Description: PUP Math – Pizza Problem (2 toppings w/halves) **Location: Harding School – Kenilworth, NJ** **Researcher: Professor Carolyn Maher**	**Transcriber(s): Private Universe Project** **Verifier(s): Sigley, Robert, Sran, Kiranjeet** **Date Transcribed: Spring 2000** **Page: 1 of 10**

Line	Time	Speaker	Transcript
1.		Student	In this pizza, in each slice I put…
2.		*Narrator*	*[We've seen some of the thoughtful and creative approaches the students used as they uncovered the mathematical similarity between the towers and the pizza problem.]*
3.		Brandon	Red- zero; yellow-one; red-zero.
4.		*Narrator*	*[In mathematics, just because everyone agrees on an answer, it doesn't mean they're right. How can you teach students the difference between feeling you're right and proving you're right?]*
5.		Carolyn	Okay. Hi, everybody. There's a problem on your table. And if you'll all take a copy - and you night want to read it yourself; then we could be sure you understand it. You might talk among yourselves.
6.		*Carolyn*	*It seems, on the surface, like a very simple problem: how many different pizzas can you make when you select from two toppings? However, as in many restaurants, you're allowed to order a different topping on half of the pizza, if you choose. So how many choices do you have? So, this was a very real problem. It was something that they would encounter in their normal, everyday life. And they never thought about all possibilities before. So it was not difficult to engage them in this problem. And they quickly saw that it got complicated very fast.*
7.		*Narrator*	*[The students were divided into two groups, and worked on the problem for about 45 minutes.]*
8.		Romina	Six?
9.		Ankur	Wait. O.K. Look, the plain pizza that's one.
10.		Bobby	Half a plain.
11.		Ankur	Then half sausage, and half pepperoni.
12.		Brian	One whole plain.
13.		Ankur	No, wait.
14.		Brian	One whole sausage, one whole pepperoni.

			There's three.
15.		Ankur	Now, half plain and half sausage.
16.		Brian	One plain, one sausage.
17.		Ankur	OK, Mike you draw the pizza, and then Amy will write underneath ... Make a pie, and make it whole plain. Just put, like cheese - a cheese pizza.
18.		Brian	Here, Ankur. Half pepperoni and sausage, half pepperoni. Half sausage and pepperoni and…
19.		Ankur	No, half plain and sausage, half pepperoni.
20.		Bobby	What are you doing?
21.		Ankur	Forget the flames, Mike. Okay? There. Now make - now put one sausage, like a sausage one.
22.		*Narrator*	*[While one group of students tried to write or draw all possible combinations, the students at the other table argued over the best way to organize their answer.]*
23.		Stephanie	Matt, that kind of graph isn't right, because it's a cheese, pepperoni, and sausage. All you're going to get there is cheese, pepperoni and sausage. You cannot put - Because it's not organized. You can't put cheese and sausage in a group. You'd have to put the cheese over here and the sausage over here. So why don't you just make - OK a little graph like this.
24.		Jeff	Because you're going to put all in one column, and then you're going to put the same amount in the next column, and then you're going to put the same amount in the next time, and then you're going to be crossing out two column's worth. It's a waste of time.
25.		*Jeff (Voice Over)*	*We didn't know if we were right, most of the time, you know? I would have an idea of how to get to a certain point, and you might have the same idea how to get to it, but we'd have to - Getting there was the hardest part, and that's what we were arguing about - the right way to get there or the right way to make sure that*

			you'd covered all the bases. You know, anyone could pick up a pen and get the right answer. But knowing how to get there, that was what we were arguing about: the right way to get there and how to make sure, how to prove. That was a big question at the time, how to prove what we needed to accomplish.
26.		Matt	Why don't you just draw it, like -cheese and.. But that isn't organized. Keep it organized, it'll be easier.
27.		Stephanie	Well, that's not - Well, how can you organize it? How do I know whether to put this under cheese or sausage? How do I know whether to put this under cheese or pepperoni
28.		Jeff	Your graph was great. Like, you said, we should make a graph with the one toppings and the two toppings and the threes.
29.		Stephanie	Yeah, because Matt
30.		Jeff	Nobody knows what that means.
31.		Stephanie	- how do you know? How do I know? You know, how do I know whether I put this under cheese or sausage? Or how do I know whether I put this under cheese or pepperoni?
32.		Matt	Put it under the column.
33.		Stephanie	But, yeah, but there's not going to be a cheese and pepperoni column, I mean, or a cheese and sausage column. That's a pizza. You don't have to make a column for that one little pizza. Do you know how many graphs that is? You know, you'd have to make, like, tons of little, separate, eeny-weeny
34.		Jeff	Eeny-weeny.
35.		Stephanie	[laughter] graphs.
36.		*Matt (Voice Over)*	*Maybe you took your idea, and put this on it. Okay. So then you go around - another person. "What do you think about this?" "Do that and that." And he'd say, "Well, what if you put this on it?" And it kind of comes into one big, whole thing*

			that you use to solve your problem.
37.		Brian	I'm saying one plain -
38.		Ankur	Do what Brian says!
39.		Brian	One sausage and pepperoni pizza.
40.		Bobby	We already have that.
41.		Ankur	We have that.
42.		Brian	Mixed!
43.		Bobby	That is mixed, almost.
44.		Brian	It's half and half! I mean mixed.
45.		Ankur	I know what he means.
46.			[Students agree.]
47.		*Narrator*	*[Even though they had some disagreement over their methods, by the end of this session, both groups had come up with a preliminary answer: ten combinations.]*
48.		Ankur	Ten. Now that's seven, eight, nine, ten, …
49.		Carolyn	Okay. I think that -
50.		Jeff	Don't tell me we're out of time!
51.		Carolyn	I know. Isn't that awful Jeff?
52.		Jeff	Ooooaaahhhh!
53.		Carolyn	It's really kind of disappointing to me that we do get out of time so fast.
54.		Jeff	Why don't we eat lunch here and come back after lunch?
55.		Carolyn	Can we come back tomorrow morning?
56.		*Carolyn (Voice Over)*	*They're so committed to working these problems out that they don't want to be disturbed, and that they say "Let us have the time." Isn't it lovely? I mean, schools aren't structured to do that. But isn't it so nice when we can do that?*
57.		Carolyn	This is a real problem, by the way. In fact, we have here Mrs. Weir, who's given the same problems for a college class. So we're not really giving you things that aren't important and the kinds of things we want you to do in the future. So think hard about this. You know, it's one thing to find them - "I think I have them all."

			Remember the towers, "I think I have them all?" But then there's the next question. How could you convince us that you have all possible ones?
58.		Jeff	Why do you always have to ask that question?
59.		Milin	Yeah.
60.		Carolyn	Because that's the mathematics of it; that's when you become mathematicians. That's when you become real problem solvers.
61.		Milin	IF everybody agrees, then - if everybody agrees in this whole class, then can you guys…?
62.		Jeff	Yeah, but this is just a class of 12 kids. If you go to ask another class, they might not all agree.
63.		Stephanie	Besides that, you know, the person that doesn't agree could be right.
64.		Carolyn	Let me say it another way. I have you on film in certain grades where you've all agreed, and you've been wrong. So that's the challenge to you now. That's what it is to do mathematics. That's what mathematicians do. You've taken it to the level of trying to convince, and that's what we're asking you to do. So kind of put your names on your papers, and leave them there, and we'll see you tomorrow. [BELL]
65.		*Shelly*	*Like, with the Rutgers, a lot of times, we found an answer. And that usually wasn't good enough. They wanted to know, well, how did you know it was the right answer? And because there was no teacher there to tell us, "Yeah, that was right" or "That was wrong," and they didn't just tell us how to do it, you had to look at it and look at it over and over again, and compare it to everybody else's answers, and see how they came about their answers and how it compared to how you got your answer. And you went through the whole process over and over again, and then you started to branch out to different answers to see if they were right. And a lot of times, in the end, you ended up with your original answer, but you*

			were more secure, knowing that was the right answer.
66.		*Narrator*	*[The next day, the students returned to the same problem for another 45 minute session.]*
67.		Alice Alston	Would you all mind if we sort of worked together, if some how we worked out a way of checking your lists and your pictures and each other's list and making sure that we all agree that everything we got is right?
68.		Brian	Here, a person can read out one of them, and we could say if
69.		Ankur	Check them.
70.		Brian	We wrote them or not.
71.		Romina	One plain. [Wait.] [Check.] One pepperoni.
72.		Brian	Check.
73.		Romina	Half pepperoni – half sausage.
74.		Matt	What are we doing?
75.		Stephanie	Figuring out our charts.
76.		Matt	Here's what we'll do.
77.		*Narrator*	*[The students spent a few minutes negotiating their justifications, and preparing charts to help them present. By now, both groups had confirmed that there were ten possible combinations.]*
78.		Carolyn	Can you sort of, in a very general way, tell me why you think ... ? You know, you really were-
79.		Stephanie	We can't get any more. We've been working, we've been -
80.		Carolyn	You should be able to have a picture in your head ... of why ... -
81.		Stephanie	We've proved everything to everybody in this group. All right. What we did is we put them into columns of one - which is a whole pie,
82.		Jeff	I just wrote mine out.
83.		Stephanie	two - which is two toppings on a pie,
84.		Jeff	Put that in you key.
85.		Stephanie	and three - which is three toppings on a pie.

			Okay?
86.		*Narrator*	*[Stephanie's group made notations to account for all of their combinations. Notice that they treated the plain, or cheese pizza, as a topping. They listed three categories of pizzas, based on the total number of toppings that were used.]*
87.		Stephanie	Now for a whole pie, you can have cheese, you can have pepperoni, and you can have sausage. You can't have it any other way. There's no other way you can get a one topping whole pie.
88.		Michelle	Why!?
89.		Stephanie	Because there's only three toppings.
90.		Jeff	Explain why.
91.		Stephanie	Because there's only three toppings.
92.		Jeff	How are you going to convince me?
93.		Stephanie	I'm not convincing you. I'm convincing her. Are you convinced?
94.		Teacher	Yes.
95.		Stephanie	See, she's convinced. Okay.
96.		Carolyn	Jeff, you're convinced too, aren't you?
97.		Jeff	Yeah.
98.		Stephanie	Two, we have halves and two toppings. Two toppings, okay? Plain old two toppings. And we have pepperoni, and then on the other side, sausage.
99.		Teacher	Now is cheese on there also?
100		Stephanie	Yeah, cheese is automatically on there.
101		Teacher	Okay.
102		Stephanie	Then we just put cheese on there to show you that there's, like, cheese on it, you know?
103		Jeff	Yeah, she was sitting there crying before, "that's not cheese, why do we call it cheese!?"
104		Stephanie	Leave me alone. You can put cheese, and then on the other half, pepperoni; cheese, and then on the other half, sausage. Or all together, mixed, no one-half the other, sausage and pepperoni.

105		Jeff	Your sure that's it?
106		Teacher	All right.
107		Milin	Jeff if you ask another-
108		Stephanie	Are you convinced? Okay. Then for three, we have sausage and pepperoni on one side, and sausage on the other.
109		Teacher	Oh, so you're allowed to mix the sausage and pepperoni on one side?
110		Stephanie	Yeah. Okay. And then we have sausage and pepperoni on one side and pepperoni on the other. Then we have sausage and pepperoni on one side and cheese on the other.
111		Michelle	Or half of the side is plain.
112		Teacher	All right. I think I got it.
113		Carolyn	OK you're convinced? You all convinced? Okay, that's great.
114		*Narrator*	*[Brian's group also divided the pizzas into categories: whole pizzas with single toppings, halves with different single toppings, and mixed. Pizzas with two toppings, both sausage and pepperoni.]*
115		Brian	We know that there's no more wholes, there can't be any more.
116		Ankur	There can't be wholes. We know there's no more halves. And no half and mixed.
117		Alice	How do you know there's no more halves?
118		Ankur	In halves, because we used all the, like, ingredients in the pizza.
119		*Ankur (Voice Over)*	*When the Rutgers program comes over here, they always ask us to convince them or they always ask us to convince the other people in our group. While we convince, we realize that we're actually learning more, we understand the concept better, and we help others understand the concept, and everyone in the group learns together.*
120		Brian	Because, uhhmm, plain, that's like considered like a topping.

121		Alice	Sure.
122		Brian	Yeah, plain, you can only use two other toppings, because that's all they give you.
123		Alice	Yeah.
124		Brian	So you use pepperoni as half and half, or half pepperoni and half plain. And then you use the other topping, which is sausage to put on half a pizza. Not mixed on whole ones, like half pepperoni and half sausage.
125		Alice	Okay. And you had one more category?
126		Ankur	Yes. We had…
127		Brian	Half pepperoni and sausage, mixed.
128		Ankur	Half one side and the other side mixed. One side is half, the other side is mixed.
129		Alice	OK, say that again.
130		Ankur	One side is like, mixed and the other is, like, a whole –no, wait.
131		Romina	Like with two colors.
132		Brian	Like with two colors. Like one side could be all different colors.
133		Ankur	And another side the same color.
134		Alice	So one side is the mixed sausage and pepperoni?
135		Brian	Yeah.
136		Ankur	And the other side is
137		Brian	Just, like, one thing.
138		Ankur	Just one thing. And so how do we write that?
139		Brian	… It could be sausage or pepperoni.
140		Alice	And that's all it could be?
141		Brian	Right.
142		Alice	Why?
143		Brian	At the end, the one's that are non-mixed. That's all the toppings.
144		Alice	Because one side is either sausage or pepperoni?
145			[Bell rings.]
146		*Carolyn*	*Always, we try to push students a little beyond*

			where they were. It was never about solving a particular problem. It was about looking at other problems, maybe, in this class, and seeing if they could come up with a generalization. So very early on, they were doing this. They might not have had the - quote - "standard notation" to do this. They sometimes did it in words. And when we thought they had the idea, we thought that would be the opportune time to now bring in the standard notation and see if they now re-represent their idea with the standard notation.