DISPERSION OF SILICON BASED MICRO- AND NANO-PHOTONIC STRUCTURES AND ITS DEVICE APPLICATIONS

by

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ABSTRACT OF THE DISSERTATION

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Significant dispersion can occur in silicon micro- and nano-photonic structures, such as photonic crystals and microresonators. These dispersions may cause the phase shift and group velocity of light to be highly wavelength dependent along a fixed propagation path, or cause the propagation direction of light to be highly sensitive to the wavelength. These two types of effects are called longitudinal dispersion and angular dispersion, respectively. The slow-light effect is due to the longitudinal dispersion and the angular dispersion is associated with the superprism effect in photonic crystals. Though, longitudinal dispersion has a less apparent influence on the superprism effect, as revealed through a more in-depth analysis. A synergistic theoretical framework of the dispersions is developed to enable a common examination of the longitudinal and angular dispersion in photonic crystal structures. These dispersive effects can lead to undesirable consequences, such as large losses and/or narrow bandwidths. For the slow-light effect, a basic proof will be shown for the scaling of random scattering losses due to fabrication
imperfections in a photonic crystal waveguide. For the superprism effect, a fundamental limit, the bandwidth-sensitivity product, will be presented that governs the maximum angular sensitivities and the achievable bandwidth. This product is the counterpart of the bandwidth-delay product for the slow-light effect.

A parallel-coupled dual racetrack silicon resonator structure is proposed and analyzed for arbitrary quadrature signal generation. The over-coupled, critically-coupled, and under-coupled scenarios are systematically studied. Simulations indicate that only the over-coupled structures can generate arbitrary quadrature signals. The effects of potential asymmetries in the coupling constants and quality factors of the two racetrack resonators are systematically studied. It is shown that these asymmetry effects can be compensated by small phase shifts in the two racetracks. The design, fabrication and characterization of silicon waveguides, resonator and periodic structures, including the parallel-coupled dual racetrack structure, will also be presented. The results have shown successful coupling of resonators.

With the high dispersion of silicon micro- and nano-photonic structures, light can be modulated, switched, and steered with higher efficiency and lower power consumption. Thus this study may contribute to saving energy in photonic devices.
DEDICATION

I dedicate this work to my parents Anthony and Suzanne, sister Alana, extended family, and those friends not mentioned here for their care and encouragement.
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1. INTRODUCTION

A brief review of silicon photonics in regards to waveguides, resonators and periodic structures is presented. The constraints on physical structures and fabrication processes influencing design are related to photonic crystal and resonator structures of interest. Some aspects of dispersion control are discussed to give a brief background, basic theory and comments on active devices. Lastly, the dissertation research related to dispersion control is introduced.

1.1 Silicon Photonics: A Brief Survey

Silicon photonics initially emerged near the mid 1980’s with a number of issues, many of which were partially resolved to various degrees depending on the application of the technology [1]. Some of these issues were related to waveguide geometry, a means to achieve low waveguide loss, fiber to waveguide mode couplers, optimum materials and index contrast, high speed Si based detectors at 1550nm and electro-optical modulation by carrier injection. Some of the possible applications that have been developed include filters, resonators, couplers, sensors, switches and Wavelength Division Multiplexing (WDM) systems. As progress has been made, additional directions for exploration have emerged, which include the advancement of locally strained Silicon devices into the nonlinear device mainstream, all optical circuits, optical interconnect with low cost and high rates, and silicon based photonic circuits embedded with photonic crystal (PC) or microresonator components.
Silicon possesses a number of important properties relevant to photonics [2]. One of silicon’s most significant optical properties is its low loss wavelength region shown in the Fig. 1.1, which includes the data communication spectrums. Other important properties include those related to high index contrast between silicon and silicon oxide, third order nonlinearities potentially enhanced when using a top oxide cladding layer, large thermal conductivity lending itself to thermal modulation or tuning applications and high tolerance to optical damage allowing for high input power applications.

Fig. 1.1. High transparency wavelength region from $\lambda \approx 1.1 \mu m$ to $\sim 6.9 \mu m$ adapted from[2]

The large index contrast between silicon ($n = 3.45$) and silicon oxide ($n = 1.45$) allow good optical confinement, large optical intensity and small mode size [2]. It should also be noted that it has a strong thermal-optic coefficient $\partial n/ \partial T =1.86 \times 10^{-4} \ K^{-1}$, allowing for thermo-optic tuning or modulation of effective index in photonic structures [3] [4]. The good optical confinement is a significant effect enabling sub-micron or nano-photonic devices to be possible. The large optical intensity in such sub-micron nanophotonic devices, in part, enables the Kerr effect and other properties of nonlinear optics to be easily observable. The small mode size on the order of approximately $0.1 \ \mu m^2$ is then compatible in size with IC fabrication, which creates the possibility for eventual
integration with mainstream manufacturing processes. Scattering losses and surface roughness are related by the index contrast when considering silicon and SiO$_2$ or air top cladding [2]. Propagation losses dependent on processing and scale, can have values within the span 0.1 dB/cm to 3 dB/cm for conventional silicon waveguides and 4.1+/-0.9 dB/cm for PC silicon waveguides [5]. Sidewall smoothing can reduce scattering losses due to surface roughness while preserving the rib waveguide profile using thermal oxidation at temperatures near 1100°C. Significant index contrast continues to be a source of potential optical device improvement specifically in regards to a reduced bending radius for silicon waveguides fabricated on SOI wafers allowing 2.5µm bending radius and recently a 1.5µm racetrack resonator radius enabling devices filters with a FSR of on the order of 80nm from a ~100µm footprint [2]. Again this leads to compact designs using nano-scale waveguides or filters for potential integration into mainstream manufacturing.

A brief review is included of some potential contributions to the development of all optical circuits by devices utilizing dispersion effects. These dispersion effects are related to the slow light effect and superprism effect in conventional waveguides, racetrack resonators, photonic crystal (PC) structures, coupling, and active control. Additionally, a short introduction to the slow light effect is provided with respect to slow light modes resulting from backscattering and omnidirectional reflection. Some basic theory is then discussed to support the concepts of slow light, the Kerr Effect and related losses. More specifically the group velocity, the group index and the Kerr Effect are described in regard to how they vary with changes in refractive index or effective index.
Then the refinements in conventional waveguides, resonators, PC structures and coupling are presented, which supports some of the work done in active control. [3,6]

Fiber interface coupling can be a significant source of loss if a modal mismatch is present. This is possible when considering that the effective modal area of a single mode fiber is $50\mu m^2$ or lensed fiber with $2.5\mu m^2$ spots size and the effective modal area of a nano scale conventional waveguide is $0.1 \mu m^2$. The use of a mode converter applying taper like structures can potentially lead a manageable loss value of 0.2 dB, whereas other methods of interface coupling include silicon etched surface straight gratings and curved gratings, which have a loss of 1dB. [2] Some of the significant applications emerging from passive silicon photonic devices include wavelength demultiplexing by arrayed waveguide gratings, add/drop filters by microring or microdisk resonators, and opto-electromechanical or opto-electroacoustic systems. [2]

Optical resonators employ one of two properties being distributed Bragg reflection (DBR) or (almost) total internal reflection (ATIR). [7] DBR is property of the interaction of light with periodic structures along the guided modes’ direction of propagation. [7,8] ATIR is a property of the interaction of light with the interface of a lower index cladding and a higher index dielectric material. Since the geometry of microresonators influences the optical mode spectra, then the exploration of these optical properties can be governed by the light confinement aspects of DBR and/or ATIR for numerous geometries. The applications possible using microresonators require single mode operation, large quality factor, large free spectral range and small mode volume, which potentially allow for small device size. Quality factor can be a metric for gauging the usefulness of a resonator structure, representing the extent of the devices capability to
recirculate coupled light with respect to dissipating it, meaning that the device would have a long decay time. Large free spectral range is needed for WDM applications, which means that there is a large wavelength spacing between notches in the transmission spectrum in the case of a single racetrack and single waveguide. In general, microresonators are challenging to fabricate with large quality factors and more so with small mode volume while maintaining low loss, but if successful the potential applications are significant when the quality factor to mode volume ratio is very large. [7,8]

Recently in the silicon photonics field, a considerable amount of work has been done on the subject of PC devices in all geometric dimensions. 2D PC slab structures strongly confine photons where in-plane confinement is attributed to the photonic band gap (PBG) and out-of-plane confinement is related to total internal reflection (TIR). The various properties explored with these devices include slow light, superprism effect, negative index of refraction, waveguides, waveguide bends and resonators. [1] The PC racetrack resonator structure marked the merging of the microresonator and PC structures, which have only recently has been explored. [9]

Silicon is the primary material for mainstream IC fabrication, but it is not the best material for electro-optic control of an optical signal such as modulation applications. This is because the centrosymmetric crystal structure of silicon prevents the Pockel effect from being observed. Since modulators traditionally make use of Pockel effect and its linear properties, then it is not possible in silicon alone to function as a modulator in that way. Though, the Pockel effect will be viable in silicon when a silicon PC waveguide is strained by a SiO$_2$ layer, allowing electro-optic modulation. Other methods for
modulation use changes in effective index, by using the carrier densities’ linear relationship to the absorption coefficient or by applying the plasma dispersion effect in relation to free carriers in the semiconductor. [2] The index can also be modulated or tuned by thermo-optic effects [3,4]. The direct relationship to traditional IC fabrication using MOS structures in the modulator design which is inline with the CMOS based fabrication processes. Furthermore, the intrachip communication or even internal system level communication could benefit from silicon photonics since the potential energy cost per bit of 100fJ. [2] This could imply less of a temperature reduction need for on chip cooling.

Silicon photodetectors operating in the band to band absorption range, which is shorter than a wavelength of 1000nm, are the earliest silicon photonic device with active control. Applications include sensors and commutations systems because of the devices’ low loss region overlapping the communication channels. Further development of photodetectors for power monitoring purposes include the improvement of silicon’s absorption properties below the bandgap by helium implantation allowing a wavelength range of 1440nm to 1590nm to be detected. [1]

Microresonators have become a fundamental building block in optical circuit design. The resonator can be composed of conventional waveguides or PC waveguides when they are integrated in devices; they can continue the trend to low power, higher sensitivity, and smaller footprint. The applications for this building block include sensors as photodetectors, thermooptic control, electrooptic control, switches, lasers, emitters, filters, Mach–Zehnder interferometers and others. The resonator itself, besides being patterned from either PC structures, conventional waveguides, spheres, disks or other
geometries, can be manifested as a broad spectrum of structures that can interact with other effects, structures or materials. [1]
1.2 Physical Design and Process Constraints

The physical relationships that govern the selection of values, conditions or constraints are discussed for the below set of parameters or concepts in designing the photonic structures with respect to wavelength and material. Some of the influencing parameters for conventional waveguide based structures and concepts are waveguide height, width and length for confined single mode propagation while considering loss effects. Other influencing parameters include the coupling separation and the coupling coefficient; the coupling length and the effective coupling length with respect to the coupling coefficient; the bending radius and the cross-section with respect to the bending loss; and the cladding thickness. Some of the influencing parameters for photonic crystal bases structures and concepts are the selection of hexagonal instead of other 2D photonic crystal patterns, PC hole diameter, and PC spacing. Design can also be influenced by the fabrication processes or equipment limitations.

Conventional waveguide height and width for confined single mode propagation with respect to the single mode condition is constrained with respect to wavelength and material. To maintain optical confinement, a rough approximation of waveguide width \( \lambda_0/2n_{\text{eff}} \) in a silicon medium of index \( n \sim 3.475 \) at room temperature and pressure with an operating wavelength (\( \lambda_o \)) of 1550nm. For a rib waveguide, the propagation angle is greater than the critical angle (\( \theta_c \)), which is influenced by the cross-sectional dimensions of the structure. The height (h) can be solved in terms of the operating wavelength and the indices of adjacent mediums at the core/cladding interface required for a single mode waveguide [10]:
For the coupling between two waveguides, the coupling separation and coupling length between two waveguides depends on the coupling coefficient. Coupling occurs when the coupling separation (s) are small enough to allow the evanescent field of one waveguide to overlap the adjacent parallel waveguide core. The coupling separation influences the coupling efficiency (κ), which expresses a relationship to other parameters such as waveguide(WG) width and propagation constant (β) [10].

\[
κ = \frac{2k_{xs}^2k_{xc}e^{-k_{xs}s}}{βw(k_{xs}^2 + k_{xc}^2)}
\]  

The decay constant in the x-dimension is \( k_{xs} = \sqrt{β^2 - n^2k^2} \). The propagation constant in the x dimension is \( k_{xc} = \sqrt{n^2k^2 - β^2} \). The coupling length for complete power transfer from one waveguide to the other is inversely proportional to the coupling coefficient. Additionally, it should be noted that the effective coupling length for a racetrack microresonator is slightly larger than that of the straight segment in the coupling region, since the increase in separation along the arc of the racetrack near the straight segment of the racetrack is small. This difference in coupling length and effective coupling length decreases as the resonator radius decreases to a limit based on WG cross-section for single mode operation [10].

\[
L_{eff} = L_π + \sqrt{2πrs}
\]  

The bending radius and corresponding WG cross-section has an influencing constraint of bending loss. Bending radius is theoretically constrained by loss when
fabrication limitations are not considered. Conceptually, the loss can be treated as a bent antenna having a propagating field inside it which will radiate light at the bends. The expression for bending loss is [9]

$$\alpha_{bend} = C_1 e^{-C_2 R}$$  \hspace{1cm} (1.4)

$$C_1 = \frac{\lambda_o \cos^2 \left( \frac{k_{wg} W}{2} \right) e^{-k_{wg} W}}{w^2 k_{wg} n_{eff} \left[ \frac{W}{2} + \frac{1}{2k_{wg}} \sin(wk_{wg}) + \frac{1}{k_{wg}} \cos^2 \left( \frac{k_{wg} W}{2} \right) \right]}$$  \hspace{1cm} (1.5)

$$C_2 = 2k_{wg} \left( \frac{\lambda_o \beta}{2m_{eff}} - 1 \right)$$  \hspace{1cm} (1.6)

The bending loss exponentially decreases as $R$ increases. The factor $C_1$, in the bending loss equation, can also be reduced by decreasing the width and height of the rib waveguide, limited by the single mode propagation condition for the rib waveguide dimensions, which linearly shrinks the bending loss. The reduced cross-section has a compounding effect, which physically allowed for a smaller bending radius, again lessening the loss [10]. Racetrack resonator racetrack path length was selected to achieve a desired free spectral range for an acceptable minimum bending loss and minimum bending radius that allowed resonance to occur without being overwhelmed by noise.

The length of the conventional waveguide was not an issue in the case of the designs implemented in this work since the length needed for testing was only near 1cm and the racetrack resonator region was always smaller than a few hundred micrometers. Propagation loss would become a constraint if the length of the waveguide was large enough to caused the cumulative effect of that loss to overwhelm the signal, then it its parameters would be adjusted.
Photonic crystal (PC) structures also have a variety of influencing factors, which include the selection of hexagonal instead of other 2D photonic crystal patterns, the PC hole diameter and the PC spacing. Though photonic crystal waveguides (PCWs) do not share of the same bending loss or waveguide roughness effects with conventional waveguides, since changes in propagation direction tend to occur on lines of symmetry of the lattice and effects of roughness are confined to the hole or rod of the PC structure. The selection of hexagonal structures instead of the other 2D photonic crystal patterns is based on the fact that the largest bandgap possible, allowing better in-plane confinement, for a 2D photonic crystal lattice with constant index contrast is hexagonal excluding honeycomb which can be reduced to hexagonal components (where the brillouin zone is closest to a circle) [11]. In general a larger bandgap allows for waveguide that is easier to design and fabricate. Again, the air hole diameter is chosen to achieve the largest bandgap. Also the location of the mode at approximately halfway in the bandgap, desired for good confinement, can affect the choice of air hole size in the lattice [11]. The photonic crystal hole spacing in silicon is related to the wavelength and the index of the medium in which it is propagating and has a value on the order of $\lambda_0/2n_{eff}$ as noted earlier. Lastly, the operating point on the dispersion contour should be in the neighborhood of the high symmetry point K. To couple to this operating point, the light direction in the incident medium must have a surface-tangential wavevector component that matches the surface-tangential component of the operating $\mathbf{k}$ point in the photonic crystal superprism.

The parameters which influence fabrication processes and concepts include electron beam lithography resolution, electron beam resist properties, etching and deposition. Resist resolution has many factors that can affect it, and some of them
include selectivity, thickness and molecular size. Higher resist selectivity during dry etching will allow for thinner resist which in turn can improve resolution. Resist resolution is limited by the molecular size of the compound and thickness of the resist layer spin coated onto the wafer. Election beam lithography resolution is limited by the system and settings used in its operation. The observed resolution limit for the system used for much of this work was ~40nm, though the Raith/eLine system could have single line resolution of ~10nm. The single line e-beam writing process is not applicable for this work, since it can not be used to build the structures designed. Moving beam selection introduces stitching which causes periodic noise that can overwhelm the signal. Stitching can be mitigated by properly aligning the electron beam lithography stigmation, aperture and focus, but not completely eliminated.

Some device design constraints from fabrication processes include the resolution limitations of e-beam lithography, the etching profile of device sidewalls and the etching rate. Sidewall roughness issues emerge as a byproduct of effects from dry etch processes and resist resolution. Stitching effects also introduce limitations causing cumulative periodic noise in photonic structures, which can overwhelm the signal. Some errors during the fabrication process can be compensated for through active control of the device, such as by thermo-optic control of the index to address unintended asymmetries or shifting the wavelength of resonance peaks closer to their intended value [12,13].
Fig. 1.2. Conventional waveguide cross section
1.3 Silicon Photonic Crystals for Dispersion Control

Photonic crystals are periodic dielectric structures on the sub-wavelength scale, possessing a wide range of extraordinary properties that are absent in conventional materials. First and foremost, the periodic structure of a photonic crystal causes photonic bands and bandgaps to form on the frequency spectrum of photons. Therefore, photonic crystals with photonic bandgaps can serve as “perfect mirrors” to confine light in small dimensions, forming ultracompact waveguides and cavities. On the other hand, there are other technologies that can also provide tight confinement of light. Compared to these alternative technologies, the uniqueness of photonic crystal-based waveguides and cavities often comes from the idea that the periodic structure of a photonic crystal provides some additional distinctive opportunities to modify the spectral property of light, leading to many dispersive effects with a wide range of applications. For example, in a photonic crystal waveguide, the dispersion relation, $\omega(k)$, generally has a portion where its slope tends to zero, implying a vanishing group velocity. Such a slow light effect, together with the tight light confinement provided by the photonic bandgap, leads to extraordinary enhancement of phase shift and time delay in such a waveguide.[14]

It will be emphasized that some of these dispersion effects can be significant in their own right, without the presence of tight light confinement. For example, the superprism effect [15] causes the beam propagation angle inside a photonic crystal to be highly sensitive to the wavelength of light. Essentially, this effect amounts to significantly enhanced angular dispersion. In the superprism effect, the light propagation is not confined. Moreover, it will show that the superprism effect does not necessarily
appear near a bandgap (or at a bandedge). It can indeed appear in the midst of a photonic band, because of the high symmetry of a photonic crystal structure. The presence of a photonic bandgap is not a necessary condition for the superprism effect.

In this section, a review some of recent theoretical and experimental works concerning these dispersive effects is presented. In these works, emphasis was placed on finding the general, *quantitative* physical laws governing these effects. For example, in the first superprism experiment, it was found that the photonic crystal could enhance the angular dispersion or sensitivity by 500 times;[15] later numerical simulations and experimental works reported varying enhancement factors.[16-20] However, a rigorous, compact mathematical form of the physical law that can express this enhancement factor in terms of the photonic band parameters is missing. As such, it is easy to obtain an instance of high sensitivity structures, but one cannot systematically predict the trend of such an effect and it is unknown whether there is a quantitative upper limit of the enhancement factor. The works were devoted to elucidating such issues. While the slow light effect is obviously due to the dispersion of a photonic crystal along the direction of light propagation, namely the longitudinal direction; the superprism effect, apparently related to angular dispersion, will be shown to also have an elusive dependence on the longitudinal dispersion. Some subtle connections and distinctions between the slow-light effect and the superprism effect will be revealed through physical analysis. This allows us to examine these two effects under a common theoretical framework based on photonic crystal dispersion function, $\omega(k)$. An example will be used to illustrate the value of this synergistic theoretical perspective on the slow-light effect and superprism effect, two seemingly distinctive phenomena in photonic crystals. Dispersive effects are
frequently accompanied by high loss and/or narrow bandwidth. These issues will be discussed for both longitudinal and angular dispersions. Similar to the bandwidth-delay product for the longitudinal dispersion, a simple, yet fundamental, limit will be introduced that governs the bandwidth and sensitivities of the angular dispersion.
1.3.1 Background

Note that the realization of controlling the speed of light, specifically being the ability to artificially reduce the speed of light or group velocity thereby exhibiting the slow light effect. This concept has been developing since some of the early experimental work, which was done by Prof. Hau et al. of Harvard. [21] This property is primarily understood as longitudinal dispersion. Many different methods of slowing light have been established, some of which are mentioned here. The electromagnetic induced transparency (EIT) method uses material dispersion. The population oscillation (PO) method is a semiconductor based system. Then the final method mentioned here, which is the focus of this brief review, employs the characteristics of periodic dielectric structures in relation to structural dispersion, such as PC devices. The general advantage of PC devices over EIT devices for slow light is that PC devices can operate at room temperature, in an unpressurized environment and the small size of the system potentially allows for PC devices to support the development of all optical circuits. [22,23] .
1.3.2 Basic Theory

The basic theory presented here, based on the papers reviewed, is to support the discussion of slow light with regard to optical loss, Kerr Effect and contributions to optical circuits. The slow light or the slow mode, also directly related to the group velocity can be described as a slow moving interference pattern generated by the interaction, at resonance, between the periodic lattice of a PC and the guided mode. In the review by Krauss and the review by Baba et al., it is suggested that PC devices using slow light enable more control of changes in phase and the spectrum bandwidth of interaction in contrast to single cavity devices. [22,23]

Here pulse-like optical signals are considered, where n is substituted with group refractive index $n_g$, and for $n_g$ significantly larger than n:

$$n_g \approx \frac{\Delta n}{\Delta \omega / \omega} \quad (1.7)$$

The group velocity is the velocity of the light pulses and its relationship to group index is shown below. Also, note that $V_g$ can be reduced by a large 1$^{st}$ order dispersion effect.

$$V_g = \frac{\partial \omega}{\partial k} = \frac{c}{n_g} \quad (1.8)$$

The maximum bandwidth can be determined from group index. [22]

$$\Delta \nu = \frac{1}{2\pi} \frac{c}{n_g} - 0.5 \frac{2\pi}{a} = \frac{c}{2n_g a} \quad (1.9)$$
For a constant change in refractive index, the bandwidth is narrow for a very large $n_g$. Slow light effects can be observed when operating near the resonance of a structure with dispersive properties such as PCWs or cascaded microresonators. Also, note that the real and imaginary parts of refractive index are very large near resonances. Consider the case where the Lorentz model resonance frequency is already close to the photonic bandgap’s (PBG) center frequency [22,23]. For each operating wavelength there is a Lorentzian linewidth, which is related to the resonance of the cavity. Furthermore, the PBG characteristics of optical operation are changed considerably by the effects of dispersive materials, such as widening of the bandgap has increased effects related to dispersion. Also, the photonic band structure is distorted when the Lorentz model’s resonant frequency becomes closer to the PBG center frequency due to the photonic band structure being encroached upon by the out-of-band properties. The out-of-band effects can be considered to be partly related to the interference between the mediums of conventional and periodic structure segments in the device. [24-26]
1.3.3 Nonlinear and Active Devices

The characteristics of the nonlinear effects are discussed in related literature as being amplified by the fact that the light remains in the material of the device for a longer duration, so that this slow light is allowed to have more interaction time with the nonlinear medium. The phase velocity divided by the group velocity is the slow down factor $S = v/\nu_g$, where the group velocity is reduced, due to changes in refractive index, causing an increase in the slow down factor and indicating a slower mode. The refractive index change ($\Delta n$) is proportional to the slow down factor ($\Delta n \propto S$). Slow light devices and their effects can be scaled with changes in refractive index, when considering dispersion and bandwidth. The slowdown has two benefits from a PC’s nonlinear interactions of a Kerr medium with light, which depend on the square of the slow down factor. These benefits are an increase in intensity and improved phase change. The Kerr effect is a 3rd order nonlinear effect where the $\Delta n \sim E^2$ square of the electric field and it is much weaker than linear effects. It should also be noted that the Pockels Effect is a 2nd order nonlinear effect, $\Delta n \sim E$, polarization $\sim E^2$.

In general, the beneficial effects of slow light in nonlinear mediums are only true for PC structures and other dielectric mediums. More specifically, material resonance based slow light does not have similar benefits. [23] The Kerr medium enables the electro-optic effect with the benefit of not having optical polarization dependence and has applications of all-optical modulation, beam steering, dynamic focusing, and beam shaping. [23,27]
The change in the refractive index conducted in a controlled manner has the capability of controlling slow light by tuning the group velocity. This change can occur through multiple methods, such as thermo-optic tuning via a micro heater [3] and injection based tuning via PIN diode [27]. In addition, a change in refractive index can be achieved by applying a high intensity optical field on a Kerr medium. The altered index of refraction is \( n = n_0 + n_2 I \), where \( n_2 \) is the Kerr constant and \( n_0 \) is the index of refraction without an applied optical field [21,90]. A method of controlling slow light is to thermally adjust the refractive index in photonic structures. Designs of such a structure would also account for parameters effecting the WG bends and the interface coupling. Potential future work could be the introduction of a tapered input WG to reduce the Fabry-Perot Noise in this device, which is due to WG interface reflection. This could help improve the accuracy of measurements. This type of work can be applied as MZI modulators with future potential application of all-optical buffers, variable optical buffers and dynamic dispersion compensators. [3]
1.4 Silicon Microresonator for Dispersion Control

Microresonators are waveguide realizations of Fabry-Perot resonators. In a typical resonator modulator, light in an input optical waveguide is side coupled into a microresonator. Optical power circulates in the microresonator before it couples back to the original optical waveguide. Microresonators have been extensively studied [28-31], and racetrack resonators also received some attention [29,32-34]. Dispersion is a wavelength dependent effect. These effects can be characterized in multiple ways, such as spreading or the shifting of the notch created by a resonator in a wavelength spectrum. Also, phase changes near resonance shows dispersion. A theory has been developed to describe the optical coupling and resonance in such a waveguide-resonator system [30,31]. Initially it is assumed there is a forward transmission coefficient, $\tau$, in the through-waveguide, the normalized output amplitude of such a waveguide-resonator can be readily expressed as [31]

$$b = \frac{\eta e^{i\theta} - \tau}{\eta e^{i\theta} \tau^* - 1},$$

(1.10)

where $\eta$ is the normalized light amplitude after one circulation (hence, $20\log_{10}|\eta|$ gives the round-trip optical loss), and $\theta$ is the phase shift per cycle. At resonance $\theta=2\pi$, the output amplitude will vanish if the critical coupling condition [30,31,35], $\eta=\tau$, is satisfied. A typical output spectrum of a microresonator critically coupled to a waveguide is shown in Fig. 1.3(a). The corresponding phase shift at the output is given in Fig. 1.3(b). Obviously, in a narrow wavelength range, the intensity exhibits a dip, whereas the phase
undergoes a sharp change. The (loaded) resonator quality factor $Q$ is defined as $\frac{\lambda_0}{\Delta \lambda}$, where $\Delta \lambda$ is the full width half maximum of the resonance peak. The delay time is given by $\tau = \frac{d \phi}{d \omega}$. For a resonator having a quality factor $Q$, a significant phase shift (up to $2\pi$) occurs in bandwidth $\Delta \omega = \omega_\phi/Q$. The corresponding time delay is on the order of $QT_0$, where $T_0$ is the oscillation period of lightwave. Obviously, the delay could be significant for a high $Q$ resonator. It is both the duration that a pulse of light is delayed for multiples of that period times $Q$ and a measure of how well light stays in the racetrack. Delay time has strong wavelength dependence and it is dispersive. It can be controlled by carrier injection shifting the phase or thermo-optically.

![Fig. 1.3. Single racetrack resonator spectra: (a) normalized intensity; (b) phase.](image)

**Modulation** refers to the ability to control a new device structure or phenomenon. In the case of racetrack resonators operating near resonance, the modulation can be enhanced or controlled by a number of factors. [1] One factor is small index changes, which can be induced by many methods, one of which is free carriers by two-photon absorption. One of the positive effects is a potential reduced power requirement for switching and deep modulation depths. The design of the racetrack resonator was affected by the need to maintain single mode guided light in the waveguide and to
achieve approximate critical coupling from the waveguide to the racetrack resonators. It is interesting to note that photonic crystals can also be used to construct microresonators. [9]
1.5 Objectives of the Dissertation Research

The objectives of the dissertation research are to understand the fundamental aspects of the dispersion in micro- and nano-photonic structures and investigate its applications. Specifically, the loss and bandwidth of slow light and superprism effects will be investigated. Their scaling with respect to the group velocity and angular sensitivity will also be analyzed. This will clarify the fundamental limit of practically achievable values for the slow light group velocity and superprism angular sensitivity.

In addition, the microresonator structure will be studied for advanced dispersion control. The specific interest of this portion of the work is to independently manipulate the intensity and phase in microresonators so as to seek applications in coherent communications. Additionally, this technology has the potential of reducing the energy usage for on chip communication and the need for cooling. Some of these micro- and nano-photonic structures that are analyzed theoretically will also be fabricated and characterized in a preliminary form.
2. LONGITUDINAL DISPERSION IN PHOTONIC CRYSTALS

Longitudinal dispersion in periodic structures in silicon is addressed with regards to its lossy processes, scattering loss calculations and applications. Coupling, absorption, radiation and scattering losses through roughness and interfaces are discussed. This is followed by the exploration of scattering loss calculations using scattering matrices and coupled mode theory. Additionally, a simple proof of scaling of roughness-induced scattering loss in the slow light regime is presented. Lastly, some slow light effect applications are examined.

A theoretical concept in reference to coupled mode theory was developed for scattering loss calculations in PCWs. The transverse field expression of PCW coupled mode theory can be used \( \psi = [E_x, E_y, H_x, H_y]^T \) [36]. A previous study Ref. [36] on the coupled mode theory for PCWs indicated the eigenmode orthogonality as a significant issue.

The roughness-induced loss in a PCW is examined for multiple factors. Roughness-correlation has a relationship to spatial phase variation that is involved with the creation of loss. The speed of light can be reduced by photonic crystal waveguides for application of modulation, switching, sensing and delay lines. Growth in subjects related to photonic crystal waveguides is hindered by the issue of optical loss in the slow light regime, including loss due to roughness. Examination of slow light loss has occurred, though for some properties conclusive and accepted understandings are unavailable or unclear. For instance, theory for roughness-induced loss through radiation has a scaling factor of \( 1/\nu_s \) and the loss through backscattering [37] has a scaling factor
of $1/v_g^2$ assuming no multiple scattering affects. The power law $v_g^{-\nu}$ is sometimes used for the fitting of loss data from experiments and $\nu$ usually has significantly varying values \([38,39]\). A universal understanding should be possible for a theory in regards to the loss and related strength due to roughness and backscatter loss, where experimental device structure and roughness characteristics are changed within some ranges. The characteristics can be represented by parameters other than $v_g$, to illustrate a varying effect from loss. In general, the theory shall have the potential of allowing methods of reducing loss to eventually emerge.

The propagation loss is distributed into radiation loss ($\alpha_1 n_g$) and backscatter loss ($\alpha_2 n_g^2$) terms. Propagation loss is expressed as $\alpha = \alpha_{\text{rad}} + \alpha_{\text{back}} = \alpha_1 n_g + \alpha_2 n_g^2$ and is shown in fig. 2.1. The 1\textsuperscript{st} and 2\textsuperscript{nd} rows of air holes mainly cause radiation loss. The 1\textsuperscript{st} rows of air holes mainly cause the backscattering loss, according to a recent study \([41]\).
2.1 Estimate of Scattering Loss for Slow Light Photonic Crystal Waveguides

The practical application of the slow-light effect is primarily limited by optical loss. For most practical applications, group velocity values of 100 or less have been currently considered. Further slowing down light causes the optical loss to increase significantly. To understand the slow light effect, a close examination of the accompanying optical loss is warranted.

The total insertion loss of a photonic crystal waveguide is given by

\[
\text{Loss (dB)} = 10 \log_{10} C_1 + 10 \log_{10} C_2 - \alpha L, \tag{2.1}
\]

where \( C_1 \) and \( C_2 \) are the coupling efficiencies at the input and output end of the photonic crystal waveguide, and \( \alpha \) is the propagation loss coefficient. Note the definition, \( 0 < C_1 < 1, \ 0 < C_2 < 1, \ \alpha > 0 \). Also the \( \alpha \) term dominates the other terms as \( L \) increases. The loss coefficient can be expressed as

\[
\alpha = \alpha_1 n_g + \alpha_2 n_g^2 + \ldots \tag{2.2}
\]

where the first term can be attributed to absorption \textit{and} out-of-plane scattering by random imperfections in the photonic crystal waveguide, and the second term can be attributed primarily to back-scattering into the reverse propagating mode with an identical group index. The backscattering can be due to random imperfections.
Several works [37,42] have theoretically discussed the scaling of scattering loss. Here a proof of Eq. (2.2) is given that does not invoke the detailed solution of the waveguide equation. Backscattering and radiation loss will be shown to have different $n_g$ dependencies. In fig. 2.2 depicts a relationship between backscatter and radiation mode as a linear transform transmission matrix.

The scattering process is considered in regards to how it is due to random imperfections in a photonic crystal waveguide. For any scattering event of interest, the initial state must be a guided mode, which is assumed to have propagation constant $\beta$. The final state can be a guided mode or a radiation mode. For a line-defect waveguide formed in a photonic crystal slab, the radiation modes propagate out of the plane. Assume the scattering amplitude between an arbitrary initial state $\beta$ and a final state $k$ is $T_{k\beta}$. In any physical situation, the incoming light is always a wave-packet with a continuous distribution of $\beta$ values, although often the $\beta$ values are within a narrow range centered around $\beta_0$. Packet is a collection of wavelengths and a wavepacket is the integration of
the scattering intensity over k. The scattering intensity is $T^2$ and $S_{\text{eff}}$ is the total scattering loss for such a wave-packet, which is roughly

$$S_{\text{eff}} \sim \int d\beta \sum_k |T_{k\beta}|^2 \delta(\omega_f - \omega_i)$$

$$\sim \int d\beta \int d\omega \frac{1}{v_g} |T_{k\beta}|^2 \delta(\omega_f - \omega_i) + C_B \int d\beta' \int d\omega |T_{\beta'\beta}|^2 \delta(\omega_f - \omega_i),$$

(2.3)

where $k'$ represents a final radiation mode, $\beta'$ represents a final guided mode, and $C_B$ is a constant. The factor $\delta(\omega_f - \omega_i)$ ensures that the frequencies of the initial and final states are the same, while satisfying energy conservation. The $(1/v_g)$ factor from $(d\beta/d\omega)$ will arise naturally from each integration of an arbitrary function with respect to $\beta$ or $\beta'$

$$\int f(\beta)d\beta = \int f(\beta(\omega))d\omega \frac{d\beta}{d\omega} = \frac{1}{v_g} \int f(\beta(\omega))d\omega.$$  

(2.4)

Therefore, find

$$S_{\text{eff}} \sim \tilde{T}_1/v_g(\beta) + \tilde{T}_2/v_g(\beta)v_g(\beta')$$

$$= \tilde{T}_1/v_g(\beta) + \tilde{T}_2/v_g^2(\beta).$$

(2.5)

where $\tilde{T}_1$ and $\tilde{T}_2$ are some constants. The second line in Eq. (2.5) follows from $\beta'=-\beta$, according to energy conservation in typical photonic crystal waveguides. Note that a similar factor $1/v_g(k')$ may arise from the integration $d\omega$ as well. However, for a narrow bandwidth, the group velocity, $v_g(k')$, of a radiation mode never vanishes. Therefore, this factor has no significance here and is absorbed into $\tilde{T}_1$. Thus, Eq. (2.2) is proved.

The above derivation clearly shows that the out-of-plane scattering has only one $n_g$ factor because only the initial state is a slow-light state; whereas the backscattering
process has a $n_g^2$ factor because both the initial and final states are slow-light states. One would be tempted to assume that the value of the second integral in Eq. (2.3) is much smaller than that of the first, because there are a large number of radiation modes that satisfy the energy conservation whereas only one backward guided mode does so. In other words, the total “scattering cross-section” of all radiation modes could be much larger than that of the backward guided mode. However, a general, rigorous proof is needed.

Experimentally, a quantitative evaluation of these scaling laws has been elusively difficult and the reported loss dependences[6,38,43,44] vary between $v_g^{-1/2}$ and $v_g^{-2}$, as discussed in Ref. [39]. It should be clarified that because the scattering events occur with statistical uniformity over a given distance, the scattering loss should have the general form $\alpha = -(1/L) \log(T_{\text{prop}}) \sim v_g^{-\tau}$, not $T_{\text{prop}} \sim v_g^{-\tau}$. On the other hand, the coupling loss coefficients should have the form $C_1, C_2 \sim v_g^{-\delta}$, where $\delta = 1$ for a normally (abruptly) terminated photonic crystal waveguide Eq. (2.1). Therefore, for a normally terminated photonic crystal waveguide, the total insertion loss is given by

$$\text{Loss (dB)} \approx B_0 - 20 \log_{10}(n_g) - (\alpha_1 n_g + \alpha_2 n_g^2)L,$$

(2.6)

where $B_0$ is a constant that gives the “baseline” insertion loss.

Here several outstanding issues are listed characterizing the optical loss in the slow light regime against the original total PCW insertion loss Eqs. (2.1) using the loss coefficient expression (2.2), as the expanded total PCW insertion loss (2.6): (1) the unknown relative magnitudes of $\alpha_1$ and $\alpha_2$; (2) the difficulties of separating the coupling loss and propagation loss in experiments; (3) the proper application of the scaling laws of
the propagation and coupling losses. Systematic and careful experimental studies over a wide range of waveguide parameters must be performed before a conclusive statement can be put forth regarding the optical loss in the slow-light regime.
2.2 Lossy Processes: Radiation and Backscatter

Using analytic theory, the case of a PCW air-bridge has been further examined for analysis of the radiation and backscattering loss calculation for TE guided modes, where the electric field is approximately in the plane of propagation. The loss coefficient is found to be [41]

\[ \alpha = \alpha_1 n_g + \alpha_2 n_g^2, \]  

(2.7)

\[ \alpha_1 \approx 2N_{x,rad}(n_1^2 - n_2^2)n_{sub}(k_0^2 \sigma^2 l_{slab}/aw_0)|e_{eff,\beta}\bar{e}_{eff,\alpha}|^2 \cdot \mathcal{R}_0(\alpha_1 r_0), \]  

(2.8)

\[ \alpha_2 \approx 2N_{x,back}(n_1^2 - n_2^2)n_{sub}(k_0^2 \sigma^2 l_{slab}/aw_0)|e_{eff,\beta}\bar{e}_{eff,\alpha}|^4 \cdot \mathcal{R}_0(2\alpha_1 r_0). \]  

(2.9)

Again, \( \alpha_1 \) is the radiation loss coefficient and the backscattering loss coefficient is \( \alpha_2 \). The group index is \( n_g \). Effective number of rows of holes associated with radiation loss is \( 2N_{x,rad} \). Effective number of rows of holes associated with backscattering loss \( 2N_{x,back} \). The index of refraction for the medium of the slab is \( n_1 \), the hole \( n_2 \), and the substrate \( n_{sub} \). The wave number is \( k_0=2\pi/\lambda \). The root mean square (rms) roughness is \( \sigma \). The correlation length is \( l_c \). The hole radius is \( r_0 \). The slab thickness is \( t_{slab} \). The PCW width is \( w_d \). The special function is \( \mathcal{R}_0(x) = I_0(x)\exp(-x) \), where \( I_0 \) is the modified Bessel function of the first kind. Lattice constant is \( a \). Normalized effective amplitudes of the radiation mode is \( e_{eff,\beta} \) and guided mode is \( \bar{e}_{eff,\alpha} \). Furthermore, it is found
The ratios terms expressed in Eq. (2.10) are near a unity order of magnitude, which shows that $\alpha_1$ and $\alpha_2$ are within an order of magnitude, where terms have values such as $N_{x,rad}, N_{x,back} = 1 - 2, n_{sub} = 1, w_d = \sqrt{3}a, t_{slab} \sim 220\text{nm}, e_{\text{eff}, \beta}, k_{\text{eff}, \beta} \sim 0.5$ and $\alpha_1 \sim 0.5(2\pi/a)$. This establishes the dominance of backscattering loss for $n_g > 10$ [41].

The analytical theory has the potential for allowing methods of reducing loss to eventually emerge [41]. Experimental results were shown to verify the suggested reduction in loss. Varying the waveguide width $w_d$ may decrease the loss significantly. Loss due to random roughness could be decreased by use of guided modes designed to have accentuated high-wavenumber Fourier components and different polarization characteristics. Lastly, the analytic research indicates that the backscattering losses are dominated by the first rows of air holes next to the PCW core.
2.3 Applications of the Slow Light Effect

The slow group velocity of light renders the phase shift in a photonic crystal structure more sensitive to refractive index changes.[14] Generally, as the refractive index changes, the dispersion relation of a photonic crystal or a PCW will be shifted by a certain amount $\Delta \omega = \sigma \omega (\Delta n/n)$ along the frequency axis. Here $\omega$ is the frequency of light, $n$ is the refractive index, and $\sigma$ is a factor typically on the order of unity. In many cases, the interest lies in a small frequency range where $\sigma$ can be regarded as a constant. In the case of a PCW, the factor $\sigma$ can be interpreted as the fraction of the mode-energy in the waveguide core region. For a given wavelength, the propagation constant changes as $\Delta \beta = \Delta \omega \nu_g$. Therefore, the phase shift induced by a refractive index change of $\Delta n$ is given by[14]

$$\Delta \phi = \Delta \beta L = \frac{n_g \sigma \Delta n}{n} \frac{2 \pi L}{\lambda}, \quad (2.11)$$

where $n_g = c/\nu_g$ is the group index. Evidently, a slow group velocity (or a high $n_g$) enhances the phase shift significantly.

To exploit such a significant slow-light enhancement, a number of physical mechanisms [3,4,23,45,46] have been employed to change the refractive index and actively tune the phase shift in a photonic crystal waveguide. Here a brief review of some previous works on thermo-optic and electro-optic tuning of the phase shift for optical modulation and switching applications. Many common semiconductor materials, such as silicon and GaAs, have appreciable thermo-optic coefficients (dn/dT>10^{-4} /K). As such,
they are suitable for making thermo-optically tunable slow-light photonic crystal devices. In these devices, thermal expansion also contributes to the tuning of the phase shift. In many cases, these two effects cumulatively produce a larger phase shift. A demonstration of thermo-optic tuning in a photonic crystal waveguide Mach-Zehnder interferometer with an interaction length of 80µm was presented.[4] A photonic crystal waveguide Mach-Zehnder interferometer with one active arm is schematically illustrated in Fig. 2.3(a). A close-up view of the arm with thermo-optic tuning is shown in fig. 2.3(b). The device was patterned on a silicon-on-insulator (SOI) wafer using a combination of e-beam nanolithography and photolithography. The switching rise time and fall time were measured to be 19.6µs and 11.4µs, respectively.

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![Diagram](image)

**Fig. 2.3.** PCW Mach-Zehnder modulators/switches (MZM/S), adapted from [47]. (a) PCW based Mach-Zehnder interferometer with one active tuning arm on a SOI wafer. (b)
Cross-sectional schematic of an active arm with thermo-optic tuning (inset: micrograph of a thermo-optic device—top view); (c) cross-sectional schematic of an active arm with an embedded silicon p-i-n diode for electro-optic modulation (inset: micrograph of an electro-optic device—top view); (d) cross-sectional schematic of an active arm with an embedded silicon MOS capacitor for electro-optic modulation.

Alternatively, it is possible to electro-optically change the refractive index by carrier injection into silicon. Soref and Bennett found the following relation between the refractive index of silicon and the carrier concentrations for wavelengths near 1.55\,\mu m [48]

\[
\Delta n = -[8.8 \times 10^{-22} \Delta N_e + 8.5 \times 10^{-18} (\Delta N_h)^{0.8}],
\tag{2.12}
\]

where $\Delta N_e$ and $\Delta N_h$ are electron and hole concentration changes, respectively. A refractive index change up to $\Delta n \sim 10^{-3}$ can be obtained with $\Delta N_e = \Delta N_h \sim 3 \times 10^{17}$ cm$^{-3}$. The first high-speed photonic crystal waveguide modulator on silicon was demonstrated in 2007.[45] A schematic of the active arm of the device is show in Fig. 2.3(c). The device was made on a silicon-on-insulator wafer through a series of micro- and nano-fabrication processes, including e-beam lithography, photolithography, dry and wet etching, ion implantation, and metal lift-off. The device had a measured modulation bandwidth in excess of 1GHz, with the lowest driving voltage for high-speed silicon modulators at the time of publication. It should be noted that the introduction of air holes does not significantly increase the electrical resistance of silicon. Fig. 2.4 is a plot of the electrical resistance of a photonic crystal waveguide made in a silicon slab for varying air hole sizes and varying number of rows of air holes.
Fig. 2.4. Electrical resistances for a PCW in a silicon slab, adapted from [47]. The horizontal axis indicates the total number of rows of air holes between two electrodes separated by $10\mu$m. The different curves correspond to different values of $r/a$ (0.05–0.4), where $r$ is the radius of the air holes and $a$ is the lattice constant (~400nm). The resistance values are normalized by original slab resistance, $R_0$.

In the simulation, the electrical contact pads were placed at a fixed separation of about $10\mu$m along the sides of the photonic crystal waveguide. The contact resistance can generally be neglected. Evidently, the electrical resistance may increase about 2.6 times for a large hole radius and for a large number of holes, but the value generally remains on the same order of magnitude as the original silicon slab. Lower resistance for smaller holes might allow for lower power costs for PIN diode modulation. In the experiments, an order-of-magnitude change of the resistance was not observed after the photonic crystal structures were etched in a silicon slab. The resistance values in fig. 2.4 were computed by 2D finite element method for the DC case.
In 2007, the AC injection current density was also derived for a Si modulator based on a forward-biased p-i-n diode [45,46]

\[ j = 2qw_i \Delta N f, \]  

(2.13)

where \( w_i \) is the intrinsic region width of the p-i-n diode, and \( f \) is the modulation frequency. Combining Eq. (2.12) and Eq. (2.13), a minimum AC current density of \( 10^4 \) A/cm\(^2\) was obtained for high speed (>1GHz) modulation in a typical SOI waveguide. In addition, it was shown that due to the non-ideal diode \( I-V \) relation \( I \sim \exp(qV/2k_B T) \) at high injection, it is possible to limit (or “lock”) the injected carrier concentrations to around \( \Delta N \sim 3 \times 10^{17} \) cm\(^{-3}\) for a diode with proper doping levels and under normal forward bias conditions. This ensures that the silicon modulator naturally works under the most desired electro-optic state. In a follow-up work, it was predicted that an RF power consumption of less than 50mW is possible for 10GHz silicon modulators.[46] Subsequently, IBM demonstrated a 10GHz silicon modulator with 50mW RF driving power in the forward bias mode;[49] MIT Lincoln Laboratory also reported similar power consumption for 10GHz silicon modulators.[50] These results affirmed the value of Eq. (2.13) in designing high speed silicon modulators.

A metal-oxide-semiconductor (MOS) type photonic crystal waveguide modulator was also developed, as illustrated in fig. 2.3(d). A MOS capacitor can be embedded into a slot photonic crystal waveguide, where the slot is filled with oxide.[51] In such a waveguide, there exists two enhancement effects: the slow light effect, and the field boost inside the low-dielectric slot due to the continuity of surface-normal displacement vector
component. Some recent results show that such a configuration can help reduce the power consumption of a silicon MOS modulator.[52]

As mentioned earlier, the slow light effect can also occur in a “bulk” photonic crystal without intentionally introducing line-defects. Such a configuration has been explored in photonic crystal slabs made of conventional electro-optic materials such as LiNbO$_3$.[53]

To summarize this chapter, optical loss associated with the slow light effect in Si photonic crystal waveguides is studied. A simple proof of the scaling of the roughness-induced scattering loss in the slow light regime is given based on the density of states for the initial and final states in the scattering process. The applications of the slow light effect in Si photonic crystal waveguides were discussed.
3. ANGULAR DISPERSION IN PHOTONIC CRYSTALS: THE SUPERPRISM EFFECT

Photonic crystal lattices as superprisms with hexagonal periodicity are examined for angular dispersion control. Emphasis will be placed on the loss and bandwidth assessments. In addition, a brief introduction to the superprism phenomenon is presented. Lastly, the loss and bandwidth scaling for the superprism effect are discussed.

3.1 Introduction of Superprism Effect

The superprism effect can be considered to be anomalous light refraction on the surface of a photonic crystal. A physical effect of a light beam incident upon a photonic crystal surface is that the refraction angle inside the photonic crystal could be ~500 times more sensitive to the changes in wavelength than in a conventional medium.[15] The superprism effect is a manifestation of the strong angular dispersion of a photonic crystal. Strong wavelength-dependent refraction in relation to the angular dispersion is represented by sharp corners on the dispersion curve, where sharp corners represent large curvature and indicate high wavelength sensitivity. For a change in wavelength of \( \Delta \lambda/\lambda = 1\% \), the refraction angle can change by \( \Delta \theta = 50^\circ \) in contrast to 0.1 deg for conventional prisms. Fig. 3.1 illustrates a factor of ~ 500 enhancement in the wavelength-sensitivity of the refraction angle, comparing conventional crystal to photonic crystal prism [15,54].
Significant progress was made in investigating the superprism effect in the first ten years after its initial discovery. A number of experiments demonstrated the potential of the superprism effect in wavelength division multiplexing, beam steering, and sensing applications. [16-18,20,55,56]

Fig. 3.2. (a) Band diagram as normalized frequency versus wave vector with inset depicting the Brillouin zone; (b) dispersion surface.
The blue star shaped curve in Fig. 3.2(b) is a dispersion surface or constant-frequency(\(\omega\)) contour on the \(k_x-k_y\) plane (in reciprocal space) for a hexagonal lattice. Curvature of the dispersion surface is large near the sharp corners where a shift in frequency is indicated by changes in color or wavelength corresponding to changes in energy with changes in \(k_y\).

![Dispersion Surface](image)

**Fig. 3.3.** The photonic band structure a hexagonal lattice structure [57].

High sensitivity is a result of the dispersion surface having large curvature or sharp bends. Then for a given 2D local curvature of the dispersion surface, the curvature \(\zeta\) is inversely proportional to the local radius \(R\) of the dispersion surface or \(\zeta \sim 1/R\). The radius reduces to zero near the Brillouin zone corner when the dispersion surface curvature increases to \(\infty\) corresponding to a singularity in the dispersion contour. Fig. 3.2 (a) shows the band diagram. Fig. 3.3 adapted from Ref. [57] shows that for the first photonic band of a hexagonal lattice, the slope of the \(\omega(k)\) curve is not equal to zero at the high symmetry point \(K\), meaning the group velocity \(v_g\) does not vanish. Hence, the first band is chosen and lower loss is possible. [57] Note at the \(K\) point, the first band has two branches intersecting each other, resulting in a double degeneracy. Lattice
symmetry contributes to this degeneracy, and according to the group theory this occurs in the cases of 2D and 3D lattices, but not 1D[58].

Many applications are possible, some of which include demultiplexers, beam steering and sensors. Wavelength demultiplexing and beam steering could emerge as application based on the high sensitivity of refraction angle to variations in wavelength [18,59]. Sensing applications are possible and would make use of the sensitivity to changes in index. Variations in index can cause large shifts in refraction angle at the input side of a super prism structure. When a super prism structure is accompanied by a 1D array of photo detectors at the output side of the photonic crystal lattice with sufficient density, then the change in angle of the beam can be detected and correlated to a change in the refractive index [60]. Other applications may emerge that make use of the combination of super prism concepts and focusing as negative refraction, which could lead to extending the resolution of conventional microscopes beyond the approximate half wavelength limitation based on the Rayleigh limit (Resolution~1.22λ/(2 x Maximum Numerical Aperture)) [20]. In general, the applications show a potential trend in enhancing the optical control of systems through wavelength sensitivity, index sensitivity and related effects in photonic crystal lattices.
3.2 Loss and Bandwidth Assessment for the Superprism Effect

Generally, the curvature of a curve on the $k_x$-$k_y$ plane is given by [57]

$$
\zeta = \frac{d^2k_x}{dk_x^2} \left[ 1 + \left( \frac{dk_y}{dk_x} \right)^2 \right]^{3/2},
$$

(3.1)

where the derivatives can be calculated from the function given by $k_y = \kappa(k_x)$.

To derive the relation between the curvature of the dispersion surface and the angular dispersion of the photonic crystal, consider the conservation of tangential wavevector component across the photonic crystal surface,

$$
\frac{1}{c} n_I \omega \sin \alpha = k_{x0} + u \sin \theta - v \cos \theta,
$$

(3.2)

where $\alpha$ is the incident angle, $n_I$ is the refractive index of the incident medium, $\theta$ is the direction of the group velocity (i.e. the beam direction) with respect to the surface normal. $u$ and $v$ are local Cartesian coordinates in the neighborhood of a point, $k_0 = (k_{x0}, k_{y0})$, on the dispersion surface.

Fig. 3.4. Schematic of a simple configuration for the superprism effect.
Here the local $u$-axis is parallel to the group velocity, and $v$-axis is tangential to the dispersion surface.

It can be shown from Eq. (3.2) that the sensitivity of the beam angle to wavelength change or the angular dispersion, is given by [57]

$$\left| \frac{d\theta}{d\lambda} \right| = \left| \frac{2\pi \zeta}{\lambda^2 \cos \theta} \left( n_g \sin \theta - n_I \sin \alpha \right) \right|. \quad (3.3)$$

In addition, the sensitivity to refractive index perturbation is given by

$$\left| \frac{d\theta}{dn_a} \right| = \left| \frac{\zeta}{c \cos \theta} \left( \frac{\partial \omega}{\partial n_a} \right)_k n_g \right|. \quad (3.4)$$

In Eq. (3.3) and Eq. (3.4), the quantities, $\zeta$, $1/\cos \theta$ (note $\tan \theta = \sin \theta / \cos \theta$), and $n_g$ are the only three factors that can grow several orders of magnitude compared to a conventional medium, which can result in significant enhancements of angular dispersion/sensitivities as observed in prior superprism experiments.

However, it results in the optical transmission across the photonic crystal surface is given by [61]

$$T \propto |t|^2 \varepsilon_{em} v_g \cos \theta, \quad (3.5)$$

where $\varepsilon_{em}$ is the cell-averaged mode energy density and $t$ is the complex coupling amplitude of the mode in question. Normalized transmission ($T$) is given by the ratio of the surface-normal component of the Poynting vector and is depicted with respect to the
high symmetry axis $\kappa$ in fig. 3.6. Although Eqs. (3.3) and (3.4) obviously indicate a linear dependence of the angular dispersion and angular sensitivity on the group index, a casual numeric analysis without the knowledge of Eqs. (3.3) and (3.4) would yield a deceptively stronger enhancement in the slow-light regime. Consider the following approximate mode dispersion near a photonic bandedge

$$\omega = \omega_0 - b(k_x^2 + k_y^2),$$

(3.6)

where $\omega_0$ is the bandedge frequency and $b$ is a constant.

The two key parameters for the slow-light effect and the superprism effect have the following frequency dependence near the bandedge

$$n_g = c/\left[2\sqrt{b(\omega - \omega_0)}\right],$$

$$\zeta = \sqrt{b/(\omega - \omega_0)}.$$  

(3.7)

Evidently, the group index and the curvature diverge at the same rate as $\omega$ approaches the bandedge $\omega_0$. Substituting $n_g$ and $\zeta$ into Eqs. (3.3) and (3.4), it can be shown

$$\frac{d\theta}{d\lambda} \sim \frac{1}{\omega - \omega_0},$$

$$\frac{d\theta}{dn_g} \sim \frac{1}{\omega - \omega_0}.$$ 

(3.8)

A straightforward numeric calculation should find that near the bandedge, when the group index increases 10 times, the angular dispersion and angular sensitivities would increase 100 times. Thus, if the analytic form of Eqs. (3.3) and (3.4) are unknown, it could be concluded that $d\theta/d\lambda \propto n_g^2$ and $d\theta/dn_g \propto n_g^2$ are appropriate in this particular
case. However, the above analysis shows that the proper dependence should be $d\theta/d\lambda \propto n_g \zeta$ and $d\theta/dn_a \propto n_g \zeta$ because $n_g$ and $\zeta$ diverge at the same rate. Although the above analysis is based on a specific photonic band structure described by Eq. (3.8), a general asymptotic analysis, given in Ch. 2. A of Ref. [57], indicates that such a slow-light induced strong angular dispersion due to equal diverging rate of $n_g$ and $\zeta$ exists in a wider range of slow-light scenarios.

The dispersion surface can exhibit ultra-high curvature values in the vicinity of certain high-symmetry points in the Brillouin zone (BZ) without the presence of the slow-light effect. [57] An example is the $K$ point of a triangular lattice. Some photonic bands have a double degeneracy at this point. Approaching such a doubly degenerate point, the curvature of the dispersion surface tends to infinity whereas the group velocity approaches a non-zero constant. Therefore, the benefits of high dispersion and high sensitivity can be reaped, according to Eqs. (3.3) and (3.4) without concern for the high optical loss that would occur in the slow-light case, shown in fig. 3.5 (b). This indicates that high angular sensitivity or angular dispersions in a hexagonal PC and high loss do not necessarily accompany each other. In this case, the mode dispersion relation is approximately

$$\omega = \omega_0 - b |k - k_0|.$$  \hspace{1cm} (3.9)

Based on this, the scaling of the group index and curvature is given by

$$n_g \rightarrow \text{const},$$

$$\zeta \rightarrow 1/(\omega - \omega_0).$$ \hspace{1cm} (3.10)
where $\omega_0$ is the frequency of the doubly degenerate point, where the curvature is singular. These differ from the slow-light induced superprism effect in Eq. (3.7). Now, again $n_g$ and $\zeta$ are substituted into Eqs. (3.3) and (3.4) to obtain $d\theta/d\lambda$ and $d\theta/dn$. Interestingly, in this case, the overall scaling of $d\theta/d\lambda$ and $d\theta/dn$ with frequency perturbation, $\Delta \omega = \omega - \omega_0$, remains the same as in Eq. (3.8) even though the $n_g$ and curvature dispersion relations are different from the slow-light induced superprism effect. However, the optical loss is almost constant in this case, independent of the angular dispersion values.

Fig. 3.5. (a) Band diagram depicting degeneracy points; (b) Low transmission curve for zero group velocity of 2nd degeneracy point in a parabolic region; (c) Greater than 50% transmission for Non-Zero Group Velocity of the 1st degeneracy point at the intersecting curve region. Adapted from Ref. [57].
Fig. 3.6. (a) Near Zero Group Velocity for 2nd Band; (b) Non-Zero Group Velocity for 1st Band. Both cases are for propagation along off high symmetry axis, where the $\Delta \lambda \rightarrow$ beam propagation angle change.

In general the 1st and 2nd bands of a band diagram are similarly affecting the sensitivity BW of the superprism effect, except the 1st band maintains higher transmission. [47]

When the optical loss is no longer a limiting factor, the angular dispersion and angular sensitivities can be enhanced to much higher values until it encounters some other limits. Now a fundamental angular sensitivity-bandwidth limit similar to the bandwidth-delay limit for the longitudinal dispersion is presented. Assume the angular dispersion is maintained at a value above $\left(\frac{d\theta}{d\lambda}\right)_0$ over a spectral range of BW with units of nanometers. The maximum beam steering range cannot exceed $180^\circ$, which means that no backward propagation is physically possible. Therefore it is found

$$\frac{d\theta}{d\lambda} \sim \frac{1}{\lambda - \lambda_0}, \quad \frac{d\theta}{d\lambda} (\lambda - \lambda_0) = const.$$ 

$$(d\theta/d\lambda)_0 \times BW < 180^\circ.$$  \hspace{1cm} (311)
This relation could be referred to as the angular dispersion correspondent of the bandwidth-delay product. Therefore, the maximum bandwidth as a limiting effect for a sustained high sensitivity \((d\theta/d\lambda)_0\) is

\[
BW < 180^\circ / (d\theta/d\lambda)_0. \tag{3.12}
\]

Initially, consider the case where a sensitivity of 100 \(^{\circ}/\text{nm}\) can be sustained over a bandwidth less than 1.8nm, and a sensitivity of 1000 \(^{\circ}/\text{nm}\) can be sustained over a bandwidth less than 0.18nm. For a laser having 1pm linewidth (~100MHz), these two cases may allow for 1800 and 180 wavelength tuning points, respectively, which are reasonable for practical applications. These performance parameters are possible with the existing laser technologies. A similar limiting property for the Slow light effect as the slow light bandwidth delay product, \((\text{Bandwidth}) \times (\text{delay}) \sim 1\), where for both the slow light effect and superprism effect have products that result in a constant as a fundamental limit.

Lastly, keep in mind that not all applications require a wide bandwidth. There are some applications that can benefit from a high angular dispersion/sensitivity in a narrow bandwidth. A detailed theoretical analysis based on group theory shows that such types of “pure” angular dispersion originates fundamentally from symmetry induced degeneracy in photonic band structures. Furthermore, such types of symmetry-induced enhancement of angular sensitivities can only occur in 2D and 3D photonic crystals, but not in 1D photonic crystals. Discovering such a crystal-symmetry induced effect exemplifies the effectiveness of utilizing the solid-state physics paradigm to shed new light on the study of periodic dielectric structures, which is the central theme of photonic
crystal research.[62] In passing, it is noted that the rigorous theory for computing the
transmission of each photonic crystal mode[61] has been extended to gratings,[63] which
can be regarded as monolayer photonic crystals, and 3D photonic crystals.[64] Formulas
similar to Eq. (3.5) can be used to assess the optical loss in the 1D and 3D cases as well.

Before concluding this section, it should be mentioned that there are a number of
applications for the superprism effect. Different applications may have additional limiting
factors specific to themselves. For example, for the widely studied wavelength
demultiplexer application, the beam width divergence is an additional limiting factor
specific to this application. Fortunately, recent works [20,65] have demonstrated a
promising method of overcoming this limit. Note that this factor is important only for
those applications that require narrow beam width, or spot size, at the receiving end of
the superprism. If a sufficiently large beam width is used, meaning that there is a small
lateral spread of the wavevector, is used, then this factor is less important.

To summarize this chapter, high angular sensitivity and high loss do not
necessarily accompany each other for the angular dispersion unlike the longitudinal
dispersion. The scaling of the beam angle with the wavelength offset is the same for
sensitivities to wavelength and refraction index in the superprism effect when analyzed at
a photonic band edge for zero group velocity and at a degenerate point for singular
curvature of the dispersion surface. The case considered at a degenerate point is not
accompanied by high optical loss due to the slow light effect. Such understanding gives
rise to an important fundamental relation as the bandwidth-sensitivity product obtained
for the superprism effect, which is the counterpart to the bandwidth-delay product for the
slow light effect.
4. LASER BEAM STEERING BASED ON LONGITUDINAL AND ANGULAR DISPERSIONS IN PHOTONIC CRYSTALS

Phonic crystal structure longitudinal dispersion, angular dispersion and beams steering concepts are discussed. First, a brief review of beam steering technologies is presented. Then a direct comparison between the two dispersions mentioned is conducted, along with a comparison of beam steering methods as a potential application.

4.1 Review of Laser Beam Steering Technologies

Beam steering is when the direction of propagation of the main lobe of the radiation pattern is altered or steered as noted in the Federal Standard 1037C. It also should be noted that the beam steering concept was originally developed for radio frequency systems. Optical systems for beam steering are considered here. The steering can be achieved by altering the transmission mediums refractive index and can make use of 1-D optical phase arrays. Additionally, beam steering can be done acoustically, by mirrors, prisms, or lenses, among other methods. Beam steering technology has the potential of being reliable and inexpensive. The applications for this technology can include wind profiling, gas cloud identification, communications, laser guidance systems and radar for target detection. The application of optical phase arrays for beam steering has significant future potential for applications such as infrared sensors, if the effects of dispersion can be reduced. [66]
A radiation pattern’s normalized intensity is described by [66]

\[
I = \frac{\sin^2(N\alpha)}{\sin^2(\alpha)}.
\]  

(4.1)

The field is measured at \( \theta \), an angle relative to array Boresight. \( \alpha \) is a function dependent on illumination profile, array shape and FWHM definition and is shown below in eq. (4.2). The array’s number of phase shifters is \( N \). The normalized intensity \( I \) has a narrow antenna pattern from the effect of all array elements’ interference patterns.

\[
\alpha = \pi \left( \frac{d}{\lambda} \right) (\sin \theta - \sin \psi)
\]  

(4.2)

The steering direction is shown above in eq. (4.2) and \( \psi(\phi) \) is the steering angle of the beam and the direction maximum intensity expressed below in eq. (4.3). The free space wavelength \( \lambda \), between the elements has a uniform phase difference \( \phi \) and spacing \( d \).

\[
\sin \psi = \left( \frac{\lambda \phi}{2 \pi d} \right)
\]  

(4.3)

The core concepts of an optical phased array are indistinguishable from those of a microwave array, though the wavelength of interest differs by multiple orders of magnitude and are applied differently. The optical phased array is steered in one dimension for its 1-D array in comparison to microwave arrays which are steered in 2 dimensions for its 2-D array. McManamon et al. described a passive 1-D, phase-only, aperture optical device [66]. Additionally, they described the device as space-fed with an assumed Gaussian profile, which means that edge tapering of the array is not required as it is in a microwave system. The steering angle and corresponding aperture size permutations that necessitate phase resets with minimum thickness for reflective-mode
design allow two passes through a given medium [66] having a full wave optical path delay (OPD) created by a structure of half the minimum thickness.

The blazed grating is applied with a stair step design for the purpose of having energy maximized in the first order and has a diffraction efficiency given by [66]

$$\eta = \left( \frac{\sin(\pi / q)}{\pi / q} \right)^2 \quad (4.4)$$

where the number of blaze profile steps represented by q. The blaze grating is a 1-D periodic structure where the ridge top surface is angled as part of the design to allow light diffraction efficiency to be maximum at a certain wavelength and could have an efficiency of ~95% [66,67]. The wavefront deflection in a prism represented by a step approximation includes phase resets of $2\pi$, where the design wavelength needs a $2\pi$ phase reset. Dispersion effects are caused by changes in phase reset from $2\pi$. Also a wavelength shift away from the design wavelength causes changes to an unfolded phase profile from a straight line.

There are various methods of beam steering including the use of different blazed gratings, liquid crystals, lenslets, mechanical systems, combinations of other methods. Beam steering can be done by acoustic-optical devices in microseconds and electro-optical devices in nanoseconds. However, both methods are usually limited in usefulness to small beam steering with fine resolution.

Obtaining high resolution steering with large angles while being able to utilize a large wavelength range is inhibited due to the single design wavelength of operation with a $2\pi$ phase shift reset via the blazed grating like structure. Liquid crystal phased array enables large steering angles where the deflection angle is modified by a voltage
controlled variable refractive index. One of the limiting factors is the fabrication thickness of the liquid crystal layer [68]. This is related to the so called true time delay in microwave systems. It would need to be made with a thickness that would remove the need for phase resets, but currently fabrication technology cannot meet this demand for large aperture beam steering. High resolution broad band beam steering with large steering angles will have image deterioration due to dispersion without true time delay [66].

A blazed grating could have an efficient deflection into a single order of diffraction from a phase profile resembling a periodic sawtooth. Enhancing the functionality beyond a static grating can be done by introducing variable period beam steering. This is enabled by a nanomechanical grating, which can be dynamic when controlled electronically. There are two methods of control: one is to change the grating period and the other is to alter the blaze of the grating. Altering the period of the grating is done by changing the number of phase shifters in a grating period, where the largest angle steered occurs for the smallest period. Changing the blaze of the grating is done by altering the top surface angle of the grating, which allows the steering of energy to different orders or discrete angles.

Microelectromechanical systems (MEMS) can be implemented to control mirrors with movement perpendicular to the surface enabling piston phase changes, where the ideal spacing is half wavelength. This allows variable period beam steering. Additionally, different sets of mirrors can be adjusted to different angles to allow variable blazed angles.
Beam steering by cascaded microlens arrays with variable blaze has no physical resets, but the exiting wavefront from the last microlens array causes resets that are virtual. Conventionally, microlenses are unable to move, which prevents some dynamic properties. If the microlenses are integrated into liquid crystal devices, then they could be controlled electronically and obtain very high efficiency. Additionally, the concept of re-writable microlenses in liquid crystal devices can enable many interesting variable properties in the realm of beam steering [68].

Beam steering in periodic structures is examined in regards photonic crystals devices and is discussed in the following section.
4.2 Photonic Crystal Laser Beam Steering Approaches: Comparison Between Two Approaches

The group velocity of light can be slowed down in various types of photonic crystal structures, especially when the wavelength of light approaches a bandedge. Two common cases shall be considered. (1) For a “bulk” photonic crystal, such a bandedge typically appears around some high symmetry points in reciprocal space. As such, in real space, slow light propagation typically occurs along certain high symmetry axes of a photonic crystal. (2) For a photonic crystal waveguide (PCW) composed of a line-defect, generally the PCW is already aligned with a high symmetry axis of the photonic crystal lattice. An example of this would be alignment with the ΓK axis of a hexagonal lattice. The original lattice periodicity remains along the longitudinal direction of the waveguide. This results in a one-dimensional (1D) photonic band structure with a maximum or minimum, or other types of extrema, at the 1D Brillouin zone (BZ) boundary β=π/a, where β is the propagation constant of the photonic crystal waveguide in question. Since the dispersion relation ω(β) is generally a smooth function, an extremum ensures ν_g=dω/dk=0 at the BZ boundary. Therefore, the periodicity along the longitudinal direction dictates that a vanishing group velocity must exist in such a photonic crystal waveguide. Soljacic et al. showed the vanishing group velocity leads to a significant enhancement of the phase shift in a photonic crystal waveguide [14], which can be used for optical modulation. However, the slow light effect also induces significant loss in a photonic crystal waveguide, and that loss may be a fundamental limit to the practically achievable dispersion through the slow light effect.
On the other hand, when a light beam is incident upon a photonic crystal surface, the refraction angle inside the photonic crystal could be 500 times more sensitive to the wavelength perturbation than in a conventional medium.[15] The super prism effect is related to photonic crystal dispersion. Nonetheless, many fundamental questions remained unanswered: (1) How is the angular dispersion or angular sensitivity of a photonic crystal expressed in terms of basic parameters of a photonic band structure as seen in the slow-light effect? (2) Is there an ultimate limit of the angular sensitivity of a photonic crystal? (3) If there is a limit, what are the limiting factors? To build a foundation for the solution of these problems, Jiang et al. developed a rigorous theoretical framework to compute the transmission and reflection coefficients for refraction across a photonic crystal surface in a 2005 work.[61] A parallel work was reported by a group at the University of Toronto in the same period.[71,72] Subsequently, Jiang et al. developed the first theory to systematically address the aforementioned general questions in a 2008 work.[57]

While the key parameter for tuning the longitudinal dispersion is the group velocity, a new parameter, the curvature of the dispersion surface, must be introduced to describe the angular dispersion. Here the dispersion surface refers to the constant-frequency surface in reciprocal space. This curvature can be calculated directly from the dispersion relation $\omega(k)$, which also represents the photonic band structure. With this theory, the sensitivity of the super prism effect can be directly express in terms of $\omega(k)$ and the fundamental limiting factors of the super prism effect can be explored.

The goal of this part of the research is to utilize the aforementioned theory to investigate two types of super prism effects: the slow-light induced angular dispersion
effect and the “pure” angular dispersion effect. Of particular are the loss and bandwidth of these two types of effects. The scaling of the loss and bandwidth with the sensitivities will be studied in detail. Such scaling may set different fundamental limits on practically achievable sensitivity values for two types of the superprism effects. In addition, the loss-limiting and bandwidth-limiting behavior of the longitudinal dispersion and angular dispersion will be compared. As longitudinal dispersion and angular dispersion often takes place in different directions with respect to the light propagation, eventually it would be interesting to find a special scenario where the outcome of these two effects can be evaluated in a common quantity.

Fig. 4.1. (a) Longitudinal Dispersion in PCWs: Fixed direction in a waveguide along high symmetry axis in bulk PC, where the $\Delta \lambda \rightarrow \Delta k_x \rightarrow$ phase shift; (b) Angular Dispersion in a PC lattice for propagation along off high symmetry axis, where the $\Delta \lambda \rightarrow$ beam propagation angle change.
**Longitudinal dispersion vs. Angular dispersion: a direct comparison**

The analyses in two preceding sections show that the two key parameters, $v_g$ and $\zeta$, of the slow-light effect and the super prism effect are entirely determined by the dispersion function $\omega(k_x,k_y)$. In other words, these two effects are manifestations of the longitudinal and angular characteristics of the dispersion function. In this section, some further connections between these two effects through an application example are discussed. Here, the beam steering application is chosen, which intends to manipulate the direction of a laser beam by changing the refractive indices of materials in certain device structures. Two approaches are considered: (1) an optical phase array[73] (OPA) composed of a 1D array of slow-light photonic crystal waveguides as shown in Fig. 4.2; (2) a super prism composed of a 2D photonic crystal. Note that to steer the output beam in free space, the super prism device cannot have a flat output surface parallel to the input surface. In this example, it is assume the output surface has a semi-circular configuration for simplicity.[16]

![Fig. 4.2. Schematic of a one-dimensional optical phase array composed of photonic crystal waveguide phase shifters.](image)
First, the phase shift induced by a refractive index change of Eq. (2.11) and the angular dispersion given in Eq. (3.4) are re-written in the following forms:

\[
\Delta \phi = L \left( \frac{n_s \sigma}{n} \frac{2\pi}{\lambda} \right) \Delta n, \tag{4.5}
\]

\[
\left| \Delta \theta \right| = \zeta \tan \left( \frac{n_s \sigma}{n_a} \frac{2\pi}{\lambda} \right) \Delta n_a. \tag{4.6}
\]

The second equation follows from

\[
\left( \frac{\partial \omega}{\partial n_a} \right)_k = \sigma_a \frac{\omega}{n_a} \tag{4.7}
\]

where \(\sigma_a\) measures the fraction of mode energy located in the medium \(a\). Note this relation, Eq. (4.7), is essentially the same as that used in an early derivation of the slow-light enhancement of the phase sensitivity in a photonic crystal waveguide.[14] Therefore, it is not surprising to see that Eq. (4.5) and Eq. (4.6) share similar factors

\[
\frac{\sigma}{n} \frac{2\pi}{\lambda},
\]

which come from \(\partial \omega / \partial n\). In the case of a photonic crystal waveguide, \(n\) refers to the refractive index of the waveguide core, and \(\sigma\) denotes the fraction of the mode-energy in the core region.[14]

More interestingly, a direct comparison of the steering angle sensitivity between the two approaches can be obtained from Eq. (4.5) and Eq. (4.6). For an optical phase array, the far-field beam angle \(\theta\) relates to the phase difference, \(\Delta \phi\), between adjacent array elements as follows
\sin \theta = \frac{\Delta \phi \lambda}{2\pi d}.

(4.8)

Therefore, the beam steering sensitivity for a slow-light based optical phase array is given as

\[ \Delta \theta = \frac{1}{\cos \theta} \left( \frac{n_g \sigma}{n} \right) \frac{L}{d} \Delta n. \]

(4.9)

To simplify the comparison with the superprism effect, it is assumed \( \sigma = \sigma_a \), and \( n = n_a \).

Then the ratio of the beam angle changes in the two cases is given by

\[ \frac{\Delta \theta_{SP}}{\Delta \theta_{SL}} = \sin \left( \frac{2\pi d}{\lambda} \right) \frac{\zeta}{L \left| n_g,SP \right| / \left| n_g,SL \right|}, \]

(4.10)

where SP denotes the superprism effect and SL denotes the slow-light effect. In most optical phase arrays, the waveguide spacing, \( d \), is on the order of the wavelength, \( \lambda \). If it is assumed \( \sin \theta > 0.1 \) and, then these two factors have an overall contribution on the order of unity (note the \( 2\pi \) factor in Eq. (4.10)). Therefore, the difference between \( \Delta \theta_{SP} \) and \( \Delta \theta_{SL} \) primarily comes from the terms, \( (\zeta/L) \) and \( (n_{g,SP} / n_{g,SL}) \). Note the curvature \( \zeta \) also has the dimension of length.

As a numeric example, a silicon photonic crystal waveguide is considered with \( n_{g,SL} \sim 30 \), and \( \Delta n \sim 10^{-3} \). This generally requires a waveguide length on the order of \( L=100\mu m \) to achieve a phase shift of \( 2\pi \) at \( \lambda=1.55\mu m \). Note that in order to extend the waveguide length far beyond this value to achieve larger \( \Delta \phi \) will cause multiple side-lobes and is not desired for many practical OPA beam steering applications. On the other hand, it is relatively easy to get \( \zeta \gg 100\mu m \) in a properly designed photonic crystal
superprism. For example, it is found that \(2\pi \xi/\lambda > 10^3\) for a hexagonal photonic crystal with \(n_g \sim 7\).\(^{[57]}\) In this particular example, the beam angle ratio \((\Delta \theta_{SP} / \Delta \theta_{SL})\) in Eq. (4.10) is around \(3\sin \theta\). For moderate \(\theta\) values, the beam steering efficiencies due to the two effects are roughly on the same order of magnitude. A more detailed investigation is beyond the scope of this work.

To summarize this chapter, the beam steering approaches based on the slow light and the superprism effects are directly compared based on this synergistic perspective of photonic crystal dispersions and this synergistic perspective is necessary for the comparison to be possible. Lastly, a numeric example was provided.
5. THEORY OF PARALLEL-COUPLED DUAL RACETRACK RESONATORS

Although photonic crystal structures offer a wide range of dispersive effects, they are considerably difficult to fabricate. In contrast, microresonator based structures are relatively easy to fabricate and characterize. The dispersion of microresonators will significantly modify both the intensity and phase of light in a narrow wavelength range. Such dispersive properties can be utilized to create novel devices. In this chapter, the design, fabrication, characterization of some novel racetrack microresonator structures will be presented. Such structures can be used for various modulation schemes using digital transmission of 2 or more bits simultaneously (M-aryl) with known bandwidths, number of symbols $M = 2^b$ and channel capacities of maximum $b$ bits per symbol. Initially, a brief introduction of racetrack resonators is provided, followed by the theory of parallel dual racetrack microresonators for $M$-ary quadrature amplitude modulation (QAM) and phase shift keying (PSK). The effect of the asymmetries between the two racetrack resonators is discussed. It is shown that such asymmetries can be compensated by appropriate phase shifts in the two racetrack resonators. Then the fabrication processes, results and fabrication barriers are discussed. Also, characterization procedures, results and some sources of noise are given. Lastly, some comments are made on potential future work and applications.
5.1 Introduction to Parallel Coupled Dual-Racetrack Microresonators

The dispersion of microresonators will significantly modify both the intensity and phase of light in a narrow wavelength range. For some coherent communications applications, it would be interesting to utilize this effect to simultaneously modulate the intensity and phase of light. To this end, it is expected that at least two microresonators, each being controlled by an independent modulation signal, are needed to modulate both the intensity and phase. As we shall see, racetrack microresonators have dispersive properties similar to ring-shaped microresonators, and provide some advantages in practical applications. Therefore, we will focus on racetrack microresonators in this part.

A preliminary study indicates that simply cascading two racetracks in series, as shown in Fig. 5.1(a), will not provide sufficient modulation freedom. As shown in Fig. 5.1(b), by the modulating the refractive index of a segment in each racetrack independently, the normalized output amplitude only covers a small portion of states in the unit circle on the complex plane of the amplitude of the output light field.

Fig. 5.1 Cascaded racetrack microresonators in series. (a) structure schematic; (b) all
possible complex amplitudes on the complex plane (within the unit circle).

![Diagram of parallel coupled dual racetrack microresonators]

Fig. 5.2. Parallel coupled dual racetrack microresonators. (a) structural schematic (b) all possible complex amplitudes on the complex plane (within the unit circle).

A novel modulator is presented as a device fabricated in the silicon layer of a silicon-on-insulator (SOI) wafer. Fig. 5.2 (a) depicts the passive elements of the design, which consist of parallel coupled dual racetrack microresonators, with one on each side of a central waveguide. The active elements of the modulator for carrier injection are PIN diodes or MOS capacitors embedded in the microresonators. The aforementioned modulator design allow quadrature phase shift keying (QPSK) in addition to other modulation formats. Some advantages of these modulation formats include tolerance to fiber nonlinearities and greater spectral efficiency [74]. The phase and amplitude of an output signal can be separately controlled for each of the identical racetrack microresonators at the resonant wavelength when injection signals applied to each microresonator are appropriately configured. This will be discussed in more depth in later sections. The proposed resonator can potentially allow for coherent communication functions when a sufficient portion of the complex plane is covered within the unit circle, which represents complex amplitudes of the output light field. The coupling between the
adjacent parallel racetrack microresonators and the central through waveguide can be illustrated by multiwaveguide coupling theory, in which the coupling region of the resonator includes three single mode waveguide segments that are parallel. The coupling region is indicated in Fig. 5.2. The propagating electric field $E_n(x,y,z)$ having a slowly varying envelope is assumed. Parallel-coupled dual racetrack microresonator coupling region is significant and the interaction of the three waveguide segments contained in that region can be represented by three coupled mode equations for the input and output fields from that region.

The multi-waveguide coupled mode theory discussed by Kim et al. [75] indicates that the three coupling segments can be analyzed with respect to the interdependencies of coupled complex amplitudes for the input and output. This will be discussed later. Evanescent coupling occurs between the three waveguide segments of the coupling region. The two straight segments of the racetrack microresonators closest to the central throughput waveguide use that central waveguide as a cross coupling medium allowing indirect coherent coupling between the closest pair of racetrack straight segments.
5.1.1 Evanescent Coupling

Evanescent coupling is a primary consideration when analyzing the structure of the racetrack resonators. The evanescent tail of a waveguide mode exponentially decays from the core into the cladding. When the evanescent tail reaches an adjacent waveguide core, it is coupled into that waveguide. This is depicted as the dashed area circled in the Fig. 5.3 of the overlapping single modes guided in waveguides I and II. The coupling coefficient can be analytically described by Eq. (5.1). Furthermore, it is noted that for weakly guided modes in a medium with low index contrast to an adjacent medium then perturbation theory can depict the evanescent coupling. [76] Though for nanophotonic waveguides with large index contrast and for widths and separations on the order of 400nm, the waveguides can be considered in very close proximity. They can no longer be assumed to be weakly coupled. Thus, it is no longer appropriate to use the perturbation theory. The method of approaching this issue is to go back to Maxwell’s equations for each region’s changing set of boundary conditions using FDTD simulation methods.

Fig. 5.3. Evanescent Coupling marked in the circled dashed region of the overlapping single modes guided in waveguide I and II and analytically described by the overlap integral. This figure was adapted from. [77]
The circled dashed region in Fig. 5.3 denotes the region where coupling occurs and the coupling coefficient describing that effect is $\kappa_{ab}$ [77]

$$\kappa_{ab} = C\int_{II} (f_a^* \Delta \varepsilon) dxdy. \quad (5.1)$$

Noting that the for a coupled waveguide system derived from first principles, the coupling coefficient supports the modal coupling equation. Where $\kappa_{ab}$ is a form of the transversal integration range of waveguide II for modes a and b and $\Delta \varepsilon$ is the infinitesimal changes in dielectric permittivity effecting waveguide I. Then when considering the normalization of functions for modes a and b, the constant C is found. [77]

FDTD Simulations in 3D with cell sizes of 35nm done by Huang et al. showed that for ~350nm width at 1550nm wavelength, a silicon rib waveguide with air cladding would have a coupling length or transfer length of ~1.4 $\mu$m for maximum coupling efficiency of ~96%. [76] Again, assuming now that the adjacent nanophotonic structures are no longer weakly coupled; other interesting effects of evanescent coupling should be noted such as high efficiency coupling, which could give rise to large percentage energy transfer between adjacent structures and optical energy recirculation for racetrack resonators.
5.2 Theory of Parallel-Coupled Dual Racetrack Resonators

A parallel-coupled dual racetrack silicon micro-resonator structure is proposed and analyzed for arbitrary quadrature signal generation. The critical coupling condition is analytically obtained first. The over-coupled, critically coupled, and under-coupled scenarios are systematically studied. Simulations indicate that only the over-coupled structures can generate arbitrary quadrature signals. Analytic study shows that the large dynamic range of amplitude and phase of a modulated over-coupled structure stems from a delicate balance between the direct sum and the coherent interaction of two individual resonances. The asymmetries in the coupling constants and quality factors of the resonators are systematically studied. Compensations for these asymmetries by phase adjustment are shown to be feasible.

Advanced optical modulation formats could offer significant advantages for optical communications [74]. For example, quadrature phase-shift keying provides higher spectral efficiency, better tolerance to fiber nonlinearity and chromatic dispersion, and enhanced receiver sensitivity compared to on-off keying. Traditional Lithium Niobate (LiNbO₃) modulators can be used for such modulation. However, LiNbO₃ modulators are relatively large in size. For a general M-ary modulation format that requires a large number of optical modulator components along with their driving signal circuitries, the overall size of the entire modulator is rather cumbersome. Recent breakthroughs in silicon photonics [1,2], particularly silicon based optical modulators [28,78], have fundamentally changed the landscape of modulator technology. Notably, micro-resonator
based silicon modulators [28,79,80] constitute an ideal candidate for optical modulation due to their compact size, low power consumption, and ease of monolithic integration with driving circuitries on the same silicon chip. Most research on silicon resonator modulators employed intensity modulation in binary formats. Recently, microresonator based modulators for differential binary phase-shift-keying and differential quadrature phase-shift keying (QPSK) have been proposed, and satisfactory performances have been predicted [81,82]. Another work employed the anti-crossing between paired amplitude and phase resonators and demonstrated enhanced sensitivity to the input drive signal [83]. A high-Q racetrack resonator quadrature modulator incorporating dual 2×2 Mach-Zehnder interferometers is also recently proposed for arbitrary quadrature signal generation [84].

A novel parallel-coupled dual racetrack micro-resonator structure is proposed, illustrated in Fig. 5.4(a), for arbitrary quadrature signal generation. Two identical racetrack resonators are symmetrically side-coupled in parallel to a through waveguide in the center. The modulator can be fabricated on a silicon-on-insulator (SOI) wafer. The carriers can be injected or depleted from the racetrack resonators using a pin diode [45] or metal-oxide-semiconductor capacitor [85] embedded in a silicon waveguide. The plasma dispersion effect [48] of the injected carriers causes a change of refractive index, Δn_1, Δn_3, in each racetrack resonator, which modifies the cross-coupled resonances of the two racetrack resonators. By carefully choosing the voltage signals applied to each resonator, the amplitude and phase of output optical signal can be controlled to generate arbitrary quadrature signal.
Fig. 5.4. Parallel-coupled dual racetrack resonators (a) schematic of the structure, and typical spectra for an over-coupled structure: (b) intensity and (c) phase.

The proposed modulator structure has two distinctive features. (1) The coherent cross-coupling between the two racetrack resonators mediated by the center waveguide drastically modifies the amplitude/phase characteristics of resonance. This enables arbitrary quadrature signal generations such as quadrature phase shift keying (QPSK). (2) The relatively long straight segments of racetracks allow more gradual, stable coupling than the circular rings, which feature a “point contact” between each racetrack and the waveguide and tend to be very sensitive to any misalignment between “coupling points.” The tolerance to misalignment is particularly important for the proposed structure, in which the cross-coupling between two resonators is essential to its unique functionality. Note that compact silicon racetrack resonators have been systematically characterized recently [86]. The structure of this paper is organized as follows. First, the cross-coupling between the racetrack resonators is analyzed and the output transfer function of the proposed structure is presented. The critical coupling condition is obtained. Systematic studies of the over-coupled, critically coupled, and under-coupled scenarios for the parallel-coupled racetrack resonator structure indicated that strong over-coupling case is
desired for arbitrary quadrature signal generation. The interaction between the resonances of two racetracks is analyzed, and its critical role in arbitrary quadrature signal generation is presented. The effects of asymmetries in the coupling strengths and quality factors of resonators are systematically studied, and phase compensations for such asymmetries are presented. Lastly, the electrical aspects of the proposed modulators are discussed.

Next, the principles of parallel-coupled racetrack resonators for a cross-coupling analysis and an output transfer function are presented. The coupling between the two racetrack resonators and the through-waveguide in Fig. 1(a) can be described by a multi-waveguide coupling theory [87-89]. Assume the fields in three identical single-mode waveguides have slowly varying envelopes $u_n(z)$

$$E_n(x, y, z) = M_n(x, y)\exp(i\beta z)u_n(z), \quad n = 1, 2, 3$$  \hspace{1cm} (5.2)

where $M_n(x, y)$ is the lateral mode profile, $\beta$ is the propagation constant along the waveguide axis $z$ for an isolated waveguide. For the parallel coupled racetrack resonator structure in Fig. 1, the input fields and output fields of the coupling segments are given by

$$E_n^{(in)}(x, y, 0) = M_n(x, y)u_n(0) \equiv M_n(x, y)a_n, \quad n = 1, 2, 3$$  \hspace{1cm} (5.3)

$$E_n^{(out)}(x, y, L) = M_n(x, y)\exp(i\beta L)u_n(L) \equiv M_n(x, y)b_n,$$

where $a_n$ and $b_n$ are the normalized input and output complex amplitudes, respectively.

The solution of the coupled mode equations yields [89]

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \exp(i\beta L) \begin{bmatrix} c_1 + 1/2 & c_2 & c_1 - 1/2 \\ c_2 & 2c_1 & c_2 \\ c_1 - 1/2 & c_2 & c_1 + 1/2 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix},$$  \hspace{1cm} (5.4)
where

\[ c_1 = \frac{1}{2} \cos(\sqrt{2}L), \quad c_2 = \frac{i}{\sqrt{2}} \sin(\sqrt{2}L). \]  
(5.5)

The coupling coefficient is \( \kappa \). The coupling length is \( L \). The strength of the cross-coupling between the two racetrack resonators mediated by the through waveguide is given by \(|c_1-1/2|\). In addition, light propagation along a racetrack gives rise to the following relations

\[ a_1 = \eta_1 \exp(i\theta_1)b_1, \]
\[ a_3 = \eta_3 \exp(i\theta_3)b_3, \]
(5.6)

where the amplitude attenuation along a racetrack is given by \( \eta_n < 1 \), and the phase shift is given by \( \theta_n \). Assuming a unity input amplitude \( a_2=1 \), the output amplitude \( b_2 \) can be solved from Eqs. (5.4) and (5.6)

\[ E_{out} = b_2 = \frac{e^{i\theta} \left[ - \frac{(1/2 - c_1)(\Delta u_1 + \Delta u_3) + 2c_1\Delta u_2}{\frac{(1/2 - c_1)(\Delta u_1 + \Delta u_3) + \Delta u_2\Delta u_3}{\eta_n^{-1}} \frac{1}{e^{i\theta}} - 1, n=1, 3. \]  
(5.7)

where

\[ \Delta u_n \equiv \frac{1}{e^{i\theta_1+b_n}} - 1, \quad n=1, 3. \]
(5.8)

Because of the symmetry of the structure shown in Fig. 1(a), the output amplitude, Eq. (5.7), only involves terms symmetric with respect to an interchange of \( \Delta u_1 \) and \( \Delta u_3 \). \( \Delta u_n \) can be described as the *normalized* change of the field amplitude (of the envelope) after one round-trip single mode propagation in a racetrack connected to WG segment \( n \). As such, the symmetry of the structure can be utilized to help simplify the understanding of
the device principles, as noted in the study of other devices [90]. Detailed analysis of a modulated symmetric dual racetrack resonator structure is given below.

The following discussion of the parallel-coupled racetrack resonators addresses the critical coupling condition and vanishing amplitude for a modulated over-coupled structure. The critical coupling condition can be obtained by setting $b_2=0$. For symmetric parallel-coupled racetracks without modulation ($\Delta u_1 = \Delta u_3$), one readily shows that the critical coupling condition for such a parallel-coupled dual racetrack resonator structure is given by

$$\eta_1 = 2c_1 = \cos \sqrt{2\kappa L}. \quad (5.9)$$

The asymmetric cases will be discussed in later.

For modulated racetracks, the phase shift $\theta_n$ in each racetrack will be a linear function of the refractive index changes, $\Delta n_n$, due to carrier injection or depletion in the respective racetrack resonator. Therefore the output amplitude $b_2$ depends on $\Delta n_n$ through the phase shift terms. To understand the modulation characteristics, it is helpful to rewrite the output amplitude in the following form

$$b_2 = e^{i\phi} \left[ -1 + \frac{(2c_1 + 1)}{(1/2 - c_1)(1/\Delta u_1 + 1/\Delta u_3 + 1)} \right]. \quad (5.10)$$

As $c_1$ is a real number, for a modulated symmetric ($\eta_1=\eta_3$) dual-racetrack structure, the output amplitude can vanish only if $\Delta u_1 = \Delta u_3^*$. Indeed, one can show that even if the critical coupling condition is not satisfied in absence of modulation, the modulated amplitude can still vanish under the following modulation condition
For real nonzero $\Delta \theta$, this requires

$$\eta_i > 2c_1 = \cos \sqrt{2\kappa L},$$

(5.13)

which corresponds to over-coupling in comparison to Eq. (5.10). The spectra of an over-coupled dual racetrack structure (without modulation) are illustrated in Fig. 5.4(b) and (c).

The Arbitrary quadrature signal generation capability for parallel-coupled racetrack resonators is now discussed. For intensity and phase modulation, the refractive index of the silicon waveguides in each racetrack is varied on the order of 0.001. Such an amount of $\Delta n$ can be achieved with relatively low carrier concentration changes $\Delta N_e, \Delta N_h \sim 3 \times 10^{17} \text{cm}^{-3}$ according to well-known plasmas dispersion relation [48]. Fig. 5.5 depicts the simulated intensity and phase variations as a function of refractive index variations $\Delta n_1$ and $\Delta n_3$ at the resonant wavelength for an over-coupled structure. The structure parameters are $r_1=r_3=3\mu\text{m}, L=3\mu\text{m}, \eta_1=\eta_3=0.994, c_1=0.4243$. Evidently, the intensity vanishes at two points $(\Delta n_1, \Delta n_3)= (\pm 3.5 \times 10^{-4}, \mp 3.5 \times 10^{-4})$, which agree well with the analytic results based on Eq. (5.12). Note in all phase plots starting from Fig. 5.5, the overall constant phase factor $e^{i\phi}$ in $b_2$ is omitted to better illustrate the symmetry of the modulated output. Note the following symmetries for the intensity and phase variations: $I_{out}(\Delta n_3,\pm \Delta n_1)=I_{out}(\Delta n_1,\pm \Delta n_3), \phi_{out}(\Delta n_3,\pm \Delta n_1)=\pm \phi_{out}(\Delta n_1,\Delta n_3).$
Fig. 5.5. Intensity (a) and phase (b) variations under refractive index modulation for the parallel-coupled dual racetrack resonators with $r_1=r_3=3\mu m$, $L=3\mu m$, $\eta_1=\eta_3=0.994$, $c_1=0.4243$.

To visualize complex amplitude, $E_{\text{out}}(\Delta n_1, \Delta n_3)$, for quadrature signal generations, the ensemble of complex $b_2$ values for all values of $\Delta n_1$ and $\Delta n_3$ are mapped onto the complex plane of the normalized output electric field. Each point in Fig. 5.6(a) gives the amplitude and phase of the output signal for a particular pair of $\Delta n_1$, $\Delta n_3$ values in the aforementioned range. Evidently, the ensemble of points covers most part of the unit circle, therefore, allowing for the access of a wide range of amplitude and phase values.

A close examination of Fig. 5.5 indicates that the intensity and phase varies widely in the second and fourth quadrants where $\Delta n_1$ and $\Delta n_3$ have opposite signs, which is equivalent to a push-pull configuration. In contrast, the intensity and phase are much less sensitive to $\Delta n_1$ and $\Delta n_3$ when they have the same sign. Indeed, simulations indicate that the push-pull configuration is usually responsible for over 90% of coverage on the complex $E$ plane. Hence, a push-pull modulation configuration is preferred for such a parallel-
coupled dual-racetrack structure. Note that a push-pull configuration is also used in another structure for quadrature signal generation [84].

Fig. 5.6. Mapping of the normalized complex output field amplitude $E_{\text{out}}$ on the complex plane for refractive index $\Delta n_1, \Delta n_3$ varying in the range of $-0.002 \sim 0.002$. (a)-(c) for parallel-coupled dual racetrack resonators; (d)-(f) for two uncoupled racetrack resonators in series. Evidently, only case (a) is suitable for arbitrary quadrature signal generation. Constellations for QPSK (brown circles) and 16-QAM (red squares) modulation formats are illustrated in (a).

The direct sum and “interaction” of cross-coupled parallel racetrack resonances is discussed. It should be noted that the broad coverage inside the unit circle observed in Fig. 5.6(a) is a signature of the strong cross-coupling between the two racetrack resonators mediated by the center waveguide. To illustrate this point, the simulated typical coverage of a critically coupled case and an under-coupled case is shown in Fig. 5.6(b) and (c), respectively, for parallel-coupled dual racetrack resonators. In addition,
the simulated typical coverage for two uncoupled racetrack resonator in series is plotted in Fig. 5.6(d)-(f). None of the cases illustrated in Fig. 5.6(b)-(f) has adequate coverage for arbitrary quadrature signal generation.

The cross-coupling present in the parallel coupled racetrack resonators helps only the over-coupling case to achieve sufficient coverage over all four quadrants inside the unit circle. It can be shown that such a behavior stems from a delicate balance between the direct sum term $\Delta u_1+\Delta u_3$ and the “interaction” term $\Delta u_1\Delta u_3$ on both the numerator and denominator in Eq. (5.7). Based on their definitions $\Delta u_n = (1 - e^{i\phi+i\theta_n}) / e^{i\phi+i\theta_n}$, $\Delta u_n$ can be regarded as the normalized change of the field amplitude after one round-trip propagation in a racetrack. Here the initial field amplitude is unity, and the amplitude change is normalized by the final field amplitude $e^{i\phi+i\theta_n}$. For a racetrack without modulation ($\Delta n_1=\Delta n_3=0$), $\Delta u_i$ is small (on the order of $1-\eta_1$) near resonance, and $\Delta u_1$ and $\Delta u_3$ are in phase. Therefore, it is found that

$$|\Delta u_1\Delta u_3| \ll |(1/2-c_1)(\Delta u_1 + \Delta u_3)|$$

(5.14)

because $1-\eta_1 \ll (1/2-c_1)$ according to the strong coupling condition. The dominance of the direct sum term in Eq. (5.7) yields an output amplitude close to $-1$. With sufficient modulation in a push-pull configuration, $\Delta u_n$ can gain large imaginary parts ($\text{Im}(\Delta u_n) \sim \Delta \theta_n$, up to $\pm 0.09$ at $\Delta n_n=0.001$) with opposite signs whereas their real parts remain small. Therefore, the product term exceeds the sum by a large margin, $|\Delta u_1\Delta u_3| > |\Delta u_1+\Delta u_3|$ such that $(1/2-c_1)(\Delta u_1 + \Delta u_3)$ and $\Delta u_1\Delta u_3$ in Eq. (5.7) become comparable. Now the output amplitude can take virtually any value. Particularly, the two
terms on the numerator can exactly cancel each other so that the output amplitude vanishes. Hence, the large dynamic range of \(|(1/2 - c_1)(\Delta u_1 + \Delta u_3) / \Delta u_1 \Delta u_3|\) in the over-coupling case causes the output amplitude given by Eq. (5.7) to vary widely, traversing a large fraction of the area in the unit circle. In contrast, for an under-coupling case, it is straightforward to show, in general,

\[ |(1/2 - c_1)(\Delta u_1 + \Delta u_3)| \leq |\Delta u_1 \Delta u_3|. \]  

The dominance of the “interaction” term limits the accessible area in the unit circle.

For asymmetry effect in parallel-coupled dual racetrack resonators being a structure with two racetrack resonators, their asymmetry due to fabrication imperfections can be a major concern for practical applications. Optical path differences between the two racetracks can usually be compensated by a proper DC bias or by additional thermo-optic heaters [91,92]. However, the asymmetries in quality factors and coupling ratios cannot be directly compensated as easily. Therefore, their impacts on the device performance must be evaluated.

For three parallel waveguides with asymmetric coupling constants, the coupled mode equations can be written as

\[
\frac{d}{dz} \begin{bmatrix} u_1(z) \\ u_2(z) \\ u_3(z) \end{bmatrix} = i \begin{bmatrix} 0 & \kappa_{12} & 0 \\ \kappa_{12} & 0 & \kappa_{23} \\ 0 & \kappa_{23} & 0 \end{bmatrix} \begin{bmatrix} u_1(z) \\ u_2(z) \\ u_3(z) \end{bmatrix},
\]

where the coupling constants between waveguide pairs (1,2) and (2,3) are \(\kappa_{12}\) and \(\kappa_{23}\), respectively. To solve such a set of differential equation, \(\frac{d}{dz}[u_m] = i[\kappa_{mn}][u_n]\), the
The coupling matrix is decomposed into the following form \( \kappa_{mn} = X \Lambda X^+ \), where \( \Lambda \) is a diagonal matrix whose diagonal elements are the eigenvalues of the matrix \( \kappa_{mn} \), the columns of \( X \) are the eigenvectors of \( \kappa_{mn} \), and \( XX^+ = I \). The original equation can then be integrated according to

\[
[u_n(z)] = \exp(i \kappa_m z) [u_n(0)] = X \exp(i \Lambda z) X^+ [u_n(0)].
\]  

(5.17)

Thus the solution of Eq. (5.16) is given by

\[
\begin{bmatrix}
    u_1(z) \\
    u_2(z) \\
    u_3(z)
\end{bmatrix} =
\begin{bmatrix}
    \cos(\kappa_+ z) \rho_1^2 + \rho_3^2 & i \sin(\kappa_+ z) \rho_1 & \cos(\kappa_+ z) \rho_1 \rho_3 - \rho_1 \rho_3 \\
    i \sin(\kappa_+ z) \rho_1 & \rho_1^2 & \cos(\kappa_+ z) \rho_1 \rho_3 \\
    \cos(\kappa_+ z) \rho_3 - \rho_1 \rho_3 & i \sin(\kappa_+ z) \rho_3 & \rho_3^2 + \rho_1^2
\end{bmatrix}
\begin{bmatrix}
    u_1(0) \\
    u_2(0) \\
    u_3(0)
\end{bmatrix}
\]  

(5.18)

where \( \kappa_+ = \sqrt{\kappa_{12}^2 + \kappa_{23}^2} \), \( \rho_1 = \kappa_{12} / \kappa_+ \), and \( \rho_3 = \kappa_{23} / \kappa_+ \). In a symmetric case, \( \kappa_+ = \sqrt{2} \kappa_{12} = \sqrt{2} \kappa_{23} = \sqrt{2} \kappa \), \( \rho_1 = \rho_3 = 1 / \sqrt{2} \), this returns to Eq. (5.4). The output amplitude \( b_2 \) can be solved in a procedure similar to that given for the symmetric case. After lengthy calculations, the final result is surprisingly simple

\[
b_2 = e^{i \theta} \left[ -1 + \frac{1 + \cos(\kappa_+ L)}{[1 - \cos(\kappa_+ L)](\rho_1^2 / \Delta u_1 + \rho_3^2 / \Delta u_3) + 1} \right],
\]  

(5.19)

where \( \Delta u_n \) are defined the same way as in the symmetric case. Comparing Eq. (5.19) and Eq. (5.10), it is evident that all asymmetry effects can be effectively factored into the term

\[
\rho_1^2 / \Delta u_1 + \rho_3^2 / \Delta u_3 = \frac{\kappa_{12}^2}{\kappa_+^2} \frac{e^{i \phi + i \theta} \eta_1}{1 - e^{i \phi + i \theta} \eta_1} + \frac{\kappa_{23}^2}{\kappa_+^2} \frac{e^{i \phi + i \theta} \eta_3}{1 - e^{i \phi + i \theta} \eta_3}.
\]  

(5.20)
As a consequence, for reasonable asymmetries in the coupling constants and resonator quality factors, there exists a pair of phases $\Delta \theta_1$ and $\Delta \theta_3$ such that the output amplitude $b_2$ vanishes. The required phase variations are plotted against the asymmetric coupling ratio, $\kappa_{23}/\kappa_{12}$, in Fig. 5.7(a) for up to 50% asymmetry. As $\Delta \theta_1$ and $\Delta \theta_3$ generally have opposite signs, $\Delta \theta_1$ and $-\Delta \theta_3$ are plotted to better illustrate the deviation from symmetry. Note that $\Delta \theta_1 = -\Delta \theta_3$ for a symmetric structure ($\kappa_{23}/\kappa_{12}=1$) according to Eq. (5.11), (5.12). The difference between $\Delta \theta_1$ and $-\Delta \theta_3$ becomes larger as the asymmetry increases. Fig. 5.7(a) shows that although it is not easy to directly compensate the asymmetric coupling constants themselves, asymmetric phase shifts can be introduced to recover the low intensity states ($b_2 \approx 0$). Where asymmetric phase shifts can be achieved by applying different DC biases to the two resonators. The un-modulated output spectrum for the worst case ($\kappa_{23}/\kappa_{12}=1.5$) is illustrated in Fig. 5.7(b) and shows no significant anomaly. However, the intensity variation upon refractive index modulation shows obvious distortion from the symmetric case. Nonetheless, two features remain: (1) there are two points with relatively small index changes ($\pm 2.2 \times 10^{-4}$, $\mp 5.4 \times 10^{-4}$) where the intensity vanishes; (2) the intensity varies significantly in the push-pull configuration and much less otherwise. The coverage on the complex $E$ plane is slightly enhanced, although a small hole exists at a large amplitude value, which may limit the maximum accessible amplitude to 0.78 for a generic $M$-ary modulation format.
Fig. 5.7: Effect of asymmetric coupling constants. (a) Required phase compensation in each racetrack for up to 50% asymmetry in the coupling ratios. The characteristics of the asymmetric dual racetrack structure for the worst case scenario ($\kappa_{23}/\kappa_{12}=1.5$) are illustrated in (b)-(d). (b) Output spectrum without modulation; (c) Intensity variation with index modulations; (d) Mapping of the output field on the complex plane. All parameters are the same as those used in Fig. 5.5 except $\kappa_{23}/\kappa_{12}$ is varied.

Asymmetric quality factors for parallel-coupled dual racetrack resonators are addressed below. The effects of asymmetric quality factors are illustrated in Fig. 5.8. The required phase shifts, $\Delta \theta_1$ and $-\Delta \theta_3$, for vanishing $b_2$, are plotted against the ratio of the quality factors in Fig. 5.7 (a). The quality factor $Q_1$ is fixed at its original value $\sim 2.5 \times 10^4$. 
Note that $\Delta \theta_1 = -\Delta \theta_3$ for the case of $Q_3/Q_1 = 1$ in accordance to the symmetric case. The unmodulated output spectrum for the worst case ($Q_3/Q_1 = 0.5$) is illustrated in Fig. 5.8(b). A small yet noticeable spike appears at the resonance due to the asymmetric quality factors of the two racetrack resonators. The modulated intensity variation upon refractive index modulation depicted in Fig. 5.8(c) shows less severe distortion compared to the distortion observed in the Fig. 5.7(c). Again, two features remain: (1) there are two points with relatively small index changes ($\pm 4.4 \times 10^{-4}$, $\mp 4.1 \times 10^{-4}$) where the intensity vanishes; (2) the intensity varies significantly in the push-pull configuration and much less otherwise. The coverage on the complex $E$ plane slightly deteriorates. There exists a small hole, which may limit the maximum accessible amplitude to 0.74 for a generic $M$-ary modulation format.
Fig. 5.8. Effect of asymmetric quality factors. (a) Required phase compensation in each racetrack for asymmetry in the quality factors. The characteristics of the asymmetric dual racetrack structure for the worst case scenario ($Q_3=0.5Q_1$) are illustrated in (b)-(d). (b) Output spectrum; (c) Intensity variation with index modulations; (d) Mapping of the output field on the complex plane. All parameters are the same as those used in Fig. 5.5 except $\eta_3$ is varied to yield different $Q_3$.

Overall, the asymmetry analysis presented above show that substantial asymmetries in coupling constants and quality factors of the two racetrack resonators can be compensated by refractive index changes on the order of $4 \times 10^{-4}$, which can be readily provided through a low-power microheater or a small change of the DC bias.

In general, an encoder is needed to convert an original $M$-ary digital signal into the driving signal for the modulator. Consider the case of a QPSK signal with four symbols shown in Fig. 5.6(a). The encoder will have a two-bit input and two output ports. Each output port has four output voltage levels. In contrast, a conventional nested Mach-Zehnder QPSK [74] modulator needs three output voltage levels for each port. The additional voltage levels required for the proposed QPSK modulator will increase the size of the driving circuitry. However, electronic devices such as transistors are generally significantly smaller than photonic devices. Therefore, the enlargement of the driving circuitry is usually negligible compared to the significant space saving offered by changing from a bulky nested Mach-Zehnder modulator to the proposed dual racetrack resonators.
Driving voltages and power consumption are important issues for silicon modulators used in optical interconnects [46,93]. For a nested Mach-Zehnder QPSK modulator which is biased across the minimum point of the transfer curve, a lower driving voltage and lower RF power consumption can be achieved at the expense of a lower maximum output intensity, which entails a trade-off with the detector sensitivity or the input laser power. For the proposed parallel-coupled dual racetrack modulator, a similar power reduction scheme is possible. For simplicity, a silicon racetrack resonator is considered with embedded MOS capacitors, whose index change is approximately linearly dependent on the voltage. As illustrated in Fig. 5.9(a), the driving power can be significantly lower at lower output intensity. It should be mentioned that the nested Mach-Zehnder modulator structure also causes substantial insertion loss because each Y-junction introduces 3dB loss if light in one branch of the outer Mach-Zehnder interferometer is suppressed. Asymmetries of the coupling constants and quality factors could entail extra power penalty but the power remains reasonable. According to Fig. 5.9, if the asymmetry is large, electrical power penalty is significantly lower when the modulator operates at a lower output intensity level. Therefore, for a modulator that happens to have a large asymmetry due to imperfection in fabrication, the balance of the power trade-off may tip towards enhancing the detector sensitivity. Overall this could indicate the potential of dispersion enabled active control of silicon micro- and nano-photonic devices and enhanced power efficiency.
Fig. 5.9. Output intensity is depicted as a function of the driving power for parallel-coupled dual racetrack modulators with varying degrees of asymmetry. (a) For various $\kappa_{23}/\kappa_{12}$ values and (b) For various $Q_3/Q_1$ values. The output intensity is normalized by the input intensity. The driving power is normalized by the power level corresponding to the case that each racetrack is driven to $\Delta n=0.001$. 
5.3 Fabrication Results

Fabrication of nano-photonic devices was conducted with various materials, processes and designs, while compensating for some barriers to fabrication. Fabrication processes limitations and design considerations are discussed. Fabrication results are presented for multiple designs. Some of the aspects of the designs considered are conventional rib waveguide cross sections accounting for single mode operation, length accounting for propagation loss and racetrack radius accounting for bending loss. Additionally, the coupling region length and coupling separation are considered for their influence on the coupling efficiency and in part, the affect on critical coupling. Also, the affect on photonic crystal structure hole radius and period on confinement was accounted for. Some barriers to fabrication include stitching, overdosing in electron beam lithography, surface contaminants, and angled sidewalls after etching. Lastly, potential future work and improvements are mentioned.
5.3.1 Fabrication Processes

The fabrication process used to create the photonic devices discussed consists of many basic steps, which include sample preparation, mask file adaptation, electron beam lithography, developing resist, dry etching, resist stripping, silicon oxide deposition and cleaving processes. The fabrication process was explored using both positive e-beam resist ZEP-520A diluted 1:1 and negative e-beam resist XR-1541 HSQ. Additionally, alternative methods of modifying the chip edge interface were explored.

Sample preparation includes cleaning, prebaking, spin coating resist and post baking. Cleaning the SOI wafer, with its stack structure of 0.26μm of silicon on 2μm of silicon oxide on silicon, begins with submersion of the sample in Buffered Oxide Etch (BOE), then into water, followed by Trichloroethylene (TCE), Acetone, and then Isopropyl alcohol (IPA) in an ultrasonic bath. The sample is then pre-baked. The positive and negative e-beam resists are both spin coated yielding a ~200nm layer of positive resist or a ~100nm layer of negative resist. Lastly, the sample is post baked. Mask file adaptation includes removing overlaps or fragmented lines in the original mask design files and proximity or dose correction. Removing overlaps prevents doubling of dose or overexposing the resist in that region. Removing fragmented lines prevents the Raith/eLine software from failing. Proximity correction prevents electron beam writing in one location from effecting another location within a ~2μm radius of that position. It should also be noted that positive e-beam resist correction patterns have doses increased near high pattern density regions and negative e-beam resist correction patterns have doses reduced in high pattern density regions. The electron beam lithography was conducted in a Raith/eLine system using a beam aperture of approximately 10μm for
positive e-beam resist and 30\(\mu\)m for negative e-beam resist. The working distance was approximately 10.9 mm. The stigmation is then initially adjusted accounting for changes in beam current and resist types being positive or negative values. This was followed by adjusting the initial aperture values, again accounting for positive or negative resist choices. The Tilt/Rotation is set to zero and then each value is adjusted as the system is focused. In positive resist, the e-beam should be focused on white spots, and in negative resist, on dark spots. These spots could be contamination particles usually near the edge.

Next, make initial focusing and alignment adjustments. The process would start at the Faraday cup, checking the beam current, then finding the origin of the sample, checking the angle offset of the samples edges to the stage, and then conducting a working distance correction. The laser height control is set by testing three corners of the sample. The write field alignment is conducted by using 5\(\mu\)m and then 1\(\mu\)m magnification settings. Finally the e-beam scan is conducted from the position list. Development process begins by submerging sample into ZED N50, then in Methyl Isobutyl Ketone (MIBK) and IPA. Blow drying must be avoided since this may damage the resist pattern. Finally, the sample is baked on a hot plate or in a vacuum oven. Dry etching is done using an ICP etching chamber (SAMCO 800). The ICP process applies a sidewall passivation step and an etching step that are repeated for multiple cycles. Some initial fabrication results are shown in Fig. 5.11. Note that for negative e-beam resist, more surface area is exposed during the etching process unless covered by conductive tape or a second layer of photo resist. The residual resist is removed or stripped using oxygen plasma etch and then BOE. The top SiO\(_2\) cladding layer is deposited by Plasma-enhanced chemical vapor deposition (PECVD). The final step is to cleave manually with diamond scribe; by
photolithography and ICP etch; or by a cleaving machine. All of these methods can be further refined when trying to obtain a consistently optically viable interface.

Fig. 5.10. Fabrication Process Steps.
5.3.2 Fabrication Results

The fabrication results presented below depict many designs which include single racetrack resonators, parallel-coupled dual racetrack resonators and photonic crystal based structures. Additionally, 1D periodic structures are integrated into WGs and single racetrack resonators along with laterally coupled dual photonic crystal racetrack resonators, cascaded racetrack micro-resonators and add/drop filters. Images of a side coupled single racetrack resonator occur at the 6th process step before depositing silicon oxide is shown below. A laterally coupled parallel dual racetrack resonators were also fabricated and shown in fig. 5.12.

Fig. 5.11. (a) Single racetrack resonator has a 2μm coupling length, ~400nm separation between racetrack and input waveguide, ~260nm thickness, ~500nm width, racetrack radius of 5μm and the adjacent channels forming the silicon rib waveguide is ~4μm wide;
(b) Coupling region of the resonator.

The constant dimensions of the structure include a ~260nm thickness, ~500nm width and 5μm radius. The design had many variations in scale including coupling length ranging from 2μm to 3μm and coupling separation ranging from 200nm to 400nm. The round-trip path length of each racetrack ranged from ~ 35.4159 μm to 42.4159um. Additionally, single and parallel dual racetrack resonators were compared. In general, the side channels are etched through to the silicon oxide. The etching tolerances for various dimensions in comparison from the designs to the actual results is in a range of + or - 15%. In the parallel coupled racetrack resonator, it should be noted that a slight asymmetry occurred due to the nature of some fabrication processes having variations within fabrication tolerances and also possibly due to proximity effects of the rastering e-beam.

Fig. 5.12. (a) Parallel Dual racetrack resonator with 5μm radius; (b) Coupling region of racetrack resonator.
A 20 μm radius parallel dual racetrack resonator was also fabricated for comparison to racetrack resonators with radii of 5 μm. The single racetrack resonator version was also fabricated but is not shown. The constant dimensions of the structure are a thickness of 260nm and rib WG width of 500nm. The round-trip path length of each racetrack ranged from ~129.6637 μm to 139.6637 μm. The design had variations in scale including coupling separation ranging from 200nm to 400nm and coupling region length ranging from 2 μm to 3μm. Some design variations fabricated but not shown here include those with straight segments ranging from 2 μm to 4 μm between each 90 degree arc.

![Parallel Dual racetrack resonator with 20μm radius](image)

Fig. 5.13. Parallel Dual racetrack resonator with 20μm radius.

The below SEM images of a hexagonal lattice line defect WG with and without a slit were examined to compare fabrication quality and results. The slit width was varied from ~120nm to ~140nm, hole radius varied from ~170nm to ~210nm, and periodicity ranged from 380nm to 420nm.
Periodic Structures Integrated into Resonators and Waveguides:

Sidewall gratings integrated into resonators and waveguides were fabricated. These structures were created to examine their potential of acting as gratings and lateral coupling enhancements. These structures were also examined in regards to disturbances in periodicity acting as another resonator [94]. The constant dimensions of the structure are a thickness of 260nm and rib WG width of 500nm. The design had variations in scale including a periodicity that ranged from approximately 100nm to 500nm, the length of the gratings protrusion into the coupling region that varied from approximately 100nm to 300nm and a racetrack radius that ranged from 5 μm to 20 μm. The structural variations included sidewall gratings on rib waveguides, sidewall gratings on racetrack resonators and interleaved sidewall gratings on WGs and racetrack structures.
Fig. 5.15. Sidewall gratings integrated into resonators and waveguides (a) 200nm periodicity; (b) 100nm periodicity.

One-dimensional periodic structures integrated into the coupling region between resonators and waveguides for single and dual racetrack resonators. The periodicity ranged from approximately 100nm to 500nm and the length or diameter varied from 50nm to 300nm. The periodic structure varied from rectangular, square to circular.

Fig. 5.16. One-dimensional periodic structures integrated into the coupling region of a racetrack resonator of 5μm radius and a 400nm coupling separation.
Additional structures included breaks in longitudinal periodicity that created cavities as potential resonators. These 1D aperiodic cavity in 1D PC structures integrated into rib WGs and racetrack resonators were created to examine the potential interaction of the various resonator structures.

**Photonic Crystal Racetrack Resonators:**

Two dimensional periodic structure racetrack resonators or photonic crystal racetrack resonators were fabricated as a laterally coupled parallel dual PC racetrack resonator which are shown below and as single PC racetrack resonators. Single PC racetrack resonators are not shown since similar structures have been done. This device could potentially allow for the exploration of the interactions between slow light effects of PCWs and resonance effects. The dimensions of the structure included 500nm width and ~260nm WG height for the input and output rib WGs. The PC racetrack resonators had a hexagonal lattice with periodicity of approximately a 400nm. The coupling region is approximately 7 periods or ~2.8um. The path length for each racetrack is approximately 38 periods or ~15.2um. The coupling separation is approximately 450nm. A ~130nm slot is in the PC line defect WG segments and the termination point in the period at the interface between the PCW and conventional rib WG is at a 0.75 multiple of the lattice period for the expectation improved loss [6]. The design had many variations in scale including hole diameters from 170nm to 210nm, periodicity from 380nm to 420nm, slot width from 120nm to 140nm and the reduced hole radius values ranged from 115nm to 140nm. The variations in structure include, cascaded PC resonators, addition
of add/drop features, PC racetrack-like defects with and without a slot and with and without reduced hole radii near the 60 degree bends, in addition to others.

![Image](image_url)

Fig. 5.17. photonic crystal racetrack resonators (a) Parallel Dual PC racetrack Resonator; (b) Coupling region, slot in line defect WG and reduced radius holes at bends.

**Add/Drop Resonator Filters:**

Add/drop resonator filters were created to examine the effect that resonators would have on other resonators when laterally coupled to each other with a through-put waveguide and a filter output waveguide. This was fabricated for the symmetric and asymmetric cases having two sets of three laterally cascaded racetracks. The dimensions of the structure included 500nm rib WG width and ~260nm WG height. The design had many variations, including the coupling separation varied from 200nm to 400nm, the coupling region length is varied from 2 \( \mu \)m to 3 \( \mu \)m, radius is varied from 5 \( \mu \)m to 20 \( \mu \)m, racetracks with horizontal straight segments between each 90 degree arc of the racetrack, the number of laterally cascaded racetracks is varied from one to three and the insertion of periodic couplers in coupling regions.
Fig. 5.18. Add/Drop Filters (a) Parallel Dual Racetrack Resonator Add/Drop Filters; (b) Racetrack Resonator Add/Drop Filters

**Cascaded Resonators:**

Longitudinally cascaded resonators were created to examine the effect that resonators would have when longitudinally coupled to a single through-put waveguide. This was fabricated for the symmetric and asymmetric cases having two sets of two longitudinally cascaded racetracks in parallel coupled to both sides of a rib WG. For the purposes of comparison the case of a single set of cascaded resonators on one side of a WG was also made. The dimensions of the structure include 500nm rib WG width, 260nm WG height, racetrack radius of 5μm. The design had many variations with a coupling separation of 200nm to 400nm, a coupling region length of 2 μm to 3 μm and separation between cascaded racetracks of 10 μm to 14 μm.
Laterally cascaded Resonators were created to examine the effect that resonators would have on other resonators when laterally coupled to each other and a single through-put waveguide. This was fabricated for the symmetric case having two sets of three laterally cascaded racetracks. The dimensions of the structure include a 500nm rib WG width, a 260nm WG height, a 2 μm coupling region lengths, racetrack radius of 5 μm. The design also has two variations with the coupling separation from 200nm to 400nm.

Some other designs which were made, but not shown here, potentially speak to the versatility of the resonator structure. One specific design change, such as altering the coupling length will allow comparisons between effects on the signal from systems operating in the critical and over coupled states. Additionally, some other design variations include racetracks with horizontal straight segments connecting the two halves of the racetrack, photonic crystal racetrack resonators filters, 1D periodic structures integrated with rib waveguides and radial periodic structures integrated into racetrack resonators. Some of the designs mentioned could potentially enhance surface light coupling, exploration of slow light effects on resonators and exploration of the interaction
between coupling region cavity resonators with racetrack resonators. Additional designs were created but not fabricated due to resource limitations. Some of these devices include parallel racetrack resonators integrated into MZI structures, thermo-optic tuning devices, electro-optic modulators with thermo-optic resonance tuning. These devices that were not fabricated could potentially allow for the exploration of thermo-optic tuning, which can compensate for fabrication errors on the scale of tens of nanometers. These issues can cause undesired asymmetries in parallel racetrack resonators or undesired resonance shifts. Moreover, electro-optic devices could allow for exploration modulation, sensing and communication applications.
5.3.3 Some Barriers to Fabrication

Some barriers to fabrication faced during this work include stitching, overdosing, surface contaminants, etch back and angled sidewalls. Flaws in device structure due to stitching misalignment of the e-beam lithography system (Raith / eLine) are caused by mechanical misalignment of the sample stage, which can occur every 100μm. The width of the gap is approximately 50nm shown in Fig. 5.21(b). The stitching produced a periodic noise with a FSR ~ 2.7nm as an effect of a linear Fabry-Perot cavity resonator with a round-trip length 200μm. A more advanced Leica/VB6 e-beam lithography system would potentially reduce this effect since its stitching error is approximately 10nm.

![Fig. 5.21. Defects due to stitching](image)

(a) Stitching in a waveguide intended as a template for imprint lithography; (b) stitching error in a rib waveguide

Overdose and improper proximity correction resulted in the destruction of the coupling region which is shown below. The necessary proximity correction is also shown for positive resist in Fig. 5.22(b) and negative resist in Fig. 5.22(c). One of the effects of this problem is a decrease in the quality factor and an increase in noise.
Another major issue was selecting the correct acceleration voltage for the e-beam system. If the proper dose topology and level were not appropriate for the voltage then overdosing or under-dosing would occur. Lastly, the orientation of the rastering e-beam spot pattern on the surface with respect to the lateral resonator to resonator symmetry of the pattern is which is crucial and must be accounted for when implementing this process.

Fig. 5.22. (a)Defects caused by overdose in the coupling region of racetrack resonator; (b)Proximity correction for positive resist; (c)Proximity correction for negative resist.

Contaminations on the surface causing defects were common regardless of processing in the cleanroom. Proper wafer cleaning and the use of the vacuum oven instead of a nitrogen gun to dry a sample can reduce contamination or pattern damage. The process is most vulnerable during the resist development steps after e-beam lithography. Furthermore, the use of some materials necessary for etching such as conductive tape or thermal grease will create surface contamination, if not properly removed. All surface contaminant increase loss, especially when near critical areas of the design, such as a particle with on the WG top surface acting as a resonator cavity or disrupting periodicity of PC structures.
Fig. 5.23: Surface contaminants (a) Defects in PC structures; (b) Defects in waveguide structures.

Etch-back and angled sidewalls both negatively affect coupling loss at various points in the design. Etch-Back at the interface, shown in the below figure, occurred when attempting to create a dry etched fiber to waveguide interface facet. Additionally, some debris can remain on the surface after the etching process as a byproduct of the passivation step in that process. The sample in this case can experience significant surface charging. An etched back interface was found to produce poor coupling between a lensed fiber and a waveguide interface facet. Angled sidewalls are often undesired since it does not allow for a strong discontinuity at the point of change in index for good index contrast which enables better confinement. Reduced index contrast may also negatively affect potential cross coupling between resonators and waveguides. Angled Sidewalls yields a trapezoidal cross section, which is not as preferred as a rectangular cross section with stronger lateral index contrast.
Overall, a discussion of various fabrication subjects were presented, which included fabrication processes, resulting designs variations and some fabrication barriers. Also, some comments are made in regards to future work and potential improvements. The procedure has proven to produce fine features with a resolution of 40nm. Additionally, the process included multiple resist types. Some optional methods were also discussed for e-beam lithography and for cleaving that could enhance results. The etching method for modifying chip interfaces was cumbersome and is point of further work. The fabrication processing steps are depicted in Fig. 5.10. The yield from the processing of functional devices was approximately 7%. Various designs were explored and introduced many possible paths for examining the interaction of resonators, conventional waveguides, PCWs, and the merging of periodic structures with resonator structures. Some of the physical manifestation of the various barriers to fabrication include stitching, overdosing, surface contaminants, etch back and angled sidewalls. All of these fabrications anomalies increase loss. Some future fabrication processes that can enable integration of functionality to vary effective index are mentioned here. Electron
beam deposition for electrodes can allow for thermal or acoustic tuning of the racetrack’s effective index. Photolithography to pattern a 20μm channel in silicon oxide as a microfluidic channel can allow for index changes using fluids. Implantation processes for creation of lateral PIN diode structures can enable carrier injection. Improvements to processes could include the use of a Leica/VB6 e-beam instead of a Raith/eLine system to reduce stitching effects, or the use of a cleaving machine to improve the optical interface at the edge of the sample.
5.4 Characterization of Resonators

A multi-input and output test bed with optical and electrical control was necessary for characterization of the various devices. The characterization of the fabricated devices is presented, along with the related procedures, followed by the results and some of the sources of noise. Lastly, some potential future work is discussed.

5.4.1 Characterization Procedures

The testing procedure for optical devices discussed consists of some basic steps which included alignment, mode profile scan, wavelength scan, and top surface scan. Additional testing that was considered or attempted included high power laser scan, interferometer phase measurement, and infrared imaging of device during operation. In general the equipment was controlled and the data was collect using MATLAB.
The testing setup in fig. 5.25 has multiple layouts (configurations) and is briefly elaborated in this section. The first layout excludes the reference arm. The second layout includes the reference arm. The third layout excludes the reference arm and includes a system to scan a lensed fiber tip over the surface of the device without touching it.

All layouts include a tunable laser source, where through various devices used could allow a scanable wavelength range of ~1350nm to ~1585nm with a resolution of ~1pm and a maximum power of ~3mW. The optical power meter has two occupied bays with detectors having a sensitivity of approximately -85dBm and -110dBm. Alternatively, an Amplified Spontaneous Emission (ASE) source and optical spectrum analyzer (OSA) could be used over a range of ~1525nm to ~1575nm with a resolution of ~0.05nm. The stages used for alignment are mechanical and piezoelectric. The piezoelectric stages have a resolution of ~40nm with a range of ~18 μm. Additionally, a piezoelectric stage with a 90 μm range with a 100nm resolution is an option when a larger
In all cases the data is captured via GPIB cables and a GPIB to USB converter, connected to a Personal Computer operating a MATLAB program for storing data, graphing data and controlling equipment.

The first layout considered with no reference arm uses a tunable laser for large wavelength scans, the sample alignment process and capturing the guided single mode profile. The tunable laser or ASE was connected via an angled polished FC/APC connector to a single mode polarization maintaining (SMPM) fiber and then the fiber was connected to a SMPM lensed fiber with a 2.5 μm spot size. The lensed fiber was then aligned to a silicon channel waveguide input facet of ~260nm x 500nm in area via microscope in the horizontal x-y plane and then aligned with a red laser in the vertical z dimension, all with mechanical stage adjustments having a possible resolution greater than 1 μm. The lensed fiber at the input needs to be approximately 10 to 15 μm from the waveguide face for the 2.5 μm spot size of the focal point to be at the facet. At the exit facet the light was coupled back into a lensed fiber to a connecting SMPM fiber via a polished connector FC/PC connector to the optical power meter or in the case of using of an ASE source an optical spectrum analyzer. Both systems allow for measurements of wavelength spectra. The lensed fiber was mounted via fiber holders to piezoelectric stages which are controlled by a PC running MATALB. The piezoelectric stages can then be used to scan the mode profile at the output facet.
Fig. 5.26. Wavelength Scan and Mode Profile Test Layout.

The interferometer layout enables phase and time delay measurements. It includes a reference arm, splitters, attenuators and two optical power meter detectors. The tunable laser again was connected via FC/APC connector through SMPM fiber to a 99:1 (or 90:10) splitter that directs 99% of the light to the device under test (DUT) and 1% to an adjustable attenuator to match the loss in the DUT when the signals were merged. The signals were combined through two cascaded 2x2 splitters where one signal went to one detector before being combined and the other went to the other detector after being combined, which allowed for comparison.

Fig. 5.27. Interferometer Test Layout.
The third layout used for surface scans involves no reference arm, but did require additional piezoelectric stages, mechanical stages, lensed fiber, two detectors and fiber holders. Potentially, this layout could also include a second high powered source to be incident on the device surface. The layout was similar to that of the first layout, but with a surface rastering lensed fiber. An angled fiber holder positioned at 60 degrees was mounted on a mechanical stage, which was secured to a piezoelectric stage controlled by a MATLAB program. The lensed fiber was connected to the second fiber, which was connected via a FC/PC connector to a second optical power meter detector and again the data was plotted by MATLAB.

![Diagram of Surface Wavelength Scan Test Layout](image)

**Fig. 5.28. Surface Wavelength Scan Test Layout.**

The alignment and test procedure consists of some initial steps which include visual alignment, coarse microscope alignment, red laser alignment, ASE-powermeter alignment and measurement scans. First, a visual alignment is done to make the fiber approximately parallel with waveguide’s longitudinal direction. A coarse alignment of the fiber with the waveguide is next and is done in the x-y dimensions at 180x microscope magnification. The red laser alignment is then conducted to align the Z
dimension by looking for the position just before the first peak of reflected red light off the chip’s edge with the laser power set at ~0.3mW. ASE powermeter alignment measurement using the ASE is a 3 axis alignment based on lowest optical loss reading with ASE input power of ~5mW. Lastly, the tunable laser-powermeter alignment is done by using a tunable laser at ~1mW and finding the lowest optical loss.

The testing process begins with a mode profile characterization in order to confirm single mode propagation and confinement. It consists of two parts and has the purpose of confirming. First, check single mode profile using piezoelectric stages at 1μm steps. Then check the single mode profile using piezoelectric stages at 0.4μm steps. The wavelength scan, which also consists of two parts, detects notch events in transmission spectrum and detects sets of notches that may nearly overlap. First, a coarse scan is conducted at .1nm steps in segments of ~20nm from ~1350nm to ~1580nm using a tunable laser with ~1mW of power. This is followed by a fine scan at .05nm steps in segments at identified notches at the same power. Top surface scan of the racetracks is conducted to image the racetracks more intensely illuminated at notch wavelengths from a tunable laser input at ~1mW and observe the reduction of intensity away from notch wavelengths of the resonator. The process includes three basic parts, the first of which are the large area surface scan of 90μm x 80μm of racetrack resonator with a 20μm radius, followed by a surface scan of the 5μm radius racetrack resonator and finally a surface scan of a single waveguide region for reference. All steps use scan resolutions of 0.5μm and 1μm for working areas of 89μm x 89μm and 18μm x 18μm.

More testing is needed to cultivate a more in-depth analysis. Other testing variations beyond those mentioned include applying a laser input at the surface on the
racetracks with measurements at the chip edge and inputs at chip edge with top surface input simultaneously. The tests recently attempted and planned for re-examination in the immediate future include those that involve a high power laser source, interferometer setup and IR camera. The high power laser wavelength scan is to see shifts in wavelength of notches in the transmission spectrum with a process similar to the previous wavelength scan, but instead using 160mW laser source for injection. The interferometer phase change measurements requires the confirmation of the polarization angle for polarization maintaining fibers, followed by adjusting the variable optical attenuator to match the loss in both arms of the interferometer allowing the phase shift to be seen (time delay $\tau = \frac{d\phi}{d\omega}$) and then measuring the branches of the interferometer separately before and after the two arms of the interferometer are combined. The last of the significant tests images the device during operation using an infrared camera through a microscope with adequate resolution.

Mode scanning at the waveguide interface or the top surface is crucial in distinguishing between noise, racetrack resonator coupling, slab mode in Si layer and the guided mode of the signal. The below image depicts the testing system.

![Fig. 5.29. Surface Scan (a)Multi stage system; (b)Surface fiber with inputs and outputs](image)

A guided mode analysis was done to determine the existence of a single mode signal with low loss by scanning the output interface with a piezoelectric mechanical stage and
recording the signal strength at each position. The loss topology at the surface of the sample provides information about racetrack resonator coupling. This is examined by scanning an area across the surface of the racetrack resonators and capturing escaping light lost through the top surfaces of the devices. This testing method allows us to show that a racetrack is operating near resonance and it also characterizes some of the relative losses in the racetrack.
5.4.2 Characterization Results

A single guided mode distinguished from the slab mode and noise is shown Fig. 5.30. Observations have indicated that signal strength has various intensities, from less than a nano-Watt to micro-Watts, depending on the quality of the device. The slab mode is visible as a wide hump that bisects the intensity graph. The peak represents the single mode signal of interest.

![Intensity vs. Position](image)

Fig. 5.30. Mode Profile Showing Single Mode Propagation.

A wavelength scan was also conducted to show notches in the transmission spectrum at approximate intervals of the free spectral range, using a tunable laser, optical spectrum analyzer and optical power meter. In addition, some effects related to the noise produced by features such as the Fabry-Perot noise introduced by stitching were identified. The wavelength scan of a single racetrack micro-resonator shows the free spectral range and the full width half maximum (FWHM) approximately as 0.9nm. The
device tested has a quality factor of 1800. Fig. 5.31 depicts more shallow notches of 2 to 4 dBm depth as an indicator of the intensity value for coupling into the single racetrack micro-resonator.

Fig. 5.31. Single racetrack resonator FSR.

A 5 μm radius parallel dual racetrack resonator was examined. The slight asymmetric nature of the parallel resonators is due to fabrication processing. The offset between the notches is ~4.9nm at wavelengths about 1550nm. This offset changes as the wavelength increases. The FSR is ~14nm. The FWHM is ~0.3nm. Alignment of the notches could occur at ~960nm, but silicon becomes less transparent at ~1100nm. Other samples show some variation in the separation between notches. In future work the examination should be pursued of the notch separation near 1100nm.
Fig. 5.32. Spectra of 5 \( \mu \text{m} \) radius parallel-coupled dual racetrack resonator (a) Free Spectral Range; (b) Full Width Half Maximum.

A 20 \( \mu \text{m} \) radius parallel dual racetrack resonator was also examined and compared to the 5 \( \mu \text{m} \) radius resonator. This structure produces a smaller FSR and the results tended to be noisier than that of the 5 \( \mu \text{m} \) radius. At \( \sim 1559 \text{nm} \) the cumulative notch separation is \( \sim 2.7 \text{nm} \), showing a shift in separation of \( \sim 0.9 \text{nm} \) for every 21 nm change in source wavelength when the effects from noise was averaged over 3 periods. A sample exhibiting less noise may be required to allow for for a more discernable signal.

![Insertion Loss (laser input = default)](image)

Fig. 5.33. 20 \( \mu \text{m} \) radius parallel dual racetrack resonator Free Spectral Range.

Longitudinally cascaded resonators were examined for parallel dual cascaded racetrack resonators and single cascaded racetrack resonators, both of which exhibited a significant amount of noise. The parallel dual cascaded racetrack resonators showed a FWHM of \( \sim 0.3 \text{nm} \).

The surface wavelength scan results for a racetrack resonator are depicted below in Fig. 5.34. The light coupled at a wavelength, where a notch occurred, shows a
resonance peak with a height ranging from 8 dB to 17 dB. The FSR is about 14.5nm, the FWHM ranges in value from ~0.4nm to ~0.8nm and the resonance peak separation ~3.2nm near 1550nm. The smaller separation detected here can indicate that at a shorter wavelength near ~1200nm where an overlap of the resonances can occur when detected at the top surface of the resonators. It should also be noted that the FWHM of the surface scan is significantly smaller than the value detected from the edge of the sample at the throughput waveguide. A decrease in FWHM based on choosing the detection point at the surface yields an increase in quality factor that can range from ~2000 to ~3900.

Fig. 5.34. Spectra obtained from the light radiating from the top surface of the racetrack resonators. (a) the FSR; (b) Resonance Separation.

The surface intensity topology scan results for a single racetrack resonator are depicted below in Fig. 5.35. The light coupled at a wavelength, where a notch occurs, is approximately 0.16nW compared to the peak background noise of ~0.08nW.
Fig. 5.35. Surface Scan Result of a Racetrack Resonator.
5.4.3 Sources of Noise

Some sources of noise appearing in the surface scan include stitching flaws, misalignment of racetracks, debris on the surface and damage to chip edge near the lensed fiber interface. Flaws due to stitching create linear cavities of a round-trip length of 200µm cause cumulative effect in the transmission spectrum, manifesting as an almost periodic noise with FSR ~ 2.7nm near 1550nm. Debris on the surface created random cavities and resonances that produce noise that cannot be predicted as easily as the effects of stitching flaws, but considered as surface resonator cavities with unknown dimensions. Damage to chip edge near the coupling interface can create significant noise effects that could potentially mask all signal information. The damage was usually created during the cleaving process, but damage to the lensed fiber tip can also contribute and this could be caused by an impact or impacts into the side of the sample. Additional noise factors could be attributed to many causes, some of which could be environmental issues of the test environment, alignment resolution limits and sidewall roughness caused by the dry etching process.

Overall, a discussion of device characterization was presented, which included characterization procedures, results and sources of noise. The procedures used equipment with a realizable resolution lower limit of ~40nm and wavelength scanning range from ~1350nm to ~1584nm. In addition, the alignment process had some limitations in magnification for visual alignment and the lack of rotational alignment functionality in the stages used for testing. The surface testing process has significant
potential and could potentially be very useful in providing significant information about the surface leakage topology and could allow the introduction of surface injection or heating capabilities. The testing results show the racetrack resonators operating and with loss levels that allowed signals to be observed with quality factors near 2000. Operation of single racetrack resonator was demonstrated along with operation of parallel coupled dual racetrack resonator. Significant loss was exhibited in measurement from multiple sources. Trends in varying structure and fabrication of devices showed reduction in noise effects. Issues related to noise were discussed and appear to mostly be attributed fabrication issues. Some of the noise sources could be identified such as the periodic noise effect from the stitching. Future possible tests mainly revolve around fabricating and characterizing modulating or shifting the value of the effective refractive index by thermal changes, applying fluid index matching at the surface, acoustic modulation, carrier injection by a high power laser source and carrier injection by a laterally integrated PIN diode.

To summarize this chapter, a parallel-coupled dual racetrack micro-resonator modulator for arbitrary quadrature signal generation was proposed and analyzed. The parallel dual racetrack resonator structure was designed, fabricated and tested in silicon. Lastly, some barriers to fabrication and related noise issues were analyzed.
6. CONCLUSIONS

In this work, silicon based nano-photonic structures, such as photonic crystal waveguides and microresonators are studied for applications such as coherent optical modulation and laser beam steering. The critical characteristics examined in regards to these applications include optical loss, dispersion, and bandwidth.

The slow-light effect and the superprism effect are discussed in one synergistic perspective based on dispersions. A rigorous analysis is provided of the phase shift sensitivities, angular sensitivities, optical loss, and bandwidth for these effects in fairly general situations. A simple proof of the scaling of the roughness-induced scattering loss in the slow light regime is given based on the density of states for the initial and final states in the scattering process. A more detailed analysis can be conducted based on the coupled mode theory, the results of which were briefly discussed.

The scaling of the beam angle sensitivities to wavelength and refraction index in the superprism effect is analyzed for two scenarios: at a photonic band edge where the group velocity vanishes and at a degenerate point where the curvature of the dispersion surface becomes singular. The beam angle sensitivities to wavelength and refraction index are shown to have the same scaling with the wavelength offset from the bandedge or the degenerate point. However, the second case is not accompanied by high optical loss due to the slow light effect. An important fundamental relation, the bandwidth-sensitivity product, \([/(d\theta/d\lambda)_0 \times BW < 180^\circ]\), is obtained for the superprism effect. This relation is the counterpart of the bandwidth-delay product in the slow light effect.
In addition, the performance of two laser beam steering approaches based on the slow light effect and the superprism effect are directly compared based on this synergistic perspective of photonic crystal dispersions. Such a direct comparison is not possible without such a synergistic theoretical perspective.

A parallel-coupled dual racetrack micro-resonator modulator for arbitrary quadrature signal generation was proposed and analyzed. The critical coupling condition is obtained for such a structure. The coverage of the complex plane of the output light field is systematically studied for over-coupling, critical-coupling, and under-coupling scenarios, and is compared to the corresponding scenarios of two decoupled racetrack-resonators in series. It is found that only the over-coupling scenario of a parallel-coupled dual racetrack resonator structure results in adequate coverage for arbitrary quadrature signal generation. The interaction between the parallel-coupled racetrack resonators is key to the coverage of the complex $E$ plane. In an over-coupled dual racetrack structure, a delicate balance is achieved between the direct sum and the interaction of the two racetrack resonances, which results in a large dynamic range of the output amplitude and phase. Particularly, the intensity can reach zero in a push-pull configuration although the intensity of the un-modulated over-coupled racetrack resonators never vanishes. The effects of asymmetries in the coupling constants and quality factors were systematically studied. Small refractive index changes, which can be readily obtained with a reasonable thermal or electrical bias, can be used to compensate the asymmetry and ensure sufficient coverage of the complex $E$ plane.

Various novel micro-resonators have been fabricated on silicon-on-insulator wafers using e-beam lithography, dry etching, followed by deposition of silicon oxide. In
addition, some other structures including cascaded single and parallel dual racetrack micro-resonators, one-dimensional periodic structures integrated into racetrack resonators, photonic crystal parallel dual racetrack resonators and add-drop filters based on photonic crystal racetrack resonator have been fabricated. The fabrication barriers have been analyzed. The operations of some of these structures have been characterized. The resonance linewidths, quality factors, free spectral ranges, and noise have been analyzed.

Overall, this work resulted in a group of achievements, where the more significant topics are noted below. A simple proof is shown of the scaling of roughness-induced scattering loss in the slow light regime. An important fundamental relation expressed as the bandwidth-sensitivity product for the superprism effect. Parallel-coupled dual racetrack micro-resonator modulator for arbitrary quadrature signal generation was proposed and analyzed. A novel parallel dual racetrack resonator structure was designed, fabricated and tested.

Future work may possibly resolve issues related to fabricating and characterizing dual-racetrack resonators, such as by using electron beam lithography systems with a reduced stitching effect, modulating or shifting the value of the effective refractive index using thermal changes, acoustic modulation, and carrier injection. Additionally, some future applications of the proposed resonator structure include filters, sensors, thermally tuned parallel coupled dual racetrack ring resonator, thermally modulated parallel coupled dual racetrack ring resonator, PIN diode modulation of a parallel coupled dual racetrack ring resonator and beam steering.
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Publications:

1. Integlia, Ryan; Song, Weiwei; Yin, Lianghong; Ding, Duo; Pan, David Z.; Gill, Douglas M.; Jiang, Wei. “Parallel-Coupled Dual Racetrack Silicon Micro-Resonators for Arbitrary Quadrature Signal Generation.” (Submitted). 2011


Conferences
