STRUCTURAL CREDIT RISK MODELS IN BANKING
WITH APPLICATIONS TO THE FINANCIAL CRISIS

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This dissertation uses structural credit risk models to analyze banking institutions during the recent financial crisis.

The first essay proposes a dynamic approach to estimating bank capital requirements with a structural credit risk model. The model highlights the fact that static measures of capital adequacy, such as a fixed percentage of assets, do not capture the true condition of a financial institution. Rather, a dynamic approach that is forward-looking and tied to market conditions is more appropriate in setting capital requirements. Furthermore, the structural credit risk model demonstrates how market information and the liability structure can be used to compute default probabilities, which can then be used as a point of reference for setting capital requirements. The results indicate that capital requirements may be time-varying and conditional upon the health of the particular bank and the financial system as a whole.
The second essay develops a structural credit risk model for computing a term-structure of default probabilities for a financial institution. The model uses current market data and the complete debt structure of the financial institution. It incorporates an endogenous default boundary in a generalized binomial lattice framework. This framework is flexible and easily permits alternate capital structure policy assumptions; for example, it can allow for the possibilities of funding maturing debt with new debt, new equity, or a mix of debt and equity. The model is implemented to analyze Lehman Brothers in the midst of the 2008 financial crisis and develop estimates of default probability under these alternate capital structure policy assumptions. The assumptions result in different levels of default probability but all predict higher default probabilities in March of 2008, well before Lehman’s bankruptcy in September of 2008. In addition to being used in a descriptive capacity, the model can be used for prescriptive purposes. The lower-bound default probabilities assume de-leveraging by the financial institution and can be used to indicate when debt should be retired with new equity rather than being rolled-over.
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Dedication

To my wife Tina, the love of my life.
# Table of Contents

Abstract ................................................................. ii
Acknowledgements ................................................... iv
Dedication ................................................................. vi

List of Tables .......................................................... ix
List of Figures ........................................................... x

1. Introduction ........................................................... 1

2. Dynamic Bank Capital ................................................. 12
   2.1. Introduction ..................................................... 12
   2.2. Related Literature ............................................. 18
       2.2.1. Capital Adequacy ........................................ 18
       2.2.2. Market Discipline ....................................... 23
       2.2.3. Another Look at Bank Capital:
               Liability Structure and Market Information .......... 25
   2.3. Model ............................................................ 29
       2.3.1. Review of Structural Credit Risk Models .......... 29
       2.3.2. A Structural Model of the Banking Firm .......... 32
               Assumptions and Model Setup ......................... 33
               Valuation .................................................. 35
               Endogenous Default and Default Probabilities ...... 40
               Dynamic Implementation ............................... 47
               Capital Requirements ................................... 49
   2.4. Data and Methodology ......................................... 53
       2.4.1. Description of Sample ................................. 53
List of Tables

2.1. Bank Holding Companies in the Sample, Ranked by Average Assets . . . . 70
2.2. Data Items Used in the Empirical Study of Bank Capital . . . . . . . . . . 71
3.1. Timeline of Events at Lehman Brothers . . . . . . . . . . . . . . . . . . . . . . . . . . 108
3.2. Lehman Brothers Debt as of January 2008 (in Millions of Dollars) . . . . . 110
List of Figures

2.1. Simplified balance sheet for a typical banking firm . . . . . . . . . . . . . . 72
2.2. State-contingent payoffs and valuation in the compound option model . . . 73
2.3. Short-Term Default Probabilities . . . . . . . . . . . . . . . . . . . . . . . . 74
2.4. Correlation Between Default Probability and Market Value Capital Ratio . 75
2.5. Capital Ratios: Risk-Based Capital vs. Market Value Capital . . . . . . . . 76
2.6. Average Leverage for the Sample Bank Holding Companies . . . . . . . . 77
2.7. Capital Infusions (as % of Book Value of Assets) – Full Sample . . . . . . 78
2.8. Capital Infusions (as % of Book Value of Assets) – Restricted Sample . . . 79
3.1. Lattice Implementation of the Model . . . . . . . . . . . . . . . . . . . . . . . 111
3.2. Numerical Example – 3-Period Asset Binomial Tree . . . . . . . . . . . . . 112
3.3. Numerical Example – Equity Values in a 3-Period Binomial Tree . . . . . 113
3.4. Lehman Brothers Stock Price and Timeline . . . . . . . . . . . . . . . . . . 114
3.5. Lehman Brothers Debt Maturity . . . . . . . . . . . . . . . . . . . . . . . . . 115
3.6. Book Value and Market Value of Lehman Brothers Equity . . . . . . . . . . 116
3.7. Lehman Brothers Equity Volatility . . . . . . . . . . . . . . . . . . . . . . . 117
3.8. Model-Implied Market Value of Lehman Brothers Debt . . . . . . . . . . . 118
3.9. Lehman Brothers 1-Year and 2-Year Default Probabilities . . . . . . . . . . 119
3.10. Lehman Brothers Cumulative Default Probabilities (2008 - 2032) . . . . . 120
Chapter 1
Introduction

The Financial Crisis of 2007-2009 gave rise to an unprecedented number of financial institutions that either failed or had to be bailed-out with taxpayer money. How could such a crisis occur? And what preventative actions can be taken to ensure that a crisis of this magnitude and severity never happens again? These are among the many questions that financial economists and banking scholars are struggling to answer.

A good starting point is to re-examine the risk management practices that financial institutions have in place. Typically, financial institutions facilitate transactions, provide liquidity, and engage in asset transformation and, in the course of doing so, take substantial risk onto their books. In addition, financial institutions are often highly leveraged, funding a significant portion of their operations with debt. In times of crisis, this debt-intensive funding model makes financial institutions particularly vulnerable to market shocks. This is one reason why banks, and other financial institutions, are mandated by regulators to maintain at least a certain level of equity capital on their balance sheets. The established regulations are intended to provide a “safety net” of sorts to minimize the social costs of the negative externalities associated with bank failures. Furthermore, studies of the Great Depression have shown that banks must find a balance between their asset risk and financial risk (i.e. leverage), especially in times of crisis (see, for example, Calomiris and Wilson (2004)). However, existing regulations focus almost entirely on the asset risk, practically ignoring the financial risk arising from the bank’s leverage and liability structure.

\(^1\)For an excellent discussion on financial institutions’ leverage, risk, and the procyclical nature of this relationship please see Adrian and Shin (2010).
As noted by Alexander (2005), risk management is not just about satisfying regulatory mandates, but should also be viewed as a strategic decision-making tool to add value for the bank’s shareholders. Indeed, the link between performance and risk management in financial institutions has been documented in the literature both empirically and in terms of theoretical models (see, for instance, Froot and Stein (1998), Zaik, Walter, Retting, and James (1996), and Stoughton and Zeichner (2007)).

The predominant metric used for risk management in financial institutions is Value-at-Risk or VaR. VaR can be defined as the projected maximum loss that will be sustained over a specified horizon with a given level of confidence. For example, a one-day VaR of $1 million at the 95% confidence level means that there is a 5% chance that the bank will lose more than $1 million today. VaR is attractive for several reasons. One reason is that it assumes asset returns are normally distributed (and asset prices are therefore lognormally distributed). The Gaussian assumption means that traditional methods of probability theory and mathematical statistics can be used for computation and analysis. Furthermore, it means that VaR is additive, which fulfills the desire of many developers and users of risk management systems, namely that risk measures be easily aggregated. Another reason for financial institutions’ widespread use and acceptance of VaR has to do with the fact that it is a single number that summarizes the level of risk in terms of the “worst-case scenario” loss. That makes VaR easy to communicate and understand, even for somebody with a nontechnical background. However, empirically, VaR does not appear to perform well in forecasting future losses as documented by Berkowitz and O’Brien (2002).

In fact, the irony is that many of the same features that make VaR attractive, have proven to be its biggest limitations, especially in times of crisis. For one, VaR is not applicable when the underlying distribution has “fat tails” and/or is skewed (see Rosenberg and Schuermann (2006)). Furthermore, for years researchers have argued that VaR lacks many of the features that are desirable in a valid risk measure (see, for example, Szegö (2002)). Perhaps one of the biggest drawbacks is that VaR

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2For more about VaR please see the book by Jorion (2006).

3For insight into the role of risk management in the recent financial crisis please see Stulz (2008) and Jorion (2009).
only focuses on the asset side of the bank’s balance sheet without taking leverage into account. What is needed is an integrated approach to measuring risk for financial institutions that takes both sides of the balance sheet into consideration.

Structural credit risk models show promising potential for widening the scope and flexibility of risk management systems for financial institutions. However, there are some hurdles that must be overcome; most notably, the fact that structural credit risk models have seen very few successful applications to banking is one hurdle. One of the overarching themes of this dissertation is that for a structural credit risk model to be successfully applied to analyze financial institutions, it must satisfy two necessary but not sufficient conditions: first, there must be endogenous default; and second, it must take the entire liability structure into account.

Structural credit risk models, which regard corporate securities as contingent claims on a firm’s underlying assets, originated with the seminal works of Black and Scholes (1973) and Merton (1974). Black and Scholes (1973) noted that, when a firm has debt in its capital structure, equity is very much like a call option on the firm’s assets. Therefore, traditional methods of contingent claims analysis and option pricing could be used to value and hedge corporate securities. Merton (1974) formalized the mathematics behind it. Adopting this framework allows the modeler to value both debt and equity in equilibrium as well as compute default probabilities. Equity is valued using the famed Black-Scholes formula with the following inputs: the market value of assets, asset volatility, the risk-free rate, time-to-maturity, and face value of the outstanding debt as the strike price of the option. Then debt can be valued simply by applying put-call-parity. Thus, debt is a covered call or equivalently a risk-free loan with a short put option attached to it. The put premium can be seen

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4 However, it should be noted that Adrian and Shin (2011) suggest that VaR is consistent with banks’ optimal contracting decision between debt and equity; i.e. the amount of leverage employed by the bank.

5 Extensions to VaR have been proposed to address various issues, not necessarily dealing with the liability structure per se. For instance, Adrian and Brunnermeier (2009) develop the measure of CoVaR which captures a financial institution’s contribution to system-wide risk (i.e. a measure of systemic risk).

6 The alternative approach to credit risk modeling views default as a jump process. These “reduced-form” credit risk models typically do not incorporate any specific information about the structure or economics of the firm in question. Rather, in practice, they are calibrated to market data (i.e. credit spreads) to estimate the model parameters. Two examples of very well-known reduced-form models are Jarrow and Turnbull (1995) and Duffie and Singleton (1999).
as the compensation for the added default risk. Therefore, the Black-Scholes-Merton structural credit risk model can be used to calculate the risky credit spread. The probability that the call option will be exercised, which is known in closed-form, is essentially the survival probability or the probability that the firm will not default. Since either the firm defaults or it survives (they are mutually exclusive events), the default probability is one minus the survival probability.

The original Black-Scholes-Merton model relies on several unrealistic assumptions including the requirement that there is only one class of debt outstanding in the form of a pure discount bond, that interest rates are constant, and that default can only occur at maturity of the debt. As a result many extensions have been developed to address these assumptions and make structural credit risk models more realistic representations of corporate debt and the default process. Loosely speaking, one might say that the extensions have gone in two directions: the compound option approach of Geske (1977) and the barrier models which began with Black and Cox (1976).

The Black and Cox (1976) model accounts for safety covenants, which if not followed would trigger default. This is modeled as a barrier option: when the asset value falls below a specified barrier – which may be exogenous or endogenously determined within the model – the firm defaults. Thus, in the model, default can occur any time before and up to maturity of the debt. The original Black and Cox (1976) model utilizes an exponential barrier and recovery is a fixed portion of the discounted face value of the debt. The Black and Cox (1976) barrier structural model was extended by Longstaff and Schwartz (1995) to include a second factor for stochastic interest rates. A flat, exogenous default barrier is used and recovery is a fixed fraction of the face value of the debt at maturity; however the model has the additional contribution in that it allows for violations of the Absolute Priority Rule (APR). Collin-Dufresne and Goldstein (2001) directly extend the Longstaff and Schwartz (1995) model with a mean-reverting default boundary (still exogenous) to account for the empirically-noted phenomenon that firms tend to actively manage leverage ratios.

Leland (1994) derived a structural credit risk model with an endogenous default
The model assumes one class of debt with infinite maturity and a fixed coupon. The result is a flat, time-invariant default barrier, which is endogenously determined as the lowest possible asset value for which equity is still positive. As noted by Leland (1994) the feature of limited liability of equity ensures that the barrier cannot be arbitrarily small. Furthermore, due to convexity of equity value, the barrier is less than the present value of the after-tax coupon payments to bondholders. Since the model includes taxes and bankruptcy costs, it can therefore be used to solve for the optimal capital structure in addition to the value of risky corporate debt, where the solutions are known in closed-form. Leland and Toft (1996) extend the Leland (1994) model to take debt maturity into account. All of the features from the original Leland (1994) model are preserved – including endogenous default, optimal capital structure, risky debt valuation, and complete closed-form solutions – with additional insights about the term structure of credit spreads and default probabilities. The Leland and Toft (1996) model assumes that every instant a fraction of the outstanding debt is retired at par and refinanced by issuing new debt with the same features (maturity, coupon, etc.) at the current market value. Essentially the firm is being modeled to systematically roll-over debt, which gives rise to a flat default barrier and a stationary debt structure. The default conditions are as follows. At any given time, the firm is paying out the instantaneous after-tax coupon plus the face value of the portion of the debt that is being retired; simultaneously, funds are coming into the firm from both the issuance of new debt and the cash-flow generated by the firm’s assets. If the instantaneous debt obligations (principal amount plus coupon payments) cannot be covered by the funds coming in, then shareholders must contribute the cash needed to fill the shortfall; when this cannot be done the shareholders choose to default. It is shown the the firm will default when the change in equity value is exactly equal to the additional amount needed to service the debt. Therefore the endogenous default condition implies that firm survival relies on the ability to raise capital; this is a feature that will prove to be very important in modeling bank default and in understanding the behavior of banking institutions during the financial crisis.

The other class of structural credit risk models is the compound option approach,
which also has endogenous default. This approach was originally proposed by Geske (1977), who noted that when there are multiple cash flows, in the form of coupon payments and/or several classes of debt outstanding, then equity is a compound call option on the firm’s assets. A compound call option is a call option on a call option. Each cash flow is therefore a strike price for the compound option. Whenever a payment is due to bondholders, the firm’s shareholders must decide whether or not to exercise the compound call option. Making the payment represents exercising the compound call option, thereby agreeing to continue holding a sequence of contingent claims on the firm’s assets. When there are two cash flows the model solution is known in closed-form using the compound option formulae from Geske (1979). This set-up is useful for valuing risky debt when the liability structure is bifurcated into short-term and long-term debt or junior and senior debt. The correct solution for junior and senior debt is given by Geske and Johnson (1984). When there are more than two cash flows, the compound option model is known in “quasi-closed-form” in that it can be written out but not solved analytically. Therefore, numerical methods must be used in these instances to both value the corporate securities as well as compute default probabilities. Default is endogenously determined at every cash flow and, as a result, the model is able to generate a complete term structure of default probabilities. This concept of a complete term structure of default probabilities will be returned to shortly.

In the compound option framework the endogenous default condition is as follows. Every time a payment is due a decision has to be made as to whether or not to default. If the asset value is sufficiently high, then new equity can be raised and the proceeds used to make the imminent payment to creditors. When asset value drops, so does the value of the equity which is a contingent claim on the assets. If the asset value gets too low, then not enough equity can be raised to satisfy the next obligation and the shareholders choose to not exercise the compound call option (i.e. default) and creditors take control of the assets in accordance with absolute priority. Thus, default is endogenously determined as the asset value that sets the value of equity just equal to the next cash flow. Note that this is similar to the endogenous default condition
in Leland and Toft (1996) described above.

The compound option structural model assumes that new equity is issued used to pay down maturing debts. This implies that the firm in question is expected to systematically reduce its leverage over time, which may be an unrealistic assumption when modeling a financial institution.

In the compound option structural credit risk model there is a default probability associated with each and every cash flow over time. As a result the compound option model explicitly gives a complete term-structure of default probabilities. The term structure of default probabilities is not just a curve of the cumulative default probabilities, which can also be generated with a barrier structural credit risk model; rather it includes both conditional and unconditional forward default probabilities (i.e. marginal and joint probabilities).

Empirically, structural credit risk models do not perform well in terms of pricing risky debt and generating credit spreads (see studies by Jones, Mason, and Rosenfeld (1984) and Eom, Helwege, and Huang (2004)). However, structural credit risk models have been reasonably successful in using market information to compute default probabilities and in predicting corporate default. The original Black-Scholes-Merton structural model computes the default probability as the likelihood that the European call option will expire out-of-the-money. The barrier structural models compute default as a first passage time, whereas the compound option approach computes the probability that a promised payment will be made.

The KMV model is a practical implementation of the original Black-Scholes-Merton structural model that is widely used in industry as well as a benchmark in academic studies. As described by Crosbie and Bohn (2003), the model uses a linear combination of the short-term and long-term debt to define the default point (i.e. strike price). Specifically, the model takes the total face value of the short-term debt plus one-half the value of the long-term debt as the input for the default point. Since the true asset value and volatility is unknown, the model is used to iteratively solve two nonlinear equations given equity value and volatility. Crosbie
and Bohn (2003) illustrate how the KMV model uses market information to generate two metrics, Expected Default Frequency (EDF) and Distance to Default (DD), which provide forward-looking estimates about default risk with impressive predictive power.\(^7\) Leland (2004) examines the ability of structural credit risk models to fit historical default frequencies. Using aggregate-level data, default probabilities are computed with two barrier structural models (endogenous versus exogenous) as well as the KMV Distance-to-Default method. Not surprisingly, all three models result in different default probability curves. In general, the endogenous barrier model fits the data fairly well for investment grade debt (Baa-rated), but underestimates default rates on speculative grade debt (single B rated). The exogenous barrier model does not fit actual default frequencies well, except when the parameters are chosen so that it coincides with the endogenous barrier. The KMV model generates default probabilities that level off over longer-term horizons, which is not observed in the empirical data.\(^8\) Furthermore, Leland (2004) shows that, contrary to popular belief, the KMV Distance-to-Default is not a sufficient statistic for probability of default. Delianedis and Geske (2003) study risk-neutral default probabilities (RNDPs) implied by market prices and computed using the original Black-Scholes-Merton model and the Geske compound option model. The analysis has several important implications for the use of market information and structural credit risk models. First, it can be shown mathematically that RNDPs serve as an upper bound to risk-adjusted default probabilities (under the physical measure) and therefore are more conservative estimates of the true likelihood of default. Second, the compound option model is able to provide information that other structural credit risk models cannot; namely, a complete term structure of default probabilities (discussed above). Third, both structural credit risk models show evidence of being able to anticipate rating migrations, although the slope of the default probability term structure from the compound option model conveys

\(^7\)Bharath and Shumway (2008) claim that hazard models perform slightly better in terms of default prediction.

\(^8\)Typically the KMV model is used to estimate one-year default probabilities, since it follows a European option setup. However, it is not uncommon for researchers to synthetically construct an ad-hoc default probability curve so as to allow for direct comparison between the KMV model and the barrier and compound option structural credit risk models which do give default probabilities over time.
more information about credit events.

Structural credit risk models have seen very limited applicability to banks. In fact, almost all of the empirical works on structural credit risk models exclude financial institutions. Leland (2009) addresses the difficulties of using structural credit risk models on financial institutions. The difficulties come from several characteristics unique to financial institutions including the high degree of leverage and a liability structure that has a disproportionate amount of short-term debt, including deposits and repurchase agreements. Furthermore, in normal times financial institutions typically have low asset volatility, an important input in any structural credit risk model, but in times of crisis system-wide uncertainty tends to increase volatility across financial institutions. As a result, Leland (2009) also emphasizes the importance of volatility assumptions when implementing structural credit risk models.

Endogenous default is an essential feature for a structural credit risk model to effectively be applied to financial institutions. One of the reasons is because a bank’s survival depends critically on its access to capital. While this is true for any firm, it is much more acute for the banking firm. Neither the compound option approach (Geske (1977), Geske (1979), Geske and Johnson (1984)) nor the endogenous barrier model (Leland (1994), Leland and Toft (1996)) are ideal for the analysis of financial institutions. For instance the compound option approach assumes systematic deleveraging over time which may not be appropriate for a financial institution. But the compound option setup does allows for complex liability structures. The endogenous barrier model assumes that debt roll-over, which is what most financial institutions look to do, especially with short-term debt. However, the Leland and Toft (1996) model only permits one homogenous class of debt. Although both models have aspects that are not entirely appropriate for financial institutions, a large part of this

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9Notable exceptions might include the classic risk-based deposit insurance models which utilize a contingent claims setup (see, e.g., Merton (1977), Marcus and Shaked (1984), Ronn and Verna (1986), Pennacchi (1987a), Pennacchi (1987b), Allen and Saunders (1993), and Cooperstein, Pennacchi, and Redburn (1995) and Mason (2005) who uses a real options approach for estimating the liquidation value of banks.

10These include the aforementioned papers by Delianedis and Geske (2003), Eom, Helwege, and Huang (2004), Leland (2004), and Bharath and Shumway (2008).
dissertation is dedicated to finding the “right” endogenous structural credit risk model that incorporates the best features of both of these approaches. There are several instances in which a structural credit risk model can be very insightful in analyzing the operations and risk of financial institutions even during times of crisis. The two key elements are a nontrivial liability structure and endogenous default. These concepts will be developed extensively in the two chapters that follow. Structural credit risk models are good in this capacity because they use market information thereby providing a forward-looking representation of participants’ expectations. Furthermore, structural credit risk models integrates both sides of the balance sheet in a way that is consistent with economic theory.

This dissertation is comprised of two essays that examine how structural credit risk models can be used to analyze financial institutions from various different perspectives. The first essay, Chapter 2 proposes a dynamic approach to estimating bank capital requirements with a structural credit risk model. Using market data along with information about the bank’s liability structure, the model can compute default probabilities for the bank in question. Then, if it appears the bank is in distress, or likely to be in the near future, the model can be used to estimate the amount of capital that would be needed to bring default probability down to a more acceptable level. After the model is developed, a small empirical study is conducted using the 15 largest bank holding companies (BHCs) in the United States over the period from 2005 up to the end of 2008. Default probabilities were computed for the entire sample, and before mid-2007 the default probabilities for the banks were close to zero. Then as the Financial Crisis unfolded the default probabilities increased dramatically; by the third quarter of 2008 all 15 BHCs were above 10% with some reaching as high as 50%. The model is then used to estimate the amount of capital that would have been needed to bring default probabilities below a 5% level. The results are compared to both the Troubled Asset Relief Program (TARP) and the Supervisory Capital Assessment Program (SCAP, better known as the “stress tests”) with all but one of the BHCs requiring more capital than both TARP and SCAP combined. Clearly, not only were the existing methods of determining bank capital
requirements slow to respond to the crisis, but the recapitalization attempts by the banks proved minimally effective.

The second essay, Chapter 3, expands upon the idea of endogenous default and examines the importance of modeling how the financial institution refines maturing debt in estimating default probabilities. A generalized binomial lattice based structural credit risk model is developed, which permits alternate capital structure policy assumptions; for instance rolling-over maturing debt versus paying down maturing debt with new equity (i.e. *de-leveraging*). The model can accommodate banks with even the most complex liability structures, and it works particularly well in times of crisis. A clinical investigation of Lehman Brothers in 2008, the year that the prestigious Wall Street firm declared bankruptcy, illustrates how the model can be used. The two refinancing assumptions result in different levels of default probability, however both predict extremely high default probability months before Lehman’s failure in September 2008. Furthermore, the model can be used in a prescriptive capacity by indicating when a financial institution should de-leverage rather than the typical practice of rolling over debt.

Both essays go a long way towards realizing the full potential of structural credit risk models in banking research and risk management. Finally, in Chapter 4, several opportunities for future research are discussed.
Chapter 2
Dynamic Bank Capital

2.1 Introduction

In the wake of the devastating Financial Crisis of 2007 - 2009, capital requirements, a key regulatory and risk management tool for financial institutions, have once again come into question. A bank’s capital is supposed to serve as a buffer against losses and thereby reduces the probability of financial distress and default. How much capital a bank should have is a much-debated topic and, indeed, there is a vast literature examining the costs and benefits as well as the effectiveness of risk-based capital requirements (see Section 2.2.1). Regardless, it is understood that the larger the amount of capital held on a bank’s balance sheet, the lower the probability of default. When asset values fall, the bank’s capital absorbs the losses so that depositors and other creditors are not impacted significantly. Thus, the solvency of any given bank relies in part on the capital position of the institution. In addition, the soundness of the financial system as a whole depends on the ability of individual banks to withstand market-wide and macroeconomic shocks. Historically, bank capital has been regulated both domestically and internationally. However, as the recent crisis illustrated, existing regulations and bank capital standards proved inept for the pace and complexity of the modern financial system.

As the global economy entered into one of the worst financial crises in history, data indicates that the biggest financial institutions were “well-capitalized” by regulatory standards. Furthermore, the tools used internally by financial institutions also showed no evidence of indicating a need for more capital. This paper proposes a dynamic approach to estimating bank capital and in doing so addresses many of the shortcomings that were illustrated in the recent financial crisis.
The accords collectively referred to as Basel II provide the current framework for auditing and monitoring the operations of financial institutions.\(^1\) The First Pillar of these accords specifies minimum capital requirements as a function of the bank’s credit, market, and operational risks. Specifically, the requirements mandate that total capital must be no lower than 8% of risk-weighted assets. Hence, the degree of capital adequacy is summarized by one number, the risk-based capital ratio, which is total capital divided by risk-weighted assets.\(^2\) Total capital refers to the sum of Tier 1 and Tier 2 capital. Tier 1 capital, also referred to as “core capital”, is comprised mostly of shareholders’ equity (common stock, perpetual preferred stock, etc.). Tier 2 capital is comprised of other residual claims (redeemable preferred stock, qualifying subordinated debt, etc.) and is limited to 100% of Tier 1 capital.\(^3\) The riskiness of the bank’s assets is accounted for in the denominator where weights are assigned to different risk classes for each type of asset on the balance sheet. The idea is that riskier assets require larger capital allocations to cover potential losses. While the underlying framework may seem logical, the existing standards for evaluating the capital adequacy of financial institutions relies on measures that are static in nature and are the result of historical analysis of the institution’s balance sheet and operations. The recent financial crisis provides striking new evidence that static, asset-based regulatory requirements are ineffective.

This paper proposes a dynamic, forward-looking approach for analyzing bank capital. A structural credit risk model is used to first compute default probabilities and then analyze the effects of recapitalizing the financial institution. It can be used as an analytical tool to find the minimum amount of capital that is needed to keep the probability of default within an acceptable range. The model is essentially a dynamic implementation of the Geske compound option model (see Geske (1977), Geske (1979), Geske and Johnson (1984)). Structural credit risk models, based on

\(^1\)For more information regarding the specific details of Basel II the reader is directed to the consultative document, Basel Committee on Banking Supervision (2003), or the comprehensive report Basel Committee on Banking Supervision (2006)

\(^2\)An alternative, stricter, and arguably more appropriate measure of capital adequacy would be to look at the core capital ratio, which is Tier 1 capital divided by risk-weighted assets.

\(^3\)The definition of Tier 1 and Tier 2 capital remains the same as the original Basel Capital Accords. See Basel Committee on Banking Supervision (2006), p. 14.
the seminal work of Black and Scholes (1973) and Merton (1974), model corporate securities as contingent claims on the firm’s assets and can therefore be used to compute default probabilities in terms of the distributional properties of the underlying asset value (as a first passage density or exercise probability) and the nature of the contingent claims’ payoffs (boundary conditions). Unfortunately, structural credit risk models have seen little success in analyzing financial institutions, attributable in part to the assumption of an exogenous default boundary and/or oversimplifying the liability structure. In contrast, the approach used in this paper explicitly incorporates information about the seniority and maturity structure of bank liabilities and, furthermore, has an endogenous default condition that provides tremendous insight into the financial institution’s ability to recapitalize.

The complex liability structure associated with financial institutions lends itself nicely to the compound option structural credit risk model. A compound option is an option on an option, i.e. the underlying asset is itself an option. This comes directly from the original insight of Black and Scholes (1973) and Merton (1974) who demonstrated that shareholders’ decision of whether or not to make a principal payment on risky debt is analogous to the exercise decision on a long call option. When there are multiple cash flows (coupon payments, different classes of debt, etc.) this is best modeled by a compound option as shown by Geske (1977). For example, when a firm has coupon bonds outstanding, each decision to make a payment is contingent on having made the last. The option to make today’s coupon payment is conditional on having exercised last period’s option. Defaulting on a coupon payment is therefore like allowing the compound option to expire.

Alternatively, Geske (1977) and Geske and Johnson (1984) show how the compound option model can be applied to institutions with subordinated debt in the capital structure. When there are just two classes of debt (short-term and long-term, junior and senior), the solution is known in closed-form and can be calculated analytically.\footnote{This is as opposed to higher-dimensional compound options where the solution can be written out in “quasi-closed-form” but must be solved numerically.} Fortunately, this specification captures the general capital structure of a
typical bank quite nicely (this point is expanded upon below), and therefore allows for the explicit calculation of short-term and long-term default probabilities as well as comparative statics.

In addition to the liability structure, the model uses the market value of the assets and the asset volatility as primary inputs. However, since the true asset value and volatility are unobservable, equity market information can be used to solve for these quantities. It should be noted that using equity market information to help gauge the value and quality of bank assets, which tend to be rather “opaque”, is common practice in the literature (see Flannery, Kwan, and Nimalendran (2004)). There is a wide body of literature that studies the ability of market participants to assess bank performance and discipline poorly managed banks. If investors are not pleased with the performance of a particular bank they can sell their information-sensitive claims on the bank (i.e. equity and subordinated debt) thereby driving prices down and the resulting cost of capital up (see Section 2.2.2). In fact, “Market Discipline” is the Third Pillar of the Basel II Accords. However, there has been a lack of congruency in efforts to integrate market discipline and capital adequacy in practice. The model proposed in this paper represents one potential way of merging the monitoring ability of market participants and the regulator as discussed by Flannery (1998). The default probabilities generated by the model are a function of both the liability structure and equity market information. Given the market value of a financial institution’s equity and the equity volatility, two equations must be solved in two unknowns to simultaneously obtain measures of the implied asset value and asset volatility. The model outputs include short-term and long-term default probabilities, as well as the total survival probability. The real insight, however, comes from examining the relationship between default probabilities and the minimum level of capital needed to reduce default probabilities for distressed financial institutions. The model is, therefore, a liability-driven approach for analyzing bank capital using market information; this is in stark contrast to the currently employed, static, asset-driven approaches. The methodology used in this paper goes a long way towards realizing the potential of
structural credit risk models to compute a practical measure of default risk and analyze capital requirements for financial institutions in a dynamic setting. Given the hundreds of billions of dollars that were injected into struggling financial institutions, the recent financial crisis gives rise to an abundance of opportunities for empirically testing the model. The empirical portion of this paper will examine the top 15 bank holding companies (BHC’s), in terms of average assets, on a quarterly basis over a four year period (2005:Q1 to 2008:Q4). The model is run using data from Federal Reserve reports and the equity markets. First, the model is used to compute model-implied market value capital ratios which are then compared to core capital ratios as defined by the existing risk-based capital regulatory specifications. The core capital ratios are calculated using balance sheet information and static risk weightings; therefore, they should be fairly constant over the sample period. If anything, one might even expect that they rise over the sample period as bank executives attempted to reassure the public that their institution was solvent. On the other hand, the model clearly shows that BHC default probabilities increase over the course of 2008. The source of this pattern comes from increasing volatility, decreasing asset values, and leverage ratios that remained high.

Since the model uses market information, as long as the liability structure was not significantly altered, the prediction is that the model-implied market value capital ratios decrease over the sample period, and do so at a faster rate than the core capital ratios increased. The results do, in fact, confirm this prediction. Furthermore, market value capital ratios tend to move in opposite directions of core capital ratios - a pattern that has profound regulatory and theoretical implications.

Next, the model is used to estimate the amount of capital infusions that would have been needed to lower default probabilities to a more acceptable level for the large banks in the sample. By the third quarter of 2008, default probabilities were significantly elevated for all 15 BHCs in the sample. It is no coincidence that this was the point where the financial crisis took a turn for the worse. Between September and October of 2008 Fannie Mae and Freddie Mac were placed into federal conservatorship, Lehman Brothers declared bankruptcy, the Federal Reserve Bank of New
York propped up AIG, and the government committed $700 billion of capital to “bail out” the U.S. financial system through the Troubled Asset Relief Program (TARP). In fact, this provides for a very interesting natural experiment: the capital requirements computed by the model are compared to the initial TARP allocations made from October 2008 through December 31, 2008. The results obtained by the model indicate that the capital infusions provided by the TARP program were substantially lower than needed in all but one of the BHCs. It is conceivable that the paucity of capital in the banking system can help explain why conditions continued to deteriorate before they got better from 2008 into 2009. Also, the estimates of the Federal Reserve’s Supervisory Capital Assessment Program (the SCAP, better known as the “stress tests”) are then incorporated into the analysis to see how the forecasted capital needs, as computed by the regulators, compare to the figures obtained from the model.

To summarize, the structural model proposed in this paper could potentially be used by bank managers and regulators alike to assess capital adequacy. The model incorporates information about the liability structure of the bank as well as risk and valuations from market data, as opposed to the static capital ratios that are driven purely by asset-based measures of risk. In addition to market risk and valuations, by including default probabilities into the analysis one can determine a flexible capital ratio that is customized to fit the particular financial institution’s needs and the current market environment. The results indicate the need for time-varying capital requirements, consistent with the recommendation of Kashyap, Rajan, and Stein (2008).

The remainder of the paper is structured as follows. The next section, Section 2, reviews the relevant literature, specifically looking at capital adequacy and market discipline in banking. After a brief review of previous structural credit risk models, Section 3 develops a structural model of the banking firm, derives the default probabilities, and proposes the dynamic implementation of the model as well as the application to capital requirements. Section 4 goes through the data and methodology. The empirical results are presented in Section 5 and Section 6 concludes.
2.2 Related Literature

2.2.1 Capital Adequacy

The role of bank capital and the importance of developing good capital adequacy standards have been studied extensively in the literature.\(^5\) Berger, Herring, and Szegö (1995) note that, as with any other firm, the presence of market frictions – taxes, distress costs, asymmetric information, etc. – leads to an optimal capital structure.\(^6\) It seems as though standard capital structure theory cannot justify the high amounts of leverage that are typically employed by financial institutions. Berger, Herring, and Szegö (1995) do point out that there is one market imperfection that is unique to banks; the “regulatory safety net” implies that financial institutions, either explicitly or implicitly, are partially protected from negative outcomes associated with higher risk-taking. This suggests that whereas traditional theories of capital structure indicate that a firm will settle on the appropriate amount of leverage so that the benefits and costs are balanced, the regulatory safety net tilts the scale toward banks having less capital and more debt. However, this could be problematic in the event that there is a financial crisis and widespread bank failures pervade the entire system. Thus, one reason that banks should hold some minimum amount of capital is that it protects the broader financial system and economy as a whole from the negative externalities associated with bank failures. This is a common theme in the literature and will be returned to shortly.

Diamond and Rajan (2000) show how capital can be introduced into a traditional model of financial intermediation, where the most appropriate method of financing is through fixed claims with a sequential service constraint (i.e. deposits, demandable debt, or other short-term, senior promissory notes that can be cashed out at any time). They show that, when there is uncertainty, the bank may choose to finance its operations in part with long-term, “softer claims”; that is a contract that gives

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\(^5\) For a comprehensive review of the bank capital regulation literature see Santos (2001).

\(^6\) As will be discussed in Section 2.3, the current paper does not attempt to find an “optimal” capital structure for financial institutions. Flannery (1994), on the other hand, develops a simple and elegant theoretical model for explaining financial institutions’ capital structures.
the holder a residual claim to the assets and the cash-flows they produce. The most natural interpretation of such a claim is as capital. In the Diamond and Rajan (2000) model, capital acts as a protective cushion against shocks to asset values, which in turn makes the bank safer. The downside, however, is that capital reduces liquidity, and one of the primary functions of a bank is its ability to create liquidity. Thus, the optimal degree of capital in a financial institution must be such that the costs, in terms of restricted ability to provide liquidity and extend credit, must be weighed against the benefits, specifically a higher survival probability and less of a chance of distress for the bank.

The negative consequences of setting too high of a capital requirement for banks have been examined by several researchers. Thakor (1996) argues that that risk-based capital requirements can potentially lead to credit rationing. The empirical evidence on this matter, however, is mixed (see, e.g. Berger and Udell (1994), Brinkmann and Horvitz (1995), and Flannery and Rangan (2008)). Hellmann, Murdock, and Stiglitz (2000) find that capital requirements, by themselves, are insufficient to mitigate bank bankruptcy risk, and that instead they can lead to the perverse result where banks gamble.

The recent financial crisis has highlighted the importance of maintaining the right balance when dealing with bank capital; while too much capital can have the undesired effect of hindering any possible recovery, too little exposes banks and hurts not only shareholders and creditors, but puts the broader economy at risk. It is because of the social costs and negative externalities associated with bank failures (e.g. contagion, fire sales, etc.), that capital requirements are at the forefront of the discussion of banking reform in response to the financial crisis. Kashyap, Rajan, and Stein (2008) and the Squam Lake Working Group (2009b) provide excellent analysis and make several compelling recommendations, many of which are consistent with the insights that are derived from the model in the current paper.

Before moving forward, it might be helpful to take a look back at the literature to see how bank capital requirements, and the perceptions about it, have evolved over
the years.\textsuperscript{7} As bank capital is crucial to a bank’s survival, Santomero and Watson (1977) were among the first to ask the fundamental questions of whether bank capital levels should be regulated and if there is an optimal level of bank capital. They find that the optimal level of bank capital is a trade-off between the cost of bank failures (as a result of undercapitalization) and the constraints placed on the bank (essentially the cost of overcapitalization).

This research on optimal bank capital levels leads to a discussion of bank capital adequacy and minimum capital requirements. Kareken and Wallace (1978) present an equilibrium model of banking under different regulatory schemes, two of which include deposit insurance and capital requirements.\textsuperscript{8} They find that capital requirements, in and of themselves, do not do much to mitigate bankruptcy risk. Sharpe (1978) examines the role of bank capital where the bank’s liabilities consist only of deposits which are insured by a third party, effectively making them risk-free. He defines capital adequacy in terms of minimizing the insurer’s liability so that it is “no larger than an insurance premium.” Buser, Chen, and Kane (1981) argue that capital requirements can serve as an implicit risk-based deposit insurance premium. Therefore, using their line of reasoning, regulating a bank’s capital levels may be used in conjunction with a flat-rate deposit insurance scheme to achieve the same results as one that sets insurance premiums according to risk.

Although Sharpe (1978) indirectly demonstrates that capital adequacy standards should be risk-based, Koehn and Santomero (1980) formally and directly argue the point. They examine how minimum capital requirements affect banks’ behavior. Their idea is that a regulator wants to ensure that the bank sets aside enough capital to keep the probability of failure low (within “acceptable levels”). They demonstrate that placing restrictions on bank capital might actually have counter-intuitive results: namely, banks with too much risk may actually become riskier as a result of

\textsuperscript{7}For a comprehensive review of the bank capital regulation literature see Santos (2001).

\textsuperscript{8}The literature frequently cites risk-based deposit insurance and minimum capital requirements as two methods for controlling financial institution risk. The current paper deals with the latter and, as such, the abundant literature on risk-based deposit insurance is not discussed here. For a good review of this literature the reader is referred to Allen and Saunders (1993). Also, Santos (2001) expounds the link between deposit insurance and capital requirements as regulatory tools.
stricter capital requirements. Kim and Santomero (1988) extend Koehn and Santomero (1980) to include capital requirements that account for the riskiness of the bank’s portfolio. Along these lines, Rochet (1992) finds that, under certain conditions, risk-based capital requirements can be effective if the weights are proportional to the bank’s systematic risk (i.e. its asset beta).

Hendricks and Hirtle (1997) provide an overview of risk-based capital requirements and present a case in favor of banks using internal models to determine regulatory capital, with the premise that this practice should help to better align the capital charge with the true risks of banks’ portfolios. In fact, the New Basel Accord, or Basel II, was created with the intention of accounting for bank risks in a more robust manner. The scope of risks to be considered was widened to incorporate operational risk and credit risk, with the latter depended on bank’s internal ratings of the loans and other credit instruments in their portfolios. These assessments were made using quantitative methodologies traditionally used in measuring and managing market risk in financial institutions (i.e. Value-at-Risk, or VaR). In fact, Jarrow (2007) offers a strong critique of Basel II and argues that the VaR measure does not account for credit risk appropriately. Altman and Saunders (2001) examine two aspects of Basel II risk-based capital regulation. First, they conclude that using agency ratings for the risk-weighting could introduce a cyclical lag in capital requirements, thereby possibly making the financial system less stable over time. Furthermore, they argue that the current risk-bucketing lacks granularity. As a substitute, they propose a revised weighting system that fits historical default loss statistics more closely.

The notion of procyclicality arising from the ratings-based approach for determining risk-based capital requirements appears in several critiques of Basel II. Basically, the argument goes as follows. When internal ratings are used in determining how much capital to set aside against a particular loan or position, a ratings downgrade will lead to a higher capital charge. By setting more capital aside the bank will be forced to pull back its extension of credit. On a large scale this will slow economic

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9 For background on Basel II please see Basel Committee on Banking Supervision (2003)

10 Two papers that attempt to reconcile the traditional VaR measure and credit risk within the context of capital requirements are Gordy (2000) and Gordy (2003).
growth and ultimately trigger more ratings downgrades in existing loans. This type of vicious cycle would ultimately be counterproductive to the financial system and the broader economy, the stability of which are supposed to be protected by capital requirements.\textsuperscript{11} Herring (2005) argues that the costs of Basel II, including the tendency to amplify macroeconomic cycles, outweigh the benefits of the New Accords.

Kashyap and Stein (2004) examine the Basel II Internal Ratings Based (IRB) capital requirements and the effect that it may have on cyclicality in the financial system.\textsuperscript{12} They, too, find that the Basel capital requirements may exacerbate business cycle effects and that this procyclicality essentially comes from the static and rigid nature of the regulatory requirements (implying constant risk over time and across loans). In their paper, Kashyap and Stein (2004) note that the regulator’s goal should not just be to minimize a bank’s default probability (a simple solution would be to require 100% capital), but also to ensure that banks are channeling funds in the most efficient way (i.e. making positive-NPV loans). They propose that the acceptable default probabilities be allowed to move as economic conditions change. This can be accomplished by either changing the risk-based capital ratio or modifying the risk-weights applied to the loans. They further argue that by formalizing and specifying this flexibility beforehand, rather than giving regulators discretion to make changes when conditions become bad, the potential for moral hazard is reduced.

Gordy and Howells (2006) use a simulation-based methodology to examine the procyclicality in Basel II and evaluate the best policy options to “dampen” the cyclical effects. They show that variation in regulatory capital charges over the business cycle can be reduced by applying adjustments to the capital formula. For example, a through-the-cycle IRB methodology would smooth the inputs, whereas an autoregressive specification for the regulatory minimum would represent a smoothing of the output. They do point out that, regardless of how the regulatory capital is smoothed, the procyclical effects may still persist in the banks’ economic capital. This highlights the

\textsuperscript{11}Interestingly, Pennacchi (2005) argues that risk-based capital requirements are inclined to yield greater procyclical effects than risk-based deposit insurance. He, therefore, suggests that Basel II incorporate some elements of risk-based deposit insurance to help moderate the procyclical nature of the existing framework.

\textsuperscript{12}A more technical version of this paper is Kashyap and Stein (2003).
importance of consistency between regulatory capital and economic capital. Gordy and Howells (2006) also take market discipline, which makes up Pillar 3 of the Basel II Accords, into account. It turns out that smoothing the inputs (i.e. the ratings) is the worst alternative from a market discipline perspective. Smoothing the inputs will intertemporally distort bank capital disclosures thereby making it difficult for market participants to make viable inferences and do their part in monitoring banks. In fact, Gordy and Howells (2006) emphasize that the success of Basel II hinges on how well Pillar 1 and Pillar 3 (capital adequacy and market discipline, respectively) reinforce each other. Herring (2004) offers an alternative to Basel II that is routed in market discipline: the mandatory issuance of subordinated debt. Successfully integrating market discipline and regulatory oversight in managing and monitoring bank capital requirements is one of the overarching themes of the present paper.

2.2.2 Market Discipline

Market discipline, which makes up Pillar 3 of the New Basel Accord (Basel II), has long been discussed in the literature as a possible alternative or complementary means of monitoring banks and enforcing bank behavior. The idea is that capital markets can “punish” the bank’s management for poor performance by driving down security prices, or in extreme situations taking over the bank or forcing liquidation. Billett, Garfinkel, and O’Neal (1998) argue that regulatory mechanisms have a tendency to limit the ability of capital markets to effectively discipline banks. However, efforts to integrate regulatory oversight with market discipline persist.

Early work on market discipline in banking began by looking at the value of traded bank liabilities, specifically subordinated debentures. In this case investors can drive up yields on subordinated bank debt in response to excessive risk-taking by the bank’s managers. This can be empirically tested by identifying how the market prices of banks’ traded bonds respond to bank risk-taking. Avery, Belton, and Goldberg (1988) perform one of the first empirical investigations on whether subordinated note and debenture investors are able to successfully discipline banks. They find that market discipline is not consistent with the factors that are typically assessed by
regulators. Specifically, the analysis shows that the default-risk premium on traded
bank debt is uncorrelated with balance sheet measures of risk and weakly related
to credit ratings. A possible explanation is that this inconsistency may arise from
the capital markets and regulators having different objective functions. Gorton and
Santomero (1990) utilize a contingent claims approach to valuing subordinated debt,
noting that previous studies of market discipline rely on two flawed assumptions:
the relationship between a bank’s risk and default premia is linear; and that static
accounting measures capture the true risk faced by banks. The contingent claims
approach uses market information to estimate implied volatility as a nonlinear risk
proxy which is then regressed on various accounting measures. The authors find
that the accounting measures do very little to explain bank risk. While Gorton and
Santomero (1990) do not provide any explicit support in favor of market discipline, it
did open doors for more sophisticated methodology in research on market discipline
in banking.

Flannery and Sorescu (1996) use market prices to study how subordinated debt
investors perceive the risks of banks using both linear and nonlinear (contingent
claims) approaches. They find that asset quality and market leverage have the most
significant effects on bank credit spreads implying that investors do price default
risk. This is one of the first papers to provide empirical evidence of value in market
discipline, although the authors stop short of calling for market discipline to replace
compare the speed and accuracy of regulatory assessments on bank financial condition
against market evaluations of the same banks. They find evidence that both regulators
and market participants seem to obtain information that would be useful to the
other in making accurate assessments. As a result, they call for more disclosure
and increased information flow between regulatory bodies and investors. Berger,
Davies, and Flannery (2000) note that regulators tend to be concerned with a bank’s
“current condition” whereas financial market participants are more concerned with
their “future condition” which supports the argument that both capital adequacy
standards and bank risk measures must be more forward-looking.
Noting that there is a lot of ambiguity in the term “market discipline”, Flannery (2001) defines it in terms of two components: market monitoring and market influence. This idea is reiterated in Bliss and Flannery (2002) where the topic is studied empirically; the extent to which investors are able to influence the behavior of bank managers is examined using market prices of traded securities (both bonds and stocks). The results are inconclusive although there is some evidence of market influence in the data.

Flannery (1998) surveys the literature to evaluate how well market participants are able to assess the financial condition of banks. He concludes that bank supervisors should regularly incorporate market information in an effort to provide the most comprehensive oversight system. Krainer and Lopez (2004) specifically advocate the use of equity market information. The fact that the model proposed in this paper uses market information, including the market value of equity and equity volatility, as key inputs illustrates how market discipline can be incorporated in the regulation of financial institutions.

2.2.3 Another Look at Bank Capital: Liability Structure and Market Information

The Financial Crisis of 2007-2009 raised new questions about bank capital. While the literature clearly indicates a role for bank capital, with some evidence in favor of risk-based capital requirements, the existing framework failed to address some very important features of modern financial institutions. First of all, current measures of risk-based capital adequacy focus on the risk of the bank’s portfolio of assets (i.e. the left-hand-side of the balance sheet); this practically ignores the bank’s own risk of default which is a function of both sides of the balance sheet. Several researchers have begun to recognize the importance of bank liabilities and funding choices in setting capital requirements. In fact, the Squam Lake Working Group (2009b) explicitly states that “capital requirements should be higher for banks that finance more of their operations with short-term debt.” Kashyap, Rajan, and Stein (2008) suggest that the financial crisis was the proverbial “perfect storm” that arose from banks’ excessive
risk-taking in low quality assets (something that previous capital requirement standards should have addressed) combined with the overreliance on short-term debt to finance these risky investments. The latter can be problematic from two perspectives. First, there is the idea of “rollover risk” where the financial institution is unable to refinance maturing debt obligations due to poor liquidity in the short-term funding markets. However, even more fundamental is the idea that maturing debt provides a hurdle that banks must get over in order to survive. If the asset values are too low and maturing debts cannot be paid, the bank will be in a state of insolvency and possibly default. Both of these dimensions, funding liquidity and short-term default, should be incorporated into a new model of bank capital.\textsuperscript{13}

Another problem that remains is determining how distressed financial institutions should recapitalize. It is well-known in the corporate finance literature that distressed firms encounter severe agency problems when trying to raise new equity (see Myers (1977)). These problems are even more pronounced for financial institutions (see Flannery (1994)). Kashyap, Rajan, and Stein (2008) advocate a capital insurance system that would payout to participating banks in distressed states, thereby providing a capital infusion without the negative externalities and agency problems of issuing new equity. The state-contingent nature of the capital insurance scheme is similar in theme to the use of Contingent Capital Certificates (CCC) as proposed by Flannery (2009). CCCs are debt securities that pay coupons like any other bond, but are junior to subordinated notes and debentures. In the event of some downturn, represented by the so-called “trigger” the CCCs automatically convert into equity, thereby replenishing the capital cushion on the bank’s balance sheet. The downside of CCCs is that there is no new cash infusion into the bank. Therefore, the CCCs must be used in prudential a priori planning for risk management and capital structure solutions, a point that is returned to later in the paper.

Contingent-capital-like securities have received a lot of attention from both policymakers and academics alike. The Squam Lake Working Group (2009a) recommends

\textsuperscript{13}The present paper focuses on the latter – i.e. default risk – in determining the amount of capital. Extending the model to include a liquidity factor is a nontrivial feat.
the use of “regulatory hybrid securities” in bank recapitalization. There are quite a few recent papers that examine contingent capital from many different perspectives including the pricing, design, and regulatory considerations of such securities (see Pennacchi (2010), McDonald (2010), Albul, Jaffee, and Tchistyi (2010), and Sundaresan and Wang (2010)). At the center of every discussion is determining what the trigger should be for these securities. Contingent capital securities with a market-based trigger represent a way of integrating market discipline into bank capital adequacy standards.

New research indicates that the use of market information analyzing and regulating bank capital is critical, especially in times of crisis. Rapidly changing market conditions are not captured in static capital ratios and historical metrics. Hart and Zingales (2009) propose a mechanism for recapitalizing large financial institutions using credit default swaps (CDS). Under the Hart and Zingales (2009) proposal, if CDS spreads for a particular financial institution were to become too high then regulators would force the financial institution in question to take action regarding recapitalization. CDS spreads do convey information about the market’s perception of a firm’s likelihood of default; however, there are a few problems that could potentially arise from relying on the CDS market in the context of bank capital (and perhaps help support the case for using equity market information). First, as it currently stands, the CDS market only covers the largest and most liquid credit names.14 As a result, a CDS-based mechanism would not work for smaller, but publicly-traded, regional banks whereas one based on the equity market would. Second, some market participants believe that CDS contracts destabilized the markets during the recent financial crisis; in these instances it becomes difficult (if not impossible) to isolate the component of CDS spread movements due to true default risk versus those based on a vicious cycle of panic and speculation (see Stulz (2010)). Third, there is not a direct structural link between CDS spreads and the economics behind a bank’s operations (i.e. degree of leverage, riskiness of the assets, and the interaction between the two).

Above all of this, even if CDS spreads are reliable indicators of bank distress and

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14 Chen, Cheng, Fabozzi, and Liu (2008) use the 60 most liquid CDS contracts in their empirical study, and note that “The liquidity drops substantially [after] the top 50 names.” (p. 131)
triggers for when to recapitalize, this last point is critical: CDS spreads cannot be used to determine how much capital is needed to reduce default probabilities because of the lack of a structural link.\textsuperscript{15} It will be shown that only a structural credit risk model (with endogenous default) can be used in this capacity.

Bond, Goldstein, and Prescott (2010) acknowledge the important role of market information in making corrective action decisions; however, they do caution the over-reliance on market information due to adverse feedback loop and dual causality between the decision-maker’s actions and market prices. It should therefore be noted that the current paper does not suggest that market-based measures should replace the existing tools that regulators and risk managers use; but rather that they can serve as useful complements to the same end.

Having identified a need for incorporating the liability structure and using market information in analyzing the capital requirements of financial institutions, and further having indicated that structural credit risk model can be of substantial value in doing so, it would be fitting to lay out the basic assumptions underlying the model developed in this paper. The appropriate structural credit risk model includes information about the liability structure to compute default probabilities, which in turn, can be used to assess the capital adequacy of a financial institution and subsequently solve for how much capital is needed, if any. This is similar to the option-theoretic approach employed by Ronn and Verma (1989). In Ronn and Verma (1989) a standard Black-Scholes-Merton model was used to solve for the capital infusion that would reduce the risk-based deposit insurance premium to a specified level. The approach presented in the present paper, however, differs from Ronn and Verma (1989) in at least two ways. First, Ronn and Verma (1989) use a single period, static model with one class of debt, whereas the this paper proposes a multi-period, dynamic model with both junior and senior debt. Second, Ronn and Verma (1989) use deposit insurance to find the risk-based capital requirement, whereas the model proposed in this paper uses default probability. The main distinctions from Ronn and Verma (1989) are nontrivial: the model developed in this paper is more robust in accounting for the

\textsuperscript{15}Along these lines CDS spreads can be viewed as “reduced-form”.
bank’s liability structure and does not rely on deposit insurance to find the capital requirements. The next section develops and discusses the model.

2.3 Model

2.3.1 Review of Structural Credit Risk Models

Structural credit risk models began with the seminal works of Black and Scholes (1973) and Merton (1974), who noted that when a firm has debt in its capital structure the equity can be viewed as a call option on the firm’s assets. Thus, the standard methods for pricing financial options can be applied to valuing corporate securities. The basic Black-Scholes-Merton (BSM) model provided an efficient, analytical method for calculating risky debt prices, credit spreads, and comparative statics, thereby taking the economics of credit risk to another level. Since then many excellent extensions have been developed, in response to some of the restrictive and unrealistic assumptions behind the basic BSM model. One class of extensions are the “barrier” structural models where default can occur at any time and is the result of the asset value process hitting a barrier. Many of these models assume an exogenous default barrier (Black and Cox (1976), Longstaff and Schwartz (1995), Collin-Dufresne and Goldstein (2001)) although the Leland (1994) and Leland and Toft (1996) models include an endogenous default barrier that is solved for within the model. Another structural credit risk model with endogenous default is the compound option model of Geske (1977). These endogenous default models are critical to understanding the structural credit risk model developed in this paper; hence, they will be discussed in more detail.

Structural credit risk models are commonly used to compute default probabilities. Within the original BSM framework the probability that the call option is not exercised at time $T$ is the probability that the firm defaults; that is because if the market value of the firm’s assets is not greater than the face value of the debt, then

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16Perhaps the most notorious of these assumptions is that the firm has only one class of debt outstanding in the form of pure discount bonds maturing at a fixed time $T$. Accordingly, many of these extensions allow for multiple classes of debt, default prior to maturity, and coupon payments, although some models also include stochastic interest rates and jumps in the asset value process.
shareholders will let their option expire and leaving the assets to the debt holders. The extensions that include a continuous default barrier compute default probabilities by the first passage density. The [discrete-time] compound option structural model actually computes default probabilities at every cash flow time (i.e. whenever a payment, principal or interest, is due to bondholders). This gives rise to a term structure of default probabilities. A practical implementation of the basic BSM model is the KMV model, detailed in Crosbie and Bohn (2003). However, despite its popularity, the KMV model has little or no applicability to financial institutions. This is partly attributable to exogenous default condition where the entire liability structure is simplified to a linear combination of short-term debt and long-term debt, not taking into account seniority or the relevance of the individual components.

Leland (2004) looks at default probabilities calculated using two structural models – the *endogenous barrier* model of Leland and Toft (1996) and the *exogenous barrier* model of Longstaff and Schwartz (1995) – and observes how well the predictions fit historical default frequencies. The results of the barrier structural models are also compared with the distance to default and expected default frequencies from the KMV model. Delianedis and Geske (2003) compute risk-neutral default probabilities (RNDP) using the original BSM model and the Geske model. They draw two interesting conclusions that have very important implications for the model developed in this paper: first, they show that RNDP’s serve as an upper bound to risk-adjusted default probabilities (for both models) and, second, the Geske model is able to provide important information that other structural models cannot; specifically, it provides a full term structure of default probabilities. Bharath and Shumway (2008) compare the distance-to-default from the BSM/KMV model with forecasts obtained using a hazard model. They find that the hazard model performs better than the distance-to-default in out-of-sample default prediction.

In almost all of the credit risk literature, including the empirical studies cited above, structural credit risk models are assumed to be inapplicable to financial institutions. Leland (2009) discusses some of the difficulties in applying structural credit risk model to financial institutions. Two of the biggest issues are the high degree
of leverage and the disproportionate use of short-term debt, including repurchase agreements.

An appropriate structural credit risk model for a banking firm must be flexible enough to account for how funding decisions are made in different market conditions but also capture the complex liability structure that is an inherent characteristic of financial institutions. Another important feature is endogenous default, especially for dealing with bank capital, as is argued below.

The Leland and Toft (1996) model has endogenous default and is attractive for several reasons. First of all, there is a nice, elegant closed-form solution for the values of debt and equity, as well as default probabilities and the default barrier. It is assumed that all debt is rolled-over which, in normal market conditions, is in line with the practices of financial institutions. However, the Leland and Toft (1996) model does not capture all of the features of a bank because it assumes that there is only one class of debt with a fixed maturity and the debt structure is fixed.

Another structural credit risk model with endogenous default is the compound option model of Geske (1977). The model allows default to occur whenever a cash flow is due, rather than the two extremes of defaulting at maturity (as with European options and the original BSM model) or defaulting anytime between now and maturity (as with barrier models). This can then be used to model coupon-paying debt, where default can occur if it is not economically feasible to make an interest and/or principal payment; or it can be used to model a complex seniority or maturity structure where debt holders are to be paid sequentially. This is a practical way to capture the realistic feature of cross-default provisions in debt contracts.

When there are just two cash flows the model can be solved analytically as shown by Geske (1979). It is also shown that there is an explicit leverage effect that causes the volatility of the compound option to change with the value of the underlying assets, which is one reason why compound options (and debt with multiple cash flows) cannot be valued with the standard BSM model. Geske and Johnson (1984) appropriately specify the default conditions when there are two distinct classes of debt, which leads to the correct closed-form expression for the intertemporal values
of equity and junior debt.

This compound option structural model provides a consistent and elegant framework for valuing corporate securities and quantifying credit risk. Furthermore, the model captures the complex and important capital structure of a bank rather nicely. The model is simple enough where it can be computed analytically, but still has the information regarding the seniority structure of debts and the inherent compound-optionality in the decision to default.

The drawback of the compound option structural model is that it is assumed that all debt obligations are satisfied by issuing new equity. This implies that the firm being analyzed will have a leverage ratio that decreases systematically over time and it will ultimately become an all-equity firm. There are very few firms that practice such extreme de-leveraging, especially in the banking industry. A structural model that preserves endogenous default but that has some mechanism for rolling-over debts (especially those that are short-term in nature), like Leland and Toft (1996) might be more appropriate for financial institutions.

The model used in this paper draws from the most promising aspects of existing structural models with the intention of developing a practical and realistic credit-risk-based approach to estimating bank capital requirements. As such, one of the key features is endogenous default. The model and its features are discussed next.

### 2.3.2 A Structural Model of the Banking Firm

In this section, a structural model for the banking firm is developed. It is based on the Geske compound option structural credit risk model, but includes a dynamic implementation. When used in this dynamic capacity, the compound option structural model has the interpretation that, as long as default probabilities are sufficiently low approaching the first cash flow (i.e. payment to short-term, senior creditors), some or all of the debts will be rolled over before the specified maturity time. As a result, this dynamic implementation preserves the economic intuition of a model such as Leland

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17 Adrian and Shin (2010) show that financial institutions tend to actively manage their leverage ratios in a procyclical manner.

In the model, default is endogenous and is triggered when the value of the bank’s assets is too low to justify the sale of any new equity. The inability to raise fresh capital can be thought of as the driving force of bank distress, especially during the recent financial crisis. Therefore, the endogenous default condition given by the model is crucial to understand the events that unfolded.

**Assumptions and Model Setup**

First, the structural credit risk model developed in this paper assumes that financial institutions exist in a world with no market frictions. Specifically, it is assumed that there are no taxes, no distress costs, no transaction costs, and perfect liquidity. The assumption of perfect liquidity implies that there is no rollover risk - a crucial point for the dynamic implementation of the model. Since there are no market frictions, it follows that the bank’s capital structure is exogenous; that is, no claim is made with respect to the optimal level of bank capital. Rather, the goal is to find the amount of capital needed to bring default probabilities down to “acceptable” levels given the bank’s current capital structure. Despite the exclusion of such frictions, the insights that are given by this model are extremely useful. The model highlights the fact that static measures of capital adequacy, such as a fixed percentage of assets, are arbitrary and do not capture the true health of a financial institution. Rather, a dynamic approach that is forward-looking and tied to market conditions is more appropriate in setting capital requirements. Furthermore, the structural credit risk model developed in this paper demonstrates how market information and the liability structure can be used to compute default probabilities which can then be used as a reference point for setting capital requirements. The result is that capital requirements may be time varying and conditional upon the health of the particular bank and the financial system as a whole. Finally, the model provides a quantitative and objective method for determining how much capital is needed to stabilize a distressed financial institution. All of this can be demonstrated without the presence of market frictions. The nontrivial task of extending the model to account for one or more of
the above market frictions is left for future research.

In addition to the fact that the bank’s capital structure is exogenous, no claim is made regarding the optimality of contracts in the model. It is taken as given that debt exists, and that banks rely on it as a primary source of funding. This assumption may be justified by casual empiricism which indicates that banks primarily rely on debt to fund their operations or theoretically where banks, as liquidity providers, convert short-term, liquid liabilities to long-term, illiquid assets. Though agency costs are not explicitly considered, the existence of debt makes them unavoidable. In the structural credit risk model there is a wealth transfer effect as described by Jensen and Meckling (1976) and Myers (1977) and discussed within the context of banking firms by Flannery (1994). As a result, there is an implicit moral hazard problem that arises from these wealth transfers.

To set up the model mathematically, several additional assumptions have to be made. First, assume that the market value of the bank’s portfolio of assets (i.e. loans, investment securities, etc.) evolves according to a Geometric Brownian Motion. That is, at time \( t \) the asset value process is given by the stochastic differential equation

\[
dV_t = r_t \, V_t \, dt + \sigma_V \, V_t \, dW_t
\]

(2.1)

where \( dW_t \) represents a standard Brownian increment under the risk-neutral probability measure.\(^{18}\) In Equation (2.1) \( r_t \) represents the risk-free interest rate at time \( t \) and \( \sigma_V \) is the asset volatility. Neither the interest rates nor asset volatility are stochastic, another assumption of the model.

Now, assume that there are two general classes of debt: senior debt with face value \( F_1 \) maturing at time \( T_1 \) and junior debt with face value \( F_2 \) maturing at time \( T_2 \), where \( T_1 < T_2 \). Here, the short-term, senior debt can be time deposits, repurchase agreements, federal funds purchased, and/or other similar claims. While many of

\(^{18}\)The risk-neutral measure is used for at least two reasons: first, it does not require the estimation of the risk premium for individual banks. Since at any given time the current risk premium is unobservable, attempting to estimate the risk premium introduces yet another degree of uncertainty and potential for model error. Rather, it is more straightforward to use the risk-free rate which is observable. Second, it can easily be shown that under certain conditions the risk-neutral measure serves as an upper bound on true default probabilities.
these claims are secured, they need not be; the key feature is that they are expected
to be re-paid by time $T_1$ or have priority in the event of a liquidation. The long-term,
junior debt can be thought of as subordinated debentures, which are issued by many
financial institutions and featured prominently in the market discipline literature (See
Section 2.2.2). The subordinated debentures lie just above equity in terms of seniority,
bu since they are unsecured, have longer maturities, and have lower priority, they
bear more risk than the senior debt. Figure 2.1 illustrates this capital structure with a
simplified balance sheet for a typical bank. This two-tiered liability structure implies
that the bank’s equity is a compound call option on the bank’s assets. First, short-
term, senior creditors must be paid; then, the junior, long-term debt is paid; and,finally, whatever is left goes to the shareholders (the residual claimants). Note that
the usual balance sheet equality must hold [in market value]: $V_t = S_t + J_t + E_t$, where
$S_t$ is the market value of the short-term, senior debt at time $t$, $J_t$ is the market value
of the long-term, junior debt at time $t$, and $E_t$ is the market value of the equity at
time $t$. Next, valuation of the bank’s corporate securities will be discussed. The value
of all three security classes—equity, junior debt, and senior debt—can be solved for in
closed-form within the framework that has been established to this point.

Valuation

To value the securities within the compound option framework it is helpful to visualize
the default process as a binomial tree. In Figure 2.2 the solid nodes represent survival
states, whereas the dashed nodes represent default states. The accompanying table
shows the value of the corporate securities (equity, junior debt, and senior debt) at
each node. Solving the expectations at node A give rise to the closed-form valuation
equations below.

Moving from node A to node B means that the bank survives to time $T_1$ and the
senior debt holders will be paid. Survival at the first cash flow time, $T_1$, depends on
the asset value relative to the default barrier, $\bar{V}$, which is endogenously determined.
Precisely how the endogenous default barrier is determined will be discussed in detail
below. Since, at node B, the asset value is greater than the default barrier, the first
cash flow obligation, $F_1$ is satisfied in full and all that remains are the junior debt and equity claims. At this point, the model is identical to the standard Black-Scholes-Merton (BSM) structural model. It is no surprise that the value of equity at time $T_1$, $E_{T_1}$, is given by their famous equation for pricing a European call option, with strike price and time-to-expiration $T_2 - T_1$. This will be an important point when deriving the endogenous default boundary. Accordingly, the value of the subordinated debt at time $T_1$, $J_{T_1}$, is equal to the value of a covered call or a put option attached to a risk-free loan of maturity $T_2 - T_1$ as per put-call parity. The probability of getting to node B, or equivalently the probability that $V_{T_1} > \bar{V}$, is the short-term survival probability, denoted as $p_1$ in Figure 2.2. All of the probabilities are known in closed-form and derived below.

On the other hand, if the value of the bank’s assets are less than or equal to the default barrier, then the bank will hit the default state at node C. There is a very important subtlety that must be pointed out here. Node C is actually further broken down into two states, $C_1$ and $C_2$, which represent partial versus total default, respectively. $C_1$ is referred to as “partial” default, because senior debt holders are paid in full but asset value is not high enough to pay junior debt holders so the latter party receive only the residual payment. In and near this distressed state, equity has no value and the bank’s subordinated debentures will behave more like equity rather than debt. $C_2$, on the other hand, represents total default where neither equity holders nor junior debt holders are paid, and the senior debt holders will receive whatever value the assets are worth at default time.

In the static compound option framework if the firm makes it to node B, then new equity is issued at the current market price and used to finance the payment to the senior creditors. Thus, once the short-term, senior debt holders are paid at node B the leverage of the firm is reduced by $F_1$ and only long-term, junior debt remains outstanding. As mentioned above, in defining $E_{T_1}$ and $J_{T_1}$, the problem reduces to the standard BSM setup. If the value of the assets at time $T_2$ is greater than the face value of the junior debt ($V_{T_2} > F_2$) then the bank will end up in the terminal survival state, represented by node D in the lattice. The assets are liquidated, junior creditors
are repaid what they are owed, and shareholders receive the residual value $V_{T_2} - F_2$. However, if the value of the assets is less than or equal to the face value of the junior debt ($V_{T_2} \leq F_2$) then the bank will end up in the terminal default state, represented by node E, with conditional probability of $1 - p_2$. This is the "conditional" default probability because it is conditional upon the bank’s survival to time $T_1$.

From the perspective at node A the model seems to be a reasonable, albeit simple, representation of the scenario faced by a typical financial institution. However, once node B is transcended, this leads to a rather unrealistic circumstance in the static world. Recall, that the traditional static compound option structural model has the unfavorable feature that, since debt is paid down by issuing new equity, leverage is systematically reduced over time ultimately leading to an all-equity firm. Fortunately, with the dynamic implementation proposed in this paper, the bank will never get to that point. Every time the model is run, the lattice effectively starts over and the analysis is from the perspective of node A all over again. Essentially, at time $T_1$ when payment to the senior creditors is due, the bank will either default or roll-over some or all of their short-term debt. This dynamic implementation is described below in Section 3.2.4.

The value of the securities can be determined at node A (time $t = 0$) by taking the discounted expected value of the future cash flows at each of the subsequent nodes. For equity and junior debt this results in nested expectations. Consequently, the assumption that Geometric Brownian Motion is the underlying stochastic process ensures that the solution is a function of the bivariate normal distribution, which is very manageable.

Now, for any time $t$, $0 \leq t < T_1$, evaluating the expectations at $t$ gives the closed-form expressions for valuing all of the bank’s claims. Equity is a compound call on the bank’s assets in that shareholders have a claim to

$$
\mathbb{E} \left[ e^{-r \tau_1} \max \{ E_{T_1} - F_1, 0 \} | V_t \right], \tag{2.2}
$$

where $E_{T_1}$ is equal to the value of a European call option at time $T_1$, which is not known at time $t$. Let $\tau_1 = (T_1 - t)$ be the time between $t$ and the short-term, senior
debt maturity $T_1$ and $\tau_2 = (T_2 - t)$ be the time between $t$ and the long-term, junior debt maturity $T_2$. The expectation given in Equation (2.2) is equal to:\(^{19}\)

$$E_t = V_t N_2 (h_1^+, h_2^+; \rho) - F_2 e^{-\tau_2} N_2 (h_1^-, h_2^-; \rho) - F_1 e^{-\tau_1} N (h_1^-) \quad (2.3)$$

where $h_1^+ = \frac{\ln(\frac{V_t}{F_1}) + (r + \frac{\sigma^2}{2}) \tau_1}{\sigma \sqrt{\tau_1}}$, 

$h_1^- = \frac{\ln(\frac{V_t}{F_1}) + (r - \frac{\sigma^2}{2}) \tau_1}{\sigma \sqrt{\tau_1}} = h_1^+ - \sigma \sqrt{\tau_1}$, 

$h_2^+ = \frac{\ln(\frac{V_t}{F_1}) + (r + \frac{\sigma^2}{2}) \tau_2}{\sigma \sqrt{\tau_2}}$, 

$h_2^- = \frac{\ln(\frac{V_t}{F_1}) + (r - \frac{\sigma^2}{2}) \tau_2}{\sigma \sqrt{\tau_2}} = h_2^+ - \sigma \sqrt{\tau_2}$.

$N (\cdot)$ denotes the cumulative standard normal distribution,

$N_2 (\cdot)$ denotes the cumulative bivariate standard normal distribution,

and correlation $\rho = \sqrt{\frac{\tau_1}{\tau_2}}$ which follows from the properties of Brownian Motion.

$\bar{V}$ is the endogenous default boundary, which was introduced above and will be derived below.

The senior debt is to be paid at time $T_1$. If the market value of the assets at that time is greater than $\bar{V}$, then senior debt holders will be paid the amount $F_1$ in full. However, if the market value of the assets is less than or equal to $\bar{V}$ and less than the face value of the senior debt, then there is [total] default at time $T_1$. In this case, the assets will be liquidated and senior debt holders will only recover a portion of what they are owed; specifically they will recover the market value of the assets at the time of default, $T_1$. Thus, the value of the short-term, senior debt at time $t$ is given by the expectation

$$E \left[ e^{-\tau_1} \min \{V_{T_1}, F_1 \} \mid V_t \right]. \quad (2.4)$$

It is pretty straightforward to show that the expectation given in Equation (2.4) is equal to

$$S_t = V_t \left[ 1 - N \left( k^+ \right) \right] + F_1 e^{-\tau_1} N \left( k^- \right), \quad (2.5)$$

where $k^+ = \frac{\ln(\frac{V_t}{F_1}) + (r + \frac{\sigma^2}{2}) \tau_1}{\sigma \sqrt{\tau_1}}$ and $k^- = \frac{\ln(\frac{V_t}{F_1}) + (r - \frac{\sigma^2}{2}) \tau_1}{\sigma \sqrt{\tau_1}} = k^+ - \sigma \sqrt{\tau_1}$.

\(^{19}\)For a derivation please see Geske (1977), Geske (1979), and Geske and Johnson (1984).
Then, finally, there is the bank’s junior debt which has the very interesting property that when asset values are well above the default boundary they are a fixed claim and their prices behave like debt, but in times of distress they are a residual claim and their prices behave like equity. The model does a very good job of capturing this characteristic of bank subordinated debentures. The value of the long-term, junior debt at time $t$ is given by the expectation:

$$
\mathbb{E} \left[ e^{-r\tau_1} \left( (J_{T_1} \cdot \mathbb{I}_{\{V_{T_1}>\bar{V}\}} + \left( \max \{V_{T_1} - F_1, 0\} \cdot \mathbb{I}_{\{V_{T_1}\leq \bar{V}\}} \right) \right) \bigg| V_t \right] \tag{2.6}
$$

where $\mathbb{I}_{\{V>\bar{V}\}}$ is the indicator function that equals 1 if $V > \bar{V}$ and 0 otherwise.

To obtain the closed-form solution for the long-term, junior debt the expectation given in Equation (2.6) can be evaluated directly or, since the expectation for short-term, senior debt is considerably more straightforward, the fact that $V_t = E_t + J_t + S_t$ can be used to solve for $J_t = V_t - E_t - S_t$. Subtracting Equation (2.3) and Equation (2.5) from $V_t$ gives:

$$
J_t = V_t \left[ N \left( k^+ \right) - N_2 \left( h_1^+, h_2^+; \rho \right) \right] + F_1 e^{-r\tau_1} \left[ N \left( h_1^- \right) N \left( k^- \right) \right] + F_2 e^{-r\tau_2} N_2 \left( h_1^+, h_2^-; \rho \right) \tag{2.7}
$$

where everything defined in Equation (2.3) and Equation (2.5) remain the same.

It should be noted that, although asset volatility is assumed to be constant, equity volatility is not constant but rather changes as a function of asset value and other parameters in the model. This was first shown by Geske (1979). Equity, being a compound option on the underlying asset, has its own stochastic dynamics which can be specified using Ito’s Lemma. From the Ito expansion the volatility term for equity is

$$
\sigma_E = \frac{\partial E}{\partial V} V \sigma \frac{V}{E}. \tag{2.8}
$$

The partial derivative $\frac{\partial E}{\partial V}$ can be found by differentiating Equation (2.3) with respect to asset value and is known as the “equity delta”:

$$
\frac{\partial E}{\partial V} = N_2 \left( h_1^+, h_2^+; \rho \right). \tag{2.9}
$$
Substituting Equation (2.9) into Equation (2.8) gives

\[
\sigma_E = N_2 \left( h_1^+, h_2^+; \rho \right) \frac{V}{E} \sigma_V. \tag{2.10}
\]

This equation says that, holding everything else constant, as more leverage is introduced the equity volatility becomes greater than the asset volatility. This leverage effect is consistent with standard corporate finance theory which says that leverage increases the riskiness of a firm’s equity as financial risk is compounded on top of the inherent total business risk (as proxied by \( \sigma_V \)).

**Endogenous Default and Default Probabilities**

In both the original [static] compound option structural model as well as the dynamic implementation presented in this paper the default condition arises from the fact that the firm must have access to capital in order to survive. At time \( T_1 \), when the short-term, senior debt is due, the firm must be able to raise enough capital to satisfy the pending obligations. Given the assumption that capital is raised in a frictionless market, equity can always be issued at the current market prices. As the market value of equity approaches zero, the firm will find itself under increasing pressure to raise the funds needed to service the outstanding debt. Recall that equity, in turn, is a function of the value of the firm’s assets. Thus, there must be some “breakeven” asset value such that the market value of equity is just high enough that a new issuance will just satisfy the next cash flow, leaving nothing for the residual claimants. In other words, the default condition maintains that equity must have positive value for survival.\(^{20}\)

Mathematically, the default boundary is the asset value, \( V_{T_1} = \bar{V} \), that is the internal solution to the integral equation

\[
E \left( \bar{V} \right) - F_1 = 0. \tag{2.11}
\]

\(^{20}\)This is the same as the default condition in the Leland-Toft model; see Leland and Toft (1996), p. 994.
It follows from Equation (2.1) that

\[ \bar{V} = F_1 + J_{T_1} \]  

(2.12)

where \( J_{T_1} \) is the market value of the long-term, junior debt at the time the senior debt holders are paid (this was defined earlier). In order to find the default boundary, Equation (2.11) can be solved so that

\[ F_1 = E(\bar{V}) = \bar{V} N(d_+) - F_2 e^{-r(T_2-T_1)} N(d_-) \]  

(2.13)

where \( d_+ = \frac{\ln(\frac{\bar{V}}{F_2}) + \left(\frac{\sigma^2}{2}\right)(T_2-T_1)}{\sigma \sqrt{T_2-T_1}} \) and \( d_- = \frac{\ln(\frac{\bar{V}}{F_2}) + \left(\frac{\sigma^2}{2}\right)(T_2-T_1)}{\sigma \sqrt{T_2-T_1}} \).

Solving for \( \bar{V} \) in Equation (2.13) gives the endogenous default boundary. If the asset value at time \( T_1 \) is less than this amount, then the firm will not be able to raise capital and will default. Once the default boundary has been found, the default probabilities can be specified in closed-form. It is no surprise that this will be a function of how close the expected future assets value will be to the endogenously determined boundary.

The first probability to define is the time \( T_1 \) survival probability; i.e. \( \Pr\{V_{T_1} > \bar{V}\} \). Under the risk-neutral measure this is equal to \( N(h^-) \) defined in the valuation equations above (see, e.g., Equation (2.3)).

Proof:

Since the future asset value is a lognormal random variable, take the natural logarithm of both sides to get

\[ \Pr \{ V_{T_1} > \bar{V} \} = \Pr \{ \ln V_{T_1} > \ln \bar{V} \} . \]  

(2.14)

\( V_{T_1} \) is not known at time \( t \), but \( V_t \) is; from the asset price dynamics given in Equation (2.1):

\[ V_{T_1} = V_t e^{\sigma(W_{T_1-W_t}) + \left(\frac{\sigma^2}{2}\right)T_1} . \]  

(2.15)

Next, define the random variable \( Y \triangleq \frac{W_{T_1-W_t}}{\sigma \sqrt{T_1-t}} \). Take note that \( Y \) is a standard normal random variable; i.e. \( Y \sim N(0,1) \). Now, Equation (2.15) can be rewritten
as

$$V_{T_1} = V_t e^{(\sigma \sqrt{\tau_1})Y + \left(r - \frac{\sigma^2}{2}\right)\tau_1}$$  \hspace{1cm} (2.16)$$

which implies

$$\ln V_{T_1} = \ln V_t + (\sigma \sqrt{\tau_1}) Y + \left(r - \frac{\sigma^2}{2}\right) \tau_1.$$  \hspace{1cm} (2.17)$$

This gives the distributional properties of the log-value:

$$\ln V_{T_1} \sim N \left(\ln V_t + \left(r - \frac{\sigma^2}{2}\right) \tau_1, \sigma^2 \tau_1\right).$$

Subtracting the mean and dividing by the standard deviation and plugging into Equation (2.14) gives

$$\Pr\left\{\ln V_{T_1} > \ln \bar{V}\right\} = \Pr\left\{Y > \frac{\ln \bar{V} - \ln V_t - \left(r - \frac{\sigma^2}{2}\right) \tau_1}{\sigma \sqrt{\tau_1}}\right\}.$$  \hspace{1cm} (2.18)$$

Now, multiply both sides by $-1$ and define the new random variable: $X_1 \triangleq -Y$ which is also a standard normal random variable. This gives

$$\Pr\left\{\ln V_{T_1} > \ln \bar{V}\right\} = \Pr\left\{X_1 < \frac{\ln \bar{V} + \left(r - \frac{\sigma^2}{2}\right) \tau_1}{\sigma \sqrt{\tau_1}}\right\}.  \hspace{1cm} (2.19)$$

Note that the right-hand-side of Equation (2.19) is equal to $h_1^{-}$ defined above. This results in the following:

$$\Pr\left\{X_1 < h_1^{-}\right\} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{h_1^{-}} e^{-\frac{(x_1)^2}{2}} \, dx_1 = N \left(h_1^{-}\right).  \hspace{1cm} (2.20)$$

Q.E.D.

The short-term default probability, $DP (T_1)$, can now be defined as $1 - N \left(h_1^{-}\right)$ or $N \left(-h_1^{-}\right)$.

Proof:

This proof is quite simple. Since the two events—surviving past the first cash flow and defaulting at time $T_1$—are mutually exclusive, the probability that the asset value at time $T_1$ is less than or equal to the endogenous default boundary is equal to one.
minus the probability given in Equation (2.20). In symbols, this is:

\[
DP (T_1) = \Pr \{ V_{T_1} \leq \bar{V} \} = 1 - \Pr \{ V_{T_1} > \bar{V} \} \tag{2.21}
\]

which, when combined with Equations (2.14) - (2.20), is equivalent to

\[
1 - \Pr \{ X_1 < h_1^- \} = 1 - N (-h_1^-) = N (h_1^-) . \tag{2.22}
\]

Q.E.D.

The next probability to define is the total survival probability, which is the joint probability of surviving to both \( T_1 \) (and) \( T_2 \). This can be done by making use of the properties of Brownian Motion and the fact that the two events— the asset value at \( T_1 \) being greater than the endogenous default boundary and the asset value at \( T_2 \) being greater than the face value of the junior debt—are jointly lognormal. It can be shown that under the risk-neutral measure this is equal to \( N_2 (h_1^-, h_2^-; \rho) \) defined in the valuation equations above (see, Equation (2.3)). \(^{21}\)

Proof:

The total survival probability is the joint probability \( \Pr \{ (V_{T_1} > \bar{V}) \ \& \ (V_{T_2} > F_2) \} \).

Both of the random variables, \( V_{T_1} \) and \( V_{T_2} \) are lognormally distributed and, in fact, since they are both generated by the same Brownian Motion they are jointly distributed with correlation \( \rho = \sqrt{\frac{T_1}{T_2}} \). The same transformations used in the above proof must also be applied to \( V_{T_2} \) at time \( t \); essentially Equations (2.14) - (2.20) have to be repeated for \( V_{T_2} \). First take the logarithm of both sides of the two inequalities, yielding:

\[
\Pr \{ (V_{T_1} > \bar{V}) \ \& \ (V_{T_2} > F_2) \} = \Pr \{ (\ln V_{T_1} > \ln \bar{V}) \ \& \ (\ln V_{T_2} > \ln F_2) \} . \tag{2.23}
\]

Although \( V_{T_2} \) is not known at time \( t \), it can be specified in terms of \( V_t \)

\[
V_{T_2} = V_t e^{\sigma (W_{T_2} - W_t) + \left( r - \frac{\sigma^2}{2} \right) T_2} . \tag{2.24}
\]

\(^{21}\)Recall, \( N_2 (a, b; \rho) \) refers to the cumulative bivariate standard normal distribution for two jointly distributed standard normal random variables with correlation \( \rho \).
Next, define the new random variable \( Z \triangleq \frac{W_{T_2} - W_t}{\sigma \sqrt{T_2}} \), which is a standard normal random variable; i.e. \( Z \sim N(0, 1) \). Equation (2.24) can be rewritten as

\[
V_{T_2} = V_t e^{(\sigma \sqrt{T_2})Z + \left( r - \frac{\sigma^2}{2} \right) T_2} \tag{2.25}
\]

which implies

\[
\ln V_{T_2} = \ln V_t + (\sigma \sqrt{T_2}) Z + \left( r - \frac{\sigma^2}{2} \right) T_2. \tag{2.26}
\]

This gives the distributional properties of the log-value:

\[
\ln V_{T_2} \sim N\left( \ln V_t + \left( r - \frac{\sigma^2}{2} \right) T_2, \sigma^2 T_2 \right).
\]

Subtracting the mean, dividing by the standard deviation, and combining with Equation (2.18) gives:

\[
\Pr \left\{ (\ln V_{T_1} > \ln V) \text{ } \& \text{ } (\ln V_{T_2} > \ln F_2) \right\} = \Pr \left\{ \left( Y > \frac{\ln \frac{V_t}{V} - \left( r - \frac{\sigma^2}{2} \right) T_1}{\sigma \sqrt{T_1}} \right) \text{ } \& \text{ } \left( Z > \frac{\ln \frac{F_2}{V} - \left( r - \frac{\sigma^2}{2} \right) T_2}{\sigma \sqrt{T_2}} \right) \right\}. \tag{2.27}
\]

Now, let \( X_2 \triangleq -Z \) and multiply all of the terms in Equation (2.27) by \(-1\) to get:

\[
\Pr \left\{ (\ln V_{T_1} > \ln V) \text{ } \& \text{ } (\ln V_{T_2} > \ln F_2) \right\} = \Pr \left\{ \left( X_1 < \frac{\ln \frac{V_t}{V} + \left( r - \frac{\sigma^2}{2} \right) T_1}{\sigma \sqrt{T_1}} \right) \text{ } \& \text{ } \left( X_2 < \frac{\ln \frac{F_2}{V} + \left( r - \frac{\sigma^2}{2} \right) T_2}{\sigma \sqrt{T_2}} \right) \right\}. \tag{2.28}
\]

The right-hand-side of the first inequality is equal to \( h_1^- \) and the right-hand-side of the second inequality is equal to \( h_2^- \), both of which were defined above. Since, \( \ln V_{T_1} \) and \( \ln V_{T_2} \) are jointly distributed normal random variables, the transformations \( X_1 \) and \( X_2 \) follow a standard bivariate normal distribution with correlation \( \rho = \sqrt{T_1/T_2} \). Therefore, Equation (2.28) can be written as

\[
\Pr \left\{ (X_1 < h_1^-) \text{ } \& \text{ } (X_2 < h_2^-) \right\} = \int_{-\infty}^{h_1^-} \int_{-\infty}^{h_2^-} \phi(X_1, X_2; \rho) \, dX_2 \, dX_1 \tag{2.29}
\]

where \( \phi(a, b; \rho) \) denotes the standard bivariate normal density;

\[
i.e. \quad \phi(X_1, X_2; \rho) = \frac{1}{2\pi\sqrt{1-\rho^2}} \exp \left\{ -\frac{1}{2(1-\rho^2)} (X_1^2 + X_2^2 - 2\rho X_1 X_2) \right\}.
\]

The right-hand-side of Equation (2.29) is, by definition, equal to \( N_2(h_1^-, h_2^-; \rho) \), as
The forward default probability is fairly easy to compute once the short-term survival probability and the total survival probability are known. The forward default probability is defined as the probability that the long-term debt obligations will not be satisfied given that the short-term debt obligations have been satisfied, or \( \Pr \{ V_{T_2} \leq F_2 | V_{T_1} > \bar{V} \} \). This is equal to one minus the conditional survival probability, or \( 1 - \Pr \{ V_{T_2} > F_2 | V_{T_1} > \bar{V} \} \). The conditional survival probability is obtained by applying Bayes’ Theorem to the short-term and total survival probabilities, which have already been proved to be \( N \left( h^- \right) \) and \( N_2 \left( h^-_1, h^-_2; \rho \right) \), respectively. It can be shown that the conditional survival probability is \( \frac{N_2 \left( h^-_1, h^-_2; \rho \right)}{N \left( h^-_1 \right)} \).

Proof:

Bayes’ Theorem says that a conditional probability is equal to the joint probability of both events occurring divided by the unconditional probability of the event upon which it is being conditioned. Thus the conditional survival probability is

\[
\Pr \{ V_{T_2} > F_2 | V_{T_1} > \bar{V} \} = \frac{\Pr \left\{ (V_{T_2} > F_2) \ & \ (V_{T_1} > \bar{V}) \right\}}{\Pr \{ V_{T_1} > \bar{V} \}}. \tag{2.30}
\]

The last proof established that the numerator in Equation (2.30) is equal to \( N_2 \left( h^-_1, h^-_2; \rho \right) \); an earlier proof established that the denominator in Equation (2.30) is equal to \( N \left( h^-_1 \right) \). Substituting these into Equation (2.30) gives:

\[
\Pr \{ V_{T_2} > F_2 | V_{T_1} > \bar{V} \} = \frac{N_2 \left( h^-_1, h^-_2; \rho \right)}{N \left( h^-_1 \right)}. \tag{2.31}
\]

Q.E.D.

From mutual exclusivity, the forward default probability must therefore be

\[
DP \left( T_2 \right) = 1 - \frac{N_2 \left( h^-_1, h^-_2; \rho \right)}{N \left( h^-_1 \right)}. \tag{2.32}
\]

This default probability should be interpreted with extreme caution and is much less relevant for the analysis of financial institutions. To see why, recall the key
assumption for the compound option structural model: when debt is due, new equity is issued and the proceeds are used to finance the debt payment. In the case where there are just two classes of debt the degree of leverage in the firm’s capital structure will fall immediately after the first payment is made. This is because that first class of debt will be replaced entirely with equity. This is especially problematic for financial institutions for two reasons. First of all, financial institutions tend to have a lot of short-term, senior debt, so assuming that it will all be paid off by issuing new equity results in a massive, very unnatural drop in leverage for a firm that is typically characterized by high leverage. Second, as mentioned earlier, financial institutions tend to rollover short-term debts. This means that the true forward default probability should not only use the junior, long-term debt in creating the forward default boundary but also some portion of the short-term debts that had been rolled over.

As a result, the forward default probabilities calculated with Equation (2.32) are biased downward. The implicit deleveraging that is built into the compound option model creates the unrealistic impression that bank default risk declines over time. Of course, this is not what is really going on but the static compound option model distorts future expectations along these lines. If, however, the model is periodically recalibrated a more accurate picture of the default risk can be created. Regardless, the short-term default probability, or \( DP(T_1) \) as given by Equation (2.22), is still a valid measure of the financial institution’s default risk since it takes the entire liability structure, as it is now, into account. And because it relies on the endogenous default boundary this is an internally consistent metric for determining the capital needs for a financial institution. In the next subsection the proposed dynamic implementation is described. Each time the model is recalibrated, new inputs are used including the most recent liability structure. In this dynamic framework a financial institution’s default risk, and subsequently the capital requirements, can be analyzed in a way that is consistent with the actual financing strategy (i.e. some of the debts are rolled-over).

\footnote{At this point, the model would collapse to the standard BSM structural model described in Section 2.3.1.}
Dynamic Implementation

There are several aspects of the original [static] compound option model that make it unattractive, especially for analyzing financial institutions. Two are theory problems and one is a practical problem. The first is the issue that was just brought up; namely that immediately after the short-term, senior debt is paid the model takes the financial institution’s leverage to drop dramatically. This leads to the next problem which is more practical in nature. As noted above, in practice financial institutions tend to rollover a good proportion of their outstanding debt at some point. While explicitly modeling this rollover decision (i.e. when, how much, and which debts to rollover, etc.) can be tricky, a good structural credit risk model for financial institutions should account for it somehow. Lastly, there is yet another problem with the static compound option model. Since equity is a compound call option as \( t \) approaches \( T_1 \) the equity value will drop purely from time decay (i.e. “theta”). That is, even if the bank fundamentals stay the same (the asset value, volatility, etc.) there will be a decline of the equity value over time. This implies that the bank’s shareholders have some incentive to arrange for the short-term debt to be rolled-over sometime before \( T_1 \) when the short-term creditors would agree to it.\(^23\) In this paper a dynamic implementation of the compound option structural model is proposed to address these three problems.

In contrast to the static compound option model, the dynamic implementation employed in this paper allows the user to run and recalibrate the model every day (or more often if there is a need) with updated data from the equity markets and about the liability structure of the bank. Every time the model is run, only the existing liabilities at that particular moment are taken into account. This implies that all previous liabilities were retired in some way that is consistent with the bank’s actual funding strategy. As the liability structure changes, the model is recalibrated and adapts to the new liabilities. In this context, the default condition says that as long as the firm can issue new equity at the current market price then it will survive to

\(^{23}\) Presumably this would be when \( DP (T_1) \) is close enough to zero since a creditor is not going to willingly extend the maturity on what they are already owed if it looks like the borrower might default in the near future.
time $T_1$. That does not mean that the bank will issue equity. Upon meeting this survival criterion, it will choose to rollover some proportion of the debts and finance the rest with equity capital. This decision is exogenous to the model and reflects the bank’s actual funding strategy.

Recall from earlier that the dynamic implementation has the interpretation that, as long as default probabilities are sufficiently low as $t$ approaches $T_1$, some or all of the debts will be rolled over before maturity. If default probabilities are not “sufficiently low” within this framework it means that there is very little chance of being able to raise capital to satisfy the pending debt obligations. Thus, the conditions that fresh capital can be raised and that debt can be rolled over are both sufficient and necessary conditions for each other in the model. Furthermore, since there is no notion of liquidity risk in the model, the rollover occurs with probability one.\(^{24}\)

The dynamic implementation is very straightforward: every period, the model is recalibrated and re-run with updated inputs (market information, liability structure, interest rates, etc.). This will give new outputs including the default probabilities. The long-term forward default probability will still be an underestimate, but the short-term default probability is a valid measure since it is a function of the entire liability structure and the endogenous default boundary. Thus, short-term default probability is the key output to focus on when assessing the health of the financial institution in question and in evaluating capital adequacy. Over time, as the financial institution retires some debts and rolls over other debt, the liability structure will change. This new liability structure combined with changing market information will, in turn, results in new default probabilities which may indicate that more equity capital has to be raised in order to ensure the solvency of the financial institution in question. Hence, the model can and should be re-run in this dynamic capacity to periodically assess the capital adequacy of financial institutions in the face of changing market conditions.

The dynamic implementation of the model still captures the fact that critical

\(^{24}\)Recent papers by He and Xiong (2011), Acharya, Gale, and Yorulmazer (2010), and Morris and Shin (2009) develop models that incorporate both credit risk and liquidity risk in an effort to quantify this “rollover risk”.
factor for a bank’s survival is its ability to satisfy the short-term, senior claims. Precisely how the long-term, junior obligations are satisfied is not a direct concern of this model. Rather, these securities play a different role: in the model, as well as in practice, bank subordinated debentures and similar debt instruments have the unique feature of behaving like debt when the firm is far from the default threshold, but behaving like equity when the firm is in distress. That is because when equity has almost no value, the junior debt becomes a residual claim rather than a fixed claim. This is a very important result of the model and has important implications for analyzing bank capital.

It is worth repeating why the short-term default probability is the appropriate metric to use as the recapitalization trigger. First, the short-term default probability depends on the entire liability structure; i.e. it is a function of not only the amount of debt in the capital structure, but the mix of short-term versus long-term debt in funding the bank’s operations. Second, the short-term default probability reflects the endogenous default condition; i.e. asset value falls below the critical level, as described above in Section 3.2.3, rather than some exogenously-specified default rule (such as asset value falling below the face value of debt or some weighted average of short-term debt and long-term debt). Next, it is shown precisely how the model can be used in this dynamic capacity to solve for minimum capital requirements.

**Capital Requirements**

The feature of endogenous default makes the model well-suited to solve for the amount of capital needed to bring default probabilities down to levels that are deemed acceptable by bank managers and/or regulators. This credit-risk-based approach for minimum capital requirements is a unique way to use the most recent data on a bank’s liability structure as well as market information about risk and valuations to analyze capital adequacy. The model is, by definition, forward-looking and therefore avoids many of the inconclusive or erroneous inferences that can be made when relying on backward-looking historical metrics.

The model works in the following way. Every period (daily, weekly, monthly,
quarterly, etc.) default probabilities are computed using the current liability structure and the most up-to-date market information. The regulator specifies the highest acceptable short-term default probability. In this paper no claim is made as to what is an acceptable default probability. It may be 10%, 5%, 1%, or 0.1%, but this is entirely up to the regulators’ discretion. Let this default threshold be represented by \( \alpha \). It has already been shown that the short-term default probability is a function of the asset value, asset volatility, liability structure (size and timing of the promised cash flows), and the risk free rate. This is denoted as \( DP_{T_1} (V, \sigma_V, F_1, F_2, T_1, T_2, r_f) \).

To isolate the effect that a cash infusion has on the default probability, assume that \( V \) and \( \sigma_V \) are the only parameters that change. Therefore, the rest of the arguments can be dropped so that the short-term default probability is denoted as \( DP_{T_1} (V, \sigma_V) \).

The solvency condition as specified by the regulator or internal risk manager is that \( DP_{T_1} (V, \sigma_V) \leq \alpha \). As soon as this condition is broken, or \( DP_{T_1} (V, \sigma_V) > \alpha \), the bank is required to obtain a capital infusion. For now assume that there is a capital infusion comprised entirely of cash and coming from some external source (i.e. dropped from a helicopter, a la Friedman (1969)). The amount is \( C \) such that the newly capitalized bank has total assets valued at

\[
V^* = V + C.
\]  

(2.33)

Since cash has no volatility and is completely uncorrelated with the risky assets in the bank’s portfolio, the new asset volatility can be computed as a weighted average:

\[
\sigma_{V^*} = \left( \frac{V}{V^*} \right) \sigma_V + \left( \frac{C}{V^*} \right) 0 = \left( \frac{V}{V^*} \right) \sigma_V.
\]  

(2.34)

Thus, the cash infusion has two effects: on the one hand, it increases the market value of the bank’s assets by an amount \( \Delta V = C \); at the same time it reduces the total risk of the bank’s portfolio. Both of these, in isolation, will result in lower default probabilities. The combination of increased asset value along with decreased asset risk gives a proverbial “one-two punch” in combating bank distress. The problem of
capital adequacy then reduces to solving the following nonlinear optimization:

\[ C = \inf \{ C : DP_{T_1} (V^*, \sigma_{V^*}) \leq \alpha \} \]  

(2.35)

subject to the constraints in Equation (2.33) and Equation (2.34).

That is, find the smallest \( C \) such that the short-term default probability is below the specified level, \( \alpha \). This can be done numerically and, in fact, a simple search algorithm was used to perform the empirical analysis in Section 5 where the model is used to find the smallest capital infusion that would have been necessary to help stabilize (in terms of reducing short-term default probabilities) the largest bank holding companies in the United States in the midst of the Financial Crisis in 2008.

The reason that an infimum is used in Equation (2.35) is because the conventional wisdom is that banks prefer to have as little equity capital as possible. The existence of minimum capital requirements is a testament to this idea. It has traditionally been argued that capital is the more expensive financing choice and that banks can keep their cost of capital lower by using more debt in the capital structure. However, there is recent evidence against this argument (see Admati, DeMarzo, Hellwig, and Pfleiderer (2010)). An alternative explanation is that too much equity capital runs counter to the essential roles of the bank as an asset transformer and liquidity provider. Regardless, the key point for the purpose of this paper has already been established: banks must maintain some specified level of equity capital to serve as a buffer against distress. Precisely how much is a matter of regulatory policy. But, for any “acceptable” level of default probability, the model provides the technology to determine the smallest capital infusion for controlling bank risk and maintaining solvency without being excessively deleterious to the bank’s core functions.

The way that the model can be used in this capacity is as follows. Suppose that the bank is operating dangerously close to the default boundary. Using information from the markets and given the current liability structure, the model will give short-term default probabilities that are “too high” by some standard. Say that regulators and internal risk managers believe that a short-term default probability of 5% is an appropriate threshold and that the model shows short-term default probabilities
of 20%. The model can be used to solve the optimization problem in Equation (2.35) with $\alpha = 5\%$ to determine how much new capital is needed to bring default probabilities below the 5% level. This is very similar to a dynamic margin requirement that may be imposed on a trader.\textsuperscript{25} Note that whether it is the regulator that issues a mandate requiring a recapitalization or the bank decides to recapitalize itself depends on who is using the model (the regulators or internal risk managers) and in what context (ex-post or ex-ante). The distinction between ex-post or ex-ante recapitalization of financial institutions is developed very nicely in Admati, DeMarzo, Hellwig, and Pfleiderer (2010).

If the model were to be used ex-post it would be as follows. Using market information and given the bank’s current liability structure the model implies that a recapitalization is necessary immediately. In other words, default probabilities are so high that cash must be raised now. It is widely known in the corporate finance literature that, because of the debt overhang problem, it is difficult to convince existing shareholders to contribute capital to a distressed firm. Two possible alternatives are mandatory equity rights offerings or the use of a capital insurance scheme. While both of these options are consistent with the model, perhaps the latter as proposed by Kashyap, Rajan, and Stein (2008) is most promising and, in fact, represents one way to extend the present model. Whereas contingent capital certificates (CCC) could alleviate the debt overhang problem, these securities are not appropriate within this particular context of the model. The reason is that, while the conversion of CCC from debt into equity certainly recapitalizes the bank and insulates it from further deterioration in asset value, it does not bring in new cash which is needed for the objective described by Equation (2.35).

However, if used ex-ante, the model can determine how much capital the bank should have before anything disastrous happens. For instance, the model can be used to perform different scenario analyses. Given the current liability structure, management can determine how much equity the bank should have in the event that asset values fall and/or volatility rises so that short-term default probabilities are

\textsuperscript{25}See Hart and Zingales (2009) for a very good analogy to dynamic margin calls.
above the threshold $\alpha$. Rather than waiting for this to happen, the bank can raise new equity now. Whereas this may be interpreted by the market as a bad signal (as per Myers and Majluf (1984)), contingent capital certificates could potentially be a more appropriate means of raising cash in this ex-ante context.

Lastly, one may make the objection that requiring banks to hold excessive amounts of cash on their balance sheet for an extended period of time would be counterproductive. First of all, in practice it need not be tied up in cash but could be used to purchase Treasury securities or other very low risk, highly liquid instruments. Furthermore, it is important to remember that at this point the model is just being proposed in an abstract manner without the presence of market frictions. This implies that there is no cost associated with raising equity, buying it back, etc. With that being said, once the bank’s default risk falls back down to acceptable levels that same cash can be used to purchase risky assets or make new loans. This allows the bank to grow its balance sheet in a safe and measured manner. In the empirical portion of the paper (specifically Section 6), some of the issues that may arise in the actual implementation of the model will be addressed.

2.4 Data and Methodology

2.4.1 Description of Sample

The empirical study looks at the top 15 American bank holding companies (BHCs) in terms of assets. The sample period covers from 2005:Q1 to 2008:Q4; this was a very interesting time for large banks in the US as it captures the transition from great prosperity to the most dramatic and devastating crisis since the Great Depression. The sample BHCs along with their average assets over the sample period are reported in Table 2.1. The first column lists the BHC name, which is then followed by the average assets in thousands of dollars. The last two columns contain the unique identifiers for the BHCs: the RSSD is the code assigned by the Federal Reserve to all financial institutions and their subsidiaries (and is also the lookup variable for the Bank Holding Company Database (BHCDB) in WRDS) and the PERMCO is the permanent company identifier for CRSP. This mapping was used to link the two
datasets, which will be described in more detail below. It is important to note that these are, in fact, the largest BHCs in the country: the average assets over the sample period are at or above $100Billion. The “smallest” bank in the sample is KeyCorp with average assets of over $97Billion. This is significantly larger than the next bank, which would have been Northern Trust Corporation (RSSD 1199611). With just under $66Billion in average assets Northern Trust falls short of KeyCorp’s average assets by more than $30Billion.

The selection procedure had to be carefully designed so as to capture the most representative sample of the largest BHCs in America. The first step was to collect the Year End Consolidated Assets from the BHC Peer Group Average Reports for 2005, 2006, 2007, and 2008. The Peer Group Average Reports summarize key performance metrics and ratios for BHC Peer Groups. The Peer Groups consist of six different categories based on BHC characteristics, and the reports provide a benchmark for analyzing and comparing a bank to others with similar traits. The first five Peer Groups are by asset size (Peer Group 1 is for BHCs with total assets greater than or equal to $10 billion; Peer Group 5 is for BHCs with less than $500 million in assets). The last one, Peer Group 9, is reserved for “atypical banks”. As it turns out, some of the BHCs in Peer Group 9 happen to be large institutions (in terms of asset size) that are relevant to the study; two particular examples being MetLife and PNC Financial Services Group. The sample therefore was restricted to just Peer Group 1 and Peer Group 9.

For each BHC in Peer Group 1 and Peer Group 9 the average assets were computed for 2005 to 2008. Several requirements were imposed to ensure a consistent and comprehensive data set. First, each BHC had to have four consecutive observations in the BHCDB; i.e. assets reported for December 2005, December 2006, December 2007, and December 2008. Then, each BHC had to be in the “matched sample”, which means that the following two conditions must be met. The BHC had to be a FR Y-9C reporter, as identified by the RSSD number (the FR Y-9C reports will

\[26\]

The specific criteria for being categorized in Peer Group 9 can be found in the User’s Guide for the Bank Holding Company Performance Report available from the Board of Governors of the Federal Reserve System.
be discussed in the next subsection) and the BHC must be publically traded and have continuous stock price data in CRSP over the sample period (1259 stock price observations).

There are a few things to be said about the landscape of the banking industry during the sample period and how they are accounted for in this study. First, and perhaps most notable in the context of this study, is the 2007 merger between Bank of New York (RSSD 1033470) and Mellon Financial (RSSD 1068762) to form the BHC Bank of New York Mellon (RSSD 3587146). Both the predecessor firm, Bank of New York, and the merged firm, Bank of New York Mellon, have the same PERMCO in CRSP allowing for a consistent time series of stock prices. Due to the fact that Mellon Financial is not in CRSP and, in the years prior to the merger, Bank of New York was more than twice the size of Mellon financial (in terms of assets), Bank of New York is used for 2005 and 2006.

Also, Wachovia Corporation (RSSD 1073551) was acquired by Wells Fargo (RSSD 1120754) in 2008. However, since there is not data for the entire sample period, Wachovia is not included in the analysis whereas Wells Fargo is; the RSSD and PERMCO for Wells Fargo are consistent for the entire sample period. Lastly, Washington Mutual Bank failed in 2008. The assets were ultimately sold off to JP Morgan Chase.

2.4.2 Data

Most of the data for this study comes from the FR Y-9C reports, which are submitted quarterly to the Federal Reserve by all bank holding companies (BHCs) with consolidated assets of $500 million or more. The FR Y-9C reports contain very detailed information about the income and condition of bank holding companies, and are intended to parallel the Call Reports, which are submitted to the FDIC by commercial banks. The difference is that the Call Reports are at the individual bank level rather than the holding company level; as such, there can be (and usually are) multiple entities that roll up into the BHC. The two reports do not reconcile even when aggregated due to non-commercial bank subsidiaries that are included in the consolidated figures for the BHCs. Since this study deals with BHCs, the FR Y-9C
reports are the most appropriate source for the bank-specific data needed.

The FR Y-9C reports summarize the bank’s operations in two sections. First is the “Consolidated Report of Income”, Schedule HI, which lists all income, expenses, and accruals, as well as net charge-offs and recoveries on bad loans over a particular quarter. A breakdown of the components of and changes in the BHC equity capital are given in Schedule HI-A. Details regarding the loan loss reserve, including charge-offs and recoveries, are given in Schedule HI-B. Then there is the “Consolidated Balance Sheet”, Schedule HC, which gives a snapshot of the bank’s assets, liabilities, and capital position as of quarter-end. The multitude of supporting sections (Schedule HC-B to HC-S) allow for a much more detailed look at the condition of the BHC.

While most of the data used in this study comes from Schedule HC, some of the more detailed information was used to supplement the basic line items from the balance sheet. Two sections were particularly useful: Schedule HC-M was used to disentangle the appropriate seniority for some ambiguous liabilities and Schedule HC-R was used to define and properly identify the components of regulatory capital.

Line Item #16 in Schedule HC is for “Other Borrowed Money”, which is not entirely clear as to these liabilities’ position in the seniority structure. However, within Schedule HC-M this “Other Borrowed Money” is further broken down into commercial paper, borrowed money not reported elsewhere that is due within one year, and other borrowed money with more than one year left to maturity. Commercial paper (Line Item #14(a) in Schedule HC-M) is included in the data as short-term, senior debt. Line Item #14(b) in Schedule HC-M is other borrowed money that is not reported elsewhere and due within one year. This item is also included the data under short-term, senior debt. It contains repurchase agreements with original maturity of more than one day, transfers of financial assets not considered ‘sales’, funds borrowed from Federal Reserve Banks with maturities under one year, and term federal funds purchased, among others. The last component of “Other Borrowed Money” (Line Item #14(c)) consists of long-term borrowing not reported elsewhere, including mortgages on fixed assets, capital leases, as well as unsecured perpetual debt that are not subordinated. In most cases this item is an insignificant portion of the BHC total
liabilities, and is therefore not included in the data.

In addition, Schedule HC-R contains a detailed breakdown of risk-based regulatory capital. Recall, that as per Basel II there are two forms of regulatory capital: Tier 1 and Tier 2 capital. Tier 1 capital is predominantly comprised of shareholders’ equity plus or minus various adjustments for unrealized and accumulated gains or losses. In addition to common stock and perpetual preferred stock, limited amounts of other qualifying claims are permitted in the calculation of Tier 1 capital. Tier 2 capital includes qualifying subordinated debt, redeemable preferred stock, trust preferred securities, and perpetual preferred stock that was not included in Tier 1 capital.

The actual FR Y-9C reports are available from the National Information Center website at http://www.ffiec.gov/nicpubweb/nicweb/NicHome.aspx. The RSSD ID can be used to search for the specific Bank Holding Company’s records and then the FR Y-9C report can be accessed for any given quarter going back several years. The FR Y-9C data can also be downloaded from the Bank Holding Company Database (BHCDB) in WRDS. This has the added benefit that FR Y-9C data can be downloaded for multiple quarters and/or multiple BHCs in one pass. However, the downside is that the data items are not clearly defined within the BHCDB and so one must still use the actual FR Y-9C report to create a mapping of the variable code to the data item. Table 2.2 shows all of the data items that were used in the analysis. The table lists the data items, the source, and the variable code. In addition to the BHCDB, CRSP and Compustat were used to obtain market data and some supplementary financial statement data for the banks in the sample.

Table 2.2 can be used to show how the model inputs and other relevant metrics were created from the BHCDB downloads. The sum of Item 1 through Item 4 gives the total deposits. Adding Total Deposits with Federal Funds Purchased (Item 5), Repurchase Agreements (Item 6), Commercial Paper (Item 7), and Other Borrowed Money with remaining maturity of 1 year or less (Item 8) gives the face value of the short-term, senior debt or \( F_1 \) in the model. The face value of the long-term, junior debt is taken to be the sum of Item 9 and Item 10, Subordinated Notes and Debentures
and Trust Preferred Securities, respectively; this is $F_2$ in the model. Another important quantity computed from the BHCDB downloads is the Risk-Based-Capital Ratio (RBC Ratio). This is Tier 1 Capital (Item 11) divided by the Total Risk-Weighted Assets (Item 12). The market value of the bank’s equity is obtained from CRSP and is Stock Price (Item 13) multiplied by Shares Outstanding (Item 14). This and equity volatility, which is computed as the three-month rolling standard deviation of daily log-price-relatives times the square root of 252, are the two primary market inputs.\(^{27}\)

Finally, the risk-free rate data is from the Federal Reserve Bank of St. Louis FRED database. The one year Constant Maturity Treasury (CMT) yield was retrieved for every month over the sample period; the data point for the last day of each quarter was then pulled in to serve as the risk-free rate in the model. Monthly yields were downloaded from the FRED database and then matched to the FR Y-9C reports by date.

### 2.4.3 Methodology

This subsection covers the methodology behind running the model. First, the model inputs are specified. This is followed by a discussion about calibration of the model, where an iterative procedure is used to solve two equations in two unknowns. Finally, the model outputs are reviewed with emphasis being placed on two specific measures that are relevant for the analysis of bank capital.

As mentioned earlier in the paper, the model uses both market information and the liability structure of the bank in question. At this point only equity market information is considered; the reason being that every public bank holding company has traded equity with market information readily available and accessible. Specifically, the market information that is used as inputs into the model are the market value of the bank’s equity and the equity volatility, both defined in terms of the data above. For any time $t$ these quantities are denoted as $E_t$ and $\sigma_E$, respectively. Recall that $E_t$ is the value of the compound call option on the bank’s assets, which is known in

\[ \sigma_E = \frac{sd}{\sqrt{\frac{1}{252}}} = sd\sqrt{\frac{252}{1}}, \]

where $sd$ is the standard deviation of $\ln \frac{P_t}{P_{t-1}}$ for each day over the previous three months.

\(^{27}\)In symbols: $\sigma_E = \frac{sd}{\sqrt{\frac{1}{252}}} = sd\sqrt{\frac{252}{1}}$, where $sd$ is the standard deviation of $\ln \frac{P_t}{P_{t-1}}$ for each day over the previous three months.
closed-form as a function of the bank’s asset value and asset volatility. Since the market value of equity and equity volatility are observable, but the true market value of the bank’s assets and asset volatility are unobservable there is something of an inverse problem. Fortunately, the model allows for this problem to be solved rather easily (calibrated using a numerical algorithm as discussed below) and the solution provides some insight about the implied market value of the bank’s assets at any given time.

The other required inputs to the model are from the bank’s liability structure. First, is a measure of the face value of the short-term, senior debt that is outstanding at a particular point in time. This amount, $F_1$, will serve as the first strike price in the dynamic compound option model. The next input is the face value of the long-term, junior debt. This amount, $F_2$, is the sum of the outstanding subordinated notes and debentures and trust preferred securities as described in the previous subsection. $F_2$ is therefore the second strike price in the dynamic compound option model. The endogenous default boundary is, in fact, explicitly a function of the values of these inputs $F_1$ and $F_2$. The model also requires as inputs the average maturities of the outstanding short-term, senior debt and the long-term, junior debt; $T_1$ and $T_2$, respectively. Due to data constraints, this study assumes $T_1 = 1$ and $T_2 = 20$, which implies that the average maturity of the short-term, senior debt is one year and the average maturity of the long-term, junior debt is 20 years. Regulators and internal risk managers could have access to data that can allow for more precise estimation of the average maturity of the debt obligations thereby allowing for more accurate estimation of default probabilities. The last input is the appropriate risk-free rate.\footnote{This results in risk-neutral default probabilities. In order to compute the “true” probabilities under the physical measure, one would have to be able to accurately estimate the risk premium for each bank holding company over the course of the entire sample period. The cost of attempting to incorporate this into the model arguably outweighs the benefits, since it can be shown (mathematically) when the risk premium is non-negative, the risk-neutral default probability will serve as an upper bound on the physical default probability. This will result in more conservative estimates of default probabilities and capital requirements, which is not necessarily bad in this context.}

In the empirical implementation of the model the risk-free rate is set equal to the one year Constant Maturity Treasury (CMT) yield at the same point in time as the FR Y-9C report (i.e. 04-01-2005 for the end of 2005:Q1).

Once the inputs are obtained, the model must be calibrated. The calibration is
necessary because the closed-form solution gives the value of an observable market value (equity) but all of the outputs are in terms of an unobservable quantity: the market value of the bank’s assets. At any given time $t$, the observable equity value and equity volatility, $E_t$ and $\sigma_E$, are related to the unobservable asset value and asset volatility, $V_t$ and $\sigma_V$, by Equation (2.3) and Equation (2.10). Let $E(V_t, \sigma_V)$ refer to the closed-form expression in Equation (2.3). Also let $\sigma_E(V_t, \sigma_V, E_t)$ refer to the expression relating equity volatility to asset volatility given in Equation (2.10). The goal is to simultaneously solve for $V_t$ and $\sigma_V$ in the following set of equations:

\begin{align*}
E(V_t, \sigma_V) &= \text{Equity} \quad (2.36) \\
\sigma_E(V_t, \sigma_V, E_t) &= \text{Volatility} \quad (2.37)
\end{align*}

where $\text{Equity}$ is the market value of equity as computed from the data (Stock Price times Shares Outstanding) and $\text{Volatility}$ is the standard deviation of the daily log-price-relatives multiplied by the square root of 252 (see the end of Section 4.2). For each quarter in the sample period there are 15 values of $\text{Equity}$ and $\text{Volatility}$ corresponding to each of the BHCs in the sample. The iterative procedure to solve for $V_t$ and $\sigma_V$ is therefore run for each of the 240 observations (15 BHCs times 16 quarters) to calibrate the model to the market data.

For each BHC-quarter the iterative procedure goes as follows. First, the initial values are set as $V_0 = \text{Assets}$ and $\sigma_{V0} = \text{Volatility} \ast \left(\frac{\text{Liabilities}}{\text{Assets}}\right)$, where $\text{Assets}$ and $\text{Liabilities}$ are the book values of total assets and total liabilities, respectively, from the bank’s balance sheet; $\text{Volatility}$ is the estimated equity volatility as described in the previous paragraph. Then, $V_t$ is numerically solved so that Equation (2.36) is satisfied. Although the compound call value computed by Equation (2.3) will equal the current market value of the bank’s equity, the value of $\sigma_E$ will change as per Equation (2.10). Consequently, $\sigma_V$ must then be numerically solved so that Equation (2.37) is satisfied. This causes the compound call as a function of the asset value and

\footnote{In the interest of brevity, the other model inputs $- F_1, F_2, T_1, T_2, \text{ and } r$ – have been dropped from the expression.}
new asset volatility to deviate from the market value of equity, so the procedure must be repeated until the results converge. Fortunately, convergence is relatively quick.

Once the conditions given by Equation (2.36) and Equation (2.37) are satisfied for all 240 of the BHC-quarters, the model outputs can be used for analysis. There are many outputs from the model: short-term default probabilities, forward default probabilities, total survival probability, equity delta, implied market value of assets, market value of junior debt, market value of senior debt (discount to par), etc. However, there are two outputs that are particularly valuable for this study. The first is the short-term default probability which is computed in closed-form and given by Equation (2.22). Since the short-term default probability takes the entire liability structure into account (it depends on \( F_1, F_2, T_1, \) and \( T_2 \)) and it is a function of the endogenous default boundary \( \bar{V} \), this is the key output and metric that should be used in determining the appropriate capital requirements. Recall, the objective of the model user (bank regulator or internal risk manager) is to find the minimum amount of capital needed to ensure that the bank will be able to fund and repay the pending debt obligations, considering the size and timing of all future promised payments to creditors. It is, therefore, of utmost importance to know the likelihood that the most imminent obligations will be satisfied. Then, if the short-term default probability is unacceptably high, the model can be used to solve for how much capital should be added to reduce the probability to a more comfortable level.

Another output that is very useful is the implied market value capital (MVC) ratio. This is given by the following equation:

\[
MVC\text{-ratio} = \frac{E_t}{V_t} \tag{2.38}
\]

where \( E_t \) is the market value of the bank’s equity at time \( t \), given by the condition in Equation (2.36), and \( V_t \) is the implied market value of the bank’s assets. It is “implied” from the equity market information and the structural credit risk model. The MVC ratio is very useful in the analysis of bank capital because it looks at the relative size of the equity buffer given the current market conditions. When the market assigns lower values to the bank’s assets and/or there is a lot of uncertainty regarding the
true value of the bank’s assets, it is vital that the equity buffer be sufficient enough to absorb additional losses. This is not to say that capital requirements should be raised during times of distress, which could lead to the undesired procyclical effects that have been a source of criticism of the prevailing bank capital standards. Rather, the MVC ratio helps make the point that capital requirements should not be fixed and should move with market conditions. The idea of time-varying capital requirements and the possibility of procyclicality in the model will be addressed at the end of the paper.

Some will argue that using market values in the management of bank capital will introduce too much noise into the process. It should be noted that the MVC ratios are not necessarily proposed to be used in lieu of RBC ratios but rather in conjunction with the existing measures of bank capital. As will be shown in the next section, when both the MVC ratio and the RBC ratio are analyzed with respect to one another there are many interesting patterns that can be identified. Furthermore, the MVC ratios are not as prone to manipulation or regulatory arbitrage and they provide a way to integrate market discipline into the management and oversight of bank capital.

2.5 Empirical Results and Analysis

After the model was run for all 15 BHCs over the 16 quarter sample period, yielding 240 data points for each output, the results were averaged cross-sectionally. This allows for more general, high-level analysis of the average condition of the largest BHCs both before and during the Financial Crisis. The time-series of cross-sectional averages (default probabilities, capital ratios, etc.) are reported and examined to get a “big picture” perspective of the condition of the largest banks in the financial system over the sample period. The model illustrates that existing regulatory measures fell short precisely when conditions began to really deteriorate in the financial system. At the end of the section a more in-depth analysis is performed on the individual BHC capital positions at the height of the Financial Crisis.

The starting point in analyzing the results is the short-term default probabilities.
Recall, that these are the most important output regarding the health of the financial institution in question. The short-term default probabilities are, by definition, forward-looking and take the entire liability structure into account. Hence, they provide an objective, quantitative measure of the solvency of the financial institution using the most recent and relevant information. Taking the cross-sectional average across the 15 BHCs in the sample gives rise to a time-series of default probabilities. Figure 2.3 plots this time-series of average short-term default probabilities over the sample period. It is interesting to see that short-term default probabilities for the 15 largest BHCs were close to zero up until the latter half of 2007. This is when the infamous “subprime crisis” began to incite panic throughout the country. Then, throughout 2008, when the full-blown financial crisis permeated the entire financial system, bank default probabilities climbed consistently and aggressively. By December 2008, when the sample period ends, average short-term default probabilities spiked at almost 50%. A crude interpretation of this result is that, on average, the probability that one of the nation’s largest banks would fail within one year was akin to flipping a fair coin!

The model uses market information, namely equity values and volatility, along with each BHC’s liability structure to compute the default probabilities and implied market value capital ratios. As such, it must be the case that any changes in the market value capital ratios over time are going to come from changes in one or more of those factors: changes in the market’s perception of value, the degree of uncertainty, and/or the mix of short-term and long-term debt in the bank’s financing. Intuition suggests that the market value capital ratios should be negatively correlated with short-term default probabilities, especially considering that they are both generated by the same structural credit risk model.

Figure 2.4 shows the correlations between the short-term default probabilities and the market value capital ratios for each BHC in the sample. All but one BHC exhibit high negative correlations between these two metrics. The lone BHC with a

---

30See Crouhy, Jarrow, and Turnbull (2008) and Gorton (2009) for excellent accounts of the causes and extreme ramifications of the “subprime crisis”.

positive correlation is MetLife (MET), with correlation of approximately 65%. By drilling down and looking at the data for MetLife more carefully, it can be seen that a spike in MetLife’s short-term default probabilities late in 2008 comes purely from volatility. The market value of equity dropped but the implied market value of assets fell by proportionately more; and at the same time, the total face value of MetLife’s senior, short-term debt obligations ($F_1$) was reduced indicating some deleveraging over this period. Meanwhile equity volatility for MET almost doubled from 2008:Q3 to 2008:Q4. Thus, it can be concluded that the large increase in MetLife’s short-term default probabilities was a result of widespread uncertainty about large financial institutions. This shows how a particular anomaly in the results can be explained by looking at which of the factors (i.e. model inputs) are behind it.

The market value capital ratios implied by the model are computed as follows. For each quarter the market value of equity is observed. The market value capital ratio would be equal to this quantity divided by the market value of the bank’s assets. However, as noted earlier, the market value of bank assets are unobservable. Fortunately, by solving two equations in two unknowns, the structural credit risk model gives the implied market value of the assets, conditional upon the market information and liability structure.\(^{31}\) Thus, the implied market value capital ratio, henceforth referred to as the MVC ratio, is computed by dividing the market value of the BHC’s equity ($E_t$) by the model-implied asset value ($V_t$). The risk-based capital ratio, or RBC ratio, is computed directly from the FR Y-9C data. It is, in fact, the “core capital” ratio which is Tier 1 Capital divided by Total Risk-Weighted Assets.

From Figure 2.5, it can be seen that on average, the MVC ratios of the BHCs declined while the RBC ratios stayed fairly constant, and even increased slightly, over the sample period. This pattern actually has some very profound regulatory implications. When there is convergence in the MVC and RBC ratios, there are two [not mutually exclusive] conclusions that one may draw: the condition of the bank in question has deteriorated so severely that the market value of its common equity has fallen to levels such that it is in line with the book value of core capital relative

\(^{31}\)This iterative procedure was described above in Section 2.4.3.
to their respective measures of assets and/or the RBC ratios have been manipulated so as to give off the impression that the bank is healthier than it really is. The most convenient way to manipulate the RBC ratio is by moving some risky assets off of the balance sheet, in a type of “regulatory capital arbitrage”.

It is rather alarming to see that while the RBC ratios hovered around 8% to 9%, which by regulatory standards is “well-capitalized”, the MVC ratios declined steadily from well over 20% to half of their highest levels. As pointed out above, these results can be coming from one of three primary factors: decreasing valuations (the market value of equity will decrease by more, in relative terms, and at a faster rate than the market value of the assets, from convexity in the compound call option value), increasing volatility, and/or increasing leverage. This last factor (increasing leverage) can be ruled out simply by looking at the BHCs’ leverage ratios and noting that, on average, there was very little variation in the degree of leverage over the sample period. Figure 2.6 plots four different measures of leverage using the collected data. Again, the reported ratios are cross-sectional averages from the 15 BHCs, to obtain a time series of leverage ratios over the sample period. The first ratio (TL/TA) is total liabilities over total assets, which captures the overall degree of leverage for the banks. This ratio remained very constant at about 0.9 over the sample period. The next ratio (Dep/TA) is total deposits over total assets, which shows the extent to which the banks rely on traditional deposits as a source of funding. Although there was more variation in the ratio of total deposits to assets, this measure of leverage was still fairly constant over the sample period staying between 0.5 and 0.6. The last two ratios (Sr/TA and Jr/TA) show the amount of senior debt and junior debt, respectively, as percentages of total assets. As would be expected for a financial institution, a greater proportion of the banks’ debt funding comes from the short-term, senior debt as opposed to the long-term, junior debt. Both of these ratios are also quite stable on average for the 15 BHCs.

Since leverage ratios remained fairly constant for the sample BHCs over this period, that can be ruled out as a factor for the declining MVC ratios. Rather, the falling MVC ratios can be attributed to a complex mix of increasing volatility and
decreasing market valuations. This is evidence of market discipline at work and could potentially have provided a good opportunity for regulators and market participants to unite in combating the financial crisis.

The third quarter of 2008 (2008:Q3) represents the first point in the sample period where all 15 BHCs had short-term default probabilities above 10%. While this might be a bit alarming it is not surprising. It was during this time that Lehman Brothers failed, AIG had to be bailed-out, Fannie Mae and Freddie Mac went into conservatorship, and the Troubled Asset Relief Program (TARP) was announced. Since all 15 BHCs exhibited elevated short-term default probabilities, 2008:Q3 provides a valuable opportunity to examine the individual bank capital positions in the sample and to implement the model as a tool for estimating capital requirements. The model can be used to solve for the minimum capital infusion that would have been needed to reduce each BHC’s default probability below 5%, as proposed earlier. It should be noted that the decision to set \( \alpha = 5\% \) is entirely arbitrary; the same procedure can be done to obtain results for different \( \alpha \)-levels. Also, it is important to reiterate that the “helicopter drop” approach is used in that no claim is made about where the capital infusion should come from. This is left open for future research. The goal here is to determine how much capital each BHC should have had in 2008:Q3 to keep default risk under control. Finally, it is assumed that the entire capital infusion is in the form of cash. Since cash has zero volatility there are two effects on the bank: first, asset values increase and, second, the weighted-average asset volatility decreases. The relationship is highly nonlinear, which is captured by the model.

Figure 2.7 and Figure 2.8 plot the capital infusions as computed by the model. To control for size the results are scaled by the book value of assets. The model results are given by the dark, solid bar. For each bank the model results are compared to three other numbers. The first is the amount of TARP funds received by each BHC at the end of 2008; if the BHC did not receive any TARP funds, then this number is zero. The second number is the result of the Supervisory Capital Assessment Program (SCAP), better known as the “stress tests” conducted by the Federal Reserve Board of Governors in early 2009. The SCAP results indicated how much capital was needed
by each BHC at the end of 2008 according to macroeconomic simulations, pro-forma forecasting, and the existing risk-based capital requirements. All 15 BHCs in the sample were included in the SCAP stress tests. The “SCAP Buffer” that was reported in the press was the amount of capital needed at the end of 2008 as indicated by the stress tests less any capital actions taken in the first part of 2009. The numbers used to construct Figure 2.7, and in the resulting analysis, are the amounts before accounting for any capital actions; this allows the comparisons to be consistent—i.e. the amount of capital needed at the end of 2008. The last number is the sum of the TARP and SCAP amounts.

Figure 2.7 shows the full sample of all 15 BHCs. According to the model, all but one BHC needed more capital than TARP and SCAP combined. The model indicated that PNC Financial Services Group actually needed less capital than the sum of TARP and SCAP; in fact, the model estimate is very close to what was distributed to the bank through TARP. However, the other 14 BHCs seem to have still been undercapitalized even after the programs that were set up to restore financial system stability. This could help explain why the Financial Crisis persisted for so long, even after the very costly government intervention, and why the economy did not experience the V-shaped recovery that so many predicted. The answer undoubtedly lies, in part, with capital requirements: banks should have had more capital on their balance sheet to begin with (this is the ex-ante approach described earlier), or should have been required to bolster capital positions more aggressively, ex-post.

There are two outliers in the sample: Regions Financial and Fifth-Third Bancorp. The results imply that these two BHCs should have had over 40% of the balance sheet as cash. This is completely unrealistic and even counterproductive for any bank, especially in times of distress. What that says, however, is that clearly something else was going on with these particular BHCs. Extreme results such as these should be interpreted as evidence that a wider variety of tools, in addition to managing capital, would have to be used to alleviate the financial distress for such a bank (perhaps asset sales). Even though the capital estimates may be hard to justify for these extreme cases, the model is still useful in identifying sever problem banks. The case-in-point:
these two BHCs, Regions Financial and Fifth-Third Bancorp, were two of the worst performers throughout the first half of 2009 with their stock prices falling to as low as 65% and 86% of the 2008 year-end levels, respectively. Figure 2.8 shows the restricted sample with the two outliers removed, and puts the capital needs of the remaining BHCs in better perspective.

2.6 Conclusion

This paper proposed a dynamic approach to estimating and analyzing bank capital requirements with a structural credit risk model. The approach has many features that make it attractive relative to the existing methods for determining the amount of capital that banks should have. There are three main innovations associated with this new model for bank capital. First, the structural credit risk model takes both sides of the balance sheet into account: both the asset risk and the liability structure (both in terms of maturity and seniority). The prevailing paradigm focuses on asset risk and practically ignores the liability structure. Second, the model uses market information. This allows for a practical way to link market discipline and capital requirements that is consistent with economic theory. Finally, the model is, by definition, forward-looking. This is in contrast to the existing static measures that use historical data.

The model gives rise to many interesting insights about financial institutions, both theoretically and empirically during the recent Financial Crisis. The model captures the complex, nonlinear relationship between volatility, default probabilities, and bank capital. It also allows for the computation of an implied market value capital ratio. Since the true value of a bank’s assets is unobservable it has been very difficult to be able to analyze the market value of equity capital relative to some market measure of asset value. However, the model uses equity market information to calculate the implied market value of the bank’s assets at a point in time. The implied market value capital ratios can then be compared with the risk-based capital ratios to identify patterns indicating that a bank might be on the verge of distress. Actions can then be taken to recapitalize, ex-ante, to correct any capital deficiencies and prepare for any future deterioration in asset values or an increase in market-wide uncertainty. If
the bank waits too long, the model can still be used ex-post to determine how much capital would have to be raised immediately to reduce default probabilities. This, however, raises some additional issues that was not dealt with in the current paper.

There is, of course, the issue of debt overhang in recapitalizing financial institutions, especially during times of crisis. Fortunately, the model allows for the quantification of the debt overhang effect very nicely. The model does not allow for asset sales, which could be one additional tool that a distressed bank could use when undercapitalized and in the midst of a crisis. However, the risk of fire sales and liquidity spirals arises. Again, these are beyond the scope of the current model. Future research should also consider how distressed banks should raise fresh capital. The idea of capital insurance seems promising and is consistent with the current model. There is also the potential for the use of contingent capital certificates which is consistent with the model being used in an ex-ante capacity.

The model could potentially be expanded to incorporate market frictions such as liquidity, explicit distress costs, and asymmetric information. However, the inclusion of any one of these market frictions would be extremely nontrivial and it was shown that, even in this primitive form, the model provides significant insight into the management and analysis of bank capital positions.

In conclusion, the model highlights the fact that static measures of capital adequacy, such as a fixed percentage of assets, are arbitrary and do not capture the true condition of a financial institution. Rather, a dynamic approach that is forward-looking and tied to market conditions is more appropriate in setting capital requirements. Furthermore, the structural credit risk model developed in this paper demonstrates how market information and the liability structure can be used to compute default probabilities which can then be used as a reference point for setting capital requirements.
Table 2.1: Bank Holding Companies in the Sample, Ranked by Average Assets

<table>
<thead>
<tr>
<th>BHC Name</th>
<th>Average Assets ($000)</th>
<th>RSSD</th>
<th>PERMCO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Citigroup, Inc.</td>
<td>1,876,114,000.00</td>
<td>1951350</td>
<td>20483</td>
</tr>
<tr>
<td>Bank of America Corporation</td>
<td>1,575,188,544.25</td>
<td>1073757</td>
<td>3151</td>
</tr>
<tr>
<td>JPMorgan Chase</td>
<td>1,571,915,250.00</td>
<td>1039502</td>
<td>20436</td>
</tr>
<tr>
<td>Wells Fargo</td>
<td>712,204,500.00</td>
<td>1120754</td>
<td>21305</td>
</tr>
<tr>
<td>MetLife, Inc.</td>
<td>517,400,453.50</td>
<td>2945824</td>
<td>37138</td>
</tr>
<tr>
<td>U.S. Bancorp</td>
<td>233,336,000.00</td>
<td>1119794</td>
<td>1645</td>
</tr>
<tr>
<td>SunTrust Banks, Inc.</td>
<td>182,646,586.00</td>
<td>1131787</td>
<td>21691</td>
</tr>
<tr>
<td>Bank of New York Mellon</td>
<td>160,275,750.00</td>
<td>3587146</td>
<td>20265</td>
</tr>
<tr>
<td>PNC Financial Services Group, Inc.</td>
<td>155,978,908.75</td>
<td>1069778</td>
<td>3685</td>
</tr>
<tr>
<td>Capital One Financial Corporation</td>
<td>138,736,129.00</td>
<td>2277860</td>
<td>30513</td>
</tr>
<tr>
<td>State Street Corporation</td>
<td>131,237,511.50</td>
<td>1111435</td>
<td>4260</td>
</tr>
<tr>
<td>Regions Financial Corp</td>
<td>128,863,667.50</td>
<td>3242838</td>
<td>1620</td>
</tr>
<tr>
<td>BB&amp;T Corporation</td>
<td>128,788,362.50</td>
<td>1074156</td>
<td>4163</td>
</tr>
<tr>
<td>Fifth Third Bancorp</td>
<td>109,154,909.50</td>
<td>1070345</td>
<td>1741</td>
</tr>
<tr>
<td>KeyCorp</td>
<td>97,426,130.75</td>
<td>1068025</td>
<td>2535</td>
</tr>
</tbody>
</table>
Table 2.2: Data Items Used in the Empirical Study of Bank Capital

<table>
<thead>
<tr>
<th>Data Item</th>
<th>Source</th>
<th>Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Non-interest-bearing deposits in domestic offices</td>
<td>BHCDB</td>
<td>BHDM6631</td>
</tr>
<tr>
<td>2 Interest-bearing deposits in domestic offices</td>
<td>BHCDB</td>
<td>BHDM6636</td>
</tr>
<tr>
<td>3 Non-interest-bearing deposits in foreign offices</td>
<td>BHCDB</td>
<td>BHFM6631</td>
</tr>
<tr>
<td>4 Interest-bearing deposits in foreign offices</td>
<td>BHCDB</td>
<td>BHFM6636</td>
</tr>
<tr>
<td>5 Federal Funds Purchased</td>
<td>BHCDB</td>
<td>BHDMB993</td>
</tr>
<tr>
<td>6 Repurchase Agreements</td>
<td>BHCDB</td>
<td>BHCKB995</td>
</tr>
<tr>
<td>7 Commercial Paper</td>
<td>BHCDB</td>
<td>BHCK2309</td>
</tr>
<tr>
<td>8 Other Borrowed Money with remaining maturity of 1 year or less</td>
<td>BHCDB</td>
<td>BHCK2332</td>
</tr>
<tr>
<td>9 Subordinated Notes and Debentures</td>
<td>BHCDB</td>
<td>BHCK4062</td>
</tr>
<tr>
<td>10 Trust Preferred Securities</td>
<td>BHCDB</td>
<td>BHCKC699</td>
</tr>
<tr>
<td>11 Tier 1 Capital</td>
<td>BHCDB</td>
<td>BHCK8274</td>
</tr>
<tr>
<td>12 Total Risk-Weighted Assets</td>
<td>BHCDB</td>
<td>BHCKA223</td>
</tr>
<tr>
<td>13 Stock Price</td>
<td>CRSP</td>
<td>PRC</td>
</tr>
<tr>
<td>14 Shares Outstanding</td>
<td>CRSP</td>
<td>SHROUT</td>
</tr>
<tr>
<td>15 Total Assets</td>
<td>Compustat</td>
<td>ATQ</td>
</tr>
<tr>
<td>16 Total Liabilities</td>
<td>Compustat</td>
<td>LTQ</td>
</tr>
</tbody>
</table>
Figure 2.1: Simplified balance sheet for a typical banking firm

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities &amp; Equity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loans &amp; Investment Securities</td>
<td>Short-term, Senior debt</td>
</tr>
<tr>
<td></td>
<td>Long-term, Subordinated debt</td>
</tr>
<tr>
<td></td>
<td>Shareholders’ Equity</td>
</tr>
</tbody>
</table>
Figure 2.2: State-contingent payoffs and valuation in the compound option model

$$V_i > P$$

$$V_i < P$$

<table>
<thead>
<tr>
<th>Time</th>
<th>Node</th>
<th>Equity</th>
<th>Junior</th>
<th>Senior</th>
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</thead>
<tbody>
<tr>
<td>0</td>
<td>A</td>
<td>$E[e^{-rT} \max {E_i - F_i, 0}] V_0] = E_0$</td>
<td>$E[e^{-rT} \left( J_i \cdot I_{[V_i \geq F_i]} \right) + \left( \max {V_i - F_i, 0} - I_{[V_i \geq F_i]} \right) V_0] = J_0$</td>
<td>$E[e^{-rT} \min {V_i, F_i}] V_0] = S_0$</td>
</tr>
<tr>
<td>$T_1$</td>
<td>B</td>
<td>$E[e^{-r(5-T_1)} \max {V_i - F_i, 0}] V_{T_1} = E_{T_1}$</td>
<td>$E[e^{-r(5-T_1)} \min {V_i, F_i}] V_{T_1} = J_{T_1}$</td>
<td>$F_i$</td>
</tr>
<tr>
<td>C</td>
<td>$C_i$</td>
<td>0</td>
<td>$V_i - F_i$</td>
<td>$F_i$</td>
</tr>
<tr>
<td></td>
<td>$C_2$</td>
<td>0</td>
<td>0</td>
<td>$V_i$</td>
</tr>
<tr>
<td>$T_2$</td>
<td>D</td>
<td>$V_i - F_i$</td>
<td>$F_i$</td>
<td>$N / A$ (Paid)</td>
</tr>
<tr>
<td></td>
<td>E</td>
<td>0</td>
<td>$V_i$</td>
<td>$N / A$ (Paid)</td>
</tr>
</tbody>
</table>
Figure 2.3: Short-Term Default Probabilities
Figure 2.4: Correlation Between Default Probability and Market Value Capital Ratio
Figure 2.5: Capital Ratios: Risk-Based Capital vs. Market Value Capital
Figure 2.6: Average Leverage for the Sample Bank Holding Companies
Figure 2.7: Capital Infusions (as % of Book Value of Assets) – Full Sample
Figure 2.8: Capital Infusions (as % of Book Value of Assets) – Restricted Sample
Chapter 3
Endogenous Modeling of Bank Default Risk

3.1 Introduction

The recent financial crisis has demonstrated the pressing need for new tools to both measure and manage the risks of financial institutions. Central banks and regulators are grappling with the need for a quick and accurate method to anticipate bank default in rapidly changing market conditions; an effort that is all-the-more crucial for distressed institutions. Existing regulations focus almost entirely on the bank’s asset risk (see e.g. Basel Committee on Banking Supervision (2001) and Altman and Saunders (2001)), practically ignoring the endogenous nature of the financial institution’s own credit risk which is quite naturally a function of its leverage and market valuations of its asset portfolio. In this paper, we develop a new, easily-implementable methodology to characterize a financial institution’s default risk. Our model uses equity market information to make a forward-looking assessment about the credit risk of financial institutions. This incorporates the recommendations of Flannery (1998) and Krainer and Lopez (2004) who have suggested that market information, especially equity market information, should be included in the oversight of financial institutions. We illustrate the model’s ease of use and power by employing it to analyze the evolution of default probabilities for Lehman Brothers during the 2008 Financial Crisis.

We propose a lattice based structural credit risk model as the solution to estimating default risk in financial institutions. To date, implementation of structural

1Structural credit risk models take a contingent claims approach to estimating default risk. This approach was pioneered by Black and Scholes (1973) and Merton (1974) who modeled equity as a call option on a firm’s assets. The basic insight behind structural credit risk models is that the probability that the call option expires out-of-the-money is a measure of the firm’s default probability. These models use the market value of equity and its volatility as key inputs, thereby allowing for a market information based estimate
credit risk models have mostly excluded financial institutions and focused on analyzing default risk of industrial firms. This is largely due to the size and complexity of the liabilities of financial institutions, which makes calibration extremely difficult or requires simplifying assumptions that limits applicability. The liability structure of financial institutions also makes it important to control for the endogeniety of default. Those structural models that oversimplify the liabilities and use an exogenously specified default condition – i.e. an exogenous default boundary as in Longstaff and Schwartz (1995) – can dramatically distort estimates of credit risk when the liability structure is complex. The endogenous default boundary in our model allows for default to be a function of firm value and its liability structure and thereby allows us to extend the application of structural credit risk models to the estimation of default risk in financial institutions.

Our model is flexible and can incorporate alternate capital structure policy assumptions. Endogenizing the default boundary in a structural credit risk framework requires implicit and explicit assumptions about how the firm finances maturing debt. Two extreme cases have been solved in the literature. The Leland (1994) model, and its extensions, assumes that the firm replaces the maturing debt with an identical new debt, which results in constant leverage. Such models, which assume that firms rollover debt, capture some elements of the real world in which financial institutions continue to operate with high degrees of leverage. On the contrary, models based on the Geske (1977) compound option pricing model assume that the firm issues equity to fund maturing debt and implicitly de-leverages the firm. The de-leveraging assumption becomes more relevant during a financial crisis when a financial institution is not able to access debt markets to fund maturing debt or the firm is required to raise equity capital by regulators. It is also plausible that financial institutions will gradually reduce the amount of leverage in a financial crisis and replace some of the

caption of default risk. Similar approaches have been employed in the banking literature with respect to risk-based deposit insurance (see, for example, Merton (1977), Marcus and Shaked (1984), Ronn and Verma (1986), and Pennacchi (1987a)). Pennacchi (1987b) and Cooperstein, Pennacchi, and Redburn (1995) extend the approach to allow for multi-period analysis. Allen and Saunders (1993) take forbearance into account and model deposit insurance as a callble put option.

See for example (Bharath and Shumway 2008) who state that “We exclude financial firms (SIC codes 6021, 6022, 6029, 6035, 6036) from the sample” (p.1351).
their maturing debt with equity. Financial institutions, faced with declining market values due to rapidly diminishing liquidity and increasing volatility, typically look to raise equity to hedge against expected losses. This was the case with several financial institutions, e.g. Bear Stearns, Lehman Brothers, etc., during the 2008 crisis period. Our model is flexible enough to allow for these and other capital structure policy assumptions.

It is important to emphasize two features of our model that make it particularly useful for the analysis of financial institutions’ default risk during a financial crisis. First, our model uses market information and can therefore incorporate rapidly evolving market conditions into the analysis. This is especially important during a financial crisis as circumstances can change dramatically over the course of a day. Second, the flexibility of specifying alternate ways to fund maturing debt allows us to address alternate market conditions in assessing default probabilities. Note that the ability of a financial institution to fund maturing debt is endogenous and is independent of the source of funding. That is the alternate capital structure assumptions preserve the endogenous nature of the default boundary in our model.

We apply our model to estimate the term structure of default probabilities for Lehman Brothers during the Financial Crisis of 2008 under two assumptions: debt-rollover and de-leveraging. We use data from FactSet to create a detailed picture of the liability structure at the end of each month from December 2007 to August 2008. We also use the market value of equity at month end and historical equity volatility to estimate the market value of assets and asset volatility. We are then able to compute Lehman Brothers’ default probability curves under both assumptions for every month leading up to the firm’s bankruptcy filing in September 2008.

Not surprisingly, we discovered that during the period of crisis for Lehman Brothers, assuming debt rollover yields a much higher default probability curve than assuming de-leveraging. If Lehman had de-levered quickly and substantially, its default probabilities would have dropped to a fraction of what rollover strategy would have indicated. Lehman Brothers did, in fact, raise equity of $4 billion in April and expressed an additional $6 billion in June. But that was too little and too late.
Our default risk analysis shows that in the month of March, the 1-year default probability soared under both rollover and de-leveraging assumptions. After the $4 billion capital infusion, the 1-year default probability came down. Then until September, had Lehman followed the de-leveraging strategy, our analysis shows that its default probability would have been relatively low (at about 20% level). Yet, the rollover assumption would indicate an increase in default probability (at a peak of 80% in June). Indeed, Lehman Brothers reacted too late and took too long to continue to raise equity and ultimately default became inevitable.

Our analysis further shows that it would have been most effective for Lehman to raise equity capital in March when the default probability soared suddenly. According to our model, Lehman should have substantially de-levered by roughly $13 billion and then continue to gradually de-lever over time; then Lehman could have likely avoided default.

The anatomy of the events surrounding the crisis at Lehman Brothers parallels the assumptions made in our model. We see no evidence of a decrease in asset levels at Lehman Brothers over this time period. This is consistent with the notion that illiquidity makes it difficult for Lehman to use asset sales to pay down debt and rely exclusively on capital market transactions. Lehman Brothers raised equity in the first half of 2008 in an attempt to recapitalize and de-lever, again consistent with our assumption that financial institutions issue equity to raise capital during times of distress. Finally, we note that equity value and equity volatility change rapidly in the crisis period. It is therefore important to incorporate market information into the analysis of default risk.

Although this analysis is an ex-post analysis, provided in “hind-sight”, it does demonstrate how important it is for the managers and shareholders of a financial institution to work to avoid default; and more importantly, for the regulators to understand the importance of maintaining the right level of adequate capital and the timing of capital infusion. We do note that the analysis is based upon economic default and does not take into account the liquidity squeeze that also played a crucial role in the past crisis.
Our analysis sheds light on capital structure in the financial industry. This can only been seen in a multi-period model such as ours and not in the single period Merton model. This is because in a single period, there is no place to allow debt rollover and de-leveraging. Our model which is based upon the option-theoretical framework pioneered by Geske (1979) and Leland (1994) provides for the first time an endogenous and structural view of credit risk in the financial industry. Furthermore, we are the first to explicitly consider the impact of refinancing assumptions on default risk, which is of critical importance.

While the level of default probabilities differ across the different capital structure policy assumptions, the pattern of default probabilities are similar. We find that market uncertainty, in addition to the market value of equity, plays a very important role in determining default risk. An increase in equity volatility reflects an increase in asset volatility. Default probability increases as a function of asset volatility, which follows from standard contingent claims models where the probability of the call option expiring out of the money increases with volatility.

Keeping in mind that early estimates of default probability are especially important for regulators, we argue that the lower bound default probability estimates from the model with the de-leveraging assumption can be a very useful tool. An increase in the lower bound estimate of default probability can be used to flag unacceptably high default risk for the financial institution. Lehman Brothers raised equity capital of $4 billion in April 2008. Markets reacted positively as evidenced by its increasing stock price, resulting in a higher market value of equity, and lower levels of volatility. This, in turn, reduced default probabilities in April and May of 2008. Another attempt to raise equity capital in June 2008 proved much less effective; we see that default probabilities rose again to unacceptably high levels in the subsequent months through August 2008. The markets clearly anticipated the crisis at Lehman Brothers well before the firm actually failed. Our model thus shows that the one-time capital infusion was not sufficient to reduce Lehman Brothers’ default risk and that Lehman

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3According to SEC filings and press releases, Lehman Brothers raised $4 billion of perpetual convertible preferred stock in April 2008 and $2 billion of mandatory convertible preferred stock in addition to $4 billion of common stock in June 2008.
Brothers required a substantial additional restructuring of its debts and assets in order to reduce default risk.

The rest of the paper is as follows. Section 2 reviews the relevant literature. Section 3 provides an overview of the model framework and uses a numerical example to help develop the intuition. Section 4 presents the liability structure and a timeline of events for Lehman Brothers. Section 5 develops estimates of default risk and clearly shows that our model can predict the higher default risk and demonstrates the need for equity capital. Section 6 presents our conclusions and the policy implications of our work.

3.2 Related Literature

Since our analysis examines the recent financial crisis from the perspectives of leverage and default risk, we focus on the literature that examines financial institutions during the 2008 financial crisis, as well as the use of market information for analyzing risk in financial institutions, and the development of structural models. Several recent papers document, examine, and explain the role of leverage and credit risk in perpetrating the financial crisis. Shin (2009) profiles one of the first victims of the crisis, the UK based Northern Rock Bank in September of 2007. He describes a new type of bank run; one that is driven by traditional liquidity issues and maturity mismatch and further exacerbated by high leverage. Excess leverage, illiquidity, and credit risk went on to affect nearly every financial institution. Duffie (2010) shows that large non-depository financial institutions were brought down by runs from capital market creditors and counterparties (e.g. Bear Stearns). For an excellent discussion of liquidity and the credit crisis please see Brunnermeier (2009).

Crouhy, Jarrow, and Turnbull (2008) provide an in-depth look at the flawed mechanisms driving the crisis, from subprime securitization to capital requirements, and even the role of ratings agencies. Hull (2009) argues that the crisis was perpetuated by multiple layers of securitization and the inability of market participants to fully evaluate the risks of these structured products. Gorton (2009) explains how the events went from a relatively localized debacle involving derivatives on subprime mortgages
to a systemic crisis that engulfed the global financial system. Stulz (2010) looks at the role of credit default swaps in the recent financial crisis and concludes that the derivative contracts did not cause the crisis. Rather they proved to be something of a double-edged sword; they led to increased uncertainty in financial markets, but at the same time provided powerful hedging tools for financial institutions. In an examination of the CDS-bond basis over the crisis period, Bai and Collin-Dufresne (2010) use market data to estimate a term structure of default probabilities. Although their work focuses on different issues than ours, it highlights the importance of using market information in assessing the default probability of financial institutions.

Measuring and managing risk is a central tenet of sound bank management and regulation. The traditional approaches for quantifying bank risk have been primarily asset-driven. These static approaches proved to be inadequate during the fast-moving financial crisis and literature has been increasingly critical. For instance, looking at commercial banks, Berkowitz and O’Brien (2002) empirically document that, in practice, the traditional Value-at-Risk measures do not perform well. Jorion (2009) notes that the recent financial crisis exposed several weaknesses in the existing architecture for financial risk management. Stulz (2008) looks more generally at risk management failures over the past decade and argues that relying on static measures that use historical data is problematic. Also, looking strictly at market risk is too narrow of an approach for a financial institution. Adrian and Brunnermeier (2009) and Adrian and Shin (2011) examine more dynamic and comprehensive approaches to measure bank risk taking into account leverage and the underlying liability structure.

Several researchers have called for the use of market information in evaluating the extent to which banks take excessive risks. Flannery (1998) surveys the literature to evaluate how well market participants are able to assess the financial condition of banks. He concludes that bank supervisors should regularly incorporate market information in an effort to provide the most comprehensive oversight system. Krainer

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and Lopez (2004) specifically advocate the use of equity market information in this capacity.

Our approach represents a liability-driven market-information-based estimation of default risk and overcomes the shortcomings of the static approach. Our model is based on the compound option approach and falls into the class of structural credit risk models. Structural models have been commonly used for predicting default probabilities for non-financial firms. These models view default as an economic event where firm value declines to a level that is too low to justify servicing outstanding debt obligations. The concept originated with Black and Scholes (1973) in their seminal option pricing paper, where they noted that when a firm has debt in its capital structure, equity is like a call option on the firm’s unlevered assets. This idea was later formalized by Merton (1974). If the value of the firm’s assets is not greater than the face value of the debt then shareholders choose to let the call option expire (i.e. default) and bondholders do not receive their promised payment but rather take ownership of the assets. In the Black-Scholes-Merton (BSM) framework, the probability that the call option is not exercised at maturity is thus a measure of default probability. Structural models use market information, specifically the market value of equity and equity volatility, as key inputs to arrive at default probability.

The KMV model, as described by Crosbie and Bohn (2001), is a modified version of the original BSM model that is very popular in practice. In order to adhere to the single-period European option framework, the KMV approach reduces the entire liability structure of a firm to a linear combination of two points — short-term and long-term debt. This approach thus assumes an exogenously specified simplistic default condition. Many extensions have been developed looking to incorporate more realistic features of debt. One class of extensions, commonly referred to as “barrier” structural models, was pioneered by Black and Cox (1976). Default probabilities are given by the first passage time density to the barrier. Longstaff and Schwartz (1995) and Madan and Unal (2000) develop models with stochastic interest rates and a flat exogenous barrier. Collin-Dufresne and Goldstein (2001) incorporate mean-reverting leverage ratios into an exogenous barrier model with stochastic interest rates. Leland
(1994) derives a barrier structural model with *endogenous default*. The model solves for both the optimal capital structure and the price of risky debt in the presence of taxes and bankruptcy costs. Leland and Toft (1996) extend the Leland (1994) model to take debt maturity into account as well. Distinct from the barrier models is the compound option model of Geske (1977) which underlies our approach. In the Geske model, shareholders own a *compound* option on the firm’s unlevered assets. In the compound option model, default probabilities are found by calculating the probability that the compound option is not going to be exercised at a particular future cash flow time. This allows us to calculate both the conditional and unconditional default probabilities and actually results in a term structure of default probabilities. Default is determined endogenously as a function of the liability structure and the associated promised cash flows.

Several papers present empirical studies on the performance of structural models. Leland (2004) looks at default probabilities for industrial firms, calculated using two structural models – the *endogenous barrier* model of Leland and Toft (1996) and the *exogenous barrier* model of Longstaff and Schwartz (1995). They compute risk-neutral default probabilities (RNDPs) using the original BSM model and the Geske model. They draw two interesting conclusions that have very important implications for our model: first, they show that RNDP’s serve as an upper bound to risk-adjusted default probabilities, i.e. the true default probabilities based on the true distribution are strictly lower than the RNDP predictions. Therefore, the models do not underestimate true default probabilities and provide conservative estimates of the firm’s true default risk. Second, the Geske model is able to provide important information that other structural models cannot; specifically, it provides a full term structure of default probabilities.

Our model, which is a lattice-based model, is an extension of the compound option model. Our model allows for a full specification of the financial institution’s liability structure and does not require calibration. This extends the application of structural

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5The Geske compound option model is also derived in a no-arbitrage setting in Geske (1979) where a leverage effect is shown to result in non-constant volatilities. The original Geske (1977) model was also corrected by Geske and Johnson (1984) to properly account for the seniority structure of debt.
models to financial institutions. Leland (2009) examines the performance of structural credit risk models during the Financial Crisis; specifically looking at Goldman Sachs and JP Morgan. He notes that, relative to industrial firms, it is more difficult to calibrate structural models for financial firms due to the high degree of leverage and the reliance on disproportionate amounts of short-term debt, including repos and deposits. As we show using Lehman Brothers as an illustration, our model is especially well-suited for measuring default risk in financial institutions.

3.3 Model Framework

In this section we discuss our dynamic lattice based structural model for estimating default risk in financial institutions. Structural models are especially well-suited for managing and monitoring credit risk, either internally or externally, as they use the most recent inputs from financial statements and market data. Our approach is based on the compound option pricing model as developed by Geske (1977) (henceforth Geske). We first review the role of capital structure assumptions in structural models and next present our lattice model.

3.3.1 Endogenous Default and Capital Structure Policy

Black and Scholes (1973) and Merton (1974) pioneered the notion that when a firm has risky debt outstanding, the equity is very much like a call option where shareholders are faced with the decision to exercise when payment is due to debtholders. Upon maturity of the debt, shareholders can choose to not make the payment, thereby letting the call option expire unexercised, and default. In the Black-Scholes-Merton (BSM) approach, the liabilities are modeled as a fixed point barrier with only one future date in which the exercise decision is made. The firm has the option to default only at one point - on the final maturity date of the debt.

Financial institutions have several different debt contracts outstanding and the fixed point barrier has limited application. The basic model has been extended to accommodate more complex liability structures consisting of debt with different maturities (see Geske (1977), Black and Cox (1976), Leland (1994), and Leland and
Toft (1996)) and allow for endogenous barrier. To allow for endogenous default, the
survival condition must be that the firm has the capability to raise capital. Such a
c Condition can be shown to be identical to the requiring that the asset value be no less
than total debt value, which is dependent upon the default barrier.

Let the firm’s assets evolve according to a diffusion process with dynamics de-
scribed by the Stochastic Differential Equation:

\[ \frac{dA_t}{A_t} = rd t + \sigma_t dW_t \tag{3.1} \]

where \( A_t \) represent the value of the firm’s assets at time \( t \), \( r \) represents the risk-free
rate, \( \sigma \) represents the volatility of the firm’s assets, and \( dW_t \) is the Wiener process
under the risk neutral measure.

Define \( E \) as the equity value of the firm. Then both Geske (Equation 1 on page 66,
1979) and Leland (Equation 2 on page 1218, 1994) show that the partial differential
equation for the equity (that is the same as that of the Black-Scholes) as follows:

\[ rE = \frac{1}{2} \sigma^2 A^2 \frac{\partial^2 E}{\partial A^2} + rA \frac{\partial E}{\partial A} + \frac{\partial E}{\partial t} \tag{3.2} \]

Also define \( D \) as the market value of debt for the firm; then by the accounting
identity \( D(t) = A(t) - E(t) \) for all \( t \). The capital structure policy that the firm is
formulated as follows. The firm has a collection of debts that have various coupons and
maturities. These debts incur a sequence of cash obligations defined as \( K_1, K_2, \ldots, K_k \)
that are paid at times \( T_1, T_2, \ldots, T_k \).

On the date of the final contractual cash flow at terminal time \( T_k \), the firm liqui-
dates. The value to equity holders and the optimal exercise decision is given by,

\[ E(T_k) = \max\{A(T_k) - K_k, 0\} \tag{3.3} \]

where \( E(T_k) \) is the equity value at the terminal time \( T_k \), \( A(T_k) \) is the asset value
at the terminal time \( T_k \) and \( K_k \) is the final redemption value of debt.

At time \( T_{k-1} \), when cash flow \( K_{k-1} \) is due, the equity holders must decide if such
a payment is worthwhile. If the equity holders decide rationally that it is, then the payment is made and the firm survives, and otherwise the firm defaults. Formally,

\[ E(T_{k-1}) = \begin{cases} 
\mathbb{E}_{T_{k-1}} [e^{-r(T_k-T_{k-1})} E(T_k)] - \mathbb{I}_{K_{k-1}} & \text{if } \mathbb{E}_{T_{k-1}} [e^{-r(T_k-T_{k-1})} E(T_k)] > K_{k-1} \\
0 & \text{otherwise}
\end{cases} \]  

(3.4)

where \( \mathbb{E}_u[\cdot] \) represents the risk-neutral expectation conditional on time \( u \) and \( \mathbb{I} \) represents the indicator function related to debt-rollover. If \( \mathbb{I} = 1 \) then all maturing debt is replaced by new equity and if \( \mathbb{I} = 0 \) then all maturing debt is financed by new debt.\(^6\) Continue to move backwards and we obtain the current equity value as:

\[ E(t) = \max \{ \mathbb{E}_t [e^{-r(T_1-t)} E(T_1)] - K_0, 0 \} \]  

(3.5)

where \( K_0 = 0 \). It is apparent that \( E(t) \) involves multi-dimensional integrals, as Geske (1977) demonstrates and difficult to implement. Leland (1994) and Leland and Toft (1996) derive closed-form solutions with the simplification that cash flows are level (fraction of the face value of outstanding debt plus a constant coupon) and occur continuously.

The default barrier where the asset value must stay above in order for the firm to survive can only be solved endogenously recursively as \( \mathbb{E}_{T_i} [e^{-r(T_i-T_{i-1})} E(T_i)] = K_{i-1} \) for \( i = 1, \ldots, k \).

While the modeling of an endogenous default barrier is similar in both the Geske and Leland approaches, financing the maturing debt differs. As a result, the boundary conditions for equation [3.2] are different for the Geske and Leland approaches, which leads to different closed form solutions. In the Geske model, maturing debt is funded by new equity, i.e. the firm goes through a deleveraging process and we refer to this as the *deleveraging* assumption. In the Leland and Leland-Toft models an identical new debt is raised to pay for the maturing debt, i.e. the firm rolls over its

\(^6\)Note that later on in our model such formulation is cast in a discrete-time, lattice framework and the expectation is taken at the lattice time steps, not the cash flow time steps. Yet the valuation described by the equation is still valid.
debt and we refer to this as the *rollover* assumption. The different assumptions essentially impound different capital structure policies for the financial institution. The Geske approach implies that the firm de-leverages over time and the Leland approach assumes that the firm maintains the same leverage over time. Indeed, the Geske and Leland approaches represent only two possible ways of modeling capital structure policy and firms clearly can follow intermediate policies, wherein they replace a part of their debt with equity and roll over the rest, as well. Our approach allows for a range of capital structure assumptions. Naturally this results in a range of default probability estimates conditional on the capital structure assumptions made.

Our model is a discrete time implementation and directly solves for the endogenous boundary without the need for the expensive iterative calculations. In our lattice model implementation, the shareholders of the firm are modeled as having several sequential exercise decisions to make. For each debt in the capital structure, we note the expiration date, taking into account the debt rollover policy. At each of the debt maturity dates, a cash flow payment is due and the shareholders have an exercise decision to make. The shareholders can default on the debt by choosing not to pay the amount due and turn over the assets to the debt holders. The decision to exercise the option to default fully takes into account the relative value of the assets and the debt of the firm. Each point at which the shareholders choose to default represents a node on the lattice; the *default boundary* refers to the asset value for which the shareholders will exercise the option to default at different points in time in the lattice. Equivalently, the default boundary represents the “cut off” asset values below which the firm is in default at each time \( t \).

Our model is a one-factor lattice model where there are \( k \) cash flows and a total number of \( n \) periods, denoted as \( L_1, L_2, \ldots, L_n \). The cash flows are due at times \( T_1, T_2, \ldots, T_k \) that are not necessarily equally spaced. In between any two cash flows, the lattice is further partitioned into \( m_1, m_2, \ldots, m_k \) periods respectively. In other words, \( m_i \) represents the number of steps between \( T_i \) and \( T_{i-1} \) and \( \sum_{i=1}^{k} m_i = n \). Note that these cash flows may contain interest and principal from various debts. Note that in our model, \( T_n \) is the liquidation date of the firm. Certainly, \( T_n \geq T_k \) as the
firm should not liquidate prior to its last cash obligation.

We estimate the default boundary, debt value, and equity value by backward recursion in the tree. The capital structure assumptions translates to the structure of the liabilities at each node of the tree. A Leland model implementation, for example, would required that the liability structure at each node be identical to the liability structure at the first node (subject to discretization approximations). A Geske model implementation, on the other hand, would only incorporate debt maturing at the node and debt maturing at all other nodes further down the tree.

We believe our approach is especially well-suited for analyzing default in financial institutions for two reasons. One, it is important to incorporate an endogenous default boundary that takes into account market conditions in estimating default probability. Two, the appropriate capital structure assumption can be dependent on the economic environment. Under normal market conditions, we would expect a financial institution to maintain its leverage and the rollover assumption is appropriate. Under adverse economic conditions when the financial institution is under distress, it may be more important to evaluate the expected default probability under the deleveraging assumption.

3.3.2 A Three Cashflow, Six Period Lattice Example

Consider a 6-period binomial lattice model where three cash flows are paid. Hence, \( n = 6 \) and \( k = 3 \). All periods are evenly spaced, although this need not be the case. Figure 1 depicts the tree structure of the model. In periods \( L_2 (T_1) \), \( L_4 (T_2) \), and \( L_6 (T_3) \), cash flows \( K_1 \), \( K_2 \), and \( K_3 \) are paid. The solid dots in Figure 3.1 represent economic states where the equity has positive value and the hollow dots represent those states where the equity has no value. In between two consequent cash flows, there is a middle time slice.

Along the lattice, the equity value is computed as the risk neutral expectation of the values in the next period, that is \( E(L_i) = E_{L_i} \left[ e^{-r\Delta t} E(L_{i+1}) \right] \) for any \( t \) in between any two cash flows. At a time of the cash flow, the following computation is executed: \( E(T_j) = E(L_i) = \max \left\{ E_{L_i} \left[ e^{-r\Delta t} E(L_{i-1}) \right] - K_j, 0 \right\} \). The above two steps
can be used to calculate the values at all other nodes in the lattice.

This lattice model permits us to track important values for the model such as survival probability curve and default barrier easily. The default barrier is the critical value where the asset value must stay above in order for the firm to stay solvent. The default barrier value at each cash flow time \( T_j \), symbolized as \( \bar{A}(T_j) \) can be captured in the lattice. In a numerical demonstration later, one can see how the barrier values can be observed and captured.

Second, we track the survival probability at each cash flow period, \( T_j^* \). The survival probability is the probability that the asset value stays above the default boundary. That is,

\[
Q(t, T_j) = \text{Pr}(A(T_1) > \bar{A}(T_1) \cap A(T_2) > \bar{A}(T_2) \cap \cdots \cap A(T_j) > \bar{A}(T_j)) \quad (3.6)
\]

This joint probability is easy to compute within the lattice. We simply trace each path in the lattice and count only those that survive.

Third, from the survival probabilities, we can track the default probability. The default probability between any two cash flow periods is simply the incremental change in survival probability, i.e.

\[
p(t, T_{j-1}, T_j) = Q(t, T_{j-1}) - Q(t, T_j) \quad (3.7)
\]

This means that to compute the \( j^{th} \) default probability, the firm must survive until time \( T_{j-1} \) and then default at time \( T_j^* \). Similar to the computation of the survival probability, we trace defaults along the lattice.

Finally, we compute the expected recovery which is the expected value of the assets for the survival states. Note that such recovery value is not obtainable in the original Geske model.
3.3.3 Numerical Example

In this section we present a numerical example of our lattice model. We construct a 6-period equally spaced binomial lattice with $\Delta t = 0.5$ year. That is, $n = 6$, $T_2 = 1, \ldots, T_6 = 3$ years. The firm faces three cash flow obligations, i.e., $k = 3$, to be paid at $T_1 = L_2 = 1$, $T_2 = L_4 = 2$, and $T_3 = L_6 = 3$. The cash obligations are $K_1 = 10$, $K_2 = 20$, and $K_3 = 275$ respectively.

Consider a firm with a current asset value of 300. Let $\sigma = 10\%$, $r = 3\%$. Using the standard binomial model of Cox, Ross, and Rubinstein (1979), we have: $u = e^{\sigma \sqrt{\Delta t}} = 1.0733$ and $d = e^{-\sigma \sqrt{\Delta t}} = 0.9317$. The risk-neutral probabilities are $q = \frac{e^{r\Delta t} - d}{u - d} = 0.5891$ and $1 - q = 0.4109$. Figure 3.2 shows the asset value binomial tree beginning at $A(t) = 300$.

Debt Rollover

As argued before, we need to make assumptions about the capital structure policy, i.e. the policy that the firm will implement with respect to maturing debt. The rollover assumption implies that the cash flow due at each time $T_j$ is paid for by a new debt due in the next cash flow payment time, i.e. $T_{j+1}$. As a result, the firm faces cash flow needs of $K_1 = $10, $K_2 = $30 (i.e. 10 + 20), and $K_3 = $305 (i.e. 30 + 275), respectively. Note however that the firm issues new debt to retire debt, so that there is no exchange of debt for equity in the tree. The value of equity under this assumption is 27.53 and the value of the firm’s debt is 272.47. The default probability at $t = 1$ is $16.9\%$.

When the liability structure is such that the firm has bonds maturing at $t = 1, 2,$ and 3 with face values 275, 10, and 20 respectively. The rollover assumption implies that the cash flows $K_1, K_2,$ and $K_3$ are 275, 285, and 305, respectively. The value of equity under this assumption is 0 and the value of the firm’s debt is 300.\(^7\) The default probability at $t = 1$ is 100%.

\(^7\)Assuming no bankruptcy cost.
The example clearly indicates the importance of the capital structure assumption and the ability of our lattice model to incorporate alternate capital structure assumptions. The case of extreme rollover is the Leland-Toft model (1996) where debt is retired continuously and an identical debt is issued in its replacement. This is a steady-state situation that results in a flat default barrier. Our lattice model can adopt any rollover assumption desired. To approximate the Leland-Toft model by the lattice model, we simply allow $T_n$ to be a very distant future (i.e. $T_n >> T_k$) and let the debt roll according to Leland and Toft (1996). In fact, in our analysis of Lehman Brothers in the next section, we let the short term debt roll every year to reflect the practice in the banking industry.

**Geske De-Leveraging**

In this example, we assume that the firm de-leverages as in the Geske (1977) model. The Geske assumption implies that the cash flow amounts that is due at each time $t$ is equal to the amount of debt maturing at time $t$ as there is no roll-over of debt. Therefore the cash flows that the firm has to pay $10, $20, and $275 at 1, 2, and 3 years respectively.

We next calculate the optimal default decision that equity holders will implement given the asset value tree and the liability structure given above. Figure 3.3 (3A, 3B, and 3C, respectively) shows the equity values in the lattice over the three time periods, working backwards from the last time step. The equity value takes into account the optimal decision to not make the debt payment and declaring bankruptcy, which is shown as zero values in the lattice. We note that we are using a small number of binomial steps in our illustration, which increases the granularity of the numerical values in the example. The granularity of the numerical values becomes less of an issue in any real world application that uses a large number of binomial steps.

The default boundary is obtained by tracing the lattice. For example, in Year 1, default occurs in the bottom state, i.e. the state with the lowest asset value. Hence, the default boundary must lie between 300 and 260.44. We assume that the firm defaults at the average of the asset values where the firm just survives and the firm
just defaults, e.g. the bottom two nodes in Year 1. Of course, in models with a higher number of steps and a smaller time period per step, the gap will narrow and converge. In Year 2, the bottom two states default and top three states survive. Hence, the default boundary falls in between 300 and 260.44. In Year 3, the default boundary falls between 300 and 260.44, but this time we know for sure it is 275, the last coupon. The default boundary curve is therefore: 280.22, 280.22, and 275 at 1, 2, and 3 years respectively.

To compute the survival probability, we trace survival through the lattice, i.e. wherever the value of equity is greater than 0. For the first cash flow, survival occurs at the asset values of 57.6 and 17.1. The survival probability is equal to $q^2 + 2q(1-q)$ or 83.1%. Similarly, in the second year, the survival probability is 74.9%, and 70.0% in the third year. The default probabilities are therefore 16.9%, 8.2%, and 4.9% in Years 1, 2, and 3 respectively.

Finally, from Figure 3, the value of equity at $t = 0$ is equal to $27.4$. Since asset value is $300$, we know the resulting debt value must be $272.6$.

**Effect of Maturity Structure**

To see the impact of the maturity structure on the default boundary and default probabilities, we modify the base numerical example by changing the sequence of the annual cash flows. We present two alternate specifications: first, for the case where the largest cash flow is due in the short-term and then, second, for the case where the largest cash flow is due in the medium-term. We examine the value of debt and equity and the default boundary to illustrate the differences for different maturity structures.

Let the bond face values be 275, 10, and 20 with maturities at $t = 1, 2, 3$, respectively; i.e. the short-term bond has the largest face value. Also assume that the firm retires debt with equity, i.e. the firm de-leverages, as before. The value of debt, equity, and assets, and the default probabilities are different from the previous case when the long-term cash flow was the largest, as would be expected. Specifically, the value of Equity drops to 14.2 and the value of Debt rises to 285.8. At time 1,
the firm defaults when the value of the assets are below 322.8 (the midpoint between 300 and 345.6). The first period survival probability is 34.7% and the first period default probability is 65.30%. If the firm survives in the first period, it also survives in the second and third period. The second and third period default probabilities are therefore zero.

Consider a third example where the intermediate maturity cash flow is the largest. Specifically, let the annual cash flows be 10, 275, and 20 at $t = 1, 2, 3$, respectively. In this case, the value of equity and debt are 21.4 and 278.6, respectively. The first period default probability is 16.9% and the second period default probability is 25.06%. If the firm survives in the second period, it does not default in the third period and the third period default probability is, therefore, zero. The default boundary is similar to our base example.

These examples are illustrative on several fronts. First, the maturity of the debt with the highest face value, whether it is short-, medium-, or long-term, is important in determining the market value of debt and equity as well as the first year default probability. A higher level of short-term debt, as is typical in a financial institution, dramatically increases the probability of bankruptcy in the near term. Second, we see that the default probability can increase in conjunction with a rise in the value of debt. If the intent is to prevent losses to creditors, regulators could take this into account in determining acceptable default probabilities for financial institutions. Finally, we see that the value of debt as a function of its maturity structure is a complex function of pure time value and default risk premium.

3.4 Analysis of Lehman Brothers

On September 15, 2008, Lehman Brothers Holding Inc. (Lehman hereafter) filed the largest bankruptcy in U.S. history.

Lehman was the 4th largest investment bank in the USA behind Goldman Sachs, Morgan Stanley, and Merrill Lynch prior to its bankruptcy in 2008. The C.E.O. of Lehman was Richard (Dick) Fuld who began his career with Lehman Brothers in 1969 and had been known as a “typical” Wall Street investment banker who is
aggressive, ruthless, and ambitious. Lehman’s fall started the credit crisis and the worst economic recession after World War II.

It is still a large debate if Lehman’s default or the entire crisis was due to liquidity or credit. In the case of Bear Sterns, the bank reported a profit of $0.83 EPS when JP Morgan bailed it out on March 15, 2008 at $2 per share. Similarly, Lehman reported earnings of $489 million for the first quarter of 2008 and was able to raise $4 billion equity capital in April. It was true that Lehman subsequently reported large losses as more subprime investment failures unfolded.\(^8\) With $600 billion worth of assets in place, it is hard to attribute Lehman’s bankruptcy to only credit losses. It is rumored that Lehman’s failure was due to its difficulty in acquiring overnight repo funding from major banks.

We regard credit default as an economic event that a company can no longer make enough profits to pay for its outstanding debts. We regard liquidity default as a phenomenon that relates to lack of cash to keep the operation running.\(^9\) It is our belief that Lehman’s default is inevitable as subprime losses mounted and real estate bubble bursted, although the trigger of default could be liquidity driven.

We did notice that immediately prior to default, Lehman had a large amount of repo obligations which could be the primary reason that triggered Lehman’s default. It was also true that the net repo position was small.\(^10\) By the end of May of 2008, Lehman had $169,684 million in reverse repurchase agreements outstanding and $127,846 in repurchase agreements resulting in a $41,838 net repo position that was quite small. This leads us to believe that the huge repo obligations would cause Lehman to default during a liquidity squeeze as it cannot net its repo positions. In other words, Lehman’s default could possibly be only liquidity-driven.

However, in our analysis, we discovered that Lehman’s credit risk was too high and its default was inevitable. Using our model, we discovered that Lehman’s default probability was as high as 75% if Lehman was permitted to roll-over its debts and

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\(^8\) Lehman lost $2.8 billion in the second quarter of 2008.

\(^9\) For more on this distinction see Davydenko (2010).

\(^10\) We regard the net repo position as the net result of repurchase agreements (repos) and reverse repurchase agreements (reverse repos).
35% if Lehman could have de-levered over time.

We implement our approach to analyze the default risk for Lehman Brothers leading up to the firm’s failure. Lehman Brothers was a major financial institution that experienced severe financial problems disastrous enough that the Wall Street powerhouse was forced to file bankruptcy. Figure 3.4 shows the trajectory of Lehman Brothers’ stock price along with a timeline of the events as they unfolded over 2007 - 2008. In this section, we show that our model predicts the substantial increase in default risk for Lehman Brothers over the first few months of 2008 and we examine whether capital infusions alone could have prevented bankruptcy.

3.4.1 The Liabilities

We compile a comprehensive data set representing the liabilities of Lehman Brothers using the FactSet database. FactSet aggregates financial data from various sources which allowed us to analyze the firm’s capital structure at extremely detailed levels, specifically looking into the debt profile of Lehman Brothers. In particular, FactSet allows us to collect data on every single debt issuance at any given point in time. The dataset includes basic information of the bonds in FactSet such as the CUSIP number, the total face value of the issue, the issue date, the coupon rate, and the maturity date. In addition, it also includes other detailed information such as the “Status” (matured/redeemed/active), redemption options (callable/putable/convertible), the redemption date (where applicable), the ratings as per Moody’s and S&P, the coupon type (fixed/floating/etc.), and the seniority from the Fixed Income Explorer. This specific data regarding the debt structure was supplemented with data from the firms’ financial statements (annual income statements from 1998 to 2007, quarterly balance sheets from 3Q.2005 to 4Q.2007, and quarterly cash flows from 3Q.2005 to 4Q.2007). This permits us to study Lehman Brothers through September 2008, right before it filed for bankruptcy.

We collate the debt of Lehman by calendar year and estimate the dollar value of Lehman’s debt in each calendar year. Table 3.2 details the notional amount of the liabilities maturing in each calendar year as of January 2008. As seen in the table,
Lehman has substantial amount of debt maturing in the short term in 2008 and 2009. The table also shows other details of the debt outstanding.

With the exception of repurchase agreements (discussed earlier in the section), we note that we do not include private debt and off-balance sheet obligations of Lehman Brothers because of a lack of data on these obligations. We note that regulators can demand access to these data and can easily incorporate the cash-flow liabilities in any real-world implementation of our model. Indeed, one advantage of our model is the ease with which additional cash-flows can be incorporated into the analysis when such cash-flow obligations are deemed to be relevant by the regulators. Our estimates, without data on any off-balance sheet obligations, provide a lower bound on the true level of default risk. If these off-balance sheet obligations were to be included, we would certainly see default probabilities rise accordingly. Our estimates are therefore conservative when assessing the default risk of a financial institution.

Figure 3.5 displays the debt maturity structure from December, 2007 until September, 2008. The figure shows the notional amount (face value) of debt maturing in each year. Table 3.1 presents more detailed information for January of 2008 to illustrate the dollar breakdown. As the figure and table show, short-term debt maturing in one to three years dominates in Lehman’s liability structure. This is typical for financial institutions which finance their operations using liquid, short-term debt. Figure 3.5 also shows a spike for debt maturing after 30 years. This reflects the fact that we aggregate all debt with maturity greater than 30 years, e.g. perpetual debt/preferred securities and 30-40 year Mortgage Backed Securities, and include it in this bucket. It is very important to note that that Lehman Brothers’ short-term debt increased dramatically after March of 2008. This reflects the constraints imposed on Lehman after the fall of Bear Stearns. However, we are able to show that these short-term debts would put even more pressure on Lehman Brothers as the financial crisis worsened.

We implement our model to determine the default probabilities at the end of each month, collating the debt based on their maturities by year. Both the time at which the liability structure is determined and the time period over which different tranches of debt are collated can be changed. For example we can calculate the debt profile
every week or every fortnight and collate the liabilities quarterly or over six-month windows. Our choice here represents a compromise between the level of detail and clarity of presentation and does not affect any of our results.

3.4.2 Market Value Inputs

Our model uses two market-determined inputs for calculating the default probability of Lehman Brothers. The first is the market value of Lehman Brothers’ equity and the second is the volatility of Lehman stock returns.

Figure 3.6 shows the book value and the market value of equity for Lehman Brothers. The market value of equity is calculated as the product of the closing stock price on the estimation day and the number of shares outstanding on that day. The book value of equity is as reported by COMPUSTAT. As the figure shows, the market value of equity had a precipitous decline over 2008. In February the equity value dropped from $35.1 billion to $26.8 billion, a drop of 23.6%. As Figure 3.6 shows, the financial crisis, which is reflected in this dramatic drop in equity value, led Lehman to raise additional funds in March 2008. Lehman obtained a $2 billion 3 year credit line from a consortium of 40 banks, including JPMorgan Chase and Citigroup. They also raised $2 billion in perpetual convertible preferred stock at the beginning of April 2008. The infusion stabilized Lehman over the next two months; however a reported loss of $2.8 billion in May 2008 led to a further stock price decline of 27.9% in May as shown in the figure.

Figure 3.7 shows the historical 30-day stock return volatility for Lehman. We calculate the volatility as the annualized standard deviation of Lehman’s daily stock returns over the prior 30-day period. We use the most recent period to focus on the most recently available market information of Lehman’s stock returns. As Figure 3.7 shows, the volatility estimate spiked in March 2008 reflecting the increase in uncertainty in February.
3.4.3 Default Probability

We use data on the liability structure, the equity market value, and the estimated volatility to determine the endogenous default boundary for Lehman Brothers at the beginning of each month for the period from January 2008 to September 2008. Figure 3.8 shows the market value and book value of Lehman’s debt. The book value of debt is the cumulative dollar value of Lehman’s publicly-traded debt. It is obtained from FactSet. The market value of debt is as estimated by our model. As Figure 3.8 shows, the implied market value of debt is substantially lower than its book value indicating a high default risk premium for the debt.

Figure 3.9 shows our model estimates of the 1-year and 2-year default probabilities. Default probability spiked in March 2008 to well over 50% after staying low in January and February of 2008. Thus, our model predicts the dramatic rise in default risk for Lehman Brothers, well before the September 2008 bankruptcy filing. The short-term funds obtained by Lehman did lower the default risk substantially in May and June. In fact, the default probability in May was lower than the default probability in April, presumably due in part to the modest amount of equity capital that was raised during this time or possibly because the market anticipates a government intervention and a rescue package.

Figure 3.10 shows the cumulative default probability over time for Lehman Brothers at the beginning of each month in 2008. As seen in the figure, the cumulative default probability is upward sloping with a steep slope in the initial period, and then leveling off. The plot of cumulative probability is typical for the Geske model, especially when looking at financial institutions; the intuition behind the behavior of cumulative default is as follows. Financial institutions are characterized by having large amounts of liabilities, a substantial amount of which typically matures in the short term. The present value of debt is therefore high, and in adverse market conditions, a financial institution faces much higher default risk in the short term. The initial rise in default probability captures the degree of financial distress faced by the financial institution. If the financial institution does not default in the initial period, the model assumes that the firm was able to raise equity to meet the debt obligation.
This lowers the subsequent default probabilities and results in the cumulative default probability that levels off in the future periods. We note that the results in Figure 3.10 reinforce the results from Figure 3.9 in that the default risk is the highest for Lehman during March 2008 when the firm began to face financial distress and experienced a high degree of uncertainty regarding whether the government would play a role in supporting financial institutions.

The pattern of default probabilities over time, shown in Figure 3.10, has important implications for the management of default risk in financial institutions. As shown in the figure, default risk rises initially and levels off in the longer term. The pattern of default probabilities is not only a function of the liability structure, but also a function of the assumptions made by the model on capital structure changes when liabilities come due. We argue that these features are important in measuring and managing default risk in financial institutions. Financial institutions are saddled by a large amount of debt and a substantial fraction of these liabilities are due in the short-term. It is therefore not surprising that the periodic default probability is the highest in the initial period and is lower in the later periods. Further, the model’s assumption that debt is retired using equity further mitigates default risk in later periods.

Our model thus indicates that the largest threat to the survival of a financial institution is in the short term. When default probability rises, the near term default is usually the driving force. Regulators therefore have to be especially vigilant when the short-term default risk is unusually high. It is clear from Figure 3.9, that the default probability for Lehman Brothers shows an upward trend from March 2008 and is unacceptably high at over 30% in July 2008.

3.4.4 Equity Infusion

We next examine how default risk changes for varying capital structures. Lehman’s market value of equity in March is $20.754 billion and the model-implied market value of assets is $115 billion. Assume that capital raised during a crisis is retained as cash or invested in riskless assets and serves as a cushion to absorb further losses
and reduce default risk. The portfolio of Lehman’s existing risky assets and the new equity that is invested in riskless assets will therefore have a lower level of risk. Since the variance of riskless assets is zero and the correlation between the riskless assets and the firm’s other risky assets is zero, volatility of assets will decrease proportionate to the amount of capital raised. Given market conditions as of March 2008, Lehman would need to raise substantial additional capital, nearly 30% of its current asset market value, to reduce default probability below 10%.

The predictions of our model are a function of the assumptions we have made on the how Lehman Brothers uses the funds raised by issuing additional equity. It is plausible that a dramatic increase in equity capital and a credible signal that the firm is going to shore up the balance sheet by investing in riskless assets until at least a time that the financial crisis is passed, will lower the volatility as we have assumed in our illustration. It is also plausible that Lehman Brothers may use the equity to retire debt with no change in the total value of the firm’s assets. The impact of such a move will of course also depend on the maturity of the debt that is retired. The default probability will differ in these alternate scenarios and we expect that the financial institution will take the approach that reduces default probability by the largest amount.

The implications for managing default risk in financial institutions are profound. Our model validates the Federal Reserve’s focus on the short-term viability of financial institutions and provides tools that the Fed (and other regulators) can use to evaluate institutions in dramatically changing market conditions.

3.5 Conclusion

This paper develops and presents a new approach to estimating the default risk of financial institutions. It allows for the accurate estimation of default risk, using an endogenous default boundary, and works for the complex liability structures that are typical for financial institutions. The model can be readily applied to estimate the default risk of financial institutions and gives regulators a powerful dynamic tool that reflects current market conditions in managing default risk of financial institutions.
The model uses several inputs that represent the anatomy of a financial institution in a crisis (the detailed liability structure of a financial institution, the market value of equity, the volatility of stock returns, the illiquidity of assets, and credit constraints on the financial institution) to estimate default risk. We implement our model on a monthly basis for 2008, the year of the financial crisis; specifically to the case of Lehman Brothers. We use hand collected data from FactSet to determine the firm’s detailed liability structure and daily stock data to estimate market value of equity and stock return volatility. We show that default risk spiked to over 50% in March 2008, well before the bankruptcy filing by Lehman Brothers in September of 2008. Furthermore, we show that the firm’s attempts to bolster their equity capital was ultimately insufficient to mitigate the default risk.

Our model uses a structural contingent claims approach to estimate default risk. While such an options-based approach has been advocated as a way to estimate default risk since the seminal works of Black and Scholes (1973) and Merton (1974), the application to financial institutions has several practical drawbacks. Traditionally, such structural models have used a single point equivalent of a firm’s liabilities and have used exogenously imposed default barriers. Our model presents an alternate approach that incorporates the entire liability structure and uses an endogenous default boundary to develop accurate estimates of default risk.

We show that financial institutions such as Lehman Brothers are characterized by rapidly rising default risk in the short term which levels off over the longer term. The pattern of cumulative default probability has important implications for the regulation of financial institutions. Our model validates the Federal Reserve’s focus on the short-term survival of financial institutions during times of crisis, since the ability to meet the large near-term cash flows is the most critical. We also show that in times of financial crisis when the short-term default risk is very high, a surprisingly large level of capital infusion is required to increase survival probability, as illustrated by the case of Lehman Brothers.

Our model represents a powerful diagnostic tool that will enable bank managers and regulators to accurately estimate default risk and analyze the impact of any
intervention. Moreover, our model uses the most recent market data, as opposed to the static approaches of measuring bank risk. This is crucial when facing fast-moving market conditions such as those that exist in the midst of a financial crisis.
Table 3.1: Timeline of Events at Lehman Brothers

<table>
<thead>
<tr>
<th>Date</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>2007- Jan. 2008</td>
<td>◇ Lehman scales back its mortgage business, cutting thousands of mortgage-related jobs and closing Mortgage origination units</td>
</tr>
<tr>
<td>Q4:2007</td>
<td>◇ Lehman Brothers shows $886 million in quarterly earnings (flat compared to 3rd quarter); reported earnings of $4.192 billion for 2007 fiscal year (a 5% increase from the previous fiscal year)</td>
</tr>
<tr>
<td>Jan. 29, 2008</td>
<td>◇ Lehman announces an increase in dividends and plans to repurchase up to 100 million shares of common stock</td>
</tr>
<tr>
<td>Q1:2008</td>
<td>◇ Lehman Brothers increases holding of Alt-A mortgages despite the prevailing troubles in the real-estate market</td>
</tr>
<tr>
<td>March 14, 2008</td>
<td>◇ Lehman obtains $2 billion 3 year credit line from consortium of 40 banks including JPMorgan Chase and Citigroup</td>
</tr>
<tr>
<td>March 14, 2008</td>
<td>◇ Federal Reserve and JPMorgan Chase begin to put together deal to “bail out” Bear Stearns</td>
</tr>
<tr>
<td>March 16, 2008</td>
<td>◇ JP Morgan announces offer to purchase Bear Stearns for $2 per share</td>
</tr>
<tr>
<td>March 18, 2008</td>
<td>◇ Lehman shares surged up almost 50% after Federal Reserve gives investment banks access to the discount window</td>
</tr>
<tr>
<td>April 1, 2008</td>
<td>◇ Lehman looks to raise $4 billion in new capital via an offering of perpetual convertible preferred stock</td>
</tr>
<tr>
<td>Q2:2008</td>
<td>◇ Lehman Brothers shows a $2.8 billion loss, which was the first loss in its history as a public firm. It admits losses came not only from mortgage-related positions but also from hedges against those positions</td>
</tr>
<tr>
<td>June 9, 2008</td>
<td>◇ Lehman Brothers announces plan to raise addition $6 billion in new capital ($4 billion in common stock, $2 billion in mandatory convertible preferred stock)</td>
</tr>
<tr>
<td>July 7 - July 11, 2008</td>
<td>◇ Lehman shares plunge more than 30% for the week amid rumors that the firm’s assets have not been priced to appropriately reflect the “true value”</td>
</tr>
<tr>
<td>Date</td>
<td>Event Description</td>
</tr>
<tr>
<td>-----------------------</td>
<td>---------------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>September 9, 2008</td>
<td>♦ Markets punish Lehman Brothers for not raising capital more aggressively; Lehman’s share price falls 45% to $7.79 on fears that the firm’s capital levels may not be sufficient to support exposure to deteriorating real-estate investments</td>
</tr>
<tr>
<td>September 10, 2008</td>
<td>♦ Lehman CEO Dick Fuld reveals plans to spin off real-estate assets and sell a portion of the asset management division, insisting that the firm is solvent enough to survive</td>
</tr>
<tr>
<td>September 11, 2008</td>
<td>♦ Talks of a Lehman takeover permeate the markets as Lehman shares fall further, closing at $4.22</td>
</tr>
<tr>
<td>September 12, 2008</td>
<td>♦ Lehman approaches several potential buyers, including Bank of America and Barclays</td>
</tr>
<tr>
<td>September 15, 2008</td>
<td>♦ Lehman Brothers officially files bankruptcy after Treasury Secretary Paulson refuses to back any takeover; Shares close at $0.21</td>
</tr>
<tr>
<td>September 16, 2008</td>
<td>♦ Lehman Brothers dropped from the S&amp;P 500</td>
</tr>
<tr>
<td>September 18, 2008</td>
<td>♦ Lehman shares close at $0.052 in OTC trading as effects of the biggest bankruptcy in history ripples through the financial markets</td>
</tr>
<tr>
<td>September 22, 2008</td>
<td>♦ Lehman’s U.S. operations re-open for business under Barclays Capital after approval for the acquisition was granted by the federal bankruptcy court presiding over the liquidation</td>
</tr>
</tbody>
</table>
Table 3.2: Lehman Brothers Debt as of January 2008 (in Millions of Dollars)

<table>
<thead>
<tr>
<th>Maturity</th>
<th>Notional</th>
</tr>
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<tbody>
<tr>
<td>2008</td>
<td>19,172</td>
</tr>
<tr>
<td>2009</td>
<td>22,138</td>
</tr>
<tr>
<td>2010</td>
<td>15,792</td>
</tr>
<tr>
<td>2011</td>
<td>17,577</td>
</tr>
<tr>
<td>2012</td>
<td>17,385</td>
</tr>
<tr>
<td>2013</td>
<td>15,560</td>
</tr>
<tr>
<td>2014</td>
<td>9,815</td>
</tr>
<tr>
<td>2015</td>
<td>10,685</td>
</tr>
<tr>
<td>2016</td>
<td>5,893</td>
</tr>
<tr>
<td>2017</td>
<td>8,999</td>
</tr>
<tr>
<td>2018</td>
<td>716</td>
</tr>
<tr>
<td>2019</td>
<td>1,778</td>
</tr>
<tr>
<td>2020</td>
<td>493</td>
</tr>
<tr>
<td>2021</td>
<td>305</td>
</tr>
<tr>
<td>2022</td>
<td>1,233</td>
</tr>
<tr>
<td>2023</td>
<td>441</td>
</tr>
<tr>
<td>2024</td>
<td>29</td>
</tr>
<tr>
<td>2025</td>
<td>13</td>
</tr>
<tr>
<td>2026</td>
<td>427</td>
</tr>
<tr>
<td>2027</td>
<td>1,136</td>
</tr>
<tr>
<td>2028</td>
<td>308</td>
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<tr>
<td>2029</td>
<td>279</td>
</tr>
<tr>
<td>2030</td>
<td>215</td>
</tr>
<tr>
<td>2031</td>
<td>151</td>
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<tr>
<td>2032</td>
<td>823</td>
</tr>
<tr>
<td>2033</td>
<td>61</td>
</tr>
<tr>
<td>2034</td>
<td>422</td>
</tr>
<tr>
<td>2035</td>
<td>1,388</td>
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<tr>
<td>2036</td>
<td>4,211</td>
</tr>
<tr>
<td>2037</td>
<td>7,831</td>
</tr>
<tr>
<td>2038+</td>
<td>4,996</td>
</tr>
<tr>
<td><strong>Grand Total</strong></td>
<td><strong>170,271</strong></td>
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<table>
<thead>
<tr>
<th>Coupon Type</th>
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<th>Seniority</th>
<th>Notional</th>
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<tbody>
<tr>
<td>Combination</td>
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<td>Senior</td>
<td>64,995</td>
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<tr>
<td>Combo - Fixed/Floating</td>
<td>5,549</td>
<td>Senior Subordinate</td>
<td>500</td>
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<tr>
<td>Combo - Floating/Fixed</td>
<td>2,600</td>
<td>Subordinate</td>
<td>7,300</td>
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<tr>
<td>Extendible Reset</td>
<td>428</td>
<td>NONE</td>
<td>3,000</td>
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<tr>
<td>Fixed Listing</td>
<td>85,296</td>
<td>N/A</td>
<td>94,477</td>
</tr>
<tr>
<td>Floating Rate</td>
<td>73,774</td>
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<tr>
<td>Graduated Rate</td>
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<td></td>
<td></td>
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<tr>
<td>Step-Up/Down</td>
<td>182</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N/A</td>
<td>96</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 3.1: Lattice Implementation of the Model

\[ E_t = E_t | e^{iD} E_{t+1} \]

\[ E_i = \max \{ E_i | e^{iD} E_{i+1} | \ - K_1, 0 \} \]

\[ E_F = \max \{ A_F - K_2, 0 \} \]

\[ E_i = \max \{ E_i | e^{iD} E_{i+1} | \ - K_2, 0 \} \]
Figure 3.2: Numerical Example – 3-Period Asset Binomial Tree

<p>| | | | |</p>
<table>
<thead>
<tr>
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<td>226.09</td>
<td>226.09</td>
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<tr>
<td>210.66</td>
<td>196.29</td>
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</tr>
</tbody>
</table>
Figure 3.3: Numerical Example – Equity Values in a 3-Period Binomial Tree
Figure 3.4: Lehman Brothers Stock Price and Timeline
Figure 3.5: Lehman Brothers Debt Maturity
Figure 3.6: Book Value and Market Value of Lehman Brothers Equity
Figure 3.7: Lehman Brothers Equity Volatility
Figure 3.8: Model-Implied Market Value of Lehman Brothers Debt
Figure 3.9: Lehman Brothers 1-Year and 2-Year Default Probabilities

Geske

Leland-Toft
Figure 3.10: Lehman Brothers Cumulative Default Probabilities (2008 - 2032)
Chapter 4

Conclusions and Future Research

This dissertation deals with the use of structural credit risk models in the analysis of financial institutions. The models developed in this dissertation are specifically applied to help understand the risks of banking firms during the recent Financial Crisis.

The first essay, Chapter 2, proposed a dynamic approach to determining a bank’s capital needs using a structural credit risk model. The model uses market data along with information about the bank’s liability structure to compute default probabilities. Then, if it appears that the bank is in distress, the model can be used to estimate the amount of capital that would be needed to reduce default probabilities back down to a level that is acceptable. The model is implemented in an empirical study of the 15 largest bank holding companies (BHCs) in the United States both before and during the Financial Crisis. While there were no signs of distress prior to the onset of the crisis, conditions took a dramatic turn for the worse over 2007. By 2008 default probabilities began to rise, and before the end of the year, the model indicates that all 15 BHCs in the sample had default probabilities above 10% with some getting close to 50%. The model is then used to estimate the amount of new capital needed to reduce default probabilities below 5%, with the results compared to TARP and SCAP. The model indicates that all but one of the 15 BHCs were still undercapitalized after both TARP and SCAP.

The second essay, Chapter 3, developed an endogenous structural credit risk model for the analysis of financial institutions that allows for different refinancing assumptions. For instance, the bank can roll-over maturing debt or pay down maturing debt with new equity. The model is implemented in a clinical investigation of Lehman
Brothers. Lehman’s default probabilities are computed under both refinancing assumptions in the months leading up to the firm’s historic bankruptcy in 2008. The two refinancing assumptions result in different levels of default probability, however both predict extremely high default probability months before Lehman’s failure in September 2008. Furthermore, the model can be used in a prescriptive capacity by indicating when a financial institution should de-leverage rather than the typical practice of rolling over debt.

While this dissertation has answered several interesting research questions, both theoretically and empirically, it has also raised some new questions that will allow for fruitful future research in banking, risk management, and structural credit risk modeling. In this chapter, several opportunities for extensions and ideas for future research are discussed.

One area of future research is to introduce liquidity into a structural credit risk model for banks. Neither of the models developed in this dissertation address liquidity. It is assumed that capital is raised in a perfect market and default is defined in terms of solvency. In both models, the bank defaults at the endogenously determined point where it can no longer raise capital in a perfect market. This condition is based purely on fundamentals: no rational market participant would invest in a bank that is on the verge of insolvency. It can be determined at what asset value this will occur. However, as noted by several recent papers, liquidity appears to have played a major role in this financial crisis (see Brunnermeier (2009), Shin (2009), Duffie (2010), and Gorton and Metrick (2011)).

In fact, “rollover risk” has become an interesting topic explored by several well-respected researchers in financial economics (see Morris and Shin (2009), Acharya, Gale, and Yorulmazer (2010), and He and Xiong (2011)). As such, it would be very interesting to develop a structural credit risk model with a liquidity factor that could
be used for a banking firm.\textsuperscript{1} Liquidity would be modeled along the lines of Brunnermeier and Pedersen (2009) where there is market liquidity and funding liquidity and an adverse feedback effect between the two. The default boundary would be a function of the liquidity parameter and would move down or up as liquidity conditions got better or worse, respectively. If the asset value falls according to a pure diffusion and simultaneously the barrier rises, it could give the effect of a jump-to-default instance.

In collecting the data and specifying the model, careful attention should be given to making the distinction between short and long-term debt. This distinction was a critical component of the model used in Chapter 2, and it was argued throughout this dissertation that appropriately accounting for the liability structure is one of the most important features in developing a structural credit risk model for a banking firm. Furthermore, there is evidence that fast-moving volatile sources of bank funding (i.e. Repurchase Agreements (Repos), Asset-Backed Commercial Paper (ABCP), etc.) contributed to the liquidity crisis that ensued in 2008.\textsuperscript{2}

Using the Brunnermeier and Pedersen (2009) model to parameterize funding liquidity, the barrier would be an explicit function of the liability structure and move up and down with liquidity conditions. This would be one way to incorporate rollover risk into a structural credit risk model in a unified manner consistent with the framework developed by Brunnermeier and Pedersen (2009). During times of low market liquidity, assets can only be sold at fire-sale prices. The downward pressure on asset prices causes counterparties to increase their margin requirements (or haircuts on repurchase agreements), thereby driving up the cost of short-term funding. This leads to a decrease in funding liquidity making it possible for a financial institution to fail, not necessarily from fundamentals but simply due to the fact that operations cannot be financed through the traditional channels.

Another potential area for future research is to examine the debt overhang problem as described by Myers (1977). The debt overhang problem refers to the instance

\textsuperscript{1}Leland (2006) proposes a structural credit risk model with jumps in asset value and a liquidity premium to account for the additional compensation required by investors on illiquid bonds. In modeling rollover risk, He and Xiong (2011) develop a structural credit risk model with random shocks to liquidity in the secondary market for debt.

\textsuperscript{2}See Brunnermeier (2009) and Krishnamurthy (2010).
where a distressed firm issues new equity and there is a transfer of wealth to the firm’s creditors. This can lead to underinvestment and the rejection of positive NPV projects simply because the shareholders do not get to accrue any of the benefits. As it turns out debt overhang was a major problem for banks trying to (or being required to) raise new capital in the midst of the financial crisis (see, for example, Philippon and Schnabl (2009), Philippon (2010), Acharya, Gujral, Kulkarni, and Shin (2011)).

The model from Chapter 2 can quantify the extent of debt overhang with respect to each of the two classes of debt (junior and senior). It could then be used to empirically study how severe the wealth transfer would have been for banks’ shareholders if more capital was raised during the financial crisis, as the results from Chapter 2 suggest should have been done.

If market frictions were to be explicitly included, in the form of bankruptcy costs and taxes, it is possible to find an analytical solution for the optimal maturity structure (long-term versus short-term) of financial institutions’ debt so as to minimize the degree of debt overhang. This could provide additional insight into the debt maturity structure for financial institutions (see Flannery (1994), Brunnermeier and Oehmke (2010), among others). In fact, exploring this avenue could shed light on some unresolved issues in the literature. For instance, Diamond and He (2011) argue that having more short-term debt increases the debt overhang problem and leads to distortions in the firm’s investment decision. On the other hand, Flannery (1994) argues that one reason why banks rely so extensively on short-term debt is because it helps to mitigate against the investment distortions brought about by asset substitutions. It would be interesting to see what an extended version of the model from Chapter 2 would indicate in regards to this issue.

The model in Chapter 3 could be extended to endogenize the funding decision of the bank by introduction a parameter that specifies the proportion of debt that is refinanced with equity or rolled-over into new debt. The model could be specified so as to choose the optimal value of the parameter (and therefore the optimal refinancing strategy) that either maximizes firm value or achieves a target leverage ratio.
Finally, there are still many questions remaining about the optimal recapitalization mechanism for distressed banks. The idea of capital insurance, as proposed by Kashyap, Rajan, and Stein (2008), seems very promising especially when there is an immediate need for a cash infusion at a distressed bank. In fact, the dynamic implementation of the compound option model, that was proposed in Chapter 2 is consistent with this idea of capital insurance as proposed by Kashyap, Rajan, and Stein (2008). Since the model is based on the compound option framework, one additional layer could be added to look at the pricing and amount of capital insurance that would be needed by distressed financial institutions. The only problem is that the solution would be in terms of a tri-variate normal distribution and would most likely not be feasible in closed-form. Additionally, the application of structural credit risk models to the pricing and design of contingent capital securities is very promising. Two examples include recent papers by Pennacchi (2010) and Albul, Jaffee, and Tchistyi (2010). However, as argued throughout this dissertation, one of the most important features will be the ability to incorporate endogenous default.
References


———, 2009, “Structural Models and the Credit Crisis,” in *China International Conference in Finance*.


Vita

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