INTRODUCING UNCERTAINTY INTO EVACUATION MODELING VIA
DYNAMIC TRAFFIC ASSIGNMENT WITH PROBABILISTIC DEMAND AND
CAPACITY CONSTRAINTS

by

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ABSTRACT OF THE DISSERTATION

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Emergency evacuations are low-probability-high-consequence events that have attracted the attention of researchers since 1960s. An evacuation process can be triggered by various natural (hurricane, flood, tsunami etc.) and man-made (industrial accidents, terrorist attack etc.) events. Regardless of the threat, the nature of the evacuation process involves a very high utilization of the transportation network and searching for plans/strategies to move large number of people to a safe place in the shortest possible time. Researchers from different disciplines approach to the evacuation problem from different perspectives. Two major components of any evacuation event are estimation of the evacuation demand and traffic analysis to make planning inferences about the evacuation performance measures such as clearance time. Although related studies and real-life practices show a significant uncertainty regarding the evacuation demand due to the unpredictability of human behavior and changing roadway as a result of disaster impacts, the state-of-the-practice does not consider this type of randomness. This
dissertation aims to address this important gap by proposing a dynamic traffic assignment formulation with probabilistic constraints that takes into account uncertainties in demand and roadway capacities. The proposed model uses a cell transmission model based system optimal dynamic traffic assignment formulation. The demand and roadway capacities are assumed to follow a discrete random distribution and the p-level efficient points approach [115] is employed to solve the proposed model. Two numerical examples regarding the use of the model are provided. The numerical examples also discuss the implications using individual chance constraints vs. joint chance constraints which provide different interpretations for the reliability of the results. Overall, the proposed formulation generates evacuation time performance measures that can be interpreted within reliability measures rather than single deterministic point estimates that would not be necessarily observed during a real life test, mainly due to high level of uncertainty created by human behavior and capacity impacts of the disaster.
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Dedication

To the best engineer I have known, my father, Mesut Yazıcı…

Taǹd̀̀ğım en iyi mühendis, babam, Mesut Yazıcı'ya…
Preface

The work conducted in this dissertation has been presented and published in several conferences and journals. Below is the list of publication derived from this dissertation with corresponding chapter numbers.

Chapter-2 & Chapter-3


Chapter-4 & Chapter-5

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CHAPTER 1. INTRODUCTION

Evacuation is defined as “mass physical movements of people, of a temporary nature, that collectively emerge in coping with community threats, damages, or disruptions” by E. L. Quarantelli [36]. There has been extensive research on evacuation, mostly motivated by the threats that are the main concerns during the time of the research. For instance, the number of nuclear power plant evacuation studies in U.S. increased considerably after the Three Mile Island nuclear power plant disaster near Middletown, Pennsylvania, on March 28, 1979. Recent interest in emergency response for man-made disasters can also be attributed to concerns about terrorism. If the worldwide research is investigated, one can find additions such as tsunami and earthquake related evacuation studies. Nevertheless, the hurricane evacuation stands as the most popular research area in the U.S. because of its regular and frequent occurrence.

Figure 1.1 Evacuation Modeling Components
Although the evacuation process is strongly affected by the nature of the threat, underlying mechanism is the same for all disasters and can be illustrated with some main components that are in connection with each other as shown in Figure 1.1. Each component is affected by the nature of the threat. For instance, the demand profile is expected to be different for a no-notice event (e.g. nuclear power plant explosion) compared to an advance-notice event (e.g. hurricane). The number of people to be evacuated may turn out to be same for different disasters, but their timing to hit the road can be different. One would expect evacuation of all affected public at once in a no-notice event whereas the evacuees will be loaded onto transportation network within a longer period (e.g. few days for hurricanes) in an advance-notice event. Likewise, the traffic network may be affected and lose capacity during hurricane evacuation because of flood, whereas no such impact is anticipated for nuclear power plant evacuation. As these points stand as direct impacts of the threat, the secondary impacts are coupled effects of capacity reduction and variation in demand profile which, in turn, adjust the clearance time of the network.

1.1 Problem Statement

Historically, the network traffic is modeled as flows between source and sink nodes (which are basically the origins and destinations for all trips) within certain defined costs (mostly travel time) incurred while traversing each link in the network. The mathematical formulations also include constraints that bound the maximum link flows by the link capacities and maintain the conservation of the number of users in the system to ensure the physical correctness of the model outcomes. In terms of the time structure,
the model can be static, which refers to calculation the flows for single period instance for the aggregate demand at the source nodes; or dynamic, in which the traffic flow is constantly updated at -preferably- smaller time intervals using time dependent link congestion and number of vehicles in source nodes. Nevertheless, regardless of the time structure of the model, the basic flow model components are:

1. The network topology
2. The number of vehicles at the sources (origins) to be loaded onto the network to travel to the sink nodes (destinations)
3. The link costs and capacities

These components also constitute the base of evacuation models; however the demand and travel patterns during evacuation are different than normal, non-emergency conditions (e.g. peak/off-peak travel patterns). Evacuation planning studies use some specific demand models developed for evacuation conditions. Furthermore, evacuation networks are composed mostly of one-way links, which connects the origins to few selected destinations (e.g. shelters) instead of employing 2-way links to account for traffic flow in both directions. In other words, although the underlying approach is similar, evacuation modeling differs from modeling of daily (or better say, non-emergency) traffic patterns with the use of evacuation specific network topology and demand profiles.
1.1.1 Sources of Uncertainty

Similar to other problems in transportation modeling area, evacuation modeling is also subject to uncertainties. The sources of uncertainty are numerous and mostly not easy to quantify or control. As shown in the simple sketch of the evacuation modeling in Figure 1.1, the human behavior (which is inherently probabilistic) plays an important role throughout the evacuation process. Human behavior can affect various stages of the analysis such as decision for the timing of evacuation, route choice, driving behavior, compliance with official notices etc. However the stochastic nature of evacuation modeling is not confined only to human behavior. The limited predictability of variation in the severity of the disaster (e.g. the intensity and track of a hurricane, or the timing of any disaster occurrence), the impacts of disasters on infrastructure (e.g. flood on network links during a hurricane, collapse of a bridge after an earthquake, long term blackout after a nuclear power plant explosion etc.) are just few factors to name few among many others. For instance, roadway capacity problems experienced in recent hurricane evacuation (Katrina, Rita) has turned attention of researchers more to the capacity degradations during disasters. Although such uncertainties in evacuation planning are mentioned in the literature, the researchers mostly ignore probabilistic considerations in the model formulations. On the other hand, the evacuation practices have proven these uncertainties to be a real problem that is in need of an urgent consideration, rather than just an academic interest. Hence, this study aims to analyze further the major uncertainties involved in evacuation modeling and come up with more realistic models that incorporate probabilistic factors into the evacuation modeling.
1.2 Dissertation Objectives

Current study approaches the evacuation problem from a planning perspective and aims to quantify evacuation performance measures for disaster scenarios by employing a probabilistic analysis approach. These performance measures can be possible severe bottleneck points, average travel or the clearance times. Clearance time is the time that all evacuees exit the evacuation zone and reach to safety and used as an important indicator that determines the required duration for the completion of an evacuation.

The flow and congestion in a road network are mainly driven by two factors: The number of vehicles at a certain time on certain routes and the available link capacities that determine how efficiently the existing users in these routes can be accommodated. During emergency conditions resulting in mass evacuation, the number of vehicles in the network is generally much higher compared to non-emergency conditions. Moreover, link capacities are prone to the risk of degradation due to disaster related disruptions. These unexpected changes in evacuation demand levels and capacity may result in significant differences in terms of the predictions of a model. Reliability of transportation planning model predictions is important because the predictions are used to make long-term planning decisions. If these predictions are not accurate then infrastructure investments based on the predictions might not generate desired outcome. This will have an adverse effect on the congestion, economy and environment. On the other hand, in evacuation planning, people's lives are at stake, hence the accounting for the uncertainty becomes even more important compared with usual planning studies.
Current study, with its stochastic formulation, provides predictions of evacuation performance measures in terms of reliability levels. In other words, the outcomes of the model can be interpreted to represent the probabilistic performance of the studied network in a future disaster within a certain level of reliability measure. For instance, a model result pointing out to certain hours of clearance time for an emergency evacuation case can be used by the planner to hold true as an upper bound in a real evacuation situation with a certain level of reliability. This kind of probabilistic approach allows the determination of the reliability of the uncertain model outcomes instead of providing deterministic results, which will more likely prove erroneous in reality due to the highly probabilistic nature of the problem.

1.3 Methodology

In this dissertation, cell transmission model (CTM) based deterministic system optimal dynamic traffic assignment (SO DTA) formulation proposed by Ziliaskopoulos [178] is used as the base model. For the proposed objective of the study, demand and roadway capacities are assumed to be random variables and the deterministic SO DTA model is extended by adding probabilistic demand and capacity constraints. System optimality refers to assigning flows in the network so that total travel time for all evacuees is minimized. In term of probability distributions, the evacuee demand is estimated at certain time intervals with the help of available demand models. In the proposed model, the estimated value is assumed to be the mean of a random variable with a probability distribution. Hence, the probabilistic model accounts for deviation from the estimated value within some pre-specified time intervals. On the capacity side, it is
assumed that the link capacity reduction probabilities in the network are known in advance.

1.4 Scope of Dissertation

Hurricane evacuations are the most studied type of disasters in the evacuation modeling literature. This is the main reason for the lack of information about the demand profiles or roadway capacity changes for disasters other than hurricanes. There are several evacuation demand models proposed for hurricane evacuations, mostly based on empirical survey data. For other disaster types, behavioral response curves (S-curves) are the most frequently used demand profile, however lack of empirical data makes it difficult to justify the use S-curves for all disasters. On the other hand, there are several evacuation demand models proposed for hurricane evacuations and computerized tools like SLOSH (Sea, Lake and Overland Surges from Hurricanes) exists to help making inferences about the road capacities. Provided that disaster specific demand profiles and roadway capacity probabilities can be determined, formulation proposed in this dissertation can be used for other disaster types without loss of generality.

The outline of the dissertation is as follows. First a literature review on human evacuation behavior and estimation of evacuation demand is provided. Selected demand generation models are studied to assess the impacts of demand uncertainty coupled with capacity reduction scenarios. Then dynamic traffic assignment and evacuation-specific models are discussed in terms of how the uncertainties related to demand and capacity are handled. An in depth numerical study of the existing evacuation demand models is concluded by the analysis of the implications of these findings on the dissertation
objectives. The system optimal dynamic traffic assignment model with probabilistic demand and capacity constraints are formulated and discussed in detail along with a specialized solution methodology. A detailed discussion regarding the interpretation of model results is then presented. Two numerical examples are provided for illustration of the proposed model’s use. Finally, the conclusions and future research directions are discussed.

CHAPTER 2. LITERATURE REVIEW

Main goal of this study is to provide a probabilistic model for preventive evacuation, which was called as “tactical evacuation” in 1950s [110]. Preventive evacuation can be defined as to remove the public out of the area they may return soon. In this literature review, first, general information about evacuation will be presented. Second, behavioral models for the evacuees, how and when they chose to evacuate and how they chose their routes and destinations will be discussed with the help of relevant literature.

2.1 Evacuation

Evacuation is defined as “mass physical movements of people, of a contemporary nature, that collectively emerge in coping with community threats, damages, or disruptions” by E. L. Quarantelli [36] and referred as a round trip by Church and Cova [27]. Perry et.al. [110] goes one step forward of this definition and they define 2 factors to have major impact on nature and conduct of evacuation; time between the evacuation
and disaster impact, and amount of time that the evacuees will spend away from their places. They also define subtypes as preventive, protective, rescue and reconstructive. Preventive and protective types are mentioned to be prior to disaster where protective requires evacuees to be away from their locations longer. Rescue and reconstructive types are post-disaster actions where rescue involve short term recovery operations such as moving the victims away from the disaster impact area. Reconstructive evacuation is for extreme cases where people are moved for a long time period to permit rehabilitation of an area, which has become uninhabitable. Wolshon et. al.[166] define categorization for hurricane evacuation regarding the official warning actions, with 3 categories:

1 **Voluntary**: Targeted towards populations that are not projected to experience serious storm surge or extreme winds. People may remain if they want and no special traffic control or transportation measures are taken.

2 **Recommended**: Issued when a storm has a high probability of endangering people living in at-risk areas. Similar to voluntary, people are free to chose not to evacuate and no traffic control measures are taken.

3 **Mandatory**: Issued for the populations that are estimated to experience serious storm surge and extreme winds. Extensive control measures are taken to avoid in-bound traffic to coastal areas. As discussed later in detail, timing of mandatory evacuation order is a challenging subject since a false warning may cause economical and psychological effects.
One of the earliest work in evacuation area is Lewis’[73] study. He asks 3 critical questions to be answered by the emergency planners:

1. What is the clearance time required to get the hurricane-vulnerable population to safe shelter?

2. Which roads should be selected?

3. What measures can be used to improve the efficiency of the critical roadway segments?

Lewis also mentions some subjects to be studied:

1. Evacuation travel patterns; trip purposes e.g. shelters, friends, shopping and supply gathering trips etc

2. Estimation of travel demand; evacuation zones and people’s evacuation behavior (to be determined by pre or post evacuation behavioral studies)

3. Calculation of clearance times; time required by evacuees to secure their homes and prepare to leave (mobilization time), time spent along the road network (travel time), waiting time because of congestion (queuing delaying time).

4. Development of traffic control measures; the operations that are performed by officials to fully utilize the road network

As mentioned by Lewis too, evacuation process is heavily dependent of behavior of people under threat which is necessary information for traffic assignment. There are many factors addressed in the literature to have effect on the evacuation behavior. Although different studies may give different factors as significant, some studies succeed
to determine the evacuation demand consistent with real-life data. However, there is no guarantee that a model that is developed with a certain regional data will be valid in other regions, or more formally, transferable. Figure 2.1 gives a general framework for the evacuation decision mechanism.

![Figure 2.1 A General Model of Evacuation Behavior [139]](image)

Behavioral studies are incorporated in transportation analysis for further analysis of:

- The specification of disaster scenarios (a separate transport analysis is conducted for each scenario);
- Definition of evacuation transport zones;
- The determination of demographic characteristics such as the size of the population at risk and characteristics of the evacuation population for each zone;
- Updating traffic conditions as they become known;
- Simulating changes in the links (roads) resulting from extreme weather conditions and fires and floods;
- Identifying roads and streets expected to be heavily used in an evacuation as well as their characteristics;
- Estimating the number of trips expected (this depends heavily on the outcome of the behavioral analysis) and trip productions and attractions are calculated for each zone;
- Distributing trips among the evacuation transport zones. In some instances, gravity models are used to show the effects of distance between pairs of production and attraction zones and the population size of likely attraction zones;
- Assigning trips to the road network connecting the zones; and
- Calculating clearance times for each scenario [5, 11, 12]

Following list of information is necessary to conduct the transportation analysis;

- An accurate description of the transport network/infrastructure;
- Size and makeup of the evacuation population including the location of subpopulations such as hospitals and schools;
- An accurate description of the spatial distribution of population by time of day and type of activity;
- Shape, size and rate of growth of the evacuation area;
- An accurate representation of vehicle utilization during an emergency. For instance, it is assumed that the number of household vehicles used during a night time evacuation is lower than the number of household vehicles used during a day time evacuation. Thus, it is expected that vehicle occupancy rates during night time evacuations will be higher than day time vehicle occupancy rates. This is an area that requires further investigation;
• An accurate representation of the timing of people’s response to an emergency;
• An accurate representation of evacuee route and destination selection behavior;
• An accurate representation of traffic management controls that may be included within the evacuation plan; and
• An accurate representation of any non-evacuation based protective actions [5]

Most of the factors summarized above are very hard to measure or represent, and lack of any data may result in significant reduction of the model’s ability to represent the reality.

2.2 Evacuation Behavior Studies

Behavioral models for the evacuees, how and when they chose to evacuate and how they chose their routes and destinations have received the attention of researchers for a long time. This is an interdisciplinary research question where studies from various fields such as engineering, psychology, planning all contribute. The results of building evacuation studies are mainly used for the architectural design of the structures or other infrastructure. However, when it comes to the subject of mass evacuation, these studies are addressed for emergency planning rather than using them in the preliminary design of the roads, highways etc. Most of the studies focus on sociological and psychological aspects, and elaborate on the preparedness for disaster or post-disaster actions. Limited number of studies found in the literature focus on the actual evacuee behavior during the disaster conditions because there is not enough actual evacuation data available. Evacuee behavior is mostly estimated using surveys conducted under non-disaster conditions. These surveys are assumed to determine the possible evacuation behavior in the future
and the findings are used for emergency demand analysis. Unfortunately, there is a heavy bias towards hurricane evacuation regarding the possible behavioral studies and demand models. The following section will also suffer from this bias. Nevertheless, it is possible, although with certain limitations, to make analogies for different disaster types, hence the behavioral studies for hurricane evacuation may still give an idea about the complexity of the human behavior under disaster conditions.

### 2.2.1 Factors Affecting Evacuation Demand

Some of the first evacuation studies were conducted for hurricane evacuation in the 1970s [6,151]. However, after the meltdown accident at Three Mile Island in Pennsylvania on March 8, 1979, the focus of evacuation modeling shifted toward the study of nuclear power plant evacuations [54,60,152]. These studies typically estimated the number of evacuating vehicles by determining the total number of people/households expected to evacuate and assigning a vehicle for each household or assuming a certain number of people would evacuate in a single vehicle. Then highway network traffic was analyzed with the estimated number of vehicles. A general travel demand forecasting process for hurricane evacuations was first described by Lewis [73]. He approached the problem by using the traditional urban travel demand forecasting methodology. Most of the post-hurricane surveys and behavioral studies were also conducted during the late eighties [28,58,110,113,122].

A behavior model should be able to reflect the following [5,11]:

- How many people will evacuate (evacuation participation rate),
- When evacuees will leave,
- What the rate of public shelter usage will be,
- How many evacuees will leave the area, and
- How many of the available vehicles will be used.

To clarify the points stated above, an individual decision process must determine (5,139):

- Whether to evacuate,
- When to evacuate,
- What to take,
- How to travel,
- What route to travel,
- Where to go, and
- When to return.

Baker [10] summarized the results of surveys conducted after 12 hurricanes from 1961 to 1989 in almost every coastal state from Texas through Massachusetts and identified the five most important variables in hurricane evacuation [10]:

- Risk level (hazardousness) of the area,
- Actions by public authorities,
- Housing,
- Prior perception of personal risk, and
- Storm-specific threat factor.

Most researchers in this area agree that these are some of the major factors that affect evacuation behavior. Baker especially mentioned that many “intuitively obvious” variables are “notoriously poor” at predicting whether people will evacuate and stated that it is almost impossible to completely model people’s evacuation decision process.
However, he cited some basic factors that can be used as a starting point. Feeling safe, although it is very difficult to explain how people feel, is identified as a major factor in evacuation decision making. People who feel safe where they are tend not to evacuate.

Whitehead et al. [160] studied hurricane evacuation behavior with data obtained from telephone surveys conducted among North Carolina coastal residents. A logit model was introduced to estimate the evacuation destination. This work was stated to be the first study to model the effect of the intensity of the storm for evacuation behavior as well as the destination patterns. It is found that information about more severe storm intensity also increases the tendency for evacuation.

Baker mentioned the expectation of damage rather than the scale of the storm to be a good predictor, giving as an example studies of Hurricane Eloise in which people who believed winds would overturn their autos or water would damage their homes were more likely to evacuate [10]. This statement relates more to the “perceived risk”, which coincides with the old literature stating that perceived risk plays an important role in deciding to evacuate. This finding also agrees with Dow and Cutter [37], which is why mobile home residents tend to evacuate more with increased perceived risk. However, this perceived risk is not very clear as households rely on perceived risk from flooding but not from wind in making evacuation decisions [160].

Baker also pointed out that people in high-risk areas tend to evacuate more. However, the reason for their actions is not clear because the evacuation action can be due to evacuee perception of high risk or to public officials’ greater efforts to evacuate residents of an area. An official evacuation notice is mentioned as an important factor affecting the decision because official notices are more likely to convince people that a
threat exists; in addition, the legal penalty for noncompliance may affect people’s decisions. Evacuation decisions can be made under a legal notice without a high perceived risk. Residents are also more likely to respond if a notice comes in a more personalized way [10].

An evacuation “shadow” is another issue in which evacuation from high-risk and moderate-risk areas influences response in nearby areas where evacuation is not necessarily needed. However, any notice to stay for low-risk areas is legally risky and ethically uncertain because the unnecessary departure of evacuees may result in longer clearance times for regions where people need to leave a high-risk area quickly; on the other hand, casualties among people who obeyed a “not evacuate” order may raise liability concerns and emergency management responsibility conflicts [10]. Gladwin and Peacock [45] mentioned that people who live in multiunit buildings are more likely to evacuate than those who live in single family dwellings and stay to protect their property. Besides protecting property, Baker cited two other reasons for not evacuating: inconvenience or the effort associated with evacuating such as gathering belongings and arranging for a place to stay, and neighbors’ decision not to evacuate impeding the subjects from leaving [10].

Whitehead et al. [160] also found that people with prior storm experience are more likely to evacuate if pet ownership restricts evacuation, because pets may not be allowed in shelters or other possible destination points [10, 160]. On the contrary Riad et al. [124] found that prior evacuation experience significantly predicts future evacuation behavior rather than prior disaster experience, because people who evacuated before know what to do and how to act.
“Crying wolf” syndrome is an issue that basically makes people reluctant to evacuate because of a false warning in the past [38,162,10]. However, there is no clear evidence that people will be reluctant to evacuate after experiencing a false alarm. After a false alarm in Panama City, few people declared that they would be less likely to evacuate in the future [10].

Whitehead [162] approached evacuation alarm from an economic perspective and stated that the anticipated opportunity cost for false evacuation is overestimated. Although time of day is not proven to be a significant deterrent to evacuation, people would prefer to evacuate in the daytime. However, Baker mentioned very successful late-night evacuations from Hurricane Eloise in northwest Florida and Hurricane Elena in the Tampa Bay area [10].

These types of conflicts are more common in work trying to relate demographic factors to evacuation behavior. Different studies present different conclusions regarding the demographic factors that affect evacuation behavior. Even surveys done at the same region for different hurricanes cite different factors as being significant for evacuation [161].

Riad and Norris [123] investigated evacuation intentions in four categories: risk perception, preparedness, social influence, and resources. They used a survey conducted during a hurricane warning and after the threat disappeared. In addition to their findings in common with other studies, they mentioned some interesting cultural issues. Their study showed that, although having children in the household was not a factor for deciding to evacuate, having a male child related to perceiving more risk. They cited some cross-cultural studies done in India and China about the importance of having male
child and a possible tendency to protect the male child. They also found that people who are more attached to their cities (Savannah in their case) are less likely to evacuate and people who are less attached to the community have a greater tendency to evacuate. Baker discussed this residence issue from two perspectives: either newcomers to a city do not appreciate the potential of hurricanes and do not know what to do, so they are less likely to evacuate; or they are less experienced about hurricanes and leave the area before the more experienced dwellers. However, no conclusion was drawn because there was no data to verify either possibility [10].

Table 2.1 summarizes the factors stated to affect evacuation behavior and demand, including the studies that cited these factors.

<table>
<thead>
<tr>
<th>Significant Factors</th>
<th>Studies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age</td>
<td>1</td>
</tr>
<tr>
<td>Gender</td>
<td>1, 161</td>
</tr>
<tr>
<td>Education</td>
<td>167; 161</td>
</tr>
<tr>
<td>Income</td>
<td>1, 167</td>
</tr>
<tr>
<td>Past experience</td>
<td>45, 161, 167</td>
</tr>
<tr>
<td>Flood risk</td>
<td>10, 42, 43, 161, 160</td>
</tr>
<tr>
<td>Personal risk perception</td>
<td>1, 10, 42, 45, 37, 167, 161</td>
</tr>
<tr>
<td>Mobile housing</td>
<td>10, 42, 167, 161</td>
</tr>
<tr>
<td>Length of residence</td>
<td>1, 45</td>
</tr>
<tr>
<td>Dissemination of orders</td>
<td>10, 42, 45</td>
</tr>
<tr>
<td>False alarms</td>
<td>38, 45</td>
</tr>
<tr>
<td>Pet ownership</td>
<td>160, 161</td>
</tr>
<tr>
<td>Presence of elderly or child</td>
<td>38, 45, 124</td>
</tr>
<tr>
<td>Storm properties (e.g., intensity, speed)</td>
<td>10, 42, 160</td>
</tr>
<tr>
<td>Time of day</td>
<td>42</td>
</tr>
<tr>
<td>------------------------------</td>
<td>----</td>
</tr>
<tr>
<td>Race, cultural issues</td>
<td>45, 167, 161</td>
</tr>
</tbody>
</table>

Impacts of various evacuation scenarios can only be estimated by using realistic time-dependent demand generation models. Table 2.1, in this sense, shows the variety and diversity of factors that are supposed to form the basis for a demand model. However, unlike traffic assignment models employed to study various evacuation strategies, demand generation has not attracted much attention of the researchers. This is mainly due to the complex nature of evacuation behavior and evacuees’ decision process, which is simply hard to model. The next section provides a review of the evacuation demand models that are proposed to capture the complex evacuation decision process.

### 2.2.1.1 Review of the Demand Generation and Loading Models

A widely used estimation method for time-dependent demand is the two-staged method used in evacuation decision support software packages. In the first stage, the number of households in pre-specified regions expected to evacuate is estimated by using participation rates. These rates are determined by the speed and type of the hurricane, type of housing, and proportion of the population that is transient. Multiplying these rates by the population in pre-specified areas gives the total number of evacuees in the area. At the second stage, the time when the calculated evacuee number will be loaded on the network is estimated [165]. Loading is typically done using a so-called response or mobilization curve that estimates the proportion of the total evacuation demand that starts evacuating in each time period. These curves are represented by mathematical functions and formulized to reproduce past evacuation behavior.
The most popular loading model is the sigmoid curve (S-curve or behavioral response curve). The S-curve represents the cumulative percentage of evacuees at every time period. It is commonly used in practice (Comprehensive Hurricane Data Preparedness Study Web Site [28] and studies therein). S-curve parameters can be adjusted to mimic different behavioral responses. However, they still do not fully reflect reality because they introduce a time-independent continuous process, whereas it is widely accepted that time of day affects evacuation decisions and rates. Moreover, S-curves do not allow for investigation of specific decision-making processes in households and produce aggregate results (43). Nevertheless, S-curves are frequently mentioned in the literature and are used in evacuation software packages, such as MASSVAC [55].

Another approach for loading transportation networks under evacuation conditions uses the planner’s knowledge and judgment to estimate departure time. Mobilization time is the time from issuing an evacuation order to the time of departure. Tweedie et al. [147] determined mobilization time parameters based on the information obtained from experts in the Civil Defense Office of Oklahoma. A specific amount of time for which given percentages of the evacuating population could normally be expected to be mobilized is determined according to expert knowledge. Tweedie et al.’s approach suggests a loading function with Rayleigh distribution that has only one independent parameter—namely, maximum mobilization time. This is assumed to be the time after which all the evacuees are assumed to leave the danger area. It has an S-like shape, like S-curves, but a proposed original loading scheme assumes total mobilization time to be 1800 minutes.
Since the dependent variable is discrete (evacuate or not), logistic regression models can also be used as alternative trip-generation models [165]. There have been attempts to model the demand with ANNs. Mei [89] conducted a detailed analysis of logistic regression and ANN models for hurricane trip generation and investigated three ANN models—namely, back-propagation neural network (BPNN), probabilistic neural network (PNN), and learning vector quantizer (LVQ). The models developed in Mei’s study are also compared with a cross-classification type model developed by consultants to estimate trip generation in Southern Louisiana (PBS&J). Overall, BPNN and logistic regression models are put forward to perform better than the other two models.

In another study [165], a feed forward neural network (FFNN) is analyzed, and although no clear preference was stated, neural network models are reported to perform marginally better than the logistic model.

Fu (43) compares four approaches for hurricane evacuation demand—namely, Cox proportional hazard models and piecewise exponential models and a sequential logit model (SLM). SLM, which successfully captures the evacuation behavior parameters presented by Baker [10], is stated to perform best among those. Moreover, this model is stated to be transferable to a certain degree; that is, the model can be applied to different situations in terms of hurricane characteristics and geographic locations with some limitations [43]. Transferability of SLM and the statistical packages supporting robust estimation of logit models are additional incentives [43] for using SLM.

Table 2.2 shows a selection of models specifically developed for evacuation with corresponding demand loading scheme that is employed. It can be seen that, although there are sophisticated evacuation demand models being developed, the available
software packages do not incorporate these demand/loading models. Moreover, some models do not use dynamic loading, which is proven by real data and observation to be the actual case, and assign static loading.

<table>
<thead>
<tr>
<th>MODEL</th>
<th>DEVELOPER</th>
<th>INCORPORATED DEMAND MODEL</th>
</tr>
</thead>
<tbody>
<tr>
<td>NETVAC (NETVAC1)</td>
<td>Sheffi et.al. [130]</td>
<td>Employs time varying user-defined O-D Tables [43, 89]</td>
</tr>
<tr>
<td>DYNEV (I-DYNEV)</td>
<td>KLD &amp; Associates [68]</td>
<td>Employs static traffic assignment. No dynamic loading is used. Traffic conditions are assumed to remain at a fixed level throughout the simulation period [24]</td>
</tr>
<tr>
<td>CLEAR</td>
<td>Moeller et.al. [93]</td>
<td>Employs static assignment. No dynamic loading is used. Traffic conditions are assumed to remain at a fixed level throughout the simulation period [24]</td>
</tr>
<tr>
<td>TEDDS, MASSVAC</td>
<td>Hobeika et.al. [54], Hobeika and Kim [53], Hobeika [52]</td>
<td>S-curve</td>
</tr>
<tr>
<td>TEVACS</td>
<td>Anthony F. Han [49]</td>
<td>Dynamic loading incorporating public transit is incorporated for large cities in Taiwan</td>
</tr>
<tr>
<td>REMS, OREMS</td>
<td>ORNL, Tufekci &amp; Kisko [98]</td>
<td>Does not estimate the timing of people's response to the perceived emergency by location or estimate the number of evacuees. Total number of evacuees and timing information are used as an input to the program. [165, 94]</td>
</tr>
<tr>
<td>ETIS</td>
<td>PBS&amp;J [112]</td>
<td>S-curve</td>
</tr>
<tr>
<td>HURREVAC</td>
<td>FEMA, Army Corps of Engineers [28]</td>
<td>Evacuee loading is an input to the program (S-curve for USACE and FEMA HES)</td>
</tr>
</tbody>
</table>

### 2.2.2 Analysis of Evacuation Demand Models

In this section, a deterministic analysis which is conducted for structuring the basis for the proposed stochastic approach is presented. The analysis aims to investigate
the differences in evacuation model outcomes with respect to the employed demand profile. First, most widely accepted evacuation demand models are analyzed in terms of their mathematical structure and parameters they use. Then the network-wide analysis of demand model choice and capacity reductions are presented. For the sensitivity analysis and network-wide assignment, three selected models—namely, S-curve, Rayleigh distribution, and sequential logit—are selected. These models represent different types of approaches that can be used with different information about the region and people’s experiences. The general difficulty in acquiring hurricane evacuation data makes it easy to use an S-curve for the analysis. Rayleigh distribution approximation for evacuation demand proposed by Tweedie et al. [147] is selected for comparison purposes because it relies mainly on expert judgment. The main reason behind the selection of the two methods is that they do not need extensive data for calibration. The only model among those selected that relies heavily on real data is the SLM proposed by Fu [43]. This model is selected to make it possible to compare a more detailed model with other relatively simple, easy to use models.

2.2.2.1 Rayleigh Distribution Approach

Tweedie et. al. [147] used Rayleigh distribution to represent evacuation loading onto the network. The formula is:

\[
F(t) = 1 - \exp\left(-\frac{t^2}{1800}\right)
\]  

(1)

In this approach, the only parameter to be investigated is the maximum mobilization time (1800 minutes) when all the people are assumed to be evacuated. This number which is determined with the help of the Civil Defense Office of Oklahoma, may
not be valid for other locations and evacuation conditions. The evacuation curves according to different maximum evacuation times are given in Figure 2.2. An important point is that these curves give time-dependent evacuation rates of total demand, which is determined exogenous to the model.

Figure 2.2 Cumulative and percentage loading graphs for Rayleigh distribution approach with changing maximum mobilization times

As shown in Figure 2.2, as the maximum evacuation time increases, the evacuee departure curves become closer to each other. It takes 460, 650, 790, and 920 minutes to complete 90% evacuation for maximum evacuation times of 900, 1800, 2700, and 3600 minutes, respectively. This means that the difference in loading pattern is more significant when the total evacuation period is shorter. Although the loading decay (assumed as loading rate < 0.0001) occurs at 620, 840, 1020, and 1150 minutes, maximum loading times are at 220th, 310th, 370th, and 430th minutes for maximum
mobilization times of 900, 1800, 2700, and 3600 minutes, respectively. Thus, it can be concluded that the time of maximum loading does not vary in the same order as maximum mobilization time parameter.

2.2.2.2 *S-Curves (Behavioral Response Curves)*

Following the recent state-of-the-practice (studies listed in Comprehensive Hurricane Data Preparedness Study Web Site [28]) behavioral response curves (S-curves) were chosen to be further analyzed as an evacuation loading model. However, it would be appropriate to mention the drawbacks of S-curves as stated by Fu [43]:

- An S-curve usually covers a shorter period of evacuation. However, actual evacuation may take several days.
- An S-curve cannot be used to evaluate the official evacuation order timing or the nature of the evacuation order (mandatory/voluntary).
- S-curves do not capture the time-of-day variation.
- Level of total demand (participation rate) must be predicted to start using the curves.
- Selection of a response curve is subjective, reflecting the perception of the analyst only.
- For hurricane evacuations, S-curves cannot capture hurricane characteristics, such as hurricane speed, intensity, track, etc.

On the other hand, behavioral response curves are popular, because they

- are mathematically simple to use and implement,
- require considerably less site-specific data,
• claimed to reproduce realistic evacuation behavior with the loading rate and half loading time constants determined based on past evacuation data, and

• are extensively mentioned in the literature regardless of the disaster/evacuation type and are used in a number of official studies. Thus, they are considered to be a credible modeling approach that is widely used by other studies.

A sigmoid curve, or S-curve, that can be mathematically expressed using the equation given by Radwan et al. [118]. S-Curve is used in some of the well-known evacuation software packages such as TEDSS and MASSVAC. The general S-curve formula is as follows:

\[
P(t) = \frac{1}{1 + e^{-\alpha(t-H)}}
\]

where,

- \( P(t) \): cumulative percentage of total trips generated at time \( t \).
- \( \alpha \): a parameter representing the response of the public to the disaster that alters the slope of the cumulative traffic-loading curve.
- \( H \): half loading time; the time when half the vehicles in the system have been loaded onto the highway network. To be more specific, \( H \) defines the midpoint of the loading curve and can be varied by the user according to disaster characteristics. These curves are shown in Figure 2.3 and Figure 2.4.
In Figure 2.3, different S-curves with varying $\alpha$ parameters are shown. All curves intersect at half loading time, which was kept fixed for all the curves. As the $\alpha$ parameter increases, the response is more concentrated near the half loading time (Figure 2.4). A low value of $\alpha$ produces more homogeneous loading percentages. The time it takes for 90% evacuation of all the demand, with a half loading time equal to 12 hours, is 13.8, 13.2, 12.9, and 12.7 hours for $\alpha$ values of 0.2, 0.3, 0.4, and 0.5, respectively. This is an expected result since the $\alpha$ value determines the response rate and, as it increases, the time to reach high loading percentages gets lower and curves become closer.
Figure 2.4 Percent evacuations with half loading time = 12 hours and varying response rate parameters

The half loading time for S-curves is an important factor because it determines the time when maximum loading will occur (see Figure 2.4). Figure 2.5 basically shows that half loading time shifts the S-curve in the horizontal direction. It also changes the time of maximum loading onto the network. Briefly, the half loading time parameter changes the timing of the evacuation without changing the behavior of the evacuees.
2.2.2.3 Sequential Logit Model

A sensitivity analysis for evacuation probabilities used in SLM was conducted by Fu [43]. In the model, each random utility function $U^c_i$ (utility of household not to evacuate at time $i$, $c = \text{continue}$) and $U^s_i$ (utility of household to evacuate at time $i$, $s = \text{stop}$) are assumed to be composed of a systematic component $x'\beta$, which represents the explanatory variables, and an error term, i.e., $U = x'\beta + \varepsilon$. Also, the utility differences $U^c_i - U^s_i$ are assumed to be independently logistically distributed. Then the probability of a household to evacuate at time $i$ given that it has not evacuated earlier can be expressed as (Fu, 2004):

$$P(i)_{s/c} = \frac{e^{x'\beta}}{1 + e^{x'\beta}}$$  \hspace{1cm} (2)

where

$$x'\beta = V = -2.8238 - 0.7995 \times \text{dist} + 1.4512(2.0244) \times \text{TOD} + 0.1463 \times \text{speed} + 0.5401 \times \text{orderper} + 0.7089 \times \text{flood} + 1.6496 \times \text{mobile}$$

where $\text{dist}$: a function of distance to the storm at time $t$
**TOD**: time of the day, periods used—night, morning, afternoon.

**speed**: forward speed of the hurricane at time \( t \)

**orderper**: 1 if perceived evacuation order, 0 otherwise

**flood**: 1 if the residence is likely to be flooded, 0 otherwise

**mobile**: 1 if a mobile home, 0 otherwise

Two coefficients, 1.4512 and 2.0244, are used for morning and afternoon, respectively. For night, since \( TOD = 0 \), the utility function is not affected. These coefficients basically state that people are more likely to evacuate in the afternoon, morning, and night, in decreasing order respectively.

Signs of the variables are consistent with intuitive expectations as increasing distance will decrease the probability of evacuation, and an increase in all other variables increases the probability of evacuation. Among all variables, \( TOD \) has the largest absolute value, and it affects evacuation considerably. “Mobile” and “flood” are the next two important parameters according to their impact on the utility function. From the data set used for model estimation, the values of \( dist \) range from 0 to 7 and have a ratio of 270 between two extreme values, making \( dist \) the most influential variable in the model [42].

One point that needs to be discussed in SLM is that high-risk households tend to leave their houses first. Fu [43] stated that high-risk households tend to live near water or low-lying areas and therefore probably have longer evacuation distances, so their early departure is reasonable. However, one would expect people to wait to make their final evacuation decisions until they are sure about the hurricane’s path and its intensity; they
may decide to stay to protect their houses. The latter points were not incorporated into this model but are stated in the literature [11].

Besides the facts stated by Fu [43], for this study, the evacuation percentage output of SLM was estimated by Monte Carlo simulation along for comparison with participation rates in the literature. For this purpose, artificial samples were generated with alternating attributes for evacuation order, flood risk, and housing type. Each sample was assumed to be subject to the same hurricane characteristics. The simulation results can be seen in Figure 2.6.

As shown in Figure 2.6, SLM gives about a 90% participation rate for mobile households with flood risk that received an evacuation order. According to behavioral studies conducted by the U.S. Federal Emergency Management Agency and the U.S. Army Corps of Engineers (Comprehensive Hurricane Data Preparedness Study Web Site [28] and studies therein), relatively higher participation rates for high-risk households are
reasonable. However, for low-risk households without an evacuation order, the participation rate predicted by the model is about 25%. This is assumed to be 10%–15% at most in behavioral studies. Although the model estimate is higher than the assumption, it still gives a value that is on the safe side. It should also be noted that these participation rates are assumptions, so it may be misleading to decide about a model’s accuracy relying only on another model’s assumptions. Nevertheless, the discrepancy between actual practice and theoretical model outcomes is worth mentioning.

Since the evacuation is assumed to last for 3 days, the beginning period is also analyzed if it has an effect on evacuation behavior. The participation rate did not change (around 1%–3%) in the case of starting the evacuation in the morning or afternoon. The most diverse evacuation numbers (12%–16%) between different starting periods are obtained for two extreme cases: high-risk (mobile home, flood risk, received evacuation order) and low-risk (not a mobile home, no flood risk, no evacuation order received) houses.

For Cape May County, which is used as a case study in the next section, the difference of 12%–16% is equal to a difference of roughly 5000–7000 households of a total of 42,148 households (U.S. Census). This is equal to 11,000–16,000 people according to U.S. Census statistics. Overall, it can be said that SLM is sensitive to the starting period of evacuation, especially for extreme cases, such as high- and low-risk households.

For further comparison, S-curves with different loading parameters are used to estimate the evacuation demand for Cape May County, and compared with curve produced by SLM. Below are some important facts about Cape May County (U.S.
Census, Cape May County Planning Department) that are used in the Monte Carlo simulation.

- Number of housing units = 93,541 → Used as total number of houses in the area in the simulation
- Number of mobile houses = 2807 → Used to determine the percentage of mobile houses together with total number of houses
- Number of households = 42,148 → Total number households to be evacuated
- Water area = 365.09 mi^2 → Used to determine households with flood risk
- Total area = 620.28 mi^2

The following assumptions were made for the demand simulation under evacuation conditions:

- Total number of evacuations is equal to 42,148 households, where there are a total of 93,541 housing units.
- Houses are evenly distributed in the county, so the proportion of mobile homes to the total number of houses applies to the whole county. Likewise, water area over the total area represents the proportion of households exposed to flood risk.
- All attributes are assigned independently, because no joint statistics such as mobile home and flood risk are present.
Figure 2.7 Different evacuation loading patterns generated for Cape May County

Figure 2.7 shows predicted evacuation patterns for Cape May obtained using various S-curves and SLM. The participation rate was predicted by the SLM to be 42% for the whole county. Four different response curves were generated. S-curves 1 and 2 represent three-day-long loading scenarios, where S-curve 3 is the commonly employed S-curve with a short loading duration. S-curve 4 is also a three-day-long loading curve where each daily part resembles S-curve 3. Thus S-curve 4 is the reproduction of S-curve 3, assuming equally divided participation rates for each day. For S-curves 1 and 2, parameters are chosen as $\alpha_1 = 0.002$ and $\alpha_2 = 0.004$, with $H = 8$ hours for both curves. For each daily loading pattern, parameters of S-curves 3 and 4 are $\alpha = 0.04$, $H = 12$ and $\alpha = 0.01$, $H = 9$, respectively. Although parameters are adjusted to replicate long evacuation times, S-curve 1 and S-curve 2 shown in Figure 2.7 do not produce curves similar to SLM curves. S-curve 1 achieves an evacuation pattern closer to that of SLM than S-curve 2 but it does not reach 100% at the end of the evacuation period. Also, it
underestimates the first half of the evacuation percentages, whereas it overestimates the second half compared with SLM. Quick loading S-curve 3 shows a distinctively different behavior compared with the first two curves, because it loads all the evacuees onto the network in less than one day. Curve 4 incorporates some kind of a time-of-day dependency. The comparison of S-Curves and SLM curve shows us that the available demand models may not necessarily produce similar curves for the same area, although both models claim to represent the reality.

2.3 Evacuation Planning Models

Evacuation networks are basically the same roadwork that is used by the public on a daily basis. However the networks that are used in evacuation studies do not necessarily include all roads but analyze major roadways which are chosen by transportation planners to be used during evacuation. This kind of “macro simulation” approach is almost inevitable from a planning perspective due to the large scale of mass evacuation. Nonetheless, micro simulation can still be used to analyze in a finer scale for smaller networks. Both macro and micro approach types have pros and cons. As discussed by Lindell and Prater [78], microscopic models simulate the behavior of individual vehicles as they merge, turn, and respond to traffic signals. For this purpose detailed data such as number of lanes, shoulder width, and traffic control devices are required for the analysis. Macroscopic models simulate vehicle flows along links from one node to another, which represent the origin and demand regions rather than single points, and vehicle flows can be analyzed with less information than is required for microsimulation. Microscopic models can model bottlenecks within emergency planning
response areas (ERPAs), whereas macroscopic models can model such phenomena only between ERPAs [133]. On the other hand, macroscopic models can be computed with a shorter run time. As mentioned by Lindell et al. [79] it is more important to have a quick and approximate model as an evacuation decision support system. Thus, it can be said that microsimulation is not suitable for large networks because of run time and extensive data detail requirements. Microsimulation models also need to be calibrated with actual data; however, such calibration data exist for a limited number of regions. On the other hand, macrosimulation cannot give output as detailed as a microsimulation model since network traffic conditions are simplified and interaction between evacuees cannot be modeled in macrosimulation models.

2.4 Evacuation Modeling Software Packages

There are numerous software packages developed specifically for evacuation modeling besides the traffic simulation and analysis tools employed for evacuation modeling. Some evacuation-specific software packages are Mass Evacuation (MASSVAC), Network Emergency Evacuation (NETVAC), Oak Ridge Evacuation Modeling System (OREMS), Dynamic Network Evacuation (DYNEV), and Evacuation Traffic Information System (ETIS). A historical sketch of evacuation software packages can be found in Figure 2.8.
<table>
<thead>
<tr>
<th>Year</th>
<th>Model/Software</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1980</td>
<td>Sheffi et al. (1981,1982) – NETVAC1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>PRC Voorhees (1982) – EVAC PLAN PACK</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Lewis (1985)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Radwan et al. (1985) – MASSVAC</td>
<td></td>
</tr>
<tr>
<td>1990</td>
<td>Han (1990) – TEVACS</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Tufekci and Kisko (1991) – REMS</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Abkowitz and Meyer (1996) – NETVAC1 w/ TIGER/line files &amp; Census data</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Hobeika and Kim (1998) – MASSVAC 4.0</td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td>Barret et al. (2000) – Framework</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Sattayhatwe and Ran (2000) – DTA implementation</td>
<td></td>
</tr>
<tr>
<td></td>
<td>PBS&amp;J (2000) – ETIS</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Franzese and Han (2001) – OREMS</td>
<td></td>
</tr>
<tr>
<td></td>
<td>FEMA – TriEPs</td>
<td></td>
</tr>
</tbody>
</table>

Figure 2.8 Chronological representation of simulation models [144]

General use transportation software packages such as PARAMICS [26,29,22], NETSIM [151,50], VISSIM [88], CORSIM [134,142,163,67], DYNASMART [70,71], VISTA [145,23] are used in evacuation modeling of the complete evacuation process or analysis of evacuation policies such as contraflow. ARENA [119], which is a general
simulation software is also employed in evacuation modeling. Since those software packages are not specifically designed for evacuation modeling, they do not include demand generation models suitable for evacuation modeling. On the other hand, the main issue of vis-a-vis evacuation-specific models turns out to be the determination of participation rates through the geographical and demographics of the evacuation area. The loading rate is mostly oversimplified or no dynamic loading of demand is employed. Loading rate has not received sufficient attention in the past and mostly traffic assignment and/or determination of the O-D pairs (or solely level of demand) are emphasized. Information regarding the scale (e.g. micro-macro) and employed demand models on selected evacuation-specific software packages are presented in Table 2.3.
### Table 2.3 A Selection of Evacuation-Specific Software Packages

<table>
<thead>
<tr>
<th>MODEL</th>
<th>DEVELOPER</th>
<th>BRIEF INFO</th>
<th>APPLIED REGIONS</th>
</tr>
</thead>
</table>
| NETVAC      | Sheffi et al. [130]     | • Macroscopic evacuation model developed for regions near nuclear power plants  
• The first evacuation package with dynamic assignment capability  
• Insensitive to evacuees’ behavior  
• Structured in a descriptive mode rather than design and planning mode  
• A deterministic model rather than a probabilistic and dynamic simulation model.  
• Time-varying O-D tables are required as input [43,89] | -                        |
| NETVAC1     | KLD & Associates [68]   | • Macroscopic evacuation model for sites near nuclear power plant  
• Improved version I-DYNEV  
• Employs static assignment  
• Has been used to determine the impacts of alternative traffic controls such as traffic signals, stop signs, and yield signs  
• It has also been used to analyze network capacity and evacuation demand  
• Does consider modal-split in its data processing, but only bus as a means of evacuation for those without access to private vehicles  
• Cannot deal with time-varying                                                                 | Seabrook Nuclear Power Plant in New Hampshire |
flows. It is assumed that traffic conditions in the network remain at a fixed level through the simulation period. [94, 116, 26]

| EVACUATION PLANING PACKAGE | PRC Voorhees [114] | • Dynamic and probabilistic model  
• Human behavior is taken into account for determining the loading and response rate of evacuees [127] | - |
| EVACUATION PLANING PACKAGE | NRC [93] | • A microsimulation tool with static assignment  
• Like DYNEV, cannot deal with time-varying flows [127,24] | - |
| TEDDS, MASSVAC | Hobeika et al. [53, 51] | • TEDSS, a macrosimulation tool, was originally developed for power plants and is based on computer simulation model MASSVAC  
• TEDSS has a knowledge-based system called the data base module that stores evacuation expert rules, disaster-related information, and area and transportation network characteristics  
• Employs quasi-dynamic traffic assignment  
• The simulation module of TEDSS is an event-type simulation designed to load evacuees onto the highway network, to determine their best evacuation route. | Virginia Beach Hurricane/Flood Transportation Evacuation Study [51] |
<table>
<thead>
<tr>
<th>Model</th>
<th>Author(s)</th>
<th>Description</th>
</tr>
</thead>
</table>
| TEVACS | Anthony F. Han [49] | - Developed to analyze large-scale evacuation (specifically for large cities in Taiwan)
- Incorporates all modes (public transit, motorcycles, and bikes) into the model by converting each mode into a universal unit called the PCU or passenger car unit [49] |
| REMS, OREMS | ORNL, Tufekci & Kisko [143] | - Based on a FORTRAN program, ESIM (Evacuation SIMulation), which combines the trip distribution and traffic assignment submodel with a detailed traffic flow simulation submodel
- Allows for conducting extensive analysis such as traffic management and control, operational assessment via its extensive data input structure
- Has a dynamic nature in which evacuation analysis can be tracked at user-specified time intervals
- Includes human behavior and weather information as inputs. It is also capable of modeling contraflow |

- Newer version of MASSVAC is introduced with improved traffic assignment schemes [53]
- Developed for large cities in Taiwan [49]
- North Carolina DOT
- Maryland DOT
- Tennessee DOT
- Oregon DOT
- Nuclear Regulatory Commission
<table>
<thead>
<tr>
<th>ETIS</th>
<th>PBS&amp;J [111,112]</th>
</tr>
</thead>
</table>
| Operations.  
- Does not estimate the timing of people’s response to the perceived emergency by location, estimate traffic control settings and parameters, estimate the number of evacuees and evacuating vehicles by location, or determine the EPZ. This information must be determined ahead of time and used as input to the software package [165,94].  
- Macroscopic model using static assignment and operates within a GIS environment. Participation rates are determined by the category and speed of the hurricane, tourist occupancy, and type of housing.  
- The model is proprietary and was developed specifically for the southeastern U.S., including Florida, Georgia, South Carolina, and North Carolina.  
- Incorporates human behavior and weather conditions as input.  
- Capable of modeling contraflow operations and using real-time information.  
- Based on assumptions regarding old data, thus not appropriate for regions lacking historical data.  
- No modal split is considered [165,94].  | Originally had been applied to North Carolina, South Carolina, Georgia, and Florida.  
- Its application has recently been introduced to Alabama, Mississippi, Louisiana, and Texas [116]. |
<table>
<thead>
<tr>
<th>HURREVAC</th>
<th>FEMA, Army Corps of Engineers [28]</th>
</tr>
</thead>
</table>
| • Specifically for hurricane evacuation and developed on behalf of FEMA by the U.S. Army Corps of Engineers.  
• Operational tool, assisting decision makers in advance of and during an evacuation.  
• Draws information from a wide variety of sources, such as National Hurricane Center.  
• Estimates the time required to evacuate an area [116]. | Developed for FEMA by the U.S. Army Corps of Engineers for use by emergency managers in hurricane-prone states (Texas to Maine), in Puerto Rico, and the Virgin Islands [28]. |
2.5 Traffic Assignment and Uncertainty

The studies on traffic assignment can be broadly categorized as static or dynamic based on time dependency of the network attributes. Static traffic assignment (STA) is the first problem that is studied by the transportation research community. After Merchant and Nemhauser's seminal work [92], dynamic traffic assignment attracted interest among researchers due to its theoretically sound approach for modeling dynamic real life traffic patterns. Although the two main components of traffic assignment problem, number of users (demand) and the link capacities (supply) exhibit a stochastic nature, the general practice is towards assuming fixed demand and capacity during the analysis. The randomness can be introduced to the model via users' perception of travel times on each path besides assuming the assigned demand to be a random variable, or treating roadway capacities as random. At this point, we have the second main categorical split in traffic assignment literature: user equilibrium (UE) vs. system optimal (SO).

STA has a better established solution method compared to DTA from both UE and SO perspectives. UE formulations are based on perfect user knowledge regarding the link travel times at all possible routes between the OD pairs in the network. Straightforward way of incorporating uncertainty to the model is to introduce user travel time perception as a random variable (for early works, please see 32, 34, 128). On the demand side, uncertainty can be introduced by assuming demand as a random variable while keeping the network attributes and user perception fixed. Then the problem becomes simply assigning different level of demands on the same network and obtaining
the optimal flows. On the capacity side, uncertainty considerations start receiving more attention from the research community following the works on transportation network reliability [20,19,8,83,46,56,39]. In early transportation reliability studies [20,19,8,56,39] the performance measures were calculated based on extensive numerical simulations which result in computationally challenging tasks to extend the methodology for larger networks [84]. In the follow up works [83,46,84,72], chance constraint programming approach is employed to obtain a deterministic equivalent of the probabilistic capacity constraint, \( P(x_a < c_a) \geq p \) where \( x_a \) and \( c_a \) represents the link flow and link capacity respectively, and \( c_a \) is assumed to follow a probability distribution. In the constraint, \( p \) represents the probability value that the assigned flows would not exceed the capacity. Via taking the inverse CDF of the random variable \( c_a \) deterministic equivalent for the flow constraint is obtained. Then, the flow inequality is plugged into travel time equation (conventionally, BPR function) for the traffic assignment.

There are not many studies in the literature regarding SO-STA with uncertain demand or capacity. The problem of uncertain demand and capacity in SO setup does not possess any additional challenge for the approaches used for its UE counterpart. The demand and capacity uncertainties can be treated in the same manner as done for UE assignment, e.g. using capacity randomness to determine the distribution of link travel times, or performing assignment with different demand levels to extract the flow pattern changes. One of the few SO-STA papers [86] study SSO (Stochastic Social Optimum - they use the name "social optimum" instead of system optimal) traffic assignment problem is formulated to complement the well-known UE, SO and SUE problems and the
relationships and similarities are investigated. As their results show, there are strong
connexions between stochastic UE and stochastic SO in terms of formulation and
interpretation of model outcomes. Other SO-STA study known to the authors is a recent
paper [150] on using SO-STA with recourse for analyzing the impacts of information on
routing patterns while network conditions are subject to change. However, the recourse
formulation they offer is different than the conventional SO-STA, hence a simple analogy
with the previously published uncertainty studies in UE-STA is not possible.

In stochastic DTA models, the literature mainly focus on demand uncertainty and
the implementation area is network design problem (NDP). First attempts for
incorporating demand uncertainty [66,14,126,158] are performed through running the
deterministic model with a large number of randomly generated demand patterns, and to
infer some rules and principles from the results. Waller and Ziliaskopoulos [156] provide
two stochastic programming approaches for modeling demand uncertainty in NDP. First
one is individual chance constraints (ICC). The second approach is the two-stage
stochastic programming problem with recourse (SLP2). They use a cell transmission
model (CTM) based system optimal dynamic traffic assignment (SO DTA) formulation
[178] as the underlying traffic assignment model. Ukkusuri et. al. [149] provide CTM
based user equilibrium (UE) DTA formulation for NDP problem using ICC and SLP2 to
incorporate demand uncertainty into their formulation. In addition to the two-stage bi-
level stochastic programming formulation for NDP, Karonsoontawong and Waller [65]
propose robust bi-level NDP formulation. Yao et. al [171] also take robust optimization
approach based on the SO DTA formulation for modeling the demand uncertainty during
evacuation, again using a CTM based SO DTA model. Waller and Ziliaskopoulos [159]
provide CTM based SO DTA formulation with individual chance constraints to model
demand uncertainty and discuss possible real-world implementations rather than focusing
on NDP. Although the capacity uncertainty is mentioned in the literature as an important
variable, the number of DTA studies modeling the stochastic capacity is considerably
less. General solution idea is to generate random samples or capacity scenarios to analyze
with the deterministic models. Peeta and Zhou [109] study demand and capacity
uncertainty related with incidents in a DTA setup for use in the context of on-line route
guidance. Their approach incorporates capacity reductions due to incidents as scenarios
where the stochasticity is a result of the probabilistic occurrence of an incident. The
solution of the traffic network assignment is calculated for the mean O-D demand and the
results are used to update the on-line routing information calculated via using several O-
D demand realizations. Hence, the provided methodology is a scenario analysis in terms
of incident induced capacity reductions rather than probabilistic treatment of the capacity.
First study that provides an analytical treatment of link capacity in the context of dynamic
traffic assignment models is given in Yazici and Ozbay [172]. In [172] the capacity
randomness due to flooding during hurricane evacuation is incorporated into CTM based
SO DTA model by formulating probabilistic capacity constraints and the impacts of
capacity uncertainty on favorable shelter locations are analyzed. Besides being first
analytical attempt on capacity uncertainty, their approach uses joint chance constraints
(JCC) (also called joint probability constraints) in addition to individual chance
constraints (ICC) approach which is used extensively for modeling demand uncertainty.
Another branch of studies tackle the uncertainty by employing robust optimization
techniques [65,171]. However, as discussed in Chen et. al. [21], robust optimization
provides “safe” approximations to chance constraints. They also discuss that robust optimization (RO) techniques are more successful while approximating the ICC, however unsatisfactory for approximation of JCC, and propose formulations that leads to better approximations for the JCC programming. However, in general, RO methodologies are proposed for the cases in which the solution of the stochastic program is intractable and RO provides approximations for the solution of the stochastic problem.

2.5.1 Dynamic Traffic Assignment in Evacuation Modeling

Barrett et. al.[12] analyze the components of a dynamic traffic management model for evacuation and summarize the model objectives for both the evacuees and system perspectives as in Table 2.4. They further propose model architecture for planning purposes (Figure 2.9) and real time operational purposes (Figure 2.10). Although Table 2.4, Figure 2.9 and Figure 2.10 are designed for hurricane evacuation, the objectives and methodologies can analogously be applied to different disaster and threat conditions. Regarding the scope of the current study, Figure 2.9 provides a better basis to discuss the proposed model use compared to Figure 2.10.
### Table 2.4 Evacuation Model Objectives

<table>
<thead>
<tr>
<th>Destination Choice</th>
<th>System Users (Evacuees)</th>
<th>System Management (Emergency Management)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Minimize perceived costs:</td>
<td>Minimize total costs:</td>
</tr>
<tr>
<td></td>
<td>• travel time to destination</td>
<td>• minimize total travel time</td>
</tr>
<tr>
<td></td>
<td>• proximity of destination to origin</td>
<td>• maximize distribution of emergency services and housing</td>
</tr>
<tr>
<td></td>
<td>• time available to reach safe area</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• availability of housing/emergency services</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• housing costs</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mode Choice</th>
<th>System Users (Evacuees)</th>
<th>System Management (Emergency Management)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Maximize perceived utility:</td>
<td>Minimize Total Costs:</td>
</tr>
<tr>
<td></td>
<td>• travel time to destination</td>
<td>• minimize total evacuation time</td>
</tr>
<tr>
<td></td>
<td>• flexibility</td>
<td>• reduce congestion</td>
</tr>
<tr>
<td></td>
<td>• storage capability (for evacuation supplies and belongings)</td>
<td>• evacuate least mobile population segments (elderly, handicapped, low income)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Departure Time Choice</th>
<th>System Users (Evacuees)</th>
<th>System Management (Emergency Management)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Maximize (cost of delay - cost of departure)</td>
<td>Minimize total costs:</td>
</tr>
<tr>
<td></td>
<td>Cost of Delay:</td>
<td>• minimize total evacuation time</td>
</tr>
<tr>
<td></td>
<td>• potential for injury or loss of life</td>
<td>• evacuate residents closest to danger area first</td>
</tr>
<tr>
<td></td>
<td>• increased travel time</td>
<td>• distribute departure times to minimize congestion</td>
</tr>
<tr>
<td></td>
<td>• reduction in destination choice</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Cost of early departure:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• loss of productivity</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• loss of comfort</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• increased total costs for food and lodging</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• increased emotional costs</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• loss of perceived safety</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• reduction in preparation time</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Route Choice</th>
<th>System Users (Evacuees)</th>
<th>System Management (Emergency Management)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>User Optimal based on perception of:</td>
<td>Minimize total costs:</td>
</tr>
<tr>
<td></td>
<td>• shortest distance to destination</td>
<td>• minimizing total travel time</td>
</tr>
<tr>
<td></td>
<td>• minimum travel time to destination</td>
<td>• maximizing traffic on planned evacuation routes</td>
</tr>
<tr>
<td></td>
<td>• safety of the route – proximity to danger</td>
<td>• maximizing traffic on routes at least likely to be impacted by the storm</td>
</tr>
<tr>
<td></td>
<td>• evacuation information</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• familiarity with route</td>
<td></td>
</tr>
</tbody>
</table>
Table 2.4 and Figure 2.9 provide a nice illustration for the basis of the current study. First, as the main contribution of the proposed formulation, road network data and evacuation demand is assumed to be probabilistic. By probabilistic assignment, the proposed loops for the roadway data in Figure 2.9 are unnecessary since the analysis results are based on probability distribution of the roadway data covering several possible cases at once. Current work also uses system optimal assignment where the destination
choice is imposed on the evacuees according to the system optimal performance. Although this can be a strong assumption for daily trip assignments, it does not come up to be as restrictive for disaster conditions, assuming that the evacuation process will be controlled by the legal authorities. More, user equilibrium assignments inherently assumes a-priori knowledge of travel times on alternative routes for the possible destinations. However, such knowledge hardly exist for emergency evacuation for which the evacuees will have very limited – if any – prior experience regarding the travel patterns. Following the overall practice in the literature, current work mainly focuses on car-based evacuation. Evacuation staging is a phenomenon in which the evacuation departures are assumed to be controlled by the authorities to maintain the optimal clearance time for the evacuated area. Evacuation staging stands as a more theoretical and academic area rather than practical because of the hardness of real life implementations as well as the insufficient legal regulations that allows authorities to impose such plans, hence, not considered in the current study’s scope.
Figure 2.10 Model architecture for real time operational purposes [12]
CHAPTER 3. IMPACTS OF THE DEMAND MODEL SELECTION AND CAPACITY REDUCTION ON EVACUATION PERFORMANCE MEASURES

The analysis in previous chapter shows that demand models may produce distinctive curves for the same study area (See Figure 2.7). Even when the same model is used, such as S-curve, change in model parameters result in different loading curves. Besides the decision of “the most realistic” curve, there is another concern of whether the predicted demand values are going to be realized exactly in real life. The final outcome of different demand models and possible variations cannot be fully understood by just studying the shape of loading curves. The total time needed to evacuate people from a danger area is one of the most important outcomes of these models. For example, two models with different loading curves but resulting in the same clearance times will have no difference as far as the transportation modeler is concerned. On the other hand, two different loading scenarios giving inconsistent results in terms of average delays can point to important theoretical problems related to these models in terms of failure to represent real-life conditions. Thus, there might be a need to use a new or improved demand model. Another approach may be assuming an inherent error in these models due to the complexity of the problem at hand, and offer a probabilistic approach. Nevertheless, before proposing such probabilistic formulations, the impact of demand curve variations on the evacuation performance measures should be investigated.

For analyzing the impacts of different demand curve patterns, CTM based SO DTA model is used for simplified Cape May County, NJ, evacuation network (Figure
3.2. SO-DTA formulation minimizes the total time that the cells are occupied and, in this setup, the final clearance time can be considered as the time when there are no occupied cells. Assuming that evacuees are not safe until they reach their destination, the algorithm minimizes the time those evacuees are considered to be unsafe. Thus, average travel time, as a period of time spent outside the shelter, corresponds to a risk exposure measure for the problem and is also used to evaluate the evacuation performance. Besides analysis of demand models, the impact of capacity reductions is also studied. The details of CTM based SO DTA model will be discussed in detail in following sections as the base model for the proposed probabilistic approach, but for time being, only the network-wide analysis results for different loading curves along with network capacity reductions are presented.

Two scenarios were investigated with the three demand models described previously:

1. **Base scenario:** The empty network is loaded with constant capacities.

2. **Reduced capacity scenario:** The network is loaded with reduced link capacities to investigate the sensitivity of different loading schemes due to capacity losses. Both flow and physical cell capacities are assumed to decrease continuously from the beginning until the end of the analysis, which can be considered the time of the hurricane landfall.
Clearance and average travel times obtained for different demand models are summarized in Table 3.1. Results of a similar analysis conducted for behavioral response
curves are given in Table 3.2. The SO-DTA formulation does not force vehicles to move forward if there is not enough time for all the loaded vehicles to leave the network. Since there is a heavy computational burden for every time step added to the analysis, the analysis period was limited to 48 hours and average travel times were not computed for scenarios that require more than 48 hours of clearance time and simply stated as “CT>48” (Table 3.1).

Table 3.1 SO-DTA clearance times and average travel times (hours) for selected demand curves

<table>
<thead>
<tr>
<th>Clearance and Average Travel Times (hours)</th>
<th>Base Scenario</th>
<th>Reduced Capacity (30%)</th>
<th>Reduced Capacity (50%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low P.R</td>
<td>Low P.R</td>
<td>High P.R</td>
</tr>
<tr>
<td>Sequential Logit Model</td>
<td>CT=28.5 ATT=1.9</td>
<td>CT=31 ATT=2.2</td>
<td>CT&gt;48 (-)</td>
</tr>
<tr>
<td>S-Curve (rapid response)</td>
<td>CT=22 ATT=4</td>
<td>CT=23.5 ATT=4.2</td>
<td>CT=46 ATT=19.7</td>
</tr>
<tr>
<td>S-Curve (slow response)</td>
<td>CT=25.5 ATT=1.7</td>
<td>CT=20 ATT=4.6</td>
<td>CT=42 ATT=20.6</td>
</tr>
<tr>
<td>Tweedie’s Approach</td>
<td>CT=19 ATT=4.5</td>
<td>CT=20 ATT=4.6</td>
<td>CT=42 ATT=20.6</td>
</tr>
</tbody>
</table>

ATT: Average Travel Time, CT: Clearance Time, PR: Participation Rate

Table 3.2 SO-DTA clearance times and average travel times (hours) for S-curves with changing parameters

<table>
<thead>
<tr>
<th>Response Rate α</th>
<th>Half Loading Time, H [hr]</th>
<th>Base Scenario</th>
<th>Reduced Capacity(25%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>6</td>
<td>9</td>
<td>12</td>
</tr>
<tr>
<td><strong>0.1</strong></td>
<td>CT=21 ATT=5.5</td>
<td>CT=23.5 ATT=5.5</td>
<td>CT=26 ATT=5.5</td>
</tr>
<tr>
<td><strong>0.3</strong></td>
<td>CT=23 ATT=8.5</td>
<td>CT=26 ATT=8.5</td>
<td>CT=29 ATT=9.75</td>
</tr>
<tr>
<td><strong>0.5</strong></td>
<td>CT=24 ATT=9.5</td>
<td>CT=27 ATT=9.5</td>
<td>CT=30 ATT=10.5</td>
</tr>
</tbody>
</table>

ATT: Average Travel Time, CT: Clearance Time

Some important observations in Table 3.1 and Table 3.2 based on the clearance time results are summarized below.
• An increase in participation rate (P.R) (Table 3.1) increases the clearance time for the curves with shorter loading periods more than the ones with longer loading periods (depicted by a lower α value). Quick loading S-curve and Tweedie’s models double the clearance time with doubled P.R. For the clearance times for the other two models, a minimum increase of 55% is observed.

• For high participation rates, the overall clearance times are close for all loading models (Table 3.1). However, for low P.R, the clearance times differ considerably. This is mainly because when the system is not congested, the clearance time depends on the loading timing of the vehicles rather than on the network capacity. For networks operating at capacity, the clearance times become more dependent on network characteristics.

• Clearance times are directly affected by the half loading time. Half loading time shifts the clearance times almost in a linear fashion, e.g., for each 3 hour increment for \( H \), the order of change in clearance time is almost the same for all scenarios.

• The difference in clearance times gets closer as the response rate increases. This means that slow-medium-quick response curves do not provide linear scales for predicting network clearance times (Table 3.2).

• A capacity decrease increases the clearance times, as expected. The order of increase depends partly on the response rate but mainly on the half loading time. The larger the half loading time, the more the system is affected by the capacity decrease since the reduction is assumed to occur and remain unchanged during the evacuation period.
The findings presented above point out some important issues related to the aim of this dissertation. It is shown that variations in demand curve, either in the shape of the curve or the level of demand values (dictated by the assumed participation rate) within the same curve, result in significantly changes for clearance and average travel times. More importantly, these changes do not show a consistent pattern that can be forecasted by using the input demand profile. Likewise, the capacity changes affect the evacuation performance measures considerably, which reveals that using deterministic, constant roadway capacity values may result in erroneous estimations for the clearance and average travel time. The analyzed demand models are finely tuned, state-of-the-art models, but they provide demand curves with varying level and timing pointing out the uncertainty in the estimation process.

3.1 Implications of Literature Review and Network-wide Analysis

Findings on the Current Study

The literature review shows that the evacuation demand is based on many factors, mostly being hard to quantify, such as risk perception. The uncertainty due to the complexity of the evacuee decision process can also be identified from the proposed demand models in the literature. It can be said that the state-of-the-practice demand models also suffer from the uncertainty. Outcomes of different models may show distinct variations for the same study region. It was shown in network wide analysis that the variations in demand estimation result in considerable change in clearance and average travel time results. While treating the demand profiles obtained from a model as is, there is a significant possibility that the model predictions will be off the target. Likewise, the
not accounting for capacity reductions during an evacuation study may change the planning outcomes of the analysis. The demand profiles and the link capacities have a probabilistic nature and their divergence from deterministic estimations result in significant differences in evacuation performance measure. Hence there is need for models that will treat the problem from probabilistic perspective and will give results based on the uncertain nature of the problem. In other words, the findings of the literature review reinforce the need for the probabilistic approach that is employed in the current study.

CHAPTER 4. MATHEMATICAL MODEL

Regarding the proposed formulation in the current dissertation, Cell Transmission Model (CTM) proposed by Daganzo [30,31] and System Optimal Dynamic Traffic Assignment (SO DTA) formulation by Ziliaskopoulos [178] requires special attention. The cell transmission model is an innovative transformation of the differential equations of Lighthill and Whitham’s [75] and Richards’[106] hydrodynamic model to simple difference equations by assuming a piecewise linear relationship between flow and density at the cell level (Figure 4.1). The model accurately describes traffic propagation on street networks and captures traffic phenomena, such as disturbance propagation and creation of shockwaves on freeways, and it can be easily adapted to account for traffic signal control and ramp metering devices. It should also be noted that, although the model assumes a piecewise linear relationship at the cell level, it captures reasonably well the non-linearities between speed density and travel time density at the link level.
Moreover, the computational requirements of the model are adjustable, depending on the discretization interval and the required accuracy.

![The equation of the state of the cell transmission model](image)

**Figure 4.1** The equation of the state of the cell transmission model [31]

Although Daganzo in his second paper of CTM [31] explicitly points out the applicability of his model for evacuation modeling, almost 10 years pass for an evacuation application study to appear in which Tuydes and Ziliaskopoulos [146, and follow-up study 145] uses CTM to evaluate optimal contraflow operations. On the other hand, the recent evacuation applications do not appear solely using CTM, but employs the SO DTA model based on CTM. Liu et. al. [82,81,80] use CTM based SO DTA model for evacuation analysis for staged evacuation and two level optimization schemes in large-scale networks. Ozbay et.al. [104] and Ozbay & Yazici [103] employ SO DTA for evaluating the network-wide impact of evacuation demand model choice on clearance and average travel times. Shen et. al. [131] propose a computationally more efficient methodology while using CTM based SO DTA in evacuation applications. Chiu and Zheng [25] also base their model on CTM SO DTA model and they define “simultaneous
mobilization destination, traffic assignment, and departure schedule for multi-priority groups” (SMDTS-MPG) for real-time emergency response in no-notice disasters. Kalafatas and Peeta [64] provide a strategic planning perspective similar to current study's objectives. They provide analysis in terms of the budget for contraflow operations, the total number of evacuees, and spatial distribution of origin-destinations in evacuation network. While doing so, reduced flow capacities are also considered that representing the decrease due to the evacuation operations. Although the popularity of CTM SO DTA model is increasing in evacuation modeling area, all the studies provide deterministic improvements or applications, and almost none undertakes a stochastic approach to the formulation. Yazici and Ozbay [172] approach the problem from stochastic programming perspective and study the impact of roadway capacity reduction probabilities. They analyze the changes in clearance and average travel time changes in case of probabilistic roadway capacities, and the spatial shelter utilization is discussed in terms of shelter management.

In this chapter, the proposed mathematical formulation for probabilistic analysis of evacuation problem is presented. First, CTM based SO DTA model is given for the completeness of the presented stochastic extension proposed in the dissertation. Then the stochastic SO DTA formulation with probabilistic demand and capacity constraints is provided along with some stochastic programming basics.
4.1 Cell Transmission Model and System Optimal Dynamic Traffic Assignment

Original single-destination LP formulation with detailed explanations can be found in [178]. Briefly, the objective of the SODTA problem is to minimize the total travel time in the network, i.e., the travel time experienced by all users of the network. At any time interval $t$, the travel time experienced by the users of cell $i$ equals to $\tau x^t_i$, $x^t_i$ being the number of vehicles in cell $i$ time $t$. According to the CTM, these users have to stay in this cell for the duration of the time interval. The travel time experienced by all users of the network during time interval $t$ is $\sum_{i \in \zeta / \zeta} \tau x^t_i$, because no users are stored at the cell connectors. $\zeta / \zeta$ is the set of all cells except the sink cells, because the sink cells do not contribute to the total system travel time. Thus, the total system travel time during the whole assignment period $T$ is

$$\sum_{i \in T} \sum_{i \in \zeta / \zeta} \tau x^t_i$$

The SODTA objective is to minimize the function (1), or,

$$\sum_{i \in T} \sum_{i \in \zeta / \zeta} x^t_i$$

because $\tau$ was assumed to be one time unit since $\tau$ can take any positive value without affecting the solution of the LP. The LP problem with complete set of constraints are given below (Equations 5-24)

$$\text{Minimize} \sum_{i \in T} \sum_{i \in \zeta / \zeta} x^t_i$$
Subject to:
Conservation for all cells except source (R) and sink cells (S):

\[ x_i^t - x_i^{t-1} - \sum_{k \in \Gamma^{-1}(i)} y_{ki}^{t-1} + \sum_{j \in \Gamma(i)} y_{ij}^{t-1} = 0, \forall i \in \zeta \setminus \{\zeta_R, \zeta_S\}, \forall t \in T \]  

(6)

Flow inequality constraints for source (R) and ordinary cells (O):

\[ y_{ij}^t - x_i^t \leq 0, \quad \forall (i, j) \in \xi_o \cup \xi_R, \forall t \in T \]  

(7)

\[ y_{ij}^t \leq Q_{ij}^t, \quad \forall (i, j) \in \xi_o \cup \xi_R, \forall t \in T \]  

(8)

\[ y_{ij}^t \leq Q_i^t, \quad \forall (i, j) \in \xi_o \cup \xi_R, \forall t \in T \]  

(9)

\[ y_{ij}^t + \delta_j^i x_j^t \leq \delta_j^i N_j^t, \quad \forall (i, j) \in \xi_o \cup \xi_R, \forall t \in T \]  

(10)

Flow inequality constraints for sink (S) cells:

\[ y_{ij}^t - x_i^t \leq 0, \quad \forall (i, j) \in \xi_S, \forall t \in T \]  

(11)

\[ y_{ij}^t \leq Q_{ij}^t, \quad \forall (i, j) \in \xi_S, \forall t \in T \]  

(12)

Flow inequality constraints for diverging (D):

\[ y_{ij}^t \leq Q_{ij}^t, \quad \forall (i, j) \in \xi_D, \forall t \in T \]  

(13)

\[ y_{ij}^t + \delta_j^i x_j^t \leq \delta_j^i N_j^t, \quad \forall (i, j) \in \xi_D, \forall t \in T \]  

(14)

\[ \sum_{j \in \Gamma(i)} y_{ij}^t - x_i^t \leq 0, \quad \forall i \in \zeta_D, \forall t \in T \]  

(15)

\[ y_{ij}^t \leq Q_{ij}^t, \forall i \in \zeta_D, \forall t \in T \]  

(16)

Flow inequality constraints for merging (M) cells:

\[ y_{ij}^t - x_i^t \leq 0, \quad \forall i \in \zeta_D, \forall t \in T \]  

(17)

\[ y_{ij}^t \leq Q_{ij}^t, \forall i \in \zeta_D, \forall t \in T \]  

(18)

\[ \sum_{i \in \Gamma^{-1}(j)} y_{ij}^t \leq Q_j^t, \forall j \in \zeta_M, \forall t \in T \]  

(19)

\[ \sum_{i \in \Gamma^{-1}(j)} y_{ij}^t + \delta_j^i x_j^t \leq \delta_j^i N_j^t, \quad \forall j \in \zeta_M, \forall t \in T \]  

(20)
Mass balance for source cells (R):

\[ x_i^t - x_i^{t-1} + y_{ij}^{t-1} = d_i^{t-1}, \quad j \in \Gamma(i), \forall i \in \zeta_R, \forall t \in T, x_i^0 = 0, \forall i \in \zeta \] (21)

Initial conditions and non-negativity constraints:

\[ y_{ij}^0 = 0, \quad \forall (i, j) \in \zeta \] (22)

\[ x_i^t \geq 0, \quad \forall i \in \zeta, \forall t \in T \] (23)

\[ y_{ij}^t \geq 0, \quad \forall (i, j) \in \zeta, \forall t \in T \] (24)

where:

\( q \): link flow

\( q_{\text{max}} \): maximum flow

\( k \): density

\( k_j \): jam density

\( v \): link free flow speed

\( w \): backward propagation speed

\( \zeta \): set of cells; ordinary (O), diverging (D), merging (M), source (R) and sink (S).

\( T \): set of discrete time intervals

\( x_i^t \): number of vehicles in cell \( i \) at time interval \( t \)

\( N_i^t \): maximum number of vehicles in cell \( i \) at time interval \( t \)

\( y_{ij}^t \): number of vehicles moving from cell \( i \) to cell \( j \) at time interval \( t \)

\( \xi \): set of cell connectors; ordinary (O), diverging (D), merging (M), source (R), sink (S).

\( Q_i^t \): maximum number of vehicles that can flow into or out of cell \( i \) during time interval \( t \)

\( \delta_i^t \): ratio \( v/w \) for each cell and time interval (Assumed \( \delta_i^t = 1 \) throughout the analysis)
\( \Gamma_i(t) \): set of successor cells to \( i \)

\( \Gamma_i^{-1}(t) \): set of predecessor cells to cell \( i \)

\( \tau \): discretization time interval

\( d_i^t \): demand (inflow) at cell \( i \) in time interval \( t \)

CTM based SODTA formulation can be simply given in the following compact standard form as below, with explicitly written capacity and demand constraints.

\[
\begin{align*}
\min & \quad \sum_t \sum_i x_i^t \\
\text{s.t.} & \quad A_{eq}v = b_{eq} \\
& \quad Rv = D \\
& \quad Av \leq b \\
& \quad Tv \leq C \\
& \quad x_i^t \geq 0, \ y_{ij}^t \geq 0, \ \forall (i, j) \in \xi, \ \forall t \in T
\end{align*}
\]

where \( v = \begin{bmatrix} x_i^t \\ y_{ij}^t \end{bmatrix} \) is the vector of system states. Regarding the matrix dimensions, let there are \( r \) and \( s \) equality and inequality constraints excluding the constraints with capacity and demand values at the right hand side (RHS), \( k \) capacity constraints and \( l \) source cell constraints with demand values at the RHS, making \( r+s+k+l=m \) constraints in total and \( n \) decision variables. Then \( A(s \times n) \) and \( A_{eq}(r \times n) \) stands for matrix for the equality and inequality constraints, with corresponding RHS \( b(s \times 1) \) and \( b_{eq}(r \times 1) \). \( R(l \times n) \) represents the demand constraints with demand vector \( D(k \times 1) \) and \( T(k \times n) \) represents the capacity constraints with capacity vector \( C(k \times 1) \). Since the constraints are formulated for each time step, \( C \) and \( D \) vectors consist of time-expanded forms of all cell capacities and demands in the network.
4.2 System Optimal Dynamic Traffic Assignment with Probabilistic Demand and Capacity Constraints

Stochastic programming has been defined in Prekopa [115] as "mathematical programming problems, where some of the parameters are random variables; either we study the statistical properties of the random optimum value or random variables that come up with the problem, or we reformulate it into a decision type problem by taking into account the joint probability distribution of random variables". Although the term stochastic clearly conveys the probabilistic nature, stochastic programming problems are solved through their underlying deterministic problem. The stochastic constraints of the problem are represented in their deterministic equivalent forms which enforce the constraints to hold for certain prescribed probabilities. The deterministic equivalent of the probabilistic constraints are plugged into the underlying deterministic formulation to obtain the results for the stochastic programming at hand. In other words, stochastic programs are solved as nothing but deterministic problems whose form is imposed by the random variables in the model. The results are interpreted within user prescribed probability level on the probabilistic constraint.

Regarding our underlying deterministic SODTA problem, when cell flow capacities are assumed to be probabilistic, we have \( k \) constraints of the form \( T_i \nu \leq \phi_j^t \) for each probabilistic cell capacity, where \( \phi_j^t \) represents the probabilistic capacity distribution of cell \( j \) at time \( t \), and \( T_i \) represents the \( i^{th} \) row of matrix \( T \) \((k \times 1)\) corresponding to the probabilistic capacity constraint with RHS \( \phi_j^t \). Similarly, we can write the demand constraint for the source cell as \( R_i \nu = \psi_j^t \) for each source cell, where
ψ_j^t$ represents the probabilistic demand at source cell $j$ at time $t$, and $R_i$ represents the $i^{th}$ row of matrix $R(l \times 1)$ corresponding to the probabilistic demand constraint with RHS $ψ_j^t$.

The form of the underlying deterministic problem is closely related with whether the desired probability levels for the constraints are imposed individually on each constraint (ICC), or jointly on multiple constraints (JCC). Choice of ICC or JCC provides different interpretations of the model results, mainly in terms of system reliability.

The SODTA problem with probabilistic demand and capacity constraints can be modeled with ICC and the formulation (Problem-1) is given below.

\[
\begin{align*}
\min \quad & \sum_{t} \sum_{i} x_i^t \\
\text{s.t.} \quad & A_{eq}v = b_{eq} \\
& P(R_i v \geq \psi_i) \geq p_l, \quad \bar{I} = 1, \ldots, l \\
& Av \leq b \\
& P(T_k v \leq \phi_k) \geq p_k, \quad \bar{k} = 1, \ldots, k \\
& x_i^t, y_{ij}^t \geq 0, \quad \forall(i,j) \in \xi, \quad \forall t \in T
\end{align*}
\]

(P1)

where $\bar{I}$ refers to the number of probabilistic demand constraints (having a total of $l$) and $\bar{k}$ refers to the number of probabilistic capacity constraints (having a total of $k$). $p_l$ and $p_k$ are the probability levels are imposed on each probabilistic constraint individually and can be assigned different values based on the desired local reliability of the constraint at hand.

When the problem is modeled with joint constraints for demand and capacity, we get:
\[
\text{min } \sum_t \sum_i x_i^t \\
\text{s.t. } A_{eq} v = b_{eq} \\
P(Rv \geq \psi) \geq p_1 \\
A^\top v \leq b \\
P(Tv \leq \phi) \geq p_2 \\
x_i^t, y_{ij}^t \geq 0, \ \forall (i, j) \in \xi, \ \forall t \in T \tag{P2}
\]

where \( \phi \) (\( k \times 1 \)) is the random capacity vector composed of capacity distributions of cell \( j \) at time \( t \) (\( \phi_j^t \)), and \( \psi \) (\( l \times 1 \)) is the random demand vector composed of demand values at source cell \( j \) at time \( t \) (\( \psi_j^t \)). We refer to this JCC approach as Problem-2 (P2).

Since the constraints are enforced jointly, \( p_1 \) and \( p_2 \) refers to the reliability levels enforced on random roadway capacities and random demand.

Solutions of P1 and P2 may have different levels of difficulty based on the type of random variable used, e.g. discrete or continuous. The DTA problem studied in the current study uses discrete parameters (e.g. number of vehicles, the flow capacity of a roadway). Still the probability distributions for both demand and capacity can be assumed to follow a continuous distribution and rounded up. However, small capacity fluctuations (e.g. 1990 vphpl instead of 2000 vphpl) will not cause a significant change on the network traffic, but changes that correspond to a more significant percentage of the existing capacity (e.g. 1750 vphpl instead of 2000 vphpl) will affect network flows. Similar argument is also valid for demand. Moreover, considering that such continuous distributions will be based on continuous fits to discrete field data, it can be argued that using known continuous distributions will just add generalized assumption to the actual measurements. In the current study, it is assumed that demand and capacity distributions follow discrete probability distributions. Naturally, the discretization of the probability
distribution is important and coarse intervals may provide rough results. Nevertheless, properly chosen discretization level will provide the desired network analysis results. On the other hand, to solve the proposed problems, solution techniques customized for discrete random variables are needed. Such a technique to be used for solving stochastic programming problem with discrete right hand sides has been provided by Prekopa [115].

In terms of interpretation of the solutions, P2 differs from P1 by providing overall reliability for network capacity and demand over all time horizon rather than meeting reliability levels individually for each link and source cell at each time step. \( p_1 \) and \( p_2 \) can be defined based on problem dynamics and modeler's risk perception, e.g. assigning higher \( p_2 \) would mean that the analysis will provide results assuming less possible capacity degradations. Similarly higher \( p_1 \) would assign higher demand on the network so that the possible realized demand would not exceed the assigned demand value. A detailed discussion of ICC and JCC approaches are provided below.

### 4.2.1 ICC vs. JCC: Theoretical Considerations

The capacity constraints in SO DTA formulation are in the form of \( y_{ij}^t \leq C_i^t \) where \( y_{ij}^t \) is the flow from cell \( i \) to cell \( j \) at time \( t \), and \( C_i^t \) is the flow capacity of cell \( i \) at time \( t \). Assuming that cell flow capacities are random, we can write \( k \) probabilistic constraints as \( T_i v \leq \phi_j^t \), where \( \phi_j^t \) is the random capacity of cell \( j \) at time \( t \), and \( T_i \) represents the \( i^{th} \) \((i=1,2,...,k)\) row of matrix \( T (k \times I) \) corresponding to the probabilistic capacity constraint with right hand side \( \phi_j^t \). We can write the probabilistic constraints individually on the inequalities as:
\[ P(T_i v \leq \phi_j^i) \geq p_i \]

where \( p_i \) is the prescribed probability that the probabilistic constraint would hold.

Following the same logic, for the stochastic demand, the probabilistic constraints can be written individually as \( P(R_i v = \psi_j^i) \geq p_i \). However, probabilistic equality constraints with random RHS is problematic since probability of a random variable being equal a certain value is either zero in case of a continuous random variable, or very small in case of a discrete random variable. A remedy proposed in the literature for this problem [115,159] is to re-write the demand constraints in inequalities form as \( P(R_i v \geq \psi_j^i) \geq p_i \) and use the demand constraint as a lower bound for the possible demand realizations. This approach was used by Waller and Ziliaskopoulos [115] to model the demand uncertainty for SODTA problem and also adopted in the current study.

The \( p_i \) value in a probabilistic constraints connects to the reliability of the system and "ensures that the state of the system remains within a subset of all possible states where its functioning is undisturbed by major failures" [115]. An optimal solution with the probabilistic constraint \( P(T_i v \leq \phi_j^i) \geq p_i \) ensures that the constraint is violated at most \((1-p_i)\) 100% of the possible realizations. Assuming that \( F_{\phi_j^i} \) is the probability distribution function of \( \phi_j^i \), then the probabilistic constraint can be re-written in its deterministic equivalent as \( T_i v \leq F_{\phi_j^i}^{-1}(p_i) \). Then the problem at hand can be solved via conventional LP solution methods after substituting deterministic equivalent of each probability constraint. This approach is, in general, called "chance constrained programming", first proposed by Charnes, Cooper and Symonds in 1958 [18].
Specifically, the above formulation is called individual chance constraint formulation because the probabilistic constraints are imposed individually. On the other hand, the problem can be also formulated using joint chance constraints (JCC) - or joint probability constraints (JPC). In JCC setup, the probabilistic capacity constraints are arranged to hold jointly with prescribed probability level $p$ as:

$$P(T_1 v \leq \phi_1^1, T_2 v \leq \phi_1^2, ..., T_i v \leq \phi_i^j, ..., T_k v \leq \phi_k^k, ) \geq p$$

and for demand, with the same $p$, we get:

$$P(R_1 v \geq \psi_1^1, R_2 v \geq \psi_1^2, ..., R_i v \geq \psi_i^j, ..., R_k v \geq \psi_k^k, ) \geq p_2$$

where $N$ is the total number of cells with random capacity.

The $p$ value associated with ICC provides local reliability, since the probability constraint is bounded for each link/cell separately. Hence, the reliability levels can be assigned based on local requirements in the system. On the other hand, JCC solution yields a system wide reliability via using the joint cases of possible system wide capacity realizations. As a direct result of Boole's inequality, the solutions satisfying ICC formulation also satisfy JCC [115]. Hence ICC provides tighter bounds for the problem compared to JCC. However, from the computational perspective, deterministic equivalent ICCs are easier to calculate by re-writing the probabilistic constraint as $T_i v \leq F_{\phi_i^j}^{-1}(p_i)$, provided that we can compute $F_{\phi_i^j}^{-1}(p_i)$. On the other hand, JCC may require high order integrals to calculate the joint probability density of the random right hand side variables. Fortunately, this computational burden can be reduced significantly if the random
variables are independent, since the joint probability density calculation boils down to multiplication of marginal densities.

The choice of joint constraints or individual chance constraints is a matter of model formulation and the dynamics of the problem. It may be meaningless to employ ICC when the constrained processes/events in the problem do not operate without affecting each other. The reservoir problem [115] in which the probabilistic constraint imposed on one reservoir nullifies the local probabilistic constraint of the other reservoir provides a good example for such problematic use of ICC. In case the problem dynamics allow both approaches, the decision of ICC or JCC is a matter of modeler's perspective on the desired reliability of system components. In this respect, the formulation discussed in this dissertation distinguishes itself from previous works [156, 149, 65] in which use only ICC approach in network design problem context and incorporates only demand uncertainty into DTA formulation.

4.2.2 Nature and Modeling of Random Roadway Capacity and Demand

The SODTA formulation is based on a “clock tick” which is a –preferably short–time interval that the flows in the network are updated. Clock tick also determines the length of each cell according to CTM setup. A link on the highway network can be composed of several cells, which will have imposed capacity constraints for each time step. The modeling of link capacity can be performed at the cell level, or link level composed of several cells. It can be assumed that the roadway capacity does not exhibit significant fluctuations for small time intervals or for small road segments. Similarly for the demand at a specific time interval would not be expected to differ significantly for
short time intervals. In other words, compared to CTM clock tick and cell length, the
demand and capacity randomness may be quantifiable for relatively larger time intervals
and for longer length of road segments. Thus, we can assume all the roadway and source
cells in a roadway link would more likely have the same random capacity and demand
realizations for certain period of time. The actual realization of the demand and link
capacity is assumed to prevail during whole time interval that the distribution is
calculated for. Thus, the capacity and demand are allowed to change at pre-determined
time intervals rather than each clock tick of the SODTA model. Let $\Phi_i^t$ be the capacity
realization of cell $i$ for time interval $\Delta t$; let $T$ is the duration that the calculated capacity
probability is valid; and assume cells $i-1$, and $i+1$ are on the same link with cell $i$, then we
can write:

$$
\Phi_i^t = \Phi_{i+1}^t = \Phi_{i-1}^t = \Phi_{i-1}^{t+\Delta t} = \Phi_{i+1}^{t+\Delta t} = \Phi_{i+1}^{t+\Delta t} \text{ with probability } 1, \text{ for all } t + \Delta t \leq T
$$

Then, if we assign the same $p$ for all constraints, we will have the RHS of
deterministic equivalent of probabilistic constraints as $F_{\Phi_i^{-1}}^{-1}(p) = F_{\Phi_{i-1}^{t+\Delta t}}\Phi_{i-1}^{-1}(p) = F_{\Phi_i^{t+\Delta t}}\Phi_i^{-1}(p) = F_{\Phi_{i+1}^{t+\Delta t}}\Phi_{i+1}^{-1}(p)$ since the random capacities are assumed
to be the same for all cells on the same link and for a certain time period. Following the
same steps for demand at each source cell, we will have $F_{\Psi_i^{-1}}^{-1}(p) = F_{\Psi_{i+1}^{-1}}\Psi_i^{-1}(p)$ at the RHS
for the deterministic equivalent of the demand constraint. The calculation of pLEPs for
ICC is straightforward since we are dealing with one dimensional probability
distributions. For solution of JCC we cannot directly use the individual distribution, and
we need the joint probability distribution of the RHS random variables to calculate the
deterministic equivalent of the probabilistic constraint. Due to space and temporal dependency of the random capacity between consecutive time steps and cells, we can calculate the joint probability density as follows. Letting $\phi^r$ and $\psi^r$ as the rth realization of random discrete capacity, we know that $P(\phi_{i+1}^{t+1} = \phi^r / \phi_{i}^{t} = \phi^r) = 1$ since the experienced capacity will be the same for consecutive time steps for the whole period that the random capacity distribution is defined. For consecutive cells we can write $P(\phi_{i+1}^{t} = \phi^r / \phi_{i}^{t} = \phi^r) = 1$ since the realized capacity will not change at short distances. Then we will have the joint probability $P(\phi_{i}^{t}, \phi_{i+1}^{t+1})$ as:

$$P(\phi_{i+1}^{t+1} = \phi^r, \phi_{i}^{t} = \phi^r) = P(\phi_{i+1}^{t+1} = \phi^r / \phi_{i}^{t} = \phi^r). P(\phi_{i}^{t} = \phi^r) = P(\phi_{i}^{t} = \phi^r)$$

or for consecutive cells, we can calculate $P(\phi_{i+1}^{t}, \phi_{i}^{t})$:

$$P(\phi_{i+1}^{t} = \phi^r, \phi_{i}^{t} = \phi^r) = P(\phi_{i+1}^{t} = \phi^r / \phi_{i}^{t} = \phi^r). P(\phi_{i}^{t} = \phi^r) = P(\phi_{i}^{t} = \phi^r)$$

Similarly, for demand at a source cell for consecutive time steps, we can compute

$$P(\psi_{i+1}^{t+1} = \psi^r, \psi_{i}^{t} = \psi^r) = P(\psi_{i+1}^{t+1} = \psi^r / \psi_{i}^{t} = \psi^r). P(\psi_{i}^{t} = \psi^r) = P(\psi_{i}^{t} = \psi^r)$$

The same procedure can be followed for higher dimensions and the joint probability distribution for several time steps for a link composed of several cells can be computed. On the other hand, we assume that a capacity decrease (say, due to an incident) on a specific link does not imply occurrence of another capacity decrease on another link in the network. Thus, the capacity decreases in the network links can be assumed as independent both in time and space. Likewise, demand at a certain source cell is independent of the demand loaded onto network from different sources. Then, the joint probability of the random capacity variables will be the product of each probability
distribution. As an example, let we have 3 links (links L1, L2, L2, each composed of certain number of cells) and the capacity distributions are calculated 4 time intervals \((T_1, T_2, T_3, T_4, \text{ each including by several clock ticks})\). Assume that the capacity probability distribution for link L at time interval \(T\) is represented by \(F^T_L\), then the joint probability of the random capacity for 3 links and for 4 time interval will be the product of each probability distribution: \(F^{T_1,T_2,T_3,T_4}_{L_1,L_2,L_3} = \prod_{T=1}^{4} F^T_L\). The resulting joint probability function will be used to determine pLEPs for JCC setup.

For a simple representation of how the random capacity is modeled, assume that we have cells \(i, j\) and \(k, m\) have random capacities at two different links in a cell network (Figure 4.2). Their discrete probability distributions are given by \(\phi^t_i\) and \(\phi^t_k\) for link-A and link-B respectively, where \(i\) is the cell number and \(t\) is the time step. For other cells, the assume deterministic capacity assigned as \(C^t_i\).

![Diagram](image.png)  
**Figure 4.2 Two Links Extracted from a Cell Network**

For Link-A, let us set random variable for capacity to be \(\phi^t_i\) where \(i\) is the cell number and \(t\) is the time step. If we write the capacity constraints for cells \(i\) and \(j\) for 2 time steps, we have:
As shown above, in ICC formulation we can assign separate reliability levels \((p_i)\) for each constraint, however that will provide the constraint to hold within the assigned local capacity reliability for that specific time and cell that the constraint is written for. If we want to have the same local reliability for all links, hence we assign the same \(p\) for all constraints, e.g. \(p_1 = p_2 = p_3 = p_4 = p_5 = p_6 = p_7 = p_8 = p\) for all constraints. Then we will have \(F_{\phi_l}^{-1}(p) = F_{\phi_{l+1}}^{-1}(p) = F_{\phi_{l+2}}^{-1}(p) = F_{\phi_{l+3}}^{-1}(p) = F_{\phi_{l+4}}^{-1}(p)\) since the random capacities are assumed to be the same for all cells on the same link and for a certain time period. Set of probabilistic capacity constraints can be written for Link-B in a similar fashion. The solution will follow substituting \(F_{\phi_l}^{-1}(p)\) values as the right hand side of the probabilistic equation for all probabilistic capacity constraints for Link-A and Link-B, thus make them deterministic, and solve the problem via conventional LP solvers. For link-B let us assign \(\phi_{l}^{i}\) for the random capacity of cell \(i\) at time \(l\). The complete set of capacity constraints for Link-A and Link-B is given below, assuming that we impose the same reliability level \(p\) on all probabilistic constraints:

\[
\begin{align*}
    y_{l-1,i}^{j} &\leq C_{l-1,i}, \quad y_{l-1,i}^{j+1} \leq C_{l-1,i}^{j+1}, \\
    y_{l-1,i}^{j} &\leq \phi_l^i, \\
    y_{l-1,i}^{j+1} &\leq \phi_l^{i+1}, \\
    y_{l,j}^{i} &\leq \phi_l^j, \\
    y_{l,j}^{i+1} &\leq \phi_l^{j+1}, \\
    y_{l,i+1}^{j} &\leq \phi_l^j, \\
    y_{l,i+1}^{j+1} &\leq \phi_l^{j+1}, \\
    y_{l+1,j}^{i} &\leq \phi_l^{j+1}, \\
    y_{l+1,j}^{i+1} &\leq \phi_l^{j+1}, \\
    y_{l+1,i}^{j} &\leq \phi_l^{j+1}, \\
    y_{l+1,i}^{j+1} &\leq \phi_l^{j+1}, \\
    y_{l+1,j+1}^{i} &\leq \phi_l^{j+1}, \\
    y_{l+1,j+1}^{i+1} &\leq \phi_l^{j+1}, \\
    y_{l+1,i+1}^{j} &\leq \phi_l^{j+1}, \\
    y_{l+1,i+1}^{j+1} &\leq \phi_l^{j+1}, \\
    y_{l+1,j+1}^{i} &\leq \phi_l^{j+1}, \\
    y_{l+1,j+1}^{i+1} &\leq \phi_l^{j+1}, \\
    y_{l+1,i+1}^{j} &\leq \phi_l^{j+1}, \\
    y_{l+1,i+1}^{j+1} &\leq \phi_l^{j+1}, \\
    y_{l+1,i+1}^{j} &\leq \phi_l^{j+1}, \\
    y_{l+1,i+1}^{j+1} &\leq \phi_l^{j+1}, \\
    y_{l+1,i+1}^{j} &\leq \phi_l^{j+1}, \\
    y_{l+1,i+1}^{j+1} &\leq \phi_l^{j+1}
\end{align*}
\]
Now, let us write the probabilistic equations for link-A in JCC setup.

\[
\begin{align*}
&y_{i-1,i}^t \leq C_{i-1}^t, \quad y_{i-1,i}^{t+1} \leq C_{i-1}^{t+1}, \\
&y_{i,j+1}^t \leq C_{j+1}^t, \quad y_{i,j+1}^{t+1} \leq C_{j+1}^{t+1}, \\
&y_{i-1,i}^t \leq F_{\phi_i}^{-1}(p) \\
&y_{i-1,i}^{t+1} \leq F_{\phi_i}^{-1}(p) \\
&y_{i,j}^t \leq F_{\phi_i}^{-1}(p) \\
&y_{i,j}^{t+1} \leq F_{\phi_i}^{-1}(p) \\
&y_{i,j+1}^t \leq F_{\phi_j}^{-1}(p) \\
&y_{i,j+1}^{t+1} \leq F_{\phi_j}^{-1}(p) \\
&y_{k-1,k}^t \leq C_{k-1}^t, \quad y_{k-1,k}^{t+1} \leq C_{k-1}^{t+1} \\
&y_{m,m+1}^t \leq C_{m+1}^t, \quad y_{m,m+1}^{t+1} \leq C_{m+1}^{t+1} \\
&y_{k-1,k}^t \leq F_{\phi_k}^{-1}(p) \\
&y_{k-1,k}^{t+1} \leq F_{\phi_k}^{-1}(p) \\
&y_{k,m}^t \leq F_{\phi_k}^{-1}(p) \\
&y_{k,m}^{t+1} \leq F_{\phi_k}^{-1}(p) \\
&y_{k,m}^t \leq F_{\phi_m}^{-1}(p) \\
&y_{k,m}^{t+1} \leq F_{\phi_m}^{-1}(p) \\
&y_{m,m+1}^t \leq F_{\phi_m}^{-1}(p) \\
&y_{m,m+1}^{t+1} \leq F_{\phi_m}^{-1}(p)
\end{align*}
\]

The difference of JCC solution is that we cannot use the marginal probability densities and need the joint probability distribution of the right hand side random variables to calculate the deterministic equivalent of the probabilistic constraint. The calculation of joint probability densities can be difficult, which stands as the trade off for receiving system reliability with JCC rather than obtaining local reliability levels with ICC. For our problem, due to space and temporal dependency of the random capacity between consecutive time steps and cells, we can calculate the joint probability density
easily. For instance, letting $\bar{\phi}$ as a single realization of random capacity, we know that 
$P(\phi_i^{t+1} = \bar{\phi} / \phi_i^t = \bar{\phi}) = 1$ since the experienced capacity will be the same for 
consecutive time steps for the whole period that the random capacity distribution is 
defined. Likewise, for consecutive cells we can write $P(\phi_{i+1}^t = \bar{\phi} / \phi_i^t = \bar{\phi}) = 1$ since 
the realized capacity will not change at short distances. Then we will have the joint 
probability $P(\phi_i^t, \phi_i^{t+1})$ as:

$$P(\phi_i^t = \bar{\phi}, \phi_{i+1}^{t+1} = \bar{\phi}) = P(\phi_i^{t+1} = \bar{\phi} / \phi_i^t = \bar{\phi}).P(\phi_i^t = \bar{\phi}) = P(\phi_i^t = \bar{\phi})$$

or similarly $P(\phi_{i+1}^t, \phi_i^{t+1})$ as:

$$P(\phi_i^{t+1} = \bar{\phi}, \phi_{i+1}^t = \bar{\phi}) = P(\phi_{i+1}^t = \bar{\phi} / \phi_i^{t+1} = \bar{\phi}).P(\phi_i^{t+1} = \bar{\phi}) = P(\phi_i^{t+1} = \bar{\phi})$$

Following the same procedure for higher dimensions, we can write our joint 
probability distribution of random capacities for a link (composed of several cells) and 
time interval (composed of several time steps) as:

$$F_A^T(\phi_i^t = \bar{\phi}^r, \phi_{i+1}^{t+1} = \bar{\phi}^r, \phi_{i+1}^t = \bar{\phi}^r, \phi_i^{t+1} = \bar{\phi}^r, ..., \phi_{i+\Delta x}^{t+\Delta t} = \bar{\phi}^r) = F(\phi_i^t = \bar{\phi}^r)$$

where $\bar{\phi}^r$ is the $r^{th}$ possible realization of the discrete random roadway capacity, 
which is assumed to prevail for cells $[i, i + \Delta x]$ and time interval $[t, t + \Delta t]$. $F_A^T$ is the 
joint probability function of random capacities for link-A at time interval $T=[t, t + \Delta t]$. 
Hence, our multi dimensional joint probability function $F_A^T$ has zero probability 
everywhere except capacity realizations for each cell and time step are equal. More, the 
probability values assigned for those points are equal to the marginal probabilities of each 
realization.

Expanding JCC formulation for two links is not as straightforward as ICC. The 
JCC formulation for link-A and link-B is given below.
For the JCC solution, we need to find the joint probability function covering both links. We can compute the joint probability function for link-B for time interval $T$, $F_{B}^{T}$, similar to $F_{A}^{T}$. Moreover, a capacity decrease (say, due to an incident) on a specific link does not imply occurrence of another capacity decrease on another link in the network. Thus, the capacity decreases in the network links can be assumed as spatially independent. It is also reasonable to assume that capacity reductions are independent for different long time intervals. This temporal and spatial independency assumption is important in terms of joint probability calculations. We can write the joint probability function for capacity of link-A and link-B for single time interval as multiplication of $F_{B}^{T}$ and $F_{A}^{T}$, e.g. $F_{A,B}^{T} = F_{A}^{T} \times F_{B}^{T}$. Now, let us assume our analysis horizon is composed of two time intervals, $T_1$ and $T_2$, for which the link capacities are defined. Then our joint probability function for JCC formulation will include $F_{A}^{T_1}$, $F_{A}^{T_2}$, $F_{B}^{T_1}$, $F_{B}^{T_2}$ and our joint
probability function of two links for two time steps will be \( F_{A,B}^{T_1,T_2} = F_A^{T_1} \times F_A^{T_2} \times F_B^{T_1} \times F_B^{T_2} \). The same approach is valid for source cells where the demand at each source cell is assumed to be independent from each other and the demand distribution is assumed to prevail for certain time period rather than fluctuating at every CTM time-step.

The resulting joint probability function can be used to determine p-level efficient points (pLEP) which is a method introduced by Prekopa [116] for solution of stochastic programming problems with discrete random RHS.

### 4.2.3 Proposed Solution Approach for Stochastic SO DTA Problem with Integer Right Hand Side

P-Level Efficient Points (pLEP) method proposed by Prekopa [116] can be used to solve the stochastic SO DTA problem. Defining \( Z_p = \{ y \in R^m | P(Tv \geq \emptyset) \geq p \} \), letting \( F \) be the probability distribution of \( \emptyset \), then p-level efficient points can be defined as follows.

**Definition:** A point \( z \in Z_p \) is called a p-level efficient point (pLEP) of a probability distribution function \( F \), if \( F(z) \leq p \) and there is no \( y, z, y < z \) such that \( F(y) \geq p \), where \( p \in [0, 1] \).

pLEPs provide discretized set of points, which give the lower bound of a specific probability distribution which ensures that the probabilistic constraint would hold with the given \( p \). They are used in the deterministic equivalent of the probabilistic constraint and assure that the constraint will satisfy the given reliability level \( p \). For a scalar random variable \( \phi \) and for every \( p \in (0, 1) \), there is exactly one \( p \)-efficient point which is the
smallest $z$ such that $F(z) \geq p$. Figure 4.3 visualizes the pLEP points for a two dimensional case.

![Figure 4.3 An Example of the set $Z_p$ with pLEPs $v'...v'$][35]

Prekopa [117] proposes a recursive algorithm to enumerate the $p$-efficient points for multidimensional discrete probability distributions. Let $Z$ be a discrete random vector, let the $i^{th}$ element of $Z$ be a discrete random variable, and let the number of possible values of multidimensional discrete random variable $\emptyset$ be $g_i$. The variable $z_{i,m}$ denotes the $m^{th}$ possible value of the $i^{th}$ element of $Z$ in the increasing order (i.e., $z_{i,g_n+1}$ is the highest possible value of the $n^{th}$ discrete random variable of $Z$). The algorithm to enumerate the $p$-efficient points for a multidimensional discrete probability distribution is as follows [117]:

**Step 0.** Initialize $k \leftarrow 0$.

Step 1. Let
\[
\begin{align*}
    z_{1,j_1} &= \arg\min \{y | F(y, z_{2,k_2+1}, \ldots, z_{n,k_n+1}) \geq p\} \\
    z_{2,j_2} &= \arg\min \{y | F(z_{1,k_2+1}, y, z_{3,k_3+1} \ldots, z_{n,k_n+1}) \geq p\} \\
    &\vdots \\
    z_{n,j_n} &= \arg\min \{y | F(z_{1,j_1}, \ldots, z_{n-1,j_{n-1}}, y) \geq p\}
\end{align*}
\]

**Step 2.** Let \( Z_p \leftarrow \{z_{1,j_1}, \ldots, z_{n,j_n}\} \)

**Step 3.** Let \( k \leftarrow k+1 \). If \( j_i + k > k_j + 1 \) then go to **Step 5.** If \( j_i + k \leq k_j + 1 \), then go to **Step 4.**

**Step 4.** Enumerate all \( p \)-efficient points of the function \( F(z_{1,j_1+k}, w) \) of the variable \( w \) and eliminate in \( Z_p \) and eliminate those that dominate at least one element in \( Z_p \). If \( Z_r \) is the set of the remaining \( p \)-efficient points, which may be empty, then let \( Z_p \leftarrow Z_p \cup Z_r \)

Go to Step 3.

**Step 5.** Stop. The variable \( Z_p \) is the set of all \( p \)-efficient points of the function \( F \).

To provide a simple example of how pLEP points are found, assume we have a 3 dimensional joint probability function \( F_{\alpha,\beta,\gamma}(\alpha = \bar{\alpha}, \beta = \bar{\beta}, \gamma = \bar{\gamma}) \) where \( \alpha, \beta \) and \( \gamma \) are the discrete random variables and \( \bar{\alpha}, \bar{\beta} \) and \( \bar{\gamma} \) are possible realizations of the corresponding discrete random variable. pLEP enumeration first finds the pLEP candidate points whose probability values are higher than the prescribed \( p \) value, e.g. \( F_{\alpha,\beta,\gamma} \geq p \). Among those pLEP candidate points, the algorithm eliminates the ones which are dominated by another candidate in terms of providing tighter bounds. For instance, if there are two points, point-1=(\( \bar{\alpha}, \bar{\beta}, \bar{\gamma} \)) and point-2=(\( \bar{\alpha} - 1, \bar{\beta}, \bar{\gamma} \)), point-1 is excluded from set of pLEP points because point-2 dominates point-1 by at least one dimension, e.g. \( \bar{\alpha} - 1 < \bar{\alpha} \). Assume we have another point, point-3=(\( \bar{\alpha}, \bar{\beta} - 1, \bar{\gamma} \)). Point-3 also
dominates point-1 but no such inference can be made between point-2 and point-3. Although point-2 dominates by $\bar{\alpha} + 1 \geq \tilde{\alpha}$, since point-3 also dominates point-2 for the $2^{nd}$ dimension ($\tilde{\beta} - 1 < \tilde{\beta}$) no absolute dominance can be mentioned and both points-2 and 3 stay in the pLEP set. The algorithm continues until there are no points that are dominated by another candidate and the resulting set of points is assigned as the pLEP set.

Once the set of pLEPs are calculated, the SO DTA problem with probabilistic capacity constraints can be re-written in its deterministic form as:

$$\min \sum_t \sum_i x_{t}^i$$

s. t. $A_{eq}v = b_{eq}$

$Rv \geq z^i, \ z^i \in Z_{p_1}$

$Av \leq b$

$Tv \leq \tilde{z}^j, \ \tilde{z}^j \in \tilde{Z}_{p_2}$

$v \in D$

where $Z_{p_1} = \{z^1, z^2, z^3, ..., z^{N_1}\}$ is the set of pLEPs for demand constraints, and $
\tilde{Z}_{p_2} = \{\tilde{z}^1, \tilde{z}^2, \tilde{z}^3, ..., \tilde{z}^{N_2}\}$ is the set of pLEPs for capacity constraints. The proof for necessary and sufficient conditions for the existence of optimality can be found in [35]. Depending on the approach (ICC or JCC) the set of pLEP point sets are either computed from marginal probability, or joint probability functions. Brute-force solution approach is to find all $p$-efficient points and to solve all corresponding LP problems. Let $v^i$ is the optimal solution to the $i^{th}$ LP problem with constraint $Tv \geq z^i$. If $c^Tv^i = \min_j c^Tv^j$, then $v^i$ is the optimal solution. Hence the use of pLEPs does not increase the complexity
of the problem but requires LP solutions as many as the number of pLEPs. It is proved that the number of pLEPs is finite a number [35], however for high-dimensional random vectors the number of pLEPs can be large. For large number of pLEP points, Dentcheva et al.[35] provide cone generation method which decreases the computational burden by using the dual of the problem and generating the pLEP points when needed instead of running the problem for each pLEP.

CHAPTER 5. NUMERICAL EXAMPLES

For the illustration of the proposed formulation’s use, 2 numerical examples are provided. First example studies the impact of capacity randomness in JCC setup and shows how the favorable shelter locations and the shelter utilization change if the roadway capacity uncertainty is taken into account. Second numerical example studies both the demand and capacity uncertainty and provides a comparison of ICC and JCC approaches.

5.1 Numerical Example-1: Effect of Roadway Capacity Uncertainties on Shelter Locations and Capacities

In this section the change in traffic flow due to capacity uncertainties is investigated regarding the impacts on favorable shelter locations and shelter utilization on Cape May, NJ evacuation network. For this purpose, the existence/necessity of a shelter at one of the destinations is analyzed by employing consecutive assignments assuming a shelter is operational, then not operational. This analysis is done in a deterministic
fashion in which all network specific attributes are kept constant. The flow patterns for each operational shelter set scenario, the network flows and the capacity utilization is calculated. Then, the same analysis is performed under stochastic SO DTA. For the numerical analysis, only the roadway capacity is assumed to be stochastic. The time step chosen for the analysis and the corresponding cells length are large for the current example, hence the capacity distribution is assigned on selected cells rather than using links composed of several cells. More, the problem is solved via JCC only. At the end, the results of deterministic and probabilistic analysis are compared to make inference about the impacts of capacity changes. Figure 5.1 shows the official evacuation routes for Cape May and Figure 5.2 shows the simplified cell representation of the Cape May evacuation network.

![Figure 5.1 Cape May Evacuation Routes](image-url)
The analyzed network is a multi-origin multi-destination network, each destination being a shelter location. However, the original SO-DTA formulation is based on single destination. Kalafatas and Peeta [62] suggest that in the evacuation problem, where all destinations are equivalent, a single super-destination cell can be added and connected to all destination cells. Destination cells and the connectors to super-destination are assigned infinite capacity so that there will be no congestion at the destination cells. This suggestion is adopted in the current setup.

The average evacuation speeds of the vehicles are assumed to be 30 mph. The cell length is set to be 5 miles. Following the requirement of the CTM that a vehicle can traverse at most one cell in one time interval, the time interval is set to be 10 minutes and loading is also performed for each 10 minute interval. Following the Highway Capacity Manual the maximum flow rate is set to be 2160 vehicles per hour per lane and about 150 vehicles are assumed to fit 1 mile road segment. The cells on Garden State Parkway (cell#1, 11, 12, 13, 14, 15, 16) have 4 lanes, whereas the other roads have 3 lanes. Overall network features and cell physical properties are shown in Table 5.1.
<table>
<thead>
<tr>
<th>Cell#</th>
<th># of Lanes</th>
<th>Max Flow (veh/τ/ln)</th>
<th>Physical Capacity, $N_i$ (veh / mile)</th>
<th>Cell Length (miles)</th>
<th>Speed (mph)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4,5,6,7,20,21,22,23,24</td>
<td>3</td>
<td>1080</td>
<td>450</td>
<td>5</td>
<td>30</td>
</tr>
<tr>
<td>11,12,13,14,15,16</td>
<td>4</td>
<td>1440</td>
<td>600</td>
<td>5</td>
<td>30</td>
</tr>
</tbody>
</table>

$\tau$: Time interval = 10 minutes,

For network loading, S-curve is used following state-of-the-practice in evacuation modeling. Mathematical representation of S-curve is as follows:

$$P(t) = \frac{1}{\alpha + \exp[-\alpha(t-H)]}$$

(25)

where $P(t)$ is the cumulative percentage of the total trips generated at time $t$. The “$\alpha$” parameter represents the response of the public to the disaster and alters the slope of the cumulative traffic-loading curve. $H$ is the half loading time; the time at which half of the vehicles in the system have been loaded onto the highway network. $H$ defines the midpoint of the loading curve and can be varied by the planner according to disaster characteristics. The loading parameter choice adjusts the overall performance thus the parameters are kept fixed during all analyses. Specifically, loading parameters are set as $\alpha=0.01$ and $H=6$. According to census data, there are 42148 households in Cape May area and assuming 1 departure from each household, approximately half of the evacuations (21000) are generated from 3 concentrated sources representing the resident and tourist population along the shore.

5.1.1 Case Studies

There are 2 basic issues that are studied. First issue is the existence/necessity of a shelter at one of the destinations. Second issue is the effect of flood probability, which
may cause decrease in roadway capacity, and eventually change the “favorable” shelter locations. In both cases, capacity of the shelters are under consideration since deciding the location of a shelter is not enough without knowing the number of people that may use that shelter. Maintaining a shelter is further complicated because of emergency supply and response personnel requirements.

Let the problem be the elimination of one shelter out of three shown in Figure 5.1 because of supply logistics and possible problems in finding sufficient number emergency response personnel that can staff all three shelters. However, let’s also assume that there is a flood risk in the area, especially near the shore, which may cause a specific link to lose part or all of its capacity and consequently alter the shelter selection plan.

First, cell capacities are assumed to be deterministic and constant. Then, the network is analyzed for all possible couples of shelters by eliminating the third one at each iteration. Same procedure is employed but this time with capacity loss probabilities for specifically chosen links. The independence of flood probabilities of cells is again assumed, while solving the simple network probabilistic assignment problem.

Case-1

For this case, Cells #20, #21 and #22 represent the roadway which is near the shore and covered with water creeks that can be visually seen from aerial photos. Probabilities of the number of lanes that are operational are set to be 0.3, 0.45, 0.20, 0.05 for 0, 1, 2 and 3 lanes respectively for cell#20 and #21. This distribution represents a severe flooding where the fully operational and 1 lane loss probabilities only sum up to
Cell#22 is assigned probabilities of 0.20, 0.30, 0.45 and 0.05 for 0, 1, 2 and 3 operational lanes, which represent less severe flooding conditions compared to other cells with flood risk. These cells are chosen since the road segments that are represented with these cells are close to the shore and lie in a water-rich area as well. All other cells are assigned fixed, deterministic capacities throughout the evacuation.

First, a complete analysis with all shelters is performed to compare the average evacuation travel time and needed shelter capacities under best conditions. Then each shelter shown in Figure 5.2 are eliminated one-by-one, and the increase travel times and shelter capacity requirements are compared with the complete network where all the shelters are operational. The same procedure is applied for probabilistic road capacity formulation and results are given in Table 5.2. Please note that for the probabilistic assignment, \( p \) is set to be 0.75. This \( p \) value results in pLEPs which correspond to 1 lane flooding (in other words 2 operational lanes) for all cells with flood risk. Also note that the base scenario is chosen to be the deterministic case and all other performance values are compared with the base scenario.

<table>
<thead>
<tr>
<th>Abandoned Shelter</th>
<th>ATT(^*) (mins)</th>
<th>ATT Change</th>
<th>Needed Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Operational</td>
<td>66 (CT(=)440)</td>
<td>78 (CT=450)</td>
<td>n/a</td>
</tr>
<tr>
<td></td>
<td>S#1</td>
<td>S#2</td>
<td>S#3</td>
</tr>
<tr>
<td>S#1</td>
<td>127 (CT=550)</td>
<td>169 (CT=640)</td>
<td>92%</td>
</tr>
<tr>
<td></td>
<td>S#1 = 6943</td>
<td>S#2 = 7000</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S#2 = 7057</td>
<td>S#3 = 5671</td>
<td></td>
</tr>
<tr>
<td>S#2</td>
<td>126 (CT=550)</td>
<td>169 (CT=640)</td>
<td>91%</td>
</tr>
<tr>
<td></td>
<td>S#1 =10369</td>
<td>S#2 =12426</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S#3 =10531</td>
<td>S#3 = 8574</td>
<td></td>
</tr>
<tr>
<td>S#3</td>
<td>128 (CT=550)</td>
<td>128 (CT=550)</td>
<td>94%</td>
</tr>
<tr>
<td></td>
<td>S#1 = 10318</td>
<td>S#2 = 10280</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S#2 = 10682</td>
<td>S#2 =10720</td>
<td></td>
</tr>
</tbody>
</table>

* ATT: Average Travel Time, ** CT: Clearance Time
As seen in Table 5.2, for the deterministic case, all the shelters appear to be equal in terms of overall evacuation performance and capacity requirements. The increases in clearance times, average travel times and capacities are equal or very close.

However, when the analysis is done with predetermined flood probabilities, the absence of shelter#1 makes a big difference in evacuation performance. Average travel time increases by 156% whereas the increase in deterministic case is 92%. Also compared to absence of other shelters, shelter#1 is distinguished as the most vital shelter. There is also another point that is of importance other than the location or absence of the shelter. Even if one assumes that all shelters will remain open, probabilistic analysis suggests different capacities for shelters. Results show that the number of evacuees in the shelters will differ by 6%, 10%, -20% for shelter#1, #2 and #3 respectively. These changes are equal up to 1386 evacuees, for instance for shelter 3. If the shelter maintenance aspects are considered, e.g. food-water supply, medical facilities, this difference can change evacuation plans.

**Case-2**

For Case-1, it should be noted that the cells, which have flood risk, are the ones near the shore and affect only the evacuees that travel from origin#3 to shelter#3. However, the results are still significant in terms of the impact of the probabilistic analysis on the overall picture. For Case-2, same analysis that is presented in Table 5.2 is repeated by assigning a flood probability to an additional cell. One can also think of this proposed probabilistic capacity decrease as a result of the probability of an accident on the road instead of possible flooding. This perspective may lead us to use probabilistic analysis for links that do not have a major flood risk but high incident risks instead. For
this purpose, cell#23 is assigned a capacity decrease probability. Cell#23 connects shelter#1 to all origins, and is an important cell. The probability distribution, which is same as cell#22 (0.20, 0.30, 0.45, and 0.05 for 0, 1, 2 and 3 operational lanes respectively) is assigned to cell#23. The results of Case-1 and Case-2 are given in Table 5.3. Note that all the increase/decrease comparisons are based on the deterministic base scenario in which all cell capacities are deterministic and all shelters are operational. Same $p$ value as in case-2 is used (0.75) and this again corresponds to 1 lane closure for all cells with flood risk.

Table 5.3 Results for Case-1 and Case-2

<table>
<thead>
<tr>
<th>Abandoned Shelter</th>
<th>ATT* (mins)</th>
<th>ATT Change</th>
<th>Needed Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3 Flooded Cells</td>
<td>4 Flooded Cells</td>
<td>3 Flooded Cells</td>
</tr>
<tr>
<td>All Operational</td>
<td>78 (CT**=450)</td>
<td>96 (CT=490)</td>
<td>24%</td>
</tr>
<tr>
<td></td>
<td>S#1 = 7585</td>
<td>S#2 = 7744</td>
<td>S#3 = 5671</td>
</tr>
<tr>
<td></td>
<td>S#1 = 6006</td>
<td>S#2 = 8703</td>
<td>S#3 = 6291</td>
</tr>
<tr>
<td>S#1</td>
<td>169 (CT=640)</td>
<td>169 (CT=640)</td>
<td>156%</td>
</tr>
<tr>
<td></td>
<td>S#2 = 12426</td>
<td>S#3 = 8574</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S#2 = 12430</td>
<td>S#3 = 8570</td>
<td></td>
</tr>
<tr>
<td>S#2</td>
<td>169 (CT=640)</td>
<td>234 (CT=800)</td>
<td>156%</td>
</tr>
<tr>
<td></td>
<td>S#1 =13399</td>
<td>S#3 =7601</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S#1 =10370</td>
<td>S#3 =10630</td>
<td></td>
</tr>
<tr>
<td>S#3</td>
<td>128 (CT=550)</td>
<td>170 (CT=640)</td>
<td>94%</td>
</tr>
<tr>
<td></td>
<td>S#1 =10280</td>
<td>S#2 =10720</td>
<td></td>
</tr>
<tr>
<td></td>
<td>S#1 =8315</td>
<td>S#2 =12685</td>
<td></td>
</tr>
</tbody>
</table>

* ATT: Average Travel Time, ** CT: Clearance Time

As seen in Table 5.3 addition of another cell with flood probability alters the overall network performance. For example, the required shelter capacities, when all shelters are operational, change with respect to both deterministic case and Case-1. Shelter#2 receives 959 more evacuees than it did in Case-1, and 1703 more compared to the deterministic case. Demand for shelter#3 increases by 620 compared to Case-1, nevertheless it still stays under the deterministic case demand. For Case-2 shelter#1 demand drops below deterministic case by 937 which was above the deterministic
capacity prediction by 642 in Case-1. This shows that not only probabilistic approach changes the overall results, but also a complete probability estimates of flooding for all the cells in the network is essential. Since the network capacities are fully utilized during the evacuation, any change in capacity, especially at merging cells can alter the flows considerably.

The addition of a new flood risk cell also changes the importance ranking of the shelters. In the deterministic case, absence of any of the shelters results more or less in the same overall consequence. In Case-1, shelters #1 and #2 are found to affect the performance of evacuation more than shelter#3. In Case-2, shelter#2 is found to be the most vital shelter and the absence of shelters #1 and #3 are found to have almost the same impact on ATT. In terms of capacity requirements in case of an abandoned shelter, the needed capacity for a shelter can be up to 3029 evacuees. This change is equal to relocating of almost 15% of the total evacuees in Cape May County to operational shelters.

5.1.2 Discussion of Results

Findings show that accounting for flood probabilities, even for links that are not used by all evacuees, can change the system-optimal flows and performance measures, as well as the favorable shelter locations and capacity requirements. Two case studies show that a complete flood risk analysis is also necessary because any new flood probability assignment to a link in an already congested network alters the evacuation pattern considerably. Since shelter allocation is not only building the shelter but also maintaining it, these kinds of shelter allocation and capacity determination models are not sufficient
on their own. Other emergency management issues such as medical equipment and personnel, food and water supply, energy supply etc. should also be considered. However, these issues can be dealt efficiently only if planners employ realistic models such as the proposed stochastic SO DTA model which not only captures time-dependent traffic flows but also various stochasticities due the events causing the emergency situation. As shown in this analysis, when flooding risk of certain links are incorporated into the model, the demand for shelters changed significantly (highest change being at shelter#2) compared with the predictions of the deterministic model. Thus, if the planners consider the predictions of the deterministic model, they face the risk of not having sufficient food, medicine and other emergency supply in shelter#2. This kind of inefficient emergency planning has already created post-disaster problems in case of major disasters such as Katrina and Tsunami in South East Asia. These recent disasters and post–disaster conditions have only increased the need for better and more realistic planning models along the possible improvements suggested in this dissertation.

5.2 Numerical Example-2: Simultaneous Use of Demand and Capacity Constraints

For the second numerical example for simultaneous use of demand and capacity constraints, a test network of 3 origins and 3 destinations is used. Is it assumed that total of 54,000 vehicles depart equally from 3 sources (=18000 vehicles from each source). The evacuees are assumed to be loaded onto the network following S-curve for 3 hours and and α=0.05 is used. As the background traffic, the network is assumed to have vehicles at the level of 50% of its physical capacity already in the network.
The cell network used for the analysis is given in Figure 5.3, along with Table 5.4 showing the time-independent parameters of the CTM network. For the numerical example representing congested traffic, $v/w$ is assumed to be 5.

![Figure 5.3 Cell Representation Of The Test Network](image)

<table>
<thead>
<tr>
<th>Cell #</th>
<th>Source/Destination</th>
<th>1-31</th>
<th>32-42</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Free Flow Speed (mil/h)</strong></td>
<td>-</td>
<td>70</td>
<td>35</td>
</tr>
<tr>
<td><strong>Cell Length (ft)</strong></td>
<td>-</td>
<td>3000</td>
<td>1500</td>
</tr>
<tr>
<td><strong>Number of Lanes</strong></td>
<td>-</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td><strong>Max Flow Capacity (veh/hr/lane)</strong></td>
<td>Infinite</td>
<td>2400</td>
<td>1800</td>
</tr>
<tr>
<td><strong>Max Cell Flow (veh/time step)</strong></td>
<td>Infinite</td>
<td>60</td>
<td>45</td>
</tr>
<tr>
<td><strong>Max Cell Physical Capacity (vehicles/lane)</strong></td>
<td>Infinite</td>
<td>316</td>
<td>108</td>
</tr>
<tr>
<td><strong>Ratio of v/w</strong></td>
<td>-</td>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>

As one of the important contributions of the current study, the demand profile is not assumed to be deterministic, but probabilistic. The randomness is introduced by assuming that the demand will follow a distribution with mean value based on the S-
curve demand values. Considering that the clock tick will be very small compared to the overall evacuation time, instead of assuming a different demand distribution for each time interval, the demand distribution will be subject to the same random variable for each half an hour time interval for total of 3 hours loading time. In other words, the demand profile will be linearized for every half hour period, yielding the same average total demand for the overall assignment. As shown in Figure 5.4, such an approach will consider an area of possible demand realizations rather than using the exact S-curve values. Regarding the probability distributions, the demand probabilities are discretized for each ±5% interval from the average value covering the range of ±15% deviations from the S-Curve estimations. The assigned probabilities can be found in Table 5.5.

![Figure 5.4 S-Curve and Probabilistic Loading Distribution Used In Numerical Example](image-url)
For the capacity distributions, the roadway capacities are defined for 10% intervals up to 100% of maximum (=deterministic) capacity. For the numerical example, cells 1-31, which represent major freeways with higher flow capacities carrying the majority of the evacuees are analyzed. Other connecting routes (cells 32-42) are not assigned any capacity probability, since they are infrequently used due to the topography of the network and their inclusion would not affect the results. The assigned probabilities can be found in Table 5.5. To present the model capabilities, probability distributions representing different levels of capacity reduction severity are used. Such a case can be realized based on the threat types, e.g. higher flood prone risk links during a hurricane, or links with more/weaker bridges that can fail after a seismic disaster, etc.

<table>
<thead>
<tr>
<th>Link #</th>
<th>Cells</th>
<th>10%</th>
<th>20%</th>
<th>30%</th>
<th>40%</th>
<th>50%</th>
<th>60%</th>
<th>70%</th>
<th>80%</th>
<th>90%</th>
<th>100%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1&amp;2</td>
<td>[1-5] &amp; [6-10]</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.05</td>
<td>0.15</td>
<td>0.25</td>
<td>0.35</td>
<td>0.15</td>
<td>0.05</td>
</tr>
<tr>
<td>3,4,5&amp;</td>
<td>6 [11-13], [14-16],</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.05</td>
<td>0.10</td>
<td>0.15</td>
<td>0.25</td>
<td>0.35</td>
<td>0.10</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>[17-19] &amp; [20-21]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7&amp;8</td>
<td>[22-24] &amp; [25-31]</td>
<td>0.00</td>
<td>0.00</td>
<td>0.05</td>
<td>0.10</td>
<td>0.25</td>
<td>0.30</td>
<td>0.20</td>
<td>0.10</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Probability of Divergence from the Estimated S-Curve Demand

<table>
<thead>
<tr>
<th></th>
<th>O1, O2, O3</th>
<th>-15%</th>
<th>-10%</th>
<th>-5%</th>
<th>0%</th>
<th>5%</th>
<th>10%</th>
<th>15%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0.075</td>
<td>0.125</td>
<td>0.175</td>
<td>0.25</td>
<td>0.175</td>
<td>0.125</td>
<td>0.075</td>
</tr>
</tbody>
</table>

The analysis aims to find out the changes in clearance and average travel times in the network, which are two important outcomes of an evacuation study. To draw conclusions about the use of probabilistic approach, apart from comparison of ICC and JCC approach, effect of different levels of reliability are also tested. For the problem, the
flow capacity probabilities are assumed to obey the same random variable for all assignment period. In other words, realized capacity is assumed prevail during whole assignment. On the other hand, the demand is assumed to obey a different random variable (based on the linearized S-Curve) at every half hour. Following the previous discussion on the joint probability calculations, the link capacity and demand changes are assumed to be independent. The results can be found in Table 5.6.

<table>
<thead>
<tr>
<th></th>
<th>JCC</th>
<th>ICC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$p_1=0.90$, $p_2=0.90$</td>
<td>$p_1=0.90$, $p_2=0.80$</td>
</tr>
<tr>
<td>Clearance Time</td>
<td>+27.7%</td>
<td>+27.7%</td>
</tr>
<tr>
<td>Average Travel</td>
<td>+141.2%</td>
<td>+155.5%</td>
</tr>
</tbody>
</table>

Essence of the proposed formulation lies in the rich interpretation possibilities of the results given in Table 5.6. For the demand constraint, higher $p_1$ value corresponds to higher assigned demand. Hence, a possible realized demand is less likely to exceed the assigned value, and the calculated evacuation performance measures are safe estimations for the real-life case. In other words, higher the $p_1$ value, the planner takes less risks in terms of having lower clearance and average travel time estimations than would possible be realized, and provide results more on the safer side. For the capacity constraints, higher $p_2$ corresponds to less likely capacity degradations hence the results are based on a higher system reliability for roadway capacities. For instance, the assigned flow capacities for $p_2=1$ would be the maximum capacity for the links since the transportation network is assumed to work without any capacity reduction. Regarding the difference of JCC and ICC approach, with ICC approach, reliability levels are assigned for each
demand and capacity constraint individually. On the other hand, JCC ensure the reliability for the set of demand and capacity constraints covering joint event space. More strict nature of ICC compared to JCC can be mathematically verified with Boole's inequality, and also reflected in analysis results. ICC approach predicts higher clearance and average travel times compared to JCC.

The modeler can use either approaches with different \( p \) values based on modeling needs. If possible exceeded capacity at each link is considered to be fatal, use of ICC would be a better choice, however the results will reflect local reliability enforcements instead of system reliability. Regarding the numerical results, it is clear that deterministic analysis estimates lower clearance and average travel times. However, the difference in clearance time is lower in magnitude compared to average travel time. Although frequently addressed in the literature, clearance time may be short of providing planning measures if not properly analyzed. Clearance time is defined as the last vehicle exiting the network and if the last vehicle hits the road considerably later than the other vehicles, then the clearance time is defined by this last vehicle exit of the system although majority of the vehicles arrive to safety earlier. In that respect, analysis of average travel time, which gives a measure of how long the vehicles stay in the danger zone before leaving the danger area and relates to the risk exposure. So, even the clearance times are close, like the results in Table 5.6 for different levels of \( p_1 \) and \( p_2 \), the risk exposure measures can be different. Assuming that the vehicles arriving at destination D1, D2 and D3 arrive their safe destinations (e.g shelters), or they represent the exit points from the danger area, Figure 5.5 shows the number of people reaching to safety for different analysis scenarios. As shown in Figure 5.5, deterministic results may underestimate the risk
exposure considerably, since the percentage of evacuation at certain time, or the time needed to evacuate a certain percentage shows significant discrepancies. Thinking in terms of emergency personnel needed for evacuation operations, such discrepancies may result in under utilization or lack of human resources during evacuation.

Overall, the proposed probabilistic approach incorporates the inherent stochastic nature of evacuation demand and roadway capacities into the evacuation modeling. The probabilistic SODTA model results are expected to be more realistic compared to deterministic approach results which most likely would not be realized during actual evacuation process. Regarding the interpretation of results, SODTA with probabilistic capacity and demand constraints gives flexibility to the planner in terms of reliability.
levels on the estimated evacuation performance measures such as clearance and average travel times. This is an important planning issue since inferences made for a highly probabilistic process cannot be provided as just point estimates of expected outcomes, but they should rather be accompanied by confidence intervals or reliability levels.

CHAPTER 6. CONCLUSIONS AND FUTURE RESEARCH

The literature on emergency evacuation and the analysis of real-world evacuation practices exhibit a discrepancy between the deterministic assumptions of main stream evacuation studies and what is actually experienced due to uncertainty in evacuation demand and network roadway capacities. The state-of-practice in evacuation planning either ignores the real-world uncertainties or attempts to address this problem by providing scenario-based solutions. This dissertation proposes an alternative solution that employs a closed form cell transmission model based dynamic traffic assignment formulation with probabilistic demand and capacity constraints. The major achievements and contributions of the dissertation can be summarized as follows:

1. A comprehensive review of evacuee behavior is done with the main objective of conducting a network-wide sensitivity analysis of evacuation performance measures (e.g. clearance time, average travel time) vis-a-vis different demand generation models for the first time in the literature. The results of the analysis show that the impact of demand uncertainty, especially coupled with capacity reduction can cause significant changes in the evacuation performance measures. The impacts of
uncertainty is found to be non-linear, hence it is not possible to make reliable inferences about a possible scenario based on the results of a deterministic or scenario analysis. These findings in turn prove the need for developing and solving probabilistic network models.

2. Based on the findings of the literature review regarding the necessity of a probabilistic evacuation model, cell transmission model based dynamic traffic assignment with probabilistic capacity and demand constraints is formulated. The novelty of the proposed formulation comes from the fact that the computed results are accompanied by a reliability measure rather than providing deterministic/scenario-based results. Real-life practices show that such deterministic models fall short of meeting the reality such as the Hurricane Katrina experience, hence the proposed formulation can be considered as a pioneering model for next generation of emergency evacuation modeling.

3. Although the literature includes probabilistic approaches for DTA, these works are in the area of network design problem and mainly focus on demand uncertainty [156, 149, 65]. Moreover, these studies employ individual chance constraint approach which provides link based reliability for the analysis. The formulation discussed in this dissertation distinguishes itself from previous works by incorporating demand and capacity uncertainty together and introducing joint chance constraint approach, which gives the modeler a better tool to assess the overall system reliability.

4. The numerical examples studied in this dissertation show that the probabilistic SO-DTA model results are more realistic compared to the results of the deterministic
models which most likely would not be experienced during actual evacuation process. The proposed SO-DTA with probabilistic capacity and demand constraints gives a certain flexibility to the planner in terms of considering reliability levels of the estimated evacuation performance measures such as clearance and average travel times. This is an important planning issue since inferences made for a highly probabilistic process cannot be provided as point estimates of outcomes, but they should rather be represented using confidence intervals and/or reliability levels. For instance a deterministic study would conclude that the clearance time of a city to be, say, 12 hours. On the other hand proposed model’s outcome would be, say 14 hours at 90% of the possible capacity and demand realizations. This type of probabilistic will help the decision maker to make a more informed decision.

6.1 Future Research

The mathematical model proposed in this dissertation is a planning model that predicts evacuation time estimates based on the given demand and capacity probability distributions. As for all DTA problems, the network size is a challenge since CTM based SO DTA has large number of constraints. However the LP structure of the formulation allows the use of computationally efficient LP solution methods provided in the literature. Due to scarcity – if ever exists – of real data for a rare event like evacuation, it is not possible to benchmark the model output with real-world data. Nevertheless, an immediate research direction is comparing simulation-based approaches with the proposed model in terms of computational efficiency and accuracy. Such a comparison will show the developed model’s power of using a closed-form approach as opposed to
multiple demand/capacity realization scenario analysis using a series of deterministic models.

Another research direction is the development of a framework for the use of the proposed model in a real-time setup. This can be achieved by updating the evacuation time estimated at certain intervals based on the real-time information obtained during the evacuation. For instance, hurricanes are known to change their track as they approach the mainland. Consequently the flood probabilities on the roadways as well as the status of an evacuation order – which affect the demand – also change in time. Since the proposed model outcomes are as good as the probability distributions, the changing environmental and emergency management actions can be fed into the model during the approach of the hurricane to the mainland. The updated results can be used for route guidance or shelter management purposes as illustrated in the second numerical example.

Another research direction lies in the generic nature of the proposed stochastic DTA model. Although the proposed formulation is developed within an emergency evacuation context, the use of the model is transferable to other transportation engineering problems, provided that the demand and capacity probability distributions can be identified. An example of such an application is emergency management. Incidents are probabilistic events and they have an impact on roadway capacity similar to a flood after hurricane or to roadway damage after an earthquake. Hence, the existing incident occurrence and duration models can provide a suitable input for the roadway capacity probability distributions. The ultimate aim of incident management policies is to reduce impacts of incidents on the roadway capacity. Thus, the proposed model can be
used to assess the network-wide effectiveness of several IM solutions and can be used to provide cost-benefit analysis of candidate policies.
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