MODELING TRAVELER BEHAVIOR

VIA

DAY-TO-DAY LEARNING DYNAMICS

by

OZLEM YANMAZ-TUZEL

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ABSTRACT OF THE DISSERTATION

Modeling Traveler Behavior via Day-to-Day Learning Dynamics

By OZLEM YANMAZ-TUZEL

Dissertation Director:

Kaan Ozbay

Travel behavior lies at the core of analysis and evaluation of transportation related measures aiming to improve urban mobility, environmental quality and a wide variety of social objectives. A better understanding of travel behavior will improve travel demand forecasting and the assessment of emerging transport policies, and will improve our means to increase road safety.

The day-to-day models reflect the travelers' learning and forecasting mechanisms. These models predict travelers' choices for any given day based on their experienced choices in the previous days. Day-to-day approaches allow the use of wide range of behavioral rules, and levels of aggregation, and capture the heterogeneity in users' learning and adaptation processes, and behavioral characteristics.

This thesis aims to develop a novel framework to model the interdependence between travelers' choice decisions, learning and adaptation behavior and the day-to-day update mechanism of traffic flows. The novelty of this thesis is that the proposed approach combines traveler heterogeneity and rationality in a single framework to predict travelers' day-to-day departure time and route decisions, and develops a novel day-to-day dynamic traffic assignment approach. The empirical results obtained from real transportation network, New Jersey Turnpike, confirm that the proposed day-to-day learning and dynamic traffic assignment framework model can successfully capture the significant learning dynamics, demonstrating the possibility of developing a psychological framework (i.e., learning models) as a viable approach to represent travel behavior.

The other contributions of this thesis include a novel route choice set generation approach based on stochastic integer programming approach. The proposed methodology takes into account travel time variability and reliability in the transportation network. The path relevance criteria are directly incorporated into the optimization model by minimizing mean travel time, travel time variability and path overlap. Unlike previous approaches in the literature, proposed methodology eliminates the filtering step from the choice set generation and generates paths sets at desired dissimilarity level while minimizing the travel time and variability of these paths. Several case studies show the applicability of the proposed methodology on real transportation networks.

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Dedication

To my dear husband Oncel and beloved son Kerem.

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CHAPTER 1. INTRODUCTION

1.1 Background

Demand for highway travel in the Unites States continues to increase as population grows, especially in metropolitan areas. The Texas Transportation Institute estimated that, in 2005, 4.2 billion vehicle hours of delay were experienced throughout the country, resulting in 2.9 gallons in wasted fuel and a congestion cost of $78.2 billion (Schrank and Lomax, 2007). Many trips are delayed both because of the excess demand of travel changing from day-to-day, week-to-week, or season-to-season, and by events that are irregular, but frequent. Crashes, vehicle breakdowns, improperly timed traffic signals, special events, work zones, and adverse weather conditions are some of the factors that cause variety of congestion problems.

In the last years, travel has increased 105 percent in big metropolitan areas while road capacity on freeways and major streets has grown by only 45 percent (Schrank and Lomax, 2007). Moreover, in many urban areas, land scarcity and environmental constraints limit construction of new roads or expansion of existing ones even if funds were available.

As a result, the importance of better management of the road network to efficiently utilize existing capacity is increasing. Addressing the congestion problem can provide substantial benefits and improvements in many sectors of society and the economy.

The decades of transportation research and practice have emphasized that effectiveness of any congestion management policy is predicated on the ability to elicit behavioral responses from the users of the transportation system. Understanding and modeling travel behavior in a variety of situations is crucial for successful management of transportation systems.

Travel behavior lies at the core of analysis and evaluation of transportation related measures aiming to improve urban mobility, environmental quality and a wide variety of social objectives. A better understanding of travel behavior will improve travel demand forecasting and the assessment of emerging transport policies, and will improve our means to increase road safety.

Modeling traveler behavior can be distinguished according to following dimensions: static versus dynamic, deterministic versus stochastic, and equilibrium based versus non-equilibrium based.

Static type models assume steady state network conditions. Link volumes are time-invariant, the time to traverse a link depends only on the number of vehicles on that link, and the vehicle queues are stacked vertically and do not traverse to the upstream links in the networks. In contrast, dynamic models consider the influence of what has happened in the previous period on what happens in the current period, and the influence on what will happen in the next period of time.

In deterministic models, travelers are assumed to be rational, exploring each alternative's relevant attributes and trading off the utilities derived from them. The decision strategy serves to generate a choice from a choice set for the alternative that provides the individual with the maximum utility (minimum travel time). The stochastic models, on the other hand, are based on imperfect or perceived knowledge, and random utility component is introduced to the modeling process. Kahneman and Tversky (1979) found that people do not necessarily maximize expected utility, but have a perception of probability of a certain outcome and the value of that outcome. Moreover, Trevesky and Kahneman (1981) showed that people exhibit risk behavior which is dependent on the way the decision is framed.

Each travel behavior model is based on either equilibrium or non-equilibrium state. In equilibrium transport models, flows are pushed towards the equilibrium by route or departure time switching. As defined by Wardrop (1952) at equilibrium no traveler can experience a lower travel time by unilaterally changing routes. Each traveler minimizes his/her own travel time or cost. Wardrop's principle states that all travelers are assigned to a shortest path between their corresponding O-D's and that travel times and volumes are consistent with each other everywhere on the network.

Classical equilibrium approaches assume rigid behavioral tendencies; categorize drivers into homogeneous classes via user equilibrium, system optimal or stochastic user-equilibrium. Moreover, these models assume that the driver behavior classes are known deterministically a priori. The estimation of equilibrium is typically achieved through the solution of some optimization, or variational inequality problem, which makes the approach restrictive in terms of generalizations.

Moreover, most route choice models in equilibrium approach assume that travelers choose the shortest route. However, travelers may "adopt some idea of alternative perception and mental map that can reduce hundreds of possible alternatives to a limited number of real potentially available path alternatives that are actually faced by travelers" (Ridwan, 2003)

In any event, realistic travel choice models should first identify relevant route choice sets available to travelers, and be able to integrate behavioral characteristics of the travelers into the models. This integration is one of the challenges in future travel behavior modeling. Unfortunately, with equilibrium approach, it is difficult to capture the heterogeneity in users' behavioral characteristics.

Non-equilibrium models (day-to-day models) aim to solve situations not in equilibrium. The behavioral dynamics in these models is based on the underlying belief, where the behavior on a given day is affected by the behavior on previous days.

In the context of day-to-day dynamics it is highly expected that drivers' knowledge and perception of the network performance will vary depending on their past experiences and personal attributes. Thus, assumptions on drivers' perception in equilibrium network models are restrictive and unrealistic.

In reality, the decision making process of traveler choice is a dynamic process. A learning process is central to the traveler's cognition, as the information acquired through earlier travel experience affect the future decisions. Moreover, the characteristics of the traveler, subjective interpretation of the traffic information (if available), trip characteristics (time of day, trip purpose, etc), and uncertain traffic conditions have an importance in the traveler's final decision. Repeated choices make the drivers better aware of the travel options, inducing them to consider a destination, choose a route or try a new transportation mode.

1.2 Motivation

Day-to-day modeling becomes crucial while investigating the potential behavioral responses of travelers to major disturbances which have recently occurred in the transportation system. Some of these disturbances are:

1. Introduction of congestion pricing
2. Change in the implementation structure of an existing congestion pricing strategy
3. Construction of new roads or other network components such as, major new interchanges / intersections
4. Long term closure of roads due to a major change in the system

Transportation networks of the New York (NY) and New Jersey (NJ) Metropolitan Area has experienced several major changes in the last few years. NJ Turnpike, a 148-mile toll road, is one of the major freeways in the State of NJ, which has experienced several significant policy changes during the last decade:

- Starting from September 2000, E-ZPass technology (a form of electronic toll collection system) was introduced to the facility
- In September 2000, New Jersey Turnpike Authority (NJTA) implemented the first stage of the time-of-day pricing application and increased the toll levels for cash users and peak E-ZPass users, while E-ZPass off-peak users continued to pay the

same toll amounts as in 1991. As part of this program, different toll levels were charged to users depending on time-of-day and vehicle type; such that, E-ZPass users started to pay discounted tolls during off-peak hours. Peak hour tolls are effective on weekdays from 7:00 to 9:00 a.m. and from 4:30 to 6:30 p.m., and on weekends, peak-hour tolls were effective throughout the day.

- In January 2003, toll levels for each time period and vehicle type were increased as the second stage of the NJ Turnpike time-of-day pricing program.
- In January 2006, discounts for E-ZPass peak users were eliminated, and E-ZPass peak users started to pay the same amount of toll as the cash users.
- After nearly three years of construction, NJTA opened the $250 million Exit 15X on the Eastern Spur (just south of EXIT 16E) on December 1, 2005. The new interchange serves the new Secaucus Junction rail transfer station. The Turnpike Authority contributed an additional $84 million to develop the $450 million adjacent Allied Junction, which will have 3.5 million square feet of combined commercial and residential development, as well as up to 2,600 new parking spaces when the development is completed. Upon full development, Exit 15X is expected to handle 40,000 vehicles per day.

Due to a major change in the transportation system, like the examples mentioned above, the network which was at equilibrium (slow-moving system) would be disrupted. Until, a new long-term equilibrium is reached, the users of the transportation system would adapt themselves to the newly imposed conditions. Thus, a transient period would occur where the travelers learn the prevailing conditions of the disturbed transportation system (fast-moving system). These intermediate stages are important for evaluation of the transportation system, because the transportation system is expected to be in a disequilibrium state due to travelers' gradual response to these unforeseen disruptions.

The day-to-day models reflect the travelers' learning and forecasting mechanisms. These models predict travelers' choices for any given day based on their experienced choices in the previous days. Day-to-day approaches allow the use of wide range of behavioral rules, and levels of aggregation.

Current day-to-day modeling approaches studied in the transportation area mainly focus on slow moving-systems, i.e. the systems with no disruptions. These methodologies usually aim to understand travel behavior learning under Advanced Traveler Information Systems (ATIS), and/ or travel time reliability. Moreover, instead of finding the learning parameters optimally, these studies impose constant values for the travelers' behavior updating mechanism. Finally, most of these studies validate their models using either laboratory experiments or stated preference travel surveys.

However, none of these studies investigated the travel behavior changes in response to real-world disruptions in the context of real transportation systems. Understanding travelers' behavioral responses to the major system changes would help both researchers and policy makers in identifying expected impacts of future transportation management strategies.

Thus, it is crucial to study the day-to-day dynamics of the transportation system; i.e. the variations occurring, day after day, on the flows and on the network performances as a result of variations in the demand and/or supply, in order to provide both the researcher and the planner with more concrete and accurate answers.

1.3 Problem Definition and Contributions

This thesis aims to develop a framework to model the interdependence between travelers' choice decisions, learning and adaptation behavior and the day-to-day update mechanism of traffic flows. The following flow chart summarizes the methodological steps considered in the proposed day-to-day modeling approach (Figure 1-1).

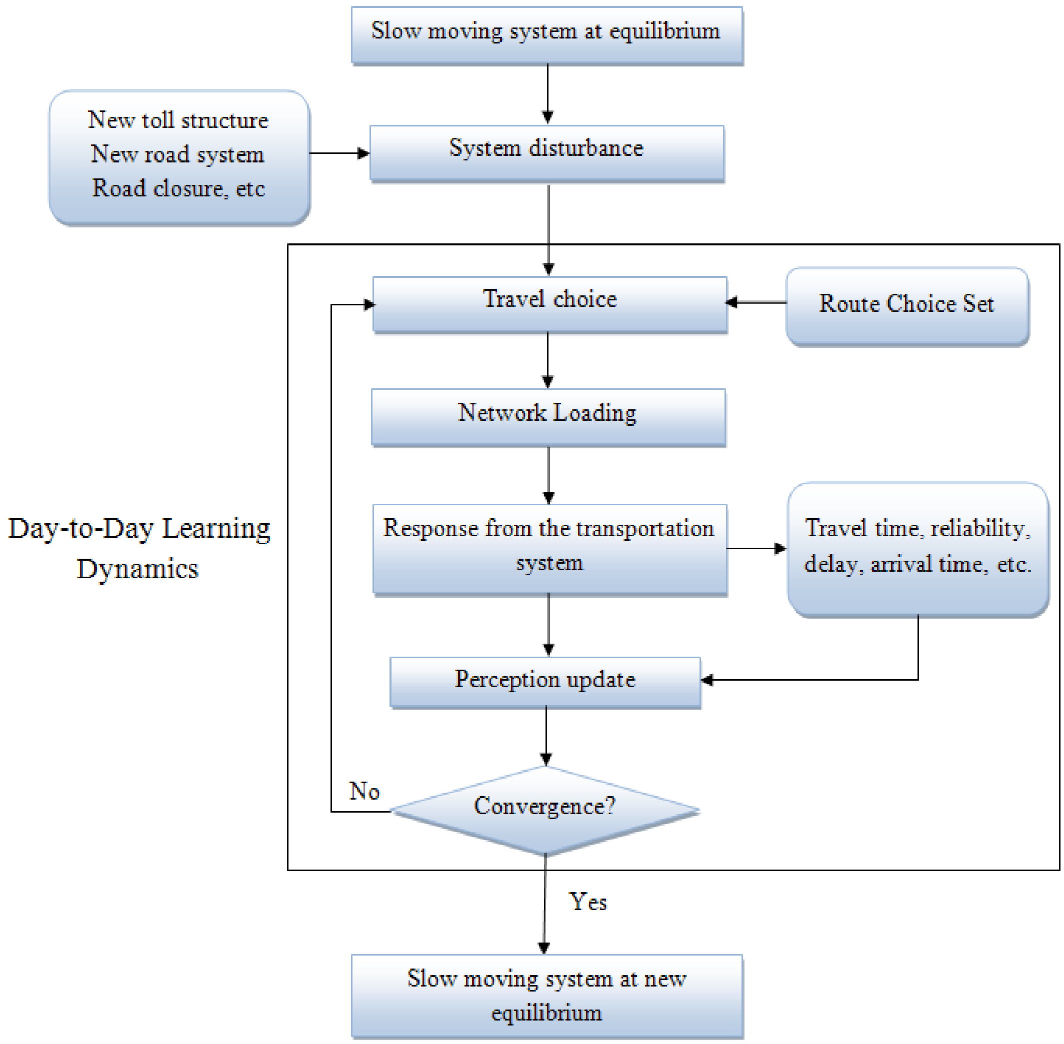


Figure 1-1 Flow chart for modeling the day-to-day learning dynamics

Major contributions of this thesis are:

1. Provide an analytical formulation for the path set selection to minimize ad-hoc nature of current path selection procedures adopted in the literature.

2. Propose SLA based learning model first proposed by Ozbay et al. (2001, 2002) by designing an agent-based learning system via Bayesian Inference theory.

3. Introduce novel estimation techniques to better capture user heterogenity and estimate the optimal learning parameters in a non-stationary environment.

4. Use real-world case studies and scenarios to understand the strengths and weaknesses of the Bayesian-SLA based learning model and propose ways to improve it.

5. Incorporate the proposed day-to-day learning framework into a network dynamic traffic assignment formulation to demonstrate user and network interaction.

The above contributions will enable the transportation community to better understand following aspects of traffic modeling problems:

1. Modeling travelers day-to-day behavior when disruptions are imposed to the system.
2. Modeling travelers' departure-time and route choice behavior on a given day; i.e. route choice set generation, the variables (other than travel time) affecting travelers' dynamic travel choice behavior
3. Modeling travelers' mechanism to update their perceptions based on their experience with the transportation system; i.e. the criteria involved in travelers' day-to-day learning behavior
4. Analyze the convergence propoerties required for the system to reach to the new slow-moving system (long-term equilibrium).

1.3.1 Application Area

The proposed day-to-day learning model will be estimated and tested using the empirical data obtained from NJ Turnpike. The system disruptions included in the estimation process are:

1. Toll increase in January 2003
2. Elimination of discounts for peak-period E-ZPass users in January 2006
3. Opening of exit location 15X in December 2005.

NJ Turnpike is a 148 mile-toll road extending from the Delaware Memorial Bridge in the South of New Jersey to George Washington Bridge in New York City. A schematic of the NJ Turnpike and the surrounding transportation network is shown in Figure 1-2.

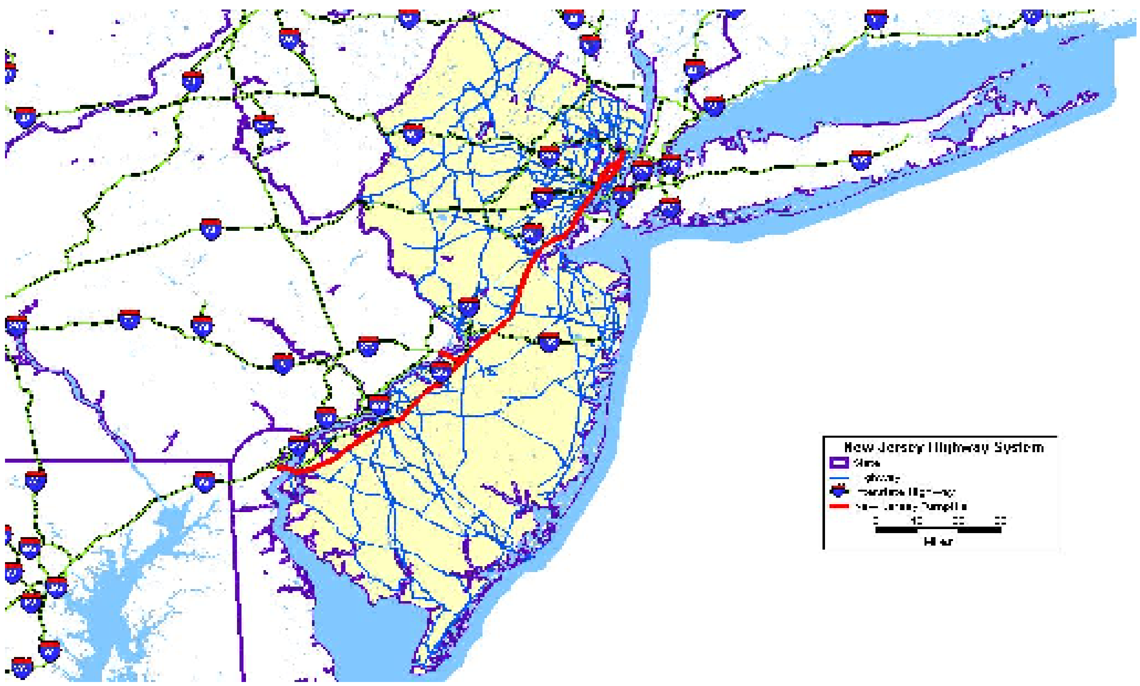


Figure 1-2 Map of NJ Turnpike (Ozbay et al., 2005)

Since its completion in 1952, NJ Turnpike has played a key role in facilitating the economic development of the State of New Jersey, its neighboring municipalities, and the entire mid-Atlantic region.

Currently, the road has 28 interchanges, commonly referred to as exits, with an average daily traffic that exceeds 700,000 vehicles. To minimize queuing delays, NJ Turnpike has minimal number of toll plazas over its 148 miles and the toll plazas are located at the exits. The interchanges connect to NJ's major highways and vast transportation network, institutions, and economic hubs.

While testing and validating the proposed day-to-day learning framework two types of datasets are considered.

First dataset covers the traffic data which include real world vehicle-by-vehicle traffic and travel time data observed from passenger cars with toll tags. The traffic data available for NJ Turnpike include vehicle-by-vehicle information regarding O-D locations, tolls paid and observed travel times of each E-ZPass vehicle, for each time-of-day and day of the week from October 2002 to March 2003 and from December 2005 to December 2006. Since exit location 15X have been operating since December 1, 2005, the vehicle-by-vehicle data for this specific exit are not available before December 2005. The travel time data include mean and standard deviations of the travel times observed for the corresponding time period. During estimation process, weekends and holidays were excluded from the database. For each month approximately 15 days were considered. Preliminary analysis of the response of travelers to disturbed conditions (toll increase on January 2003) can be found in a study by Ozbay et al. (2006). The results of this analysis revealed that travelers do not choose their travel choices solely based on toll differentials, but travelers' individual preferences affect their travel behavior.

The second dataset covers the individual travel survey data which were used to estimate the utility functions and to provide information regarding users' departure time choices and their socio-economic characteristics. The survey was conducted by the Eagleton Institute of Rutgers University (Ozbay et al. 2005). The data set contains 513 observations, 483 (94.2%) of which are current regular users residing in NJ. The survey participants were asked in detail about their most recent trips in the am and pm peaks. The questions include origin, destination, toll, departure time, desired/actual arrival time of each trip, as well as the socio-economic characteristics such as; income, education, employment, age and gender.

1.4 Thesis Outline

The main consideration in this thesis is to provide a day-to-day learning framework to understand the travelers' learning and adaptation behavior to the disruptions in the transportation system.

Chapter 2 reviews the available methodologies used to model traveler (commuter) behavior. The literature review chapter is divided into three main categories: (a) route choice set generation methods; (b) static/dynamic, deterministic/stochastic network loading (traffic assignment) methods; and (c) day-to-day learning methods.

Chapter 3 presents a novel path choice set generation approach based on mathematical programming approach. Specifically a stochastic integer programming model is proposed, where the relevant route choice set is determined via minimizing travel time, travel time variability, and path overlap.

Chapter 4 introduces a new novel day-to-day learning framework to model travelers' departure time and route choice behavior under non-equilibrium network conditions due to major disturbances, such as changes in the congestion pricing policies, and building of new road sections. An agent-based learning system via Bayesian-SLA is designed which can learn the best possible actions and model travelers' day-to-day travel choices in a non-stationary stochastic environment. The developed learning framework reflects travelers' perception about the system and their response to the experienced traffic conditions. Then the proposed day-to-day learning framework was tested and verified using extensive vehicle-by-vehicle real traffic data obtained from NJ Turnpike, to understand the traveler responses to real changes in the transportation system. In particular, two different major disruptions are considered; January 2003 toll increase and December 2005 15X Interchange installation.

Chapter 5 develops a day-to-day dynamic traffic assignment methodology to capture dynamic traffic flow evolution and network-level interactions of driver departure time and route choice decisions. The approach uses microscopic simulation to model the behavior of drivers on the demand side, and uses macroscopic simulation to obtain system variables such as link travel time, volume and density. Bayesian-SLA framework developed in Chapter 4 is used to model day-to-day update mechanism of the transportation network.

Chapter 6 summarizes the conclusions and future work.

CHAPTER 2. LITERATURE REVIEW

This chapter reviews the available methodologies related to modeling travel behavior. The literature review chapter is divided into three main categories: (a) path choice set generation methods; (b) static/dynamic, deterministic/stochastic network loading (traffic assignment) methods; and (c) day-to-day learning methods.

2.1 Path Choice Set Generation Approaches

Identification of fully or partially disjoint paths between origin and destination (OD) pairs has been at the heart of general network problems for decades. In most of the transportation related problems, such as traffic assignment, route choice, vehicle dispatching, or advanced traveler information systems, the initial step is to generate a relevant path choice set. In fact, correctness and accuracy of travel demand estimates and predictions depend on the quality of the adopted choice sets (Swait and Ben-Akiva, 1985, 1987; VanderWaerden et al., 2004). However, in dense networks, the universal set of all available path sets between an OD pair is usually very large (Bovy, 2007).

To correctly predict path flows, the adopted method to generate a path subset should consider all relevant alternatives which may attract travelers. The generated set should exclude unrealistic paths that no traveler would ever consider, and highly similar paths that no traveler would ever differentiate between (Prato and Bekhor, 2006). Moreover, most of the time travelers have limited awareness about all the alternatives, and will not always consider all known alternatives to be actual travel options due to several constraints influenced by their travel preferences and travel experiences. Bliemer and Bovy (2008) defined the relevant path sets as the paths that would be chosen by most of the travelers due to their characteristics, such as travel time, distance, or cost. The authors investigated the impact of path choice set size and composition on the prediction quality of route choice models, and emphasized that it is crucial to create path choice sets consisting of relevant alternatives only, excluding the irrelevant paths that may cause problems due to path overlap.

Several different approaches to generate path sets have been adopted in the literature. Generation approaches can be a deterministic or a randomization method. While most of these approaches propose heuristics, there are some studies in the literature which develop mathematical programming models to tackle with path set generation. The proposed approaches can be grouped in five categories:

1. Labeling algorithms
2. K-Shortest path algorithms
3. Branch-and-bound algorithms
4. Randomization methods
5. Mathematical programming models

Figure 2-1 summarizes the studies proposing different methods to generate route choice sets.

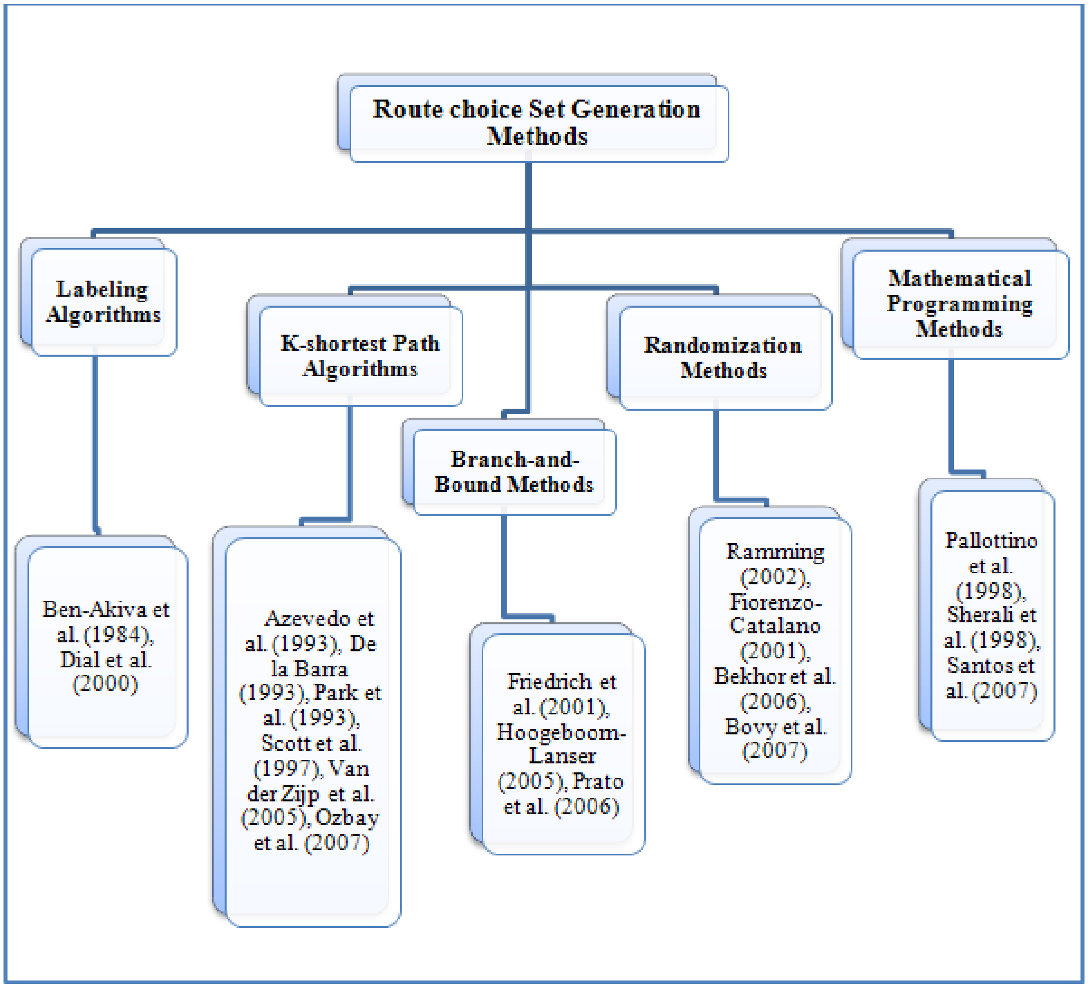


Figure 2-1 Summary of different route-choice set generation algorithms

2.1.1 Labeling Algorithms

The largest group of deterministic generation methods is based on labeling algorithms. In this approach, shortest paths are successively generated by changing one or more of the input variables such as the search criterion, link attributes, and constraints. The generation function is the path search criterion (travel time, distance, etc.), which is strongly preference driven. This approach was first proposed by Ben-Akiva et al. (1984). More recently, Dial (2000) generalized the labeling method by constructing a set of efficient paths by minimizing a linear combination of label costs. These methods generate a selective set, such that not all potentially relevant routes are generated. Thus, the generated choice sets highly depend on the objective function chosen by the analyst.

2.1.2 K-Shortest Path Algorithms

K-shortest path algorithms are generalizations of shortest path algorithms to generate path sets. Several different approaches to generate path sets via exact *k*-shortest path algorithms have been adopted in the literature including computer science, operations research and engineering (Dreyfus, 1969; Shier, 1979; Ziliaskopoulos, 1994). These algorithms are generally extensions of the label-setting and label-correcting approaches used to determine a single shortest path, such as Dijkstra's Algorithm. The best result known to-date is an algorithm by Yen (1971, 1972) (generalized by Lawler (1972) to provide a uniform framework for solving additional problems). Eppstein (1998) provides an excellent review of the different algorithms to find *k*-shortest paths. More recently, Katoh et al. (1982), Hershberger and Suri (2001), Van der Zijp and Fiorenzo-Catalano (2005) and Hershberger et al. (2007) computationally improved Yen's algorithm.

In the transportation field, under the assumption of deterministic user equilibrium Bar-Gera (2006) and Bar-Gera and Boyce (2007) presented a method to identify the universal path choice set in a user-equilibrium traffic assignment solution. However, under stochastic conditions finding the path choice set is still a challenging research problem. Other approaches implemented to generate path sets can be based on deterministic or randomization methods. While most of these approaches are heuristic, there are some studies in the literature which develop mathematical programming models to generate path sets.

Heuristic solutions for *k*-shortest path algorithms are very common and can be categorized into two groups. The first group is based on the link-elimination techniques, where all or some of the links of the "shortest" path between a given origin and destination pair on the network are removed to identify more paths. A review of this approach and inclusion of constraints such as overlap and detour can be found in Van der Zijp and Fiorenzo-Catalano (2005). Azevedo et al. (1993) described an algorithm where all the links on the shortest path are removed to find the next shortest path. One major drawback of removing all links is related to disconnecting centroid connectors and major junctions, which would result in a disconnected network and thus possible failure to find other paths between certain OD pairs. A variant to this approach is to eliminate individual links or a combination of links from the shortest path instead of removing all the links along the shortest path. Recently, Ozbay et al. (2007) proposed an algorithm based on constrained link elimination approach. In particular, after finding the first shortest path, several arcs on this path are eliminated randomly, and the next shortest path which satisfies the disjointness rate and travel time criteria is generated. This procedure is repeated until the desired number of paths is generated between each OD pair. However, even this approach does not guarantee network continuity. Thus, there are not any established methodologies in the literature to determine links to be removed from the original shortest path to increase the probability of finding paths between the selected OD pairs. The second group covers the link-penalty techniques, where instead of removing the links along the shortest path; impedance on these links is increased. De la Barra et al. (1993) illustrated this approach by increasing the link impedances of the shortest path to calculate the next best path. Later, Park and Rilett (1997) modified this approach by not increasing the impedance of links within a certain distance from the origin or the destination. Scott et al. (1997) proposed an optimization problem to determine the rate of increase of the shortest path link impedances to generate the next shortest path that overlaps with the original shortest path by no more than a given number of links.

One major drawback of the *k*-shortest path algorithms, even though the overlap rate between the next generated path and the first shortest path is controlled, there is no guarantee that the generated path will not be very similar to the paths other than the first shortest path. Moreover, since these methods are based on heuristic approaches, they do not guarantee optimality. Finally, in order to find *k* acceptable paths, the shortest path algorithm has to be run at least *k* times, increasing the complexity of these approaches compared with mathematical programming models.

2.1.3 Branch-and-Bound Algorithms

Branch-and-bound technique is another deterministic approach for generating an exhaustive master set of paths given a set of constraints. The generation function builds a tree between the origin and the destination of a trip by processing sequences of links according to a branching rule. Each sequence of links connecting the origin and the destination, and satisfying all the constraints is included in the choice set as a feasible alternative. In the transportation field, Friedrich et al. (2001) applied branch-and-bound technique in transit networks. Later, Hoogeboorn-Lanser (2005) used the same procedure in multi-modal networks, while Prato and Bekhor (2006) adapted it in the route choice modeling by incorporating several constraints into the original algorithm. Inclusion of these constraints increased the complexity of the algorithm resulting in 40 hours of estimation time to generate path sets in a network with 419 nodes and 1 427 links.

2.1.4 Randomization Methods

Randomization methods develop heuristics by randomly selecting cost parameters and simulating large number of path decisions. Link properties of the network at hand are randomized around their measured values. Then, by successively generating new shortest paths, alternative paths are produced. Impedances are drawn from different probability distributions of these link properties whose parameters are determined based on the measured values. The selected probability distributions can be Gaussian, Gumbel, Poisson, etc. Ramming (2002) provides an overview of randomization based models. Fiorenzo-Catalano and Van der Zijpp (2001) implemented the Monte Carlo technique by gradually increasing the variance of the random components in the model to keep the new path finding frequency at constant rate. Later, Bovy and Fiorenzo-Catalano (2007) developed a doubly stochastic choice set generation approach. In this approach not only the link properties, but also the parameters of the generating function of travelers are simultaneously randomized, as well. Bekhor et al. (2006) developed randomization methods to generate path sets and compare the results with labeling and link-penalty methods. The authors stated that the randomization approach had a computational time close to those experienced by link elimination approaches, however the network coverage was observed to be less compared with labeling and link-penalty approaches. A typical property of this approach, like the labeling methods, even with different cost values the same path may be found several times as the best one.

Link elimination/penalty, labeling algorithms and simulation methods first generate an exhaustive base path set, called the master set. Then this master set is further filtered in order to establish relevant choice sets that will be used in the analysis. Thus, further constraints such as maximum detour, logical sequence of links by road type, or maximum overlap, etc. are applied to the master set. Prato et al. (2006), Bovy et al. (2007) and Fiorenzo-Catalano, S. (2007) developed filtering processes to remove the largely overlapping routes from the master set. The results showed that via filtering around 50% of the routes in the master set were removed due to these additional constraints.

2.1.5 Mathematical Programming Methods

Path set generation via integer programming has been studied by several researchers in the literature. Most of these studies focus on finding only one shortest path under additional constraints (e.g. capacity, travel time). A review of these studies can be found in Pallottino et al. (1998) and Santos et al. (2007). Since additional constraints result in an NP-hard, or NP-complete problem, several approaches are proposed to relax the constraint set, such as Lagrangian relaxation. Sherali et al. (1998) extended the original shortest path formulation to find the shortest pair of fully disjoint paths. In this formulation first a shortest path for an OD pair is found, then the arcs in the first shortest pair are flipped, and the second shortest path is found via binary integer programming. One major situation that can arise in this approach is that splitting of the initial shortest path solution may eliminate the accessibility of some nodes, and might fail to identify a feasible solution to problem. Moreover, with this approach at most two paths can be generated for each OD pair.

2.2 Traffic Assignment Approaches

The dominant approach followed to capture the interaction between traveler choice and the network performance has been to solve an equilibrium assignment problem that can be any combination of static/dynamic and deterministic/stochastic models. Traffic assignment is concerned with the selection of routes between origins and destinations in transportation networks. The basic problem in traffic assignment is to find the link flows given the origin-destination (O-D) trip rates, the networks, and the link performance functions.

In this approach, the only state of interest is the fixed point in which the system is at equilibrium, where individual travelers have no incentive to change their decisions. Travelers are assumed to be rational, exploring each alternative's relevant attributes and trading off the utilities derived from them. The decision strategy serves to generate a choice from a choice set for the alternative that provides the individual with the maximum utility.

Traffic assignment models can be classified into several major categories: static or dynamic, and deterministic or stochastic. Static assignment models assume that traffic is in a steady state, link volumes are time-invariant, the time to traverse a link depends only on the number of vehicles on that link, and that the vehicle queues are stacked vertically and do not traverse to the upstream links in the networks.

There are two different approaches for determining steady-state flows. The system optimal (SO) approach, which minimizes the total system travel time over the planning horizon, and the user equilibrium (UE) approach, which seeks user path assignments that satisfy the Wardropian UE condition. Since the focus in this thesis user equilibrium only studies related the UE is provided.

Figure 2-2 summarizes the different models developed in traffic assignment and solution methods designed to solve these models. Similarly, Figure 2-3 and Figure 2-4 depict the historical timeline of static and dynamic traffic assignment models.

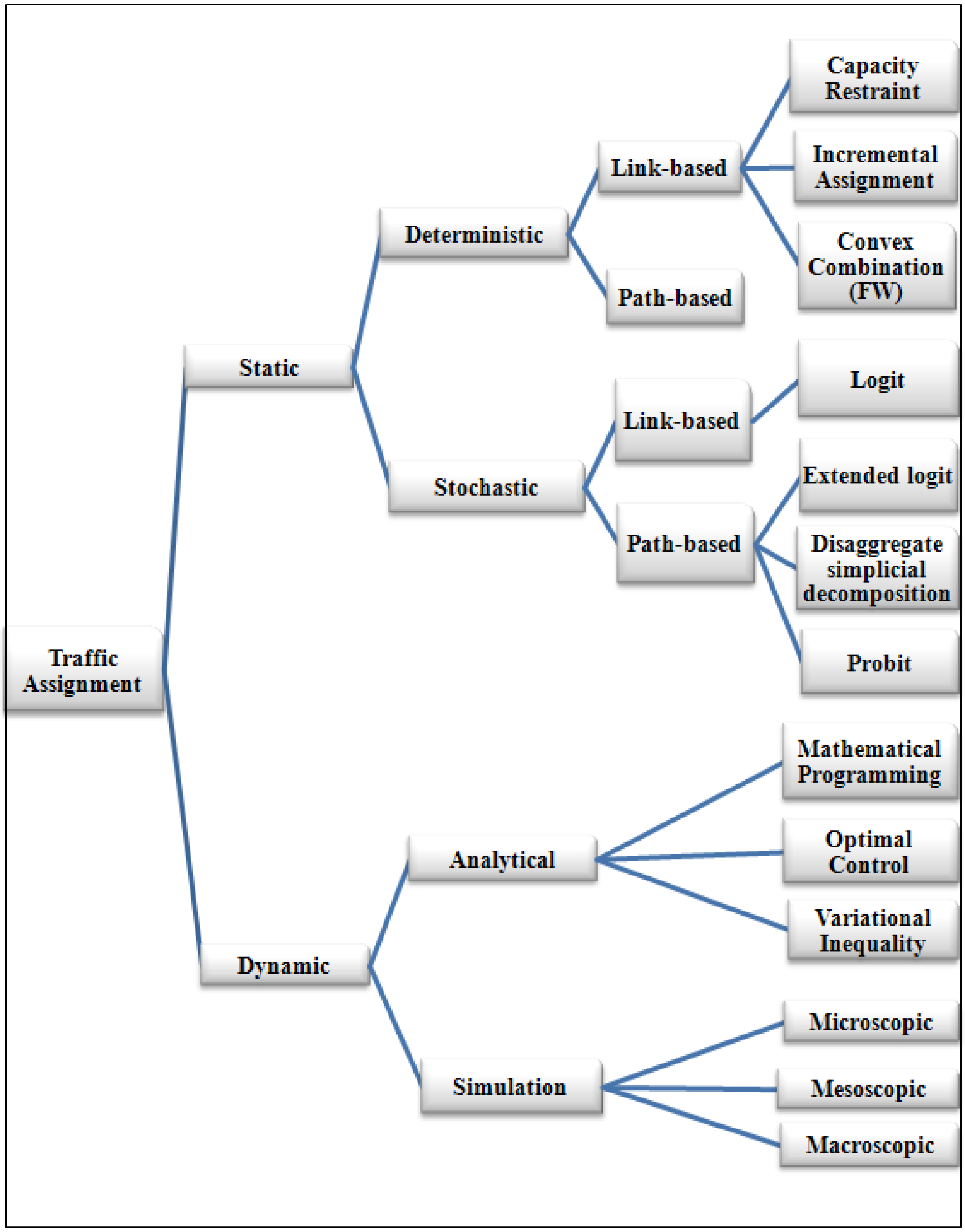


Figure 2-2 Traffic assignment models

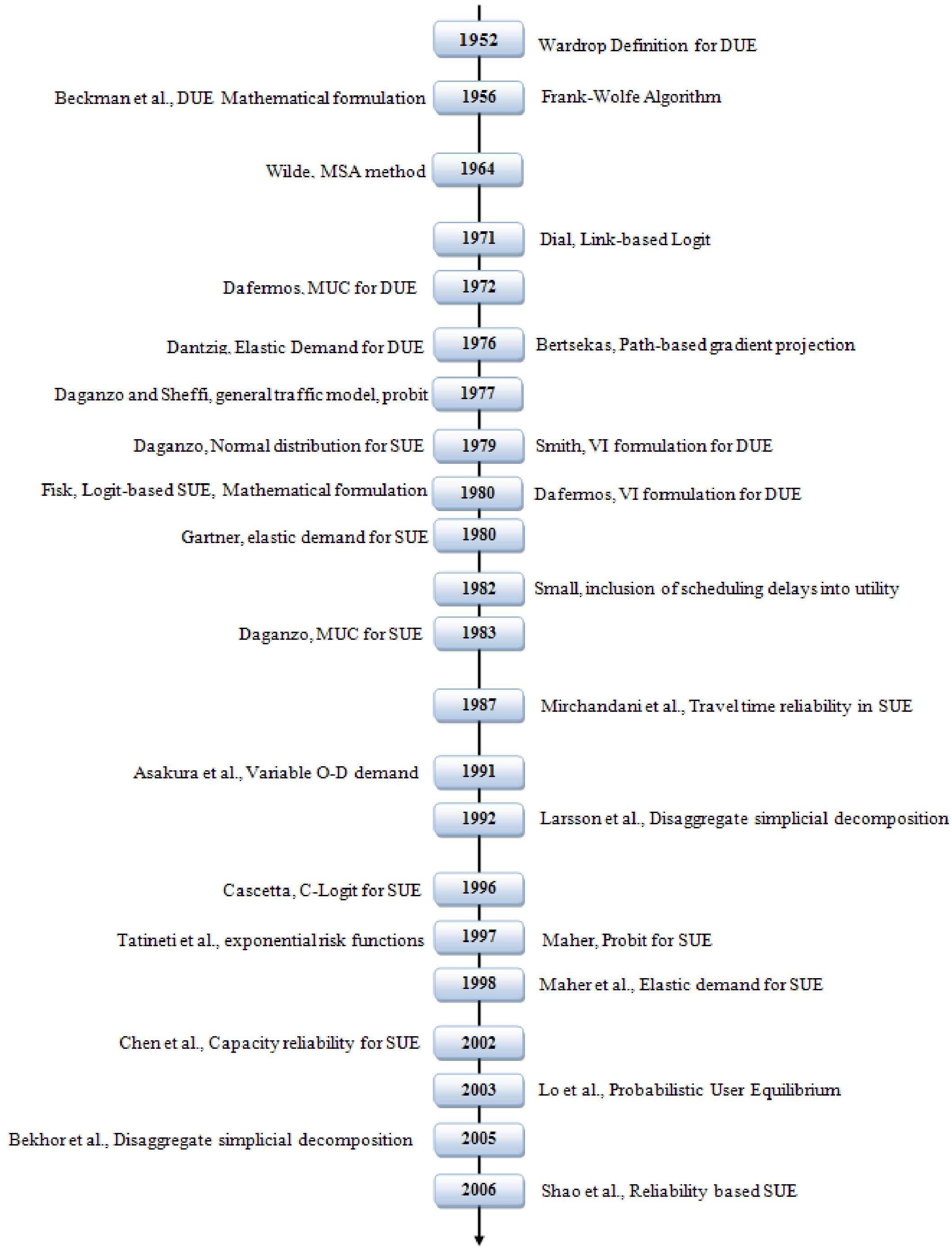


Figure 2-3 Historical timeline for Static Traffic Assignment

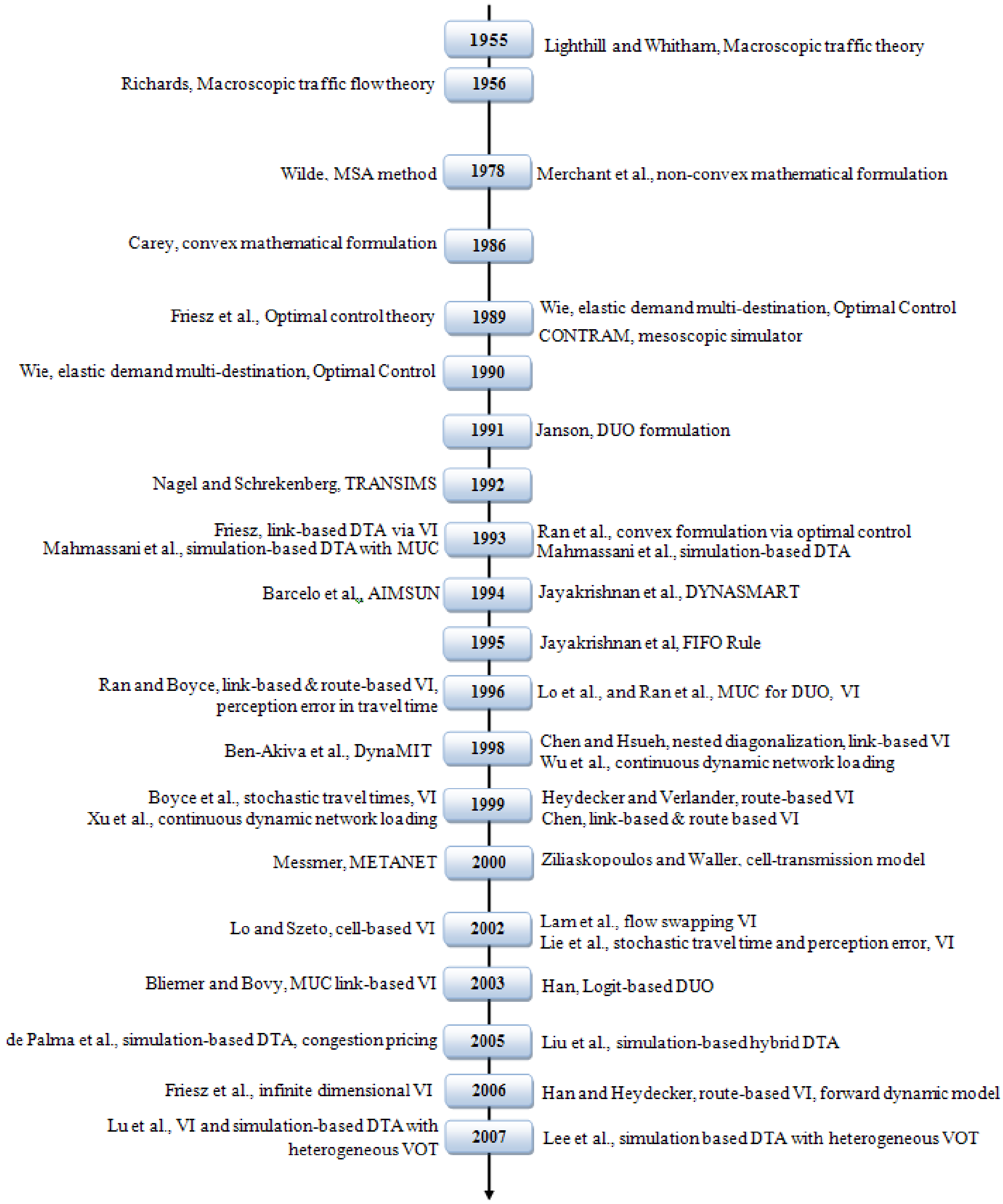


Figure 2-4 Historical timeline for Dynamic Traffic Assignment

2.3 Static Traffic Assignment

2.3.1 Deterministic User Equilibrium

The solution of the user equilibrium problem is based on the behavioral assumption that each traveler on the path that minimizes the travel time from origin to destination. This choice rule implies that at equilibrium the travel times on all used paths connecting any given O-D pair will be equal, and the travel time on all of these used paths will be less than or equal to the travel time on any of the unused paths (Sheffi, 1985). This point is defined as the user-equilibrium (Wardrop, 1952); i.e. no traveler can experience a lower travel time by unilaterally changing routes. Each traveler minimizes his/her own travel time or cost. Wardrop's principle states that all travelers are assigned to a shortest path between their corresponding O-D's and that travel times and volumes are consistent with each other everywhere on the network. Later, Beckmann et al. (1956) formulated the UE problem as a mathematical program, and proved the equivalency, existence and uniqueness of the solution.

The mathematical programming formulation of the UE problem can be presented as (Beckman et al., 1956):

$$\min z(x) = \sum_a \int_0^{x_a} t_a(w)dw \tag{2.1.a}$$

subject to

$$\sum_k f_k^{rs} = q_{rs} \qquad \forall r,s \tag{2.1.b}$$

$$f_k^{rs} \geq 0 \qquad \forall k,r,s \tag{2.1.c}$$

The definitional constraints

$$x_a = \sum_r \sum_s \sum_k f_k^{rs} \delta_{a,k}^{rs} \quad \forall a \tag{2.1.d}$$

where;

r: Origin index $r \in \mathcal{R}$

s: Destination index $s \in \mathcal{S}$

a: Link index $a \in \mathcal{A}$

k: Path index $k \in \mathcal{K}_{rs}$

x_a: Flow on link *a*

q_{rs}: Trip rate between origin *r* and destination *s*

f_k^{rs}: Flow on path *k* connecting origin *r* and destination *s*

$\delta_{a,k}^{rs} = \begin{cases} 1 \; if \; link \; a \; is \; apart \; of \; path \; k \; connecting \; O-D \; pair \; r-s \\ 0 \; otherwise \end{cases}$

$t_a(.)$: Link performance (travel time function) on link *a*

In this formulation, the objective function is the sum of integrals of the link performance functions. First constraint (2.1.b) represents the set of flow conservation constraints, stating that the flow on all paths connecting each O-D pair has to be equal to the O-D trip rate. The nonnegativity condition in equation (2.1.c) ensures that the solution of the program will be physically meaningful. And the definitional constraint (2.1.d) means that the flow on each link is the sum of the flows on all paths going through that link.

For illustration purposes, consider the network depicted in Figure 2-5. This network includes two paths connecting origin *r* and destination *s*. The link performance functions for the two links are given by:

$$t_1 = 2 + x_1 \tag{2.2.a}$$

$$t_2 = 1 + 2x_2 \tag{2.2.b}$$

The O-D flow, q, is 5 units of flow:

$$x_1 + x_2 = 5 \tag{2.2.c}$$

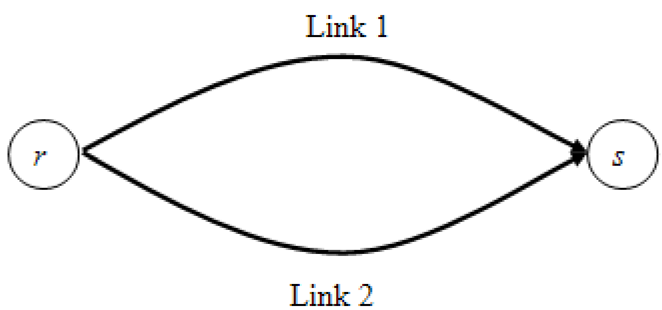


Figure 2-5 User equilibrium example network

The problem can be formulated mathematically as follows:

$$\min z(x) = \int_0^{x_1} (2 + w)dw + \int_0^{x_2=5-x_1} (1 + 2w)dw$$

subject to

$$x_1 + x_2 = 5$$

$$x_1, x_2 \geq 0$$

This problem attains its minimum at $x_1^* = 3$ and $x_2^* = 2$.

2.3.2 Solution Approaches to Static User Equilibrium Problem

Solution algorithms to user equilibrium problem can be categorized as link-based, path-based, and origin based.

Link-based algorithms include heuristic and optimization approaches. Heuristic approaches include capacity restraint methods and incremental assignment techniques. In all these algorithms, the key step is the network loading mechanism; i.e. assignment of the O-D trip rates to the network for specific link travel times. The network loading mechanism used in all the algorithms assigns each O-D flow to the shortest travel time path connecting the corresponding O-D pair; i.e. all-or-nothing assignment. This assignment method does not recognize the dependence between flows and travel time; thus ignores the equilibrium problem all together (Sheffi, 1985).

2.3.3 Link-based Approaches

2.3.3.1 Capacity Restraint Method

This method involves a repetitive all-or-nothing assignment in which the travel times resulting from the previous assignment are used in the current iteration. The initial algorithm does not converge. To remedy this situation, first instead of using the travel time obtained in the previous iteration for the new loading, a combination of the last two travel times obtained is used. Next, if the desired convergence is not obtained, the algorithm is terminated after a given number of iterations *N*. The steps of the capacity restraint algorithm are as follows:

Step 0: *Initialization*. Perform an all-or-nothing assignment based on $t_a^0 = t_0(a)$. Obtain $\{x_a^0\}$. Set *n=1*.

Step 1: *Update*. Set $\tau_a^n = t_a(x_a^{n-1}), \ \forall a$

Step 2: *Smoothing*. Set $t_a^n = 0.75 t_a^{n-1} + \tau_a^n, \ \forall a$

Step 3: *Network Loading*. Perform all-or-nothing assignment based on travel times $\{t_a^n\}$. This yields $\{x_a^n\}$.

Step 4: *Stopping rule*. If *n=N*, go to step 5. Otherwise, set *n=n+1* and go to Step 1.

Step 5: *Averaging*. Set $x_a^* = \frac{1}{4}\sum_{n=0}^{3} x_a^{n-1} \quad \forall a$ and stop. $\{x_a^*\}$ are the link flows at equilibrium.

2.3.3.2 Incremental Assignment Method

Incremental assignment method loads a portion of the O-D matrix at each iteration. The travel times are then updated and an additional portion of the O-D matrix is assigned onto the network. The main steps of the algorithm are as follows:

Step 0: *Preliminaries.* Divide each O-D entry into *N* equal portions, i.e. $q_{rs}^n = q_{rs}/N$. Set *n*=1 and $x_a^0 = 0, \forall a$.

Step 1: *Update*. Set $t_a^n = t_a(x_a^{n-1}), \ \forall a$

Step 2: *Incremental loading*. Perform all-or-nothing assignment based on $\{t_a^n\}$, but using only the trip rates q_{rs}^n for each O-D pair. This yields a flow pattern $\{w_a^n\}$

Step 3: *Flow summation*. $x_a^n = x_a^{n-1} + w_a^n, \forall a$

Step 4: *Stopping rule*. If *n=N*, stop, the current set of link flows is the solution; otherwise, *n=n+1* and go to Step 1.

In conclusion, these heuristic methods either do not converge or produce a set of flows that is not in agreement with the UE criterion. Next subsection provides a way of solving the UE problem via convex combinations method

2.3.3.3 Convex Combinations Method

The Frank-Wolfe (FW) algorithm, also known as the convex combinations algorithm was originally developed by Frank and Wolfe (1956) as a procedure for solving quadratic programming problems with linear constraints. At each step the objective function is linearized and then a step is taken in a direction that reduces the objective while maintaining feasibility.

The steps of the FW algorithm are as follows:

Step 0: *Initialization*. Perform all-or-nothing assignment based on $t_a^0 = t_0(a)\ \forall a$. Obtain $\{x_a^1\}$. Set *n=1*.

Step 1: *Update*. $t_a^n = t_a(x_a^n),\ \ \forall a$

Step 2: *Direction finding*. Perform all-or-nothing assignment based on $\{t_a^n\}$. This yields a set of auxiliary flows $\{y_a^n\}$

Step 3: *Line search*. Find α_n that solves

$$\min_{0\le\alpha\le 1} \sum_a \int_o^{x_a^n+\alpha(y_a^n - x_a^n)} t_a(w)dw$$

Step 4: *Move*. Set $x_a^{n+1} = x_a^n + \alpha_n(y_a^n - x_a^n), \forall a$

Step 5: *Convergence test*. If the convergence criterion is met, stop the current set of link flows is the solution; otherwise, *n=n+1* and go to Step 1.

Different convergence criteria are proposed in the literature. Some of these criteria are:

$$\sum_{rs} \frac{|u_{rs}^{n} - u_{rs}^{n-1}|}{u_{rs}^{n}} \leq \varepsilon \tag{2.3.a}$$

$$\frac{\sqrt{\left(\sum_{a} x_{a}^{n+1} - x_{a}^{n}\right)^{2}}}{\sum_{a} x_{a}^{n}} \leq \varepsilon' \tag{2.3.b}$$

where;

u_{rs}^{n}: Minimum path travel time between O-D pair r-s at the n^{th} iteration.

In solving the UE program over a large network via FW algorithm, each iteration requires a significant computational cost, due to time required to calculate shortest paths in the direction-finding step. Moreover, the convergence speed of this method is very slow. Many algorithms have been proposed to accelerate the speed of the search direction (LeBlanc et al., 1955; Fukushima, 1984; Lee et al., 2001), and to accelerate the step-size (Anders et al., 1985; Gao et al., 2004).

All these algorithms can conveniently and efficiently solve the mathematical problem. However, these methods provide only link traffic flows. In order to obtain the route flows, one has to solve the equations associating all route flows with the link flows produced by them.

2.3.4 Path-based and Origin-based Approaches

Path-based approaches aim to determine the route traffic flows directly. The main path-based algorithms are disaggregated simplicial decomposition algorithm (Larsson et al., 1992), gradient projection algorithm (Bertsekas et al., 1976; Jayakrishnan et al., 1994), O-D based Frank-Wolfe algorithm (Chen et al., 2002), and conjugate gradient projection algorithm (Lee et al., 2003). The main drawback of these algorithms is that, subset of concerned routes has to be stored at each iteration. At the worst case the size of this subset will contain all possible routes of the network at hand.

Origin-based algorithm proposed by Bar-Gera (2002) can provide both link and route traffic flows for the UE traffic assignment. However, similar to path-based approaches this algorithm is computationally expensive. In particular, it requires to enumerate all the routes in the subnetwork, and to determine the last common nodes of all routes. Hence there is no efficient algorithm to find the last common nodes of all routes of the network.

2.4 Deterministic User Equilibrium Extensions

The formulation of the UE problem summarized in previous section assumes that the demand between O-D pairs is fixed and known. However, in reality the demand may be influenced by the level of service on the network; e.g. congestion level (Sheffi, 1985).

In order to consider variable demand, the fixed trip rate q_{rs} between any O-D pair r-s can be assumed to be a function of travel time between r and s. Then;

$$q_{rs} = D_{rs}(u_{rs}) \quad \forall r, s \tag{2.4}$$

where;

$D_{rs}(.)$: The demand function between *r* and s.

Typically, the demand function would include O-D specific parameters, such as population size, income distribution, vehicle ownership, employment levels, or levels of service for different modes. In the following formulations it is assumed that $D_{rs}(.)$ is inversely proportional with travel time.

A straightforward change in the representation of the network can make it possible for the solution of the UE problem via FW convex combinations algorithm. This modification is defined as the zero-cost overflow formulation (Murchland, 1970; Dantzig et al., 1976).

Consider the network depicted in Figure 2-6. The figure shows the modified network in which every O-D pair is augmented to include a "virtual" destination node *r'* and a "zero-cost overflow link *rr'* leading directly from the origin to the new virtual destination node.

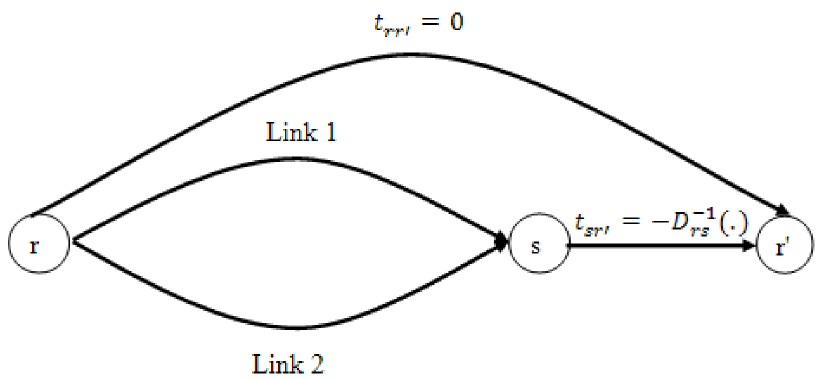


Figure 2-6 The zero-cost overflow network representation

The performance curves for these new links are:

$$t_{sr'} = -D_{rs}^{-1}(.) \quad \forall r, s \tag{2.5a}$$

$$t_{rr'} = 0 \ \ \forall r, s \tag{2.5b}$$

Assume now that a fixed number of trips $\bar{q}_{rs}$ as to be assigned between each O-D pair of the modified network. Then, the equivalent mathematical model for the UE problem with variable demand would be:

$$\min z(\boldsymbol{x}, \boldsymbol{q}) = \sum_a \int_0^{x_a} t_a(w)dw + \sum_{rs} \int_0^{x_{sr'}} t_{sr'}(w)dw + \sum_{rr'} \int_0^{x_{rr'}} t_{rr'}(w)dw \tag{2.6a}$$

subject to

$$\sum_k f_k^{rs} + x_{rr'} = \bar{q}_{rs} \qquad \forall r, s \tag{2.6b}$$

$$f_k^{rs} \geq 0 \qquad \forall k, r, s \tag{2.6c}$$

$$x_{rr'} \geq 0 \qquad \forall r, s \tag{2.6d}$$

In this formulation, $t_{sr'} = -D_{rs}^{-1}(.)$, $x_{sr'} = q_{rs}$, and $t_{rr'} = 0$. The excess demand, $x_{rr'} = \bar{q}_{rs} - q_{rs}$ overflows onto the virtual link *rr'*. Consequently the problem provided above is identical to the original UE formulation with fixed demand. Thus problem (2.6) can easily be solved via original FW convex combinations algorithm.

Dafermos (1971, 1972) introduced the concept of multi user classes into traffic assignment. The author proposed that all classes should be assigned to the network in such a way that at equilibrium no one in any class would improve his/ her travel cost by unilaterally changing his/her route. This proposition is an extension to Wardrop's single class UE principle. Later, Wynter (1995) proposed extensions and improvements to the theory of multiclass UE.

UE with asymmetric cost-flow functions and elastic demand has been first formulated as a variational inequality (VI) problem by Smith (1979) and Dafermos (1980).

The UE definition implies that travelers have full information about all possible routes, they consistently make the correct decisions regarding route choice, and all individuals are identical in their behavior (Sheffi, 1985). These assumptions can be relaxed by making a distinction between travel time that the travelers perceive and the actual travel time. The perceived travel time can be defined as a random variable distributed across the population of travelers. UE is then reached when no travelers believe that his/her travel time can be improved by unilaterally changing routes. This equilibrium point is defined as the stochastic user equilibrium (SUE) condition. Next section provides a detailed literature review of SUE models.

2.5 Stochastic User Equilibrium

The deterministic traffic assignment assumes that travelers choose the minimum cost or travel time path from their origin to destination. Moreover, this approach assumes that travelers have full information about all possible routes, they consistently make the correct decisions regarding route choice, and all individuals are identical in their behavior (Sheffi, 1985). Stochastic user equilibrium (SUE) relaxes these assumptions by including a random component in travelers' perception of travel time.

SUE approach, first proposed by Daganzo and Sheffi (1977) generalizes the all-or-nothing network loading mechanism via discrete choice models. To apply these models, the probability distribution function of the perceived travel time on each path has to be known to calculate the path choice probability. This requires determination of route choice sets connecting the O-D pairs. The review of different approaches and proposed model for the route choice set generation will be explained in the preceding sections.

Depending on the followed route choice model, different SUE problem can be generated. In the literature, the most common models are logit and probit-based SUE solutions.

2.5.1 Logit –based SUE formulation

Logit based SUE modeling was first proposed by Fisk (1980). Summary of different logit-based algorithms can be found in a study by Mather (1998).

In logit-based SUE formulation the utility of the k^{th} path between origin r and destination s, $V_k^{rs}$is:

$$V_k^{rs} = -\beta c_k^{rs} + \varepsilon_k^{rs} \tag{2.7}$$

where;

c_k^{rs}: Measured travel cost

β: Positive parameter

ε_k^{rs}: Random error term

The probability of route choice is represented by;

$$P_k^{rs} = \frac{e^{-\beta c_k^{rs}}}{\sum_l e^{-\beta c_l^{rs}}} \quad \forall k, \forall r, s \tag{2.8}$$

Logit based SUE can be formulated mathematically as follows (Fisk, 1980):

$$\min Z(f) = \frac{1}{\beta} \sum_{r,s} \sum_k f_k^{rs} log f_k^{rs} + \sum_a \int_0^{x_a} t_a(w)dw \tag{2.9a}$$

subject to

$$\sum_k f_k^{rs} = q_{rs} \qquad \forall r, s \tag{2.9b}$$

$$f_k^{rs} \geq 0 \qquad \forall k, r, s \tag{2.9c}$$

$$x_a = \sum_{r,s} \sum_k f_k^{rs} \delta_{a,k}^{rs} \quad \forall a \tag{2.9d}$$

An equivalent formulation is presented by Sheffi (1985):

$$\min Z(x) = -\sum_{rs} q_{rs} E[\min_k\{C_k^{rs}\}] \sum_a x_a t_a(x_a) - \sum_a \int_0^{x_a} t_a(w)dw \tag{2.10}$$

For the logit model, the satisfaction function becomes:

$$E[\min_k\{C_k^{rs}\}] = -\frac{1}{\beta} ln \sum_k \exp\left(-\beta C_k^{rs}\right) \tag{2.11}$$

For $\beta < \infty$ the objective function Z is strictly convex, hence equilibrium route flows are unique. The parameter β may be interpreted as a measure of travelers' sensitivity to the route travel times, or the degree of information that is available to a traveler. In the limit when $\beta \rightarrow \infty$ the UE solution is obtained.

Since the number of routes is very large in general, most of the algorithms proposed for this problem have mainly been based on generating the optimal link flows instead of optimal route flows; i.e. paths are generated implicitly.

Dial (1971) proposed a logit route choice model, where choice probabilities are not assigned to all paths connecting each O-D pair, but to a subset called "reasonable" paths. Dial's method does not require path enumeration; instead it operates on the set of efficient paths which include only those links taking travelers away from their origins and to their destinations. However, in congested networks, since the efficiency of a path would depend on the level of service, stability problems with the iterative SUE assignment might rise (Bell, 1995).

Recently, Russo et al. (2003) proposed an implicit algorithm through a specification of C-Logit choice model (Cascetta et al., 1996) based on the Dial structure (Dial, 1971). The modified algorithm avoids the IIA problem associated with logit models, by simulating the overlapping effect among alternative paths, and eliminates the explicit path enumeration.

With the methodological improvements in route choice set generation models, path-based approaches, i.e. explicit path enumeration techniques were started to gain interest. In the Method of Successive Averages (MSA) (Wilde, 1964; Fisk, 1980; Sheffi

1985) link performance costs are simulated iteratively, and the resulting shortest route flows are weighted with the current flow solution. Since in MSA, efficient paths alter from iteration to iteration, it is not possible to guarantee convergence via this method.

Cascetta et al. (1996, 1997) incorporated the labeling algorithm proposed by Ben-Akiva et al. (1984) in solving the SUE problem. The authors proposed a C-logit formulation

Later Bell et al. (1993) proposed an iterative balancing procedure embedded in the route generation scheme. Similarly, Huang (1995) developed the set of paths in a preliminary phase, where the paths found via Dial's approach are combined with the ones obtained through standard UE. Larsson and Patriksson (1992) proposed a disaggregate simplicial decomposition algorithm to find the optimal route flows in SUE. Later, Damberg et al. (1996) improved this algorithm. The proposed approach approximately solves the SUE problem, given a subset of the complete set of routes. The summary of the steps of the proposed algorithm is as follows:

Step 0: *Initialization*. Compute initial set of routes for all O-D pairs *r-s*, and the initial route flows $\{f^0\}$. Set *n=0*.

Step 1: *Restricted master problem phase*. Set *i=0,* and $\bar{f}^i = f^n$. Repeat

1. Compute route costs $(c_k^{rs})^i = c_k^{rs}(\bar{f}^i)\ \forall k, \forall r, s$
2. Compute the auxiliary route flow $\underline{f}^i$ according to the formula

$$\left(\underline{f}_k^{rs}\right)^i = q_{rs}\frac{e^{-\beta(c_k^{rs})^i}}{\sum_l e^{-\beta(c_l^{rs})^i}}\ \ \forall k, \forall r, s$$

3. Terminate if $\underline{f}^i = \bar{f}^i$

4. Solve the line search problem
5. $t_i = \arg\min_{t\in[0,1]} Z(t) \stackrel{\text{def}}{=} Z[\bar{f}^i + t(\underline{f}^i - \bar{f}^i)]$

Let the new point be $\bar{f}^{i+1} = \bar{f}^i + t_i(\underline{f}^i - \bar{f}^i)$, *i*=*i*+*1*, and $\overline{f} = \bar{f}^i$ be the output

Step 2: *Column generation phase*. Generate a set of routes and augment the sets. Terminate if no new routes are found.

Let $f^{n+1} = \overline{f}$, *n*=*n*+*1*, and go to Step 1.

Bekhor et al. (2005) developed a path-based SUE algorithm using disaggregate simplicial decomposition algorithm and gradient projection on the linear manifold of active constraints.

Moreover, Mussone et al. (2005) proposed a deterministic UE algorithm based on Ant Colony Optimization, and compared with the original FW algorithm. Later, D'Acierno et al. (2006) improved this approach via introducing stochasticity. The authors showed that the proposed algorithm has an MSA framework, with less computation times and same accuracy as the traditional MSA algorithms.

2.5.2 Probit –based SUE formulation

The main drawbacks of logit-based formulation are its inability to take account of overlapping or correlated paths, and inability to account for perception variance with respect to trips of different lengths. Probit-based methods, on the other hand, do not suffer from these kinds of weaknesses. However, since these models cannot be represented in a closed form formulation, they require Monte Carlo techniques or of complete path enumeration and numerical integration of multivariate Normal distribution (Daganzo, 1979). A summary of this methodology can be found in Maher et al. (1997). Recently, Connors et al. (2007) provides an SUE formulation for multiple user classes and variable demand via probit model.

2.5.3 Extended Logit-based SUE formulation

With the recent improvements in logit models, several modifications and generalizations have been proposed to relax the IID assumption in the logit model. These extended logit models include C-logit (Cascetta et al., 1996), path-size logit (Ben-Akiva and Bierlaire, 1999; Ramming, 2002; Frejinger and Bierlaire; 2007), cross-nested logit (Prashker and Bekhor, 1998; Vovsha and Bekhor, 1998; Bierlaire, 2006), paired combinatorial logit (Bekhor and Prashker, 1999; Gliebe et al., 1999; Prashker and Bekhor, 2000), generalized nested logit (Bekhor and Prashker, 2001), logit kernel (Bekhor et al., 2002; Paag et al., 2004), and link-based path-multilevel Logit (Marzano and Papola, 2004).

2.5.4 SUE formulation – Extensions

The SUE models described in the previous section assume travelers with homogeneous characteristics with fixed demand. Thus, inclusion of multiple user classes (MUC) and elastic demand are two important improvements to the original SUE solution.

MUC considers heterogeneous population of travelers, where travelers within each user class have homogeneous properties. Each user class is then modeled in equilibrium. MUC framework was first proposed by Daganzo (1982, 1983). The author considered *M* user classes, each with its O-D matrix and network costs. The standardized link flow v_a is obtained as:

$$v_a = \sum_M \alpha^m x_a^m \tag{2.12}$$

The parameter α^m in the above equation refers to the coefficient to standardize the flow of vehicles of the user class m, and x_a^m is the flow on link a for user class m. similarly the cost on link *a* for user class m, c_a^m, is defined as:

$$c_a^m = c_{0a}^m + \theta^m b_a(v_a) \tag{2.13}$$

In this formulation, $c_{0a}^m$is the fixed part of the cost, θ^m is the user class specific coefficient, and $b_a(v_a)$ is an increasing function common to all user classes. Daganzo (1982) showed the minimum of MUC SUE problem is unique, and solved the equilibrium problem via MSA in the probit case. Later, several researches used this formulation to formulate the impacts of traveler information on the SUE problem (Van Vuren et al., 1991; Maher et al., 1996).

Elastic demand, which allows the O-D trips to vary with the network conditions, has been studied via bi-level programming approach. In this approach, two separate objective functions are considered; one for SUE and one for elastic demand (Maher and Hughes, 1998). The authors proposed feasible descent direction algorithms that move iteratively along separate search directions for the network flows and the demand. Later, Maher et al. (1999) proposed a Balance Demand Algorithm which uses a single search direction, and showed that when demand and flow is balanced, the model reduces to regular SUE problem. In this formulation the link flows are represented as:

$$v_a = \sum_M \alpha^m \sum_{rsk} q_{rs}^m\big(c^m(v)\big)\, P_{rsk}^m\big(c^m(v)\big) \quad \forall a \tag{2.14}$$

2.5.5 Travel Time Reliability

In SUE problem, the aim is to minimize the perceived travel time of the travelers. In this approach, network uncertainty is ignored, all users are assumed to be risk neutral, and mean travel times are considered in the route choice. In reality, travel times are distributed probabilistically. These variations in the network travel times can be due to differences in driver reactions under different conditions, accidents in the network, or differences in the delays experienced by different vehicles. Thus, variability in travel time introduces uncertainty to the transportation system, which prevents travelers to know their exact arrival to their destination. Asakura and Kashiwadani (1991) defines the travel time reliability as the probability that a traveler can arrive at the destination within a given travel time threshold. This uncertainty in travel time can be included as risk to the traveler making the trip. Depending on the interpretation of risk, travelers can be risk averse or risk seeking. Risk averse travelers, among travel time distributions with equal expectations, often choose the route with smaller variability. On the other hand, risk seeking travelers, among travel time distributions with equal expectations, often choose the route with the larger variability. Travel time variability can be divided into two categories:

1. Non-recurrent variability: This kind of variability occurs due to changes in the capacity which resulted from incident.
2. Recurrent variability: This kind of variability is generated from the fluctuations in the demand. Travel demand varies between times of day, days of the week, and seasons of the year.

Recent empirical studies found that travelers are not only interested in saving travel time but also in reducing the travel time variability (Abdel-Aty et al., 1996; Ghosh, 2001; Kazimi et al., 2000; Lam and Small, 2001; Katsikopoulos et al., 2002). The earliest theoretical contribution to travel time variability was by Gaver (1968). The author incorporated travel time variability into the utility maximization, and found that travelers select a “slack” time by departing earlier than they would with no travel time variability.

The impact of travel time reliability on the network performance was evaluated via stochastic traffic assignment by several studies, as well (Mirchandani and Soroush 1987; Lo and Tung, 2000; Yin and Ieda, 2001; Gordon et al., 2001, Watling 2002; Chen et al., 2002; Chen et al., 2003; Lo and Tung, 2003; Clark and Watling, 2005).

In the past years, capacity changes and network reliability due to non-recurrent congestion have been investigated by many researchers. In recent years, Chen et al. (2002) proposed the capacity reliability as the probability that the network capacity can accommodate a certain traffic demand. Lo and Tung (2003) developed a probabilistic user equilibrium (PUE) model to account for travel time reliability due to capacity changes.

On the other hand, travel time variability due to demand variations, have been received very little attention (Shao et al., 2006).

Mirchandani and Soroush (1987) introduced probabilistic travel times and variable perceptions into the traffic equilibrium via following formulation:

$$C_k^{rs} = E[U(T_k^{rs})] = -\beta \int U(t_k^{rs}) pdf(t_k^{rs}) dt_k^{rs} \tag{2.15}$$

In this formulation $C_k^{rs} = E[U(T_k^{rs})]$ is the expected utility for traveling on route *k* between *r* and *s*. $U(t_k^{rs})$ utility function describing the risk preference of the traveler on route *k*. T_k^{rs} is the random travel time on route *k*. Similarly, $pdf(t_k^{rs})$ is the probability density function for route *k*.

Mirchandani and Soroush (1987) assume that each traveler has a variable perception error $\varepsilon_i \sim N(\mu_i, \theta_i)$ with $\mu_i \sim N(0, \tau)$ and $\theta_i \sim G(\alpha, \gamma)$. This perception error allows each traveler to experience different travel times for a given set of flows.

In terms of departure time Small (1982) developed a specific utility model for the scheduling choicer:

$$U(C_k^{rs}) = \alpha_1 T_k^{rs} + \alpha_2 SDE_k^{rs} + \alpha_3 SDL_k^{rs} + \alpha_4 DL_k^{rs} \tag{2.16}$$

In this formulation utility, U, is a function of travel time, T_k^{rs}, schedule delay-early, SDE_k^{rs} , schedule delay-late, SDL_k^{rs}, and a fixed penalty for arriving late, DL_k^{rs}. With empirical estimates, Small (1982) found that travelers prefer to arrive early than arriving late. Thus, $\alpha_2 > \alpha_1 > \alpha_3$.

Later, Noland and Small (1995), extended the theory and proposed a simple expected utility model:

$$E[U(C_k^{rs})] = \alpha_1 E(T_k^{rs}) + \alpha_2 E(SDE_k^{rs}) + \alpha_3 E(SDL_k^{rs}) + \alpha_4 PL_k^{rs} \qquad (2.17)$$

In the above equation, $E[U(C_k^{rs})]$ is the expected utility, as a function of expected travel time $E(T_k^{rs})$, expected schedule delay-early, $E(SDE_k^{rs})$, expected schedule delay-late, $E(SDL_k^{rs})$, and probability of arriving late to the destination, PL_k^{rs}.

The calculation of the model parameters in the above equations depend on the selection of the travel time distributions. Giuliano (1989) determined log-normally distributed delays for incidents in the transportation system.

Noland and Small (1995), decomposed travel time into free flow travel time, $(T_f)_k^{rs}$, extra delay due to recurrent congestion, $(T_x)_k^{rs}$, and mean travel time due to non-recurrent congestion, b:

$$cost = \alpha_1\left(\left(T_f\right)_k^{rs} + (T_x)_k^{rs} + b\right) + b\left\{\alpha_2 ln\left[\frac{\alpha_4+b(\alpha_2+\alpha_3)}{b(\alpha_2-\alpha_1\Delta)}\right] - \frac{\alpha_4(\alpha_2-\alpha_1\Delta)}{\alpha_4+b(\alpha_2+\alpha_3)} - \alpha_1\Delta\right\} + \alpha_4(PL_k^{rs})^* \qquad (2.18)$$

Δ is the change in the profile of the recurrent congestion, and $(PL_k^{rs})^*$ is the optimal probability of arriving late:

$$(PL_k^{rs})^* = \frac{b(\alpha_2 - \alpha_1 \Delta)}{\alpha_4 + b(\alpha_2 + \alpha_3)} \tag{2.19}$$

Later Bates et al. (2000) added additional terms to the model of Noland and Small (1995); adherence to schedule early, ASE_k^{rs}, and adherence to schedule late, ASL_k^{rs}.

$$E[U(C_k^{rs})] =$$

$$\alpha_1 E(T_k^{rs}) + \alpha_2 E(SDE_k^{rs}) + \alpha_3 E(SDL_k^{rs}) + \alpha_4 PL_k^{rs} + \alpha_5 E\left[\left(ASE(t_h)\right)_k^{rs}\right] +$$

$$\alpha_6 E\left[\left(ASL(t_h)\right)_k^{rs}\right] + \alpha_7 PS_k^{rs} \tag{2.20}$$

where t_h is the departure time, and PS_k^{rs} is the probability of not adhering to schedule.

Tatineti et al. (1997) and Chen et al. (2003) used an exponential functional form to describe the risk averse and risk prone traveler behavior:

$$Risk\ averse \quad U(T_k^{rs}) = a_1(e^{a_2 T_k} - 1) \tag{2.21a}$$

$$Risk\ prone \quad U(T_k^{rs}) = b_1(1 - e^{b_2 T_k}) \tag{2.21b}$$

where a_1, a_2, b_1, and b_2 are the positive parameters to be estimated. Using the exponential form risk behavior function utility associated with a route can be estimated by summing the link utilities on a given route.

Recently, Clark and Watling (2005) proposed an analytical model to estimate the probability distribution of the total network travel time under day-to-day variations in travel demand. However, the authors focused mostly on total network travel time distribution and paid little attention to the travelers' route choice problems under demand variations.

Shao et al. (2006) defined a reliability-based stochastic user equilibrium (RSUE) model. This formulation deduces travel time reliability from daily demand variations including travelers' perception errors in their route choice decisions. The problem was then solved via Variational Inequality (VI) model formulation.

In order to illustrate the idea behind RSUE, a two-link simple network with one O-D pair is considered (Figure 2-7). The travel time functions of these two links are provided in Figure 2-7.

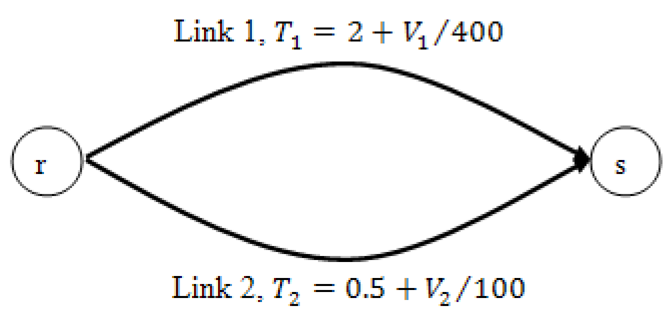


Figure 2-7 Example network for RSUE problem

The O-D travel demand is assumed to be random and follow a normal distribution (Asakura and Kashiwadani, 1991; Chen et al., 2003):

$$Q \sim N\left(q, \left(\sigma_q\right)^2\right) = N(275, (137.5)^2) \tag{2.22}$$

where q is the mean travel demand, and $\left(\sigma_q\right)^2$ is the variance of the demand.

Assuming that route flows follow the same type of distribution as the travel demand, coefficient of variation of the route flow is equal to that of the O-D demand, and route flows are mutually independent, the route flows can be formulated as:

$$V_1 \sim N(\vartheta_1, (\sigma_\vartheta^1)^2) = N(\vartheta_1, (cv \times \vartheta_1)^2) \tag{2.23a}$$

$$V_2 \sim N\left(\vartheta_2, \left(\sigma_\vartheta^2\right)^2\right) = N(\vartheta_2, (cv \times \vartheta_2)^2) \tag{2.23b}$$

where $cv = \sigma_q/q$, ϑ_1 and ϑ_2 are mean route flows, σ_ϑ^1 and σ_ϑ^2

Similarly, the travel time distributions become normal with the following specific forms:

$$T_1 \sim N(t_1, (\sigma_t^1)^2) = N\left(2 + \frac{\vartheta_1}{400}, \frac{(cv \times \vartheta_1)}{400}\right) \tag{2.24a}$$

$$T_2 \sim N(t_2, (\sigma_t^2)^2) = N\left(0.5 + \frac{\vartheta_2}{100}, \frac{(cv \times \vartheta_2)}{100}\right) \tag{2.24b}$$

where t_1 and t_2 are the mean travel times, σ_t^1 and σ_t^2 are the standard deviations of the route travel times.

Due to travel time variability, the travelers could not know the exact time of arrival to the destination point. Thus, they include a safety margin (s_k) to their travel time (Hall, 1983). Then the traveler would try to minimize their effective travel time, i.e.

summation of mean travel time and safety margin ($c_k = t_k + s_k$), by choosing a route with travel time not less than the desired confidence level, ρ:

$$\min_{s_k} c_k = t_k + s_k \tag{2.25a}$$

subject to

$$\int_0^{t_k+s_k} \frac{1}{\sqrt{2\pi}\sigma_t^k} exp\left(-\frac{(x-t_k)^2}{2\left(\sigma_t^k\right)^2}\right) dx \geq \rho = 0.95 \tag{2.25b}$$

where;

$s_k = \sigma_t^k \phi^{-1}(0.95)$

$c_k = t_k + \sigma_t^k \phi^{-1}(0.95)$

Assuming that travelers' perception errors of route effective travel time follow IID Gumbel distributions with mean zero and route standard deviation, the route flows becomes:

$$\vartheta_a = q \frac{exp(-\beta c_k)}{exp(-\beta c_1) + exp(-\beta c_2)} \tag{2.26}$$

2.6 Dynamic Traffic Assignment

Conventional static traffic assignment models assume a fixed point steady-state equilibrium condition, where individual travelers have no incentive to change their decisions. Travelers are assumed to choose the alternative that maximizes their utility. Unfortunately, in reality traffic is a dynamic process, and these models fail to capture the dynamic nature of the traffic. Thus, in the past years, the traffic assignment models started to progress from static to dynamic. Dynamic models can successfully represent the time-varying nature of the congestion during different times of the day, help to understand travelers' responses to time-varying transportation system policies (e.g. congestion pricing) including departure-time choice, pre-trip route choice, and en-route response to traffic information.

The commonly adopted approach for dynamic travel choice is the dynamic extension of Wardrop's (1952) principle called the Dynamic user optimal (DUO), or its stochastic extension stochastic dynamic user optimal (SDUO) (Ran and Boyce, 1996)

DUO with multiple origins and destinations is a nonlinear programming problem with nonlinear constraints. Dynamic traffic assignment (DTA) is not a solution algorithm for DUO, but was designed to produce assignments that approximate the optimality conditions of DUO.

Fixed-demand DTA model consists of route choice model, and dynamic network loading model. Dynamic network loading provides the propagation of the route flows through the network, On the other hand, in the elastic-demand case, departure time choice needs to be modeled, as well.

Capturing the actual traffic behavior is one of the most crucial requirements of DTA models. Past efforts have focused on the following requirements of the traffic behavior:

1. Flow propagation
2. Flow conservation

3. First-in-first-out (FIFO)
4. Causality
5. Queue Spillback

A comprehensive overview of these requirements is provided in Mun (2001) and Carey (2004). Following section gives detailed description of these requirements.

2.7 Traffic Flow Requirements for DTA

2.7.1 Flow Propagation

In static traffic assignment, flow is assumed to be constant along each route between an origin and destination. Thus, flow propagation is guaranteed implicitly. On the other hand, in DTA flow should propagate through a link consistent with speed of the vehicles; i.e. the minimum time taken for a vehicle to traverse a link should not be less than the free flow travel time. The flow propagation can be mathematically represented as follows:

$$E(t) = G\big(\tau(t)\big) \tag{2.27}$$

In this formulation $\tau(t)$ refers to the link exit time for a vehicle that entered at time *t*, $E(t)$ is the accumulated inflow at time *t*, and $G\big(\tau(t)\big)$ accumulated outflow at time $\tau(t)$. The inflow, *u*(.), and outflow, *v*(.), rates can be determined by differentiating the above equation with respect to entry time *t*:

$$u(t) = v\big(\tau(t)\big)\dot{\tau}(t) \tag{2.28}$$

2.7.2 Flow Conservation

Flow conservation guarantees that no traveler leaves the network before reaching to their destination, or outflow rate from a link never exceeds the inflow rate to that link. Mathematically,

$$E(t) = G(t) + x(t) \tag{2.29}$$

The outflow rate can be calculated using the flow propagation eqn-2.28:

$$g\big(\tau(t)\big) = \frac{u(t)}{\dot{\tau}(t)} \tag{2.30}$$

2.7.3 FIFO Rule

In static assignment, it is assumed the time taken to traverse a link is the same for all vehicles. Thus, automatically FIFO rule is satisfied. On the other hand, in DTA time varying traffic demand is considered. Hence, a vehicle entered to a link earlier than others is expected to leave that link before those travelers. FIFO rule has been studied by many researchers in the past (Jayakrishnan et al., 1995; Astarita, 1996; Heydecker and Addison, 1998; Tong and Wong, 2000; Huand and Lam, 2002; Carey et al., 2003; Szeto and Lo, 2004). The FIFO rule is equivalent to satisfying the constraint $\dot{\tau}(t) \geq 0$.

2.7.4 Causality

Causality means that a traveler's speed and travel time is affected from the speed of the travelers ahead, but is independent of the any traveler entering to the network in the future (Friesz et al., 1993; Heydecker and Addison, 1998; Carey et al., 2003).

2.7.5 Queue Spillback

Queue spillback refers to the end of queue spilling backwards in the network (Daganzo, 1994, 1995; Tong and Wong, 2000; Kuwahara and Akamatsu, 2001; Szeto and Lo, 2004; Ziliaskopoulos et al., 2004)

2.8 Approaches to DTA

The current approaches to dynamic network modeling include analytical and simulation models. Analytical models provide a sound mathematical structure and solution algorithms (Bliemer, 2001).

Analytical approaches to DTA model can be divided into three categories:

1. Mathematical programming
2. Optimal control theory
3. Variational inequality theory

2.8.1 Analytical DTA

2.8.1.1 Mathematical Programming

In the past years, many formulations and solution algorithms have been proposed to solve DTA models via mathematical programming approach. Although existing DUO traffic assignment models vary in details, the basic models usually extend the static UE assignment by including the time dimension along with a set of additional constraints.

Merchant and Nemhauser (1978a, 1978b) proposed the first models to formulate DTA problem via mathematical programming approach. Their formulation was developed for deterministic networks with fixed-demand and single origin and destination, system optimum (SO) case. The proposed model was a discrete-time, nonlinear non-convex optimization model which was solved by a piecewise linearization.

Carey (1986, 1987, 1992) proved that Merchant and Nemhauser's model satisfies the optimality conditions, reformulated the model as a convex non-linear model, and introduced multiple destinations. However, even the proposed extensions could not handle the non-convexity resulting from FIFO requirements. Similarly, Birge and Ho (1993) extended the original SO dynamic model proposed by Merchant and Nemhauser. The authors developed a multi-stage non-convex stochastic mathematical programming model assuming finite number of scenarios of random realizations. Recently; Ziliaskopoulos (2000) developed SO-DTA model via cell-transmission (Daganzo, 1994). Even though the proposed model is not suitable for real-world applications, it provides some insights on the properties of the DTA model.

Earliest efforts to model DTA via UE approach was by Janson (1991). The proposed approach described the equilibrium in terms of experienced travel times, and formulated the problem as a non-linear mixed integer bi-level programming model. This model can be summarized in a continuous time mathematical model:

$$\min z(x) = \int_t \sum_a \int_{w=0}^{x_a(t)} t_a(w) dw dt \tag{2.31a}$$

subject to

$$\sum_k f_k^{rs}(t) = q_{rs}(t) \qquad \forall r, s,\ \forall t \tag{2.31b}$$

$$f_k^{rs}(t) \geq 0 \qquad \forall k, r, s, \forall t \tag{2.31c}$$

$$x_a(t) = \int_s \sum_r \sum_s \sum_k f_k^{rs} \delta_{a,k}^{rs}(s) ds \quad \forall a, \forall t \tag{2.31d}$$

where $x_a(t)$ is the time-varying link flow, $q_{rs}(t)$ is the demand for travel between O-D pair *r-s* at time *t*, and $\delta_{a,k}^{rs}(s)$ is 1 if traffic on route *k* departing at time *s* is present on link *a* at time *t*, and 0 otherwise.

The upper level of the model solved a multi-interval, time-varying demand traffic assignment, while the lower level determined the reachability of each node in each time interval. These two sub-problems were solved interdependently until the convergence was satisfied. The solution to Janson's method, also known as the predictive-user equilibrium, suffers from serious violation of traffic flow requirements, such as FIFO rule. This type of mathematical formulation has been adopted in many studies including Jayakrishnan et al. (1995), and Ron and Boyce (1996). However, Lin and Lo (2000) showed that this type of generalization might not converge or approximate the UE defined for the static case, and might violate the FIFO rule.

Wu et al. (1998) formulated the continuous dynamic network loading problem as a system of functional equations. Based on the proposed system, authors showed that FIFO rule was respected under reasonable assumption. The finite dimensional system of equations was developed to approximate the model as an optimization problem. A similar mathematical formulation was proposed by Xu et al. (1999), where the continuous dynamic network problem with nonlinear travel delays was formulated via mathematical model.

Recently Han (2003) and Lim and Heydecker (2005) proposed a logit-based dynamic SUE model. In the solution algorithm, using Dial's (1971) concept of reasonable path, the authors eliminated the unreasonable paths. Moreover, it the algorithm allowed for the cost of travel at a certain time to be affected by the flows that enter the network from other origins at later times.

The substantial research on DTA via mathematical programming approach still suffers from mathematical limitations of non-convexity due to FIFO requirement.

2.8.2 Optimal Control

Optimal control theory was first proposed by Friesz et al. (1989) to model dynamic traffic networks. The authors discussed link-based optimal control formulations for both SO and UE objectives for the single destination case. Wie et al. (1989, 1990) extended this formulation and included multi-destinations and elastic-time varying travel demand.

Ran et al. (1993) proposed a convex model for instantaneous DUO problem via optimal control theory. Recognizing the inability of usual cost functions to account for dynamic queuing, the authors splitted the link travel cost into moving and queuing components.

Although this approach provides an explicit representation of outflows and link flows, it does not always satisfy FIFO requirement. Moreover, the strong assumptions based on instantaneous travel time formulation usually result in unrealistic behavioral processes.

Because of these limitations of optimal control theory, in recent years, VI formulation has gained increasing attention among researchers. Compared with mathematical programming and optimal control theory models provide a more attractive approach to formulate dynamic traffic assignment.

2.8.3 Variational Inequality

VI supplies a general framework for several classes of DTA formulation such as optimization, fixed point, and complementarity. Nagurney (1998) and Chen (1999) provide a detailed description of VI, and explain various equilibrium problems.

Based on the requirements of DTA, the constraint set for a typical VI formulation can be summarized as follows:

1. *Relationship between link status and link flow variables:* The number of vehicles on a link a at time t can be calculated via inflow and outflow rates on that particular link:

$$\frac{dx_{ak}^{rs}}{dt} = u_{ak}^{rs}(t) - v_{ak}^{rs}(t) \quad \forall a, k, r, s \tag{2.32}$$

where:

x_{ak}^{rs}: Number of vehicles on link *a* at time *t*

$u(t)$: Inflow rate of link *a* on route *k* between O-D pair *r-s* at time *t*

$v_{ak}^{rs}(t)$: Outflow rate link *a* on route *k* between O-D pair *r-s* at time *t*

Similarly, the cumulative number of travelers arriving at destination *s* from origin *r* on route *k* at time *t*, $E_k^{rs}(t)$, can be used to calculate the arrival flow rate at destination *s* from origin *r* on route *k* at time *t*:

$$\frac{dE_k^{rs}}{dt} = e_k^{rs}(t) \qquad \forall k, r, s \neq r \tag{2.33}$$

2. *Flow conservation constraints:* The flow conservation requirement states that the departure flow rate from origin *r* toward destination *s* at time *t*, $d^{rs}(t)$ is equal to the sum of all inflow rates to all paths *k* and links *a* corresponding to that O-D pair:

$$d^{rs}(t) = \sum_a \sum_k u_{ak}^{rs}(t) \tag{2.34}$$

Similarly, the sum of inflow rate to the set of links outgoing from node *j*, $B(j)$, is equal to the sum of outflow rate from the set of links incoming to the same node *j*, $A(j)$:

$$\sum_{a \in B(j)} v_{ak}^{rs}(t) = \sum_{a \in A(j)} u_{ak}^{rs}(t) \tag{2.35}$$

$$\sum_a \sum_k v_{ak}^{rs}(t) = e_k^{rs}(t) \tag{2.36}$$

3. *Flow propagation constraints:* The flow propagation constraint ensures that the flow on link *a* at time *t* due to flow on route *k* between O-D pair *r-s*, x_{ak}^{rs}, is equal to:

$$x_{ak}^{rs} = \sum_{b \in \tilde{k}} \{x_{bk}^{rs}[t + \tau_a(t)] - x_{bk}^{rs}(t)\} + \{E_k^{rs}[t + \tau_a(t)] - E_k^{rs}(t)\} \tag{2.37}$$

$$\forall a \in B(j); j \neq r; k, r, s$$

In this formulation $\tilde{k}$ refers to the subroute from node j to destination s, and $\tau_a(t)$ is the actual travel time over link a for flows entering link a at time t.

4. Definitional Constraints:

$$\sum_{rsk} u_{ak}^{rs}(t) = u_a(t) \quad \forall a \tag{2.38}$$

$$\sum_{rsk} v_{ak}^{rs}(t) = v_a(t) \quad \forall a \tag{2.39}$$

$$\sum_{rsk} x_{ak}^{rs}(t) = x_a(t) \quad \forall a \tag{2.40}$$

5. Nonnegativity Constraints:

$$x_{ak}^{rs}(t), v_{ak}^{rs}(t), u_{ak}^{rs}(t) \geq 0 \ \ \forall a, k, r, s \tag{2.41}$$

$$e_k^{rs},\ E_k^{rs} \ \geq 0 \ \ \forall k, r, s$$

6. Boundary Constraint:

$$E_k^{rs}(0) = 0 \quad \forall k, r, s \tag{2.42a}$$

$$x_{ak}^{rs}(0) = 0 \quad \forall a, k, r, s \tag{2.42b}$$

2.8.3.1 Model Formulation

The models developed to solve DUO problem via VI formulation are either link-based or route-based. Route-based discrete-time VI formulation was first proposed by Wie et al. (1995) for simultaneous route/departure time problem. The authors showed that the solution existed under certain regularity conditions. The problem was solved exit flow functions instead of exit time functions as proposed by Friesz et al. (1993). Since the formulation is route-based it required complete path enumeration.

In route-based models, the complementary inequality in DUO can be written mathematically as follows (Beckmann et al., 1956):

$$f_k^{rs}(t) = \begin{cases} > 0 \ \Rightarrow C_k^{rs}(t) = C_{rs}^*(t) \\ = 0 \ \Rightarrow C_k^{rs}(t) \geq C_{rs}^*(t) \end{cases} \quad \forall k,\ \forall rs,\ \forall t \tag{2.43}$$

where $f_k^{rs}(t)$ is the instantaneous flow entering route k at time t, $C_k^{rs}(t)$ is the cost incurred on route k starting at time t, and $C_{rs}^*(t)$ is the minimum travel cost from origin r to destination s starting at time t. Smith (1979) transformed this complementary inequality into VI form with demand feasible route flow vectors. This formulation can accommodate both separable and non-separable cost functions, and was first adopted by Friesz et al. (1993):

$$f_k^{rs}(t) \geq 0 \qquad \forall rs, \quad \forall t \tag{2.44a}$$

$$\sum_k f_k^{rs}(t) = q^{rs} \ \ \forall rs, \quad \forall t \tag{2.44b}$$

Ran and Boyce (1996) showed that solving in a route-based DUO route choice state is in equilibrium if and only if it satisfies the following VI formulation:

$$\int_0^T \sum_{rs} \sum_a \eta_k^{rs*}(t)\, [f_k^{rs}(t) - f_k^{rs*}(t)]dt \geq 0 \tag{2.45}$$

where:

$\eta_k^{rs*}(t)$: Route travel time for path k between r and s at optimal conditions

The route travel time for each path was calculated via following recursive formulation (Ran and Boyce, 1996):

$$\eta_k^{ri}(t) = \eta_k^{r(i-1)}(t) + \tau_a\left[t + \eta_k^{r(i-1)}\right] \quad \forall k, r, i; i = 1,2, \dots s; a = (i-1, i) \quad (2.46)$$

According to Ran and Boyce (1996) at equilibrium the following conditions are satisfied:

$$\eta_k^{rs*}(t) - \pi^{rs}(t) \geq 0 \quad \forall k, r, s \quad (2.47)$$

$$f_k^{rs*}(t)[\eta_k^{rs*}(t) - \pi^{rs}(t)] = 0 \quad \forall k, r, s \quad (2.48)$$

$$f_k^{rs*} \geq 0 \quad \forall k, r, s \quad (2.49)$$

In this formulation $\pi^{rs}(t)$ is the functional of all link flow variables at time *t*, and can be computed as $\min_k \eta_k^{rs}(t)$.

Heydecker and Verlander, (1999) expressed the traffic assignment at time *t* as a column vector of route inflows $\mathbf{f}(t) \in D(t)$, and showed that the assignment would be in equilibrium if and only if:

$$-[\mathbf{h} - \mathbf{f}(t)]^T . \mathbf{C}(t) \leq 0 \quad \forall \mathbf{h} \in D(t) \quad (2.50)$$

where $C(t)$ is the column vector of travelers' route flows departing at time *t*, and $D(t)$ is the set of route inflows. The time varying costs can be calculated using the route in flows **f**, time at which traffic entered at route *k* arrives to link a, $\tau_a^k(t)$, and link-route incidence matrix δ as:

$$C_k(t) = \sum_a c_a[\tau_a^k(t)]\delta_a^k \tag{2.51}$$

The authors showed that the VI in eqn-44 can be solved as a forward dynamic programming model. For any $\mathbf{f}(t)$, the left-hand side of eqn-44 would be zero when $\mathbf{h} = \mathbf{f}(t)$. Thus, at equilibrium, the left-hand side will be zero. Then, the equilibrium solution can be obtained by solving the equivalent minimization problem (Heydecker and Verlander, 1999):

$$f^*(t) = \text{Arg} \min_{f \in D(t)} z(\mathbf{f}) \tag{2.52}$$

where;

$z(f) = \min_{h \in D(t)} -[\mathbf{h} - \mathbf{f}]^T . \mathbf{C}(t)$

In recent years, Lam et al. (2002) proposed a flow swapping method to obtain travelers' route/ departure time on congested road networks with queues. The DUO was expressed as a discrete-time, finite dimensional VI formulation, and was converted to an equivalent "zero-extreme value" minimization problem. The generalized disutility considered in travelers' decision making process included schedule delay penalty of arrival, utility of performing non-work activity and the cost of travel time. Using link travel time functions Jang et al. (2005) developed a discrete time route-based VI dynamic flow model was developed, and determined time-dependent network states. VI formulation was formed based on alternative cost mapping derived from a route swapping heuristic approach. Garcia et al. (2005) developed a multi-modal assignment model for the case of asymmetric costs. The authors formulated the assignment as a VI problem in the space of the hyper-path flows and then solved by the disaggregate simplicial decomposition algorithm.

Recently, Han and Heydecker, (2006) formulated the route-based VI formulation in a mathematical programming model for both discrete- and continuous time. Based on the VI formulations provided in Heydecker and Verlander (1999), the authors showed that the optimization models could be solved via forward dynamic programming approach. Provided that, the objective function given in eqn-46 will be exactly zero in equilibrium state, the DUO problem over time was presented in the form of dynamic programming model as follows:

$$\int_t \; z\big(\mathbf{f}(t)\big)dt \tag{2.53a}$$

subject to

$$\mathbf{f}(t) \in D(t) \;\; \forall t \tag{2.53b}$$

The flows $\mathbf{f}(t)$ assigned to each route influence the future travel cost from the same origin but not past ones. Thus, optimal solution at time t can be found without knowledge of the inflows at future times. Then, the optimal solution of the above dynamic programming flow can be found by solving the individual problem in eqn-2.52, iteratively in increasing order of time. The proper DUO solution can be obtained by calculating the flow pattern that minimizes the objective function in eqn-2.52 for each instant sequentially from the earliest time to the latest one.

In the case of a discrete time formulation, the flows $\mathbf{f}(t)$ can be identified as the route inflows during the time interval $[t, t + \Delta t)$. Thus, in discrete time, route inflows $\mathbf{f}(t) \in D(t)$ are in equilibrium when:

$$-[\mathbf{h} - \mathbf{f}(t)]^T . \mathbf{C}(t + \Delta t) \leq 0 \quad \forall \mathbf{h} \in D(t) \tag{2.54}$$

The discrete time DUO problem for the single origin *s* case can be mathematically formulated as follows (Han and Heydecker, 2006, 107):

$$\min_{e,f} z(t) = \sum_a \int_{e=0}^{e_a} c_a[\tau_{ra}^*(t + \Delta t)]\dot{\tau}_{ra}^*(t)de \tag{2.55a}$$

subject to
$$e_a = \sum_d \sum_k \delta_a^k f_k / \dot{\tau}_{ra}^*(t) \quad \forall a \tag{2.55b}$$

$$\sum_k f_k = q^{rs}(t) \quad \forall s \tag{2.55c}$$

$$f_k \geq 0 \quad \forall s,\ \forall k \tag{2.55d}$$

Link-based VI formulation was first proposed by Friesz et al. (1993). The author formulated a continuous-time VI model to solve the departure time/ route choice by equilibrating the experienced travel times. The proposed model uses link-based performance functions, penalty functions for early/late arrivals, travel demands, desired arrival times, and all possible paths between O-D pairs. The route cost was formed as a combination of travel cost determined by the link-based performance function and the penalty of early/late arrival. Since the proposed formulation was a continuous-time infinite dimensional VI problem, the solution required solution of a complex system of simultaneous integral equations. Later, Ran and Boyce (1996) proposed a link-based discrete-time VI formulation with fixed departure time. The authors included a queuing delay component to partially alleviate the traffic realism issues arising in the context of analytical models. Ran et al. (1996) extended this model for simultaneous departure time/route choice problem. Chen and Hsueh (1998) showed that for link-based VI formulation, the travel time on a link could be represented as a function of solely link inflow, and proposed a solution algorithm based on nested diagonalization procedure. However, the method was still very expensive to implement on real world. Lo and Szeto (2002) developed a cell-based VI formulation using Daganzo's cell transmission model. To solve the VI formulation, the authors employed an alternating direction method proposed by Han and Lo (2002) for co-coercive VI problems. Bliemer and Bovy (2003) extended the single user-class macroscopic DTA model by including multiple user-classes. To deal with different asymmetries due to interaction between user classes, link-based (quasi) VI approach was considered. The assignment problem was solved via nested modified projection method. Friesz et al. (2006) considered infinite dimensional VI formulation for DUO. The authors showed how the theory of optimal control and the infinite dimensional VIs are combined to create a simple and effective fixed point algorithm for calculating DUO network flows. Jeihani et al. (2007) proposed a modification of the convex-simplex method for determining DUO. The proposed method disposed with the line search step, and controlled the subset of travelers to be re-routed at each step while updating the link travel times after each assignment.

2.8.4 Extensions to Analytical DTA Model

Extensions to analytical DTA model cover inclusions of MUC, and travel time reliability to the modeling process.

Inclusion of MUC users to DTA problem was first proposed by Lo et al. (1996) and Ran et al. (1996). In these studies, travelers were categorized into three user classes; travelers who follow predetermined routes, travelers who follow a stochastic DUO assignment and travelers who follow DUO assignment. The problem was formulated via route-based continuous time VI formulation, and solved based on a combination of relaxation procedure, F-W algorithm and MSA. In this solution, time-space network expansion was a required step of the solution process. Later, Ran et al. (2002) developed an improved continuous time VI formulation, and eliminated the time-space network expansion step. Ran and Boyce (1996) stratified travelers based on route diversion willingness, income, age and driving behavior. Like the previous studies, link travel times were calculated for all travelers, not for individual classes. Bliemer and Bovy (2003) extended the single user-class macroscopic DTA model by including multiple user-classes. The authors proposed link-based (quasi) VI approach was considered. For simplicity, only route choice was modeled dropping the departure time choice.

Inclusion of travel time reliability to stochastic DTA models was proposed by Boyce et al. (1999) and Ran et al. (1996). Boyce et al. (1999) considered stochastic route travel times ignoring the traveler perception error, while Ran et al. (1996) considered the traveler perception error ignoring the stochastic route travel times. Later, Liu et al. (2002) included both stochastic travel times and perception errors in DTA model, and formulated the problem via continuous time VI formulation. Travelers' risk taking behavior was represented by exponential disutility function (Tatineni et al., 1997). To solve the VI formulation, first continuous time problem was converted to discrete-time VI model. Then, this discrete-time VI problem was solved by using a combination of relaxation, stochastic network loading, and MSA techniques.

2.8.5 Simulation-based DTA

Simulation-based DTA models develop a traffic simulator to imitate the complex traffic flow dynamics. In this approach, constraints such as traffic propagation, flow conservation, spatio-temporal interactions, link-path incidence relationships and vehicular movements are addressed through simulation instead of analytical evaluation. Due to their flexibility to develop realistic traffic modeling, simulation-based approaches have gained interest in the past years.

Most of the DTA micro-simulators represent the driver behavior regarding car following, gap acceptance and lane choice. These are microsimulation models such as CORSIM (http://www.fhwa-tsis.com/corsim_page.htm), INTEGRATION (Van Aerde, 1999), AIMSUN (Barcelo´ et al., 1994) (http://www.tss-bcn.com), VISSIM (http://www.ptv.de), PARAMICS (http://www.quadstone.com) and DRACULA (http://www.its.leeds.ac.uk/software/dracula/). MITSIM (Yang, 1997) (http://web.mit.edu/its/products.html) is an academic research model that has been used in several studies in Boston, Stockholm and elsewhere.

Micro-simulation models use large number of heuristic rules added to the basic car-following theory. Moreover their application is limited to relatively small size networks.

In order to handle larger size networks and to reduce computational times mesoscopic approaches introduced to traffic simulation. Most microscopic simulators and some mesoscopic simulators use a fixed-time step approach, where the simulation period is divided into small time intervals. After each interval, all vehicles in the network are moved and new position of each vehicle is calculated.

One of the first examples of this type of simulator was CONTRAM developed by Leonard et al., (1989). Ghali and Smith (1992) and Smith (1994) addressed UE and SO DTA problem and implemented their solutions using CONTRAM (Leonard et al., 1989; Taylor, 1996). Other major developments in mesoscopic simulators are DYNASMART (Jayakrishnan et al., 1994; Mahmassani et al., 2001; Mahmassani, 2001), DynaMIT (Ben-Akiva et al. 1998), TRANSIMS (Nagel and Schrekenberg (1992), and mesoscopic simulation model based on cell transmission (Ziliaskopoulos and Waller 2000). The software, DynaMIT, consists of a demand and supply simulator. Demand simulator predicts the O-D demand using Kalman Filter method, while supply simulator is mesoscopic, which determines the flow patterns based on demand, and moves the vehicles in packets.

Simulators based on macroscopic traffic flow theory were first developed during 1950s (Lighthill and Whitham, 1955; Richards, 1956). These simulators describe the evolution of traffic over time and space using a set of differential equations. Macroscopic traffic simulators include METACOR (Papageourgiou, 1990; Diakaki and Papageorgiou, 1996) and METANET (Messmer, 2000a; Messmer, 2000b). The route choice in METANET was achieved by splitting proportions at the nodes of the network. The number of arcs originating at a given node was limited to two, and the network-loading method was based on a second order (p.d.e.) traffic flow model.

Mahmassani and Peeta (1993, 1995) and Peeta and Mahmassani (1995) developed DTA models using DYNASMART. The developed deterministic DTA model assumed complete a priori knowledge of O-D demands, and single class of users. Mahmassani et al. (1993) extended this formulation by including multiple classes in terms of information availability.

Barcelo and Casas (2002) described a heuristic approach to solve DTA problem. The path flow rates were determined based on stochastic route choice model, while network loading was done by microscopic simulation using AIMSUN. Mirchandani et al. (2003) proposed an iterative simulation model in CORSIM, where travelers' experience in a period was used as input to MSA to provide traffic assignment for the next period. Mahut et al. (2004) calibrated and application of microscopic simulation-based DTA model, where path flows were reassigned using MSA (Mahut, 2000) and Florian et al. (2001). O-D matrix was created from trip generation/distribution model and a matrix adjustment algorithm. Path choices were modeled as a decision variable, where each traveler tried to minimize his/her travel time. At each step first the time-dependent paths flows were determined based on the previous path travel times, and then the actual travel times were calculated given the path flow sets. Liu et al. (2005) proposed a hybrid DTA model. The authors combined DUO technique with micro-simulator Paramics. The time dependent path flows were generated by analytical DUO model using VI methodology, whereas the link and path travel times were simulated. The convergence was assured via MSA method. Similarly, Florian et al. (2007) described a simulation-based, iterative DTA model. Time dependent flows were determined via MSA method, and path travel times were calculated using the space-time queuing approach by Mahut (2000).

In all these models MSA approach was used to ensure convergence of the DTA model. Recently, Sbayti et al. (2007) developed a methodology to improve the performance of MSA heuristic for UE and SO DTA problems on large networks. In this approach local information made available in the results of vehicle based simulation models were used to provide the mapping between path assignment and experienced travel cost.

2.8.6 Extensions to Simulation-based DTA

A few studies in the literature have proposed simulation-based models to analyze value pricing applications. Using DYNASMART Abdelghany et al. (2000) and Murray et al. (2000) evaluated the impacts of different pricing programs. Kwon et al. (2000) combined analytical DTA model with PARAMICS to study road pricing. More recently, de Palma et al. (2005) used METROPOLIS simulator to investigate congestion pricing on road networks. However, in all these models constant value of time (VOT) was considered, and user heterogeneity was ignored. Lee et al. (2007) relaxed the constant VOT assumption, and investigated the drivers' behavior in the presence of high occupancy toll lane value pricing system. The proposed DTA model was implemented in

TRANSIMS on a large transportation network. Lu et al. (2007) developed a bi-criterion DUO model to capture travelers' route choices in response to time-varying toll charges. The authors considered heterogeneous travelers with different VOT preferences. After formulating an infinite dimensional VI formulation, a simulation-based heuristic approach was developed to find route flows.

2.9 Day-to-Day Learning Approach

As described in the previous section, the dominant approach followed to capture the interaction between travel choice and network performance has been to solve an equilibrium problem obtained from static/dynamic, deterministic/stochastic traffic assignment models. In this approach, the only state of interest is the fixed point in which the system is at equilibrium and individual travelers have no incentive to change their decisions. Travelers are assumed to be rational, exploring each alternative's relevant attributes and trading off the utilities derived from them. The decision strategy serves to generate a choice from a choice set for the alternative that provides the individual with the maximum utility. When choices are made under uncertainty, this utility is usually defined through maximum expected utility (Von Neumann and Morgenstern, 1947). However, Kahneman and Tversky (1979) found that people do not necessarily maximize expected utility, but have a perception of probability of a certain outcome and the value of that outcome. Moreover, Trevesky and Kahneman (1981) showed that people exhibit risk behavior which is dependent on the way the decision is framed.

Classical equilibrium approach assumes rigid behavioral tendencies. It categorizes driver behavior into homogeneous classes via UE, SO, or SUE. Moreover, these models assume that the driver behavior classes are known deterministically a priori. The estimation of equilibrium is typically achieved through the solution of some optimization, or VI problem, which makes the approach restrictive in terms of generalizations. With this approach, it is also difficult to capture the heterogeneity in users' behavior, learning and adaptation processes, and behavioral characteristics.

Thus, classical equilibrium approach needs to be improved to represent day-to-day and within-day dynamics. In day-to-day modeling, behavioral approaches are integrated into the equilibrium paradigm, where the sequences of states that occur as the system reaches to equilibrium are linked through a learning model based on travelers' past experiences. These intermediate stages are important for evaluation of the transportation system, because the transportation system is often in disequilibrium due to travelers' gradual response to non-standard conditions. The day-to-day models reflect travelers' learning and forecasting mechanisms. These models predict travelers' choices for any given day based on their experienced choices in the previous days. Day-to-day approaches allow the use of wide range of behavioral rules, and levels of aggregation.

The fundamental difference between classical equilibrium approach and day-to-day approach lies in the underlying hypothesis. In traditional approach, market is assumed to be in equilibrium, where users are rational and have perfect knowledge about the system, whereas in day-to-day approach, the behavioral dynamics is based on the underlying belief, where the behavior on a given day is affected by the behavior on previous days.

The day-to-day modeling of driver travel choice requires incorporating travelers' decisions which are made according to their perceptions of travel choices, their experience with the system, perceptions of traffic information provided (if available), and the impact of any perception change in the travel choice.

This section provides detailed description of day-to-day learning models and their application to traffic assignment.

2.9.1 Day-to-Day Route Choice Models

Travelers' current route-choice decisions are influenced from past experiences with the transportation system. Thus travelers' route-choice behavior may be represented by an iterative process, in which the travelers update their choices based on previous experiences.

Horowitz (1984) described a simple route-choice dynamic model in order to treat decisions over time. The author assumed that in each time period *t* the route-choice decisions were based on the average of measured travel utilities in previous time period. The route-choice model was formulated as follows:

$$\widehat{U}_k(t) = \sum_{r=1}^{t-1} w_{r,k}(t-1)U_k(r) + \varepsilon_{kt} \quad k = 1, \ldots\ldots m \tag{2.56}$$

In the above formulation $\widehat{U}_k(t)$ is the perceived utility of route *k* at time period *t*, $w_{r,k}(t-1)$ is a nonnegative weight describing the relative influences of recent and past route performances on current utility, $U_k(r)$ is the measured utility of route *k*, and ε_{kt} is a random error variable whose probability distribution is independent of t. For each route k and time period t, the following equality should hold:

$$\sum_{r=1}^{t-1} w_{r,k}(t-1) = 1 \qquad \forall t = 1, \dots . T \tag{2.57}$$

Based on the above utility function the probability of choosing route k in time period t can be formulated via multinomial logit model:

$$p_k(t) = \frac{e^{\mu \sum_{r=1}^{t-1} w_{r,k}(t-1) U_k(r)}}{\sum_{j=1}^{m} e^{\mu \sum_{r=1}^{t-1} w_{r,j}(t-1) U_j(r)}} \tag{2.58}$$

Under myopic adjustment approach, Mahmassani and Chang (1986) updated the travel time estimate by driver i, $\tau_{i,t}$, based on the following equation:

$$\tau_{i,t} = T_{i,t-1}^{*} + a_i \gamma_{i,t-1}^{c} E_{i,t-1}^{*} + b_i \gamma_{i,t-1}^{l} E_{i,t-1}^{*} \tag{2.59}$$

In the above formulation, $T_{i,t-1}^{*}$ is the experienced trip travel time by driver i on day (*t-1*), $E_{i,t-1}^{*}$ is the schedule delay, $\gamma_{i,t-1}^{c}$ is a binary variable with value -1 for early arrival, and 0 otherwise, $\gamma_{i,t-1}^{l}$ is also a binary variable with value 1 for late arrival, and 0 otherwise, and a_i and b_i are nonnegative weight parameters for early and late arrival.

Similarly, Ben-Akiva et al. (1991) proposed a model where perceived travel time of users were updated based on the weighted average of historically perceived travel time and the time provided by ATIS. Iida et al. (1992) conducted a laboratory experiment to analyze route-choice behavior and dynamic adjustment over time. The focus of these dynamic network models is on the final state of the network assuming equilibrium or steady state conditions. Such restrictions prevent realistic and detailed modeling of dynamic driver behavior. Friesz et al. (1994) proposed a tatonnement adjustment process model to estimate the day-to-day adjustment processes of network flows. The authors showed that when traffic information was incomplete, it took around 260 time units for the traffic assignment model to converge SUE conditions. On the other hand when perfect information was available, the convergence to SUE condition was much faster.

Dynamic learning models aiming to update travelers' perception from one day to the next based on the experience with the system can be categorized into three types:

1. Bayesian learning models
2. Reinforcement learning models
3. Stochastic automata models

The studies related to the above learning models are summarized in the following flow chart, shown in Figure 2-8. Next sections provide detailed description of these methodologies and summarize the studies performing these approaches

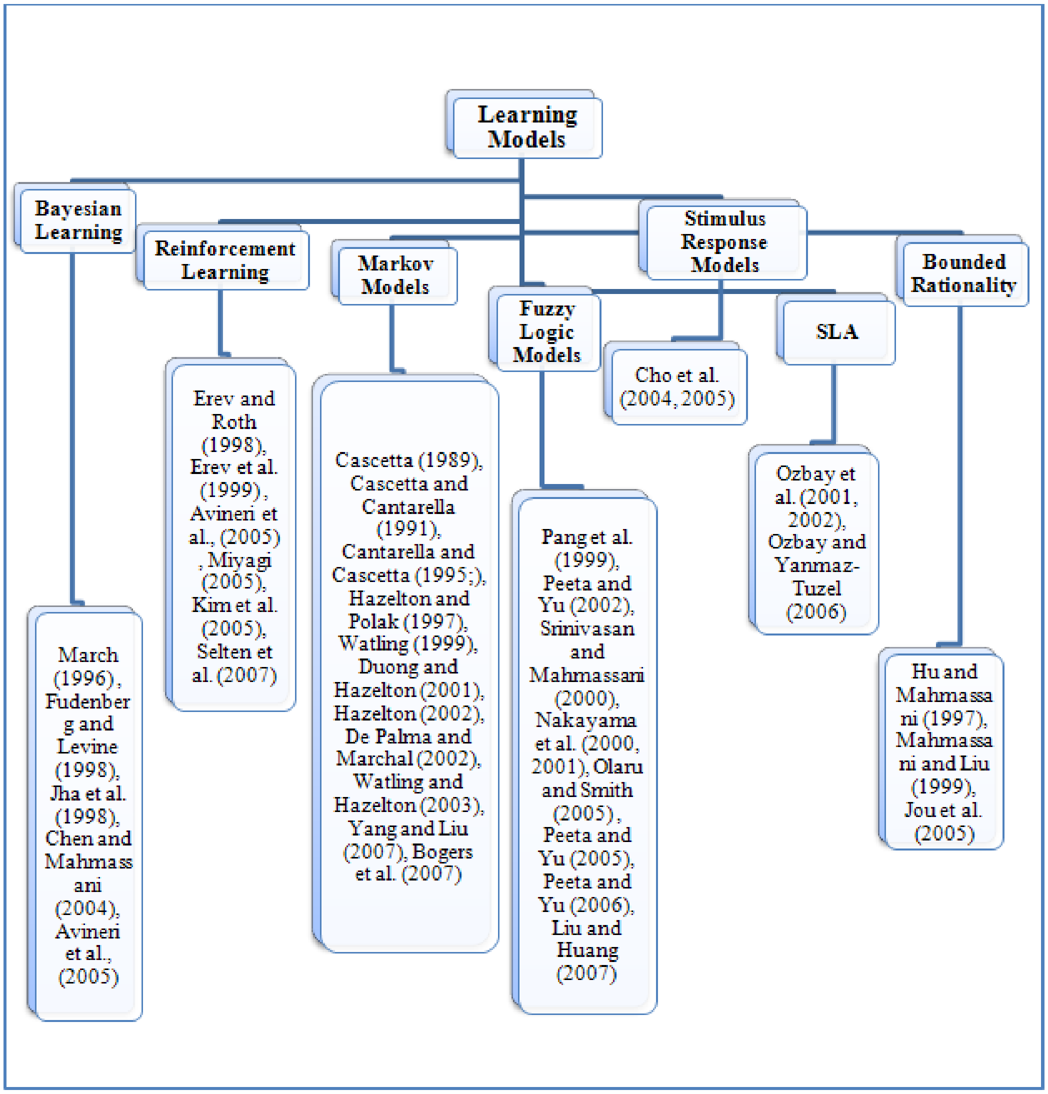


Figure 2-8 Summary of different learning models

2.9.2 Bayesian Learning Approach

In Bayesian learning (BL) approach, travelers' route choice is described as an iterative process. At each step the travelers form a belief about routes' expected travel times based on the historical frequencies of travel times. Then they choose the route which minimizes their expected (or random) utility, given their beliefs. The update mechanism in this approach is based on Bayes theorem.

March (1996) proposed different learning models to explain risk aversion in lotteries. In the first model, the average return model, the probability of choosing a strategy is calculated based on the average outcomes obtained from selecting that strategy. In particular, if strategy k was chosen in time period n and a payoff of x was obtained, then the propensity to choose strategy j is updated as follows:

$$q_j(n+1) = \begin{cases} q_j(n) + x(n) & if\ j = k \\ q_j(n) & if\ j \neq k \end{cases} \tag{2.60}$$

Then the probability of choosing strategy k, in time period n, $p_k(n)$, becomes:

$$p_k(n) = \frac{q_k(n)/c_k(n)}{\sum_{j=1}^{m} q_j(n)/c_j(n)} \tag{2.61}$$

where $c_k(n)$ is the number of time strategy n was selected in the past.

In the second model, the weighted return model, the probability of each strategy to be chosen was represented as a function of weighted past responses.

$$q_j(n+1) = \begin{cases} (1-\varphi).\, q_j(n) + \varphi.\, x(n) & if\ j = k \\ q_j(n) & if\ j \neq k \end{cases} \tag{2.62}$$

where φ is the rate of learning and forgetting. Based on this update mechanism, the probability of choosing strategy *k*, in time period *n* is:

$$p_k(n) = \frac{q_k(n)}{\sum_{j=1}^{m} q_j(n)} \tag{2.63}$$

Later, Fudenberg and Levine (1998) used random utility theory to model the learning process. The authors proposed that the effectiveness of each strategy is remembered and determined the probability to choose each strategy. The propensity to choose strategy *j*:

$$q_j(n+1) = \begin{cases} q_j(n) + \left[x(n) - q_j(n)\right]c_j(n) & if\ j = k \\ q_j(n) & if\ j \neq k \end{cases} \tag{2.64}$$

Then, the probability of choosing strategy *k* in time period *n* is:

$$p_k(n) = \frac{e^{(1/\lambda)q_k(n)}}{\sum_{j=1}^{m} e^{\sum_{r=1}^{t-1}(1/\lambda)q_j(n)}} \tag{2.65}$$

In terms of day-to-day learning in transportation, Jha et al. (1998) proposed a framework, where travelers update their perceptions under real time information in two stages based on Bayes rule. In the first stage, pre-trip beliefs are updated based on historical perceptions and real time information. In the second stage post-trip beliefs are updated based on pre-trip beliefs and perceived experienced travel time on the current day. The model is formulated for single O-D network:

1. Pre-trip update mechanism:

$$tp_{k,r}^{i,n} = ti_{k,r}^{n} + \varepsilon_{2,k,r}^{i,n} \tag{2.66}$$

where $ti_{k,r}^{n}$ and $tp_{k,r}^{i,n}$ are the travel time information provided by ATIS, and perceived travel time by individual *i* for path *k* and departure time interval *r*, on day *n*, respectively. And $\varepsilon_{2,k,r}^{i,n}$ is normally distributed perception error as a function of traveler's past experience with traffic information.

The updated best estimate, $\tau_{k,r}^{2,i,n}$, was given by the following Bayesian formulation (Ang and Tang, 1975):

$$\tau_{k,r}^{2,i,n} = E\left[T_{k,r}^{2,i,n}\right] = \frac{tp_{k,r}^{i,n}.Var\left(T_{k,r}^{1,i,n}\right)+T_{k,r}^{1,i,n}.Var\left(tp_{k,r}^{i,n}\right)}{Var\left(T_{k,r}^{1,i,n}\right)+Var\left(tp_{k,r}^{i,n}\right)} \tag{2.67}$$

where $T_{k,r}^{1,i,n}$ is the mean perceived travel time, and $T_{k,r}^{2,i,n}$ is the updated distribution of $T_{k,r}^{1,i,n}$ in light of information. The updated variance of $T_{k,r}^{2,i,n}$ was formulated as:

$$Var\left(T_{k,r}^{2,i,t}\right) = \frac{Var\left(T_{k,r}^{1,i,n}\right).Var\left(tp_{k,r}^{i,n}\right)}{Var\left(T_{k,r}^{1,i,n}\right)+Var\left(tp_{k,r}^{i,n}\right)} \tag{2.68}$$

And the updated mean travel time was given by:

$$T_{k,r}^{2,i,n} = \tau_{k,r}^{2,i,n} + \varepsilon_{3,k,r}^{i,n} \tag{2.69}$$

2. Post-trip update mechanism:

$$tx_{k,r}^{i,n} = te_{k,r}^{i,n} + \varepsilon_{4,k,r}^{i,n} \tag{2.70}$$

where $tx_{k,r}^{i,n}$ and $te_{k,r}^{i,n}$ are the experienced travel time as perceived by individual i, and experienced travel time, respectively. Then the best estimate, $\tau_{k,r}^{3,i,n}$, and the updated variance of $T_{k,r}^{3,i,t}$ was given by:

$$\tau_{k,r}^{3,i,n} = \frac{tx_{k,r}^{i,n}.Var\left(T_{k,r}^{2,i,n}\right)+T_{k,r}^{1,i,n}.Var\left(tx_{k,r}^{i,n}\right)}{Var\left(T_{k,r}^{2,i,n}\right)+Var\left(tx_{k,r}^{i,n}\right)} \tag{2.71}$$

$$Var\left(T_{k,r}^{3,i,n}\right) = \frac{Var\left(T_{k,r}^{2,i,n}\right).Var\left(tx_{k,r}^{i,n}\right)}{Var\left(T_{k,r}^{2,i,n}\right)+Var\left(tx_{k,r}^{i,n}\right)} \tag{2.72}$$

And the updated mean travel time was given by:

$$T_{k,r}^{3,i,n} = \tau_{k,r}^{3,i,n} + \varepsilon_{5,k,r}^{i,n} \tag{2.73}$$

3. Route and departure time choice model

While performing choice model, Jha et al. (1998) assumed a nested structure, where the traveler first chooses his/her departure time, then based on the departure time he/she selects the route:

$$P_{k,r}^{i,n} = \frac{e^{\mu_r V_r^{i,n*}}}{\sum_{r=T^{i,n}}^{r=T^{i,n+1}} e^{\mu_r V_r^{i,n*}}} \cdot \frac{e^{\mu_k V_{k,r}^{i,n}}}{\sum_k e^{\mu_k V_{k,r}^{i,n}}} \tag{2.74}$$

where $V_{k,r}^{i,n}$ is the utility as a function of perceived travel time, anticipated early/late arrival.

Recently, Chen and Mahmassani (2004) proposed a BL framework for updating perceived mean travel time and variance in the presence of experience of information. Unlike Jha et al. (1998) assuming that individuals consider all the new information in their decision making mechanism, Chen and Mahmassani (2004) proposed a selective updating process.

In this formulation it was assumed that, depending on the outcome of a choice for any given day, the traveler accepts the perceived travel time or not. The acceptability or tolerance was defined as the difference between the current and the best travel time:

$$\delta_{i,n} = \begin{cases} 1 \ \ if \ T_k^{i,n} - T_{k,best}^{i,n} \geq \Delta^i T_k^{i,n} \\ 0 \qquad\qquad\qquad\qquad otherwise \end{cases} \tag{2.75}$$

where $T_k^{i,n}$ is the perceived travel time for path *k* on day *t* for individual *i*, $T_{k,best}^{i,n}$ is the perceived best travel time on day *n*.

Then using the same Bayesian update mechanism (Jha et al., 1998), the authors calculated the mean and variance of the perceived travel time:

$$\tau_k^{i,n} = \frac{te_k^{i,n}.Var\left(T_k^{i,n}\right)+\left(\sum_n T_k^{i,n}\right).Var\left(te_k^{i,n}\right)}{N.Var\left(T_k^{i,n}\right)+Var\left(te_k^{i,n}\right)} \tag{2.76}$$

$$Var\left(\tau_k^{i,n}\right) = \frac{Var\left(T_k^{i,n}\right).Var\left(te_k^{i,n}\right)}{N.Var\left(T_k^{i,n}\right)+Var\left(te_k^{i,n}\right)} \tag{2.77}$$

where $te_k^{i,n}$ is the experienced perceived travel time, $T_k^{i,n}$ is prior updated travel time, $\tau_k^{i,n}$ is the posterior mean updated travel time, and *N* is the number of experienced travel times in sample.

The authors performed a sensitivity analysis in order to investigate the impacts of different number of days between updates on the convergence. The results found that as the number of days between updates increased, the number of days until convergence decreased initially and then increased, and the number of updates required for convergence decreased. These trends are a consequence of Bayesian updating as expressed through Equations 2.76 and 2.77. As the number of days between updating increases, users are obtaining a larger sample of experienced travel times.

2.9.3 Reinforcement Learning Models

Reinforcement learning (RL) models include types of learning where repetition of the relationship between stimulus and response patterns leads to memory (Bogers et al., 2007). RL is a form of trial-and-error learning, where an agent starts interacting with the environment with a random action, and receives rewards when this action leads to successful performance. As the agent explores the environment and finds actions to high reward, its behavior changes (Sutton and Barto, 1998). RL system is slower than other approaches, since it requires a detailed model for the relationship between the environment, available actions, and the rewards that accrue over a period of time (Sutton and Barto, 1998).

Erev and Roth (1998) and Erev et al. (1999) studied Reinforcement Learning (RL) models. The model specifications can be summarized as follows (Avineri et al., 2005):

1. Initial propensities: the traveler has an initial propensity, $q_k(1)$ to choose a route k on day 1
2. Average updating: The propensity to choose route k on day $n+1$ is given by:

$$q_k(n+1) = q_k(1)\frac{N(1)}{C_k(n)+N(1)} + AVE_k(n)\frac{C_k(n)}{C_k(n)+N(1)} \tag{2.78}$$

In the above formulation, $N(1)$ is the "strength of initial propensities", and $C_k(n)$ is the number of times route k is selected, and $AVE_k(n)$ is the average payoff obtained from choosing route k at first n days.

3. Exponential response rule: The probability $p_k(n)$ of choosing route k in day n is given by:

$$p_k(n) = \frac{e^{\lambda q_k(n)/S(n)}}{\sum_{j=1}^{m} e^{\sum_{r=1}^{t-1} e^{\lambda q_j(n)/S(n)}}} \tag{2.79}$$

In this formulation $S(n)$ is the standard deviation of the payoffs that the traveler has experienced up to travel n, and estimated as follows:

$$S(n+1) = S(n)W'(n) + |A(t) - x|(1 - W'(n)) \tag{2.80}$$

where;

$W'(n) = \frac{n+mN(1)}{n+mN(1)+1}$

$A(n+1) = A(n)W'(n) + x(1 - W'(n))$

For simplicity, the authors assumed that (Erev et al., 1999) uniform initial propensities equal to the expected payoff from random route choice, $A(1)$.

Another RL model was proposed by Miyagi (2005) under complete and incomplete route information. In incomplete information, it was assumed that the travelers do not know the travel time of the routes that they have not chosen, and in complete information it was assumed that travelers were informed about the travel times of the unselected routes after the selection was made. Under these assumptions propensity of route *k* for traveler *i* on day *n+1* was formulated as:

$$q_{k,i}(n+1) = (1 - \alpha_i^{n+1})q_{k,i}(n) + \alpha_i^{n+1}\pi_{k,i}(n) \tag{2.81}$$

where α_i^{n+1} user specific update parameter, and $\pi_{k,i}(n)$ is the payoff from path *k* on day n.

Under incomplete information $\pi_{k,i}(n)$ was calculated as:

$$\pi_{k,i}(n) = \begin{cases} \tau_{k,i}^{n} p_{k,i}^{n} & if\ path\ k\ is\ the\ maimum\ utility\ path \\ \gamma\pi_{k,i}(n-1) & otherwise \end{cases} \tag{2.82}$$

where γ is the weakening rate.

Similarly, when complete information was available, the payoff was calculated as:

$$\pi_{k,i}(n) = \begin{cases} \tau_{k,i}^{n} p_{k,i}^{n} & if\ path\ k\ is\ the\ maimum\ utility\ path \\ \gamma \tau_{k,i}^{n} p_{k,i}^{n} & otherwise \end{cases} \tag{2.83}$$

Kim et al. (2005) proposed an inductive learning process to formulate the day-to-day dynamic traffic assignment model. In this approach the concept of route preference was introduced as follows:

$$q_{k,i}(n+1) = q_{k,i}(n) + \gamma\left(te_{k}^{i,n}, \tau_{k,i}^{n}\right) \tag{2.84}$$

where $\gamma\left(te_{k}^{i,n}, \tau_{k,i}^{n}\right)$ is a function of difference between expected and experienced travel time. The expected travel time was calculated as the weighted sum of experienced travel times up to day *n.* This preference function took the value of the product of travel time difference and the value of driver's sensitivity, when the travel time difference was greater than the indifference band, and zero otherwise.

Based on this inductive learning formulation, on each day, travelers select the best route depending on their travel time expectation and preference:

Traveler *i*'s route choice on day *n*: $\arg\min_{k} q_{k,i}(n). te_{k}^{i,n}$ (2.85)

This routine was repeated every day, until convergence was reached. The convergence criterion was set as the state where more than 99% of the travelers do not change their route for the predefined number of days (5 days).

A similar RL model was proposed by Selten et al. (2007) with a two route choice scenario. 18 subjects were requested to choose two roads. Two different types of feedbacks were given to the subjects: (a) only experienced travel time, and (b) travel time information on both roads. The propensity of each road was calculated as the sum of all previous propensities, and the probability of selecting a road was the ration of each road's propensity. The study results showed at after 2000 periods, which was unusually long, the mean number of travelers on both roads were very near to the equilibrium.

2.9.4 Stimulus-Response Formulation

Several researchers proposed stimulus-response formula to represent the interacted day-to-day network dynamics under the assumption of a learning and adaptive behavioral process (Cho et al., 2004, 2005). The basic equation of this model is of the form:

$$Response(t+T) = Sensitivity \times Stimulus(t) \tag{2.86}$$

In the day-to-day learning process, the stimuli was described as the discrepancy between experienced travel time and predicted (expected) travel time, and resulted in the path flow diversion in the next day, denoted as the response. Sensitivity was the tendency to adaption of predicted travel time based on the corresponding stimuli. A general model to specify the network dynamics on a day-to-day basis was specified as:

$$Path\ flow\ diversion\ at\ day\ (t+T) = F\left(Sensitivity_p, (experienced\ travel\ time - predicted\ travel\ time\ at\ day\ (t))\right) \tag{2.87}$$

$$Predicted\ travel\ time\ adjustment\ at\ day\ (t+T) =$$

$$F\left(Sensitivity_w, (actual\ sum\ of\ path\ flows - predicted\ travel\ demand(t))\right) \quad (2.88)$$

where $F(.)$ and $G(.)$ are functional. $Sensitivity_p$ and $Sensitivity_w$ of path flow dynamics and predicted travel time evolutions, respectively. Based on this formulation, quasi-user equilibrium was derived from steady state network dynamics. This specific equilibrium point was defined as the situation when all users feel indifferent between the experienced path travel time and the predicted O-D travel time.

2.9.5 Markov Learning Models

Cascetta (1989) and Cascetta and Cantarella (1991) attempted to describe it as a Markov chain and showed that the system converged to one stochastic distribution. In this model, day-to-day evolution of the traffic flow was represented, where each traveler's route choice was stochastically dependent on network flows and costs during the finite past. This specific structure of Markov models allowing learning mechanism has attracted many researchers in the past (Cantarella and Cascetta, 1995; Hazelton and Polak, 1997; Watling, 1999; Duong and Hazelton, 2001; Hazelton, 2002; Watling and Hazelton, 2003; Yang and Liu, 2007).

De Palma and Marchal (2002) modeled day-to-day learning as an Order-1 exponential Markov process. The following rule was considered:

$$T_{H,a}^{n+1}(t) = (1-\lambda)T_{H,a}^{n}(t) + \lambda T_{S,a}^{n}(t) \quad (2.89)$$

where;

H: Historical information

a: Index of the link

t: Time of day

S: Value of computed travel time in the simulation or the experienced travel time

Unfortunately, de Palma and Marchal (2002) did not test this model empirically. Later, Bogers et al. (2007) proposed a similar order-1 exponential Markov learning model, and tested the proposed model using a traffic simulator. The authors used the following travel time perception updating rule:

$$PTT_{k,i}^{n+1} = (1-\lambda)PTT_{k,i}^{n} + \lambda TT_{k,i}^{n}\delta_{k,i}^{n} + \lambda PTT_{k,i}^{n}\left(1-\delta_{k,i}^{n}\right) \tag{2.90}$$

In the above formulation, $PTT_{k,i}^{n}$ and $TT_{k,i}^{n}$ are respectively the perceived and experienced travel times by individual i, on path *k* on day *n*. If the individual travels on route *k* on day *n*, then $\delta_{k,i}^{n}$ becomes one, otherwise it is zero.

Recently, Lim et al. (2007) proposed a one-dependent Markov Chain model to analyze the sudden changes on the road networks. Pre-trip traffic information was provided in advance for travelers (agents) prior to the beginning of trips, and the effects of the information were assessed. The steps of the model can be summarized as follows:

1. Compute the average travel cost, c_k, as the weighted sum of experienced costs on previous days:

$$c_k^n = \sum_{m=1}^{d-1} w_m c_k^{n-m} \tag{2.91}$$

2. Compute the route choice probability of travelers:

$$p_k(n) = \frac{e^{-\beta c_k(n)}}{\sum_{j=1}^{m} e^{-\beta c_j(n)}} \tag{2.92}$$

3. Compute the transition probability, p_{ij}, by one-dependent Markov chain:

$$p_{ij} = \frac{D!}{j!(D-j)!}(p_i)^j(1-p_i)^{D-j} \tag{2.93}$$

where *D* is the total travel demand

4. Calculate the stationary distribution π^n recursively from:

$$\pi^{n+1} = \pi^n p \tag{2.94}$$

5. Compute the average path flow matrix F^n:

$$F^n = s\pi^n \qquad , s = 1, \dots ..D \tag{2.95}$$

6. Compute the link-flow matrix υ^nbased on the link-route incidence matrix, *A*:

$$\upsilon^n = AF^n \tag{2.96}$$

7. If $\upsilon^{n+1} = \upsilon^n$ stop, otherwise, n=n+1

2.9.6 Fuzzy-Logic (Rule-based) Approaches

Fuzzy logic based day-to-day modeling approach combines qualitative and quantitative information, and considers a rule-based route choice modeling. Several fuzzy logic models have been proposed for driver route decisions under information (Pang et al., 1999; Peeta and Yu, 2002).

In this approach, concepts like inertia (propensity to remain on the current path), compliance (tendency to choose the path recommended by the information system), delusion (biased perception about a travel choice), freezing (habit), bounded rationality, and perception of traffic information become meaningful.

Srinivasan and Mahmassani (2000) examined inertia and compliance based on nested multinomial probit models.

Nakayama et al. (1999), proposed a formulation using Genetic Algorithms, and calculated the weights in the Horowitz's (1984) travel time formulation endogenously. The results of the microscopic simulation analysis indicated that network flow does not necessarily converge to UE and may reach "deluded equilibrium," which was caused by a driver's false perception of travel time. The model formulation in this did not necessarily have psychological bases, and represented only a limited variety of cognitive processes underlying route choice behavior. Nakayama et al. (2000, 2001) developed an inductive rule-based model to examine delusion and freezing under traffic information. Four different rules were included in the route choice model: no switching (driver continued to travel on the same route), random switching (driver switched routes purely randomly), experienced based (driver evaluated the alternative routes based on the experience, and chose the best one), and rational (driver evaluated the alternative routes rationally and chose the best one). The general form of the experience based rule was:

$$t_{k,i}(n+1) = t_{k,i}^{avg}(n_m) + \gamma_{k,i}\left(t_{k,i}^{max} - t_{k,i}^{min}\right) \tag{2.97}$$

In the above formulation, $t_{k,i}(n+1)$ is the expected travel time on path k on day $n+1$for traveler i, $t_{k,i}^{avg}(n)$ is the average experienced travel time on the most recent m days, $\gamma_{k,i}$ is the risk parameter, and $t_{k,i}^{max}$ and $t_{k,i}^{min}$ are the maximum and minimum travel times experienced by the traveler. Based on this rule-based system, the traveler behavior was explored through Monte Carlo simulation without adoption of the assumptions underlying network equilibrium. Based on the estimation results, the authors concluded that a driver supplied with limited or incorrect information on a route might form a delusion, i.e., a biased perception of that route. If this delusion continued over time, the driver might develop the habit of excluding that route from consideration, resulting in a habit called freezing.

Olaru and Smith (2005) modeled the impacts of travel time variability on travelers re-scheduling of daily activities based on fuzzy logic rules. The fuzzy model is used to handle the imprecision of the data which is unstructured. The developed model is tested via travel diary data collected for academics and students from ten universities in Bucharest in 1998.

Liu and Huang (2007) proposed a similar rule-based day-to-day traffic assignment model and compared the results with SUE conditions. The expected travel time on day $n+1$ was calculated as the weighted average of experienced and actual travel time on day *n*. The simulation results showed that, when information about routes that were not chosen, was not available to the traveler, the system started to reach to SUE conditions after 1400 days. On the other hand, when this information was available to the traveler, convergence to SUE was much faster.

Peeta and Yu (2005) formulated the en-route traveler route-choice behavior via fuzzy logic approach. A hybrid model was proposed to combine logit-based route choice with qualitative if-then rules. The qualitative information was transformed into fuzzy variables via center of sums method defuzzification (Tsoukalas and Uhrig, 1997). Then these variables were included in the utility function along with the quantitative variables.

Peeta and Yu (2006) proposed a behavior-based consistency seeking model to investigate the traffic systems under information provision. Heterogeneous driver classes were elicited from surveys, and fuzzy logic based if-then rules were used to model various driver behavior classes. The authors used DYNASMART to perform the traffic simulation.

2.9.7 Bounded Rationality

Hu and Mahmassani (1997) proposed a simulation-assignment approach to investigate real-time dynamics in terms of en-route switching decisions and day-to-day evolution of the traffic system under real-time information provision. Route and departure time selection were based on the driver's scheduled delays experienced on the previous day. En-route switching was assumed to be based on boundedly rational behavior under information provision. Later, Mahmassani and Liu (1999) extended this work by using an interactive dynamic traveler simulator to generate data through laboratory experiments. They developed a multinomial probit framework to model driver departure time and route choices. Recently, Jou et al. (2005) investigated the route switching behavior on freeways in response to different types of real-time traffic information. The authors designed the experiments based on stated preference surveys. Travel time and travel cost were included in the bounded rationality framework, and through the variance–covariance matrix, the correlation of travelers' node-to-node route switching was studied.

2.9.8 Stochastic Learning Automata

Stochastic Learning Automata (SLA) mimics drivers' day-to-day learning by updating travelers' choice probabilities based on their experience with the system. The main advantage of the SLA over RL models is that in SLA theory, no specific relationship between the environment and actions is required. In fact, in SLA the environment is treated as an unknown random media in which an automaton operates, and the response of the environment to a specific action rather than the environment itself is considered. In simple terms, SLA approach is an inductive inference mechanism that updates the probabilities of its actions occurring in a stochastic environment to improve a certain performance index. This process is naturally closely related to BL, in which the distribution function of a parameter is updated at each instant on the basis of new information. However, in BL updating takes place according to Bayes' rule, while it is more general in SLA (Narendra and Thathachar, 1989). SLA is first proposed to model the day-to-day learning behavior of drivers by (Ozbay et al., 2001) and (Ozbay et al., 2002).

Ozbay et al. (2001) calibrated the SLA model using an Internet-based route choice traffic simulator. The simulator composed of one O-D pair and two routes. The travelers were assumed to be rational users trying to minimize their travel times. Moreover, the authors (Ozbay et al., 2001) assumed constant reward and punishment parameters and via trial-and-error approach determine the values of these parameters. This model was specific to route choice behavior. Later, Ozbay et al. (2002) improved this methodology by introducing departure time choice into the learning framework first proposed by Ozbay et al. (2001).

2.9.9 Summary

This section has focused on the detailed description of traffic assignment methodologies, and provided a comprehensive literature review of static and dynamic traffic assignment methodologies and day-to-day approaches.

The approach to solve traffic assignment problem in the literature was mostly based on the notion of equilibrium. The earlier approaches to this problem involved assignment of O-D flows to the network links such that the travel time on all used paths for any O-D pair equals the minimum travel time between the O-D pair. This type of deterministic approach assumes that travelers choose the least cost or minimum travel time path from their origin to destination. Moreover, it presumes that all travelers have perfect knowledge regarding the transportation network, and that they make consistently correct decisions.

Stochastic traffic assignment models relax some of the assumptions of deterministic approaches, and introduce random utility maximization methodology into the traffic assignment. In particular, SUE approach assumes that each traveler tries to minimize his/her perceived travel time, generalizes the all-or-nothing network loading mechanism via discrete choice models.

Conventional static traffic assignment models a fixed point steady-state equilibrium condition is assumed, where individual travelers have no incentive to change their decisions. Travelers assume to choose the alternative that maximizes their utility. Unfortunately, in reality traffic is a dynamic process, and these models fail to capture the dynamic nature of the traffic. Thus, in the past years, traffic assignment models started to progress from static to dynamic. Dynamic models can successfully represent the time-varying nature of the congestion during different times of the day, help to understand travelers' responses to time-varying transportation system policies (e.g. congestion pricing) including departure-time choice, pre-trip route choice, and en-route response to traffic information.

Even though, DTA models provide some insight in terms of modeling traveler behavior, it assumes rigid behavioral tendencies; categorizes drivers into homogeneous classes via UE. Moreover, these models assume that the driver behavior classes are known deterministically a priori. The estimation of equilibrium is typically achieved through the solution of some optimization, or VI problem, which makes the approach restrictive in terms of generalizations. With this approach, it is difficult to capture the heterogeneity in users' behavior, learning and adaptation processes, and behavioral characteristics.

In reality, the decision making process of driver choice is a dynamic process. A learning process is central to the driver's cognition as the information acquired through earlier travel experience affect the future decisions. Moreover, the characteristics of the driver, subjective interpretation of the traffic information (if available), trip characteristics (time of day, trip purpose, etc), and uncertain traffic conditions have an importance in the driver's final decision. Repeated choices make the drivers better aware of the travel options, inducing them to consider a destination, choose a route or try a new transportation mode.

CHAPTER 3. ROUTE CHOICE GENERATION

The route set generation approach proposed in this thesis provides an alternative to the algorithms mentioned Section 2.1. The proposed approach develops a stochastic integer programming model to generate *k* partially disjoint paths. Unlike previous approaches, in this thesis the relevant route choice set is determined via minimizing travel time, travel time variability, and route overlap. The several advantages of the proposed approach can be summarized as follows:

1. The proposed mathematical programming model, limits the number of paths using each link by a user defined overlap constraint set. In particular, the links on the network are labeled based on their functional characteristics. Since road type criterion covers important characteristics of each link such as speed, travel time, capacity, and accessibility; generating path overlap set via this criterion allows us take into account several crucial parameters at once. Based on these label-constraints, the set of overlapping links is determined.
2. The proposed mathematical programming model is an exact method to determine probabilistic *k*-PDP. The generated paths are not only disjoint with the "first" shortest path but they can be completely or partially disjoint with respect to each other as well. Instead of first generating a master set then applying various filters to this set to generate a final feasible path set, constraints considered in filtering the paths are explicitly incorporated into the optimization problem. This approach

eliminates the filtering process altogether from the set generation resulting in reduced computational costs.

3. Unlike most of the previous models which can compute paths from only one origin to all destinations, the proposed model can simultaneously compute the paths between multiple OD pairs; which improves the computational costs.
4. Finally, the proposed model includes stochasticity of travel times via travel time variability and network reliability components. Variability of travel times is necessary when trying to generate paths that are considered by travelers who do not only consider travel times but also variability of travel times. In fact, there is increasing evidence based on the recent research studies, travelers associate a high value to travel time variability (Noland and Polak, 2002; Brownstone and Small, 2005). Thus, a path generation method that considers travel time variability while generation path choice set is highly desirable.

Next section provides the details of the stochastic integer programming model proposed to indentify *k* partially disjoint paths in a network. The proposed mathematical model is compared with the existing mathematical models. After, the proposed model is applied to a real network. Finally, in the last section conclusions and discussions are provided.

3.1 Model Formulation

The fundamental problem we consider in this chapter is to find k partially disjoint directed paths between multiple origin and destination nodes. Unlike other studies that developed heuristic algorithms to find k-PDP, this chapter formulates this problem via stochastic integer programming approach. The proposed approach considers probabilistic travel times and network reliability while finding the desired number of paths.

First, we will briefly describe the existing mathematical formulations to find one and two shortest paths. Then, we will propose the mathematical formulation to find k-PDP, and finally we will extend this formulation to introduce stochastic travel times and network reliability.

Let $G = (M, A)$ be a directed graph, where M is the set of nodes $\{1,2, \ldots m\}$, and A is the set of links whose size is equal to the total number of links, a. Each link $(i, j) \in A$ has a nonnegative cost c_{ij}. Moreover, suppose that we have a designated pair of origin and destination nodes o and d, respectively. A path from node o to node d is shortest if there is no path from o to d of shorter length.

In the following formulations $FS(i)$ and $RS(i)$ denote respectively, the forward and reverse stars of node i. That is, $FS(i)$ refers to the set of links exiting node i, and $RS(i)$ refers to the set of links entering node i, $\forall i \in M$. Moreover, x_{ij} is equal to 1 if link $(i, j) \in A$ is used in the shortest path and 0 otherwise.

Finding the single shortest path is formulated via binary integer programming where the objective function minimizes the cost of the path, and the constraints guarantees the path flow balance. Sherali et al. (1998) extended this formulation to find the shortest pair of fully disjoint paths. In this formulation the objective function minimizes the cost of two paths, while constraint sets ensure path flow balance and path disjointness. Sherali et al. (1998) simplified this formulation for a special case. In this formulation, first a shortest path from an origin to a destination is found via original shortest path problem formulation, then the links in the first shortest path are reversed, and the second shortest path is found via binary integer programming problem. One major problem that can arise in this formulation is that splitting initial shortest path solution from the network may eliminate the accessibility of some nodes and might fail to identify a feasible solution to the problem. Apart from the connectivity issues faced in these approaches, none of these models consider more than two completely disjoint paths, or partially disjoint paths. Furthermore, the travel time variability and network reliability issues are not considered in the aforementioned approaches, while determining the possible paths.

To overcome these issues, first we propose the following mathematical model to determine k-PDP.

$$\text{(P-1)} \qquad \min \quad \sum_{n=1}^{N} \sum_{h=1}^{k^n} \sum_{(i,j)\in A} t_{ij} x_{ij}^{h,n} \tag{3.1a}$$

subject to

$$\sum_{j\in FS(i)} x_{ij}^{h,n} - \sum_{j\in RS(i)} x_{ji}^{h,n} = \begin{cases} 1 & \text{for } i \equiv o^n \\ -1 & \text{for } i \equiv d^n \\ 0 & \text{otherwise} \end{cases} \quad \forall h = 1,2,\ldots k^n \quad \forall n = 1,2,\ldots N \tag{3.1b}$$

$$\sum_{h=1}^{k^n} x_{sj}^{h,n} \le k^n \qquad \forall (s,j) \in C^n \qquad \forall n = 1,2,\ldots N \tag{3.1c}$$

$$\sum_{h=1}^{k^n} x_{if}^{h,n} \le k^n \qquad \forall (i,f) \in C^n \qquad \forall n = 1,2,\ldots N \tag{3.1d}$$

$$\sum_{h=1}^{k^n} x_{ij}^{h,n} \le 1 \qquad \forall (i,j) \in D^n \qquad \forall n = 1,2,\ldots N \tag{3.1e}$$

$$\sum_{h=1}^{k^n} x_{ij}^{h,n} \le s^n \qquad \forall (i,j) \in \{S^n\} = \{A - C^n - D^n\} \qquad \forall n = 1,2,\ldots N \tag{3.1f}$$

$$x_{ij}^{h,n} \in \{0,1\} \qquad \forall (i,j) \in A \qquad \forall h = 1,2,\ldots k^n \qquad \forall n = 1,2,\ldots N \tag{3.1g}$$

where,

n = Index for the OD pair (n=1,2,…N)

N = Total number of OD pairs

o^n = Origin node for OD pair n

d^n = Destination node for OD pair n

k^n = Total number of partially disjoint paths for OD pair n

h = Path index, $h = 1,2, \ldots k^n$

t_{ij} = Travel time of arc (i, j)

$$x_{ij}^{h,n} = \begin{cases} 1 & \text{if link } (i,j) \text{ is used by path } h \text{ and OD pair } n \\ 0 & \text{otherwise} \end{cases}$$

C^n = Set of zone connectors (i, j) from the origin and to the destination nodes for OD pair n

D^n = Set of links (i, j) which can be used by at most one path (user defined) for OD pair n

S^n = Set of links (i, j) which can be overlapped by s^n s^npaths, where $s^n \le k^n$ $s^n \le k^n$ (user defined) for OD pair n

In this formulation the deterministic objective function (3.1a) aims to simultaneously minimize the total travel time of all k-PDP for a set of OD pairs, while constraint (3.1b) guarantees the network flow balance. Constraints (3.1c) and (3.1d) ensure that the zone connectors (i, j) connecting the zones (origins and destinations) to the real network can be overlapped by all possible k-PDP. Similarly, constraint (3.1e) ensures that the links (i, j) in set D^n can be used by at most one path. The remaining set of links $(i,j) \in S^n$ can be overlapped by up to s^n paths. This condition is satisfied via constraint (3.1f). Finally, constraint (3.1g) is the binary variable constraint.

The optimization model presented in problem (P-1) can be formulated in a more compact form as shown in problem (P-2). In this formulation, previous constraint (3.1b) is represented in a vector form in constraint (3.2a) where matrix $\boldsymbol{Z^n}$ represents the node-to-arc incidence matrix, and vector $\mathbf{b^n}$ which takes value of 1 and -1 for the origin and destination nodes, and takes value of zero elsewhere. Similarly, constraints (3.1c), (3.1d) and (3.1e) are combined together in constraint (3.2b), where vector $\boldsymbol{d^n}$ takes value of k^n for links exiting origin node and entering destination node, takes value of s^n for overlapping links, and takes value of 1 for non-overlapping links.

$$\text{(P-2)} \qquad \min \quad \sum_{n=1}^{N}\sum_{h=1}^{k^n} \mathbf{t}^{\mathbf{T}}\mathbf{x}^{\mathbf{h,n}} \tag{3.2a}$$

subject to

$$\mathbf{Z}^{\mathbf{n}}\mathbf{x}^{\mathbf{h,n}} = \mathbf{b}^{\mathbf{n}} \quad \forall h = 1,2,....k^n \quad \forall n = 1,2,...N \tag{3.2b}$$

$$\sum_{h=\mathbf{1}}^{k^n} \mathbf{x}^{\mathbf{h,n}} \le \mathbf{d}^{\mathbf{n}} \quad \forall n = 1,2,...N \tag{3.2c}$$

$$\mathbf{x}^{\mathbf{h,n}} - binary \quad \forall h = 1,2,....k^n \quad \forall n = 1,2,...N \tag{3.2d}$$

where,

$\boldsymbol{t}$ = Travel time vector

$\boldsymbol{x^{h,n}}$ = The a-dimensional binary decision vector which takes value 1 if link (i, j) is in the optimal problem, and 0 otherwise.

$\mathbf{t}^{\mathbf{T}}\mathbf{x}^{\mathbf{h,n}} = \sum_{(i,j)\in A} t_{ij} x_{ij}^{h,n}$

Next, we extend problem (P-2) to incorporate random travel times and path travel time reliability where link travel times are obtained from a probability distribution function. The proposed model is formulated as follows:

(P-3) $$min \sum_{n=1}^{N} \sum_{h=1}^{k} \underline{\boldsymbol{t}}^{T} \boldsymbol{x^{h,n}} \quad (3.3a)$$

subject to

$$\boldsymbol{Z^n x^{h,n}} = \boldsymbol{b^n} \forall h = 1,2, \ldots k^n \forall n = 1,2, \ldots N \quad (3.3b)$$

$$\sum_{h=1}^{k} \boldsymbol{x^{h,n}} \leq \boldsymbol{d^n}\ \forall n = 1,2, \ldots N \quad (3.3c)$$

$$\boldsymbol{x^{h,n}} - binary \forall h = 1,2, \ldots k^n \forall n = 1,2, \ldots N \quad (3.3d)$$

where;

$\underline{\mathbf{t}}$ = Random vector of travel time vector with a continuous probability distribution

$\underline{\mathbf{t}}^{\mathbf{T}}\mathbf{x}^{\mathbf{h,n}} = \sum_{(i,j)\in A} \underline{t}_{ij} x_{ij}^{h,n}$

In this formulation the probabilistic objective function (3.3a) aims to minimize the nondeterministic travel time for all k-PDP with the same set of constraints developed in problem (P-2).

3.1.1 From Stochastic NL-IP to Deterministic NL-IP

The proposed Stochastic NL-IP problem is a highly complex mathematical model. In this section, via a novel transformation we obtain its equivalent deterministic convex NL-IP model which can be solved in polynomial time.

The problem of how to deal with random objective function can be tackled in several ways. One possibility is to introduce a new constraint and a new objective function so that the stochastic programming problem becomes as follows (Kataoka, 1963) (In the following formulations, in order to simplify the transformation of the proposed mathematical model, index for the OD pair is omitted):

(P-4) $$\min \quad \sum_{h=1}^{k} g^h \tag{3.4a}$$

$$\boldsymbol{Z}\boldsymbol{x}^{\boldsymbol{h}} = \boldsymbol{b} \forall h = 1,2, \ldots k \tag{3.4b}$$

$$\sum_{h=1}^{k^n} \mathbf{x}^{\mathbf{h}} \leq \mathbf{d} \qquad \sum_{h=1}^{k} \boldsymbol{x}^{\boldsymbol{h}} \leq \boldsymbol{d} \tag{3.4c}$$

$$\mathrm{P}\left(\mathbf{t}^{\mathbf{T}}\mathbf{x}^{\mathbf{h}} \leq g^h\right) \geq \rho^{\mathrm{h}} \quad \forall h = 1,2, \ldots k \tag{3.4d}$$

$$\mathbf{x}^{\mathbf{h}} - binary \quad \forall h = 1,2, \ldots . k \qquad \boldsymbol{x}^{\boldsymbol{h}} - binary \forall h = 1,2, \ldots k \tag{3.4e}$$

where;

g^h = Deterministic variable for path h

ρ^{h} = Reliability of path h

In this new optimization problem (P-4), the objective function is deterministic and minimizes the total travel time of k-PDP. The additional constraint (3.4d) ensures that travel time on each path is minimized at a certain level of path reliability ρ^h. Unlike the other deterministic constraints, this constraint is a probabilistic constraint with a continuous distribution.

If it is assumed that the probabilistic constraint (3.4d) has a normal distribution, i.e. travel time of each link (*i, j*) is independent random variables that are normally distributed with mean μ_{ij} and standard deviation σ_{ij}, $\left(N\left(\mu_{ij}, \sigma_{ij}\right)\right)$constraint (3.4d) can be rewritten as follows (Prekopa, 1995):

$$P\left(\left(\frac{(\boldsymbol{t}-\boldsymbol{\mu})^{\boldsymbol{T}}\boldsymbol{x}}{\sqrt{\boldsymbol{x}^{\boldsymbol{T}}\boldsymbol{\Omega}\boldsymbol{x}}}\right)^h \leq \left(\frac{\boldsymbol{g}-(\boldsymbol{\mu}^{\boldsymbol{T}}\boldsymbol{x})}{\sqrt{\boldsymbol{x}^{\boldsymbol{T}}\boldsymbol{\Omega}\boldsymbol{x}}}\right)^h\right) \geq \rho^h h = 1,2, \ldots k \tag{3.5a}$$

$$\phi\left(\left(\frac{g-(\boldsymbol{\mu}^{\boldsymbol{T}}\boldsymbol{x})}{\sqrt{\boldsymbol{x}^{\boldsymbol{T}}\boldsymbol{\Omega}\boldsymbol{x}}}\right)^h\right) \geq \rho^h h = 1,2, \ldots k \tag{3.5b}$$

$$g^h - (\boldsymbol{\mu}^{\boldsymbol{T}}\boldsymbol{x})^h \geq \phi^{-1}(\rho^h)\left(\sqrt{\boldsymbol{x}^{\boldsymbol{T}}\boldsymbol{\Omega}\boldsymbol{x}}\right)^h h = 1,2, \ldots k \tag{3.5c}$$

$$(\boldsymbol{\mu}^{\boldsymbol{T}}\boldsymbol{x})^h + \phi^{-1}(\rho^h)\left(\sqrt{\boldsymbol{x}^{\boldsymbol{T}}\boldsymbol{\Omega}\boldsymbol{x}}\right)^h \leq g^h h = 1,2, \ldots k \tag{3.5d}$$

where;

$\boldsymbol{\mu}$ = The n – dimensional vector for mean travel time = $\begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ \boldsymbol{\mu_{ij}} \\ \cdot \\ \cdot \\ \cdot \end{bmatrix}$

$\phi^{-1}(.)$= Inverse of the standard normal distribution, $N(0,1)$.

$\mathbf{\Omega}$ = The *nxn* diagonal matrix such that each diagonal entry is equal to the variance of the corresponding link (*i, j*).

Thus, our probabilistic programming problem can be formulated in the following manner:

(P-5) $$\min \quad \sum_{h=1}^{k} g^h \tag{3.6a}$$

$$\boldsymbol{Z}\boldsymbol{x}^{\boldsymbol{h}} = \boldsymbol{b} \forall h = 1,2,\ldots k \tag{3.6b}$$

$$\sum_{h=\mathbf{1}}^{k^n} \mathbf{x}^{\mathbf{h}} \leq \mathbf{d} \qquad \sum_{h=1}^{k} \boldsymbol{x}^{\boldsymbol{h}} \leq \boldsymbol{d} \tag{3.6c}$$

$$\left(\boldsymbol{\mu}^{\mathbf{T}}\mathbf{x}\right)^h + \Phi^{-1}\left(\rho^h\right)\left(\sqrt{\mathbf{x}^{\mathbf{T}}\mathbf{\Omega}\mathbf{x}}\right)^h \leq g^h \quad \forall h = 1,2,\ldots k \tag{3.6d}$$

$$\mathbf{x}^{\mathbf{h}} - binary \quad \forall h = 1,2,\ldots.k \qquad \boldsymbol{x}^{\boldsymbol{h}} - binary \forall h = 1,2,\ldots k \tag{3.6e}$$

Problem (P-5) is a stochastic NL-IP problem with a nonlinear travel time constraint (3.6d). This constraint limits the path travel time between a specific OD pair at a lower bound given by an exponential function. The very existence of transportation problems indicates that the travel time g^h will not be more than the minimum required time; thus, constraint (3.6d) is a binding constraint. Mathematically,

$$\left(\boldsymbol{\mu}^{\mathbf{T}}\mathbf{x}\right)^h + \Phi^{-1}\left(\rho^h\right)\left(\sqrt{\mathbf{x}^{\mathbf{T}}\mathbf{\Omega}\mathbf{x}}\right)^h = g^h \quad \forall h = 1,2,\ldots k \tag{3.7}$$

Then, the equality constraint (3.7) can be included into the objective function of the problem (P-5) and the stochastic optimization problem can be rewritten in such a way that both the objective function and the probabilistic constraint with continuous distribution are transformed to their equivalent deterministic forms. We are left with an optimization problem with nonlinear objective function and two sets of linear constraints:

(P-6) min $$\sum_{n=1}^{N}\sum_{h=1}^{k}\left(\boldsymbol{\mu}^{\mathbf{T}}\mathbf{x}\right)^{h,n}+\Phi^{-1}\left(\rho^{h,n}\right)\sqrt{\left(\mathbf{x}^{h,n}\right)^{\mathbf{T}}\boldsymbol{\Omega}\mathbf{x}^{h,n}} \quad (3.8a)$$

subjectto

$$\boldsymbol{Zx^{h}}=\boldsymbol{b}\forall h=1,2,\dots k \quad (3.8b)$$

$$\sum_{h=1}^{k^{n}}\mathbf{x}^{\mathbf{h}}\leq\mathbf{d} \qquad \textstyle\sum_{h=1}^{k}\boldsymbol{x^{h}}\leq\boldsymbol{d} \quad (3.8c)$$

$$\mathbf{x}^{\mathbf{h}}-binary \quad \forall h=1,2,\dots.k \qquad \boldsymbol{x^{h}}-binary\forall h=1,2,\dots k \quad (3.8d)$$

Note that, the objective function (3.8a) in problem (P-6) is the weighted sum of mean path travel times $(\mu)^{T}\boldsymbol{x^{h,n}}$ and their standard deviations $\sqrt{\left(\mathbf{x}^{h,n}\right)^{\mathbf{T}}\boldsymbol{\Omega}\mathbf{x}^{h,n}}$ for each OD pair set. If $(\rho^{h,n})\geq 1/2$, then $\Phi^{-1}\left(\rho^{h,n}\right)\geq 0$, and the objective function becomes convex (Prekopa, 1995). Since the constraint set is linear, i.e. convex constraint set, the resulting deterministic nonlinear integer programming problem is a convex NL-IP.

When the variance of each link (*i, j*) is set to zero, the objective function omits the travel time variability, where only the mean total path travel times are minimized $\left(\sum_{n=1}^{N}\sum_{h=1}^{k}\boldsymbol{\mu^{T}x^{h,n}}\right)$with respect to constraints (3.8b)-(3.8d).

The basic structure of the resulting deterministic NL-IP model is:

$$\text{(P-7)}\quad \begin{array}{ll}\min & f(x)=c^{T}x+\left(x^{T}Bx\right)^{1/2}\\ \text{subject to} & g(x)\geq 0\end{array} \quad (3.9)$$

where the objective function is a convex nonlinear function with positive semi-definite matrix B and linear constraint set $g(x)$. Even though primal of this convex NL-IP problem involves non-differentiability of the terms in the objective function, dual of this problem, which is again a convex programming problem (nonlinearity is in the constraints in the dual formulation) can easily be solved (Sinha 1966). Thus, even though the original stochastic programming model is highly complex, after the theoretical transformation applied to the stochastic programming model, we obtain a nicely structured convex NL-IP model which can be solved in polynomial time. Next section focuses on the solution algorithm.

3.1.2 Solution Algorithm

In order to solve the proposed NL-IP problem KNITRO 6.0 solver is used. KNITRO is an optimization package developed for solving nonlinear optimization problems. It is designed for large-scale applications, but it is also effective on small and medium scale problems.

The basic form of the constrained nonlinear programming models is as follows:

$$\text{(P-8)} \qquad \begin{array}{ll} \min\limits_{x} & f(x) \\ \textit{subject to} & h(x) = 0 \\ & g(x) \le 0 \end{array} \qquad (3.10)$$

where $f : R^n \to R$ is a nonlinear objective function, $g : R^n \to R^m$ is the vector of inequality constraints, and $h : R^n \to R^t$ is the vector of equality constraints.

In order to solve the NL-IP problem, first branch-and-bound approach is employed to build a tree where each node is associated with a relaxation of problem P-8. Each node has a set of upper and lower bounds for variable *x*. Resulting nonlinear programming relaxation is solved via interior-point algorithm.

The algorithm implemented in this chapter is based on interior-point (or barrier) method proposed by Byrd et al. (1996). The algorithm incorporates the interior-point methods with line search and Newton approaches. Here, we provide a brief description of the interior-point algorithm. A more detailed explanation of the algorithm can be found in Byrd et al. (2006).

In the interior-point algorithm used in this chapter, each barrier sub-problem is formulated in the form of:

$$
\text{(P-9)} \qquad \begin{aligned} &\min_{x} && f(x) - \mu \sum_{i=1}^{m} \ln(sl_i) \\ &\textit{subject to} && h(x) = 0 \\ & && g(x) - sl = 0 \end{aligned} \tag{3.11}
$$

where sl is the vector of slack variables and $\mu > 0$ is the barrier parameter. The interior-point method consists of finding solutions of the barrier problem for a sequence of positive barrier parameters $\{\mu\}$ that converges to zero.

The Karush Kuhn Tucker (KKT) conditions for the above problem can be written as:

$$\begin{aligned} \nabla f(x) - \lambda_h A_h^T(x) - \lambda_g A_g^T(x) &= 0 \\ -\mu e + S\lambda_g &= 0 \\ h(x) &= 0 \\ g(x) - sl &= 0 \end{aligned} \tag{3.12}$$

where $e = (1,...,1)^T$, $S = diag(sl_1,...,sl_m)$, A_h and A_g are the Jacobian matrices corresponding to the equality and inequality constraint vectors respectively, and λ_h and λ_g represent vectors of Lagrange multipliers. In the line search approach, Newton's method is applied to the above problem given in eqn-3.12, backtracking if necessary so that the variables sl remain positive, and so that the merit function is sufficiently reduced.

To control the quality of the steps, the interior point algorithm makes use of the non-differentiable merit function:

$$\phi_\upsilon(x, sl) = f(x) - \mu \sum_{i=1}^{m} \log sl_i + \upsilon \|h(x)\|_2 + \upsilon \|g(x) - sl\|_2 \tag{3.13}$$

where $\upsilon > 0$. A step is acceptable only if it provides a sufficient decrease in ϕ_υ.

In this algorithm in each iteration, line search step is computed using direct linear algebra as described in Waltz et al. (2003). In particular, applying Newton's method to the above problem line search step is conducted:

$$\begin{bmatrix} \nabla_{xx}^2 L & 0 & -A_h^T(x) & -A_g^T(x) \\ 0 & \lambda_g & 0 & S \\ A_h(x) & 0 & 0 & 0 \\ A_g(x) & -I & 0 & 0 \end{bmatrix} \begin{bmatrix} d_x \\ d_{sl} \\ d_{\lambda_h} \\ d_{\lambda_g} \end{bmatrix} = - \begin{bmatrix} \nabla f(x) - \lambda_h A_h^T(x) - \lambda_g A_g^T(x) \\ S\lambda_g - \mu e \\ h(x) \\ g(x) - sl \end{bmatrix} \tag{3.14}$$

where L denotes the Lagrangian

$$L(x,s,\lambda_h,\lambda_g) = f(x) - \lambda_h^T h(x) - \lambda_g^T (g(x) - sl) \tag{3.15}$$

then the step d obtained from eqn-3.14 can be guaranteed to be a descent direction for the merit function (eqn-3.13). In this case the scalars $\alpha_{sl}^{\max}$ and $\alpha_z^{\max}$ are computed as:

$$\begin{aligned} \alpha_{sl}^{\max} &= \max\{\alpha \in (0,1] : sl + \alpha d_{sl} \geq (1-\tau)sl\}, \\ \alpha_{\lambda_g}^{\max} &= \max\{\alpha \in (0,1] : \lambda_g + \alpha d_{\lambda_g} \geq (1-\tau)\lambda_g\} \end{aligned} \tag{3.16}$$

with $\tau = 0.995$. If $\min(\alpha_{sl}^{\max}, \alpha_{\lambda_g}^{\max})$ is not too small, the line search algorithm computes the step lengths:

$$\begin{aligned} x^+ &= x + \alpha_{sl} d_x & sl^+ &= sl + \alpha_{sl} d_{sl} \\ \lambda_h^+ &= \lambda_h + \alpha_{\lambda_g} d_{\lambda_h} & \lambda_g^+ &= \lambda_g + \alpha_{\lambda_g} d_{\lambda_g} \end{aligned} \tag{3.17}$$

where α are the step lengths.

If in the resulting solution, some integer variables take non-integer values (e.g. x_i with value $\hat{x}_i$), the algorithm then selects one of these integer variables and branches on it. Branching generates two new NL-P problems by adding simple bounds $x_i \leq [\hat{x}_i]$ and $x_i \geq [\hat{x}_i] + 1$, respectively to the NL-P relaxation. One of the two new NL-P problems is selected and solved next. If the integer variables take non-integer values then branching is repeated, thus generating a branch-and-bound tree whose nodes correspond to NL-P problems and where an edge indicates the addition of a branching bound. If one of the following conditions are satisfied the corresponding node is abandoned: (1) an infeasible node is detected (then the whole subtree at this node is infeasible); (2) an integer feasible node is detected; (3) a lower bound on the NL-P solution of a node is greater or equal than the current upper bound. Once a node has been explored the algorithm backtracks to another node which has not been explored until all nodes are explored.

The main steps of the interior-point algorithm are given below (Byrd et al. (2006):

Choose x_0, sl_0 and parameters $0 < \alpha_{min} < 1$ and $0 < \eta$. Compute initial values for the multipliers $\lambda_h, \lambda_g > 0$, and the barrier parameter $\mu > 0$. Set k=0.
Repeat until a stopping test for the nonlinear program is satisfied:
  Repeat until the perturbed KKT conditions (eqn-12) are approximately satisfied:
    Factor the primal-dual system (eqn-14) and record the number `neig` of negative eigenvalues of its coefficient matrix.
    If `neig` $\leq m + t$
      Solve eqn-14 to obtain the search direction $d = (d_x, d_{sl}, d_{\lambda_h}, d_{\lambda_g})$
      Define $w = (x_k, sl_k)$ and $d_w = (d_x, d_{sl})$
      Compute $(\alpha_{sl}^{max}, \alpha_{\lambda_g}^{max})$ using eqn-16
      If $\min(\alpha_{sl}^{max}, \alpha_{\lambda_g}^{max}) > \alpha_{min}$
        Update the penalty parameter υ_k
        Compute a step length $\alpha_{sl} = \bar{\alpha}\alpha_{sl}^{max}, \bar{\alpha} \in (0,1]$ such that
        $\phi_\upsilon(w + \alpha_{sl} d_w) \leq \phi_\upsilon(w) + \eta D\phi_\upsilon(w; d_w)$
        If $\alpha_{sl} \geq \alpha_{min}$
          Set $\alpha_{\lambda_g} = \bar{\alpha}\alpha_{\lambda_g}^{max}$
          Set $(x_{k+1}, sl_{k+1}, (\lambda_h)_{k+1}, (\lambda_g)_{k+1})$ using eqn-17
        Endif
      Endif
    Endif
    Set $k \leftarrow k+1$
  End
Choose a smaller value for the barrier parameter μ
End

3.1.3 Simple Example

To illustrate the impacts of travel time variability on the shortest path selection, the proposed optimization model is implemented using the example network shown in Figure 3-1. For each link *(i, j)* travel times are assumed to be normally distributed. Mean and standard deviation of each link travel time are shown in the parenthesis above the corresponding link.

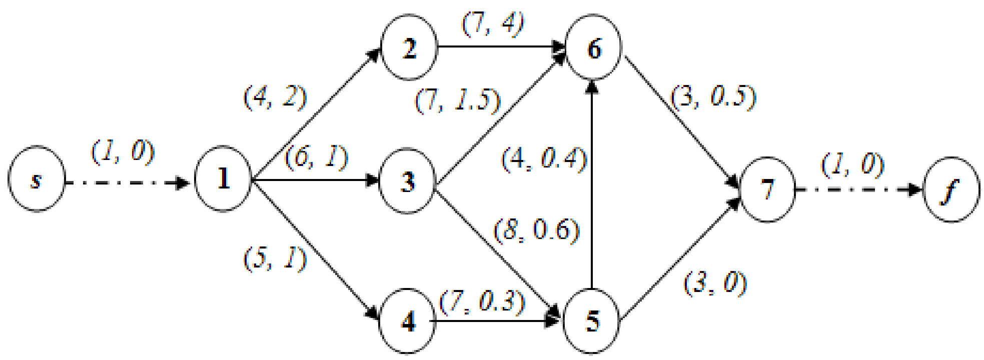


Figure 3-1 Example network with stochastic travel times

First, the k-PDP is found in deterministic case via problem (P-2). For simplicity, N is set to 1. $S = \{(1,3),(5,7)\}$, k is set to three, and s is set to two. The solution of the optimization problem yields to the following set of paths:

$x^1 = \{(s,1),(1{,}2),(2{,}6),(6{,}7),(7,f)\}$ (Travel time = 16 units)

$x^2 = \{(s,1),(1{,}4),(4{,}5),(5{,}7),(7,f)\}$ (Travel time = 17 units)

$x^3 = \{(s,1),(1{,}3),(3{,}5),(5{,}7),(7,f)\}$ (Travel time = 19 units)

Next, stochasticity introduced into the mathematical model, ρ is set to 0.9 for each path. The k-PDP is found in stochastic case via solving problem (P-5).

After inserting the corresponding values into the problem (P-5), the optimization model is solved in CPLEX. The solution yields to the following paths:

$x^1 = \{(s,1),(1{,}4),(4{,}5),(5{,}7),(7,f)\}$ (Travel time including variability = 18.34 units)

$x^2 = \{(s,1),(1{,}3),(3{,}5),(5{,}7),(7,f)\}$ (Travel time including variability = 20.49 units)

$x^3 = \{(s,1),(1,3),(3,6),(6,7),(7,f)\}$ (Travel time including variability = 20.39 units)

The results show that when travel time variability is introduced into the objective function; both mean and variance of path travel time are minimized simultaneously, allowing us to obtain set of paths with the least travel time and variability.

Note that, while solving probabilistic k-PDP problem above, set S is formed visually. However, in a very large network, set of rules are required to generate this set. Next section provides the details of the selection of set S for large transportation networks.

3.2 Disjointness Rate

The mathematical formulation presented in problem (P-6) finds the probabilistic k-PDP between a set of OD pairs, while ensuring that zone connectors always remain connected to the real network, and a pre-determined percentage of each path is disjoint from each other.

One of the key elements of this formulation lies in the determination of the size and elements of the overlapping link set S^n. This step is crucial in the creation of relevant path choice sets. The generated set should exclude highly similar paths that no traveler would ever differentiate between. Depending on the desired level of disjointness the modeler can modify the number of links in the overlapping link set s^n. In particular, if the modeler requires a high level of disjointness rate, the magnitude of s^n can be set to a number smaller than k^n. In the extreme case, when s^n is set to 1, the almost fully disjoint paths are determined, where only the zone connectors are overlapped among different paths. Moreover, links (*i, j*) to be included in the overlap set S^ncan be specified depending on the type of a specific application and scenario. For instance, a particular section of the network may have a major crossing, which most of the paths have to cover. Then, the modeler may wish to include these links in the set S^n.

Alternatively, during certain periods of the day, some road sections may experience recurring bottlenecks, and the transportation modeler may wish to generate paths which circumvent these bottlenecks. Then it would be desirable to prevent these links to be shared by certain paths, thus include them in set D^n.

In a small network, it is straight-forward to determine the size and elements of the set S^n. On the other hand, in large and very dense networks, manually creating this set can be tricky. The proposed model allows the modeler to form a user defined overlap constraint set. We determine the size and the elements of the set S^nbased on functional characteristics of the links. Since road type criterion covers important characteristics of each link such as speed, travel time, accessibility, generating path overlap set via this criterion allows us take into account several crucial parameters at once. Categories (labels) considered in the estimation process are expressway, freeway, principal arterial, major/minor arterial, local roads, and zone connectors. Based on these label-constraints, the set of overlapping links are determined.

To determine the size of set S^n, initially the first shortest path between the O-D pair *n* is determined. Then the ordered list of links are stored in the shortest path set, SP_1^n, where the size of set SP_1^n denotes the total number of links $\left(M^n\right)$ along this path. Then, disjointness rate is set to be a percentage of M^n.

Similarly, links (*i, j*) to be included in the set S^n are determined via label-constraints. In particular, each link in the first shortest path is labeled according to their functional characteristics and stored in the label set, L_1^n. Depending on the purpose of the path generation problem, the links which can be overlapped among different paths are included in the set S^n.

3.3 Sensitivity Analysis

In this section the proposed model is tested using several different networks using KNITRO 6.0 solver. The test networks include Sioux-Falls Network, a small-size network, and Northern New Jersey network, a large transportation network.

3.3.1 Sioux-Falls Network

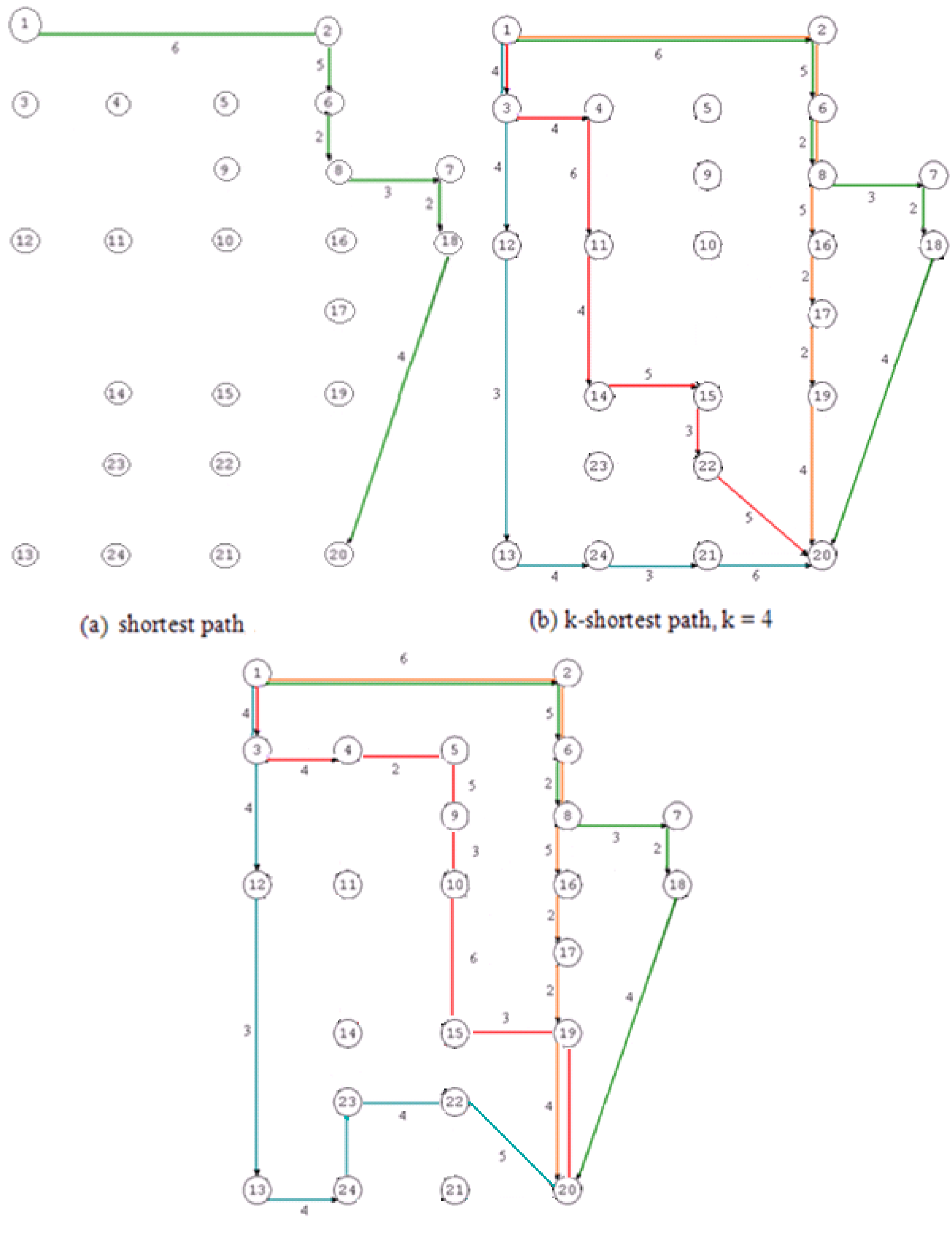
The first test network is selected from "Transportation Network Test Problems" database generated by Bar-Gera (2007). The network for Sioux-Falls has 24 nodes and 76 links. The origin node is selected as node 1, and destination node is selected as node 20, and k is set to be 4. Unfortunately, this database does not provide any information regarding the link labels. Thus, for the sake of illustration, all the outgoing links from node 1, and all the incoming links to node 20 are assumed to be zone connectors.

First, set of paths are found in the deterministic case via solving problem (P-2). While generating k-PDP, initially, the first shortest path is found. Figure 3-2-a highlights the links belonging to the first shortest path. The mean travel time of this path is 22 units.

Next, based on the link information for the first shortest path, sets *C, S* and *D* are determined. As mentioned before, set *C* consists of all the outgoing and incoming links from and to the origin and destination nodes, respectively. Hence, $C = \{(1,2), (1,3), (18,20), (19,20),$. Moreover, since link label information is not available in this formulation, the second and third links are included in the set $S = \{(2,6), (6,8)\}$, and all other links are included in set *D*. Figure 3-2-b illustrates the new set of paths generated by the model. Total elapsed time to generate these paths is recorded as one second.

Next, travel time variability and network reliability properties are included into the model via problem (P-6). Unfortunately, the database for the Sioux-Falls does not provide any information regarding the network reliability or the travel time variability. Thus, for the sake of illustration, a random number between zero and twice the link travel time are assigned as the variance of each link's travel time. Furthermore, ρ is set to 0.9 for each path, k is set to four, and *s* is set to two. Using these assumptions, initially, the first shortest path is found. The model successfully generates four different partially disjoint shortest paths between node 1 and 20. Total computational time required to generate these paths is recorded as one second. Figure 3-2-c highlights the probabilistic *k*-PDP generated using the proposed model. The results of the optimization problem indicate that when the variability of the link travel time is considered, paths are selected based on the combination of minimum travel time and variability. For instance, candidate path composed of links $\{(1,3), (3,12), (12,13), (13,24),$ has not been selected as a legitimate path, even though total travel time (24 units) is lower compared with path x^2

(26 units), due to the high travel time variability on links $\{(24,21),(21,20)\}$. Thus, path x^2 with lower weighted sum of mean travel time and variance is selected by the proposed model. Similarly, paths x^4 and candidate path $\{(1,3),(3,4),(4,11),(11,14),(14,15),(15,22),(22,20)\}$ both have the same travel time. However, due to high travel time variability of links $\{(4,11),(11,14)\}$, this candidate path is not included in the final path set.



(a) shortest path

(b) k-shortest path, k = 4

(c) k-shortest path, k = 4

Figure 3-2 *K*-shortest path problem results

3.3.2 Northern New Jersey Network

Next, the proposed stochastic integer programming model is tested on a large real network. The Northern New Jersey network, shown in Figure 3-3 consists of 5 418 nodes, 1 451 of which are zonal nodes and a total of 15 387 links. The input data required for the path set generation process are obtained from TP+, transportation planning software. These input data include: (1) loaded travel time of each link resulting from the assignment of separate OD demand matrices for am peak, pm peak and off-peak periods; (3) Node and link ID's, (4) highway and residential area type; (5) length, number of lanes, capacity and free flow travel time of each link.

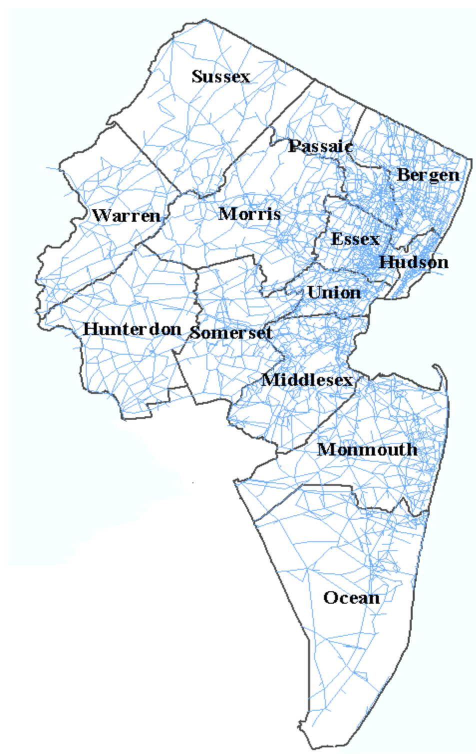


Figure 3-3 Map of Northern New Jersey

Using the link and node information provided for this network, proposed path set generation model is applied for different OD pairs. To observe the impacts of network reliability, for each case the mathematical model is solved both including and excluding travel time variability. Since, the database does not provide any information regarding link travel time variability, a random number between zero and twice the link travel time is assigned as the standard deviation of each link's travel time. While generating the path set for each OD pair, the links in the first shortest path labeled as freeways or expressways are included in set *D*. For illustration purposes disjointness rate constraint is chosen as 0.5, i.e. each pair of generated relevant paths will not share more than 50% arcs.

The overall disjointness rate is calculated as the lowest disjoint rate among all the paths between an O-D pair, and calculated using the following formulation

$$disjointess\ rate\ (i,j) = \min_{i,j}\left(\frac{Number\ of\ nonoverlapping\ links\ between\ paths\ i\ and\ j}{Total\ number\ of\ links\ on\ path\ i}\right)$$

(3.18)

Figure 3-4, Figure 3-5, and Figure 3-6 illustrate the path sets for sample OD pairs in Bergen, Middlesex and Monmouth Counties, respectively. In each figure, figure-a illustrates the set of paths excluding travel time variability. Similarly, figure-b presents the set of paths including travel time variability. Total elapsed time to generate different path sets ranges between 26 and 33 seconds depending on the network size and the type of the optimization model employed (Table 3.1). As the network size increases the time to find the optimal path set increases, as well. Similarly, the time required to solve the nonlinear integer model is slightly higher compared with the time required to solve the linear integer model. As expected, in each case, inclusion of variability results in longer trips with links that have lower travel time variances (Table 3.1). Overall, the mean travel time of each path set increase by 10% when network reliability is considered. Moreover, overall disjointness rate of each path with the shortest path is at least 60%. Similarly, overall disjointness rate among all paths ranges between 70% and 88%.

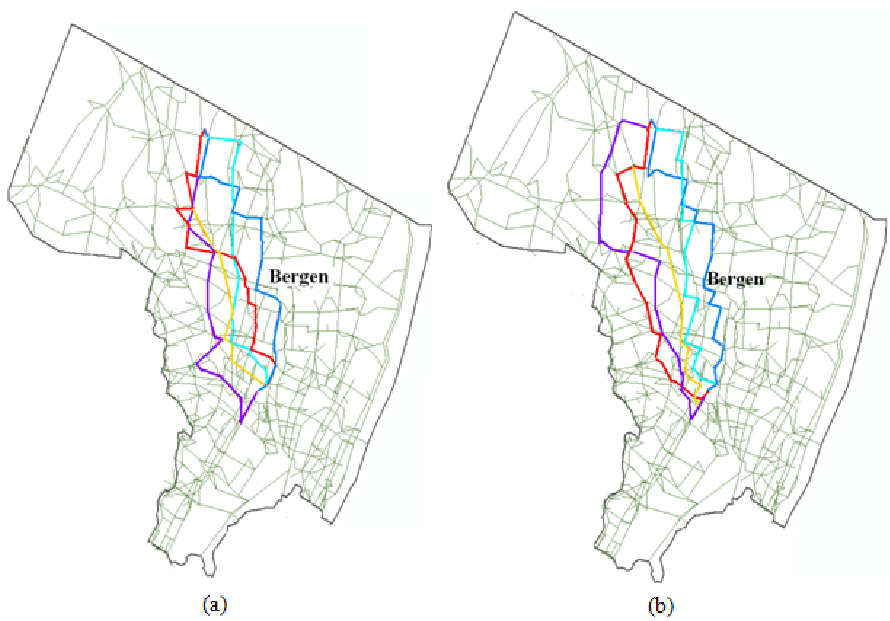


Figure 3-4 Path set – Bergen County (a) Excluding variability, (b) Including variability

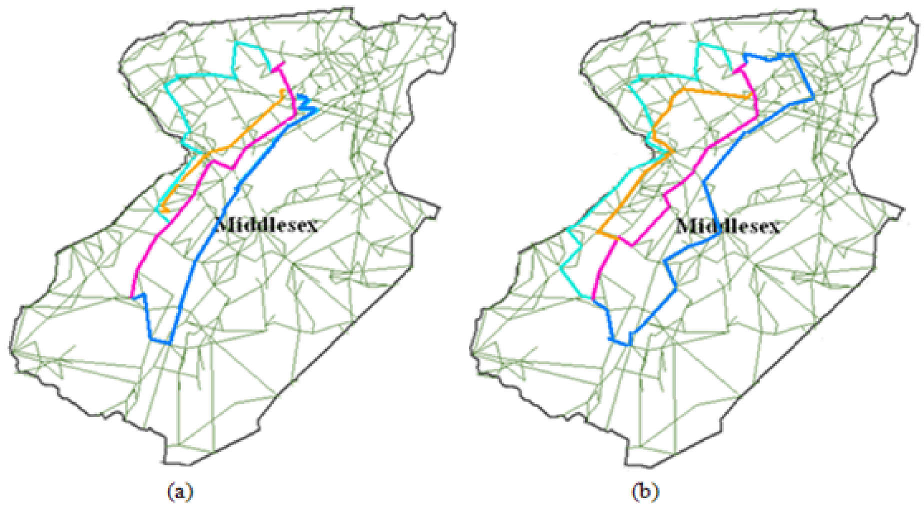


Figure 3-5 Path set – Middlesex County (a) Excluding variability, (b) Including variability

Figure 3-6 Path set – Monmouth County (a) Including variability, (b) Excluding variability

Table 3.1 Travel time and disjoint rate information for selected OD pairs

Path #	Bergen County O-D pair 63-136				Middlesex County O-D pair 591-734				Monmouth County O-D pair 742-882			
	Time inc. var.	**Disj. rate**	**Time exc var**	**Disj. rate**	**Time inc. var.**	**Disj. rate**	**Time exc. var.**	**Disj. rate**	**Time inc. var.**	**Disj. rate**	**Time exc. var.**	**Disj. Rate**
1	58	0.82	54	0.75	83	0.75	77	0.73	72	0.73	71	0.6
2	62	0.93	55	0.9	88	0.95	78	0.74	73	0.73	68	0.75
3	67	0.91	65	0.8	99	0.75	89	0.73	82	0.77	75	0.85
4	73	0.82	68	0.75	100	0.95	87	0.82	89	0.77	83	0.6
5	82	0.9	70	0.9	-	-	-	-	-	-	-	-
Mean	***68***	***0.88***	***62***	***0.82***	***92***	***0.85***	***83***	0.76	***79***	***0.75***	***74***	***0.7***
Std dev	***9.45***	***0.05***	***7.44***	***0.08***	***8.36***	***0.12***	***6.25***	0.04	***7.9***	***0.02***	***6.69***	***0.12***
CPU time	***33 sec***		***3 sec***		***28 sec***		***3 sec***		***26 sec***		***3 sec***	

3.4 Computational Performance

In evaluating different path set generation approaches, computational performance of the proposed approach needs to be evaluated. For this purpose, random OD pairs are generated from Northern New Jersey network and computational performance experiments are conducted. For each OD pair, total number of paths, average travel time per path, and disjointness rate values are calculated. A total of 75 different OD pairs are analyzed. Summary of results, shown in Table 3.2, exhibits similar computational performance with and without travel time variability. Including travel time variability leads to slightly higher travel times, while both approaches find the same number of total paths between different OD pairs. The average travel time per OD pair ranges between 17.20 minutes to 118.5 minutes with a standard deviation of 22.41

minutes. Furthermore, the minimum disjointness rate between every pair of paths ranges between 55% and 88%.

Table 3.2 Travel time and disjoint rate information for selected OD pairs

Statistics	**# of paths**	**Travel Time (min)**		**Disjointness Rate**	
		with variation	**without variation**	**with variation**	**without variation**
Total	404	-	-	-	-
Avg/ OD pair	5.39	69.49	67.32	0.68	0.66
Std	1	24.56	22.41	0.1	0.08
Max	7	124.53	118.5	0.88	0.85
Min	3	17.2	17.2	0.55	0.55

3.5 Comparison with Existing Methods

This section compares our proposed methodology with the existing approaches in the literature. In order to perform a proper comparison with the existing methodologies, the parameters of these methods were adjusted. In particular, zone connectors were not removed in order to increase the probability of finding more paths. Moreover, since none of the above algorithms consider travel time variability, while comparing these approaches with the proposed methodology, only the results obtained without the travel time variability are considered.

As mentioned in the introduction section, while calculating the path sets applying the above algorithms, first the master choice set needs to be generated. Then, this master set is further filtered according to desired disjointness rates among the paths. In order to investigate the significance of disjointness rate on the number of paths generated, two different types of filtering are applied. In the first filtering process, for each OD pair the paths with disjoint rates are less than 0.1 are eliminated, and no constraint is imposed on the travel time of the paths. This process allows us to eliminate all the paths that are exactly (or almost) the same, and the paths that share at least 90% of their links. In the second filtering process, for each OD pair, the paths with disjoint rates that are less than 0.5 and paths that have travel times that are double of the first shortest path, are eliminated. This process allows us to generate relevant path choice sets, and compare their results with ours.

The model specifications and assumptions imposed to the comparison algorithms are as follows:

A1. Labeling approach (Ben-Akiva et al., 1984): The shortest paths with respect to path attributes such as minimum travel time, distance, and free flow travel time; and maximum freeway, arterials, and expressway paths are calculated. The cost functions required for maximizing freeways, arterials and expressways are taken from Ben-Akiva et al. (1984).

A2. Link elimination approach (Azevedo et al., 1993): A modified version of the original formulation by Azevedo et al. (1993) is applied by repeating for 15 iterations of the following steps: (a) Compute the first shortest path by minimizing travel time, (b) Eliminate all the links on the shortest path except the zone connectors, (c) Recalculate the next shortest path.

A3. Link penalty approach (De la Barra et al., 1993): The original formulation proposed by De la Barra et al. (1993) is applied. The following steps are repeated for 15 iterations: (a) Compute the first shortest path by minimizing travel time, (b) Increase the travel time of all the links on the shortest path except the zone connectors by 40%, (c) Calculate the next shortest path.

A4. Randomization approach (Ramming, 2002): Shortest paths are computed by drawing impedances from normal distribution. The mean parameter of the normal distribution is taken as the link travel time, and variance is taken as the twice of the mean link travel time. A total of 16 draws are extracted.

A5. Doubly stochastic approach (Bovy and Fiorenzo-Catalano, 2007): In this approach the cost function in the form of $\beta * travel\ time$ is used. The parameter β is normally distributed with mean 1 and standard deviation 0.4 (Bovy and Fiorenzo-Catalano, 2007). Similarly, link travel times are normally distributed, where mean is the link travel time, and variance is the twice of the mean link travel time.

Table 3.3 summarizes the results when disjointness rate is set to 0.1, and no constraint is imposed on the travel time of the paths. Each method except the randomization approach on average generates three to four paths between each OD pair. The highest number of paths generated by these methods is seven, while the least number of paths is one, which is the first shortest path. The highest number of paths is generated via doubly stochastic approach (A5) followed by link penalty (A3) and link elimination

(A2) approaches. The least number of paths are generated by the simulation approach (A4). Furthermore, the highest disjointness rate is obtained from link elimination approach (A2) followed by link penalty (A3). This result is expected, since to calculate the next shortest path all the links in the previous path except the zone connectors are removed from the network. The most similar paths are generated by the simulation approach (A4) followed by doubly stochastic (A5) and labeling (A1) approaches. Provided that randomization approaches only change the magnitude of the link travel time, and the link attributes in the labeling approach are highly correlated, the paths generated via these methods are likely to be similar to each other.

Table 3.3 Path set generation results, disjointness rate = 0.1

Method	**Paths**					**Travel Time**				**Disjointness Rate**			
	Total	***Avg/ OD pair***	***Std***	***Max***	***Min***	***Avg/ OD pair***	***Std***	***Max***	***Min***	***Avg/ OD pair***	***Std***	***Max***	***Min***
Labeling	258	3.48	1.14	6	1	60.82	21.03	105.46	15.9	0.22	0.18	0.92	-
Link Elimination	265	3.58	1.03	5	1	72.32	20.3	117.4	21.9	0.896	0.12	0.96	-
Link Penalty	308	4.16	1.25	5	1	72.28	22.01	118.53	17.2	0.386	0.21	0.8	-
Simulation	170	2.29	1.51	7	1	53.75	17.78	93.44	14.8	0.13	0.15	0.75	-
Doubly Stochastic	348	4.71	2.35	7	1	64.04	21.71	138.34	18.8	0.18	0.09	0.52	-

When the disjointness rate criterion is increased and travel time constraint is imposed, almost 50% of the generated paths are eliminated except for link elimination approach (Table 3.4). Compared with the proposed *k*-PDP generation model, link elimination and the proposed approach generate the closest set of paths. However, even this method provides 35% less number of paths. Among all approaches, proposed *k*-PDP method performs the fastest followed by labeling, link penalty and link elimination approaches. Randomization methods, on the other hand, require more additional time to generate paths.

Table 3.4 Path set generation results, disjointness rate = 0.5

Method		***k*-PDP**	**Labeling**	**Link Elimination**	**Link Penalty**	**Simulation**	**Doubly Stochastic**
Paths	***Total***	404	134	265	172	84	118
	Avg/ OD pair	5.39	1.81	3.58	2.32	1.135	1.6
	Std	1	1.28	1.03	0.74	0.38	1.8
	Max	7	6	5	3	3	14
	Min	3	1	1	1	1	1
Travel Time (min)	***Avg/ OD pair***	67.32	62.6	72.32	72.28	53.75	64.04
	Std	22.41	21.28	20.3	22.01	17.78	21.71
	Max	118.5	110.3	117.4	118.5	93.44	138.3
	Min	17.2	16.5	21.9	17.2	14.8	18.8
Disjointness Rate	***Avg/ OD pair***	0.66	0.63	0.91	0.62	0.66	0.5
	Std	0.08	0.13	0.06	0.09	0.11	0.1
	Max	0.85	0.92	0.96	0.8	0.88	0.81
	Min	0.55	-	-	-	-	-
Computation Time (sec)	***Total***	215.25	410.84	678.61	574.91	2524.1	2648
	Avg/ OD pair	2.87	5.29	9.17	7.77	34.11	35.8
	Std	1.96	2.19	4.81	3.91	9.06	7.82
	Max	6.42	10.95	19.84	14.65	63.47	56.2
	Min	2.11	1.02	0.99	0.71	19.57	22.8

Figure 3-7 illustrates the distribution of cumulative number of paths among different approaches. The number of unique paths generated by the proposed *k*-PDP methodology is distinctively high compared with all the previous approaches. Overall results of computational experiments show a superior performance of the proposed methodology over previous approaches provided by Ben-Akiva et al. (1984), Azevedo et al. (1993), De la Barra et al. (1993), Ramming (2002) and Bovy and Fiorenzo-Catalano (2007).

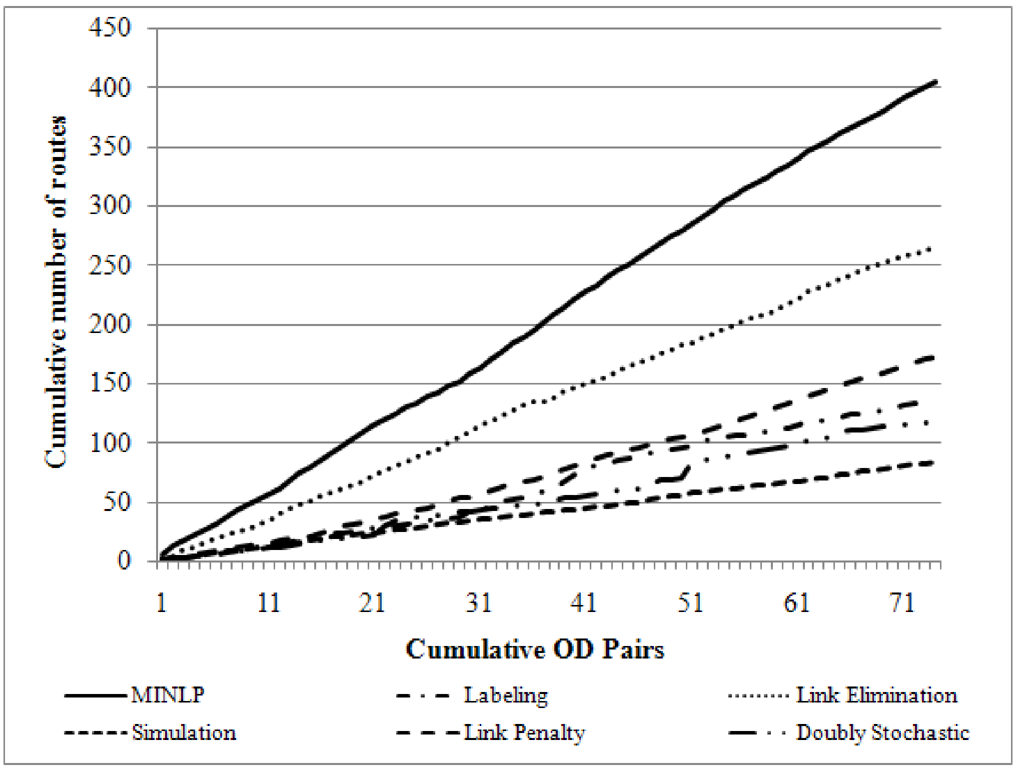


Figure 3-7 Cumulative number of paths

3.6 Conclusions and Discussions

Identification of path choice sets is crucial in transportation related problems such as traffic assignment, route choice, vehicle dispatching, or advanced traveler information systems. Correctness and accuracy of travel demand estimates and predictions depend on the quality of the adopted choice sets (Swait and Ben-Akiva, 1985, 1987; VanderWaerden et al., 2004).

The generated paths in the choice set should include all relevant paths, while excluding unrealistic paths that no traveler would ever consider, and highly similar paths that no traveler could differentiate between.

In this chapter a mathematical programming model is proposed to optimally generate probabilistic *k*- partially disjoint paths between different origin and destination pairs. In particular this approach proposes a stochastic nonlinear integer programming model that takes into account travel time variability and network reliability of the transportation network, while limiting the links overlapped among different paths. Then, via a novel transformation we obtain its equivalent deterministic convex nonlinear integer programming model which can be solved in polynomial time.

The path relevance criteria are directly incorporated into the optimization model by minimizing mean travel time, travel time variability and path overlap. Unlike most of the previous models which can compute paths from only one origin to all destinations, the proposed model can simultaneously compute the paths between multiple OD pairs; which improves the computational costs.

Unlike previous approaches in the literature, proposed methodology is not based on heuristics, eliminates the need for the filtering step from the choice set generation and generates path sets at a desired dissimilarity level while minimizing the travel time and variability of these paths. Dissimilarity among different paths is determined via user defined constraints defined according to functional types of the links. Dissimilarity constraint imposed into the optimization model generates paths that are not only disjoint with respect to the first shortest path but among each other as well.

Several computational experiments using small and large real-life networks (Sioux-Falls and Northern New Jersey Networks) show that when network reliability is included in the mathematical model, weighted sum of path travel time and its standard deviation is minimized and slightly longer trips with less variability are generated. Case studies confirm the applicability of the proposed methodology on real transportation networks.

Computational experiments also show that proposed probabilistic *k*- partially disjoint paths model performs better than previous approaches for the test cases presented in this chapter. The overlap rate of the generated path sets among different origin and destination pairs are significantly lower compared with the existing methodologies. The developed stochastic nonlinear integer programming formulation for determining probabilistic *k*-partially disjoint paths can be an important tool for researchers who wish to generate path choice sets that also take into account probabilistic distribution of link travel times.

CHAPTER 4. DAY-TO-DAY LEARNING

In this chapter, a new methodology is proposed to understand travelers' day-to-day learning behavior under steady-state conditions and when changes are imposed into the transportation system.

This thesis introduces a new novel day-to-day learning framework to model travelers' departure time and route choice behavior under non-equilibrium network conditions due to major disturbances, such as changes in the congestion pricing policies, and building of new road sections. An agent-based learning system via Bayesian-SLA is designed which can learn the best possible actions and model travelers' day-to-day travel choices in a non-stationary stochastic environment. In the proposed approach, each traveler maintains a choice probability profile for the available alternatives, and updates his/her probability profile based on previous travel choices, exhibiting a tendency to search for satisfying choices rather than the best behavior. Thus, bounded rationality is included into the probability profile, where travelers learn selectively. This reflects travelers' capability in predicting future traffic conditions and their inertia in changing behavior. To overcome unrealistic assumption of identical traveler attributes, the proposed approach introduces user heterogeneity into day-to-day learning modeling via Bayesian inference approach, and assumes that learning parameters follow probabilistic distributions across the population. The estimated learning parameters reflect travelers' perception about the system and their response to the experienced traffic conditions.

The novelty of this work is that the proposed learning approach combines traveler heterogeneity and rationality in a single framework to predict travelers' day-to-day departure time and route decisions. Moreover, for the first time in the transportation science, based on Bayesian framework and Bounded Rationality approach, this learning model estimates distribution of the optimal learning parameters. Imposing probability distributions to the learning parameters allows us to investigate the differences across the individuals rather than the differences between classes of users. Moreover, the proposed day-to-day learning model is applied to real world data, which allows us to investigate the evaluation, convergence, and stability of dynamic systems.

We believe that the proposed work will give both practitioners and researchers an insight on how to understand traveler choice mechanism, and model the impacts of various changes in the transportation system on the day-to-day adaptation of travelers.

4.1 Day-to-Day Learning Model

In this chapter, commuters' day-to-day learning behavior on the basis of experienced travel choices and user-specific characteristics is modeled via Bayesian-SLA theory. In SLA, automaton attempts a solution to the problem without any information on the optimal action. One action is selected at random, response from the environment is observed, action probabilities are updated based on that response, and the procedure is repeated. Stochastic automaton acting as described to improve its performance is called a learning automaton (Narendra and Thathachar 1989). The objective in the design of the automaton is to determine how the choice of the action at any stage should be guided by past actions and responses.

Learning automata is concerned with the analysis and synthesis of the automata which operate in random environments. In this section we describe the random environment, structure and characteristics of the automata, and the mathematical tools that are applicable to the analysis of such systems.

4.2 Environment

In SLA, environment, in our case the transportation system, is defined as a large class of unknown random media in which an automaton (traveler) can operate. Mathematically, an environment is represented by a triple$\{\alpha, c, \beta\}$, where $\underline{\alpha}$ represents a finite action/input set (travel choice in our model), $\underline{\beta}$ represents an output set (utility experienced from a choice), and *c* is a set of penalty probabilities, where each element c_i corresponds to one action α_i of the set $\underline{\alpha}$. The action $\alpha(n)$ of the automaton is then applied to the environment at time *t*=*n*. Consequently, c_i represents the probability that the action α_i will result in a penalty output. The elements of *c* are defined as:

$$Pr(\beta(n) = 1 | \alpha(n) = \alpha_i) = c_i \qquad (i = 1,2, \dots r) \tag{4.1}$$

Several models are defined by the response set of the environment. Models, in which the output can take only, 0 or 1, are referred to as P-models. In this case, response value of 1 corresponds to an “unfavorable” (failure) response, while output of 0 means the action is “favorable.” The focus of this chapter is P-models.

4.3 The Stochastic Automaton

The automaton takes in a sequence of inputs and puts out a sequence of actions. Mathematically, the automaton can be represented by a quintuple $\left\{\underline{\Phi}, \underline{\alpha}, \underline{\beta}, F(.), H(.)\right\}$ (Figure 4-1) (Narendra and Thathachar, 1989):

- $\underline{\Phi}$ is asset of internal states
- $\underline{\alpha}$ is a set of actions (or outputs of the automaton)
- $\underline{\beta}$ is a set of responses (or inputs from the environment)
- $F(.): \underline{\Phi} \times \beta \rightarrow \underline{\Phi}$ is a stochastic function that maps the current state and input into the next state
- $H(.): \underline{\Phi} \times \beta \rightarrow \underline{\alpha}$ is a function that maps the current state and input into the current output

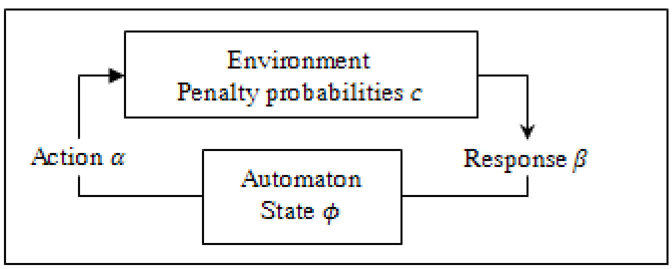


Figure 4-1 The automaton and the environment (Narendra and Thathachar, 1989)

4.4 Behavior Updating Mechanism

An automaton generates a sequence of actions on the basis of its interaction with the environment. Each action results in a favorable or unfavorable response. In this chapter, whether a choice is favorable or unfavorable is determined via experienced utility value and deviation from desired arrival time of the selected choice. Moreover, it is assumed that travelers exhibit a tendency to search for satisfying choices rather than the best behavior; thus they do not have the cognitive ability to process all the information simultaneously and are happy with a good solution. To incorporate this kind of behavior, bounded rationality (BR) approach (Simon 1955) is included in the behavior updating mechanism. Various assumptions considered in this chapter while applying BR in departure time choices are:

- Based on former choices, a traveler has personal experiences. From these experiences, s/he can learn about the characteristics of the actions. Information about the actions not chosen by the traveler can only be updated, if s/he has chosen them in the past.
- To account for BR, we use indifference bands. As long as the outcome of the travel choice falls within the indifference bands, the travelers will not update their choice. In calculating the indifference bands, $\pm$10% confidence intervals are used.

The utility values employed in the behavior updating mechanism are estimated via Hierarchical Bayesian Mixed Logit (HB-ML) models. Next section provides the details of this framework.

4.4.1 Hierarchical Bayesian Mixed Logit Model

One of the important contributions of this chapter is the introduction of the population heterogeneity through the use of HB-ML models. This is especially important in the case of a learning model where different users have different learning behavior that can be represented through varying the coefficients of the model.

Most discrete choice models are based on random utility theory (Ortuzar and Willumsen, 2001). Suppose a sample of N independent decision makers are observed, and let the y_n be the observed choice of decision maker n, where $Y = \{y_1, \dots, y_N\}$ is the set of observed choices for the entire sample. The utility that person n obtains from alternative j can be represented by:

$$U_{nj} = \sum_{k=1}^{K} \theta_{jk} x_{njk} + \varepsilon_{nj} \tag{4.2}$$

where U_{nj} is the utility associated with alternative j for individual n, x_{njk} is the value of the attribute k of alternative j for individual n, θ_{jk} is the parameter associated with this attribute, and ε_{nj} is a stochastic component that reflects everything that the modeler cannot measure or observe.

The parameters θ_{jk} are usually considered constant for all individuals although they can vary across alternatives. If we allow for random variations in the taste parameters (θ_n) and use the same distribution for the error terms (ε_{nj}), we get a mixed logit (ML) random parameters model, in which utility of alternative j is given by the following expression (Train, 2003):

$$U_{nj} = \theta_n x_{nj} + \varepsilon_{nj} \tag{4.3}$$

where $\varepsilon_{nj} \sim Gumbel(0, \sigma^2)$ and $f(\theta_n) \sim Normal_k(\theta_n | b, \Sigma)$, and b is the vector of means for the k explanatory variables, Σ is the $k \times k$ variance-covariance matrix.

This model can be further be extended by imposing hyper-parameters on parameters b and Σ via Bayesian approach, and we obtain Hierarchical Bayesian Mixed Logit (HB-ML) model. With recent developments in statistics, Bayesian procedures have become powerful techniques for estimating discrete choice models. From Bayesian point of view, logit model is based on the posterior distribution of unknowns of interest instead of maximum likelihood estimation. The Bayesian framework avoids two of the most important difficulties associated with classical approaches. First, unlike probit and some mixed logit models which require maximization of the simulated likelihood function, Bayesian procedures do not require maximization of any function. Second, desirable estimation properties, such as consistency and efficiency, can be attained under more relaxed conditions with Bayesian procedures than classical ones (Train, 2003).

In HB-ML, coefficients of the utility function are assumed to vary in the population rather than being fixed at the same value for each person. Thus, unlike the classical approach, in Bayesian statistics, parameters are treated as random variables, and prior knowledge about parameter vector θ_n (such as coefficients of the logit model) is represented by a prior distribution, $f(\theta_n)$. The prior distribution can either be based on previous empirical work, or researcher's subjective beliefs. For instance, while it is difficult to impose even simple non-negativity constraints (e.g. negativity constraint of travel time or travel cost coefficients) on standard discrete choice model estimators, in case of the Bayesian estimators, the specified prior distribution quantifies all the previous knowledge on the model parameters by prior distribution $f(\theta_n)$ (Andrews, 1999). In fact, Poirier (1988) claims that the subjective Bayesian approach is the only approach consistent with the usual rational model adopted by economists and transportation researchers to explain consumers' choices under uncertainty.

As mentioned above, while estimating HB-ML model, the researcher imposes priors on parameters b and Σ. The elicitation of any available prior information and its formulation into a prior distribution are usually difficult for the multi-parameter case, as in multinomial regression models. Different types of noninformative and informative prior distributions have been proposed for Bayesian binary regression models in the literature. Some of these priors are improper uniform prior, Jeffreys prior, hierarchical prior structures, Laplace priors, and empirical Bayes approaches (Ibrahim and Laud, 1991; Bedrick et al., 1996). In this study, to specify the values of distribution for b and Σ, normal distribution with large variance is assumed for b, and inverted Wishart (IW) distribution with m degrees of freedom and inverse scale matrix ψ, the m- dimensional identity matrix is assumed for the prior distribution of Σ. In general, these parameters are called the hyper-parameters:

$$f(b|b_0,S_0) = Normal(b|b_0,S_0) = \frac{1}{(2\pi)^{k/2}}\frac{1}{|S_0|^{1/2}}exp\left\{-\frac{1}{2}(b-b_0)^T S_0^{-1}(b-b_0)\right\} \quad (4.4)$$

$$f(\Sigma|\psi,m) = IW(\Sigma|\psi,m) = \frac{|\psi|^{m/2}|\Sigma|^{-(m+k+1)/2}\exp\left(-trace(\psi\Sigma^{-1})/2\right)}{2^{mk/2}\Gamma_k(m/2)} \quad (4.5)$$

where;

b_0 = k-dimensional mean vector

S_0= kxk dimensional large covariance matrix

m =degree of freedom

ψ = Positive definite mxm dimensional inverse scale matrix (assumed to be Identity matrix)

$\Gamma_k(.)$= Multivariate gamma function

Given the parameter vector θ, the probability of traveler n's observed choices, conditional on θ is represented by:

$$P(y_n|\theta) = \frac{e^{\theta^T x_{njy_n}}}{\sum_j e^{\theta^T x_{nj}}} \tag{4.6}$$

Then, the probability not conditional on θ is the integral of $P(y_n|\theta)$ over all θ, depending on the prior distribution:

$$P(y_n|b, \Sigma) = \int P(y_n|\theta)\, \phi(\theta|b, \Sigma) d\theta \tag{4.7}$$

where $\phi(\theta|b, \Sigma)$ is the normal density with mean b and variance Σ. This $P(y_n|b, \Sigma)$ is the HB-ML probability function. This probability is the behavioral model that relates the explanatory variables and parameters to the outcome. There is a precise relationship between prior and the posterior distribution linked through the likelihood function. Let $L(Y|\theta)$ be the likelihood function of the observed data, formulated as:

$$L(Y|\theta) = \prod_n P(y_n|\theta_n) \tag{4.8}$$

Then, based on Bayes' rule, the posterior distribution of the parameter vector, $f(\theta|Y)$, is represented as:

$$f(\theta|Y) = \frac{L(Y|\theta)\phi(\theta|b,\Sigma)}{\int L(Y|\theta) d\theta} \tag{4.9}$$

The posterior distribution summarizes the knowledge about the unknown parameter, θ, given the information contained in the data (represented by the likelihood function) and the prior information. Since $L(Y) = \int L(Y|\theta) d\theta$ is the normalizing constant independent of parameter θ, which assures that the posterior distribution integrates to 1, equation (8) can be stated in a more concise way, such that the posterior distribution is proportional to the multiplication of the prior distribution and the likelihood function:

$$f(b, \Sigma, \theta_n \ \forall n | Y) \sim \prod_n P(y_n|\theta_n)\phi(\theta_n|b, \Sigma)\, f(b) f(\Sigma) \tag{4.10}$$

where $f(b)$ is $Normal(b|b_0, S_0)$, with large variance and $f(\Sigma)$ is $IW(\Sigma|\psi, m)$.

Unfortunately, the above posterior distribution is a complex multidimensional function with no closed form function. We thus require computation methods to integrate the posterior distribution via sampling methods such as modern Bayesian Markov Chain Monte Carlo (MCMC) algorithms. In this study, using Random Walk Metropolis (RWM) algorithm we produce MCMC samplers to estimate HB-ML model for the travel choice at PANYNJ and NJ Turnpike facilities. Based on the prior information and likelihood function, RWM algorithm approximates the asymptotic normal distribution:

$$f(\theta|Y) \sim |H|^{\frac{1}{2}} \exp\{\tfrac{1}{2}(\theta - \hat{\theta})^T H(\theta - \hat{\theta})\} \tag{4.11}$$

The RWM proposal distribution is centered at the current value of θ and has variance-covariance matrix $H = T(B_0^{-1} + C^{-1})^{-1}T$. In this formulation T is the diagonal positive definite matrix of the RWM tuning parameter (set to a constant value such that the acceptance probability, α, is between 0.2 and 0.5), and C is the large sample variance-covariance matrix of the maximum likelihood estimates. The Metropolis class of algorithms is a general-purpose approach to producing Markov chain samplers. The main idea behind the Metropolis approach is to generate a Markov chain with the posterior, as its invariant distribution by appropriate modifications to a related Markov

Chain that is relatively easy to simulate from (Rossi, 2005). In this study RWM algorithm is used to obtain all unknown parameters. The goal of the RWM algorithm is to construct a MCMC sampler that has a specified equilibrium distribution π. The summary of the steps RWM algorithm is as follows:

1. Define a Markov Chain as follows. Start with θ_0
2. Draw a candidate value $Z = \theta + \varepsilon$ (where $\varepsilon \sim Normal(0, s^2)$)
3. Compute $\alpha = min\left(1, \frac{\pi(Z)}{\pi(\theta)}\right)$
4. With probability α, accept the candidate and set $\theta_1 = Z$, otherwise set $\theta_1 = \theta_0$
5. Repeat as necessary

The main advantage of Metropolis algorithms over Gibbs sampler is that, Gibbs sampler is useful for models built up from hierarchies of relatively standard distributions. However, there are many problems for which the conditional distributions are not of a known form that is easy to simulate from (Rossi, 2005). For this reason, it is useful to have a more general-purpose tool, such as Metropolis algorithms.

4.5 Reinforcement Schemes

In stochastic systems, after determining whether an observed action is favorable or not, probability values for each action are updated at every state using a reinforcement scheme. In general terms a reinforcement scheme can be represented as (Narendra and Thathachar 1989):

$$p(n+1) = T[p(n), \alpha(n), \beta(n)] \tag{4.12}$$

where T is mapping. If $p(n+1)$ is a linear function of $p(n)$, the reinforcement scheme is said to be linear, otherwise it is termed nonlinear. Since this chapter utilizes a linear reinforcement scheme with multi-actions, the update process of action probabilities in a linear environment is discussed in detail. This kind of learning scheme is called linear reward-penalty learning scheme and denoted by $L_{R-\epsilon P}$.

For an r-action learning automaton, the linear reinforcement scheme is given as (Narendra and Thathachar 1989):

$$If\ \alpha(n) = \alpha_i,$$

$$\beta(n) = 0 \Rightarrow \begin{cases} p_j(n+1) = (1-a).\,p_j(n) \quad \forall j \neq i \\ p_i(n+1) = p_i(n) + a.\,[1 - p_i(n)] \end{cases} \tag{4.13}$$

$$\beta(n) = 1 \Rightarrow \begin{cases} p_j(n+1) = \frac{b}{r-1} + (1-b).\,p_j(n) \ \forall j \neq i \\ \qquad p_i(n+1) = (1-b).\,p_i(n) \end{cases}$$

where 0<a<1 is the reward parameter, and 0<b<1 is the penalty parameter of the reinforcement scheme.

The concepts associated with the convergence of SLA require sophisticated mathematical tools, and the nature of convergence depends on the kind of reinforcement scheme employed (Narendra and Thathachar 1989). The multi-action automaton using linear reward-penalty scheme$L_{R-\epsilon P}$, is expedient for all initial action probabilities and in all stationary random environments, i.e. the automaton will behave better than the pure chance automaton. The details of the derivation for the expedient criterion can be found in Narendra and Thathachar (1989).

In previous studies, learning parameters (*a, b*) were estimated via trial and error approach (Narendra and Thathachar 1989, Ozbay et al. 2001, Ozbay et al. 2002, Ozbay and Yanmaz-Tuzel, 2006). This study, on the other hand, utilizes Bayesian Inference theory, and estimates the posterior distribution of these parameters. Next section provides the details of the estimation process.

4.6 Posterior Distribution of Learning Parameters

In this chapter, learning parameters (*a, b*) are estimated via Bayesian Inference approach. In particular, given the likelihood of the observations and the prior information regarding parameters (*a, b*), joint posterior distribution of the learning parameters are estimated. Unlike maximum likelihood analysis, the aim of a Bayesian analysis is not to provide so-called point estimates of the model parameters; the result of the analysis is the posterior probability distribution itself. With this approach, it is possible to introduce user heterogeneity into the estimation process, and to investigate distribution of the learning parameters among different users. The proposed likelihood function of the observations is estimated via following equation:

$$p(D|a,b) = \prod_{k=1}^{K}\prod_{n=1}^{N}\prod_{i=1}^{r} p_{ki}(n)$$

$$p(D|a,b) = \prod_{k=1}^{K}\prod_{n=1}^{N}\prod_{i=1}^{r}\Big\{\big[p_{ki}(n-1) + a\big(1-p_{ki}(n-1)\big)\big]^{\alpha_{ki}(n-1)\left(1-\beta_k(n-1)\right)} \cdot$$

$$[(1-a)p_{ki}(n-1)]^{\left(1-\alpha_{ki}(n-1)\right)\left(1-\beta_k(n-1)\right)} \cdot$$

$$[(1-b)p_{ki}(n-1)]^{\alpha_{ki}(n-1)\beta_k(n-1)} \cdot \tag{4.14}$$

$$\left[\frac{b}{r-1}+(1-b)p_{ki}(n-1)\right]^{(1-\alpha_{ki}(n-1))\beta_k(n-1)}\Bigg\}$$

where;

$p(D|a,b)$: Likelihood function of the observations *D* given learning parameters (*a, b*)

k: index for users (*K*: total number of users)

n: index for days (*N*: total number of days)

i: index for choices (*r*: total number of choices)

$p_{ki}(n-1)$: probability of selecting choice *i* for user *k* on day (*n-1*)

$$\alpha_{ki}(n-1)=\begin{cases}1 & \text{if user } k \text{ selects choice } i \text{ on day } (n-1)\\ 0 & \text{otherwise}\end{cases}$$

$$\beta_k(n-1)=\begin{cases}0 & \text{if user } k \text{ experiences a favorable action on day } (n-1)\\ 1 & \text{otherwise}\end{cases}$$

Similarly, the prior distribution of the learning parameters (*a, b*) can be represented by $p(a,b)$. In this chapter, Dirichlet, Normal and Uniform distributions are tested as prior distributions. For illustration purposes Normal prior distribution of $p(a)$ and $p(b)$ is provided here. In this case, functional form of joint prior distribution is as follows:

$$p(a,b)=Dirichlet(a,b|\alpha_a,\alpha_b)=\frac{\Gamma(\alpha_a+\alpha_b)}{\Gamma(\alpha_a)\Gamma(\alpha_b)}a^{\alpha_a-1}b^{\alpha_b-1} \tag{4.15}$$

$$p(a,b)=\frac{1}{2\pi|\boldsymbol{\Sigma}|^{1/2}}exp\left\{-\frac{1}{2}(\boldsymbol{x}-\boldsymbol{\mu})^T\boldsymbol{\Sigma}^{-1}(\boldsymbol{x}-\boldsymbol{\mu})\right\} \tag{4.16}$$

$$(a,b) = U(a,b|x_a,y_a,x_b,y_b) = \begin{cases} \frac{1}{(y_a-x_a)(y_b-x_b)} & x_a \leq a \leq y_a \wedge x_b \leq b \leq y_b \\ 0 & other\ wise \end{cases}$$

(4.17)

where;

α_a, α_b = Hyper-parameters for *a* and *b*

$\Gamma(.)$ = Gamma distribution

$\boldsymbol{\mu} = \begin{bmatrix} \mu_a \\ \mu_b \end{bmatrix}$

$\boldsymbol{\Sigma} = \begin{bmatrix} \sigma_a^2 & 0 \\ 0 & \sigma_b^2 \end{bmatrix}$

x= (*a, b*)

x_a, y_a= Lower and upper limits for learning parameter *a*

x_b, y_b= Lower and upper limits for learning parameter *b*

Finally, the posterior distribution of the learning parameters given the observations, $p(a,b|D)$, can be calculated using Bayes' theorem:

$$p(a,b|D) = \frac{p(D|a,b)p(a,b)}{\iint_{a,b} p(D|a,b)p(a,b)} \tag{4.18a}$$

$$p(a,b|D) = \frac{p(D|a,b)p(a,b)}{p(D)} \tag{4.18b}$$

Since $p(D)$ is independent of (*a, b*) the posterior distribution of the learning parameters will be proportional to the multiplication of the likelihood function and the prior distributions of the learning parameters:

$$p(a, b|D) \sim p(D|a, b)p(a, b) \quad (4.19)$$

Posterior distribution of the learning parameters is very complex multidimensional function which requires integrating. Thus, in order to obtain the mean and variance of the parameters (a, b) Metropolis-Hastings (M-H) algorithm is used. The Metropolis-Hastings algorithm is a rejection sampling algorithm used to generate a sequence of samples from a probability distribution that is difficult to sample from directly. The details of the algorithm can be found in Gelman et al. (2003). Since inference of the posterior distribution of (a, b) is based on the simulation of the posterior distribution by construction of a MCMC via M-H algorithm, the chain needs to be monitored and tested for convergence. To ensure MCMC convergence, Heidelberger and Welch (1983) first test diagnostic was employed. This diagnostic compares the observed sequence of MCMC samples to a hypothetical stationary distribution, using the Cramer-von-Mises statistic. The test iteratively discards the first 10% of the chain until the null hypothesis is not rejected (i.e. the chain is stationary), or until 50% of the original chain remains. If the null hypothesis is rejected each time, the stationarity test fails. For those samples which pass the stationarity test, a second test which calculates a $(1-\alpha)\times 100\%$ confidence interval on the sample mean is executed. The half-width of this interval is compared to the mean over the same interval; if the ratio of the mean to the half-width is larger than some threshold, the test fails. In the final estimation only the parameters which have passed both tests are included. Given these specifications, next section focuses on the empirical testing of the proposed model.

4.7 Calibration and Validation of the Learning Parameters

Calibration and validation are important processes in the development and application of day-to-day DTA models. These processes are developed to ensure that the models accurately replicate the observed traffic condition and driver behavior.

Model calibration is a process whereby the values of model parameters are adjusted so as to match the simulated model outputs with observations from the study site. It is usually formulated as an optimization problem to determine the best set of model parameter values in order to minimize the discrepancies between the observed and simulated values (Toledo, 2003). The calibration process is then to modify the values of the model parameters $\{\theta\}$, so to find the best set of values which minimizes *F*. The proposed objective function *F* minimizes the difference between observed and simulated volumes:

$$\min F = \sum_{\theta} \sum_{n} \left[\left(\frac{q_n^{sim} - q_n^{obs}}{q_n^{obs}} \right)^2 \right] \tag{4.20}$$

where;

q_n^{sim}: Simulated link flows in day *n*

q_n^{obs}: Observed link flows in day *n*

After determining the optimal set of parameters from the calibration process, a validation process is performed in order to determine whether the simulation model replicates the real system. Mean standard errors (MSE) are calculated for each day the validation process:

$$MSE = \frac{1}{L}\sum_{l=1}^{L}\frac{|q_l^{sim}-q_l^{obs}|}{q_l^{obs}} \tag{4.21}$$

4.8 Applications

The proposed Bayesian-SLA framework is tested on two different case studies. The first case study focuses on the impacts of January 2003 toll increase on the day-to-day departure time choice behavior of NJ Turnpike travelers. The second case study focuses on the impacts of December 2005 15X Interchange installation, on the day-to-day departure time and route choice behavior of NJ Turnpike travelers.

4.8.1 Case Study 1: NJ Turnpike Toll Increase

The first case study focuses on the impacts of January 2003 toll increase on the day-to-day departure time choice behavior of NJ Turnpike travelers. In January 2003 NJ Turnpike Authority has increased the toll levels on NJ Turnpike by 5-10% for E-ZPass users, and by 17% for cash users. Table 4.1 summarizes the changes in the toll levels for interchanges between 1 and 18E.

Table 4.1 Toll levels at NJ Turnpike

	Passenger Cars				Tractor Trailers			
Toll	**1991**	**Sept. 2000**	**Jan. 2003**	**Jan. 2006**	**1991**	**Sep. 2000**	**Jan. 2003**	**Jan. 2006**
Cash all day	70 % ($4.60)	20% ($5.50)	17% ($6.45)	0% ($6.45)	100 % ($18.20)	13% ($20.55)	13% ($23.20)	0% ($23.20)
E-ZPass peak	-	8% ($4.95)	10% ($5.45)	18% ($6.45)	-	8% ($19.65)	8% ($21.20)	10% ($23.20)
E-ZPass off peak	-	0% ($4.60)	5 % (%4.85)	5 % (%4.85)	-	8% ($19.65)	8% ($21.20)	0% ($21.20)
E-ZPass (weekend)	-	8% ($4.95)	10 % ($5.45)	18 % ($6.45)	-	8 % ($19.65)	8 % ($21.20)	10 % ($23.20)

The proposed Bayesian-SLA model is tested via two road sections from Exit 15E and Exit 18E and from Exit 15W to 18W located between Newark and George Washington Bridge along the NJ Turnpike. The main reason to select these road sections is that NJ Turnpike road sections from an exit to another exit include both the demand between these two exits and the demand from that particular exits to other exits located further away. Thus, any change in the latter demand will affect the traffic conditions in the selected road section. To minimize these outside effects we select road sections isolated from the other portions of NJ Turnpike, i.e., more than 90% of the traffic observed on these section is due to the demand between these particular exits.

While training and testing the proposed learning model two types of datasets were considered. First dataset covers the traffic data which include real world vehicle-by-vehicle traffic and travel time data observed from passenger cars with toll tags. The traffic data contain the counts for each 1 hour time interval from 6:00 am to 10:00 am from January 2003 to March-2003, three months after the toll increase at NJ Turnpike (Ozbay et al., 2005). The travel time data include mean and standard deviations of the travel times observed for the corresponding time period. During estimation process, weekends and holidays were excluded from the database. For each month approximately 15 days were considered. Preliminary analysis of the response of travelers to disturbed conditions (toll increase on January, 2003) can be found in a study by Ozbay et al. (2006). The results of this analysis revealed that travelers do not choose their travel choices solely based on toll differentials, but travelers' individual preferences affect their travel behavior.

The second dataset covers the individual travel survey which was used to estimate the utility functions and to provide information regarding users' departure time choices and their socio-economic characteristics. The surveys were conducted by the Eagleton Institute of Rutgers University (Ozbay et al., 2005). The data set contains 513 observations, 483 (94.2%) of which are current regular users residing in NJ. The survey participants were asked in detail about their most recent trips in the am and pm peaks. The questions include origin, destination, toll, departure time, desired/actual arrival time of each trip, as well as the socio-economic characteristics such as; income, education, employment, age and gender.

4.8.1.1 HB-ML Model Estimation

Utility function of each choice is estimated via revealed-preference traveler surveys conducted as a part of the Evaluation Study of NJ Turnpike Authority's Time-of-day Pricing Initiative (Ozbay et al. 2005).

For the proposed model, an input set $\boldsymbol{X}$ composed of the explanatory variables is considered. Output set $\boldsymbol{D} = \{d_1, d_2, d_3\}$ includes actions composed of three choices (1: pre-peak from 6:00 am to 7:00 am, 2: peak from 7:00 am to 9:00 am, and 3: post-peak from 9:00 am to 10:00 am). Using the explanatory variables obtained from traveler survey, a user-specific utility function is derived for each choice based on the proposed Bayesian framework. Summary of the explanatory variables along with the choice set is provided in Table 4.2. Estimation process is conducted via statistical software "R" (www.r-project.org/).

Table 4.3Table 4.3 shows the mean and standard deviation of the coefficients for each utility function. Each parameter follows a normal distribution, with the mean and standard deviations provided in Table 4.3.

Table 4.2 Definition of variables

Variable	Description
Choice Variables	
E-ZPass pre-peak	Respondents traveling at pre-peak periods
E-ZPass peak	Respondents traveling at peak periods
E-ZPass post-peak	Respondents traveling at post-peak periods
Explanatory Variables	
Time	Travel time, in hours
Toll	Toll paid per occupancy, in dollars
Early	Amount of early arrival time, in minutes
Late	Amount of late arrival time, in minutes
Dep. Time	(Departure time) – (Desired arrival time), in min
Income	Income level, in $10,000
Age	Age
Female	1 if female, 0 otherwise
Education	1, if user has at least bachelor degree,0 otherwise
Employment	1, if user is manager or professional,0 otherwise

Table 4.3 HB-ML estimation results

	E-ZPass pre-peak		**E-ZPass peak**		**E-ZPass post-peak**	
	Mean	**SD**	**Mean**	**SD**	**Mean**	**SD**
Constant	-3.621	0.122	-4.251	0.160	-4.025	0.110
DepTime	-0.069	0.045	-0.083	0.029	-0.082	0.025
tr time	-1.861	0.691	-1.853	0.350	-1.960	0.220
Toll	-0.673	0.112	-0.680	0.121	-0.650	0.150
Early	-0.086	0.019	-0.105	0.028	-0.109	0.023
Income	0.235	0.033	0.242	0.012	0.238	0.013
Late	-0.125	0.041	-0.114	0.035	-0.118	0.047
income*tr time	-0.218	0.051	-0.198	0.064	-0.215	0.054
income*toll	-0.217	0.025	-0.218	0.011	-0.282	0.026
toll*tr time	-0.895	0.055	-0.713	0.035	-0.887	0.089
Education	0.712	0.062	1.241	0.055	0.520	0.085
Age	0.088	0.055	0.112	0.036	0.098	0.039
Employment	0.667	0.219	0.861	0.025	0.620	0.063
Gender	0.751	0.123	0.951	0.062	0.439	0.050

4.8.1.2 Estimation of the Learning Parameters

Estimation process updates the action probabilities $p(n+1)$ at the end of each day n based on $L_{R-\epsilon P}$ scheme, such that at the end of the estimation process, the difference between the observed and the calculated $p_j(N)$ values at day N (last day of the calibration period) is minimized. Unlike previous SLA models in the literature (Ozbay et al. 2001, Ozbay et al. 2002, Ozbay and Yanmaz-Tuzel 2006), we combine Bayesian approach with SLA theory and estimate Bayesian posterior probability distributions for reward and penalty parameters. For the normal distribution case, joint prior distribution for the learning parameters is selected as:

$$N\left(\boldsymbol{\mu} = \begin{bmatrix} 0.02 \\ 0.02 \end{bmatrix}, \boldsymbol{\Sigma} = \begin{bmatrix} 0.006 & 0 \\ 0 & 0.006 \end{bmatrix}\right) \tag{4.22}$$

In order to determine the joint posterior distribution which represents the traveler behavior the best, mean standard deviations (MSD) for each day were calculated as the percent difference between observed traffic values and the assigned traffic volumes using the converged learning parameters. The parameters which minimize the MSD value were selected. Figure 4-2 provides the samples from joint posterior distribution of the converged learning parameters (*a, b*). The samples are obtained from Metropolis-Hastings algorithm with 10, 000 iterations coded in Matlab. The sensitivity analysis revealed that Normal prior distribution resulted in lowest mean MSD at a value of 0.07. These results show that the proposed Bayesian-SLA model can successfully mimic NJ Turnpike travelers' day-to-day travel behavior.

Figure 4-3 shows the histogram of the each learning parameter. The estimation process resulted in Beta distribution for the posterior distribution of each parameter. Since beta distribution always lies within [0, 1], the constraints on the learning parameters will be satisfied at all times. Mean values for the parameters (*a, b*) are (0.062, 0.0067), and standard deviations are (0.0046, 0.0021), respectively.

$$p(a) = \frac{1}{B(2.29,1.73)} \frac{(a-0.003)^{1.29}(0.106-a)^{0.73}}{1.03^{3.02}} \tag{4.23a}$$

$$p(b) = \frac{1}{B(49.35,4.75)} \frac{(b+0.041)^{1.29}(0.011-b)^{0.73}}{0.052^{53.1}} \tag{4.23b}$$

where;

B(.): Beta distribution.

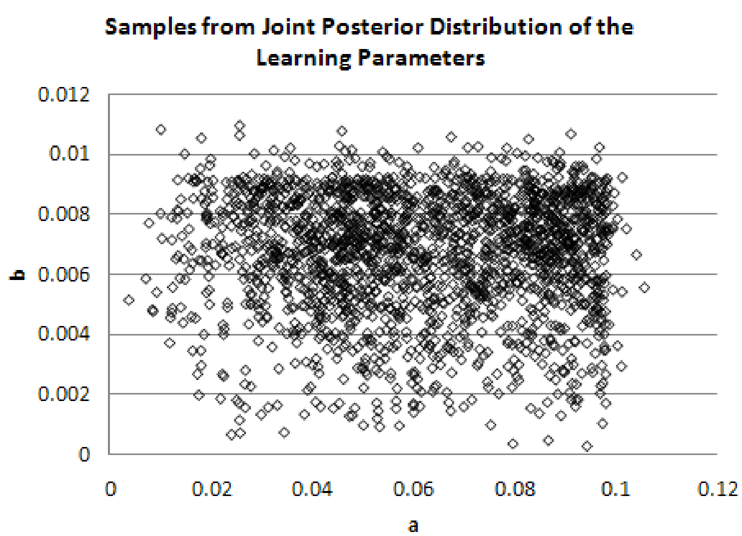


Figure 4-2 Samples from joint posterior distribution

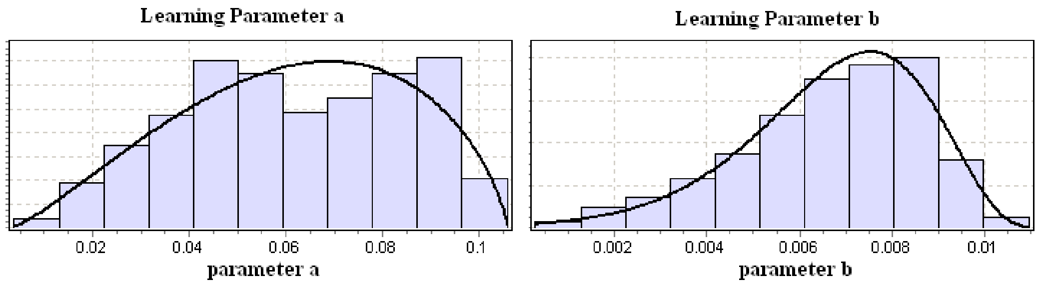


Figure 4-3 Histograms for the posterior distributions of the learning parameters

Mean values of the reward and penalty parameters estimated for NJ Turnpike users are different than the ones in other disciplines. These relatively different values can be due to the fact that NJ Turnpike commuters are familiar with the system, thus can adapt themselves to the changes in the system rather quickly. This is in fact an expected behavior for NJ Turnpike users since most of the E-ZPass users are frequent users of NJ Turnpike and are familiar with the daily traffic conditions.

4.8.2 Case Study 2: NJ Turnpike 15X Interchange Installation

The second case study focuses on the impacts of December 2005 15X Interchange installation, on the day-to-day departure time and route choice behavior of NJ Turnpike travelers.

After nearly three years of construction, NJ Turnpike Authority (NJTA) opened the $250 million Interchange 15X on the Eastern Spur (just south of Interchange 16E) on December 1, 2005. The new interchange serves the new Secaucus Junction rail transfer station. The Turnpike Authority contributed an additional $84 million to develop the $450 million adjacent Allied Junction, which will have 3.5 million square feet of combined commercial and residential development, as well as up to 2,600 new parking spaces when the development is completed. Upon full development, Interchange 15X is expected to handle 40,000 vehicles per day.

The proposed Bayesian-SLA model is applied to the road sections from Interchanges 11 (Garden State Parkway) and 14 (Jersey City – Holland Tunnel) to Interchanges 15X (Secaucus Junction and 16E (Lincoln Tunnel). Before opening of Interchange 15X, the only alternative traveling to Lincoln Tunnel area was to travel through Interchange 16E. However, since December 2005, Interchange 15X became a viable alternative for these travelers (Figure 4-4).

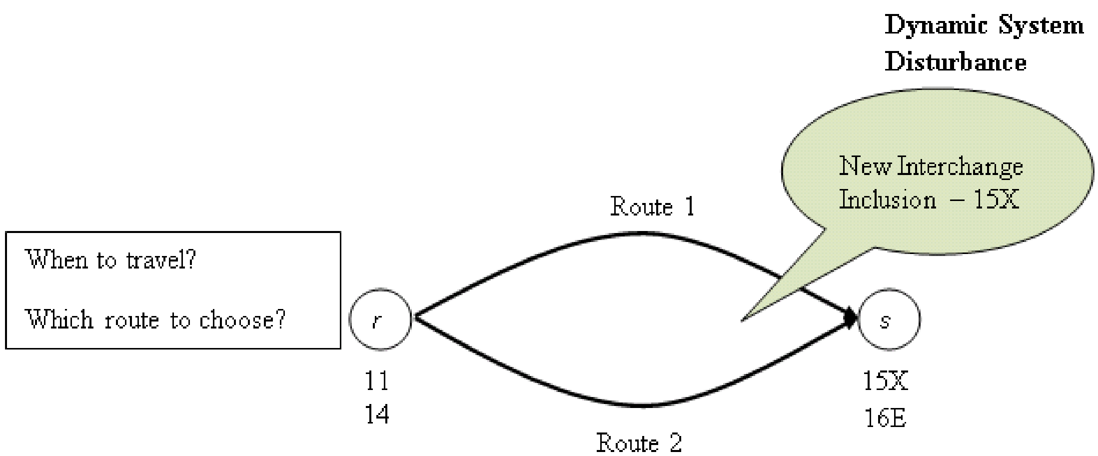


Figure 4-4 Schematic view of possible alternatives

On average, the travel time from Interchanges 11 and 14 to Interchange 15X is 15-25 percent lower compared with the travel time to Interchange 16E. On the other hand, the traffic volume between Interchanges 11-14 and Interchange 15X is much lower compared with the traffic volume between Interchanges 11-14 and Interchange 16E (Figure 4-5). On December 2005, only 3 percent of the travelers prefer to use Interchange 15X, while on December 2006, one year later, this value increases up to almost 20 percent. This preliminary analysis indicates that habit has a strong influence on travelers' route and departure time choice. In particular, instead of searching new alternatives travelers might prefer to reuse past solutions to make their behavior easier and less risky.

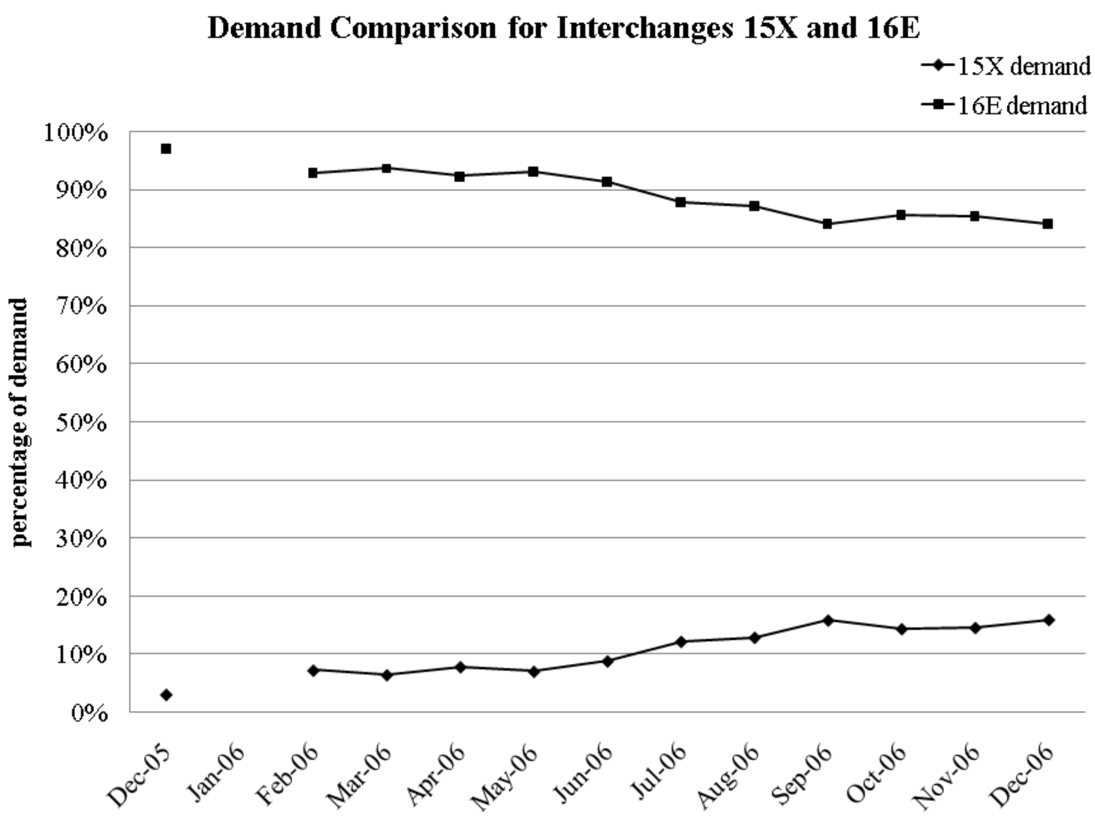


Figure 4-5 Demand comparison for Interchanges 15X and 16E

While training and testing the proposed learning model traffic and travel time data which include real world vehicle-by-vehicle traffic and travel time data observed from passenger cars with toll tags were considered. The traffic data contain the counts for each 1 hour time interval from 6:00 am to 10:00 am from December 2005 to December 2006. The travel time data include mean and standard deviations of the travel times observed for the corresponding time period. During estimation process, weekends and holidays were excluded from the database. For each month approximately 15 days were considered.

4.8.2.1 Day-to-Day Learning Model

4.8.2.1.1 Travel Choice

After the construction of Interchange 15X, travelers exiting from Interchange 16E started choose both departure time and destination (either interchange 16E or 15X). The viable options for travelers making simultaneous destination and departure time choice are composed of 6 choices. $\boldsymbol{D}$ = {d_1,d_2,d_3,d_4,d_5,d_6} includes actions composed of six choices (1: pre-peak Interchange 16E, 2: peak Interchange 16E, 3: post-peak Interchange 16E, 4: pre-peak Interchange 15X, 5: peak Interchange 15X, 6: post-peak Interchange 15X).

In the developed day-to-day learning framework, each traveler holds a probability profile for each departure time period and destination (route) choice. On day n=1 it is assumed that the probability profile represents a random variation around the observed departure time period (peak vs. off peak) and destination frequency on day n=0:

$$p_{k,i}^{o,d}(n = 1) = \left(u_k^{o,d}\right) * p_i^{o,d}(n = 0) \quad \forall i, j \; and \; \forall k \tag{4.24}$$

where;

o: Origin index (i = 1, 2, ….27)

d: Destination index (j = 16E, 15X)

k: Individual traveler index (k = 1, 2,….N_n)

n: Day index

i: Travel choice index (i = 1,2,…6)

$p_{k,i}^{o,d}(n=1)$: Probability of selecting choice i at day n=1 for individual k traveling from origin o to destination d

$u_k^{o,d}$: Random variation for individual k traveling from origin o to destination d (uniform distribution with limits [0.9, 1.1]*$p_i^{o,d}(n=0)$)

$p_i^{o,d}(n=0)$: Observed frequency of choice i from origin o to destination d

Depending on the individual probability profiles Monte Carlo simulation is used to generate discrete choices, and travel demand at each departure time period and interchange is generated.

After each traveler makes a choice, s/he experiences a travel cost and early/late arrival amount. The travel cost function, $C_{k,i}(n)$, considered in this chapter includes, travel time, $tt_{k,i}(n)$, travel time variability, $tvar_{k,i}(n)$, departure time, $dt_{k,i}(n)$, early arrival amount, $ea_{k,i}(n)$, late arrival amount, $la_{k,i}(n)$, inertia effect on choice i for driver k, $I_{k,i}$ (takes value of 1 if choice i is the current route; 0 otherwise); where $\theta_{..}$ are the coefficient for each corresponding variable (to simplify the notation, origin and destination indices are omitted).

$C_{k,i}(n) = \theta_{tt} tt_{k,i}(n) + \theta_{tvar} tvar_{k,i}(n) + \theta_{dt} dt_{k,i}(n) + \theta_{ea} ea_{k,i}(n) + \theta_{la} la_{k,i}(n) + \theta_I I_{k,i} + \varepsilon_{k,i}$ (4.25)

Travel time variable considered in the above formulation is the total travel time which is represented as the sum of travel time on NJ Turnpike and travel time outside NJ Turnpike. Note that, travel time experienced on NJ Turnpike is obtained from the vehicle-by-vehicle travel time data, whereas the remaining travel time is obtained from traveler surveys. Similarly, information regarding users' departure times and desired arrival times are obtained from travel surveys, as well. Accordingly $dt_{k,i}$ is defined as the difference between desired arrival time and departure time to travel on choice i, and $ea_{k,i}(n)$ and $la_{k,i}(n)$ parameters are defined as the difference between actual arrival time and desired arrival time to the destination.

4.8.2.1.2 Behavior Updating Mechanism

After each traveler makes a choice, a response from the transportation system is observed, i.e. experienced cost (from equation 4.23) and deviation from the desired arrival time (from traveler surveys). Depending on the experienced cost and deviation from the desired arrival time, this response may be favorable or unfavorable. In this mechanism favorable actions are rewarded, while unfavorable actions are punished.

While determining the whether an action is favorable or not, it is assumed that travelers exhibit a tendency to search for satisfying choices rather than the best behavior; thus they do not have the cognitive ability to process all the information simultaneously and are happy with a good solution. To incorporate this kind of behavior, bounded rationality (BR) approach (Simon, 1955) is included in the behavior updating mechanism. In particular, the travelers will switch routes and/or departure times if the difference between experienced costs on the selected choice and the travel cost on the best choice that day. If the difference is acceptable ($\delta_{k,i}(n) = 1$), selected choice is marked as favorable ($\beta(n) = 0$) and the traveler increases the probability to select the current choice for the next day. On the other hand, if the difference is not acceptable ($\delta_{k,i}(n) =$

0), selected choice is marked as unfavorable ($\beta(n) = 1$) and the traveler decreases the probability to select the current choice, and increases the probability to select the other choices on the next day. The mechanism for the indifference threshold can be formulated as follows:

$$\delta_{k,i}(n) = \begin{cases} 1 & if\ C_{k,i}(n) - C_{k,best}(n) \geq \Delta_k C_{k,i}(n), where\ 0 \leq \Delta_k \leq 1 \\ 0 & otherwise \end{cases} \tag{4.26}$$

where $\delta_{k,i}(n)$ is a binary variable that takes value of 1 if the difference between the cost of the current choice i, $C_{k,i}(n)$, and the cost of the best choice, $C_{k,best}(n)$ is acceptable for individual k on day n, and 0 otherwise; Δ_k is the acceptability threshold. Behaviorally, small Δ_k value indicates less tolerance of small cost differences compared with large Δ_k value. If Δ_k takes zero value, traveler is intolerant of any difference in travel cost, and would switch for even the smallest cost difference. The acceptability threshold Δ_k reflects individual attitudes and preferences, and thus should vary across the population (Chen and Mahmassani, 2004). In this study, a normally distributed acceptability threshold with mean μ_{Δ_k} and standard deviation σ_{Δ_k} is assumed.

4.8.2.2 Estimation of the Learning Parameters

Estimation process updates the action probabilities $p(n+1)$ at the end of each day n based on $L_{R-\epsilon P}$ scheme, such that at the end of the estimation process, the difference between the observed and the calculated $p_j(N)$ values at day N (last day of the calibration period) is minimized. Unlike previous SLA models in the literature (Ozbay et al. 2001, Ozbay et al. 2002, Ozbay and Yanmaz-Tuzel 2006), we combine Bayesian approach with SLA theory and estimate Bayesian posterior probability distributions for reward and penalty parameters.

In order to determine the joint posterior distribution which represents the traveler behavior the best, mean standard deviations (MSD) for each day were calculated as the percent difference between observed traffic values and the assigned traffic volumes using the converged learning parameters. The parameters which minimize the MSD value were selected. Figure 4-6 provides the samples from joint posterior distribution of the converged learning parameters (*a, b*). The samples are obtained from Metropolis-Hastings algorithm with 10, 000 iterations coded in Matlab. The sensitivity analysis revealed that Normal prior distribution resulted in lowest mean MSD at a value of 0.12. These results show that the proposed Bayesian-SLA model can successfully mimic NJ Turnpike travelers' day-to-day travel behavior.

Figure 4-7 shows the histogram of the each learning parameter. The estimation process resulted in Beta distribution for the posterior distribution of each parameter. Since beta distribution always lies within [0, 1], the constraints on the learning parameters will be satisfied at all times. Mean values for the parameters (*a, b*) are (0.029, 0.0029), and standard deviations are (0.011, 0.00093), respectively.

$$p(a) = \frac{1}{B(5.649,5.764)} \frac{(a+0.0076)^{4.649}(0.066-a)^{4.764}}{0.0736^{10.413}} \tag{4.27a}$$

$$p(b) = \frac{1}{B(47.56,35.18)} \frac{(b+0.0069)^{46.56}(0.0101-b)^{34.18}}{0.017^{81.74}} \tag{4.27b}$$

where;

$B(.)$: Beta distribution.

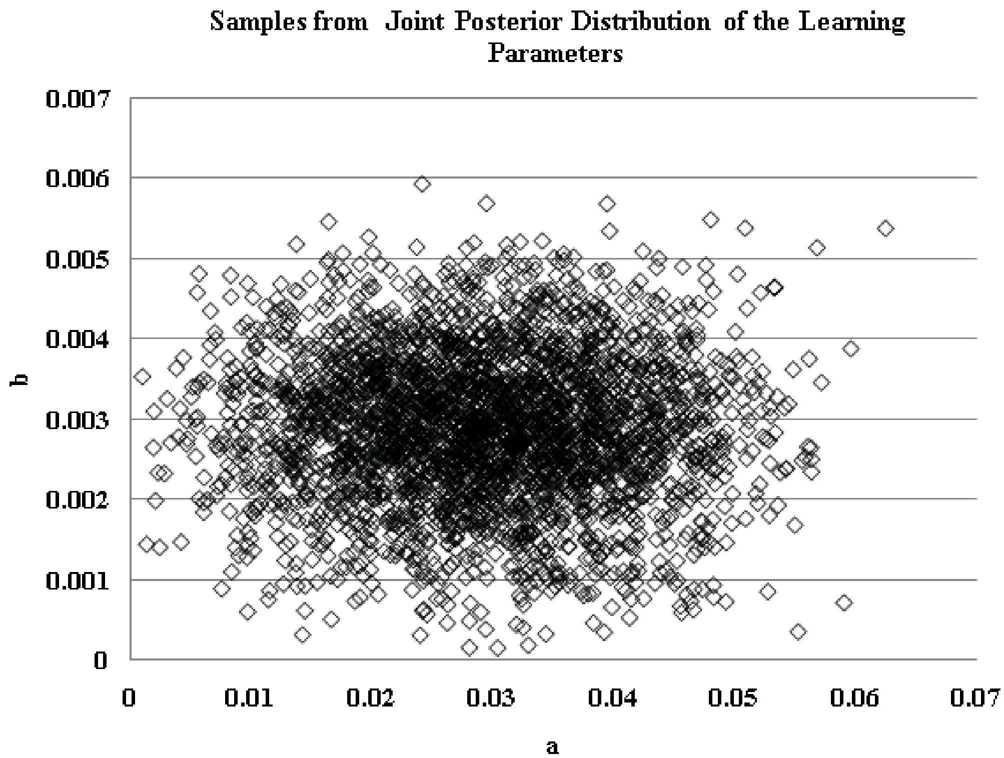


Figure 4-6 Samples from joint posterior distribution

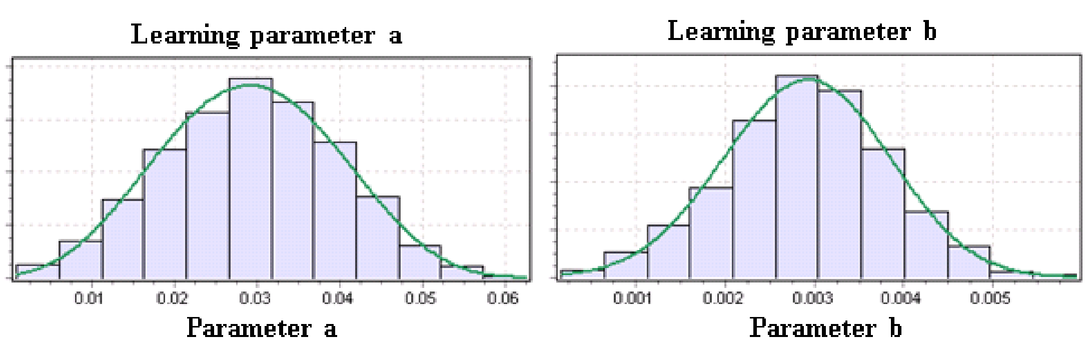


Figure 4-7 Histograms for the posterior distributions of the learning parameters

Mean values of the reward and penalty parameters estimated for NJ Turnpike users are different than the ones in other disciplines. These lower learning parameters indicate that even though the travel time from Interchanges 11 and 14 to Interchange

15X is lower compared with the travel time to Interchange 16E, high percentage of travelers put higher utility to Interchange 16E continue to use this choice. These results confirm the strong effect of habitual behavior on traveler choice, consistent with the preliminary analysis findings.

4.9 Conclusions

This chapter focuses on behavioral mechanisms for updating route and departure time choices in light of new route inclusions to the transportation system. The proposed model extends the existing SLA theory by using it in a Bayesian framework and bounded rationality (BR), while considering the impacts of habitual behavior.

1. Day-to-day learning behavior is modeled based on Bayesian-SLA theory, where each user updates his/her choice based on the rewards/punishments received due to selected actions in previous days. A linear reward-penalty reinforcement scheme is considered to represent day-to-day behavior of NJ Turnpike users as a response inclusion of a new Interchange.
2. The original SLA model proposed by Ozbay et al. (2001, 2002) considered only travel times with some random perception error, and assumed the same reward/penalty parameters for each user. In this chapter, via Bayesian Inference theory we introduced user heterogeneity into the SLA modeling process.
3. Instead of using just travel times, utility functions were introduced into the learning model. These functions were estimated via BRC models, considering a wide variety of explanatory variables, including travel time, toll, departure time, early/late arrival, income, education, employment, age, and gender.

4. In order to account for travelers' resistance to switch routes, concept of habitual behavior (inertia) is included in the proposed model, such that the travelers switch to the new route only if it has significantly less cost.
5. Finally, learning parameters were modeled as probability distributions rather than deterministic values, and Bayesian posterior distributions are estimated.

To the best of our knowledge, this is the first attempt to dynamically model the variations in perception in a day-to-day travel choice model. The estimation process conducted via Bayesian Inference approach resulted in Beta distribution for the posterior distribution of both of the learning parameters. Mean values for the learning parameters (*a, b*) of the first case study are (0.062, 0.0067), and standard deviations are (0.0046, 0.0021). Mean values for the parameters (*a, b*) for the second case study are (0.029, 0.0029), and standard deviations are (0.011, 0.00093), respectively. These results show that learning parameters are not the constant among different users of the transportation system; rather they exhibit variations in perception among the population.

This chapter has attempted to gain insights into commuters' learning behavior in uncertain and dynamic environments, when a new alternative is provided to travelers. The empirical results obtained from real transportation network, NJ Turnpike, confirm the strong effect of habitual behavior on traveler choice. The proposed Bayesian-SLA model can successfully capture the significant learning dynamics, demonstrating the possibility of developing a psychological framework (i.e., learning models) as a viable approach to represent travel behavior.

The present framework does not incorporate traffic assignment into the modeling process; rather it uses observed travel times to model the learning behavior. Integrating the proposed day-to-day update mechanism into dynamic traffic assignment would demonstrate the possibility of developing a psychological framework (i.e., learning models) as an alternative to represent traveler behavior. Next chapter proposes a novel day-to-day dynamic traffic assignment framework which integrates driver learning behavior into the modeling process.

CHAPTER 5. DAY-TO-DAY DYNAMIC TRAFFIC ASSIGNMENT

This chapter presents a new dynamic traffic assignment framework to examine the day-to-day evolution of travel patterns in a traffic network when major disturbances are introduced into the transportation system. Differences among travelers and days are explicitly modeled within this framework. The dynamic traffic flow evolution and network-level interactions of driver departure time and route choice decisions are captured within the traffic flow simulator. Proposed approach uses a microscopic simulation to model the behavior of drivers on the demand side while using a macroscopic traffic simulation model to update system variables such as link travel time, volume and density. Bayesian-SLA framework developed in the previous chapter is used to model day-to-day update mechanism of travelers' learning and adaptation to the changes in the transportation network.

5.1 Introduction

Modeling of traffic flows and travel times on congested road networks is crucial for predicting, controlling or managing congestion, and analyzing the need for infrastructure provision or improvement. Dynamic traffic assignment (DTA) refers to the assignment process that incorporates the traffic flow dynamics varying over time. The problem of predicting these dynamics for a road network is referred to as the DTA problem. DTA approach is useful to analyze how congestion forms and dissipates under time-varying conditions. Currently, many analytical models and simulation-based models are under development in attempt to understand the evolution of traffic congestion over a given time period.

The most common approach employed to capture the interaction between travel choice and network performance has been to solve an equilibrium DTA problem where a time-dependent yet pre-determined trip matrix for the design period to be evaluated is assigned onto a network. Under equilibrium conditions the travel choice is assumed to be governed by the Wardrop principle which states that all used routes have equal and minimum travel time costs. At this equilibrium point no user can decrease his/her travel time cost by switching to another choice since travel costs on all the used routes are equal.

These approaches assume that network conditions from day-to-day and within different periods of a day are in steady-state, predicate rigid behavioral tendencies a priori, and try to attain either UE or SO (Peeta and Yu, 2006). Moreover, travelers are assumed to be rational, exploring each alternative's relevant attributes and trading off the utilities derived from them. The decision strategy serves to generate a choice from a choice set for the alternative that provides the individual with the maximum utility. The question of whether equilibrium actually takes place or is a mathematical construct is a very old question (Peeta and Ziliaskopoulos, 2001). Horowitz (1984) showed that day-to-day link and path flow dynamics may oscillate around SUE, or may even converge to some non-equilibrium point other than SUE. Chang and Mahmassani (1988) and Friesz et al. (1994) performed experiments for modeling transition of disequilibria from one state to another. The results lead to the conclusion that a day-to-day adjustment process can lead to an equilibrium state under fixed supply and demand conditions. However, the fact that there are always changes in supply, demand, and traffic propagation, in combination with the stochasticity of all the involved parameters and drivers' day-to-day disequilibrium route choice behavior, makes the notion of a unique equilibrium highly questionable (Peeta and Ziliaskopoulos, 2001). In addition there is strong evidence that by ignoring most sources of day-to-day and within-day variabilities, conventional equilibrium models tend to over-estimate network performance and therefore to produce biased results (Mutale, 1992). Moreover, these models exclude driver learning (based on past experiences and personal characteristics) which can significantly affect traveler choice behavior on a specific day. In reality, the natural mechanism of traveler choice is based on traveler's behavioral tendencies, past experiences, and the traffic conditions encountered (Peeta and Yu, 2006). This issue implies the need for day-to-day modeling of users' learning mechanism through a day-to-day traffic assignment model.

Dynamic day-to-day learning models can successfully integrate travelers' learning behavior into the modeling process and represent the time-varying nature of the congestion during different times of the day and among different days. These models become more crucial in understanding travelers' responses to time-varying transportation system policies (e.g. congestion pricing) including departure-time choice, pre-trip route choice, and en-route response to traffic information.

Day-to-day DTA models aim to model traveler's day-to-day learning and adaptation behavior and provide insight on how the traffic flow pattern evolves over time. In day-to-day modeling, behavioral approaches are integrated into the equilibrium paradigm, where the sequences of states that occur as the system reaches to equilibrium are linked through a learning model based on travelers' past experiences. These intermediate stages are important for evaluation of the transportation system, because the transportation system is often in disequilibrium due to travelers' gradual response to continuously changing conditions. These models predict travelers' choices for any given day based on his/her experienced choices in the previous days. Day-to-day approaches also allow the use of wide range of behavioral rules, and levels of aggregation. Day-to-day models thus reflect the travelers' learning and forecasting mechanisms.

Several studies have focused on modeling drivers' day-to-day route choice behavior adjustment (Smith, 1984; Cascetta, 1989; Smith, 1993; Friesz et al., 1994; Zhang and Nagurney, 1996; Hu and Mahmassani, 1997; Mahmassani et al., 2001; Mahmassani, 2001; Sandholm, 2001; Hazelton, 2002; Huang and Lam, 2002; Peeta and Yang, 2003; Yang, 2005; Peeta and Yu, 2006). An extensive review of these models can be found in Yang and Zhang (2009). Moreover, several simulation tools have been proposed to model day-to-day learning behavior of travelers including DYNASMART which is based on a mesoscopic simulator that treats traffic individually but moves them according to macroscopic flow principles (Jayakrishnan et al., 1994; Hu and Mahmassani, 1997; Mahmassani et al., 2001; Mahmassani, 2001) and DRACULA (Liu et al., 1995; Liu et al., 2006) which is based on a microscopic simulator both on demand and supply levels.

This chapter presents a new DTA framework to examine the day-to-day evolution of travel patterns in a transportation network when major disturbances are introduced into the transportation system. Proposed day-to-day DTA framework combines three main sub-models. The first sub-model namely, demand model represents the day-to-day variability in total demand. It takes the observed total demand values within the analysis period and simulates for each traveler the preferred departure time choice and route to be taken. This information is then passed to the traffic simulator, the second sub-model, which calculates macroscopic flow conditions for each simulation interval. At the end of the day (the analysis period), a learning model, the third sub-model, stores the experienced travel history, and updates each individual's probability profile based on this experience.

The developed day-to-day DTA model is tested using the NJ Turnpike network. The case study investigates the impacts of the addition of a new interchange (15X) on the Eastern Spur (just south of EXIT 16E) on day-to-day departure-time and route choice behavior of NJ Turnpike travelers, and the impacts of toll structure change on day-to-day departure-time behavior of the travelers. Next section presents the details of the day-to-day DTA framework, and the results of the application.

5.2 Data Sources

In order to test and verify the proposed day-to-day DTA model, the vehicle-by-vehicle database between December 2005 and December 2006 is considered. This database includes the observed trips and travel times between each interchange after two major disruptions imposed to NJ Turnpike network.

The first major disruption is the installation of 15X Interchange. After nearly three years of construction, NJ Turnpike Authority opened the $250 million Interchange 15X on the Eastern Spur (just south of EXIT 16E) on December 1, 2005. The new interchange serves the new Secaucus Junction rail transfer station. Before opening of Interchange 15X, the only alternative traveling to Lincoln Tunnel area was to travel through Interchange 16E. However, since December 2005, Interchange 15X became a viable alternative for these travelers (Figure 4-4).

The second major disruption was imposed one month later. In January 2006, NJ Turnpike Authority eliminated the E-ZPass peak period discounts and E-ZPass peak users started to pay the same amount of toll as the cash users. In this new toll structure, toll amounts were increased by around 18% and 5% for E-ZPass peak and off-peak users, respectively; while cash tolls were kept the same.

5.3 Evolution of Day-to-Day Traffic

Consider a transportation network [*M; L; S*] with nodes m, (m=1,2,…*M*), links l, (l=1,2,….L) and OD pairs s = (i, j) (s=1,2,…..*S*). We consider a time period [0, *T*] for departures; and a termination time *T'* at which all traffic is assumed to have been served. The demand functions for each OD pair are time-dependent, given by $G_t(s), s \in S$, and give rise to path flows $h_p(t)$ The experienced travel time for a path p which carries a flow generated at time t is $\tau_p(t, h)$.

As with conventional simulation models, we start with the concept of demand and supply submodels which interact with each other. Demand side is modeled via microscopic simulation to describe individual decisions, and supply side is modeled via macroscopic simulation to depict movement of vehicles and both sub models evolve over time from one day to another. In other words, the demand side predicts the level of demand at each time period for the day n, and the supply model determines the resulting travel conditions. The travel costs experienced by the travelers are then re-input to the demand model for the next day. Thus, there is no pre-determined requirement for the process to converge to a stable "equilibrium" state. In fact, the transportation system never reaches to a single state but continuously changes and evolves from one day to the next as the travelers learn about the system and react to changes within the system. The general form of the day-to-day dynamic traffic assignment simulator is as follows:

1. **Initialization:** For each traveler in the network set individual characteristics, and an initial departure time choice probability profile. Set day counter $n = 0$
2. **OD demand generation:** Increment day counter $n = n+1$. Select the total am peak demand for each origin-destination pair.
3. **Departure time and/or route choice:** For each individual traveling on day n select a departure time period (pre-peak, peak or post-peak) and/or route (interchange 15X or interchange 16E), based on their choices, travel costs, and arrival time to their destinations on previous days.
4. **Loading:** Load the O-D matrix on day n depending on departure time and/or route choice of each individual.
5. **Supply:** For each time interval calculate travel time, traffic flow, speed and density variables.
6. **Individual Travel Experience:** For each traveler calculate experienced travel cost and determine whether the selected action is favorable or unfavorable.
7. **Learning:** Update the probability profile of each traveler using the learning parameters. In particular, increase the probability of selecting favorable actions, and decrease the probability of selecting unfavorable actions for the next day. Return to step 2.

The overall structure of the day-to-day DTA framework is presented in Figure 5-1. Our objective is to determine the probability distribution of individual day-to-day states as proposed and discussed in (Cascetta (1989), Cantarella and Cascetta (1995), Watling (1996), Hazelton and Watling (2004), Liu et al. (2006), and Peeta and Yu (2006).

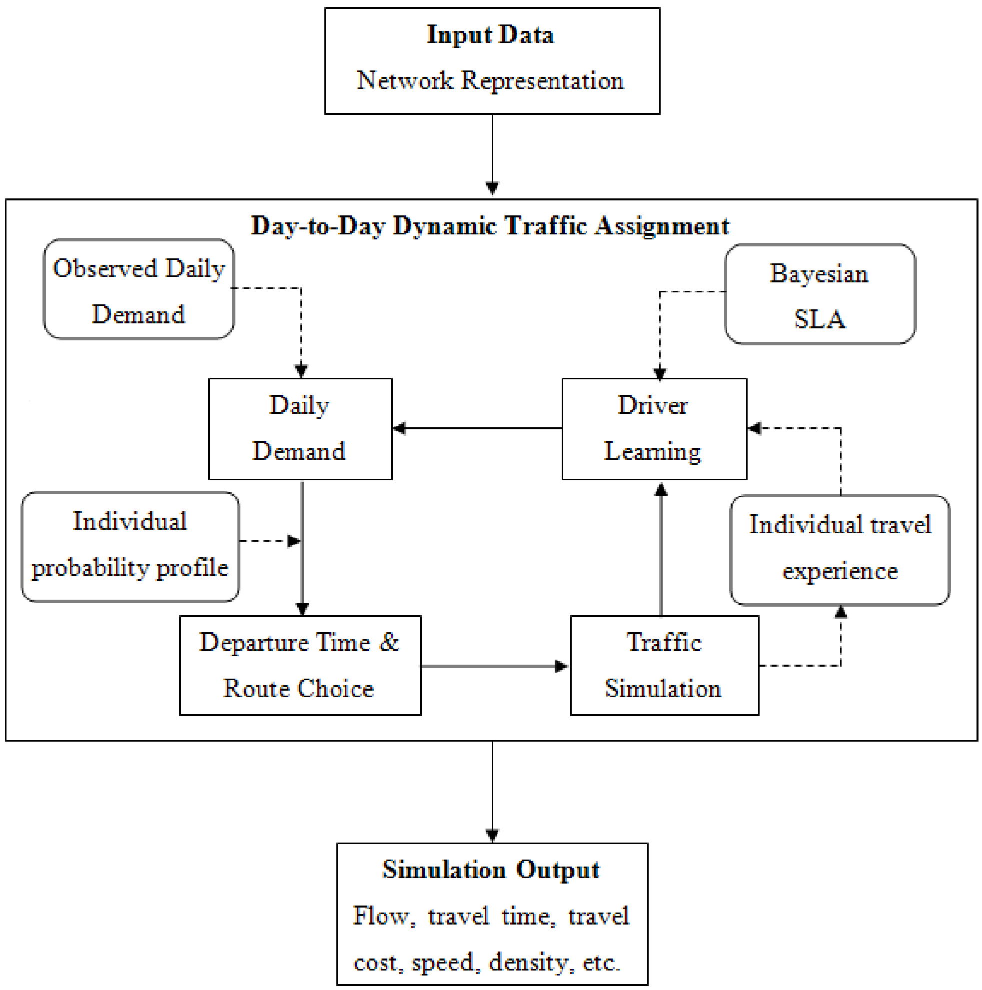


Figure 5-1 Flow chart of the day-to-day dynamic traffic assignment framework

5.4 Traffic Demand

The proposed traffic simulator performs a day-to-day DTA for the time period between 6 am and 10 am. Each traveler within this period has three travel choices: pre-peak (6-7am), peak (7-9 am) and post-peak (9-10 am). The demand that enters to the transportation network during this time period is obtained from observed O-D trip matrices.

5.5 Travel Choice

On any given day, each driver traveling from origin *i* to destination *j* maintains a probability profile for the available alternatives and updates his/her probability profile based on previous travel choices, exhibiting a tendency to search for satisfying choice rather than the best behavior.

Annual and daily traffic trends observed at NJ Turnpike indicate that traffic demand continues to increase and shows similar behavior before and after the implemetation of time-of-day pricing (Ozbay et al., 2006). Based on the results of the same study by Ozbay et al. (2006), it can be safely assumed that majority of the NJ Turnpike users make departure time choices (between peak and off-peak periods), or route choice between interchanges 15X and 16E. This is in fact an expected outcome since NJ Turnpike is practically the only alternative for a large number of trips to various important employment centers in and outside the state including New York City. NJ has an excellent rail system, but it does not provide a viable alternative to most of the NJ Turnpike users who reside far from train and who want to travel at peak periods can shift to peak hours.

Thus in this chapter two different choice mechanisms are considered. Travelers exiting from interchanges 16E or 15X make both route (which interchange to take) and departure time (which period to travel) choice. Next sections provide in depth description of both choice models.

5.5.1 Departure Time Choice

For the departure time choice model, an input set ***X*** composed of the explanatory variables is considered. Output set ***D*** = {d_1, d_2, d_3} includes actions composed of three choices (1: pre-peak from 6:00 am to 7:00 am, 2: peak from 7:00 am to 9:00 am, and 3: post-peak from 9:00 am to 10:00 am).

Each traveler has a probability profile for each departure time choice. On day *n*=1 it is assumed that the probability profile represents a random variation around the observed departure time frequency on day *n*=0:

$$p_{i,j}^{k,n=1}(r) = \left(u_{i,j}^{k}\right) * p_{i,j}^{n=0}(r) \quad \forall i, j \; and \; \forall k \tag{5.1}$$

where;

i: Origin index (i = 1, 2, ….27)

j: Destination index (j = 1, 2, ….27)

k: Individual traveler index (k = 1, 2,….N_n)

n: Day index

r: Travel choice index (1: pre-peak, 2: peak, 3: post-peak)

$p_{i,j}^{k,n=1}(r)$: Probability of selecting departure time period *r* at day *n*=1 for individual *k* traveling from origin *i* to destination *j*

$u_{i,j}^{k}$: Random variation for individual *k* traveling from origin *i* to destination *j*

$p_{i,j}^{n=0}(r)$: Observed frequency of departure time period r from origin i to destination j

Depending on the individual probability profiles Monte Carlo simulation is used to generate discrete choices, and travel demand at each departure time period is also generated.

5.5.2 Route Choice:

Before the opening of Interchange 15X, the travelers exiting through Interchange 16E were making only departure time choice. However, after the opening of 15X, this exit became a viable option for 16E travelers. Thus, these travelers were making both departure time and route (interchange 16E or 15X) after December 2005, opening of interchange 15X.

The viable options for travelers making simultaneous route and departure time choice consist of 6 choices. $\boldsymbol{D} = \{d_1, d_2, d_3, d_4, d_5, d_6\}$ includes actions composed of six choices (1: pre-peak Interchange 16E, 2: peak Interchange 16E, 3: post-peak Interchange 16E, 4: pre-peak Interchange 15X, 5: peak Interchange 15X, 6: post-peak Interchange 15X).

Similar to departure time choice model, each traveler has a probability profile for each departure time choice. On day n=1 it is assumed that the probability profile represents a random variation around the observed departure time frequency on day n=0:

$$p_{i,j}^{k,n=1}(r) = \left(u_{i,j}^{k}\right) * p_{i,j}^{n=0}(r) \quad \forall i, j \text{ and } \forall k \tag{5.2}$$

where;

i: Origin index (i = 1, 2, ….27)

j: Destination index (j = 16E, 15X)

k: Individual traveler index (k = 1, 2,….N_n)

n: Day index

r: Travel choice index (r = 1,2,…6)

$p_{i,j}^{k,n=1}(r)$: Probability of selecting choice r at day n=1 for individual k traveling from origin i to destination j

$u_{i,j}^{k}$: Random variation for individual k traveling from origin i to destination j

$p_{i,j}^{n=0}(r)$: Observed frequency of choice r from origin i to destination j

Depending on the individual probability profiles Monte Carlo simulation is used to generate discrete choices, and travel demand at each departure time period and interchange is generated.

5.6 Traffic Loading

Generated travel demand at each departure time period is loaded to the transportation network. The traffic loading (supply) model is a mesoscopic simulation of vehicle movement on the network. The model uses a small time step so that congestion details can be accurately modeled. During each small time interval, vehicles keep their variables constant (position, and speed). At time each interval, the vehicle variables are computed for all vehicles and the simulation proceeds to the next time step. The major steps of the traffic simulation on day n are as follows:

1. **Initialization**: Set simulation clock t=0, generate vehicle macro-particles by distributing travel demand at each departure time period equally within each time interval

2. **Vehicle movement**: Load the macro-particles to the transportation network
3. **Simulation Results**: Store aggregate measures of traffic volume, travel time, speed, density for each link and O-D pair
4. **Termination**: If all drivers have finished their journey, terminate the day; otherwise increment the simulation clock and return to step 2.

The traffic simulation algorithm satisfies the following constraints:

1. Flow conservation: The flow conservation require that for any given time instant, the flow entering to any node, together with the demand generated at that node, must all exit from this node to the next link unless the node is a destination. Mathematically, this constraint can be expressed as:

$$\sum_{l \in A(i)} \varphi_l^j(t) = G^{ij}(t) + \sum_{l \in B(i)} v_l^j(t) \tag{5.3}$$

In the above equation $\varphi_l^j(t)$ denotes the inflow rate to link *l* to destination *j* at time instant *t*, $G^{ij}(t)$ denotes the travel demand from node *i* to destination *j* at time *t*, and $v_l^j(t)$ is the exit flow rate from link *l* to destination *j* at time *t*. Moreover, $A(i)$ is the set of links whose starting nodes are *i*, and $B(i)$ is the set of links whose ending nodes are *i*.

Then, total flow on link *l* is the sum of vehicles entering to link *l* from previous link and demand generated at link *l* minus the number of vehicles exiting link *l*:

$$h_l^{ij}(t) = \varphi_l^{ij}(t) - v_l^{ij}(t) + G_l^{ij}(t) \quad \forall l \in L, \ \forall (i,j) \in S \tag{5.4}$$

Then total flow on link *l* at time *t*, $h_l(t)$:

$$\sum_{(i,j)} h_l^{ij}(t) = h_l(t) \tag{5.5}$$

where;

$h_l^{ij}(t)$: Number of vehicles on link *l* between OD pair (i, j) at time *t*.

$\varphi_l^{ij}(t)$: Number of vehicles entering to link *l* at time *t*

$v_l^{ij}(t)$: Number of vehicles exiting to link *l* at time *t*

$G_l^{ij}(t)$: Demand generated at link *l* between OD pair (i, j) at time *t*.

2. FIFO constraint: FIFO condition states that a traffic stream generated at time *t"* cannot overtake another stream that has started earlier at time *t* $(t < t")$. In order to satisfy FIFO condition when path costs are additive, certain conditions must be satisfied by the link cost functions. The FIFO condition may be stated as:

$$t" > t \Rightarrow t" + \tau_l(t") > t + \tau_l(t) \tag{5.6}$$

At each time interval, link travel time function, τ_l^t, is linked to instantaneous inflow φ_l^t, link volume h_l^t, density k_l^t and speed V_l^t, and other link parameters $\vec{P}_l^t$ such as length and capacity:

$$\tau_l^t = f\left(\varphi_l^t, h_l^t, k_l^t, V_l^t; \vec{P}_l^t\right) \tag{5.7}$$

This mesoscopic description can also be seen as a cellular automata description where each link and node corresponds to a unique cell. Congestion occurs on link cells, and route decisions occur in node cells. Inside any given link, there is no underlying representation of the vehicles, except the fact that they follow FIFO rule. The vehicle interaction is described only by travel time functionτ_l^t. This approach implies vertical queuing since there is no restriction imposed on φ_l^t or h_l^t.

Travel time at each time interval and link is calculated based on well-known speed-concentration relationship. The speed-flow relationships are:

$$k_l^{t+1} = k_l^t + \left(\frac{1}{\gamma_l}\Delta x_1\right)(\varphi_l^{t+1} - v_l^{t+1} + G_l^{t+1}) \tag{5.8}$$

$$V_l^t = \left(V_f - V_o\right)\left(1 - \frac{k_l^t}{k_o}\right)^{\alpha} + V_o \tag{5.9}$$

$$(\tau)_l^t = \frac{\Delta x_l}{V_l^t} \tag{5.10}$$

where

l = Index for links (l = 1,2,…27)

k_l^{t+1} = Concentration on link l at time interval t+1

γ_l= Number of lanes

Δx_1 = Length of link l

V_l^t= Mean speed at link l during t-th time step

V_f= Free flow speed (Assumed to be 70 mph)

V_o = Minimum speed on the facility (Assumed to be 5 mph)

k_o = Maximum concentration (assumed to be capacity*Δx_1*5, capacity = 2000 veh/hr/lane)

$(\tau)_l^t$= Travel time at link l during t^{th} time step

α = a parameter to be calibrated

G_l^{t+1}: Demand generated at link l at time $t+1$.

Note that, the travel time equation formulated above is an average travel time for each simulation time interval. In order to evaluate individual perceived travel times a random error was added to the aggregate travel times:

$$(\tau)_{l,i,j,k}^{t} = (\tau)_{l}^{t} + (\tau)_{l}^{t} * \left(u_{i,j}^{k}\right) \tag{5.11}$$

The observed travel times for NJ Turnpike include the link travel times, as estimated in eqn-5.10, and service and waiting times at the exit toll plazas. To model the toll plaza delays at each interchange, we have used a macroscopic toll plaza delay model developed by Lin (2001). Even though, this model is a relatively simple formulation, it has been validated by Ozmen-Ertekin et al. (2008) that macroscopic model results are comparable (within average error of 2.6% to 6.4%) with the PARAMICS microscopic model for the NJ Turnpike toll plazas.

According to this macroscopic model the total delay experienced at toll plazas by each vehicle can be expressed as follows:

$$d = d_d + d_i + d_p + d_a + d_q \tag{5.12}$$

where;

d_d: deceleration delay (s/veh)

d_i: incremental delay (s/veh)

d_p: service time (s/veh)

d_a: acceleration delay (s/veh)

d_q: initial queue delay (s/veh)

Deceleration delay is the extra travel time incurred while drivers decelerate before reaching a toll booth (Lin, 2001):

$$d_d = \frac{(V - V_b)^2}{2dV} \tag{5.13}$$

where,

V: Speed of the approaching vehicle (m/s) (set to 30 mph (13.3 m/s), Ozbay et al. (2006))

d: Deceleration rate (m/s^2) (set to 4.87 m/ s^2, Ozbay et al. (2006))

V_b: Speed at toll booth (m/s) (set to 12 mph (5.36 m/s), Ozbay et al. (2006))

Acceleration delay depends on the free flow speed and the acceleration characteristics of vehicles (Lin, 2001):

$$d_a = \frac{(V - V_b)^2}{2aV} \tag{5.14}$$

where,

a: Acceleration rate (m/s^2) (set to 2.71 m/ s^2, Ozbay et al. (2006))

Incremental delay experienced by each vehicle refers to the random variations in toll processing times and vehicle arrivals (Lin, 2001):

$$d_i = 900\mathrm{T}\left((\mathrm{X} - 1) + \sqrt{(\mathrm{X} - 1)^2 + \frac{4\mathrm{X}}{\mathrm{C.T.N}}}\right) \tag{5.15}$$

where,

T: Analysis period (h) (4 hours)

X: Volume-to-capacity ratio

C: Capacity (veh/lane/hour) (set to 1,150 veh/hr, Ozbay et al. (2006))

N: number of toll lanes

Similarly, initial queue delay can be formulated as (Lin, 2001):

$$d_q = \frac{1800Q_b(1+u)t}{cT} \tag{5.16}$$

where,

Q_b: Total number of vehicles present at toll lanes at beginning of T (veh) (set to 5 veh/lane, Ozbay et al. (2006))

C: Toll lane group capacity (vph) (set to 1,150 veh/hr, Ozbay et al. (2006))

t: Duration of oversaturation within T (h)

u: Delay parameter

Service time for toll plazas was estimated as 3 seconds in a study by Ozbay et al. (2006) focusing on toll plaza service times for NJ Turnpike.

5.7 Travel Experience

At the end of each day n the following measures are calculated:

a. Travel choice r at day n for each individual k traveling from origin i to destination j
b. Travel time for each individual k traveling from origin i to destination j
c. Utility of each individual k traveling from origin i to destination j
d. Early/ late arrival amount for each individual k traveling from origin i to destination j
e. Travel choice with maximum utility
f. Favorable/ unfavorable actions

After determining the performance measures for each traveler, depending on the experienced utility and deviation from the desired arrival time, whether the action is favorable or not is determined. Next sections describe the utility value calculation for departure time and route choice, and classification of favorable and unfavorable actions

5.7.1 Utility Functions

Using the explanatory variables obtained from traveler survey, a user-specific utility function is derived for each departure time choice based on the proposed Bayesian framework. Eqn 5.7 shows the mean and standard deviation of the coefficients for each utility function. Each parameter, $\beta_{..}$, follows a normal distribution, with mean, $\mu_{\beta_{..}}$, and standard deviation values, $\sigma_{\beta_{..}}$, provided in Table 4.3. Thus, parameters of the utility function for each traveler are different. This departure time choice model considers travelers' trip characteristics (travel time, toll), desired arrival time characteristics (departure time, early / late arrival amount) and socio economic characteristics (income, education, age, employment, gender).

$$U_{k,n}^{r} = \beta_{tt}tt_{k,n}^{r} + \beta_{dt}dt_{k,n}^{r} + \beta_{ea}ea_{k,n}^{r} + \beta_{la}la_{k,n}^{r} + (\beta_{toll}toll)_{k,n}^{r} + (\beta_{in}income)_{k,n}^{r} +$$
$$(\beta_{intt}income * tt)_{k,n}^{r} + (\beta_{intoll}income * toll)_{k,n}^{r} + (\beta_{tttoll}tt * toll)_{k,n}^{r} + (\beta_{ed}ed)_{k,n} +$$
$$(\beta_{gen}gender)_{k,n} + (\beta_{emp}emp)_{k,n} + (\beta \quad gender)_{k,n} + \varepsilon_{k,n} \quad (5.17)$$

where;

$$\beta_{..} \sim Normal(\mu_{\beta_{..}}, \sigma_{\beta_{..}})$$

The travel utility function for route choice, $U_{k,n}^{r}$, includes travel time, $tt_{k,n}^{r}$, departure time, $dt_{k,n}^{r}$, early arrival amount, $ea_{k,n}^{r}$, late arrival amount, $la_{k,n}^{r}$, inertia effect on choice r for driver k, I_k^r (takes value of 1 if choice r is the current choice ; 0 otherwise); where $\beta_{..}$ are the coefficient for each corresponding variable.

$$U_{k,n}^r = \beta_{tt} tt_{k,n}^r + \beta_{dt} dt_{k,n}^r + \beta_{ea} ea_{k,n}^r + \beta_{la} la_{k,n}^r + (\beta_{toll} toll)_{k,n}^r + \beta_I I_k^r + \varepsilon_{k,n} \quad (5.18)$$

5.7.2 Favorable vs. Unfavorable Actions

After calculating the experienced travel utility value and early/late arrival amount associated with each individual travel choice; whether the selected choice is favorable or not is determined. In particular, it is assumed that travelers exhibit a tendency to search for satisfying choices rather than the best behavior; thus they do not have the cognitive ability to process all the information simultaneously and are happy with a good solution. To incorporate this kind of behavior, bounded rationality (BR) approach (first introduced by Simon 1955 and used in many studies in transportation field including Chen and Mahmassani (2004)) is included in the behavior updating mechanism. In particular, the travelers will switch routes and/or departure times if the difference between experienced costs on the selected choice and the travel cost on the best choice that day. If the difference is acceptable the traveler increases the probability to select the current choice for the next day, otherwise decrease the probability to select the current choice, and increases the probability to select the best choice of that day. The mechanism for the indifference threshold can be formulated as follows:

$$\delta_{k,n} = \begin{cases} 1 & if\ U_{k,n}^r - U_{best,n} \geq \Delta^k U_{k,n}^r, where\ 0 \leq \Delta^k \leq 1 \\ 0 & otherwise \end{cases} \quad (5.19)$$

where $\delta_{k,n}$ is a binary variable that takes value of 1 if the difference between the utility of the current choice r, $U_{k,n}^{r}$, and the utility of the best choice, $U_{best,n}$ is acceptable for individual k on day n, and 0 otherwise; Δ^k is the acceptability threshold. Behaviorally, small Δ^k value indicates less tolerance of small cost differences compared with large Δ^k value. If Δ^k takes zero value, traveler is intolerant of any difference in travel cost, and would switch for even the smallest cost difference. The acceptability threshold Δ^k reflects individual attitudes and preferences, and thus should vary across the population (Chen and Mahmassani, 2004). In this study, a normally distributed acceptability threshold with mean μ_{Δ^k} and standard deviation σ_{Δ^k} is assumed (Chen and Mahmassani, 2004).

5.8 Learning Model

In this chapter, commuters' day-to-day learning behavior on the basis of experienced travel choices and user-specific characteristics is modeled via Bayesian-SLA theory. Specifically, each user updates his/her choice based on the rewards/punishments received due to selected actions in the previous days. At the end of each day, favorable actions are rewarded, while unfavorable actions are punished. Whether an action is favorable or not is determined using bounded rationality approach via indifference bands calculated around the traveler's experienced cost function value and deviation from desired arrival time. After determining favorable and unfavorable actions, a linear reward-penalty reinforcement scheme is considered to update day-to-day learning behavior of NJ Turnpike users, and to investigate commuters' response to new route inclusion while selecting their departure times and routes.

As discussed in Chapter 4, the learning parameters for departure time choice follow Beta distribution. Mean values for the parameters (*a, b*) are (0.062, 0.0067), and standard deviations are (0.0046, 0.0021), respectively.

$$p(a) = \frac{1}{B(2.29,1.73)} \frac{(a-0.003)^{1.29}(0.106-a)^{0.73}}{1.03^{3.02}} \tag{5.20a}$$

$$p(b) = \frac{1}{B(49.35,4.75)} \frac{(b+0.041)^{1.29}(0.011-b)^{0.73}}{0.052^{53.1}} \tag{5.20b}$$

where;

B(.): Beta distribution.

Similarly, the learning parameters for departure time and route choice follow Beta distribution. Mean values for the parameters (*a, b*) are (0.029, 0.0029), and standard deviations are (0.011, 0.00093), respectively.

$$p(a) = \frac{1}{B(5.649,5.764)} \frac{(a+0.0076)^{4.649}(0.066-a)^{4.764}}{0.0736^{10.413}} \tag{5.21a}$$

$$p(b) = \frac{1}{B(47.56,35.18)} \frac{(b+0.0069)^{46.56}(0.0101-b)^{34.18}}{0.017^{81.74}} \tag{5.21b}$$

5.9 Calibration and Validation

Calibration and validation are important processes in the development and application of day-to-day DTA models. These processes are to ensure that the models accurately replicate the observed traffic condition and driver behavior.

Model calibration is a process whereby the values of model parameters are adjusted so as to match the simulated model outputs with observations from the study site. It is usually formulated as an optimization problem to determine the best set of model parameter values in order to minimize the discrepancies between the observed and simulated values (Toledo, 2003). The calibration process is then to modify the values of the model parameters {β}, so to find the best set of values which minimizes F. The proposed objective function *F* minimizes the difference between observed and simulated link volumes and travel times:

$$\min F = \sum_{\beta} \sum_{t} \left[\left(\frac{q_t^{sim} - q_t^{obs}}{q_t^{obs}} \right)^2 + \left(\frac{tt_t^{sim} - tt_t^{obs}}{tt_t^{obs}} \right)^2 \right] \tag{5.22}$$

where;

β: Set of parameters to be calibrated

t: Time interval (one hour)

q_t^{sim}: Simulated link flows in *t*

q_t^{obs}: Observed link flows in *t*

tt_t^{sim}: Simulated link travel time in *t*

tt_t^{obs}: Observed link travel time in *t*

Figure 5-2 illustrates the solution algorithm for the calibration process. It is an iterative procedure to try to match the simulated results with those observed from the study site.

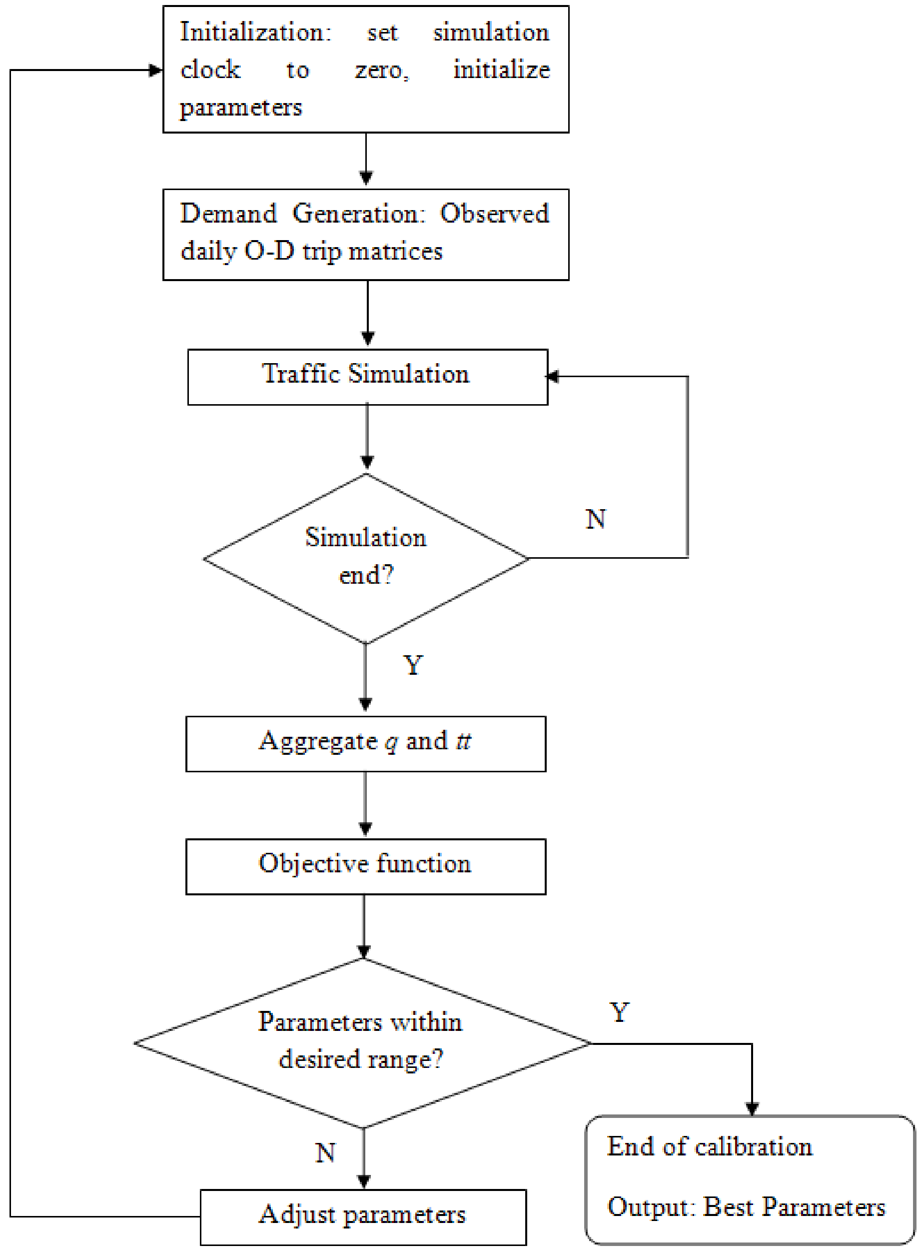


Figure 5-2 Flowchart for the calibration process

5.9.1 Model Validation

After determining the optimal set of parameters from the calibration process, a validation process is performed in order to determine whether the simulation model successfully replicates the real system. Two different measures were considered in the validation process:

1. Root mean square error (RMSE)
2. Mean percentage error (MPE)

The formulations of these measures are as follows:

$$RMSE = \sqrt{\frac{1}{N*T}\sum_{t=1}^{T}\sum_{l=1}^{N}\left(\frac{q_{t,l}^{sim}-q_{t,l}^{obs}}{q_{t,l}^{obs}}\right)^2} \quad (5.23)$$

$$MPE = \frac{1}{N*T}\sum_{t=1}^{T}\sum_{l=1}^{N}\frac{|q_{t,l}^{sim}-q_{t,l}^{obs}|}{q_{t,l}^{obs}} \quad (5.24)$$

where $q_{t,l}^{sim}$ and $q_{t,l}^{obs}$ are the simulated and observed measurements for link l during aggregated time period t, $\bar{q}$ is the sample mean, σ is the sample standard deviation, N is the total number of links and T is the total number of time periods (equal to three). RMSE measure penalizes large errors at a higher rate than small errors, while MPE indicates the existence of systematic under or over-prediction in the simulated variables (Toledo, 2003).

5.10 Applications

This section analyzes the effectiveness of the proposed day-to-day DTA framework in evaluating the impacts of major disruption on day-to-day traffic flows of real transportation networks.

5.10.1 NJ Turnpike Road Network

As mentioned before, NJ Turnpike network is composed of 27 interchanges. Figure 5-2 depicts the NJ Turnpike network and the location of each interchange. Next section provides the detailed results of the calibration and validation of the day-to-day DTA framework applied to NJ Turnpike

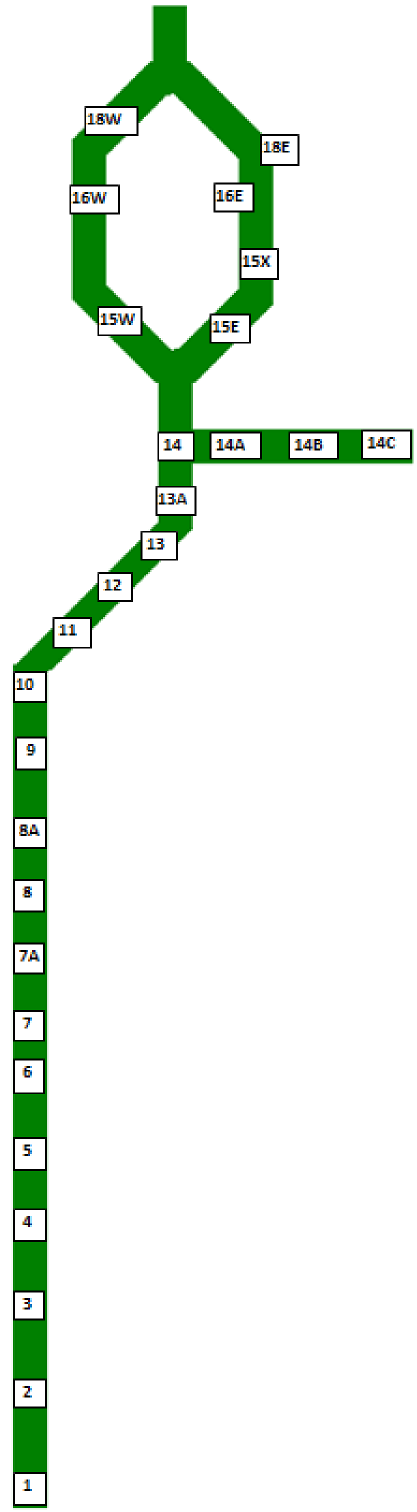


Figure 5-3 NJ Turnpike network (NJTA, 2007)

5.10.2 Analysis Results

This section provides results of the validation process. In particular, after determining the optimal set of parameters from the calibration process, a validation process is performed in order to determine whether the simulation model replicates the real system.

Two different measures were considered in the validation process namely, root mean square error (RMSE), and mean percentage error (MPE) performance measures are calculated for the entire analysis period from December 2005 to December 2006. Figure 5-4 and Figure 5-5 summarize the MPE and RMSE plots, respectively for traffic volume calculations. The proposed day-to-day DTA assignment framework has a good performance with MPE values ranging around 0.107, and RMSE values ranging around 0.235 between December 2005 and December 2006. Similarly, Figure 5-6 and Figure 5-7 summarize the MPE and RMSE plots, respectively for travel time calculations. These results are fairly consistent regardless of network congestion levels, and the simulation model and the real system show fairly good agreement. The relative magnitude of errors is similar to well expected day-to-day DTA simulation softwares, such as DynaMIT, developed for FHWA. (Antoniou, 2004; Park et al., 2008) and AIMSUN (Barceló and Casas, 2005). Both softwares obtained RMSE values between 0.2 and 0.3 when the DTA on real transportation networks was calibrated with observed day-to-day traffic and travel time data.

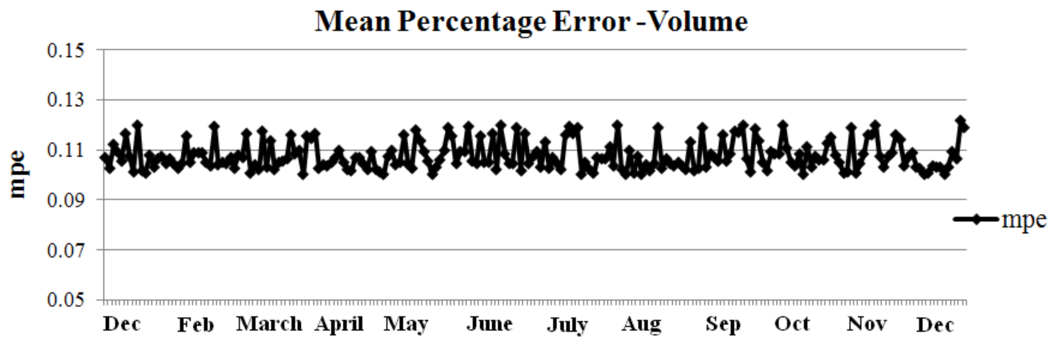


Figure 5-4 Mean percentage error for traffic volume

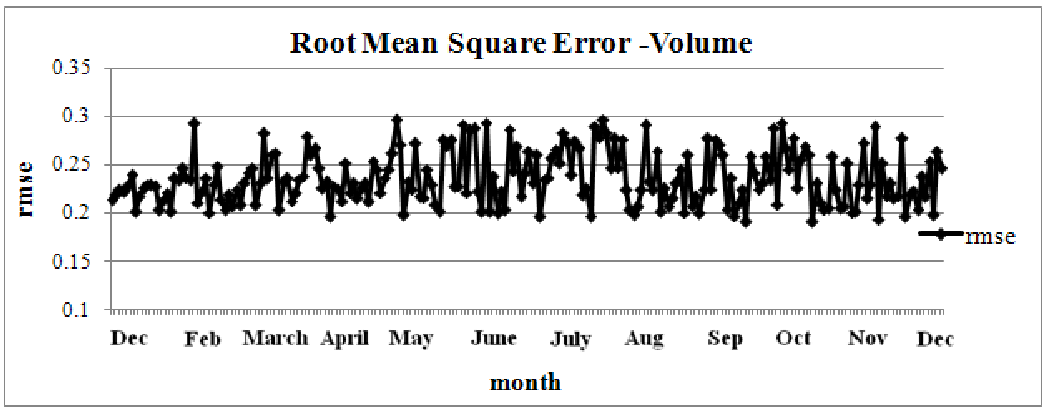


Figure 5-5 Root mean square error for traffic volume

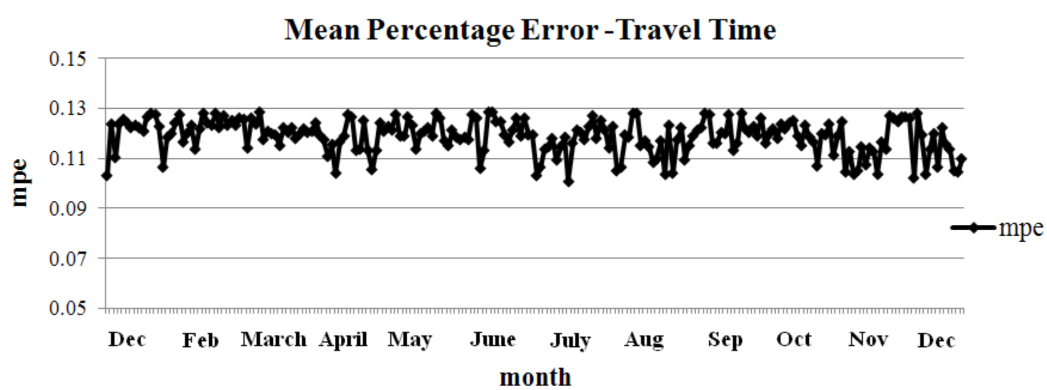


Figure 5-6 Mean percentage error for travel time

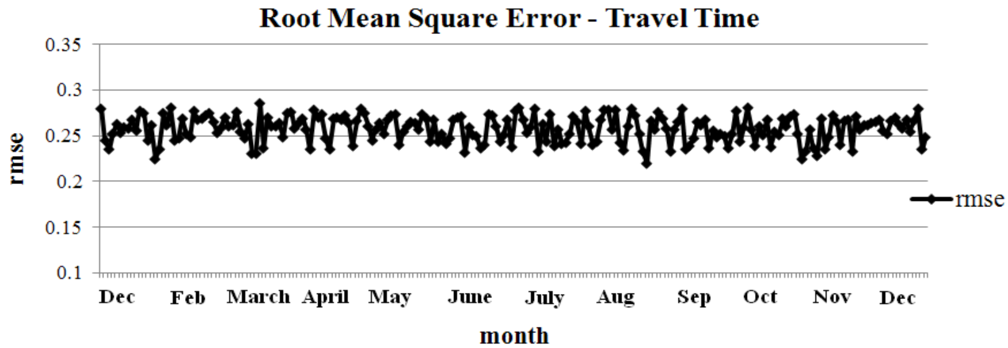


Figure 5-7 Root mean square error for travel time

Figure 5-8 shows the comparison of estimated and observed hourly link volumes during peak period on March 9, 2006. Congestion levels show a similar trend for both estimated and observed traffic volumes. Highest congestion levels are observed between links between Exit 11 and Exit 12, and between Exit 9 and Exit 10. Similarly, Figure 5-9 shows the vehicle flow trajectory at each 5 min interval from 6 am to 11 am for links 1-2, 8-8A and 9-10. Since the link between exit locations 1 and 2 are the starting link of the network, the vehicle trajectory shows a similar trend during pre-peak (6-7 am), peak (7-9 am) and post-peak (9-10 am) hours. Thus, discharge rates are mode stable compared with other links. On the other hand, the intermediate links experience several increase in the vehicle flow trajectory. Since travelers who entered the network from earlier exit locations experience the congestion levels on the previous links, their arrival rate to these particular links (8-8A and 9-10) varies. Thus, vehicle flow trajectory shows significant variability and fluctuates more compared with previous links. Moreover, since the demand during peak hours is higher compared with other periods, around 7 am and 8 am we observe sharp increases in the volume levels. As the travelers exit to their destinations, the flow levels start to diminish until all the vehicles are discharged from the network.

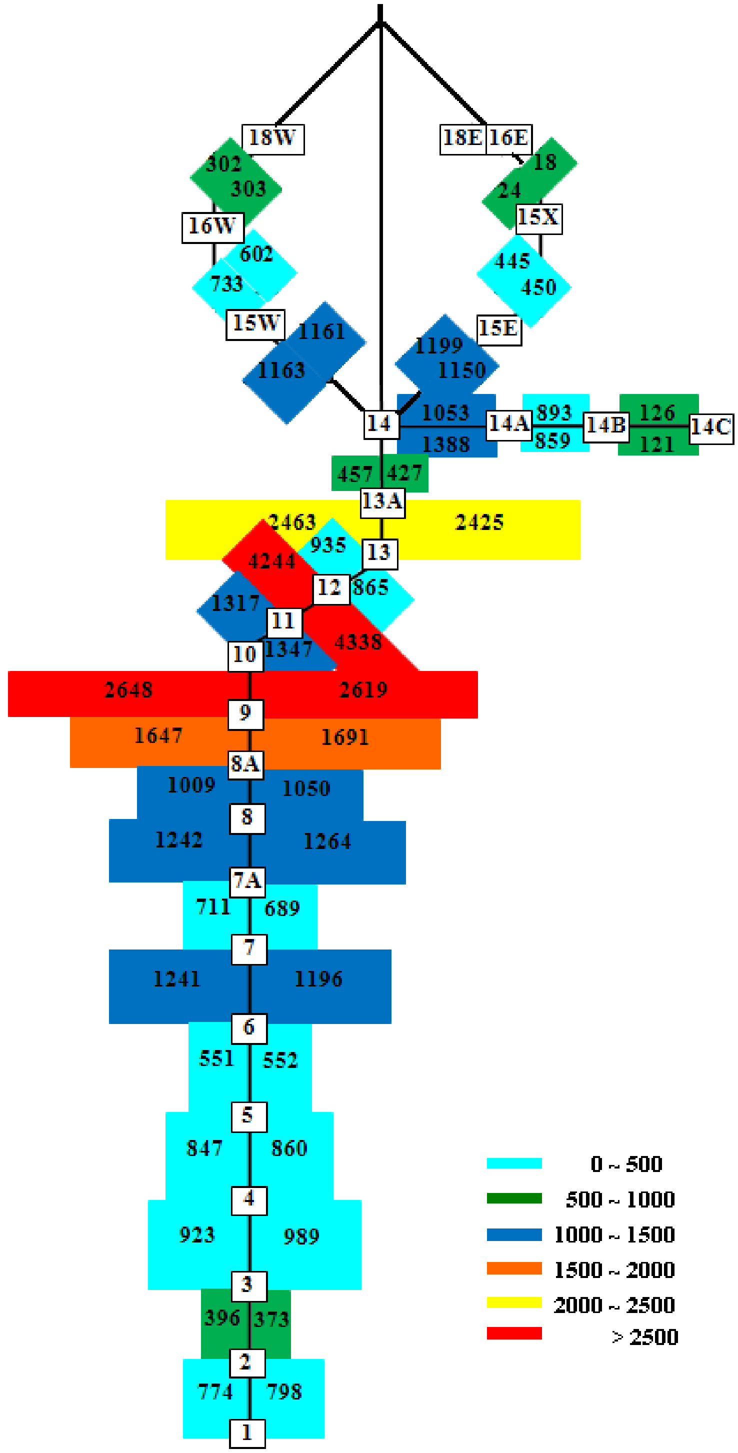


Figure 5-8 Estimated and observed traffic volumes for the peak period

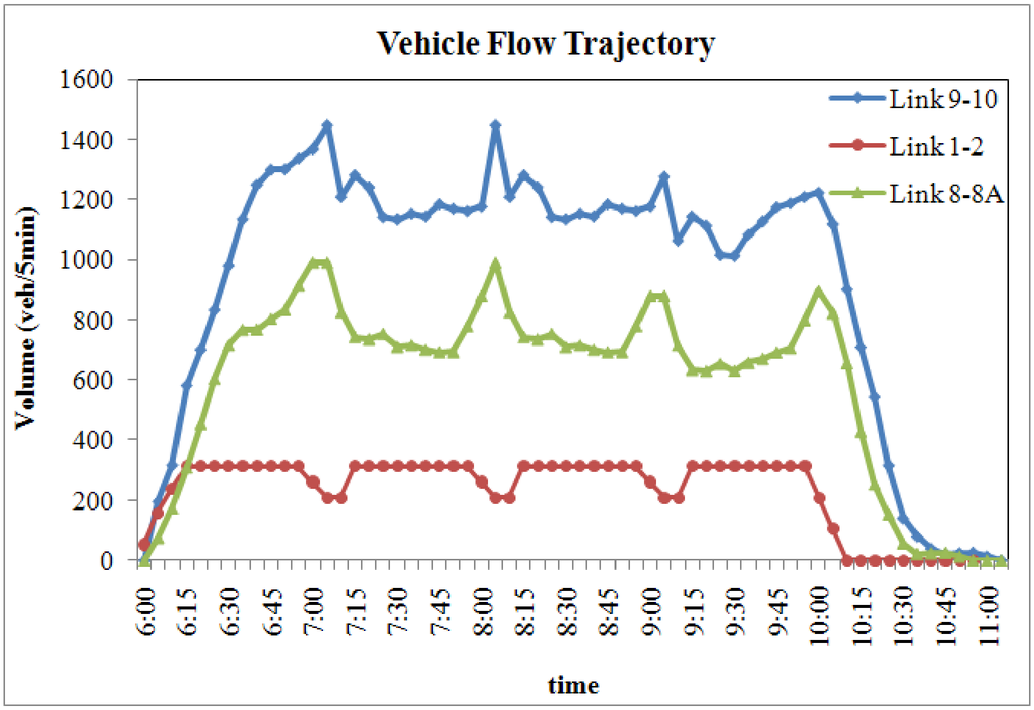


Figure 5-9 Simulated vehicle flow trajectory

Figure 5-10, Figure 5-11, and Figure 5-12 show the evolution of day-to-day traffic volume exiting from Interchange 15X from December 2005 and December 2006, for pre-peak, peak and post-peak periods, respectively. The day-to-day traffic volume analysis result exhibit an increasing trend hourly volumes for pre-peak, peak and post-peak periods. The highest traffic volume levels are observed during peak periods followed by pre-peak and post-peak periods. Moreover, the comparison between estimated and observed traffic volumes shows that the proposed day-to-day learning framework can successfully capture the increasing trend for Interchange 16E traffic behavior. Similarly, Figure 5-13, Figure 5-14, Figure 5-15 show the evolution of day-to-day traffic volume exiting from Interchange 16E from December 2005 and December 2006, for pre-peak, peak and post-peak periods, respectively. Unlike Interchange 15X, traffic volume levels exiting from Interchange 16E exhibit a decreasing trend for pre-peak peak and post-peak periods. The highest traffic volume levels are observed during pre-peak periods followed by peak and post-peak periods. Moreover, the comparison between estimated and observed traffic volumes shows that the proposed day-to-day learning framework can successfully capture the increasing trend for Interchange 15X traffic behavior.

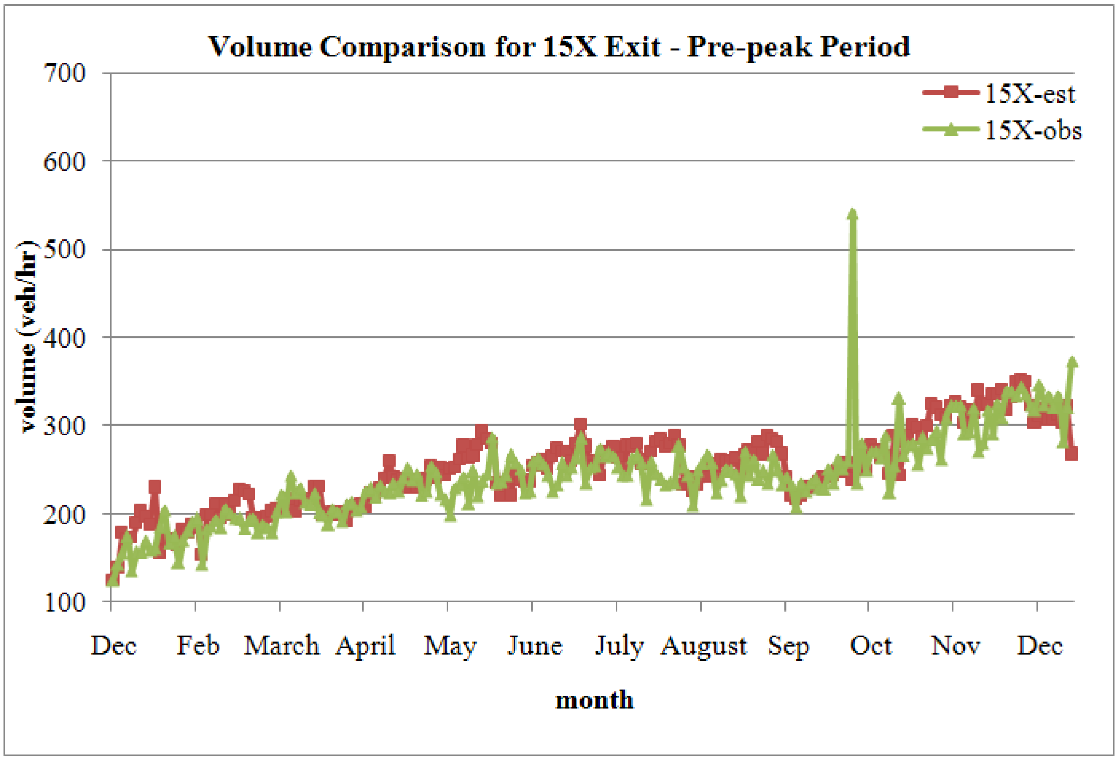


Figure 5-10 Volume comparison for Exit 15X, pre-peak period

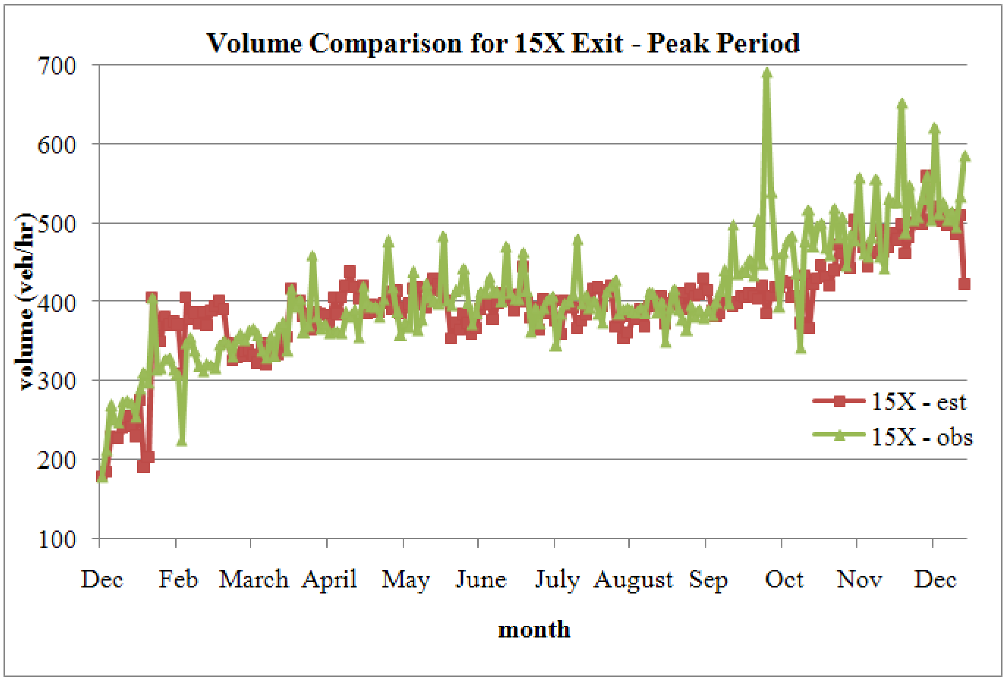


Figure 5-11 Volume comparison for Exit 15X, peak period

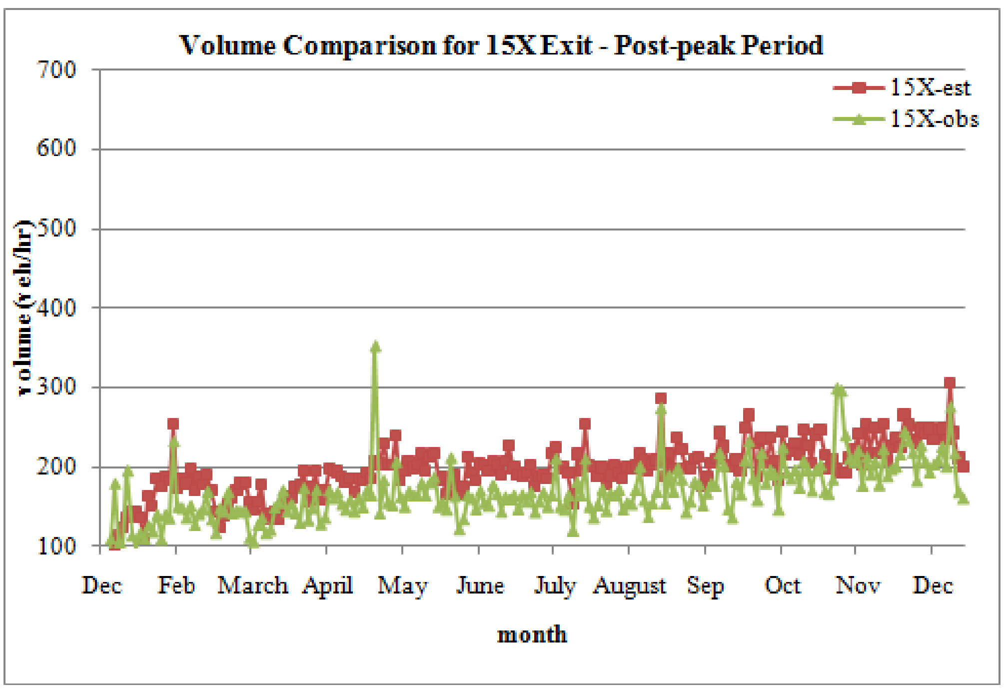


Figure 5-12 Volume comparison for Exit 15X, post-peak period

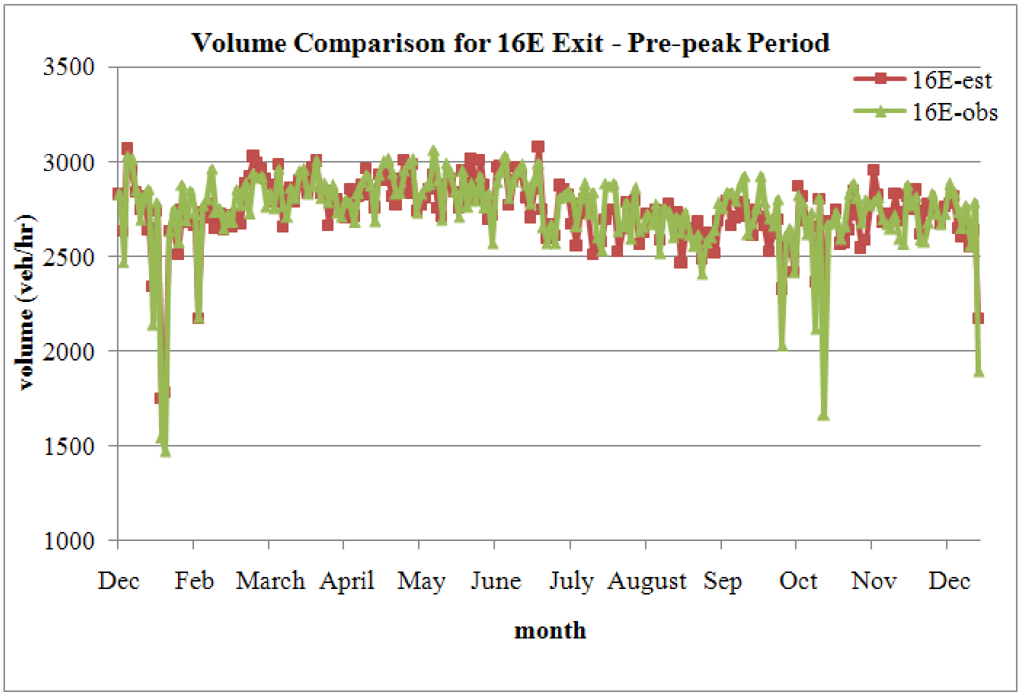


Figure 5-13 Volume comparison for Exit 16E, pre-peak period

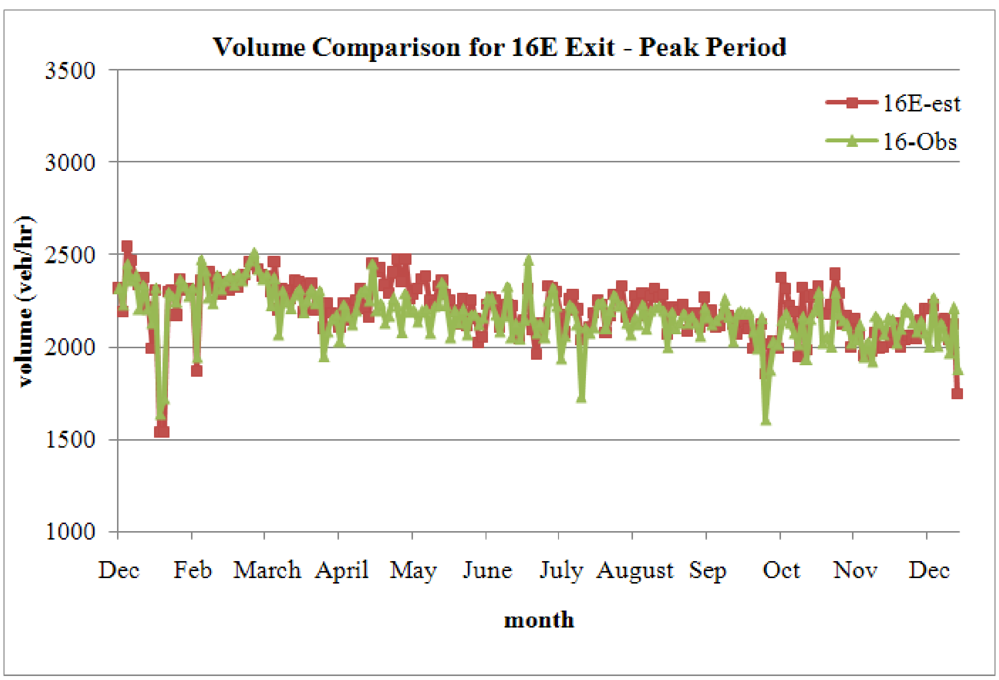


Figure 5-14 Volume comparison for Exit 16E, peak period

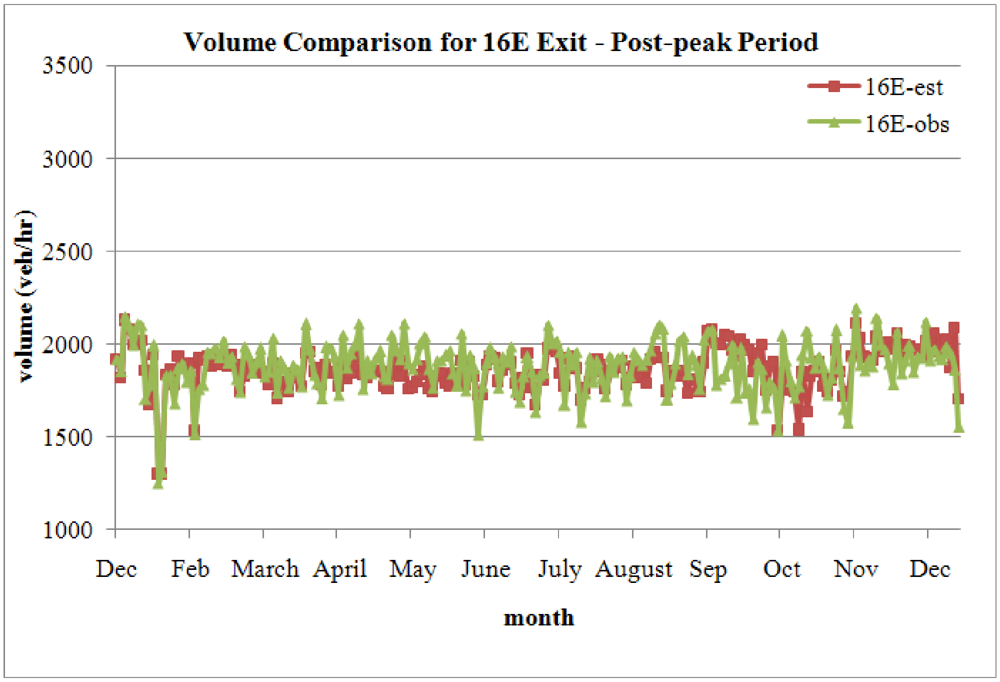


Figure 5-15 Volume comparison for Exit 16E, post-peak period

A similar analysis is conducted to investigate the evolution of day-to-day path travel times between Exit 1 and Exit 18E, the longest trip at NJ Turnpike, and between Exit 11 and Exit 16E, a shorter trip at NJ Turnpike. Table 5.1 summarizes the mean and standard deviation of travel times for these particular links during pre-peak, peak and post-peak periods. Similarly, Figure 5-16, Figure 5-17 and Figure 5-18 show the evolution of travel time between exits 1 and 18E, exits 11 and 16E and exits 9 and 14 from December 2005 and December 2006, respectively. The summary statistics show that mean travel time levels for the proposed day-to-day learning model and the observed travel time levels are very similar.

On the other hand standard deviation levels are slightly higher for observed travel time values. The reason for higher standard deviation values can be due to several reasons. Given that traffic flow counts were matching quite well, it seems that these differences were in part due to supply parameters. Moreover, the observed travel time information for each interchange includes travel time on the NJ Turnpike, and waiting time and service time at the exit toll plaza. Since the exact toll plaza delays are not available, to calculate the waiting time and service time at the toll plazas we have used a travel delay formulation proposed by Lin (2001) and later validated by Ozmen-Ertekin et al. (2008) on NJ Turnpike toll plazas, which have affected the estimation results. Lastly, the proposed framework does not consider accidents or different adverse weather conditions that are likely to affect demand and traffic. Instead this framework aims to model the changes in the traffic volume levels for a typical day at NJ Turnpike due to major disturbances in the system. However, as seen from the plots of the path travel times during different periods, on several days the travel time levels are more than twice of the regular travel time levels. This observation indicates that an unusual event has happened on that particular day, increasing the standard deviation levels for the path travel time of the interchanges. On the other hand, the estimated traffic flow values excluding the spikes are comparable to the observed values during the calibration process. In order to incorporate these unexpected trends in the traffic flows, within-day dynamics should be included in the modeling process. Via this approach, traffic irregularities caused by accidents and other minor disturbances can be modeled.

Table 5.1 Travel time statistics

Path	Statistics	Pre-peak		Peak		Post-peak	
		Est.	Obs.	Est.	Obs.	Est.	Obs.
1-18E	Mean (min)	130.26	128.94	123.26	125.17	116.32	113.37
	St. dev (min)	16.49	23.61	12.645	17.79	9.47	14.81
11-16E	Mean (min)	22.39	20.79	24.18	26.07	28.23	27.557
	St. dev (min)	1.12	1.44	2.96	3.93	3.63	5.499
9-14	Mean (min)	22.26	24.41	26.26	27.89	27.92	29.43
	St. dev (min)	2.44	4.81	3.11	4.12	4.55	6.51

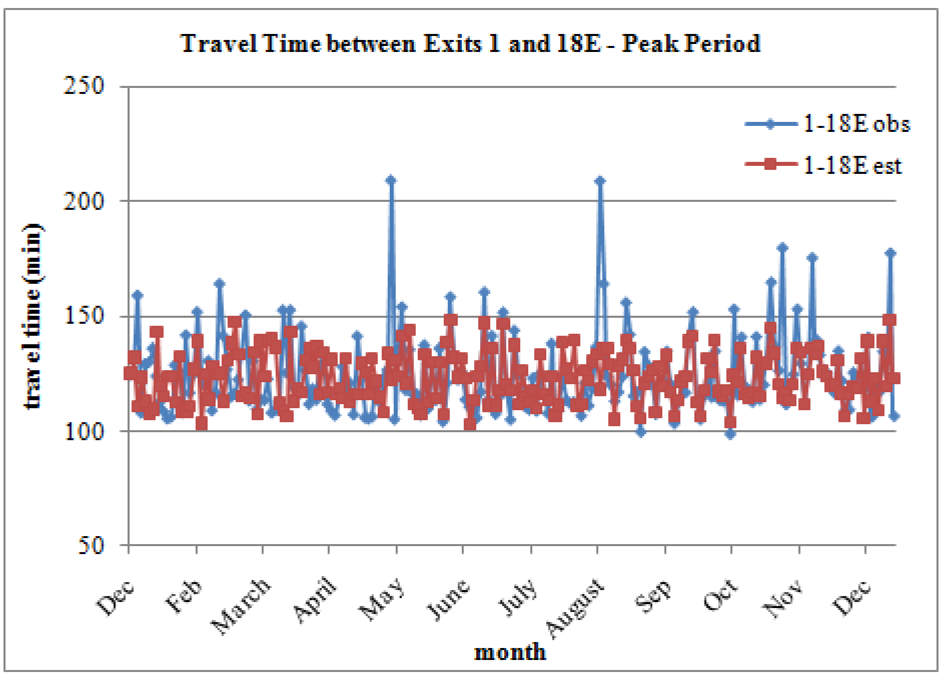


Figure 5-16 Travel time comparison for Exit 1-18E, peak period

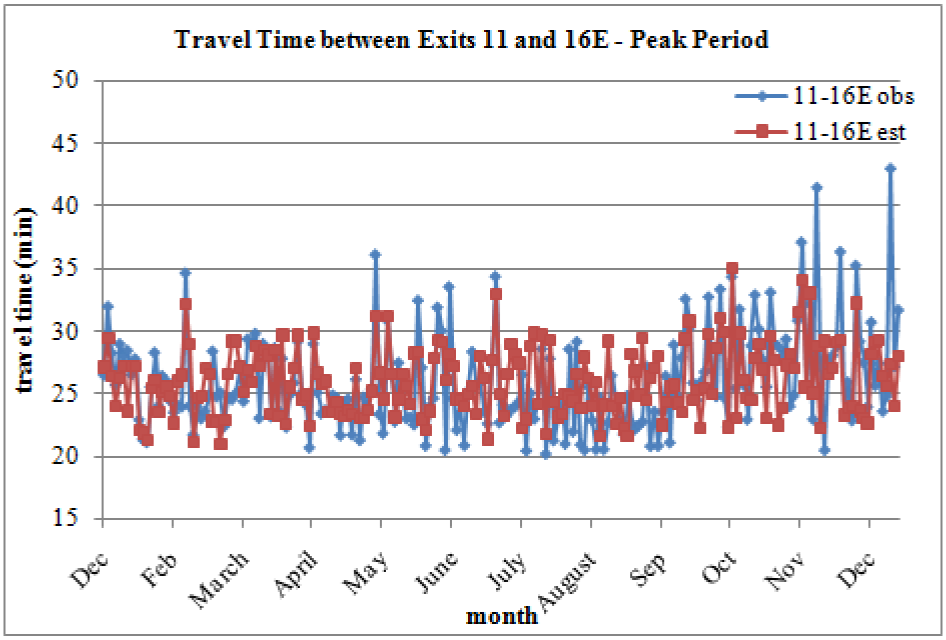


Figure 5-17 Travel time comparison for Exit 11-16E, peak period

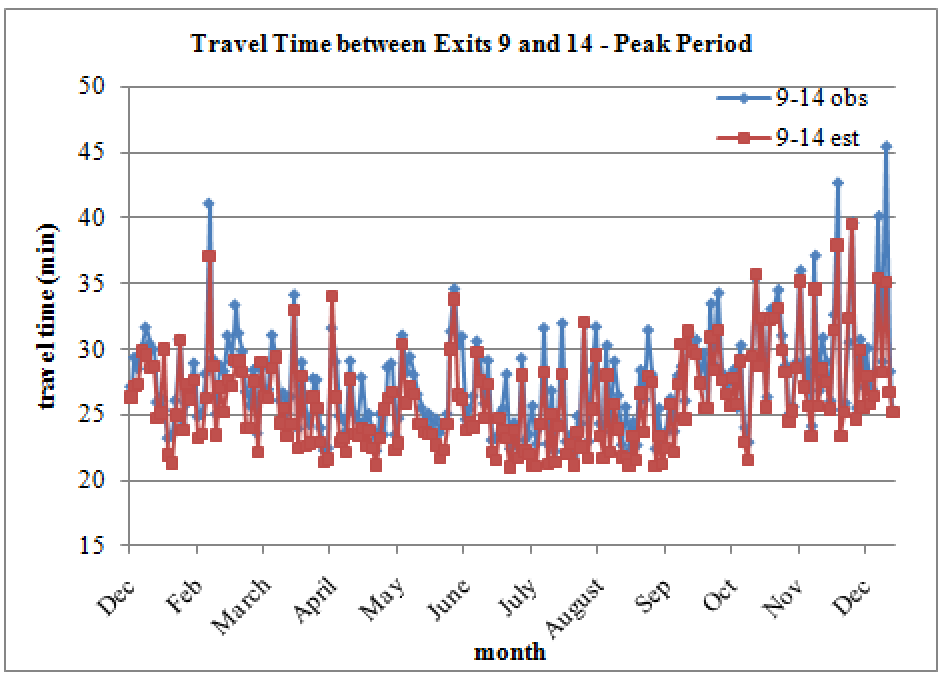


Figure 5-18 Travel time comparison for Exit 9-14, peak period

5.11 Convergence Properties

One major issue while investigating the day-to-day impacts of policy implications is to understand how long it takes for the transportation system to converge to a new steady state. This information is crucial in terms of understanding travelers' behavioral responses to the major system changes that would help both researchers and policy makers in identifying expected impacts of future transportation management strategies.

To this extent, this section focuses on the overall system changes in terms of departure time and route choice after January 2006 toll structure change and December 2005 Interchange 15X installation, respectively. For each major disturbance, changes in the average traffic volume during peak and peak shoulder (pre-peak and post-peak) periods were analyzed. To evaluate the performance of day-to-day DTA framework in terms of predicting traveler departure time and route choice behavior, simulated and observed traffic volumes were compared. In order to reduce the impacts of seasonal changes each monthly average volume is normalized.

Figure 5-19 and Figure 5-20 summarize the behavioral changes (simulated and observed) after January 2006 toll structure change for peak and peak shoulder periods, respectively. Analysis results reveal that day-to-day DTA framework can successfully capture the trends in peak and peak shoulder periods after the disturbance. Furthermore, the observed and simulated traffic volume results show that February 2006 is the transient period where the travelers learn the prevailing conditions of the disturbed transportation system (fast-moving system). In this period, a rapid decrease for peak period and rapid increase for peak shoulder periods is observed. After February 2006, experienced and simulated traffic conditions exhibit a more steady state until November

2006. Between November 2006 and December 2006 the change in the traffic conditions exhibits an increased rate. This trend may be explained due to winter conditions and increased leisure trips due to holiday season. Since travelers are familiar with the transportation system this short transient period and rapid changes in this period is expected, and supported by larger learning parameters. Travelers being aware of the structure of the whole transportation system can adapt themselves to the new conditions and exhibit faster learning behavior.

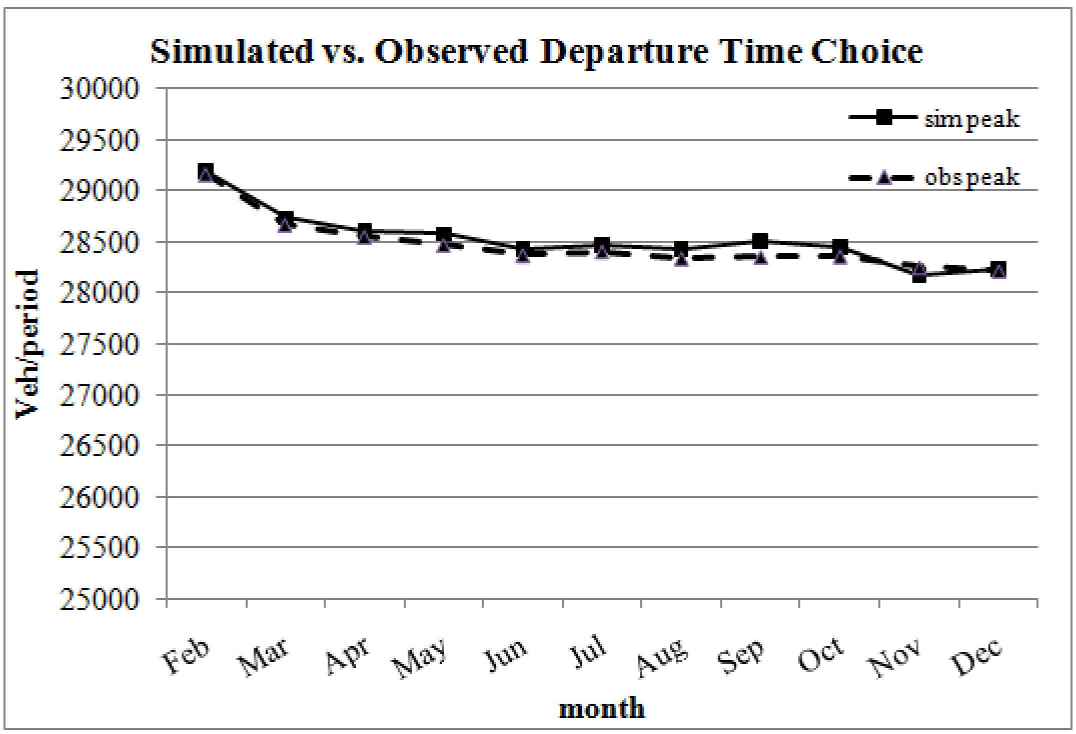


Figure 5-19 Simulated vs. observed departure time choice, peak period

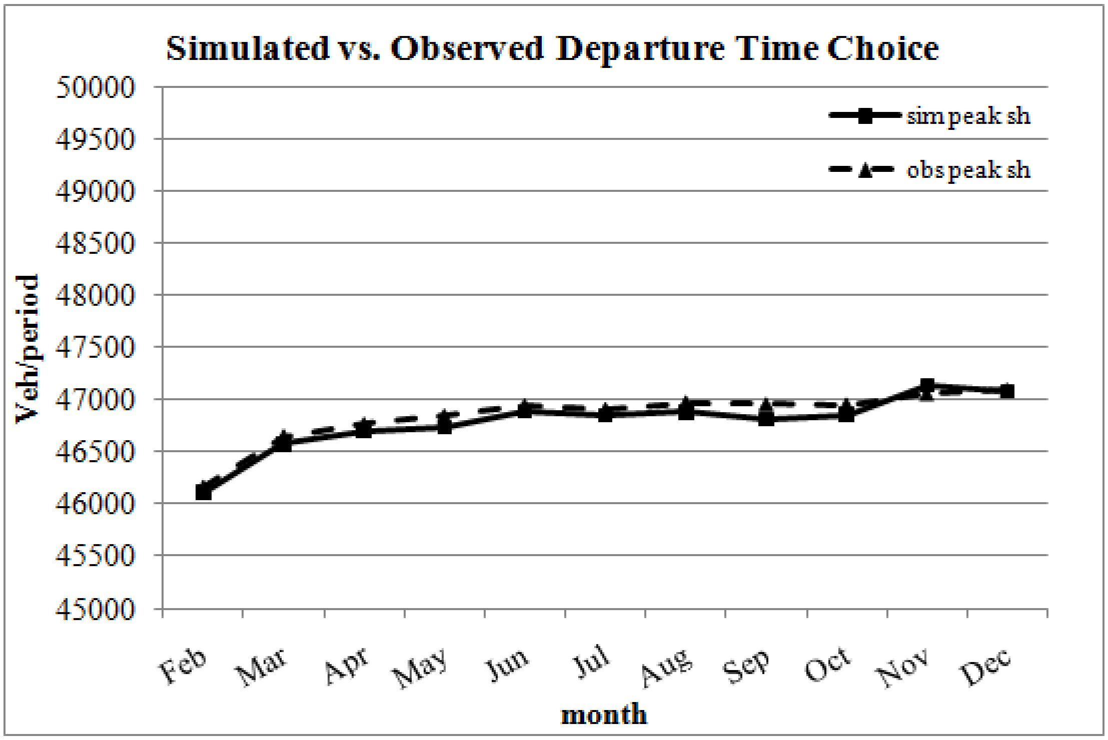


Figure 5-20 Simulated vs. observed departure time choice, peak shoulder

A similar analysis is conducted to investigate the impacts of Interchange 15X installation. To determine the duration of the transient period and understand the convergence behavior to the new equilibrium, traveler choice behavior is analyzed by observing the changes in average traffic volume exiting interchanges 15X and 16E during am period in the northbound direction.

Figure 5-21 and Figure 5-22 summarize the behavioral changes (simulated and observed) at Interchange 15X and Interchange 16E, respectively. Analysis results reveal that day-to-day DTA framework can successfully capture the trends in demand at interchanges 15X and 16E after March 2006. Furthermore, the observed and simulated traffic volume results show that the transient period after this major disturbance is much longer compared with January 2006 toll structure change. In fact, by September 2006, we observe that the demand for Interchange 15X is still increasing at a positive rate. Within this transit period, instead of a rapid change we observe slower responses from the travelers. As pointed out in the previous chapter, travelers of NJ Turnpike exhibit resistance to change their behavior resulting in slower learning and adapting rates to the new conditions. This type of behavioral pattern causes a longer transient period where no rapid changes in the demand is observed. After September 2006, the transient period diminishes and transportation system starts to reach to a new steady state. In particular, the demand for Interchange 15X still continuous to increase, however rate of increase diminishes resulting in reduced fluctuations in traffic flow conditions.

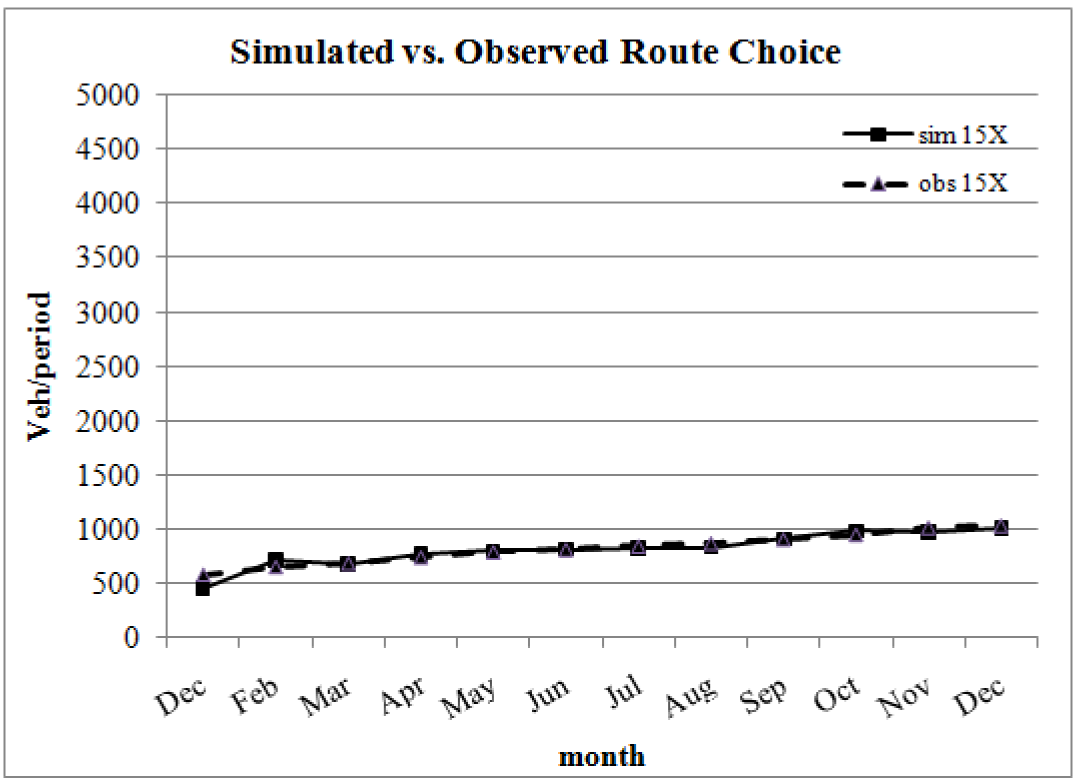


Figure 5-21 Simulated vs. observed route choice, Interchange 15X

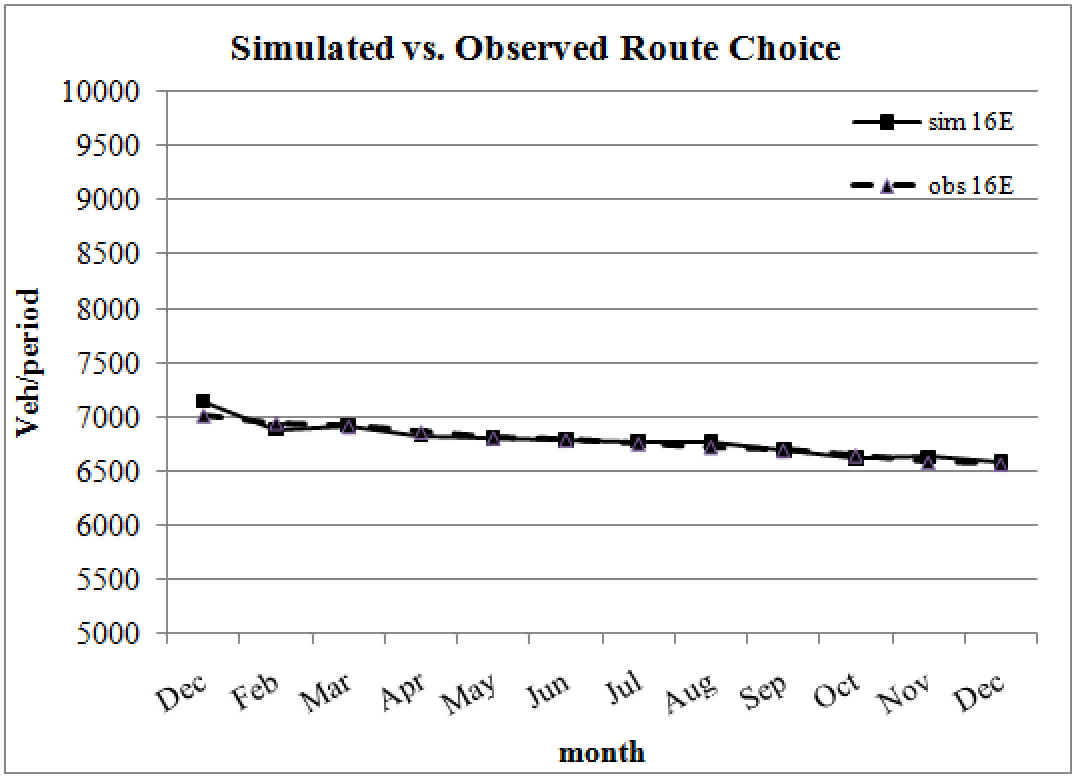


Figure 5-22 Simulated vs. observed route choice, Interchange 16E

The overall convergence properties of observed and simulated traffic conditions reveal that, when the travelers are familiar with the form of the disturbance imposed to the system, such as changes in an existing pricing application, the transient period is rather short and we observe rapid changes and fast learning rates during this period. On the other hand, when the disturbance is more significant, such as an infrastructural change in the transportation system, the transient period becomes longer, and we observe lower learning rates where travelers are hesitant to make drastic behavioral changes.

5.12 Sensitivity Analysis

In this section several sensitivity analyses are conducted to investigate the impacts of changes in the model specifications on the validity of the proposed day-to-day DTA framework. In particular, the following changes are considered:

1. Initial probability profile: in order to see the impacts of initial probability profile on the convergence and validity of the proposed model, instead of using observed frequency values as the initial probability profile, we start with equal initial probability profile, i.e.

$$p_{i,j}^{k,n=1}(r) = \left(u_{i,j}^{k}\right) * \frac{1}{r} \quad \forall i, j,\ \forall r \ and \ \forall k \tag{5.25}$$

2. Learning component: In order to see the impacts of learning component on the performance of the proposed model, learning component is removed from the proposed framework, and initial frequency values are used for daily probability profile:

$$p_{i,j}^{k,m}(r) = \left(u_{i,j}^{k}\right) * p_{i,j}^{k,n=1}(r) \quad m = 1,2, \dots N \quad \forall i, j \ \ \forall r \ and \ \forall k \tag{5.26}$$

Figure 5-23 and Figure 5-24 show the trend in MPE and RMSE values for traffic volume when equal initial probability profile is considered for each individual. As expected, for the first couple of weeks very high MPE and RMSE values are observed. However, as the travelers learn the system the error values start to diminish. After two months (21 days) the MPE and RMSE values start to follow the same trend with the proposed framework. Similarly, Figure 5-25 and Figure 5-26 show the trend in MPE and

RMSE values for travel time when equal initial probability profile is considered for each individual. The trend in the MPE and RMSE values are similar to the analysis results of the traffic volume, except the fact that error values are slightly lower for travel time.

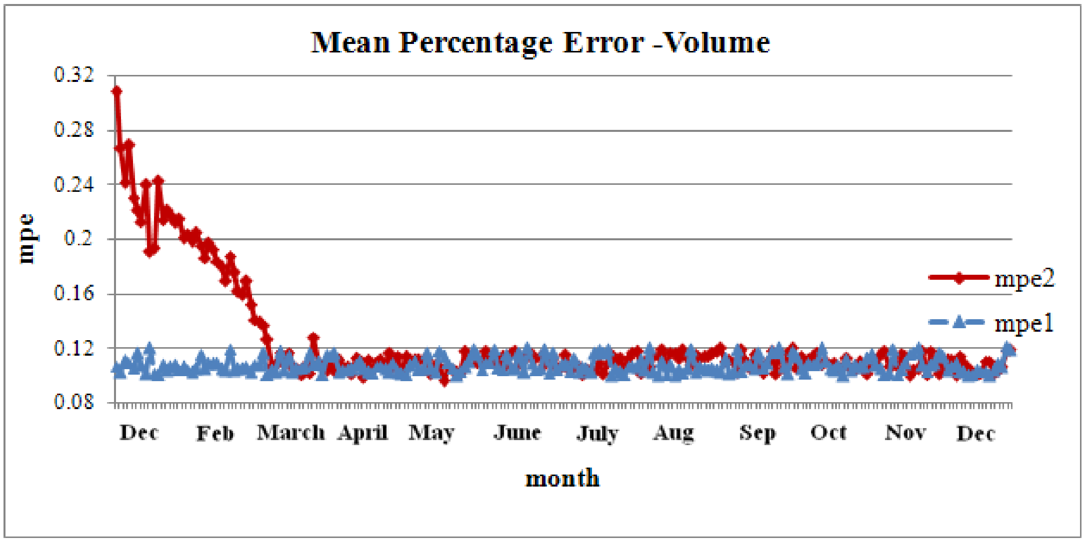


Figure 5-23 Mean percentage error for traffic volume, equal initial probability

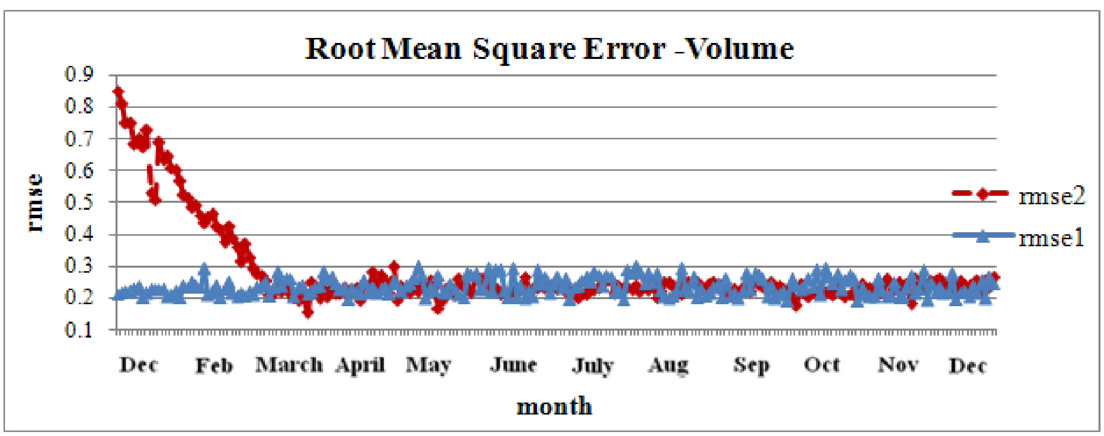


Figure 5-24 Root mean square error for traffic volume, equal initial probability

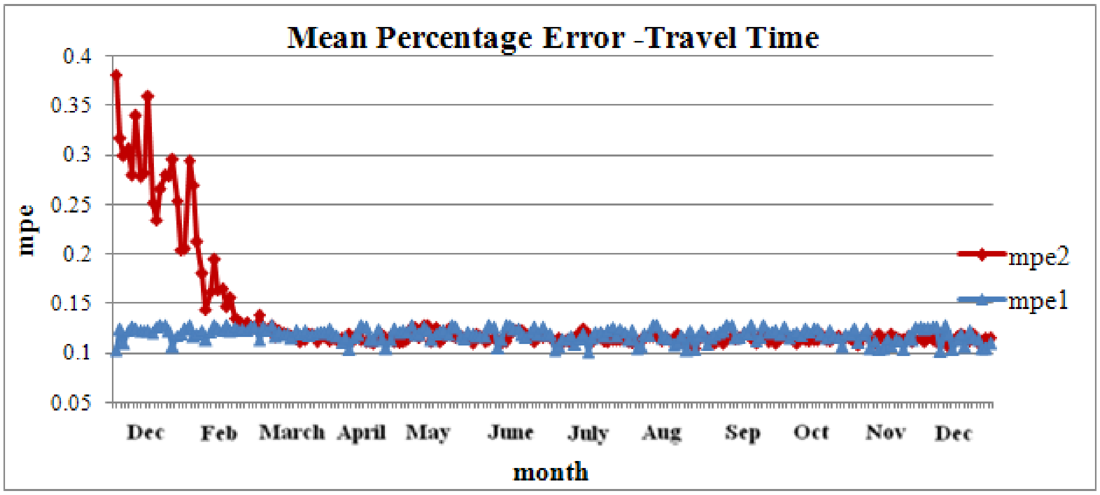


Figure 5-25 Mean percentage error for travel time, equal initial probability

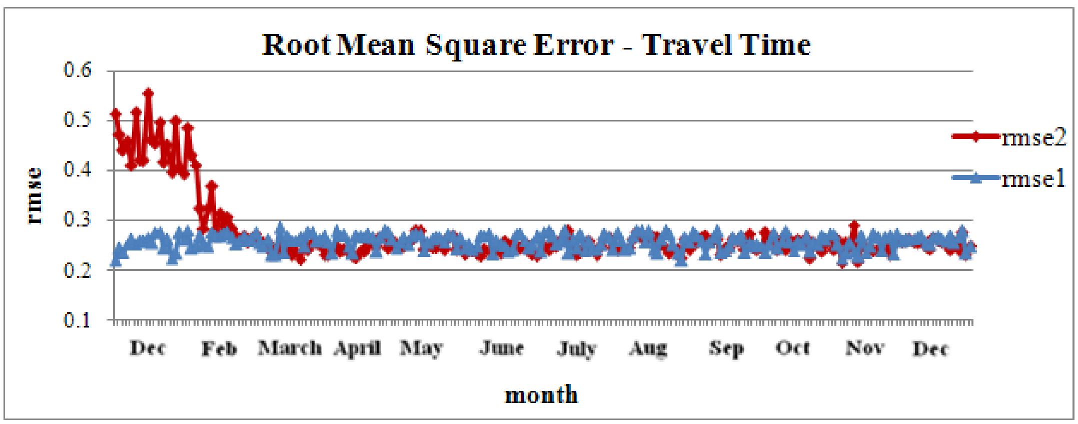


Figure 5-26 Root mean square error for travel time, equal initial probability

The second sensitivity analysis is conducted in order to observe the impacts of the learning component on the performance of the proposed model. Figure 5-27 and Figure 5-28 show the trend in MPE and RMSE values for traffic volume when learning component is excluded from the simulation model. Figure 5-29 and Figure 5-30 show the trend in MPE and RMSE values for travel time when learning component is excluded from the simulation model.

When learning component is removed from the day-to-day DTA framework, it is observed that both MPE and RMSE values increase compared with the error values calculated when day-to-day learning behavior of individuals is included into the model. Since the initial probability profile was assumed to be the observed frequency values, the estimated error values are smaller during first 10 days. However, as the transportation system evolves from day-to-day and drivers learn the new system conditions the error values start to increase. Moreover, neither MPE nor RMSE values show any tractable trend.

This sensitivity analysis confirms that proposed day-to-day learning framework is a crucial component of the DTA framework, particularly while investigating the traveler behavior during the transient period after a major disruption. Ignoring the impacts of travel experiences and travelers' learning behavior on the evolution of traffic conditions results in lower prediction capabilities, and failure to capture the day-to-day evolution of travel trends.

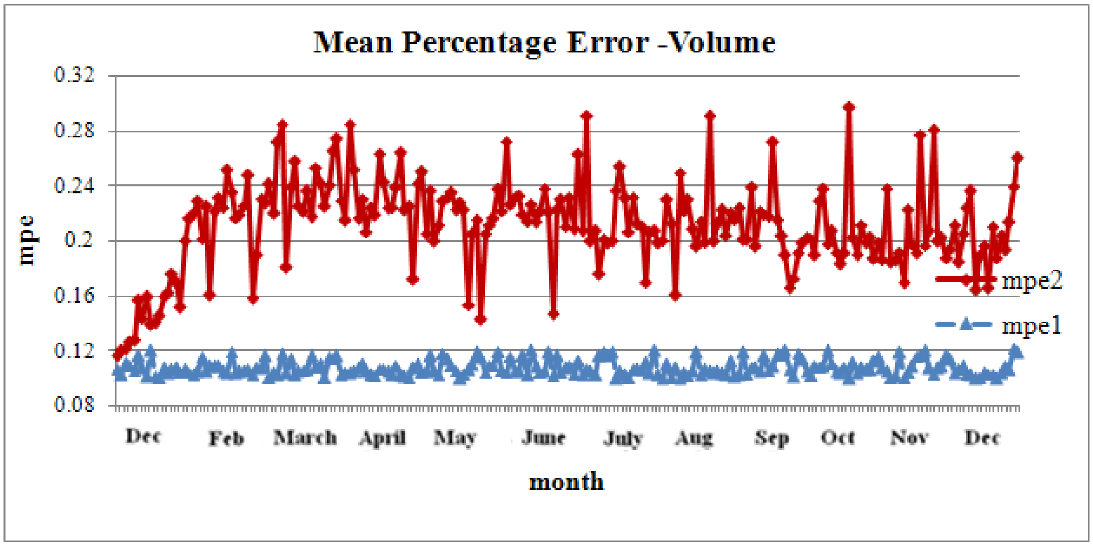


Figure 5-27 Mean percentage error for traffic volume, no learning component

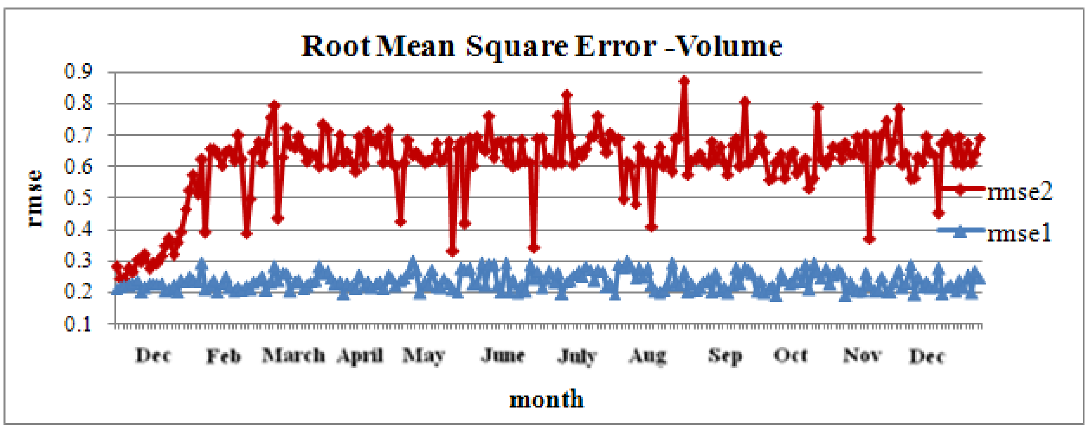


Figure 5-28 Root mean square error for traffic volume, no learning component

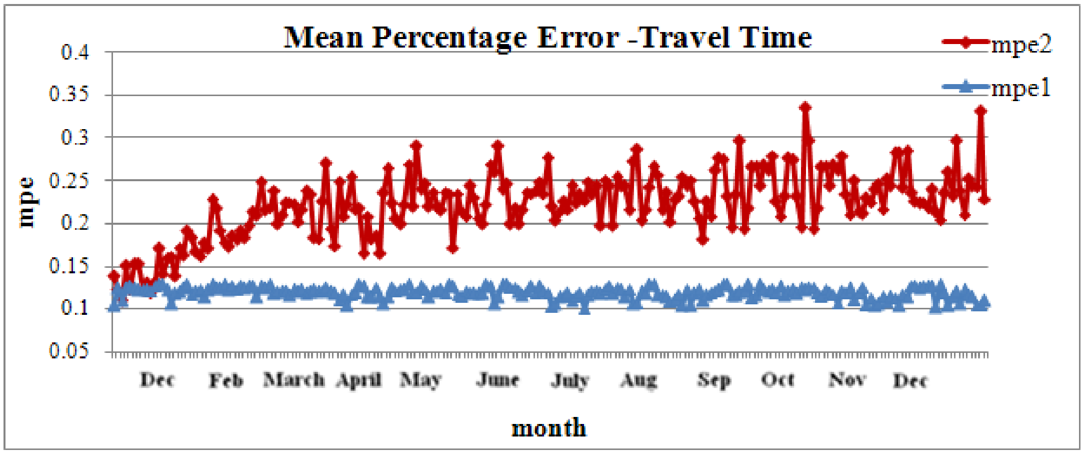


Figure 5-29 Mean percentage error for travel time, no learning component

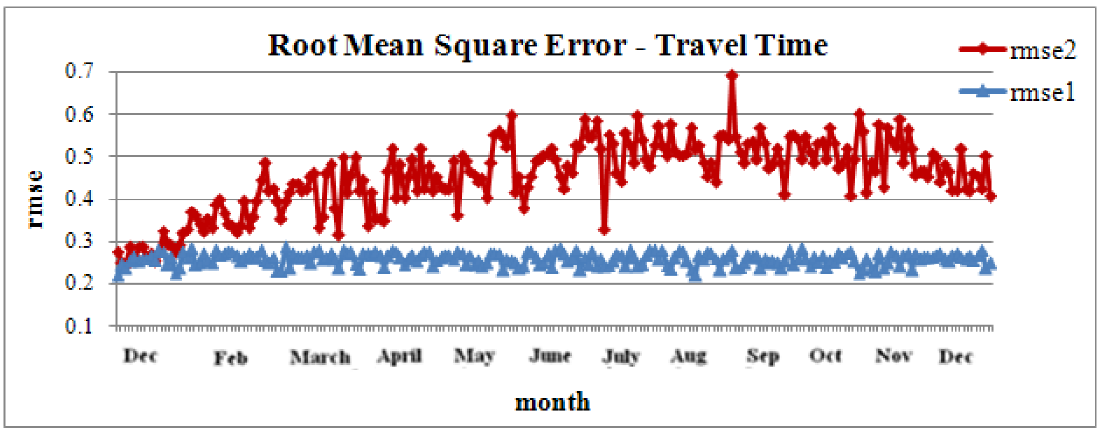


Figure 5-30 Root mean square error for travel time, no learning component

5.13 Conclusions and Discussions

This chapter presented a new DTA framework to examine the day-to-day evolution of travel patterns in a traffic network when major disturbances are introduced into the transportation system. The dynamic traffic flow evolution and network-level interactions of driver departure time and route choice decisions are captured via a traffic flow simulator. The approach uses microscopic simulation to model the behavior of drivers on the demand side, and uses macroscopic simulation to obtain system variables such as link travel time, volume and density. Bayesian-SLA framework developed in the previous chapter is used to model day-to-day update mechanism of the transportation network.

The proposed model has been tested and verified on NJ Turnpike. In particular, two major disruptions were considered. The first major disruption is the installation of 15X Interchange on December 2005. The second major disruption was imposed one month later. In January 2006, NJ Turnpike Authority eliminated the E-ZPass peak period discounts and E-ZPass peak users started to pay the same amount of toll as the cash users.

The calibration and validation results have shown that the proposed day-to-day dynamic traffic assignment framework can successfully capture day-to-day update of traffic flow after the imposed disruptions. The proposed day-to-day DTA assignment framework performed reasonably well with MPE values ranging around 0.107, and RMSE values ranging around 0.235 between December 2005 and December 2006 for traffic. Similarly, the MPE values range around 0.118 and RMSE values range around 0.257 between December 2005 and December 2006 for travel time. These results are fairly consistent regardless of network congestion levels, and the relative magnitude of errors is similar to the ones observed in other studies (Antoniou, 2004; Barceló and Casas, 2005; Park et al., 2008).

The overall convergence properties of observed and simulated traffic conditions reveal that, when the travelers are familiar with the form of the disturbance imposed to the system, such as changes in an existing pricing application, the transient period is rather short and we observe rapid changes and fast learning rates during this period. On the other hand, when the disturbance is more significant, such as an infrastructural change in the transportation system, the transient period becomes longer, and we observe lower learning rates where travelers are hesitant to make drastic behavioral changes.

Next, day-to-day learning component is removed from the DTA framework in order to investigate the impacts of traveler learning behavior on capturing the day-to-day evolution of the travel trends. The sensitivity analysis confirmed that proposed day-to-day learning framework is a crucial component of the DTA framework, particularly while investigating the traveler behavior during the transient period after a major disruption. Ignoring the impacts of travel experiences and travelers' learning behavior on the evolution of traffic conditions resulted in lower prediction capabilities, and failure to capture the day-to-day evolution of travel trends.

CHAPTER 6. CONCLUSIONS AND FUTURE WORK

This thesis has proposed a novel framework to model the interdependence between travelers' choice decisions, learning and adaptation behavior and the day-to-day update mechanism of traffic flows. The day-to-day models predict travelers' choices at any given day based on their experienced choices in the previous days. Day-to-day approaches allow the use of wide range of behavioral rules, and levels of aggregation, and capture the heterogeneity in users' learning and adaptation processes, and behavioral characteristics.

We introduce a new novel day-to-day learning framework to model travelers' departure time and route choice behavior under non-equilibrium network conditions due to major disturbances, such as changes in the congestion pricing policies, and building of new road sections. An agent-based learning system via Bayesian-SLA is designed which can learn the best possible actions and model travelers' day-to-day travel choices in a non-stationary stochastic environment. The developed learning framework reflects travelers' perception about the system and their response to the experienced traffic conditions.

Next, the proposed day-to-day learning framework is integrated into dynamic traffic assignment problem to capture the dynamic traffic flow evolution and network-level interactions of driver departure time and route choice decisions. The approach uses microscopic simulation to model the behavior of drivers on the demand side, and uses macroscopic simulation to obtain system variables such as link travel time, volume and density.

The novelty of this thesis is that the proposed approach combines traveler heterogeneity and rationality in a single framework to predict travelers' day-to-day departure time and route decisions, and develops a novel day-to-day dynamic traffic assignment approach.

In order to test the performance of the proposed day-to-day learning framework, and to understand the traveler responses to real changes in the transportation system two different major disruptions imposed on NJ Turnpike were investigated. The empirical results obtained from real transportation network, NJ Turnpike, confirm the strong effect of habitual behavior on traveler choice. The proposed Bayesian-SLA model can successfully capture the significant learning dynamics, demonstrating the possibility of developing a psychological framework (i.e., learning models) as a viable approach to represent travel behavior.

The overall convergence properties of observed and simulated traffic conditions reveal that, when travelers are familiar with the form of the disturbance imposed to the system, such as changes in an existing pricing application, the transient period is rather short and we observe rapid changes and fast learning rates during this period. On the other hand, when the disturbance is more significant, such as an infrastructural change in the transportation system, the transient period becomes longer, and we observe lower learning rates where travelers are hesitant to make drastic behavioral changes.

Several sensitivity analyses conducted to investigate the impacts of traveler learning behavior on capturing the day-to-day evolution of the travel trends confirmed that proposed day-to-day learning framework is a crucial component of the DTA framework, particularly while investigating the traveler behavior during the transient period after a major disruption. Ignoring the impacts of travel experiences and travelers' learning behavior results in lower prediction capabilities, and failure to capture the day-to-day evolution of travel trends.

The overall results of this thesis have shown that, major changes in the transportation system disrupts the network equilibrium and causes a dynamic disequilibrium transient state where travelers adjust their choices to adapt the prevailing conditions of the disturbed transportation system. In this dynamic traffic network disequilibrium state, travelers exhibit a learning process where experiences in a previous day affect their expectations and decisions in subsequent days. Travelers' familiarity or unfamiliarity with the newly imposed network conditions affect the evolution of the network conditions from day to day as travelers continually adjust their behavior based on prior experiences and a new steady state (equilibrium) flows are approached as a result of this learning period.

Unlike traditional equilibrium analysis, which only pays attention to the final "steady-state" while ignoring how travelers dynamically adjust their behavior and how traffic flow evolves over "days", disequilibrium approach developed in this thesis, focuses on how travelers respond to changes in the transportation system, day-to-day evolution of the traffic flows and the convergence properties of the disrupted transportation system. This type of day-to-day learning approach is of great importance in transportation network analysis, both for a better understanding of the properties of the standard traffic equilibrium model, and for practical reasons related to the monitoring and management of traffic flows.

6.1 Future Research Directions

For future research on day-to-day learning topic, several directions should be worth of attempts.

This thesis focuses on day-to-day dynamic and within-day static transportation networks and does not consider the impacts of within-day dynamics on travelers' day-to-day travel choice. An interesting research direction would be to extend the methodology proposed in this thesis to within-day dynamic context to capture more realistic traffic flow dynamics.

Moreover, this thesis aims to model day-to-day travel choice behavior as a result of experienced choices. With recent advances in intelligent transportation systems, route guidance and pre-trip information offer promising system efficiency. Another future research direction is to model the effects of route departure time switching dynamics under advanced traveler information systems on travelers' day-to-day choice behavior.

The proposed day-to-day learning framework is a trip-based model aiming to predict travelers' day-to-day departure time and route choice. However, with recent advances in transportation field, activity-based models have gained attention from researchers. Unlike, trip-based models, activity-based models consider the linkage among trips. Travel demand is a derived demand on the basis of travel behaviors where travelers arrange their travel to perform their activities. Thus, to fully understand and predict the travel demand, it is crucial to understand what drives people to travel, i.e. why, where and when activities are engaged in, and how activity engagement is related to the spatial and institutional organization of a transportation system. To this, extent including the trip chains via activity models would be an important improvement to the proposed day-to-day learning model.

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Curriculum Vita

Ozlem Yanmaz-Tuzel

Education

2005-2010 Ph.D., Civil and Environmental Engineering, Rutgers University

2003-2005 Master of Science, Civil and Environmental Engineering, Rutgers University

1998-2003 Bachelor of Science, Civil and Environmental Engineering, Double Major in Industrial Engineering, Bogazici University, Turkey

Publications
Book Chapters

- Berechman J., Yanmaz-Tuzel O., Ozbay K., Bartin B. (2010) "The Full Marginal Costs of Highway Travel: Methods and Empirical Estimation for North America", Handbook in Transport Economics, Chapter 12. In print.
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