ESSAYS ON LATE BIDDING IN INTERNET AUCTIONS

by

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A dissertation submitted to the
Graduate School—New Brunswick
Rutgers, The State University of New Jersey
in partial fulfillment of the requirements
for the degree of
Doctor of Philosophy
Graduate Program in Economics

Written under the direction of
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New Brunswick, New Jersey
January, 2011
Empirical studies have documented that buyers in internet auctions tend to bid immediately before the scheduled closing time. This practice of “late bidding” is often called “sniping”. My dissertation investigates this phenomenon from both theoretical and empirical aspects.

The first chapter of my dissertation studies a second-price independent private value auction with two risk-neutral buyers who can submit a single bid either early or late. Early bids are received by the auctioneer with probability one, but late bids are received with a probability less than one. I prove that there can exist a symmetric cutoff equilibrium in which high-value buyers always bid early and low-value buyers bid late as long as their opponent has not bid early. A sufficient condition is then identified for the existence of the cutoff equilibrium with late bidding. Finally, I illustrate the equilibrium with examples from the beta distribution on values.

The second chapter of my dissertation empirically investigates the impact of late bidding on the final price of internet auctions. Using a dataset collected from eBay, I estimate the time after which late bidding has a differential impact on the final price. The time is estimated as a structural break in the data using a method proposed by Bai (1997). The break point occurs approximately three minutes before the close of the auction. The empirical findings indicate that buyers who won auctions by bidding in the last three minutes were typically more experienced than other winners and that they won the auctions for used items at lower prices.
The third chapter of my dissertation extends the first chapter by introducing an uncertainty on the number of buyers through a Poisson arrival process. In addition, I investigate the impact of late bidding on the seller’s expected revenue and provide numerical comparative statics for the equilibrium.
Acknowledgements

I could not have accomplished my dissertation and the PhD degree without the guidance and support from an extraordinary group of people. Special thanks go to Professor Martin K. Perry for his role as my academic and professional mentor. Marty encouraged me to learn the nuances and beauties of economic research and taught me how to think as an economist. He spent countless hours on discussions about my research as well as reading drafts of my dissertation, which have greatly improved the clarity of my work.

It would have been impossible for me to write my dissertation without Professor Colin Campbell’s guidance. He spent hours listening to my immature ideas and offered plenty of insightful advice regarding the models, the proofs as well as the improvements on the presentation of the results.

It has been a great pleasure to work with Professor Roger Klein. He taught me the fundamentals of econometrics that were essential to my research and guided me through the empirical work throughout my years at Rutgers. I thank him for all his invaluable advice, patience and encouragement.

I acknowledge Professor Richard McLean and Roberto Burguet for their very careful notes and comments on my research.

Finally, I thank all of my professors, colleagues, the administrative assistants at Rutgers University, especially Dorothy Rinaldi. They made my graduate study at Rutgers enjoyable.

Last, but certainly not the least, I would like to thank my husband Xiaofeng Mi and my parents for their assistance and support during this long journey.
Dedication

To my husband and my parents
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Chapter 1
Introduction

The volume of online auction trades has exploded in recent years. According to InternetRetailer.com, merchandise sales on the largest online auction site eBay increased from $23.8 billion in 2004 to $59.3 billion in 2007, representing a 35.6% compound annual growth rate. This tremendous success of online auctions has stimulated a growing interest among economists. One reason for this interest is the enormous amount of field data recorded electronically from online auctions. Such data would enable researchers to empirically test the various theories of auctions and market design. Another reason is that many empirical regularities observed in online auctions seem to defy the expectations based on the existing models of auctions. Thus, researchers are motivated to search for new theoretical explanations for these regularities.

An especially interesting regularity reported in online auctions is that bidders strategically place their bids just seconds before the closing time even though the auction has been open for days. Such practice is called “late bidding” or “sniping”. A survey by Roth and Ockenfels (2002) suggests that late bids in eBay auctions have the risk of not being received by the sellers. Despite of this potential risk, late bidding is common among eBay buyers. This dissertation investigates “late bidding” from theoretical and empirical aspects attempting to suggest an explanation for this puzzle as well as to evaluate the consequence of late bidding.

The second chapter of my dissertation studies a second-price independent private value auction with two risk-neutral buyers who can submit a single bid either early or late. Early bids are received by the auctioneer with probability one, but late bids are received with a probability less than one. I prove that there can exist a symmetric cutoff equilibrium in which high-value buyers always bid early and low-value buyers bid late as long as their opponent has not bid early. A sufficient condition is then identified for the existence of the cutoff equilibrium with late bidding. Finally, I illustrate the equilibrium with examples from the beta distribution on
values.

The third chapter of my dissertation empirically investigates the impact of late bidding on the final price of internet auctions. Using a dataset collected from eBay, I estimate the time after which late bidding has a differential impact on the final price. The time is estimated as a structural break in the data using a method proposed by Bai (1997). The break point occurs approximately three minutes before the close of the auction. The empirical findings indicate that buyers who won auctions by bidding in the last three minutes were typically more experienced than other winners and that they won the auctions for used items at lower prices.

The fourth chapter of my dissertation extends the second chapter by introducing an uncertainty on the number of buyers through a Poisson arrival process. In addition, I investigate the impact of late bidding on the seller’s expected revenue and provide numerical comparative statics for the equilibrium.
Chapter 2
Late Bidding with Two Bidders

2.1 Introduction

Late bidding occurs when a bidder submits a bid as late as possible in an online auction with a fixed closing time. This phenomenon cannot be easily explained by the standard auction models, so researchers have recently developed various new theories to justify the rationality behind late bidding. Bajari and Hortacsu (2004) review the explanations for late bidding and classify them into a few categories. The first category is characterized in terms of learning or concealing information on the value of the good. For example, Bajari and Hortacsu (2003) develop a common value model in which all bidders bid late in the online auction to avoid providing other bidders with information about the common value. Rasmusen (2006) develops a private value model in which bidders have to pay a fixed fee to discover their private values. Late bidding can occur because bidders wish to economize on the costs of acquiring information about their private values. The second category is characterized by the fact that identical goods may be separately and simultaneously listed for sale. The ability to bid across auctions creates incentives for bidders to wait until the closing time in order to learn how prices develop across the auctions (Peters and Severinov, 2006; Wang, 2003; Anwar et. al., 2006; and Stryszowska 2005a, 2005b). The third category is characterized by situations in which late bidding is a best response to incremental bidding. In an auction with a fixed closing time, bidding near the close of the auction will not leave sufficient time for incremental bidders to respond. Therefore, a late bidder competing with an incremental bidder may win the auction at the initial low bid of the incremental bidder (Roth and Ockenfels 2002).

Roth and Ockenfels (2002) also offer a fourth explanation for late bidding based on tacit collusion by the bidders against the seller. They illustrate the idea with an example, in which
bidders can choose to bid either early or late. Since a late bid may not be successfully transmitted due to network traffic, there is a risk of bidding late. On the other hand, late bidding softens competition by probabilistically reducing the number of bidders. Our model is motivated by this explanation. Using a two-period model of an independent private value auction with two buyers, the paper proves that late bidding near the fixed closing time of an auction can be a rational strategy for both buyers, even when these late bids may not be received by the auction websites.

2.2 Model Setup

A single indivisible good is offered for sale in a second-price independent private value (IPV) auction with two risk neutral buyers. Each buyer knows his own value for the good. He also knows that the other buyer’s value is drawn from a distribution \( F(v) \) on the interval \([0, 1]\). Assume that \( F(v) \) is continuous and strictly increasing with \( F(0) = 0 \) and \( F(1) = 1 \). Let \( f(v) \) denote the density function of values.

The auction starts at time \( t = 0 \) and closes at time \( t = T \). If a buyer bids at time \( t < T \), his bid is received with probability 1. If a buyer bids at time \( t = T \), his bid is received with probability \( p < 1 \). To simplify the analysis, we make several assumptions. First, we assume that each buyer can submit a single bid at any time in the continuum \([0, T]\). Second, the seller sets no reserve price and imposes no restrictions on the bid increment.

2.3 A Cutoff Equilibrium

We consider a Bayesian equilibrium in which buyer \( i \), where \( i \in \{1, 2\} \), with value \( v_i \) uses the following cutoff strategy:

At time \( t < T \), buyer \( i \) submits a bid \( b_i = v_i \) if his value \( v_i \) is greater than his cutoff value \( v_i^* \) or his opponent buyer \( j \) bid at any time \( t' < t \). He does not submit a bid if \( v_i < v_i^* \) and buyer \( j \) has not bid. At time \( t = T \), buyer \( i \) attempts to bid \( b_i = v_i \) if he has not submitted his bid beforehand.

When both buyers follow this cutoff strategy, they will both bid their values early in \([0, T]\) unless \( v_i < v_i^* \) and \( v_j < v_j^* \), in which case they will both attempt to bid their values late.
at \( t = T \). Since bids submitted at \( t = T \) are not received with certainty, the buyer with the higher value does not always win the object. Thus, both the allocative efficiency and the seller’s revenue are affected.

We begin the discussion by comparing the benefit and the cost of bidding late for each buyer. Then, we prove that the cutoff values for the two buyers must be the same. The remaining discussion focuses on the existence of a pure-strategy equilibrium where both buyers use the same cutoff value. We identify a sufficient condition for the existence of this kind of cutoff equilibrium.

**Lemma 1.** If buyers follow the proposed cutoff strategy in equilibrium, they must use the same cutoff value i.e. \( v_1^* = v_2^* \).

**Proof.** Since \( v_i^* = 0 \) implies bidding early, we focus on \( v_i^* > 0 \). Suppose that buyer \( i \) uses a cutoff value \( v_i^* \) with \( F(v_i^*) > 0 \) and her opponent buyer \( j \) uses a cutoff value \( v_j^* > v_i^* \). Given buyer \( j \)'s strategy, buyer \( i \) with value \( v_i^* \) benefits from bidding late rather than early when buyer \( j \)'s value \( v_j \) is less than or equal to \( v_j^* \), buyer \( i \)'s bid is received, and buyer \( j \)'s late bid is not received. The benefit of bidding late for buyer \( i \) in this case is

\[
p (1 - p) \left\{ v_i^* \left[ F(v_j^*) - F(v_i^*) \right] + \int_{0}^{v_i^*} zdF(z) \right\}, \tag{2.1}
\]

where the first term in the brackets represents buyer \( i \)'s benefit when \( v_i^* < v_j < v_j^* \) and the second term represents her benefit when \( v_j < v_i^* \). Bidding late rather than early costs when buyer \( j \)'s value \( v_j \) is less than or equal to \( v_i^* \) and buyer \( i \)'s late bid is not received. The cost of bidding late for buyer \( i \) in this case is

\[
(1 - p) \int_{0}^{v_i^*} (v_i^* - z) dF(z). \tag{2.2}
\]

Given buyer \( i \)'s strategy, the benefit of bidding late rather than early for buyer \( j \) with value \( v_j^* \) is simply

\[
p (1 - p) \int_{0}^{v_j^*} zdF(z), \tag{2.3}
\]

while his cost is again

\[
(1 - p) \int_{0}^{v_j^*} (v_j^* - z) dF(z). \tag{2.4}
\]
Since \( v_j^* > v_i^* \), we can see from (2.1) and (2.3) that the benefit of bidding late is strictly smaller for buyer \( j \) with value \( v_j^* \) than for buyer \( i \) with value \( v_i^* \). Furthermore, we can also see from (2.2) and (2.4) the cost of bidding late is strictly higher for buyer \( j \) with value \( v_j^* \) than for buyer \( i \) with value \( v_i^* \). Buyer \( i \) with value \( v_i^* \) receives a greater benefit than buyer \( j \) with value \( v_j^* \), because each buyer receives the same benefit from bidding late when the opponent’s value is less than or equal to \( v_i^* \), but buyer \( i \) also receives an additional benefit when \( v_j \in (v_i^*, v_j^*) \). Therefore, the total benefit of bidding late for buyer \( i \) is greater than that for buyer \( j \). Buyer \( i \) with value \( v_i^* \) also has a lower cost of bidding late than buyer \( j \) with value \( v_j^* \), because buyer \( i \) only loses \( v_i^* \) rather than \( v_j^* \) when she would have won the good by bidding early but her late bid was not received. Therefore, it is impossible that both buyer \( i \) with value \( v_i^* \) and buyer \( j \) with value \( v_j^* \) are indifferent between bidding early and late. That is, the necessary condition for a cutoff equilibrium cannot hold with \( v_j^* > v_i^* \) where \( i, j \in \{1, 2\} \) and \( j \neq i \).

Lemma 1 implies that any equilibrium with the proposed cutoff strategy must have the property that \( v_1^* = v_2^* \). The equilibrium must be symmetric, despite of the fact that the buyer who makes the decision later during \([0, T]\) can observe the initial decision of his opponent. To understand the symmetry, suppose that buyer \( j \) can observe buyer \( i \)'s decision before deciding whether to bid early or late. While buyer \( j \) is able to infer whether buyer \( i \)'s value is above or below \( v_i^* \) from observing buyer \( i \)'s initial bid decision, buyer \( i \) effectively makes her initial decision under the assumption that \( v_j < v_i^* \), since otherwise she would have bid \( v_i \) early and her initial decision will not matter for buyer \( j \).

The remainder of this paper will focus on a symmetric cutoff value \( v_1^* = v_2^* = v^* \) and will identify a sufficient condition on the distribution function \( F(v) \) for the existence of a cutoff equilibrium with late bidding. The equilibrium outcome with late bidding emerges only when both buyers have values below the cutoff value \( v^* \). In this case, a buyer with value \( v < v^* \) prefers bidding late rather than early if his expected payoff from bidding late is higher than from bidding early, that is

\[
p(1-p)v + p^2 \int_0^v (v-z) d \frac{F(z)}{F(v^*)} > \int_0^v (v-z) d \frac{F(z)}{F(v^*)}.
\]

(2.5)
The left-hand side of the inequality (2.5) is the buyer’s expected payoff from bidding late given that his opponent’s value is below \( v^* \). In particular, the first term on the left-hand side is his
expected payoff when his late bid is received but his opponent’s late bid is not. The second term is his expected payoff when both his late bid and his opponent’s late bid are received but his bid is higher than his opponent’s bid. The right-hand side is the buyer’s expected payoff from bidding early given that his opponent’s value is below \( v^* \). The inequality (2.5) is thus the condition that buyers with low values prefer bidding late rather than early. This condition can be rewritten as

\[
\frac{p}{1 + p} v > \frac{1}{F(v^*)} \int_0^{v^*} F(z) \, dz, \quad \text{for } v < v^*. 
\]  

(2.6)

A necessary condition for the existence of a cutoff equilibrium with late bidding is that there exists a \( v^* \in (0, 1) \) solving the following equality

\[
\frac{p}{1 + p} v^* = \frac{1}{F(v^*)} \int_0^{v^*} F(z) \, dz. 
\]  

(2.7)

Since the difference between a buyer’s expected payoff from bidding early and that from bidding late is continuous in the buyer’s value, \( v^* \notin (0, 1) \) implies that either all types of buyers bid early or all types bid late.

The zero value \( v^* = 0 \) is a degenerate solution for (2.7), so we are interested in finding an interior cutoff \( v^* \in (0, 1) \) which satisfies (2.7) and for which the inequality (2.6) holds. If there exists an interior cutoff \( v^* \) such that the inequality

\[
\frac{p}{1 + p} v > \frac{1}{F(v)} \int_0^v F(z) \, dz \quad \text{for } v < v^* \]

is satisfied, then the inequality (2.6) must hold because \( F(v) < F(v^*) \) for \( v < v^* \). Therefore, it is sufficient to examine (2.8) together with (2.7).

Even if there exists no interior \( v^* \) which solves (2.7), late bidding can still arise in equilibrium if buyers with any value \( v \) prefer to wait in the early period. That is, if the following inequality holds for all \( 0 < v \leq 1 \), then \( v^* = 1 \) effectively

\[
\frac{p}{1 + p} v > \frac{1}{F(v)} \int_0^v F(z) \, dz, \quad \forall v \in (0, 1]. 
\]  

(2.9)

Considering these cases together, we have the following sufficient condition for late bidding to occur in equilibrium

\[
\frac{p}{1 + p} v > \frac{1}{F(v)} \int_0^v F(z) \, dz \quad \text{for } v \in (0, \varepsilon) 
\]  

(2.10)
for some $\varepsilon > 0$. This inequality guarantees that there exists a positive cutoff value $v^*$ such that late bidding can be an equilibrium strategy. The inequality (2.10) only guarantees that the smallest $v^*$ satisfying (2.7) constitutes a cutoff equilibrium, but there could exist a greater $v^{**}$ that satisfies (2.7) but violates (2.10).

Solving for $v^*$ analytically from (2.7) can be difficult. However, from the Intermediate Value Theorem, an interior cutoff value $v^* \in (0, 1)$ must exist if the following inequality holds together with (2.10)

$$\frac{1}{1 + p} > E(V) \quad (2.11)$$

where $E(V)$ is the expectation of the value. The inequality (2.11) guarantees the condition that the expected payoff from bidding early exceeds that from bidding late for a buyer with the highest value $v = 1$.

We can now identify a natural property on the distribution function for which condition (2.10) is satisfied.

**Proposition 1.** If $F(v)^p$ is strictly convex for $v \in (0, \varepsilon)$ for some $\varepsilon > 0$, a cutoff equilibrium with late bidding exists.

**Proof.** A sufficient condition for the existence of a cutoff equilibrium with late bidding is (2.10), which is equivalent to

$$\frac{p}{1 + p} > \frac{\int_0^v F(z) \, dz}{vF(v)} \quad \text{for } v \in (0, \varepsilon). \quad (2.12)$$

Furthermore, we have that

$$\int_0^v \frac{z^p}{v^{p+1}} \, dz = \frac{p}{1 + p} \quad \text{for all } v > 0. \quad (2.13)$$

Thus, substituting (2.13) into the left-hand side of (2.12) and re-expressing the right-hand side of (2.12) yields

$$\int_0^v \frac{z^p}{v^{p+1}} \, dz > \int_0^v \frac{v^p F(z)}{F(v)} \, dz. \quad (2.14)$$

The convexity of $F(z)^p$ ensures that $\frac{F(z)^p}{z}$ is monotonically increasing, that is, $\frac{F(z)^p}{z} < \frac{F(v)^p}{v}$ for $z < v$. The convexity of $F(z)^p$ in turn implies that $\frac{1}{z^p} > \frac{v^p F(z)}{F(v)}$ for $z < v$. This inequality in turn implies that condition (2.14) must be satisfied. Thus, (2.12) holds and the cutoff equilibrium with late bidding exists. $\square$
Note that $F(v)^p$ is strictly convex for $v \in (0, \varepsilon)$ implies that $F(v)$ is strictly convex in the neighborhood, since $0 < p < 1$. Equivalently, it suggests that the probability density function $f(v)$ is strictly increasing in the neighborhood $v \in (0, \varepsilon)$.

Although solving for $v^*$ analytically from (2.7) is difficult in general, the following proposition provides some intuition on the comparative statics of $v^*$.

**Proposition 2.** If for a given distribution function $F(v)$, there exists a cutoff equilibrium with an interior cutoff value $v_p^*$, then for any distribution function $G(v)$ which dominates $F(v)$ in terms of reversed hazard rate, i.e. $F(v)$ and $G(v)$ satisfy $\frac{g(v)}{G(v)} > \frac{f(v)}{F(v)}$ for all $v \in (0, 1)$, there must also exist a cutoff equilibrium with late bidding such that the cutoff value $v_G^* > v_p^*$.

**Proof.** Suppose that for a given distribution function $F(v)$, there exists a cutoff equilibrium with the interior cutoff $v_p^*$ defined as the value that satisfies (2.7) and (2.8). Then, we have

$$\frac{p}{1 + p}v > v - \frac{1}{F(v)} \int_0^v zf(z)dz \text{ for } v \in (0, v_p^*) \quad (2.15)$$

$$\frac{p}{1 + p}v_p^* = v_p^* - \frac{1}{F(v_p^*)} \int_0^{v_p^*}zf(z)dz \quad (2.16)$$

where $f(z)$ is the density function of the distribution $F(z)$. (2.15) and (2.16) follow from rewriting the right-hand side of (2.8) and (2.7), respectively.

Suppose that there exists another distribution $G(v)$ which dominates $F(v)$ in terms of reversed hazard rate, that is, $\frac{g(v)}{G(v)} > \frac{f(v)}{F(v)}$ for all $v \in (0, 1)$. Then, the following stochastic order holds

$$\frac{1}{G(v)} \int_0^v zg(z)dz > \frac{1}{F(v)} \int_0^v zf(z)dz, \text{ for all } v \in (0, 1). \quad (2.17)$$

The appendix explains that the reverse hazard rate dominance implies (2.17). That is, for every $v$, the expectation of a random draw from $G(\cdot)$ conditional on being less than or equal to $v$ is strictly greater than the expectation of a random draw from $F(\cdot)$ conditional on being less than or equal to $v$. Substituting (2.17) into (2.15) and (2.16), we have the following two conditions

$$\frac{p}{1 + p}v > v - \frac{1}{G(v)} \int_0^v zg(z)dz \text{ for } v \in (0, v_G^*) \quad (2.18)$$

$$\frac{p}{1 + p}v_p^* > v_p^* - \frac{1}{G(v_p^*)} \int_0^{v_p^*}zf(z)dz \quad (2.19)$$
which together imply that there must be a cutoff equilibrium with late bidding for the distribution \( G(\cdot) \). Furthermore, any cutoff \( v^*_G \) that constitutes such an equilibrium must be strictly greater than \( v^*_F \).

Informally, the stochastic order (2.17) implies that high draws under \( G(v) \) are more likely than under \( F(v) \). Since \( v^*_G > v^*_F \), Proposition 2 suggests that distributions weighted toward higher types are more conducive to a symmetric cutoff equilibrium with late bidding and with more types intending to bid late. To see the intuition, consider the cost and benefit of bidding late for buyer \( i \) with value \( v_i \) as a function of buyer \( j \)'s value \( v_j \). Suppose that \( v_j < v^* \) so that buyer \( i \)'s decision will determine whether both buyers bid early or late. If \( v_i > v_j \) and buyer \( j \)'s value \( v_j \) is relatively high, then buyer \( i \)'s benefit from bidding late will be large because when her late bid is received and buyer \( j \)'s late bid is not received, she will pay substantially less than if she bids early. If \( v_i < v_j \), buyer \( i \) also benefits from bidding late because she will win when she would not have won by bidding early. Furthermore, when buyer \( j \)'s value \( v_j \) is relatively high, the cost for buyer \( i \) to bid late is small, because her expected payoff from bidding early would have been small. In other words, the difference between buyer \( i \)'s expected payoff from bidding late and that from bidding early conditional on \( v_j \) is increasing in \( v_j \). Thus, a distribution that places more weight on higher \( v \) makes bidding late relatively more attractive for all types of buyers.

The sufficient condition for late bidding in Proposition 1 can be satisfied by many distribution functions. The next section shows that this sufficient condition is fulfilled by the beta distribution family over a wide range of parameter values.

### 2.4 Beta Distribution Family

Suppose the values are drawn from the beta distribution with parameter \( \alpha \) and \( \beta \).

\[
F(v; \alpha, \beta) = \frac{\int_0^v x^{\alpha-1} (1-x)^{\beta-1} \, dx}{\int_0^1 x^{\alpha-1} (1-x)^{\beta-1} \, dx} \quad \text{for } v \in [0, 1], \; \alpha > 0 \text{ and } \beta > 0
\]

If \( \alpha > \frac{1}{\beta} \), then the sufficient condition for late bidding in Proposition 1 is satisfied and a cutoff equilibrium with late bidding exists. The proof is as follows.
Proof. Begin by considering the second derivative of $F(v)^p$

$$\frac{\partial^2 F(v)^p}{\partial v^2} = pF(v)^{p-2} \left[ F(v) f'(v) - (1 - p) f(v)^2 \right].$$

In order for $F(v)^p$ to be strictly convex in the neighborhood $v \in (0, \varepsilon)$, the term in the brackets must be positive for $v \in (0, \varepsilon)$. Dividing this term by $f(v) F(v)$ yields

$$\frac{f'(v)}{f(v)} - (1 - p) \frac{f(v)}{F(v)} = \frac{d[\ln f(v)]}{dv} - (1 - p) \frac{f(v)}{F(v)} > 0 \text{ for } v \in (0, \varepsilon).$$

(2.18)

After substituting the beta distribution, the inequality (2.18) becomes

$$(\alpha - 1) - (\beta - 1) \frac{v}{1 - v} - (1 - p) \frac{v^\alpha (1 - v)^{\beta - 1}}{\int_0^v x^{\alpha-1} (1 - x)^{\beta-1} dx} > 0 \text{ for } v \in (0, \varepsilon).$$

In order for the above inequality to hold, the following must be true

$$\lim_{v \to 0^+} \frac{v^\alpha (1 - v)^{\beta - 1}}{\int_0^v x^{\alpha-1} (1 - x)^{\beta-1} dx} < \frac{\alpha - 1}{1 - p}.$$

Using L’Hôpital’s rule, we obtain

$$\lim_{v \to 0^+} \frac{\alpha v^{\alpha-1} (1 - v)^{\beta - 1} - (\beta - 1) v^\alpha (1 - v)^{\beta - 2}}{v^{\alpha-1} (1 - v)^{\beta-1}} < \frac{\alpha - 1}{1 - p}.$$

Cancelling $v^{\alpha-1} (1 - v)^{\beta - 2}$ in the numerator and the denominator on the left-hand side and noting that $\lim_{v \to 0^+} \frac{v}{1 - v} = 0$, we find that the left-hand side equals $\alpha$ and the inequality simplifies to

$$\alpha > \frac{1}{p}.$$ 

(2.19)

Therefore, if the parameter $\alpha > \frac{1}{p}$, then $F(v; \alpha, \beta)^p$ is strictly convex in the neighborhood $v \in (0, \varepsilon)$. That is, the sufficient condition in Proposition 1 is satisfied.

If an interior $v^*$ exists, it can be found by solving (2.7). In general, it is not easy to solve for $v^*$, but since the expectation for the beta distribution is $E(V) = \frac{\alpha}{\alpha + \beta}$, (2.11) together with (2.19) guarantees that if $\frac{1}{\alpha} < p < \frac{\beta}{\alpha}$, an interior $v^*$ must exist when $\beta > 1$.

Our first example shows that late bidding can occur in equilibrium even if there is no interior $v^*$ that solves (2.7).

Example 1. $\alpha > \frac{1}{p}$ and $\beta = 1$ (Power distribution)
The power distribution $F(v) = v^\alpha$ with $\alpha > \frac{1}{p}$ is a special case of the beta distribution with $\beta = 1$. It satisfies the sufficient condition in Proposition 1, because
\[
\frac{\partial^2 F(v)^p}{\partial v^2} = \alpha p (\alpha p - 1) v^{\alpha p - 2} > 0.
\]
Solving (2.7) does not give an interior $v^*$, but the following proves that this is the case when all types of buyers prefer to wait in the equilibrium.

Given $\alpha > \frac{1}{p}$, we have
\[
\frac{p}{p + 1} > \frac{1}{\alpha + 1}.
\]
Multiplying both sides by any $v \in (0, 1]$ yields
\[
\frac{p}{p + 1} v > \frac{1}{\alpha + 1} v^\alpha
\]
which can then be written as
\[
\frac{p}{1 + p} v > \frac{1}{F(v)} \int_0^v F(z) \, dz \text{ for } v \in (0, 1]
\]
for the power distribution $F(v) = v^\alpha$. This inequality is exactly the sufficient condition (2.9) for the special equilibrium where all types wait to bid late.

The following two examples show that late bidding can occur in equilibrium with an interior $v^*$ that solves (2.7).

**Example 2.** $\alpha = \beta = 2$ and $p > \frac{1}{2}$

When $\alpha = \beta = 2$ and $p > \frac{1}{2}$, (2.19) holds together with (2.11). Hence, an interior $v^*$ must exist. Substituting the distribution function
\[
F(v; 2, 2) = \frac{\int_0^v (x - x^2) \, dx}{\int_0^1 (x - x^2) \, dx} = 3v^2 - 2v^3
\]
into (2.7) and solving for $v^*$, we obtain
\[
v^*(p, 2, 2) = \frac{2 - 4p}{1 - 3p}
\]

**Example 3.** $\alpha = \beta = 3$ and $p > \frac{1}{3}$

When $\alpha = \beta = 3$ and $p > \frac{1}{3}$, (2.19) holds together with (2.11). Hence, an interior $v^*$ must exist. Substituting the distribution function
\[
F(v; 3, 3) = \frac{\int_0^v (x - x^2)^2 \, dx}{\int_0^1 (x - x^2)^2 \, dx} = 10v^3 - 15v^4 + 6v^5
\]
into (2.7) and solving for $v^*$, we obtain

$$v^* (p, 3, 3) = \frac{3 - 12p + \sqrt{-6p^2 + 8p - 1}}{2 (1 - 5p)}$$

Figure 2.1 shows the $v^*$ as a function of $p$ in Examples 2 and 3.

The change in $p$ has two affects. On one hand, a buyer’s cost of bidding late decreases as $p$ increases, because the probability of his own late bid being received is higher. On the other hand, his benefit of bidding late also decreases, because the probability of his opponent’s late bid being received is higher as well. As shown in Figure 2.1, the cutoff value $v^*$ increases in $p$ for both beta distributions $F (v; 2, 2)$ and $F (v; 3, 3)$. Therefore, for a buyer with value $0 < v < 1$, he eventually will prefer to bid late as $p$ approaches 1, because $v^*$ will exceed $v$ when $p$ is high enough. This relationship between $v^*$ and $p$ actually implies that although both the cost and the benefit of bidding late decreases when $p$ increases, the cost actually decreases faster than the benefit.

A general result for the comparative statics of $v^*$ follows from the monotone likelihood dominance. Consider two beta distributions $F (v; a_1, \beta_1)$ and $F (v; a_2, \beta_2)$ with $F (v; a_2, \beta_2)$ dominating $F (v; a_1, \beta_1)$ in terms of monotone likelihood. Let $f (v; a_1, \beta_1)$ and $f (v; a_2, \beta_2)$ denote their density functions respectively. Suppose that there exists a cutoff equilibrium with
an interior cutoff value $v_1^*$ for $F (v; a_1, \beta_1)$. The monotone likelihood dominance implies that the following ratio is increasing in $v$ for all $v \in (0, 1]$:

$$\frac{f(v; a_2, \beta_2)}{f(v; a_1, \beta_1)} = v^{a_2-a_1} (1 - v)^{\beta_2-\beta_1}$$

The corresponding first order condition suggests that

$$\frac{a_2 - a_1}{(\beta_2 - \beta_1) + (a_2 - a_1)} > v$$

which holds for all $v \in (0, 1]$ if and only if $a_2 \geq a_1$ and $\beta_2 \leq \beta_1$. That is, for any beta distribution $F (v; a_2, \beta_2)$ such that $a_2 \geq a_1$ and $\beta_2 \leq \beta_1$, there must also exist a cutoff equilibrium with late bidding such that the cutoff value $v_2^* > v_1^*$.

### 2.5 Conclusions

In this paper, I studied a second-price independent private value auction with two buyers. Focusing on the pure strategies that buyers always bid their values, I have proved that late bidding near the fixed closing time of an auction can be a rational strategy for buyers, even when these late bids may not be received by the auction websites. In particular, there can exist a symmetric cutoff equilibrium in which high-value buyers always bid early and low-value buyers bid late as long as their opponent has not bid early. The examples from the beta distribution family illustrate the cutoff equilibrium and the comparative statics of $v^*$.

### 2.6 Appendix

This appendix proves that $G (v)$ dominates $F (v)$ in terms of reversed hazard rate implies the inequality (2.17).

Let $V_F$ and $V_G$ denote the random variables with the distribution functions $F (\cdot)$ and $G (\cdot)$ respectively. Define the conditional distribution functions $F (z | V_F < v) \equiv \frac{F(z)}{F(v)}$ and $G (z | V_G < v) \equiv \frac{G(z)}{G(v)}$. Furthermore, let $f (z | V_F < v)$ and $g (z | V_G < v)$ denote the corresponding conditional density functions, that is, $f (z | V_F < v) \equiv \frac{dF(z | V_F < v)}{dz}$ and $g (z | V_G < v) \equiv \frac{dG(z | V_G < v)}{dz}$.

Since $G (\cdot)$ dominates $F (\cdot)$ in terms of reversed hazard rate, according to the definition, we have $\frac{g(z)}{G(z)} > \frac{f(z)}{F(z)}$, which can be rewritten as $\frac{g(z | V_G < v)}{G(z | V_G < v)} > \frac{f(z | V_F < v)}{F(z | V_F < v)}$. That is, the conditional
distribution $G(z \mid V_G < v)$ dominates $F(z \mid V_F < v)$ in terms of the reversed hazard rate. Since the reversed hazard rate dominance implies the first order dominance, we have

$$G(z \mid V_G < v) < F(z \mid V_F < v)$$

Integrating both sides over $z$ for $z \in (0, v)$, we get $\int_0^v G(z \mid V_G < v) \, dz < \int_0^v F(z \mid V_F < v) \, dz$, which is equivalent to (2.17). To see the equivalence, one can move $\frac{1}{G(v)}$ and $\frac{1}{F(v)}$ into the integration and apply the integration by parts to both sides of (2.17).
Chapter 3
Price Impact of Late Bidding

3.1 Introduction

The volume of online auction trades has exploded in recent years. According to InternetRetailer.com, merchandise sales on the largest online auction site eBay increased from $23.8 billion in 2004 to $59.3 billion in 2007, representing a 35.6% compound annual growth rate. This tremendous success of online auctions has stimulated a growing interest among economists. One reason for this interest is the enormous amount of field data recorded electronically from online auctions. Such data would enable researchers to empirically test the various theories of auctions and market design. Another reason is that many empirical regularities observed in online auctions seem to defy the expectations based on the existing models of auctions. Thus, online auction data provides an opportunity to test new theoretical explanations for these regularities.

An especially interesting regularity reported in online auctions is that bidders strategically submit their bids just seconds before the closing time of the auction. These late bids occur even though the auction has been open for days. Ockenfels and Roth (2003), Wilcox (2000), Bajari and Hortacsu (2003), and Schindler (2003) have observed late bids in their datasets and considered the possible explanations and implications. The practice of placing a winning bid at the last possible moment is often called “sniping”. In simple settings, Ockenfels and Roth (2006) have shown that sniping affects the final price, hence the seller’s expected revenue, and the allocative efficiency in online auctions. However, most empirical papers studying various aspects of online auctions either ignore the presence of late bidding or arbitrarily select a time to define when late bidding has begun.

1The term “sniping” is derived from the action of a military sniper who targets commanders in the opposing army. The term sniper is derived in turn from the species of wading birds which are difficult to hunt or catch.
This paper contributes to the empirical literature by proposing a way of estimating the time at which late bidding begins to have an impact on the final price. The methodology is based on the hypothesis that the price model for auctions won by early bids may differ from the price model for auctions won by late bids. This paper estimates and compares the two price models and highlights the resulting empirical findings. Using a dataset collected from eBay on automobile Global Positioning System (GPS) navigators, I estimate the time after which late bidding has a differential impact on the final price. The time is estimated as a structural break in the data with a method proposed by Bai (1997). The break point occurs approximately three minutes before the close of the auction. The empirical findings indicate that buyers who won auctions by bidding in the last three minutes were typically more experienced than other winners and that they won the auctions for used items at lower prices by bidding late in those auctions.

This paper does not estimate the underlying distribution of bidders’ values, which represents the demand for the goods. However, the methodology can be adopted by researchers who are interested in the structural estimation of demand using auction data in which a significant fraction of the auctions are won by late bidders. If the estimation procedure does not account for the effect of late bidding, the empirical results will incorrectly estimate the seller’s revenues in auctions that close without late bidding, thus creating a bias in the estimation of the demand. The method proposed in this paper allows researchers to decide which auctions are affected by late bidding and thus should be excluded from their structural estimation of the demand.

The rest of this chapter is organized as follows. Section 3.2 reviews the related literature concerning online auctions and late bidding. Section 3.3 gives a brief introduction of eBay auctions. Section 3.4 describes the dataset. Section 3.5 presents the regression models and the variables. Section 3.6 explains the methodology to estimate the time for late bidding. Section 3.7 summarizes the estimation results. Section 3.8 provides the conclusions.

### 3.2 Literature

eBay auctions have the attributes of both an English auction and a second-price sealed-bid auction. This section discusses the attributes and reviews the standard theories on these two
auction formats within the independent private value framework. The conflicts between the theoretical predictions and the practice of eBay bidders then lead to a discussion of the late bidding literature.

Online auctions often operate with a second-price format, where the bidder with the highest bid wins the good and pays a price equal to the second highest bid plus the minimum bid increment. The bidding process is similar to an English or oral ascending auction, where each bidder observes the current highest bid and decides whether to outbid it by at least the minimum bid increment. Although the button auction interpretation studied by Milgrom and Weber (1982) dominates the literature of open ascending auctions, Haile and Tamer (2003) have argued that bidding data do not provide empirical support for the button auction interpretation. In a button auction, bidders indicate their willingness to pay by holding down their buttons as the price of the good is increased continuously and exogenously. In the dominant strategy equilibrium, each bidder exits irrevocably by releasing his button when either the price reaches his value for the good or all of his opponents have exited. As a result, the observed exit price reveals the value of the bidder who exits at that point. The only exception is the winning bidder, who stops bidding as soon as all the other bidders have exited.

In contrast, researchers studying English auctions have observed that the last bid of a bidder is not a good measure of his value. For example, a bidder with a relatively high value may appear to exit at a low price if his opponents bid the price beyond his value before he can submit an additional bid. The use of a minimum increment in the auction and the prevalence of jump bidding beyond the minimum increment also create difficulties in measuring a bidder’s value by his last bid. Instead of strictly following the equilibrium outcomes predicted by the basic theoretical models, Haile and Tamer (2003) make only two assumptions: (i) bidders do not bid more than they are willing to pay and (ii) bidders do not allow an opponent to win at a price they are willing to match. These two weak assumptions allow them to identify the upper and lower bounds for the distribution of bidders’ values and to infer the bounds for the optimal reserve price. The remainder of this paper refers to the bidding process which satisfies these

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2My discussion is restricted to the independent private value framework for two reasons. First, the prices of the goods tend to decline over time, so there is very little resale value. Second, the goods are mostly purchased for personal use. Third, buyers are likely to be well informed about the retail prices of the goods through internet searching.
two assumptions as “competitive bidding”.

Many online auction sites offer a proxy bidding service so that bidders do not need to remain vigilant throughout the auctions. A bidder can set his maximum bid through the proxy and let the software agent bid incrementally on his behalf. The maximum bid is only known to this bidder and can only be modified by him while the auction is still in progress. When outbid by a competitor, the bidder will be notified through email. If proxy bidding were used by bidders in the way that eBay recommends, the online auctions would be similar to the second-price sealed-bid auctions. Vickrey (1961) has shown that it is a weakly dominant strategy for bidders to bid their values in the independent private value auctions with this format. Thus, bidders should set their maximum bids equal to their values. Consequently, it is the second highest value that determines the payment of the winner.

A similar result is proved by Haile and Tamer (2003) for independent private value English auctions where their assumptions (i) and (ii) hold and the minimum bid increment is small. According to the above theories, late bidding would occur only when the bidder with the highest value is the last to arrive at the auction. In this case, there should be no systematic price difference between the auctions won by early bids and those won by late bids. On the contrary, finding such a price difference would be evidence that the predictions of standard auction theories cannot be naively applied to the online auctions. The methodology in my paper is based on the premise that the final prices in the auctions won by early bids are formed differently from the final prices in the auctions won by late bids, so the corresponding econometric models should be different as well.

Since late bidding is not readily explained by the standard auction theories, new models that can explain this phenomenon have been proposed by researchers. One paper which explores for an answer to late bidding is Roth and Ockenfels (2002). They first note that an important difference between online auctions is the rule that governs the close of the auction. For example, eBay uses fixed deadlines to close their auctions, but Amazon automatically extends the close of the auction beyond the scheduled closing time until a specified number of minutes have passed without a bid. The authors argue that bidders in fixed deadline auctions have an incentive to bid late, even though late bids run the risk of not being recorded by the online auction sites due to network congestion.
In their subsequent paper, Ockenfels and Roth (2006) provide an example of the existence of a perfect Bayesian equilibrium in which bidders only bid at the last moment even in auctions with private values. The intuition behind this late bidding equilibrium is that bidding early can trigger a bidding war when there is time for other bidders to react. The bidding war will raise the expected final price paid by the ultimate winner. On the contrary, mutual delay until the last moment may increase the expected payoff of all bidders by probabilistically suppressing some bids, and thus soften the competition. This kind of tacit collusion among bidders cannot be achieved easily in auctions with the closings that extend automatically. In addition, the authors prove that late bidding may also arise in out-of-equilibrium behavior as a best response to bidders who bid incrementally. These arguments are supported by the empirical finding that sniping is more prevalent in eBay auctions than in Amazon auctions and by the experimental results in Ariely et. al. (2005). Relative to the equilibrium in which every bidder bids his true value early, the late bidding equilibrium yields higher expected payoffs for the bidders but lower revenues for the sellers.

Haile and Tamer (2003) have recognized that their assumption (ii) is violated in Internet auctions with fixed closing times, where it may be possible for a bidder to submit a winning bid at the last moment, leaving no time for an opponent to respond. Hence, applying their methodology to online auctions without considering late bidding can also be problematic. If auction theory offered no specific prediction on the magnitude of the expected price in the auctions won by late bids, a natural solution to this problem would be to eliminate those auctions won by late bids from the sample. The methodology proposed in this paper can be applied for that purpose. However, if auction theory had a specific prediction on the expected price in auctions won by late bids, the empirical work could incorporate those predictions and take advantage of all the data.

3.3 eBay Auctions

Sellers on eBay can designate whether they want their goods listed for sale on an auction or listed at a fixed price.\(^3\) A listing contains a description of the good with photographs. In the

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\(^3\)The seller can add an option called “Buy-It-Now” to his auction. Buy-It-Now gives buyers an opportunity to purchase at a fixed price before the auction closes. This buy-it-now price is different from the price in a fixed price
appendix, I provide an example of an eBay listing including the auction page and the bidding history page (Figure 3.3 in the appendix). All listings remain publicly available on eBay’s website for at least one month after the auction closes. This is an eBay service which allows both buyers and sellers to research the recent prices at which similar goods were sold on eBay.

When creating an auction listing, a seller can choose a starting bid and an optional secret reserve price. If a reserve price is set and the highest bid remains below the reserve price, the seller need not sell the good to the high bidder. The seller can schedule his listing to start at a specific date and time. The seller can also choose the duration of the auction to be 1, 3, 5, 7, or 10 days. Since eBay charges an extra fee for 10-day auctions, few sellers choose the 10-day duration. In order to sell multiple units of a good on eBay, the seller can sell all units in one listing or create multiple listings with one listing for each unit. The latter practice is more common.

At any point of time during the auction, the current price of the good is usually set at one minimum bid increment above the current second highest bid. If one minimum bid increment above the second highest bid exceeds the current highest bid, the current price is set equal to the current highest bid. The minimum bid increment is specified by eBay and varies according to the range of the current price. Most eBay sellers choose to disclose the bidding history, which includes the magnitude of each bid, when the bid was made, which user ID made the bid (see Figure 3.3 in the appendix). The auction closing date and time is also announced to the buyers. Potential buyers can bid as many times as they want and at any time before the auction closing time. Bids can be placed by the buyer himself or by the proxy service provided by eBay.

listing, because it is available only for a limited time. For example, the buy-it-now option is usually available at the beginning of the auction until someone bids on the good or the reserve price is exceeded. Once this buy-it-now option expires, the auction proceeds and no bidder can identify whether a buy-it-now price had ever existed for the auction. The buy-it-now option could potentially decrease a seller’s revenue. When exercised, the buy-it-now option eliminates the possibility of higher prices that could be reached through competitive bidding. Furthermore, when exercised by a buyer with a valuation below the highest valuation, the buy-it-now option leads to an allocative inefficiency. Unfortunately, the data collection procedure for this GPS receivers dataset could not collect any information on the existence or the amount of a buy-it-now price.

4The bid increment is $0.05, when the current price increases from $0.01 to $0.99; $0.25, when the current price increases from $1.00 to $4.99 and so on. The largest bid increment in my dataset is $10.00, when the current price increases from $500.00 to $999.99.

5In 2007, eBay started to disguise the losing bidders’ user IDs in the auctions of expensive goods. Two random characters from the real user ID are chosen to compose a disguised ID, for example, a***b. The purpose is to prevent scammers from contacting losing bidders with fake second chance offers. Therefore, it is now impossible to identify whether a buyer is bidding on multiple auctions in our dataset. Within an auction, the disguised ID of a buyer remains the same, so multiple bids placed by one buyer can still be identified.
eBay has a reputation mechanism designed to help buyers and sellers determine the possibility of default by either side of a transaction. A typical default by a seller would be the failure to deliver the good in the described condition, while a typical default by a buyer would be the failure to pay the final price. For each transaction, the buyer and the seller can record feedback about each other. Feedback can consist of a positive, neutral or negative rating and a short comment. These ratings are used to calculate a feedback score, which is displayed in parentheses next to every user ID. A positive rating increases the feedback score by one point, and a negative rating decreases the feedback score by one point. A neutral rating leaves the feedback score the same. The detailed feedback profile of each buyer or seller can be viewed by clicking on his user ID. This profile publishes all the comments and the exact number of positive, negative, and neutral ratings that the buyer or seller received over the past one, six, and twelve months. All information in this profile is accessible to the public and is continuously updated. eBay also computes the percentage of positive ratings received by a seller, which is the ratio of the positive ratings to the total number of ratings. The auction page displays both the seller’s feedback score and his percentage of positive ratings as the primary indicators of his reputation.

3.4 Dataset

The primary dataset includes 1273 completed single-object eBay auctions between December 2007 and mid March 2008. Auctions that led to no transaction were not collected. An auction without a transaction can occur when the seller received no bid or the reserve price was not met. The data was collected between one day to one month after the auction closing dates. This ensures that the feedback scores will be nearly the same as they were during the auction process. In fact, the feedback score changed about 1% over a month for the seller with the largest transaction volume in the dataset. The goods for sale in these auctions were Garmin Nuvi 650 and Nuvi 660 automobile GPS navigators. The choice of these two goods was determined by their similar features and the high transaction volume over that period. Thus, it

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Both of the goods are portable compact GPS navigators with wide touchscreen. A standard package of such navigators includes the navigation system with preloaded maps, a USB cable, a car charger, a car mount with a dashboard disk and a quick reference guide.
is easy to control for the heterogeneity of the goods while maintaining a reasonable sample size for examining the impact of late bidding. Another advantage of this dataset is that Nuvi 660 has all the features of Nuvi 650. Thus, the value of the additional features can then be estimated.

The details of each auction were collected from the auction page and the bidding history page. The general information on the auction page includes the opening date and time, the closing date and time, the number of bids received, the magnitude of the winning bid, the user ID of winning bidder and his feedback score, the feedback score and the percentage of positive ratings of the seller ID, and the shipping cost and shipping method. Information on the good was collected from the description on the auction page. This information includes the model, condition, warranty and accessories. The bidding history of each auction contains the starting bid, the number of prior bidders, the date, time, and magnitude of each bid, and the bidder’s user ID and his feedback score.

For the analysis of late bidding, I use a subset of the data that only contains auctions with starting bids of $1 or less. This smaller dataset with 909 auctions is referred to as the dataset, while the full dataset with 1273 auctions is referred to as the primary dataset. The auctions with high starting bids are excluded from the analysis, because the price formation and the timing of bids in those auctions are likely to be different. High starting bids were often specified by sellers who had low feedback scores and were selling used items in working condition. On the one hand, a high starting bid may serve as a signal to the buyers that the used item for sale is of high quality. On the other hand, a high starting bid may discourage buyers from participating the auction. In the primary dataset, the average feedback score of the sellers who set starting bids above $1 was 23416 and 31% of these auctions were selling used goods. The average feedback score of the sellers who set starting bids of $1 or less was 94165 and only 11% of these auctions were selling used goods. Since the starting bids are set by the sellers instead of endogenously determined by the bidding process, it is reasonable to assume the selection based on the starting bids is exogenous.

7When the auction is won by a proxy bid higher than the second highest bid plus a minimum increment, the magnitude of the winning bid refers to the second highest bid plus a minimum increment instead of the amount of the proxy bid.
Table 3.1 shows the number of auctions with different length in the dataset. The 395 auctions listed for one day accounted for about 43% of the auctions in the dataset, but 236 of these were listed by the two largest sellers. Seven days is the second most popular auction duration among sellers in this dataset. Less than 3% of the auctions were listed for ten days. This is probably caused by the small surcharge for 10-day auction listings. The summary statistics of these variables for the primary dataset and the smaller dataset are presented in the appendix.

<table>
<thead>
<tr>
<th>Auction Length</th>
<th># of Auctions</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 day</td>
<td>395</td>
<td>43.45%</td>
</tr>
<tr>
<td>3 days</td>
<td>155</td>
<td>17.05%</td>
</tr>
<tr>
<td>5 days</td>
<td>95</td>
<td>10.45%</td>
</tr>
<tr>
<td>7 days</td>
<td>239</td>
<td>26.29%</td>
</tr>
<tr>
<td>10 days</td>
<td>25</td>
<td>2.75%</td>
</tr>
</tbody>
</table>

Table 3.2 and 3.3 reflect the prevalence of late bidding. More than half of the winning bids were received in the last 5 minutes of the auctions. From the total bids received (22995) approximately 55% of these bids (12704) were received on the day of closing. For the auctions that lasted for 3 to 10 days, approximately 44% of the total bids (11660) were received on the day of closing (5185).

<table>
<thead>
<tr>
<th># of Auctions</th>
<th>10 sec</th>
<th>1 min</th>
<th>5 mins</th>
<th>10 mins</th>
<th>1 hour</th>
<th>1 day</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percentage</td>
<td>157</td>
<td>394</td>
<td>506</td>
<td>555</td>
<td>714</td>
<td>891</td>
</tr>
<tr>
<td></td>
<td>17.27%</td>
<td>43.34%</td>
<td>55.67%</td>
<td>61.06%</td>
<td>78.55%</td>
<td>98.02%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th># of Bids</th>
<th>10 sec</th>
<th>1 min</th>
<th>5 mins</th>
<th>10 mins</th>
<th>1 hour</th>
<th>1 day</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percentage</td>
<td>272</td>
<td>811</td>
<td>1402</td>
<td>1719</td>
<td>3194</td>
<td>12704</td>
</tr>
<tr>
<td></td>
<td>1.18%</td>
<td>3.53%</td>
<td>6.10%</td>
<td>7.48%</td>
<td>13.89%</td>
<td>55.25%</td>
</tr>
</tbody>
</table>

8 The surcharge for a 10-day auction listing is $0.40, but eBay also mentions that durations of longer than seven days may change some other fees for selling on eBay.
3.5 Regression Model

The regression model is specified as

\[ y = x\beta + \varepsilon, \ E[\varepsilon|x] = 0 \text{ and } Cov[\varepsilon|x] = \begin{bmatrix} \sigma_1^2 & 0 & 0 & \cdots & 0 \\ 0 & \sigma_2^2 & 0 & \cdots & 0 \\ \vdots & & & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \sigma_n^2 \end{bmatrix}. \] (3.1)

In (3.1), \( E[\varepsilon|x] \) and \( Cov[\varepsilon|x] \) are the conditional expectation and the diagonal covariance matrix of the error term \( \varepsilon \) respectively, and \( n \) is the sample size. The dependent variable \( y \) is the natural logarithm of the final price (\( LN_P \)) and the vector \( x \) of independent variables includes the reputation variables of the seller (\( LN_POS, LN_NEG \)), the experience of the winning buyer (\( LN_WINNER \)), the characteristics of the good (\( N660, WORK, REFURB, WARRANTY, EXTRA \)), the monthly dummy variables (\( MTH1, MTH2 \)), the number of bidders (\( \#BIDDER \)) and the constant. Each of these variables is discussed in the following subsections.

3.5.1 Characteristics of goods

The characteristics of a good affect its final price. For example, buyers should have a higher value for the GPS navigator that is has more features, is in better condition, has a warranty, and includes extra accessories. Therefore, dummy variables \( N660, REFURB, WORK, WARRANTY \) and \( EXTRA \) are included in the regression.

The dummy variable \( N660 \) captures the value of the additional features in the Nuvi 660. Both of the goods in my dataset are compact GPS navigators with a wide touchscreen. In addition to all the features of the Nuvi 650, the Nuvi 660 has Blue Tooth technology, an integrated traffic navigator and an FM audio transmitter. The Nuvi 660 also comes with a leather case. Hence, the Nuvi 660 is sold at a higher retail price than the Nuvi 650. In the dataset, the average price is about $354 for a new Nuvi 650 and $419 for a new Nuvi 660. On eBay, the goods are classified into new, refurbished and used, depending on the description by the sellers. Dummy variables \( NEW, REFURB \) and \( WORK \) are created accordingly. The variable \( NEW \) is dropped from the regression to avoid the problem of perfect collinearity among these variables.
The coefficients of \textit{REFURB} and \textit{WORK} are thus expected to be negative.

Some sellers add extra accessories, such as memory cards or screen protector kits to the standard package as a bonus. A dummy variable \textit{EXTRA} is created to examine whether these extra accessories affect the final prices of the GPS navigators. GPS navigators in new or refurbished condition often have a one-year manufacturer warranty or even an extended warranty. Used GPS navigators may also have an unexpired warranty if they have been used for a short period of time. However, not every seller mentioned the warranty information of their goods in the description, so the dummy variable \textit{WARRANTY} actually captures the affect of advertising the warranty on the goods.

3.5.2 Reputation of sellers

A seller’s feedback score and percentage of positive ratings are the two primary indicators of his reputation. Both are displayed on the auction page for any good he lists for sale. Section 3.5.3 describes how these two indicators are calculated by eBay using the number of positive, neutral and negative ratings. Some studies use both the positive ratings and the negative ratings to measure the reputation of a seller. Other studies only use the feedback score of a seller as a measure of his reputation. The feedback score is the sum of his positive ratings minus his negative ratings. The model presented in this paper follows the first approach, since that approach provides more information than the second one. Furthermore, measuring a seller’s reputation by his feedback score does not change the other results in this paper.

Empirical evidence is mixed on the relationship between the reputation of a seller and the final price that he receives from the auction. A few studies have concluded that the reputation of sellers had no significant effect on the final price. With data provided by eBay, Resnick and Zeckhauser (2002) find that neither the positive ratings nor the negative ratings had a statistically significant effect on the final price. However, they do find that the ratings affect the probability of the good being sold. Jin and Kato (2004, 2005) obtain similar results in their experiment on eBay. On the other hand, many studies indicate that positive ratings increase the final price, while negative ratings reduce the final price (Houser and Wooders 2005, Lucking-Reiley et. al. 2007, Melnik and Alm 2002). Using an eBay auction dataset on computer CPU,
Houser and Wooders (2005) find that a 10% increase in the number of positive ratings will increase the final price of a computer CPU by about 0.17%, while a 10% increase in the number of neutral or negative ratings reduces the final price by about 0.24%. McDonald and Slawson (2002) measure the reputation of a seller by his feedback score. They discover that sellers with higher feedback scores received higher final prices and their auctions also attracted more bids. Livingston (2005) finds that the marginal return to the initial positive ratings of a seller is high, but declines once the seller has established a good reputation. Thus, it is appropriate to apply a specification with diminishing effect to the variables representing the sellers’ reputation.

The number of positive ratings and negative ratings were not collected in my dataset, but they can be derived from the sellers’ feedback scores and percentage of positive ratings. Following the most common specification in the literature, I include the natural logarithm of one plus sellers’ positive ratings $LN_{POS}$ and the natural logarithm of one plus sellers’ negative ratings $LN_{NEG}$ in the regression model. As a test for the logarithm specification, the appendix presents the alternative estimation result from a partial linear model which allows a seller’s positive and negative ratings to enter additively through an unknown function $g(\cdot)$.

It is difficult to disentangle the reputation of a seller from his experience. Summary statistics in Table 3.5 indicate that sellers received many more positive ratings than negative ratings. As a result, their feedback scores are nearly equal to the number of eBay transactions for which they received feedback from buyers. Thus, a high feedback score implies that the seller has extensive experience in selling on eBay. More experienced sellers are often more adept at constructing a well-designed webpage and responding to buyers’ questions. These actions reduce buyers’ uncertainty about the good for sale (Yin 2003). As a result, more experienced sellers will receive higher final prices for reasons other than their reputations.

### 3.5.3 Experience of bidders

Bajari and Hortacsu (2003) have examined an eBay dataset on coin auctions and discovered in their dataset that bidder experience has no impact on the final price of an auction. In contrast, experimental research on auctions suggests that bidders’ performance improves with their

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9The feedback score = The number of positive ratings - The number of negative ratings
The percentage of positive ratings = \(\frac{\text{The number of positive ratings}}{\text{The number of positive and negative ratings}}\)
experience (Kagel 1995). The feedback score of a buyer can reflect his relative experience in some sense. Therefore, the natural logarithm of one plus the feedback score of the auction winner $LN_{WINNER}$ is also included in the regression. Based the previous discussion, this variable is expected to have either no impact or a negative impact on the final price.

3.5.4 Number of bidders

Auction theory predicts that the seller’s expected revenue increases as more bidders participate in the auction. Thus, it is natural to include the number of bidders in the regression model. However, a concern common to any the empirical study of online auctions is that the number of observed bidders can be endogenous. That is, there may be a correlation between $\#BIDDER$ and the error term $\epsilon$. This endogeneity can be caused by measurement error or omitted variables. Unlike an open live auction, the number of bidders participating in an online auction is seldom observed with accuracy. A potential bidder might arrive late at an online auction only to find that the current high bid for the item is already above his value. When this happens, using the number of bidders who place bids instead of the number of potential bidders poses a measurement error problem. The omitted variable problem can arise because it is difficult to assess the attractiveness of the goods listed. Bidders may be attracted to an online auction for reasons which are unobservable to the econometrician. For example, some auction listings are more professional in their design and description than others. Irrespective of the reason for the endogeneity, the coefficients from OLS regression will be biased. Therefore, the instrumental variable (IV) approach must be applied in order to capture the possible correlation. Dewally and Ederington (2004) find empirical evidence that auctions lasting for more days are usually noticed by more potential bidders. Therefore, an instrument for the number of bidders can be the length of the auction $AUCLTH$. This variable satisfies the exclusion restriction\(^\text{10}\) and has been used as an instrument for the number of bidders in the existing literature of online auctions (Yin 2003).

High starting bids can also cause a few complications in the number of bidders. These

\(^{10}\)For two identical auctions with the same number of bidders, the expected price will not differ simply because one auction is longer than the other. In other words, the length of auction only affects the expected price through the number of bidders.
complications are why the analysis in this paper is restricted to the dataset which only includes auctions with starting bids of $1 or less. The complications arise from the fact that the starting bid is equivalent to a reserve price known to all other bidders. Since only bidders with values above the starting bid will participate in the auction, a high starting bid truncates the final price from below and decreases the number of observed bidders. In addition, Simonsohn and Ariely (2008) have discovered that bidders herd into auctions with more existing bids, even if these existing bids have been made not because of higher quality of the goods but because of lower starting bids which are already outbid. Their finding implies that even after accounting for the truncation effect, the auctions with high starting bids still have fewer bidders than those with low starting bids. The pattern that fewer bidders are observed in auctions with high starting bids is clear in this dataset on GPS navigators, as shown in Figure 3.1. Restricting the analysis to the auctions with low starting bids will eliminate both the truncation and the herding effects of high starting bids on the number of bidders. However, as noted in Section 3.4, using this smaller dataset may introduce a selection bias to the analysis, so the results need to be interpreted with caution. Furthermore, this dataset must still have a reasonable number of observations to ensure a consistent estimation of $s^*$. 

Figure 3.1. The number of bidders and the starting bid
Lucking-Reiley et al. (2007) managed to avoid the endogeneity problem by replacing the number of bidders with variables which are relevant to the bidder’s participation and bid choices. For example, one such variable is the length of the auction, because longer auctions attract more bidders and thus result in higher final prices. The authors conceded that omitted variables might exist in their model, because quantifying the attractiveness of the auction listing was impossible. Then, they applied a censored-normal maximum likelihood estimation procedure to account for the effect of the high starting bids. This procedure is similar to a standard Tobit regression except that the censoring point is different across observations. Since the analysis in this paper is restricted to the smaller dataset with low starting bids, the censoring problem is minimal and thus will not be addressed here.\footnote{Besides the censoring problem, high starting bids create another issue that cannot be addressed simply by correcting for the censoring biases. As discussed in Section 3.4, auctions with high starting bids and low starting bids are likely to differ in their price formation as well as the timing of bids. Therefore, restricting the analysis to the small dataset with starting bids of $1 or less is a preferred solution.}

### 3.5.5 Time

The retail prices of the two goods in the sample decrease over time because of economic obsolescence. This leads to a decrease in buyers’ values and consequently a decreasing trend in the final price in eBay auctions. In the dataset, the average price of a new Nuvi 660 was $446 in December 2007, $432 in January 2008 and $366 since February 2008. For a new Nuvi 650, the average price had reduced from $357 to $323 over these three months. Since the data is collected over a three-month time span, I include two monthly dummy variables $MTH_1$ and $MTH_2$ in the regression model.

The description of all variables in the regression model are summarized in Table 3.4.
Table 3.4 Description of variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>LN_P</td>
<td>ln(price), where the price equals the winning bid plus the shipping cost</td>
</tr>
<tr>
<td>LN_WINNER</td>
<td>ln(the winner’s feedback score + 1)</td>
</tr>
<tr>
<td>LN_POS</td>
<td>ln(the number of the seller’s positive ratings + 1), where the number of</td>
</tr>
<tr>
<td></td>
<td>the seller’s positive ratings is derived from his feedback score and the</td>
</tr>
<tr>
<td></td>
<td>percentage of his positive ratings*</td>
</tr>
<tr>
<td>LN_NEG</td>
<td>ln(the number of the seller’s negative ratings + 1), where the number of</td>
</tr>
<tr>
<td></td>
<td>the seller’s negative ratings is derived from his feedback score and the</td>
</tr>
<tr>
<td></td>
<td>percentage of his positive ratings*</td>
</tr>
<tr>
<td>#BIDDER</td>
<td>The number of bidders who placed one or more bids in the auction</td>
</tr>
<tr>
<td>MTH1, MTH2,</td>
<td>Dummy variables for the month in which an auction closed: MTH1 for Dec 2007,</td>
</tr>
<tr>
<td>MTH3</td>
<td>MTH2 for Jan 2008 and MTH3 for Feb 2008 and after. MTH3 is dropped in the</td>
</tr>
<tr>
<td></td>
<td>regressions.</td>
</tr>
<tr>
<td>N660</td>
<td>Dummy variable for a Nuvi 660 GPS receiver</td>
</tr>
<tr>
<td>NEW</td>
<td>Dummy variable for a new GPS receiver (dropped in the regressions)</td>
</tr>
<tr>
<td>REFURB</td>
<td>Dummy variable for a refurbished GPS receiver</td>
</tr>
<tr>
<td>WORK</td>
<td>Dummy variable for a used GPS receiver of working condition</td>
</tr>
<tr>
<td>WARRANTY</td>
<td>Dummy variable for a GPS receiver advertised with manufacturer warranty</td>
</tr>
<tr>
<td>STD</td>
<td>Dummy variable for a GPS receiver with standard accessories (dropped in</td>
</tr>
<tr>
<td></td>
<td>the regressions)</td>
</tr>
<tr>
<td>EXTRA</td>
<td>Dummy variable for a GPS receiver with extra accessories</td>
</tr>
<tr>
<td>AUCLTH</td>
<td>The length of the auction in days, which can take the value 1, 3, 5, 7,</td>
</tr>
<tr>
<td></td>
<td>or 10.</td>
</tr>
</tbody>
</table>

Table 3.5 presents summary statistics for the smaller dataset. The average winning bid is $345 and the average shipping cost is $19. The average starting bid is 67 cents with most values centered on 1 cent and 99 cents. The dataset also shows that eBay bidders often submit multiple bids. The number of bidders in an auction ranges from 3 to 22 with a mean of 11, but the number of bids ranges from 4 to 49 with a mean of 21.
### Table 3.5 Summary statistics for the dataset

<table>
<thead>
<tr>
<th>Variable</th>
<th>Min</th>
<th>Max</th>
<th>Median</th>
<th>Mean</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Winning bid</td>
<td>180</td>
<td>485</td>
<td>341.8</td>
<td>345.44</td>
<td>52.43</td>
</tr>
<tr>
<td>Shipping cost</td>
<td>0</td>
<td>40</td>
<td>19.99</td>
<td>19.28</td>
<td>5.47</td>
</tr>
<tr>
<td>Starting bid</td>
<td>0.01</td>
<td>1</td>
<td>0.99</td>
<td>0.67</td>
<td>0.62</td>
</tr>
<tr>
<td>LN_WINNER</td>
<td>0</td>
<td>8.61</td>
<td>3.30</td>
<td>3.30</td>
<td>1.73</td>
</tr>
<tr>
<td>LN_SELLER</td>
<td>0</td>
<td>12.87</td>
<td>9.05</td>
<td>8.51</td>
<td>3.28</td>
</tr>
<tr>
<td>LN_POS</td>
<td>0</td>
<td>12.87</td>
<td>9.05</td>
<td>8.51</td>
<td>3.28</td>
</tr>
<tr>
<td>LN_NEG</td>
<td>0</td>
<td>7.11</td>
<td>4.03</td>
<td>3.51</td>
<td>2.63</td>
</tr>
<tr>
<td>#BIDDER</td>
<td>3</td>
<td>22</td>
<td>11</td>
<td>11.19</td>
<td>2.84</td>
</tr>
<tr>
<td>MTH1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0.41</td>
<td>0.49</td>
</tr>
<tr>
<td>MTH2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0.29</td>
<td>0.45</td>
</tr>
<tr>
<td>N660</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0.53</td>
<td>0.50</td>
</tr>
<tr>
<td>REFURB</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0.37</td>
<td>0.48</td>
</tr>
<tr>
<td>WORK</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0.11</td>
<td>0.31</td>
</tr>
<tr>
<td>WARRANTY</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0.70</td>
<td>0.46</td>
</tr>
<tr>
<td>EXTRA</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0.48</td>
<td>0.50</td>
</tr>
<tr>
<td># of bids per auction</td>
<td>4</td>
<td>49</td>
<td>20</td>
<td>21.10</td>
<td>7.62</td>
</tr>
<tr>
<td># of auctions per seller</td>
<td>1</td>
<td>153</td>
<td>16</td>
<td>45.73</td>
<td>56.67</td>
</tr>
<tr>
<td># of observations</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>909</td>
</tr>
</tbody>
</table>

#### 3.6 Searching for Late-bidding Window

A late bid is defined as a bid that is received no more than $s^*$ seconds before the closing time. Similarly, a late winning bid refers to the winning bid that is received no more than $s^*$ seconds before the closing time. $s^*$ is the length of the late-bidding window. There are several reasons that auctions won by late bids should on average have lower final prices than auctions won by early bids. First, when a late winning bid is received immediately before the closing time, other buyers who were bidding incrementally will have no time to respond to this bid even if their values are higher than the winning bid. Second, the erratic internet traffic near the closing time of the auction may cause some late bids not to be received by the closing time. Third, a bidder with high value who had planned to bid late might not be available to submit his bid at the close of the auction because of an unexpected event in his life.

On the other hand, when the winning bid arrives early, this usually indicates that buyers were bidding competitively against each other using the proxy bidding program or raising the bids as in an English auction. In either case, the final prices must have already reached the second highest value of the participating buyers, and no other bidder is willing to outbid the
current high bid even when there is still time to do so. In summary, the final prices in the auctions with late winning bids and those with early winning bids are formed in different ways. This implies that there should be a potential structural break in the final prices based on the time at which the winning bid is received. This section describes the method used to estimate that structural break $s^*$. 

3.6.1 Methodology

The length of late-bidding window $s^*$ is estimated by the structural break estimator proposed by Bai (1997). The dataset is grouped into two subsamples I and II according to whether the winning bid arrived earlier or later than $s$ seconds before the auction closed. Then, for every possible value of $s$, each of the two subsamples is fitted with the regression model in Section 3.5. The break-point estimator is defined as the time that minimizes the residual sum of squares from the regressions:

$$
\hat{s}^* = \arg \min_s \{\text{RSS}_I(s) + \text{RSS}_{II}(s)\}
$$

In this paper, the regression model is specified to be log-linear, so there is little computational burden in searching for $s^*$. If the regression model for each subsample were complex and there are many possible values for $s$, the estimation of $s^*$ could be computationally intensive. Chong (2003) has shown that Bai’s break-point estimator remains consistent even with certain types of model misspecifications. Therefore, when the sample size is large, estimation of a less complex model should generate the same break-point estimate $\hat{s}^*$ as estimation of a more complex model with a more accurate specification of the nature of the auction.

3.7 Results

To estimate $s^*$ as a break point, I apply the regression equation (3.1) and use Bai’s method of minimizing the residual sum of squares described in Section 3.6. The searching begins with $s = 15$ minutes and stops at $s = 8$ seconds. When $s$ is less than 8 seconds, the late bid

---

12 The only exception occurs when every late bid fails to be received in an auction. This type of auction cannot be separated from the auctions without late bidding, but they should not create a great concern here. Given the length of the late-bidding window estimated from my dataset, it would be a very rare event that every bid in the window was not received. All existing studies on late bidding have this inherent problem, yet there is no obvious solution to it.
subsample becomes very small. For the OLS regression, the minimum residual sum of squares occurs at $\hat{s}_{OLS} = 191$ seconds. This estimate should be close to the truth, even though the OLS regression does not account for the potential endogeneity problem. The reason is that Bai’s break-point estimator is consistent even with certain model misspecifications as discussed in Section 3.6. As a robustness check, I also examine a two-stage least squares (2SLS) regression with the auction length $AUCLTH$ as the instrument for the number of bidders. For the 2SLS regression, the minimum residual sum of squares occurs at $\hat{s}_{2SLS} = 183$ seconds, a result similar to $\hat{s}_{OLS}$.\footnote{According to econometric theory, the break point estimated with the OLS regression should be close to that with the 2SLS regression. The results here are consistent with this theoretical prediction, since only 4 auctions have the winning bids arriving between 191 seconds and 183 seconds before the auctions closed.} Figure 3.2 plots the residual sum of squares from the 2SLS regressions for different $s$ with the minimum clearly shown at 183 seconds.

![Figure 3.2 Residual sum of squares from 2SLS for different $s$](image)

There are 471 auctions with winning bids received within 183 seconds before the closing time. This is the late bid group. The other 438 auctions are in the early bid group. Table
Table 3.6 Regression results
OLS IV (two-step GMM)

<table>
<thead>
<tr>
<th>Group</th>
<th>Early</th>
<th>Late</th>
<th>Early</th>
<th>Late</th>
</tr>
</thead>
<tbody>
<tr>
<td># of observations</td>
<td>438</td>
<td>471</td>
<td>438</td>
<td>471</td>
</tr>
<tr>
<td>Wald-Chi2</td>
<td>1364.01</td>
<td>2207.65</td>
<td>155.12</td>
<td>218.51</td>
</tr>
<tr>
<td>R²</td>
<td>0.7946</td>
<td>0.8376</td>
<td>0.7498</td>
<td>0.8253</td>
</tr>
<tr>
<td>C-statistic</td>
<td>2.6046</td>
<td>0.1237</td>
<td>0.0056</td>
<td>0.0017</td>
</tr>
</tbody>
</table>

A comparison of the estimates from the late bid and early bid groups reveals notable differences in the coefficients on WORK and LN_WINNER at the 5% significance level and the coefficient of EXTRA at the 10% level. The coefficient of the dummy variable for used items (WORK) is more negative in the late bid group than in the early bid group. In the auctions

\[^{14}\text{For } s^2_{OLS} = 191 \text{ seconds, 434 auctions are included in the early group and 457 auctions in the late group. The estimated coefficients for this case are not presented here, because they are nearly the same as those for } s^2_{2SLS} = 191 \text{ seconds.}\]

\[^{15}\text{The two-step GMM estimation is equivalent to the 2SLS estimation with a covariance matrix adjusted for heteroskedasticity in this case.}\]
won by early bids, used items in working condition were sold at a price 11% lower than new items. However, in the auctions won by late bids, used items were sold at a price 19% lower than new items. This implies that the negative impact of late winning bids on the final price occurs mainly on used items. Consider a used Nuvi 660 without warranty or extra accessories which was sold in December 2007 by an average seller and 10 bidders submitted bids in the auction. When no one bid within the last 3 minutes and a bidder with a feedback score of 100 won the auction, the final price would have been $388 on average.\textsuperscript{16} If the winning bid came in the last 3 minutes, everything else equal, the final price would have been $359 on average, which is about $29 lower than it should have been without late bids.

This empirical finding coincides with the explanation for late bidding by Rasmusen (2006). Rasmusen (2006) studies a second-price independent private-value auction in which some bidders are uncertain about their own values for the good. The uncertain bidder would be reluctant to incur the cost of learning his own value early in the auction, when knowing his value did not seem to affect the success of his bidding. However, he would not have time to learn such information in the last moment of an auction. Thus, by bidding late, his opponent could avoid inducing him to learn more about his value. There is more uncertainty about the quality of used items than the quality of new or refurbished items. Hence, a good judgment on the quality of used items requires more expertise than new or refurbished ones. Based on their past transactions on eBay, experienced buyers are better than novice buyers at identifying the quality of such items according to the photos and descriptions. By bidding in the last moment, experienced buyers leave no time for novice buyers to raise the prices. Thus, the expected final prices in the auctions with late winning bids are lower.

Among the auctions won by early bids, a 10% increase in the winner’s feedback score \( LN\_WINNER \) corresponds to a 0.06% decrease in the final price and this negative impact is statistically significant at 1% level. Since long-time eBay buyers in general have higher feedback scores than new buyers, this result is consistent with the finding of Kagel (1995) that bidders’ performance improves with their experience. Among the auctions won by late bids, the impact of \( LN\_WINNER \) on the final price becomes positive, but not statistically significant.

\textsuperscript{16}Since the analysis is restricted to the small dataset with starting bids of $1 or less, this revenue prediction may not be generalized to the auctions with high starting bids.
Two sample t-tests for a difference in the mean indicate that $LN_{\text{WINNER}}$ is about 0.4782 higher in the auctions won by late bids than in those won by early bids. Since late winners are more experienced than early winners, the differential impacts of $LN_{\text{WINNER}}$ in the late bid and early bid groups could be a consequence of the diminishing returns to experience for the buyers.

Lucking-Reiley et al. (2007) discovered in their dataset on eBay coin auctions that the effect of sellers’ negative ratings on the expected price works in the opposite direction of positive ratings and is much larger. They conclude that eBay bidders focus on sellers’ negative ratings. A similar pattern seems to appear in both the late bid and early bid groups in the dataset on GPS navigators. The coefficient estimates on $LN_{\text{POS}}$ and $LN_{\text{NEG}}$ are statistically significant with the expected signs. However, the linear hypothesis testing on the sum of these two coefficients cannot support the disparity in the effects of positive and negative ratings found by Lucking-Reiley et al. (2007).

Auction theory predicts that sellers’ expected revenue increases when more buyers participate in the auction. However, the empirical results on the number of bidders in this dataset are not robust across different model specifications. Since the number of bidders who place bids does not accurately measure the number of potential bidders (see the discussion in Section 3.5.4), these empirical results should not be interpreted as a contradiction to the theoretical prediction.

### 3.8 Conclusions

Late bidding is a common practice among online bidders. This study indicates that the final price of the auctions in which the winning bids arrived in the last 3 minutes is generally lower than when winning bids arrived earlier. Furthermore, the negative impact of late winning bids on the final price is mainly on used items. Previous studies have suggested several explanations for late bidding, but few have examined the impact of late bidding on the final prices in online auctions. This study bridges the gap. Unlike the previous research, I draw from the structural break literature and provide a method to estimate this cutoff time as a break point. Then, I examine the differences in the coefficients of the model before and after the break point.
The empirical evidence in this paper suggests that late winning bids are often submitted by experienced buyers and the benefit from late bidding occurs mainly from purchasing used goods at lower prices. I also investigated other determinants of the final price in online auctions. First, the variation in the final prices is mainly determined by the condition and characteristics of the goods as well as the economic obsolescence. In contrast, the effects of other factors on the final price are relatively small. Second, experience helps eBay buyers obtain lower prices in auctions of used items. Third, eBay buyers focus on both sellers’ negative and positive ratings, but the negative ratings matter slightly more than positive ones. Fourth, longer auctions on eBay tend to attract more bidders and generate higher final prices. The last two findings are not statistically significant.

This paper does not test the different theoretical explanations for late bidding. Future work can address questions such as when late bidding is likely to happen and how eBay sellers can increase their expected revenues by preventing their auctions from being sniped.
3.9 Appendix

3.9.1 eBay page

Figure 3.3 eBay auction page and bidding history page
### 3.9.2 Summary statistics of the dataset

Table 3.7 Summary statistics for the primary dataset

<table>
<thead>
<tr>
<th>Variable</th>
<th>Min</th>
<th>Max</th>
<th>Median</th>
<th>Mean</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Winning bid</td>
<td>180</td>
<td>520</td>
<td>341</td>
<td>346.37</td>
<td>54.63</td>
</tr>
<tr>
<td>Shipping cost</td>
<td>0</td>
<td>73.75</td>
<td>19.99</td>
<td>18.59</td>
<td>5.95</td>
</tr>
<tr>
<td>Starting bid</td>
<td>0.01</td>
<td>520</td>
<td>0.99</td>
<td>64.25</td>
<td>124.70</td>
</tr>
<tr>
<td>LN_WINNER</td>
<td>0</td>
<td>8.61</td>
<td>3.37</td>
<td>3.31</td>
<td>1.72</td>
</tr>
<tr>
<td>LN_SELLER</td>
<td>0</td>
<td>12.87</td>
<td>7.31</td>
<td>7.68</td>
<td>3.42</td>
</tr>
<tr>
<td>LN_POS</td>
<td>0</td>
<td>12.87</td>
<td>7.31</td>
<td>7.68</td>
<td>3.42</td>
</tr>
<tr>
<td>LN_NEG</td>
<td>0</td>
<td>7.15</td>
<td>2.26</td>
<td>2.88</td>
<td>2.64</td>
</tr>
<tr>
<td>#BIDDER</td>
<td>1</td>
<td>22</td>
<td>10</td>
<td>9.64</td>
<td>4.04</td>
</tr>
<tr>
<td>MTH1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0.41</td>
<td>0.49</td>
</tr>
<tr>
<td>MTH2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0.30</td>
<td>0.46</td>
</tr>
<tr>
<td>N660</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0.54</td>
<td>0.50</td>
</tr>
<tr>
<td>REFURB</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0.30</td>
<td>0.46</td>
</tr>
<tr>
<td>WORK</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0.16</td>
<td>0.37</td>
</tr>
<tr>
<td>WARRANTY</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0.59</td>
<td>0.49</td>
</tr>
<tr>
<td>EXTRA</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0.44</td>
<td>0.50</td>
</tr>
<tr>
<td># of bids per auction</td>
<td>1</td>
<td>49</td>
<td>18</td>
<td>18.06</td>
<td>9.30</td>
</tr>
<tr>
<td># of auctions per seller</td>
<td>1</td>
<td>153</td>
<td>7</td>
<td>35.47</td>
<td>52.70</td>
</tr>
<tr>
<td># of observations</td>
<td></td>
<td></td>
<td></td>
<td>1273</td>
<td></td>
</tr>
</tbody>
</table>

### 3.9.3 Additional regression results

**Sample without auctions by large sellers**

In the dataset with 909 auctions, 148 auctions were listed by the largest seller and 88 by the second largest seller. Besides the high transaction volumes, the feedback scores of these two sellers are also much higher than those of others. Table 3.8 presents the regression results after excluding these 236 auctions from the dataset in order to examine whether they have introduced any bias into the estimates in Table 3.6. As one can see, the estimation results below are close to those in Table 3.6.
Table 3.8 Regression results after excluding 236 auctions by the two largest sellers

<table>
<thead>
<tr>
<th>Group</th>
<th>OLS Early</th>
<th>OLS Late</th>
<th>IV (two-step GMM) Early</th>
<th>IV (two-step GMM) Late</th>
</tr>
</thead>
<tbody>
<tr>
<td># of observations</td>
<td>346</td>
<td>327</td>
<td>346</td>
<td>327</td>
</tr>
<tr>
<td>LN_WINNER</td>
<td>-0.0064***</td>
<td>0.0034</td>
<td>-0.0064***</td>
<td>0.0026</td>
</tr>
<tr>
<td></td>
<td>(0.0022)</td>
<td>(0.0027)</td>
<td>(0.0023)</td>
<td>(0.0033)</td>
</tr>
<tr>
<td>LN_POS</td>
<td>0.0196***</td>
<td>0.0049</td>
<td>0.0195***</td>
<td>0.0032</td>
</tr>
<tr>
<td></td>
<td>(0.0062)</td>
<td>(0.0042)</td>
<td>(0.0062)</td>
<td>(0.0051)</td>
</tr>
<tr>
<td>LN_NEG</td>
<td>-0.0239***</td>
<td>-0.0093*</td>
<td>-0.0225***</td>
<td>-0.0066</td>
</tr>
<tr>
<td></td>
<td>(0.0069)</td>
<td>(0.0053)</td>
<td>(0.0076)</td>
<td>(0.0066)</td>
</tr>
<tr>
<td>#BIDDER</td>
<td>0.0027**</td>
<td>0.0015</td>
<td>0.0117</td>
<td>0.0166</td>
</tr>
<tr>
<td></td>
<td>(0.0014)</td>
<td>(0.0014)</td>
<td>(0.0104)</td>
<td>(0.0179)</td>
</tr>
<tr>
<td>MTH1</td>
<td>0.1666***</td>
<td>0.1783***</td>
<td>0.1607***</td>
<td>0.1583***</td>
</tr>
<tr>
<td></td>
<td>(0.0111)</td>
<td>(0.0118)</td>
<td>(0.0137)</td>
<td>(0.0258)</td>
</tr>
<tr>
<td>MTH2</td>
<td>0.1467***</td>
<td>0.1751***</td>
<td>0.1394***</td>
<td>0.1759***</td>
</tr>
<tr>
<td></td>
<td>(0.0123)</td>
<td>(0.0147)</td>
<td>(0.0155)</td>
<td>(0.0166)</td>
</tr>
<tr>
<td>N660</td>
<td>0.1854***</td>
<td>0.1820***</td>
<td>0.1794***</td>
<td>0.1581***</td>
</tr>
<tr>
<td></td>
<td>(0.0135)</td>
<td>(0.0135)</td>
<td>(0.0158)</td>
<td>(0.0338)</td>
</tr>
<tr>
<td>REFURB</td>
<td>-0.1010***</td>
<td>-0.1338***</td>
<td>-0.0952***</td>
<td>-0.1323***</td>
</tr>
<tr>
<td></td>
<td>(0.0135)</td>
<td>(0.0192)</td>
<td>(0.0148)</td>
<td>(0.0199)</td>
</tr>
<tr>
<td>WORK</td>
<td>-0.1176***</td>
<td>-0.1855***</td>
<td>-0.1141***</td>
<td>-0.1990***</td>
</tr>
<tr>
<td></td>
<td>(0.0174)</td>
<td>(0.0229)</td>
<td>(0.0185)</td>
<td>(0.0291)</td>
</tr>
<tr>
<td>WARRANTY</td>
<td>0.0160</td>
<td>0.0263***</td>
<td>0.0114</td>
<td>0.0196</td>
</tr>
<tr>
<td></td>
<td>(0.0099)</td>
<td>(0.0087)</td>
<td>(0.0118)</td>
<td>(0.0137)</td>
</tr>
<tr>
<td>EXTRA</td>
<td>-0.0049</td>
<td>0.0321</td>
<td>-0.0060</td>
<td>0.0386</td>
</tr>
<tr>
<td></td>
<td>(0.0157)</td>
<td>(0.0164)</td>
<td>(0.0156)</td>
<td>(0.0207)</td>
</tr>
<tr>
<td>Constant</td>
<td>5.6236***</td>
<td>5.6493***</td>
<td>5.5355***</td>
<td>5.5004***</td>
</tr>
<tr>
<td></td>
<td>(0.0362)</td>
<td>(0.0303)</td>
<td>(0.0959)</td>
<td>(0.1783)</td>
</tr>
<tr>
<td>F-statistic</td>
<td>89.31</td>
<td>112.64</td>
<td>864.76</td>
<td>860.72</td>
</tr>
<tr>
<td>Wald-Chi2</td>
<td>0.7489</td>
<td>0.8034</td>
<td>0.7173</td>
<td>0.7367</td>
</tr>
<tr>
<td>R²</td>
<td></td>
<td></td>
<td>0.7325</td>
<td>0.6804</td>
</tr>
<tr>
<td>C-statistic</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Partial linear regression

The model presented in Section 3.5 specifies the sellers’ reputation variables to be logarithm transformed, which is common in literature. To examine whether such a specification is proper, I estimate the following partial linear model which allows the sellers’ reputation variables enter additively through an unknown function \( g(\cdot) \).

\[
y = x_1 \beta + g(z) + \varepsilon
\]

where \( y \) is the natural logarithm of the final price \( LN_P \), \( z \) includes the reputation variables of the seller (\( LN_POS, LN_NEG \)), \( x \) includes the experience of the winning buyer (\( LN_WINNER \)), the characteristics of the good (\( N660, WORK, REFURB, WARRANTY, EXTRA \)), the monthly dummy variables (\( MTH1, MTH2 \)). The two specifications in Table
3.9 differ on whether the number of bidders ($\#BIDDER$) is included in the linear term or the non-linear term.

The partial linear regression results are similar to OLS and IV in Table 3.6. Comparison of the estimates from the late bid and early bid groups reveals notable differences in the coefficients on $LN\_WINNER$ and $WORK$ at 5% significance level and the coefficients of $REFURB$ and $EXTRA$ at 10% level.

<table>
<thead>
<tr>
<th>Group</th>
<th>Early</th>
<th>Late</th>
<th>Early</th>
<th>Late</th>
</tr>
</thead>
<tbody>
<tr>
<td>LN_WINNER</td>
<td>-0.0054***</td>
<td>0.0017</td>
<td>-0.0055***</td>
<td>0.0017</td>
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<tr>
<td></td>
<td>(0.0019)</td>
<td>(0.0018)</td>
<td>(0.0019)</td>
<td>(0.0018)</td>
</tr>
<tr>
<td>#BIDDER</td>
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<td>0.0016</td>
<td>0.0028</td>
<td>0.0016</td>
</tr>
<tr>
<td></td>
<td>(0.0012)</td>
<td>(0.0011)</td>
<td>(0.0012)</td>
<td>(0.0011)</td>
</tr>
<tr>
<td>MTH1</td>
<td>0.1825***</td>
<td>0.1883***</td>
<td>0.1814***</td>
<td>0.1892***</td>
</tr>
<tr>
<td></td>
<td>(0.0090)</td>
<td>(0.0084)</td>
<td>(0.0091)</td>
<td>(0.0084)</td>
</tr>
<tr>
<td>MTH2</td>
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<td>0.1778***</td>
<td>0.1578***</td>
<td>0.1785***</td>
</tr>
<tr>
<td></td>
<td>(0.0093)</td>
<td>(0.0087)</td>
<td>(0.0096)</td>
<td>(0.0087)</td>
</tr>
<tr>
<td>N660</td>
<td>0.1846***</td>
<td>0.1869***</td>
<td>0.1854***</td>
<td>0.1840***</td>
</tr>
<tr>
<td></td>
<td>(0.0094)</td>
<td>(0.0093)</td>
<td>(0.0094)</td>
<td>(0.0093)</td>
</tr>
<tr>
<td>REFURB</td>
<td>-0.0948***</td>
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<td>-0.0985***</td>
<td>-0.1272***</td>
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<tr>
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<td>(0.0144)</td>
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<td>(0.0146)</td>
<td>(0.0133)</td>
</tr>
<tr>
<td>WORK</td>
<td>-0.1130***</td>
<td>-0.1822***</td>
<td>-0.1145***</td>
<td>-0.1821***</td>
</tr>
<tr>
<td></td>
<td>(0.0118)</td>
<td>(0.0125)</td>
<td>(0.0117)</td>
<td>(0.0123)</td>
</tr>
<tr>
<td>WARRANTY</td>
<td>0.0152***</td>
<td>0.0227***</td>
<td>0.0145</td>
<td>0.0193***</td>
</tr>
<tr>
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<td>(0.0090)</td>
<td>(0.0085)</td>
<td>(0.0089)</td>
<td>(0.0086)</td>
</tr>
<tr>
<td>EXTRA</td>
<td>0.0029</td>
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<td>0.0029</td>
<td>0.0294</td>
</tr>
<tr>
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<td>(0.0100)</td>
<td>(0.0102)</td>
<td>(0.0100)</td>
<td>(0.0102)</td>
</tr>
<tr>
<td>F-statistic</td>
<td>138.367</td>
<td>213.480</td>
<td>139.136</td>
<td>218.504</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.744</td>
<td>0.806</td>
<td>0.721</td>
<td>0.791</td>
</tr>
</tbody>
</table>
Chapter 4

Sniping as Rational Behavior in Hard-Close Auctions

4.1 Introduction

Online bidding often looks similar to an English oral ascending auction, where each bidder observes the current highest bid and decides whether to bid higher by at least a minimum increment. Haile and Tamer (2003) proved that as long as no one bids more than his willingness to pay or allows an opponent to win at a price below his willingness to pay, the payment of the winner will be determined by the second highest value just as in a standard second-price auction. Hence, there seems to be no theoretical reason for bidders to postpone their bids. However, contrary to this theoretical prediction, empirical studies have found that online bidders often place their bids just moments before the closing time even though the auction has been open for days (Ockenfels and Roth 2006, Wilcox 2000, Bajari and Hortacsu 2003, Schindler 2003). This practice is called “sniping”.

Bajari and Hortacsu (2004) review the various explanations for late bidding. The first category of explanations are motivated by the information on values. Bajari and Hortacsu (2003), for example, develop a common value model in which all bidders bid late in the online auction to avoid giving other bidders information. Rasmusen (2006) considers a model in which bidders are uncertain about their own private value for an item. He assumes that bidders have to pay a fixed fee to discover their private values, thus late bidding can occur because bidders wish to economize on the costs of acquiring information. The second category of explanations are based on simultaneous listings of identical objects. The ability to bid across auctions create incentives for bidders to wait until the end in order to see how prices develop across auctions (Peters and Severinov 2006, Wang 2003, Anwar et. al. 2006 and Stryszowska 2005a, 2005b). The third category of explanations assert that late bidding is a best response to incremental bidding. In an auction with a fixed closing time, bidding at the end of the auction will not leave
sufficient time for incremental bidders to respond. Therefore, a late bidder competing with an incremental bidder may win the auction at the initial low bid of the incremental bidder (Roth and Ockenfels 2002).

Roth and Ockenfels (2002) also offer a fourth explanation based on “tacit collusion” by the bidders against the seller. Our model is closely related to this explanation. Roth and Ockenfels (2002) compared the timing of bids for computers and antiques on eBay and Amazon. They discovered that late bidding was more prevalent on eBay than on Amazon. They attribute this finding to the different rules for ending the auctions. For eBay auctions, there is a fixed closing time for submitting bids, which is referred to as a hard close. Conversely, for Amazon auctions, there is a flexible deadline which automatically extends beyond the scheduled closing. If bidding occurs near the scheduled closing, the auction closes only after no bids have been received for some period of time thereafter. To support this view, Ockenfels and Roth (2006) examine a model of an independent private value auction with a fixed closing time. They find that late bidding can be the best response against bidders who bid incrementally. Furthermore, they argue that even without incremental bidding, sniping can occur in equilibrium. Their model incorporates a positive probability that late bids may not be successfully received either because of erratic internet traffic or some other reasons. The probability that some bids will not be received at the end of auction results in a higher expected profit to the bidders whose bids were received. Such “tacit collusion” among bidders increases the expected profit of all bidders but lowers the seller’s expected revenue.

The model in Ockenfels and Roth (2006) assumes that there are only two bidders with identical values. With a similar model, Gonzalez et. al. (2004) shows that late bidding can be an equilibrium even with a large number of bidders whose values are distributed according to a general distribution function. A major limitation to his model is that unlike a live open auction, the number of bidders participating in an online auction is seldom observed. Although the number of participating bidders may not affect the amount of each bidder’s bid in a second-price auction, it can affect his decision on when to submit a bid if the auction is hard-close. Gonzalez et. al. (2004) avoids this problem by requiring all bidders with values above the reserve price to bid the reserve price at the beginning of the auction. This requirement is inconsistent with the fact that online bidders randomly arrive during the auction and choose
whether and when to submit a bid thereafter.

In this paper, we examine a second-price auction model with a hard close. We incorporate the random arrivals of potential bidders. With this model, we demonstrate that late bidding can exist in equilibrium, even though there is no uncertainty about one’s own value and late bids may not be received. Numerical examples are included to illustrate the sniping equilibrium and how it is sensitive to the distribution of values. In addition, we discuss the reduction in the expected revenue of the seller caused by late bidding.

4.2 A Model of Hard-Close Auction

A single indivisible good is offered for sale in an auction with potential bidders arriving randomly. Each bidder knows his own value for the good. He also knows that the other bidders’ values are independently drawn from a distribution on the interval [0, 1]. Let \( F(v) \) be the distribution of values and assume that \( F(v) \) is strictly increasing.

The auction begins at time \( t = 0 \) and ends with a hard close at time \( t = S \). To simplify the analysis, we assume that the seller sets no reserve price and imposes no restrictions on the bid increment. Bids are submitted by the potential bidders, but not all bids may be received by the seller. The seller awards the good to the bidder making the highest received bid at a price equal to the second highest received bid. Thus, the auction is a second-price auction. If there is a tie for the winning bid, the good is awarded to the bidder who bid that amount first.

The distinction between submitted bids and received bids is captured in the following manner. The auction time period is divided into an early and a late period. The early period begins at \( t = 0 \) and ends at \( t = T \), where \( T < S \). Bidders arrive randomly during this early period and any bid that is submitted in this period will be received by the seller with certainty. However, during the late period, which begins at \( t = T \) and ends at \( t = S \), no bidder arrives and any bid is received with only a probability of \( p < 1 \). Thus, not all bids submitted during the late period may be received. We denote this late period as the sniping period.

There are several alternative interpretations for this probability. First, there may be delay caused by network congestion or computer malfunctions. If so, the bids submitted may simply
not be transmitted before the hard close at \( t = S \). Another interpretation is that random exogenous events prevent bidders from submitting their bids before the hard close. For example, bidders may arrive early in the auction choose to wait for the sniping period, but then they are unable to return to the auction during the sniping period. This may occur due to unanticipated personal commitments or simply forgetfulness.

The probability of receiving bids provides the substance for the sniping period. Since this probability is independent of the magnitude of the bid, high bids are equally likely to be lost as are low bids. When high bids are lost, bidders with values below the highest value now have a chance of winning the auction and earning a surplus. However, there is also a probability that the bids of these bidders will also be lost. The net gains from postponing one’s bid until the sniping period depend on the trade-off between these two forces. Furthermore, with a smaller number of received bids, the expected payment of the winner is also lower.

In eBay auctions, a bidder can submit multiple bids and he can bid at anytime as long as the auction is open. However, for simplicity, we restrict each bidder to submit one and only one bid, but they can choose when to submit their bids. This assumption implies that a bidder still submits his bid when the current price is already above his bid. Bidders have an incentive to do so, because there is a slight chance that the winning bidder might withdraw from the auction and thus the good would be awarded to the bidder with the second highest bid.

The sniping period differs from the early period in several aspects. If a bidder has not bid prior to the sniping period, he will certainly want to do so during the sniping period. The sniping period is very short, so no bidder would have time to submit his own bid during this period if he waits to observe the sniping bids of others. The interesting question is whether any bidder would choose to wait until the sniping period to submit his bid.

Now consider the early period of the auction and the bidding behavior of bidders. To mirror the online environment, we assume that each bidder is uncertain about how many other bidders will participate in the auction. Instead, he approximates their arrivals by a Poisson process with a rate parameter \( \lambda \). Hence, the probability that \( n \) bidders have arrived before time \( t \) is

\[
P_n(t) = \frac{(\lambda t)^n}{n!} e^{-\lambda t}, \quad n = 0, 1, 2 \ldots
\]

Since we assume that all bidders arrive by time \( T \) when the early period ends, the probability that \( n \) bidders have arrived to participate in the auction is \( P_n(T) \). A bidder can submit his bid anytime after he arrives. When a bidder arrives,
he can observe whether the seller has received any bid from others. He can also observe the current price which is the second highest standing bid.

This model has multiple equilibria as shown in the following section. However, we do not attempt to characterize the set of all possible Bayesian equilibria in this paper. Instead, we focus on the equilibrium in which bidders bid their true value and we prove that bidding in the sniping period can be an equilibrium strategy for some bidders.

4.2.1 A Cutoff Equilibrium

Even if we only focus on the equilibrium where bidders bid their values, there still exist multiple equilibria for the model. One natural equilibrium occurs when every bidder bids his own value in the early period.

Proposition 3. (Early bidding equilibrium) There exists a perfect Bayesian Nash equilibrium in which every bidder bids his true value before the sniping period.

Proof. Consider bidder \( i \) with valuation \( v_i \). Suppose all other bidders arriving at the auction bid their valuations before the sniping period. If bidder \( i \) does the same, his expected payoff is denoted by \( \Pi(v_i) \). If he deviates by bidding during the sniping period, his expected payoff will become \( p \cdot \Pi(v_i) \). Since \( p < 1 \), bidder \( i \) has no incentive to deviate from this early bidding equilibrium.

Since online auctions last for several days, most online auction sites offer a service called proxy bidding so that bidders can submit incremental bids without actually observing the incoming bids from others. With proxy bidding, a bidder can set his maximum bid and the proxy agent bids incrementally on his behalf in response to higher arriving bids that are still below his maximum bid. This maximum bid is not revealed to others and it can be modified by the bidder during the auction. When the current highest bid exceeds his maximum bid, the bidder will be notified by email. eBay recommends that bidders use proxy bidding and set their maximum willingness to pay for the good early in the auction to avoid being outbid in the last minute of the auction. The automatic bids submitted by the eBay proxy are received by the seller with certainty. This recommended strategy is consistent with the early bidding equilibrium and also maximizes the seller’s revenue.
The prevalence of late bidding on eBay indicates that many bidders are not following this recommended strategy. This raises the question whether there exists another type of equilibrium in which late bidding is rational behavior so that sniping can emerge as an equilibrium outcome. Roth and Ockenfels (2002) surveyed 368 eBay bidders who successfully bid at least once in the last minute of an auction. Most of the respondents reported that they stay vigilant near the closing time of the auction. Some respondents reported using sniping software. The existence of market for sniping software suggests that sniping is at least a best response for some bidders, and thus could be an equilibrium outcome. The sniping software supplied by the third parties allows a bidder to submit a proxy bid near the closing time of an auction. With this software, bidders can choose their maximum bid early in the auction, enter when the auction is scheduled to end, and decide how many minutes or seconds before the end of the auction the sniping agent should submit the bid.

In order to identify an equilibrium with sniping, we study a simple strategy based on the values of bidders. Consider the cutoff strategy in which a bidder with a high value bids before the sniping period irrespective of what other bidders have done, but a bidder with a low value waits to bid during the sniping period if no other bidders have bid. Intuitively, a bidder with a high value should be less willing to take the risk of having his bid lost during the sniping period and thus bids early before the sniping period. Conversely, a bidder with a low value should be more willing to snipe, because his probability of winning the auction at a lower price is increased if some sniping bids from other bidders are not received. Therefore, we examine the existence of an equilibrium with the cutoff strategy in which high-value bidders bid before the sniping period and low-value bidders wait to snipe.

We first describe the cutoff strategy of a bidder, called “sniping-or-war” strategy and then prove that an equilibrium with this strategy profile can exist. We focus on the case in which bidders bid their values whenever they bid. The cutoff strategy for a bidder takes the following form:

1) If other bids have been submitted when a bidder arrives, the bidder bids his value before the sniping period.

2) If no other bids have been submitted when a bidder arrives:
a) if his value \( v \geq v^* \), the bidder bids his value before the sniping period;

b) if his value \( v < v^* \), the bidder:

   i) bids his value before the sniping period if any subsequent bids are submitted before the sniping period;

   ii) bids his value during the sniping period.

For the rest of the presentation, we will use \textit{sniping} to refer to the case in which all bidders wait to bid during the sniping period and \textit{war} for the case in which all bidders bid their values before the sniping period. A few things are worth mentioning about this cutoff strategy. First, as long as one bidder with value above \( v^* \) arrives at the auction, he immediately triggers a war by bidding his value and leaves no incentive for other bidders to snipe. The bidders who arrived earlier would immediately bid their values and the subsequent bidders would bid their values when they arrive. Consequently, a sniped auction emerges only when no bidder with value above \( v^* \) arrives before the end of the early period. Second, a bidder with the cutoff value \( v^* \) or above will initiate a war if no other bidder has done so. Thus, finding \( v^* = 0 \) implies the early bidding equilibrium in Proposition 3.

**Proposition 4.** (Sniping-or-war equilibrium) A symmetric Bayesian Nash equilibrium with the sniping-or-war strategy can exist.

**Proof.** Constructing a sniping-or-war equilibrium will complete the proof for Proposition 4. Let \( G(z) \) denote the probability that all bidders arriving at the auction have valuations below \( z \). With the Poisson process, \( P_k(T) = \frac{(\lambda T)^k}{k!} e^{-\lambda T} \) is the probability that \( k \) bidders arrive before the end of the auction. \( F(z)^k \) is the joint probability that all of these \( k \) bidders have valuations below \( z \). Thus, we can express \( G(z) \) as the joint probability that \( k \) bidders arrive all with valuations below \( z \):

\[
G(z) = \sum_{k=0}^{+\infty} P_k(T) F(z)^k = \sum_{k=0}^{+\infty} \frac{(\lambda T)^k}{k!} e^{-\lambda T} F(z)^k = e^{-\lambda T[1-F(z)]}.
\]

The last equality follows from the fact that \( \sum_{k=0}^{+\infty} \frac{x^k}{k!} \) is the Taylor series expansion of \( e^x \) around \( x = 0 \). The distribution function \( G(z) \) will be important for determining the incentive to bid during the early period or wait until the sniping period.
In contrast, let \( H(z; v^*) \) denote the probability that all bidders whose late bids are received during the sniping period have a valuation below \( z \) conditional on their valuations being below \( v^* \). Thus, we can express \( H(z; v^*) \) as

\[
H(z; v^*) = \sum_{k=0}^{+\infty} \left\{ P_k(T) \sum_{j=0}^{k} \binom{k}{j} p^j (1-p)^{k-j} \left[ \frac{F(z)}{F(v^*)} \right]^j \right\}
\]

\[
= \sum_{k=0}^{+\infty} \left( \frac{\lambda T}{k!} \right)^k e^{-\lambda T} \left[ \frac{F(z)}{p F(v^*)} + (1-p) \right]^k = e^{-\lambda T p \left[ 1 - \frac{F(z)}{F(v^*)} \right]}. \]

Again, \( P_k(T) \) is the probability that \( k \) bidders arrive before the end of auction. If all of these \( k \) bidders wait until the sniping period to bid, then \( \binom{k}{j} p^j (1-p)^{k-j} \) is the probability that \( j \) out of these \( k \) late bids are received. Finally, \( \left[ \frac{F(z)}{F(v^*)} \right]^j \) is the joint probability that all of these \( j \) bidders whose late bids are received have a valuations below \( z \), conditional on their valuations all being below \( v^* \). The second equality follows from the fact that \( \sum_{j=0}^{+\infty} \frac{x^j}{j!} \) is the Taylor series expansion of \( e^x \) around \( x = 0 \). The conditional distribution function \( H(z; v^*) \) will be important for determining the incentive to wait until the sniping period to submit a bid.

Any bidder’s deviation from this “sniping-or-war” equilibrium makes a difference in his expected payoff only when no other arriving bidder triggers a war. This happens with probability \( G(v^*) \). The following discussion is conditioned on the happening of this event.

**Low-value bidder:** First, we prove that a bidder with valuation \( v < v^* \) has no incentive to deviate by bidding early before the sniping period if no other bidder has done so. In other words, the cutoff strategy to wait and bid in the sniping period yields a higher expected payoff for him. Consider bidder \( i \), who has valuation \( v_i < v^* \). Let \( X \) denote the highest valuation among all other bidders whose bids are received in the sniping period. The expected payoff of
bidder $i$ by following the cutoff strategy to snipe under this circumstance is

$$
\Pi_L (\text{snipe} \mid X < v^*) = p \int_0^{v_i} (v_i - z) \, dH (z; v^*) \\
= p \left[ \int_0^{v_i} H (z; v^*) \, dz - v_i H (0; v^*) \right].
$$

(4.1)

The second equality follows from integration by parts. In contrast, the expected payoff from deviation by bidding in the early period to start a war is:

$$
\Pi_L (\text{war} \mid X < v^*) = \int_0^{v_i} (v_i - z) \frac{G(z)}{G(v^*)} \, dz - v_i \frac{G(0)}{G(v^*)}.
$$

(4.2)

Again, the second equality follows from integration by parts. Therefore, his net gain from deviation equals

$$
\Delta \Pi_L (v_i; v^*) = \Pi_L (\text{war} \mid X < v^*) - \Pi_L (\text{snipe} \mid X < v^*) \\
= \int_0^{v_i} \pi_l(z; v^*) - \pi_l(0; v^*) \, dz,
$$

(4.3)

where

$$
\pi_l(z; v^*) = \frac{G(z)}{G(v^*)} - pH (z; v^*) = e^{-\lambda T [F(v^*) - F(z)]} - pe^{-\lambda T \{1 - F(z)\}}.
$$

(4.4)

Conditional on no other bidders having started a war, bidders with valuation $v^*$ have the same expected payoff whether they bid during the early period and start a war or wait to bid late during the sniping period. If an interior solution exists, we can solve for $v^*$ from the following equation:

$$
\Delta \Pi_L (v^*; v^*) = 0, \quad v^* \in (0, 1).
$$

(4.5)

Bidder $i$ has no incentive to deviate by bidding early as long as $\Delta \Pi_L (v_i; v^*) < 0$ for $v_i < v^*$. We will provide numerical evidence for the existence of such a $v^*$ in Section 4.2.2, since identifying an analytical condition is difficult.

**High-value bidder:** Now, we prove that a bidder with valuation $v > v^*$ has no incentive to wait to bid during the sniping period. In particular, bidding in the early period yields a higher expected payoff for him. Suppose bidder $i$ has a valuation $v_i > v^*$. His expected payoff from
deviation by waiting to bid in the sniping period is

\[
\Pi_H (\text{snipe} \mid X < v^*) = p \left( v_i - \int_0^{v^*} zdH (z; v^*) \right) = p \left[ v_i - v^* \cdot H (v^*; v^*) + \int_0^{v^*} H (z; v^*) dz \right] = p (v_i - v^*) + p \int_0^{v^*} H (z; v^*) dz.
\]

The second equality follows from integration by parts. The last equality holds because \( H (v^*; v^*) = 1 \). His expected payoff by following the cutoff strategy to bid in the early period is

\[
\Pi_H (\text{war} \mid X < v^*) = v_i - \int_0^{v^*} zd G (z) G (v^*) = v_i - v^* + \int_0^{v^*} G (z) dz.
\]

Therefore, the net gain from deviation is

\[
\Delta \Pi_H (v_i; v^*) = \Pi_H (\text{snipe} \mid X < v^*) - \Pi_H (\text{war} \mid X < v^*) = -(v_i - v^*) (1 - p) + \int_0^{v^*} \frac{G (z)}{G (v^*)} - p H (z; v^*) dz < 0.
\]

The last equality follows from the fact that \( \int_0^{v^*} \frac{G (z)}{G (v^*)} - p H (z; v^*) dz = 0 \). The sign of \( \Delta \Pi_H (v_i; v^*) \) is opposite to the sign of \( (v_i - v^*) \). Since the net gain from deviation by sniping is negative, bidder \( i \) with valuation \( v_i > v^* \) would bid in the early period.

So far, we have proved that there can exist an equilibrium where both low-value bidders with \( v < v^* \) and high-value bidders with \( v > v^* \) would follow the cutoff strategy. With the cutoff strategy, the outcome of the auction can have all bidders sniping or all bidders with a war bidding before the sniping period. The following corollaries state that these two outcomes can arise in equilibrium.

**Corollary 1.** If \( v_i < v^* < 1, \forall i \), the outcome of the sniping-or-war equilibrium is that all bidders wait to bid during the sniping period and the auction ends with all bidders sniping.

**Corollary 2.** If \( \exists j \) such that \( v_j > v^* \), the outcome of the sniping-or-war equilibrium is that all bidders bid in the early period and the auction is a war with all bidders submitting bids before the sniping period.
The above process of constructing the “sniping-or-war” equilibrium relies heavily on the existence of \( v^* \in (0, 1) \) such that \( \Delta \Pi_L (v^*; v^*) = 0 \). However, it is still not obvious under what circumstance such a \( v^* \) exists. Solving for the cutoff value \( v^* \) from equation (4.5) can be very complicated even with very simple functional forms for \( F(\cdot) \). Instead, we will show through numerical results for a few common functional forms that such a \( v^* \) can exist for wide range of parameters \( p \) and \( \lambda T \). Furthermore, we can illustrate the effect of \( p \) and \( \lambda T \) on the cutoff value \( v^* \) and on the probability of an auction being sniped. For example, the benefits from late bidding diminish as \( p \) approaches to 1 reducing the incentive to snipe. This should lower the probability of an auction being sniped. If more bidders are expected to arrive during the auction, it is more likely that a bidder with value above \( v^* \) arrives and starts a war. Therefore, as \( \lambda T \) increases, the probability of an auction being sniped should become smaller.

The sniping-or-war equilibrium is not the only equilibrium where sniping can emerge. An equilibrium with all bidders sniping can also exist.

**Proposition 5.** (All-sniping equilibrium) An equilibrium that all bidders snipe can exist.

**Proof.** Suppose \( \pi_l (0; 1) < 0 \). If solving \( \Delta \Pi_L (v^*; 1) = 0 \) yields \( v^* > 1 \), then all bidders snipe in the equilibrium, because \( \Delta \Pi_L (v_i; 1) < 0 \) for all \( v_i \in (0, 1) \).

In the all-sniping equilibrium, the only outcome is that all bidders wait to bid in the sniping period. Thus, it is always the second highest received bid that decides the payment of the winner and the revenue of the seller. Hence, it is the least attractive equilibrium from the seller’s point of view.

### 4.2.2 Numerical Evidence

Our construction of the sniping-or-war equilibrium depends on the existence of \( v^* \in (0, 1) \) that solves (4.5). This general non-linear equation has value \( v^* \) both as the integration interval and in the expression to be integrated. To the best of the author’s knowledge, exact analytical solutions to such equations do not generally exist. And this is true even for simple cumulative distribution functions such as \( F(v) = v \) in our case. Thus, we turn to numerical methods to solve for \( v^* \) on the interval \((0, 1)\).
For the selection of the distribution functions defined on interval \([0, 1]\), we are particularly interested in the beta distribution (Evans 2000), which is a family of distribution functions parameterized by two positive shape parameters \(\alpha\) and \(\beta\). The probability density function of the beta distribution is:

\[
f(v; \alpha, \beta) = \frac{v^{\alpha-1}(1-v)^{\beta-1}}{\int_0^1 u^{\alpha-1}(1-u)^{\beta-1} \, du}
\]

This density function takes on different shapes depending on the parameters of \(\alpha\) and \(\beta\):

- If \(\alpha = \beta = 1\), \(f(v; \alpha, \beta) = 1\) is the uniform \([0, 1]\) distribution.
- If \(\alpha < 1, \beta < 1\), \(f(v; \alpha, \beta)\) is U-shaped, decreasing if \(v < \frac{\alpha-1}{\alpha-1+\beta-1}\) and then increasing if \(v \geq \frac{\alpha-1}{\alpha-1+\beta-1}\).
- If \(\alpha < 1, \beta \geq 1\) or \(\alpha = 1, \beta > 1\), \(f(v; \alpha, \beta)\) is strictly decreasing.
- If \(\alpha = 1, \beta < 1\) or \(\alpha > 1, \beta \leq 1\), \(f(v; \alpha, \beta)\) is strictly increasing.
- If \(\alpha > 1, \beta > 1\), \(f(v; \alpha, \beta)\) is unimodal, where the mode \(v_m = \frac{\alpha-1}{\alpha-1+\beta-1}\).

For a more intuitive interpretation of how the existence of \(v^*\) changes based on the density of bidders’ values, we can parameterize the family of beta distributions by the expected value \(E(V)\) and the variance \(Var(V)\) according to the following relationship:

\[
\begin{align*}
\alpha &= \left[ \frac{1 - E(V)}{Var(V)} E(V) - 1 \right] E(V) \\
\beta &= \left[ \frac{1 - E(V)}{Var(V)} E(V) - 1 \right] [1 - E(V)]
\end{align*}
\]

Figure 4.1 explains the plane of \(E(V)\) and \(Var(V)^{-1}\), which will be used to illustrate the existence of \(v^*\) in Figure 4.2. Figure 4.1(a) shows how the shape of the beta density function changes with \(E(V)\) and \(Var(V)^{-1}\). Any point on the plane of \(E(V)\) and \(Var(V)^{-1}\) corresponds to a particular beta density function. This plane is divided into four regions and each represents one kind of shape of the density function. An exception is point \((0.5, 12)\), which corresponds to the uniform \([0, 1]\) distribution. We choose the plane of \(E(V)\) and \(Var(V)^{-1}\) instead of \(E(V)\) and \(Var(V)\) simply for an easier presentation.

To illustrate the mapping from the plane of \(\alpha\) and \(\beta\) to the plane of \(E(V)\) and \(Var(V)^{-1}\), we select two groups of functions: \(f_a\) to \(f_e\) which have identical expectation, and \(f_1\) to \(f_5\).
which have identical variance. Their corresponding positions in the plane of \( \alpha \) and \( \beta \) are shown in Figure 4.1(b). The shape of functions \( f_a \) to \( f_e \) are graphed in Figure 4.1(c). In this figure, a flatter shape with wider area underneath implies more dispersed values among bidders, and vice versa. Figure 4.1(d) shows functions \( f_1 \) to \( f_5 \). A function with its underneath area more to the left implies that on average bidders have lower values for the good. As we see from Figure 4.1(c) and (d), \( f_1 \) is monotonically decreasing; \( f_a \) and \( f_5 \) are strictly increasing; and the remainder are all unimodal distributions.

Figure 4.1 Beta family

Figure 4.2 shows the existence of a non-zero \( v^* \) for the beta functions on both the plane of \( \alpha \) and \( \beta \) and the plane of \( E(V) \) and \( Var(V)^{-1} \). We are interested in how the existence of \( v^* \) may depend on the shape of the bidders’ value density, its mean and variance, \( \lambda T \) in the Poisson arrival process, and the probability \( p \) of sniping bids being received. While fixing \( \lambda T = 6 \) and varying \( p \), we numerically search for \( v^* \in (0, 1) \) that satisfies equation (4.5) for each beta function on the plane of \( \alpha \) and \( \beta \) and the plane of \( E(V) \) and \( Var(V)^{-1} \). If such \( v^* \) is
not found, we continue to check whether it belongs to the case in Proposition 5. Figure 4.2(a) and (c) presents these results. For each different $p$, we sketch out the regions that a non-zero cutoff value $v^*$ does not exist, labeled as I; $v^* \in (0, 1)$ exists, labeled as II; and the all-sniping equilibrium exists, labeled as III. Figure 4.2(b) and (d) presents the results for $p = 0.6$ while $\lambda T$ is changing. Both the plane of $\alpha$ and $\beta$ and the plane of $E(V)$ and $Var(V)^{-1}$ are divided into three regions by each set of boundaries corresponding to the same $\lambda T$. Similar to Figure 4.2(a) and (c), a non-zero cutoff value $v^*$ does not exist for the beta distributions in region I; $v^* \in (0, 1)$ exists in region II and the all-sniping equilibrium exists in region III.

![Figure 4.2 Existence of sniping-related equilibria](image)

These figures suggest that certain relationships may exist between sniping and the shape of the density function of bidders’ values. First, if bidders’ values are distributed according to a decreasing density function, the sniping equilibria as we described in Proposition 4 and 5 do
not exist. Second, if bidders’ values are distributed according to an increasing or a unimodal density function, both the sniping-or-war equilibrium and the all-sniping equilibrium can exist. Third, if bidders’ values are distributed according to a U-shape density function, the sniping-or-war equilibrium can exist, but the all-sniping equilibrium cannot. Figure 4.2 (c) and (d) indicate that if bidders’ values are distributed according to the density functions with the same variance \( \text{Var}(V) \), the sniping equilibria can exist for those density functions with high expectation \( E(V) \).

Figure 4.3 graphs the contour of \( F(v^*) \) on the plane of \( \alpha \) and \( \beta \) for four pairs of \( p \) and \( \lambda T \). The figures indicate that the region of beta functions for the all-sniping equilibrium exists with relatively low \( p \) and small \( \lambda T \) and this region decreases as \( p \) or \( \lambda T \) increases. For \( p = 0.6 \), as \( \lambda T \) increases from 6 to 12, the region of beta functions for the all-sniping equilibrium disappears and the region for the sniping-or-war equilibrium decreases. These observations indicate that a sniped auction is unlikely to exist if the probability of late bids being received is high or more bidders arrive at the auction.
4.2.3 Impact on Seller’s Revenue

If the winning bid in the auction is a sniping bid, the seller’s revenue is generally lower than it would be if all bidders bid early. We ask how large this impact might be, depending on the parameters of the model. If \( n \) bidders arrive at the auction and all have values less than \( v^* \), the seller’s expected revenue is

\[
R_{s,n} = \sum_{i=2}^{n} \binom{i}{n} p^i (1 - p)^{n-i} E \left[ v_{(i-1:i)} \mid v < v^* \right]
\]  

(4.6)
where \( E \left[ v(i-1:b) \mid v < v^* \right] \) is the expectation of the second highest order statistic of \( i \) bidders’ values conditional on their values being below \( v^* \):

\[
E \left[ v(i-1:b) \mid v < v^* \right] = \int_0^{v^*} \left( \sum_{i=2}^{n} \binom{i}{n} p^i (1-p)^{n-i} \int_0^{v^*} z \cdot i (i-1) \left[ \frac{F(z)}{F(v^*)} \right]^{i-2} \frac{F(z)}{F(v^*)} \right) \frac{f(z)}{F(v^*)} dz
\]

Substituting into (4.6) gives

\[
R_{s,n} = \sum_{i=2}^{n} \binom{i}{n} p^i (1-p)^{n-i} \int_0^{v^*} z \cdot i (i-1) \left[ \frac{F(z)}{F(v^*)} \right]^{i-2} \frac{F(z)}{F(v^*)} \right) \frac{f(z)}{F(v^*)} dz
\]

\[
= \sum_{i=2}^{n} \binom{i}{n} p^i (1-p)^{n-i} \left\{ \int_0^{v^*} z \cdot d \left[ \frac{F(z)}{F(v^*)} \right]^{i-1} - (i-1) \int_0^{v^*} z \cdot d \left[ \frac{F(z)}{F(v^*)} \right]^{i} \right\}
\]

\[
= \sum_{i=2}^{n} \binom{i}{n} p^i (1-p)^{n-i} \left\{ 1 - i \int_0^{v^*} \left[ \frac{F(z)}{F(v^*)} \right]^{i-1} dF + (i-1) \int_0^{v^*} \left[ \frac{F(z)}{F(v^*)} \right]^{i} dF \right\}
\]

\[
= 1 - (1-p)^n - np (1-p)^{n-1}
\]

\[
- np \int_0^{v^*} \left\{ \frac{F(z)}{F(v^*)} + (1-p) \right\}^{n-1} - (1-p)^n \right\} \frac{F(z)}{F(v^*)} \right) \frac{f(z)}{F(v^*)} \right) \frac{f(z)}{F(v^*)} \right) \frac{f(z)}{F(v^*)} \right) \frac{f(z)}{F(v^*)} \right) \frac{f(z)}{F(v^*)} \right)
\]

\[
= 1 + \int_0^{v^*} (n-1) \left[ \frac{F(z)}{F(v^*)} + (1-p) \right]^{n} - n \left[ \frac{F(z)}{F(v^*)} + (1-p) \right]^{n} - dF \geq 0.
\]

\( R_{s,n} \) is non-decreasing in \( p \): \( \frac{\partial R_{s,n}}{\partial p} = \int_0^{v^*} np \right) - np \right) \right) \right) \right) \frac{F(z)}{F(v^*)} + (1-p) \right]^{n-2} dF \geq 0 \). This implies that late bidding has a negative impact on the sellers’ revenue due to the fact that some of the late bids are not received. It can be easily verified that the seller’s revenue in the early bidding equilibrium conditional on all \( n \) bidders having values below \( v^* \) is given by substituting \( p = 1 \) into (4.7), that is,

\[
R_{e,n} = 1 + \int_0^{v^*} (n-1) \left[ \frac{F(z)}{F(v^*)} \right]^{n} - n \left[ \frac{F(z)}{F(v^*)} \right]^{n-1} dF.
\]

In the appendix, we prove that \( R_{s,n} \) becomes closer to \( R_{e,n} \) as \( n \to \infty \), suggesting that the negative impact of sniping on seller’s expected revenue decreases as the number of bidders increases.
4.3 Conclusions

In this paper, we study a second-price auction with bidders arriving randomly. We demonstrate that late bidding can exist in equilibrium, even when there is no uncertainty about one’s own value and late bids have the risk of not being received. This model has multiple equilibria, including one that only bidders with values below a certain cutoff value snipe. Although this cutoff value is generally not solvable with analytical methods, we provide some numerical evidence to support its existence. At the same time, these numerical results indicate that sniping tends to occur when there are fewer bidders and the expectation of a bidder’s value is high. We also find that the impact on seller’s revenue is negative, but it diminishes as the number of bidders increases.

One limitation to our model is that eBay bidders are allowed to submit multiple bids, which is simplified in our model by restricting bidders to submitting a single bid. A more general model should allow for multiple bids and dynamics in continuous-time framework. Another limitation is that we assume that $v^*$ is time invariant, which makes the comparative statics on $p$ and $\lambda T$ feasible. However, $v^*$ may actually vary according to the time remaining in the auction.

4.4 Appendix

To prove the difference between $R_{e,n}$ and $R_{s,n}$ decreases as $n \to \infty$, we need to prove that $\Delta R_{e-s,n+1} \equiv (R_{e,n+1} - R_{s,n+1}) - (R_{e,n} - R_{s,n}) < 0$ as $n \to \infty$. Let $K (z; v^*) \equiv \frac{F(z)}{F(v^*)}$ and $L (z; v^*) \equiv pK (z; v^*) + (1 - p)$. Note that

\[
R_{e,n+1} - R_{e,n} = \int_0^{v^*} nK (z; v^*)^{n+1} - (n + 1) K (z; v^*)^n \, dz \\
- \int_0^{v^*} (n - 1) K (z; v^*)^n - nK (z; v^*)^{n-1} \, dz \\
= n \int_0^{v^*} K (z; v^*)^{n} \left[ -2K (z; v^*) + 1 + K (z; v^*)^2 \right] \, dz \\
= n \int_0^{v^*} [K (z; v^*) - 1]^2 K (z; v^*)^{n-1} \, dz
\]
and $R_{s,n+1} - R_{s,n} = n \int_0^{v^*} [L(z; v^*) - 1]^2 L(z; v^*)^{n-1} \, dz$. Therefore,

$$\Delta R_{e-s,n+1} = (R_{e,n+1} - R_{s,n+1}) - (R_{e,n} - R_{s,n})$$

$$= (R_{e,n+1} - R_{e,n}) - (R_{s,n+1} - R_{s,n})$$

$$= n \int_0^{v^*} [K(z; v^*) - 1]^2 K(z; v^*)^{n-1} - [L(z; v^*) - 1]^2 L(z; v^*)^{n-1} \, dz. \tag{4.9}$$

Let $\gamma(x) \equiv (x - 1)^2 x^{n-1}$. $\gamma(x)$ is strictly increasing on the interval $(0, 1)$ as $n \to \infty$, because

$$\frac{d\gamma(x)}{dx} = (x - 1) x^{n-2} [2x + (x - 1) (n - 1)] > 0, \forall x \in (0, 1), \text{ if } n > \frac{1 + x}{1 - x}.$$

Therefore, $[K(z; v^*) - 1]^2 K(z; v^*)^{n-1} < [L(z; v^*) - 1]^2 L(z; v^*)^{n-1}$ for $z \in (0, v^*)$ due to the fact that $K(z; v^*) < L(z; v^*)$. Furthermore, $K(0; v^*) < L(0; v^*)$ and $K(v^*; v^*) = L(v^*; v^*)$. Thus, $\lim_{n \to \infty} \Delta R_{e-s,n+1} < 0$ follows.
References


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