Description: PUP Math - Shirts and Pants Location: Harding School – Kenilworth, NJ Researcher: Professor Carolyn Maher	Transcriber(s): Private Universe Project Verifier(s): Sigley, Robert, Sran, Kiranjeet Date Transcribed: Spring 2000 Page: 1 of 9

Line	Time	Speaker	Transcript
1.		*Narrator*	*[Stephen has a white shirt, a blue shirt, and a yellow shirt. He has a pair of blue jeans and a pair of white jeans. How many different outfits can he make?* *Let's see how the students in the Rutgers' study approached this problem.]*
2.		Stephanie	I'm going to make a shirt and I'm going to put a "W" for white.
3.		Michael	Yeah, white shirt, white pants.
4.		*Narrator*	*[The students spontaneously begin to make drawings.]*
5.		Dana	Blue and… a yellow shirt.
6.		Stephanie	He has a pair of blue jeans and a pair of white jeans. How many different outfits can he make?
7.		Michael	Well, he can only make two outfits.
8.		Stephanie	No, how many different outfits? He can make a lot of different outfits. Look, he can make white and white...
9.		Dana	He can make all three of these shirts with that outfit.
10.		Stephanie	Shh! You can do it in a lot of different ways. You can do white and white and that's one. By doing "W" and "W".
11.		Stephanie	Two. Blue. Blue jeans and a white shirt. Shh! Yellow shirt...number three could be a yellow shirt-
12.		Dana	It can't be yellow. A yellow shirt can't go with the white.
13.		Stephanie	Yeah but how many outfits can he make? It doesn't matter if it doesn't match, as long as it can make outfits.
14.		Stephanie	It doesn't have to go with each other, Dana.
15.		Dana	What outfits can it be? It can make more if you put them mixed up. Look, I'm on my fourth one. Number four, it could be a blue shirt and blue pants.

16.		Stephanie	Number five. It can be a white shirt, and...It can be a blue shirt...wait, did I do blue and white?
17.		Dana	What's two?
18.		Stephanie	It can be a yellow?
19.		Dana	What's two?
20.		Stephanie	Two's a blue shirt and white pants. Wait a yellow shirt... Wait did I do yellow and white? A yellow shirt and blue pants. Yellow shirt and blue pants...
21.		Michael	Well I'm gonna do it the way you want.
22.		Stephanie	'Cause look. There's five combinations. There's only five combinations.
23.		Dana	Lemme see your paper…
24.		Stephanie	You can do this. Listen, Michael. Michael, will you listen for once? Five combinations: Number 1 - white and white. Number 2 - Blue and white. Number 3 - Yellow and white. Number 4 - Blue and blue. Number 5 - Yellow and blue.
25.		Michael	(Inaudible)
26.		Stephanie	You can do four combinations, I'm sure of it.
27.		Dana	Five!
28.		Stephanie	I mean five.
29.		Dana	Amy. Amy, we're done. We made five combinations.
30.		Amy	What did you do? What are your combinations?
31.		Stephanie	[to Amy] I've got white and white…
32.		*Narrator*	*[Let's look at this segment again and see what the researchers found out about each of the students' thinking:* *Very early in the tape, Dana drew lines to show combinations.* *At that time, the school's curriculum did not include teaching multiplication in the second grade.* *But Dana's approach to the problem shows that she is capable of the kind of mathematical thinking*

			that underlies multiplication. *Dana's initial graph shows three shirts, with all but one, the yellow shirt, connected to two pairs of jeans. At this point Dana was influenced by her sense of fashion!]*
33.		Dana	It can't be yellow. A yellow shirt can't go with the white.
34.		Stephanie	Yeah but how many outfits can it make? It doesn't matter if it doesn't match, as long as it can make outfits....
35.		*Narrator*	*[She believed that yellow and white don't "go together."]*
36.		Stephanie	They don't have to go with each other, Dana.
37.		*Narrator*	*[As teachers, how often have we seen cases like this - where students come up with a logic that makes sense to them but is completely different from what we expected?* *Stephanie made drawings and wrote letters and numbers to keep track of her random attempts to find outfits. Using these representations, she found five outfits. Notice that Stephanie, on her fifth combination, first wrote "W" over the "B", then wrote a "Y". Does this notation stand for one or two outfits?* *For Michael, perhaps his sense of style required that outfits have matching colors. He created a new color of pants - yellow - that wasn't included in the problem.* *The researchers were wondering: How were these students influenced by each others' ideas?]*
38.		*Amy*	*What was fascinating was that nobody seemed to be truly bothered by the fact that they'd come up with different numerical answers to the question. To me that was very important, because it really did say you do need to give children time to build, just to build and to think. And that's what that*

			session was for. That was really the purpose. So we really kind of left it open.
39.		Amy	(to students) Is this all the ways you can make it?
40.		Dana	… Yes, that's all the ways you can make it. I have the same thing.
41.		Amy	I see that. That's really good! Okay, Stephanie, put your name on this, okay? And write to me that you found five ways, okay?
Line	**Time**	**Speaker**	**Transcript**
42.		*Narrator*	*[The first opportunity to revisit the problem came four and a half months later, when the students were in third grade.]*
43.		Amy	Are we ready to start?
44.		Students	Yes!
45.		*Narrator*	*[In the meantime, the student's classroom teachers, who cooperated with the study, were careful not to tell the students how to solve the problem.* *The wording of the problem was identical. Let's see how their thinking has grown.]*
46.		Stephanie	Want me to read it out loud?
47.		Dana	No, I'll do it... he has a pair of white jeans and a pair of blue jeans? How many outfits can he make?
48.		Stephanie	Why don't we draw a picture?
49.		Dana	He has a white shirt, a blue shirt, and a yellow shirt.
50.		Stephanie	He has a pair of blue jeans and a pair of white jeans.
51.		*Narrator*	*[First the students made drawings and used letters to show the colors.]*
52.		Stephanie	All right, let's find out how many different outfits you can make. Well, you can make white and white. That would be one…
53.		*Narrator*	*[When they started counting the outfits, it was Stephanie, not Dana, who started drawing lines to connect the different shirts and pants, and both of them solved the problem this way.]*

54.		Dana	… the blue and the white…
55.		Stephanie	You mean the blue pants and the white shirt?
56.		Dana	Now we could have three with the blue and the blue pants. And the yellow could go with the....
57.		Stephanie	Hold on Dana, you're going too fast. And we could have the blue and the white. That would be three. The blue and the blue, that could be four. We could have the yellow and the white, that would be five...
58.		Dana	One, two, three, four, five, six. I have six so far.
59.		Stephanie	I've got one, two, three, four, five. What are your other combinations? I have white and blue. I've got white and white. I've got blue and white. I've got yellow and white. What were your two other combinations?
60.		Dana	I mean I have six. Six.
61.		Stephanie	What where your two other combinations?
62.		Dana	You mean one other combination. The yellow and the blue.
63.		Stephanie	The yellow and the blue.
64.		Dana	And the yellow and the white.
65.		Stephanie	I put the yellow and the white.
66.		Dana	All right… Then the blue and the white?
67.		Stephanie	Four, five, six. I have six too. I have six.
68.		Dana	Amy! We're done. It's six.
69.		Amy	(off camera) Are you both convinced of that?
70.		Dana and Stephanie	Yes
71.		Amy	Can you explain it to me?
72.		Stephanie	Well OK. If we have three shirts and two pants-
73.		Dana	- and we just drew lines they all made combinations. Six combinations.
74.		Stephanie	We made white and blue. We made blue and white.
75.		Dana	We made yellow – blue. And yellow – white. Six.
76.		Amy	Is that six
77.		Dana	Yes
78.		Amy	What are these lines that you drew? You drew lines between the shirts and the pants.

79.		Stephanie	So that we could make sure; so instead of we didn't do that again and say, "Oh, that would be seven, eight, nine, 10." We just drew lines so that we can count our lines and say, "Oh we can't do that again, we can't do that again."
80.		Amy	Oh. That's very nice... And you're sure that there are six?
81.		Dana and Stephanie	Yeah.
82.		Amy	Positive that there aren't seven or five?
83.		Dana	No six!
84.		Amy	OK, now what I want you to do then, as a pair, is to open this up and take a fat marker and take either the blue or the red and draw this for me.
85.		Dana	How about we ach do one thing?
86.		Stephanie	Dana, you can write and I'll draw, 'cause you're a better writer...
87.		*Narrator*	*[Michael solved the problem with lines too. But he didn't make drawings. Rather, he drew lines between the words. This is interesting because last time, when he came up with a completely different answer, it wasn't clear whether Michael was hearing or noticing the solutions of his partners.]*
88.		*Amy*	*The second time around is interesting. Say, a strategy that one child had come up with in second grade, in third grade, another child had modified that strategy and was making use of it. And it's fascinating, because most of the class, that time, many children did not even recall having done that exact problem. The ones who did recall it all remembered having come up with the exact, correct number of combinations, which goes to show that, really, that first session was just so important for getting them to build strategies and to modify. But those two tapes had a heavy impact on me.*
89.		*Narrator*	*[Since the students were able to come up with a way to solve this problem on their own, would they be able to use this strategy to solve a more*

			complex problem? *Six months later, when the students were in April of the third grade, the researchers presented an extension of the Shirts and Pants problem.]*
90.		Amy	...first, we're going to pretend that there's a certain situation. We're going to pretend that today is somebody's birthday in your class.
91.		*Narrator*	*[Called "Cups, Bowls, and Plates," the problem adds another choice to be considered.* *"Let's pretend that there's a birthday party in your class today. It's your job to set the places with cups, bowls, and plates. The cups and bowls are blue or yellow. The plates are either blue, yellow, or orange. Is it possible for 10 children at the party each to have a different combination of cup, bowl, and plate?"]*
92.		Amy	... it's supposed to be finger food, so we're not going to have forks and knives today. And what your job is going to be to do is that you're to make different combinations...
93.		Dana	B, yellow, yellow.
94.		*Narrator*	*[Stephanie and Dana worked together in the problem.]*
95.		Stephanie	B, B, Y.
96.		Dana	No, it isn't "B, yellow, yellow…"
97.		Stephanie	B, Y, Y, B, yellow, yellow.
98.		Dana	B, yellow, blue?
99.		*Narrator*	*[Using their own system of notation based on letters, they came up with the answer that there <u>are</u> at least 10 different combinations.]*
100.		Stephanie	...and we found 10 by recording our answers. We found 12 by looking at the recording from the first problem...
101.		*Narrator*	*[The next day, researcher Alice Alston interviewed Stephanie to explore her thinking in more depth. After reviewing her previous work with Shirts and Pants, Alice went on to ask how Stephanie could*

			justify her total number of combinations - 12. Stephanie used the same strategy that she had used with Shirts and Pants - linking choices by drawing lines, then counting the lines.]
102.		Stephanie	..so then, I guess what you'd do is that you'd go like this...
103.		*Narrator*	*[Then she had an idea for a different way to solve the problem: multiplication!]*
104.		Stephanie	...and then I just thought of it at the end when I was done figuring out that that was 12 I thought, "Hey, there's three fours and four times three equals 12."
105.		Alice	OK, what does the three represent?
106.		Stephanie	OK. The three represents the three fours that we got. Or the three plates.
107.		Alice	OK, the three represents the one, two, three plates? What does the four represent?
108.		Stephanie	The four represents, see each one could have could have four different ways, so you then multiply three times four and then you got 12.
109.		Alice	Ah…
110.		Carolyn	Students, we have found, in a very natural way , like to represent their ideas symbolically. Algebraic thinking must begin early. In our first task, shirts and pants, the origins of algebraic thinking are there. The notion of the generalized solution is there. The notion of controlling for variables is there. The notion of extending that particular solution structure to more complicated problems like cups, bowls, and plates is there. So that's very, very important.
111.		*Narrator*	*[We've just seen the teacher/researchers repeating the same, or similar problems, three times over the course of almost a year.* *What can we say about the changes in the students' methods over time?]*
112.		Amy	(to student) ..why, why couldn't you find some more, couldn't you just look for more?

113.		Dana	Well…
114.		Stephanie	Before we had recorded what we had found already...