*p*-val)	ψ_1 Coeff.(*p*-val)	$H_0: \psi_1=1$ LR χ^2 (*p*-val)	ψ_2 Coeff.(*p*-val)
Panel B. using moneyness between 0.95 and 1.05							
Sigma, and square term	<= 90 days	0.3576(0.343)	2.90(0.088)	0.5563(0.111)	0.4436(0.203)	2.99(0.084)	0.6423(0.089)
	> 90 days	0.2142(0.194)	22.74(0.000)	0.5528(0.008)	0.4471(0.031)	31.74(0.000)	0.7857(0.000)
	All Maturities	0.4543(0.024)	7.39(0.006)	0.5263(0.006)	0.4736(0.014)	7.88(0.005)	0.5456(0.006)
Sigma, Mu, and square terms	<= 90 days	0.2901(0.500)	2.72(0.099)	0.4320(0.118)	0.5679(0.040)	2.58(0.108)	0.7098(0.100)
	> 90 days	0.0130(0.931)	41.99(0.000)	0.5335(0.006)	0.4664(0.017)	31.16(0.000)	0.9869(0.000)
	All Maturities	0.5318(0.010)	5.09(0.024)	0.3941(0.044)	0.6058(0.002)	5.82(0.016)	0.4681(0.024)
Panel C. using moneyness not between 0.95 and 1.05							
Sigma, and square term	<= 90 days	0.9156(0.000)	0.14(0.703)	0.0478(0.808)	0.9521(0.000)	0.54(0.462)	0.0843(0.704)
	> 90 days	0.5937(0.007)	3.42(0.064)	0.3278(0.184)	0.6721(0.006)	5.67(0.017)	0.4063(0.065)
	All Maturities	0.8518(0.000)	0.80(0.372)	0.1193(0.463)	0.8806(0.000)	1.06(0.303)	0.1481(0.372)
Sigma, Mu, and square terms	<= 90 days	0.9845(0.000)	0.01(0.940)	-0.0360(0.815)	1.0360(0.000)	1.79(0.180)	0.0154(0.940)
	> 90 days	0.4363(0.060)	5.91(0.015)	0.3226(0.160)	0.6773(0.003)	4.26(0.039)	0.5636(0.015)
	All Maturities	0.9444(0.000)	0.14(0.711)	0.0401(0.754)	0.9598(0.000)	0.14(0.709)	0.0555(0.711)

Table VI: Results from Regressing Level of Mu on the Indicated Variables

This table presents results from regression of $\mu_{t,T}$ levels for different days-to-maturity groups using S&P500 index all moneyness call option data. We use generalized method of moments for this estimation. Days to maturity groups are formed based on option days-to-maturities. For example, if days to maturity is less than or equal to 90 days then the observation is in '<=90' days-to-maturity group. If days to maturity is greater than 90 days it is in '> 90' days-to-maturity group. The values in parenthesis are the *t*-statistics. AvgMoneyness is average of the stock price divided by the strike price of options used to compute $\mu_{t,T}$. For S&P500, stock price is the level of the index. AbsRet, LAbsRet, L2AbsRet are the current and first two lagged daily absolute returns of the S&P 500 index. AvgSpread is average of (offer-bid)/call price of all option records used to compute $\mu_{t,T}$. TotalOpnInt is the total option interest of the options used to compute $\mu_{t,T}$. AvgVolume is the average volume, and RecCount is the number of records used to compute $\mu_{t,T}$. Ret, LRet, L2Ret are the current and first two lagged daily returns of the S&P 500 index. ** and * represent the *p*-values of less than 0.01, and between 0.01 and 0.05 respectively.

Days-to-maturity groups	**<= 90 Days**	**> 90 Days**	**All Maturities**
Intercept	-0.3439(-1.95)	-0.0548*(-2.31)	-0.1674**(-5.56)
AvgMoneyness	0.6083**(3.42)	0.1732**(7.49)	0.2641**(8.64)
DaysToMaturity	-0.002**(-8.16)	-1.10E-04**(-9.55)	-1.10E-04**(-9.11)
AbsRet	2.6419**(4.64)	1.2907**(5.76)	1.3217**(3.69)
LAbsRet	2.3448**(4.25)	1.1145**(4.64)	1.1902**(3.58)
L2AbsRet	1.3331**(2.65)	-0.3074(-1.57)	0.122(0.37)
AvgSpread	-0.0264(-0.65)	0.0989*(2.54)	0.1085**(2.73)
TotalOpnInt	-1.14E-07**(-3.91)	-1.04E-07**(-2.71)	-2.10E-07**(-6.34)
AvgVolume	-1.00E-05(-1.74)	1.25E-06(0.47)	3.00E-06(1.02)
RecCount	0.0011(0.94)	1.85E-04(0.25)	0.0061**(7.25)
Ret	-0.1874(-0.46)	-0.0695(-0.4)	0.0063(0.03)
LRet	-0.219(-0.62)	-0.0729(-0.48)	-0.0236(-0.11)
L2Ret	-0.0687(-0.2)	0.0872(0.63)	-0.0751(-0.35)
Adj-R2	0.5142	0.4554	0.5843

Table VII: Input Data Summary Statistics of S&P500 Index Balanced Call and Put Options

This table presents the summary statistics of all moneyness month-end S&P 500 index call and put balanced options having positive trading volume based on the month-end observations for the period of January 1996- April 2006. Balanced options means we take only the options that have both call and put with same strike price. If any call (put) does not have a corresponding put (call) with same strike price we do not take that option. Days-to-maturity groups are formed based on option days-to-maturity. For example, if days to maturity is less than or equal to 90 days then the observation is in '<=90' days-to-maturity group. If days to maturity is greater than 90 days it is in '> 90' days-to-maturity group. Moneyness we define as the stock price divided by the strike price. For S&P500, stock price is the level of the index. Avg. volume is the average of volume of options used for a $\mu_{t,T}$ and $\sigma_{t,T}$ pair estimate. Avg. spread is the average of spread of options used for a $\mu_{t,T}$ and $\sigma_{t,T}$ pair estimate. Spread is defined as (offer - bid)/option price. Option price is the midpoint of bid and offer or the European option price whichever is lower. European option price is computed from Black-Scholes implied volatility in the data. Number of options used is the number of option records that are used to compute a $\mu_{t,T}$ and $\sigma_{t,T}$ pair.

Days-to-maturity groups	**<= 90 Days**	**> 90 Days**	**All Maturities**
Number of observations	4206	2036	6242
Days-to-maturity Mean	56.5615	316.1805	213.7511
Avg. moneyness Mean	1.0102	1.0036	1.0062
Std. Dev.	0.031	0.0685	0.0568
Min	0.9087	0.8142	0.8142
Max	1.1554	1.8598	1.8598
Median	1.008	0.9972	1.0031
Number of options used Mean	16.1769	5.1028	9.4719
Std. Dev.	10.37	3.7946	8.9653
Min	2	2	2
Max	48	30	48
Median	16	4	6
Avg. spread Mean	0.0366	0.0182	0.0254
Std. Dev.	0.0284	0.0159	0.0235
Min	0.003	0.0015	0.0015
Max	0.2632	0.129	0.2632
Median	0.0325	0.0128	0.0181
Avg. volume Mean	688.7162	349.5335	483.3537
Std. Dev.	733.9706	607.5059	680.2962
Min	1.5	1	1
Max	7839	8311	8311
Median	533.35	180	300.9167
Total open interest Mean	152587.6346	51103.0727	91142.5053
Std. Dev.	213038.8693	69700.8886	152535.4084
Min	0	0	0
Max	1660937	511561	1660937
Median	77180	23191	34169

Table VIII: Implied and Realized Summary Statistics Using S&P500 Balanced Call and Put Options

This table presents the implied (using risk-adjusted model) and realized summary statistics using moneyness month-end S&P 500 index call and put balanced options having positive trading volume based on the month-end observations for the period of January 1996- April 2006. Balanced options means we take only the options that have both call and put with same strike price. If any call (put) does not have a corresponding put (call) with same strike price we do not take that option. Days to maturity groups are formed based on option days-to-maturities. For example, if days to maturity is less than or equal to 90 days then the observation is in '<=90' days-to-maturity group. If days to maturity is greater than 90 days it is in '> 90' days-to-maturity group. We use all the options on the same CUSIP, days-to-maturity, and trade date to compute the implied expected return and implied volatility by a grid search method of global optima that minimizes the square of the difference between the observed and computed option prices. Realized volatility is computed based on actual return of the index from trade date to maturity date of the option. Implied standard deviation ($\overline{\sigma}_{t,T}^{BS}$) is the Black-Scholes implied volatility. Results are shown in decimals.

Days-to-maturity groups	**<= 90 Days**	**> 90 Days**	**All Maturities**
Implied expected return $\mu_{t,T}$			
Mean	0.1553	0.0983	0.1208
Std. Dev.	0.1012	0.0331	0.074
Min	-0.2589	-0.0639	-0.2589
Max	0.8189	0.2169	0.8189
Median	0.1584	0.0961	0.1109
Implied volatility $\sigma_{t,T}$			
Mean	0.2129	0.2168	0.2153
Std. Dev.	0.0565	0.0501	0.0527
Min	0.0992	0.1191	0.0992
Max	0.4028	0.4796	0.4796
Median	0.2143	0.2157	0.2153
Implied standard deviation ($\overline{\sigma}_{t,T}^{BS}$)			
Mean	0.2171	0.2054	0.2133
Std. Dev.	0.08	0.0627	0.075
Min	0.0876	0.0948	0.0876
Max	1.3255	1.1215	1.3255
Median	0.2087	0.2005	0.2063
Realized volatility			
Mean	0.1722	0.1694	0.1705
Std. Dev.	0.0679	0.0541	0.0599
Min	0.0632	0.0871	0.0632
Max	0.4119	0.3255	0.4119
Median	0.1601	0.1767	0.169

Figures

Figure I: Term Structures of Mu, Sigma, and Black-Scholes Implied Volatility Using All Moneyness S&P500 Index Call Options.

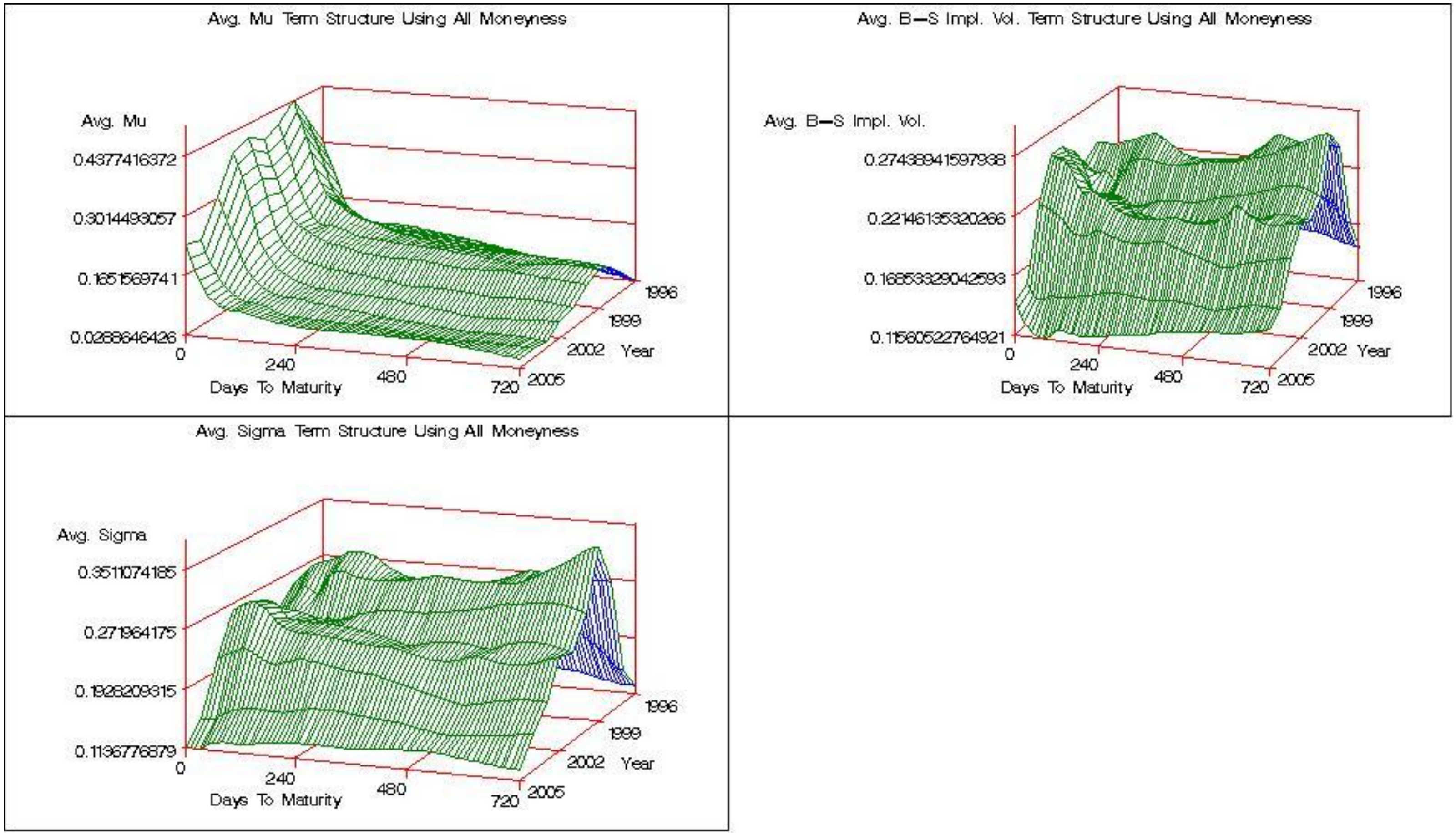


Figure II: 90 Days or Less Predictability of Realized Sigma by Sigma, Mu, and B-S Implied Volatility of S&P500 Index Using All Moneyness Call Options.

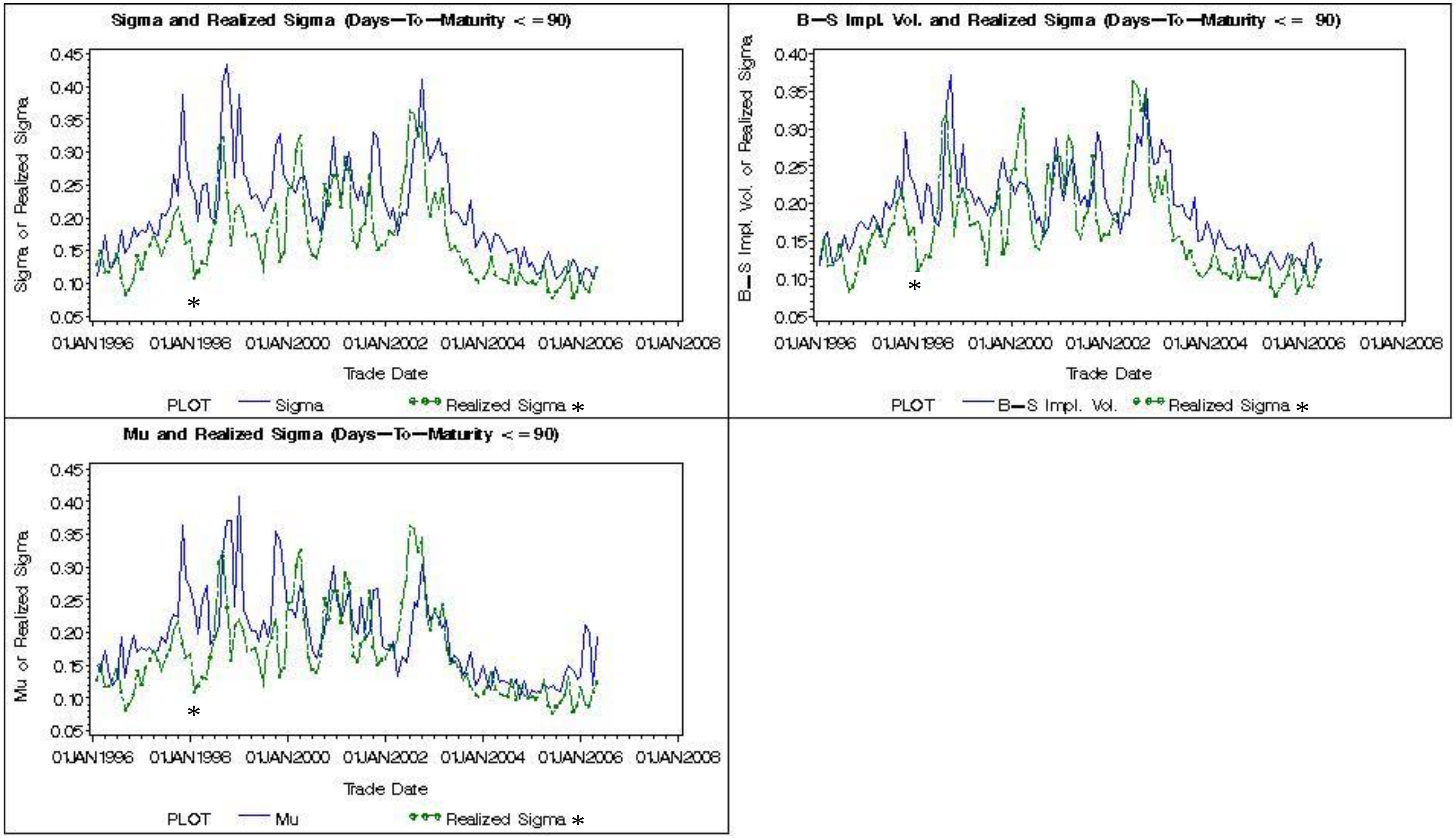


Figure III: More Than 90 Days Predictability of Realized Sigma by Sigma, Mu, and B-S Implied Volatility of S&P500 Index Using All Moneyness Call Options.

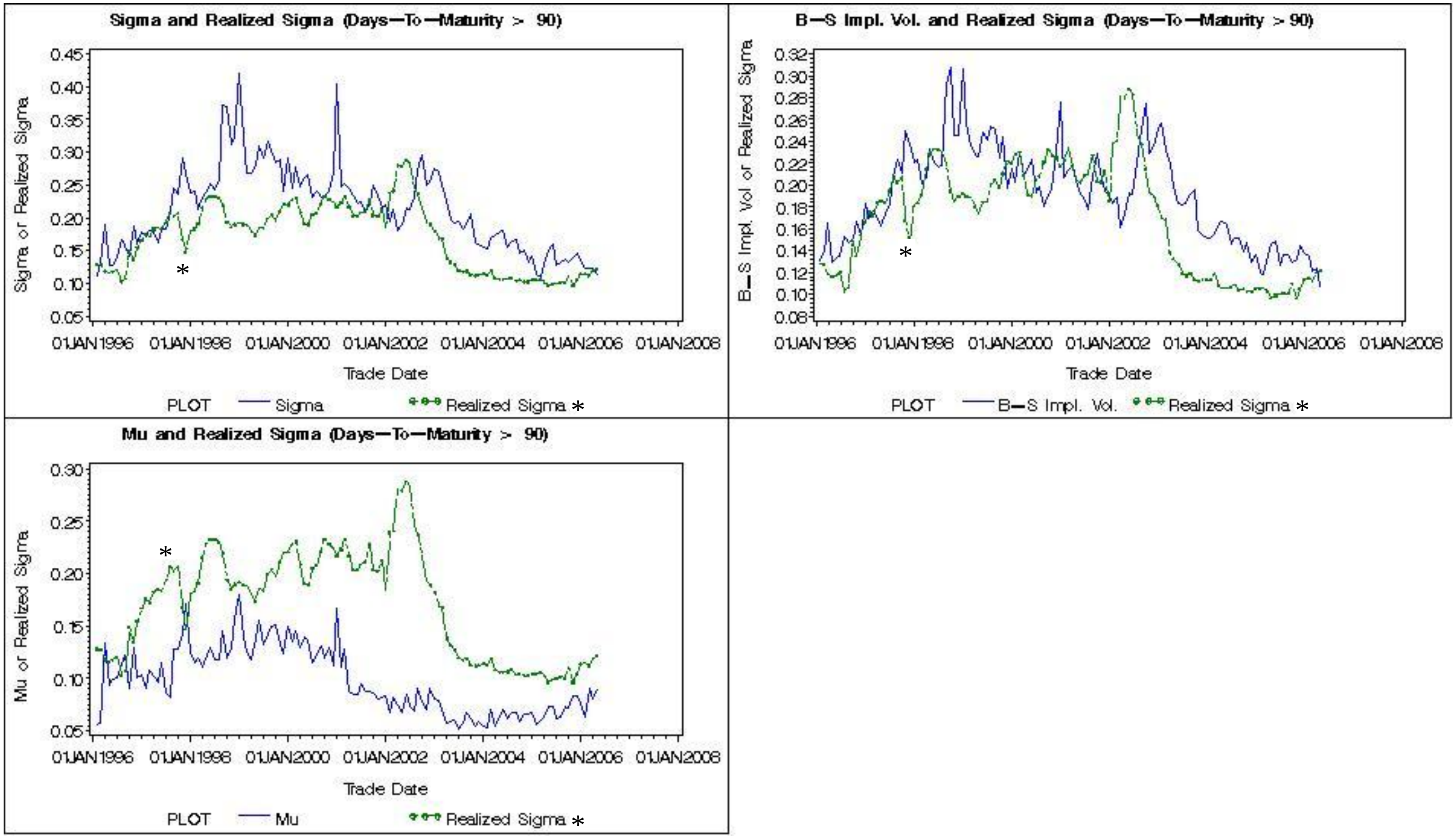
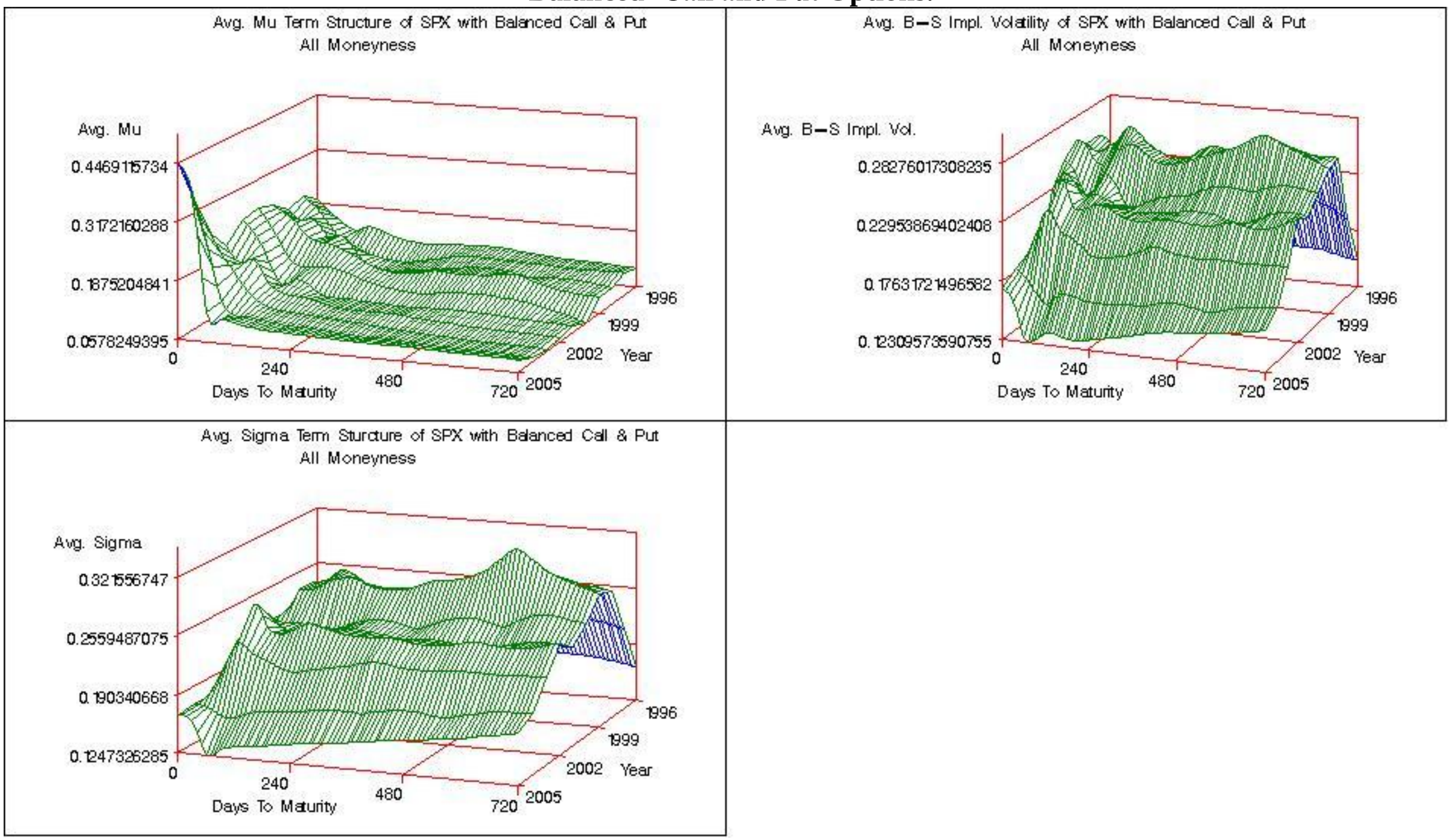


Figure IV: Term Structures of Mu, Sigma, and Black-Scholes Implied Volatility Using All Moneyness S&P500 Index 'Balanced' Call and Put Options.



Chapter-4

Cost of Equity Estimate Using Risk Adjusted Expected Return

Estimation of forward-looking expected stock return is an important part of financial research for at least two reasons. First, it helps determine the cost of capital of an investment of a firm. Second, it is useful for portfolio allocation and balancing. The commonly used methods of estimating the cost of equity (and expected returns) are based on a relationship between one or more factor expected returns and the asset expected return. In particular, the capital asset pricing model (CAPM) of Sharpe (1964), Lintner (1965) and Mossin (1966); the three-factor model of Fama and French (1993); the momentum factor of Jegadeesh and Titman (1993), and the macroeconomic factor model of Chen, Roll, and Ross (1986) provide different fundamental factors that can explain the expected return of assets. The common feature of these models is that they use historical data to estimate the expected returns of the assets into the future. However, as we discussed in chapter 2, use of historical returns may not be a good substitute for ex ante expected returns. Unless return distributions are stationary and precise over time, the cost of equity estimated by these methods may not perform well as the discount rate of the future cash flows of an investment project. Moreover, these models do not explain any relationship between the time horizon of investment and the expected return.

Recent research has used forward-looking options data to estimate the cost of equity. McNulty et al. (2002) uses a heuristic approach to compute the ‘real cost of equity capital' from option prices. Their finding of higher implied expected return in short term and lower implied expected return in long term matches with our finding, however their approach lacks the theoretical support. Another recent paper, which computes the stock implied expected return from option prices, is by Camara, Chung, and Wang (2007). Interestingly, even though they assume a different utility structure they find a downward sloping term structure of expected return similar to our finding.[1] In contrast to the above papers, we follow a discrete time approach that has the advantage of being consistent with single period standard CAPM. Using our approach, the expected return of a stock can be computed without using any information of option of all companies like the Camara, Chung, and Wang (2007), or of the market portfolio such as the market risk premium. This implies we do not have to define what the 'market' consists of, and we do not have to estimate the risk premium of the market, which is required in traditional asset pricing models to estimate the expected return.

This chapter is organized as follows. Section 4.1 provides the descriptive statistics using all stock options, Section 4.2 discusses estimates of cost of equity, and Section 4.3 provides the concluding remarks.

4.1 Descriptive Statistics Using All Stock Options

In the previous chapter we showed the results of expected return using S&P500 index data. In this chapter we show similar results using all stock options and then examine

[1] Our approach is consistent with the standard CAPM and thus consistent with a specific utility structure of hyperbolic absolute risk aversion (HARA) preference family namely the quadratic utility structure. Camara, Chung, and Wang (2007) assume a specific utility structure (see chapter 2).

the cost of equity for different industry groups. We jointly estimate expected stock return and implied volatility using the risk-adjusted model as explained in the previous chapters. Before we analyze the cost of equity for different industry groups we show in Table I the descriptive statistics of the input data used for our estimation of option implied expected stock return (μ) and implied volatility (σ) using all stock options. To analyze the results we group the data into different days-to-maturities. Thus the options whose days to maturity is less than or equal to 30 days are grouped in 30-days-to-maturuty group. The options whose days-to-maturity is greater than 30 days but less then or equal to 60-days are grouped in 60 days-to-maturity groups and so on. As we see from Table I, the number of observations is higher for lower days to maturity than for higher days to maturity. For example, for 30 days-to-maturity, the number of observations is 11565, and for 720 days to maturity, it is 730 observations. Since we take only near-the-money options, the average moneyness mean is around 0.99. In most cases we have around two option records to compute a μ and σ pair. The spread in our data is defined as (offer-bid)/call price. Interestingly even though all our options are near the money we see the average spread is mostly higher for lower days-to-maturity. Since closer to maturity options are most actively traded, it is not surprising to see the average volume to be higher for lower days-to-maturity. Table II shows the descriptive statistics of μ and σ estimated using all stock options. This table and Figure I show the term structure of μ for all stock options which is similar to the results we got using the S&P500 index options. Mcnulty et al. (2001), and Camara Chung and Wang (2008) find similar term structure of expected returns using options data.

4.2 Estimates of Cost of Equity

The option implied expected stock return (μ) can be used as an estimate of cost of equity (COE) of projects. The traditional approach is to use the capital asset pricing model (CAPM) to estimate the COE. The most popular market-based alternative to the CAPM is the Fama and French (1993 and 1996) three-factor model, which is shown to be better than the CAPM in explaining expected returns of stocks. Therefore, in this study we compare the Fama and French approach with our option implied method of estimation of the cost of equity for one-year into the future for different industry portfolios grouped by standard industry classification (SIC) codes. We obtain the industry group to SIC code mapping from Kenneth French website. For this comparison, we take six industry groups namely, Consumer Products and Services, Manufacturing, Information Technology, Healthcare, Utilities and Finance. To have a smooth estimate of cost of equity by the Fama and French model, we take previous three-years of historical data. Using this historical data, first we estimate the loadings of the three factors[2]. We use the historical average of the factors with these loading to estimate the cost of equity. Then by rolling over the sample period month by month, we obtain the time series of cost of equity estimates for the entire period of January 1996-April 2006. The option implied cost of equity that we use to compare with the Fama and French is based on options with 360 days-to-maturity group. We take equally-weighted average to compute the cost of equity for different industry groups, although the comparison properties are similar when we use the value-weighted average. The results

[2] A similar method was used in Fama and French (1997) to estimate the cost of equity.

of the comparison are given in Table III. As we see, the mean of option implied expected return for Consumer Products and Services is about same for our method, and Fama andFrench method. Whereas for Manufacturing, Utilities and Finance our approach produces lower, and for Information Technology and Healthcare our approach produces slightly higher cost of equity than the Fama and French approach. It should be noted that the option implied cost of equity shown in Table III is based on the options in 360 days-to-maturity group. As we see from Table I, the average days-to-maturity for this group is 239 days. An alternative comparison could have the interpolated option implied expected returns for one-year (365 days). Since we know the estimates of option implied cost of equity decrease with days-to-maturity, an interpolation to one-year will provide lower of cost of equity with lower standard deviation, than the values shown in this table.

As we see from Table III, the standard deviations of cost of equity by our option implied method are lower compared to the Fama and French method for all industry groups. For example, the standard deviation of Fama and French cost of equity estimates varies from 5.48% to 19.59% across different industry groups. Whereas the standard deviation of our option implied cost of equity varies from 3.15% to 4.39%. Moreover, the cost of equity estimate of all industries by Fama and French method varies from -44.64% to 53.78%, whereas by our option implied method it varies from 0.89% to 26.99%. More specifically, the standard deviation by option implied method for Consumer Products and Services is 3.54%, whereas it is 10.23% by the Fama and French method. It clearly shows that our option implied estimates are less volatile compared to the Fama and French estimates. Moreover, the Fama and French method may produce negative estimates of cost of equity for some industry groups, or for some time periods, which cannot be used as a discount rate for the projects. For example at some point in time Consumer Products and Services industry cost of equity was -10.07%. Clearly the Fama and French cost of equity estimates are very volatile and in some cases it produces negative cost of equity. This finding is consistent with the Fama and French (1997) argument that CAPM or three-factor model produces imprecise estimate of cost of equity due to the uncertainty about the true factor risk premiums, and the imprecise estimate of the factor loadings. Figure II depicts the time series process of cost of equity by our option implied, and by the Fama and French method. This supports the previous observation that cost of equity estimate by our option implied method is stable over time, compared to the Fama and French method.

4.3 Conclusion

In this chapter we extended our results to all stock options. The finding of term structure of μ using all stock options is similar to the results using S&P500 index options. The μ estimated from our risk-adjusted model can be used to estimate the cost of equity for different industry groups. The cost of equity estimate by our approach has at least two advantages. First, our approach uses observed option and stock prices to extract expected returns, whereas the traditional models such as the CAPM and the Fama and French model need the unobservable market risk premium. Second, unlike the CAPM or the Fama and French, our approach does not use historical information to compute the forward-looking expected return. There are two empirical findings in this chapter.

First, the option implied expected returns are more stable over time than the Fama and French estimates. In fact the Fama and French cost of equity estimates in some case become negative, which is not the case using our model. Second, our result shows even using all stock options the downward sloping term structure of μ is maintained.

Tables
Table I: Input Data Summary Statistics of All Options

The sample consists of all month-end near-the-money U.S. exchange traded call options for the period of January 1996- April 2006. Days to maturity groups are formed based on option days to maturity. For example, if days to maturity is less than or equal to 30 days then the observation is in 30 days to maturity group. If days to maturity is greater than 30 but less than or equal to 60 then the observation is in 60 days to maturity group and so on. Moneyness we define as the stock price divided by the strike price. Volume is the call option volume. Spread is defined as (offer - bid)/call price. Call price is the mid point of bid and offer or the European option price whichever is lower. European option price is computed from Black-Scholes implied volatility in the data. Number of calls used is the number of option records that have same days to maturity on the same CUSIP with different strike prices on the same trade date.

Days-to-maturity groups	**30**	**60**	**90**	**120**	**180**	**360**	**540**	**720**
Number of observations	11565	8881	3240	2763	4373	3371	642	730
Days-to-maturity Mean	19.48249	49.99257	80.2608	110.8064	155.5751	239.0389	449.9486	708.0863
Avg. moneyness Mean	0.998656	0.997899	0.997736	0.998565	0.997883	0.998086	0.997091	0.998257
Std. Dev.	0.010261	0.010452	0.010799	0.010609	0.0107	0.01087	0.010648	0.010765
Min	0.960968	0.954817	0.958715	0.959115	0.959948	0.962419	0.965038	0.962616
Max	1.038239	1.036293	1.034318	1.038687	1.034853	1.044099	1.030084	1.032731
Median	0.998601	0.997899	0.997665	0.998693	0.997958	0.99825	0.997163	0.998202
Number of calls used Mean	2.364462	2.312577	2.357099	2.211726	2.1866	2.179769	2.160436	2.156164
Std. Dev.	1.638055	1.274061	1.154322	0.754696	0.662989	0.599965	0.570293	0.465839
Min	2	2	2	2	2	2	2	2
Max	24	21	14	13	11	10	7	6
Median	2	2	2	2	2	2	2	2
Avg. spread Mean	0.135416	0.080799	0.068048	0.059072	0.052442	0.044398	0.03364	0.033451
Std. Dev.	0.129671	0.067326	0.060767	0.046554	0.047561	0.030124	0.031041	0.024066
Min	-0.46121	-0.36306	-0.20916	-0.15284	-1.33368	-0.10263	-0.52822	-0.03165
Max	1.625	1.227679	1.714286	1.153846	1.645161	0.62079	0.209304	0.253661
Median	0.102403	0.068027	0.058277	0.052668	0.04739	0.04028	0.032072	0.029731
Avg. volume Mean	547.0359	266.7459	206.621	157.6575	131.0213	139.379	121.7841	104.5247
Std. Dev.	1786.683	814.9684	519.0864	433.2877	459.6463	708.1293	369.9922	453.1043
Min	1	1	1	1	1	1	1	1
Max	53213	25245	10729.25	8100	13579.5	27849	6054	10015.33
Median	122	61.66667	48.5	38.5	31.33333	27	30.5	21
Total open interest Mean	12396.11	7452.418	11179.92	9480.218	7398.776	8462.374	16587.81	8422.127
Std. Dev.	42459	27743.11	29327.43	24452.97	20893.22	25813.37	37569.38	24622.19
Min	0	0	0	0	0	0	0	0
Max	984687	1003966	541457	499967	514526	445689	321969	293838
Median	3161	1344	3406.5	2698	2053	1314	4715.5	1667

Table II: Implied and Realized Summary Statistics Using All Options

The sample consists of all month-end near-the-money U.S. exchange traded call options for the period of January 1996- April 2006. Days-to-maturity groups are formed based on option days-to-maturity. For example, if days-to-maturity is less than or equal to 30 days then the observation is in 30 days-to-maturity group. If days-to-maturity is greater than 30 but less than or equal to 60 then the observation is in 60 days-to-maturity group and so on. We use all the call options on the same CUSIP, days-to-maturity, and trade date to compute the implied stock expected return (μ) and implied volatility (σ) by a grid search method that minimizes the square of difference between the observed and computed option price. The grid search for μ, is in the interval of 0.00%-200.00%, and for σ it is in the interval of 0.00%-100.00%. Realized volatility is computed based on actual return of the stock from trade date to maturity date of the option. Results are shown in decimals.

Days-to-maturity groups	**30**	**60**	**90**	**120**	**180**	**360**	**540**	**720**
Implied expected return Mean	0.430065	0.274133	0.21481	0.187903	0.159334	0.129949	0.107797	0.092186
Std. Dev.	0.230455	0.131472	0.099885	0.085866	0.075132	0.05942	0.0477	0.040843
Min	0	0	0	0	0	0	0	0
Max	2	0.820043	0.582205	0.483588	1.504852	0.374305	0.240838	0.219323
Median	0.3818	0.255299	0.201111	0.180111	0.15197	0.125633	0.107531	0.09107
Implied volatility Mean	0.443168	0.432945	0.411373	0.416079	0.413623	0.390927	0.393844	0.41534
Std. Dev.	0.236102	0.225957	0.216578	0.214337	0.216488	0.206093	0.189718	0.199014
Min	0.04648	0.031193	0.047858	0.032757	0.037692	0.038889	0.058111	0.081111
Max	1	1	1	1	1	1	1	1
Median	0.371658	0.372446	0.35557	0.36157	0.35987	0.342989	0.361167	0.38752
Realized volatility Mean	0.410876	0.402699	0.384182	0.397161	0.402098	0.383391	0.400312	0.434903
Std. Dev.	0.329419	0.299004	0.286341	0.285333	0.286419	0.269453	0.268378	0.28585
Min	0.020839	0.031882	0.025105	0.025999	0.012713	0.016992	0.077382	0.03674
Max	3.957029	3.281549	2.15659	2.071244	2.547094	2.522718	2.414214	2.90283
Median	0.309298	0.310205	0.301039	0.311524	0.318625	0.310317	0.337483	0.374236

Table III: Summary Statistics of Cost of Equity Estimates by Different Methods

This table presents the equally-weighted cost of equity by our option implied method and the Fama and French method for various standard industry classification (SIC) code industry groups based on option data for the period of January 1996- April 2006. By the Fama and French method, first we estimate factor loadings based on previous three-year of historical data and then we use average of historical factor values along with these factor loadings to get the cost of equity estimate for each point in time. Option implied expected returns for different industries are estimated using the options in 360 days-to-maturity group. Results are shown in decimals.

Panel A: Option implied expected return

Industry Group	Consumer Products and Services	Manufacturing	Information Technology	Healthcare	Utilities	Finance
Mean	0.1312	0.1179	0.1602	0.1387	0.0776	0.1283
Median	0.1263	0.1151	0.1590	0.1367	0.0644	0.1159
Std. Dev.	0.0354	0.0315	0.0344	0.0413	0.0367	0.0439
Minimum	0.0653	0.0489	0.0760	0.0097	0.0241	0.0089
Maximum	0.2224	0.2357	0.2699	0.2616	0.1752	0.2663

Panel B: Fama and French expected return

Industry Group	Consumer Products and Services	Manufacturing	Information Technology	Healthcare	Utilities	Finance
Mean	0.1322	0.1444	0.1002	0.1035	0.1416	0.1538
Median	0.1273	0.1442	0.1599	0.0901	0.1547	0.1683
Std. Dev.	0.1023	0.1062	0.1959	0.1043	0.0548	0.1484
Minimum	-0.1007	-0.0702	-0.4464	-0.1010	0.0206	-0.1219
Maximum	0.3805	0.3939	0.5378	0.3663	0.2110	0.4119

Figures

Figure I: Term Structures of Equally-Weighted Average of Mu, Sigma, and B-S Implied Volatility of All Stocks.

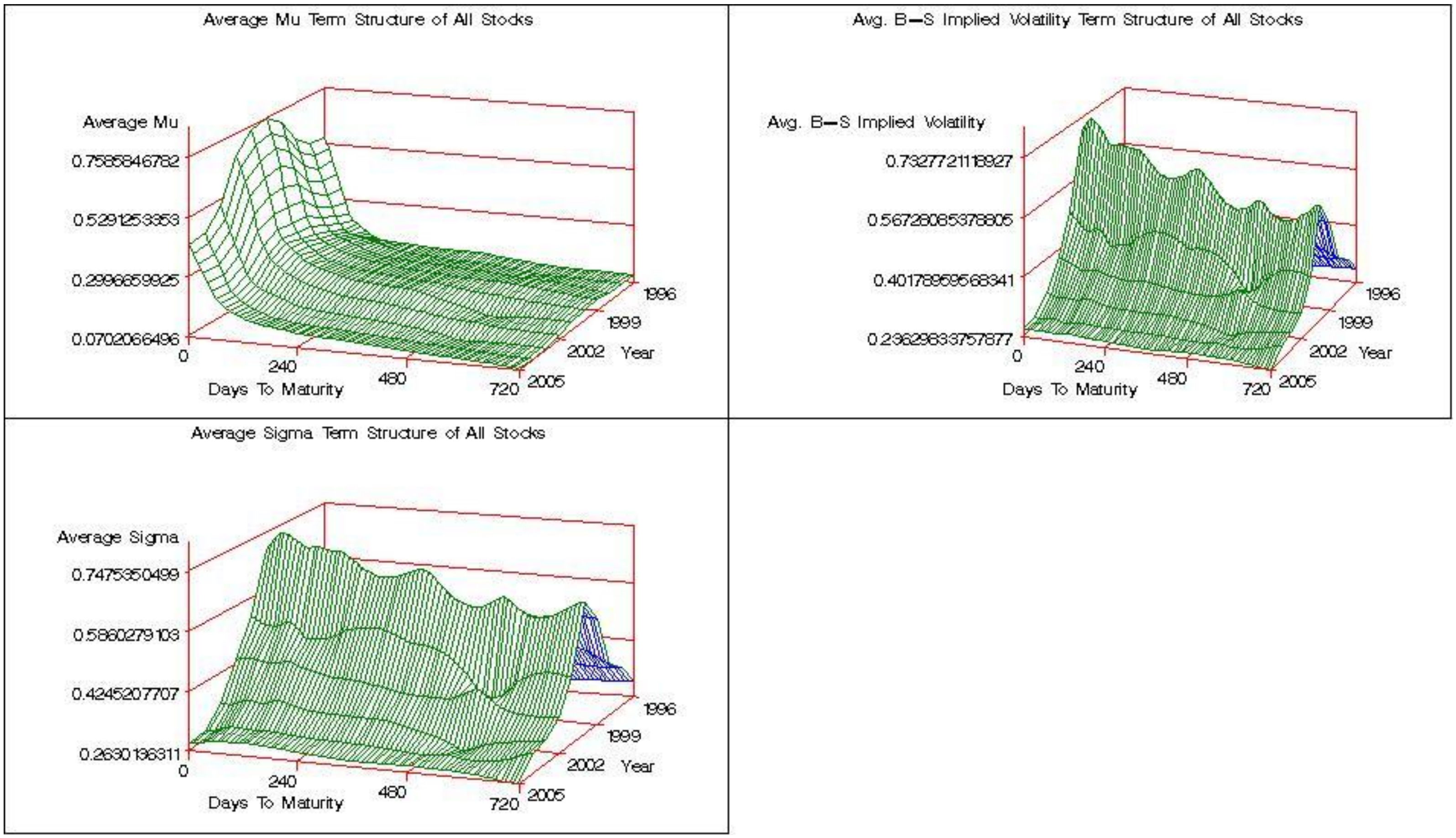
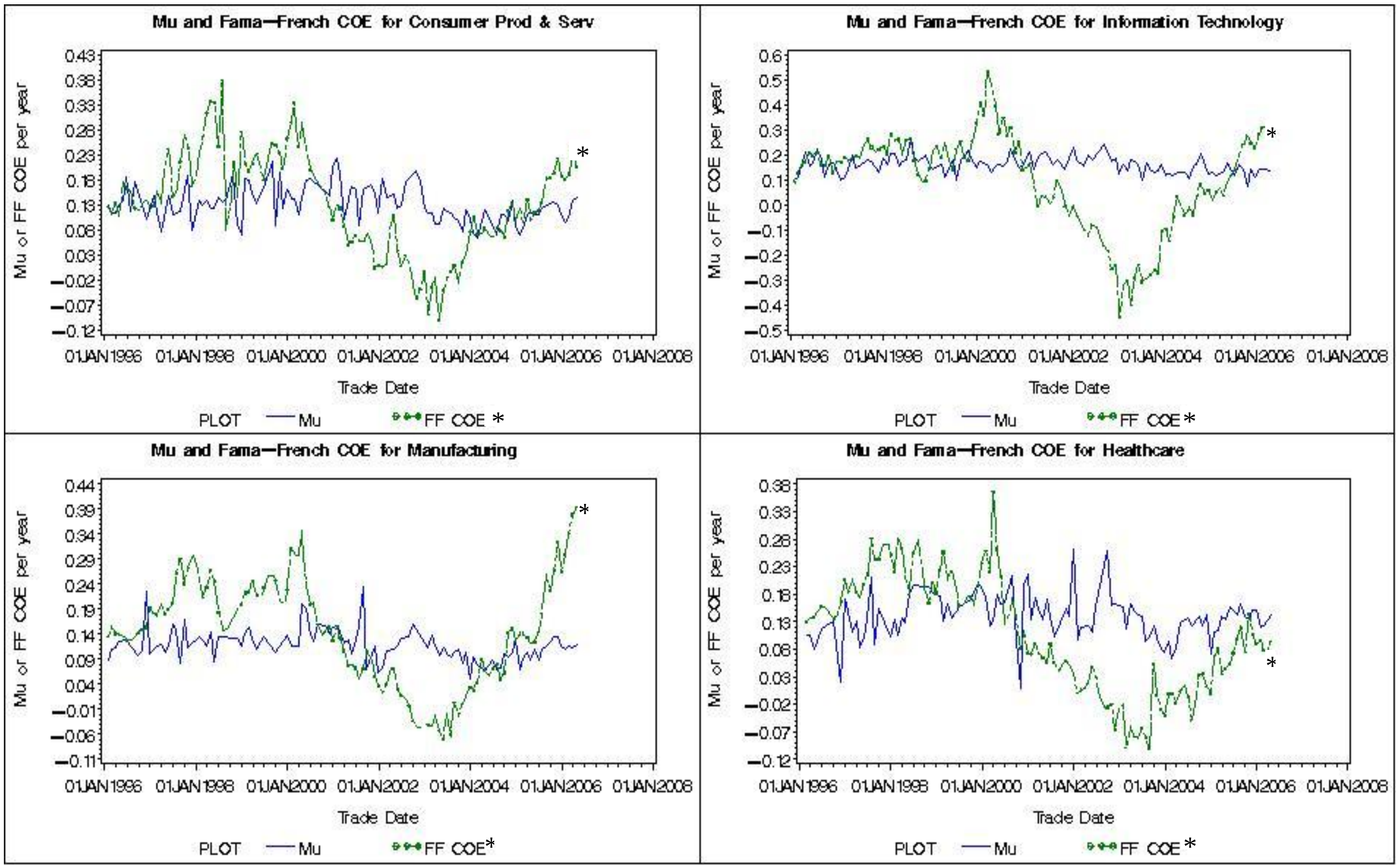


Figure II: Time Series Cost of Equity for Different Industry Groups



Chapter-5

On the Ex-Ante Cross-Sectional Relation Between Risk and Return Using Option-Implied Information[1]

One of the most fundamental issues in finance is what is the appropriate amount of return expected (or required) by investors when they bear risk. The first and most prominent model among others to address this issue is the Capital Asset Pricing Model (CAPM) by Sharpe (1964), Lintner (1965), and Black (1972). This model posits a linearly positive relationship between systematic risk (or market beta) and expected return on a risky asset. Indeed, the CAPM applies to all areas: computation of the cost of capital, measurement of investment performance, determination of fair returns for regulated industry, etc. Numerous investment institutions, such as Value Line, Standard & Poor's, and Merrill Lynch, use beta as the appropriate risk index and report beta to their customers. Due to the importance of the model, many researchers have been testing its validity since it was introduced. Empirical testing of the validity of the CAPM is the most heavily investigated area in finance.[2]

---

[1] This chapter is based on a joint paper with my dissertation committee members Dr. Ren-Raw Chen (advisor), and Dr. Dongcheol Kim.

[2] There are many obstacles to the test of traditional CAPM. First, the model needs the ex ante expected return and beta. Second, there should be time horizon matching of this information for the CAPM test; alternatively researchers assume that the beta and risk premium are stationary. Finally, Roll's critique (1977) applies for this test. As Roll points out "the only legitimate test of the CAPM is whether or not the market portfolio is mean-variance efficient" and "If performance is measured relative to an index that is ex post efficient, then from the mathematics of the efficient set no security will have abnormal performance when measured as a departure from the security market line." This means that the efficiency of the market portfolio and the validity of the CAPM are joint hypothesis that are almost impossible to test because of the difficulty of measuring the true market portfolio.

Contrary to the prediction of the CAPM, however, most empirical results have found that idiosyncratic risk factors have significant explanatory power for stock returns, while market beta has little power. For example, Fama and French (1992) reports that firm size and book-to-market explain well the cross-section of average stock returns, while market beta has no explanatory power. This challenges the validity of the CAPM, one of the most important models in finance. In this chapter we examine the test of the CAPM using the ex ante expected return that we extract from risk-adjusted option prices.

This chapter is organized as follows. Section 5.1 discusses various sources of ex ante information and the findings, Section 5.2 describes the risk-adjusted option pricing model for implied return and volatility, Section 5.3 describes the data, and Section 5.4 explains the computational details for the implied variables. Section 5.5 presents empirical results, and Section 5.6 sets forth our conclusion.

5.1 Sources of Ex-Ante Expected Return

The CAPM determines the equilibrium risk–return relationship on an *ex ante* basis. Thus, empirical test of the CAPM should be performed on an ex ante basis. It is difficult, however, to empirically test the CAPM on an ex ante basis, since the future expected return and beta are unavailable at the beginning of the investment period. Because of this empirical difficulty, most previous tests have been done on an ex post historical basis, implicitly assuming that historical realized average returns are good estimates of future expected returns. However, there is ample evidence that average realized return does not converge to expected return in finite samples. One of the features, which work against the convergence of average realized return to expected return, is the time-variation of expected returns and market risk premium (i.e., nonstationarity). Unless return distributions are stable and precise over time, the expected returns estimated by these methods may not perform well as a true representation of ex ante market expectations.[3] In his presidential address, Elton (1999) notes that "there are periods longer than 10 years during which stock market realized returns are on average less than the risk-free rate (1973 to 1984). There are periods longer than 50 years in which risky long-term bonds on average underperform the risk-free rate (1927 to 1981)." In these circumstances, the use of realized returns for expected returns and market betas could lead to biased estimation and to rejection of the CAPM. Despite the problems caused by the use of realized returns, most results in the empirical asset pricing literature are obtained from such returns.

Elton (1999) also notes that "developing better measures of expected return and alternative ways of testing asset pricing theories that do not require realized returns have a much higher payoff than any additional development of statistical tests that continue to rely on realized returns as a proxy for expected returns." In this vein, several studies construct alterative proxies for expected returns. Gebhardt, Lee, and Swaminathan (2001), Fama and French (2002), Botosan and Plumlee (2005), and Easton and Monahan (2005) use valuation models to estimate expected returns. Brav, Lehavy, and Michaely (2005) construct estimates of expected returns using financial

[3] Fama and French (1997) and Pastor and Stambaugh (1999) find that both the CAPM and the Fama and French three-factor model are imprecise owing to the uncertainty about true factor risk premiums and imprecise estimates of the factor loadings that are based on historical returns.

analysts' target prices from Value Line, and Campello, Chen, and Zhang (2008) use corporate bond yields to estimate expected equity returns.[4] In particular, Brav, Lehavy, and Michaely (2005) and Campello, Chen, and Zhang (2008) conduct cross-sectional tests for the relation between market beta and expected return by using their own measures of expected returns, and find that market beta is significantly priced.

However, the measures of expected returns used in the previous studies have several problems. The most frequently used approach to obtain estimates of expected returns is to use valuation models and calculate internal rates of return for the estimates. Most valuation models use unrealistic assumptions for the future evolution of accounting variables, such as constant dividend growth. Furthermore, most models use indirect measures for expected stock returns. For example, the Brav, Lehavy, and Michaely (2005) approach of using analyst target prices from Value Line adopts similar assumptions. Another popular measure of investors' expected return is bond yields, which are used in Campello, Chen, and Zhang (2008). Bond yields are forward-looking expected returns over the life of the bonds, under the conditions that the bonds do not default, the yields do not change in the next periods, and coupon payments are reinvested at the same rate as the yield until maturity. However, although bond yields reflect the expected risk premium for default risk, which is the financial side of systematic risk, bond yields may not reflect the expected risk premium caused by an uncertain business environment, which is the business side of systematic risk. It would be difficult to say, therefore, that bond yields fully reflect the expected risk premium of

[4] Levy (1997) conducts a classroom experiment to estimate ex ante parameters.

all systematic risks of a firm. Another problem inherent in using bond yields as a proxy for ex ante expected return is that many firms' bond trade prices are unavailable.

To overcome the shortcomings of the above-mentioned measures, we use option prices to extract information regarding ex ante expected returns and market beta of the underlying asset. Since option prices reflect investor expectations for future stock price movements, option data are an excellent information source for ex ante parameters. Option data have many advantages over other information sources for expected returns used in the previous studies. Option data are observed market prices, are not obtained from any specified model, and expected returns implied from option prices might reflect investor expectations for all systematic risk of the underlying asset. We extract implied mean return and implied volatility of the underlying asset from forward-looking option prices using a risk-adjusted approach. We use this implied mean return as a proxy for ex ante expected return.

5.1.1 The Risk-Adjusted Approach

The approach we follow is a risk-adjusted option pricing model that prices an option in discrete time and that retains the expected return of the underlying asset in the pricing equation. The Black-Scholes (1973) risk-neutral model prices options by taking advantage of the interesting feature that a particular portfolio of the stock and the option can cancel out the unknowns—namely the expected mean returns of the option and its underlying stock in continuous time.[5] However, if our objective is to extract expected return given the market price of options, we should form the corresponding risk-adjusted valuation model that will retain the expected returns in the pricing model.

Option pricing models that embed mean stock returns are not new. The early option pricing models of Sprenkle (1961), Ayres (1963), and Boness (1964) have implicitly or explicitly assumed some form of risk-adjusted framework such that investors who employ a buy and hold strategy could be linked to expected stock returns. However, none of these models provides an adequate theoretical structure that relates option returns and stock returns, hence they lack the ability to extract stock returns from option prices. Our risk-adjusted model, however, provides the pricing equations necessary to jointly estimate the expected returns of both the stock and the option.

The main purpose of this study is to examine the CAPM relation on an ex ante basis. While more complex versions of the CAPM may include quite a number of parameters, in the standard CAPM that we study in this paper, two ex ante parameters are needed in this test: expected return and market beta. In order to obtain these two parameters on an ex ante basis, we must derive a risk-adjusted option pricing model that contains these two parameters and at the same time the model is consistent with the standard CAPM.

---

[5] Black and Scholes (1973) show that if the market is complete, the expected return of the stock should disappear from the valuation of the option as dynamic hedging (known as continuous rebalancing, price by no arbitrage, or risk neutral pricing) effectively removes the dependence of the option price on the stock return. This is true, however, only if the market is truly complete in reality. In other words, if the reality is exactly described by the Black-Scholes model, it is impossible to theoretically solve for both expected return and volatility. However, it has been empirically shown that the Black-Scholes model cannot explain all option prices (known as the volatility smile and volatility term structure).

Our model to obtain the expected return and market beta follows a two-step process. First, at the end of each month (i.e., at the last trading day of each month), we observe the prices of a stock option with a particular maturity and compute implied returns of the underlying stock from the observed option prices based on the risk-adjusted option model. We regard these option-implied returns (or simply, implied returns) as ex ante expected returns. At the same time, we also observe the prices of a market index option such as Standard & Poor's 500 index option whose maturity is matched with that of the stock option. Then we compute implied market returns from the observed market index options based on the risk-adjusted option model. Thus, each implied return of a stock has its counterpart implied market return.

Second, there is no explicit way to directly extract expected market betas. The literature is limited in the area of extraction of implied betas from option prices. To our knowledge, there are only two papers in this area. Siegel (1995) proposes a new "exchange option," the price of which is based on the number of units of a specific stock that can be exchanged for one unit of an index. Thus, he argues that the price of this exchange option can reveal the implied beta of the stock. However, such exchange options do not exist in current capital markets. More recently, Christoffersen, Jacob, and Vainberg (2006) show that implied beta can be extracted from option prices without using this new derivative. The beta in their model is computed using forward-looking variances and the skewnesses of the stock and the market. However, the main limitation in their approach is the internal conflict between the assumption of the CAPM where returns of the stocks follow a multivariate normal distribution, and the existence of skewness in stock returns. Furthermore, their approach does not generate the unique implied beta in that an implied beta can be obtained by using kurtosis (or any moment), which can differ from the one obtained by using skewness. Because of these problems, we simply estimate expected market betas by regressing option-implied returns of the underlying stock on option-implied returns of the market index, the Standard & Poor's 500 Index.

Option-implied monthly returns for a total of 4,078 stocks are obtained over the period January 1996 through April 2006. One feature of our implied returns is that it portrays how investor expectations differ for different investment horizons. We find that there is apparently a downward sloping term structure of implied returns. That is, the longer the investment horizon, the smaller the expected return. The term structures of implied volatility and implied market beta are also downward sloping.[6]

In month-by-month, cross-sectional regressions of ex ante implied returns on implied market betas, which is an ex ante version of the CAPM test, we find that there is a significantly positive relation between these two ex ante variables. Even though firm characteristics such as firm size, book-to-market, and momentum are included in the model, this positively significant relation is strongly maintained. We also examine whether implied market betas have explanatory power for ex post realized stock returns and find that implied market betas are significantly priced. Since there is apparently a non-constant term structure of expected returns, we repeat the cross-sectional asset pricing tests for each maturity group. In all maturity groups, we find results similar to those obtained from using the whole sample.

[6] The downward sloping term structure of volatilities is well documented in the literature. See Hull (2002).

Since we have implied returns with various investment horizons at a given time, it is possible to compute forward-implied returns and betas and to examine cross-sectional relations between these two forward variables. We find that forward-implied returns also have a positive and significant relation with forward market betas.

Another way to test whether our CSR estimate of ex ante market risk premium has economic significance is to examine whether the ex ante market risk premia estimate contains forward-looking information on macroeconomic conditions. We find that the ex ante market risk premium has a significant positive relation with the future default premium. And, it has a significant negative relation with future dividend yield and a generally negative relation with the future growth of real economic activity as measured by consumption, GDP, and labor income. These results indicate that as more cash flows (from more dividends and expanding real economic activity) are expected in the future, the stock price level increases and then the subsequent ex ante expected return is lowered. In sum, the ex ante market risk premium contains significant forward-looking information on future macroeconomic conditions. When the implied market returns (from S&P 500 Index options) are used instead of the ex ante market risk premium estimate, we obtain stronger but similar results. However, when the CRSP value-weighted market returns are used in the regression, we find that the realized market returns have no significant forward-looking information on future macroeconomic conditions.

5.2 A Model for the Forward-Looking Implied Return and Volatility

As mentioned in the previous section, in order to test the CAPM on a true ex ante basis, we need an option pricing model that contains expected return and market beta and at the same time is consistent with the CAPM to be tested. While the seminal Black-Scholes model is consistent with the standard CAPM [see page 645 in Black-Scholes (1973)], it is well-known that the Black-Scholes model contains only the volatility of the stock.[7] As a result, we must derive an option pricing model that contains the parameters desired and is also consistent with the Black-Scholes model and the standard CAPM.

Black and Scholes (1973) first showed that if continuous rebalancing is possible then the expected return will be replaced by the risk-free rate as continuous rebalancing effectively removes any risk in option prices (known as no arbitrage trading). Furthermore, they demonstrated that their model is consistent with the CAPM over the infinitesimal time period. In this paper, we derive the option pricing model under the standard CAPM where there is no rebalancing before maturity and the return period is not infinitesimal. Note that if either assumption holds (i.e., continuous rebalancing is permitted, or the return period is infinitesimal), our option pricing model reduces to the Black-Scholes model. We shall note that our option pricing model is consistent with the standard CAPM only. If one tests other versions of the CAPM, then different option pricing models must be derived. For example, if one tests the CAPM under random volatility, then one must extend the Heston model (1993) to derive an option pricing model with the expected return.

[7] Volatility is the only unknown in the Black-Scholes equation.

We derived risk-adjusted pricing equations in chapter 3. For ease of exposition we will restate the propositions in this chapter. However, for detail derivations, readers are requested to look in appendix of chapter 3. We show several propositions required to compute the implied return and implied volatility. Our objective is not to price options as the Black-Scholes model does, but to have a closed-form solution in which the expected risk-adjusted return is retained. Using this framework, we jointly estimate implied return and implied volatility through the market prices of options. Proposition 1, below, describes how the implied mean return and volatility can be simultaneously estimated from option prices.

Proposition 1:

Assume stock price S follows a geometric Brownian motion with an expected instantaneous return of μ_s and volatility of σ_s. Let a call option on the stock at any point in time t be given by $C(S,t)$ that matures at time T. Let μ_c be the expected instantaneous return on this option. Then for a small interval of time, Δt, the relationship between the expected returns on the underlying stock and the option, μ_s and μ_c, can be given by:[8]

(1) $$\mu_c = r_f + \beta_{cs}(\mu_s - r_f)$$

where

(2) $$\beta_{cs} = \frac{\text{Cov}\,(r_c, r_s)}{\text{Var}\,(r_s)},$$

[8] The notations used for the variables are optimized for presenting the story in this chapter independent of other chapters. However, the notations could be different from other chapters. Therefore we explain each notation as and when they are introduced.

and $r_s = \Delta S/S$ and $r_c = \Delta C/C$ are two random variables representing the stock return and call option return, respectively, over the period Δt. And, r_f is the instantaneous risk-free rate of return for the period Δt. Note that Proposition 1 can be proved without assuming the CAPM. Also, note that all returns and volatility are annualized, otherwise mentioned.

Proof: See chapter 3 appendix 3.A.1.1.[9]

If the CAPM holds, then the expected returns on the underlying stock and call option are expressed, respectively, as:

(3) $\mu_s = r_f + \beta_s(\mu_m - r_f)$ and

$\mu_c = r_f + \beta_c(\mu_m - r_f)$,

where μ_m is the instantaneous expected return on the market portfolio, and β_s and β_c are the market betas of the underlying stock and the call option, respectively, which are defined as:

$\beta_s = \frac{\text{Cov}(r_s, r_m)}{\text{Var}(r_m)}$ and

$\beta_c = \frac{\text{Cov}(r_c, r_m)}{\text{Var}(r_m)}$.

Thus, it can be seen that

(2a) $\beta_{cs} = \frac{\beta_c}{\beta_s}$.

[9] Chapter 3 Appendix 3.A.2.1 provides a similar derivation for put options.

Equation (1) holds for a small interval of time Δt. We assume the distributions of stock and option returns, r_s and r_c, are both Gaussian and stationary over the life of the option. This implies that β_{cs} is constant over this period. Since our approach is to price the option in a discrete setting, we approximate β_{cs} over the discrete time from *t* to *T* as:

$$\text{(2b)} \qquad \beta^*_{cs} = \frac{\text{Cov}\,(C_T/C_t, \quad S_T/S_t)}{\text{Var}\,(S_T/S_t)}$$

$$= \left(\frac{S_t}{C_t}\right) \frac{\text{Cov}\,(C_T, S_T)}{\text{Var}\,(S_T)}\,.$$

The linear relation between μ_s and μ_c in discrete time is the same as in continuous time when r_s and r_c are stationary over the life of the option. Since we use the risk-adjusted model for pricing the option where the expectation of the pricing kernel is based on the entire life of the option, β^*_{cs} as given in equation (2b) is more appropriate for our equations.

Equation (1) in continuous time and equation (2b) in discrete time can also be proved using the CAPM. For these two equations to hold, however, it is not necessary that the CAPM should hold. The assumptions of the CAPM are much stronger, so that all return distributions are stationary. However, here we need only the stationarity and Gaussian distribution assumption of the stock and option returns to obtain these two equations. Hence, the stationarity assumption of r_s and r_c is weaker than what is needed for the CAPM. Furthermore, Galai (1978) demonstrates many similarities between the continuous time and discrete time properties of r_c that support our stationarity assumption for the return distribution.[10] We also note that the right hand side of equation (2b) is a close approximation of β_{cs} under the stationarity of r_s and r_c .

Proposition 2:

Under the physical measure, the risk-adjusted price of the call option over the discrete time period from *t* to *T* is given by:

$$(4) \qquad C_t = e^{(\mu_s - r_f)(1-\beta_{cs})(T-t)} S_t N(h_1) - e^{-\mu_c (T-t)} K\, N(h_2),$$

where *K* is the strike price of the option, *N*(·) is the standard normal probability density function, and

$$\mu_c = r_f + \beta_{cs}(\mu_s - r_f)$$

$$h_1 = \frac{\ln S_t - \ln K + (\mu_s + \sigma_s^2/2)(T-t)}{\sigma_s \sqrt{T-t}}$$

$$h_2 = h_1 - \sigma_s \sqrt{T-t}\,.$$

Proof: See chapter 3 section 2.

Equation (4) is obtained based on the assumption that the expected return of the option, μ_c, the expected return of the stock, μ_s, and the volatility of stock price, σ_s, are constants. We approximate β_{cs} by β_{cs}^*, based on the discrete time period of the option

---

[10] Note that our assumption of stationarity of r_s and r_c is applicable only to the options with the same days-to-maturity. This means that the distributional properties of r_s and r_c are allowed to differ for different days-to-maturity.

from t to T as explained above. Furthermore, we assume that the stock price follows a geometric Brownian motion.

Proposition 3:

The ratio of the market beta of the stock to the option, β^*_{cs}, over the life of the option can be written as:

$$(5) \qquad \beta^*_{cs} = \frac{S_t\left[e^{\sigma_s^2(T-t)}\,N(h_3) - \left(\frac{K}{S_t}\right)e^{-\mu_s(T-t)}\{N(h_1) - N(h_2)\} - N(h_1)\right]}{C_t\left[e^{\sigma_s^2(T-t)} - 1\right]},$$

where

$$h_3 = \frac{\ln S_t - \ln K + \left(\mu_s + \frac{3}{2}\sigma_s^2\right)(T-t)}{\sigma_s\sqrt{T-t}}.$$

Proof: See chapter 3 appendix 3.A.1.2.[11]

Substituting equations (1), (2b), and (5) into (4), we arrive at an option pricing model as a function of the known variables S_t (current stock price), C_t (current call option price), K (strike price), r_f (risk-free interest rate), and T-t (time to maturity), along with two unknown variables, μ_s and σ_s. If we observe two or more call option prices with the same days-to-maturity but different strike prices, we can simultaneously solve the option pricing model for μ_s and σ_s for each individual stock and days-to-

[11] Appendix 3.A.2.2 provides the corresponding derivation for put options.

maturity.[12] Through this approach, for each stock, we obtain different μ_s and σ_s pairs for different days-to-maturity. Similarly, we can estimate the market expected return (μ_m) and market volatility (σ_m) using S&P 500 Index call options.

Note that the implied return here indicates investors' forward-looking ex ante expected return of the stock over the period from the current time, *t*, to the maturity date, *T*. We therefore obtain different implied returns and volatilities for different maturities at a given trade date, *t*. This is consistent with investor expectations of return and volatility, which could differ according to their investment horizon.

5.3 Data

In order to extract forward-looking information on implied return and volatility from option trading prices, we obtain daily close transaction data of the options of individual stocks listed on NYSE, NASDAQ, and AMEX from OptionMetrics for the last trading day of each month for the period from January 1996 to April 2006. This data file contains CUSIP, trade date, strike price, offer price, bid price, trading volume, option open interest, Black-Scholes implied volatility, and maturity date for each option. This data set also contains the daily closing data of S&P 500 Index options.

For the corresponding stocks whose option data are available, we obtain daily stock prices and returns from the CRSP. To match the stock price with option records, we use the CUSIP and trade date of the stock. A total of 4,078 stocks are found to have

[12] With prices for options with more than two strike prices, we can find values for μ_s and σ_s that produce option prices closest to the observed prices in the least squares sense. A similar least-squares methodology is used by Melick and Thomas (1997).

both option and stock price data. We also obtain information of firm characteristics, such as firm size and book-to-market, from CRSP and Compustat.

For the risk-free interest rates, we use the St. Louis Fed's 3-month, 6-months, 1-year, 2-year, 3-year, and 5-year Treasury Constant Maturity Rates. Assuming a step-function of interest rates, we match the days-to-maturity in the option record with its corresponding constant maturity rate. For example, if the days-to-maturity of the option is less than or equal to 3 months, we use 3-month rates, and if the days-to-maturity is between 3 months and 6 months, we use the 6-month rate, and so on.

5.4 Computation of the Implied Return, Volatility, and Market Beta

We jointly estimate implied mean return (or implied return) and implied volatility of the underlying stock, μ_s and σ_s, by using the risk-adjusted option pricing model through equations (4) and (5). At a given trade date (i.e., the last trading day of each month), we obtain the market prices of only near-the-money call options with same maturity date but different strike prices. We define the near-the-money option as any option whose ratio of stock price to strike price (S_t/K) falls between 0.9 and 1.3. By using all these options, we compute the implied return and implied volatility via a method of grid search to look for global optima that minimizes the error square. The error is defined as the difference between the observed option price and the right hand side of equation (4) using market observed values along with implied return and implied volatility. For the grid search, we set the implied return search range from 0 to 175.00 percent, and the implied standard deviation search range from 0 to 100 percent. The reason we take only near-the-money options is to minimize the effect of measurement error in estimating implied returns and volatilities, since measurement error could be caused by failing to adjust for jumps and the stochastic behavior of volatilities, such as the volatility smile, which are observed in deep-out-of-money options.[13] Options with zero trading volume are excluded. Put options are not used only because our models are designed for call options.

We use the closing bid/ask mid-point as the closing American option price. The option dataset also has the Black-Scholes implied volatility adjusted for any stock dividends during the life of the option. Using this information along with interest rates, we reverse to compute the corresponding European option price. If the computed European option price is higher than the American option price, we take the American option price as the option price. Otherwise, we take the European price as the option price. Our results are based on the last trading day observations of option prices of each calendar month. Taking any other day of the month produces similar results. For example, we verify our results by taking the first working day, second Thursday, and third Friday of each month. The results are qualitatively similar.

Since one pair of the estimated implied return and volatility is obtained for each maturity and there are several different maturity dates at a given trade date, we obtain several sets of implied return and volatility pairs at a given trade date. That is, we obtain term structures of implied returns and implied volatilities of a stock at a given date.

[13] According to Canina and Figlewski (1993), measurement errors may also be systematically affected by time-to-maturity, even though there are no jumps and stochastic behavior of volatilities. To mitigate these errors, options with the same maturity are used to compute implied return and implied volatility.

Similarly, at a given trade date, we also obtain similar term structures for S&P 500 Index options.

If there are no such market index options available at a given trade date, we interpolate the value of market implied return and volatility using other days-to-maturity information of the market index options. For example, suppose that for a particular trade date, we have three different implied market returns corresponding to three different days-to-maturities: 90 days, 120 days, and 150 days. For the implied return of an underlying stock whose option has 140 days to maturity, the corresponding market implied return will be obtained from a linear interpolation using the market implied returns of 120 days and 150 days. If days-to-maturity of stock implied return is more than 150 days, the corresponding market implied return will be the market implied return of 150 days. Therefore, there is one-to-one correspondence between the implied return of an underlying stock and the market implied return. Hence, we obtain the matched implied market returns and implied stock returns.

Since options whose payoffs are determined by the correlation between the underlying stock and the market portfolio do not exist, it would be difficult to directly extract information regarding implied market betas like the implied mean return. Therefore, we estimate implied market betas of an underlying stock by regressing implied returns of the stock on implied market returns.

5.5 Empirical Results

5.5.1 Basic Statistics of the Implied Variables

Table 1 presents the basic statistics of the three key implied variables of all pooled sample obtained from all 4,078 firms' individual stock call options over the period from January 1996 to April 2006: implied return, μ_i, implied volatility, σ_i, and implied beta estimate, $\hat{\beta}_i^{\text{imp}}$. Note that for the implied variables of individual stock options, now we use subscript *i* instead of *s*. These implied variables are computed from the option prices observed at the last trading day of each month. The total number of firm-month observations is 179,048. Days to maturity of the sample ranges from 3 days to 1,027 days. μ_i and σ_i are implied instantaneous return (or continuously compounded return; CCR) and volatility, respectively. As seen in Table 1, the number of firm-year observations is much greater for short-term options than for long-term options. [14] This is because the near-the-money options of most of the stocks are actively traded on short maturities.

Table 1 shows that implied return decreases with maturity; that is, the term structure of implied returns is apparently downward sloped. Specifically, when days to maturity are less than or equal to 30 days ($0 < T \leq 30$), between 30 and 60 days ($30 < T \leq 60$), between 60 and 120 days ($60 < T \leq 120$), between 120 and 210 days ($120 < T \leq 210$), and longer than 210 days ($T > 210$), the averages of implied returns are 0.538, 0.336 0.243, 0.178, and 0.122, respectively. The average of the whole implied returns is

[14] Among these, the numbers of firm-month observations whose days to maturity are between 0 and 30 days, between 30 and 60 days, between 60 and 120 days, between 120 and 210 days, and longer than 210 days are 47863, 41838, 31188, 34171, and 23988, respectively.

0.315. This indicates that investors have high expectation in a short-term horizon, while they are more subdued and hold more reasonable expectation in a long-term horizon. In previous chapters we show that this downward term structure is robust to market friction proxies such as option volume, open interest, and bid-ask spread. Furthermore, this term structure is found for both European and American option prices.[15] Our findings on this term structure indicate that expected returns are affected by investment time horizon. These findings are consistent with McNulty et al. (2002). They argue that shorter-horizon investments should be discounted at a higher rate and that the marginal risk of an investment declines as a function of the square root of time. This falling marginal risk should be reflected in the annual discount rate for longer-horizon investments. A recent paper by Camara et al. (2007) also shows the similar result that short-term expected returns are higher than long-term expected returns when using market-observed option prices.[16]

Implied volatility also shows a downward sloping term structure. That is, implied volatility is higher for a shorter maturity than for a longer maturity. However,

---

[15] This downward sloping term structure of the implied returns is also found in deep-in-the-money call options. We separately estimate implied returns and volatilities by using deep-in-the-money call options where stock price divided by strike price is greater than 1.20 and deep-out-of-the-money call options where stock price divided by strike price is less than 0.90. In both cases, we obtain a similar downward term structure of implied returns (not reported).

[16] However, there are at least two differences between our approach and theirs. First, they assume a specific utility structure for the representative agent such that the marginal utility of wealth of the representative investor is:

$$U'(W_T) = W_T^{\alpha} + \beta$$

where α and β are risk aversion parameters.

Based on this utility structure, they show that their option pricing equation contains implied stock return as one of the parameters to be estimated. Our approach instead uses a risk-adjusted version of option pricing with no explicit assumption about the utility structure. Second, their approach requires an intermediate parameter that needs to be computed using options of all companies, before computing the implied return of any individual firm. On the other hand, our model does not need information about other companies to compute the expected return and volatility. Our model jointly computes implied volatility using all stock options and S&P 500 Index options.

the decreasing rate of the slope over days-to-maturity is smaller than the case of implied returns. The averages of the implied standard deviations are 0.515, 0.497, 0.474, 0.456, and 0.423 over the above-mentioned five intervals of maturity, respectively.

Since we observe a downward sloping term structure of implied returns and volatilities, the risk-return structure differs across maturities (or investment horizon). It is appropriate, therefore, that implied returns be matched with implied market betas in the tests, which are both in the same maturity group. As mentioned above, we classify the whole sample into five maturity groups: $0 < T \leq 30$, $30 < T \leq 60$, $60 < T \leq 120$, $120 < T \leq 210$, and $T > 210$. In each maturity group, implied betas are estimated by regressing implied returns of an underlying stock on implied market returns over the whole period contained in the maturity group. For any stock, therefore, there can be up to five implied betas according to the availability of implied returns. Since the CAPM is a one-period model, holding period return (HPR) should be used in the tests. Thus, implied HPRs are used in estimating implied market betas, $\hat{\beta}_i^{\text{imp}}$, instead of CCRs. Implied HPR is computed as $e^{\mu} - 1$, where μ is implied CCR. The implied beta also shows a similar downward pattern across maturities. The averages of the implied beta over the five maturity groups are 1.146, 0.959, 0.542, 0.530, and 0.467, respectively. The longer is the investment horizon, the smaller is the beta. These results are somewhat consistent with Levhari and Levy (1977), who show theoretically that market beta is a function of investment horizon.

Table 1 also reports the correlation coefficients between the implied variables and their historical counterparts. Using the whole pooled sample, the correlation coefficient ($\rho_{\mu,\bar{r}}$) between the implied return (μ) and its historical counterpart (annualized CCR of the underlying stock over the option life, ($\bar{r}$)) is 0.100. There is no particular pattern in this correlation coefficient across the five maturity groups. The correlation coefficient ($\rho_{\sigma,s}$) between the implied volatility (σ) and its historical counterpart (annualized sample standard deviation over the option life is 0.695, and the correlation coefficient ($\rho_{\beta,\hat{\beta}}$) between the implied beta (β) and its historical market beta (Scholes-William's (1977) beta estimate using daily returns over the option life) is 0.114. The correlation coefficients, $\rho_{\sigma,s}$ and $\rho_{\beta,\hat{\beta}}$, tend to increase with length to maturity, which indicates that implied volatility and beta could be more informative in predicting their historical counterparts.

Table 2 presents the basic statistics of the implied variables of the market index option, S&P 500 Index call option. The number of firm-month observations of the market-implied variables is exactly matched with the number of observations of individual stock options. The term structure of the implied market returns is also apparently downward across investment horizons, although its slope is less steep than the case of implied returns for individual stocks. The averages of the implied market return and standard deviation are 0.169 and 0.202, respectively, using the whole pooled sample. These are much smaller in magnitude than those of individual stock options. The term structure of the volatility of S&P 500 Index option is almost flat.

5.5.2 Cross-Sectional Regression Tests Using *Ex-Ante* Implied Returns and Implied Betas

As mentioned above, the forward-looking implied variables obtained from option prices can be used as investors' ex ante expectation on the risk and return. In this sense, implied return and implied beta are the most plausible proxies for ex ante return and risk. By using the computed implied returns and betas, we examine the ex ante risk–return relationship by using the classical Fama and MacBeth methodology. In order to do this, we estimate the following cross-section regression (CSR) model at month *t*,

(6) $$\mu_{i,[t,T]} - r_{f,[t,T]} = \gamma_{0t} + \gamma_{1t}\hat{\beta}_{it}^{\text{imp}} + \Gamma_t\ (\text{Control variables}) + \varepsilon_{it},$$

where $\mu_{i,[t,T]}$ is the implied annualized HPR on underlying stock *i* over the option life ([t, *T*]) from the last trading day of month *t* to maturity *T*, and $r_{f,[t,T]}$ is the Treasury bill annualized holding period yield over the period [*t*,*T*]. In fact, $\mu_{i,[t,T]}$ is the expected return over the period from the first trading day of month *t*+1 to the maturity, *T*. $\hat{\beta}_{it}^{\text{imp}}$ is the OLS implied beta estimate of stock *i* obtained from regressing implied HPRs of stock *i* on implied market HPRs over the whole period in each maturity group. The control variables used in the CSR tests are firm characteristics such as firm size, book-to-market, and momentum (past six-month returns), which are the variables for the widely known market anomalies that the CAPM fails to explain.

Table 3 shows the CSR estimation results of equation (6) over the period from January 1996 to April 2006. The upper panel presents time series averages of the gammas (or the risk premium estimates) with implied market beta alone in the model, and the bottom panel presents those of the full model including the control variables. The estimates of the risk premium ($\bar{\bar{\gamma}}_1$) are positively significant regardless of the inclusion of the control variables. When the implied market beta is alone in the model, the risk premium estimate is 11.30 percent per year (with t-statistic of 13.67), using the whole sample. Its significance is also maintained in each maturity group, although it is weakened. That is, the risk premium estimates are 6.12 percent (t=7.43), 2.45 percent (t=5.09), 0.75 percent (t=1.89), 0.57 percent (t=1.73), and 1.06 percent (t=4.18), respectively, in the five maturity groups. However, the intercept estimates are strongly positive in all cases, which means that the implied ex ante returns may not be fully explained by the implied market beta. The large positive intercept estimates may be from a large value of the implied mean returns.

Even when the control variables (firm size, book-to-market, and momentum) are added to the model, the estimates of the risk premium are even more positively significant; using the whole sample, it is 12.31 percent (t=14.80). The risk premium estimates in the five maturity groups are 5.10 percent (t=5.95), 3.53 percent (t=7.32), 1.93 percent (t=4.83), 1.98 percent (t=5.68), and 2.03 percent (t=6.72), respectively. The above results indicate that the implied market beta has a significant explanatory power for ex ante expected returns in all maturity groups.

Table 3 also presents the estimation results on the control variables. The CSR coefficient estimates on the firm size variable (log(ME)) are all negative and statistically strongly significant. That is, investors have high (low) ex ante expected returns on small (large) firms. The CSR coefficient estimates on the book-to-market variable (log(BM)) are all negative and statistically significant, which implies that investors have high ex ante expected returns on low book-to-market stocks, while they have low ex ante expected returns on high book-to-market stocks. These results are consistent with the Lakonishok, Shleifer, and Vishny (1994) explanation that low book-to-market stocks are in fact growth stocks whose ex ante expected return tends to be high. The opposite holds for high book-to-market value stocks. The CSR coefficient estimate on the momentum variable (annualized past six-month return) is overall insignificant, which implies that investors may not have an a priori, ex ante expectation based on past intermediate-term stock performance. These ex ante results on momentum are interesting because they contrast with the ex post results in which the presence of momentum is significant.[17]

5.5.3 Cross-Sectional Regression Tests Using *Ex-Ante* Implied Betas and *Realized* Returns

In order to examine whether implied betas explain the cross-section of realized ex post returns, we also cross-sectionally regress realized ex post returns on the implied betas and the control variables. The CSR model to be estimated at month *t* is given by:

$$(7) \qquad R_{i,[t,t+H]} - r_{f,[t,t+H]} = \gamma_{0t} + \gamma_{1t}\hat{\beta}_{it}^{\text{imp}} + \Gamma_t \text{ (Control variables)} + \varepsilon_{it},$$

where $R_{i,[t,t+H]}$ is the ex post HPR of an underlying stock *i* over the period *H* (i.e., from one day after the last trading day of month *t* to *H* days after the last trading day of month *t*), and $r_{f,[t,T]}$ is the Treasury bill annualized holding period yield over the

[17] The above results on the control variables are also similar when each of the control variables is alone in the CSR model.

corresponding measurement period $R_{i,[t,T]}$. We consider two different holding periods, *H*. The first holding period is up to maturity (*H*=*T*), which means that investors invest in each stock at the last trading day of every month according to the value of the implied betas and hold the stock until the option maturity date. The second holding period is one month (*H*=one month), which means that investors invest in each stock at the last trading day of each month according to the value of the implied betas and hold each stock for one month. Thus, the investment period overlaps in the first scheme, while it does not overlap in the second scheme.

Table 4 presents the time series averages of the CSR coefficients ($\bar{\hat{\gamma}}$) of equation (7) when the holding period is up to the maturity (in Panel A; $R_{i,\cdot}$ is annualized retutn) and up to one month (in Panel B; $R_{i,\cdot}$ is monthly return), respectively. Panel A shows that implied market betas have cross-sectionally significant explanatory power for average realized returns over the option life. That is, the coefficient estimate ($\bar{\hat{\gamma}}_1$) on the implied betas is 9.49 percent per year, with t-statistic of 8.44, using the whole sample. It is also positive and statistically significant in all maturity groups except for the shortest maturity group. That is, it is 1.61 percent (*t*=1.27), 5.75 percent (*t*=3.68), 6.35 percent (*t*=3.71), 6.50 percent (*t*=3.89), and 10.67 percent (*t*=4.36), respectively, for the five maturity groups. Even when the three control variables are added to the model, the risk premium estimates are more strongly significant. They are 12.11 percent (*t*=9.72) for the whole sample, 3.06 percent (*t*=2.22), 7.43 percent (*t*=4.89), 6.95 percent (*t*=3.60), 13.21 percent (*t*=6.74), and 15.04 percent (*t*=6.46), respectively, for the five maturity groups.

Panel B of Table 4 also presents the time series average of the gammas when the holding period is one month. The results indicate that implied market betas also have a significant explanatory power for the cross-section of average realized returns over the next 1-month period. That is, the coefficient estimate ($\bar{\bar{\gamma}}_1$) on the implied betas is 0.21 percent per month, with t-statistic of 2.74, using the whole sample. It is also positive and statistically significant in all maturity groups except for the shortest maturity group; -0.02 percent (*t*=-0.43), 0.25 percent (*t*=2.01), 0.32 percent (*t*=2.15), 0.65 percent (*t*=2.17), and 0.99 percent (*t*=1.91), respectively, for the five maturity groups. Even when the control variables are added to the model, the risk premium estimates are more strongly significant. The intercept estimates are insignificant in all cases.

Table 4 also presents the CSR estimation results of ex post realized returns on the control variables. The CSR coefficient estimates on the firm size variable are also negative and statistically significant, as ex ante expected returns are used. It could be argued, therefore, that investors' ex ante expected return based on firm size tends to be realized as expected. However, investors' ex ante expectation based on book-to-market and momentum tends to be realized differently from their expectation. That is, the CSR coefficient estimates on the book-to-market variable are overall positive and marginally significant, which is opposite when ex ante expected returns are used. The CSR coefficient estimates on the momentum variable are positive and significant, which means that momentum does not exist a priori but appears significant a posteriori. Note that even when each of the control variables is alone in the model, the estimated coefficients on the control variable are similar.

5.5.4 Forward Relationships Between *Ex-Ante* Implied Betas and Implied *Ex-Ante* Returns

Since implied returns and volatilities observed at any given time have a variety of maturities (from short to long), it is possible to compute forward-implied returns and volatilities for an underlying stock. That is, the forward-implied return, observed at time *t*, on an underlying stock over the forward period $[T_1, T_2]$ is computed as:

$$(8) \qquad \mu^f_{t,[T_1,T_2]} = \frac{\mu_{[t,T_2]}(T_2-t) - \mu_{[t,T_1]}(T_1-t)}{(T_2-T_1)},$$

where $\mu_{[t,T_1]}$ and $\mu_{[t,T_2]}$ are the implied (annualized) returns on the underlying stock over the option lives $[t, T_1]$ and $[t, T_2]$, respectively. These implied returns are observed at time *t* (i.e., at the last trading day of each month), and T_1 and T_2 are the shorter and longer maturities of the option, respectively. Note that implied returns in equation (8) are CCRs, but their HPRs are used in estimating forward-implied betas and in the CAPM tests. Similarly, the forward-implied standard deviation over the forward period $[T_1, T_2]$ is computed as:

$$(9) \qquad \sigma^f_{t,[T_1,T_2]} = \sqrt{\frac{\sigma^2_{[t,T_2]}(T_2-t) - \sigma^2_{[t,T_1]}(T_1-t)}{(T_2-T_1)}},$$

where $\sigma_{[t,T_1]}$ and $\sigma_{[t,T_2]}$ are the implied standard deviations of the underlying stock over the option lives $[t, T_1]$ and $[t, T_2]$, respectively. When there are more than two options with different maturities at a given time, say, T_1, T_2, and T_3, we compute the forward-implied variables over nonoverlapped forward periods, such as over the periods $[T_1, T_2]$ and $[T_2, T_3]$, not $[T_1, T_3]$.

Table 5 presents the basic statistics of the forward-implied returns, standard deviation, and betas. Note that forward-implied betas are estimated by regressing the forward-implied HPRs of an underlying stock on the forward-implied market HPRs in each forward period length group over the whole sample period. Forward period length groups are classified as four groups: $0 < [T_1, T_2] \leq 30$, $30 < [T_1, T_2] \leq 90$, $90 < [T_1, T_2] \leq 120$, and $[T_1, T_2] > 120$ days. As shown in Table 5, the forward-implied return also decreases with the length of the forward period; that is, the term structure of forward-implied returns is downward shaped, although its slope is slower than that of the implied returns. The forward-implied volatility and forward-implied beta estimates also show a modestly downward term structure across the length of the forward period.

It would be interesting to examine whether there is a positive forward relation between ex ante expected returns and betas. To do this, we estimate the following CSR model at month *t*,

$$(10) \qquad \mu^{f}_{it,[T_1,T_2]} - r_{ft,[T_1,T_2]} = \gamma^{f}_{0t} + \gamma^{f}_{1t}\hat{\beta}^{f,imp}_{it} + \varepsilon_{it},$$

where $\mu^{f}_{it,[T_1,T_2]}$ is the implied *forward* annualized HPR on underlying stock *i* over the forward period $[T_1, T_2]$, $r_{ft,[T_1,T_2]}$ is the Treasury bill annualized holding period yield over the same forward period, and $\hat{\beta}^{f,imp}_{it}$ is the forward-implied estimate of stock *i* obtained from regressing forward-implied HPRs of stock *i* on forward-implied market

HPR returns over the whole sample period; both forward returns are contained in each forward period length group.

Table 6 reports the time series averages of the gamma estimates of equation (10), which are the forward risk premium estimates ($\hat{\gamma}_{0t}^{f}$ and $\hat{\gamma}_{1t}^{f}$); these are positively significant in all cases. Using the whole forward sample, the forward market risk premium estimate is 1.88 percent per year (with *t*-statistic of 5.42). This positive significance holds regardless of the length of the forward period. That is, the forward market risk premium estimates are 1.12 percent (*t*=2.41), 0.75 percent (*t*=1.87), 1.05 percent (*t*=2.70), and 1.58 percent (*t*=4.16), respectively, for the four forward period length groups.

5.5.5 Do the Ex-Ante Market Risk Premia Estimates Contain the Forward-Looking Information of Macroeconomic Conditions?

Investors' ex ante returns reflect their forward-looking expectation for individual stocks and the market as a whole. Therefore, another way to test whether our CSR estimate of ex ante market risk premium (presented in Table 3) has an economic significance is to examine whether the ex ante market risk premium estimates contain forward-looking information on macroeconomic conditions. To do so, we regress the ex ante market risk premia estimate on the future macroeconomic variables. That is, we estimate the following time-series regression model:

(11) $\hat{\gamma}_{1t} = b_0 + b_1\mathrm{TB}_{t+1,t+L} + b_2\mathrm{TERM}_{t+1,t+L} + b_3\mathrm{DEF}_{t+1,t+L} + b_4\mathrm{DIV}_{t+1,t+L} + b_5\mathrm{CONSUME}_{t+1,t+L} + b_6\mathrm{GDP}_{t+1,t+L} + b_7\mathrm{LABOR}_{t+1,t+L} + \varepsilon_t,$

where $\hat{\gamma}_{1t}$ is the estimate of ex ante market risk premium (i.e., the CSR coefficient estimates) at month *t*, $\mathrm{TB}_{t+1,t+L}$ is the three-month Treasury bill yield from month *t*+1 through month *t*+*L* (*L* is the number of months of the forward-looking period), TERM is the term spread defined as the difference between the yield on 10-year government bonds and the yield on the three-month Treasury bill, DEF is the default spread defined as the difference between the yield on Moody's BAA rated bonds and the yield on Moody's AAA rated bonds, DIV is the dividend yield on the value-weighted market index, CONSUME is the growth rate of personal consumption expenditures, GDP is the growth rate of GDP, and LABOR is the growth rate of personal labor income.[18] The value of each macroeconomic variable is its geometric average (i.e., compounded value) over *L* forward-looking months from *t*+1 to *t*+*L*. [19] The sample period is from January 1996 to April 2006.

Table 7 presents the regression estimation results of the ex ante market risk premium estimated using each maturity group on the future macroeconomic variables with *L* = 1 month (Panel A), *L* = 2 months (Panel B), *L* = 4 months (Panel C), and *L* = 6 months (Panel D), respectively. The results apparently show that the ex ante market risk premium reflects the forward-looking information on future macroeconomic conditions.

---

[18] The dividend yield (DIV) is obtained by using the CRSP value-weighted market returns with and without dividends through the method in Fama and French (1988).

[19] The minimum number of forward-looking months is one month. Over the last L months from the last sample period, therefore, we calculate the geometric average value of the macroeconomic variables by using the remaining observations up to the last month of the sample period.

The association between the ex ante market risk premium and the future macroeconomic variables becomes stronger with the length of the forward-looking period (L) and with the maturity of implied mean returns used in estimating the ex ante market risk premium. Specifically, the adjusted R-squares of equation (11) using all maturities are 0.329, 0.357, 0.432, and 0.454 for L = 1 month, 2 months, 4 months, and 6 months, respectively. For a particular length of the forward-looking period, say L = 4 months, (in Panel C), the adjusted R-squares are 0.201, 0.295, 0.247, 0.401, and 0.427 for the maturities of $0 < T \leq 30$, 30 < $T \leq$ 60, 60 < $T \leq$ 120, 120 < $T \leq$ 210, and T > 210 days, respectively. These R-squares are quite high.

The ex ante market risk premium also has a significant forward-looking relation with individual macroeconomic variables. In all regressions (all 24 regressions), it has a strongly significant positive relation with future default premium (DEF). This indicates that investors' ex ante risk premium is *proactively* increased as the default premium will be increased in the future (at least one month through six months later). In turn, option-implied returns contain important information about future defaults. The ex ante market risk premium also has a clear relation with future dividend yield (DIV). It has a strongly significant negative relation with DIV in most regressions. This indicates that as dividend yield increases in the future, the stock price level increases and the subsequent expected return (i.e., ex ante market risk premium) is lowered. The negative magnitude of the regression coefficients tends to decrease with the length of maturity.

The ex ante market risk premium has generally negative relations with the future growth of real economic activity as measured by consumption, GDP, and labor income (CONSUME, GDP, and LABOR), although the estimated coefficients are not as statistically significant as those on DEF and DIV. This indicates that as real economic activity is expected to be in expansion, the stock price level increases and then the ex ante market risk premium declines. The ex ante market risk premium is insignificantly related to future short-term interest rates (TB). This may be because the riskless rate of return is already adjusted in the market risk premium; however, it generally has a significant positive relation with future term structure (TERM). Since the coefficient on TERM can also be the coefficient on long-term interest rates (10-year Treasury bond yield), these results indicate that the ex ante market risk premium is positively associated with future long-term interest rates.

In sum, the CSR estimates of the ex ante market risk premium are significantly associated with forward-looking economic conditions and are rationally consistent with our perception. These results support that the CSR estimates have economic significance as well as statistical significance.

Table 8 presents the estimation results of the time-series regression model of equation (11) by using the implied market returns (extracted from S&P 500 Index options) as the dependent variable, rather than the CSR estimates of the ex ante market risk premium. The results are stronger than but overall similar to those using the ex ante market risk premium estimates (Table 7), except for the results for future short-term interest rates (TB). The coefficient estimates on TB are mostly positively significant, which means that the ex ante market return increases with future short interest rates. In sum, implied market returns contain significant information on future macroeconomic conditions. In order to compare these ex ante results with ex post results, we regress the CRSP value-weighted market returns on the forward-looking economic variables. The results are reported in Table 9. Most of the estimated coefficients are insignificant. The R-squares are quite low, compared with the R-squares from the regressions using the ex ante values. It is difficult to say that the realized market returns contain information on future macroeconomic conditions.

5.6 Conclusion

This chapter examines the CAPM relation on an ex ante basis. That is, we investigate the cross-sectional relation between ex ante expected returns and ex ante betas. As a proxy for ex ante expected returns, we use implied mean returns obtained from the risk-adjusted option pricing model that we suggest in this paper. Ex ante betas are estimated by regressing implied returns of an underlying stock on implied market returns.

We find that the ex ante cross-sectional relation between ex ante expected returns and betas is positive and statistically strongly significant. This significant relation is maintained regardless of the inclusion of the well known firm characteristics such as firm size, book-to-market, and momentum. Since there is an apparent downward term structure of implied mean returns and betas across investment horizons, we examine the ex ante relation in each maturity group and find there is still a strongly significant ex ante cross-sectional relation. We also find a significant positive forward relation between these two ex ante variables.

In order to examine whether ex ante betas have explanatory power for realized ex post returns, we estimate cross-sectional regressions of realized returns on ex ante betas and find that ex ante betas have a positive and statistically significant relation with ex post realized returns, regardless of the inclusion of the firm characteristics mentioned above. That is, ex ante betas are significantly priced in realized returns.

We also find an interesting difference between ex ante and ex post market anomalies such as firm size, book-to-market and momentum. Investors' ex ante expected return based on firm size tends to be realized as expected. However, investors' ex ante expectation based on book-to-market and momentum tends to be realized differently from their expectation. That is, investors' ex ante expected returns are negatively associated with book-to-market, but their realized returns are positively related with book-to-market. Investors' ex ante expected returns are not associated with past stock returns, but their realized returns are positively related with past stock returns.

In order to test whether our CSR estimate of ex ante market risk premium contains forward-looking information on future macroeconomic conditions, we regress the ex ante market risk premia estimate on the future macroeconomic variables. We find that the ex ante market risk premium has a significant positive relation with future default premium. Further, it has a significant negative relation with future dividend yield and also has generally negative relations with the future growth of real economic activity as measured by consumption, GDP, and labor income. These results indicate that as more cash flows (from increasing dividends and expanding real economic activity) are expected in the future, the stock price level increases and then the subsequent ex ante expected return is lowered. In sum, the ex ante market risk premium contains significant forward-looking information on future macroeconomic conditions. When the implied market returns (from S&P 500 Index options) are used instead of the ex ante market risk premium estimate, we obtain stronger but similar results. However, when the CRSP value-weighted market returns are used in the regression, we find that realized market returns contain no significant forward-looking information on future macroeconomic conditions.

Tables

Table 1: Basic Statistics of the Implied Variables for Individual Stock Options

This table presents the basic statistics of the pooled implied data of individual stock options. By using the risk-adjusted option pricing model, the implied mean returns (μ_i) and standard deviations (σ_i) of individual stocks (all 4,078 stocks) are computed with call option prices of various maturities observed at the last trading day of each month from January 1996 to April 2006. The implied beta OLS estimate of stock i ($\hat{\beta}_i^{\text{imp}}$) is obtained from regressing the implied holding period mean returns on stock i on the implied holding period market mean returns in each maturity group (with at least 10 implied observations). Implied mean returns are measured at the end of every month. "Correlation" is the correlation coefficient between the implied variable and its historical counterpart. The historical counterpart of the implied return is the annualized continuously compounded return of the stock over the option life, that of the implied standard deviation is the annualized sample standard deviation, and that of the implied beta is the Scholes-William (1977) beta estimate using daily returns over the option life. "NSAM" is the number of all available firm-month observations.

Implied variable	Maturities (in days)	Mean	Standard deviation	Correlation	Min	1%	10%	Median	90%	99%	Max	NSAM
Implied return (μ_i)	All maturities	0.315	0.234	0.100	0.001	0.039	0.101	0.245	0.633	1.133	1.750	179,048
	$0 < T \leq 30$	0.538	0.293	0.006	0.003	0.071	0.197	0.486	0.959	1.329	1.750	47,863
	30 < T ≤ 60	0.336	0.156	0.047	0.001	0.064	0.149	0.313	0.558	0.729	1.056	41,838
	60 < T ≤ 120	0.243	0.107	0.068	0.001	0.048	0.116	0.231	0.391	0.522	0.704	31,188
	120 < T ≤ 210	0.178	0.074	0.030	0.001	0.029	0.089	0.171	0.280	0.370	0.483	34,171
	T > 210	0.122	0.051	0.013	0.001	0.011	0.062	0.118	0.188	0.266	0.366	23,988
Implied volatility (σ_i)	All maturities	0.480	0.206	0.695	0.030	0.143	0.237	0.446	0.790	0.964	0.990	17,9048
	$0 < T \leq 30$	0.515	0.212	0.615	0.046	0.161	0.259	0.481	0.836	0.971	0.990	47,863
	30 < T ≤ 60	0.497	0.206	0.713	0.030	0.151	0.250	0.464	0.806	0.968	0.990	41,838
	60 < T ≤ 120	0.474	0.200	0.741	0.032	0.142	0.236	0.444	0.772	0.959	0.990	31,188
	120 < T ≤ 210	0.456	0.200	0.765	0.038	0.134	0.222	0.421	0.754	0.956	0.990	34,171
	T > 210	0.423	0.189	0.727	0.039	0.133	0.204	0.389	0.700	0.935	0.990	23,988
Implied OLS beta ($\hat{\beta}_i^{\text{imp}}$)	All maturities	0.792	1.043	0.114	-9.521	-1.969	-0.087	0.684	1.847	4.385	9.866	148,973

	$0 < T \leq 30$	1.146	1.378	0.049	-9.521	-2.543	-0.119	0.997	2.673	5.787	9.805	40,910
	30 < T ≤ 60	0.959	1.111	0.200	-7.441	-2.578	-0.055	0.974	2.033	3.900	9.866	34,581
	60 < T ≤ 120	0.542	0.825	0.248	-5.348	-2.103	-0.308	0.606	1.288	2.872	5.209	24,630
	120 < T ≤ 210	0.530	0.614	0.299	-3.438	-1.346	-0.069	0.540	1.064	2.188	8.161	27,793
	T > 210	0.467	0.369	0.398	-3.794	-0.609	0.069	0.489	0.856	1.409	1.794	21,059
# days to maturity	All maturities	125	155	-	3	16	18	53	261	785	1027	179,048

Table 2: Basic Statistics of the Implied Variables for Standard and Poors 500 Index Options

This table presents the basic statistics of the pooled implied data of Standard & Poor's 500 Index options as the market index options. By using the risk-adjusted option pricing model, the implied market mean returns (μ_m) and standard deviations (σ_m) of the market index options are computed with call option prices of various maturities observed at the last trading day of each month from January 1996 to April 2006. For each observed individual stock option, we find the corresponding Standard & Poor's 500 Index option whose maturity is the same as the stock option. Implied market mean returns are measured at the end of every month. "Correlation" is the correlation coefficient between the implied variable and its historical counterpart. The historical counterpart of the implied market return is the annualized continuously compounded return of the market index over the option life, and that of the implied standard deviation is the annualized sample standard deviation. "NSAM" is the number of all available firm-month observations.

Implied variable	Maturities (in days)	Mean	Standard deviation	Correlation	Min	1%	10%	Median	90%	99%	Max	NSAM
Implied market return (μ_m)	All maturities	0.169	0.085	0.139	0.008	0.054	0.087	0.150	0.283	0.508	0.590	179,048
	$0 < T \leq 30$	0.246	0.099	0.184	0.109	0.127	0.139	0.229	0.375	0.577	0.590	47,863
	$30 < T \leq 60$	0.169	0.059	0.096	0.091	0.091	0.109	0.153	0.243	0.334	0.371	41,838
	$60 < T \leq 120$	0.145	0.053	0.129	0.074	0.079	0.092	0.133	0.214	0.328	0.353	31,188
	$120 < T \leq 210$	0.130	0.047	0.044	0.062	0.063	0.083	0.115	0.194	0.275	0.323	34,171
	$T > 210$	0.101	0.039	0.026	0.008	0.017	0.059	0.093	0.157	0.211	0.272	23,988
Implied market volatility (σ_m)	All maturities	0.202	0.075	0.643	0.079	0.107	0.118	0.190	0.304	0.437	0.517	17,9048
	$0 < T \leq 30$	0.200	0.078	0.604	0.079	0.103	0.111	0.191	0.310	0.430	0.463	47,863
	$30 < T \leq 60$	0.202	0.075	0.644	0.107	0.107	0.118	0.197	0.314	0.448	0.456	41,838
	$60 < T \leq 120$	0.202	0.073	0.617	0.108	0.109	0.120	0.190	0.295	0.417	0.517	31,188
	$120 < T \leq 210$	0.202	0.073	0.682	0.110	0.111	0.124	0.189	0.300	0.440	0.482	34,171
	$T > 210$	0.206	0.076	0.748	0.101	0.106	0.130	0.182	0.308	0.431	0.499	23,988

Table 3: Time-Series Averages of Cross-Sectional Regressions of *Ex Ante* Implied Returns on Implied Beta Estimates

This table presents the time-series averages (in percent, ×100) of the Fama-MacBeth month-by-month cross-sectional regression coefficients:

$$\mu_{i,[t,T]} - r_{f,[t,T]} = \gamma_{0t} + \gamma_{1t}\hat{\beta}_{it}^{\text{imp}} + \Gamma_t \text{ (Control variables)} + \varepsilon_{it},$$

where $\mu_{i,[t,T]}$ is the *implied* annualized holding period mean return on underlying stock *i* over the option life, measured at the end of each month (t). $r_{f,[t,T]}$ is the Treasury bill annualized holding period yield measured at the end of each month (t). $\hat{\beta}_{it}^{\text{imp}}$ is the OLS implied beta estimate of stock *i* obtained from regressing the implied mean returns of stock *i* on the implied market mean returns in each maturity group over the whole sample period. Maturity groups are classified as 5 groups: $0 < T \leq 30$, $30 < T \leq 60$, $60 < T \leq 120$, $120 < T \leq 210$, and $T > 210$ days. Control variables are as follows: ME is the market value of common equity measured one month before the option trading day, BM is the book-to-market ratio and the earnings–price ratio, which is most recently available six months before the option trading day, and "Momentum" is the stock return over the past six months before the option trading day. Numbers in parentheses indicate *t*-statistics. The sample period is from January 1996 to April 2006.

Maturity (in days)	Intercept	$\hat{\beta}_{it}^{\text{imp}}$	Control Variables log (ME)	log(BM)	Momentum
All maturities	29.95 (62.25)	11.30 (13.67)			
$0 < T \leq 30$	68.78 (55.78)	6.12 (7.43)			
$30 < T \leq 60$	35.83 (84.65)	2.45 (5.09)			
$60 < T \leq 120$	24.23 (79.30)	0.75 (1.89)			
$120 < T \leq 210$	16.21 (74.61)	0.57 (1.73)			
$T > 210$	8.84 (59.41)	1.06 (4.18)			
All maturities	44.70 (60.19)	12.31 (14.80)	-8.84 (-44.65)	-3.44 (-13.84)	-0.87 (-1.44)
$0 < T \leq 30$	85.96 (53.51)	5.10 (5.95)	-12.92 (-33.03)	-5.95 (-12.02)	-3.25 (-2.53)
$30 < T \leq 60$	41.68 (71.56)	3.53 (7.32)	-5.68 (-34.41)	-3.46 (-14.73)	-3.46 (-1.19)
$60 < T \leq 120$	26.71 (67.73)	1.93 (4.83)	-3.21 (-29.26)	-2.61 (-13.87)	0.52 (1.17)
$120 < T \leq 210$	17.24 (58.94)	1.98 (5.68)	-1.84 (-25.87)	-1.66 (-15.59)	0.08 (0.30)
$T > 210$	10.31 (38.08)	2.03 (6.72)	-1.08 (-16.56)	-0.96 (-12.35)	0.37 (2.43)

Table 4: Time-Series Averages of Cross-Sectional Regressions of *Ex-Post* Returns on the Implied Beta Estimates

This table shows the time-series averages (in percent, ×100) of the Fama-MacBeth month-by-month cross-sectional regression coefficients:

$$R_{i,[t,t+H]} - r_{f,[t,t+H]} = \gamma_{0t} + \gamma_{1t}\hat{\beta}_{it}^{\text{imp}} + \Gamma_t \text{ (Control variables)} + \varepsilon_{it},$$

where $R_{i,[t,t+H]}$ is the ex post annualized holding period return of underlying stock *i* over the period *H*. The period *H* is the option life from the following day of the end of each month (t) to its maturity date (T) (in Panel A) or is one month from the day following the end of each month (t) to the end of the next month (in Panel B). The option trading day is the last day of each month. Thus, the realized ex post return is measured from the first day of the month following the option trade month to the option maturity. $r_{f,[t,T]}$ is the Treasury bill annualized holding period yield over the same measurement period of $R_{i,[t,T]}$, and $\hat{\beta}_{it}^{\text{imp}}$ is the OLS implied beta estimate of stock *i* obtained from regressing implied mean returns of stock *i* on implied market mean returns in each maturity group. Maturity groups are classified as follows: $0 < T \leq 30$, 30 < T ≤ 60, 60 < T ≤ 120, 120 < T ≤ 210, and T > 210 days. Control variables are as follows: ME is the market value of common equity measured one month before the option trading day, BM is the book-to-market ratio and the earnings-price ratio, which is most recently available six months before the option trading day, and "Momentum" is the stock return over the past six months prior to the option trading day, Numbers in parentheses indicate *t*-statistics. The sample period is from January 1996 to April 2006.

Maturity (in days)	Intercept	$\hat{\beta}_{it}^{\text{Imp}}$	Control Variables		
			log (ME)	log(BM)	Momentum
Panel A: Y-variable = Realized returns over the option life (H = *T*)					
All maturities	31.64 (10.52)	9.49 (8.44)			
$0 < T \leq 30$	61.98 (10.53)	1.61 (1.27)			
30 < T ≤ 60	44.11 (8.94)	5.75 (3.68)			
60 < T ≤ 120	25.03 (8.66)	6.35 (3.71)			
120 < T ≤ 210	18.37 (6.00)	6.50 (3.89)			
T > 210	6.98 (3.11)	10.67 (4.36)			
All maturities	46.74 (10.60)	12.11 (9.72)	-5.80 (-6.92)	2.47 (1.93)	7.29 (2.81)
$0 < T \leq 30$	63.49 (9.77)	3.06 (2.22)	1.41 (1.09)	4.38 (1.92)	6.15 (1.41)
30 < T ≤ 60	54.94 (8.06)	7.43 (4.89)	-6.07 (-4.95)	-0.50 (-0.23)	3.13 (0.68)
60 < T ≤ 120	44.64 (8.22)	6.95 (3.60)	-7.09 (-7.60)	0.08 (0.00)	11.54 (3.21)
120 < T ≤ 210	28.76 (6.42)	13.21 (6.74)	-5.41 (-6.61)	1.35 (1.00)	10.90 (5.03)
T > 210	22.86 (6.74)	15.04 (6.46)	-4.58 (-5.46)	1.27 (1.26)	7.60 (4.87)

Panel B: Y-variable = Realized returns over the next one month (H = 1 month)

All maturities	0.66 (1.26)	0.21 (2.74)			
$0 < T \leq 30$	0.56 (1.03)	-0.02 (-0.43)			
30 < T ≤ 60	0.61 (1.12)	0.25 (2.01)			
60 < T ≤ 120	0.52 (1.06)	0.32 (2.15)			
120 < T ≤ 210	0.40 (0.68)	0.65 (2.17)			
T > 210	0.39 (0.62)	0.99 (1.91)			
All maturities	0.79 (1.17)	0.26 (3.07)	-0.04 (-0.32)	0.24 (1.25)	0.90 (2.19)
$0 < T \leq 30$	0.75 (1.15)	0.07 (1.47)	0.07 (0.53)	0.36 (1.99)	0.77 (1.87)
30 < T ≤ 60	0.61 (0.92)	0.35 (2.87)	-0.04 (-0.27)	0.21 (1.10)	0.66 (1.61)
60 < T ≤ 120	0.68 (1.13)	0.46 (2.28)	-0.13 (-0.91)	0.04 (0.17)	0.92 (2.14)
120 < T ≤ 210	0.63 (0.87)	0.97 (3.27)	-0.17 (-1.09)	0.22 (1.04)	1.10 (2.48)
T > 210	1.52 (1.70)	1.41 (2.30)	-0.32 (-1.74)	0.18 (0.75)	0.98 (2.01)

Table 5: Basic Statistics of the *Forward* Implied Variables for Individual Stock Options

This table presents the basic statistics of the pooled implied forward variables for individual stock options. $\mu^{f}_{i,[T_1,T_2]}$ is the forward-implied annualized holding period return (HPR) on underlying stock i over the forward period $[T_1, T_2]$, which is from the next of the first option maturity (T_1) to the maturity of the second option (T_2). This forward-implied return is measured at the end of every month(t) from January 1996 to April 2006. $\sigma^{f}_{i,[T_1,T_2]}$ is the forward-implied annualized standard deviation of the underlying stock i over the forward period $[T_1, T_2]$. $\hat{\beta}^{f,imp}_{it}$ is the forward-implied beta estimate of stock i obtained from regressing the forward-implied HPRs of stock i on the forward-implied market HPRs in each forward period length group over the whole sample period. Forward period length groups are classified as follows: $0 < T \leq 30$, $30 < T \leq 90$, $90 < T \leq 120$, and $T > 120$ days. "NSAM" is the number of all available firm-month observations.

Implied forward variable	Forward period (in days)	Mean	Standard deviation	Min	1%	10%	Median	90%	99%	Max	NSAM
Forward implied return $\left(\mu^{f}_{i,[T_1,T_2]}\right)$	All forward periods	0.145	0.160	-2.053	-0.286	-0.003	0.126	0.332	0.631	2.018	106,082
	$0 < [T_1,T_2] \leq 30$	0.208	0.214	-2.053	-0.394	-0.024	0.203	0.451	0.791	2.018	25,643
	$30 < [T_1,T_2] \leq 90$	0.171	0.161	-1.280	-0.271	-0.001	0.163	0.360	0.616	1.454	28,843
	$90 < [T_1,T_2] \leq 120$	0.106	0.102	-0.626	-0.211	-0.001	0.107	0.217	0.374	0.717	30,777
	$[T_1,T_2] > 120$	0.084	0.063	-0.531	-0.102	0.018	0.083	0.153	0.259	0.419	17,877
Forward implied volatility $\left(\sigma^{f}_{i,[T_1,T_2]}\right)$	All forward periods	0.464	0.211	0.002	0.109	0.215	0.431	0.773	0.985	1.551	106,082
	$0 < [T_1,T_2] \leq 30$	0.493	0.216	0.002	0.111	0.237	0.460	0.804	1.029	1.551	25,643
	$30 < [T_1,T_2] \leq 90$	0.468	0.205	0.003	0.108	0.226	0.440	0.764	0.985	1.271	28,843
	$90 < [T_1,T_2] \leq 120$	0.437	0.197	0.002	0.105	0.205	0.408	0.721	0.955	1.330	30,777
	$[T_1,T_2] > 120$	0.413	0.187	0.014	0.115	0.196	0.383	0.674	0.933	1.335	17,877
Forward implied beta $(\hat{\beta}^{f,imp}_{i})$	All forward periods	0.283	0.879	-12.407	-2.183	-0.403	0.276	0.996	2.785	13.380	89,547
	$0 < [T_1,T_2] \leq 30$	0.316	1.337	-12.407	-3.990	-0.708	0.307	1.378	3.823	12.488	21,636
	$30 < [T_1,T_2] \leq 90$	0.273	0.858	- 4.124	-2.114	-0.508	0.293	0.983	2.682	13.380	24,699
	$90 < [T_1,T_2] \leq 120$	0.290	0.557	- 3.310	-1.277	-0.257	0.277	0.859	2.184	4.769	26,080
	$[T_1,T_2] > 120$	0.175	0.330	- 2.185	-0.678	-0.149	0.148	0.529	1.037	2.263	15,037
# of forward days	All forward periods	98	104	28	28	28	63	245	462	945	106,082

Table 6: Forward Relationship: Time-Series Averages of Cross-Sectional Regressions of Implied Forward Returns on Implied Forward Beta Estimates

This table shows the time-series averages (in percent, ×100) of the Fama-MacBeth month-by-month cross-sectional regression coefficients:

$$\mu^{f}_{it,[T_1,T_2]} - r_{ft,[T_1,T_2]} = \gamma^{f}_{0t} + \gamma^{f}_{1t}\hat{\beta}^{f,imp}_{it} + \varepsilon_{it},$$

where $\mu^{f}_{it,[T_1,T_2]}$ is the forward-implied annualized holding period return (HPR) on an underlying stock *i* over the forward period $[T_1, T_2]$ which is from the day following the first option maturity (T_1) to the maturity of the second option (T_2), and $r_{ft,[T_1,T_2]}$ is the Treasury bill annualized holding period yield over the forward period. Both $\mu^{f}_{it,[T_1,T_2]}$ and $r_{ft,[T_1,T_2]}$ are measured at time t (i.e., the last trading day of each month). $\hat{\beta}^{f,imp}_{it}$ is the forward-implied beta estimate of stock *i* obtained from regressing the forward-implied HPRs of stock *i* on the forward-implied market HPRs in each forward period length group over the whole sample period. Forward period length groups are classified as follows: $0 < T \leq 30$, 30 < T ≤ 90, 90 < T ≤ 120, and T > 120 days.

Forward periods (in days)	Intercept $(\bar{\hat{\gamma}}^{f}_{0t})$	$\hat{\beta}^{f,imp}_{it}$ $(\bar{\hat{\gamma}}^{f}_{1t})$
All forward periods	16.66 (91.83)	1.88 (5.42)
$0 < [T_1, T_2] \leq 30$	24.61 (52.78)	1.23 (2.41)
30 < $[T_1, T_2]$ ≤ 90	18.99 (45.99)	0.75 (1.87)
90 < $[T_1, T_2]$≤ 120	11.58 (81.62)	1.05 (2.70)
$[T_1, T_2]$>120	8.44 (49.63)	1.58 (4.16)

Table 7: Relationship Between Estimated Ex Ante Market Risk Premium and Forward-Looking Macroeconomic Variables

This table presents the results of the following time-series regression model:

$\hat{\gamma}_{1t} = b_0 + b_1\mathrm{TB}_{t+1,t+L} + b_2\mathrm{TERM}_{t+1,t+L} + b_3\mathrm{DEF}_{t+1,t+L} + b_4\mathrm{DIV}_{t+1,t+L} + b_5\mathrm{CONSUME}_{t+1,t+L} + b_6\mathrm{GDP}_{t+1,t+L} + b_7\mathrm{LABOR}_{t+1,t+L} + \varepsilon_t$,

where $\hat{\gamma}_{1t}$ is the CSR coefficient estimates (or estimated ex ante market risk premia) at month t of ex ante implied returns (with various maturities) of individual stocks on their implied beta estimates. The macroeconomic variables used as explanatory variables are as follows: $\mathrm{TB}_{t+1,t+L}$ is the 3-month Treasury bill (geometric average) yield from month t+1 through month t+L (L is the number of months of the forward-looking period), TERM is the term spread defined as the difference between the yield on 10-year government bonds and the yield on the three-month Treasury bill, DEF is the default spread defined as the difference between the yield on Moody's BAA rated bonds and the yield on Moody's AAA rated bonds, DIV is the dividend yield on the value-weighted market, CONSUME is the growth rate of personal consumption expenditures, GDP is the growth rate of GDP, and LABOR is the growth rate of personal labor income.

	All maturities		Maturities (in days) of ex ante implied returns used in estimating ex ante market risk premia									
			0<T≤30		30<T≤60		60<T≤120		120<T≤210		210<T	
Panel A: Forward-looking period (L) = 1 month												
Intercept	0.05	(0.55)	0.18	(1.74)	0.13	(2.10)	0.17	(3.76)	0.03	(0.85)	0.04	(1.19)
TB	0.97	(1.03)	-0.97	(-0.87)	-0.88	(-1.36)	-1.40	(-2.67)	0.04	(0.08)	-0.47	(-1.31)
TERM	-0.60	(-0.48)	-0.76	(-0.54)	-0.03	(-0.04)	-1.13	(-1.80)	0.45	(0.84)	0.04	(0.09)
DEF	19.62	(4.32)	5.52	(1.04)	1.48	(0.48)	-0.22	(-0.10)	3.46	(1.79)	4.55	(2.94)
DIV	-38.72	(-2.30)	-48.50	(-2.53)	-34.41	(-3.26)	-40.71	(-4.90)	-18.43	(-2.58)	-11.29	(-2.19)
CONSUME	-3.80	(-0.93)	-4.26	(-0.92)	-2.61	(-0.95)	0.37	(0.16)	-2.12	(-1.11)	-2.88	(-2.17)
GDP	-3.22	(-2.03)	-1.47	(-0.83)	-1.34	(-1.49)	-1.98	(-2.75)	-2.34	(-3.92)	-1.66	(-3.72)
LABOR	-2.00	(-1.58)	-1.69	(-1.10)	-1.31	(-1.46)	-0.78	(-1.19)	0.01	(0.01)	-0.01	(-0.03)
Adj R^2	0.329		0.119		0.227		0.310		0.276		0.408	
Panel B: Forward-looking period (L) = 2 months												
Intercept	0.04	(0.46)	0.09	(1.13)	0.04	(1.14)	0.05	(1.76)	0.00	(0.18)	0.00	(0.06)
TB	1.13	(1.33)	-0.18	(-0.21)	0.11	(0.26)	-0.04	(-0.10)	0.61	(2.04)	-0.09	(-0.42)
TERM	-0.06	(-0.05)	0.18	(0.14)	1.33	(2.00)	0.41	(0.75)	0.96	(2.40)	0.46	(1.64)
DEF	20.97	(4.65)	10.56	(2.14)	6.19	(2.60)	4.58	(2.38)	5.59	(3.85)	6.31	(5.85)
DIV	-45.59	(-2.05)	-55.02	(-2.16)	-46.78	(-3.43)	-43.08	(-3.61)	-27.78	(-2.91)	-12.55	(-1.75)
CONSUME	-6.33	(-1.21)	-9.56	(-1.56)	-5.65	(-1.67)	-0.38	(-0.13)	-2.02	(-0.89)	-2.16	(-1.24)

GDP	-3.74	(-2.14)	-0.60	(-0.30)	-0.67	(-0.67)	-1.10	(-1.22)	-2.27	(-3.35)	-1.56	(-2.86)
LABOR	-1.49	(-1.08)	-1.22	(-0.71)	-0.77	(-0.82)	-0.91	(-1.21)	-0.47	(-0.88)	0.00	(-0.01)
Adj R^2	0.357		0.132		0.279		0.226		0.342		0.406	

	All maturities		Maturities (in days) of ex ante implied returns used in estimating ex ante market risk premium									
			0<T≤30		30<T≤60		60<T≤120		120<T≤210		210<T	
					Panel C: Forward-looking period (L) = 4 month							
Intercept	0.02	(0.28)	0.01	(0.17)	-0.01	(-0.49)	0.00	(-0.17)	-0.01	(-0.65)	0.00	(0.31)
TB	1.23	(1.78)	0.03	(0.04)	0.26	(0.69)	0.47	(1.45)	0.90	(3.58)	-0.13	(-0.66)
TERM	0.63	(0.57)	0.38	(0.31)	2.07	(2.84)	1.08	(1.84)	1.13	(2.70)	0.43	(1.34)
DEF	24.86	(5.74)	19.01	(3.85)	9.27	(3.98)	7.45	(3.84)	7.05	(4.98)	7.18	(6.53)
DIV	-58.56	(-2.29)	-68.49	(-2.15)	-55.52	(-2.96)	-61.17	(-3.68)	-33.12	(-2.62)	-19.44	(-1.95)
CONSUME	-16.66	(-2.15)	-26.81	(-2.84)	-10.05	(-1.95)	2.77	(0.62)	-1.21	(-0.36)	-4.27	(-1.45)
GDP	-4.63	(-2.19)	1.26	(0.51)	0.42	(0.30)	0.14	(0.11)	-2.60	(-2.76)	-1.14	(-1.43)
LABOR	0.22	(0.14)	0.80	(0.39)	0.91	(0.80)	-0.43	(-0.48)	-0.61	(-1.00)	-0.10	(-0.21)
Adj R^2	0.432		0.201		0.295		0.247		0.401		0.427	
					Panel D: Forward-looking period (L) = 6 months							
Intercept	0.00	(0.04)	-0.05	(-0.93)	-0.04	(-1.58)	-0.02	(-1.07)	-0.01	(-0.44)	0.01	(1.16)
TB	1.25	(1.99)	-0.33	(-0.51)	0.27	(0.69)	0.37	(1.07)	1.02	(3.65)	-0.18	(-0.81)
TERM	0.93	(0.82)	0.35	(0.28)	2.92	(3.46)	1.23	(1.82)	1.04	(2.16)	0.16	(0.42)
DEF	26.77	(6.17)	26.62	(5.11)	9.48	(3.84)	8.30	(4.09)	7.14	(4.80)	7.90	(6.91)
DIV	-55.34	(-1.92)	-85.71	(-2.47)	-79.49	(-3.55)	-74.01	(-3.92)	-35.17	(-2.46)	-12.86	(-1.14)
CONSUME	-27.01	(-2.75)	-55.00	(-4.62)	-6.91	(-1.03)	5.45	(0.96)	2.43	(0.57)	-10.08	(-2.52)
GDP	-4.67	(-1.75)	5.12	(1.79)	2.16	(1.24)	1.58	(1.02)	-3.01	(-2.64)	-1.36	(-1.39)
LABOR	1.01	(0.52)	3.81	(1.58)	2.34	(1.64)	0.24	(0.21)	-1.08	(-1.43)	-0.84	(-1.35)
Adj R^2	0.454		0.298		0.308		0.268		0.411		0.453	

Table 8: Relationships Between Implied Market Returns on S&P500 Index and Forward-Looking Macroeconomic Variables

This table presents the results of the following time-series regression model:

$$\mu_{mt} = b_0 + b_1\text{TB}_{t+1,t+L} + b_2\text{TERM}_{t+1,t+L} + b_3\text{DEF}_{t+1,t+L} + b_4\text{DIV}_{t+1,t+L} + b_5\text{CONSUME}_{t+1,t+L} + b_6\text{GDP}_{t+1,t+L} + b_7\text{LABOR}_{t+1,t+L} + \varepsilon_t,$$

where μ_{mt} is the implied market return on S&P500 Index option (with various maturities) obtained at the end of month t. The macroeconomic variables used as explanatory variables are as follows: $\text{TB}_{t+1,t+L}$ is the 3-month Treasury bill (geometric average) yield from month t+1 through month $t+L$ (L is the number of months of the forward-looking period), TERM is the term spread defined as the difference between the yield on 10-year government bonds and the yield on the three-month Treasury bill, DEF is the default spread defined as the difference between the yield on Moody's BAA rated bonds and the yield on Moody's AAA rated bonds, DIV is the dividend yield on the value-weighted market, CONSUME is the growth rate of personal consumption expenditures, GDP is the growth rate of GDP, and LABOR is the growth rate of personal labor income. The sample period is from January 1996 to April 2006.

	All maturities		Maturities (in days) of S&P500 Index options									
			0<T≤30		30<T≤60		60<T≤120		120<T≤210		210<T	
Panel A: Forward-looking period (L) = 1 month												
Intercept	0.13	(2.60)	0.34	(2.73)	0.31	(4.64)	0.27	(5.05)	0.18	(3.89)	0.13	(3.06)
TB	1.59	(2.84)	1.27	(0.99)	0.34	(0.45)	0.34	(0.56)	0.70	(1.23)	0.50	(1.01)
TERM	-0.21	(-0.28)	1.16	(0.68)	1.06	(1.15)	0.89	(1.22)	0.90	(1.41)	1.30	(2.34)
DEF	3.96	(1.48)	1.73	(0.28)	-1.94	(-0.59)	-3.52	(-1.35)	-0.26	(-0.11)	1.69	(0.86)
DIV	-24.65	(-2.51)	-63.16	(-3.00)	-53.27	(-4.50)	-46.75	(-4.90)	-30.53	(-3.78)	-32.83	(-5.15)
CONSUME	-5.53	(-2.33)	-12.44	(-2.49)	-7.32	(-2.28)	-2.31	(-0.89)	-2.00	(-0.92)	-4.62	(-2.98)
GDP	-0.45	(-0.50)	-1.97	(-1.08)	-2.68	(-2.66)	-2.26	(-2.78)	-3.16	(-4.51)	-1.73	(-3.35)
LABOR	-1.36	(-1.80)	-1.84	(-1.02)	-0.91	(-0.94)	-0.76	(-1.04)	0.17	(0.30)	0.84	(2.13)
Adj R^2	0.295		0.239		0.262		0.282		0.357		0.505	
Panel B: Forward-looking period (L) = 2 months												
Intercept	0.14	(3.29)	0.18	(2.25)	0.15	(3.86)	0.14	(4.35)	0.09	(3.64)	0.06	(3.89)
TB	1.65	(3.65)	2.73	(2.90)	1.99	(3.82)	1.89	(4.50)	1.91	(5.39)	1.44	(6.02)
TERM	0.16	(0.25)	2.61	(1.88)	2.77	(3.71)	2.48	(4.25)	2.12	(4.56)	2.30	(6.97)
DEF	5.06	(2.04)	10.86	(2.19)	5.51	(2.10)	2.07	(1.00)	4.67	(2.71)	6.29	(5.12)
DIV	-47.43	(-3.79)	-67.73	(-2.41)	-55.11	(-3.51)	-49.60	(-3.71)	-34.77	(-3.11)	-48.88	(-5.46)
CONSUME	-8.01	(-2.68)	-15.71	(-2.42)	-7.46	(-1.88)	-1.22	(-0.37)	-2.47	(-0.92)	-3.91	(-2.04)

GDP	0.14	(0.14)	-0.78	(-0.38)	-2.21	(-1.92)	-1.67	(-1.70)	-3.04	(-3.60)	-1.10	(-1.77)
LABOR	-1.03	(-1.28)	-1.19	(-0.62)	-0.70	(-0.67)	-1.00	(-1.26)	-0.30	(-0.53)	0.32	(0.85)
Adj R^2	0.354		0.168		0.291		0.262		0.381		0.582	

	All maturities		Maturities (in days) of S&P500 Index options									
			0<T≤30		30<T≤60		60<T≤120		120<T≤210		210<T	
					Panel C: Forward-looking period (L) = 4 months							
Intercept	0.13	(4.05)	0.17	(3.44)	0.12	(4.15)	0.10	(4.66)	0.06	(3.83)	0.05	(6.87)
TB	1.74	(5.03)	2.32	(3.19)	2.27	(5.18)	2.21	(6.53)	2.15	(7.85)	1.53	(8.29)
TERM	0.55	(0.91)	2.00	(1.48)	2.71	(3.36)	2.56	(4.29)	1.69	(3.79)	2.10	(6.63)
DEF	8.22	(3.53)	18.26	(4.16)	8.85	(3.45)	5.77	(2.92)	8.79	(5.78)	8.33	(7.45)
DIV	-72.91	(-4.88)	-123.46	(-3.50)	-66.68	(-3.12)	-62.65	(-3.62)	-43.29	(-3.17)	-54.45	(-5.30)
CONSUME	-15.98	(-3.69)	-39.10	(-4.13)	-7.35	(-1.26)	-3.13	(-0.68)	-2.44	(-0.64)	-1.53	(-0.60)
GDP	0.93	(0.77)	5.10	(1.97)	-0.82	(-0.50)	-0.54	(-0.42)	-2.31	(-2.10)	-1.38	(-1.80)
LABOR	-0.12	(-0.13)	-0.55	(-0.25)	-0.64	(-0.51)	-0.66	(-0.76)	-0.36	(-0.55)	0.00	(0.00)
Adj R^2	0.441		0.303		0.333		0.370		0.540		0.699	
					Panel D: Forward-looking period (L) = 6 months							
Intercept	0.11	(3.99)	0.15	(3.23)	0.09	(3.41)	0.10	(4.89)	0.05	(3.54)	0.06	(9.83)
TB	1.78	(5.82)	1.98	(2.87)	2.02	(4.45)	2.13	(5.80)	1.83	(6.15)	1.32	(7.21)
TERM	0.73	(1.24)	2.55	(1.78)	3.06	(3.40)	2.23	(3.36)	1.06	(2.17)	1.87	(5.31)
DEF	11.37	(4.83)	22.92	(5.10)	10.40	(3.88)	6.55	(3.22)	10.11	(6.76)	9.76	(8.61)
DIV	-85.12	(-5.38)	-171.17	(-4.44)	-99.95	(-4.05)	-64.75	(-3.38)	-51.92	(-3.53)	-66.99	(-6.09)
CONSUME	-27.39	(-5.27)	-58.99	(-4.95)	-5.61	(-0.76)	-1.00	(-0.17)	1.55	(0.31)	-3.98	(-1.21)
GDP	2.09	(1.50)	9.70	(3.20)	2.23	(1.11)	-0.24	(-0.15)	-1.00	(-0.77)	-0.36	(-0.43)
LABOR	0.71	(0.66)	1.88	(0.76)	1.02	(0.67)	-0.64	(-0.60)	0.23	(0.27)	-0.21	(-0.54)
Adj R^2	0.532		0.373		0.368		0.404		0.601		0.734	

Table 9: Relationships Between Realized Market Returns and Forward-Looking Macroeconomic Variables

This table presents the results of the following time-series regression model:
$R_{mt} = b_0 + b_1 \mathrm{TB}_{t+1,t+L} + b_2 \mathrm{TERM}_{t+1,t+L} + b_3 \mathrm{DE}F_{t+1,t+L} + b_4 \mathrm{DIV}_{t+1,t+L} + b_5 \mathrm{CONSUME}_{t+1,t+L} + b_6 \mathrm{GDP}_{t+1,t+L} + b_7 \mathrm{LABOR}_{t+1,t+L} + \varepsilon_t$,
where R_{mt} is the CRSP value-weighted market return at month t. The macroeconomic variables used as explanatory variables are as follows: $\mathrm{TB}_{t+1,t+L}$ is the 3-month Treasury bill (geometric average) yield from month t+1 through month $t+L$ (L is the number of months of the forward-looking period), TERM is the term spread defined as the difference between the yield on 10-year government bonds and the yield on the three-month Treasury bill, DEF is the default spread defined as the difference between the yield on Moody's BAA rated bonds and the yield on Moody's AAA rated bonds, DIV is the dividend yield on the value-weighted market, CONSUME is the growth rate of personal consumption expenditures, GDP is the growth rate of GDP, and LABOR is the growth rate of personal labor income. The sample period is from January 1996 to April 2006.

Explanatory Variables	Forward-looking period (L)											
	L = 1 month		L = 2 months		L = 3 months		L = 4 months		L = 5 months		L = 6 months	
Intercept	-0.01	(-0.25)	-0.01	(-0.10)	0.00	(-0.10)	0.01	(0.28)	0.01	(0.40)	0.02	(0.47)
TB	0.01	(0.02)	-0.04	(-0.07)	0.06	(0.12)	0.11	(0.24)	0.16	(0.36)	0.08	(0.18)
TERM	0.40	(0.51)	0.36	(0.49)	0.47	(0.65)	0.61	(0.83)	0.72	(0.97)	0.61	(0.80)
DEF	-1.02	(-0.38)	-0.58	(-0.22)	-1.04	(-0.41)	-2.70	(-1.08)	-3.56	(-1.41)	-3.52	(-1.38)
DIV	-2.73	(-0.26)	-12.17	(-0.88)	-14.10	(-0.86)	-21.59	(-1.29)	-21.92	(-1.24)	-20.07	(-1.07)
CONSUME	-0.25	(-0.10)	-3.59	(-1.10)	-0.80	(-0.19)	3.17	(0.62)	3.15	(0.52)	-0.82	(-0.12)
GDP	1.00	(1.03)	1.49	(1.37)	1.37	(1.10)	1.85	(1.27)	2.14	(1.27)	2.56	(1.36)
LABOR	1.29	(1.77)	1.55	(1.92)	1.33	(1.48)	0.59	(0.59)	0.30	(0.26)	0.42	(0.33)
Adj R^2	0.088		0.023		0.009		0.019		0.022		0.024	

Chapter- 6

Future Extension of the Risk-Adjusted Model: A Stochastic Volatility Approach

In the previous chapters we provided a framework for discrete time risk-adjusted option pricing model that is consistent with the capital asset pricing model (CAPM) of Sharpe (1964), Lintner (1965), and and Black (1972). Similar to the standard CAPM equilibrium model, this option pricing can be regarded as a single period model where the investor buys and holds till maturity that does not need rebalancing. Also, we argued that the risk-adjusted model is consistent with the Black and Scholes (1973) continuous time model. In this setting our risk-adjusted model can be thought of as the generalization of the Black and Scholes model of no-arbitrage. In the presence of continuous rebalancing or with instantaneous holding, our model will collapse to the Black-Scholes model. However, if continuous rebalancing or instantaneous holding is not imposed then our model can still be used. Therefore our model is consistent with the standard CAPM and does not conflict with Black-Scholes pricing model when their assumptions hold. Since our model is consistent with the standard CAPM, the expected return from our model can be used to have an ex ante test of the standard CAPM. Extending this idea, if we want to test the asset pricing model under random volatility, first we would need to extract expected return from an option pricing equation that is consistent with random volatility. In fact, this concept can be extended for testing numerous other asset pricing models by extracting the information from a corresponding option pricing model with similar assumptions. More specifically in this chapter we suggest a risk adjusted option pricing model with stochastic volatility to extract the ex ante expected stock returns. The parameter estimates from this model along with the risk-adjusted characteristic function can then be used to understand the higher moments of the stock return distribution, and to test a single period asset pricing model with random volatility.

This chapter is organized as follows. Section 6.1 provides the motivation for stochastic volatility, Section 6.2 describes different stochastic volatility processes, Section 6.3 discusses stochastic volatility option pricing models, Section 6.4 derives the risk-adjusted stochastic volatility option pricing model, Section 6.5 provides estimation methodology, and Section 6.6 provides the conclusion.

6.1 Motivation for Stochastic Volatility

Unlike the Black-Scholes model, our risk-adjusted model of previous chapters is a discrete time model. On the other hand, similar to Black-Scholes model our model assumes a geometric Brownian price process with constant volatility.[1] Even though the models with constant volatility are simple and elegant they do not capture all the important distributional characteristics of stock returns. In this section we briefly discuss few empirical studies that imply stock price process does not follow the constant volatility assumptions in real life and hence we need a stochastic volatility model.

[1] The volatility of the stock price process is usually quoted as the standard deviation of continuously compounded return per year (see Hull 2002). Therefore study of stochastic volatility is the study of how the standard deviation of return distribution changes over time.

Early papers by Mandelbrot (1963), Fama (1965), and Blattberg and Gonedes (1974) found the stationary (log)normal distribution to be an inadequate descriptor of stock returns, and have fitted various alternate stationary distributions to the data. These papers find that the probability that extreme events will occur is greater than the corresponding probability calculated under the normal distribution. In other words, the empirical distribution of returns exhibits excess kurtosis. Therefore a leptokurtic distribution describes the return distribution better than a normal distribution. Subsequent studies by Black (1976) and Christie (1982) have uncovered an inverse correlation between stock returns and changes in volatility, at least partly attributable to financial leverage effects. Black (1976), Poterba and Summers (1986), and Beckers (1983) provide evidence that shocks to volatility persist but tend to decay over time. Mandelbrot (1963) and Fama (1965) also report evidence that periods of high (low) volatility are followed by periods of high (low) volatility. Mandelbrot (1963) has called this phenomenon "the clustering effect" of volatility. Volatility clustering suggests heteroscedasticity in volatility that is autocorrelated. Based on these findings, papers by Engle (1982), Bollerslev (1986), Bollerslev, Chou, and Kroner (1992), and Taylor (1994) have introduced models to capture volatility clustering in form of ARCH, GARCH, and SV models in time-series data. Furthermore, studies by Scott (1987), Poterba and Summers (1986), Stein (1989), and Harvey and Whaley (1992) have found that volatility oscillates around a constant value. This phenomenon is termed as "mean reversion," indicating that volatility tends to revert to a long-run mean.

Another interesting finding using the Black-Scholes model is the volatility smile and volatility skew. For a given day, for a specific maturity deep in-the-money and deep out of the money options might have higher volatility than near-the-money options. This U-shaped pattern is the volatility smile mostly observed in foreign currency options. It has been argued (Hull 2002) that volatility smile in currency options are a result of stochastic volatility or jump in the price process. Also, it is found that volatility decreases gradually with strike prices (called volatility skew), most frequently found in equity options. Volatility skewness in implied distributions has heavier left tail and a less heavy right tail than the lognormal distribution. If the assumption of the price process with constant volatility would have been valid in real life then we would not have observed the volatility smile or volatility skew. Early paper by Rubinstein (1985) and Taylor and Xu (1994) provide evidence of volatility smile and volatility skew in the observed options data. As shown in Heston (1993a) stochastic volatility model, if volatility is uncorrelated with the price process, then increasing the volatility of volatility increases the kurtosis of stock returns, not the skewness. In this case, random volatility is associated with increases in the prices of far-from-the-money options relative to near-the-money options. In contrast, the correlation of volatility with the price process produces skewness. And positive skewness is associated with increases in the prices of out-of-the-money options relative to in-the-money options. Therefore, it is essential to choose properly the correlation of volatility with spot returns as well as the volatility of volatility to model the stochastic volatility. Papers by Rubinstein(1985), Stein (1989), Xu and Taylor (1994), and Canina and Figulewski (1993) provide evidence of term structure of implied volatility. Volatility tends to be increasing function of maturity when short-dated volatilities are historically low.

Similarly volatility tends to be a decreasing function of maturity when short-dated volatility is historically high. This is consistent with mean reversion in volatility.

6.2 Different Stochastic Volatility Processes

We start with a generic process for the volatility and discuss how specific characterization of this process could provide various effects that are observed in real life. Let the volatility process be given by:

(1) $$dV = a(V,t)dt + b(V,t)dW$$

where *V* is the instantaneous volatility at time *t*, and *W* is a standard Weiner process at time *t*. The drift term in *a*, and the volatility of volatility term in *b* could be in general functions of both *V* and *t*. The specific models of volatility processes that could be explained by the above equation are: (1) the geometric Brownian process; (2) the mean reverting Gaussian process; and (3) the mean reverting square-root process.

The Geometric Brownian motion Process

This process is similar to the price process in Black-Sholes (1973) model. Under this process the evolution of volatility is given by:

(2) $$dV = \alpha V dt + \gamma V dW$$

where α is the constant drift term or the expected growth rate of the volatility, and γ is the volatility of the volatility process per unit time. Solving equation (2) for V_T (the volatility at any time T in the future) yields:

$$
\begin{aligned}
&V_T = V_0 e^{\left(\alpha - \frac{1}{2}\gamma^2\right)T + \gamma W} \\
(3) \qquad &E[V_T] = V_0 e^{\alpha T} \\
&\operatorname{var}[V_T] = V_0^2 e^{2\alpha T}\left(e^{\gamma^2 T} - 1\right)
\end{aligned}
$$

where V_0 is the initial volatility, and V_T is the volatility at time T. The properties of the process show that the volatility is unbounded and does not conform to the observed pattern in real life namely the mean-reversion.

The Mean-Reverting Gaussian Process

The mean-reverting Gaussian process also known as the Ornstein-Uhlenbeck process is a continuous time version of the AR1 process. Nelson (1990) has shown that this process is the diffusion limit of the GARCH (1,1) process. The mean-reverting Gaussian process is modeled as:

$$
(4) \qquad dV = \alpha\left(\beta - V\right)dt + \gamma dW ,
$$

where β is the long-run mean, α is the speed of mean reversion. The above process implies:

(5)
$$V_T = \beta + (V_0 - \beta)e^{-\alpha T} + \gamma \int_0^T e^{-\alpha s} dW_s$$
$$E[V_T] = \beta + (V_0 - \beta)e^{-\alpha T}$$
$$\mathrm{var}[V_T] = \frac{\gamma^2}{2\alpha}\left(1 - e^{-2\alpha T}\right)$$

Equation (4) and (5) show a small value of α indicates strong autocorrelation in volatility, whereas a greater value of α implies a faster convergence to the long-run mean. In this model the mean and variance of the volatility are bounded from above. Scott (1987), Hull and White (1987) and Wiggins (1987) use the mean-reverting Gaussian process to price the options. Using this model Stein and Stein (1991) derive an exact closed-form solution for the stock price process and show with suitable value to the parameters this process can explain the 'fat-tail' pattern that is observed in stock returns. However, the limitation of this process is that, V_T is normally distributed which means volatility can take negative values.

The Mean-Reverting Square-Root Process

This process has the feature of mean-reversion and it does not allow negative values. The evolution of variance, V under this process is given by:

(6)
$$dV = \alpha\left(\beta - V\right)dt + \gamma\sqrt{V}dW$$

The above process implies:

$$V_T = \beta + (V_0 - \beta) e^{-\alpha T} + \gamma e^{-\alpha T} \int_0^T e^{-\alpha s} \sqrt{V_s} dW_s$$

$$E[V_T] = V_0 e^{-\alpha T} + \beta \left(1 - e^{-\alpha T}\right), \tag{7}$$

$$\text{var}[V_T] = V_0 \left(\frac{\gamma^2}{\alpha}\right)\left(e^{-\alpha T} - e^{-2\alpha T}\right) + \left(\frac{\gamma^2}{\alpha}\right)\left(1 - e^{-\alpha T}\right)^2$$

In this process, volatility is no longer normally distributed. Cox, Ingersoll, and Ross (1985b) have shown that the probability density function follows a noncentral chi-square distribution using this process. Heston (1993a) and Hull and White (1987) use this assumption for variance process in deriving and analyzing option pricing models, given the close resemblance of real life data with this process. Our risk-adjusted stochastic volatility model in this chapter also assumes this variance process to price the option.

6.3 Stochastic Volatility Option Pricing Models

The earlier papers before Heston (1993a) used the equilibrium argument to price the options with stochastic volatility. For example Hull and White (1987), Scott (1987), and Wiggins (1987) use the Garman (1976) differential equation for the security process, Cox, Ingersoll, and Ross (1985a) intertemporal capital asset pricing model or the Merton (1973) equilibrium model. These option pricing models imply that the solution of the differential equation is independent of the risk preference only if (a) the volatility is a traded asset or (b) the volatility is uncorrelated with aggregate consumption. Hull and White (1987) assume that the volatility is uncorrelated with the aggregate consumption which means that volatility has a zero systematic risk. However it is difficult to see how this assumption could be realistic, in general given the empirical evidence of stock return and volatility correlations. They also state that constant correlation between rate of change in volatility and aggregate consumption can be used but do not discuss the preference restrictions that are required. To simplify the model Scott (1987) also had similar assumptions. Wiggings (1987) show that the logarithmic utility assumption is consistent with options on the market portfolio so that the price of risk of the hedge portfolio is zero. Thus the author argues the logarithmic utility assumption could be used to price the option on a market portfolio. These prior models provide the motivation for addressing the stochastic volatility issue; nonetheless they did not provide a close form general solution. Jarrow and Eisenberg (1994) and Stein and Stein (1991) assume the volatility is uncorrelated with the spot asset, thus it could not capture the important skewness effect in the option prices that arise from such correlation. Using a square-root process for variance, Heston (1993a) provided a closed-form solution where the risk preference is accommodated through the market price of volatility risk as we will explain in the following sections. Heston's model can explain the skewness and kurtosis of the return distribution with suitable adjustment of the volatility process parameters. We also assume a square-root process and follow the Fourier inversion approach to obtain a close form solution similar to Heston (1993a) for the risk-adjusted model. In our approach the option is held till maturity without rebalancing so as to extract the expected stock return along with other parameters.

6.4 The Risk-Adjusted Stochastic Volatility Option Pricing Model

We assume the stock price process follows a geometric Brownian and the variance process follows a mean-reverting square root process at time *t*,

(8a) $$dS = \mu S dt + \sqrt{V} S dW_1$$

(8b) $$dV = \alpha(\beta - V)dt + \gamma\sqrt{V} dW_2$$

where μ is the expected growth rate of the stock in risk-adjusted world, S is the stock price, V is the variance, α is the speed of mean reversion, and β is the long-run mean variance, $dW_1 dW_2 = \rho dt$, γ is the volatility of volatility, and W_1 and W_2 are the two Weiner processes. The mean-reverting square-root variance process has the nice feature that volatility will be positive and will revert to mean level when it moves above or below this level. The speed of reversion will depend on α. As α increases, the level of volatility stays close to the long-run mean level. In the Black-Scholes model we had a single uncertainty so that the stock with one call option can form the riskless portfolio and the partial differential equation (PDE) does not contain the Weiner process; here we need two call options and the stock to form the PDE that will not contain any of the above two Weiner process. If *C(S,V,t)* is the call price, then with Ito's Lemma we have:

(9)
$$\begin{aligned}
dC &= C_t dt + C_S dS + \frac{1}{2}C_{SS}\ dS^{\ 2} + C_V dV + \frac{1}{2}C_{VV}\ dV^{\ 2} + C_{SV}(dS)(dV) \\
&= C_t dt + C_S\left[\mu S dt + \sqrt{V} S dW_1\right] + \frac{1}{2}C_{SS}\ VS^2 dt\ + C_V\left[\alpha(\beta - V)dt + \gamma\sqrt{V} dW_2\right] \\
&\quad + \frac{1}{2}C_{VV}\ V\gamma^2 dt\ + C_{SV}(\rho\gamma S V dt) \\
&= \left[C_t + \mu S C_S + \frac{1}{2}VS^2 C_{SS} + \alpha(\beta - V)C_V + \frac{1}{2}V\gamma^2 C_{VV} + \rho\gamma S V C_{SV}\right]dt \\
&\quad + \sqrt{V} S C_S dW_1 + \gamma\sqrt{V} C_V dW_2
\end{aligned}$$

In the following equations, we form a portfolio X that contains two options (C^* and C^{**}) and the stock, and set the terms having dW_1 and dW_2 separately to zero to get two equations that we use to compute the portfolio weights h_1 and h_2.

(10)
$$X = C^* + h_1 S + h_2 C^{**}$$
$$dX = dC^* + h_1 dS + h_2 dC^{**}$$

Solving for h_1 and h_2 as explained above, we get:

(11)
$$h_1 = -C_S^* + \frac{C_V^*}{C_V^{**}} C_S^{**}$$
$$h_2 = -\frac{C_V^*}{C_V^{**}}$$

Since the portfolio is riskless we have:

(12)
$$dX = dC^* + h_1 dS + h_2 dC^{**}$$
$$= rXdt$$

Substituting the values of h_1 and h_2 from equation (11) and the value of X from equation (10) in equation (12) and equalizing the terms containing the same options (C^* or C^{**}) on left and right hand side we get the following PDE (after the adjustment of market price of volatility risk):

(13)
$$rSC_S + \frac{1}{2}VS^2C_{SS} + \ \alpha(\beta - V) - \lambda V \ C_V + \frac{1}{2}V\gamma^2 C_{VV} + \rho\gamma SVC_{SV} + C_t - rC = 0$$

The corresponding volatility process after the adjustment for market price of risk for the risk-neutral valuation is given by:

(14) $$dV = \left[\alpha(\beta - V) - \lambda V\right]dt + \gamma\sqrt{V}dW_2$$

where λ is the market price of volatility risk.

To process the PDE in risk-neutral world we adjust the volatility process for market price of risk as shown in equation (14). In the Black-Scholes one-dimensional PDE where the uncertainty comes from the single traded asset (the stock process), the market price of risk is such that the stock and the option growth rate can be set to the risk-free rate to value the option in traditional risk-neutral world. However when call option price is a function of additional uncertainties, the growth rate of these processes may not be the risk-free growth rate (using change of measure). This implies in general the stochastic volatility option pricing equation will be dependent on the risk preference parameter (via λ). Alternatively the PDE will be independent of risk preference if (a) the volatility is a traded asset or (b) the volatility is uncorrelated with aggregate consumption. For example if the underlying asset is a hypothetical market portfolio or aggregate wealth, then volatility risk will be orthogonal (to market risk) in which case the price of volatility risk will be zero for this portfolio. However, in general the price of volatility risk of an individual stock will not be zero and a risk-neutral PDE will have an adjustment for market price of risk in equation (13). In general when there are multiple processes, Cox, Ingersoll, Ross (1985a) provide the necessary framework to show the link between the option expected return and the stock expected return in equilibrium. From their paper using equation (13) and (22) with change of notation to our paper and assuming constant relative risk aversion (CRRA), the relationship between the option return and the stock return can be given as:

(15)
$$kC = rC + C_S\left[\left(\frac{-J_{SS}}{J_S}\right)\text{var}(S) + \left(\frac{-J_{SC}}{J_S}\right)\text{cov}(\text{cov}(S,V)\right] + C_V\left(\frac{-J_{SS}}{J_S}\right)(\text{cov}(S,C))$$
$$= rC + C_S\left[(\mu - r)S\right] + C_V\lambda V$$

where *J* is the indirect utility function, *k* is the expected growth rate of the option in risk adjusted world (same as expected return of the option), and

(16)
$$J_S \equiv \frac{\partial J}{\partial S}$$
$$J_{SS} \equiv \frac{\partial^2 J}{\partial S^2}$$
$$J_{SC} \equiv \frac{\partial^2 J}{\partial S \partial C}$$

Furthermore, with the assumption of CRRA we have:

(17)
$$-\frac{J_{SS}}{J_S} = \frac{\phi}{S},$$
$$\lambda = \phi\gamma\rho$$

where ϕ is a constant of the CRRA that measures the degree of risk aversion of the representative agent, and λ is the market price of risk for volatility process. For example, $\phi=0$ implies a logarithmic utility function which makes the market price of risk zero. A value of $\phi > 0$ will imply a lower relative risk aversion than a log utility function, and a value of $\phi < 0$ will imply the opposite. Therefore λ depends on the utility structure of the representative agent and it is an empirical issue to estimate the constants ϕ, ρ, and σ that are required to estimate λ. In our risk-adjusted approach the PDE does not need a market price of risk adjustment. By analogy of risk-adjusted model with risk-neutral model in constant volatility, the risk-adjusted PDE in stochastic volatility can be written as:

(18a) $$\mu SC_S + \frac{1}{2}VS^2C_{SS} + \alpha(\beta - V)C_V + \frac{1}{2}V\gamma^2C_{VV} + \rho\gamma SVC_{SV} + C_t - kC = 0$$

Since the above model is a risk-adjusted model we don't have the market price of risk (λ) term in the equation. The risk-adjusted volatility process is given by:

(18b) $$dV = \left[\alpha(\beta - V)\right]dt + \gamma\sqrt{V}dW_2$$

To solve the above equation we follow the Fourier inversion approach of Chen (2009). The risk adjusted option pricing model is given by:

(19) $$\begin{aligned} C &= e^{-k(T-t)}E[\max\{S(T) - K, 0\}] \\ &= e^{-k(T-t)}U \\ &= e^{-k(T-t)}E[S]\Pi_1 - e^{-k(T-t)}K\Pi_2 \\ &= Se^{-(k-\mu)(T-t)}\Pi_1 - e^{-k(T-t)}K\Pi_2 \end{aligned}$$

where (via Fourier inversion)

$$\Pi_j = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \mathrm{Re}\left[\frac{e^{-iu\ln K}f_j(u)}{iu}\right]du;\ j = 1 \text{ or } 2$$

$$U = Se^{\mu(T-t)}\Pi_1 - K\Pi_2$$

$f_j(u)$ is the characteristic function, and *C* is the call price at time *t*. We guess the form of the solution in second and third line of equation (19).

Before we solve the PDE of equation (18) we do the following transformation:

(20) $$x = \ln S$$

Using the above transformation the original PDE of equation (18a) can be transformed to:

(22) $$\left(\mu - \frac{1}{2}V\right)C_x + \frac{1}{2}VC_{xx} + \alpha(\beta - V)C_V + \frac{1}{2}V\gamma^2 C_{VV} + \rho\gamma V C_{xV} + C_t - kC = 0$$

From equation (19) using the partial first and second order derivatives of *C* with respect to x, *V*, and *t* in equation (22) we have:

(23) $$\left[\mu - \frac{V}{2}\right]U_x + \frac{V}{2}U_{xx} + [\alpha\beta - \alpha V]U_V + \frac{\gamma^2 V}{2}U_{VV} + \rho\gamma V U_{Vx} + U_t = 0$$

Furthermore, *U* of equation (19) can be written as:

(24) $$U = e^{x+\mu(T-t)}\Pi_1 - K\Pi_2$$

Before we transform the PDE of equation (23) to ordinary differential equations we compute the partials of *U* with respect to x, *V*, and *t* using equation (24):

(25) $$\begin{cases} U_x = e^{x+\mu(T-t)}\Pi_1 + e^{x+\mu(T-t)}\Pi_{1x} - K\Pi_{2x} \\ U_{xx} = e^{x+\mu(T-t)}\Pi_1 + 2e^{x+\mu(T-t)}\Pi_{1x} + e^{x+\mu(T-t)}\Pi_{1xx} - K\Pi_{2xx} \\ U_V = e^{x+\mu(T-t)}\Pi_{1V} - K\Pi_{2V} \\ U_{VV} = e^{x+\mu(T-t)}\Pi_{1VV} - K\Pi_{2VV} \\ U_{xV} = e^{x+\mu(T-t)}\Pi_{1V} + e^{x+\mu(T-t)}\Pi_{1xV} - K\Pi_{2xV} \\ U_t = e^{x+\mu(T-t)}(-\mu)\Pi_1 + e^{x+\mu(T-t)}\Pi_{1t} - K\Pi_{2t} \end{cases}$$

Using equation (25), PDE of equation (23) is written as two partial differential equations corresponding to the partials of Π_1 and Π_2 as follows:

(26) $$\begin{cases} \frac{V}{2}\Pi_{1xx} + \frac{\gamma^2 V}{2}\Pi_{1VV} + \rho\gamma V\Pi_{1xV} + [\alpha\beta - (\alpha - \rho\gamma)V]\Pi_{1V} + \left[\mu + \frac{V}{2}\right]\Pi_{1x} + \Pi_{1t} = 0 \\ \frac{V}{2}\Pi_{2xx} + \frac{\gamma^2 V}{2}\Pi_{2VV} + \rho\gamma V\Pi_{2xV} + [\alpha\beta - \alpha v]\Pi_{2V} + \left[\mu - \frac{V}{2}\right]\Pi_{2x} + \Pi_{2t} = 0 \end{cases}$$

The following characteristic function satisfies the above partial differential equations:

(27) $$f_j = e^{C_j(T-t)+D_j(T-t)V+iux}, \ J=1,2.$$

C_j and D_j are functions of the parameters $\alpha, \beta, \gamma, \lambda,$ and ρ, and the time to maturity.

Using an approach similar to Chen (2009) we can solve the above PDEs, and write the solutions to D_j and C_j as follows:

(28) $$D_1 = \frac{d_1}{\gamma^2}\left[\frac{\left(\frac{d_1+\rho\gamma(1+iu)-\alpha)}{d_1-\rho\gamma(1+iu)+\alpha)}\right)e^{-d_1(T-t)} - 1}{\left(\frac{d_1+\rho\gamma(1+iu)-\alpha)}{d_1-\rho\gamma(1+iu)+\alpha)}\right)e^{-d_1(T-t)} + 1} - \frac{\rho\gamma(1+iu)-\alpha}{d_1}\right]$$

and

(29) $$D_2 = \frac{d_2}{\gamma^2}\left[\frac{\left(\frac{d_2+\rho\gamma iu-\alpha}{d_2-\rho\gamma iu+\alpha}\right)e^{-d_2(T-t)} - 1}{\left(\frac{d_2+\rho\gamma iu-\alpha}{d_2-\rho\gamma iu+\alpha}\right)e^{-d_2(T-t)} + 1} - \frac{\rho\gamma iu-\alpha}{d_2}\right]$$

where

$$d_1 = \sqrt{[\rho\gamma(1+iu)-\alpha]^2 - \gamma^2 \ \ iu - u^2}$$

$$d_2 = \sqrt{[\rho\gamma iu-\alpha]^2 + \gamma^2 \ \ iu + u^2} \ ;$$

(30)

$$C_1 = -\frac{\alpha\beta}{\gamma^2}\Big[-(\alpha - \rho\gamma(1 + iu)) + \sqrt{(\alpha - \rho\gamma(1 + iu))^2 + \gamma^2(u^2 - iu)}\Big](T - t)$$
$$-\frac{2\alpha\beta}{\gamma^2}\left\{\begin{array}{l}\ln\left[1 + \frac{-(\alpha-\rho\gamma(1+iu))+\sqrt{(\alpha-\rho\gamma(1+iu))^2+\gamma^2(u^2-iu)}}{(\alpha-\rho\gamma(1+iu))+\sqrt{(\alpha-\rho\gamma(1+iu))^2+\gamma^2(u^2-iu)}}e^{-\sqrt{(\alpha-\rho\gamma(1+iu))^2+\gamma^2(u^2-iu)}(T-t)}\right] \\ -\ln\left[1 + \frac{-(\alpha-\rho\gamma(1+iu))+\sqrt{(\alpha-\rho\gamma(1+iu))^2+\gamma^2(u^2-iu)}}{(\alpha-\rho\gamma(1+iu))+\sqrt{(\alpha-\rho\gamma(1+iu))^2+\gamma^2(u^2-iu)}}\right]\end{array}\right\}$$

and

(31)

$$C_2 = -\frac{\alpha\beta}{\gamma^2}\Big[-\alpha + \sqrt{\alpha^2 + \gamma^2(u^2 + iu)}\Big](T - t)$$
$$-\frac{1}{\gamma^2}\left\{\ln\left[1 + \frac{-\alpha+\sqrt{\alpha^2+\gamma^2(u^2+iu)}}{\alpha+\sqrt{\alpha^2+\gamma^2(u^2+iu)}}e^{-\sqrt{\alpha^2+\gamma^2(u^2+iu)}(T-t)}\right] - \ln\left[1 + \frac{-\alpha+\sqrt{\alpha^2+\gamma^2(u^2+iu)}}{\alpha+\sqrt{\alpha^2+\gamma^2(u^2+iu)}}\right]\right\}$$

As we note unlike the risk-neutral approach in this risk-adjusted model C_j and D_j do not contain λ, the market price of volatility risk. Instead, λ is in the expected growth rate of the option price using the ICAPM equation (15). Therefore, using equation (15) equation (19) can be written as:

(32)
$$\begin{aligned} C &= e^{-k(T-t)}\Big[Se^{\mu(T-t)}\Pi_1 - K\Pi_2\Big] \\ &= e^{-\left(r+\frac{C_S S}{C}(\mu-r)+\frac{C_V}{C}\lambda V\right)(T-t)}\Big[Se^{\mu(T-t)}\Pi_1 - K\Pi_2\Big] \end{aligned}$$

where Π_1 and Π_2 can be solved using the values of C_j and D_j from equation (28) through (31).

6.5 Methods of Estimation

Unlike the risk-adjusted constant volatility model there are many unknown parameters in additions to the ex ante expected stock return (μ), in this model. More specifically the unknown parameters in equation (32) are μ, α, β, γ, λ, and ρ. In one extreme, when all the parameters are considered ex ante, we need six or more near the money call options with same maturity for our estimation. Similar to the risk-adjusted constant volatility, we use a grid search for global optima to estimate these parameters. The limitation of this method is that for many stocks we may not have six options that are actively traded for a given day with same maturity. Therefore, this method will discard many options for most of the stocks. An alternative method to estimate these parameters is to have a three step approach. In the first step we estimate α, β and γ using the physical world historical volatilities along with the econometric model corresponding to equation (18b). In the second step we compute the risk-neutral probability density for the stock price. To obtain the risk-neutral probability density, we use the Black-Scholes implied volatility smile. From the smile curve, we extract a set of call price and the corresponding strike prices. Then we use the following Breeden and Litzenberger (1978) result to extract the risk-neutral density:

(34) $$\frac{\partial^2 C}{\partial K^2} e^{-r(T-t)} = p(K, T \mid S, t)$$

where $p(K, T \mid S, t)$ is the conditional risk-neutral probability of an underlying price to reach the level K at time T given the price at time t is S. From this risk-neutral density we can compute the volatilities in risk-neutral world for any time t. By this approach we obtain a time-series of risk-neutral volatilities. Using these risk-neutral volatilities with the estimates of α, β and γ from the previous step, along with the econometric model corresponding to equation (18b) we can estimate λ. We use the maximum likelihood estimator for the above two steps. In the last step we use the above estimated $\alpha, \beta, \gamma,$ and λ along with equation (33) to compute ρ and μ. Using this approach we need only two traded options to estimate ρ and μ. When the parameters are stationary the second approach is a better approach that does not need many option records. A third method could be to identify the specific parameters that can be considered stationary. This would be a separate interesting empirical study. Once the stationary parameters are identified, these parameters can be estimated using the econometric models as in the previous method. Then the remaining parameters can be estimated along with μ using equation (33).

6.6 Conclusion

In this chapter we used a mean-reverting square root volatility process in addition to the geometric Brownian price process, to estimate risk-adjusted expected return. Similar to the prior chapters, using this expected return, we can examine whether the term structure of expected return remains with stochastic volatility approach. We can also use the information content of risk-adjusted stochastic volatility to forecast ex post volatilities. Similar to the ex ante test of standard CAPM we can derive a single period CAPM with stochastic volatility and then pursue an ex ante test of this version of CAPM using the expected return estimates. Our approach provides a starting point for estimation of risk-adjusted expected return and volatility from option prices using different assumption of stochastic processes. These risk-adjusted parameters can be used to understand the ex ante underlying return distribution and to examine different versions of capital asset pricing models that are consistent with these risk-adjusted model assumptions.

References

Andersen, T. G., and T. Bollerslev, F. X. Diebold, and. H. Ebens, 2001, "The Distribution of Realized Stock Return Volatility," *Journal of Financial Economics*, 61, 43-76.

Aït-Sahalia, Y., P. A. Mykland, and L. Zhang, 2005, "How Often to Sample a Continuous-Time Process in the Presence of Market Microstructure Noise," *Review of Financial Studies*, 18, 351-416.

Ayres, Herbert F., 1963, “Risk Aversion in the Warrants Market,” *Industrial Management Review*, 4, 497-505. Reprinted in Cootner (1967), 497-505.

Bates, David S., 1991, “The crash of '87: Was it expected? The evidence from option markets,” *Journal of Finance*, 46, 1009-1044.

Beckers, S., 1981, "Standard Deviations Implied in Option Prices as Predictors of Future Stock Price Variability," *Journal of Banking and Finance*, 5, 363-381.

Beckers, S., 1983, "Variances of security price returns based on high, low. and closing prices," *Journal of Business*, 56, 97-112.

Black, Fischer, 1972, “Capital Market Equilibrium with Restricted Borrowing,” *Journal of Business*, 45, 444-455.

Black, F. and M. Scholes, 1972, “The Valuation of Option Contracts and a Test of Market Efficiency,” *Journal of Finance* 27, 399-417.

Black, Fisher, and Myron Scholes, 1973, “The Pricing of Options and Corporate Liabilities,” *Journal of Political Economy*, 81, 637-654.

Black, F., 1976, "Studies of stock price volatility changes," *Proceedings of the 1976 Meetings of the American Statistical Association*, 177-181.

Blair, B. , S.-H. Poon and S. Taylor, 2001, “Forecasting S&P100 Volatility: The Incremental Information Content of Implied Volatility and High Frequency Index Returns,” *Journal of Econometrics*, 105, 5-26.

Blattberg, R. C., and N.J. Gonedes, 1974, "A Comparison of the Stable and Student Distributions as Statistical Models for Stock Prices," *Journal of Business*, 47, 244-280.

Bollerslev, T., 1986, "Generalized Autoregrcssive Conditional Heteroskedasticity," *Journal of Econometrics*, 31, 307-327.

Bollersiev, T., R.Y. Chou, and K.F. Kroner, 1992, "ARCH Modeling in Finance: A Review of the Theory and Empirical Evidence," *Journal of Econometrics*, 52, 5-59.

Boness, James A., 1964, "Elements of a Theory of Stock-Option Values," *Journal of Political Economy*, 72, 163-175.

Botosan, C., and M. Plumlee, 2005, "Assessing alternative proxies for the expected risk premium," *The Accounting Review* 80, 21-54.

Brav, A., R. Lehavy, and R. Michaely, 2005, "Using Expectations to Test Asset Pricing Models." *Financial Management* Autumn:5–37.

Breeden, D. and Litzenberger, R., 1978, "Prices of State Contingent Claims Implicit in Options Prices," *Journal of Business*, 51, 621-651.

Brennan, Michael J., 1979, "The pricing of contingent claims in discrete time models," *Journal of Finance,* 34, 53-68.

Camara, Antonio, 2003, "A Generalization of the Brennan-Rubinstein Approach for the Pricing of Derivatives," *Journal of Finance*, 58, 805-819.

Camara, A., S. Chung, and Y. Wang, 2009, "Option Implied Cost of Equity and Its Properties," *Journal of Futures Markets*, 29, 599-629.

Campello, M., L. Chen, and L. Zhang, 2008, "Expected Return, Yield Spreads, and Asset Pricing Tests," *Review of Financial Studies* 21, 1297-1338.

Canina, L. , and S. Figlewski, 1993, "The Informational Content of Implied Volatility," *Review of Financial Studies*, 6, 659-681.

Chen, Ren-Raw, 2009, “Financial Mathematics,” *Global Social Science Institute*.

Chen, N. F., Roll, N., and Ross, S. A., 1986, "Economic Forces and the Stock Market," *Journal of Business*, 59, 383–404.

Chiras, D. P. , and S. Manaster, 1978, "The Information Content of Option Prices and a Test of Market Efficiency," *Journal of Financial Economics*, 6, 213-234.

Christensen, B. and N. Prabhala, 1998, “The Relation Between Implied and Realized Volatility,” *Journal of Financial Economics*, 50, 125-150.

Christie, A., 1982, "The stochastic behavior of common stock variances: Value. leverage, and interest rate effects," *Journal of Financial Economics* 10, 407-432.

Christoffersen, Peter., Jacobs, Kris., Vainberg Gregory., 2006, “Forward-Looking Betas,”Faculty of Management, McGill University, Working Paper Series.

Cox, J. C., J. E. Ingersoll, and S. A. Ross, 1985a, "An Intertemporal General Equilibrium Model of Asset Prices," *Econometrica*, 53, 363-384.

Cox, J. C., J. E. Ingersoll, and S. A. Ross, 1985b, "A Theory of the Term Structure of Interest Rates," *Econometrica*, 53, 385-408.

Cox, John C., and Ross Stephen A. , 1976a, “The Valuation of Options for Alternative Stochastic Processes,” *Journal of Financial Economics*, 3, 145-166.

Cox, John C., and Ross Stephen A., 1976b, “A Survey of Some New Results in Financial Option Pricing Theory,” *Journal of Finance* 31, 383-402.

Cox, John C., Stephen A. Ross, and Mark Rubinstein, 1979, “Option pricing: A simplified approach,” *Journal of Financial Economics*, 7, 229-264.

Davidson, Russell, and James MacKinnon, 1981, "Several Tests for Model Specification in the Presence of Alternative Hypotheses,” *Econometrica*, 49, 781-793.

Day, T. E., and C. M. Lewis, 1992, "Stock Market Volatility and the Information Content of Stock Index Options," *Journal of Econometrics*, 52, 267-287.

Duffie, D., and C-F. Huang, 1985 (November), "Implementing Arrow-Debreu Equilibria by Continuous Trading of Few Long-lived Securities," Econometrica. Vol. 53 (6). p 1337-56.

Easton, Peter., 2003, “ PE Ratios, PEG Ratios and Estimating the Implied Expected Rate of Return on Equity Capital,” *SSRN Working Paper Series*.

Easton, P., and S. Monahan, 2005, “An evaluation of accounting-based measures of expected returns,” *The Accounting Review* 80, 501-538.

Eisenberg, L. K., and Jarrow, R., 1994, “Option Pricing with Random Volatilities in Complete Markets,” *Journal of Quantitative Finance and Accounting*, 4, 5-17.

Elton, Edwin, 1999, “Expected Return, Realized Return, and Asset Pricing Tests,” *Journal of Finance*, Vol. 54, 1199-1220.

Engle, R.F., 1982, "Auroregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation," *Econometrica*, 50, 987-1006.

Fama, E., 1965, "The behavior of stock market prices," *Journal of Business*, 38, 34-105.

Fama, E.F., and K.R French, 1988, “Dividends Yields and Expected Stock Returns,” *Journal of Financial Economics* 22, 3-25.

Fama, E.F., and K.R French, 1992, “The Cross-Section of Expected Stock Returns,” *Journal of Finance* 47, 427-465.

Fama, E.F. and K.R French, 1993, “Common Risk Factors in the Returns on Stocks and Bonds,” *Journal of Financial Economics* 33, 3-56.

Fama, E.F. and K.R. French, 1996, “Multifactor Explanations of Asset Pricing Anomalies,” *Journal of Finance*, 51, 1121-1152.

Fama, E.F. and K.R. French, 1997, “Industry Costs of Equity,” *Journal of Financial Economics* 43, 153-193.

Fama, E.F. and K.R. French, 2002, “The Equity Premium,” *Journal of Finance* 57, 637-659.

Fama. E.F., and J. MacBeth. 1973. “Risk Return and Equilibrium: Empirical Tests,” *Journal of Political Economy* 81, 607-636.

Galai, Dan, 1978, “On the Boness and Black-Scholes Models for Valuation of Call Options,” *Journal of Financial and Quantitative Analysis*, 13, 15-27.

Garman, M., 1976, "A General Theory of Asset Valuation under Diffusion State Processes," *Working Paper No. 50*, University of California, Berkeley.

Gebhardt, W., C. Lee, and B. Swaminathan, 2001, Toward an Implied Cost of Capital, *Journal of Accounting Research* 39, 135-176.

Granger, C. and S.-H. Poon, 2005, “Practical Issues in Forecasting Volatility,” *Financial Analysts Journal*, 61, 45-56.

Harvey, C.R., and R.E. Whaley, 1992, "Market Volatility Prediction and the Efficiency of the S&P 100 Index Option Market," *Journal of Financial Economics*, 31, 43-74.

Hentschel Ludger, 2003, "Errors in Implied Volatility Estimation," *Journal of Financial and Quantitative Analysis*, 38, 779-810.

Heston, Steven L., 1993a, "A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options," *Review of Financial Studies*, 6, 327-343.

Heston, Stephen L., 1993b, "Invisible Parameters in Option Prices," *Journal of Finance*, 48 3, 933-947.

Hull, J., 2002, "Options, Futures, and Other Derivatives," *5th edition, Prentice Hall*, Upper Saddle River, NJ.

Hull, J., and A. White, 1987, "The Pricing of Options on Assets with Stochastic Volatilities," *Journal of Finance*, 42, 281-300.

Jarrow, Robert, 1980, "Heterogeneous Expectations, Restrictions on Short Sales, and Equilibrium Asset Prices," *Journal of Finance*, 35, 1105-1113.

Jegadeesh, N. and S. Titman, 1993, "Returns on Buying Winners and Selling Losers: Implications for Stock Market Efficiency," *Journal of Finance*, 48, 65-91.

Kothari, S.P., J. Shanken, and R. Sloan, 1995, "Another Look at the Cross Section of Expected Stock Returns," *Journal of Finance,* 50, 185-224.

Lakonishok, J., Shleifer, A., and Vishny, R. W., 1994, "Contrarian Investment, Extrapolation, and Risk," *Journal of Finance* 49, 1541-1578.

Lamoureux, G. and W. D. Lastrapes, 1993, "Forecasting Stock-Return Variance: Toward an Understanding of Stochastic Implied Volatilities," *Review of Financial Studies*, 6, 293-326.

Latané, H. A., and R. J. Rendleman, 1976, "Standard Deviations of Stock Price Ratios Implied in Option Prices," *Journal of Finance,* 31, 369-381.

Leland, Hayne E., 1985, "Option Pricing and Replication with Transactions Costs," *Journal of Finance,* 40, 1283-1301.

Levhari, D., and H. Levy, 1977, "The Capital Asset Pricing Model and the Investment Horizon," *Review of Economics and Statistics* 59, 92-104.

Levy, Haim, 1985,"Upper and Lower Bounds of Put and Call Option Value: Stochastic Dominance approach," *Journal of Finance*, 40, 1197-1217.

Levy, M., 1997, "Computer-assisted language learning: Context and conceptualization," Oxford University Press, New York.

Lintner, John, 1965, “The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets,” *Review of Economics and Statistics*, 47, 13-37.

Longstaff, Francis A., 1995, “Option Pricing and the Martingale Restriction,” *Review of Financial Studies*, 8, 1091-1124.

Macbeth, J. D., and L. J. Merville, 1980, "Tests of the Black-Scholes and Cox Call Option Valuation Models," *Journal of Finance*, 35, 285-300.

Mandelbrot, B.B., 1963, "The Variation of Certain Speculative Prices," *Journal of Business*, 36, 394-416.

McNulty, J. J., T. D. Yeh, W. S. Schulze and M. H. Lubatkin, 2002, “What’s Your Real Cost of Capital?,” *Harvard Business Review*, 114-121.

Melick, W. R., and C. P. Thomas, 1997, “Recovering an Asset's Implied PDF from Option Prices: An Application to Crude Oil During the Gulf Crisis,” *Journal of Financial and Quantitative Analysis*, 32, 91-115.

Merton, Robert C., 1973, “Theory of Rational Option Pricing,” *Bell Journal of Economics and Management Science*, 4, 141-183.

Merton, Robert C., 1976a, “Option Pricing When Underlying Stock Returns are Discontinuous,” *Journal of Financial Economics*, 3, 125-144.

Merton, Robert C., 1976b, “The Impact on Option Pricing of Specification Error in the Underlying Stock Price Returns,” *Journal of Finance*, 31, 333-350.

Mossin, J., 1966, “Equilibrium in a capital asset market,” *Econommetrica*, 34, 768-783.

Nelson, D. B., 1990, "ARCH Models as Diffusion Approximations," *Journal of Econometrics*, 45, 7-38.

Pastor, L., M. Sinha, and B. Swaminathan, 2008, "Estimating the Intertemporal Risk-Return Tradeoff using the Implied Cost of Capital," *Journal of Finance*, 63, 2859-2897.

Pastor, L., and Stambaugh, R. F., 1999, "Costs of Equity Capital and Model Mispricing," *Journal of Finance* 54, 67-121.

Perrakis, Stylianos and Peter J. Ryan, 1984, “Option pricing bounds in discrete time,” *Journal of Finance,* 39, 519-525.

Poterba, J., and L. Summers, 1986, "The Persistence of Volatility and Stock Market Fluctuations," *American Economic Review,* 76, 1142-1151.

Roll, R., 1977, “A Critique of the Asset Pricing Theory’s Tests,” *Journal of Financial Economics,* 4, 129-176.

Ross, Stephen A., 1976, “Options and Efficiency,” *Quarterly Journal of Economics*, 90, 75-89.

Rubinstein, Mark, 1976, "The valuation of uncertain income streams and the pricing of options," *Bell Journal of Economics,* 7, 407-425.

Rubinstein, Mark, 1985, "Nonparametric Tests of Alternative Option Pricing Models Using All Reported Trades and Quotes on the 30 Most Active CBOE Option Classes from August 23, 1976 Through August 31, 1978," *Journal of Finance,* 40, 455-480.

Samuelson, Paul A., 1965, "Rational Theory of Warrant Pricing," *Industrial Management Review,* 6, 13-31. Reprinted: 1967, Cootner, 506-532.

Scott, Louis O. , 1987, "Option Pricing When the Variance Changes Randomly: Theory, Estimation, and an Application, " *Journal of Financial and Quantitative Analysis*, 22, 419-438.

Sprenkle, Case, 1961, "Warrant Prices as Indications of Expectations," *Yale Economic Essays*, 1, 179-232. Reprinted in Cootner (1967), 412-474.

Sharpe, William F., 1964, "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk," *Journal of Finance*, 19, 425-442.

Sharpe, W.F., 1978, "New Evidence on the Capital Asset Pricing Model: Discussion," *Journal of Finance,* 33, 917-920.

Shefrin, H. and M. Statman, 2003, "The Style of Investor Expectations," *The Handbook of Equity Style Management*, Thomas Coggin and Frank Fabozzi, Editors.

Siegel, A., 1995, “Measuring Systematic Risk Using Implicit Beta,” *Management Science*,” 41, 124-128.

Sprenkle, Case, 1961, “Warrant Prices as Indications of Expectations.” *Yale Economic Essays*, 1, 179-232. Reprinted in Cootner (1967), 412-474.

Stein, J., 1989, "Overreactions in the Options Market," *Journal of Finance*, 44, 1011-1023.

Stein, E.M., and J. C. Stein, 1991, “Stock Price Distributions with Stochastic Volatility: An Analytic Approach,” *Review of Financial Studies*, 4, 727-752.

Taylor, S.J., 1994, "Modeling Stochastic Volatility; A Review and Comparative Study," *Mathematical Finance*, 4, 183-204.

Taylor, S.J., and X. Xu, 1994, "The Magnitude of Implied Volatility Smiles: Theory and Empirical Evidence for Exchange Rates," *Review of Futures Markets*, 13, 355-380.

Vanden, Joel M., 2004, “Options Trading and the CAPM,” *Review of Financial Studies*, 17, 207-238.

Whaley, Robert E., 1986, "Valuation of American Futures Options: Theory and Empirical Tests," *Journal of Finance,* 41, 127-150.

Wiggins, J.B., 1987,"Option Values under Stochastic Volatilities," *Journal of Financial Economics,* 19, 351-372.

VITA

1986	Bachelor of Technology (Mechanical Engineering), Regional Engineering College, Calicut, India.
1987-1988	P.G.D.I.E. (Industrial Engineering), National Institute of Industrial Engineering, Mumbai. India.
1988	Internship/ Management Trainee, Voltas, Mumbai, India.
1989-1990	Software consultant for client companies including Credit Rating Information Services of India and Continental Insurance Company.
1991-1996	Software consultant and Systems Analysts for the client, American International Group.
1996-2004	Software consultant and Systems Analysts for the client, JP Morgan Chase.
2005	Master of Quantitative Finance, Rutgers Business School, Rutgers University, Newark, New Jersey.
2005-2009	Teaching Assistantship, Rutgers Business School, Rutgers University, Newark, New Jersey.
2009-2010	Dissertation Fellowship, Rutgers Business School, Rutgers University, Newark, New Jersey.
2010	Ph.D. in Finance, Rutgers Business School, Rutgers University, Newark, New Jersey.