MULTI-OBJECTIVE STOCHASTIC MODELS FOR

ELECTRICITY GENERATION EXPANSION PLANNING PROBLEMS

CONSIDERING RISK

By

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ABSTRACT OF THE DISSERTATION

Multi-Objective Stochastic Models for Electricity Generation Expansion Planning

Problems Considering Risk

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Prof. David W. Coit

This dissertation is focused on the development of mathematical models to solve electricity generation expansion planning problems where important problem objectives, such as cost, greenhouse gas and pollutant emissions and reliability are explicitly considered under an uncertain environment. Generation expansion planning problems are solved to determine what, when and where to built the new technologies. The main objective of the power grid is to provide an economic and reliable energy supply to consumers. Due to the increasing awareness for clean air and global warming, the power grid should also be designed to be environmental friendly. In this research, an approach is proposed to determine critical components for the grid with regard to reliability, cost and gas emissions, and an optimization approach is proposed to select a set of availability scenarios which represent the stochastic characteristics of the system and to determine the associated probabilities. The problem is formulated as a two stage multi-objective stochastic optimization problem considering the generated scenarios. There are also some

other technological developments, called “Smart Grid Technologies” which can affect the grid. The impacts of “Smart Grid Technologies” on the grid are that (*i*) shift/reduce energy demand, (*ii*) increase the effective availability of the system components, and (*iii*) reduce the energy loss during transmission. This research is the first comprehensive attempt to include the Smart Grid technologies, affecting the availabilities and transmission loss, into the generation expansion planning problem. This research also leads to the contributions for developing models where risk aversion is incorporated into the model, improving solution efficiency by extending Benders decomposition and improving solution techniques for multi-objective optimization problems.

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1. Introduction

This dissertation is concerned with the development of mathematical models to solve electricity generation expansion planning (GEP) problems which explicitly consider important problem objectives, such as cost, greenhouse gas and pollutant emissions and reliability under an uncertain environment. Intelligent planning is essential to meet consumers increasing need for electricity while minimizing the harm to the environment. This is a critically important problem that requires knowledge and models from different fields of engineering. The problem is formulated as a stochastic optimization problem with multiple objectives, and research results indicate that this is an effective approach. There are meaningful research contributions, considering both the specific problem domain and the solution of stochastic multiple objective problems generally.

Uninterrupted access to electricity becomes critically important considering the economic growth and the development in industry worldwide. Everyday life, as well as most production and service systems depend on electricity, and the demand for electricity is increasing every year. Expansion planning for the electricity generation system must be analyzed carefully and implemented correctly, considering many important objectives and decision makers' preferences. Accurate and timely optimization methods are critically needed.

To keep energy costs reasonable and acceptable to consumers and the business community, the generation planning process must always consider cost as a primary part of the decision making criteria. However, only considering the cost is insufficient, because the electricity system must also be reliable and it should also provide adequate electricity even under adverse conditions.

The global warming problem is commonly believed by the scientific community to be caused or greatly influenced by the greenhouse gas emissions. Electricity production is one of the major contributors to greenhouse gas emissions, and if the electricity generation system is expanded without considering the impact of the greenhouse gas emissions, future generations may be exposed to further irreversible global warming. These investments are very large, and it is not easy to make changes in the future. Therefore, long term expansion plans should include greenhouse gas emissions as a part of planning objectives.

There is also an increasing awareness for clean air. Nitrogen oxides, or NO_x, are one of the major pollutants which cause a wide range of health and environmental problems. Some of the problems caused by the emission of NO_x are listed by US Environmental Protection Agency [1]. NO_x together with volatile organic compounds causes the formation of ground-level ozone. Ozone can damage lung tissue, vegetation and crop yield. NO_x causes acid rain, deterioration in water quality and visibility impairment. Electricity generation is one of the primary sources for NO_x emissions, and therefore, it should also be part of the planning process.

Demand for electricity is increasing, so the electricity generation network must be expanded. However, only considering the available generation plants as investment options in the problem is inadequate since there are other technologies which can provide energy conversation or economical usage of energy. Since resources for the electricity generation is limited, energy conversation is also a very important issue. Some of these new technologies are grouped together and labeled as “Smart Grid Technologies.” Therefore, the plans neglecting these options are not entirely appropriate to build a modern network.

Another research challenge involves the uncertainties in the load growth rates, fuel prices, environmental constraints and so on. Considering only the expected value of the objective functions means that the decision makers are risk neutral. However, since the electricity expansion plans are long-term commitments and affects people’s living conditions and business prosperity deeply, it is reasonable that the decision makers may also be risk averse. Therefore, expansion plans found by the models considering only expected values might be unrealistic and inapplicable.

In this research, a two-stage multi-objective stochastic optimization problem is used to solve the electricity generation expansion planning problem and allow for trade-offs with respect to all important aspects. Instead of using average capacities for the components like most all other research in this area, component availability scenarios are generated to model the uncertainties of available generation units, transmission and distribution lines. The two stage stochastic multi-objective optimization problem is constructed considering these scenarios so that a reliable electricity supply can be achieved. The objective function includes the cost (including the cost for unmet demand), greenhouse gas emissions and pollutant emissions. Scenarios for uncertain parameters are generated and risk measures are introduced in order to incorporate risk aversion into the planning problem.

This research provides many benefits both in the content of electric power system planning, and industrial engineering and operation research fields. Since the developed model considers all critical factors such as cost, emissions and reliability, and incorporates the expansion, dispatching and reliability decisions, the plans obtained by this approach have the ability to be more reliable, robust and environmental friendly. Moreover, the recent developments in the electric power system are addressed in this research. This means that the plans obtained are more realistic, applicable and better reflect the often conflicting needs of diverse communities. Since trade-off solutions are provided, the decision makers will be able to compare the plans, access the relative impact of decisions, and choose the ones which are more appropriate. There are also distinct contributions related to the efficient selection of scenarios for stochastic programming problems including selection of the critical components of the system, their availability states and the associated probabilities; the application of Benders decomposition; the extensions to the Benders decomposition for problems with integer variables; and further development of solution techniques for multi-objective optimization problems.

1.1. Electric Power System Planning

The electricity network can be divided into three parts: (*i*) generation, (*ii*) transmission and (*iii*) distribution. The electricity generation system consists of the plants producing the electricity using very different technologies. The transmission system consists of the transmission lines which are used to transfer the electricity from plants to substations, and the distribution system consists of the lines which distribute the electricity to the final user. The focus of this dissertation is the electricity generation network. Since the demand for electricity is increasing, the electricity generation network must be expanded. Therefore, electricity generation expansion planning can be defined as selecting the time, location and technology type for the investments in a long term planning horizon. In this research, the investments can be made for the generation units such as nuclear plants, wind turbines, solar panels; or for Smart Grid technologies. Smart Grid technologies are the technological developments in information technology, material science and engineering that may significantly improve the security, reliability, efficiency and effectiveness of the electric system.

The GEP problem intends to find the schedule for expansion investments to supply the anticipated demand for electricity in the future. One main objective is finding the most cost efficient expansion plans. The cost consists of two main parts; investment cost and operations cost. Investment cost includes the construction costs for the generation units and other technologies. The operations cost is the electricity production cost which mainly depends on the fuel costs. There are mainly two categories of decision variables in the GEP problem; investment decisions and operations decisions. In some previous research, investment decisions are considered as continuous variables representing the capacity required to be added, while in some other research, the decisions are considered as binary variables representing whether the investment in corresponding technology should be made or not. The operations variables represent the production amount of each generation unit to meet demand as it occurs. The constraints for the GEP problem include energy demand constraints, reliability constraints, capacity constraints, some fuel type related constraints, etc.

Hobbs [2] stated that until the 1970s, GEP problems are defined as the determination of the type, the size, and the time and location of large central generation plants to meet growing electric demand. From this perspective, there have been effective models already developed. However, the problem is now distinctly different due to the three following reasons:

- There are more investment options other than complete dependence on centralized plants.
- The objective of utilities has expanded beyond the cost to include environmental concerns.
- There are greater uncertainties in the system, such as uncertainties in load growth, fuel prices and so on.

The GEP problem is well studied, with many of the studies focusing on finding the least cost expansion plan. Kagiannas *et al.* [3], Zhu and Chow [4], and Hobbs [2] provide surveys of modeling techniques developed for GEP. These authors provide detailed lists of previous research using dynamic programming approaches, decomposition techniques, stochastic optimization, Genetic Algorithm (GA), fuzzy set theory, artificial neural networks, network flows, simulated annealing, etc. Nara [5] presents a systematic survey for applied simulated annealing, genetic and evolutionary algorithms, and tabu search applied to power systems planning problems. A review of some of the most important and/or relevant papers is included in Section 2.2 of this dissertation.

1.2. Motivation

There are already many studies and previous research done to solve GEP problems. Most of the previous studies focus on finding the least cost GEP plans. However, the expansion plan must also guarantee very high reliability, and environmental issues must be considered as a critical concern as well. Since electricity production has a significant impact on greenhouse gas emissions and pollutant emissions, the environmental impact of the expansion and operations should be an integral part of the GEP problem. Also, the resources to produce electricity are being depleted and they are more expensive. Therefore, energy conversation considerations are also very important. The GEP problem should also include modern technological developments to capture their benefits and include them in the model.

There is a high demand for a cost effective and highly reliable electric power system. Since electricity is very critical for many operations, the shortage of electricity can cause billions of dollars lost. Therefore, the GEP problem should also consider how the system works in adverse conditions. These adverse conditions occur when some of the main generation units, transmission lines or distribution lines are simultaneously down. Most previous research includes the availability of the system components by updating the available unit capacities using an availability factor and assuming that these units are operating with derated capacities all the time. This approach does not properly address the adverse conditions when several critical components may be unavailable during peak demand. Some models assume that the load is a random variable and use production cost simulation methods to calculate the expected generation from each generation unit and associated reliability measures. In this approach, although the capacity losses due to the failure of the generation components are considered, it still finds the expected generation amount for each generation unit, and does not consider the system's response to adverse conditions. Therefore, there is a need for a model which can explicitly consider the availability of the system components (i.e., generation units, transmission and distribution lines, fuel supply infrastructure).

Greenhouse gas and pollutant emissions are critical national and world-wide concerns that will impact all phases of energy and business policies. Approximately 40% of US greenhouse gas emissions are due to the production, transmission and distribution of electricity [6]. In a carbon-constrained world, the electric power system needs to be transformed from a reliance on large fossil fuel power plants to a more distributed system using renewable energy, energy efficiency and other non-carbon emitting technologies while maintaining high reliability at affordable costs. For example in New Jersey, the state has established goals to rely extensively on energy efficiency and distributed generation to meet a stated objective of reducing greenhouse gas emissions by 20% by 2020 and 80% by 2050 [6].

The electric network is designed to satisfy the peak demand plus some reserve. Therefore, most of the time, the electricity demand is lower than the generation capacity. However, the demand is increasing generally and so is the peak demand. Since reliability is very important for the system, the network should be designed to satisfy this increasing peak demand. One option is to expand the generation system, but another intriguing option is to implement the technologies which increase energy conversation and/or shift demand such as demand side management (DMS), plug-in-hybrid vehicles (PHEVs) or storage devices. In this dissertation, mathematical models are developed to integrate Smart Grid technologies into the expansion planning process.

When expanding or upgrading the electric grid, new generation technologies are either *centralized* or *distributed*. Historically, large centralized power generation units, such as nuclear or coal burning, were used. Distributed generation units are smaller units that can be located closer to the load so that long distance transmission from generation units to the distribution system is not necessary. It is often possible to use renewable energy sources such as a solar or small-scale wind as distributed generation units. Additionally there are reliability benefits. There is generally more generation capacity than the demand, so when there is unmet demand, it is mostly due to the failure of the distribution lines. Since distributed generation units do not require the same distribution lines, there are potential reliability benefits as well. There are also available distributed energy sources having co-generation capabilities, i.e., the generation of heat in addition to electricity. Since these units are located close to demand, useful heat can also be generated and used so the efficiency of the electricity production is increased.

Risk is very critical in planning for electric power systems. Since there are many uncertainties, it is important to incorporate these into the model. However, the expected value is only for risk neutral decision makers. Therefore, it is required to model the risk into the model for risk-averse decision makers and provide trade-off solutions according to the level of decision makers' risk aversion.

1.3. Research Contributions

There are several contributions of this research. Some of them are related to the problem domain and specific to the electric power generation system. Those contributions include (*i*) modeling the GEP problems to simultaneously address issues such as environmental impacts of electricity generation, reliability of the system and uncertain characteristic of the problem; (*ii*) considering the availability of the system components explicitly while maintaining the problem linearity; (*iii*) incorporating the recent technological developments such as Smart Grid technologies into the GEP problems; (*iv*) presenting the risk with more efficient measures such as conditional value at risk (CVaR) or maximum excess (regret), etc. and providing trade-off solutions with respect to the risk for utilities in the system; (*v*) providing a systematic way to reduce the size of the problem with respect to the demand levels which should be considered and the representation of the demand.

There are some other research contributions which are related to the efficient solution of applied operations research or industrial engineering problems. These kinds of improvements resulting from this research can be extended to problems in other fields. These are (*i*) presenting a methodology to find the critical components with respect to more than one dimension in a large system; (*ii*) finding the availability vector of the critical components and the associated probability to represent the possible working states of the critical components explicitly; (*iii*) using Benders decomposition to solve the multi-objective optimization problem; (*vi*) utilizing the parallel solution technique for Benders decomposition to solve larger problems; and (*v*) providing a new method to find Pareto solutions for mixed integer multi-objective optimization problems.

The first contribution is development of a unified model to address all the issues previously mentioned by proposing a stochastic multi-objective optimization problem, which incorporates expansion, dispatching and reliability. There are many studies for GEP problems; however, most of them focus on the finding the least cost expansion plans. In this research, the CO_2 and NO_x emission amounts are defined as objectives in the multi-objective optimization problem so that trade-off solutions can be provided. Also, there are several measures used to define the reliability of the system including the expected unmet demand. In most previous studies, expected unmet demand is defined as a constraint, but in this research plan, expected unmet demand is incorporated into the objective function. There are uncertainties regarding the forecasted demand, fuel costs, construction time and so on. A set of scenarios with the corresponding probabilities are defined to represent the uncertainties, and a two-stage stochastic programming model is used to solve the problem. This comprehensive model offers greater capabilities than other models.

The second contribution is providing a rigorous method to represent the availability of the critical system components explicitly in the electric power system problem. There are two main advantages of this newly developed; namely (1) it models how the system reacts under adverse conditions so that more robust plans can be obtained, and (2) it explicitly models the availability of the critical system components while maintaining a linear model so that large problems can be efficiently solved. Most of studies use derated capacities for the system components in order to represent forced outages. These methods assume that all the components are always working at their average levels throughout the year. Therefore, the system states where several critical components are simultaneously failed are not explicitly considered in the model. On the other hand, there are some studies that use probabilistic cost simulation methods to incorporate forced outages explicitly. However, the expansion problems using the probabilistic cost simulation methods become nonlinear. With the proposed approach, it is possible to consider the each critical component availability explicitly and still have a linear model. Another issue is that the dispatching in the probabilistic cost simulations is done according to the some predefined rules. The proposed research approach incorporates the dispatching directly into the optimization problem with decision variables dedicated to dispatching decisions.

The third contribution is that this research is one of the first attempts to integrate Smart Grid technologies into the problem. There are some studies where the technologies which affect the demand, such as demand side management (DSM), are introduced into the model. However, this research effort is the first time where the Smart Grid technologies which affect the failure rate of the system components, repair rate and the transmission losses are integrated directly into the planning problem.

Another contribution with respect to the electricity generation planning problem domain is the utilization of several risk measures such as conditional value at risk (CVaR) or maximum excess (regret), etc. There are some studies which use the variance as a risk measure. However, the variance has a disadvantage of penalizing the deviation from both sides. Therefore, more efficient risk measures are needed.

In the models developed as part of this research, the availabilities of the critical components are considered explicitly. Scenarios are defined and used to represent the availability of the critical components. These scenarios are defined as availability scenarios. The demand level can be at various levels for each of the availability scenario options. That is, it is possible to have the same availability state when the demand is at its peak or lowest level. However, considering all the demand levels for all component states is not computationally efficient. Since the reliability is more critical when the demand level is high, the newly developed approach partitions the load into subsections and uses more availability scenarios for the subsection with higher demand and less availability scenarios for the ones with lower demand level.

The first contribution which is related to reliability and industrial engineering research fields is a new methodology to identify critical components in the system. In this research effort, the criticality of the components is based on both how the component availability affects the operations cost, and also on how the availability of the components affects the reliability of the system and the gas emissions. As a part of this study, a method was developed to find the critical components with respect to all the dimensions. This method can also be extended to other fields.

Another contribution is the proposed optimization method to select the availability vectors for critical components which are used in the expansion problem. If there are N critical components, then there are 2^N availability vectors. To consider all the availability vectors is either impossible or inefficient depending on the value of N. Therefore, it is desirable to select a subset which represents the system well enough. As a part of this research, a new optimization model was developed to select the subset and corresponding probabilities.

To improve the computational efficiency, Benders decomposition is utilized for multi-objective optimization problems. Benders decomposition consists of one master problem and multiple subproblems. The subproblems are independent from each other. That is, the solution of one subproblem does not affect the solution of another subproblem. Therefore, there is no need to solve these subproblems sequentially, i.e. they can be solved in parallel. In this research, a plan is also developed to implement parallel solution techniques together with the Benders decomposition in order to improve efficiency for multi-objective optimization problems.

There are several methods which can be used to solve multi-objective optimization methods such as the weighted sum method, the normal boundary intersection method and augmented weighted Chebychev method. New approaches which capture the best characteristics of these methods are developed. The new methods can be used to solve any mixed integer multi-objective optimization problem to find a Pareto set.

2. Background/ Literature Survey

In this chapter, the electrical power system is described in detail together with a concise explanation of the metrics used for reliability and gas emissions. In addition, some risk measures are defined and their mathematical representations for the optimization problems are explained. The technological developments which can improve the efficiency and the effectiveness of the electric power system are introduced together with their potential benefits.

Moreover, a detailed summary of literature survey is provided for electricity generation expansion planning problems. The existing research is grouped with respect to the domain of the problem and the methodology used to solve the problem and presented accordingly. Finally, three multi-objective optimization methods, namely weighted sum method, normal boundary intersection method, and augmented weighted Chebychev method, are described and their advantages and drawbacks are presented.

2.1. Electric Power System

In this section, the electric power system is explained in more detail and some basic terminology and their definitions are given. Additionally, the metrics which are used for the reliability and emissions are introduced. There are also some technological developments which can help to construct more modern, effective, efficient and reliable electric power systems, called Smart Grid technologies. A brief summary of these technologies and their benefits is presented. Finally, some of the risk measures are defined and their mathematical representation for the two-stage stochastic programming models is also explained.

2.1.1. System Description and Definitions

The primary purpose of electricity power system is to provide a reliable and economic supply of electricity. In this section, a description of the power system is provided and the definitions of the components are given.

An electric power system consists of mainly three major components which are (*i*) the electricity generation system, (*ii*) transmission lines which consists of high voltage lines and connect the generation unit to substations, and (*iii*) distribution lines which consist of lower voltage lines and connect the substations to the final customers. A simple representation of a power system can be given as in Figure 2.1.

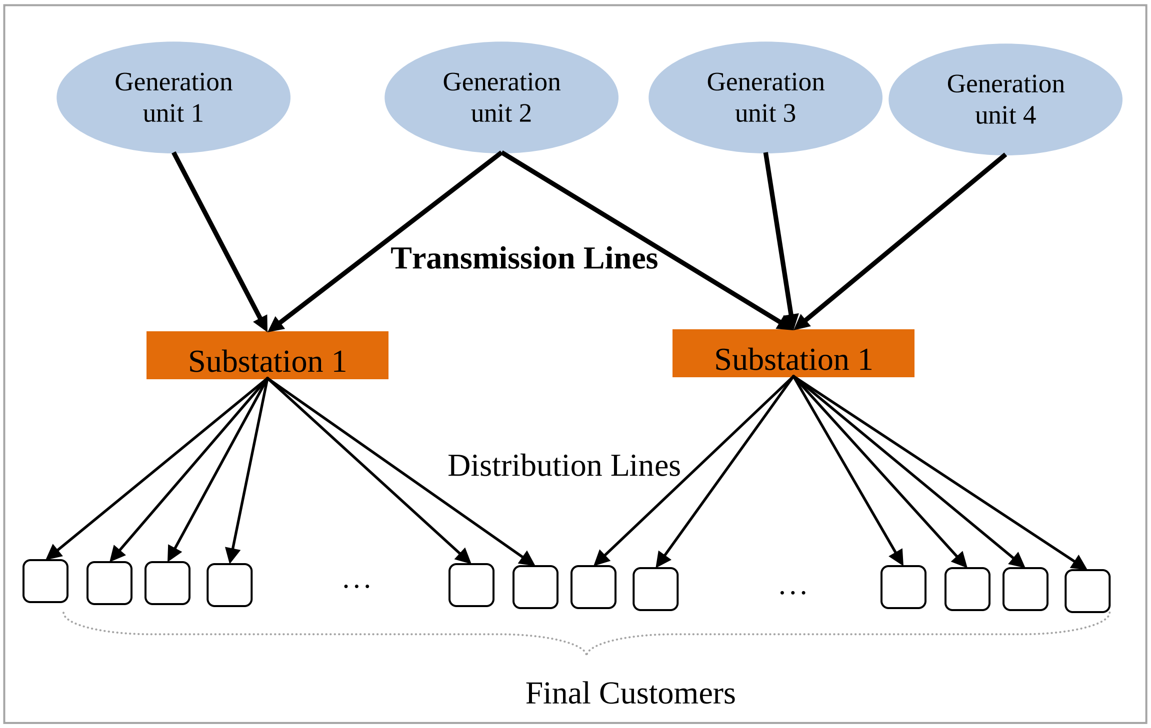

Figure 2.1. Simple representation of a power system

The power system traditionally uses large-scale centralized generation units relying on technologies such as nuclear or coal burning. However, the power system can also include distributed generation units which are small scale generation units located close to the final consumers. Some of the technologies available for the distributed generation are fuel cells, micro-turbines, photovoltaic systems, small wind turbines and so on. Some of the distributed generation units have the co-generation capability. Co-generation means that they can use the generated heat together with the electricity.

Each transmission and distribution line in the system has a capacity constraint and some of the energy is lost when it is transferred through the line. In addition, the generation network is represented above a simple representation which uses one directional flow. In reality, there are other flows such as loop or parallel flows.

2.1.2. Load Duration Curve

Load Duration Curve (LDC) is used to depict the demand for power over some defined time periods. The demand is arranged in descending order to form the LDC. Each point in the *x*-axis denotes the amount of time in the period during which demand is equal to or greater than the corresponding load value on the *y*-axis. In general, peak demand in each day or peak demand in each hour are used to construct the LDC. The area under the LDC represents the total electricity power demand in the period. An illustrative example is given in Figure 2.2 where L_{max} and L_{min} are the maximum and minimum load in the period involved and T is the total number of time units considered.

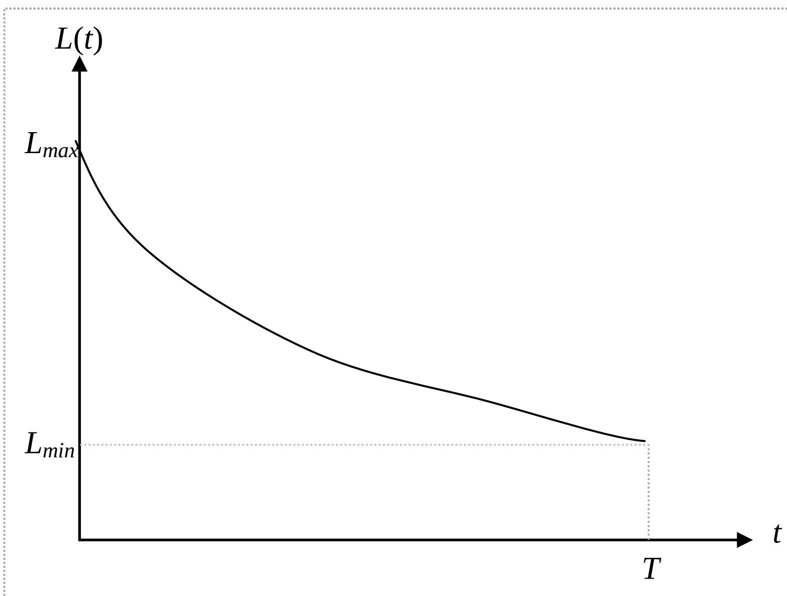

Figure 2.2: Typical load duration curve

2.1.3. Metrics

In the power industry, the term “reliability” is not used in the conventional way as a probability of a failure. The “reliability” of the electric power system is defined as the ability of the system to satisfy the electricity demand. There are several indices used to represent the reliability of the system. In this section, the detailed definitions are presented. Also, the metric which is used to represent gas emissions is introduced.

2.1.3.1. Reliability Indices

In the literature, there are several indices which are commonly used to measure the reliability of the electric power system, such as (1) Loss of Load Expectation (LOLE), (2) Loss of Load Probability (LOLP), and (3) Expected Energy Not Supplied (EENS) or Loss of Energy Expectation (LOEE). More detailed explanation can be found in [7].

A loss of load occurs when the system load level, or demand level, exceeds the available generation capacity in the system. LOLE is defined as the expected number of time units in the specified period in which the demand exceeds the available capacity. LOLP is defined as the probability that there is a loss of load. EENS or LOEE are defined as the expected amount of unmet energy in the period.

There are different approaches to calculate these indices. One approach is to combine the system capacity outage probability table with the system load. Capacity outage indicates a loss of generation. A capacity outage results in a loss of load only if the demand level exceeds the available capacity, which is the difference between the maximum capacity and capacity outage level.

System capacity outage probability tables can be constructed based on the availabilities of each generation unit. This table consists of the capacity outage level and the corresponding probability. A capacity outage is the amount of generation capacity that is not available because of failures. As an example, consider a system with two generation units with the following capacities and availabilities; (1) 100MW and 0.96, and (2) 400MW and 0.88. The capacity outage levels can be 0 with the probability of 0.8448, (when all the components are working); 100 with the probability of 0.0352, (when the first component fails and the second one is working); 400 with the probability of 0.1152, (when the first component is working, the second one is failed); and 500 with the probability of 0.0048, when both components fail. A cumulative probability for each outage level can also be obtained. This cumulative probability means the probability of a capacity outage in the system which is equal or greater than the indicated amount. For a large system, the size of the table is very large. In practice, the table size can be truncated by omitting all the capacity outages whose cumulative probability is smaller than some predefined small value. Another approach to reduce the size of the table is rounding the capacity outages by using equal increments.

Consider that the system consists of n capacity outage levels, and O_k is the magnitude of the capacity outage level k. A representation for the system load-capacity relationship is given in the Figure 2.3. As it can be seen from the Figure 2.3, any outage less than the reserve does not cause a loss of load. The available capacity remaining in the system is calculated by subtracting the outage level from the maximum capacity. The intersection of the horizontal line representing the available capacity with the LDC indicates the amount of time in the interval that an outage magnitude of O_k results in a loss of load, that is t_k.

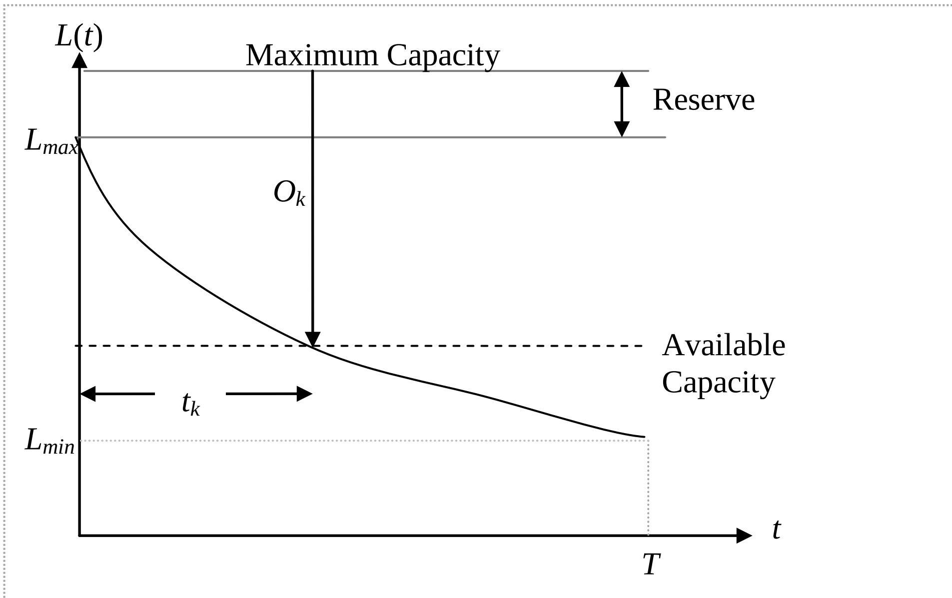

Figure 2.3: Relationship between load and capacity

LOLE is calculated as follows;

$$LOLE = \sum_{k=1}^{n} p_k t_k = \sum_{k=1}^{n} (t_k - t_{k-1}) P_k$$

where, p_k is the individual probability of the capacity outage level k and P_k is the cumulative probability of the level k.

LOLP is calculated by dividing the *LOLE* with the total number of time units in the period (i.e., T).

$$LOLP = \frac{\sum_{k=1}^{n} p_k t_k}{T}$$

ENNS or LOEE is calculated as follows;

$$ENNS = LOEE = \sum_{k=1}^{n} E_k P_k$$

where E_k represent energy not supplied or energy curtailed when an outage magnitude of O_k occurs. Figure 2.4 illustrates this concept.

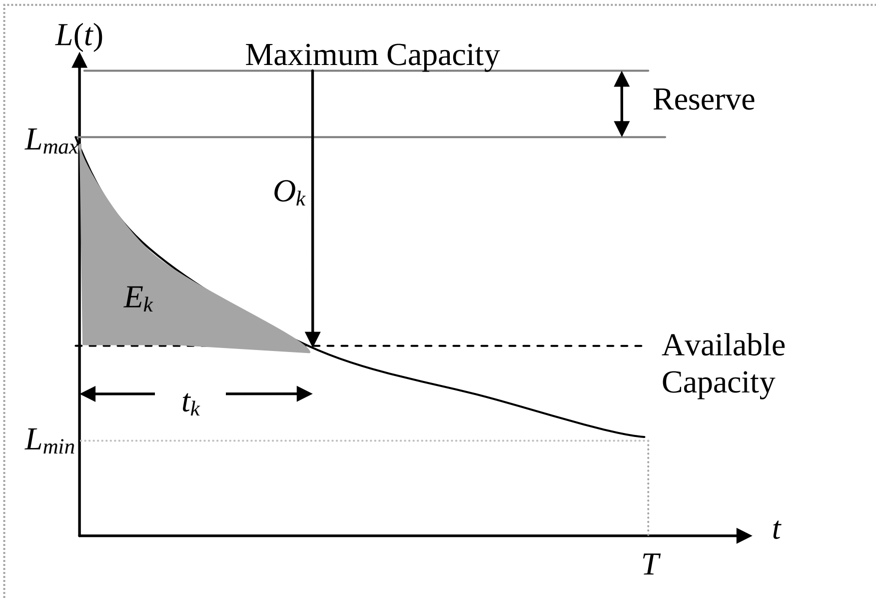

Figure 2.4. Illustration of loss of energy expectation

The concept of capacity outage and equations to calculate the reliability indices, with respect to these outage levels for a given system, have been presented. There are several approaches used in previous research to incorporate capacity outages, which also can also be termed as unit operation uncertainty or forced outages. Two approaches commonly used in the literature are the use of derated capacities or probabilistic simulation.

One approach is using derated capacities for each generation unit as an available capacity. The derated capacities are calculated by multiplying the installed capacity by the availability of the corresponding generation unit. This approach makes the GEP problem easier to solve; however, it assumes that all the generation units are working at their average or expected levels all the time. Therefore, these approaches fail to consider the state when multiple generation units are not working at the same time. Therefore, the plans produced by this approach may not result in robust expansion plans which will still be effective when the system has multiple component failures. Similarly, the use of derated capacities means that the model never considers the case of possibility when the system is fully operational.

Another approach to consider outages is using probabilistic simulation (costing) method proposed by Baleriaux [8] and Booth [9]. These methods are used to calculate the expected energy served by each plant, along with the associated cost by considering all possible availability states of the system for a given number of plants installed. The available plants are ordered according to their operating cost; that is, the plant with cheapest operating cost is first. This is called merit order sequence. Then, equivalent load duration curve (ELDC) faced by each plant in a merit order sequence is recursively calculated by convolving the ELDC faced by the previous plant in merit order and the outage distribution of the present plant. For the first plant, ELDC is equal to LDC. Moreover, expected unserved energy can also be calculated by the area under the inverse of the ELDC curve beyond the total capacity of available plants. The associated fraction of time for which this unserved load exists gives the LOLP. Therefore, the production costing model is solved to find expected production, expected unserved energy and LOLP for each year in the planning horizon. The production costing method requires more computational time due to the numerical convolution process. Rau *et al.* [10] and Stremel and Rau [11] propose an approximation method to simplify the convolution process by approximating load distribution and outage distribution by the Gram-Charlier expansion. This approach make the GEP problem nonlinear, and, it still fails to consider the availability of the other components in the system such as transmission lines and distribution lines.

2.1.3.2. Emission Metrics

Each generation uses different types of resource to generate electricity. Based on the resource used, each generation units produce different amount of emissions. Generation of electricity generally produce emissions such as CO_2, SO_2, and NO_x that are harmful the environment. To properly compare different generation expansion plans, it is necessary to compute gas emission metrics. In this section, the metrics used for the gas emission are given.

Each fuel type has its own heat rate. The heat rate is commonly given in terms of Btu/kWh. Btu unit represents the heat value of the fuel. The gas emission for each fuel type is given in terms of lbs/MMBtu. In order to incorporate gas emissions into the GEP problem, it is required to know the amount of emission (lbs) produced to generate 1MWh of energy. This can be obtained by multiplying the heat rate with the emission amount (per heat value).

2.1.4. Risk Measures

Electricity planning is subject to a large degree of uncertainty due to the uncertainties in forecasted demand, cost and availability of fuels/technologies, reliability of generation groups, environmental regulation, weather condition and so on. Therefore, the GEP problem should be modeled as a stochastic optimization problem. An effective approach is to generate scenarios for uncertain parameters and assigning probabilities to each of them. This approach is applied thorough out this dissertation. The mathematical model proposed in this study is a stochastic programming model with recourse. More specifically, the model is a two-stage mixed integer linear programming model. Additionally, risk metrics are introduced into the optimization model to explicitly minimize risk as decision-maker preferences. In this section, some of these risk measures are explained.

Most studies are focusing on determining the optimum solution on average with respect to all scenarios by optimizing the expected value of the objective function. This approach assumes that every decision makers are risk neutral. However, there often are risk averse decision makers. For this kind of decision makers, some kind of risk measures are introduced to the mathematical programming model. There are several types of risk measures. As a part of this study, a mathematical model is developed which provides trade-offs between the expected objective functions and corresponding risk measures. In this section, some risk measures and their corresponding representations for two-stage mixed integer linear programming model are explained.

For the purpose of illustration, consider a two stage optimization model under uncertainty where the objective is to minimize the expected cost for a planning problem.

$$\min \left\{ \mathbf{cx} + \sum_{s \in \delta} \mathbf{p}_s \mathbf{f}_s \mathbf{y}_s \right\}$$

$$\text{s.t.}$$

$$B_s \mathbf{x} + D_s \mathbf{y}_s = \mathbf{z}_s$$

$$\mathbf{x} \in X, \mathbf{y} \in Y$$

A set of scenarios $s \in \delta$ is introduced for uncertain parameters with the probability p_s. $\mathbf{x}$ and $\mathbf{y_s}$ are stage one and stage two decision variables. $\mathbf{f_s}$ represents the objective function parameters associated with stage two variables for each scenario, and B_s, D_s, and $\mathbf{z_s}$ represents the parameters associated with the constraints for each scenario. Stage one variables must be selected considering the distribution of uncertainty, while stage two variables can be selected after observing the uncertain outcome. This model can now be extended by including an additional risk objective.

Variance

Variance is one of the most common risk measure used in the literature. It is defined as the expected square of the deviation from the expected value [12]. However, the variance is subjected to criticism for it represents the risk. Drawbacks for variance as a risk measure can be stated such that ups and downs (or highs and lows) are penalized equally and fat tails are not adequately represented. If the variance is used as a risk measure, two-stage optimization model can be represented as follows to minimize the cost objective and the risk objective.

$$\min \left\{ \mathbf{cx} + \sum_{s\in\delta} p_s \mathbf{f}_s \mathbf{y}_s , \sum_{s\in\delta} p_s \left((\mathbf{cx} + \mathbf{f}_s \mathbf{y}_s) - (\mathbf{cx} + \sum_{s'\in\delta} p_{s'} \mathbf{f}_{s'} \mathbf{y}_{s'}) \right)^2 \right\}$$

s.t.

$$B_s \mathbf{x} + D_s \mathbf{y}_s = \mathbf{z}_s$$
$$\mathbf{x} \in X, \mathbf{y} \in Y$$

Expected Excess (Expected Regret)

The expected excess can be defined as the expected value over a predefined target level η; which is equal to $\mathrm{E}[\max\{\mathbf{cx}+\mathbf{f}_s\mathbf{y}_s-\eta,0\}]$. Märkert and Schultz [13] show that the expected excess is introduced to two-stage stochastic programming model as follows;

$$
\begin{aligned}
&\min\left\{\mathbf{cx}+\sum_{s\in\delta} p_s\mathbf{f}_s\mathbf{y}_s, \sum_{s\in\delta} p_s\mathbf{v}_s\right\}\\
&\text{s.t.}\\
&\quad B_s\mathbf{x}+D_s\mathbf{y}_s=\mathbf{z}_s\\
&\quad \mathbf{cx}+\mathbf{f}_s\mathbf{y}_s-\eta\le\mathbf{v}_s\\
&\quad \mathbf{x}\in X, \mathbf{y}\in Y, \mathbf{v}_s\ge 0
\end{aligned}
$$

Excess Probability

The excess probability can be defined as the probability of exceeding a predefined target level η; which is equal to $P[s:\mathbf{cx}+\mathbf{f}_s\mathbf{y}_s>\eta]$. Schultz and Tiedemann [14] show that for a bounded *X*, there exists a constant *M* > 0 such that the two-stage stochastic programming model with excess probability can be written as;

$$
\begin{aligned}
&\min\left\{\mathbf{cx}+\sum_{s\in\delta} p_s\mathbf{f}_s\mathbf{y}_s, \sum_{s\in\delta} p_s\theta_s\right\}\\
&\text{s.t.}\\
&\quad B_s\mathbf{x}+D_s\mathbf{y}_s=z_s\\
&\quad \mathbf{cx}+\mathbf{f}_s\mathbf{y}_s-\eta\le M\theta_s\\
&\quad \mathbf{x}\in X, \mathbf{y}\in Y, \theta_s\in\{0,1\}
\end{aligned}
$$

Conditional Value-at-Risk

Value-at-Risk (VaR) is another risk measure used commonly, especially in the finance field. VaR is defined as the potential value loss of a portfolio over a predefined period for a given confidence interval. If underlying risk factors are distributed normally (or log-normally), it can be efficiently estimated. For other cases, there are multiple methodologies for modeling VaR. Linsmeier and Person [15] describe the VaR concept and they provide three methods to calculate VaR. VaR has some undesirable mathematical characteristics such as lack of subadditivity and convexity shown by Artzner *et al.*, [16, 17]. Therefore, Rockafellar and Uryasev [18] propose a new approach for portfolio optimization which minimizes conditional value at risk (CVaR). CVaR can be defined as the conditional expectation of losses above the VaR. Mean excess loss, mean shortfall and tail-VaR are other names used for CVaR. CVaR reflects the expectation of the $(1-\alpha)\times 100\%$ worst outcomes for a given probability level where $0<\alpha<1$. There are different formulations for CVaR [19]; one possible way used in Schultz and Tiedemann [20] is as follows.

$$\text{CVaR}_{\alpha} = \min_{\eta\in\mathbb{R}} g(\eta, \mathbf{x}, \mathbf{y})$$

where

$$g(\eta, \mathbf{x}, \mathbf{y}) = \eta + \frac{1}{1-\alpha} E[\max\{\mathbf{cx} + \mathbf{f}_s \mathbf{y}_s - \eta, 0\}].$$

Schultz and Tiedemann [20] show how to model CVaR for two-stage stochastic programming models as follows;

$$\min\left\{\mathbf{cx}+\sum_{s\in\delta}p_s\mathbf{f}_s\mathbf{y}_s,(\eta+\frac{1}{1-\alpha}\sum_{s\in\delta}p_s\mathbf{v}_s)\right\}$$

s.t.

$$B_s\mathbf{x}+D_s\mathbf{y}_s=\mathbf{z}_s$$
$$\mathbf{cx}+\mathbf{f}_s\mathbf{y}_s-\eta\le\mathbf{v}_s$$
$$\mathbf{x}\in X,\mathbf{y}\in Y,\mathbf{v}_s\ge 0$$

2.1.5. Smart Grid Technologies

There are technological developments which may be used to construct a more reliable, effective and energy efficient power grid. In this section, the key technologies and the benefits obtained from these technologies is presented. Also, the characteristics of the grid constructed by using the Smart Grid technologies are explained briefly. In this research, Smart Grid technologies are included in the optimization model as investment options and their benefits are modeled in the optimization models.

The Smart Grid is defined as a broad range and collection of technology solutions that optimize the energy value chain [21]. The energy value chain consists of the generation, transmission and distribution utilities and the final customers. These utilities can gain benefits from deploying parts of a Smart Grid technologies.

Amin and Stringer [22] define Smart Grid as an intelligent system which produces an autonomous digital system. This system is expected to be capable of identifying surges, downed lines and outages; providing instantaneous damage control and dynamic load balancing; accommodating new alternative energy sources; and minimizing vulnerability to terrorist or other attacks.

According to the National Energy Technology Laboratory (NETL) there are five key smart grid technologies. One of the main objectives of Smarter Grid is provide real time information to all the utilities in the energy chain. In order to provide this, it is required to have technologies providing two-way communication. These technologies are called as "integrated communication technologies", and they will support to integrate smart sensors, control devices and other intelligent technologies into grid.

In order to increase the reliability and efficiency of the grid, it is required to know the state of the system. Therefore, new digital technologies are developed; these are called "sensing and measurement" technologies. These technologies collect the data and transfer the data to be analyzed by using two-way communication system. By means of such technologies, it is possible to get the information about the state of the grid component and overall system, provide outage detection and response; enable utilizing demand response programs and so on.

There are many developments in new materials technologies, nanotechnologies, advanced digital designs and so on. In order to obtain modern grid, it is necessary to consider these technologies as an available technologies for expansion. The general name for this group of technologies is "advanced grid components." Some of the advanced technologies are superconducting transmission cable, fault current limiters, advanced energy storage, distributed generation, advanced transformers and so on.

Taking good decisions in a short time is very important for operating the power grid. Therefore, there are some new technologies, called "Decision Support and Human Interfaces" technologies, to convert the complex power system data into information which can be understood by the operator easily. Some of them are visualization tools and systems, operator decision support systems (what-if tolls, alerting tools, etc.), semi-autonomous agent software, real-time dynamic simulator training and so on.

Another group of technologies required to construct a Smart Grid are called “advanced control methods (ACM).” ACM technologies are the devices and algorithms that will monitor the essential grid components. They collect data (Sensing and Measurement), analyze the data and provide rapid diagnosis (Improved Interfaces and Decision Support), determine and take automated or provide appropriate response (Integrated communications, Advanced Components). Therefore, ACM technologies rely on and contribute to each of the four key technology areas. The key technologies and relations are given in Figure 2.5.

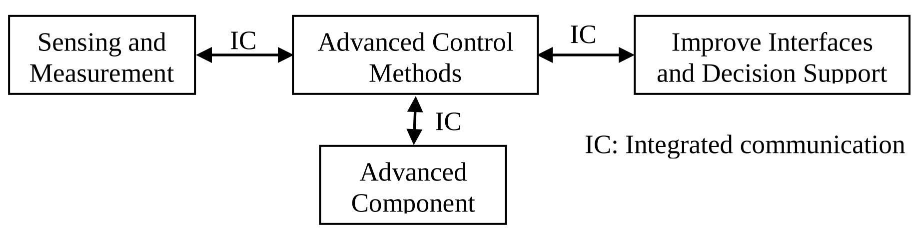

Figure 2.5. Smart Grid five key technologies [23]

There are numerous benefits of transforming the current electric power system into a Smart Grid. The Electric Power Research Institute (EPRI) states [24] that there will be $1.8 trillion in annual additive revenue by 2020 with a substantially more efficient and reliable grid. The Galvin Electricity Initiative states [25] that there will be reduction in power disturbance costs by $40 billion per year by means of Smart Grid technologies. They also estimate a reduction in infrastructure investments by between $46 billion and

$117 billion over the next 20 years by the Smart Grid. National Renewable Energy Laboratory [26] estimates that the carbon emissions would rise from 1,700 million tons of carbon per year today to 2300 by the year 2030 if nothing is done. They presented in the same study that if energy efficiency programs are implemented and renewable energy sources are used, the carbon emission growth can be prevented and reduced to 1,000 million tons of carbon by 2030.

Some of the benefits which are expected to be obtained by modernization of the power grid can be listed as (*i*) improved reliability, (*ii*) reduction in investments for generation, transmission and distribution, (*iii*) reduced operation and maintenance cost, (*iv*) integration of renewable energy and distributed resources, and (*v*) consumption management.

By means of two-way commutations all across the grid, power outages are remotely identified, located, isolates and restored more quickly. In addition, the frequency and duration of power outages are expected be reduced via proactive grid management and automated response. By using remote monitoring and control devices throughout the system, the outages can be restored and prevented and the life of substation equipment and distribution assets can be extended. All of these may improve the reliability of the grid.

Demand response and load management programs can be used to reduce the peak demand. This can lead a reduction for additional transmission lines and power plants. Enhanced asset management methodologies about the system component can prolong the life of the existing assets. This also can result in a reduction in capital investments.

Since some of the Smart Grid technologies enable remote and automated disconnections and reconnections, they can also eliminate unneeded field trips and reduce costumer outages and high-bill class, which can reduce O&M costs. Also, near real-time asset monitoring makes it possible to move from time based maintenance practices to equipment-condition-based maintenance which reduce the risk of overloading problematic equipments. This can also reduce O&M costs.

Another benefit which Smart Grid technologies provide is the ability of controlling energy flows. By means of integrated monitoring and control, it is possible to control differing energy flows and planning a standby capacity to supplement intermittent generation from renewable energy sources such as solar and wind. Moreover, the cost of distributed energy resources such as geothermal, biomass, carbon-free hydrogen fuel cells, photovoltaic panels, small-scale wind turbines, plug-in-hybrid vehicles (PHEVs) and batteries for energy storage declines while the cost of traditional energy sources increase. Smart Grid provide consumers to generate their own electricity and sell the surplus back to grid which can provide cost saving for consumers.

One of the main benefits of Smart Grid is that consumers can be more involved. Advanced meters inform the consumer how the energy is used in their home/business, what is the cost of usage, what kind of impact that usage has on the environment. Therefore, they can interactively manage their usage or set some preferences and let the grid manage. Home area networks consisting of smart appliance, thermostats, security systems and electronic which can communicate with the grid and send the information to consumer can be created. The appliances and security systems can be initiated for the conversation by the means of two-way communication systems. Smart metering and communication technologies would enable the consumers use energy more efficiently. All of these will lead better consumption management. Smart Grid also provides implementing demand response/load management programs, communication peak prices to consumer, integrating smart appliances and consumer storage and distributed generation to reduce the peak load demand which result in cost savings.

To provide an answer how to upgrade from existing system to Smart Grid, the GEP problem must be redefined. New GEP problems should be solved, not only to determine the type of technology, timing and the location of the new generation units or transmission lines, but also to determine which groups of Smart Grid technologies should be implemented and the time of implementation. As a part of this research, optimization models have been developed and solved where some subset of Smart Grid solutions are represented as the decision variables in the problem and the optimum introduction time for these technologies is selected. The benefits of the technologies are incorporated into the model by,

- Increase in the availability of the component
- Affect on the energy demand
- Reduction in the energy losses in the transmission lines.

Chapter 4 of this dissertation presents decision variable definitions and updated formulations to represent the impact of Smart Grid technologies in the GEP model.

2.2. Generation Expansion Planning

The electricity generation expansion planning (GEP) problem involves the selection of generation technology options to be added to an existing system, and when and where they should be constructed to meet the growing energy demand over a planning horizon time. Most studies focus on minimizing the cost. However, GEP includes many conflicting objective such as environmental impacts of generation or imported fuel and so on. Therefore, there are also some studies for solving multi-objective versions of these problems. Moreover, expansion plans are developed according to the estimates about the future, and this yields many uncertainties in the system. Therefore, there is some research done to consider the stochastic characteristics of the GEP problems. Besides the studies which consider different aspects of the problems, there are also many methods (i.e., mathematical programming, metaheuristic, decomposition techniques, etc) that have been used solve these problems.

In this section, a detailed summary of the literature on the GEP problems is presented. Benders decomposition is one of the most common decomposition technique used in GEP literature and as a part of this dissertation, Benders decomposition is used to increase the solution efficiency. Therefore, a detailed description about the usage of Benders decomposition for general cases is also presented.

2.2.1. Problem Formulation and Definitions

Before presenting the literature survey about the GEP problems, it is useful to provide an overview of the problem and some related definitions. GEP problems start with an existing power network. Consider the power network presented in Figure 2.6 as an example for existing network. This network consists of power groups where large central generation units are located. Electricity generated in these power groups are transmitted to area grid by transmission lines. For simplicity, the loop flows in transmission is not presented here. Demand is presented by load blocks and distribution lines are used to distribute electricity to load blocks. Since the electricity demand keeps increasing, the existing network will be insufficient in the future. Therefore, the existing system should be expanded by new technologies in order to provide economic and reliable energy supply in the future. Expansion schedule is determined by solving the GEP problems for a long term planning horizon.

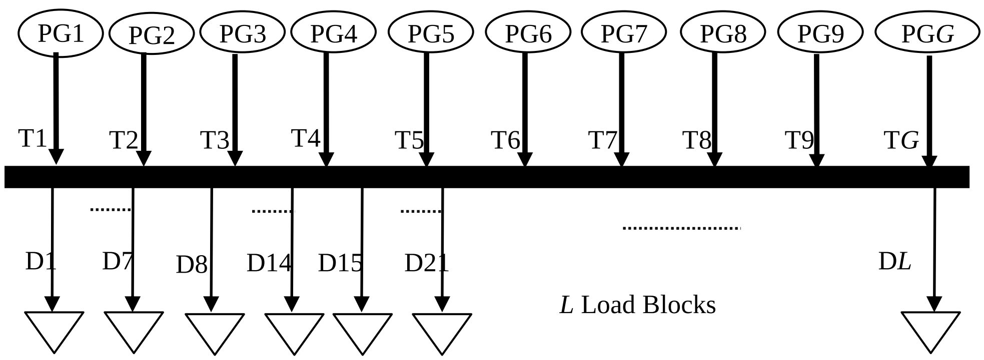

Figure 2.6. Example for existing power system

There are four main group of technologies can be added to the system; generation units, transmission lines, distribution lines and Smart Grid technologies. The literature review presented in this section focus on the studies where generation expansion is the main objective. In some these research, transmission lines are also considered together with generation units. A few researchers also consider demand side management and distributed generation units (Smart Grid technologies) as an expansion options. Expansion plans can include both building large central generation units such as nuclear plants, coal burning plants, large wind turbines, etc. in the power groups and small distributed generation units close to the load blocks.

Table 2.1 lists objectives considered in GEP problems; decision variables used and their definitions; some constraints commonly used and stochastic parameters which can be part of the GEP problems.

Table 2.1: Generation Expansion Planning Problem

<table>
<tr><th colspan="4">Objective Functions</th></tr>
<tr><td colspan="2">1. Cost
• Investment
• Fixed O&M Cost
• Generation Cost
• Unmet demand Cost</td><td colspan="2">2. Gas emissions
• Greenhouse gases (CO_2)
• Pollutants
(NO_x, SO_2)</td></tr>
<tr><th colspan="4">Decision Variables</th></tr>
<tr><td colspan="2">1. Investment Decisions
• Large Central Generation Units (Nuclear, coal burning, oil, etc.)
• Renewable Generation Units (Wind, Solar, etc.)
• Distributed Generation Units (Internal Combustion engine, micro-turbines, fuel cells, etc.)
• Smart Grid Technologies</td><td colspan="2">2. Dispatching Decisions
• Amount of energy (MWh) produced by central units
• Amount of energy (MWh) produced by renewable energy sources
• Amount of energy (MWh) produced by distributed generation units</td></tr>
<tr><th colspan="4">Constraints</th></tr>
<tr><td>1. Demand Constraints</td><td>2. Capacity Constraints</td><td>3. Reliability Constraints</td><td>4. Fuel Type Related Constraints</td></tr>
<tr><th colspan="4">Stochastic Parameters</th></tr>
<tr><td>1. Demand Growth Rate</td><td>2. Fuel Prices</td><td>3. Construction Time</td><td>4. Environmental regulations</td></tr>
</table>

2.2.2. Existing Research for GEP Problems

The GEP problem has been an extensively studied problem. Although most of the studies focus on cost minimization, there are some studies addressing other conflicting objectives such as minimization of the cost, minimization of the green gas emissions and so on. In addition, since there are uncertainties associated with the input data, some researchers provide models to represent the stochastic characteristics of the system. In this section, relevant research is summarized.

2.2.2.1. Least-Cost Generation Expansion Planning Problems

GEP research often focuses on finding the least cost expansion plan which satisfies a predefined reliability target such as LOLP, ENNS, and so on. In this section, some of the research which focuses on finding deterministic least-cost expansion plan is reviewed.

One of the earliest works where least cost GEP is solved is Masse and Gibrat [27]. Anderson [28] provides a survey for earlier research to determine the least cost expansion plan. Beglari and Laugton [29] solve the least cost expansion plan for generation units and transmission lines where the objective is to minimize the total capital cost and operations cost. In their research, the planning period is divided into intervals which are represented by the peak demand in the corresponding interval. They increase the peak load demand by a reserve margin for each interval and use this as a reliability constraint by forcing available capacity to be greater than the peak load demand plus reserve capacity. Sawey and Zinn [30] also provide a model to choose the minimum cost expansion plan for generation units and transmission lines over a planning horizon.

Noonan and Giglio [31] model the least-cost generation planning problem as a large scale, chance constrained, mixed integer programming model. They divide the year into weeks in order to model seasonal variations in demand and available capacity. They also use different demand levels for each weekly demand pattern. They aggregate plants into three generation classes; thermal, conventional hydro and pumped hydro. The plants in the same class have similar characteristic in terms of operations costs, capacity and so on.

They provide the model with the alternative investment projects in each year from each class. Therefore, they define integer variables defining the number of projects selected from the available investment projects for each year and generation class. The objective function is to minimize the investment and operations cost. To represent the system reliability constraints, they use chance constraints which ensure that the probability that annual peak demand for each year will not be satisfied must be less than or equal to some specified level of risk. They provide an equivalent deterministic constraint for this chance constraint when the probability function of available capacity, at peak demand hour, minus peak demand is normal. The other constraints are for demand for each week and demand level in each year and capacity constraints for each generation class for each week and demand level in each year.

Bloom [32] models long range least cost GEP problems using production costing modeling. The objective includes the investment cost and expected operational cost. He uses ENNS as a reliability measure by defining a constraint for each year which forces the expected unmet energy in each year to be smaller than a predefined value. Sherali *et al.* [33] propose a model to solve least cost GEP problem where renewable energy sources are also considered as investment options. They consider discrete sizes in which plants are available for expanding capacity. They use expected unserved energy as a reliability criteria along with additional reserve margin reliability constraints. They impose some upper and lower bounds for decision variables due to some practical considerations such as the lead time on construction or algorithmic considerations. They also impose an upper bound on the cumulative capacity of each renewable energy source. Ramos *et al.* [34] find the single period least cost expansion plan where the model also considers the technical minima of thermal technologies; that is, the minimum percentage of the capacities connected to the power grid which must operate. They also use detailed operations models for storage-hydro and pumped-hydro technologies and calculate the fixed cost for storage-hydro and pumped-hydro accordingly.

Park *et al.* [35] solve a least-cost GEP problem where loss of load probability (LOLP) is used as a reliability criterion. In this model, the objective is to find a set of optimal decision vectors, representing the capacity extension plan for each time period, which minimizes the objective function under an LOLP constraint and maximum construction capability. The objective function is the summation of three discounted costs; capital cost, O&M cost and salvage value.

Su *et al.* [36] formulate the GEP problem to determine the long term expansion plan where the objective is to minimize the capital, maintenance and fuel cost. They use LOLE as a reliability index. Although the objective is to find least cost expansion plan, they incorporate the reliability and environmental issues by applying fuzzy theory. They apply the fuzzy theory to represent the generation mix constraints and environmental protection constraint. The suitability of different types of units to serve base load and to serve emergency load are represented as a fuzzy set and they are used to construct the fuzzy constraint for the generation mix. The constraints are used to make sure that the system has enough base load units like nuclear, units with lower operation cost, but probably with higher starting time, and enough emergency load units like combustion turbine, units with rapid-starting but probably with higher operation cost. The degree of each type of units to produce different kinds of pollutants are also represented as a fuzzy set and used to construct the fuzzy constraint for environmental protection.

Kannan *et al.* [37] solve the least cost GEP problem where the objective is to minimize total cost which consists of outage costs as well as the investment cost O&M cost, and salvage costs to satisfy reliability, fuel mix and demand constraints.

Sirikum and Techanitisawad [38] model a power generation expansion planning problem as a mixed integer nonlinear programming problem. The objective of the model is to determine an optimal generation expansion plan which minimizes the expected sum of discounted investment cost and variable costs containing fuel costs, operating costs, environmental costs and unserved energy cost under the demand constraints, capacity constraints, reliability constraints (reserve margin constraints and LOLP constraints), emission constraints and location constraints. The load duration curve for each period is divided into segments. Demand constraints are used for each segment in each period. The power output generated by each generation unit is limited with its available capacity which is found by rating the available capacity with the availability factors of the corresponding unit in the corresponding period. They use two discrete decision variables; one to show whether or not the plant type is selected to be constructed in time period t and the second one is to show whether or not the DSM program is implemented at the beginning of time period t. They model the DSM program such that if the DSM program is implemented, it provides a power saving by the efficient energy using equipment under DSM program type d in time period t. They use a parameter showing this power saving for each type DSM program in each time period t.

2.2.2.2. Multi-Objective Generation Expansion Planning

Power generation expansion planning problem includes conflicting objectives. Therefore, models which exclusively consider these distinct objective models become more realistic than the ones where the other objectives functions are encompassed by a single economic indicator. Multi-objective models provide an opportunity for decision makers to comprehend the conflicting nature and trade-offs among different objectives to select satisfactory compromise solutions.

Mavrotas *et al.* [39] solve single period GEP for the Greek electricity generation industry where the objectives are to minimize cost and SO_2 emissions. The load duration curve is divided into subperiods and the units are grouped together based on their technologies. Therefore, the expansion decisions are represented as integer variables and they correspond to the number of units operating in the examined year for each unit type; and the operation variables are represented as continuous variables corresponding to the output level of each unit type in each subperiod. The forced outages are introduced into the model by using derated capacities for each unit type. The specific technical operational level for each unit type is formulated as the minimum load requirement constraint. Other constraints are the demand, reserve margin, natural gas supply limit and some other constraints specific for the case studied.

Karaki *et al.* [40] develop a generation expansion planning tool to minimize either the cost or the environmental impact or some weighted function of the two. They include the environmental impact into the objective function by adding the costs for cleaning the pollutants emitted. They use probabilistic production cost simulation to obtain expected energy not supplied and expected yearly energy produced by each unit in order to calculate the expected production cost for each unit.

Antunes *et al.* [41] formulate the GEP as multi-objective mixed integer linear programming (MOMILP) problem. They consider peak shaving as a demand-side option. In their model, an important part of the load is supplied in a franchise environment and generation capacity expansion is mostly centrally planned. They use integer variables to represent the number of capacity modules for each group type which prevent the shortcoming of using continuous variables for expansion decisions, and then, discretizing them in a post-processing phase without considering the effects on the obtained solution.

Antunes *et al.* [41] use improved *z*-substitute method to represent the load demand. The planning horizon is divided into periods and each period has a load duration curve. Periods are composed of a specified number of intervals where each interval corresponds to a power demand value. Decision variable z^j_{is} represent the reduction in power output of type *i* occurring from interval *s*-1 to *s* in period *j*.

Antunes *et al.* [41] consider three objective functions; (*i*) total expansion cost, investment and operational and maintenance cost, (*ii*) the environmental impact associated with the installed power capacity, and (*iii*) the environmental impact associated with the energy generation. DSM programs are modeled as an equivalent DSM generating unit. However, it is allowed to be effective only for the highest demand values in each period in order to provide peak shaving properties. They model reliability by defining reserve margin for peak load demand. They impose upper bounds on the total capacity of each generation technology to be installed in each period and limitations for pollutants and green gas emissions. They also have constraints that the power that can be generated by any type unit in each period cannot exceed its rated capacity by an availability factor. They also have constraints for supplying demand and an upper bound on penetration of DSM at each period.

Meza *et al* [42] proposes a model to optimize simultaneously multiple objectives to determine the number and the location of generating units of each type to be constructed in each period for a multi-period planning horizon. The first objective is to minimize the investment, operation and transmission cost. The second objective is to minimize the amount of carbon dioxide emission calculated by multiplying the generation amount from each type by corresponding emission amount for that type of generation unit. The third objective function is to minimize the imported fuel which is calculated by fuel used in each year by the corresponding forecasted price. The last objective function is to minimize the energy price risks which are calculated by multiplying the generation amount using the fuel type k by the expected coefficient of variation in prices of fuel type k. They use multiple constraints such as flow balance constraints for each supply/demand nodes, transmission limits on arcs, generation and investment capacities, and availability of fuel types. They did not model forced outages for generation units in their model.

Meza *et al.* [43] solve a single period GEP problem where they minimize the same objective functions as in Meza *et al.* [42]. However, in this study they include the Kirchoff's second law into the model, making the problem nonlinear. Therefore, the GEP problem is solved to determine the number of generating units, the number of new circuits on the network and the voltage angle at each node.

2.2.2.3. Stochastic Generation Expansion Planning

There are many uncertainties in the generation expansion planning such as load growth rates, fuel costs, fuel availability, construction time, interest rate, financial constraints, environmental constraints and so on. Therefore, the models representing these uncertainties are required to represent the actual generation system accurately. In this section, research is presented where uncertainties are considered as a part of the expansion plan.

Scenario-based approaches are commonly used to model uncertainties where some of the realizations of uncertain parameters are represented by the scenarios. There is a tradeoff between the accuracy and the complexity of the problem since the complexity of the problem is increased as the number of the scenarios is increased.

One of the first applications of stochastic programming to GEP problem is presented in Dapkus and Bowe [44]. They model the GEP problem as a stochastic dynamic programming problem. They consider the uncertainties in demand, the commercialization date of new technologies and the possible loss of existing nuclear capacity due to the accident, regularity action or lack of fuel. By means of this approach, they provide contingency plans for decision makers to wait until uncertainty is resolved before committing to construction.

Levin *et al.* [45] consider the uncertainty in the prices of primary energy resources. They assume that the fixed costs are deterministic and the variable costs are random numbers due to the uncertainty in the fuel prices. They solve the problem for a system consisting of two units. First, they show how to determine the probability distribution of the installed capacity for any distribution of the fuel prices. Then, they show how to derive a variety of performance measures such as expected value, the mode and the variance of the installed capacity and the expected costs under the assumption of normally distributed fuel prices. They first determine the capacities of both units for a known realization of the fuel prices by applying the breakeven approach, and then, they derive the distribution for installed capacity of each unit.

The breakeven approach is a recursive algorithm to find the capacity of each unit to meet the demand for power at a minimum cost where the problem is formulated to minimize the fixed and variable costs by satisfying a constraint that total installed capacity is greater than or equal to the peak load demand in the period. When the fixed costs and variable costs are deterministic, the breakeven approach can be explained simply as calculating a ratio of the fixed cost difference over variable cost for each generation unit pair to find breakeven points and determining the capacities to be installed for each unit by projecting corresponding breakeven points onto the LDC.

Sanghvi and Shavel [46] consider the uncertainties in the load growth and in hydro energy availability. The uncertainty in hydro energy availability is introduced into the model by defining a number of states, each of which corresponds to a certain steam flow conditions. For each season, available energy from hydro plants is used to characterize the state. They use seasonal load duration curves for each period under different load growth scenarios to represent the load growth uncertainties.

Mo *et al.* [47] also model least cost GEP as a stochastic dynamic programming problem. The uncertainties included in their models are the uncertainties in energy demand and oil price as well as delays in construction. Gorenstin *et al.* [48] describe a methodology for least cost GEP under several scenarios representing the uncertainties such as demand growth, fuel cost, construction delay, financial constraints and so on. They propose to minimize the maximum regret associated with each scenario instead of minimizing the expected cost. They claim that using the mean cost as the decision criterion in stochastic problems is adequate only if all the possible scenarios have a similar probability to occur. If some scenarios have much less probability to occur than the others, minimizing the mean cost is not adequate. Therefore, they calculate the regret associated with each combination of decision and scenario, and they minimize the maximum regret. To do this, they define a variable and add constraints for each scenario such that this new variable should be equal to or greater than the cost occur if all the scenarios are considered minus the optimal cost when only the corresponding scenario is considered.

Malcolm and Zenios [49] develop a robust optimization model for single period GEP problems considering uncertain demand. Two kinds of robustness are defined in the paper; solution and model robustness. Solution robustness is defined as the optimum solution obtained from the model that is almost optimal for any realization of the demand scenarios. Model robustness is defined as the optimal solution obtained from the model that has almost no excess capacity for any realization of the demand scenarios. The LDC is divided into subperiods and two kinds of continuous variables are used as decision variables. The expansion decisions are independent from the uncertainty and defined as the total capacity assigned to each plant. The design variables are dependent on the uncertainty and defined as the allocation of capacity to each subperiod from each plant type under each demand scenario. The forced outages are not considered in this paper.

In order to satisfy the robustness, the objective function is composed of three terms; the expected cost of the system over all demand scenarios, the weighted variance of the cost and a weighted function penalizing the deviations from feasibility. The deviation from feasibility is defined in two ways; the surplus capacity of each plant type under each scenario and the unmet demand in each subperiod under each scenario. The trade-off between solution and model robustness is examined by varying the weights of the second and third terms.

Pokharel and Ponnambalam [50] formulate a single period GEP problem to find the minimum cost expansion plan. They minimize the annualized capital costs and operations costs. They include the capacity constraints, technological limits, budget constraint and pollution constraints into their model. They also include a stochastic model where demand in each operational mode and availability of the system components are uncertain.

Marín and Salmerón [51] present a nonlinear stochastic model for electric capacity expansion planning under demand uncertainty. In their model, periods in the planning horizon are divided into smaller subperiods whose demand is uncertain and modeled as a continuous probability distribution function. Therefore, the objective function also includes risk-cost for each subperiod due to the uncertainty, as well as, the investment and operations cost. For each period in each year, a penalty occurs if the total production is less than the demand. They define risk cost as an expected penalty for a given production level. This means that the penalty function and probability distribution of demand for each period in each year should be known. Demand constraints are adjusted to reflect the uncertain demand. Instead of forcing production level to be greater than the demand for each subperiod in each year, they define a lower and upper bound for production level in each subperiod. Lower bound is defined as the minimum requirement for demand supply, such as expected demand, and upper bound is defined as the upper bound of the probability distribution of demand for each subperiod. They also include budget constraints for each period, an investment capacity bound for each type of unit in each period and coupling constraints which force generation from each unit type in each subperiod to be less than or equal to the available capacity of the corresponding unit. They use derated capacity of each unit as an available capacity.

Jirutitijaroen and Singh [52] model multi-area power systems as capacity flow networks to determine the generation capacity requirement in each area. However, the problem is not solved to determine the type and size of the new technologies to invest. Therefore, it is not generation expansion planning, but multi-area generation adequacy planning. The problem is stochastic due to the random uncertainties in area generation, transmission lines and area loads. They model the problem as a mixed-integer stochastic programming model with a two-stage recourse model. The first stage decision variables are the number of generators to be invested in each area (variables responding to the distribution of uncertainty) and the second stage variables are the actual flows in the network (variables responding to observed uncertainty). The objective is to minimize the expansion cost and maximize the reliability. The reliability index used in the paper is the expected loss of load.

Jirutitijaroen and Singh [52] construct the network such that each node in the network represents an area with corresponding area generation capacity and area load. The nodes are connected with tie-lines with the corresponding line capacity. They construct discrete probability distribution functions for generation in each area by using a sequential unit addition approach based on the generation unit parameters such as the failure rate, mean repair time and capacity assuming two-stage Markov process. They round the generation capacity to a fixed increment to decrease the number of states for generation capacity in each area. They obtain the area generation states with the corresponding probabilities. For the area load, they cluster the similar load together. Therefore, they have an appropriate number of states with corresponding probabilities for the area load. For the tie-line, they also construct line capacity states based on the tie-line parameters, capacity, forced outage rate and repair rate, assuming a two-stage Markov process. For the new generators, they use their effective capacities. They did not consider the generators individually or as groups with the same technologies. They just simply investigated the system in terms of available capacity. Therefore, it is not possible to consider dispatching or operational cost or environmental impact. Although this problem is not a generation expansion problem, it provides some insight about how stochastic programming with recourse can be used.

2.2.3. Methodologies used to Solve Generation Expansion Planning

In this section, solution methodologies for this problem are summarized including mathematical programming approaches, metaheuristics and decomposition approaches. Benders decomposition is used to increase solution efficiency in this dissertation; therefore, a detailed explanation for Benders decomposition is also presented.

Kagiannas *et al.* [3], Zhu and Chow [4], and Hobbs [2] provide a survey of modeling techniques developed for GEP. These authors provide detailed lists of previous research using dynamic programming approaches, decomposition techniques, stochastic optimization, Genetic Algorithm (GA), fuzzy set theory, artificial neural networks, network flows, simulated annealing, etc. Nara [5] presents a systematic survey for applied simulated annealing, genetic and evolutionary algorithms, and tabu search applied to power systems planning problems.

2.2.3.1. Mathematical Programming Approaches

In this section, linear and nonlinear mathematical programming approaches are presented. One of the earliest research studies where GEP is modeled as a linear programming problem is presented in Masse and Gibrat [27]. Anderson [28] provides a survey for earlier linear and nonlinear programming models where the derating technique is used to account for forced outages.

Petersen [53] develops a dynamic programming model to determine the optimal expansion plan for the electric power system. The problem is to find the least cost capacity expansion for system consisting of hydro, nuclear, thermal and peaking turbine plants and the timing of these expansions. They also show how to computationally simplify the formulation to get a computationally feasible model.

Beglari and Laugton [29] model the least cost GEP problem as a linear programming problem. They proposed a method where they can remove the operating constraints from the mathematical formulation by making assumptions on possible plant operation conditions and optimizing the problem given these assumptions. They write the production cost in terms of total capacity of each technology. They assume that energy produced by plant type i in period n is equal to the multiplication of load factor for plant *i* in period *n* (θ_{in}), utilization factor for plant *i* in period *n* (α_{in}), duration of each interval *t* and total capacity of plant *i* in interval *n.* By means of this transformation, they can write the total capital and generation cost in terms of only expansion decisions. They started solving the problem with estimated α and θ values. Once they obtained the best expansion plan for given α and θ values, they perform the simulation to obtain new estimates of α and θ values. They iteratively solve LP problem and use simulation to get α and θ values until there is a convergence. They apply the same procedure for the transmission expansion planning problem and combined generation and transmission expansion planning problem.

Sawey and Zinn [30] model the problem as a mixed integer GEP problem. They represent the electric utility system as a network where nodes in the network are demand or supply points and the links between them represent the transmission lines. The objective is to minimize the sum of discounted capital and operations cost. Besides the commonly used constraints, they also include capacity constraints for power flow for transmission lines.

Levin *et al.* [54] preset the conditions to indicate whether the solutions obtained by using the time-step approach to solve the generation capacity planning in power system, are the same as the solutions obtained by an equivalent dynamic model. The dynamic model they consider is to minimize the discounted fixed and operations cost given the constraints of utilization of each unit type in each year cannot exceed its cumulative installed capacity prior to the corresponding year. They assume that it is unlikely that a large unit is built in a given year and utilized at smaller capacity at the same year. Therefore, they assume that all of the new capacities installed are utilized in the same year. They calculate the energy produced for each unit by assuming that the units are loaded in merit order. They calculate the loading point for each unit, and then, they find the energy produced by each unit by calculating the area under the inverse of the LDC between the loading point of the corresponding unit and the loading point of the next unit in merit order. Since the inverse of the LDC is a nonlinear function, the mathematical problem is nonlinear. The time step approach is an alternative approach where a series of related one year problems are solved, and the expansion plan of a given year is used as an input to the optimization problem of the following year. Time step approach provides significant computational savings. However, it is not guaranteed to be the optimal solutions in the dynamic sense.

Dapkus and Bowe [55] use a stochastic dynamic programming approach to solve the stochastic GEP problem. They define the state of the system $S(k)$ for each period k by the number of units of each type of technology, the availability status of each technology and the peak level of demand. The availability status of each technology represents the situation which affects the availability of entire technology such as the loss of service of all existing nuclear plants due to the regulatory actions or having an embargo which leads to shortage of fuel making plants unavailable, or delay in the introduction of new technology. Unavailabilities due to the planned or forced outages which only affect individual plants are considered by means of using production costing methods. They solve an optimization problem to find the best expansion plan for each state $S(k)$ in period k which minimizes the production cost for state $S(k)$, the capital cost of expansion plan $U(k)$ and expected cost in period $k+1$ given decision $U(k)$ is made. They define the probability of the system being in state $S(k+1)$ given the system in state $S(k)$, $U(k)$ and k

to calculate the expected cost in period $k+1$. The performance of the method depends on the number of states which makes it difficult to consider additional uncertainties and individual generation units instead of using generation technology types.

Park *et al.* [56] develop an analytical approach for the production costing model with an assumption of a Gaussian probabilistic distribution for random load fluctuation and plant outages. The production costing models are solved to determine the operations cost and annual reliability measures. They show that this problem can be solved analytically with the Gaussian assumption.

Sherali *et al.* [33] model the least cost GEP as a nonlinear programming model with discrete decision variables. They use branch and bound algorithm to solve the problem. At each node in the enumeration tree, a continuous relaxation of the problem is solved with a proposed two-phase procedure. In the first phase, the problem is solved by using derated capacities to determine a near optimal solution. In the second phase, outer linearization is employed in the vicinity of the resulting solution of the first phase to accelerate the convergence of the accurate solution (i.e., the solution that would be found by using production costing model).

David and Rongda [57] develop interactive software to integrate the engineering experience and judgment of the decision makers with dynamic programming. The number of states and the transition between states are reduced by rules which are defined based on the expert, which leads them to use dynamic programming for large problems. David and Rongda [58] improve their previous research by using fuzzy set theory as a mechanism for incorporating the qualitative judgments. The model presented in Ramos *et al.* [34] is nonlinear programming problem. The objective function becomes a nonlinear due to the fixed cost for storage-hydro and pumped-hydro. The constraints in the model are linear. Therefore, they state that this problem can be solved with nonlinear optimization code such as MINOS.

Mo *et al.* [47] describe how to use stochastic dynamic programming to solve GEP problems. They model the uncertainties as a Markov chain and use backward stochastic dynamic programming to solve the problem. Malcolm and Zenios [49] model a robust GEP optimization problem as a linear programming problem by representing the demand uncertainties by means of corresponding scenarios.

Pokharel and Ponnambalam [50] formulate the GEP problem as a linear programming problem. They include the availability of the generation units by decreasing the total capacity with availability factor. To include uncertainties, they form scenarios and they calculate the expected costs. They adjust the operational constraint for each technology. The electricity delivered by each technology should be smaller than the availability factor multiplied by the capacity of that technology.

Mavrotas *et al.* [39] propose a new approach to solve mixed 0-1 multiple objective linear programming (MOLP) models. The proposed model is based on the branch-and-bound algorithm which is modified for the multi-objective case in order to generate the efficient set in mixed 0-1 multiple objective linear programming models. The algorithm applies the depth first search. Therefore, the process moves from the root node to final node. When the process reaches the final node, the efficient solutions for the current combination of binary variables are calculated. They call these efficient solutions "partially efficient solutions." Since these solutions are candidates for being the efficient solutions for the general problem, they are stored in the database. The partially efficient set is updated every time the new partially efficient solutions are obtained at the final node. Updates are based on the comparison done to check the dominance between the new generated solutions and the existing ones. In the intermediate nodes, the ideal vector for the problem is calculated by optimizing each objective function separately. If the ideal vector is infeasible or dominated by any of the solutions from the partially efficient set, then this branch is fathomed. This process is continued until all of the possible combinations of binary variables are examined.

Karaki *et al.* [40] use tunnel dynamic programming to solve the GEP problem. Each year is represented as stage which each stage having several states. At each stage, the tunnel dynamic programming model determines the states of the next stage by adding units to the states of the present stage. At each state, probabilistic production costing simulation is run to obtain the cost incurred up to the current year. They apply tunnel-heuristic rules at each stage to limit the number of options analyzed and saved for further expansion. The rules applied are *i*) defining the maximum number of feasible transition from each state as the number of available technologies; *ii*) selecting the best *N* states from the states analyzed at each stage to determine the feasible transitions to the states of next stage. At each stage, the tunnel dynamic programming adds units to the system when expected energy not served exceeds a predefined upper bound until the expected energy not served drops below the predefined lower bound.

Su *et al.* [36] use the forward dynamic programming as the optimization method for a long term GEP problem. They define a path as the newly installed units and the state as the existing units plus the new units. They use the fuzzy theory to make qualitative judgment about the path and the states which allows them to delete unnecessary paths and states to reduce the computational time.

Antunes *et al.* [59] propose an interactive method to find nondominated solutions. The proposed algorithm starts with finding the individual optima of each objective function for a relaxed MOMILP (MOLP), where the binary variables are relaxed. The individual optima solutions are used to form ideal solution for the relaxed problem. This ideal solution is used as reference point for the first iteration. The nondominated solution to MOLP is found by minimizing the Tchebycheff distance to reference point. If decision maker found the solution to be satisfactory, then the solution of MOLP is taken as a reference point and the nondominated solution to the MOMILP which minimizes the Tchebycheff distance to this new reference point is calculated. If the decision maker finds the solution to be unsatisfactory, then the decision maker is asked to input his/her preferences into the procedure by either specifying a new reference point or indicating which objective functions can be relaxed and by how much. Then the same procedure is carried out until a satisfactory solution is found.

Meza *et al.* [42] model the multi-objective GEP problem as a linear programming model. They propose a solution methodology consisting of two phases. In the first phase, they generate *m* nondominated solutions. Three of these solutions are obtained by using min-max, max-min and compromise programming methods. The remaining is obtained by solving the weighted summation of scaled objective functions for a large number of random weights and combining the similar ones by using *k*-means clustering algorithm.

In the second phase these nondominated solutions are ranked by using the Analytical Hierarchy Process (AHP).

2.2.3.2. Metaheuristics

Metaheuristics are an iterative search procedure which explores and exploits solutions in the search space. In this section, studies are presented which use metaheuristics to solve the GEP problem.

GA can be defined as a search mechanism based on the hypothesis of natural selection. GA produces solutions by generating a set of chromosomes where each of them has its own fitness measure which reflects the solution quality. The new sets of chromosome, referred to, as a generation, is produced through three genetic operations; reproduction, crossover and mutation. Reproduction is copying individual chromosomes based on their fitness measure into the next generation. Crossover is generating a new chromosome from two parent chromosomes. Mutation is altering the generated chromosome. There are many types of crossover and mutation methods.

Fukuyama and Chiang [60] apply a parallel genetic algorithm (PGA) to optimal long range least cost GEP problem. PGA performs GA in parallel. They mention two types of PGA; coarse-grain and fine-grain. In coarse-grain PGA, the total population is distributed into sub-populations and each sub-population is allocated to each process. At each process conventional GA is applied and during the optimization, the strings are sometimes exchanged between the sub-populations. In this kind of PGA, a parallel program consists of a few processes with intensive computation and little communication demands. In fine-grain PGA, each string is mapped into each process and each process exchanges the string information. In this kind of PGA, a large number of processes are needed with low computation and high communication demands. They apply coarse-grain PGA to solve GEP problem. At each process, conventional GA procedures are applied with addition of migration procedure. Migration procedure allows the process to migrate strings with highest fitness values the neighboring processes at every iteration of the GA. They applied the proposed method to a test system and they conclude that the method provide fast and accurate solutions.

Park *et al.* [35] propose an evolutionary programming algorithm to solve the GEP problem. The evolutionary programming method is based on mutation, competition and selection. An initial population is randomly selected from a feasible region to be initial parents. Each individual in the parent population creates a new population called offspring. This step is called mutation. A new set of offspring created by mutation and the original population constitute a competing pool. Each individual in this competing pool competes with others in the pool to be selected as parent for the next generation. Park *et al.* [35] propose a method to create offspring based on a Gaussian method and quadratic approximation technique. Mutation by the Gaussian method means creating offspring by adding a Gaussian random variable with zero mean and predetermined standard deviation. They chose a random point and generate a point near the symmetry point on a straight line based on the two selected points. Then, an approximated quadratic extreme point is calculated by approximated quadratic functions based on the individual solutions of the population. Then, the orderly selected point, symmetry point and extreme point are mutated and the one having best cost value and satisfying LOLP constraint is selected and inserted into the competing pool.

Park *et al.* [61] propose an improved GA to solve the least cost GEP problem. They convert the decision variables (which have dimensions of MW) into the vectors which represent the decision variables as the number of units in each plant type. They represent the solution of generation expansion by a string where decisions are represented as binary variables for each type of plant and each year. They provide some improvements for GA methods. The first improvement they propose is related to the fitness function. One way to calculate the fitness of each string is by dividing a pre-specified constant value by the objective function value plus 1. However, they claim that using this as a fitness measure causes premature convergence and duplications among strings in a population. Therefore, they use a modified fitness measure based on the minimum and maximum fitness value in the generation. The second improvement is for creating the initial population. They suggest a new artificial initial population scheme which distributes the strings throughout the solution space. The third improvement is using stochastic crossover where one of three crossovers is selected from a biased roulette wheel, where each crossover method has different sized slot on the wheel. The three crossovers considered in this paper are 1-point crossover, 2-point crossover and 1-point substring crossovers. 1-point crossover means that a point is randomly selected in the parent strings and the left-side of the strings are exchanged to generate new chromosomes. 2-point crossover means that two random points are selected and the part between these points are exchanged. 1 point substring crossover means that a random point for each substring, which represent the decision for each type, is selected and the part on the left is exchanged. It is stated that 1-point substring crossover is good for exploring the solution space but it is easy to destroy the string structure which has partial information about the optimal structure. 1-point and 2-point crossovers cannot explore the solution space as widely as 1-point substring crossover. However, they are preferred for keeping the structure which has partial information about the optimal structure.

Chung *et al.* [62] use the GA to solve the optimal least cost GEP problem of an all-thermal power system modeled as a single period mixed integer nonlinear programming model. The system consists of the nuclear power units, oil-fired units and coal-fired units where coal-fired units are committed during peak hours. Therefore, coal-fired units have different capacity blocks with varying incremental cost.

Kannan *et al*. [37] apply the five variants of particle swarm optimization (PSO) techniques to solve the GEP problem. PSO techniques are based on the inherent rule, followed by the member of birds and fish in the swarm, which enables them to move without colliding. PSO techniques use a population of potential solutions to search the solution space by using physical movement of the individuals in the swarm. Each individual is represented by a position and velocity vector. Bird flocking optimizes the objective functions. Each individual knows its best fitness value so far (individual intelligence) which contains the information on the position and velocities. They also know the best fitness value among the group (group intelligence). Each individual tries to modify its position by considering current positions and velocities, individual intelligence and group intelligence. The fitness function used in the paper is the objective function plus a penalty function for unsatisfied constraints. They apply different variants of PSO techniques which differ in the way that they modify the velocity or selection mechanism used.

Kannan *et al.* [63] present the application and the comparison of the metaheuristic techniques such as GA, Differential Evolution (DE), Evolutionary Programming (EP), Evolutionary Strategy (ES), Ant Colony Optimization (ACO), PSO, Tabu Search (TS), Simulated Annealing (SA) and Hybrid Approach (HA) to the GEP problem. In their paper, GEP is formulated to minimize the cost with an upper bound on the construction capacity at each period, minimum and maximum reserve margin, minimum and maximum fuel mix ratio constraints and LOLP as the reliability criterion. They modify the GEP problem to increase the effectiveness of the metaheuristic techniques. The first modification is to introduce a novel mapping procedure which transforms the each combination of the candidate units into a dummy variable showing the total capacity of each combination. The second modification is applied to the objective function to apply a penalty factor approach which makes it possible to investigate infeasible solutions during intermediate steps. They also provide a method to generate the initial population. They apply the nine metaheuristic techniques to three test cases and compare their performances in terms of the success rate and execution time.

Sirikum and Techanitisawaw [38] propose a GA-based heuristic to solve nonlinear power GEP problem. The proposed method decomposes the problem into two parts; combinatorial and continuous linear programming problem. GA search is used to solve the combinatorial problem where a feasible generation mix is determined. The constraints are related to the only expansion decisions. Reserve margin constraints, LOLP and location constraints are considered in the first part to find feasible generation mix. For a given generation mix, the variable cost for each year is calculated by solving an LP problem where the objective is to minimize the variable cost under the constraints for demand constraints, capacity constraints, emission constraints. They use string representation where the solution of the combinatorial problem is represented as a chromosome. The length of the chromosomes is set to be the number of candidate generation units and DSM programs. String values indicate the period numbers in which the candidate generation units and DSM program is to be introduced. The initial solution is randomly generated. Then, each chromosome is transformed to an expanded chromosome where binary variables are used for expansion decisions in each year. Each chromosome is checked for reserve margin and location constraints. The solutions are adjusted until feasibility constraints are satisfied. They use a fitness function consisting of investment cost, a variable cost which is the result of LP problem solved, unserved energy cost and penalty cost. The penalty cost is used to penalize the solutions which fail to satisfy LOLP constrains.

The GEP problem can also be modeled as a stochastic nonlinear optimization problem. In order to reduce the complexity of the problem, decomposition techniques have been used. Firmo and Legey [64] use a decomposition scheme based on Benders cuts where the problem is divided into investment and operations subproblem. It is an iterative procedure where a relaxed investment subproblem is solved to obtain a lower bound for the GEP problem. Then the solution of the investment subproblem is used as an input to the operations subproblem which provides sensitivity vectors to generate a new investment subproblem and an upper bound to the GEP problem. This procedure is continued until the optimal solution is found or the gap between upper and lower bounds are within a specified limit.

Firmo and Legey [64] propose an iterative GA approach to solve the investment subproblem in Benders decomposition. In their proposed approach, the investment subproblem is transformed from an integer constrained problem into an unconstrained one by using pointer based chromosomes (PBC) to represent the candidate solutions for the investment subproblem. The decision variables are binary and there are singleness constraints which ensure that when a variable in one singleness constraint is equal to one, then all the other constraints in the same constraint are necessarily equal to zero. The second observation is about the variable (α) representing operational cost in relaxed investment subproblem. This variable should be greater than all the Benders cuts. For a given investment plan, the value of all the cuts can be calculated, and then α is the maximum of them. Based on these observations, they construct PBC such that the length of the chromosome is equal to the number of singleness constraint. If a variable in a singleness constraint is equal to one, then the value of the gene associated with that singleness constraint would point to the position of that variable. If all variables are zero, then the value of gene is also zero. If the investment subproblem only includes the singleness constraints and Benders cuts, then the problem would be transformed into an unconstrained one. If not, the problem with many constraints would be transformed to an equivalent one with fewer constraints. They also provide maximum and minimum admissible values for each gene to guarantee feasible solutions after mutation.

Meza *et al.* [43] model the single period GEP problem as a mixed integer, bilinear multiobjective GEP problem. The proposed method is similar the one presented in Meza *et al.* [42] except in how they generate a large number of non-dominated solutions. In this paper, they propose a multi-objective evolutionary programming algorithm based on a multi-objective genetic algorithm. The individuals (solutions) are represented by two arrays; the first one shows the number of new generation units in the system and the second one shows the number of new circuits in the system. For a given population, each individual receives a rank based on non-dominance. The individuals that are non-dominant are assigned the lowest (best) rank and individuals that are highly dominated are assigned the highest (worst) rank. The reciprocal of the rank of each solution is given as a fitness value for the corresponding solution. To preserve the diversity of the population along the approximated Pareto front, the fitness for each solution is adjusted by niche count. For every solution with the same rank, the sharing function value is computed and the niche count for solution *i* is calculated by summing the sharing function for each solution *j* which has the same rank as solution *i*. Reproduction is made through mutations of the parent solutions and best the *N* solutions from the parents and offspring are selected as a new population. Current non-dominated solutions are compared with the known non-dominated solutions from earlier iterations and known non-dominated solutions are updated accordingly. This procedure is continued until the stopping criterion is achieved.

2.2.3.3. Decomposition Approaches

Benders decomposition is the most common decomposition approach used to solve GEP problems. Noonan and Giglio [31] propose a solution method depending on Benders decomposition. They apply Benders decomposition to solve a nonlinear mixed integer generation planning problem which minimizes the cost. Since they define a single set of linearized constraints to represent nonlinear reliability constraints, the proposed technique becomes a heuristic. They state that the computational results show that the simplification made has given satisfactory convergence. The algorithm starts with solving a linearized master problem to find the investment plant, and then for each week in each year, the production subproblem is solved. If the required convergence is not satisfied, the dual variables obtained from the subproblems are used to update the Benders constraints. The linearization of reliability constraints are also updated and master problem is solved with a new set of constraints. The procedure is continued until the convergence is achieved.

Bloom [65] discusses the application of Benders decomposition for the least cost GEP problem subject to probabilistic reliability constraints. Gorenstin *et al.* [48] describe how to apply Benders decomposition to solve GEP problems considering several uncertainty factors such as demand, fuel cost, delay and so on. He applies the proposed method to solve the stochastic GEP problem for the Brazilian energy system under the uncertainties of inflows to hydro plants and demand growth. Kenfack *et al.* [66] solve the least-cost GEP problem in a hydro dominated environment by using Benders decomposition. Sirikum *et al.* [67] propose a methodology where GA is combined with Benders decomposition to solve mixed integer nonlinear GEP problem which minimizes the cost.

Sanghvi and Shavel [46] formulate the GEP problem as a multi-period stochastic programming model and solve the problem by using the Dantzig-Wolfe decomposition principle to the dual of the problem. Although Benders decomposition and Dantzig-Wolfe decomposition are duals of one another for a linear programming problem [68], they choose to use Dantzig-Wolfe decomposition to the dual of the problem because of the linear programming code they used (Roy Marsten's XMP [69]). In this code, adding new columns is more natural, and therefore, column generation become more efficient than the row generation. The details for Dantzig-Wolfe decomposition can be found in [70].

Marín and Salmerón [51] implement decomposition approaches to solve their proposed stochastic nonlinear model with a nonlinear convex objective function and linear constraints. They consider Benders decomposition, Lagrangean relaxation and Lagrangean decomposition methods. In Lagrangean relaxation model, the coupling constraint, which forces generation from each unit to be less than the available capacity, is relaxed and corresponding constraints are multiplied with a weight (Lagrangean multiplier) and added to the objective function. This results in separating the investment and operations variables into two submodels. The corresponding subproblems can be solved relatively easily. In Lagrangean decomposition, the problem is divided into separable models by splitting the operation variables into z and z' and adding a constraint forcing $(z = z')$. Then, this constraint is relaxed and added to objective function after multiplied by Lagrangean multiplier u. Then problem is divided into two parts where the objective is to minimize investment cost and minus uz' under the coupling constraint and investment related constraints. The second part is to minimize operation and risk cost plus uz under the production level constraints. Marín and Salmerón [51] conclude that in their computational experiment, generalized Benders decomposition performs better.

1.1.1.1.1. Benders Decomposition

Benders decomposition technique was first published by Benders [71], and then, Geoffrion [72] reviewed the method. Freund [73] describes how to apply Benders decomposition for structured optimization problems that also include two-stage stochastic optimization problems under uncertainty.

Benders decomposition technique is very efficient for solving problems which have block ladder structure. A flow chart describing the steps of Benders decomposition is depicted in Figure 2.7. A simple example can is represented as follows.

$$\begin{aligned}
&P: \min \mathbf{c}x + \mathbf{f}\mathbf{y} \\
&\text{s.t.} \\
&\quad A\mathbf{x} = \mathbf{b} \\
&\quad B\mathbf{x} + D\mathbf{y} = \mathbf{d} \\
&\quad \mathbf{x} \geq 0, \mathbf{y} \geq 0
\end{aligned}$$

This problem P can be written as;

$$\begin{aligned}
&P1: \min \mathbf{c}\mathbf{x} + z(\mathbf{x}) \\
&\text{s.t.} \\
&\quad A\mathbf{x} = b \\
&\quad \mathbf{x} \geq 0
\end{aligned}$$

where

$$\begin{aligned}
P2: z(\mathbf{x}) = \min \mathbf{f}\mathbf{y} \\
\text{s.t.} \\
D\mathbf{y} = d - B\mathbf{x} \\
\mathbf{y} \geq 0
\end{aligned}$$

and the dual of the $P2$ is

$$\begin{aligned}
D2: z(\mathbf{x}) = \max\ p(d - B\mathbf{x}) \\
\text{s.t.} \\
\mathbf{p}D \leq \mathbf{f}
\end{aligned}$$

For a given **x**, if it is assumed that the feasible region for $P2$ is bounded (for the other case, see Freund [73]), and if all the extreme points p_i of $D2$ are enumerated, it is possible to write $D2$ as

$$\begin{aligned} D2: z(\mathbf{x}) = \min z \\ \text{s.t.} \\ p_i(d - B\mathbf{x}) \le z \quad \forall i \end{aligned}$$

If this formulation of $z(\mathbf{x})$ is placed into the original problem $P1$, the new problem is called the full master problem, as follows

$$\begin{aligned} FMP: \min \mathbf{cx} + z \\ \text{s.t.} \\ A\mathbf{x} = b \\ p_i(d - B\mathbf{x}) \le z \quad \forall i \\ \mathbf{x} \ge 0 \end{aligned}$$

In the Benders decomposition technique, each of the $p_i(d - B\mathbf{x}) \le z$ constraints are called Benders cut and Benders decomposition provides solving FMP using only a subset of the constraints (i.e., Benders cut) and checking whether any of the non-included constraints are violated. The problem with only subset of constraints is called the restricted master problem RMP. For the k^{th} iteration of Benders decomposition, RMP(k):

$$\begin{aligned} RMP(k): \min \mathbf{cx} + z \\ \text{s.t.} \\ A\mathbf{x} = b \\ p_i^T(d - B\mathbf{x}) \le z \quad \forall i = 1, \ldots, k-1 \\ \mathbf{x} \ge 0 \end{aligned}$$

Consider that $\bar{x}$ and $\bar{z}$ are the optimal solutions of $RMP(k)$. Then,

- $Val_k = \mathbf{c}\bar{\mathbf{x}} + \bar{z}$ is the lower bound on the solution of FMP.
- If $\bar{\mathbf{x}}$ and $\bar{z}$ do not violate any of non-included constraints, then $\bar{x}$ and $\bar{z}$ is the optimal solution of FMP.
- If $\bar{\mathbf{x}}$ and $\bar{z}$ violate any of non-included constraints, then a new Benders cut should be added to the problem.

To check whether or not $\bar{\mathbf{x}}$ and $\bar{z}$ violate any of non-included constraints, it is required to solve the following problem for $\bar{\mathbf{x}}$,

$$
\begin{aligned}
P2(\bar{\mathbf{x}}):\ & \min \mathbf{f}\mathbf{y} \\
& \text{s.t.} \\
& D\mathbf{y} = d - B\bar{\mathbf{x}} \\
& \mathbf{y} \geq 0
\end{aligned}
$$

Consider that $\bar{\mathbf{p}}$ and $\bar{\mathbf{y}}$ are the dual and primal solutions of $P2(\bar{\mathbf{x}})$. It can be observed that $\bar{\mathbf{p}}$ is the extreme point which has the maximum value of $\bar{\mathbf{p}}(d - B\bar{\mathbf{x}})$. Then,

- If $\bar{z} \geq \bar{\mathbf{p}}(d - B\bar{\mathbf{x}})$, then $\bar{\mathbf{x}}$ and $\bar{z}$ are the optimum solution of FMP and $\bar{\mathbf{x}}$ and $\bar{\mathbf{y}}$ are the optimum solution to *P*. Else construct a Benders cut, $z \geq \bar{\mathbf{p}}(d - B\mathbf{x})$ and add to the RMP.
- If $\mathbf{c}\bar{\mathbf{x}} + \mathbf{f}\bar{\mathbf{y}} \leq UB,$ then $\bar{\mathbf{x}}$ and $\bar{\mathbf{y}}$ are assigned as the best solution has been found so far and the upper bound on the solution of FMP is updated as $\min\{UB, \mathbf{c}\bar{\mathbf{x}} + \mathbf{f}\bar{\mathbf{y}}\}$
- It is also possible to terminate the algorithm by construction stopping criteria based on the UB and LB.

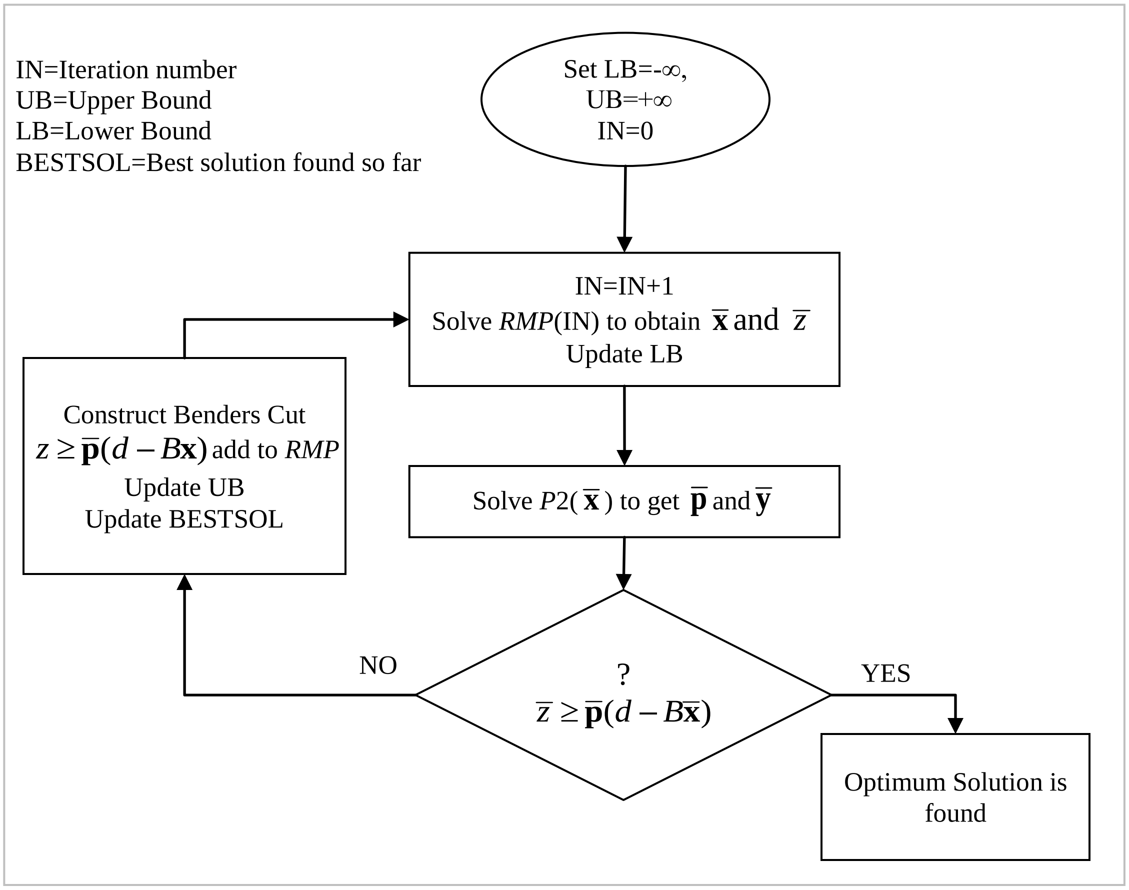

Figure 2.7. Benders decomposition

The GEP has two important structural properties that make Benders decomposition particularly attractive. They are:

- Constraints containing integer variables only concern the investment decisions
- The problem can be divided into master problem containing investment decisions and subproblems containing operational decisions.

2.3. Multiple Objective Optimizations

GEP problems include conflicting objectives. In order to provide trade-offs solutions among the different objectives, multi-objective optimization methods can be used. In this section, the formulation for multi-objective optimization problems is given and some exact approaches to solve multi-objective optimization problems are presented.

2.3.1. Formulation of Multi-Objective Optimization Problem

The multi-objective optimization problem is when there are two or more objectives that should be satisfied simultaneously. Often, the objectives are in conflict with each other. Therefore, there is usually no one optimal solution which optimizes all the objective functions. Instead, there are a set of nondominated solutions called efficient solution or Pareto optimal solutions. Without loss of generality, for the minimization problem, multi-objective optimization problem can be defined as follows [74].

$$\begin{aligned}&\min\ f(\mathbf{x}) = (f_1(\mathbf{x}), \ldots, f_p(\mathbf{x}))\\&\text{s.t. } \ \mathbf{x} \in X\end{aligned}$$

Where $\mathbf{x}$ is a vector of decision variables, and $X \subseteq \mathbb{R}^n$ represent the feasible set, $f : X \to \mathbb{R}^p$ represents the p real-valued objective functions, and $\mathbb{R}^n$ and $\mathbb{R}^p$ represents the vector spaces.

The efficient set consists of the nondominated solutions where a solution $\mathbf{x}_1$ dominates a solution $\mathbf{x}_2$, if and only if two following conditions are satisfied:

- $f_i(\mathbf{x}_1) \leq f_i(\mathbf{x}_2), \forall i \in \{1, \ldots, p\}$ where p is the number of objective functions.
- $f_i(\mathbf{x}_1) < f_i(\mathbf{x}_2)$ for at least one objective function i.

The efficient set X_E is defined as

$$X_E := \{\mathbf{x} \in X;\ \text{there is no } \bar{\mathbf{x}} \in X \ \text{where } f_i(\bar{\mathbf{x}}) \leq f_i(\mathbf{x}) \forall i \ \text{and } f_i(\bar{\mathbf{x}}) < f_i(\mathbf{x}) \text{ for at least one } i\}.$$

The efficient set, $Y_N = f(X_E)$, is called the Pareto set or Pareto front. For two objectives, the Pareto front can be illustrated as in Figure 2.8.

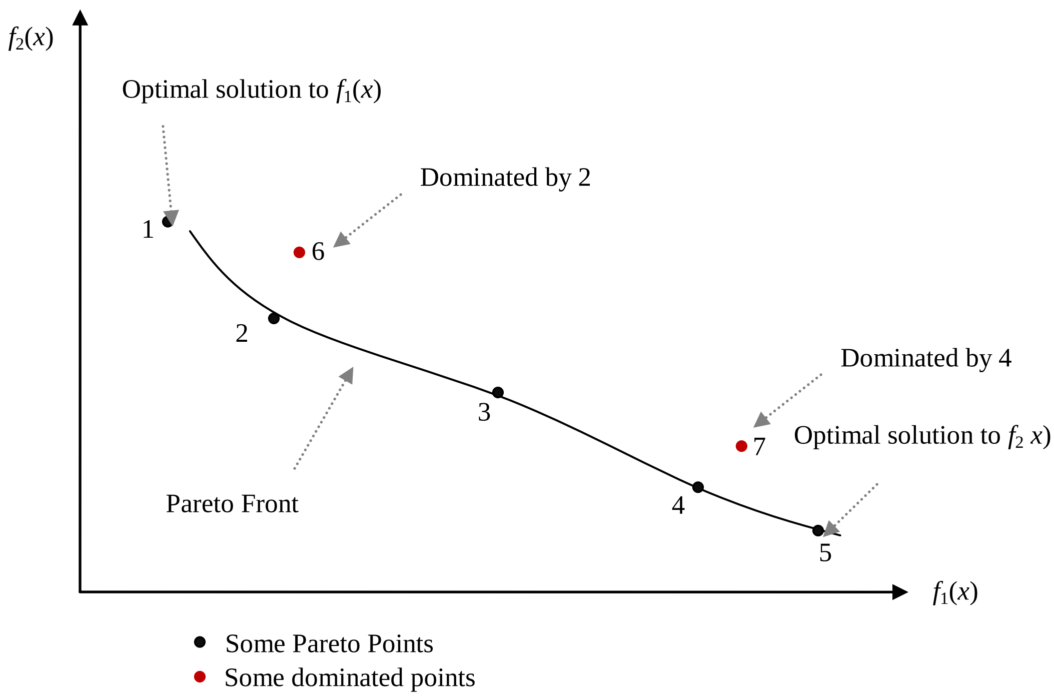

Figure 2.8. Representation of a Pareto Front

2.3.2. Solution Methods to Solve Multi-Objective Optimization Problems

There are many methods developed to solve multi-objective optimization problems. Ruzika and Wiecek [74] provide a survey of the exact methods developed to find the Pareto set and Ehrgott [76] provides a discussion for the scalarization techniques for multiple objective integer programming problems. In this section, three exact methods to find a Pareto set for multi-objective optimization problems are described; namely the weighted sum method, the augmented weighted Chebychev method and the normal Boundary Intersection (NBI) method.

2.3.2.1. Weighted Sum Method

The weighted sum method is one of the scalarization technique used to find a Pareto Set. However, it has deficiencies. The scalarization is transforming multi-objective optimization problem into a single objective problem that is solved repeatedly. In the weighted sum method, a single objective function is obtained by convex combination of the p objective functions which can be represented as,

$$\begin{aligned} &\min \sum_{i=1}^{p} w_i f_i(\mathbf{x}) \\ &\text{s.t. } \mathbf{x} \in X \end{aligned}$$

where $\sum_{i=1}^{p} w_i = 1$ and $w_i \in [0,1], \forall i \in \{1,\ldots,p\}$.

The weighted sum method has two main drawbacks. The objective functions should be normalized to remove the effect of the relative scales of the objective functions. The normalization procedure may affect the solution quality. Moreover, efficient solutions located in the interior of the convex hull cannot be found by the weighted sum method because they are dominated by a convex combination of vertex solutions. This concept is illustrated in Figure 2.9.

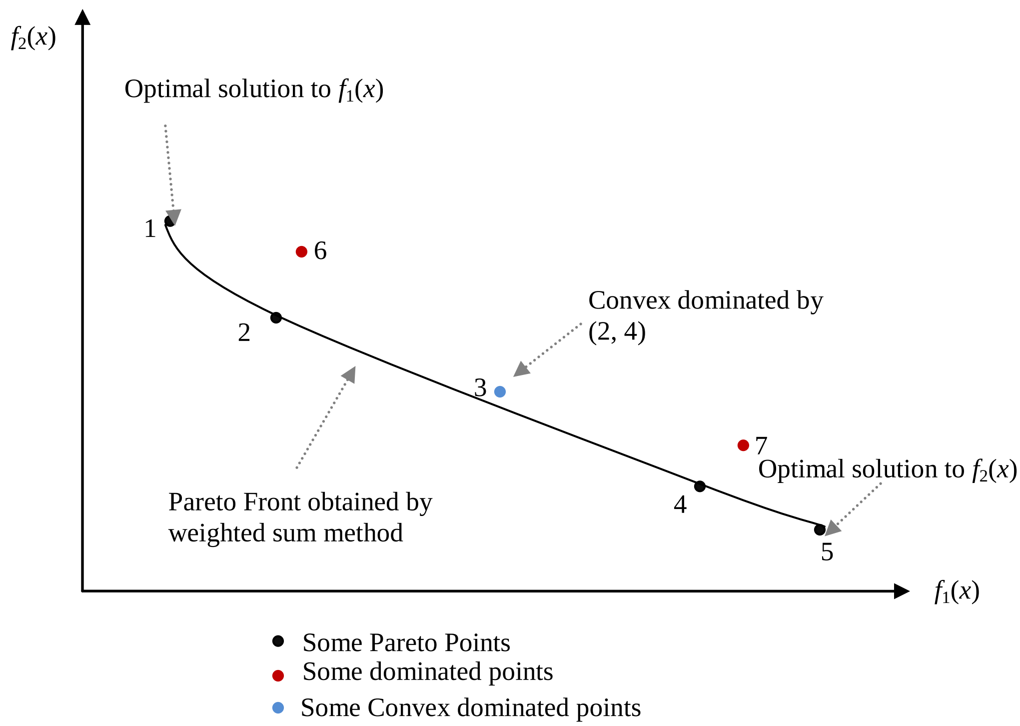

Figure 2.9. Illustration of convex domination

2.3.2.2. Normal Boundary Intersection Method

An alternative method proposed by Das and Dennis [76] is the normal boundary intersection (NBI) method. NBI is proven that it is independent of the relative scales of the objective functions and is quite successful in producing an evenly distributed set of solutions in the Pareto set [76].

The idea behind this method is to find the solutions located at the intersection of the normal vector emanating from a point in the convex hull of individual minima (CHIM) and the boundary of the objective space. Figure 2.10 presents the concept of NBI. Das and Dennis [76] define the following notation

- $\mathbf{x}_i^*$ is the optimal solution to objective function $f_i(\mathbf{x})$ for every objective function i, $i \in \{1,\ldots, p\}$.
- F_i^* is defined as the vector containing the objective functions values for the solution $\mathbf{x}_i^*$, that is, $F_i^* = F(\mathbf{x}_i^*), i = 1,\ldots, p$.
- F^* is defined as the vector containing the individual global optima of the objective functions, that, is,

$$F^* = \begin{bmatrix} f_1(\mathbf{x}_1) \\ f_2(\mathbf{x}_2) \\ \vdots \\ f_p(\mathbf{x}_p) \end{bmatrix}$$

- Φ is the $p \times p$ matrix whose i^{th} column is defined as $F_i^* - F^*$. Therefore, $\Phi(i,i) = 0$ and $\Phi(i, j) \geq 0,\ j \neq i$
- $\mathbf{w}$ is defined as a vector where $\mathbf{w} \in \mathrm{R}^p, \sum_{i=1}^{p} w_i = 1, w_i \geq 0 \forall i$.

Then, $\Phi\mathbf{w}$ is defined as the CHIM. Therefore, for a given convex weighting $\mathbf{w}$, $\Phi\mathbf{w}$ represents a point in CHIM. If $\hat{\mathbf{n}}$ denote the unit normal vector emanating from the CHIM and pointing towards the origin, then the sets of points on that normal is represented by $\Phi\mathbf{w} + t\hat{\mathbf{n}}, t \in R$. The intersection point between the normal and the boundary of the objective space for a given convex weighting $\mathbf{w}$, $\mathrm{NBI}_\mathbf{w}$ can be found mathematically as follows;

$$
\begin{aligned}
&\max t \\
&\text{s.t.} \\
&\quad \Phi\mathbf{w} + t\hat{\mathbf{n}} = F(\mathbf{x}) \\
&\quad \mathbf{x} \in X
\end{aligned}
$$

where $\Phi\mathbf{w} + t\hat{\mathbf{n}} = F(\mathbf{x})$ guarantees that the solution $\mathbf{x}$ is actually mapped by F to a point on the normal vector. The graphical representation is given in Figure 2.10.

Das and Dennis [76] show that using quasi-normal vectors is also efficient in identifying desired boundary points. They choose an equally-weighted linear combination of the columns of Φ as the quasi-normal direction. That is, $\hat{\mathbf{n}}$ is calculated by $-\Phi\mathbf{e}$ where $\mathbf{e}$ is the column vector of all ones. Since all components of Φ are nonnegative, all components of the $\Phi\mathbf{e}$ are also nonnegative.

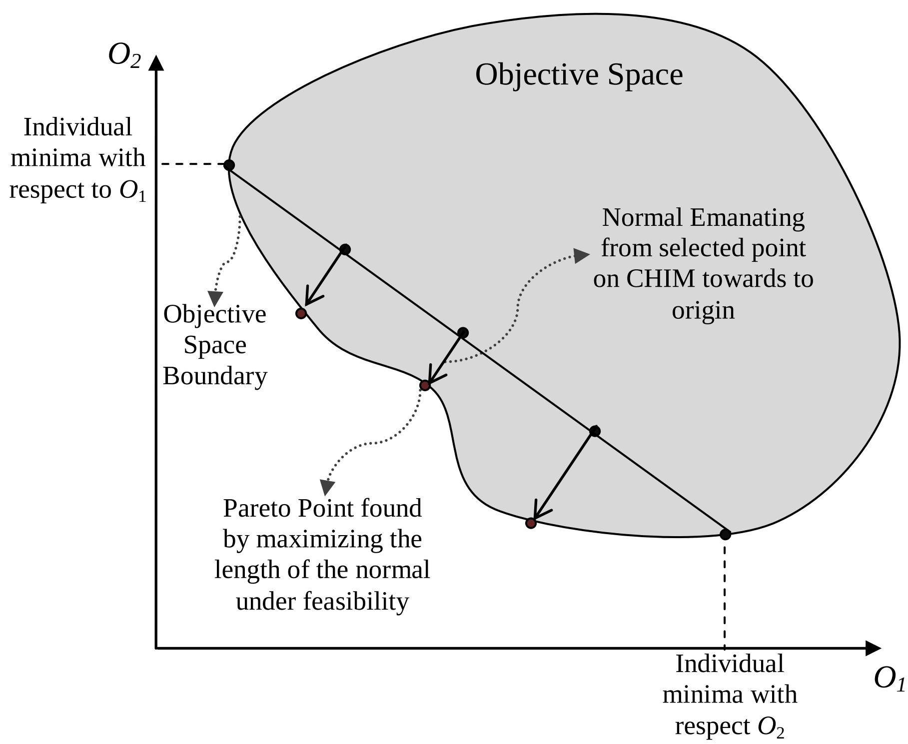

Figure 2.10. Illustration of Normal Boundary Intersection Method

If the points obtained lie on the sufficiently concave part of the boundary, there is a possibility that this point is not a Pareto solution. But if these points in the concave part are Pareto optimal, then NBI overcomes the drawback of weighted sum method which fails to obtain the points in the non-convex parts.

The solution obtained by NBI method may not be the Pareto point if the trade-off surface in the objective space is folded as illustrated in Figure 2.11. If NBI started at point 1 is used, NBI will find the point 2 as a Pareto Solution, whereas the corresponding globally efficient point is point 3. However, this is a very rare occurrence, and it is not anticipated to happen for the problems being studied as part of this research plans.

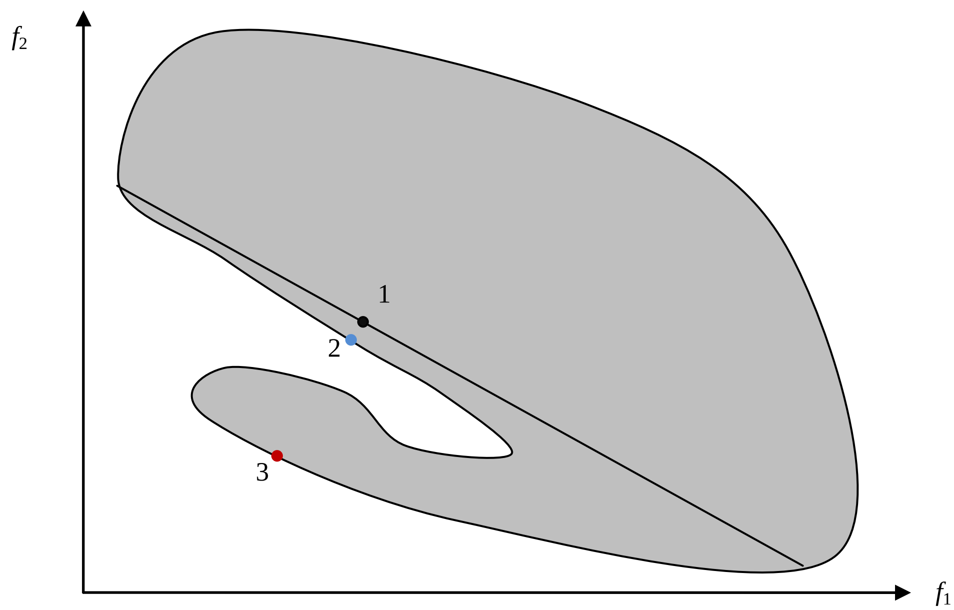

Figure 2.11. Illustration of folded trade-off surface in a objective space

2.3.2.3. Augmented Weighted Chebychev Method

Another method, which is also a scalarization technique, is the augmented weighted Chebychev method proposed by Steuer and Choo [77]. This method minimizes the maximum weighted Chebychev distance to an ideal point. The "ideal" point is found by minimizing (or maximizing) all objective functions individually. The intent is to find solutions close to this ideal point by minimizing the maximum weighted distance. Efficient solutions located in the interior of the convex hull can be found by this method. Although, this method also uses weights, these weights do not directly reflect the preference. The Pareto set is obtained by appropriately specifying the parameters of the method. The mathematical formulation of the method is as follows

$$\min_{x \in X} \max_i v_i (f_i(\mathbf{x}) - f_i(\mathbf{x}_i^*)) + \gamma \sum_{i=1}^{p} (f_i(\mathbf{x}) - f_i(\mathbf{x}_i^*))$$

where $\mathbf{v}>0$ is a vector of weights. If $\gamma>0$, then the solution is Pareto optimum, but otherwise, it is possible to get a weak Pareto optimum solution. The weak Pareto solution occurs when there is no solution $\mathbf{x}$ which has lower objective function value for all objectives.

3. Multi-Objective Generation Expansion Planning Model

Mathematical models have been developed and tested to find a Pareto front for the multi-objective generation expansion planning problem that explicitly considers availability of the system components over the planning horizon and operational dispatching decisions. In this newly developed model, scenarios are used to explicitly represent the availability of the system components. Two approaches are presented that were used to generate numerous scenarios based on the component availabilities and anticipated demand for energy. The first approach is based on Monte Carlo simulation and the second approach is entirely new approach which offers distinct advantages compared to other method for this problems, based on scenario optimization. In addition, different power system topologies are presented and example problems are formulated as a mixed integer linear programming problem, and optimal solutions are found based on the generated scenarios with a combined objective function considering the multiple problem objectives. The different objectives are combined using dimensionless weights and a Pareto front is determined by varying these weights. The results demonstrate how expansion decisions vary depending on whether minimizing cost or minimizing greenhouse gas emissions or pollutants is given higher priority. Moreover, Benders decomposition for multi-objective optimization problems is explained in detail and then it is utilized in a new formulation of the problem to increase the solution efficiency for the later problem which is a large scale stochastic mixed integer optimization problem.

Limitations on greenhouse gas emissions and conservation of energy are critical national and world-wide concerns that impact all phases of energy and business policies. Approximately 40% of US greenhouse gas emissions are due to the production, transmission and distribution (T&D) of electricity [6]. In a carbon-constrained world, the electric power system needs to be transformed from its existing structure of primarily large fossil fuel power plants to a more distributed system using renewable energy, energy efficiency, and other non-carbon emitting technologies while maintaining high reliability standards at affordable costs. For example in New Jersey, the state has established goals to rely extensively on energy efficiency and distributed generation to meet the Governor's stated objective of reducing greenhouse gas emissions by 20% by the year 2020 and 80% by the year 2050 [6]. Therefore, a new approach is proposed for the electricity generation expansion problem to minimize simultaneously multiple objectives, such as cost and air emissions, including CO_2 and NO_x, over a long term planning horizon. In this problem, system expansion decisions are made to select the type of power generation, such as coal, nuclear, wind, etc., where the new generation asset should be located, and at which time period expansion should take place.

In Section 3.1, the mathematical model to integrate the reliability, expansion and dispatching is introduced and two different power system topologies are presented. A general formulation is developed that is demonstrated on two specific examples. The first power system is a simpler representation of the real life power grid where the loop flows in transmission and limit on transmission capacity are not considered. The second power grid is a more realistic representation of the real life power grid where loop flows, transmission capacity limits and transmission losses are considered.

The availability of the system components is incorporated in the model via scenarios. In Section 3.2, two methodologies used to generate availability scenarios are presented. In Section 3.2.1, a description for Monte Carlo simulation based approach is presented. In this approach, the scenarios are randomly generated based on the availability of system components and demand level. A numerical example for the simpler representation of the real life power grid is solved and the results are presented. Section 3.2.2 presents a description of the scenario optimization based approach. By means of this approach, a subset of all the availability scenarios is selected which sufficiently represent the uncertainty in the system. This section also includes a numerical example to demonstrate the steps of the approach. In Section 3.3, the mathematical model is presented for the GEP problems involving more realistic representation of the transmission system. In Section 3.4, the procedure to use Benders decomposition for multi-objective optimization problems is given. Finally, a numerical example is solved by using Benders decomposition. In this example, the second scenario generation approach is utilized to find a subset of scenarios which sufficiently represent the more realistic power grid.

3.1. Mathematical Model to Integrate Reliability, Expansion and Dispatching

The electricity GEP problem involves the selection of the generation technology options (coal, wind, etc.) to add to an existing system and when and where they should be constructed to meet the increasing energy demand over a planning time horizon.

Dispatching decisions can be defined to assign how much energy to produce from each generation units to meet hourly demand. In the literature, this is included in the expansion problem by calculating the expected energy generated from each unit/technology. In this research, the availability of the components is explicitly considered, which is a better and more accurate approach to the problem in many ways. Therefore, expansion planning problem should also determine how to dispatch based on the system condition. In this section, a model is proposed to integrate reliability, expansion and dispatching.

It is not desirable to simply assume that each component (generating unit, line, etc.) is available at its expected amount at all times, although this is the approach used by many analysts. This approach would preclude the possibility that the system is operating at full capacity at any time, but more importantly, it precludes the possibility that there are several simultaneous outages, that are unlikely but critical to consider. In this dissertation, two approaches are proposed and these approaches are preferable by considering much greater diversity of reliability and availability behavior.

Stochastic and multiple-objective optimization models are proposed to address this important problem with specific objectives to minimize relevant costs, and to minimize the environmental impact, e.g., CO_2 and NO_x emissions. The CO_2 and SO_2 emissions are largely generated from coal burning. Therefore the SO_2 emission is highly and positively correlated with the CO_2 emission. Even though SO_2 emission is not explicitly considered, it is implicitly considered by minimizing CO_2 emission.

Scenarios are defined based on the uncertainty of the system components availability and selected scenarios are used to characterize the uncertainty of user demand and the availability of the system components, including generation units, lines, gas supplies, etc. A two-stage stochastic programming model is proposed to solve the electricity generation expansion planning problem. There are two levels of decision variables, which are:

i. Variables responding to the distribution of uncertainty (expansion investment decisions) *ii.* Variables responding to observed uncertainty (energy dispatching decisions).

The problem examined is a multi-objective generation expansion plan over the multi-period planning horizon given an existing centralized power system. The idea is to integrate reliability, generation expansion and dispatching decisions while reducing air emissions, and to consider supplementing the existing centralized system with distributed and central power generation. Distributed power generation involves the use of smaller generating units located closer to energy users.

An example network and the corresponding mathematical model are considered to demonstrate the model. The topology for existing central system studied is the same as in Zerriffi, *et al.* [80] with one directional electricity transmission. A graphical representation of network topology is presented in Figure 3.1. In this network, it is considered that the system has sufficient transmission capacity. The system has *K* central generation units consisting of different technologies. These generation units are distributed among *G* power groups. Some of the generation units use natural gas as fuel. The transmission pipelines from natural gas storage feed the power groups which contain natural gas burning generation units. The energy generated in these power groups is transmitted to the distribution system via transmission lines. There are *L* independent local load blocks and these load blocks are connected to the area grid by the distribution lines. It is also assumed that there is a similar natural gas network as in Zerriffi *et al.* [80] providing natural gas to these load blocks. The transmission pipelines are used to transmit natural gas from storage areas to 13 city-gates. Each city-gate has three sub-transmission mains, each of which feeds seven micro-grids. The distribution pipelines are used to distribute natural gas from city-gate to sub-transmission mains.

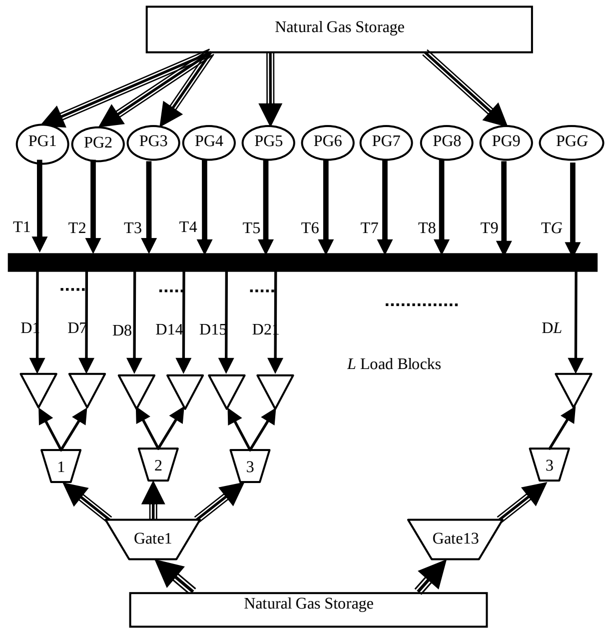

Figure 3.1. Network topology

As a part of this dissertation, a mathematical model is also developed and solved for more complex power grid. Such power system has two-directional energy flow, capacity limits on transmission lines, energy losses due to the transmission and better representation of how the power is distributed among the transmission lines. The IEEE Reliability Test System [87] is used as an example network topology in Figure 3.2.

As illustrated in Figure 3.2, the nodes can represent the supply points, demand points, both supply and demand points and neither supply nor demand points. Demand points are assumed to have a distribution system. Therefore, the IEEE Reliability Test System was modified to assume that each demand point i consists of L_i load blocks.

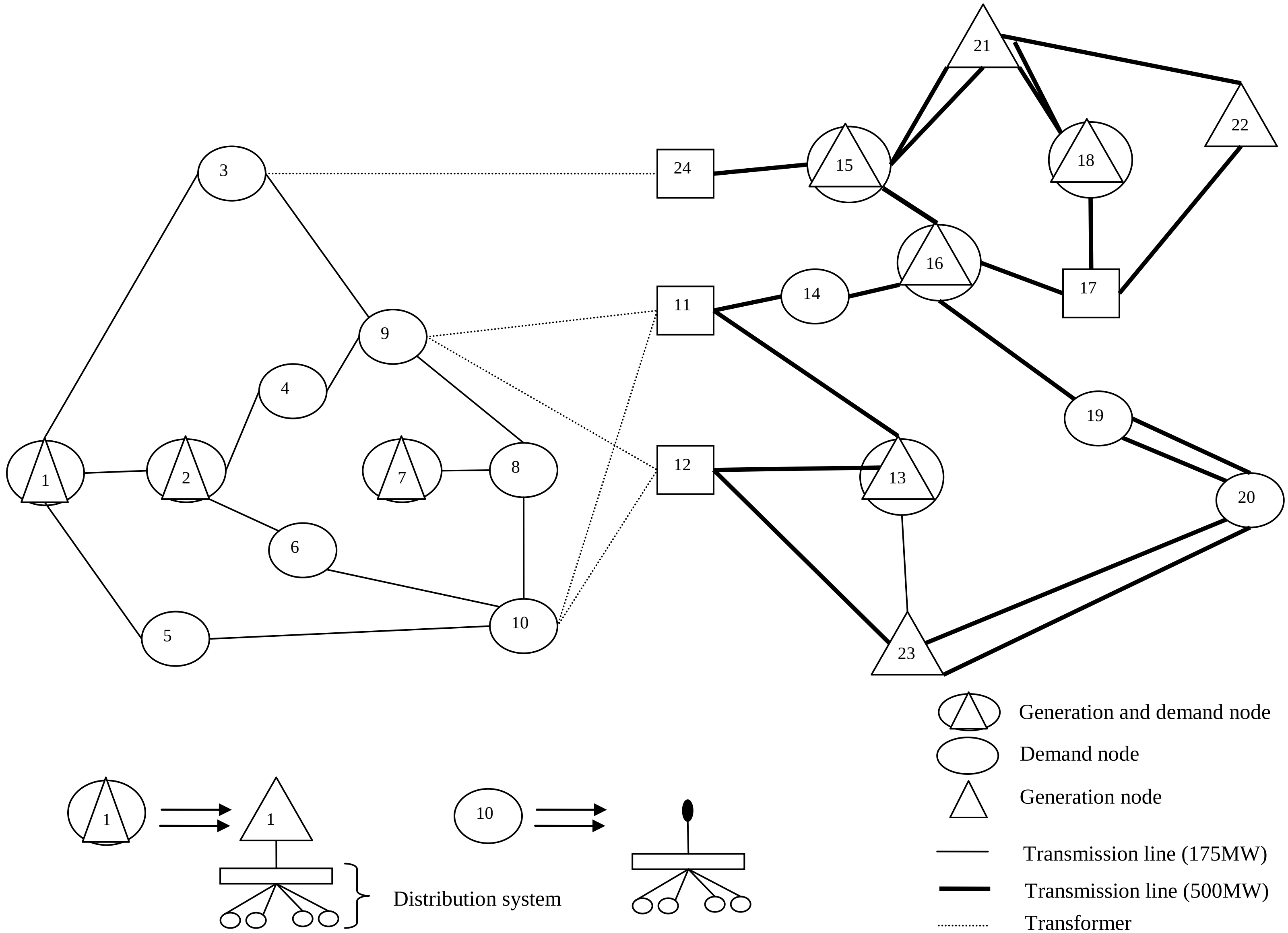

Figure 3.2. The IEEE Reliability Test System

In the following sub-sections, the mathematical model for the power system as Figure 3.1 is given to demonstrate general multi-period multi-objective two-stage stochastic GEP problems. In the later sections, a more complex mathematical model for the power system as Figure 3.2 is also presented.

3.1.1. Multiple Objective Functions

The problem is to determine an optimal expansion plan given the objectives of minimizing cost and minimizing undesirable air emissions. The individual objectives are scaled and combined so that a single objective function problem can be solved. The weights to combine the individual objective functions are varied to determine a Pareto front. This is an effective approach if the Pareto front is convex. In this manner the trade-offs involved can be explicitly considered. For example, a relative increase in cost corresponds to a decrease in greenhouse gas emissions within the Pareto front. The detailed explanation of each objective function is as follows.

3.1.1.1. Total Cost (O_1)

The total cost objective function include the investment cost for new generation units, fixed operation and maintenance cost for all generation units, variable energy generation cost, unmet demand cost (due to unreliability) minus revenue from the steam used in co-generation. In the model, there is a cost associated with unmet demand. The system reliability is implicitly maximized by minimizing the unmet demand. All future costs are discounted to the present time (with a discount rate r) to yield a net present value (NPV).

Investment Cost:

$$O_{11}=\sum_{t=1}^{T}(1+r)^{-t}\sum_{q=1}^{Q}s_{tq}a_{tq}+\sum_{t=1}^{T}(1+r)^{-t}\sum_{l\in\Lambda}\sum_{j=1}^{J_l}w_{tlj}b_{tlj}$$

s_{tq} is the investment decision of a central unit type q in time period t. That is, s_{tq} is equal to 1 if central unit type q is built in time period t and 0 otherwise. w_{tlj} is the investment decision of a distributed unit j located at load block l in time period t. That is, w_{tlj} is equal to 1 if distributed unit type j is built at load block l in time period t and 0 otherwise. a_{tq} is the investment cost ($) of a central unit type q in time period t. b_{tlj} is the investment cost ($) of a distributed unit type j located at load block l in time period t. Here, r is the interest rate, T is the total number of time periods, Q is the total number of centralized generation investment options, and J_l is the total number of distributed generation investment options available at local load block l.

Fixed Operational and Maintenance Cost:

$$O_{12}=\sum_{t=1}^{T}(1+r)^{-t}\sum_{k=1}^{K}g_{tk}+\sum_{t=1}^{T}(1+r)^{-t}\sum_{q=1}^{Q}(\sum_{\tau=1}^{t}s_{\tau q})h_{tq}+\sum_{t=1}^{T}(1+r)^{-t}\sum_{l\in\Lambda}\sum_{j=1}^{J_l}(\sum_{\tau=1}^{t}w_{\tau lj})m_{tlj}$$

g_{tk}, h_{tq} and m_{tlj} are the fixed operational and maintenance cost ($) for a existing central unit type k, new central unit type q and distributed unit type j located at load block l in time period t respectively. K represents the total number of centralized generation units existing in the system.

Generation Cost:

$$O_{13}=\sum_{t=1}^{T}(1+r)^{-t}\sum_{n=1}^{N}\sum_{k=1}^{K}\varpi_n x_{tnk}c_{tk}+\sum_{t=1}^{T}(1+r)^{-t}\sum_{n=1}^{N}\sum_{q=1}^{Q}\varpi_n u_{tnq}e_{tq}+\sum_{t=1}^{T}(1+r)^{-t}\sum_{n=1}^{N}\sum_{l\in\Lambda}\sum_{j=1}^{J_l}\varpi_n(y_{tnlj}+z_{tnlj})d_{tj}$$

x_{tnk} is the generation amount (MW) of existing central unit type k for scenario n in time period t. u_{tnq} is the generation amount (MW) of new central unit type q for scenario n in time period t. y_{tnlj} is the generation amount (MW) of distributed unit type j located at load block l to satisfy satisfiable demand. z_{tnlj} is the generation amount (MW) of distributed unit type j located at load block l to satisfy local demand. c_{tk}, e_{tq} and d_{tj} are the generation cost ($/MW) of existing central unit type k, new central unit type q and distributed unit type j in time period t respectively. The adjustment factor, ϖ_n is defined as the number of hours represented by each scenario n.

<u>Unmet Demand Cost</u>:

$$O_{14} = \sum_{t=1}^{T} (1+r)^{-t} \sum_{n=1}^{N} \varpi_n v_{tn} f_t + \sum_{t=1}^{T} (1+r)^{-t} \sum_{n=1}^{N} \sum_{l \in \Lambda} \varpi_n \pi_{tnl} f_t$$

v_{tn} and π_{tnl} are the unmet satisfiable demand (MW) for scenario n in time period t and unmet local demand at load block l for scenario n in time period t respectively. f_t is the cost of not satisfying the demand in time period t ($/MW).

<u>Revenue from Steam</u>:

$$O_{15} = \sum_{t=1}^{T} (1+r)^{-t} \sum_{n=1}^{N} \sum_{l \in \Lambda} \sum_{j \in R} \varpi_n (y_{tnlj} + z_{tnlj}) p_t r_t$$

R is the set of distributed generation units with co-generation capabilities. p_t is the proportion of generated energy can be used to receive benefit and r_t is the revenue obtained from the usage of steam ($/MW). Co-generation is the process of capturing and using generated steam that would otherwise be lost.

The total cost objective function (O_1) is the sum of the NPV for investment cost for new generation units, fixed operation and maintenance cost for all generation units, variable energy generation cost, unmet demand cost (due to unreliability) minus revenue from the steam used in co-generation.

$$O_1 = O_{11} + O_{12} + O_{13} + O_{14} - O_{15}$$

3.1.1.2. CO_2 Emission (O_2)

The amount of CO_2 emissions is the second objective function. It can be determined based on the emission rates of the different generating units.

$$O_2 = \sum_{t=1}^{T}\sum_{n=1}^{N}\sum_{k=1}^{K}\varpi_n x_{tnk} C_{tk} + \sum_{t=1}^{T}\sum_{n=1}^{N}\sum_{q=1}^{Q}\varpi_n u_{tnq} E_{tq} + \sum_{t=1}^{T}\sum_{n=1}^{N}\sum_{l\in\Lambda}\sum_{j=1}^{J_l}\varpi_n (y_{tnlj} + z_{tnlj}) D_{tj}$$

C_{tk}, E_{tq} and D_{tj} are the amounts (lbs) of CO_2 per MW generated by existing central unit type k, new central unit type q and distributed unit type j in time period t respectively.

3.1.1.3. NO_x Emission (O_3)

The amount of NO_x emissions is the third objective function. It can be determined based on the emission rates of the different generating units.

$$O_3 = \sum_{t=1}^{T}\sum_{n=1}^{N}\sum_{k=1}^{K}\varpi_n x_{tnk} F_{tk} + \sum_{t=1}^{T}\sum_{n=1}^{N}\sum_{q=1}^{Q}\varpi_n u_{tnq} G_{tq} + \sum_{t=1}^{T}\sum_{n=1}^{N}\sum_{l\in\Lambda}\sum_{j=1}^{J_l}\varpi_n (y_{tnlj} + z_{tnlj}) H_{tj}$$

F_{tk}, G_{tq} and H_{tj} are the amounts (lbs) of NO_x per MW generated by existing central unit type k, new central unit type q and distributed unit type j in time period t respectively.

3.1.1.4. Combined Objective Function

The three individual objective functions have different units and scaling. Therefore, the objective function for our optimization model is the weighted sum of normalized values of these three objective functions. The objective functions are linearly scaled between 0 and 1 via the following scaling function. For each of the three objectives, a minimum and maximum value is required. These minimum and maximum values (min O_f, max O_f) are determined based either on physical constraints and restrictions or based on experimentation with the model.

$$\bar{O}_f = \frac{O_f - \min O_f}{\max O_f - \min O_f}$$

Once the objective functions have been scaled, they can be combined into a composite objective function. w_i represents the relative weight assigned to each objective function. The weights represent the relative importance of the particular objective function compared to the others. For example if w_1=1, w_2=w_3=0, then the problem is to only minimize costs. However, the weights can be varied to reflect many different decision maker preferences or to explore the possible trade-offs between different plans and preferences.

$$\min z = w_1\bar{O}_1 + w_2\bar{O}_2 + w_3\bar{O}_3$$

3.1.2. Problem Formulation

The mathematical formulation for the GEP model is presented below. As stated before, the objective is to minimize the weighted summation of three scaled objective functions over 16 sets of constraints.

Some of the parameters used in the mathematical model are scenario-based. The definition of those parameters for the power network as in Figure 3.1 is given in the following paragraphs.

Satisfiable Demand (Ψ): Total demand of local load blocks whose distribution lines are operational, i.e., working.

Locally Satisfiable Demand (Φ): Demand of local load block with a failed distribution line but demand can be met locally and locating distributed generation units to that load block is possible.

Available Capacity of Central Units (γ): The available capacity for units in the centralized system is determined based on the availability of the generation unit, the transmission line and the availability of gas.

Available Capacity of Distributed Units (W): These are capacities of units which can be used to satisfy satisfiable demand.

Available Local Capacity of Distributed Units (F): These are capacities of units which can be used to satisfy only local demand.

$$\min z = w_1\bar{O}_1 + w_2\bar{O}_2 + w_3\bar{O}_3$$

s.t.

$$\sum_{k=1}^{K} x_{tnk} + \sum_{q=1}^{Q} u_{tnq} + \sum_{l\in\Lambda}\sum_{j=1}^{J_l} y_{tnlj} + v_{tn} \geq \Psi_{tn} \qquad \forall t, n \qquad (1)$$

$$\sum_{j=1}^{J_l} z_{tnlj} + \pi_{tnl} \geq \Phi_{tnl} \qquad \forall t, n, \forall l \in \Lambda \qquad (2)$$

$$x_{tnk} \leq \gamma_{tnk} \qquad \forall t, n, k \qquad (3)$$

$$u_{tnq} \leq \gamma_{tnq} \sum_{\tau=1}^{t} s_{\tau q} \qquad \forall t, n, q \qquad (4)$$

$$y_{tnlj} \leq W_{tnlj} \sum_{\tau=1}^{t} w_{\tau lj} \qquad \forall t, n, \forall l \in \Lambda, \forall j \in J_l \qquad (5)$$

$$z_{tnlj} \leq F_{tnlj} \sum_{\tau=1}^{t} w_{\tau lj} \qquad \forall t, n, \forall l \in \Lambda, \forall j \in J_l \qquad (6)$$

$$\sum_{t=1}^{T} s_{tq} = 1 \qquad \forall q \qquad (7)$$

$$\sum_{t=1}^{T} w_{tlj} = 1 \qquad \forall l \in \Lambda, \forall j \in J_l \qquad (8)$$

$$s_{tq} \in \{0,1\} \qquad \forall t, q \qquad (9)$$

$$w_{tlj} \in \{0,1\} \qquad \forall t, \forall l \in \Lambda, \forall j \in J_l \qquad (10)$$

$$x_{tnk} \geq 0 \qquad \forall t, n, k \qquad (11)$$

$$u_{tnq} \geq 0 \qquad \forall t, n, q \qquad (12)$$

$$y_{tnlj} \geq 0 \qquad \forall t, n, \forall l \in \Lambda, \forall j \in J_l \qquad (13)$$

$$z_{tnlj} \geq 0 \qquad \forall t, n, \forall l \in \Lambda, \forall j \in J_l \qquad (14)$$

$$v_{tn} \geq 0 \qquad \forall t, n \qquad (15)$$

$$\pi_{tnl} \geq 0 \qquad \forall t, n, \forall l \in \Lambda \qquad (16)$$

Equation 1 is for satisfiable demand constraints. For each scenario in each time period, the total generation and unmet demand should be at least as much as the satisfiable demand for the corresponding scenario in that time period. Equation 2 is for locally satisfiable demand. If the distributed line for the load block has failed, then the demand can only be satisfied from distributed units located in that load block. Therefore, for those local load blocks where distributed units can be located, the total generation from distributed units and unmet local demand should be at least as much as the local demand in that load block each scenario in each time period.

Equations 3 to 6 pertain to generating unit capacities. Equation 3 represents the existing capacity for central units. Although each generation units has a predefined capacity, they can be unavailable. Therefore, available central unit capacity is calculated for each scenario in each year. Generation from each existing central unit for each scenario in each year should be smaller than the available capacity. In order to generate from new existing units at time period *t*, they should be built before or in time period *t*. Because they can also be unavailable, the generation from new central units in each scenario in each year should be smaller than the available capacity. Therefore, the model includes the fourth equation presented. Distributed generation units can be used for satisfiable demand or locally satisfiable demand. The capacity of the distributed generation units for satisfiable units is presented in the fifth equation. In order to generate from distributed units at time period *t*, they should be built before or in time period *t*. Because they can also be unavailable, the capacity for each scenario in each year should be smaller than the available capacity to serve satisfiable demand. The capacity of the distributed generation units for locally satisfiable demand is presented in the sixth equation. In order to generate from distributed units at time period *t*, they should be built before or in time period *t*. Because they can also be unavailable, the capacity for each scenario in each year should be smaller than the available capacity to serve local demand.

Equations 7 and 8 are for expansion for each investment choice. Each specific investment choice can only be built once over the planning horizon. Equation 9 and 10 show that the expansion decisions are binary variables. The remaining constraints are for nonnegativity constraints on dispatching decisions.

The proposed approach is implemented in Tekiner *et al*. [78, 79] and the results provide credible indications that this proposed modeling approach does offer improved capabilities.

3.2. Methodology to Generate Scenarios

In this dissertation, the GEP problems are modeled as multi-objective stochastic mixed integer programming problems. The most common approach to solve stochastic programming problems is to define scenarios which collectively represent uncertainty. For this research, the availability of the system components are explicitly represented in the model by means of scenarios. Scenarios are also used to represent the uncertainty in the energy demand.

Two approaches are presented to generate scenarios. The first approach is based on Monte Carlo simulation. This approach is accurate and easy to implement; however, since a large number of scenarios must be generated to sufficiently represent the stochastic nature of the power grid, this approach is more efficient and useful for smaller power grids. When discrete scenarios are used to represent uncertainty, the problem size can become very large for larger power grid. Therefore, a new efficient approach has been developed. In this new approach, it is necessary to generate an efficient set of scenarios which contain the minimum number of scenarios to adequately represent the uncertainty in the grid. For this reason, the second approach which involves scenario optimization to efficiently select such scenarios. The second approach represents an important research contribution because (*i*) it involves the systematic selection of critical components, and (*ii*) it demonstrates how problem-specific information can be exploited to make the problem more efficient.

3.2.1. Monte Carlo Simulation Based Approach

In this section, Monte Carlo simulation based approach is described in more detail. This approach is easy to implement and produce scenarios which represent the stochastic nature of the system sufficiently. However, for the larger problems it requires large number of scenarios. Therefore, it is not efficient to use for such problems. The mathematical representations for the scenario-based parameters for the power system as in Figure 3.1 are also provided in this section.

Numerous scenarios are generated by considering the availability of the system components. Each scenario represents a random hour of consumer demand and asset availability. For each scenario, demand is chosen randomly from the load demand curve, and then Monte Carlo simulation is used to randomly assign whether the system assets (lines, generation units, etc.) are available for that scenario based on estimated component availability metrics. As the planning horizon is extended, the demand increases for each year in the model. The demand increases for each year are applied by annually increasing the peak load demand. The IEEE Reliability Test System [87] presents a load model where the load for each hour is calculated in terms of the percentage of the peak load demand.

The system components whose failures are considered are generation units, transmission lines, distribution lines, transmission pipelines providing natural gas to centralized units, transmission pipelines providing natural gas to city gates, and sub-transmission pipelines delivering natural gas to sub-mains. If these components fail, they are not available within that scenario. It is assumed that the backbone of the transmission and distribution grids and the micro-grids, which transfer natural gas from sub-transmission mains to local load blocks, are always available, similar to Zerriffi *et al.* [80].

Scenario-based parameters are obtained from Monte Carlo simulation as follows. In each scenario, uniform distributed random numbers between 0 and 1 are selected for each component. If the chosen number is smaller than the unavailability of the corresponding component, this component is assumed unavailable for that scenario.

The mathematical calculations of the scenario-based parameters are given as follows:

Satisfiable Demand

$$\Psi_{tn} = \sum_{l=1}^{L} P_{tl}\alpha_{tnl} I_{tn}(l)$$

where

Ψ_{tn} : Satisfiable demand for scenario n in time period t.

P_{tl} : Peak demand at local load block l in time period t.

α_{tnl} : Proportion of peak demand for local load block l for scenario n in time period t.

L: Total number of local load blocks.

$$I_{tn}(l) = \begin{cases} 1, & \text{if distribution line of load block } l \text{ is working for scenario } n \text{ in time period } t \\ 0, & \text{Otherwise} \end{cases}$$

Locally Satisfiable Demand

$$\Phi_{tnl} = P_{tl}\alpha_{tnl} J_{tn}(l) \qquad l \in \Lambda$$

where

Φ_{tnl} : Local demand at load block *l* for scenario *n* in time period *t*.

Λ : Set of load blocks where distributed generation units can be located

$$J_{tn}(l) = \begin{cases} 1, & \text{if distribution line of load block } l \text{ failed for scenario } n \text{ in time period } t \\ 0, & \text{Otherwise} \end{cases}$$

Available Capacity of Central Units

$$\gamma_{tnk} = \lambda_k Z_{tn}(k) R_{tn}(g_k) Y_{tn}(g_k) \qquad k \in \Theta$$

$$\gamma_{tnk} = \lambda_k Z_{tn}(k) R_{tn}(g_k) \qquad k \in \Theta'$$

where

γ_{tnk} : Available capacity of central unit *k* for scenario *n* in time period *t*.

λ_k : Capacity of central unit *k*.

g_k : Power group number where generation unit *k* is located.

Θ : Set of generation units using natural gas.

Θ' : Set of generation units not using natural gas

$$Z_{tn}(k) = \begin{cases} 1, & \text{if generation unit } k \text{ is working for scenario } n \text{ in time period } t \\ 0, & \text{Otherwise} \end{cases}$$

$$R_{tn}(g_k) = \begin{cases} 1, & \text{if transmission line from power group } g_k \text{ is working for scenario } n \text{ in time period } t \\ 0, & \text{Otherwise} \end{cases}$$

$$Y_{tn}(g_k) = \begin{cases} 1, & \text{if natural gas transmission pipe to power group } g_k \text{ is working for scenario } n \text{ in time period } t \\ 0, & \text{Otherwise} \end{cases}$$

Available Capacity of Distributed Units

$$W_{tnlj} = \mu_{l_j} Q_{tnl}(j) H_{tn}(t_l) K_{tn}(d_l) I_{tn}(l) \qquad \forall j \in H, l \in \Lambda$$
$$W_{tnlj} = \mu_{l_j} Q_{tnl}(j) I_{tn}(l) \qquad \forall j \in H', l \in \Lambda$$

where

W_{tnlj} : Available capacity of distributed unit *j* located at load block *l* in scenario *n* in time period *t* which can be used to satisfy satisfiable demand.

μ_{l_j} : Capacity of distributed unit *j* located at load block *l*.

H : Set of distributed generation units using natural gas.

H' : Set of distributed generation units not using natural gas.

$$Q_{tnl}(j) = \begin{cases} 1, & \text{if distributed generation unit } j \text{ located at load block } l \text{ is working for scenario } n \text{ in time period } t \\ 0, & \text{Otherwise} \end{cases}$$

$$H_{tn}(t_l) = \begin{cases} 1, & \text{if natural gas transmission pipe serving load block } l \text{ is working for scenario } n \text{ in time period } t \\ 0, & \text{Otherwise} \end{cases}$$

$$K_{tn}(d_l) = \begin{cases} 1, & \text{if natural gas sub-main transmission pipe serving load block } l \text{ is working for scenario } n \text{ in time period } t \\ 0, & \text{Otherwise} \end{cases}$$

Available Local Capacity of Distributed Units

$$F_{tnlj} = \mu_{l_j} Q_{tnl}(j) H_{tn}(t_l) K_{tn}(d_l) J_{tn}(l) \qquad \forall j \in H, l \in \Lambda$$
$$F_{tnlj} = \mu_{l_j} Q_{tnl}(j) J_{tn}(l) \qquad \forall j \in H', l \in \Lambda$$

where

F_{tnlj} : Available capacity of distributed unit *j* located at load block *l* in scenario *n* which can be used to satisfy only local demand.

3.2.1.1. Numerical Example

In this section, an example problem with the topology as in Figure 3.1 is solved for a 15 year planning horizon to demonstrate the model. In the example system, there are 50 load blocks where the distributed units can be located. The planning horizon is divided into three time periods of five years each. Therefore, if a new generation unit is to be installed, the options are to install it as soon as possible, in five years, or in ten years for the current period. In each year there are 100 different demand and availability scenarios that are randomly generated to reflect the range of possible conditions. Therefore, the optimization is based on a total of 1,500 different scenarios.

The topology for existing central system studied here is the same as in Zerriffi *et al.* [80]. The existing network has 32 generation units consisting of different technologies. These generation units are distributed among 10 power groups. Some of the generation units use natural gas as fuel. The existing generation units are listed in Table 3.1 with corresponding capacity, unavailability, fixed operation and maintenance cost, variable cost, CO_2, NO_x and SO_2 emissions. In the table, the source of the various data elements is noted with a footnote. The energy generated in these power groups is transmitted to the distribution system via transmission lines. There are 273 independent local load blocks and these load blocks are connected to the area grid by the distribution lines.

Table 3.1. Existing generation units and their capacities, unavailability, cost and emission characteristics

		Capacity[1]	Unav.[1]	Var.Cost[1]	Fixed OM[1]	CO_2 [2]	SO_2 [2]	NO_x [2]
		(MW)		($/MW)	($)	(lbs/MW)		
PG1	Oil/CT[4]	20	0.1	18.89	2044000	1362.5	3.27	13.08
	CCGT	20	0.065	10.95	2452000	889	0.7	0.56
	CCGT	76	0.021	10.95	9317600	889	0.7	0.56
	Coal/Steam	76	0.02	7.07	18635200	1840	13.8	3.68
PG2	Oil/CT	20	0.1	18.89	2044000	1362.5	3.27	13.08
	CCGT	20	0.065	10.95	2452000	889	0.7	0.56
	Coal/Steam	76	0.02	7.07	18635200	1840	13.8	3.68
	Coal/Steam	76	0.02	7.07	18635200	1840	13.8	3.68
PG3	Oil/Steam	100	0.04	18.89	10220000	1638	7.7	2.66
	Oil/Steam	100	0.04	18.89	10220000	1638	7.7	2.66
	CCGT[4]	100	0.058	10.95	12260000	889	0.7	0.56
PG4	Oil/Steam	197	0.05	18.89	20133400	1638 [3]	7.7 [3]	2.66 [3]
	Oil/Steam	197	0.05	18.89	20133400	1638 [3]	7.7 [3]	2.66 [3]
	Oil/Steam	197	0.05	18.89	20133400	1638 [3]	7.7 [3]	2.66 [3]
PG5	Oil/Steam	12	0.02	18.89	1226400	1638 [3]	7.7 [3]	2.66 [3]
	Oil/Steam	12	0.02	18.89	1226400	1638 [3]	7.7 [3]	2.66 [3]
	Oil/Steam	12	0.02	18.89	1226400	1638 [3]	7.7 [3]	2.66 [3]
	CCGT	12	0.065	10.95	1471200	889	0.7	0.56
	CCGT	12	0.065	10.95	1471200	889	0.7	0.56
	CCGT	155	0.058	10.95	19003000	889	0.7	0.56
PG6	Coal/Steam	155	0.04	7.07	38006000	1840	13.8	3.68
PG7	Nuclear	400	0.12	0.83	2.34E+08	0	0	0
PG8	Nuclear	400	0.12	0.83	2.34E+08	0	0	0
PG9	Oil/CT	50	0.1	18.89	5110000	1362.5	3.27	13.08
	Oil/CT	50	0.1	18.89	5110000	1362.5	3.27	13.08
	Oil/CT	50	0.1	18.89	5110000	1362.5	3.27	13.08
	CCGT	50	0.021	10.95	6130000	889	0.7	0.56
	CCGT	50	0.021	10.95	6130000	889	0.7	0.56
	CCGT	50	0.021	10.95	6130000	889	0.7	0.56
PG10	Coal/Steam	155	0.04	7.07	38006000	1840	13.8	3.68
	Coal/Steam	155	0.04	7.07	38006000	1840	13.8	3.68
	Coal/Steam	350	0.08	7.07	85820000	1840	13.8	3.68

Notes: [1] Zerriffi *et al.* [80]
[2] New Jersey Draft Energy Master Plan Modeling Report [81]
[3] EPA eGrids database [82]
[4] CT = combustion turbine, CCGT = combined cycle gas turbine

Internal combustion (IC) engines are considered with availability of 0.953 and capacity of 0.5 MW as distributed generation units. The engines use natural gas as fuel and have co-generation capabilities. In order to minimize the number of binary decision variables, 25 engines are assumed to be built together so the capacity of the distributed generation can be considered as binomial variables. Table 3.2 lists their cost characteristics, gas emissions and unavailabilities.

As expansion options for centralized units, wind turbines, oil/steam, coal/steam, nuclear and combined cycle gas turbines (CCGT) are considered. The cost characteristics, gas emissions and unavailabilities are also provided in Table 3.2. Due to the environmental issues and the uncertainty in fuel supply and fuel prices, using renewable energy sources is attractive. In most parts of the world, there are opportunities to penetrate renewable energy sources into power generation system. Therefore, wind turbines are also considered as an expansion option. It is assumed that 30% of wind generation capacity can be used to generate electricity based upon *US Capacity Factors by Fuel Types* [83]. Also, the system is considered to have sufficient transmission line capacity. However, installation of wind turbines may require adding new transmission lines to the system. As a result, the capital investment cost for wind turbine is increased by 30%.

The unavailabilities for the transmission lines and distribution lines are estimated to be 0.01. The unavailability for the natural gas transmission pipelines and sub-transmission pipelines are 9.5×10^{-5} and 9.5×10^{-6} respectively, as in Zerriffi, et.al [80].

Table 3.2. Available technologies for expansion

	Cap.	Unav.	Var. Cost	Capital Cost	Fixed OM	CO_2	SO_2	NO_X
	(MW)		($/MW)	($)	($)	(lbs/MW)		
Oil/Steam	197	0.05	18.89	80573000	20133400	1638	7.7	2.66
Coal/Steam	155	0.04	7.07	1.79E+08	38006000	1840	13.8	3.68
Wind	50	0.05	1	69736800 [2]	11622000 [2]	0	0	0
Nuclear	400	0.12	0.83	8.47E+08	2.34E+08	0	0	0
CCGT	76	0.02	10.95	40736000	9317600	889	0.7	0.56
IC Engines	12.5	0.04	26.82	11250000	312500	1231	0.7	8.09

Notes:[1] Study of the Costs of the Offshore Wind Generation [84]
[2] Electricity Market Module of the National Energy Modeling System, 2007 [85]

The load model presented in the IEEE Reliability Test System [87] is used, similar to Zerriffi *et al.* [80]. The load model is divided into demand intervals and the probabilities are calculated for each interval and they are presented in Table 3.3.

Table 3.3. Demand interval and corresponding probabilities

Percentage of peak load	Probability	Percentage of peak load	Probability
1.00	0.01	(0.60, 0.70)	0.23
(0.95, 1.00)	0.01	(0.50, 0.60)	0.21
(0.90, 0.95)	0.02	(0.40, 0.50)	0.22
(0.80, 0.90)	0.11	(0.33, 0.40)	0.03
(0.70, 0.80)	0.16		

Demand for each scenario is randomly determined by using the interval and the probabilities presented in Table 3.3. The peak load demand in this problem is 2850 MW and it is assumed that demand increases 1% in each year. The cost of not satisfying demand is assumed as 10,000 $/MW. It is also considered that 50% of energy produced by distributed generation units can be used to gain benefits from the steam, and in the model, the profit per MW by using steam is approximately 60% of energy generation cost from IC, i.e., 15.91 $/MW. 100 scenarios are used to represent the year, and the corresponding cost parameters are adjusted to reflect the equivalent amount of time represented by each scenario. In this formulated model, all the scenarios generated have the same probability. Therefore, the probability for each scenario is calculated by dividing the probability of demand interval by the number of availability scenarios generated for the corresponding demand interval. The higher the demand level, the more availability scenarios are generated. Then, the adjustment factor, ϖ_n is calculated by multiplying the probability of each scenario by the total number of hours in the time period, which they represent. The adjustment factor for each scenario is presented in Table 3.4. Since each time period represents five years, the cost parameters for fixed

O&M cost are also adjusted. The expansion decision for each time period means that the expansion is made at the beginning of the time period and the new technology is available for that time period (5 years) and the following time periods.

Table 3.4. Adjustment factors for each scenario

Percentage of peak load demand	# of Scenarios	Adj. Factor	Percentage of peak load demand	# of Scenarios	Adj. Factor
1.00	20	4.38	(0.60,0.70)	10	201.48
(0.95, 1.00)	15	5.84	(0.50,0.60)	5	367.92
(0.90, 0.95)	15	11.68	(0.40,0.50)	5	385.44
(0.80,0.90)	15	64.24	(0.33,0.40)	5	52.56
(0.70,0.80)	10	140.16			

The problem was solved for 26 cases representing different objective function weight combinations as presented in Table 3.5. The objective function values for Pareto front solutions are presented in Table 3.6. Although SO_2 emission is not as a part of the optimization model, the SO_2 emission level for each solution is calculated. Decision makers can choose the solution that is the most appropriate given their preferences. This analysis is providing the trade-offs between each objective function.

Table 3.5. Cases and the corresponding weight combinations

Case	Cost	CO_2	NO_x	Case	Cost	CO_2	NO_x	Case	Cost	CO_2	NO_x
1	0.5	0	0.5	11	0.7	0	0.3	21	0.9	0	0.1
2	0.5	0.125	0.375	12	0.7	0.075	0.225	22	0.9	0.025	0.075
3	0.5	0.25	0.25	13	0.7	0.15	0.15	23	0.9	0.05	0.05
4	0.5	0.375	0.125	14	0.7	0.225	0.075	24	0.9	0.075	0.025
5	0.5	0.5	0	15	0.7	0.3	0	25	0.9	0.1	0
6	0.6	0	0.4	16	0.8	0	0.2	26	1	0	0
7	0.6	0.1	0.3	17	0.8	0.05	0.15				
8	0.6	0.2	0.2	18	0.8	0.1	0.1				
9	0.6	0.3	0.1	19	0.8	0.15	0.05				
10	0.6	0.4	0	20	0.8	0.2	0				

Table 3.6. Objective function solutions for Pareto front for GEP solutions

Cases	Cost (billions of $)	CO_2 (thousands of tons)	NO_x (thousands of tons)	SO_2 (thousands of tons)
1	17.99	79,051.23	71.91	144.52
2	18.43	76,766.70	69.12	137.20
3,4	22.02	59,180.90	49.23	87.76
5	22.02	56,757.17	83.13	45.42
6,7,8,9,11,12,13,14	17.99	79,051.33	71.91	144.52
10	18.01	74,511.35	135.79	64.42
15	17.86	75,418.91	151.00	67.65
16,17,18	17.83	80,698.87	76.72	161.19
19	17.35	85,427.11	96.63	266.50
20	17.22	81,291.70	232.15	113.99
21	16.42	99,833.18	143.59	457.01
22	16.32	102,109.60	151.06	486.18
23	16.27	103,286.00	154.90	501.50
24	16.17	105,757.70	163.05	533.18
25	16.12	105,187.30	194.80	517.35
26	15.88	126,398.10	270.38	803.70

Figure 3.3 presents the Pareto front for the numerical example. It can be difficult to observe trade-offs in three-dimensional graph so the trade-off between cost and CO_2, cost and NO_x, cost and SO_2, CO_2 and NO_x, and CO_2 and SO_2, is presented in Figures 3.4 through 3.8 respectively.

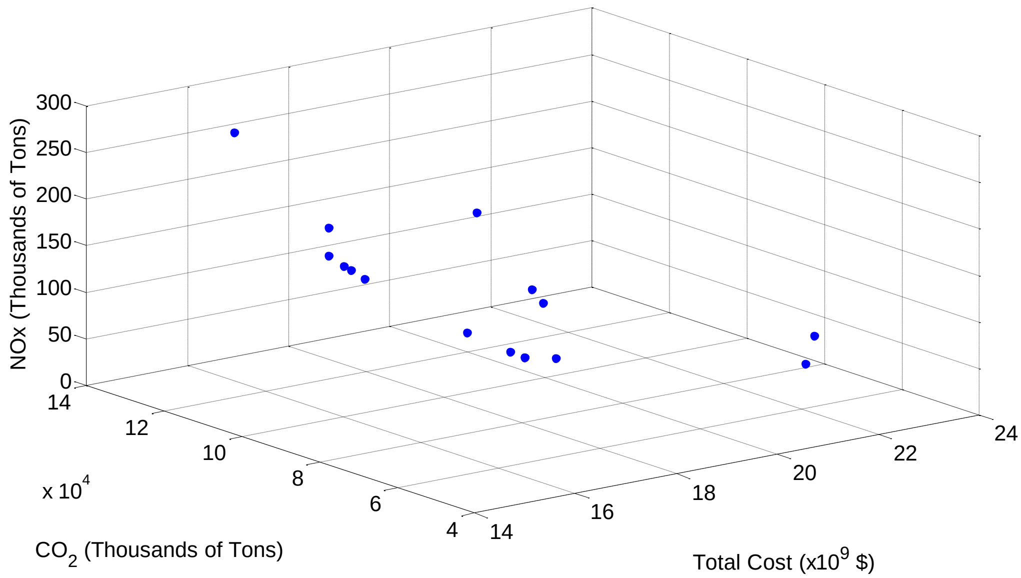

Figure 3.3. Trade-offs between Cost, NO_x and CO_2

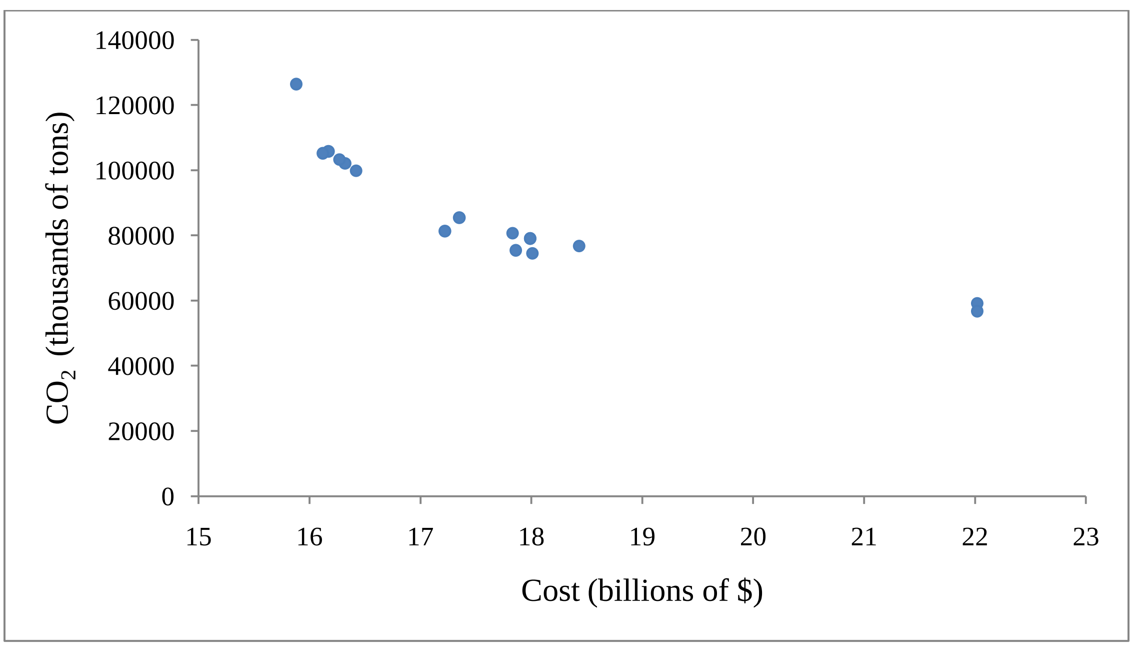

Figure 3.4. Trade-offs between Cost and CO_2

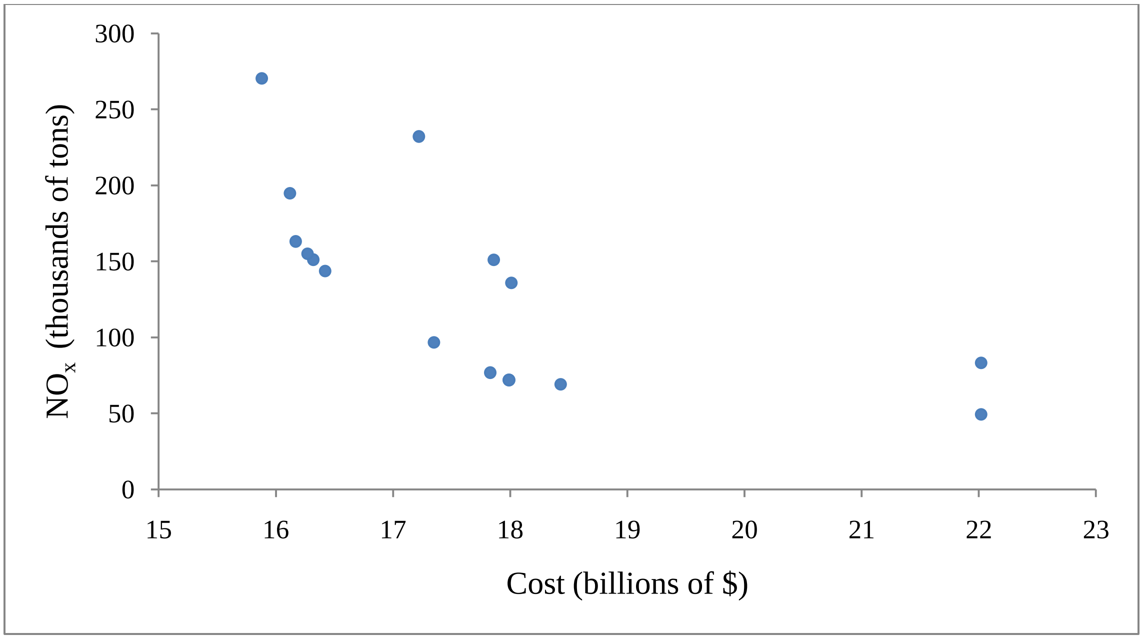

Figure 3.5. Trade-offs between Cost and NO$_x$

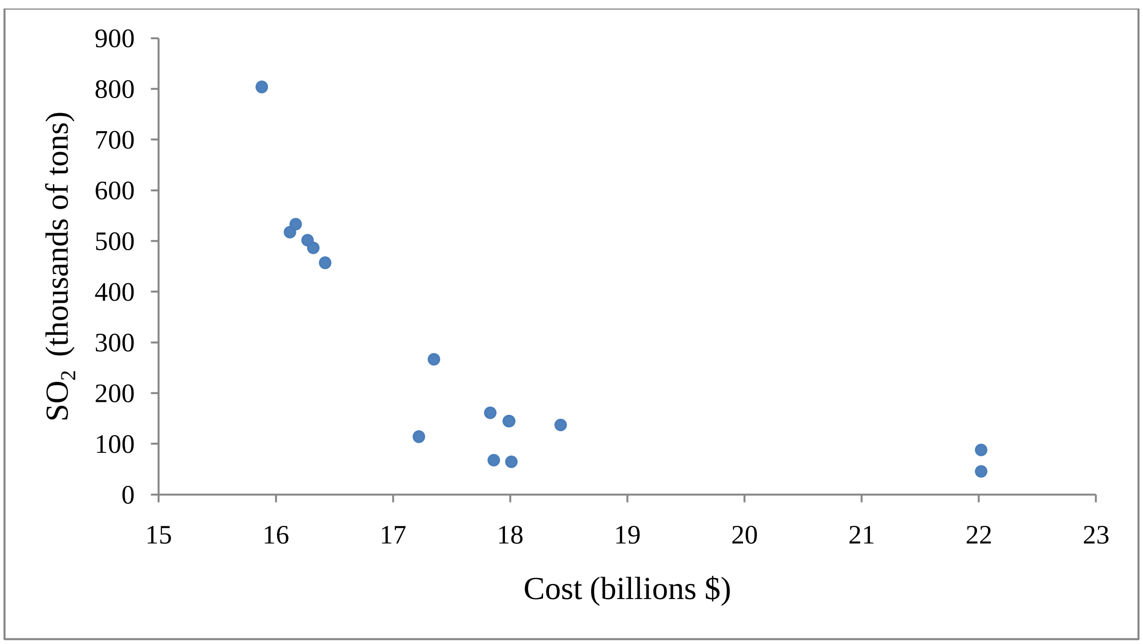

Figure 3.6. Trade-offs between Cost and SO$_2$

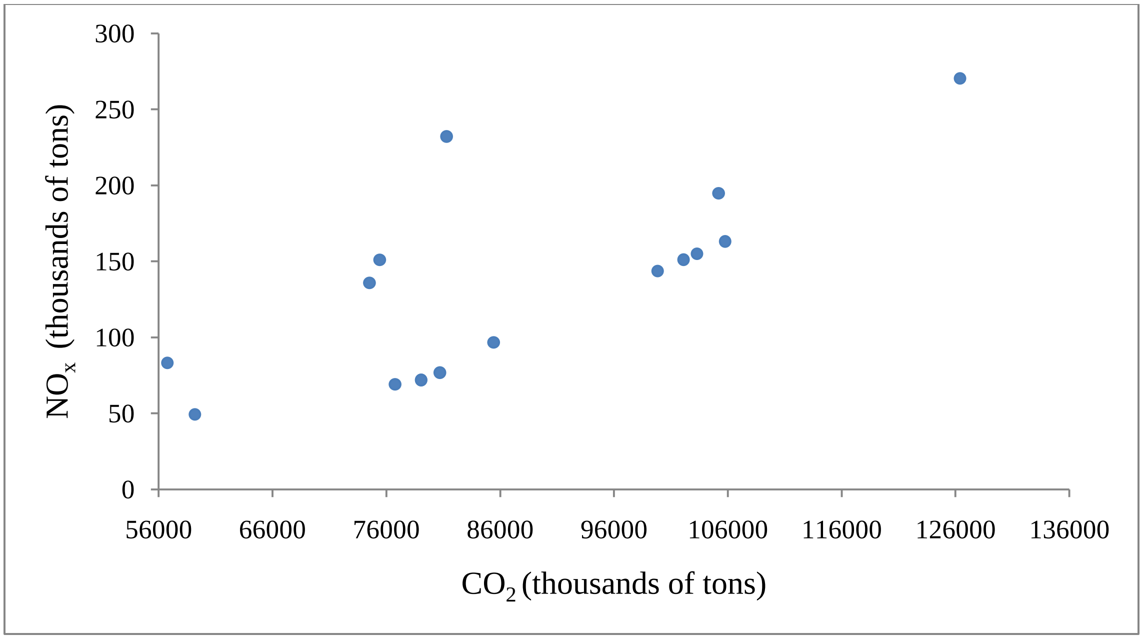

Figure 3.7. Trade-offs between CO_2 and NO_x

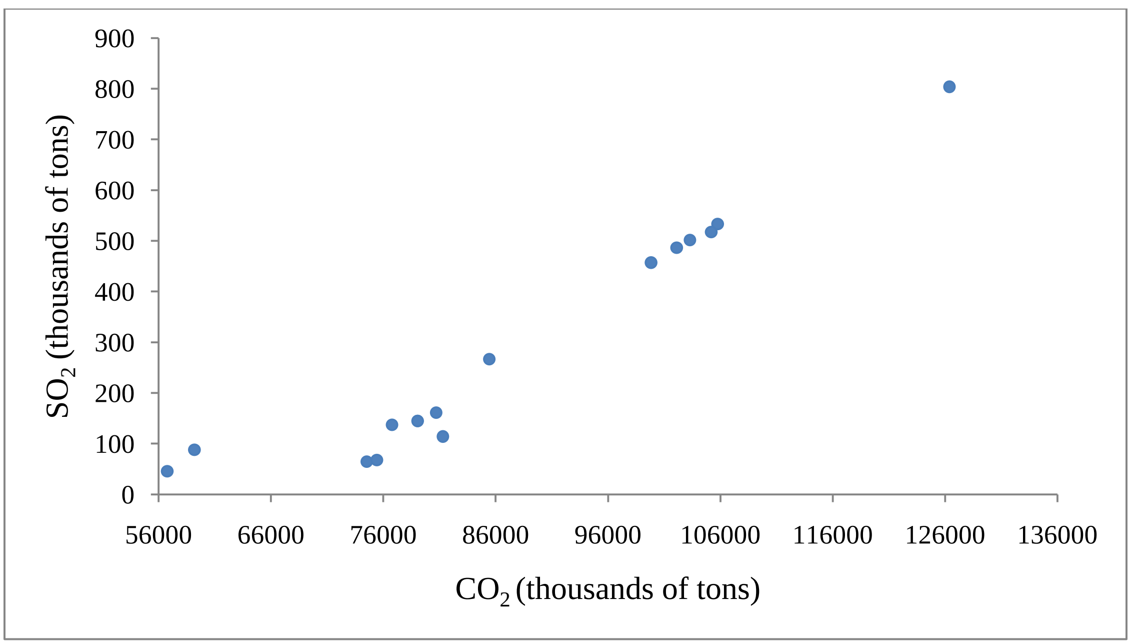

Figure 3.8. Trade-offs between CO_2 and SO_2

For every weight combination, each solution is to build distributed generation units at all allowable locations. At the first time period, 47 distributed units are constructed, and in five years, the remaining ones are to be installed in the optimal plan. The main reason for this is unmet demand attributed to distribution line failures. Building distributed generation units is cost beneficial, assuming that there is a buyer for their steam at the equivalent of 15.91 $/MW. Since in this example, distributed generation units are using natural gas, they are also environmental friendly compared to coal and oil.

The expansion plan for central units is presented in Table 3.7. For each weight combination, the investment decision is presented for each time period. The investment decisions are changed based on relative weights assigned to cost, CO_2 and NO_x. As the table indicates, for the last combination where the objective is to find the least cost expansion plan, there is no central unit investment. This is because the existing reserve of the system with built distributed generation is high enough to accommodate the expected demand. When the weight for cost is decreased, the expansion decisions change towards more environmental friendly technologies. CCGTs (combined cycle gas turbines) are introduced to the system even when the weight of the cost is decreased by 0.1. When only cost and CO_2 is considered (no NO_x), CCGTs are introduced into the system to reduce the production mainly from coal burning units. When NO_x is introduced to the objectives, more CCGTs are constructed to allow for a reduction in production from Oil/CT and IC engines for satisfiable demand. For the combinations where the cost has relatively high priority, the reduction on emissions is accomplished by using CCGTs. When the weight of the cost reaches its lowest levels, wind turbines and nuclear plants are also included to expansion plan to reduce CO_2 emission. When the objective includes only cost and NO_x emission, only CCGTs are built.

Table 3.7. Central units expansion solutions for Pareto front

Cases				Technology	Number/Total Capacity Added for Each Technology		
#	Cost	CO_2	NO_x		T1 year 1	T2 year 6	T3 year 11
1	0.5	0	0.5	CCGT	10/760MW		
2	0.5	0.125	0.375	CCGT Wind	10/760MW 2/100MW		
3 4	0.5 0.5	0.25 0.375	0.25 0.125	CCGT Wind Nuclear	10/760MW 10/500MW		1/400MW
5	0.5	0.5	0	CCGT Wind Nuclear	10/760MW 10/500MW		1/400MW
6 7 8 9 11 12 13 14	0.6 0.6 0.6 0.6 0.7 0.7 0.7 0.7	0 0.1 0.2 0.3 0 0.075 0.15 0.225	0.4 0.3 0.2 0.1 0.3 0.225 0.15 0.075	CCGT	10/760MW		
10	0.6	0.4	0	CCGT	10/760MW		
15	0.7	0.3	0	CCGT	9/684MW		
16 17 18	0.8 0.8 0.8	0 0.05 0.1	0.2 0.15 0.1	CCGT	9/684MW		
19	0.8	0.15	0.05	CCGT	8/608MW		
20	0.8	0.2	0	CCGT	5/380MW		
21	0.9	0	0.1	CCGT	2/152MW	1/76MW	1/76MW
22	0.9	0.025	0.075	CCGT	2/152MW		1/76MW
23	0.9	0.05	0.05	CCGT	1/76MW	1/76MW	1/76MW
24	0.9	0.075	0.025	CCGT		1/76MW	2/152MW
25	0.9	0.1	0	CCGT			3/228MW
26	1	0	0				

In addition to the change in investment plans based on the relative objective function weight differences, the dispatching decisions also change with the Pareto solutions. In Figure 3.9, the percentage usage of the technologies is given for each Pareto solution. IC engines are used for distributed generation option and they are mainly used to meet locally satisfiable demand. They are also used for central satisfiable demand when NO_x emission is not part of the objective function. When the objective function includes NO_x

emission, IC engines are used only to meet locally satisfiable demand. This is because IC engines have relatively high NO_x emission rates compared to the other technologies, except Oil/CT. Oil/Steam technologies have the highest variable cost among the central units. Therefore, Oil/Steam units are not used when the cost is assigned a high weight. However, since Oil/Steam units have the second lowest NO_x emissions, for the combination where the objective function includes NO_x, Oil/Steam units are used to satisfy the demand despite their high variable costs. For the combinations where the objective is only to minimize the cost and CO_2 emission, Oil/Steam units are not used as much. Nuclear plants and wind turbines are both economically and environmentally beneficial (in terms of air emissions), and therefore, they are used to their capacity limits when they are available. Coal burning units have low variable cost. Therefore, when the objective is only to minimize the cost; they are used as much as possible. However, they also have the highest CO_2 emissions and relatively high NO_x emission. Therefore, the usage of coal burning units is lower while the relative importance of the gas emissions is increased. CCGT usage increases with an increase in the importance of reduction in gas emissions and its highest usage occurs when the objective is to minimize cost and NO_x and the cost has its lowest level of importance.

In this section, Monte Carlo based approach is explained to generate the scenarios which are used to represent the stochastic nature of the power grid in the mathematical model. Although this approach is very efficient and easy to implement, it requires generating large number of scenario for sufficient representation. Therefore, a second approach is presented in the next section which is used to select a subset of possible scenarios to sufficiently represent the uncertainty in the system.

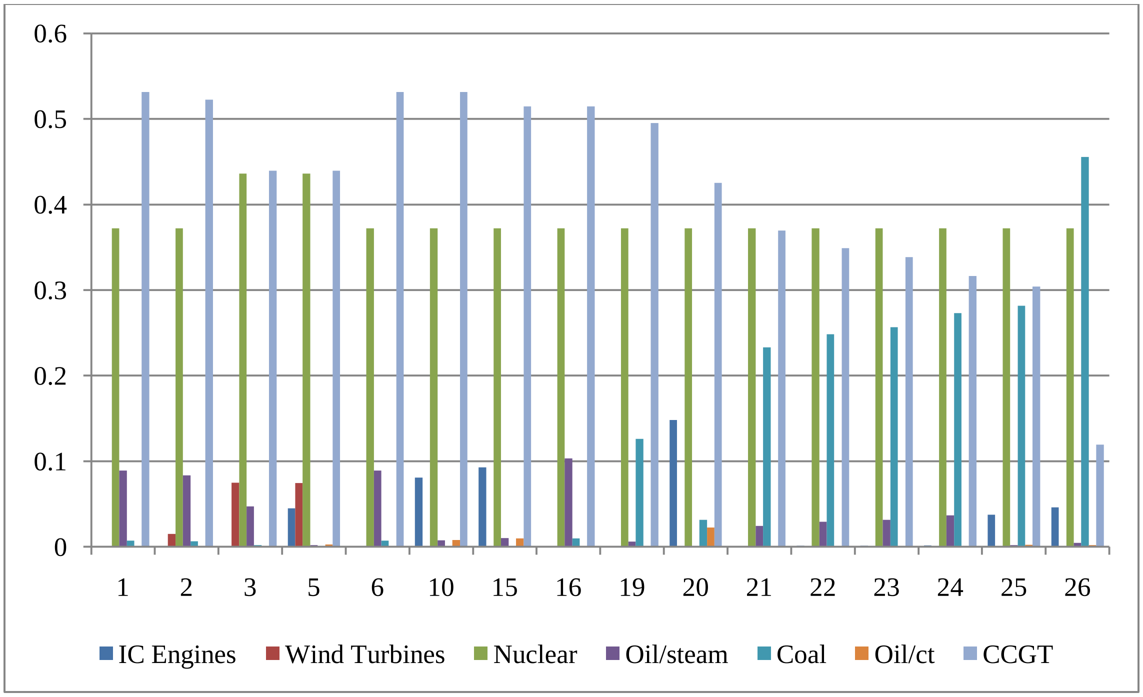

Figure 3.9. Percentage usage of technologies with Pareto solutions

3.2.2. Scenario Optimization Based Approach

In the Monte Carlo based approach, a subset of scenarios is selected randomly to reduce the size of the problem. This approach is very efficient; however, it requires large number of scenarios for the larger system such as presented in Figure 3.2. Therefore, as a part of this research plan, a methodology is developed to generate an efficient set of scenarios to characterize the uncertainty. In this section, optimization based approach is presented to select a subset of scenarios which can sufficiently represent the uncertainty in the system.

There are two sources of uncertainty in the model; component availability and user demand. For a large system, there are numerous combinations of possible failed components that can have differing impact on the objective functions. In practice, the demand scenarios and availability scenarios can be considered separately, but due to the specific nature of the problem, it is advantageous to consider them together. The objective is to develop a general scenario development approach to minimize the number of required scenarios for complex problems.

Demand for electricity fluctuates from season-to-season, day-to-day and hour-to-hour in a generally predictable manner, often modeled using a load-demand curve. Additionally demand for electricity is anticipated to increase at a rate of 1 to 2 % per year. A year can be segmented into many small duration time periods with a specified demand level and associated probability. Furthermore, for each of these time periods, there is a corresponding availability vector indicating the availability (0 or 1) for every component in the system, including generation units, transmission lines, and distribution lines. If there are N_1 different demand levels and N_2 components, then there will be $N_1 \times (2^{N_2})$ possibilities. The objective of scenario optimization is to determine a representative subset of those, to assign a probability to each, and to accurately characterize the uncertainty with a minimum number of scenarios selected.

The system components whose failures considered are generation units, transmission lines and distribution system. If these components fail, they are not available for that scenario.

Step 1: **Critical Component Determination**

The first step is to prioritize system components based on reliability or availability importance metrics. These metrics provide an indication of the importance each individual *component* as it relates to *system* reliability or availability. Components with importance values below some defined threshold can be removed from the availability vector, and assumed to always perform at their expected value. This will reduce the number of components in the scenarios.

Step 2: **Selecting Availability Scenarios and Their Corresponding Probabilities**

Once a set of critical components is selected, availability vectors for these components can be generated. An optimization problem, with the objective of collectively minimizing the deviation from the first moment for component availabilities, can be solved to find the availability scenarios and their associated probabilities. Consider that N is the number of availability scenarios are obtained together with the corresponding probabilities, $Pr(n)$.

Step 3: **Selecting Distribution System Scenarios and Their Corresponding Probabilities**

Load blocks in each demand node are classified into groups based on their peak load levels and the availability of the distribution line. If you consider the system at some specific instance of time, it is possible to observe that all the distribution lines in each load group are working or several of them are not working. Considering all of these states is not computationally efficient. Therefore, a method is proposed to select a subset of scenarios which can be used to best represent this stochastic nature. Two states for each group are defined; all the distribution lines to this group are working, or there is at least one distribution line failed. Since there are two cases defined for each group, there should be 2^G scenarios if there are G load groups, which is still a very large number. In order to select a subset from these scenarios, an optimization problem is formulated and solved to minimize the deviation from the first moment of the expected number of failed distribution lines for each load group. Consider that M is the number of scenarios to represent the stochastic nature of the distribution system. These scenarios are used to determine "satisfiable demand" and "locally satisfiable demand" for each demand node. The detail information how to calculate them is given in the following sections. At the end of this step, the corresponding probabilities, $Pr(m)$ are also obtained for each scenario m.

Step 4: **Selecting Demand Level**

Another uncertainty about the demand is the demand level for each load group throughout the year. Although peak load level differs between the load groups, the load groups have the same load duration curve. The load duration curve is partitioned into distinct segments to capture this stochastic behavior. In the Monte Carlo based approach, a demand level for each availability scenario is randomly chosen from the corresponding segment. Here, we suggest using the percentage level which is at the center point of the each segment to calculate the demand level to represent the corresponding segment. That is, the demand level of each segment is calculated by multiplying the peak load demand by the selected percentage level of the corresponding segment.

Step 5: Reducing the Size of the Availability-Demand Scenarios

If there are q number of segments selected and if all the combinations between availability scenarios and distribution system scenarios are considered, then there will be $N \times M \times q$ scenarios. When the demand is low, the system is more robust, and therefore, it is not required to consider all the availability scenarios for the lower demand level, or for some scenario demands, it is possible to select some subset of availability scenarios for each scenario demand. One way to do this is to use the capacity reserve of each combination. Consider that for each segment q, N_q availability and distribution system scenarios are chosen, and then there are $\mathbb{N} = \sum_q N_q$ availability-demand scenarios, which is called as the set of scenarios. The probability for each scenario, $P(n)$ is calculated by multiplying the probabilities of demand level, availability scenario and distribution system scenario.

3.2.2.1. Step1: Critical Component Determination

In Step 1, a simulation based approach is used to determine the component reliability importance metrics. The components considered are the existing central generation units and transmission lines.

Consider that there are C number of components in the existing system (central generation units, and transmission lines). A Monte Carlo simulation based approach is used to select the most critical components with respect to cost, CO_2 and NO_x. The idea is to observe how the system is affected when the component is assumed to be working all the time and when it is assumed to be not working all the time. As a part of the procedure, the dispatching problem is solved to satisfy the defined load (P_i) in each demand node i. The dispatching problem can be defined as the determination of the amount of energy produced by each generation unit given the existing power system, without considering expansion. P_i is defined as peak load demand in demand node i when the objective is to minimize the cost. When the objective is to minimize the CO_2 or NO_x emissions, P_i is defined as the 90% of the peak load demand in each demand node i. When the demand is too high, all the generation units are generally used. Therefore, there is limited flexibility to use different technologies to reduce the gas emissions. In order to provide some flexibility, we consider using 90% of the peak load demand in each demand node i as the demand. Availability scenarios used in the procedure are generated by using the Monte Carlo based approach. The procedure is given below.

Sub-step 1.1: Choose a component c from the component list.

Sub-step 1.2:

- Assume that the availability of the component c is equal to 1 and generate N availability scenarios for other components.
- Calculate for each scenario n, assuming that demand is equal to P_i.
    - Operational cost, $OC_n(c,1)$ by solving dispatching problem with the objective of minimizing the cost.
    - CO_2 emission, $CO_{2,n}(c,1)$ by solving dispatching problem with the objective of minimizing the CO_2 emission.
    - NO_x emission, $NO_{x,n}(c,1)$ by solving dispatching problem with the objective of minimizing the NO_x emission.
- Calculate
    - Total Operational Cost, $\mathrm{TOC}(c,1) = \sum_{n=1}^{N} OC_n(c,1)$
    - Total CO_2 emission, $\mathrm{TCO}_2(c,1) = \sum_{n=1}^{N} \mathrm{CO}_{2,n}(c,1)$
    - Total NO_x emission, $\mathrm{TNO}_x(c,1) = \sum_{n=1}^{N} \mathrm{NO}_{x,n}(c,1)$

Sub-step 1.3: Repeat the *Sub-step* 1.2 assuming that the availability of the component c is equal to 0 and obtain

$$\mathrm{TOC}(c,0)=\sum_{n=1}^{N} OC_n(c,0)\,;\mathrm{TCO}_2(c,0)=\sum_{n=1}^{N}\mathrm{CO}_{2,n}(c,0)\,;\mathrm{TNO}_\mathrm{x}(c,0)=\sum_{n=1}^{N}\mathrm{NO}_{\mathrm{x},n}(c,0)$$

Sub-step 1.4: Calculate the differences and normalize them as follows.

$$\mathrm{DC}(c)=\frac{\left|\mathrm{TOC}(c,1)-\mathrm{TOC}(c,0)\right|}{\sum_j\left|\mathrm{TOC}(j,1)-\mathrm{TOC}(j,0)\right|},\ \mathrm{DCO}_2(c)=\frac{\left|\mathrm{TCO}_2(c,1)-\mathrm{TCO}_2(c,0)\right|}{\sum_j\left|\mathrm{TCO}_2(j,1)-\mathrm{TCO}_2(j,0)\right|}$$

$$\mathrm{DNO}_\mathrm{x}(c)=\frac{\left|\mathrm{TNO}_\mathrm{x}(c,1)-\mathrm{TNO}_\mathrm{x}(c,0)\right|}{\sum_j\left|\mathrm{TNO}_\mathrm{x}(j,1)-\mathrm{TNO}_\mathrm{x}(j,0)\right|}$$

Sub-step 1.5: Rank the components with respect to their DC(c), $\mathrm{DCO_2}$(c), and $\mathrm{DNO_x}$(c) and choose the components with importance values above a threshold as critical components.

3.2.2.2. Step 2: Selecting Availability Scenarios and Their Corresponding Probabilities

Once the critical components are selected, all the availability vectors are generated in Step 2. That is, if we choose R critical components, then, 2^R availability vectors are generated. The probabilities for some of these vectors are very small, and 2^R is still potentially a very large number. Therefore, it is necessary to select a subset of these vectors in an intelligent way so that the subset reflects the characteristics of the system. Therefore, an optimization method has been devised to select an efficient set of availability scenarios. The model is selecting a subset of all availability scenarios which collectively minimizes the deviation from the first moment for component availabilities and it is given below.

$$\min \sum_{c=1}^{R} (s_c + e_c)$$

s.t.

$$\sum_{j \in F} \pi_j a(c, j) + s_c - e_c = p_c \qquad \forall c \tag{17}$$

$$\sum_{j \in F} \pi_j = 1, \pi_j \geq 0 \qquad \forall j \tag{18}$$

$$s_c, e_c \geq 0 \qquad \forall c \tag{19}$$

where $a(c, j) = \begin{pmatrix} 1, \text{ if the component } c \text{ is working in availibility vector } j \\ 0, \text{ otherwise} \end{pmatrix}$,

p_c is the estimated availability of the system component c. s_c and e_c are the deviation from the availability of the component. π_j is the decision variable which represents the probability of the availability scenario j. If this value is non-zero, it means that the corresponding availability scenario is a part of the subset. F is the set of all the availability scenarios. Equation 17 guarantees that the availability of the component c calculated by the selected scenarios and the deviation is to be equal to the availability of the component c. Equation 18 guarantees that the sum of the probabilities is one. Assume that we obtain N availability scenarios at the end of this step.

This approach gives a subset of scenarios with the corresponding probabilities which approximates the availability of each component best. Therefore, it is possible to represent the uncertainty of the component availabilities by a subset instead of using all the availability scenarios.

3.2.2.3. Step 3: Selecting Distribution System Scenarios and Their Corresponding Probabilities

In Step 3, a subset of scenarios is selected to represent the uncertainty in the distribution system. For each load group g, there are two states defined: State 0 means that there are no failed distribution lines in the load group and State 1 means that there is at least one distribution line failed. Assuming that there are S_g number of load blocks (distribution line) in each load group, the expected number of failed distribution lines for each load group, D_g, is equal to $S_g \times \alpha_g$, where α_g is the unavailability of the distribution lines of the load group g. Then, the following optimization problem can be solved to select a subset of scenarios which reflects the characteristic of the system. The model collectively minimizes the deviation from the first moment of the expected number of failed distribution lines for each load group.

$$\min \sum_{g=1}^{G} (x_g + y_g)$$

s.t.

$$\sum_{j \in W} \eta_j b(g, j) + x_g - y_g = D_g \qquad \forall g \tag{20}$$

$$\sum_{j \in W} \eta_j = 1, \eta_j \geq 0 \qquad \forall j \tag{21}$$

$$x_g, y_g \geq 0 \qquad \forall g \tag{22}$$

where $b(g, j) = \begin{pmatrix} 1, \text{ if there are at least one failed distribution line} \\ 0, \text{ otherwise} \end{pmatrix}.$

x_g and y_g are the deviation from the expected number of failed distribution lines for each load group g. η_j is the decision variable which represent the probability of the scenario j. If this value is non-zero, it means that the corresponding distribution system scenario is a part of the subset. W is the set of all the scenarios. That is, if there are G load groups, W includes all 2^G scenarios. Equation 20 guarantees that the expected number of failed distribution lines calculated by the selected scenarios and the deviation is to be equal to expected number of failed distribution lines. Equation 21 guarantees that the sum of the probabilities is one.

This optimization approach provides a subset which approximates the expected number of failed distribution lines in each load group the best. Therefore, it is possible to sufficiently represent the stochastic nature of the distribution system by this subset instead of using all the scenarios.

3.2.2.4. Step 4 and 5: Selecting Demand Level and Reducing the Size of the Availability-Demand Scenarios

In Step 4, the load duration curve is divided into distinct demand segment and a representative demand level is selected for each demand segment. For each demand segments, using all the combinations of the availability and distribution system scenarios is not necessary. The reliability is much more important when the demand is high or there are multiple failures in the critical system components. Therefore, Step 5 provides an intelligent way to reduce the size of the scenarios. The procedure is based on the capacity reserve of each combination. Assume that C is the total capacity available and D is the total demand can be served. Then, the capacity reserve is calculated as $(C-D)/D$. The steps of the procedure are as follows:

For each demand segment *q*;

- All the distribution system scenarios are considered.
- All the availability scenarios in which there is at least one failed transmission line are considered.
- It is possible to ignore some availability scenarios in which all the transmission lines are working. Define N' is the set of availability scenarios which can be ignored. Then,

Sub-step 5.1: Choose the distribution system scenario which has the highest demand and the availability scenario from the set N' which has the lowest capacity.

Sub-step 5.2: Calculate the capacity reserve for the combination of the distribution system scenario and the availability scenario selected in Step 1. If the capacity reserve of this combination is larger than the predefined value, it means that all the combinations for the corresponding demand segment have the capacity reserve which is larger than the predefined value. If this is true,

- Ignore all the availability scenarios in N' and add the total probability of ignored scenarios to the scenario in which all the generation units and the transmission lines are working.

If not, go to *Sub-step* 5.3.

Sub-step 5.3: Choose the distribution system scenario which has the largest demand, then, calculate the capacity reserve for each combination between the availability scenario from the set N' and the selected distribution system scenario. If the calculated capacity reserve is larger than the predefined value, ignore the availability scenario, otherwise keep it.

3.2.2.5. Numerical Example for Scenario Optimization

In this section, the scenario optimization procedure is applied to the modified IEEE Reliability Test System, shown in Figure 3.2. There are 32 generation technologies which are distributed among ten power nodes. The existing generation units in each power node are the same as the one presented in Table 3.1. However, the node numbers are different. The power group numbers listed in Table 3.1 are now replaced by 1, 2, 7, 13, 15, 16, 18, 21, 22, and 23 respectively. There are 38 transmission lines and transformers. The capacity, impedance, loss factor and unavailability of the lines are presented in Table 3.8 and obtained from [87]. The unavailability of the lines is calculated by using the outage rate and outage duration of the lines presented in [87]. Since these outage rates do not include the planned maintenance, the unavailability is actually higher, and in the example increased by 10%. The unavailabilities presented in Table 3.8 are the values after the adjustment. There are 12 transmission lines with the capacity of 175MW, five transformers with the capacity of 400MW and 17 transmission lines with the capacity of 500MW in the system. There are 17 demand nodes which are given in Table 3.9 along with the number of load blocks in each demand node and their peak load demands.

Table 3.8. Capacity, impedance, loss factor and unavailability date for the transmission lines in IEEE Reliability Test System

Transmission Lines			Capacity (MW)	Impedance	Loss Factor	Unavailability
I	*j*	*m*				
1	2	1	175	0.0139	0.93	0.004
1	3	1	175	0.2112	0.93	0.005
1	5	1	175	0.0845	0.93	0.003
2	4	1	175	0.1267	0.93	0.004
2	6	1	175	0.192	0.93	0.005
3	9	1	175	0.119	0.93	0.004
3	24	1	400	0.0839	1	0.017
4	9	1	175	0.1037	0.93	0.004
5	10	1	175	0.0883	0.93	0.003
6	10	1	175	0.0605	0.93	0.013
7	8	1	175	0.0614	0.93	0.003
8	9	1	175	0.1651	0.93	0.005
8	10	1	175	0.1651	0.93	0.005
9	11	1	400	0.0839	1	0.017
9	12	1	400	0.0839	1	0.017
10	11	1	400	0.0839	1	0.017
10	12	1	400	0.0839	1	0.017
11	13	1	500	0.0476	0.95	0.005
11	14	1	500	0.0418	0.95	0.004
12	13	1	500	0.0476	0.95	0.005
12	23	1	500	0.0966	0.95	0.006
13	23	1	500	0.0865	0.95	0.006
14	16	1	500	0.0389	0.95	0.004
15	16	1	500	0.0173	0.95	0.004
15	21	1	500	0.049	0.95	0.005
15	21	2	500	0.049	0.95	0.005
15	24	1	500	0.0519	0.95	0.005
16	17	1	500	0.0259	0.95	0.004
16	19	1	500	0.0231	0.95	0.004
17	18	1	500	0.0144	0.95	0.004
17	22	1	500	0.1053	0.95	0.006
18	21	1	500	0.0259	0.95	0.004
18	21	2	500	0.0259	0.95	0.004
19	20	1	500	0.0396	0.95	0.004
19	20	2	500	0.0396	0.95	0.004
20	23	1	500	0.0216	0.95	0.004
20	23	2	500	0.0216	0.95	0.004
21	22	1	500	0.0678	0.95	0.005

In the IEEE Reliability Test System, there is no distribution system. Therefore, a modified system was considered with unavailability of the distribution lines equal to 0.01 and load blocks in each demand node are grouped into one load group. The modified demand data for each demand node is shown in Table 3.9.

Table 3.9. Modified demand data for IEEE Reliability Test System

Node Number	Number of load blocks	Peak Load Demand in each load block (MW)	Peak Load Demand in each load group (MW)	Node Number	Number of load blocks	Peak Load Demand in each load block (MW)	Peak Load Demand in each load group (MW)
1	10	10.8	108	10	19	10.26	195
2	9	10.78	97	13	16	16.56	265
3	18	10.00	180	14	19	10.21	194
4	7	10.57	74	15	31	10.23	317
5	7	10.14	71	16	10	10.00	100
6	13	10.46	136	18	33	10.09	333
7	12	10.42	125	19	18	10.06	181
8	17	10.06	171	20	12	10.67	128
9	17	10.29	175				

The scenario optimization based approach is applied to generate and select a subset of scenarios which sufficiently represent the uncertainty in the system. The results with respect to each step of the procedure are given below.

Step 1: Critical Component Determination

In this step, Monte Carlo simulation is used to generate scenarios for the dispatching problem. Therefore, 10,000 scenarios are generated and the procedure to determine the critical component is applied. The peak load demand for each load group in Table 3.9 is used as the peak load demand in each demand node for the dispatching problems. The sorted importance values for each component are given in Appendix A. The threshold value is selected as 0.03; and therefore, 15 components are selected as the critical components. The critical components are given in Table 3.10. TL stands for the transmission line and CU stands for the central generation unit in the table.

Table 3.10. Critical components selected

CU 23,3	CU 18,1	CU 13,1	TL 14,16,1	TL 7,8, 1	CU 23,1	TL 2,6, 1	TL, 6,10,1
CU 21,1	CU 13,3	CU 13,2	CU 16,1	CU 23,2	TL 16,19,1	TL 4,9,1	

Step 2: Selecting Availability Scenarios and Their Corresponding Probabilities

The procedure to select a subset of availability scenarios for the critical components is applied. At the end of this procedure, 16 availability scenarios are selected to represent the availabilities of the selected critical components, which is a very small number compare to 2^{15}. The selected scenarios and the corresponding probabilities are given in Table 3.11.

Table 3.11. Availability scenarios and corresponding probabilities for the critical components

Selected Scenario Numbers	Scenario numbers	Critical Components															Prob.
		CU 13 1	CU 13 2	CU 13 3	CU 23 1	CU 23 2	CU 23 3	CU 16 1	CU 18 1	CU 21 1	TL 14 16	TL 16 19	TL 2 6	TL 4 9	TL 6 10	TL 7 8	
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0.65
2	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1		0.003
3	3	1	1	1	1	1	1	1	1	1	1	1	1	1		1	0.008
4	75	1	1	1	1	1	1	1	1		1	1		1		1	0.005
5	129	1	1	1	1	1	1	1		1	1	1	1	1	1	1	0.096
6	321	1	1	1	1	1	1		1		1	1	1	1	1	1	0.04
7	517	1	1	1	1	1		1	1	1	1	1	1		1	1	0.004
8	529	1	1	1	1	1		1	1	1	1		1	1	1	1	0.004
9	1537	1	1	1	1			1	1	1	1	1	1	1	1	1	0.022
10	3137	1	1	1			1	1	1		1	1	1	1	1	1	0.018
11	4609	1	1		1	1		1	1	1	1	1	1	1	1	1	0.047
12	4673	1	1		1	1		1	1		1	1	1	1	1	1	0.003
13	8257	1		1	1	1	1	1	1		1	1	1	1	1	1	0.05
14	16481		1	1	1	1	1	1	1			1	1	1	1	1	0.004
15	16513		1	1	1	1	1	1		1	1	1	1	1	1	1	0.024
16	18433		1	1		1	1	1	1	1	1	1	1	1	1	1	0.022

Step 3: Selecting Distribution System Scenarios and Their Corresponding Probabilities

The procedure to select a subset of scenarios to represent the stochastic characteristic of the distribution system is applied. In order to assure that the scenario where all the distribution lines are working is selected, a constraint was added forcing the probability of this scenario to be greater than zero. 19 scenarios are selected, which is a very small number compare to 2^{17}. In Table 3.12, the selected scenarios and the corresponding probabilities are given.

Table 3.12. Selected scenarios to represent the distribution system in the IEEE Reliability Test System

Selected Scenario Numbers	Scenario Numbers	Demand Node Numbers																	Probabilities
		1	2	3	4	5	6	7	8	9	10	13	14	15	16	18	19	20	
1	53278	1																	0.1
2	66146		1	1					1	1	1	1	1	1	1	1	1	1	0.035
3	78375		1				1	1	1	1	1	1	1	1		1	1	1	0.055
4	86991			1	1		1		1	1	1	1	1	1		1	1	1	0.003873
5	94052			1			1	1	1	1	1	1	1	1	1	1	1		0.065
6	98144			1															0.076127
7	106136				1														0.066127
8	110218					1													0.07
9	112266						1												0.006127
10	113802								1										0.011128
11	114058									1									0.011128
12	114186										1								0.031127
13	114250											1							0.001128
14	114282												1						0.031127
15	114298													1					0.151127
16	114310															1			0.171127
17	114312																1		0.021127
18	114313																	1	0.026127
19	114314																		0.0676

Step 4: Selecting Demand Level

The load duration curve was divided into six segments and the proportion for each demand segment was determined to represent the energy demand for the corresponding segment. The proportions of the peak load demand used are given in Table 3.13.

Table 3.13. Demand segments and the corresponding percentage levels

Segments	Percentage of peak load demand	Probability	Segments	Percentage of peak load	Probability
1.00	1.00	0.01	(0.80,0.90)	0.85	0.11
(0.95, 1.00)	0.975	0.01	(0.60,0.80)	0.70	0.39
(0.90, 0.95)	0.925	0.02	(0.33,0.60)	0.465	0.46

Step 5: Reducing the Size of the Availability-Demand Scenarios

The total number of scenarios for each year is equal to $19 \times 16 \times 6 = 1{,}824$ if all the distribution system scenarios and availability scenarios are used in each segment. However, it is possible to reduce the number of scenarios by applying the procedure described in Section 3.2.2 A predefined value of 20% is used in the procedure, 16 availability scenarios were selected for the first three segments, 14 for the forth segment and seven for the remaining ones. The total number of scenarios for each year is reduced to 1,311. The selected availability scenarios and the corresponding probabilities for each demand segment are given in Table 3.14.

Table 3.14. Selected availability scenarios for each demand segment

Segments	Scenario numbers and probabilities
1.00	**1** (0.65); **2** (0.003); **3** (0.008); **4** (0.005); **5** (0.096); **6** (0.04); **7** (0.004); **8** (0.004); **9** (0.022); **10** (0.018); **11** (0.047); **12** (0.003); **13** (0.05); **14** (0.004); **15** (0.024); **16** (0.022)
(0.95, 1.00)	**1** (0.65); **2** (0.003); **3** (0.008); **4** (0.005); **5** (0.096); **6** (0.04); **7** (0.004); **8** (0.004); **9** (0.022); **10** (0.018); **11** (0.047); **12** (0.003); **13** (0.05); **14** (0.004); **15** (0.024); **16** (0.022)
(0.90, 0.95)	**1** (0.65); **2** (0.003); **3** (0.008); **4** (0.005); **5** (0.096); **6** (0.04); **7** (0.004); **8** (0.004); **9** (0.022); **10** (0.018); **11** (0.047); **12** (0.003); **13** (0.05); **14** (0.004); **15** (0.024); **16** (0.022)
(0.80,0.90)	**1** (0.768); **2** (0.003); **3** (0.008); **4** (0.005); **6** (0.04); **7** (0.004); **8** (0.004); **9** (0.022); **10** (0.018); **11** (0.047); **12** (0.003); **13** (0.05); **14** (0.004); **15** (0.024)
(0.60,0.80)	**1** (0.972); **2** (0.003); **3** (0.008); **4** (0.005); **7** (0.004); **8** (0.004); **14** (0.004);
(0.33,0.60)	**1** (0.972); **2** (0.003); **3** (0.008); **4** (0.005); **7** (0.004); **8** (0.004); **14** (0.004);

3.3. Mathematical Model Considering More Realistic Transmission System Representation

The problem examined in this section is a multi-objective generation expansion plan over the multi-period planning horizon given an existing centralized power system with bi-directional energy flow and loop flow. It is an extension of the model presented in Section 3.1 to include the transmission system more realistically and allow loop flow. The objective is to minimize cost and gas emissions simultaneously and integrate the reliability with expansion and dispatching decisions. Weighted sum approach is utilized to combine multiple objectives into a single objective function.

In the model presented in Section 3.1, it is assumed that the system has sufficient transmission capacity, and so transmission lines are considered as a single line from central power groups to the area grid. Moreover, the availability of the transmission lines are embedded into the availability of the central generation units. This model can be improved by representing the central generation and transmission system as a network

$G=(N, A)$. The IEEE Reliability Test System [87] is used as an example network topology in Figure 3.2. N represents the set of nodes in the system. As illustrated in Figure 3.2, the nodes can represent the supply points, demand points, both supply and demand points and neither supply nor demand points. Demand points are assumed to have a distribution system. Therefore, the IEEE Reliability Test System was modified to assume that each demand point i consists of L_i load blocks. A represents the set of transmission lines. Since it is possible to have multiple transmission lines between two nodes as shown in Figure 3.2, the arcs in the system are represented as (i,m,r), which is the r^{th} transmission line between the nodes (i,m). Each transmission line has a capacity and is subject to failure. Moreover, based on the availability of the transmission lines of each node, each transmission line has a power distribution factor. This factor is calculated according to the impedance of the transmission lines and it shows the ratio of the capacity usage of transmission lines from the corresponding node if the energy is sent from it. Therefore, the availability scenarios is needed to include the transmission lines explicitly, that is, the availability of the transmission lines and corresponding power distribution factors. Moreover, transmission losses are also included in the model.

Each demand point has a distribution system which consists of load blocks, distribution lines connecting load blocks to the area grid and a local transmission system which connects area grid to the power. It is assumed that this local transmission system is 100% reliable which means that it is not subject to failure. Furthermore, if there are distributed generation units located at the demand points, generation from these units can only be used to satisfy the demand for the corresponding demand point. It means that it is not possible to send the energy produced by distributed generation units to the other nodes in the system.

Some of the parameters in the model are scenario-based. The scenario optimization based approach is utilized to calculate these parameters. In the following subsections, the mathematical representation for scenario-based parameters, the objective functions of the problem and the mathematical formulation are presented.

3.3.1. Calculating Scenario-Based Parameters

In this section, the definitions and mathematical representation for the scenario-based parameters are given. The scenario optimization based approach is used to generate and select the scenarios to be used in the mathematical model. Each scenario *n* can be represented a vector which has the following:

- $t(n,(i,m,r))$: Availability of the r^{th} transmission line between nodes (i,m) where (i,m,r) is defined as a critical transmission line.
- $c(n,(i,k))$: Availability of the existing central generation unit *k* in the node *i*, where *k* is defined as a critical generation unit.
- $w(n,l_i)$: State of the distribution system for the load group l_i in the node *i*,
- $\rho(n,l_i)$: Proportion of peak demand selected for the load group l_i for the node *i*,

The availability of the noncritical transmission lines, the noncritical existing central generation units, the new central and distributed generation units are assumed to be one in all of the scenarios and they assumed to be working at their expected levels. The expected capacity levels are calculated by multiplying the actual capacities and estimated component availabilities.

The scenario-based parameters are calculated as follows.

Power Distribution Factor:

$$\beta_{tnimr} = \frac{z(i,m,r) \times a(n,(i,m,r))}{\sum_{j,k} z(i,j,k) \times a(n,(i,j,k))}$$

where $z(i,m,r)$ is the impedance of the r^{th} transmission line between nodes (i,m), and

$$a(n,(i,m,r)) = \begin{pmatrix} t(n,(i,m,r), \text{ if } (i,m,r) \text{ is defined as critical tranmission line} \\ 1, \text{ Otherwise} \end{pmatrix}.$$

Available Critical Transmission Line capacity:

$$\Delta_{tnimr} = t(n,(i,m,r)) \times \delta_{imr}$$

where δ_{imr} is the actual transmission line capacity.

Available Critical Existing Central Unit Capacity:

$$\gamma_{tnik} = c(n,(i,k)) \times \mu_{ik}$$

where μ_{ik} is the actual central unit capacity.

Satisfiable Demand and Locally Satisfiable Demand:

Considering a problem with S_{l_i} number of load blocks (distribution lines) in each load group, the expected number of failed distribution lines for each load group is $S_{l_i} \times \alpha_{l_i}$, where α_{l_i} is the unavailability of the distribution lines of the load group *g*. Then, expected local demand can be calculated by multiplying the expected number of failed distribution lines by the demand level of each load block in the group. Since each load block has an equal demand level, the demand level of each load block can be calculated by dividing the demand for the load group, *D*, by the number of load blocks, $D(l_i)/S_{l_i}$. Therefore, expected local demand for the load group l_i, $E(l_i)$ is calculated as

$$E(l_i) = S_{l_i} \times \alpha_{l_i} \times \frac{D(l_i)}{S_{l_i}} = \alpha_{l_i} \times D(l_i).$$

By using above formulation, satisfiable and locally satisfiable demand is defined for each scenario. If $P(t,l_i)$ and $\rho(n,l_i)$ are the peak demand at time period *t* and the proportion of the peak demand selected for the load group l_i in the node *i* respectively, then the demand level selected for the load group l_i for each scenario, $D(t,n,l_i)$ is equal to $P(t,l_i) \times \rho(n,l_i)$. Moreover, the expected local demand for load group l_i for each scenario, $E(t,n,l_i)$, is equal to $D(t,n,l_i) \times \alpha_{l_i}$.

The satisfiable demand for each scenario in each demand node is calculated as follows:

$$\Psi_{t,n,i} = \sum_{l_i \in L_i} \left(D(t,n,l_i) \times (1 - w(n,l_i)) \right) + \left((D(t,n,l_i) - E(t,n,l_i)) \times w(n,l_i) \right)$$

The locally satisfiable demand for each scenario is calculated as follows:

$$\Phi_{t,n,l_i} = E(t,n,l_i) w(n,l_i)$$

3.3.2. Objective Functions

The objective is to minimize the cost, CO_2 emissions and NO_x emissions simultaneously. Each objective is normalized and their weighted sum is used as a single objective function in the model. Since the power grid is represented as a network $G=(N, A)$, decision variables are redefined. The individual objective functions are as follows.

Total Cost (O_1):

Total cost consists of investment cost, fixed operation and maintenance cost, electricity generation cost, unmet demand cost and revenue obtained from the distributed generation units.

$$
\begin{aligned}
O_1 = \sum_{t=1}^{T}(1+r)^{-t} \Bigg(& \Bigg(\sum_{i\in(N_1 \text{ or } N_3)} \sum_{q\in\Xi_i} (s_{tiq} - s_{(t-1)iq})a_{tiq} + \sum_{i\in(N_1 \text{ or } N_2)} \sum_{l_i\in L_i} \sum_{j\in J_{l_i}} w_{tl_i j} b_{tl_i j} \Bigg) \\
& + \Bigg(\sum_{i\in(N_1 \text{ or } N_3)} \sum_{k\in\Theta_i} g_{tik} + \sum_{i\in(N_1 \text{ or } N_3)} \sum_{q\in\Xi_i} s_{tiq} h_{tiq} + \sum_{i\in(N_1 \text{ or } N_2)} \sum_{l_i\in L_i} \sum_{j\in J_{l_i}} \sum_{\tau=1}^{t} w_{\tau l_i j} m_{tl_i j} \Bigg) \\
& + \Bigg(\sum_{n=1}^{\mathbb{N}} \varpi_n \Bigg(\sum_{i\in(N_1 \text{ or } N_3)} \sum_{k\in\Theta_i} x_{tnik} c_{tik} + \sum_{i\in(N_1 \text{ or } N_3)} \sum_{q\in\Xi_i} u_{tniq} e_{tiq} + \sum_{i\in(N_1 \text{ or } N_2)} \sum_{l_i\in L_i} \sum_{j\in J_{l_i}} (y_{tnl_i j} + z_{tnl_i j}) d_{tl_i j} \Bigg)\Bigg) \\
& + \Bigg(\sum_{n=1}^{\mathbb{N}} \varpi_n \Bigg(\sum_{i\in(N_1 \text{ or } N_2)} v_{tni} f_t + \sum_{i\in(N_1 \text{ or } N_2)} \sum_{l_i\in L_i} \pi_{tnl_i} f_t - \sum_{i\in(N_1 \text{ or } N_2)} \sum_{l_i\in L_i} \sum_{j\in J'_{l_i}} (y_{tnl_i j} + z_{tnl_i j}) p_t r_t \Bigg)\Bigg)\Bigg)
\end{aligned}
$$

N_1, N_2, N_3 and N_4 represent the nodes which are both power and demand nodes (N_1), just the demand nodes (N_2), just the power nodes (N_3), and neither supply nor demand nodes (N_4) respectively.

In each power node, there are existing generation units, Θ_i and there are generation expansion options, Ξ_i. s_{tiq} is defined the investment decision for central unit q in power node i at time period t. x_{tnik} is the generation amount (MW) of existing central unit type k

in power node i for scenario n in time period t. u_{tniq} is the generation amount (MW) of new central unit type q in power node i for scenario n in time period t.

Load blocks in each demand node are classified into groups based on their peak load levels and the availability of the distribution line. Since the load blocks in the same group have the same characteristics, a decision variable is defined for each group to represent the amount of distributed generation capacity to be built for the corresponding demand group in each demand point. $w_{tl_i j}$ is the decision variable for the investment amount of a distributed generation technology j located at load group l_i in time period t. $y_{tnl_i j}$ is the generation amount (MW) of distributed generation type j located at load group l_i to satisfy satisfiable demand in demand node i and $z_{tnl_i j}$ is the generation amount (MW) of distributed generation type j located at load group l_i to satisfy local demand of load group l_i in demand node i.

v_{tni} and π_{tnl_i} are the unmet satisfiable demand (MW) for scenario n in demand node i in time period t and unmet local demand at load group l_i in demand node i for scenario n in time period t respectively. f_t is the cost of not satisfying the demand in time period t ($/MW).

a_{tiq} is the investment cost ($) of a central unit type q in power node i in time period t. $b_{tl_i j}$ is the investment cost ($) of a distributed unit type j located at load group l_i in time period t. g_{tik}, h_{tiq} and $m_{tl_i j}$ are the fixed operational and maintenance cost ($) for a existing central unit type k, new central unit type q and distributed unit type j located at load group l_i in node i in time period t respectively. c_{tik}, e_{tiq} and $d_{tl_i j}$ are the generation cost

(\$/MW) of existing central unit type *k*, new central unit type *q* and distributed unit type *j* located in load group l_i in node *i* in time period *t* respectively. J_{l_i} is the set of distributed generation units with co-generation capabilities available for load group l_i. p_t is the proportion of generated energy can be used to receive benefit and r_t is the revenue obtained from the usage of steam (\$/MW). *r* is the interest rate, and *T* is the total number of time periods.

CO_2 Emission (O_2):

$$O_2 = \sum_{t=1}^{T} \left(\sum_{n=1}^{\mathbb{N}} \varpi_n \left(\sum_{i \in (N_1 \text{ or } N_3)} \sum_{k \in \Theta_i}^{\mathbb{N}} x_{tnik} C_{ik} + \sum_{i \in (N_1 \text{ or } N_3)} \sum_{q \in \Xi_i} u_{tniq} E_{iq} + \sum_{i \in (N_1 \text{ or } N_2)} \sum_{l_i \in L_i} \sum_{j \in J_{l_i}} (y_{tnl_i j} + z_{tnl_i j}) D_{l_i j} \right) \right)$$

where C_{ik}, E_{iq} and $D_{l_i j}$ are the amounts (lbs) of CO_2 per MW generated by existing central unit type *k*, new central unit type *q* and distributed unit type *j* located at load group l_i in node *i* respectively.

NO_x Emission (O_3):

$$O_3 = \sum_{t=1}^{T} \left(\sum_{n=1}^{\mathbb{N}} \varpi_n \left(\sum_{i \in (N_1 \text{ or } N_3)} \sum_{k \in \Theta_i}^{\mathbb{N}} x_{tnik} F_{ik} + \sum_{i \in (N_1 \text{ or } N_3)} \sum_{q \in \Xi_i} u_{tniq} G_{iq} + \sum_{i \in (N_1 \text{ or } N_2)} \sum_{l_i \in L_i} \sum_{j \in J_{l_i}} (y_{tnl_i j} + z_{tnl_i j}) H_{l_i j} \right) \right)$$

where F_{ik}, G_{iq} and $H_{l_i j}$ are the amounts (lbs) of NO_x per MW generated by existing central unit type *k*, new central unit type *q* and distributed unit type *j* located at load group l_i in node *i* respectively.

3.3.3. Problem Formulation

The mathematical formulation for the GEP model with more realistic transmission system representation is presented below. The objective is to minimize the weighted summation of three scaled objective functions.

$$\min w_1\bar{O}_1 + w_2\bar{O}_2 + w_3\bar{O}_3$$

s.t.

$$\left[\sum_{m,r}\Gamma_{tnmir}\alpha_{mir} + \sum_{k\in\Theta_i} x_{tnik} + \sum_{q\in\Xi_i} u_{tniq} - \vartheta_{tni}\right]\beta_{tnimr} = \Gamma_{tnimr} \qquad \forall i\in N_1, \forall(i,m,r)\in A, \forall(t,n) \quad (23)$$

$$\left[\sum_{m,r}\Gamma_{tnmir}\alpha_{mir} - \vartheta_{tni}\right]\beta_{tnimr} = \Gamma_{tnimr} \qquad \forall i\in N_2, \forall(i,m,r)\in A, \forall(t,n) \quad (24)$$

$$\left[\sum_{m,r}\Gamma_{tnmir}\alpha_{mir} + \sum_{k\in\Theta_i} x_{tnik} + \sum_{q\in\Xi_i} u_{tniq}\right]\beta_{tnimr} = \Gamma_{tnimr} \qquad \forall i\in N_3, \forall(i,m,r)\in A, \forall(t,n) \quad (25)$$

$$\left[\sum_{m,r}\Gamma_{tnmir}\alpha_{mir}\right]\beta_{tnimr} = \Gamma_{tnimr} \qquad \forall i\in N_4, \forall(i,m,r)\in A, \forall(t,n) \quad (26)$$

$$\vartheta_{tni} + \sum_{l_i\in L_i}\sum_{j\in J_{l_i}} y_{tnl_ij} + v_{tni} = \Psi_{tni} \qquad \forall i\in(N_1 \text{ or } N_2), \forall(t,n) \quad (27)$$

$$\sum_{j\in J_{l_i}} z_{tnl_ij} + \pi_{tnl_i} = \Phi_{tnl_i} \qquad \forall i\in(N_1 \text{ or } N_2), \forall l_i\in L_i, \forall(t,n) \quad (28)$$

$$\Gamma_{tnimr} - \Gamma_{tnmir} \le \Delta_{tnimr} \qquad \forall(i,m,r)\in A', \forall(t,n) \quad (29)$$

$$\Gamma_{tnmir} - \Gamma_{tnimr} \le \Delta_{tnimr} \qquad \forall(i,m,r)\in A', \forall(t,n) \quad (30)$$

$$\Gamma_{tnimr} - \Gamma_{tnmir} \le \delta_{imr}p_{imr} \qquad \forall(i,m,r)\in A'', \forall(t,n) \quad (31)$$

$$\Gamma_{tnmir} - \Gamma_{tnimr} \le \delta_{imr}p_{imr} \qquad \forall(i,m,r)\in A'', \forall(t,n) \quad (32)$$

$$x_{tnik} \le \gamma_{tnik} \qquad \forall i\in(N_1 \text{ or } N_3), k\in\Theta_i', \forall(t,n) \quad (33)$$

$$x_{tnik} \le \mu_{ik}p_{ik} \qquad \forall i\in(N_1 \text{ or } N_3), k\in\Theta_i', \forall(t,n) \quad (34)$$

$$u_{tniq} \le \mu_{iq}p_{iq}s_{tiq} \qquad \forall i\in(N_1 \text{ or } N_3), q\in\Xi_i, \forall(t,n) \quad (35)$$

$$y_{tnl_ij} + z_{tnl_ij} \le \sum_{\tau=0}^{t} w_{tl_ij}p_{l_ij} \qquad \forall i\in(N_1 \text{ or } N_2), \forall l_i\in L_i, \forall j\in J_{l_i}, \forall(t,n) \quad (36)$$

$$s_{tiq} \ge s_{(t-1)iq} \qquad \forall i\in(N_1 \text{ or } N_3), q\in\Xi_i, \forall t=2,\dots,T \quad (37)$$

$$s_{tiq}\in\{0,1\} \qquad \forall i\in(N_1 \text{ or } N_3), q\in\Xi_i, \forall t \quad (38)$$

$$w_{tl_ij} \ge 0 \qquad \forall i\in(N_1 \text{ or } N_2), \forall l_i\in L_i, \forall j\in J_{l_i}, \forall t \quad (39)$$

$$x_{tnik} \ge 0 \qquad \forall i\in(N_1 \text{ or } N_3), k\in\Theta_i, \forall(t,n) \quad (40)$$

$$u_{tniq} \ge 0 \qquad \forall i\in(N_1 \text{ or } N_3), q\in\Xi_i, \forall(t,n) \quad (41)$$

$$y_{tnl_ij} \ge 0, z_{tnl_ij} \ge 0 \qquad \forall i\in(N_1 \text{ or } N_2), \forall l_i\in L_i, \forall j\in J_{l_i}, \forall(t,n) \quad (42)$$

$$\Gamma_{tnimr} \ge 0 \qquad \forall(i,m,r)\in A, \forall(t,n) \quad (43)$$

$$v_{tni} \ge 0 \qquad \forall i\in(N_1 \text{ or } N_2), \forall(t,n) \quad (44)$$

$$\pi_{tnl_i} \ge 0 \qquad \forall i\in(N_1 \text{ or } N_2), \forall l_i\in L_i, \forall(t,n) \quad (45)$$

Γ_{tnimr} is the flow (MW) through r^{th} transmission line between nodes (i, m) for scenario n in time period t. A decision variable ϑ_{tni} is defined to represent the amount of energy sent to load area grid. These decision variables represent the flow on the local transmission system between the power and the area grid in the corresponding node. Θ_i' represents the set of critical existing central units and Θ_i'' represents the set of noncritical existing central units. It is considered that all the expansion options are working at their expected levels. A' is the set of critical transmission lines and A'' is the set of noncritical transmission lines.

Equation 23 is to satisfy energy conservation constraint for the nodes which are both supply and demand points. That is, for each such node the energy transmitted to the node multiplied by loss factor plus electricity generated in the corresponding node minus the energy sent to local transmission system represents the excess energy in the corresponding node. This excess energy is distributed among the available transmission lines proportional to their power distribution factor. Equations 24 to 26 are to satisfy energy conservation for the nodes which are demand points, supply points and neither demand nor supply points respectively.

Equation 27 is for satisfiable demand for each load group. There are L_i load groups in each demand nodes. There are also J_{l_i} numbers of distributed generation technologies available for expansion for each load group. This constraint guarantees that the energy sent trough local transmission system plus energy generated from the distributed generation units located at the load groups plus unmet demand is equal to the satisfiable demand for the corresponding demand node. Equation 28 is for locally satisfiable demand for each load group in each demand node.

Equations 29 and 30 are to satisfy the capacity constraints for each critical transmission lines. These constraints guarantee that the net flow on the corresponding line is smaller or equal to the line capacity in each scenario in each time period. For each scenario, Δ_{tnimr} is equal to the line capacity δ_{imr} if the critical transmission line is working, and it is equal to zero if the line is not working. Equations 31 and 32 are for the capacity constraint for noncritical transmission lines. Here, the available line capacity is equal to the expected line capacity. Therefore, the available line capacity for each scenario in each time period is calculated by multiplying the line capacity (δ_{imr}) by the availability factor of the corresponding transmission line (p_{imr}). Equation 33 represents the available capacity for the critical central units. The generation from the unit cannot exceed the available capacity of the unit. The available capacity for those units is equal to its actual capacity of the units (μ_{ik}) if the corresponding generation unit is working in the particular scenario and it is zero if the unit is not working. Equation 34 represents the available capacity limit for noncritical generation units. For those units, available capacity in each scenario is assumed to be the expected capacity which is calculated by multiplying the actual capacity of the unit (μ_{ik}) by the availability of the unit (p_{ik}). Equation 35 is for the available capacity for the new generation units. In order to generate from new central units at time period *t*, they should be built before or in time period *t*. Therefore, the expected capacities are multiplied by the corresponding decision variables for these generation units. Equation 36 states that total generation from distributed generation units should be smaller or equal to total expected distributed generation available.

Equation 37 is for expansion for each central investment choice. Each investment choice can only be built once over the planning horizon. Equation 38 shows that the expansion decisions for new central units are binary variables. The remaining constraints are for nonnegativity constrains on dispatching decisions and expansion amount of distributed generation technologies.

An example problem is solved and the results are presented in Section 3.5. Benders decomposition is utilized to solve this large scale optimization problem. Therefore, first the procedure of Benders decomposition for multi-objective optimization problems is given in the next section and then the numerical example is demonstrated.

3.4. Benders Decomposition for Multi-Objective Optimization Problems

One effective approach used in the literature to solve GEP problems is applying Benders decomposition. Benders decomposition is mostly utilized to solve least cost GEP problems. In this section, a procedure is explained to impellent Benders decomposition for multi-objective optimization problems.

Before presenting the procedure for multi-objective optimization problems, insights about the benefits of implementing Benders decomposition to solve GEP problems obtained by solving least cost GEP is provide below.

Benders decomposition is implemented to solve the least cost GEP problems. In this problem, the objective is to find the expansion plan which minimizes the investment cost, operational and maintenance cost, unmet demand cost and profit obtained from the heat produced by the units with co-generation capability. A numerical example with 6,000 scenarios has been generated and the GEP problem with 6,000 scenarios was attempted to be solved with and without Benders decomposition using GAMS. A solution is obtained when Benders decomposition is used, but often could not be solved without Benders decomposition because the problem was too large for the computer being used. This analysis reveals that, if Benders decomposition is implemented in a computer with larger capacity, even much larger problems can be solved. The numerical analysis indicates that there are ways to improve the model and solution efficiency. These insights are listed below.

1. The subproblems are independent from each other. This means that they can be solved in parallel. For a numerical example with 6,000 scenarios, the solution is obtained in three iterations. Therefore, if the subproblems can be solved in parallel, the solution time can decrease dramatically.
2. The subproblems are small linear programming problems. The model with more realistic transmission system can be solved efficiently.

3. It is possible to reduce the solution time by initializing the problem with a good starting point. Expert judgment and/or preprocess analyses can be done before solving the expansion problem and at the initialization step; the model can be initialized with a good starting solution.

The objective of this section is to provide the procedure on how to implement Benders decomposition for multi-objective optimization problem. Benders decomposition enables dividing a large problem into two subproblems: master problem and operational subproblem. The master problem includes the expansion decision variables and the operational objective function is represented by a new continuous variable. The operational objective function is approximated by the constraints, called Benders cut, which obtained by solving operational subproblems

Consider that O_1, O_2, O_3, and O_4 are the discounted investment and fixed operational and maintenance cost, discounted operational cost, CO_2 and NO_x emissions respectively. Then, the normalized objectives and weighted sum of these objectives can be stated as follows:

$$\min w_1\left(\frac{(O_1+O_2)-\min(O_1+O_2)}{\max(O_1+O_2)-\min(O_1+O_2)}\right)+w_2\left(\frac{O_3-\min O_3}{\max O_3-\min O_3}\right)+w_3\left(\frac{O_4-\min O_4}{\max O_4-\min O_4}\right)$$

which can be written as;

$$\begin{aligned}\min w_1&\left(\frac{(O_1+O_2)}{\max(O_1+O_2)-\min(O_1+O_2)}\right)-w_1\left(\frac{\min(O_1+O_2)}{\max(O_1+O_2)-\min(O_1+O_2)}\right)\\ +w_2&\left(\frac{O_{33}}{\max O_3-\min O_3}\right)-w_2\left(\frac{\min O_3}{\max O_3-\min O_3}\right)\\ +w_3&\left(\frac{O_4-\min O_4}{\max O_4-\min O_4}\right)-w_3\left(\frac{\min O_4}{\max O_4-\min O_4}\right)\end{aligned}$$

It is known that adding or subtracting a constant to objective function does not change the solution. Therefore, the constants are omitted from the above formulation and the objectives can be rewritten to separate the objectives with respect to the first stage and second stage variables. Assume that $\bar{w}_1 = \left(\dfrac{w_1}{\max(O_1 + O_2) - \min(O_1 + O_2)} \right)$ and $\bar{w}_i = \left(\dfrac{w_i}{\max O_i - \min O_i} \right)$ for i=2, 3. Then, the new objective function is as follows.

$$\min \bar{w}_1 O_1 + \bar{w}_1 O_2 + \bar{w}_3 O_3 + \bar{w}_4 O_4$$

3.4.1. Restricted Investment Master Problem for Multi-Objective Optimization Problem

The Restricted Investment Master Problem (RIMP) for the GEP problem is solved to find the investment decision solutions. RIMP only includes the first stage variables. The new continuous variables are defined to represent the weighted sum of discounted operational cost, CO_2 and NO_x emissions. The constraints included in this problem are related only to investment decisions and Benders cuts which are the linear constraints formed with the dual variables obtained by solving subproblems. These constraints are used to obtain a new and more accurate approximation for the weighted sum of discounted operational cost, CO_2 and NO_x emissions and a new and better investment decision in RIMP in each iteration.

Define the following objective functions:

$$O_{1t} = (1+r)^{-t}\left(\left(\sum_{i\in(N_1 \text{ or } N_3)}\sum_{q\in\Xi_i}(s_{tiq}-s_{(t-1)iq})a_{tiq} + \sum_{i\in(N_1 \text{ or } N_2)}\sum_{l_i\in L_i}\sum_{j\in J_{l_i}} w_{tl_ij}b_{tl_ij}\right)\right.$$

$$\left.+\left(\sum_{i\in(N_1 \text{ or } N_3)}\sum_{k\in\Theta_i} g_{tik} + \sum_{i\in(N_1 \text{ or } N_3)}\sum_{q\in\Xi_i} s_{tiq}h_{tiq} + \sum_{i\in(N_1 \text{ or } N_2)}\sum_{l_i\in L_i}\sum_{j\in J_{l_i}}\sum_{\tau=1}^{t} w_{\tau l_ij}m_{tl_ij}\right)\right)$$

$$O_{2t} = (1+r)^{-t}\left(\sum_{n=1}^{\mathbb{N}}\varpi_n\left(\sum_{i\in(N_1 \text{ or } N_3)}\sum_{k\in\Theta_i} x_{tnik}c_{tik} + \sum_{i\in(N_1 \text{ or } N_3)}\sum_{q\in\Xi_i} u_{tniq}e_{tiq}\right.\right.$$

$$+\sum_{i\in(N_1 \text{ or } N_2)}\sum_{l_i\in L_i}\sum_{j\in J_{l_i}}(y_{tnl_ij}+z_{tnl_ij})d_{tl_ij} + \sum_{i\in(N_1 \text{ or } N_2)} v_{tni}\, f_t$$

$$\left.\left.+\sum_{i\in(N_1 \text{ or } N_2)}\sum_{l_i\in L_i}\pi_{tnl}\, f_t - \sum_{i\in(N_1 \text{ or } N_2)}\sum_{l_i\in L_i}\sum_{j\in J'_{l_i}}(y_{tnl_ij}+z_{tnl_ij})p_t r_t\right)\right)$$

$$O_{3t} = \sum_{n=1}^{\mathbb{N}}\varpi_n\left(\sum_{i\in(N_1 \text{ or } N_3)}\sum_{k\in\Theta_i}^{\mathbb{N}} x_{tnik}C_{ik} + \sum_{i\in(N_1 \text{ or } N_3)}\sum_{q\in\Xi_i} u_{tniq}E_{iq} + \sum_{i\in(N_1 \text{ or } N_2)}\sum_{l_i\in L_i}\sum_{j\in J_{l_i}}(y_{tnl_ij}+z_{tnl_ij})D_{l_ij}\right)$$

$$O_{4t} = \sum_{n=1}^{\mathbb{N}}\varpi_n\left(\sum_{i\in(N_1 \text{ or } N_3)}\sum_{k\in\Theta_i}^{\mathbb{N}} x_{tnik}F_{ik} + \sum_{i\in(N_1 \text{ or } N_3)}\sum_{q\in\Xi_i} u_{tniq}G_{iq} + \sum_{i\in(N_1 \text{ or } N_2)}\sum_{l_i\in L_i}\sum_{j\in J_{l_i}}(y_{tnl_ij}+z_{tnl_ij})H_{l_ij}\right)$$

Then the weighted objective function for GEP problems is as follows:

$$\min \bar{w}_1\sum_{t=1}^{T}O_{1t} + \bar{w}_1\sum_{t=1}^{T}O_{2t} + \bar{w}_3\sum_{t=1}^{T}O_{3t} + \bar{w}_4\sum_{t=1}^{T}O_{4t} = \min\sum_{t=1}^{T}\bar{w}_1O_{1t} + \sum_{t=1}^{T}\left(\bar{w}_1O_{2t} + \bar{w}_3O_{3t} + \bar{w}_4O_{4t}\right)$$

A new continuous variable μ_t is defined which represents the weighted sum of operational cost, CO_2 and NO_x emissions for year t, $\mu_t = \left(\bar{w}_1O_{2t} + \bar{w}_3O_{3t} + \bar{w}_4O_{4t}\right)$. Then, RIMP at iteration P can be demonstrated as follows:

$$\min \sum_{t=1}^{T} \overline{w}_1 O_{1t} + \sum_{t=1}^{T} \mu_t$$

s.t.

$$s_{tiq} \geq s_{(t-1)iq} \qquad \forall i \in (N_1 \text{ or } N_3), q \in \Xi_i, \forall t = 2,...,T$$

$$s_{tiq} \in \{0,1\} \qquad \forall i \in (N_1 \text{ or } N_3), q \in \Xi_i, \forall t$$

$$w_{tl_i j} \geq 0 \qquad \forall i \in (N_1 \text{ or } N_2), \forall l_i \in L_i, \forall j \in J_{l_i}, \forall (t,n)$$

$$\mu_t \geq \sum_{n=1}^{\mathbb{N}} \left(\sum_{\forall i \in (N_1 \text{ or } N_2)} \Psi_{ni} A_{tni}^p + \sum_{\forall i \in (N_1 \text{ or } N_2)} \sum_{\forall l_i \in L_i} \Phi_{tnl_i} B_{tnl_i}^p + \sum_{(i,m,r) \in A'} \Delta_{tnimr} (C_{tnimr}^p + D_{tnimr}^p) \right.$$
$$+ \sum_{(i,m,r) \in A''} \delta_{imr} p_{imr} (E_{timr}^p + F_{timr}^p) + \sum_{\forall i \in (N_1 \text{ or } N_3)} \sum_{k \in \Theta_i^{'}} \gamma_{tnik} G_{tnik}^p + \sum_{\forall i \in (N_1 \text{ or } N_3)} \sum_{k \in \Theta_i^{''}} \mu_{ik} p_{ik} H_{tnik}^p$$
$$\left. + \sum_{\forall i \in (N_1 \text{ or } N_3)} \sum_{q \in \Xi_i} \mu_{ik} p_{ik} s_{tiq} M_{tniq}^p + \sum_{\forall i \in (N_1 \text{ or } N_2)} \sum_{\forall l_i \in L_i} \sum_{\forall j \in J_{l_i}} \sum_{\tau=0}^{t} w_{\tau l_i j} p_{l_i j} R_{tnl_i j}^p \right) \qquad \forall t, \forall p = 1,..,P-1$$

where

A, *B*, *C*, *D*, *E*, *F*, *G*, *H*, *M*, and *R* are the dual variables associated with the satisfiable demand constraints, locally satisfiable demand constraints, transmission capacity constraints, and generation capacity constraints respectively.

3.4.2. Operational Subproblems for Multi-Objective Optimization Problem

Operational subproblems are the dispatching problems given the expansion decisions. These subproblems are independent from each other. That is, the solution of one does not affect the other one.

Consider that the optimal solutions obtained by solving the RIMP at iteration *p* are $\overline{s}_{tiq}$, $\overline{w}_{tl_i j}$ and $\overline{\mu}_t$. For a given *t* and $\overline{s}_{tiq}$ and $\overline{w}_{tl_i j}$, the subproblem at iteration *p* can be demonstrated as follows. Since *t* is given, the index for it can be omitted in model.

$$\min\left(\bar{w}_1 O_{2t} + \bar{w}_3 O_{3t} + \bar{w}_4 O_{4t}\right)$$

s.t.

$$\left[\sum_{m,r} \Gamma_{nmir}\alpha_{mir} + \sum_{k\in\Theta_i} x_{nik} + \sum_{q\in\Xi_i} u_{niq} - \vartheta_{ni}\right]\beta_{nimr} = \Gamma_{nimr} \qquad \forall i \in N_1, \forall(i,m,r)\in A, \forall n$$

$$\left[\sum_{m,r} \Gamma_{nmir}\alpha_{mir} - \vartheta_{ni}\right]\beta_{nimr} = \Gamma_{nimr} \qquad \forall i \in N_2, \forall(i,m,r)\in A, \forall n$$

$$\left[\sum_{m,r} \Gamma_{nmir}\alpha_{mir} + \sum_{k\in\Theta_i} x_{nik} + \sum_{q\in\Xi_i} u_{niq}\right]\beta_{nimr} = \Gamma_{nimr} \qquad \forall i \in N_3, \forall(i,m,r)\in A, \forall n$$

$$\left[\sum_{m,r} \Gamma_{nmir}\alpha_{mir}\right]\beta_{nimr} = \Gamma_{nimr} \qquad \forall i \in N_4, \forall(i,m,r)\in A, \forall n$$

$$\vartheta_{ni} + \sum_{l_i\in L_i}\sum_{j\in J_{l_i}} y_{nl_ij} + v_{ni} = \Psi_{ni} \qquad \forall i \in (N_1 \text{ or } N_2), \forall n$$

$$\sum_{j\in J_{l_i}} z_{nl_ij} + \pi_{nl_i} = \Phi_{nl_i} \qquad \forall i \in (N_1 \text{ or } N_2), \forall l_i \in L_i, \forall n$$

$$\Gamma_{nimr} - \Gamma_{nmir} \le \Delta_{nimr} \qquad \forall(i,m,r)\in A', \forall n$$

$$\Gamma_{nmir} - \Gamma_{nimr} \le \Delta_{nimr} \qquad \forall(i,m,r)\in A', \forall n$$

$$\Gamma_{nimr} - \Gamma_{nmir} \le \delta_{imr} p_{imr} \qquad \forall(i,m,r)\in A'', \forall n$$

$$\Gamma_{nmir} - \Gamma_{nimr} \le \delta_{imr} p_{imr} \qquad \forall(i,m,r)\in A'', \forall n$$

$$x_{nik} \le \gamma_{nik} \qquad \forall i \in (N_1 \text{ or } N_3), k \in \Theta_i', \forall n$$

$$x_{nik} \le \mu_{ik} p_{ik} \qquad \forall i \in (N_1 \text{ or } N_3), k \in \Theta_i'', \forall n$$

$$u_{niq} \le \mu_{iq} p_{iq} \bar{s}_{iq} \qquad \forall i \in (N_1 \text{ or } N_3), q \in \Xi_i, \forall n$$

$$y_{nl_ij} + z_{nl_ij} \le \sum_{\tau=0}^{t} \bar{w}_{\tau l_i j} p_{l_i j} \qquad \forall i \in (N_1 \text{ or } N_2), \forall l_i \in L_i, \forall j \in J_{l_i}, \forall n$$

$$x_{nik} \ge 0 \qquad \forall i \in (N_1 \text{ or } N_3), k \in \Theta_i, \forall n$$

$$u_{niq} \ge 0 \qquad \forall i \in (N_1 \text{ or } N_3), q \in \Xi_i, \forall n$$

$$y_{nl_ij} \ge 0, z_{nl_ij} \ge 0 \qquad \forall i \in (N_1 \text{ or } N_2), \forall l_i \in L_i, \forall j \in J_{l_i}, \forall n$$

$$\Gamma_{nimr} \ge 0 \qquad \forall(i,m,r)\in A, \forall n$$

$$\vartheta_{ni}, v_{ni} \ge 0 \qquad \forall i \in (N_1 \text{ or } N_2), \forall n$$

$$\pi_{nl_i} \ge 0 \qquad \forall i \in (N_1 \text{ or } N_2), \forall l_i \in L_i, \forall n$$

The primal solution of the subproblem for t are $\bar{\Gamma}^p_{nimr}, \bar{x}^p_{nik}, \bar{u}^p_{niq}, \bar{\vartheta}^p_{ni}, \bar{v}^p_{ni}, \bar{y}^p_{nl_i j}, \bar{z}^p_{nl_i j}$, and $\bar{\pi}^p_{nl_i}$ which can also be written as $\bar{\Gamma}^p_{tnimr}, \bar{x}^p_{tnik}, \bar{u}^p_{tniq}, \bar{\vartheta}^p_{tni}, \bar{v}^p_{tni}, \bar{y}^p_{tnl_i j}, \bar{z}^p_{tnl_i j}$, and $\bar{\pi}^p_{tnl_i}$.

The dual solutions of the subproblem for t at iteration p are:

$A^p_{nimr}, B^p_{nimr}, C^p_{nimr}, D^p_{nimr}, E^p_{ni}, F^p_{nl_i}, G^p_{nimr}, H^p_{nimr}, M^p_{nik}$, and $R^p_{nl_i j}$,

which can also be written as ;

$A^p_{tnimr}, B^p_{tnimr}, C^p_{tnimr}, D^p_{tnimr}, E^p_{tni}, F^p_{tnl_i}, G^p_{tnimr}, H^p_{tnimr}, M^p_{tnik}$, and $R^p_{tnl_i j}$.

At each iteration, the subproblems for every t are solved.

Consider $\bar{f}_t^{\,p}$ is equal to:

$$\begin{aligned}\bar{f}_t^{\,p} = \sum_{n=1}^{\mathbb{N}} \Bigg(& \sum_{\forall i \in (N_1 \text{ or } N_2)} \Psi_{ni} A^p_{ni} + \sum_{\forall i \in (N_1 \text{ or } N_2)} \sum_{\forall l_i \in L_i} \Phi_{tnl_i} B^p_{tnl_i} + \sum_{(i,m,r)\in A'} \Delta_{tnimr} (C^p_{tnimr} + D^p_{tnimr}) \\ & + \sum_{(i,m,r)\in A''} \delta_{imr} p_{imr} (E^p_{timr} + F^p_{timr}) + \sum_{\forall i \in (N_1 \text{ or } N_3)} \sum_{k \in \Theta'_i} \gamma_{tnik} G^p_{tnik} + \sum_{\forall i \in (N_1 \text{ or } N_3)} \sum_{k \in \Theta''_i} \mu_{ik} p_{ik} H^p_{tnik} \\ & + \sum_{\forall i \in (N_1 \text{ or } N_3)} \sum_{q \in \Xi_i} \mu_{ik} p_{ik} \bar{s}_{tiq} M^p_{tniq} + \sum_{\forall i \in (N_1 \text{ or } N_2)} \sum_{\forall l_i \in L_i} \sum_{\forall j \in J_{l_i}} \sum_{\tau=0}^{t} \bar{w}_{\tau l_i j} p_{l_i j} R^p_{tnl_i j} \Bigg)\end{aligned}$$

If $\bar{\mu}_t^{\,p} \geq \bar{f}_t^{\,p}$ for every t, then the optimal solution is found, and $\bar{s}_{tiq}$, $\bar{w}_{tl_i j}$, $\bar{\Gamma}^p_{tnimr}, \bar{x}^p_{tnik}, \bar{u}^p_{tniq}, \bar{\vartheta}^p_{tni}, \bar{v}^p_{tni}, \bar{y}^p_{tnl_i j}, \bar{z}^p_{tnl_i j}$, and $\bar{\pi}^p_{tnl_i}$ are the optimal solutions. If not, then construct a Benders cut with dual variables obtained in iteration p and add to the restricted master problem and continue the procedure.

It is also possible to form lower and upper bound for the objective function and terminate the procedure based on some criterion related to them.

3.4.3. Benders Procedure for Multi-Objective Optimization Problem

In this section, a general description for the procedure is provided.

***Step*1:** Initialization

- Assign $\bar{s}_{tiq}{}^{0} = 0, \forall i \in (N_1 \text{ or } N_3), q \in \Xi_i, \forall t$, and

  $\bar{w}_{tlj}{}^{0} = 0, \forall i \in (N_1 \text{ or } N_2), \forall l_i \in L_i, \forall j \in J_{l_i}, \forall (t,n)$

- Solve subproblem for every t

- Construct a Benders cut and add to RIMP

- p=0

Step* 2*:

- p=p+1

- Solve RIMP to obtain $\bar{s}_{tiq}{}^{p}, \bar{w}_{tl_i j}{}^{p}$ and $\bar{\mu}_t{}^{p}$

- For every t solve the subproblem to obtain $\bar{\Gamma}^{p}_{tnimr}, \bar{x}^{p}_{tnik}, \bar{u}^{p}_{tniq}, \bar{\vartheta}^{p}_{tni}, \bar{v}^{p}_{tni}, \bar{y}^{p}_{tnl_i j}, \bar{z}^{p}_{tnl_i j}, \bar{\pi}^{p}_{tnl_i}$

  and $\bar{f}_t{}^{p}$

***Step* 3:**

- If $\bar{\mu}_t{}^{p} \geq \bar{f}_t{}^{p}$ for every t, then the optimal solution is found, and $\bar{s}_{tiq}{}^{p}$, $\bar{w}_{tl_i j}{}^{p}$,

  $\bar{\Gamma}^{p}_{tnimr}, \bar{x}^{p}_{tnik}, \bar{u}^{p}_{tniq}, \bar{\vartheta}^{p}_{tni}, \bar{v}^{p}_{tni}, \bar{y}^{p}_{tnl_i j}, \bar{z}^{p}_{tnl_i j}$ and $\bar{\pi}^{p}_{tnl_i}$ are optimal solutions.

- Otherwise, construct a Benders cut as

$$\mu_t \geq \sum_{n=1}^{\mathbb{N}} \Bigg(\sum_{\forall i \in (N_1 \text{ or } N_2)} \Psi_{ni} A_{tni}^{p} + \sum_{\forall i \in (N_1 \text{ or } N_2)} \sum_{\forall l_i \in L_i} \Phi_{tnl_i} B_{tnl_i}^{p} + \sum_{(i,m,r) \in A'} \Delta_{tnimr} (C_{tnimr}^{p} + D_{tnimr}^{p})$$
$$+ \sum_{(i,m,r) \in A''} \delta_{imr} p_{imr} (E_{timr}^{p} + F_{timr}^{p}) + \sum_{\forall i \in (N_1 \text{ or } N_3)} \sum_{k \in \Theta_i^{'}} \gamma_{tnik} G_{tnik}^{p} + \sum_{\forall i \in (N_1 \text{ or } N_3)} \sum_{k \in \Theta_i^{''}} \mu_{ik} p_{ik} H_{tnik}^{p}$$
$$+ \sum_{\forall i \in (N_1 \text{ or } N_3)} \sum_{q \in \Xi_i} \mu_{ik} p_{ik} s_{tiq} M_{tniq}^{p} + \sum_{\forall i \in (N_1 \text{ or } N_2)} \sum_{\forall l_i \in L_i} \sum_{\forall j \in J_{l_i}} \sum_{\tau=0}^{t} w_{\tau l_i j} p_{l_i j} R_{tnl_i j}^{p} \Bigg)$$

and add to RIMP and go to *Step* 2.

3.4.4. Development of Parallel Solution Technique for Benders Decomposition

Throughout the dissertation, the model has been solved by using GAMS. GAMS has a grid computing facility which provides solving the subproblems in parallel. GAMS carries out three operations to solve a problem [86]: (*i*) problem generation, (*ii*) problem solution and (*iii*) update in GAMS data base. Grid computing facility enables generate problems without waiting the solution of the previous one. Therefore, two types of loop are defined in the GAMS code: (*i*) submission loop and (*ii*) collection loop [86]. In the submission loop, models are generated and submitted for solutions and in the collection loop the solutions of the models which are submitted previously are collected. In most problems, generation of the model takes shorter time compare to solution times. Since multiple models are solved in parallel, the execution time is expected to be shorter. A GAMS code was written for parallel solution and used to solve the problem described in Section 3.5. The code is implemented on a single computer, but based on [86], it does not need any changes if a massive grid network is used such as Condor.

3.5. Numerical Example for Multi-Period Multi-Objective GEP with more Realistic Transmission System Representation

To demonstrate how the Benders decomposition can be used to solve the multi-objective optimization problems, an example problem is solved for a 15 year planning horizon. The existing power system topology is the modified IEEE Reliability Test System and the model is GEP problem with more realistic transmission system representation presented in Section 3.3.

Scenario optimization based approach was before implemented for the modified IEEE Reliability System. In this section, the availability scenarios generated in the Section 3.2.2 are used. There are three demand nodes where distributed generation units can be located, namely these demand nodes are 10, 14 and 19. The load blocks in these demand nodes have the similar characteristics; therefore, they can be represented by one load group in each demand node. The unavailability of the distribution lines are considered as 0.01. Then, the distribution system scenarios for these four demand node is found by implementing the procedure described in Step 3 in Section 3.2.2. The distribution system scenarios and the corresponding probabilities are given in Table 3.15.

Table 3.15. Distribution system scenarios selected for numerical example

Selected Scenario Numbers	Scenario Numbers	Demand Node where DU available			Probabilities
		10	14	19	
1	1	1	1	1	0.0648
2	4	1			0.1252
3	6		1		0.1252
4	7			1	0.1152
5	8				0.5696

Furthermore, the load duration curve is divided into four segments; peak demand, high demand, medium demand and low demand. The corresponding segments and their probabilities are given in Table 3.16.

Table 3.16. Demand segments and the corresponding probabilities for numerical example

Segments	Percentage of peak load demand	Probability	Segments	Percentage of peak load	Probability
1.00	1.00	0.01	(0.60,0.90)	0.75	0.5
(0.90, 1.00)	0.95	0.03	(0.33,0.60)	0.46	0.46

The procedure to reduce the number of availability-distribution system scenario is implemented which is described as Step 5 in Section 3.2.2. The selected scenarios and their probabilities are given in Table 3.17. The total number of scenarios considered in each year is equal to 220, which is calculated as $((16\times5\times2)+(6\times5\times2))$. This is very small number compare to $2^{15}\times2^{3}\times4$. Therefore, the optimization is based on a total of 3,300 different scenarios.

Table 3.17. Availability scenarios selected for each demand segment for the numerical example

Segments	Scenario numbers and probabilities
1.00	**1** (0.65); **2** (0.003); **3** (0.008); **4** (0.005); **5** (0.096); **6** (0.04); **7** (0.004); **8** (0.004); **9** (0.022); **10** (0.018); **11** (0.047); **12** (0.003); **13** (0.05); **14** (0.004); **15** (0.024); **16** (0.022)
(0.90, 1.00)	**1** (0.65); **2** (0.003); **3** (0.008); **4** (0.005); **5** (0.096); **6** (0.04); **7** (0.004); **8** (0.004); **9** (0.022); **10** (0.018); **11** (0.047); **12** (0.003); **13** (0.05); **14** (0.004); **15** (0.024); **16** (0.022)
(0.60,0.90)	**1** (0.972); **2** (0.003); **3** (0.008); **4** (0.005); **7** (0.004); **8** (0.004); **14** (0.004);
(0.33,0.60)	**1** (0.972); **2** (0.003); **3** (0.008); **4** (0.005); **7** (0.004); **8** (0.004); **14** (0.004);

Internal combustion (IC) engines are considered with availability of 0.953 as distributed generation units. Table 3.2 lists the cost characteristics, gas emissions and unavailabilities for the distributed generation units. The cost listed in Table 3.2 is for the generation units with the capacity of 12.5MW. Since the investment for distributed generation is defined as a continuous variable, the cost information per MW is calculated based on the cost parameter given in Table 3.2.

As expansion options for centralized units, wind turbines, nuclear and combined cycle gas turbines (CCGT) are considered. The cost characteristics, gas emissions and unavailabilities are also provided in Table 3.2.

The peak load demand for each demand node is given in Table 3.9 and it is considered that demand increases 1% in each year. The cost of not satisfying demand is 10,000 \$/MW. It is also considered that 50% of energy produced by distributed generation units can be used to gain benefits from the steam, and in the model, the profit per MW by using steam is approximately 60% of energy generation cost from IC, i.e., 15.91 \$/MW.

In order to find a Pareto front, 16 different weight combinations (cases) are used and they are presented in Table 3.18.

Table 3.18. Weight Combinations used for numerical example

Cases	Cost	CO_2	NO_x	Cases	Cost	CO_2	NO_x
	w_1	w_2	w_3		w_1	w_2	w_3
1	1	0	0	9	0.7	0.15	0.15
2	0.9	0.1	0	10	0.7	0	0.3
3	0.9	0.05	0.05	11	0.6	0.4	0
4	0.9	0	0.1	12	0.6	0.2	0.2
5	0.8	0.2	0	13	0.6	0	0.4
6	0.8	0.1	0.1	14	0.5	0.5	0
7	0.8	0	0.2	15	0.5	0.25	0.25
8	0.7	0.3	0	16	0.5	0	0.5

The objective function values for Pareto front solutions are presented in Table 3.19. Although SO_2 emission is not as a part of the optimization model, the SO_2 emission level for each solution is calculated. Decision makers can choose the solution that is the most appropriate given their preferences. This analysis is providing the trade-off solution between each objective function. Figure 3.10 presents the Pareto front for the numerical example.

Table 3.19. Objective function values for the numerical example

Cases	Cost (billions $)	CO_2 (thousands of tons)	NO_x (thousands of tons)	SO_2 (thousands of tons)
1	16.71	154,408.23	350.42	944.74
2	17.33	121,060.36	310.60	493.73
3	17.28	134,243.19	213.67	651.98
4	17.41	137,873.52	200.24	630.78
5	17.64	115,729.78	297.43	379.78
6	17.39	133,628.47	203.15	613.14
7	17.69	136,946.53	186.67	573.75
8	21.62	79,520.54	219.48	173.63
9	21.61	89,912.61	112.08	263.07
10	18.75	130,616.70	160.73	371.15
11	22.54	74,493.31	215.11	155.07
12	22.11	87,265.49	105.53	242.57
13	22.12	91,479.98	101.38	228.53
14	22.88	72,986.69	211.66	141.60
15	23.12	83,252.66	94.66	201.09
16	22.78	88,072.70	95.42	213.86

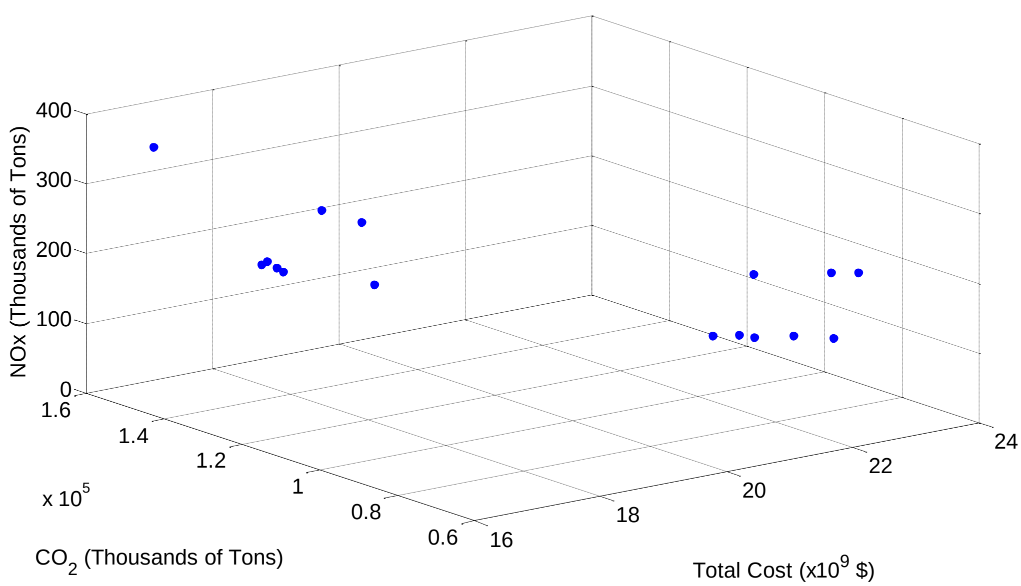

Figure 3.10. A Pareto Front for the numerical example

The expansion plans for the distributed and the central generation units are also presented. Table 3.20 presents the expansion plan for the distributed generation units. Distributed generation units are built heavily, when the cost is very important and energy generated from the distributed generation units are used both locally satisfiable demand and demand for the corresponding demand node. There are two reasons; since the distributed generation units are located closer to the demand point; 1) the energy losses due to the transmission of energy is smaller; 2) unmet demand due to the transmission line unavailability is smaller. When the objective function includes the NO_x, distributed generation units are built only as much as to satisfy locally satisfiable demand since IC engines have relatively high NO_x emissions.

Table 3.20. Expansion plan for distributed generation units for different weight combinations

Cases	Demand Nodes		
	10	14	19
1	219.05	204.00	166.91
2	171.21	150.34	123.73
3	142.55	124.62	106.66
4	143.75	123.84	106.89
5	174.65	127.05	117.99
6	142.91	121.59	105.44
7	119.70	110.75	96.09
8	166.18	138.54	141.57
9	2.26	65.82	43.08
10	59.41	96.69	85.10
11	171.21	149.59	144.64
12	2.24	65.27	40.87
13	2.24	46.94	33.42
14	174.66	158.97	163.73
15	2.23	59.31	45.46
16	2.24	50.16	31.40

Table 3.21 presents the expansion plan for the central generation units. In this table C, W and N represent the CCGT, wind turbines and nuclear plants respectively. The numbers in the table shows in which year the corresponding technology is built for each power

group. For the first weight combination, the objective is only to minimize the cost, three CCGTs are built. In power group 2 and 18, the CCGTs are built in the first year and in power group 1, the CCGT is built in year 9. When the gas emissions are introduced to the objective function, more CCGTs are built to reduce the production from the technologies which have larger gas emissions such as coal burning units and Oil/CT. When the importance of the cost is 0.7 and the objective function includes CO_2 emissions, the wind turbines are introduced to the system. Further decrease in the importance of the cost results in the introduction of the nuclear plant into the system. As you see from the Table 3.21, when CO_2 is a part of the objective system, more wind turbines are built in the earlier years of the planning horizon.

Table 3.21. Expansion plan for central generation units for different weight combinations

Cases	Power Nodes														
	1			2			7			13			15		
	C	W	N	C	W	N	C	W	N	C	W	N	C	W	N
1	9			1											
2	4			1			1			1					
3	1			1			1			1					
4	4			1			1			1					
5	1			1			1			1					
6	1			1			1			1					
7	1			1			1			1			1		
8	1			1	1		1			1					
9	1			1	1		1			1			1		
10	1			1	5		1			1			1		
11	1	1		1	1		1	1		1	1				
12	1			1	1		1			1			1		
13	1			1	1		1			1			1		
14	1	1		1	1		1	1		1	1	1			
15	1	1		1	1		1	1		1	1		1		
16	1			1	1		1			1	1		1		

Cases	16			18			21			22			23		
	C	W	N	C	W	N	C	W	N	C	W	N	C	W	N
1				1											
2				6									1		
3	1			7									1		
4	1			7									1		
5	1			8									1		
6	1			6									1		
7	1			6									1		
8				8									1		
9	1			7									1		
10	1			1			1			1			1		
11				7									1	1	1
12	1	1		2									1	1	1
13	1			1			1			1			1	1	
14	1	1											1	1	
15	1	1		6			1			1			1	1	1
16	1	1		1			1			1			1	1	1

4. GEP with Smart Grid Technologies

There are technological developments, called as Smart Grid technologies, which can be implemented and used within the power grid to manage the grid more efficiently and conserve energy usage. The GEP model has been extended to incorporate the usage of Smart Grid technologies to determine their optimal implementation planning and to observe how Smart Grid technologies can impact other expansion and dispatching decisions.

In the models presented in the previous chapter, distributed generation units are introduced into the model as one example of a Smart Grid technology. However, there are more technological developments than just distributed generation units. Smart Grid technologies are the collection of many types of technologies such as demand side management tools, advanced meters, advanced control devices and so on. In this chapter; the GEP model is extended to reflect effects of Smart Grid technologies on the availability of the components, the demand and the transmission losses. Investment decision variables, with associated costs are introduced into the GEP models. It is possible for Smart Grid technologies have additive effects, but, it is also possible that combinations of feasible or likely Smart Grid technologies affect the system more than just the summation of their individual effects. Decision variables are defined to indicate whether a particular combination of Smart Grid technology is selected. These are called state variables, and constraints are added to the model to incorporate the impact of selected combinations of Smart Grid technologies.

In the following sections, the models have been developed how to incorporate Smart Grid technologies into the GEP model for both single and multi-objective optimization problems. Smart Grid technologies are divided into three categories based on their impacts on the power grid and in each section the model is presented to incorporate each category of technologies into the generation expansion planning optimization model. In Section 4.1, Smart Grid technologies which increase the effective availability of the system components are considered. The new formulation of the GEP model to incorporate this type of technologies is presented. In Section 4.2, Smart Grid technologies which shift or reduce the energy demand are considered and in Section 4.3, Smart Grid technologies which reduce the energy loss during transmission are considered. Additionally, for each section example problems which have different levels of Smart Grid impacts are solved for different cost levels of Smart Grid technologies to observe how the expansion plan changes with respect to the level of impact and associated cost. Finally, in Section 4.4, a summary of how these technologies are affecting the expansion plan of the generation units and the dispatching decision is given.

4.1. GEP with Smart Grid Technologies Affecting the Component Availability

The first category of Smart Grid technologies considered is those that impact and improve the effective availability of the system components. This type of Smart Grid technologies includes remote monitoring and control devices throughout the system, so that, the system components can be monitored in real time and preventive maintenance can be done based on the condition of the components. Moreover, since Smart Grid technologies also include devices for remote and automated disconnections and reconnections, so that the service time can be decreased. All of these effects can be represented in terms of the availability of the system components. In this section, a model has been developed to incorporate the effects on the availability of the components.

In the GEP model presented in the previous Chapter, the operational cost (or gas emissions) is calculated by multiplying the cost (emission) of each scenario with their corresponding adjustment factors (ϖ_n) and summing over all the scenarios. Therefore, if a technology (or a group of technologies) which affects the availability of the component is selected, then this impact can be incorporated into the model by updating the adjustment factors accordingly. A two step method was developed to incorporate the Smart Grid technologies affecting the availability of the system components into GEP problem.

Consider that $\mathbb{N}$ number of scenarios are generated with their associated probabilities $P(n)$. Then, the adjustment factor is calculated by multiplying the probability of the scenario and the total number of hours in the period. The probability of the scenario consists of three components; the probability of the availability scenario, the probability of the distribution system scenario and the probability of the demand level. Since the Smart Grid technologies affecting the component availabilities are considered, these technologies affect only the probability of the availability scenario and the probability of the distribution system scenario. In the first step, the new probabilities associated with each combination of Smart Grid technologies are obtained and the adjustment factors are updated. In the second step, mathematical model is modified to incorporate these technologies.

Step 1: Pre-Processing

Consider that there are H number of technologies (projects/groups of technologies) affecting component availability. First, the availability scenarios and distribution system scenarios are generated considering there are no Smart Grid technologies in the system. Represent these sets as A^0 (availability scenarios) and B^0 (distribution system scenarios), respectively. For every combination of the Smart Grid technologies, the availability of the system component c is calculated for when the s^{th} combination of Smart Grid technologies is implemented, which is represented by (p_c^s). The following problem is solved to find availability scenario probabilities (π_j^s) when the s^{th} combination of Smart Grid technologies is implemented.

$$\min \sum_{c=1}^{R} \left(s_c + e_c \right)$$

s.t.

$$\sum_{j \in A^0} \pi_j^s a(c, j) + s_c - e_c = p_c^s \qquad \forall c$$

$$\sum_{j \in A^0} \pi_j^s = 1, \pi_j^s \geq 0 \qquad \forall j, s_c, e_c, g_c, x_c \geq 0 \qquad \forall c$$

The intent of this model is to assign new probabilities to the availability scenarios selected considering that there are no Smart Grid technologies in the system. The availability scenarios with the new probabilities collectively minimize the deviation from the first moment of the component availabilities which are adjusted based on the impact of the s^{th} combination of the Smart Grid Technologies.

The next step is to calculate the unavailability of the distribution lines of the load group g when the s^{th} combination of Smart Grid technologies is implemented (α_g^s). The expected number of failed distribution lines for each load group when the s^{th} combination of Smart

Grid technologies is implemented (D_g^s), is calculated by using α_g^s. Then, the following problem is solved to find distribution system scenario probabilities (η_j^s) when the s^{th} combination of Smart Grid technologies is implemented. This optimization model assigns probabilities to the distribution system scenarios, so that the scenarios with the new probabilities collectively minimize the deviation from the first moment of the expected number of failed distribution lines for each load group.

$$\begin{aligned}
&\min \sum_{g=1}^{G} (x_g + y_g) \\
&\text{s.t.} \\
&\quad \sum_{j \in B^0} \eta_j^s b(g,j) + x_g - y_g = D_j^s \qquad \forall g \\
&\quad \sum_{j \in B^0} \eta_j^s = 1, \eta_j^s \geq 0 \qquad \forall j \\
&\quad x_g, y_g \geq 0 \qquad \forall g
\end{aligned}$$

These new probabilities for availability scenarios and the distribution system scenarios can be used to calculate new probabilities for the scenarios. At the end of this step, $\mathbb{N}$ number of scenarios can be used with their new corresponding probabilities $P_s(n)$, which is the probability of the scenario n when the s^{th} combination of the Smart Grid technologies is implemented. Therefore, modified adjustment factors (ϖ_{sn}) can be obtained for each scenario n and each combination s, $s \in S$, where S is the set of all combinations.

Step 2: Modifying the Mathematical Model

The second step is to provide modifications required on the GEP to incorporate the Smart Grid technologies affecting the component availability into the model. Define a new variable $z(t, s)$ as the operational cost in year t in the case when all the technologies in the combination s (denoting this set as a_s) is in operation in year t. Define $O(t,n)$ as the operational cost for scenario n in year t, then $z(t, s)$ can be stated as follows:

$$z(t,s)=\sum_{n=1}^{\mathbb{N}}\varpi_{sn}O(t,n)$$

The calculation of investment and fixed operational and maintenance cost remains the same. Consider that $I(t)$ is the total investment and fixed operational and maintenance cost in year t.

Two types of new binary variables are now defined; (*i*) the investment decision (w_{th}) for Smart Grid technology h in year t, (*ii*) decision variables representing the combination of Smart Grid technologies which is in operation in year t, b_{ts}.

As an example, consider that there are only two Smart Grid technology options, then, H={1, 2} and S = {1, 2, 3, 4} where '1' means no Smart Grid technologies are in operation, '2' means only first Smart Grid technology is in operation, '3' means only second Smart Grid technology is in operation, and '4' means first and second Smart Grid technologies are in operation. Therefore, there are two investment decision variables g_{t1} and g_{t2} and four state variables b_{t1}, b_{t2}, b_{t3}, and b_{t4}.

The modified mathematical model is as follows. The constraints which stay the same are not explicitly demonstrated here.

$$\min z = \sum_{t=1}^{T}(1+r)^{-t}\left(I(t) + \sum_{h=1}^{H}(w_{th} - w_{(t-1)h})m_{th} + \sum_{h=1}^{H} w_{th}r_{th} + \sum_{s=1}^{S} z(t,s) \right)$$

s.t.

$$z(t,s) \geq \sum_{n=1}^{\mathbb{N}} \varpi_{sn} O(t,n) - M \times \sum_{\tau \in S, \tau \neq s} b_{t\tau} \qquad \forall t,s \tag{46}$$

$$b_{ts} \geq \left(\sum_{h \in a_S} w_{th} - (\lVert a_s \rVert - 1) \right) - \sum_{h \notin a_S} w_{th} \qquad \forall t,s \tag{47}$$

$$\sum_{s=1}^{S} b_{ts} = 1 \qquad \forall t \tag{48}$$

$$w_{th} \geq w_{t-1,h} \qquad \forall t = 2,..,T,h \tag{49}$$

$$w_{th} \in \{0,1\} \qquad \forall t,h, \qquad b_{ts} \in \{0,1\} \qquad \forall t,s \tag{50}$$

In this model, m_{th} and r_{th} are the capital and fixed operational and maintenance cost of the Smart Grid technology h in year t. The M is a very big number, and therefore, Equation 46 forces operational cost to be equal to the cost that would occur when the combination s is in operation. Equation 47 forces the state variables s to be equal to one when all the technologies in the combination s are in operation. Equation 48 makes sure that in each year, the system is only in one state. Equation 49 makes sure that once the Smart Grid technology is in operation, it is in operation for the rest of the planning horizon.

In the model presented above, all the components are assumed to be critical ones. For larger systems availability scenarios are only generated for critical components (Section 3.2.2). Therefore, the modification presented above is effective only for those components. However, Smart Grid technologies can improve the availability of the noncritical components too. For those components, it is assumed that they are working at their expected level. The following constraints should be included into the model to incorporate the change in the expected capacity of the noncritical components. The expected capacity of noncritical components is calculated by multiplying the full capacity of the component by its availability. Therefore, for each noncritical component k, the availability p_k is replaced by $\left(p_k + \sum_{s=1}^{S} p_k f_{ks} b_{ts} \right)$ where f_{ks} is the additional impact of the s^{th} combination on the availability of the component k, and the constraints are updated by using this new availability to force the model to calculate the effective capacity by considering the impacts of Smart Grid technologies.

This modification can also be applied to the multi-objective optimization problem. In that case, three variables should be defined: a variable for operational cost $z_1(t, s)$, a variable for CO_2 emissions $z_2(t, s)$, and a variable for NO_x emissions $z_3(t, s)$, for each year t and s. Define $O_1(t,n)$, $O_2(t,n)$ and $O_3(t,n)$ as the operational cost, CO_2 emissions and NO_x emissions for scenario n in year t, respectively. Then, $z_i(t, s)$ for i=1,2,3 can be stated as follows:

$$
\begin{aligned}
z_1(t,s) &= \sum_{n=1}^{\mathbb{N}} \varpi_{sn} O_1(t,n) \\
z_2(t,s) &= \sum_{n=1}^{\mathbb{N}} \varpi_{sn} O_2(t,n) \\
z_3(t,s) &= \sum_{n=1}^{\mathbb{N}} \varpi_{sn} O_3(t,n)
\end{aligned}
$$

The objective functions are normalized, and the following constraints are added to the mathematical model.

$$
\begin{aligned}
z_1(t,s) &\geq \left(\sum_{n=1}^{\mathbb{N}} \varpi_{sn} O_1(t,n) \right) - M \times \sum_{\tau \in S, \tau \neq s} b_{t\tau} \qquad \forall t, s \\
z_2(t,s) &\geq \left(\sum_{n=1}^{\mathbb{N}} \varpi_{sn} O_2(t,n) \right) - M \times \sum_{\tau \in S, \tau \neq s} b_{t\tau} \qquad \forall t, s \\
z_3(t,s) &\geq \left(\sum_{n=1}^{\mathbb{N}} \varpi_{sn} O_3(t,n) \right) - M \times \sum_{\tau \in S, \tau \neq s} b_{t\tau} \qquad \forall t, s
\end{aligned}
$$

4.1.1. Numerical Examples for Smart Grid Technologies Affecting the Availability of the Generation Units

In this section, numerical examples considering the Smart Grid technologies affecting availability of the generation units are solved and their results are presented. Three different impact levels of the Smart Grid technologies are considered and for each impact level, multiple problems are solved for different cost levels of Smart Grid technologies to demonstrate how the expansion plans change based on the impact and cost level of the Smart Grid technologies.

To demonstrate the model, example problems are solved for a 15 year planning horizon to minimize the total cost. The planning horizon is divided into three time periods of five years each. Therefore, if a new generation unit is to be installed, the options are to install it as soon as possible, in five years, or in ten years for the current period. Electricity demand is considered to be increasing 1% in each year.

The topology for existing central system is the part of the IEEE Reliability Test System [87] presented in Figure 3.2. The nodes 13 though 23 of the IEEE Reliability Test System are considered as the nodes used in this example and the transmission lines between these nodes. In this example, the distribution system and transmission system availabilities are not considered. The focus is given on the availability of the generation units. It is also assumed that the system has enough transmission capacity; therefore, transmission capacity constraints are not included into the model. The generation units existing in the selected nodes for the test system are used as the existing generation units. The cost characteristics for the existing units can be found in Table 3.1, however, the power group numbers listed in Table 3.1 are now replaced by 1, 2, 7, 13, 15, 16, 18, 21, 22, and 23 respectively.

As expansion options, only one type of generation unit, namely CCGT, and two types of Smart Grid technologies which affect the availability of the generation units are considered. Four feasible combinations for the Smart Grid technologies are defined. Combination or State 1 means that the system does not have any Smart Grid technologies; State 2 means that the first type of technology is operational in the system; State 3 means that the second type of technology is operational in the system; and State 4 means both technologies are in the system. The cost characteristics for CCGT units can be found in Table 3.2.

Since only a subset of the IEEE Reliability Test System is used as an example, the peak load level in the demand nodes are modified to make the system have the same capacity reserve margin as in the original IEEE Reliability Test System. The modified peak load level for each demand node in the system is given in Table 4.1. The load duration curve is divided into four segments and a representative load level is chosen for each segment. The segments, the percentage of the peak load level selected for each segment and the probability of each segment is the same as the ones presented in Table 3.16.

Table 4.1. Modified peak load demand levels for the demand nodes

Node Number	Peak Load Level	Node Number	Peak Load Level	Node Number	Peak Load Level	Node Number	Peak Load Level
13	374	15	426	18	442	20	237
14	303	16	209	19	290		

Since the focus is on the generation units, the availability scenarios are generated only for the critical generation units by using the scenario optimization based approach presented previously. For each segment, all the availability scenarios generated are used. Therefore, each year is represented by 44 scenarios. Then, the mathematical model is solved to find the least cost generation expansion plan over 660 scenarios. The availability scenarios selected are given in Table 4.2.

Table 4.2. Availability scenarios selected for the critical generation units in the example problems

Scenario Numbers	Critical Generation Units (power node number generation unit number)								
	13 1	13 2	13 3	23 1	23 2	23 3	16 1	18 1	21 1
1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	
3	1	1	1	1	1	1	1		1
4	1	1	1	1	1	1		1	
5	1	1	1	1	1		1	1	1
6	1	1	1	1			1	1	1
7	1	1	1			1	1	1	
8	1	1		1	1		1	1	1
9	1		1	1	1	1	1	1	
10		1	1	1	1	1	1	1	
11		1	1		1	1	1	1	1

The mathematical model for example problems is presented as follows.

$$\min \sum_{t=1}^{T}(1+r)^{-t}\left(\left(\sum_{i\in N}\sum_{q\in\Xi_i}(s_{tiq}-s_{t-1iq})a_{tiq}\right)+\left(\sum_{i\in N}\sum_{k\in\Theta_i}g_{tik}+\sum_{i\in N}\sum_{q\in\Xi_i}s_{tiq}h_{tiq}\right)\right.$$
$$\left.+\left(\sum_{h\in H}(w_{th}-w_{t-1h})m_{th}+\sum_{h\in H}w_{th}r_{th}\right)+\sum_{s=1}^{S}z(t,s)\right)$$

s.t.

$$z(t,s)\geq \sum_{n=1}^{\mathbb{N}}\varpi_{sn}\left(\sum_{i\in N}\sum_{k\in\Theta_i}x_{tnik}c_{tik}+\sum_{i\in N}\sum_{q\in\Xi_i}u_{tniq}e_{tiq}+\sum_{i\in N}v_{tni}f_t\right)-M\times\sum_{\tau\in S,\tau\neq s}b_{t\tau}\qquad \forall t,s$$

$$b_{ts}\geq\left(\sum_{h\in a_s}w_{th}-(\|a_s\|-1)\right)-\sum_{h\notin a_s}w_{th}\qquad \forall t,s,\qquad \sum_{s=1}^{S}b_{ts}=1\qquad \forall t$$

$$w_{th}\geq w_{t-1,h}\qquad \forall t,h,\qquad w_{th}\in\{0,1\}\qquad \forall t,h,\qquad b_{ts}\in\{0,1\}\qquad \forall t,s$$

$$\sum_{(m,i,r)\in A}\Gamma_{tnmir}\alpha_{mir}-\sum_{(i,m,r)\in A}\Gamma_{tnimr}+\sum_{k\in\Theta_i}x_{tnik}+\sum_{q\in\Xi_i}u_{tniq}+v_{tni}=D_{tni}\qquad \forall i\in N,\forall(t,n)$$

$$x_{tnik}\leq\gamma_{tnik}\qquad \forall i\in N,k\in\Theta_i^{'},\forall(t,n)$$

$$x_{tnik}\leq\mu_{ik}(p_{ik}+\sum_{s=1}^{S}p_{ik}f_{sik}b_{ts})\qquad \forall i\in N,k\in\Theta_i^{''},\forall(t,n)$$

$$u_{tniq}\leq\mu_{iq}s_{tiq}\qquad \forall i\in N,q\in\Xi_i,\forall(t,n)$$

$$u_{tniq}\leq\mu_{iq}(p_{iq}+\sum_{s=1}^{S}p_{ik}f_{siq}b_{ts})\qquad \forall i\in N,q\in\Xi_i,\forall(t,n)$$

$$s_{tiq}\geq s_{(t-1)iq}\qquad \forall i\in N,q\in\Xi_i,\forall t=2,...,T,\qquad s_{tiq}\in\{0,1\}\qquad \forall i\in N,q\in\Xi_i,\forall t$$

$$x_{tnik}\geq 0\qquad \forall i\in N,k\in\Theta_i,\forall(t,n),\qquad u_{tniq}\geq 0\qquad \forall i\in N,q\in\Xi_i,\forall(t,n)$$

$$\Gamma_{tnimr}\geq 0\qquad \forall(i,m,r)\in A,\forall(t,n),\qquad v_{tni}\geq 0\qquad \forall i\in N,\forall(t,n)$$

where ϖ_{ns} are calculated before solving the mathematical model for each scenario n and for each combination of Smart Grid technologies, s, as described before.

Multiple example problems with different assumptions on the investment cost and the impact of each Smart Grid technology and the combination of them are solved. The first example examines how the expansion plan for Smart Grid technologies and generation units change if the impact of Smart Grid technologies is relatively small. In the second example, the impact level of Smart Grid technologies is larger than the first example. The third example is solved to observe the effects of Smart Grid technologies which have relatively high impact. For each example, five investment cost cases are defined for each Smart Grid technology and all the combination of these cases are solved. That is, 25 problems are solved in each example. The objective is to demonstrate that model can be used to obtain solutions, to study trade-offs between the expansion with Smart Grid technologies and generation units, and to investigate the effect of Smart Grid technologies on the expansion and dispatching decisions.

Example 1: In this example, two Smart Grid technologies which have the relatively lower impact on the component availabilities are considered with different investment cost cases.

Consider following assumptions for the impact and investment cost level of Smart Grid technologies:

- The first type of technology alone increases the availability of the generation units by 0.1%.
- The second type of technology increases the availability of the generation units by 0.5%.
- If both first type and second type of technologies are operational, the availability of the generation units is increased by 1%.
- Five investment cost cases are defined for each Smart Grid technology as presented in Table 4.3.

Table 4.3. Investment cost cases for Example Problem 1

Case Number	Investment Cost ($)	
	Smart Grid Technology 1	Smart Grid Technology 2
1	10,000,000	40,000,000
2	20,000,000	80,000,000
3	50,000,000	150,000,000
4	100,000,000	300,000,000
5	150,000,000	400,000,000

The procedure described in Section 4.1 is applied to obtain the probabilities of the availability scenarios for each combination. These new probabilities are then used to calculate the adjustment factor, ϖ_{ns}, for each scenario *n* and for each combination *s*. The probabilities calculated for Example Problem 1 is presented in Table 4.4. In the availability scenario 1, all the system components are working. Therefore, when the system includes Smart Grid technologies, the probability of this scenario increases based on the level of the impact. As presented in Table 4.4, the probability of the availability scenario 1 is largest when both Smart Grid technologies are in operation (*s*=4) which has the largest Smart Grid impact, and it is lowest when there are no Smart Grid technologies are in operation (*s*=1).

Table 4.4. Probabilities of the availability scenarios for Example Problem 1

s	Availability Scenarios for Critical Generation Units										
	1	2	3	4	5	6	7	8	9	10	11
1	0.65	0	0.12	0.04	0	0.03	0.01	0.05	0.05	0.02	0.03
2	0.65415	0	0.11912	0.03904	0.0015	0.02853	0.01051	0.04905	0.04905	0.02052	0.02853
3	0.66325	0.015	0.1156	0.0352	0.03015	0.00505	0.04525	0.04525	0.0151	0.03015	0
4	0.6765	0.03	0.1112	0.0304	0	0.0303	0.0001	0.0405	0.0405	0.0102	0.0303

For all 25 combinations of the investment cost cases, the problem is solved. From this point forward, "Case" is used to refer to the problem solved for each combination. That is, if the problem is solved considering the first investment cost case of the first Smart Grid technology and the third investment cost case of the second Smart Grid technology, this problem is referred as Case (1,3).

The objective function values are presented in Table 4.5 for each combination of the investment cost cases and the expansion plan is presented in Table 4.6. In Table 4.5 and 4.6, SGT represents the Smart Grid technology, ICC represents the investment cost case and C represents the case.

As it can be seen from the Table 4.6, when investing on Smart Grid technologies are more economical, the model chooses to invest in them. In those cases, fewer generation units are built. For example, in problem Case (1,1) both Smart Grid technologies are invested and this results in not building CCGT in power node 13 and postponing to build CCGT in power node 23 from year 6 to year 11. The results indicate that Smart Grid technologies affecting the availability of the system components generally decrease the investment in the generation units.

Table 4.5. Objective function values for the investment cost cases in Example 1

ICC for SGT 1	Cost (in terms of billions of $)				
	ICC for SGT 2				
	1	2	3	4	5
1	14.038	14.078	14.100	14.100	14.100
2	14.048	14.088	14.100	14.100	14.100
3	14.078	14.100	14.100	14.100	14.100
4	14.100	14.100	14.100	14.100	14.100
5	14.100	14.100	14.100	14.100	14.100

Table 4.6. Expansion plan for Example Problem 1

Problems solved	Expansion Plan (year)		Expansion Plan (year: power nodes)
	SGT1	SGT2	CCGT
C(1,1)-C(1,2)-C(2,1)-C(2,2)-C(3,1)	1	1	(1: 15,16,18,21) (6: 22) (11: 23)
Other 20 cases	-	-	(1: 15,16,18,21) (6: 22,23) (11: 13)

Example 2: The second example is solved to demonstrate how the expansion decision changes if the impact of the Smart Grid technologies is higher. Since the impact is higher, the investment costs are also increased in this example. Similar to Example 1, five investment cost cases are defined for each Smart Grid technology and a problem is solved for each combination. That is, 25 problems are solved.

As a second example, consider following assumptions:

- The first type of technology alone increases the availability of the generation units by 0.1%.
- The second type of technology increases the availability of the generation units by 1%.
- If both first type and second type of technologies are operational, the availability of the generation units is increased by 1.5%.
- Five investment cost cases are defined for each Smart Grid technology as presented in Table 4.7.

Table 4.7. Investment cost cases for Example Problem 2

Case Number	Investment Cost ($)	
	Smart Grid Technology 1	Smart Grid Technology 2
1	10,000,000	80,000,000
2	20,000,000	150,000,000
3	50,000,000	300,000,000
4	100,000,000	450,000,000
5	150,000,000	500,000,000

The procedure described in Section 4.1 is applied to obtain the probabilities of the availability scenarios for each combination. The probabilities calculated for Example Problem 2 is presented in Table 4.8. Since the impact is higher in this example, the probability of the first availability scenario where all the components are working is higher than the ones in Example 1.

Table 4.8. Probabilities of the availability scenarios for Example Problem 2

s	Availability Scenarios for Critical Generation Units										
	1	2	3	4	5	6	7	8	9	10	11
1	0.65	0	0.12	0.04	0	0.03	0.01	0.05	0.05	0.02	0.03
2	0.66325	0.015	0.1156	0.0352	0.03015	0.00505	0.04525	0.04525	0.0151	0.03015	0
3	0.6765	0.03	0.1112	0.0304	0	0.0303	0.0001	0.0405	0.0405	0.0102	0.0303
4	0.70552	0.03836	0.10416	0.02272	0.00782	0.02272	0	0.0329	0.0329	0.01018	0.02272

For each combination of the investment cost case, the problem is solved. The objective function values are presented in Table 4.9 for each combination and the expansion plan is presented in Table 4.10. In some cases, Smart Grid technologies provide cost benefits due to the fact that their presence leads to fewer generation unit investments, in other cases, cost benefits are due to improvement in the operational cost such as less unmet demand, more utilization of least cost generation units and so on. For example, in problem Case (4,1) where the problem is solved considering the fourth investment cost case of the first Smart Grid technology and the first investment cost case of the second Smart Grid technology, the second Smart Grid technology is introduced into the system since this results in postponing the investment of CCGTs in power nodes 22 and 23 from year 6 to year 11. Also, the investment of Smart Grid technology provides not building the CCGT in power nodes 13. The comparison between problem Case (1,1) and problem Case (4,1) revealed that there is no difference in expansion plan for the generation units. However, in Case (1,1) the first type of Smart Grid technology is also introduced into the system. This shows that Smart Grid technologies can also be invested due to the benefits obtained due to improvement in the operational cost.

The comparison of the results between Example 1 and Example 2 shows that when the impact of the Smart Grid technologies on the availability of the generation units is larger, the expansion plans changes more and the model is capable of providing expansion solutions which minimizes the cost under different assumptions on the impact and investment cost level of the Smart Grid technologies.

Table 4.9. Objective function values for the investment cost cases in Example 2

ICC for SGT 1	Cost (in terms of billions of $)				
	ICC for SGT 2				
	1	2	3	4	5
1	14.066	14.100	14.100	14.100	14.100
2	14.066	14.100	14.100	14.100	14.100
3	14.085	14.100	14.100	14.100	14.100
4	14.085	14.100	14.100	14.100	14.100
5	14.085	14.100	14.100	14.100	14.100

Table 4.10. Expansion plan for Example Problem 2

Problems solved	Expansion Plan (year)		Expansion Plan (year: power nodes)
	SGT1	SGT2	CCGT
C(1,1)-C(2,1)	1	1	(1: 15,16,18,21) (11: 22,23)
C(3,1)-C(4,1)-C(5,1)	-	1	(1: 15,16,18,21) (11: 22,23)
Other 20 cases	-	-	(1: 15,16,18,21) (6: 22,23) (11: 13)

Example 3: This example is solved to demonstrate how the expansion plans changes if the impact of Smart Grid technologies is much higher. The availability of the system components with respect to the states are defined as presented in Table 4.11 and investment cost cases are presented in Table 4.12. The procedure described in Section 4.1 is applied to obtain the probabilities of the availability scenarios for each combination. The probabilities calculated for Example Problem 3 is presented in Table 4.13. Since the impact of the Smart Grid probabilities are much higher, the availability scenario 1 where all the components are working is much higher. As it can be seen from Table 4.13, the probability is increased to 0.829 for state 4 where both Smart Grid technologies are in operation.

For each combination of the investment cost cases, the problem is solved. The objective function values are presented in Table 4.14 for each combination and the expansion plan is presented in Table 4.15. The results shows that the combination of Smart Grid technologies are selected according to the cost benefits they provide. In some cases, the presence of Smart Grid technologies decrease the number of generated units to be built, and in other cases, the investment time for the required generation units are postponed due the impact of the Smart Grid technologies invested. If no Smart Grid technologies are invested, four CCGTs are built in year 1, 2 in year 6 and 1 in year 1. However, if both Smart Grid technologies are invested like in Case (2,2), only two CCGTs are invested in year 1, 2 in year 6 and 1 in year 11. The investments done in early stage of the planning horizon is now unnecessary due to the impact of Smart Grid technologies on the availability of the generation units since these units can now utilized more.

Table 4.11. Availabilities defined for each state in Example 3

Generation unit (*i*, *k*)		States			
		1	2	3	4
13	1	0.95	0.96	0.98	0.999
13	2	0.95	0.96	0.98	0.999
13	3	0.95	0.96	0.98	0.999
13	4	0.979	0.989	0.999	0.9999
15	1	0.95	0.96	0.98	0.999
15	2	0.95	0.96	0.98	0.999
15	3	0.95	0.96	0.98	0.999
15	4	0.98	0.99	0.999	0.9999
15	5	0.98	0.99	0.999	0.9999
15	6	0.98	0.99	0.999	0.9999
17	7	0.979	0.989	0.999	0.9999
22	1	0.935	0.95	0.965	0.985
22	2	0.935	0.95	0.965	0.985
22	3	0.942	0.955	0.975	0.99
22	4	0.96	0.97	0.989	0.999
22	5	0.88	0.9	0.91	0.93
22	6	0.88	0.9	0.91	0.93
22	7	0.979	0.989	0.999	0.9999
16	1	0.96	0.97	0.989	0.999
16	2	0.979	0.989	0.999	0.9999
18	1	0.88	0.9	0.91	0.93
18	2	0.979	0.989	0.999	0.9999
21	1	0.88	0.9	0.91	0.93
21	2	0.979	0.989	0.999	0.9999
23	1	0.96	0.97	0.989	0.999
23	2	0.96	0.97	0.989	0.999
23	3	0.92	0.93	0.95	0.97
23	4	0.979	0.989	0.999	0.9999

Table 4.12. Investment cost cases for Example Problem 3

Scenario Number	Investment Cost ($)	
	Smart Grid Technology 1	Smart Grid Technology 2
1	50,000,000	100,000,000
2	150,000,000	300,000,000
3	400,000,000	800,000,000
4	650,000,000	1,300,000,000
5	1,000,000,000	2,000,000,000

Table 4.13. Probabilities of the availability scenarios for Example Problem 3

s	Availability Scenarios for Critical Generation Units										
	1	2	3	4	5	6	7	8	9	10	11
1	0.65	0	0.12	0.04	0	0.03	0.01	0.05	0.05	0.02	0.03
2	0.7	0.02	0.1	0.03	0	0.03	0	0.04	0.04	0.01	0.03
3	0.759	0.05	0.09	0.011	0.019	0.011	0	0.02	0.02	0.009	0.011
4	0.829	0.068	0.07	0.001	0.028	0.001	0	0.001	0.001	0	0.001

Table 4.14. Objective function values for the investment cost cases in Example 3

ICC for SGT 1	Cost (in terms of billions of $)				
	ICC for SGT 2				
	1	2	3	4	5
1	13.795	13.995	14.019	14.019	14.019
2	13.895	14.095	14.100	14.100	14.100
3	13.938	14.100	14.100	14.100	14.100
4	13.938	14.100	14.100	14.100	14.100
5	13.938	14.100	14.100	14.100	14.100

Table 4.15. Expansion plan for Example Problem 3

Problems solved	Expansion Plan (year)		Expansion Plan (year: power nodes)
	SGT1	SGT2	CCGT
C(1,1)-C(1,2)-C(2,1)-C(2,2)	1	1	(1: 15,16) (6: 18,21) (11: 22)
C(1,3)-C(1,4)-C(1,5)	1	-	(1: 15,16,18,21) (6: 22) (11: 23)
C(3,1)-C(4,1)-C(5,1)	-	1	(1: 15,16,18) (6: 21) (11: 22,23)
Other 15 cases	-	-	(1: 15,16,18,21) (6: 22,23) (11: 13)

The results show that if the impact of the Smart Grid technologies on the component availability is larger, the expansion plan of generation units is affected more. Consider the cases where both Smart Grid technologies are invested in the three examples. The impact is at its highest level in Example 3 and so fewer number of generation units are built in the early stage of the planning horizon compare to the other examples. Also, since it is possible to utilize the generation units more, the least cost generation units are used more. Additionally, the unmet demand cost is much lower when the impact is higher. Therefore, the total cost is much lower when the impact of the Smart Grid technologies on the component availability is higher.

4.2. GEP with Smart Grid Technologies Affecting the Demand

In this section, a methodology how to incorporate the impact of Smart Grid technologies on the energy demand has been developed. There are many kinds of Smart Grid technologies which can reduce the demand or shift the demand. For example, by means of advanced meters together with a two-way communication system, consumers can observe the energy prices and change their energy usage or smart appliances can be connected to the grid so they can operate according to the system condition and preferences of the consumers. Energy storage devices or plug-in hybrid electric cars can store energy during low demand hours and then consumer can use that energy during peak load demand hours. More energy efficient devices such as programmable thermostats, home automation systems and so on can be included to the grid for energy conservation.

After the preprocessing $\mathbb{N}$ number of scenarios is available, scenarios can be divided into two groups: (*i*) N_1: a set of scenarios where the capacity margin is smaller than predefined value. These types of scenarios are the ones with high demand or high number of failed components. Smart Grid technologies such as grid-friendly appliances, smart meters or technologies which increase the energy efficiency can be used to reduce or shift the demand in these scenarios; (*ii*) N_2: a set of scenarios where the capacity margin is larger than a predefined value. Smart Grid technologies for reducing the demand are also effective in these scenarios. However, the demand in these scenarios increases due to the fact that the technologies for shifting the demand are shifting the demand from the first group to the second group.

Consider two groups of Smart Grid technologies; H_1, set of technologies which reduce the demand, and H_2, set of technologies which shift the demand. Decision variables are defined for both groups such that x_{th} is equal to 1 if the Smart Grid technology h is selected, and 0 otherwise. The satisfiable and locally satisfiable demand are modified as follows.

$$
\begin{aligned}
&\Psi_{tn}{}' = \Psi_{tn} - R_{tn} - S_{tn} && \forall t, n \in N_1 \\
&\Phi_{tnl}{}' = \Phi_{tnl} - W_{tnl} - Q_{tnl} && \forall t, \forall n \in N_1, \forall l \in \Lambda \\
&\Psi_{tn}{}' = \Psi_{tn} - U_{tn} + I_{tn} && \forall t, n \in N_2 \\
&\Phi_{tnl}{}' = \Phi_{tnl} - M_{tnl} + E_{tnl} && \forall t, \forall n \in N_2, \forall l \in \Lambda
\end{aligned}
$$

where R_{tn}, U_{tn}, S_{tn} and I_{tn} are the demand reductions and demand shift for satisfiable demand in year t for scenario n, respectively. W_{tnl}, M_{tnl}, Q_{tnl} and E_{tnl} are the demand reductions and demand shift for the locally satisfiable demand respectively.

In the mathematical model, Ψ_{tn} and Φ_{tnl} are replaced by $\Psi_{tn}{}'$ and $\Phi_{tnl}{}'$ and following constraints are added to the model.

$$
\begin{aligned}
&R_{tn} \le \sum_{h\in H_1} a_h \Psi_{tn} x_{th} && \forall t, n \in N_1 \\
&U_{tn} \le \sum_{h\in H_1} b_h \Psi_{tn} x_{th} && \forall t, n \in N_2 \\
&W_{tnl} \le \sum_{h\in H_1} c_h \Phi_{tnl} x_{th} && \forall t, \forall n \in N_1, \forall l \in \Lambda \\
&M_{tnl} \le \sum_{h\in H_1} d_h \Phi_{tnl} x_{th} && \forall t, \forall n \in N_2, \forall l \in \Lambda \\
&S_{tn} \le \sum_{h\in H_2} e_h \Psi_{tn} x_{th} && \forall t, n \in N_1 \\
&I_{tn} = \frac{\sum_{n\in N_1} \omega_{tn} S_{tn}}{\sum_{n\in N_2} \omega_{tn}} && \forall t, n \in N_2 \\
&Q_{tnl} \le \sum_{h\in H_2} f_h \Psi_{tn} x_{th} && \forall t, \forall n \in N_1, \forall l \in \Lambda \\
&E_{tnl} = \frac{\sum_{n\in N_1} \omega_{tn} Q_{tnl}}{\sum_{n\in N_2} \omega_{tn}} && \forall t, \forall n \in N_2, \forall l \in \Lambda
\end{aligned}
$$

where a_h, b_h, c_h, d_h, e_h, and f_h are the proportional effect of Smart Grid technology h to the corresponding demand. The constraints above make sure that the demand reduction and shift can only occur when the related technologies are implemented; and total demand shift from the scenarios in N_1 should be equal to total demand shift to the scenarios in N_2. Here, it is assumed that the demand increase for the scenarios due to the shift is equally divided into all of the scenarios in the second group.

4.2.1. Numerical Examples for Smart Grid Technologies Affecting the Demand

To demonstrate the model for investigating Smart Grid technologies that shift or reduce the demand, example problems are solved for a 15 year planning horizon. The planning horizon is divided into three time periods of five years each. Therefore, if a new generation unit is to be installed, the options are to install it as soon as possible, in five years, or in ten years for the current period. Electricity demand is considered to be increasing 1% in each year.

The topology for existing central system is the IEEE Reliability Test System [87] presented in Figure 3.2. The generation units for the test system are used as the existing generation units. The cost characteristics for the existing units can be found in Table 3.1, however, the power group numbers listed in Table 3.1 are now replaced by 1, 2, 7, 13, 15, 16, 18, 21, 22, and 23 respectively.

In this example, the distribution system availabilities are not considered. The availability scenarios for the critical generation units and transmission lines presented in Table 3.11 are used for each segment. Therefore, each year is represented by 64 scenarios and the model is solved over 960 scenarios.

As expansion options, one type of generation unit, namely CCGT, and two types of Smart Grid technologies, which reduce the demand, and two types of Smart Grid technologies which shift the demand are considered. The cost characteristics for CCGT units can be found in Table 3.2.

The peak load level in the demand nodes in the IEEE Reliability Test System is given in Table 4.16. The load duration curve is divided into four segments and a representative load level is chosen for the segment. The segments, the percentage of the peak load level selected for each segment and the probability of each segment is the same as the ones presented in Table 3.16.

Table 4.16. Peak load levels for demand nodes in IEEE Reliability Test System

Node Number	Peak Load Level	Node Number	Peak Load Level	Node Number	Peak Load Level	Node Number	Peak Load Level
1	108	6	136	13	265	19	181
2	97	7	125	14	194	20	128
3	180	8	171	15	317		
4	74	9	175	16	100		
5	71	10	195	18	333		

It is considered that the technology reducing the electricity demand is effective for all scenarios; and the technology shifting the demand is working as follows.

- This technology reduces the demand of all scenarios used for first and second segments. N_1 represents these scenarios in the model.
- The reduced electricity is distributed equally between the third and fourth segment scenarios where all the components and transmission lines are working. N_2 represents these scenarios in the model.

The mathematical model for example problems is presented as follows.

$$\min \sum_{t=1}^{T}(1+r)^{-t}\left(\left(\sum_{i\in N}\sum_{q\in\Xi_i}(s_{tiq}-s_{t-1iq})a_{tiq}\right)+\left(\sum_{i\in N}\sum_{k\in\Theta_i}g_{tik}+\sum_{i\in N}\sum_{q\in\Xi_i}s_{tiq}h_{tiq}\right)\right.$$

$$\left.+\left(\sum_{h\in H}(w_{th}-w_{t-1h})m_{th}+\sum_{h\in H}w_{th}r_{th}\right)+\sum_{n=1}^{\mathbb{N}}\varpi_n\left(\sum_{i\in N}\sum_{k\in\Theta_i}x_{tnik}c_{tik}+\sum_{i\in N}\sum_{q\in\Xi_i}u_{tniq}e_{tiq}+\sum_{i\in N}v_{tni}f_t\right)\right)$$

s.t.

$$w_{th}\ge w_{t-1,h}\qquad \forall t,h$$

$$w_{th}\in\{0,1\}\qquad \forall t,h,\qquad b_{ts}\in\{0,1\}\qquad \forall t,s$$

$$\sum_{(m,i,r)\in A}\Gamma_{tnmir}\alpha_{mir}-\sum_{(i,m,r)\in A}\Gamma_{tnimr}+\sum_{k\in\Theta_i}x_{tnik}+\sum_{q\in\Xi_i}u_{tniq}+v_{tni}=D_{tni}-R_{tni}-S_{tni},\forall i\in N_1,\forall(t,n)$$

$$\sum_{(m,i,r)\in A}\Gamma_{tnmir}\alpha_{mir}-\sum_{(i,m,r)\in A}\Gamma_{tnimr}+\sum_{k\in\Theta_i}x_{tnik}+\sum_{q\in\Xi_i}u_{tniq}+v_{tni}=D_{tni}-U_{tni}+I_{tni},\forall i\in N_2,\forall(t,n)$$

$$R_{tni}\le\sum_{h\in H_1}a_hD_{tni}w_{th}\qquad \forall t,i,\forall n\in N_1$$

$$U_{tni}\le\sum_{h\in H_1}b_hD_{tn}w_{th}\qquad \forall t,i,\forall n\in N_2$$

$$S_{tn}\le\sum_{h\in H_2}e_hD_{tni}x_{th}\qquad \forall t,i,\forall n,\in N_1$$

$$I_{tni}=\frac{\sum_{n\in N_1}\omega_{tn}S_{tni}}{\sum_{n\in N_2}\omega_{tn}}\qquad \forall t,i,\forall n\in N_2$$

$$R_{tni},S_{tni}\ge 0\qquad \forall t,i,\forall n,\in N_1\qquad U_{tni},I_{tni}\ge 0\qquad \forall t,i,\forall n,\in N_2$$

$$\Gamma_{tnimr}-\Gamma_{tnmir}\le\Delta_{tnimr}\qquad \forall(i,m,r)\in A^{'},\forall(t,n)$$

$$\Gamma_{tnmir}-\Gamma_{tnimr}\le\Delta_{tnimr}\qquad \forall(i,m,r)\in A^{'},\forall(t,n)$$

$$\Gamma_{tnimr}-\Gamma_{tnmir}\le\delta_{imr}p_{imr}\qquad \forall(i,m,r)\in A^{''},\forall(t,n)$$

$$\Gamma_{tnmir}-\Gamma_{tnimr}\le\delta_{imr}p_{imr}\qquad \forall(i,m,r)\in A^{''},\forall(t,n)$$

$$x_{tnik}\le\gamma_{tnik}\qquad \forall i\in N,k\in\Theta_i^{'},\forall(t,n)$$

$$x_{tnik}\le\mu_{ik}p_{ik}\qquad \forall i\in N,k\in\Theta_i^{''},\forall(t,n)$$

$$u_{tniq}\le\mu_{iq}p_{iq}s_{tiq}\qquad \forall i\in N,q\in\Xi_i,\forall(t,n)$$

$$s_{tiq}\ge s_{(t-1)iq}\qquad \forall i\in N,q\in\Xi_i,\forall t=2,...,T$$

$$s_{tiq}\in\{0,1\}\qquad \forall i\in N,q\in\Xi_i,\forall t$$

$$x_{tnik}\ge 0\qquad \forall i\in N,k\in\Theta_i,\forall(t,n)$$

$$u_{tniq}\ge 0\qquad \forall i\in N,q\in\Xi_i,\forall(t,n)$$

$$\Gamma_{tnimr}\ge 0\qquad \forall(i,m,r)\in A,\forall(t,n)$$

$$v_{tni}\ge 0\qquad \forall i\in N,\forall(t,n)$$

Multiple example problems are solved with different assumptions on the investment cost and the impact of each Smart Grid technology.

<u>Example 1</u>: In the first example, Smart Grid technologies which have relatively lower impact on the demand are considered with different investment cost cases to investigate how the expansion plan changes. Consider following assumptions for the impact and investment cost level of Smart Grid technologies:

- The first type of technology (for shifting) can shift the demand by 0.1%. The second type can shift the demand by 0.5%.
- The first type of technology (for reducing) can reduce the demand by 0.1%. The second type can reduce the demand by 0.5%.
- Six investment cost cases are defined for each Smart Grid technology as presented in Table 4.17.

Table 4.17. Investment cost cases for the technologies which can shift/reduce the demand in Example Problem 1

	Investment Cost ($)			
Case Number	Technologies shifting the demand		Technologies shifting the demand	
	SGT 1	SGT 2	SGT 1	SGT 2
1	10,000,000	40,000,000	10,000,000	40,000,000
2	10,000,000	75,000,000	10,000,000	75,000,000
3	15,000,000	60,000,000	15,000,000	60,000,000
4	15,000,000	150,000,000	15,000,000	150,000,000
5	50,000,000	200,000,000	50,000,000	200,000,000
6	100,000,000	500,000,000	100,000,000	500,000,000

For all 36 combinations of the investment cost cases, the model is solved to find the least cost expansion plan for the example problem. The objective functions are given for each combination in Table 4.18. The expansion plan is given in Table 4.19. Both Smart Grid technologies which can reduce the demand is chosen for the problem cases (1,1)-(2,1)-(3,1)-(4,1)-(5,1) and (6,1). No demand shifting technology is invested. The investment on the demand reducing technologies results in fewer investments in the generation units and also postpones the investments for some generation units. For example, in the problem Case (1,1), the CCGTs are built in nodes 2, 13 and 16 in year 1 and in nodes 1, 15 and 23 in year 6. However, for the cases where no demand shifting technologies are invested, the CCGTs are built in nodes 2, 13, 15 and 16 in year 1, in nodes 1 and 23 in year 6 and in node 21 in year 11. The result shows that introduction of the demand reducing technologies leads to not building a CCGT in node 21 and postponing to build a CCGT in 15 from year 1 to year 6.

Table 4.18. Objective function values for each investment cost scenarios in Example 1

ICC for SGT (shifting)	Cost (in terms of billions of $)				
	ICC for SGT (reducing)				
	1	2	3	4	5
1	16.814	16.816	16.816	16.816	16.816
2	16.814	16.816	16.816	16.816	16.816
3	16.814	16.816	16.816	16.816	16.816
4	16.814	16.816	16.816	16.816	16.816
5	16.814	16.816	16.816	16.816	16.816
6	16.814	16.816	16.816	16.816	16.816

Table 4.19. Expansion plans for shifting/reducing technologies and generation units in Example 1

Problems solved	Expansion Plan (year)				Expansion Plan (year: power nodes)
	Demand Shifting Technologies		Demand Reducing Technologies		
	SGT1	SGT2	SGT1	SGT2	CCGT
C(1,1)-C(2,1)-C(3,1)-C(4,1)-C(5,1)-C(6-1)	-	-	1	1	(1: 2,13,16) (6: 1,15,23)
Other 30 cases	-	-	-	-	(1:2,13,15,16) (6: 1,23) (11: 21)

Example 2: The second example is solved to demonstrate how the expansion decision changes if the impact of the Smart Grid technologies on the demand is greater. The objective is to investigate the effect of the impact level on the expansion plan. For the second example, consider following assumptions:

- The first type of technology (for shifting) can shift the demand by 1%. The second type can shift the demand by 5%.
- The first type of technology (for reducing) can reduce the demand by 1%. The second type can reduce the demand by 5%.
- Six investment cost cases are defined for each Smart Grid technology as presented in Table 4.17.

For all 36 combinations of investment cost cases, the model is solved to find the least cost expansion plan for the example problem. The objective function values are given for each combination in Table 4.20. The expansion plan is given in Table 4.21. These results show that the investment plan can change dramatically if there are Smart Grid technologies available for investment which have relatively high impact on demand and their investment costs are relatively lower. In almost all the problem cases, all the demand shifting technologies and demand reducing technologies are invested. The results show that if the demand shifting technologies are economical, they are introduced into the system and reduce the number of generation units to be built. Since the system is designed to satisfy peak load demand, reducing the demand in peak hours results in fewer investments in generation units. For example, for all 30 cases, only two CCGTs are built. Investing in only demand reducing technologies results in building four CCGTs, two of which is build in year 1.

Table 4.20. Objective function values for each investment cost cases in Example 2

ICC for SGT (shifting)	Cost (in terms of billions of $)				
	ICC for SGT (reducing)				
	1	2	3	4	5
1	15.862	15.897	15.887	15.977	16.062
2	15.897	15.932	15.922	16.012	16.097
3	15.887	15.922	15.912	16.002	16.087
4	15.977	16.012	16.002	16.092	16.177
5	16.062	16.097	16.087	16.177	16.262
6	16.223	16.258	16.248	16.338	16.423

Table 4.21. Expansion plans for shifting/reducing technologies and generation units in Example 2

Problems solved	Expansion Plan (year)				Expansion Plan (year: power nodes)
	Demand Shifting Technologies		Demand Reducing Technologies		
	SGT1	SGT2	SGT1	SGT2	CCGT
C(6,1)-C(6,2)-C(6,3)-C(6,4)-C(6,6)-C(6-7)	-	-	1	1	(1: 1,16) (6: 13) (11: 15)
Other 30 cases	1	1	1	1	(6: 2) (11: 16)

In this section, example problems are solved for different impact and cost level of Smart Grid technologies which can shift or reduce the demand. The results shows that if investing in Smart Grid technologies is economical and they reduce the number of generation units to be built dramatically as in Example 2. Consider the cases where demand reducing technologies are invested. In Example 1, the introducing them leads to fewer investments and postpones. In Example 2 where their impact is larger, they provides even fewer investments and greater postpones.

4.3. GEP with Smart Grid Technologies Affecting the Transmission Loss

In this section, the technologies which can reduce the energy loss due to the transmission are considered. There are technologies which can reduce the energy loss in transmission.

Advanced conductors, low loss substation equipments and transformers or other technologies enabling better voltage control and upgrade can reduce the transmission losses in the system.

Consider that for each line (i, j), the loss factor is $\alpha_{i,j}$, the flow on line (i, j) is $f_{i,j}$. In this section, a loss factor is defined as the percentage of the energy transmitted through transmission line. That is, if 100 MW of electricity is sent from i to j and if the loss factor is 0.95, then 95MW electricity reaches to j. The transmission loss is simply incorporated into the model as follows.

$$\sum_{j,i} f_{t,j,i}\alpha_{i,j} + G_{t,i} - \sum_{i,j} f_{t,i,j} = d_{t,i} \qquad \forall t, i$$

where G_i and d_i are the total generation and the demand in node i.

Consider that there are L numbers of Smart Grid technologies available and there are S numbers of feasible combinations. Two types of new binary variables are defined; (i) the investment decision (g_{th}) for Smart Grid technology h in year t, (ii) decision variables representing the combination of Smart Grid technologies which is in operation in year t, b_{ts}. Then, a loss factor for each combination of Smart Grid technologies is calculated. That is, if the s^{th} combination of Smart Grid technologies are implemented, then the loss factor is $\alpha^s_{i,j}$. Then, the following updates are made in the model to incorporate the Smart Grid technologies. $C(t)$ represent the total cost but not including the cost due to the investment on the Smart grid technologies.

$$\min z = \sum_{t=1}^{T} (1+r)^{-t} \left(C(t) + \sum_{h=1}^{H} (g_{th} - g_{(t-1)h}) m_{th} + \sum_{h=1}^{H} g_{th} z_{th} \right)$$

s.t.

$$\sum_{j,i} f_{t,j,i} \alpha_{i,j}^{s} + G_{t,i} - \sum_{i,j} f_{t,i,j} \le d_{t,i} + M(\sum_{\tau \in S, \tau \ne s} b_{t\tau}) \qquad \forall t,i,s \tag{51}$$

$$\sum_{j,i} f_{t,j,i} \alpha_{i,j}^{s} + G_{t,i} - \sum_{i,j} f_{t,i,j} \ge d_{t,i} - M(\sum_{\tau \in S, \tau \ne s} b_{t\tau}) \qquad \forall t,i,s \tag{52}$$

$$b_{ts} \ge \left(\sum_{h \in a_s} g_{th} - (\|a_s\| - 1) \right) - \sum_{h \notin a_s} g_{th} \qquad \forall t,s \tag{53}$$

$$\sum_{s=1}^{S} b_{ts} = 1 \qquad \forall t \tag{54}$$

$$g_{th} \ge g_{t-1,h} \qquad \forall t,h \tag{55}$$

$$g_{th} \in \{0,1\} \qquad \forall t,h, \qquad b_{ts} \in \{0,1\} \qquad \forall t,s \tag{56}$$

In this model, m_{th} and z_{th} are the capital cost and the fixed operational and maintenance cost corresponding to technology h in year t, respectively. The M is very big number, and therefore, Equation 51 and 52 guarantee that if the combination s is not in operation, then the related constraints does not have any impact on the solution. Also, these constraints guarantee that, if the combination s is in operation, the constraints with the corresponding loss factor are the ones used. a_s represents the technologies in the s^{th} combination. Equation 53 forces the state variables s to be equal to one when all the technologies in the combination s are in operation. Equation 54 makes sure that in each year, the system is only in one state. Equation 55 makes sure that once the Smart Grid technology is in operation, it is in operation for the rest of planning horizon. The remaining constraints of the model stay the same.

4.3.1. Numerical Examples for Smart Grid Technologies Affecting the Transmission Losses

In this section, an example problem is modeled and solved with the proposed approach to demonstrate the approach to incorporate the Smart Grid technologies into the model.

In this example, IEEE Reliability Test System [87] (Figure 3.2) is used as existing system. The generation units existing in the nodes for the test system are used as the existing generation units. The cost characteristics for the existing units can be found in Table 3.1, however, the power group numbers listed in Table 3.1 are now replaced by 1, 2, 7, 13, 15, 16, 18, 21, 22, and 23 respectively. The objective is to find the investment plan for 15 years of planning horizon. The planning horizon is divided into three time periods of five years each. Therefore, if a new generation unit is to be installed, the options are to install it as soon as possible, in five years, or in ten years for the current period. Each year divided into four segments and a representative demand level is chosen for each segment (Table 3.16). All the components are considered to be working at their expected capacity in each segment. The demand is considered to increase 1% in each year. It is also considered that there are three new generation units for each power nodes, wind turbines, nuclear plants and CCGTs (information of the new generation units can be found in Table 3.2) and there are three Smart Grid technologies which can reduce the transmission losses. It is considered that all eight combinations of Smart Grid technologies are possible. These combinations can also be referred as the state *s*. The states and the Smart grid technologies in each state is given in Table 4.22.

Table 4.22. States and corresponding Smart Grid technologies

State	Smart Grid Technologies	State	Smart Grid Technologies
1	none	5	1, 2
2	1	6	1, 3
3	2	7	2, 3
4	3	8	1, 2, 3

The mathematical model for the numerical example is as follows.

$$\min \sum_{t=1}^{T}(1+r)^{-t}\left(\left(\sum_{i\in N}\sum_{q\in\Xi_i}(s_{tiq}-s_{t-1iq})a_{tiq}\right)+\left(\sum_{i\in N}\sum_{k\in\Theta_i}g_{tik}+\sum_{i\in N}\sum_{q\in\Xi_i}s_{tiq}h_{tiq}\right)\right.$$
$$\left.+\left(\sum_{h\in H}(w_{th}-w_{t-1h})m_{th}\right)+\sum_{n=1}^{\mathbb{N}}\varpi_n\left(\sum_{i\in N}\sum_{k\in\Theta_i}x_{tnik}c_{tik}+\sum_{i\in N}\sum_{q\in\Xi_i}u_{tniq}e_{tiq}+\sum_{i\in N}v_{tni}f_t\right)\right)$$

s.t.

$$\sum_{(m,i,r)\in A}\Gamma_{tnmir}\alpha^{s}_{mir}-\sum_{(i,m,r)\in A}\Gamma_{tnimr}+\sum_{k\in\Theta_i}x_{tnik}+\sum_{q\in\Xi_i}u_{tniq}+v_{tni}\le D_{tni}+M(\sum_{\tau\in S,\tau\ne s}b_{t\tau})\qquad \forall t,i,n,s$$

$$\sum_{(m,i,r)\in A}\Gamma_{tnmir}\alpha^{s}_{mir}-\sum_{(i,m,r)\in A}\Gamma_{tnimr}+\sum_{k\in\Theta_i}x_{tnik}+\sum_{q\in\Xi_i}u_{tniq}+v_{tni}\ge D_{tni}-M(\sum_{\tau\in S,\tau\ne s}b_{t\tau})\qquad \forall t,i,n,s$$

$$b_{ts}\ge\left(\sum_{h\in a_s}w_{th}-(\|a_s\|-1)\right)-\sum_{h\notin a_s}w_{th}\qquad \forall t,s$$

$$\sum_{s=1}^{S}b_{ts}=1\qquad \forall t$$

$$w_{th}\ge w_{t-1,h}\qquad \forall t,h$$

$$w_{th}\in\{0,1\}\qquad \forall t,h,\qquad b_{ts}\in\{0,1\}\qquad \forall t,s$$

$$\Gamma_{tnimr}-\Gamma_{tnmir}\le\delta_{imr}p_{imr}\qquad \forall(i,m,r)\in A,\forall(t,n)$$

$$\Gamma_{tnmir}-\Gamma_{tnimr}\le\delta_{imr}p_{imr}\qquad \forall(i,m,r)\in A,\forall(t,n)$$

$$x_{tnik}\le\mu_{ik}p_{ik}\qquad \forall i\in N,k\in\Theta_i,\forall(t,n)$$

$$u_{tniq}\le\mu_{iq}p_{iq}s_{tiq}\qquad \forall i\in N,q\in\Xi_i,\forall(t,n)$$

$$s_{tiq}\ge s_{(t-1)iq}\qquad \forall i\in N,q\in\Xi_i,\forall t=2,\ldots,T$$

$$s_{tiq}\in\{0,1\}\qquad \forall i\in N,q\in\Xi_i,\forall t$$

$$x_{tnik}\ge 0\qquad \forall i\in N,k\in\Theta_i,\forall(t,n)$$

$$u_{tniq}\ge 0\qquad \forall i\in N,q\in\Xi_i,\forall(t,n)$$

$$v_{tni}\ge 0\qquad \forall i\in N,\forall(t,n)$$

where w_{th} is the investment decision for the Smart Grid technology h in time period t and b_{ts} is the decision variables which show the state of the system with respect to Smart Grid technologies operating in the system.

As it can be seen, reduction in the transmission losses means increase in the loss factor. Therefore, the effect of the Smart Grid technologies is reflected with respect to α_{imr}. That is, if a Smart Grid technology has the effect of a%, this means that α_{imr} is increased by a%. The effects of the Smart Grid combinations are given in Table 4.23.

Table 4.23. The impact of the combinations of the Smart Grid technologies on transmission losses

State	Smart Grid Technologies	Combination Effect	State	Smart Grid Technologies	Combination Effect
1	None	0	5	1, 2	3
2	1	1	6	1, 3	3.5
3	2	1.5	7	2, 3	4
4	3	2	8	1, 2, 3	5

For the example problem, ten investment cost cases are considered which are presented in Table 4.24.

Table 4.24. Investment cost scenarios for the Smart Grid technologies affecting transmission losses

ICC	Smart Grid Technologies			ISC	Smart Grid Technologies		
	1	2	3		1	2	3
1	10,000,000	20,000,000	30,000,000	6	60,000,000	120,000,000	180,000,000
2	20,000,000	40,000,000	60,000,000	7	70,000,000	140,000,000	210,000,000
3	30,000,000	60,000,000	90,000,000	8	80,000,000	160,000,000	240,000,000
4	40,000,000	80,000,000	120,000,000	9	90,000,000	180,000,000	270,000,000
5	50,000,000	100,000,000	150,000,000	10	100,000,000	200,000,000	300,000,000

The objective function values are given in Table 4.25 and the expansion plan is given in Table 4.26. This example problem is solved to show that the model chooses the Smart

Grid technologies over the generation units if they are economically beneficial. For this problem, in the first four cases, investing on all of the Smart Grid technologies is found to be more beneficial. Since there is less loss with this investment, in those cases a fewer number of CCGTs are introduced into the system. For the cases of 5, 6, 7, and 8, Smart Grid technology 1 and 2 are introduced into the system. In this case, the system has relatively more energy losses. In the last two cases, no Smart Grid technologies are introduced, which results in more investment in the generation units. The expansion plan only includes the CCGTS. The power nodes where these CCGTs are invested and the time periods are given in Table 4.26. For example, for the investment cost case 2, only one CCGT is built in node 2 in year 11; for the investment cost case 10, one CCGT is built in node 2 in year 6 and two CCGTs are built in nodes 15 and 16 in year 11.

Table 4.25. Objective function values and the investment plan of Smart Grid technologies for each investment cost scenario

Cost Scenarios	System State (Smart Grid Technologies)	Cost (in billions of $)	State	Smart Grid Technologies	Cost (in billions of $)
1	8 (1, 2, 3)	15.079	6	5 (1, 2)	15.338
2	8 (1, 2, 3)	15.139	7	5 (1, 2)	15.368
3	8 (1, 2, 3)	15.199	8	5 (1, 2)	15.398
4	8 (1, 2, 3)	15.259	9	1	15.406
5	5 (1, 2)	15.308	10	1	15.406

Table 4.26. Expansion plan of the generation units for each investment cost scenario

ICC	System State	Years for CCGT investments	
		6	11
1-2-3-4	8		2
5-6-7-8	5		2, 16
9-10	1	2	15,16

The results show that if such technologies are economical, they are introduced into the system and they change the expansion plans for generation costs dramatically. Since less energy is lost, less electricity is generated which reduce the operational cost too.

4.4. Smart Grid Technologies and GEP Problems

In this chapter, the mathematical models have been developed to introduce the impacts of the Smart Grid technologies into the GEP problems. There are many types of technologies which can impact the operation of the power grid to obtain a more reliable, effective and efficient power grid. The Smart Grid technologies could impact the power grid by increasing the effective availability of the system components, by reducing or shifting the demand or by reducing the energy loss. In this Chapter, a model to incorporate each impact is given in each separate section.

Example problems are solved to demonstrate the models. The results shows that the expansion plans change based the impact level of the Smart Grid technologies. Technologies which have one of these three impacts are effective in reducing the investment in the generation units or reducing the operational cost. In the first type where the technologies are increasing the effective component availabilities, the investments in generation units are reduced since it is possible to utilize the existing units more. Additionally, operational cost is reduced since least cost generation units are now more available and unmet demand is now less.

In the second case, technologies which reduce or shift the energy demand, the technologies are affecting the demand and the expansion and dispatching plans change accordingly. The technologies which reduce the demand changes the expansion plan of the generation units since the energy demand now is lower. On the other hand, the technologies shifting the demand change the expansion plan since the network is designed to satisfy peak load demand. The reduction in the peak load demand would require fewer investments in the generation units. Additionally, the operational cost is also reduced since the demand shifted is satisfied by less costly generation units in the off-peak hours.

In the third case, the technologies reducing the transmission loss are considered. Since less amount of energy get lost in the system, less energy generation is required and therefore, less number of generation units are built and the operational cost is reduced if these technologies are invested

The results show that implementing Smart Grid technologies offer many benefits. These technologies not only provide a more reliable and efficient power grid but also impact energy expansion decision that can provide for inexpensive and environmentally friendly future.

5. GEP Problems including Uncertainties in the Input Data and Risk

In this chapter, a GEP optimization model is presented to represent the uncertainties associated with the input data such as demand forecasts, input fuel prices, the real effect of technologies on user behavior and others. Since the future cannot be known fully and perfectly, the input data for the expansion plans includes uncertainty. Models that explicitly consider the uncertainty are advantageous because instead of finding an expansion plan based on the expected value of the input data, these models provide solutions responding to the distribution of the uncertainty. In this section, the uncertainties in the demand forecasts are specifically considered. However, the other types of uncertainties can be handled similarly.

Relevant risk measures are also presented for the expansion planning problem and the corresponding mean-risk stochastic integer programming models. Examples models are solved to show how the investment decisions are affected if the risk is introduced into the model.

5.1. GEP Problems with Uncertainty in the Demand Growth Rate

In this section, models are developed to explicitly consider the uncertainty associated with the energy demand. The demand growth rate is an estimate; therefore, it includes the uncertainties. In the previous chapters, an expected or anticipated growth rate is used to uniformly increase the energy demand. However, there are two possible risks involved with this. If the real demand growth is lower than the one used, then there is a potential risk of building large generation units even though there is no need for them. On the other hand, if the real demand rate is higher than the one used, then there is a risk of not being able to satisfy the demand. Therefore, the GEP problem is modeled by explicitly representing the possible growth rates. Scenarios are generated for each growth rate and the expected objective function is minimized. The growth rate scenario δ means that the demand level is increasing by x_δ % in each year.

To show how the model is modified to include the uncertainties in the demand, the IEEE Reliability Test System [87] is used as an example system topology. It was previously depicted in Figure 3.2. In this model, the system expansion is only considered for the central generation units (i.e., not distributed generation) and the distribution system failures are not included in the model, although the failures of the generation units and transmission lines are still considered. Then, the expected total cost is as follows:

$$O_1 = \sum_{t=1}^{T}(1+r)^{-t}\left(\left(\sum_{i\in N}\sum_{q\in\Xi_i}(s_{tiq}-s_{t-1iq})a_{tiq}\right)+\left(\sum_{i\in N}\sum_{k\in\Theta_i} g_{tik}+\sum_{i\in N}\sum_{q\in\Xi_i} s_{tiq}h_{tiq}\right)\right.$$
$$\left.+\sum_{\delta} p_\delta\left(\sum_{n=1}^{\mathbb{N}}\varpi_n\left(\sum_{i\in N}\sum_{k\in\Theta_i} x_{t\delta nik}c_{tik}+\sum_{i\in N}\sum_{q\in\Xi_i} u_{t\delta niq}e_{tiq}+\sum_{i\in N} v_{t\delta ni} f_t\right)\right)\right)$$

Investment decision variables are the first stage variables of the two-stage stochastic programming model. The variables considering the distribution of the uncertainty, the investment and operational and maintenance cost are the same as the previous models. In this model, the second stage variables are the dispatching decisions, and they are affected by the demand change. Therefore, the dimension for those variables is increased and new decision variables are defined as:

$x_{t\delta nik}$: Amount of energy produced by the existing unit k in the power node i for the scenario n in the year t under the growth rate scenario δ.

$u_{t\delta nik}$: Amount of energy produced by the new unit *k* in the power node *i* for the scenario *n* in the year *t* under the growth rate scenario *δ*.

$v_{t\delta ni}$: Amount of unmet energy in the power node *i* for the scenario *n* in the year *t* under the growth rate scenario *δ*.

The expected operational cost is calculated by multiplying the operational cost for each growth rate by the corresponding growth rate probability. It is also required to modify the decision variables for the flow, $\Gamma_{t\delta nimr}$, as the flow (MW) through the r^{th} transmission line between nodes (*i*, *m*) for scenario *n* in time period *t* under the growth rate scenario *δ*. The dimension for operations related constraints are increased. The modified constraints are as follows:

$$\sum_{(m,i,r)\in A}\Gamma_{t\delta nmir}\alpha_{mir}-\sum_{(i,m,r)\in A}\Gamma_{t\delta nimr}+\sum_{k\in\Theta_i}x_{t\delta nik}\sum_{q\in\Xi_i}u_{t\delta niq}+v_{t\delta ni}=D_{t\delta ni}\qquad\forall i\in N,\forall(t,\delta,n)$$
$$\Gamma_{t\delta nimr}-\Gamma_{t\delta nmir}\le\Delta_{tnimr}\qquad\forall(i,m,r)\in A^{'},\forall(t,\delta,n)$$
$$\Gamma_{t\delta nmir}-\Gamma_{t\delta nimr}\le\Delta_{tnimr}\qquad\forall(i,m,r)\in A^{'},\forall(t,\delta,n)$$
$$\Gamma_{t\delta nimr}-\Gamma_{t\delta nmir}\le\delta_{imr}p_{imr}\qquad\forall(i,m,r)\in A^{''},\forall(t,\delta,n)$$
$$\Gamma_{t\delta nmir}-\Gamma_{t\delta nimr}\le\delta_{imr}p_{imr}\qquad\forall(i,m,r)\in A^{''},\forall(t,\delta,n)$$
$$x_{t\delta nik}\le\gamma_{tnik}\qquad\forall i\in N,k\in\Theta_i^{'},\forall(t,\delta,n)$$
$$x_{t\delta nik}\le\mu_{ik}p_{ik}\qquad\forall i\in N,k\in\Theta_i^{''},\forall(t,\delta,n)$$
$$u_{t\delta niq}\le\mu_{iq}p_{iq}s_{tiq}\qquad\forall i\in N,q\in\Xi_i,\forall(t,\delta,n)$$
$$s_{tiq}\ge s_{(t-1)iq}\qquad\forall i\in N,q\in\Xi_i,\forall t=2,...,T$$
$$s_{tiq}\in\{0,1\}\qquad\forall i\in N,q\in\Xi_i,\forall t$$
$$x_{t\delta nik}\ge 0\qquad\forall i\in N,k\in\Theta_i,\forall(t,\delta,n)$$
$$u_{t\delta niq}\ge 0\qquad\forall i\in N,q\in\Xi_i,\forall(t,\delta,n)$$
$$\Gamma_{t\delta nimr}\ge 0\qquad\forall(i,m,r)\in A,\forall(t,\delta,n)$$
$$v_{t\delta ni}\ge 0\qquad\forall i\in N,\forall(t,\delta,n)$$

where $D_{t\delta ni}$ is the demand level in the node *i* for the scenario *n* with the growth rate δ in time period *t*.

5.2. Risk measures for the GEP problem

Two risk measures are presented for the GEP problem to model uncertainty behavior and to reflect decision makers risk preferences. In Chapter 2, some risk measures for general problems are presented so that the trade-off solutions for risk averse decision makers can be found. Here, the mathematical formulation is presented for minimizing the maximum regret and CVaR. The models presented and subsequent results represent a fundamentally new approach to the problem.

Regret or excess is defined as the value over a predefined target level. CVaR is defined as the expectation of the $(1-\alpha)\times 100\%$ worst outcomes for a given probability level $\alpha \in (0,1)$.

Define *I* as the investment and fixed operational and maintenance costs and O_δ as the operational costs associated to the growth rate scenario δ.

$$I = \sum_{t=1}^{T}(1+r)^{-t}\left(\left(\sum_{i\in N}\sum_{q\in\Xi_i}(s_{tiq}-s_{(t-1)iq})a_{tiq}\right)+\left(\sum_{i\in N}\sum_{k\in\Theta_i}g_{tik}+\sum_{i\in N}\sum_{q\in\Xi_i}s_{tiq}h_{tiq}\right)\right.$$

$$O_\delta = \sum_{t=1}^{T}(1+r)^{-t}\left(\sum_{n=1}^{\mathbb{N}}\varpi_n\left(\sum_{i\in N}\sum_{k\in\Theta_i}x_{t\delta nik}c_{tik}+\sum_{i\in N}\sum_{q\in\Xi_i}u_{t\delta niq}e_{tiq}+\sum_{i\in N}v_{t\delta ni}f_t\right)\right)$$

An adjustment is made to calculate the regret. Instead of defining the regret as value over a predefined target level, the regret is defined as the value over the cost that would have been occurred if it is known which scenario would take place as in [48]. Therefore, if z_δ

is the optimal cost when the model is solved by considering the demand growth rate is x_δ % in each year. Then, the following constraints are added to the model in Section 5.1.

$$\begin{aligned} & I + O_\delta - z_\delta \leq \lambda \qquad \forall \delta \\ & \lambda_\delta \geq 0 \qquad \forall \delta \end{aligned}$$

Then, to obtain the mean-risk stochastic model to minimize the expected cost and maximum regret, the objective function is modified as follows.

$$\min I + p_\delta O_\delta + \rho\lambda$$

where $\rho \geq 0$.

For the CVaR, z_δ is replaced by a continuous first stage variable. Therefore, the constraints added to the model in Section 5.1 are as follows.

$$\begin{aligned} & I + O_\delta - \eta \leq \lambda_\delta \qquad \forall \delta \\ & \lambda_\delta \geq 0 \qquad \forall \delta \end{aligned}$$

The objective function to minimize the expected cost and CVaR for a given probability level $\alpha \in (0,1)$ is as follows:

$$\min I + p_\delta O_\delta + \rho\left(\eta + \frac{1}{(1-\alpha)}\sum_\delta p_\delta \lambda_\delta\right).$$

5.2.1. Numerical Examples for Determination of Trade-off Solutions between Risk Measures and Expected Cost

To demonstrate the approach, three example problems are solved by using the mean-risk stochastic integer GEP models. The IEEE Reliability Test System (Figure 3.2) is used as the existing system topology. In these examples, the intent is to find the investment plan for a 15 year planning horizon. The planning horizon is divided into three time periods of five years each. Therefore, if a new generation unit is to be installed, the options are to install it as soon as possible, in five years, or in ten years for the current period. Each year divided into four segments and a representative demand level is chosen for each segment (Table 3.16). All the components are considered to be working at their expected capacity in each segment (cost information of the generation units in the existing system is in Table 3.1). It is also considered that there are three new generation units for each power node, namely wind turbines, nuclear plants and CCGTs (cost information is in Table 3.2). The constraint which restricts the number of nuclear plants built to one is also added to the model. The demand growth rate is considered as uncertain and the demand growth rate scenarios are generated. In Table 5.1, 5.2, and 5.3, the demand growth rate scenarios and their probabilities are presented for example problems 1, 2 and 3 respectively.

In Example 1, a small problem is solved with four scenarios. The idea is to investigate the impact on the optimal expansion plan, if there is one large demand growth rate with very small probability. In this example, the demand growth rate is smaller or equal to 2.5 with the probability of 99%. Example Problem 2 is a larger problem. In this example, the demand growth rate is varying between 1% and 6%, but this time, the probability of having growth rate larger than 2.5 is higher than in the previous example. In addition, instead of having one large demand growth rate with small probability, multiple scenarios are generated for the larger demand growth rate. In Example 3, a larger problem is solved to see how the investment decisions change if there is a high variation in the growth rate and the probability of having demand growth rate larger than 2.5% is increased to 0.48.

Table 5.1. Demand growth rates and the probabilities for demand growth scenarios used in Example Problem 1

Demand Growth Rate Scenarios	Growth Rate (%)	Probability
1	1	0.4
2	2	0.36
3	2.5	0.23
4	6	0.01

Table 5.2. Demand growth rates and the probabilities for demand growth scenarios used in Example Problem 2

Demand Growth Rate Scenarios	Growth Rate (%)	Probability
1	1	0.3
2	1.5	0.15
3	2	0.25
4	2.5	0.17
5	3	0.05
6	3.5	0.05
7	4	0.01
8	4.5	0.01
9	5	0.005
10	6	0.005

Table 5.3. Demand growth rates and the probabilities for demand growth scenarios used in Example Problem 3

Demand Growth Rate Scenarios	Growth Rate (%)	Probability
1	0.5	0.01
2	1	0.25
3	1.5	0.1
4	2	0.08
5	2.5	0.08
6	3	0.08
7	3.5	0.05
8	4	0.05
9	4.5	0.05
10	5	0.04
11	5.5	0.04
12	6	0.04
13	6.5	0.02
14	7	0.02
15	7.5	0.02
16	8	0.02
17	8.5	0.0125
18	9	0.0125
19	9.5	0.0125
20	10	0.0125

5.2.1.1. Numerical Examples for Determination of Trade-off Solutions for Maximum Regret and Expected Cost

In this section, the example problems are solved to minimize the expected cost and the maximum regret for different ρ values and the results are presented.

Example problem 1 is solved for each demand growth rate $x_\delta\%$ in each year to find the corresponding z_δ. The demand growth scenarios and the optimum costs, when the model is solved by when the demand growth rate is $x_\delta\%$ in each year, are given in Table 5.4.

Table 5.4. Optimal objective function value obtained solving the problem for each demand growth scenario individually

Demand Growth Rate Scenarios	z_δ
1	15.41
2	16.40
3	17.51
4	91.12

Then, Example 1 is solved to minimize the expected cost and the maximum regret with different ρ factors. The expected cost, maximum regret and the investment plan are given in Table 5.5. When ρ is increased, the cost increases while the maximum regret decreases. There are more investments for the higher ρ values and the investments are done in the earlier stage of the planning horizon. Since the probability of having 6% of demand growth rate is very small, when the objective is to minimize the expected cost, the effect of this growth rate is very small. However, even though the probability of this scenario is very small, it might occur and if the investment is done without considering this possibility, the regret can be very large. Therefore, for the larger ρ values, more generation units are built in the earlier time periods to minimize this regret.

Table 5.5. Cost, maximum regret and expansion plan for Example 1

ρ	Cost	Maximum Regret	Expansion Plan (year: power nodes)		
			CCGT	Wind	Nuclear
0	18.31	Not defined	(1: 2) (6: 1,13,1516,18,21,23) (11: 7,22)	-	-
0.001	18.31	91.66	(1: 2) (6: 1,13,1516,18,21,23) (11: 7,22)	-	-
0.1	20.45	4.28	(1: 2,13,16) (6: 1,7,15,18,21,22,23)	(11: 1,2,7,13,15,16,18,21,22,23)	(11: 16)
0.5,1,10,100	20.49	4.18	(1: 2,7,13,16) (6: 1,15,18,21,22,23)	(11: 1,2,7,13,15,16,18,21,22,23)	(11: 16)

Example 2 is solved for each growth rate scenario to find the corresponding objective values z_δ, and the results are given in Table 5.6. The mean-risk stochastic model is solved for different ρ for Example 2 and the solutions are given in Table 5.7. The similar results are obtained as in Example 1. The cost presented in the following tables is in term of billions of dollars.

Table 5.6. Optimal objective function value obtained solving the problem for each demand growth scenario individually

Demand Growth Rate Scenarios	z_δ
1	15.41
2	15.86
3	16.40
4	17.51
5	19.12
6	21.27
7	24.79
8	29.18
9	36.19
10	91.12

Table 5.7. Cost, maximum regret and expansion plan for Example 2

ρ	Cost	Maximum Regret	Expansion Plan (year: power nodes)		
			CCGT	Wind	Nuclear
0	18.833	Not defined	(1: 2) (6: 1,13,15,16,18,21,23) (11: 7,22)	-	-
0.001	18.834	89.810	(1: 2) (6: 1,13,15,16,18,21,22,23) (11: 7)	-	-
0.1	20.415	4.283	(1: 2,13,16) (6: 1,7,15,18,21,22,23)	(11: 1,2,7,13,15,16,18,21,22,23)	(11: 16)
0.5	20.453	4.186	(1: 2,13,16,23) (6: 1,7,15,18,21,22)	(11: 1,2,7,13,15,16,18,21,22,23)	(11: 16)
1,10,100	20.453	4.185	(1: 2,7,13,16) (6: 1,15,18,21,22,23)	(11: 1,2,7,13,15,16,18,21,22,23)	(11: 16)

Example Problem 3 is solved for each growth rate scenario to find the corresponding objective values z_δ, and the results are given in Table 5.8. The mean-risk stochastic model is solved for different ρ and the solutions are given in Table 5.9. For this example, if the risk is introduced to the model, the investments are done in the later time of the planning horizon. The results show that the main risk encountered here is having more investment than needed. This also shows that introducing risk measures into the model changes the investment decisions and can minimize the impact of risk.

Table 5.8. Optimal objective function value obtained solving the problem for each demand growth scenario individually

Demand Growth Rate Scenarios	z_δ	Demand Growth Rate Scenarios	z_δ
1	15.017	11	56.918
2	15.406	12	91.116
3	15.857	13	137.095
4	16.399	14	193.076
5	17.509	15	260.058
6	19.121	16	334.848
7	21.272	17	418.063
8	24.786	18	517.400
9	29.181	19	632.397
10	36.194	20	758.634

Table 5.9. Cost, maximum regret and expansion plan for Example 3

P	Cost	Maximum Regret	Expansion Plan (year: power nodes)		
			CCGT	Wind	Nuclear
0	71.807	Not defined	(1: 1,2,13,15,16,18,23) (6: 7,21,22)	(6: 1,2,7,13,15,16,18,21,22,23)	(6: 16)
0.001	71.809	6.156	(1: 1,2,13,15,16,23) (6: 7,18,21,22)	(6: 1,2,7,13,15,16,18,21,22,23)	(6: 16)
0.1	71.837	6.058	(1: 1,2,13,15,16) (6: 7,18,21,22,23)	(6: 1,2,13,15,16,18,21,22,23) (11: 7)	(6: 16)
0.5	71.908	5.959	(1: 2,13,15,16) (6: 1,7,18,21,22,23)	(6: 1,2,13,15,16,18,21,23) (11: 7,22)	(6: 16)
1	72.056	5.924	(1: 2,13) (6: 1,7,15,16,18,21,22,23)	(6: 1,2,13,15,16,18,21,22,23) (11: 7)	(6: 16)
10,100	72.075	5.922	(1: 2,13) (6: 1,7,15,16,18,21,23)	(6: 1,2,7,13,15,16,18,21,22,23) (11: 22)	(6: 16)

5.2.1.2. Numerical Examples for Determination of Trade-off Solutions for CVaR and Expected Cost

In this section, the example problems are solved to minimize the expected cost and the CVaR for different ρ and α values and the results are presented.

Example Problem 1 is solved to minimize the expected cost and CVaR with different ρ and α values and the results are given in Tables 5.10 and 5.11 for α=0.95 and α=0.99 respectively. In the case of α=0.95, more generation units are built to decrease the CVaR while the ρ value increases.

As a rule, an increase in the α value results in a corresponding increase in the CVaR. Therefore, for the α=0.99, larger CVaR values are found. However, to decrease the CVaR, more generation units are built in the earlier time of the planning horizon as can be seen in Table 5.11.

Table 5.10. Cost, CVaR and expansion plan for Example 1 where α=0.95

P	Cost	Maximum Regret	Expansion Plan (year: power nodes)		
			CCGT	Wind	Nuclear
0	18.312	Not defined	(1: 2) (6: 1,13,15,16,18,21,23) (11: 7,22)	-	-
0.001	18.312	50.580	(1: 2) (6: 1,13,15,16,18,21,23) (11: 7,22)	-	-
0.1	19.312	37.785	(1: 2) (6: 1,7,13,15,16,18,21,23) (11: 22)	=	(11: 16)
0.5	20.304	35.191	(1: 2,16) (6: 1,7,13,15,18,21,22,23)	(11: 1,2,13,15,16,18,21,22,23)	(11: 16)
1	20.446	34.962	(1: 2,13,16) (6: 1,7,15,18,21,22,23)	(11: 1,2,7,13,15,16,18,21,22,23)	(11: 16)
10,100	20.530	34.929	(1: 1,2,13,15,16) (6: 7,18,21,22,23)	(11: 1,2,7,13,15,16,18,21,22,23)	(11: 16)

Table 5.11 Cost, CVaR and expansion plan for Example 1 where α=0.99

ρ	Cost	Maximum Regret	Expansion Plan (year: power nodes)		
			CCGT	Wind	Nuclear
0	18.312	Not defined	(1: 2) (6: 1,13,15,16,18,21,23) (11: 7,22)	-	-
0.001	18.312	182.781	(1: 2) (6: 1,13,15,16,18,21,23) (11: 7,22)	-	-
0.1	20.446	95.400	(1: 2,13,16) (6: 1,7,15,18,21,22,23)	(11: 1,2,7,13,15,16,18,21,22,23)	(11: 16)
0.5	21.570	91.618	(1: 1,2,13,15,16,23) (6: 7,18,21,22)	(11: 1,2,7,13,15,16,18,21,22,23)	(6: 16)
1	21.725	91.436	(1: 1,2,13,15,16,23) (6: 7,18,21,22)	(6: 1,2,16) (11: 7,13,15,18,21,22,23)	(6: 16)
10,100	22.181	91.116	(1: 1,2,13,15,16,18,21,23) (6: 7,16,18,21,22,23)	(6: 1,2,7,13,15,16,22)	(6: 16)

Example Problem 2 is also solved to minimize the expected cost and CVaR with different ρ and α values and the results are given in Tables 5.12 and 5.13 for α=0.95 and α=0.99 respectively. For both α values, an increase in ρ results in more generation units in earlier time periods. In the case where α=0.99, more generation units are built in the earlier time periods to decrease the CVaR compared to the case where α is 0.95.

Table 5.12. Cost, CVaR and expansion plan for Example 2 where α=0.95

ρ	Cost	Maximum Regret	Expansion Plan (year: power nodes)		
			CCGT	Wind	Nuclear
0	18.833	Not defined	(1: 2) (6: 1,13,15,16,18,21,23) (11: 7,22)	-	-
0.001	18.834	51.864	(1: 2) (6: 1,13,15,16,18,21,22,23) (11: 7)	-	-
0.1	19.398	35.645	(1: 2,16) (6: 1,7,13,15,18,21,22,23)	-	(11: 16)
0.5	20.306	33.043	(1: 2,13,16) (6: 1,7,15,18,21,22,23)	(11: 1,2,13,15,16,18,21,22,23)	(11: 16)
1	20.454	32.870	(1: 2,13,15,16) (6: 1,7,18,21,22,23)	(11: 1,2,7,13,15,16,18,21,22,23)	(11: 16)
10,100	21.464	32.606	(1: 1,2,13,15,16) (6: 7,18,21,22,23)	(11: 1,2,7,13,15,16,18,21,22,23)	(6: 16)

Table 5.13. Cost, CVaR and expansion plan for Example 2 where α=0.99

ρ	Cost	Maximum Regret	Expansion Plan (year: power nodes)		
			CCGT	Wind	Nuclear
0	18.833	Not defined	(1: 2) (6: 1,13,15,16,18,21,23) (11: 7,22)	-	-
0.001	18.833	131.195	(1: 2) (6: 1,13,15,16,18,21,22,23) (11: 7)	-	-
0.1	20.306	67.500	(1: 2,13,16) (6: 1,7,15,18,21,22,23)	(11: 1,2,13,15,16,18,21,22,23)	(11: 16)
0.5	21.464	63.984	(1: 1,2,13,15,16) (6: 7,18,21,22,23)	(11: 1,2,7,13,15,16,18,21,22,23)	(6: 16)
1	21.507	63.917	(1: 1,2,13,15,16,23) (6: 7,18,21,22)	(11: 1,2,7,13,15,16,18,21,22,23)	(6: 16)
10,100	21.977	63.680	(1: 1,2,13,15,16,23) (6: 7,18,21,22)	(6: 1,2,13,15,16,18,21,22,23) (11: 7)	(6: 16)

The solutions for Example Problem 3 is given in Tables 5.14 and 5.15 for the cases of α=0.95 and α=0.99 respectively. When the risk is introduced into the model, either less generation units are built, or the investments are done in the generation units in the earlier stage of the planning horizon.

Table 5.14. Cost, CVaR and expansion plan for Example 3 where α=0.95

ρ	Cost	Maximum Regret	Expansion Plan (year: power nodes)		
			CCGT	Wind	Nuclear
0	71.807	Not defined	(1: 1,2,13,15,16,18,23) (6: 7,21,22)	(6: 1,2,7,13,15,16,18,21,22,23)	(6: 16)
0.001	71.807	760.178	(1: 1,2,13,15,16,18,23) (6: 7,21,22)	(6: 1,2,7,13,15,16,18,21,22,23)	(6: 16)
0.1	71.841	581.972	(1: 1,2,13,15,16,18,21,22,23) (6: 7)	(6: 1,2,7,13,15,16,18,21,22,23)	(6: 16)
0.5,1,10	71.869	581.789	(1: 1,2,7,13,15,16,18,21,22,23)	(6: 1,2,7,13,15,16,18,21,22,23)	(6: 16)
100	71.967	581.787	(1: 1,2,7,13,15,16,18,21,22,23)	(6: 1,7,13,15,16,18,21,22,23)	(1: 16)

Table 5.15. Cost, CVaR and expansion plan for Example 3 where α=0.99

ρ	Cost	Maximum Regret	Expansion Plan (year: power nodes)		
			CCGT	Wind	Nuclear
0	71.807	Not defined	(1: 1,2,13,15,16,18,23) (6: 7,21,22)	(6: 1,2,7,13,15,16,18,21,22,23)	(6: 16)
0.001	71.807	760.178	(1: 1,2,13,15,16,18,23) (6: 7,21,22)	(6: 1,2,7,13,15,16,18,21,22,23)	(6: 16)
0.1	71.841	759.496	(1: 1,2,13,15,16,18,21,22,23) (6: 7)	(6: 1,2,7,13,15,16,18,21,22,23)	(6: 16)
0.5,1	72.061	758.666	(1: 1,2,7,13,15,16,18,21,22,23)	(6: 1,2,7,13,15,16,18,21,22,23)	(6: 15)
10	72.260	758.642	(1: 1,2,7,13,15,16,18,21,22,23)	(1: 2,15,16,18) (6: 1,7,13,21,22,23)	(6: 15)
100	72.361	758.634	(1: 1,2,7,13,15,16,18,21,22,23)	(1: 1,2,13,15,16,18) (6: 7,21,22,23)	(6: 15)

6. New Approaches to solve Multi-Objective Optimization Problems

In this chapter, new approaches are presented for the multi-objective GEP problems which are based on normal boundary and weighted Chebychev method. In Chapter 2, NBI and Augmented Weighted Chebychev methods were described together with the associated advantages and drawbacks. The main advantages of NBI method are that there is no need for scaling the objective functions and it does not require the convexity assumption for the objective functions space. In the example model solved in Chapter 3, it is observed that the scaling impacts the resulting solutions for GEP problems. Therefore, two new approaches are provided to obtain benefits from this property of NBI. The Chebychev method helps to resolve the drawback associated with NBI, that is NBI does not guarantee to find the solution when there are integer variables in the model.

In the first approach, the GEP problem is relaxed by replacing the binary variable for each investment decision with a continuous variable bounded by 0 and 1. The NBI approach is applied on this problem to obtain uniformly distributed Pareto solutions. Chebychev method is then exploited to search integer solutions around the solutions found in the first step. The second approach is based on relaxing the constraint for the total cost in NBI method which guarantees that the solution is actually on the normal vector. By this approach, not only the solutions on the normal vector but also the solutions around of the bounded area of the normal vector are considered.

6.1. New Method 1: Combination of NBI and Augmented Weighted Chebychev Methods

In this section, a procedure is presented to solve multi-objective integer optimization problems. In Chapter 2, NBI and Augmented Weighted Chebychev methods were described separately with the associated advantages and drawbacks. There is another drawback associated with the NBI method for problems that involve integer variables. When problems have integer variables, there is no guarantee that there will be a feasible integer solution along the normal vector. This means that for some **w**, the problem $\text{NBI}_{\mathbf{w}}$ could be infeasible. **w** is defined as the vector of the weights used to determine the point in the convex hull of individual minima for NBI approach and $\text{NBI}_{\mathbf{w}}$ is the problem solved by using the corresponding point. The GEP problem defined here is an integer optimization problem, but NBI method can still be used to get benefit from its advantages. The NBI method provides the independence of the relative scales of the objective functions and the ability of producing evenly distributed set of solutions in the Pareto set. A method was developed for multi-objective GEP problems, which is a combination of NBI and Chebychev method. The procedure is as follows:

Step 1: Solve Relaxed Problem with NBI method

- Replace the constraint for binary variables with a constraint set stating that the binary variables are continuous and bounded between 0 and 1. In the case of GEP problem, the constraint $s_{tiq} \in \{0,1\}, \forall i \in N, q \in \Xi_i, \forall t$ is replaced by $0 \le s_{tiq} \le 1, \forall i \in N, q \in \Xi_i, \forall t$.

- For each weight combination **w**, solve the $\text{NBI}_{\mathbf{w}}$. Define $\bar{f}_{\mathbf{w}}^{i}$ as the value of the i^{th} objective function obtained by solving $\text{NBI}_{\mathbf{w}}$. If the solution is not integer, go to Step 2.

Step 2: Use the solution obtained in NBI as reference point for Augmented Weighted Chebychev method and find the integer solutions around the reference point.

- Define the Augmented Weighted Chebychev problem for the weight combination **w** as $\min_{x \in X} \max_{i} v_i (f_i(\mathbf{x}) - \bar{f}_i^{\mathbf{w}}) + \gamma \sum_{i=1}^{p} (f_i(\mathbf{x}) - \bar{f}_i^{\mathbf{w}})$ where $\bar{v}_i = \dfrac{v_i}{\max f_i - \min f_i}$, where $v_i = \dfrac{1}{p}$ and p is the number of objective functions; $\bar{\gamma} = \dfrac{\gamma}{\max f_i - \min f_i}$, where γ is the selected number.

6.1.1. Numerical Example for the Proposed Approach 1

To demonstrate the presented method, an example problem is solved for 15 years planning horizon. The planning horizon is divided into three time periods of five years each. Therefore, if a new generation unit is to be installed, the options are to install it as soon as possible, in five years, or in ten years for the current period. Each year is divided into four segments and a representative demand level is chosen for each segment (Table 3.16). The topology used for this problem is obtained by modifying the IEEE Reliability Test System [87] and previously depicted in Figure 3.2. It is considered that there is enough transmission capacity available and there are no transmission losses in the system. It is also considered that it is possible to expand the system with three central generation units, namely wind turbines, nuclear plants and CCGTs. The corresponding data for these units was previously presented in Table 3.2. The model also includes a constraint to restrict the number of nuclear plants build throughout the planning horizon.

The availability scenarios for the nine critical generation units are generated and presented in Table 6.1. After applying the procedure to reduce the number of scenarios, a subset of these availability scenarios is chosen for each segment. The selected scenarios and their probabilities are presented in Table 6.2. It is assumed that the critical generation units in the segment four is working at their expected capacity level.

Table 6.1. The availability scenarios used for the examples solved by the proposed method

Avail. Scenarios	Critical Generation Units (power node number generation unit number)								
	13	13	13	23	23	23	16	18	21
	1	2	3	1	2	3	1	1	1
1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	
3	1	1	1	1	1	1	1		1
4	1	1	1	1	1	1		1	
5	1	1	1	1	1		1	1	1
6	1	1	1	1			1	1	1
7	1	1	1			1	1	1	
8	1	1		1	1		1	1	1
9	1	1		1	1		1	1	
10	1		1	1	1	1	1	1	
11		1	1	1	1	1	1	1	
12		1	1	1	1	1	1		1
13		1	1		1	1	1	1	1

Table 6.2. Availability scenarios and their probabilities for each segment

Segments	Availability Scenarios												
	1	2	3	4	5	6	7	8	9	10	11	12	13
1	0.661	0.005	0.096	0.04	0.008	0.022	0.018	0.047	0.003	0.05	0.004	0.024	0.022
2	0.661	0.005	0.096	0.04	0.008	0.022	0.018	0.047	0.003	0.05	0.004	0.024	0.022
3	0.997	-	-	-	-	-	-	-	0.003	-	-	-	-
4	1	-	-	-	-	-	-	-	-	-	-	-	-

The objective of the problem is to minimize the cost and air emissions (CO_2 and NO_x) simultaneously. The cost consists of the investment cost, fixed operation and maintenance cost, generation cost and unmet demand. For the air emissions, the CO_2 and NO_x emissions are considered. The mathematical representations of the objective functions are as follows.

Total Cost

$$O_1 = \sum_{t=1}^{T} (1+r)^{-t} \left(\left(\sum_{i \in N} \sum_{q \in \Xi_i} (s_{tiq} - s_{t-1iq}) a_{tiq} \right) + \left(\sum_{i \in N} \sum_{k \in \Theta_i} g_{tik} + \sum_{i \in N} \sum_{q \in \Xi_i} s_{tiq} h_{tiq} \right) \right.$$
$$\left. + \sum_{n=1}^{\mathbb{N}} \varpi_n \left(\sum_{i \in N} \sum_{k \in \Theta_i} x_{tnik} c_{tik} + \sum_{i \in N} \sum_{q \in \Xi_i} u_{tniq} e_{tiq} + \sum_{i \in N} v_{tni} f_t \right) \right)$$

Total CO_2 Emissions

$$\sum_{t=1}^{T} \sum_{n=1}^{\mathbb{N}} \varpi_n \left(\sum_{i \in N} \sum_{k \in \Theta_i} x_{tnik} A_{ik} + \sum_{i \in N} \sum_{q \in \Xi_i} u_{tniq} B_{iq} \right)$$

where A_{ik} and B_{iq} are the CO_2 emissions of the existing and new generation units.

Total NO_x Emissions

$$\sum_{t=1}^{T} \sum_{n=1}^{\mathbb{N}} \varpi_n \left(\sum_{i \in N} \sum_{k \in \Theta_i} x_{tnik} C_{ik} + \sum_{i \in N} \sum_{q \in \Xi_i} u_{tniq} D_{iq} \right)$$

where C_{ik} and D_{iq} are the NO_x emissions of the existing and new generation units.

The shadow minimum F^* is defined as the vector containing the individual global minima of the objectives. Therefore, the problem is solved separately for each individual objective function to get individual minima, f_i^*. If x_i^* is the solution obtained when the objective is to minimize f_i individually, then the matrix F_i^* is equal to $F(x_i^*)$ where i=1,2,3. Then, the matrix Φ is defined as to be 3×3 matrix whose i^{th} column is $F_i^* - F^*$.

The equal step-sizes are used on all weights (w_i) and the procedure presented [76] is conducted to find the uniformly spaced weights. The weights obtained are given in Table 6.3.

Table 6.3. Weights selected for the example problem

Weight comb.	Cost	CO_2	NO_x	Weight comb.	Cost	CO_2	NO_x
1	0.9	0.1	0	11	0.6	0.3	0.1
2	0.9	0	0.1	12	0.6	0.2	0.2
3	0.8	0.2	0	13	0.6	0.1	0.3
4	0.8	0.1	0.1	14	0.6	0	0.4
5	0.8	0	0.2	15	0.5	0.5	0
6	0.7	0.3	0	16	0.5	0.4	0.1
7	0.7	0.2	0.1	17	0.5	0.3	0.2
8	0.7	0.1	0.2	18	0.5	0.2	0.3
9	0.7	0	0.3	19	0.5	0.1	0.4
10	0.6	0.4	0	20	0.5	0	0.5

The first step is to relax the constraints and solve the problem with $NBI_{\mathbf{w}}$ for each weight combination **w**. Define z_i as the value in the i^{th} row of the vector of $\Phi\mathbf{w}$ and β_i as the value in the i^{th} row of the vector of $\hat{n} = -\Phi e$ where e is the column vector of all ones. Then, the mathematical model for the weight combination **w** is as follow.

max $\quad t$

s.t.

$$z_1 + t\beta_1 = \sum_{t=1}^{T}(1+r)^{-t}\left(\left(\sum_{i\in N}\sum_{q\in\Xi_i}(s_{tiq}-s_{t-1iq})a_{tiq}\right)+\left(\sum_{i\in N}\sum_{k\in\Theta_i}g_{tik}+\sum_{i\in N}\sum_{q\in\Xi_i}s_{tiq}h_{tiq}\right)\right.$$
$$\left.+\sum_{n=1}^{\mathbb{N}}\varpi_n\left(\sum_{i\in N}\sum_{k\in\Theta_i}x_{tnik}c_{tik}+\sum_{i\in N}\sum_{q\in\Xi_i}u_{tniq}e_{tiq}+\sum_{i\in N}v_{tni}f_t\right)\right)-f_1^*$$

$$z_2 + t\beta_2 = \sum_{t=1}^{T}\left(\sum_{n=1}^{\mathbb{N}}\varpi_n\left(\sum_{i\in N}\sum_{k\in\Theta_i}x_{tnik}A_{ik}+\sum_{i\in N}\sum_{q\in\Xi_i}u_{tniq}B_{iq}+\sum_{i\in N}v_{tni}f_t\right)\right)-f_2^*$$

$$z_3 + t\beta_3 = \sum_{t=1}^{T}\left(\sum_{n=1}^{\mathbb{N}}\varpi_n\left(\sum_{i\in N}\sum_{k\in\Theta_i}x_{tnik}C_{ik}+\sum_{i\in N}\sum_{q\in\Xi_i}u_{tniq}D_{iq}+\sum_{i\in N}v_{tni}f_t\right)\right)-f_3^*$$

$$\sum_{(m,i,r)\in A}\Gamma_{tnmir}-\sum_{(i,m,r)\in A}\Gamma_{tnimr}+\sum_{k\in\Theta_i}x_{tnik}+\sum_{q\in\Xi_i}u_{tniq}+v_{tni}=D_{tni} \qquad \forall i\in N,\forall(t,n)$$

$$x_{tnik}\le\gamma_{tnik} \qquad \forall i\in N,k\in\Theta_i^{'},\forall(t,n)$$

$$x_{tnik}\le\mu_{ik}p_{ik} \qquad \forall i\in N,k\in\Theta_i^{''},\forall(t,n)$$

$$u_{tniq}\le\mu_{iq}p_{iq}s_{tiq} \qquad \forall i\in N,q\in\Xi_i,\forall(t,n)$$

$$s_{tiq}\ge s_{(t-1)iq} \qquad \forall i\in N,q\in\Xi_i,\forall t=2,...,T$$

$$0\le s_{tiq}\le 1 \qquad \forall i\in N,q\in\Xi_i,\forall t$$

$$x_{tnik}\ge 0 \qquad \forall i\in N,k\in\Theta_i,\forall(t,n)$$

$$u_{tniq}\ge 0 \qquad \forall i\in N,q\in\Xi_i,\forall(t,n)$$

$$\Gamma_{tnimr}\ge 0 \qquad \forall(i,m,r)\in A,\forall(t,n)$$

$$v_{tni}\ge 0 \qquad \forall i\in N,\forall(t,n)$$

In this formulation, t represent the distance and the objective is to maximize t to find the solution located at the intersection of the normal vector emanating from the selected point in the convex hull of individual minima and the boundary of the objective space.

The solution obtained in the first step is given in Table 6.4. The table presents the objective function values for each weight combinations.

Table 6.4. Objective functions obtained in the first step of the proposed approach

Weight. Comb.	Cost (billions $)	CO_2 (thousands of tons)	NO_x (thousands of tons)	Weight. Comb.	Cost (billions $)	CO_2 (thousands of tons)	NO_x (thousands of tons)
1	15.73	112,344.94	201.48	11	16.82	78,367.41	120.24
2	15.72	112,355.35	200.99	12	16.81	78,377.14	119.74
3	15.85	100,111.68	172.34	13	16.81	78,391.23	119.25
4	15.84	100,120.33	171.84	14	16.81	78,402.30	118.75
5	15.84	100,130.75	171.35	15	17.81	69,358.04	99.54
6	16.10	88,392.20	144.47	16	17.81	69,372.03	99.06
7	16.10	88,400.13	143.96	17	17.81	69,385.19	98.57
8	16.10	88,412.44	143.47	18	17.81	69,400.13	98.08
9	16.10	88,424.76	142.98	19	17.81	69,415.89	97.60
10	16.82	78,353.23	120.72	20	17.81	69,426.75	97.10

If the solution for the weight combination **w** is integer, then there is no need for the second step. If not, for those cases, define $\overline{f}_i^{\mathbf{w}}$ as the value of the i^{th} objective function for the weight combination **w** and solve the following Chebychev model. For our example, the solution for every weight combinations includes investment decisions which are not integer. Therefore, the following model is solved for each weight combination.

Chebychev Model:

$$\min \quad \alpha + \bar{\gamma}\left(\sum_{t=1}^{T}(1+r)^{-t}\left(\left(\sum_{i\in N}\sum_{q\in\Xi_i}(s_{tiq}-s_{t-1iq})a_{tiq} \right) + \left(\sum_{i\in N}\sum_{k\in\Theta_i} g_{tik} + \sum_{i\in N}\sum_{q\in\Xi_i} s_{tiq}h_{tiq} \right) + \sum_{n=1}^{\mathbb{N}}\varpi_n\left(\sum_{i\in N}\sum_{k\in\Theta_i} x_{tnik}c_{tik} + \sum_{i\in N}\sum_{q\in\Xi_i} u_{tniq}e_{tiq} + \sum_{i\in N} v_{tni} f_t \right)\right) - f_1^w \right)$$

$$+\bar{\gamma}\left(\sum_{t=1}^{T}\sum_{n=1}^{\mathbb{N}}\varpi_n\left(\sum_{i\in N}\sum_{k\in\Theta_i} x_{tnik}A_{ik} + \sum_{i\in N}\sum_{q\in\Xi_i} u_{tniq}B_{iq} \right)\right) - f_2^w \right)$$

$$+\bar{\gamma}\left(\sum_{t=1}^{T}\sum_{n=1}^{\mathbb{N}}\varpi_n\left(\sum_{i\in N}\sum_{k\in\Theta_i} x_{tnik}C_{ik} + \sum_{i\in N}\sum_{q\in\Xi_i} u_{tniq}D_{iq} \right)\right) - f_3^w \right)$$

s.t.

$$\alpha \geq \bar{v}_1\left(\sum_{t=1}^{T}(1+r)^{-t}\left(\left(\sum_{i\in N}\sum_{q\in\Xi_i}(s_{tiq}-s_{t-1iq})a_{tiq} \right) + \left(\sum_{i\in N}\sum_{k\in\Theta_i} g_{tik} + \sum_{i\in N}\sum_{q\in\Xi_i} s_{tiq}h_{tiq} \right) + \sum_{n=1}^{\mathbb{N}}\varpi_n\left(\sum_{i\in N}\sum_{k\in\Theta_i} x_{tnik}c_{tik} + \sum_{i\in N}\sum_{q\in\Xi_i} u_{tniq}e_{tiq} + \sum_{i\in N} v_{tni} f_t \right)\right) - f_1^{\mathbf{w}} \right)\right)$$

$$\alpha \geq \bar{v}_2\left(\sum_{t=1}^{T}\sum_{n=1}^{\mathbb{N}}\varpi_n\left(\sum_{i\in N}\sum_{k\in\Theta_i} x_{tnik}A_{ik} + \sum_{i\in N}\sum_{q\in\Xi_i} u_{tniq}B_{iq} \right)\right) - f_2^{\mathbf{w}} \right)$$

$$\alpha \geq \bar{v}_3\left(\sum_{t=1}^{T}\sum_{n=1}^{\mathbb{N}}\varpi_n\left(\sum_{i\in N}\sum_{k\in\Theta_i} x_{tnik}C_{ik} + \sum_{i\in N}\sum_{q\in\Xi_i} u_{tniq}D_{iq} \right)\right) - f_3^{\mathbf{w}} \right)$$

$$s_{tiq} \in \{0,1\} \qquad \forall i \in N, q \in \Xi_i, \forall t$$

The additional constraints for demand and generation capacity are the same and they are not repeated here.

The Chebychev model is solved for each weight combinations and the trade-off solutions are obtained. The objective function values are presented in Table 6.5. The table shows the trade-off solutions with respect to cost and air emissions.

Table 6.5. Objective function values obtained in the second step of the proposed approach

Weight. Comb.	Cost (billions $)	CO_2 (thousands of tons)	NO_x (thousands of tons)	Weight. Comb.	Cost (billions $)	CO_2 (thousands of tons)	NO_x (thousands of tons)
1	15.71	112,269.97	192.61	11	16.82	78,367.87	120.24
2	15.71	112,287.05	192.66	12	16.82	78,384.05	119.75
3	15.82	99,895.16	152.01	13	16.82	78,397.83	119.26
4	15.82	99,913.90	152.07	14	16.82	78,413.29	118.78
5	15.82	99,930.97	152.12	15	17.95	70,403.95	102.06
6	16.11	88,404.69	135.22	16	17.95	70,417.80	101.57
7	16.10	88,419.75	134.92	17	17.95	70,432.09	101.08
8	16.10	88,431.63	134.69	18	17.95	70,445.41	100.60
9	16.10	88,443.50	134.45	19	17.95	70,458.27	100.10
10	16.82	78,354.13	120.72	20	17.95	70,473.84	99.62

The expansion plans are given in the Table 6.6. The number presented in Table 6.6 represents the year in which the corresponding generation unit is built. The column labeled W stands for the wind turbines, C stands for the CCGT. In the NBI model, the weights are not used or considered as the importance of the objective functions. They are used to scatter the objective function space and find the trade-off solutions. The results indicate that the expansion and dispatching decisions are changing between the weight combinations. When the weight for the cost is relatively higher, the system is expanded by CCGT units to decrease the production from the units with high gas emissions. Further increase in the weights for the gas emissions results in building wind turbines. No nuclear units are built for our example problem.

Table 6.6. Expansion plan obtained by proposed approach

Weight Comb.	Power Nodes									
	1		2		7		13		15	
	C	W	C	W	C	W	C	W	C	W
1			1						11	
2			6						1	
3					1				1	
4	6		11				1		1	
5			1						6	
6	1				1		6			
7	1		1		6					
8	1		1						1	
9	6				1		1		1	
10	1		1		1		1		1	
11	1		1		1		1		1	
12	1				1		1		1	
13			1		1		1		1	
14			1		1		1		1	
15	1	1	1	1	1		1		1	1
16	1		1	6	1	1	1	1	1	1
17	1	6	1	1	1		1		1	1
18	1		1		1	1	1	1	1	
19	1		1	1	1	6	1		1	1
20	1	1	1	1	1	6	1		1	

Weight Comb.	16		18		21		22		23	
	C	W	C	W	C	W	C	W	C	W
1	1				6				6	
2	1		11				6			
3	6						11		6	
4	6									
5	11						6		1	
6	1						6		1	
7	1		1				6			
8	1						6		6	
9	6								1	
10	1		1		1		1			
11	1		1				1		1	
12	1		1		1		1		1	
13	1		1		1		1		1	
14	1		1		1		1		1	
15	1	6	1		1		1		1	
16	1		1		1		1		1	
17	1	1	1		1		1		1	
18	1	6	1	1	1		1		1	
19	1	1	1		1		1		1	
20	1	1	1		1		1		1	

6.2. New Method 2: Relaxation of the NBI constraints

A relaxation method was also developed to overcome the drawback of the NBI method when there are integer variables in the problem. In this method, a new continuous decision variable, η, is defined to represent the total investment cost. Then, the NBI$_\mathbf{w}$ is modified as follows.

$$\max \quad t$$
s.t.

$$z_1 + t\beta_1 = \left(\eta + \sum_{n=1}^{\mathbb{N}} \varpi_n \left(\sum_{t=1}^{T} (1+r)^{-t} \sum_{i\in N} \sum_{k\in\Theta_i} x_{tnik} c_{tik} + \sum_{i\in N} \sum_{q\in\Xi_i} u_{tniq} e_{tiq} + \sum_{i\in N} v_{tni} f_t \right) \right) - f_1^* \tag{57}$$

$$z_2 + t\beta_2 = \sum_{t=1}^{T} \left(\sum_{n=1}^{\mathbb{N}} \varpi_n \left(\sum_{i\in N} \sum_{k\in\Theta_i} x_{tnik} A_{ik} + \sum_{i\in N} \sum_{q\in\Xi_i} u_{tniq} B_{iq} + \sum_{i\in N} v_{tni} f_t \right) \right) - f_2^* \tag{58}$$

$$z_3 + t\beta_3 = \sum_{t=1}^{T} \left(\sum_{n=1}^{\mathbb{N}} \varpi_n \left(\sum_{i\in N} \sum_{k\in\Theta_i} x_{tnik} C_{ik} + \sum_{i\in N} \sum_{q\in\Xi_i} u_{tniq} D_{iq} + \sum_{i\in N} v_{tni} f_t \right) \right) - f_3^* \tag{59}$$

$$\eta \geq \sum_{t=1}^{T} (1+r)^{-t} \left(\left(\sum_{i\in N} \sum_{q\in\Xi_i} (s_{tiq} - s_{t-1iq}) a_{tiq} \right) + \left(\sum_{i\in N} \sum_{k\in\Theta_i} g_{tik} + \sum_{i\in N} \sum_{q\in\Xi_i} s_{tiq} h_{tiq} \right) \right. \tag{60}$$

$$\eta \leq q \left(\sum_{t=1}^{T} (1+r)^{-t} \left(\left(\sum_{i\in N} \sum_{q\in\Xi_i} (s_{tiq} - s_{t-1iq}) a_{tiq} \right) + \left(\sum_{i\in N} \sum_{k\in\Theta_i} g_{tik} + \sum_{i\in N} \sum_{q\in\Xi_i} s_{tiq} h_{tiq} \right) \right) \right) \tag{61}$$

$$\sum_{(m,i,r)\in A} \Gamma_{tnmir} - \sum_{(i,m,r)\in A} \Gamma_{tnimr} + \sum_{k\in\Theta_i} x_{tnik} + \sum_{q\in\Xi_i} u_{tniq} + v_{tni} = D_{tni} \qquad \forall i \in N, \forall (t,n) \tag{62}$$

$$x_{tnik} \leq \gamma_{tnik} \qquad \forall i \in N, k \in \Theta_i^{'}, \forall (t,n) \tag{63}$$

$$x_{tnik} \leq \mu_{ik} p_{ik} \qquad \forall i \in N, k \in \Theta_i^{''}, \forall (t,n) \tag{64}$$

$$u_{tniq} \leq \mu_{iq} p_{iq} s_{tiq} \qquad \forall i \in N, q \in \Xi_i, \forall (t,n) \tag{65}$$

$$s_{tiq} \geq s_{(t-1)iq} \qquad \forall i \in N, q \in \Xi_i, \forall t = 2,...,T \tag{66}$$

$$s_{tiq} \in \{0,1\} \qquad \forall i \in N, q \in \Xi_i, \forall t \tag{67}$$

$$x_{tnik} \geq 0 \qquad \forall i \in N, k \in \Theta_i, \forall (t,n) \tag{68}$$

$$u_{tniq} \geq 0 \qquad \forall i \in N, q \in \Xi_i, \forall (t,n) \tag{69}$$

$$\Gamma_{tnimr} \geq 0 \qquad \forall (i,m,r) \in A, \forall (t,n) \tag{70}$$

$$v_{tni} \geq 0 \qquad \forall i \in N, \forall (t,n) \tag{71}$$

where q represents the relaxation level.

The decision variables which are binary are used to calculate the investment and fixed operational and maintenance cost. Therefore, a new continuous variable defined to represent this cost and to search only the areas which are closer to the normal vector, the defined variable is bounded by the fourth and fifth type of constraints. In original NBI, the first set of constraints guarantees that the solution is on the normal vector. The modified constraints set presented above (Equations 57-61) guarantee that the solution is in the bounded area of the normal vector. The other constraints are the same as the original NBI problem.

6.2.1. Numerical Example for the Proposed Approach 2

The same example problem presented in the Section 6.1.1 is solved with the proposed approach. The trade-off solutions for (q=1.001) are presented in Table 6.7 and the expansion plan is given in Table 6.8. Even though this method could not find the Pareto front solutions for the first nine weight combinations, it does find the solutions very close to the Pareto front. In fact, the number of generation units built for these weight combinations is the same as the one found by the method presented before.

Table 6.7. Objective functions obtained by the second proposed method

Weight. Comb.	Cost (billions $)	CO_2 (thousands of tons)	NO_x (thousands of tons)	Weight. Comb.	Cost (billions $)	CO_2 (thousands of tons)	NO_x (thousands of tons)
1	15.73	112,362.10	201.53	11	16.82	78,367.70	120.24
2	15.73	112,372.51	201.03	12	16.82	78,381.60	119.75
3	15.85	100,127.08	172.38	13	16.82	78,395.49	119.26
4	15.85	100,137.49	171.89	14	16.82	78,409.38	118.77
5	15.85	100,147.90	171.39	15	18.00	70,048.69	101.24
6	16.12	88,458.46	144.63	16	18.00	70,062.59	100.75
7	16.12	88,468.87	144.13	17	18.00	70,076.48	100.27
8	16.12	88,479.28	143.64	18	18.00	70,090.37	99.78
9	16.12	88,489.70	143.14	19	18.00	70,104.27	99.29
10	16.82	78,353.81	120.72	20	18.00	70,118.16	98.80

Table 6.8. Expansion plan obtained by the second proposed method

Weight comb.	Power Nodes									
	1		2		7		13		15	
	C	W	C	W	C	W	C	W	C	W
1	1		11		6					
2	1		11						6	
3	1		11							
4	6								6	
5	6		1							
6	1		1		1					
7	1		1						6	
8	1		6		1					
9	1		1				6			
10			1		1		1		1	
11	1		1		1		1		1	
12	1		1		1		1		1	
13	1		1		1		1		1	
14	1		1		1		1		1	
15	1		1	1	1	1	1	1	1	
16	1	1	1		1		1	1	1	
17	1	1	1		1	1	1		1	1
18	1	1	1		1		1		1	
19	1		1	1	1	1	1		1	
20	1	1	1		1		1	1	1	

Weight comb.	16		18		21		22		23	
	C	W	C	W	C	W	C	W	C	W
1	6						1			
2	6						1			
3	6		1				6			
4	1						1		11	
5			11				6		1	
6	6						6		1	
7			1				6		1	
8			1				6		1	
9	6						1		1	
10	1		1		1		1		1	
11	1		1		1		1		1	
12	1		1		1		1		1	
13	1		1		1		1		1	
14	1		1		1		1		1	
15	1		1		1	1	1		1	
16	1		1		1	1	1	1	1	
17	1	1	1		1		1		1	
18	1		1	1	1		1	1	1	1
19	1		1	1	1	1	1		1	
20	1		1		1	1	1	1	1	

7. Future Research

There are several interesting opportunities where this research can be expanded or extended with. Even though there is already a wide collection of articles focused on the GEP problems, and this research provides additional contribution to GEP literature in terms of both the domain and the method to solve the problem, there are still areas to be investigated further and there are opportunities for further contribution to this literature.

One possible subject for the future research is to include the uncertainties associated with the renewable energy sources especially wind and solar and determine a model which can also incorporate the risk into the problem. In this dissertation, the incorporation of the risk in the presence of the demand uncertainties is presented, and for some risk levels wind turbines are introduced into the model. The uncertainty of the availability of the wind turbines is incorporated by simply reducing the average capacity of the wind turbines. However, during the year, the amount of energy which can be produced from the wind turbines can change based on the availability of the wind, which is affected by the season and the time of the day. One possible expansion can be to represent this uncertainty in the model together with a risk measure, and observe the expansion decision changes with respect to the risk behavior of the decision maker. There are also incentives to increase the use of renewable energy sources in the power system. For this purpose, renewable portfolio standards are forced into the model. That is, the generation from the renewable energy sources should be greater than some predefined level. The uncertainty in the availability of the wind and solar would affect these constraints too. Stochastic model can be proposed to include the uncertainties and mean-risk model can be proposed to find trade-off solutions with respect to cost and some risk measures.

In Chapter 2 and 4, the advantages of the Smart Grid technologies are presented and as a part of this research, the effects are categorized into three principle groups and models are provided to incorporate the effects into the model. One possible extension of this work is to include other impacts of Smart Grid technologies such as the reduction in the operational and maintenance cost and the improvement in the energy quality. The operational and maintenance cost is expected to be lower since Smart Grid provides real-time information about the system. Therefore, this can eliminate unneeded field trips, unnecessary maintenance actions or provide equipment-condition-based maintenance which reduces the risk of overloading problematic equipments. As a result, the operational and maintenance cost could be reduced. The model presented to include the impact on the availability of the system components can be modified and expanded to include such effects. Another important issued is there is an increasing importance for energy quality. Energy quality is very important especially for the manufacturing companies who produce high-tech products. One possible research subject can be providing a model to represent the quality measures (in the GEP model) and investigating how the expansion plans would be affected in the presence of such measures.

There are also some expansion opportunities in terms of the solution procedures. In this research, Benders decomposition is used to solve our large-scale problems. During the numerical experiments, a slow divergence in Benders decomposition is encountered when there are continuous distributed generation options available in the master problem. Mode detailed observations/modifications can be done to increase the convergence speed. More detailed analysis can be done specific to the GEP problems to investigate the conditions where Benders cuts should or should not be generated and added to the master problem.

In the numerical analysis for multi-objective GEP problems, it is realized that scaling is very important. Therefore, new approaches are presented to overcome this issue. However, there is still a need for intense numerical analysis for the proposed methods to demonstrate their effectiveness. Future research can be done to compare the new approaches presented with the existing approaches to provide an efficiency table with respect to solution quality and the solution time.

Appendix A: Sorted Components with Respect to Importance Values

Table A.1. Sorted components with respect to importance values

With Respect to COST		With Respect to CO_2		With Respect to NO_x	
Component Name	Importance Value	Component Name	Importance Value	Component Name	Importance Value
CU 23,3	0.1032246	TL 14,16,1	0.1107508	TL 14,16,1	0.1074776
CU 21,1	0.091838	TL 16,19,1	0.0957318	TL 16,19,1	0.0816888
CU 18,1	0.0917435	CU 18,1	0.0925618	CU 18,1	0.0815459
CU 13,3	0.0454883	CU 21,1	0.0917947	CU 21,1	0.078988
CU 13,1	0.0454651	TL 2,6,1	0.071522	TL 2,6,1	0.052134
CU 13,2	0.0453759	TL 4,9,1	0.0446697	TL 4,9,1	0.0345645
TL 14,16,1	0.0405157	TL 6,10,1	0.0438404	TL 6,10,1	0.0334809
CU 16,1	0.0366482	CU 23,3	0.0299749	CU 23,3	0.0324577
TL 7,8,1	0.036636	TL 7,8,1	0.0283205	TL 20,23,2	0.0280933
CU 23,2	0.0331666	TL 16,17,1	0.0280721	TL 20,23,1	0.0280501
CU 23,1	0.0331313	TL 3,24,1	0.0268495	CU 15,6	0.025493
CU 15,6	0.0269325	TL 20,23,2	0.024956	TL 16,17,1	0.0217266
TL 16,19,1	0.0234641	TL 20,23,1	0.0248957	CU 13,2	0.0211131
CU 2,4	0.0215891	TL 15,24,1	0.0220374	CU 13,1	0.0210983
CU 2,3	0.0215688	CU 2,4	0.0174372	CU 13,3	0.0210276
TL 2,6,1	0.0191528	CU 2,3	0.0174247	CU 2,4	0.0203994
CU 7,1	0.0187098	TL 10,12,1	0.0164882	CU 2,3	0.0203582
CU 7,2	0.0186677	CU 7,3	0.013592	TL 11,14,1	0.019187
CU 7,3	0.018573	CU 15,6	0.0122506	CU 7,3	0.0184097
TL 3,24,1	0.0183196	TL 11,14,1	0.0111417	TL 3,24,1	0.0161162
TL 15,24,1	0.0148399	TL 11,13,1	0.0095344	CU 1,3	0.0139476
CU 1,4	0.0145903	CU 1,3	0.0092558	CU 16,1	0.0137809
CU 1,3	0.0145598	TL 12,23,1	0.008466	TL 7,8,1	0.0131113
TL 16,17,1	0.0136362	CU 16,1	0.0084426	TL 15,24,1	0.012659
TL 20,23,2	0.0099484	TL 21,22,1	0.0081676	TL 10,12,1	0.0106847
TL 20,23,1	0.009896	CU 23,1	0.0080324	TL 11,13,1	0.0105695
TL 4,9,1	0.0097594	CU 23,2	0.0079962	CU 23,2	0.0103915
TL 6,10,1	0.0091981	CU 2,2	0.0079171	CU 23,1	0.0103106
TL 10,12,1	0.0076709	TL 13,23,1	0.0071368	CU 7,2	0.0089172
CU 22,6	0.0073469	TL 15,16,1	0.0061222	CU 7,1	0.0089144
CU 22,5	0.007346	TL 8,10,1	0.0060638	CU 22,4	0.0086975
CU 22,4	0.0073444	CU 2,1	0.0059029	CU 22,5	0.0086903
CU 22,1	0.0072425	CU 13,3	0.0049478	CU 22,6	0.0086845
CU 22,2	0.007237	CU 13,1	0.004871	CU 2,2	0.0065347
CU 22,3	0.0072368	CU 13,2	0.0047997	TL 21,22,1	0.0060803
TL 11,14,1	0.0051023	TL 19,20,2	0.0045052	TL 15,16,1	0.0051804
TL 11,13,1	0.004806	TL 19,20,1	0.0043525	TL 19,20,2	0.0050847

With Respect to COST		With Respect to CO_2		With Respect to NO_x	
Component Name	Importance Value	Component Name	Importance Value	Component Name	Importance Value
CU 2,2	0.0047189	TL 8,9,1	0.0042037	TL 19,20,1	0.0049495
CU 2,1	0.004682	TL 2,4,1	0.0041977	TL 13,23,1	0.0041859
CU 1,2,	0.0032269	TL 1,5,1	0.004122	CU 22,2	0.0041656
CU 1,1	0.0032027	CU 22,4	0.0039677	CU 22,3	0.0041589
TL 13,23,1	0.0030404	CU 22,5	0.0039635	CU 22,1	0.0041223
TL 9,12,1	0.0026432	CU 22,6	0.00396	TL 12,23,1	0.0040093
TL 21,22,1	0.0025683	TL 1,3,1	0.0037316	CU 1,4	0.0037615
TL 1,5,1	0.0024544	CU 1,4	0.0035065	CU 1,2,	0.0033576
TL 15,16,1	0.0018494	TL 10,11,1	0.0030082	TL 8,10,1	0.0032888
CU 15,4	0.0016963	CU 1,2,	0.0023956	TL 8,9,1	0.0032694
CU 15,2	0.0016955	TL 12,13,1	0.0022578	TL 12,13,1	0.0029779
CU 15,5	0.0016953	CU 7,1	0.0020019	TL 5,10,1	0.0028068
CU 15,3	0.0016952	CU 7,2	0.0019721	TL 9,11,1	0.0027891
CU 15,1	0.0016952	TL 5,10,1	0.0019042	TL 1,3,1	0.0024241
TL 12,23,1	0.0015856	TL 9,11,1	0.0017026	TL 1,5,1	0.0021911
TL 3,9,1	0.0015851	TL 3,9,1	0.0015159	TL 18,21,2	0.002125
TL 19,20,2	0.0014089	CU 15,5	0.0010215	TL 18,21,1	0.0021033
TL 19,20,1	0.0013886	CU 15,4	0.0010188	CU 15,5	0.002073
TL 8,10,1	0.0011795	TL 1,2,1	0.0010002	CU 15,4	0.0020705
TL 12,13,1	0.001054	CU 22,3	0.0009498	TL 17,22,1	0.0016961
TL 10,11,1	0.0010497	CU 22,2	0.0009233	TL 2,4,1	0.0015936
TL 2,4,1	0.0010339	CU 22,1	0.000902	CU 1,1	0.0014331
TL 8,9,1	0.0008616	TL 17,22,1	0.0005789	TL 15,21,2	0.0012233
TL 5,10,1	0.0007141	TL 17,18,1	0.0005724	TL 15,21,1	0.0012175
TL 18,21,1	0.0005441	CU 15,2	0.000466	TL 17,18,1	0.0010973
TL 9,11,1	0.0005436	CU 15,3	0.0004658	CU 15,2	0.0009323
TL 18,21,2	0.0005303	CU 15,1	0.0004657	CU 15,1	0.0009319
TL 1,2,1	0.0004784	CU 1,1	0.0004288	CU 15,3	0.0009318
TL 1,3,1	0.0004596	TL 15,21,1	0.0004285	TL 1,2,1	0.0008892
TL 17,22,1	0.0003028	TL 15,21,2	0.0004151	CU 2,1	0.0005924
TL 17,18,1	0.0002681	TL 9,12,1	0.0004068	TL 9,12,1	0.00048
TL 15,21,1	9.604E-05	TL 18,21,2	0.0003687	TL 10,11,1	0.0003971
TL 15,21,2	8.038E-05	TL 18,21,1	0.0003512	TL 3,9,1	0.0001058

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