Description: PUP Math - Towers **Location: Harding School – Kenilworth, NJ** **Researcher: Professor Carolyn Maher**	**Transcriber(s): Private Universe Project** **Verifier(s): Sigley, Robert, Sran, Kiranjeet** **Date Transcribed: Spring 2000** **Page: 1 of 11**

Line	Time	Speaker	Transcript
1		*Narrator*	*The "Towers" problem that the Englewood teachers were working on came out of the Rutgers long-term study. The researchers originally presented this problem to the Kenilworth students in October, 1990.*
1.		Amy Martino	All right. We're going to do something a little different today. We're going to build towers today that have four stories to them. You're going to get two colors of Unifix® cubes. Your job is-
2.		*Narrator*	*The question was - How many different towers four blocks tall can you build when selecting from blocks of two colors?*
3.		Teacher	..and again, it's like the shirts and pants. You have to be convinced that you've found them all…
4.		*Narrator*	*On this particular day, the students spent about an hour working on the problem. The researchers wanted to find out how the focus group of students would build mathematical ideas, not just today, but over a long period of time. And this was the first in a series of carefully linked activities.*
5.		*Carolyn Maher*	*We start with at least four tall. The students keep trying to do the problem until they can't find any more, even if they haven't come to organize their findings in a way that would account for all possibilities. We do not believe that you start them building towers one tall, two tall, three tall, four tall, and then they see this pattern. That was not what we were trying to do. That, to me, is a programmed way of proceeding.*
6.		*Narrator*	*The researchers always tried to challenge the students with problems that would force them to invent new strategies.*
7.		*Carolyn*	*For instance, in the four tall tower problem,*

		Maher	*when you have two of a color inside that tower, and you produce all possible towers with two of a color, making the argument that you have them all is demanding of some interesting reasoning, like controlling for variables, keeping one row constant and changing the other. So, it pushes them to invent other approaches, heuristics - methods of solving problems - like "guess and check" is a heuristic; a random method is a heuristic; working backwards is a heuristic.*
8.		Amy Martino	You guys working together? Do you have any of the same towers as each other?
9.		Student	Yeah.
10.		Amy Martino	Yeah? Which ones are the same?
11.		Student	This one...
12.		*Narrator*	*After spending less than five minutes making random combinations, the students started to compare their towers and eliminate duplicates.*
13.		Stephanie	Everything we make, we have to check. Everything we make... Let's make a deal. Everything we make we have to check.
14.		Dana	All right. I'll always make it and you'll always check it.
15.		Stephanie	Okay, you make it and I'll check it.
16.		Amy Martino	How's it going guys?
17.		Jeff, Brian	We're done.
18.		Amy Martino	Okay, you- What'd you get?
19.		Brian	We found 17 towers.
20.		Amy Martino	17? Is there a way that you can check to be sure?
21.		Brian	No.
22.		Amy	Is the way that you could, you know-

		Martino	
23.		Brian	We like laid them down and we saw if they're the same or not, and they weren't. They weren't the same.
24.		Amy Martino	Stephanie, what makes you sure that you got everything?
25.		Stephanie	I don't know.
26.		Dana	Well, we just test it, like we used all of our blocks and then we had matches and the ones that matched - because one of them that matched, and we eliminate them.
27.		Amy Martino	Could you have missed one?
28.		Dana	No.
29.		Amy Martino	How come? How do you know?
30.		Dana	Because we double-checked about four times.
31.		Stephanie	Okay, Dana, I'm going to try and make one more.
32.		*Narrator*	*The students recorded their findings, and most agreed that there were 16 combinations. The following day, the researchers returned to the problem. In a whole group discussion, they asked the students whether there would be more, fewer, or the same amount of combination for towers three tall.*
33.		Alice Alston	So, do you think there'd be more than 16 or fewer than 16?
34.		Student	More.
35.		Alice Alston	You think there'd be more?
36.		*Carolyn Maher*	*It is very interesting to say to students - "Now you've built all towers four tall; you've convinced us, or you haven't, that you've found them all. What about three tall?" The three tall is interesting because very young children predict that there will be more towers three tall,*

			and that surprises many teachers, many researchers, who expect them to think there are fewer.
37.		*Narrator*	*Matt suggested taking off one cube from every tower.*
38.		Matt	Take one block off each pattern. And then count up how many of these things...
39.		Alice Alston	Could everybody with your partners see if you can figure out how many there would be if there were only three. Remember that each one is to be different from all the others at your place...
40.		*Carolyn Maher*	*If you really listen carefully to what the students are saying and ask them why do they think there are more, well, what they're imagining is removing one of the rows of cubes from their four-tall towers and having more cubes to create new towers. And until they try to do that, they are really unaware that they end up duplicating towers of three that are already built. And that's very important for them to recognize why there are fewer rather than more or the same. It's important for them to understand that reversibility in their thinking.*
41.		Dana	Amy, we figured out that it's less.
42.		Amy Martino	You think so? Why?
43.		Stephanie	Because if we took one away, we had these. If we took one away from these, so-
44.		Amy Martino	And you can't have those?
45.		Stephanie	Well, we can't have them the same because-
46.		Amy Martino	That's right. They have to be different.
47.		Brian	...'cause, if you take like one off the bottom or one off the top, you might have another one that's the same as that. And then you have to

			make like - then you can't use that because it's a match, and they have to be different...
48.		*Narrator*	*Jeff shared an interesting way to look at the problem.*
49.		Jeff	You could link - first of all, you could do this as a math problem, because you could do 16 minus 8 is 8.
50.		Alice Alston	You mean, there's something about math to it?
51.		Jeff	Yeah, because 16 minus 8 or 8 plus 8 equals 16. And when you took the one away from each, it would be minus 1, so instead of saying minus 1, minus 1, minus 1, you could have just said 16 minus 8 or 8 plus 8.
52.		Alice Alston	I see. So it has something to do with 16 minus 8.
53.		Carolyn Maher	...and so let's talk about what they should be. First of all, how tall should they be...
54.		*Narrator*	*16 months later, in the fourth grade, the Kenilworth students investigated towers five cubes tall.*
55.		Carolyn Maher	...and you have to be able to convince us that you have found all possibilities - that there are no more or no less. Got the problem? Have fun!
56.		Stephanie	Okay, we'll start out with the easiest one. One, two, three, four, five reds and five yellows.
57.		Dana	One, two, three, four, five.
58.		Stephanie	I only have four. Okay, well, stand them up straight so we know what we have.
59.		Carolyn Maher	In towers five tall, to make a convincing argument that you found them all is harder except for when you have all of a color or one of a color.
60.		Shelly	Now we take one of these, one of these.
61.		*Narrator*	*Building towers five tall offered a richer, more*

			complex challenge for the students to investigate. Students spontaneously invented strategies, such as making a tower and then building its "opposite".
62.		Brian	...this one matches with this.
63.		Romina	Put the pairs.
64.		Brian	Like the opposites.
65.		Dana	And then I got another idea.
66.		Stephanie	Well, tell me it so I can do the opposite.
67.		Dana	I'm going to do the red - this, that-
68.		Stephanie	Show me. Oh, okay, and I'll do the red - and I'll do it with the red at the top.
69.		*Carolyn Maher*	*They were holding one variable fixed, constant, and then varying the other. It was exciting that these children at a very early age were showing evidence of controlling for variables. It's lovely. And they were being exhaustive.*
70.		Brian	I have to do the opposite. I'll do this-
71.		Stephanie	We made a pair!
72.		Dana	No, look. Look, that's fine. That goes with this one.
73.		Stephanie	No it doesn't because if you turn it around, it's the same, so that doesn't go with that one.
74.		Dana	That one goes with that one.
75.		Stephanie	Wait, let me check. Let me make sure...No that doesn't because...
76.		Romina	I think we have them all.
77.		Carolyn Maher	Do you think it's possible to have an odd number?
78.		Student	No.
79.		Carolyn Maher	They have an odd number - 35.
80.		Mike	You're not supp- You can't because when you have a number, you could have the opposite. And if you would have one of this, you have

			another one because it's the opposite. And if you have 10 of these, you have another one that's opposite, so that makes 20.
81.		Shelly	We found 32.
82.		Carolyn Maher	You found 32? How did you do that?
83.		Jeff	Easy. You just go this way and then-
84.		Carolyn Maher	You're tired, Jeff. Jeff, how do I know that you don't have duplicates?
85.		Jeff	You can check: all you want.
86.		Carolyn Maher	Because you checked it. How? ... That's how you checked it, you compared? How do you know you there're not 34?
87.		Jeff	I can't make any more. My brain is tired!
88.		Carolyn Maher	Your brain is tired?
89.		*Carolyn Maher*	*So you might ask us - Why did we ask them to convince us? Why do we ask them to justify? Well, we do that because beginning when they start, they solve their problem randomly. It's sort of guess and they try something. When you don't know what to do, you try something, so you'll build something. And maybe you'll notice certain kinds of patterns in your building; maybe you won't. You might just do trial and error, trial and error, trial and error. We want students to get past trial and error.*
90.		Carolyn Maher	Okay, let's take another set and try to convince me the same way.
91.		Jeff	We'll show you the other set.
92.		Carolyn Maher	Okay. I believe this one, too - you can have one red, right? And you have the other possibilities. I buy that.
93.		Jeff	There's only two kinds of these because there are alternates.
94.		Carolyn Maher	Okay, I buy that. All right. You're convincing me. That's great.

95.		Jeff	This, we just ... How are we going to convince her about this one?
96.		Carolyn Maher	You've got to convince me about this one. Why don't you think about this? I'm convinced about these that there are no other possibilities when you have one of a color - either one yellow or one red. Okay? I'm not so sure I'm convinced if there's two reds or if there's two yellows, so why don't you work on convincing me of that? You think about it and I'll be back, and you can call me.
97.		*Carolyn Maher*	*But they're thinking was still very, very exhaustive and it was very organized - when they had to justify their solutions. What it does, then, is it enables them to look at what they have, that they did just by hard work and drive, which we skip in school; we skip that piece of it. How awful - because we don't have time. You know, we skip that drudgery of that going through this hard, hard work we might not see the point of. We don't look enough. Because as they're going through this real intense, hard work, they're noticing things about the structure of the problem - maybe not seeing it overall, but they begin to notice relationships, they begin to notice sub-patterns, and they invent names for these. They really get to know the task well. This is what we expect mathematicians to do in their work.*
98.		Jeff	...we tried to make like patterns...
99.		*Narrator*	*The following day, the researchers interviewed the students about their thinking on the towers problem.*
100		Jeff	First we did this - we started out by moving the block up in each one.
101		Carolyn	What did you come up with as a solution to

		Maher	the problem? Did you finally decide on how many?
102		Jeff	We decided on 32. And we kept on going up -
103		Carolyn Maher	Right.
104		Jeff	- And then we did the opposite of it.
105		Carolyn Maher	So, you decided on this pattern, that there were how many like that?
106		Jeff	It would go up to five.
107		*Carolyn Maher*	*The interviews served multi purposes for us. It validated some of what, to us, were just theories. We had certain theories about what they were doing and what they were thinking based upon what we observed, what the cameras caught. But we weren't sure how aware they were of that thinking.*
108		Stephanie	Well, 'cause this one, we had the pattern, the two, and then the two blocks up, and then the two blocks up.
109		Carolyn Maher	Yes.
110		Stephanie	And then we had the two in the middle, and we had the two here, and the two here.
111		Carolyn Maher	Okay. So, that was the other pattern you had. I'm confused though. How do you know that some of these aren't these?
112		Stephanie	Oh, that’s right. This one is this one. So, this one's -
113		Carolyn Maher	How did you deal with that yesterday? Did you end up counting things?
114		Stephanie	We ended up counting a lot over. We had 34 and we had it so we subtracted, I think, three groups, because we were down to 28 and then we added two groups.
115		Carolyn Maher	So, so, you think that that's what was happening yesterday?
116		Stephanie	Yeah, we kept, we kept finding different

			patterns, but we didn't check it with the other patterns.
117		Carolyn Maher	Uh-huh, okay.
118		Stephanie	It's really the same pattern in different places.
119		Carolyn Maher	Right.
120		Stephanie	It's taking one - building on one pattern. It's, okay, say I started with the pattern at the top. You're taking that pattern and you're moving it down one and then you're moving it down another and another.
121		Carolyn Maher	I'm just wondering if we can come up with a way that would make it easier to remember, because it's a nice way of trying to find them. I like your patterns, but I worry about if we're missing some or counting some twice. That's tricky. You might want to think about that: a way of trying to come up with a way to do that.
122		Stephanie	Yeah. How could you be absolutely, positively... A guess, a very lucky guess.
123		Carolyn Maher	Yeah, but math isn't a guess. Math you should be able to figure it out and be convinced. And you promise you're going to work on this?
124		Stephanie	Yeah.
125		Carolyn Maher	Okay, that's going to be great. I can't wait to talk to you about it some more. So, imagine the four towers and imagine the five towers and if you have time, imagine six.
126		Stephanie	All right, and I'll work on a way to be definite about your answer.
127		Carolyn Maher	Yeah, that would be exciting. That would be really fun. Terrific. Well, thank you so much.
128		Stephanie	You're welcome.
129		Carolyn Maher	You have a good weekend. I enjoyed this very much.

Description: PUP Math - Towers **Location: Harding School – Kenilworth, NJ** **Researcher: Professor Carolyn Maher**	**Transcriber(s): Private Universe Project** **Verifier(s): Sigley, Robert, Sran, Kiranjeet** **Date Transcribed: Spring 2000** **Page: 11 of 11**

130		Stephanie	Thank you.
131		Carolyn Maher	It was good thinking, very good.