

Description: PUP Math – Pizza Problem (4 toppings wholes and halves) Location: Harding School – Kenilworth, NJ Researcher: Professor Carolyn Maher	Transcriber(s): Private Universe Project Verifier(s): Sigley, Robert, Sran, Kiranjeet Date Transcribed: Spring 2000 Page: 1 of 7
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Line	Time	Speaker	Transcript
1.		Narrator	<i>[One month after working on the problem of pizzas with two toppings and halves, the same group of 12 students met for an extended session, lasting approximately 2-1/2 hours. This time, the researchers began with the simpler problem: How many different combinations could be made when selecting from four toppings, with no half pizzas?]</i>
2.		Alice Alston	We have to make a decision. Did they say anything about halves or is this just pizzas?
3.		Jeff	Oh, wait there's no halves. Yes, hallelujah!
4.		Alice	Read it, what do you think it says?
5.		Ankur	Wait, but it says how many different choice does...
6.		Jeff	I don't think they make halves there. [Wholes! Wholes! Wholes!]
7.		Alice	I think it's just whole pizzas.
8.		Class	YEAHHHHH!!!!
9.		Jeff	Thank the Lord!
10.		Matt	Cross it out!
11.		Alice	OK, do you all want to work for a couple minutes, and see if you can come up with something?
12.			[Student conversation and writing]
13.		Romina	... and the plain, too.
14.		Ankur	What about the mixed?
15.		Jeff	The plain!
16.			[Student conversation]
17.		Alice	And then what was this pattern?
18.		Ankur	I started with the first one, and mixed it with the second one. That's "P" slash "S." Start with the first one mix it with the third one: "P" slash "M." And then "P" slash "PE." And then start with "S": "S" slash "M", "S" slash "PE," then "M" slash "PE."
19.		Narrator	<i>[Approximately 15 minutes later, the students</i>

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			<i>were confident that they had found all possible combinations.]</i>
20.		Alice	Did everyone come up with a solution to this one?
21.		Ankur	Yes.
22.		Class	Sixteen.
23.		Alice	Okay. If you're going to do 16, who's going to convince me of it?
24.		Ankur	I will. I already did.
25.		Alice	Stephanie and Matt?
26.		Stephanie	All right, uhhmm. Well, we have whole and then we have a mixed column.
27.		Matt	Well, we have - They're thinking we have
28.		Student	Sub-titles.
29.		Matt	the whole column and the mixed column. The sub-title.
30.		Student	That's what we got, too.
31.		Alice	Okay. Whole and then mixed, and then sub-titles? Is that what you're saying?
32.		Stephanie	And when we started out, we did, like, ... And then cheese, we did pepperoni, we did sausage, we did peppers and we did mushrooms. And each one of them was all by themselves. You know, nothing was ...
33.		Alice	Okay. This was in your singleton category?
34.		Stephanie	Yeah.
35.		Alice	How many were in that category?
36.		Stephanie	Five.
37.		Alice	Five?
38.		Student	Yeah.
39.		Alice	So that was an easy one, wasn't it?
40.		Student	Yep.
41.		Alice	Okay. Now, Stephanie and Matt, you're saying that your second category was sub-divided? Tell me what your first sub-division was.
42.		Matt	Our subtitle was "the mixed ones." And what we did for the mixed ones was we started with the

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			topping, and we added a topping. So we had -
43.		Alice	Ankur, this is sounding a little bit like the way you described it to me, too. How did you do it?
44.		Ankur	I had a pattern.
45.		Alice	What was your pattern?
46.		Ankur	I started with the first one and mixed it with the second. Like, so my first one was peppers and sausage. So I took peppers slash sausage. So I skipped the second - I started with the first one again, skipped the second one, and took the third one, "P" slash "M". And then I put peppers and skipped the second and third, and I went with the fourth one, "P" slash "PE." And then I started with the "S" and -
47.		Alice	And then you're sure you were finished then. And what did you do?
48.		Ankur	And then I started with the next, the second one. I started with S, sausage, and mixed it with mushrooms. And then sausage and pepperoni. Then I went down to the next one, mushrooms - mushrooms and pepperoni.
49.		<i>Narrator</i>	<i>[Ankur's idea of holding one topping constant and changing the others is a strategy that Matt noticed and will use again in the next problem.]</i>
50.		Matt	We started with peppers and pepperoni, and added.
51.		Alice	Okay. You say peppers and pepperoni?
52.		Matt	And then we added.
53.		Alice	And you added -
54.		Matt	Sausage. Peppers and pepperoni, with mushrooms. Then we had - then we couldn't do any more with peppers and pepperoni. So then we figured out a peppers, sausage and mushrooms.
55.		Alice	Peppers, sausage, and mushrooms. Yeah. Is that all?
56.		Matt	No. And there was no more for peppers. We

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---	---

			were convinced there was no more for peppers.
57.		Alice	That was all you could do with peppers? Yeah.
58.		Matt	There was only one thing you should do with pepperoni -
59.		Alice	Which was?
60.		Matt	Pepperoni, sausage, and mushrooms.
61.		Alice	And then you were done?
62.		Matt	And then you have the big one, the four topping pizza, which was the pepperoni, the peppers, the sausage and the mushrooms.
63.		Carolyn	<i>In most of our other sessions, and even with adults, and even with college students and high school students, if you give students the pizza problem and you ask them to account for all possibilities, that takes at least a session to do. These students did it in a matter of 10, 15 minutes, if that long. And I suspect that the complexity of the pizza with halves made this a very trivial problem for them. Just had to write it up and tell us what it was. They had to think about the idea of whole pizzas in solving the pizza with halves. And generalizing it to four toppings was very easy.</i>
64.		Alice	Pizza Hut feels like they didn't get their money's worth from their consultants, and so
65.		Student	Another pizza problem.
66.		Alice	They're saying, OK, [groans from students] now I want to see if...
67.		Narrator	<i>[About half an hour into this session, the researchers introduced a final problem, one that included half pizzas.]</i>
68.		Alice	Sure, Robert, would you read it for us?
69.		Robert	At customer request, Pizza Hut has agreed to fill orders with different choices for each half of a pizza. Remember that they offer a cheese pizza with tomato sauce. A customers can then select from the following toppings: pepper, sausage, mushrooms, and pepperoni. There's a choice of

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			crusts: regular, thin or Sicilian, thick. How many different choices with pizzas does the customer have? List all the possible choices. Find a way to convince each other that you have accounted for all possible choices.
70.		Alice	Is this going to be more? Or is this going to be less?
71.		Student	It's going to be more.
72.		Student	What you do is you times it by two.
73.		<i>Narrator</i>	<i>[The researchers deliberately chose this problem to stretch the students' thinking. The number of combinations is much larger than in the previous problems, too large to accurately count out, using trial and error. The students built on their past work, and Matt immediately came up with a system that could find the answer.]</i>
74.		Alice	Before you start working on it, Matt, you have an answer?
75.		Matt	Well, I'm going to start with – What you could do is you start with the cheese, and then you put a half, then you add all the rest of the toppings, the peppers, all the rest of the toppings, the pepperoni, all the rest of the toppings, the mushrooms, all the rest of the toppings.
76.		Alice	OK, you all want to work on it for a little while? Remember...
77.		<i>Narrator</i>	<i>[The other students ignored Matt's solution at first, and attempted to find their own answer.]</i>
78.		<i>Narrator</i>	<i>[A few minutes later, the researchers asked Matt to explain his strategy in more detail.]</i>
79.		Matt	We got 120 pizzas. I figured it out. I figured it out. Some way I thought I might have been right. What I did was I got the half cheese, the half cheese- divided it in half; then I took each topping and I put it in the half. Then I went to the peppers, each topping, put it on that, put it on the side. Then to pepperoni, same thing.

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---	---

80.		Alice	Okay, Matt, explain to me what you're saying. You're saying that you started with your cheese, and it could be with all of the others? Okay, that was how many?
81.		Matt	That was 15. It's like Ankur, it's like Ankur did ..with the last problem. He moved down the line, and added all the other toppings as he went. So it was like this.
82.		Carolyn	If you think about, you know, Matt's solution, and if you think about Matt's reference to the idea that he gives credit to Ankur for presenting in the two topping choice of the earlier problem, think of what he does. You know, he makes use of all of the ideas, from the more complex problem to the simpler problem, to, again, a more complex problem, and he introduces a strategy of controlling for variables. Now he says "Well I have all the sixteen, you know?" But he talks about holding one topping constant. And then you can, on the half, you have all your choices.
83.		<i>Narrator</i>	<i>[Matt knew, from the previous problem, that there are 16 possible combinations of toppings for whole, undivided pizzas. Matt next considered all the possible pizzas that are made up of two different halves.]</i>
84.		Matt	So it's half cheese, and half -
85.		Alice	And half each of those other things.
86.		<i>Narrator</i>	<i>[He started with a pizza that is half cheese and half other toppings. Since he had already counted a whole cheese pizza, he couldn't use cheese on the other half, and so he had to count only 15 possible combinations.]</i>
87.		Alice	And on this page, what do I have here?
88.		Matt	Half pepperoni.
89.		<i>Narrator</i>	<i>[Then he moved on to his second topping, pepperoni, holding that constant on one side of his pizza. Since he couldn't repeat either cheese</i>

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			<i>or pepperoni, he counted the remaining 14 toppings.]</i>
90.		Matt	<i>... pepperoni and sausage – like that...</i>
91.		<i>Narrator</i>	<i>[Going through his list, he eliminated the toppings that would have made duplicates, eventually accounting for each of the possible remaining combinations. Finally, he added up the numbers in each column: 16 plus 15 plus 14 plus 13, and so on, all the way down to one.]</i>
92.		Ankur	Is it possible to write out all different combinations?
93.		Matt	Well, if you wrote out all the different combinations that I had -
94.		Milin	You'd die!
95.		Matt	- your hand would be pretty sore.
96.		Brian	All right Matt.
97.		Alice	Are there any duplicates in Matt's approach?
98.		Class	No.
99.		Alice	Is everybody convinced that you've got a solution?
100		Class	Yes.
101		<i>Carolyn</i>	<i>Matt's notation was particular to Matt. You know, he had his elaborate lines to show the detail of the possibilities. He said, "Well look, you know, if you keep this constant you could have it with this topping, with this, with this - Notice the care. Now an adult might say "You could have it with any of those 15 toppings," or "Now you have 14 left." Now Matt eventually said that, but Matt, remember, was part of a group, and he had to express his idea to others. And in order to do that, he had to provide detail. And the detail was provided in the notation he used.</i>