Forcing awareness of mathematics: Self, mind, and content in dialogue

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Forcing awareness of mathematics: self, mind and content in dialogue

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Introduction
In education, the ideas of ‘forcing awareness’ and, more generally, ‘awareness’ are due to Caleb Gattegno (1988, 1987, 1974, 1970), who has elaborated on the psychology and recursive property of awareness. Insistently and persuasively, he claims that “only awareness is educable.”1 The notion of awareness and the pedagogical utility of forcing awareness inform my selection of pedagogical activities and techniques as well as how I reflect on my teaching practice.2 By structuring these activities and techniques in specific ways, I have found them useful for forcing awareness.

In this brief exploratory paper, I am concerned with communication or inner dialogue between the active self and the observing mind of a learner that results in awareness of mathematical ideas. I am especially interested in the role that forcing awareness can play in helping learners whose mathematical powers have been underdeveloped owing to material and moral conditions of oppressive societies.

Notwithstanding objectionable denotations and connotations of the verb ‘to force,’ Gattegno’s definition of forcing awareness has compelling and useful pedagogical value. Gattegno makes clear that, for learners, learning or generating knowledge occurs not as a teacher narrates information but rather as learners employ their will to focus their attention so that the mind observes the content of one’s experience and, through dialogue with the self, becomes aware of particularities of one’s experience. Specifically, in mathematics, the content of experiences, whether internal or external to the self, can be feelings, objects, relations among objects, and dynamics linking different relations (1987, p. 14; my emphasis).

Forcing awareness of exponents
Educating awareness leads to generating mathematical knowledge. I will attempt to substantiate this claim and indicate specialized and generalized awarenesses with the following example: Suppose that one calculates the integral powers of 2 from 2 to 5 and onward (see Figure 1).

In Figure 1, the exponential expressions or numerical objects in the first row are related to the objects in the second row. The double-head arrows between rows indicate that each exponential expression in the first row is equivalent to the number in the second row.

By focusing one’s attention on and, importantly, engaging one’s self in dialogue about the content of Figure 1, what can one observe and what awarenesses result?3 Dialogue between one’s self and the content of Figure 1 by different learners may result in possible different sequences of observations and interpretations.

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1For a general, theoretical treatment of awareness and his claim that “only awareness is educable,” see Gattegno (1987); and for discussions of educating the awareness of mathematics learners, see, for example, Gattegno (1974, 1988). Also, see Wheeler (1975) for further discussion and examples of awareness.
2These activities and techniques include circle expressions and equations (Hoffman & Powell, 1991), expressive writing (Powell & López, 1989), multiple-entry logs (Powell & Ramlauth, 1992), and graphing calculators with transactional writing (Powell, in press).
3You might wish to consider these questions before reading my discussion.
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awarenesses. The following sequence represents just one:

\[ 2^3 \rightarrow 2^4 \]

**Figure 2** 3 increased by 1 is 4 and 4 decreased by 1 is 3

What can be seen? First, one can become aware that, from left to right, the base of each exponential expression is the same and that the objects in the first row are related by increasing the exponent by one or, from right to left by decreasing the exponent by one (see Figure 2). Holding the base invariant and increasing or decreasing each successive exponent by 1 are operations relating objects in the first row.

Second, starting with the first object in the second row, the 4, from left to right, then each successive object is the double of the previous one or, from right to left, each successive object is half the previous one (see Figure 3). Halving and doubling are operations relating objects in the second row.

\[ 16 \rightarrow 32 \]

**Figure 3** 16 doubled is 32 and 32 halved is 16.

Combining the first and second set of observations, one can notice and articulate a link in Figure 1 between the relations among objects in the two rows. From right to left, as the exponent of an object in the first row decreases by one, its equivalent object in the second row is halved. One may preserve this awareness in the mind for future use while capturing other awarenesses or, equivalently, educating further one's awareness.

Having captured this awareness may positively affect one's affective relation to the experience. Awareness of this feeling may be sufficient to provide the requisite emotional energy to sustain focused attention on the text of Figure 1 to notice other particularities in the experience.

Using the awareness just realized and maintaining invariant the link between the relations of the two rows in Figure 1, one can further notice that starting with the first object in the first row and decreasing its exponent by one, then 2 to the first power is half of 2 to the second or is equivalent to two; that, decreasing the last exponent by one, 2 to the zeroth power is half of 2 to the first power or is equivalent to one; that, decreasing the last exponent by one, 2 to the negative first power is half of 2 to the zeroth power or is equivalent to one-half; that, decreasing the last exponent by one, 2 to the negative second power is half of 2 to the negative first power or is equivalent to one-fourth, and so forth (see Figure 4).

Moreover, awarenesses prompted by one's contact with the situation represented in Figures 1 and 4 can lead to noticing other mathematical items. For example, if one examines these awarenesses considering exponential bases other than 2, perhaps first integral and afterward rational bases, then the collection and correspondence of these specialized awarenesses can lead to more generalized mathematical awarenesses such as \( a^\infty = \frac{1}{a^n} \) and \( a^0 = 1 \).

**Discussion**

The above experience included noticing objects and unpacking relations among objects and dynamics linking these relations. Since particular noticing occurred, these events help us to construe the notion of 'awareness' and the recursive notion of 'awareness of awareness.' Stimulated by the active self, an awareness occurs when the mind perceives and captures some item or particularity of experience; hence, an awareness is a particularity that the perceiving mind preserves and that avails itself for future use. As suggested at the end of the experience above, a particularity may correspond in some way to similar particularities or specialized occurrences that the mind perceived and captured from experiences, and thus one may articulate the invariant awareness in words and signs as a generalization (Wheeler, 1975).

Whether specialized or generalized, one articulates statements about the particularities of the content of Figures 1 and 2. Each statement

\[ \ldots 2^{-2}, 2^{-1}, 2^0, 2^1, 2^2, 2^3, 2^4, 2^5 \ldots \]

\[ \ldots \frac{1}{2}, \frac{1}{3}, 1, 2, 4, 8, 16, 32 \ldots \]

**Figure 4**

\[ 16 \text{ doubled is } 32 \text{ and } 32 \text{ halved is } 16. \]

As a generalization, this statement is typical of what students write or say after the an experience such as the one I have described. Although the statement, strictly speaking, requires qualifications such as \( a = 0 \) and \( n > 0 \), here I am first concerned with clarity rather than precision. The latter requires the former. After students achieve clarity, they can reflect on the statement and determine more precisely its domain of applicability.
represents an awareness. When one's observing mind notices from a "distance" that the active self is articulating an awareness, then one achieves awareness of one's awareness. Furthermore, when the mind remains in contact with a transpiring experience and sustains dialogue with one's self about the content of the experience, one can force specialized or generalized mathematical awarenesses.

"Forcing awareness", according to Gattegno, has two meanings: "One is concerned with what we do to ourselves, and the other with what can be done to us so that we become aware of what has escaped us, or might escape us" (1987, p. 210). On the one hand, one can force their own awareness by directing their attention to perceive particularities in the content of their experience, and of course, this process necessitates discipline. Though mathematical awarenesses can come to mind exactly when one is involved in other matters, one progresses mathematically through a disciplined search for awarenesses in the content of one's experience. On the other hand, it is possible for someone else to offer guidance in forcing one's own awareness, and naturally, this requires the willingness of an individual to be guided. Nevertheless, becoming aware only can be accomplished by the individual and, therefore, is the responsibility of the individual not the guide. Clearly, for someone to force the awareness of an individual necessarily presupposes a natural relationship of trust or, at the very least, a social contract that institutionally stipulates the guide as someone with whom the individual should or must trust. Indeed, this is the relational starting point between teachers and students since the institution of schooling imposes such a social contract.

Whether prompts for forcing of awareness are internal or external, one can become aware of mathematical particularities by attending to attributes, objects, patterns, relations, and dynamics among relations contained in one's experience. In forcing awareness, learners generate mathematical knowledge. Dialogue, which may occur either among individuals or between an individual and, broadly speaking, a text, is necessary for awareness to manifest or be forced. As seen in the above example, by educating one's awareness, one may indeed generate mathematical knowledge.

**Conclusion**

The challenge to reconstruct the role of mathematics in the struggle to empower learners whose mathematical powers have been underdeveloped demands pedagogical changes. In the context of schooling, one direction of change pertains to the roles of teachers and of students. Briefly, a role of a teacher involves structuring an environment, presenting situations, creating constraints, stimulating dialogue, and posing questions, in short, providing experiences, with the purpose of drawing students' attention to aspects of experiences that contain mathematical particularities that ought not to escape their attention, be ignored, or overlooked. A corresponding role of teachers is to provide further experiences for students to obtain facility. How this can be done and how to structure instructional interventions to force the awareness of students lead to interesting questions worthy of investigation. For instance, besides using one's will to focus one's attention and engaging the self and mind in dialogue about content, the structure for writing the powers of 2 in Figure 1, I would argue, proved useful in prompting and facilitating the awarenesses that I have discussed.

The responsibility of students is both different and similar. Students must give of their time and use their will to focus their attention and enter into a dialogue with situations to educate their awareness. Correspondingly, students must further give of their time to gain facility in the area of their acquired awareness. Most importantly, mathematics educators must help students realize how to structure appropriately learning environments and use a variety of tools for forcing awareness so that they may develop the habit of forcing their own awareness to learn mathematics. In this context, it is crucial to make students aware that, as they examine the content of their experience, along with becoming aware of what is explicitly present, they must force their awareness about what is also present but hidden. Here an important instrument is the question, which can initiate dialogue between self and mind about experiential content, is "What do I see?" As students assume the responsibility and habit of forcing their awareness in the field of mathematics, the question emerges whether students become independent and autonomous learners of mathematics.

**References**


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