Essays on Optimal Portfolio Decisions for Long-term Investors

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ABSTRACT OF THE THESIS

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This dissertation contains two essays on the optimal portfolio decision for long-term investors. The first essay studies the optimal asset allocation for long-horizon investors with non-tradable labor income when multiple risky asset returns are predictable. It finds that more risk-averse investors hold a higher bond/stock ratio in their risky portfolios when labor income is positively correlated with stock return or independent of risky asset returns, but the reverse is true when labor income is positively correlated with bond return. The allocation to stock inherits the inverted U-shaped pattern of labor income growth with respect to expected time until retirement. These results suggest that popular recommendations of investment advisors that more conservative investors should hold a higher bond/stock ratio and that the portfolio allocation to stock should equal 100 minus age may both lack theoretical justification. In the out-of-sample performance test, the dynamic portfolio shows the highest mean returns and Sharpe ratio than two benchmark portfolios,

justifying the economic significance of incorporating the time-variation of investment opportunities and nontradable labor income into investors' portfolio choice.

The second essay studies employees' optimal portfolio in their defined contribution pension plans. Assuming a discrete time model with predictable risky asset returns, the essay finds that the employees' optimal portfolio decision can be greatly affected by the employees' time to retirement, risk preference, contribution rate as well as the correlation between labor income and asset returns. Performance test shows that the gains from adopting the dynamic portfolio strategy relative to several benchmark strategies, including the 1/*n* rule, the optimal static strategy with and without the consideration of asset return predictability, all stock strategy, and all company stock strategy, are economically significant and the economic gain increases with employees' risk aversion. The empirical evidence that employees invest significantly in their company stock in pension plans is difficult to be justified, even after the consideration of short-sale constraints, higher expected company stock return, employees' familiarity with their company, and employers' exclusive match policy. Over allocation to company stock can be very costly, especially to conservative employees.

Dedication

To my grandparents and parents

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Contents

List of Tables

Chapter 1

Introduction

Optimal consumption and portfolio choice is an important topic in finance research and has attracted many financial economists' attention. Markowitz's (1952) mean-variance analysis set up the pioneer work in portfolio theory. His famous mutual fund separation theory shows that given that investors care only about the mean and variance of portfolio return one period ahead, investors with different risk preferences will optimally allocate between riskless asset and market portfolio, while holding the same market portfolio. This conclusion can be extended into a long horizon context when asset returns are independently and identically distributed (i.i.d.) or when investors' preferences can be described by a log utility function (see Samuelson, 1969; and Merton, 1969, 1971).

These special circumstances, however, are not consistent with empirical findings (see Keim and Stambaugh, 1986; and Fama and French, 1988, 1989). Markowitz (1952) also assumes that all assets are tradable. This assumption may be plausible to financial institutions, but not to individual investors who possess nontradable human capital due to moral hazard and adverse selection problem. Thus, recent development in financial theory is to look at the portfolio and consumption choices when either time-varying investment opportunity or nontradable human capital is incorporated. Studies assuming time-varying investment opportunities show that the optimal portfolio contains a myopic demand as well as an intertemporal hedging demand. The myopic demand is related to the mean and variance of asset returns, while the intertemporal hedging demand is held due to time-varying investment opportunities (see Kim and Omberg, 1996; Brennan, *et al.*, 1997; Balduzzi and Lynch, 1999; and Campbell and Viceira, 1999). Papers that allow nontradable human capital find that the presence of human capital can have a significant effect on investors' optimal strategy (see Bodie, *et al*., 1992; Heaton and Lucas, 1997; Koo, 1998, 1999; Viceira, 2001; and Boyle and Guthrie, 2005).[1] However, few papers have addressed the optimal portfolio and consumption problem when both time-varying investment opportunities and nontradable human capital are considered. And they either consider only one risky asset or ignore the consumption decision (see Michaelides, 2001; and Koijen, *et al*., 2005).

Chapter 2 studies the optimal consumption and portfolio decision for long-term investors with nontradable labor income when asset returns are predictable. It studies the optimal consumption and portfolio choices faced by an investor with nontradable labor income when asset returns follow a vector autoregressive (VAR) process. It then

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[1] Bodie, *et al.* (1992) show that the investor invests more in stocks in the presence of certain labor income and becomes even more aggressive when she has labor supply flexibility. Heaton and Lucas (1997) find that with labor income, investors optimally borrow to invest in stocks. Koo (1998, 1999) shows that with noninsurable labor income risk and liquidity constraints, both consumption and investment in risky assets are greatly reduced. Viceira (2001) demonstrates that in a life-cycle model, the investor invests more in stock when employed than when retired and an increase in labor income risk reduces her optimal allocation to stocks. Boyle and Guthrie (2005) incorporate labor income into mean-variance analysis and provide a justification for the popular investment advice that the stock/bond and stock/wealth ratios increase with risk tolerance.

examines how nontradable labor income, time until retirement, the degree of investor's risk aversion, and the investor's time preference can affect the optimal portfolio. Finally, we conduct a performance test to examine the economic importance of adopting a dynamic portfolio strategy that considers asset return predictability in the presence of nontradable human capital.

Chapter 3 studies the optimal portfolio problem faced by employees in their defined contribution pension plans when asset returns are predictable. Investment in pension accounts can be very different from that in regular wealth accounts.[2] During the past decade, defined contribution pension plans have become employees' most important saving channel for retirement in the U.S. However, empirical studies seem to suggest that employees are not portfolio optimizers in their pension investment (e.g., Waggle and Englis, 2000; Benartzi, and Thaler, 2001; Meulbroek, 2005; Mitchell and Utkus, 2005; Huberman and Jiang, 2006; and Beshears, Choi, Laibson, and Madrian, 2007). Since investment in pension plans is for long-term purposes, the loss from adopting a suboptimal strategy can be large. It is thus important to study employees' optimal portfolio strategy in their defined contribution pension plans. Chapter 4 concludes and outlines future research directions.

[2] For instance, before retirement, employees cannot consume their pension wealth without paying costly penalty. In addition, investment in pension accounts faces short sale constraints.

Chapter 2

Optimal Consumption and Portfolio Decision for Long-horizon Investors with Nontradable Labor Income When Asset Returns Are Predictable

2.1 Introduction

Optimal consumption and portfolio choice has been a fundamental topic in finance research. Markowitz's (1952) pioneering work shows that investors with different degrees of risk tolerance should have different allocations between the risky portfolio and the risk-free asset, while holding the same risky portfolio, a result known as the mutual fund separation theorem. However, his model is a one-period model and assumes that all assets are tradable. As investors may make decisions over long horizons and due to asset return predictability (e.g., Keim and Stambaugh, 1986; and Fama and French, 1988, 1989), their optimal portfolios can be different from that in the one-period model (Samuelson, 1969; and Merton, 1969, 1971). Furthermore, investors possess nontradable human capital due to moral hazard and adverse selection problems. The objective of this paper is to investigate the optimal consumption and portfolio decision problem for long-horizon investors with nontradable labor income when asset returns are predictable.

Numerous researchers study separate effects of time-varying investment opportunities or nontradable human capital on consumption and portfolio choice. Papers considering time-varying investment opportunities report that in addition to the myopic demand related to the mean and variance of asset returns, the optimal portfolio contains an additional term called intertemporal hedging demand (e.g., Kim and Omberg, 1996; Brennan, *et al*., 1997; Balduzzi and Lynch, 1999; and Campbell and Viceira, 1999). These studies, however, do not consider labor income. Another group of researchers find that the existence of nontradable human capital has a significant impact on the optimal consumption and portfolio choice, but they assume a constant investment opportunity (e.g., Bodie, *et al*., 1992; Heaton and Lucas, 1997; Koo, 1998, 1999; Viceira, 2001; and Boyle and Guthrie, 2005).

Michaelides (2001) and Koijen, *et al*. (2005) are two exceptions that consider both time-varying investment opportunities and nontradable labor income. Michaelides (2001) studies the optimal portfolio choice for an infinite-horizon investor with nontradable labor income when there is one risk-free asset and one risky stock with a predictable risk premium. However, the assumption of one risky asset limits his model's ability to examine the effects of correlations between human capital and different classes of risky assets on investment decisions. Koijen, *et al*. (2005) assume time variation in bond risk premium faced by an investor receiving inflation-indexed labor income each period and find that the optimal allocation to bond is greatly reduced in the presence of labor income. However, they ignore the well-documented evidence of equity return predictability. Furthermore, abstracted from a more general consumption and portfolio choice problem, their model defines the investor's utility over terminal wealth and thus does not study the optimal consumption over time.

This paper jointly studies time-varying investment opportunities and nontradable labor income for long-horizon investors. Unlike Michaelides (2001) and Koijen, *et al*. (2005) who assume that the investor receives labor income each period, we take into account the life cycle effect by assuming that the investor receives labor income when employed but faces a probability of jumping to the retirement state during which she must live off her wealth. The investor faces a probability of death once retired. These two probabilities determine the expected investment horizon. We assume that there are multiple assets available for investment, and to incorporate return predictability, the excess returns of risky assets are assumed to follow a vector autoregressive (VAR) process. Following Campbell and Viceira (1999), we use a log linear approximation to obtain an analytical solution, which intuitively provides nice economic insight while greatly reducing the computational burden. With this solution technique, we show conveniently that the optimal portfolios in both the employment and retirement states are linear in state variables, consisting of a myopic demand and an intertemporal hedging demand. Furthermore, the optimal log consumption-wealth ratio in the retirement state and the log consumption-labor income ratio in the employment state are both quadratic functions of state variables.

Our model calibration demonstrates that with a time-varying investment opportunity, the investor with independent nontradable labor income holds a higher bond/stock ratio in her risky portfolio when she becomes more risk averse. This pattern becomes more pronounced when shock to labor income is positively correlated with stock return. On the other hand, when shock to labor income is positively correlated with bond return, a more risk-averse investor will hold a lower bond/stock ratio in her risky portfolio. Canner, *et al*. (1997) report that, inconsistent with the mutual fund separation theorem, financial professionals generally suggest that more conservative investors should hold more bonds relative to stocks in their portfolios (see Table 2.1), and they call this the “asset allocation puzzle.” Recent papers show that time-varying investment opportunities can help resolve this puzzle (Bajeux-Besnainou, *et al*., 2001, 2003). Our finding indicates that given the predictability of asset returns, the presence of labor income may either help resolve or deepen the asset allocation puzzle, depending on the correlations between labor income and asset returns.

We show that the allocation to risky assets inherits the inverted-U-shaped pattern of labor income growth with respect to the expected time until retirement, consistent with Viceira (2001), who considers one risky asset and a constant investment opportunity. That is, investors with a lower income growth, namely, younger workers or those near retirement, should invest less in risky assets than those who are in the middle of their career and have a higher income growth, a well-known life cycle pattern of investment (Heaton and Lucas, 2000; and Ameriks and Zeldes, 2004). Thus, our results show that popular recommendations from financial advisors that the optimal portfolio weight in risky stocks should be equal to 100 minus age (e.g., Kintzel, 2007) may lack theoretical justification.

To evaluate the economic significance of incorporating the time-variation of investment opportunities and nontradable labor income into investors' portfolio choice, we compare the out-of-sample performance of the portfolio constructed using our dynamic optimal trading strategy with that of two benchmark portfolios. The first benchmark portfolio is constructed from the optimizing model under a constant investment opportunity, while the second is the popular $1/n$ rule (e.g., Benartzi and Thaler, 2001; and Huberman and Jiang, 2006). We find that our dynamic optimal portfolio yields significantly higher mean return and Sharpe ratio than the two benchmark portfolios. We also find the welfare loss of ignoring the predictability of asset returns to be economically significant.

Previous research on asset allocation assumes that the investor has one wealth account and makes her consumption and portfolio choice accordingly. However, in practice most investors also save through their pension plans for retirement. Because a pension account is different in nature from a regular wealth account (for instance, investors cannot withdraw pension wealth for consumption before retirement and short sale is not allowed in a pension portfolio), the portfolio decision made for each account should be investigated separately. We address this issue by incorporating an exogenous pension account into our baseline model. We find that the optimal portfolio in the employment state contains an additional component that comes from the exogenous pension account. The higher the pension wealth relative to total wealth or the more sensitive the consumption in the employment state to the pension wealth, the stronger the effect of the pension portfolio on the regular portfolio.

Section 2.2 specifies the model and Section 2.3 solves the optimal consumption and portfolio choice problem. Model calibrations are conducted in Section 2.4. Section 2.5 compares the out-of-sample performance of our dynamic strategy with two benchmark strategies. Section 2.6 extends the model by incorporating an exogenous pension account. Concluding remarks are given in Section 2.7. All proofs are provided in the Appendix.

2.2 Model Specification

2.2.1 Assumptions

Following Viceria (2001), we assume two states for individual investors, namely, the employment state and the retirement state, with probabilities π^e and $\pi^r = 1 - \pi^e$, respectively. The process is irreversible, i.e., once the retirement state occurs, the individual is not allowed to go back to the employment state. In the employment state, the individual's labor income follows the process: $Y_{t+1} = Y_t \exp(g + \varsigma_{t+1})$, where Y_t is individual labor income received at time *t*, *g* is the expected growth rate of labor income, and ς_{t+1} is labor income shock, assumed to be normally distributed with mean zero and variance σ_ς^2. In the retirement state, labor income is set to zero and the investor must live off her wealth for the rest of her life.

There are *n* assets available for investment, and the first asset is risk-free. Therefore, the portfolio gross return can be written as $R_{p.t+1} = \sum_{i=2}^{n} \alpha_{i,t} (R_{i,t+1} - R_f) + R_f$, where R_f is the constant risk-free interest rate, $R_{i,t}$ is the return on asset *i* at time *t*, and $\alpha_{i,t}$ denotes the portfolio weight of asset *i* at time *t*. Following the setting in Campbell, *et al*. (2003), we define $\mathbf{X}_{t+1} \equiv \begin{bmatrix} r_{2,t+1} - r_f \\ r_{3,t+1} - r_f \\ \vdots \\ r_{n,t+1} - r_f \end{bmatrix}$, where $r_f = \ln(R_f)$, and $r_{i,t+1} = \ln(R_{i,t+1}),\ i = 2, ..., n$. $\mathbf{X}_{t+1}$ is an $(n\text{-}1)\times 1$ vector of excess risky log returns. Let $\mathbf{S}_{t+1}$ represent the vector of other state variables that can predict excess risky returns. Stacking $\mathbf{X}_{t+1}$ and $\mathbf{S}_{t+1}$ yields an $m\times 1$ state vector $\mathbf{Z}_{t+1}$, which is assumed to follow the first-order vector autoregressive (VAR(1)) process:

$$\mathbf{Z}_{t+1} = \mathbf{\Phi}_0 + \mathbf{\Phi}_1 \mathbf{Z}_t + \mathbf{v}_{t+1}, \tag{2.1}$$

where $\mathbf{\Phi}_0$ is an $m\times 1$ constant vector of intercepts, $\mathbf{\Phi}_1$ is an $m\times m$ constant matrix of slope coefficients, $\mathbf{v}$ is an $m\times 1$ vector of shocks to state variables and $\mathbf{v} \sim N(0, \mathbf{\Sigma}_v)$. We allow shocks to state variables to be correlated with labor income shock and denote their covariance by $\sigma(\mathbf{v}_{t+1}, \varsigma_{t+1}) = \boldsymbol{\sigma}_{v\varsigma}$.

The investor has a standard, time separable power utility function defined on consumption: $U(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma}$, where γ is the coefficient of relative risk aversion and $1/\gamma$ represents the elasticity of intertemporal substitution of consumption. The investor's optimization problem at time *t* is:

$$\underset{\left\{C_s;\left[\alpha_{i,s}\right]_{i=2}^{n}\right\}_{s=t}^{\infty}}{\mathrm{Max}} \mathrm{E}_t\left[\sum_{s=t}^{\infty}\delta^{s-t}U\left(C_s\right)|\,Y_t,\mathbf{v}_t,\varsigma_t\right], \tag{2.2}$$

subject to the budget constraints

$$W_{t+1}^{e}=\left(W_t^{e}+Y_t-C_t^{e}\right)R_{p,t+1} \text{ in the employment state, and}$$

$$W_{t+1}^{r}=\left(W_t^{r}-C_t^{r}\right)R_{p,t+1} \text{ in the retirement state.}$$

Following Viceira (2001), we obtain two sets of Euler equations:

$$1=\mathrm{E}_t\left\{\left[\pi^{e}\delta^{e}\left(\frac{C_{t+1}^{e}}{C_t^{e}}\right)^{-\gamma}+\left(1-\pi^{e}\right)\delta^{r}\left(\frac{C_{t+1}^{r}}{C_t^{e}}\right)^{-\gamma}\right]R_{i,t+1}\right\} \text{ in the employment state,}$$

and

$$1=\mathrm{E}_t\left\{\left[\delta^{r}\left(\frac{C_{t+1}^{r}}{C_t^{r}}\right)^{-\gamma}\right]R_{i,t+1}\right\} \text{ in the retirement state,}$$

where $i=1,\ 2,\ ...,\ n,$ and p, $\delta^{\mathrm{e}}=\delta$ and $\delta^{r}=\left(1-\pi^{d}\right)\delta$.

Since in the employment state the realized state in the next period is unknown, the Euler equation in the employment state has a probability form.

2.2.2 Approximation Framework

The log-normality assumption of excess risky asset returns preserves the limited liability nature of financial assets, but it cannot be extended to portfolio returns. We use an approximation relation between log portfolio return and log returns on individual assets (e.g., Campbell, *et al*., 2003), which can be expressed as

$$r_{p,t+1} \approx r_f + \boldsymbol{\alpha}_t{}' \mathbf{X}_{t+1} + \frac{1}{2}\boldsymbol{\alpha}_t{}'(\boldsymbol{\sigma}_X^2 - \boldsymbol{\Sigma}_{XX}\boldsymbol{\alpha}_t), \tag{2.3}$$

where $\boldsymbol{\Sigma}_{XX}$ is the variance-covariance matrix of shocks to excess returns and $\boldsymbol{\sigma}_X^2$ is the diagonal of $\boldsymbol{\Sigma}_{XX}$. The relation holds exactly in continuous time and is very accurate for short time intervals.

From Viceira (2001), we obtain the log linear approximations of budget constraints in both states as well as the corresponding Euler equations. Using lowercase letters to denote variables in log, we get the approximate budget constraint in the employment state: [1]

$$w_{t+1}^e - y_{t+1} \approx k^e + \rho_w^e(w_t^e - y_t) - \rho_c^e(c_t^e - y_t) - \Delta y_{t+1} + r_{p,t+1}^e, \tag{2.4}$$

where $k^e = -(1+\rho_c^e - \rho_w^e)\log(1+\rho_c^e - \rho_w^e) - \rho_w^e \log \rho_w^e + \rho_c^e \log \rho_c^e$,

$$\rho_c^e = \frac{\exp\left[\mathrm{E}(c_t - y_t)\right]}{1+\exp\left[\mathrm{E}(w_t^e - y_t)\right] - \exp\left[\mathrm{E}(c_t - y_t)\right]} \text{ and } \rho_w^e = \frac{\exp\left[\mathrm{E}(w_t^e - y_t)\right]}{1+\exp\left[\mathrm{E}(w_t^e - y_t)\right] - \exp\left[\mathrm{E}(c_t - y_t)\right]}.$$

The budget constraint in the retirement state can be approximated by

$$w_{t+1}^r - w_t^r = k^r - \rho_c^r(c_t^r - w_t^r) + r_{p,t+1}^r, \tag{2.5}$$

[1] The log linear budget constraint in the employment state is obtained by conducting a first-order Taylor expansion around the long-term mean of log consumption-labor income and log wealth-labor income ratio, while that in the retirement state is obtained by conducting a first order Taylor expansion around the long-term mean of log consumption-wealth ratio.

[illegible] and $\rho_c^r = \dfrac{\exp\left[\mathrm{E}(c_t^r - w_t^r)\right]}{1-\exp\left[\mathrm{E}(c_t^r - w_t^r)\right]}$.

The log-linear approximations of the Euler equations are

$0 = \sum_{s=e,r} \pi^s \left\{ \log \delta^s - \gamma \mathrm{E}_t \left(c_{t+1}^s - c_t^e \right) + \mathrm{E}_t \left(r_{i,t+1} \right) + \frac{1}{2} \mathrm{Var}_t \left[r_{i,t+1} - \gamma \left(c_{t+1}^s - c_t^e \right) \right] \right\}$ in the employment state,

and $0 = \log \delta^r - \gamma \mathrm{E}_t \left(c_{t+1}^r - c_t^r \right) + \mathrm{E}_t \left(r_{i,t+1} \right) + \frac{1}{2} \mathrm{Var}_t \left[r_{i,t+1} - \gamma \left(c_{t+1}^r - c_t^r \right) \right]$ in the retirement state,

where $i = 1, 2, \ldots, n$, and p.

2.3 Optimal Portfolio and Consumption Choice

2.3.1 Retirement State

We first guess a solution in the retirement state and then verify that the solution satisfies the Euler equation and solve for the unknown parameters. In the retirement state, the optimal portfolio and consumption choice can be written as

$$\boldsymbol{\alpha}_t^r = \mathbf{A}_0 + \mathbf{A}_1 \mathbf{Z}_t , \qquad (2.6)$$

$$c_t^r - w_t^r = b_0^r + \mathbf{B}_1' \mathbf{Z}_t + \mathbf{Z}_t' \mathbf{B}_2 \mathbf{Z}_t , \qquad (2.7)$$

where b_0^r is a constant coefficient and $\mathbf{A}_0$, $\mathbf{A}_1$, $\mathbf{B}_1$, and $\mathbf{B_2}$ are coefficient matrices with dimensions $(n-1)\times 1$, $(n-1)\times m$, $m\times 1$ and, $m\times m$, respectively.

Optimal Portfolio Choice in the Retirement State

The optimal portfolio rule in the retirement state can be written as

$$\boldsymbol{\alpha}_t^r = \frac{1}{\gamma}\boldsymbol{\Sigma}_{XX}^{-1}\left[\mathrm{E}_t\left(\mathbf{X}_{t+1}\right)+\frac{1}{2}\mathrm{Var}_t\left(\mathbf{X}_{t+1}\right)\right]-\boldsymbol{\Sigma}_{XX}^{-1}\left(\boldsymbol{\Lambda}_0+\boldsymbol{\Lambda}_1\mathbf{Z}_t\right), \qquad (2.8)$$

where $\boldsymbol{\Lambda}_0 = \left(\boldsymbol{\Sigma}_v\mathbf{H}_X'\right)' B_1 + \left(\boldsymbol{\Sigma}_v\mathbf{H}_X'\right)'\left(\mathbf{B}_2+\mathbf{B}_2'\right)\boldsymbol{\Phi}_0$, $\boldsymbol{\Lambda}_1 = \left(\boldsymbol{\Sigma}_v\mathbf{H}_X'\right)'\left(\mathbf{B}_2+\mathbf{B}_2'\right)\boldsymbol{\Phi}_1$, and $\mathbf{H}_X$ is a selection matrix that picks elements corresponding to excess risky log returns $\mathbf{X}$.

Proof: See Appendix

The above result shows that the optimal portfolio comprises two components, the "myopic demand" and the "intertemporal hedging demand." The myopic demand, the first term on the right hand side of equation (2.8), is proportional to the risk premium relative to the variance-covariance matrix, scaled by the reciprocal of the coefficient of relative risk aversion. The intertemporal hedging demand, the second term on the right hand side of equation (2.8), comes from the covariance between risky asset returns and log consumption-wealth ratio. Since log consumption-wealth ratio is a constant when the investment opportunity set is constant, the intertemporal hedging demand vanishes when asset returns are independently and identically distributed. Substituting $\mathrm{E}_t\left(\mathbf{X}_{t+1}\right)=\mathbf{H}_X\boldsymbol{\Phi}_0+\mathbf{H}_X\boldsymbol{\Phi}_1\mathbf{Z}_t$ into the above result gives

$$\boldsymbol{\alpha}_t^r = \mathbf{A}_0 + \mathbf{A}_1 \mathbf{Z}_t ,$$

where $\mathbf{A}_0 = \frac{1}{\gamma} \boldsymbol{\Sigma}_{XX}^{-1} \left(\mathbf{H}_X \boldsymbol{\Phi}_0 + \frac{1}{2} \boldsymbol{\sigma}_X^2 \right) - \boldsymbol{\Sigma}_{XX}^{-1} \boldsymbol{\Lambda}_0$ and $\mathbf{A}_1 = \frac{1}{\gamma} \boldsymbol{\Sigma}_{XX}^{-1} \left(\mathbf{H}_X \boldsymbol{\Phi}_1 \right) - \boldsymbol{\Sigma}_{XX}^{-1} \boldsymbol{\Lambda}_1$.

This shows that the optimal portfolio is linear in state variables and depends on the coefficients of optimal consumption. We now solve the optimal consumption choice.

Optimal Consumption in the Retirement State

To solve the optimal consumption problem in the retirement state, we set $i = p$ in the log-linearized Euler equation to obtain

$$\mathrm{E}_t \left(c_{t+1}^r - c_t^r \right) = \frac{1}{\gamma} \left\{ \log \delta^r + \mathrm{E}_t \left(r_{p,t+1} \right) + \frac{1}{2} \mathrm{Var}_t \left[r_{p,t+1} - \gamma \left(c_{t+1}^r - c_t^r \right) \right] \right\} . \quad (2.9)$$

Equation (2.9) shows that the expected consumption growth increases when the investor becomes more patient or when she expects a higher portfolio return. The variance term on the right hand side indicates a precautionary saving motive. The investor reduces current consumption when the variance is higher, increasing the expected consumption growth rate. Also, the precautionary saving motive increases when portfolio return and consumption growth are negatively correlated (see also Campbell, *et al*., 2003).

The optimal log consumption-wealth ratio can be written as

$$c_t^r - w_t = \Xi_0^r + \boldsymbol{\Xi}_1^r \mathbf{Z}_t + \boldsymbol{\Xi}_2^r vec(\mathbf{Z}_t \mathbf{Z}_t') , \quad (2.10)$$

where Ξ_0^r, $\boldsymbol{\Xi}_1^r$, and $\boldsymbol{\Xi}_2^r$ are determined in Appendix B.

Proof: See Appendix B.

We can compare these coefficients to our initial guess and solve for the unknown parameters. The optimal consumption and thus portfolio choice in the retirement state are then determined accordingly.

2.3.2 Employment State

As in the retirement state, we first guess a solution in the employment state and then solve for the unknown coefficients. The optimal portfolio and consumption choice can be expressed as

$$\boldsymbol{\alpha}_t^e = \mathbf{M}_0 + \mathbf{M}_1 \mathbf{Z}_t , \tag{2.11}$$

$$c_t^e - y_t = b_0^e + \mathbf{N}_1' \mathbf{Z}_t + \mathbf{Z}_t' \mathbf{N}_2 \mathbf{Z}_t + b_1^e \left(w_t^e - y_t \right), \tag{2.12}$$

where b_0^e and b_1^e are constant coefficients and $\mathbf{M}_0$, $\mathbf{M}_1$, $\mathbf{N}_1$, and $\mathbf{N}_2$ are coefficient matrices with dimensions $(n-1)\times 1$, $(n-1)\times m$, $m \times 1$ and, $m \times m$, respectively.

<u>**Optimal Portfolio Choice in the Employment State**</u>

The optimal portfolio choice in the employment state can be written as

$$\boldsymbol{\alpha}_t^e = \frac{1}{\gamma \bar{b}_1^e} \boldsymbol{\Sigma}_{XX}^{-1} \left[\mathrm{E}_t\left(\mathbf{X}_{t+1}\right) + \frac{1}{2} \mathrm{Var}_t\left(\mathbf{X}_{t+1}\right) \right]$$

$$- \frac{1}{\bar{b}_1^e} \boldsymbol{\Sigma}_{XX}^{-1} \left\{ \pi^e \boldsymbol{\Lambda}_0^e + \left(1-\pi^e\right)\boldsymbol{\Lambda}_0^r + \left[\pi^e \boldsymbol{\Lambda}_1^e + \left(1-\pi^e\right)\boldsymbol{\Lambda}_1^r\right]\mathbf{Z}_t + \pi^e (1-b_1^e) \boldsymbol{\sigma}_{\varsigma X} \right\}$$

(2.13)

where $\bar{b}_1^e = \pi^e b_1^e + \left(1-\pi^e\right)$ and $\boldsymbol{\Lambda}_0^e$, $\boldsymbol{\Lambda}_1^e$, $\boldsymbol{\Lambda}_0^r$, and $\boldsymbol{\Lambda}_1^r$ are determined in Appendix C.

Proof: See Appendix C.

Similar to the retirement state, the optimal portfolio choice in the employment state comprises the myopic demand and the intertemporal hedging demand. However, the intertemporal hedging demand includes an additional covariance term between risky asset returns and labor income shock, which is held for hedging labor income risk. This hedging demand for labor income risk is negative when labor income shock and risky asset returns are positively correlated but vanishes when labor income is independent of asset returns.

Substituting $\mathrm{E}_t\left(\mathbf{X}_{t+1}\right) = \mathbf{H}_X \boldsymbol{\Phi}_0 + \mathbf{H}_X \boldsymbol{\Phi}_1 \mathbf{Z}_t$ into the above result and rearranging terms yields $\boldsymbol{\alpha}_t^e = \mathbf{M}_0 + \mathbf{M}_1 \mathbf{Z}_t$, where

$$\mathbf{M}_0 = \frac{1}{\gamma \bar{b}_1^e} \boldsymbol{\Sigma}_{XX}^{-1} \left(\mathbf{H}_X \boldsymbol{\Phi}_0 + \frac{1}{2} \boldsymbol{\sigma}_X^2 \right) - \frac{1}{\bar{b}_1^e} \boldsymbol{\Sigma}_{XX}^{-1} \left[\pi^e \boldsymbol{\Lambda}_0^e + \left(1-\pi^e\right)\boldsymbol{\Lambda}_0^r + \pi^e \left(1-b_1^e\right) \boldsymbol{\sigma}_{\varsigma X} \right], \text{ and}$$

$$\mathbf{M}_1 = -\frac{1}{\gamma \bar{b}_1^e} \boldsymbol{\Sigma}_{XX}^{-1} \left\{ \gamma \left[\pi^e \boldsymbol{\Lambda}_1^e + \left(1-\pi^e\right)\boldsymbol{\Lambda}_1^r \right] - \mathbf{H}_X \boldsymbol{\Phi}_1 \right\}.$$

Thus, as in the retirement state, the optimal portfolio is linear in the state variables and depends on consumption decisions. Furthermore, the portfolio decision in the employment state depends on the consumption and portfolio choice made in the retirement state.

Optimal Consumption in the Employment State

The optimal log consumption-labor income ratio can be written as

$$c_t^e - y_t = \Theta_0 + \boldsymbol{\Theta}_1 \mathbf{Z}_t + \boldsymbol{\Theta}_2 vec(\mathbf{Z}_t \mathbf{Z}_t') + \Theta_3 (w_t^e - y_t), \tag{2.14}$$

where Θ_0, $\boldsymbol{\Theta}_1$, $\boldsymbol{\Theta}_2$, and Θ_3 are determined in Appendix D.

Proof: See Appendix D.

The above result confirms our initial conjecture that the optimal log consumption-labor income ratio is a quadratic function of state variables. Then we can obtain the optimal consumption and portfolio choice by solving this non-linear equations system.

2.4 Model Calibration

We assume that there are three assets, namely, risk-free asset, stock, and 20-year bond, and that the excess log risky asset returns follow a VAR(1) process. Using annual return data from Campbell, *et al*. (2003) for years 1890-1998 and updating it to year 2005, we estimate risk-free rate and stock returns with 6-month commercial paper returns and

S&P 500 index, respectively.[2] Also, following Campbell, *et al*. (2003), we use Moody's Aaa bond yields to compute the corresponding 20-year bond returns and convert them to real terms with the Producer Price Index (PPI).

The estimated annual log risk-free interest rate r_f is 1.819 percent, and the estimated VAR process is

$$\mathbf{Z}_{t+1} = \begin{bmatrix} 0.0455 \\ 0.0021 \end{bmatrix} + \begin{bmatrix} 0.0617 & 0.0164 \\ 0.0954 & 0.1210 \end{bmatrix} \mathbf{Z}_t + \mathbf{v}_{t+1}, \tag{2.15}$$

where $\mathbf{Z}_t = \begin{bmatrix} r_{2,t} - r_f \\ r_{3,t} - r_f \end{bmatrix}$ with $r_{2,t}$ and $r_{3,t}$ being log stock and bond returns, respectively, and $\mathbf{v}_t \sim N\left(0, \begin{bmatrix} 0.0324 & 0.0004 \\ 0.0004 & 0.0039 \end{bmatrix}\right)$. These parameters imply the unconditional expected excess stock and bond returns equal to 4.87 percent and 0.77 percent, respectively. The implied unconditional variance-covariance matrix for the excess returns is $\begin{bmatrix} 0.0326 & 0.0005 \\ 0.0005 & 0.0043 \end{bmatrix}$.

In our baseline model, we assume the following parameter values: the annual labor income risk $\sigma_\varsigma = 0.1$, the income growth rate g is set such that $\mathrm{E}_t(Y_{t+1}/Y_t)$ =1.03, the time discount factor δ = 1/1.1, the coefficient of relative risk aversion γ = 8, and the expected lifetime after retirement is 10 years (i.e., $\pi^d = 0.1$).

[2] Since after year 1998 returns on the 6-month commercial paper are not available, we update the data with 3-month commercial paper returns.

2.4.1 Relative Risk Aversion and Expected Time until Retirement

First we consider how the optimal portfolio varies with the investor's level of relative risk aversion and the expected time until retirement. We consider cases where the coefficients of relative risk aversion $\gamma = \{4, 6, 8, 10, 12\}$ and the expected time until retirement equal to {30, 20, 10, 5} years. For simplicity, we first assume that labor income is independent of risky asset returns and will relax this assumption later.

Table 2.2 Panel A displays the optimal portfolio weights under different combinations of relative risk aversion and expected time before retirement when labor income is independent of asset returns. Consistent with Viceira (2001), the optimal allocation to risky assets is systematically larger in the employment state than in the retirement state. This agrees with the argument that when labor income is independent or weakly correlated with asset returns, human capital, the present value of labor income, is more like an implicit investment in the risk-free asset and thus the investor may optimally allocate more to risky assets to satisfy her target portfolio in her investment of total wealth (financial wealth + human capital) (Bodie, *et al*., 1992). When the employed investor comes closer to the retirement state, the optimal allocation to risky assets decreases and approaches the level in the retirement state. This is also consistent with financial advisors' suggestion that investors should decrease their allocation to risky assets when they are close to retirement. For instance, with $\gamma = 8$, the investor allocates 33 percent of her wealth to stock when her expected time until retirement is 30 years, but the allocation decreases to 24 percent when she expects to retire in 5 years, which is close to the optimal portfolio weight of 21 percent when she actually retires. As the investor becomes more risk averse, her optimal investment in stock in the employed state decreases and approaches that in the retirement state. Intuitively, more risk-averse investors save more such that labor income plays a less important role, making the asset allocation decision in employment state similar to that in the retirement state. In both the retirement and employment states, the bond/stock ratio increases as the investor becomes more risk averse. This agrees with financial advisors' recommendation that conservative investors should invest more in bonds relative to stocks than aggressive investors.

Table 2.3 Panel A shows the hedging demands for stock and bond as a percentage of their respective total demands. Both hedging demands are negative and increase in absolute magnitude with the investor's degree of relative risk aversion. When labor income is uncorrelated with asset returns, the percentage hedging demands for both risky assets in absolute magnitude are systematically lower in the employment state than in the retirement state and decrease when expected time until retirement is longer. Thus, the presence of independent labor income in general reduces the investor's hedging demands for risky assets.

Table 2.4 Panel A reports the exponential of mean log wealth-labor income ratio $\exp \mathrm{E}\left(w_t^e - y_t\right)$ under different combinations of relative risk aversion and expected time before retirement. The mean wealth-labor income ratio increases with the investor's degree of risk aversion. Intuitively, the investor's degree of risk aversion can affect the wealth-labor income ratio in two ways. When the investor is more risk averse, she invests less in risky asset, leading to a lower wealth-labor income ratio since risky assets have higher expected returns. On the other hand, a higher degree of relative risk aversion implies a stronger precautionary saving motive and thus a higher mean wealth-labor income ratio. Our model calibration result suggests that the second effect dominates the first.

Now we relax the independence assumption and allow labor income to be correlated with risky asset returns. We consider the cases where labor income is correlated either with stock or bond return, with the correlation coefficient set to 25 percent. The resulting portfolio weight, hedging demand and mean wealth-labor income are displayed in Panels B and C of Tables 2.2, 2.3 and 2.4 respectively.

As Table 2.2 Panel B shows, when labor income is positively correlated with stock return, the investor decreases her investment in stock and allocates more wealth to bond and the risk-free asset. Furthermore, the bond/stock ratio increases at a higher rate when the investor becomes more risk averse than in the case where labor income and asset returns are independent. On the other hand, when labor income is positively correlated with bond returns (Table 2.2 Panel C), the investor increases her investment in stock and risk-free asset and decreases her investment in bond. Contrary to the cases where labor income is independent or positively correlated with stock returns, the bond/stock ratio decreases as the investor becomes more risk averse. Thus, complementing the prior literature that horizon effect from time-varying investment opportunities can explain the asset allocation puzzle (Bajeux-Besnainou, *et al*., 2001), our work shows that introducing labor income may qualitatively affect the basic result. When labor income is positively correlated with bond returns, the effect that the more risk-averse investor decreases her investment in bond dominates the horizon effect which increases the bond/stock ratio, resulting in an overall lower bond/stock ratio. This result also uncovers the complicated interactions between labor income and multiple risky asset returns, which are hidden when only one risky asset is considered.

Table 2.3 Panels B and C illustrate the effects of labor income correlations with risky assets on investors' hedging demand for risky assets. Although the previous result shows that the presence of independent labor income decreases the investor's hedging demands for risky assets (in absolute terms), the pattern is reversed when labor income is correlated with risky asset returns. For instance, when labor income is positively correlated with stock returns, the hedging demand for stock (in absolute terms) is larger in the employment state than in the retirement state and increases when expected time until retirement is longer. The same pattern is observed when labor income is positively correlated with bond returns.

In both cases where labor income is positively correlated with stock and bond returns, the mean wealth-labor income ratio is lower than in the case where labor income is independent of asset returns (Table 2.4 Panels B and C). Intuitively, when labor income is positively correlated with risky asset returns, the investor reduces her allocation to the correlated asset and invests instead more in the risk-free asset. Since the risk-free asset has a lower expected return, the mean wealth-labor income ratio is lower.

2.4.2 Labor Income Growth and Expected Retirement Horizon

Table 2.5 reports the effect of labor income growth on optimal portfolio decisions in the employment state. We consider the cases where the labor income growth $\mathrm{E}_t(Y_{t+1}/Y_t)$ = {1.02, 1.04, 1.06, 1.08} and the expected time until retirement = {30, 20, 10, 5} years. For simplicity, labor income is assumed to be independent of risky asset returns. Other parameters are set as follows: $\gamma = 8$, $\sigma_\varsigma = 0.1$, $\delta = 1/1.1$, and $\pi^d = 0.1$. As Table 2.5 shows, for a given expected year to retirement, investment in risky assets increases with labor income growth. Since labor income is assumed to be independent of risky asset returns, human capital is like an implicit investment in the risk-free asset. An increase in labor income growth implies a higher investment in risk-free asset, encouraging the investor to hold more risky financial assets.

There has been much empirical evidence showing an inverted U-shaped relationship between labor income growth and the expected time to retirement (Rosen and Taubman, 1982; and Klevmarken, 1982). That is, an investor's labor income growth rate is lower when she is either at the beginning of her career or near retirement. We thus compare the asset allocation under short or long expected time until retirement with low labor income growth to that under a medium time to retirement with high labor income growth. Consistent with the single asset case in Viceira (2001), we find that an investor with a medium time to retirement and high income growth holds a riskier portfolio. For instance, an investor with 20 years to retirement and labor income growth $\mathrm{E}_t(Y_{t+1}/Y_t)$=1.08 allocates 48 percent of total wealth to stock and 57 percent to bond, while an investor with 5 (30) years to retire and labor income growth rate

$\mathrm{E}_t(Y_{t+1}/Y_t)=1.02$ holds 23 (28) percent in stock and 28 (34) percent in bond. Thus, the allocation to risky assets inherits the inverted U-shaped pattern of labor income growth with respect to time until retirement. That is, young workers or those near their retirement ages should invest less in risky assets, while those with a higher labor income growth during their medium career life should increase their risk exposure by holding more risky assets.

2.4.3 Time Preference Rate

Table 2.6 examines the effects of time preference on optimal asset allocation. We consider the cases where the time discount factor δ is set such that -lnδ = {0.02, 0.04, 0.06, 0.08, 0.1} in combination with 10, 20 and 30 years expected time until retirement. Labor income is assumed to be independent of risky asset returns, and the other model parameters are set as follows: $\gamma=8$, $\sigma_{\varsigma}=0.1$, $\delta=1/1.1$, and $\pi^d=0.1$.

In general, as the investor becomes more patient (i.e., with a lower value of -lnδ), her portfolio weights in risky assets decrease but her wealth-labor income ratio increases, suggesting that she accumulates wealth in a more conservative way but saves more. Not surprisingly, the longer the expected time until retirement, the greater the effect of time preference on the optimal portfolio in the employment state. For instance, in our example, the optimal portfolio weight in stock (bond) ranges between 25-26 (29-31) percent when the expected time until retirement is 10 years, but the range increases to 28-33 (34-40) percent when the expected time until retirement is 30 years.

2.4.4 Labor Income Risk

Table 2.7 investigates how the optimal portfolio varies with labor income risk. In our model, an increase in labor income risk σ_ζ also raises the expected labor income growth rate because $\mathrm{E}_t(Y_{t+1}/Y_t) = e^{g+\frac{1}{2}\sigma_\zeta^2}$. To make the comparison meaningful, we increase the *uncompensated labor income risk* by varying the growth rate *g* such that $\mathrm{E}_t(Y_{t+1}/Y_t)$ =1.03 is maintained (see Viceira, 2001). We consider the cases where labor income is independent of asset returns, $\sigma_\varsigma = \{0,\ 0.04,\ 0.08,\ 0.12,\ 0.16,\ 0.20\}$, $\gamma = 8$, $\pi^e = 0.05$, and $\pi^d = 0.1$.

Consistent with Viceira (2001), as the uncompensated labor income risk increases, the allocation to risky assets decreases while the mean wealth-labor income ratio increases. The uncompensated labor income risk affects the mean wealth-labor income ratio in opposite directions. On the one hand, when labor income risk increases, the investor invests less in risky assets, leading to a lower mean wealth-labor income ratio since risky assets have higher expected returns. On the other hand, as the uncompensated labor income risk increases, the growth rate *g* has to be decreased to keep $\mathrm{E}_t(Y_{t+1}/Y_t)$ =1.03, thereby raising the mean wealth-labor income ratio. Our calibration results suggest that the second effect dominates the first such that the mean wealth-labor income ratio increases with uncompensated labor income risk.

2.5 Performance Test and Economic Significance

In this section, we measure the performance of our dynamic strategy and evaluate the economic significance of incorporating time-varying investment opportunities into investors' portfolio choice problem. Firstly, we compare the out-of-sample performance of our strategy with two benchmark strategies: (1) the optimal strategy from a model that assumes constant investment opportunities; and (2) the "1/*n* rule" which allocates wealth evenly across different investment objects. If there are *n* assets available for investment, the portfolio weight assigned to each asset is equal to 1/*n*. This strategy is found to be used by many investors in their pension plans (Benartzi and Thaler, 2001; and Huberman and Jiang, 2006). We use mean portfolio return and Sharpe ratio as comparison criteria. Secondly, to evaluate the economic significance, we estimate the investor's welfare loss by computing the cost that the investor is willing to pay if she can change from the model with a constant investment opportunity to the VAR model when the underlying return generating process follows VAR.

2.5.1 Out-of-sample Performance Test

When the investor believes that the investment opportunity is constant, i.e., the excess risky asset returns are i.i.d. over time, the optimal portfolio and consumption choice can be shown as follows.

The optimal portfolio and consumption choices in the retirement state:

$$\boldsymbol{\alpha}_t^r = \mathbf{A}_0, \tag{2.16}$$

$$c_t^r - w_t^r = b_0^r, \tag{2.17}$$

where $\mathbf{A}_0 = \frac{1}{\gamma b_1^r} \mathbf{\Sigma}_{XX}^{-1} \left(\boldsymbol{\mu} + \frac{1}{2} \boldsymbol{\sigma}_X^2 \right)$, and

$$b_0^r = \log \left\{ 1 - \exp \left[\left(\frac{1}{\gamma} - b_1^r \right) \mathrm{E}\left(r_{p,t+1}^r \right) + \frac{1}{\gamma} \log \delta^r + \frac{1}{2\gamma} (1-\gamma)^2 \operatorname{Var}\left(r_{p,t+1}^r \right) \right] \right\}.$$

The optimal portfolio and consumption choices in the employment state:

$$\boldsymbol{\alpha}_t^e = \mathbf{M}_0, \tag{2.18}$$

$$c_t^e - y_t = b_0^e + b_1^e \left(w_t^e - y_t \right), \tag{2.19}$$

where $\mathbf{M}_0 = \frac{1}{\gamma \bar{b}_1^e} \mathbf{\Sigma}_{XX}^{-1} \left(\boldsymbol{\mu} + \frac{1}{2} \boldsymbol{\sigma}_X^2 \right) - \frac{\pi^e \left(1 - b_1^e \right)}{\bar{b}_1^e} \mathbf{\Sigma}_{XX}^{-1} \boldsymbol{\sigma}_{\varsigma X}$,

$$b_1^e = \frac{-\left[1 - \pi^e \rho_w^e + \left(1 - \pi^e \right) b_1^r \rho_c^e \right] + \sqrt{\left[1 - \pi^e \rho_w^e + \left(1 - \pi^e \right) \rho_c^e \right]^2 + 4 \pi^e \rho_c^e \left(1 - \pi^e \right) \rho_w^e}}{2 \pi^e \rho_c^e},$$

$$b_0^e = \frac{-1}{\left(1 - \pi^e \right) \bar{b}_1^e \rho_c^e} \left[\left(\frac{1}{\gamma} - \bar{b}_1^e \right) \mathrm{E}\left(r_{p,t+1}^e \right) + \frac{1}{\gamma} \sum_{s=e,r} \log(\delta^s) + \frac{1}{2\gamma} V^e - \pi^e \left(1 - b_1^e \right) g - \left(1 - \pi^e \right) b_0^r - k^e \right],$$

and

$$\begin{aligned} V^e = & \left[\pi^e \left(1 - \gamma b_1^e \right)^2 + \left(1 - \pi^e \right) (1-\gamma)^2 \right] \operatorname{Var}\left(r_{p,t+1}^e \right) \\ & + \pi^e \gamma^2 \left(1 - b_1^e \right)^2 \operatorname{Var}\left(\Delta y_{t+1} \right) - 2 \pi^e \gamma \left(1 - \gamma b_1^e \right) \left(1 - b_1^e \right) \operatorname{Cov}\left(r_{p,t+1}^e, \Delta y_{t+1} \right). \end{aligned}$$

We assume three asset categories: risk-free asset, CRSP value-weighted stock market portfolio, and 5-year government bond. The risk-free asset is approximated by the 90-day Treasury bill. We use quarterly return data from Campbell, *et al*. (2003) for the period 1952.Q2-1999.Q4 and update it to 2007.Q3. Nominal returns are converted to real terms using the Consumer Price Index (CPI). We use returns for the first 20 years (1952.Q2-1971.Q4) to estimate the model and compare the out-of-sample performance of our dynamic strategy with two benchmark strategies using returns for the period 1972.Q1-2007.Q3. The benchmark strategies are: (1) the optimal strategy from a model that assumes constant investment opportunities; and (2) the "1/*n* rule."

In our dynamic model, the excess returns on stock and bond are assumed to follow a VAR(1) process, whose estimate is reported below:

$$\mathbf{Z}_{t+1} = \begin{bmatrix} 0.0162 \\ 0.0016 \end{bmatrix} + \begin{bmatrix} 0.1133 & 0.3547 \\ -0.0506 & 0.0223 \end{bmatrix} \mathbf{Z}_t + \mathbf{v}_{t+1},$$

where $\mathbf{Z}_t = \begin{bmatrix} r_{2,t} - r_f \\ r_{3,t} - r_f \end{bmatrix}$ with $r_{2,t}$ and $r_{3,t}$ being the log stock and bond returns, respectively, and $\mathbf{v}_t \sim N\left(0, \begin{bmatrix} 0.0052 & 0.0002 \\ 0.0002 & 0.0004 \end{bmatrix}\right)$, and. The estimated risk-free rate r_f is 0.28 percent per quarter. For the model with a constant investment opportunity, the estimated mean excess returns are equal to 1.85 and 0.07 percent per quarter for stock and bond, respectively, and the variance-covariance matrix is $\begin{bmatrix} 0.0053 & 0.0001 \\ 0.0001 & 0.0004 \end{bmatrix}$.

We assume that the expected time to retirement and lifetime after retirement are equal to 20 and 10 years, respectively.[3] Labor income is assumed to be independent of asset returns. We adjust the other baseline parameter values used in Section 2.4 to quarterly rates. Labor income growth rate g is set such that $\mathrm{E}_t(Y_{t+1}/Y_t)$ =1+0.03/4, σ_ς = 0.05, γ = 8, and $\delta = (1/1.1)^{0.25}$.

The results are summarized in Table 2.8. The portfolio strategy under the VAR model has the highest mean return and Sharpe ratio. Its mean annual return is 9.60 percent, significantly higher than that of 5.32 percent from the strategy assuming constant investment opportunities and than that of 3.14 percent from the 1/*n* rule. The Sharpe ratio of 0.61 under the dynamic strategy is again substantially higher than that of 0.36 under the strategy assuming constant investment opportunities and than that of 0.25 under the 1/*n* rule. These results suggest that investors can significantly improve their investment performance by exploiting asset return predictability.

2.5.2 Economic Significance Evaluation

In this section, we conduct the performance test by comparing the conditional value functions between the VAR and the static model, assuming that the VAR process is the true underlying return generating process. The parameters $\mathbf{\Phi}_0$, $\mathbf{\Phi}_1$ and $\mathbf{\Sigma}_v$ are assumed to be equal to those in Section 2.4. The coefficient of relative risk aversion γ is

[3] Since we use quarterly data to estimate the model, the corresponding expected time before retirement and lifetime after retirement are 80 and 40 periods, respectively. The corresponding probabilities are π^e =0.9875 and π^d =0.025.

equal to 8. The state variable $\mathbf{Z}_t$ is set equal to the unconditional mean implied in the process. We assume that the investor has 20 years before retirement and 10 years lifetime after retirement. We set the initial labor income equal to \$64,400 and the initial wealth level equal to \$60,600, following the estimates in Huberman and Jiang (2006).

According to Viceira (2001), the conditional value function in the employment state can be expressed as

$$J^e(W_t, Y_t, \mathbf{Z}_t) = \max_{C,\boldsymbol{\alpha}} \left\{ U(C_t^e) + \mathrm{E}_t\left[\pi^e \delta^e J^e(W_{t+1}, Y_{t+1}, \mathbf{Z}_{t+1}) + (1-\pi^e)\delta^r J^r(W_{t+1}, \mathbf{Z}_{t+1})\right]\right\}, \tag{2.20}$$

where $J^r(W_{t+1}, \mathbf{Z}_{t+1}) = (1-\gamma)^{-1}\exp\left\{(1-\gamma)\left[b_0^r + \mathbf{B}_1'\mathbf{Z}_{t+1} + \mathbf{Z}_{t+1}'\mathbf{B}_2\mathbf{Z}_{t+1}\right]\right\}W_{t+1}^{1-\gamma}$ is the value function in the retirement state and $U(C_t^e)$ is the utility function over consumption in the employment state. If C_t^{e*} is the optimal consumption in the employment state, then

$$U(C_t^{e*}) = (1-\gamma)^{-1}\exp\left\{(1-\gamma)\left[b_0^e + \mathbf{N}_1'\mathbf{Z}_t + \mathbf{Z}_t'\mathbf{N}_2\mathbf{Z}_t + b_1^e\left(w_t^e - y_t\right)\right]\right\}Y_t^{1-\gamma}. \tag{2.21}$$

We simulate time series of asset returns and labor income with 15,000 replications and then take the average of the discounted values as the estimates of conditional value function. We compute the certainty equivalent consumption C corresponding to each model by equating $\sum_{i=1}^{30}\delta^{i-1}\frac{C^{1-\gamma}}{1-\gamma}$ to the conditional value function. Given that the true return generating process follows the VAR, we find that the certainty equivalent consumption in our dynamic model is 7.60 percent higher than the certainty equivalent consumption in the model that assumes constant investment opportunities.

Thus, the investor's welfare loss can be economically significant if she ignores the predictability of asset returns.

2.6 Asset Allocation with an Exogenous Pension Account

Classical models on long-horizon asset allocation usually assume that the investor has only one financial account and makes her portfolio decision accordingly. However, in reality, in addition to the regular financial account, the employed investor saves for retirement through her pension account. These two saving channels are different in some perspectives. For instance, the investor usually does not finance her consumption through pension wealth before retirement. In addition, the investment in pension accounts is in general subject to borrowing and short sale constraints. In this section, we investigate how the existence of a pension account affects the investor's optimal consumption and portfolio decision. To simplify the analysis, we assume that pension wealth investment is exogenously given and study how the investor optimally adjusts her regular portfolio. Modeling the optimal investment strategy for pension account is beyond the scope of this paper and is left for future research.

2.6.1 Model Assumptions

We maintain the same assumptions as in Section 2.2 except that in the employment state, in addition to the regular financial account, the investor has a pension account. Each period in the employment state, the investor contributes a proportion (1-H)

of her labor income to her pension account and cannot consume her pension wealth before retirement.[4] For simplicity, we assume that the portfolio decision in pension account is exogenously given. Thus, the budget constraint in the pension account before retirement can be described as

$$W_{t+1}^{p} = \left[W_t^{p} + (1-H)Y_t\right]R_{p,t+1}^{p}, \tag{2.22}$$

where (1-*H*) is the exogenous contribution rate, W_t^{p} is the pension wealth, Y_t is labor income, and $R_{p,t+1}^{P}$ is the portfolio return in the pension account. Similarly, the budget constraint in the regular financial account can be described as:

$$W_{t+1}^{e} = \left(W_t^{e} + HY_t - C_t\right)R_{p,t+1}^{e}, \tag{2.23}$$

where W_t^{e} is the regular wealth in the employment state, C_t is the consumption made at time *t*, and $R_{p,t+1}^{e}$ is the portfolio return in the regular financial account.

Once the investor jumps to the retirement state, her total wealth is the sum of her regular financial wealth and pension wealth, i.e., $W_{t+1}^{r} = W_{t+1}^{e} + W_{t+1}^{p}$.

2.6.2 Approximation Framework

Once the investor jumps to the retirement state, there is no distinction between pension wealth and regular financial wealth and the investor makes her decision based on

[4] For simplicity, we assume that the contribution rate is exogenous Michaelides, *et al*. (2004) show that the utility cost of an exogenous fixed contribution rate is small when the saving rate is set appropriately.

total wealth. Thus, the optimal portfolio and consumption choice in the retirement state is the same as that shown in Section 2.3. In the following section, we only show the optimal portfolio and consumption decision in the employment state given the exogenous pension account.

Similar to Section 2.2, we use log linear approximation to solve this problem. Using the first-order Taylor expansion around the long-term mean of *log consumption-(H$\times$labor income)* and *log wealth-(H$\times$labor income)* ratio, we obtain the approximate regular financial account budget constraint in the employment state:

$$w_{t+1}^{e}-\tilde{y}_{t+1}\approx-\Delta\tilde{y}_{t+1}+r_{p,t+1}+\tilde{k}^{e}+\tilde{\rho}_{w}^{e}\left(w_{t}^{e}-\tilde{y}_{t}\right)-\tilde{\rho}_{c}^{e}\left(c_{t}-\tilde{y}_{t}\right), \tag{2.24}$$

where lowercase letters denote variables in log,

$$\tilde{k}^{e}=-(1+\tilde{\rho}_{c}^{e}-\tilde{\rho}_{w}^{e})\log(1+\tilde{\rho}_{c}^{e}-\tilde{\rho}_{w}^{e})-\tilde{\rho}_{w}^{e}\log\tilde{\rho}_{w}^{e}+\tilde{\rho}_{c}^{e}\log\tilde{\rho}_{c}^{e},\quad \tilde{y}_{t}=\log(HY_{t}),$$

$$\tilde{\rho}_{w}^{e}=\frac{\exp\left[\mathrm{E}\left(w_{t}^{e}-\tilde{y}_{t}\right)\right]}{1+\exp\left[\mathrm{E}\left(w_{t}^{e}-\tilde{y}_{t}\right)\right]-\exp\left[\mathrm{E}\left(c_{t}-\tilde{y}_{t}\right)\right]}\quad\text{and}\quad\tilde{\rho}_{c}^{e}=\frac{\exp\left[\mathrm{E}\left(c_{t}-\tilde{y}_{t}\right)\right]}{1+\exp\left[\mathrm{E}\left(w_{t}^{e}-\tilde{y}_{t}\right)\right]-\exp\left[\mathrm{E}\left(c_{t}-\tilde{y}_{t}\right)\right]}.$$

Similarly, we take the first-order Taylor expansion around the long-term mean of *log pension wealth-(H$\times$labor income)* ratio to obtain the approximate pension wealth budget constraint:

$$w_{t+1}^{p}-\tilde{y}_{t+1}\approx-\Delta\tilde{y}_{t+1}+r_{p,t+1}^{p}+\tilde{k}^{p}+\tilde{\rho}_{w}^{p}\left(w_{t}^{p}-\tilde{y}_{t}\right), \tag{2.25}$$

where $\tilde{\rho}_w^p = \dfrac{\exp\left[\mathrm{E}\left(w_t^p - \tilde{y}_t\right)\right]}{\exp\left[\mathrm{E}\left(w_t^p - \tilde{y}_t\right)\right] + \dfrac{1}{H} - 1}$, and

$$\tilde{k}^p = \left(1 - \tilde{\rho}_w^p\right)\log\left(\frac{1}{H} - 1\right) - \tilde{\rho}_w^p \log \tilde{\rho}_w^p - \left(1 - \tilde{\rho}_w^p\right)\log\left(1 - \tilde{\rho}_w^p\right).$$

The constraint $W_{t+1}^r = W_{t+1}^p + W_{t+1}^e$, can be approximated by

$$w_{t+1}^r - \tilde{y}_{t+1} \approx k^* + \rho^*\left(w_{t+1}^p - \tilde{y}_{t+1}\right) + \left(1 - \rho^*\right)\left(w_{t+1}^e - \tilde{y}_{t+1}\right), \tag{2.26}$$

where $k^* = \log\left(\dfrac{G}{\rho^*}\right) - \rho^* \log G - \left(1 - \rho^*\right)\log\left(\dfrac{G}{\rho^*} - G\right)$, $G = \dfrac{\tilde{\rho}_w^p\left(\dfrac{1}{H} - 1\right)}{1 - \tilde{\rho}_w^p}$, and

$$\rho^* = \frac{\exp\left[\mathrm{E}(w_{t+1}^p - \tilde{y}_{t+1})\right]}{\exp\left[\mathrm{E}(w_{t+1}^p - \tilde{y}_{t+1})\right] + \exp\left[\mathrm{E}\left(w_{t+1}^e - \tilde{y}_{t+1}\right)\right]}.$$

The parameter ρ^* measures the weight of pension wealth relative to total wealth in the retirement state and must be less than one.

Proof: See Appendix E.

2.6.3 Optimal Portfolio and Consumption Choice in the Employment State

We first guess the optimal solution of the following form:

$$\boldsymbol{\alpha}_t^e = \tilde{\mathbf{M}}_0 + \tilde{\mathbf{M}}_1 \mathbf{Z}_t, \tag{2.27}$$

$$c_t^e - \tilde{y}_t = \tilde{b}_0^e + \tilde{\mathbf{N}}_1' \mathbf{Z}_t + \mathbf{Z}_t \tilde{\mathbf{N}}_2 \mathbf{Z}_t + \tilde{b}_1^e\left(w_t^e - \tilde{y}_t\right) + \tilde{b}_2^e\left(w_t^p - \tilde{y}_t\right), \tag{2.28}$$

where $\tilde{b}_0^e$, $\tilde{b}_1^e$, and $\tilde{b}_2^e$ are constant coefficients and $\tilde{\mathbf{M}}_0$, $\tilde{\mathbf{M}}_1$, $\tilde{\mathbf{N}}_1$, and $\tilde{\mathbf{N}}_2$ are coefficient matrices with dimensions $(n-1)\times 1$, $(n-1)\times m$, $m\times 1$ and, $m\times m$, respectively. The optimal portfolio decision is linear in state variables, while the optimal *log consumption-(H*$\times$*labor income)* ratio is a quadratic function of state variables.

We can also rewrite the optimal policy in the retirement state as

$$c_{t+1}^r - \tilde{y}_{t+1} = b_0^r + \left(w_{t+1}^r - \tilde{y}_{t+1}\right) + \mathbf{B}_1'\mathbf{Z}_{t+1} + \mathbf{Z}_{t+1}'\mathbf{B}_2\mathbf{Z}_{t+1}. \tag{2.29}$$

The optimal portfolio choice can be expressed as

$$\begin{aligned}\boldsymbol{\alpha}_t^e &= \frac{1}{\gamma\tilde{\tilde{b}}_1^e}\boldsymbol{\Sigma}_{XX}^{-1}\left[\mathrm{E}_t\left(\mathbf{X}_{t+1}\right) + \frac{1}{2}\mathrm{Var}_t\left(\mathbf{X}_{t+1}\right)\right] \\ &- \frac{1}{\tilde{\tilde{b}}_1^e}\boldsymbol{\Sigma}_{XX}^{-1}\left\{\pi^e\tilde{\boldsymbol{\Lambda}}_0^e + \left(1-\pi^e\right)\tilde{\boldsymbol{\Lambda}}_0^r + \left[\pi^e\tilde{\boldsymbol{\Lambda}}_1^e + \left(1-\pi^e\right)\tilde{\boldsymbol{\Lambda}}_1^r\right]\mathbf{Z}_t + \pi^e\left(1-\tilde{b}_1^e - \tilde{b}_2^e\right)\boldsymbol{\sigma}_{\varsigma X}\right\} \\ &- \frac{1}{\tilde{\tilde{b}}_1^e}\boldsymbol{\Sigma}_{XX}^{-1}\left[\pi^e\tilde{b}_2^e\boldsymbol{\Sigma}_{XX}\boldsymbol{\alpha}_t^p + \left(1-\pi^e\right)\rho^*\boldsymbol{\Sigma}_{XX}\boldsymbol{\alpha}_t^p\right],\end{aligned} \tag{2.30}$$

where $\tilde{\tilde{b}}_1^e = \pi^e\tilde{b}_1^e + \left(1-\pi^e\right)$, $\boldsymbol{\alpha}_t^p$ denotes the exogenous portfolio choice made in the pension account, and $\tilde{\boldsymbol{\Lambda}}_0^e$, $\tilde{\boldsymbol{\Lambda}}_1^e$, $\tilde{\boldsymbol{\Lambda}}_0^r$, and $\tilde{\boldsymbol{\Lambda}}_1^r$ are determined in Appendix F.

Proof: See Appendix F.

The optimal portfolio choice contains three components. The first term on the right hand side of equation (2.30) is called the myopic demand, which is proportional to the expected excess risky asset returns relative to the variance-covariance matrix, scaled by the level of relative risk aversion. When the investor becomes more risk averse, the myopic demand of risky assets decreases. The second term is called the intertemporal hedging demand which contains two elements, one for hedging changes in the investment opportunity and the other for hedging labor income risk. When consumption is not very sensitive to the pension as well as regular wealth accounts such that $\left(1-\tilde{b}_1^e-\tilde{b}_2^e\right)>0$, a positive correlation between labor income and asset returns will reduce the optimal investment in risky assets. The last term comes from the exogenous pension account. It shows a substitution effect between the regular financial account and the pension account: a higher portfolio weight in the pension account will reduce the corresponding allocation in the regular financial account. In addition, asset allocation in the pension account influences portfolio decisions in the regular financial account through a probability structure, depending on the realized state in the next period. With probability $1-\pi^e$ that the investor will jump to the retirement state, the influence depends on the importance of pension wealth relative to total wealth, which is measured by the log-linearized constant ρ^*. On the other hand, with probability π^e that the investor will stay in the employment state, the influence depends on the sensitivity of optimal consumption in the employment state relative to the pension wealth, which is measured by $\tilde{b}_2^e$. The higher the pension wealth relative to total wealth in the retirement state or the more sensitive the consumption in the employment state to the pension wealth, the greater the substitution effect in the portfolio choice between the regular financial and pension accounts.

With some arrangement, it can be shown that

$$\boldsymbol{\alpha}_t^e = \tilde{\mathbf{M}}_0 + \tilde{\mathbf{M}}_1 \mathbf{Z}_t, \tag{2.31}$$

where

$$\tilde{\mathbf{M}}_0 = \frac{1}{\gamma \tilde{\tilde{b}}_1^e} \boldsymbol{\Sigma}_{XX}^{-1} \left(\mathbf{H}_X \boldsymbol{\Phi}_0 + \frac{1}{2} \boldsymbol{\sigma}_X^2 \right)$$
$$- \frac{1}{\tilde{\tilde{b}}_1^e} \boldsymbol{\Sigma}_{XX}^{-1} \left[\pi^e \tilde{\boldsymbol{\Lambda}}_0^e + \left(1 - \pi^e\right) \tilde{\boldsymbol{\Lambda}}_0^r + \pi^e \left(1 - \tilde{b}_1^e - \tilde{b}_2^e\right) \boldsymbol{\sigma}_{\zeta x} + \pi^e \tilde{b}_2^e \boldsymbol{\Sigma}_{XX} \boldsymbol{\alpha}_t^p + \left(1 - \pi^e\right) \rho^* \boldsymbol{\Sigma}_{XX} \boldsymbol{\alpha}_t^p \right], \text{ and}$$

$$\tilde{\mathbf{M}}_1 = -\frac{1}{\gamma \tilde{\tilde{b}}_1^e} \boldsymbol{\Sigma}_{XX}^{-1} \left\{ \mathbf{H}_X \boldsymbol{\Phi}_1 + \gamma \left[\pi^e \tilde{\boldsymbol{\Lambda}}_1^e + \left(1 - \pi^e\right) \tilde{\boldsymbol{\Lambda}}_1^r \right] \right\}.$$

Thus, as initial guess, the optimal portfolio decision is linear in state variables and depends on the parameters of optimal consumption. Now we proceed to solve the optimal consumption decision.

It can be shown that the optimal consumption can be expressed as

$$c_t^e - \tilde{y}_t = \tilde{\Theta}_0 + \tilde{\boldsymbol{\Theta}}_1 Z_t + \tilde{\boldsymbol{\Theta}}_2 vec(\mathbf{Z}_t \mathbf{Z}_t') + \tilde{\Theta}_3 \left(w_t^e - \tilde{y}_t\right) + \tilde{\Theta}_4 \left(w_t^p - \tilde{y}_t\right), \tag{2.32}$$

where $\tilde{\Theta}_0$, $\tilde{\boldsymbol{\Theta}}_1$, $\tilde{\boldsymbol{\Theta}}_2$, $\tilde{\Theta}_3$, and $\tilde{\Theta}_4$ are determined in Appendix G.

Proof: See Appendix G.

Thus, the *log consumption-($H \times$ labor income)* ratio is quadratic in state variables and depends on the wealth level of both pension and regular financial accounts. Solving the above nonlinear equations obtains the coefficients for the optimal consumption and portfolio decisions.

2.6.4 Calibration - Asset Allocation with Exogenous Pension Account

We assume that there are three asset categories. The first asset is riskless and the other two are risky assets with excess returns following a VAR process as described in Section 2.4. The investor expects to work for 20 years and after retirement she incurs a probability of death π^d = 0.1. Labor income is assumed to be independent of asset returns. Other parameter values are given as follows: labor income growth rate g is set such that $\mathrm{E}_t(Y_{t+1}/Y_t) = 1.06$, $\sigma_\varsigma = 0.1$, H = 0.95, δ = 1/1.1, and γ = 8. We consider different cases where the investor allocates (0, 0.5), (0.25, 0.25), (0, 0), and (0.5, 0) of her pension wealth in stock and bond, respectively, and investigate how her optimal portfolio in the regular financial account is changed. The result is shown in Table 2.9.

Not surprisingly, when the investor allocates more in risky assets in her pension account, the *pension wealth-(H*x*labor income)* ratio is higher, reflecting the fact that risky assets have higher mean returns. For instance, the investor allocating (0.5, 0) of her pension wealth in stock and bond has $\exp \mathrm{E}\left(w_t^p - \tilde{y}_t\right)$ equal to 7.84, compared to 1.47 when the investor allocates all her pension wealth in the risk-free asset. As the theoretical result shows, there exists some substitution effect between the allocation in regular financial and pension accounts. For instance, when the investor allocates (0.5, 0) of her pension wealth in stock and bond, it is optimal for her to hold a position in bond and stock with weights equal to 202 percent and 50 percent, respectively. On the other hand, when the investor allocates (0, 0.5) of her pension wealth in stock and bond, the optimal portfolio weight in bond decreases to 75 percent while the weight in stock increases to 77 percent.

2.7 Conclusion

Using log linear approximation, this paper solves analytically the optimal consumption and asset allocation problem for an investor with nontradable labor income when there are multiple risky assets whose excess returns follow a VAR process. The investor receives risky labor income each period when employed but must live off her wealth after retirement. We show conveniently that the optimal portfolios in both the employment and retirement states are linear in state variables, consisting of a myopic demand and an intertemporal hedging demand. Furthermore, the optimal log consumption-wealth ratio in the retirement state and the log consumption-labor income ratio in the employment state are both quadratic functions of state variables.

Our model calibration shows that the more risk-averse investor holds a higher bond/stock ratio in her risky portfolio when labor income is independent of asset returns or positively correlated with stock return, but the reverse is true when labor income is positively correlated with bond return. Our finding indicates that given the predictability of asset returns, depending on the correlations between labor income and asset returns, the presence of labor income may either help resolve or deepen the asset allocation puzzle documented in Canner, *et al*. (1997).

We find that the allocation to risky stock inherits the inverted U-shaped pattern of labor income growth with respect to expected time until retirement. That is, investors with lower income growth, namely, younger workers or those near retirement, should invest less in risky assets than those who are in the middle of their career and have a higher income growth. Thus, our results show that popular recommendations from financial advisors that the optimal portfolio weight in risky stocks should be equal to 100 minus age may lack theoretical justification.

Out-of-sample test demonstrates that our dynamic optimal portfolio yields higher mean return and higher Sharpe ratio than two benchmark portfolios. Furthermore, the certainty equivalent consumption under the VAR is 7.6 percent higher than that under the model with a constant investment opportunity when the underlying return generating process follows VAR. These results justify the economic significance of incorporating the time-variation of investment opportunities in the presence of nontradable labor income into investors' portfolio choice.

When an exogenous pension account is incorporated into our model, the optimal portfolio in the employment state contains an additional component that comes from the exogenous pension account. The higher the pension wealth relative to total wealth or the more sensitive the consumption in the employment state to the pension wealth, the higher the effect of the pension portfolio on the regular portfolio.

Chapter 3

Optimal Dynamic Portfolio Strategy in Defined Contribution Pension Plans with Company Stock

3.1 Introduction

Over the past two decades, defined contribution plans have become the most important pension plans that Americans use to save for their retirement. For instance, in year 2005 more than 90 percent of all pension plans in the U.S. are defined contribution plans, covering more than 75 million workers and managing assets of more than $2.8 trillion.[1] In defined contribution plans, plan sponsors make a specific contribution to the employees' accounts and employees themselves make the asset allocation decision and bear the investment results. That is, asset allocation autonomy is rendered to the employees. Due to the popularity of defined contribution plans, how employees should make their portfolio choices in their pension plans becomes an important research topic.

The importance of research in optimal pension investment is further strengthened by recent studies showing that employees are not portfolio optimizers: they tend to adopt some heuristic strategies and have inertia in their pension investment. For instance,

[1] Source: U.S. Department of Labor.

Benartzi and Thaler (2001) and Huberman and Jiang (2006) find that employees tend to adopt the 1/*n* strategy that allocates contribution evenly across either all available or some chosen investment objects in their pension plans. The most interesting example comes from Nobel Economics laureate Harry Markowitz - a founding father of classic portfolio theory, who admitted that he split his contribution evenly between bond and stock fund in his TIAA-CREF account during the period when TIAA-CREF had only two options (Benartzi and Thaler, 2007). Empirical studies also find that employees have inertia in their pension management. Beshears, Choi, Laibson, and Madrian (2007) point out that the default option has a wide influence on employed investors' participation rate as well as asset allocation decision. A study by Waggle and Englis (2000) also reports that the average number of trades in pension plans during a year is zero for more than 87 percent of the participants and only 7 percent of plan participants trade more than once a year.[2]

Another sub-optimizing evidence about employees' pension investment is that they tend to allocate a significant amount of their contribution to company stock when company stock is an investment option in their pension plans. Table 3.1 shows the percentage of defined contribution plan assets invested in company stock in some well-established companies. General Electric (GE), for instance, has more than 75 percent of its defined contribution plan assets invested in company stock. This is puzzling because according to portfolio diversification theory, employees should not invest in company

[2] This is contrary to Odean (1999) who uses data from the trading record in a brokerage firm and finds that investors are quite active traders. This difference may be due to sample selection bias. People who open brokerage accounts are more likely to be active traders. Another reason for this difference may be that while investment targets in a brokerage account could be individual stocks, most assets in pension plans are mutual funds that may reduce investors' incentives to trade actively.

stock for more than what a diversified market portfolio suggests. In addition, individual stock can crash and employees may lose their pension wealth if they invest too much in company stock. For instance, Enron's employees lost 60 percent of their wealth in their 401(k) plan when Enron's stock collapsed. Furthermore, when the correlation between labor income and company stock return is considered, employees' high allocation to company stock becomes even more puzzling.

This paper studies employees' optimal investment in their pension plans under a dynamic investment opportunity set with company stock available for investment. Several prior papers look at the optimal portfolio decision in defined contribution plans. For example, Vigna and Haberman (2001, 2002) consider the optimal investment strategy in defined contribution plans under a discrete time model. Assuming a constant investment opportunity with two assets available for investment, they derive the optimal dynamic trading strategy by minimizing the deviation from a series of interim and retirement targets. Under a continuous time model with stochastic interest rates, Boulier, Huang, and Taillard (2001) obtain the optimal portfolio strategy by maximizing the expected utility over terminal wealth with a minimum guarantee. Deelstra, Grasselli, and Koehl (2003) extend Boulier *et al.*'s (2001) framework by assuming a more general interest rate process and allowing the contribution as well as the guarantee to be random. Studying a closely related problem, Battocchio and Menoncin (2004) consider the optimal pension investment problem in a model that defines utility over terminal wealth in the presence of non-hedgable labor income and inflation risk. Assuming an investment opportunity of multiple risky assets with a stochastic risk-free interest rate and correlated labor income, Cairns, Blake, and Dowd (2006) show that the optimal portfolio in defined contribution plans is composed of three efficient mutual funds, namely, cash, bond, and equity fund. These papers, however, do not address the issue of company stock. Empirically the literature has found company stock to be an important asset that can greatly affect employees' portfolio choices. Douglass, Wu, and Ziemba (2004) consider the optimal company stock investment in pension plans, but they assume a static model. Due to the long-horizon property of pension investment and recent evidence of asset return predictability (e.g., Keim and Stambaugh, 1986; and Fama and French, 1988, 1989), the result from a static model cannot be generalized into a long-term context.

This paper is to fill the research gap by studying the optimal pension portfolio under a dynamic investment opportunity set with short sale constraints and company stock available for investment. Assuming a discrete time model with multiple risky asset returns described by a vector autoregressive (VAR) process, this paper uses the simulation methodology proposed by Brandt, Goyal, Santa-Clara, and Stroud (2005). This methodology is used instead of the traditional value function iteration method because it can better handle the dynamic investment opportunity assumption described by a large number of state variables and produces lower biases when investment horizon is long.

The result shows that when company stock return is positively correlated with labor income, employees should generally invest less in company stock but more in stock market when time to retirement increases, but the pattern is reversed when the degree of employees' risk aversion is low. This shows the trade-off between higher company stock expected return and its higher risk in terms of variance and correlation with labor income.

When the exogenous contribution rate increases, employees in general should decrease their investment in company stock while investing more in the other risky assets because the return of company stock has a higher correlation with labor income than the returns of the other risky assets. This paper also finds that the substitution effect exists among risky assets investment even when the short sale constraints are not hit. The performance test shows that when asset returns are predictable, the gains from adopting the dynamic portfolio strategy relative to several benchmark strategies, including the $1/n$ rule, the optimal static strategy with and without the consideration of asset return predictability, all stock strategy, and all company stock strategy, are economically significant and the economic gains increase with employees' level of risk aversion.

The second part of this paper examines if employees' high allocation to company stock can be justified with economic reasons.[3] The literature has proposed several economic motivations to explain employees' high allocation to company stock (see Douglass, Wu, and Ziemba, 2004; Meulbroek, 2005; Mitchell and Utkus, 2005; and Cohen, 2009). Firstly, due to short sale constraints in pension investment, aggressive employees may want to earn a higher expected return by investing in company stock because a leveraged position in market portfolio is not allowed. Secondly, employees may expect a higher return from their company stock investment than the general market does. The reason is that employees may be able to buy company stock at a discounted price or that they are more optimistic about their own company's future. The third argument is related to employees' familiarity with their own company. Employees may

[3] Behavioral arguments such as loyalty or peer pressure can also possibly explain investors' high allocation to company stock (see Cohen, 2009). For simplicity, we do not consider these factors in this paper and we leave it for future research.

think that they are more familiar with their own company than outsiders and have less uncertainty in company stock's return generating process. Employees' high allocation to company stock can also be attributed to employers' match to employees' contribution exclusively with company stock. Employees may view this as an endorsement of company stock investment from the management and thus are willing to invest more. The simulation results suggest that none of these reasons alone can justify employees' high allocation to company stock. In addition, these economic motives have small effects on the portfolio decisions of more risk-averse employees. It is usually optimal for these conservative employees to invest most of their pension wealth in riskless asset, and over allocating to company stock can be very costly to them.

Section 3.2 specifies the model and describes the methodology used to solve the optimal portfolio choice problem. Model calibrations are conducted in Section 3.3. Section 3.4 compares the performance of our dynamic strategy with some benchmark strategies. Section 3.5 examines if employees' high investment in company stock can be justified with economic reasons. Concluding remarks are given in Section 3.6.

3.2. Optimal Portfolio Strategy in Pension Plans

3.2.1 Model Specification

We assume that employees have a power utility over terminal pension wealth maturing at time *T*:

$$V_t\left(W_t, Y_t, \mathbf{Z}_t\right) = \underset{\{\mathbf{x}_s\}_{s=t}^{T-1}}{\mathrm{Max}} \mathrm{E}_t\left[\frac{W_T^{1-\gamma}}{1-\gamma}\right], \tag{3.1}$$

subject to the budget constraint

$$W_{s+1} = \left(W_s + HY_s\right) \times \left(\mathbf{x}_s' \mathbf{R}_{s+1}^e + R^f\right), \quad \forall s \geq t, \tag{3.2}$$

where γ is the coefficient of relative risk aversion, W_s denotes pension wealth, Y_s is labor income, H is the contribution rate, R^f represents the riskless (gross) interest rate, $\mathbf{x}_s$ is the vector of portfolio weights in risky assets, and $\mathbf{R}_{s+1}^e$ is the vector of excess risky asset returns.

Each period before retirement, employed investors contribute a proportion H of their labor income to their pension accounts, and receive their pension wealth at maturity date T. We assume that labor income process follows $Y_{t+1} = Y_t \exp(g + \varsigma_{t+1})$, where g is labor income growth rate and $\varsigma_{t+1} \sim N(0, \sigma_\varsigma^2)$ denotes labor income risk. For simplicity, we do not consider non-pension wealth and the corresponding consumption and asset allocation decision. There are n assets available for investment and the first asset is riskless. We assume that excess risky asset returns follow a VAR process:

$$\mathbf{Z}_{t+1} = \mathbf{\Phi}_0 + \mathbf{\Phi}_1 \mathbf{Z}_t + \mathbf{v}_{t+1}, \tag{3.3}$$

where $\mathbf{Z}_{t+1} \equiv \begin{bmatrix} r_{2,t+1} - r_f \\ r_{3,t+1} - r_f \\ \vdots \\ r_{n,t+1} - r_f \end{bmatrix}$ is the vector of excess log risky asset returns with $r_{i,t+1} = \log(R_{i,t+1})$ and $r_f = \log(R_f)$, $\mathbf{\Phi}_0$ and $\mathbf{\Phi}_1$ are constant matrices with dimensions (n-1)×1 and (n-1)×(n-1), respectively, and $\mathbf{v}_{t+1}$ is an (n-1)×1 vector of shocks to excess returns with $\mathbf{v}_{t+1} \sim N(0, \mathbf{\Sigma}_v)$.

3.2.2 Methodology

We use the simulation methodology proposed by Brandt, *et al.* (2005) to solve this optimal portfolio problem. This methodology is used instead of the traditional value function iteration method because it can better handle the portfolio problem under a dynamic investment opportunity set described by a large number of state variables and produces lower biases when investment horizon is long. We firstly rewrite the value function as

$$V_t(W_t, Y_t, \mathbf{Z}_t) = Y_t^{1-\gamma} V_t\left(\frac{W_t}{Y_t}, \mathbf{Z}_t\right), \tag{3.4}$$

where

$$V_t\left(\frac{W_t}{Y_t},\mathbf{Z}_t\right)=\frac{V_t\left(W_t,Y_t,\mathbf{Z}_t\right)}{Y_t^{1-\gamma}}$$

$$=\underset{\{\mathbf{x}_s\}_{s=t}^{T-1}}{\mathrm{Max}}\frac{\mathrm{E}_t\left\{\left(\frac{W_t}{Y_t}+H\right)\prod_{s=t}^{T-1}\left(\mathbf{x}_s'\mathbf{R}_{s+1}^e+R^f\right)+H\sum_{s=t+1}^{T-1}\exp\left(\sum_{v=t+1}^{s}\left(g+\varsigma_v\right)\right)\prod_{k=s}^{T-1}\left(\mathbf{x}_k'\mathbf{R}_{k+1}^e+R^f\right)\right\}^{1-\gamma}}{1-\gamma}$$

We approximate the value function by taking a second order Taylor expansion around the terminal wealth $\hat{W}_T$, obtained from the current wealth and contribution growing at the risk-free interest rate for one period and then at the expected portfolio return given the optimal portfolio decisions in future periods:

$$V_t\left(\frac{W_t}{Y_t},\mathbf{Z}_t\right)$$
$$\approx\underset{\mathbf{x}_t}{\mathrm{Max}}\,\mathrm{E}_t\left[\frac{1}{1-\gamma}\left(\frac{\hat{W}_T}{Y_t}\right)^{1-\gamma}+\left(\frac{\hat{W}_T}{Y_t}\right)^{-\gamma}\left(\frac{W_T}{Y_t}-\frac{\hat{W}_T}{Y_t}\right)-\frac{\gamma}{2}\left(\frac{\hat{W}_T}{Y_t}\right)^{-\gamma-1}\left(\frac{W_T}{Y_t}-\frac{\hat{W}_T}{Y_t}\right)^2\right], \tag{3.5}$$

where

$$\frac{W_T}{Y_t}=\left(\frac{W_t}{Y_t}+H\right)\left(\mathbf{x}_t'\mathbf{R}_{t+1}^e+R^f\right)\prod_{s=t+1}^{T-1}\left(\hat{\mathbf{x}}_s'\mathbf{R}_{s+1}^e+R^f\right)+H\sum_{s=t+1}^{T-1}\exp\left(\sum_{v=t+1}^{s}\left(g+\varsigma_v\right)\right)\prod_{k=s}^{T-1}\left(\hat{\mathbf{x}}_k'\mathbf{R}_{k+1}^e+R^f\right),$$

and $\frac{\hat{W}_T}{Y_t}=\left(\frac{W_t}{Y_t}+H\right)R^f\prod_{s=t+1}^{T-1}\left(\hat{\mathbf{x}}_s'\mathbf{R}_{s+1}^e+R^f\right)+H\sum_{s=t+1}^{T-1}\exp\left(\sum_{v=t+1}^{s}\left(g+\varsigma_v\right)\right)\prod_{k=s}^{T-1}\left(\hat{\mathbf{x}}_k'\mathbf{R}_{k+1}^e+R^f\right).$

We now solve for the optimal portfolio decision backward from time *T*-1 to *t*. The algorithm is described below:

Step 1: Simulate *K* sample paths on realized asset returns and labor income with length (*T*-*t*).

Step 2: Given a portfolio weight at time T-1, we compute realized values in equation (3.5) on each sample path and conduct an ordinary linear regression of these realized values on functions of state variables $\mathbf{Z}_{T-1}$ to obtain the conditional expectation for each sample path. Repeating the process on different portfolio weights, we can obtain the corresponding optimal portfolio choice for each sample path at time T-1 that gives the highest expected utility. Given the optimal portfolio choice on each sample path, we can compute the corresponding expected portfolio return at time T-1.

Step 3: Given a portfolio weight at time T-2, we use the realized asset returns at time T-2 and the expected portfolio return at time T-1 obtained from *step 2* to compute realized values in equation (3.5) for each sample path. We then regress these realized values on functions of state variables $\mathbf{Z}_{T-2}$ to get the expected utility over terminal wealth. Repeating the process on different portfolio weights at time T-2, we can obtain the optimal portfolio choice corresponding to each sample path at time T-2 that gives the highest expected utility. With the optimal portfolio decision at time T-2, we can compute the corresponding expected portfolio return at time T-2. The process is repeated backward until time t so that a dynamic portfolio decision over time is obtained.

We should note that each period the realized utility depends on the current wealth level. However, the current wealth level depends on past portfolio choices that are not determined yet because we solve the model backward. To make adjustments, we conduct the above process at different wealth levels and then use a linear regression method to solve for the optimal portfolio decision.[4]

3.3 Model Simulation

3.3.1 Effect of Labor Income

We assume that there are three assets available for investment: riskless asset, stock market portfolio and company stock, and that excess returns on risky assets follow a VAR(1) process. We use annual 90-day T-Bill returns and CRSP value-weighted market portfolio returns for the period 1930-2005 to estimate riskless asset and stock market portfolio returns, respectively. To estimate company stock returns, we choose General Electric (GE) as the representative stock. The riskless asset has the average log annual return of 0.8 percent and the VAR process follows:

$$\begin{bmatrix} r_{s,t+1} - r_f \\ r_{c,t+1} - r_f \end{bmatrix} = \begin{bmatrix} 0.05620 \\ 0.05271 \end{bmatrix} + \begin{bmatrix} 0.17430 & -0.10593 \\ 0.18670 & 0.06799 \end{bmatrix} \begin{bmatrix} r_{s,t} - r_f \\ r_{c,t} - r_f \end{bmatrix} + \mathbf{v}_{t+1},$$

where $r_{s,t+1}$, $r_{c,t+1}$, and r_f are log returns of stock market portfolio, company stock, and riskless asset, respectively, and $\mathbf{v}_{t+1} \sim N(0, \mathbf{\Sigma_v})$ with $\mathbf{\Sigma_v} = \begin{bmatrix} 0.03616 & 0.03715 \\ 0.03715 & 0.05310 \end{bmatrix}$. These parameters imply the unconditional expected excess stock market and company stock returns equal to 5.93 percent and 6.84 percent, respectively.

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[4] This method is modified from Binsbergen and Brandt (2007) who suggest solving the process under different wealth levels and using linear interpolation to obtain the optimal portfolio decision.

The labor income growth rate g is set such that $\mathrm{E}_t\left(\frac{Y_{t+1}}{Y_t}\right)=1.03$ and $\varsigma_{t+1} \sim N(0,\ 0.01)$. We assume labor income to be independent of risky asset returns at first and later we will consider the case where labor income is correlated with risky asset returns. The annual contribution rate H is assumed to be 6 percent. We consider the cases where employees have 5, 10 and 15 years to retirement with the initial *wealth/labor income* ratio equal to 0.55. Under these assumptions, we simulate 1000 times and use a grid search over portfolio weights [0,1]×[0,1] in step 0.01×0.01 to find the optimal portfolio decision for investors with relative risk aversion γ ={2, 4, 6, 8, 10, 12}.

The result is shown in Table 3.2. When employees have longer time to retirement, they become more aggressive by increasing their investment in company stock. Employees with longer time to retirement expect to receive more labor income in the future and are more tolerant of investment loss. When labor income is independent of risky asset returns, company stock is like an investment object alternative to stock market but with a higher expected return and risk. Thus, employees are willing to invest more in company stock to earn a higher expected return when they have longer time to retirement. For instance, when γ = 4, employed investors with 5 years to retirement allocate 28 percent of their contribution to company stock, but increasing the portfolio weight to 40 percent when they have 15 years before retirement.

Now we relax the assumption of independence between labor income and risky asset returns and assume that the correlation coefficients between labor income and stock market and company stock returns are equal to 5 and 25 percent, respectively. Table 3.3

shows the simulation result. When employees become more risk averse, they put less weights in risky assets and may invest nothing in company stock due to its higher risk in terms of variance and correlation with labor income. While employees' total investment in risky assets increases with time to retirement, their investment within each risky asset, however, may increase or decrease with time to retirement, depending on their risk attitude. More risk-averse employees invest less in company stock but more in stock market portfolio when time to retirement increases. For example, in the case where $\gamma = 6$, employees with 5 years to retirement allocate 20 (11) percent of their contribution to stock market portfolio (company stock), but increase (decrease) the portfolio weight to 39 (6) percent when they have 15 years before retirement. Because employees with longer time to retirement expect to receive more labor income, they become more risk tolerant by investing more in stock market portfolio, but decreasing their company stock investment since company stock returns have a higher correlation with labor income. For employees with a low degree of risk aversion, however, the pattern is reversed. Their investment in company stock increases with time to retirement. To these employees, the benefit of higher expected company stock return out-weights the cost of higher company stock investment risk. For instance, when $\gamma = 2$, employees increase their allocation to company stock from 43 percent to 53 percent when their time to retirement increases from 5 years to 15 years.

3.3.2 Effect of Contribution Rate

Since terminal pension wealth is also determined by the contribution made each period to the pension account, it is interesting to study the effect of contribution rate on employed investors' asset allocation decision. To do so, we increase the contribution rate from 6 to 12 percent while maintaining the initial *wealth/labor income* ratio.[5] We assume that employees have 10 years to retirement and the correlation coefficients between labor income and stock market and company stock returns are equal to 5 and 25 percent, respectively. The simulation result is shown in Table 3.4.

When the exogenous contribution rate increases, employees become more aggressive such that their optimal portfolio weight of total risky assets increases. For less risk-averse employees (for instance, when $\gamma = 2$), since they already invest total pension wealth to risky assets when the contribution rate is 6 percent, they become more aggressive by allocating more wealth to company stock when the contribution rate increases to 12 percent. On the other hand, for more risk-averse employees, they become more aggressive by investing more in stock market but decreasing their allocation to company stock. This is because when contribution rate is higher, employees put more of their labor income to their pension accounts that is highly correlated with company stock returns and thus making investment in company stock less attractive.

[5] When the initial *wealth-labor income* ratio is zero, changing contribution rate does not affect employees' optimal portfolio decisions.

3.3.3 Effect of Correlation between Company Stock Returns and Labor Income

Since one main argument against employees' allocation to their company stock is the high correlation between company stock returns and labor income, it is worthy to examine how the correlation between company stock returns and labor income can affect employees' optimal portfolio decision. To do so, we increase the correlation coefficient between company stock returns and labor income from 25 percent to 55 percent. The result is shown in Table 3.5.

When company stock returns are more correlated with labor income, employees will decrease their investment in company stock but increase their investment in stock market. For instance, when $\gamma = 4$, employees' portfolio weight in stock market (company stock) is 61 (0) percent when the correlation between company stock returns and labor income is equal to 55 percent, but decreases (increases) to 40 (19) percent when the correlation decreases to 25 percent. Employees' portfolio weight in total risky assets, however, does not change much given different correlation scenarios. In the case where $\gamma = 4$, for example, total risky assets investment goes from 59 percent to 61 percent when the correlation between labor income and company stock returns increases from 25 percent to 55 percent. This also indicates the existence of some substitution effect between stock market and company stock investment even when the short sale constraints are not hit.

3.4 Performance Test

To evaluate the economic significance of adopting the optimal dynamic trading strategy relative to some heuristic trading strategies or some optimal strategies that maintain constant portfolio weights, we compare the performance of the optimal dynamic trading strategy with five benchmark strategies: (1) the strategy that allocates all pension wealth to stock market; (2) the strategy that allocates all pension wealth to company stock; (3) the 1/*n* rule that allocates the contribution evenly across all available investment objects; (4) the optimal strategy that assumes a time-varying investment opportunity but does not change the portfolio decision over time; and (5) the static optimal strategy that assumes a constant investment opportunity.

We assume that the excess risky asset returns follow a VAR(1) process as described in Section 3.3.1. Employees have 10 years to retirement, and labor income is correlated with stock market and company stock returns with correlation coefficients equal to 5 and 25 percent, respectively. To evaluate the economic significance of adopting the dynamic trading strategy, we estimate the contribution rate in each benchmark strategy that employees need to make in order to obtain the same expected utility as in the dynamic trading strategy that has an exogenous contribution rate of 6 percent. A strategy that is superior to the dynamic trading rule should require a contribution rate smaller than 6 percent. On the other hand, an inferior strategy will commend a higher contribution rate to obtain the same expected utility as in the dynamic trading strategy. Table 3.6 shows the results.

The simulation results show that the dynamic strategy is the best strategy since in all benchmark strategies the contribution rates required to give the same expected utility as in the dynamic trading strategy are higher than 6 percent. In addition, except in the case where the level of employees' risk aversion is low (e.g., $\gamma = 2$), the strategy of investing all pension wealth to company stock commends the highest contribution rate, indicating that it is the worst strategy. The loss of investing all pension wealth to company stock can be quite high especially when employees are very risk averse. For example, when $\gamma = 12$, employees need to contribute 29.75 percent of their income to pension in order to obtain the same expected utility as in the dynamic trading strategy that has a contribution rate of 6 percent. Compared to the strategy of all company stock, the strategy of all stock market is much better. For instance, when $\gamma = 12$, employees investing all pension wealth to stock market only need to contribute 12.01 percent of their income, as compared to 29.75 percent if they allocate all pension wealth to company stock. The reason is that stock market has already provided some diversification to employees and it is less correlated with labor income than company stock.

When employees have a low degree of risk aversion, $1/n$ is the worst among all trading strategies. The reason is that $1/n$ strategy allocates one third of all pension wealth to the riskless asset and it can be costly to employees with low degrees of risk aversion who may want to adopt a more aggressive trading strategy. In the cases where employees have a medium or high level of risk aversion, however, $1/n$ strategy is a better strategy than the strategy of all company stock or all stock market. This partially justifies the popularity of $1/n$ strategy in employees' pension investment. The employees, however, can greatly improve their welfare if they adopt some "optimal" trading strategies. Both strategies (4) and (5) are optimal strategies that maintain constant portfolio rules throughout the entire investment period, but strategy (5) ignores asset return predictability. As it is expected, strategy (4) is better than strategy (5): The contribution rate of strategy (4) is strictly less than that of strategy (5). However, the differences in the contribution rates between these two strategies are very small. For example, when $\gamma = 6$, the contribution rate of strategy (4) is 6.28 percent, which is not very much different from 6.36 percent of strategy (5). This suggests that the gain from considering the underlying dynamic investment opportunity is not significant if employees maintain a constant portfolio strategy.

Table 3.6 also shows that the gain from adopting the dynamic trading strategy instead of the optimal strategies (4) and (5) decreases with employees' degree of risk aversion but the reverse is true for the $1/n$ rule and the strategies that allocate all wealth either to stock market or company stock. This suggests that when employees become more risk averse, it is important for them to invest strategically instead of adopting some simplified trading rules.

3.5 Why Do Employees Hold Company Stock?

Empirical studies show that employees tend to allocate a significant amount of their pension wealth to company stock when company stock is available for investment in their pension plans. For instance, Meulbroek (2005) finds that company stock accounts for about 27 percent of total assets in those plans that have company stock in their investment menu (see also Mitchell and Utkus, 2005). This is puzzling because according to portfolio diversification theory, employees should not invest in company stock for more than what the market portfolio suggests. Besides, individual stock can crash and employees may lose their pension wealth if they invest too much in company stock. For instance, Enron's employees lost 60 percent of their wealth in their 401(k) plan when Enron's stock collapsed. Also, investing in a single stock can be very costly since investors may not be compensated for the incurred idiosyncratic risk. And the positive correlation between company stock returns and labor income makes the investment in company stock become even less attractive.

The literature has proposed several economic reasons to explain employees' high allocation to company stock. Firstly, since the investment in pension accounts generally has borrowing and short sale constraints, employees may invest in company stock to earn a higher expected return because they cannot hold a leveraged position in the market portfolio. Secondly, employees may expect a higher return from company stock investment than the market does. Plausible reasons are that employees can buy company stock at a discounted price or that they are more optimistic about their own company's future. Thirdly, employees may think that they know better about their own company and want to invest in what they are more familiar with. Fourthly, some companies have a restricted match policy that matches employees' contribution exclusively with company stock and employees may view this as an endorsement from the management that company stock is a good investment target. In addition, companies with restricted match policies are likely to prohibit their employees from reallocating the matched company stock investment, leading to employees' higher company stock investment. In this section, we examine if any of these reasons can justify employees' high allocation to company stock.

3.5.1 Higher Expected Return

Employees may invest in company stock because they expect a higher return from their company stock investment than the overall stock market does. The reason can be that employees are more optimistic about their own company or that they can buy company stock at a discounted price. To see how higher expected company stock return affects employees' portfolio strategy, we consider the cases where employees expect a unconditional mean company stock return that is higher than that implied in the original model described in Section 3.3 by 1, 2, and 4 percent while holding the same expected stock market return.[6] The other baseline assumptions are: employees have 10 years to retirement, time discount factor δ is 1/1.1, labor income growth rate g is set such that $\mathrm{E}_t\left(\frac{Y_{t+1}}{Y_t}\right) = 1.03$, and the correlation between labor income and stock market and company stock returns are equal to 5 percent and 25 percent, respectively. The result is shown in Table 3.7.

Not surprisingly, when company stock has a higher expected return, employees optimally increase their allocation to company stock and reduce their investment in the

[6] To do so, we adjust the parameter $\mathbf{\Phi}_0$ such that the unconditional mean company stock return is 1, 2, and 4 percent higher than that implied in the original case while maintaining the unconditional mean returns of the other risky assets. The unconditional mean return implied in the process is $\mathbf{u}_z = (\mathbf{I} - \mathbf{\Phi}_1)^{-1}\mathbf{\Phi}_0$.

overall stock market. However, consistent with Douglass, *et al.* (2003), higher expected company stock return can explain employees' high allocation to company stock only when employees' degree of risk aversion is low or when employees have extremely high expectation of company stock return. [7] For instance, when the coefficient of employees' relative risk aversion is 4 and they expect a mean company stock return that is higher than that implied in the original model by 4 percent, the optimal portfolio weight in company stock is 58 percent.

Table 3.7 also demonstrates a substitution effect between stock market and company stock investment. Increasing the company stock expected return by 1 percent can switch employees' investment preference from stock market to company stock. For instance, when γ = 2, the portfolio weight in stock market (company stock) is equal to 51 (49) percent in the original case but decreasing (increasing) to 16 (84) percent when the expected company stock return increases by 1 percent. This switching effect is more pronounced for employees with low degrees of risk aversion. For those extremely conservative employees, higher expected company stock return does not affect a lot on their portfolio decisions because they invest most of their pension wealth in risk-free asset. For example, employees with γ = 12 increase their allocation to company stock from 0 percent to 17 percent but decreasing their portfolio weight of stock market from

[7] Assuming a mean-variance utility function, Douglass, *et al.* (2003) show that the risk aversion parameter needs to be below 0.5 or the employees believe that the expected company stock return to be as high as 50 percent such that it is optimal for the investors to hold company stock above 50 percent. When considering other retirement savings outside pension plans, they show that investors with relative risk aversion equal to 8 need to have 50 percent of their savings outside pension plans for them to optimally hold 50 percent of their pension wealth in company stock.

18 percent to 0 percent when the expected company stock return is higher than that implied in the original model by 4 percent.

3.5.2 Parameter Uncertainty

It has been argued that employees invest much of their pension wealth in company stock because they think that they know better about their companies and want to invest in what they are more familiar with. To examine if familiarity can justify employees' high allocation to company stock, we now incorporate parameter uncertainty to the VAR model described in Section 3.2.1. For simplicity, we assume that there is parameter uncertainty in the coefficients $\mathbf{\Phi}_0$ and $\mathbf{\Phi}_1$, but the riskiness of shocks to state variables $\mathbf{\Sigma}_\mathbf{v}$ is known to the investors. We can rewrite equation (3.3) as

$$\begin{pmatrix} \mathbf{z}_2' \\ \mathbf{z}_3' \\ \vdots \\ \mathbf{z}_T' \end{pmatrix} = \begin{pmatrix} 1 & \mathbf{z}_1' \\ 1 & \mathbf{z}_2' \\ \vdots & \\ 1 & \mathbf{z}_{T\text{-}1}' \end{pmatrix} \begin{pmatrix} \mathbf{\Phi}_0' \\ \mathbf{\Phi}_1' \end{pmatrix} + \begin{pmatrix} \mathbf{v}_2' \\ \mathbf{v}_3' \\ \vdots \\ \mathbf{v}_T' \end{pmatrix}, \tag{3.6}$$

where $\mathbf{Z}_{t+1} \equiv \begin{bmatrix} r_{2,t+1} - r_f \\ r_{3,t+1} - r_f \\ \vdots \\ r_{n,t+1} - r_f \end{bmatrix}$ denotes log risky excess returns and $\mathbf{v}_{t+1} \sim N(0,\ \mathbf{\Sigma}_{\mathbf{v}})$ is the shock to excess risky returns. Let $\mathbf{X} \equiv \begin{pmatrix} 1 & \mathbf{z}_1' \\ 1 & \mathbf{z}_2' \\ \vdots & \\ 1 & \mathbf{z}_T' \end{pmatrix}$. Given realized historical asset returns and $\mathbf{\Sigma}_v$, we can compute the posterior distribution of $\mathbf{C} \equiv \begin{pmatrix} \mathbf{\Phi}_0' \\ \mathbf{\Phi}_1' \end{pmatrix}$:

$$vec(\mathbf{C}) \mid \mathbf{\Sigma}, \mathbf{z} \sim N\left(vec(\hat{\mathbf{C}}), \mathbf{\Sigma}_v \otimes \left(\mathbf{X}'\mathbf{X}\right)^{-1}\right), \tag{3.7}$$

where $\hat{\mathbf{C}} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Z}$. That is, the posterior distribution of $\mathbf{\Phi}_0$ and $\mathbf{\Phi}_1$ is simply the result of multivariate regression.[8] We then simulate sample paths in asset returns from the posterior distribution and follow the algorithm in Section 3.2.2 to compute the optimal portfolio decision with parameter uncertainty.

To examine how familiarity in company stock can change employees' portfolio decision, we compare two scenarios: (1) employees have parameter uncertainty in the return generating processes of both stock market and company stock, and (2) employees have parameter uncertainty only in the return generating process of stock market, but not of company stock. We use the same historical data to estimate the VAR model as discussed in Section 3.3, but allowing parameter uncertainty in $\mathbf{\Phi}_0$ and $\mathbf{\Phi}_1$ here. The other parameter values are borrowed from the baseline model in Section 3.3. Table 3.8

[8] See Zellner (1971) and Barberis (2000).

shows the results under different combinations of relative risk aversion and time to retirement.

Compared to employees who have uncertainty in the return generating processes of both stock market and company stock, employees who have no parameter uncertainty in company stock will allocate more wealth to company stock while reducing their investment in the stock market. For instance, employees with $\gamma = 6$ and 10 years to retirement will allocate 25 (9) percent to stock market (company stock) when they have parameter uncertainty in both assets, but their investment in stock market (company stock) will decrease (increase) to 15 (19) percent when they have uncertainty only in stock market.

Employees' familiarity in company stock, however, has only a marginal effect on their investment in company stock, and the effect decreases with their level of risk aversion. For extremely risk-averse employees, their optimal allocation to company stock is small even when they have no uncertainty in the return generating process of company stock. For example, employees with 10 years to retirement and $\gamma = 12$ optimally allocate only 4 percent of their contribution to company stock. As to those employees with a low level of risk aversion, the effect of parameter uncertainty is also limited. For employees with 10 years to retirement and $\gamma = 4$, their optimal portfolio weight in company stock increases only 13 percent (from 20 percent to 33 percent) when they have no uncertainty in company stock's return generating process. Thus, the possibility that employees are more familiar with their own company cannot fully justify the observed high allocation to company stock.

According to Meulbroek (2005), familiarity bias can be very costly to employees. Assuming a holding period of 5 years and stock's volatility of 40 percent, Meulbroek (2005) shows that employees who invest all pension wealth to company stock may incur a 35 percent loss in the stock's market value due to under-diversification. It is interesting to ask even if employees are truly more familiar with company stock's return generating process, how costly it is when they over-allocate to company stock. To answer this question, we compare the dynamic trading strategy with three benchmark strategies that allocate 50, 75, and 100 percent of contribution to company stock and the rest to riskless asset, given that employees have no parameter uncertainty in company stock's return generating process. We estimate the cost by computing the contribution rate made in each benchmark strategy such that the expected utility is the same as that in the dynamic trading strategy which has an exogenous contribution rate of 6 percent.

The result is shown in Table 3.9. Over-allocating to company stock can be costly to investors even when they are more familiar with company stock. In the case of $\gamma = 6$, employees allocating 75 percent of their contribution to company stock should contribute 10.5 percent of their income to pension in order to obtain the same expected utility as that in the dynamic trading strategy. In other words, they have to contribute 4.5 percent more of their income to pension than the 6 percent contribution rate in the dynamic trading strategy. The gain from adopting the dynamic trading strategy also increases with employees' level of risk aversion. For example, the contribution rate for employees investing all pension wealth in company stock should be 7.43 percent if $\gamma = 2$, but it increases to 28.34 percent if $\gamma = 12$. In other words, it is costly to conservative employees to put much of their pension wealth to company stock even when they have no uncertainty in company stock's return generating process. Adding to Meulbroek's (2005) conclusion that familiarity bias can be very costly to employees, our result indicates that even when employees are more familiar with their company stock, high allocation to company stock can still be very costly.[9]

3.5.3 Employer's Match

Another possible reason for employees' high allocation to company stock is related to employers' exclusive match in company stock. Employees may view this match as an endorsement from management that company stock is a good investment target. In addition, employees are usually prohibited from re-allocating employers' match in company stock before reaching certain ages or service years.[10] This kind of restriction becomes even more popular among large companies. Here we are unable to test directly how the endorsement effect can affect employees' optimal portfolio decision, but we can examine how employers' restricted match in company stock can affect employees' optimal allocation to company stock.

[9] Assuming a holding period of 5 years and stock's volatility of 40 percent, Meulbroek (2005) shows that investors who invest all their pension wealth in company stock may incur 35 percent loss in the stock's market value due to under-diversification. This non-diversification cost is found to be decreasing with company capitalization. For instance, Evan and Macpherson (2004) report that the non-diversification cost is 13 percent for companies in the top one third of market value groups but increases to 54 percent for those firms with market value in the bottom one third. They also find that plan participants tend to decrease their company stock investment if non-diversification cost increases.

[10] For many companies, it is usually until the age of 55 or 65 or after retirement that investors can freely allocate their employers' match in company stock (see Meulbroek, 2005).

Let M be employers' match rate and for simplicity we assume that employees are not allowed to reallocate employers' match. The value function with employers' match in company stock can be written as

$$
\begin{aligned}
& V_t\left(\frac{W_t}{Y_t}, \mathbf{Z}_t\right) = \frac{V_t(W_t, Y_t, \mathbf{Z}_t)}{Y_t^{1-\gamma}} \\
& = \underset{\{\mathbf{x}_s\}_{s=t}^{T-1}}{\mathrm{Max}} \frac{1}{1-\gamma} \mathrm{E}_t \Bigg\{ \left(\frac{W_t}{Y_t} + H\right) \prod_{s=t}^{T-1} \left(\mathbf{x}_s' \mathbf{R}_{s+1}^e + R^f\right) + H \sum_{s=t+1}^{T-1} \exp\left(\sum_{v=t+1}^{s} (g + \varsigma_v)\right) \prod_{k=s}^{T-1} \left(\mathbf{x}_k' \mathbf{R}_{k+1}^e + R^f\right) \\
& + \left(\frac{W_t^m}{Y_t} + M\right) \prod_{s=t}^{T-1} \left(R_{s+1}^{e,company} + R^f\right) + \left(\frac{W_t^m}{Y_t} + M\right) \sum_{s=t+1}^{T-1} \exp\left(\sum_{v=t+1}^{s} (g + \varsigma_v)\right) \prod_{k=s}^{T-1} \left(R_{k+1}^{e,company} + R^f\right) \Bigg\}^{1-\gamma},
\end{aligned} \tag{3.8}
$$

where W_t^m is the accumulated wealth level from employers' match at time t, and $R_t^{e,company}$ is the excess company stock return at time t.

We assume that each period the employers match employees' contribution dollar to dollar up to 6 percent of employees' annual income, and the employees face the baseline case as described in Section 3.3. Employees are assumed to have 10 years to retirement, and labor income is correlated with stock market and company stock returns with correlation coefficients equal to 5 and 25 percent, respectively. Similarly to the model without employers' match, we approximate the value function by taking a second order Taylor expansion around terminal wealth and then solve for the optimal portfolio decision backward. Table 3.10 shows the result of employees' optimal portfolio decision and total portfolio decision that includes employers' exclusive match in company stock.[11]

[11] Since both employers and employees contribute 6 percent of annual labor income to the pension plans, the total portfolio decision is simply the equal average of employers and employees' portfolio decisions.

Employees' optimal portfolio decision when there is no employers' match is also presented.

When employers restrict their match in company stock and employees are not allowed to reallocate employers' match, employees with a medium or high level of risk aversion should optimally decrease their investment in company stock while increasing their allocation to stock market. Also, their total investment in risky assets increases with employers' match but the pattern is less pronounced when employees are very risk averse. For instance, employees with $\gamma = 4$ increase their investment in total risky assets from 59 to 74 percent, while employees with $\gamma = 12$ do not change their portfolio weight in total risky assets, which is 18 percent. For employees whose degree of risk aversion is low (i.e., $\gamma = 2$), employers' match in company stock makes them become more aggressive by allocating more to company stock because they already invest all pension wealth in total risky assets when there is no employers' match.

Employees' total portfolio weight shows how employers' exclusive match in company stock can affect employees' portfolio when they are not allowed to reallocate. For instance, employees with $\gamma = 6$ optimally invest nothing in company stock, but due to employers' exclusive match, their total portfolio weight of company stock is 50 percent. It seems that employers' exclusive match in company stock may provide some explanations to company stock investment in pension plans. This result, however, is based on the assumption that employees are not allowed to reallocate employers' match, but in reality employees usually can do so after reaching certain ages or service years. It thus raises another puzzle that employees choose not to reallocate their employers' match when they can do so. Thus, employers' exclusive match in company stock cannot fully justify employees' high allocation to company stock in their pension plans.

3.6. Conclusion

Due to the popularity of defined contribution plans and the empirical evidence that employees are not portfolio optimizers in their pension investment, how employees should make their portfolio decision for retirement thus becomes an important research topic. Assuming a discrete time model with multiple risky asset returns described by a VAR process, this paper studies the optimal portfolio decision in defined contribution plans with short sale constraints and company stock being available for investment. We find that a positive correlation between company stock returns and labor income decreases employees' optimal allocation to company stock and the effect is larger when employees have longer time to retirement. In addition, when the exogenous contribution rate increases, employees become more aggressive by investing more in total risky assets. Employees with a low level of risk aversion may optimally increase their allocation to company stock, while employees with a medium or high level of risk aversion will increase their allocation to the other risky assets but decreasing their allocation to company stock due to its higher correlation with labor income. Our performance test suggests that when asset returns are predictable, the welfare gains from adopting a dynamic portfolio strategy instead of some simplified trading strategies or some optimal strategies that maintain constant portfolio weights are economically significant. Besides, the gains from adopting a dynamic portfolio strategy instead of some simplified trading strategies increase with employees' level of risk aversion, suggesting the importance for conservative employees to invest strategically. The performance test shows that investing all pension wealth to company stock can be extremely costly to conservative employees.

This paper also examines if employees' high allocation to their company stock in pension plans can be justified with economic motivations. It is still puzzling after considering the factors including higher expected company stock return, familiarity with company stock, and employers' exclusive match policy in company stock. Higher expected company stock return can explain employees' high allocation to company stock only when employees have a low degree of risk aversion or when employees have extremely high expectation of company stock return. Familiarity in general has a small effect on employees' allocation to company stock and the effect decreases with employees' degree of risk aversion. When employers provide exclusive match in company stock, it is generally optimal for employees to decrease their allocation to company stock while increasing their investment in the other risky assets. Although the employers' match in company stock may explain employees' company stock investment, it still cannot justify employees' high allocation to company stock and raises another puzzle that employees choose not to reallocate their employers' match when they can do so.

Chapter 4

Conclusion

This dissertation contains two essays on the optimal portfolio decision for long-term investors.

The first essay studies the optimal consumption and portfolio decision for long-term investors with nontradable labor income when asset returns are predictable. I find that when labor income is positively correlated with stock return or is independent of all risky asset returns, more conservative investors hold a higher bond/stock ratio in their risky portfolio. However, the reverse pattern is true when labor income is positively correlated with bond return. In addition, the optimal investment in risky assets inherits the inverted U-shaped pattern of labor income growth with respect to time until retirement. The performance test proves the importance of adopting a dynamic portfolio strategy that considers asset return predictability in the presence of nontradable human capital.

The second essay addresses the topic of optimal portfolio strategy in defined contribution pension plans under a dynamic investment opportunity set with company stock available for investment. It shows that employees' optimal portfolio decision can be greatly affected by their time to retirement, risk preference, contribution rate, and the correlation between labor income and risky asset returns. This chapter then examines if employees' high investment in company stock can be justified with economic motivations, including short sale constraints, higher expected company stock return, employees' familiarity with company stock, and employers' exclusive match in company stock. I find none of these arguments can explain the observed high allocation to company stock. Furthermore, it can be costly for employees to over allocate their pension wealth to company stock. The more conservative the employees are, the higher the welfare loss.

This dissertation assumes that there is no transaction cost to investors when they adopt a dynamic portfolio strategy. Future research can incorporate transaction cost into the model to see how transaction cost can affect the result. In addition, for simplicity, I do not consider housing decision in these two essays. However, it is admitted that housing is an important factor that will affect each household's consumption as well as investment decision. It is also reserved for future research.

Appendices

Appendix A. Optimal portfolio choice in the retirement state

First we guess that the optimal solution in the retirement state can be expressed as

$$\boldsymbol{\alpha}_t^r = \mathbf{A}_0 + \mathbf{A}_1\mathbf{Z}_t,$$
$$c_t^r - w_t^r = b_0^r + \mathbf{B}_1'\mathbf{Z}_t + \mathbf{Z}_t'\mathbf{B}_2\mathbf{Z}_t.$$

Subtracting the log Euler equation for r_f from the general log Euler equation obtains

$$\mathrm{E}_t\left(r_{i,t+1} - r_f\right) + \frac{1}{2}\mathrm{Var}_t\left(r_{i,t+1}\right) = \gamma\mathrm{Cov}_t\left[r_{i,t+1}, \left(c_{t+1}^r - c_t^r\right)\right]. \tag{A1}$$

We have the trivial equality that

$$c_{t+1}^r - c_t^r = \left(c_{t+1}^r - w_{t+1}\right) - \left(c_t^r - w_t\right) + \left(w_{t+1} - w_t\right). \tag{A2}$$

Substituting equation (A2) into equation (A1) obtains

$$\begin{aligned}\mathrm{E}_t\left(r_{i,t+1} - r_f\right) + \frac{1}{2}\mathrm{Var}_t\left(r_{i,t+1}\right) &= \gamma\mathrm{Cov}_t\left(r_{i,t+1}, c_{t+1}^r - w_{t+1}\right) + \gamma\mathrm{Cov}_t\left(r_{i,t+1}, w_{t+1}^r - w_t^r\right) \\ &= \gamma\mathrm{Cov}_t\left(r_{i,t+1}, c_{t+1}^r - w_{t+1}\right) + \gamma\mathrm{Cov}_t\left(r_{i,t+1}, r_{p,t+1}^r\right).\end{aligned} \tag{A3}$$

Substituting the guessed relation $c_t^r - w_t^r = b_0^r + \mathbf{B}_1'\mathbf{Z}_t + \mathbf{Z}_t'\mathbf{B}_2\mathbf{Z}_t$ into equation (A3) obtains

$$\begin{aligned}\mathrm{E}_t\left(r_{i,t+1} - r_f\right) + \frac{1}{2}\mathrm{Var}_t\left(r_{i,t+1}\right) &= \gamma\mathrm{Cov}_t\left(r_{i,t+1}, \mathbf{B}_1'\mathbf{Z}_{t+1} + \mathbf{Z}_{t+1}'\mathbf{B}_2\mathbf{Z}_{t+1}\right) + \gamma\mathrm{Cov}_t\left(r_{i,t+1}, r_{p,t+1}\right) \\ &= \gamma\mathrm{Cov}_t\left[r_{i,t+1} - r_f, \mathbf{B}_1'\mathbf{v}_{t+1} + \left(\boldsymbol{\Phi}_0 + \boldsymbol{\Phi}_1\mathbf{Z}_t + \mathbf{v}_{t+1}\right)'\mathbf{B}_2\left(\boldsymbol{\Phi}_0 + \boldsymbol{\Phi}_1\mathbf{Z}_t + \mathbf{v}_{t+1}\right)\right] + \gamma\mathrm{Cov}_t\left(r_{i,t+1}, r_{p,t+1}\right) \\ &= \gamma\left[\boldsymbol{\Sigma}_v^{(i)\prime}\mathbf{B}_1 + \boldsymbol{\Sigma}_v^{(i)\prime}(\mathbf{B}_2 + \mathbf{B}_2')\left(\boldsymbol{\Phi}_0 + \boldsymbol{\Phi}_1\mathbf{Z}_t\right)\right] + \gamma\boldsymbol{\alpha}_t'\boldsymbol{\Sigma}_{XX}^{(i-1)}.\end{aligned}$$

Stacking the above result from i = 2 to n and rearranging terms give

$$\boldsymbol{\alpha}_t^r = \frac{1}{\gamma}\boldsymbol{\Sigma}_{XX}^{-1}\left[\mathrm{E}_t\left(\mathbf{X}_{t+1}\right)+\frac{1}{2}\mathrm{Var}_t\left(\mathbf{X}_{t+1}\right)\right]-\boldsymbol{\Sigma}_{XX}^{-1}\left(\boldsymbol{\Lambda}_0+\boldsymbol{\Lambda}_1\mathbf{Z}_t\right),$$

where $\boldsymbol{\Lambda}_0 = \left(\boldsymbol{\Sigma}_v\mathbf{H}_X'\right)'\mathbf{B}_1 + \left(\boldsymbol{\Sigma}_v\mathbf{H}_X'\right)'\left(\mathbf{B}_2+\mathbf{B}_2'\right)\boldsymbol{\Phi}_0$ and $\boldsymbol{\Lambda}_1 = \left(\boldsymbol{\Sigma}_v\mathbf{H}_X'\right)'\left(\mathbf{B}_2+\mathbf{B}_2'\right)\boldsymbol{\Phi}_1$.

Appendix B. Optimal consumption choice in the retirement state

Setting $i = p$ in the log Euler equation and rearranging terms obtains

$$\mathrm{E}_t\left[\left(c_{t+1}^r - c_t^r\right)\right] = \frac{1}{\gamma}\left\{\log\delta^r + \mathrm{E}_t\left(r_{p,t+1}\right)+\frac{1}{2}\mathrm{Var}_t\left[r_{p,t+1}-\gamma\left(c_{t+1}^r - c_t^r\right)\right]\right\}. \tag{B1}$$

With the trivial equality (A2) and the log-linear budget constraint in the retirement state, we obtain

$$\mathrm{E}_t\left(\Delta c_{t+1}^r\right) = \mathrm{E}_t\left(c_{t+1}^r - w_{t+1}\right) - \left(c_t^r - w_t\right) + k^r - \rho_c^r\left(c_t^r - w_t\right) + \mathrm{E}_t\left(r_{p,t+1}^r\right) \tag{B2}$$

Combining equation (B1) and equation (B2) gives

$$c_t^r - w_t = \frac{1}{1+\rho_c^r}\left\{\mathrm{E}_t\left(c_{t+1}^r - w_{t+1}^r\right)+k^r+\mathrm{E}_t\left(r_{p,t+1}^r\right)-\frac{1}{\gamma}\left\{\log\delta^r+\mathrm{E}_t\left(r_{p,t+1}^r\right)+\frac{1}{2}\mathrm{Var}_t\left[r_{p,t+1}^r-\gamma\left(c_{t+1}^r-c_t^r\right)\right]\right\}\right\} \tag{B3}$$

From the appendix of Campbell, *et al*.(2003), we have

$$\mathrm{E}_t\left(r_{p,t+1}^r\right) = \Gamma_0^r + \boldsymbol{\Gamma}_1^r\mathbf{Z}_t + \boldsymbol{\Gamma}_2^r vec\left(\mathbf{Z}_t\mathbf{Z}_t'\right), \tag{B4}$$

where

$$\Gamma_0^r = \mathbf{A}_0'\mathbf{H}_X\mathbf{\Phi}_0 + r_f + \frac{1}{2}\mathbf{A}_0'\boldsymbol{\sigma}_X^2 - \frac{1}{2}\mathbf{A}_0'\mathbf{\Sigma}_{XX}\mathbf{A}_0,$$

$$\mathbf{\Gamma}_1^r = \mathbf{\Phi}_0'\mathbf{H}_X\mathbf{A}_1 + \mathbf{A}_0'\mathbf{H}_X\mathbf{\Phi}_1 + \frac{1}{2}\boldsymbol{\sigma}_X^{2\,\prime}\mathbf{A}_1 - \mathbf{A}_0'\mathbf{\Sigma}_{XX}\mathbf{A}_1, \text{ and}$$

$$\mathbf{\Gamma}_2^r = vec(\mathbf{A}_1'\mathbf{H}_X\mathbf{\Phi}_1)' - \frac{1}{2}vec(\mathbf{A}_1'\mathbf{\Sigma}_{XX}\mathbf{A}_1)'.$$

It can be shown that

$$\begin{aligned}
&\mathrm{Var}_t\left[r_{p,t+1} - \gamma\left(c_{t+1}^r - c_t^r\right)\right] \\
&= \mathrm{Var}_t\left[r_{p,t+1}^r - \gamma\left(\mathbf{B}_1'\mathbf{Z}_{t+1} + \mathbf{Z}_{t+1}'\mathbf{B}_2\mathbf{Z}_{t+1} + r_{p,t+1}^r\right)\right] \\
&= \mathrm{Var}_t\left\{(1-\gamma)r_{p,t+1}^r - \gamma\left[\mathbf{B}_1'\mathbf{v}_{t+1} + \left(\mathbf{\Phi}_0 + \mathbf{\Phi}_1\mathbf{Z}_t + \mathbf{v}_{t+1}\right)'\mathbf{B}_2\left(\mathbf{\Phi}_0 + \mathbf{\Phi}_1\mathbf{Z}_t + \mathbf{v}_{t+1}\right)\right]\right\} \\
&= \mathrm{Var}_t\left[\left(\boldsymbol{\pi}_1 + \mathbf{Z}_t'\boldsymbol{\pi}_2\right)\mathbf{v}_{t+1} - \gamma vec(\mathbf{B}_2)'vec(\mathbf{v}_{t+1}\mathbf{v}_{t+1}')\right],
\end{aligned}$$

where *vec* denotes the vectorization factor, $\boldsymbol{\pi}_1 = (1-\gamma)\mathbf{A}_0'\mathbf{H}_X - \gamma\mathbf{B}_1' - \gamma\mathbf{\Phi}_0'\left(\mathbf{B}_2 + \mathbf{B}_2'\right)$,

and $\boldsymbol{\pi}_2 = -\gamma\mathbf{\Phi}_1'\left(\mathbf{B}_2 + \mathbf{B}_2'\right) + (1-\gamma)\mathbf{A}_1'\mathbf{H}_X$.

Therefore,

$$\begin{aligned}
&\mathrm{Var}_t\left[r_{p,t+1}^r - \gamma\left(c_{t+1}^r - c_t^r\right)\right] = \left(\boldsymbol{\pi}_1 + \mathbf{Z}_t'\boldsymbol{\pi}_2\right)\mathbf{\Sigma}_v\left(\boldsymbol{\pi}_1 + \mathbf{Z}_t'\boldsymbol{\pi}_2\right)' + \gamma^2 vec(\mathbf{B}_2)'\mathrm{Var}_t\left[vec\left(\mathbf{v}_{t+1}\mathbf{v}_{t+1}'\right)\right]vec(\mathbf{B}_2) \\
&= \left(\boldsymbol{\pi}_1 + \mathbf{Z}_t'\boldsymbol{\pi}_2\right)\mathbf{\Sigma}_v\left(\boldsymbol{\pi}_1' + \boldsymbol{\pi}_2'\mathbf{Z}_t\right) + \gamma^2 vec(\mathbf{B}_2)'\mathrm{Var}_t\left[vec\left(\mathbf{v}_{t+1}\mathbf{v}_{t+1}'\right)\right]vec(\mathbf{B}_2) \\
&= \boldsymbol{\pi}_1\mathbf{\Sigma}_v\boldsymbol{\pi}_1' + \gamma^2 vec(\mathbf{B}_2)'\mathrm{Var}_t\left[vec\left(\mathbf{v}_{t+1}\mathbf{v}_{t+1}'\right)\right]vec(\mathbf{B}_2) + \left(2\boldsymbol{\pi}_1\mathbf{\Sigma}_v\boldsymbol{\pi}_2'\right)\mathbf{Z}_t + vec\left(\boldsymbol{\pi}_2\mathbf{\Sigma}_v\boldsymbol{\pi}_2'\right)'vec\left(\mathbf{Z}_t\mathbf{Z}_t'\right) \\
&= V_0 + \mathbf{V}_1\mathbf{Z}_t + \mathbf{V}_2 vec\left(\mathbf{Z}_t\mathbf{Z}_t'\right),
\end{aligned}$$

where

$$V_0 = \boldsymbol{\pi}_1 \boldsymbol{\Sigma}_v \boldsymbol{\pi}_1' + \gamma^2 vec(\mathbf{B}_2)' \mathrm{Var}_t \left[vec(\mathbf{v}_{t+1}\mathbf{v}_{t+1}') \right] vec(\mathbf{B}_2),$$

$$\mathbf{V}_1 = 2\boldsymbol{\pi}_1 \boldsymbol{\Sigma}_v \boldsymbol{\pi}_2', \text{ and}$$

$$\mathbf{V}_2 = vec(\boldsymbol{\pi}_2 \boldsymbol{\Sigma}_v \boldsymbol{\pi}_2')'. \tag{B5}$$

From the guessed relationship,

$$\mathrm{E}_t \left(c_{t+1}^r - w_{t+1}^r \right) = b_0^r + \mathrm{E}_t \left(\mathbf{B}_1' \mathbf{Z}_{t+1} + \mathbf{Z}_{t+1}' \mathbf{B}_2 \mathbf{Z}_{t+1} \right), \tag{B6}$$

where

$$\mathrm{E}_t \left(\mathbf{B}_1' \mathbf{Z}_{t+1} \right) = \mathbf{B}_1' \boldsymbol{\Phi}_0 + \mathbf{B}_1' \boldsymbol{\Phi}_1 \mathbf{Z}_t, \text{ and}$$

$$\mathrm{E}_t \left(\mathbf{Z}_{t+1}' \mathbf{B}_2 \mathbf{Z}_{t+1} \right) = \boldsymbol{\Phi}_0' \mathbf{B}_2 \boldsymbol{\Phi}_0 + \left[\boldsymbol{\Phi}_0' \left(\mathbf{B}_2 + \mathbf{B}_2' \right) \boldsymbol{\Phi}_1 \right] \mathbf{Z}_t + vec\left(\boldsymbol{\Phi}_1{}' \mathbf{B}_2 \boldsymbol{\Phi}_1 \right)' vec\left(\mathbf{Z}_t \mathbf{Z}_t' \right) + vec(\mathbf{B}_2)' vec(\boldsymbol{\Sigma}_v).$$

Substituting equations (B4)-(B6) into equation (B3) obtains

$$\begin{aligned} c_t^r - w_t^r = \frac{1}{1+\rho_c^r} \Big\{ & b_0^r + k^r + \mathbf{B}_1' \boldsymbol{\Phi}_0 + \mathbf{B}_1' \boldsymbol{\Phi}_1 \mathbf{Z}_t + \boldsymbol{\Phi}_0' \mathbf{B}_2 \boldsymbol{\Phi}_0 + \left[\boldsymbol{\Phi}_0' \left(\mathbf{B}_2 + \mathbf{B}_2' \right) \boldsymbol{\Phi}_1 \right] \mathbf{Z}_t \\ & + vec(\boldsymbol{\Phi}_1' \mathbf{B}_2 \boldsymbol{\Phi}_1)' vec\left(\mathbf{Z}_t \mathbf{Z}_t' \right) + vec\left(\mathbf{B}_2 \right)' vec\left(\boldsymbol{\Sigma}_v \right) - \frac{1}{\gamma} \log \delta^r + \left(1 - \frac{1}{\gamma} \right) \Gamma_0^r + \left(1 - \frac{1}{\gamma} \right) \boldsymbol{\Gamma}_1^r \mathbf{Z}_t \\ & + \left(1 - \frac{1}{\gamma} \right) \boldsymbol{\Gamma}_2^r vec\left(\mathbf{Z}_t \mathbf{Z}_t' \right) - \frac{1}{2\gamma} V_0 - \frac{1}{2\gamma} \mathbf{V}_1 \mathbf{Z}_t - \frac{1}{2\gamma} \mathbf{V}_2 vec\left(\mathbf{Z}_t \mathbf{Z}_t' \right) \Big\}. \end{aligned}$$

Thus,

$$c_t^r - w_t = \Xi_0^r + \boldsymbol{\Xi}_1^r \mathbf{Z}_t + \boldsymbol{\Xi}_2^r vec\left(\mathbf{Z}_t \mathbf{Z}_t' \right),$$

where

$$\Xi_0^r = \frac{1}{1+\rho_c^r}\left[b_0^r + \mathbf{B}_1'\mathbf{\Phi}_0 + \mathbf{\Phi}_0'\mathbf{B}_2\mathbf{\Phi}_0 + vec(\mathbf{B}_2)' vec(\mathbf{\Sigma}_v) - \frac{1}{\gamma}\log\delta^r + \left(1-\frac{1}{\gamma}\right)\Gamma_0^r - \frac{1}{2\gamma}V_0 + k^r\right],$$

$$\mathbf{\Xi}_1^r = \frac{1}{1+\rho_c^r}\left[\mathbf{B}_1'\mathbf{\Phi}_1 + \mathbf{\Phi}_0'(\mathbf{B}_2 + \mathbf{B}_2')\mathbf{\Phi}_1 + \left(1-\frac{1}{\gamma}\right)\mathbf{\Gamma}_1^r - \frac{1}{2\gamma}\mathbf{V}_1\right], \text{ and}$$

$$\mathbf{\Xi}_2^r = \frac{1}{1+\rho_c^r}\left[vec(\mathbf{\Phi}_1'\mathbf{B}_2\mathbf{\Phi}_1)' + \left(1-\frac{1}{\gamma}\right)\mathbf{\Gamma}_2^r - \frac{1}{2\gamma}\mathbf{V}_2\right].$$

Appendix C. Optimal portfolio choice in the employment state

First we guess that the optimal solution in the employment state can be expressed as

$$\boldsymbol{\alpha}_t^e = \mathbf{M}_0 + \mathbf{M}_1\mathbf{Z}_t,$$
$$c_t^e - y_t = b_0^e + \mathbf{N}_1'\mathbf{Z}_t + \mathbf{Z}_t'\mathbf{N}_2\mathbf{Z}_t + b_1^e(w_t^e - y_t).$$

The optimal policy in the retirement state can be re-written as

$$c_{t+1}^r - y_{t+1} = b_0^r + (w_{t+1}^r - y_{t+1}) + \mathbf{B}_1'\mathbf{Z}_{t+1} + \mathbf{Z}_{t+1}'\mathbf{B}_2\mathbf{Z}_{t+1}. \tag{C1}$$

Subtracting the log Euler equation for the risk-free asset from the general log Euler equation in the employment state obtains

$$\mathrm{E}_t(r_{i,t+1} - r_f) + \frac{1}{2}\mathrm{Var}_t(r_{i,t+1}) = \gamma\left[\pi^e \mathrm{Cov}_t(r_{i,t+1}, c_{t+1}^e - c_t^e) + (1-\pi^e)\mathrm{Cov}_t(r_{i,t+1}, c_{t+1}^r - c_t^e)\right] \tag{C2}$$

Also, we have the trivial equality

$$c_{t+1}^s - c_t^e = (c_{t+1}^s - y_{t+1}) - (c_t^e - y_t) + \Delta y_{t+1} \text{ , where } s = e \text{ or } r. \tag{C3}$$

With the budget constraint, the guessed consumption rule in the employment state and equation (C3), we get

$$
\begin{aligned}
&\mathrm{Cov}_t\left(r_{i,t+1}, c^e_{t+1} - c^e_t\right) = \mathrm{Cov}_t\left(r_{i,t+1}, c^e_{t+1} - y_{t+1} + \Delta y_{t+1}\right) \\
&= \mathrm{Cov}_t\left[r_{i,t+1}, \mathbf{N}_1'\mathbf{Z}_{t+1} + \mathbf{Z}_{t+1}'\mathbf{N}_2\mathbf{Z}_{t+1} + b_1^e\left(w^e_{t+1} - y_{t+1}\right) + \Delta y_{t+1}\right] \\
&= \mathrm{Cov}_t\left[r_{i,t+1}, \mathbf{N}_1'\mathbf{Z}_{t+1} + \mathbf{Z}_{t+1}'\mathbf{N}_2\mathbf{Z}_{t+1} + b_1^e\left(-\Delta y_{t+1} + r_{p,t+1}\right) + \Delta y_{t+1}\right] \\
&= \mathrm{Cov}_t\left[r_{i,t+1}, \mathbf{N}_1'\mathbf{v}_{t+1} + \left(\mathbf{\Phi}_0 + \mathbf{\Phi}_1\mathbf{Z}_t + \mathbf{v}_{t+1}\right)'\mathbf{N}_2\left(\mathbf{\Phi}_0 + \mathbf{\Phi}_1\mathbf{Z}_t + \mathbf{v}_{t+1}\right)\right] \\
&\quad + \mathrm{Cov}_t\left[r_{i,t+1}, b_1^e r_{p,t+1} + \left(1 - b_1^e\right)\Delta y_{t+1}\right] \\
&= \mathbf{\Sigma}_v^{(i)'}\mathbf{N}_1 + \mathbf{\Sigma}_v^{(i)'}\left(\mathbf{N}_2 + \mathbf{N}_2'\right)\left(\mathbf{\Phi}_0 + \mathbf{\Phi}_1\mathbf{Z}_t\right) + b_1^e\boldsymbol{\alpha}_t^{e'}\mathbf{\Sigma}_{XX}^{(i-1)} + \left(1 - b_1^e\right)\boldsymbol{\sigma}_{\varsigma i}.
\end{aligned}
\tag{C4}
$$

Using equations (C1), (C3) and the guessed consumption rule in the retirement state, we obtain

$$
\begin{aligned}
&\mathrm{Cov}_t\left(r_{i,t+1}, c^r_{t+1} - c^e_t\right) = \mathrm{Cov}_t\left(r_{i,t+1}, c^r_{t+1} - y_{t+1} + \Delta y_{t+1}\right) \\
&= \mathrm{Cov}_t\left[r_{i,t+1}, \left(w^r_{t+1} - y_{t+1}\right) + \mathbf{B}_1'\mathbf{Z}_{t+1} + \mathbf{Z}_{t+1}'\mathbf{B}_2\mathbf{Z}_{t+1} + y_{t+1} - y_t\right] \\
&= \mathrm{Cov}_t\left(r_{i,t+1}, -\Delta y_{t+1} + r_{p,t+1} + \mathbf{B}_1'\mathbf{Z}_{t+1} + \mathbf{Z}_{t+1}'\mathbf{B}_2\mathbf{Z}_{t+1} + \Delta y_{t+1}\right) \\
&= \mathbf{\Sigma}_v^{(i)'}\mathbf{B}_1 + \mathbf{\Sigma}_v^{(i)'}\left(\mathbf{B}_2 + \mathbf{B}_2'\right)\left(\mathbf{\Phi}_0 + \mathbf{\Phi}_1\mathbf{Z}_t\right) + \boldsymbol{\alpha}_t^{e'}\mathbf{\Sigma}_{XX}^{(i-1)}.
\end{aligned}
\tag{C5}
$$

Substituting equations (C4) and (C5) into equation (C2) and stacking gives

$$
\begin{aligned}
&\mathrm{E}_t\left(\mathbf{X}_{t+1}\right) + \frac{1}{2}\mathrm{Var}_t\left(\mathbf{X}_{t+1}\right) \\
&= \gamma\left\{\pi^e\left[\mathbf{\Lambda}_0^e + \mathbf{\Lambda}_1^e\mathbf{Z}_t + b_1^e\mathbf{\Sigma}_{XX}\boldsymbol{\alpha}_t^e + (1 - b_1^e)\boldsymbol{\sigma}_{\varsigma x}\right] + \left(1 - \pi^e\right)\left[\mathbf{\Lambda}_0^r + \mathbf{\Lambda}_1^r\mathbf{Z}_t + \mathbf{\Sigma}_{XX}\boldsymbol{\alpha}_t^e\right]\right\},
\end{aligned}
$$

where

$$
\mathbf{\Lambda}_0^e = \left(\mathbf{\Sigma}_v\mathbf{H}_X'\right)'\mathbf{N}_1 + \left(\mathbf{\Sigma}_v\mathbf{H}_X'\right)'\left(\mathbf{N}_2 + \mathbf{N}_2'\right)\mathbf{\Phi}_{0,}
$$

$$
\mathbf{\Lambda}_1^e = \left(\mathbf{\Sigma}_v\mathbf{H}_X'\right)'\left(\mathbf{N}_2 + \mathbf{N}_2'\right)\mathbf{\Phi}_1,
$$

$$
\mathbf{\Lambda}_0^r = \left(\mathbf{\Sigma}_v\mathbf{H}_X'\right)'\mathbf{B}_1 + \left(\mathbf{\Sigma}_v\mathbf{H}_X'\right)'\left(\mathbf{B}_2 + \mathbf{B}_2'\right)\mathbf{\Phi}_0, \text{ and}
$$

$$
\mathbf{\Lambda}_1^r = \left(\mathbf{\Sigma}_v\mathbf{H}_X'\right)'\left(\mathbf{B}_2 + \mathbf{B}_2'\right)\mathbf{\Phi}_1.
$$

Appendix D. Optimal consumption choice in the employment state

Substituting $i = p$ into the log Euler equation in the employment state obtains

$$0 = \sum_{s=e,r} \pi^s \left\{ \log \delta^s - \gamma \mathrm{E}_t \left(c_{t+1}^s - c_t^e \right) + \mathrm{E}_t \left(r_{p,t+1}^e \right) + \frac{1}{2} \mathrm{Var}_t \left[r_{p,t+1}^e - \gamma \left(c_{t+1}^s - c_t^e \right) \right] \right\}. \tag{D1}$$

Substituting the trivial equality (C3) into equation (D1) yields

$$\begin{aligned} &\pi^e \left[\mathrm{E}_t \left(c_{t+1}^e - y_{t+1} \right) + \mathrm{E}_t \left(y_{t+1} - y_t \right) \right] + \left(1 - \pi^e \right) \left[\mathrm{E}_t \left(c_{t+1}^r - y_{t+1} \right) + \mathrm{E}_t \left(y_{t+1} - y_t \right) \right] - \left(c_t^e - y_t \right) \\ &= \frac{1}{\gamma} \sum_{s=e,r} \pi^s \left\{ \log \delta^s + \mathrm{E}_t \left(r_{p,t+1}^e \right) + \frac{1}{2} \mathrm{Var}_t \left[r_{p,t+1}^e - \gamma \left(c_{t+1}^s - c_t^e \right) \right] \right\} \end{aligned} \tag{D2}$$

In addition, we have

$$\mathrm{E}_t \left(c_{t+1}^e - y_{t+1} \right) + \mathrm{E}_t \left(y_{t+1} - y_t \right) = b_0^e + \mathrm{E}_t \left(\mathbf{N}_1' \mathbf{Z}_{t+1} + \mathbf{Z}_{t+1}' \mathbf{N}_2 \mathbf{Z}_{t+1} \right) + b_1^e \mathrm{E}_t \left(w_{t+1}^e - y_{t+1} \right) + g, \tag{D3}$$

where

$\mathrm{E}_t \left(\mathbf{N}_1' \mathbf{Z}_{t+1} \right) = \mathbf{N}_1' \mathbf{\Phi}_0 + \mathbf{N}_1' \mathbf{\Phi}_1 \mathbf{Z}_t$, and

$$\mathrm{E}_t \left(\mathbf{Z}_{t+1}' \mathbf{N}_2 \mathbf{Z}_{t+1} \right) = \mathbf{\Phi}_0' \mathbf{N}_2 \mathbf{\Phi}_0 + \left[\mathbf{\Phi}_0' \left(\mathbf{N}_2 + \mathbf{N}_2' \right) \mathbf{\Phi}_1 \right] \mathbf{Z}_t + vec\left(\mathbf{\Phi}_1' \mathbf{N}_2 \mathbf{\Phi}_1 \right)' vec\left(\mathbf{Z}_t \mathbf{Z}_t' \right) + vec(\mathbf{N}_2)' vec(\mathbf{\Sigma}_v) .$$

Similarly,

$$\mathrm{E}_t \left(c_{t+1}^r - h y_{t+1} \right) + \mathrm{E}_t \left(y_{t+1} - y_t \right) = b_0^r + \mathrm{E}_t \left(\mathbf{B}_1' \mathbf{Z}_{t+1} + \mathbf{Z}_{t+1}' \mathbf{B}_2 \mathbf{Z}_{t+1} \right) + \mathrm{E}_t \left(w_{t+1}^r - y_{t+1} \right) + g, \tag{D4}$$

where

$\mathrm{E}_t \left(\mathbf{B}_1' \mathbf{Z}_{t+1} \right) = \mathbf{B}_1' \mathbf{\Phi}_0 + \mathbf{B}_1' \mathbf{\Phi}_1 \mathbf{Z}_t$, and

$$\mathrm{E}_t \left(\mathbf{Z}_{t+1}' \mathbf{B}_2 \mathbf{Z}_{t+1} \right) = \mathbf{\Phi}_0' \mathbf{B}_2 \mathbf{\Phi}_0 + \left[\mathbf{\Phi}_0' \left(\mathbf{B}_2 + \mathbf{B}_2' \right) \mathbf{\Phi}_1 \right] \mathbf{Z}_t + vec\left(\mathbf{\Phi}_1' \mathbf{B}_2 \mathbf{\Phi}_1 \right)' vec\left(\mathbf{Z}_t \mathbf{Z}_t' \right) + vec(\mathbf{B}_2)' vec(\mathbf{\Sigma}_v) .$$

Now we consider the variance term in equation (D1). Firstly, we consider $\mathrm{Var}_t \left[r_{p,t+1}^e - \gamma \left(c_{t+1}^e - c_t^e \right) \right]$.

$$
\begin{aligned}
&\mathrm{Var}_t\left[r^e_{p,t+1}-\gamma\left(c^e_{t+1}-c^e_t\right)\right]=Var_t\left\{r^e_{p,t+1}-\gamma\left[\left(c^e_{t+1}-y_{t+1}\right)+\left(y_{t+1}-y_t\right)\right]\right\}\\
&=\mathrm{Var}_t\left\{\boldsymbol{\alpha}'_t\mathbf{X}_{t+1}-\gamma\left[\mathbf{N}'_1\mathbf{Z}_{t+1}+\mathbf{Z}'_{t+1}\mathbf{N}_2\mathbf{Z}_{t+1}+b^e_1\left(w^e_{t+1}-y_{t+1}\right)\right]-\gamma\Delta y_{t+1}\right\}\\
&=\mathrm{Var}_t\left\{\boldsymbol{\alpha}'_t\mathbf{H}_X\mathbf{v}_{t+1}-\gamma\left[\ \mathbf{N}'_1\mathbf{v}_{t+1}+\left(\boldsymbol{\Phi}_0+\boldsymbol{\Phi}_1\mathbf{Z}_t+\mathbf{v}_{t+1}\right)'\mathbf{N}_2\left(\boldsymbol{\Phi}_0+\boldsymbol{\Phi}_1\mathbf{Z}_t+\mathbf{v}_{t+1}\right)+b^e_1\left(-\Delta y_{t+1}+r_{p,t+1}\right)\ \right]-\gamma\Delta y_{t+1}\right\}\\
&=\mathrm{Var}_t\left[\boldsymbol{\alpha}'_t\mathbf{H}_X\mathbf{v}_{t+1}-\gamma\left(\ \mathbf{N}'_1\mathbf{v}_{t+1}+\boldsymbol{\Phi}'_0\mathbf{N}_2\mathbf{v}_{t+1}+\mathbf{Z}'_t\boldsymbol{\Phi}'_1\mathbf{N}_2\mathbf{v}_{t+1}+\mathbf{v}'_{t+1}\mathbf{N}_2\boldsymbol{\Phi}_0+\mathbf{v}'_{t+1}\mathbf{N}_2\boldsymbol{\Phi}_1\mathbf{Z}_t+\mathbf{v}'_{t+1}\mathbf{N}_2\mathbf{v}_{t+1}\right.\right.\\
&\quad\left.\left.+b^e_1\boldsymbol{\alpha}'_t\mathbf{H}_X\mathbf{v}_{t+1}\ \right)-\gamma\left(1-b^e_1\right)\Delta y_{t+1}\ \right].
\end{aligned}
$$

Substituting $\boldsymbol{\alpha}'_t=\mathbf{M}'_0+\mathbf{Z}'_t\mathbf{M}_1$ into the above equation obtains

$$
\begin{aligned}
&\mathrm{Var}_t\left[r^e_{p,t+1}-\gamma\left(c^e_{t+1}-c^e_t\right)\right]\\
&=\mathrm{Var}_t\left[\ \boldsymbol{\psi}_1\mathbf{v}_{t+1}+\mathbf{Z}'_t\boldsymbol{\psi}_2\mathbf{v}_{t+1}-\gamma vec\left(\mathbf{N}_2\right)'vec\left(\mathbf{v}_{t+1}\mathbf{v}'_{t+1}\right)-\gamma\left(1-b^e_1\right)\Delta y_{t+1}\ \right],
\end{aligned}
\tag{D5}
$$

where

$$
\boldsymbol{\psi}_1=\left(1-\gamma b^e_1\right)\mathbf{M}'_0\mathbf{H}_X-\gamma\mathbf{N}'_1-\gamma\boldsymbol{\Phi}'_0\left(\mathbf{N}_2+\mathbf{N}'_2\right),\text{ and }\boldsymbol{\psi}_2=\left(1-\gamma b^e_1\right)\mathbf{M}'_1\mathbf{H}_X-\gamma\boldsymbol{\Phi}'_1\left(\mathbf{N}_2+\mathbf{N}'_2\right).
$$

Therefore,

$$
\begin{aligned}
&\mathrm{Var}_t\left[r^e_{p,t+1}-\gamma\left(c^e_{t+1}-c^e_t\right)\right]=\mathrm{Var}_t\left[\ \boldsymbol{\psi}_1\mathbf{v}_{t+1}+\mathbf{Z}'_t\boldsymbol{\psi}_2\mathbf{v}_{t+1}-\gamma vec\left(\mathbf{N}_2\right)'vec\left(\mathbf{v}_{t+1}\mathbf{v}'_{t+1}\right)-\gamma\left(1-b^e_1\right)\Delta y_{t+1}\ \right]\\
&=\boldsymbol{\psi}_1\boldsymbol{\Sigma}_v\boldsymbol{\psi}'_1+\mathbf{Z}'_t\boldsymbol{\psi}_2\boldsymbol{\Sigma}_v\boldsymbol{\psi}'_2\mathbf{Z}_t+\gamma^2 vec\left(\mathbf{N}_2\right)'Var_t\left[vec\left(\mathbf{v}_{t+1}\mathbf{v}'_{t+1}\right)\right]vec\left(\mathbf{N}_2\right)+\gamma^2\left(1-b^e_1\right)^2\sigma^2_\varsigma\\
&\quad+2\boldsymbol{\psi}_1\boldsymbol{\Sigma}_v\boldsymbol{\psi}'_2\mathbf{Z}_t-2\gamma\left(1-b^e_1\right)\left(\boldsymbol{\psi}_1+\mathbf{Z}'_t\boldsymbol{\psi}_2\right)\boldsymbol{\sigma}_{\varsigma v}\\
&=\boldsymbol{\psi}_1\boldsymbol{\Sigma}_v\boldsymbol{\psi}'_1+\gamma^2 vec\left(\mathbf{N}_2\right)'Var_t\left[vec\left(\mathbf{v}_{t+1}\mathbf{v}'_{t+1}\right)\right]vec\left(\mathbf{N}_2\right)+\gamma^2\left(1-b^e_1\right)^2\sigma^2_\varsigma-2\boldsymbol{\psi}_1\gamma\left(1-b^e_1\right)\boldsymbol{\sigma}_{\varsigma v}\\
&\quad+\left[2\boldsymbol{\psi}_1\boldsymbol{\Sigma}_v\boldsymbol{\psi}'_2-2\gamma\left(1-b^e_1\right)\boldsymbol{\sigma}'_{\varsigma v}\boldsymbol{\psi}'_2\right]\mathbf{Z}_t+vec\left(\boldsymbol{\psi}_2\boldsymbol{\Sigma}_v\boldsymbol{\psi}'_2\right)'vec\left(\mathbf{Z}_t\mathbf{Z}'_t\right)\\
&=\Omega_0+\boldsymbol{\Omega}_1\mathbf{Z}_t+\boldsymbol{\Omega}_2 vec\left(\mathbf{Z}_t\mathbf{Z}'_t\right)
\end{aligned}
\tag{D6}
$$

where

$$
\Omega_0=\boldsymbol{\psi}_1\boldsymbol{\Sigma}_v\boldsymbol{\psi}'_1+\gamma^2 vec\left(\mathbf{N}_2\right)'Var\left[vec\left(\mathbf{v}_{t+1}\mathbf{v}'_{t+1}\right)\right]_t vec\left(\mathbf{N}_2\right)+\gamma^2\left(1-b^e_1\right)^2\sigma^2_\varsigma-2\boldsymbol{\psi}_1\gamma\left(1-b^e_1\right)\boldsymbol{\sigma}_{\varsigma v},
$$

$$
\boldsymbol{\Omega}_1=2\boldsymbol{\psi}_1\boldsymbol{\Sigma}_v\boldsymbol{\psi}'_2-2\gamma\left(1-b^e_1\right)\boldsymbol{\sigma}'_{\varsigma v}\boldsymbol{\psi}'_2,\text{ and}
$$

$$
\boldsymbol{\Omega}_2=vec\left(\boldsymbol{\psi}_2\boldsymbol{\Sigma}_v\boldsymbol{\psi}'_2\right)'.
$$

Now we consider the variance term $\mathrm{Var}_t\left[r^e_{p,t+1} - \gamma\left(c^r_{t+1} - c^e_t\right)\right]$.

$$\begin{aligned}
&\mathrm{Var}_t\left[r^e_{p,t+1} - \gamma\left(c^r_{t+1} - c^e_t\right)\right] = \mathrm{Var}_t\left\{r_{p,t+1} - \gamma\left[\left(c^r_{t+1} - y_{t+1}\right) + \left(y_{t+1} - y_t\right)\right]\right\} \\
&= \mathrm{Var}_t\left\{\boldsymbol{\alpha}'_t \mathbf{X}_{t+1} - \gamma\left[\mathbf{B}'_1 \mathbf{Z}_{t+1} + \mathbf{Z}'_{t+1}\mathbf{B}_2 \mathbf{Z}_{t+1} + \left(w^r_{t+1} - y_{t+1}\right) - \gamma \Delta y_{t+1}\right]\right\} \\
&= \mathrm{Var}_t\left\{\boldsymbol{\alpha}'_t \mathbf{H}_X \mathbf{v}_{t+1} - \gamma\left[\mathbf{B}'_1 \mathbf{v}_{t+1} + \left(\boldsymbol{\Phi}_0 + \boldsymbol{\Phi}_1 \mathbf{Z}_t + \mathbf{v}_{t+1}\right)' \mathbf{B}_2 \left(\boldsymbol{\Phi}_0 + \boldsymbol{\Phi}_1 \mathbf{Z}_t + \mathbf{v}_{t+1}\right) + \boldsymbol{\alpha}'_t \mathbf{H}_X \mathbf{v}_{t+1}\right]\right\}.
\end{aligned}$$

Again, substituting $\boldsymbol{\alpha}'_t = \mathbf{M}'_0 + \mathbf{Z}'_t \mathbf{M}_1$ into the above equation obtains

$$\mathrm{Var}_t\left[r^e_{p,t+1} - \gamma\left(c^r_{t+1} - c^e_t\right)\right] = \mathrm{Var}_t\left[\boldsymbol{\pi}^e_1 \mathbf{v}_{t+1} + \mathbf{Z}'_t \boldsymbol{\pi}^e_2 \mathbf{v}_{t+1} - \gamma vec(\mathbf{B}_2)' vec(\mathbf{v}_{t+1}\mathbf{v}'_{t+1})\right], \tag{D7}$$

where $\boldsymbol{\pi}^e_1 = (1-\gamma)\mathbf{M}'_0 \mathbf{H}_X - \gamma \mathbf{B}'_1 - \gamma \boldsymbol{\Phi}'_0\left(\mathbf{B}_2 + \mathbf{B}'_2\right)$ and $\pi^e_2 = (1-\gamma)\mathbf{M}'_1 \mathbf{H}_X - \gamma \boldsymbol{\Phi}'_1\left(\mathbf{B}_2 + \mathbf{B}'_2\right)$.

Thus, we get

$$\mathrm{Var}_t\left[r^e_{p,t+1} - \gamma\left(c^r_{t+1} - c^e_t\right)\right] = V^e_0 + \mathbf{V}^e_1 \mathbf{Z}_t + \mathbf{V}^e_2 vec(\mathbf{Z}_t \mathbf{Z}'_t), \tag{D8}$$

where

$$V^e_0 = \boldsymbol{\pi}^e_1 \boldsymbol{\Sigma}_v \boldsymbol{\pi}^{e\,\prime}_1 + \gamma^2 vec(\mathbf{B}_2)' Var_t\left[vec(\mathbf{v}_{t+1}\mathbf{v}'_{t+1})\right] vec(\mathbf{B}_2),$$

$$\mathbf{V}^e_1 = 2\boldsymbol{\pi}^e_1 \boldsymbol{\Sigma}_v \boldsymbol{\pi}^{e\,\prime}_2, \text{ and}$$

$$\mathbf{V}^e_2 = vec(\boldsymbol{\pi}^e_2 \boldsymbol{\Sigma}_v \boldsymbol{\pi}^{e\,\prime}_2)'.$$

Now we consider the term $\mathrm{E}_t\left(w^e_{t+1} - y_{t+1}\right)$. It can be shown that

$$\begin{aligned}
&\mathrm{E}_t\left(w^e_{t+1} - y_{t+1}\right) = k^e + \rho^e_w\left(w_t - y_t\right) - \rho^e_c\left(c_t - y_t\right) + \mathrm{E}_t(r^e_{p,t+1}) - g \\
&= k^e + \rho^e_w\left(w_t - y_t\right) - \rho^e_c\left(c_t - y_t\right) + \Gamma^e_0 + \boldsymbol{\Gamma}^e_1 \mathbf{Z}_t + \boldsymbol{\Gamma}^e_2 vec(\mathbf{Z}_t \mathbf{Z}'_t) - g,
\end{aligned} \tag{D9}$$

where

$$\Gamma^e_0 = \mathbf{M}'_0 \mathbf{H}_X \boldsymbol{\Phi}_0 + r_f + \frac{1}{2}\mathbf{M}'_0 \boldsymbol{\sigma}^2_X - \frac{1}{2}\mathbf{M}'_0 \boldsymbol{\Sigma}_{XX} \mathbf{M}_0,$$

$$\boldsymbol{\Gamma}^e_1 = \boldsymbol{\Phi}'_0 \mathbf{H}_X \mathbf{M}_1 + \mathbf{M}'_0 \mathbf{H}_X \boldsymbol{\Phi}_1 + \frac{1}{2}\boldsymbol{\sigma}^{2\,\prime}_X \mathbf{M}_1 - \mathbf{M}'_0 \boldsymbol{\Sigma}_{XX} \mathbf{M}_1, \text{ and}$$

$$\mathbf{\Gamma}_2^e = vec\left(\mathbf{M}_1'\mathbf{H}_X\mathbf{\Phi}_1\right)' - \frac{1}{2}vec\left(\mathbf{M}_1'\mathbf{\Sigma}_{XX}\mathbf{M}_1\right)'.$$

Substituting the results from equations (D3)-(D8) into equation (D2) obtains

$$\begin{aligned}
c_t^e - y_t &= \pi^e\left[\mathrm{E}_t\left(c_{t+1}^e - y_{t+1}\right) + \mathrm{E}_t\left(y_{t+1} - y_t\right)\right] + (1-\pi^e)\left[\mathrm{E}_t\left(c_{t+1}^r - y_{t+1}\right) + \mathrm{E}_t\left(y_{t+1} - y_t\right)\right] \\
&\quad - \frac{1}{\gamma}\left\{\sum_{s=e,r}\pi^s\left(\log\delta^s\right) + \mathrm{E}_t\left(r_{p,t+1}^e\right) + \frac{1}{2}\sum_{s=e,r}\pi^s \mathrm{Var}_t\left[r_{p,t+1} - \gamma\left(c_{t+1}^s - c_t^e\right)\right]\right\} \\
&= \Theta_0 + \mathbf{\Theta}_1\mathbf{Z}_t + \mathbf{\Theta}_2 vec\left(\mathbf{Z}_t\mathbf{Z}_t'\right) + \Theta_3\left(w_t^e - y_t\right),
\end{aligned}$$

where

$$\begin{aligned}
\Theta_0 &= \left[\pi^e b_0^e + \left(1-\pi^e\right)b_0^r\right] + \left[\pi^e b_1^e + \left(1-\pi^e\right)\right]k^e - \bar{b}_1^e\rho_c^e b_0^e + \left(1-\bar{b}_1^e\right)g + \left(\bar{b}_1^e - \frac{1}{\gamma}\right)\Gamma_o^e \\
&\quad + \left[\pi^e\mathbf{N}_1' + \left(1-\pi^e\right)\mathbf{B}_1'\right]\mathbf{\Phi}_0 + \pi^e\mathbf{\Phi}_0'\mathbf{N}_2\mathbf{\Phi}_0 + \left(1-\pi^e\right)\mathbf{\Phi}_0'\mathbf{B}_2\mathbf{\Phi}_0 + \\
&\quad \left[\pi^e vec(\mathbf{N}_2)' + \left(1-\pi^e\right)vec(\mathbf{B}_2)'\right]vec(\mathbf{\Sigma}_v) - \frac{1}{\gamma}\left[\pi^e\log\delta^e + \left(1-\pi^e\right)\log\delta^r\right] \\
&\quad - \frac{1}{2\gamma}\left[\pi^e\mathbf{\Omega}_0 + \left(1-\pi^e\right)\mathbf{V}_0^e\right],
\end{aligned}$$

$$\begin{aligned}
\mathbf{\Theta}_1 &= \bar{b}_1^e\mathbf{\Gamma}_1^e - \bar{b}_1^e\rho_c^e\mathbf{N}'_1 + \left[\pi^e\mathbf{N}'_1 + (1-\pi^e)\mathbf{B}_1'\right]\mathbf{\Phi}_1 + \pi^e\left[\mathbf{\Phi}_0'\left(\mathbf{N}_2 + \mathbf{N}_2'\right)\mathbf{\Phi}_1\right] \\
&\quad + \left(1-\pi^e\right)\left[\mathbf{\Phi}_0'\left(\mathbf{B}_2 + \mathbf{B}_2'\right)\mathbf{\Phi}_1\right] - \frac{1}{\gamma}\mathbf{\Gamma}_1^e - \frac{1}{2\gamma}\left[\pi^e\mathbf{\Omega}_1 + \left(1-\pi^e\right)\mathbf{V}_1^e\right],
\end{aligned}$$

$$\begin{aligned}
\mathbf{\Theta}_2 &= \bar{b}_1^e\mathbf{\Gamma}_2^e - \bar{b}_1^e\rho_c^e vec(\mathbf{N}_2)' + \pi^e vec(\mathbf{\Phi}_1'\mathbf{N}_2\mathbf{\Phi}_1)' + \left(1-\pi^e\right)vec(\mathbf{\Phi}_1'\mathbf{B}_2\mathbf{\Phi}_1)' - \frac{1}{\gamma}\mathbf{\Gamma}_2^e \\
&\quad - \frac{1}{2\gamma}\left[\pi^e\mathbf{\Omega}_2 + \left(1-\pi^e\right)\mathbf{V}_2^e\right],
\end{aligned}$$

and

$$\Theta_3 = \bar{b}_1^e\rho_w^e - \bar{b}_1^e\rho_c^e b_1^e.$$

Appendix E. Log-linear approximation of the budget constraint $W_{t+1}^{r} = W_{t+1}^{p} + W_{t+1}^{e}$.

Dividing both sides of the equation $W_{t+1}^{r} = W_{t+1}^{p} + W_{t+1}^{e}$ by HY_{t+1} and then taking log obtains

$$w_{t+1}^{r} - \tilde{y}_{t+1} = \log\left[\exp(w_{t+1}^{p} - \tilde{y}_{t+1}) + \exp(w_{t+1}^{e} - \tilde{y}_{t+1})\right], \tag{E1}$$

where lowercase letters denote variables in log and $\tilde{y}_{t+1} = \log\left(HY_{t+1}\right)$.

Taking the first-order Taylor expansion of equation (E1) around $\mathrm{E}\left(w_{t+1}^{p} - \tilde{y}_{t+1}\right)$ and $\mathrm{E}\left(w_{t+1}^{e} - \tilde{y}_{t+1}\right)$ obtains

$$\begin{aligned} \log\left[\exp\left(w_{t+1}^{p} - \tilde{y}_{t+1}\right) + \exp\left(w_{t+1}^{e} - \tilde{y}_{t+1}\right)\right] &= \log\left[\exp \mathrm{E}\left(w_{t+1}^{p} - \tilde{y}_{t+1}\right) + \exp \mathrm{E}\left(w_{t+1}^{e} - \tilde{y}_{t+1}\right)\right] \\ &+ \frac{\exp\left[\mathrm{E}(w_{t+1}^{p} - \tilde{y}_{t+1})\right]}{\exp\left[E(w_{t+1}^{p} - \tilde{y}_{t+1})\right] + \exp\left[\mathrm{E}\left(w_{t+1}^{e} - \tilde{y}_{t+1}\right)\right]}\left[w_{t+1}^{p} - \tilde{y}_{t+1} - \mathrm{E}\left(w_{t+1}^{p} - \tilde{y}_{t+1}\right)\right] \\ &+ \frac{\exp\left[\mathrm{E}(w_{t+!}^{e} - \tilde{y}_{t+1})\right]}{\exp\left[\mathrm{E}\left(w_{t+1}^{p} - \tilde{y}_{t+1}\right)\right] + \exp\left[\mathrm{E}\left(w_{t+1}^{e} - \tilde{y}_{t+1}\right)\right]}\left[w_{t+1}^{e} - \tilde{y}_{t+1} - \mathrm{E}\left(w_{t+1}^{e} - \tilde{y}_{t+1}\right)\right]. \end{aligned}$$

Rearranging the above result obtains

$$w_{t+1}^{r} - \tilde{y}_{t+1} = k^{*} + \rho^{*}\left(w_{t+1}^{p} - \tilde{y}_{t+1}\right) + \left(1 - \rho^{*}\right)\left(w_{t+1}^{e} - \tilde{y}_{t+1}\right),$$

where $\rho^{*} = \dfrac{\exp\left[\mathrm{E}(w_{t+1}^{p} - \tilde{y}_{t+1})\right]}{\exp\left[\mathrm{E}(w_{t+1}^{p} - \tilde{y}_{t+1})\right] + \exp\left[\mathrm{E}\left(w_{t+1}^{e} - \tilde{y}_{t+1}\right)\right]}$,

$$k^{*} = \log\left(\frac{G}{\rho^{*}}\right) - \rho^{*}\log G - \left(1 - \rho^{*}\right)\log\left(\frac{G}{\rho^{*}} - G\right), \text{ and } G = \frac{\tilde{\rho}_{w}^{p}\left(\dfrac{1}{H} - 1\right)}{1 - \tilde{\rho}_{w}^{p}}.$$

Appendix F. Optimal portfolio choice in the employment state with exogenous pension account

First we guess the optimal solution in the employment state can be expressed as

$$\boldsymbol{\alpha}_t^e = \widetilde{\mathbf{M}}_0 + \widetilde{\mathbf{M}}_1 \mathbf{Z}_t,$$
$$c_t^e - \widetilde{y}_t = \widetilde{b}_0^e + \widetilde{\mathbf{N}}_1' \mathbf{Z}_t + \mathbf{Z}_t' \widetilde{\mathbf{N}}_2 \mathbf{Z}_t + \widetilde{b}_1^e \left(w_t^e - \widetilde{y}_t\right) + \widetilde{b}_2^e \left(w_t^p - \widetilde{y}_t\right).$$

The optimal policy in the retirement state can be re-written as

$$c_{t+1}^r - \widetilde{y}_{t+1} = b_0^r + \left(w_{t+1}^r - \widetilde{y}_{t+1}\right) + \mathbf{B}_1' \mathbf{Z}_{t+1} + \mathbf{Z}_{t+1}' \mathbf{B}_2 \mathbf{Z}_{t+1}. \tag{F1}$$

The log-linear approximation of the budget constraints in the regular wealth account and pension account can be expressed as

$$w_{t+1}^e - \widetilde{y}_{t+1} = r_{p,t+1}^e - \Delta\widetilde{y}_{t+1} + \widetilde{k}^e + \widetilde{\rho}_w^e \left(w_t^e - y_t\right) - \widetilde{\rho}_c^e \left(c_t - \widetilde{y}_t\right), \tag{F2}$$

and

$$w_{t+1}^p - \widetilde{y}_{t+1} = -\Delta\widetilde{y}_{t+1} + r_{p,t+1}^p + \widetilde{k}^p + \widetilde{\rho}_w^p \left(w_w^p - \widetilde{y}_t\right). \tag{F3}$$

Subtracting the log linear Euler equation for r_f from the general log Euler equation obtains

$$\mathrm{E}_t\left(r_{i,t+1} - r_f\right) + \frac{1}{2}\mathrm{Var}_t\left(r_{i,t+1}\right) = \gamma\left[\pi^e \mathrm{Cov}_t\left(r_{i,t+1}, c_{t+1}^e - c_t^e\right) + \left(1 - \pi^e\right)\mathrm{Cov}_t\left(r_{i,t+1}, c_{t+1}^r - c_t^e\right)\right]. \tag{F4}$$

We also have the trivial equality

$$c_{t+1}^s - c_t^e = \left(c_{t+1}^s - \widetilde{y}_{t+1}\right) - \left(c_t^e - \widetilde{y}_t\right) + \Delta\widetilde{y}_{t+1}, \text{ where } s = e, r. \tag{F5}$$

Substituting equation (F5) into the covariance terms in equation (F4) obtains

$$\begin{aligned}
&\mathrm{Cov}_t\left(r_{i,t+1}, c_{t+1}^e - c_t^e\right) = \mathrm{Cov}_t\left(r_{i,t+1}, c_{t+1}^e - \widetilde{y}_{t+1} + \Delta\widetilde{y}_{t+1}\right) \\
&= \mathrm{Cov}_t\left[r_{i,t+1}, \widetilde{\mathbf{N}}_1' \mathbf{Z}_{t+1} + \mathbf{Z}_{t+1}' \widetilde{\mathbf{N}}_2 \mathbf{Z}_{t+1} + \widetilde{b}_1^e \left(w_{t+1}^e - \widetilde{y}_{t+1}\right) + \widetilde{b}_2^e \left(w_{t+1}^p - \widetilde{y}_{t+1}\right) + \Delta\widetilde{y}_{t+1}\right] \\
&= \mathrm{Cov}_t\left[r_{i,t+1}, \widetilde{\mathbf{N}}_1' \mathbf{Z}_{t+1} + \mathbf{Z}_{t+1}' \widetilde{\mathbf{N}}_2 \mathbf{Z}_{t+1} + \widetilde{b}_1^e r_{p,t+1}^e + \widetilde{b}_2^e r_{p,t+1}^p + \left(1 - \widetilde{b}_1^e - \widetilde{b}_2^e\right)\Delta\widetilde{y}_{t+1}\right] \\
&= \mathrm{Cov}_t\left[r_{i,t+1}, \widetilde{\mathbf{N}}_1' \mathbf{v}_{t+1} + \left(\boldsymbol{\Phi}_0 + \boldsymbol{\Phi}_1 \mathbf{Z}_t + \mathbf{v}_{t+1}\right)' \widetilde{\mathbf{N}}_2 \left(\boldsymbol{\Phi}_0 + \boldsymbol{\Phi}_1 \mathbf{Z}_t + \mathbf{v}_{t+1}\right)\right] \\
&\quad + \mathrm{Cov}_t\left[r_{i,t+1}, \widetilde{b}_1^e r_{p,t+1}^e + \widetilde{b}_2^e r_{p,t+1}^p + \left(1 - \widetilde{b}_1^e - \widetilde{b}_2^e\right)\Delta\widetilde{y}_{t+1}\right] \\
&= \boldsymbol{\Sigma}_v^{(i)\prime} \widetilde{\mathbf{N}}_1 + \boldsymbol{\Sigma}_v^{(i)} \left(\widetilde{\mathbf{N}}_2 + \widetilde{\mathbf{N}}_2'\right)\left(\boldsymbol{\Phi}_0 + \boldsymbol{\Phi}_1 \mathbf{Z}_t\right) + \widetilde{b}_1^e \boldsymbol{\alpha}_t^{e\prime} \boldsymbol{\Sigma}_{XX}^{(i-1)} + \widetilde{b}_2^e \boldsymbol{\alpha}_p' \boldsymbol{\Sigma}_{XX}^{(i-1)} + \left(1 - \widetilde{b}_1^e - \widetilde{b}_2^e\right)\sigma_{\varsigma i}.
\end{aligned} \tag{F6}$$

Similarly,

$$
\begin{aligned}
&\mathrm{Cov}_t\left(r_{i,t+1}, c_{t+1}^{r} - c_t^{e}\right) = \mathrm{Cov}_t\left(r_{i,t+1}, c_{t+1}^{r} - \tilde{y}_{t+1} + \Delta\tilde{y}_{t+1}\right) \\
&= \mathrm{Cov}_t\left[r_{i,t+1}, \left(w_{t+1}^{r} - \tilde{y}_{t+1}\right) + \mathbf{B}_1'\mathbf{Z}_{t+1} + \mathbf{Z}_{t+1}'\mathbf{B}_2\mathbf{Z}_{t+1} + \Delta\tilde{y}_t\right] \\
&= \mathrm{Cov}_t\left[r_{i,t+1}, \rho^*\left(w_{t+1}^{p} - \tilde{y}_{t+1}\right) + \left(1-\rho^*\right)\left(w_{t+1}^{e} - \tilde{y}_{t+1}\right) + \mathbf{B}_1'\mathbf{Z}_{t+1} + \mathbf{Z}_{t+1}'\mathbf{B}_2\mathbf{Z}_{t+1} + \Delta\tilde{y}_{t+1}\right] \\
&= \mathrm{Cov}_t\left[r_{i,t+1}, \rho^*\left(-\Delta\tilde{y}_{t+1} + r_{p,t+1}^{p}\right) + \left(1-\rho^*\right)\left(-\Delta\tilde{y}_{t+1} + r_{p,t+1}^{e}\right) + \mathbf{B}_1'\mathbf{v}_{t+1} + \mathbf{Z}_{t+1}'\mathbf{B}_2\mathbf{Z}_{t+1} + \Delta\tilde{y}_{t+1}\right] \\
&= \mathrm{Cov}_t\left[r_{i,t+1}, \rho^*\boldsymbol{\alpha}_t^{p\,\prime}\boldsymbol{\Sigma}_{XX}^{(i-1)} + \left(1-\rho^*\right)\boldsymbol{\alpha}_t^{e\,\prime}\boldsymbol{\Sigma}_{XX}^{(i-1)} + \boldsymbol{\Sigma}_v^{(i)\,\prime}\mathbf{B}_1 + \boldsymbol{\Sigma}_v^{(i)}\left(\mathbf{B}_2 + \mathbf{B}_2'\right)\left(\boldsymbol{\Phi}_0 + \boldsymbol{\Phi}_1\mathbf{Z}_t\right)\right].
\end{aligned}
\tag{F7}
$$

Substituting equations (F6) and (F7) into equation (F5) and stacking obtains

$$
\begin{aligned}
&\mathrm{E}_t\left(\mathbf{X}_{t+1}\right) + \frac{1}{2}Var_t\left(\mathbf{X}_{t+1}\right) \\
&= \gamma\Big\{\pi^e\left[\tilde{\boldsymbol{\Lambda}}_0^e + \tilde{\boldsymbol{\Lambda}}_1^e\mathbf{Z}_t + \tilde{b}_1^e\boldsymbol{\Sigma}_{XX}\boldsymbol{\alpha}_t^e + \tilde{b}_2^e\boldsymbol{\Sigma}_{XX}\boldsymbol{\alpha}_t^p + \left(1 - \tilde{b}_1^e - \tilde{b}_2^e\right)\boldsymbol{\sigma}_{\varsigma x}\right] \\
&\quad + \left(1-\pi^e\right)\left[\tilde{\boldsymbol{\Lambda}}_0^r + \tilde{\boldsymbol{\Lambda}}_1^r\mathbf{Z}_t + \rho^*\boldsymbol{\Sigma}_{XX}\boldsymbol{\alpha}_t^p + \left(1-\rho^*\right)\boldsymbol{\alpha}_t^e\right]\Big\},
\end{aligned}
$$

where

$$
\tilde{\boldsymbol{\Lambda}}_0^e = \left(\boldsymbol{\Sigma}_v\mathbf{H}_X'\right)'\tilde{\mathbf{N}}_1 + \left(\boldsymbol{\Sigma}_v\mathbf{H}_X'\right)'\left(\tilde{\mathbf{N}}_2 + \tilde{\mathbf{N}}_2'\right)\boldsymbol{\Phi}_0,
$$

$$
\tilde{\boldsymbol{\Lambda}}_1^e = \left(\boldsymbol{\Sigma}_v\mathbf{H}_X'\right)'\left(\tilde{\mathbf{N}}_2 + \tilde{\mathbf{N}}_2'\right)\boldsymbol{\Phi}_1,
$$

$$
\tilde{\boldsymbol{\Lambda}}_0^r = \left(\boldsymbol{\Sigma}_v\mathbf{H}_X'\right)'\mathbf{B}_1 + \left(\boldsymbol{\Sigma}_v\mathbf{H}_X'\right)'\left(\mathbf{B}_2 + \mathbf{B}_2'\right)\boldsymbol{\Phi}_0, \text{ and}
$$

$$
\tilde{\boldsymbol{\Lambda}}_1^r = \left(\boldsymbol{\Sigma}_v\mathbf{H}_X'\right)'\left(\mathbf{B}_2 + \mathbf{B}_2'\right)\boldsymbol{\Phi}_1.
$$

Appendix G. Optimal consumption in the employment state with exogenous pension account

Substituting $i = p$ into the log linear Euler equation obtains

$$
0 = \sum_{s=e,r}\pi^s\left\{\log\delta^s - \gamma\mathrm{E}_t\left(c_{t+1}^s - c_t^e\right) + \mathrm{E}_t\left(r_{p,t+1}^e\right) + \frac{1}{2}\mathrm{Var}_t\left[r_{p,t+1}^e - \gamma\left(c_{t+1}^s - c_t^e\right)\right]\right\}.
$$

Substituting equation (F5) to the above log Euler equation obtains

$$\begin{aligned}&\pi^e\left[\mathrm{E}_t\left(c^e_{t+1}-\tilde{y}_{t+1}\right)+E_t\left(\tilde{y}_{t+1}-\tilde{y}_t\right)\right]+\left(1-\pi^e\right)\left[\mathrm{E}_t\left(c^r_{t+1}-\tilde{y}_{t+1}\right)+\mathrm{E}_t\left(\tilde{y}_{t+1}-\tilde{y}_t\right)\right]-\left(c^e_t-\tilde{y}_t\right)\\&=\frac{1}{\gamma}\sum_{s=e,r}\pi^s\left\{\log\delta^s+\mathrm{E}_t\left(r^e_{p,t+1}\right)+\frac{1}{2}\mathrm{Var}_t\left[r^e_{p,t+1}-\gamma\left(c^s_{t+1}-c^e_t\right)\right]\right\}.\end{aligned}\tag{G1}$$

Also, we have

$$\begin{aligned}&\mathrm{E}_t\left(c^e_{t+1}-\tilde{y}_{t+1}\right)+\mathrm{E}_t\left(\tilde{y}_{t+1}-\tilde{y}_t\right)\\&=\tilde{b}^e_0+\mathrm{E}_t\left(\tilde{\mathbf{N}}_1'\mathbf{Z}_{t+1}+\mathbf{Z}_{t+1}'\tilde{\mathbf{N}}_2\mathbf{Z}_{t+1}\right)+\tilde{b}^e_1\mathrm{E}_t\left(w^e_{t+1}-\tilde{y}_{t+1}\right)+\tilde{b}^e_2\mathrm{E}_t\left(w^p_{t+1}-\tilde{y}_{t+1}\right)+g,\end{aligned}\tag{G2}$$

where

$$\mathrm{E}_t\left(\tilde{\mathbf{N}}_1'\mathbf{Z}_{t+1}\right)=\tilde{\mathbf{N}}_1'\mathbf{\Phi}_0+\tilde{\mathbf{N}}_1'\mathbf{\Phi}_1\mathbf{Z}_t\text{, and}$$

$$\mathrm{E}_t\left(\mathbf{Z}_{t+1}'\tilde{\mathbf{N}}_2\mathbf{Z}_{t+1}\right)=\mathbf{\Phi}_0'\tilde{\mathbf{N}}_2\mathbf{\Phi}_0+\left[\mathbf{\Phi}_0'\left(\tilde{\mathbf{N}}_2+\tilde{\mathbf{N}}_2'\right)\mathbf{\Phi}_1\right]\mathbf{Z}_t+vec\left(\mathbf{\Phi}_1'\tilde{\mathbf{N}}_2\mathbf{\Phi}_1\right)'vec\left(\mathbf{Z}_t\mathbf{Z}_t'\right)+vec(\tilde{\mathbf{N}}_2)'vec(\mathbf{\Sigma}_v)\,.$$

Similarly,

$$\mathrm{E}_t\left(c^r_{t+1}-\tilde{y}_{t+1}\right)+\mathrm{E}_t\left(\tilde{y}_{t+1}-\tilde{y}_t\right)=b^r_0+\mathrm{E}_t\left(\mathbf{B}_1'\mathbf{Z}_{t+1}+\mathbf{Z}_{t+1}'\mathbf{B}_2\mathbf{Z}_{t+1}\right)+\mathrm{E}_t\left(w^r_{t+1}-\tilde{y}_{t+1}\right)+g\,,\tag{G3}$$

where

$$\mathrm{E}_t\left(\mathbf{B}_1'\mathbf{Z}_{t+1}\right)=\mathbf{B}_1'\mathbf{\Phi}_0+\mathbf{B}_1'\mathbf{\Phi}_1\mathbf{Z}_t\text{, and}$$

$$\mathrm{E}_t\left(\mathbf{Z}_{t+1}'\mathbf{B}_2\mathbf{Z}_{t+1}\right)=\mathbf{\Phi}_0'\mathbf{B}_2\mathbf{\Phi}_0+\left[\mathbf{\Phi}_0'\left(\mathbf{B}_2+\mathbf{B}_2'\right)\mathbf{\Phi}_1\right]\mathbf{Z}_t+vec\left(\mathbf{\Phi}_1'\mathbf{B}_2\mathbf{\Phi}_1\right)'vec\left(\mathbf{Z}_t\mathbf{Z}_t'\right)+vec(\mathbf{B}_2)'vec(\mathbf{\Sigma}_v)\,.$$

Now we compute the variance terms in equation (G1). First we look at $\mathrm{Var}_t\left[r^e_{p,t+1}-\gamma\left(c^e_{t+1}-c^e_t\right)\right]$.

$$
\begin{aligned}
&\mathrm{Var}_t\left[r^e_{p,t+1}-\gamma\left(c^e_{t+1}-c^e_t\right)\right]=Var_t\left\{r^e_{p,t+1}-\gamma\left[\left(c^e_{t+1}-\tilde{y}_{t+1}\right)+\left(\tilde{y}_{t+1}-\tilde{y}_t\right)\right]\right\}\\
&=\mathrm{Var}_t\left\{\boldsymbol{\alpha}^{e\,\prime}_t\mathbf{X}_{t+1}-\gamma\left[\tilde{\mathbf{N}}_1'\mathbf{Z}_{t+1}+\mathbf{Z}'_{t+1}\tilde{\mathbf{N}}_2\mathbf{Z}_{t+1}+\tilde{b}^e_1\left(w^e_{t+1}-\tilde{y}_{t+1}\right)+\tilde{b}^e_2\left(w^p_{t+1}-\tilde{y}_{t+1}\right)\right]-\gamma\Delta\tilde{y}_{t+1}\right\}\\
&=\mathrm{Var}_t\{\ \boldsymbol{\alpha}^{e\,\prime}_t\mathbf{H}_X\mathbf{v}_{t+1}-\gamma[\ \tilde{\mathbf{N}}_1'\mathbf{v}_{t+1}+\boldsymbol{\Phi}_0'\left(\tilde{\mathbf{N}}_2+\tilde{\mathbf{N}}_2'\right)\mathbf{v}_{t+1}+\mathbf{Z}_t'\boldsymbol{\Phi}_1'\left(\tilde{\mathbf{N}}_2+\tilde{\mathbf{N}}_2'\right)\mathbf{v}_{t+1}+vec(\tilde{\mathbf{N}}_2)'vec(\mathbf{v}_{t+1}\mathbf{v}'_{t+1})\\
&\quad+\tilde{b}^e_1\boldsymbol{\alpha}^{e\,\prime}_t\mathbf{H}_X\mathbf{v}_{t+1}+\tilde{b}^e_2\boldsymbol{\alpha}^{p\,\prime}_t\mathbf{H}_X\mathbf{v}_{t+1}\ ]-\gamma\left(1-\tilde{b}^e_1-\tilde{b}^e_2\right)\Delta\tilde{y}_{t+1}\}.
\end{aligned}
$$

Substituting $\boldsymbol{\alpha}^{e\,\prime}_t=\tilde{\mathbf{M}}_0'+\mathbf{Z}_t'\tilde{\mathbf{M}}_1'$ into the above equation obtains

$$
\mathrm{Var}_t\left[r^e_{p,t+1}-\gamma\left(c^e_{t+1}-c^e_t\right)\right]=\mathrm{Var}_t\left[\tilde{\boldsymbol{\psi}}_1\mathbf{v}_{t+1}+\mathbf{Z}_t'\tilde{\boldsymbol{\psi}}_2\mathbf{v}_{t+1}-\gamma vec\left(\tilde{\mathbf{N}}_2\right)'vec\left(\mathbf{v}_{t+1}\mathbf{v}'_{t+1}\right)-\gamma\left(1-\tilde{b}^e_1-\tilde{b}^e_2\right)\Delta\tilde{y}_{t+1}\right],\text{(G4)}
$$

where

$$
\tilde{\boldsymbol{\psi}}_1=\left(1-\gamma\tilde{b}^e_1\right)\tilde{\mathbf{M}}_0'\mathbf{H}_X-\gamma\tilde{\mathbf{N}}_1'-\gamma\boldsymbol{\Phi}_0'\left(\tilde{\mathbf{N}}_2+\tilde{\mathbf{N}}_2'\right)+\gamma\tilde{b}^e_2\boldsymbol{\alpha}^{p\,\prime}_t\mathbf{H}_X\text{, and}
$$

$$
\tilde{\boldsymbol{\psi}}_2=\left(1-\gamma\tilde{b}^e_1\right)\tilde{\mathbf{M}}_1'\mathbf{H}_X-\gamma\boldsymbol{\Phi}_1'\left(\tilde{\mathbf{N}}_2+\tilde{\mathbf{N}}_2'\right).
$$

We then obtain

$$
\begin{aligned}
&\mathrm{Var}_t\left[r^e_{p,t+1}-\gamma\left(c^e_{t+1}-c^e_t\right)\right]=\tilde{\boldsymbol{\psi}}_1\boldsymbol{\Sigma}_v\tilde{\boldsymbol{\psi}}_1'+\mathbf{Z}_t'\tilde{\boldsymbol{\psi}}_2\boldsymbol{\Sigma}_v\tilde{\boldsymbol{\psi}}_2'\mathbf{Z}_t+\gamma^2 vec\left(\tilde{\mathbf{N}}_2\right)'\mathrm{Var}_t\left[vec\left(\mathbf{v}_{t+1}\mathbf{v}'_{t+1}\right)\right]vec\left(\tilde{\mathbf{N}}_2\right)\\
&+\gamma^2\left(1-\tilde{b}^e_1-\tilde{b}^e_2\right)^2\sigma^2_\varsigma+2\tilde{\boldsymbol{\psi}}_1\boldsymbol{\Sigma}_v\tilde{\boldsymbol{\psi}}_2'\mathbf{Z}_t-2\gamma\left(1-\tilde{b}^e_1-\tilde{b}^e_2\right)\left(\tilde{\boldsymbol{\psi}}_1+\mathbf{Z}_t'\tilde{\boldsymbol{\psi}}_2\right)\boldsymbol{\sigma}_{\varsigma v}\qquad\text{(G5)}\\
&=\tilde{\Omega}_0+\tilde{\boldsymbol{\Omega}}_1\mathbf{Z}_t+\tilde{\boldsymbol{\Omega}}_2 vec\left(\mathbf{Z}_t\mathbf{Z}_t'\right),
\end{aligned}
$$

where

$$
\tilde{\Omega}_0=\tilde{\boldsymbol{\psi}}_1\boldsymbol{\Sigma}_v\tilde{\boldsymbol{\psi}}_1'+\gamma^2 vec\left(\tilde{\mathbf{N}}_2\right)'Var_t\left[vec\left(\mathbf{v}_{t+1}\mathbf{v}'_{t+1}\right)\right]vec\left(\tilde{\mathbf{N}}_2\right)+\gamma^2\left(1-\tilde{b}^e_1-\tilde{b}^e_2\right)^2\sigma^2_\varsigma-2\tilde{\boldsymbol{\psi}}_1\gamma\left(1-\tilde{b}^e_1-\tilde{b}^e_2\right)\boldsymbol{\sigma}_{\varsigma v}
$$

$$
\tilde{\boldsymbol{\Omega}}_1=2\tilde{\boldsymbol{\psi}}_1\boldsymbol{\Sigma}_v\tilde{\boldsymbol{\psi}}_2'-2\gamma\left(1-\tilde{b}^e_1-\tilde{b}^e_2\right)\boldsymbol{\sigma}_{\varsigma v}\tilde{\boldsymbol{\psi}}_2'\text{, and}
$$

$$
\hat{\boldsymbol{\Omega}}_2=vec\left(\tilde{\boldsymbol{\psi}}_2\boldsymbol{\Sigma}_v\tilde{\boldsymbol{\psi}}_2'\right)'.
$$

Similarly,

$$
\begin{aligned}
&\mathrm{Var}_t\left[r^e_{p,t+1}-\gamma\left(c^r_{t+1}-c^e_t\right)\right] \\
&=\mathrm{Var}_t\left\{\boldsymbol{\alpha}^{e\,\prime}_t\mathbf{X}_{t+1}-\gamma\left[\mathbf{B}_1'\mathbf{Z}_{t+1}+\mathbf{Z}_{t+1}'\mathbf{B}_2\mathbf{Z}_{t+1}+\left(w^r_{t+1}-\tilde{y}_{t+1}\right)-\gamma\Delta\tilde{y}_{t+1}\right]\right\} \\
&=\mathrm{Var}_t\left\{\boldsymbol{\alpha}^{e\,\prime}_t\mathbf{H}_X\mathbf{v}_{t+1}-\gamma\left[\mathbf{B}_1'\mathbf{v}_{t+1}+\mathbf{Z}_{t+1}{}'\mathbf{B}_2\mathbf{Z}_{t+1}+(1-\rho^*)\left(\boldsymbol{\alpha}^{e\,\prime}_t\mathbf{H}_X\mathbf{v}_{t+1}\right)+\rho^*\left(\boldsymbol{\alpha}^{p\,\prime}_t\mathbf{H}_X\mathbf{v}_{t+1}\right)\right]\right\}.
\end{aligned}
$$

Substituting $\boldsymbol{\alpha}_t'=\mathbf{M}_0'+\mathbf{Z}_t'\mathbf{M}_1'$ into the above formula obtains

$$\mathrm{Var}_t\left[r^e_{p,t+1}-\gamma\left(c^r_{t+1}-c^e_t\right)\right]=\mathrm{Var}_t\left[\tilde{\boldsymbol{\pi}}^e_1\mathbf{v}_{t+1}+\mathbf{Z}_t'\tilde{\boldsymbol{\pi}}^e_2\mathbf{v}_{t+1}-\gamma vec(\mathbf{B}_2)'vec(\mathbf{v}_{t+1}\mathbf{v}_{t+1}')\right], \qquad \text{(G6)}$$

where

$$\tilde{\boldsymbol{\pi}}^e_1=\left(1-\gamma\rho^*\right)\tilde{\mathbf{M}}_0'\mathbf{H}_X-\gamma\mathbf{B}_1'-\gamma\boldsymbol{\Phi}_0'\left(\mathbf{B}_2+\mathbf{B}_2'\right)-\left(1-\rho^*\right)\boldsymbol{\alpha}^{p\,\prime}_t\mathbf{H}_X \text{, and}$$

$$\tilde{\boldsymbol{\pi}}^e_2=\left(1-\gamma\rho^*\right)\tilde{\mathbf{M}}_1'\mathbf{H}_X-\gamma\boldsymbol{\Phi}_1'\left(\mathbf{B}_2+\mathbf{B}_2'\right).$$

We then have

$$\mathrm{Var}_t\left[r^e_{p,t+1}-\gamma\left(c^r_{t+1}-c^e_t\right)\right]=\tilde{V}^e_0+\tilde{\mathbf{V}}^e_1\mathbf{Z}_t+\tilde{\mathbf{V}}^e_2 vec(\mathbf{Z}_t\mathbf{Z}_t'), \qquad \text{(G7)}$$

where

$$\tilde{V}^e_0=\tilde{\boldsymbol{\pi}}_1\boldsymbol{\Sigma}_v\tilde{\boldsymbol{\pi}}_1'+\gamma^2 vec(\mathbf{B}_2)'Var_t\left(vec(\mathbf{v}_{t+1}\mathbf{v}_{t+1}')\right)vec(\mathbf{B}_2),$$

$$\tilde{\mathbf{V}}^e_1=2\tilde{\boldsymbol{\pi}}_1\boldsymbol{\Sigma}_v\tilde{\boldsymbol{\pi}}_2' \text{, and}$$

$$\tilde{\mathbf{V}}^e_2=vec(\tilde{\boldsymbol{\pi}}_2\boldsymbol{\Sigma}_v\tilde{\boldsymbol{\pi}}_2')'.$$

Lastly, we have

$$\mathrm{E}_t\left(w^p_{t+1}-\tilde{y}_{t+1}\right)=\tilde{k}^p+\tilde{\rho}^p_w\left(w^p_t-\tilde{y}_t\right)+\mathrm{E}_t(r^p_{p,t+1})-g \qquad \text{(G8)}$$

and

$$
\begin{aligned}
\mathrm{E}_t\left(w_{t+1}^e - \tilde{y}_{t+1}\right) &= \tilde{k}^e + \tilde{\rho}_w^e\left(w_t - \tilde{y}_t\right) - \tilde{\rho}_c^e\left(c_t - \tilde{y}_t\right) + \mathrm{E}_t(r_{p,t+1}^e) - g \\
&= \tilde{k}^e + \tilde{\rho}_w^e\left(w_t - \tilde{y}_t\right) - \tilde{\rho}_c^e\left(c_t - \tilde{y}_t\right) + \tilde{\Gamma}_0^e + \tilde{\mathbf{\Gamma}}_1^e \mathbf{Z}_t + \tilde{\mathbf{\Gamma}}_2^e vec(\mathbf{Z}_t\mathbf{Z}_t') - g,
\end{aligned} \tag{G9}
$$

where

$$
\tilde{\Gamma}_0^e = \tilde{\mathbf{M}}_0'\mathbf{H}_X\mathbf{\Phi}_0 + r_f + \frac{1}{2}\tilde{\mathbf{M}}_0'\boldsymbol{\sigma}_X^2 - \frac{1}{2}\tilde{\mathbf{M}}_0'\mathbf{\Sigma}_{XX}\tilde{\mathbf{M}}_0,
$$

$$
\tilde{\mathbf{\Gamma}}_1^e = \mathbf{\Phi}_0'\mathbf{H}_X\tilde{\mathbf{M}}_1 + \tilde{\mathbf{M}}_0'\mathbf{H}_X\mathbf{\Phi}_1 + \frac{1}{2}\boldsymbol{\sigma}_X^{2\,\prime}\tilde{\mathbf{M}}_1 - \tilde{\mathbf{M}}_0'\mathbf{\Sigma}_{XX}\tilde{\mathbf{M}}_1, \text{ and}
$$

$$
\tilde{\mathbf{\Gamma}}_2^e = vec\left(\tilde{\mathbf{M}}_1'\mathbf{H}_X\mathbf{\Phi}_1\right)' - \frac{1}{2}vec\left(\tilde{\mathbf{M}}_1'\mathbf{\Sigma}_{XX}\tilde{\mathbf{M}}_1\right)'.
$$

Substituting the results from equations (G2)-(G9) into equation (G1), we get

$$
c_t^e - \tilde{y}_t = \tilde{\Theta}_0 + \tilde{\mathbf{\Theta}}_1\mathbf{Z}_t + \tilde{\mathbf{\Theta}}_2 vec(\mathbf{Z}_t\mathbf{Z}_t') + \tilde{\Theta}_3\left(w_t^e - \tilde{y}_t\right) + \tilde{\Theta}_4\left(w_t^p - \tilde{y}_t\right),
$$

where

$$
\begin{aligned}
\tilde{\Theta}_0 = \pi^e\{ & \tilde{b}_0^e + \tilde{\mathbf{N}}_1'\mathbf{\Phi}_0 + \mathbf{\Phi}_0'\tilde{\mathbf{N}}_2\mathbf{\Phi}_0 + vec(\tilde{\mathbf{N}}_2)'vec(\mathbf{\Sigma}_\mathbf{v}) + \tilde{b}_1^e\left(\tilde{k}^e - g + \tilde{\Gamma}_0^e\right) + \tilde{b}_2^e\left[\tilde{k}^p - g + \mathrm{E}_t\left(r_{p,t+1}^p\right)\right] + g \; \} \\
& + \left(1-\pi^e\right)\{ \; b_0^r + \mathbf{B}_1'\mathbf{\Phi}_0 + \mathbf{\Phi}_0'\mathbf{B}_2\mathbf{\Phi}_0 + vec(\mathbf{B}_2)'vec(\mathbf{\Sigma}_v) + k^* + \rho^*\left[\tilde{k}^p + E_t(r_{p,t+1}^p) - g\right] \\
& + (1-\rho^*)\left(\tilde{k}^e - g + \tilde{\Gamma}_0^e\right) + g \; \} - \frac{1}{\gamma}\left[\sum_{s=e,r}\pi^s \log\delta^s + \tilde{\Gamma}_0^e + \frac{1}{2}\pi^e\tilde{\Omega}_0 + \frac{1}{2}\left(1-\pi^e\right)\tilde{V}_0^e\right],
\end{aligned}
$$

$$
\begin{aligned}
\tilde{\mathbf{\Theta}}_1 = \pi^e\{ & \; \tilde{\mathbf{N}}_1'\mathbf{\Phi}_1 + \left[\mathbf{\Phi}_0'\left(\tilde{\mathbf{N}}_2 + \tilde{\mathbf{N}}_2'\right)\mathbf{\Phi}_1\right] + \tilde{b}_1^e\tilde{\mathbf{\Gamma}}_1^e \; \} + \\
& \left(1-\pi^e\right)\{ \; \mathbf{B}_1'\mathbf{\Phi}_1 + \left[\mathbf{\Phi}_0'\left(\mathbf{B}_2 + \mathbf{B}_2'\right)\mathbf{\Phi}_1\right] + \left(1-\rho^*\right)\tilde{\mathbf{\Gamma}}_1^e \; \} - \frac{1}{\gamma}\left[\tilde{\mathbf{\Gamma}}_1^e + \frac{1}{2}\pi^e\tilde{\mathbf{\Omega}}_1 + \frac{1}{2}\left(1-\pi^e\right)\tilde{\mathbf{V}}_1^e\right],
\end{aligned}
$$

$$
\begin{aligned}
\tilde{\mathbf{\Theta}}_2 = & \; \pi^e\left[vec(\mathbf{\Phi}_1'\tilde{\mathbf{N}}_2\mathbf{\Phi}_1)' + \tilde{b}_1^e\tilde{\mathbf{\Gamma}}_2^e\right] + \left(1-\pi^e\right)\left[vec\left(\mathbf{\Phi}_1'\mathbf{B}_2\mathbf{\Phi}_1\right)' + \left(1-\rho^*\right)\tilde{\mathbf{\Gamma}}_2^e\right] \\
& - \frac{1}{\gamma}\left[\tilde{\mathbf{\Gamma}}_2^e + \frac{1}{2}\pi^e\tilde{\mathbf{\Omega}}_2 + \frac{1}{2}\left(1-\pi^e\right)\tilde{\mathbf{V}}_2^e\right],
\end{aligned}
$$

$$\hat{\Theta}_3 = \pi^e \tilde{b}_1^e \tilde{\rho}_w^e + \left(1-\pi^e\right)\left(1-\rho^*\right)\tilde{\rho}_w^e, \text{ and}$$

$$\tilde{\Theta}_4 = \pi^e \tilde{b}_2^e \tilde{\rho}_w^p + \left(1-\pi^e\right)\rho^* \tilde{\rho}_w^p .$$

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Table 2.1

Asset Allocations Recommended by Financial Advisors (source: Canner, *et al*., 1997)

Advisor and investor type	Portfolio (in percent)			Ratio of bonds to stocks
	Cash	Bonds	Stocks	
A. Fidelity				
Conservative	50	30	20	1.50
Moderate	20	40	40	1.00
Aggressive	5	30	65	0.46
B. Merrill Lynch				
Conservative	20	35	45	0.78
Moderate	5	40	55	0.73
Aggressive	5	20	75	0.27
C. Jane Bryant Quinn				
Conservative	50	30	20	1.50
Moderate	10	40	50	0.80
Aggressive	0	0	100	0.00
D. New York Times				
Conservative	20	40	40	1.00
Moderate	10	30	60	0.50
Aggressive	0	20	80	0.25

Table 2.2

Optimal Portfolio Choice in Retirement and Employment States

This table reports the optimal portfolio weights in stock and bond in both the retirement and employment states under different combinations of expected time until retirement and relative risk aversion. Panel A assumes labor income to be independent of risky asset returns. Panels B and C show the optimal portfolio choices when labor income is correlated with stock and bond returns, respectively. The correlation coefficient is assumed to be 25 percent. The estimated VAR process of excess risky asset returns is $\mathbf{Z}_{t+1}=\begin{bmatrix}0.0455\\0.0021\end{bmatrix}+\begin{bmatrix}0.0617 & 0.0164\\0.0954 & 0.1210\end{bmatrix}\mathbf{Z}_t+\mathbf{v}_{t+1}$, with $\mathbf{v}_t \sim N\left(0,\begin{bmatrix}0.0324 & 0.0004\\0.0004 & 0.0039\end{bmatrix}\right)$. The other baseline values are: $r_f=0.01819$, σ_ς =0.1, g is set such that $\mathrm{E}_t(Y_{t+1}/Y_t)=1.03$, $\delta=1/1.1$, and $\pi^d=0.1$. The expected portfolio shares are given in percentage points.

Expected time	30			20			10			5			Retirement		
until retirement (years)	stock	bond	bond/stock	stock	bond	bond/stock	stock	bond	bond/stock	stock	bond	bond/stock	stock	bond	bond/stock
A. Labor income is uncorrelated with asset returns															
γ=4	64.57	76.62	1.1866	59.62	70.82	1.1879	53.37	63.48	1.1894	49.00	58.33	1.1904	43.76	52.13	1.1913
6	43.67	52.05	1.1919	39.86	47.54	1.1927	35.11	41.92	1.1940	32.03	38.26	1.1945	28.75	34.37	1.1955
8	32.85	39.24	1.1945	29.88	35.72	1.1954	26.14	31.27	1.1963	23.77	28.45	1.1969	21.41	25.63	1.1971
10	26.25	31.40	1.1962	23.87	28.58	1.1973	20.81	24.93	1.1980	18.89	22.64	1.1985	17.05	20.44	1.1988
12	21.81	26.12	1.1976	19.86	23.79	1.1979	17.28	20.72	1.1991	15.67	18.80	1.1997	14.16	16.99	1.1999
B. Correlation coefficient between stock and labor income = 0.25															
γ=4	59.22	78.44	1.3246	55.44	72.08	1.3001	50.73	64.09	1.2634	47.50	58.59	1.2335	43.76	52.13	1.1913
6	37.54	53.92	1.4363	35.21	48.80	1.3860	32.36	42.50	1.3133	30.57	38.49	1.2591	28.75	34.37	1.1955
8	26.29	41.07	1.5622	24.96	36.93	1.4796	23.33	31.81	1.3635	22.34	28.66	1.2829	21.41	25.63	1.1971
10	19.43	33.15	1.7061	18.78	29.73	1.5831	17.96	25.44	1.4165	17.47	22.84	1.3074	17.05	20.44	1.1988
12	14.84	27.78	1.8720	14.65	24.90	1.6997	14.41	21.20	1.4712	14.27	18.99	1.3308	14.16	16.99	1.1999
C. Correlation coefficient between bond and labor income = 0.25															
γ=4	66.08	59.96	0.9074	60.74	58.05	0.9557	54.00	55.65	1.0306	49.31	54.00	1.0951	43.76	52.13	1.1913
6	45.48	33.72	0.7414	41.16	33.85	0.8224	35.81	34.02	0.9500	32.35	34.14	1.0553	28.75	34.37	1.1955
8	34.80	20.25	0.5819	31.28	21.63	0.6915	26.87	23.36	0.8694	24.10	24.45	1.0145	21.41	25.63	1.1971
10	28.27	12.15	0.4298	25.32	14.29	0.5644	21.56	17.02	0.7894	19.22	18.71	0.9735	17.05	20.44	1.1988
12	23.87	6.80	0.2849	21.34	9.42	0.4414	18.04	12.82	0.7106	16.00	14.93	0.9331	14.16	16.99	1.1999

Table 2.3

Hedging Demand in Retirement and Employment States

This table reports the optimal hedging demand as a percentage of total portfolio weights in stock and bond in both the retirement and employment states under different combinations of expected time until retirement and relative risk aversion. Panel A assumes labor income to be independent of risky asset returns. Panels B and C show the percentage hedging demand when labor income is correlated with stock and bond returns, respectively. The correlation coefficient is assumed to be 25 percent. The estimated VAR process of excess risky asset returns is $\mathbf{Z}_{t+1} = \begin{bmatrix} 0.0455 \\ 0.0021 \end{bmatrix} + \begin{bmatrix} 0.0617 & 0.0164 \\ 0.0954 & 0.1210 \end{bmatrix} \mathbf{Z}_t + \mathbf{v}_{t+1}$, with $\mathbf{v}_t \sim N\left(0, \begin{bmatrix} 0.0324 & 0.0004 \\ 0.0004 & 0.0039 \end{bmatrix}\right)$ .The other baseline values are: $r_f = 0.01819$, σ_ς =0.1, g is set such that $\mathrm{E}_t(Y_{t+1}/Y_t) = 1.03$, δ =1/1.1, and $\pi^d = 0.1$. The values shown are given in percentage points.

Expected time until	30		20		10		5		Retirement	
retirement (years)	stock	bond	stock	bond	stock	bond	stock	bond	stock	bond
A. Labor income is uncorrelated with asset returns										
γ = 4	-11.09	-8.46	-11.56	-8.81	-12.16	-9.26	-12.57	-9.57	-12.93	-9.84
6	-13.14	-9.99	-13.55	-10.29	-14.04	-10.64	-14.33	-10.87	-14.61	-11.09
8	-14.28	-10.83	-14.59	-11.06	-15.00	-11.35	-15.27	-11.56	-15.46	-11.71
10	-15.01	-11.37	-15.25	-11.55	-15.57	-11.79	-15.83	-11.97	-15.95	-12.08
12	-15.50	-11.72	-15.71	-11.90	-15.97	-12.11	-16.21	-12.23	-16.31	-12.36
B. Correlation coefficient between stock and labor income = 0.25										
γ = 4	-22.91	-7.52	-21.25	-8.05	-18.59	-8.75	-16.32	-9.27		
6	-34.50	-8.48	-30.47	-9.06	-24.54	-9.86	-20.05	-10.44		
8	-46.63	-8.74	-39.70	-9.37	-29.83	-10.31	-22.92	-10.99		
10	-60.11	-8.72	-49.52	-9.42	-35.02	-10.46	-25.53	-11.25		
12	-75.40	-8.57	-60.34	-9.28	-40.32	-10.47	-27.96	-11.37		
C. Correlation coefficient between bond and labor income = 0.25										
γ = 4	-9.02	-39.19	-9.81	-33.11	-10.98	-24.76	-11.90	-18.39		
6	-9.26	-70.76	-10.33	-55.42	-11.95	-36.54	-13.26	-24.34		
8	-8.59	-116.20	-9.91	-84.14	-12.02	-49.32	-13.73	-29.90		
10	-7.53	-189.96	-9.12	-124.00	-11.73	-63.98	-13.89	-35.54		
12	-6.33	-332.50	-8.15	-183.76	-11.31	-81.44	-13.88	-41.39		

Table 2.4

Optimal Wealth-Labor Income Ratio in the Employment State

This table shows the optimal exponential mean log wealth-labor income ratio $\exp \mathrm{E}(w_t^e - y_t)$ in the employment state under different combinations of expected time until retirement and relative risk aversion. Panel A assumes labor income to be independent of risky asset returns. Panels B and C show the optimal portfolio choices when labor income is correlated with stock and bond returns, respectively. The correlation coefficient is assumed to be 25 percent. The estimated VAR process of excess risky asset returns is $\mathbf{Z}_{t+1} = \begin{bmatrix} 0.0455 \\ 0.0021 \end{bmatrix} + \begin{bmatrix} 0.0617 & 0.0164 \\ 0.0954 & 0.1210 \end{bmatrix} \mathbf{Z}_t + \mathbf{v}_{t+1}$, with $\mathbf{v}_t \sim N\left(0, \begin{bmatrix} 0.0324 & 0.0004 \\ 0.0004 & 0.0039 \end{bmatrix}\right)$. The other baseline values are: $r_f = 0.01819$, $\sigma_\varsigma = 0.1$, g is set such that $\mathrm{E}_t(Y_{t+1}/Y_t) = 1.03$, $\delta = 1/1.1$, and $\pi^d = 0.1$.

Expected time until retirement (years)	30	20	10	5
A. Labor income is uncorrelated with asset returns				
γ=4	19.73	22.31	24.93	25.70
6	20.87	23.62	26.84	28.30
8	22.43	24.96	28.20	29.95
10	23.99	26.14	29.21	31.10
12	25.46	27.19	30.00	31.96
B. Correlation coefficient between stock and labor income = 0.25				
γ=4	19.00	21.54	24.27	25.27
6	19.86	22.55	25.93	27.72
8	21.16	23.63	27.10	29.28
10	22.48	24.59	27.97	30.36
12	23.74	25.45	28.65	31.16
C. Correlation coefficient between bond and labor income = 0.25				
γ=4	19.48	22.09	24.77	25.59
6	20.53	23.34	26.64	28.17
8	22.02	24.62	27.97	28.81
10	23.50	25.76	28.97	30.96
12	24.90	26.76	29.74	31.80

Table 2.5

Effect of Labor Income Growth Rate and Expected Time until Retirement on Optimal Portfolio Choice in the Employment State

This table reports the optimal portfolio weights in stock and bond in the employment state under different combinations of labor income growth rate and expected time until retirement. The estimated VAR process of excess risky asset returns is $\mathbf{Z}_{t+1} = \begin{bmatrix} 0.0455 \\ 0.0021 \end{bmatrix} + \begin{bmatrix} 0.0617 & 0.0164 \\ 0.0954 & 0.1210 \end{bmatrix} \mathbf{Z}_t + \mathbf{v}_{t+1}$, with $\mathbf{v}_t \sim N\left(0, \begin{bmatrix} 0.0324 & 0.0004 \\ 0.0004 & 0.0039 \end{bmatrix}\right)$. The other baseline values are: $r_f = 0.01819$, $\sigma_{\varsigma} = 0.1$, $\delta = 1/1.1$, $\gamma = 8$, and $\pi^d = 0.1$. Labor income is assumed to be independent of risky asset returns. Portfolio shares are given in percentage points.

Expected time until retirement (years)	30		20		10		5	
	stock	bond	stock	bond	stock	bond	stock	bond
$\mathrm{E}_t(Y_{t+1}/Y_t)$								
1.02	28.11	33.62	26.57	31.79	24.45	29.27	22.99	27.52
1.04	37.85	45.16	33.29	39.75	27.83	33.28	24.55	29.38
1.06	48.97	58.22	40.46	48.21	31.23	37.30	26.08	31.19
1.08	62.00	73.38	48.23	57.31	34.66	41.35	27.58	32.97

Table 2.6

Effect of Time Preference on Optimal Portfolio Choice

This table reports optimal portfolio weights in stock and bond in retirement and employment states under different combinations of time preference rate and expected time until retirement. The estimated VAR process of excess risky asset returns is $\mathbf{Z}_{t+1} = \begin{bmatrix} 0.0455 \\ 0.0021 \end{bmatrix} + \begin{bmatrix} 0.0617 & 0.0164 \\ 0.0954 & 0.1210 \end{bmatrix} \mathbf{Z}_t + \mathbf{v}_{t+1}$, with $\mathbf{v}_t \sim N\left(0, \begin{bmatrix} 0.0324 & 0.0004 \\ 0.0004 & 0.0039 \end{bmatrix}\right)$. The other baseline values are: $r_f = 0.01819$, $\sigma_\varsigma = 0.1$, $\gamma = 8$, and $\pi^d = 0.1$. The labor income growth rate g is set such that $E_t(Y_{t+1}/Y_t) = 1.03$, and labor income is assumed to be independent of risky asset returns. The last row in each panel shows the exponential of mean log wealth-labor income ratio $\exp E(w_t^e - y_t)$ in the employment state. Portfolio shares are given in percentage points.

Time preference rate (in percent)	-ln(δ)=2	-ln(δ)=4	-ln(δ)=6	-ln(δ)=8	-ln(δ)=10
A. Expected time until retirement=10 years					
Retirement state_stock	21.38	21.38	21.39	21.40	21.41
Retirement state_bond	25.61	25.61	25.62	25.63	25.63
Employment state_stock	24.54	24.96	25.38	25.81	26.24
Employment state_bond	29.38	29.88	30.38	30.88	31.39
$\exp E(w_t^e - y_t)$	44.97	39.13	34.46	30.66	27.51
B. Expected time until retirement=20 years					
Retirement state_stock	21.38	21.38	21.39	21.40	21.41
Retirement state_bond	25.61	25.61	25.62	25.63	25.63
Employment state_stock	26.77	27.57	28.39	29.23	30.09
Employment state_bond	32.03	32.98	33.96	34.95	35.96
$\exp E(w_t^e - y_t)$	43.77	37.03	31.79	27.62	24.22
C. Expected time until retirement=30 years					
Retirement state_stock	21.38	21.38	21.39	21.40	21.41
Retirement state_bond	25.61	25.61	25.62	25.63	25.63
Employment state_stock	28.38	29.52	30.69	31.90	33.14
Employment state_bond	33.95	35.30	36.69	38.12	39.59
$\exp E(w_t^e - y_t)$	42.30	35.02	29.48	25.15	21.69

Table 2.7

Effect of Labor Income Risk on Optimal Portfolio Choice and Wealth-Labor Income Ratio in the Employment State

This table reports the optimal portfolio weights in stock and bond in employment state and the exponential of the mean log wealth-labor income ratio $\exp \mathrm{E}(w_t^e - y_t)$ given different labor income risks. To make the comparison meaningful, g is set such that $\mathrm{E}_t(Y_{t+1}/Y_t) = 1.03$ is maintained when labor income risk σ_ς is increased. Labor income is assumed to be independent of risky asset returns. The estimated VAR process of excess risky asset returns is $\mathbf{Z}_{t+1} = \begin{bmatrix} 0.0455 \\ 0.0021 \end{bmatrix} + \begin{bmatrix} 0.0617 & 0.0164 \\ 0.0954 & 0.1210 \end{bmatrix} \mathbf{Z}_t + \mathbf{v}_{t+1}$, with $\mathbf{v}_t \sim N\left(0, \begin{bmatrix} 0.0324 & 0.0004 \\ 0.0004 & 0.0039 \end{bmatrix}\right)$. The other baseline parameters are $r_f = 0.01819$, $\pi^e = 0.95$, $\pi^d = 0.1$, γ = 8, and $\delta = 1/1.1$. Portfolio shares are given in percentage points.

Standard deviation of labor income	0.00	0.04	0.08	0.12	0.16	0.20
Stock	32.51	32.04	30.75	28.92	26.79	24.51
Bond	38.83	38.27	36.75	34.58	32.06	29.33
$\exp \mathrm{E}(w_t^e - y_t)$	18.33	19.29	22.38	28.50	40.46	70.95

Table 2.8

Out-of-Sample Performance Test among Three Investment Strategies

This table reports the annual mean return and Sharpe ratio from out-of-sample performance test for three investment strategies: (1) the strategy that considers time-varying investment opportunities as described by a VAR process; (2) the strategy under a static model; and (3) a naïve investment strategy that simply allocates wealth evenly to all investment options. Three asset categories are used: stock, 5-year government bond, and 90-day Treasury bill. Quarterly return data for the period 1952.Q3-1999.Q4 are from Campbell, *et al*. (2003) and are updated to 2007.Q3. The nominal returns are converted to real terms using the Consumer Price Index (CPI). Excess stock and bond returns are assumed to follow a VAR(1) process and its parameters are estimated using data from1952.Q2-1971.Q4. Labor income is assumed to be independent of asset returns. The labor income growth rate g is set such that $\mathrm{E}_t(Y_{t+1}/Y_t) = 1 + 0.03/4$. Other baseline values are: σ_ς =0.05, $\pi^e = 0.9875$, $\pi^d = 0.025$, γ =8, and $\delta = (1/1.1)^{0.25}$. The mean return and Sharpe ratio are annualized.

	VAR model	Static model	1/*n* rule	VAR - Static	VAR - 1/*n*
Mean return (annualized percentage)	9.60	5.32	3.14	4.28	6.46
t-ratio	4.37	3.04	2.90	2.55	3.80
Sharpe ratio (annualized)	0.61	0.36	0.25		

Table 2.9

Effect of Exogenous Pension Account on Asset Allocation in the Employment State

This table shows the optimal portfolio weights, the exponential of mean log wealth-labor income ratio $\exp \mathrm{E}[w_t^e - \tilde{y}_t]$ in the employment state, the exponential of mean log consumption-labor income ratio $\exp \mathrm{E}[c_t - \tilde{y}_t]$, and the exponential of mean of log pension wealth-labor income ratio $\exp \mathrm{E}[w_t^p - \tilde{y}_t]$ given the exogenous asset allocation in the pension account. We consider the cases where the portfolio weights in stock and bond in the pension accounts are (0, 0.5), (0.25, 0.25), (0, 0), and (0.5, 0), respectively. Labor income is assumed to be independent of asset returns. The labor income growth rate g is set such that $\mathrm{E}_t(Y_{t+1}/Y_t) = 1.06$. Other baseline values are: σ_ς = 0.1, $\pi^e = 0.95$, $\pi^d = 0.1$, γ =8, and $\delta = 1/1.1$. In the employment state, the investor contributes 5 percent of her labor income to her pension account each year. The optimal portfolio weights in the retirement state in stock and bond are 21.41 percent and 25.6 percent, respectively. Portfolio shares are given in percentage points.

Pension account				
Stock (in percent)	0	25	0	50
Bond (in percent)	50	25	0	0
Risk-free asset (in percent)	50	50	100	50
Regular wealth account				
Employment_stock	76.90	75.10	75.70	50.11
Employment_bond	75.40	91.60	89.33	201.64
$\exp \mathrm{E}[w_t^e - \tilde{y}_t]$	5.64	5.27	5.73	3.42
$\exp \mathrm{E}[c_t - \tilde{y}_t]$	0.66	0.71	0.65	0.90
$\exp \mathrm{E}[w_t^p - \tilde{y}_t]$	1.69	2.96	1.47	7.84

Table 3.1

Percentage of DC Plan Assets in Company Stock

Company	**Company Stock Percentage in 2001**
Procter & Gamble (PG)	94.7
Pfizer (PFE)	85.5
Coca-Cola (KO)	81.5
General Electric (GE)	77.4
Enron (ENRNQ)	57.7
Texas Instruments (TXN)	75.7
McDonald's (MCD)	74.3
FORD (F)	57.0
Qwest (Q)	53.0
AOL Time Warner (TWX)	52.0

Source: The Economist, December 15, 2001, Morningstar.com.

Table 3.2

Optimal Asset Allocation When Labor Income Is Independent of Asset Returns

This table shows the optimal portfolio decision in defined contribution pension plans when labor income is independent of risky asset returns. Three assets, namely, riskless asset, stock market and company stock, are available for investment. The riskless asset has a log annual return of 0.8 percent. Excess returns on stock market and company stock are assumed to follow the VAR process:

$$\begin{bmatrix} r_{s,t+1} - r_f \\ r_{c,t+1} - r_f \end{bmatrix} = \begin{bmatrix} 0.05620 \\ 0.05271 \end{bmatrix} + \begin{bmatrix} 0.17430 & -0.10593 \\ 0.18670 & 0.06799 \end{bmatrix} \begin{bmatrix} r_{s,t} - r_f \\ r_{c,t} - r_f \end{bmatrix} + \mathbf{v}_{t+1}$$

where $\mathbf{v}_{t+1} \sim N\left(0,\ \mathbf{\Sigma}_{\mathbf{v}}\right)$ and

$$\mathbf{\Sigma}_{\mathbf{v}} = \begin{bmatrix} 0.03616 & 0.03715 \\ 0.03715 & 0.05310 \end{bmatrix}.$$

Labor income Y_t follows $Y_{t+1} = Y_t \exp(0.03 + \varsigma_{t+1})$, where $\varsigma_{t+1} \sim N(0, 0.01)$, and is independent of risky asset returns. The contribution rate is assumed to be 6 percent.

	γ=2	**4**	**6**	**8**	**10**	**12**
5 years to retirement						
Stock market	0.49	0.19	0.12	0.09	0.06	0.05
Company stock	0.49	0.28	0.19	0.14	0.12	0.10
10 years to retirement						
Stock market	0.38	0.22	0.11	0.07	0.04	0.03
Company stock	0.62	0.35	0.26	0.20	0.17	0.14
15 years to retirement						
Stock market	0.29	0.25	0.13	0.08	0.05	0.03
Company stock	0.71	0.40	0.30	0.24	0.20	0.17

Table 3.3

Optimal Asset Allocation When Labor Income Is Correlated with Asset Returns

This table shows the optimal portfolio decision in defined contribution pension plans when labor income is correlated with risky asset returns. Three assets, namely, riskless asset, stock market and company stock, are available for investment. The riskless asset has a log annual return of 0.8 percent. Excess returns on stock market and company stock are assumed to follow the VAR process: $\begin{bmatrix} r_{s,t+1}-r_f \\ r_{c,t+1}-r_f \end{bmatrix} = \begin{bmatrix} 0.05620 \\ 0.05271 \end{bmatrix} + \begin{bmatrix} 0.17430 & -0.10593 \\ 0.18670 & 0.06799 \end{bmatrix} \begin{bmatrix} r_{s,t}-r_f \\ r_{c,t}-r_f \end{bmatrix} + \mathbf{v}_{t+1}$ where $\mathbf{v}_{t+1} \sim N\left(0,\ \mathbf{\Sigma}_{\mathbf{v}}\right)$ and $\mathbf{\Sigma}_{\mathbf{v}} = \begin{bmatrix} 0.03616 & 0.03715 \\ 0.03715 & 0.05310 \end{bmatrix}$. Labor income Y_t follows $Y_{t+1} = Y_t \exp(0.03 + \varsigma_{t+1})$, where $\varsigma_{t+1} \sim N(0, 0.01)$, and is correlated with stock market and company stock returns with correlation coefficients equal to 5 and 25 percent, respectively. The contribution rate is assumed to be 6 percent.

	γ = 2	**4**	**6**	**8**	**10**	**12**
5 years to retirement						
Stock market	0.55	0.28	0.20	0.17	0.15	0.14
Company stock	0.43	0.20	0.11	0.06	0.03	0.01
10 years to retirement						
Stock market	0.51	0.40	0.31	0.25	0.22	0.18
Company stock	0.49	0.19	0.08	0.03	0.00	0.00
15 years to retirement						
Stock market	0.47	0.49	0.39	0.34	0.27	0.22
Company stock	0.53	0.18	0.06	0.00	0.00	0.00

Table 3.4

Optimal Asset Allocation under Different Contribution Rates

This table shows the optimal portfolio decision in defined contribution pension plans when the contribution rate is equal to 6 and 12 percent, respectively. Three assets, namely, riskless asset, stock market and company stock, are available for investment. The riskless asset has a log annual return of 0.8 percent. Excess returns on stock market and company stock are assumed to follow the VAR process:
$\begin{bmatrix} r_{s,t+1}-r_f \\ r_{c,t+1}-r_f \end{bmatrix} = \begin{bmatrix} 0.05620 \\ 0.05271 \end{bmatrix} + \begin{bmatrix} 0.17430 & -0.10593 \\ 0.18670 & 0.06799 \end{bmatrix} \begin{bmatrix} r_{s,t}-r_f \\ r_{c,t}-r_f \end{bmatrix} + \mathbf{v}_{t+1}$ where $\mathbf{v}_{t+1} \sim N\left(0,\ \mathbf{\Sigma_v}\right)$ and $\mathbf{\Sigma_v} = \begin{bmatrix} 0.03616 & 0.03715 \\ 0.03715 & 0.05310 \end{bmatrix}$.
Labor income Y_t follows $Y_{t+1} = Y_t \exp(0.03+\varsigma_{t+1})$, where $\varsigma_{t+1} \sim N(0, 0.01)$, and is correlated with stock market and company stock returns with correlation coefficients equal to 5 and 25 percent, respectively. The investors are assumed to have 10 years to retirement.

	γ=2	**4**	**6**	**8**	**10**	**12**
Contribution rate = 0.06						
Stock market	0.51	0.40	0.31	0.25	0.22	0.18
Company stock	0.49	0.19	0.08	0.03	0.00	0.00
Contribution rate = 0.12						
Stock market	0.39	0.58	0.46	0.37	0.28	0.23
Company stock	0.61	0.18	0.04	0.00	0.00	0.00

Table 3.5

Optimal Asset Allocation under Different Correlations between Labor Income and Company Stock Return

This table shows the optimal portfolio decision in defined contribution pension plans when the correlation coefficient between labor income and company stock return is equal to 55 and 25 percent, respectively. Three assets, namely, riskless asset, stock market and company stock, are available for investment. The riskless asset has a log annual return of 0.8 percent. Excess returns on stock market and company stock are assumed to follow the VAR process:

$$\begin{bmatrix} r_{s,t+1}-r_f \\ r_{c,t+1}-r_f \end{bmatrix} = \begin{bmatrix} 0.05620 \\ 0.05271 \end{bmatrix} + \begin{bmatrix} 0.17430 & -0.10593 \\ 0.18670 & 0.06799 \end{bmatrix} \begin{bmatrix} r_{s,t}-r_f \\ r_{c,t}-r_f \end{bmatrix} + \mathbf{v}_{t+1} \text{ where } \mathbf{v}_{t+1} \sim N\left(0,\ \boldsymbol{\Sigma}_{\mathbf{v}}\right) \text{ and } \boldsymbol{\Sigma}_{\mathbf{v}} = \begin{bmatrix} 0.03616 & 0.03715 \\ 0.03715 & 0.05310 \end{bmatrix}.$$

Labor income Y_t follows $Y_{t+1} = Y_t \exp(0.03 + \varsigma_{t+1})$, where $\varsigma_{t+1} \sim N(0, 0.01)$, and has a 5 percent correlation with stock market. The investors are assumed to have 10 years to retirement and to contribute 6 percent of their labor income to their pension account each year.

	γ = 2	**4**	**6**	**8**	**10**	**12**
Correlation (labor income, stock market return) = 5 percent; Correlation (labor income, company stock return) = 25 percent						
Stock market	0.51	0.40	0.31	0.25	0.22	0.18
Company stock	0.49	0.19	0.08	0.03	0.00	0.00
Correlation (labor income, stock market return) = 5 percent; Correlation (labor income, company stock return) = 55 percent						
Stock market	0.72	0.61	0.41	0.31	0.25	0.20
Company stock	0.28	0.00	0.00	0.00	0.00	0.00

Table 3.6

Gains of Dynamic Trading Strategy Relative to Benchmark Strategies

This table shows the contribution rate in each benchmark strategy that employees need to make in order to obtain the same expected utility as in the dynamic strategy that has an exogenous contribution rate of 6 percent. The five benchmark trading strategies are: (1) the strategy that allocates all pension wealth to stock market; (2) the strategy that allocates all pension wealth to company stock; (3) the $1/n$ rule that allocates the contribution evenly across all available investment objects; (4) the strategy that takes into account the time variation in investment opportunity but does not change the portfolio decision over time, denoted "Optimal 1"; and (5) the static strategy that assumes a constant investment opportunity, denoted "Optimal 2". Three assets, namely, riskless asset, stock market and company stock, are available for investment. The riskless asset has a log annual return of 0.8 percent. Excess returns on stock market and company stock are assumed to follow the VAR process:
$\begin{bmatrix} r_{s,t+1} - r_f \\ r_{c,t+1} - r_f \end{bmatrix} = \begin{bmatrix} 0.05620 \\ 0.05271 \end{bmatrix} + \begin{bmatrix} 0.17430 & -0.10593 \\ 0.18670 & 0.06799 \end{bmatrix} \begin{bmatrix} r_{s,t} - r_f \\ r_{c,t} - r_f \end{bmatrix} + \mathbf{v}_{t+1}$ where $\mathbf{v}_{t+1} \sim N\left(0,\ \boldsymbol{\Sigma}_{\mathbf{v}}\right)$ and $\boldsymbol{\Sigma}_{\mathbf{v}} = \begin{bmatrix} 0.03616 & 0.03715 \\ 0.03715 & 0.05310 \end{bmatrix}$.
Labor income Y_t follows $Y_{t+1} = Y_t \exp(0.03 + \varsigma_{t+1})$, where $\varsigma_{t+1} \sim N(0, 0.01)$, and is correlated with stock market and stock market with correlation coefficients equal to 5 and 25 percent, respectively. The investors are assumed to have 10 years to retirement. Contribution rates are given in percentage points.

	All stock market (1)	**All company stock (2)**	**1/*n* rule (3)**	**Optimal 1 (4)**	**Optimal 2 (5)**
γ=2	7.03	7.48	7.66	6.84	6.92
4	7.06	10.53	6.72	6.45	6.54
6	8.33	15.94	7.44	6.28	6.36
8	9.69	21.59	8.59	6.20	6.26
10	10.94	26.20	9.86	6.16	6.19
12	12.01	29.75	11.08	6.13	6.15

Table 3.7

Optimal Portfolio Decision When Investors Have Higher Expected Company Stock Return

This table shows the optimal asset allocations under the baseline case and the cases where investors expect their company stock returns to be 1, 2, and 4 percent higher than in the baseline case, while the expected return in stock market remains the same. Three assets, namely, riskless asset, stock market and company stock, are available for investment. The riskless asset has a log annual return of 0.8 percent. In the baseline case, excess returns on stock market and company stock are assumed to follow the VAR process:
$\begin{bmatrix} r_{s,t+1}-r_f \\ r_{c,t+1}-r_f \end{bmatrix} = \begin{bmatrix} 0.05620 \\ 0.05271 \end{bmatrix} + \begin{bmatrix} 0.17430 & -0.10593 \\ 0.18670 & 0.06799 \end{bmatrix} \begin{bmatrix} r_{s,t}-r_f \\ r_{c,t}-r_f \end{bmatrix} + \mathbf{v}_{t+1}$ where $\mathbf{v}_{t+1} \sim N\left(0,\ \boldsymbol{\Sigma}_{\mathbf{v}}\right)$ and $\boldsymbol{\Sigma}_{\mathbf{v}} = \begin{bmatrix} 0.03616 & 0.03715 \\ 0.03715 & 0.05310 \end{bmatrix}$.
Labor income Y_t follows $Y_{t+1} = Y_t \exp(0.03 + \varsigma_{t+1})$, where $\varsigma_{t+1} \sim N(0, 0.01)$, and is correlated with stock market and company stock with correlation coefficients equal to 5 and 25 percent, respectively. The investors are assumed to have 10 years to retirement and to contribute 6 percent of their labor income to their pension account each year.

	γ=2	**4**	**6**	**8**	**10**	**12**
Baseline case						
Stock market	0.51	0.40	0.31	0.25	0.22	0.18
Company stock	0.49	0.19	0.08	0.03	0.00	0.00
1 percent higher in company stock return						
Stock market	0.16	0.18	0.15	0.14	0.13	0.12
Company stock	0.84	0.38	0.22	0.13	0.08	0.05
2 percent higher in company stock return						
Stock market	0.00	0.00	0.00	0.02	0.04	0.05
Company stock	1.00	0.53	0.34	0.23	0.16	0.11
4 percent higher in company stock return						
Stock market	0.00	0.00	0.00	0.00	0.00	0.00
Company stock	1.00	0.58	0.38	0.28	0.21	0.17

Table 3.8

Optimal Portfolio Decision with Parameter Uncertainty

This table shows the optimal portfolio decision when the investors have parameter uncertainty about the return generating process. Three assets, namely, riskless asset, stock market and company stock, are available for investment. The riskless asset has a log annual return of 0.8 percent. In the baseline case, excess returns on stock market and company stock are assumed to follow the VAR. Labor income Y_t follows $Y_{t+1} = Y_t \exp(0.03 + \varsigma_{t+1})$, where $\varsigma_{t+1} \sim N(0, 0.01)$, and is correlated with stock market and company stock returns with correlation coefficients equal to 5 and 25 percent, respectively. The investors contribute 6 percent of their labor income to their pension account each year. Two cases are considered: (1) Investors have parameter uncertainty in both the stock market and company stock return generating processes and (2) Investors have parameter uncertainty only in stock market return generating process.

5 years to retirement

	γ= 2	**4**	**6**	**8**	**10**	**12**
With parameter uncertainty in both stock market and company stock return generating processes						
Stock market	0.48	0.22	0.17	0.15	0.14	0.13
Company stock	0.44	0.21	0.11	0.06	0.03	0.01
With parameter uncertainty only in stock market return generating process						
Stock market	0.40	0.16	0.12	0.10	0.10	0.09
Company stock	0.54	0.28	0.17	0.11	0.07	0.05

10 years to retirement

	γ= 2	**4**	**6**	**8**	**10**	**12**
With parameter uncertainty in both stock market and company stock return generating processes						
Stock market	0.51	0.33	0.25	0.21	0.20	0.16
Company stock	0.49	0.20	0.09	0.04	0.00	0.00
With parameter uncertainty only in stock market return generating process						
Stock market	0.40	0.20	0.15	0.13	0.12	0.12
Company stock	0.60	0.33	0.19	0.12	0.07	0.04

15 years to retirement

	γ= 2	**4**	**6**	**8**	**10**	**12**
With parameter uncertainty in both stock market and company stock return generating processes						
Stock market	0.45	0.39	0.32	0.28	0.24	0.20
Company stock	0.55	0.21	0.08	0.02	0.00	0.00
With parameter uncertainty only in stock market return generating process						
Stock market	0.32	0.22	0.17	0.16	0.15	0.16
Company stock	0.68	0.37	0.22	0.13	0.08	0.04

Table 3.9

Gains of Dynamic Trading Strategy When There Is Parameter Uncertainty

This table shows the contribution rate in some benchmark strategies that employees need to make in order to obtain the same expected utility as in the dynamic strategy that has an exogenous contribution rate of 6 percent, given that investors have parameter uncertainty in stock market return generating process but not in company stock. The benchmark strategies are those that invest 50, 75, or 100 percent of contribution in company stock and the rest in riskless asset. Three assets, namely, riskless asset, stock market and company stock, are available for investment. The riskless asset has a log annual return of 0.8 percent. Excess returns on stock and company stock are assumed to follow the VAR process. Labor income Y_t follows $Y_{t+1} = Y_t \exp(0.03 + \varsigma_{t+1})$, where $\varsigma_{t+1} \sim N(0, 0.01)$, and is correlated with stock market and company stock returns with correlation coefficients equal to 5 and 25 percent, respectively. Investors have 10 years time to retirement. Contribution rates are given in percentage points.

	0.5 in company stock 0.5 in riskless asset	**0.75 in company stock 0.25 in riskless asset**	**100 percent in company stock**
γ=2	8.24	7.52	7.43
4	7.05	8.15	10.47
6	7.67	10.50	15.51
8	8.74	13.27	20.71
10	9.92	15.80	25.00
12	11.04	17.91	28.34

Table 3.10

Optimal Portfolio Decision with Employer's Matching

This table shows the optimal portfolio decision with and without employer's matching. Three assets, namely, riskless asset, stock market and company stock, are available for investment. The riskless asset has a log annual return of 0.8 percent. In the baseline case, excess returns on stock market and company stock are assumed to follow the VAR process:

$$\begin{bmatrix} r_{s,t+1} - r_f \\ r_{c,t+1} - r_f \end{bmatrix} = \begin{bmatrix} 0.05620 \\ 0.05271 \end{bmatrix} + \begin{bmatrix} 0.17430 & -0.10593 \\ 0.18670 & 0.06799 \end{bmatrix} \begin{bmatrix} r_{s,t} - r_f \\ r_{c,t} - r_f \end{bmatrix} + \mathbf{v}_{t+1}$$

where $\mathbf{v}_{t+1} \sim N\left(0,\ \mathbf{\Sigma}_{\mathrm{v}}\right)$ and

$$\mathbf{\Sigma}_{\mathrm{v}} = \begin{bmatrix} 0.03616 & 0.03715 \\ 0.03715 & 0.05310 \end{bmatrix}.$$

Labor income Y_t follows $Y_{t+1} = Y_t \exp(0.03 + \varsigma_{t+1})$, where $\varsigma_{t+1} \sim N(0, 0.01)$, and is correlated with stock market and company stock with correlation coefficients equal to 5 and 25 percent, respectively. The investors are assumed to have 10 years to retirement and to contribute 6 percent of their labor income to their pension account each year. In the case with matching, each period the employer matches the employee's contribution dollar to dollar up to 6 percent of the employee's annual labor income and the employee is not allowed to reallocate employer's contributions from company stock to other assets.

	γ = 2	**4**	**6**	**8**	**10**	**12**
With employer's matching						
Employees' Portfolio weights						
Stock market	0.40	0.62	0.48	0.34	0.24	0.18
Company stock	0.60	0.12	0.00	0.00	0.00	0.00
Portfolio weights includes employers' match						
Stock market	0.20	0.31	0.24	0.17	0.12	0.09
Company stock	0.80	0.56	0.50	0.50	0.50	0.50
Without employer's matching						
Stock market	0.51	0.40	0.31	0.25	0.22	0.18
Company stock	0.49	0.19	0.08	0.03	0.00	0.00

VITA

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