Description: PUP Math – Rominas Proof **Location: David Brearley High School – Kenilworth, NJ** **Researcher: Professor Carolyn Maher**	**Transcriber(s): Private Universe Project** **Verifier(s): Sigley, Robert, Sran, Kiranjeet** **Date Transcribed: Spring 2000** **Page: 1 of 7**

Line	Time	Speaker	Transcript
1.		CAROLYN MAHER	So you have to convince me that you've found them all.
2.		NARRATOR	About one month later, the students met again with Caroline Maher after school. On the agenda were variations of the towers problems. As a warm-up exercise they looked at the problem: Choosing from two colors, red and yellow, how many total combinations exist for towers five tall that each contain two red?
3.		ANKUR	the first number, it's a one there, and then puts a one here, and then the rest are zeros.
4.		NARRATOR	Mike and Ankur quickly solve the problem by counting combinations.
5.		ROMINA	You guys proved it already.
6.		ANKUR	Yeah.
7.		BRIAN	...Throw it out.
8.		CAROLYN MAHER	No, we're going to wait to hear what you do, and you're going to hear what they did
9.		BRIAN	- tell us now.
10.		NARRATOR	While they were waiting for the rest of the group, Ankur started playing around with a new problem that he had just invented: How many combinations can you make with towers four tall, selecting from a choice

			of three colors, and using at least one of each color in every tower?
11.		ROMINA	Let's use one's, zero's, and x's.
12.		JEFF	One's, zero's, x's?
13.		CAROLYN MAHER	Notice, Ankur's problem is not trivial; it is really rather complicated and it is challenging, I'm sure, for those who are watching these tapes- will find the problems, themselves, challenging. But mind you, this is a problem posed by one of the group. So, I think this notion of problem-posing of students is something that we ought to think about and ask ourselves: What are the problems that our students pose to each other to solve, and can they solve them?
14.		NARRATOR	Building on their experience with counting towers, they worked on Ankur's new problem for fifteen minutes, arriving at the answer to the simpler problem: The number of combinations for towers four tall, choosing from three colors.
15.		ANKUR	There's eighty-one total.
16.		JEFF	Of these?
17.		ANKUR	No, of, like, everything.
18.		ROMINA	How did you get eighty-one?
19.		ANKUR	Do it and you'll figure it out.
20.		ROMINA	No Ankur.
21.		JEFF	X to the y. X is three?
22.		ANKUR	It's three to the fourth. Because look-
23.		JEFF	Three times three is nine, times three is twenty-seven, times three is eighty-one.
24.		NARRATOR	After calculating that there were eighty-one total towers when selecting

			from three colors, Mike and Ankur returned to the conditions of Ankur's problem and came up with thirty-nine combinations, which is close but is not the correct answer. Brian, Jeff, and Romina approached the problem differently.
25.		ROMINA	You have to have two of the same color, right, in one of them, if we're going to have all three colors, right?
26.		ANKUR	I have no idea what you just said.
27.		JEFF	You need to have 2 of every one.
28.		ANKUR	Okay, okay.
29.		ROMINA	So you have to organize them so you don't have any doubles. You can have them next to each other, you can have them separating, one on the end, in the middle, then two in the fourth spot, and the third in the fourth spot, right?
30.		ANKUR	Yes.
31.		ROMINA	So that's six.
32.		ANKUR	Yes.
33.		ROMINA	Now, in the other spots, you have an O and an X- those are colors, these are three different colors- an O and an X and an X and an O. So you have to multiply each of these six by two.
34.		JEFF	And you couldn't have, like, XX, because that wouldn't meet the requirements. So you multiply each one by two, so that would give you twelve, correct? Because that means you could have either the bottom or the top? So that's 12.
35.		ANKUR	Hold up. I just want to think about it for a second.
36.		ROMINA	Six times 2 is 12, 6 times 2 is 12, 6 times 2 is 12, 6 times 2 is 12, 6 times 2

			is 12, 6 times 2 is 12.
37.		JEFF	Why you keep crossing that out?
38.		ROMINA	Because that's wrong.
39.		BRIAN	Yeah, it is.
40.		ROMINA	You multiply all this by two, right, and you multiply all that by three because of the three different colors. That's what we were trying to say, but we wrote it bad.
41.		JEFF	She wrote it funny.
42.		ROMINA	So you can multiply these all by two, right, because you have one-color or the other, right? And then you have to multiply all by the three because ones can be any color, can be the three colors.
43.		JEFF	There's twelve this way and there'd be twelve if you took the Xs, put them here, took the 1s, put them there- that's twelve more, and there's twelve more if you took the 0s and put them here and put the Xs back over there with the ones.
44.		ROMINA	So it's thirty-six.
45.		NARRATOR	Romina visualized the possible set of towers being divided into six groups. since every tower would have two of one color, Romina focused on the placement of the duplicate color. This gave her six placements. Romina understood that for each placement of the first, or duplicate color, there would be two possible combinations for the second and third colors. She also realized that these combinations would have two opposite arrangements for the second and third colors. So then 12 is multiplied by 3 to

			represent every color, making a total of thirty-six.
46.		MICHAEL	Explain the thirty-six one more time because I was not paying attention. [simultaneous conversations] Now I want to know. Was it a good explanation? I wasn't paying attention.
47.		ROMINA	I knew you weren't paying attention. All right. We have all three colors, right?
48.		MICHAEL	What's 1, what's 0, and what's X?
49.		ROMINA	The three different colors.
50.		MICHAEL	I understand.
51.		ROMINA	We have three different colors, and then you know that they have to be paired up, like the fourth color being added has to be the same as one that's already there, right-
52.		MICHAEL	The fourth color has to be the same, yes-
53.		ROMINA	Because you have-
54.		MICHAEL	Yeah. Okay, so what we did, we said you have, let's say these are your four different ones, and we came up with six different possibilities where the match could be. It could be here and here, here and here-
55.		CAROLYN MAHER	Romina provides a solution that is a proof, and she uses, notice, some of the notations that were introduced earlier by Michael, and she accounts for all possibilities. This is a beautiful moment; this is spontaneously working for a very short period of time, the student provides a solution that is an elegant proof.
56.		ROMINA	Okay, do you agree with me? And then each one, this is either going to remain

			0 and an X or an X and an 0.
57.		ANKUR	So there's two of each one. You can't have an X and an X.
58.		MICHAEL	I get that.
59.		ROMINA	You get that?
60.		MICHAEL	Yeah.
61.		ROMINA	So, so far we have six and we have to multiply the six by the two for all these, so you get twelve, right? You multiply the twelve times the three, to get thirty-six, you multiply it because it's three different colors. So each one can be- you multiply that to get 36.
62.		CAROLYN MAHER	You know, when we started working with the students, we never knew how far they would get. We just accepted what they did. So, teachers who are starting to do this with their students now in their classroom should be encouraged to know that it's helped the students enormously to have begun the investigations and tasks in their earlier years. It should also help them to know that our students, just as theirs, did not all get to the same place. All of them didn't come up equally with the same always-new, brilliant idea. That varied, and that's okay, because they do learn from each other and ideas travel within the community. The students are doing mathematics, they're working as mathematicians, they are building for themselves very powerful images, they are having an opportunity to use and develop and try strategies, and they're having such fun doing it. They're enjoying it. They feel good about that enjoyment, they feel

			good about themselves and their success.
63.		NARRATOR	Is Romina's argument convincing? Why, or why not?