| Description: Night Session - Pascal's Identity, Clip 3 of 7: Further explorations of factorials and combinations <br> Parent Tape: Night Session - Pascal's Identity <br> Date: 1999-05-12 <br> Location: David Brearley High School <br> Researcher: Professor Carolyn Maher |  |  |  | ```Authors: Uptegrove, Elizabeth B. Verified: Poprik, Brad Date Transcribed: }200 Page: 1 of 8``` |
| :---: | :---: | :---: | :---: | :---: |
| Line | Time | Name | Transcript |  |
| 1. | 00:01 | Jeff: | Yeah but say, all right, say go five factorial. Which is | we're doing five choose two, right, with this. Then we what? |
| 2. |  | Michael: | That'll give you all the con | binations they can put everybody in. |
| 3. |  | Jeff: | Uh, twenty times three. |  |
| 4. |  | Ankur: | OK. Sixty. |  |
| 5. |  | Jeff: | Would be sixty times two. |  |
| 6. |  | Ankur: | One-twenty. |  |
| 7. |  | Jeff: | One-twenty? That would | e; it's one-twenty, right, Romina? |
| 8. |  | Romina: | Yeah. |  |
| 9. |  | Jeff: | We're faster than the calcu that. So that'd be one-twe | ator, around here. [Romina laughs.] We're good like ty. |
| 10. |  | Michael: | And, and if you're doing chat where those two are going | oose two, obviously there's going to be a lot of times to be in the same spot as the other three are going to be- |
| 11. |  | Romina: | What are you doing, five | hoose two? |
| 12. |  | Michael: | -you know, I guess movin | around different spots. |
| 13. |  | Jeff: | Yeah. |  |
| 14. |  | Michael: | That's why you want to ge | rid of the, the $n$ minus $x$ thing. |
| 15. |  | Jeff: | Yeah, we got, that makes | ense. |
| 16. |  | Michael: | Yeah, that, that makes sen | e to you? |
| 17. |  | Jeff: | That, that part right here, why this is all happening? | this all good? Up to this point? Do you understand |
| 18. |  | R1: | I'm waiting for the whole | ing. |
| 19. |  | Michael: | Whole thing? Oh we're not | done with that yet. |
| 20. |  | Jeff: | Then, um, then you multip | y. Well, at this point here you have three. |
| 21. |  | Romina: | That's six. |  |
| 22. |  | Jeff: | Yeah, it's six. So you hav | one-twenty over six times five factorial. |
| 23. |  | Romina: | No isn't it- |  |
| 24. |  | Michael: | Oh I think its the repeats- |  |
| 25. |  | Jeff: | Or- |  |


| Description: Night Session - Pascal's Identity, Clip 3 of 7: Further explorations of factorials and combinations <br> Parent Tape: Night Session - Pascal's Identity <br> Date: 1999-05-12 <br> Location: David Brearley High School <br> Researcher: Professor Carolyn Maher |  |  |  | Authors: Uptegrove, Elizabeth B. <br> Verified: Poprik, Brad <br> Date Transcribed: 2003 <br> Page: 2 of 8 |
| :---: | :---: | :---: | :---: | :---: |
| Line | Time | Name | Transcript |  |
| 26. |  | Michael: | Would, would be like- |  |
| 27. |  | Romina: | Isn't it three factorial, two | actorial? |
| 28. |  | Jeff: | Three factorial. Oh two, oh | , it's act-, all right, yeah. Two. |
| 29. |  | Michael: | Yeah, I guess the, the $x$ - |  |
| 30. |  | Jeff: | That's the number you wer | raising- |
| 31. |  | Michael: | That $x$. |  |
| 32. |  | Jeff: | -and, and five choose $x$, sa | and there was- |
| 33. |  | Michael: | That's what. Since you- M | mm h. |
| 34. |  | Jeff: | And this was- |  |
| 35. |  | Ankur: | I get it. I get it. I get it. I | get it. [Romina laughs.] |
| 36. |  | Michael: | I, I got it now. |  |
| 37. |  | Jeff: | Like that. |  |
| 38. | 01:25 | Michael: | All right, then the last num | ber would be- |
| 39. |  | Jeff: | Because this just gives you | the number. |
| 40. |  | Michael: | You have- Yeah. |  |
| 41. |  | Jeff: | You're going to multiply b | the number. |
| 42. |  | Michael: | Those, those, you want to are moved around and thos | get rid of those. The, all the combinations that the three e, those two aren't. |
| 43. |  | Jeff: | Yeah, they- |  |
| 44. |  | Michael: | But then those two themse | ves will be repeat- |
| 45. |  | Jeff: | Yeah- |  |
| 46. |  | Michael: | You will be mixed up. |  |
| 47. |  | Jeff: | Be repeating that's what you | u- that's why you |
| 48. |  | Michael: | That's why you want to get | rid of that, too. |
| 49. |  | Jeff: | Exactly. And then, so that | would be just two. |
| 50. |  | Michael: | Yeah. |  |
| 51. |  | Jeff: | So it would be one-twenty | divided by twelve and you get ten. Is that what it is? |
| 52. |  | Michael: | Yeah it is. Do you get like know, you, you get that? | why we divide by the $n$ minus $x$ and the, the $x$ ? You |

of 7: Further explorations of factorials and combinations
Parent Tape: Night Session - Pascal's Identity
Location: David Brearley High School
Researcher: Professor Carolyn Maher

Michael: Would, would be like-
Romina: Isn't it three factorial, two factorial?
Michael: Yeah, I guess the, the $x$ -
Jeff: That's the number you were raising-
Jeff: $\quad$-and, and five choose $x$, say and there was-
Michael: That's what. Since you- Mm hm.
Jeff: And this was-
I get it. I get it. I get it. I get it. Romina laughs.]

Michael: All right, then the last number would be-
Jeff: Because this just gives you the number.
Michael: You have- Yeah.
You're going to multiply by the number
Those, those, you want to get rid of those. The, all the combinations that the three
are moved around and those, those two aren't.
Jeff: Yeah, they-
Jeff: Yeah
Michael: You will be mixed up.
Be repeating that's what you- that's why you

Exactly. And then, so that would be just two.
Michael: Yeah.
Jeff: So it would be one-twenty divided by twelve and you get ten. Is that what it is?
Michael: Yeah it is. Do you get like why we divide by the $n$ minus $x$ and the, the $x$ ? You know, you, you get that?

| Description: Night Session - Pascal's Identity, Clip 3 of 7: Further explorations of factorials and combinations <br> Parent Tape: Night Session - Pascal's Identity <br> Date: 1999-05-12 <br> Location: David Brearley High School <br> Researcher: Professor Carolyn Maher |  |  |  | Authors: Uptegrove, Elizabeth B. <br> Verified: Poprik, Brad <br> Date Transcribed: 2003 <br> Page: 3 of 8 |
| :---: | :---: | :---: | :---: | :---: |
| Line | Time | Name | Transcript |  |
| 53. |  | R3: | I don't get that. Could you | [Inaudible.]? |
| 54. |  | Michael: | You don't get that? |  |
| 55. |  | R1: | Ankur, did you have that? |  |
| 56. |  | Jeff: | What, what part don't, don |  |
| 57. |  | R1: | I wonder if Ankur has that? | I wonder if Ankur could explain. |
| 58. |  | Romina: | I don't think the $x$ [Inaudib |  |
| 59. |  | Michael: | All right. The top thing, the you how many? | $n$ to the, the $n$ to the, uh, factorial was going to give |
| 60. |  | Romina: | That's all the combinations. |  |
| 61. |  | Michael: | That's every single combin | ation. |
| 62. |  | Romina: | I got that. That I got. |  |
| 63. |  | Michael: | Right? Now you're, you're So there's going to be som same place and those three | only worried about them, those two people in that line. instances where those two people are going to be in the |
| 64. |  | Jeff: | Are the ones changing. |  |
| 65. |  | Michael: | Will be, you know, will be | switch, you know, changing. |
| 66. |  | Jeff: | And that's- |  |
| 67. |  | Michael: | So that's, that would be the that. You want to get rid of | , the three factorial. You want to, you want to get rid of f them. |
| 68. |  | Ankur: | Wait, say that again. |  |
| 69. |  | Romina: | Hold on. Well, we- |  |
| 70. |  | Michael: | Don't worry about that thre | e, we're doing like five. |
| 71. |  | Romina: | No, we're doing this one so | the two- |
| 72. |  | Ankur: | All right, so you have the five | five minus two, is that what you're explaining on there? |
| 73. |  | Romina: | Five minus two, that's- |  |
| 74. |  | Michael: | So you have the hundred a | nd twenty different combinations. |
| 75. |  | Ankur: | Yeah. |  |
| 76. |  | Jeff: | Total. |  |
| 77. |  | Michael: | All right. But you don't thi | nk like when those two people are going to be in these |

of 7: Further explorations of factorials and combinations
Parent Tape: Night Session - Pascal's Identity : 1999-05-12

Locaton: Dars Brer Caroly Maner
Researcher: Professor Carolyn Maher
53. R3: I don't get that. Could you [Inaudible.]?
54.
56.

57

59
60.

R.
got
Right? Now you're, you're only worried about them, those two people in that line. So there's going to be some instances where those two people are going to be in the ree-
Are the ones changing
Jeff: And that's-
Michael: So that's, that would be the, the three factorial. You want to, you want to get rid of that. You want to get rid of them.
Ankur: Wait, say that again.
Romina: Hold on. Well, we-
Michael: Don't worry about that three, we're doing like five.
Romina. No, we're doing this one so the two-

Romina: Five minus two, that's-
Michael: So you have the hundred and twenty different combinations.
Jeff:
Michael: All right. But you don't think like when those two people are going to be in these

| Description: Night Session - Pascal's Identity, Clip 3 of 7: Further explorations of factorials and combinations <br> Parent Tape: Night Session - Pascal's Identity <br> Date: 1999-05-12 <br> Location: David Brearley High School <br> Researcher: Professor Carolyn Maher |  |  |  | ```Authors: Uptegrove, Elizabeth B. Verified: Poprik, Brad Date Transcribed: 2003 Page: 4 of 8``` |
| :---: | :---: | :---: | :---: | :---: |
| Line | Time | Name | Transcript |  |
|  |  |  | two spots- |  |
| 78. |  | Jeff: | And everyone else is chan |  |
| 79. |  | Michael: | -not those other three. |  |
| 80. |  | Jeff: | And those are, those are, are where those two peopl | ose make no difference because all we're worried about are. |
| 81. |  | Romina: | Oh like when, oh, oh, okay, | , okay, okay. |
| 82. |  | Michael: | All right, those two people they're like- | are going to be moving around and it- you know, |
| 83. | 02:59 | Jeff: | These people are going to going- | stay the same and every, all the three people, they're just |
| 84. |  | Michael: | -the two people staying in | he same place. So that's why you get rid of that. |
| 85. |  | Jeff: | You know, going nuts. |  |
| 86. |  | Michael: | But then those two people | hemselves could switch places too. |
| 87. |  | Ankur: | Yeah. [Ankur nods.] |  |
| 88. |  | Michael: | You know what I'm saying? |  |
| 89. |  | Ankur: | Um-huh. |  |
| 90. |  | Michael: | Or if- |  |
| 91. |  | Ankur: | So then you got to get rid | f those, too. |
| 92. |  | Michael: | -there were three that coul | go on. |
| 93. |  | Jeff: | So that's why you get rid of | the three. |
| 94. |  | Ankur: | That's why you do the $x$ fa | torial |
| 95. |  | Michael: | Then you get rid of the, you | know- |
| 96. |  | Jeff: | The other one. |  |
| 97. |  | Ankur: | Yeah, so you get rid of tho |  |
| 98. |  | Romina: | OK. |  |
| 99. |  | Jeff: | And then, then- |  |
| 100. |  | Romina: | Oh, there you go. That ma | kes sense. |
| 101. |  | Michael: | Because you're not worried | about every, each person. |
| 102. |  | Romina: | Just the two. |  |

of 7: Further explorations of factorials and combinations
Parent Tape: Night Session - Pascal's Identity
Location: David Brearley High School
Researcher: Professor Carolyn Maher
Line Time Name Transcript

Jeff
Michael: -not those other three.
Jeff: And those are, those are, those make no difference because all we're worried about are where those two people are.
Romina: Oh like when, oh, oh, okay, okay, okay.
Michael: All right, those two people are going to be moving around and it- you know, they're like-
ichael: $\quad$ going- $\quad$-the two people staying in the same place. So that's why you get rid of that.
Jeff: You know, going nuts.
But then those two people themselves could switch places too.

Michael: You know what I'm saying?
Ankur: Um-huh.

Ankur: So then you got to get rid of those, too.
Michael: -there were three that could go on.
Ankur: That's why you do the $x$ factorial
Michael: Then you get rid of the, you know-
Ankur: Yeah, so you get rid of those.
Romina: OK.
And then, then-

Michael: Because you're not worried about every, each person.
Romina: Just the two.

| Description: Night Session - Pascal's Identity, Clip 3 of 7: Further explorations of factorials and combinations <br> Parent Tape: Night Session - Pascal's Identity <br> Date: 1999-05-12 <br> Location: David Brearley High School <br> Researcher: Professor Carolyn Maher |  |  |  | Authors: Uptegrove, Elizabeth B. <br> Verified: Poprik, Brad <br> Date Transcribed: 2003 <br> Page: 5 of 8 |
| :---: | :---: | :---: | :---: | :---: |
| Line | Time | Name | Transcript |  |
| 103. |  | Michael: | Just worry about two, righ |  |
| 104. |  | Jeff: | Just those two. Exactly. |  |
| 105. |  | Romina: | Yeah, we all have, I got it. | I'm good. |
| 106. |  | Michael: | Extension? |  |
| 107. |  | R1: | Ankur? Can you explain this, and she's not following | is because poor Researcher 3 is trying to understand Michael. |
| 108. |  | Ankur: | Something like, I understo | d it but- |
| 109. |  | Jeff: | Just go through it dude. |  |
| 110. |  | Ankur: | All right. The top number for, for five, for five peopl | is five factorial, that's the total number of possibilities |
| 111. |  | Michael: | One twenty |  |
| 112. |  | Ankur: | And then the five minus two everyone, you're just worri the five minus two. Those all the possibilities of just | o comes, comes in where you're not worried about d about two people at a time. So we need to subtract get, that gives you and you do factorial, that gives you wo people, right? |
| 113. |  | Michael: | No, that gives you |  |
| 114. |  | Romina: | Three people. |  |
| 115. |  | Jeff: | No, three extras. |  |
| 116. |  | Michael: | The three that you don't, y | u're not worried about. |
| 117. |  | Jeff: | That's going to eliminate e | eryone except the two people you're worried about. |
| 118. |  | Ankur: | OK. Everyone except the eliminates, except the- | wo people you're worried about. And then the $x$ factorial |
| 119. |  | Michael: | When the two people- |  |
| 120. |  | Romina: | Two people, yeah. |  |
| 121. |  | Ankur: | Yeah. When the two people ones over again. [Romina | are switched back and forth when you have the same aughs]. |
| 122. |  | Jeff: | OK, [Inaudible.]. |  |
| 123. | 04:29 | R3: | It's, it's getting better. It's saying and with your finge | etting better. So they switch back and forth you're <br> s. I think I'm getting switch back- So could you give |

of 7: Further explorations of factorials and combinations
Parent Tape: Night Session - Pascal's Identity
Location: David Brearley High School
Researcher: Professor Carolyn Maher
103. Michael: Just worry about two, right.
104. Jeff: Just those two. Exactly.
105. Romina: Yeah, we all have, I got it. I'm good.

106
Michael: Extension?
Ankur? Can you explain this because poor Researcher 3 is trying to understand this, and she's not following Michael.
Ankur: Something like, I understood it but-
Jeff: Just go through it dude.
Ankur: All right. The top number is five factorial, that's the total number of possibilities for, for five, for five people.
Michael: One twenty
Ankur: And then the five minus two comes, comes in where you're not worried about everyone, you're just worried about two people at a time. So we need to subtract the five minus two. Those get, that gives you and you do factorial, that gives you all the possibilities of just two people, right?
Michael: No, that gives you
Romina: Three people.
Jeff: No, three extras.
The three that
Ankur: OK. Everyone except the two people you're worried about. And then the $x$ factorial eliminates, except the-
Michael: When the two people-
Romina: Two people, yeah.
Yeah. When the two people are switched back and forth when you have the same ones over again. [Romina laughs].
122. Jeff: OK, [Inaudible.].
123. $04: 29$ It's, it's getting better. It's getting better. So they switch back and forth you're saying and with your fingers. I think I'm getting switch back- So could you give

| Description: Night Session - Pascal's Identity, Clip 3 of 7: Further explorations of factorials and combinations <br> Parent Tape: Night Session - Pascal's Identity <br> Date: 1999-05-12 <br> Location: David Brearley High School <br> Researcher: Professor Carolyn Maher |  |  |  | Authors: Uptegrove, Elizabeth B. <br> Verified: Poprik, Brad <br> Date Transcribed: 2003 <br> Page: 6 of 8 |
| :---: | :---: | :---: | :---: | :---: |
| Line Time Name Transcript |  |  |  |  |
|  |  |  | me an example? |  |
| 124. |  | Ankur: | Like when you have, when | you have like person $A$ and, over here. |
| 125. |  | Michael: | You want to stand up and | how them? |
| 126. |  | Ankur: | And person $B$ over here. | nd then you have person $B$ and person $A$. |
| 127. |  | Michael: | You want to be in a line and | we'll show them? |
| 128. |  | R1: | Michael, start from the beg | inning very slow. |
| 129. |  | Michael: | All right. You have five p | ople. |
| 130. |  | R1: | Stand up and show us. |  |
| 131. |  | R3: | Stand up and show us. |  |
| 132. |  | Jeff: | All right, I'm going to sit i | your seat cause I can't see. |
| 133. |  | Michael: | I'm going to write it nice a people, in a line. You agr can put those five people. | d clear so you all can see. All right. You got five with me that's how many different combinations you |
| 134. |  | R3: | That part I understand. |  |
| 135. |  | Michael: | All right. |  |
| 136. |  | R3: | I understood the multiplica | ion that you showed. |
| 137. |  | Michael: | Now in, you're only worri put those two people. All they're going to be repeate certain place and you know, one of them switches, that that. | , you want to know how many different places you can ght? So, in all the combinations you're going to have, a lot. A lot. When you have like, the two people in a , those three. If the three are, are like this. And then another combination. And you get a lot of repeats like |
| 138. |  | R3: | Oh, I see. OK. |  |
| 139. |  | Michael: | So by eliminating that, yo people moving around. | eliminate the combinations that repeat by the three |
| 140. |  | R3: | Uh-hum. |  |
| 141. |  | Michael: | Then let's say you just have one, if this guy switches the combinations, but in this | those two people in, in any given combination. If, if place with this guy it's the, they're different e're not worried about where they are. We just, you |

of 7: Further explorations of factorials and
combinations
Parent Tape: Night Session - Pascal's Identity
Location: David Brearley High School
Researcher: Professor Carolyn Maher
me an example?

| Description: Night Session - Pascal's Identity, Clip 3 of 7: Further explorations of factorials and combinations <br> Parent Tape: Night Session - Pascal's Identity <br> Date: 1999-05-12 <br> Location: David Brearley High School <br> Researcher: Professor Carolyn Maher |  |  |  | ```Authors: Uptegrove, Elizabeth B. Verified: Poprik, Brad Date Transcribed: 2003 Page: 7 of 8``` |
| :---: | :---: | :---: | :---: | :---: |
| Line Time Name Transcript |  |  |  |  |
| 142. ${ }^{\text {143. }} 066: 01$ |  | R3: <br> Michael: | understand? |  |
|  |  | Mm hm . |  |
|  |  | That's why we get rid of th as many times as you cou people. Right? Like the could put those three peop would repeat because tho move around in the, in the all that, you just get, um, you're not worried if, like guy has a switch with this how that eliminates. | , the two factorial to, to, uh, eliminate the amount like , as many combinations as you could put those two ree would, would be to eliminate the combinations you e that you're not worried about. Then the two, they people too, they move around. They, they could, they line also. And then when, when, when you're done with ou get how many places you can just put that two. Like ou don't care who they are. You don't care like if this uy. You understand like why you would eliminate, |
| 144. |  |  | R1: | OK. I don't want to think what Jeff said? And now | people. I want to think of the tower now. Isn't that m thinking of towers that are five tall? |
| 145. |  |  | Jeff: | Yeah. You can, we just- |  |
| 146. |  | R1: | And we're talking of those | that have two reds? |
| 147. |  | Jeff: | Yeah. Well. [Inaudible.] |  |
| 148. |  | R1: | Explain it to me with that. |  |
| 149. |  | Jeff: | All right. Say, say we're Towers of five tall with two possibilities is the five fac high with the combination the three factorial on the b the two spots that you're con | ing, we're doing towers that were, were five tall. different colors in it. Then that's the total amount of orial that you could have. All right, in, with, with five So that's where, that's the five factorial on top. Then ttom would be five different, five different spots minus ncerned about, leaving you with the three other spots- |
| 150. |  | Romina: | You could say- |  |
| 151. |  | Jeff: | -that you don't care about. | That's going to eliminate all of them. |
| 152. | 07:29 | Romina: | That's like, if you say like in the same place, and lik | he reds. Let's say reds are our two colors that they stay |
| 153. |  | Jeff: | Reds. |  |

of 7: Further explorations of factorials and combinations
Parent Tape: Night Session - Pascal's Identity
Location: David Brearley High School
Researcher: Professor Carolyn Maher
understand?

| Description: Night Session - Pascal's Identity, Clip 3 of 7: Further explorations of factorials and combinations <br> Parent Tape: Night Session - Pascal's Identity <br> Date: 1999-05-12 <br> Location: David Brearley High School <br> Researcher: Professor Carolyn Maher |  |  |  | Authors: Uptegrove, Elizabeth B. <br> Verified: Poprik, Brad <br> Date Transcribed: 2003 <br> Page: 8 of 8 |
| :---: | :---: | :---: | :---: | :---: |
| Line | Time | Name | Transcript |  |
| 154. |  | Romina: | They're. Like yeah, the t switching while they're in | stay in the same place and then the other three are just taying in the same place. |
| 155. |  | Jeff: | Yeah, they're staying in th | same spot. |
| 156. |  | Romina: | But we're not concerned w | th them. |
| 157. |  | Jeff: | That's why you're not con | erned with those. |
| 158. |  | Michael: | It's going to repeat like six | times. |
| 159. |  | Jeff: | Yeah. So that's where the by the two factorial. Tho | hree factorial comes from, and you're multiplying that are what you're- |
| 160. |  | Romina: | That's to say like the first | lace and the third place and then they just switch. |
| 161. |  | Michael: | Yeah, like- this way |  |
| 162. |  | Jeff: | Exactly. |  |
| 163. |  | Michael: | They just don't have a nam | on them so the, they're the same thing. |
| 164. |  | Romina: | Yeah. |  |
| 165. |  | Jeff: | And then that's where the each other and that gives | ottom number comes from and then you divide them by u what we're looking for. |
| 166. |  | R1: | OK, so I think I follow wh | t you said. |

of 7: Further explorations of factorials and combinations
Parent Tape: Night Session - Pascal's Identity
Location: David Brearley High School
Researcher: Professor Carolyn Maher
Line Time

Romina:
Jeff: Yeah, they're staying in the same spot.
Romina: But we're not concerned with them.
Jeff: That's why you're not concerned with those.
Michael: Its going to repeat like six times.
Yeah. So that's where the three factorial comes from, and you're multiplying that by the two factorial. Those are what you're-
Romina: That's to say like the first place and the third place and then they just switch.
Yean, like- this way

Romina: Yeah
And then that's where the bottom number comes from and then you divide them by each other and that gives you what we're looking for.
166.

R1:

