Description: PUP Math – Brandon interview Location: Conover Road School – Colts Neck, NJ Researcher: Professor Carolyn Maher	Transcriber(s): Private Universe Project Verifier(s): Sigley, Robert, Sran, Kiranjeet Date Transcribed: Spring 2000 Page: 1 of 8

Line	Time	Speaker	Transcript
1.		*Narrator*	*When the researchers gave them the pizzas with four toppings problem, most of the students made lists of toppings and counted their combinations. But researcher Amy Martino noticed that one student, Brandon, used a highly unusual and insightful method of keeping track of his combinations. Brandon first made a chart with the toppings arranged vertically in columns. Moving down the page, he worked methodically row by row to create his pizzas. He wrote a one in each column to represent the inclusion of a topping and a zero to indicate when a topping was not present.*
2.		Brandon	...I'm making a graph.
3.		Amy Martino	What does that mean, one-zero, one-zero?
4.		Brandon	Well, instead of using, like, you have pepper down, or sausage down, I'm just going to put, like, a one, for like, "Yes, it's going on," and zero for "No I'm not."
5.		*Narrator*	*One month later, in an interview with Amy Martino, Brandon was asked to recreate his chart and account for all possibilities.*
6.		*Carolyn*	*The interview was to validate what we already found in the classroom, and Amy wanted to push it further. We did not expect Brandon to do what he did. It was spontaneous.*
7.		Amy	Okay. You want to tell me about what you're doing here, and how these turn out to be pizzas, these zeroes and ones?
8.		Brandon	Well, since there are three, four toppings, that is. Nothing on the pizza. And you could have one pepper on the pizza with nothing else, one mushroom on the pizza with nothing else. Then you could have a

			couple sausages on the pizza with nothing else, maybe a couple pepperonis. And if you don't want to have that, you could start getting fancy and go into twos. So have a pepperoni and mushroom, nothing else, then a pepperoni-sausage, nothing else.
9.		Amy	Mm-hmmm.
10.		Brandon	Pepper and pepperoni, nothing else, and so on. Then, since we're all done with pepperoni, you could have a mushroom and sausage with nothing else.
11.		Amy	What do these zeroes and ones mean? Like what does the zero represent here?
12.		Brandon	You have nothing on that - that's nothing. I don't know why I chose to use zeroes and ones.
13.		Amy	Mm-hmm. I was going to ask you about that, where you got this idea from?
14.		Brandon	I don't know how I got it. It just popped into my head.
15.		Amy	Oh.
16.		*Carolyn*	*Some of my colleagues were saying to me, at the time, "Maybe his father is a computer scientist, and he is exposed to binary numbers, and that's how he knows his ones and zeroes." Well, his father is a businessman. His mother was a homemaker. And as we pushed that, nope, we eliminated that possibility. Brandon didn't have a computer at home. He wasn't a person who worked on the computer all of the time. Literally, the idea of zero and one popped into his head, just as he said it.*
17.		Amy	Can you show me what - you have them in groups here - can you show me what those groups are on here?
18.		*Narrator*	*Brandon divided his chart into groups,*

			organized by the number of toppings.
19.		Amy	Okay. And what group is that?
20.		Brandon	Okay. Here's the "ones" group.
21.		Amy	Okay, and what does that mean, the "ones" group?
22.		Brandon	You only have one topping in the group.
23.		Amy	Okay.
24.		Brandon	Then you could have the "twos" group, which will go about - The "twos" group is like the most.
25.		Amy	What do you mean, "the most"?
26.		Brandon	You get the most out of two, because you get more choices than one, and you get more choices: pepperoni and mushroom, pepperoni-sausage, pepper-pepperoni, and that so on ... So the "two" group is, like, the biggest.
27.		Amy	Can you convince me that there aren't any more in the "twos" group, that there aren't seven or eight?
28.		Brandon	You go, pepper-mushroom, that's one. Pepper-sausage, that's two. Pepper-pepperoni, three. Then you can't do any more, because you already used sausage once and mushrooms once. And to tell that you already - And to see that you made duplicate, look over there, and "one." Because if you just look there, you'll see another one. But if you see a zero there, that means it's not a duplicate, because you've got nothing there.
29.		Amy	Okay.
30.		Brandon	So if there's a "one/one", then that would be the same as there. Then you get into mushrooms...
31.		*Carolyn*	*He decided to keep track of his pizzas by saying it either had a particular topping, or it did not. And he did it in a very systematic*

			way. And as his chart reveals, he accounted for all possible pizzas, and he had 16. It was the notation he used that helped him.
32.		Brandon	So then your only choice left is having an "all" pizza, with everything.
33.		Amy	Interesting. And what are we calling this group?
34.		Brandon	The "all"...I don't know what I call that. The "total."
35.		Amy	Okay, the total. You call these the "zeros," the "one toppings," right?
36.		Brandon	Yeah. "Two toppings," "three toppings," "four toppings."
37.		Amy	You call it four toppings, right? Sure. Does this problem with pizzas remind you of any other problems we've done this year?
38.		Brandon	It kind of a little reminds me of the blocks, because you ...
39.		*Narrator*	*When Amy asked Brandon if this problem reminded him of any other problem. He asked for manipulatives, and started making towers. He showed how each topping column in his chart corresponded to one position on the tower, with a "one" on his chart representing yellow, and orange represented by a "zero." Brandon organized his answer by categories, based on the number of blocks of each color.*
40.		Brandon	It's kind of like the pizza problem. You start off with the group. Like this would be the "ones" group. Oh yeah, I see this now. This is like the "ones" group. You only have one of the opposite color in there. This isn't how I did it, but I just noticed this.
41.		Amy	This is fascinating to me.
42.		Brandon	I just noticed it. Then you would have - that would be the "ones" group - you only

			have one...
43.		*Carolyn*	*He did exactly the same rebuilding of towers at that interview session as he did in the classroom. He found the tower and an opposite, the tower and an opposite. And he found all 16. But something happened; something happened in his head. Because he said, "Wait, I just thought of something. Just a minute." And he had these tower models right in front of him, and he reorganized them in a way that they mapped into his chart for pizzas.*
44.		Brandon	... you have one pepperoni. That would be like - one pepperoni is like. Since we were looking at yellows, a yellow would be "one", the reds would be "zeroes." You could have one pepper, like I chose here, and right there. Then it's like stairs. If I draw a line down -
45.		Amy	You need a pen?
46.		Brandon	If I draw a line down here like this, it would go like - sort of look like stairs.
47.		Amy	I see.
48.		Brandon	Then you'd go across, draw a line down there, go across, draw a line down there, across, draw a line down there - across - So you would have, like, "one," "one," "one," "one." It's sort of like here. You have one pepperoni, one mushroom, one sausage, one pepper.
49.		Amy	Oh! Is what you're saying to me then that, like, the yellow cube here is like a number one on your chart?
50.		Brandon	Yes. If we were focusing on red, a red would be a number one.
51.		Amy	Okay. Well let's continue with yellow. This is interesting. I think this is really neat. Now, what would come next, with what we have here, if we want to reorganize.

			You said these would be like the one - yellows.
52.		Brandon	Yeah. These are the "ones" group.
53.		Amy	Okay. What about -
54.		Brandon	Now you would start with the "two" yellow group.
55.		Amy	Okay.
56.		*Narrator*	*Brandon referred to his notations, and demonstrated an exact correspondence between each tower he had built and each pizza on his chart.*
57.		Brandon	Yellow-yellow, red-red. Same here. Because if you wanted to stand them up, it would be harder to have to stand up the paper. So it's yellow-yellow, one-one...
58.		Amy	I understand.
59.		Brandon	That would be a "two." Then you could have 'em
60.		Amy	Yeah, what would the tower be that would like this pizza?
61.		Brandon	Right here you would have yellow stand for "one." So you would have a yellow "one," red "zero", yellow "one," red "zero."
62.		Amy	I see.
63.		Brandon	That would be another one.
64.		*Narrator*	*When two problems that might look different on the surface, like towers four high and pizzas with four toppings, have the same underlying mathematical structure, this is called isomorphism.*
65.		*Carolyn*	*Brandon recognized the isomorphism after working on pizzas. What students sometimes do is they think of one problem one way, they think of the other problem the other way, and don't see the equivalence in structure. So to recognize the isomorphism is to disclose that*

			equivalence by looking at both problems in very deep ways.
66.		Brandon	If we're just focusing on yellows, then the pizza with everything.
67.		Amy	Oh, I see. Okay. And are we missing any?
68.		Brandon	No.
69.		Amy	You know what I'm wondering? We have this guy left, right?
70.		Brandon	Yeah, because we're not focusing...
71.		Amy	Because he's the opposite of this guy?
72.		Brandon	Yeah, we're not focusing on red.
73.		Amy	If we had to call him a name, though -
74.		Brandon	Oh, this will be the "zero." Oh yeah. Since the reds would stand for "zero," this would be a "zero" guy.
75.		Amy	This is neat. This is really neat, Brandon.
76.		Brandon	I finally found out what the red would be. Red: "zero" guy.
77.		Amy	I wanted to ask you. Could we have done it the other way around? Could we have focused on red and gotten it to work the same way?
78.		Brandon	Same way. It would just look like this. Here's the "ones" group, "twos" group -
79.		Amy	One red. Okay.
80.		Brandon	The "twos" group would be the same. And then all you'd do is -
81.		Amy	What would these be? What would these things be?
82.		Brandon	That would be the "threes" group. And just switch those around. Same thing.
83.		Amy	Neat! Now, would we be changing the number names for red and yellow? In other words, when we did this -
84.		Brandon	Yeah. Now the reds would be "one" and the yellow would be "zero."
85.		Amy	This is really nice. Are you convinced that

			you found all the towers and all the pizzas?
86.		Brandon	Yeah. All the towers, all the pizzas. Yeah.
87.		Amy	They both come out to how many?
88.		Brandon	It's 16. Two, four, six, eight, ten, twelve, fourteen, sixteen.
89.		Amy	Are you convinced of this now?
90.		Brandon	Yeah.
91.		Amy	Yeah? This is really very nice.
92.		*Carolyn*	*Brandon had an opportunity to think deeply about a problem. And he had an opportunity to talk to someone about his ideas. I think we have to remember - We see Brandon and we all so impressed with what he did. And what he did was very impressive. But at that time, the schools grouped students according to math ability. They don't do that anymore. This was many years ago. And Brandon was in the lowest group. And when later we went to the teachers with what we found, with our interview of Brandon, and we said, "Look. Look at this! This is just absolutely brilliant. This is wonderful; this is amazing!" And they hadn't seen anything like that, they told us.* *Well, I think we don't see these things because we don't give students an opportunity to show us their thinking. I think the world is full of Brandons. We just don't take the time to find them and to listen to them. We don't have mechanisms to pull them out. I think they're all over.*