A STUDY ON MIDDLE SCHOOL STUDENTS' USE OF COMPUTER-GENERATED REPRESENTATIONS AS THEY SOLVE PROBABILITY TASKS

BY

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ABSTRACT OF THE DISSERTATION:

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This study examined the problem-solving behavior of four students from an urban, middle school as they used computer simulation software to solve probability tasks, by generating and interpreting computer data and representations to make decisions about fairness and adequacy of sample size. The questions that guided the study were: (1) How are data generated by the students from computer simulations interpreted with respect to (a) fairness and (b) significance of sample size? (2) What decisions about fairness and adequacy of sample size do students make on the basis of evidence that they collect? and (3) How are student ideas influenced, if at all, by their computer-generated representations and others?

The students were video-taped during five sessions which occurred on two days of a summer institute, a component of the Informal Mathematical Learning (IML) Project at Rutgers University. Data consisted of discussions between and among students as they worked in pairs on the task, conversations between students and researchers, screen-shots of computer representations that students selected and discussed, and students’ written work recorded on CDs. These were analyzed using the Powell, Francisco & Maher
(2003) model for investigating the development of mathematical knowledge using video data.

Analysis of the data revealed that the simulation software, together with social interaction, resulted in students' making and testing conjectures about a sophisticated concept, the Law of Large Numbers. The type of representations that were chosen by students also influenced their arguments. The students agreed that fair dice have a uniform frequency distribution; however, they also agreed that a fair die could have an outcome that alternated between having the highest and then lowest frequencies in two separate experiments.

This study contributes to the data base that documents the building of mathematical ideas as students work on investigations in supportive environments, and addresses a gap in the probability education literature for studies of middle-school students using simulation software to generate data and representations that support their claims.
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DEDICATION

For Bob, Lisa and Bobby,
the loves of my life.
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Chapter 1 INTRODUCTION

1.1 Statement of the Problem and Significance

This research examines four middle-school students as they used computer simulation software to solve probability tasks. Through careful analysis of video data and written documents, this qualitative study investigates how the students generated and interpreted computer data and representations, and made decisions about fairness and adequacy of sample size. This study considers how those interpretations and decisions emerged from and were influenced by the students' interactions with the computer, researchers, and each other. It is argued that this research addresses an empirical gap in the literature, because it provides a window into the ideas of middle-school students as they solve probability tasks when computer-simulation software is available.

In the document Principles and Standards for School Mathematics (NCTM, 2000), the National Council of Teachers of Mathematics (NCTM) specifies five content standards that all students should learn in prekindergarten through grade 12. Data Analysis and Probability is one of the recommended content standards. This NCTM document suggests that knowledge about data analysis and other probability concepts is needed for statistical reasoning, which is an essential skill for informed citizens and educated consumers (p. 48).

In the middle-school curriculum, typical problems in data analysis and probability can connect mathematics to students’ personal experiences as well as other disciplines. For example, the NCTM (2000) recommends that middle-school students design surveys to collect relevant data with a goal of comparing characteristics within a population or
between populations (p. 249). Moreover, because the middle-school science curriculum includes experiments with data collection, the design of these experiments should include input from mathematics teachers. Data analysis can be situated in statistical investigations, a process with four non-sequential components: pose a question, gather data, analyze data, and develop and communicate conclusions (Friel & Bright, 1998). The natural curiosity in children provides an easy segue into the study of data analysis and probability.

Applications involving probabilistic reasoning can be used to stimulate discussion of different ways of thinking about the same problem and possible different conclusions. In other content areas such as algebra, geometry and measurement, students are accustomed to seeing one correct answer that is the result of an algorithmically-driven process. The work of students is validated when they can find “the correct answer.” In studying data analysis and probability, students learn to recognize that some problems may have several solutions, each of which can be justified by making certain assumptions. Students have the opportunity to decide whether there is evidence to support a conclusion when they learn how to reason statistically and understand how to use data analysis and probability concepts (NCTM, 2000).

Given the importance of the NCTM (2000) Data Analysis and Probability content standard, there is a problem. The kind of reasoning used in statistics and probability is not always intuitive (p. 48). In fact, there are many studies that relate the different misconceptions that students have about probability (Fischbein & Schnarch, 1997; Kahneman & Tversky, 1982; Konold, 1991). In an attempt to overcome the non-intuitive nature of this reasoning, the NCTM (2000) has recommended that students need to work
directly with data. The study of data and statistics gives students the opportunity to learn that some solutions depend on assumptions (p. 48), and that there is not always one correct and definitive answer. The NCTM further recommends that as a student progresses through each grade, examples of the probability for events with predictable outcomes should evolve to experiments that require sampling and simulations, so that students can quantify the probability of an uncertain outcome (p. 51).

The computer can serve as a helpful tool for sampling and simulations, and to help avoid or overcome erroneous probabilistic reasoning. As such, the NCTM (2000) endorses the use of technology in the classroom (p. 254). Particularly in explorations involving data, computer software can organize data and assist students in the analysis of data. Computers can provide simulation experiences that can help students learn mathematics. Students can use computers to examine more examples or representations than they would experience by hand (p. 25). Toward that end, recent studies have investigated the impact of using technology in the classroom to dynamically simulate probability experiments and visually represent and manage the resulting data (Ben-Zvi, 2000; McClain & Cobb, 2001; Pratt, 2000; Stohl & Tarr, 2002).

The purpose of this study is to examine how students interpret data and make decisions about fairness and adequacy of sample size when computer simulation tools are available. As noted, the NCTM (2000) supports the use of computer simulations because they give students the option of quickly and easily generating large samples, so that students can focus on analyzing the data (p.254). The use of technology also provides students with an easy mechanism for testing their conjectures.
There is limited research in the probability education literature studying the effect of computer simulation tools when they are used by middle school students to solve a probability task, particularly in situations where students have the opportunity to generate and collect what they consider to be “enough” data and decide which, if any, computer representations help convince others of their solutions. The results of this study will have implications for curriculum design and practice using computer software in middle school mathematics units involving probability.

Studying the building of probability ideas in children is particularly difficult. As Greer (2001) questions, how can a researcher pose a question on probability in order to determine the development of probabilistic understanding if the student has not yet learned about probability? Yet Shaughnessy (1992) points out that a logical method is to give related tasks to students and attempt to interpret students’ results. This dissertation is a report on students’ use of computer representations, situated in probability tasks. This is a basic study which attempts to provide a window into the ideas of children as they work collaboratively on probability tasks when simulation software is available, and to provide evidence of how students make data-based decisions about sample size and fairness of dice.

1.2 Background

Research on the development of mathematical ideas has been the focus at Rutgers University for many years. Findings from a longitudinal study and other projects using videotaped data indicate that students can build mathematical ideas and construct convincing arguments within supportive environments (Maher, 2010). These findings
have been extensively documented (Kiczek, 2000; Kiczek & Maher, 2001; Maher & Martino, 1996; Powell, 2003; Speiser, Walter, & Maher, 2003; Steffero, 2010) and, building on these experiences, researchers at Rutgers proposed an Informal Mathematical Learning (IML) Project. The IML Project was supported by the National Science Foundation (ROLE Grant REC0309062, directed by Carolyn A. Maher, Arthur B. Powell and Keith H. Weber). Beginning in fall, 2003, this three year research project focused on the mathematical thinking of minority middle school students in Plainfield, an urban community in central New Jersey. The IML project followed previous professional development partnerships with the Plainfield Public School system that involved after school programs.

During the three years, IML students worked on well-defined tasks in the areas of counting and combinatorics, algebra, and probability. As in the previous studies at Rutgers, students worked collaboratively, listened carefully to their peers, justified their ideas, and decided whether they were convinced by the reasoning of their peers (Maher, 2010). The IML setting was a supportive classroom environment in which all student contributions to discussions were acknowledged. Students were encouraged to make claims, and they were asked to give justifications based on evidence. All justifications were respected. Students were never told if their answer was correct. Rather, insufficient justifications or incorrect reasoning resulted in gently-probing questions from the researcher and an invitation for further discussion among the students. In fact, student initiated questions often spontaneously became the next task for the entire group to explore.
As in the previous Rutgers' studies, video and written data were acquired and analyzed (Maher & Uptegrove, 2010). Following each IML session there was a debriefing session, during which researchers, graduate students and teachers discussed what had been observed and provided input into the planning of the next session. The IML study differed from previous studies in that just middle school students were participants. Moreover, students engaged voluntarily in open-ended investigations only during informal after school sessions and summer institutes. At some IML sessions, students had the opportunity to use technology, such as computer software that simulated probability experiments. As a result, the IML data provides the opportunity to study which, if any, computer representations students selected to provide evidence that justified their solutions to probability tasks.

1.3 Focus and Research Questions

This dissertation is a case study of four student participants in the IML project as they worked in pairs on a well-defined probability task during a four day summer institute in 2005, just before they entered eighth grade. During this summer session, students spent time each day in a computer classroom where they could use Probability Explorer (Probability Explorer (c) Hollylynne Stohl Lee, 1999-2005), a software tool that simulates experiments and provides graphic and table representations of data. This dissertation focuses on probability tasks that the students worked on during the last two days of the session. (The task descriptions are included in Appendix A.) In one of the tasks, the Schoolopoly task, students were asked to decide if the dice manufactured by different companies were fair. Some of the probability ideas that had the potential to be addressed in the design of this task were sample size, inference, theoretical and
experimental probability, and fairness. Students could use Probability Explorer (PE) to simulate rolling a die as many times as they decided were needed in order to be convinced that the dice were fair. The computer-generated representations included an actual dynamic model of the face of the die that was rolled, as well as a pie chart, a bar graph and a data table that provided the frequency and the relative frequency displayed as a fraction, a decimal and a percent. The graphs and table were dynamically changing during the simulation, and students came to think of different numbers as “catching up to” or “falling behind” other numbers, similar to a game. In order to justify their determination of the fairness of the die, students could choose to use some, all, or none of these computer-generated representations.

This study examines the choices and decisions these students made as they generated computer data and representations to solve the probability tasks. The research questions guiding this study are:

1) How are data generated by the students interpreted with respect to (a) fairness and (b) significance of sample size?

2) What decisions about fairness and adequacy of sample size do students make on the basis of evidence that they collect from computer simulations?

3) How are students' ideas influenced, if at all, by their computer-generated representations?

4) How are students' ideas about fairness and sample size influenced, if at all, by others?
In earlier Rutgers’ studies on the development of statistical ideas (Benko, 2006; Kiczek, 2000), computer-generated simulations were not accessible. Shay (2008) examined the IML middle-school students as they explored experimental data, created sample spaces, and made decisions about fairness based on the data they generated as they played games with dice that were assumed to be fair. A conclusion from that study is that these IML students needed additional experience with experimental probability.

What is unique in this study is that the students experimented with gathering data from experiments with a posteriori sample spaces. The inclusion of computer-generated representations to build on explorations in which students generated sample spaces and reasoned from the data they generated, makes this study different from previous longitudinal work at Rutgers. Hence, a goal of this dissertation is to extend prior Rutgers longitudinal research documenting students’ construction of mathematical ideas without the use of computer-generated representations.
Chapter 2 THEORETICAL FRAMEWORK AND LITERATURE REVIEW

2.1 Introduction

The framework for this case study is derived from a constructivist perspective on learning, which is a theory of knowing based on a person’s experiences (von Glasersfeld, 1995). In the classroom, this translates to creating experiences that engage the learner in the learning process by making a conjecture, testing that conjecture, sharing and defending results, and reflecting on personal results as well as results of others (Davis, Maher & Noddings, 1990; Yackel & Cobb, 1996). This study recognizes the power of the environment on building understanding (Noddings, 1990), and is guided by research on the development of representations (Maher & Davis, 1990) as well as external representational systems (Goldin, 1998; Kaput, 1998). The related literature includes studies on the building of probability ideas such as fairness, significance of sample size, and reasoning from data in computer environments, and the use of representations selected from computer-generated graphs and tables in order to provide evidence and convince others.

2.2 Ways to view probability

An individual may consider the probability of an event, such as getting heads or tails when flipping a coin, in three non-distinct ways, according to Borovcnik, Bentz & Kapadia (1991). The classical approach includes the theoretical perspective resulting from the sample space, a priori doing the experiment. Theoretically, when tossing a coin, the probability of getting heads is \( \frac{1}{2} \) because there is one head out of two possible
outcomes (heads or tails) in the sample space. While the classical approach depends on the correct establishment of the sample space (Horvath, 1998), the frequentist actually does the experiment, and makes a decision based on outcomes a posteriori conducting trials.

The frequentist approach can be supported by the data that is the result of conducting trials. However, the question that is of concern for the frequentist, is whether enough trials have been made to substantiate a final decision. For example, if an individual tosses a coin ten times, it is possible to get 10 heads, yielding the experimental result that the probability of getting heads when tossing a coin is 100%. This result is, of course, false. While it is still possible to get 100 heads from 100 tosses, it is much less probable. In fact, with 100 trials, it is more likely to have an outcome close to 50 heads and 50 tails. Here, the Law of Large Numbers becomes relevant. The Law of Large Numbers connects the frequentist view to the classical view by stating that if you repeat a random experiment a large number of times, your outcomes on average should be the same as the theoretical average. However, studies have demonstrated that this connection is not intuitive (Fischbein and Schnarch, 1997). Moreover, Wagner (2006) presented college students with a series of isomorphic probability tasks, and has argued that the Law of Large Numbers is not transferable. In his detailed case study of one undergraduate student, Maria, Wagner found that initially she perceived tasks which were all instances of this mathematical principle as "different." Eventually, through engagement with materials developed to support her learning, Maria was able to identify these tasks as being "alike" (p. 3).
The subjectivist view of the probability of an event results from personal experience. For example, based on personal experience, an individual may simply not believe that if a coin is tossed 100 times it is possible to get 100 heads. To address this, a recommendation from the Chinn and Samarapungavan (2001) study of the difference between students’ understanding and belief, is to create tasks that enable students to compare and contrast their thoughts to those of other students to enable understanding. Another possible belief for the subjectivist is called "the negative recency effect" or "the gambler's fallacy" (Fischbein and Schnarch, 1997). An individual may think that if a coin is tossed 10 times and that a head has come up each time, then tails is more likely to come up on the next toss. In actuality, the probability of getting tails on the eleventh trial is still ½. These are two common misconceptions that can occur when a subjective decision regarding outcomes is made.

This research study draws from frequentist and subjectivist views. The students in this study repeatedly performed an experiment, at first by hand and then later using computer simulation software which permitted them to run as many trials as they considered necessary. With computer simulation software, instead of a student just being asked “What do you think would happen if we did this 10 times, 500 times or even 1000 times?”, the student can be engaged in actually doing the experiment and getting immediate results. The students in this study also used their own personal experiences and beliefs when interpreting outcomes of experiments and making conclusions.

In addition to these ways of considering the probability of an event, this study recognizes the influence of intuition. One of the seminal theorists on probabilistic thinking in children, Efraim Fischbein, noted that probabilistic reasoning incorporates
intuition (1975). He further defined primary intuitions which come from individual experience, and secondary intuitions which can result from what is learned in a school environment (p. 117). The misconceptions outlined previously because of personal beliefs, intuitions or experiences, as well as Fischbein’s definition of secondary intuitions, call attention to the importance of a supportive school environment in the process of building ideas about probability.

2.3 **Elements that promote the building of probability ideas**

2.3.1 **Providing time for collaborative work**

Recommendations for a school environment that supports learning have emerged from constructivist theories and studies. Of primary importance is the design of an activity or task that will interest and therefore engage students (Hiebert et al., 1997; Derry et al., 1998). In Fischbein’s studies, children were attentive to activities that simulated games, but which are at the same time valid scientific experiments. “The most surprising finding was that even those children who had previously been weak in mathematics were attracted by this type of activity” (1975, p.95). Using the computer to simulate game-like experiments and designing activities that interest students are arguably important factors in engaging students and promoting student understanding. However, those elements would not be as effective without an environment that includes time for students to work collaboratively, communicate, and build social communities (Davis, Maher & Noddings, 1990; Yackel & Cobb, 1996). Children gradually internalize the talk that occurs in groups (Vygotsky, 1978).
2.3.2 The importance of teacher education

When a student answers a question, it is more important to listen for the level of understanding that the student has demonstrated, rather than simply evaluating the content of the response (Pirie & Kieren, 1994). Findings from Kahneman and Tversky (1982) support the importance of an environment where the lead researcher does not judge responses or in any way make students feel that there is an expectation for only acknowledging correct responses. Moreover, Konold (1991) supports the position that teachers should refrain from just giving the correct answer. These reactions to students’ responses are not necessarily natural for the classroom teacher. Maher & Alston (1990) note the importance of professional development for teachers. Of particular need is the education of classroom teachers in the art of accepting students' ideas and suggestions with an open mind without judging, and using student input to give direction to additional tasks. This type of spontaneous reaction comes with belief and confidence in the approach, as well as practice and experience.

2.3.3 Convincing others by providing evidence to support a claim

In addition to giving students the responsibility to resolve conflicts themselves and not acquire resolution from the teacher when solving a task, a supportive environment is one in which students are asked to convince others by giving evidence (Pirie & Kieren, 1994). Yackel and Hanna (2003) posit that there is a strong link between the nature of the mathematical explanations and justifications that students give in a classroom and the mathematical learning that takes place (p. 234). The evidence that students use to convince others can come from their own representations. When students are given time to work collaboratively on tasks – particularly those that seem to have
more than one possible solution, and build ideas in an environment that supports answers
that are given with justification, then students are more likely to build representations that
help justify their solution (Maher & Martino, 1996).

2.3.4 Representations and computer-generated representations

Maher and Davis (1990) found that observing and analyzing students’
representations provides a window into assessing their understanding of mathematical
concepts. The relation between representations and the student’s ability to communicate
mathematical ideas, and the importance of the teacher’s role in recognizing and building
upon those representations has been well-documented (Benko, 2006; Davis & Maher,
1997; Francisco & Maher, 2005; Kiczek, Maher & Speiser, 2001; Maher & Martino,
1996; Muter & Uptegrove, 2010; Uptegrove & Maher, 2004). When situated in data
analysis, representations help students connect data to the experiment, possibly helping
them reason about the data. Yet one type of representation is not superior to another - bar
graphs are not necessarily better than pie graphs or tables; each may be a better
representation than another, depending on the goal (Konold & Higgins, 2003).

The use of technology provides an easy and quick way to generate these
representations for data analysis. Goldin (1998) noted that computer technology provides
dynamic configurations that are "no longer static like pencil-and-paper productions" (p.
146). Kaput (1998) recognized that as new analytical tools become available, we are
entering a new era in the study of representation. The advent of dynamic computer
environments and the increased ability to explicitly display external representational
structures, place an additional responsibility on educators to "render ever more
mathematics learnable" (p. 279).
Teacher use of technology in the classroom was relevant in a study involving twenty-five 2nd grade students. Horvath and Lehrer (1998) found that the general shape of the distribution and frequencies for each outcome from rolling a pair of dice became obvious when students graphed their individual results and then shared their findings. When the teacher used technology to graph the final class results on one bar graph that represented the aggregate findings, the differences between outcomes for rolling the dice became very visual on the resulting bar graph. As a result, the students began to understand the mathematical/theoretical relationship between the sample space and the probabilities of outcomes. Without the graphs, they were less able to reason about relationships between individual results and patterns of results (p. 138).

During the fall of 1994, Friel and Bright (1998) studied the ways that middle school students made sense of information in graphical representations of ungrouped and then grouped data. The students found line graphs easier to interpret than bar graphs, particularly when the bar graphs were paired as ungrouped and grouped data. Although they were able to read frequencies from a stem-and-leaf plot and histogram, the students had difficulty making inferences about the data. “Making correct interpretations of increasingly sophisticated representations requires more fully developed cognitive functioning” (p. 82). One recommendation from the Friel and Bright study was that students should have practice comparing multiple representations of the same data set. This recommendation is addressed in this case study. The four students in this case study were able to use Probability Explorer simulation software to generate multiple representations of the same data set.
There are considerably fewer studies that examine the role of computer-generated representations, particularly for students in middle school. In a study on collective mathematical learning, Cobb (1999) introduced computer “minitools” that he and other researchers have designed, based on research, to offer students several ways to structure data. While working with twenty-nine 7th grade students, Cobb noted that students spent the first four classes of thirty-four lessons over a period of ten weeks relaying results of instructional tasks that had been assigned, without consideration of input from listening students. When computer minitools were introduced in the fifth lesson, a major shift began to occur in the class discourse. The initial minitool allowed students to sort the data by size and color, so that students could compare data sets or even hide a data set to consider one set. A second minitool introduced ways to structure the data that involved multiplicative reasoning. When the use of minitools became available, students began to use data-based arguments to support their claims and critique the claims of others, and some students demonstrated evidence of progressing from additive reasoning about data to the more sophisticated multiplicative reasoning. Additional studies on the effects of computer representations in the development of mathematical and probabilistic reasoning (Ben-Zvi, 2000; Groth, 2006; Lavy, 2006; McClain & Cobb, 2001; Pijls, Dekker, & van Hout-Wolters, 2003; Pratt, 2000; Schorr, 2003; Stohl & Tarr, 2002; Wu, 2003) will be discussed further in the next section.
2.4 Probabilistic Reasoning

2.4.1 Reasoning about fairness, significance of sample size, and reasoning from data in studies without technology

While Fischbein noted the importance of intuition in probabilistic reasoning, he and many other researchers have brought to light the many misconceptions that can result from this type of reasoning. For example, when tossing a coin, if ten heads really did come up in ten tosses, the fairness of the coin may be suspect. An individual may question whether the coin is weighted, based on personal experience and the expectation that heads should only show up approximately 5 out of 10 times. In a study that addresses this perception, middle school students were interviewed to learn their beliefs about fairness of dice and their strategies to establish fairness (Watson & Moritz, 2003). Students were given actual weighted dice instead of the computer simulation of a weighted die. As a result, students had differing thoughts, including that some numbers were “luckier” or that all the dice may not be fair. Some students proceeded to roll the dice and note results, yet retained their own beliefs (even three to four years later), such as expecting that outcomes of one and six occur least frequently. Recommendations from this study included adding questions on fairness as a topic in the school curriculum, giving more time to experiments that students can perform, and instructing teachers on the best way to encourage an understanding of fairness. It is interesting to note that computer simulation tools can provide the experience of performing the experiments, but in a much shorter amount of classroom time.

In another study on fairness of dice in which students did not use technology (Horvath & Lehrer, 1998), students noticed that individuals rolled dice in differing ways, which could affect the outcome. In an attempt to keep conditions of the experiment
constant, the students decided that each student should roll dice from a cup. This particular problem is avoided when computer simulation is used.

Shay (2008) studied urban, middle-school students who attended an Informal Mathematical Learning (IML) after school program. She traced the development of probabilistic reasoning through two years, grades 6 and 7. Students were engaged in open-ended tasks designed to build ideas about chance. They investigated dice games to determine if they were fair, and devised strategies to make the games fair. Four of the students in Shay's study are the subjects of this case study: Kianja, Chanel, Chris and Jerel.

During the Grade 6 IML after school sessions on probability, students were given games to analyze which involved a pair of standard dice. Kianja, Chanel, Chris and Jerel participated in that activity. In one game, Player A scored a point if the sum was 2, 3, 4, 10, 11, or 12. Player B scored a point for a sum of 5, 6, 7, 8, or 9. Students had to decide whether or not the game was fair. According to Shay, initially Chris and Jerel thought the game was unfair because Player A had more numbers than Player B. But, after playing the game, Chris noted “Because after we played the game we realized that um Player B had, since it had larger numbers it had more chance of getting ‘em” (p. 82). Moreover, Chris and Jerel observed that 7 appeared frequently in their games, because, as Jerel noted, “Oh because it had a better chance, because it had three ways to get it” (p. 79). When asked to consider if a sum of 6 could appear as frequently as a sum of 7, since there were three ways to obtain a 6, Chris theorized that the “small” numbers on a die, 1, 2, and 3, were less likely than the “large” numbers, 4, 5, and 6. The boys tested both theories for tossing a pair of dice: 1) small numbers are less likely to occur than large
numbers, and 2) it is as likely to roll a sum of 6 as it is to roll a sum of 7. The result of 22 rolls yielded 12 small numbers and 10 large numbers. Jerel concluded that “the big numbers don’t always show up” (p. 80). Shay noted that Jerel used experimental evidence and trusted the results even if the sample size was small. However, Chris viewed experimental data as untrustworthy (page 185). Shay recognized that larger samples may convince Chris. Subsequently, during the summer following grade 7, these students used Probability Explorer to generate larger samples, and decided how large to make the samples in order to make decisions about fairness.

According to Shay (2008), during the grade 6 dice games, Jerel expressed an interesting perception of fairness: if either player is able to win a game, then the game must be fair. Shay created the term hybrid heuristic for chance events to describe this perception.

An instructional curriculum designed to determine whether simulations of random phenomena impact students' understanding of the role of sample size in experimental probability, was studied by Aspinwall and Tarr (2001). A cognitive framework (Jones, Thornton, et al., 1999) was used to evaluate the influence of simulation games and discussions on students' probabilistic reasoning. Twenty-three 6th grade students were assessed prior to the instruction to determine awareness of the role of sample size in experimental probability, and then after the 5-day instructional program. Following a series of problem tasks in which students drew inferences from experimental data at various stages, the study found that the majority of case study students had demonstrated a higher level of thinking in their understanding of experimental probability. Small and whole-group discussions were recognized as contributing to this growth. However, not
all students experienced growth in understanding. In fact, some students were distracted by irrelevant aspects of the games, and other students were misled by erroneous intuitions. Two of the games: The Race Game and To Sum It Up, were found to connect data and chance. The games in this study were played manually, without the use of computer software.

While interviewing high school students, Groth (2006) asked students to compare results from two samples and to relate which results were more valid. Students did not all acknowledge that the sample with more data resulted in conclusions that could be trusted more. In fact, one student who had a year of statistics noted that there would be no difference between the mean of the smaller sample and the mean of the larger sample. Groth recommended the use of computer-simulation software in the classroom, and noted the importance of understanding how students reason about probability in order to encourage growth in that domain.

2.4.2 Reasoning about fairness, significance of sample size, and reasoning from data in studies with technology

Pratt (2000) investigated the effect computer tools had on students’ internal resources for stochastic understanding. Students aged 10 and 11 worked on tasks involving the sum of two dice and spinners. These "gadgets" were generated by computer software (p. 609). Pratt found that in a supportive environment, where students can make conjectures as well as share and compare results, the technology gave students the opportunity to experience the consequences of their beliefs and provided the best evidence so that they could restructure their beliefs and connect to what they are experiencing externally through the computer tools.
Computer simulation tools also permit students to replicate experiments as many times as they consider necessary. For some students, tossing a die 6 times provides enough information to determine fairness. Other students are not convinced of the fairness of a die unless it is rolled 1000 times. Technology gives students the opportunity to try both approaches and compare results. The significance of sample size is related to the Law of Large Numbers, which is not an intuitive concept (Konold, 1991). Biehler (1991) concurs that without computer support it is difficult to work with large numbers, and long run frequencies remain mysterious (p. 194).

In a study involving high school students, Pijls, Dekker, and van Hout-Wolters (2003) developed learning materials for 16-year-old students in the domain of probability theory, consisting of computer simulations based on a gambling game. Three groups were studied: those who used the computer simulations before learning the concept, during the learning, and after the learning. The “during” group increased to the level of perception (a higher level of understanding), indicating that enhancing instruction with computer simulation tools may provide the best strategy for probabilistic learning.

2.4.3 Studies on making inferences from data when using computer tools

The NCTM (2000) recommends that high school students should use simulations to learn about sampling distributions and make informal inferences (p. 50). Several researchers have introduced computer simulation software to children and middle school students, and observed how the opportunity to use the software affected their probabilistic reasoning and understanding of inference.
A pilot study (Drier, 2000) using Probability Explorer (PE) computer simulation software with fourth-grade students, resulted in students observing an "evening out" of the dynamic representations. Subsequently, in 2001, Stohl and Tarr led a twelve-day teaching experiment with 23 average-level sixth grade students. The students worked in pairs on six problem-based tasks designed to elicit student’s understandings of fairness, randomness and theoretical probability (Stohl & Tarr, 2002) using PE. Two of the tasks – mystery marble bags and Schoolopoly (see Appendix A) were replicated during the IML summer session, and the Schoolopoly task is analyzed in this study.

The findings from Stohl and Tarr (2002) indicate that students used the PE tools to help them make informal inferences about the content of marbles in the bag when samples were taken with replacement, as well as fairness of a die, and that they supported their inferences with data. The two students in the case study, Brandon and Manuel, learned that trends in data from sufficiently large samples can be used for making inferences, and that small samples can lead to erroneous conclusions (p. 334). An initial constraint of using the software was noted, when one of the students entered a small sample size for the marbles. As a result, he received information about the quantity of red marbles but no useable information from the computer-generated tables and graphs about the quantity of the other two colors. This led to the development of the student’s recognition of the importance of running a large number of trials. It is interesting to note that although the student did run a large number of trials to infer the number of marbles of each color, he used the results of running 12 trials as evidence for his inference perhaps since he knew that the bag only contained 12 marbles.
In another related study, middle school students were given the opportunity to work on genuine data analysis problems during two twelve-week semesters (McClain & Cobb, 2001). Researchers worked with a group of seventh grade students, and some of the same students were included when the experiment continued the next year. Each semester, students received instructional activities and could use computer tools to assist in data analysis. The purpose of the sequence of instructional activities was to determine if students developed an understanding of important statistical ideas and supported conclusions with data-based arguments when computer tools were available. Students could use the computer tools to explore the data and create graphs to complement the computational methods of statistical inference (p. 108).

In one of the instructional activities, the researcher asked students for advice on which type of car to buy if safety was a major factor in the decision-making process. Students were able to enter different braking speeds of cars and discuss and compare resulting graphs. The computer tools and class discussions in which the researcher used the computer-generated graphs to discuss different braking speeds of cars provided a strong imagery of the situation.

Computer tools also made it convenient to investigate larger data sets in subsequent tasks. In the 8th grade, students were asked to recommend one of two ambulance companies to the school board, based on response time. Students discussed the data collection process and worked in pairs to analyze the data. Most of the students were able to use the computer tools to hide individual data points in order to infer trends and patterns in the data from the resulting graphs.
A desirable outcome for this study was observing a progression from reasoning additively (where students focus on the absolute frequency) to multiplicative reasoning (where students focus on relative frequency), which is considered necessary in creating traditional graphs such as histograms and box-and-whiskers plots. Findings indicated that students with more sophisticated arguments included more graphs in their descriptions, and that there was a shift to multiplicative ways of reasoning among students.

2.4.4 Computer-generated representations used in mathematical and probabilistic reasoning

Technology tools allow students to be more active participants in exploring data and making observations about tasks which can be situated in the context of other disciplines. Computer simulations are found to work well with discovery learning (deJong & vanJoolingen, 1998). Schorr (2003) gave middle school children the opportunity to explore ideas related to rates of change using dynamic software that generated graphical representations of velocity and motion. The findings of this study indicate that it is possible for middle school students to reason about mathematical ideas that are not usually part of their math curriculum, when these ideas are introduced in a supportive environment combined with the opportunity to observe and analyze the ideas in a dynamic way. In a related study, ten 7th-grade students met after school in the computer lab. The conversations of two students, who discussed out loud the computer images they generated as they worked with geoboards, were captured on video and studied by Lavy (2006). She analyzed their understanding by studying their ensuing arguments. Lavy looked at different modes of work, categorizing by environment-
dependent and environment-independent, and noted that the computerized environment contributed to their ability to support claims and mediate differences.

Technology tools provide visualization and simulation, which help students understand statistical concepts and methods. Ben-Zvi (2000) studied the effects of continuous and immediate access to technological tools on students' construction of meaning in mathematics and statistics (p. 131). Middle school students used spreadsheets to collect and analyze data, construct graphs and make inferences in a supportive classroom environment. The dynamic nature of the multi-representational software allowed students to add or delete data points to consider the immediate effect on the graph (p. 135). The ability to design and alter graphs effortlessly and immediately helped the students develop “scale sensitivity” (p. 137). This study suggests that technology tools extend cognition and enhance the sociocultural environment.

Wu’s (2003) work with determining if certain visual depictions are more influential than others in improving mental stimulation in the context of probability tasks (p. 25), resulted in findings that were most promising - visual representations have positive effects on students’ learning performance and the ability of transferring what they have learned to more complex tasks (p. 87). Unfortunately, his work was with graduate students at Teachers College, Columbia University, and may be difficult to generalize to younger students. Nevertheless, it contributes to the studies which demonstrate the impact that computer software and graphing technology can have on the teaching and learning of probability concepts because of the visualization they provide, as well as the opportunities to discuss data that immediately and effortlessly results from experiments with large samples.
2.5 The intent of this study

The collective studies in this literature review have a common strand - they acknowledge and emphasize the importance of the inclusion of probability concepts in the K-12 curriculum. Although there are studies that document how students reason about fairness and significance of sample size, there is a need for studies which investigate the role of technology in facilitating the connection between experimental and theoretical probability (Jones, 2005). This study addresses a call in the literature to examine the result of providing students with experiences that generate experimental probability, when technology is used to repeatedly produce random experiments.
Chapter 3 METHODOLOGY

3.1 Research Design

This dissertation is a qualitative study because, according to Marshall and Rossman (2006), “context matters” (p. 53). The study takes place in an after-school classroom setting, and that environment is critical to analyzing and interpreting how students working in pairs reason about probability concepts. Several sources of data were used for the analysis. These include documents such as the students’ work and graduate student notes, as well as video recordings of the students working collaboratively, their discussions with the researchers and other students, and the representations captured on their computer screens. The video data provides the least intrusive yet most inclusive way to study students' growth in understanding, particularly when that data is analyzed along with documents (Goldman-Segall, 1998).

This dissertation studies four of the eleven students who attended an IML summer institute that extended from August 1, 2005 through August 4, 2005. The IML summer session was held in Washington Elementary School in Plainfield, New Jersey. The four middle school students included two girls, Chanel and Kianja, and two boys, Jerel and Chris. Chanel, Kianja and Jerel are African-American, and Chris is Hispanic. Chanel and Jerel are twins. These four students were beginning their third year of the IML after-school program. They were purposely selected because both pairs of students were consistently video-recorded throughout each session, and there are screen-shot videos of the computer work generated by both pairs of students. Moreover, their collaborative work represents different perspectives on how to justify the solution to the probability tasks. The study qualifies as a case study since, according to Creswell (1998) it is
bounded by time and place. In addition, extensive, multiple sources of information in data collection were analyzed to provide insight into the students’ work (p. 37).

3.2 Setting

Plainfield, New Jersey is a city in north-eastern New Jersey, located twenty-eight miles west of New York City. With a population of approximately 47,000, Plainfield is an urban city that has around 16,000 houses of which half are renter occupied. The estimated median income is about 15% lower than that of the state. Approximately 62% of the residents are black, 25% are Hispanic, and 11% are white-non-Hispanic. Plainfield has one high school, two middle schools and eight elementary schools. Within the public school system, 98% of the students are African-American or Latino.

In August 2005, sessions of the IML summer institute were held in a Plainfield elementary school that had air-conditioning and was a relatively modern facility. Two rooms were used - a computer classroom and the music room. The music room had large tables. There was an area in each room that was used for students to bring their chairs together when they were called to gather and discuss their results. The atmosphere was relaxed. Sessions ran from 9:30 AM – 1 PM, and the eleven participating students were given opportunities for lunch and breaks with snacks. At times there were as many as twenty adults in the classroom, including the research team, graduate students, teacher interns and video recording staff.

A research team had planned the summer sessions by creating probability tasks which were designed to engage students and invite different solutions. Hollylynne Stohl Lee, Assistant Professor at North Carolina State University and an IML grant consultant, led each session with the IML grant Principal Investigator Carolyn Maher, and Co-Pi's
Arthur Powell, and Keith Weber. Alice Alston was a lead researcher. Several probability tasks were assigned in sequence that week. A description of the tasks is in Appendix A.

This dissertation focuses on five sessions that occurred during the last two days of the summer institute. Three sessions occurred on August 3, and two sessions took place on August 4, 2005. Each session was approximately 45 minutes long.

On Wednesday August 3, Hollylynne Stohl Lee discussed fairness of dice with the students, and they had time to use Probability Explorer (PE) software to simulate rolling dice. Stohl Lee also showed students how to use the “Weight Tool” to permit certain numbers to come up more than others, depending on the weight that was assigned to each number. Students had time to experiment with putting different weights into the Weight Tool, and to see the resulting representations for dice which were “unfair.” Stohl Lee then introduced the Schoolopoly task, giving students time to read the task and ask questions. By the end of the first day, each pair of students had received the names of three different companies. Students were instructed to determine if the dice that each company manufactured were fair. They were asked to summarize their justifications and save representations generated by the software in a Microsoft Word summary document. Each company was investigated by at least two pairs of students. The last activity for the Wednesday session involved grouping students who had analyzed dice from the same company, so they could discuss and compare their results. On the second day, students created a poster for each company they had examined. The posters displayed their claim about fairness, and included computer-generated representations that they selected to support their claims.
3.3 Data/Collection Methods

The data for this study come from several sources acquired during the IML summer institute. Students worked in pairs at computers, recorded notes, and saved their typed solutions on disks. As the middle school students worked, researchers circulated and/or sat with a pair of students. Graduate students and teacher interns were assigned to specific pairs of students and recorded notes of their observations. These observation notes were collected at the end of each day by a post-doctoral associate. Researchers would sometimes interject a question or seek clarification on something that was stated by a student, but graduate students were instructed to only observe quietly and take notes. On the last day, students constructed posters, prior to discussing conclusions as a whole group.

All of these data – records of the discussions between students as they worked on the tasks in pairs, conversations between pairs of students and the visiting researchers, the written work, the posters, and discussions among the entire group, is captured on video that was recorded by three or four videographers each day. A boom operator worked with each videographer, holding a microphone to pick up these conversations. The discourse video data for this study is contained on ten CDs that have been accumulated and collated at the Robert B. Davis Institute of Learning (RBDIL) at the Rutgers University Graduate School of Education. The posters which were displayed on the walls of the classroom are captured on an additional video. In addition, this study analyzes five screen-shot videos that recorded what students were viewing as they used PE to solve probability tasks.
The analyzed discourse video data and synchronized screen-shot video data of the computer monitor, along with the task, participants, dates, and sessions during which they occurred are summarized in Table 3-1.

Table 3-1. Summary of video data

<table>
<thead>
<tr>
<th>Session</th>
<th>Date</th>
<th>Discourse CD</th>
<th>Screen-shot CD</th>
<th>Task and participants</th>
</tr>
</thead>
<tbody>
<tr>
<td>Morning 1</td>
<td>8/3/05</td>
<td>136B</td>
<td>Kianja/Chanel 1 of 3</td>
<td>Group discussion on fairness and sample size; Kianja and Chanel use PE to roll a die; introduction to the Weight Tool.</td>
</tr>
<tr>
<td>Late morning 1</td>
<td>8/3/05</td>
<td>137B</td>
<td>Kianja/Chanel 2 of 3</td>
<td>Group introduction to Schoolopoly task; Kianja and Chanel investigate Dazzling Dice.</td>
</tr>
<tr>
<td>After lunch 1</td>
<td>8/3/05</td>
<td>138B</td>
<td>Kianja/Chanel 3 of 3</td>
<td>Kianja and Chanel investigate Dice Depot and begin Dice R Us; they compare results for Dice Depot with Keisha and Tiffany.</td>
</tr>
<tr>
<td>Morning 2</td>
<td>8/4/05</td>
<td>139B 140B</td>
<td></td>
<td>Kianja and Chanel make a claim for Dice R Us and select representations for the three posters.</td>
</tr>
<tr>
<td>Morning 1</td>
<td>8/3/05</td>
<td>136C</td>
<td>Chris/Jerel 1 of 3</td>
<td>Group discussion on fairness and sample size; Chris and Jerel use PE to roll a die; introduction to the Weight Tool.</td>
</tr>
<tr>
<td>Late morning 1</td>
<td>8/3/05</td>
<td>137C</td>
<td>Chris/Jerel 2 of 3</td>
<td>Group introduction to Schoolopoly task; Chris and Jerel investigate Dice R Us, Delta's Dice and Calibrated Cubes.</td>
</tr>
<tr>
<td>After lunch 1</td>
<td>8/3/05</td>
<td>138C</td>
<td>Chris/Jerel 2 of 3</td>
<td>Chris and Jerel defend their claims to a researcher and use the Weight Tool to duplicate Calibrated Cubes' outcomes; they compare results for Delta's Dice with Jarae.</td>
</tr>
<tr>
<td>Morning 2</td>
<td>8/4/05</td>
<td>139D 140D</td>
<td></td>
<td>Chris and Jerel create posters for their three companies.</td>
</tr>
<tr>
<td>Late morning 2</td>
<td>8/4/05</td>
<td>142E</td>
<td></td>
<td>This is the video recording of all of the posters displayed on the classroom wall.</td>
</tr>
</tbody>
</table>
Hard copies of written work and notes were also saved and are analyzed in this study as data in the form of documents. I was a graduate student participant at all IML sessions in this study. Included as a data source are my own notes as a direct observer, as well as observation notes written by other graduate students. The written notes, students' posters, and graduate student observations are also available in hard copy and are stored at the RBDIL.

3.4 Method of Analysis

Analysis of the data adhered to the Powell, Francisco & Maher (2003) model for investigating the development of mathematical knowledge using video data. This model involves seven interacting, non-linear steps: attentively viewing the data; describing the data; identifying critical events; transcribing; coding; constructing a storyline; and composing a narrative (p. 413). A description of how I used these steps follows.

3.4.1 Viewing and Describing the Data; Identifying Critical Events

Although I attended each session of the IML summer 2005 institute, I did not focus on the individual students in this case study at that time. Therefore it was essential that I become very familiar with the video data that focused on Kianja, Chanel, Chris and Jerel. As such, I rigorously viewed the ten discourse CDs many times, and wrote descriptions. Several graduate students also viewed the CDs and provided input. Three of the discourse CDs in this study were described by graduate students in an Introduction to Math Education course. The remaining discourse CDs were described by graduate students during a summer Practicum course. Each description listed critical events, or situations in which the four students selected computer-generated representations that
they used as evidence to justify claims. The critical events also took note of students' ideas or misconceptions regarding fairness and significance of sample size, as well as how the students’ ideas were influenced by computer-generated representations and others.

3.4.2 Transcribing and Verifying

The audio component of the video data consists of the researchers' discussions with students as a large group and in smaller groups, as well as the student-to-student conversations and interactions. The audio component has been transcribed (see Appendices B – K) and verified by this researcher, graduate students, or an independent researcher, in order to address the research questions. The creation of the transcript and the verification were done by two different individuals. In addition, screen-shot videos of what students were actually viewing on the computer monitor as they used PE, were synchronized to the discourse transcripts. Images from the screen-shot videos are situated within the discourse transcripts in the appendices.

Examining screen-shot video of the computer monitor was an essential component of the video analysis. There were many periods of time when, although the group was not talking, students were pointing to the monitor and making facial expressions or excited exclamations. Occasionally when students discussed ideas about sample size, the discourse video camera was focused on the group of students rather than the monitor. By synchronizing the screen-shot videos to the discourse videos, I was able to see what the students were seeing and pointing to, and therefore identify the cause of the students’ excitement or what sample size they actually ran for a particular experiment. For example, Kianja and Chanel had originally thought that the dice from one company,
Dazzling Dice, were fair. In their first three experiments, an outcome of *five* came up almost as many times as the other sides. However, those experiments had small samples (ten, twenty, and forty, respectively). In their next experiment with sixty trials, the girls noticed that *five* did not come up nearly as many times as the other sides.

*Kianja*  Dag. *Five* not on there. Dag. Okay now that’s, that’s

*Chanel*  *Five* was on there the last time.

*Kianja*  Whoa. *Five* only got one.

*Chanel*  *Five* was on there last time though.

*Kianja*  Wow. We need to copy this whole screen. (lines C285 - C289)

In this excerpt, the computer monitor is not in the camera view of the discourse video. The discourse video camera is focusing on the girls. By synchronizing the monitor video to the discourse video, this researcher was able to see what the girls are excited about and pinpoint which computer-generated representations were on the “whole screen” that Kianja wanted to copy. The "whole screen" included a pie chart which showed that the outcome for *five* was represented by a very slim sector, and stacked outcomes which visually depicted the low frequency for an outcome of *five*.

*Figure 3-1. "Five not on there!"*
The transcripts provided evidence for claims. Because six of the discourse CDs correspond to five screen-shot videos, I decided that it would be most efficient to separate each chronologically recorded session (of approximately 45 minutes in length) into different appendices. This organization was helpful as I referenced line numbers. Videos of Kianja and Chanel are transcribed in Appendices B through F, and transcripts of videos of Chris and Jerel are in Appendices G through K.

3.4.3 Coding

As I repeatedly viewed the video data, wrote the description of each CD, identified critical events, and shared my interpretations with other graduate students and my advisor, I noticed certain themes emerging. I created an initial set of codes which was too detailed. Through suggestions that I received at additional student/advisor sharing sessions, I was able to fine tune the codes so I could connect the critical events and see a story emerge. According to the Powell, Francisco and Maher (2003) model, this process helps the researcher interpret data. In fact, this process also helped me refine my research questions so that I could address the recurring themes in my data. Many of the initial codes that were used to guide my study of the research questions were then either eliminated or collapsed during this circular process. The codes that I focused on for my findings are included in Table 3-2.

Table 3-2. Codes used for this study

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Determining fairness (before, during, after computer-generated representations)</td>
<td>DF</td>
</tr>
<tr>
<td>2</td>
<td>Determining sample size (before, during, after computer-generated representations)</td>
<td>SS</td>
</tr>
<tr>
<td>3</td>
<td>Influenced by collaborators - comparing results with others and keeping or changing decisions</td>
<td>IC</td>
</tr>
</tbody>
</table>
For example, during the morning session on the first day in this study, students were asked for their ideas about the characteristics of a fair die, and how many rolls it would take to get a certain outcome on a fair die. A selection of the responses which represent the DFB (determining fairness before) code and SSB (sample size before) code, are included in Table 3-3.

**Table 3-3. Coding examples**

<table>
<thead>
<tr>
<th>CODE</th>
<th>LINES</th>
<th>SPEAKER</th>
<th>ROLE 136B TRANSCRIPT - Appendix B</th>
</tr>
</thead>
<tbody>
<tr>
<td>DFB</td>
<td>17</td>
<td>Jerel</td>
<td>That <em>four</em> come up. That <em>four</em> and <em>three</em> come up a lot.</td>
</tr>
<tr>
<td>DFB</td>
<td>54-56</td>
<td>Jerel</td>
<td>Wait, I think it would come up even... They would all come up about the same.</td>
</tr>
<tr>
<td>DFB</td>
<td>66-67</td>
<td>Chanel</td>
<td>It has six sides. I mean it can’t have an odd or even number of sides.</td>
</tr>
<tr>
<td>DFB</td>
<td>85-87</td>
<td>Lorrin</td>
<td>I’m saying like if every other number comes up once and then <em>four</em> comes up like four times, I’m saying they’re might be more than one <em>four</em> on the thing</td>
</tr>
<tr>
<td>DFB</td>
<td>137</td>
<td>Jerel</td>
<td>You expect to see all of the numbers come up.</td>
</tr>
<tr>
<td>SSB</td>
<td>141</td>
<td>Jerel</td>
<td>About ten turns.</td>
</tr>
<tr>
<td>SSB</td>
<td>157-158</td>
<td>Jarae</td>
<td>I would like to roll the number <em>six</em> and to get to the number <em>six</em> it would take you about fifteen, twenty times to rolls.</td>
</tr>
</tbody>
</table>

3.4.4 Constructing a Story

Once I had written a thorough description that identified critical events, and created a coding scheme that categorized them, I realized that the best way to merge the events into a story was to proceed chronologically. I also made the decision to address the work of each pair of students separately, within each session. I worked with both the transcripts and the notes from graduate student observers, to get a better picture of how the two pairs of students were building mathematical ideas. I continued to carefully
watch and listen to the video data, to ensure that I was interpreting the text in the transcripts correctly. I also continued to share my interpretations with my advisor and other graduate students to get their feedback.

### 3.4.5 Composing a Narrative

Every step in this process is intertwined with composing the narrative. This composition actually begins when the research study begins (Powell, Francisco & Maher, 2003). As I linked the critical events, I also wrote a summary for each important segment of the findings and my interpretations. The final chapter on conclusions emerged from the research questions and the summaries.

### 3.5 Verification of Validity

Verification procedures employed in this case study include rich, thick description of the participants and setting, which enables transferability of aspects of this study (Creswell, 1998). Triangulation of data with multiple documents, including researcher/observer notes and the students' written work, as well as the video data and transcripts, was used to create a valid narrative of the events. Finally, portions of this study were shared at student/advisor debriefing sessions to provide an external check of the process I followed.
Chapter 4  EXPERIMENTS WITH FAIR AND WEIGHTED DICE

4.1  Introduction

The purpose of this study is to investigate how four middle school students from an urban community built ideas about probability when computer simulation tools were available. The girls in this study, Kianja and Chanel, and one boy, Jerel, are African-American. A second boy, Chris, is Latino. The students attended sessions that took place on two days during the 2005 Summer Institute of the Informal Mathematical Learning (IML) after-school program, a component of the National Science Foundation supported research study. The students worked in pairs, using Probability Explorer (PE) computer software to solve open-ended probability tasks that were designed to stimulate discussions about fairness and sample size.

This study examines the choices and decisions these students made as they generated computer data and representations to solve the probability tasks. The research questions guiding this study are: (1) How are data generated by the students interpreted with respect to (a) fairness and (b) significance of sample size? (2) What decisions about fairness and adequacy of sample size do students make on the basis of evidence that they collect from computer simulations? (3) How are students' ideas influenced, if at all, by their computer-generated representations? and (4) How are students' ideas about fairness and sample size influenced, if at all, by others?

This study presents chronologically, the analysis of the video-recorded and written work of each pair of students, as they solved probability tasks. This chapter addresses the first task, during which students used PE to simulate tossing dice, made
claims regarding the fairness of dice, and provided evidence that justified their claims. This chapter also examines the students’ work on an extension of the task designed for how to use the PE software to introduce bias. When the data are presented, italics are used to represent a die outcome.

The names of the teacher/researchers and graduate student/researchers are omitted from the transcripts and are replaced with the codes in Table 4-1, listed in the order in which they appeared in the transcripts. The key in Table 4-1 is the same for all of the transcripts.

Table 4-1. Key for researchers and graduate students codes

<table>
<thead>
<tr>
<th>Code</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>Hollylynne Stohl Lee, North Carolina State University</td>
</tr>
<tr>
<td>R2</td>
<td>Keith Weber, Rutgers University</td>
</tr>
<tr>
<td>R3</td>
<td>Arthur B. Powell, Rutgers University</td>
</tr>
<tr>
<td>G1</td>
<td>John M. Francisco, post-doctoral associate</td>
</tr>
<tr>
<td>G2</td>
<td>Kathy Shay, graduate student/researcher</td>
</tr>
</tbody>
</table>

4.1.1 Background and Setting

The IML program began in fall 2003 with the first cohort group of students, volunteers from two middle schools of the Plainfield Public Schools. The research project focused on the mathematical behavior and ways of reasoning of minority middle school students in the urban city in central New Jersey. During this three-year project, IML students worked on well-defined tasks in the areas of counting and combinatorics, algebra, and probability. The IML setting was designed to be a comfortable classroom environment in which all student contributions to discussions were acknowledged, valued
A classroom community was designed in which the students were encouraged to make claims, and provide justifications based on evidence.

This chapter examines the first session in this study, which took place on August 3, 2005, the third day of an IML summer institute. The session was held in an air-conditioned computer classroom. Several teacher/researchers and graduate student/researchers were present. Four videographers recorded the session. Each videographer had a sound technician. For this session, one camera focused on R1, another camera focused on Kianja and Chanel, a third on Jerel and Chris, and the remaining camera was roving among other pairs of children as they used software to solve probability tasks at their computers.

### 4.1.2 Computer simulation software – Probability Explorer

*Probability Explorer (PE)* was the software tool used by the students. It is designed to simulate probability experiments. In so doing, data collected can be represented on the screen as tables and graphs. In particular, the screen options were: (1) stacking in columns, lining up in rows, or displaying without organization the outcome of each event; (2) creating a data table that listed the frequency for each outcome along with the relative frequency - the ratio of the frequency divided by the total number of trials for each experiment, listed as a fraction, decimal and percent; (3) generating colorful graphical representations – a pie chart and a bar graph. The results of an experiment with 100 trials are captured in Figure 4-1. This figure shows the various representations which could be selected when 100 fair dice were tossed, including the lined-up outcomes in rows, the pie chart, bar graph, and table that included the frequency and relative frequency for each outcome.
4.2 Task 1: Simulating Fair Dice

The first session in this study began with a whole group introduction to the task, Simulating Fair Dice (Appendix A). Then students worked in pairs using PE at their computers to simulate tossing fair dice. At the end of their investigation, they shared results. Data are presented in a corresponding order.

4.2.1 Introducing the Problem – large group discussion

In preparation for the Simulating Fair Dice task, R1 brought the eleven students in attendance together and asked the whole group what they already knew about the characteristics of a die, how they can tell that a die is fair, and how many times a die needs to be tossed before fairness can be determined.

Figure 4-1. Representations produced by Probability Explorer
4.2.1.1 What do you know about a die?

R1 began the first session by gathering the students into a semi-circle near the front board, and asking them what they already knew about dice. Several students responded immediately and noted that a standard die is six-sided, has dots that represent numbers which range from one to six, and that dice are easy to roll. According to Jerel, “four and three come up a lot” (line B17). Jerel’s response hints at an experience during the IML grade 6 probability strand, when he and other students determined whether a game played with two dice that resulted in specific sums was fair (Shay, 2008). R1 asked students if they agreed that if you roll one die, four and three will come up more often than the other sides. A student recognized that four and three come up more often if there are two dice. This gave R1 the opportunity to clarify that, for this task, only one die would be considered.

4.2.1.2 How can you tell that a die is fair?

R1 initiated a discussion about fairness by asking the whole group of students what it meant for a die to be fair. A student replied that a die is fair if it is “not cheating” and further explained that to mean the die has six sides and it doesn’t have three of the same numbers (lines B46 - B48). Jerel claimed that a fair die would come up “even”, and stated that the numbers “would come all up about the same” (lines B52 - B56). R1 questioned the students about whether ”even” means the same as having the numbers come up the same amount of times, and Jerel replied that they do not have the same meaning. Although “even” is not a statistical term, it is conjectured that Jerel used the term “even” to describe an experiment in which the outcomes have an equal probability of occurring.
Students continued to note characteristics of a fair die. When a student said that you can tell that a die is fair if you “shake it in your hand” (line B75), another student, Terrill, pointed out that if you could be certain of the outcome, then that would “defeat the purpose of Las Vegas. Then you could call out a number and you would never lose” (lines B121 - B123). Here, Terrill contributed a practical application of fairness as applied to a gambling situation.

4.2.1.3 How long would it take you to see all of the numbers on a die?

When a student commented that if a die was fair then all numbers would appear as outcomes, this provided a natural segue into a discussion of sample size. One student volunteered that if a die was fair, it could take ten tosses to see a specific outcome. Another student postulated that to roll a six, she would have to toss the die about fifteen times. Following these claims, a student asked if she could toss a die to see how many trials it would take to roll a six. R1 gave her the die, and it actually took two trials to roll a six, providing experimental probability data.

A student noted that if it takes 50 tries to roll a six, then the die is not fair. Terrill proposed another reason – if you roll the die fifty times and never get a six, that would indicate “you don’t know how to roll” (line B172). Students laughed at this comment, and then Terrill corrected his statement and recognized that the roller cannot control the outcome. At the conclusion of this discussion and hands-on experience with tossing a die, R1 informed students that they would next break into pairs and use Probability Explorer to roll a die.
4.2.1.4 Summary of large group discussion

The large group discussion at the beginning of this session engaged students in probability ideas and gave them an opportunity to share with the group their ideas about the characteristics of a die, fairness, and sample size. During this introduction to the task, students also witnessed an experiment that had quite a different outcome from a student’s conjecture. Initially, one student had claimed that it would take fifteen tosses to roll a six. When another student actually rolled the die, it only took two tosses to roll a six.

4.2.2 Students work in pairs using PE

After the students dispersed into pairs, their first challenge was to follow the directions that R1 verbalized as she explained how to use PE to roll a die. R1 established that all students had correctly followed the directions, by asking each group for their outcomes and remarking that “about half of us got a six” (lines B206 - B207). R1 repeated that task and told the group that she got a five. She asked students to explore using the software and decide how many times to roll the die. R1 reminded students about the available computer-generated representations.

4.2.2.1 Kianja and Chanel decide how many times to roll the die

Following the introduction to using PE to roll a die, R1 circulated among the pairs of students. She asked Kianja and Chanel what they think will happen if they roll the die thirty times. Chanel replied that two will come out more “cuz my favorite number” (lines B223 - B227). Here, Chanel presented her own subjective preference for an outcome, without providing mathematical evidence or justification.
R1 was looking at their monitor when the girls decided to roll the die thirty times. However, Kianja actually ran the experiment 36 times. When R1 asked Kianja why she changed the sample size from 30 to 36, Kianja replied that she did not know why. R1 asked her if it was because 36 is six times six, and Kianja nodded her head, adding that “the highest number on the dice is six” (lines B228 - B233). It is noted that initially, Kianja did not provide a reason for running the experiment 36 times. Following R1’s suggestion that it was related to the square of six, Kianja did acknowledge that fact, and recognized that each outcome was not greater than six. It is conjectured that Kianja’s response was influenced by R1’s suggestion. Of additional importance is that in PE, once an initial sample size is typed, it is very easy to run multiples of that sample size by simply clicking on the Run key. The outcomes are accumulated. An alternative interpretation is that the ease of running trials that are multiples of 6 may be the reason why Kianja ran 36 trials in her experiment, rather than the reason (36 is the square of 6) that R1 introduced.

Chanel took over the keyboard and tried to roll the die six times to get her favorite number, the two. For this small sample size, R1 noticed that Chanel did not get any two’s, as depicted in Figure 4-2.

Figure 4-2. Outcome for six trials
Chanel laughed at this outcome (line B240). She and Kianja clicked on Run again so that their next three experiments had sample sizes which were multiples of six: 12, 24, and 36. The girls represented the results for 36 trials as a stacked group of dice within a column for each outcome. As the experiment ran, Chanel began to cheer for a \( \text{two} \): “Come on, \text{two}!” (line B242). However, in this experiment, the frequency for an outcome of \text{two} was lowest. Kianja questioned why the column depicting the outcome of \text{three} was so high, and Chanel noted that the \text{six} and \text{four} also came out more frequently than the other numbers, as seen in Figure 4-3.

![Figure 4-3. Outcome for thirty-six trials](image)

Kianja ran another experiment with 36 trials, and once again the outcome of \text{two} came out the lowest. Chanel suggested they begin to summarize their findings in a Microsoft Word document. As the students used \textit{PE} on their computer, they also had access to a Word document. They used this computer tool to record their findings and claims. Students selected computer-generated representations from \textit{PE} which provided evidence for their claims, and copied and pasted them directly into their Word summary document.

Kianja initially placed the results for 30 trials into the document, but Chanel reminded her that prior to that experiment, they ran an experiment in which the sample size was 12. Because they had not saved the results of their experiment with 12 trials,
they ran another experiment with 12 trials and there were no outcomes for six. They laughed at this result, but they decided to include the stacked-outcomes representation for a sample size of 12 in their report. They next ran an experiment with 24 trials. Chanel exclaimed: “I told you there’s more six. I knew it, I knew it.” (lines B265 - B266).

Chanel typed in their report that outcomes of six and one occurred most frequently in an experiment with 24 trials. They next ran 25 trials. In this experiment, the one did not come out at all and the outcome of six occurred only once. In fact, the outcome of two occurred thirteen times! This striking result is captured in the screen shot in Figure 4-4.

![Figure 4-4. Outcome for twenty-five trials](image)

Kianja admitted that she had run 25 trials by accident. Next, Kianja ran an experiment with 30 trials and this time the frequencies for outcomes of one, two, three, four, five and six were 6, 6, 5, 3, 5, and 5, respectively. Chanel told Kianja to “print that one. That’s good” (line B286). Chanel was indicating the representation in Figure 4-5 which showed that the frequencies were close, and the girls copied and pasted that representation in their summary report.

![Figure 4-5. Outcome for thirty trials](image)
It should be noted that, although Kianja and Chanel had pasted the representations for outcomes of several experiments and typed their observations, they had not yet discussed fairness. However, the outcomes for this last experiment depicted in Figure 4-5, was the one that Kianja referred to when the girls were questioned about the fairness of the dice during the subsequent whole-group discussion.

### 4.2.2.2 Chris and Jerel decide how many times to roll the die

As the boys began to explore the outcomes of rolling a die on PE, their first experiment had ten trials. They selected the lined-up outcomes of the dice as they were tossed, as well as the pie chart, bar graph, and table representations. They acquired more evidence by running another experiment with 100 trials, and then they ran that size experiment again. The frequencies for the first three experiments are listed in Table 4-2 below.

**Table 4-2. Frequencies for three experiments**

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Sample Size</th>
<th>Frequency for each outcome:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>17</td>
</tr>
<tr>
<td>3</td>
<td>100</td>
<td>17</td>
</tr>
</tbody>
</table>

Jerel observed that the last experiment looked “even” (line G46). He may have been referring to the closeness of the frequencies, supporting the previous conjecture (in Chapter 4.2.1.2) that Jerel used the term “even” to describe an experiment in which the outcomes had an equal probability of occurring. Another possibility is that Jerel could have been looking at the heights of the bars in the bar graph, noting that they were almost level, or even. He suggested that they run a larger sample size, and they ran 500 trials.
As the experiment was running, initially the outcome of *five* was highest. The boys rooted for various outcomes or colors as the bars in the bar graph increased in height and the sectors in the pie chart changed size. Chris and Jerel were looking at the representations in Figure 4-6.

*Figure 4-6. Running 500 trials*

Chris decided to run a sample size of 1500. He accomplished this by typing 500 as the number of trials, and clicking on Run so that the results accumulated. At 1250 trials, Jerel claimed that “six is losing” (G:105). Chris pointed to the pie chart and bar graph, and noted that they were really all coming out the same. The boys began to talk off task after they realized the number of trials may go to 4000 or possibly 6000, depending how many times they clicked Run. Chris and Jerel had each clicked Run several times, but neither boy was certain of the total number of times that Run had been clicked. The actual results are listed in Table 4-3.
Table 4-3. Frequencies that occurred while running 4500 trials

<table>
<thead>
<tr>
<th>Sample Size</th>
<th>Frequency for each outcome:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>1250</td>
<td>208</td>
</tr>
<tr>
<td>2720</td>
<td>412</td>
</tr>
<tr>
<td>3842</td>
<td>633</td>
</tr>
<tr>
<td>4500</td>
<td>729</td>
</tr>
</tbody>
</table>

4.2.3 Students compare thoughts about fairness and sample size as a large group

After providing approximately ten minutes for students to explore how to use PE to roll a die, R1 asked the whole group to share their findings. Jerel made a claim that the die was fair and submitted the percents in the PE data table as evidence. Chris added that they ran the experiment 4500 times.

Other students volunteered that they also thought the die was fair. R1 asked Chanel and Kianja if they thought the die was fair. Chanel initially replied that it was fair because her favorite number, two, won. She then noted that it was fair because all of the numbers had a chance of coming out the highest. Kianja referred to the screen shot in Figure 4-5, and added that the outcomes were “really close” (lines B309 - B316). When R1 asked how many times they ran the experiment, the girls replied that they ran 12 trials followed by 24 and then 30. Although they had not typed a conclusion about fairness into their Word document, immediately after listening to the number of trials that other students used and their decisions about the fairness of the die, Chanel suggested that Kianja “do um some high numbers to see” (lines B328 - B330). Kianja typed 500 for the sample size. After using this larger sample size and opening additional computer-
generated representations (the bar graph and pie chart), Kianja and Chanel were convinced that the die was fair and typed this claim in their report.

4.2.4 **What computer-generated representations do students prefer?**

*Probability Explorer* produces colorful representations that dynamically change as an experiment runs. Bars in the bar graph get taller; area of the sectors in the pie graph can vary greatly if the die is unfair. As a die is tossed, there are several ways to look at the outcomes. The actual die can be stacked in a column and sorted by the value on its face. Each column can stack 17 dice. If the frequency for a face value is greater than 17, then the stacking representation cannot be used. If an experiment has a sample size of 150 or less, then each tossed die can be lined-up horizontally. Dice are not sorted by face value in this representation and if the sample size is greater than 150, then the remaining dice are scattered across the screen. Finally, dice can simply be scattered across the screen as they are tossed. The students in this study had a preference for the representations they selected, which was sometimes limited by the sample size they ran.

4.2.4.1 **Kianja and Chanel – preferences for computer-generated representations**

When Kianja and Chanel started using *PE*, they used the stacked table of outcomes for each roll to determine whether a die was fair. The representation of stacked outcomes was similar to a bar graph. Following the whole group discussion, Kianja and Chanel ran 500 trials and they noticed that as each die was rolled, the outcomes for that experiment could no longer be stacked – there were too many dice. They selected the option to line-up the outcomes, but that only worked for the first 150 dice that were tossed. Each die that depicted the outcome of a roll showed up on the monitor, but now
the dice were scattered across the screen as they were rolled. As a result, Kianja and Chanel opened other representations. They selected the data table which showed the relative frequency for each event as a fraction, decimal and percent. In addition, they opened the bar graph and pie chart representations.

Chanel began to summarize their findings in their Word document and prepared to copy the screen with the experiment's outcomes, the bar graph and pie chart. In the bar graph, the bar that represented an outcome of two was visually lower than the other bars. Kianja suggested that they replace the bar graph with the table, and she rearranged the screen so that it showed the scattered outcomes for each roll, the table with numerical outcomes, and the pie chart, as seen in Figure 4-7.

![Figure 4-7](image)

*Figure 4-7. Multiple representations selected by Kianja and Chanel*

Kianja read aloud what Chanel was typing: “Now we’re really sure that it’s….a fair dice because the numbers came up even” (lines B343 - B345). It is conjectured that "even" was used here to refer to the approximately equal areas of the sectors of the pie graph, as well as the closeness of the frequencies in the table. It is interesting to note that the girls did not include the bar graph as evidence for their claim.
4.2.4.2 Chris and Jerel – preferences for computer-generated representations

From the very first experiment that the boys ran with a sample size of 10 trials, they selected the option to have $PE$ horizontally line up the dice as they were tossed. The boys also opened the table, the pie chart and the bar graph to make decisions from their experiments. Jerel told R1 that he used the pie chart to determine whether the outcomes were “even” (lines G74 - G75). For an experiment with 500 trials, R1 asked the boys why the pie chart looked "even" but the bar graph heights were different, in the screen shot captured in Figure 4-8.

![Figure 4-8. Comparing the pie chart with the bar graph](image)

The boys replied that they did not know why there was a difference in this case. However, the boys used the dynamic color changes in both the pie chart and the bar graph as a game, enthusiastically rooting for and observing various outcomes such as “Yo, six is beasting” (line G35) or “It’s about to lose to six. Oh snap!” (line G50).

When asked for what evidence was used to make a decision about fairness, Chris pointed to the pie graph (line G105) to make his claim that the die was fair. When Jerel shared their claim about fairness during the whole-group discussion, he read percents from the table as evidence: “Its only two percents in the seventeen's, and the rest is in the
sixteen's" (lines G157 – G158). It is interesting to note that the boys did not refer to the bar graph when discussing the evidence for their claim.

4.2.5 Summary of students working in pairs to simulate fair dice

When the students first separated into pairs, they experimented with the number of trials and representations to select. Kianja and Chanel typed 6 trials initially, and then clicked on Run to get multiples of 6 up to 36. With these small sample sizes, they used the stacked outcomes as evidence, but they did not type a claim about fairness. After listening to the shared results from students who had used larger sample sizes, Kianja and Chanel ran an experiment with 500 trials and they typed their claim about fairness. This larger sample size forced them to open other representations, and they looked at the bar graph, pie chart and table to provide evidence for their final decision. However, because they decided that the bar graph did not support their claim, the representations that they copied into their summary document as evidence for their claim were the scattered dice, the table and the bar graph.

Jerel and Chris had a different approach. They started with 10 trials, and then ran two experiments with 100 trials. For their very first experiment, they opened the pie graph, bar graph and table. During an experiment with 500 trials, they wanted more data and so they kept clicking on Run. That experiment actually had 4500 trials. R1 provided an interesting intervention. She asked the boys why the heights on the bar graph were different, and yet the pie chart sectors looked very close in size. The boys had no answer for this question, and it is conjectured that continued discussion may have provided the boys with additional insight into how to define closeness of frequencies when comparing representations.
4.3  *Task 1 extension: Introduction to Bias*

During the process of working on the Simulating Fair Dice task, R1 introduced students to the idea of bias. She asked students to recall that Chanel was interested in getting as an outcome, her favorite number – *two*, and introduced the Weight Tool in *PE* which can be used to favor certain outcomes. A snapshot of the Weight Tool is captured in Figure 4-9.

![Screen shot of the PE Weight Tool](image)

*Figure 4-9. Screen shot of the PE Weight Tool*

4.3.1  *How to Use the Weight Tool in PE*

R1 described the icon on the *PE* toolbar that was called the Weight Tool, and told students to click on the scale to open it. She asked students what it meant to change the weight for *two* from 1 to 3 (lines G217 – G218). Previously, each side had a weight of 1. Students volunteered their ideas about assigning a weight of 3 to an outcome of *two*, if the other outcomes still have a weight of 1.

4.3.1.2  *Kianja and Chanel work with the Weight Tool*

In response to R1’s question about the Weight Tool, Kianja volunteered that the weight is related to the number of sides on a die, and she recognized that increasing the
weight to 3 for an outcome of two would result in a higher frequency for two. According to Kianja, “It means that they have three dice, I mean three sides that have two” (lines B368 – B369).

R1 asked students to proceed with weighting the die so that two had the greatest weight. Kianja told Chanel to run an experiment with 50 trials. It is noted that for this experiment, there was an increase in the initial sample size that the girls ran. In their previous experiments related to tossing a die, they had used sample sizes that were multiples of 6, beginning with 6 trials.

Chanel pointed to the pie chart. Kianja became excited about the visual evidence that the pie chart provided, stating “oh that is really pretty. I like it. I like it a lot. I like it” (lines B387 - B388). Kianja pointed to the table, which also gave evidence that the side with the greater weight had the highest frequency. From the table, she read “46%” (line B395). The representations they selected for their first experiment using the Weight Tool to favor an outcome of two are depicted in Figure 4-10.

![Figure 4-10](image)

Figure 4-10. Kianja and Chanel’s representations for 50 trials

R1 walked over to the girls and commented that they only ran the experiment 50 times. Kianja replied that “it’s a eight-sided dice now” (line B397). R1 nodded her head in response and walked away. It is noted that additional questioning may have resulted in
clarifying Kianja’s reference to the connection between the Weight Tool and the number of sides on the die. Following R1’s remark about their sample size, Kianja decided to run 50 trials again, yielding a total of 100 trials. She typed her conclusion that the die was definitely not fair because now it was eight-sided. Kianja copied the outcomes for all 100 trials, as well as the table and pie chart, and placed that evidence to support her claim into a summary document.

G1 had been observing the girls and recording notes for the entire morning session. He read the conclusion that Kianja had typed, and asked Kianja what she meant by eight-sided. Kianja pointed to the Weight Tool and a box that listed the total for the weights, which was 8. She said “See Total Weight – eight sided” (lines B420 - B421). It can now be interpreted that Kianja used the sum of the weights as an indicator for representing the number of faces on the die.

Kianja experimented with using the Weight Tool to give more weight to an outcome of six. She entered weights of 1-3-1-1-1-47, respectively. The girls ran 60 trials and as Kianja expected, the largest sector on the pie graph was for the outcome of six. Kianja noted “Watch six come out like every time. What I told you. I told you” (line B429).

The girls continued to experiment, setting the number of trials to 30. Kianja and Chanel changed the weight for an outcome of two to 30, and entered a weight of 3 for an outcome of six. They compared the resulting pie chart to the computer game character Pac-Man, stating:

*Kianja* Two is gonna win. That’s beautiful. I like those colors. That like Pac-Man.

*Chanel* It do, and here’s the eye right here. (lines B442 - B444)
Kianja and Chanel ran another 30 trials. The computer-generated representations which they selected for the total 60 trials were the lined-up outcomes, the table and the pie chart. They decided to include these in their typed summary report, and they are in Figure 4-11.

Figure 4-11. Comparing the pie chart to Pac-Man

Kianja walked away for a moment, and Chanel continued to run experiments on her own. She changed the weight of the two to 30 and she changed the weight of the remaining sides to 0. Chanel ran a sample size of 60, and called Kianja back to see the monitor, which is depicted in Figure 4-12. Kianja was excited to see only one color in the pie graph, and she said: “That’s wonderful. That is excellent. That’s beautiful. I like it a lot. I like it a lot” (lines B449 - B450).

Figure 4-12. The only possible outcome is two
G1 asked Chanel what she did to get the results in Figure 4-12. Chanel explained that she had "just put thirty, and ten and then the rest zero" (line B454). G1 asked “So what does that tell you? Does that tell you anything?” (line B455). The girls did not respond to the graduate student/researcher and G1 did not pursue getting a response. Next, Chanel changed the weight values to 1 for all of the outcomes except two. She kept the weight for two at 30. The resulting representations for 60 trials in which two is favored and the other outcomes are equally weighted, are in Figure 4-13.

![Figure 4-13](image)

Figure 4-13. Favoring an outcome of two

The girls looked at the pie graph as well as the table and typed their final comments before the end of this session, noting that Pac-Man was now only a little bit more than half of the circle, and that all of the other outcomes had occurred.

### 4.3.1.3 Chris and Jerel work with the Weight Tool

When R1 first introduced the Weight Tool, she asked the whole group for their ideas about what the numbers in the Weight Tool mean. Chris compared changing the weight to a marble bag – “like how many you put in” (line G208). This response hints at an experiment with marbles of three different colors, done earlier in the week (Appendix A). Students picked one marble at a time from a bag, replaced it, and were asked to
guess how many marbles of each color were in the bag. It is conjectured that Chris was indicating that if the outcome *two* had a weight of 3, then it was more likely to roll a *two*, which was comparable to there being more of, for example, the blue marbles in the bag. It is further conjectured that Chris recognized that the person setting up the experiment could control which color had the most marbles, or which side had the greatest frequency.

In response to R1's question when she asked students what would happen if the weight for the outcome of *two* were changed to 3, Jerel suggested that this meant that there were three dice (line G222). Kianja corrected him, stating: "There are three sides that have *two* on it" (G223). Jerel agreed with Kianja, and he recognized that changing the weight meant that an outcome of *two* would occur most frequently (line G230). As the boys explored using *PE* to change the weights, they heard Kianja and Chanel talk about Pac-Man. Chris mentioned that he liked Pac-Man (line G241). Chris typed 10 for the weight of *two* but he left the weight of the other outcomes as 1. He told R1 that by giving only the outcome of *two* a weight of 10, the relative frequency for *two* was 143 out of 210 trials. Chris was successful designing an experiment that favored an outcome of *two*, however he was not successful replicating Pac-Man. The resulting representations for Chris' experiment are depicted in Figure 4-14.

*Figure 4-14. Trying to get Pac-Man*
Jerel acknowledged that the die was unfair when the weights were not equal. Next, Jerel created his own experiment by adjusting the Weight Tool so that four was highest. When no one was watching, Jerel opened the Weight Tool and replaced the 40 for the weight of four with a 1. Jerel ran 210 trials with the weights in Table 4-4.

Table 4-4. Adjusting weights so that four "beats" two

<table>
<thead>
<tr>
<th>Sample Size</th>
<th>Weight for each outcome:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>210</td>
<td>1</td>
</tr>
</tbody>
</table>

Jerel saved the window that had representations which depicted that an outcome of four had the greatest frequency, as seen in Figure 4-15.

Figure 4-15. Representations selected to show that four "beats" two

Jerel tried to get the attention of R1 by exclaiming: "Two got beat!" (line G257). When R1 looked at the boys’ monitor to find out if an outcome of two had the greatest frequency, Jerel did not admit to R1 that he had created his own experiment and that he had changed the weight to favor an outcome of four. He even re-opened the Weight Tool and changed the weight for four back to 1. He tried to show Kianja and R1 the Weight Tool and representations, all the while claiming that an outcome of four came out the
most but only had a weight of 1. Chris acknowledged to R1 that they had typed 40 as the weight for *four*. However, Jerel insisted that *two* had a weight of 10 and the other outcomes were equal, and yet *four* beat *two*. R1 laughed as she reminded Jerel that she could figure out that he had given the greatest weight to *four*, and that she could not be tricked.

### 4.3.2 Summary of using the Weight Tool

At the conclusion of this first session, the students in this study were introduced to the *PE* Weight Tool. They used the Weight Tool to create experiments that favored outcomes, and they made claims about which outcome would occur most frequently. They selected multiple *PE* representations to support their claims. Both pairs of students opened the pie chart, bar graph and table, and were able to determine fairness of dice based on the graphs and the data table. When the sum of the weights was a small total (8), Kianja interpreted the number written in the Total Weight box to be the number of sides of the die that was being rolled. Kianja and Chanel were able to produce only one outcome color in the pie chart by setting the weights of the other outcomes to 0. The girls created an experiment from which they compared the pie chart representation to a computer game character called Pac-Man.

When Chris and Jerel heard the girls talking about Pac-Man, they also tried to introduce weights that would produce a Pac-Man-like pie graph. They were not as successful as the girls. However, they were able to use the Weight Tool to predict which outcome would have the highest frequency. Chris compared using the Weight Tool to being able to bias the Marble Task by placing more marbles of the same color into a bag. At the end of the session, Jerel tried to trick R1 by denying that he had adjusted the
weight for an outcome of *four*. He admitted that the weight for *two* was changed to 10, but he would not admit that he had also changed the weight of *four* to 40. Jerel deliberately kept the representations in Figure 4-15 on his monitor, and opened the Weight Tool. He kept the weight for *two* at 10, but changed the weight for *four* from 40 back to 1. R1 laughed, and let Jerel know that she could figure out that he changed the Weight Tool after he ran his data. It is conjectured that Jerel is building correct ideas about how to make dice unfair, as evidenced by his attempt to “trick” R1 and other students.
Chapter 5 THE SCHOOLOPOLY TASK

5.1 Introduction

This chapter analyzes the work of two pairs of students as they used computer software to solve a task entitled Schoolopoly (Appendix A). The students were attending an IML Summer Institute. This chapter examines chronologically, two sessions held on August 3, 2005. The first session occurred during the late morning. Students were introduced to the Schoolopoly task and then had time to solve the task. In the second (after lunch) session, students continued to work on the task. During these sessions, students were asked to defend their claims to other students and researchers. Both sessions took place in the same air-conditioned computer classroom as previously described in Chapter 4.1.1.

5.1.1 Reading and Understanding the Schoolopoly Task

At the beginning of this late morning session, R1 brought the whole group of eleven students together in a semi-circle near the board. She distributed the Schoolopoly task handout, and asked for volunteers to read what was written. One student began reading, and had difficulty saying “Schoolopoly”. R1 explained that it was a word that she had created. Students were curious about that title, and Terrill asked if R1 had typed the problem. She replied that she did. He then questioned how to spell the die that represents one of a pair of dice stating, “I’m not talking about dead die” (line C16). R1 took this opportunity to discuss other meanings for die as well as the homonym, dye. After students read the task, R1 asked students to reiterate it in their own words.
Time for discussion and questions was built into the introduction of this task, to ensure that the directions were clear. Students were told that they would need to make a really important business decision and to recommend, possibly to a visitor from the School Board, which companies produced fair dice. When one student suggested that the dice could be purchased from all of the companies, another student pointed out that if the dice were purchased from all of the companies under consideration, then it would cost more money. This suggests that the students were beginning to consider ideas about the cost implications that could result from making their recommendations, and the importance of solving the problem.

5.2 Students work in pairs and investigate dice made by several companies

Following the introduction to the Schoolopoly task, students separated into pairs and returned to the same computer that they had used during the earlier session. Each pair of students was asked to investigate the fairness of dice produced by two or three specific companies. Kianja and Chanel were given Dazzling Dice, Dice Depot and Dice R Us. Chris and Jerel received Delta’s Dice, Calibrated Cubes and Dice R Us.

5.2.1 Kianja and Chanel investigate Dazzling Dice

When Kianja and Chanel were seated at their computer, Chanel suggested that they examine Dazzling Dice first. Kianja decided that the first experiment should have 10 trials (line C202). The reader is reminded that during the earlier session that morning, Kianja and Chanel were not able to make a decision about fairness until after the whole group discussion in Chapter 4.2.3, during which they heard that other groups had larger sample sizes. Prior to that, the girls had been running trials that were multiples of 6 up to
and including 36. Following the whole group discussion, Kianja ran 500 trials. After
viewing the results from that experiment, the girls typed their claim about fairness.
Following that successful experience, during the late morning session when students were
experimenting with the Weight Tool, Kianja and Chanel ran 50 trials in their first
experiment. It is interesting to note that they were beginning the new task with a smaller
sample size.

For their first experiment, the girls began by simply listing the outcomes. Kianja
decided they should use, as evidence for their claim, the representation of stacked
outcomes that looked like a bar graph (line C204). From 10 trials, the frequencies were
1, 2, 2, 2, 2, and 1 for each face of the die, respectively. The columns of stacked
outcomes were approximately the same height. Based on this representation, the girls
typed their claim that the dice were fair in their Word document summary. The
representation in Figure 5-1 was included as evidence for the claim about fairness in their
summary.

Figure 5-1. Dazzling Dice outcome from 10 trials

Kianja opened the table, the bar graph, and the pie chart. She mentioned that she
would not continue to paste the stacked columns of outcomes in the summary document
because it would “take up too much space” (lines C212 - C213). She and Chanel decided
that they would include the graphs, and Kianja remarked that the circle graphs are
“pretty” (line C217).
Chanel suggested that their next experiment have 20 trials, and Kianja simply clicked "Run" again. In this second experiment, the frequencies of 20 trials were 3, 3, 5, 4, 2 and 3, respectively. Kianja began to type their claim about fairness in the Word document summary. Chanel decided that they did not need the bar graph. She included the numerical table as evidence, as well as the pie chart and typed “From this data it looks as though this is a fair 6-sided die because the numbers are all close together” (lines C232 - C233), as captured in Figure 5-2.

![Figure 5-2. Dazzling Dice outcome from 20 trials](image)

For the next experiment the girls ran 40 trials. When R1 questioned why they ran 20 trials followed by 40, Kianja replied that she did not know (line C251). R1 pursued this line of questioning and asked why they had typed 10 by the running man and then clicked; Kianja replied that it is easy because you simply have to click repeatedly on the Run key. She tapped on the table as she stated: "If we put the 10 there we do da da da da da…” (lines C260 - C261). The frequencies for this experiment of 40 trials were 8, 7, 12, 4, 2, and 7, respectively. The pie chart representation was different from what the girls had seen for the two previous experiments. The sector for three was larger than the others, and the sector for five was smaller. Chanel typed their findings into their Word
document: “Yes it all looks good but we are still thinking because three showed up higher and five being the lowest” (line C266 – C268). It is noted that at 40 trials, the girls were beginning to question the fairness of the dice but they maintained their decision that the dice produced by Dazzling Dice were fair.

The next experiment had 60 trials. Chanel ran it the “easy way” (line C282), and she directly typed the number 60 for trials. As the computer generated the outcomes, Kianja began to get excited and exclaimed “Dag. Five not on there” (line C285). As the number of trials approached 60, Kianja indicated surprise and exclaimed “Whao. Five only got one” (line C287). The frequencies for this experiment with 60 trials were 8, 13, 10, 15, 1 and 13, respectively. Kianja and Chanel showed astonishment that there was only one outcome for five, as captured in Figure 5-3.

Figure 5-3. “Whao. Five only got one”

Kianja suggested that they copy the “whole screen” (line C289), which included the stacked outcomes, the table and the pie chart. Chanel included all of the representations in their summary, and typed their conclusion: “We really don’t think it is fair now because five keeps being the lowest of all while everything else is higher.”
Their next experiment had 80 trials. There was not enough room in the columns to stack all of the dice that were being tossed for this number of trials, and so the dice were scattered on the screen. The girls selected the table and pie chart as evidence for their word document. Chanel typed that this evidence supported their claim that the dice from Dazzling Dice were not fair because “five hit the bottom with only eight appearances” (lines C307 - C308).

In their final experiment of 100 trials for Dazzling Dice, the girls showed excitement to see that five rarely came out. In fact, Kianja paused the experiment at 76 trials because up to that point, five had only come out twice (line C318). Kianja noted that this was an important piece of evidence which was needed to justify their claim that the dice were unfair. She inserted the output for 76 trials into their word document and typed: “We see now that this die is absolutely almost unfair because about three-fourths of the way to 100 there are only 2 fives.” For representations, she selected the lined-up outcomes, the table, and the pie chart as seen in Figure 5-4.

Figure 5-4. Kianja paused the experiment at 76 trials
Chanel resumed the run of 100 trials and when it was done, an outcome of five had only occurred three times. Kianja told Chanel to copy the table and the pie chart, and to include that as evidence in their summary. G1 questioned Kianja about why she stopped at 76 and included that outcome in her report, rather than printing the result of the complete experiment which had 100 trials. Kianja replied that they included both the results for 76 trials as well as 100 trials in their summary, and further stated “But I wanted to show that at 76, almost three-fourths of the way to 100, you only had 2 fives appear. Meaning that I don’t think that this dice is very fair. Because only twice did five show up and we are at 76. How likely is that on a fair die?” (lines C353 - C357)

When she finished typing, Kianja tried to determine the number of pages in their summary for Dazzling Dice. G2 provided information on how to open some additional views, and then asked a pertinent question about what could be left out of their summary if there was not enough room (lines C380 - C381). A response may have clarified what the girls considered to be the most important components when providing evidence that supported their claims. Kianja spent time placing her preferred representations, the table and pie chart, side-by-side in the summary document. This made the document shorter. However, she included results from the earlier experiments with a smaller sample size, which had resulted in an initial conclusion that the dice were fair. That claim eventually changed when the girls ran 60 trials.

R1 approached the girls and asked for their conclusion for their first company – Dazzling Dice. Chanel replied that the dice were not fair, and Kianja explained that “five didn’t come up as much as the rest of the numbers” (line C418). Chanel then contributed more specific information from the table, stating that they ran 100 trials and five “only
came up three times and everybody else came out like twenty-something” (lines C420 - C421). R1 reminded the girls that when they began this experiment, they had decided the dice were fair. She questioned why they changed their minds. Chanel admitted that it wasn’t until they used the larger number of trials that they realized that five “just stayed down there at the bottom” (lines C427 - C428). Kianja acknowledged that they changed their opinion and R1 validated her response by noting that it is fine, “when you have different data, to change your opinion” (lines C430 – C431).

5.2.2 Chris and Jerel investigate Dice R Us, Delta’s Dice and Calibrated Cubes

Following the introduction and explanation of the Schoolopoly task at the beginning of the late morning session, the boys immediately opened Dice R Us and ran 500 trials. Chris told R1 that he thought four and three both “got 2” (lines H198 – H199). After analyzing Chris’ work and discussions, I can note here that Chris was referring to the Weight Tool and he was indicating that four and three have twice the weight of the other sides. R1 asked Chris if he meant that four and two have “too” much (line H208), and Chris nodded his head in agreement. This exchange suggests that at this time, R1 was not aware that Chris was describing outcomes with a greater frequency in terms of the weights entered in the Weight Tool. In fact, R1 interpreted Chris’ use of the word “two” as “too”.

Chris told Jerel that that the weights for outcomes of two, three, four and five were more than for one and six. The actual frequencies for two experiments of 500 trials were: 120, 186, 194, 214, 183 and 103, respectively. The boys saved their results in PE rather than saving to a word document. Then, after they made decisions about the three
companies, they typed their claims into a summary document. Their typed claim was that Dice R Us dice are unfair because “2 of the dice barely are rolled and the rest are rolled a lot.” The representations they selected and saved for evidence that dice produced by Dice R Us are unfair, are included in Figure 5-5.

![Figure 5-5. Dice R Us outcome from 500 trials](image)

Next, the boys looked at Delta’s Dice. Chris suggested they run 500 twice (line H215). Jerel opened the table, bar graph and pie chart and claimed that Delta’s Dice were fair (line H223). The frequencies for 1000 trials were: 173, 147, 180, 162, 155 and 183, respectively, and their selected representations that they saved in PE to show that dice produced by Delta's Dice are fair, are depicted in Figure 5-6.

![Figure 5-6. Delta’s Dice outcome from 1000 trials](image)
The final company for the boys to investigate was Calibrated Cubes. Initially, Jerel claimed it was not fair – “six is beating em” (line H238). He had opened the table, bar graph and pie chart. Chris countered back that four was also occurring more frequently than the other outcomes. Bantering took place regarding whether six or four was highest, and whether the die was fair or not. Chris noted that the percentages in the table were “not even close” (line H251) and made the claim that the dice from Calibrated Cubes are unfair. Chris said to Jerel “See, three of them got 2 in them; the other three got 1. I’m so smart” (lines H254 - H255). Again Chris was referring to the bias introduced by using the Weight Tool, which made the dice unfair. He was further observing that the ratio of weights was 2:1, favoring outcomes of two, four and six. The boys ran 1000 trials with the outcomes having the following frequencies: 130, 204, 126, 204, 134, and 202, respectively. The results of that experiment, which the boys provided as evidence that Calibrated Cubes' dice are unfair, are included in Figure 5-7.

![Figure 5-7. Calibrated Cubes outcome from 1000 trials](image)

The boys had run two experiments for each company, with sample sizes that started at 500. For the three companies, they clicked on "Run" again and based their decisions on the results for a total of 1000 trials. They had saved their evidence on PE. After they completed investigating all three companies, Chris began to type their findings...
Chris asked Jerel to type their findings for Dice R Us. Jerel displayed all representations in Figure 5-5 on the monitor, and typed “We think that it is unfair because every other number stayed at 200 or lower but the die number went on to 214. The percents are not even close” (lines H295 - H297). Chris looked over their work. He pointed to the table and provided further evidence for their conclusion, stating that “214 is for four and the other ones are like way off” (lines H318 - H319).

Chris suggested that they finish their summary by typing their findings for Calibrated Cubes. Jerel claimed this company made unfair dice and included the representations from Figure 5-7 in the summary document.

R2 approached the boys and questioned whether Delta’s Dice were fair, pointing to the lowest bar on the bar graph which represented the outcome for two. During the experiment for Delta's Dice with 1000 trials (Figure 5-6), two had the lowest frequency, and the boys had concluded that the dice were fair. Jerel pointed out the percents in the table, and Chris added that the percents were close. R2 asked if the result would be the same if they ran the experiment again with 1000 trials. The boys proceeded to run another 1000 trials. For this experiment, Jerel noted “See, look. See now two is winning. This convinces us that it is fair” (line H367). It is conjectured that Jerel made his claim...
that that the die was fair because the lowest outcome became the highest outcome in a
second experiment. This suggests that Jerel may have the idea that dice are fair when the
same outcome alternates between having the highest or lowest frequency.

Next, R2 asked about the dice from Calibrated Cubes. To defend their claim
about Calibrated Cubes, Chris pointed to the table and the percents column and said that
“the percents are not even. They’re not close” (line H372). Chris then mentioned the
numerical values in the Weight Tool that may have affected the frequencies for
Calibrated Cubes. He stated: “One, three and five and two, four and six… wait, I think
that one, three and five got 1 in them, and then two, four and six got 2 in them” (lines
H374 – H376). I conjecture that when Chris used the table to make a claim about
fairness, if the dice were unfair then he tried to determine the ratio between the
frequencies of highest outcomes to lowest. His evidence for unfair dice also included his
conjecture for which numbers may have been typed in the PE Weight Tool to produce
those results. Jerel agreed that the dice from Calibrated Cubes were unfair, but he
referred to the number of sides and stated: “Yeah, I think like two, four and six got two
sides, and they like came out the most like that” (line H378 - H379). Jerel's statement
suggests an idea mentioned earlier - if the die is unfair then the numbers two, four and six
may be on more than one side of the die. Students had used this terminology regarding
the number of sides on a fair versus unfair die previously, during the introduction to how
you can tell if a die is fair (section 4.2.1.2). Based on Jerel's suggestion regarding the
sides on the unfair dice from Calibrated Cubes, it is plausible that Jerel did not realize
that Chris was referring to the weights entered into the Weight Tool when he made his
claim that the dice were unfair.
For the final company, Dice R Us, R2 noted that the bar graph looked pretty even. Chris countered that the “percents aren’t close” (line H383), and maintained his claim that their dice were unfair.

R2 asked the boys to look at the fair company, Delta’s Dice, one more time. They ran another experiment with 1000 trials for Delta’s Dice, and this time five was winning at the beginning. Jerel sustained his claim that dice produced by Delta’s Dice were fair, even though when the experiment started and they looked at the results of a partial run with a smaller sample size, the five clearly had a greater frequency. Jerel explained that the other numbers will “catch up” (line H396). During an exchange with R2, Jerel mentioned that initially one number may have a higher frequency, but if the die is fair, after a large enough sample size the outcomes will be close. He noted that one outcome “could start out by a big lead but then probably the other numbers come back” (lines H410 – H411).

5.2.3 Summary of late morning session–Students work in pairs on the Schoolopoly task

At the beginning of this session, students were introduced to the Schoolopoly task. Initial whole group discussion was designed to clarify what students were being asked to do, and to identify the importance of solving the task. As Kianja and Chanel began to investigate Dazzling Dice, they ran their first experiment with a small sample size of 10 trials. They did not discuss the reason for this decision about the number of trials for their first experiment. Perhaps this indicates that they did not yet build an idea about the significance of sample size, despite their recognition in the earlier session that they were only able to type a claim about fairness after they ran an experiment with 500
trials, as indicated in Chapter 4.2.3. During that earlier session, the girls did not type a claim for their experiments with smaller sample sizes.

The experience was almost replicated when Kianja and Chanel investigated Dazzling Dice. However, for this company, the girls did make claims when they ran experiments with smaller sample sizes of 10, 20 and 40 trials. Following each of these three experiments, they claimed the dice were fair. At 40 trials, they started to question their findings because *five* had a lower frequency than the other outcomes. At 60 trials, they expressed surprise with an enthusiastic outburst as they observed that a *five* had an extremely low frequency. At this time they changed their claim. Their new decision that the dice were unfair was further supported by an experiment with 100 trials, in which the outcome of *five* had a frequency of 3. It is interesting that Kianja paused that experiment at 76 trials, when she found it significant that *five* had a frequency of 2 at this point, and indicated that it occurred at "three-fourths" (line C354) of the completed experiment.

During this session, this Kianja and Chanel investigated one company – Dazzling Dice. As they ran an experiment, they typed their findings into their Word summary document. They selected and pasted the stacked outcomes, pie chart and table as evidence in their Word document. Although their earlier findings with smaller sample sizes contradicted their final claim, they included the result and representations for each experiment in their summary document.

Chris and Jerel had a different approach. They made claims based on a sample size of 1000 for their assigned companies. As they looked at the outcomes for 1000 trials, the boys discussed the computer-generated representations and made a verbal decision about fairness. They saved all representations in *PE*. When they typed their
claim for each company in the Word summary document, they copied and pasted the rolled outcomes, table, bar graph and pie chart for 1000 trials.

Chris and Jerel solved the Schoolopoly task faster than any other group of students. Consequently, R2 approached them and questioned their claims. Chris and Jerel quoted information in the table to support their decisions about fairness. For unfair dice, Chris went further to indicate how the Weight Tool could have been used to affect the frequency of each outcome. For example, he concluded that the ratio of frequencies for even to odd outcomes for Calibrated Cubes was 2 to 1. During this session, there is no evidence that the researchers or even Jerel realized that Chris was discussing the PE Weight Tool, and that Chris was building mathematical ideas about bias.

This session also provides evidence for how Jerel interpreted fairness of dice and the significance of sample size. When Chris discussed the possible adjustments in the Weight Tool as being 2 to 1 for unfair dice from Calibrated Cubes, Jerel referred to the number of sides for an outcome, indicating that outcomes with the highest frequencies had "two sides", which is why they "came out the most" (line H378). Jerel's comment reveals that he is retaining the idea that dice are unfair if the same outcome appears on more than one side. Students had discussed this interpretation of unfair dice during the morning session (lines B46-B48).

An additional window into Jerel's ideas about fairness was provided during his exchange with R2 regarding dice produced by Delta's Dice. When R2 asked if the results for Delta's Dice would be the same if the boys ran another experiment with 1000 trials, the boys were able to immediately find out. After he ran a second experiment with 1000 trials, Jerel stated that Delta's Dice were still fair because in the first experiment, two had
the greatest frequency and in the second experiment, two had the lowest frequency. This implies that an additional interpretation of fairness, that is, dice are fair if outcomes alternate between having the highest and lowest frequency.

Regarding sample size, Jerel stated that if a die is fair then, as an experiment with a large sample size runs, it is possible that one or two outcomes may be much higher than others initially. This suggests the idea that the results of an experiment with a small sample size may not be an indicator of fairness. Moreover, in a smaller sample size the same outcome may have the highest frequency for one experiment, and then the lowest frequency for another experiment. Finally, Jerel noted that after 1000 trials, if the die is fair then the percentages will be close. Perhaps this is evidence that he has confidence in the result of an experiment with a sample size of 1000.

5.3 After lunch session – Students continue to work on the Schoolopoly task in pairs

Following lunch, the students returned to the classroom and sat at the computers they had been using all day. During this session, the students worked in pairs. There was time built into this session for exchanging ideas with other students and defending claims during discussions with teacher/researchers and graduate student/researchers.

5.3.1 Kianja and Chanel work on the Schoolopoly task

During the after-lunch session, Kianja and Chanel continued to determine fairness of dice for two additional companies – Dice Depot and Dice R Us, and they compared their findings for Dazzling Dice and Dice Depot with another pair of students, Keisha and Tiffany.
5.3.1.1 Kianja and Chanel investigate Dice Depot

When the girls were seated at their computer, they opened Dice Depot. For their first experiment, they entered 20 trials. It is interesting to note that this is the third set of experiments analyzed in this research study, for which this pair of students were exploring a task related to fairness of dice. In the first set of experiments, the girls entered a sample size of 6 and ran multiples of 6 in additional experiments. In the second set of experiments, when the girls were exploring Dazzling Dice, the first experiment had a sample size of 10 and additional experiments were multiples of 10. For this experiment, the first sample size was 20 trials. This observation suggests that the girls were following a process when they explored the fairness of dice. Although the initial sample size was gradually increasing, the girls began each set of experiments with a small number of trials and subsequent experiments included multiples of that initial sample size.

After studying the table and pie chart for an experiment with 20 trials, Kianja typed the claim that dice produced by Dice Depot were unfair because the outcome of four had the highest frequency. Chanel was not sure, and stated that “maybe it is fair” (line D79). Kianja selected as evidence, the representations depicted in Figure 5-8.

Figure 5-8. Dice Depot outcome from 20 trials
The girls next ran an experiment with 40 trials. The frequencies were 7, 6, 6, 7, 9, and 5, respectively. Kianja was uncertain as to the fairness, and she typed in the summary that they questioned their claim because “the numbers came out closer than before. The numbers may have gotten closer because of the height of our total.” They typed the claim that the dice were fair in their summary, and included the representations in Figure 5-9 as evidence.

![Figure 5-9. Dice Depot outcome from 40 trials](image)

During the next experiment with 60 trials, the frequencies were 5, 10, 11, 14, 11, and 9, respectively. Chanel changed her claim regarding fairness and stated “Deeng. I don’t think it’s fair” (line D98). However, Kianja typed “We think that we are almost positive that this is a fair die. The numbers like we said get closer as the height.” The girls were viewing the representations in Figure 5-10.
Kianja and Chanel ran 80 trials next. The girls looked at the monitor display shown in Figure 5-11, and Chanel typed “Now we are a little hesitant to say it is fair because the one, two, and seven are lower than 15 and the others are higher.” It is noted that although Chanel typed seven, she meant the outcome for six. With this sample size of 80, the girls were beginning to change their claim that the dice produced by Dice Depot were fair. It is also noted that in this experiment, the frequencies for two outcomes were greater than 17, and so $PE$ could not stack all of the outcomes.
5.3.1.2  **Kianja and Chanel compare findings with Keisha and Tiffany**

Another pair of students, Keisha and Tiffany, had also explored Dice Depot. R1 asked them to join Kianja and Chanel to compare their findings. Keisha asked Kianja and Chanel why they did an experiment with 80 trials. Kianja and Chanel replied they were “going up by 20” (lines D124 – D125), and that they were going to 100. Tiffany observed that they were doing “five graphs for each one” (line D131), and agreed that Dazzling Dice were unfair. Chanel asked what Keisha and Tiffany claimed for Dice Depot, and Keisha replied that they were not fair. Chanel replied that they would “probably get that too” (line D140).

Tiffany pointed out that in their document, Kianja and Chanel had typed that dice from Dice Depot were fair and now they just said the dice were unfair. Chanel replied that was before they made their “final decision” (line D147). Kianja explained that for Dazzling Dice, they had also changed their initial claim. Chanel ran an experiment for Dice Depot with 100 trials, and the girls viewed the representations in Figure 5-12. An outcome of *four* is the light blue (largest) sector on the pie chart, and *four* had a frequency of 30.

![Figure 5-12. Dice Depot outcome from 100 trials](image)
Kianja and Tiffany continued to discuss the results which Tiffany had found. Tiffany had made her decision that Dice Depot was unfair because all of the sides were “in the hundred” (line D229) but one outcome was different. It is conjectured that Kianja and Chanel were influenced by Tiffany's rationale and decision. After hearing Tiffany's comments, Chanel typed "Now we really don't think it is fair because how did four get 30 as his high and the rest was in its teens." Then, Kianja typed “In conclusion we don’t think the die is fair because we see that the 4th side always comes out on top.”

5.3.1.3 Summary of Kianja and Chanel's investigation of Dice Depot

By studying how the girls began this investigation and experiments from two earlier sessions, I conjecture that Kianja and Chanel have established a process for determining fairness of dice. They began solving each task by running an experiment that contained a small sample size. They ran additional experiments with sample sizes that were multiples of the initial run. This process actually hindered their ability to make a claim, because their final conclusions contradicted earlier findings with smaller sample sizes. Moreover, it took several experiments (and ultimately larger sample sizes) before they were able to type a final decision.

The girls were present at interventions that had the potential to influence a change in their process. At the whole group discussion during the first session, the boys mentioned that they based their claim on 4500 trials. Following that discussion, Kianja and Chanel were finally able to make a decision about fairness when they ran 500 trials. After conversing with Tiffany and comparing findings, Tiffany was questioned by G1. During that conversation which took place just behind Kianja, Tiffany noted that she ran experiments with 1000 trials and she was able to defend her claims. Kianja was aware
that Tiffany had used a sample size of 1000 trials to make her decisions about fairness. When R1 asked Kianja and Chanel how Tiffany and Keisha had determined fairness, Kianja noted that their strategy was “a little bit different” (line D367). She acknowledged that Tiffany and Keisha started with 1000 trials, while her experiments with Chanel “start smaller” (line D371). Notwithstanding these interventions, the girls chose to begin with a small sample size as they investigated each company for the Schoolopoly task. In fact, immediately following Tiffany’s statement about sample size and Kianja’s explanation to R1 about Tiffany’s strategy that used the results from 1000 trials, Kianja still ran 20 trials for her first experiment to investigate Dice R Us.

5.3.1.4 Kianja and Chanel investigate Dice R Us

Chanel and Kianja completed the first four experiments for Dice R Us with sample sizes of 20, 40, 60 and 80. The outcomes for the first three experiments are listed in Table 5-1. Following these experiments, Kianja and Chanel could not agree on a decision about fairness.

Table 5-1. Frequencies for Dice R Us experiments

<table>
<thead>
<tr>
<th># Trials</th>
<th>Outcome:</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>Frequencies:</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>7</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>40</td>
<td></td>
<td>4</td>
<td>8</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>4</td>
</tr>
<tr>
<td>60</td>
<td></td>
<td>6</td>
<td>7</td>
<td>11</td>
<td>11</td>
<td>16</td>
<td>9</td>
</tr>
</tbody>
</table>

Chanel typed the conclusion that the dice are unfair because the numbers "aren't close enough." Kianja noted the dice were fair and stated "No cause they are close enough" (line D403). Although the recording sound on the video CDs ended, the screen-
shot video established that the girls had time to run 80 trials and saved their results in their Word document summary.

The next morning, the girls completed their investigation of Dice R Us. They looked at the results from the experiment with 80 trials which Kianja had completed the previous day. These results are in Table 5-2. For this experiment, Chanel had typed the conclusion: "We think it is unfair because while the other numbers are the teens the dice #1 was a single digit number."

Table 5-2. Dice R Us outcome from 80 trials

<table>
<thead>
<tr>
<th>Outcome</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequencies</td>
<td>7</td>
<td>10</td>
<td>20</td>
<td>20</td>
<td>12</td>
<td>11</td>
</tr>
</tbody>
</table>

As the girls read the experiment's outcomes and their typed conclusion, G2 intervened and asked if they will "stick with the same sizes" as on the previous day (line E23). The girls replied they would follow their previous procedure, and they ran an experiment with 100 trials. The outcomes for 100 trials are listed in Table 5-3.

Table 5-3. Dice R Us outcome from 100 trials

<table>
<thead>
<tr>
<th>Outcome</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequencies</td>
<td>14</td>
<td>20</td>
<td>10</td>
<td>20</td>
<td>24</td>
<td>12</td>
</tr>
</tbody>
</table>

Kianja and Chanel discussed the computer-generated data, and how it could be used to make a decision. At first Chanel did not make a decision about fairness. Kianja suggested they write "we're not sure" (line E58) because "it looked fair and then it looked unfair" (line E61). Kianja typed the conclusion for 100 trials: "We are unsure whether this die is fair or unfair." She then stated: "because in the beginning three was one of the highest and then it was the lowest" (lines E77 - E78). G2 intervened and asked "Well
since you're unsure do you think you can get more evidence?" (line E79) Chanel replied that she did not need more evidence. She changed her decision, and said "I think it's fair" (line E82). She explained that "at first, the um, three had won then, at the end it lost and then in the middle it um, won and then lost again and then twelve, six, six got more than the first than it did in the last" (lines E92 - E94). I would interpret that Chanel's explanation implies that the dice are fair if each outcome had an equal chance of occurring. G2 interpreted Chanel's reply as "So everybody had a chance to win" (line E97). Following this exchange, Kianja changed her conclusion and typed that the dice were fair. It is conjectured that G2's intervention presented the girls with a possible definition for fairness that was similar to Jerel's idea stated in Chapter 5.2.2 - dice are fair when outcomes alternate between having the highest or lowest frequency.

G2 continued the intervention by reminding the girls that on the previous day, Keisha and Tiffany had mentioned that they made their decision from an experiment with 1000 trials. Both Kianja and Chanel acknowledged that Keisha and Tiffany did "a thousand but we did like, a different number of trials." (line E106 - E109). However, Kianja and Chanel agreed that their result would remain the same with a larger sample size, since their conclusions on the previous day matched Tiffany and Keisha's conclusions for the same two companies. It is conjectured that comparing notes with Tiffany and Keisha actually validated Kianja's and Chanel's decision to begin with smaller sample sizes and make claims based on their outcomes, because the results come out to be the same as for Tiffany and Keisha's experiments, which began with a large sample size.
Next the girls ran 500 trials for Dice R Us. Perhaps they were influenced by their conversation with G2. After the first 500 trials, Chanel looked at the bar chart and noted “See they're all close” (line E126). Looking at the pie chart, Kianja stated, “And they look even, right?” (line E129) They ran an additional 500 trials for Dice R Us. The frequencies for the total 1000 trials are in Table 5-4.

**Table 5-4. Dice R Us outcome from 1000 trials**

<table>
<thead>
<tr>
<th>Outcome</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequencies</td>
<td>116</td>
<td>167</td>
<td>184</td>
<td>203</td>
<td>180</td>
<td>150</td>
</tr>
</tbody>
</table>

After 1,000 trials, the girls opened the data table and read the frequencies. Kianja stated, “Put the pie chart on there. I want to see. Yeah, still almost equal. I think our decision was right. What do you think?” (lines E137 - E138). Chanel agreed, “I do think it’s fair” (line E140). The girls deleted the data from this experiment without adding it to their Word summary document. These two experiments supported the girls' idea that if dice are fair, then using a large sample size such as 500 or 1000 trials will have the same outcome as experiments with smaller sample sizes. It is interesting to note, however, that the girls' were not certain that the dice were fair after 100 trials. They made their claim that the dice were fair after talking with G2, who reiterated their idea about fairness, which was that the outcomes with highest and lowest frequencies were alternating and that each side had an equal chance to have the highest outcome.

**5.3.1.5 Summary of Kianja and Chanel's investigation of Dice R Us**

Kianja and Chanel began their investigation of Dice R Us with an experiment that had 20 trials. They continued to run experiments with sample sizes that were multiples of 20, ending at 100. During these experiments, Chanel referred to the table and decided
that the dice were unfair. Kianja looked at the pie chart and decided that the dice were fair. However, after the experiment with 100 trials, Kianja said she was "unsure" and Chanel changed her mind and said the dice were fair.

G2 provided two significant interventions during this investigation. The first intervention was related to the justification of Chanel's decision following the experiment with 100 trials. After G2 asked Chanel to defend her decision that the dice were fair, G2 then gave a stronger interpretation of Chanel's response by noting that the dice were fair because different outcomes took turns having the highest frequency. Kianja was influenced by this conversation. She changed her decision and typed the conclusion that the dice were fair.

A second intervention regarding sample size occurred when G2 reminded the girls that on the previous day, Keisha and Tiffany made decisions based on experiments with 1000 trials. At first Kianja and Chanel stated this was not necessary because they got the same results as Keisha and Tiffany. However, after some quiet conversation that hinted at the largeness of 1000 when Chanel noted "I don't even have a thousand dollars in my bank account" (line E132), Kianja and Chanel ran two experiments that totaled 1000 trials for Dice R Us. They looked at the table and the pie chart, and concluded that their previous decision that the dice were fair was the correct decision because the sectors were "almost equal" (line E137).

### 5.3.2 Chris and Jerel work on the Schoolopoly task

Chris and Jerel had investigated their three assigned companies during the previous session. For this session, they defended their claims with one of the researchers, and discussed their findings with another student.
5.3.2.1 Chris' assignment of weights to unfair dice emerges

When the students returned to their computers after lunch, Chris and Jerel were the only pair who had completed investigating all three companies. R3 initiated a conversation with them about their results. They opened their files in PE and discussed their experiment for Delta's Dice, which they ran 1000 times. Jerel noted "what made it look even is that the percents are close, the bar, the pie graph looks even, even though it wasn’t and uh the bar graph, where it showed the exact numbers were very close together" (lines I79 - I82). R3 asked to see their data table, and this time Chris confirmed that "all the percents were close" (Line I92) and so the dice were fair.

The boys next opened their findings for Dice R Us, and noted these dice were not fair. Jerel stated that some outcomes are "taking up more shades on the pie graph" (line I109) and that "It’s like they made all the even numbers two-sided" (line I111). Chris corrected him noting that "number three is two-sided" (line I116), and not just the even numbers. Jerel had previously made a reference to "two-sided." During a discussion with R2 (Chapter 5.2.2), Jerel was defending his claim for dice from Calibrated Cubes. This meaning for "two-sided" hinted at the greater probability of rolling certain outcomes because the same outcome may be on two different sides of the die.

Chris corrected Jerel's use of the idea of two-sided, stating "They call it, not two-sided. They call it more than one of them in, so that four has two, three has two, and two has two" (lines I118 - I120). R3 followed Chris' statement by recognizing "So you’re talking about like that Weight Tool that you guys looked at earlier" (lines I121-I122). Chris and Jerel both nodded their heads in agreement. This was the first time that a researcher acknowledged that Chris was analyzing the unfairness of the dice by
describing the numbers that could have been entered as weights in the Weight Tool. R3 asked Chris which outcomes were weighted differently, and Chris replied the "two, three, four and five" (line I128).

The next company that the boys discussed with R3 was Calibrated Cubes. The boys had determined that their dice were unfair. Chris pointed to the pie chart as evidence, saying "You see that some of them are bigger than others" (lines I148 - I149). Chris continued to defend his decision that the dice were unfair by giving the weights that may have been entered into the Weight Tool, saying "two, four and six they have two, and three and five only have one" (lines I151 - I152). R3 verified that Chris was referring to the Weight Tool, and Chris agreed that he was.

5.3.2.2 Researcher intervention - an extension of the task

R3 proposed an extension of the Schoolopoly task. He asked the boys if they could duplicate the representations for the Calibrated Cubes outcomes by opening the Weight Tool and making adjustments. The boys typed the weights displayed in Figure 5-13.
After Chris changed the Weight Tool, he ran a total of 1000 trials for his attempted duplication of the outcomes for Calibrated Cubes. While the experiment ran, R1 joined the boys and R3 and asked for an update. Chris explained that "one, three and five have one in them and two, four and six have two" (line I181 - I182). R1 asked what Chris meant, and Chris replied "Like on the Weight Tool" (line I184). It is noted that this is the first time that Chris described his response about unfair dice by specifying that he was thinking about possible weights that could have been entered into the Weight Tool to favor certain outcomes. R1 asked why he assigned the same numbers to three of the sides, and Chris replied that those three outcomes had percentages which were close.

Chris observed that the percentages which resulted from his experiment were not the same percentages as the outcomes for Calibrated Cubes. However, R1 asked "so, they’re not exactly but they’re close. Are they close enough?" (lines I208 - I209) The boys stated that the outcomes from their experiment were close enough to the Calibrated Cubes outcomes. The outcomes for Calibrated Cubes as well as the outcomes for Chris' experiment, for which he had entered weights of 2:1 for the ratio of even outcomes to odd outcomes, are in Figure 5-14.

![Figure 5-14. Calibrated Cubes (left) and Chris' experiment](image-url)
R1 questioned the boys' decision that the dice were unfair for both Calibrated Cubes and Chris' experiment using the Weight Tool. She asked the boys if the dice were actually fair, since "I have the same chance of getting one, three and five. And I’ve got the same chance of getting two, four and six. Isn’t that fair?" (lines I219 - I221). The boys disagreed with her, and Chris replied it was unfair "cause you’re gonna keep getting two, four and six more" (line I225). It seems that Chris has rejected the fairness idea that R1 introduced: dice are fair if the outcomes are balanced, that is, three of the sides always have the lowest frequency, and the other three always have the greatest frequency.

5.3.2.3 Student intervention - comparing findings for Delta's Dice

Another student, Jarae, had investigated Delta's Dice, and R1 suggested that she compare her findings with the boys. Jarae explained that another group of students, Justina and Lorrin, ran 55 trials, and "each number had got to win at least one time" (line I243). Because the sides took turns coming out the highest, the girls decided that Delta's Dice were fair. When Jarae tried to determine fairness, she used 6 trials but each side did not come out. However, based on "evidence from Justina and Lorrin’s" experiments (line I258), Jarae seemed confident that the dice were fair.

When Chris and Jerel explained their findings for Delta's Dice, they stated that the dice were fair. Chris noted that "the pie graph looks even" (line I268). R1 pointed to the two in the bar graph and on the table. She said: "That pie graph doesn’t look even to me" (line I269). Chris and Jerel disagreed with her opinion. R1 asked the boys to read the percent for two, and they replied: "14.7." Then she asked them if that is close to 18. The boys stated that it was, and Jerel said: "five or more apart is not close but four is close" (line I279). This hints at a definition of "close" for Jerel. In looking at the table of
percents, Jerel further noted that some of the percents were approximately 15, two were 18, and one was 17. The difference between the percents in the table was less than 4%, and Jerel stated: "If you’re trying to say that four is not close, that’s trying to say three is not close" (lines 1283 - 1284). For this experiment, Jerel had determined that if there was a difference of 4% or less between the percents in the table, then the dice were fair. It is unknown how he had decided on 4%, and why he had clearly stated that percents that differ by 5% are not close.

Jarae asked the boys why they ran 1000 trials. Jerel replied that they "wanted to make sure that one number wouldn't be way in front of the other number" (lines 1358 - 1361). Jarae was not convinced that the boys needed to run that many trials. She said that she ran 600 trials, and then corrected herself and said she ran 6 trials. While she was asking questions about sample size, Chris ran some additional experiments for Delta's Dice. Figure 5-15 shows the results for Delta's Dice, 100 trials.

*Figure 5-15. Delta's Dice outcome from 100 trials*

Next Chris duplicated Jarae's experiment with 6 trials. The outcomes for this experiment are in Figure 5-16. For this experiment, three outcomes had zero frequency.
Jerel noted "I guarantee you if we would have ran it 1000 times it would have been even" (lines I395 - I396). When R1 asked why the bar graph wasn't showing up even (which I interpret as meaning that the bars have the same heights) for 100 trials, Jerel said "because you’re not running it enough times" (line I402). Jerel continued, "When you run it 100 times you will never know if it’s gonna catch up, cause you stopping too early. But then, if you run it 1000 times you know that you have a better chance of having it be even and another number catching up to it" (lines I416 - I423). Based on his responses, it is plausible that Jerel had a definite idea about sample size and that he had confidence in outcomes from 1000 trials.

Jarae was not convinced that the larger sample size would result in a decision. The boys noted that for fair dice, the outcomes became "even" (line I444) after 1000 trials. It is conjectured that the use of the term "even" suggests that the outcomes are close or almost equal. Chris said "you're not just gonna roll six times and yeah, okay I'm done"(lines I445 - I446). Jerel agreed, suggesting a practical application "cause if you're losing your money, you're gonna keep going" (lines I447 - I448). The boys continued to run experiments with smaller sample sizes to convince Jarae. For another experiment
with 100 trials, the outcomes were not close. For 500 trials, blue (two) had a much lower frequency than the other outcomes. However, during a second experiment with 500 trials the two had the greatest frequency. The boys ran experiments with 1000 trials. Each time the five was highest. But the boys did not change their decision that the dice were fair. They used the tables rather than the bar graph, and interpreted the percents as close. Finally, at 1500 trials, the bars in the bar graph looked very close in height. Figure 5-17 shows the outcomes for Delta's Dice, 1500 trials. The boys and Jarae agreed that the dice were fair, and they agreed that the bar graph, pie chart and table supported their decision.

\[\text{Figure 5-17. Delta's Dice outcome from 1500 trials}\]

R2 had observed the different experiments that Chris had been running, and stated "I bet if you just do it 12 times one of the numbers is gonna win by a lot" (lines 1523 - 1524). Jerel explained that it could happen because "it's only a little bit of number you ran" (line 1525). R2 asked why sample size mattered if dice are fair. Jerel replied that if you "roll a little bit, of course a number is gonna blow another number out" (lines 1527 - 1528). This hints at Jerel's idea that some numbers could have higher frequencies if the
sample size is less than 1000 trials, and provides additional evidence that he is confident in making a claim when the sample size is 1000 trials.

5.3.2.4 Summary of after-lunch session for Chris and Jerel

Because Chris and Jerel completed the Schoolopoly task during the late morning session, they had time to discuss their findings with the researchers. During these discussions, as well as their conversation with another student, Jarae, their ideas about fairness of dice and sample size emerged. According to Chris and Jerel, when dice are fair, the percents in the table are close. Jerel interpreted his idea of close by considering the relative frequencies of all outcomes as a percent. In one experiment, he decided that outcomes had an even (or equal) chance of occurring if the difference between the percents was less than or equal to 4%. When a researcher suggested that the dice were fair if half of the outcomes had low frequencies and the other half had high frequencies, Chris and Jerel rejected this idea about fairness. In fact, they stated that if the same outcome was consistently highest or lowest, then the die was not fair. To determine fairness, Chris and Jerel ran experiments with 1000 trials. Jerel noted that as an experiment ran, it was possible for a number to have a higher frequency than other numbers. However, after 1000 trials, if a die is fair then the percents in the table will be close. In fact, it took 1500 trials to convince Jarae that Delta's Dice were fair because with this sample size, the bars on the bar graph were level and all of the PE representations provided evidence to support that claim.

For unfair dice, R3 was able to interpret what Chris was referring to when he talked about an outcome followed by a number such as 1 or 2. Chris was assigning a weight to each outcome for unfair dice, as if he were designing the task and using the PE
Weight Tool to introduce bias. After R3 verified that Chris was talking about which numbers represented weights in the Weight Tool, Chris was able to clearly explain that he was predicting the weights used in the Weight Tool to R1, using the correct name of the tool. R3 spontaneously created an extension of the Schoolopoly task, and asked Chris to duplicate the outcomes of Calibrated Cubes by entering his guessed weights into the Weight Tool. The relative frequency percents in his duplicated experiment were not exactly the same as in Calibrated Cubes, but the outcomes with the highest frequency were the same as in Calibrated Cubes, as were the outcomes that had the lowest frequency. Moreover, the bar graph and pie chart representations were similar. The boys were convinced that this experiment validated their idea that the even numbers had a greater chance of occurring, because the weight entered for the even numbers was twice the weight of the odd numbers. Initially, Jerel used $x$-sided terminology to describe why an outcome had a greater frequency (because that outcome appeared on $x$-sides of the die, and the die had more than 6 sides). Chris corrected him, and it is conjectured that this task extension experiment may have influenced Jerel to also describe unfair dice by considering weights in the Weight Tool.
Chapter 6  STUDENTS CREATE POSTERS

6.1  Introduction

This chapter presents an analysis of how the four students who are the focus of this study created posters that illustrated their solutions for the Schoolopoly task. The sessions in this chapter took place on the morning of Thursday, August 4, 2005, the last day of an IML Summer Institute. During that institute, students had worked collaboratively in pairs solving probability tasks using Probability Explorer (PE) software to generate tables, bar graphs and pie charts, as described in Chapter 4.1.2. On this final morning, students were asked to answer questions related to the fairness of dice manufactured by three companies, which they had investigated on the previous day. Students were asked to support their answers with evidence, and to put their answers and the evidence on posters.

At the start of the first morning session, R1 welcomed students in an air-conditioned classroom on the second floor, which had large tables and moveable chairs. Four videographers and audio technicians, as well as teacher/researchers and graduate student/researchers were present. One of the video cameras was focused on each pair of students in this study. A third video camera circulated among the other students as they worked on their posters, and the fourth camera followed R1. R1 gave students directions for working on the posters, and clarified the questions they were asked to address on their posters. Students were to create one poster for each company they investigated. Each student taped a poster to his or her own table. The posters had an adhesive strip on the back of the top. Students were given colored markers to use. R1 explained that they
should not write their names on their posters. Each poster would be given a number, and later that day students would have an opportunity to judge each anonymous poster.

6.2.1 Kianja and Chanel create posters

At the beginning of the session, Kianja and Chanel completed the investigation of their third assigned company, Dice R Us, as described in Chapter 5.3.1.4. They worked in the computer classroom where they had worked the previous day. It took about 15 minutes for them to make a decision about the fairness of dice produced by Dice R Us, and then to save their work. When they arrived in the poster classroom, R1 answered their questions about what they were expected to do. Chanel said that she preferred to create a poster for the company which they had decided was fair, Dice R Us. Kianja selected Dice Depot.

6.2.1.1 Dice Depot Poster

Kianja started by working on scratch paper. She added the frequency of one for each Dice Depot experiment. For that investigation, the girls had run five experiments with samples sizes of 20, 40, 60, 80 and 100, or a total of 300. Kianja found that the relative frequency for the outcome of one from all of her experiments was 42/300. Initially, she wrote her results on scratch paper and did the calculations by hand. She then asked for a calculator to convert the fraction to the decimal, to obtain 0.14. Her calculations for an outcome of one are in Figure 6-1.
Figure 6-1. Relative frequency of an outcome of one in Dice Depot

Once she began using a calculator, Kianja created a different representation, as captured in Figure 6-2. It is conjectured that she created this representation so that she could summarize her findings and enter the frequencies into the calculator in a more efficient manner.

Figure 6-2. Kianja's representation for summarizing frequencies

Kianja began to list the percents. At first, Kianja's percents were 14, 13, 19, 25, 19, and 12. She added the percents and observed: "A hundred and two, woah. So then, we’ve been doin’ this wrong. 18 and 11? I guess. Ok " (lines E266 - E268). Once she rounded the relative frequencies for the outcomes of five and six down to 18% and 11%, respectively, she noted that the total was now 100%. She seemed to have recognized that
the total of the probabilities should equal 100%. Kianja gave her notes to Chanel, and Chanel created the Dice Depot poster. She used Kianja's calculations to construct a summary table which listed the relative frequency for each outcome as a percent.

Chanel's summary table for the Dice Depot poster is seen in Figure 6-3.

Figure 6-3. The relative frequency for each outcome in Dice Depot

As noted in Chapter 5.3.1, during their first three experiments with trials of 20, 40 and 60, the girls were initially uncertain about the fairness of the dice produced by Dice Depot. In fact, their final decision followed two events. First, Kianja asked Tiffany what she and Keisha had decided, and Tiffany said that they claimed the dice were unfair. Tiffany told Kianja that they had run a sample size of 1000, and that one side had a frequency which was very different from the other outcomes. Second, Kianja ran experiments with 80 and 100 trials, and for both experiments she noted that the frequency for six was consistently lower than the frequency for the other outcomes. After these two events, Kianja was convinced that the dice were unfair and she typed her claim into the computer summary document. As evidence for the poster, Chanel cut and pasted the computer-generated tables that listed the results for 40 and 80 trials, as seen in Figure 6-4. At 40 trials, the girls were uncertain about fairness, but at 80 trials (and then again at
100 trials), Kianja and Chanel were convinced that the dice were unfair. Chanel circled the outcome with the highest frequency on the table with data from each experiment.

![Figure 6-4. Dice Depot outcomes for 40 trials (left) and 80 trials](image)

A snapshot of the claim regarding fairness of dice produced by Dice Depot, written by Chanel, is in Figure 6-5. Chanel wrote that she recognized that the outcome for six was lowest when the die was tossed 40 times, and that both one and six were lowest compared to the other outcomes which were in "double digits already" as a result of 80 trials.

![Figure 6-5. Claim that dice produced by Dice Depot are unfair](image)
6.2.1.2 Dice R Us Poster

As Kianja was calculating the probability of outcomes for dice produced by Dice Depot, Chanel worked on the Dice R Us poster. R1 started to read Chanel's poster. Chanel told R1 that "we found out the dice were fair because at first like, um, just at first like, three, just this as an example, like three went from being the highest from the first one, to being the lowest what it is, the last one" (lines F10 - F13). Perhaps Chanel was persuaded that the dice were fair if an outcome alternated between having the highest frequency and then the lowest frequency, for different experiments. Based on these outcomes, Chanel told R1 that there is "a possibility that all of them can get rolled the same amount" (lines F19 - F20). The reader is reminded that in Chapter 5.3.1.4, the girls ran experiments with 500 and then 1000 trials, perhaps as a consequence of their discussion with G2 regarding sample size. They did not save the representations from those experiments, and they decided that the result of the experiments with larger samples validated the findings of their series of experiments with smaller samples. Figure 6-6 shows the two tables that Chanel pasted on the Dice R Us poster to provide evidence that the dice are fair, because in 80 trials an outcome of three had the highest frequency, and yet it had the lowest frequency in 100 trials.

![Figure 6-6](image)

*Figure 6-6. Dice are fair because the outcome of three alternates*
R1 pointed out that Chanel needed to provide an explanation on her poster to indicate why she included both tables in Figure 6-6 as evidence to support her claim that dice from Dice R Us are fair. Chanel added a sentence under the tables, which summarized her findings that the dice were fair because three went from being the highest rolled outcome to the lowest rolled outcome, perhaps averaging the frequencies. In her answer for Schoolopoly task question 3 regarding the probability of rolling each number, Chanel wrote that "the chances of rolling all of the numbers on the dice are good because just like in my example at (and she drew an arrow pointing to the tables in Figure 6-6), first 3 was the highest rolled then the last 3 was the lowest rolled overall."

6.2.1.3 Dazzling Dice Poster

When Kianja finished her relative frequency calculations for Dice Depot, she started working on the poster for Dazzling Dice. She made a table that was similar to her previous representation for listing the frequency for each experiment, seen in Figure 6-2. Kianja's table of frequencies for Dazzling Dice is in Figure 6-7.

![Figure 6-7. Dazzling Dice summary of frequencies](image)
Next, Kianja wrote the quotient of the sum of the frequencies for each outcome, divided by the total number of trials. For Dazzling Dice, the girls ran trials of 20, 40, 60, 80 and 100, so that once again the total number of trials was 300. Kianja used a calculator to convert the fractions in Figure 6-8 to percents but when she checked the total of her percents, they did not add up to 100.

Figure 6-8. Percents for Dazzling Dice do not add up to 100

Kianja checked her work as she talked to herself, saying softly "16, daaamn. Ok, all right. Ok. 64, 65, 50 over 300 equals 0 point what, clear. 47 divided by 300 equals, I agree. (Continues working.) All right, all right, all right, all right. Let’s see, maybe round this out to 16, this to 21, this to 22, this to 19, this to 53, and this to 17, maybe I don’t know, I’m not sure, clear, clear…” (lines F38 - F43). After a few minutes of talking to herself, Kianja caught her mistake in converting 16/300 to the decimal 0.053, and then to 53% instead of 5.3%. She rounded that to 5% and her percents now totaled 100. Kianja wrote on her poster, the summary table in Figure 6-9.
Kianja circled the outcome of five and the probability 5% and drew a line to her written words: "makes this die unfair." Kianja stated to G2 that she had "put my little proof for the five" (lines F159 - F160) on the poster. Next to her probability summary table, Kianja pasted the pie chart and table for 100 trials, generated by the computer. These are included in Figure 6-10. The data from this experiment seemed to convince Kianja about the unfairness of the dice, and might explain why she referred to the computer-generated representations from this experiment as her "proof." In fact, when R3 asked Kianja how unfair the company was, she replied "Well this one is so unfair that it’s only 5% chance that you’ll get, you’ll roll a five" (lines F185 - F186).

Figure 6-9. The relative frequency for each outcome in Dazzling Dice

Figure 6-10. Kianja's "little proof for the five"
When Kianja described her method of adding the frequencies and dividing by the total number of trials to R3, he asked her "Is there just one trial that you could have used, to determine whether it was fair or unfair?" (lines G338 - F339) Kianja pointed to the trial depicted in Figure 6-10 and explained: "one has 13, two has 22, um three has 20, four has 25 and then five has 3. I wrote, ... we are now sure that this die is unfair because, in the hundred trial we ran, only 3 times did the five show up. I don’t think that’s very likely on a fair die. I mean, you would get at least ten tries, I mean" (lines F348 - F353). Next, G1 approached Kianja and asked her why she selected the experiment with 100 trials as evidence for her poster. Kianja and Chanel agreed that the experiment with 100 trials had results that were "more clear about the fairness" (lines F366 - F367). Moreover, Kianja noted: "I think it’s more clear because, if you look on this pie chart, you see this small little wedge…” (lines F370 - F371). This suggests that the pie graph for 100 trials had a comparatively slim sector which represented the frequency of five. This visual representation convinced Kianja that there were very few outcomes for that side compared to the other sides, and therefore the dice were unfair. It also suggests that Kianja is using both the pie chart and the table to support her claim.

G1 continued to talk to Kianja about their experiments, and noted that she ran experiments with different sample sizes. He asked why she added the frequencies. Kianja replied that she wanted to "see the overall percentage...and then that would help with the decision of how fair and how unfair it is" (lines F304 - F308). Kianja stated emphatically: "Oh I'm confident" (line F316) in her decision because the girls had worked on Dazzling Dice initially, but they lost their findings "because our computer wasn’t working" (line F404). Kianja reminded G1 that they ran their experiments again,
"and when we did it over I mean we had the same conclusions and stuff like that but different numbers" (lines F409 - F410). Perhaps the girls were aware that, although the percents in the relative frequency table may not have been identical to outcomes of previous experiments, the final decision about fairness would be the same, regardless of the sample size.

As the session was coming to an end, R3 talked to Kianja one more time about which representations provided the best evidence.

R3 I was wondering whether or not this (referring to the experiment with 100 trials) would be sufficient, data for you to use, to show that this company’s dice is unfair. Did you really need to, go through the act of accumulating all of the different trials?

Kianja Like you can’t but, you, I mean I thought that, you know you might need a picture.

R3 Uh huh.

Kianja So this circle graph, this pie chart…

R3 Uh huh.

Kianja Could sort of be a picture, for you to look at…

R3 I see.

Kianja And relate it to the percents that you put in there.

R3 Right. So which, which one of these representations are you asking the viewer to focus on? Which one is, is providing the evidence for you?

Kianja This (pointing to the table).

R3 That one, uh huh. This one is, this one is providing the…

Kianja The percents. (lines F447 - F464)

This exchange substantiates Kianja's preference for the table and the pie chart when she selected computer-generated representations for evidence. The percents in the table provided the numerical evidence, while the pie chart provided the visual "picture."

Further, it seems that Kianja gradually built her sample sizes in different experiments to
tell a story about the fairness or unfairness of dice. It is plausible that, as the story evolved, she deliberately included the representations and claims from each experiment, to build a complete picture which provided the evidence for whether or not the dice are fair.

6.2.1.4 Summary of Kianja and Chanel creating posters

As the girls worked on their posters, they talked with the researchers about what was the most convincing evidence that helped them decide whether dice were fair. For Dice Depot, Kianja made her own percents table by adding the frequencies for all experiments and dividing by the total number of trials, 300. She claimed that Dice Depot dice were unfair. As evidence, the girls selected two computer-generated tables from experiments with 40 trials and 80 trials. In both experiments, the frequency for an outcome of six was the lowest or one of the lowest. Chanel added a summary table to their poster which included the relative frequency for each outcome, and she used the percents which Kianja had calculated by adding the data from all of their experiments.

Chanel worked on the Dice R Us poster. She claimed their dice were fair, and for evidence she included two computer-generated tables. These tables illustrated that for an experiment with 80 trials an outcome of three came out the highest, and for an experiment with 100 trials three came out the lowest. It is conjectured that Chanel maintained the interpretation of this balancing of outcomes as indicating fairness, even though she and Kianja had run two experiments with 500 and then 1000 trials. They decided that the frequencies for these experiments were close, and that therefore the result from the experiments with larger sample sizes validated their findings for fairness based on an outcome alternating between highest and lowest frequency.
Kianja created the poster for Dazzling Dice. She added the frequencies for each experiment, and divided by the total of the trials. She claimed that dice produced by Dazzling Dice were unfair, and as evidence she included her "proof" for five, which consisted of a computer-generated pie chart and table representing the outcomes for an experiment with 100 trials. The sector on the pie graph which represented an outcome of five was a thin sliver in comparison to the other sectors, and the table indicated that the frequency for five was indeed very low. For this company, Kianja told R3 that the computer-generated representation that provided the evidence for her was the table with percents.

Kianja was the recipient of several interventions from researchers and other students. R3 asked her which experiment she would choose if she did not use the total of all experiments, and she indicated that she would use the experiment with 100 trials. However, Kianja did not change her decision to tell the story by running several experiments and using a gradually increasing number of trials in multiples of 20. Even after talking with Tiffany and Keisha, Kianja did not indicate that one experiment with 1000 trials was more accurate than running five smaller experiments.

### 6.2.2 Chris and Jerel create posters

Chris and Jerel were the first pair of students to complete the investigation of their assigned companies. They were in the poster classroom at the start of the August 4 morning session, along with two other pairs of students who were also ready to work on posters. Chris decided to work on Delta's Dice, and Jerel selected Calibrated Cubes, which he called "Calaborated Cubes" (line J118).
6.2.2.1 Calibrated Cubes Poster

When the boys ran a sample size of 1000 for Calibrated Cubes on the previous day, they made the claim that the dice were unfair. They based their claim on the outcomes, because the odd sides had almost one-half the frequency of the even sides. For this reason, Chris had stated that the Weight Tool possibly had 2 for the even outcomes, and 1 for the odd outcomes. The reader is reminded that R3 had asked the boys to duplicate the experiment by adjusting the Weight Tool (2:1 = even : odd) to see if the outcome for that experiment would be the same as the Calibrated Cubes outcome. Chris stated that although the probabilities for the duplicated experiment were not identical to those of the original problem, the bar graph and pie chart resulting from their experiment did look similar to those representing Calibrated Cubes.

For the poster, Jerel cut out the entire computer screen with the rolled dice, the pie chart, bar graph and table, as seen in Figure 6-11. Here, he is making use of multiple representations to back his claim.

Figure 6-11. Dice produced by Calibrated Cubes are unfair.
Jerel had written on the poster: "We ran it 1000 times and all the odd number had a low percent and high number had a 20+%". R1 read aloud from Jerel's poster, "So we ran a thousand times, and all the odd numbers had a low percent and high number… What do you mean by high number?" (lines J255 - J258) Chris corrected Jerel and said "Even number" (line J259) and Jerel wrote the corrected statement (in Figure 6-12) on the poster with a marker.

![Poster image](image)

*Figure 6-12. Jerel's substantiation for why Calibrated Cubes were unfair*

### 6.2.2.2 Delta's Dice Poster

Chris started working on the Delta's Dice poster and answered the three Schoolopoly task questions. He wrote the claim that the dice were fair "because if you look at the percents below you can see that they are real close. If you look at the pie graph, you can see that they are very even." For evidence, Chris drew a pie chart by hand. For his first attempt, he made eight sectors instead of six. He crossed out that pie graph and drew another one alongside it. R1 looked at his poster and suggested that observers may question whether he "made up that pie graph" (line J200). Then she asked "What information doesn’t it tell you?" (line J209) and guided Chris to the realization that the pie graph does not give numerical information about sample size or frequency. R1 recommended that Chris cut out actual data that the computer generated. Chris cut
out the entire computer screen which showed the tossed dice, the pie chart, bar graph, and frequency table, and pasted it over his original, crossed-out pie graph, as seen in Figure 6-13.

*Figure 6-13. Chris' evidence that Delta's Dice are fair*

In addition, Chris made by hand, a table on the poster that listed the relative frequency for each outcome as a fraction, decimal and percent. He copied that information from the screen shot of the computer monitor that he had pasted onto the poster. Chris' table is in Figure 6-14.

*Figure 6-14. Chris' hand-written data table for Delta's Dice*
6.2.2.3 Dice R Us Poster

Chris and Jerel worked on the poster for Dice R Us together. Chris wrote the answers to the Schoolopoly task questions as Jerel cut out the computer representations. Chris wrote that they would not recommend dice from Dice R Us because "2 of the dice barely are rolled and the rest are rolled a lot." For evidence, Chris wrote that "the percent are not close. Only one number reaches 200. The bar graph is not close. Almost one number takes up the pie graph." R1 read the poster and commented that it was "nice that you’re pointing to the different representations, but again, I don’t know, which 200. Out of how many trials?" (lines K246 - K248) R1 noted that their answers on the poster did not indicate how the evidence proved that the dice were unfair. She said that the boys needed to interpret the data. Jerel replied that "some percents were at 21.4 and others were at 10.3" (lines K55 - K257). R1 questioned this further in the following exchange, which is an example of how Chris' explanation for unfair dice included his speculation about the weights which were entered into the Weight Tool, when the experiment was created.

R1 But why does that, why does that mean that it’s not fair?
Chris Because, right there you know that one and six weren’t rolled that much, so you know there’s not going to be a lot of one and six. So there’s probably one of one and six and probably two of two, three, four, and five.
R1 When you say there’s probably two of, you’re thinking about the Weight Tool?
Chris Yeah the Weight Tool. (lines K263 - K270)

R1 asked the boys to write more about this idea on the poster. They drew an arrow from their answer to Schoolopoly task question 2 and wrote "200 out of 1000 trials 2 numbers have very low percent and the others have high percent." Although the boys
now specified the sample size, it is plausible that they did not realize that R1 was asking them to write about how they were convinced that the dice were unfair, or to explain how Chris speculated about the use of weights in the design of the experiment. Chris' method of representing the weight of sides by suggesting numbers that could have been entered into the Weight Tool was made clear through an exchange between G3 and Chris.

G3  Chris I have a question for you. Uh, ok you see this dice here?
Chris    Yeah
G3  What’s the number here?
Chris    One.
G3  Ok. What do you think the numbers of the other sides of this dice?
Chris    For this one?
G3  Yeah, ok, you see, look at this dice?
Chris    Yeah one.
G3  Ok, well, what do you think were the numbers of the other sides?
Chris    Oh I don’t know.
G3  Well what do you think?
Chris    Hmmm, let’s see. I think it’s…
G3  What do you think?
Chris    One, it’s one…
G3  Yeah.
Chris    Then it’s…
G3  Do you think it’s, if it’s one, what, what do you think on the other side is there?
Chris    It would be two, two.
G3  mm hmm.
Chris    Three, four, I don’t know.
G3  So they would be all different, right?
Chris    Yeah, yeah they’d be a lot different, I think. (lines K214 - K236)

After reading this passage many times and listening to Chris' justification for unfair dice, I conjecture that G3 is asking Chris about the weight that may have been
entered into the Weight Tool for Dice R Us. It is interesting to note that Chris recognized that the lowest weight, 1, was entered for an outcome of one, and that the weight for an outcome of two was twice that amount, or 2. But then, the bar graph indicated that the frequencies for the other outcomes were different. Chris acknowledged this bar graph representation by agreeing with G3 that the weights would be "all different."

For Dice R Us representations, the boys pasted to the poster the screen shot seen in Figure 6-15, with the rolled dice, bar graph, pie chart and table.

![Figure 6-15. Dice R Us outcome from 1000 trials](image)

**6.2.2.3 Summary of Chris and Jerel creating posters**

At first, Chris did not use computer printouts for evidence. He drew a pie graph by hand, directly on the poster. When R1 pointed out that no one may believe his claim without computer evidence, Chris then cut out the screen shot with multiple representations. However, he still created a hand-drawn pie graph with equal sectors, as well as a hand-written table that duplicated the computer-generated table.
An intervention by R1 was designed to inform Chris that the table was the representation that needed to be included so that poster viewers could see the sample size of the boys' experiment. Following this intervention, Chris and Jerel included the entire screen shot for each company on all of their posters. The screen shot captured the tossed dice, bar graph, pie chart and table.

As the boys worked on their posters, Chris was asked to discuss possible weights in the PE Weight Tool for the two companies that manufactured unfair dice. It is conjectured that Chris was now able to communicate his numerical interpretation of the weights for each side, because of a discussion in Chapter 5.3.2.1, during which R3 recognized that Chris was referring to the "Weight Tool."
Chapter 7  CONCLUSIONS

7.1  Introduction

This study examines the selected data, statistical claims, and justifications made by four students as they used simulation software to solve probability tasks. Video recordings of the students' discourse were synchronized to screen shot video of the computers that these students used, to provide this researcher with a detailed view of the students as they worked in pairs. The video data also includes posters that the students created to display their findings.

This study analyzes five sessions that occurred on the last two days of an Informal Mathematical Learning (IML) summer institute. The sessions and tasks are summarized in Table 7.1, along with the approximate time length of each session.

Table 7-1  Sessions analyzed in this study

<table>
<thead>
<tr>
<th>Date</th>
<th>Session</th>
<th>Task</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>August 3, 2005</td>
<td>1.</td>
<td>Simulating Fair Dice – Students used Probability Explorer (PE) to produce various representations for the outcomes of tossing a fair die, and created their own experiments using the PE weight tool to introduce bias when tossing a die.</td>
<td>45 min</td>
</tr>
<tr>
<td></td>
<td>2.</td>
<td>Schoolopoly – Introduction to the task; students began to explore assigned companies.</td>
<td>45 min</td>
</tr>
<tr>
<td></td>
<td>3.</td>
<td>Schoolopoly – Students continued to explore companies; they compared and justified findings with other students and researchers.</td>
<td>45 min</td>
</tr>
<tr>
<td>August 4, 2005</td>
<td>4.</td>
<td>Schoolopoly – Students prepared posters.</td>
<td>45 min</td>
</tr>
<tr>
<td></td>
<td>5.</td>
<td>Schoolopoly – Students prepared posters.</td>
<td>45 min</td>
</tr>
</tbody>
</table>
In this chapter I will discuss the findings of this study which relate to the research questions:

1) How are data generated by the students interpreted with respect to (a) fairness and (b) significance of sample size?

2) What decisions about fairness and adequacy of sample size do students make on the basis of evidence that they collect from computer simulations?

3) How are student ideas influenced, if at all, by their computer-generated representations?

4) How are student ideas about fairness and sample size influenced, if at all, by others?

7.2 How are data generated by the students interpreted?

This study examines four students as they used computer software to generate data. The students in this study made decisions that were primarily the result of viewing probability in two ways: as a frequentist and as a subjectivist, as defined by Borovcnik, Bentz & Kapadia (1991). The frequentist actually does the experiment and assesses the likelihood of an event based on experimental data. Students in this study used computer simulation software to run trials for different experiments designed to elicit their ideas about fairness of dice. The sample size these students selected is particularly important to this study, as well as the interpretations they made from experiments which had various sample sizes.

As students interpreted data, their ideas can be traced to previous IML after-school classroom activities which they had experienced. In fact, the second way of
viewing probability, as a subjectivist, involves making decisions based on personal experiences. These experiences can range from subjective views held by the students, to actual experiments performed by the students from which they have built ideas. An example of a subjective view is illustrated when Chanel was asked what would happen if a die were rolled 30 times. Chanel replied that an outcome of \textit{two} would occur most frequently because \textit{two} was her "favorite number" (lines B206 - B210). There are also examples of subjectivist ways to view probability in this study, which are based on previous experiences with playing dice games prior to the computer simulation probability tasks during the summer session.

Shay (2008) found that, in previous IML experiences during grades 6 and 7, Jerel used experimental evidence quite heavily and trusted the results, even if the sample size was small. Shay further notes that as part of the IML probability strand in grade 7, students were given a task that asked them to determine whether a specified game played with two pyramidal dice was fair. Pyramidal dice have four sides (faces). Each face is a triangle that contains three numbers – one number is written along each edge. These dice were also called triangle dice.

The IML sessions in this study took place the summer following grade 7. Evidence of these previous IML experiences surfaced when students were questioned about properties of a standard die in the first session analyzed in this study (Chapter 4.2.1.1). Students were quick to recognize that a standard die is six-sided, has dots that represent numbers which range from one to six, and are easy to roll (lines B4 - B15). According to Jerel, \textit{four} and \textit{three} come up a lot (line B16). When R1 questioned Jerel’s comment, Chanel stated that only one number will come up (line B25). Shay’s (2008)
findings regarding the grade 6 experience of determining the fairness of a game played with two dice as described in Chapter 2.4.1, may be a contributing factor in Jerel’s response to the question regarding what he knows about a die. Perhaps, when he said that outcomes of four and three were likely to occur, Jerel recalled his grade 6 conclusion that a sum of 7 had the highest probability of occurring when dice are rolled. It is interesting to note that, because Jerel volunteered his thoughts about the properties of a die during the first introductory session, R1 was able to clarify to the students that the tasks in this study would only involve one die rather than a pair of dice.

7.2.1 Interpretation of data with respect to fairness

During the first session in this study (Chapter 4.2.1.1), R1 asked students for their ideas about fairness of dice. A student noted that a die is fair if it is not cheating – that is, it has six sides and it doesn’t have three of the same numbers (lines B43 - B45). Terrill noted that a fair die has a different number on each side (line B59), and Chanel noted that a fair die can’t have an odd number of sides (line B62). These answers hinted at recalling the grade 7 experience of playing with dice that did not have six faces, and had more than one number on each face. In fact, a student specifically mentioned the triangle dice during that session, and noted that they had a bottom (lines B64 - B65).

7.2.1.1 Unfair dice have the same outcome on more than one side

Jerel discussed the number of sides on a die during conversations about fairness, particularly when finding that dice were unfair. For example, when analyzing the dice produced by Dice R Us for the Schoolopoly task, Jerel and Chris noticed that the frequency of the outcomes for even-numbered sides on a die was approximately twice the
frequency of the outcomes for odd-numbered sides, and therefore these dice were unfair. Jerel's interpretation considered that the person who designed the dice from Dice R Us "made all the even numbers two-sided" (line 1111). Jerel had a similar interpretation for Calibrated Cubes, which also produced unfair dice. Jerel noted that certain outcomes had more sides on the die, when he said “Yeah, I think like two, four and six got two sides, and they like came out the most like that” (line H378 - H379). Chris contributed to this description, by adding that the outcome for an additional side, three, also had a higher frequency than outcomes for one and five. Chris said the "number three is two-sided" (line I116).

Kianja and Chanel also interpreted the outcomes for unfair dice in terms of the number of faces that one outcome may have on a die. Through thoughtful questioning, G1 elicited how the girls used the total weight that was depicted on the PE Weight Tool, to indicate how many sides were on a die. For example, in one experiment, the girls entered a weight of 3 for an outcome of two. Kianja pointed to the total column on the Weight Tool and said: “See total weight – 8 sided” (lines B379 - B380). For this experiment, the ratio of the outcomes to weights was: 1:2:3:4:5:6 = 1:3:1:1:1:1. Here, Kianja interpreted the total of the weights as indicating the number of sides on a die. In this experiment, the Total Weight box on the PE Weight Tool may have validated the idea that unfair dice have the same outcome on more than one side, and that they have more than 6 sides.

7.2.1.2 The probabilities of outcomes of fair dice are "even"

Additional ideas about fairness emerged as R1 encouraged students to contribute their ideas during the first session in this study. Jerel said that if a die is fair it would
come up even (line B50), which he defined as meaning that the numbers would come all up about the same number of times (line B52). R1 questioned whether even means the same as having each face come up the same number of times, and Jerel recognized that they do not have the same meaning (line B55). It is noted that "even" is not a statistical term. The substitution of the word "even" for meaning that the outcomes would have an "equal" probability of occurring was used by many of the children who attended the IML summer session and all four of the students in this study. One possibility is that the students' interpretation of representations produced by the PE software may have contributed to the dual use of the term "even". For example, when the bars on the bar graph were of the same height, or level, Jerel said that representation depicted even outcomes (line G46). In this case, "even" may have been used to describe that the bars were level.

Later that first day, during the afternoon session, Jerel explained what he meant by "even" when he said "what made it look even is that the percents are close, the bar, the pie graph looks even, even though it wasn’t and uh the bar graph, where it showed the exact numbers were very close together" (lines I79 - I82). Chris agreed with this use of even to determine fairness of dice. He further noted that the dice were fair because "all the percents were close" (Line I92). Kianja and Chanel also used the term "even" to describe the outcomes of fair dice, during the initial introductory experiment. Following an experiment with a sample size of 500, the girls produced a pie chart and bar graph. The areas of the sectors were approximately equal, and the bars on the bar graph had the same height. The girls typed a conclusion into their summary document which stated:
"Now we’re really sure that it’s….A fair dice, yeah, because the numbers came up even" (lines B309 - B311).

### 7.2.1.3 An outcome on fair dice can alternate between highest and lowest frequency

The students in the IML summer session worked on probability tasks that were designed to elicit ideas about fairness of dice, based on how students interpreted the data they generated using simulation software. If the frequency of an outcome was the highest in one experiment and then lowest in another experiment, several students interpreted this result as indicating that the dice were fair. For example, Chanel and Kianja decided that dice produced by Dice R Us were fair, because the outcomes of three and six from two experiments alternated between having the highest and then the lowest frequencies. Chanel stated that "at first, the um, three had won then, at the end it lost and then in the middle it um, won and then lost again and then twelve, six, six got more than the first than it did in the last" (lines E92 - E94). Kianja agreed with Chanel's interpretation that if a die is fair, then the same outcome can "win" and "lose." Kianja changed her decision from unfair to fair. G1 verified this reasoning when she interpreted Chanel's statement by stating: "So everybody had a chance to win" (line E97). It is interesting to note that in Chanel's statement, the implication was that each outcome had an equal chance of "winning." In fact, when Chanel provided evidence on her poster for the claim that dice produced by Dice R Us were fair, she pasted the table that depicted an outcome of three with the lowest frequency in one experiment, and the table from a second experiment which showed that an outcome of three had the greatest frequency. Chanel explained to
R1 that the tables provided evidence for the "possibility that all of them can get rolled the same amount" (lines F19 - F20).

Jerel had a similar interpretation for fair dice. As he ran 1000 trials for Delta's Dice, Jerel noticed that the outcome of two had the lowest frequency. During a second experiment with 1000 trials, the outcome of two had the highest frequency. Jerel concluded: “See, look. See now two is winning. This convinces us that it is fair” (line H367). Shay (2008) created the term hybrid heuristic for chance events to describe Jerel's perception of fairness: if either player is able to win a game, then the game must be fair. She concluded that this belief dominated his judgments about fairness. In this study, one idea about fairness that all four students shared, was that dice are fair if the same outcome can alternate between having the highest frequency in one experiment and the lowest frequency in a second experiment.

R1 proposed a variation of this definition when she noted that half of the outcomes for dice produced by Delta's Dice had the same probability. She told the boys: "I have the same chance of getting one, three and five. And I’ve got the same chance of getting two, four and six. Isn’t that fair?" (lines I219 - I221). Both Chris and Jerel rejected this balancing definition for fairness, and Chris explained that it was unfair "cause you’re gonna keep getting two, four and six more" (line I225).

Kianja and Chanel also recognized that if an outcome consistently had the highest (or lowest) frequency, then the dice were unfair. They provided two computer-generated tables as evidence on their poster that claimed dice produced by Dice Depot are unfair. In both tables, an outcome of six had a frequency that was lower than the other outcomes.
7.2.1.4 Fair dice have a uniform frequency distribution - but how close is close?

The four students in this study were able to identify fair dice if the outcomes were close. When Kianja and Chanel were questioned about dice produced by Dice R Us as being fair because the outcome of three alternated between having the lowest and then the highest frequency, they ran an experiment with 1000 trials. The frequencies for this experiment were 116, 167, 184, 203, 180 and 150, respectively. The girls maintained their decision about fairness, and noted that the frequencies for this experiment were "almost equal" (line E138). For Kianja and Chanel, if the frequency distribution was reasonably uniform, then they determined that the dice were fair. However, they were liberal in their determination of the meaning for reasonably uniform.

Jerel provided a description for a reasonably uniform frequency distribution, when he explained what he meant by "close" during one experiment. R1 questioned the decision that dice produced by Delta's Dice were fair, which had been claimed by Jerel and Chris. She compared the various representations generated by PE that the boys had selected, noting that the "pie graph doesn’t look even to me" (line I269). The boys disagreed, and Jerel referred to the table as definitive evidence, stating that the difference between the probabilities for each outcome was less than 4%. He declared that "five or more apart is not close but four is close" (line I279). In this experiment, Jerel used a sample size of 1000 to provide evidence for his claim.

7.2.2 Interpretation of data with respect to sample size

In the first session analyzed in this study, as students were asked for their ideas about dice, one student conjectured that it would take fifteen trials to obtain an outcome
of *six*. When a student actually rolled a die, the desired outcome (*six*) was obtained in only two trials. It is interesting to note that if a Geometric probability model is used to determine how long it will take to achieve a particular outcome, then the expected value is $1/p$ where $p$ is the probability of success. Because the probability of rolling an outcome of *six* is $1/6$, the expected number of rolls required to obtain a *six* on a fair die is 6 rolls.

Initially, students drew on their own ideas regarding the sample size needed for an expected outcome. However, because of the convenience of the *PE* software to run experiments with any sample size up to a maximum of 500 trials (which could then be accumulated), Chris and Jerel quickly established a preference for running experiments with an accumulated sample size of at least 1000. Terrill subjectively criticized this approach, noting that it was impractical. A discussion took place in the last IML summer session, which is not included in this study (ROLE 142B). I had the opportunity to observe this discussion and take notes as a student/researcher. My observations, backed by video data, indicated that Terrill argued that realistically, no one in a gambling situation would ever toss a die 1000 times. Terrill further observed that a more reasonable number of trials was smaller, indicating 80 as an example and rationale for why he ran 80 trials in his experiments. The reader is reminded that Terrill had provided another practical application of gambling earlier, in Chapter 4.2.1.2.

Kianja and Chanel developed their own process for running trials, which included running five discrete experiments with sample sizes that gradually increased. Although they used larger sample sizes to make decisions, Kianja and Chanel continued to run a small number of trials for each experiment in this study.
7.2.2.1 Large sample sizes can be trusted - but how large is large?

As noted previously in this chapter, the students in this study played dice games during an after-school IML probability strand in grade 7. In her conclusions' chapter, Shay (2008) observed that at times Chris viewed experimental data as untrustworthy (page 184), but she noted that this may reflect the recognition that larger samples were needed.

In this study, after the first three experiments with sample sizes of 10, 100, 100, and 500 respectively, Chris ran experiments with 1500 trials and made claims about fairness of dice. Moreover, he and Jerel confidently defended their decisions about fairness. Chris and Jerel compared findings for Delta's Dice with Jarae - who questioned why they ran a sample size of 1000. Jarae had, in fact, run her experiment with 6 trials. Chris generated a visual justification for his decisions about sample size. First he showed Jarae the results from his experiment with 1000 trials, and explained to her that the outcomes were close. He said that "the pie graph looks even" (line 1268). Even after R1 pointed out "That pie graph doesn’t look even to me" (line 1269), Chris and Jerel convinced R1 and Jarae that the dice produced by Delta's Dice were fair based on the percents in the table. The greatest difference between any two percents was less than 4%, and Jerel explained that for the probability percents in the table, "five or more apart is not close but four is close" (line 1279). Then, Chris ran another experiment with 1000 trials for Delta's Dice, to show Jarae that the frequencies were still close. Perhaps this demonstrated Chris' trust that the outcomes from different experiments with a sample size of 1000 would be consistent. This represents a change from the Shay (2008) study with small sample sizes, which noted Chris' uncertainly about trusting experimental data (page
It is interesting to note that looking for consistency in outcomes from different experiments with large sample sizes was also observed in a case study involving two sixth grade boys, Brandon and Manuel. Stohl and Tarr (2002) found that as the boys used PE to solve the Schoolopoly task, they made confident decisions about fairness and convincing arguments when they ran sets of large trials (1500 or more), compared empirical probabilities, and analyzed trends to find the results were consistent.

Next, Chris ran 100 trials to show Jarae that a smaller sample size would not give as accurate an analysis of fairness as the sample size of 1000. In fact, the bars on the bar graph were not as close in height for this experiment as they had been for the sample size of 1000. R1 questioned "why isn’t it showing up even for 100?" (line I300). Jerel explained that the heights were not equal "because you’re not running it enough times" (line I402). He considered 100 trials to be a partial run, and said: "if you run it 1000 times you know that you have a better chance of having it be even and another number catching up to it" (lines I416 - I423). Finally, Chris ran an experiment with the sample size that Jarae had used: 6 trials. In this experiment, the frequency for outcomes of two, three and five was zero.

Chris and Jerel continued to run experiments for Delta's Dice. At 1500 trials, Jarae was convinced that the frequencies were almost equal, based on the bar graph and pie chart. R2 intervened and made the bet that if the sample size were 12, then one of the numbers may have a frequency that is significantly higher than the others. Jerel agreed with this statement, noting that if you "roll a little bit, of course a number is gonna blow another number out" (lines I527 - I528). Throughout these discussions, Jerel maintained confidence in the outcomes resulting from experiments with 1000 trials.
7.2.2.2 Small discrete sample sizes tell a story

When Kianja and Chanel used *PE* to determine if a die was fair in the first session, they ran an experiment with 6 trials. Their goal was to obtain an outcome of *two*, because that was Chanel's favorite number. However, an outcome of *two* had a frequency of zero in this experiment. Chanel continued to click on run, so that the next experiment had 12 trials, and subsequent experiments were multiples of 6. Kianja and Chanel made inferences from each experiment, and considered the result of each experiment to have equal weight in their decision-making process.

As a result of their first set of experiments, Kianja and Chanel saved the representation that depicted the outcomes for 30 trials. Chanel indicated that the frequencies (6, 6, 5, 3, 5, and 5, respectively) were close. As students shared their findings with the whole-group, Kianja and Chanel listened to Chris say that he and Jerel ran an experiment with 4500 trials and that they thought the dice were fair. After the whole-group discussion which provided the opportunity of learning about other students' findings, Kianja and Chanel then ran an experiment with 500 trials. The representations that they selected in this experiment, the bar graph and pie chart, convinced the girls that their earlier decision based on 30 trials was indeed correct - the dice were fair. This outcome may have convinced the girls that the results from a smaller sample size are valid, which points to the problem of relying on experimental data - unusual samples can support incorrect theories.

For the Schoolopoly task, the girls investigated three companies. In each case, they started by running an experiment that contained a small sample size. They ran additional experiments with sample sizes that were multiples of the initial run. This
process actually hindered their ability to make a claim for two companies that produced unfair dice, because their final conclusions contradicted earlier findings with smaller sample sizes. Moreover, it took several experiments (and ultimately larger sample sizes) before they were able to produce their final decision. However, Kianja asserted during the last IML summer session (ROLE 142B) that if dice are unfair, it does not matter how large the sample size is. The percents in the table will be the same for unfair dice, regardless of the sample size. Kianja’s belief is an example of representativeness, which is the belief that a sample, no matter how small, should be representative of the larger population (Kahneman & Tversky, 1982). Kianja's statement also illustrates the common misconception that the Law of Large Numbers applies to small numbers as well (Kahneman & Tversky, 1982).

Interestingly, Kianja also noted during that last session (ROLE 142B) that the percents in the table could be different for experiments with fair dice, because of her idea that fair dice can have an outcome that alternates between having the highest and lowest frequency. It is observed that she is recognizing that fair dice may not have a uniform distribution because of the alternating frequencies idea for fairness, which is that the same outcome can win and lose in a fair die. Here, Kianja is supporting what Shay (2008) had termed the hybrid heuristic for chance events: if either player is able to win a game, then the game must be fair.

When Kianja created posters that depicted evidence which supported claims regarding the fairness of dice produced by their three assigned companies, she created a table for each poster in which she added the frequencies for each of the experiments. For example, to determine if dice produced by Dice Depot were fair, Kianja and Chanel had
run five experiments with 20, 40, 60, 80 and 100 trials. Kianja created a table that summarized the total frequency for each outcome. She wrote that the total number of times an outcome of one occurred in the five experiments was 42. Kianja considered this outcome as if it were the outcome of a larger experiment with 300 trials. It is interesting to note that the percents used in the tables on her posters came from Kianja's summary table of accumulated frequencies, rather than from running an experiment with a larger sample size and using a computer-generated table.

7.3 What decisions about fairness and adequacy of sample size do students make on the basis of evidence that they collect from computer simulations?

During the introductory session, students were asked to share their ideas about dice. Jerel had volunteered that four and three come up a lot (line B16), possibly recalling a grade 6 IML experience of playing with dice to determine the fairness for games that involved sums of dice, in which there was the greatest chance of rolling a 7. In an earlier study, Chris had concluded that 4, 5, and 6 are more likely to occur than 1, 2, and 3 when a single die is rolled (Shay, 2008). After R1 clarified that the students would be considering only one die for these tasks, Jerel proceeded to experiment using PE to simulate tossing dice. During an experiment with 3000 trials, Jerel reacted strongly to the representations that he viewed, as illustrated in the excerpt below.

<table>
<thead>
<tr>
<th>08:02</th>
<th>Jerel</th>
<th>I don’t know. It's going to go on to like 3,000.</th>
</tr>
</thead>
<tbody>
<tr>
<td>08:02</td>
<td>Jerel</td>
<td>If this one stops, six will catch up. I told you 4 and 3 come up the most!</td>
</tr>
<tr>
<td>08:17</td>
<td>Jerel</td>
<td>Four is in second place. Yo I never knew two came up that much!</td>
</tr>
<tr>
<td>08:17</td>
<td>Chris</td>
<td>two</td>
</tr>
<tr>
<td>08:17</td>
<td>Chris</td>
<td>But those are close.</td>
</tr>
<tr>
<td>08:17</td>
<td>Jerel</td>
<td>I never knew one came up that much. (lines G110 - G116)</td>
</tr>
</tbody>
</table>
Perhaps, within this 15 second time span, the dynamic and colorful representations that were generated by the PE simulation software may have convinced Jerel and Chris that indeed, an outcome of one or two was as likely to occur as his original idea that three and four had the highest frequencies.

### 7.3.1 Comparing the simulation software to a computer game

Students responded to the PE simulation software as if they were playing a game. The bright display of outcomes and colorful graphical representations dynamically changed as each die was tossed. Bars in the bar graph grew in height, almost appearing to race each other; the area of the sectors in the pie graph varied greatly if the die was unfair. The boys often enthusiastically rooted for and observed various outcomes such as “Yo, six is beasting” (line G35) or “It’s about to lose to six. Oh snap!” (line G50). Kianja and Chanel cheered for favored outcomes. In the first session, Chanel was hoping that her favorite number, two, would have the greatest frequency when she said "Come on, two" (line B242). Stohl and Tarr (2002) also found that two sixth-grade boys, Brandon and Manuel, cheered for certain outcomes as they used PE to explore the fairness of dice.

During the morning session, when Chris and Jerel were introduced to using PE to simulate tossing a die, by their fourth experiment they ran the maximum 500 trials and repeatedly clicked on run. This resulted in an experiment which had a sample size that totaled 4500 trials. At 1250 trials, Jerel claimed that “six is losing” (line G105). Chris pointed to the pie and bar graph, and said that they were really all coming out the same. It is noted that from the beginning of using the simulation software, Chris trusted the results for a large sample size and made a decision about fairness.
7.3.2 Using the *PE* Weight Tool to bias outcomes

The simulation software enhanced the classroom experience in many ways. Students were engaged. They had the freedom to make and test conjectures, and even change the rules of the task. Then they could immediately observe the results. This was obvious when the students were introduced to the Weight Tool in Chapter 4.3. Kianja and Chanel adjusted the weights so that Chanel's favorite number, *two* would have the greatest frequency and "win." In the process of changing the weights, the girls noticed that the pie chart looked like the computer game character "Pac-Man" (lines B442 - B444). Chanel continued to experiment, and was successful making an experiment with only one outcome, and therefore a solid color pie chart. Chris overheard the girls talking about Pac-Man, and tried (unsuccessfully) to replicate their experiment.

Chris appeared to recognize the potential that the Weight Tool had in favoring outcomes. When he first learned about the weight tool, Chris compared it to the Marbles task (Appendix A) that the IML students had worked on earlier in the week. In the Marbles task, a bag contained marbles of three different colors. Students picked one marble at a time from a bag, replaced it, and were asked to guess how many marbles of each color were in the bag. Students then used *PE* to create experiments which favored the outcome of a specific color marble. Chris realized that using the Weight Tool could affect the outcome of tossing a die in a way similar to "how many (marbles) you put in" (line G208). In fact, as he worked on the Schoolopoly task, every time he determined that dice were unfair he tried to guess the possible weights that were entered into the Weight Tool in the design of the experiment. This was recognized by R3 in Chapter 5.3.2.1. On the next day in Chapter 6.2.2.1, R3 challenged the boys to replicate the
weights for Calibrated Cubes, a company that they had claimed produced unfair dice. Chris and Jerel had decided that because the frequency of the odd outcomes was approximately one-half the frequency of the even outcomes, the weights had a ratio of $\text{odd:even} = 1:2$. The simulation software made R3's challenge easy to test. Chris and Jerel were satisfied with their replication of Calibrated Cubes, even though the probabilities for the duplicated experiment were not identical to those of the original problem. However, in the boys' opinion, the percentages were "close," and the bar graph and pie graph resulting from their experiment did look similar to those representing Calibrated Cubes (lines 1192 - 1201).

### 7.4 How are student ideas influenced, if at all, by their computer-generated representations?

The students in this study used all of the representations that $PE$ generated. Throughout the transcripts of the recorded sessions, there is evidence of students pointing to multiple computer-generated representations: stacked or lined-up outcomes, the data table, the bar graph and the pie chart. Stohl and Tarr (2002) found that students used all of the visual displays to support their inferences. In this study, the type of representations that were chosen by students seemed to influence their reasoning. The examples that follow illustrate the preference that students had in attending to particular representations in their experiments.

#### 7.4.1 Representations selected by Chris and Jerel

When Chris and Jerel were initially experimenting with using $PE$ to toss dice, they were asked to provide the evidence they used to make a decision about fairness. Chris pointed to the pie graph (as Jerel audibly specified that representation) (line G76) to
make his claim that the die was fair. R1 provided an interesting intervention, asking why
the heights on the bar graph were different, and yet the pie graph sectors looked equal
(line G82). The boys gave no answer for this question, and it is conjectured that
continued discussion may have provided the boys with additional insight into how to
consider closeness of frequencies when comparing the graphical representations. Jerel
followed this question by quoting from the data table: “It's only two percents in the
seventeens, and the rest is in the sixteens” (lines G157 – G158). There are many
examples in the transcripts in which Jerel quoted the percents from the table as evidence,
and it is conjectured that the availability of this numerical representation may have
contributed to his description of closeness during one experiment (Chapter 7.2.1.4).

7.4.2 Representations selected by Kianja and Chanel

The girls seemed to have a preference for the pie chart because, as Kianja noted,
"the circle graphs be pretty” (line C217). When the girls were asked which
representations provided evidence for bias in an experiment, Chanel pointed to the pie
chart. The girls had created an experiment that favored an outcome of three. Kianja
became excited about the visual evidence that the pie chart provided because almost half
of the circle was blue, which represented the outcome of three. Kianja stated: “oh that is
really pretty. I like it. I like it a lot. I like it” (lines B387 - B388). As the girls
experimented further, Chanel was able to adjust the weights so that the resulting pie
graph looked like the computer game character Pac-Man, which Kianja remarked was
"beautiful. I like those colors." (line B440). Kianja also pointed to the table, which
provided additional evidence that the outcome with the greatest weight had the highest
frequency.
When the girls were investigating the fairness of dice produced by Dazzling Dice, their original claim was that these dice were fair. However, they were astonished to observe that, as they increased their sample size, the outcome of five remained low. In their final experiment of 100 trials, Kianja stopped the run at 76 trials because five had only occurred twice and the experiment was three-fourths of the way completed. It is conjectured that the dynamically changing representations made this outcome more obvious and significant than it may have been if this experiment were done manually. In fact, the girls acknowledged that it was only because they did an experiment with 100 trials that they realized five “just stayed down there at the bottom” (lines C427 - C428), possibly referring to the bar graph display. It is noted that the software provided the opportunity for the girls to easily run and pause an experiment with 100 trials. During last two sessions in this study, Kianja pasted the computer-generated pie chart and data table on her poster, as evidence that the dice produced by Dazzling Dice were unfair. She referred to those representations as her "little proof for the five" (lines F159 - F160). Kianja explained that she selected evidence from the experiment with 100 trials, and she said: "I think it’s more clear because, if you look on this pie chart, you see this small little wedge…” (lines F370 - F371). The computer-generated representations truly had a powerful influence on students' ideas, and even contributed to changing their decisions about fairness of dice.

7.5 How are student ideas about fairness and sample size influenced, if at all, by others?

The classroom environment in this study replicated the supportive environment in previous Rutgers research studies, in recognizing the importance of acknowledging and
respecting all student ideas and contributions (Maher, 2010). In the first session analyzed in this study, R1 listened carefully to student contributions to discussions. She repeated their comments in affirmation, and invited other students to contribute their ideas. The value of a supportive environment that encourages experimentation and sharing is well documented, as described in Chapter 2.1. It is therefore plausible that the classroom environment contributed to the comfort level of students as they discussed their ideas with adult researchers and other students. The opportunity to share and learn was conducive to trying new experiments or expanding on previous ideas.

Stohl and Tarr (2002) indicated that the use of PE tools coupled with social interaction, enabled students to make connections between simulation data (empirical probabilities) and weights in the Weight Tool. Moreover, the social activity which included gathering data and judging the validity of data-based observations, prompted reflection and often required further data collection, analysis, and argumentation. They indicated that this laid a foundation for understanding the power of larger samples, and argued that without the social interaction, the students' limited understanding may have gone unchallenged.

### 7.5.1 Influence from researchers

The IML teacher/researchers were experienced and skilled in asking thoughtful, open-ended questions to promote student collaboration. The IML students were not instructed on how to carry out their experiments. Rather, they worked in pairs on the well-designed probability tasks, and were given the latitude to experiment and test their own ideas about fairness and sample size. Graduate student/researchers were advised to observe quietly and take notes. Despite these instructions, there were some interesting
interventions provided by the researchers. For example, on several occasions researchers and graduate students specifically questioned the preference of Kianja and Chanel to run experiments with a small sample size. In the very first session in this study, R1 proposed that perhaps the reason that the girls began with a sample size of 36 was because a die has six sides, and 36 is the square of 6 (line B231). Kianja agreed with this rationale, and she next ran an experiment with 6 trials. Subsequently, Kianja ran trials that were multiples of 6. It is interesting to note that in a comparable study, Stohl & Tarr (2002) found that a student who used PE tools to solve the Marbles Task (Appendix A) discovered that entering a small sample size for the marbles resulted in no useable information about the quantity of the other two colors. This resulted in the student's recognition of the need for a larger sample size. However, although the student made inferences from a large number of trials, he used the results of running 12 trials as evidence. A possible rationale for using the smaller sample size was because the bag only contained 12 marbles. Similarly, one might conjecture that Kianja began with a sample size of 6 because of the number of faces on a die, and then ran multiples of 6 by simply clicking on the RUN key in PE. Later that morning when the girls were experimenting with the Weight Tool, R1 commented that they "only ran the experiment 50 times" (line B396). Following that statement, the girls clicked on RUN for an additional 50 trials and made inferences from the larger sample size.

As Kianja and Chanel explored their assigned companies for the Schoolopoly Task, G2 provided several thoughtful interventions that may have provided the girls with the opportunity to consider running larger sample sizes or even just a single experiment with a large sample size. During the late morning session, as the girls examined the dice
produced by Dazzling Dice, Kianja noted that the evidence for their decision of that exploration was 6 pages long (line C379) in their summary document. The girls had run 6 experiments for Dazzling Dice, with trials of 10, 20, 40, 60, 80 and 100. G2 asked a pertinent question: "If you don’t have room on your poster, would you leave any of this out?" (lines C380 - C381) Kianja and Chanel did not answer that question. Rather, Kianja continued to organize the document and succeeded in using fewer pages for their summary.

G2 provided additional interventions regarding sample size at the end of the third session. Kianja and Chanel were not able to reach consensus for the fairness of dice produced by Dice R Us. The next morning, at the beginning of session 4, G2 asked: "Now are you gonna stick with the same sizes that you did yesterday, 80 and then a hundred?" (lines E23 - E24). The girls said they would follow that procedure. Subsequently, the girls were still not able to make a decision about fairness. G2 followed up by asking: "Well since you’re unsure do you think you can get more evidence?" (line E79). The girls came to the conclusion that dice were fair because the outcome of three alternated between having the highest and lowest frequency. G2 continued to question their decision about sample size and said: "Yesterday when Keisha and Tiffany were talking to you, they had like a thousand trials didn’t they? What do think about that" (lines E101 - E103). Chanel noted that it was not necessary to run 1000 trials, and Kianja explained that on the previous day, they did a different number of trials and the end result of their experiments for testing dice from both Dazzling Dice and Dice Depot were the same as the results that Keisha and Tiffany had found from running 1000 trials. However, following G2’s reminder about a sample size of 1000, Kianja and Chanel did
run one experiment with 1000 trials, testing dice produced by Dice R Us. The results indicated that their original decision was correct - the dice produced by Dice R Us were fair. These questions from G2 inspired the girls to think about the results for their three Schoolopoly companies, even though these results seemed to validate their misconception that using a large sample size such as 500 or 1000 trials would have the same outcome as experiments with smaller sample sizes.

Other researchers and graduate students posed questions designed to elicit the girls' ideas about sample size. R3 specifically asked if only one trial could be used to demonstrate fairness, which experiment would be selected out of a group with sample sizes that were multiples of 20 up to 100 (lines F338 - F339). Kianja decided that the experiment with 100 trials was clearest, because an outcome of five had a frequency of 3. Despite these attempts by researchers to challenge their ideas about sample size, Kianja and Chanel continued to follow their process of beginning with a small sample size and running a total of five experiments with trials that were multiples of the first sample size. In fact, this researcher was made aware of Kianja's desire to show the story that evolved from the series of experiments because of a dialogue between R3 and Kianja (lines F447 - F464). Kianja was building a story about her decision for fairness by including each data table for numerical evidence and each pie chart to visually show how the decision emerged.

The researchers also posed thoughtful questions to the boys. R1 asked why the pie graph sectors looked equal, and yet the bar graph heights were different (line G77 - G78). Perhaps this influenced Jerel to read the percents in the table as evidence, because Jerel seemed to prefer that representation when defending claims about fairness.
The researchers shared a wonderful rapport with the IML students. In fact, at the end of the first morning session, Jerel created an experiment by adjusting the Weight Tool to significantly favor an outcome of four, altering the assigned task which was to favor an outcome of two. Jerel then tried to trick R1. He displayed the representations for his experiment favoring four on his computer, along with the Weight Tool, in which he replaced his original weight of 30 for an outcome of four, with a 1. R1 laughed and reminded Jerel that she could figure out the trick.

One of the most interesting interventions occurred when R3 spontaneously created an extension of the Schoolopoly task, and asked Chris and Jerel to duplicate the design of the unfair dice produced by Calibrated Cubes (Chapter 5.3.2.2). This provided the boys with an easy and quick opportunity to test Chris' theory that the frequency of even outcomes was twice the frequency of odd outcomes. The skillful researcher and the available simulation software contributed to the success of this intervention.

### 7.5.2 Influence from other students

Built into the design of the IML summer sessions, was time for students to have whole-group discussions. Students had the opportunity to share their own ideas and learn what others were finding. In the first morning session, Kianja and Chanel used PE to determine if a die was fair. They ran several experiments with samples sizes that were multiples of 6, and the only computer-generated representation they selected was the stacking of the dice by outcome, as they were tossed. During the whole-group discussion, Kianja and Chanel heard Jerel and Chris say that they used a sample size of 4500 to determine fairness of the die. Following that discussion, Kianja and Chanel ran an experiment with 500 trials. With this large sample size they could no longer use the
stacking representation they had previously selected. They opened the bar graph, pie chart and table, which provided evidence for their final decision about fairness. In their next experiment they used the Weight Tool. For this experiment they started with a sample size of 50. They were not consistent in beginning with a larger sample size, however. When the girls started to work on the Schoolopoly Task during the late morning session, they ran five experiments to determine if dice produced by Dazzling Dice were fair. The first experiment had 10 trials, which was a significantly smaller sample size than they had been using, as noted in Chapter 5.2.1. Despite the success they encountered when they ran 500 trials following the whole-group discussion, Kianja and Chanel maintained their process for telling the story of fairness as it evolved from experiments that began with sample sizes of 10 or 20 and gradually increased through 100.

A second example of a student contribution that influenced the girls was made by Tiffany in Chapter 5.3.1.2. Kianja and Chanel had been running experiments with trials that were multiples of 20. Prior to running an experiment with 80 trials, the girls were not certain that dice produced by Dice Depot were fair. As they looked at the representations for 80 trials, they were beginning to think that the dice were unfair. Tiffany explained her rationale for deciding that dice produced by Dice Depot were unfair, based on the percents in the data table. Chanel agreed that they would probably get the same result, and she ran one final experiment with 100 trials. Following the sharing of their findings and Chanel's final experiment, Kianja typed the decision that the dice were unfair. It is likely that listening to Tiffany's rationale convinced Kianja and Chanel that the dice were unfair.
Chris and Jerel were also influenced by other students. For example, when Kianja and Chanel were able to use the PE Weight Tool to make the pie chart look like the computer game character Pac-Man, Chris overheard their enthusiasm and remarked that he also like Pac-man (line G241). He tried, unsuccessfully, to replicate their experiment and produce Pac-Man on his computer. Chris and Jerel influenced another student, Jarae, when they described through a visual demonstration, their rationale for using a large sample size to determine fairness. Jarae had run an experiment with 6 trials to determine if dice produced by Delta's Dice were fair. The boys ran experiments with 6 trials, 100 trials, and finally 1000 trials to compare results. The most consistent results came from the experiments that had a sample size of 1000. However, the bars on the bar graph did not have the same height until the boys ran an experiment with 1500 trials. At that point, the boys and Jarae were convinced that the dice were fair, and that all of the computer-generated representations supported their decision about fairness.

7.6 Conclusions

This study provides a window into the ideas of four students as they used simulation software to solve probability tasks during two days of a summer IML institute. The dynamic visualization of the multiple representations generated by the simulation software was found to influence their ideas about fairness and sample size. The students worked collaboratively in a supportive environment, in which all of their contributions were acknowledged by researchers and other students. Time was provided for whole-group discussions, and for the comparison of findings with other students. This study found that the students' ideas about fairness and sample size were influenced by the researchers and other students.
The students used the simulation software to explore the fairness of dice produced by different companies. They ran experiments for which they determined the sample size, and they selected the representations that provided evidence to support their decisions about fairness. Through their collaborative work in pairs, group-sharing discussions, and discussions with researchers, the students in this study recognized that fair dice have a uniform frequency distribution.

Other ideas about fairness of dice emerged in this study. Jerel determined that dice are fair if the probabilities of outcomes are close. In one experiment, Jerel further explained that the outcomes are "close" if the difference between the percents was 4% or less. This idea may have resulted from viewing the data table representation in the PE simulation software, which listed the relative frequency of the outcomes for each experiment as a percent. Jerel, Kianja, and Chanel agreed that dice were fair if they had an outcome that alternated between having highest and then lowest frequencies in different experiments. A researcher asked the students to consider another idea about fairness: dice are fair if half of the outcomes were always lowest and the other half were always highest. All four students in this study rejected this "balancing" definition, and recognized that if an outcome was consistently highest or lowest, then the dice were unfair.

All of the students in this study interpreted an unfair die as having the same outcome on more than one face, and therefore the die could have more than 6 sides. For example, if an outcome of two had a weight of 3, then the students interpreted this as meaning that three of the faces on the die had a value of two written on them. This resulted in students referring to an 8-sided die.
As students explored using PE to determine the fairness of dice situated in well-defined tasks, sample size became relevant. Students had clear preferences with regard to their process for running an experiment. Kianja and Chanel ran an initial experiment with a small sample size, such as 6 or 20. Then, they ran subsequent experiments with sample sizes that were multiples of the original experiment. For evidence to support the claim about fairness, Kianja made a table that totaled the frequency for each outcome in all of the experiments. Kianja revealed to a researcher, that she was telling a story about the fairness of the dice as her decision about fairness evolved, based on observing and recording the evidence from computer-generated representations she selected from each experiment. Researchers, graduate student researchers, and even other students implicitly and explicitly provided the suggestion to consider running an experiment with a larger sample size, such as 1000 trials. In fact, when Kianja and Chanel tested their decision that dice produced by Dice R us were fair by running 1000 trials, they found that their decision which was based on the smaller sample size was a correct decision. This seemed to validate Kianja's misconception that the Law of Large Numbers applies to small numbers as well. It is interesting to note that when Kianja and Chanel investigated Dazzling Dice, the results of larger sample sizes contradicted earlier findings with smaller sample sizes. It was not until they ran 60 trials that they were astonished to find that an outcome of five only had a frequency of 1. R1 supported their decision to change their claim, because it was based on data. The availability of the simulation software provided the opportunity for students to experiment and possibly change their claims about fairness.
Jerel quantified his idea for an adequate sample size. He ran experiments with 1000 trials to show that the percents consistently remained close. Jerel and Chris were very confident in the results of experiments with 1000 trials. In an earlier study (Shay, 2008), Chris was found to be cautious about relying on experimental data to make inferences. Perhaps this was due to his use of small sample sizes in running the experiments by hand, without simulation software. When challenged about the need for running 1000 trials, Jerel and Chris defended their process by using computer-generated representations to provide a visual proof that large samples yield consistent results.

The students in this study had preferences for the representations they selected as evidence, and the type of representations that they chose seemed to influence their reasoning. Chris and Jerel relied on the data table and pie chart, and recognized that an experiment with fair dice may not necessarily generate a bar graph with bars that have exactly the same height, even when the pie chart had sectors with equal area. In one experiment with fair dice, the bar graph did not seem to support their claim about fairness until they increased the sample size to 1500. Kianja and Chanel also quoted frequencies from the data table. In fact, when Kianja created a poster to display evidence for a claim, she produced her own data table by adding the results from her small samples. The girls preferred the pie chart, and noted how pretty it looked during different experiments.

7.7 Implications

The students in this study viewed probability from both frequentist and subjectivist perspectives. This study follows an earlier study (Shay, 2008) which recommended that these same students needed more practice with experimental data (page 196). In this earlier study, Chanel was found to rely on the sample space to make
judgments. Because of her difficulties in creating the sample space for some games, Chanel was unable to make a reasonable judgment about fairness. In this study, the nature of the tasks and the simulation software removed that obstacle. Shay (2008) found that Kianja began each game by writing the sample space. In a game where she only wrote combinations, she did listen to an intervention from a researcher and then added permutations to the sample space. According to Shay, following that intervention, Kianja admitted that she saw permutations as different events but said that the sample space could still be based on combinations if that was preferred. This may help explain why, after interventions that suggested larger sample spaces when determining the fairness of dice in the Schoolopoly game, Kianja consistently used her preferred process of beginning with a small number and running additional experiments in which the number of trials was a multiple of the original sample space. In the Shay (2008) study, Jerel did not rely on the sample space, and even ignored the arguments from other students which were based on the sample space. He preferred to play the game to determine fairness. Shay created the term hybrid heuristic for chance events to describe Jerel's perception of fairness: if either player is able to win a game, then the game must be fair (page 187). This belief seemed to have dominated his judgments about fairness. In this study, Jerel decided that dice produced by Delta's Dice were fair because the outcome of two alternated between having the highest, and then the lowest frequency in different experiments. Shay (2008) also found that Jerel relied heavily on experimental data to make judgments. In this study, Jerel changed his idea about which number occurs most frequently when a die is tossed, because of the results of experimental data. Finally, Shay (2008) noted that: "Chris’ level of reasoning about experimental probability is difficult to
pin down. At times it seems that he regards data from experimental trials as irrelevant or untrustworthy, but this may reflect the recognition that larger samples are needed" (page 185). In this study, Chris was able to make judgments from experimental data because of the simulation software and its ability to generate large sample sizes. This study also supports findings consistent with Pratt (2000), Drier (2000), and Stohl & Tarr (2002), which suggest that when young adolescents have the opportunity to use simulation tools, they use data displays to support their claims and inferences.

7.8 Limitations

No research project is perfectly designed; all have limitations (Marshall & Rossman, 2006). This is inherent in a case study. In this study, four students were purposely selected because they attended all analyzed sessions. These four students were consistently videotaped, and screen shots of their computer work were also video-recorded.

This study is bounded by time and place (Creswell, 1998). It took place over a span of two days, and in an air-conditioned elementary school in Plainfield, New Jersey. The results of this study are specific to the participants, and cannot be generalized even to the other IML students. Those students may be the subject of a future study.

The students in this study were voluntary participants in an after school program. The classroom environment, which replicated the environment in other research studies at Rutgers, may be a contributing factor to the students' engagement. Moreover, the well-designed tasks and dynamic simulation software may have encouraged students' data-based decisions and justifications. Finally, the social interaction among students and
other researchers provided opportunities for introspection and experimentation. These aspects are not typical of a traditional classroom.

Given these limitations, there is evidence that a collection of case studies which provide a window into the probabilistic reasoning of middle school students can stimulate ideas that can benefit learners and teachers of mathematics.

7.9 Future Studies

In light of the progress demonstrated by the students in this study, as compared to findings of these students' ideas about fairness and sample size in grades 6 and 7 (Shay 2008), it is proposed that the opportunity of using simulation software can have a dramatic impact on influencing and convincing students to make data-based decisions and justifications. The simulation software provided an opportunity for students to make and test conjectures, create their own experiments, and immediately obtain results. The students in this study were engaged, and compared running various experiments to playing games on the computer.

This study focused on four students during two days of a summer institute. It would be interesting to document their ideas about fairness and sample size as they learned about probability the following year or years, in a traditional classroom. There were eleven students who participated in the 2005 IML summer institute. It may be of interest to study the data for pairs of other students and perhaps compare to the findings of the students in this study. All of the students who attended the summer institute were interviewed 10 months later, in June of their 8th grade. Those interviews may provide information related to the students' perception of the intervention.
The researchers and graduate student/researchers challenged the students in this study, as they made decisions and provided justifications for their claims. A study of interest may be examining further the consequence of these researcher interventions, as well as the collaborative nature of listening to other students.

This study focuses on the students' use of simulation software as they solve probability tasks. The four students were participants in other strands of the IML research project, some of which utilized graphing calculators. It may be of interest to study these students as they investigated the counting and combinatorics, and algebra strands.

While the technology used to capture screen-shots of what the students were doing with the simulation software was ground-breaking in 2005, this technology is much improved today, only five years later. Sharper images of screen shots are now more easily available, and it may be interesting to see this study replicated with other groups of middle school students.

This study along with other similar studies has the potential to impact curriculum design in middle school mathematics. As Powell (2010) notes: "If we agree that students must be actively and purposely engaged in their learning so that they can take ownership and be proud of their accomplishments, we need to create opportunities for this to occur" (p. 203). A necessary ingredient in this process, is the education of teachers and the design of curriculum which provides opportunities for students work collaboratively on well-designed tasks. In the area of probability, the use of simulation software and social interaction provides the opportunity for students to gather data and judge the validity of
data-based observations. This encourages reflection and often requires further data collection, analysis, and argumentation.

As a Professor of Mathematics at Brookdale Community College, I have taught calculus in a computer classroom since the early 1990s, using software that enables students to visualize challenging concepts in calculus. Since the middle 90s, I have used graphing calculators to present developmental algebra from a functions approach, using calculator-generated graphs and tables to provide multi-representations. Group work and group sharing time have been essential elements as the learning and teaching of mathematics unfolds in my classrooms. Studying the middle school students in this case study as they worked with simulation software, and made and defended decisions about fairness and sample size was exciting for me. It would be interesting to investigate the ideas about fairness and sample size which could result if adult learners were given the opportunity to use simulation software to solve probability tasks.

I remain extremely appreciative that I had the privilege to learn from Kianja, Chanel, Jerel and Chris. It my wish that every teacher could have the opportunity to carefully observe and listen to students as they work collaboratively on well-designed tasks, and provide an in-depth analysis of what they have studied. It is hoped that this research contributes to other studies on the potential for the building of mathematical ideas in curriculum that supports creating and testing conjectures by using dynamic simulation software, and justifying those conjectures within a supportive, social community.
Appendix A

The Tasks:

1. Marbles Task - Mystery Marble Bags

Your school is giving away a free computer. Every student will get to choose one of the bags (A or B). Then, he or she can pick a marble 7 times (with replacement) out of that bag. After each marble is picked, it is then replaced back into the bag for the next draw. For each time (out of 7) that the student picks a blue marble, their name will go into a drawing for a chance to win the computer.

The Prize Committee has decided to allow your class to do some experimentation with the bags before the contest begins. You are not allowed to look in the bags, but you can collect some data by pulling one marble at a time with replacement.

On the day of the contest, which bag would you prefer to choose from?

2. Simulating Fair Dice

a) Students set up simulation in Probability Explorer to explore what happens when rolling a fair die, to give them some experience of the behavior in short term and long term of a fair die.

b) Students are introduced to the Weight Tool, and explore introducing bias into experiments.
3. **Schoolopoly – Is the die fair or biased?**

**Background:** Suppose your school is planning to create a board game modeled on the classic game of Monopoly. The game is to be called Schoolopoly and, like Monopoly, will be played with dice. Because many copies of the game expect to be sold, companies are competing for the contract to supply dice for Schoolopoly. Some companies have been accused of making poor quality dice and these are avoided since players must believe the dice they are using are actually “fair.” Each company has provided dice for analysis and you will be assigned one company to investigate:

<table>
<thead>
<tr>
<th>Company</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perfect Polyhedra</td>
</tr>
<tr>
<td>Calibrated Cubes</td>
</tr>
<tr>
<td>Delta’s Dice</td>
</tr>
<tr>
<td>Dice R Us</td>
</tr>
<tr>
<td>Dazzling Dice</td>
</tr>
<tr>
<td>Dice Depot</td>
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</tbody>
</table>

**Your assignment:**

Working with a partner, investigate whether the dice sent to you by the company is fair or biased. That is, collect data to infer whether all six outcomes are equally likely and answer the following questions:

1. Do you believe the dice you tested are fair or biased? Would you recommend that dice be purchased from the company you investigated?
2. What compelling evidence do you have that the dice are fair or unfair?
3. Use your data to estimate the probability of each outcome, 1 – 6, of the dice you tested.

Collect data about the dice supplied to you. Note that each single trial represents the outcome of one roll of a “new” virtual die provided by the company.

Copy any graphs and screen shots you want to use as evidence and print them for your poster. Give a presentation pointing out the highlights of your group’s poster.
Appendix B

Complete Transcript of CD: ROLE 136B
with Synchronized Kianja/Chanel Monitor CD 1 of 3

Date: 03 August 2005  Grade 7
Location: Washington Elementary School
Transcribed by: Barbara Tozzi
Verified by: Anat Even Zahav

<table>
<thead>
<tr>
<th>Time</th>
<th>Speaker</th>
<th>Transcription</th>
</tr>
</thead>
<tbody>
<tr>
<td>01:03</td>
<td>R1</td>
<td>[to class] We’ll be actually doing some things with dice. And,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>we’ll be doing a couple of different things but I want to start off</td>
</tr>
<tr>
<td></td>
<td></td>
<td>with, just talking about what you guys know about, about dice. All</td>
</tr>
<tr>
<td></td>
<td></td>
<td>right.</td>
</tr>
<tr>
<td>01:04</td>
<td>Student</td>
<td>They’re six-sided.</td>
</tr>
<tr>
<td>01:05</td>
<td>R1</td>
<td>They’re six-sided. And what are on each of the sides?</td>
</tr>
<tr>
<td>01:06</td>
<td>Students</td>
<td>Numbers. Dots.</td>
</tr>
<tr>
<td>01:07</td>
<td>R1</td>
<td>There are dots and what do those dots symbolize.</td>
</tr>
<tr>
<td>01:08</td>
<td>Students</td>
<td>Numbers.</td>
</tr>
<tr>
<td>01:09</td>
<td>R1</td>
<td>Numbers.</td>
</tr>
<tr>
<td>01:10</td>
<td>Terrill</td>
<td><em>One</em> through <em>six</em>. Okay, I think you guys have done some stuff</td>
</tr>
<tr>
<td>01:11</td>
<td>R1</td>
<td>with dice games this year and last year.</td>
</tr>
<tr>
<td>01:12</td>
<td>Students</td>
<td>(respond affirmatively)</td>
</tr>
<tr>
<td>01:13</td>
<td>R1</td>
<td>What kinds of things did you find out about dice?</td>
</tr>
<tr>
<td>01:14</td>
<td>Students</td>
<td>That they are easy to roll.</td>
</tr>
<tr>
<td>01:15</td>
<td>Jerel</td>
<td>That <em>four</em> come up, that <em>four</em> and <em>three</em> come up a lot.</td>
</tr>
<tr>
<td>01:16</td>
<td>R1</td>
<td><em>Four</em> and <em>three</em> come up a lot. How do you know.</td>
</tr>
<tr>
<td>01:17</td>
<td>Jerel</td>
<td>Cuz when you play dice, when you play dice…(others laughing)</td>
</tr>
<tr>
<td>01:18</td>
<td>R1</td>
<td>Okay, so you’re saying that if I roll this die that you think <em>four</em> and</td>
</tr>
<tr>
<td>01:19</td>
<td></td>
<td><em>three</em> will come up a lot. What do the rest of you think about that?</td>
</tr>
<tr>
<td>01:20</td>
<td>Students</td>
<td>(some agree, some don’t)</td>
</tr>
<tr>
<td>01:21</td>
<td>R1</td>
<td>(points to a student) You say you don’t agree? Oh, you do agree –</td>
</tr>
<tr>
<td>01:22</td>
<td></td>
<td><em>four</em> and <em>three</em> come up a lot. Okay. When I roll one die, <em>four</em> and</td>
</tr>
<tr>
<td>01:23</td>
<td></td>
<td><em>three</em> are gonna come up a lot.</td>
</tr>
<tr>
<td>01:24</td>
<td>Chanel</td>
<td>No only one, only one number will come up.</td>
</tr>
<tr>
<td>01:25</td>
<td>R1</td>
<td>So it could be…but I heard…</td>
</tr>
<tr>
<td>01:26</td>
<td>Student</td>
<td>If you have two.</td>
</tr>
<tr>
<td>01:27</td>
<td>R1</td>
<td>Oh you’re talking about if you have two die. Okay. I see, I see.</td>
</tr>
<tr>
<td>01:28</td>
<td></td>
<td>And um so we’re only talking about rolling one die. All right, so</td>
</tr>
<tr>
<td>01:29</td>
<td></td>
<td>get your two-die games out of your head for a second. All right, so</td>
</tr>
<tr>
<td>01:30</td>
<td></td>
<td>if I rolled one die, do you think that uh each of these has the same</td>
</tr>
<tr>
<td>01:31</td>
<td></td>
<td>chance of coming up?</td>
</tr>
<tr>
<td>01:32</td>
<td>Students</td>
<td>No. Yes (different comments)</td>
</tr>
</tbody>
</table>
R1: No?

Student: So six will come up, four will come up, and three.

R1: What about those numbers, six, four and three. And Chris you said something...you said yes, that they would come up the same? I don’t remember what you said.

3:00 Chris: Either one could come up.

R1: Either one could probably come up. And then, Jerel, you said no…

Terrill: You can’t know based on that because it depends on how you roll it.

R1: Depends on how you roll it. Okay. So what does it, what does it mean if I say that this die is fair?

Students: It’s not cheating.

R1: It’s not cheating. What’s the it – the dice isn’t cheating?

Students: No because it’s six sides. It’s not like there’s three of the same numbers.

R1: Not like there’s three of the same numbers. All right. One of the things that Chris said is that you don’t know that yet. Well, I think if we inspect this (hands the die to a student) could you tell me if this is true on that die. Every side has a different number. Okay.

Jerel: Wait, I think it would come up even.

R1: It would come up even.

Jerel: They would all come up about the same.

R1: They’d all come up about the same. Is about the same the same thing as even?

Jerel: No

R1: No, okay.

Students: (provide ideas)

R1: So, how do you know if a die is fair.

Terrill: If it has a different number on every side.

R1: If it has a different number on every side it’s fair. Is there any other way that you know that a die is fair?

Chanel: It has six sides. I mean it can’t have an odd or even number of sides.

R1: It can’t have an odd number of sides. Okay.

Student: That’s just like when we play with the triangle dice. Only that they had a bottom. Well, yeah well they had a bottom and top. And if you rolled it (inaudible)

R1: Had to have an even amount of sides. Okay. Is there any other way that you know that a die is fair. What if you weren’t actually able to inspect the die.

Students: (provide answers) you shake it in your hand;

R1: Okay, so you’re talking about feeling it. And then you said something else, Lorrin.

5:12 Lorrin: If you can’t look at it, just keep rolling it and seeing the numbers you get.

R1: And how would that help you decide whether or not it was fair.
Lorrin: Cuz like, if you see *four* come up four times then you know that it’s not fair.

R1: Okay. What do you think about what she just said. If you see *four* come up a lot of times then you know it’s not fair.

Lorrin: I’m saying like if every other number comes up once and then *four* comes up like four times, I’m saying they’re might be more than one *four* on the thing.

R1: There might be one more than *four*.

Lorrin: Or you might never get a couple of certain numbers.

Student: What do you mean by fair. When you say fair do you mean like when you roll the dice and a number comes up, every time you roll the numbers.

R1: That’s exactly what I’m asking you all. What do you think I mean whenever I say: fair dice.

Student: You can’t pick what number you are gonna roll.

R1: All right. You can’t pick what number you’re gonna roll.

Student: What if you just dropped it.

R1: If you roll it the way you’re supposed to, you’re saying, you can’t pick. So some of you are giving me some arguments about the fair die based on how the die looked. All right. How it felt and whether or not it had an even number of sides, things like that. But Lorrin was giving me a different kind of argument about if she rolls the die, she may be able to tell something. Does anybody…

Student: If I roll it out I can tell that I get a *four*.

R1: You can tell that you’re gonna get a *four*. How.

Student: Cuz that cuz I said so. (laughs)

R1: (rolls the die) So let's go ahead and roll it.

Student: Oh, *five’s* close enough to *four* (laughs). So we probably wouldn’t be very good at just calling out and picking a specific number and making sure that that’s the one that I rolled.

Terrill: That would defeat the purpose of…

R1: Hold on. He’s saying that would defeat the purpose of

Terrill: Las Vegas.

R1: Las Vegas. Why.

Terrill: Because, say if you’re playing like, craps, or whatever. If you can just call out the number, you gonna be…

Students: You can call like *three* or whatever.

R1: Okay, and Jarae, you had your hand up. Did you want to say…(she is interrupted by a request to let Terrill finish) Okay, Terrill, sorry.

Terrill: Um, what was I saying, oh yeah, if you was playing craps and you could just call out a number you would never lose.

R1: So if you could just call out a number, if that worked, then you would never lose. All right. Hold on a second, Jarae what was your comment.
Okay, so what we’re gonna do is we are able to simulate rolling die on the computer with Probability Explorer. So, so far we’ve pulled marbles out of a bag, and we’ve actually been able to kind of pull out the sports balls from the gym class bag. We can also simulate rolling dice. So I’m gonna have you guys go back to your computer and we’re gonna simulate rolling a fair die on the computer, okay. When I, if we’re gonna roll a fair die on the computer, what do you expect to happen if you know that the die is fair. What kind of things do you expect to see?

You expect to see all of the numbers come up.

You expect to see all the numbers come up. Anything else you might expect? Jerel, how long do you think it would take you, so that all the numbers come up?

About ten turns.

About ten times, ten rolls?

Well maybe, the number would be six and I have to try maybe about fifteen times.

All right. When you say the number would have to be six, what does that six refer to. Tell me what you were referring to about the number six.

That’s the number I would like to roll. And to get to that number, I would have to roll about fifteen, twenty times.

Okay. So, does everybody hear what Jarae said. So Jerel, You said you heard what she said, but Chris didn’t. Say it again Jarae. Cuz I think your answer is a little different than what Jerel said, so I want him to hear, I want to make sure that we all hear it.

I don’t remember what I said.

Well, I remember a little bit of it. About you were looking for the number six.

I would like to roll the number six and to get to the number six it would take you about fifteen, twenty times to rolls.

Okay. All right. What if you had a six, you were looking for the number six, and it took you fifty times to get to the number six. What do you think about that.

It won’t. (some one else) exactly.

What do you mean it won’t? Why won’t it?

may be is gonna, may be if you go fifty time before you get to fifty, a couple of six would come up.

Yeah, but what if I went the whole way to fifty, and I never got a six. I’m doing a suppose. What would you think about that. What would you think about that.

That dice is not fair.

That that dice is not fair. Do you think that would be an indication of that.
Terrill: No it would be an indication that you don’t know how to roll.

R1: So it could be an indication that you don’t know how to roll the die.

Terrill: No, it doesn’t have anything to do with you don’t know how to roll. But if you roll it maybe you can’t help it if you don’t get six.

Danielle: Can I try.

R1: You want to try rolling it. Okay. All right, so Danielle wants to see if she can get a six. (Danielle rolls.) Uh oh, got a one. (to Tiffany)

So let’s see if you can try to get a six. (Tiffany rolls the die.) Hold on, don’t touch it. Oh, she got a six. So there we only rolled it twice and we got a six. Hand me the dice. Okay, so we’re going to go back to the computers and I’m gonna actually show you how to pull up and be able to roll dice in the computers. So go ahead back and don’t touch anything because we’re gonna have to do that synchronize – being able to sync everything up. So go ahead and some of you may need to take your chairs. (each pair goes to his station).

12:35 R1: (The camera focuses on Chanel and Kianja) On your desktop, there should be a short cut to Probability Explorer and it’s the red and yellow circles that say H and T on them. Can everybody find that? (The girls in low voice say: “lets go.”) You double click it. You should get this little screen, you click on let’s go. When you come to this next one that says what do you want to do today, I want you to choose the option for dice. Select dice, does everybody have that selected. Okay. And then in this next screen, you say OK, this next screen says how many at a time, that’s how many die you want to roll at a time. We’re only gonna roll one die at a time. All right so the option to select is number one, you just have to select OK. So now, hit the run man once. You should see the result of rolling the die.

13:44 Chanel (whisper) We got a six.

R1: So I got a six. Who else got a six.

(and Kianja) We did too.

13:46 Chanel: Anybody else get a six. (Kianja raises her hand.) You guys got a six back there. So, about half of us got a six, the rest of us didn’t.

And if you hit run again, this will show you what you rolled the next time. I got a five again. Well not again, I got a five.

14:15 Kianja: (to Chanel) I thought she had another six. I was gonna say what…

R1: So, some of you are talking about what you might expect if you roll a fair die. What I would like you to is just explore a little bit. You can choose how many times you want to roll the die. You can look at any of the tables and graphs you want. And just see what happens when you do roll a fair die. You do know that this die is fair.
R1 (Kianja and Chanel are looking at the bar graph)….Well you can
explore what ever you want.
Chanel Wait, erase, erase it. Wait, stop, don’t run them yet.
R1 (to Chanel and Kianja) So, if you guys, what do you thinks gonna
happen?
Chanel Of what, if we roll thirty.
R1 Uh huh (nods her head) If you roll thirty, what do you think is
gonna happen?
Chanel Two will come out more.
R1 Why do you think two is gonna come out more.
Chanel Cuz my favorite number.
R1 Cuz it’s your favorite number. (to Kianja) Why’d you change it
from 30 to 36?
Kianja I don’t know, cuz of the way it (inaudible)
R1 Cuz 36 is six times six?
Kianja (nods her head) and the highest number on the dice is six and the
numbers is high to six.
R1 Okay, and now Chanel, you turned it back to six. If you only roll it
six times, what do you thinks gonna happen.
Chanel I don’t know.
R1 You didn’t get any two’s.
Chanel (laughs)
(15:45) Outcome for thirty-six trials – three is highest, two is lowest.
R1 (the girls continue to explore)
Chanel Come on, two.
Kianja Uhh, why the three’s like that.
(16:06) There’s a lot of threes, and a lot of these too (points to the bar
graph and the numbers four and six.)
Kianja Six, four and three. Let’s do it again.
Chanel Wait. Go to Word.
Kianja Look at that. *Two* ain’t coming up once and now it’s on the last
*six*.

Chanel Go on to Word...

Kianja (types their findings)

17:35 Chanel Why do Print Screen. Wait, you just run 12 the first time.

Kianja Run twelve. All right. Twelve?(laugh)

17:35 (syncK/C1-6:55) Twelve trials – only 1 *five* and 0 *six*.

Chanel O.K run Thirty.

Kianja Thirty? Oh you want me to print this one first and then run thirty
and put it on there?

Chanel Yeah. Let me see.

Kianja Let it end. I don’t see the end part.

Chanel Wait hold on. Sorry

18:50 Kianja Ha ha looking , I was (typing)

19:55 Chanel Let me type. (mumbling something) I told you there’s more *six*. I
knew it, I knew it. Here (Chanel gives Kianja a candy). Make it
smaller. A little bit smaller, move it up. That’s good.

20:40 Chanel (begins to type)

Kianja All right this time try the 24. Let’s see what you think.

Chanel (Typing)

22:10 Chanel What number you think?

Kianja *six* and *one*.

Chanel Un uh, I think *six* and *three*. *Six*, oh let’s see , *six* comes and *one*.

(continues typing).

Kianja (mumbling) *three* and *one*.

22:40 Kianja (begins to type)

Chanel I said um, thirty

Kianja What’s, wait, that’s twelve, it’s twenty-four

Chanel Wait, twelve plus twelve, I mean twenty-four plus twelve is
thirty…

23:10 Kianja Whoooa…I did it by accident. Look at the *two’s*.
Twenty-five trials – two is highest; no one’s; only 1 six.

That was good.

Ah, and I did it by accident.

You should have kept that one.

Twenty.

Do that one. Print that one. That’s thirty? That’s a set of thirty.

That’s good. Make it a little bit bigger.

Thirty trials – outcomes are close; four is lowest.

O.K, I want everybody to go ahead and stop what they’re doing; take your hands off the mouse or off the keyboard if you’re typing and I want to hear what are some of the things that you found when you were rolling this die. I mean I told you that this die was fair, do you think I’m right. Do you think that this die that you’re rolling here is fair.

yeah

(in background) Jarae, did you have your hand up.

(inaudible)

(typing – girls are talking to each other)

(in background) What did anyone else find out. Do you think that this die is fair? Jerel?

Yeah, because ...(inaudible).

(students are all agreeing that die is fair)

4000 times.

What did you guys find out (to Chanel and Kianja). Do you think that this die is fair.

Um, yeah. Cuz two won.

You think it’s fair because two won. All right. So what makes that the same as fair.

Um. I just think it’s fair because all the numbers are really coming up ahead sometimes

They really close

Yeah.
They’re really close.

All right. How many times did you run it?

We ran…the first one we ran twelve.

Then 24 and then 30.

Okay. 12, 24 and 30.

(nods her head)

(continues to ask for input from other groups.)

Now… can I finish typing now. (types)

Do um, do some high numbers to see…Oh wait, erase erase it. Go now.

(runs a trial of 500)

Oh look (points to end of die lining up)

(laughs) Let me see. (and she opens the table to see how many times each side is coming up.)

There’s 500 right there.

What do you think.

Two came out, Oh, there’s three.

(opens the bar graph and pie chart – mumbles something about five)

Why’d you make it…

Oopsies…a little better (she enlarges the picture of the dice as they are rolled, the bar graph and the pie chart)

That’s good. Wait let me touch (she begins to type)

Now we’re really sure that it’s…. (pauses and looks at Chanel)

even dice, what she say? (looks at what Chanel has typed) A fair dice, yeah, because the numbers came up even.

(they discuss what Chanel had typed in the Word document) Wait, (Chanel points to the monitor) take this one off, take this one off.

Representations – Kianja and Chanel select the die outcomes, the table and the pie graph. (B:348-350)

(begins to talk about the weight tool)

That’s good. That’s all right now. (looking at the screen captured into the Word document.)

Kianja, she’s introducing a tool. You want to go to that screen.

(continues to type)

Okay. (tries to type on the keyboard.)
Chanel Wait. (and finishes typing)
Kianja (opens the weight tool)
Chanel You didn’t let me finish typing.
Kianja Oh, (mumbles something about a side)
Chanel (in background) What do you think would happened if I amm.. So
Kianja Chanel was interested in the number two. What would happen if
Chanel we change this number one to be a different number. So Chanel
Kianja was interested in the number 2 earlier. Let’s say I change the
Chanel number two to be 3. Can everybody do that for me. Change the
Kianja number underneath the icon for two, change that to a 3. What do
Chanel you think that means?
Kianja (makes the change) It means that they have three dice, I mean
Chanel three sides that have two.
Kianja (and Chanel) Okay, you think the same thing. So what do you think is gonna
Chanel happen now if we go back and roll the die.
Kianja You'll get more two’s.
Chanel You think you’re gonna get more two’s.
Kianja (and Chanel) Yah. Okay, let’s change this number.
Chanel How many people think there’ll be more two’s now.
Kianja I do.
Chanel (to Kianja) did you say x. did you say x.
Kianja That’s a eight-sided die. Wait.
Chanel You can do…
R1 Let’s try it, click the x to clear the weight tool, and clear any other
data. Use the eraser tool. Now, hit and see if you get more 2’s.
Kianja (Chanel begins to bring up the table and bar graph. Kianja points
to the run man.) Go to 50.
Chanel (Points to the pie chart.) Look at three.
Kianja Oh that is really pretty (pointing to the pie chart.) I like it. I like it
Chanel a lot. I like it.
Kianja and Chanel select the die outcomes, the table and the pie
graph; two has greatest weight, fifty trials.
Chanel Dang two came out…
R1 (walks over to the girls) You guys are getting excited about something over here.

Kianja 46%.

R1 You only ran it 50 times.

33:05 Kianja It’s a eight-sided dice now

R1 Um hm (nods her head and walks away)

Kianja How can…is that possible…can you make it… All right (points to the run man) run 50 again.

Kianja (remarks at the pie chart at 39) don’t fail me now (while the chart is moving). Beautiful.

Chanel Look don’t you see those gray things...

Kianja Yeah, I see it. That’s probably because of all the dots and stuff

Chanel You gonna print this.

Kianja Yeah. (makes noises as she arranges the screen) Okay, I’m gonna type...(types) This die is definitely not fair because now it is eight-sided. We’re not taking no snack today.

Chanel Yeah.

R1 What I want you to do is stop running and go ahead and clear your data and then we’re gonna come back up here for a couple of minutes. How much time to we have left of film. Nine minutes?

35:20 G1 Kianja, what do you mean by eight-sided.

Kianja What you say.

G1 I say, can I see the composition that you wrote.

Kianja (points to screen) This is eight-sided.

G1 What do you mean eight-sided.

Kianja I got to put the weight, wait, no, no. (Chanel types) Wait, I want to show him how it look. Wait a minute. I got to show him what sided, I mean eight. (points to the weight tool) See total weight – eight sided.

G1 Oh. (nods his head) Okay.

Chanel We know what we’s talking about. He always doin that. You know what we talking about.

Kianja It’s 47 now (to Chanel), 47.

Chanel (to G1) Yes you do you know what we talking about.

Kianja Watch 6 come out, watch 6 come out like every time.

Chanel go to 60.

Kianja Oh, 60. Watch 6 come out like every time. What I told you. I told you.

Chanel Dang.

Kianja I told you (repeats). 57 it took. I told you.

G1 What’s the weight you that you give.

Kianja 57 and 3, a matter of fact. I told you.

36:25 G1 What weight did you give.

Kianja 47.

G1 47. When you changed the what.

Kianja Over here. 46, of the six’s.
Chanel: Let’s do this twice and run 30
Kianja: Two is gonna win. That’s beautiful. I like those colors. That like Pacman.
Chanel: It do, and here’s the eye right here.
Kianja: Huh.
Kianja and Chanel compare the pie graph to Pacman.

R1: (in background) Okay, so did anyone find anything interesting when they were doing their two’s. (students respond)
Chanel: (types) look.
Kianja: (points at screen) That’s wonderful. That is excellent. That’s beautiful. I like it a lot (repeats)
The pie graph is only one color, reflecting the outcome for two.

G1: What did you do to come up with that.
Chanel: Who me? I just put thirty, and ten and then the rest zero
G1: So what does that tell you? Does that tell you anything?
Chanel: It’s almost half.
Kianja: It’s a little bit mo than half.
Chanel: Yeah, change them all to 1.
Kianja: (continues typing – Chanel leaves for snack)
Appendix C

Complete Transcript of CD: ROLE 137B
with Synchronized Kianja/Chanel Monitor CD 2 of 3

Date: 03 August 2005    Grade 7
Location: Washington Elementary School
Transcribed by: Barbara Tozzi
Verified by: Anat Even Zahav

<table>
<thead>
<tr>
<th>Time</th>
<th>Speaker</th>
<th>Transcription</th>
</tr>
</thead>
<tbody>
<tr>
<td>00:00</td>
<td>Lorrin</td>
<td>(reads Schoolopoly task) Plainfield Middle Schools are planning to create a board game modeled on the classic game of Monopoly. The game is to be called “Schoolopoly”</td>
</tr>
<tr>
<td>01:08</td>
<td>Terrill</td>
<td>I have a question. Are you positive that die is spelled like that?</td>
</tr>
<tr>
<td>01:09</td>
<td>R1</td>
<td>Am I positive that…</td>
</tr>
<tr>
<td>01:10</td>
<td>Terrill</td>
<td>Die is spelt like that.</td>
</tr>
<tr>
<td>01:11</td>
<td>R1</td>
<td>Die? D-i-e.</td>
</tr>
<tr>
<td>01:12</td>
<td>Terrill</td>
<td>As dice. I’m not talking about dead die, but I thought it was d-y-e.</td>
</tr>
<tr>
<td>01:13</td>
<td>R1</td>
<td>Actually, there are two different ways to spell...(She writes on the board.) There is die or dye. This one (as she points to d-y-e), can anyone tell me…</td>
</tr>
<tr>
<td>01:14</td>
<td>Students</td>
<td>That’s a color. Like color dye.</td>
</tr>
<tr>
<td>01:15</td>
<td>R1</td>
<td>This one (pointing to d-i-e) is, if you talk about, that somebody is going to die, you spell it that way. If you talk about the idea of coloring something, you are going to dye something, dye it a certain color – you spell it like this (points to dye). And when we talk about dice, dice is a plural form, meaning more than one. So if I only talk about one, I talk about a die. And we spell it like that (points to d-i-e). It’s a good question, because oftentimes in the English language, I’m sure you guys have found this, that two things that sound exactly alike are spelled very differently. So that’s a good question. Okay, so first of all let’s talk about what</td>
</tr>
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</table>
Lorrin just read to us. Can somebody recap what’s going on here with the Schoolopoly? Kianja?

2:40 Kianja I don’t know. Something about the companies…some of the companies are making the die unfair. And we want the company that makes the die fair, and we have to try to figure out which one is not fair and which one is fair.

Terrill You can’t make a die unfair because a die has six sides and six different numbers, so when you roll it six different ways.

(inaudible)

R1 But I think earlier some people were talking about ways that the die could be unfair.

Chanel There is ways that the die can be unfair. If you put two of the same numbers on the dice.

R1 Put two of the same numbers on the dice. I think somebody was talking about the corners…if the corners weren’t particularly straight. If they were rounded or something. Lorrin?

Lorrin Oh yeah, I was gonna as you. Have you ever seen a die that um, has more than six sides?

R1 Yes I have (nodding her head). Yes, they make die, they have die that have a hundred sides on them. I personally…

Student Is it like flat? Is it flat though, like the side is flat.

R1 The side is flat but because there’s a hundred of them, the sides are very small.

(More student comments.)

R1 So, I actually have a collection of different kinds of dice and so I’ve got lots of different kinds. Some of them have way more than six sides on em. So, we’re kind of worried that some of these companies may produce unfair dice. And, since your school wants to create the Schoolopoly game, we have a really important business decision to make because these companies want to provide the dice for the game, but if they’re unfair, we don’t want those die in our game (pauses, looks around at students). So, we want to try to find out, we need to make a recommendation to the School Board as far as which company they should go ahead and give the contract to.

4:55 Chris Just get all the dice.

R1 Just get all of the, all the companies? Why would we get all the companies?

Chanel There’s more money.

R1 So, Chanel, you’re saying that it would cost more money to have all of them, to have all the companies. So we want to find the company that’s the best, to make a recommendation. So we’ve got six different companies that are trying to get this contract. And they’re called Perfect Polyhedra, Dice ‘R Us, Delta’s Dice, Calibrated Cubes, Dazzling Dice, and Dice Depot. So, what we’re gonna do is…
Student: What’s a polyhedra.

R1: What is a polyhedra? That’s a good question.

Chanel: A square-sided figure.

R1: So Chanel said that it’s a square-sided figure, (Another student notes it’s a polygon.)

R1: It’s a polygon.

(other student comment)

R1: What is a polyhedra?

(more talk off topic)

R1: Okay, so the question was what’s a polyhedra. So, a die is an example of a polyhedra.

Tiffany: It’s a polygon.

R1: Go ahead, let her finish. Go ahead, it’s a polygon…

Tiffany: I was gonna say a hexagon has six sides.

R1: Okay, a hexagon has six sides. A hexagon is an example of a polygon. So, this is an example of a polyhedra (holds up die). This is a polyhedra, but there are many different kinds of polyhedra with different numbers of sides. This is a polyhedra that we typically call a cube. Right, we typically call this shape a cube because it’s made up of squares. And we’ve got, each side is a square and then we have six of those squares kind of folded up together to make this cube. And it’s an example of a polyhedra. Some of you were asking – let’s take the example of a hexagon. If I took different hexagons and tried to fold them up together (uses hands to make a shape) I could make a polyhedra where each of the sides looked like a hexagon rather than a square.

(8:05) R1: And so, there’s lots of different kinds of polyhedra; this just happens to be one example of it. All right. Now, when you go, when you go to explore your companies, I’m going to give you your assignments. I’ve assigned different companies to different groups and you’re gonna actually, you’re gonna actually explore more than one. You’re gonna have two or three to explore. All right. And what you’re gonna do, because each of us are gonna have two or three to explore, after you have some time to explore um the different companies to find out whether you think that company produces fair dice, you’ll get a chance to get together with another group to see what they found out. Because a different group may be exploring your same company. So you’ll have a chance to talk, and say “oh, what did you find out?” All right. And then eventually, eventually, each of your groups are gonna have to make a poster that tells us something about the companies that you were assigned. And you’ll have to make one poster per company. We won’t get to make posters probably, well, we definitely won’t get to it until tomorrow. But, it is very important that you know – take a look what’s in that box. Take a look what’s
in the box down here. These are the questions that you’re gonna have to answer on your poster. So it’s important that you keep those in mind when you’re doing your exploration. So, Terrill, go ahead. Terrill, read number one for us.

9:45 Terrill Would you recommend that dice be purchased from the company you investigated?

R1 Okay. So that’s the first thing you’re going to have to answer on your poster. Do you think we should buy die from this company?

Danielle Number 2 – What evidence do you have that the die is fair or unfair?

R1 Okay. So what evidence…you’re gonna have to show us some evidence. All right. Jerae, number three.

Jerae Three. What do you think the chances are for rolling each of the six numbers.

R1 Okay, so that’s the third question you’re gonna have to answer on your poster. Okay, Kianja go ahead, read the bottom.

Kianja Use *Probability Explorer* to collect data from simulated rolls of the die. Copy any graphs and screen shots you want to use as evidence and paste them in a Word document. Later, you will be able to print these to use on a poster. You will give a brief presentation pointing out the highlights of your group’s poster.

R1 Okay. So tomorrow, when we actually do presentations, it will be a little different because you’re gonna have posters. All right. So we’re gonna have posters hung up here. And, it’s gonna be very different because we’re gonna actually have somebody from the School Board come in, and you’ve got to tell them what you found out about these dice. (Some students are talking.) So, you want to make sure that what you write on your poster, that you’re ready to make that presentation tomorrow. Okay. Does anybody have any questions about this?

Jarae I do.

R1 Who said that? Tiffany? Sombody said that. Jarae. Listen up – Jarae has a question.

Jarae Do you have to do a poster?

R1 Yes you do.

Jarae And, if you don’t have a partner do you have to do it by yourself?

R1 Or will you get somebody to work with?

R1 Um. You will get somebody to work with. All right, so I have got the assignments of which companies you’re gonna be exploring. So, (begins to hand out papers) Chanel and Kianja, this is your assignment. Those are the three companies you’ve got. Chris and Jerel, Lorrin and Justina, Jarae, Terrill and Danielle, and Keisha and Tiffany. (Students compare companies.)

Girl Do we got both of these.
Yes, you’ve got both of them. Okay. So, let’s go ahead and go back to the computers and I’m going to show you where to find these files. Okay.

12:10 R1 Draw a circle. Use your mouse. (synchronizing computer monitor video) And we’re okay. Now, all of you on desktop let’s go back into the folder of probability files. And there is a folder called Plainfield Schoolopoly. Go ahead and open up that folder, and in that folder you’ll find all six companies. You only need to look at the ones that you were assigned to.

Chanel Which one you going to.

Kianja Dice R Us

R1 (continues in background) You can do them in any order you want. It doesn’t matter. But remember, you want to have a Word document and you want to put things in your Word document.

R1 And you want to keep it organized so you know which company you’re collecting data from.

Chanel Go to Dice R Us. Hm. Instead of putting it on disc I’m gonna write it.

Kianja Hold on, wait a second.

R1 (continues in background) So you can organize your Word document any way you want but make sure you always know which company your data is coming from. You can use separate documents, you can use one document; I don’t care.

Chanel Can we write on paper?

R1 Um, you can write on paper but you want to think about the kinds of things you want to print because you’re gonna get to print stuff later. But you can certainly use paper.

13:35 Kianja (begins to type) Let’s see Kianja and Chanel…No (erases) Thank you. Enter.

Chanel Let me type. (the girls discuss who should type.)

Chanel Do Dazzling Dice first.

Kianja Okay, let’s go ten times.

Chanel No no. Let me see, let me see.

Kianja Five…there you want to stack it.

Chanel yeah.

Kianja Yeah they don’t have it cuz…(inaudible) Let me see.

Chanel Print that one.

(syncK/C2-18:50) Outcome for ten trials; claim that die is fair.
From now on, I’m not gonna print the graphs. (She is referring to the stacked columns of output for each side.) I’m not gonna print this, I’m just gonna put this on there. Cuz it will take up too much space.

All right.

I’m just putting whatever graphs we want.

The graphs and um circle graph.

The circle graphs be pretty.

All right we start with twenty time.

Twenty? Okay.

Twenty, forty, sixty,

Enter. Paste. (begins to type) Our first trial of twenty… I don’t want the underlined…tab, copy, then we’re gonna put this here, tab...

Outcome for twenty trials; claim die is still fair.

We don’t need to do the bar graph, sometimes we do a bar graph… Scroll down. Let me type.

From this data it looks like it is even, I mean uh, a fair die. (Chanel typing what Kianja says) It’s a fair die because the numbers are close. (She inserts the table and pie graph into the document)

Word document with conclusion for twenty trials.

(Kianja starts to type again) Schoolopoly. (the girls try to say the word) Kianja, do forty.
Oh, yeah, it is an odd word Schoolopoly.

Kianja: (continues to type) Our next … Oh too many spaces, one two three four I should just keep going, till forty. Beautiful. Yeah, I was right. We don’t need this. (She points to the stacked outcomes.) We’re not getting this anyway. So it don’t matter.

R1: Do you know how to copy the whole screen if you want?

Girls: yeah.

R1: Okay.

Kianja: But we don’t want to because it take up too much space.

R1: Okay. That’s fine I just want to make sure you know how to do it.

Kianja: (Copied into the word document) (inaudible) and I’ll do the bar graph.

R1: So, I saw you guys did twenty and then forty. What was your thinking on that. Why those number of trials.

Kianja: For real. We really don’t know.

R1: You don’t know.

Girls: (shake their heads) no.

Kianja: No we just picked numbers.

R1: Okay. But I also noticed that you put ten over here by the running man and then you were clicking it four times.

Kianja: Oh because um it’s easier that way.

R1: It’s easier that way, okay.

Kianja: Cuz then if we want to do another number we just gonna have to type it. But if we put the 10 there we do da da da da da…(taps the table) What’s our next step. Oh wait, from this data, it looks as though

R1: So does that mean you always want to do multiples of ten

Kianja: There are more, three sides. Yes it still looks even but we’re questioning the number of sides with three dice.

(syncK/C2-23:38) Questioning why three came out so much higher then the others at a trial of forty runs

G1: You said even? Or good?

Kianja: Cuz three showed up the most. Three showed up higher.

G1: (points to monitor) You said good or even. Did you say even. I noticed that. That’s okay. I thought that said even.
Kianja (talks to Chanel) It was sixty. And then we’re gonna stop at 100 and then do 200.
Chanel 100! Geez.
Kianja Yeah. 60, 80, 100.
Chanel I know, that’s a lot.
Kianja (laughs – points to the monitor) Go to the second one. No just click it six times.
Chanel Why don’t we just do it the easy way.
Kianja Cuz it’s harder. Wait wait no no.
Chanel Run, man, run. Ding.
Kianja Dag. *Five* not on there. Dag. Okay now that’s, that’s
Chanel *Five* was on there the last time.
Kianja Whoa. *Five* only got one.
Chanel *Five* was on there last time though.
Kianja Wow. We need to copy this whole screen. Go here. I Click print screen. Oh my gosh.
Chanel (types)
Girls (talking off task)

But *five*
Oh my gosh. I definitely. So put, we definitely think that there are less like, oh my gosh, do you see that.
We think it’s not right because *five* is being
Because the last time *five* came out a lot more than this time
(types)
(talking off task)
24:00 Kianja See this is why we’re not gonna use this (She points to the stacked outcomes on the monitor.)...six, one, three...

Chanel 3, 6, ....2 (dos), 4 (quarto)...

Kianja Dag there’s a lot of one’s left. But they can fit right. Oh beautiful, that’s excellent... We’re just gonna get this two this time cuz... and then go here (Chanel is typing) copy...you want a get it right... yet and still five, five hit the bottom. With only eight appearances, eighty appearances, oh yeah, all right


(Kianja takes the keyboard) and six over 100. Now just click past eight past eight.

Chanel (increases the sample size to eighty and clicks on run)

Kianja Look. It ain’t coming till the last...

Chanel It’s only got two. Keep going, keep going. Don’t paste it yet.

Kianja No I’m pasting to show something.

Chanel Why? Don’t paste it, don’t paste it.

26:50 Kianja Yeah, I’m pasting to show that um on 76 was only two.

Chanel You should have let it kept going

Kianja On 76 there’s only two (inaudible) So we could prove our point and then we can still

Chanel We gonna prove our point when we get to 100.

Kianja wait, wait....Oh Chanel

Chanel I’m sorry I really didn’t know. I clicked the x button on it.

Kianja (typing) We see now that five is definitely ...this die is almost absolutely unfair because about three fourths of the way (continues to type)...there two

(synK/C2-32:00) Kianja’s reason for stopping at 76 trials – three fourths of the way to 100 the five only came out twice.

28:15 Chanel out of 76...but you didn’t have, you don’t have a hundred there.

You only got a 76.
Kianja I know, but we wanted to go to 100, so I set up to a 100.

Chanel So that’s how you were right?

Kianja So now we’re gonna go back here and print 100. Um here. Okay. Just copy these two. (points to the table and pie chart)

(syncK/C2-32:27) Outcome for 100 trials.

G1 So, Chanel, what was the decision on the 76. Because she stopped at 76, right Kianja? You ran another one?

Chanel We finished it. We was running up to a 100, but she stopped at 76. To print it, it showed that out of 76

Kianja Only twice did 5 show up

G1 So what happened for 100

Kianja For 100, only three times did it show up.

Chanel So we don’t think it’s fair.

G1 And then you got another one?

Kianja We don’t think this die is fair is what we’re trying to say.

G1 Yeah but my point Kianja is this. You stopped at 76 correct, and the five had appeared only two times right? Suppose you had run up to 100. There are still more to come, right.

G1 And did you run 100 next?

Kianja Yes.

G1 And what happens now.

Kianja Three only. Only three times did it show up. (types) The die is unfair because…

G1 All right, we’re done. We’re done with this one. I don’t know how many pages this is.

Kianja I do.

Chanel How many.

Chanel Two. Watch out. (Chanel scrolls through the document)

Kianja Where that tool bar at.
Chanel: It just showed there were two pages over there…this stupid computer.
Kianja: Wait, no, no, no, no, if we do it like. I’m show you something.
G2: You want to put it in Print format. See where under View it says Print Layout.
Chanel: Print Layout. Print Layout.
G2: Higher up on that View box. Third one down. Then you see the page breaks.
Kianja: It’s six pages.
Chanel: (points to monitor) Well it says.
Kianja: It’s six pages. Oh wow.
G2: 32:55 If you don’t have room on your poster, would you leave any of this out?
Chanel: (talking to another student) Get out of here. (She covers the monitor.)
Kianja: Who that, Jerel?
Chanel: Yeah. (walks away)
G1: Kianja, let me see what you wrote for 100. I just want to get that.
Kianja: (scrolls down) Wait a minute please. Hold on I got to fix this right quick. Beautiful. (talking to herself)
Chanel: What are you doing, printing it?
Kianja: I’m trying to put it up here.
Kianja: 34:00 Beautiful. I like it a lot.
Chanel: Here Kianja hold this for me. Thank you.
Kianja: There we go. Okay, Chanel. That’s a lot of space.
Chanel: Come on, Kianja.
Kianja: I’m trying to fix this so it doesn’t look so unorganized and sloppy, you know what I mean.
Chanel: No I don’t.
Kianja: You should.
Chanel: Why.
Kianja: I don’t know but you should. So it’s less pages.
Chanel: Now we only got three pages cuz of you.
Kianja: Yes and that is excellent. Now we got three.
Chanel: Just go to…that’s Dazzling Dice. (They discuss which file represents their earlier work, and which file is for Dazzling Dice.)
G2: Chanel would you grab that blue floppy disk so you can save on that?
Kianja: (exhales)
Kianja: What? I can’t help the perfection. I can’t help the need for perfection. All right, let’s exit this.
R1: 36:30 Are you guys movin on to another company? What did you find out about your first company. Which one is it.
Chanel: We did Dazzling Dice first, we think it’s not fair.
It’s not fair.
You don’t think it’s fair? Why not?
Because
Because five didn’t come up as much as the rest of the numbers. So we figured it’s more of the other numbers.
We ran, we ran it for 100 and it only came up three times and everybody else came out like twenty-something.
But I, correct me if I’m wrong but I thought I was here earlier and you were doing like your ten and your thirty and you thought it was fair.
Yeah at first we thought it was fair because all the number was coming out, it was coming out small and stuff so we think it all is good but when we got to higher numbers five just stayed down there at the bottom.
Yep so we changed our opinion at the end.
At the end. And you are perfectly able to, when you have different data, to change your opinion.
(answers student question) Okay, what I need you guys to do. If you have not saved your Word document yet, I want you to put your disk in and save your Word document. You can also save…
Appendix D

Complete Transcript of CD: ROLE 138B
with Synchronized Kianja/Chanel Monitor CD 3 of 3

Date: 03 August 2005    Grade 7
Location: Washington Elementary School
Transcribed by: Anthony Smaldone
Verified by: Anat Even Zahav

<table>
<thead>
<tr>
<th>Time</th>
<th>Speaker</th>
<th>Transcription</th>
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<tbody>
<tr>
<td>00:53</td>
<td>R1</td>
<td>Circle; nice and slow; take it around the circle; all right, we should</td>
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<td></td>
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<td>be ready; OK, first of all, I want to know, how many of you have</td>
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<td>explored, um, um, more than one company so far? You’ve looked</td>
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<td>at more than one company?</td>
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<td></td>
<td>Chanel</td>
<td>Oh, we doing it now; look, we’re doing it now</td>
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<td></td>
<td>R1</td>
<td>More than one; Chanel and Kianja, you’ve just done your one.</td>
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<td></td>
<td>Both</td>
<td>We’ve finished one, so we’re…(inaudible)</td>
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<td></td>
<td>R1</td>
<td>So, you’re going to start your second. Ok. Justina and Mark: Did</td>
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<td>you do more than one or did you do one so far? You’re working on</td>
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<td>number one? You’re working on your second one, ok. All right.</td>
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<td>Because what I want, what I want to do, there are, and I don’t have</td>
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<td>my list in front of me; Dr. Weber has it; Does anyone know where</td>
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<td></td>
<td>the sheet of paper went that Dr. Weber had, that has red ink on it?</td>
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<td></td>
<td>Does anyone see that anywhere? No? Ok. So, I don’t have my list.</td>
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<td>Dr. Weber, the piece of paper that I gave you, do you have that?</td>
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<td></td>
<td>R2</td>
<td>Yes</td>
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<td></td>
<td>R1</td>
<td>Can I have it please? Thank you.</td>
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<td></td>
<td>R2</td>
<td>Sure could; sorry, I hid it.</td>
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<td></td>
<td>R1</td>
<td>That’s ok, thanks. Ok, so, um, what I’d…do any of you have some</td>
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<td>of the same companies? I want to give you a chance to talk to other</td>
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<td>people who are exploring the same company; all right,</td>
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<td></td>
<td>Chanel</td>
<td>My gosh…</td>
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<td></td>
<td>R1</td>
<td>So we’re going to have a little bit of informal time where if you</td>
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<td>still have one person that wants to (inaudible) that’s fine, but we’re</td>
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<td>going to be able to go around the room and talk to other people.</td>
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<td>So, for example, ah, Chris and Jarel, you did, um, Delta’s Dice,</td>
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<td>Dice R Us and Calibrated Cubes, is that right?</td>
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<td></td>
<td>Chris</td>
<td>Yeah.</td>
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<td>R1</td>
<td>Who else has done those companies? Delta’s Dice, Dice R Us or</td>
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<td></td>
<td></td>
<td>Calibrated Cubes? Any body else have any of those companies?</td>
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<td></td>
<td></td>
<td>Did anybody else do Delta Dice?</td>
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<td></td>
<td>Student</td>
<td>Oh, ah</td>
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<tr>
<td></td>
<td>R1</td>
<td>You mean you have Delta’s Dice?</td>
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<tr>
<td></td>
<td>Student</td>
<td>No we don’t, We don’t have Delta’s Dice? Oh yeah….</td>
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Jarae We do.
R1 Ok, Jarae has that one, ok, so, and um, and Warren and Justina, at some point, you guys are gonna want to talk to Chris and Jarel to see what they did with Delta’s Dice, ok?
Chanel/Kianja (mumbling to each other)
R1 Dice R Us. Who else has Dice R Us?
Jeral We do.
R1 Who else besides them?
Chanel/Kianja (inaudible)
G2 You girls have Dice R Us.
R1 Chanel and Kianja, don’t you have Dice R Us?
Chanel Oh yeah, yeah, we didn’t do it yet, though
R1 You haven’t done it yet? Ok. We’ll hold off, we’ll hold off on that one. What about Calibrated Cubes? Who else has that one?
(inaudible) Who else did Calibrated Cubes? Tiffany and Kiesha, do you have that one? Calibrated Cubes? (disagreement) Somebody else besides you guys should have it.
Chanel Jarel
Jarel What?
Chanel Jarel and them.
R1 Someone besides them; do you have it?
Chanel Calibrated Cubes? No.
R1 Oh, you guys have Calibrated Cubes…..so…(background mumbling).….So, Justina and Lorrin, have you guys looked at Delta’s Dice yet? (inaudible) so, Chris and Jarel, I’d like you to come over here and be able to talk to Lorrin and Justina, about their findings and what I’d want you to do, hold on guys, so that you have your data, I want you to bring your disk up here. We’re gonna print out what you did for Delta’s Dice and you can take it with you…(background mumbling)…that’s fine…um, actually, yeah we’ll do that…Chris and Jarel, go ahead back to your station and we’ll have Justina and Lorrin come over there – good idea. Justina and Lorrin, do you have your data from Delta’s Dice yet? (background discussions…inaudible)
05:01 Chanel Why did you put that next to it?
G1 You’re doing Dice Depot, right?
Kianja What do we think?
(syneK/C3-5:33) Sample size is twenty; initial claim - Dice Depot die is unfair.
5:25 Chanel So you can see it better, cause I can’t….We think (oh, sorry), we think this die might be (Kianja typing…..a lot of background noise…Chanel reading) they’re all close to each other….four…..four side dice…..came up more than any other dice…..are (inaubible)Oh no….ooo, that’s pretty…..

6:50 Chanel ok, maybe it is fair……

Sample size of forty – outcomes are closer, sectors on the pie graph look closer in area size, and the girls think the die is fair.

Kianja Can I see the…what do you call…
Chanel These?
Kianja Ok, hold on, go down…make it smaller….(Chanel working on computer taking over from Kianja) Just go to the edges…..(Kianja takes over)
Chanel hmmm, that’s pretty. (Kianja typing on the computer; a lot of background noise; apparently setting up an experiment and one can see the results being seen on the screen (for example, seeing the results being displayed on the pie chart))
Chanel The numbers came {inaudible}…getting..don’t, yeh do that, oh, enter…(Kianja continues typing)
07:54 Chanel Oooh. That’s pretty. (Kianja is typing on the computer. Background noise.)
08:12 Chanel The numbers came close to zero. (Kianja continues typing; Chanel gives occasional suggestions.)
09:48 Chanel Deeng. I don’t think it’s fair…..hmmmm….copy…..(Kianja presses a button on the key board).
(syncK/C3-9:06) At sixty trials, different opinions about fairness.
183

102 10:20 Kianja (Chanel is using the mouse..) Go to here (Kianja points to the
103 bottom of the screen)….click that; go.
104 105 (syncK/C3-11:12) At eighty trials, Chanel types that they are hesitant to say the die is
106 fair.

107 11:11 Chanel (begins to type, then both look at their work)
108 11:45 Chanel Um um, it’s not. Look, most of ‘em came up 5, 8, 7, but yet these
109 come up 22, 18, 20.
110 111 Kianja How did it….
112 12:10 Kianja You got to um, make the um pie charts big; o.k (starts typing)
113 12:21 Chanel Let me type [takes over typing]
114 12:57 Tiffany Hallo.
115 116 Gl Hey. What’s your name.
117 Tiffany [Tiffany sits down]
118 119 Kianja (looks around) Pass her a chair, somebody. (Keisha comes over
120 and sits down. There is a lot of inaudible mumbling among the
121 girls.)
122 13:42 Chanel Hold on, we’re doing Dice Depot now [inaudible mumbling among
123 the girls continues. Chanel continues to type their results]
124 14:04 Kianja Are lower than 15 and others are higher than 15; right there lower
125 126 127 than 15 and others are higher. (Chanel continues to type.)
128 129 130 131 132 133 134 135 136 137 138 14:36 Keisha Can I ask you a question? Why did you do 80?
14:36 Keisha Can I ask you a question? Why did you do 80?
14:16 Keisha What about, what’s the last one? Dice Depot – what y’all get for
14:16 Keisha What about, what’s the last one? Dice Depot – what y’all get for
14:16 Keisha What about, what’s the last one? Dice Depot – what y’all get for
14:16 Keisha What about, what’s the last one? Dice Depot – what y’all get for
Keisha  Not fair
Chanel  Yeah, we’re probably gonna get that too. Go to 100.
Tiffany  Wait, can you scroll up, ‘cause you said it’s not fair, but at the
top you said it was a fair sided dice
Chanel  This is Dazzling Dice
Tiffany  Oh, so…
Chanel  This was the first one
Kianja  This was before
Chanel  This was before we made our final decision
Kianja (has hand in front of mouth while reading from the screen – most
inaudible)….because……5 being the lowest….we don’t think it’s
fair because 5 keeps being the lowest while everything else is
higher, like, it was only one out of like 60…Dazzling Dice….Five
is still at the bottom with only eight out of 80…we was going to
100, but, um, at 76 there was only two of ‘em
Chanel  Why you talking like that?
Kianja  Cause my hand’s here (in front of mouth)
Keisha  (Checks her own notes) 5 is lower for, um, Dazzling Detail
Chanel  Dazzling Detail???
Kianja  And there was only…..[everyone laughs]
Keisha  Whatever. That’s what it is.
All  Dazzling Dice
Chanel  You said Dazzling Detail [mumbling]
Keisha  Go back to Dice Depot…so I can finish…[mumbling]
Tiffany  What time is it?
Kianja  It’s 12:15
Keisha  What time do we leave outta here?
Chanel  1:30
Kianja  2 o’clock (inaudible mumbling); We go upstairs at 1:30.
Keisha  I’m leaving at 1:30
Chanel  …bathroom in a few minutes…..(mumbling conversation between
the girls)
Chanel  Hey, you know the microphone’s on
Keisha  I don’t care; I’m telling my story…[continues telling story softly]
Kianja  I’m gonna put this on the next page
Chanel  No
Kianja  Yes
Chanel  No
Kianja  Cause that’s gonna look retarded
Chanel  So?
Kianja  Ok, now I can do that
Keisha  She want everything to fit on one page (pointing to Tiffany)
Tiffany  Me too
Kianja  Who me?
Keisha  No, her (pointing to Tiffany)
Chanel  Oops
Kianja: We’ve got three pages for each thing.
Keisha: They’re not going to let you use staplers, so
Chanel: Why?
Keisha: Ms. Tozzi gave us, um, um, paper clips
Chanel: I thought they letting us use staplers.
Keisha: Um, Ms., Dr. Lee
Tiffany: What you mean?
Keisha: [Laughs] I said Ms. Dr. Lee
She said that we don’t need no staplers, just number the page
[mumbling conversation between the girls]
Tiffany: So, what did y’all pick for your choice?
Kianja: We all don’t know yet
19:20 Chanel: I don’t think it’s fair, cause if this got all the way up to 30, and all
in the teens…[inaudible mumbling]
(syneK/C3-18:30) At 100 trials, Chanel notes the frequency for four is thirty, and
makes a claim.

Keisha: What did we pick?
Tiffany: Um, what was the last one we did?
19:27 G1: Keisha, which ones did you do? Did you do Dazzling, or…..you
done with Dazzling?
Keisha: We did Dazzling Depot before…[inaudible]
Tiffany: We did Perfect Polyhedra
Keisha: I mean Dazzling Dice. And we did Dice Depot
G1: Ok
20:01 Tiffany: We did Perfect Polyhedra because it’s so so, cause only one of the
um numbers is higher than everything else; um, …five of the
numbers are in the 100’s… but there’s one that said 200…so it’s
so, so
[Chanel continues to type her report]
21:55 Chanel: Hmmm, now which one do we have to do? Dice-R-Us. Only got
like 5 minutes, I’m going to the bathroom. I’ll be back [Chanel
gets up and leaves]
[Chanel continues to type]
Kianja: All right…all right, now what was I saying,
[Keisha gets up and leaves]
[mumbling between Kianja and Tiffany – discussing their findings but quite inaudible]
[Chanel returns]
23:10 Kianja (to Tiffany) You guys said that Dice Depot was unfair. Right?
Why?
Tiffany Because. We was doing it like this. It had to be close. Like, they was all in the hundred. Like the one hundred. (inaudible) Out of all of them that was different…
Chanel returns
23:57 G1 What did you find out about the dice, the die
Tiffany That
G1 What was your decision?
Tiffany Did I do that one? I don’t think I did that one
G1 You didn’t do Dazzling?
Tiffany I don’t know. Where Keisha at? Keisha did it
Kianja You did it. It’s right there
Tiffany No, that’s Dice Depot
G1 You didn’t do it together
Tiffany No, we did, I’m just saying…[inaudible]
Kianja Yeah, Dazzling Dice right here
G1 So, what was your final decision? Is it a fair die, or?
Tiffany No.
G1 No? On which grounds. Why?
24:30 Tiffany Because, these two, these three, one, two and four was the only ones that was in 100, then 3 was in 200, and that was 136 and that was in 200, so
G1 But that was for which, for how many, for trials over 1000
Tiffany Umm Hmmm
G1 you just did once?
Tiffany No
G1 You did one trial?
Tiffany We did, then we did the same thing for that
G1 For what
Tiffany It sort of came up the same way; it was always three, then one higher, no, two higher, then one low one
G1 Slowly, slowly, I missed that; How many times did you try; one, two, right? This was one (pointing to her report), then you did it again here, then here again. No, this was different
Tiffany No, this was a another one
G1 So, you did two times, right? And then what did you, what’s your conclusion? What are you saying?
Tiffany That
G1 That both times you tried, what are you concluding
Tiffany That, um, Dazzling Dice isn’t fair because there was three that was in 100 and there was one that was higher, wait, two that was
higher, and one that was lower for both tries, so we said that it wasn’t fair.

G1: Which one in this try is higher, what you say? Which one on that table, I see a table that you have [looking at her report]

Tiffany: Right here

G1: Ohhm, um, this is Dazzling, ok. What’s going on…

Tiffany: There’s three that’s in 100, there’s two that’s higher, and there’s one that is lower

G1: Uh huh

Tiffany: And it’s the same thing with this one [inaudible] three that’s in 100, two that’s higher, and then one that’s lower, so we decided that it was unfair if it happened both times

G1: Hmmm. So, you, if it were fair, what would have happened? What would we have gotten, then?

Tiffany: Then that…it would have been fair, it would be [inaudible] like the other one

G1: Kiesha begins to work on Dice R Us.

(Initial result for twenty trials.)

G1: (continues his conversation with Tiffany) Would you say if you have a fair die, right, can you run and still see the differences? Show me the run with the 1000, show me the 1000 again. Can I see the 1000? So, this is what you did, right (looking at her report).

G1: First time, 1000 tries, right? And then you got 157, 108, and what’s the problem here

Tiffany: That it is not all in the same place it has to be like, for instance, it has to be like, either all lower than 100, all in the 100s, in 100, all in 200s or all in 300s [inaudible]…2 in 100…[inaudible]; it’s not fair
Let’s not move on, I want to ask you a question here; put it here maybe so I can see; So, if this was; let’s forget the 36 for a while; so what about that one, would it be fair?

No. These two would still be higher than everything

Ok, so they’re higher than everything. So, and then the second one [flips through Tiffany’s report] this is the second one right? So what is the problem here for you?
The same thing, one is lower, two is higher, and three is the same

Suppose we didn’t have this one, would that be a problem for you?

What would be the problem for you?

That these two are still higher

So, suppose you had this perfect die, right, this, ah, ah, fair die?

What would you expect if you had 15 hundred if you run it [inaudible]

It should be, like, either all close together or all even

Oh you’re saying all close together or

Or even, ah, I see, I see, I see…Hmmmm, hmmmmm, hmmmmm. Ok, so in other words, if you forget about this one, so you don’t think this is close enough, is it close enough? What you get, it’s not close enough for you? Ok, I see. Ummm, and here, in the first one? If you close this one, is it close enough for you? Suppose we didn’t have that, would it be close enough [inaudible]

No

Ah, I see. Let’s see about the Depot then, what you got for Depot?

What’s your conclusion on that one?

It’s not fair, ‘cause, it’s basically

How many times did you try?

Two

Two times; show me about the first time

For this, these, um, four was in 100

The only [Researcher looks up; appears Tiffany was being called]

And these two

Tiffany, ah, well, she was here, but if you need her, that’s fine [Tiffany gets up and leaves], ok. [inaudible as he talks to another researcher]

Kianja’s result for forty trials.
R1 [R1 returns] How are you guys doing with your companies? I know you had some company over here. You had some people talking to you; what did you guys talk about?

Kianja Um

R1 Which company were you talking about?

Kianja Both of ‘em

R1 Both of them? The Dazzling Dice

Kianja Um-hmm

R1 And the Dice Depot

Kianja Um-hmm

R1 Ok; and I think you’ve got Dice-R-Us and Chris and Jarel have Dice-R-Us. Are you done, oh you’re still doing that one

Kianja Yeah

R1 Ok; when you guys are done with that one

Kianja Um-hmm

R1 Ah, I’m going to have you come over here and talk to Chris and Jarel

Kianja All right

R1 About what you’re finding out about that company

Kianja All right

R1 Ok, I’m curious with the Dazzling Dice, cause that was the first company you guys did, right?

Kianja Uh-hmm

R1 And that was the one where you changed your mind?

Kianja Uh-hmm

R1 How did that match up with what the other group did?

Kianja They thought it was unfair too

R1 They thought it was unfair too? And how did they how did, did, did the strategy that they used, was it the same as what you used or did they use something different?

Kianja It was a little bit different

R1 It was a little different. Do you remember what they did?

Kianja I thought they started out with a 1000

R1 Oh, they did a 1000, ok.

Kianja [inaudible] we start smaller

R1 Ok. Go ahead and finish what you’re doing with Delta’s Dice, I mean, no, Dice-R-Us, and then, um, and then I’ll have you come talk to Chris and Jarel, ok?

Kianja OK

R1 Ok. And, actually, we’ll print this paper out for you.

[Dr. Lee leaves]

[Chanel resumes typing, inaudible mumbling]

32:20 Chanel I got the rest, I did the rest

[Dr. Lee reaches over] I got it…[inaudible]...

32:46 Chanel you only clicked it three times.

Kianja Um-hmm
Chanel: Why you only click it three times?

Kianja: Cause you’re only going to sixty

Chanel: [a lot of background noise]


Chanel: All right

Kianja: Oh, wait, put that um arrow first [inaudible]

Chanel: How you do that?

Kianja: No wait, go back, put it right there [works keyboard] all right. Now make that smaller so we that we can put, um, that other thing in there, next to it. Now make that smaller.

Chanel: [laughs] He’s a dummy, yeah

Danielle: [arrives] You guys, which one you think is fair?

Chanel: [inaudible response]

Kianja: We got Dice-R-Us and [inaudible]

Chanel: Don’t put your hand on the keyboard or nothing [inaudible]

[Danielle was called away]

Kianja: [claps hands] close enough, close enough

Chanel: Ooo

Kianja: No cause they are close enough

Chanel: No, it’s unfair

Kianja: You said you really don’t think it’s unfair?

Chanel: Ah

Kianja: Oh well, using [inaudible] sixty, eighty

R1: Ok, we’re going to wrap it up for today. So, I want you to save everything on your disk and what I’m gonna do is I’m going to print out your results so you’ll have them when we first start tomorrow morning. But what I need you to tell me…..[end of sound]
Appendix E

Complete Transcript of CD: ROLE 139B with Images from ROLE 142E

Date: 04 August 2005    Grade 7
Location: Washington Elementary School
Transcribed by: Robert Tozzi
Verified by: Barbara Tozzi

<table>
<thead>
<tr>
<th>Time</th>
<th>Speaker</th>
<th>Transcription</th>
</tr>
</thead>
<tbody>
<tr>
<td>00:02</td>
<td>Kianja</td>
<td>All right, probability files. It’ll open faster, oh wait a minute,</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>cancel, or ok. Go to open…</td>
</tr>
<tr>
<td>3</td>
<td>Chanel</td>
<td>I know.</td>
</tr>
<tr>
<td>4</td>
<td>Kianja</td>
<td>Cause I saved it.</td>
</tr>
<tr>
<td>5</td>
<td>Chanel</td>
<td>(silly) I low.</td>
</tr>
<tr>
<td>6</td>
<td>Kianja</td>
<td>Uh, ok.</td>
</tr>
<tr>
<td>7</td>
<td>Chanel</td>
<td>This is not 80.</td>
</tr>
<tr>
<td>8</td>
<td>Kianja</td>
<td>Yes it is 80.</td>
</tr>
<tr>
<td>9</td>
<td>Chanel</td>
<td>Just this?</td>
</tr>
<tr>
<td>10</td>
<td>Kianja</td>
<td>I did that before we left. Uh uh, line it up (mumbles) I’m saying…</td>
</tr>
<tr>
<td>11</td>
<td>Chanel</td>
<td>I like just doing this though.</td>
</tr>
<tr>
<td>12</td>
<td>Kianja</td>
<td>Hold up. So we could um, copy these (mumbles). Too many to</td>
</tr>
<tr>
<td></td>
<td></td>
<td>stack them all, ha ha ha. I never saw that there.</td>
</tr>
<tr>
<td>14</td>
<td>Chanel</td>
<td>Me either.</td>
</tr>
<tr>
<td>15</td>
<td>Kianja</td>
<td>Until just now.</td>
</tr>
<tr>
<td>16</td>
<td>Chanel</td>
<td>All right um. Line them up.</td>
</tr>
<tr>
<td>17</td>
<td>Kianja</td>
<td>Huh?</td>
</tr>
<tr>
<td>18</td>
<td>Chanel</td>
<td>Line them up.</td>
</tr>
<tr>
<td>19</td>
<td>Kianja</td>
<td>These?</td>
</tr>
<tr>
<td>20</td>
<td>Chanel</td>
<td>No line them up.</td>
</tr>
<tr>
<td>21</td>
<td>Kianja</td>
<td>Oh, um, copy, Dice R’s Us. Oh wait! (whispers) oh wait. Ok</td>
</tr>
<tr>
<td>22</td>
<td></td>
<td>umm…</td>
</tr>
<tr>
<td>1:30</td>
<td>G2</td>
<td>Now are you gonna stick with the same sizes that you did</td>
</tr>
<tr>
<td>24</td>
<td></td>
<td>yesterday, 80 and then a hundred?</td>
</tr>
<tr>
<td>25</td>
<td>Kianja</td>
<td>Uh huh. To make it go faster. We think it’s even dice.</td>
</tr>
<tr>
<td>26</td>
<td>Chanel</td>
<td>You think it’s even?</td>
</tr>
<tr>
<td>27</td>
<td>Kianja</td>
<td>Oh.</td>
</tr>
<tr>
<td>28</td>
<td>Chanel</td>
<td>I think it’s unfair.</td>
</tr>
<tr>
<td>29</td>
<td>Kianja</td>
<td>I don’t know.</td>
</tr>
<tr>
<td>30</td>
<td>Chanel</td>
<td>Oh what ever.</td>
</tr>
<tr>
<td>31</td>
<td>Kianja</td>
<td>Well just write what you think.</td>
</tr>
<tr>
<td>32</td>
<td>G2</td>
<td>What did you say about this company yesterday?</td>
</tr>
<tr>
<td>33</td>
<td>Kianja</td>
<td>We were thinking it wasn’t fair.</td>
</tr>
<tr>
<td>34</td>
<td>G2</td>
<td>Not fair.</td>
</tr>
<tr>
<td>35</td>
<td>Kianja</td>
<td>Oh wait, hold on don’t write nothing. Ok.</td>
</tr>
</tbody>
</table>
(Chanel writes on the screen)

Kianja  Ok.  What is this a hundred?

Chanel  Uh huh.

Kianja  Put this next, put it next to the zero.

Chanel  I tell you that was pretty weird it came out like the same. So.

Kianja  Copy this first.

Chanel  Oh look, three now, three has 10 (mumbles)

3:18

Kianja  Well now it looks even.  Don’t it look even there?

Chanel  Not to me.

Kianja  Because…

Chanel  Look, look how there…

Kianja  But it’s six, it’s six numbers though so, it’s not going to be even.

Chanel  But look, one is still, last.

Kianja  No it’s not. Six is the last integral, no actually three is last.

Chanel  Watch out.

Kianja  Control V.

Chanel  I know I always get something wrong.

Kianja  Well you can make this smaller.  Other side.

Chanel  Ooh.

(Students work on computer)

4:53

Kianja  All right, ok, what do we think?

Chanel  Hmm, I don’t think.  I still don’t think it’s fair, but that’s just me.

Kianja  Let’s just write we’re not sure.

Chanel  All right.

G2  If you’re not sure…

Kianja  Cause it looked fair and then it looked unfair, you know what I mean?

Chanel  Yeah.

G2  So it could go either way.

Chanel  But see, yeah look 10 out of these…

Kianja  Yeah cause I’m saying that’s 10 right…

Chanel  10, 12, and 14

Kianja  And on here it was one of the highest, and on here it was almost the highest, you know what I mean?

Chanel  Yeah.

Kianja  And this was like almost the highest and then, now it’s the lowest, so I’m thinking…(whispering as she types) we think that…

Chanel  It is fair.

5:40

Kianja  This die, no, we are unsure whether this die’s…

Chanel  That it looks (unintelligible)

Kianja  (still whispering as she types) Because in the beginning, oh I spelled it right, beginning, three was one of the highest and then it…was the lowest.

G2  Well since you’re unsure do you think you can get more evidence?

Chanel  I don’t know.

G2  Cause it’d be a good idea to make a decision about this company.
7:15 Chanel I think it’s fair.
Kianja You do?
Chanel mm hmm.
G2 Are you, how sure are you about that Chanel? Are you confident?
Chanel Yeah.
G2 Yeah?
Chanel mm hmm.
G2 So you don’t think you need more evidence?
Chanel No.
G2 Kianja what about you? You’re saying you’re unsure.
Chanel But see um, cause see at first, the um, three had one then, at the
end it lost and then in the middle it um, one and then lost again
and then twelve, six, six got more than the first than it did in the
last.
G2 So you’re saying different, different numbers won different times?
Chanel Uh huh.
G2 So everybody had a chance to win.
(Kianja types on the computer)
8:12 G2 So I guess you’ve convinced Kianja now cause you’ve changed
her mind, huh? One thing I was wondering, you know, yesterday
when Keisha and Tiffany were talking to you, they had like a
thousand trials didn’t they? What do think about that, do you think
it’s necessary to…
Chanel No I don’t think so.
Kianja I don’t think it’s necessary I mean, cause they it a thousand but we
did like, different trials.
G2 Did different numbers.
Kianja Yeah. We did, a different number of trials.
G2 Do you think you’d come up with anything different if you did a
thousand?
Kianja No.
Chanel No.
Kianja Because we had the same answers.
G2 You did have the same answers.
Kianja Yes they both said that the um ones that we did, I mean they both
said that these two here, were unfair and so did we.
G2 Right. Did you get a chance to talk to anybody who did Dice R’
Us?
Kianja Nope cause we weren’t done with Dice R’ Us.
G2 Yeah, yeah.
Kianja So (mumbles) Put that the um, little um…
Chanel Wait hold on, cause I don’t know if it’s all gonna stay in the thing.
Kianja It’s probably not, I’m gonna guarantee it’s not.
(Chanel works on the computer. The girls run 500 trials.)
9:54 Kianja Now let’s see the chart. Not that one.
Chanel See they all close.
Chanel: And now look, look at three.
Kianja: And they look even right? Mm hmm, So, yeah. All right.
Chanel: Wait put that up there and see if it’s…
Kianja: I don’t even have a thousand dollars in my bank account. I don’t.
Kianja: I think we got like 947 something like that. 950.
Chanel: (off-topic discussion)
Kianja: Wait put that up there and see if it’s…
Chanel: (Reading the frequencies for 1000 trials) 116, 167, 184, 184, 203, 180 and 150, so that means it’s second…
Kianja: Put the pie chart on there. I want to see. Yeah, still almost equal.
Chanel: I think our decision was right what do you think?
Kianja: Ok. All right. Now we gotta print this right?
G2: Yeah, I have the other um, computer setup to print after you put it on your disk. Did you add that last one to your word document?
Chanel: (uh uh – negative sound)
G2: Do you want to?
Chanel: (uh uh – negative sound) We already erased it.
G2: Oh you erased it already. Ok.
Kianja: I already erased it.
Chanel: (Chanel removes disk)
Kianja: Oh wait put that back in there.
Chanel: What’d you just save?
Kianja: The um, nah when it closes it got to make sure all of that is on there. All right, I think we’re done.
G2: Ok you know how to print it or do you need…(camera cuts to different shot)
Kianja: (in mid sentence) cause I can’t really the difference in color.
G2: Yeah you’re right.
Kianja: So I think I should label this so I can…
G2: Yeah, too bad they don’t have color printers.
Kianja: That would look pretty.
G2: Yeah.
Chanel: (In the background) We done. I’m just waitin’ for her. She writing the names or something.
G2: (Pointing to the pie graph) Oh so it goes around in order, 1, 2, 3, 4, 5, 6, right?
Kianja: Ha ha, I didn’t even realize…
G2: Yeah I didn’t realize it either.
Kianja: Oh, I did not realize that. Ok so I really don’t need to label this.
G2: As long as you know where one starts, it goes counterclockwise.
Kianja: All right, I’m done. All right let’s go. Exit that.
G2 You want to copy the problem? So you can remember these are the things you have to address on the poster.
Kianja I think we’re ready for this.
(Kianja and Chanel walk upstairs to the other classroom)
R1 You ladies are done? You got all your stuff with you? Wow. Ok, let’s put you in that back table back there and I’ll get you some of your supplies.
(R1 goes to get supplies)
R1 Did you guys finish your last one?
Chanel Our last, yeah?
R1 Cool, do you have, oops stop rolling, the sheet of paper that has the task on it, I had that downstairs for you.
Kianja Yes.
R1 Great! Ok, before you get started on your poster, I want you to review, these three questions because these are the three questions you have to answer on, on your poster.
Kianja All right Dice Depot, um, question 1, wait…
R1 And you’ll be making three different posters. One for each company. So you may want to split up to do this.
Chanel We make them on here?
R1 No, on here.
Chanel Oh.
R1 Now, on your poster…
Chanel Uh huh.
R1 We’re not putting your names on your poster, the only thing that matters is that you put the name of the company.
Chanel Ok.
R1 Ok. So yeah you’ve got three of them. And, if you, if you’re running out of room and you need to spread out onto some of those back tables, you can. And this is, this is, we’ve got scratch paper here if you want to use it to write…
Kianja We’re just gonna write it on the back of the um…
R1 That’s fine. That’s fine. Can you do me a favor and write in a black marker rather than the pencil? Let me get you a black one, because then we can see…
Chanel We got one right here.
R1 Well that ones too thick.
(R1 gets them a marker)
R2 19:00 Hey guys, is it clear what you’re doing?
Chanel Yeah.
G2 So you know…
Chanel Wait we don’t draw no pictures we just write what we got to write right?
G2 Yeah you got to write, and you can paste this on too, see this is your work.
Chanel I know.
(As she writes on the scratch paper) What evidence did you use to
(unintelligible) you tested is fair or unfair. What evidence? Such
annoying question.

(to someone else) Yeah I’m about to go to the table over there
(mumbles). What color marker you want?

I don’t care. (thinking out loud) Each of the six numbers.

(Chanel continues to work)

Here, let me see the sheet right quick Kianja?

Which one?

(Chanel reads the sheet)

Which one did we say was fair? Dice R Us, I’m putting that on
top.

Let me see this paper right here. Cause I want mine straight and I
know I’m gonna mess up.

(speaking what she’s writing) 7 out of 40, 5 out of 60, 5 out of 80,
18 out of a hundred. Ok, 1, 2, 3, 4, 300, 2 plus 7 equals 9 plus 4 is
what 13, oh wait not that’s even 11, 13, 14 plus 5 equals 19 plus 5
equals 24 plus 18 equals…

Yeah I messed up my “D”. I messed up my “D”.

Yeah 42, so 42 over 300, um, do you have a calculator or
anything? I don’t have my phone on me.

Chanel you know what 300 divided by 42 is, no 42 divided by 300
is?

No.

42 divided by 300. No the other way.

300 divided by 42?

That’s what I meant. Cause you’re taking 300 at 42
(unintelligible)

Um, I don’t know if I’m right or wrong. Too bad. (throws paper)

(Chanel works on Dice R Us)

(reading the directions) Would you recommend that, would you
recommend (mumbles) Oh, what’d you get?

.14, so 14 out of…

I got .12, ha, I was close.
256 25:55 Kianja Ok, so .14 equals one, ok. 3, 6, 4, 6, 7, and 1, 9, and 3, 5, ok. 10, 11, 14, 11, 9, 8, 2, 3, 4, 5, 6, (noise) 18, 20, 1, 2, 3, 4, 5 ok that’s the percent. 12, 14, 30, 15, 11, ok.

259 (Kianja inputs numbers into the calculator)

260 28:00 Kianja’s representation for the frequencies of all outcomes

261 Kianja Ok, so. two is 39 over 300, three, 57 over 300, four equals 76 over 300. And six, ok cleared of it. 39 divided by 300 enter, .13. 57 divided by 300, .19, 76 divided by 300, gosh (writes down .253 repeating)

266 30:30 Kianja This should be let’s see, .25, .19 and .12. Ok. (mumbles) A hundred and two, woah. So then, we’ve been doin’ this wrong. 18 and 11? I guess. Ok.

269 (Kianja continues to work by hand and then picks up the calculator to add the frequencies for each side.)

271 32:05 Kianja Um, ok, 14 plus 13 plus 19 plus 25 plus 18 plus 11 plus 100, ok. Ok, so then it would, for one it would be, 14 percent chance. two would be 13%, 19%, 25%, 18%, and 11%. All right! Ok, so,

274 where’s that paper with the questions on it? Ok what evidence do you have, ok, what do you think the chances are for rolling each of the six numbers?

277 Kianja’s answers on the Dice Depot poster – ROLE 142E – 29:04

278 Chanel Do we have to do another company like um, Dazzling Dice or something because I already started Dice R’ Us?

281 Kianja I’m doing Dice Depot.

282 Chanel Ok, whatever.

283 Kianja All right, um.
(Kianja and Chanel continue to create their posters)

Kianja (mumbling) Ok so, 4, 5, 4, 5, 4, 3, and 4.

Dice Depot table for forty trials - ROLE 142E – 30:06

Dice Depot table for eighty trials - ROLE 142E – 30:00

Kianja’s table of totals - ROLE 142E – 30:10

Kianja’s claim - ROLE 142E – 30:20

35:27 Chanel What’s the third question say?

Kianja Probability.

Let me see the third question.
Kianja: Let me do that. Cause that’s, that’s a lot, that probability thing.
Kianja: Each of the six numbers coming, the six numbers coming up.
Chanel: (reading the sheet) What do you think the chances…well, I think each are never gonna come up in this one. A even amount of times.
Kianja: Well you could write that.
Chanel: I am. Ha ha.
Kianja: (thinking out loud) kept coming up as, what? Ok, um I’m done with our…You sure you don’t need this, Chanel?
Chanel: I used it. Look at the back.
Kianja: I know but I’m saying you don’t need this anymore?
Chanel: (negative sound)
Kianja: All right next one, let’s see. Oh I need this, ok, so…1, 2, 3, 4, 5, all right, um.
(video ends)
## Appendix F

Complete Transcript of CD ROLE 140B with Images from ROLE 142E

Date: 04 August 2005    Grade 7  
Location: Washington Elementary School  
Transcribed by: Robert Tozzi  
Verified by: Barbara Tozzi

<table>
<thead>
<tr>
<th>Time</th>
<th>Speaker</th>
<th>Transcription</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>R1</td>
<td>I am very curious how you got those blue and purple markers, cause those are not the markers I gave you.</td>
</tr>
<tr>
<td>3</td>
<td>Chanel</td>
<td>I like blue and purple.</td>
</tr>
<tr>
<td>4</td>
<td>Kianja</td>
<td>Yep.</td>
</tr>
<tr>
<td>5</td>
<td>R1</td>
<td>All right, (reading the poster) I’d recommend that the dice would be bought from the store because we investigated and found out that their dice were fair, those were fair unlike the others. Ok.</td>
</tr>
<tr>
<td>9</td>
<td>Chanel</td>
<td>And then just like over here, I just said that we found out the dice were fair because at first like, um, just at first like, three, just this as an example, like three went from being the highest from the first one, to being the lowest what it is, the last one.</td>
</tr>
<tr>
<td>15</td>
<td>Chanel</td>
<td>Chanel’s table with three highest - ROLE 142E – 31:35</td>
</tr>
<tr>
<td>16</td>
<td>Chanel</td>
<td>Chanel’s table with three lowest - ROLE 142E – 31:50</td>
</tr>
<tr>
<td>18</td>
<td>R1</td>
<td>Ok.</td>
</tr>
</tbody>
</table>
Chanel: So I’m saying like, it’s a possibility that all of them can get rolled the same amount.
R1: All right, you need to write something like that.
Chanel: All right.
R1: On here because that’s not, because what, what you just said was, the evidence is posted right to my writing but you don’t tell me how this is evidence.
Chanel: added the statement under the tables - ROLE 142E – 30:40

Kianja: (mumbling) 16, daaamn. Ok, all right. Ok. 64, 65, 50 over 300 equals 0 point what, clear. 47 divided by 300 equals, I agree. (Continues working) All right, all right, all right, all right. Let’s see, maybe round this out to 16, this to 21, this to 22, this to 19, this to 53, and this to 17, maybe I don’t know, I’m not sure, clear, clear…(mumbles as she adds)
Kianja calculates percentages for outcomes of 300 trials.
6:00 Kianja   OH! Ooopsies. So, seven, oooh. Oh yes!

6:30 Kianja’s percents now total 100.

Chanel No, she didn’t care. I just asked, I asked the other lady (mumbles) I don’t know if I got (mumbles) I asked the other lady, she didn’t (mumbles)

Kianja Ok what did I write for this? This dice is definitely not fair because we ran our trials three and four…the die is not fair…

(crosstalk)

G1 Those are nice colors.

Chanel Thank you.

Kianja Cause out of a hundred trials, only 3…

G1 That’s only one poster right?

Chanel Uh huh.

G1 Just chose one, like Dice R’ Us, is it how it works, because you worked on two of them, is that right?

Chanel Huh?

G1 Didn’t you work on two, uh dice companies?

Chanel No I got to work on this, the, second one next then…

G1 Oh there is a second poster.

Chanel Yeah I got to work on Dice Depot.

G1 Oh I see.

Kianja …times did five appear. It did not appear very much in any of the other sets either. One equals sixteen percent. Two, 16%, 21%, 22%, 19%. All right I’m done! Ok here.

8:35 Kianja’s evidence.
Chanel: Yeah.
Kianja: When you do this one and I'll do this one, the one that I just write.
Chanel: Which?
Kianja: A buy, a buy. I love this color.
Chanel: Yeah I’m puttin’ blue and red.
Kianja: Purple’s my favorite color.
Chanel: Purple? I just picked it.
Kianja: Huh?
Chanel: I just picked it.
(Both students begin to work on their posters)
10:00 G2: Wow that’s dazzling all right.
Kianja: Dazzling Dice.
(Off-topic discussion)
Kianja: Question 1, I think I’ll answer it in bold.
Chanel: What is question 1 again?
Kianja: Where’s that question I gave you?
Chanel: You got the paper. Let me see.
12:00 Kianja: That this, dice, what does it say?
Chanel: Uh would you recommend that the dice be purchased from the company you investigated.
Kianja: That this, this dice, be purchased, be purchased from this company. Ok, all right, ok.
(Both students continue to work on their posters)
Kianja: 3 times this…
Chanel: (unintelligible)…What do the (mumbles)
Kianja: 16:38 (working on her poster) our, what’s the word I’m looking for? Discoveries! The…
(Off-topic discussion)
Kianja: According to our discoveries these are percents of how…
17:35 R1: Did you guys do Delta’s Dice?
Chanel: Um no we only did Dice Depot, Dazzling Dice and (mumbles).
R1: Ok, you got Dazzling Dice, I’m gonna call your poster number 3.
Chanel: Sixteen percent. Nineteen.
19:15 Chanel made a table of Kianja's percentages for Dice Depot
Kianja’s percentages for Dazzling Dice

Kianja’s poster for Dazzling Dice – ROLE 142E – 21:52

Question 1: No, I would not recommend that this dice be purchased from this company.

Question 2: The dice is not fair because when we did our set of 100 trials, only 3 times did 5 appear. It did not appear very much in any of the other sets either. In our set of 100 it appeared 33 times, 60-1 time, 80-9 times, 100-3 times.

Question 3: According to our discoveries these are the percents of how much each number would appear.

1 - 16%
2 - 21%
3 - 19%
4 - 19%
5 - 20%
6 - 5%

Makes this dice unfair.
R1 Chanel I am just going to write at the very tip-top of your poster…
Chanel Ok.
R1 That this is going to be poster number 9. There we go.
Chanel And there’s one up there too.
R1 Oh, ok, um, yeah I want to make this one number 10.
G1 Is that yours Kianja?
Kianja This is ours.
(Chanel continues to work on her poster)
G1 You can put a…
Kianja What does a die look like?
G1 Do they put a comma after this?
Kianja Huh?
Chanel What are you talking about put a comma after…
G1 Is there a comma or something…
Chanel After no?
G1 Is that right? No? Something like that.
Kianja Thank you.
G1 Just trying to help.
Kianja Um, what does a die look like?
Chanel A dice?
Kianja Yeah. I’m trying to think…
Chanel Square.
Kianja Ha. No I mean.
Chanel Hold on wait.
Kianja A four sided I mean a six sided die because you know I’m a little
(unintelligible).
G1 This is a fair one?
Chanel No.
G1 Not, not fair.
(Off-topic discussion, graduate student/researchers discuss the
posters in the background)
G1 Oh, very nice. Ok I thought it was based on one sample.
G2 No, no she pulled all the data.
G1 So she’s basically saying (inaudible)
(Off-topic discussion)
Kianja I think it’s beautiful, what do you think?
G2 Very nice, I like the way you did that 5.
Kianja I do it too. Yeah.
G2 Looks very authentic.
Kianja Yeah.
Chanel Ha ha.
Kianja (unintelligible) and I put my little proof for the five (inaudible).
Kianja’s hand written proof for the five – ROLE 142E – 22:44
Kianja’s computer-generated proof for the five – ROLE 142E – 23:19

(Off-topic discussion)
(R1 mentions there is 10-12 minutes left.)

Kianja And then we come back we got to share?

(More R1 dialog)

Kianja Huh, oh my God!

Chanel Uh our poster’s almost, our poster’s almost done.

Kianja Ok I think we’ll be done.

(Both students continue to work on posters)

Chanel Give me that black marker.

30:08 R3 How are you guys doing?

Kianja We’re doing fine.

Chanel Fine.

R3 Yeah.

Kianja Yep.

R3 I don’t recall, which, which company did you, makes a fair dice in your opinion? Dice R’ Us, uh huh, and these do not?

Chanel No.

R3 The ones you’re working on now.

Kianja Right.

R3 How badly, how unfair are they?

Kianja Well this one is so unfair that it’s only 5% chance that you’ll get, you’ll roll a five.

R3 Is that right?

Kianja Yes.

R3 5% chance, where do you get percentages from?

Kianja Um, well…

Chanel The computer.

Kianja This what I did, um, where’s my paper I put all that stuff on? Oh here it is, well to get my percent right, you know how we did different trials?
Kianja: Like we did sets of 20, 40, 60, and 80 and then a hundred?

R1: Do I have all of yours labeled?

Kianja: (agrees, yes), um I took the numbers like say for five, right. And say I’m trying to find a percent for five.

R3: Right.

Kianja: So I would take how many out of 5 right…

R3: Um hm. (agrees)

Kianja: How many out of this 20 so that’s two five’s out of the twenty, they came up, and then two five’s out of the… I mean, yeah, two five’s out of the forty came up, so that’s 4, out of what, 60?

R3: Ok.

Kianja: Well, first I added up all of these, the 20 the 40 the 60 the 80 and then 100, and I have 300.

R3: Ok.

Kianja: So the denominator would be six, I mean 300.

R3: And the 300 represents what again?

Kianja: The total of trials that…

R3: Ok.

Kianja: The total numbers of trials that I did.

R3: Uh huh.

Kianja: And then that was 300, and then…

R3: And by the way is trials the same as throwing the dice?

Kianja: Each time we threw the die.

R3: Uh huh that’s a trial.

Kianja: Yes.

R3: Ok.

Kianja: And then after I did that, I added up all the numbers, all of the numbers, wait, where’s the other one? Right here I put up all the numbers, this one was from twenty, forty…

R3: Uh huh.

Kianja: And then sixty and then so on.

R3: Uh huh.

Kianja: Until we got to a hundred.

R3: But you went up by 20?

Kianja: Twenties yeah.

R3: Is that right? So that’s…

Kianja: When we did our sets of trials, we did, we went up by 20.

R3: Uh huh.

Kianja: So we did 20, 40, 60…

R3: Uh huh.

Kianja: And then when I added all these up I got 16.

R3: Uh huh.

Kianja: Then I did 16 divided by a hundred.
208

R3 Uh huh.

Kianja I mean 300.

R3 Uh huh.

Kianja And I got my percentage of .053.

R3 And...

Kianja I rounded it off to 5%.

R3 Ok, mm hmm. And with, where’s the calculation, let’s, this calculation?

Kianja Right here.

R3 I see. Uh huh

Kianja five each time I had, this is the number of times that five showed up out of the 300 runs that we did.

R3 Right.

Kianja So, 16 out of 300, and then this is my percent that I did.

R3 I see, now what is, open, what does that mean?

Kianja Huh?

R3 The 5% what does it mean?

Kianja That’s how likely it is for you to roll a five. 5% chance that you can roll a five.

R3 Ok.

Kianja Whew all right.

R3 I understand yeah, so that’s the least fair?

Kianja Yes.

R3 Um hm. And what about, Dice Depot?

33:23 Kianja That one was, which one is that? Unfair?

R3 That’s the Chanel’s working on now.

Kianja That unfair Chanel?

Chanel What this?

Kianja Yeah.

Chanel Yeah.

Kianja Um...

R3 In what ways is it unfair?

Kianja In what way is it unfair?

R3 Uh huh.

Kianja Because the same numbers kept coming up, like um, the same numbers kept coming up ahead, and the same numbers were in the bottom. But in that one, that’s why we called this one fair because some numbers came up in, in the lead certain times and then others num, other numbers came up in the lead other times, so it showed that there was an equal opportunity for all of them to be in the lead.

R3 I see, whereas in this case, which one...

Kianja It wasn’t that way, 3, 4, and 5 kept coming up the highest numbers and then 1, 2, and 6 kept coming up the least numbers.

R3 I see, uh huh.

34:14 Kianja So we thought that was unfair.
And the percentages that Chanel has there, you calculated the same way?

Yes, I did it the same way. We didn’t do percents for that one because we thought it was fair. So I didn’t think we needed to show percents.

Oh I see. So you used percents to show why...

It was unfair.

You determined these were unfair.

Yes.

Um hm.

I am just curious Kianja, because I was watching you do it, I noticed that you did many times right? You, ha…

What’d you say?

You did trials many times?

Yes.

And I noticed that, why did you decide to add up all of those things and then get…

Because I didn’t really do like a (inaudible) because if I did, if I did it that way, if I added them all up, then I can see the overall percentage. You know what I mean?

Uh huh, ok. And that you…

And then that would help with the decision of how fair and how unfair it is. If we knew overall percentage, I mean we can’t just take like, well, this one it was fair because such and such happened and then this one, it was fair because such and such happened but this one, it was unfair because, this didn’t happen.

Ok.

We had to put it all together so that we could make one conclusion.

Uh huh. I see. Ah, I see I see. And how confident are you on your, in what you did?

Oh I’m confident.

And what makes you confident about it?

What makes me confident um, because um…

That you answered…

Each time we, um, when we did it?

Uh huh.

Well I’m confident that it’s unfair because if, I mean if I didn’t do the percents…

Uh huh.

I would still be confident that it was unfair because each time we did it, the numbers still didn’t add up.

Uh huh.

Like, like five, it was always the lowest number, it only had like a few numbers. Like, I mean, it was one time when it had eight, like the other numbers had twenties and stuff like that.

hmm.
Kianja: So, it really didn’t matter that it had 8.
G1: Uh huh.
Kianja: And all the rest of the times it had 1 through 3, it didn’t have like 5, 6, you know, 15 maybe, it didn’t have any of those numbers. It had 1, 2, 3, and then that one time it had 8.
G1: Um hm.
R3: If, if you could just with one trial, is there just one trial that you could have used, to determine whether it was fair or unfair?
Kianja: (nods yes)
R3: Which one?
Kianja: This one.
R3: What is that one?
Kianja: Because this is a hundred, this is a trial, this is the set of a hundred trials that we did.
R3: Uh huh.
Kianja: And I’m thinking that we can um, determine that because, all of these other numbers like, 6, I mean one has 13, two has 22, um three has 20, four has 25 and then five has 3. I wrote, we are now not show, we are now sure that this die is unfair because, in the hundred trial we ran, in the hundred trial, we ran, only 3 times did the five show up. I don’t think that’s very likely on a fair die. I mean, you would get at least ten tries, I mean.
R3: Hmm.
Kianja: I mean I think that you would at least see five at least ten times, you wouldn’t see it like, 3 and then all the other numbers you know higher than that.
R3: Hmm, hmm hmm, so that one really helps you see…
Kianja: Yeah that’s why I put it on here as an example (inaudible)
G1: You did different sets right? This is a hundred, right?
Kianja: Yes.
G1: You did other ones that are not a hundred and you picking the hundred, why, why this one? Why the hundred?
Kianja: Because I think, I think this is, this is more clear, as too…
G1: In which sense? In which sense is it more clear?
Kianja: I think this is more clear about how…
Chanel: The fairness.
Kianja: Yeah.
G1: Say it again?
Kianja: Like the fairness of it. I think it’s more clear because, if you look on this pie chart, you see this small little wedge…
G1: Ok.
Kianja: And it’s suppose to be, how many times the five showed up.
G1: Yes.
Kianja: And you see these big wedges and you’re like, why is it this much and then, all of these are so much bigger.
So you’re saying that this is sort of like, close to this, to what you’re saying here? Because this is overall...

Yeah, this is closer to what I’m saying here.

Like the 5%, you know you can tell like, that can be like, that can be represented by, 5%.

Uh huh.

So…

And this was the highest, uh, trial that you did?

Yes.

The hundred was the highest?

Um hm.

Ok, I see. Is there a slight chance that somebody did it, right, say for 200, right, and may have gotten a slightly different result.

If they do, wait, wait, say that again?

If somebody, uh, did it again, a trial say for another with another number, you think they could have gotten a different result?

Because some people, people could have continued to do trials trials trials, right?

I don’t think so…

Uh huh.

Because as we did it…

Uh huh. What makes you…

We did it more and more and more…

Yeah.

Like I remember this one was the one that got lost…

Yeah.

On the computer like because our computer wasn’t working.

Right.

And this one got lost so all our information was lost and we didn’t have anything.

Right.

And when we did it over I mean we had the same conclusions and stuff like that but different numbers.

Right.

And when we did it over it was the same thing.

Ok.

It wasn’t really the, the numbers didn’t really change.

Right, right, right.

You know what I mean?

So you’re saying that basically, it, if somebody did most likely they would be getting the same kind of…

Yeah they would get, they would be getting the same thing.

All right.

(Kianja goes back to working on her poster)

You know I had one other question.
R3 May I ask you?
Kianja Yes.

R3 Um, ha, Chanel says no. You said to me that you could, that you’re using this one to show, why, this company’s dice is unfair…
Kianja Yes.
R3 And I understand…
Kianja Oh wait you weren’t here when I said the thing about it can, it, it um, kind of, relates to this right here, even though it’s not the same data.
R3 Well, no maybe I wasn’t. Why don’t you say it.
Kianja Um, I said that this right, this can be like a, um, even though this is not the same data, it, it can be…
R3 This being which?
Kianja This right here is not the same data as this…
R3 Ok.
Kianja Because this is just a hundred and this was taken out of the 300 that we added up and all of that.
R3 Uh huh.
Kianja So I was thinking that this…
R3 Uh huh.
Kianja Can show representation of this, like show, another part of the data.
R3 Ah, ok.
Kianja Like you can relate…
R3 So you anticipated my question. I was wondering whether or not this would be sufficient, data for you to use, to show that this company’s dice is unfair. Did you really need to, go through the act of accumulating all of the different trials?
Kianja Like you can’t but, you, I mean I thought that, you know you might need a picture.
R3 Uh huh.
Kianja So this circle graph, this pie chart…
R3 Uh huh.
Kianja Could sort of be a picture, for you to look at…
R3 I see.
Kianja And relate it to the percents that you put in there.
R3 Right. So which, which one of these representations are you asking the viewer to focus on? Which one is, is providing the evidence for you?
Kianja This.
R3 That one, uh huh. This one is, this one is providing the…
Kianja The percents.
R3 Uh huh.
Kianja And I’m done.
Appendix G

Complete Transcript of CD: ROLE 136C
with Synchronized Chris/Jerel Monitor CD 1 of 3

Date: 03 August 2005 Grade 7
Location: Washington Elementary School
Transcribed by: Doreen Nuzzolese
Verified by: Barbara Tozzi

<table>
<thead>
<tr>
<th>Time</th>
<th>Speaker</th>
<th>Transcription</th>
</tr>
</thead>
<tbody>
<tr>
<td>00:21</td>
<td>R1</td>
<td>You shouldn’t have anything, but if you have, you need to close what you’re doing. The slow circle, again, ready go, slow circle. All right and that should be good. Okay, now on your desktop, there should be a shortcut to Probability Explorer. And its the red and the yellow circles that say H &amp; T on them. Can everybody find that? If you double click it, you should get here, where you’ve got this little screen and you click on “Let’s Go” When you come up to this next one it says, “What do you want to do today?” I want you to choose the option for dice. (inaudible background voices)</td>
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<tr>
<td></td>
<td>R1</td>
<td>We’re going to do six-sided dice. Does everybody have that selected? (inaudible background voices)Yes.</td>
</tr>
<tr>
<td></td>
<td>R1</td>
<td>Okay. And then in this next screen if you say okay, this next screen says how many at a time. That’s how many die you want to roll at a time. We’re only going to roll one die at a time. Okay, so the option should already be selected for number one. You just have to select okay. So now, hit the run man once. You should get a result of rolling a die. So I got a six. Who else got a six? Did anybody else get a six?</td>
</tr>
<tr>
<td></td>
<td>J &amp; C</td>
<td>No. No.</td>
</tr>
<tr>
<td></td>
<td>R1</td>
<td>You guys got a six back there. About half of us got a six. The rest of us didn’t. And if you hit run again, this will show you what you roll the next time. I got a five again.</td>
</tr>
<tr>
<td>2:10</td>
<td>Jerel</td>
<td>I got a 3.</td>
</tr>
<tr>
<td>2:22</td>
<td>Chris</td>
<td>Well, not again, I got a five. So some of you were talking about what you might expect if you roll a fair die. What I would like you to do…</td>
</tr>
<tr>
<td></td>
<td>R1</td>
<td>…is just explore a little bit. You can choose how many times you want to roll the die. You can look at any of the tables or the graphs that you want. And just see what happens when you do roll a fair die. We do know that this die is fair.</td>
</tr>
<tr>
<td></td>
<td>Jerel</td>
<td>Dang. Six beast it.</td>
</tr>
<tr>
<td></td>
<td>R1</td>
<td>You could do, you could explore however you want.</td>
</tr>
</tbody>
</table>
First experiment – 10 trials.

Second experiment – 100 trials.

Six and five

Yo, six is beasting.

It’s the same thing. It’s the same thing as two. (Chris points out that the results for six are the same as for two.)

Third experiment – 100 trials again.

Yo. Dang, six sucks.

Why do you think two is going to come out more? It’s your favorite number?

It look even.

We got to run it more times. This sucks. Yo, red is beasting.

For 500 trials, the frequency for red (five) is higher.
It turned back to five.

Red is five.

(background) If you only roll it six times, what do you think is going to happen?

It’s about to lose to six. Oh snap!

Two is beasting. Two and four, oh and 5.

So what are you guys noticing?

That two and five

and six

and six

Are the even numbers

Yeah.

Are usually even?

They usually come out more.

Yeah.

They come out more?

Yeah.

Two, five and six?

Uh-huh

Six doesn’t look like its coming out.

Before it did.

Look! Then it was all even.

Two, three and five.

What are you looking at that’s telling you even?

The pie graph.

The pie graph? I think I agree with you when I look at that pie graph. But when I look at the bar graph, it doesn’t look even. Why is that?

I don’t know. Because its bars.

I don’t know. Because it’s a bar? Or maybe we can’t see the colors under it. Maybe the colors are going under it.

Comparing the pie graph and the bar graph.

Under what?

Like when it pushes it, maybe the colors go under it.

No, actually they don’t. It just rescales it. But see now, I think that pie graph looks even. And you’re only at about 200.

No it don’t look even to me.

Doesn’t look even to you?

Yeah it does.
Jerel No, gray is losing. Nuh-uh, gray is losing!

Chris No blue is.

Jerel Look! Gray is losing! Listen to your project. (inaudible background voices)

Chris Was that 500 times?

Jerel Yeah. You like Ben Franklin.

Chris Who?

Jerel You. Let’s play some more. Cause we ain’t got no special surprises.

Chris So what’s going on here? We’re saying that this die is fair.

Jerel I think it’s fair.

Chris We’re going 1500 times just to tell you. Watch, how much you want to bet?

Jerel You clicked it again?

Chris Yeah (laughing) All right. See they’re all even.

Six is losing. To every color. Six, sorry.

Chris But they are like the same, you see?

Jerel Dang, six came back!

Chris Look at these and look at that one. Look how it changed. How many times did you click it.

Jerel I don’t know. It's going to go on to like 3,000.

Chris If this one stops, six will catch up. I told you 4 and 3 come up the most!

Jerel You clicked it again?

Chris Yeah (laughing) All right. See they’re all even.

Six is losing. To every color. Six, sorry.

Chris But they are like the same, you see?

Jerel Dang, six came back!

Chris Look at these and look at that one. Look how it changed. How many times did you click it.

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Six is losing. To every color. Six, sorry.

Chris But they are like the same, you see?

Jerel Dang, six came back!

Chris Look at these and look at that one. Look how it changed. How many times did you click it.

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Chris But they are like the same, you see?

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Chris Look at these and look at that one. Look how it changed. How many times did you click it.

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Chris If this one stops, six will catch up. I told you 4 and 3 come up the most!

Six is losing. To every color. Six, sorry.

Chris But they are like the same, you see?

Jerel Dang, six came back!

Chris Look at these and look at that one. Look how it changed. How many times did you click it.

Jerel I don’t know. It's going to go on to like 3,000.

Chris If this one stops, six will catch up. I told you 4 and 3 come up the most!

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Chris But they are like the same, you see?

Jerel Dang, six came back!

Chris Look at these and look at that one. Look how it changed. How many times did you click it.
And I want to hear what are some of the things you found when you were rolling this die. I mean, I told you this die was fair. Do you think I'm right? Do you think that this die that you are rolling on computer here is fair? Jerae?

Jerae Hmm?

Did you have your hand up? Yeah?

Um, what I know is um, is that one was the most and it came over and over again. And the other number was going to go over but actually one was in the lead by a lot.

No.

One was in the lead by a lot? Wow. How many times did you roll the die?

Four is in the lead!

So what do you think that tells you?

(whispers) Stop typing!

You don’t know? What did everybody else find out? Do you think that this die that I have in here is fair?

Yeah! Yeah!

Why?

Because none of them is like beating another number by a lot and that the percents are very very close. It's only two percents in the seventeens, the rest is in the sixteens.

Sample size 4500 trials.

Okay, so two percents are in the seventeens, and the rest in the sixteens.

And 4 is winning.

We ran it four thousand five hundreded times.

4,000 times! What did you guys find out, do you think the die is fair?

Um, Yeah.

Why, Chanel? You think its fair because two won. So what makes that mean that its fair.

I didn’t think that its fair because all the numbers are really close

Really close? How many times did you run it?

Twelve.

Okay, 12, 24 and 30. Okay. Lauren, Justina, what did you guys find out? You think this die is fair? You think it is? What told you that its fair?
218

175 Jerel Let me get some.
176 R1 They all have the same amount of chance? Terrill and Danielle?
177 What do you think about your die? Is it fair?
178 Chris We still got a long way to go. (inaudible background voices)
179 15:04 R1 And what about in the back, Tiffany and Keisha? Do you think your
die was fair? And why do you think so?
180 Girl Yeah, its fair.
181 R1 You think its fair?
182 Girl Yeah.
183 R1 Why? What tells you that its fair?
184 Girl Because (inaudible)
185 Jerel I’m trying to looks at your pictures.
186 R3 What about so far, what do you guys have?
187 16:09 R1 So all the counts are in the 300s.
188 R3 Do you agree with what she said?
189 Girl Yes
190 16:31 R1 Okay, so what I’d like you to do, up on your toolbar, see if you can
find the icon that looks like a balance scale. Yes, Jarae.
191 Jarae On the second try all of them are over, ahead of one.
192 R1 Wow, and before you said one had both. (inaudible background
voices) When you tried the second time did you erase your old data?
193 You’re starting from fresh? Got it. What do you think that tells you?
194 Jarae Sometimes you don’t get it on your first try.
195 17:51 R1 Okay, all right. So I want everyone to take a look, thank you for
sharing that, Jarae, take a look at your toolbar and find the icon that
looks like a balance scale, there’s two red pans. And I want you to
click it, and open it. Up on the toolbar, there’s an icon that looks like a
balance scale. Did everybody find that?
196 Jerel Yeah
197 R1 All right, so, could you tell me what you think these numbers mean?
198 This is called the weight tool.
199 Chris Oh! How many of each are...how many of each there are.
200 R1 How many of each there are.
201 Chris Its like a marble bag. Like how many you put in.
202 R1 Okay, so then interpret for me what you think those exact numbers
mean.
203 Chris It means that there’s only one one dice, once two dice, one three
dice...like that.
204 18:28 R1 Okay, anyone else have another interpretation of that?
205 Jerel Its the fraction of the dice. Oh, never mind.
206 18:55 R1 So what would you think would happen if I’m going to go with
Chanel’s special number to be a number two. So Chanel was
interested in the number 2 earlier. Let’s say I change the number two
to be three. Can everybody do that for me?
207 Jerel Turn it to three.
R1 Change the number underneath the icon for two, change that to a three.
What do you think that means?
Jerel That it has three dice.
Kianja There are three sides that have two on it.
R1 There are three sides that have two on it.
Student I think the same thing.
R1 Okay, you think the same thing? SO what do you think is going to happen now, if we go back and we roll the die.
Kianja You’ll get more twos.
R1 You think you’re going to get more twos?
Jerel Yea, me too!

20:08 R1 How many people think you’re going to get more twos now? Terrill and Danielle Do you guys think we’re going to get more twos? Less twos or about the same? So let’s go ahead and try this. Hit “x” to close the weight tool. Clear whatever date you have. So clear out any of your data with the erase tool. And now do some exploration and see if we do indeed get more twos.
Chris We’ll run it like a thousand times.
Jerel Do you know how many times I clicked it? Like seven!
R1 (background) You guys are getting excited about something over here.
You only ran it 50 times?
Jerel No! NO I’m putting it in. Man, that’s cheating.
Chris Dang! I like pacman.

21:55 R1 Is everybody getting a lot of twos?
Chris Yeah! We put ten twos!
R1 You got ten twos? Out of how many?
Chris No, I mean we put ten twos.
R1 Oh, you put ten instead of two. You’re getting a lot of twos I bet.
Chris We got 143 two out of 210.

(syncC/J1-21:26) Trying to get Pacman.

So do you think this die is fair?
Jerel No.

No, Jerel says no, that this die is not fair. All right, so what I want you guys to do is to stop whatever you’re running, and to go ahead and clear your data and then we’re going to come back up here. Um, how much time do we have left on film? Nine minutes. Okay. Um.
Jerel: Two got beat! Even with three! Even with ten sides of the two. Two got beat!

(syncC/J1-22:26) Adjusting the weights so that four “beats” two.

R1: 210 times? So when you ran it 210 times, even though 2 had a lot of weights, 4 was able to beat it?

Jerel: Yeah.

Chris: Yeah, because it has more fours.

R1: Did anyone find anything interesting when they were looking at their twos? Yes, Tiffany, what did you find? The two was always higher than…than ten?

Jerel: Than the other number?

R1: Okay, and how many times did you run it? 75 times. The two always came higher than ten. Okay.

R1: Wait, we ran ours 210 times, and then 4 beat two.

R1: Yes, but I already know why four beat two. Because you already told me. You did something in the weight tool.

R1: (laughing) Okay, I want you to go ahead and clear your data and we’re going to go ahead and take your snack break. So I’m not sure who’s in charge of snack time this time.
Appendix H

Complete Transcript of CD:  ROLE 137C
with Images from Chris/Jerel Monitor CD 2 of 3

Date: 03 August 2005   Grade 7
Location: Washington Elementary School
Transcribed by: Barbara Tozzi
Verified by: Doreen Nuzzolese

<table>
<thead>
<tr>
<th>Time</th>
<th>Speaker</th>
<th>Transcription</th>
</tr>
</thead>
<tbody>
<tr>
<td>00:00</td>
<td>R1</td>
<td>Okay, I just gave you all a piece of paper and I would like a volunteer to read…Many students volunteer.</td>
</tr>
<tr>
<td>3</td>
<td>R1</td>
<td>All right, Lorrin. Let’s follow along…let’s follow along as Lorrin reads this, all right.</td>
</tr>
<tr>
<td></td>
<td>Lorrin</td>
<td>(reads Schoolopoly task) Plainfield Middle Schools are planning to create a board game modeled on the classic game of Monopoly. The game is to be called “Schoolopoly” “Schoolopoly” – that’s a word I made up.</td>
</tr>
<tr>
<td></td>
<td>Terrill</td>
<td>You typed this.</td>
</tr>
<tr>
<td></td>
<td>R1</td>
<td>I did.</td>
</tr>
<tr>
<td></td>
<td>Lorrin</td>
<td>(continues reading) and, like Monopoly, will be played with dice. Because many copies of the game will be sold as a fundraiser, several companies are competing for the contract to supply dice for Schoolopoly. Several companies, however, have been accused of making poor quality dice. These companies are to be avoided since players of Schoolopoly need to know that the dice they are using are actually “fair.” Each dice company has provided a sample die for analysis. You will be assigned one company to investigate.</td>
</tr>
<tr>
<td>1:40</td>
<td>Terrill</td>
<td>I have a question. Are you positive that die is spelled like that?</td>
</tr>
<tr>
<td></td>
<td>R1</td>
<td>Am I positive that…</td>
</tr>
<tr>
<td></td>
<td>Terrill</td>
<td>Die is spelt like that.</td>
</tr>
<tr>
<td></td>
<td>R1</td>
<td>Die? D-i-e.</td>
</tr>
<tr>
<td></td>
<td>Terrill</td>
<td>As dice. I’m not talking about dead die, but I thought it was d-y-e.</td>
</tr>
<tr>
<td></td>
<td>R1</td>
<td>Actually, there are two different ways to spell...(She writes on the board.) There is die or dye. This one (as she points to d-y-e), can anyone tell me…</td>
</tr>
<tr>
<td></td>
<td>Many</td>
<td>That’s a color. Like color dye.</td>
</tr>
</tbody>
</table>
| | R1 | This one (pointing to d-i-e) is, if you talk about, that somebody is going to die, you spell it that way. If you talk about the idea of coloring something, you are going to dye something, dye it a certain color – you spell it like this. And when we talk about dice, dice is a plural form, meaning more than one. So if I only talk about one, I talk about a die. And we spell it like that (she points to d-i-e). It’s a good question, because oftentimes in the English
language, I’m sure you guys have found this, that two things that sound exactly alike are spelled very differently. So that’s a good question. Okay, so first of all let’s talk about what Lorrin just read to us. Can somebody recap what’s going on here with the Schoolopoly? Kianja?

Kianja I don’t know. Something about the companies…some of the companies are making the die unfair. And we want the company that makes the die fair, and we have to try to figure out which one is not fair and which one is fair.

Terrill You can’t make a die unfair because a die has six sides and six different numbers, so when you roll it six different ways (inaudible)

R1 But I think earlier some people were talking about ways that the die could be unfair.

Chanel There is ways that the die can be unfair. If you put two of the same numbers on the dice.

R1 Put two of the same numbers on the dice. I think somebody was talking about the corners…if the corners weren’t particularly straight. If they were rounded or something. Lorrin?

Lorrin Oh yeah, I was gonna as you. Have you ever seen a die that um, has more than six sides?

R1 Yes I have (nodding her head). I have. They make die, they have die that have a hundred sides on them. I personally…

Student Is it like flat? Is it flat though, like the side is flat.

R1 The side is flat but because there’s a hundred of them, the sides are very small.

(R1 nods her head and looks around at students.)

R1 So, I actually have a collection of different kinds of dice and so I’ve got lots of different kinds. Some of them have way more than six sides on em. So, we’re kind of worried that some of these companies may produce unfair dice. And, since your school wants to create the Schoolopoly game, we have a really important business decision to make because these companies want to provide the dice for the game, but if they’re unfair, we don’t want those die in our game (pauses, looks around at students). So, we want to try to find out, we need to make a recommendation to the School Board as far as which company they should go ahead and give the contract to.

Chris Just get all the dice.

R1 Just get all of the, all the companies? Why would we get all the companies?

Chanel There’s more money.

R1 So, Chanel, you’re saying that it would cost more money to have all of them, to have all the companies. So we want to find the company that’s the best, to make a recommendation. So we’ve got six different companies that are trying to get this contract. And
they’re called Perfect Polyhedra, Dice ‘R Us, Delta’s Dice, Calibrated Cubes, Dazzling Dice, and Dice Depot. So, what we’re gonna do is…

Student What’s a polyhedra.

R1 What is a polyhedra? That’s a good question.

Chanel A square-sided figure.

R1 So Chanel said that it’s a square-sided figure. (Another student notes it’s a polygon.)

R1 It’s a polygon. (other student comments) What is a polyhedra?

(more talk off topic) Okay, so the question was what’s a polyhedra.

So, a die is an example of a polyhedra. (Student mentions it’s a polygon.)

R1 Go ahead, let her finish. Go ahead, it’s a polygon…

Tiffany I was gonna say a hexagon has six sides.

R1 Okay, a hexagon has six sides. A hexagon is an example of a polygon. So, this is an example of a polyhedra (holds up die). This is a polyhedra, but there are many different kinds of polyhedra with different numbers of sides. This is a polyhedra that we typically call a cube. All right, we typically call this shape a cube because it’s made up of squares. And we’ve got, each side is a square and then we have six of those squares kind of folded up together to make this cube. And it’s an example of a polyhedra.

Some of you were asking – let’s take the example of a hexagon. If I took different hexagons and tried to fold them up together (uses hands to make a shape) I could make a polyhedra where each of the sides looked like a hexagon rather than a square. (off topic question)

8:05 R1 And so, there’s lots of different kinds of polyhedra; this just happens to be one example of it. All right. Now, when you go, when you go to explore your companies, I’m going to give you your assignments. I’ve assigned different companies to different groups and you’re gonna actually, you’re gonna actually explore more than one. You’re gonna have two or three to explore. Okay. And what you’re gonna do, because each of us are gonna have two or three to explore, after you have some time to explore um the different companies to find out whether you think that company produces fair dice, you’ll get a chance to get together with another group to see what they found out. Because a different group may be exploring your same company. So you’ll have a chance to talk, and say “oh, what did you find out?” All right. And then eventually, eventually, each of your groups are gonna have to make a poster that tells us something about the companies that you were assigned. And you’ll have to make one poster per company.

We won’t get to make posters probably, well, we definitely won’t get to it until tomorrow. But, it is very important that you know – take a look what’s in that box. Take a look what’s in the box down
here. These are the questions that you’re gonna have to answer on your poster. So it’s important that you keep those in mind when you’re doing your exploration. So, Terrill, go ahead. Tell me number one.

10:25 Terrill Would you recommend that dice be purchased from the company you investigated?

R1 Okay. So that’s the first thing you’re going to have to answer on your poster. Do you think we should buy die from this company?

Girl Number 2 – What evidence do you have that the die is fair or unfair?

R1 Okay. So what evidence…you’re gonna have to show us some evidence. All right. Jerae, number three.

Jarae What do you think the chances are for rolling each of the six numbers.

R1 Okay, so that’s the third question you’re gonna have to answer on your poster. Okay, Kianja go ahead, read the bottom.

Kianja Use Probability Explorer to collect data from simulated rolls of the die. Copy any graphs and screen shots you want to use as evidence and paste them in a Word document. Later, you will be able to print these to use on a poster. You will give a brief presentation pointing out the highlights of your group’s poster.

R1 Okay. So tomorrow, when we actually do presentations, it will be a little different because you’re gonna have posters. All right. So we’re gonna have posters hung up here. And, it’s gonna be very different because we’re gonna actually have somebody from the School Board come in, and you’ve got to tell them what you found out about these dice. (Positive response from students.) So, you want to make sure that what you write on your poster, that you’re ready to make that presentation tomorrow. Okay. Does anybody have any questions about this?

Jarae I do.

R1 Who said that? Tiffany? Sombody said that. Jarae. Listen up – Jarae has a question.

Jarae Do you have to do a poster?

R1 Yes you do.

Jarae And, if you don’t have a partner do you have to do it by yourself? Or will you get somebody to work with?

R1 Um. You will get somebody to work with. All right, so I have got the assignments of which companies you’re gonna be exploring. So, (begins to hand out papers) Chanel and Kianja, this is your assignment. Those are the three companies you’ve got. Chris and Jerel, Lorrin and Justina, Jarae, Terrill and Danielle, and Keisha and Tiffany. (Students compare companies.)

Girl Do we got both of these.
Yes, you’ve got both of them. Okay. So, let’s go ahead and go back to the computers and I’m going to show you where to find these files. Okay.

All right, go. (synchronizing computer screen shots) And we’re okay. Now, all of you on desktop let’s go back into the folder of probability files. And there is a folder called Plainfield Schoolopoly. Open up that folder, and in that folder you’ll find all six companies. You only need to look at the ones that you were assigned to. You can do them in any order you want. It doesn’t matter. But remember, you want to have a Word document and you want to put things in your Word document. And you want to keep it organized so you know which company you’re collecting data from. So you can organize your Word document any way you want but make sure you always know which company your data is coming from. You can use separate documents, you can use one document; I don’t care.

Can we write on paper?

Um, you can write on paper but you want to think about the kinds of things you want to print because you’re gonna get to print stuff later. But you can certainly use paper.

I think all of them are…like three, four and five, got more, well that should be four. (inaudible)

So which company are you guys doing first?

Dice R Us.

Like Toys R Us.

Like Toys R Us.

We were thinking…right now I’m thinking that four got two…four and three.

Four

and three

Yeah I think they both got two in them.

You think they both have both have two? What do you mean by they have two?

Three, four. I think two, three and four. I don’t know. I’m confused. No, three and four I think they both have two in them.

You mean they both have too much? Is that what you said?

nods his head.

(to Chris) You said three and two?

I said three and four. Two, three and four; two, three, four and five.
Jerel: Let’s look at this one. (they begin to look at Delta’s Dice)

Chris: Run 500, twice. See, it’s still running.

Jerel: Again?

Chris: No, run it twice.

Jerel: (Opens the table, bar graph and pie chart.)

G3: What company is that?

Jerel: Delta’s Dice. Sings: Delta’s Dice, Delta’s Dice. Yeah, me too cuz

I’m gonna

Chris: There it is.

Jerel: I think Delta’s Dice is fair.

(syncC/J2-12:24) Delta’s Dice Results

R1: (in the background) Make sure on your Word document you put your name on it. On your Word document put your name and date.

R1: (in the background) Make sure you put your name, your date, you want to put Schoolopoly in there or if you want to put which company name you’re currently investigating.

R1: If we’re gonna print these things out later, you want to make sure…

Chris: (Begins to investigate Calibrated Cubes. He opens the table, bar graph and pie chart and places them so they do not overlap.)


Chris: Is this Delta Dice or is this something else.

Chris: It’s Calibrated.

Jerel: It’s not fair. Unfair. Six is beating em.

Chris: It’s not six, four.

Jerel: Six.

Chris: Four. Chris fakes a jab at Jerel and types on the keyboard.

Jerel: No. What is wrong with you.

Jerel: Let’s click again. You just told me that like ten minutes ago.
19:45 Jerel       Oh, this might be fair.
244             Chris       It’s not fair.
245             Jerel       How is it not fair.
246             Chris       It’s not fair.
247             Jerel       How.
248             Chris       (inaudible)
249             Jerel       What do you mean.
250 20:00 Chris       The percentage are not even close.
251             (syncC/J2-15:08) Calibrated Cubes results.
252
253             Chris       (leans forward) See, three of them got two in them; the other three
got one. I’m so smart. (Goes to another screen. Talks to himself.)
254             Jerel       Dice R Us. This. (scrolls to Delta’s Dice representations) We said
this one was fair.
255             Jerel       Which one. I don’t know which one you’re talking about.
256             Chris       Delta’s Dice.
257             Jerel       Yeah, Delta’s Dice is fair.
259             Chris       All right erase it.
260             Jerel       What is that from before? Make the font smaller. (inaudible)
261 21:00 R1       I don’t know if those special keys work so well on these machines.
262             Jerel       Is it locked?
263             Jerel       (types)
264             Chris       What did you do? You screwed up.
265             Chris       (takes over the typing)
266             Jerel       Stop. (inaudible)
267 21:35 Chris       (Beginning to type their claims for the three companies into a Word
document.) Which one is the first one? (checks his paper)
268             Jerel       Delta’s Dice.
269             Chris       (continues to type)
270             Jerel       It is fair.
271             Chris       No it isn’t.
272             Jerel       Yes it is.
273 22:29 Kianja       Wow! (is reacting to her computer screen with the larger sample
size)
274             Chris       Displays the dice, table, bar graph and pie chart for Delta’s Dice.
275             Chris       He types “We think that Delta’s Dice is fair because the percents
are very close.
24:24 Chris  (has typed) the pie graph looks even, and the even though the bar
24 graph looks different it is still close.
284 Chris  (continues to type) Dice R Us
285 Jerel  Wait. Which one was that. You sure that’s Dice R Us? Dice R Us
286 is the first one that we did.
287 Chris  (presses the space bar)
288 Jerel  Why you do that?
289 Chris  I’m skipping spaces.
290 Jerel  Why you do that?
291 Chris  Because I felt like it. See look. (Chris has moved to the next page.)
292 Dice R Us. You go.
293 25:10 Jerel  What. Oh, I know. I know what to write. I know what to write.
294 (All representations are on the screen.)
295 Jerel  (types) We think that it is unfair because every other number
296 stayed at 200 or lower but the die number went to two fourteen.
297 The percents are not even close. (Jerel clicks on another company)
298 (inaudible conversation between Chris and Jerel – Chris takes disk
299 in and out of computer.)
300 27:35 G3  (talks to Chris and points to screen – inaudible)
301 Chris  Is that good? (He shows the representations and what Jerel had
302 written.)
303 28:00 Jerel  Okay.
304 Chris  We did that one already.
305 Jerel  Uh uh.
306 Chris  Hold up. Calaborated Cubes. (The boys say calaborated instead of
307 calibrated.)
308 Jerel  That’s the one…we did that one already.
309 Chris  No we didn’t. That’s the last one.
310 Jerel  Oh yeah. That’s right. We did Delta’s Dice. Sings: Delta’s Dice.
311 Chris  (types)
312 Jerel  Why don’t you just put it on the next page?
313 Chris  (moves the mouse)
314 Jerel  (continues to sing) calaborated…calaborated…calaborated…Isn’t
315 it time for lunch.
316 G3  (Points to what is on screen for Dice R Us.)
317 28:42 Chris  (reads the typed claim, realizes there is a typo and changes it)
318 …The percents are not even close. He points to the table: 214 is
319 for four and the other ones are like way off.
320 Jerel  yeah. All right. Go to Calaborated Cubes. (continues to type)
321 Chris  Now you got to hit enter all the way down there, right? That dog
322 has an idea. (Camera zooms in on dog with helpful hints.)
323 Jerel  reads what is in the dog box: type your question here and then
324 click on Search.
325 Chris  (begins to type a question - why does Jerel’s neck hurt)
327 Jerel  What? (they laugh)
Chris (types another question off subject)
Jerel (inaudible) We’ll get out of here (and closes the dog search box)
Chris (all representations are on the screen) This one was unfair, right?
Jerel We’re saying we don’t think this is fair because this is 202, 204, and 204, and the other ones are in the 130’s. (Chris is typing) Jerel is talking off subject.

32:00 Chris That’s it. (and he puts the keyboard back on the table.) On screen, he has written: We found that his store is unfair because the percents are not even close and two of the dice are way off the other dice.
Jerel But the numbers are…three numbers are in the 120 and 130.
Chris Oh, it’s three
Jerel Yeah, and put, three numbers are in the 200’s. That’s why we …because how are three numbers between 120 and 135 and the other three numbers are between 200 and 204 or 200 and …. Chris Like the odd numbers got two and the even numbers got one.

33:37 R2 You guys got an answer for me? Jerel, come here buddy. Let me see which…were any of them fair or were they all unfair?
Jerel Well one of them was fair.
R2 Which one was fair?
Chris Delta’s Dice
R2 Can I see your Delta’s thing.
Chris (gets all representations for Delta’s Dice on the screen.)
R2 Oh, that’s Delta’s. That was the one that was fair?
Jerel Yes. Oh no, it’s this one right here.

34:17 R2 It looks like there’s less two’s.
Chris Oh well.
Jerel Still, look at the percents.
Chris The percents are close. The pie graph looks even. (pointing at the screen) The bar graph, you can see that one is less than the other, but they are still kind of close.
R2 If you ran it again do you think you’d get less two’s again or do you think it would be about the same.
Chris I don’t know. We think they’re even. So we’re saying this already. How many times did we run it, 1000?
Jerel yeah.
Chris (runs another 1000 trials)
R2 What would you think if you got less two’s again. Would you still think they were fair?

35:00 Jerel See, look. See now two is winning. This convinces us that it is fair.
R2 I don’t know if it’s fair or not, I was just asking. All right, what about the other ones. Why are they unfair – while this one’s running.
Chris (points to the table and the percents column) Right here, we saw…See right here, the percents are not even. They’re not close.
R2 They’re not close.
Chris: One, three and five are close, and two, four and six... wait, I think that one, three and five got 1 in them, and then two, four and six got two in them.

R2: What do you think, Jerel.

Jerel: Yeah, I think like two, four and six got two sides, and they like came out the most like that.

R2: So it’s not fair because there are too many even numbers.

Chris: This is Calaborated... and Dice R Us.

R2: Say, that looks pretty even

Chris: But the percents aren’t close

R2: Oh not, they’re not. So, what do you think. They give you more... they make it harder to get...

Jerel: They give more fours than everything.

Chris: The fours... (inaudible) So, Delta’s Dice.

Jerel: Go back to Delta’s Dice. Go back to Delta. Go.

Chris: (shows all representations) That’s Delta.

R2: Oh, yeah, look at that time...

J37:42 Jerel: Yeah, they still even. I told you.

Chris: Okay. (runs Delta again)

R2: You’re running Delta again? Yeah look, five is winning.

Delta’s Dice – 1000 trials, five is winning.

Jerel: They’ll catch up.

R2: Look, five’s still winning with Delta.

Jerel: But look, they caught up. Didn’t they.

R2: mmm, catching up.

Chris: (runs it again) It’s possible. Cuz the other ones are going too. I’m gonna have to do them one by one.

Jerel: Why didn’t you leave them on there.

Chris: Cuz they’re going slow.

Jerel: So. I told you they’d caught up.

R2: You did tell me they would catch up. How do you know they’re going to catch up. Because they did the last two times?

Jerel: Because they even. One number would never be winning forever, on this, on Delta’s Dice.

R2: Could one number start out winning?
Yes, it could start out by a big lead but then probably the other numbers come back.

So if I just rolled it like ten times, then, if I just rolled it like ten times I might win with two or something.

Yeah but that’s with ten. But if you rolled it more times, you will see...you will see...

I can’t pause it

Click the eraser maybe

Let me see, let me see, let me see...no, cancel, cancel, cancel,

cancel...

Let me see, let me see, let me see...no, cancel, cancel, cancel,

cancel...

We still got to do the rest, that’s why.

Oh. No it stopped, it stopped.

This is Calibrated. We already did that one, right? You got to reopen that one. Go ahead and reopen it.

Isn’t it lunch time.

Almost, Jerel. Look the three won.

That’s because (inaudible)

Look, the three won again. You’re running it more and more.

That’s very interesting. Do you guys want to tell that to Hollylynne when she comes by?

(gives closing directions for saving file to disk or in Probability Explorer)
Appendix I

Complete Transcript of CD: ROLE 138C
with Synchronized Chris/Jerel Monitor CD 2 of 3

Date: 03 August 2005    Grade 7
Location: Washington Elementary School
Transcribed by: Barbara Tozzi
Verified by: Doreen Nuzzolese

<table>
<thead>
<tr>
<th>Time</th>
<th>Speaker</th>
<th>Transcription</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 00:35</td>
<td>R1</td>
<td>[to class] Who has looked at more than one company so far. Put your hand up if you have looked at more than one.</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>(Chris and Jerel raise their hands.)</td>
</tr>
<tr>
<td>3</td>
<td>R1</td>
<td>Chanel and Kianja, you just looked at one? You finished one, and just started your second one. Justina and Lorrin, did you do more than one or you’ve just done one so far? You’re working on number one? You’re working on your second one. Okay. All right. Because what I’d like to do, is …I don’t have my list in front of me… Dr. Weber has. Does anyone know where the sheet of paper was that Dr. Weber had that had some red ink on it? Do you see it anywhere? No, okay. So I don’t have my list. Dr. Weber, the piece of paper that I gave you, do you have that?</td>
</tr>
<tr>
<td>13</td>
<td>R2</td>
<td>Yeah.</td>
</tr>
<tr>
<td>14</td>
<td>R1</td>
<td>Can I have it please? Thank you.</td>
</tr>
<tr>
<td>15</td>
<td>R2</td>
<td>You sure could.</td>
</tr>
<tr>
<td>16</td>
<td>R1</td>
<td>Okay. So um, some of you have some of the same companies. I want to give you a chance to talk to other people who were exploring the same company. All right. So, we’re going to have a little bit of informal time where if you still have one person that wants to be (inaudible) some data that’s fine, but we’re gonna be able to float around the room to talk to other people. So for example, Chris and Jerel, you did Delta’s Dice, Dice R Us and Calibrated Cubes. Is that right?</td>
</tr>
<tr>
<td>24</td>
<td>Chris</td>
<td>Yeah.</td>
</tr>
<tr>
<td>25</td>
<td>R1</td>
<td>Who else has done those companies? Delta’s Dice, Dice R Us, or Calibrated Cubes. Anybody else have any of those companies? Did anybody else do Delta’s Dice?</td>
</tr>
<tr>
<td>28</td>
<td>Terrill</td>
<td>Oh us.</td>
</tr>
<tr>
<td>29</td>
<td>R1</td>
<td>You have Delta’s Dice?</td>
</tr>
<tr>
<td>30</td>
<td>Danielle</td>
<td>No we don’t.</td>
</tr>
<tr>
<td>31</td>
<td>Terrill</td>
<td>We don’t have Delta’s Dice? Oh yeah.</td>
</tr>
<tr>
<td>32</td>
<td>Jarae</td>
<td>We do.</td>
</tr>
</tbody>
</table>
| 33   | R1    | Okay, so Jarae has that one. And Lorrin and Justina? So at some point, you guys are gonna want to talk to Chris and Jerel to see...
what they did with Delta’s Dice. Okay? Um, Dice R Us. Who else has Dice R Us?

(raises his hand) We do.

Who else besides them? Chanel and Kianja, don’t you have Dice R Us?

Oh yeah, but we haven’t done it yet.

You haven’t done it yet, so okay, we’ll hold off on that one. And what about Calibrated Cubes. Who else did Calibrated Cubes?

(Discussion ensues and Terrill and Danielle realize they have that company.) Oh you guys have Calibrated Cubes. Have you done it yet?

Terrill No.

We have.

We finished.

We finished.

All three.

All three.

So, Chris and Jerel, I’d like you to come over here and be able to talk to Lorrin and Justina about...Justina and Lorrin, do you have your data from Delta’s Dice yet? (inaudible)

That, um, Delta’s Dice was fair.

Yeah.

How did you determine that? Can you show me? I’m pretty interested.(Chris and Jerel type on the keyboard.)

Is this your Word document that you’re opening up?

Yeah. (Chris nods his head.)

How many companies did you have?

Three.

Which ones?

Delta’s Dice,

Delta’s Dice, Dice R Us and Calaborated Cubes.

Calibrated or…

I don’t know how to say it.

Calibrated

Calibrated

So, this is your information. Okay, so explain to me what you did.

Oh, all right. We…

So you first looked at Delta Dice.

I’ll make it bigger.

Um hm. That’s big enough.

All right. First we, the first one we looked at was Delta’s Dice.

Um hm.

We ran this one 1000 times

Okay

and to us, what made it look even is that the percents are close, the bar, the pie graph looks even, even though it wasn’t and uh the bar
graph, where it showed the exact numbers were very close together.

R3 So, okay, so go back. So you said the pie chart looked even, even though it wasn’t. You mean like because this blue area (points to the monitor) looks a little less than the others.

Delta’s Dice

Jerel Yeah.

Chris Sometimes, but if you like, if you like see it, it’s kind of all even.

R3 Can I see your data table? (Chris clicks on the table.) Uh huh. So what did you get from the data table?

Chris We saw that the percents, all the percents were close…

R3 Were close.

Chris yeah, and that’s it.

R3 Okay. So what did you conclude about this one?

Jerel That it was uh fair.

Chris (nods his head) Fair.

R3 Okay. So Delta’s Dice seem to be fair.

Chris (scrolls down their Word document) This is

R3 Which one is this?

6:18 Chris Dice R Us.

Jerel Dice R Us.

Chris I don’t think it’s fair.

R3 You don’t think it’s fair.

Chris No.

Jerel Nope.

Chris (points to the monitor) This right here see, you see that some are not bigger than others.

Jerel They’re taking up more shades on the pie graph.

Chris And you see the percentages aren’t close (points to the table)

Jerel It’s like they made all the even numbers two-sided.

R3 All the even numbers…

Jerel It like they made

Chris No, not all the even numbers. They just took some

Jerel Yeah, they made number *four* two-sided

Chris And number *three* is two-sided

R3 When you say two-sided, do you mean
Two, three and four, they call it, not two-sided, they call it more than one of them in so that four has two, three and two and two has two.

So you're talking about like that weight tool that you guys looked at earlier.

(nods his head) yeah.

Yeah.

So you think that number four and which other one?

Number two, three and four.

Two, three and four, that there might be a two in those boxes

Two, three, four and five

And five, um hm

They see that those percentages are close but one and six, those percentages aren't close.

Okay, so one and six aren't close…

So we said they were unfair

So unfair. So wait, hang on, go back for one minute. Unfair, but which number has the advantage or disadvantage?

Two, three, four and five have the advantage.

Yeah.

Yeah.

And then, Calibrated Cubes. Is that the next one?

(scrolls down) We said this was unfair too.

Calibrated Cubes unfair?

(points to the monitor) You see the pie graph.

Calibrated Cubes

Um hm.
Chris You see that some of them are bigger than others. The percentages are like: one, three and five.

Jerel are between thirteen and twelve. And, two, four and six they have
two and three and five only have one.

R3 So when you say two again, you mean that weight tool.

Chris Yeah, that weight tool.

R3 Uh huh, uh huh. Suppose you ran, um, I think Dr. Lee would allow this, if you, if you were to, um, go to the dice and set up the weight tool according to what you think this is. Remember how you worked on the weight tool before.

Jerel yeah.

R3 Could you set up a dice to produce this result (points to the table).

Chris yeah.

R3 Let’s see.

Chris (leans over to type)

Jerel How we gonna do it?

R3 I think Lorren and Justina may be coming over soon, but until they come over see what you can do.

Jerel What are ya doin

Chris We said two, right

Jerel yeah

Chris (types in the weights into the weight tool)

R3 Adjusting the weight tool

Jerel Na ah, that was one. Cuz remember it was the even numbers that…(sings)

Chris (With the new weights, he runs 500 trials and produces the table, pie graph and bar graph.)

R3 So what did you do? (He explains what the boys are doing to R1.)

R1 (to Chris) What do you think’s going on in Calibrated Cubes? I wasn’t here for that.

Chris Well, that one, three and five have one in them and two, four and six have two.

R1 When you say one and two, what do you mean?

Chris Like on the weight tool.

R1 Oh, on the weight tool. Okay.

R1 What makes you…why did you assign the same numbers to one, three and five?

Chris Because the percentages were, like real close. And two, four and six – those percentages were close. So,
R1 And so you gave one, three and five a one…
Chris a one, and two, four and six a two.
R1 a two. Okay. Do you think that that’s giving you what you had
before?
Chris (nods his head)
Jerel yeah
11:40 Chris They’re close
R1 What do you mean they’re close
Chris (points to monitor) Cause one, three and five are all in tens, and
two, four and six are in like the twenties – 22 and 23.
R1 And is that what you got with Calibrated Cubes?
Jerel yeah.
Chris It’s not the same percentages but,
Jerel They’re very close
Chris (reads from the table) thirteen, twelve, thirteen, twenty, twenty and
twenty.
syncC/J2-18:44 Chris and Jerel’s experiment with weights of even:odd as 2:1.

Okay. So do you, so, they’re not exactly but they’re close, are they
close enough
Jerel yeah
Chris yeah
R1 So ten’s close enough to thirteen, and twenty-two’s close enough
to twenty.
Jerel yeah.
Chris (nods his head)
R1 So you are, you’re pretty sure that Calibrated Cubes is not fair.
Chris (nods his head)
Jerel Um hm.
R1 But, I seem to be getting, I have the same chance of getting one,
three and five. And I’ve got the same chance of getting two, four
and six. Isn’t that fair?
Jerel No.
Chris (shakes his head no)
R1 Why isn’t it fair.
Chris Cause you’re gonna keep getting two, four and six more.
R1 Oh I’ll get two, four and six more. But even though two, four and six have, you think have the same chance…

(nods his head)

yeah. I think two, four and six have the same chance.

So, are you ready to talk about Delta’s Dice. All right, I want you to go and print out – do you have your disk so you can print out your stuff? (talks to Jerae)

So, she was also looking at Delta’s Dice. So, why don’t you guys pull up what you did on Delta’s Dice. All right. So Jarae, why don’t you tell them what you did first.

Well, I just I just started and um well when we talked to Lorrin and Justina they said that it wasn’t fair but then they tied again and they changed their mind and they said it was fair because they every time they did it they did 55 times, well not 55 times but a certain amount of trials, and 55 dice each time and so every time they did it they got a certain amount of number from the dice was had one each time and so it was fair because it was a different amount. And each number had got to win at least one time.

And what did you find out about Delta’s Dice.

She was telling us what Justia and Lorrin did.

And then, um we went back to our computer and we tried that and we tried it six times but the thing was that each number didn’t get um, each number didn’t get to win because four, four didn’t get to win. It was five won two times, two won once, six won once, three wait, six won twice, three won once, and one won once.

So, what do you think that means about Delta’s Dice

That means, it is fair.

You think it is fair.

yeah, it might

I thought you said four didn’t have a chance to win.

Yeah, they didn’t have a chance to win on ours but we had evidence from Lorrin’s and Justina’s and maybe we didn’t do it a certain amount of times that they did it cuz we only did six trials, and they probably did ten or more.

All right, so, what did you guys do? Jarae, take a listen to what they did.

Well, we ran it a thousand times and after we ran it a thousand times we looked at the percentages and since they were close we thought it was fair.

yeah.

You said you thought it was close. Be more specific

Like it’s close. The percentages on – the pie graph looks even,

That pie graph doesn’t look even to me.

Yes it does.

It does (laughs)
So, I’m looking at the number two in the table, and look at the percent for number two. Could you tell me what that is.

Chris 14.7
Jerel 14.7
R1 You think that’s close to 18.
Jerel yeah.
Chris yeah.
Jerel Five or more apart is not close but four is close
Chris It’s close to fifteen
R1 You’ve got some fifteen
Jerel We’ve got some fifteen. We only got two eighteens and one seventeen. And we’re done. All right. If you’re trying to say that four is not close that’s trying to say three is not close. But, we concluded that it was fair.
R1 But, so you only ran it one thousand times once.
Jerel No, then we did it again but we didn’t print it out.
Chris yeah. We did it again but it was around the same thing.
Jerel It was around the same thing but two was kinda higher. It was about fifteen or sixteen, and the rest of them was like the same.
R1 I don’t know. I’m really not convinced that this is showing that it was fair.
Jerel all right.
R1 (points to Jarae) But now you said that you thought it WAS fair.
Jarae I thought it was fair on that one.
Jerel Isn’t it the same thing?
Jarae (shakes her head no) No.
Jerel Yes.
Jarae Did you guys do it 500 at a time or did you
Jerel We double clicked at 500. Well I, can I explain something?
R1 Sure, go ahead Jerel
Jerel (points to the monitor) Look do you see like the bar graph all the way in front of it. Isn’t that close.
R1 I don’t think those look close to me.
Jarae Especially the pink one. It’s a little bit higher than…except for green.
Jerel Exactly.
Jarae It looks like the same, only.
Jerel All right, let me know you something.
R1 Actually, if you go into the probability files you can up Delta company. Yeah that one. Oops, double click it.
Jerel Delta’s Dice, right.
R1 Um hm.
Chris Run it 500, I mean 1000 times.
Jerel See, this is, we started doing this before and then I forgot what happened.
R1: It looks like you only have seven on there. You only ran it seven times.

20:00 R1: So tell me what you’re doing and what you’re looking for.

Jerel: We’re running it 100 times and we’re looking to show y’all that it’s fair.

R1: Okay.

Jarae: Can you line that up. That looks sloppy.

Jerel: (Lines up the outcomes.)

Chris: Open that up – the data table.

Jarae: Until six.

Jerel: (tries to make the data table larger)

R1: Grab it on the bottom – there you go.

R1: 20:45 R1: So what do you think this data’s showing you. What do you think, Chris.

Chris: It’s showing you, how many of a thousand is gonna be a certain number.

R1: Okay, and do you think that it’s giving you more evidence that it’s fair, or is it showing you something different?

Jerel: It’s giving more evidence that it’s fair. See, see, don’t the bar graphs look close?

R1: No, they don’t.

Jarae: (shakes her head no) No they really don’t. Only the gray, the dark blue and the other blue.

Chris: Just let it finish. Watch.

R1: Now right there it looks a little closer.

Jarae: yeah.

(syncC/J2-29:14) Delta’s Dice second run of 1000 trials.

R1: But, it didn’t to me before. So I think you guys need to articulate a little bit more what you mean close. Cuz I’m not convinced about…

Jerel: All right. How about we

R1: How close is close enough, or what close is good…It doesn’t make any sense to me.

Jarae: Why did y’all pick one thousand?

Jerel: (moans as Chris laughs)

R1: That’s a good question, Jerel.
Chris: yeah, Jerel.

R1: I mean because you guys (pointing to Jerae) were doing sixty, right.

Jerae: We did 600.

21:54 Jerel: To make, look Jarae, we did 1000 because we wanted to make sure that, we wanted to make sure that it wasn’t like, if we ran it one more time, it wouldn’t be, one number wouldn’t be way in front of the other number. That’s why we ran it 1000 times.

Jarae: Well then what’s wrong with a number being way over a different part.

Jerel: Then it wouldn’t be close. All right, we gonna run it 100, how many times did you run it Jarae.

Jarae: I did it 600.

Jerel: 600, why do 600?

Chris: Yeah, why do 600?

Jarae: Because, I want, wait, actually, we did 6.

Jerel: You did six.

Jarae: Yes.

Jerel: For real.

R1: So what does that tell you.

Jarae: Hm. It’s not very close.

Jerel: All right Jarae, you picked the same thing.

Jarae: No I didn’t.

Jerel: Yes you did.

Jarae: I picked six trials.

R1: Chris, tell me why you picked 100. I kind of missed that.

(syncC/J2-29:54) Delta’s Dice 100 trials.

Chris: all right.

Jerel: Look

R1: Hold on. I want to hear why Chris put it down to 100.

Chris: That’s because she (Jarae) said why you use 1000. So I just put it down lower to see if it was the same thing.

R1: Okay, what happened.

Chris: And it wasn’t.

Jerel: But then, if you would have kept going, you would have known if it would have changed, Jarae.
Jarae: But I did six times and it was close because every, almost every number won except for one number.

 Jerel: But look, I guarantee you if we would have ran it 1000 times it would have been even.

 R1: You said that if you ran it 1000 times...

 Jerel: It would be even.

 R1: But why isn’t it showing up even for 100.

 Jerel: Because you’re not running it enough times.

 R1: You’re not running it enough times? Then why is that important.

 Jerel: Because you will never know if anything is gonna catch up to anything.

 R1: Do you agree with that, Chris?

 Chris: No. (Chris is focused on running different experiments with 100 trials.)

 R1: Delta’s Dice – another 100 trials.
Jerel: You don’t.

Chris: Say again.

Jerel: You never know if another color, you never know if another die is gonna catch up to another die.

R1: When, when. Tell him the condition.

Jerel: When you run it 100 times you will never know if it’s gonna catch up, cause you stopping too early. But then, if you run it 1000 times you know that, you know that…

Jarae: I have a reason why.

Jerel: Wait.

R1: Go ahead, Jerel and finish your statement.

Jerel: That you have a better chance of having it be even and another number catching up to it.

R1: So now do you (Chris) understand what he (Jerel) was saying? I know you were a little bit busy right there.

Chris: (nods his head)

R1: Do you agree?

Chris: (nods his head)

Jarae: (doesn’t agree)

Jerel: Jarae, can I ask, why don’t you agree

Jarae: No you can’t ask.

Jerel: You got to answer it.

R1: Do you understand what he (Jerel) was saying.

Jarae: (shakes her head no)

Chris: Answer, please.

Jarae: I don’t understand.

R1: So Chris, recap what he was saying for Jarae. See if you can say it in a different way.

Chris: I can’t really say it in a different way.

R1: It was something about the difference between what happens with 100 versus 1000.

Chris: Now if you keep going for it to a thousand, you gonna see that they gonna like

Jerel: They gonna finally be even to each other.

Chris: yeah. Cause you’re not just gonna roll six times and yeah, okay I’m done.

Jerel: Yeah, exactly. Cause if you’re losing your money, you’re gonna keep going.

Jerel: All right, now let’s see. All right, we gonna erase. We gonna run it 500 times, not a thousand, for your sake.

Jarae: Can you line them up?

Jerel: (moans) They gonna get messy anyway, right Jarae?

Jarae: Yep.

Jerel: Oh see, look he’s catching up. (inaudible) like when you run it 100 times. Well, except the gray.

(comments among the three)
Chris
Now is it lined up.

Jarae
(shrugs her shoulders) I don’t care.

26:45 R2
I don’t know, that doesn’t look fair. Blue lost.

Delta’s Dice – blue (two) lost.

R2
Oh look blue, light blue is winning...

Tipo
No it’s not, no it’s not...

R2
Look it’s still winning

Jarae
The light blue is winning

Jerel
So Jarae, you picked the same thing. I don’t get it. You picked

Delta’s

Jarae
I didn’t pick anything.

Chris
Then which one you pick

R2
Look, it’s catching up

Jerel
Yeah, yeah. Told ya.

R2
Oh look it caught up.

R2
See, ain’t no number beating another number by at least twenty.

R2
Look how bad one is. One’s really far behind. (they laugh)

Jerel
Well one is the worst number you can get.

Delta’s Dice – 500 trials again for a sample size of 1000.

Jerel
Now watch when you run it again. I guarantee…(sings)

R2
One’s catching up.

Jerel
Yeah, one caught up. Now, who’s good. Who’s good. Please don’t
talk. Just congratulate and don’t talk.
In 1000 trials, one appears to be catching up.

(pokes Jerel in the neck) I congratulate.
Oh see, ain’t those percentages close.
Yep. No.
No. Five.
Jarae it’s four. It’s not even really five. It’s like four.
Five has 190, and all the rest of them have like 150 or 160.
Yeah, five has 190.
Outcome for 1000 trials – five is much higher.

But still.
But five is winning by a lot.
No it’s not. Look. Ain’t that close, ain’t that close, ain’t that close.
No.
Yes it is. Look, it looks close. Jarae, you getting on my nerves.
(runs it again) Let’s see if it’s close now.
Yeah let’s see if it’s close now, Jarae.
Two is losing.
I’m picking Delta’s and I’m sticking with my decision.
But two is losing.
You can stick with yours, I can stick with mine.
You gonna pick Delta. Look if she looks at the other ones she’ll see. Delta’s the fairest one you can get.
But how do you know.
All right. Watch.
I didn’t say that Delta wasn’t fair.
Yes you did.
Jarae: I said it was fair because in the trials that I did it was fair.
Jerel: Exactly that’s the same thing, Jarae. They’re all the same thing.
Jarae: I said it was fair when I got over here. I said it was fair when I got over here.
Jerel: They all the same thing.
Jarae: I said MINE’s was fair. I never said…
Jerel: Jarae on the same computer. All right, you said, when Lorrin had that four, you said uh you said that it’s still fair even though a four didn’t win on mine. So get outa here.
R2: Yeah, but I bet if you just do it 12 times one of the numbers is gonna win by a lot.
Jerel: Cuz it’s only a little bit of number you ran.
R2: Why should that matter. If it’s fair, it’s fair.
Jerel: No, cuz if you roll a little bit, of course a number is gonna blow another number out.
R2: Yeah, they look pretty similar, huh.
Jerel: (comments to Jarae)
(syncC/J2-39:05) Outcomes look close as the number of trials approaches 1500.

31:50 R1: Okay, we’re going to wrap it up for today. So I want you to save everything on your disks and what I’m going to do, I’ll print everything on your disks so you will have them in the morning. So, what I need you to tell me before you leave is if there’s a company that you still need more time on tomorrow to investigate.
Jerel: All right, otherwise when we come in tomorrow we’re gonna start making some of our posters. Okay. So if you’re not done with a company you need to let me know.
Appendix J

Complete Transcript of CD: ROLE 139D with Images from ROLE 142E

Date: 04 August 2005    Grade 7
Location: Washington Elementary School
Transcribed by: Robert Tozzi
Verified by: Barbara Tozzi

<table>
<thead>
<tr>
<th>Time</th>
<th>Speaker</th>
<th>Transcription</th>
</tr>
</thead>
<tbody>
<tr>
<td>00:25</td>
<td>R1</td>
<td>Ok. So what I’d like you to do is on, on your um, on your table, take a look at the piece of paper that has the task written on it. Then look at the piece of paper that has the task written on it, specifically what’s in the box.</td>
</tr>
<tr>
<td>00:25</td>
<td>Jerel</td>
<td>The questions.</td>
</tr>
<tr>
<td>00:26</td>
<td>R1</td>
<td>Ok? Those things that are in the box are the questions you need to answer on your poster. Ok and you want to make a poster for each separate company. So if, so if you did, um, Dice R Us, you’ll do a, you’ll do a, a poster for Dice R Us and then if you did, Dice Depot, then you’ll make a separate poster for that. Ok? Because what we’re, what we’re gonna do, is we’re gonna have these um, the posters hung up around the room for us to be able to look at and you need to have the title of the comp…(cross talk) Danielle. You’ll need to have the title of the company that you’re presenting data about at the very top of the poster, written kind of big, all right? So that it’s pretty easy to see from far away. Um, you have your, um papers printed out from yesterday so that you can um, um, copy, uh so that you can cut them out and paste them and I’ll give you scissors to do that. And you have markers if you want to write anything, um, but there are the three things that need to be on your poster in that box. Let’s review what’s in that box. Can somebody read me the first question? Jerel?</td>
</tr>
<tr>
<td>1:46</td>
<td>Jerel</td>
<td>Would you recommend that dice be purchased from the company you investigated?</td>
</tr>
<tr>
<td>1:47</td>
<td>R1</td>
<td>Ok, so the first thing you need to do is to make a recommendation. Would you buy dice from this company or not? And number, number 2, Danielle what’s number 2 say?</td>
</tr>
<tr>
<td>1:48</td>
<td>Danielle</td>
<td>What evidence do you have that the die that you tested is fair or not?</td>
</tr>
<tr>
<td>1:49</td>
<td>R1</td>
<td>So what does that, what do you, what do you think, are the kinds of things you should be doing to answer question number 2? What evidence do you have?</td>
</tr>
<tr>
<td>1:50</td>
<td>Jerel</td>
<td>Your graphs and your uh…</td>
</tr>
<tr>
<td>1:51</td>
<td>Chris</td>
<td>Your tables and your graphs.</td>
</tr>
<tr>
<td>1:52</td>
<td>R1</td>
<td>Your graphs.</td>
</tr>
</tbody>
</table>
Chris And your tables.

Danielle And the description that you wrote about.

R1 And your description. Right, and you are more than welcome to write more description than what you did before. Because what you wrote before, was more about kind of your notes as you were going along. So if you formulated a different way of saying it or something more than you wanted to say, use the markers and you can go ahead and write that on the poster. Ok? Now what about question number 3? Keisha what’s question number 3?

Keisha What do you think the chances are for rolling each of the six numbers? (off camera)

R1 All right, what do you think the chances are for rolling each of the six numbers? Terrill what does that question, what does that question mean? How would you answer such a question?

Terrill Um, you can use um, the charts and the pie graphs and the (inaudible) used yesterday on the computer.

R1 Ok?

3:05 Jerel The percents.

R1 You can use the pie graphs, all right, so if I say, what are the chances of rolling a one, what kind of answer might you, give me, for, for a certain company? (cross talk)

Jerel of 17.3%.

Terrill If you look at your, um, data tables.

R1 Look at your data tables.

Jerel And find the percents.

Terrill ...and use the fractions um, either like 8, 83 out of 500, something like that.

R1 Ok, all right. So you could, I’ve heard using fractions using percents, you can even use words to describe them. Whichever, however you want; you need to describe for me what you think the chances are of getting the numbers one through six. All right, so there’s 3 questions that you need to answer for this. Ok? Does anybody have any questions?

Student (asks question off camera)

R1 So actually, Danielle, Terrill you need to decide which poster you’re going to do and which one he’s going to do…

Both Calibrated Cubes!

R1 So let’s see, Danielle’s name is poster 2 calibrated, the D is closer to calibrated cubes, so…

Chris (laughs)

Terrill No, our, can you pick a number in your head and we guess which one it is?

R1 Ha, I was just, you know what, alphabet, so then you, so what, what’s your other company? Dice…

Terrill Dice Depot.

R1 Dice Depot, so…
Terrill But I don’t want to do Dice Depot.

Jerel (laughs)

R1 So you do Dice Depot please…

Jerel (continues laughing)

Terrill (mumbles something)

R1 Shhh, Shhh, Shhh, Ok, so you guys you can split up how you’re gonna do it. All right, so make sure…

Danielle Why don’t we just work on the same poster? Would you do that?

R1 You want to work on the same poster? Together?

Terrill Ok fine, we’ll both do the same page and then when we’ll just…

R1 Yeah.

Danielle No we’ll do the same poster just come on this poster.

R1 So you’ll both work on Calibrated Cubes?

Chris (laughs)

Danielle No we gon do both of them, Depot and…

R1 He’ll do Dice Depot?

Danielle Yeah I’m doing um, Calibrated Cubes down here and he’ll go…

R1 Well I really want to see separate posters.

Terrill Ok fine, we’ll both do Calibrated Cubes and we’ll both do that one.

R1 That’s fine with me.

Danielle All right we’ll both do Calibrated Cubes and we’ll both do that here.

R1 That sounds good, all right. Now, first of all, markers, are only to be marked on, the paper. All right? So when you’re using the markers, the markers only get, only write on the paper. Don’t write on the desk, don’t write on another thing, don’t write on your nametags, don’t color each other. Ok? Don’t color your shirts, hehe, ok? Now, scissors, were…

Terrill Can we us a ruler? Do we have a ruler? (cross talk)

R1 Um, ooh I don’t think I have any rulers. Oh we do have rulers, yes we do!

(Chris and Jerel talk off task)

R1 So which one are you guys going to do first?

5:50 Chris Uh, I’m gonna do Delta’s Dice.

R1 You’re gonna do Delta’s Dice?

Chris Yeah.

Jerel I’m gonna do Collaborated Cubes.

Chris And then afterward we’re gonna do Dice R Us, together.

R1 Ok. That’s fine. So Chris do you think you’ll need more, do you need, you need more room?

Chris Yeah.

R1 All right so let’s…

Jerel Grab that table!

(Chris moves to another table)

R1 We’ll move you here for now, so Jerel, you can get started on this one. And I’ll get you another pair of scissors. (answers a question
from assistant: Sure, yeah, if they need it.) Chris do you have a, the black marker? Oh you have a red marker. You have a red one and a black one, here’s a green one.

(Students begin to work)

R1 So, Pat has given each of you a folder, if you want to use the straight edge of the folder as a ruler, you can.

(Students work)

Terrill Do you have a pencil I can use to write my name? Can I borrow it...

R1 Actually, I tell you what, today we’re not gonna put names on the posters.

(Talking off camera.)

R1 It’s ok. It’s ok. The only, the only name that should be on is the company you are investigating.

Jerel Oh.

R1 Not who did it. That’s ok, I forgot, that was somebody. I almost forgot to tell you that. That could be my fault.

(Students work.)

8:50 R1 I’ll tell you what, let me borrow somebody’s. I’m gonna write the questions up here. Jerel I’m gonna borrow this for a second.

(Students work.)

(Off-topic discussion)

12:24 R1 So, Pat has given you all, some scratch paper. That if you want to, if you want to practice what you’re gonna write before you write it, if you haven’t already written your sentences, you can do that. Ok? It’s up to you, if you need the scratch paper you got it.

(Students work.)

(Background talk. Jerel talks to Terrill off task.)


Chris Thank you.

R3 Hmm. Nice job Jarae.

Jerel I’m not Jarae.

R3 Jerel. (smiles)

(Students work.)

22:00 G1 Wow. That’s impressive. Chris.

Chris Huh?

G1 Hmm, where’s your partner?

Chris Over there. He working on the other, on the other uh…

G1 But for the same poster?

Chris No.

G1 You did two different posters?

Chris Yeah.

G1 But you work together right?

Chris Uh huh.

G1 Ah, so you doing one company? So which dice are you doing?

Chris I’m doing Delta’s Dice.
174  G1  Which one?
175  Chris  Delta’s Dice.
176  G1  Delta Dice, all right. All right.
177  (Chris continues to work.)
178  G1  Which company are you doing?
179  Jerel  Collaborated Cubes.
180  G1  Collaborated, what is it?
181  Jerel  Collaborated Cubes.
182  (Jerel continues to work. He cuts out the entire screen with the
dice, pie graph, bar graph and table, along with the typed claim
from his summary document.)
185  Jerel’s answers for Calibrated Cubes - ROLE 142E – 28:16

186
187  Answers (continued) for Calibrated Cubes - ROLE 142E – 28:22

188
189  Representations for Calibrated Cubes - ROLE 142E - 28:30

190
191  For Calibrated Cubes, what Jerel wrote - ROLE 142E – 28:36

192
(Chris realizes he divided pie graph into too many sections.)

I’m gonna read what you wrote.

Chris’ Answers for Delta’s Dice - ROLE 142E – 18:13

(Chris continues to work)

Chris’ Pie Graph - ROLE 142E – 18:54

So one thing that, one thing that I’m wondering that people may wonder about when they look at your poster, is, if you made up that pie graph. Ok? So you may want to actually, um, maybe even, even at the bottom you could, you could glue sti…glue stick this, even if it hangs a little bit off the poster, so that they can actually see your data.

Oh, ok, like this whole thing right here?

Well eh, eh, up to you. You could just cut out the pie graph. That would be fine too. All right, but one thing that’s going to be important with the pie graph, the pie graph, if you just put a pie graph up there, it doesn’t tell you, some important information.

All right, what, what information doesn’t it tell you?

It doesn’t tell you like, how much do each have out of a thousand.

Ok. It’s the out of the thousand. Just putting the pie graph on here doesn’t tell me that you did it a thousand times, all right? And if you, and if you, if you think it’s important for people to know how many times you did it, you want to make sure that you’ve got that clear on your poster, all right. When I think about the evidence, you say, the evidence I have, um, that I think um, makes it fair is the pie graph and the data table. So you’re telling people to come down here and look at this, but you’re leaving the interpretation up to them. So you need to be a little bit more clear, and you may have, you probably have some room down here to still write, and
we can, we can actually get you another poster and extend it down if you want, um, as to why this table tells you that it’s fair. Because you just point me to the table I can look at it and go, well I think it’s unfair. So you have to convince me, ok?

(R1 leaves; Chris cuts out the print out of the computer screen.)

Chris’ table - ROLE 142E – 18:35

<table>
<thead>
<tr>
<th>Dice</th>
<th>Fraction</th>
<th>Decimal</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3/1000</td>
<td>0.003</td>
<td>1.0%</td>
</tr>
<tr>
<td>2</td>
<td>33/1000</td>
<td>0.0133</td>
<td>7.8%</td>
</tr>
<tr>
<td>3</td>
<td>57/1000</td>
<td>0.0057</td>
<td>4.7%</td>
</tr>
<tr>
<td>4</td>
<td>100/1000</td>
<td>0.0100</td>
<td>1.0%</td>
</tr>
<tr>
<td>5</td>
<td>52/1000</td>
<td>0.0052</td>
<td>6.2%</td>
</tr>
<tr>
<td>6</td>
<td>13/1000</td>
<td>0.0013</td>
<td>1.3%</td>
</tr>
</tbody>
</table>

33:15 Chris Can I have a glue stick? Anybody have a glue stick?
33 G3: What’s that?
34 Chris A glue stick.
35 G3: You need one?
36 Chris A glue stick, yeah.
37 (G3 gets Chris a glue stick.)
38 Chris Jerel!
39 Jerel What?!
40

34:25 Chris I’m finished. Let me see. Let me see. (Starts laughing.) I’m finished.
41 Jerel What’s the questions?
42 (Chris sits down next to Jerel.)
43 Chris (laughs) Yo you suck, I’m sorry, you suck. (Chris helps glue the typed claim from the summary document above the computer-generated representations.)
44 Jerel (inaudible) Yeah you start doing that. I’m not a drawer, I’m a lover. Remember that, and you just heard that, I know you did. I’m not even a mathematician.
45
46 Chris You can’t even cut.
47 Jerel So?
48 Chris You can’t even write.
49 Jerel So? You can’t even streak your head.
50 Chris Aight, watch. (Jerel starts singing.)
51 (more off-topic discussion)
52 36:42 R1 So how you y’all, how you doing over here? Are you done with this one?
53 Jerel Yeah.
54 R1 (Reads) So we ran a thousand times, and all the odd numbers had a low percent and high number…
55 Jerel (inaudible)
56 R1 What do you mean by high number?
57 Chris Even number.
262  Jerel  Yeah that’s it. (laughs)
263  R1    Ok, so, can you go over that with a marker?
264  Jerel All right.
265  R1    And, um, let me see if I can get a thinner one for you.
266  (Jerel starts singing again.)
267  R1    There you go.
268  Chris What happened?
Appendix K

Complete Transcript of CD  ROLE 140D with Images from ROLE 142E

Date 04 August 2005  Grade 7
Location Washington Elementary School
Transcribed by Robert Tozzi
Verified by Barbara Tozzi

<table>
<thead>
<tr>
<th>Time</th>
<th>Speaker</th>
<th>Transcription</th>
</tr>
</thead>
<tbody>
<tr>
<td>00:22</td>
<td>(The boys are working on their posters. Jerel sings.)</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Jerel</td>
<td>How you spell by?</td>
</tr>
<tr>
<td>4</td>
<td>Chris</td>
<td>Which by?</td>
</tr>
<tr>
<td>5</td>
<td>Jerel</td>
<td>By like by, not this buy, by.</td>
</tr>
<tr>
<td>6</td>
<td>Chris</td>
<td>B-Y.</td>
</tr>
<tr>
<td>7</td>
<td>Jerel</td>
<td>Oh, ok. (laughs)</td>
</tr>
<tr>
<td>8</td>
<td>Chris</td>
<td>You can’t spell.</td>
</tr>
<tr>
<td>9</td>
<td>Jerel</td>
<td>I spell good didn’t I?</td>
</tr>
<tr>
<td>10</td>
<td>Chris</td>
<td>Aight.</td>
</tr>
<tr>
<td>11</td>
<td>Jerel</td>
<td>How you spell be?</td>
</tr>
<tr>
<td>12</td>
<td>Chris</td>
<td>(Off topic discussion.)</td>
</tr>
<tr>
<td>13</td>
<td>Jerel</td>
<td>(Students take break.)</td>
</tr>
<tr>
<td>14</td>
<td>Jerel</td>
<td>Wait, I got to put a period, hold on.</td>
</tr>
<tr>
<td>15</td>
<td>Chris</td>
<td>(Chris writes on poster.)</td>
</tr>
<tr>
<td>16</td>
<td>Jerel</td>
<td>(Someone calls Chris’s name.)</td>
</tr>
<tr>
<td>17</td>
<td>Chris</td>
<td>What?</td>
</tr>
<tr>
<td>18</td>
<td>Chris</td>
<td>(Chris leaves the room)</td>
</tr>
<tr>
<td>6:56</td>
<td>R1</td>
<td>You guys done?</td>
</tr>
<tr>
<td>20</td>
<td>Jerel</td>
<td>No.</td>
</tr>
<tr>
<td>21</td>
<td>R1</td>
<td>No, you got one more question to do don’t you? Oh you’re writing, you have to finish this reasoning.</td>
</tr>
<tr>
<td>23</td>
<td>Jerel</td>
<td>Uh huh.</td>
</tr>
<tr>
<td>24</td>
<td>R1</td>
<td>And then you’ve got to do your next poster.</td>
</tr>
<tr>
<td>25</td>
<td>Jerel</td>
<td>Yeah, they called Chris…</td>
</tr>
<tr>
<td>26</td>
<td>R1</td>
<td>Are you, putting this back on?</td>
</tr>
<tr>
<td>27</td>
<td>Jerel</td>
<td>Yeah.</td>
</tr>
<tr>
<td>28</td>
<td>R1</td>
<td>Ok, let’s see where. It actually might fit down here.</td>
</tr>
<tr>
<td>29</td>
<td>Jerel</td>
<td>Yeah.</td>
</tr>
<tr>
<td>30</td>
<td>R1</td>
<td>Ok.</td>
</tr>
<tr>
<td>31</td>
<td>Jerel</td>
<td>But they called Chris out the room for something.</td>
</tr>
<tr>
<td>32</td>
<td>R1</td>
<td>Ok, hmm. I’m sure you can handle it on your own over here.</td>
</tr>
<tr>
<td>33</td>
<td>Jerel</td>
<td>Yeah, well where was he at? Oh yeah he was up here.</td>
</tr>
<tr>
<td>34</td>
<td>G4</td>
<td>Um, I need to see you real quick.</td>
</tr>
<tr>
<td>35</td>
<td>Jerel</td>
<td>All right.</td>
</tr>
</tbody>
</table>
Whenever you finish that sentence.

(Jerel leaves the room, cameraman moves over to R1)

Ok, I’m, I want you to go around and when the posters are done, I’m gonna put a number on them and I need somebody to, to keep track for me, what number poster goes with what kid. Because we’re not putting their names on the posters.

Oh so if somebody writes the name as you put the number, can you do that?

We could. I, we’ll do that. I think, after today.

So he, so basically he’ll follow you, as you write and he writes the name.

That’s fine. And actually you can just…

Yes.

Start a new mm hmm, page.

Yeah.

So you’ll follow her (mumbles).

So what, so the information that I’ll need is that this is poster, 1, and it’s Delta’s Dice…

And who’s doing it.

And then Chris and Jerel.

Yeah.

Right. So, so…

Do you want me to write this as only Chris because Chris prepared it and then Jerel prepared something else?

I’m, no, no, Chris and Jerel.

Ok.

Yeah, um, then you, um, the, so poster number, company name, Yeah.

And actually, yeah, yeah, could we maybe do it in a column, in a table?

Yeah, but I think he can probably later on do it in a more…

Yeah, thank you. And then there’s another Delta’s Dice…

(R1 talks to Tiffany and Keisha)

(Chris and Jerel return)

You finished this?

Yeah. Where’s the other poster?

Over there. Look.

What’s your, what’s your third company?

Uh.

Delta’s. That’s the one that was fair.

No that’s the one that was…

Dice R’ Us.

Dice R’ Us. Ok.

That's the unfair one. There it is on the…
257

Chris It’s not like yours.
Jerel Ask me if I care.
Chris Do you care? Yes. You see what happened to me, I made 8 instead
of 6. (referring to the sectors on the pie graph)
Jerel We need new paper.
R1 Oh you need another sheet of paper? It’s right here.
Jerel Where? Oh there it is.
R1 This pose, this is gonna be poster number 8.
Chris Poster number 8?
R1 Yep.
Jerel But it said number 3? Oh no, the paper said number 3. (Talks off
task and sings)
Chris Where’s the marker? Give me my marker.
Jerel Why you always writing everything in red. Oh let me find out.
(G1 engages in football conversation with Jerel)
G1 So are you done with your work?
Jerel Almost.
G1 How is work going here? Jerel?
Jerel Huh.
G1 How is work going.
Jerel Good.
G1 Well, what did you, what did you finish, any?
Jerel Huh?
G1 Any posters at all? How many you doing?
Jerel Uh, three.
G1 Three posters?
Jerel Yeah this our last one.
G1 For three like, three, three…
Jerel Different, different…
G1 Different companies?
Jerel Yeah.
G1 And um, what was the conclusions on all of them?
Jerel One was unfair.
G1 Uh huh.
Jerel And one was fair. And this one is unfair, yeah this too, one is
unfair too. So you got two unfair…
G1 And how did you, how many did you sample?
Jerel Huh?
G1 Did you take trials? Or did runs? Or trials, what do you call it?
Jerel Yeah we did trials.
G1 Uh, how many, how many did you do by the way, on average?
Chris A thousand.
Jerel Yeah one thousand.
G1 A thousand?
Chris Yeah.
G1 So like for each company? Is that what you’re saying?
128  Jerel  Yep.
129  G1   How many times, a thousand, I mean you did a thousand how
130      many times?
131  Chris  Once.
132  G1   Once. I see.
133      (G1 leaves)
134  Jerel  What are you doing? (To Chris)  You writing it like…
135      (Off-topic conversation)
136  15:58  (Jerel walks away)
137  Chris  That's so neat.
138  17:06  Jerel  What are you doing? The lines are all close together. I can’t, you
139      cannot write that small. Yo, you got to write everything.
140  Chris  Who said I got to write everything?
141  Jerel  I can’t write that small.
142  Chris  Yes you can.
143      (Off-topic conversation)
144  17:56  Jerel  What was the first question?
145      (Off-topic conversation)
146  R1    Guys…
147  Chris  Go ahead write.
148      (Cross talk)
149  Chris  Write!
150  Jerel  I can’t write that small. I told you that.
151  R1    Ten minutes, twelve minutes all right?
152  Chris  Ten minutes (cross talk) that’s it. Sit here, sit here for ten minutes.
153  R1    You have ten to twelve minutes to finish your posters.
154  Student Before we leave?
155  R1    Before you take your break, for snack.
156  Chris  And then after break? Then after break we got uh, we got uh…
157  Student  And then we play games, after the break if we done with our
158      posters?
159  R1    No, no, after break then we’re gonna take a look at the posters.
160  Student  Oh.
161  R1    Ok? So posters need to be done. Ten minutes...(cross talk) get
162      these things done. All right?
163  Jerel  C’mon let’s do (inaudible) You write the second question.
164  R1    This is the last day so we can’t run over.
165  Jerel  Write the second question, I mean the first one.
166  Chris  Would you recommend that…
167  Jerel  Oh I’ll write that cause it’s easy and short.
168  Chris  All right. Where’s the paper?
169  Jerel  Huh?
170  Chris  Where’s the paper for uh, the Dice R’ Us? What’s that?
171  Jerel  Dice R’ Us.
172  Chris  Are you sure it’s Dice R’ Us?
173  Jerel  Yeah. Ain't it say it?
2022 G1 You have twelve minutes, not an appropriate time…
177 Jerel Shut up! (Jerel is holding a rubber band) Say one more word and…
178 Terrill One more word. (Gets hit with rubber band)
179 (Off-topic conversation)
180 21:41 Jerel Why you got to do that for?
181 Chris Do it with green, hurry up!
182 Jerel What’s the second question?
183 Chris It’s, what evidence do you have that the die is tested is fair or unfair?
184 Jerel (Rhyming) I got to look at the second question. (Off topic) What evidence do we have?
185 (Off topic conversation)
186 23:58 G3 You have 5 minutes now.
187 Chris Yo, we got 5 minutes, yo.
188 (Off-topic conversation)
189 Jerel All right I’m finished.
190 Chris No you’re not.
191 Jerel Yes I am. Give me some more evidence thing.
192 Chris The evidence…
193 Jerel Oh yeah.
194 Evidence for Dice R Us - ROLE 142E - 32:15
195 (laughs; reads from the poster) The evidence we have is that, the what, the percent were not close. Only one number reached 200.
196 Answers for Dice R Us – ROLE 142E – 32:30
197 3 QUESTION
198 1) I would not recommend this company because 2 of the die barely are rolled, and the rest are rolled alot.
199 2) The evidence we have is that the percent are not close, any one number doesn’t get more than other numbers. 4 numbers tops up the pie graph.
200 3) I think the chances of getting all six number is 12% / 100%, 10.62% /100%, 12.30% /100%, 18.4% /100%.
202 Jerel All right go ahead.
203 Chris No, no don’t write it in green.
204 Jerel Shut up, I already started.
205 Chris I should slap you. We got to write one more. Move. Move before I hit you.
206 Jerel Yeah all right.
207 26:00 (Chris begins finishing the poster; Jerel walks away.)
208 R2 If he doesn’t finish them you can’t go to lunch.
209 Jerel Suuurrree.
210 (Off-topic conversation)
211 28:02 Chris We finished. Go get the other thing, over there. We got to put it over here. We got to hang them up.
212 28:30 G3 …Chris I have a question for you. Uh, ok you see this dice here?
213 Chris Yeah
214 G3 What’s the number here?
215 Chris One.
216 G3 Ok. What do you think the numbers of the other sides of this dice?
217 Chris For this one?
218 G3 Yeah, ok, you see, look at this dice?
219 Chris Yeah one.
220 G3 Ok, well, what do you think were the numbers of the other sides?
221 Chris Oh I don’t know.
222 G3 Well what do you think?
223 Chris Hmmmm, let’s see. I think it’s…
224 G3 What do you think?
225 Chris One, it’s one…
226 G3 Yeah.
227 Chris Then it’s…
228 G3 Do you think it’s, if it’s one, what, what do you think on the other side is there?
229 Chris It would be two, two.
230 G3 mm hmm.
231 Chris Three, four, I don’t know.
232 G3 So they would be all different, right?
233 Chris Yeah, yeah they’d be a lot different, I think.
234 29:36 (R1 looks over the posters)
235 R1 So, guys, where’s Chris?
236 Jerel Right here.
237 R1 Oh, Chris, Jerel, come here.
238 Chris Yes.
239 R1 So, I’m looking at your last poster, and, again, so, here, we, is that the percents are not close, only
240 Chris I told him he wasn’t done yet.
241 Jerel Shut up!
242 R1 One number reached, only one number reached 200. Um, what does this say? The bar graph is not close, almost one number takes
up, one number takes up the pie graph. Ok. So first of all, it’s nice
that you’re pointing to the different representations, but again, I
don’t know, which 200. Out of how many trials?

Chris A thousand.

R1 Ok, and, uh, why does, what you’re saying here, indicate that it’s
not fair?

Chris Jerel?

Jerel Righ…

R1 What do you think? Why is this an indicator that it’s not fair?

Jerel Uh, because, uh, because some percents were at 21.4 and

Chris Yeah.

Jerel others was at 10.3.

Chris Yeah like 2 percent were at the low numbers, well like the lower
10’s and then others were at like the, closer to like the…

Jerel The highs.

Chris Yeah the highs.

Jerel The highs.

R1 But why does that, why does that mean that it’s not fair?

Chris Because, right there you know that one and six weren’t rolled that
much, so you know there’s not going to be a lot of one and six. So
there’s probably one of one and six and probably two of two, three,
four, and five.

R1 When you say there’s probably two of, you’re thinking about the
weight tool?

Chris Yeah the weight tool.

R1 Ok. All right. So I want you guys to add, just a little bit more about
this, all right?

Chris Yeah Jerel.

R1 And, it’s really about, it’s really about number 2 so you can either
add it over here or you can add it down here, draw an arrow or
something, ok?

Chris Draw a arrow. Draw arrow…

Jerel Shut up!

Chris Like this!

Jerel You draw it! And you write it. What the…

Chris Write it!

Added answer for Dice R Us – ROLE 142E – 32:40
She said, "I don't know. (Off-topic) Oh and then two of the percents are low and then the other percents are high. And the others have a high percent. All right come on."
References


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