UNDERSTANDING FINANCIAL NETWORKS THROUGH MINORITY GAME

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The recent financial crisis has highlighted the fact that financial systems are complex in nature and understanding financial systems is not a trivial task. One of the major factors that contribute to the complexity of financial systems is the presence of human traders in these systems. Also, the interaction between these human traders leads to traders getting influenced by other trader’s decisions and thus adding another layer of complexity. It has been observed that, social interactions among traders form a social network that influences economic decisions made by these traders. This has lead to a study of a new branch of economics namely “social economics” which takes into consideration the fact that traders are influenced by their social circle. “Social economics” is gaining popularity because the study of psychological aspects of human behavior is able to enhance the modeling of economic models especially the allocation of resources. W.B. Arthur introduced the El Farol Bar Problem in 1994, which shows how agents with bounded rationality and no interaction can show emergent behavior.
The Minority game is a variant of the El Farol Bar Problem and it basically shows how agents collectively behave in an ideal situation while competing, by means of adaptation, for a scarce resource even without interacting with each other. In order to extract more characteristics of financial markets from the Minority Game, we modify it by introducing communication among the agents, thereby letting them get influenced by each other’s decisions. We aim to observer herding phenomena as agents are likely to take actions similar to that of their neighbors. Financial markets are quite dynamic in nature and the interacting traders form a network whose topology dynamically evolves. We study the effect on the network topology due to dynamic updates of the link weights of a financial network. We start with two initial network topologies namely a torus network and a random network. We also observe herding phenomenon among the agents. The degree distribution of the nodes of this financial network is fat-tailed, which is characteristic of a network that is robust against random fluctuations.
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Chapter 1

Introduction

1.1 Motivation

Many Financial Market models have been proposed in order to study the market behavior. Some models try to simulate the stock price behavior, some focus on the market environment, others focus on the agents’ behavior, while some try and observe the effect of all these variables on market behavior. The study of such models has enabled a better understanding of, as well as prediction of the behavior of financial markets. Even the random spikes and bursts in stock prices have been analyzed and understood.

However, there are still many aspects of agent behavior both individually as well as an aggregate behavior that needs to be studied.

In this thesis, we study the behavior of financial networks. By creating a dynamic regime for agents to interact, we attempt to simulate the stock market and observe the market behavior. We use network analysis metrics like degree distribution to observe the network dynamics of financial networks. This will help traders understand how the market players evolve as a group and thereby reduce their chances of falling prey to systemic risk. Systemic risk has been described in section 1.3.1 of this chapter.
1.2 Financial Markets

The financial meltdown has had a tremendous impact on all of us. This has emphasized the importance of financial markets and its impact on our lives. Financial markets have been the subject of research for a long time.

People look to save a part of their income for future use like while buying a house or for a retirement fund. While keeping this income aside, they look to gain a profit on it so that the value of that money does not depreciate due to the rising inflation [22]. The same scenario exists for companies that invest with future profits in mind. People and institutions invest their money in financial markets like the stock market in order to gain a profit. However, neither can predict with certainty how the markets would behave at a point of time. This introduces a level of risk in these transactions. The aim of any investment is to maximize the profits while minimizing the risks. In other words take calculated risks. Financial institutions have been striving over the years to make accurate and reliable predictions in order to maximize their profits. Millions of dollars have been spent on research, to unearth the rich set of patterns, behavior and trends of the markets. Analytical methods have been traditionally used to validate and affirm various behaviors of the markets. The presence of human traders brings in a level of complexity and uncertainty in the way the market behaves and performs. Effect of information from sources, optimism in certain ventures and pessimism in others, natural disasters, national events and many other unexpected scenarios tend to impact the markets. It becomes difficult to comprehend and do mathematical modeling in such scenarios.
Not so long ago, the Efficient Market Hypothesis (EMH) [22] was widely accepted and followed. According to Fama [5],

“A market in which prices always “fully reflect” available information is called “efficient” “.

Rational Expectation (RE) as suggested by Joseph Stiglitz [9],

"the expectations of individuals are rational if they take fully into account the available and relevant information".
The EMH is an application of RE to asset prices. It says that traders, collectively use all the available information regarding an asset, while arriving at an expectation of the asset price. EMH suggests that if any new information comes into the market, its effect gets completely incorporated into the asset price. Any fluctuation in asset prices is purely a random event.

However, this hypothesis has lost prominence of late. First it is not possible for all agents to have complete knowledge about the markets. This has given rise to the concept of bounded rationality wherein agent has some (bounded) capability to access and process information.

However, there has been a lot of research been done that invalidate the Efficient Market Hypothesis. There are a lot of technical and fundamental analyses done which have shown that patterns do exist in the asset prices. Even trader and market behavior depict peculiar and a rich set of properties that can be useful to understand the market dynamics. Traders use these patterns and knowledge of behavior in order to obtain an upper hand while trading and while making decisions.

1.3 Agent Based Modeling

Over time, financial systems have grown increasingly in complexity. Analysis of such systems has becoming a non-trivial task. Also decision making has undergone a radical
change from the hierarchical and centralized approach to the distributed and decentralized mode of operation. This has contributed to the complexity of financial systems and hence it has become increasingly difficult to analyze them. Arthur says in [27] that Agent Based Modeling came into existence in the 1980s when economists had just started using desktops. Because of quick and easy access to a computer, economists could now visualize how equilibrium is formed rather than just study equilibrium. Agents are discrete and autonomous as described in [18]. They have their own state and behavior. They also have a capability to modify their behavior and adapt to changes. The decision rules for each agent vary. This variation can be in terms of the size of memory of the agent or number of neighbors of an agent as well as many other such parameters. This heterogeneity of agents tends to create phenomena like domination of certain types of agents and thresholds where equilibrium is attained.

Again as in [6], Agent Based Modeling [ABM] is described as systems based on interactions of agents, who are described as autonomous, decision-making entities. Decisions are made on the basis of a set of rules, however each agent makes their own decisions. Also, interaction between agents takes place in a fashion that best models the system under observation. An interesting feature of ABM is that simulations are repeatedly run in order to extract dynamics and dynamic behavior which might be difficult to achieve using analysis or actual experimentation. This feature has unraveled many interesting behavioral patterns and properties of complex systems. The beauty of ABM is that even a simple simulation when run for varying amount of time can deliver
complex behavior, patterns and characteristics.

One of the advantages of ABM is that it captures emergent behavior [6]. Emergent behavior is defined as:

“*Emergent behavior is the phenomenon of how small, simple rules can expand into very sophisticated and interesting high-level behavior*”

in [12]. It can also be defined as:

“*a system can provide more complexity than the sum of its parts - leading to what scientists call “emergent behavior”*”

as in [13] where the example of cathedral termites is cited to explain emergent behavior. The standard looking individual cathedral termites when combined with others construct a complex hive to house their colony. Emergent behavior is observed due to the interactions between individual parts of a system. The system behavior as a whole is different than the sum of the behavior of the individual parts. It is this emergent behavior that we will observe in the El Farol Bar Problem and Minority Game later. This behavior is observed in many situations like financial markets, traffic jams and many situations where optimization of resource allocation is encountered. Bonabeau classifies emergent phenomena into four areas of application [6], namely flows -traffic and customer flow management, markets-stock markets and strategic simulations, organizational design and operational risk and diffusion. Our focus in this thesis is on market models and how traders as simulated by agents, devise strategies, have interactions and give a rich set of behavioral patterns and emergent phenomena.
Agent based modeling is characterized by the following elements as described in [24].

The first element is *expectation* wherein each agent has an expectation which is based on bounded rationality as opposed to the Rational Expectation of the Efficient Market Hypothesis. These expectations are explicitly spelt out in Agent Based Models.

The second element is *heterogeneity* of agents. Each agent has his own expectation and own strategies which may or may not be similar to other agents. This models the market much closely as compared to the homogeneous agents when we consider rational expectations.

Finally, agents evolve in agent based modeling depending on the benefit they get from their existing strategies. So if a strategy is performing well, agents re-enforce that strategy while doing away with poorly performing strategies. This is typically what happens in financial markets where traders evolve their strategies based on the evolving market conditions.

1.4 Networks in Finance

1.4.1 Systemic Risk

Systemic risk is the risk that is inherent in the market. This risk cannot be removed even by diversification of the portfolios. As discussed in [14], systemic risk is described in terms of the inter-linkages and interdependencies in the market and it could be as grave as causing bankruptcy of the market due to failure of a single institution. The recent crisis is an excellent example of systemic risk.
The financial meltdown has posed many a challenges to economists and the policymakers. This has actually created a lot of opportunity for networking experts to apply network analysis to these challenges and come up with efficient and profitable solutions.

The *avalanche effect* shown by financial institutions during the meltdown depicts a level of connectedness and interdependence that is seen in the market. It is very evident then that traders and financial institutions communicate with each other and form networks in order to predict asset prices. This means that each of them is also affected by the actions and decisions taken by the other.

This scenario gives us an opportunity to represent these entities as nodes of a network and their association as edges of the network. We can then evaluate what network properties and characteristics these entities depict. The complex structure of the network can be analyzed. We can also try to observe that by varying parameters like the number of links and link weights how topology the network evolves.

Network analysis will help us understand both the impact that the network structure will have on financial markets as well as the way financial networks form and evolve [5].

*Financial contagion* refers to the fact that some information related to certain institutions affects those few financial institutions initially but later starts spreading and affecting other financial institutions like a disease. Consider a network of financial institutions and each institution has a set of links connected to it. Now financial institutions having more number of sources withstand contagion better.
Today, social networking has become such an integral part of our lives that we cannot
discard the effect of social networks on financial markets. Studies such as Hong, Kubik
and Stein (2005), Feng and Seasholes (2005) suggest that demands from a social cluster
that is densely populated are highly correlated as compared to loosely connected social
clusters [15].
So network analysis will help us understand the interconnections of financial institutions
as well as the structure, formation and evolution of financial networks.

1.4.2 Scale–Free Networks

Recently, scale free networks have become very popular in explaining real world
networks. Many complex systems have shown a common property. In these systems,
some nodes have a large number of links to the other of thousands or even millions of
links whereas other nodes only have a handful of links [2]. This indicates the lack of a
scale for the degree distribution of these complex networks. This structure of a network
makes them extremely robust to accidental failures. However, if they are attacked in a
coordinated fashion, they are unable to efficiently absorb the failures.
1.5 Financial Networks

1.5.1 Current Situation

Financial Networks have been the subject of research recently. The impact that social networks has had on society is being observed in financial circles as well. Financial institutions are using social networks to extract information as well as advertise and market their products. The study of markets in terms of financial networks has helped researchers discover many empirical facts. For example, Topa in [10], studies local interactions among agents discussing job opportunities within their social circle. Topa infers that agents whose social network consists of agents with jobs are more likely to get jobs. Many large networks are characterized by connectivity among vertices that depict a scale free, power law distribution [3]. This behavior means that self organizing behavior governs the evolution of large and complex networks. Many such studies have emphasized the importance of the study of financial networks to understand the market behavior.

There are two aspects of financial networks that we need to consider. One is the network topology as a whole and the other is individual agents that are the vertices of the network. It would be interesting to observe how the network topology evolves by varying the agent strategies and their payoff. This will help us utilize the topology in order to gain an advantage while trading. Also, the degree distribution will help us evaluate how well connected the agents. The degree distribution will also suggest the robustness of the
Another important aspect to consider is that if we keep the network topology constant and have the agents interact in this fixed topology. We will observe herding behavior as agents get influenced by their neighbors.

1.5.2 Roadmap Ahead

Currently, there are many important issues being tackled using market modeling. These issues include market models for Inflation derivatives [8], modeling market mechanism using evolutionary games [28] and many such models. Financial networks have also been modeled to understand the market behavior [25]. However we have not seen a comparison between different financial networks for a range of the number of traders. The aim of this thesis is to study the behavior of traders as a group, the effect of interactions among traders i.e. financial network characteristics and the characteristic behavior of dynamically changing markets. To that effect we have designed three systems. The first system is the implementation of the Basic Minority Game described in Chapter 3 in which we will observe the evolving nature of a group of traders even though they make decisions independently and at the same time. In the second system, which is discussed in Chapter 4, we have a system similar to the first system but with agents interacting between each other. The network topology of traders remains fixed for this system i.e. it is a static system. This system will show how herding behavior is observed due to interaction between some traders who influence each other and so tend to herd towards the same decisions. The third system has both interacting agents and dynamic update of the connections between the traders. This dynamic setup helps us
understand how financial networks evolve and also how this new topology is resilient to fluctuations in the stock market. Finally, we have a comparison of these systems.

1.6 Outline

The thesis is organized as follows. In Chapter 1, we saw the motivation for this thesis, an introduction to financial markets, agent based modeling and networks in finance. In Chapter 2 we will see what stylized facts are. We will also study a few of the stylized facts that we will be using in this thesis to observe market behavior. In Chapter 3, the El Farol Bar Problem is introduced. The Minority Game by Chalet and Zhang which is a game created from the El Farol Bar Problem is then discussed in detail. A discussion on the design, implementation and results of the Basic Minority Game follow. In Chapter 4, we study the Introduction of Communication in the Minority Game. Our implementation of the same is included in Chapter 4. The herding effect observed due to the introduction of communication is discussed and validated with the results obtained in Chapter 4. Chapter 5 discusses a dynamic regime of a market model. The market mechanism is discussed. First minority game agents with variable memory trade in this dynamic regime. Then we have only noise agents trading in the dynamic regime. The stylized facts obtained are discussed and interpreted. Chapter 6 discusses the results, the conclusions and interpretations arrived at. Chapter 7 gives a brief conclusion of the thesis.
Chapter 2

Stylized Facts

2.1 Motivation

The aim of any model is to explain the behavior of system or process it is modeling in terms of the characteristic features of that system in actuality [22]. In order to understand any theory or behavior we need to provide a customized set of properties or characteristics that are unique to that process (for e.g. economic behavior). If we go for an analytical analysis, we might not be able to model that process as close to its actual behavior due to the constraints, like adherence to certain axioms or theorems, imposed by mathematical modeling. Nicholas Kaldor in [22] suggests that we should start with a “stylized” view of the facts. These “stylized facts” would represent the actual behavior of the process (e.g. economic behavior).

Sewell defined Stylized Facts as empirical facts that are considered to be true due to their consistency across markets and time periods by many [20]. As stylized facts are so generalized, they are usually qualitative in nature.

There have been many stylized facts that have been established as in [1] like volatility clustering, fat-tailed distribution, absence of auto correlation, gain/loss asymmetry and leverage effect. We shall see two stylized facts namely herding and fat-tailed degree distribution in this thesis because they better characterize networks.
2.2 Herding

It is human tendency to flock where the apparent maximum profit seems to be. Traders tend to observe other traders, financial institutions and sources of information like newspapers, price trends and other technical tools and try and follow those who have had success in earlier times. This tends to introduce a herding behavior in the markets.

Fig 2.1 Herding Behavior observed on Black Monday


We can attribute this herding behavior to sudden surges or sudden plunges in the stock market prices. For example as seen in Fig 2.1, on Black Monday there was a tremendous volume of trade observed. By studying and identifying the herding behavior, traders can
avoid crashes and maximize their profits. The famous American author Isaac Asimov once said [23],

“*The reaction of one man can be forecast by no known mathematics; the reaction of a billion is something else again*”

The herding behavior is very evident in the recent financial crisis situation where banks and investors undertook risky ventures without doing the appropriate cost/benefit analysis and then deserted these ventures when trouble surfaced. The concern raised by policymakers in [23] that,

“*herding by market participants exacerbates volatility, destabilizes the markets and increases the fragility of the financial markets*”

In order to follow other investors, an investor should have knowledge of the actions of other investors and also be influenced by it. It is human tendency to flock and take similar decisions. Historical data in terms of stock prices as well as performance and reputation of financial institutions lure traders to follow certain predictions. Investors have an intrinsic preference for conformity. Investors get influenced by the decisions of other investors. This influence is either due to an understanding that other investors have some specific knowledge about the returns on that stock or it’s just that sometimes investors have an intrinsic preference for conformity [23].
2.3 Fat-tail Degree Distribution

Degree Distribution of a network is an important property that helps us understand the network and its behavior. We define the degree of a node as the number of nodes it is connected to. We would get a normal distribution of the degrees of network nodes for a random network. This means that we would have a bell structure with a fat center and tapering and thin at the extremes.

However, many real world networks have fat or heavy tails. This indicates that there are a lot of nodes with either very few links or with many links [5].

![Fig 2.2 Fat-tail degree distribution](http://oddlee.blogspot.com/2007/09/eight-properties-of-social-network-14.html)

As we can see in Fig 2.2, the degree distribution has a long tail. Few nodes have many links while others have lesser number of links. This fat-tailed distribution shields the network from random disturbances, however the network is more likely to be weak against targeted attacks [15].
As suggested in [5], this is due to the fact that the impact of random disturbances is distributed at the tails. This enables the tails to absorb the random disturbances easily. On the other hand, if a densely connected node fails, it puts a lot of load onto the other connected nodes and even can cause failure of other nodes in the network.

The fat-tail distribution of the degree distribution is also an indication that a small world network exists. The concept of six-degrees of separation can cause nodes to connect to remote and far away nodes with minimal path lengths. However, now a local disturbance can also be spread to a rather far away node connected to it.

Another interesting comparison in [5] discusses the relationship between the heterogeneity of nodes and the stability of networks. If the nodes are homogeneous, then all the nodes would employ a similar strategy causing a situation where the entire system or market can be attacked in one area and be significantly damaged. However when the network nodes employ varied and diverse set of strategies then the risk is spread out and the impact of a risk will not be as severe.
Chapter 3

El Farol Bar Problem and Minority Games

3.1 Motivation for use of a Game Theory Problem

The market behavior like herding and self-emergence is quite complex to model and understand because of the number of variables involved like market crashes, bullish market and to a great extent because many decisions are made by traders who are human and hence heterogeneous. The heterogeneous traders especially contribute a lot to the complexity of the market behavior. The interaction between traders and the influence of these interactions makes it necessary to consider the social aspects like social networking and influence. In order to understand traders’ behavior we consider the branch of Game Theory. Game theory as defined in [21] as

“A tool that can help explain and address social problems. Since games often reflect or share characteristics with real situations -- especially competitive or cooperative situations -- they can suggest strategies for dealing with such circumstances.”.

It is one such Game Theory Problem that forms the base for this thesis-The El Farol Bar Problem.
3.2 El Farol Bar Problem

The El Farol Bar Problem as proposed by W. Brian Arthur [26] is a game theory problem that depicts the emergent behavior. El Farol is a Bar in Sante Fe where people go every Thursday for a drink and to have an enjoyable time. 100 People independently decide whether to go to the bar or not. However if the bar’s attendance is more than 60, the bar is crowded and hence not enjoyable. As stated, people independently would decide whether to go to the bar or not without interaction with other visitors of the bar. The only knowledge they had was the bar’s attendance for the previous few weeks. Based on this information, which is not complete information about the environment and hence bounded rationality, they would decide whether the bar would be enjoyable or crowded. Arthur suggested that he would have an alphabet soup of predictor models what had replications and would pull randomly k models for each of the 100 people. People would have this varied set of models to decide whether to go to the bar or not. This could include assuming that the attendance would be the same as the previous week, average of the attendance at the bar for the last four weeks, bar’s attendance 2 weeks ago and similar models and strategies.

Every week, people would go to the bar or decide to stay home based on the mental strategies they had devised. If a strategy would help them predict the Bar’s Attendance accurately, that strategy would gain prominence and if the strategy would cause people to go to the bar when it was crowded and stay at home when the bar was enjoyable, then
that strategy would be less credible or even discarded. Also people would not have the same set of strategies; otherwise everyone would decide the same decision. There is no deductive solution to this problem. People would learn from their mistakes of earlier weeks and either change their strategy or the way they would decide. Arthur noticed that over a period of time the bar attendance would emerge to 60. It would be interesting to understand why the bar attendance converges to 60 and not any other value. One explanation is that 60 is a natural attractor in the problem [26]. Again as [26] suggests, let us assume that 70% of the predictors suggested above 60 for an extended period of time. Then the bar attendance would be 30 on an average. This would in turn make those predictors, who suggest attendance to be 30, to be more accurate and the ecological balance is maintained. This is a great example of emergent phenomena in day to day life. Arthur thus proved that inductive reasoning and bounded rationality could still give us important characteristics and phenomena like emergent behavior.

3.3 Minority Game

In 1997, D. Challet and Y-C. Zhang [4], introduced a game based on the El Farol Bar Problem. It is called the Minority Game. It is a binary game, where there are N (odd number) players and each player has to take a side, either side A or side B for each round. The side that is in the minority at each round wins that round. Each player does not have the bar attendance of previous weeks but only binary sequences of 1s and 0s that suggests the last few rounds suggestions. A player decides whether to choose side A (1) or side B
The memory length was varied to observe the effect on bar attendance and the fluctuation in the same. It was observed that players with longer memory depict lesser fluctuations in the bar attendance.

This can be compared to financial markets where traders look for fluctuations in asset prices and try to profit from arbitrages. However after a certain point, this arbitrage is taken away.

The differential head count at the bar is given by:

\[ A(t) = \sum_{j=1}^{N} a_j(t) \] (3.1)

A(t) => the differential head count at the bar at time t,

j => the range of people attending the bar,

N => the total number of players and

\( a_j(t) \) => the decision of agent j at time t.

If the player decides to go to the bar, \( a_j(t) \) is 1 and if the player decides to stay home then \( a_j(t) \) is -1.

If \( A(t) < 0 \), majority of the players did not show up at the bar and the bar was enjoyable for those that did end up going to the bar.

If \( A(t) > 0 \), then the bar was crowded as majority players went to the bar.

The payoff for the agent would be:

\[ g_i(t) = -a_i(t) \cdot A(t) \] (3.2)
where,

\[ i \Rightarrow \text{the agent number,} \]

\[ g_i(t) \Rightarrow \text{the payoff for agent } i, \]

\[ a_i(t) \Rightarrow \text{the decision of agent } i \text{ and} \]

\[ A(t) \Rightarrow \text{the differential head count of the bar as calculated in equation (3.1).} \]

If a player wins a round, \( g_i(t) \) is positive and if she loses the round then \( g_i(t) \) is negative.

The minority game is a simple model that gives us understanding of complex systems and phenomena. We can use the minority game to model financial markets and learn and discover patterns and behavior of both the market and traders. The length of the memory also determines the duration of fluctuation and so more the length of the memory, lesser the fluctuation which can be translated to smaller limits to arbitrage.

In this thesis, the Minority Game is first implemented in order to observe the emergent phenomena and used as a benchmark. We have N agents each having a short term memory and a long term memory. The short term memory is a sequence of bits that hold the decisions of the past few rounds of that agent and the long term memory consists of strategies.

The notation in binary is 1 to go to the bar and 0 to stay at home. The short term memory is a sequence of bits denoting the last few short-term actions of each agent. The Strategy set is a combination of bits that tells the agent whether to go (1) to the bar or stay (0) based on the input sequence (short term memory) bits. Each agent has a set of strategies with her.
An example strategy set for a short term memory of 3 is as follows in Table 3.1:

<table>
<thead>
<tr>
<th>Input Bits(memory of agent)</th>
<th>Decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 0 0</td>
<td>0</td>
</tr>
<tr>
<td>0 0 1</td>
<td>0</td>
</tr>
<tr>
<td>0 1 0</td>
<td>1</td>
</tr>
<tr>
<td>0 1 1</td>
<td>0</td>
</tr>
<tr>
<td>1 0 0</td>
<td>1</td>
</tr>
<tr>
<td>1 0 1</td>
<td>1</td>
</tr>
<tr>
<td>1 1 0</td>
<td>0</td>
</tr>
<tr>
<td>1 1 1</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 3.1 A Strategy Set for an Agent

This means that if the short term memory is 101 then the agent’s decision for the next round is 1 i.e. go to the bar. Let us have a look at the brain of the agent and see how she makes decisions in Fig 3.1.
The agent has many such strategy sets and each strategy set gets a score as explained in equations 3.1 and 3.2.

Initially all the strategies have a score of 0. Let $s_{ij}(t)$ be the strategy score of $j^{\text{th}}$ strategy of agent $i$.

**Fig 3.1 Agent Brain Functioning**

![Diagram showing Short Term Memory and Long Term Memory strategy sets with a decision arrow pointing to a choice between "go to bar" and "stay at home".](image)
If the decision taken from a strategy actually helps the agent to end up in the minority group, then the agent increases the strategy score by 1 otherwise we do not increment the strategy score.

So,

\[ s_{ij}(t) = s_{ij}(t) + 1 \quad \text{if strategy recommendation helped agent be in the minority} \]
\[ s_{ij}(t) = s_{ij}(t) \quad \text{if strategy recommendation made the agent be in the majority} \quad \text{… (3.3)} \]

At each round, the agent picks up the strategy set with the highest score. If there is a tie in the scores then the agent randomly selects one of the best performing strategies.

In the basic model there was no communication between the agents. At each round, each agent independently decides whether to go to the bar or to stay at home. We ran the simulation by varying the number of agents and number of rounds for which the simulation is run. We observe that indeed there is an emergent behavior wherein the bar attendance self-organizes to approximately 60% of the total number of agents playing the round as suggested by the El Farol Bar problem suggested by Arthur in [26].

In order to empirically prove this, we ran the simulations over a wide range of values of N and at each instance, the attendance would converge to approximately 60% of the total agents. The results have been shown in section 3.6.

**The Algorithm for the Basic Minority Game is as follows:**

1) User first enters the number of agents playing the game say N and the number of rounds to be played say R.
2) Agents are created in the following manner:

i) Each agent has a short term memory which is uniformly randomly initialized with 0’s and 1’s. This will be the input combination for the strategies.

ii) Each Agent has a Long Term Memory which comprises of a set of strategies. Each strategy set will have a corresponding output for all the input combinations of the Short Term Memory. Each strategy set also has a score attached to it. Initially all agent scores are initialized to 0. This agent score will decide which strategy will be selected for each round i.e. the strategy with the highest score will be selected.

3) At each round, all the agents independently decide whether to go to the bar or to stay at home.

4) At the end of the round, the total number of agents that went to the bar (Nb) are computed.

i) If this number, Nb is in the minority group i.e. less than 50% of the total number of agents(N) participating, then the agents who are part of Nb win that round. If not then those agents who did not go to the bar win that round. Thus we have the group in the minority winning each round.

ii) The agent scores are updated based on Equation 3.2. The agent’s strategy for that round are updated based on Equation 3.3.
5) Finally, after all the rounds are completed the mean attendance at the bar is computed.

3.4 Design of our Minority Game

Please refer to Appendix A for the design of our Basic Minority Game.

3.5 Working of our Minority Game Application

First, the application prompts the user to enter the number of agents and the number of rounds for the game.

Then the Agents are instantiated and short term and long term memory for each Agent is created.

Each agent independently decides whether to go to the bar or not based on her short term memory contents and her best strategy’s tactic which matches the short term memory.

At the end of each round, the minority is calculated and agent scores are updated based on whether their decision was the minority decision or not. Agent short term memories are also updated.
3.6 Results obtained for Basic Minority Game

We ran the basic minority game by varying the number of agents participating for 100 rounds and the results are as shown below:

As shown in Fig3.1- Fig3.4, even by varying the number of players the mean attendance converges to around 60% of the total attendance. For our simulation, we varied N for a wide range of values 101, 501, 1001 and 5001. We also tried varying N for other values and came up with similar results.

Fig 3.2 Attendance Plot for N=101, No. of Rounds=100, Mean Attendance=59.8%
Fig 3.3 Attendance Plot for N=501, No. of Rounds=100, Mean Attendance=60.1%
Fig 3.4 Attendance Plot for N=1001, No. of Rounds=100, Mean Attendance=60%
We observe that even though the agents are making decisions independently, the attendance will evolve and self emerge to a certain value. The aim of understanding the Minority Game as a market model is to understand the characteristics of traders from a macroscopic level. This we have achieved by varying the number of agents playing the game as shown in Fig 3.2-3.5. Even the variation in the number of agents does not change the macroscopic behavior of the agents as they self emerge.

We also wanted to observe the effect of the threshold that we set on the Attendance.
So we varied the Comfort threshold from 50-60 in steps of 2 and observed the attendance mean.

The results are indicated below.

Fig 3.6 Attendance Mean vs Comfort Threshold Plot for N=1001, R=10000
Fig 3.7 Attendance Mean vs Comfort Threshold Plot for N=2001, R=10000

As seen in Fig 3.6 and 3.7, the Mean Attendance for the Threshold=50 is 60. A comparison with the other thresholds suggests that after this threshold the Bar attendance declines and hence lesser agents have been able to benefit by changing their strategies. For stronger majority threshold, the attendance mean flips to a lower value.

We also observed how the mean attendance initially is below the mean for the first 8-10 rounds and then it fluctuates around the mean attendance. We ran the simulations for different values of number of agents and observed that on an average the initialization period is 8-10 rounds.
For example,

\[ \text{Fig 3.8 Initialization period of 8 round for Attendance} \]

The above figure shows that initial attendance to be below the mean for about 8 rounds and then fluctuates around the mean.

The Basic Minority Game helped us understand self-emergence in Financial markets. However, it does not represent many important aspects of financial markets namely communication among the traders and the network of traders that influence these traders’
decisions. In the next chapter, we shall see the effect of communication among the agents in two different network topologies namely random networks and grid based networks.

3.7 Drawbacks and Shortcomings of the Basic Minority Game

In our Basic Minority Game, we observed large fluctuation of attendance around mean because all agents have same small memory size and are homogeneous. Also we did not observe herding behavior because of there was no interaction between Agents. However, financial market is characterized by variable number of traders trading at each time step. Traders communicate with each other and other financial institutions and these communications influence the final decisions of traders [9]. In [9], we see the importance of financial networks i.e. communication networks between traders that prove that those traders or financial institutions having a strong and robust network of sources are better positioned to do asset trading and improve their trading performance in comparison to those traders who have a scarce communication network of sources. Also, some traders look at shorter duration of the history of the stock prices compared to others.

We want to observe the stylized fact of herding behavior, as described in Section 2.2, of the agents due to the introduction of communication.

We will now see the effect of heterogeneous agents i.e. agents with different memory lengths on the mean attendance. We had agents of varying memory lengths i.e. 0 to 5 playing the game and observe their behavior.
Fig 3.9 Comparison of fluctuations in Attendance Mean for fixed and variable memory size

We observe that there is much lesser variation in the attendance for variable size memory agents as compared to the fixed size memory agents.
Chapter 4

Modified Minority Game for different topologies

4.1 Importance of Communication between Traders

In financial markets, traders don’t take decisions independently. Rather they communicate with other traders, they have reliable sources that analyze market trends and predict its future behavior. This has an enormous impact on the decisions that the traders make, the strategies they favor and also other traders are also impacted by each other’s decisions. In [16], social interactions are introduced in the modeling of financial markets. They observe in [16], that agents belonging to same clusters tend to make similar decisions and herd whereas agents from other clusters vary in their decision making. Also, the hierarchal observations of prices lead to differential influences in forming those security pricings. As proved in [16], certain traders have more influence on the security pricing than others even if their predictions are not precise. These above characterize a realistic market scenario.

Hence the basic minority game does not give us a realistic market model. The basic minority game has each agent independently deciding whether to buy or sell. So we introduce a friend list for each agent and each agent consults this friend list while deciding whether to buy or sell.
4.2 Introducing Communication between the Minority Game Agents

We observed, by implementing the basic minority game, that there was a self-emergence of the population to 60% of the total number of agents. Interestingly as described in section 3.2, this was achieved without the agents interacting with each other as each agent was unaware of the decisions of the other agents. Each agent independently decided whether to go to the bar or not. As Traders discuss with others about their decisions and get influenced by the decisions of others as discussed in section 4.1, we now will look at the scenario where we introduce communication between the agents.

As discussed earlier in section 1.3, networks are of utmost importance in financial markets due to the interconnectedness and intertwined nature of financial systems. This development of making connections between agents will enable each agent know about the decisions of her neighboring agents. The neighboring agents will give a sense of the market sentiment and also act as a source of information for each agent.
4.3 Implementation of Communication between agents in the Minority Game

We have modified the basic minority game by introducing communication between the minority game agents. A link between two agents is defined as a connection over which the agents communicate their decision to each other. This communication takes place after each agent has independently made his decision of whether to buy or sell a stock. Links can be directed links or undirected links. Directed links are those links wherein information is exchanged only in a specific direction whereas undirected links will have bidirectional exchange of information. In our simulation, each agent has a fixed number of links to some of the other agents playing the game. The link weight is constant and uniform, so each friend or neighbor of an agent gets equal importance and their decisions are weighed equally. An agents’ friend list consist of all the agents that are connected to that agent i.e. all the agents that make recommendations to that agent regarding buying of selling a stock. The agent makes his final decision by taking a consensus of the decisions made by her and her friend list and goes by the majority decision.
Fig 4.1 Agents’ decisions based on neighbor suggestion

The agent makes a decision as follows:

\[
\text{MAX } \{\text{count(number of “go”), count(number of “don’t go”)}\} \ldots \text{ equation 4.1}
\]

Referring to Fig 4.1, suppose central nodes’ decision is to go(1), then

\[
\text{count(go)} = 3, \text{count(stay)} = 2 \text{ and agent decides to go.}
\]

There are many topologies to decide how the neighbors of each agent are selected. Two of the most popular used are the Regular networks which has a ring lattice topology and Random networks. The importance of these two topologies is that real world networks
like biological, social and financial networks have a structure that is in between these extremes of regularity and randomness. For example, we can have each agent in the center of a grid and the four agents touching this agent in the grid can be in his friend list. So if we have 21 agents playing the minority game the grid would look something like as shown below in Fig 4.2:

![Grid Topology of Network](image)

**Fig 4.2 Grid Topology of Network**

As shown in Fig 4.2, Agent 8 will have in its friend list Agents 3, 7, 9 and 13 thus forming a neighborhood of 5 agents. Similarly, the other agents also form their friend list.
We need to decide the neighbors for the boundary agents. We achieve that by folding the rectangular grid into a torus as shown in Fig 4.3 below. So now for Agent 1 the neighbors would be 5, 2, 16 and 6. For agent 6 the neighbors would be 10, 7, 1 and 11. Since our game has an odd number of agents, the last agents’ neighbors will be the corner most agents in the grid, so for Agent 21 the neighbors would be 1, 5, 16 and 20.

Fig 4.3 Torus structure obtained by folding a rectangular grid

Once the network is formed using the above method, the minority game is played in a similar fashion to that described in Section 3.3. However after each agent has independently made their decision, agents consult their friend list about each of their friends’ decisions. The Agent then makes a decision which is either the majority of all the decisions or minority of all the decisions. The Results of the simulations for both the
majority and minority scenarios have been discussed in section 4.5. We observe herding behavior because of the interaction between agents and their neighbors.

The next network topology we use is a random network. We have a random network which acts as a simple social network [19] to see how the results will change from the Basic Minority Game when we have interaction among agents. The aim of the connections is to ask a simple question to the neighbor, will you decide to buy(1) or sell(-1). This network topology is shown in Fig 4.4:

![Fig 4.4 Random network with directed links](image-url)
4.4 Working of Communication between agents in the Minority Game

For both the network topologies discussed in the previous section, the game begins in a similar fashion to the Basic Minority Game as explained in section 3.3, where each agent decides whether to buy (+1) or sell (-1) independently. After this the agent looks at the decisions of agents in his friend list. The agent then chooses whether to go or to stay based on what the majority of his friends have chosen using the equation 4.1:

\[
\text{If } S_{\text{buy}}(t) > S_{\text{sell}}(t), \text{ the agent decides to buy the stock}
\]

\[
\text{If } S_{\text{buy}}(t) < S_{\text{sell}}(t), \text{ the agent decides to sell the stock} \quad \ldots \quad (4.1)
\]

This simulates the situation where traders are influenced by other traders and financial institutions in their social circle.

We expect to observe herding behavior, due to interaction between agents. Agents will tend to herd towards one decision as they choose the majority of their friend’s decision.

It is important to note that this is an asynchronous communication as all agents first decide independently and then they look into their friend list decisions.

Agent scores are updated in a similar fashion using equation (3.2). For the strategy scores best strategy gets a point if its decision matched what the agent finally chose for that round else the strategy score is not updated. This is shown in equation 3.3.
4.5 Results of Modified Minority Game

Let us now observe the results of this modified Minority Game. Again as in the Basic Minority Game we ran the game for a varied set of values of $N$ ranging from 101 to 1001.

For the Modified Minority Game with the torus topology, the results are as follows:

![Graph showing attendance for $N=101$, $R=100$, Torus and Random Network.]

**Fig 4.5 Attendance for $N=101$, $R=100$, Torus and Random Network**

During sudden news like the debt ceiling, people started selling. Others panicked and followed suit.

During normal market working, traders have close knit set of sources whom they follow to understand market trends.
Fig 4.6 Attendance for N=501, R=100, Torus and Random Network
Fig 4.7 Attendance Mean for N=1001, R=100, Torus and Random Network

The x-axis for each of the above graphs is each round of the game, the y-axis is the ratio $N_b/N$, where

$N_b$=> Number of agents that decided to buy the stock

$N$=> Number of agents participating in the market

We observe in Fig 4.5-4.7 that the agents tend to herd because of the communication between them. If we consider the torus network, we observe this network behavior during normal market operations where traders have close knit set of sources whom they follow to understand market trends. We observe random network behavior when there is a
sudden news like the debt ceiling issue recently, wherein traders start selling their stocks aggressively. Other traders panic and follow suit.

We now observe the results for the random network. Again here we ran simulations for $N$ ranging from 101 to 1001.

We observe a high degree of herding due to communication between the agents who are connected in a random fashion. A comparison of the results for the torus network and random network tells us that due to the random nature of the network, there is more herding in the random network.

We now see a comparison chart of the Attendance mean between the Basic and Modified Minority Game.
Fig 4.8 Attendance for Basic vs Modified Minority Game, N=101, R=100

As seen in Fig 4.8, there is a considerable amount of herding when agents communicate with each other in comparison to when they make decisions independently. We varied the number of agents and the number of rounds over a range of values and still observed the same behavior.

In the above systems we have static linkages among the agents. It implies that each agent takes advice from the same set of agents. However financial markets are ever evolving and traders keep changing their sources based on their experiences. Also, traders give preferential treatment to certain sources. In order to account for these properties of financial networks and markets, we have designed a system in Chapter 5 that dynamically
updates the links to an agent as well as gives preferential treatment to some agents over the others.

### 4.6 Drawbacks of Modified Minority Game for different topologies

There are two drawbacks that we observe in the Modified Minority Game that we developed for different network topologies. First we still observe large fluctuation around the mean because all agents have same memory size and are homogenous. Also the network topology remains the same for the duration of the game i.e. it is a static network.

We have also kept the influence of each neighbor to be uniform i.e. all neighbors decisions are equally influential. However in real world markets we observe that traders rely more on the decision of some sources compared to others.
Chapter 5

Heterogeneous Agents with Dynamic Link Updates

5.1 Motivation

In the implementation of the Basic Minority Game in Chapter 3 and again in the implementation of the Modified Minority Game in Chapter 4, we had Agents that were homogenous in terms of the memory each agent held. We had a fixed memory length for all the agents playing the game.

Also, the Modified Minority Game as described in Chapter 4, had a static network topology i.e. the network topology did not change for the entire duration of the game. This means that each agent has the same set of neighbors from whom she took advice while making her decision at each time step. Each link’s weight age also is uniform and constant throughout the duration of the game.

By implementing the heterogeneous nature of agents and the dynamic link updates, we aim to build a more realistic market model which will be validated by the stylized facts namely herding and a fat-tailed degree distribution. Also as we saw in Chapter 3, due to the heterogeneous nature of agents i.e. varying memory size for each agent, there is a lesser amount of fluctuation in the mean attendance.
5.2 Heterogeneous Agents

We run our simulations on a dynamic regime discussed in section 5.4 with two different types of traders.

First we run the simulations on agents whose memory length is varied from 0-\(M_{\text{max}}\). For our simulation we used \(M_{\text{max}}\) as 5 since a memory size larger than 5 does not give any improvement in agent performance. This range helps us have a varied set of agents with different memory lengths. Each trader remembers different lengths of past market data and hence varying the memory length of agents models the agents more realistically.

We then have zero-memory agents who randomly decide at each time step whether to buy or sell an asset. As the name suggests, the memory length is zero, so each agent chooses to buy or sell the asset with equal probability.

We observe how herding is affected by these two different types of agents.

5.3 Dynamic Link Updates

The regime described in the modified Minority Game in Chapter 4 is still a static market model because we have the same set of links for each agent throughout its run. However, financial markets are dynamic in nature and ever evolving based on various factors. Financial institutions and traders have different confidence levels regarding information and financial instruments. These institutions have preferences in their local social networks. Credibility is another factor that contributes to the amount of weight age traders give to their sources of information. A good experience with a client will make traders give more credibility and repose more faith in their clients. On the other hand, an
unfavorable event might discredit a source or even cause a trader to completely discard information from a source. Based on these above observations, agents need to constantly evaluate the recommendations made by her neighbors and give priority to those neighbors whose recommendation is profitable to the agent. This is achieved by updating the link’s weight as described in Section 5.4.

5.4 Financial Network/Market Mechanism

We introduce a dynamic regime of a market model here as in [25] but we have two different initial networks. First we look at the scenario where we have the torus shaped network as the initial network. Then we look at the scenario where we have the random network as the initial network.

The agents have a variable number of neighbors as opposed to the fixed number of links in our modified Minority Game from Chapter 4. Also, the link weights of each neighbor are different.

We initially assign link weights uniformly random between 0 and 1.0 for each of the links. We use two types of agents in our simulations. First we have minority game agents whose short term memory we vary from 0 to Mmax due to reasons explained in section 5.1. Then we only have noise agents i.e. zero-memory agents.

We will now see the Algorithm for creating the dynamic network starting with the initial torus network:

1) Initially, the agents are setup with their short term and long term memories as described in section 3.3.
2) Then the initial network is formed in the following way:

i) For the torus network, the neighbors for each agent are selected as seen in Chapter 4. So each agent will have 4 neighbors. We also assign a link weight for each neighbor. The link weight is assigned in a uniform random way for weights ranging from 0 to 1.0.

ii) For the random network, we need to first decide the number of neighbors for each agent. For each agent we choose in a uniform random manner a number between 1 and Dmax, where Dmax is the maximum indegree of the network. Unlike the torus network, each agent will have a variable number of neighbors. We also assign a link weight for each neighbor. The link weight is assigned in a uniform random way for weights ranging from 0 to 1.0.

3) At the beginning of each round, each agent independently decides whether to buy or sell a stock.

4) Once all the agents have made their decision, each agent checks with her friends about their decisions. The agent does not equally weigh the suggestions of her neighbors. The friends connected to that agent with higher link weight have a greater influence on the agents’ decision. Then each agent takes a consensus of the majority or minority of the sum of the decisions of her decision and that of her neighbors.

5) Agents who end up in the minority group win that round.

6) The strategy scores and agent scores are updated as in Section 3.3.

7) The link weight updates are done in the following manner:

i) If the decision of the neighbor agent connected to an agent helped the agent win that round, the weight of the link between those two agents is incremented by small value $R^+$. 

ii) If the decision of the neighbor agent connected to an agent caused the agent to lose that round, the weight of the link between those two agents is decremented by small value $R^-$. 

8) After the link weights have been updated, we check if the link weights have reached or crossed the upper threshold, $R_{max}$ or the lower threshold $R_{min}$.
i) If the link weight is greater than \( R_{\text{max}} \), we set the link weight to \( R_{\text{max}} \) i.e. the maximum link weight that a link can have is \( R_{\text{max}} \).

ii) If the link weight is lesser than \( R_{\text{min}} \), we remove the link between the two agents and randomly select another neighbor for the agent.

iii) The selection of the new neighbor happens differently in the torus network and the random network. For the torus network, referring to the grid structure in Fig 4.1, we uniformly randomly select a neighbor from the same row or column as that in which the agent is placed. For the random network, we uniformly randomly select any agent from the network as the new neighbor.

Let us first see a block diagram that explains the flow of the market mechanism.

![Block Diagram](image)

**Fig 5.1 Overview Block Diagram of Dynamic Market Mechanism**
We now look into the working of each of the blocks in Fig 5.1.

**Fig 5.2 Agent and Financial Network Setup**

As shown in Fig 5.2, the user provides the number of agents (N) playing in the market and also the number of rounds i.e. the time steps for which we need to run the market. Once we have this information, the Agents are created with their short term memory and long term strategies. Each Agent then generates a “friend list”, a list of agents that are going to communicate their decision at each round with this agent. Each of these friends are also assigned a link weight, so that some friends’ decisions are more influential than others. For the random network we uniformly randomly assign link weights to the neighbors. For the torus network, link weights are assigned as shown in Fig 5.3.
So for agent 78, first order neighbors 66, 77, 79 and 90 are assigned link weight 1.0, the first order neighbors’ neighbors, i.e. 65, 54 and 67 for agent 66 then 65, 76 and 89 for agent 77 and 67, 80 and 91 for agent 79 and finally 89, 91 and 102 for agent 90 are assigned link weight 0.5 and so on.

Fig 5.3 Neighbor creation for Torus network
Fig 5.4 Agents' Initial Decision Making

Fig 5.4 describes the Initial Decision making rationale for each agent. This decision making mechanism is identical to the mechanisms explained in Chapter 3 and Chapter 4.
Once the agents have made their initial decision, they consult their friend list in order to obtain a consensus. The agents consider their neighbors’ actions only with weight $W$ of that link.

Finally, the agent decides to buy or sell based on whichever side got a higher sum as in equation 4.1.

![Diagram]

**Fig 5.6 Dynamic Link Updates**

The link weights are evaluated and updated in this fashion. If the neighbor agent’s recommendation was the same as that of the minority of the group, then that neighbors’ link weight is incremented. The increments are capped at an upper limit. If the neighbor
agent’s recommendation was different than that of the minority of the group, then that neighbors’ link weight is decremented. If the decrement causes the link weight of that neighbor agent goes below a pre defined lower limit then that link is destroyed and a new agent is selected as a neighbor. The new agent selection is a uniformly random selection from the other agents playing the game.

Fig 5.7 Strategy Score Updates

The rule for the strategies is that if the strategy outcome, helped the agent win that round, the strategy score is incremented. If the outcome suggested by the strategy made the agent end up on the majority side then that strategy score is decremented. The strategy
scores are vital since while making decisions, the agent chooses the strategy with the highest score to make a decision.

Let us now assume the following scenario in order to understand the working of this model using Fig 5.1. For example, suppose Agent 1 has 3 friends namely Agent 4, Agent 6 and Agent 7 with corresponding link weights 0.4, 0.7 and 0.9. The decisions of the friends are Agent 4 decided to buy(1), Agent 6 and 7 decided to sell(0) the stock for that round. Now Agent 1 while making her final decision, will sum up the buy sides and the sell sides with her friends i.e. Since Agents 1 and 4 decided to buy, the buy sum is 1.4. Similarly Agents 6 and 7 decided to sell so the sell sum is 1.6. Since the sell side is greater than the buy side, Agent 1 finally decides to sell the stock.

After each round, the agent evaluates the strategies that are a part of her Long term memory. The Agent will also review the link weight of each of her neighbors. Due to this dynamic link updates, the network structure dynamically gets updated. We will observe the evolution of the network and some stylized facts.

We observed herding behavior because the agents were communicating with each other. This is quite true in financial markets, where traders are influenced by local sources and traders. Traders tend to follow other traders in the markets. A detailed discussion on herding has already been done in Section 2.2.

We define the Degree of an agent as the number of incoming links for each agent i.e. the number of agents whose friend list contains that particular agent. We will see some characteristics of this degree distribution.
The degree distribution showed a *fat-tailed distribution*. This result helps us understand the dynamics of the network. This network is robust to random and accidental failures i.e. stock market spikes. However this network is susceptible, if a coordinated attack is attempted on it. A detailed discussion on fat-tailed distribution has already been done in Section 2.3.

The network which was initially a random network evolves into a star or hub shaped network. This is typically another indication that small world networks are getting formed.
5.5 Results

We now show the results using minority game agents only for the initial torus network.

The network topology before and after the dynamic updates for the torus network are shown in Fig 5.8 for N=101, R=500:

![Initial Network Topology](image1) ![Final Network Topology with Dynamic Link Updates](image2)

- Agent  Link between Agents

**Fig 5.8 Torus Network Topology before and after dynamic link updates**

This network is observed in intra market traders where the traders cluster around some strong influential traders. In static case nearly everyone is doing the same thing i.e. buy so we observe herding behavior of bull markets. In dynamic case nearly everyone is doing the same thing i.e. sell so we observe herding behavior of bear markets.
Similarly we now observe how the network topology evolves for a random network:

![Initial Network Topology](image1.jpg)

![Final Network Topology with Dynamic Link Updates](image2.jpg)

**Fig 5.9 Random Network Topology before and after dynamic link updates**

As seen in Fig 5.9 we observe that the initial random network evolves into a hub formation after the dynamic link updates. Fewer influencers and not so prominent in random network compared to torus network. This network is observed in inter market traders where the influential traders have clustered around some strong influential traders.

Dynamic Random network makes the market behavior maximally diverse i.e nearly equal number decide to buy or sell (no herding).
Path Length is defined as the number of directed edges between two nodes. As suggested by [25],

“the hub formation enhances the cohesiveness of the system by reducing the shortest average path length between agents relative to random graphs as network size increases and the network connections become sparse”

Fig 5.10 Attendance N=101, Rounds=500, Static Random Nw vs Dynamic Random Nw

Here we have the number of rounds to be 500, so that the network has sufficient time to evolve and overcome the initial random changes in its structure and finally emerge.
We observe that there are 6 clusters formed in this topology.

Fig 5.1

In case of the torus network with dynamic link updates, few influential nodes essentially make decisions for all other nodes and therefore the attendance becomes low

The degree distribution of the initial torus network and the Final Network is shown in Fig 5.12 and Fig 5.13:
Fig 5.32 Initial Degree Distribution for Torus nw for N=101, Rounds=500, Dynamic Link Updates

Fig 5.13 Fat-Tailed Distribution for Torus nw for N=101, Rounds=500, Dynamic Link Updates
If we analyze the tail of the degree distribution we observe that it depicts power law distribution with $\alpha=1.524$ as seen in Fig 5.14.

![Graph showing power law distribution](image)

*Fig 5.14 Power Law Distribution of Tail for $N=101$, Rounds=500, Dynamic Link Updates*

We have also shown more results in Chapter 6 by varying the number of Agents participating in the game.
We will now see the network phenomena, when we have the initial network as a random network of agents.

As observed in figure 5.15, we see how the network topology evolves from a random network to a network that has hubs of clustered agents.

The initial degree distribution and the degree distribution after dynamic updates for the above network is shown below:

Fig 5.15 Initial random network and final network topology=101, Rounds=500

As observed in figure 5.15, we see how the network topology evolves from a random network to a network that has hubs of clustered agents.

The initial degree distribution and the degree distribution after dynamic updates for the above network is shown below:
Fig 5.16 Initial Degree Distribution for Random Network with N=101, R=500

Fig 5.17 Final Degree Distribution for Random Network with N=101, R=500
Fig 5.18 Power Law Distribution of tail of Degree Distribution for Random Network with N=101, R=500

The tail again depicts a power law property and the value of $\alpha = 1.545$ as seen in Fig 5.18.

Fig 5.19 shows the initial degree distribution of the random network for N=101 and 500 rounds. Fig 5.20 shows the Degree distribution after the dynamic updates and again we see that the distribution is a Fat-tailed distribution.
Fig 5.19 Initial Degree Distribution, N=51, Rounds=500

Fig 5.20 Final Degree Distribution, N=51, Rounds=500
The tail again shows a power law distribution with $\alpha=0.981$ as seen in Fig 5.21.

We shall see more results and a comparison of all our systems in Chapter 6.

We also did a comparison of the agent’s degree vs her short term memory length and the results were as follows:

**Fig 5.21 Power Law Distribution, N=51, Rounds=500**

The tail again shows a power law distribution with $\alpha=0.981$ as seen in Fig 5.21.

We shall see more results and a comparison of all our systems in Chapter 6.

We also did a comparison of the agent’s degree vs her short term memory length and the results were as follows:
Fig 5.22  Agent’s degree vs Short Term Memory Length

For the Torus network we observed that the degree of the agent remained constant irrespective of her memory length. For the random network we observe that the degree decreases as the memory increases.
Chapter 6

Results and Interpretations

In this Chapter, we do a review and a comparison of the results obtained from the three systems that have been designed in this thesis.

6.1 Basic Minority Game Results

In this section, we see a few more results of the Basic Minority Game. The Mean attendance self emerges to a specific value.

![Graph showing attendance vs time](image)

Fig 6.1 N=501, Rounds=100, Basic Minority Game
In Fig 6.1, we observe the Mean Attendance that converges to nearly 60% of the total Agents. The Number of Agents participating is 501 for the above diagram.

Fig 6.2 N=1001, Rounds=100, Basic Minority Game

In Fig 6.2, we observe the Mean Attendance that converges to exactly 60% of the total Agents. The Number of Agents participating is 1001 for Fig 6.2.
6.2 Modified Minority Game

Here we present a few more results of the modified minority game.

Fig 6.3 N=501, Rounds=100, Links=5, Modified Minority Game
Fig 6.3 shows the Mean attendance graph for N=501 for the Modified Minority Game. We can see that there is tremendous amount of herding with the mean being 86%.

**6.3 Comparison between the Basic Minority Game and the Modified Minority Game**

In the Basic Minority Game we observed the self emergence of the Bar Attendance to 60% of the total attendance. We have displayed the results for various combinations of number of agents and number of rounds. When we compare the Minority Game with Communication with the Basic Minority Game, we immediately observe the herding behavior.

For example, let us compare the mean attendance for both the basic Minority Game and the Minority Game with Communication (fixed number of links).

We ran the simulation for N=5001 agents for both our simulations for 100 rounds. For the minority game with communication we had 5 links which remained constant throughout the simulation.

Below is the comparison between the attendance for the Basic and the Modified Minority Game:
As seen in Fig 6.4, we clearly see the herding effect after the introduction of communication among the agents. This tends to happen when the agent tries to follow the flock while making their decisions.

6.4 Dynamic Regime with Link Updates for Minority Game Agents

We now display the fat-tailed distribution for the degrees of the network before and after dynamic updates for the minority game agents.
Fig 6.5 Initial Degree Distribution N=501, Rounds=100

Fig 6.6 Final Degree Distribution N=501, Rounds=100
As we can see in Fig 6.5, the random network degree distribution is Gaussian in nature. After the dynamic updates have been applied the degree distribution changes and becomes fat-tailed as seen in Fig 6.6. Fig 6.5 and Fig 6.6 show the degree distribution when N=501.

The tail depicts a power law which is typical of fat tailed distributions as shown in Fig 6.7:

![Power law exhibited by tail for N=501, R=100, α=0.894](image)

**Fig 6.7** Power law exhibited by tail for N=501, R=100, $\alpha=0.894$
As we can see in Fig 6.8, the random network degree distribution is Gaussian in nature.

After the dynamic updates have been applied the degree distribution changes and
becomes fat-tailed as seen in Fig 6.9. Fig 6.8 and Fig 6.9 show the degree distribution when N=1001.

![Graph showing Power Law Distribution](image)

**Fig 6.10 Power Law Distribution of tail for N=1001, Rounds=100**

Again we observe the tail depicting a power law distribution with $\alpha=0.828$ as seen in Fig 6.10.

We will now see the degree distribution for N=1001, for the initial torus network.
Fig 6.11 Initial Degree Distribution N=5001, Rounds=100

Fig 6.12 Part 1 of Final Degree Distribution N=5001, Rounds=100
Fig 6.13 Part 2 of Final Degree Distribution N=1001, Rounds=100

Fig 6.12 and 6.13 show the final degree distribution in parts.

Fig 6.14 Power Law Distribution of tail for N=5001, Rounds=100
After the dynamic updates have been applied, the degree distribution changes and becomes fat-tailed as seen in Fig 6.12 and Fig 6.13. Fig 6.14 shows the power law distribution of the tail.
Chapter 7

Conclusions

In the minority game we learnt that even though the players were making their decisions independently, self emergence is observed and the attendance converges to 60% of the total players playing the game. 60% emerges as a “natural attractor” as discussed in Section 3.2.

The Minority Game model was a very simplified model of a market and it gave us a complex market dynamic of emergence. However, in order to simulate a realistic market model to understand more characteristics of the markets, we needed to modify the Basic Minority Game. We chose to introduce communication among the agents as our first modification and thus create a financial network of agents. We wanted to observe how agents would influence each other by sharing their decisions whether to buy or sell a stock. We expected that those agents who interact with each other would tend to make a similar decision.

As seen in Section 4.5, the agents showed herding tendency as the agents got influenced by each other’s decisions confirming our expectation.

The above Modified Minority Game had static links i.e. each agent had the same friend list and also each of the links had equal weight age. Financial markets are very dynamic in nature with a lot of information coming in and traders keep updating their decisions
based on the prevailing market conditions. So the next modification would be to have a dynamically updating model. We also consider the fact that each trader has a different memory length with regards to the stock data. So our agents have variable memory lengths. This makes our agents heterogeneous and diverse.

In Chapter 5, section 5.5, we observe that the two stylized facts namely, herding behavior and fat-tailed distribution shown by the agents collectively. We again observed herding because of the communication among agents. The degree distribution was fat-tailed after the dynamic updates were done as seen by the power law distribution of the tail. This helps us understand that this market model can sustain random fluctuations and failures. However this network is susceptible to a coordinated attack. These stylized facts are important characteristics depicted by financial markets as seen in Section 2.2. and Section 2.3. Also the network topology reorganizes itself from a random network to a hub shaped or star shaped topology which suggests that these networks may have some small world properties i.e. it is possible for any two nodes to communicate with each other quite easily.
Appendix A
Design of our Basic Minority Game

Figure A-1 Class Diagram of Basic Minority Game
The Class diagram consists of the following classes:

**PlayGame:** The Play game class has the main method that initiates the game. It takes the number of agents and number of rounds to be played.

```
+main
```

**Agent:** The Agent class contains a short term memory, a long term memory which has 5 strategies and agent scores. It contains methods that decide whether the agent goes to the bar or stays home.

```
-number:integer
-agent_score:integer
+current_round_decision:integer
+stm:ShortTermMemory
+ltm:LongTermMemory
+Agent()
+decide():void
+displayMemory():void
+update_scores(cnt:int):void
+get_agent_score():void
```

**ShortTermMemory:** The ShortTermMemory class defines that short term memory of each agent. The Short Term memory is a shift register that gets updated every round.

```
-sTermMem : Boolean
-random : Random
```
ShortTermMemory

ShortTermMemory(num : int)

+getSTermMem() : []Boolean

LongTermMemory: The Long term Memory contains the strategies for each agent. It randomly sets the result set for each strategy. It also contains methods that return the best strategy and the corresponding result for that strategy.

+strategy : Strategy

+current_result_set : Boolean

LongTermMemory(num : int)

+getBestStrategyIndex() : int

+getDecision(bs : int, shortterm : [] Boolean) : Boolean

Strategy: The strategy class generates the strategies for the agents. It also has scores for each strategy and methods to get results of strategies and both getter and setter methods for strategy scores.

-s_score : integer

-s_set : Boolean

-s_result_set : Boolean

+s_number : integer

+Strategy(num : integer)

+get_s_score() : integer

+set_s_score(score : integer) : void

+update_s_score(change : integer) : void
+set_s_set() : void

+getResult(smem : []boolean) : boolean

The Use Case Diagram for the Basic Minority Game

Figure A-2 Use Case Diagram for Basic Minority Game
<table>
<thead>
<tr>
<th>Use Case Name</th>
<th>Play Game</th>
</tr>
</thead>
<tbody>
<tr>
<td>Participating Actors</td>
<td>User</td>
</tr>
</tbody>
</table>
| Flow of Events | 1) The User enters the number of Agents and the number of Rounds to be played.  
2) The Agents are created, each agent has a short term and long term memory.  
3) For each round, the player checks his short term memory and based on its value looks up his/her best strategy in the long term memory.  
4) Based on the strategy result, the agent decided whether to go or not to go to the bar.  
5) At the end of each round, the agents’ scores are updated and a new round is played.  
6) At the end of all the rounds, each player’s scores are displayed and the player with the highest score is the winner of the game.  
7) The game is repeated for various combinations of agents and rounds. |
<table>
<thead>
<tr>
<th>Entry Condition</th>
<th>User enters number of agents and number of rounds to be played</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exit Condition</td>
<td>All the rounds have completed and player’s scores displayed.</td>
</tr>
<tr>
<td>Quality Requirements</td>
<td>The user should be able to configure the number of agents and number of rounds.</td>
</tr>
</tbody>
</table>

Table A1 Use Case Scenario for Basic Minority Game
Fig A-3 Sequence Diagram for Basic Minority Game

The above sequence diagram describes the sequence of events that takes place when the user initiates the game.
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