Description: Clip 4 of 5: Explaining the Taxicab and Towers Isomorphism **Parent Tape: Taxicab Geometry** **Date: 2000-05-05** **Location: David Brearley High School** **Researcher(s): Professor Carolyn Maher**	**Transcriber(s): Powell, Arthur; Milonas, Jeremy** **Verifier(s): McGowan, Will; Brookes, Elijah** **Date Transcribed: Spring 2010** **Page:** 1 **of** 10

1	01;09:36 begin Clip 4	T/R1	You're showing me that's Pascal's triangle but I don't see it. Help me see it.
2		JEFF	You don't see it?
3		T/R1	Right. Can you //show it to me?
4		JEFF	//All right, well here. The one, one two one, one three three one, one four six four one- [Pointing to transparency grid with marker]
5		T/R1	Okay let- show me again where's the four?
6		JEFF	All right. We're on- all right. The- that one right there- [Pointing to grid]
7		T/R1	Mm hm.
8		JEFF	-that we added in and this two is the three. The two in that one there is a three and there's two ones on the outside- [Placing his finger on each number as he speaks.]
9		T/R1	Mm hum.
10		JEFF	So you get one three three one. And then one- the one and the three for the four. Three and the three for the six. The three and the one for the other four and then the other one on the end and then continuing through the four and the one together is the five. The four and the six is the ten. Six and the four is the other ten. Four and the one is the five. //Do you see it? [Pointing with his finger and marker to the numbers on the transparency grid.]
11		T/R1	//Okay. So can you explain to me, for instance, why that works? Why- where this ten comes from when you know- you're just saying well there's a pattern here because you found them, but is there a way where you haven't found them that makes sense to predict the number of

			paths of one you haven't found?
12		JEFF	Well like to here, I mean we //would say-
13		T/R1	//You understand my question?
14		JEFF	Well like to here we would say it was thirty-five. [Pointing to a square on the transparency grid with marker]
15		T/R1	Right. How would you- how- where //would the thirty-five come from?
16		MICHAEL	//You can't justify it because- You can justify these because you can say you counted. You can't justify that because you can't say you counted.
17		JEFF	Yeah because we didn't count it. We're saying we're following the pattern- [Waving his hand.]
18		T/R1	Right.
19		JEFF	That's- that is our justification as of now.
20		ROMINA	[Inaudible].
21		T/R1	Right.
22		JEFF	That we're just following //the pattern.
23		T/R1	Do you understand my next question Jeff? What I'm asking?
24		JEFF	//Yeah.
25		ROMINA	//What if three- what if Pascal-
26		T/R1	Because so you notice this pattern and the pattern fits Pascal's triangle.
27		BRIAN	So does that mean there's //thirty-five for the red one? [Romina and Mike are counting. Mike writes something.]
28		MICHAEL	//Only these are zeros. This is like one topping- you know on the pizza? [With Jeff looking at the transparency grid. Jeff pointing to a number on the grid.]
29		ROMINA	Yeah, one topping, two toppings.
30		BRAIN	Remember how- Mike you had thirty-four for the red one, right?

31		MICHAEL	Um- Yeah I think that was the problem.
32		BRIAN	It's thirty-five.
33		JEFF	Yeah, it's thirty-five.
34		MICHAEL	Oh, I probably missed one.
35		JEFF	Good, uh, deduction.
36		T/R1	So you counted thirty-four by brute force //and you're saying that by this pattern, um, you would feel more comfortable with the pattern in saying thirty-five.
37		JEFF	//Yeah.
38		BRIAN	But-
39		T/R1	Right?
40		ROMINA	Did you actually get thirty-five?
41		MICHAEL	I got //thirty-four.
42		BRIAN	//He got thirty-four but you know he's been off by like one cause you know. Yeah, it could- it could of //been one.
43		ROMINA	//Natural tendencies? Um,-
44		MICHAEL	Stop that.
45		T/R1	Okay. So why is- why is that-
46		ROMINA	All right. [With Jeff studying the transparency grid.]
47		T/R1	Why do you think that- Why do those numbers seem to work? How could you explain those numbers? That's that's really- isn't that interesting?
48		JEFF	Yeah. It- it hurts though. It really does.
49		ROMINA	Yeah, I'm having trouble seeing Pascal's triangle. [Rewriting the triangle the way she is used to seeing it.]
50		T/R1	It's hard to see the other way, isn't it?
51		ROMINA	Mmhmm. All right. So for this one the two comes from when there's- [Pointing to numbers in the triangle with her marker]
52		JEFF	One block.
53		ROMINA	One-

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54		JEFF	Block.
55		ROMINA	Is that-
56		JEFF	One block.
57		ROMINA	Isn't that two blocks?
58		JEFF	One, two.
59		ROMINA	No. Um, let's go back to towers. The two comes from this is one block. This is two blocks with two colors. [Continuing to point to numbers in the triangle with her marker.]
60		JEFF	I have to leave. I'm kind of out.
61		ROMINA	Alright. Hold on. How's this go? Just tell me where this comes from.
62		MICHAEL	What happened?
63		ROMINA	Okay. This is with- with just one block?
64		MICHAEL	This is nothing.
65		ROMINA	This is nothing? This is one block?
66		MICHAEL	This is like- yeah, //one. All right.
67		ROMINA	//One block, two- this one tells how many blocks.
68		MICHAEL	One block. Two blocks. [Pointing to the 1 and 2 in the triangle Romina redrew.] Not two blocks but like- [He points to the numbers on the transparency grid.] [Inaudible.]
69		ROMINA	One block, two blocks, three blocks- Oh no, this is zero block, one block, two block?
70		MICHAEL	For one block you get two. Right? For two blocks-
71		ROMINA	All right.
72		MICHAEL	Three- three- three blocks. One- So you can't really say it because there's three for three and then you get four here. You can't really- I don't think you can use that. That- that row thing. [Pointing to the numbers on the transparency grid.]
73		ROMINA	All right. Yeah. I know. I'm just trying to- because for like-

74		MICHAEL	There's got to be some type of, you know, way. Be nice if I could see-
75		ROMINA	Can't you just go one, two, three, four?
76		MICHAEL	Uh hum [nodding his head in agreement]
77		ROMINA	That's what it goes one, two, three, four? Because then okay for this one for the three. If we name all the ones going horizontal- *A*′s and ones going down same with *B*. And this would be with two *A*′s and one *B* there's three and then there's two *B*′s with one *A*, three. [Pointing with a green marker at the intersections Points † $(3,2)$ and † $(3,1)$ on the transparency grid.] And for this one remember like two *A*s two *B*s- //six. [Now pointing to the intersections point † $(4,2)$ and on the transparency grid.]
78		MICHAEL	//You could say, um-
79		ROMINA	Do you understand what I'm saying?
80		MICHAEL	Like yeah, these are like this row is everything with perimeter two. I mean I half the perimeter, like. [Pointing with his marker to numbers on the transparency grid.]
81		ROMINA	//Well no I'm saying so to get that-
82		MICHAEL	//In order to get to that point you have to go over one and down, uh, one or down one and over one. [Pointing to the intersection point † $(2,1)$.] Just like that row. Everything in this row, over two and down two and over one.
83		ROMINA	Yeah but like I'm just saying like if she were to pick anything like right there we could say it's like eight *A*′s and like six *B*′s. [Tracing a rectangle on the transparency grid.] You know like- and then we could tell you where you it is in this one. [Pointing to the redrawn Pascal's triangle on the piece of paper.]

84		MICHAEL	Well you could say all- everything in this row, the shortest route is two. Everything in this row shortest route is three. This one shortest route is four. [Pointing to a diagonal of numbers—1 4 6 4 1—in the transparency grid.]
85		ROMINA	Yeah.
86		MICHAEL	The shortest route is five, six and so on. So that's how you could, you know, name them. This is row six because it has everything in the row has shortest route of six. [Pointing with a marker to diagonals of numbers on the transparency grid.]
87		ROMINA	No, I understand. I'm just saying like-
88		MICHAEL	There's a, you know-
89		ROMINA	To get it-
90		MICHAEL	//To- to say it like, oh I'll pick this block-
91		ROMINA	//Because isn't that how- isn't that how we get like the *B*'s? Like doesn't the two- there's- that I mean, that's one- that means it's one of *A* color, one of *B* color. [Pointing to the 2 on the redrawn triangle on paper.] Here's one- it's either one- either way you go. It's one of across and one down. [Pointing to a number on the transparency grid and motions with her pen to go across and down.] And for three that means there's two *A* color and one *B* color [pointing to the 3 on the redrawn triangle], so here it's two across, one down or the other way [tracing across and down on the transparency grid] you can get three is //two down- [Pointing to the grid.]
92		MICHAEL	//You mean like one *A* color and two-
93		ROMINA	Yeah.
94		MICHAEL	This is one-
95		ROMINA	Like two blues, one red. Two across, one down or this is two reds, one blue, two down, one

			across. And that's how we would get the Pascal's triangle. [Pointing to numbers on the redrawn grid and transparency grid.]
96		MICHAEL	But there's like- you know, there's got to be a way that we could just say, all right this one's three. //So five down this has to be this because of some kind of-
97		ROMINA	//I know, I'm just saying-
98		ROMINA	So if it were-
99		MICHAEL	Pattern- I mean like, you know, reasoning. You can't just say I counted them.
100		ROMINA	I know. I'm just saying so like- and then that could relate back to this but that is this, so it's relateable- and for-
101		MICHAEL	So what- what are you looking for right now?
102		ROMINA	Yeah like-
103		T/R1	I think Romina knows what I'm looking for. I think she's said it very articulately. That if I were to pick any point right on-
104		MICHAEL	Mm hmm.
105		T/R1	If I were to make a larger grid- right Brian? I think he //knows what I'm looking for.
106		BRIAN	//Yeah.
107		T/R1	She's looking for a way to come up with a particular pattern that she's identifying that. I think I'm hearing you say- you're trying to look at blocks-
108		1227. RROMINA	Mm hmm.
109		T/R1	Colors?
110		1229. ROMINA	Yeah.
111		T/R1	And then you're doing A's and B's.
112		1231. ROMINA	Mm hmm.
113		T/R1	That's what I'm hearing you say? And you were trying to say maybe that could get you to some general point. Why don't you try saying that again? I- I thought I followed you, but I'm

			not so sure that Brian and Michael followed what you said.
114		1233. ROMINA	Like why-
115		T/R1	So say it again. What you were-
116		1235. ROMINA	Like why this and this are related?
117		T/R1	Yeah.
118		1237. ROMINA	Well-
119		T/R1	Throw out your idea //again for them so we can hear it.
120		ROMINA	//When we look- whenever we do this we always- we always talk about towers and how this is like a tower of two high with two different colors and there's one- one tower you can make that makes all one color and one and one and then all the other color. And- and then for this one it's three high and this is all one color. There's two of one color and one of the other, whatever. And for this it's basically the same thing because this is- let's see. This is- this is two but usually you go one across and one down so there's two different ways to get to that one. And for this one there's going to be two across and one down. Or to go down here it's two down and one across which is basically the same thing and it just goes on. Do you understand? Understand? Was that good? Or, do you want more? [Connecting the data from the grid and the triangle drawings by pointing to the numbers on each back and forth.]
121		BRIAN	Yeah.
122		ROMINA	Or do you want more?
123		T/R1	I don't know. I don't know if Michael-
124		BRIAN	Mike do you understand?
125		MICHAEL	Yeah, I understand what you're talking about.
126		ROMINA	Yeah. Yeah.

127		MICHAEL	This would be, um, one- we'll think of it as pizza because that's the thing I like but-
128		ROMINA	Think of towers.
129		MICHAEL	Or towers. I mean this will be a tower of three-
130		T/R1	Think of it as pizzas.
131		MICHAEL	A pizza. A pizza with, um, three possible choices for toppings and- I like the tower.
132		ROMINA	Yeah, the tower is easier.
133		MICHAEL	You have, you have a tower of three and you have, you know, two colors. So one- it's either you know- Color *x* and two of color *y*. Well this is direction *x* and two, two directions of *y*, you know-// [Pointing with a marker to the redrawn Pascal's triangle.]
134		ROMINA	//Yeah.
135		MICHAEL	//of *y*. So that makes- that makes sense.
136		ROMINA	So for like the three, it would be two *x*, one *y* or two *y*, one *x*// [Referring to the taxi grid.]
137		MICHAEL	//Yeah, I got that.
138		ROMINA	And this would be
139		T/R1	Okay. Well- where I'm still having a little trouble is, um, - Okay, so you're talking about these blocks, right?
140		ROMINA	Mm hmm.
141		T/R1	So what are you labeling them? These blocks? [Referring to blocks on the taxi grid.] Which is the *A* and which is the *B* and why is it //okay to call them *A*'s and *B*'s?
142		ROMINA	//We'll do it- how about x and y?
143		T/R1	Sure.
144		ROMINA	*x* will be the ones that go horizontal. [Motioning across with her marker on transparency grid.]
145		T/R1	Okay.
146		ROMINA	And y will be the ones that go up and down, basic graphing skills. [Moving her marker

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			down.]
147		T/R1	Does that make any sense Brian?
148		BRIAN	Yeah.
149		T/R1	Brian, do you think so?
150	01;19:20 end clip 4	BRIAN	I think so. Yeah. I'm- I'm hanging out. I'm doing good now. You know what I'm saying. Oh, I was like what is that? A research paper.