UNDERGRADUATES' (MIS) UNDERSTANDING OF PERCENTAGES

by

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ABSTRACT OF THE THESIS

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Children and adults alike continue to have difficulty with rational numbers, and, as shown in this study, percentages. Adult undergraduate students ($N = 27$) completed an untimed open-ended study that had a pretest-training-posttest design. Questions in the pretest and posttest focused on percent increases and decreases (e.g., *The price x was increased by 20%*). They were formatted in one of three representations: decimal, fraction, or bar graph. Subjects were randomly assigned to one of two training groups (decimal or fraction) that involved guiding them through the conceptual and procedural component processes that contribute to the solutions to problems within a given representation (i.e., decimal or fraction). Additional questions were asked in the pretest and posttest pertaining to successive percent increases and decreases. Overall, students performed best on bar graph representations. While decimal training had no effect on performance, fraction training improved performance from pretest to posttest in every category (decimal, fraction and bar graph). Students who had not yet taken Calculus were shown to have benefited more from the fraction training than those students who had taken that course. In contrast, prior math experience did not seem to have any effect on those questions dealing with two percent changes within one problem; less than 25% of these types of problems were answered correctly by the undergraduate students.

Acknowledgments

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Undergraduates' (Mis) Understanding of Percentages

Individuals differ radically from each other in the way they choose to approach and solve problems, especially those of a mathematical nature. Admittedly, there is usually one theoretically correct answer, but that answer can almost *always* be represented in numerous mathematical formats and can be derived using multiple algorithms.

To begin, consider the following problem:

The cost $x of a pair of skis has increased by 50%. When the ski season is over, that new cost is *then* *decreased by 50%. What is the final cost of the skis after this reduction, in terms of x?*

What would your response be? Are you be surprised to learn that a sample of undergraduate Psychology majors (N=27) answered less than 25% of 54 of these types of problems correctly? If you arrived at an answer of 75% of the original cost, .75*x*, (¾)*x*, or some variation of that, then you were correct. However, even many more educated individuals make the same mistakes as these undergraduates. Throughout the paper, this type of problem will be referred to as the "50% increase/decrease problem".

Every day, people are inundated with rational numbers all around them, from the "% Daily Values" on nutrition labels to sales and discounts in our favorite stores to the constant ups and downs of gas prices. And yet, there is a great deal of evidence demonstrating that students of all ages continue to have difficulty with fractions and rational numbers (Humberstone & Reeve, 2007). This is relevant to the professional and educational communities' concern about the rather high levels of math illiteracy. My earlier pilot data has shown that not only do college students persistently make mathematical errors, but they also have difficulty recognizing errors across different representations of a rational number. Some mistakes are due to computational errors, but many reflect the absence of conceptual understanding or a failure of learning. If one has difficulty with rational number representations, what do they understand about the rise and fall of the stock market, the interest on their loans, and the degree of risk associated with their choices? My research falls within the realm of both perceptual and conceptual understanding. In everyday life, how do people calculate cost increases or decreases so that they are able to make educated decisions in comparing products out on the market?

School math practices

School math practices are informed by the discipline, by psychological theories concerning how math is learned, and by the conventions of the school, at least in the United States. Exams influence what is taught. As a result, school math has an uneven relationship with meaning. Teaching to the stated curriculum or to the test can stand in the way of a student's ability to represent concepts in multiple representations. For example, if an instructor is concentrating primarily on teaching students the most time-efficient way to solve a problem, (s)he will be encouraging repetition through practice with the mechanics, not necessarily the concepts involved. It limits what educators might feel is necessary to teach, particularly when time constraints become an issue.

In the classroom, the steps to solve a math word problem are commonly outlined as follows: 1) read the problem, 2) use variables to represent known quantities, 3) use these symbols to write an expression to satisfy the conditions stated in the problem, 4) solve for unknown variables, 5) respond to the question asked (which may involve multiple steps after solving for the variables), and 6) verify the solution by substituting values into the equation (Hawkes, Luby, & Touton, 1929; Koedinger & MacLaren, 1997). It is almost as important to make sure that the answer makes SENSE in the context of the problem as it is to arrive at the correct answer. In the study presented here, a participant should confirm that her answer to a percent *decrease* question yields a value *less than* the original value. For instance, if a $200 dollar item is on sale for 10% off the original price, then the participant should anticipate a value less than $200. Checking that an answer makes sense emerges in other mathematical contexts as well (e.g. a negative number multiplied by a negative number yields a positive answer; the square of a negative number is positive; a correlation cannot be greater than 1.00, a sum of squares cannot be negative; etc.).

Math equations and information processing theories

The processes used in configuring mathematical algorithms can sometimes be described in information processing terms. Paige (1966) suggests the move to "postulat[ing] in detail a precise set of mechanisms to account for the observed behavior". It certainly would be ideal to be able to identify the mechanisms with which adults solve problems, especially in the area of mathematics. In solving any type of mathematical problem, there are multiple steps to be taken. The more complex the problem is, the more steps that are usually involved. In the classroom, traditional textbooks teach students to first translate sentences from word problems into equations before attempting to solve those equations (Hawkes, Luby, & Touton, 1929, as cited in Paige, 1966; Zhu & Simon, 1987).

Previous research

While little attention has been paid to problems with percentages, there *has* been some research on problems with fractions, which is one of the representations I am studying. Researchers have found results emphasizing the importance of understanding the concept of a unit. Activities relating the unit to fractions can strengthen this concept (Stafylidou & Vosniadou, 2004). We already know that proportions dealing with a fraction of ½ are much easier for children than other proportions (Spinillo & Bryant, 1991). Most work focuses on proper fractions such as ½, but not on problems with improper fractions and mixed numbers (Humberstone & Reeve, 2007). The latter requires understanding the concepts of proper fractions and the ability to use fractions in roles other than fraction addition tasks. Understanding alternative representational formats, such as fractions, is critical for effectively working with algebraic techniques. Yet, it has been shown that children acquire knowledge about natural numbers and their successors much more easily than with numbers containing fractional notation (Hartnett & Gelman, 1998). This evidence conveys the difficulties that exist in understanding equivalent forms of mathematical representations.

Recognizing multiple representations to a solution of a problem is relevant to both the acquisition of understanding and the pedagogy for teaching mathematical concepts. Past studies have led participants to ask questions and discuss responses with others, which, in turn, enabled them to evaluate their own existing concepts. Rittle-Johnson & Star (2007) have also shown that comparing alternative solution methods has great value conceptually and procedurally for seventh-grade students. This articulation accounts for higher-order thinking (Zaslavsky & Shir, 2005), which entails critical thinking and problem solving skills. Furthermore, providing explanations allows for recognition of gaps in knowledge; acknowledging these missing links paves the path toward achieving a greater understanding of the concept (Mills & Keil, 2004).

Teaching students to recognize different methods for solving a problem helps them develop a greater understanding of concepts (Gelman, 1986), as has been a long-held common practice in high-performing countries like Japan and Hong Kong. There, instructors allow time for students to produce and discuss various ways to problem solve amongst each other (Stigler & Hiebert, 1999, Rittle-Johnson & Star, 2007). While the solution processes might be different, they must share the common mathematical principles that underlie understanding. It therefore would seem to follow that students should be taught to share and compare multiple representations. This recommendation is slowly becoming part of the curriculum recommended in the United States by the National Council of Teachers of Mathematics *Standards* (1989, 2000), although it was not based on evidence. Nevertheless, there is surprisingly little relevant research. In fact, Piñon (2000) failed to show an advantage of teaching a graphic and algebraic format for multiplying two equations. This aside, other researchers have shown that students of all ages become more proficient at understanding concepts when examples accompany practice problems (Atkinson, Derry, Renkl, & Wortham, 2000, as cited by Rittle-Johnson & Star, 2007).

Current study

How do people calculate cost increases or decreases so that they are able to make educated decisions in their daily life? By presenting problems of the kind that occur every day, I explored the extent of difficulty undergraduates have with percentages that have equivalent fraction and decimal formats. In examining the approaches to problems, kinds of errors and the patterns of these, I expected to gain insight into the kinds of syntax of the mathematics that are at risk. By syntax, I refer to the rules and principles that help us construct mathematical models and algorithms. This syntax is about "representing one structure in terms of another and figuring out what relationships obtain among elements of structures" (Kaput, 1987, as cited in Lampert, 1991).

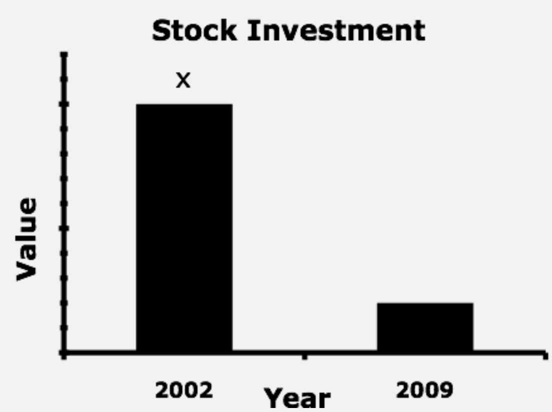
The three representations used throughout this study were chosen deliberately. Most often, we see percentages represented in terms of decimals. Less frequently, but still somewhat often, we see them represented as fractions. The decision to use bar graphs, instead of another type of "picture" such as a pie chart or a line graph, etc., was in effort to make the problems across representations as similar in structure as possible.

Figure 1. Examples of formats used in experiment

Decimal: The stock you invested in in 2002 has now fallen by **.80** of its original value *x*.

Fraction: The stock you invested in in 2002 has now fallen by **4/5** of its original value *x*.

*Percentages & Graphs**: The stock you invested in in 2002 has now fallen by **80%** of its original value *x*.



*Bar graphs were used in this experiment

These representations of percentages, of rational number, are encountered everyday, as can be seen in one example above (Figure 1). These examples may pose a problem if there is a lack of understanding. Some formats might, in fact, be easier to understand than others.

Errors committed might indicate a weakness in the facility for one representation over another. Or they might only identify a preference for a specific representation. Thus, it would be ideal to pinpoint whether there exists a lack in ability versus kinds of preferences.

Method

Participants

Twenty-eight adult volunteers were recruited from undergraduate Psychology laboratory courses at Rutgers University in New Brunswick, New Jersey. One subject was excluded due to her incomplete data, leaving $N = 27$. For the analyses, age was not a grouping factor as all subjects were within 19-24 years old, with the exception of one 35 year old female ($M = 22$, $Md = 21$, [25 females]). Human subjects' approval and all relevant consents were obtained before the study began.

Materials and Design

A pretest-training-posttest design was used. Subjects completed an untimed paper and pencil task. For the pretest and posttest, they were asked to follow a series of questions in order. The training materials and the relevant features of the pre- and posttests are as follows:

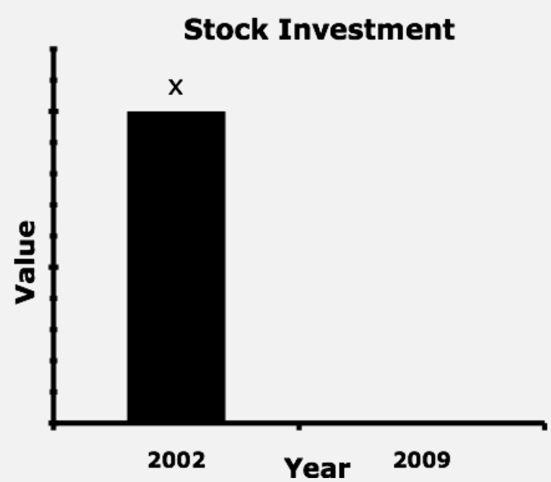
Formats of the pretest and posttest: Each consisted of six percentage problems. Of these, there were three representations of problems: (a) *decimal*, (b) *fraction*, and (c) *bar graph*. In the pretest and posttest, each representation appeared twice in the form of an *increase* problem and a *decrease* problem.

The format of the questions in the pretest and posttest can be seen in Figure 2. Only one format appeared per question. As explained in the procedure below, participants were asked to give their answer in the same representation in which the question was asked.

Figure 2. Pretest/Posttest Questions

Decimal & Fraction
Ex: The stock you invested in in 2002 has now fallen / increased by ***.80 / (4/5)*** of its original value *x*. **What is the new value of the stock? Please represent your final answer in *decimal / fractional* form.**

Percentages & Graphs
Ex: The stock you invested in in 2002 has now fallen/ increased by ***80%*** of its original value *x*. **Please draw a new *bar on the graph* below so that its height shows the stock's value now (in 2009).**



Percentage values. For this experiment, the values used were percentages frequently seen by the layperson, or more specifically, consumers. Percentages included but were not limited to ones commonly used or divisible by 5, numbers such as 20%, 25%, 30%, 50%, 75%, etc. One goal was to determine whether some values were better understood than others.

Variables. Variables (i.e. *x*, *y*, etc.) were used instead of numerical values (i.e.

$350, 168 beats/second, 471 miles). This decision was intentional. An objective of the design was to lessen the "burden" on both the subjects and the experimenter. A pilot study showed that, when responding with number to open-ended questions, raw data were more difficult to code and error patterns were much harder to find. It was also important that subjects stay focused on the conceptual process instead of getting caught up in the mechanics of the procedural process. With numerical values, there most likely would have been more individual differences and more room to err.

Direction. Analyses of previous pilot data also demonstrated that there were little if no performance differences between problems involving percent increase and problems involving percent decrease. However, as earlier data primarily used natural numbers instead of variables, the possibility of a direction effect here could not be ruled out. Therefore, it was included.

Additional "word problem" outside of main design. Another type of problem that appeared in the pretest and in the posttest was the 50% increase/decrease problem mentioned at the beginning of this paper. Psychology undergraduate students (N=27, 25 females) were presented problems (1 increase, 1 decrease) similar to the ski problem in two forms: 1) 50% increase followed by a 50% decrease, and 2) 50% decrease followed by a 50% increase. Unlike the original three types of questions, this type dealt with *two* successive percentage processes within one problem.

General Instructions.

Participants were asked to complete the "workbook" packet sequentially; they were strongly discouraged from returning to a previously completed section (especially after completing the Training task). The directions were:

"The purpose of this study is to give the experimenter an idea of undergraduates' general knowledge about percentage problems. We are not testing your individual abilities, but instead we are interested in what information Rutgers students as a GROUP can provide. This experiment consists of several sections. **It is very important that you take your time and do not rush through the material- you are not being timed!**"

Pretest. Participants received 6 questions: 1 decimal-increase, 1 decimal-decrease, 1 fraction-increase, 1 fraction-decrease, 1 bar graph-increase, 1 bar graph-decrease. They also received 1 problem dealing with a 50% increase followed by a decrease or vice versa.

Training. Students were randomly assigned to one of two groups (decimal or fraction). A student assigned to the "decimal" training condition received two "guided practice" lessons (one *decrease* and one *increase*) with corresponding examples. The format was that of fill-in-the-blanks and incorporated concrete & generic information (i.e. numbers) with algebraic information (i.e. variables). The nature of the explanations provided was conceptual *and* procedural.

Each lesson was structured as follows: A question was asked (i.e. *A t-shirt is 25% off. What is the final cost of the t-shirt?*); subjects were then guided through "workbook-type" questions as paper and pencil tasks. These questions were organized in seven parts.

1) Preparation of generic information (*see Appendix: Section B: Parts I & II: Question 1)*
2) Application of generic information (*Appendix: Section B: Parts I & II: Question 2)*
3) Representation of algebraic information (*Appendix: Section B: Parts I & II: Question 3)*

4) Application of algebraic information (*Appendix: Section B: Parts I & II: Question 4)*
5) Comparison of generic and algebraic information (*Appendix: Section B: Parts I & II: Question 5)*
6) Outline of the overall algorithm (*Appendix: Section B: Parts I & II: Question 6)*
7) Corresponding increase or decrease practice problem (*Appendix: Section B: Parts I & II: "Now Try This")*

At the end of each lesson, subjects were asked and encouraged to check their work. Answers were given (upside-down) at the bottom of the end of the lesson section.

Posttest. Participants received 6 questions very similar in format, but by no means identical, to the questions in the pretest: 1 decimal-increase, 1 decimal-decrease, 1 fraction-increase, 1 fraction-decrease, 1 bar graph- increase, 1 bar graph- decrease. A different ordering of the problems was presented and the word problems differed in content and percentage values. They also received 1 problem dealing with a 50% increase followed by a decrease or vice versa (whichever one they had not seen in the pretest).

Coding

Training Condition. Participants were in the "decimal lesson" training condition *or* the "fraction lesson" training condition.

Previous Math Knowledge. Participants' previous mathematical knowledge was obtained for subsequent data analyses. Students were grouped in one of two categories: below Calculus 1 (considered "lower level math courses" here) or Calculus 1 and above. If a student only listed a basic statistics course as a previous mathematics course (such as the Quantitative Methods course offered by the Rutgers Psychology department), (s)he was placed into the low math knowledge group.

*Group (*confounded variable).* The ordering of the questions was counterbalanced between the pretest and the posttest sections. 14 of the participants received the questions in Section A as their pretest and the questions in Section B as their posttest. 13 of the participants received the opposite: Section B as their pretest and Section A as their posttest. It was later realized that this variable was confounded: The first group (A/B) was comprised of Psychology students enrolled in a Behavioral Neuroscience laboratory course, and the second group (B/A) was comprised of Psychology students enrolled in an Infant & Child Development laboratory course.

Math SAT scores. 14 of the 27 subjects did not report their Math SAT I scores so, unfortunately, that information could not be used.

Scoring. All open-ended responses on the pretest and posttest were sorted as correct or incorrect answers. As mentioned earlier, students were asked to respond using the same representation in which the question was posed. If a correct response was given in a different representation, the subject was not penalized; a note was simply recorded by the experimenter. While subjects' often responded with mathematically lengthy or "showy" responses, an effort was made to score the answers as simply correct or incorrect. Using the same example as seen earlier, Figure 3 aims to briefly denote what the experimenter deemed as acceptable correct answers; please note that it is *not* an exhaustive list. Other representational variations of these responses also fit the criteria for "correct".

Figure 3. Scoring

Earlier example*:

The stock you invested in in 2002 has now fallen by .80 of its original value *x*. What is the new value of the stock?

Acceptable Correct Answers (Corresponding to Type of Question**)

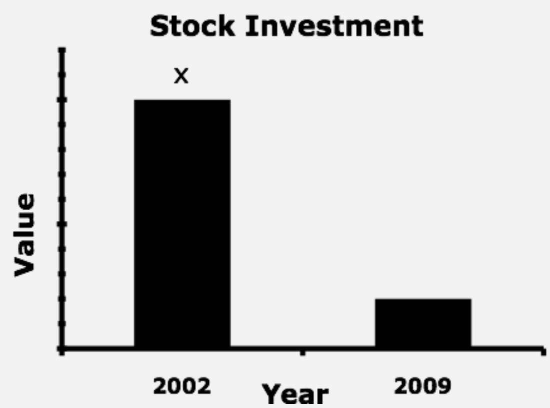
Decimal:

x – .80x **or** .20x

Fraction:

x – (4/5) x **or** (1/5) x

Bar graph:



*Note: Subjects received only 1 form per problem
**Students' additional comments and notes were also recorded

Analyses and the problem of missing data

A 2 (Pretest vs. Posttest) x 3 (Question Type) x 2 (Training Type) x 2 (Group Order) x 2 (Direction) analysis was initially run; later, direction was removed. A logistic regression was used to evaluate the participants' overall performance (percent correct) and the effect of the training models (i.e. fraction training versus decimal training). Additionally, from the subjects' total percent correct on performance, the mean was computed in a separate analysis, as depicted in the figures below. All responses were coded as *0 (Incorrect), 1 (Correct)*, or left blank. We planned to collapse the factors of Direction and Group Order unless initial analyses revealed a difference on these fronts.

Cases of missing data. Missing data was accounted for by extrapolating data where possible. For example, Subject Z was missing the data for both bar graph problems in the posttest. Data was extrapolated from the pretest where the subject was correct for both questions.

In other cases, data were collapsed when finding correlations with no significant difference between them. Doing so allowed us to compensate for the fact that some participants ($n = 3$) did not answer some of the questions in pretest and/or posttest.

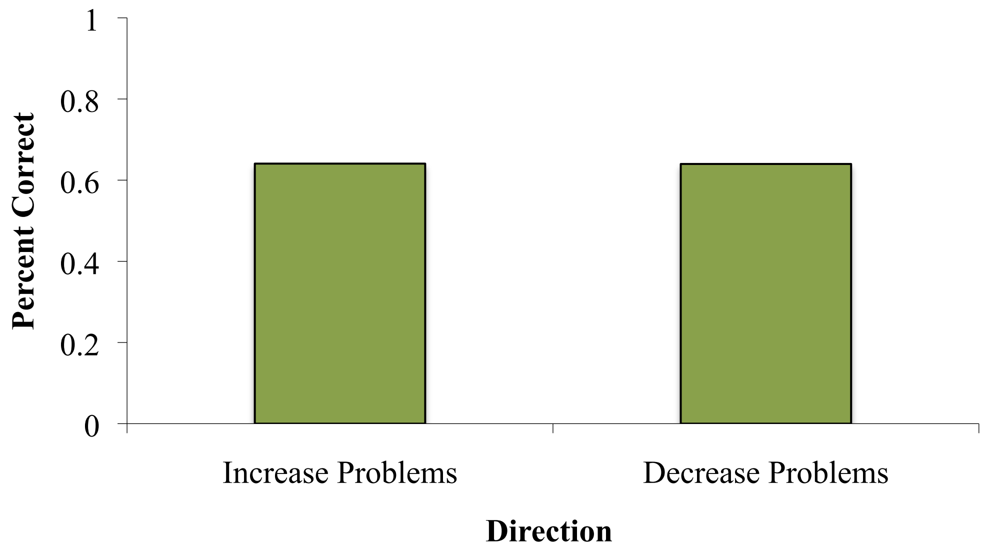
Results

There were three major findings in this study. First, undergraduates performed significantly better on questions with bar graph representations than on those with decimals or fractions across pretest and posttest (bar graphs: M =80.6% correct, decimals: M = 53.3%, fractions: M = 60.1%). Second, upon looking at students with lower math knowledge, *only* training on fraction tasks significantly improved performance across all representations from pretest to posttest (on decimal representations: M =27.3% to M = 45.5%, on fraction: M = 36.4% to M = 59.1%, on bar graph: M = 72.7% to M = 90.9%. Third, when presented with one value of a percent increase or a percent decrease, there was no significant difference of direction. Within the logistic regression, a Chi-Square test examined the relation between the two directions (increase and decrease). As predicted, the relation between these variables was not significant, $\chi^2(1, N = 27) = 0.118$, $p = .731$ (see Figure 4).

Yet, when there exists more than one process within a problem, as in the 50%

increase/decrease problem, participants encounter a significantly greater deal of trouble (i.e. less than 25% of these problems were answered correctly).

Figure 4. Performance on directional problems across Pretest and Posttest ($N = 27$)

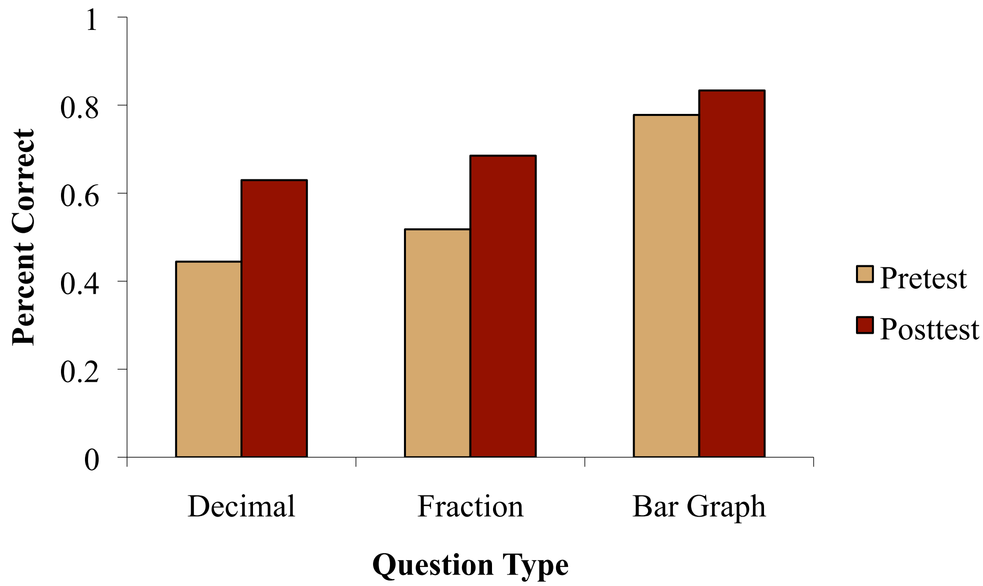


All subjects

Pretest to Posttest. Participants in both group orders (i.e. A/B or B/A as described in *Coding* of the Methods section) consistently performed better on the posttest ($M = 70.4\%$ correct) than on the pretest ($M = 57.9\%$ correct). The breakdown of the representations from the pretest to the posttest (Figure 5*)* demonstrates subjects improved across the board from pretest to posttest; there was a significant main effect for Test here ($\chi^2(1, N = 27) = 5.68, p = .017$).

Question Type (decimal, fraction, bar graph). The analysis showed a main effect of Question Type ($\chi^2(2, N = 27) = 11.41, p = .003$) as can be seen in Figure 5. Subjects performed better on bar graph questions than on other question types across both the pretest and the posttest.

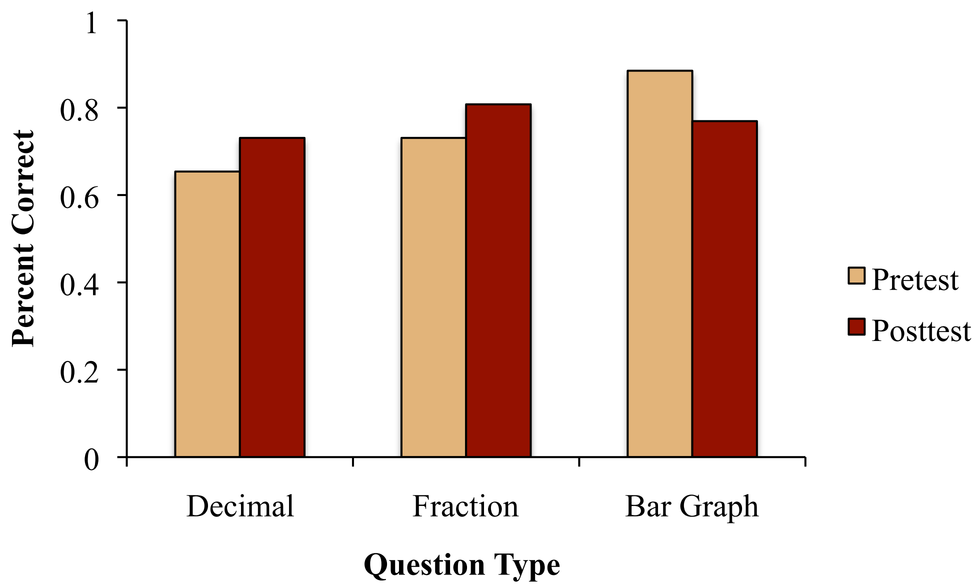
Figure 5. Overall performance across all representations (N = 27)



Training. There were only two training formats in the experiment, as mentioned earlier: participants were either randomly assigned to a decimal training group or to a fraction training group. Results showed that there seemed to be a main effect of Type of Training ($\chi^2(1, N = 27) = 3.86$, $p = .049$). From pretest to posttest, participants increased their overall performance on decimal questions by 18.2%, on fraction questions by 22.7%, and on bar graph questions by 18.2%. There was also an interaction of Test (Pre vs. Post) x Type of Training (($\chi^2(1, N = 27) = 4.66$, $p = .031$); only fraction training seemed to contribute to an increase in performance from pretest to posttest, (see Figures 6 & 7).

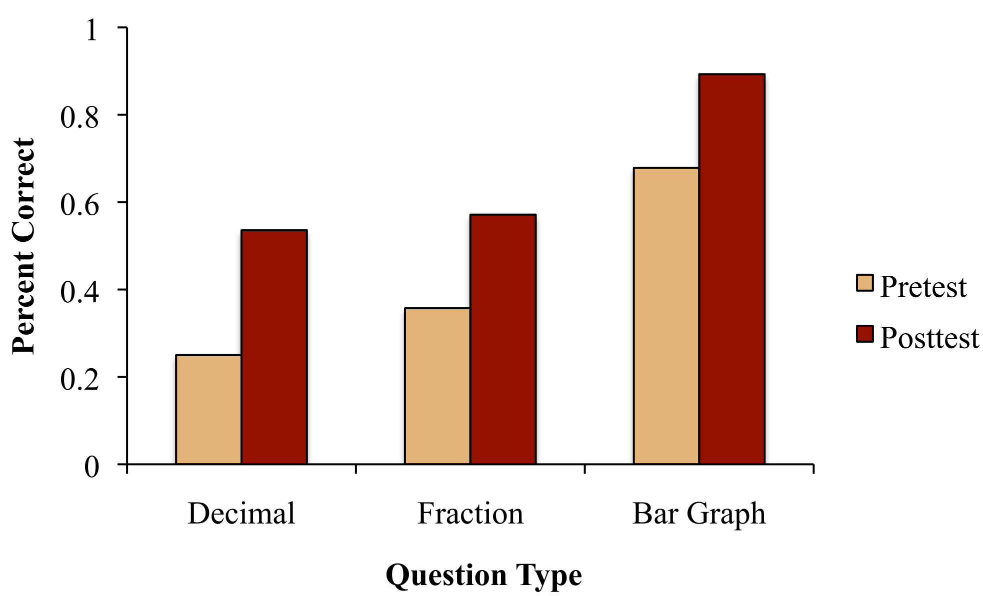
Decimal training (in Figure 6) had no effect on performance. Participants in this condition increased their performance slightly on questions having representations with decimals (M = 65.4% to M = 73.1%) and fractions (M = 73.1% to M = 80.8%), and actually decreased in questions with bar graphs (M = 88.5% to M = 76.9%).

Figure 6. Decimal Training (*n* = 13)



Fraction training (in Figure 7) showed significant improvements from pretest to posttest in every category: decimal (M = 25.0% to M = 53.6%), fraction (M = 35.7% to M = 57.1%), and bar graph (M = 67.9% to M = 89.3%).

Figure 7. Fraction Training (*n* = 14)



Confounded variable

There were more people with higher math experience in the decimal training group than in the fraction training group. It just so happened that the majority of subjects given the fraction training had less math experience than those who were in the decimal experience. This might explain why fraction training had an effect while decimal training did not. The data set (the *N*) is too small here (see Table 1) for use in statistics.

Table 1. Confounded variable: Uneven distribution of participants ($N = 27$)

		TRAINING		Total
		Decimal Training	**Fraction Training**	
PREVIOUS MATH KNOWLEDGE	**Below Calc I**	7	11	18
	Calc I & Above	6	3	9
Total # of Subjects		13	14	27

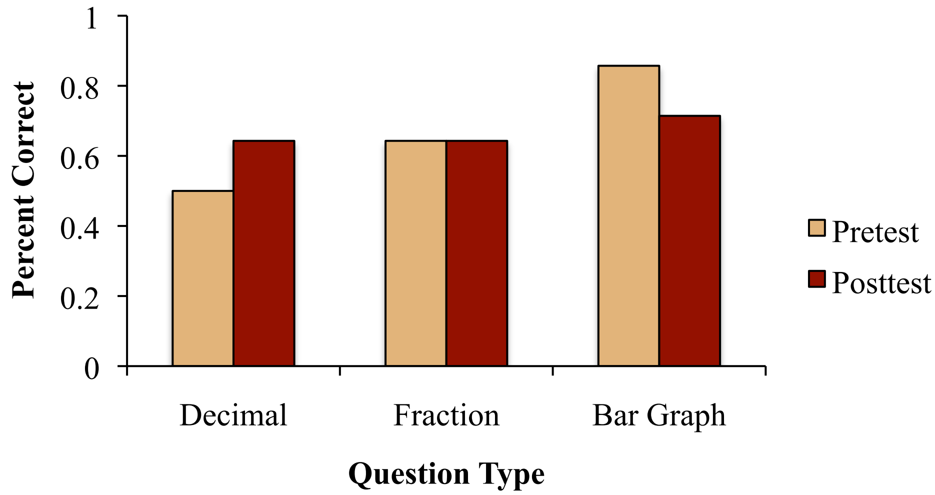
Due to the uneven distribution of math knowledge between Training groups, the higher level math group was removed and analyses with just the lower math group were run. Now, there was no main effect of Type of Training across categories.

Participants with lower level math knowledge

Once high level math subjects were removed from the analysis, overall scores for the decimal training group (in Figure 8) were lower than when these subjects were included (in Figure 6). Similarly, for the analysis that included all subjects, there was no effect for decimal training (Figure 8) within the low-level math group. Participants in this group increased their scores for the decimal representation only a bit ($M = 50.0\%$ to $M = 64.3\%$). Performance remained unchanged on problems dealing with fractions ($M = 64.3\%$) and decreased on problems with bar graphs ($M = 85.7\%$ to $M = 71.4\%$).

Figure 8. Decimal Training- Low math group

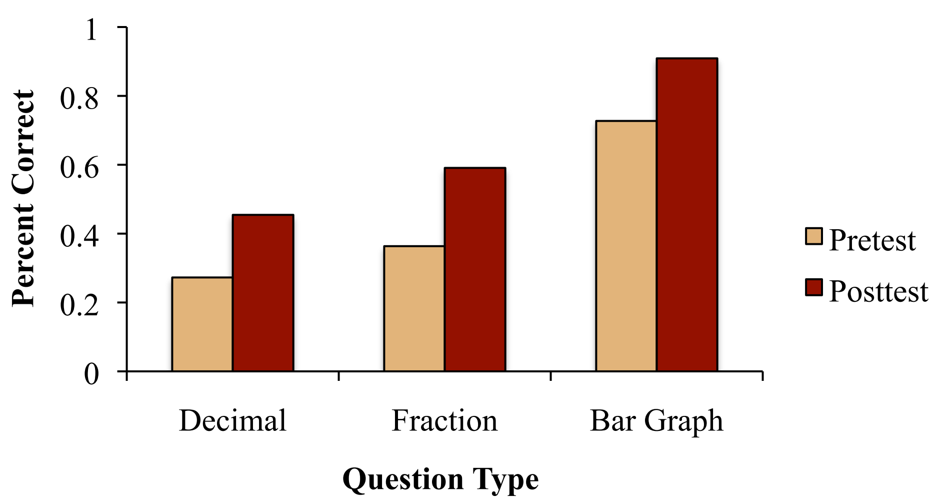
($n = 7$)



Fraction training (Figure 9), on the other hand, positively influenced all formats: decimal (M = 27.3% to M = 45.5%), fraction (M = 36.4% to M = 59.1%), and bar graph (M = 72.7% to M = 90.9%). Looking only at the low math group, there was a marginally significant interaction of Test (Pre vs. Post) x Type of Training (($\chi^2(1, n = 18) = 3.06$, p = .080), which might be attributed to the small sample size.

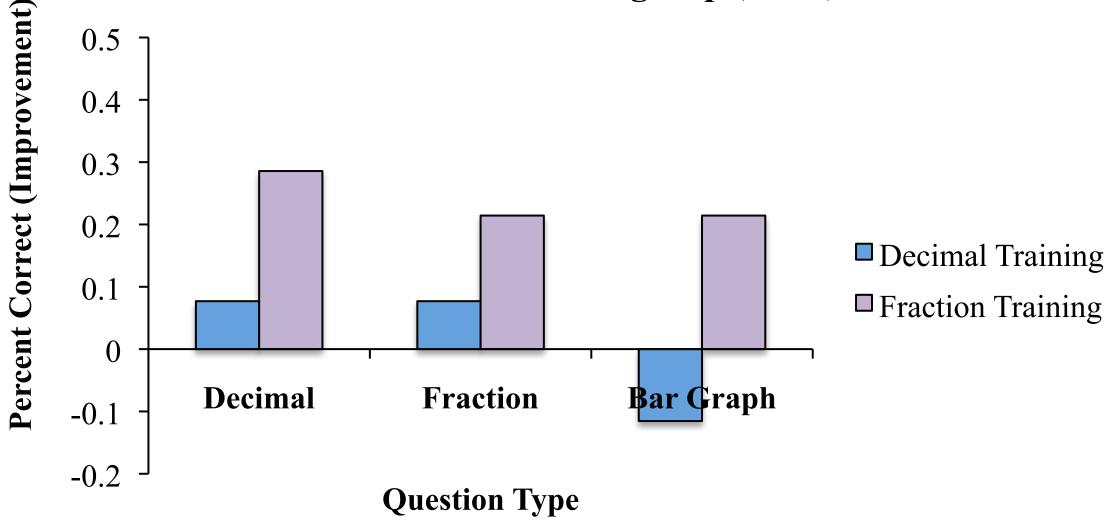
Figure 9. Fraction Training- Low math group

($n = 11$)



Additionally, within the low math group, the average change in performance from pretest to posttest across all question types is shown in Figure 10. Improvement is consistently seen over fraction training. Alternatively, there were too few students with higher previous math knowledge to run a comparative analysis.

Figure 10. Average change in performance from Pretest to Posttest- Low math group (n = 18)



50% increase/decrease additional problems

While previous math knowledge did seem to play a role throughout the experiment, there was one area where it was shown to be insignificant: in problems dealing with more than one percent increase or decrease. When two percentages existed within one problem, participants did not perform nearly as well (Table 2) as they had when percentages were presented separately.

It is possible that the variables in these types of problems may interact. Be that as it may, there was not enough power in this study to investigate that possibility. Unfortunately, the sample size was not sufficient to further test interaction patterns.

Table 2. Performance on 50% increase/decrease questions

	PERFORMANCE # of Questions Answered Correctly			
MATH GROUP	**0 out of 2**	**1 out of 2**	**2 out of 2**	Total # of Subjects
LOW	12	3	3	18
HIGH	6	2	1	9
Total# of Subjects	18	5	4	N=27

Each student received one question in the pretest and one question in the posttest. The "scoring" was out of 2 points; this value was converted to a percentage during the analysis. It can be observed (in green and yellow) that 2/3 of subjects in both math groups failed to answer *any* of these problems correctly.

Discussion

Overall, adults performed significantly better in the posttest than the pretest, regardless of the group condition to which they were randomly assigned (A/B or B/A). However, this variable could not be collapsed due to confounding variables. In one of the group conditions, A/B, there was a design error with one of the fraction problems in the pretest. It is possible that this might have lowered the overall score on the pretest along with students' performance on fraction problems. Thus, this problem was omitted from the analysis, causing some imbalance between the two group conditions.

Across representations, participants' performance was significantly higher for questions with bar graph representations than for those representations with decimals or fractions. By definition, a bar graph is a visual display of data; the length of its bars is equal to the frequency of each element in a set of data. Thus, a plausible explanation might very well be that converting problems to frequency, in the form of bar graphs in this study, causes the problem to be easier to comprehend (Hoffrage, Gigerenzer, Krauss, Martignon, 2002; Brase, Cosmides & Tooby, 1998). The other two formats dealing with rational number (i.e., decimals and fractions) presented great difficulty to students.

Together, these results *confirm* the idea that many adults are treating rational numbers as novel cases for natural number (Hartnett & Gelman, 1998).

When increase and decrease problems were presented individually, there was no significant difference in participants' performances (see Figure 4). *However*, the results from the 50% successive increase and decrease problems hint at more difficulty with problems involving multiple processes. What are the reasons for this discrepancy?

On the one hand, if a student is asked to *increase x* by a given percent, he simply needs to tack on that number to 100%. For example, if he is increasing a cost by 40%, the new amount will be 140% of the original cost. The final answer bears some resemblance to the percent of increase. In contrast, if a student is decreasing a cost by 40%, the new amount will be 60% of the original cost, which may not be as easily linked to the percent of decrease. For this reason, I had anticipated that percent decrease problems might be more difficult than percent increase problems when presented as percentages. For that very same reason, I chose to represent the percentages as various representations to put them on an even playing field. But there is still the problem of fractions- one may speculate that fractions might pose the most difficulty because of the conversion from percent to a rational number. Indeed, results showed that there existed the greatest disparity between percent increases and decreases on fraction problems than on any other representation (see Table 3).

Table 3. Percent correct across direction

		DIRECTION	
		Increase	**Decrease**
REPRESENTATION	**Decimal**	53.4	46.6
	Fraction	51.7	65.5
	Bar Graph	74.1	75.9

Perhaps this result helps to demonstrate the complications that arise when dealing with more than one percent increase or decrease. The final answer rarely looks like the percent of increase/decrease. Let us say that an item's cost increases by 40% of the original cost during the first week of September. Then, in October, that new price has been spiraled downwards by 40% due to the supply and demand of the market. The first part of the problem seems simple because it "contains" 40% within it: The new cost is 140% of the original cost. Yet, when it is time to apply a percent decrease to this amount, that 40% is still in mind and the tendency to subtract is an all-too-common mistake among many adults.

For this type of problem that I have referred to as the 50% increase/decrease problem in the study, the most frequent incorrect responses given were "x", "It remains 'x' dollars, the original cost at the start of [season]", or "Back to the original price". An interpretation of this result stems from subjects incorrectly treating these problems as natural number arithmetic problems where $x + y - y = x$, or more specifically, $x + 50 - 50 = x$. This error was illustrated in some students' incorrect raw answers:

" $x - .50 + .50 = x$ "

" $x + (1/5)x - (1/5)x = x$ "

"$x - (1/2)x = (1/2)x$ " then " $(1/2)x + (1/2)x = 1x$ "

These results provide a very strong clue that these adults would not be very good at dealing with a form of the problem: *(cost)* decreased by *x*% <u>and</u> (*cost*) increased by *x*%. In addition, the amount of previous math knowledge does not seem to matter here.

The answers given to the 50% increase/decrease problems are quite astounding. This kind of example occurs in everyday life and yet, the results here strongly suggest a lack of understanding, quite possibly due to the influence of simple rules of arithmetic. The errors committed reflect a real problem with comprehending figures about the stock market, a budget and almost anything that is numerically-related to a person's daily life. It is also a significant demonstration of failure to understand the logic of multiplying and dividing by a percent.

Future Directions

The participants in this study were from a very narrow population: female, Psychology undergraduate students. I expect to follow up on this experiment in classrooms enrolling a more heterogeneous population, such as an undergraduate Pre-calculus course. The version of this experiment will also contain a bar graph training task, as this is the representation in which subjects performed best. Certain aspects of the experiment (i.e. where there were design errors or variable confounds) will be remedied and more closely controlled.

I also plan to collect additional data with a heterogeneous population on percentage problems with successive increases and/or decreases as in the 50% increase/decrease finding. In this realm, I will be using different values across problems (not only 50%) but the same values within a problem (i.e. increase by 40% then decrease by 40%) to detect if the same types of errors are made.

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Appendix: Stimuli

ID CODE: _______________

GROUP: D F P

The purpose of this study is to give the experimenter an idea of undergraduates' general knowledge about percentage problems. We are not testing your individual abilities, but instead we are interested in what information Rutgers students as a GROUP can provide.

This experiment consists of several sections. **It is very important that you take your time and do not rush through the material- you are not being timed!**

Once you have completed a section, please <u>DO NOT</u> go back to it.

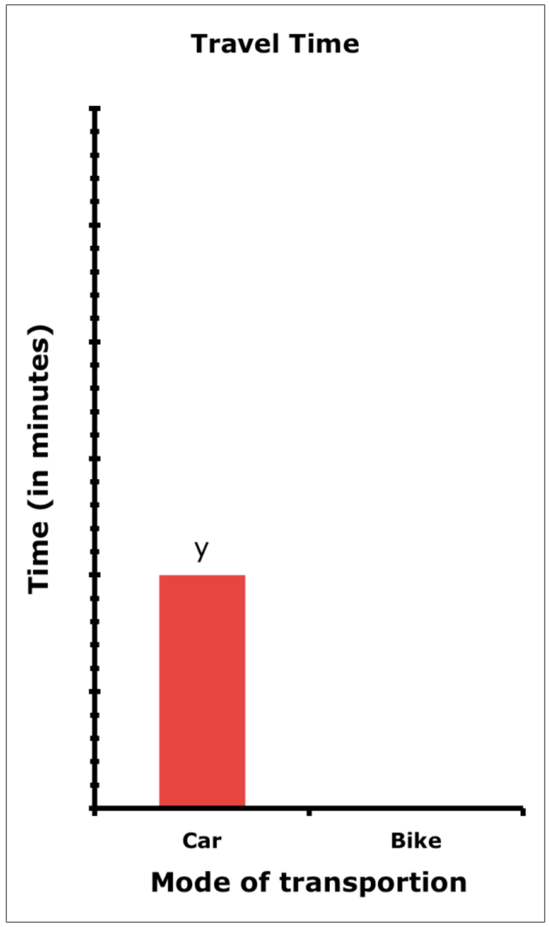
SECTION A

Please answer the following questions. Be sure to show all of your work.

1) Tables at a furniture store are now reduced in price by .25 of the original cost "y". What is the new price that you will pay for *one* table? Please represent your final answer in <u>decimal</u> form.

2) Gas prices are on the rise. This week, gas costs "y" dollars per gallon. An economics magazine predicts that, next week, they will be 1/8 higher in cost. What is the predicted cost of gas per gallon for next week? Please represent your final answer in <u>fractional</u> form.

3) During the winter months, Jeannie drives to school. It takes her "y" minutes to get there. Once the weather becomes nicer, around April, Jeannie rides her bike to school instead. Because she doesn't have to sit in traffic, it actually takes her 10% LESS the amount of time! Please draw a new bar on the graph below so that its height shows the amount of time it takes Jeannie to get to school by bike.

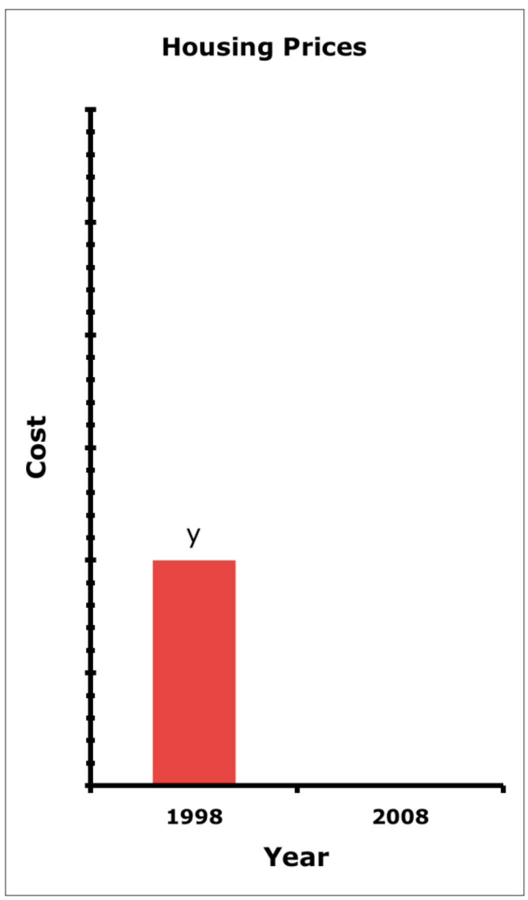


4) The stock you have invested in has fallen by 4/5 of its original value "y". What is the new value of this stock? Please represent your final answer in <u>fractional</u> form.

5) At the start of spring, outside fountains cost “y” dollars. By mid-spring, the original cost is *increased* by 50%. In early summer, that new cost is *reduced* by 50%. What is the cost of fountains after this reduction?

6) Joe’s body mass index (BMI) shows that he is underweight. Gaining .09 **OF** his current weight “y” will put him at a healthy weight and BMI. What would his healthy weight be? Please represent your final answer in <u>decimal</u> form.

7) The Jones’ house costs “y” in 1998. By 2008, the house had increased in value by 30%. Please draw a new bar on the graph below so that its height shows the house’s value in 2008.



SECTION B
('Lesson D')
Part I

In this section, please follow along with the lesson and do the guided tasks that go along with it. (Please do not go back to another section.)

A t-shirt is on sale at 25% off. What is the final cost of the t-shirt?

<u>Guided Tasks</u>: Check your answers at the bottom of the page when you are done.

1) a) Should the final cost of the t-shirt be greater than or less than the original amount? Please use a plus sign (+) to show "greater than" OR a minus sign (–) to show "less than". _____________

b) 100% is the WHOLE cost before the percent decrease. What is 100% in decimal form? ______________

c) 25% OF a WHOLE means there is a change in original cost, more specifically a <u>decrease</u>. What is 25% in decimal form? ______________

2) If you have 100% of an amount (1b), which decreases (1a) by 25% (1c), how much is left? Represent your answer in decimal form. *(Hint. Use the corresponding answers in question 1 to fill out the table below.)*

(1b)	(1a)	(1c)		
________	___________	__________	=	___________
Whole	(+/ – sign)	% change in decimal form		Simplify

3) If y = original cost of the t-shirt, then the <u>change</u> to the original cost "y" is:

(1a)	x	(1c)		
_____	x	_____________ y	=	___________
(+/ – sign)		% change in decimal form		Simplify

4) To get the new cost, we need to *subtract* the discount from the original cost. Fill in the blanks below to write out this formula in decimal form.

(1c)

y – ______________________ y = _____________
% change in decimal form Simplify

5) Compare steps (2) and (4) above.

6) **To conclude**, a shortcut when dealing with percentages is to do the (1.00 - .25) step first so you know how much of the total should be LEFT: In this case, .75y.

<u>A mathematical approach would be to look at the algorithms used:</u>

- We begin with the answer you arrived at in #6 *y - .25y*
- And then FACTOR OUT the common variable* *y (1 - .25)*
- *Don't forget to leave the 1 there!
- And then combine to get *.75y*

NOW TRY THIS:

7) Rollerblades are on sale at 30% off. How would you represent the new cost in decimal form? *(Hint: Let y = the original cost)*

SECTION B
('Lesson D')
Part 2

In this section, please follow along with the lesson and do the guided tasks that go along with it. (Please do not go back to another section.)

A diamond ring has increased in cost by 25%. What is its final cost?

Guided Tasks: Check your answers at the bottom of the page when you are done.

1) a) Should the final cost of the ring be greater than or less than the original amount? Please use a plus sign (+) to show "greater than" OR a minus sign (–) to show "less than". _____________

b) 100% is the WHOLE cost before the percent increase. What is 100% in decimal form? ______________

c) 25% OF a WHOLE means there is a change in original cost, more specifically an increase. What is 25% in decimal form? ______________

2) If you have 100% of an amount (1b), which increases (1a) by 25% (1c), how much is there now? Represent your answer in decimal form. *(Hint. Use the corresponding answers in question 1 to fill out the table below.)*

(1b)	(1a)	(1c)		
________	___________	__________	=	___________
Whole	(+/ – sign)	% change in decimal form		Simplify

3) If y = original cost of the ring, then the change to the original cost "y" is:

(1a)	x	(1c)		
_____	x	_____________ y	=	___________
(+/ – sign)		% change in decimal form		Simplify

4) To get the new cost, we need to *add* the increase to the original cost. Fill in the blanks below to write out this formula in decimal form.

(1c)

y + ______________________ y = ______________
% change in decimal form Simplify

5) Compare steps (2) and (4) above.

6) **To conclude**, a shortcut when dealing with percentages is to do the (1.00 + .25) step first so you know how much of the total you should wind up with: In this case, 1.25y.

A mathematical approach would be to look at the algorithms used:

- We begin with the answer you arrived at in #6 *y + .25y*
- And then FACTOR OUT the common variable* *y (1 + .25)*
- *Don't forget to leave the 1 there!
- And then combine to get *1.25y*

NOW TRY THIS:

7) Muffins now cost 30% more than they used to be. How would you represent the new cost in decimal form? *(Hint: Let y = the original cost)*

SECTION B
('Lesson F')
Part I

In this section, please follow along with the lesson and do the guided tasks that go along with it. (Please do not go back to another section.)

A t-shirt is on sale at 25% off. What is the final cost of the t-shirt?

Guided Tasks: Check your answers at the bottom of the page when you are done.

1) a) Should the final cost of the t-shirt be greater than or less than the original amount? Please use a plus sign (+) to show "greater than" OR a minus sign (–) to show "less than". _____________

b) 100% is the WHOLE cost before the percent decrease. What is 100% in fractional form? ______________

c) 25% OF a WHOLE means there is a change in original cost, more specifically a decrease. What is 25% in fractional form? ______________

2) If you have 100% of an amount (1b), which decreases (1a) by 25% (1c), how much is left? Represent your answer in fractional form. *(Hint. Use the corresponding answers in question 1 to fill out the table below.)*

(1b)	(1a)	(1c)		
________	___________	__________	=	___________
Whole	(+/ – sign)	% change in fractional form		Simplify

3) If y = original cost of the t-shirt, then the change to the original cost "y" is:

(1a)	x	(1c)		
_____	x	_____________ y	=	___________
(+/ – sign)		% change in fractional form		Simplify

4) To get the new cost, we need to *subtract* the discount from the original cost. Fill in the blanks below to write out this formula in fractional form.

(1c)

y – ______________________ y = _____________

% change in fractional form Simplify

5) Compare steps (2) and (4) above.

6) **To conclude**, a shortcut when dealing with percentages is to do the $(1 - \frac{1}{4})$ step first so you know how much of the total should be LEFT: In this case, $\frac{3}{4}$ y.

<u>A mathematical approach would be to look at the algorithms used:</u>

- We begin with the answer you arrived at in #6 *$y - \frac{1}{4} y$*
- And then FACTOR OUT the common variable* *$y (1 - \frac{1}{4})$*
- *Don't forget to leave the 1 there!
- And then combine to get *$\frac{3}{4} y$*

NOW TRY THIS:

7) Rollerblades are on sale at 30% off. How would you represent the new cost in fractional form? *(Hint: Let y = the original cost)*

SECTION B
('Lesson F')
Part 2

In this section, please follow along with the lesson and do the guided tasks that go along with it. (Please do not go back to another section.)

A diamond ring has increased in cost by 25%. What is its final cost?

Guided Tasks: Check your answers at the bottom of the page when you are done.

1) a) Should the final cost of the ring be greater than or less than the original amount? Please use a plus sign (+) to show "greater than" OR a minus sign (–) to show "less than". _____________

b) 100% is the WHOLE cost before the percent increase. What is 100% in fractional form? ______________

c) 25% OF a WHOLE means there is a change in original cost, more specifically an increase. What is 25% in fractional form? ______________

2) If you have 100% of an amount (1b), which increases (1a) by 25% (1c), how much is there now? Represent your answer in fractional form. *(Hint. Use the corresponding answers in question 1 to fill out the table below.)*

(1b)	(1a)	(1c)		
________	___________	__________	=	___________
Whole	(+/ – sign)	% change in fractional form		Simplify

3) If y = original cost of the ring, then the change to the original cost "y" is:

(1a)	x	(1c)		
_____	x	_____________ y	=	___________
(+/ – sign)		% change in fractional form		Simplify

4) To get the new cost, we need to *add* the increase to the original cost. Fill in the blanks below to write out this formula in fractional form.

(1c)

y + ______________________ y = ______________
% change in fractional form Simplify

5) Compare steps (2) and (4) above.

6) **To conclude**, a shortcut when dealing with percentages is to do the (1 + ¼) step first so you know how much of the total you should wind up with: In this case, (1¼)y or (5/4)y.

<u>A mathematical approach would be to look at the algorithms used:</u>

- We begin with the answer you arrived at in #6 *y + ¼ y*
- And then FACTOR OUT the common variable* *y (1 + ¼)*
- *Don't forget to leave the 1 there!
- And then combine to get *(5/4) y*

NOW TRY THIS:

7) Muffins now cost 30% more than they used to be. How would you represent the new cost in fractional form? *(Hint: Let y = the original cost)*

SECTION C

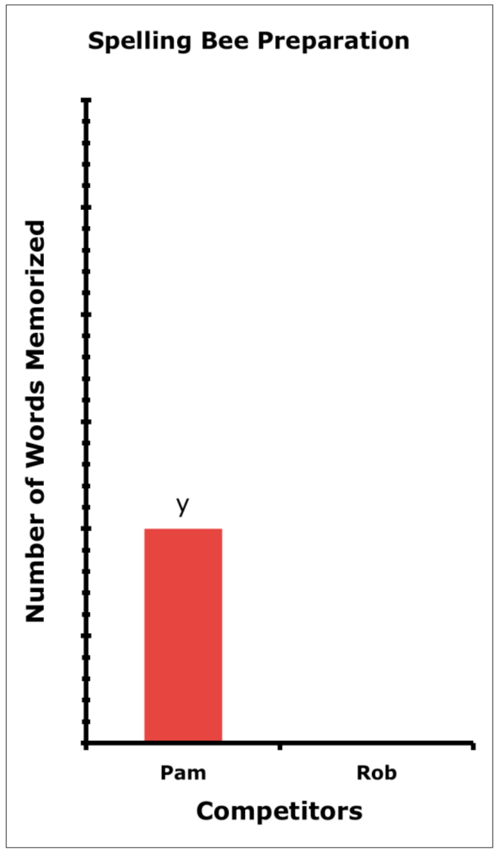
Please answer the following questions. Be sure to show all of your work.

1) The original price "x" of a watch is reduced by .20. What is the new price that you will pay for this watch? Please represent your final answer in <u>decimal</u> form.

2) Sean has finished law school. He now puts in "x" hours per week studying for his BAR exam. In another month, he plans on increasing that amount of time by 2/5. By the end of next month, how much study time will he be putting in per week, in terms of "x"? Please represent your final answer in <u>fractional</u> form.

3) In the winter, snowboarding equipment costs "y" dollars. By early spring, the original cost is *reduced* by 50%. But by the time fall arrives, that new cost has *increased* by 50%. What is the cost of snowboarding equipment by the fall?

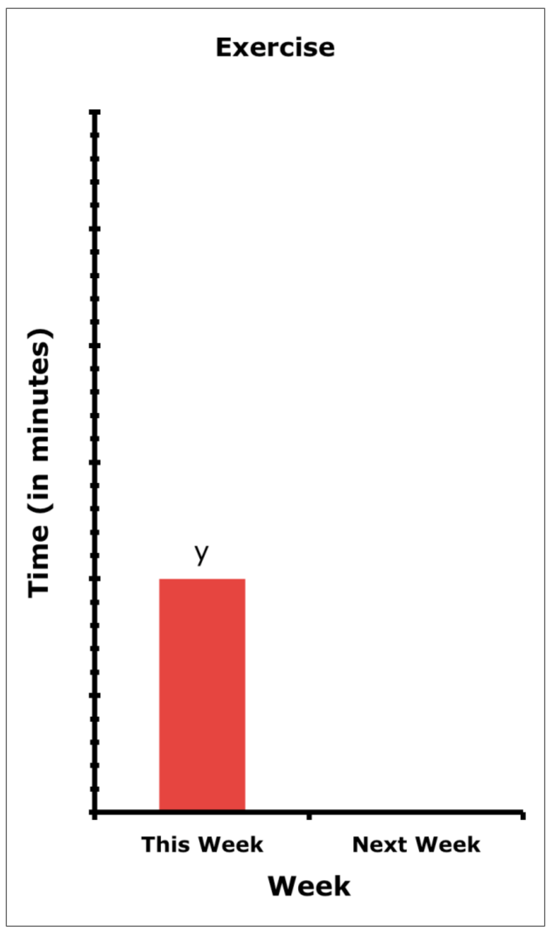
4) Rob and Pam are competitors in this year's spelling bee. They each have been memorizing words from the dictionary. Pam already has the spelling of "y" words memorized. Rob needs to study more as he soon realizes that he knows 60% less words than Pam. Please draw a new bar on the graph below so that its height shows the number of words that Rob has memorized.



5) The original cost of a high definition flat screen TV was "x" dollars. The price was then reduced by ¾ . What is the new cost, in terms of x? Please represent your final answer in <u>fractional</u> form.

6) Ralph got his football signed by his all-time favorite player last year. This year, the team is going to the playoffs. His signed football is now worth .40 more than its original value "y". Please represent your final answer in decimal form.

7) Next week, Sally will increase the amount of time "y" (in minutes) that she spends exercising by 40%. Please draw a new bar on the graph below so that its height shows the amount of time Sally will be exercising next week as compared to this week.



SECTION D

Now please answer the following questions about yourself. (Please do not go back to another section.)

1) Sex (M/F): ______

2) Age: _______

3) What was your Math SAT I score? ____________

4) Number of semesters completed at Rutgers so far (do not count this summer semester): ___________________________

5) List the math/ statistics courses you are currently taking or have taken as an undergraduate. If you have not taken any at Rutgers, please list the last high school math course you took. (For example: Pre-calculus, Calculus, Statistics, etc.)

____________________________ ____________________________

____________________________ ____________________________

____________________________ ____________________________

6) What is your major or intended major? ________________________

7) Is English your native/primary language? _______________________

8) Did you learn anything new in this experiment? If yes, what was it?

____

____

____

Thank you for your participation.