Three Essays of Firm's Fundamentals and Asset Pricing

by

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Abstract of the Dissertation

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Dissertation Directors: Professor Cheng-Few Lee and Professor Suresh Govindaraj

In my dissertation research, I have three essays discussing the firm's fundamentals and their asset pricing implications. In the first essay entitled "*Alternative Equity Valuation Models*", we use simultaneous equations estimation and combined forecasting methods to examine future stock prices forecast ability of Ohlson (1995) Model, Feltham and Ohlson (1995) Model, and Warren and Shelton (1971) Model. We also investigate whether comprehensive earnings can provide incremental price-relevant information beyond net income. Overall, we find that the simultaneous equations estimation procedure can produce more accurate future stock price forecasts than the traditional single equation estimation method, and combined forecast method can further reduce the prediction errors by using combination of individual forecasts. We also find supporting evidence that investors can use comprehensive earnings to more accurately forecast future stock prices in these valuation models.

My second essay entitled *"Technical, Fundamental, and Combined Information for Separating Winners from Losers"* jointly use fundamental and technical information to improve the technical momentum strategy. We examine how fundamental accounting information can be used to supplement the technical information, such as past returns and

past trading volume data, to separate momentum winners from losers. More specifically, we propose a unified framework of incorporating fundamental indicators FSCORE (Piotroski, 2000) and GSCORE (Mohanram, 2005) into the technical momentum strategy. Our empirical results suggest that the combined momentum strategy outperforms technical momentum strategy for both growth and value stocks.

My third essay entitled "*The Economic Consequences of Real Earnings Management*" examines how real activities based earnings management affect firm's payout and investment decisions. Our paper focuses on real earnings management in a general equilibrium production (GEP) economy setting, and studies the economic implications of this phenomenon on the economy. To formalize the notion of real earnings management, we propose that risk-averse managers "manage" earnings through investment-payout decisions that are conditioned by their history and habits. In addition, we permit habits to change randomly which introduces another source of risk. We explicitly solve for the endogenous asset prices and interest rate, and show how this additional risk from managerial habits is priced in the production economy.

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Chapter 1

Introduction

In my dissertation research, I have three essays discussing the firm's fundamentals and their asset pricing implications. My first essay entitled "*Alternative Equity Valuation Models*" investigates new estimation procedures and combined forecasting methods of alternative equity valuation models, namely the Ohlson (1995) Model, the Feltham-Ohlson (1995) Model (FO Model henceforth), and the Warren and Shelton (1971) Model (WS Model henceforth). We also examine whether comprehensive earnings can provide incremental price-relevant information beyond net income in these valuation models. The Ohlson Model and the FO Model are valuation models based on information obtained from income statement and balance sheet, i.e. earnings and book value per share while the WS Model accounts for the overall operating and financial environment of the firm. More specifically, we first use simultaneous equations estimation procedures to estimate the information dynamics in these models and compare their forecast ability of future stock prices. We then use comprehensive income, rather than net income, in these models to re-examine their ability to forecast future stock prices because it is more theoretically consistent with the clean surplus relation (CSR) which is the building block of these valuation models. Finally, we employ forecast combination methods to integrate the individual stock prices forecasts from these models and explore possible improvement in terms of producing smaller prediction errors.

Our empirical results suggest that the simultaneous equation estimation of the information dynamics improve the explanatory power of the models. Prior literature in testing the accounting based valuation model did not consider the feedback effect between the variables of interest. In other words, the traditional linear information dynamics conjectured that the earnings dynamics is an AR(1) process which is determined by past earnings and other value relevant information such as the analysts earnings forecasts. However, following Tsay et al. (2008) we conjecture that incorporating the feedback effect between the earnings and value relevant information variables, such as analysts earnings forecasts, improve the predictability of future stock prices in terms of better forecast accuracy. We find that simultaneous equation estimation procedure produces smaller mean forecast errors than the single equation estimation procedure by 5.14% (3.14%) on average in our sample period for the Ohlson (FO) model. Moreover, with the addition of other accounting information variables in the WS model, we find further improvement in forecasting future stock prices. The future stock price forecasts from the Warren-Shelton model are smaller than those predicted by Ohlson (FO) model by 5.26% (2.23%) annually in our sample period. By considering firm's overall operational and financial environment, the model produces smaller prediction errors to the previous two models considering only earnings and book value per share.

Given the future stock prices forecasts from these models, we employ forecast combination methods to integrate these individual forecasts in order to generate more accurate future stock price forecasts. We find that the combined future stock price forecasts based on weighted least square regression methods with the geometric weighting scheme produce smaller prediction errors than individual forecasts from

Ohlson, FO, and WS model. The combined forecasts using Ohlson (FO) model with the WS model generate smaller prediction errors than the individual forecasts by 7.16% (4.07%) annually. These results suggest superior accuracy of the combined forecast methods comparing to the individual forecasts in terms of future stock prices prediction errors. If comprehensive (operating) earnings are used in the Ohlson (FO) Model, we find that the prediction errors of future stock price forecast can be reduced up to 3.30% (2.44%) compared to using bottom-line earnings measure. Moreover, when comprehensive (operating) earnings based Ohlson (FO) Model forecasts are combined with WS Model forecasts, the reduction in pricing errors can reach up to 17.85% (15.96%). We find that comprehensive earnings indeed has incremental price-relevant information beyond bottom-line earnings measure, in terms of generating more accurate stock price forecasts in these valuation models.

My second essay entitled “Technical, Fundamental, and Combined Information for Separating Winners from Losers” investigates investment strategy that integrates fundamental and technical information in separating winner stocks from loser stocks. Prior literature on fundamental analysis and technical analysis framework has provided substantial evidence on their respective ability to explain the cross section of stock prices or to forecast future price movement. However, the literature is relatively silent on the integration of both analyses frameworks in equity valuation and in making investment decision. In the current study, we developed a combined momentum strategy employing past returns, trading volume, and firm's fundamentals and examine its profitability comparing to the technical momentum strategy.

The technical information of the stocks has been frequently used by securities analysts and portfolio managers as well as academic researchers. One of the most notable line of research using technical information in studying stock prices behavior is the momentum returns. By using stock's past performances, Jegadeesh and Titman (1993, 2001) documented that based on the cumulative returns in the past three to twelve months, the highest return decile portfolio outperform the lowest decile portfolio in the following three to twelve months. A large body of follow-up literature showed the presence of the price momentum across asset classes and countries. In addition to past returns, past trading volume has also been documented to predict stocks future returns and (Conrad, Hameed, and Niden (1994), Datar, Naik, and Radcliffe (1998)) and to provide information about the magnitude and persistence of the momentum returns (Lee and Swaminathan (2000), Chan, Hameed, and Tong (2000)). These findings suggest that there exist joint effect of these technical information on future stocks returns.

In addition to the technical information, the fundamental information about the firms also provides investors with guidance in making investment decision. Financial statement information such as inventory, account receivables, and gross margin have also been employed to construct fundamental signals about the firms (Ou and Penman (1989), Abarbanell and Bushee (1997), Lev and Thiagarajan (1993)). Researchers also construct aggregated measurement to examine overall performance of the firms. Piotroski (2000) and Mohanram (2005) developed fundamental indicators FSCORE and GSCORE in which firm specific information have been employed in evaluating value stocks and growth stocks respectively. These authors found that the portfolio consisting of financially healthier firms, i.e. firms with higher FSCORE or GSCORE, outperform those consisting of low scores firms up to two years after the portfolios are formed.

Based upon combined forecasting models developed by Granger and Newbold (1974), Granger and Ramanathan (1984), Lee, Newbold, Finnerty, and Chu (1986), and Lee and Cummins (1998), we propose a combined momentum strategy based on firm's past returns, past trading volume, and its composite fundamental scores. Oour results suggest that the combined momentum strategy outperforms the technical momentum strategy on average by 1.63% (1.85%) monthly among high (low) book-to-market stocks. The combined momentum strategy also generates higher information ratio than that of the technical momentum strategy. We consider our findings contributing to both technical momentum and accounting-based fundamental strategy literature. We believe the combined momentum strategy could provide investors with different performance metrics to separate the momentum winners from losers.

In my third essay entitled “The Economic Consequences of Earnings Management and Smoothing”, we introduce earnings management in a general equilibrium production economy, and to study the economic implications of this phenomenon on asset prices and interest rates. In our model, the risk-averse managers, whose decisions are conditioned by their history dependent habits, (and whose habits could randomly change), are permitted to manage earnings by their selection of projects. We explicitly solve for asset prices and interest rate endogenously in this economic setting. We show that given the deterministic habit formation, if the manager is more (less) conservative in managing the earnings, the firm will invest less (more) in risky asset, the withdrawals dynamics becomes less (more) volatile, and the equilibrium interest rate is lower (higher) which causes higher (lower) asset prices than the benchmark model. When the earnings habit formation has an extra stochastic component, the firm will invest less in risky asset, the withdrawals dynamics becomes more volatile, and the equilibrium interest rate becomes higher which causes a lower asset price than those found in deterministic case. Our results can potentially serve as a benchmark for studying the phenomenon of earnings management in more complicated economic settings, such as economies with hidden information, hidden actions, and other frictions.

The general consensus in the literature is that earnings management manifests itself in two forms: accrual management or real activities management. While there is extensive literature on the former, research on the latter is picking up. Our focus in this paper is the economic consequences of earnings manipulation through real activities management. We develop a dynamic equilibrium model with a representative risk averse manager who has the power to decide on firm investments, and whose consumption and compensation are tied to the firm's earnings (wealth) generated from the investments. Earnings net of withdrawals for consumption and compensation is reinvested in the firm, and contributes to the net worth and market value of the firm. We make two behavioral assumptions regarding the manager that capture the four characteristics for real activities management provided by Graham et al. (2005): (1) The manager will pursue actions to meet some earnings target; and (2) The manager is conditioned by past compensation habits, and is averse to fluctuations in these, especially, falling below past compensation levels. The idea is that managers are used to certain styles and levels of lifestyles. In addition, we permit the manager's weights on past compensation and standards of living to randomly change over time. While it makes our model more general, this innovation captures the reality that managers, while being conditioned by history, could react in unpredictable ways to external shocks. This adds an economic dimension that, to the best of our knowledge, has not been studied in the prior literature.

The main insights from our model yield are twofold: (1) Managers may forgo or avail of investment opportunities that they normally would not in the benchmark model; (2) Smoothing and stochastic habit changes add extra risks in the economy that affect market prices of firms and interest rates. The contributions of our paper are to provide an equilibrium framework for asset pricing with earnings management through real activities management, study the consequences of this phenomenon, quantify the risks associated with real earnings management, provide theoretical support for some prior empirical findings, and provide results for further empirical testing. The model developed by us and our results can potentially serve as a benchmark for studying the phenomenon of earnings management in more complicated economic settings, such as economies with hidden information, hidden actions, and other frictions.

The remainders of this dissertation discuss my each essay in greater details, providing the research methodology and summary of findings. The review and contribution to the related literature will also be presented.

Chapter 2

Alternative Equity Valuation Models

(*Joint work with Professor Cheng-Few Lee)*

2.1 Introduction

In this study we investigate new estimation procedures and combination of forecast methods of alternative equity valuation models, namely the Ohlson (1995) Model, the Feltham-Ohlson (1995) Model (FO Model henceforth), and the Warren and Shelton (1971) Model (WS Model henceforth). The Ohlson Model and the FO Model are valuation models based on information obtained from income statement and balance sheet, i.e. earnings and book value per share while the WS Model accounts for the overall operating and financial environments of the firm. This paper first uses simultaneous equations estimation procedures to estimate the information dynamics in these models and compare their forecast ability of future stock prices. Moreover, we examine the stock price forecasts ability of Ohlson and FO Model by using other form of earnings. Given the Ohlson Model and FO Model are derived based on the clean surplus relation (CSR), the *earnings* or *income* in CSR should include all changes in equity during a period except those resulting from investments by owners and distributions to owners. The *earnings* under this concept is closer to *comprehensive income* rather than the bottom-line or net income that had been frequently used in previous empirical studies. Given its consistency with the accounting-based valuation theory, we further investigate the stock price forecast ability of the Ohlson and FO Model by using the comprehensive earnings in the empirical model specification. Finally, we employ forecast combination methods to integrate the individual stock prices forecasts from these models and explore possible improvement in terms of producing smaller prediction errors.

Our empirical results suggest that the simultaneous equation estimation of the information dynamics improve the explanatory power of the models. Prior literature in testing the accounting based valuation model did not consider the feedback effect between the variables of interest. In other words, the traditional linear information dynamics conjectured that the earnings dynamics is an AR(1) process which is determined by past earnings and other value relevant information such as the analysts earnings forecasts. However, following Tsay et al. (2008) we incorporate the feedback effect between the earnings and value relevant information variables, such as analysts earnings forecasts, improve the predictability of future stock prices in terms of better forecast accuracy. We find that simultaneous equation estimation procedure produces smaller mean forecast errors than the single equation estimation procedure by 5.14% (3.14%) on average in our sample period for the Ohlson (FO) model. Moreover, with the addition of other accounting information variables in the WS Model, we find further improvement in forecasting future stock prices. The future stock price forecasts from the WS Model are smaller than those predicted by Ohlson (FO) Model by 5.26% (2.23%) annually in our sample period. By considering firm's overall operating and financial environment, the model produces smaller prediction errors to the previous two models considering only earnings and book value per share.

Given the future stock prices forecasts from these models, we employ forecast combination methods to integrate these individual forecasts in order to generate more accurate future stock price forecasts. We find that the combined future stock price forecasts based on weighted least square regression methods with the geometric weighting scheme produce smaller prediction errors than individual forecasts from Ohlson, FO, and WS Model. The combined forecasts using Ohlson (FO) model with the WS Model generate smaller prediction errors than the individual forecasts by 7.16% (4.07%) annually. These results suggest superior accuracy of the combined forecast methods comparing to the individual forecasts in terms of future stock prices prediction errors. If comprehensive (operating) earnings are used in the Ohlson (FO) Model, we find that the prediction errors of future stock price forecast can be reduced up to 3.30% (2.44%) compared to using bottom-line earnings measure. Moreover, when comprehensive (operating) earnings based Ohlson (FO) Model forecasts are combined with WS Model forecasts, the reduction in pricing errors can reach up to 17.85% (15.96%). We find that comprehensive earnings indeed has incremental price-relevant information beyond bottom-line earnings measure, in terms of generating more accurate stock price forecasts in these valuation models. Our results provides new evidence in the value-relevance of comprehensive earnings and shed light on how the issuance of Statement of Financial Accounting Standard No.130, *Reporting Comprehensive Income*, can help investors better assess the overall performance of the corporation.

The remainder of the essay is organized as follows. Section 2 provides literature review in accounting based valuation models and the financial planning and forecast models. The theoretical development and empirical implementation of these models are discussed. Section 3 presents the sample selection criteria, model specification of the linear information dynamics, and the research hypotheses for the empirical tests. Section

4 discusses the empirical results of individual forecasts and combined forecasts for future stock prices from these models. Section 5 provides the summary of this paper.

2.2 Literature Review

In this section, we first review the theoretical development and the empirical assessment of the Ohlson Model and the FO Model. We then review the financial planning and forecasting model developed by Warren and Shelton (1971). We also provide the background and prior academic research on the comprehensive earnings reporting issues. Finally, we will review the combined forecasting methods proposed by Granger and Newbold (1973), Granger and Ramanathan (1984), and Diebold and Pauly (1987).

2.2.1 Ohlson Model (1995) and Feltham-Ohlson Model (1995)

The Ohlson Model provides a theoretical framework linking the valuation to the reported financial statement variables. The traditional dividends discount model states the following relations

$$P_t = \sum_{\tau=1}^{\infty} R_f^{-\tau} \left(\tilde{d}_{t+\tau} \right) \qquad (2.1)$$

where P_t is the price of the firm's equity at time *t*, $\tilde{d}_{t+\tau}$ is the dividends paid at time t, and R_f is the risk free rate plus one. The restrictive nature of this relation is that equation (2.1) does not relate the reported financial statement numbers to firm value. In equation (2.1), the value depends on the accounting data that influences the present value of expected future dividends. Since this distribution of wealth eventually converge with the creation of wealth, Ohlson Model considers how the current value depends on accounting measures of wealth creation process. Ohlson Model introduced the clean surplus relations (CSR) assumption requiring that income over a period equals net dividends and the change in book value of equity. CSR ensures that all changes in shareholder equity that do not result from transactions with shareholders (such as dividends, share repurchases or share offerings) are reflected in the income statement. In other words, CSR is an accounting system recognizing that the periodically value created is distinguished from the value distributed.

Let x_t denote the earnings for period (t-1,t), y_t denote the book value at time t, and $x_t^a = x_t - \left(R_f - 1\right) y_{t-1}$ denote the abnormal earnings at time t. The clean surplus relations $y_t = y_{t-1} + x_t - d_t$ implies that

$$P_t = y_t + \sum_{\tau=1}^{\infty} R_f^{-\tau} E_t\left[\tilde{x}_{t+\tau}^a\right] \qquad (2.2)$$

the a firm's value is equal to its book value adjusted for the present value of expected future abnormal earnings. The variables on the right hand side of (2.2) are still forecasts, not past realizations. To deal with this problem, Ohlson Model introduced the information dynamics to link the value to the contemporaneous accounting data.[1] Assume $\left\{\tilde{x}_t^a\right\}_{\tau\geq 1}$ follows the stochastic process

$$\begin{aligned}\tilde{x}_{t+1}^a &= \omega x_t^a + v_t + \tilde{\varepsilon}_{1,t+1} \\ \tilde{v}_{t+1} &= \gamma v_t + \tilde{\varepsilon}_{2,t+1}\end{aligned} \qquad (2.3)$$

where v_t is value relevant information other than abnormal earnings and $0\leq\omega, \gamma\leq 1$. Based on equations (2.2) and (2.3), Ohlson Model demonstrated that the value of the equity is a function of contemporaneous accounting variables as follows.

$$P_t = y_t + \hat{\alpha}_1 x_t^a + \hat{\alpha}_2 v_t \qquad (2.4)$$

where $\hat{\alpha}_1 = \hat{\omega}/\left(R_f - \hat{\omega}\right)$ and $\hat{\alpha}_2 = R_f/\left(R_f - \hat{\omega}\right)\left(R_f - \hat{\gamma}\right)$. Or equivalently,

$$P_t = \kappa\left(\varphi x_t - d_t\right) + \left(1-\kappa\right) y_t + \alpha_2 v_t \qquad (2.5)$$

where $\kappa = \left(R_f - 1\right)\hat{\omega}/\left(R_f - \hat{\omega}\right)$ and $\varphi = R_f/\left(R_f - 1\right)$. Equations (2.4) and (2.5) imply that the market value of the equity is equal to the book value adjusted for (i) the current profitability as measured by abnormal earnings and (ii) other information that modifies the prediction of future profitability. One major limitation of the Ohlson Model is that it

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[1] Earlier literature on the time series analysis application in accounting research can be found in Hopwood, McKeown, and Newbold (1981, 1982), Bao et al. (1983), Brown and Griffin (1983), Givoly (1985), and Morton (1998).

assumed unbiased accounting. In equation (2.3), since both abnormal earnings and other information follow an AR(1) process, over time their averages are zero and thus the average abnormal earnings is zero as well. If given biased (conservative) accounting, the average abnormal earnings will be nonzero. Consequently the future growth in book value will become an important factor. This motivated Feltham and Ohlson (1995) to introduce additional dynamics to deal with this issue.

The FO Model analyzes how firm value relates to the accounting information that discloses the results from both operating and financial activities. For the financial activities, there are relatively perfect markets and the accounting measures for book value and market value of these assets are reasonably close. However for the operating assets, accrual accounting usually results in difference between the book value and the market value of these assets since they are not traded in the market. Accrual accounting for the operating assets consequently results in discrepancy between their book value and market value and thus influences the goodwill of the firm. Similar to Ohlson Model, the information dynamics in the FO Model is

$$
\begin{aligned}
\widetilde{ox}^{a}_{t+1} &= \omega_{10} + \omega_{11} ox^{a}_{t} + \omega_{12} oa_{t} + \omega_{13} v_{1t} + \tilde{\varepsilon}_{1t+1} \\
\widetilde{oa}_{t+1} &= \omega_{20} + \omega_{22} ox^{a}_{t} + \omega_{24} v_{2t} + \tilde{\varepsilon}_{2t+1} \\
\tilde{v}_{1t+1} &= \omega_{30} + \omega_{33} v_{1t} + \tilde{\varepsilon}_{3t+1} \\
\tilde{v}_{2t+1} &= \omega_{40} + \omega_{44} v_{2t} + \tilde{\varepsilon}_{4t+1}
\end{aligned}
\qquad (2.6)
$$

where ox_t^a is the abnormal operating earnings, oa_t is the operating assets, v_{1t} and v_{2t} are the other value relevant information variables for firm at time *t* respectively. The derived implied pricing function is

$$P_t = y_t + \hat{\lambda}_0 + \hat{\lambda}_1 ox_t^a + \hat{\lambda}_2 oa_t + \hat{\lambda}_3 v_{1t} + \hat{\lambda}_4 v_{2t} \qquad (2.7)$$

where

$$\begin{aligned}
\hat{\lambda}_0 &= \frac{(1+r)\begin{bmatrix}\hat{\omega}_{10}(1+r-\hat{\omega}_{22})(1+r-\hat{\omega}_{33})(1+r-\hat{\omega}_{44}) \\ +\hat{\omega}_{12}\hat{\omega}_{20}(1+r-\hat{\omega}_{33})+\hat{\omega}_{13}\hat{\omega}_{30}(1+r-\hat{\omega}_{22}) \\ +\hat{\omega}_{14}\hat{\omega}_{40}(1+r-\hat{\omega}_{44})\end{bmatrix}}{r(1+r-\hat{\omega}_{11})(1+r-\hat{\omega}_{22})(1+r-\hat{\omega}_{33})(1+r-\hat{\omega}_{44})} \\
\hat{\lambda}_1 &= \frac{\hat{\omega}_{11}}{r(1+r-\hat{\omega}_{11})} \\
\hat{\lambda}_2 &= \frac{(1+r)\hat{\omega}_{12}}{(1+r-\hat{\omega}_{11})(1+r-\hat{\omega}_{22})} \\
\hat{\lambda}_3 &= \frac{(1+r)\hat{\omega}_{13}}{(1+r-\hat{\omega}_{11})(1+r-\hat{\omega}_{33})} \\
\hat{\lambda}_4 &= \frac{(1+r)\hat{\omega}_{14}}{(1+r-\hat{\omega}_{11})(1+r-\hat{\omega}_{44})}
\end{aligned} \qquad (2.8)$$

Or equivalently,

$$P_t = k(\phi x_t - d_t) + (1-\kappa) y_t + \hat{\alpha}_2 oa_t + \hat{\lambda}_3 v_{1t} + \hat{\lambda}_4 v_{2t} \qquad (2.9)$$

where $\kappa = (R_f - 1)\hat{\omega}_{11} / (R_f - \hat{\omega}_{11})$ and $\phi = R_f / (R_f - 1)$. The implied valuation function in equations (2.7) and (2.9) is a weighted average of firm's operating earnings, firm's book value, and the other value-relevant information with an adjustment for the understatement of the operating assets resulting from accrual accounting. The major contribution of the FO Model is that it considered the accounting conservatism in the equity valuation.

The line of research in empirically testing the Ohlson Model and FO Model has been growing large since its introduction. Previous empirical literature focused on either the value-relevant information variables (Abarbanell and Bushee 1997, Myers 1999, Dechow et al. 1999, Liu and Thomas 2000, and Begley and Feltham 2002) or the dynamics of the earnings process (Morel 2001, Callen and Morel 2003). However, none of them documented empirical validity of the Ohlson Model. Callen and Segal (2005) showed that the nested Ohlson Model is rejected in favor of the FO Model but it did not improve the predictability power of the future stock prices. Based on these previous studies, we will examine whether the simultaneous equation approach in estimating the information dynamics can improve the predictability power of the Ohlson and FO Model.

The other potential cause of the lack of empirical validity of the residual income valuation models is the use of net income as the earnings measure in the linear information dynamics. Given these models are based on the clean surplus relation, the earnings measure should include all changes in equity except those resulting from investments by owners and distributions to owners. Consequently, the determination of stock price in the valuation functions of equations (2.4) and (2.7) cannot be complete unless the earnings include all the value-added activities in the firm (Linsmeier et al., 1997a). Comprehensive income is defined as the change in equity (net assets) of a business enterprise during a period from transactions and other events and circumstances from non-owners sources (Statements of Financial Accounting Concepts No. 6, *Elements* *of Financial Statements,* 1985). Accounting standards in US sometimes allow non-owners changes in asset and liabilities, such as foreign currency translation adjustments, available-for-sales marketable securities adjustments, and minimum required pension liability adjustments, to bypass the income statement. The exclusion of these value-relevant items in financial reporting might mislead the users of financial statement information in assessing the value of the firm.

The debate of whether the firms should report comprehensive income or more streamlined bottom-line income can be traced back to 1930's (Brief and Peasnell, 1996). The supporters of reporting comprehensive income argue that it captures all sources of value creation within a firm. Comprehensive income allows the users of the financial statement information to consider all relevant factors for earnings forecasting and firm value assessment. It also grants less leeway for the managers to engage in earnings management which could potentially distort the actual performance of the firms.[2] On the other hand, the opponents to the comprehensive income reporting point out that it includes many items that are transitory in nature which are not representative of the core operation of the firms. The inclusion of these nonrecurring and extraordinary items hinder income measure to reflect firm's long-term cash flows prospects (Dhaliwal et al., 1999). It is also argued that these items add noises to reported earnings and make it difficult for forecasts. The users of the financial statements should be able to focus on a single measurement that summarizes all the value-relevant information without much manager's discretion in reporting this figure.[3]

[2] Previous literature in advocating comprehensive income reporting include Robinson (1991), Johnson et al. (1995), Beresford et al. (1996), and Smith and Reither (1996).

[3] Previous literature in advocating current operating performance concepts of reporting income include Kiger and Williams (1977), Black (1993), Brief and Peasnell (1996), and Holthausen and Watts (2001).

The debate of whether firms in US should employ comprehensive income reporting led to the issuance of Statement of Financial Accounting Standard No.130 (SFAS 130), *Reporting Comprehensive Income*, by Financial Accounting Standards Board. SFAS 130 requires firms to report comprehensive income in their primary financial statements. In the pre- and post-SFAS 130 era, there were many studies examining the value-relevance of this requirement to report comprehensive income. Cheng et al. (1993) evaluates the usefulness of different earnings definition and finds that the conventional income measures such as operating income and net income provide better explanatory power for residual security returns than comprehensive income. Dhaliwal et al. (1999) examines the value-relevance of the major three components in comprehensive income required by SFAS 130. Their results suggest that only the marketable securities adjustment item improves the income and returns association. More importantly, they fail to find support to show comprehensive income is a better measurement for firm performance than net income and raise questions about the reporting requirement in SFAS 130. Biddle and Choi (2006) on the other hand find that comprehensive income outperforms other income measure in explaining equity returns and predicting future income and operating cash flows. Chambers et al. (2007) argue that prior studies in the pre-SFAS 130 era suffer from the measurement errors problem in calculating the comprehensive earning. These authors show that the aforementioned three major components in the comprehensive income are indeed priced by the market in the post-SFAS 130 era when these items are specifically reported by requirement. In addition to the US findings, there are also many other international studies regarding the value-relevance of comprehensive income.[4] The results, however, are mixed because of the

[4] International evidence of the debate of comprehensive income reporting in countries such as UK and New

different local accounting standards and time period within which the requirements are implemented.

2.2.2 Warren and Shelton Model (1971)

In addition to the accounting based valuation model discussed above, operational financial planning models can also be used to forecast stock prices. One of such mathematical models is the Warren and Shelton (1971) Model. The WS Model use a simultaneous equation approach to analyze important operating and financial variables. The WS Model considers the overall operational and financial environment of the firm. It is flexible so that it can be adapted and extended to meet various circumstances. The model accounts for the interrelations between investment activity, financing activity, dividend policy, and the production decision of the firm and their influences on the market value of the firm.

The critical inputs in the WS Model are the sales growth rate forecast and several operating ratios. The WS Model has four segments including twenty equations simultaneously determining twenty unknowns. The four segments are corresponding to firm's sales, investments, financing, and return to investment concepts in the financial theory. The model first generates the sales and earnings forecasts given the historical data. Further, the model calculates the total assets required to support these sales and earnings forecasts, and the venue through which these assets are to be financed. Finally, given these operation and financing decisions, the models determines the stock per share data.[5]

Zealand can be found in O'Hanlon and Pope (1999), Cahan et al. (2000), Brimble and Hodgson (2005), and Lin (2006).

[5] The derivation of stock price per share forecast in WS Model can be found in Appendix A.

Essentially the WS Model generates the price per share estimate from the assumptions about the firm's future growth in revenue, its financial and operating policies, and the overall economic environment. The model provides a valuation framework that includes many different parameter inputs which generate a wide range of estimates that can be presented statistically by a distribution with a mean and variance. The user of the model can easily examine the implication of alternative underlying firm-level and economy-wide environment changes on the equity share prices.

2.2.3 Forecast Combination

Forecast evaluation are of interest in many areas of empirical finance research, such as market efficiency (Fama 1970, 1991), volatility of the observed asset returns (Shiller 1979, LeRoy and Porter 1981, Fama and French 1988), and forward exchange rates (Hansen and Hodrick 1980). Borrowing from Diebold and Lopez (1996) and Yee (2009), we consider two groups of combining forecast methods, i.e. the variance-covariance method and the regression method. Bates and Granger (1969) first proposed the variance-covariance method for forecast combination. Denote the one-period ahead stock price forecast at time t, $\hat{y}^i_{t,t+1}$, from model $i \in \{1,2\}$, the combined forecast can be formed as the weighted average between the two forecasts

$$\hat{y}^c_{t,t+1} = \omega \hat{y}^1_{t,t+1} + (1-\omega)\hat{y}^2_{t,t+1}$$

which is an unbiased forecast if the weights sums up to unity. Moreover, the composite forecast error has the same relation as the combined forecast

$$\varepsilon_{t,t+1}^{c} = \omega \varepsilon_{t,t+1}^{1} + (1-\omega)\varepsilon_{t,t+1}^{2}$$

and the variance of the combined forecast

$$\sigma_c^2 = \omega^2 + \sigma_{11}^2 + (1-\omega)^2 \sigma_{22}^2 + 2\omega(1-\omega)\sigma_{12}$$

where σ_{11}^2, σ_{22}^2, and σ_{12}^2 are the variance of the forecast from model one, model two, and their covariance. The optimal weight to minimize the forecast error ω can be derived as

$$\omega = \frac{\sigma_{22}^2 - \sigma_{12}}{\sigma_{11}^2 + \sigma_{22}^2 - 2\sigma_{12}}$$

which is determined by the variances of each individual forecast and the covariance between them. The asymptotic properties of the optimal weights are

$$\lim_{\sigma_{11}^2 \to \infty} \omega = 0 \quad \lim_{\sigma_{22}^2 \to \infty} \omega = 1 \quad \lim_{\sigma_{11}^2 \to 0} \omega = 1 \quad \lim_{\sigma_{22}^2 \to 0} \omega = 0$$

Therefore, the variance-covariance method place larger weight on the more reliable forecast in forecast combination.

The regression method in forecast combination suggest a regression model in which the realization of y_{t+1} is regressed on the past forecasts of y_{t+1} to determine the optimal weights (Chong and Hendry 1986), Fair and Shiller 1989, 1990),

$$y_{t+1} = \alpha_0 + \alpha_1 \hat{y}^1_{t,t+1} + \alpha_2 \hat{y}^2_{t,t+1} + \varepsilon_{t,t+1} \qquad (2.10)$$

Granger and Ramanathan (1984) showed that the optimal weight determined in the variance-covariance method has a regression interpretation as the coefficient vector in regression model (2.10) of a linear projection of y_{t+1} onto the forecasts y^i_{t+1} subject to the constraints that the weights α_i sums to unity and the exclusion of the intercept term. However, as a number of researchers have recognized that the true but unknown variance-covariance matrix in determining the optimal weight ω are not fixed over time. Therefore the ensuing research in this literature focused on the time-varying combining weights (Granger and Newbold 1973, Diebold and Pauly 1987) which can be achieved by using the technique of weighted least square (WLS). Diebold and Pauly (1987) proposed a WLS estimator

$$\hat{\alpha}_{WLS} = \left(X'W'X \right)^{-1} X'WY$$

where the weighting matrix can be considered for the following schemes

1. Equal weight (standard regression-based combining): w_{tt} =1 for all t. (W1)

2. Linear weighting: $w_{tt} = t$ for all t. (W2)

3. Geometric weighting: $w_{tt} = \lambda^{T-t}$, 0<λ≤1, or $w_{tt} = \lambda^{t}$, λ>1. (W3)

4. t^{λ} (t-lamda): $w_{tt} = t^{\lambda}$, λ≥0. (W4)

The geometric weighting has the appealing property that the weights increases at an increasing rate as we get closer to the present time. This yields heavy weighting on the more recent observation which might provide better accuracy for forecast values. Moreover, the geometric weighting can provide a weighting scheme that dies out fairly quickly which might be useful in modeling forecasts under an unstable environment. Similarly, the t-lamda (t^{λ}) specification can also produce weights that dies out quickly but it has a even more appealing fundamental characteristic that its weighting scheme can increase either at an increasing or decreasing rate as we get closer to the present time. When λ=0 one obtains the constant weighting scheme in case (1) while λ=1 the linear weighting scheme in case (2) emerges.

The forecast combination equation with two primary individual forecasts can be written as

$$\underset{(T\times 1)}{\mathrm{Y}} = \underset{(T\times 3)}{\mathrm{f}}\ \underset{(3\times 1)}{\alpha} + \underset{(T\times 1)}{\varepsilon}$$

where

$$\alpha^{i} = P^{i}(t) = p_0^{i} + p_1^{i}t + \cdots + p_{\gamma}^{i}, \quad i == 0,1,2$$

$f_t = \left(1, f_t^1, f_t^2\right), \alpha = \left(\alpha^0, \alpha^1, \alpha^2\right)'$, and **f** is the matrix with *t*-th row f_t. The time-varying combining weights are deterministic nonlinear polynomial functions of time. The advantage of this regression-based deterministically time-varying parameters model over the weighted least square approach is that this method can explicitly model any parameter evolution in the forecast combination equation. This approach can also project the evolution in when the forecasts are combined. The general polynomial and unrestricted regression-based combination is the following

$$\begin{aligned} y_{t+1} &= \left(p_0^0 + p_1^0(t+1)\right) + \left(p_0^1 + p_1^1(t+1)\right)_t f_{t+1}^1 + \left(p_0^2 + p_1^2(t+1)\right)_t f_{t+1}^2 \\ &= p_0^0 + p_1^0(t+1) + p_{0t}^1 f_{t+1}^1 + p_1^1\left((t+1)_t f_{t+1}^1\right) + p_{0t}^2 f_{t+1}^2 + p_1^2\left((t+1)_t f_{t+1}^2\right) \end{aligned} \tag{2.11}$$

Similarly, the forecast can be obtained after estimating the parameters $\hat{p}_0^i$ and $\hat{p}_1^i$

$${}_t\hat{y}_{t+1} = \left(\hat{p}_0^0 + \hat{p}_0^1\left(t+1\right)\right) + \left(\hat{p}_0^1 + \hat{p}_1^1\left(t+1\right)\right)_t f_{t+1}^1 + \left(\hat{p}_0^2 + \hat{p}_1^2\left(t+1\right)\right)_t f_{t+1}^2 \tag{2.12}$$

The weighted least square approach can be further combined with the time-varying parameters to determine the optimal weight in the forecast combination equation. For example, one can use geometric weighting scheme λ^{T-t} to construct the weighting matrix W and then estimate the parameters in equation (2.11). The estimated parameters $\hat{p}_0^i$ and $\hat{p}_1^i$ are then used in equation (2.12) to compute the forecast values. We will explore more possible combination of different weighting schemes and the time-varying parameters model to examine the combination of primary individual forecasts.

2.3 Data and Methodology

In this section, we first introduce and the sample selection criteria. Then we introduce the research hypotheses and model specifications for the empirical tests presented in the next section.

2.3.1 Data

The data used in this paper is obtained from the intersection of the following four data sets between 1980 and 2007: annual Compustat for historical accounting data, monthly Center for Research in Security Prices (CRSP) for stock returns, and analyst forecast file from I/B/E/S. The following Compustat data items are used to construct the variables required in the empirical analysis in this study. cash and cash equivalent (# 1), total assets (# 6), long term debt (# 9), interest expense (# 15), investments and advancements (# 32), debt in current liabilities (# 34), interest income (# 62), preferred shares (# 130), short term investments (# 193), total liabilities (# 181), and notes payable (# 206). Moreover, the book value per share and price data are obtained from Compustat as well. For the Ohlson Model, the data required are already available from the data obtained from the aforementioned sources. However for empirically testing the FO Model, further distinction between the net operating assets and the net financial assets, and between the operating earnings and the financial earnings have to be conducted. As discussed previously, the FO Model assume that the conservative accounting only applies to the operating assets while financial activities are all zero net present value investments. Consequently only operating assets generate the differences between their book value and the market value which is the goodwill. However, neither theory nor the empirical rules demonstrate how to distinguish between the financial and operating assets. We follow the procedure outlined in Penman (2000) and Callen and Segal (2005) to calculate the operating assets and the financial assets.

Financial Assets = Cash and Cash Equivalent (# 1) + Investments and Advancements (# 32) + Short term Investments (# 193)

Financial Liabilities = Long term debt (# 9) + Debt in Current Liabilities (# 34) + Notes Payable (# 206)

Operating Assets = total assets (# 6) - Financial Assets

Operating Liabilities = Preferred Shares (# 130) + Total Liabilities (# 181) - Financial Liabilities

Net Operating Assets = Operating Assets - Operating Liabilities

Net Financial Assets = Financial Assets - Financial Liabilities

We also use comprehensive (operating) earnings in the linear information dynamics and examine how its effects the accuracy in forecasting future stock prices. SFAS 130 is effectively adopted in 1998 before which firms were not required to report comprehensive income. We follow Cheng et al. (1993), Dhaliwal et al. (1999) , and Biddle and Choi (2006) to measure the comprehensive income. We did not use the actual reported comprehensive income given the lack of consistency in reporting of firms in our sample period.[6] The definition of comprehensive income by SFAS 130 is the net income

[6] In the post-SFAS 130 periods, Compustat has not yet completely disclosed all components in comprehensive income. Currently, Compustat only reports some of the items related to comprehensive

adjusted for "other comprehensive income" items. These items include: (1) the change in the balance of unrealized and losses on available-for-sale marketable securities (MSA), (2) the change in cumulative foreign currency translation adjustments (RECTA), and (3) the change in additional minimum pension liability in excess of unrecognized prior service costs (PENADJ). All these variables are scaled by the beginning-of-period market value of equity and they are calculated by using the Compustat data. *MSA* and *RECTA* are items *marketable securities adjustment* and *Retained Earnings – Cumulative Translation Adjustment* obtained directly from Compustat. *PENADJ* is calculated as *Pension-Additional Minimum Liability (PADDML) – Pension-Unrecognized Prior Service Cost (PCUPSO)*. The comprehensive income defined in SFAS 130, NI_{130}, is equal to *NI+MSA+RECTA+PENADJ*.

Comprehensive Income (x_{130}) = Net Income (# 172) + *MSA* (# 238) + *RECTA* (# #230) + *PENADJ* (#297-# 298)

Given the adjustments in calculating the comprehensive income, we further define the comprehensive operating income (ox_{130}) which is used in the information dynamics of FO Model. Following Nissim and Penman (2001), we define the comprehensive operating income as follows.

Comprehensive Operating Income (ox_{130})

---

income and these data are only complete after year 2001. Given that our empirical tests require sufficient time-series to conduct forecasting, we employ the measurement methodology in Cheng et al. (1993) and Dhaliwal et al. (1999) to estimate comprehensive income.

= Comprehensive Income (x_{130}) + Comprehensive Net Financial Expenses (*NFE*) – Minority Interest in Income (# 49)

where

Comprehensive Net Financial Expenses (*NFE*)

= Core Net Financial Expenses (*Core NFE*) + Unusual Financial Expenses (*UFE*)

= After Tax Interest Expense (# 15 $\times$ (1-marginal tax rate[7])) + Preferred Dividends (# 19) – After Tax Interest Income (# 62 $\times$ (1-marginal tax rate))) + Change in MSA (Lag # 238 - # 238)

Several other variables used in this study is discussed below. The earning used in our empirical analysis are the earnings from the continued operations obtained from I/B/E/S. We follow Callen and Segal (2005) to use the earnings reported in the I/B/E/S because of their comparability with the analysts earnings forecasts. Moreover, the interest rate on debt are computed as the interest expense (DATA 15) divided by the average financial liabilities. The cost of equity capital is calculated by the Fama-French three factor model and the annualized three month treasury bill rate. Finally, we exclude the financial institution (SIC 6000) from the sample because of their minimal level of operating assets and the additional regulatory requirements. Observations with market value of equity less than $10 million, with negative operating and financial assets (liabilities), and with

[7] Borrowing from Nissim and Penman (2001), the marginal tax rate is the top statutory federal tax rate plus 2% average state tax rate. For our sample periods, the top statutory federal tax rate was 46% in 1979-1986, 40% in 1987, 34% in 1988-1992, and 35% in 1993-1999, 40% in 2000-2002, and 35% in 2003-2008.

negative net operating and financial earnings are excluded as well. Finally, firms whose empirical variables are less than two firm years are deleted.

2.3.2 Research Hypotheses

Following Dechow et al. (1999) and Callen and Segal (2005), the empirical works in testing the Ohlson Model and FO Model employ the analysts earnings forecast to be the proxy for the other value-relevant information variable. FO Model further supplements the Ohlson Model with the adjustment for conservative accounting towards which U.S. GAPP is biased. By using the data in the U.S. market, FO Model is expected to produce better stock price forecast accuracy in terms of smaller prediction errors. As a result, we formulate our first testable hypothesis as follows.

H_{10}: *Feltham-Ohlson (1995) Model provides more accurate future stock price forecasts, in terms of smaller mean forecast errors, than the Ohlson (1995) Model.*

The use of analysts earnings forecasts is commonly used by the practitioner as well since they capture the forward looking estimation of the performance of the firm. Should one can use the analysts earnings forecasts to predict the future earnings, it is possible that there exist a relation between the two variables. Moreover, for the FO Model, it is also possible that there exist feedback relations of the operating earnings and operating assets with the short-term and long-term analysts earnings forecasts. As a result, we follow Tsay et al. (2008) to use a simultaneous equation approach to estimate the linear information dynamics in both Ohlson Model and FO Model. By employing the simultaneous equation estimation in the linear information dynamics, we expect to capture the interaction of the future period earnings with the current period analysts earnings forecasts. We conjecture the earnings forecasts influence the future period earnings and thus the valuation of the equity. At the same time, the current period earnings also affect the earnings forecasts produced by the analysts. It is the inter-relationships between these variables that determine the fundamental value of the equity shares. Thus we develop our second testable hypothesis as follows.

H_{20}: *Simultaneous equations estimation of the linear information dynamics generate more accurate future stock price forecasts, in terms of smaller mean forecast errors, than single equation estimation in both Ohlson (1995) Model and Feltham-Ohlson (1995) Model.*

In addition to the Ohlson Model and FO Model discussed above, we next focus on the stock price forecasts ability of the WS Model. The WS Model uses a simultaneous equation approach to forecast future stock price by considering both operating and financing decision of the firms. The WS Model is more comprehensive than the residual income valuation models because it accounts for the interrelations between investment activity, financing activity, dividend policy, and the production decision of the firm. Given its flexibility, the WS Model is expected to better predict future stock prices than the Ohlson/FO Model discussed previously. Thus our third testable hypothesis is the following.

H_{30}: *The Warren and Shelton (1971) Model can generate more accurate future stock price forecasts, in terms of smaller mean forecast errors, than both Ohlson (1995) Model and Feltham-Ohlson (1995) Model.*

After considering the future stock price forecasts from these valuation models, we further investigate whether they can be combined to form more accurate forecasts. We thus employ the combined forecasts methods (Granger and Newbold, 1973, Granger and Ramanathan, 1984, and Diebold and Pauly, 1987) to examine whether forecasts combination are more accurate than individual forecasts in terms of mean forecasts errors. Therefore, the fourth testable hypothesis in the current paper is the following.

H_{40}: *The combination of individual forecasts from Ohlson (FO) Model and the WS Model can generate more accurate future stock price forecasts, in terms of smaller mean forecast errors, than each individual forecasts.*

Finally, we investigate whether comprehensive (operating) earnings can provide incremental price-relevant information beyond bottom-line earnings. We employ the comprehensive (operating) earnings in the linear information dynamics of Ohlson (FO) Model and examine its effects on the future stock price forecasts. These forecasts are further combined with the WS Model forecasts to and their forecasts accuracy is examined. Thus the fourth testable hypothesis can be stated as the follows.

H_{50}: *Using comprehensive earnings as the earnings measure in the linear information dynamics of the Ohlson Model and Feltham-Ohlson Model can generate more accurate future stock price forecasts, in terms of mean forecast error, than bottom-line earnings as the earnings measure.*

2.3.3 The Model Specifications

In this section, we propose two sets of the linear information dynamics in the Ohlson Model and FO Model. The estimated coefficients from these information dynamics are further used in the valuation function to forecast future stock prices. The first set of specifications includes the single equation and the simultaneous equations estimation with the analyst forecast of earnings in the linear information dynamics of Ohlson Model. The single equation approach specified in this set is essentially the model tested in Dehow et al. (1999). Moreover, as Tsay et al. (2008) stated that there are feedback relations between the other value-relevant information and the earnings, we further employ the simultaneous equation linear information dynamics. More specifically, we examine whether the feedback effect between the current earnings, analyst forecasts, and the book value improve the predictability of the model.

1. Model Set I: In the first model specification we test the Ohlson Model with the other value-relevant information variable. This variable essentially summarizes information that is captured in a firm's stock because it can predict future abnormal earnings but is not yet reflected in the financial statements. Here we test a modified version of Dechow et al.

(1999) model in which the other value-relevant information variable is the analysts' earnings forecasts. The linear information dynamic is

$$\begin{aligned}\tilde{x}^a_{i,t+1} &= \omega_{10} + \omega_{11}x^a_{i,t} + \omega_{12}v_{i,t} + \tilde{\varepsilon}_{i1,t+1} \\ \tilde{v}_{i,t+1} &= \omega_{20} \qquad\qquad\quad + \omega_{22}v_{i,t} + \tilde{\varepsilon}_{i2,t+1}\end{aligned} \qquad (2.13)$$

where $\tilde{x}^a_{i,t+1}$ is the abnormal earnings of firm *i* at time *t*, $v_{i,t}$ is the difference between the conditional expectation of abnormal earnings for firm *i* at time period t+1 based on all available information and the expectation of abnormal earnings, i.e. $v_{i,t} = E_t\left[\tilde{x}^a_{i,t+1}\right] - \omega_{11}x^a_{i,t}$. Following Dechow et al. (1999), the period t conditional expectation of period t+1 earnings are the median consensus analyst forecast of period t+1 earnings denoted by f_t, i.e. $E_t\left[\tilde{x}^a_{i,t+1}\right] = f^a_{i,t} = f_{i,t} - ry_{i,t}$, where $f_{i,t}$ is the median consensus analyst earnings forecasts of next year's earnings measured at the first month after the publication of the annual financial report. Consequently, the other value-relevant information can thus be written as $v_{i,t} = f^a_{i,t} - \omega_{11}x^a_{i,t}$.

The simultaneous equations specification of the linear information dynamic on the other hand is

$$\begin{aligned}\tilde{x}^a_{i,t+1} &= \omega_{10} + \omega_{11}x^a_{i,t} + \omega_{12}v_{i,t} + \tilde{\varepsilon}_{i1,t+1} \\ \tilde{v}_{i,t+1} &= \omega_{20} + \omega_{21}x^a_{i,t} + \omega_{22}v_{i,t} + \tilde{\varepsilon}_{i2,t+1}\end{aligned} \qquad (2.14)$$

where the coefficient ω_{21} represents the feedback effect from current period abnormal earnings to next period analyst earnings forecasts. Given the specification of the information dynamics in (2.13) and (2.14), the implied valuation function can be written as

$$P_{i,t} = y_{i,t} + \hat{\beta}_0 + \hat{\beta}_1 x^a_{i,t} + \hat{\beta}_2 v_{i,t} \qquad (2.15)$$

where the estimated coefficients are

$$\hat{\beta}_0 = \frac{\left(1+\hat{r}_{i,t}\right)\left[\hat{\omega}_{10}\left(1+\hat{r}_{i,t}-\hat{\omega}_{22}\right)+\hat{\omega}_{12}\hat{\omega}_{20}\right]}{\hat{r}_{i,t}\left(1+\hat{r}_{i,t}-\hat{\omega}_{11}\right)\left(1+\hat{r}_{i,t}-\hat{\omega}_{22}\right)}$$

$$\hat{\beta}_1 = \frac{\hat{\omega}_{11}}{\left(1+\hat{r}_{i,t}-\hat{\omega}_{11}\right)}$$

$$\hat{\beta}_2 = \frac{\left(1+\hat{r}_{i,t}\right)\hat{\omega}_{12}}{\left(1+\hat{r}_{i,t}-\hat{\omega}_{11}\right)\left(1+\hat{r}_{i,t}-\hat{\omega}_{22}\right)}$$

and $\hat{r}_{i,t}$ is the cost of equity capital for firm *i* at time *t*. Or equivalently,

$$P_{i,t} = \kappa\left(\varphi x_{i,t} - d_{i,t}\right) + \left(1-\kappa\right) y_{i,t} + \hat{\beta}_2 v_{i,t} \qquad (2.16)$$

where $\kappa = \hat{r}_{i,t}\hat{\omega}_{11} / \left(1+\hat{r}_{i,t}-\hat{\omega}_{11}\right)$ and $\varphi = \left(1+\hat{r}_{i,t}\right) / \hat{r}_{i,t}$.

In the second set of the model specification, we test the FO Model with single equation and simultaneous equation linear information dynamics. The FO Model argues that it is important to separate the financial assets and the operating assets in the valuation function since only operating assets generate goodwill. The FO Model considers the practice of accrual accounting and how it influences the equity valuation.

2. Model Set II: The single equation linear information dynamics in the FO Model is the following.

$$
\begin{aligned}
\widetilde{ox}^{a}_{i,t+1} &= \omega_{10} + \omega_{11} ox^{a}_{i,t} + \omega_{12} oa_{i,t} + \omega_{13} v_{1i,t} + \tilde{\varepsilon}_{1i,t+1} \\
\widetilde{oa}^{a}_{i,t+1} &= \omega_{20} + \omega_{22} oa_{i,t} + \omega_{24} v_{2i,t} + \tilde{\varepsilon}_{2i,t+1} \\
\tilde{v}_{1i,t+1} &= \omega_{30} + \omega_{33} v_{1i,t} + \tilde{\varepsilon}_{3i,t+1} \\
\tilde{v}_{2i,t+1} &= \omega_{40} + \omega_{44} v_{2i,t} + \tilde{\varepsilon}_{4i,t+1}
\end{aligned} \tag{2.17}
$$

and the simultaneous linear information dynamics is

$$
\begin{aligned}
\widetilde{ox}^{a}_{i,t+1} &= \omega_{10} + \omega_{11} ox^{a}_{i,t} + \omega_{12} oa_{i,t} + \omega_{13} v_{1i,t} + \omega_{14} v_{2i,t} + \tilde{\varepsilon}_{1i,t+1} \\
\widetilde{oa}^{a}_{i,t+1} &= \omega_{20} + \omega_{21} ox^{a}_{i,t} + \omega_{22} oa_{i,t} + \omega_{23} v_{1i,t} + \omega_{24} v_{2i,t} + \tilde{\varepsilon}_{2i,t+1} \\
\tilde{v}_{1i,t+1} &= \omega_{30} + \omega_{31} ox^{a}_{i,t} + \omega_{32} oa_{i,t} + \omega_{33} v_{1i,t} + \omega_{34} v_{2i,t} + \tilde{\varepsilon}_{3i,t+1} \\
\tilde{v}_{2i,t+1} &= \omega_{40} + \omega_{41} ox^{a}_{i,t} + \omega_{42} oa_{i,t} + \omega_{43} v_{1i,t} + \omega_{44} v_{2i,t} + \tilde{\varepsilon}_{4i,t+1}
\end{aligned} \tag{2.18}
$$

where $ox^{a}_{i,t}$ is the abnormal operating earnings for firm *i* at time *t*, $oa_{i,t}$ is the operating assets for firm *i* at time *t.* Moreover, the value relevant information variables $v_{1i,t}$ and $v_{2i,t}$ are the growth in expected operating earnings and the expected growth in operating assets respectively. The expected operating earnings are measured as the difference between the median consensus analyst earnings forecast for next year and the expected net interest revenue (product of end of current year financial liabilities and the interest on debt). The growth in expected operating earnings is defined as the expected change in operating earnings divided by operating assets, i.e. $v_{1i,t} = E_t\left[\Delta\widetilde{ox}_{t+1}\right]/oa_t = E_t\left[\widetilde{ox}_{t+1} - ox_t\right]/oa_t$ where the current period operating earnings ox_t is calculated as actual earnings reported by the I/B/E/S minus the interest revenue (product of beginning of current year financial liabilities and the interest on debt). The expected growth in net operating asset is defined as the change in expected operating asset divided by the operating asset, i.e. $v_{2i,t} = E_t\left[\Delta\widetilde{oa}_{t+1}\right]/oa_t$. Following Liu and Ohlson (2001), we use the analyst earnings forecasts of long-term earnings growth rates as a proxy for the expected growth in net operating assets.[8]

Given the specification of the information dynamics in equations (2.16) and (2.17), the implied valuation function is

$$P_{i,t} = y_{i,t} + \hat{\lambda}_0 + \hat{\lambda}_1 ox^a_{i,t} + \hat{\lambda}_2 oa_{i,t} + \hat{\lambda}_3 v_{1i,t} + \hat{\lambda}_4 v_{2i,t} \qquad (2.19)$$

where

[8] The Long Term Growth Forecast generally represents an expected increase in operating earnings over the company's next full business cycle. Usually these forecasts refer to a period of between three to five years. Thomson Financial recommends the median value for long term growth forecast rather than the mean. The median value is less affected by outlier forecasts.

$$\hat{\lambda}_0 = \frac{(1+\hat{r}_{i,t})\begin{bmatrix} \hat{\omega}_{10}\left(1+\hat{r}_{i,t}-\hat{\omega}_{22}\right)\left(1+\hat{r}_{i,t}-\hat{\omega}_{33}\right)\left(1+\hat{r}_{i,t}-\hat{\omega}_{44}\right) \\ +\hat{\omega}_{12}\hat{\omega}_{20}\left(1+\hat{r}_{i,t}-\hat{\omega}_{33}\right)+\hat{\omega}_{13}\hat{\omega}_{30}\left(1+\hat{r}_{i,t}-\hat{\omega}_{22}\right) \\ +\hat{\omega}_{14}\hat{\omega}_{40}\left(1+\hat{r}_{i,t}-\hat{\omega}_{44}\right) \end{bmatrix}}{\hat{r}_{i,t}\left(1+\hat{r}_{i,t}-\hat{\omega}_{11}\right)\left(1+\hat{r}_{i,t}-\hat{\omega}_{22}\right)\left(1+\hat{r}_{i,t}-\hat{\omega}_{33}\right)\left(1+\hat{r}_{i,t}-\hat{\omega}_{44}\right)}$$

$$\hat{\lambda}_1 = \frac{\hat{\omega}_{11}}{\left(1+\hat{r}_{i,t}-\hat{\omega}_{11}\right)}$$

$$\hat{\lambda}_2 = \frac{\left(1+\hat{r}_{i,t}\right)\hat{\omega}_{12}}{\left(1+\hat{r}_{i,t}-\hat{\omega}_{11}\right)\left(1+\hat{r}_{i,t}-\hat{\omega}_{22}\right)}$$

$$\hat{\lambda}_3 = \frac{\left(1+\hat{r}_{i,t}\right)\hat{\omega}_{13}}{\left(1+\hat{r}_{i,t}-\hat{\omega}_{11}\right)\left(1+\hat{r}_{i,t}-\hat{\omega}_{33}\right)}$$

$$\hat{\lambda}_4 = \frac{\left(1+\hat{r}_{i,t}\right)\hat{\omega}_{14}}{\left(1+\hat{r}_{i,t}-\hat{\omega}_{11}\right)\left(1+\hat{r}_{i,t}-\hat{\omega}_{44}\right)}$$

Or equivalently,

$$P_{i,t} = k\left(\phi x_{i,t} - d_{i,t}\right) + \left(1-\kappa\right) y_{i,t} + \hat{\lambda}_2 oa_{i,t} + \hat{\lambda}_3 v_{1i,t} + \hat{\lambda}_4 v_{2i,t} \qquad (2.20)$$

where $\kappa = \hat{r}_{i,t}\widehat{\omega}_{11} / \left(1+\hat{r}_{i,t}-\widehat{\omega}_{11}\right)$ and $\phi = \left(1+\hat{r}_{i,t}\right)/\hat{r}_{i,t}$.

On the basis of these model specifications, we empirical test the research hypotheses constructed in the previous section. For the model's ability to explain the cross section of the stock prices, we first estimate the parameters ω_{ii} in the linear information dynamics under both single equation and simultaneous equation model. Given these estimated coefficients ω_{ii}, we compute the theoretical stock prices implied by the pricing equations and compare the results to the observed prices. Pricing errors of the implied valuation function will be calculated and we examine the model's ability to explain the cross section of stock prices under different model specification. For the predictability of for future stock prices on the other hand, we run the regression of each valuation function to obtain the estimated coefficients. These estimated coefficients are then used with the observation in the future periods to compute the theoretical value of the equity in the future periods. Similarly the prediction errors of these prices calculated from the implied valuation function are calculated for the comparison between various model specifications.

2.4 Empirical Results

Based on the research hypotheses constructed in the previous section, the current study empirically tests the Ohlson Model and the FO Model under different linear information dynamics. We conjecture that the linear information dynamic including the value-relevant information variables such as analysts earnings forecasts and the insider transaction activity improve the power of the model to explain the cross section of stock prices and to predict future price movement. Furthermore, we also expect the simultaneous linear information dynamic to exhibit superior ability in pricing the equity share than its single equation counterpart.

In the following empirical analysis, we first use single equation approach and the simultaneous equation approach to estimate the coefficient in each linear information dynamic specification. The estimated coefficients are then used to compute the theoretical price of the equity implied by the valuation function. These implied values are then compared to the prices actually being observed to examine whether the model explain the cross section of the stock prices. If the Ohlson Model and the FO Model have empirical content, then the pricing errors produced by these specifications will be statistically insignificant. Furthermore, we test the model's ability to forecast the future periods stock prices. We use the panel data random effect model in each implied valuation functions to estimate the coefficients. These estimated coefficients are then used with the future periods firm level data to compute the forecasts of the stock prices. Similarly if the models have empirical content, they should produce minimal pricing errors compared to the model tested in the previous literature.

2.4.1 Summary Statistics

Table 2.2.1 provides the summary statistics of the variables used in our empirical analysis. We note that the mean earnings per share in our sample period is 0.415 while the abnormal earnings per share is -0.294. This indicates that the firms on average earned less than the required cost of equity capital of 0.146 in our sample period. Moreover, the analysts are on average optimistic about future earnings performance of the firms given that the mean analysts earnings per share forecasts is a positive 0.172. The two value relevant information variables in the FO Model, expected growth in operating earnings and in operating assets, have mean values of 0.031 and 0.018 respectively in our sample period. Finally, the mean of the book value per share and stock price per share in our sample period is 9.015 and 21.361 respectively.

2.4.2 Time Series Behavior of Linear Information Dynamics

We next examined the time series behavior of the linear information dynamics in Ohlson Model and FO Model in Table 2.2 and Table 2.3 respectively[9]. Panel A1 in Table 2.2 shows that the abnormal earnings follow a stationary process since the coefficients of the lagged variables sum up less than one. Moreover, the first lag of the abnormal earnings accounts for the most serial correlation in the abnormal earning process. Even though the estimated coefficients for the other lagged variables are statistically significant at five percent, the adjusted R^2 is approximately the same after considering these lags. Therefore we found that the AR(1) process for abnormal earnings is sufficient for the linear information dynamics in the Ohlson Model and the FO Model as documented by previous literature. Furthermore, Panel B1 in Table 2.2 showed that after accounting for the analyst forecasts in the linear information dynamics for the abnormal earnings, the adjusted R^2 increases to 69 percent from 40 percent. This indicates that the analysts earnings forecast serve to be an appropriate value relevant variable that adds explanatory power to the linear information dynamics.

Given that Ohlson Model and FO Model are based on clean surplus relation, we also use comprehensive (operating) earnings in estimating the linear information dynamics and deriving implied valuation function. In Panel A2 of Table 2.2, we first investigate the autoregressive property of abnormal comprehensive earnings. Similar to the results found in Panel A1, we find strong statistical and economic magnitude of lag one comprehensive earnings, i.e. ranging from 0.7411 to 0.8012. The higher-ordered lags do not provide additional information given the adjusted R^2 does not improve after

[9] Since the linear information dynamics contains lagged dependent variables, the OLS estimation is inconsistent. We proceed our estimation by the IV estimation and panel GMM proposed by Anderson and Hsiao (1981) and Arellano and Bond (1991) respectively. The panel GMM is more efficient than the IV estimator because of additional lags of dependent variable as instruments. The results from the two estimation methods are similar and we reported the results from panel GMM estimator.

including additional lagged comprehensive earnings. The analysts earnings forecasts also retain its value relevance in the presence of the comprehensive earnings. Panel B2 of Table 2.2 shows that including analysts earnings forecasts improves the adjusted R^2 in the regression, from 0.4121 to 0.4363, with only lagged comprehensive earnings in Panel B1. These results confirm our previous finding that lagged one comprehensive earnings and analysts earnings forecasts are sufficient to estimate the earnings dynamics in the Ohlson Model. More interestingly, using comprehensive earning, instead of the bottom-line earnings, also seems to provide explanatory power given the high adjusted R^2 in the regression. Finally, Panel C showed the extended autoregressive process for the analysts earnings forecasts. We note that the AR(1) process again is sufficient for the linear information dynamics given that the further lagged variables does not contribute to the overall explanatory power of the model. In summary, we found that AR(1) process is sufficient for both the aerial correlation in abnormal earnings and analysts earnings forecasts. Incorporating the analysts earnings forecasts into the linear information dynamics indeed improved the ability of the model to explain the abnormal earnings process in addition to its own serial dependence.

Table 2.3 summarizes the autoregressive behavior of the abnormal operating earnings, operating assets, expected growth of operating earnings, and expected growth of operating assets in the linear information dynamics of FO Model. We found that the AR(1) process is sufficient for all four variables given that the adjusted R^2 is approximately the same after more lagged variables are considered. In Panel A1 for example, when the lag 2 abnormal earnings is added the estimated coefficient for lag one is still statistically significant. Although the lag two abnormal earnings is also statistically significant, lag one abnormal earnings accounts for most of the serial dependence of the abnormal earnings process. Moreover, when additional lag three abnormal earnings is considered, the significance of the lag one abnormal earnings and adjusted R^2 are not affected. Similar results can be found for operating assets, expected growth in operating earnings, and expected growth in operating assets. Thus we note that the AR(1) process is sufficient for the aforementioned variables in the linear information dynamics of the FO Model. In Panel A2 of Table 2.3, we investigate the dynamics of comprehensive operating earnings. Similar to the operating earnings dynamics in Panel A1, we find that lag one comprehensive earnings is sufficient in explaining the dynamics and additional lagged variables do not provide more information.

Panel C1 in Table 2.3 showed that incorporating more value relevant variables in the linear information dynamics increase the ability of the model to explain the variation of abnormal operating earnings. With only lagged abnormal earnings and operating assets as the independent variables in the model, both of the variables are statistically significant with estimated coefficients of 0.2895 and 0.4023 respectively. When the expected growth of operating earnings is incorporated, lagged abnormal operating earnings and operating assets are still statistically significant and moreover the adjusted R^2 increased from 0.6433 to 0.7236. This indicates that the additional value relevant variable, the expected growth in appearing earnings indeed increased the explanatory power of the linear information dynamics. Moreover, the results in Panel C2 indicate that under the single equation estimation, incorporating operating assets and analysts earnings forecasts into the comprehensive operating earnings dynamics provide more information given the higher adjusted R^2 in the regression. In Panel D, we examine the addition of value relevant variable, the expected growth in operating assets, in the autoregressive property of the operating assets. Incorporating the expected growth in operating assets as the additional value relevant variable increased the adjusted R^2 from 0.7941 to 0.8321. Therefore, for the linear information dynamics of the operating assets, the expected growth in operating assets further improves the explanatory of the model. In summary, we find that similar to the Ohlson Model, the value relevant variables incorporated in the FO Model provide additional information beyond the accounting variables. We next examine how different specification of these linear information dynamics affects the implied pricing function in evaluating the stock prices.

2.4.3 Estimation of Linear Information Dynamics

We start our empirical analysis by estimating the linear information dynamics using both the single equation estimation and simultaneous equations estimation in Ohlson Model and FO Model.[10] We then use these estimated coefficients along with the observed inputs in the implied pricing functions to compute the theoretical price of the shares. Our conjecture is that given there exist feedback relations between the accounting variables and the value relevant information variables, the simultaneous equations estimation more accurately estimate the linear information dynamics and thus the resulting pricing function produces smaller pricing errors than those under the single equation estimation.

[10] The single equation estimation is conducted by the panel GMM estimator as in Table 2.2A and Table 2.2B. Since our system of simultaneous equations specification of information dynamics involve endogenous regressors from other equations, thus we use the more efficient error component three stage least square (3SLS) estimator proposed by Baltagi (1981) to conduct the estimation. Essentially the 3SLS is a combination of the two stage least square (2SLS) estimator and the seemingly unrelated regression (SUR) estimator. 3SLS considers both the simultaneous equations bias and the cross equation correlation of the errors.

Panel A1 in Table 2.4 provides the estimated coefficients from both the single equation specification and the simultaneous equations specification of the Ohlson Model information dynamics with the analysts earnings forecasts. The single equation specification is essentially those shown in Panel B and C1 in Table 2.2 by using the panel GMM estimator. Following Tsay et al. (2008), we conjecture that there is a feedback relation between the current period abnormal earnings x^a_{t+1} and current period analysts forecasts for the next period earnings $v_{i,t+1}$. Comparing to the single equation specification, the simultaneous equations specification also estimate the feedback effect from the abnormal earnings to the analysts earnings forecasts. By jointly estimating the coefficient, we found that the coefficients ω_{21} is statistically significant (0.3791) indicating that the abnormal earnings indeed affects the analysts earnings forecasts for next period. In Panel A2 of Table 2.4, we estimate the linear information dynamics in Ohlson Model by using the comprehensive earnings dynamics. The estimated ω_{21} is 0.4369 which is higher than 0.3791 in Panel A1 in which bottom-line easrnings are used as the earnings measure in the information dynamics. This result suggest that abnormal comprehensive earnings provides a stronger feedback effect to the analysts earnings forecasts while the other estimated coefficeints retain their statistical and economic magnitude. The statistical significant ω_{21} suggests that the single equation specification in traditional Ohlson Model linear information dynamics is not correctly identified and the simultaneous equation estimation of the linear information dynamics might yield more price-relevant information in forecasting future stock price.

Panel B1 in Table 2.4 compares the single equation and simultaneous equations specification of the linear information dynamic with bottom-line earnings in the FO

Model. In the single equation specification, all the coefficients associated with the accounting variables and the value relevant information variables are statistically significant. This indicates that the linear information dynamics in the FO Model indeed possess empirical content to capture the variation in abnormal operating earnings and operating assets. We further extended the single equation specification to simultaneous equations specification to examine whether there exist feedback relations between the accounting variables and the value relevant information variables. For example, the coefficient ω_{31} which measures how current period abnormal operating earnings affect the expected growth in abnormal operating earnings is statistically significant of 0.2725. Moreover, the coefficients ω_{42} which measures how current period operating asset affect the next period expected growth in operating asset, is also statistically significant of 0.3895. These results suggest that there exist feedback relations of abnormal operating earning and operating assets with their expected growth in the future periods. By estimating the linear information dynamics by the simultaneous equations approach improve the information content provided of these variables in computing the implied value of the shares. In Panel B2 of Table 2.4, we use comprehensive operating earnings to estimate the linear information dynamics in FO Model. Compared to the results in Panel B1, we find stronger feed back effects given the larger estimated coefficients ω_{31} (0.2931) and ω_{42} (0.4873) under the estimation with abnormal comprehensive operating earnings. In the next section, we will further examine the pricing errors of the implied valuation function by employing these estimated coefficients with the observed inputs.

2.4.4 Prediction Errors of Stock Prices Forecasts

Table 2.5 provides the summary of prediction errors of stock prices from Ohlson Model and FO Model using different estimation method for linear information dynamics (single equation vs. simultaneous equations estimation) and different earnings measures (bottom-line earnings and vs. comprehensive earnings). More specifically, we estimated the coefficients in the linear information dynamics ω_{ij} and used them with the observed inputs abnormal earnings and analysts earnings forecasts to compute the theoretical price of the share at end of each year t given the implied valuation function for Ohlson Model in equation (2.14) and for FO Model in equation (2.18). We then measure how these implied value of the share differ form the observed current market price per share, i.e. the prediction errors for the stock prices. The prediction errors are represented by the mean forecast errors, which are calculated as the observed market price per share minus the implied price form the model divided by the market price per share at end of each period t.

In Panel A of Table 2.5, we first observe that simultaneous equations estimation of the linear information dynamics indeed improve the future stock forecast accuracy by producing significantly smaller prediction errors than those generated by the single equation estimation. The prediction errors difference, $\Delta_{Simul-Single}$, are both significantly negative for both abnormal earnings and abnormal comprehensive earnings at -0.0514 and -0.0305 respectively. We then discuss whether using abnormal comprehensive earnings can improve the stock price forecast ability of each individual model. Under both single equation and simultaneous equations estimation specifications, we calculate the forecast error differences between using abnormal earnings and comprehensive abnormal earnings as the earnings measures in the information dynamics, i.e. $\Delta_{x^a_{130}-x^a}$. We are expected to observe smaller forecast errors when using abnormal comprehensive earnings because of its consistency with the clean surplus relation which is used in deriving the implied valuation function in Ohlson Model. The result in Panel A of Table 2.5 shows that the forecast errors differences $\Delta_{x_{130}^{a}-x^{a}}$ is statistically significant at -0.0334 (-0.0225) under single equation (simultaneous equations) estimation. Our results suggest that under both estimation specifications of the linear information dynamics, abnormal comprehensive earnings outperforms the abnormal earnings in terms of predicting future stock prices by generating smaller average forecast errors. Similar improvement in prediction accuracy can also be found in Panel B of Table 2.5 where comprehensive operating earnings are used in estimating the linear information dynamics in FO Model. The result in Panel B of Table 2.5 suggests that the forecast errors differences $\Delta_{ox_{130}^{a}-ox^{a}}$ is statistically significant at -0.0244 (-0.0186) under single equation (simultaneous equations) estimation. This indicates that abnormal comprehensive operating earnings provides more value-relevant information than abnormal operating earnings in estimating linear information dynamics and computing the one-year ahead model implied stock prices. In sum, the empirical results we have shown in the current study further demonstrate that comprehensive (operating) earnings can also produce more accurate future stock price forecasts in the residual valuation models.

Table 2.6 summarizes the mean forecast errors for one- to five-year forecasts of the WS Model. At each year from 1980 to 2002, financial data of each firm are used as the base year information in the WS Model to forecast the future period stock prices in the next five years. In particular, for each firm at year *t*, the sales growth rate is estimated by the linear regression model using all the past sales available in Compustat. We then calculated the mean forecast errors of future stock prices by each number of years ahead forecast errors across all rolling periods. We note that the mean forecasts errors mostly increase monotonically with number of years ahead forecasts. We compare WS Model forecasts to Ohlson/FO Model forecasts with both bottom-line earnings and comprehensive earnings. We calculate the mean forecast errors difference between WS Model and Ohlson Model with bottom-line earnings ($\Delta_{WS-Ohlson}$) and comprehensive earnings ($\Delta_{WS-Ohlson(x_{130})}$). We also calculate the mean forecast errors difference between WS Model and FO Model with operating earnings (Δ_{WS-FO}) and comprehensive operating earnings ($\Delta_{WS-FO(ox_{130})}$). Our results suggest that the WS Model produces more accurate stock prices than the Ohlson/FO Model in the shorter-term. The average one-year ahead stock prices forecasts errors is 0.3418 which is significantly lower than those forecasted by the Ohlson and FO Model using either bottom-line earnings or comprehensive earnings. The mean forecast errors differences are all significantly negative for the one-year forecast in Table 2.6. However, in the longer-term, the model produces relatively less accurate forecasts. Our findings suggest that the WS Model considering the interrelations between investment activity, financing activity, dividend policy, and the production decision of the firm provide better forecast accuracy in terms of the stock prices than the residual income valuation models discussed in the previous sections. Nonetheless the forecasts accuracy crucially depends on the model inputs such as sales growth rate, current assets as a percent of sales, fixed asset as a percent of sales, dividend payout ratio, and the leverage ratio. Therefore we next examined how the sensitivity of these model inputs affects the resulting forecasts.

Table 2.7 summarizes the sensitivity analysis of several model inputs in the WS Model, they are sales growth rate, total assets as a percent of sales, dividend payout ratio, and leverage ratio. In Panel A we showed how sensitive the stock prices forecasts are to the changes in the sales growth rate. It is expected that higher sales growth rate leads to higher stock prices because higher future earnings. When the sales growth rate changed from its median to its third quantile, the one year ahead mean forecast errors reduced from 0.4238 to 0.3954. Since the WS Model underestimate the stocks prices as shown in Table 2.6, increases in sales growth rate results in higher stock prices and thus smaller pricing errors. On the other hand, when the sales growth rate decreases from its median to the first quartile, the implied stock prices decreases and thus the pricing errors increases accordingly. Similar patterns of the changes of the pricing errors can be observed in longer years ahead forecasts. Furthermore, the sensitivity analysis regarding the total assets as a percent of sales is shown in Panel B. The stock prices are expected to decrease in assets as a percent of sales because more equity funds were required to support asset requirement and thus larger the pricing errors. The base case inputs for the total assets as a percent of the sales are the median value of each firm's available historical data. We examined the sensitivity of the stock prices forecast to the first and third quantile value of firm's total asset as a percent of sales. The mean forecast errors of one year ahead stock prices increased (decreased) to 0.4756 (0.4048) from 0.4238 if the third (first) quantile of total assets as a percent of sales is used. Similar patterns are observed in two and three years ahead forecasts but results for longer years ahead forecasts are not obvious.

Panel C summarizes the sensitivity of stock prices forecasts to the firm's payout ratio. We expect the pricing errors to decrease in the dividend payout ratio because as more dividends were paid to shareholders, the firm relies more on new issues of common stocks for the financing requirement which leads to potential decline in the stock prices. The mean forecast errors increased (decreased) to 0.4451 (0.4198) if the third (first) quantile of firm's historical dividend payout ratio is used as the model input. Panel D summarized the sensitivity of stock prices to the changes in firm's leverage. We expect the stock prices to increase in leverage and thus smaller pricing errors. The mean forecast errors decrease (increase) to 0.4187 (0.4408) if the third (first) quantile of the historical leverage ratio is used as the model input in forecasting future stock prices. The longer years ahead forecasts are again not as apparent as the shorter term forecasts suggesting model's lack of ability of accurate forecast in the long run. In summary, our results suggest that the stock prices forecasts are most sensitive to the changes in sales growth rate given the largest change in the mean forecast errors This is not a surprising result since the starting point of the WS Model is the sales growth estimate. The sensitivity of the stock prices forecasts to the other exogenously given variables such as the total assets as a percent of sales, dividend payout ratio, and the leverage ratio do not have as significant impact on the forecasts as the sales growth rate. Overall the results are consistent with the model conjecture that stock prices forecasts are influenced by the aforementioned inputs and sales growth rate is the most important factor in producing the accurate forecasts from the model.

2.4.5 Forecast Combination

In the previous section, the implied valuation functions of the Ohlson Model and FO Model were investigated for their ability in forecasting future stock prices. In this section, we examine how the stock price forecasts from these models can be combined with the Warren-Shelton model to further improve their forecastability in terms of prediction errors. More specifically, we employ the forecast combination proposed by Bates and Granger (1969), Granger and Newbold (1973), Granger and Ramanathan (1984), and Diebold and Pauly (1987) to examine the combined predictability of future stock prices from different primary individual forecasts.

We use the different weighting schemes in equations (W1) through (W4) in Section 2.3 to construct the weighting matrix W in the WLS estimation. Moreover, we employ the linear and quadratic deterministic time-varying parameters model to produce time-varying weights. Similar to the regression model in equation (2.12), the estimator can be written as

$$\text{Linear: } {}_t\hat{y}_{t+1} = \left(\hat{p}_0^0 + \hat{p}_1^0\left(t+1\right)\right) + \left(\hat{p}_0^1 + \hat{p}_1^1\left(t+1\right)\right)_t f_{t+1}^{RI} + \left(\hat{p}_0^2 + \hat{p}_1^2\left(t+1\right)\right)_t f_{t+1}^{WS}$$

$$\begin{aligned}\text{Quadratic: } {}_t\hat{y}_{t+1} = &\left(\hat{p}_0^0 + \hat{p}_1^0\left(t+1\right) + \hat{p}_2^0\left(t+1\right)^2\right) + \left(\hat{p}_0^1 + \hat{p}_1^1\left(t+1\right) + \hat{p}_2^1\left(t+1\right)^2\right)_t f_{t+1}^{RI} \\ &+ \left(\hat{p}_0^2 + \hat{p}_1^2\left(t+1\right) + \hat{p}_2^2\left(t+1\right)^2\right)_t f_{t+1}^{WS}\end{aligned}$$

where ${}_t f_{t+1}^{RI}$ and ${}_t f_{t+1}^{WS}$ are the one-year stock price forecast from the residual income valuation models (Ohlson/FO Models) and the WS Model respectively. We consider the following estimators for forecasting future stock prices.

M1. WLS, geometric weights, linear deterministic time-varying parameters.

M2. WLS, geometric weights, quadratic deterministic time-varying parameters.

M3. WLS, t^{λ} weights, linear deterministic time-varying parameters.

M4. WLS, t^{λ} weights, linear quadratic time-varying parameters.

M5. OLS (simple unrestricted regression-based combination).

M6. Variance-Covariance Combination.

Panel A1 (A2) in Table 2.8 provides the prediction errors of one-year ahead stock prices from the forecast methods M1 through M6 combining the Ohlson Model forecasts using bottom-line (comprehensive) earnings and WS Model forecasts. We also compare the forecast errors of these combined forecasts to those individual forecasts under the Ohlson Model and WS Model alone. If the combined forecast methods indeed improve model's ability to provide better accuracy, then the combined methods are expected to produce smaller pricing errors. Our results suggest that method M3 yields the best one year ahead stock price forecast in terms of smallest mean forecast errors. Method M3 employs the *t*-lamda (t^{λ}) weighting specification with the linear deterministic time-varying parameters in the WLS estimator to generate optimal weights for each individual forecast from Ohlson Model and WS Model. The M3 forecast combination method produces a mean forecast error of 0.2841 (0.2779) in Panel A1 (A2) which is significantly lower than either the individual forecast from Ohlson Model or the WS Model. For the Ohlson Model using comprehensive earnings, the mean forecast errors differences between the M3 method and the Ohlson Model (WS Model) $\Delta_{M_i - M_7}$ ($\Delta_{M_i - M_8}$) is significant at -0.1785 (-0.0639), suggesting forecast combination indeed lower the prediction error of individual forecasts. These results show that the WLS estimator with t-lamda (t^{λ}) weighting specification and linear deterministic time-varying parameters generate the lower prediction errors for stock price forecasts than other forecast combination methods and the individual forecasts. Moreover, we find that in general the forecast combination methods M1 through M4 generate smaller pricing errors than the individual forecasts from Ohlson Model and WS Model. The OLS (M5) and Variance-Covariance method (M6) on the other hand do not improve each model's ability in forecasting future stock prices given the insignificant prediction error differences.

In a similar fashion, Panel B1 (B2) in Table 2.8 provides the prediction errors of forecast combination methods M1 through M6 by combining the FO Model forecast using (comprehensive) operating earnings and WS Model forecasts. Comparing to the individual forecasts from FO Model and WS Models, methods M1 through M4 again produce significantly lower prediction errors while M5 and M6 fail to obtain improvement in forecasting one-year ahead stock prices in the sample. The M4 forecast combination method produces lowest mean forecast error, i.e. 0.2284 (0.2045) in Panel B1 (B2) among all different forecast combination methods. The mean forecast error generated by M4 is also significantly lower than that produced by either the FO Model or WS Model. For example, in Panel B2 the mean forecast errors differences between the M4 method forecast and the FO Model (WS Model) forecast, or $\Delta_{M_i - M_7}$ ($\Delta_{M_i - M_8}$), is significant at -0.1596 (-0.1373). This suggests that forecast combination indeed lower the prediction error of individual forecasts. Overall our findings demonstrate the appealing features of geometric weighting schemes and time-varying parameters in forecast combination provide superior accuracy in predicting future stock prices. The WLS estimator with geometric weighting schemes and time-varying parameters place more weights on the better forecast technique over time. Given the different structural designs of the residual income valuation models and the WS Model, each of them could provide superior forecast than the other under specific market condition.

2.5 Summary

This paper investigates the stock prices forecast ability of three alternative valuation models, namely the Ohlson (1995) Model, Feltham-Ohlson (1995) Model, and the Warren-Shelton (1971) Model. These alternative valuation models incorporate financial statements information in the equity valuation. The Ohlson Model and FO Model introduce the linear information dynamics for equity valuation while the WS Model investigates the overall operating and financing decisions of the firm to estimate stock prices. In this paper, we have developed five research hypotheses to test whether different earnings measures, estimation techniques and combined forecasts methods can improve these models' ability in predicting future stock prices. In the first hypothesis, we test whether FO Model can produce smaller prediction errors for future stock prices than Ohlson Model. The second hypothesis examines whether simultaneous equations estimation of the linear information dynamics in Ohlson and FO Model can generate smaller prediction errors for future stock prices than those produced by single equation estimation. In the third hypothesis, we investigate whether WS Model can generate more accurate future stock price forecasts than both Ohlson and FO Model. The fourth hypothesis examines whether combination of individual forecasts from Ohlson Model, FO Model, and WS Model can produce more accurate future stock price forecast than each individual model. Finally, the fifth hypothesis tests whether the use of comprehensive income, in stead of net income, can generate more accurate future stock price forecasts in these valuation models.

We first use simultaneous equation estimation approach to estimate the information dynamics for Ohlson Model and FO Model and to forecast future stock prices. Our empirical results suggest that the simultaneous equation estimation of the information dynamics improves the ability of the Ohlson Model and FO Model in capturing the dynamic of the abnormal earnings process. The predictability of the one year-ahead stock prices is also more accurate under the simultaneous equation estimation in terms of smaller prediction errors. We then use the WS Model to predict stock price per share and find that WS Model can generate smaller future stock prices prediction errors than those predicted by the Ohlson Model and FO Model. These findings indicate a better stock price forecast ability of the WS Model in determining future stock prices. The superior accuracy comparing to the Ohlson Model and FO Model are due to the incorporation of both operation and financing decisions of the firms. We also combine these different stock price forecasts by using various time-varying parameters models proposed by Granger and Newbold (1973) and Diebold and Pauly (1987) to examine whether forecast combination provide better prediction accuracy. The combined forecasting methods generally produce more accurate stock price forecasts than those made by individual models.

Previous literature found supporting evidence that comprehensive earnings can provide price-relevant information beyond bottom-line earnings measure (Cheng et al., 1993, Dhaliwal et al, 1999, and O'Hanlon and Pope, 1999). Given that the Ohlson Model and FO Model are based on the clean surplus relation, we further investigate the price-relevance of comprehensive (operating) earnings in these valuation models. Our results suggest that using comprehensive (operating) earnings in the Ohlson (FO) Model can further reduce the prediction errors of future stock price forecasts under both single equation and simultaneous equation estimation of linear information dynamics. Moreover, this superior predictability also leads to smaller prediction errors in the combined forecasting in which Ohlson (FO) Model forecasts are combined with WS Model forecasts. Evidence shown in our study demonstrates that comprehensive (operating) earnings indeed provide incremental price-relevant information beyond bottom-line earnings.

In sum, we investigate the empirical validity in terms of stock price forecast accuracy of alternative equity valuation models. By employing the simultaneous equation estimation and combined forecasting methods, we find that these models can produce higher estimate accuracy in predicting future stock prices. Our findings contribute to the literature in residual income valuation models as well as the setting of accounting standard on reporting comprehensive financial performance.

Chapter 3

Technical, Fundamental, and Combined Information for Separating Winners from Losers

(*Joint work with Professor Cheng-Few Lee*)

3.1 Introduction

This study investigates investment strategy that integrates fundamental and technical information in separating winner stocks from loser stocks. Prior literature on fundamental analysis and technical analysis framework has provided substantial evidence on their respective ability to explain the cross section of stock prices or to forecast future price movement. However, the literature is relatively silent on the integration of both analyses frameworks in equity valuation and in making investment decision. In the current study, we provide a unified framework in which the fundamental analysis using the financial statements information can be integrated with the technical analysis using past returns and past trading volume. More specifically, we developed a combined momentum strategy employing past returns, trading volume, and firm's fundamentals and examine its profitability comparing to the technical momentum strategy.

The technical information of the stocks has been frequently used by securities analysts and portfolio managers as well as academic researchers. Technical analysts focus primarily on the short term price and volume information. One of the most notable line of research using technical information in studying stock prices behavior is the momentum investment strategy. By using stock's past performances, Jegadeesh and Titman (1993, 2001) documented that based on the cumulative returns in the past three to twelve months, the highest return decile portfolio outperform the lowest decile portfolio in the following three to twelve months. This pricing anomaly is based solely on the past returns and investors do not use firm specific information in separating the winner stocks from the loser stocks. A large body of follow-up literature showed the presence of the price momentum across asset classes and countries. In addition to past returns, past trading volume has also been documented to predict stocks future returns and (Conrad, Hameed, and Niden (1994), Datar, Naik, and Radcliffe (1998)) and to provide information about the magnitude and persistence of the momentum returns (Lee and Swaminathan (2000), Chan, Hameed, and Tong (2000)). These findings suggest that there exist joint effect of these technical information on future stocks returns.

In addition to the technical information, the fundamental information about the firms also provides investors with guidance in making investment decision. The linear information model (Ohlson (1995), Feltham and Ohlson (1995)) used book value and earnings per share of the firm to estimate the stock prices. Other financial statement information such as inventory, account receivables, and gross margin have also been employed to construct fundamental signals about the firms (Ou and Penman (1989), Abarbanell and Bushee (1997), Lev and Thiagarajan (1993)). In addition to individual signals, researchers also construct aggregated measurement to examine overall performance of the firms. Piotroski (2000) and Mohanram (2005) developed fundamental indicators FSCORE and GSCORE in which firm specific information have been employed in evaluating value stocks and growth stocks respectively. These authors found that the portfolio consisting of financially healthier firms, i.e. firms with higher FSCORE or GSCORE, outperform those consisting of low scores firms up to two years after the portfolios are formed. Since both technical information (past returns and past trading volume) and fundamental information (firm-specific financial statement information) have been documented to identify winners and losers, we investigate whether the combination of two methods can improve the investor's ability in analyzing stocks and making investment decision.

Based upon combined forecasting models developed by Granger and Newbold (1974), Granger and Ramanathan (1984), Lee, Newbold, Finnerty, and Chu (1986), and Lee and Cummins (1998), we propose a combined momentum strategy based on firm's past returns, past trading volume, and its composite fundamental scores. More specifically, we form the long-short investment strategy with long position in past winners with high fundamental scores and low covariance between returns and trading volume, and short position in past losers with low fundamental scores and high covariance between returns and trading volume. Our combined momentum strategy not only outperforms the technical momentum strategy, which is based solely on technical information such as past returns and trading volume, on average by 1.63% (1.85%) monthly among high (low) book-to-market stocks but also generates higher information ratio. We also find that the returns to technical momentum strategy and accounting-based fundamental strategy are negatively correlated. This suggests that the higher information ratio generated in our combined momentum strategy results not only from the higher monthly abnormal returns but also the lower tracking errors from the integration of different sorting variables. We consider our results contributing to both technical momentum and accounting-based fundamental strategy literature. The findings in this paper also provide insights to the investment community using technical momentum strategy. These quantitative fund managers experienced significant losses during the overall market turnarounds in the months of March and April in 2009. Our combined momentum strategy could provide these managers with different performance metrics to separate the momentum winners from losers.

The remainder of this paper is organized as follows. Section 2 provides the literature review of the accounting-based investment strategies and technical momentum strategies. Section 3 presents the sample selection criteria and portfolio formulation methods to be used for the empirical test. Section 4 presents the empirical results of testing the performance of both technical momentum strategy and combined momentum strategy. Section 5 provides the summary and conclusion of this paper.

3.2 Literature Review

In the section we will first review literature related to fundamental analysis which include both value stocks and growth stocks. Then we will review literature related to technical momentum strategy.

3.2.1 Financial Statement Analysis

The root of fundamental analysis for the share price valuation can be dated back to Graham and Dodd (1934) in which the authors argued the importance of the fundamental factors in share price valuation. The dividend discount model developed by Gordon (1962) provided another building block for the fundamental analysis.

Subsequently, Ohlson (1995) residual income valuation model further extended the dividend discount model to express the share prices in terms of the contemporaneous book value and earnings per share. Although the residual income model is relatively easy to implement, the empirical results of testing the Ohlson's model are mixed (Dechow, Hutton, and Sloan (1999), Myers (1999)). Other research focuses on the fundamental analysis by calculating certain multiples for a set of benchmark firms and finding the implied value of the firm of interest by these benchmark multiples. Ou and Penman (1989), Kaplan and Ruback (1995), Gilson, Hotchkiss, and Ruback (2000), Liu, Nissim, and Thomas (2002)). However, single financial multiple or ratio might not capture the complete aspects of the firm and thus researchers also constructed composite indicators using various fundamental information of the firms to examine future performance of the share prices. Two such evaluation systems, namely the FSCORE and GSCORE fundamental indicators developed by Piotroski (2000) and Mohanram (2005) respectively, are discussed in the next two sections.

Financial Statement Analysis for Value Stocks

Previous literature showed that the investment strategy with long position in low book-to-market stocks and short position in high book-to-market stocks generate significantly abnormal returns in the periods after the portfolio formation. Fama and French (1992) argued that book-to-market ratio is a proxy for financial distress of the firms and the abnormal returns generated from this investment strategy represent investors' compensation for this financial distress risk factor. However, there exist substantial returns variation among these values stocks and further performance metrics is required to identify the stocks exhibiting higher returns. Following Piotroski (2000), we used the FSCORE system to separate winners from the losers among high book-to-market stocks. Piotroski (2000) used nine signals to proxy measure the overall financial health of the high book-to-market firms and they can be categorized in three groups: profitability-related signals, operating efficiency signals, and change in solvency/liquidity signals.

The profitability-related fundamental signals are those to measure firm's ability to generate profits. The four profitability indicators are *ROA* (return on assets), *AROA* (change in return on assets), *CFO* (cash flow from operation scaled by total assets), and *Accrual* (difference between *ROA* and *CFO*). *ROA* and *CFO* are assigned a value equal to one if they are positive, zero otherwise. Similarly, if firms experience positive change in return on assets, the variable *AROA* is assigned a value of one and zero otherwise. Finally, given the negative relation between firms' accrual and future expected returns documented by Sloan (1996), the variable Accrual is assigned a value of one if Accrual is negative and zero otherwise. The second group of fundamental variables is operating efficiency-related, e.g. *DMargin* (change in gross margin) and *DTurn* (change in asset turnover). Positive changes in gross margin and asset turnover represent improvement in generating profits and efficient employment of firm's asset. Thus the variables *DMargin* and *DTurn* are assigned a value of one if positive and zero otherwise. The third group of fundamental indicators are related to firm's solvency and liquidity, e.g. *DLever* (change in leverage), *DLIQUD* (change in current ratio), and *EQOFFER* (equity issuance). Firms issue debt when the internally generated funds are not available (Myers and Majluf (1984)) and thus the increases in financial leverage indicate firm's difficulty in generating internal capital. Therefore the variable *DLever* is assigned a value of one if negative and zero otherwise. Similarly, the variable *DLIQUD* is assigned a value of one if the firm decreases its current ratio from last year and zero otherwise. The last signal related to firm's solvency and liquidity is *EQOFFER* which is indicator variable equal to one if the firm had no equity issuance in the previous year and zero otherwise. Equity issuance by a firm suggests its difficulty raising capital from its own operation or long-term debt and thus is considered a bad signal for the future prospects of a firm.

Given these nine signals discussed above, Piotroski (2000) constructed a composite score to assess the financial soundness of a firm, i.e. the FSCORE. The sum of these nine indicator variables ranges from zero to nine with nine (zero) indicating a firm with more (less) good signals.

$$\begin{aligned} FSCORE = ROA + AROA + CFO + Accrual + DMargin + DTurn \\ + DLever + DLIQUID + EQOFFER \end{aligned} \quad (3.1)$$

Firms with higher FSCORE indicates a better overall financial health than ones with low FSCORE. Piotroski (2000) found that an investment strategy with long position in high FSCORE firms and short position in low FSCORE firms generates significant excess return up to two years after the portfolio formation. Therefore, for the high book-to-market stocks (value stocks), FSCORE seems to be an appropriate candidate for the fundamental analysis indicator in our unified valuation framework.

Financial Statement Analysis for Growth Stocks

Although FSCORE separates the winners from the losers among the value stocks, it does not work well for the low book-to-market ratio stocks as documented by Mohanram (2005). Mohanram (2005) thus extended the FSCORE to construct the GSCORE measurement to examine the fundamentals for the low book-to-market stocks (the growth stocks). He argued that GSCORE is appropriate for the growth stocks because it accounts for the growth fundamentals of these firms. Growth firms are usually those with stable earnings and sales growth, larger R&D expenses and capital expenditure, and more analysts following. His results showed that for the low book-to-market stocks, high GSCORE firms are more likely to beat the earnings forecasts and thus earn higher excess return than the low GSCORE firms. The composite GSCORE is constructed by eight fundamental signals related to firm's profitability, earnings stability, sales stability, and accounting conservatism. GSCORE emphasizes on firm's future performance and accounts for its growth factor. The GSCORE is constructed by three categories of eight signals.

The first category is the profitability-related signals which include *ROA*, *CFO*, and *Accrual*. The definition of these variables is identical to those used in FSCORE but with the difference in assigning indicator values. These profitability related variables are assigned a value of one if they are larger than that of the industry median, and zero otherwise. The second group of fundamental signals is related to earnings stability and sales stability of the firms. Firms with stable earnings and sales convey to the investors that they can consistently deliver superior performance in the future. Previous literature in earnings management documented that investors prefer stocks with stable earnings to those with volatile earnings stream (Trueman and Titman (1988), Goel and Thakor

(2003)). The indicator variable for earnings stability σ_{NI} (variance of a firm's *ROA* in the past five years) and sales growth stability σ_{SG} (variance of a firm's sales growth in the past five years) are assigned a value of one if they are less than the median of all firms in the same industry, zero otherwise. The third group of fundamental indicator variables is related to accounting conservatism. In the low book-to-market firms, the large amount of research and development expenses, advertising expenses, and capital expenditure in current period generate unrecorded intangible assets because of accounting conservatism. These low book-to-market firms are currently undervalued but better future growth is expected. Thus the last three indicator variables *RDINT* (R&D expenses scaled by total assets), *ADINT* (advertising expenses scaled by the total assets), and *CAPINT* (capital expenditure scaled by the total assets) are assigned a value of one if they are larger than the industry median, zero otherwise.

Similar to the construction of the FSCORE, the composite GSCORE is the sum of these eight fundamental signals.

$$\begin{aligned} GSCORE = {} & ROA_I + CFO_I + Accrual + \sigma_{NI} + \sigma_{SG} \\ & + RDINT + ADINT + CAPINT \end{aligned} \qquad (3.2)$$

A higher (lower) GSCORE indicate more (less) good fundamental signals of a firm and thus better financial health for the growth stocks. Mohanram (2005) showed that an investment strategy with long position in high GSCORE stocks and short position in the low GSCORE stocks generate excess returns up to two years after the portfolio formation. In our model, we employ the FSCORE and GSCORE as the fundamental analysis indicator for value stocks and growth stock respectively. These fundamental scores are expected to improve investors' ability in separating winners from losers in addition to the technical information such as past returns and trading volume.

3.2.2 Technical Momentum Strategies

The momentum returns in which past winner stocks keep winning and past loser stocks keep losing is a well known anomaly in asset pricing. Jegadeesh and Titman (1993) showed that an investment strategy with long position of past winner stocks and short position in past loser stocks in the past three to twelve month generate significantly positive return in the ensuing three to twelve months. Momentum returns has also been documented in international markets (Rouwenhorst (1998), Chui, Titman, and Wei (2003)) and researchers have examined the causes of such phenomenon (Barberis, Shleifer, and Vishny (1998), Daniel, Hirshleifer, and Subrahmanyam (1998), and Hong and Stein (1999)). Moreover, the past trading volume, along with past returns, have been documented to be associated with future returns (DeBondt and Thaler (1985), Lee and Swaminathan (2000), Chan, Hameed and Tong (2000), Grinblatt and Moskowitz (2004)). In this study, we focus on one particular trading volume related variable, the BOS ratio, developed by Wu (2007) and examine how it improves investors' ability to separate momentum winners from losers.

Wu (2007) argued that the momentum returns arises because of the asymmetric information between the informed and uninformed investors. The empirical proxy for the degree of asymmetric information developed by Wu (2007) is the BOS ratio which measures by the covariance of past returns and past trading volume of each individual stock. The BOS ratio for the *i*th stock in our portfolio is calculated as

$$BOS\ Ratio = \text{cov}(r_t^i, \pi_t^i) \qquad (3.3)$$

where

$$\pi_t^i = \frac{\left|v_t^i\right|}{E\left[\left|v_t^i\right|\right]} \qquad (3.4)$$

r_t^i is the monthly rate of return of stock *i* in month *t*, π_t^i is the relative trading volume of stock *i* in month *t*, v_t^i is the sum of daily dollar trading volume for stock *i* in month t, and $E\left[\left|v_t^i\right|\right]$ is the cross section average of the monthly dollar trading volume for all stocks in the same quintile portfolio in period (*t*-1,*t*). BOS ratio will be used to examine the strength of momentum winners and losers in the future periods.

Now we discuss how BOS ratio can be used examine the strength of momentum winners and losers in the future periods. For the winner stocks, a small or negative BOS indicates that when the informed investors try to sell their excessive long position, the informed are not in the market to buy. Negative adjustments in the prices are expected to compensate the uninformed and thus the winner momentum arises. For the loser stocks, a large and positive BOS indicates that when the informed try to close out their short position by purchasing back the shares, the uninformed are not in the market to sell. The informed investors have to raise the bid price and thus loser momentum is expected in the next period. For the purpose of further separating winners from losers, the long-short investment strategy with long position in past winners with lowest BOS ratio and short position in past losers with high BOS ratio is expected to generate larger abnormal returns than the technical momentum strategy. Prior literature in examining trading volume and momentum returns, such as Lee and Swaminathan (2000), found that momentum returns is more pronounced in high volume stocks. However, the BOS ratio allows us to further study the strength of momentum returns in the low volume stocks because the return predictability is determined by the covariance between past trading volume and past returns. In sum, we will construct our combined momentum strategy based on past returns, the BOS ratio, and the fundamental composite scores to examine the improvement of investors' ability in separating winner stocks from the loser stocks.

3.3 Sample Selection and Data Description

In this section, we will first introduce and the sample selection criteria. Then we will discuss the portfolio formulation methods used for the empirical tests.

3.3.1 Sample Selection and Methodology

Our sample includes all non-financial firms listed on NYSE and AMEX with sufficient monthly return data on CRSP and price and book value data on Compustat from January 1982 to December 2008. The firms listed on Nasdaq are not included because of the multiple counting of dealer trades. The trading volume of Nasdaq listed shares are not accurately measured due to the multiple counting trading when dealers are making the market. Therefore the Nasdaq listed shares are excluded from our sample to maintain consistency across different markets. Our sample excludes firms that are a foreign company, a closed-end fund, a real estate investment trust (REIT), and an American Depository Receipt (ADR). Firms with price less than one dollar and negative book-to-market ratio are excluded. The monthly stock data on returns, prices, and trading volumes are obtained from CRSP. Other annual financial data required to construct the

FSCORE and GSCORE[11] are obtained from Compustat. We delete all firms with insufficient time-series data required to compute the scores. Also, the returns to the sample firms delisted during the return measurement period are set to equal zero.

We conduct our empirical testing with respect to high book-to-market stocks (value stocks) and low book-to-market stocks (growth stocks) separately given their differences with respect to growth opportunity. At the end each month from January 1982 to December 2008, the stocks within the top (bottom) quintile portfolio based on the distribution of book-to-market ratio twelve months ago are selected as the value (growth) stocks sample.[12] We further sort the stocks sequentially by cumulative returns in the past twelve months, the BOS ratio, and the fundamental scores.[13][14] The resulting portfolios are denoted by (Q_{Mi},Q_{Bi},Q_{Fi}) where Q_{Mi}, Q_{Bi}, and Q_{Fi} are the *i*th quintile portfolio sorted by the past returns, the BOS ratio, and the fundamental indicator FSCORE/GSCORE respectively. For example, the portfolio consisting of the top winners, lowest BOS ratio, and highest FSCORE is denoted by (Q_{M1},Q_{B5},Q_{F5}). Similarly (Q_{M5},Q_{B5},Q_{F1}) contains stocks that are the top losers, highest BOS ratio, and lowest FSCORE.

The performance of the combined momentum strategy involving the extreme portfolios, i.e. portfolios (Q_{M1},Q_{B5},Q_{F5}) and (Q_{M5},Q_{B5},Q_{F1}), for holding periods of one,

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[11] Following Mohanram (2005), we require that there exist at least three other firms in the same industry defined by the two-digit SIC code in constructing the GSCORE. Moreover, for earnings stability σ_{NI} and sales growth stability σ_{SG}, if adequate quarterly information is not available, the information from the most recent fiscal year end is used.

[12] The book-to-market ratios are based on the market price at the portfolio construction date at the end of each month and the most recent fiscal year-end reported book value of equity.

[13] The fundamental scores are calculated based on the financial statements information in the previous fiscal year. For example, for a firm with fiscal year in June 1995, the FSCORE/GSCORE used in portfolio construction in May 1995 is based on information of the firm in the fiscal year ended in June 1994.

[14] Our dependent sorting might cause our empirical results specific to the sorting order employed. Independent sort cannot be applied in our sample due to the small number of securities in some of the intersection portfolios. We repeat our test with the reverse sorting order and the empirical results are qualitatively the same.

three, and six months after the portfolio formation date are examined. Following the literature in price momentum strategy, the monthly return of *K*-month holding period is based on an equally-weighted portfolio consisting of portfolio constructed in the current month and previous *K* -1 months. More specifically, the monthly return for *K* -month holding period return are calculated from an overlapping portfolio that in each month contains portfolios of the momentum strategy selected in the past *K* months. For example, the monthly return for a holding period of three months is calculated by averaging the returns of portfolios from momentum strategy in current month, previous month, and two months ago. Finally, there is a one month difference between the portfolio formation period and the investment period to avoid the short-term return predictability resulting from the microstructure issue.

3.3.2 Correlation between Sorting Variables

Table 3.3.1 provides the summary of the financial characteristics of the high book-to-market ratio stocks (value stocks) and the low book-to-market ratio stocks (growth stocks). The mean (median) of the book-to-market ratio are 2.2430 (1.6912) and 0.2313 (0.1765) for the value and growth stocks respectively. The growth stocks have larger assets and market value of equity comparing to the value stocks. The sales and sales growth for the growth stocks are higher than those of the value stocks and entire sample. This confirms that the growth firms grow at faster rates than the other firms in the sample. Moreover, the R&D intensity of the growth firms is also higher than other firms in the sample, indicating larger future potential growth opportunities for these firms.

We next examine the correlation between the variables based on which the investment strategies are constructed. Table 3.2 and 3.3 present the average Spearman rank-order correlations between past returns, BOS ratio, one and three-month future returns, composite fundamental scores, and the fundamental signals for value stocks and growth stocks in the sample period respectively. Consistent with the previous findings in Piotroski (2000) and Mohanram (2005), future performance of the stock returns are positively related to firms' financial health measured by the fundamental scores. The fundamental scores and one- and three-month future returns are positively correlated (0.171/0.184 and 0.114/0.123) for value and growth stocks respectively. These correlations are also stronger than those between individual signals and the future returns. Compared to the correlation of future returns with the composite GSCORE (0.114/0.123), this suggests that the investment strategy based on the aggregate information of the firm might outperform those based on individual signals. Moreover, the past cumulative returns are also positively correlated with the future performance of the stocks in our sample, i.e. the correlation is 0.423/0.397 and 0.452/0.411 for value and growth stocks respectively. More importantly, we did not find significant correlation between the past returns and the fundamental scores, and between BOS ratio and fundamental scores, while the past returns and BOS ratio are negatively correlated. We thus expect that the combined momentum strategy can generate better performance than the technical momentum strategy.

3.4 Performances of Alternative Investment Strategies

In this section, we will first investigate the technical momentum strategy based on past returns and trading volume. Then we will study the combined momentum strategy based on both technical and fundamental information. We have found combined momentum strategy outperforms technical momentum strategy in terms of larger returns.

3.4.1 Technical Momentum Strategy Based on Past Returns and Trading Volume

The combined momentum strategy constructed in the current study is based on the past returns, trading volume, and the fundamental indicators FSCORE/GSCORE. We conjecture that the combination of the technical information (past returns and trading volume) and fundamental information (composite fundamental scores) is useful to separate momentum winners from losers. More specifically, we expect that the returns to our combined momentum strategy will be significantly larger than those to the technical momentum strategy.

Table 3.4 provides the technical momentum strategy returns of one, three, and six month holding periods from a long-short portfolio formed by past twelve months winners and losers from January 1982 to December 2007. In Table 3.4, the average monthly returns of the five quintile portfolios constructed by the past twelve month cumulative returns in percentage terms are reported for both value stocks and growth stocks. Value (growth) stocks are those in the top (bottom) tercile book-to-market portfolio at the end of each year. Portfolio Q_{M5} (Q_{M1}) represents the winners (losers) which is constructed by stocks with the highest (lowest) cumulative returns in the past twelve months. The average monthly returns in excess of the three month Treasury-Bill rate for the value (growth) stocks are 0.6058 (0.9103), 0.5778 (0.9776), and 0.4905 (0.7656) percent for holding period one, three, and six month respectively. Our results showed that the momentum returns is generally stronger in the shorter holding period of one month and three month than the longer holding period of six months. Our results are consistent with those found in Jegadeesh and Titman (1993) in which they showed that the trading strategies based on past twelve months winners/losers and one-month and three-month holding periods exhibited strongest momentum returns ignoring the effect of trading costs. Moreover, Table 3.4 reports the monthly Fama-French Three Factors Model adjusted returns of each winners and losers portfolio and long-short investment strategy for the same holding periods. The risk-adjusted return of the portfolio relative to the three factors are the estimated intercept coefficients from the following time-series regression using monthly portfolio returns:

$$(r_i - r_f) = \alpha_i + \beta_i (r_m - r_f) + \phi_i SMB + \varphi_i HML + e_i \qquad (3.5)$$

where r_i is the monthly return for the long-short portfolio i, r_f is the monthly return on three month T-bill, r_m is the value-weighted return on the NYSE/AMEX/Nasdaq market index, *SMB* is the Fama-French small firm factor, *HML* is the Fama-French book-to-market factor, β_i, φ_i, ψ_i are the corresponding factor loadings. In general, the results are consistent with those previously found in the average excess returns. The returns to the long-short investment strategy generate significantly positive momentum returns for value (growth) stocks in different holding periods. We also find that the momentum returns is relatively stronger in one and three month holding period and with declining returns six months after portfolio formation. The results in Table 3.4 suggest that the momentum returns documented in the prior literature also exist in our sample period. We next examine the strength of momentum returns when past trading volume is also considered.

As previously discussed, one of the explanation to the cause of momentum returns is proposed by Wu (2007) which argues it arises due to the asymmetric information between the informed and uninformed investors in the market. More importantly, stronger momentum returns are expected for stocks which are subject to larger degree of asymmetric information. Using the BOS ratio as an empirical proxy, the winner (loser) stocks with lower (higher) BOS ratio are the ones which are subject to a larger degree of asymmetric information and expected to generate larger momentum returns. In other words, the returns to this BOS momentum strategy, which is based on both past returns and BOS ratio, are expected to be higher than those found in Table 3.4.

In Table 3.5, the returns to the BOS momentum strategy are reported. At the end of each month during the sample period, we sort the stocks based on their past twelve months returns to form five quintile portfolios Q_{M1} to Q_{M5}. We also independently sort all the sample stocks based on their BOS ratio, which is the covariance between their past twelve months returns and trading volume, to form five quintile portfolios Q_{B1} to Q_{B5}. The portfolio Q_{B5} (Q_{B1}) consists of those stocks that are subject to largest (smallest) degree of asymmetric information. More specifically, for the winner (loser) stocks, the Q_{B5} portfolio consists of stocks having lowest (highest) covariance between past cumulative returns and past trading volume.

Panel A in Table 3.5 show that controlling for winner momentum, the long-short investment strategy with long position in quintile portfolio Q_{B5} and short position in quintile portfolio Q_{B1} generate significantly positive return indicating using the additional sorting variable, the BOS ratio, allows the investors to obtain the best winners among the winners. Similarly, controlling for loser momentum, the portfolio (Q_{B5}-Q_{B1}) among losers generate significantly negative returns suggesting that BOS ratio further separates the worst losers among the losers in the technical momentum strategy. More importantly, we are interested in the BOS momentum strategy which is based on both past returns and BOS ratio, i.e. long top winners with lowest BOS ratio(Q_{M1}, Q_{B5}) and short top losers with highest BOS ratio (Q_{M5}, Q_{B5}). If the asymmetric information between informed and uninformed investors causes the momentum returns, the trading strategy constructed by these extreme portfolios is expected to generate larger long-short portfolio returns than technical momentum strategy based solely on past returns. Following Wu (2007), we formulate the testable hypothesis.

> *H_{10}: The BOS momentum strategy based on both past cumulative returns and BOS ratio generates larger returns than the technical momentum strategy based solely on past cumulative returns.*

Using our notation, we can view this hypothesis in the following manner:

$$\Delta_{BOS-MOM} = \left[\left(Q_{M1}, Q_{B5} \right) - \left(Q_{M5}, Q_{B5} \right) \right] - \left[Q_{M1}, Q_{M5} \right] \geq 0 \quad (3.6)$$

where $\Delta_{BOS\text{-}MOM}$ is the return differences between the BOS momentum strategy and the technical momentum strategy, $\left[\left(Q_{M1}, Q_{B5} \right) - \left(Q_{M5}, Q_{B5} \right) \right]$ is the return to the BOS momentum strategy, and $\left[Q_{M1}, Q_{M5} \right]$ is the return to the technical momentum strategy. In Panel A of Table 3.5, the one-, three-, and six-month average monthly excess returns difference between the BOS momentum strategy and the technical momentum strategy $\Delta_{BOS\text{-}MOM}$ are statistically significant at 0.9114, 0.9964, and 0.9856 percent respectively for high book-to-market stocks. The outperformance of the BOS momentum strategy is also found by using the Fama-French adjusted returns across in Panel A of Table 3.5. However, we observe neither the further separability among winners and losers nor the outperformance of the BOS momentum strategy among the growth stocks. The return differences $\Delta_{BOS\text{-}MOM}$ among the growth stocks is either insignificantly positive or negative as shown in Panel B of Table 3.5. In general, our results demonstrate that BOS ratio indeed help investors measure the strength of momentum returns and identify the best (worst) among the winners (losers) among the value stocks but not among the growth stocks.

In the prior research in trading volume literature, Datar, Naik, and Radcliffe (1998) found a negative relation between past trading volume and future returns for the stocks. These authors documented that stocks with low trading volume in the recent past generate higher future returns than those with high trading volume. Lee and Swaminathan (2000) found that low volume stocks outperform high volume after controlling for price momentum and momentum is stronger among high volume stocks. Simple trading volume could be proxy for many different factors such as size, liquidity, and degree of asymmetric information. However, the BOS ratio provides a proxy for asymmetric information by measuring the covariance between past returns and past trading volume, and therefore narrows down the subsets concerning our investment strategy. In general, the momentum returns is stronger when past trading volume is incorporated into separating winners from losers when forming investment strategy. However, since these winners and losers stocks could have fundamentally different financial characteristics, we ask the question that whether further analyses regarding firm's fundamentals could aid investors in selecting the best (worst) among the winners (losers) stocks. We next examine the combined momentum strategy when the fundamental analysis indicators FSCORE/GSCORE are incorporated.

3.4.2 Combined Momentum Strategy Based on Technical and Fundamental Information

In this section, we propose combined momentum strategy in term of the combined forecasting models developed by Granger and Newbold (1974), Lee, Newbold, Finnerty, and Chu (1986), and Lee and Cummins (1998). The combined momentum strategy is constructed by past returns, past trading volume, and fundamental scores (FSCORE/GSCORE). The fundamental indicators FSCORE and GSCORE developed by Piotroski (2000) and Mohanram (2005) helped investors to separate the winner stocks from the loser stocks based on firm specific financial characteristics for value stocks and growth stock respectively. Their results indicate that the financially healthier firms outperform their counterparts with more financial constraints. We expect to observe larger momentum returns when the fundamental aspects of the firms are accounted in constructing the combined momentum strategy. Since the value stocks and growth stocks have different financial characteristics, we discuss their implication for momentum strategy separately.

Table 3.6 provides the summary of the returns to the combined momentum strategy with respect to the high book-to-market (value) stocks. We empirically test whether the combined momentum strategy generate significantly larger returns than the BOS momentum strategy which is based solely on technical information among the value stocks.

> *H_{20}: The combined momentum strategy based on portfolios sorted by past cumulative returns, BOS ratio, and the FSCORE generates larger returns than the BOS momentum strategy among the high book-to-market (value) stocks.*

This testable hypothesis can be written as the following using our notation.

$$\Delta_{CS-BOS} = \left[\left(Q_{M1}, Q_{B5}, Q_{F5}\right) - \left(Q_{M5}, Q_{B5}, Q_{F1}\right)\right] - \left[\left(Q_{M1}, Q_{B5}\right) - \left(Q_{M5}, Q_{B5}\right)\right] \geq 0 \quad (3.7)$$

where $\Delta_{CS\text{-}BOS}$ is the return differences between the combined momentum strategy and the BOS momentum strategy, $\left[\left(Q_{M1}, Q_{B5}, Q_{F5}\right) - \left(Q_{M5}, Q_{B5}, Q_{F1}\right)\right]$ is the return to the combined momentum strategy, and $\left[\left(Q_{M1}, Q_{B5}\right) - \left(Q_{M5}, Q_{B5}\right)\right]$ is the return to the BOS momentum strategy. From Table 3.6 we first observe that the financially healthier firms indeed outperform those with more financial constraints. In both panels, the top quintile portfolio outperforms the bottom quintile portfolio sorted by the FSCORE after controlling for price momentum and BOS ratio. In Panel A of Table 3.6, the one month holding period average excess return of (Q_{F5}-Q_{F1}) is 1.2125 percent and 1.2561 percent after controlling for loser and winner momentum respectively. The same can be found for three month and six month holding period average excess returns as well as the Fama-French Three Factor model adjusted returns in Panel B. More importantly, the combined momentum strategy with long position in top winners with low BOS ratio and high FSCORE, and short position in top losers with high BOS ratio and low FSCORE generate statistically significant returns. The combined momentum strategy constructed by the three-way sorted portfolios produce 1.7817, 3.3598, and 2.9584 percent monthly average excess returns for one, three, and six month holding periods respectively. Comparing with the returns to the technical momentum strategy shown in Table 3.5, we find that our combined momentum strategy produce significantly higher returns in terms of significantly positive return difference ($\Delta_{CS\text{-}BOS}$) across all holding periods and return calculation. The significantly larger return to our combined investment strategy indicates a stronger momentum returns when fundamental indicators are considered to identify winners and losers. In general, our results in Table 3.6 suggest that for the value stocks, incorporating the fundamental indicators improve investors' ability in separating winners from losers.

Given the fundamentally different characteristics between the growth firms and value firms, Mohanram (2005) developed the GSCORE system for identifying firms with better overall financial soundness. Firms with higher GSCORE indicate better financial health than the ones with lower scores. In this section, we further examine whether the incorporation of the GSCORE in the technical momentum strategy can help investors separate winners from the losers. Table 3.7 summarizes the returns to our combined momentum strategy with respect to growth stocks. We note that similar to the value stocks, the financially healthier growth stocks outperform those with more financial constraints. The long-short investment strategy with long position in financially healthier firms and short position in financially constrained firms, (Q_{G5}-Q_{G1}) all produce significantly positive returns across different holding periods and return calculation. If the GSCORE further separates winners from losers along with past returns and BOS ratio, our proposed combined momentum strategy is expected to produce larger returns between extreme portfolio (Q_{M1}, Q_{B5}, Q_{G5})-(Q_{M1}, Q_{B5}, Q_{G1}) than those found in Table 3.5. In other words, we test whether the combined investment strategy generate significantly larger returns than the BOS momentum strategy based solely on technical information among the growth stocks.

H_{30}: The combined momentum strategy based on portfolios sorted by past cumulative returns, BOS ratio, and the GSCORE generates larger returns than the BOS momentum strategy among the low book-to-market (growth) stocks.

This hypothesis can be written as follows using our notation.

$$\Delta_{CS-BOS} = \left[\left(Q_{M1}, Q_{B5}, Q_{G5} \right) - \left(Q_{M5}, Q_{B5}, Q_{G1} \right) \right] - \left[\left(Q_{M1}, Q_{B5} \right) - \left(Q_{M5}, Q_{B5} \right) \right] \geq 0 \quad (3.8)$$

where $\Delta_{CS\text{-}BOS}$ is the return differences between the combined momentum strategy and the BOS momentum strategy, $\left[\left(Q_{M1}, Q_{B5}, Q_{G5} \right) - \left(Q_{M5}, Q_{B5}, Q_{G1} \right) \right]$ is the return to the combined momentum strategy, and $\left[\left(Q_{M1}, Q_{B5} \right) - \left(Q_{M5}, Q_{B5} \right) \right]$ is the return to the BOS momentum strategy. In Table 3.7, the return difference between our combined momentum strategy and BOS momentum strategy ($\Delta_{CS\text{-}BOS}$) are all significantly positive. For example, the return differences ($\Delta_{CS\text{-}BOS}$) are significantly positive at 2.4686, 1.9368, and 1.2436 percent for one-, three-, and six-month holding period excess returns. Similar results can be found among the Fama-French adjusted returns in Panel B. In sum, our results show that the investment strategy based on past returns, BOS ratio, and the GSCORE generate larger momentum returns than those to technical momentum strategy in Table 3.4 and Table 3.5.

We further compare the risk-return characteristics across three different investment strategies. Table 3.8 provides the summary of comparison of three month average excess returns for the investment strategies based on different sorting for value stocks (Panel A) and growth stocks (Panel B). For both value stocks and growth stocks, our proposed combined momentum strategy outperforms the strategy based on the past returns alone, and on the past returns with the BOS ratio. Further, we report the information ratio (IR) of the three different strategies. The information ratio is defined as the active return divided by tracking error,

$$IR = \frac{(r_i - r_m)}{\sigma_{(r_i - r_m)}} \qquad (3.9)$$

where active return (r_i-r_m) is the difference between the return on the different strategies and the return on the NYSE/AMEX/Nasdaq value-weighted return, and tracking error $\sigma_{(r_i - r_m)}$ is the standard deviation of the active return. We find that our combined momentum strategy produces the highest information ratio in all three different holding periods for both value and growth stocks. For example, our combine momentum strategy generates a higher information ratio (0.57) than the technical momentum strategy (0.37) and the BOS momentum strategy (0.51) in one-month excess return for the value stocks.

The higher information ratio is generated not only because of the higher expected return but also lower portfolio risk. The last column of Table 3.8 reports the correlation of the fundamental strategy returns with that of the momentum strategy. More specifically, we construct investment strategies based on each factor alone and examine their correlation across our sample period for both value and growth stocks. For both the value and growth stocks in our sample, the momentum returns is negatively correlated with the returns of fundamental strategy based on FSCORE/GSCORE. The negative correlation between the two strategies implies that the combination of them might reduce the overall riskiness of the portfolios while achieving higher expected rate of return. In summary, our findings suggest that the combination of technical information and the fundamental information improve the ability of the investors to further separate the winner stocks from the loser stocks.

3.5 Summary

In this paper, we have developed three hypotheses to test whether combined momentum strategy outperforms the technical momentum strategy or not. In the first hypothesis, we test whether the BOS momentum strategy (Wu (2007)) outperforms the technical momentum strategy among both value and growth stocks. In the second hypothesis, we test whether the combined momentum strategy outperforms the BOS momentum strategy among value stocks. Finally, in the third hypothesis, we test whether the combined momentum strategy outperforms the BOS momentum strategy among the growth stocks.

We construct our combined momentum strategy by incorporating the FSCORE (Piotroski, 2000) and GSCORE (Mohanram, 2005) system into the technical momentum strategy. We first find that the profits for the technical momentum strategy exist persistently during the sample period from 1982 to 2008. More importantly, we find that combined momentum strategy outperforms the technical momentum strategy in terms of higher returns. The larger profitability generated by the combined momentum strategy indicates that the composite fundamental scores can be used by investors to separate the best (worst) among the winners (losers) stocks. Our findings suggest that fundamental analysis indeed provides information to investors in addition to the technical information for selecting winner and loser stocks.

We also consider our results contributing to the security analysts and portfolio managers using technical momentum strategy. These technical momentum investors usually had success during the period when the performances of the winners are distinguishable from the losers. However, when the market experiences an overall rally like the one in the months of March and April of 2009, these technical momentum investors suffer substantially from the loss on the short side of their portfolio (Xydias and Thomasson, 2009). By incorporating the fundamental analysis into the technical momentum strategy, we believe our results should be useful for the security analysis and portfolio management to these investors.

Chapter 4

The Economic Consequences of Real Earnings Management

(*Joint work with Professor Suresh Govindaraj*)

4.1 Introduction

To many cynics, the idea that managers and firm insiders "manage" earnings, may seem self evident.[15] However, among academics in the fields of finance and accounting, *proving* that earnings are managed has been a subject of considerable research over the last two or three decades. The interest among academics is probably justified given the importance attached to earnings by different parties that have or plan to have contractual agreements with the firm, and the consequences of earnings management for all interested parties. Part of the problem in establishing that earnings are managed is that it is difficult to pin down a benchmark of what earnings may have been had they not been managed (a point that has been emphasized by Healy and Wahlen, 1999 and Beaver, 2004). Despite this, researchers have documented, and continue to document, persuasive evidence that earnings management does indeed exist.

Broadly speaking, earnings are deemed to have been "managed" when specific actions, (through selection of real opportunities, and/or selection of recording and reporting procedures), have been taken to influence reported earnings. This includes managing earnings numbers to meet specific earnings targets and benchmarks, or managing earnings to ensure that earnings do not fluctuate too much over time (income

[15]Perhaps the recent economic crisis has served to strengthen this belief.

smoothing). The motivations for earnings management are diverse. Commonly cited reasons are trying to influence the capital markets and financial analysts, influencing earnings relating contractual obligations of the firm (such as managerial compensation, debt contracts etc.), influencing regulators, and maintaining dividend payments. Healy and Wahlen (1999), and, Ronen and Yaari (2008), provide a survey of research in this area.

The general consensus in the literature is that earnings management manifests itself in two forms. First, the management of reported earnings by manipulating accruals with no direct consequences for the cash flows of the firm (Accrual Management). One example of such management would be the under recognition of bad debt expenses[16]. Second, the management of reported earnings by manipulating the real economic activities of the firm that have tangible impact on the cash flows of the firm (Real Activities Management). An example of this type of management would be reducing discretionary expenses for advertising or research (Graham et al., 2005, Roychowdhury, 2006). While there is extensive literature on the former, research on the latter is picking up.

Our focus in this paper is the economic consequences of earnings manipulation through real activities management. Graham et al. (2005), identify, and find support for four characteristics that lead to real activities management: (1) Managers believe that earnings are informative for assessing firm value; (2) Managers are interested in meeting or beating some benchmark (be it analysts' forecasts, their own forecasts, or even avoiding negative earnings) primarily to influence stock prices, advance their own careers,

[16]The interested reader is directed to the survey paper by Healy and Wahlen (1999) for research studies relating to Accrual Management and its consequences.

for political reasons, or credit ratings; (3) Managers care about smoothing earnings paths, or equivalently, managers dislike earnings volatility; and, (4) Managers are willing to sacrifice economic value to manage financial reporting. We postulate an additional characteristic of real earnings management, that is, (5) managers may unpredictably change real earnings management practices, perhaps in response to exogenous shocks, such as, managerial regime changes, or random shocks to the economy.

We develop a dynamic equilibrium model with a representative risk averse manager who has the power to decide on firm investments, and whose consumption and compensation are tied to the firm's earnings (wealth) generated from the investments.[17] Given the persuasive evidence of real activities management laid out in Graham et al. (2005) and Roychowdhury (2006), we take, as given, that this is a phenomenon to be common practice. Earnings net of withdrawals for consumption and compensation are reinvested in the firm, and these contribute to the net worth (and market value) of the firm. Since we are modeling real earnings management behavior, we make two behavioral assumptions regarding the manager that capture the five characteristics for real activities management provided by Graham et al. (2005)[18]: (1) The manager will pursue actions to meet some earnings target; and (2) The manager is conditioned by past compensation and withdrawals, and is averse to fluctuations in these, especially, falling below past compensation levels. The idea is that managers are used to certain styles and

[17]One could think of this manager as having a stakeholder interest in the firm. This kind of manager is becoming more commonplace now with managers being compensated with stocks rather than cash.

[18]The extra complication, introduced through earnings management, changes the problem from a straightforward consumption investment optimization model to one that allows for this additional dimension.

levels of lifestyles. In addition, we permit the manager's weights on past withdrawals and standards of living to randomly change over time. While it makes our model more general, this innovation captures the reality that managers, while being conditioned by history, could react in unpredictable ways to external shocks.[19] This adds an economic dimension that, to the best of our knowledge, has not been studied in the prior literature.[20] As we show, in our model managers will tend to produce relatively less or more volatile earnings streams compared to the benchmark model where managers do not practice earnings management, do not have earnings targets, and have no history dependent choices of withdrawals and re-investments. The main insights from our model are twofold: First, managers may forgo or avail of investment opportunities that they normally would not in the benchmark model; Second, and perhaps the compelling result in our paper, is that earnings management with history dependent choices of withdrawals and re-investments of earnings, add extra risks in the economy that affect market prices of firms (including stocks, bonds, and other derivatives), and economy wide interest rates as well. In other words, real earnings management carries consequences not only for the firm itself, but on the economy as a whole. We provide an explicit characterization of these risks. While prior empirical research has shown the existence of the former insight, there are practically no quantification of the risks or empirical tests of impact of real earning management on asset prices.

In section 4.3, we develop a model of real earnings management in a production economy where current managerial policies on investment and withdrawals of firm

[19]These could be exogenous economic, political, or environmental shocks, or shocks induced by sudden change in management structure.

[20]History and habit models of this kind have been studied by Constantinides (1990) and Sundaresan (1989) in the context of the equity premium puzzle. However, our work goes further by allowing for time dependent and random habit changes.

profits depends on past practices, but without random changes in these policies. This creates policies that are temporally related but eventually converge to a target policy. The associated sub-sections provide the optimal investment-withdrawal policy paths, the prices of contingent claims against the firm, and the effects on the interest rates in the economy. In section 4.4, and its sub-sections, we obtain similar results for the case where investment-withdrawal policies are permitted to change due to random exogenous shocks. Here we do not get the asymptotic convergence to desired target policies studied in Section 4.3. In addition, these exogenous shocks add an extra dimension of risk that absent in the model of Section 4.3. This has broad impact on the firm and the economy as a whole. Section 4.5 concludes the paper.

4.2 Prior Literature on Real Earnings Management

Perhaps the earliest studies that documented evidence of earnings management are by Hayn (1995), and Burgstahler and Dichev (1997) who find that firms, when confronted with the possibility of falling on the negative side of zero earnings will attempt to manage earnings up to the positive side. Since then, there have been a large number of papers either trying to prove or disprove these results.[21] Since our interests are in real activities management, we cite a few well known papers in this area alone.

On the empirical side, most studies have on real activities management focus mostly on Research & Development (R&D) related expenses. Baber, Fairfield, and Haggard (1991), who show that firms commit to less R&D spending when it could

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[21]We do no attempt to be comprehensive in our citations of prior literature on earnings management. The reader is referred to Healy and Wahlen (1999), and, Ronen and Yaari (2008), for a survey of the literature and associated citations.

prevent the firms from reporting positive earnings, or an increase in earnings. Another example is Dechow and Sloan (1991), who use a sample of firms which have significant R&D expenditures, find that CEOs towards the end of their appointment spend less on R&D in order to increase the short-term earnings performance. Bushee (1998) documents that large institutional ownerships increase the probability of managers decreasing R&D expenditure to reverse previous earnings decline. Bens, Nagar, and Wong (2002) show that managers reduce R&D expenditure to finance the shares repurchase because they are concerned about the earnings-per-share dilution accompanied the employee stock options exercises.

There are also a number of empirical studies that find evidence of earnings management through discretionary choices relating to earnings. Healy (1985), and Gaver, Gaver, and Austin (1995), find that compensation plans can influence managers in their discretionary selection of the timing and magnitude of items that influence earnings magnitude, as well as earnings volatility. Sloan (1996), Collins and Hribar (2000), Beneish and Vargus (2002), Bergstresser and Philippon (2009), and Teoh, Welch, and Wong (1998a, 1998b) show a similar pattern of behavior by managers who attempt to influence the market prices of the firm for personal benefit.

The recent papers by Graham et al. (2005), and by Roychowdhury (2006), provide the most compelling evidence of real activities management. Graham et al. (2005) uses a interview and survey analysis of over 400 executives to arrive at persuasive evidence of real activities management. Roychowdhury (2006) analyses a large sample of public disclosures to show that managers manipulate real activities to avoid reporting losses, or meet other benchmarks such as analysts' forecasts. Cohen et al. (2007)

document evidence of decrease in accrual management post Sarbanes-Oxley ACT (SOX) in 2002, but an increase in real activities management after SOX, thus confirming the findings of Graham et al. (2005).

Despite this voluminous collection of empirical papers on the subject of earnings management, there is not too much by way of analytical theory that studies the economic consequence of such behavior. Trueman and Titman (1988) develop a model where managers can shift income inter-temporally to explain why and how income smoothing might occur, and how it increases the stock prices of the income smoothing firms. They argue that managers have incentives to report a smoother income stream to the reduce earnings volatility, to lower the estimates of the probability of bankruptcy of the firms by potential lenders and claimants. This results in lower borrowing costs, more favorable terms of agreement between the firm and its customers and suppliers, and increases the market value of the firm. Continuing this line of research, Goel and Thakor (2003) develop a model showing that firms smooth earnings because high earnings volatility may scare away uniformed or under-informed investors from investing in the firm, because it puts them at a disadvantageous position against better informed investors. Consequently, managers resort to income smoothing behavior that reduces earnings volatility, does not scare away any potential investor (even if they are disadvantaged), increases demand for the firm's stocks, and increases the stock prices.

Guttman, Kadan, and Kandel (2004) develop a signaling model where managers of relatively poorly performing firms manage earnings to ensure that they cannot be distinguished from better firms. Somewhat related is the work by Povel, Singh, and Winton (2007), who show that exogenous economic conditions could influence earnings management by poorly performing firms. They show that earnings management may be more pronounced in good economic times, because investors believe that a good economy means that the firms are mostly good, and do not deeply scrutinize firm disclosures. This allows managers of bad firms to manage earnings up and get away with it. Another such model is by Kedia and Philippon (2009), who show that bad firms which have low productivity may not only manage and mis-report earnings, but also hire and invest more, in order to mimic the good firms. They demonstrate that this could have real effects on the economy by causing a misallocation of resources.

Wang (2006) models the effects of earnings management on assets return and return volatility. In his model, managers inefficiently shift assets and liabilities across periods to report noisy earnings stream to maximize their personal objectives. The investors, who are not privy to this earnings management, can only observe the managed noisy earnings reported by managers to infer true earnings process of the firm, and use this to estimate the price the firms. It is further shown that this type of earnings management leads to GARCH type behavior of asset returns.

Fudenberg and Tirole (1995) examine the income smoothing in a principal-agency setting. In their model the manager have private benefit from running the firm, and the firm owner (principal) evaluates the manager based on the reported income. The manager could be fired if they perform poorly. They assume that recent income observation are more informative than the older ones when the firm owners evaluate the performance of the manager. Given this "information decay", they show that the reporting strategies that maximize managers benefit do not necessarily maximize the firm value. This provides managers with incentives to smooth income in order to maximize the expected length of their tenure. Managers tend to increase reported income in bad times because they are concerned about short-term future prospect and keeping their jobs; and tend to decrease reported income in good times,because they want to save for bad times in the future.

The objectives of our paper mirror some of the same goals that have been articulated in earlier analytical work, which is to develop a model of asset pricing and analyze the economic consequences of earnings management. However, we emphasize that we differ from prior work by imbedding earnings management in a dynamic (not static as in prior work) equilibrium framework developed by Cox, Ingersoll and Ross (CIR) (1985a). As a consequence, we do not concern ourselves with economic frictions that naturally arise in signaling and principal agent problems. The dynamic part is crucial because firms are assumed be long lived and earnings management is intrinsically an intertemporal phenomenon.

4.3 Asset Pricing with Earnings Smoothing and Deterministic Habit Changes

In this section, we consider a model of asset pricing where the representative manager has the responsibility for an investments and withdrawals of the firm. Our goal is not to develop the most general model, but to focus on a relatively simple economic setting that will provide the necessary insights.

Further we consider a production economy with one risky technology that has following earnings return dynamics

$$\frac{d\eta}{\eta} = \alpha dt + \sigma d\zeta_t \qquad (4.1)$$

where t is time, α is the expected return (drift), σ is the volatility, and ζ_t is a standard Brownian motion.

The manager is assumed to be risk-averse. Without loss of generality, it is further assumed that the manager's compensation (or consumption) is the earnings withdrawn from the firm, $\hat{\mathcal{P}}_{t,\det}$.[22] We make the additional assumption that the source of the manager's current withdrawals is sum of current and past earnings retained by the firm. The manager maximizes expected utility at time t from the current and expected future withdrawals,

$$E_t\left[\int_t^{\infty} e^{-\rho u} U(\hat{\mathcal{P}}_{u,\det}, u)du\right] \qquad (4.2)$$

where $U(\hat{\mathcal{P}}_{t,\det}, t) = \hat{\mathcal{P}}^{\gamma}_{t,\det}/\gamma,\ \ 0 < \gamma < 1.$ [23],[24] To capture the idea that the withdrawals $\hat{\mathcal{P}}_{t,\det}$ depends on the past history of compensation (withdrawals), and the fact that the manager has become accustomed to past levels of compensation, we specify

$$\hat{\mathcal{P}}_{t,\det} = \mathcal{P}_{t,\det} - bz_{t,\det} \qquad (4.3)$$

where $b > 0$ is a constant (finite) real number, $z_{t,\det}$ is the exponentially weighted sum of past withdrawals given by

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[22]The manager partly or wholly shares with other investors in the withdrawals, $\hat{\mathbf{P}}_{t,\det}$, can also represent the total withdrawals. We do not pursue the question of the optimality of this compensation plan in this paper, but simply assume it to be a given arrangement. One could also think of the manager as a part owner.

[23]In our model the manager maximizes utility from current and future withdrawals over an infinitive horizon. We do not introduce a bequest function at a terminal time, because it really does not add much by way of insights to our model.

[24]This specific utility form has been used extensively in prior work. By using this form for the utility, we can compare our results with findings in the literature.

$$z_{t,\det} = e^{-a_t t} z_0 + \int_0^t e^{-a_u (t-u)} \mathcal{P}_{u,\det} du \tag{4.4}$$

where a_t is the weight assigned to past withdrawals. Equations (4.2) and (4.3) suggest that the manager receives positive utility from withdrawals that are in excess of past withdrawals to which the manager has become accustomed to induced by $z_{t,\det}$. It is to be noted that while the manager marginal utility increases in current excess withdrawals, it also increases the levels of withdrawals the manager becomes habituated to. Therefore excessive current withdrawals would put pressure on future withdrawals.

So far we are consistent with the time inseparable utility specifications of Constantinides (1990), and Sundaresan (1989). But deviating from their work, where, the weight a_t was assumed a constant, we specify a time dependent (deterministic) mean reverting process for the

$$a_t = a_0 e^{-\kappa t} + (1 - e^{-\kappa t})\bar{a} \tag{4.5}$$

which is the weighted average of initial weight a_0 and long run average weight $\bar{a}$. Both a_0 and $\bar{a}$ are assumed positive, and we assume that $\bar{a} > b > 0$. The dynamics of the weight a_t can be written as

$$da_t = -\kappa(a_t - \bar{a})dt \tag{4.6}$$

where $\kappa > 0$ captures the speed of convergence of current weight a_t to a constant target, $\bar{a}$ (the constant assumed in Constantinides, 1990, and Sundaresan, 1989 assigned to past withdrawals). This innovation captures the idea that managers often have target withdrawals in mind while deciding investment-withdrawal policies for the firm. Since withdrawals are the whole (or a proportion) of earnings, it would imply that managers strive to meet earnings targets. While the current weight at time t , or a_t , may not be at that ideal level, $\bar{a}$, yet, the target $\bar{a}$ would be reached asymptotically in large time.[25]

The proposition below clarifies this further. Essentially, it provides the dynamic evolution of weight $z_{t,\det}$ of past withdrawals to the manager given the mean reverting dynamics of the weight a_t . It shows that over large time, the weight will reach its equilibrium value of $\bar{a}$, and asymptotically $z_{t,\det}$ will be the same as in Constantinides (1990), and Sundaresan (1989).

Proposition 1 *Given equation (4.4) and (4.6),*

$$\frac{dz_{t,\det}}{dt} = z_0 e^{-a_t t}\left[e^{-\kappa t}\left(a_0 - \bar{a}\right)\left(\kappa t - 1\right)\right] + \mathbf{P}_{t,\det} - \bar{a} z_{t,\det} \qquad (4.7)$$

and

$$\lim_{t\to\infty} \frac{dz_{t,\det}}{dt} = \mathcal{P}_{t,\det} - \bar{a} z_{t,\det}$$

$$\lim_{\kappa\to\infty} \frac{dz_{t,\det}}{dt} = \mathcal{P}_{t,\det} - \bar{a} z_{t,\det} \qquad (4.8)$$

Proof. See Appendix.

4.3.1 The Manager's Optimization Program (the HJB Equation)

In this sub-section, we layout the optimization program of the manager for selecting the withdrawals and the reinvestment in the risky technology. This involves the

[25]In the next section we permit the weights assigned to be stochastic.

development of the so-called Hamilton-Jacobi-Bellman (HJB) equation, and solving a partial differential equation (PDE). We provide a closed form solution for the PDE and characterize the optimal strategy of the manager explicitly.

Assume the firm's wealth (accumulated earnings) at time t is W_t and invests $w_{t,\det}$, $0 \leq w_{t,\det} \leq 1$, of it in the risky technology η. The withdrawal amount in the period $[t, t+\Delta t]$ is denoted by $\mathcal{P}_{t,\det}$. The dynamics of firm's earnings can be written as (see Merton, 1992)

$$dW_t = [W_t w_{t,\det}\alpha - \mathcal{P}_{t,\det}]dt + W_t w_{t,\det}\sigma d\zeta_t \tag{4.9}$$

Denoting the discounted indirect utility function as $J(W_t, z_{t,\det}, t)$, the manager's optimization problem can be written as

$$\max_{\{\mathcal{P}_{u,\det}, w_{u,\det}\}} E_t\left[\int_t^\infty e^{-\rho u}\frac{[\mathcal{P}_{u,\det} - bz_{u,\det}]^\gamma}{\gamma}du\right] = e^{-\rho t}J(W_t, z_{t,\det}, t) \tag{4.10}$$

subject to the intertemporal budget constraint in equation (4.9). Given the above we can derive the continuous time HJB equation

$$0 = \max_{\{\mathcal{P}_{t,\det}, w_{t,\det}\}}\{U(\mathcal{P}_{t,\det}, z_{t,\det}, t) + L[e^{-\rho t}J]\}$$

where, $U(\mathcal{P}_{t,\det}, z_{t,\det}, t) = e^{-\rho t}\dfrac{(\mathcal{P}_{t,\det} - bz_{t,\det})^\gamma}{\gamma}$ and L, (the Dynkin operator) is given by

$$L[e^{-\rho t}J] = \left\{\begin{array}{l} e^{-\rho t}\{J_W(W_t w_{t,\det}\alpha - \mathcal{P}_{t,\det}) + \frac{1}{2}J_{WW}(W_t w_{t,\det}\sigma)^2 \\ +J_z[z_0 e^{-a_t t}(\kappa t - 1)(a_t - \bar{a}) + \mathcal{P}_{t,\det} - \bar{a}z_{t,\det}] \\ -\rho J + J_t\} \end{array}\right\} \tag{4.11}$$

The first-order conditions of equation (4.11) with respect to the withdrawals $\mathcal{P}_{t,\det}$ and the optimal investment weight $w_{t,\det}$ are

$$0 = (\mathcal{P}_{t,\det} - bz_{t,\det})^{\gamma-1} - J_W + J_z$$
$$0 = J_W W_t \alpha + W_t^2 J_{WW} w_{t,\det} \sigma^2$$

implying that the optimal withdrawals $\mathcal{P}^*_{t,\det}$, and optimal investment in the risky technology $w^*_{t,\det}$ are given by

$$\mathbf{P}^*_{t,\det} = bz_{t,\det} + \left[J_W - J_z\right]^{\frac{1}{\gamma-1}} \quad (4.12)$$

$$w^*_{t,\det} = \frac{-J_W}{J_{WW} W_t} \frac{\alpha}{\sigma^2} \quad (4.13)$$

Substituting equation (4.12) and (4.13) into equation (4.11) and simplifying yields the equilibrium partial differential equation

$$\begin{aligned} 0 = &\frac{1-\gamma}{\gamma}[J_W - J_z]^{\frac{\gamma}{\gamma-1}} - \rho J + J_t - \frac{1}{2}\frac{J_W^2}{J_{WW}}\frac{\alpha}{\sigma^2} + bz_{t,\det}(J_z - J_W) \\ &+ J_z\{z_0 e^{-a_t t}[e^{-\kappa t}(a_0 - \bar{a})(\kappa t - 1)] - \bar{a} z_{t,\det}\} \end{aligned} \quad (4.14)$$

The next proposition provides an explicit closed form solution for the indirect utility The next proposition provides an explicit closed form solution for the indirect utility function $J(W_t, z_{t,\det}, t)$. As may be expected, the indirect utility depends not only on the current wealth, but also the pattern of past withdrawals to the manager. It is to be noted that the coefficients are time dependent, which captures the fact that the weight given to past withdrawals a_t is changing with time.

Proposition 2 *Assume* $\overline{a} > b > 0$ *. Also assume that* $\rho > \frac{1}{2}\frac{\alpha^2}{\sigma^2}\frac{\gamma}{1-\gamma}$ *. Then the solution to the partial differential equation in (4.14) is given by*

$$J(W_t, z_{t,\det}, t) = k_0(t)[W_t + k_1(t) z_{t,\det} + k_2(t)]^{\gamma} \tag{4.15}$$

where

$$k_0(t) = \frac{1}{\gamma}\left\{ e^{(\frac{\bar{a}}{\bar{a}-b}h)t} \int_0^t e^{-(\frac{\bar{a}}{\bar{a}-b}h)u} \left[\gamma^{\frac{1}{\gamma-1}} \left[1 - \left(\frac{b}{\bar{a}-b} \right) \left[1 - e^{-(\bar{a}-b)u} \right] \right]^{\frac{\gamma}{\gamma-1}} \right] du \right\}^{1-\gamma} \tag{4.16}$$

$$k_1(t) = \left(\frac{b}{\bar{a}-b} \right) \left[1 - e^{-(\bar{a}-b)t} \right] \tag{4.17}$$

$$k_2(t) = \left(\frac{-b}{\bar{a}-b} \right) \int_0^t \left[1 - e^{-(\bar{a}-b)s} \right] \left[z_0 e^{-a_s s} (a_s - \bar{a})(\kappa s - 1) \right] ds \tag{4.18}$$

$$h = \frac{\bar{a}-b}{\bar{a}(1-\gamma)} \left(\rho - \frac{1}{2} \frac{\alpha^2}{\sigma^2} \frac{\gamma}{1-\gamma} \right) > 0 \tag{4.19}$$

Proof. See Appendix.

4.3.2 Optimal Investment

Once we have obtained the indirect utility function in Proposition 1, it is straightforward to obtain the optimal withdrawals and investment in the risky technology using equations (4.12) and (4.13) respectively for the manager. This is what we proceed to do in the next proposition.

Proposition 3 *Given the coefficients* $k_0(t)$ *,* $k_1(t)$ *, and* $k_2(t)$ *in Proposition 2 the investment in the risky technology is obtained from equation (4.13)*

$$w^*_{t,\det} = (\frac{1}{1-\gamma}) \frac{\alpha}{\sigma^2} [1 - h_1(t) + h_2(t)] \text{ where} \tag{4.20}$$

$$h_1(t) = \frac{z_{t,\det}}{W_t}\left(\frac{b}{\bar{a}-b}\right)\left[1-e^{-(\bar{a}-b)t}\right] \text{ and}$$

$$h_2(t) = \left(\frac{-b}{\bar{a}-b}\right)\int_0^t \left[1-e^{-(\bar{a}-b)u}\right]\left[z_0 e^{-a_u u}(\kappa u-1)(a_u-\bar{a})\right]du$$

Proof. See Appendix.

For the case when the mean reverting pull, κ, or time, t, are large, we already know that we recover the optimal investments in Constantinides (1990), and Sundaresan (1989). For small κ, say near zero, we can say more. For this case, it follows from the above Proposition that when $a_0 > \bar{a}$ (a constant) the manager will begin with and continue to put less weight on the history past withdrawals patterns of withdrawals and investments. The consequence of this is that the manager will tend to invest more in the risky technology compared to the case where $a = \bar{a}$. The opposite holds for the case when a_0, is smaller than $\bar{a}$. The next two Corollaries state and prove this. As we show later, this has consequences for the economy as a whole.

Corollary 1: Let $\overline{w}_t^*$ be the optimal investment when the weight assigned to past withdrawals has reached its equilibrium position over long time, and $w_{t,\det}^*$ be the optimal investment at time t. In (4.20) when initial weight of past withdrawals, a_0, is smaller (larger) than $\bar{a}$, then manager will invest less (more) in risky assets.

Proof. See Appendix.

Corollary 2: Identify $\bar{a}$ as the constant in Constantinides (1990), and Sundaresan (1989). Then the manager in our setting will invest more aggressively (conservatively) in risky assets than that in their settings when $a_0 > \bar{a}$, $(a_0 < \bar{a})$. Over a long time as a_t is finally equal to $\bar{a}$ in our setting, we arrive at the same optimal investment in risky assets as in Constantinides (1990).

Proof. Follows from Corollary 1.

4.3.3 Optimal Withdrawals and its Dynamics

The optimal withdrawals by the manager can now be characterized, that is,

$$\begin{aligned}
\mathcal{P}^*_{t,\det} &= bz_{t,\det} + [J_W - J_z]^{\frac{1}{\gamma-1}} \\
&= bz_{t,\det} + [\gamma k_0(t)]^{\frac{1}{\gamma-1}}[W_t + k_1(t)z_{t,\det} + k_2(t)][1 - k_1(t)]^{\frac{1}{\gamma-1}} \\
&= bz_{t,\det} + \{\gamma k_0(t)[1 - k_1(t)]\}^{\frac{1}{\gamma-1}}[W_t + k_1(t)z_{t,\det} + k_2(t)]
\end{aligned} \tag{4.21}$$

where $k_0(t)$, $k_1(t)$, and $k_2(t)$ are as in Proposition 2. Given this optimal withdrawals the dynamics of the optimal withdrawals is then given by the following Proposition.

Proposition 4 *The dynamics of the optimal withdrawals is given by*

$$d\mathcal{P}^*_{t,\det} = m(.) + W_t w^*_{t,\det} \sigma d\zeta_t \tag{4.22}$$

where

$$
\begin{aligned}
m(.) &= \left\{ b + k_1(t)[\gamma k_0(t)[1-k_1(t)]]^{\frac{1}{\gamma-1}} \right\} dz_{t,\det} \\
&+ \left\{ \begin{array}{l} \frac{\gamma}{\gamma-1} J^{\frac{1}{\gamma-1}} [\gamma k_0(t)[1-k_1(t)]]^{\frac{2-\gamma}{\gamma-1}} [k_0'(t) - k_0(t)k_1'(t) + \gamma k_0(t) k_1(t)] \\ +[\gamma k_0(t)[1-k_1(t)]]^{\frac{1}{\gamma-1}} \left[(W_t w_{t,\det}^* \alpha - \mathcal{P}_{t,\det}^* + k_1(t) z_{t,\det}) + k_2'(t) \right] \end{array} \right\} dt
\end{aligned}
$$

Proof. See Appendix.

Proposition 3 explicitly emphasizes the point made earlier and shows that the volatility of withdrawals dynamics in our model (compared to Constantinides, 1990, and Sundaresan, 1989) depends on the initial position of the weight a_0 and its long run average weight $\bar{a}$ assigned to past withdrawals. As before assume that mean reverting pull, κ is near zero. Then, if a_0 is smaller (larger) than $\bar{a}$, the manager invests less (more) in risky assets causing a smoother (more volatile) withdrawals pattern than that in benchmark models of Constantinides (1990) and Sundaresan (1989). In other words, when the manager invests less (more) in risky production process than in the benchmark models, i.e. $w_{t,\det}^* \le w_t^*$ ($w_{t,\det}^* \ge w_t^*$), the volatility of the withdrawals pattern, $W_t w_{t,\det}^* \sigma d\zeta_t$, becomes smaller (larger) than $W_t w_t^* \sigma d\zeta_t$, and consequently, results in a less (more) volatile withdrawals pattern.[26]

4.3.4 Contingent Claim Prices

The value of any contingent claim (including stock and bonds) will depend on the

[26]In Constantinides (1990), and Sundaresan (1989), the weight assigned to past history is a constant. One could think of our time varying weight a_t given to past payouts as asymptotically converging to this constant, which is $\bar{a}$ in our setting.

firm's retained earnings W_t, the exponentially weighted past withdrawals $z_{t,\det}$, and time t. Let $C_{\det}(W_t, z_{t,\det}, t)$ represent the value of the contingent claim. Then a straightforward application of Ito's lemma (suppressing the arguments) shows that

$$dC_{\det} = uC_{\det}dt + C_{W,\det}W_t w^*_{t,\det}\sigma d\zeta_t \tag{4.23}$$

where

$$\begin{aligned} uC_{\det} = {} & C_{t,\det} + C_{W,\det}[W_t w^*_{t,\det}\alpha - \mathcal{P}^*_{t,\det}] + \frac{1}{2}C_{WW,\det}W_t^2(w^*_{t,\det})^2\sigma^2 \\ & + C_{z,\det}\left\{z_0 e^{-a_t t}[e^{-\kappa t}(a_0 - \bar{a})(\kappa t - 1)] + \mathcal{P}^*_{t,\det} - \bar{a}z_{t,\det}\right\} \end{aligned} \tag{4.24}$$

In addition, the expected rate of return of any contingent claim must satisfy Merton's ICAPM relation (see CIR, 1985a)

$$u = r_{t,\det} + \frac{W}{A}cov(\frac{dC_{\det}}{C_{\det}}, \frac{dW_t}{W_t}) - \frac{H}{A}cov(\frac{dC_{\det}}{C_{\det}}, dz_{t,\det}) \tag{4.25}$$

where $r_{t,\det}$ is the equilibrium interest rate, $A = \frac{-J_W}{J_{WW}}$, and $H = \frac{-J_{Wz}}{J_{WW}}$. Substituting equations (4.7), (4.9), and (4.23) in (4.25) yields

$$\begin{aligned} u &= r_{t,\det} + \frac{W_t}{A}E_t[(\frac{dC_{\det}}{C_{\det}})(\frac{dW_t}{W_t})] - \frac{H}{A}E_t[(\frac{dC_{\det}}{C_{\det}})(dz_{t,\det})] \\ &= r_{t,\det} + \frac{W_t}{A}E_t[(\frac{C_{W,\det}}{C_{\det}}W_t w^*_{t,\det}\sigma d\zeta_t)(\frac{1}{W_t}W_t w^*_{t,\det}\sigma d\zeta_t)] \\ &\quad - \frac{H}{A}E_t\left\{\begin{array}{l} (\frac{C_{W,\det}}{C_{\det}}W_t w^*_{t,\det}\sigma d\zeta_t). \\ \left[z_0 e^{-a_t t}[e^{-\kappa t}(a_0 - \bar{a})(\kappa t - 1)] + \mathcal{P}^*_{t,\det} - \bar{a}z_{t,\det}\right] \end{array}\right\} \\ &= r_{t,\det} + \frac{1}{A}\frac{C_{W\det}}{C_{\det}}W_t^2(w^*_{t,\det})^2\sigma^2 \end{aligned}$$

Implying

$$uC_{\det} = r_{t,\det} C_{\det} + \frac{1}{A} C_{W,\det} W_t^2 (w_{t,\det}^*)^2 \sigma^2 \tag{4.26}$$

Equating equation (4.24) and (4.26) yields the partial differential equation

$$\begin{aligned} 0 = {} & C_{t,\det} - r_{t,\det} C_{\det} + C_{W,\det}[W_t w_{t,\det}^* \alpha - \mathcal{P}_{t,\det}^* - \frac{1}{A} W_t^2 (w_{t,\det}^*)^2 \sigma^2] \\ & + \frac{1}{2} C_{WW,\det} W_t^2 (w_{t,\det}^*)^2 \sigma^2 \\ & + C_{z,\det} \left\{ z_0 e^{-a_t t} [e^{-\kappa t}(a_0 - \bar{a})(\kappa t - 1)] + \mathcal{P}_{t,\det}^* - \bar{a} z_{t,\det} \right\} \end{aligned} \tag{4.27}$$

The above equation (4.27) admits the following solution (see CIR, 1985a).

$$C_{\det}(W, z_{\det}, t, T) = E_t \left[\int_t^T \delta(W_s, z_{s,\det}, s) \left(\frac{J_W(W_s, z_{s,\det}, s)}{J_W(W_t, z_{t,\det}, t)} \right) ds \right]$$

where $\delta(W_s, z_{s,\det}, s)$ are the cash flows at future time *s*, and the stochastic discount factor $\frac{J_W(W_s, z_{s,\det}, s)}{J_W(W_t, z_{t,\det}, t)}$ is given by

$$\begin{aligned} \frac{J_W(W_s, z_{s,\det}, s)}{J_W(W_t, z_{t,\det}, t)} = {} & \left[\exp\left(-\int_t^s r_{u,\det}(W_u, z_{u,\det}, u) du \right) \right]. \\ & \left[\exp\left(\int_t^s (-\psi' \Pi) d\zeta_u - \frac{1}{2} \int_t^s |\psi' \Pi|^2 du \right) \right] \end{aligned} \tag{4.28}$$

where

$$\psi' = \left[\left(\frac{-J_{WW}}{J_W} \right) \; \left(\frac{-J_{Wz}}{J_W} \right) \right] \qquad \Pi = \begin{bmatrix} w_{t,\det}^* \sigma W \\ 0 \end{bmatrix} \tag{4.29}$$

Note that there are two terms in equation(4.28). The first term concerns the interest rate that has to be endogenously determined, and the second term is a function of the market price of risk intrinsic to the contingent claim.[27] Given the conjectured solution (4.15) and its associated coefficients and the optimal portfolio weight in equation (4.20), we compute the market price of risk,

$$\begin{aligned}\psi'\Pi &= \left(\frac{-J_{WW}}{J_W}\right) w^*_{t,\det} \sigma W_t \\ &= \frac{\alpha}{\sigma}\end{aligned} \qquad (4.30)$$

We observe that the market price of risk and thus the second term of equation (4.28) is the same for economies in which the manager does or does not smooth withdrawals (in other words, is or is not affected by past history of withdrawals). This is because we do not introduce any additional risk in our weighting of past habits. But this does not imply that the price of the contingent claim will be unaffected; there is also the impact of past withdrawals habits on the endogenously derived interest rates $r^*_{t,\det}$ that has to be assessed. We turn to this next.

4.3.5 Equilibrium Interest Rate

Proposition 5 *It follows from Theorem 1 of CIR (1985a), the equilibrium interest rate for an economy in which the manager whose investment-withdrawals decisions are influenced by past history, is given by*

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[27] Note that we, at this stage, introduce no additional sources of risk compared to the CIR (1985a), the Constantinides (1990), and Sundaresan (1989) types of model. Therefore, the market price of risk in our setting will be the same as in the other models. However, we show in the next subsection that the interest rate may be different across these models. This will, in turn, induce different prices for contingent claims compared to the prices in their settings. Over long time, our equilibrium interest will converge to the constant rates in Constantinides (1990) and Sundaresan (1989).

$$r^*_{t,\det} = \frac{\alpha^2}{\sigma^2} - \frac{1}{2}\left(\frac{\gamma-1}{\gamma-2}\right)\frac{\alpha^2}{\sigma^2} - \frac{\bar{a}}{\bar{a}-b}h(1-\gamma) + \frac{(1-\gamma)\left\{\left[\frac{b}{b-\bar{a}} - k_1(t)\right]z_0 e^{-a_t t}(a_t - \bar{a})(\kappa t - 1)\right\}}{[W_t + k_1(t)z_{t,\det} + k_2(t)]} \tag{4.31}$$

Proof. See Appendix.

The equilibrium interest rate in Proposition 5 can be compared to an economy where the weight given to the history of past withdrawals is a constant, that is to say, $a_t = \bar{a}$ (as in benchmark models of Constantinides, 1990, or Sundaresan, 1989). When mean reverting pull, κ, or time, t, is very large, it is easily seen that the equilibrium interest rate is a constant, not time varying (once again as in Constantinides, 1990, or Sundaresan, 1989). For the case κ is near zero (very small), and the manager assigns less weight to history, i.e., $(a_t > \bar{a})$, history matters less, and the managers are more aggressive in investing in the risky technology. Therefore the economy as whole becomes more risky, which results in a higher equilibrium interest rate than that in the benchmark models. Conversely, $(a_t < \bar{a})$, the manager cares more about history, becomes more conservative in investing in risky technology, and the equilibrium interest rate is lower that in the benchmark model.

Given that the market price of risk $\psi'\Pi$ in equation (4.30) in our model is the same as that in the benchmark model, the comparison of the contingent claims prices in our model with the benchmark model depends on the equilibrium interest rate $r^*_{t,\det}$. If the equilibrium interest rate $r^*_{t,\det}$ is higher (lower) in our model, then the equilibrium current price of the contingent claim will be lower (higher) than that in the benchmark model. This suggests that the firms with more (less) smooth streams will command higher (lower) prices of their contingent claims. These results are consistent with Trueman and Titman (1988) and Goel and Thakor (2003) where stock prices for firms with smoother earnings stream are higher. While the results are similar to ours, the reasons are different. In their models the result follows from the fact that investors dislike volatile earnings streams because they increase information risk. In our model, the time dependent weight a_t makes the earnings stream more or less volatile depending on whether it is greater than or less than $\bar{a}$. Thus the prices of contingent claims written on these firms could be lower or higher than that in the benchmark model.

4.4 Habit Process with both Deterministic and Stochastic Components

4.4.1 The Habit Process

In this section, we turn our attention to the case where the history dependent withdrawals are subject to random shocks. This can happen, for example, when the managerial environment changes, or there are shocks to the firm's workforce, etc. To capture these realities, we model the history of withdrawals with the stochastic component as

$$z_{t,stoch} = z_0 e^{-a_t t} + \int_0^t e^{-a_u(t-u)} \mathcal{P}_{u,stoch} du + e^{\sigma_z B_t} \tag{4.32}$$

where the weight assigned to past withdrawals is defined as in equation (4.5) and B_t is a standard Brownian motion which is not correlated with ζ_t . We introduce the stochastic component as an exponential function to ensure that habit persistence remains positive.

When B_t assumes very negative values, equation (4.32) converges to the lower bound equation (4.4) assumed in the earlier case with deterministic habit process (which will be the benchmark model for this case).[28]

Given equation(4.32), the dynamics for the habit level of withdrawals is given by

$$dz_{t,stoch} = \left[z_0 e^{-a_t t}(\kappa t - 1)(a_t - \bar{a}) + \mathbf{P}_{t,stoch} - \bar{a} z_{t,stoch} + \frac{1}{2}\sigma_z^2 e^{\sigma_z B_t} \right] dt \qquad (4.33)$$
$$+ \sigma_z e^{\sigma_z B_t} dB_t$$

It can be immediately verified that when $\sigma_z = 0$, $dz_{t,stoch}$ simplifies to equation (4.7) which is the habit formation process of only deterministic term. Furthermore when t (or κ) goes to infinity and $\sigma_z = 0$, the habit process in (4.33) becomes

$$\begin{aligned}
\lim_{t\to\infty} dz_{t,stoch} &= -\bar{a} z_0 e^{-\bar{a}t} + \mathcal{P}_{t,stoch} - \bar{a}[z_{t,stoch} - z_0 e^{-\bar{a}t}] \\
&= \mathcal{P}_{t,stoch} - \bar{a} z_0 e^{-\bar{a}t} - \bar{a} \int_0^t e^{-\bar{a}(t-s)} \mathcal{P}_{s,stoch} ds \\
&= \mathcal{P}_{t,stoch} - \bar{a}\left[z_0 e^{-\bar{a}t} + \int_0^t e^{-\bar{a}(t-s)} \mathcal{P}_{s,stoch} ds \right] \\
&= (\mathcal{P}_{t,stoch} - \bar{a} z_{t,stoch})
\end{aligned}$$

which is the dz_t process in Constantinides (1990), and Sundaresan (1989), with a_t replaced by $\bar{a}$.

4.4.2 The Manager's Optimization Program (the HJB Equation)

Following the earlier case, we once again write down the HJB equation for this optimization program and compute the first order conditions for the optimal withdrawals

[28]Since our objective in this paper is simply to show the possible effects of random changes in the weights given to past payout history and habits, this perturbation will suffice.

and investments in the risky production technology. Denote the discounted indirect utility function as $\hat{J}(W_t, z_{t,stoch}, t)$, the manager's optimization problem is again

$$\max_{\{\mathcal{P}_{u,stoch}, w_{u,stoch}\}} E_t\left[\int_t^\infty e^{-\rho u}\frac{[\mathcal{P}_{u,stoch} - bz_{u,stoch}]^\gamma}{\gamma}du\right] = e^{-\rho t}J(W_t, z_{t,stoch}, t) \tag{4.34}$$

subject to the intertemporal budget constraint in equation (4.9). Given the dz_t term in equation (4.33), we can derive the HJB equation,

$$0 = \max_{\{\mathcal{P}_{t,stoch}, w_{t,stoch}\}}[U(\mathcal{P}_{u,stoch}, z_{t,stoch}, W_t, t) + L[e^{-\rho t}J]] \tag{4.35}$$

where $U(\mathcal{P}_{t,stoch}, z_{t,stoch}, W_t, t) = e^{-\rho t}\frac{(\mathcal{P}_{t,stoch} - bz_t)^\gamma}{\gamma}$ and L, (the Dynkin operator) is given by

$$L[e^{-\rho t}J] = \left[\begin{array}{c} e^{-\rho t}\{J_W(W_t w_{t,stoch}\alpha - P_{t,stoch}) + J_z\left[\begin{array}{c} z_0 e^{-a_t t}(\kappa t - 1)(a_t - \bar{a}) \\ +\mathcal{P}_{t,stoch} - \bar{a}z_{t,stoch}\end{array}\right] \\ +\frac{1}{2}\sigma^2_{z,stoch}e^{\frac{1}{2}\sigma_z^2 t} + \frac{1}{2}J_{WW}(W_t w_{t,stoch}\sigma)^2 + \frac{1}{2}J_{zz}(\sigma^2_{z,stoch}e^{\sigma_z^2 t}) - \rho J + J_t\}\end{array}\right]$$

The first-order conditions of equation (4.35) with respect to the withdrawals $\mathcal{P}_{t,stoch}$ and the optimal portfolio weights $w_{t,stoch}$ are

$$0 = (\mathcal{P}_{t,stoch} - bz_{t,stoch})^{\gamma-1} - J_W + J_z$$
$$0 = J_W W_t\alpha + W_t^2 J_W w_{t,stoch}\sigma^2$$

implying

$$\mathbf{P}^*_{t,stoch} = bz_{t,stoch} + \left[J_W - J_z\right]^{\frac{1}{\gamma-1}} \tag{4.36}$$

$$w^*_{t,stoch} = \frac{-J_W}{J_{WW}W_t}\frac{\alpha}{\sigma^2} \tag{4.37}$$

The next Proposition provides an explicit solution for the optimal indirect utility function. Though the functional form of the solution looks similar to the earlier (deterministic) case, the coefficients are not the same.

Proposition 6 *Substituting equation (4.36) and (4.37) into equation (4.35) yields the equilibrium partial differential equation for the case of history dependent stochastic withdrawal process (the counterpart of the equation (4.14) i.e., the case of non-stochastic time varying weight changes in the history of withdrawals)*

$$\begin{aligned} 0 = {} & \frac{1-\gamma}{\gamma}[J_W - J_z]^{\frac{\gamma}{\gamma-1}} - \rho J + J_t - \frac{1}{2}\frac{J_W^2}{J_{WW}}\frac{\alpha^2}{\sigma^2} + bz_{t,stoch}(J_z - J_W) \\ & + J_z\left[z_0 e^{-a_t t}(\kappa t - 1)(a_t - \bar{a}) - \bar{a} z_{t,stoch} + \frac{1}{2}\sigma_z^2 e^{\frac{1}{2}\sigma_z^2 t} \right] \\ & + \frac{1}{2}J_{zz}\sigma_z^2 e^{\sigma_z^2 t} \end{aligned} \tag{4.38}$$

Assume $\bar{a} > b > 0$. *Also assume that* $\rho > \frac{1}{2}\frac{\alpha^2}{\sigma^2}\frac{\gamma}{1-\gamma}$ *.Then the solution for the equilibrium partial differential equation* (4.38)*, is given by*

$$J(W_t, z_{t,stoch}, t) = k_{0s}(t)\left[W_t + k_1(t) z_{t,stoch} + k_{2s}(t)\right]^{\gamma} \tag{4.39}$$

where

$$k_{0s}(t) = \frac{(\bar{a}-b)^2}{\gamma^2}\left\{ e^{(\frac{\bar{a}}{\bar{a}-b}h)t} \int_0^t e^{-(\frac{\bar{a}}{\bar{a}-b}h)u}\left[\bar{a} - be^{-(\bar{a}-b)u}\right]^{\frac{\gamma}{\gamma-1}} du \right\}^{1-\gamma} \tag{4.40}$$

$$k_1(t) = \left(\frac{b}{\bar{a}-b}\right)\left[1 - e^{-(\bar{a}-b)t}\right] \tag{4.41}$$

$$k_{2s}(t) = \left(\frac{-b}{\bar{a}-b}\right)\int_0^t \left[1 - e^{-(\bar{a}-b)u}\right]\left[z_0 e^{-a_u u}(a_u - \bar{a})(\kappa u - 1) + \frac{1}{2}\sigma_z^2 e^{\frac{1}{2}\sigma_z^2 u}\right] du \tag{4.42}$$

where

$$h = \frac{\bar{a}-b}{\bar{a}(1-\gamma)}\left(\rho - \frac{1}{2}\frac{\alpha^2}{\sigma^2}\frac{\gamma}{1-\gamma}\right)$$

Proof. See Appendix.

4.4.3 Optimal Investment

Once the indirect utility function is known, the optimal investment can be obtained explicitly using equation (4.37). The next Proposition provides the optimal investment for this case.

Proposition 7 *Given the coefficients* $k_{0s}(t)$ *,* $k_1(t)$ *and* $k_{2s}(t)$*, the optimal investment in the risky technology for the case of history dependent stochastic withdrawal process can be written as*

$$w^*_{t,stoch} = (\frac{1}{1-\gamma})\frac{\alpha}{\sigma^2}[1-h_{1s}(t)+h_{2s}(t)] \quad (4.43)$$

where $$h_{1s}(t) = \frac{z_{t,stoch}}{W_t}(\frac{b}{\bar{a}-b})\left[1-e^{-(\bar{a}-b)t}\right] \text{ and}$$

$$h_{2s}(t) = (\frac{-b}{\bar{a}-b})\int_0^t \left[1-e^{-(\bar{a}-b)u}\right]\left[z_0 e^{-a_u u}(\kappa u-1)(a_u-\bar{a})+\frac{1}{2}\sigma_z^2 e^{\frac{1}{2}\sigma_z^2 u}\right]du$$

When $\sigma_z = 0$, *the investment becomes equation (4.20), i.e. the optimal portfolio weight of the deterministic habit process.*

Proof. See Appendix.

Comparing the optimal investment in the risky production process for the case with non-stochastic time varying weight changes in the history of withdrawals, we note that $w^*_{t,stoch} \leq w^*_{t,\det}$. This is primarily due to the additional risk σ_z in the habit process here that makes $z_{t,stoch} \geq z_{t,\det}$. Therefore, the manager is more conservative and invests less in the risky assets when there is additional risk in the habit level of firm's withdrawals. We state and prove this in the next Corollary.

Corollary 3: $w^*_{t,stoch} \leq w^*_{t,\det}$.

Proof. Given a_u and κ , we can show that

$$\begin{aligned}
w^*_{t,stoch} &= \left[\left(\frac{1}{1-\gamma}\right)\frac{\alpha}{\sigma^2} \right] \left\{ \begin{array}{l} 1 - \frac{z_{t,stoch}}{W_t}\left(\frac{b}{\bar{a}-b}\right)[1 - e^{-(\bar{a}-b)t}] \\ -\left(\frac{b}{\bar{a}-b}\right)\int_0^t [1 - e^{-(\bar{a}-b)u}]\left[z_0 e^{-a_u u}(\kappa u - 1)(a_u - \bar{a}) + \frac{1}{2}\sigma_z^2 e^{\frac{1}{2}\sigma_z^2 u} \right] du \end{array} \right\} \\
&\leq \left[\left(\frac{1}{1-\gamma}\right)\frac{\alpha}{\sigma^2} \right] \left\{ \begin{array}{l} 1 - \frac{z_{t,\det}}{W_t}\left(\frac{b}{\bar{a}-b}\right)[1 - e^{-(\bar{a}-b)t}] \\ -\left(\frac{b}{\bar{a}-b}\right)\int_0^t [1 - e^{-(\bar{a}-b)u}][z_0 e^{-a_u u}(\kappa u - 1)(a_u - \bar{a})] du \end{array} \right\} \\
&= w^*_{t,\det}
\end{aligned}$$

4.4.4 Optimal Withdrawals and its Dynamics

After we have the optimal investment in the risky assets, we proceed to obtain the optimal withdrawals and its dynamics. The next Proposition provides this.

Proposition 8 *The optimal withdrawals in the case of history dependent stochastic withdrawal process has the same functional form as in the case of non-stochastic time varying weight changes in the history of withdrawals (4.21)*

$$\mathcal{P}^*_{t,stoch} = bz_{t,stoch} + \{\gamma k_{0s}(t)[1 - k_1(t)]\}^{\frac{1}{\gamma-1}} [W_t + k_1(t) z_{t,stoch} + k_{2s}(t)]$$

where $k_0(t)$, $k_1(t)$, *and* $k_2(t)$ *are those derived in equations (4.40), (4.41), and*

(4.42). The dynamics of the optimal withdrawals is then given by

$$d\mathcal{P}_{t,stoch} = m_1(.) + W_t w^*_{t,stoch}\sigma d\zeta_t + b\sigma_z e^{\sigma_z B_t}\left\{1 + k_1(t)[\gamma k_{0s}(t) - \gamma k_{0s}(t)k_1(t)]^{\frac{1}{\gamma-1}}\right\}dB_t$$

where

$$m_1(.) = m(.) + [\gamma k_{0s}(t)[1 - k_1(t)]]^{\frac{1}{\gamma-1}} k_1(t)\frac{1}{2}\sigma_z^2 e^{\sigma_z B_t}dt$$

Proof. See Appendix.

It can be see from the above Proposition that the dynamics of the withdrawals pattern is more volatile than that in the case of non-stochastic time varying weight changes in the history of withdrawals, because of the additional shock dB_t term. This results from the additional risk σ_z that prevents the managers from smoothing the withdrawals as in the deterministic case. This makes the withdrawals pattern more volatile. Naturally this would have an impact on the firms financial claims as we show next.

4.4.5 Contingent Claim Prices

The dynamics of the contingent claim whose value depends on the firm's retained earnings W_t, the exponentially weighted past withdrawals z_t, and time t is

$$\begin{aligned} dC_{stoch} &= C_{t,stoch}dt + C_{W,stoch}(dW) + C_{z,stoch}(dz_{t,stoch}) + \frac{1}{2}C_{WW,stoch}(dW)^2 \\ &\quad + \frac{1}{2}C_{z,stoch}(dz_{t,stoch})^2 + C_{Wz,stoch}(dW)(dz_{t,stoch}) \\ &= uC_{stoch}dt + C_{W,stoch}W_t w^*_{t,stoch}\sigma d\zeta_t + C_{z,stoch}\left(\sigma_z e^{\frac{1}{2}\sigma_z^2 t}\right)dB_t \end{aligned} \tag{4.44}$$

where

$$uC_{stoch} = C_{t,stoch} + C_{W,stoch}[W_t w^*_{t,stoch}\alpha - \mathcal{P}^*_{t,stoch}] + C_{z,stoch}\left[\begin{array}{l} \mathcal{P}^*_{t,stoch} + z_0 e^{-a_t t}(\kappa t - 1)(a_t - \bar{a}) \\ -\bar{a}z_{t,stoch} + \frac{1}{2}\sigma_z^2 e^{\frac{1}{2}\sigma_z^2 t} \end{array}\right]$$
$$+ \frac{1}{2}C_{WW,stoch}W_t^2(w^*_{t,stoch})^2\sigma^2 + \frac{1}{2}C_{z,stoch}\left(\sigma_z^2 e^{\frac{1}{2}\sigma_z^2 t}\right)$$

(4.45)

Recall that the expected rate of return of any contingent claim must satisfy Merton's ICAPM relation as

$$u = r_{t,stoch} + \frac{W_t}{A}cov(\frac{dC_{stoch}}{C_{stoch}}, \frac{dW_t}{W_t}) - \frac{H}{A}cov(\frac{dC_{stoch}}{C_{stoch}}, dz_{t,stoch}) \tag{4.46}$$

where $r_{t,stoch}$ is the equilibrium interest rate, $A = \frac{-J_W}{J_{WW}}$, and $H = \frac{-J_{Wz,stoch}}{J_{WW}}$. Substituting equations (4.9), (4.33), and (4.44) in (4.46) yields

$$u = r_{t,stoch} + \frac{W_t}{A}E_t[(\frac{dC_{stoch}}{C_{stoch}})(\frac{dW_t}{W_t})] - \frac{H}{A}E_t[(\frac{dC_{stoch}}{C_{stoch}})(dz_{t,stoch})]$$
$$= r_{t,stoch} + \frac{1}{A}E_t\left\{[(\frac{C_{W,stoch}}{C_{stoch}}W_t w^*_{t,stoch}\sigma d\zeta_t) + (\frac{C_{z,stoch}}{C_{stoch}}\sigma_z e^{\frac{1}{2}\sigma_z^2 t}dB_t)]\left[w^*_{t,stoch}\sigma d\zeta_t\right]\right\}$$
$$- \frac{H}{A}E_t\left\{\begin{array}{l} [(\frac{C_{W,stoch}}{C_{stoch}}W_t w^*_{t,stoch}\sigma d\zeta_t) + (\frac{C_{z,stoch}}{C_{stoch}}\sigma_z e^{\frac{1}{2}\sigma_z^2 t}dB_t)] \\ \left[\sigma_z e^{\frac{1}{2}\sigma_z^2 t}dB_t\right] \end{array}\right\}$$

implying

$$\begin{aligned} uC_{stoch} &= \frac{1}{A}E_t\left\{[(C_{W,stoch}W_t w^*_{t,stoch}\sigma d\zeta_t) + (C_{z,stoch}t\sigma_z a_t z_{t,stoch}dB_t)]\left[w^*_{t,stoch}\sigma d\zeta_t\right]\right\} \\ &\quad -\frac{H}{A}E_t\left\{[(C_W W_t w^*_{t,stoch}\sigma d\zeta_t) + (C_z\sigma_z e^{\frac{1}{2}\sigma_z^2 t}dB_t)]\left[\sigma_z e^{\frac{1}{2}\sigma_z^2 t}dB_t\right]\right\} \\ &= \frac{1}{A}E_t\left[C_W W_t(w^*_{t,stoch})^2\sigma^2 dt\right] - \frac{H}{A}E_t\left[C_z\sigma_z^2 e^{\frac{1}{2}\sigma_z^2 t}dt\right] \end{aligned} \tag{4.47}$$

Equating equation (4.45) and (4.47) yields

$$\begin{aligned} uC_{stoch} &= r_{t,stoch}C_{stoch} + \frac{1}{A}E_t\Big\{[(C_{W,stoch}W_t w^*_{t,stoch}\sigma d\zeta_t) + (C_{z,stoch}t\sigma_z a_t z_{t,stoch}dB_t)]\big[w^*_{t,stoch}\sigma d\zeta_t\big]\Big\} \\ &\quad - \frac{H}{A}E_t\Big\{[(C_W W_t w^*_{t,stoch}\sigma d\zeta_t) + (C_z\sigma_z e^{\frac{1}{2}\sigma_z^2 t}dB_t)]\big[\sigma_z e^{\frac{1}{2}\sigma_z^2 t}dB_t\big]\Big\} \\ &= r_{t,stoch}C_{stoch} + \frac{1}{A}E_t\big[C_W W_t(w^*_{t,stoch})^2\sigma^2 dt\big] - \frac{H}{A}E_t\big[C_z\sigma_z^2 e^{\frac{1}{2}\sigma_z^2 t}dt\big] \end{aligned}$$

(4.48)

The partial differential equation in (4.48) can be solved by the following valuation equation.

$$C(W, z_{t,stoch}, t, T) = E_t\left[\int_t^T \delta(W_s, z_{t,stoch}, s)\left(\frac{J_W(W_s, z_{t,stoch}, s)}{J_W(W_t, z_{t,stoch}, t)}\right)ds\right] \quad (4.49)$$

where $\delta(W_s, z_{t,stoch}, s)$ are the cash flows at future time s and the stochastic discount factor is

$$\begin{aligned} \frac{J_W(W_s, z_{t,stoch}, s)}{J_W(W_t, z_{t,stoch}, t)} &= \left[\exp\left(-\int_t^s r(W_u, z_{t,stoch}, u)du\right)\right] \\ &\quad \left[\exp\left(\int_t^s(-\psi'\Pi)d\zeta_u - \frac{1}{2}\int_t^s|\psi'\Pi|^2 du\right)\right] \end{aligned}$$

(4.50)

The market price of risk is

$$\begin{aligned} \psi' &= \left[\left(\frac{-J_{WW}}{J_W}\right)\ \left(\frac{-J_{Wz_{t,stoch}}}{J_W}\right)\right] \\ \Pi &= \begin{bmatrix} w^*_{t,stoch}\sigma W \\ \sigma_z e^{\frac{1}{2}\sigma_z^2 t} \end{bmatrix} \end{aligned} \qquad (4.51)$$

Given the conjectured solution (4.39) and its associated coefficients and the optimal portfolio weight in equation (4.43), we know that the market price of risk is equal to

$$\psi'\Pi = \frac{\alpha}{\sigma} + \frac{(1-\gamma)\sigma_z e^{\frac{1}{2}\sigma_z^2 t}}{[W_t + k_1(t)z_{t,stoch} + k_{2s}(t)]}$$

(4.52)

Comparing to the market price of risk when the habit process only contains the deterministic part, $\frac{\alpha}{\sigma}$, the market price of risk in equation (4.52) is higher which indicates that the economy as a whole becomes more risky. The equation (4.52) provides an explicit characterization of this additional risk caused by the randomness in the investment-withdrawals policies of the manager.

4.4.6 Equilibrium Interest Rate

As a logical progression, we characterize the endogenous interest rates in this economy with stochastic changes in the history of withdrawals and then we compare it to the earlier non-stochastic case.

Proposition 9 *Following the proof in equation (A18) in Appendix, the equilibrium interest rate for the case of history dependent stochastic withdrawal process has the following functional form.*

$$r^*_{t,stoch} = -\frac{L[J_W]}{J_W} \qquad (4.53)$$

and given the optimal portfolio weight $w^*_{t,stoch}$, *optimal withdrawals* $\mathcal{P}^*_{t,stoch}$ *, and the coefficients of the conjectured solution to the value function* J *, we showed that the equilibrium interest rate is.*

$$
\begin{aligned}
r^*_{t,stoch} &= \frac{\alpha^2}{\sigma^2} - \frac{1}{2}\left(\frac{\gamma-1}{\gamma-2}\right)\frac{\alpha^2}{\sigma^2} - \frac{\bar{a}}{\bar{a}-b}h(1-\gamma) \\
&+ \frac{(1-\gamma)\left\{\left[\frac{b}{b-\bar{a}} - k_1(t)\right]z_0 e^{-a_t t}(a_t-\bar{a})(\kappa t-1) + \frac{1}{2}\sigma_z^2 e^{\frac{1}{2}\sigma_z^2 t}\right\}}{[W_t + k_1(t)z_{t,stoch} + k_{2s}(t)]} \\
&+ \frac{\frac{1}{2}(1-\gamma)(2-\gamma)\left\{(\frac{b}{\bar{a}-b})[1-e^{-(\bar{a}-b)t}]\right\}^2 \sigma_z^2 e^{\frac{1}{2}\sigma_z^2 t}}{[W_t + k_1(t)z_{t,stoch} + k_{2s}(t)]^2}
\end{aligned}
$$

The equilibrium interest rate for the stochastic case is larger than that obtained for the of case of non-stochastic time varying weight changes in the history of withdrawals, equation (4.31). The random shock in the history of withdrawals results in more risky withdrawals pattern for the firm, and consequently, results in a more risky economy. The implication of the higher market price of risk and the equilibrium interest rate on the price of the contingent claim is the following. The price of the contingent claim $C_{stoch}(W,z,t,T)$ in equation (4.49) is lower than that in the case of non-stochastic time varying weight changes in the history of withdrawals. The additional risk in the habit process σ_z causes a lower stock price because firm's withdrawals cannot be smoothed exactly to the desired level. In other words, the less stable withdrawals stream results in lower security prices.

Proof. See Appendix.

4.5 Conclusions

In this paper, we have embedded the concept of real earnings management within an inter-temporal equilibrium production economy setting. We have provided a characterization of earnings management by introducing a representative manger whose decision to manage earnings depends on the past history of withdrawals from earnings. In our model managers are motivated to manage earnings because they share in the withdrawals from the earnings of the firm. This makes the managerial utility time inseparable and history dependent. A key innovation in the paper is to allow the weight given to the past withdrawals by managers to change randomly. This captures the idea that there could be shocks that influence managerial policies on withdrawals and re-investments of earnings to change unpredictably. We then solve for the optimal earnings management path we show how real earnings management with random shocks would affect all contingent claims (including stocks, bonds, and other claims) against the firm for a specific functional form of managerial utility. We also show how real earnings management could affect the economy wide interest rates. Specifically, we show how this extra risk from random changes to earnings management is priced. Explicit solutions are provided to show the incremental effects of real earnings management.

To the best of our knowledge, this is the first paper to have incorporated real earnings management in a dynamic general equilibrium setting. Future research could try and incorporate not just real earnings management, but also earnings management that have no direct impact on cash flows (cosmetic management of earnings). This may eventually lead to a theory of accruals within an equilibrium setting. In addition, it may be worthwhile to empirically investigate and isolate the value effects of real earnings management developed in this paper.

Chapter 5

Summary

My dissertation research focuses on the area of investment strategy, portfolio management, equity valuation models, and earnings management. I am interested in both the fundamental and technical information of the firms and their joint effect on equity valuation. These three essays in my dissertation present empirical evidences as well as theoretical development in investigating firm's fundamentals and their asset pricing implications. In my first essay entitled "Alternative Equity Valuation Models", we study the stock prices forecast ability of three alternative valuation models, namely the Ohlson (1995) Model, Feltham-Ohlson (1995) Model, and the Warren-Shelton (1971) Model. We have developed five research hypotheses to test whether different earnings measures, estimation techniques and combined forecasts methods can improve these models' ability in predicting future stock prices. Overall, we find that the simultaneous equations estimation procedure can produce more accurate future stock price forecasts than the traditional single equation estimation method in terms of smaller prediction errors. Our results also show that combined forecast method can further reduce the prediction errors by using combination of individual forecasts. We also find supporting evidence that investors can use comprehensive earnings to more accurately forecast future stock prices in these valuation models.

In my second essay entitled "Technical, Fundamental, and Combined Information for Winners and Losers", we use both fundamental and technical information to improve the technical momentum strategy. We have developed three hypotheses to test whether combined momentum strategy outperforms the technical momentum strategy or not. We construct our combined momentum strategy by incorporating the FSCORE (Piotroski, 2000) and GSCORE (Mohanram, 2005) system into the technical momentum strategy. From the empirical results of these three hypotheses, we conclude that the combined momentum strategy outperforms technical momentum strategy by generating significantly larger returns for both growth and value stocks. The larger profitability generated by the combined momentum strategy indicates that the composite fundamental scores can be used by investors to separate the best (worst) among the winners (losers) stocks. Our findings suggest that fundamental analysis indeed provides information to investors in addition to the technical information for selecting winner and loser stocks.

Finally, given the importance of firm's fundamental information in asset pricing, I investigate managers' behavior in reporting this information. In my third essay entitled "The Economic Consequences of Earnings Management and Smoothing", we focus on real earnings management in a general equilibrium production (GEP) economy setting, and studies the economic implications of this phenomenon on the economy as a whole. In our model, managers will tend to produce relatively smooth earnings streams compared to the benchmark model where managers do not smooth earnings, do not have earnings targets, and have no habits. Managers may forgo or avail of investment opportunities that they normally would not in the benchmark model. Moreover, smoothing and stochastic habit changes add extra risks in the economy that affect market prices of firms and economy wide interest rates. We explicitly solve for the endogenous asset prices, and interest rate, and show how this additional risk from managerial habits is priced in the production economy.

In summary, we consider our results contributing to the literature in asset pricing, portfolio management, financial statement analysis, and earnings management. I also believe that these findings are helpful for the security analysis and portfolio management to the investors.

Appendix A

The WS Model has four segments including twenty equations simultaneously determining twenty unknowns. The first segment is with respect to the generation of sales and earnings before interest and taxes (*EBIT*). Equation (A.1) shows that the sales for period t are simply the product of sales in the prior period multiplied by the growth rate in sales for the period t.

$$SALES_t = SALES_{t-1}\left(1 + GCALS_t\right) \quad \text{(A.1)}$$

where $SALES_t$ is the sales in period t and $GCALS_t$ is the rate of growth in sales from period t-1 to t. Equation (A.2) derive $EBIT_t$ in period t as a percentage of the sales in the same period.

$$EBIT_t = REBIT_t\left(SALES_t\right) \qquad \text{(A.2)}$$

where $REBIT_t$ is the operating income as percentage of sales. The second segment is concerning the generation of total assets required. Equations (A.3) and (A.4) use the ratios of current assets to sales RCA_t and the fixed assets to sales $RFA_t / SALES_t$ to calculate the current assets and fixed assets in period t.

$$CA_t = RCA_t\left(SALES_t\right) \quad \text{(A.3)}$$

$$FA_t = RFA_t\left(SALES_t\right) \quad \text{(A.4)}$$

Equation (A.5) states that the total asset in period t is determined by the sum of the current assets and the fixed assets in period t.

$$A_t = CA_t + FA_t \quad \text{(A.5)}$$

The third segment is concerning the financing the desired level of assets. Equation (A.6) derives the current liabilities in period t from the ratio of RCA_t and $SALES_t$.

$$CL_t = RCL_t\left(SALES_t\right) \quad \text{(A.6)}$$

Equation (A.7) is the new funds required for the firm (NF_t). Equation (A.7) shows that the assets for period t are the basis for financing needs. Current liabilities are therefore subtracted from asset levels because it is one source of funds. Moreover, preferred stock is constant and therefore must be subtracted as well. Also included in Equation (A.7) are the financing that must come from internal sources (retain earnings) and long-term external sources (debt and stock issues). Finally the funds provided by operations during period t must be subtracted to arrive at the new funds required for firm in period t.

$$\begin{aligned} NF_t = &\left(A_t - CL_t - PFDSK_t\right) - \left(L_{t-1} - LR_t\right) - S_{t-1} - R_{t-1} \\ &- b_t\left\{(1-T_t)\left[EBIT_t - i_{t-1}\left(L_{t-1} - LR_t\right)\right] - PFDIV_t\right\} \end{aligned} \quad \text{(A.7)}$$

where $PFDSK_t$ is the balance of the preferred stock issued, L_t is the total, LR_t is the debt repayment, S_t is the balance of common stock, R_t is the balance of retained earnings, b_t is retention rate, T_t is the effective tax rate, i_t is the average interest rate, and $PFDIV_t$ is the preferred stock dividends in period t. Equation (A.8) specifies that new funds, after taking into account underwriting costs and additional interest costs from new debt, are to come from long-term debt and new stock issues.

$$NF_t + b_t\left\{(1-T_t)\left(i_t^e NL_t + U_t^l NL_t\right) - PFDIV_t\right\} = NL_t + NS_t \qquad \text{(A.8)}$$

where i_t^e is the expected interest rate on new debt, NL_t is the new debt issued, NS_t is the new common stock issued, and U_t^l is the underwriting cost of debt in period t. Equations (A.9) updates the new debt issuance in period t while equation (A.10) updates the new common stock issuance.

$$L_t = L_{t-1} - LR_t + NL_t \qquad \text{(A.9)}$$

$$S_t = S_{t-1} + NS_t \qquad \text{(A.10)}$$

Equation (A.11) updates the retained earnings account by earnings available to common shares as a result of operations during period t.

$$R_t = R_{t-1} + b_t\left\{\left(1-T_t\right)\left[EBIT_t - i_t L_t - U_t^l NL_t\right] - PFDIV_t\right\} \qquad \text{(A.11)}$$

Equation (A.12) calculates the new weighted-average interest rate for the firm's debt.

$$i_t L_t = i_{t-1}\left(L_{t-1} - LR_t\right) + i_t^e NL_t \quad \text{(A.12)}$$

Equation (A.13) demonstrates that the firm wants to maintain the same debt-to-equity ratio K_{t} for period t.

$$L_t = \left(S_t + R_t\right) K_t \quad \text{(A.13)}$$

Finally the fourth segment in the WS Model generates per share data for period t. Equation (A.14) computes the earnings available for common dividends by using the complement of the retention rate multiplied by the earnings available for common dividends.

$$EAFCD_t = \left(1 - T_t\right)\left[EBIT_t - i_t L_t - U_t^l NL_t\right] - PFDIV_t \quad \text{(A.14)}$$

where $EAFCD_t$ is the earnings available for common dividends. Equation (A.15) calculates the common stock dividends available to the shareholders in period t.

$$CMDIV_t = \left(1 - b_t\right) EAFCD_t \quad \text{(A.15)}$$

where $CMDIV_t$ common dividends available for shareholders in period t. Equations (A.16) updates the number of common shares for new issues.

$$NUMCS_t = NUMCS_{t-1} + NEWCS_t \quad \text{(A.16)}$$

where $NUMCS_t$ is the number of common shares outstanding and $NEWCS_t$ is the new common shares issued in period t. Equation (A.17) shows that the number of new common shares issued is determined by the total amount of the new stock issue divided by the stock price after discounting for issuance costs.

$$NEWCS_t = \frac{NS_t}{\left(1 - U_t^s P_t\right)} \quad \text{(A.17)}$$

where U_t^s is the underwriting cost of common stock issuance and P_t is the market price per share for the common stock in period t. Equation (A.18) determines the price of the stock by multiplying the earnings per share with the exogenously determined price-earnings ratio m_t of the stock purchase.

$$P_t = m_t EPS_t \quad \text{(A.18)}$$

Equation (A.19) determines EPS_t by dividing earnings available for common by the number of common shares outstanding.

$$EPS_t = \frac{EAFCD_t}{NUMCS_t} \quad \text{(A.19)}$$

Finally equation (A.20) determines dividends per share by dividing the common dividends available for the shareholders by the number of shares outstanding in period t.

$$DPS_t = \frac{CMDIV_t}{NUMCS_t} \quad \text{(A.20)}$$

This system of equations is complete because twenty equations are used to solve for twenty unknowns.

Appendix B

Proof of Proposition 1: From equation (4.4) and (4.5), the habit formation is given by

$$z_{t,\det} = z_0 e^{-\left[a_0 e^{-\kappa t} + \bar{a}(1-e^{-\kappa t})\right]t} + \int_0^t e^{-\left[a_0 e^{-\kappa u} + \bar{a}(1-e^{-\kappa u})\right](t-u)} \mathcal{P}_{u,\det} du \quad \text{(B1)}$$

Differentiating with respect to t yields

$$\begin{aligned}\frac{dz_{t,\det}}{dt} &= z_0 e^{-[a_0 e^{-\kappa t} + \bar{a}(1-e^{-\kappa t})]t}[e^{-\kappa t}(a_0 - \bar{a})(\kappa t - 1) - \bar{a}] + \mathcal{P}_{t,\det} \\ &\quad + \int_0^t -a_0 e^{-\kappa u} e^{-[a_0 e^{-\kappa u} + \bar{a}(1-e^{-\kappa u})](t-u)} \mathcal{P}_{u,\det} du \\ &\quad + \int_0^t -\bar{a} e^{-[a_0 e^{-\kappa u} + \bar{a}(1-e^{-\kappa u})](t-u)} \mathcal{P}_{u,\det} du \\ &\quad + \int_0^t \bar{a} e^{-\kappa u} e^{-[a_0 e^{-\kappa u} + \bar{a}(1-e^{-\kappa u})](t-u)} \mathcal{P}_{u,\det} du \end{aligned} \quad \text{(B2)}$$

Noting from (B1) that

$$\begin{aligned}&\int_0^t e^{-[a_0 e^{-\kappa u} + \bar{a}(1-e^{-\kappa u})](t-u)} \mathcal{P}_{u,\det} du \\ &= z_{t,\det} - z_0 e^{-[a_0 e^{-\kappa t} + \bar{a}(1-e^{-\kappa t})]t}\end{aligned}$$

and integrating by parts shows that

$$\int_0^t e^{-\kappa u} e^{-\left[a_0 e^{-\kappa u} + \bar{a}(1-e^{-\kappa u})\right](t-u)} \mathcal{P}_{u,\det} du = 0 \quad \text{(B3)}$$

Using(B3), equation (B1) reduces to

$$\begin{aligned}\frac{dz_{t,\det}}{dt} &= z_0 e^{-[a_0 e^{-\kappa t} + \bar{a}(1-e^{-\kappa t})]t}[e^{-\kappa t}(a_0 - \bar{a})(\kappa t - 1) - \bar{a}] + \mathcal{P}_{t,\det} \\ &\quad - \bar{a}[z_t - z_0 e^{-[a_0 e^{-\kappa t} + \bar{a}(1-e^{-\kappa t})]t}] \\ &= z_0 e^{-a_t t}[e^{-\kappa t}(a_0 - \bar{a})(\kappa t - 1)] + \mathcal{P}_{t,\det} - \bar{a} z_t \end{aligned} \quad \text{(B4)}$$

When t or κ tends infinity in equation (B4)

$$\lim_{t\to\infty}\frac{dz_{t,\det}}{dt}=\mathbf{P}_{t,\det}-\bar{a}z_{t,\det}$$

$$\lim_{\kappa\to\infty}\frac{dz_{t,\det}}{dt}=\mathbf{P}_{t,\det}-\bar{a}z_{t,\det}$$

which recovers the dynamics of the habit dynamics dz_t term in Constantinides (1990).

Proof of Proposition 2: The conjectured solution in equation (4.15) implies that

$$\begin{aligned}
J_t &= k_0'(t)[W_t+k_1(t)z_{t,\det}+k_2(t)]^{\gamma} \\
&\quad +\gamma k_0(t)[W_t+k_1(t)z_{t,\det}+k_2(t)]^{\gamma-1}[k_1'(t)z_{t,\det}+k_2'(t)] \\
J_W &= \gamma k_0(t)[W_t+k_1(t)z_{t,\det}+k_2(t)]^{\gamma-1} \\
J_{WW} &= \gamma(\gamma-1)k_0(t)[W_t+k_1(t)z_{t,\det}+k_2(t)]^{\gamma-2} \\
J_z &= \gamma k_0(t)k_1(t)[W_t+k_1(t)z_{t,\det}+k_2(t)]^{\gamma-1}
\end{aligned}$$

Plug these back into the equilibrium PDE in equation (4.14), multiply by $[W_t+k_1(t)z_{t,\det}+k_2(t)]^{1-\gamma}$, collecting terms and simplifying yields

$$\begin{aligned}
0 &= W_t\left\{\frac{1-\gamma}{\gamma}[\gamma k_0(t)(1-k_1(t))]^{\frac{\gamma}{\gamma-1}}-\frac{1}{2}\frac{\alpha^2}{\sigma^2}\frac{\gamma}{\gamma-1}k_0(t)-\rho k_0+k_0'(t)\right\} \\
&\quad + z_{t,\det}\left\{\begin{array}{l}\frac{1-\gamma}{\gamma}[\gamma k_0(t)(1-k_1(t))]^{\frac{\gamma}{\gamma-1}}k_1(t)-\frac{1}{2}\frac{\alpha^2}{\sigma^2}\frac{\gamma}{\gamma-1}k_0(t)k_1(t) \\ -\rho k_0(t)k_1(t)-b\gamma k_0(t)-b\gamma k_0(t)k_1(t)+\gamma k_0(t)k_1(t)\bar{a} \\ +\gamma k_0(t)k_1'(t)+k_0'(t)k_1(t)\end{array}\right\} \\
&\quad + \left\{\begin{array}{l}\frac{1-\gamma}{\gamma}[\gamma k_0(t)(1-k_1(t))]^{\frac{\gamma}{\gamma-1}}k_2(t)+\gamma k_0(t)k_2'(t)-\frac{1}{2}\frac{\alpha^2}{\sigma^2}\frac{\gamma}{\gamma-1}k_0(t)k_2(t) \\ +\gamma k_0(t)k_1(t)z_0e^{-a_t t}[(a_t-\bar{a})(\kappa t-1)]-\rho k_0(t)k_2(t)+k_0'(t)k_2(t)\end{array}\right\}
\end{aligned} \tag{B5}$$

Equation (B5) implies that each term in curly brackets must be equate to zero, that is,

$$0=\frac{1-\gamma}{\gamma}\left[\gamma k_0(t)(1-k_1(t))\right]^{\frac{\gamma}{\gamma-1}}-\frac{1}{2}\frac{\alpha^2}{\sigma^2}\frac{\gamma}{\gamma-1}k_0(t)-\rho k_0+k_0'(t) \tag{B6}$$

$$0=\begin{Bmatrix} \frac{1-\gamma}{\gamma}\left[\gamma k_0(t)(1-k_1(t))\right]^{\frac{\gamma}{\gamma-1}} k_1(t)-\frac{1}{2}\frac{\alpha^2}{\sigma^2}\frac{\gamma}{\gamma-1}k_0(t)k_1(t)-\rho k_0(t)k_1(t) \\ -b\gamma k_0(t)-b\gamma k_0(t)k_1(t)+\gamma k_0(t)k_1(t)\overline{a}+\gamma k_0(t)k_1^{'}(t)+k_0^{'}(t)k_1(t) \end{Bmatrix} \qquad \text{(B7)}$$

$$0=\begin{Bmatrix} \frac{1-\gamma}{\gamma}\left[\gamma k_0(t)(1-k_1(t))\right]^{\frac{\gamma}{\gamma-1}} k_2(t)+\gamma k_0(t)k_2^{'}-\frac{1}{2}\frac{\alpha^2}{\sigma^2}\frac{\gamma}{\gamma-1}k_0(t)k_2(t) \\ +\gamma k_0(t)k_1(t)z_0e^{-a_t t}\left[\left(a_t-\overline{a}\right)\left(\kappa t-1\right)\right]-\rho k_0(t)k_2(t)+k_0^{'}(t)k_2(t) \end{Bmatrix} \qquad \text{(B8)}$$

To solve for $k_0(t)$, $k_1(t)$, and $k_2(t)$ multiply equation (B6) by $k_1(t)$ and subtract from equation (k1), and rearrange to obtain the ordinary differential equation.

$$k_1^{'}(t)+k_1(t)\left(\overline{a}-b\right)-b=0 \qquad \text{(B9)}$$

To solve for $k_1(t)$, let $\mu_1(t)$ denote the integrating factor. Multiply both sides of equation (B9) by $\mu_1(t)$ to get

$$\mu_1(t)k_1^{'}(t)+\mu_1(t)k_1(t)\left(\overline{a}-b\right)=\mu_1(t)b \qquad \text{(B10)}$$

implying

$$d[\mu_1(t)k_1(t)] = b\mu_1(t)$$

Thus the integrating factor $\mu_1(t)$ is

$$\mu_1(t) = e^{(\bar{a}-b)t}$$

From which we obtain

$$\begin{aligned} k_1(t) &= e^{-(\bar{a}-b)t}\int_0^t be^{(\bar{a}-b)s}ds \\ &= \left(\frac{b}{\bar{a}-b}\right)[1-e^{-(\bar{a}-b)t}] \end{aligned}$$

Plugging $k_1(t)$ in equation (B8) and using equation (B6) gives

$$k_2^{\prime}(t)+k_1(t)[z_0e^{-a_t t}(a_t-\bar{a})(\kappa t-1)] = 0$$

We can then solve for $k_2(t)$ as

$$k_2(t) = \left(\frac{-b}{\bar{a}-b}\right)\int_0^t [1-e^{-(\bar{a}-b)s}][z_0 e^{-a_s s}(a_s-\bar{a})(\kappa s-1)]ds$$

Plug $k_1(t)$ in equation (B6) yields

$$k_0'(t) + k_0(t)^{\frac{\gamma}{\gamma-1}}(\frac{1-\gamma}{\gamma})\gamma^{\frac{\gamma}{\gamma-1}}[1-k_1(t)]^{\frac{\gamma}{\gamma-1}} - k_0(t)\left(-\frac{1}{2}\frac{\alpha^2}{\sigma^2}\frac{\gamma}{1-\gamma}+\rho\right) = 0$$

Let $v(t) = k_0(t)^{-\frac{1}{\gamma-1}}$ and

$$v'(t) = (\frac{1}{1-\gamma})k_0(t)^{\frac{-\gamma}{\gamma-1}}k_0'(t) \qquad \text{(B11)}$$

Multiplying both sides of equation (B10) by $k_0(t)^{\frac{-\gamma}{\gamma-1}}$ and using equation (B11) yields

$$v'(t)(1-\gamma) + v(t)\left[\frac{1}{2}\frac{\alpha^2}{\sigma^2}\frac{\gamma}{1-\gamma}+\rho\right] + \gamma^{\frac{1}{\gamma-1}}[1-k_1(t)]^{\frac{\gamma}{\gamma-1}} = 0 \qquad \text{(B12)}$$

Let $\mu_2(t)$ be the integrating factor to the above ODE. Multiplying both sides of equation (B12) by this integrating factor and rearranging gives

$$v'(t)\mu_2(t) + \mu_2(t)v(t)\left[\frac{-\frac{1}{2}\frac{\alpha^2}{\sigma^2}\frac{\gamma}{\gamma-1}-\rho}{1-\gamma}\right] = \mu_2(t)\left[\gamma^{\frac{1}{\gamma-1}}[1-k_1(t)]^{\frac{\gamma}{\gamma-1}}\right] \qquad \text{(B13)}$$

implying

$$\mu_2'(t) = \mu_2(t)\left[\frac{-\frac{1}{2}\frac{\alpha^2}{\sigma^2}\frac{\gamma}{\gamma-1}-\rho}{1-\gamma}\right]$$

Thus the integrating factor $\mu_2(t)$ is

$$\mu_2(t) = \exp\left\{\int\left[\frac{-\frac{1}{2}\frac{\alpha^2}{\sigma^2}\frac{\gamma}{\gamma-1}-\rho}{1-\gamma}\right]dt\right\}$$
$$= e^{-(\frac{\bar{a}}{\bar{a}-b}h)t}$$

where h is as defined in equation (4.19). Plugging $\mu_2(t)$ in equation (B13) yields

$$v(t) = e^{(\frac{\bar{a}}{\bar{a}-b}h)t}\int_0^t e^{-(\frac{\bar{a}}{\bar{a}-b}h)u}\left[\gamma^{\frac{1}{\gamma-1}}[1-k_1(u)]^{\frac{\gamma}{\gamma-1}}\right]du$$
$$= e^{(\frac{\bar{a}}{\bar{a}-b}h)t}\int_0^t e^{-(\frac{\bar{a}}{\bar{a}-b}h)u}\left[\gamma^{\frac{1}{\gamma-1}}\left[1-\left(\frac{b}{\bar{a}-b}\right)[1-e^{-(\bar{a}-b)u}]\right]^{\frac{\gamma}{\gamma-1}}\right]du$$

and so

$$k_0(t) = \frac{1}{\gamma}\left\{e^{(\frac{\bar{a}}{\bar{a}-b}h)t}\int_0^t e^{-(\frac{\bar{a}}{\bar{a}-b}h)u}\left[\gamma^{\frac{1}{\gamma-1}}\left[1-\left(\frac{b}{\bar{a}-b}\right)[1-e^{-(\bar{a}-b)u}]\right]^{\frac{\gamma}{\gamma-1}}\right]du\right\}^{1-\gamma}$$

Proof of Proposition 3: We know from equation (4.13)

$$\begin{aligned} w^*_{t,\det} &= \frac{-J_W}{J_{WW}W_t}\frac{\alpha}{\sigma^2} \\ &= (\frac{-1}{\gamma-1})\left[W_t + k_1(t)z_{t,\det} + k_2(t)\right]\frac{1}{W_t}\frac{\alpha}{\sigma^2} \\ &= \left[(\frac{1}{1-\gamma})\frac{\alpha}{\sigma^2}\right]\left\{\begin{array}{l} 1-\frac{z_{t,\det}}{W_t}(\frac{b}{\bar{a}-b})\left[1-e^{-(\bar{a}-b)t}\right] \\ -(\frac{b}{\bar{a}-b})\int_0^t\left[1-e^{-(\bar{a}-b)u}\right]\left[z_0 e^{-a_u u}(\kappa u-1)(a_u-\bar{a})\right]du \end{array}\right\} \end{aligned} \tag{B14}$$

Proof of Corollary 1: When the mean reverting force κ is small, the weight assigned to the past withdrawals by the manager takes a long time to converge to its long run equilibrium $\bar{a}$. In the extreme case, set $\kappa = 0$ and we know that the weight assigned to the past withdrawals becomes

$$
\begin{aligned}
a_t &= a_0 e^{-\kappa t} + (1 - e^{-\kappa t})\bar{a} \\
&= a_0
\end{aligned}
$$

which is a constant. We first examine the case where the initial position of the weights a_0 is smaller than its long run average $\bar{a}$ (this can be considered the constant used in Constantinides, 1990). Given that $a_0 < \bar{a}$, the habit formation z_t in equation (4.4) becomes

$$
\begin{aligned}
z_{t,\det} &= z_0 e^{-a_0 t} + \int_0^t e^{-a_0(t-u)} \mathcal{P}_u du \\
&> z_0 e^{-\bar{a}t} + \int_0^t e^{-\bar{a}(t-u)} \mathcal{P}_u du \\
&= \bar{z}_t
\end{aligned}
$$

where $\bar{z}_t$ denote the habit formation when $a_t = \bar{a}$ (the long term equilibrium constant). Furthermore, the optimal portfolio weight in equation (B14) becomes

$$
\begin{aligned}
w^*_{t,\det} &= \left[(\frac{1}{1-\gamma}) \frac{\alpha}{\sigma^2} \right] \left\{ \begin{array}{l} 1 - \frac{z_{t,\det}}{W_t} \left(\frac{b}{\bar{a}-b} \right) [1 - e^{-(\bar{a}-b)t}] \\ - \left(\frac{b}{\bar{a}-b} \right) \int_0^t [1 - e^{-(\bar{a}-b)s}][z_0 e^{-a_s s}(a_s - \bar{a})(\kappa s - 1)] ds \end{array} \right\} \\
&\leq \left[(\frac{1}{1-\gamma}) \frac{\alpha}{\sigma^2} \right] \left[1 - \frac{\bar{z}_t}{W_t} \left(\frac{b}{\bar{a}-b} \right) \right] \\
&= \overline{w}^*_t
\end{aligned}
$$

where the second equality follows from the fact that when mean reverting force κ is 0, the weight assigned to past withdrawals is equal to the constant weight as in

Constantinides (1990). When $a_0 > \bar{a}$ then the reverse is true. In summary, the optimal portfolio weight in our model is larger (smaller) than that in Constantinides (1990) if a_0 is smaller (larger) than its long run average $\bar{a}$. When a_0 is smaller (larger) than $\bar{a}$, the manager is more conservative (adventurous) and investing less (more) in risky assets.

When the mean reverting force κ is very large, the weight assigned to the past withdrawals by the manager converge to its long run equilibrium $\bar{a}$ instantly. In the extreme case, set $\kappa = \infty$ and we know that the weight assigned to the past withdrawals becomes

$$\begin{aligned} a_t &= a_0 e^{-\kappa t} + (1 - e^{-\kappa t})\bar{a} \\ &= \bar{a} \end{aligned}$$

Given that $a_t = \bar{a}$, the habit formation $z_{t,\det}$ in equation (4.4) collapse back to that under the benchmark model Constantinides (1990)

$$\begin{aligned} z_{t,\det} &= z_0 e^{-a_0 t} + \int_0^t e^{-a_0(t-u)} \mathcal{P}_u du \\ &= z_0 e^{-\bar{a} t} + \int_0^t e^{-\bar{a}(t-u)} \mathcal{P}_u du \\ &= \bar{z}_t \end{aligned}$$

which leads the optimal portfolio weight $w^*_{t,\det}$ collapse back to the one under the benchmark model in Constantinides (1990)

$$w_{t,\det}^* = \left[(\frac{1}{1-\gamma})\frac{\alpha}{\sigma^2} \right] \left\{ \begin{array}{l} 1 - \frac{z_{t,\det}}{W_t}\left(\frac{b}{\bar{a}-b}\right)[1 - e^{-(\bar{a}-b)t}] \\ -\left(\frac{b}{\bar{a}-b}\right)\int_0^t [1 - e^{-(\bar{a}-b)s}][z_0 e^{-a_s s}(\bar{a}-\bar{a})(\kappa s - 1)]ds \end{array} \right\}$$

$$= \left[(\frac{1}{1-\gamma})\frac{\alpha}{\sigma^2} \right] \left\{ 1 - \frac{\bar{z}_t}{W_t}\left(\frac{b}{\bar{a}-b}\right)[1 - e^{-(\bar{a}-b)t}] \right\}$$

$$= \overline{w}_t^*$$

Proof of Proposition 4: Applying Ito's lemma, the optimal withdrawals $\mathcal{P}_{t,\det}^*$ gives the following.

$$d\mathcal{P}_{t,\det}^* = b(dz_{t,\det}) + d\left\{ [\gamma k_0(t)[1-k_1(t)]]^{\frac{1}{\gamma-1}} [W_t + k_1(t)z_{t,\det} + k_2(t)] \right\}$$

$$= b\left[\mathcal{P}_{t,\det}^* + z_0 e^{-a_t t}(\kappa t - 1)(a_t - \bar{a}) - \bar{a}z_{t,\det} \right]dt$$

$$+ \left\{ \begin{array}{l} d[\gamma k_0(t) - \gamma k_0(t)k_1(t)]^{\frac{1}{\gamma-1}} [W_t + k_1(t)z_{t,\det} + k_2(t)] \\ +d[W_t + k_1(t)z_{t,\det} + k_2(t)][\gamma k_0(t)[1-k_1(t)]]^{\frac{1}{\gamma-1}} \end{array} \right\}$$

$$= b(dz_{t,\det})$$

$$+ \left\{ \begin{array}{l} \frac{\gamma}{\gamma-1}[\gamma k_0(t) - \gamma k_0(t)k_1(t)]^{\frac{2-\gamma}{\gamma-1}} \\ [k_0'(t) - k_0(t)k_1'(t) + \gamma k_0(t)k_1(t)][W_t + k_1(t)z_{t,\det} + k_2(t)]dt \\ +[\gamma k_0(t)[1-k_1(t)]]^{\frac{1}{\gamma-1}} \\ \left\{ \begin{array}{l} \left[W_t w_{t,\det}^* \alpha - \mathcal{P}_{t,\det}^* \right]dt + W_t w_{t,\det}^* \sigma d\zeta_t + k_1'(t)z_{t,\det}dt \\ +k_1(t)(dz_{t,\det}) + k_2'(t)dt \end{array} \right\} \end{array} \right\}$$

$$= \left\{ b + k_1(t)[\gamma k_0(t)[1-k_1(t)]]^{\frac{1}{\gamma-1}} \right\} dz_{t,\det}$$

$$+ \left\{ \begin{array}{l} \frac{\gamma}{\gamma-1} J^{\frac{1}{\gamma-1}} [\gamma k_0(t)[1-k_1(t)]]^{\frac{2-\gamma}{\gamma-1}} [k_0'(t) - k_0(t)k_1'(t) + \gamma k_0(t)k_1(t)] \\ +[\gamma k_0(t)[1-k_1(t)]]^{\frac{1}{\gamma-1}} \left[(W_t w_{t,\det}^* \alpha - \mathcal{P}_{t,\det}^* + k_1(t)z_{t,\det}) + k_2'(t) \right] \end{array} \right\} dt$$

$$+ W_t w_{t,\det}^* \sigma d\zeta_t$$

$$= m(.) + W_t w_{t,\det}^* \sigma d\zeta_t$$

where

$$m(.) = \left\{ b + k_1(t)[\gamma k_0(t)[1 - k_1(t)]]^{\frac{1}{\gamma-1}} \right\} dz_{t,\det}$$
$$+ \left\{ \begin{array}{l} \frac{\gamma}{\gamma-1} J^{\frac{1}{\gamma-1}} [\gamma k_0(t)[1-k_1(t)]]^{\frac{2-\gamma}{\gamma-1}} [k_0'(t) - k_0(t)k_1'(t) + \gamma k_0(t)k_1(t)] \\ +[\gamma k_0(t)[1-k_1(t)]]^{\frac{1}{\gamma-1}} \left[(W_t w_{t,\det}^* \alpha - \mathcal{P}_{t,\det}^* + k_1(t) z_{t,\det}) + k_2'(t) \right] \end{array} \right\} dt$$

Proof of Proposition 5: Rewrite the dynamics of wealth in equation (4.9) as

$$dW_t = [W_t w_t^* (\alpha - r) + rW_t - \mathbf{P}_t]dt + W_t w_t^* \sigma d\zeta_t \qquad \text{(B15)}$$

Given the rewritten dynamics of wealth in equation (B15), we can derive the dt term of the value function J, $L[J]$, as

$$L[J] = J_t + J_W[W_t w_{t,\det}^* (\alpha - r) + rW_t - \mathcal{P}_{t,\det}] + \frac{1}{2} J_{WW} W_t^2 w_{t,\det}^* \sigma^2$$
$$+ J_z[z_0 e^{-a_t t}[e^{-\kappa t}(a_0 - \bar{a})(\kappa t - 1)] + \mathcal{P}_{t,\det} - \bar{a} z_{t,\det}] \qquad \text{(B16)}$$

The first-order conditions of $L[J]$ with respect to the portfolio weights w_t^* are

$$0 = J_W W_t (\alpha - r) + J_{WW} W_t^2 w_{t,\det}^* \sigma^2 \qquad \text{(B17)}$$

Differentiating the Bellman equation (4.11) with the rewritten wealth dynamics in equation (B15) with respect wealth W yields

$$0 = J_{Wt} + J_{WW}\left[W_t w_{t,\det}^* (\alpha - r) - \mathcal{P}_{t,\det} \right] + rJ_W + \frac{1}{2} J_{WWW} W_t^2 w_{t,\det}^* \sigma^2$$
$$+ J_{Wz}[z_0 e^{-a_t t}[e^{-\kappa t}(a_0 - \bar{a})(\kappa t - 1)] + \mathcal{P}_{t,\det} - \bar{a} z_{t,\det}]$$

where the last equality follows from the first order condition in equation (B17). Therefore, the interest rate can be written as

$$
\begin{aligned}
r &= -\left\{\begin{array}{l} J_{Wt} + J_{WW}\left[W_t w^*_{t,\det}(\alpha - r) - \mathbf{P}_{t,\det}\right] \\ + J_{WW} W_t w^*_{t,\det}\sigma^2 + \tfrac{1}{2} J_{WWW} W_t^2 w^*_{t,\det}\sigma^2 \\ + J_{Wz}\left[z_0 e^{-a_t t}\left[e^{-\kappa t}\left(a_0 - \bar{a}\right)\left(\kappa t - 1\right)\right] + \mathbf{P}_{t,\det} - \bar{a} z_{t,\det}\right] \end{array}\right\} / J_W \\
&= -\frac{L\left[J_W\right]}{J_W}
\end{aligned}
\qquad \text{(B18)}
$$

The conjectured solution of $J = k_0(t) \quad [W_t + k_1(t) z_{t,\det} + k_2(t)]$ implies that

$$
\begin{aligned}
J_W &= \gamma k_0(t)[W_t + k_1(t) z_{t,\det} + k_2(t)]^{\gamma - 1} \\
J_{Wt} &= \gamma k_0'(t)[W_t + k_1(t) z_{t,\det} + k_2(t)]^{\gamma - 1} \\
&\quad + \gamma(\gamma - 1) k_0(t)[W_t + k_1(t) z_{t,\det} + k_2(t)]^{\gamma - 2}[k_1'(t) z_{t,\det} + k_2'(t)] \\
J_{WW} &= \gamma(\gamma - 1) k_0(t)[W_t + k_1(t) z_{t,\det} + k_2(t)]^{\gamma - 2} \\
J_{WWW} &= \gamma(\gamma - 1)(\gamma - 2) k_0(t)[W_t + k_1(t) z_{t,\det} + k_2(t)]^{\gamma - 3} \\
J_{Wz} &= \gamma(\gamma - 1) k_0(t) k_1(t)[W_t + k_1(t) z_{t,\det} + k_2(t)]^{\gamma - 2}
\end{aligned}
$$

Plug the above partial derivatives in equation (B18) yields

$$
\begin{aligned}
r^*_{t,\det} &= -(\gamma-1)k_2'(t)[W_t + k_1(t)z_{t,\det} + k_2(t)]^{-1} + \frac{\alpha^2}{\sigma^2} \\
&\quad - (\gamma-1)k_1'(t)z_t[W_t + k_1(t)z_{t,\det} + k_2(t)]^{-1} \\
&\quad + (\gamma-1)[W_t + k_1(t)z_{t,\det} + k_2(t)]^{-1}\{bz_{t,\det} + h[W_t + k_1(t)z_{t,\det} + k_2(t)]\} \\
&\quad - \frac{1}{2}\left(\frac{\gamma-1}{\gamma-2}\right)\frac{\alpha^2}{\sigma^2} \\
&\quad - (\gamma-1)[W_t + k_1(t)z_{t,\det} + k_2(t)]^{-1}\left(\frac{b}{\bar{a}-b}\right) \\
&\quad \{z_0 e^{-a_t t}[(a_t-\bar{a})(\kappa t-1)] + bz_{t,\det} + h[W_t + k_1(t)z_{t,\det} + k_2(t)] - \bar{a}z_{t,\det}\} \\
&= \frac{\alpha^2}{\sigma^2} - \frac{1}{2}\left(\frac{\gamma-1}{\gamma-2}\right)\frac{\alpha^2}{\sigma^2} - \frac{\bar{a}}{\bar{a}-b}h(1-\gamma) \\
&\quad + (1-\gamma)[W_t + k_1(t)z_{t,\det} + k_2(t)]^{-1}\left\{\begin{array}{c} -k_1(t)z_0 e^{-a_t t}(a_t-\bar{a})(\kappa t-1) \\ -\left(\frac{b}{\bar{a}-b}\right)z_0 e^{-a_t t}(a_t-\bar{a})(\kappa t-1) \end{array}\right\} \\
&= \frac{\alpha^2}{\sigma^2} - \frac{1}{2}\left(\frac{\gamma-1}{\gamma-2}\right)\frac{\alpha^2}{\sigma^2} - \frac{\bar{a}}{\bar{a}-b}h(1-\gamma) \\
&\quad + \frac{(1-\gamma)\left\{\left[\frac{b}{b-\bar{a}} - k_1(t)\right]z_0 e^{-a_t t}(a_t-\bar{a})(\kappa t-1)\right\}}{[W_t + k_1(t)z_{t,\det} + k_2(t)]}
\end{aligned}
$$

When the mean reverting force κ is very large, the weight assigned to the past withdrawals by the manager converge to its long run equilibrium $\bar{a}$ instantly. In the extreme case, set $\kappa = \infty$ and we know that the weight assigned to the past withdrawals becomes

$$
\begin{aligned}
a_t &= a_0 e^{-\kappa t} + (1-e^{-\kappa t})\bar{a} \\
&= \bar{a}
\end{aligned}
$$

The equilibrium interest rate collapses to that under the benchmark model in Constantinides (1990)

$$
\begin{aligned}
r^*_{t,\det} &= \frac{\alpha^2}{\sigma^2} - \frac{1}{2}\left(\frac{\gamma-1}{\gamma-2}\right)\frac{\alpha^2}{\sigma^2} - \frac{\bar{a}}{\bar{a}-b}h(1-\gamma) \\
&\quad + \frac{(1-\gamma)\left[\left(\frac{-b}{\bar{a}-b}\right)z_0 e^{-a_t t}(\bar{a}-\bar{a})(\kappa t-1)\right]}{[W_t + k_1(t)z_t + k_2(t)]} \\
&= \frac{\alpha^2}{\sigma^2} - \frac{1}{2}\left(\frac{\gamma-1}{\gamma-2}\right)\frac{\alpha^2}{\sigma^2} - \frac{\bar{a}}{\bar{a}-b}h(1-\gamma)
\end{aligned}
$$

Proof of Proposition 6: The conjectured solution in equation (4.39) implies that

$$
\begin{aligned}
J_t &= k'_{0s}(t)[W_t + k_{1s}(t)z_{t,stoch} + k_{2s}(t)]^{\gamma} \\
&\quad + \gamma k_{0s}(t)[W_t + k_{1s}(t)z_{t,stoch} + k_{2s}(t)]^{\gamma-1}[k'_{1s}(t)z_{t,stoch} + k'_{2s}(t)] \\
J_W &= \gamma k_{0s}(t)[W_t + k_{1s}(t)z_{t,stoch} + k_{2s}(t)]^{\gamma-1} \\
J_{WW} &= \gamma(\gamma-1)k_{0s}(t)[W_t + k_{1s}(t)z_{t,stoch} + k_{2s}(t)]^{\gamma-2} \\
J_z &= \gamma k_{0s}(t)k_1(t)[W_t + k_{1s}(t)z_{t,stoch} + k_{2s}(t)]^{\gamma-1} \\
J_{zz} &= \gamma(\gamma-1)k_{0s}(t)k_{1s}(t)[W_t + k_{1s}(t)z_{t,stoch} + k_{2s}(t)]^{\gamma-2}
\end{aligned}
$$

Plug these back into the equilibrium partial differential equation in (4.38) yields

$$
\begin{aligned}
0 = &\frac{1-\gamma}{\gamma}\left\{\gamma k_{0s}(t)\left[W_t + k_{1s}(t)z_{t,stoch} + k_{2s}(t)\right]^{\gamma-1}\left[1-k_{1s}(t)\right]\right\}^{\frac{\gamma}{\gamma-1}} \\
&+k'_{0s}(t)\left[W_t + k_{1s}(t)z_{t,stoch} + k_{2s}(t)\right]^{\gamma} \\
&+\gamma k_{0s}(t)\left[W_t + k_{1s}(t)z_{t,stoch} + k_{2s}(t)\right]^{\gamma-1}\left[k'_{1s}(t)z_{t,stoch} + k'_{2}\right] \\
&-\frac{1}{2}\frac{\alpha^2}{\sigma^2}\frac{\gamma}{\gamma-1}k_{0s}(t)\left[W_t + k_{1s}(t)z_{t,stoch} + k_{2s}(t)\right]^{\gamma} \\
&+bz_{t,stoch}\left\{\gamma k_{0s}(t)\left[W_t + k_{1s}(t)z_{t,stoch} + k_{2s}(t)\right]^{\gamma-1}\left[k_{1s}(t)-1\right]\right\} \\
&+\gamma k_{0s}(t)k_{1s}(t)\left[W_t + k_{1s}(t)z_{t,stoch} + k_{2s}(t)\right]^{\gamma-1} \\
&\left[z_0 e^{-a_t t}(\kappa t-1)(a_t-\bar{a}) - \bar{a}z_{t,stoch} + \frac{1}{2}\sigma_z^2 e^{\frac{1}{2}\sigma_z^2 t}\right] \\
&+\frac{1}{2}\sigma_z^2 e^{\sigma_z^2 t}\gamma(\gamma-1)k_{0s}(t)k_{1s}^2(t)\left[W_t + k_{1s}(t)z_{t,stoch} + k_{2s}(t)\right]^{\gamma-2} \\
&-\rho k_{0s}(t)\left[W_t + k_{1s}(t)z_{t,stoch} + k_{2s}(t)\right]^{\gamma}
\end{aligned}
\qquad \text{(B19)}
$$

Multiply both sides of equation (B19) by $[W_t + k_{1s}(t)z_{t,stoch} + k_{2s}(t)]^{2-\gamma}$ and collecting terms yields

$$
\begin{aligned}
0 = {} & \frac{1-\gamma}{\gamma}\{\gamma k_{0s}(t)[1-k_{1s}(t)]\}^{\frac{\gamma}{\gamma-1}}[W_t + k_{1s}(t)z_{t,stoch} + k_{2s}(t)]^2 \\
& + \gamma k_{0s}(t)[W_t + k_{1s}(t)z_{t,stoch} + k_{2s}(t)][k'_{1s}(t)z_{t,stoch} + k'_{2s}(t)] \\
& + k'_{0s}(t)[W_t + k_{1s}(t)z_{t,stoch} + k_{2s}(t)]^{\gamma} \\
& - \frac{1}{2}\frac{\alpha^2}{\sigma^2}\frac{\gamma}{\gamma-1}k_{0s}(t)[W_t + k_{1s}(t)z_{t,stoch} + k_{2s}(t)]^2 \\
& + bz_{t,stoch}\gamma k_{0s}(t)[k_{1s}(t)-1][W_t + k_{1s}(t)z_{t,stoch} + k_{2s}(t)] \\
& + \gamma k_{0s}(t)k_{1s}(t)[W_t + k_{1s}(t)z_{t,stoch} + k_{2s}(t)] \\
& \left[z_0 e^{-a_t t}(\kappa t - 1)(a_t - \bar{a}) - \bar{a}z_{t,stoch} + \frac{1}{2}\sigma_z^2 e^{\frac{1}{2}\sigma_z^2 t}\right] \\
& + \frac{1}{2}\sigma_z^2 e^{\sigma_z^2 t}\gamma(\gamma-1)k_{0s}(t)k'_{1s}(t) \\
& - \rho k_{0s}(t)[W_t + k_{1s}(t)z_{t,stoch} + k_{2s}(t)]^2
\end{aligned}
$$

implying

$$0 = W_t^2 f_1(t) + W_t f_2(t) + z_{t,stoch}f_3(t) + z_{t,stoch}^2 f_4(t) + W_t z_{t,stoch} f_4(t) + f_5(t) \qquad \text{(B20)}$$

where

$$
\begin{aligned}
f_1(t) &= \left(\frac{1-\gamma}{\gamma}\right)\{\gamma k_{0s}(t)[1-k_{1s}(t)]\}^{\frac{\gamma}{\gamma-1}} - \frac{1}{2}\frac{\alpha^2}{\sigma^2}\frac{\gamma}{\gamma-1}k_{0s}(t) - \rho k_{0s}(t) + k'_{0s}(t) \\
f_2(t) &= \begin{aligned}[t] & 2k_{2s}(t)f_1(t) + \gamma k_{0s}(t)k'_{2s}(t) \\ & + \gamma k_{0s}(t)k_{1s}(t)\left[z_0 e^{-a_t t}(\kappa t - 1)(a_t - \bar{a}) + \tfrac{1}{2}\sigma_z^2 e^{\frac{1}{2}\sigma_z^2 t}\right] \end{aligned} \\
f_3(t) &= k_{2s}(t)f_5(t) + k_{1s}(t)f_2(t) - 2k_{2s}(t)f_1(t) \\
f_4(t) &= k_{1s}(t)f_5(t) - k_{1s}^2(t)f_1(t) \\
f_5(t) &= 2k_{1s}(t)f_1(t) - b\gamma k_{0s}(t) - b\gamma k_{0s}(t)k_{1s}(t) + \bar{a}\gamma k_{0s}(t)k_{1s}(t) + \gamma k_{0s}(t)k'_{1s}(t) \\
f_6(t) &= (\gamma-1)k_{2s}(t)f_2(t) - (\gamma-1)k_2^2(t)f_1(t)
\end{aligned}
$$

In order to hold the equality of equation (sshabitpde3), it must be that

$$0 = f_1(t) \quad \text{(B21)}$$

$$0 = f_2(t) \quad \text{(B22)}$$

$$0 = f_5(t) \quad \text{(B23)}$$

To solve for the coefficients of the conjectured solution, first note that from equations (B21) and (B23) we know that

$$0 = \gamma k_{0s}(t)k'_{1s}(t) - b\gamma k_{0s}(t) - b\gamma k_{0s}(t)k_{1s}(t) + \bar{a}\gamma k_{0s}(t)k_{1s}(t)$$

implying

$$k'_{1s}(t) + k_{1s}(t)\left(\bar{a} - b\right) = b \qquad \text{(B24)}$$

which yields the same solution as in the deterministic habit process as shown in equation (k).

$$\begin{aligned} k_{1s}(t) &= \left(\frac{b}{\bar{a} - b}\right)[1 - e^{-(\bar{a}-b)t}] \\ &= k_1(t) \end{aligned}$$

Substitute $k_1(t)$ in equation (B22) and use equation (B21) to get

$$0 = \gamma k_{0s}(t)k'_{2s}(t) - \gamma k_{0s}(t)k_1(t)\left[z_0 e^{-a_t t}(a_t - \bar{a})(\kappa t - 1) + \frac{1}{2}\sigma_z^2 e^{\frac{1}{2}\sigma_z^2 t}\right]$$

Solving for $k_{2s}(t)$ yields

$$k_{2s}(t) = \left(\frac{-b}{\bar{a} - b}\right)\int_0^t [1 - e^{-(\bar{a}-b)u}]\left[z_0 e^{-a_u u}(a_u - \bar{a})(\kappa u - 1) + \frac{1}{2}\sigma_z^2 e^{\frac{1}{2}\sigma_z^2 u}\right]du$$

We note that when $\sigma_z = 0$, $k_{2s}(t)$ simplifies to

$$k_{2s}(t) = \left(\frac{-b}{\bar{a}-b}\right)\int_0^t [1-e^{-(\bar{a}-b)u}][z_0 e^{-a_u u}(a_u-\bar{a})(\kappa u-1)]du$$

which corresponds to the $k_{2s}(t)$ coefficient in the conjectured solution of the deterministic habit formation.

Substituting $k_1(t)$ and $k_{2s}(t)$ in equation (B21) yields

$$k_{0s}'(t) + k_{0s}(t)^{\frac{\gamma}{\gamma-1}}(\frac{1-\gamma}{\gamma})\gamma^{\frac{\gamma}{\gamma-1}}\left[1-k_1(t)\right]^{\frac{\gamma}{\gamma-1}} - k_{0s}(t)\left(-\frac{1}{2}\frac{\alpha^2}{\sigma^2}\frac{\gamma}{1-\gamma}+\rho\right)=0 \qquad \text{(B25)}$$

from which $k_{0s}(t)$ can be solved (following the procedure in the deterministic habit process),

$$\begin{aligned} k_{0s}(t) &= \frac{1}{\gamma}\left\{e^{(\frac{\bar{a}}{\bar{a}-b}h)t}\int_0^t e^{-(\frac{\bar{a}}{\bar{a}-b}h)u}\left[\gamma^{\frac{1}{\gamma-1}}\left[1-\left(\frac{b}{\bar{a}-b}\right)[1-e^{-(\bar{a}-b)u}]\right]^{\frac{\gamma}{\gamma-1}}\right]du\right\}^{1-\gamma} \\ &= \frac{(\bar{a}-b)^2}{\gamma^2}\left\{e^{(\frac{\bar{a}}{\bar{a}-b}h)t}\int_0^t e^{-(\frac{\bar{a}}{\bar{a}-b}h)u}[\bar{a}-be^{-(\bar{a}-b)u}]^{\frac{\gamma}{\gamma-1}}du\right\}^{1-\gamma} \end{aligned}$$

Proof of Proposition 7: Plug the derived coefficients $k_{0s}(t)$, $k_1(t)$, and $k_{2s}(t)$ in equation (4.37) and the optimal portfolio weight can written as

$$\begin{aligned} w^*_{t,stoch} &= \frac{-J_W}{J_{WW}W_t}\frac{\alpha}{\sigma^2} \\ &= \left[(\frac{1}{1-\gamma})\frac{\alpha}{\sigma^2}\right] \\ &\quad \left\{\begin{array}{l} 1-\frac{z_{t,stoch}}{W_t}\left(\frac{b}{\bar{a}-b}\right)[1-e^{-(\bar{a}-b)t}] \\ -(\frac{b}{\bar{a}-b})\int_0^t[1-e^{-(\bar{a}-b)u}]\left[z_0e^{-a_u u}(\kappa u-1)(a_u-\bar{a})+\frac{1}{2}\sigma_z^2 e^{\frac{1}{2}\sigma_z^2 u}\right]du \end{array}\right\} \end{aligned}$$

Proof of Proposition 8: From Ito's lemma

$$
\begin{aligned}
d\mathcal{P}^*_{t,stoch} &= b(dz_{t,stoch}) + d\left\{[\gamma k_{0s}(t)[1-k_1(t)]]^{\frac{1}{\gamma-1}}[W_t + k_1(t)z_{t,stoch} + k_{2s}(t)]\right\} \\
&= b\left[z_0 e^{-a_t t}(\kappa t - 1)(a_t - \bar{a}) + \mathcal{P}^*_{t,stoch} - \bar{a}z_{t,stoch} + \frac{1}{2}\sigma_z^2 e^{\sigma_z B_t}\right]dt + b\sigma_z e^{\sigma_z B_t} dB_t \\
&\quad + \left\{\begin{array}{l} d[\gamma k_{0s}(t) - \gamma k_{0s}(t)k_1(t)]^{\frac{1}{\gamma-1}}[W_t + k_1(t)z_{t,stoch} + k_{2s}(t)] \\ +d[W_t + k_1(t)z_{t,stoch} + k_{2s}(t)][\gamma k_{0s}(t)[1-k_1(t)]]^{\frac{1}{\gamma-1}} \end{array}\right\} \\
&= b\left[z_0 e^{-a_t t}(\kappa t - 1)(a_t - \bar{a}) + \mathcal{P}^*_{t,stoch} - \bar{a}z_{t,stoch} + \frac{1}{2}\sigma_z^2 e^{\sigma_z B_t}\right]dt + b\sigma_z e^{\sigma_z B_t} dB_t \\
&\quad + \left\{\begin{array}{l} \frac{\gamma}{\gamma-1}[\gamma k_{0s}(t) - \gamma k_{0s}(t)k_1(t)]^{\frac{2-\gamma}{\gamma-1}} \\ [k'_{0s}(t) - k_{0s}(t)k'_1(t) + \gamma k_{0s}(t)k_1(t)][W_t + k_1(t)z_{t,stoch} + k_{2s}(t)]dt \\ +[\gamma k_{0s}(t)[1-k_1(t)]]^{\frac{1}{\gamma-1}} \\ \left\{\begin{array}{l} \left[W_t w_t^* \alpha - \mathcal{P}^*_{t,stoch}\right]dt + W_t w_t^* \sigma d\zeta_t + k'_1(t)z_{t,stoch}dt + k'_{2s}(t)dt \\ +k_1(t)\left[z_0 e^{-a_t t}(\kappa t - 1)(a_t - \bar{a}) + \mathcal{P}^*_{t,stoch} - \bar{a}z_{t,stoch} + \frac{1}{2}\sigma_z^2 e^{\sigma_z B_t}\right]dt \\ +k_1(t)b\sigma_z e^{\sigma_z B_t} dB_t \end{array}\right\} \end{array}\right\} \\
&= b\left[z_0 e^{-a_t t}(\kappa t - 1)(a_t - \bar{a}) + \mathcal{P}^*_{t,stoch} - \bar{a}z_{t,stoch} + \frac{1}{2}\sigma_z^2 e^{\sigma_z B_t}\right]dt \\
&\quad + \left\{\begin{array}{l} \frac{\gamma}{\gamma-1}[\gamma k_{0s}(t) - \gamma k_{0s}(t)k_1(t)]^{\frac{2-\gamma}{\gamma-1}} \\ [k'_{0s}(t) - k_{0s}(t)k'_1(t) + \gamma k_{0s}(t)k_1(t)][W_t + k_1(t)z_{t,stoch} + k_{2s}(t)] \\ +[\gamma k_{0s}(t)[1-k_1(t)]]^{\frac{1}{\gamma-1}} \\ \left\{\begin{array}{l} W_t w^*_{t,stoch}\alpha + [k_1(t) - 1]\mathcal{P}^*_{t,stoch} + [k'_1(t) - k_1(t)\bar{a}]z_{t,stoch} \\ +k_1(t)z_0 e^{-a_t t}(\kappa t - 1)(a_t - \bar{a}) + k'_{2s}(t) + k_1(t)\frac{1}{2}\sigma_z^2 e^{\sigma_z B_t} \end{array}\right\} \end{array}\right\} dt \\
&\quad + W_t w^*_{t,stoch}\sigma d\zeta_t \\
&\quad + b\sigma_z e^{\sigma_z B_t}\left\{1 + bk_1(t)[\gamma k_{0s}(t)[1-k_1(t)]]^{\frac{1}{\gamma-1}}\right\}dB_t \\
&= m_1(.) + W_t w^*_{t,stoch}\sigma d\zeta_t + b\sigma_z e^{\sigma_z B_t}\left\{1 + k_1(t)[\gamma k_{0s}(t) - \gamma k_{0s}(t)k_1(t)]^{\frac{1}{\gamma-1}}\right\}dB_t
\end{aligned}
$$

where

$$m_1(.) = m(.) + [\gamma k_{0s}(t)[1 - k_1(t)]]^{\frac{1}{\gamma-1}} k_1(t)\frac{1}{2}\sigma_z^2 e^{\sigma_z B_t} dt$$

Proof of Proposition 9: Given the derived value function J, we know the functional form of the interest rate is

$$\begin{aligned} r^*_{t,stoch} &= -\frac{J_{Wt}}{J_W} + \frac{\alpha^2}{\sigma^2} + \left[\frac{J_{WW}}{J_W} - \frac{J_{Wz}}{J_W}\right] bz_{t,stoch} \\ &+ \left[\frac{J_{WW}}{J_W} - \frac{J_{Wz}}{J_W}\right][J_W - J_z]^{\frac{1}{\gamma-1}} - \frac{1}{2}\frac{J_{WWW}J_W}{J_{WW}^2}\frac{\alpha^2}{\sigma^2} \\ &- \frac{J_{Wz}}{J_W}\left[\mathcal{P}^*_{t,stoch} + z_0 e^{-a_t t}(\kappa t - 1)(a_t - \bar{a}) - \bar{a}z_{t,stoch} + \frac{1}{2}\sigma_z^2 e^{\frac{1}{2}\sigma_z^2 t}\right] \\ &- \frac{1}{2}\frac{J_{Wzz}}{J_W}\left[\sigma_z^2 e^{\frac{1}{2}\sigma_z^2 t}\right] \end{aligned}$$

where

$$\begin{aligned} J_W &= \gamma k_{0s}(t)[W_t + k_1(t)z_{t,stoch} + k_{2s}(t)]^{\gamma-1} \\ J_{Wt} &= \gamma(\gamma-1)k_{0s}(t)k'_{2s}(t)[W_t + k_1(t)z_{t,stoch} + k_{2s}(t)]^{\gamma-2} \\ J_{WW} &= \gamma(\gamma-1)k_{0s}(t)[W_t + k_1(t)z_{t,stoch} + k_{2s}(t)]^{\gamma-2} \\ J_{WWW} &= \gamma(\gamma-1)(\gamma-2)k_{0s}(t)[W_t + k_1(t)z_{t,stoch} + k_{2s}(t)]^{\gamma-3} \\ J_{Wz} &= \gamma(\gamma-1)k_{0s}(t)k_1(t)[W_t + k_1(t)z_{t,stoch} + k_{2s}(t)]^{\gamma-2} \\ J_{Wzz} &= \gamma(\gamma-1)(\gamma-2)k_{0s}(t)k_1^2(t)[W_t + k_1(t)z_{t,stoch} + k_{2s}(t)]^{\gamma-3} \end{aligned}$$

Plug the coefficients $k_{0s}(t)$, $k_1(t)$, and $k_{2s}(t)$ derived previously yields the following.

$$
\begin{aligned}
r^*_{t,stoch} &= -(\gamma-1)k'_{2s}(t)[W_t + k_1(t)z_{t,stoch} + k_{2s}(t)]^{-1} + \frac{\alpha^2}{\sigma^2} \\
&\quad - (\gamma-1)k'_1(t)z_{t,stoch}[W_t + k_1(t)z_{t,stoch} + k_{2s}(t)]^{-1} \\
&\quad + (\gamma-1)[W_t + k_1(t)z_{t,stoch} + k_{2s}(t)]^{-1}\{bz_{t,stoch} + h[W_t + k_1(t)z_{t,stoch} + k_{2s}(t)]\} \\
&\quad - \frac{1}{2}\left(\frac{\gamma-1}{\gamma-2}\right)\frac{\alpha^2}{\sigma^2} \\
&\quad - (\gamma-1)[W_t + k_1(t)z_{t,stoch} + k_{2s}(t)]^{-1}\left(\frac{b}{\bar{a}-b}\right) \\
&\quad \left\{\begin{array}{l} z_0 e^{-a_t t}[(a_t-\bar{a})(\kappa t-1)] + bz_{t,stoch} \\ +h[W_t + k_1(t)z_{t,stoch} + k_{2s}(t)] - \bar{a}z_{t,stoch} + \frac{1}{2}\sigma_z^2 e^{\frac{1}{2}\sigma_z^2 t} \end{array}\right\} \\
&\quad + \frac{1}{2}(\gamma-1)(\gamma-2)k_1^2(t)[W_t + k_1(t)z_{t,stoch} + k_{2s}(t)]^{-2}\sigma_z^2 e^{\frac{1}{2}\sigma_z^2 t} \\
&= \frac{\alpha^2}{\sigma^2} - \frac{1}{2}\left(\frac{\gamma-1}{\gamma-2}\right)\frac{\alpha^2}{\sigma^2} - \frac{\bar{a}}{\bar{a}-b}h(1-\gamma) \\
&\quad + (1-\gamma)[W_t + k_1(t)z_{t,stoch} + k_{2s}(t)]^{-1} \\
&\quad \left[\begin{array}{l} -k_1(t)z_0 e^{-a_t t}(a_t-\bar{a})(\kappa t-1) \\ -\left(\frac{b}{\bar{a}-b}\right)z_0 e^{-a_t t}(a_t-\bar{a})(\kappa t-1) - \frac{1}{2}\sigma_z^2 e^{\frac{1}{2}\sigma_z^2 t} \end{array}\right] \\
&\quad + [W_t + k_1(t)z_{t,stoch} + k_{2s}(t)]^2 \frac{1}{2}(\gamma-1)(\gamma-2)k_1^2(t)\sigma_z^2 e^{\frac{1}{2}\sigma_z^2 t} \\
&= \frac{\alpha^2}{\sigma^2} - \frac{1}{2}\left(\frac{\gamma-1}{\gamma-2}\right)\frac{\alpha^2}{\sigma^2} - \frac{\bar{a}}{\bar{a}-b}h(1-\gamma) \\
&\quad + \frac{(1-\gamma)\left\{\left[\frac{b}{b-\bar{a}} - k_1(t)\right]z_0 e^{-a_t t}(a_t-\bar{a})(\kappa t-1) + \frac{1}{2}\sigma_z^2 e^{\frac{1}{2}\sigma_z^2 t}\right\}}{[W_t + k_1(t)z_{t,stoch} + k_{2s}(t)]} \\
&\quad + \frac{\frac{1}{2}(1-\gamma)(2-\gamma)\left\{\left(\frac{b}{b-\bar{a}}\right)[1-e^{-(b-\bar{a})t}]\right\}^2\sigma_z^2 e^{\frac{1}{2}\sigma_z^2 t}}{[W_t + k_1(t)z_{t,stoch} + k_{2s}(t)]^2} \\
&\geq \frac{\alpha^2}{\sigma^2} - \frac{1}{2}\left(\frac{\gamma-1}{\gamma-2}\right)\frac{\alpha^2}{\sigma^2} - \frac{\bar{a}}{\bar{a}-b}h(1-\gamma) \\
&\quad + \frac{(1-\gamma)\left[\frac{b}{b-\bar{a}} - k_1(t)\right]z_0 e^{-a_t t}(a_t-\bar{a})(\kappa t-1)}{[W_t + k_1(t)z_{t,stoch} + k_{2s}(t)]} \\
&= r^*_{t,\det}
\end{aligned}
$$

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Table 2.1 Descriptive Statistics for Variables Used in Sample (1980-2008)

Our sample contains all available data in Compustat, CRSP, and I/B/E/S from 1980 to 2008. Firm's beta β is calculated based on the market model using the monthly rate of returns in the past 60 months. The cost of equity r is calculated based on the CAPM. Abnormal Earnings per share (x^a) is the current period earnings per share less the previous earnings per share growing at the cost of equity capital r. v is the analyst earnings forecasts of abnormal earnings per share. v_1 is the growth in expected operating earnings per share, measured as the difference between the median consensus analyst earnings per share forecast for next year and the expected net interest revenue (product of end of current year financial liabilities and the interest on debt), divided by operating asset oa. v_2 is the expected growth in operating assets which is proxied by the analyst earnings per share forecasts of long-term earnings growth rate, divided by operating asset oa. x_{130} is the comprehensive earnings per share which is calculated as earnings per share adjusted for marketable securities adjustments (*MSA*) in Compustat, cumulative foreign currency translation adjustments (*RECTA*) in Compustat, and pension requirement adjustments (*PENADJ*). *PENADJ* is calculated as Pension-Additional Minimum Liability (*PADDML*) -- Pension-Unrecognized Prior Service Cost (*PCUPSO*) in Compustat. *MSA*, *RECTA*, and *PENADJ* are all scaled by the beginning-of-period total shares outstanding. Comprehensive operating income (ox_{130}) is equal to comprehensive income (x_{130}) adjusted for comprehensive net financial expenses (*NFE*) and minority interest in income.

Variables	Mean	Std. Dev.	Q1	Median	Q3
Beta (β)	0.9926	2.4995	0.30	0.85	1.56
Cost of Equity Capital (r)	0.0342	0.0436	0.01	0.02	0.04
Earnings Per Share (x)	0.5545	1.0330	-0.02	0.60	1.24
Abnormal Earnings Per share (x^a)	0.3198	1.0269	-0.24	0.37	1.00
Growth in Expected Operating Earnings (v_1)	0.1721	0.0357	-0.09	-0.04	0.05
Expected Growth of Operating Assets (v_2)	0.2253	0.0362	-0.09	-0.04	0.05
Book Value Per Share (b)	8.0327	5.4195	3.74	7.13	11.41
Analyst Earnings Forecasts of Abnormal Earnings Per Share (v)	1.3031	0.0895	0.25	0.66	1.20
Stock Price Per Share (P)	15.5368	11.0900	6.75	13.75	23.13
Comprehensive Earnings per share (x_{130})	15.6134	13.2147	5.47	14.03	24.33
Marketable Securities Adj. (*MSA*)	0.0743	0.2161	0.02	0.09	0.16
Foreign Currency Translation Adj. (*RECTA*)	0.0000	0.0087	-0.01	0.01	0.04
Pension Requirement Adj. (*PENADJ*)	0.0765	0.1357	0.02	0.08	0.05
Comprehensive Operating Earnings per share (ox_{130})	16.4327	12.3154	8.11	14.54	25.67

Table 2.2 Time Series Behavior of Ohlson Model Linear Information Dynamics

Panel GMM methodology proposed by Arellano and Bond (1991) is used to examine the autoregressive property of earnings and other value-relevant information variable in Ohlson Model. The estimated coefficients ω_{ij} and the adjusted R^2 from the regression are provided in each panel. $x^a_{i,t}$ is the abnormal earnings per share for firm i at year t. $x^a_{130i,t}$ is the abnormal comprehensive earnings per share for firm i at time t. $v_{1i,t}$ is the consensus analyst earnings per share forecasts for year $t+1$ at year t. The p-values associated with each estimated coefficients are reported in the parenthesis below. Our sample contains all available data in Compustat, CRSP, and I/B/E/S from 1980 to 2008.

	Estimated Regression Coefficients				**Adj. R^2**
	ω_{10}	ω_{11}	ω_{12}	ω_{13}	
Panel A1: Autoregressive property of abnormal earnings with different lags					
Model: $\tilde{x}^a_{i,t+1} = \omega_{10} + \omega_{11} x^a_{i,t} + \tilde{\varepsilon}_{i1,t+1}$					
	0.0338	0.7788			0.3719
	(0.0425)	(<0.0001)			
Model: $\tilde{x}^a_{i,t+1} = \omega_{10} + \omega_{11} x^a_{i,t} + \omega_{12} x^a_{i,t-1} + \tilde{\varepsilon}_{i1,t+1}$					
	0.0057	0.6111	0.2864		0.3811
	(0.0412)	(<0.0001)	(<0.0001)		
Model: $\tilde{x}^a_{i,t+1} = \omega_{10} + \omega_{11} x^a_{i,t} + \omega_{12} x^a_{i,t-1} + \omega_{13} x^a_{i,t-2} + \tilde{\varepsilon}_{i1,t+1}$					
	-0.1312	0.4889	0.2798	0.1622	0.3566
	(0.0312)	(<0.0001)	(<0.0001)	(0.0655)	
Panel A2: Autoregressive property of abnormal comprehensive earnings with different lags					
Model: $\tilde{x}^a_{130i,t+1} = \omega_{10} + \omega_{11} x^a_{130i,t} + \tilde{\varepsilon}_{i1,t+1}$					
	0.0155	0.8012			0.4114
	(0.0231)	(0.0001)			
Model: $\tilde{x}^a_{130i,t+1} = \omega_{10} + \omega_{11} x^a_{130i,t} + \omega_{12} x^a_{130i,t-1} + \tilde{\varepsilon}_{i1,t+1}$					
	0.0121	0.7411	0.1642		0.3978

	(0.0013)	(<0.0001)	(0.0654)		
Model: $\tilde{x}^{a}_{103i,t+1} = \omega_{10} + \omega_{11} x^{a}_{103i,t} + \omega_{12} x^{a}_{103i,t-1} + \omega_{13} x^{a}_{130i,t-2} + \tilde{\varepsilon}_{i1,t+1}$					
	-0.0004	0.7841	0.1681	0.0914	0.4001
	(0.1124)	(0.0003)	(0.0241)	(0.1511)	
Panel B1: Autoregressive property of abnormal earnings with other information variables					
Model: $\tilde{x}^{a}_{i,t+1} = \omega_{10} + \omega_{11} x^{a}_{i,t} + \omega_{12} v_{i,t} + \tilde{\varepsilon}_{i1,t+1}$					
	0.0524	0.6054	0.3052		0.4121
	(<0.0001)	(0.0105)	(0.0211)		
Panel B2: Autoregressive property of abnormal comprehensive earnings with other information variables					
Model: $\tilde{x}^{a}_{130i,t+1} = \omega_{10} + \omega_{11} x^{a}_{130i,t} + \omega_{12} v_{i,t} + \tilde{\varepsilon}_{i1,t+1}$					
	0.0214	0.6327	0.1864		0.4364
	(0.0015)	(0.0064)	(0.0514)		
Panel C: Autoregressive property of analysts earnings per share forecast with different lags					
Model: $\tilde{v}_{i,t+1} = \omega_{10} + \omega_{11} v_{i,t} + \tilde{\varepsilon}_{i2,t+1}$					
	0.1754	0.4185			0.6884
	(<0.0001)	(<0.0001)			
Model: $\tilde{v}_{i,t+1} = \omega_{10} + \omega_{11} v_{i,t} + \omega_{12} v_{i,t-1} + \tilde{\varepsilon}_{i2,t+1}$					
	0.2374	0.4339	0.2634		0.6791
	(0.0050)	(<0.0001)	(<0.0001)		
Model: $\tilde{v}_{i,t+1} = \omega_{10} + \omega_{11} v_{i,t} + \omega_{12} v_{i,t-1} + \omega_{13} v_{i,t-2} + \tilde{\varepsilon}_{i2,t+1}$					
	0.1499	0.3866	0.2561	0.1735	0.7015
	(0.0015)	(<0.0001)	(<0.0001)	(0.0745)	

Table 2.3 Time Series Behavior of Feltham-Ohlson Model Linear Information Dynamics

Panel GMM methodology proposed by Arellano and Bond (1991) is used to examine the autoregressive property of earnings and other value-relevant information variable in FO Model. The estimated coefficients ω_{ij} and the adjusted R^2 from the regression are provided in each panel. $ox^a_{i,t,}$ is the abnormal operating earnings per share for firm i at year t. $ox^a_{130i,t}$ is the abnormal comprehensive operating earnings per share for firm i at year t. $oa_{i,t}$ is operating asset scaled by total assets for firm i at year t. $v_{1i,t}$ is the growth in expected operating earnings, measured as the difference between the median consensus analyst earnings forecast for next year and the expected net interest revenue (product of end of current year financial liabilities and the interest on debt). $v_{2i,t}$ is the expected growth in operating assets, measured by the analyst earnings forecasts of long-term earnings growth rate. The p-values associated with each statistics are reported in the parenthesis below. Our sample contains all available data in Compustat, CRSP, and I/B/E/S from 1980 to 2008.

	Estimated Regression Coefficients				Adj. R^2
	ω_{10}	ω_{11}	ω_{12}	ω_{13}	
Panel A1: Autoregressive property of abnormal operating earnings with different lags					
Model: $\widetilde{ox}^a_{i,t+1} = \omega_{10} + \omega_{11} ox^a_{i,t} + \tilde{\varepsilon}_{1i,t+1}$					
	-0.0099	0.2897			0.6434
	(0.1241)	(<0.0001)			
Model: $\widetilde{ox}^a_{i,t+1} = \omega_{10} + \omega_{11} ox^a_{i,t} + \omega_{12} ox^a_{i,t-1} + \tilde{\varepsilon}_{1i,t+1}$					
	-0.0227	0.2567	0.0935		0.6413
	-0.1762	(<0.0001)	(<0.0001)		
Model: $\widetilde{ox}^a_{i,t+1} = \omega_{10} + \omega_{11} ox^a_{i,t} + \omega_{12} ox^a_{i,t-1} + \omega_{13} ox^a_{i,t-2} + \tilde{\varepsilon}_{1i,t+1}$					
	-0.0339	0.1886	0.1048	0.0992	0.6505
	(0.0513)	(<0.0001)	(<0.0001)	(<0.0001)	
Panel A2: Autoregressive property of abnormal comprehensive operating earnings with different lags					
Model: $\widetilde{ox}^a_{103i,t+1} = \omega_{10} + \omega_{11} ox^a_{130i,t} + \tilde{\varepsilon}_{1i,t+1}$					
	0.0132	0.4231			0.7211

	(0.0845)	(<0.0001)			
Model: $\widetilde{ox}^{a}_{130i,t+1} = \omega_{10} + \omega_{11} ox^{a}_{130i,t} + \omega_{12} ox^{a}_{130i,t-1} + \tilde{\varepsilon}_{1i,t+1}$					
	-0.0017	0.3978	0.1412		0.6877
	(0.0512)	(<0.0001)	(0.0014)		
Model: $\widetilde{ox}^{a}_{130i,t+1} = \omega_{10} + \omega_{11} ox^{a}_{130i,t} + \omega_{12} ox^{a}_{130i,t-1} + \omega_{13} ox^{a}_{130i,t-2} + \tilde{\varepsilon}_{1i,t+1}$					
	-0.1687	0.4067	0.0874	0.1021	0.7158
	(0.0647)	(0.0014)	(0.0008)	(0.0214)	
Panel B: Autoregressive property of operating assets with different lags					
Model: $\widetilde{oa}_{i,t+1} = \omega_{10} + \omega_{11} oa_{i,t} + \tilde{\varepsilon}_{1i,t+1}$					
	0.0903	0.2432			0.7941
	(0.0714)	(0.0015)			
Model: $\widetilde{oa}_{i,t+1} = \omega_{10} + \omega_{11} oa_{i,t} + \omega_{12} oa_{i,t-1} + \tilde{\varepsilon}_{1i,t+1}$					
	0.3092	0.2417	0.1274		0.8004
	(0.1150)	(0.0023)	(0.3543)		
Model: $\widetilde{oa}_{i,t+1} = \omega_{10} + \omega_{11} oa_{i,t} + \omega_{12} oa_{i,t-1} + \omega_{13} oa_{i,t-2} + \tilde{\varepsilon}_{1i,t+1}$					
	0.0979	0.1767	0.0601	0.0148	0.8111
	(0.0691)	(0.0056)	(0.1245)	(0.2234)	
Panel C1: Autoregressive property of abnormal operating earnings with other information variables					
Model: $\widetilde{ox}^{a}_{i,t+1} = \omega_{10} + \omega_{11} ox^{a}_{i,t} + \omega_{12} oa_{i,t} + \tilde{\varepsilon}_{1i,t+1}$					
	0.0699	0.2895	0.4023		0.6433
	(0.1186)	(<0.0001)	(0.0049)		

Model: $\widetilde{ox}^{a}_{i,t+1} = \omega_{10} + \omega_{11} ox^{a}_{i,t} + \omega_{12} oa_{i,t} + \omega_{13} v_{1i,t} + \tilde{\varepsilon}_{1i,t+1}$

0.0708	0.2886	0.2744	0.1125	0.7236
(0.1161)	(<0.0001)	(0.0151)	(0.0278)	

Panel C2: Autoregressive property of abnormal comprehensive operating earnings with other information variables

Model: $\widetilde{ox}^{a}_{130i,t+1} = \omega_{10} + \omega_{11} ox^{a}_{130i,t} + \omega_{12} oa_{i,t} + \tilde{\varepsilon}_{1i,t+1}$

0.0422	0.3115	0.5104		0.7144
(0.0848)	(<0.0001)	(0.0001)		

Model: $\widetilde{ox}^{a}_{130i,t+1} = \omega_{10} + \omega_{11} ox^{a}_{130i,t} + \omega_{12} oa_{i,t} + \omega_{13} v_{1i,t} + \tilde{\varepsilon}_{1i,t+1}$

0.0214	0.2871	0.3481	0.3978	0.8412
(0.0214)	(<0.0001)	(<0.0001)	(0.0008)	

Panel D: Autoregressive property of operating assets with other information variables

Model: $\widetilde{oa}_{i,t+1} = \omega_{10} + \omega_{11} oa_{i,t} + \omega_{12} v_{2i,t} + \tilde{\varepsilon}_{1i,t+1}$

-0.0602	0.2204	0.3052		0.8321
(<0.0001)	(<0.0001)	(0.0420)		

Panel E: Autoregressive property of growth of expected operating earnings with different lags

Model: $\tilde{v}_{1i,t+1} = \omega_{10} + \omega_{11} v_{1i,t} + \tilde{\varepsilon}_{1i,t+1}$

-0.0436	-0.0587			0.3125
(0.6591)	(<0.0001)			

Model: $\tilde{v}_{1i,t+1} = \omega_{10} + \omega_{11} v_{1i,t} + \omega_{12} v_{1i,t-1} + \tilde{\varepsilon}_{1i,t+1}$

-0.0422	-0.0674	0.1324		0.3016
(0.6673)	(<0.0001)	(0.0745)		

Model: $\tilde{v}_{1i,t+1} = \omega_{10} + \omega_{11} v_{1i,t} + \omega_{12} v_{1i,t-1} + \omega_{13} v_{1i,t-2} + \tilde{\varepsilon}_{1i,t+1}$					
	-0.0394	-0.0778	0.1238	0.0747	0.3214
	(0.6882)	(<0.0001)	(0.1023)	(0.0621)	

Panel F: Autoregressive property of expected growth of operating assets with different lags					
Model: $\tilde{v}_{2i,t+1} = \omega_{10} + \omega_{11} v_{2i,t} + \tilde{\varepsilon}_{2i,t+1}$					
	0.6493	0.6685			0.4874
	(0.0004)	(<0.0001)			
Model: $\tilde{v}_{2i,t+1} = \omega_{10} + \omega_{11} v_{2i,t} + \omega_{12} v_{2i,t-1} + \tilde{\varepsilon}_{2i,t+1}$					
	0.3393	0.1599	0.2912		0.4905
	(0.0356)	(<0.0001)	(0.0647)		
Model: $\tilde{v}_{2i,t+1} = \omega_{10} + \omega_{11} v_{2i,t} + \omega_{12} v_{2i,t-1} + \omega_{13} v_{2i,t-2} + \tilde{\varepsilon}_{2i,t+1}$					
	0.2717	0.1180	0.1568	0.2004	0.4951
	(0.0771)	(<0.0001)	(0.0487)	(0.1311)	

Table 2.4 Estimation of the Linear Information Dynamics: Single Equation and Simultaneous Equations Estimation

In both Ohlson and FO Model, Panel GMM methodology proposed by Arellano and Bond (1991) is used to estimate the single equation estimation of the linear information dynamics. The simultaneous equations estimation of the linear information dynamics are estimated by the error component three stage least square (3SLS) estimator proposed by Baltagi (1981). $x^a_{i,t}$ is the abnormal earnings per share for firm i at year t. $x^a_{130i,t}$ is the abnormal comprehensive earnings per share for firm i at time t. $v_{1i,t}$ in the Ohlson Model is the consensus analyst earnings per share forecasts for year $t+1$ at year t. $ox^a_{i,t}$ is the abnormal operating earnings per share for firm i at year t. $ox^a_{130i,t}$ is the abnormal comprehensive operating earnings per share for firm i at year t. $oa_{i,t}$ is operating asset scaled by total assets for firm i at year t. $v_{1i,t}$ in FO Model is the growth in expected operating earnings, measured as the difference between the median consensus analyst earnings forecast for next year and the expected net interest revenue (product of end of current year financial liabilities and the interest on debt). $v_{2i,t}$ is the expected growth in operating assets, measured by the analyst earnings forecasts of long-term earnings growth rate. The p-values associated with each statistics are reported in the parenthesis to the right of the estimated coefficients. Our sample contains all available data in Compustat, CRSP, and I/B/E/S from 1980 to 2008.

Panel A1: Ohlson Model with Analysts Earnings Forecasts

	Single Equation		Simultaneous Equations	
	$\tilde{x}^a_{i,t+1} = \omega_{10} + \omega_{11} x^a_{i,t} + \omega_{12} v_{i,t} + \tilde{\varepsilon}_{1i,t+1}$		$\tilde{x}^a_{i,t+1} = \omega_{10} + \omega_{11} x^a_{i,t} + \omega_{12} v_{i,t} + \tilde{\varepsilon}_{1i,t+1}$	
Est. Coeff.	$\tilde{v}_{i,t+1} = \omega_{20} + \omega_{22} v_{i,t} + \tilde{\varepsilon}_{2i,t+1}$		$\tilde{v}_{i,t+1} = \omega_{20} + \omega_{21} x^a_{i,t} + \omega_{22} v_{i,t} + \tilde{\varepsilon}_{2i,t+1}$	
$\boldsymbol{\omega_{10}}$	0.0524	(<0.0001)	0.1431	(<0.0001)
$\boldsymbol{\omega_{11}}$	0.6054	(<0.0001)	0.4005	(<0.0001)
$\boldsymbol{\omega_{12}}$	0.3052	(0.0002)	0.3341	(0.0449)
$\boldsymbol{\omega_{20}}$	0.1754	(<0.0001)	0.1301	(0.0088)
$\boldsymbol{\omega_{21}}$			0.3791	(<0.0001)
$\boldsymbol{\omega_{22}}$	0.4185	(<0.0001)	0.2866	(0.0156)

Panel A2: Ohlson Model (x_{130}) with Analysts Earnings Forecasts

	Single Equation		Simultaneous Equations	
	$\tilde{x}^a_{130i,t+1} = \omega_{10} + \omega_{11} x^a_{130i,t} + \omega_{12} v_{i,t} + \tilde{\varepsilon}_{1i,t+1}$		$\tilde{x}^a_{130i,t+1} = \omega_{10} + \omega_{11} x^a_{130i,t} + \omega_{12} v_{i,t} + \tilde{\varepsilon}_{1i,t+1}$	
Est. Coeff.	$\tilde{v}_{i,t+1} = \omega_{20} + \omega_{22} v_{i,t} + \tilde{\varepsilon}_{2i,t+1}$		$\tilde{v}_{i,t+1} = \omega_{20} + \omega_{21} x^a_{130i,t} + \omega_{22} v_{i,t} + \tilde{\varepsilon}_{2i,t+1}$	
$\boldsymbol{\omega_{10}}$	0.0492	(0.0003)	0.1534	(0.0103)
$\boldsymbol{\omega_{11}}$	0.6231	(<0.0001)	0.4138	(0.0021)
$\boldsymbol{\omega_{12}}$	0.2878	(<0.0001)	0.3687	(0.0009)

ω_{20}	0.1977	(0.0008)	0.1544	(<0.0001)
ω_{21}			0.4369	(<0.0001)
ω_{22}	0.5013	(<0.0001)	0.2920	(0.0006)

Panel B1: FO Model with Analysts Earnings Forecasts

Est. Coeff.	Single Equation		Simultaneous Equations	
	$\widetilde{ox}^a_{i,t+1} = \omega_{10} + \omega_{11} ox^a_{i,t} + \omega_{12} oa_{i,t} + \omega_{13} v_{1i,t} + \tilde{\varepsilon}_{1i,t+1}$ $\widetilde{oa}_{i,t+1} = \omega_{20} + \omega_{22} oa_{i,t} + \omega_{24} v_{2i,t} + \tilde{\varepsilon}_{2i,t+1}$ $\tilde{v}_{1i,t+1} = \omega_{30} + \omega_{33} v_{1i,t} + \tilde{\varepsilon}_{3i,t+1}$ $\tilde{v}_{2i,t+1} = \omega_{40} + \omega_{44} v_{1i,t} + \tilde{\varepsilon}_{4i,t+1}$		$\widetilde{ox}^a_{i,t+1} = \omega_{10} + \omega_{11} ox^a_{i,t} + \omega_{12} oa_{i,t} + \omega_{13} v_{1i,t} + \omega_{14} v_{2i,t} + \tilde{\varepsilon}_{1i,t+1}$ $\widetilde{oa}_{i,t+1} = \omega_{20} + \omega_{21} ox^a_{i,t} + \omega_{22} oa_{i,t} + \omega_{23} v_{1i,t} + \omega_{24} v_{2i,t} + \tilde{\varepsilon}_{2i,t+}$ $\tilde{v}_{1i,t+1} = \omega_{30} + \omega_{31} ox^a_{i,t} + \omega_{32} oa_{i,t} + \omega_{33} v_{1i,t} + \omega_{34} v_{2i,t} + \tilde{\varepsilon}_{3i,t+1}$ $\tilde{v}_{2i,t+1} = \omega_{40} + \omega_{41} ox^a_{i,t} + \omega_{42} oa_{i,t} + \omega_{43} v_{1i,t} + \omega_{44} v_{1i,t} + \tilde{\varepsilon}_{4i,t+1}$	
ω_{10}	0.0708	(0.1161)	0.0222	(0.0015)
ω_{11}	0.2886	(<0.0001)	0.4241	(<0.0001)
ω_{12}	0.2744	(0.0151)	0.1287	(0.0003)
ω_{13}	0.1125	(0.0278)	0.0204	(0.0234)
ω_{14}			0.2267	(0.0459)
ω_{20}	0.0602	(<0.0001)	0.1269	(<0.0001)
ω_{21}			0.1197	(0.0018)
ω_{22}	0.2204	(<0.0001)	0.1066	(<0.0001)
ω_{23}			0.0803	(0.0486)
ω_{24}	0.3052	(0.0402)	0.1872	(0.0339)
ω_{30}	-0.0436	(0.6591)	-0.0382	(0.7187)
ω_{31}			0.2725	(0.0016)
ω_{32}			0.1762	(0.0432)
ω_{33}	-0.0587	(<0.0001)	-0.0143	(0.0287)

ω_{34}			0.0508	(<0.0001)
ω_{40}	0.6493	(0.0004)	0.6244	(0.0008)
ω_{41}			0.0331	(0.7453)
ω_{42}			0.3895	(0.0108)
ω_{43}			-0.2142	(<0.0001)
ω_{44}	0.6685	(<0.0001)	0.6947	(<0.0001)

Panel B2: FO Model (ox_{130}) with Analysts Earnings Forecasts

	Single Equation		Simultaneous Equations	
Est. Coeff.	$\widetilde{ox}^{a}_{130i,t+1} = \omega_{10} + \omega_{11} ox^{a}_{130i,t} + \omega_{12} oa_{i,t} + \omega_{13} v_{1i,t} + \tilde{\varepsilon}_{1i,t+1}$ $\widetilde{oa}_{i,t+1} = \omega_{20} + \omega_{22} oa_{i,t} + \omega_{24} v_{2i,t} + \tilde{\varepsilon}_{2i,t+1}$ $\tilde{v}_{1i,t+1} = \omega_{30} + \omega_{33} v_{1i,t} + \tilde{\varepsilon}_{3i,t+1}$ $\tilde{v}_{2i,t+1} = \omega_{40} + \omega_{44} v_{1i,t} + \tilde{\varepsilon}_{4i,t+1}$		$\widetilde{ox}^{a}_{130i,t+1} = \omega_{10} + \omega_{11} ox^{a}_{130i,t} + \omega_{12} oa_{i,t} + \omega_{13} v_{1i,t} + \omega_{14} v_{2i,t} + \tilde{\varepsilon}$ $\widetilde{oa}_{i,t+1} = \omega_{20} + \omega_{21} ox^{a}_{130i,t} + \omega_{22} oa_{i,t} + \omega_{23} v_{1i,t} + \omega_{24} v_{2i,t} + \tilde{\varepsilon}_{2}$ $\tilde{v}_{1i,t+1} = \omega_{30} + \omega_{31} ox^{a}_{130i,t} + \omega_{32} oa_{i,t} + \omega_{33} v_{1i,t} + \omega_{34} v_{2i,t} + \tilde{\varepsilon}_{3i,t}$ $\tilde{v}_{2i,t+1} = \omega_{40} + \omega_{41} ox^{a}_{130i,t} + \omega_{42} oa_{i,t} + \omega_{43} v_{1i,t} + \omega_{44} v_{1i,t} + \tilde{\varepsilon}_{4i,}$	
ω_{10}	0.0345	(0.0845)	0.0301	(0.0012)
ω_{11}	0.3654	(0.0003)	0.5011	(<0.0001)
ω_{12}	0.3054	(0.0021)	0.1512	(<0.0001)
ω_{13}	0.0788	(0.0122)	0.0113	(0.0212)
ω_{14}			0.2548	(0.0341)
ω_{20}	0.0987	(<0.0001)	0.1054	(<0.0001)
ω_{21}			0.1364	(0.0002)
ω_{22}	0.2541	(<0.0001)	0.1278	(<0.0001)
ω_{23}			0.0811	(0.0387)
ω_{24}	0.3654	(<0.0001)	0.2013	(0.0498)
ω_{30}	0.0024	(0.0874)	-0.0541	(0.2141)

ω_{31}			0.2931	(0.0004)
ω_{32}			0.2348	(0.0632)
ω_{33}	-0.1021	(0.1654)	-0.0354	(<0.0001)
ω_{34}			0.1087	(<0.0001)
ω_{40}	0.7113	(<0.0001)	0.7146	(0.0009)
ω_{41}			0.0663	(0.2227)
ω_{42}			0.4873	(<0.0001)
ω_{43}			-0.3561	(<0.0001)
ω_{44}	0.6987	(<0.0001)	0.7328	(<0.0001)

Table 2.5 Prediction Errors of One-Year Ahead Stock Prices of Ohlson and Feltham-Ohlson Model

In both Ohlson and FO Model, Panel GMM methodology proposed by Arellano and Bond (1991) is used for single equation estimation of the linear information dynamics. The simultaneous equations estimation of the linear information dynamics are estimated by the error component three stage least square (3SLS) estimator proposed by Baltagi (1981). The estimated coefficients ω_{ij} are used with the observed inputs abnormal earnings and analysts earnings forecasts to compute the estimated price of the share at end of each year *t*. The mean forecast errors (*M.F.E.*) is the average of the difference between observed market price and model estimated price divided by the market price per share at end of each period *t*. $\Delta_{FO\text{-}Ohlson}$ represents the difference of forecast errors between the Ohlson Model and FO Model. $\Delta_{Simul\text{-}Single}$ represents the difference of forecast errors between simultaneous equation estimation and single equation estimation of the linear information dynamics in a given valuation model. $\Delta_{x^a_{130}-x^a}$ represents the Ohlson Model forecast errors difference between using earnings and comprehensive earnings as the earnings measure. $\Delta_{ox_{130}-ox}$ represents the FO Model forecast errors difference between using operating earnings and comprehensive operating earnings as the earnings measure. The p-values associated with each estimated coefficients are reported in the parenthesis below. Our sample contains all available data in Compustat, CRSP, and I/B/E/S from 1980 to 2008.

Panel A: Ohlson Model

	Single Equation	Simultaneous Equations	$\Delta_{Simul\text{-}Single}$
	$\tilde{x}^a_{i,t+1} = \omega_{10} + \omega_{11} x^a_{i,t} + \omega_{12} v_{i,t} + \tilde{\varepsilon}_{1i,t+1}$ $\tilde{v}_{i,t+1} = \omega_{20} + \omega_{22} v_{i,t} + \tilde{\varepsilon}_{2i,t+1}$	$\tilde{x}^a_{i,t+1} = \omega_{10} + \omega_{11} x^a_{i,t} + \omega_{12} v_{i,t} + \tilde{\varepsilon}_{1i,t+1}$ $\tilde{v}_{i,t+1} = \omega_{20} + \omega_{21} x^a_{i,t} + \omega_{22} v_{i,t} + \tilde{\varepsilon}_{2i,t+1}$	
Abnormal Earnings (x^a)	0.5078	0.4564	-0.0514
	(<0.0001)	(<0.0001)	(0.0007)
Abnormal Comprehensive	0.4744	0.4339	-0.0305
Earnings (x^a_{130})	(<0.0001)	(<0.0001)	(<0.0001)
$\Delta_{x^a_{130}-x^a}$	-0.0334	-0.0225	
	(<0.0001)	(0.0005)	

Panel B: FO Model

	Single Equation	Simultaneous Equations	$\Delta_{Simul\text{-}Single}$
	$\widetilde{ox}^a_{130i,t+1} = \omega_{10} + \omega_{11} ox^a_{130i,t} + \omega_{12} oa_{i,t} + \omega_{13} v_{1i,t}$ $\widetilde{oa}_{i,t+1} = \omega_{20} + \omega_{22} oa_{i,t} + \omega_{24} v_{2i,t} + \tilde{\varepsilon}_{2i,t+1}$	$\widetilde{ox}^a_{130i,t+1} = \omega_{10} + \omega_{11} ox^a_{130i,t} + \omega_{12} oa_{i,t} + \omega_{13} v_{1i,t} + \omega_{14} v_{2i,t} + \tilde{\varepsilon}_{1i,t+1}$ $\widetilde{oa}_{i,t+1} = \omega_{20} + \omega_{21} ox^a_{130i,t} + \omega_{22} oa_{i,t} + \omega_{23} v_{1i,t} + \omega_{24} v_{2i,t} + \tilde{\varepsilon}_{2i,t+1}$ $\tilde{v}_{1i.t+1} = \omega_{30} + \omega_{31} ox^a_{130i.t} + \omega_{32} oa_{i.t} + \omega_{33} v_{1i.t} + \omega_{34} v_{2i.t} + \tilde{\varepsilon}_{3i.t+1}$	

Abnormal Operating Earnings (ox^a)	0.4059	0.3745	-0.0314
	(<0.0001)	(<0.0001)	(<0.0001)
Abnormal Comprehensive Operating Earnings (ox^a_{130})	0.3815	0.3559	-0.0256
	(<0.0001)	(<0.0001)	(<0.0001)
$\Delta_{ox^a_{130}-ox^a}$	-0.0244	-0.0186	
	(0.0138)	(0.0159)	
$\Delta_{\text{FO-Ohlson}}$	-0.1019	-0.0818	
	(<0.0001)	(<0.0001)	

Table 2.6 Prediction Errors of Stock Prices Forecast of Warren-Shelton Model

Table 2.6 summarizes the mean forecast errors (*M.F.E.*) for one to five years ahead stock price forecasts of the WS model. From 1980 to 2002, financial data of each firm are used as the base year information in the WS model to forecast the future period stock prices in the next five years. For each firm at year *t*, the sales growth rate is estimated by the linear regression model using all the past sales available in Compustat. We then calculated the mean forecast errors of the stock prices by each number of years ahead forecast errors across all rolling periods. $\Delta_{WS-Ohlson}$ represents the difference of forecast errors between WS Model and Ohlson Model with its linear information dynamics estimated by simultaneous equations method. $\Delta_{WS-Ohlson(x_{130})}$ represents the difference of forecast errors between WS Model and Ohlson Model with comprehensive earnings as its earnings measure and its linear information dynamics estimated by simultaneous equations method. Δ_{WS-FO} represents the difference of forecast errors between WS Model and FO Model with its linear information dynamics estimated by simultaneous equations method. $\Delta_{WS-FO(ox_{130})}$ represents the difference of forecast errors between WS Model and FO Model with comprehensive operating earnings as its earnings measure and its linear information dynamics estimated by simultaneous equations method. The p-values associated with each estimated coefficients are reported in the parenthesis below. Our sample contains all available data in Compustat, CRSP, and I/B/E/S from 1980 to 2008.

Number of Years Ahead Forecasts	M.F.E.	$\Delta_{WS-Ohlson}$	Δ_{WS-FO}	$\Delta_{WS-Ohlson(x_{130})}$	$\Delta_{WS-FO(ox_{130})}$
1	0.3418	-0.1146	-0.0327	-0.0921	-0.0141
	(<0.0001)	(<0.0001)	(0.0050)	(<0.0001)	(0.0123)
2	0.4132	-0.0751	-0.0157	-0.0051	-0.0513
	(0.0003)	(0.0451)	(0.0122)	(0.0741)	(0.0523)
3	0.5981	0.0354	-0.0084	-0.0003	-0.0024
	(<0.0001)	(0.3214)	(0.0645)	(0.1522)	(0.0841)
4	0.7016	-0.0641	0.0123	0.0874	0.0845
	(0.0005)	(0.0354)	(0.2147)	(0.0987)	(0.1137)
5	0.9142	0.0687	-0.0011	0.0783	0.0746
	(<0.0001)	(0.3329)	(0.2457)	(0.0874)	(0.1068)

Table 2.7 Sensitivity Analysis of Stock Price Forecasts Errors of Warren-Shelton Model

Table 2.7 summarizes the sensitivity analysis of the model inputs: sales growth rate, total assets as a percent of sales, dividend payout ratio, and leverage ratio in the WS model. We calculate the mean forecast errors (*M.F.E.*) of stock price forecasts from WS model by using the first quartile, median, and the third quartile of the respective model inputs. The p-values associated with each estimated coefficients are reported in the parenthesis below. Our sample contains all available data in Compustat, CRSP, and I/B/E/S from 1980 to 2008.

	Panel A: Sales Growth Rate					
	Q1		Median		Q3	
Number of Years Ahead Forecasts	*M.F.E.*	*V/P*	*M.F.E.*	*V/P*	*M.F.E.*	*V/P*
1	0.6101	0.5436	0.4238	0.6381	0.2954	0.6741
2	0.7187	0.4981	0.5002	0.5402	0.365	0.5632
3	0.8354	0.4026	0.5841	0.4783	0.4899	0.5271
4	0.9163	0.3156	0.7136	0.4297	0.5153	0.5012
5	1.0125	0.2103	0.9362	0.3882	0.7041	0.4132

	Panel B: Total Asset as a Percent of Sales					
	Q1		Median		Q3	
Number of Years Ahead Forecasts	*M.F.E.*	*V/P*	*M.F.E.*	*V/P*	*M.F.E.*	*V/P*
1	0.4048	0.6541	0.4238	0.6381	0.4756	0.6018
2	0.5015	0.5564	0.5011	0.5344	0.2321	0.4987
3	0.6136	0.4541	0.5841	0.4783	0.6328	0.4655
4	0.6654	0.4465	0.7136	0.4297	0.7843	0.4415
5	0.8972	0.4125	0.9362	0.3882	0.9637	0.4213

Panel C: Dividend Payout Ratio

Number of Years Ahead Forecasts	Q1		Median		Q3	
	M.F.E.	*V/P*	*M.F.E.*	*V/P*	*M.F.E.*	*V/P*
1	0.4198	0.6608	0.4238	0.6381	0.4451	0.6231
2	0.4936	0.5412	0.4981	0.5138	0.5215	0.5155
3	0.5787	0.5121	0.5841	0.4783	0.6123	0.4338
4	0.6897	0.4511	0.7136	0.4297	0.7122	0.4136
5	0.8987	0.4015	0.9362	0.3882	0.9345	0.3788

Panel D: Leverage Ratio

Number of Years Ahead Forecasts	Q1		Median		Q3	
	M.F.E.	*V/P*	*M.F.E.*	*V/P*	*M.F.E.*	*V/P*
1	0.4408	0.5389	0.4238	0.6381	0.4187	0.6658
2	0.5321	0.5128	205144	0.5312	0.5108	0.5974
3	0.6087	0.4569	0.5841	0.4783	0.5941	0.4568
4	0.7136	0.4158	0.7136	0.4297	0.6543	0.4412
5	0.9302	0.4111	0.9362	0.3882	0.8741	0.4123

Table 2.8 Prediction Errors of One-Year Ahead Stock Prices of Combined Forecasts Methods

Table 2.8 provides the mean forecast errors (*M.F.E.*) of one-year ahead stock price forecast from methods (M1) through (M6) combining the Ohlson model and WS model (Panel A1 and A2) and FO model and WS model (Panel B1 and B2) forecasts. The linear (*LIN TVPM*) and quadratic (*Qd TVPM*) time-varying parameters model are

$$\text{Linear: } {}_t\hat{y}_{t+1} = \left(\hat{p}_0^0 + \hat{p}_1^0(t+1)\right) + \left(\hat{p}_0^1 + \hat{p}_1^1(t+1)\right){}_t f_{t+1}^{RI} + \left(\hat{p}_0^2 + \hat{p}_1^2(t+1)\right){}_t f_{t+1}^{WS}$$

$$\text{Quadratic: } {}_t\hat{y}_{t+1} = \left(\hat{p}_0^0 + \hat{p}_1^0(t+1) + \hat{p}_2^0(t+1)^2\right) + \left(\hat{p}_0^1 + \hat{p}_1^1(t+1) + \hat{p}_2^1(t+1)^2\right){}_t f_{t+1}^{RI} + \left(\hat{p}_0^2 + \hat{p}_1^2(t+1) + \hat{p}_2^2(t+1)^2\right){}_t f_{t+1}^{WS}$$

M7 and M8 are the individual forecasts f^{Ohlson} (f^{FO}) and f^{WS} from the Ohlson (FO) Model and WS Model respectively. $\Delta_{M_i - M_7}$ represents the prediction errors differences between the forecast combination method M_i and the individual forecast from Ohlson model. $\Delta_{M_i - M_8}$ represents the prediction errors differences between the forecast combination method M_i and the individual forecast from WS model. The mean forecast errors (*M.F.E.*) is the average of the difference between observed market price and model estimated price divided by the market price per share at end of each period t. The p-values associated with each estimated coefficients are reported in the parenthesis below. Our sample contains all available data in Compustat, CRSP, and I/B/E/S from 1980 to 2008.

	Panel A1: Ohlson & WS Combined			*Panel A2: Ohlson(x_{130}) & WS Combined*		
Forecast Methods	M.F.E.	$\Delta_{M_i - M_7}$	$\Delta_{M_i - M_8}$	M.F.E.	$\Delta_{M_i - M_7}$	$\Delta_{M_i - M_8}$
M1: WLS, λ^t	0.3115	-0.1449	-0.0303	0.2981	-0.1583	-0.0437
(*Lin TVPM*)	(0.0002)	(0.0001)	(<0.0001)	(<0.0001)	(0.0150)	(0.0005)
M2: WLS, λ^t	0.3321	-0.1243	-0.0097	0.3425	-0.1139	0.0007
(*Qd TVPM*)	(0.0003)	(0.0764)	(0.0987)	(<0.0001)	(0.0254)	(0.1325)
M3: WLS, t^λ	0.2841	-0.1723	-0.0577	0.2779	-0.1785	-0.0639
(*Lin TVPM*)	(0.0231)	(0.0429)	(0.0008)	(<0.0001)	(0.0110)	(<0.0001)
M4: WLS, t^λ	0.3014	-0.1550	-0.0404	0.2845	-0.1719	-0.0573
(*Qd TVPM*)	(<0.0001)	(0.0745)	(0.0321)	(<0.0001)	(0.0005)	(0.0152)
M5: OLS	0.3614	-0.0950	0.0196	0.3554	-0.101	0.0136
	(0.0001)	(0.0712)	(0.1123)	(<0.0001)	(0.0656)	(0.0327)
M6: Var-Cov	0.3841	-0.0723	0.0423	0.3551	-0.1013	0.0133
	(0.0001)	(0.0432)	(0.0611)	(<0.0001)	(0.0841)	(0.0674)
M7: f^{Ohlson} alone	0.4564			0.4564		
	(<0.0001)			(<0.0001)		

M8: f^{WS} alone	0.3418			0.3418		
	(<0.0001)			(<0.0001)		

	Panel B1: FO & WS Combined			*Panel B2: FO(ox_{130}) & WS Combined*		
	M.F.E.	$\Delta_{M_i - M_7}$	$\Delta_{M_i - M_8}$	M.F.E.	$\Delta_{M_i - M_7}$	$\Delta_{M_i - M_8}$
M1: WLS, λ^t	0.3152	-0.0593	-0.0266	0.3057	-0.0584	-0.0361
(*Lin TVPM*)	(0.0005)	(0.0421)	(0.0742)	(0.0063)	(0.0205)	(<0.0001)
M2: WLS, λ^t	0.3444	-0.0301	0.0026	0.2874	-0.0767	-0.0544
(*Qd TVPM*)	(<0.0001)	(0.0611)	(0.0887)	(<0.0001)	(0.0050)	(0.0026)
M3: WLS, t^λ	0.2411	-0.1334	-0.1007	0.2331	-0.1310	-0.1087
(*Lin TVPM*)	(0.0008)	(0.0013)	(0.0447)	(<0.0001)	(0.0085)	(0.0154)
M4: WLS, t^λ	0.2284	-0.1461	-0.1134	0.2045	-0.1596	-0.1373
(*Qd TVPM*)	(0.0011)	(0.0003)	(0.0011)	(<0.0001)	(0.0035)	(0.0005)
M5: OLS	0.4125	0.0380	0.0707	0.3845	0.0204	0.0427
	(0.0006)	(0.0844)	(0.0334)	(0.0001)	(0.0321)	(0.0085)
M6: Var-Cov	0.3841	0.0096	0.0423	0.4012	0.0371	0.0594
	(<0.0001)	(0.1002)	(0.1245)	(<0.0001)	(0.0509)	(<0.0001)
M7: f^{Ohlson} alone	0.3641			0.3641		
	(0.0002)			(0.0002)		
M8: f^{WS} alone	0.3418			0.3418		
	(<0.0001)			(<0.0001)		

Table 3.1 Financial Characteristics of Value and Growth Stocks (1982-2008)

The value (growth) stocks are those stocks in the top (bottom) quintile portfolio based on the distribution of book-to-market ratio twelve months ago. *ROA*=1 if a firm's (Return on Asset/Total Assets) is positive, 0 otherwise. *AROA*=1 if a firm's change in ROA is positive, 0 otherwise. *CFO*=1 if a firm's (Cash Flow from Operation/Total Assets) is positive, 0 otherwise. *Accrual*=1 if a firm's (current year net income before extraordinary items less cash flow from operation) is positive, 0 otherwise. *DMargin*=1 if a firm's (gross profit scaled by total sales) is positive, 0 otherwise. *DTurn*=1 if a firm's (total sales scaled by average total assets) is positive, 0 otherwise. *DLever*=1 if the historical changes in the ratio of total long term debt to average assets is negative, 0 otherwise. *DLIQUD*=1 if a firm's change in current ratio between the current year and previous year is positive, 0 otherwise. *EQOFFER*=1 if the firm did not have equity issuance in the previous year, 0 otherwise. *RDINT*=1 if a firm's R&D expenses scaled by the average total assets is larger than that of the median of all firms in the same industry. *ADINT*=1 if a firm's advertising expenses scaled by the total assets is larger than that of the median of all firms in the same industry. *CAPINT*=1 if a firm's capital expenditure scaled by the total assets is larger than that of the median of all firms in the same industry.

	Panel A: Value Stocks (N=27091)			***Panel B: Growth Stocks (N=28274)***			***Panel C: All Firms (N=145632)***		
Variables	Mean	Median	Std Dev	Mean	Median	Std Dev	Mean	Median	Std Dev
MV Equity ($mil)	192.352	15.322	1002.364	2845.452	150.321	1512.121	1189.412	100.021	9012.125
Assets ($mil)	1465.312	60.214	4832.121	1508.121	89.126	8645.215	2215.415	136.874	15649.215
Net Income ($mil)	9.562	0.512	142.321	75.523	3.536	523.212	60.147	2.945	402.168
Book/Market	2.243	1.691	29.356	0.231	0.176	0.649	0.945	0.741	2.513
ROA	-0.025	0.019	0.134	-0.048	0.039	0.31	-0.052	0.046	0.216
AROA	-0.009	-0.003	0.198	-0.004	0.002	0.265	-0.006	0.005	0.197
CFO	0.051	0.068	0.135	0.036	0.012	0.29	0.045	0.052	0.302
Accrual	-0.061	-0.05	0.15	-0.032	0.01	0.186	-0.043	0.006	0.165
DMargin	-0.04	-0.004	1.356	0.021	0.102	1.842	-0.002	0.008	1.564
DTurn	0.026	0.007	0.602	0.041	0.102	0.846	0.035	0.043	0.987
DLever	0.003	0.002	0.081	0.015	0.021	0.156	0.009	0.013	0.231
DLIQUID	-0.009	0.001	0.132	-0.005	-0.003	0.154	-0.008	-0.003	0.187

Table 3.2 Correlation among Fundamental Signals, FSCORE, BOS Ratio, and Past Returns for Value Stocks

Table 3.3.2 presents the average Spearman rank-order correlation between the fundamental signals, the FSCORE index, future stocks return, cumulative past returns, and the BOS ratio for the value stock in our sample from 1982 to 2007. The FSCORE is the sum of nine fundamental signals which is assigned a value 1 otherwise 0 if the following criteria are met: F1:*ROA*≥0, F2:*AROA*≥0, F3: *CFO*≥0, F4: *Accrual*≤0, F5: *DMargin*≥0, F6: *DTurn*≥0, F7: *DLever*≤0, F8: *DLIQUD*≥0, and F9: *EQOFFER*=0. The definitions of these variables are described in Section 2. Ret_1 and Ret_2 are the future one- and three-month holding period monthly returns of each stocks after the portfolio formation. Cum.Ret. is the cumulative returns over the twelve months period immediately before the portfolio formation. BOS ratio is defined as the covariance between the monthly return and the adjusted trading volume $cov(r_i, \pi_i)$ over the twelve month period immediately before the portfolio formation.

	Ret_1	Ret_2	FSCORE	BOS Ratio	Cum. Ret.	F1	F2	F3	F4	F5	F6	F7	F8	F9
Ret1	1	0.521	0.171	0.003	0.423	0.084	0.031	0.102	0.049	0.042	0.052	0.051	0.025	0.015
Ret2		1	0.184	-0.022	0.397	0.091	0.029	0.114	0.061	0.037	0.029	0.054	0.031	0.038
FSCORE			1	0.067	0.005	0.471	0.549	0.512	0.341	0.416	0.371	0.41	0.347	0.297
BOS Ratio				1	-0.007	0.06	0.047	0.106	0.068	0.012	0.024	0.081	0.0871	0.034
Cum. Ret.					1	0.012	0.007	0.046	0.071	0.004	-0.002	0.102	0.031	0.054
F1:*ROA*≥0						1	0.241	0.357	-0.019	0.687	-0.017	0.141	0.114	-0.051
F2:*AROA*≥0							1	0.125	-0.023	0.411	0.009	0.128	0.124	0.031
F3: *CFO*≥0								1	0.514	0.061	0.039	0.087	0.141	-0.027
F4: *Accrual*≤0									1	-0.002	0.059	0.014	0.067	-0.013
F5: *DMargin*≥0										1	0.001	0.067	0.079	0.011
F6: *DTurn*≥0											1	0.08	0.049	0.029
F7: *DLever*≤0												1	-0.004	-0.019
F8: *DLIQUD*≥0													1	-0.021
F9: *EQOFFER*=0														1

Table 3.3 Correlation among Fundamental Signals, GSCORE, BOS Ratio, and Past Returns for Growth Stocks

Table 3.3 presents the average Spearman rank-order correlation between the fundamental signals, the GSCORE index, future stocks return, cumulative past returns, and the BOS ratio for the growth stock in our sample from 1982 to 2007. The GSCORE is the sum of eight fundamental signals which is assigned a value 1 otherwise 0 if the following criteria are met: G1:*ROA*≥Ind$_M$, G2:*CFO*≥ Ind$_M$, G3:Accrual≤0, G4:σ_{NI}≤ Ind$_M$, G5: σ_{SG} ≤ Ind$_M$, G6:*RDINT*≥ Ind$_M$, G7:*ADINT*≥ Ind$_M$, and G8:*CAPINT*≥ Ind$_M$. The definitions of these variables are described in Section 2. Ret$_1$ and Ret$_2$ are the future one- and three-month holding period monthly returns of each stocks after the portfolio formation. Cum.Ret. is the cumulative returns over the twelve months period immediately before the portfolio formation. BOS ratio is defined as the covariance between the monthly return and the adjusted trading volume cov(r_i,π_i) over the twelve month period immediately before the portfolio formation.

	Ret$_1$	Ret$_2$	GSCORE	BOS Ratio	Cum.Ret.	G1	G2	G3	G4	G5	G6	G7	G8
Ret$_1$	1	0.578	0.114	0.011	0.452	0.048	0.051	0.028	0.071	0.059	0.041	0.021	0.023
Ret$_2$		1	0.123	-0.007	0.411	0.051	0.063	0.034	0.081	0.063	0.051	0.029	0.034
GSCORE			1	0.073	0.008	0.601	0.712	0.201	0.541	0.611	0.168	0.513	0.351
BOS Ratio				1	-0.011	0.088	0.064	0.014	0.121	0.097	0.074	0.011	0.007
Cum.Ret.					1	0.004	0.016	0.007	0.013	0.024	0.031	0.007	-0.002
G1:*ROA*≥Ind$_M$						1	0.554	-0.189	0.315	0.31	-0.135	0.098	0.064
G2:*CFO*≥ Ind$_M$							1	0.061	0.341	0.321	-0.114	0.078	0.063
G3:Accrual≤0								1	0.113	0.051	-0.071	0.009	0.034
G4:σ_{NI}≤ Ind$_M$									1	0.501	-0.154	0.056	0.027
G5: σ_{SG} ≤ Ind$_M$										1	-0.112	0.083	0.071
G6:*RDINT*≥ Ind$_M$											1	0.103	-0.027
G7:*ADINT*≥ Ind$_M$												1	0.009
G8:*CAPINT*≥ Ind$_M$													1

Table 3.4 Returns to Technical Momentum Strategy (1982-2008)

Table 3.4 provides the momentum returns of one, three, and six month holding periods returns from a long-short portfolio constructed from past twelve months winner and loser stocks from January 1982 to December 2007. We reported the average monthly excess return and Fama-French Three Factors Model monthly adjusted returns in percentage term (associated White heteroskedasticity corrected t-statistics are reported below the returns). The monthly excess return is the difference between portfolio return and the monthly return on Three-Month Treasury-Bill (r_i - r_f). The Fama-French risk-adjusted return of the portfolio relative to the three factors is the estimated intercept coefficients from the following time-series regression using monthly portfolio returns:

$$(r_i - r_f) = \alpha_i + \beta_i(r_m - r_f) + \phi_i SMB + \varphi_i HML + e_i$$

where r_i is the monthly return for the long-short portfolio i, r_f is the monthly return on three month T-bill, r_m is the value-weighted return on the NYSE/AMEX/Nasdaq market index, *SMB* is the Fama-French small firm factor, *HML* is the Fama-French book-to-market factor, β_i, φ_i, ψ_i are the corresponding factor loadings. At the end each month t, the stocks are sorted into five quintile portfolios independently by the cumulative returns in the previous year, from month (t-12) to (t-1). Q_{M1} (Q_{M5}) is the portfolio consisting of the stocks with the past twelve months cumulative returns in the top (bottom) twenty percent. (Q_{M1}- Q_{M5}) is the profits from the long-short investment strategy in which the long position consisting of the past winners and short position consisting of the past losers. We measured the difference in average one, three, and six months return between the monthly rebalanced extreme portfolios. The difference between the extreme portfolios are calculated by averaging monthly profits on an overlapping portfolio that in each month contains an equally weighted portfolio of the long-short momentum portfolios selected in the past six months.

Panel A: Value Stocks

Average Monthly Excess Returns (%)

	Q_{M5}	Q_{M4}	Q_{M3}	Q_{M2}	Q_{M1}	Q_{M1}- Q_{M5}
1-Month(K=1)	0.3370	0.4851	0.5225	0.7244	0.9428	0.6058
	2.32	2.58	2.65	2.76	2.91	2.66
3-Month(K=3)	0.3219	0.3728	0.4836	0.5308	0.8997	0.5778
	2.21	2.33	2.37	2.52	2.88	2.69
6-Month(K=6)	0.2451	0.3668	0.4887	0.556	0.7356	0.4905
	1.95	2.25	2.55	2.66	2.77	2.59

Fama-French 3-Factor Model Monthly Adj. Returns (%)

	Q_{M5}	Q_{M4}	Q_{M3}	Q_{M2}	Q_{M1}	Q_{M1}- Q_{M5}
1-Month(K=1)	0.0791	0.2685	0.3724	0.5072	0.7939	0.7184
	1.42	1.88	1.93	2.53	2.82	2.78
3-Month(K=3)	0.0066	0.2271	0.3886	0.4853	0.7737	0.7671
	0.25	1.7	1.92	2.08	2.77	2.75
6-Month(K=6)	-0.0187	0.2124	0.3767	0.4093	0.6487	0.6674
	-0.53	1.71	1.90	1.95	2.69	2.65

Panel B: Growth Stocks

Average Monthly Excess Returns (%)

	Q_{M5}	Q_{M4}	Q_{M3}	Q_{M2}	Q_{M1}	Q_{M1}- Q_{M5}
1-Month(K=1)	0.0331	0.142	0.3371	0.6702	0.9434	0.9103
	1.25	1.52	1.78	2.28	2.85	2.8
3-Month(K=3)	-0.0196	0.5218	0.7565	0.8673	0.958	0.9776
	-1.15	2.15	2.37	2.53	2.91	2.98
6-Month(K=6)	0.1519	0.1784	0.2987	0.6759	0.9176	0.7656
	1.61	1.66	1.70	2.22	2.81	2.41

Fama-French 3-Factor Model Monthly Adj. Returns (%)

	Q_{M5}	Q_{M4}	Q_{M3}	Q_{M2}	Q_{M1}	Q_{M1}- Q_{M5}
1-Month(K=1)	-0.8206	-0.2347	0.2001	0.3349	0.4602	1.2808
	-2.41	-1.62	1.53	1.8	1.91	3.21
3-Month(K=3)	-0.8471	-0.1418	0.1906	0.3296	0.492	1.3391
	-2.48	-1.48	1.63	1.75	2.02	3.41
6-Month(K=6)	-0.6608	-0.2037	0.2379	0.3862	0.4655	1.1263
	-2.20	-1.50	1.65	1.83	1.97	3.02

Table 3.5 Returns to BOS Momentum Strategy (1982-2008)

Table 3.5 provides the returns of one, three, and six month holding periods from a long-short investment strategy based on past twelve months returns and BOS ratio from January 1982 to December 2007. We reported the average monthly excess return and Fama-French Three Factors Model monthly adjusted returns in percentage term (associated White heteroskedasticity corrected t-statistics are reported below the returns). The monthly excess return is the difference between portfolio return and the monthly return on Three-Month Treasury-Bill (r_i - r_f). The Fama-French risk-adjusted return of the portfolio relative to the three factors is the estimated intercept coefficients from the following time-series regression using monthly portfolio returns:

$$(r_i - r_f) = \alpha_i + \beta_i(r_m - r_f) + \phi_i SMB + \varphi_i HML + e_i$$

the definition of these variables are identical to those described in Table 3.4. At the end each month t from January 1982 to December 2007, we sort the stocks sequentially by cumulative returns in the past twelve months and the BOS ratio. Q_{M1} (Q_{M5}) is the portfolio consisting of the stocks with the past twelve months cumulative returns in the top (bottom) twenty percent. Q_{B1} (Q_{B5}) is the portfolio consisting of the winners (losers) with the lowest (highest) BOS ratio. $\Delta_{BOS\text{-}MOM}$ is the difference between the returns to the BOS momentum strategy in Table 3.5 and the technical momentum strategy in Table 3.4, i.e. $\Delta_{BOS\text{-}MOM}$ =[(Q_{M1}, Q_{B5})-(Q_{M5}, Q_{B5})]-[Q_{M1}- Q_{M5}]. We use the paired difference t-test to test whether $\Delta_{BOS\text{-}MOM}$ is statistically significant from zero. The t-statistics is $t=x_D/(s_D/\sqrt{n})$ where x_D is the mean of differences between BOS and MOM strategy, s_D is the sample standard deviation of the differences, and n is the number of months from Jan.1982 to Dec. 2007 in our sample.

	Panel A: Value Stocks			***Panel B: Growth Stocks***		
1-Month Average Excess Returns (%)						
	Q_{B1}	Q_{B5}	(Q_{B5}- Q_{B1})	Q_{B1}	Q_{B5}	(Q_{B5}- Q_{B1})
Q_{M5} (Losers)	2.6619	1.2073	-1.4546	1.2999	0.3881	-0.9118
	4.02	2.13	-2.40	1.81	0.89	-1.34
Q_{M1}(Winners)	1.7960	2.1187	0.3226	1.0297	1.2249	0.1952
	5.51	5.65	1.72	3.06	2.59	0.46
(Q_{M1}, Q_{B5})		0.9114			0.8368	
-(Q_{M5}, Q_{B5})		1.74			1.73	
$\Delta_{BOS\text{-}MOM}$		0.3056			-0.0735	
		1.95			-0.86	
3-Month Average Excess Returns (%)						
	Q_{B1}	Q_{B5}	(Q_{B5}- Q_{B1})	Q_{B1}	Q_{B5}	(Q_{B5}- Q_{B1})
Q_{M5} (Losers)	2.3017	1.3797	-0.9220	0.9414	0.1486	-0.7928
	3.65	2.54	-1.72	1.50	0.37	-1.37
Q_{M1}(Winners)	1.6533	2.3761	0.7228	1.0315	1.2411	0.2095
	5.37	6.79	2.52	3.05	2.72	0.53
(Q_{M1}, Q_{B5})		0.9964			1.0925	
-(Q_{M5}, Q_{B5})		2.05			2.44	
$\Delta_{BOS\text{-}MOM}$		0.4186			0.1149	
		2.11			1.62	
6-Month Average Excess Returns (%)						
	Q_{B1}	Q_{B5}	(Q_{B5}- Q_{B1})	Q_{B1}	Q_{B5}	(Q_{B5}- Q_{B1})
Q_{M5} (Losers)	2.0233	1.2071	-0.8162	1.4095	0.1708	-1.2388
	3.56	2.33	-1.68	2.26	0.44	-2.21
Q_{M1}(Winners)	1.5756	2.1927	0.3427	0.9459	1.1302	0.1843
	5.33	6.63	2.13	2.75	2.63	0.51
(Q_{M1}, Q_{B5})		0.9856			0.9594	

-(Q_{M5}, Q_{B5})		2.20			2.36	
$\Delta_{BOS\text{-}MOM}$		0.4951			0.1938	
		1.99			1.77	

1-Month FF3 Adj. Returns (%)

	Q_{B1}	Q_{B5}	(Q_{B5}- Q_{B1})	Q_{B1}	Q_{B5}	(Q_{B5}- Q_{B1})
Q_{M5} (Losers)	1.6209	-0.2836	-1.9045	0.4807	-0.4124	-0.8931
	2.82	-0.67	-3.16	0.74	-1.38	-1.31
Q_{M1}(Winners)	0.8560	1.2547	0.3987	0.7787	0.5516	-0.2271
	3.88	4.17	1.63	4.06	1.47	-0.54
(Q_{M1}, Q_{B5})		1.5383			0.9640	
-(Q_{M5}, Q_{B5})		2.97			1.97	
$\Delta_{BOS\text{-}MOM}$		0.8199			-0.3168	
		2.25			-1.33	

3-Month FF3 Adj. Returns (%)

	Q_{B1}	Q_{B5}	(Q_{B5}- Q_{B1})	Q_{B1}	Q_{B5}	(Q_{B5}- Q_{B1})
Q_{M5} (Losers)	1.4091	-0.0547	-1.4639	0.2501	-0.5915	-0.8416
	2.55	-0.14	-2.73	0.45	-2.29	-1.46
Q_{M1}(Winners)	0.7536	1.4667	0.7131	0.7981	0.6540	-0.1441
	4.07	5.76	2.43	4.46	1.92	-0.39
(Q_{M1}, Q_{B5})		1.5214			1.2455	
-(Q_{M5}, Q_{B5})		3.24			2.82	
$\Delta_{BOS\text{-}MOM}$		0.7544			-0.0835	
		2.01			-0.91	

6-Month FF3 Adj. Returns (%)

	Q_{B1}	Q_{B5}	(Q_{B5}- Q_{B1})	Q_{B1}	Q_{B5}	(Q_{B5}- Q_{B1})
Q_{M5} (Losers)	1.2860	-0.1102	-1.3962	0.6172	-0.5516	-1.1688
	2.64	-0.30	-2.86	1.15	-2.31	-2.08
Q_{M1}(Winners)	0.6727	1.3380	0.6653	0.7187	0.5793	-0.1394
	3.99	5.86	2.49	4.25	1.90	-0.42
(Q_{M1}, Q_{B5})		1.4482			1.1309	
-(Q_{M5}, Q_{B5})		3.34			2.85	
$\Delta_{BOS\text{-}MOM}$		0.7807			0.0046	
		2.08			0.65	

Table 3.6 Returns to Combined Momentum Strategy – Value Stocks (1982-2008)

Table 3.6 provides the summary of the momentum returns when the value stocks are sorted by past returns, BOS ratio, and the fundamental indicator FSCORE. We reported the average monthly excess return and Fama-French Three Factors Model monthly adjusted returns in percentage term (associated White heteroskedasticity corrected t-statistics are reported below the returns). The monthly excess return is the difference between portfolio return and the monthly return on Three-Month Treasury-Bill (r_i - r_f). The Fama-French risk-adjusted return of the portfolio relative to the three factors is the estimated intercept coefficients from the following time-series regression using monthly portfolio returns:

$$(r_i - r_f) = \alpha_i + \beta_i(r_m - r_f) + \phi_i SMB + \varphi_i HML + e_i$$

where the definition of these variables are identical to those described in Table 3.4. At the end each month t from January 1982 to December 2007, we sort the stocks sequentially by cumulative returns in the past twelve months, the BOS ratio, and the fundamental scores. Portfolios Q_{Mi} and Q_{Bi} have the same definition as in previous tables. Q_{F1} (Q_{F5}) is the portfolio consisting of the stocks with lowest (highest) FSCORE. (Q_{M1}, $Q_{B5,}$ Q_{F5})-(Q_{M5}, $Q_{B5,}$ Q_{F1}) is the profits generated from the long-short investment strategy with long position in top winners-lowest BOS-highest FSCORE stocks and short position in top losers-highest BOS-lowest FSCORE stocks. $\Delta_{CS\text{-}BOS}$ is the difference of long-short portfolio returns between the BOS momentum strategy in Table 3.5 and the combined strategy based on past returns, BOS ratio, and FSCORE, i.e. $\Delta_{CS\text{-}BOS}$ = [(Q_{M1}, $Q_{B5,}$ Q_{F5})-(Q_{M5}, $Q_{B5,}$ Q_{F1})] - [(Q_{M1}, Q_{B5})-(Q_{M5}, Q_{B5})]. We use the paired difference t-test to test whether $\Delta_{CS\text{-}BOS}$ is statistically significant from zero. The t-statistics is $t = x_D/(s_D/\sqrt{n})$ where x_D is the mean of differences between CS and BOS strategy, s_D is the sample standard deviation of the differences, and n is the number of months from Jan.1982 to Dec. 2007 in our sample.

Panel A: Monthly Average Excess Returns (%)

	1-Month Average Excess Returns (%)			*3-Month Average Excess Returns (%)*			*6-Month Average Excess Returns (%)*		
	Q_{F1}	Q_{F5}	(Q_{F5}- Q_{F1})	Q_{F1}	Q_{F5}	(Q_{F5}- Q_{F1})	Q_{F1}	Q_{F5}	(Q_{F5}- Q_{F1})
(Q_{M5}, Q_{B5})	-0.6941	0.5184	1.2125	-2.1933	0.0778	2.2711	-2.0617	0.2355	2.2971
	-2.22	2.33	2.95	-4.02	1.61	4.52	-3.71	1.93	4.55
(Q_{M1}, Q_{B5})	0.0315	1.0876	1.2561	-0.3498	1.1665	1.5163	0.1026	0.8968	0.7942
	0.26	2.71	3.02	-1.88	2.76	3.11	1.71	2.41	2.35
(Q_{M1}, Q_{B5},Q_{F5})		1.7817			3.3598			2.9584	
-(Q_{M5}, Q_{B5}, Q_{F1})		2.04			4.54			4.67	
$\Delta_{CS\text{-}BOS}$		0.8703			2.3634			1.9728	
		2.71			4.43			3.64	

Panel B: Fama-French Three Factors Model Adjusted Returns (%)

	1-Month FF3 Adj. Returns (%)			*3-Month FF3 Adj. Returns (%)*			*6-Month FF3 Adj. Returns (%)*		
	Q_{F1}	Q_{F5}	(Q_{F5}- Q_{F1})	Q_{F1}	Q_{F5}	(Q_{F5}- Q_{F1})	Q_{F1}	Q_{F5}	(Q_{F5}-

									Q_{F1})
(Q_{M5}, Q_{B5})	-0.8502	0.4323	1.2825	-3.0083	-0.4263	2.9656	-1.7089	0.1682	1.8771
	-2.61	2.11	2.85	-4.8	-0.07	4.35	-3.41	1.63	3.51
(Q_{M1}, Q_{B5})	0.2194	1.2928	1.0735	-1.4489	0.9097	2.3587	0.6712	1.9100	1.2388
	1.65	3.05	2.75	-2.45	2.05	3.53	2.33	3.61	3.08
(Q_{M1}, Q_{B5},Q_{F5})		2.1431			3.918			3.6189	
-(Q_{M5}, Q_{B5}, Q_{F1})		2.33			5.14			5.66	
$\Delta_{CS\text{-}BOS}$		0.6047			2.3966			2.1706	
		2.69			4.11			4.34	

Table 3.7 Returns to Returns to Combined Momentum Strategy – Growth Stocks (1982-2008)

Table 3.6 provides the summary of the momentum returns when the growth stocks are sorted by past returns, BOS ratio, and the fundamental indicator GSCORE. We reported the average monthly excess return and Fama-French Three Factors Model monthly adjusted returns in percentage term (associated White heteroskedasticity corrected t-statistics are reported below the returns). The monthly excess return is the difference between portfolio return and the monthly return on Three-Month Treasury-Bill (r_i - r_f). The Fama-French risk-adjusted return of the portfolio relative to the three factors is the estimated intercept coefficients from the following time-series regression using monthly portfolio returns:

$$(r_i - r_f) = \alpha_i + \beta_i(r_m - r_f) + \phi_i SMB + \varphi_i HML + e_i$$

Where the definition of these variables are identical to those described in Table 3.4. At the end each month t from January 1982 to December 2007, we sort the stocks sequentially by cumulative returns in the past twelve months, the BOS ratio, and the fundamental scores. Portfolios Q_{Mi} and Q_{Bi} have the same definition as in previous tables. Q_{G1} (Q_{G5}) is the portfolio consisting of the stocks with lowest (highest) GSCORE. (Q_{M1}, $Q_{B5,}$ Q_{G5})-(Q_{M5}, $Q_{B5,}$ Q_{G1}) is the profits generated from the long-short investment strategy with long position in top winners-lowest BOS-highest GSCORE stocks and short position in top losers-highest BOS-lowest GSCORE stocks. $\Delta_{CS\text{-}BOS}$ is the difference of long-short portfolio returns between the BOS momentum strategy in Table 3.5 and the combined strategy based on past returns, BOS ratio, and GSCORE, i.e. $\Delta_{CS\text{-}BOS}$ = [(Q_{M1}, $Q_{B5,}$ Q_{G5})-(Q_{M5}, $Q_{B5,}$ QG_{F1})] - [(Q_{M1}, Q_{B5})-(Q_{M5}, Q_{B5})]. We use the paired difference t-test to test whether $\Delta_{CS\text{-}BOS}$ is statistically significant from zero. The t-statistics is $t=x_D/(s_D/\sqrt{n})$ where x_D is the mean of differences between CS and BOS strategy, s_D is the sample standard deviation of the differences, and n is the number of months from Jan.1982 to Dec. 2007 in our sample.

Panel A: Monthly Average Excess Returns (%)	*1-Month Average Excess Returns (%)*			*3-Month Average Excess Returns (%)*			*6-Month Average Excess Returns (%)*		
	Q_{G1}	Q_{G5}	(Q_{G5}- Q_{G1})	Q_{G1}	Q_{G5}	(Q_{G5}- Q_{G1})	Q_{G1}	Q_{G5}	(Q_{G5}- Q_{G1})
(Q_{M5}, Q_{B5})	-1.2545	0.2357	1.4902	-1.0906	0.1133	1.2039	-0.9180	0.1193	1.0373
	-2.61	1.67	2.82	-2.41	1.18	2.77	-2.34	1.23	2.62
(Q_{M1}, Q_{B5})	0.4123	2.051	1.6387	0.0715	1.9387	1.8671	0.4166	1.4851	1.0685
	1.82	3.66	2.95	1.12	3.54	3.21	1.99	3.16	2.90
(Q_{M1}, Q_{B5},Q_{G5})		3.3055			3.0293			2.2031	
-(Q_{M5}, Q_{B5}, Q_{G1})		2.53			2.94			2.51	
$\Delta_{CS\text{-}BOS}$		2.4686			1.9368			1.2436	
		3.12			2.78			2.88	

Panel B: Fama-French Three Factors Model Adjusted Returns (%)	*1-Month FF3 Adj. Returns (%)*			*3-Month FF3 Adj. Returns (%)*			*6-Month FF3 Adj. Returns (%)*		
	Q_{G1}	Q_{G5}	(Q_{G5}- Q_{G1})	Q_{G1}	Q_{G5}	(Q_{G5}- Q_{G1})	Q_{G1}	Q_{G5}	(Q_{G5}- Q_{G1})

(Q_{M5}, Q_{B5})	-2.7527	0.0935	2.8462	-2.2433	-0.1034	2.1399	-1.9265	-0.0580	1.8686
	-4.52	1.02	4.61	-4.15	-1.20	4.01	-3.71	-1.11	3.69
(Q_{M1}, Q_{B5})	-0.1459	0.9571	1.1030	-0.4804	0.9498	1.4302	-0.1530	0.7552	0.7082
	-1.31	2.53	2.81	-1.94	2.43	3.15	-1.40	2.27	2.47
(Q_{M1}, Q_{B5},Q_{G5})		3.7097			3.1931			2.1817	
-(Q_{M5}, Q_{B5}, Q_{G1})		2.89			3.02			2.44	
$\Delta_{CS\text{-}BOS}$		2.7457			1.9476			1.0508	
		4.23			3.10			2.77	

Table 3.8 Returns to Different Momentum Strategies

Table 3.8 provides the comparison of the holding period monthly excess returns to the investment strategies based on different sorting variables for value (Panel A) and growth stock (Panel B). *MOM* is the technical momentum investment strategy based solely on past returns. *BOS* is the investment strategy based on past returns and the BOS ratio. *CS* is the combined investment strategy based on past returns, BOS ratio, and the fundamental scores FSCORE/GSCORE. (*H-L*) represents the returns from the long-short investment strategies using the extreme portfolio. For example, when the portfolios are formed based on past returns, BOS ratio, and the FSCORE, (*H-L*) represents the returns generated from the long-short investment strategy with long position in top winners-lowest BOS-highest FSCORE stocks and short position in top losers-highest BOS-lowest FSCORE stocks. We also report the information ratio which is defined as the active return divided by tracking error $IR=(r_i-r_m)/\sigma_{(r_i-r_m)}$ where active return (r_i-r_m) is the difference between the return on the different strategies and the return on the NYSE/AMEX/Nasdaq value-weighted return, and tracking error is the standard deviation of the active return. The returns from the long side (high minus middle) and from the short side (low minus middle) of the portfolio are also reported. The percentage of contribution to the returns of the extreme portfolios from the long side and short side are separately reported.

Panel A: Value Stocks

	MOM			*BOS*			*CS*			*Corr(FSCORE,MOM)*		
	H-L	H-M	L-M	H-L	H-M	L-M	H-L	H-M	L-M	H-L	H-M	L-M
		long side	*short side*		*long side*	*short side*		*long side*	*short side*		*long side*	*short side*
Excess Return 1m (%)	0.6058	0.4203	-0.1855	0.9114	0.6520	-0.2594	1.7817	1.4193	-0.3624	-0.29	-0.30	-0.14
t-stat	2.66	2.78	-1.75	1.74	2.63	-2.23	2.04	2.77	-2.61			
Information Ratio	0.37	0.51	-0.24	0.51	0.54	-0.34	0.57	0.49	-0.33			
Excess Return 3m (%)	0.5778	0.4161	-0.1617	0.9964	0.7510	-0.2454	3.3598	2.3757	-0.9840	-0.31	-0.21	-0.18
t-stat	2.69	2.78	-1.54	2.05	3.04	-1.74	4.54	4.22	-2.41			
Information Ratio	0.41	0.39	-0.29	0.47	0.46	-0.36	0.66	0.47	-0.42			
Excess Return 6m (%)	0.4905	0.2469	-0.2436	0.9856	0.7631	-0.2225	2.9584	2.2634	-0.6949	-0.34	-0.19	-0.20
t-stat	2.71	2.84	-1.81	2.20	2.76	-2.41	4.67	2.71	-2.62			
Information Ratio	0.38	0.34	-0.19	0.43	0.39	-0.34	0.60	0.63	-0.42			

Panel B: Growth Stocks

	MOM	*BOS*	*CS*	*Corr(GSCORE,MOM)*

	H-L	H-M *long side*	L-M *short side*	H-L	H-M *long side*	L-M *short side*	H-L	H-M *long side*	L-M *short side*	H-L	H-M *long side*	L-M *short side*
Excess Return 1m (%)	0.9103	0.6063	-0.3040	0.8368	0.7261	-0.1107	3.3055	2.5300	-0.7755	-0.37	-0.29	-0.17
t-stat	2.81	2.74	-2.21	1.73	2.22	-1.56	2.53	2.78	-2.35			
Information Ratio	0.39	0.41	-0.29	0.44	0.48	-0.25	0.67	0.64	-0.43			
Excess Return 3m (%)	0.9776	0.2015	-0.7761	1.0925	0.9245	-0.1680	3.0293	2.2556	-0.7737	-0.33	-0.25	-0.14
t-stat	2.98	2.46	-1.87	2.44	2.64	-1.51	2.94	3.03	-2.02			
Information Ratio	0.43	0.47	-0.35	0.48	0.44	-0.37	0.63	0.48	-0.39			
Excess Return 6m (%)	0.7656	0.6189	-0.1468	0.9594	0.7727	-0.1867	2.2031	1.9107	-0.4924	-0.26	-0.31	-0.22
t-stat	2.41	2.69	-1.86	2.36	2.71	-1.84	2.51	2.80	-2.54			
Information Ratio	0.31	0.33	-0.16	0.41	0.48	-0.21	0.58	0.60	-0.41			

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