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MODELING STAKEHOLDER INTERACTIONS IN AN INTERNATIONAL FREIGHT TRANSPORT SYSTEM

By

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Dr. Maria Boilé

and approved by

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New Brunswick, New Jersey

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ABSTRACT OF THE DISSERTATION

MODELING STAKEHOLDER INTERACTIONS IN AN INTERNATIONAL FREIGHT TRANSPORT SYSTEM

BY
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Dissertation Director:
Dr. Maria Boilé

This dissertation presents a three-level hierarchical approach which models carrier interactions in international maritime freight transportation networks. Ocean carriers, land carriers and port terminal operators are considered. Port terminal operators, providing transportation services within a port complex, are regarded as a special type of carrier, based on their behavior. The three types of carriers make pricing and routing decisions at different parts of the multimodal network, having hierarchical relationships. Ocean carriers are regarded as the leaders in a maritime shipping market. Port terminal operators are the followers of ocean carriers as well as the leaders of land carriers. The individual carrier problem is formulated at each level using Nash equilibrium in the competitive environment and compensation principle in the collusive environment, respectively. Interactions among different types of carriers are captured in three-level models according to the different carrier environment. Subsequently, shipper-carrier
interactions are captured in bi-level models when carriers are the leaders and shippers are the followers. Shippers determine the production, consumption and distribution of goods using spatial price equilibrium. Shippers are users of transportation services, therefore they choose a sequence of carriers based on the carriers’ decisions. The concepts of Stackelberg game and multi-leader-follower game are applied to multi-level games. Heuristic algorithms are developed to solve the carrier and shipper problems. This work contributes to the body of the multimodal freight network problem literature by providing a novel methodology to formulate multi-level hierarchical interaction models. The dissertation assists in understanding of the dynamics and decision-making processes of various stakeholders involved in international freight transportation via waterways.
Acknowledgement and Dedication

I would like to express sincere appreciation to my advisor, Dr. Maria Boilé, who gave me academic guidance, encouragement and support during my study. She advised me to choose and explore an interesting topic for my dissertation. I would also like to thank Dr. Theofanis for his insightful comments and advises on this work. I would like to extend my gratitude to Dr. Kaan Ozbay, Dr. Trefor Williams and Dr. Mihalis Golias for serving in my doctorate dissertation committee.

I would like to appreciate faculty and staff in Center for Advanced Infrastructure and Transportation and my fellow graduate students. Special thanks to my family. Their continuous supports and prayers encouraged me to study tirelessly and finish. I would like to express my love. This dissertation is dedicated to my parents.
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NOMENCLATURE

Carrier problem

Indices

<Ocean carrier>

\( O \) set of ocean carriers
\( I \) set of ocean carrier origins
\( J \) set of ocean carrier destinations
\( IJ \) set of ocean carrier O-D pairs
\( LK_o \) set of ocean links
\( PH_o \) set of ocean paths
\( o \) ocean carrier \((o \in O)\)
\( i \) ocean carrier origin \((i \in I)\)
\( j \) ocean carrier destination \((j \in J)\)
\( ij \) ocean carrier O-D pair \((ij \in IJ)\)
\( lk_o \) ocean link \((lk_o \in LK_o)\)
\( ph_o \) ocean path \((ph_o \in PH_o)\)

<Port terminal operator>

\( P \) set of port terminal operators
\( J \) set of port terminal origins
\( K \) set of port terminal destinations
\( JK \) set of port terminal O-D pairs
\( LK_p \) set of port links
\( PH_p \) set of port paths
\( p \) port terminal operator \((p \in P)\)
\( j \) port terminal origin \((j \in J)\)
\( k \) port terminal destination \((k \in K)\)
\( jk \) port terminal O-D pair \((jk \in JK)\)
\( lk_p \) port link \((lk_p \in LK_p)\)
\( ph_p \) port path \((ph_p \in PH_p)\)
<Land carrier>

$L$ set of land carriers
$K$ set of land carrier origins
$W$ set of land carrier destinations
$KW$ set of land carrier O-D pairs
$LK_{l}$ set of land links
$PH_{l}$ set of land paths
$l$ land carrier ($l \in L$)
$k$ land carrier origin ($k \in K$)
$w$ land carrier destination ($w \in W$)
$kw$ land carrier O-D pair ($kw \in KW$)
$lk_{l}$ land link ($lk_{l} \in LK_{l}$)
$ph_{l}$ land path ($ph_{l} \in PH_{l}$)

Data and Parameters

$Q_{iw}$ transportation demand from $i$ via $j$ to $w$
$vot^{o}$ value of time of ocean carrier $o$
$vot^{p}$ value of time of port terminal operator $p$
$vot^{l}$ value of time of land carrier $l$
$att^{lk_{o}}$ average travel time on the ocean link $lk_{o}$
$ast^{lk_{o}}$ average shift time on the port-entrance link $lk_{o}$
$ast^{lk_{p}}$ average service time on the port link $lk_{p}$
$ast^{lk_{l}}$ average shift time on the port-exit link $lk_{l}$
$att^{lk_{l}}$ average travel time on the land link $lk_{l}$
$cap^{lk_{o}}$ capacity of the ocean link $lk_{o}$
$cap^{lk_{p}}$ capacity of the port link $lk_{p}$
$cap^{lp}$ capacity of port terminal operator $p$
$cap^{lk_{l}}$ capacity of the land link $lk_{l}$
$\alpha_{0}, \alpha_{1}$ coefficient parameters in the ocean carrier demand function
$\omega_{0}, \omega_{1}, \omega_{2}$ coefficient parameters in the average ocean link operating cost function
$\mu_{0}, \mu_{1}$ coefficient parameters in the average ocean link travel (shift) time function
\( \beta_0, \beta_0 \) coefficient parameters in the port throughput function

\( \pi_0, \pi_1, \pi_2 \) coefficient parameters in the average port link operating cost function

\( v_0, v_1 \) coefficient parameters in the average port link service time function

\( \gamma_0, \gamma_1, \nu_0, \nu_1 \) coefficient parameters in the average port service charge function

\( \lambda_0, \lambda_1 \) coefficient parameters in land carrier demand function

\( \tau_0, \tau_1, \tau_2 \) coefficient parameters in the average land link operating cost function

\( \theta_0, \theta_1, \theta_2, \theta_3 \) coefficient parameters in the average land link travel (shift) time function

Decision Variables

<Ocean carrier>

\( c_{ij}^o \) continuous variable, the service charge of ocean carrier \( o \) between \( i \) and \( j \)

\( c_{ij}^{o-} \) continuous variable, the service charges of other ocean carriers, except ocean carrier \( o \), between \( i \) and \( j \)

\( c_{ij} \) continuous variable, the ocean carrier service charge between \( i \) and \( j \)

\( f_{ij}^o \) continuous variable, the commodity flow of ocean carrier \( o \) between \( i \) and \( j \)

\( f_{ij}^{ph_o.o} \) continuous variable, the commodity flow of ocean carrier \( o \) on the ocean path \( ph_o \) connecting O-D pair \( ij \)

\( f_{ij}^{ph_o} \) continuous variable, the commodity flow on the ocean path \( ph_o \) connecting O-D pair \( ij \)

\( f_{lk_o}^{o} \) continuous variable, the commodity flow on the ocean link \( lk_o \)

\( \xi_{ij}^{lk_o,ph_o} \) binary variable, 1, if the ocean link \( lk_o \) is on the ocean path \( ph_o \)

0, otherwise

\( \psi_{ph_o,p}^{o} \) binary variable, 1, if the ocean path \( ph_o \) is connected to port terminal operator \( p \)

0, otherwise

\( \eta_{o,l} \) binary variable, 1, if alliances exist between ocean carrier \( o \) and land carrier \( l \)

0, otherwise
<Port terminal operator>

\( c^p \) continuous variable, the service charge of port terminal operator \( p \)

\( c^{-p} \) continuous variable, the service charges of other port terminal operators, except port terminal operator \( p \)

\( f^p \) continuous variable, the commodity flow of port terminal operator \( p \)

\( f_{jk}^{ph, p} \) continuous variable, the commodity flow on the port path \( ph_j \) connecting O-D pair \( jk \)

\( f_{jk}^{ph, p, o} \) continuous variable, the commodity flow on the port path \( ph_j \) connecting O-D pair \( jk \), delivered by carrier \( o \)

\( f_{kw}^{l} \) continuous variable, the commodity flow of land carrier \( l \) between \( k \) and \( w \)

\( f_{kw}^{-l} \) continuous variable, the service charges of other land carriers, except land carrier \( l \), between \( k \) and \( w \)

\( c_{kw} \) continuous variable, the land carrier service charge between \( k \) and \( w \)

\( f_{kw}^{ph, l} \) continuous variable, the commodity flow of land carrier \( l \) on the land path \( ph_j \) connecting O-D pair \( kw \)

\( f_{kw}^{ph, l} \) continuous variable, the commodity flow on the land path \( ph_j \) connecting O-D pair \( kw \)

\( f_{kw}^{l} \) continuous variable, the commodity flow on the land link \( lk_j \)

\( \delta_{kw, ph, l} \) binary variable, 1, if the land link \( lk_j \) is on the land path \( ph_j \)

0, otherwise

<Land carrier>

\( c_{kw}^{l} \) continuous variable, the service charge of land carrier \( l \) between \( k \) and \( w \)

\( c_{kw}^{-l} \) continuous variable, the service charges of other land carriers, except land carrier \( l \), between \( k \) and \( w \)

\( c_{kw} \) continuous variable, the land carrier service charge between \( k \) and \( w \)

\( f_{kw}^{l} \) continuous variable, the commodity flow of land carrier \( l \) between \( k \) and \( w \)

\( f_{kw}^{ph, l} \) continuous variable, the commodity flow of land carrier \( l \) on the land path \( ph_j \) connecting O-D pair \( kw \)

\( f_{kw}^{ph, l} \) continuous variable, the commodity flow on the land path \( ph_j \) connecting O-D pair \( kw \)

\( f_{kw}^{l} \) continuous variable, the commodity flow on the land link \( lk_j \)

\( \delta_{kw, ph, l} \) binary variable, 1, if the land link \( lk_j \) is on the land path \( ph_j \)

0, otherwise
Functions

<Ocean carrier>

\( U^{o}_{ij} \)  
profit function of ocean carrier \( o \) between \( i \) and \( j \)

\( I^{o}_{ij} \)  
income function of ocean carrier \( o \) between \( i \) and \( j \)

\( D^{o}_{ij} \)  
service demand function of ocean carrier \( o \) between \( i \) and \( j \)

\( D_{ij} \)  
service demand function between \( i \) and \( j \)

\( AC^{ph-o}_{ij} \)  
average transportation cost function on the ocean path \( ph_o \) connecting O-D pair \( ij \)

\( TC^{o}_{ij} \)  
transportation cost function of ocean carrier \( o \) between \( i \) and \( j \)

\( TC^{ph-o-o}_{ij} \)  
transportation cost function of ocean carrier \( o \) on the ocean path \( ph_o \) connecting O-D pair \( ij \)

\( TC^{ph-o}_{ij} \)  
transportation cost function on the ocean path \( ph_o \) connecting O-D pair \( ij \)

\( TC^{lk-o}_{ij} \)  
transportation cost function on the ocean link \( lk_o \)

\( AOC^{lk-o}_{ij} \)  
average operating cost function on the ocean link \( lk_o \)

\( ATT^{lk-o}_{ij} \)  
average travel (shift) time function on the ocean link \( lk_o \)

\( MOC^{lk-o}_{ij} \)  
marginal operating cost function on the ocean link \( lk_o \)

\( MTT^{lk-o}_{ij} \)  
marginal travel (shift) time function on the ocean link \( lk_o \)

<Port terminal operator>

\( U^{p} \)  
profit function of port terminal operator \( p \)

\( G^{p} \)  
port throughput function of port terminal operator \( p \)

\( C^{p} \)  
average port service charge function of port terminal operator \( p \)

\( AC^{ph-p}_{jk} \)  
average service cost function on the port path \( ph_p \) connecting O-D pair \( jk \)

\( SC^{p}_{jk} \)  
port service cost function of port terminal operator \( p \)

\( SC^{ph-p}_{jk} \)  
port service cost function on the port path \( ph_p \) connecting O-D pair \( jk \)

\( SC^{lk-p}_{jk} \)  
port service cost function on the port link \( lk_p \)

\( AOC^{lk-p}_{jk} \)  
average operating cost function on the port link \( lk_p \)

\( AST^{lk-p}_{jk} \)  
average service time function on the port link \( lk_p \)

xvi
\( MOC^{lk,p} \) marginal operating cost function on the port link \( lk_p \)

\( MST^{lk,p} \) marginal service time function on the port link \( lk_p \)

<Land carrier>

\( U^{kw}_{lw} \) profit function of land carrier \( l \) between \( k \) and \( w \)

\( I^{kw}_{lw} \) income function of land carrier \( l \) between \( k \) and \( w \)

\( D^{lw}_{kw} \) service demand function of land carrier \( l \) between \( k \) and \( w \)

\( D_{lw} \) service demand function between \( k \) and \( w \)

\( AC^{ph,l}_{kw} \) average transportation cost function on the land path \( ph_j \) connecting O-D pair \( kw \)

\( TC^{l} \) transportation function of land carrier \( l \)

\( TC^{lw}_{kw} \) transportation cost function of land carrier \( l \) between \( k \) and \( w \)

\( TC^{ph,l,j}_{kw} \) transportation cost function of land carrier \( l \) on the land path \( ph_j \) connecting O-D pair \( kw \)

\( TC^{ph,l}_{kw} \) transportation cost function on the land path \( ph_j \) connecting O-D pair \( kw \)

\( TC^{lk,l}_{kw} \) transportation cost function on the land link \( lk_j \)

\( AOC^{lk,l}_{kw} \) average operating cost function on the land link \( lk_j \)

\( ATT^{lk,l}_{kw} \) average travel (shift) time function on the land link \( lk_j \)

\( MOC^{lk,l}_{kw} \) marginal operating cost function on the land link \( lk_j \)

\( MTT^{lk,l}_{kw} \) marginal travel (shift) time function on the land link \( lk_j \)

**Shipper problem**

Indices

\( S \) set of shippers

\( X \) set of shipper origins

\( Y \) set of shipper destinations

\( XY \) set of shipper O-D pairs

\( LK_s \) set of shipper links
$PH_s$ set of shipper paths
$s$ shipper ($s \in S$)
$x$ shipper origin market ($x \in X$)
$y$ shipper destination market ($y \in Y$)
$xy$ shipper O-D pair ($xy \in XY$)
$lk_s$ shipper link ($lk_s \in LK_s$)
$ph_s$ shipper path ($ph_s \in PH_s$)

Data and Parameters

$\sigma_0, \sigma_1$ coefficient parameters in the inverse supply function
$\rho_0, \rho_1$ coefficient parameters in the inverse demand function
$\kappa_0, \kappa_1$ coefficient parameters in the shipper travel time function
$vot^s$ value of time of the shipper $s$
$att^lk_s$ average travel time on the shipper link $lk_s$
$cap^lk_s$ capacity of the shipper link $lk_s$
$t_c$ tariff of commodity $c$

Decision Variables

$s_x$ continuous variable, the amount of supply at the origin market $x$
$d_y$ continuous variable, the amount of demand at the destination market $y$
$f^lk_s$ continuous variable, the commodity flow on the shipper link $lk_s$
$f^ph_s$ continuous variable, the commodity flow on the shipper path $ph_s$
$\xi_{lk_s,o}$ binary variable, 1, if the shipper link $lk_s$ is served by ocean carrier $o$
0, otherwise
$\xi_{lk_s,p}$ binary variable, 1, if the shipper link $lk_s$ is served by port terminal operator $p$
0, otherwise
$\xi_{lk_s,l}$ binary variable, 1, if the shipper link $lk_s$ is served by land carrier $l$
0, otherwise
Functions

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$IS_x$</td>
<td>inverse supply function at the origin market $x$</td>
</tr>
<tr>
<td>$ID_y$</td>
<td>inverse demand function at the destination market $y$</td>
</tr>
<tr>
<td>$ATC^{ph_{xy}}$</td>
<td>average transaction cost function on the shipper path $ph_{xy}$, connecting O-D pair $xy$</td>
</tr>
<tr>
<td>$ATC^{lk_{s}}$</td>
<td>average transaction cost function on the shipper link $lk_{s}$</td>
</tr>
<tr>
<td>$ATT^{lk_{s}}$</td>
<td>average travel time function on the shipper link $lk_{s}$</td>
</tr>
</tbody>
</table>
CHAPTER 1

INTRODUCTION

1.1 Background

In today’s global market place, setting competitive yet profitable service charges and minimizing transportation costs, while providing good quality services, are challenges facing transportation service providers. Through the 1980s, many modeling approaches for predicting freight movements have focused on intercity freight transportation at national or regional levels, mostly regarding land carriers (i.e., motor carriers and railroads) as transportation service providers. The research efforts to study freight transportation via waterways emerged in the 1990s following the rapid growth of the international maritime freight industry and change in the freight transport system.

Maritime shipment plays a vital role in global trade as the most cost and energy effective mode of transport. The world maritime container traffic has increased significantly over the past two decades with the increased size of container ships, advances in information technology, the expansion of global economy and trade agreements among countries. According to the Bureau of Transportation Statistics (BTS, 2010), the world maritime shipment grew from 137.2 million TEUs (20 foot equivalent units) in 1995 to 432.0 million TEUs in 2009, showing an increase of 214.9%. There was sustained growth between 1995 and 2008. On the other hand, the global recession caused a relatively significant decline in 2009, as depicted in Figure 1.1
TEUs = Twenty-foot equivalent units. One 20-foot container equals one TEU and one 40-foot container equals two TEUs.

Source: Bureau of Transportation Statistics (BTS)

**Figure 1.1 World Maritime Container Traffic Trends**

The growth forecasts of container movements are ultimately derived from the economic trends. International Monetary Fund (IMF, 2011) published World Economic Outlook for the 2010-2012 periods. In 2010, global Gross Domestic Product (GDP) increased 5.0%, experiencing the economic recovery. The forecast data indicate growth of 4.4% in 2011 and 4.5% in 2012 each. Therefore, world maritime container traffic is expected to grow continuously.

The maritime traffic increase has resulted in fundamental changes in the roles of seaports as major nodes in the intermodal freight transport system. Ports provide transportation services for outbound, inbound and transshipment freight at sea-land interfaces or transshipment points. They have become an integral part of the freight logistics link, particularly with regard to the international market. Ports have been
confronted with more containers to be treated effectively and productively, and forced to think how to improve the service quality for maintaining and serving current and new business. Hence, various efforts and policies have been established to improve the overall service environment in ports.

There exist two types of major stakeholders involved in port operations: Port Authorities and port terminal operators. For the competitive power of a port, in terms of capacity, competitive service charge and efficient operations, decisions of the two stakeholders are quite important. Port Authorities determine various port policies associated with finance, tariff and labor and are responsible for port transportation infrastructure at the supply side of the port. Port Authorities attempt to take advantage of the marine transportation market for the prosperity of ports and local communities as public agencies. In general, particularly in the common case of a landlord port, a Port Authority controls the entire port and leases the marine terminals to private terminal operators. Port terminal operators provide transportation services of vehicles and containers within a port complex. Port terminal operators can be regarded as a special type of carrier based on their behavior.

In the given port layouts by Port Authorities, port terminal operators attempt to improve port services in a more cost and time efficient manner. Particularly, port service charges determined by port terminal operators are an integral factor in port choices by ocean carriers. For example, port service charges including wharf charge, pilot charge, handling charge, etc. in Japan ports is more than twice higher than those in Pusan. Hence, many ocean liners delivering containers from China to America and to/from western Japan has shifted to Busan (Zan, 1999).
Throughputs of top 10 world’s busiest container ports handled freight ranging from 974 TEUs to 2,587 TEUs, as of 2009. Industrializing countries in Asia experienced fast regional economic development and port growth. Nine Asian ports were ranked for top 10 world container ports, containing top 9: Singapore, Shanghai, Hong Kong, Shenzhen, Busan, Guangzhou, Dubai, Ningbo and Qingdao. Table 1.1 shows top 10 world container ports and their throughputs in 2009.

**Table 1.1 Top 10 World Container Ports**

<table>
<thead>
<tr>
<th>Rank</th>
<th>Port</th>
<th>Country</th>
<th>TEUs (thousand)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Singapore</td>
<td>Singapore</td>
<td>2,587</td>
</tr>
<tr>
<td>2</td>
<td>Shanghai</td>
<td>China</td>
<td>2,500</td>
</tr>
<tr>
<td>3</td>
<td>Hong Kong</td>
<td>China</td>
<td>2,098</td>
</tr>
<tr>
<td>4</td>
<td>Shenzhen</td>
<td>China</td>
<td>1,825</td>
</tr>
<tr>
<td>5</td>
<td>Busan</td>
<td>South Korea</td>
<td>1,195</td>
</tr>
<tr>
<td>6</td>
<td>Guangzhou</td>
<td>China</td>
<td>1,119</td>
</tr>
<tr>
<td>7</td>
<td>Dubai</td>
<td>United Arab Emirates</td>
<td>1,112</td>
</tr>
<tr>
<td>8</td>
<td>Ningbo</td>
<td>China</td>
<td>1,050</td>
</tr>
<tr>
<td>9</td>
<td>Qingdao</td>
<td>China</td>
<td>1,026</td>
</tr>
<tr>
<td>10</td>
<td>Rotterdam</td>
<td>Netherlands</td>
<td>974</td>
</tr>
</tbody>
</table>

Source: American Association of Port Authorities (AAPA)

It is well known that the inter port competition is a dominating issue in the international maritime shipping market. As maritime shipment increases significantly and ports develop rapidly, port competition becomes more intense to attract more ocean carrier companies. In addition, the decentralized ownership of port instigates the competition. Most port terminals in a port complex are also competitive since typically the terminals are managed by private port terminal operators. Therefore, both inter and
intra port competitions occur simultaneously in many regions. The intra port terminal competition is considered beneficial for port terminals and local or national economies, although the competition could cause the waste of facility and more congestion (Langen and Pallis, 2005). On the other hand, port terminals collude in a port complex or ports in competitive regions make alliances due to the port conditions or regional situations. Therefore, three port (port terminal operator) competition/collusion cases can be classified: 1) inter/intra port competition 2) inter port competition/intra port collusion and 3) inter/intra port collusion, as depicted in Figure 1.2.

**Figure 1.2 Port Competition/Collusion Cases**
Ocean carriers provide maritime transportation services between a departure port terminal and an arrival port terminal. 5,890 ships active on ocean liner trades delivered 13,636,710 TEUs, as of 2009. Top 10 ocean carriers delivered 57.7% of the total ocean freight movement. In particular, big 3 ocean carriers such as APM-Maersk, Mediterranean Shg Co and CMA CGM Group transported 33.5%, showing relatively significant market shares. Table 1.2 shows top 10 ocean carriers and their operated fleets in 2009.

Table 1.2 Top 10 Ocean Carriers

<table>
<thead>
<tr>
<th>Rank</th>
<th>Ocean carrier</th>
<th>TEUs (thousand)</th>
<th>Ships</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>APM-Maersk</td>
<td>2,047</td>
<td>538</td>
</tr>
<tr>
<td>2</td>
<td>Mediterranean Shg Co</td>
<td>1,488</td>
<td>388</td>
</tr>
<tr>
<td>3</td>
<td>CMA CGM Group</td>
<td>1,031</td>
<td>353</td>
</tr>
<tr>
<td>4</td>
<td>Evergreen Line</td>
<td>559</td>
<td>151</td>
</tr>
<tr>
<td>5</td>
<td>APL</td>
<td>545</td>
<td>138</td>
</tr>
<tr>
<td>6</td>
<td>Hapag-Lloyd</td>
<td>458</td>
<td>112</td>
</tr>
<tr>
<td>7</td>
<td>COSCO Container L.</td>
<td>454</td>
<td>135</td>
</tr>
<tr>
<td>8</td>
<td>CSCL</td>
<td>450</td>
<td>124</td>
</tr>
<tr>
<td>9</td>
<td>Hanjin Shipping</td>
<td>440</td>
<td>99</td>
</tr>
<tr>
<td>10</td>
<td>NYK</td>
<td>407</td>
<td>106</td>
</tr>
</tbody>
</table>

Source: AXS-Alphaliner

The big ocean carrier companies cover most of the primary maritime routes and port terminals over the world. Ocean carriers providing transportation services via similar routes are fundamentally competitive to serve more users by offering high quality services in terms of time, money, safety, etc. On the other hand, some ocean transportation firms are engaged in global alliances against huge firms for surviving in
the competitive market environment. For example, 4 ocean firms (Hapag-Lloyd, MISC Berhad, Nippon Yusen Kaisha and Orient Overseas Container Line) and 3 ocean firms (APL, Hyundai Merchant Marine and Mitsui OSK Lines Ltd) began cooperating on key trade lanes in 2006 via Grand Alliance (GA) and New World Alliance (NWA), respectively. They initially exchanged slots on loops in the Asia-Europe and Asia-Mediterranean trades and added a new jointly operated loop on the Asia-East Coast of North America trade via the Panama Canal. All ocean carriers covering a section can form alliances to promote efficiency and avoid over competition. Therefore, three competition/collusion cases can be defined: 1) perfect competition, 2) partial competition and 3) perfect collusion, as shown in Figure 1.3. Land carriers have the similar competition/collusion patterns with ocean carriers.

![Figure 1.3 Ocean Carrier Competition/Collusion Cases](image-url)
There also exist relationships among different types of carriers. The competitive environment may be the normal case. On the other hand, many ocean carriers provide multimodal transportation services nowadays by combining maritime shipping with rail and truck services, for providing the seamless transportation service. Therefore, special negotiations and cooperation frequently occur between ocean carriers and land carriers. In these cases, direct interactions exist between them. The total amount of freight to be delivered by land carriers is decided by the routing pattern of their cooperating ocean carriers. Land transportation service costs charged to ocean carriers are predetermined through contracts. Alliances also can occur between ocean carriers and port terminal operators. Some port terminals suggest flexible service charges due to the total amount of freight arriving in their port terminals to attract more ocean carriers. As the total freight arriving in the port terminal by an ocean carrier increases, the unit port service charge decreases, within the port terminal capacity. To get more discounts, ocean liners try to make good deals with port terminals, by considering not only port conditions but also landside shipping conditions.

In short, a set of carriers in international maritime freight transportation networks includes ocean carriers, land carriers and port terminal operators. The carriers make pricing and routing (port service) decisions at different parts of the multimodal network, interacting with each other. To date, little attention has been paid to modeling maritime freight transport systems. A few models have captured relationships between shippers and ocean carriers or port terminal operators for international trade, assuming the behavior of the other carriers is known. Very few studies have considered the relationships among different types of carriers. The dissertation proposes a predictive network model, which
captures interactions among different types of carriers in a maritime shipping market using multi-level optimization programming.

Subsequently, a shipper - carrier model is formulated to capture interactions between shippers and carriers. Shippers determine the production, consumption and distribution of goods. Shippers are users of transportation services provided by carriers.

The rest of this chapter is organized as follows. Section 1.2 describes the concept of the freight transport system. Section 1.3 discusses the research problem and section 1.4 gives the organization of the dissertation.

1.2 Concept of the Freight Transport System

Understanding decisions and interactions of stakeholders in the freight transportation system is important to predict freight movements accurately. A set of stakeholders involved in freight transportation contains producers, consumers, shippers, brokers, freight-forwarders, motor carriers, railroads, ocean liners and regulatory agencies. To simplify this complex picture, Harker (1985), Crainic (2002) and Wang (2001) defined key stakeholders and their relationships.

Harker (1985) classified major stakeholders into producers, consumers, shippers, carriers and governments in the intercity freight transport system. Producers are the firms that produce goods and consumers are the individuals who consume these goods in various regions. Producers and consumers interact via market prices of goods which they are selling and buying. Shippers are firms that coordinate freight movements to explore
the potential economic benefit from the price differences between different regions. Carriers are firms (i.e., transportation companies or Third Party Logistics Service Providers) that operate transportation facilities and provide transportation services. Carriers determine service charges, levels of service and delivery routings to maximize their profits. Typically, shippers choose a carrier or a sequence of carriers to deliver goods. Governments are federal, state and local agencies that determine the regulation and provision of transportation infrastructure by investigating the behavior of carriers and shippers and expecting resulting effects, while their decisions influence the carriers and shippers’ reactions.

Some researchers did re-define and re-group the stakeholders for their modeling purposes. Crainic (2002) presented stakeholders involved in long-haul freight transportation, such as producers, consumers, shippers, carriers and governments. According to his definition, shippers are producers of goods or some intermediary firms (brokers). Wang (2000) combined the stakeholders into three major decision groups; shippers, carriers and regulatory agencies. Shippers were regarded as a generalized notation for anyone among the producer, consumer, freight-forwarder or broker.

Most former researchers regarded carriers as a decision group ignoring the interactions among different types of carriers frequently occurring in maritime shipping. For landside freight transportation, land carriers provide all transportation services from an origin to a destination. Land carriers include alternative modes such as truck, rail and combined truck/rail. On the other hand, for maritime freight transportation, ocean carriers need the connected transportation services by port terminal operators and land carriers to reach customer locations, as exhibited in Figure 1.4.
The dissertation presents distinctive roles of ocean carriers, port terminal operators, land carriers and their relationships in the maritime freight transport system as follows:

**Ocean carriers**

Ocean carriers manage ocean transportation facilities and provide ocean transportation services. Ocean carriers aim to maximize their profits. Decision variables are the transportation service charge, level of service and routing pattern (i.e., ocean link flow) on the ocean carrier network connecting the departure and arrival ports.

**Land carriers**

Land carriers control land transportation facilities and provide land transportation services. Land carriers aim to maximize their profits. Land carriers contain all land modes such as truck, rail and combined truck/rail. Decision variables are the transportation service charge, level of service and routing pattern (i.e., land link flow) on
the land carrier’s multimodal network.

Port terminal operators

Port terminal operators manage port facilities and offer port transportation services. Port operations such as loading, unloading, moving and storing containers are conducted along the port facilities. Port terminal operators aim to maximize their profits. Decision variables are the port service charge, level of service and port service pattern (i.e., port link flow) on the port sub-network.

Interactions of carriers

The three types of carriers make pricing and routing decisions depending on information on the shippers’ flow and governments’ decisions as well their relationships and reactions. Ocean carriers departing from a port terminal choose an arrival port terminal of the alternative ones by considering the transportation network and port determinants such as the port location, service charge and terminal efficiency (i.e., work practices, crane efficiency, inland transportation, etc.). On the other hand, port throughputs are determined by ocean carrier routings, therefore port terminal operators attempt to suggest competitive service charges and improve port service levels to serve more ocean liners. Port terminal operators have information on the inland transportation services, while land carrier service demands are determined from the port throughputs. Figure 1.5 depicts the interactions among three types of carriers.
Based upon the literature and carriers’ interactions above, Figure 1.6 shows a conceptual framework addressing roles and relationships of key stakeholders in the maritime freight transport system: shippers, ocean carriers, port terminal operators, land carriers and governments.

In this freight system, Port Authorities can be considered a type of government since they determine port policies and investment strategies as public or quasi-public agents associated with a city, county or state government. Port Authorities make operational decisions on financial, tariff and labor policies and change port transportation infrastructure to maximize the net social benefit derived from the regions they serve.
1.3 Research Problem

This research presents a novel multi-level hierarchical approach which models carrier interactions in international freight transportation networks. Ocean carriers, land carriers and port terminal operators are considered. Port terminal operators, providing transportation services within a port complex, are regarded as a special type of carrier, based on their behavior. Subsequently, a shipper - carrier model is formulated to capture interactions between shippers and carriers by a bi-level modeling approach. The consideration of port Authority’s decisions is beyond the scope of the dissertation.

The research questions are described below. The developed models will answer the following questions.
The carrier problem

- What are optimal pricing and routing decisions of the three types of carriers at an international maritime shipping market?
- How do the carriers interact with each other?
- What are impacts of the competitive or collusive market environment?

The shipper-carrier problem

- What are optimal decisions of shippers on the production, shipment and consumption pattern in an international maritime shipping market?
- How does the shipper network correspond to the carrier network defining the service charge and routing set.

1.4 Organization of the Dissertation

The dissertation is organized as follows. Chapter 2 presents a critical literature review of predictive freight network models using various equilibrium concepts. Chapter 3 defines the research problem with structures of the shipper’s aggregated network and the carrier’s multimodal network and modeling approaches. Chapter 4 focuses on the carrier’s pricing and routing problem that captures the behavior of three types of carriers - ocean carriers, port terminal operators and land carriers - and their interactions. The individual carrier problem is formulated at each level using Nash equilibrium in the competitive environment and compensation principle in the collusive environment, respectively. Interactions among different types of carriers are captured in three-level models.
according to the different carrier environment. Chapter 5 presents the shipper’s spatial price equilibrium problem. The shipper-carrier interactions are captured in bi-level models. Chapter 6 shows a case study to demonstrate the validity of the developed models using the MATLAB software version R2008a. Finally, Chapter 7 concludes the dissertation and suggests possible modeling extensions and future research directions.
CHAPTER 2

PREDICTIVE NETWORK MODELS: A REVIEW

Predictive network models forecast freight movements by representing the transportation network explicitly. The models capture decisions and interactions of key stakeholders involved in freight transportation, such as producers, consumers, shippers, carriers and governments, commonly using the four modeling methodologies: freight network equilibrium models, spatial price equilibrium models, integrated network equilibrium models and other equilibrium models (i.e., Nash equilibrium models and compensation equilibrium models).

For modeling, the decision sequence and relationships among different groups of stakeholders or within the same stakeholder group should be defined by considering the actual market circumstances or study purposes. Two general types of models exist; simultaneous models and sequential models. For modeling purposes, the decision sequence and relationships among different groups of stakeholders or within the same stakeholder group should be defined by considering the actual market circumstances or special study purposes. Two general types of models exist; simultaneous models and sequential models. In simultaneous models, regarded as normal game models, stakeholders are assumed to make decisions simultaneously. On the other hand, in many real cases, competitors make decisions at different times since some have priority in making decisions of policy over others (2010). In the sequential models, stakeholders are considered to make decisions hierarchically, defining the leader and the follower in the decision making process. If there is a single leader, the market is modeled as a
Stackelberg game and if there are multiple leaders, the market is modeled as a multi-leader-follower game.

Freight network equilibrium models chiefly capture simultaneous decisions among shippers or carriers, or simultaneous/sequential interactions between shippers and carriers, via user equilibrium models or/and system optimal models. Spatial price equilibrium models capture interactions of producers, consumers and shippers at the same time. Integrated network equilibrium models find interactions of producers, consumers, shippers and carriers simultaneously by adding Cournot-Nash models in the spatial price equilibrium models. Nash equilibrium models and compensation equilibrium models capture the simultaneous behavior of shippers or carriers. These models are combined with spatial price equilibrium models or freight network equilibrium models for sequential relationships between different types of stakeholders.

These modeling techniques have been used extensively in the freight modeling literature. Most of the models have focused on the intercity freight transport system at national or regional levels. Beginning in the late 1990s, researchers started to study international freight transportation via waterways, as the maritime freight industry and transport system changed rapidly. This Chapter reviews the existing predictive network models and compares their modeling approaches with the dissertation in detail.

Chapter 2 is organized as follows. Section 2.1 describes equilibrium concepts and main modeling advances of the modeling methodologies. Section 2.2 reviews predictive network models that have been formulated over the two decades, concerning the intercity freight transport system and the international maritime freight transport system,
respectively. Section 2.3 presents features and directions of the dissertation by comparing with the closely relevant papers.

2.1 Modeling Methodologies

Harker (1985) presented a good summary of major researches on predictions of intercity freight movements, according to the three modeling categories: econometric models, spatial price equilibrium models and freight network equilibrium models. Crainic (2002) provided an overview of freight transport systems and planning issues regarding long-haul transportation and presented an extensive review of modeling methodologies at strategic, tactical and operational planning levels. The strategic system planning that identifies the fundamental system components and their interactions uses modeling methodologies such as input/output models, spatial price equilibrium models and freight network optimization models. Valsaraj (2008) presented taxonomy of freight transportation models with two big categories: predictive models and design models. The predictive models were classified into spatial equilibrium models, freight network equilibrium models and integrated network equilibrium models. In particular, he addressed simultaneous or sequential decisions between the stakeholders through the freight network equilibrium models.

Based on the categories by Harker (1985), Crainic (2002) and Valsaraj (2008), freight network models have four types of modeling approaches: econometric models, spatial price equilibrium models, freight network equilibrium models and integrated network equilibrium models. The econometric models determine the production and the
attraction of goods and analyze impacts of various policies on labor, capital, etc., depending on the simple description of transportation network, while the other three types of models, called predictive network models, focus on freight movement predictions, representing transport systems by actual networks explicitly.

In addition to the three predictive network models, other equilibrium models such as Nash equilibrium models and compensation principle models were also used to formulate alternative stakeholder behavior and decision making process Figure 2.1 summarizes modeling approaches of predictive network models.

**Figure 2.1** Modeling Approaches of Predictive Network Models

The dissertation briefly describes equilibrium concepts and main modeling advances according to the four modeling methodologies and then discusses mixed equilibrium models generated by combining some equilibrium models.
2.1.1 Freight network equilibrium models

Freight network equilibrium models focus on the behavior of shippers or/and carriers. The models predict freight movements using Wardrop’s First Principle which is usually referred to as user equilibrium, and Wardrop’s Second Principle which is referred to as system optimal. User equilibrium states that all used paths between the same O-D (Origin-Destination) pair for the same commodity have the equalized lowest transportation cost. No player is capable of reducing his/her cost by unilaterally changing routes, at equilibrium. System optimal states that all used paths between the same O-D pair for the same commodity have the equalized lowest marginal cost to minimize the total transportation cost in a transport system. Generally, shippers determine trip distribution and modal split through user equilibrium, while carriers decide the trip assignment via system optimal. However, these network equilibrium models don’t include trip generation.

A system wide view of modeling freight flows was presented in the Harvard-Brookings model (1971). This model took into account the behavior of shippers. Friesz, et al. (1981a, 1981b) developed a predictive freight model called the Freight Network Equilibrium Model (FNEM), which initially recognized two distinct stakeholders, shippers and carriers. The model assumed sequential decisions, first solving the shipper sub-model and then sending the results to the carrier sub-models. Harker (1981) and Friesz, et al. (1985) proposed simultaneous models to improve the inconsistency of the sequential approach. Afterward, Friesz et al. (1986) formulated a sequential shipper-carrier model to show better model performance than simultaneous models, whereas many researchers focused on simultaneous decisions of major stakeholders.
2.1.2 Spatial price equilibrium models

Spatial price equilibrium models capture interactions of producers, consumers and shippers. The models find the optimal solutions on the production, consumption and distribution of goods, satisfying the following two equilibrium principles: 1) If the price of commodity \( i \) at the origin market plus the transportation cost between the origin and destination markets are equivalent to the price of commodity \( i \) at the destination market, the commodity flow between the markets will be positive. 2) If the price of commodity \( i \) at the origin market plus the transportation cost between the origin and destination markets are greater than the price of commodity \( i \) at the destination market, there will be no commodity flow.

The behavior of carriers is not directly considered in spatial price equilibrium models. Instead, arc and node cost functions are defined on the elements of the network. Samuelson (1952) initially presented the spatial price equilibrium problem by an optimization formulation approach with the fixed transportation cost function and linear supply and demand functions in an over-simplified transport system. Takayama and Judge (1964) extended Samuelson’s model to cover the cases of multiple commodities and multiple time periods. Subsequently, numerous researchers focused on modeling methodologies, solution algorithms and applications to various commodity and industry studies.

2.1.3 Integrated network equilibrium models

Integrated network equilibrium models capture interactions of producers, consumers, shippers and carriers simultaneously. In the traditional spatial price equilibrium problem,
the behavior of carriers is not explicitly included. On the other hand, this integrated concept incorporates spatial price equilibrium models with carrier behavior models. In the demand side model, trip generation, distribution and modal split are determined using spatial price equilibrium, while trip assignment is decided via the cost minimizing behavior of carriers in the supply side model. Harker (1983) proposed an integrated network equilibrium model and applied it to an analysis of the United States’ domestic coal economy. Harker and Freisz (1986a, 1986b, 1986c) extended and improved modeling and solution techniques.

2.1.4 Other equilibrium models

In addition to user equilibrium, system optimal and spatial price equilibrium, Nash equilibrium and compensation principle were used in literature. Devarajan (1981) and Haurie and Marccotte (1985) proved that if there are many players and they are competitive seeking the shortest path, based on other players’ choices, the game becomes user equilibrium. On the other hand, if there is only one player in charge of all the users, that attempts to minimize the total system travel cost, the game corresponds to system optimum. Therefore, players are either completely competitive or collusive in the Wardropian sense. The fundamental concepts of user equilibrium and system optimum are similar with those of Nash equilibrium in the competitive environment and compensation principle in the collusive environment. Nash equilibrium models make pricing and routing decisions for which each player obtains the greatest profit. No one can get better profit by changing his/her decisions unilaterally, at equilibrium. Compensation principle states that if prospective gainers could compensate prospective
losers and leave no one worse off, the other state is to be selected (Chipman, 1987). According to Hicks and Kalder (Tirole, 1988), if the total surplus increases, the winners can compensate the losers and everyone is made better off. With the compensation principle, better collective outcomes of collusive players could be obtained, compared to the sum of individual outcomes of competitive players. Hence, the total profit should be better than the sum of individual profits of competitive players, at equilibrium. Zhang et al. (2008) used Nash equilibrium for the competitive game of ports. Wang (2001) applied both Nash equilibrium and compensation principle to capture the behavior of carriers in the competitive and collusive market environment.

2.1.5 Mixed equilibrium models

Mixed equilibrium models are generated by combining some equilibrium models. The mixed equilibrium concept is used for capturing interactions between different groups of stakeholders or within the same stakeholder group, simultaneously or sequentially. For simultaneous decisions, freight network models unite user equilibrium models and system optimal models and integrated network equilibrium models integrates spatial price equilibrium models and Cournot-Nash models, respectively. These two combinations have been frequently applied by many researchers. Figure 2.2 depicts simultaneous modeling approaches using mixed equilibrium concepts.
Figure 2.2 Simultaneous Modeling Approaches Using Mixed Equilibrium Concepts

Considering sequential decisions, three types of combinations exist, as shown in Figure 2.3. Individual equilibrium models are combined via Stackelberg game or multi-leader-follower game, according to the number of leaders. If there is a single leader, the market is modeled as a Stackelberg game, while if there are multiple leaders, the market is modeled as a multi-leader-follower game. In both games, the leader (s) makes decisions from information on the follower’s rational reactions and the followers have knowledge of the leader’s decisions. The leader attempts to exercise power to manipulate the freight transport market for its own advantage. More complexity exists in multi-leader-follower game by identifying the decisions and relationships of individual leader firms. Several researchers, e.g. Miller et al. (1991), Zan (1999), Wang (2001), Yang et al. (2007), and Zhang et al. (2008), used these mixed concepts.
2.2 Predictive Network Models

This section reviews predictive network models that have been used for making predictions concerning the intercity freight transport system and the international maritime freight transport system over the past two and half decades. Harker (1985) provides a good summary of major works in this field up to 1985. Table 2.1 summarizes different assumptions of the models, regarding freight transport system, stakeholder, model type, equilibrium concept and modeling approach.
2.2.1 Intercity freight transport system

Most of the works have focused on one or two stakeholder problems considering shippers or/and carriers in the intercity freight transport system. On the other hand, Xiao and Yang (2007) considered relationships among three stakeholder groups such as one shipper, multiple carriers and infrastructure companies. The developed models used freight network equilibrium, spatial price equilibrium and integrated network equilibrium, for national or regional freight movements. Some studies applied Stackelberg game to analyze the multiple equilibrium behavior on networks.

Friesz et al. (1986), Hurley and Petersen (1994), Fernandez et al. (2003) and Agrawal and Ziliaskopoulos (2006) captured simultaneous or sequential interactions between shippers and carriers using freight network equilibrium models. Some forms of user equilibrium were used for modeling the shipper behavior, whereas carriers were assumed to behave according to system optimum. Friesz et al. (1986) proposed a sequential shipper-carrier model to show better model performance than simultaneous models. The shipper model estimates O-D pairs and routing patterns and the composition algorithm translates the shipper path flow configuration into modal specific O-D pairs. Then, the carrier model determines a system optimized flow pattern. The model showed better accuracy than simultaneous models however flow predictions still needed improvement due to the highly idealized treatment of the supply side. Hurley and Petersen (1994) considered nonlinear tariffs at an equilibrium market where shippers and carriers aim to maximize their profits. The authors allowed carriers to choose tariffs depending on the volume. It was demonstrated that if the carrier or coalition of carriers uses vertically efficient pricing schedules, the vertically efficient flows are the
equilibrium flows, via the developed Integrated System Optimization Problem (ISP). Fernandez et al. (2003) proposed a new approach to intercity freight transport system modeling, based upon simultaneous demand-supply network equilibrium. Shippers determines destinations, modes and carriers at the demand side, while carriers determined routing patterns over a multi-modal, multi-product and multi-operator network at the supply side. The supply and demand models were integrated within an equilibrium framework. Agrawal and Ziliaskopoulos (2006) presented a dynamic freight assignment model that captures the shipper-carrier interactions. Carriers optimize their operations and derive the transportation service charge. Shippers decide the carriers that offer the lowest shipping cost to minimize their cost. There is a feedback between carriers and shippers by an iterative procedure that carriers recognize the commodity flow and revise the transportation service charge based on the optimized cost.

Harker (1998) Guelat, et al. (1990) and Cheng and Wu (2006) focused on the shipper or carrier problem, respectively. Harker (1998) proposed a mixed user behavior model for capturing the situations which do not fit properly into either the user equilibrium or system optimal framework because typically the smaller shippers behave according to user equilibrium and larger shippers attempt to minimize the transportation cost via system optimal. If a single price taking player controls all O-D pairs, one is able to get the user equilibrium model. On the other hand, if all O-D pairs are handled by a single Cournot-Nash player, one can obtain the system optimal model. The two types of models were united in a mixed behavior model. Guelat et al. (1990) developed a multimode multiproduct network assignment model for modeling national and regional freight transport systems. Carriers make routing decisions for the shipper demands
provided by exogenous assumptions to minimize the total generalized cost. Cheng and Wu (2006) presented an analytical framework to attain optimal performance on a supply-demand network involving multiple products and multiple criteria cost functions. When the supply and demand are known, the shippers choose one of the delivery paths at minimum cost via user equilibrium. Former network equilibrium models considered a single criterion that may not be reasonable. On the other hand, the authors proposed a multiproduct network equilibrium model with a vector-valued cost functions. The necessary and sufficient conditions of the model were derived from an equivalent vector variational inequality.

On the other hand, Xiao and Yang (2007) considered three types of stakeholders, one shipper, carriers and infrastructure companies, who act as profit maximizing agents, respectively, except that carriers and infrastructure companies are assumed to work cooperatively. First, infrastructure companies determine a tariff to carriers according to their cost function and information on the shipper and carriers. Then, the carriers decide another tariff to the shipper, based on their cost function and information on the shipper and the tariff given by infrastructure companies. Finally, the shipper makes decision for the production to maximize the profit. The three-stage sequential decision process was captured in game-theoretical models.

Miller et al. (1991) and Yang et al. (2007) formulated a Stackelberg network equilibrium model, each, for the multiple equilibrium behavior. Miller et al. (1991) discussed a spatial Stackelberg-Nash-Cournot competitive network equilibrium problem by combining the behavior of two types of players, the Stackelberg firm and Cournot firms. The Stackelberg firm acts as a leader firm, taking into account the expected
reactions of the other firms, whereas Cournot firms react to the production and shipping decisions of the leader as the followers. Yang et al. (2007) presented both competitive and collusive behavior among users through user equilibrium, system optimal and Cournot-Nash players. The authors formulated a mixed behavior network equilibrium model via a Stackelberg routing game on the network in which the system optimal player is a leader and the user equilibrium and Cournot-Nash players are the followers.

Harker and Friesz (1985a, 1985b, 1985c) focused on integrated network equilibrium considering producers, consumers, shippers and carriers. Harker and Friesz (1985a) presented an integrated network equilibrium model for the simultaneous shipper-carrier interactions. The authors discussed an economic mechanism that integrates demand and supply models in an intercity freight transport system. At the demand side, the production, consumption and routing pattern are determined using spatial price equilibrium, while, on the supply side, the profit maximization behavior of carriers is captured. Harker and Friesz (1985b) proposed two alternative mathematical representations of the model developed by Harker and Friesz (1985a). A variation inequality formulation and a nonlinear complimentary formulation were developed. In particular, the variation inequality formulation was transformed into an equivalent optimization formulation to find the relationship between the two modeling approaches. Harker and Freisz (1985c) reviewed the mathematical formulations (1985a, 1985b) and focused on computational techniques for solving the models. Then, an application to the U.S. coal economy was discussed by extending the general framework.

Dafermos and Nagurney (1987) established the connection between Cournot oligopoly and perfect competition. They showed that a fairly general oligopoly model
with spatially separated markets generates a general spatial price equilibrium model as an extreme case. First of all, the generalized versions of an oligopoly model and a spatial price equilibrium model were described and the relationship between the two models was found subsequently. Xu and Holguin-Veras (2009) considered the integrated shipper-carrier operations when shipping and transporting functions are part of the same company, for estimating urban freight flows. The authors developed an analytical formulation to find equilibrium production-routing patterns and commodity flows with spatial price equilibrium principles and investigated the dynamics of production and transportation.

2.2.2 International maritime freight transport system

Maritime freight transportation involves port operations to change transportation service providers from ocean carriers to land carriers, and vice versa. Several studies capturing interactions of the key stakeholders in this system have been done by Zan (1999), Wang (2001), Kuroda et al. (2005) and Zhang et al. (2008). The developed models considered a type of carrier, in given scenarios of the other carriers. All assumed the hierarchical interactions between two types of stakeholders, shippers and ocean carriers or port terminal operators. The individual stakeholder problem was formulated at each level using user equilibrium, Nash equilibrium, spatial price equilibrium, or compensation principle. For multilevel games, Stackelberg game model by Zan (1999) and Wang (2000) and multi-leader-follower game by Zhang et al. (1999) were formulated. Kuroda et al. (2005) developed a shipper model considering the navigation time determined by ocean carriers.

Zan (1999) proposed a sequential shipper-carrier model to seek flows of the
foreign trade container cargo in an equilibrium shipping market. The interactions between an ocean carrier and domestic shippers were captured by a Stackelberg game in given decisions of the port administrator. Due to a global alliance, an ocean carrier was regarded as the leading player which has the complete information about the shippers. The carrier aims to maximize their profits, considering the optimal behavior of shipper and competition with rival companies. On the other hand, domestic shippers determine ports and ocean carriers to minimize the total cost in an inland transport network as the followers in the game. Wang (2001) discussed a hierarchical game between oligopolistic carriers and shippers. At the upper level, carriers decide service charges and link flows and at the lower level, shippers make decisions on supply, demand and commodity flows. Various investment strategies of the Port Authority as the supply side of the port were evaluated and compared to show their policy impacts. Both carrier competitive and collusive markets were considered using Nash equilibrium and compensation principle, respectively. On the other hand, the shipper’s behavior was captured by the concept of spatial price equilibrium. The individual equilibrium problems were combined into bi-level models via Stackelberg game. Kuroda et al. (2005) presented container transportation network analysis considering Post-Panamax vessels. They proposed a network competition model concerning the supply-demand interaction in an international maritime transport market. Shippers optimize inland transport routes by choosing importing and exporting ports and assigning cargo volumes at each port, while ocean carriers find the optimal routing pattern including service frequency and vessel size among ports. Carriers were assumed to provide the same quality of service for the given cargo volume. Zhang et al. (2008) analyzed the behavior of participants involved in an
international container transport system. At the upper level, ports compete against each other to maximize their profits by attracting more shippers through Nash equilibrium. On the other hand, at the lower level, shippers attempt to find the minimum shipping cost via user equilibrium. Each level model was combined into a bi-level model by the concept of multi-leader-follower game. Both deterministic and stochastic container transport network equilibrium models were proposed, respectively.
### Table 2.1 Different Assumptions of Predictive Network Models

<table>
<thead>
<tr>
<th>No.</th>
<th>Authors</th>
<th>Freight transport system</th>
<th>Stakeholder</th>
<th>Model type</th>
<th>Equilibrium concept</th>
<th>Mathematical formulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Harker and Freisz (1985)</td>
<td>Intercity</td>
<td>Shipper, Carrier</td>
<td>Simultaneous model</td>
<td>Integrated network equilibrium</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>Harker and Friesz (1986a)</td>
<td>Intercity</td>
<td>Shipper, Carrier</td>
<td>Simultaneous model</td>
<td>Integrated network equilibrium</td>
<td>Nonlinear constrained optimization problem</td>
</tr>
<tr>
<td>3</td>
<td>Harker and Friesz (1986b)</td>
<td>Intercity</td>
<td>Shipper, Carrier</td>
<td>Simultaneous model</td>
<td>Integrated network equilibrium</td>
<td>Variational inequality problem Nonlinear complementarity problem</td>
</tr>
<tr>
<td>4</td>
<td>Friesz et al. (1986)</td>
<td>Intercity</td>
<td>Shipper, Carrier</td>
<td>Sequential model</td>
<td>Freight network equilibrium</td>
<td>Nonlinear constrained optimization problem</td>
</tr>
<tr>
<td>5</td>
<td>Dafermos and Narguney (1987)</td>
<td>Intercity</td>
<td>Shipper</td>
<td>Simultaneous model</td>
<td>Spatial price equilibrium</td>
<td>Variational inequality problem</td>
</tr>
<tr>
<td>6</td>
<td>Harker (1988)</td>
<td>Intercity</td>
<td>Shipper</td>
<td>Simultaneous model</td>
<td>Freight network equilibrium</td>
<td>Variational inequality problem</td>
</tr>
<tr>
<td>7</td>
<td>Guelat et al. (1990)</td>
<td>Intercity</td>
<td>Carrier</td>
<td>Simultaneous model</td>
<td>Freight network equilibrium</td>
<td>Nonlinear constrained optimization problem</td>
</tr>
<tr>
<td>8</td>
<td>Miller et al. (1991)</td>
<td>Intercity</td>
<td>Shipper</td>
<td>Sequential model</td>
<td>Stackelberg game</td>
<td>Variational inequality problem</td>
</tr>
<tr>
<td>9</td>
<td>Hurley and Peterson (1994)</td>
<td>Intercity</td>
<td>Shipper, Carrier</td>
<td>Simultaneous model</td>
<td>Freight network equilibrium</td>
<td>Various theoretical formulations</td>
</tr>
<tr>
<td>10</td>
<td>Zan (1999)</td>
<td>International Maritime</td>
<td>Shipper, Carrier</td>
<td>Sequential model</td>
<td>Freight network equilibrium Stackelberg game</td>
<td>Nonlinear constrained optimization problem</td>
</tr>
<tr>
<td>12</td>
<td>Fernandez et al. (2003)</td>
<td>Intercity</td>
<td>Shipper, Carrier</td>
<td>Simultaneous model</td>
<td>Freight network equilibrium</td>
<td>Variational inequality problem</td>
</tr>
<tr>
<td>13</td>
<td>Kuroda et al. (2005)</td>
<td>International Maritime</td>
<td>Shipper, Carrier</td>
<td>Sequential model</td>
<td>Freight network equilibrium</td>
<td>Nonlinear constrained optimization problem</td>
</tr>
</tbody>
</table>
Table 2.1 (Continued) Different Assumptions of Predictive Network Models

<table>
<thead>
<tr>
<th>No.</th>
<th>Authors</th>
<th>Freight transport system</th>
<th>Stakeholder</th>
<th>Model type</th>
<th>Equilibrium concept</th>
<th>Mathematical formulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>Yang et al. (2007)</td>
<td>Intercity</td>
<td>Shipper/Carrier</td>
<td>Sequential model</td>
<td>Stackelberg game</td>
<td>Variational inequality problem Nonlinear constrained optimization problem</td>
</tr>
<tr>
<td>17</td>
<td>Xiao and Yang (2007)</td>
<td>Intercity</td>
<td>Shipper, Carrier, Infrastructure company</td>
<td>Sequential model</td>
<td>Freight network equilibrium</td>
<td>Various theoretical formulations</td>
</tr>
<tr>
<td>19</td>
<td>Xu and Holguin-Veras (2009)</td>
<td>Intercity</td>
<td>Shipper, Carrier</td>
<td>Simultaneous model</td>
<td>Spatial price equilibrium</td>
<td>Nonlinear constrained optimization problem</td>
</tr>
</tbody>
</table>
2.3 Features and Directions of the Dissertation

As yet, researches on the predictions of maritime freight movements have been quite limited due to the complicated interactions of stakeholders. The relevant studies haven’t perceived distinctive roles of ocean carriers, land carriers and port terminal operators and their relationships fully. The dissertation proposes predictive network models which capture interactions among different types of carriers in a maritime shipping market by three-level optimization programming. Subsequently, a shipper - carrier problem is formulated to capture relationships between shippers and carriers by bi-level optimization programming.

This dissertation is closely related with the works by Zan (1999), Wang (2001) and Zhang et al. (2008) who captured sequential decisions between two types of stakeholders, shippers and ocean carriers or port terminal operators, for international trade. The three papers are compared with the dissertation in Table 2.2.

Table 2.2 Comparison between the Most Related Literature and the Dissertation

<table>
<thead>
<tr>
<th>No.</th>
<th>Author</th>
<th>Stakeholder</th>
<th>Equilibrium Concept for Individual Stakeholder</th>
<th>Equilibrium Concept for Different Types of Stakeholders</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Zan (1999)</td>
<td>Carrier (leader) Shipper (follower)</td>
<td>Carrier - NE Shipper - UE</td>
<td>Stackelberg game</td>
</tr>
<tr>
<td>3</td>
<td>Zhang et al. (2008)</td>
<td>Port (leader) Shipper (follower)</td>
<td>Port - NE Shipper - UE</td>
<td>Multi-leader-follower game</td>
</tr>
<tr>
<td>*</td>
<td>The dissertation</td>
<td>Three types of carriers (leader) Shipper (follower)</td>
<td>Carrier - NE, CP Shipper - SPE</td>
<td>Multi-leader-follower game Stackelberg game</td>
</tr>
</tbody>
</table>

Note: NE (Nash Equilibrium), UE (User Equilibrium), CP (Compensation Principle), SPE (Spatial Price Equilibrium)
There are similarities and differences between the papers and the dissertation. First, Zan (1999), Wang (2001) and Zhang et al. (2008) captured interactions between shippers and carriers, while this dissertation considers interactions both among three types of carriers and between shippers and carriers.

Second, Zan (1999) and Wang (2001) regarded the carrier as the leader and the shipper as the follower, assuming carriers are more powerful than shippers. The dissertation also regards carriers as the leaders in a realistic aspect. The carrier problem formulating the pricing and routing model is the upper level problem and the shipper problem constructing the spatial pricing model is the lower level problem. For the carrier problem, three-level models are formulated by defining the leader and the follower. Ocean carriers are regarded as the leaders in a maritime shipping market. Port terminal operators are the followers of ocean carriers as well as the leaders of land carriers.

Third, the former studies used equilibrium concepts such as user equilibrium, Nash equilibrium, compensation principle and spatial price equilibrium for the individual stakeholder problems. In particular, equilibrium concepts applied in this dissertation are similar with those used by Wang (2001) in that Nash equilibrium and compensation principle are applied according to the different market environment of carriers and spatial price equilibrium is used for the shipper problem.

Finally, Zan (1999) and Wang (2001) considered a Stackelberg game that assumes a carrier has a certain power in setting price and routing decisions. On the other hand, Zhang et al. (2008) set the equal powers for multiple leaders, via a multi-leader-follower-game. The dissertation employs both games. If ocean transportation firms are competitive,
a multi-leader-follower-game is applied and if the firms are collusive via alliances, a
Stackelberg game is used, respectively.

The research makes significant contributions, both from a theoretical and a
practical point of view. From a theoretical standpoint, the proposed work is the first
attempt to capture relationships of the different types of carriers, particularly regarding
port terminal operator as a special type of carrier. This work contributes to the body of the
multimodal freight network problem literature by providing a novel methodology to
formulate multi-level hierarchical interaction models. From a practical point of view, the
paper assists in the understanding of the dynamics and the decision-making processes of
various stakeholders involved in international freight transportation via waterways.
CHAPTER 3

PROBLEM DEFINITION

Carriers and shippers have their distinctive roles as decision markers in multimodal freight transportation networks. Carriers provide transportation services between an origin and a destination, determining service charges, levels of service and delivery routings, on the carrier network. On the other hand, shippers produce goods and deliver them using the transportation services by carriers. Shippers make decisions on the production in an origin market, the consumption in a destination market and the shipment pattern between these markets, on the shipper network.

A set of carriers in international maritime freight transportation networks includes ocean carriers, land carriers and port terminal operators. Port terminal operators, providing transportation services within a port complex, are regarded as a special type of carrier based on their behavior. The carriers make decisions on prices and delivery routes (or port services) at different parts of the multimodal network, interacting with each other hierarchically. Ocean carriers typically transport freight between a departure and an arrival port terminal via waterways; port terminal operators handle the freight within a port complex; and land carriers transport freight between the port terminal and inland destinations.

The dissertation captures interactions among three types of carriers and between shippers and carriers in an international maritime shipping market, using novel multi-level hierarchical modeling approaches. Three-level models are formulated for the carrier problem and bi-level models are developed for the shipper-carrier problem, considering
The chapter is organized as follows. Section 3.1 presents structures and attributes of the shipper and carrier networks. Section 3.2 defines basic assumptions for the definiteness and simplicity of modeling. Section 3.3 presents modeling approaches of the carrier and shipper problems. Section 3.4 discusses equilibrium concepts applied to modeling. Section 3.5 summarizes Chapter 3.

3.1 Structures and Attributes of Networks

The shipper network shows an aggregated network and the carrier network represents a physically detailed multimodal transportation network. As a part of the carrier network, the port terminal sub-network depicts physical locations and port operations.

The shipper network includes nodes expressing origin markets, destination markets and transshipment points. The supply and demand are determined in origin and destination nodes, respectively, and delivery routings are shown along transshipment nodes located between the origin and destination nodes. Links connecting these nodes show a delivery route of the alternative ones.

The carrier network shows actual transportation routes and modes. The carrier network contains nodes representing origins, destinations and intermediate points for changes in types of services, modes or routes. Links depict all alternative routes between these nodes. The ocean carrier network, land carrier network and port sub-network coexist in the maritime freight network. Ocean and land carrier networks have port-
entrance links to access port terminals and port-exit links to egress from port terminals, respectively.

The port sub-network represents not only physical locations and but also port operations such as loading, unloading, moving and storing freight. Nodes express change points of port transportation services. Therefore, different port service processes are shown along links connecting the nodes.

The shipper link corresponds to the carrier O-D pair defining the service charge and routing set. The shipper network, carrier network and samples of the port sub-network are depicted in Figure 3.1 ~ Figure 3.3.

**Figure 3.1 Shipper Network**
Figure 3.2 Carrier Network

Figure 3.3 Port Sub-Network (Samples)
3.2 Assumptions

Basic assumptions for the definiteness and simplicity of modeling are presented below.

1) Freight is shipped from the departure port terminal via the arrival port terminal to the ultimate inland destination. For simplicity, there is no movement of land carriers to reach a port since commodities are assumed to be produced near the departure port terminal.

2) Ocean carriers may select an arrival port terminal of the alternative ones located in competitive regions. A destination of the ocean carriers is assumed to have the alternative set.

3) In selecting a marine terminal, the location and service charge are key determinants, assuming that other port service conditions are similar.

3.3 Modeling Approaches

The dissertation formulates predictive freight network models that capture interactions among ocean carriers, land carriers and port terminal operators and between carriers and shippers in an international shipping setting. The consideration of port Authority’s decisions is beyond the scope of the research.

3.3.1 Carrier problem

Ocean carriers are regarded as the leaders in an international maritime shipping market. Port terminal operators are the followers of ocean carriers as well as the leaders of land
carriers. The individual carrier problem is formulated at each level. Interactions among different types of carriers are captured in three-level models.

At the first level, ocean carriers aim to maximize individual profits. Each ocean carrier determines the profit based upon the ocean carrier service demand function depending on the service charge and the ocean transportation cost function relying on the routing pattern. The ocean transportation cost contains the ocean carrier operating cost and travel (shift) time and the port service charge. At the second level, port terminal operators attempt to maximize individual profits. Each port terminal operator decides the profit from the port throughput function depending on the port service charge and the port service cost function relying on the port service pattern. The port service cost includes the port operating cost and service time. At the third level, land carriers aim to maximize individual profits. Each land carrier determines the profit based on the land carrier service demand function depending on the service charge and the land transportation cost function relying on the routing pattern. For the land transportation cost, the land carrier operating cost and travel (shift) time are considered. Different types of modes such as truck, rail and combined truck/rail are considered according to the network structure.

Assuming independent operations of the three types of carriers without alliances, hierarchical relationships occur between ocean carriers and port terminal operators and between port terminal operators and land carriers, respectively. At the upper level interaction, port service charges affect ocean carrier routings, while port throughputs are influenced by ocean carrier decisions. Ocean carriers departing from a port terminal choose a destination port terminal of the alternative ones by considering the ocean carrier operating cost and travel time on the transportation network as well as port determinants.
such as the port location and the service charge. Therefore, port terminal operators suggest competitive prices and services to attract more ocean liners. Interactions between ocean carriers and port terminal operators are captured in the ocean path transportation cost function, the port throughput function and the feasible region of the port terminal operator objective function. At the lower level interaction, land carrier service demands are determined from the port throughputs. Land carriers deliver the total amount of freight treated in each port terminal to land destinations. Interactions between port terminal operators and land carriers are captured in the feasible region of the land carrier objective function. Figure 3.4 depicts the interactions among three types of carriers for modeling.

![Figure 3.4 Interactions among Three Types of Carriers for Modeling](image)
For the ocean and land carrier problems, both perfect competition and collusion among the same types of carriers are considered. The partial competition is included in the perfect competition by regarding the collusive firms as a competitive player since the firms are still competitive against the other firms. However, the collusive firms utilize their resources to save the total cost. For the port terminal operator problem, perfect competition is considered because inter/intra port competition usually occurs in many regions. When port terminal operators are collusive in a port, the port is regarded as a competitive port terminal operator. The collusive port terminal operators share port facilities and operate the entire port efficiently. According to the different environment of carriers, four types of three-level models are formulated, as shown in Table 3.1.

<table>
<thead>
<tr>
<th>Carrier model</th>
<th>Ocean carrier</th>
<th>Port terminal operator</th>
<th>Land carrier</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carrier model 1</td>
<td>Competitive</td>
<td>Competitive</td>
<td>Competitive</td>
</tr>
<tr>
<td>Carrier model 2</td>
<td>Competitive</td>
<td>Competitive</td>
<td>Collusive</td>
</tr>
<tr>
<td>Carrier model 3</td>
<td>Collusive</td>
<td>Competitive</td>
<td>Competitive</td>
</tr>
<tr>
<td>Carrier model 4</td>
<td>Collusive</td>
<td>Competitive</td>
<td>Collusive</td>
</tr>
</tbody>
</table>

Subsequently, alliances between ocean carriers and port terminal operators or land carriers are considered when individual carriers within same groups are competitive.

### 3.3.2 Shipper-carrier problem

Shippers are users of transportation services provided by carriers. Therefore, shippers choose a sequence of carriers by considering their service charges and transportation routes. The shipper-carrier interactions are captured in a bi-level modeling approach
when the carrier problem is the upper level problem and the shipper problem is the lower level problem. At the upper level, carriers make pricing and routing decisions based upon the knowledge of the shipper flow. At the lower level, shippers determine the production, consumption and routing pattern with the transportation service information presented by carriers. Figure 3.5 shows the interactions between carriers and shippers for modeling.

![Figure 3.5 Interactions between Carriers and Shippers for Modeling](image)

**Figure 3.5 Interactions between Carriers and Shippers for Modeling**

### 3.4 Equilibrium Concepts

The carrier and shipper problems are formulated using Nash equilibrium, compensation principle and spatial price equilibrium. For multi-level games, Stackelberg game or multi-leader-follower game are applied according to the number of leaders.

In the carrier competitive game, Nash Equilibrium is used to find the optimal service charge and routing pattern for which each carrier obtains the greatest profit. No carrier can get better profit by changing its decisions unilaterally, at equilibrium. On the
other hand, in the carrier collusive game, compensation principle is used to determine the optimal service charge and routing set of the colluded carriers, by maximizing the total profit. The total profit should be greater than the sum of individual profits of competitive carriers, at equilibrium.

Ocean carriers are regarded as the leaders. If there is a single leader, the market is modeled as a Stackelberg game, whereas if there are multiple leaders, the market is modeled as a multi-leader-follower game. Hence, a multi-leader-follower game is used for ocean carrier competition, while a Stackelberg game is applied for ocean carrier collusion, assuming a single ocean shipping company through alliances.

Shippers determine the optimal quantity supplied and the supply cost at an origin market, the quantity demanded and the demand cost at a destination market and the delivery routes and costs between these markets, via spatial price equilibrium.

Figure 3.6 summarizes equilibrium concepts used for modeling.
3.5 Summary of Chapter 3

Chapter 3 defined the research problem by presenting 1) structures and attributes of the shipper’s aggregated network and the carrier’s multimodal network, 2) basic assumptions for modeling and 3) modeling approaches with the applied equilibrium concepts.

The individual carrier problem is formulated at each level using Nash equilibrium in the competitive environment and compensation principle in the collusive environment. Interactions among three types of carriers are captured by a three-level modeling approach. The shipper problem is formulated using spatial price equilibrium. The shipper-carrier interactions carriers are captured by a bi-level modeling approach. For multi-level games, Stackelberg game and multi-leader-follower game are applied according to the number of leaders (ocean carriers).

Chapter 4 presents the pricing and routing problem of carriers. Based upon the problem descriptions in Chapter 3, predictive freight network models capturing decisions and interactions of ocean carriers, land carriers and port terminal operators are formulated.
CHAPTER 4

CARRIER PROBLEM

Chapter 4 presents a multi-level hierarchical approach which captures interactions among three types of carriers - ocean carriers, land carriers and port terminal operators - in international maritime freight transportation networks. The individual carrier problem is formulated at each level using Nash equilibrium in the competitive environment and compensation principle in the collusive environment, respectively. Interactions among different types of carriers are captured in three-level models. Stackelberg game and multi-leader-follower game are applied to multi-level games according to the number of leaders.

Subsequently, two types of alliances between different groups of carriers are considered when individual carriers within same groups are competitive: 1) cooperation between ocean carriers and port terminal operators and 2) cooperation between ocean carriers and land carriers.

Chapter 4 is organized as follows. Section 4.1 presents the ocean carrier problem for transporting freight between a departure port (origin market) and an arrival port. Section 4.2 focuses on the port terminal operator problem for handling freight arriving in a port complex. Section 4.3 presents the land carrier problem for transporting freight from a port terminal to an inland destination. Section 4.4 formulates three-level models according to the different carrier environment, without alliances. Section 4.5 considers cooperation between ocean carriers and port terminal operator or carriers. Section 4.6 develops a heuristic algorithm to solve the problems.
4.1 Ocean Carrier Problem

Ocean carriers provide transportation services between an origin port terminal and a destination port terminal via waterways. Ocean carriers departing from a port terminal choose an arrival port terminal of the alternative ones by considering the transportation network and the port location and service charge. In the competitive environment, each ocean carrier aims to maximize the individual profit, while in the collusive environment, the colluded ocean carriers attempt to maximize the total profit. This section is organized as follow: equilibrium conditions, objective functions, assumptions and properties of the objective functions, feasible region, mathematical formulations, and existence and uniqueness of the solution.

4.1.1 Equilibrium conditions of the ocean carrier problem

1) Competitive environment

In the competitive game, Nash Equilibrium is used to find the optimal service charge and routing pattern for which each ocean carrier obtains the greatest profit. No ocean carrier can get better profit by changing its decisions unilaterally, at equilibrium. The equilibrium condition of Nash Equilibrium for ocean carrier $o$ between $i$ and $j$ is shown in Eq. (4.1).

$$U_{ij}^o(c_{ij}^*, c_{ij}^{-o*}, f_{ij}^o) \geq U_{ij}^o(c_{ij}^o, c_{ij}^{-o*}, f_{ij}^o) \quad \forall \ o \in O, \ ij \in IJ$$  \hspace{1cm} (4.1)

Eq. (4.1) indicates that if ocean carrier $o$ between $i$ and $j$ changes pricing and routing decisions from its optimal $(c_{ij}^*, f_{ij}^*)$ to any other feasible decisions $(c_{ij}^o, f_{ij}^o)$,
while other ocean carrier keep their service charges constant, its profit will be worse.

2) Collusive environment

In the collusive game, compensation principle is used to determine the optimal pricing and routing set of the colluded ocean carriers, by maximizing their total profit. If the total profit increases, the gainers compensate the losers and everyone is better off. The total profit should be greater than the sum of individual profits of competitive ocean carriers, at equilibrium. The equilibrium condition of compensation principle for the ocean carriers between \( i \) and \( j \) is shown in Eq. (4.2).

\[
\max_{c,f} \sum_{o \in O} U^o_{ij} (c^o_{ij}, c^{-o}_{ij}, f^o_{ij}) \geq \sum_{o \in O} \max_{c,f} U^o_{ij} (c^o_{ij}, c^{-o}_{ij}, f^o_{ij}) \quad \forall \ ij \in IJ
\]

Eq. (4.2) represents that the total profit of the colluded ocean carriers must be greater than the sum of individual profits of the competitive ocean carriers, between \( i \) and \( j \).

4.1.2 Objective functions of ocean carriers

The objective functions of competitive and colluded ocean carriers are defined in this sub-section. The objective functions are profit maximization functions since ocean carriers are generally private firms to maximize their outcomes.

1) Competitive environment

In the competitive game, each ocean carrier aims to maximize the profit by maximizing the income and minimizing the transportation cost, for O-D pair \( ij \). The objective function of ocean carrier \( o \) \( (U^o_{ij}) \) is defined in Eq. (4.3).
Eq. (4.3) expresses the profit function of ocean carrier $o (U_{ij}^o)$ composed of the income function ($I_{ij}^o$) and the transportation cost function ($TC_{ij}^o$). The income of ocean carrier $o$ is determined by the service demand function ($D_{ij}^o$) depending on the service charge ($c_{ij}^o$), while other ocean carriers keep their current level of service charges ($\bar{c}_{ij}^o$) constant. The transportation cost of ocean carrier $o$ is decided by the average ocean path transportation cost function ($AC_{ij}^{ph-o,o}$) relying on the flow ($f_{ij}^{ph-o,o}$) on all used paths.

2) Collusive environment

In the collusive game, the objective of the ocean carriers is to maximize the total profit, for O-D pair $ij$. The objective function of the ocean carriers is defined in Eq. (4.4).

\[
\sum_{o}^{O} U_{ij}^o = \sum_{o}^{O} \sum_{ij}^{IJ} D_{ij}^o (c_{ij}^o, \bar{c}_{ij}^o) \times c_{ij}^o - \sum_{o}^{O} \sum_{ij}^{IJ} \sum_{ph-o,o}^{PH} AC_{ij}^{ph-o,o} (f_{ij}^{ph-o,o}) \times f_{ij}^{ph-o,o} \quad \forall \ ij \in IJ
\] (4.4)

Eq. (4.4) expresses the total profit function of the colluded ocean carriers ($\sum_{o}^{O} U_{ij}^o$), composed of the total income function ($\sum_{o}^{O} I_{ij}^o$) and the total transportation cost function ($\sum_{o}^{O} TC_{ij}^o$). The total income is determined by the service demand set of ocean carriers
depending on their service charges. The total transportation cost is decided by the average ocean path transportation cost set relying on the flow of ocean carriers on all used paths.

4.1.3 Assumptions for the objective functions

The properties of the ocean carrier demand function \( D_{ij}^o \) and the ocean carrier transportation cost function \( TC_{ij}^o \) are assumed for defining objective function properties. The demand function is assumed to be strictly monotone decreasing in the service charge since if ocean carrier \( o \) increases the service charge, customers who want use ocean transportation services attempt to find chipper prices from the other ocean carries. On the other hand, the transportation cost function is assumed to be strictly monotone increasing in the flow because as the flow passing an ocean route grows, the transportation cost and time on the route increase accordingly. Wang (2000) used similar assumptions for the carrier demand and cost functions in her study.

1) Demand function

The ocean carrier demand function \( D_{ij}^o = \alpha_o - \alpha \times c_{ij}^o \) is assumed to be linear and strictly monotone decreasing in the service charge \( c_{ij}^o \) and satisfies:

\[
\frac{\partial D_{ij}^o}{\partial c_{ij}^o} < 0 \quad \forall \ o \in O, \ ij \in IJ
\]  

Eq. (4.5) expresses that as the service charge of ocean carrier \( o \) between \( i \) and \( j \) increases, its service demand between the same regions decreases.
2) Transportation cost function

The ocean path transportation cost is the expense to deliver the total amount of freight via the ocean path \((ph_o)\) connecting \(i\) and \(j\). The ocean link transportation cost is the expense to deliver the freight via the ocean link \((lk_o)\). The ocean path transportation cost function \((TC_{ij}^{ph_o})\) is a linear combination of the following two attributes: a) the sum of ocean link transportation cost functions \((TC_{ij}^{lk_o})\) if the ocean link \((lk_o)\) is on the path \((ph_o)\) and b) the sum of port service charges \((c_p)\) if the ocean path \((ph_o)\) is connected to port terminal operator \(p\). A mathematical form of the ocean path transportation cost function is shown in Eq. (4.6).

\[
TC_{ij}^{ph_o} (f_{ij}^{ph_o}) = \sum_{lk_o} \xi_{ij}^{lk_o,ph_o} \times TC_{ij}^{lk_o} (f_{ij}^{lk_o}) + \psi_{ij}^{ph_o,p} \times c_p \times f_{ij}^{ph_o}
\]

\(\forall \ ij \in IJ, ph_o \in PH, p \in P\) \hspace{1cm} (4.6)

If the ocean link \((lk_o)\) is on the path \((ph_o)\) connecting \(i\) and \(j\), the incidence index \(\xi_{ij}^{lk_o,ph_o}\) is 1, and the ocean path transportation cost function combines the ocean link transportation cost functions. Otherwise, \(\xi_{ij}^{lk_o,ph_o}\) is 0. If the ocean path \((ph_o)\) is connected to the destination port terminal operator \(p\), the incidence index \(\psi_{ij}^{ph_o,p}\) is 1, and the average port service charge presented by port terminal operator \(p\) is adjusted. Otherwise, \(\psi_{ij}^{ph_o,p}\) is 0.

The ocean link transportation cost function \((TC_{ij}^{lk_o})\) comprises: a) the average ocean link operating cost function \((AOC_{ij}^{lk_o})\) and b) the average ocean link travel (shift)
time function ($ATT_{lk-o}$). The travel (shift) time is changed to monetary value using the value of time ($vot^o$) for unit consistency. A mathematical form of the ocean link transportation cost function is shown in Eq. (4.7).

$$TC_{lk-o} (f_{lk-o}) = AOC_{lk-o} (f_{lk-o}) \times f_{lk-o} + vot^o \times ATT_{lk-o} (f_{lk-o}) \times f_{lk-o}$$

\[ \forall lk_o \in LK_o \] (4.7)

where

$$AOC_{lk-o} (f_{lk-o}) = \omega_0 + \omega_1 \left( \frac{f_{lk-o}}{cap_{lk-o}} \right) + \omega_2 \left( \frac{f_{lk-o}}{cap_{lk-o}} \right)^2 \quad \forall lk_o \in LK_o \] (4.8)

$$ATT_{lk-o} (f_{lk-o}) = att_{lk-o} + ast_{lk-o} \left( 1 + \mu_0 \left( \frac{f_{lk-o}}{cap_{lk-o}} \right) \right) \quad \forall lk_o \in LK_o \] (4.9)

Eq. (4.8) expresses the average ocean link operating cost function ($AOC_{lk-o}$) considering both fixed and flexible costs. The fixed cost is defined regardless of the flow, while the flexible cost is affected by the flow. The first element shows the fixed cost to deliver a unit of freight on the ocean link. The second and third elements represent the flexible costs due to the transportation demand beyond the link capacity. The link capacity is maximum flows of freight on the link to handle the freight efficiently in a unit of time. In Eq. (4.8), $cap_{lk-o}$ denotes the capacity of the ocean link and $\omega_0$, $\omega_1$ and $\omega_2$ express coefficient parameters with a positive value. Eq. (4.9) represents the average ocean link travel (shift) time function ($ATT_{lk-o}$). The first element shows the average travel time on the ocean navigation link. The second element expresses the average shift time on the port-entrance link. The third element represents the average shift time due to
the transportation demand beyond the capacity of the port entrance link. In Eq. (4.9), \( \mu_0 \) and \( \mu_1 \) denote coefficient parameters with a positive value.

The average ocean link operating cost and travel (shift) time functions are assumed to be continuous and strictly monotone increasing in the flow, as shown in Eq. (4.10) and Eq. (4.11).

\[
\frac{\partial AOC_{lk,o}}{\partial f_{lk,o}} = \omega_3 \frac{1}{cap_{lk,o}} + 2 \omega_2 \frac{f_{lk,o}}{(cap_{lk,o})^2} > 0 \quad \forall \ lk_o \in LK_{o} \quad (4.10)
\]

\[
\frac{\partial ATT_{lk,o}}{\partial f_{lk,o}} = ast_{lk,o} \cdot \mu_0 \cdot \mu_1 \left( \frac{f_{lk,o}}{(cap_{lk,o})^2} \right) > 0 \quad \forall \ lk_o \in LK_{o} \quad (4.11)
\]

The marginal ocean link operating transportation cost and travel (shift) time functions in Eq. (4.12) and Eq. (4.13) are obtained by differentiating total costs with respect to the flow. The marginal costs are assumed to be continuous and strictly monotone increasing in the flow.

\[
MOC_{lk,o} (f_{lk,o}) = \omega_0 + 2 \omega_2 \left( \frac{f_{lk,o}}{cap_{lk,o}} \right) + 3 \omega_2 \left( \frac{f_{lk,o}}{cap_{lk,o}} \right)^2 > 0 \quad \forall \ lk_o \in LK_{o} \quad (4.12)
\]

\[
MTT_{lk,o} (f_{lk,o}) = att_{lk,o} + awt_{lk,o} \left( 1 + \mu_1 \cdot \left( f_{lk,o} \right) \right) > 0 \quad \forall \ lk_o \in LK_{o} \quad (4.13)
\]

The link transportation cost functions of ocean carrier \( o \) (\( TC_{lk,o,o} \)) can be defined by player \( o \)'s behavior alone, each.
4.1.4 Properties of the objective functions

Base on the function assumptions in sub-section 4.1.3, the properties of the objective functions are defined. The objective functions in Eq. (4.3) and Eq. (4.4) are continuous since the sum or the product of various continuous functions is also continuous.

In order to find the properties of the objective function in Eq. (4.3), first and second derivatives of the function with respect to the ocean carrier service charge \( c_{ij}^{o} \) and the flow \( f_{ij}^{ph-o,o} \) are obtained, respectively. First derivatives of the objective function are shown in Eq. (4.14) and Eq. (4.15).

\[
\frac{\partial U_{ij}^{o}}{\partial c_{ij}^{o}} = D_{ij}^{o}(c_{ij}^{o}, c_{ij}^{o}) + \frac{\partial D_{ij}^{o}(c_{ij}^{o}, c_{ij}^{o})}{\partial c_{ij}^{o}} \times c_{ij}^{o} \quad \forall \ o \in O, \ ij \in IJ
\]  

(4.14)

\[
\frac{\partial U_{ij}^{o}}{\partial f_{ij}^{ph-o,o}} = MC_{ij}^{ph-o,o}(f_{ij}^{ph-o,o}) \quad \forall \ o \in O, \ ij \in IJ, \ ph_{a,o} \in PH_{a,o}
\]  

(4.15)

Eq. (4.14) is a continuous function since the first element is linear by the ocean carrier demand function assumption in Eq. (4.5) and the second element is constant. Eq. (4.15) is also a continuous function based upon the marginal cost function assumptions in Eq. (4.12) and Eq. (4.13). Therefore, the vector of first derivatives of the objective function defined in Eq. (4.16) is continuous.

\[
\left(\cdots, \frac{\partial U_{ij}^{o}}{\partial c_{ij}^{o}}, \cdots, \frac{\partial U_{ij}^{o}}{\partial f_{ij}^{ph-o,o}}, \cdots\right) \quad \forall \ o \in O, \ ij \in IJ
\]  

(4.16)

Second derivatives of the objective function are expressed by Hessian matrix in Eq. (4.17).
\[ \nabla^2 (U_{ij}^o) = \begin{bmatrix} \nabla e_{ij}^o D_{ij}^o (c_{ij}^o, c_{ij}^{-o}) & 0 \\ 0 & -\nabla f_{ij}^{ph_{-o}o} MC_{ij}^{ph_{-o}o} (f_{ij}^{ph_{-o}o}) \end{bmatrix} \quad \forall o \in O, ij \in IJ \quad (4.17) \]

Hessian matrix is negative definite, therefore the objective function in Eq. (4.3) is strictly concave in \((c_{ij}^o, f_{ij}^{ph_{-o}o})\). Appendix A provides proofs of concave properties of the ocean carrier objective function. By the similar steps, the objective function in Eq. (4.4) is also a concave function.

### 4.1.5 Feasible region of the objective functions

The feasible region of the ocean carrier objective functions \((OFR)\) is defined by linear equality and non-negativity constrains as follows:

\[ \sum_{w} Q_{iw} = \sum_{o} D_{ij}^o = \sum_{o} \sum_{ph_{-o}o} f_{ij}^{ph_{-o}o} = \sum_{ph_{-o}} f_{ij}^{ph_{-o}} \quad \forall ij \in IJ \quad (4.18) \]

\[ Sc_{ij}^o < c_{ij}^o < Bc_{ij}^o \quad \forall o \in O, ij \in IJ \quad (4.19) \]

\[ c_{ij}^o, f_{ij}^{ph_{-o}o}, f_{ij}^{ph_{-o}} \geq 0 \quad \forall o \in O, ij \in IJ, ph_{-o}o \in PH_{-o}o, ph_{-o} \in PH_{-o} \quad (4.20) \]

Eq. (4.18) ensures that the total transportation demand from \(i\) via \(j\) to \(w\) is equivalent to the sum of individual ocean carrier service demands between \(i\) and \(j\). Also, the total ocean carrier service demand is equivalent to the sum of ocean carrier flows on all used ocean paths between \(i\) and \(j\). These linear equality constrains define a closed and convex feasible region. Eq. (4.19) ensures that the ocean carrier service charge ranges from a small number to a large number. Eq. (4.20) states non-negativity of the ocean carrier service charge and flow.
4.1.6 Mathematical formulations for the ocean carrier problem

As a modeling methodology, the dissertation uses variational inequality (VI) formulation by Nagurney (1999). The theory is a powerful unifying methodology for studying various equilibrium problems. It qualitatively analyzes equilibrium problems in terms of the existence and the uniqueness of solutions, stability and sensitivity analysis and provides algorithms with accompanying convergence analysis for computational purposes. The definition of VI problem is shown below.

**Definition 1.1**

The finite-dimensional variational inequality problem, $\text{VI}(F,K)$, is to determine a vector $x^* \in K \subset \mathbb{R}^n$, such that

$$F(x^*)^T(x - x^*) \geq 0, \quad \forall x \in K$$

(4.21)

where $F$ is given continuous function $(K \rightarrow \mathbb{R}^n)$ and $K$ is a given closed convex set.

1) Competitive environment

Based upon the objective function properties and feasible region above, ocean carrier models are formulated by a VI problem. In the competitive game, ocean carrier $o$ finds the optimal service charge and routing pattern to obtain the maximum profit. For the new service charge, the income and the transportation cost are updated to compare the profit. Each ocean carrier attempts to minimize the total transportation cost of the vessels belonging to this ocean carrier. Thus, ocean carrier $o$ exhibits a system equilibrium-like behavior through the marginal transportation cost function. Table 4.1 illustrates ocean carrier VI formulations for the competitive game.
Table 4.1 Ocean Carrier VI Formulations for the Competitive Game

<table>
<thead>
<tr>
<th>Expression</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(- \sum_{o} \sum_{ij} \left( D_{ij}^o (c_{ij}^{o*}, c_{ij}^{-o}) \times c_{ij}^{o} \right) \geq 0 )</td>
<td>Eq. (4.22) expresses a VI formulation for the profit of ocean carrier ( o ), depending on the service charge and the transportation cost. Due to the properties of the VI problem defining the minimization function, the (-) profit formulation is developed. Eq. (4.23) represents a VI formulation for the marginal transportation cost of ocean carrier ( o ), relying on the flow on all used ocean paths connecting ( i ) and ( j ). The VI formulations are shown as a vector form as follows:</td>
</tr>
<tr>
<td>(- \left( D_{ij}^o (c_{ij}^{o*}, c_{ij}^{-o}) \times c_{ij}^{o} \right) \geq 0 )</td>
<td>Eq. (4.24)</td>
</tr>
<tr>
<td>where ( \nabla f_{ij}^{o} TC_{ij}^{ph(o-o)} \geq 0 )</td>
<td>Eq. (4.25)</td>
</tr>
<tr>
<td>\forall (c_{ij}^{o}, f_{ij}^{ph(o-o)}) \in OFR</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.2 Ocean Carrier VI Formulations for the Competitive Game (Vector form)

<table>
<thead>
<tr>
<th>Expression</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(- \left( D_{ij}^o (c_{ij}^{o*}, c_{ij}^{-o}) \times c_{ij}^{o} \right) \geq 0 )</td>
<td>Eq. (4.24)</td>
</tr>
<tr>
<td>where ( \nabla f_{ij}^{o} TC(f_{ij}^{o}) \geq 0 )</td>
<td>Eq. (4.25)</td>
</tr>
<tr>
<td>\forall (c_{ij}^{o}, f_{ij}^{o}) \in OFR</td>
<td></td>
</tr>
</tbody>
</table>
2) Collusive environment

In the collusive game, the ocean carriers find the optimal service charge set to obtain the maximum total income and the optimal routing set to spend the minimum total transportation cost. The formulation approach is different from the competitive game because pricing and routing decisions of individual ocean carriers are not considered separately. Table 4.2 exhibits ocean carrier VI formulations for the collusive game.

**Table 4.3 Ocean Carrier VI Formulations for the Collusive Game**

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( -\sum_{ij} \left( D_{ij}(c_{ij}^<em>) \times c_{ij}\right)(c_{ij} - c_{ij}^</em>) \geq 0 )</td>
<td>Eq. (4.26) expresses a VI formulation for the income of the ocean carriers depending on the service charge between ( i ) and ( j ). Due to the properties of the VI problem, the (-) income formulation and the (+) transportation cost formulation are developed, respectively. Eq. (4.27) represents a VI formulation for the marginal transportation cost relying on the flow of the ocean carriers on all used paths connecting ( i ) and ( j ). The VI formulations are expressed as a vector form.</td>
</tr>
<tr>
<td>( \sum_{ij} \sum_{PH_{\neq o}} \nabla_{f_{ij}^{PH_{\neq o}}} \text{TC}<em>{ij}^{PH</em>{\neq o}}(f_{ij}^{PH_{\neq o}})(f_{ij}^{PH_{\neq o}} - f_{ij}^{PH_{\neq o}^*}) \geq 0 )</td>
<td></td>
</tr>
<tr>
<td>( \forall (c_{ij}, f_{ij}^{PH_{\neq o}}) \in OFR )</td>
<td></td>
</tr>
</tbody>
</table>

Eq. (4.26) expresses a VI formulation for the income of the ocean carriers depending on the service charge between \( i \) and \( j \). Due to the properties of the VI problem, the (-) income formulation and the (+) transportation cost formulation are developed, respectively. Eq. (4.27) represents a VI formulation for the marginal transportation cost relying on the flow of the ocean carriers on all used paths connecting \( i \) and \( j \). The VI formulations are expressed as a vector form.
Table 4.4 Ocean Carrier VI Formulations for the Collusive Game (Vector form)

\[
\begin{align*}
-\left(D_{ij}(c_{ij}) \times c_{ij} - TC_{ij}\right)^T (c_{ij} - c_{ij}^*) & \geq 0 \\
\nabla_{f_{ij}} TC (f_{ij})^T (f_{ij} - f_{ij}^*) & \geq 0 \\
\forall (c_{ij}, f_{ij}) & \in OFR
\end{align*}
\] (4.28)

\[
\begin{align*}
\nabla_{f_{ij}} TC (f_{ij})^T (f_{ij} - f_{ij}^*) & \geq 0 \\
\forall (c_{ij}, f_{ij}) & \in OFR
\end{align*}
\] (4.29)

4.1.7 Existence and uniqueness of the solution

The existence and uniqueness of the solution are guaranteed by Nagurney’s theorems (1999).

**Theorem 1.4**

If $K$ is a compact convex set and $F(x)$ is continuous on $K$, then the variational inequality problem admits a least one solution $x^*$.

As discussed in sub-section 4.1.4, the formulations in Table 4.1 are continuous and linear equality constraints of $OFR$ define a closed convex feasible region. Hence, the existence of the solution is proved.

Let $B_R(0)$ denote a closed ball with radius $R$ centered at 0 and let $K_R = K \cap B_R(0)$. $K_R$ is then bounded. Determine $x_R^* \in K_R$, such that

\[
F(x_R^*)^T(y - x_R^*) \geq 0, \quad \forall y \in K_R
\] (4.30)

**Theorem 1.5**

$VI(F,K)$ admits a solution if and only if there exists an $R>0$ and a solution of $VI_R$. 
\( x^*_R \), such that \( \| x^*_R \| < R \).

A solution exists when \( (c^*_i, f_{ij}^{\text{ph}}) \) satisfies \( \|c^*_i, f_{ij}^{\text{ph}}\| < R \). The service charge \( c_i^o \) is bounded in Eq. (4.19), therefore the demand \( D_{ij}^o \) is also bounded. With the bounded \( D_{ij}^o \), the flow \( f_{ij}^{\text{ph},o} \) is bounded due to the flow conservation constraint in Eq. (4.18), as shown in Eq. (4.31).

\[
S_{ij}^{\text{ph},o} < f_{ij}^{\text{ph},o} < B_{ij}^{\text{ph},o} \quad \forall \ ij \in IJ, \ ph_{o,o} \subseteq PH_{o,o} \tag{4.31}
\]

Based on the bounded values, \( R \) is constructed as \( R = \|Bc_{ij}^o, Bf_{ij}^o\| \). Hence,

\[
\|c_{ij}^o, f_{ij}^{\text{ph},o}\| < \|Bc_{ij}^o, Bf_{ij}^{\text{ph},o}\|.
\]

**Theorem 1.6**

Suppose that \( F(x) \) is strictly monotone on \( K \). Then the solution is unique if one exists.

The formulations are strictly monotone in \( (c_{ij}^o, f_{ij}^{\text{ph},o}) \) based on the concave properties of the objective function. Therefore, the uniqueness of the solution is demonstrated.

The existence and uniqueness of the solution in the ocean carrier collusive game can be demonstrated by the similar procedures.
4.1.8 Summary of section 4.1

Section 4.1 formulated ocean carrier models to capture the decisions and interactions of competitive and collusive ocean carriers, between an origin port and a destination port. In the competitive game, Nash Equilibrium was used to find the optimal service charge and the routing pattern of each ocean carrier. On the other hand, in the collusive game, the optimal service charge and routing set of the colluded ocean carriers was determined via compensation principle. Based upon the objective function properties and feasible region, ocean carrier models were formulated by a VI problem according to the different market environment.

Section 4.2 presents the port terminal operator problem focusing on the decisions and relationships of port terminal operators located in competitive regions.

4.2 Port Terminal Operator Problem

Port terminal operators handle freight arriving in a port complex to provide transshipment services between ocean carriers and land carriers. As discussed in Chapter 3, perfect competition among port terminal operators is considered in this research. Therefore, each port terminal operator aims to maximize the individual profit in the competitive environment. The port terminal operator problem is organized as follow: equilibrium condition, objective function, assumptions and properties of the objective function, feasible region, mathematical formulations, and existence and uniqueness of the solution.
4.2.1 Equilibrium condition of the port terminal operator problem

In the competitive game, Nash equilibrium is used to find the optimal port service charge and pattern for which each port terminal operator has the greatest profit. No port terminal operator can be better off by changing its decisions unilaterally, at equilibrium. The equilibrium condition of Nash equilibrium for port terminal operator \( p \) is shown in Eq. (4.32).

\[
U^p(c^p*, c^{-p*}, f^{p*}) \geq U^p(c^p, c^{-p*}, f^p) \quad \forall p \in P
\]  

(4.32)

Eq. (4.32) indicates that if port terminal operator \( p \) changes pricing and routing decisions from its optimal \((c^p*, f^{p*})\) to any other feasible decisions \((c^p, f^p)\), while other port terminal operator keep their service charges unchanged, its profit will be worse.

4.2.2 Objective function of port terminal operators

The objective of each port terminal operator is to maximize the profit. The objective function of port terminal operator \( p \) \((U^p)\) is defined in Eq. (4.33)

\[
U^p = G^p(c^p, c^{-p}) \times c^p - \sum_{jk} \sum_{ph \neq p} AC_{jkph}^p (f_{jkph}^p) \times f_{jkph}^p \quad \forall p \in P
\]  

(4.33)

Eq. (4.33) expresses the profit function of port terminal operator \( p \) \((U^p)\) composed of the income function \((I^p)\) and the port service cost function \((SC^p)\). The income of port terminal operator \( p \) is determined by the port throughput function \((G^p)\) depending on the port service charge \((c^p)\), while other port terminal operators keep their
current level of service charges \( (\overline{c^p}) \) unchanged. The port service cost of port terminal operator \( p \) is determined by the average port path cost \( (AC_{jk-p}) \) set relying on the flow \( (f_{jk-p}) \) on all used paths.

### 4.2.3 Assumptions for the objective function

The properties of the port throughput function \( (G^p) \) and the port service cost function \( (SC^p) \) are assumed for defining objective function properties. The port throughput function is assumed to be strictly monotone decreasing in the port service charge since if port terminal operator \( p \) increases the service charge, ocean carriers attempt to find cheaper prices from the alternative port terminals. The port service cost function is assumed to be strictly monotone increasing in the flow because as the port flow grows, the port service cost and time increase accordingly.

1) Port throughput function

The port throughput function \( (G^p = \beta_0 - \beta_1 \times c^p) \) is assumed to be linear and strictly monotone decreasing in the port service charge \( (c^p) \) and satisfies:

\[
\frac{\partial G^p}{\partial c^p} < 0 \quad \forall p \in P
\]  \hspace{1cm} (4.34)

Eq. (4.34) expresses that as the service charge of port terminal operator \( p \) increases, the throughput of port terminal operator \( p \) decreases.

2) Port service cost function

The port path and link service costs are the expenses to deal with the total amount of
freight via the port path \((ph_p)\) and the port link \((lk_p)\), respectively. The port path service cost function \((SC^p_{ph} - p)\) is the sum of port link service cost functions \((SC^p_{lk} - p)\) if the port link \((lk_p)\) is on the path \((ph_p)\). A mathematical form of the port path service cost function is shown in Eq. (4.35).

\[
SC^p_{ph} - p (f_{ph} - p) = \sum_{lk \in PH_p} \phi^p_{lk} \times SC^p_{lk} - p (f_{lk} - p) \quad \forall jk \in JK, ph_p \in PH_p \tag{4.35}
\]

If the port link \((lk_o)\) is on the port path \((ph_p)\) connecting \(k\) and \(w\), the incidence index \(\phi^p_{lk} \times ph\) is 1 and the port path service cost function unites the port link service cost functions. Otherwise, \(\phi^p_{lk} \times ph\) is 0.

The port link service cost function \((SC^p_{lk} - p)\) includes the following two attributes:

a) the average port link operating cost function \((AOC^p_{lk} - p)\) and b) the average port link service time function \((AST^p_{lk} - p)\). For unit consistency, the value of time \((vot_p)\) is utilized.

A mathematical form of the port link service cost function is shown in Eq. (4.36).

\[
SC^p_{lk} - p (f_{lk} - p) = AOC^p_{lk} - p (f_{lk} - p) \times f_{lk} - p + vot_p \times AST^p_{lk} - p (f_{lk} - p) \times f_{lk} - p \quad \forall lk_p \in LK_p \tag{4.36}
\]

where

\[
AOC^p_{lk} - p (f_{lk} - p) = \pi_o + \pi_1 \left( \frac{f_{lk} - p}{cap_{lk} - p} \right) + \pi_2 \left( \frac{f_{lk} - p}{cap_{lk} - p} \right)^2 \quad \forall lk_p \in LK_p \tag{4.37}
\]

\[
AST^p_{lk} - p (f_{lk} - p) = ast^p_{lk} \left( 1 + \nu_o \left( \frac{f_{lk} - p}{cap_{lk} - p} \right)^\nu_1 \right) \quad \forall lk_p \in LK_p \tag{4.38}
\]
Eq. (4.37) represents the average port link operating cost function \( AOC^{lk-p} \) considering both fixed and flexible costs. The first element shows the fixed cost to deal with a unit of freight on the port link. The second and third elements express the flexible costs due to the transportation demand beyond the link capacity. In Eq. (4.37), \( cap^{lk-p} \) denotes the capacity of the port link and \( \pi_0, \pi_1, \text{ and } \pi_2 \) represent coefficient parameters with a positive value. Eq. (4.38) expresses the average port service time function \( AST^{lk-p} \). The first element shows the average service time \( \ast^{lk-p} \) on the port link. The second element represents the average port service time due to the service demand beyond the link capacity. In Eq. (4.38), \( \nu_0 \) and \( \nu_1 \) denote coefficient parameters with a positive value.

The average port link operating cost and service time functions are assumed to be continuous and strictly monotone increasing in the flow, as shown in Eq. (4.39) and Eq. (4.40).

\[
\frac{\partial AOC^{lk-p}(f^{lk-p})}{\partial f^{lk-p}} = \pi_1 \frac{1}{cap^{lk-p}} + 2\pi_2 \frac{f^{lk-p}}{(cap^{lk-p})^2} > 0 \quad \forall lk_p \in LK_p \quad (4.39)
\]

\[
\frac{\partial AST^{lk-p}(f^{lk-p})}{\partial f^{lk-p}} = \ast^{lk-p} \cdot \nu_0 \cdot \nu_1 \frac{(f^{lk-p})^{\nu_2 - 1}}{(cap^{lk-p})^{\nu_1}} > 0 \quad \forall lk_p \in LK_p \quad (4.40)
\]

The marginal port link operating cost and service time functions in Eq. (4.41) and Eq. (4.42) are assumed to be continuous and strictly monotone increasing in the flow.

\[
MOC^{lk-p}(f^{lk-p}) = \pi_0 + 2\pi_1 \left( \frac{f^{lk-p}}{cap^{lk-p}} \right) + 3\pi_2 \left( \frac{f^{lk-p}}{cap^{lk-p}} \right)^2 > 0 \quad \forall lk_p \in LK_p \quad (4.41)
\]
\[
MST^{lk_p} (f^{lk_p}) = att^{lk_p} + awt^{lk_p} \left( 1 + V_0 \cdot (V_i + 1) \cdot \frac{(f^{lk_p})^{V_i}}{(cap^{lk_p})^{V_i+1}} \right) > 0 \ \forall \ lk_p \in LK_p \quad (4.42)
\]

### 4.2.4 Properties of the objective function

With the assumptions of port throughput function and port service cost function, the objective function in Eq. (4.33) is continuous. For analyzing the objective function properties, first and second derivatives of the function with respect to the port service charge \(c_p\) and flow \(f_{jk}^{ph_p}\) are obtained. First derivatives of the objective function are shown in Eq. (4.43) and Eq. (4.44).

\[
\frac{\partial U^p}{\partial c^p} = G^p (c^p, c^{-p}) + \frac{\partial G^p (c^p, c^{-p})}{\partial c^p} \cdot c^p \quad \forall \ p \in P \quad (4.43)
\]

\[
\frac{\partial U^p}{\partial f_{jk}^{ph_p}} = MC_{jk}^{ph_p} (f_{jk}^{ph_p}) \quad \forall \ p \in P, \ jk \in JK, \ ph_p \in PH_p \quad (4.44)
\]

Eq. (4.43) is a continuous function since the first element is linear by the port carrier throughput function assumption in Eq. (4.34) and the second element is constant. Eq. (4.44) is also a continuous function based upon the marginal cost function assumptions in Eq. (4.41) and Eq. (4.42). Hence, the vector of first derivatives of the objective function defined in Eq. (4.45) is continuous.

\[
\left\{ \left( \frac{\partial U^p}{\partial c^p}, \ldots, \frac{\partial U^p}{\partial f_{jk}^{ph_p}}, \ldots \right) \right\} \quad \forall \ p \in P \quad (4.45)
\]

Second derivatives of the objective function are expressed by Hessian matrix in Eq. (4.46).
\[ \nabla^2 (U^p) = \begin{bmatrix} \nabla^T \cdot c^p G^p (c^p, c^{-p}) & 0 \\ 0 & -\nabla f_{jk}^{ph^{-p}} MC_{jk}^{ph^{-p}} (f_{jk}^{ph^{-p}}) \end{bmatrix} \quad \forall p \in P \quad (4.46) \]

Hessian matrix is negative definite and symmetric. Hence, the objective function in Eq. (4.33) is strictly concave in \((c^p, f_{jk}^{ph^{-p}})\). Appendix A provides proofs of concave properties of the port terminal operator objective function.

### 4.2.5 Feasible region of the objective function

The feasible region of the port terminal operator objective function \((PFR)\) is defined by linear equality and non-negativity constraints as follows:

\[ \sum_{ij} \sum_{ph_{-o}} \psi_{ij}^{ph_{-o},p} f_{ij}^{ph_{-o}} = G^p = \sum_{jk} \sum_{ph_{-p}} f_{jk}^{ph_{-p}} \quad \forall p \in P; jk \in JK \quad (4.47) \]

\[ Sc^p < c^p < Bc^p \quad \forall p \in P \quad (4.48) \]

\[ c^p, f_{jk}^{ph_{-p}} \geq 0 \quad \forall p \in P; jk \in JK; ph_{-p} \in PH_{-p} \quad (4.49) \]

Eq. (4.47) ensures that the total amount of freight transported via ocean paths is equivalent to the port throughput of the port terminal operator \(p\) when ocean paths are connected to the port terminal. Also, the throughput is equivalent to the sum of port flows on all used paths in the port complex. These linear equality constraints define a closed and convex feasible region. Eq. (4.48) ensures that the port service charge ranges from a small number to a large number. Eq. (4.49) states non-negativity of the port service charge and flow.
4.2.6 Mathematical formulations for the port terminal operator problem

Based on the objective function properties and feasible region, the port terminal operator model is formulated by a VI problem. In the competitive game, port terminal operator \( p \) finds the optimal service charge and routing pattern to obtain the maximum profit. For a new service charge of port terminal operator \( p \), the income and the port service cost are updated to compare the profit. Each port terminal operator attempts to minimize the total transportation cost in its port terminal, showing a system equilibrium-like behavior. Table 4.5 shows port terminal operator VI formulations for the competitive game.

**Table 4.5** Port Terminal Operator VI Formulations for the Competitive Game

| Determinant: \( c^p, f^p_{jk} \),  
| \[ \sum_p (G^p(c^p, c^{-p}) \times c^p - SC^p) \geq 0 \]  
| *(4.50)*  
| \[ \sum_{jk} \sum_{ph \in p} \nabla (SC_{jk}^p)(f^p_{jk} - f^{p-}_{jk}) \geq 0 \]  
| *(4.51)*  
| \( \forall (c^p, f^p_{jk}) \in PFR \)  

Eq. (4.50) expresses a VI formulation for the profit of port terminal operator \( p \), depending on the port service charge and cost. Eq. (4.51) represents a VI formulation for the marginal service cost of port terminal operator \( p \), relying on the flow on all used paths connecting \( j \) and \( k \). The VI formulations are defined as a vector form.
Table 4.6 Port Terminal Operator VI Formulations for the Competitive Game (Vector form)

\[
-\left( G^p (c^p*, c_{-p}^*) \times c^p - SC^p \right)^T (c^p - c^p*) \geq 0
\]  
(4.52)

where \( \nabla_{f^p} SC(f^p)^T (f^p - f^p*) \geq 0 \)  
(4.53)

\[ \forall (c^p, f^p) \in PFR \]

4.2.7 Existence and uniqueness of the solution

The existence and uniqueness of the solution are guaranteed by the theorems (theorem 1.4, theorem 1.5 and theorem 1.6), introduced in sub-section 4.1.7. The formulations in Table 4.5 are continuous and linear equality constraints of \( PFR \) define a closed convex feasible region. Hence, the existence of the solution is proved by theorem 1.4.

The port terminal operator game has a solution when \( (c^p*, f_{jk}^{ph-p*}) \) satisfies

\[
\|c^p*, f_{jk}^{ph-p*}\| < R \text{ by theorem 1.5.}
\]

The service charge \( (c^p) \) is bounded in Eq. (4.48), therefore the port throughput \( (G^p) \) is also bounded. With the bounded \( G^p \), the flow \( (f_{jk}^{ph-p}) \) is bounded due to the flow conservation constraint in Eq. (4.47), as shown in Eq. (4.54).

\[
S f_{jk}^{ph-p} < f_{jk}^{ph-p} < B f_{jk}^{ph-p} \quad \forall jk \in JK, ph_p \in PH_p
\]  
(4.54)

Based on the bounded values, \( R \) is constructed as \( R = \|B c^p, B f_{jk}^{ph-p}\| \). Hence,

\[
\|c^p*, f_{jk}^{ph-p*}\| < \|B c^p, B f_{jk}^{ph-p}\|.
\]
The formulations are strictly monotone in \((c^p, f^p_{jk-})\) from the concave properties of the objective function. Therefore, the solution is unique through theorem 1.6.

### 4.2.8 Summary of section 4.2

Section 4.2 proposed a port terminal operator model to capture the decisions and interactions of competitive port terminal operators. Nash Equilibrium was used to find the optimal service charge and routing pattern of each port terminal operator. Based on the objective function properties and feasible region, the model was formulated by a VI problem.

Section 4.3 presents the land carrier problem focusing on the decisions and relationships of land carriers between a destination port and a land destination in the competitive and collusive environment.

### 4.3 Land Carrier Problem

Land carriers deliver freight treated in a port terminal to a land destination via roadways or railways by truck, rail or combined truck/rail. In the competitive environment, each land carrier attempt to maximize the individual profit. On the other hand, in the collusive environment, the colluded land carriers aim to maximize the total profit. The land carrier problem is organized as follow: equilibrium conditions, objective functions, assumptions and properties of the objective functions, feasible region, mathematical formulations, and existence and uniqueness of the solution.
4.3.1 Equilibrium conditions of the land carrier problem

1) Competitive environment

In the competitive game, Nash Equilibrium is used to find the optimal service charge and routing pattern for which each land carrier obtains the greatest profit. No land carrier can get better profit by changing its decisions unilaterally, at equilibrium. The equilibrium condition of Nash equilibrium for land carrier $l$ between $k$ and $w$ is shown in Eq. (4.55).

$$U_{kw}^l (c_{kw}^l, c_{kw}^{-l}, f_{kw}^l) \geq U_{kw}^l (c_{kw}^l, c_{kw}^{-l'}, f_{kw}^{l'}) \quad \forall l \in L, k \in Kw$$ (4.55)

Eq. (4.55) indicates that if land carrier $l$ between $k$ and $w$ changes pricing and routing decisions from its optimal $(c_{kw}^l, c_{kw}^{-l}, f_{kw}^l)$ to any other feasible decisions $(c_{kw}^l, c_{kw}^{-l'}, f_{kw}^{l'})$, while other land carrier keep their service charges constant, its profit will be worse.

2) Collusive environment

In the collusive game, compensation principle is applied to find the optimal pricing and routing set of the colluded land carriers, by maximizing their total profit. The total profit should be greater than the sum of individual profits of the competitive land carriers, at equilibrium. The equilibrium condition of compensation principle for the land carriers between $k$ and $w$ is shown in Eq. (4.56).

$$Max \sum_{c, f} U_{kw}^l (c_{kw}^l, c_{kw}^{-l}, f_{kw}^l) \geq \sum_{c, f} Max U_{kw}^l (c_{kw}^l, c_{kw}^{-l}, f_{kw}^l) \quad \forall k \in Kw$$ (4.56)

Eq. (4.56) expresses that the total profit of the colluded land carriers must be greater than the sum of individual profits of the competitive land carriers, between $k$ and $w$. 
4.3.2 Objective functions of land carriers

The objective functions of the individual and colluded land carriers are defined as profit maximization functions for the competitive and collusive environment, as follows.

1) Competitive environment

In the competitive game, each land carrier aims to maximize the profit, for O-D pair \( kw \).

The objective function of land carrier \( l \) \((U_{kw}^l)\) is defined in Eq. (4.57).

\[
U_{kw}^l = D_{kw}^l (c_{kw}^l, \overline{c}_{kw}^l) \times c_{kw}^l - \sum_{ph_{-kw}, l, j} AC_{kw}^{ph_{-kw}, l, j} (f_{kw}^{ph_{-kw}, l, j}) \times f_{kw}^{ph_{-kw}, l, j} \quad \forall \ l \in L, \ kw \in KW
\]  

Eq. (4.57) expresses the profit function of land carrier \( l \) \((U_{kw}^l)\) composed of the income function \((I_{kw}^l)\) and the transportation cost function \((TC_{kw}^l)\). The income of land carrier \( l \) is determined by the service demand function \((D_{kw}^l)\) depending on the service charge \((c_{kw}^l)\), while other land carriers keep their current level of service charges \((\overline{c}_{kw}^l)\) constant. The transportation cost of land carrier \( l \) is decided by the average land path transportation cost function \((AC_{kw}^{ph_{-kw}, l, j})\) relying on the flow \((f_{kw}^{ph_{-kw}, l, j})\) on all used paths.

2) Collusive environment

In the collusive game, the objective of the land carriers is to maximize their total profit, for O-D pair \( kw \). The objective function of land carriers is defined in Eq. (4.58)
\[
\sum_{l} U_{kw}^l = \sum_{l} D_{kw}^l (c_{kw}^l, c_{kw}^{-l}) \times c_{kw}^l - \sum_{ph_{kw}}^{l} TC_{kw}^{ph_{kw}} (f_{kw}^{ph_{kw}}) \times f_{kw}^{ph_{kw}} \quad \forall \ kw \in KW
\] (4.58)

Eq. (4.58) expresses the total profit function of the colluded land carriers (\(\sum_{l} U_{kw}^l\)), composed of the total income function (\(\sum_{l} I_{kw}^l\)) and the total transportation cost function (\(\sum_{l} TC_{kw}^l\)). The total income is determined by the service demand set of land carriers depending on their service charges. The total transportation cost is decided by the average land path transportation cost set relying on the flow of land carriers on all used paths.

4.3.3 Assumptions for the objective functions

The properties of land carrier demand function (\(D_{kw}^l\)) and land carrier transportation cost function (\(TC_{kw}^l\)) are assumed. The demand function is assumed to be strictly monotone decreasing in the service charge, while the transportation cost function is assumed to be strictly monotone increasing in the flow.

1) Demand function

The land carrier demand function (\(D_{kw}^l = \lambda_0 - \lambda_1 \times c_{kw}^l\)) is assumed to be linear and strictly monotone decreasing in the service charge (\(c_{kw}^l\)) and satisfies:

\[
\frac{\partial D_{kw}^l}{\partial c_{kw}^l} < 0 \quad \forall l \in L, kw \in KW
\] (4.59)
Eq. (4.59) expresses that as the service charge of land carrier \( l \) between the \( k \) and \( w \) increases, its service demand between the same regions decreases.

2) Transportation cost function

The land path and link transportation costs are the expenses to deliver the total amount of freight via the land path \((p_{h,j})\) and the land link \((l_{k,j})\), respectively. The land path transportation cost function \((TC^{ph_{-l}}_{kw})\) is the sum of land link transportation cost functions \((TC^{lk_{-l}}_{k})\) if the land link \((l_{k,j})\) is on land path \((p_{h,j})\). A mathematical form of the land path transportation cost function is shown in Eq. (4.60).

\[
TC^{ph_{-l}}_{kw}(f_{kw}^{ph_{-l}}) = \sum_{l_{k,j}} \delta^{l_{k,j},p_{h,l}}_{kw} \times TC^{lk_{-l}}_{k}(f^{lk_{-l}}) \quad \forall \ kw \in KW, \ p_{h,j} \in PH_j \quad (4.60)
\]

If the land link \((l_{k,j})\) is on the land path \((p_{h,j})\) connecting \( k \) and \( w \), the incidence index \( \delta^{l_{k,j},p_{h,l}}_{kw} \) is 1 and the land path transportation cost function combines the land link transportation cost functions. Otherwise, \( \delta^{l_{k,j},p_{h,l}}_{kw} \) is 0.

The land link transportation cost functions contain the following two attributes: a) the average land link operating cost function \((AOC^{lk_{-l}}_{k})\) and b) the average land link travel (shift) time function \((ATT^{lk_{-l}})\). For unit consistency, the travel (shift) time is changed to monetary value using the value of time \((vot')\). A mathematical form of the land path transportation function is shown in Eq. (4.61).

\[
TC^{lk_{-l}}_{k}(f^{lk_{-l}}) = AOC^{lk_{-l}}_{k}(f^{lk_{-l}}) \times f^{lk_{-l}} + vot' \times ATT^{lk_{-l}}(f^{lk_{-l}}) \times f^{lk_{-l}}
\]

\[
\forall \ l_{k,j} \in LK_j \quad (4.61)
\]
where

\[
AOC_{lk-j}(f^{lk-j}) = \tau_o + \tau_1 \left( \frac{f^{lk-j}}{\text{cap}_{lk-j}} \right) + \tau_2 \left( \frac{f^{lk-j}}{\text{cap}_{lk-j}} \right)^2 \quad \forall lk_j \in LK_j \tag{4.62}
\]

\[
ATT_{lk-j}(f^{lk-j}) = ast_{lk-j} \left[ 1 + \theta_0 \left( \frac{f^{lk-j}}{\text{cap}_{lk-j}} \right)^\theta_1 \right] + att_{lk-j} \left[ 1 + \theta_2 \left( \frac{f^{lk-j}}{\text{cap}_{lk-j}} \right)^\theta_3 \right] \quad \forall lk_j \in LK_j \tag{4.63}
\]

Eq. (4.62) expresses the land link operating cost function including both fixed and flexible costs. The first element shows the fixed cost to deliver a unit of freight on the land link. The second and third elements represent the flexible costs due to the transportation demand beyond the link capacity. In Eq. (4.62), \( cap_{lk-j} \) represents the capacity of the land link and \( \tau_o, \tau_1 \) and \( \tau_2 \) denote coefficient parameters with a positive value. Eq. (4.63) shows the average land link travel (shift) time function. The first element expresses the average shift time and the second element represents the average shift time due to the transportation demand beyond the link capacity, on the port-exit link. The third element shows the average travel time and the last element exhibits the average travel time due to the transportation demand beyond the link capacity, on the land link. In Eq. (4.63), \( \theta_0, \theta_1, \theta_2 \) and \( \theta_3 \) denote coefficient parameters with a positive value.

The average land link operating cost and travel (shift) time functions are assumed to be continuous and strictly monotone increasing in the flow, as shown in Eq. (4.64) and Eq. (4.65).
\[
\frac{\partial AOC^{lk,-l}}{\partial f^{lk,-l}} = \tau_1 \frac{1}{cap^{lk,-l} - c_0} + 2\tau_2 \frac{f^{lk,-l}}{(cap^{lk,-l})^2} > 0 \quad \forall lk_j \subseteq LK, \tag{4.64}
\]

\[
\frac{\partial ATT^{lk,-l}}{\partial f^{lk,-l}} = ast^{lk,-l} \cdot \theta_0 \cdot \theta_1 \left( \frac{f^{lk,-l}}{cap^{lk,-l}} \right)^{\theta_1-1} + att^{lk,-l} \cdot \theta_2 \cdot \theta_3 \left( \frac{f^{lk,-l}}{cap^{lk,-l}} \right)^{\theta_3-1} > 0 \quad \forall lk_j \subseteq LK, \tag{4.65}
\]

The marginal land link operating cost and travel (shift) time functions in Eq. (4.66) and Eq. (4.67) are assumed to be continuous and strictly monotone increasing in the flow.

\[
MOC^{lk,-l}(f^{lk,-l}) = \omega_0 + 2\omega_i \left( \frac{f^{lk,-l}}{cap^{lk,-l}} \right) + 3\omega_2 \left( \frac{f^{lk,-l}}{cap^{lk,-l}} \right)^2 > 0 \quad \forall lk_j \subseteq LK, \tag{4.66}
\]

\[
MTT^{lk,-l}(f^{lk,-l}) = ast^{lk,-l} \left( 1 + \theta_0 (\theta_1 + 1) \left( \frac{f^{lk,-l}}{cap^{lk,-l}} \right)^{\theta_1} \right) + att^{lk,-l} \left( 1 + \theta_2 (\theta_3 + 1) \left( \frac{f^{lk,-l}}{cap^{lk,-l}} \right)^{\theta_3+1} \right) > 0 \quad \forall lk_j \subseteq LK, \tag{4.67}
\]

The land link transportation cost functions of land carrier \( l \) (\( TC^{lk,-l} \)) can be defined by player \( l \)'s behavior alone, each.

### 4.3.4 Properties of the objective functions

The land carrier demand function and transportation functions assumed to be continuous above, therefore the objective functions of land carriers are also continuous. To analyze the properties of the objective function in Eq. (4.57), first and second derivatives of the function with respect to the land carrier service charge (\( c_{kw}^{l} \)) and flow (\( f_{kw}^{ph,-l,l} \)) are obtained. First derivatives of the objective function with are shown in Eq. (4.68) and Eq. (4.69).
\[
\frac{\partial U_{kw}^l}{\partial c_{kw}} = D_{kw}^l(c_{kw}^l, c_{kw}^{-l}) + \frac{\partial D_{kw}^l(c_{kw}^l, c_{kw}^{-l})}{\partial l_{kw}} \times l_{kw} \quad \forall l \in L, \, kw \in KW
\]  

\[
\frac{\partial U_{kw}^l}{\partial f_{kw}^{ph_{-l},l}} = MC_{kw}^{ph_{-l},l}(f_{kw}^{ph_{-l},l}) \quad \forall l \in L, \, kw \in KW, \, ph_{,j} \in PH_{,j}
\]  

Eq. (4.68) is a continuous function since the first element is linear by the land carrier demand function assumption in Eq. (4.59) and the second element is constant. Eq. (4.62) is also a continuous function based on the marginal transportation cost function assumptions in Eq. (4.66) and Eq. (4.67). Hence, the vector of first derivatives of the objective function defined in Eq. (4.70) is continuous.

\[
\left(\ldots, \frac{\partial U_{kw}^l}{\partial c_{kw}}, \ldots, \frac{\partial U_{kw}^l}{\partial f_{kw}^{ph_{-l},l}}, \ldots\right) \quad \forall l \in L, \, kw \in KW
\]  

Second derivatives of the objective function are expressed by Hessian matrix, as shown in Eq. (4.71).

\[
\nabla^2(U_{kw}^l) = \begin{bmatrix}
\nabla_{c_{kw}} D_{kw}^l(c_{kw}^l, c_{kw}^{-l}) & 0 \\
0 & -\nabla_{f_{kw}^{ph_{-l},l}} MC(f_{kw}^{ph_{-l},l})
\end{bmatrix} \quad \forall l \in L, \, kw \in KW
\]  

Hessian matrix is negative definite therefore the objective function in Eq. (4.57) is strictly concave in \((c_{kw}^l, f_{kw}^{ph_{-l},l})\). Appendix A provides proofs of concave properties of the land carrier objective function. By the similar steps, the objective function in Eq. (4.58) is also a concave function.
4.3.5 Feasible region of the objective functions

The feasible region of the land carrier objective functions (LFR) is defined by linear equality and non-negativity constraints as follows:

\[
\sum_{j}^{L} \sum_{ph \in p}^{PH} f_{jk}^{ph \in p} = \sum_{l}^{W} D_{kw}^{l} \quad \forall jk \in JK, kw \in KW \tag{4.72}
\]

\[
\sum_{l}^{L} D_{kw}^{l} = \sum_{l}^{PH} \sum_{ph \in l}^{PH} f_{kw}^{ph \in l} = \sum_{l}^{PH} f_{kw}^{ph \in l} \quad \forall kw \in KW \tag{4.73}
\]

\[
Sc_{kw}^{l} < c_{kw}^{l} < Bc_{kw}^{l} \quad \forall l \in L, kw \in KW \tag{4.74}
\]

\[
c_{kw}^{l}, f_{kw}^{ph \in l}, f_{kw}^{ph \in l} \geq 0 \quad \forall l \in L, kw \in KW, ph \in PH \tag{4.75}
\]

Eq. (4.72) ensures that the total amount of freight handled at \( k \) is equivalent to the sum of land carrier service demands departing from \( k \). Eq. (4.73) ensures that the total land carrier service demand for O-D pair \( kw \) is equivalent to the sum of land carrier flows on all used land paths between \( k \) and \( w \). These linear equality constraints define a closed and convex feasible region. Eq. (4.74) ensures that the land carrier service charge ranges from a small number to a large number. Eq. (4.75) states non-negativity of the land carrier service charge and flow.

4.3.6 Mathematical formulations for the land carrier problem

From the properties of objective functions and feasible region, land carrier models are formulated by a VI problem for the competitive and collusive environment.
1) Competitive game

In the competitive game, land carrier $l$ finds the optimal service charge and routing pattern to obtain the maximum profit. When the land carrier considers a new service charge, the income and the transportation cost are updated to compare the profit. Each land carrier attempts to minimize the total transportation cost of the modes (i.e., truck, rail, etc.) belonging to this land carrier. Thus, each land carrier exhibits a system equilibrium-like behavior. Table 4.7 shows land carrier VI formulations for the competitive game.

**Table 4.7 Land Carrier VI formulations for the Competitive Game**

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{Determine } c_{kw}^l, f_{kw}^{ph_{-l}}, \ast, \ast, \ast, \ast, \ast$</td>
<td></td>
</tr>
<tr>
<td>$-\sum_{l}^{L} \sum_{kw}^{KW} \left( D_{kw}^{l} \left( c_{kw}^l, c_{kw}^l \ast, c_{kw}^l \ast \right) \times c_{kw}^l \ast - T C_{kw}^{l} \right) \left( c_{kw}^l - c_{kw}^l \ast \right) \geq 0$</td>
<td>Eq. (4.76) expresses a VI formulation for the profit of land carrier $l$, depending on the service charge and the transportation cost.</td>
</tr>
<tr>
<td>$\sum_{kw}^{KW} \sum_{ph_{-l},l}^{PH_{-l,l}} \nabla f_{kw}^{ph_{-l},l} T C_{kw}^{ph_{-l},l} \left( f_{kw}^{ph_{-l},l}, f_{kw}^{ph_{-l},l} \ast \right) \left( f_{kw}^{ph_{-l},l} - f_{kw}^{ph_{-l},l} \ast \right) \geq 0$</td>
<td>Eq. (4.77) represents a VI formulation for the marginal transportation cost land carrier $l$, relying on the flow on all used paths connecting $k$ and $w$.</td>
</tr>
</tbody>
</table>

| $\forall \left( c_{kw}, f_{kw}^{ph_{-l}} \right) \in LFR$ | |

Eq. (4.76) expresses a VI formulation for the profit of land carrier $l$, depending on the service charge and the transportation cost. Eq. (4.77) represents a VI formulation for the marginal transportation cost land carrier $l$, relying on the flow on all used paths connecting $k$ and $w$. The VI formulations are expressed as a vector form as follows:
Table 4.7 Land Carrier VI Formulations for the Competitive Game (Vector form)

\[- \left( D^l (c^l_*, c^{-l}_*) \times c^l_* - TC^l \right)^T (c^l - c^l_*) \geq 0 (4.78)\]

where \( \nabla_f TC(f^l)^T (f^l - f^l_*) \geq 0 \) \( (4.79) \)

\[ \forall (c^l, f^l) \in LFR \]

2) Collusive game

In the collusive game, the colluded land carriers seek the optimal service charge set to have the maximum total income and the optimal routing set to obtain the minimum total transportation. Table 4.9 illustrates land carrier VI formulations for the collusive game.

Table 4.9 Land Carrier VI formulations for the Collusive Game

\[
\text{Determine } c_{kw}^*, f_{kw}^{ph_{-l}}^*; \\
- \sum_{kw}^{KW} (D_{kw}(c_{kw}^*) \times c_{kw}^*) (c_{kw} - c_{kw}^*) \geq 0 \quad (4.80) \\
\sum_{kw}^{KW} \sum_{ph_{-l}}^{PH} \nabla_{f_{kw}^{ph_{-l}}} TC_{kw}^{ph_{-l}} (f_{kw}^{ph_{-l}})^* (f_{kw}^{ph_{-l}} - f_{kw}^{ph_{-l}})^* \geq 0 \quad (4.81) \\
\forall (c_{kw}, f_{kw}^{ph_{-l}}) \in LFR
\]

Eq. (4.80) expresses a VI formulation for the marginal income depending on the service charge between \( k \) and \( w \). Eq. (4.81) represents a VI formulation for the marginal transportation cost relying on the flow of the land carriers on at all used paths connecting \( k \) and \( w \). The VI formulations are defined as a vector form as follows:
Table 4.10 Land Carrier VI Formulations for the Competitive Environment (Vector form)

\[-(D_{kw}(c_{kw}^*)\times c_{kw}^* - TC_{kw}^T)(c_{kw} - c_{kw}^*) \geq 0 \]  
(4.82)

\[\nabla_{f_{kw}} TC(f_{kw})^T(f_{kw} - f_{kw}^*) \geq 0 \]  
(4.83)

\[\forall (c^l, f^l) \in LFR\]

4.3.7 Existence and uniqueness of the solution

The existence and uniqueness of the solution are guaranteed by theorem 1.4, theorem 1.5 and theorem 1.6. The formulations in Table 4.7 are continuous and linear equality constraints of \(LFR\) define a closed convex feasible region. Therefore, the existence of the solution is proved by theorem 1.4.

The land carrier game has a solution when \((c_{kw}^l, f_{kw}^{ph-l}*\) satisfies

\[\|c_{kw}^l, f_{kw}^{ph-l}*\| < R \] by theorem 1.5. The service charge \((c_{kw}^l)\) is bounded in Eq. (4.74), therefore the demand \((D_{kw}^l)\) is also bounded. With the bounded \(D_{kw}^l\), the path flow \((f_{kw}^{ph-l,l})\) is bounded due to the flow conservation constraint in Eq. (4.73), as shown in Eq. (4.84)

\[S_{f_{kw}^{ph-l,l}} < f_{kw}^{ph-l,l} < B_{f_{kw}^{ph-l,l}} \quad \forall kw \in KW, ph_{j,l} \in PH_{j,l} \]  
(4.84)

Based upon the bounded values, \(R\) is constructed as \(R = B_{c_{kw}^l, B_{f_{kw}^{ph-l,l}}}\). Hence,

\[\|c_{kw}^l, f_{kw}^{ph-l,l}\| < B_{c_{kw}^l, B_{f_{kw}^{ph-l,l}}}\|.
The formulations are strictly monotone in \( (c_{kw}^*, f_{kw}^{ph-*}) \) due to the concave properties of the objective function. Therefore, the uniqueness of the solution is demonstrated by theorem 1.6.

The existence and uniqueness of the solution in the land carrier collusive game can be proved by the similar procedures.

### 4.3.8 Summary of section 4.3

Section 4.3 formulated land carrier models to capture the decisions and interactions of competitive and collusive land carriers, between a port terminal and a land destination. In the competitive game, Nash Equilibrium was applied to find the optimal service charge and routing pattern of each land carrier, while compensation principle was used to decide the optimal service charge and routing set of the colluded land carriers in the collusive environment. Based upon the objective function properties and feasible region, land carrier models were formulated by a VI problem according to the different market environment.

Section 4.4 presents a multi-level hierarchical approach which models interactions among the three types of carriers in international maritime freight transportation networks.
4.4 Three-Level Models

Based upon the individual carrier problems in section 4.1 ~ section 4.3, relationships among different types of carriers are captured in three-level models. According to the different environment of carriers in Figure 3.1, four types of three-level models are formulated.

A three-hierarchical level is defined among ocean carriers, port terminal operators and land carriers. Ocean carriers are regarded as the leaders in an international shipping market via waterways. Port terminal operators are the followers of ocean carriers as well as the leaders of land carriers. Without special alliances, hierarchical relationships occur between ocean carriers and port terminal operators at the upper level and between port terminal operators and land carriers at the lower level, respectively. The former interactions are captured in the ocean path transportation cost function, the port throughput function and the feasible region of the port terminal operator objective function. The latter interactions are captured in the feasible region of the land carrier objective function.

Table 4.12 shows a three level model concerning carrier model 1 when all carriers are competitive. Carrier model 2 ~ carrier model 4 are provided Table B.1 ~ Table B.3 in Appendix B.
Table 4.11 Three-level Model (Carrier Model 1)

\[-\left(D^o(c^o*, \overline{c^o*}) \times c^o* - TC^o \right)^T (c^o - c^o*) \geq 0\]  \hspace{1cm} (4.85)

where \( \nabla f^o TC(f^o)^T (f^o - f^o*) \geq 0 \)  \hspace{1cm} (4.86)

\[\forall (c^o, f^o) \in OFR\]

s.t.

\[-\left(G^p(c^{p*}, \overline{c^{p*}}) \times c^p - SC^p \right)^T (c^p - c^{p*}) \geq 0\]  \hspace{1cm} (4.87)

where \( \nabla f^p TC(f^p)^T (f^p - f^{p*}) \geq 0 \)  \hspace{1cm} (4.88)

\[\forall (c^p, f^p) \in PFR\]

s.t.

\[-\left(D^l(c^{l*}, \overline{c^{l*}}) \times c^{l*} - TC^l \right)^T (c^l - c^{l*}) \geq 0\]  \hspace{1cm} (4.89)

where \( \nabla f^l TC(f^l)^T (f^l - f^{l*}) \geq 0 \)  \hspace{1cm} (4.90)

\[\forall (c^l, f^l) \in LFR\]
4.5 Alliances between Different Groups of Carriers

To reflect some feasible cases, this research considers two types of alliances between different groups of carriers when individual carriers within same groups are competitive. First, collusion exists between ocean carriers and port terminal operators. Port terminal operators suggest different and flexible port service charges according to ocean carriers, considering their performances and conditions, etc. and agree on discount rates by arrangements with ocean carriers. Second, ocean carriers and land carriers cooperate, having direct interactions. The transportation demands of land carriers are influenced by routing patterns of their cooperating ocean carriers.

4.5.1 Alliances between ocean carriers and port terminal operators

Port terminal operators suggest flexible port service charges according to ocean carriers and agree on discount rates. These service charges depend on the total amount of freight arriving in their port terminals. As the total amount of freight transported by ocean carrier \( o \) increases, the port service charge per a unit of freight decreases, within the port terminal capacity.

1) Transportation cost function of the ocean carrier problem

In the ocean path transportation cost function \( (TC^{ph}_{ij}^o) \) in Eq. (4.6), the last element with respect to the port service charge is changed, as shown in Eq. (4.91). The average port service charge \( (c^p) \) becomes a function \( (C^p) \) of the total amount of freight arriving in port terminal operator \( p \).
\[ TC_{ij}^{ph-o} (f_{ij}^{ph-o}) = \sum_{l \in PH-o} \sum_{o} z_{ij}^{l-o-0} \times TC_{ik-o} (f_{ik-o}) + \psi_{ij}^{ph-o-p} \times C_{ij}^{PH-o-p} (f_{ij}^{ph-o}) \times f_{ij}^{ph-o} \]

\[ \forall \ ij \in I, \ ph_{ij} \in PH_{op}, \ p \in P \]  

(4.91)

where

i) if \( \sum_{ph-o} f_{ij}^{ph-o} < cap^p \)

\[ C_{ij}^{p} \left( \sum_{ph-o} f_{ij}^{ph-o} \right) = \gamma_0 - \gamma_1 \times \frac{\sum_{ph-o} f_{ij}^{ph-o}}{cap^p} \]  

(4.92)

ii) if \( \sum_{ph-o} f_{ij}^{ph-o} \geq cap^p \)

\[ C_{ij}^{p} \left( \sum_{ph-o} f_{ij}^{ph-o} \right) = \nu_0 + \nu_1 \times \frac{\sum_{ph-o} f_{ij}^{ph-o}}{cap^p} \]  

(4.93)

In Eq. (4.91), if the ocean path \( (ph-o) \) is connected to port terminal operator \( p \), the incidence index \( \psi_{ij}^{ph-o-p} \) is 1. Otherwise, \( \psi_{ij}^{ph-o-p} \) is 0. Eq. (4.92) expresses the average port service charge function \( (C_{ij}^{p}) \) when the total amount of freight arriving in the terminal managed by port terminal operator \( p \) do not exceed its capacity. The first element shows the fixed port service charge presented by port terminal operator \( p \). The second element represents the port service charge discount due to the total amount of freight beyond the port terminal capacity. Eq. (4.93) shows the average port service charge function \( (C_{ij}^{p}) \) when the total amount freight exceeds the port capacity. In the equations, \( \nu_0, \nu_1, \nu_o \) and \( \nu_1 \) denote coefficient parameters with a positive value.
2) Profit function of the port terminal operator problem

In the port terminal operator profit function, the port throughput is determined by the port service charge depending on the total amount of freight arriving in the port terminal. Eq. (4.94) shows a mathematical form the profit function of the port terminal operator problem.

\[ U^p = G^p \left( \sum_{p \in P} \left( \sum_{j \in J} \sum_{k \in K} \sum_{w \in W} \eta^{o,l} \times f^{o,l,p,o}_{j,k} \right) \right)^2 \sum_{p \in P} \left( \sum_{j \in J} \sum_{k \in K} \sum_{w \in W} \eta^{o,l} \times f^{o,l,p,o}_{j,k} \right) + \sum_{p \in P} \left( \sum_{j \in J} \sum_{k \in K} \sum_{w \in W} \eta^{o,l} \times f^{o,l,p,o}_{j,k} \right)^2 \]

\[ \forall p \in P \quad (4.94) \]

4.5.2 Alliances between ocean carriers and land carriers

Many ocean carriers provide multimodal transportation services by combining maritime shipping with rail and truck services. In these cases, direct interactions exist between ocean carriers and land carriers. Land carrier service demands are decided by the routing pattern of their cooperating ocean carriers.

1) Feasible region of the land carrier problem

The feasible region of the land carrier objective functions when there are alliances between ocean carriers and land carriers (LFRA) is defined by linear equality and non-negativity constrains.

\[ \sum_{j} \sum_{p} \sum_{o,l} \eta^{o,l} \times f^{o,l,p,o}_{j,k} = \sum_{w} D^{l}_{kw} \quad \forall o \in O, l \in L, kw \in KW \quad (4.95) \]
\[ D_{kw}^l = \sum_{ph_{-l,l}} f_{kw}^{ph_{-l,l}} \quad \forall \ l \in L, \ kw \in KW \] (4.96)

\[ f_{kw}^{ph_{-l,l}} \geq 0 \quad \forall \ l \in L, \ kw \in KW, \ ph_{-l,l} \in PH_{-l,l} \] (4.97)

Eq. (4.95) ensures that the total flow of port terminal operator \( p \), transported by land carrier \( l \)'s cooperating ocean carrier \( o \), is equivalent the service demand of land carrier \( l \). Eq. (4.96) ensures that the service demand of land carrier \( l \) is equivalent to the sum of flows of land carrier \( l \) on all used land paths between \( k \) and \( w \). Eq. (4.97) states non-negativity of the flow.

2) Mathematical formulations for the land carrier problem

In the competitive game, each land carrier attempts to minimize its total transportation cost through a system equilibrium-like behavior for the fixed transportation demand by its cooperating ocean carrier \( o \). Table 4.12 shows a land carrier VI formulation for the competitive game.

**Table 4.12** Land Carrier VI formulation for the Competitive Game with Alliances

<table>
<thead>
<tr>
<th>Determine ( f_{kw}^{ph_{-l,l}} )*.</th>
</tr>
</thead>
</table>

\[ \sum_{kw} \sum_{ph_{-l,l}} \nabla f_{kw}^{ph_{-l,l}} \text{TC}_{kw}^{ph_{-l,l}} (f_{kw}^{ph_{-l,l}})(f_{kw}^{ph_{-l,l}} - f_{kw}^{ph_{-l,l}}) \geq 0 \] (4.98)

\[ \forall \ (f_{kw}^{ph_{-l,l}}) \in LFR \]
Eq. (4.98) represents a VI formulation for the marginal transportation cost of land carrier $l$, relying on the flow on all used paths connecting $k$ and $w$. The VI formulations are expressed as a vector form as follows:

\[ \text{Eq. (4.98)} \]

4.6 Solution Algorithms

Nagurney (1999) provides algorithms for solving VI problems, such as the projection algorithm, the relaxation (diagonalization) algorithm, decomposition algorithm, etc. The projection algorithm and the relaxation algorithm that resolve the VI problem into a sequence of sub-problems by the general iterative scheme have been widely applied. The decomposition algorithm solves the VI problem defined over a Cartesian product of sets and has efficiency for large-scale problems.

The dissertation employs the projection algorithm to solve the carrier problems in section 4.1 ~ section 4.4. The extragradient algorithm, an improved version of the projection algorithm, is utilized. A heuristic algorithm is developed to solve the three-level model, based on solution algorithms of the individual carrier models.

The projection algorithm resolves VI into a sequence of sub-problems which are equivalent to quadratic programming problems. At each step $k$, the sub-problem is solved by Eq. (4.99).

\[ \min_{x \in C} \frac{1}{2} x^T \left( x - (x^{k-1} - \rho F(x^{k-1})) \right)^T x \]  

(4.99)

where $\rho$ denotes a judiciously chosen positive steplength.
The optimal solution of the VI sub-problem at each step $k$ is expressed in Eq. (4.100).

$$x^* = P_c(x^{k-1} - \rho F(x^{k-1}))$$  \hspace{1cm} (4.100)

where $P_c$ denotes the orthogonal projection map onto the feasible region.

The approach starts from any $x^0 \in C$ and iteratively updates until the convergence point. The solution is obtained when the difference between $x^k$ and $x^{k-1}$ is smaller than an appointed number ($\sigma$), a preset tolerance, as shown in Eq. (4.101).

$$|x^k - x^{k-1}| < \sigma$$  \hspace{1cm} (4.101)

The extragradient algorithm was proposed in the first by Korpelevich (1987) to enlarge the class of the solvable problems by relaxing the strong hypotheses of the projection algorithm. The fundamental idea is to obtain the solution by the double projection formula, as written in Eq. (4.102).

$$x^k = P_c(x^{k-1} - \alpha F(P_c(x^{k-1} - \alpha F(x^{k-1}))))$$  \hspace{1cm} (4.102)

### 4.6.1 Ocean carrier problem

1) Competitive game

**Step 0:** Define the initial ocean carrier service charge $c_{ij}^{o0} = c_{ij}^{o1}, c_{ij}^{o2}, ..., c_{ij}^{om}$ and flow $f_{ij}^{o0} = f_{ij}^{ph_{-o1},o0} + f_{ij}^{ph_{-o2},o0} + ... + f_{ij}^{ph_{-on},o0} + ... + f_{ij}^{ph_{-om},o0}$, for O-D pair $ij$. Let $g$ denote the order of ocean carrier $o$ and $u$ denote the order of ocean path $ph_{-o}$. Let $v$ and $z$ denote the order of the iteration each. Set $g:=1$, $u:=1$, $v:=1$, $z:=1$. $\sigma_1$ and $\sigma_2$ are preset tolerances.
Step 1: Determine $c_{ij}^o$ for $g^{th}$ ocean carrier with $c_{ij}^{-o} = c_{ij}^{o,v-1}$.

Step 2: Determine $f_{ij}^{o_{ph-o}}$ for $u^{th}$ ocean path of $g^{th}$ ocean carrier. Then, decide the next ocean path flow by setting $u:=u+1$ until $u<n$. If $|f_{ij}^{o_{ph-o}1,z} - f_{ij}^{o_{ph-o}1,z-1}| < \sigma_1$, $|f_{ij}^{o_{ph-o}2,z} - f_{ij}^{o_{ph-o}2,z-1}| < \sigma_1$, ..., $|f_{ij}^{o_{ph-o}n,z} - f_{ij}^{o_{ph-o}n,z-1}| < \sigma_1$, stop and go to Step 3; otherwise, $z:=z+1$ and repeat Step 2.

Step 3: Solve the next ocean carrier problem via Step 1 and Step 2, by setting $g:=g+1$ until $g<m$.

Step 4: If profit differences of ocean carriers are smaller than $\sigma_2$, stop and denote solutions as $(c_{ij}^*, f_{ij}^*)$; otherwise, set $v:=v+1$ and turn to Step 1.

2) Collusive game

Step 0: Define the initial ocean carrier service charge and flow, $c_{ij}^0 = c_{ij}^1, c_{ij}^2, ..., c_{ij}^m$ and $f_{ij}^{o_{ph-o}0} = f_{ij}^{o_{ph-o}1}, f_{ij}^{o_{ph-o}2}, ..., f_{ij}^{o_{ph-o}n}$, for O-D pair $ij$. Let $v$ denote the order of iteration of ocean carrier O-D pair $ij$. Let $u$ denote the order of ocean path $o_{ph-o}$ and $z$ the order of iteration. Set $v:=1, u:=1, z:=1$. $\sigma_3$ and $\sigma_4$ are preset tolerances

Step 1: Determine $c_{ij}^*$ for O-D pair $ij$. If $|c_{ij}^v - c_{ij}^{v-1}| < \sigma_3$, denote the solution as $(c_{ij}^*)$; otherwise, $v:=v+1$ and repeat step 1.

Step 2: Determine $f_{ij}^{o_{ph-o}}$ for $u^{th}$ ocean path for ocean carrier O-D pair $ij$. Then, decide the next ocean path flow by setting $u:=u+1$ until $u<n$. If $|f_{ij}^{o_{ph-o}1,z} - f_{ij}^{o_{ph-o}1,z-1}| < \sigma_4$, $|f_{ij}^{o_{ph-o}2,z} - f_{ij}^{o_{ph-o}2,z-1}| < \sigma_4$, ..., $|f_{ij}^{o_{ph-o}n,z} - f_{ij}^{o_{ph-o}n,z-1}| < \sigma_4$, denote the solution as $(f_{ij}^*)$;
otherwise, \( z := z + 1 \) and repeat step 2.

### 4.6.2 Port terminal operator problem

**Step 0:** Define the initial port terminal operator service charge \( c^{p0} = c^{p1}, c^{p2}, \ldots, c^{pm} \) and flow \( f^{p0} = f^{p1} + f^{p2} + \ldots + f^{pm} \). Let \( g \) denote the order of port terminal operator \( p \) and \( u \) denote the order of port path \( ph_p \). Let \( v \) and \( z \) denote the order of the iteration each. Set \( g := 1, u := 1, v := 1, z := 1 \). \( \sigma_5 \) and \( \sigma_6 \) are preset tolerances.

**Step 1:** Determine \( c^p \) for \( g \)th port terminal operator with \( c^{-p} = c^{-p,v-1} \).

**Step 2:** Determine \( f^{ph_p} \) for \( u \)th port path of \( g \)th port terminal operator. Then, decide the next port path flow by setting \( u := u + 1 \) until \( u < n \). If \( |f^{ph_1} - f^{ph_1,z-1}| < \sigma_5 \), \( |f^{ph_2} - f^{ph_2,z-1}| < \sigma_5 \), \( \ldots \), \( |f^{ph_{n-1}} - f^{ph_{n-1},z-1}| < \sigma_5 \), stop and go to Step 3; otherwise, \( z := z + 1 \) and repeat Step 2.

**Step 3:** Solve the next port terminal operator problem via Step 1 and Step 2, by setting \( g := g + 1 \) until \( g < m \).

**Step 4:** If profit differences of port terminal operators are smaller than \( \sigma_6 \), stop and denote solutions as \((c^*, f^*)\); otherwise, set \( v := v + 1 \) and turn to Step 1.

### 4.6.3 Land carrier problem

1) Competitive game

**Step 0:** Define the initial land carrier service charge \( c^{kw10} = c^{kw11}, c^{kw12}, \ldots, c^{kw1m} \) and flow
\( f_{kw}^{o0} = f_{kw}^{ph_{11},l0} + f_{kw}^{ph_{12},l0} + \ldots + f_{kw}^{ph_{ln},l0} + \ldots + f_{kw}^{ph_{ln},ln} \), for O-D pair \( kw \). Let \( g \) denote the order of land carrier \( l \) and \( u \) denote the order of land path \( ph_{j} \). Let \( v \) and \( z \) denote the order of the iteration each. Set \( g:=1, u:=1, v:=1, z:=1 \). \( \sigma_7 \) and \( \sigma_8 \) are preset tolerances.

**Step 1:** Determine \( c_{kw}^{l} \) for \( g \)th land carrier with \( c_{kw}^{l} = c_{kw}^{l,v-1} \).

**Step 2:** Determine \( f_{kw}^{ph_{1},l} \) for \( u \)th land path of \( g \)th land carrier. Then, decide the next land path flow by setting \( u:=u+1 \) until \( u<n \). If \( |f_{kw}^{ph_{11},l1} - f_{kw}^{ph_{11},z-1}| < \sigma_7 \), \( |f_{kw}^{ph_{12},l2} - f_{kw}^{ph_{12},z-1}| < \sigma_7 \), \( \ldots \), \( |f_{kw}^{ph_{ln},ln} - f_{kw}^{ph_{ln},z-1}| < \sigma_7 \), stop and go to Step 3; otherwise, \( z:=z+1 \) and repeat Step 2.

**Step 3:** Solve the next land carrier problem via Step 1 and Step 2, by setting \( g:=g+1 \) until \( g<m \).

**Step 4:** If profit differences of land carriers are smaller than \( \sigma_8 \), stop and denote solutions as \((c_{kw}^{*}, f_{kw}^{*})\); otherwise, set \( v:= v+1 \) and turn to Step 1.

2) Collusive game

**Step 0:** Define the initial land carrier service charge and flow, \( c_{kw}^{0} = c_{kw}^{1}, c_{kw}^{2}, \ldots, c_{kw}^{m} \) and \( f_{kw}^{ph_{1},l0} = f_{kw}^{ph_{11},l1}, f_{kw}^{ph_{12},l2}, \ldots, f_{kw}^{ph_{ln},ln} \), for O-D pair \( kw \). Let \( v \) denote the order of iteration of land carrier O-D pair \( kw \). Let \( u \) denote the order of land path \( ph_{j} \) and \( z \) the order of iteration. Set \( v:=1, u:=1, z:=1 \). \( \sigma_g \) and \( \sigma_{i0} \) are preset tolerances.

**Step 1:** Determine \( c_{kw}^{*} \) for land carrier O-D pair \( kw \). If \( |c_{kw}^{v} - c_{kw}^{v-1}| < \sigma_g \), denote the solution as \((c_{kw}^{*}, f_{kw}^{*})\); otherwise, \( v:=v+1 \) and repeat step 1.

**Step 2:** Determine \( f_{kw}^{ph_{1},l} \) for \( u \)th land path for land carrier O-D pair \( kw \). Then, decide the
next land path flow by setting \( u := u + 1 \) until \( u < n \). If \( \left| f_{kw}^{ph_{-1}, z} - f_{kw}^{ph_{-1}, z-1} \right| < \sigma_{10} \),
\[
\left| f_{kw}^{ph_{-2}, z} - f_{kw}^{ph_{-2}, z-1} \right| < \sigma_{10}, \ldots, \left| f_{kw}^{ph_{-n}, z} - f_{kw}^{ph_{-n}, z-1} \right| < \sigma_{10},
\]
denote the solution as \( (f_{kw}^{*}) \); otherwise, \( z := z + 1 \) and repeat step 2.

### 4.6.4 Heuristic algorithm

First, port terminal operators determine the port service charge and pattern, based on the reactions of land carriers. Port terminal operators examine the reactions of land carriers for every feasible scenario of them. From all these reactions, port terminal operators decide the best reaction which gives them the maximum profit. After this step, ocean carriers determine the service charge and the routing pattern, based on the reactions of port terminal operators. Ocean carriers examine the reactions of port terminal operators for feasible scenarios and finally choose the best reaction. Ocean carriers apply the scenario which gives them the maximum profit.
CHAPTER 5

SHIPPER - CARRIER PROBLEM

Chapter 5 presents a bi-level modeling approach which captures interactions between shippers and carriers. Different types of carriers such as ocean carriers, port terminal operators and land carriers provide transportation services in international maritime freight transportation networks. The carriers determine service charges and delivery routes at different parts of the multimodal network through competitions/collusions and interactions among them. Shippers are users of the transportation services. Shippers choose a sequence of carriers based upon the carriers’ decisions for international maritime shipping.

Hence, hierarchical relationships occur between shippers and carriers. These relationships are captured in bi-level models when the carrier model is the upper level problem and the shipper model is the lower level problem. First of all, a shipper model is formulated to find the production, consumption and distribution of goods using spatial price equilibrium. Afterward, bi-level models are developed according the different carrier environment.

Section 5.1 describes equilibrium principles of spatial price equilibrium. Section 5.2 presents assumptions and mathematical forms of the cost functions used for the spatial price equilibrium problem. Section 5.3 defines the feasible region of the shipper problem. Section 5.4 presents a mathematical formulation by a VI problem and section 5.5 proves the existence and uniqueness of the solution. Section 5.6 develops an algorithm to solve the shipper model. Section 5.7 formulates bi-level models for the
shipper-carrier problem and develops a heuristic algorithm.

5.1 Equilibrium Conditions of Spatial Price Equilibrium

Shippers make decisions on the production, shipment, and consumption pattern using spatial price equilibrium, satisfying the following two principles.

1) If the commodity price (supply cost) in the origin market $x$ plus the average transaction cost on path $ph_s$ connecting markets $x$ and $y$ is equivalent to the commodity price (demand cost) in the destination market $y$, there will be a commodity flow on the path.

$$IS_x^* + ATT_{xy}^{ph_s} = ID_y^* \quad f_{xy}^{ph_s} > 0 \quad \forall x \in X, y \in Y, xy \in XY, ph_s \in PH_s \quad (5.1)$$

2) If the commodity price in the origin market $x$ plus the average transaction cost on path $ph_s$ connecting markets $x$ and $y$ is greater than the commodity price in the destination market $y$, there will be no commodity flow on path.

$$IS_x^* + ATT_{xy}^{ph_s} > ID_y^* \quad f_{xy}^{ph_s} = 0 \quad \forall x \in X, y \in Y, xy \in XY, ph_s \in PH_s \quad (5.2)$$

The two principles are rewritten in Eq. (5.3).

$$IS_x^* + ATT_{xy}^{ph_s} - ID_y^* \times f_{xy}^{ph_s} = 0 \quad \forall x \in X, y \in Y, xy \in XY, ph_s \in PH_s \quad (5.3)$$
5.2 Cost Functions

The spatial price equilibrium problem involves the inverse supply function, inverse demand function and transaction cost function. Assumptions and mathematical forms of the functions are described as follows.

5.2.1 Inverse supply and demand functions

The inverse supply function \( IS \) and the inverse demand function \( ID \) are derived from the general supply and demand function as invertible forms of the functions. They find supply and demand costs relying on the amount of supply at the origin market \( x \) \( (s_x) \) and demand at the destination market \( y \) \( (d_y) \). The mathematical forms are shown in Eq. (5.4) and Eq. (5.5).

\[
\text{IS}(s_x) = \chi_o + \chi_1 s_x \quad \forall x \in X \tag{5.4}
\]

\[
\text{ID}(d_y) = \rho_0 - \rho_1 d_y \quad \forall y \in Y \tag{5.5}
\]

Eq. (5.4) expresses the inverse supply function strictly monotone increasing in the supply. In Eq. (5.4), \( \chi_o \) and \( \chi_1 \) denote coefficient parameters with a positive value. Eq. (5.5) represents the inverse demand function strictly monotone decreasing in the demand. In Eq. (5.5), \( \rho_0 \) and \( \rho_1 \) denote coefficient parameters with a positive value.
5.1.2 Transaction cost function

The transaction cost function comprises the transportation cost and tariff. The average shipper path transaction cost is the expense to deliver a unit of freight via the shipper path \((ph_s)\) connecting \(x\) and \(y\). The average shipper link transaction cost is the expense to deliver a unit of freight via the shipper link \((lk_s)\). The average shipper path transaction cost function \((ATC_{ph_s}^{xy})\) is a linear combination of the following two attributes: a) the sum of the average shipper link transaction cost functions \((ATC_{lk_s}^{xy})\) if the shipper link \((lk_s)\) is on the ocean path \((ph_s)\) and b) tariff \((t_c)\) according to the commodity type. A mathematical form of the average shipper path transaction cost function is shown in Eq. (5.6).

\[
ATC_{xy}^{ph_s} (f_{xy}^{ph_s}) = \sum_{lk_s} \frac{f_{xy}^{lk_s, ph_s}}{\sum_{xy}^{PH_s, PH_s}} ATC_{lk_s}^{xy} (f_{xy}^{lk_s}) + t_c \times f_{xy}^{ph_s}
\]

\[\forall xy \in XY, ph_s \in PH_s, c \in C \quad (5.6)\]

If the shipper link \((lk_s)\) is on the shipper path \((ph_s)\) connecting \(x\) and \(y\), \(\sum_{xy}^{lk_s, ph_s}\) is 1. Otherwise, 0.

The average shipper link transaction cost function \((ATC_{lk_s}^{xy})\) comprises: a) the shipper link transportation service charge \((c_{lk_s}^{xy})\) and b) the average shipper link travel time function \((ATT_{lk_s}^{xy})\). Shippers consider three different transportation service charges, ocean carrier service charge \((c^o)\), port service charge \((c^p)\) and land carrier service charge \((c^l)\), on the shipper link. For unit consistency, the travel time is changed to money using the value of time \((vot')\). The mathematical form of the link transaction cost function is
shown in Eq. (5.7).

\[
ATC^{lk_s} (f^{lk_s}) = c^{lk_s} + \nu T \times ATT^{lk_s} (f^{lk_s}) \quad \forall \, lk_s \in LK_s
\]  

(5.7)

where

\[
c^{lk_s} = \xi^{lk_s,o} \times (c^o)^T + \xi^{lk_s,p} \times (c^p)^T + \xi^{lk_s,l} \times (c^l)^T \quad \forall \, lk_s \in LK_s, \, o \in O, \, p \in P, \, l \in L
\]  

(5.8)

\[
ATT^{lk_s} (f^{lk_s}) = att^{lk_s} \left( 1 + \kappa_0 \left( \frac{f^{lk_s}}{\text{cap}^{lk_s}} \right)^{\kappa_1} \right) \quad \forall \, lk_s \in LK_s
\]  

(5.9)

Eq. (5.8) expresses link transportation service charges presented by the three types of carriers on the shipper link. Different transportation service charges are earmarked according to the carriers and O-D pairs. If the shipper link corresponds to the ocean carrier network, the incidence index \( \xi^{lk_s,o} \) is 1. Otherwise, 0. If the shipper link corresponds to the port sub-network, the incidence index \( \xi^{lk_s,p} \) is 1. If the shipper link corresponds to the land carrier network, the incidence index \( \xi^{lk_s,l} \) is 1. Eq. (5.9) represents the average shipper link travel time function. The first element represents the average travel time on the shipper link and the second element expresses the average travel time due to the transportation demand beyond the link capacity. In Eq. (5.9), \( \kappa_0 \) and \( \kappa_1 \) denotes coefficient parameters with a positive value.

The average shipper link travel time function is assumed to be continuous and strictly monotone increasing in the flow, as shown in Eq. (5.10).

\[
\frac{\partial ATT (f^{lk_s})}{\partial f^{lk_o}} = att^{lk_o} \times \kappa_0 \times \kappa_1 \times \left( \frac{f^{lk_o}}{\text{cap}^{lk_o}} \right)^{\kappa_1-1} > 0 \quad \forall \, lk_s \in LK_s
\]  

(5.10)
5.3 Feasible Region

The feasible region of the shipper problem \((SFR)\) is defined by linear equality and non-negativity constrains as follows:

\[
\sum_{s} s_{x} = \sum_{y} d_{y} = \sum_{xy} \sum_{ph_{s}} f_{xy}^{ph_{s}}
\] (5.11)

\[
Sd_{y} < d_{y} < Bd_{y} \quad \forall y \in Y
\] (5.12)

\[
d_{y}, f_{xy}^{ph_{s}} > 0 \quad \forall y \in Y, xy \in XY, ph_{s} \in PH_{s}
\] (5.13)

Eq. (5.11) ensures that the total supply in origin market \(x\) is equivalent to the total demand in destination market \(y\) and the total flow between these two markets. Eq. (5.12) ensures that the demand ranges from a small number to a large number. Eq. (5.13) states non-negativity of the demand and the flow.

5.4 Mathematical Formulation

The spatial price equilibrium problem is formulated by a VI problem utilizing theorem 3.1 by Nagurney (1999).

\[K\] denote the closed convex set where \(K = \{(s, f, d)\}\).

**Theorem 3.1**

A commodity production, shipment, and consumption pattern \((s^{*}, f^{*}, d^{*}) \in K\) is in equilibrium if and only if it satisfies the VI problem:
\[ IS(s^*) \cdot (s - s^*) + ATC(f^*) \cdot (f - f^*) - ID(d^*) \cdot (d - d^*) \geq 0 \quad \forall (s, f, d) \in K \] (5.14)

**Proof**  For a fixed market pair \( xy \),

\[ (IS_x(s^*) + ATC_{xy}^{ph-s}(f^*) - ID_y(d^*)) \times (f_{xy}^{ph-s} - f_{xy}^{ph-s^*}) \geq 0 \] (5.15)

i) If \( f_{xy}^{ph-s^*} > 0 \) for \( \forall \ f_{xy}^{ph-s} > 0 \),

\[ (IS_x(s^*) + ATC_{xy}^{ph-s}(f^*) - ID_y(d^*)) \times (f_{xy}^{ph-s} - f_{xy}^{ph-s^*}) = 0 \text{ because } IS_x(s^*) + ATC_{xy}^{ph-s}(f^*) - ID_y(d^*) = 0 \text{ by the first principle in Eq. (5.1). Hence, it satisfies } \]

\[ (IS_x(s^*) + ATC_{xy}^{ph-s}(f^*) - ID_y(d^*)) \times (f_{xy}^{ph-s} - f_{xy}^{ph-s^*}) \geq 0 . \]

ii) If \( f_{xy}^{ph-s^*} = 0 \) for \( \forall \ f_{xy}^{ph-s} > 0 \),

\[ (IS_x(s^*) + ATC_{xy}^{ph-s}(f^*) - ID_y(d^*)) \times f_{xy}^{ph-s} \geq 0 \text{ by the second principle in Eq. (5.2), therefore it satisfies } (IS_x(s^*) + ATC_{xy}^{ph-s}(f^*) - ID_y(d^*)) \times (f_{xy}^{ph-s} - f_{xy}^{ph-s^*}) \geq 0 . \]

By summing all market pairs, one can get Eq. (5.16).

\[ \sum_x \sum_y \sum_{ph-s} (IS_x(s^*) + ATC_{xy}^{ph-s}(f^*) - ID_y(d^*)) \times (f_{xy}^{ph-s} - f_{xy}^{ph-s^*}) \geq 0 \] (5.16)

By substituting conservation constraints in Eq. (5.17) and Eq. (5.18),

\[ s_x = \sum_x \sum_y f_{xy}^{ph-s} \] (5.17)

\[ d_y = \sum_x \sum_{ph-s} f_{xy}^{ph-s} \] (5.18)

Eq. (5.16) is transformed into:
Table 5.1 shows a vector form of VI formulation for the shipper’s spatial price equilibrium problem.

**Table 5.1 Shipper VI Formulation (Vector Form)**

| \( IS(s^*) \times (s_s - s_s) + \sum_{x} \sum_{y} \sum_{p_{h,s}} TC_{xy}^{ph,s}(f^*) \times (f - f^*) - \sum_{y} ID_y(d^*) \times (d_y - d_y) \) | (5.19) |

5.5 Existence and Uniqueness of the Solution

The existence and uniqueness of the solution are guaranteed through Nagurney (1999)’s theorems.

**Theorem 3.2**

\( F(x) \) is monotone, strictly monotone, or strongly monotone if and only if \( IS(s) \), \( ATC(f) \) and \( ID(d) \) are each monotone, strictly monotone, or strongly monotone in \( s, f, d \), respectively.

Eq. (5.20) in Table 5.1 is strictly monotone since the three functions - supply cost function, transaction cost function and demand cost function - are assumed to be continuous and strictly monotone curves, respectively. Hence, at least a solution exists by the theorem 1.4.
A solution is admitted when \( \|s^*, f^*, d^*\| < R \) is satisfied by theorem 1.5. With the bounded demand \( (d_x) \) in Eq. (5.12), the supply \( (s_x) \) and the path flow \( (f_{xy}^{ph,x}) \) are also bounded due to the flow conservation constraint in Eq. (5.11), as shown in Eq. (5.21) and Eq. (5.22).

\[
S_s < s_x < B_s, \quad \forall x \in X \tag{5.21}
\]

\[
S_{f_{xy}}^{ph,x} < f_{xy}^{ph,x} < B_{f_{xy}}^{ph,x}, \quad \forall x, y \in X, ph_x \in PH_x \tag{5.22}
\]

Based on the bounded values, \( R \) is constructed as \( R = \|B_s, B_{f_{xy}}^{ph,x}, Bd_y\| \) therefore \( \|s^*, f^*, d^*\| < \|B_s, B_{f_{xy}}^{ph,x}, Bd\| \).

Eq. (5.20) in Table 5.1 is strictly monotone in \((s, f, d)\), therefore the unique of solution is demonstrated by the theorem 1.6.

### 5.6 Solution Algorithm

The algorithm for solving the shipper model has the following steps.

**Step 0:** Define \( x \) and \( F(x) \).

\[
x = \begin{pmatrix} f^T \\ d^T \\ s^T \end{pmatrix} \quad \text{and} \quad F(x) = \begin{pmatrix} ATC(f)^T \\ -ID(d)^T \\ IS(s)^T \end{pmatrix}
\]

**Step 1:** Define the initial feasible solution \( x^o = (f^o, d^o, s^o) \). Let \( v \) denote the order of iteration. Set \( v := 1 \). \( \sigma_{11} \) is preset tolerances.
Step 2: Determine \((f^v, d^v, s^v)\) by solving VI sub-problems.

Step 3: If \(\left| TC_{xy}^{p^h_s,x,v} - TC_{xy}^{p^h_s,x,v-1} \right| < \sigma_{11}, \left| D^v - D^{v-1} \right| < \sigma_{11}, \left| S^v - S^{v-1} \right| < \sigma_{11}\), stop and denote the solutions as \((f^*, d^*, s^*)\); otherwise, set \(v:=v+1\) and go to step 2.

5.7 Bi-level Models and Heuristic Algorithm

Carriers make decisions on the service charge and routing pattern through competitions/collusions and interactions, with the information on shipper flow. Shippers determine the production, shipment, and consumption pattern, considering ocean/land transportation or port service charges and carrier link flows.

The shipper link is a path or a sequence of links between an O-D pair of carriers. Therefore, the capacity of the shipper link \((lk_s)\) is derived from the capacity of the carrier link. Ocean link \((lk_o)\) flows between \(i\) and \(j\), port link \((lk_p)\) flows between \(j\) and \(k\) and land link \((lk_l)\) flows between \(k\) and \(w\) define the capacity of the shipper link.

According to the different market environment of shippers and carriers, four types of bi-level models are formulated, as shown in Table 5.2.

<table>
<thead>
<tr>
<th>Bi-level model</th>
<th>Shippers</th>
<th>Ocean carriers</th>
<th>Port terminal operators</th>
<th>Land carriers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bi-level Model 1</td>
<td>Competitive</td>
<td>Competitive</td>
<td>Competitive</td>
<td>Competitive</td>
</tr>
<tr>
<td>Bi-level Model 2</td>
<td>Competitive</td>
<td>Competitive</td>
<td>Competitive</td>
<td>Collusive</td>
</tr>
<tr>
<td>Bi-level Model 3</td>
<td>Competitive</td>
<td>Collusive</td>
<td>Competitive</td>
<td>Competitive</td>
</tr>
<tr>
<td>Bi-level Model 4</td>
<td>Competitive</td>
<td>Collusive</td>
<td>Competitive</td>
<td>Collusive</td>
</tr>
</tbody>
</table>
Table 5.3 shows a bi-level model concerning bi-level model 1 when all carriers and shippers are competitive. Bi-level model 2 ~ bi-level model 4 are provided Table B.4 ~ Table B.6 in Appendix B.

A heuristic algorithm is developed to solve the bi-level model based upon the solution algorithms for the carrier problem in section 4.6 and the shipper problem in section 5.6. Carriers make pricing and routing decisions based on the reactions of shippers. Carriers examine the reactions of shippers for every feasible scenario of them. From all these reactions, carriers decide the best reaction which gives them the maximum profit.
### Table 5.3 Bi-level model 1

**Upper level: Carrier problem**

\[
-\left(D^o(c^o*, \overline{c^o*}) \times c^o* - TC^o\right)^T (c^o - c^o*) \geq 0 \tag{5.23}
\]

where \( \nabla_f TC(f^o)^T (f^o - f^o*) \geq 0 \) \tag{5.24}

\( \forall (c^o, f^o) \in OFR \)

s.t.

\[
-\left(G^p(c^p*, \overline{c^p*}) \times c^p - SC^p\right)^T (c^p - c^p*) \geq 0 \tag{5.25}
\]

where \( \nabla_f SC(f^p)^T (f^p - f^p*) \geq 0 \) \tag{5.26}

\( \forall (c^p, f^p) \in PFR \)

s.t.

\[
-\left(D^l(c^l*, \overline{c^l*}) \times c^l* - TC^l\right)^T (c^l - c^l*) \geq 0 \tag{5.27}
\]

where \( \nabla_f TC(f^l)^T (f^l - f^l*) \geq 0 \) \tag{5.28}

\( \forall (c^l, f^l) \in LFR \)

**Lower level: Shipper problem**

\[
IS(s^*)^T (s - s*) + ATC(f^*)^T (f - f*) - ID(d^*)^T (d - d*) \geq 0
\]

\( \forall (s, f, d) \in SFR \) \tag{5.29}
CHAPTER 6

CASE STUDY

Chapter 6 presents a case study to show the application and capability of the multi-level hierarchical models which capture interactions of ocean carriers, port terminal operators, land carrier and shippers in the competitive and collusive environment. The developed models are solved by the mathematical programming software package MATLAB, version R2008b. All numerical examples are executed on a PC with Pentium IV 2.00 GHz CPU with 3.00 GB of RAM.

The chapter is organized as follow. Section 6.1 describes structures and attributes of international maritime freight networks in the example. Section 6.2 present results of the developed carrier and shipper models, respectively, and verifies equilibrium conditions. Section 6.3 discusses the computational efficiency of the solution algorithm.

6.1 Structures and Attributes of Networks

The shipper network in the example is depicted in Figure 6.1. The shipper network consists of 8 nodes \( (n1 \sim n8) \) including an origin market, a destination market and 6 transshipment points and 9 links \( (lk_s1 \sim lk_s9) \) containing 3 ocean links, 3 port links and 3 land links.
The carrier network in the example is shown in Figure 6.2. The carrier network comprises 26 nodes \((i_1, j_1, j_2, j_3, k_1, k_2, k_3, x_1 \sim x_{18}, w_1)\), 14 ocean links \((l_{k,o1} \sim l_{k,o14})\), 18 port links \((l_{k,p1} \sim l_{k,p18})\) and 13 land links \((l_{k,l1} \sim l_{k,l13})\). Ocean carriers has an O-D pair including 3 alternative sets of \((i_1, j_1), (i_1, j_2)\) and \((i_1, j_3)\), each port terminal has an O-D pair of \((j_1, k_1), (j_2, k_2)\) and \((j_3, k_3)\), and land carriers has 3 O-D pairs of \((k_1, w_1), (k_2, w_1)\) and \((k_3, w_1)\), respectively.
Figure 6.2 Carrier Network in the Example

Port sub-networks of the three arrival port terminals are depicted in Figure 6.3.
6.2 Results

6.2.1 Carrier problem

The unit of freight is expressed by 20 TEUs (twenty-foot equivalent units). The total transportation demand is assumed to be 1,230 TEUs during a time period. 5 ocean carrier companies make pricing and rouging decisions to maximize their individual profits or the total profit. They transport freight from a port terminal to one of the three alternative port terminals, via 11 waterways. The ocean carrier service charge ranges from $800 to $950
per a unit of freight. The value of time \( (\text{vot}^o) \) is set at $5/day. Table C.1 in Appendix C shows parameters in the ocean carrier demand function and Table C.2 illustrates the transportation capacity of ocean carrier \( o \). Table C.3 and table C.4 exhibit parameters in the average link operating cost and travel (shift) time functions, based upon the behavior of ocean carrier \( o \). Table C.5 and Table C.6 show parameters in the average link operating cost and travel (shift) time functions of the colluded ocean carriers.

Port terminal operators determine port service charges and processes to maximize their individual profits. The fixed port service charge ranging from $300 to $400 per a unit of freight is considered in each port terminal. The value of time \( (\text{vot}^p) \) is set at $2/hour. Table C.7 and Table C.8 show parameters in the average port link operating cost and service time functions.

Land carriers transport freight from port terminals to a land destination for 3 O-D pairs \((k1, w1), (k2, w1)\) and \((k3, w1)\). 2, 4 and 6 land carrier companies decide transportation service charges and delivery routings to maximize their individual profits for the O-D pairs. The land carrier service charge ranges from $250 to $320 per a unit of freight. The value of time \( (\text{vot}^l) \) is set at $2/hour. Table C.9 shows parameters in the land carrier demand function and Tables C.10 illustrates the transportation capacity of land carrier \( l \). Table C.11 and Table C.12 exhibit parameters in the average link operating cost and travel (shift) time functions, based upon the behavior of land carrier \( l \). Table C.13 and Table C.14 show parameters in the average link operating cost and travel (shift) time functions of the colluded land carriers.

1) Ocean carrier problem

Ocean carriers find the optimal service charge and routing set in the competitive and
collusive environment. Table 6.1 compares the ocean carrier revenue, transportation cost and profit under the competitive and collusive games. Ocean carriers 1~5 in the competitive game have profits ranging from $68,564 to $100,582. The collusive ocean carriers earn $8,474 more, compared with the sum of individual profits of competitive ocean carriers.

**Table 6.1** Ocean Carrier Revenue, Transportation Cost and Profit under Competitive and Collusive Games

<table>
<thead>
<tr>
<th>Game</th>
<th>Ocean carrier ( o )</th>
<th>Revenue ($)</th>
<th>Transportation cost ($)</th>
<th>Profit ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Competitive</td>
<td>Ocean carrier 1</td>
<td>285,000</td>
<td>184,418</td>
<td>100,582</td>
</tr>
<tr>
<td></td>
<td>Ocean carrier 2</td>
<td>256,500</td>
<td>165,319</td>
<td>91,181</td>
</tr>
<tr>
<td></td>
<td>Ocean carrier 3</td>
<td>228,000</td>
<td>146,399</td>
<td>81,601</td>
</tr>
<tr>
<td></td>
<td>Ocean carrier 4</td>
<td>209,000</td>
<td>133,886</td>
<td>75,114</td>
</tr>
<tr>
<td></td>
<td>Ocean carrier 5</td>
<td>190,000</td>
<td>121,436</td>
<td>68,564</td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>1,168,500</td>
<td>751,459</td>
<td>417,041</td>
</tr>
<tr>
<td>Collusive</td>
<td>1,168,500</td>
<td>742,996</td>
<td></td>
<td>425,504</td>
</tr>
</tbody>
</table>

Ocean carriers aim to minimize their transportation costs through a system equilibrium-like behavior each. Hence, the marginal transportation cost to deliver a unit of freight via any ocean path is the equalized lowest. Table 6.2 illustrates the flow and the marginal and total transportation costs of ocean carrier 1 under the competitive game, as a sample. The marginal transportation cost is decided at $612 on the ocean link. Data for the other four competitive ocean carriers and the collusive ocean carriers are shown in Table D.1 and Table D.2.
Table 6.2 Flow and Transportation Cost of Ocean carrier 1

<table>
<thead>
<tr>
<th>Path</th>
<th>Link</th>
<th>Flow (TEUs)</th>
<th>Marginal cost ($)</th>
<th>Total cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ph_{o1,o1}$</td>
<td>$lk_{o1,o1} \rightarrow lk_{o12,o1}$</td>
<td>30</td>
<td>612</td>
<td>18,446</td>
</tr>
<tr>
<td>$ph_{o2,o1}$</td>
<td>$lk_{o2,o1} \rightarrow lk_{o12,o1}$</td>
<td>13</td>
<td>612</td>
<td>8,259</td>
</tr>
<tr>
<td>$ph_{o3,o1}$</td>
<td>$lk_{o3,o1} \rightarrow lk_{o12,o1}$</td>
<td>40</td>
<td>612</td>
<td>24,425</td>
</tr>
<tr>
<td>$ph_{o4,o1}$</td>
<td>$lk_{o4,o1} \rightarrow lk_{o12,o1}$</td>
<td>24</td>
<td>612</td>
<td>14,848</td>
</tr>
<tr>
<td>$ph_{o5,o1}$</td>
<td>$lk_{o5,o1} \rightarrow lk_{o13,o1}$</td>
<td>15</td>
<td>612</td>
<td>9,319</td>
</tr>
<tr>
<td>$ph_{o6,o1}$</td>
<td>$lk_{o6,o1} \rightarrow lk_{o13,o1}$</td>
<td>42</td>
<td>612</td>
<td>25,358</td>
</tr>
<tr>
<td>$ph_{o7,o1}$</td>
<td>$lk_{o7,o1} \rightarrow lk_{o13,o1}$</td>
<td>19</td>
<td>612</td>
<td>11,714</td>
</tr>
<tr>
<td>$ph_{o8,o1}$</td>
<td>$lk_{o8,o1} \rightarrow lk_{o14,o1}$</td>
<td>42</td>
<td>612</td>
<td>25,633</td>
</tr>
<tr>
<td>$ph_{o9,o1}$</td>
<td>$lk_{o9,o1} \rightarrow lk_{o14,o1}$</td>
<td>28</td>
<td>612</td>
<td>17,270</td>
</tr>
<tr>
<td>$ph_{o10,o1}$</td>
<td>$lk_{o10,o1} \rightarrow lk_{o14,o1}$</td>
<td>25</td>
<td>612</td>
<td>15,472</td>
</tr>
<tr>
<td>$ph_{o11,o1}$</td>
<td>$lk_{o11,o1} \rightarrow lk_{o14,o1}$</td>
<td>22</td>
<td>612</td>
<td>13,674</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>300</td>
<td>-</td>
<td>184,418</td>
</tr>
</tbody>
</table>

2) Port terminal operator problem

Port terminal operators find the optimal service charge and routing set in the competitive environment. Port throughputs are determined by ocean carrier routings that consider the port service charge as a part of the ocean transportation cost. Table 6.3 compares the port terminal operator revenue, transportation cost and profit under the competitive game according to the different ocean carrier games. Port terminal operators have profits ranging from 26,518$ to 34,613$ if ocean carriers are competitive. On the other hand, the profits become $22,707~$37,618 if ocean carriers form an alliance. Port terminal operators 1 and 2 are better off for the ocean carrier competitive game, while port terminal operator 3 shows more savings for the ocean carrier collusive game.
Table 6.3 Port Terminal Operator Revenue, Service Cost and Profit under the Competitive Game

<table>
<thead>
<tr>
<th>Ocean carrier game</th>
<th>Port terminal operator 1 Revenue ($)</th>
<th>Service cost ($)</th>
<th>Profit ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Competitive</td>
<td>132,600</td>
<td>98,468</td>
<td>34,132</td>
</tr>
<tr>
<td></td>
<td>96,000</td>
<td>69,482</td>
<td>26,518</td>
</tr>
<tr>
<td></td>
<td>140,400</td>
<td>105,787</td>
<td>34,613</td>
</tr>
<tr>
<td>Collusive</td>
<td>128,400</td>
<td>94,680</td>
<td>33,720</td>
</tr>
<tr>
<td></td>
<td>78,000</td>
<td>55,293</td>
<td>22,707</td>
</tr>
<tr>
<td></td>
<td>162,600</td>
<td>124,982</td>
<td>37,618</td>
</tr>
</tbody>
</table>

With the port demands, port terminal operators attempt to minimize port service costs in their port terminals. Therefore, the marginal port service cost to deal with a unit of freight via any port path in a port terminal is the equalized lowest. Table 6.4 illustrates the flow and the marginal and total service costs of port terminal operator 1. The flow of port terminal operator 1 is determined when the marginal port service cost is $240 or $238. Data for other two port terminal operators are shown in Table D.2 and Table D.3.

Table 6.4 Flow and Service Cost of Port Terminal Operator 1

<table>
<thead>
<tr>
<th>Ocean carrier game</th>
<th>Path</th>
<th>Link</th>
<th>Throughput (TEUs)</th>
<th>Marginal cost ($)</th>
<th>Total coat ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Competitive</td>
<td>$p_{h,p1}$</td>
<td>$l_{k,p1} \rightarrow l_{k,p2}$</td>
<td>219</td>
<td>240</td>
<td>48,148</td>
</tr>
<tr>
<td></td>
<td>$p_{h,p2}$</td>
<td>$l_{k,p1} \rightarrow l_{k,p4} \rightarrow l_{k,p6}$</td>
<td>94</td>
<td>240</td>
<td>22,235</td>
</tr>
<tr>
<td></td>
<td>$p_{h,p3}$</td>
<td>$l_{k,p3} \rightarrow l_{k,p4} \rightarrow l_{k,p6}$</td>
<td>129</td>
<td>240</td>
<td>28,084</td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td></td>
<td>442</td>
<td>-</td>
<td>98,468</td>
</tr>
<tr>
<td>Collusive</td>
<td>$p_{h,p1}$</td>
<td>$l_{k,p1} \rightarrow l_{k,p2}$</td>
<td>211</td>
<td>238</td>
<td>46,103</td>
</tr>
<tr>
<td></td>
<td>$p_{h,p2}$</td>
<td>$l_{k,p1} \rightarrow l_{k,p4} \rightarrow l_{k,p6}$</td>
<td>92</td>
<td>238</td>
<td>21,486</td>
</tr>
<tr>
<td></td>
<td>$p_{h,p3}$</td>
<td>$l_{k,p3} \rightarrow l_{k,p4} \rightarrow l_{k,p6}$</td>
<td>125</td>
<td>238</td>
<td>27,091</td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td></td>
<td>428</td>
<td>-</td>
<td>94,680</td>
</tr>
</tbody>
</table>
3) Land carrier problem

The land carrier demands for O-D pairs of \((k_1, w_1)\), \((k_2, w_1)\) and \((k_3, w_1)\) are determined from the port throughputs of port terminal operator 1, 2, and 3, respectively. Table 6.5 compares the land carrier revenue, transportation cost and profit under competitive and collusive games, according to the competitive games of ocean carriers and port terminal operators. Competitive land carriers earn $7,020~$17,444 for each O-D pair. The total profit of the colluded land carriers is determined at $35,647~$47,495. Therefore, land carriers increase savings through cooperation.

**Table 6.5 Land Carrier Revenue, Transportation Cost and Profit under Competitive and Collusive Games**

<table>
<thead>
<tr>
<th>Ocean carrier Game</th>
<th>Port terminal operator game</th>
<th>Land carrier game</th>
<th>Land carrier O-D</th>
<th>Revenue ($)</th>
<th>Transportation cost ($)</th>
<th>Profit ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Competitive</td>
<td>Competitive</td>
<td>Competitive (Each carrier)</td>
<td>(k_1 - w_1)</td>
<td>66,300</td>
<td>48,856</td>
<td>17,444</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(k_2 - w_1)</td>
<td>25,600</td>
<td>17,588</td>
<td>8,012</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(k_3 - w_1)</td>
<td>23,400</td>
<td>16,380</td>
<td>7,020</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Collusive (Total carriers)</td>
<td>(k_1 - w_1)</td>
<td>132,600</td>
<td>85,105</td>
<td>47,495</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(k_2 - w_1)</td>
<td>102,400</td>
<td>66,753</td>
<td>35,647</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(k_3 - w_1)</td>
<td>140,400</td>
<td>93,159</td>
<td>47,241</td>
</tr>
</tbody>
</table>

Land carriers aim to minimize their transportation costs. Table 6.6 illustrates the flow and the marginal and transportation costs of land carrier 1 for an O-D pair of \((k_1, w_1)\).

The marginal transportation cost is decided at $180 on the land link. Data for other two O-D pairs are shown in Table D.4.
Table 6.6 Flow and Transportation Cost of Land Carrier 1

<table>
<thead>
<tr>
<th>Land carrier O - D</th>
<th>Path</th>
<th>Link</th>
<th>Flow (TEUs)</th>
<th>Marginal cost ($)</th>
<th>Total cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>k1 - w1</td>
<td>( ph_{-11,1} )</td>
<td>( lk_{-11,1} \rightarrow lk_{-14,1} \rightarrow lk_{-15,1} )</td>
<td>77</td>
<td>231</td>
<td>16,806</td>
</tr>
<tr>
<td></td>
<td>( ph_{-12,1} )</td>
<td>( lk_{-11,1} \rightarrow lk_{-14,1} \rightarrow lk_{-16,1} )</td>
<td>70</td>
<td>231</td>
<td>15,791</td>
</tr>
<tr>
<td></td>
<td>( ph_{-13,1} )</td>
<td>( lk_{-11,1} \rightarrow lk_{-14,1} \rightarrow lk_{-17,1} )</td>
<td>74</td>
<td>231</td>
<td>16,259</td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td></td>
<td>221</td>
<td>-</td>
<td>48,856</td>
</tr>
</tbody>
</table>

4) Flexible port service charge impacts

We examine an example considering flexible port service charges and analyze their impacts. Port terminal operator 1 determines the fixed port service charge and discount rate by an arrangement with ocean carrier 1 in the collusive environment. The fixed port service charge is set at $300. The price discount rate of 56.2 is adjusted due to the total amount of freight beyond the port terminal capacity. Table 6.7 illustrates the transportation cost of ocean carrier 1 according to fixed and flexible port service charges. Ocean carrier 1 saves $6,799, in alliance with port terminal operator 1.

Table 6.7 Ocean Carrier Transportation Cost according to Fixed and Flexible Port Service Charges

<table>
<thead>
<tr>
<th>Ocean carrier ( o )</th>
<th>Transportation cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fixed port service charge</td>
</tr>
<tr>
<td>Ocean carrier 1</td>
<td>184,418</td>
</tr>
</tbody>
</table>

Port terminal operators also experience revenue changes. Table 6.8 shows the port terminal operator revenue according to fixed and flexible port service charges. Port terminal operator 1 has more income of $5,362 via cooperation with ocean carrier 1, while port terminal operators 2 and 3 loose $2,219 and $3,143, each.
Table 6.8 Port Terminal Operator Revenue according to Fixed and Flexible Service Charges

<table>
<thead>
<tr>
<th>Port terminal operator</th>
<th>Revenue ($)</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fixed port service charge</td>
<td>Flexible port service charge</td>
<td></td>
</tr>
<tr>
<td>Port terminal operator 1</td>
<td>132,600</td>
<td>137,962</td>
<td></td>
</tr>
<tr>
<td>Port terminal operator 2</td>
<td>96,000</td>
<td>93,781</td>
<td></td>
</tr>
<tr>
<td>Port terminal operator 3</td>
<td>140,400</td>
<td>137,257</td>
<td></td>
</tr>
</tbody>
</table>

6.2.2 Shipper Problem

Shippers deliver their products using transportation services provided by the carriers. The shipper network corresponds to the ocean carrier network, the port sub-network and the land carrier network. The value of time \( (vot^*) \) is set at $0.3/hour on the ocean link and $2/hour on the port and land links. Table C.15 shows parameters in the inverse demand and supply functions and table C.16 illustrates parameters in the average shipper link travel time function.

Table 6.9 shows supply in an origin market and demand in a destination market and their costs. Supply and demand are 1,114 TEUs, respectively, and equivalent to the flow on all used shipper paths between origin and destination markets. The supply cost is $1,102 and the demand cost is $2,554 each.

Table 6.9 Demand and Supply and their Costs

<table>
<thead>
<tr>
<th>Network component</th>
<th>Flow (TEUs)</th>
<th>Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supply</td>
<td>1,114</td>
<td>1,102</td>
</tr>
<tr>
<td>Demand</td>
<td>1,114</td>
<td>2,554</td>
</tr>
</tbody>
</table>

According to the equilibrium conditions of the spatial price equilibrium problem, the supply cost in an origin market plus the average transaction cost on path between
these markets are equivalent to the demand cost in a destination market, at equilibrium. As results, the average transaction cost at any paths is same as $1,452 and the supply cost plus the transaction cost ($1,102 + $1,452 = $2,554) is equivalent to the demand cost ($2,554). Table 6.10 shows the flow and the transaction cost of shippers

**Table 6.10 Flow and Transaction Cost of Shippers**

<table>
<thead>
<tr>
<th>Path</th>
<th>Link</th>
<th>Flow (TEUs)</th>
<th>Transaction cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ph_s1</td>
<td>lk_s1 → lk_s2 → lk_s3</td>
<td>400</td>
<td>1,452</td>
</tr>
<tr>
<td>ph_s2</td>
<td>lk_s4 → lk_s5 → lk_s6</td>
<td>300</td>
<td>1,452</td>
</tr>
<tr>
<td>ph_s3</td>
<td>lk_s7 → lk_s8 → lk_s9</td>
<td>414</td>
<td>1,452</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>1,114</td>
<td>-</td>
</tr>
</tbody>
</table>

**6.3 Computational Efficiency**

Heuristic algorithms based on the projection algorithm solved the three-level models of carriers and the bi-level model between shippers and carriers, respectively. The projection algorithm resolves the VI problem into a sequence of sub-problems by the general iterative scheme. In the case study, the algorithms are implemented in MATLAB input files by developing a set of MATLAB models.

The numerical example explored the ocean carrier competitive and collusive games, port terminal operator competitive game, land carrier competitive and collusive games and shipper competitive game. Heuristic algorithms converge after 581 ~ 859 iterations. The developed models were solved in 2.37 to 3.37 seconds on a PC with Pentium IV 2.00 GHz CPU with 3.00 GB of RAM. Table 6.11 shows the number of iterations and elapsed times of heuristic algorithms.
Table 6.11 Convergence of Heuristic Algorithms

<table>
<thead>
<tr>
<th>Ocean carrier game</th>
<th>Number of Iterations</th>
<th>Elapsed time (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Competitive</td>
<td>859</td>
<td>3.37</td>
</tr>
<tr>
<td>Collusive</td>
<td>581</td>
<td>2.37</td>
</tr>
</tbody>
</table>
CHAPTER 7

CONCLUSIONS AND FUTURE RESEARCH DIRECTIONS

The dissertation proposed a conceptual framework that addresses roles and relationships of key stakeholders involved in international maritime freight transportation networks, such as ocean carriers, land carriers and, port terminal operators, shippers and governments. Based on the framework, three-level models capturing interactions among three types of carriers and bi-level models capturing interactions between shippers and carriers were formulated, respectively, according the different market environment. For the carrier problem, a three-hierarchical level was defined as the following decision sequence: ocean carriers, port terminal operators and land carriers. In the shipper-carrier problem, the dissertation assumed that carriers the leaders and shippers are the followers. Heuristic algorithms were developed to solve the multi-level hierarchical models and implemented in the mathematical programming software package MATLAB, version R2008b. The proposed work contributes to the body of the multimodal freight network problem by providing novel methodologies to formulate multi-level hierarchical interaction models. The research will assist decision-making processes of various stakeholders in the maritime freight transport system.

Possible extensions to the models presented in this dissertation include three main aspects: decisions of the Port Authority, the market environment of shippers and different types of commodities.

In this dissertation, the consideration of Port Authority’s roles is beyond the scope of the research. Port Authorities determine operational policies associated with finance,
tariff and labor, and are responsible for port transportation infrastructure at the supply side of the port. They set various port policies and investment strategies for the prosperity of ports and local communities by investigating the behavior of carriers and shippers and forecasting resulting effects. According to the Port Authority’s decisions, carriers and shippers react and show different interactions. The research can be extended to a study that considers Port Authorities as another decision maker in international maritime freight transportation networks. Wang (2000) evaluated Port Authority’s decisions partially by comparing alternative port investment strategies.

Based on the real world circumstances, the dissertation explored the competitive and collusive market environment of carriers. For the ocean and land carrier problems, perfect competition and collusion were considered using Nash equilibrium and compensation principle, respectively. The partial competition is included in the perfect competition by regarding the collusive firms as a competitive player since the firms are still competitive against the other firms. The collusive firms utilize their resources to save the total cost. For the port terminal operator problem, perfect competition was considered because inter/intra port competition usually occurs in many regions. When port terminal operators are collusive in a port, the port is regarded as a competitive port terminal operator. The collusive port terminal operators share port facilities and operate the entire port efficiently. On the other hand, in the shipper problem, the competitive environment was considered via spatial price equilibrium. In reality, a finite number of shippers compete due to the limited resources and some shippers can make alliances for growth in productivity and efficiency. Therefore, future researches may focus on these cases by using the spatial oligopolistic model by Nagurney (1999) or compensation principle.
The dissertation defined a unit of freight expressed by 20 TEUs (twenty-foot equivalent units) for predicting international freight movements. The different types of commodities were not distinguished. Numerous types of commodities exist, however the consideration of the detailed types of commodities brings about a very complex optimization problem. Future studies need to categorize of main commodities and apply to modeling.
REFERENCES


46. Murphy, P.R and Daley, J.M (1997). Carrier selection: do shippers and carriers agree,


1. Ocean carrier problem

The objective function of the ocean carrier problem in Eq. (4.3) is as follows:

$$U_{ij}^o = D_{ij}^o (c_{ij}^o, c_{ij}^{-o}) - \sum_{ph_{-o,o}}^{PH_{-o,o}} AC_{ij}^{ph_{-o,o}} (f_{ij}^{ph_{-o,o}}) * f_{ij}^{ph_{-o,o}}$$

\[ \forall o \in O, \ ij \in IJ \]

The function is strictly concave in \((c_{ij}^o, f_{ij}^{ph_{-o,o}})\) since the Hessian matrix is negative definite and symmetric.

$$\nabla^2 (U_{ij}^o) = \begin{bmatrix}
\nabla_{c_{ij}^o} D_{ij}^o (c_{ij}^o, c_{ij}^{-o}) & 0 \\
0 & -\nabla_{f_{ij}^{ph_{-o,o}}} MC_{ij}^{ph_{-o,o}} (f_{ij}^{ph_{-o,o}})
\end{bmatrix}$$

\[ \forall o \in O, \ ij \in IJ \]

$$\frac{\partial^2 U_{ij}^o}{\partial c_{ij}^o \partial c_{ij}^o} = \frac{\partial D_{ij}^o (c_{ij}^o, c_{ij}^{-o})}{\partial c_{ij}^o}$$

\[ \forall o \in O, \ ij \in IJ \]

$$\frac{\partial^2 U_{ij}^o}{\partial c_{ij}^o \partial f_{ij}^{ph_{-o,o}}} = \frac{\partial^2 U_{ij}^o}{\partial f_{ij}^{ph_{-o,o}} \partial c_{ij}^o} = 0$$

\[ \forall o \in O, \ ij \in IJ, \ ph_{-o,o} \in PH_{-o,o} \]

$$\frac{\partial^2 U_{ij}^o}{\partial f_{ij}^{ph_{-o,o}} \partial f_{ij}^{ph_{-o,o}}} = \frac{\partial MC_{ij}^{ph_{-o,o}} (f_{ij}^{ph_{-o,o}})}{\partial f_{ij}^{ph_{-o,o}}} < 0$$

\[ \forall o \in O, \ ij \in IJ, \ ph_{-o,o} \in PH_{-o,o} \]

Based upon the function assumptions in sub-section 4.1.3, \(\nabla_{c_{ij}^o} D_{ij}^o (c_{ij}^o, c_{ij}^{-o})\) and \(-\nabla_{f_{ij}^{ph_{-o,o}}} MC_{ij}^{ph_{-o,o}} (f_{ij}^{ph_{-o,o}})\) are negative definite, respectively. The Hessian matrix consists of along the diagonal and zero elsewhere is negative definite and symmetric.
2. Port terminal operator problem

The objective function of the port terminal operator in Eq. (4.33) is as follows:

$$U^p = G^p(c^p, c^{-p}) \times c^p - \sum_{jk}^{JK} \sum_{p}^{PH} AC_{jk}^{ph-p} (f_{jk}^{ph-p}) \times f_{jk}^{ph-p} \quad \forall p \in P$$

The function is strictly concave in \((c^p, f_{jk}^{ph-p})\) since the Hessian matrix is negative definite and symmetric.

$$\nabla^2 (U^p) = \begin{bmatrix} \nabla c^p G^p(c^p, c^{-p}) & 0 \\ 0 & -\nabla f_{jk}^{ph-p} MC(f_{jk}^{ph-p}) \end{bmatrix} \quad \forall p \in P$$

$$\frac{\partial^2 U^p}{\partial c^p \partial c^p} = \frac{\partial D^p(c^p, c^{-p})}{\partial c^p} \quad \forall p \in P$$

$$\frac{\partial^2 U^p}{\partial c^p \partial f_{jk}^{ph-p}} = \frac{\partial^2 U^p}{\partial f_{jk}^{ph-p} \partial c^p} = 0 \quad \forall p \in P, jk \in JK$$

$$\frac{\partial^2 U^p}{\partial f_{jk}^{ph-p} \partial f_{jk}^{ph-p}} = \frac{\partial MC_{jk}^{ph-p}(f_{jk}^{ph-p})}{\partial f_{jk}^{ph-p}} < 0 \quad \forall p \in P, jk \in JK$$

Based on the function assumptions in sub-section 4.2.3, \(\nabla c^p G^p(c^p, c^{-p})\) and \(-\nabla f_{jk}^{ph-p} MC(f_{jk}^{ph-p})\) are negative definite, respectively. The Hessian matrix consists of along the diagonal and zero elsewhere is negative definite and symmetric.
3. Land carrier problem

The objective function of the land carrier problem in Eq. (4.57) is as follows:

\[
U^l = D_{kw}^l (c_{kw}^l, \bar{c}_{kw}^l) \times c_{kw}^l - \sum_{ph_{kw}^l,l} AC_{kw}^{ph_{kw}^l,l} (f_{kw}^{ph_{kw}^l,l}) \times f_{kw}^{ph_{kw}^l,l} \quad \forall l \in L, kw \in KW
\]

The function is strictly concave in \((c_{kw}^l, f_{kw}^{ph_{kw}^l,l})\) since the Hessian matrix is negative definite and symmetric.

\[
\nabla^2 (U^l_{kw}) = \begin{bmatrix}
\nabla^2 D_{kw}^l (c_{kw}^l, \bar{c}_{kw}^l) & 0 \\
0 & -\nabla^2 MC_{kw}^{ph_{kw}^l,l} (f_{kw}^{ph_{kw}^l,l})
\end{bmatrix} \quad \forall l \in L, kw \in KW
\]

\[
\frac{\partial^2 U^l_{kw}}{\partial c_{kw}^l \partial c_{kw}^l} = \frac{\partial D_{kw}^l (c_{kw}^l, \bar{c}_{kw}^l)}{\partial c_{kw}^l} \quad \forall l \in L, kw \in KW
\]

\[
\frac{\partial^2 U^l_{kw}}{\partial c_{kw}^l \partial f_{kw}^{ph_{kw}^l,l}} = \frac{\partial^2 U^l_{kw}}{\partial f_{kw}^{ph_{kw}^l,l} \partial c_{kw}^l} = 0 \quad \forall l \in L, kw \in KW, ph_{kw}^l,l \in PH_{kw}^l,l
\]

\[
\frac{\partial^2 U^l_{kw}}{\partial f_{kw}^{ph_{kw}^l,l} \partial f_{kw}^{ph_{kw}^l,l}} = \frac{\partial MC_{kw}^{ph_{kw}^l,l} (f_{kw}^{ph_{kw}^l,l})}{\partial f_{kw}^{ph_{kw}^l,l}} < 0 \quad \forall l \in L, kw \in KW, ph_{kw}^l,l \in PH_{kw}^l,l
\]

Based upon the function assumptions in sub-section 4.3.3, \(\nabla_{c_{kw}^l} D_{kw}^l (c_{kw}^l, \bar{c}_{kw}^l)\) and \(-\nabla_{f_{kw}^{ph_{kw}^l,l}} MC_{kw}^{ph_{kw}^l,l} (f_{kw}^{ph_{kw}^l,l})\) are negative definite, respectively. The Hessian matrix consists of along the diagonal and zero elsewhere is negative definite and symmetric.
### Table B.1 Carrier Model 2

\[- \left( D^o (c^o\ast, c^{-o\ast}) \times c^o - TC^o \right)^\top (c^o - c^o\ast) \geq 0 \]

where \( \nabla_{f^o} TC(f^o)^\top (f^o - f^o\ast) \geq 0 \)

\[ \forall (c^o, f^o) \in OFR \]

s.t.

\[- \left( G^p (c^p\ast, c^{-p\ast}) \times c^p - SC^p \right)^\top (c^p - c^p\ast) \geq 0 \]

where \( \nabla_{f^p} SC(f^p)^\top (f^p - f^p\ast) \geq 0 \)

\[ \forall (c^p, f^p) \in PFR \]

s.t.

\[- \left( D_{kw} (c_{kw}\ast) \times c_{kw} - TC_{kw} \right)^\top (c_{kw} - c_{kw}\ast) \geq 0 \]

\[ \nabla_{f_{kw}} TC(f_{kw})^\top (f_{kw} - f_{kw}\ast) \geq 0 \]

\[ \forall (c_{kw}, f_{kw}) \in LFR \]
| Table B.2 Carrier Model 3 |

\[-\left( D_y (c_{ij}^*, \overline{c_{-ij}^*}) - TC_y \right)^T (c_{ij} - c_{ij}^*) \geq 0\]

\[\nabla_{f_y} TC(f_{ij})^T (f_{ij} - f_{ij}^*) \geq 0\]

\[\forall \, (c_{ij}, f_{ij}) \in OFR\]

s.t.

\[-\left( G^p (c^p*, \overline{c^{p*}}) \times c^p - SC^p \right)^T (c^p - c^p*) \geq 0\]

where  \[\nabla_{f^p} SC(f^p)^T (f^p - f^p*) \geq 0\]

\[\forall \, (c^p, f^p) \in PFR\]

s.t.

\[-\left( D^l (c^l*, \overline{c^{-l*}}) \times c^l - TC^l \right)^T (c^l - c^l*) \geq 0\]

where  \[\nabla_{f^l} TC(f^l)^T (f^l - f^l*) \geq 0\]

\[\forall \, (c^l, f^l) \in LFR\]
Table B.3 Carrier Model 4

\[-\left(D_{ij} (c_{ij}^*) \times c_{ij}^* - TC_{ij} \right)^T (c_{ij} - c_{ij}^*) \geq 0\]

\[\nabla f_{ij} TC(f_{ij})^T (f_{ij} - f_{ij}^*) \geq 0\]

\[\forall (c_{ij}, f_{ij}) \in OFR\]

s.t.

\[-\left(G^p (c^p*, \overline{c^p*}) \times c^p - SC^p \right)^T (c^p - c^p*) \geq 0\]

where \[\nabla f^p SC(f^p)^T (f^p - f^p*) \geq 0\]

\[\forall (c^p, f^p) \in PFR\]

s.t.

\[-\left(D_{kw} (c_{kw}^*) \times c_{kw}^* - TC_{kw} \right)^T (c_{kw} - c_{kw}^*) \geq 0\]

\[\nabla f_{kw} TC(f_{kw})^T (f_{kw} - f_{kw}^*) \geq 0\]

\[\forall (c_{kw}, f_{kw}) \in LFR\]
Table B.4 Bi-level Model 2

**Upper level: Carrier problem**

\[-\left(D^o(c^o*, c^{-o}*) \times c^o* - TC^o\right)^T (c^o - c^o*) \geq 0\]

where \(\nabla f^o TC(f^o)^T (f^o - f^o*) \geq 0\)

\(\forall (c^o, f^o) \in OFR\)

s.t.

\[-\left(G^p(c^p*, c^{-p}*) \times c^p - SC^p\right)^T (c^p - c^p*) \geq 0\]

where \(\nabla f^p SC(f^p)^T (f^p - f^p*) \geq 0\)

\(\forall (c^p, f^p) \in PFR\)

s.t.

\[-\left(D_{kw}(c_{kw}*, c^{-kw}*) \times c_{kw} - TC_{kw}\right)^T (c_{kw} - c_{kw}*) \geq 0\]

\(\nabla f_{kw} TC(f_{kw})^T (f_{kw} - f_{kw}*) \geq 0\)

\(\forall (c_{kw}, f_{kw}) \in LFR\)

**Lower level: Shipper problem**

\(IS(s^*)^T (s - s^*) + ATC(f^*)^T (f - f^*) - ID(d^*)^T (d - d^*) \geq 0\)

\(\forall (s, f, d) \in SFR\)
Table B.5 Bi-level Model 3

Upper level: Carrier problem

\[-(D_{ij}(c_{ij}^*) \times c_{ij}^* - TC_{ij})^T (c_{ij} - c_{ij}^*) \geq 0\]

\[\nabla_{f_{ij}} TC(f_{ij})^T (f_{ij} - f_{ij}^*) \geq 0\]

\[\forall (c_{ij}, f_{ij}) \in OFR\]

s.t.

\[-(G^p(c^p, c^-p^*) \times c^p - SC^p)^T (c^p - c^p^*) \geq 0\]

where \[\nabla_{f^p} SC(f^p)^T (f^p - f^p^*) \geq 0\]

\[\forall (c^p, f^p) \in PFR\]

s.t.

\[-(D^l(c^l, c^-l^*) \times c^l - TC^l)^T (c^l - c^l^*) \geq 0\]

where \[\nabla_{f^l} TC(f^l)^T (f^l - f^l^*) \geq 0\]

\[\forall (c^l, f^l) \in LFR\]

Lower level: Shipper problem

\[IS(s^*)^T (s - s^*) + ATC(f^*)^T (f - f^*) - ID(d^*)^T (d - d^*) \geq 0\]

\[\forall (s, f, d) \in SFR\]
Table B.6 Bi-level Model 4

Upper level: Carrier problem

\[-\left((D_{ij}(c_{ij}^*) \times c_{ij}^* - TC_{ij})\right)^T (c_{ij} - c_{ij}^*) \geq 0\]

\[\nabla_{f_{ij}} TC(f_{ij})^T (f_{ij} - f_{ij}^*) \geq 0\]

\[\forall (c_{ij}, f_{ij}) \in OFR\]

s.t.

\[-\left((G_p(c^p) \times c^p - SC_c)\right)^T (c^p - c^p) \geq 0\]

where \[\nabla_{f_{p}} SC(f^p)^T (f^p - f^p) \geq 0\]

\[\forall (c^p, f^p) \in PFR\]

s.t.

\[-\left((D_{kw}(c_{kw}^*) \times c_{kw}^* - TC_{kw})\right)^T (c_{kw} - c_{kw}^*) \geq 0\]

\[\nabla_{f_{kw}} TC(f_{kw})^T (f_{kw} - f_{kw}^*) \geq 0\]

\[\forall (c_{kw}, f_{kw}) \in LFR\]

Lower level: Shipper problem

\[IS(s^*)^T (s - s^*) + ATC(f^*)^T (f - f^*) - ID(d^*)^T (d - d^*) \geq 0\]

\[\forall (s, f, d) \in SFR\]
APPENDIX C. INPUT DATA IN THE NUMERICAL EXAMPLE

Table C.1 Parameters in the Ocean Carrier Demand Function

\[ D_{ij}^o = \alpha_0 - \alpha_1 \times c_{ij}^o \]

<table>
<thead>
<tr>
<th>( \alpha_0 )</th>
<th>( \alpha_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>259</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Table C.2 Transportation Capacity of Ocean Carrier \( o \)

<table>
<thead>
<tr>
<th>Ocean carrier ( o )</th>
<th>Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ocean carrier 1</td>
<td>300</td>
</tr>
<tr>
<td>Ocean carrier 2</td>
<td>270</td>
</tr>
<tr>
<td>Ocean carrier 3</td>
<td>240</td>
</tr>
<tr>
<td>Ocean carrier 4</td>
<td>220</td>
</tr>
<tr>
<td>Ocean carrier 5</td>
<td>200</td>
</tr>
</tbody>
</table>

Table C.3 Parameters in the Average Link Operating Cost Function of Ocean Carrier \( o \)

\[ AOC_{lk_{o},o}^o(f_{lk_{o},o}) = \omega_0 + \omega_1 \left( \frac{f_{lk_{o},o}}{\text{cap}_{lk_{o},o}} \right) + \omega_2 \left( \frac{f_{lk_{o},o}}{\text{cap}_{lk_{o},o}} \right)^2 \]

<table>
<thead>
<tr>
<th>Link ( lk_{o1,o} )</th>
<th>( \omega_0 )</th>
<th>( \omega_1 )</th>
<th>( \omega_2 )</th>
<th>Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>lk_{o2,o}</td>
<td>180</td>
<td>2.2</td>
<td>-</td>
<td>50</td>
</tr>
<tr>
<td>lk_{o3,o}</td>
<td>184</td>
<td>2.3</td>
<td>-</td>
<td>50</td>
</tr>
<tr>
<td>lk_{o4,o}</td>
<td>188</td>
<td>2.8</td>
<td>-</td>
<td>50</td>
</tr>
<tr>
<td>lk_{o5,o}</td>
<td>180</td>
<td>2.9</td>
<td>-</td>
<td>50</td>
</tr>
<tr>
<td>lk_{o6,o}</td>
<td>174</td>
<td>3.1</td>
<td>-</td>
<td>50</td>
</tr>
<tr>
<td>lk_{o7,o}</td>
<td>181</td>
<td>2.9</td>
<td>-</td>
<td>50</td>
</tr>
<tr>
<td>lk_{o8,o}</td>
<td>183</td>
<td>3.2</td>
<td>-</td>
<td>50</td>
</tr>
<tr>
<td>lk_{o9,o}</td>
<td>186</td>
<td>2.6</td>
<td>-</td>
<td>50</td>
</tr>
<tr>
<td>lk_{o10,o}</td>
<td>182</td>
<td>2.4</td>
<td>-</td>
<td>50</td>
</tr>
<tr>
<td>lk_{o11,o}</td>
<td>184</td>
<td>2.2</td>
<td>-</td>
<td>50</td>
</tr>
<tr>
<td>lk_{o12,o}</td>
<td>28</td>
<td>3.5</td>
<td>2.5</td>
<td>200</td>
</tr>
<tr>
<td>lk_{o13,o}</td>
<td>36</td>
<td>3.7</td>
<td>2.75</td>
<td>200</td>
</tr>
<tr>
<td>lk_{o14,o}</td>
<td>32</td>
<td>3.3</td>
<td>2.4</td>
<td>200</td>
</tr>
</tbody>
</table>
Table C.4 Parameters in the Average Link Travel (Shift) Time Function of Ocean Carrier $o$

\[
ATT_{lk_{-o},o}^{o}(f_{lk_{-o},o}) = att_{lk_{-o},o}^{o} + ast_{lk_{-o},o}^{o} \left(1 + \mu_0 \left(\frac{f_{lk_{-o},o}^{o}}{cap_{lk_{-o},o}^{o}}\right)^{\mu_1}\right)
\]

<table>
<thead>
<tr>
<th>Link</th>
<th>$att$ (day)</th>
<th>$ast$ (day)</th>
<th>$\mu_0$</th>
<th>$\mu_1$</th>
<th>Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$lk_{-01,o}$</td>
<td>17.8</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>50</td>
</tr>
<tr>
<td>$lk_{-02,o}$</td>
<td>17.1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>50</td>
</tr>
<tr>
<td>$lk_{-03,o}$</td>
<td>16.8</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>50</td>
</tr>
<tr>
<td>$lk_{-04,o}$</td>
<td>16.2</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>50</td>
</tr>
<tr>
<td>$lk_{-05,o}$</td>
<td>15.8</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>50</td>
</tr>
<tr>
<td>$lk_{-06,o}$</td>
<td>15.9</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>50</td>
</tr>
<tr>
<td>$lk_{-07,o}$</td>
<td>15.5</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>50</td>
</tr>
<tr>
<td>$lk_{-08,o}$</td>
<td>16.3</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>50</td>
</tr>
<tr>
<td>$lk_{-09,o}$</td>
<td>16.2</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>50</td>
</tr>
<tr>
<td>$lk_{-10,o}$</td>
<td>17.1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>50</td>
</tr>
<tr>
<td>$lk_{-11,o}$</td>
<td>16.8</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>50</td>
</tr>
<tr>
<td>$lk_{-12,o}$</td>
<td>-</td>
<td>0.3</td>
<td>3.8</td>
<td>2</td>
<td>200</td>
</tr>
<tr>
<td>$lk_{-13,o}$</td>
<td>-</td>
<td>0.8</td>
<td>4.2</td>
<td>2</td>
<td>200</td>
</tr>
<tr>
<td>$lk_{-14,o}$</td>
<td>-</td>
<td>0.2</td>
<td>3.8</td>
<td>2</td>
<td>200</td>
</tr>
</tbody>
</table>
Table C.5 Parameters in the Average Link Operating Cost Function of the Colluded Ocean Carriers

\[ AOC^{lk-o}(f^{lk-o}) = \omega_o + \omega_1 \left( \frac{f^{lk-o}}{cap^{lk-o}} \right) + \omega_2 \left( \frac{f^{lk-o}}{cap^{lk-o}} \right)^2 \]

<table>
<thead>
<tr>
<th>Link</th>
<th>( \omega_o )</th>
<th>( \omega_1 )</th>
<th>( \omega_2 )</th>
<th>Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>lk_o1</td>
<td>175</td>
<td>2.2</td>
<td>-</td>
<td>200</td>
</tr>
<tr>
<td>lk_o2</td>
<td>180</td>
<td>2.2</td>
<td>-</td>
<td>200</td>
</tr>
<tr>
<td>lk_o3</td>
<td>179</td>
<td>2.3</td>
<td>-</td>
<td>200</td>
</tr>
<tr>
<td>lk_o4</td>
<td>183</td>
<td>2.8</td>
<td>-</td>
<td>200</td>
</tr>
<tr>
<td>lk_o5</td>
<td>175</td>
<td>2.9</td>
<td>-</td>
<td>200</td>
</tr>
<tr>
<td>lk_o6</td>
<td>169</td>
<td>3.1</td>
<td>-</td>
<td>200</td>
</tr>
<tr>
<td>lk_o7</td>
<td>176</td>
<td>2.9</td>
<td>-</td>
<td>200</td>
</tr>
<tr>
<td>lk_o8</td>
<td>178</td>
<td>3.2</td>
<td>-</td>
<td>200</td>
</tr>
<tr>
<td>lk_o9</td>
<td>181</td>
<td>2.6</td>
<td>-</td>
<td>200</td>
</tr>
<tr>
<td>lk_o10</td>
<td>177</td>
<td>2.4</td>
<td>-</td>
<td>200</td>
</tr>
<tr>
<td>lk_o11</td>
<td>179</td>
<td>2.2</td>
<td>-</td>
<td>200</td>
</tr>
<tr>
<td>lk_o12</td>
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<td>3.5</td>
<td>2.5</td>
<td>1,000</td>
</tr>
<tr>
<td>lk_o13</td>
<td>31</td>
<td>3.7</td>
<td>2.75</td>
<td>1,000</td>
</tr>
<tr>
<td>lk_o14</td>
<td>27</td>
<td>3.3</td>
<td>2.4</td>
<td>1,000</td>
</tr>
</tbody>
</table>
Table C.6 Parameters in the Average Link Travel (Shift) Time Function of the Colluded Ocean Carriers

\[
ATT^{lk-o}(f^{lk-o}) = att^{lk-o} + ast^{lk-o} \left(1 + \mu_0 \left(\frac{f^{lk-o}}{cap^{lk-o}}\right)^{\mu_i}\right)
\]

<table>
<thead>
<tr>
<th>Link</th>
<th>(att) (day)</th>
<th>(ast) (day)</th>
<th>(\mu_0)</th>
<th>(\mu_i)</th>
<th>Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>lk_o1</td>
<td>17.8</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>200</td>
</tr>
<tr>
<td>lk_o2</td>
<td>17.2</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>200</td>
</tr>
<tr>
<td>lk_o3</td>
<td>16.8</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>200</td>
</tr>
<tr>
<td>lk_o4</td>
<td>16.2</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>200</td>
</tr>
<tr>
<td>lk_o5</td>
<td>15.8</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>200</td>
</tr>
<tr>
<td>lk_o6</td>
<td>15.9</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>200</td>
</tr>
<tr>
<td>lk_o7</td>
<td>15.5</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>200</td>
</tr>
<tr>
<td>lk_o8</td>
<td>16.3</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>200</td>
</tr>
<tr>
<td>lk_o9</td>
<td>16.2</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>200</td>
</tr>
<tr>
<td>lk_o10</td>
<td>17.1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>200</td>
</tr>
<tr>
<td>lk_o11</td>
<td>16.8</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>200</td>
</tr>
<tr>
<td>lk_o12</td>
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<td>3.8</td>
<td>2</td>
<td>1,000</td>
</tr>
<tr>
<td>lk_o13</td>
<td>-</td>
<td>0.8</td>
<td>4.2</td>
<td>2</td>
<td>1,000</td>
</tr>
<tr>
<td>lk_o14</td>
<td>-</td>
<td>0.2</td>
<td>3.8</td>
<td>2</td>
<td>1,000</td>
</tr>
</tbody>
</table>
Table C.7 Parameters in the Average Port Link Operating Cost Function

\[
AOC_{lk-p} (f^{lk-p}) = \pi_o + \pi_1 \left( \frac{f^{lk-p}}{cap^{lk-p}} \right) + \pi_2 \left( \frac{f^{lk-p}}{cap^{lk-p}} \right)^2
\]

<table>
<thead>
<tr>
<th>Link</th>
<th>(\pi_o)</th>
<th>(\pi_1)</th>
<th>(\pi_2)</th>
<th>Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>lk_p1</td>
<td>78</td>
<td>2.7</td>
<td>0.85</td>
<td>400</td>
</tr>
<tr>
<td>lk_p2</td>
<td>102</td>
<td>2.1</td>
<td>0.78</td>
<td>400</td>
</tr>
<tr>
<td>lk_p3</td>
<td>68</td>
<td>3.1</td>
<td>0.65</td>
<td>300</td>
</tr>
<tr>
<td>lk_p4</td>
<td>26</td>
<td>2.3</td>
<td>0.55</td>
<td>300</td>
</tr>
<tr>
<td>lk_p5</td>
<td>56</td>
<td>1.4</td>
<td>0.58</td>
<td>300</td>
</tr>
<tr>
<td>lk_p6</td>
<td>52</td>
<td>1.7</td>
<td>0.74</td>
<td>300</td>
</tr>
<tr>
<td>lk_p7</td>
<td>112</td>
<td>2.2</td>
<td>0.75</td>
<td>400</td>
</tr>
<tr>
<td>lk_p8</td>
<td>42</td>
<td>2.6</td>
<td>0.62</td>
<td>400</td>
</tr>
<tr>
<td>lk_p9</td>
<td>32</td>
<td>3.2</td>
<td>0.63</td>
<td>300</td>
</tr>
<tr>
<td>lk_p10</td>
<td>29</td>
<td>2.2</td>
<td>0.82</td>
<td>300</td>
</tr>
<tr>
<td>lk_p11</td>
<td>74</td>
<td>1.9</td>
<td>0.53</td>
<td>800</td>
</tr>
<tr>
<td>lk_p12</td>
<td>120</td>
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<td>0.87</td>
<td>400</td>
</tr>
<tr>
<td>lk_p13</td>
<td>63</td>
<td>1.6</td>
<td>0.47</td>
<td>400</td>
</tr>
<tr>
<td>lk_p14</td>
<td>88</td>
<td>2.6</td>
<td>0.85</td>
<td>300</td>
</tr>
<tr>
<td>lk_p15</td>
<td>95</td>
<td>1.8</td>
<td>0.72</td>
<td>300</td>
</tr>
<tr>
<td>lk_p16</td>
<td>43</td>
<td>0.9</td>
<td>0.62</td>
<td>400</td>
</tr>
<tr>
<td>lk_p17</td>
<td>59</td>
<td>1.2</td>
<td>0.58</td>
<td>400</td>
</tr>
<tr>
<td>lk_p18</td>
<td>85</td>
<td>2.1</td>
<td>0.77</td>
<td>400</td>
</tr>
</tbody>
</table>
### Table C.8 Parameters in the Average Port Link Service Time Function

\[
\text{AST}^{lk-p} (f^{lk-p}) = ast^{lk-p} \left( 1 + \nu \left( \frac{f^{lk-p}}{\text{cap}^{lk-p}} \right)^{\nu_1} \right)
\]

<table>
<thead>
<tr>
<th>Link</th>
<th>ast (hour)</th>
<th>(\nu_0)</th>
<th>(\nu_1)</th>
<th>Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>lk_p1</td>
<td>4.5</td>
<td>1.2</td>
<td>2</td>
<td>400</td>
</tr>
<tr>
<td>lk_p2</td>
<td>3.8</td>
<td>2.2</td>
<td>2</td>
<td>400</td>
</tr>
<tr>
<td>lk_p3</td>
<td>4.2</td>
<td>2.1</td>
<td>2</td>
<td>300</td>
</tr>
<tr>
<td>lk_p4</td>
<td>8.0</td>
<td>1.7</td>
<td>2</td>
<td>300</td>
</tr>
<tr>
<td>lk_p5</td>
<td>3.8</td>
<td>2.3</td>
<td>2</td>
<td>300</td>
</tr>
<tr>
<td>lk_p6</td>
<td>2.8</td>
<td>1.6</td>
<td>2</td>
<td>300</td>
</tr>
<tr>
<td>lk_p7</td>
<td>5.6</td>
<td>2.2</td>
<td>2</td>
<td>400</td>
</tr>
<tr>
<td>lk_p8</td>
<td>3.8</td>
<td>2.4</td>
<td>2</td>
<td>400</td>
</tr>
<tr>
<td>lk_p9</td>
<td>3.2</td>
<td>2.6</td>
<td>2</td>
<td>300</td>
</tr>
<tr>
<td>lk_p10</td>
<td>2.3</td>
<td>1.2</td>
<td>2</td>
<td>300</td>
</tr>
<tr>
<td>lk_p11</td>
<td>3.5</td>
<td>1.8</td>
<td>2</td>
<td>800</td>
</tr>
<tr>
<td>lk_p12</td>
<td>7.2</td>
<td>3.2</td>
<td>2</td>
<td>400</td>
</tr>
<tr>
<td>lk_p13</td>
<td>6.2</td>
<td>3.4</td>
<td>2</td>
<td>400</td>
</tr>
<tr>
<td>lk_p14</td>
<td>4.9</td>
<td>3.6</td>
<td>2</td>
<td>300</td>
</tr>
<tr>
<td>lk_p15</td>
<td>6.3</td>
<td>2.2</td>
<td>2</td>
<td>300</td>
</tr>
<tr>
<td>lk_p16</td>
<td>3.4</td>
<td>2.8</td>
<td>2</td>
<td>400</td>
</tr>
<tr>
<td>lk_p17</td>
<td>4.6</td>
<td>3.1</td>
<td>2</td>
<td>400</td>
</tr>
<tr>
<td>lk_p18</td>
<td>5.3</td>
<td>3.5</td>
<td>2</td>
<td>400</td>
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</tbody>
</table>
Table C.9 Parameters in the Land Carrier Demand Function

\[ D_{kw}^l = \lambda_0 - \lambda_1 \times c_{kw}^l \]

<table>
<thead>
<tr>
<th>Land carrier O - D</th>
<th>( \alpha_0 )</th>
<th>( \alpha_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k1 ) - ( w1 )</td>
<td>234</td>
<td>0.044</td>
</tr>
<tr>
<td>( k2 ) - ( w1 )</td>
<td>93</td>
<td>0.040</td>
</tr>
<tr>
<td>( k3 ) - ( w1 )</td>
<td>88</td>
<td>0.034</td>
</tr>
</tbody>
</table>

Table C.10 Transportation Capacity of Land Carrier \( l \)

<table>
<thead>
<tr>
<th>Land carrier O - D</th>
<th>Land carrier ( l )</th>
<th>Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k1 ) - ( w1 )</td>
<td>Land carrier 1</td>
<td>300</td>
</tr>
<tr>
<td></td>
<td>Land carrier 2</td>
<td>300</td>
</tr>
<tr>
<td>( k2 ) - ( w1 )</td>
<td>Land carrier 1</td>
<td>150</td>
</tr>
<tr>
<td></td>
<td>Land carrier 2</td>
<td>150</td>
</tr>
<tr>
<td></td>
<td>Land carrier 3</td>
<td>120</td>
</tr>
<tr>
<td></td>
<td>Land carrier 4</td>
<td>120</td>
</tr>
<tr>
<td>( k3 ) - ( w1 )</td>
<td>Land carrier 1</td>
<td>300</td>
</tr>
<tr>
<td></td>
<td>Land carrier 2</td>
<td>200</td>
</tr>
<tr>
<td></td>
<td>Land carrier 3</td>
<td>200</td>
</tr>
<tr>
<td></td>
<td>Land carrier 4</td>
<td>150</td>
</tr>
<tr>
<td></td>
<td>Land carrier 5</td>
<td>150</td>
</tr>
<tr>
<td></td>
<td>Land carrier 6</td>
<td>100</td>
</tr>
</tbody>
</table>
Table C.11 Parameters in the Average Link Operating Cost Function of Land carrier \( l \)

\[
AOC_{lk_{-},l}^{\text{ACC},l} (f_{lk_{-},l}^{\text{ACC},l}) = \tau_o + \tau_1 \left( \frac{f_{lk_{-},l}^{\text{ACC},l}}{\text{cap}_{lk_{-},l}} \right) + \tau_2 \left( \frac{f_{lk_{-},l}^{\text{ACC},l}}{\text{cap}_{lk_{-},l}} \right)^2
\]

<table>
<thead>
<tr>
<th>Link</th>
<th>( \tau_o )</th>
<th>( \tau_1 )</th>
<th>( \tau_2 )</th>
<th>Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>lk_{-}1,l</td>
<td>35</td>
<td>1.23</td>
<td>0.54</td>
<td>300</td>
</tr>
<tr>
<td>lk_{-}2,l</td>
<td>52</td>
<td>2.2</td>
<td>0.65</td>
<td>200</td>
</tr>
<tr>
<td>lk_{-}3,l</td>
<td>38</td>
<td>1.08</td>
<td>0.55</td>
<td>200</td>
</tr>
<tr>
<td>lk_{-}4,l</td>
<td>52</td>
<td>1.16</td>
<td>0.36</td>
<td>300</td>
</tr>
<tr>
<td>lk_{-}5,l</td>
<td>65</td>
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<td>0.68</td>
<td>150</td>
</tr>
<tr>
<td>lk_{-}6,l</td>
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<td>0.72</td>
<td>150</td>
</tr>
<tr>
<td>lk_{-}7,l</td>
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<td>0.76</td>
<td>150</td>
</tr>
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<td>lk_{-}8,l</td>
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<td>0.48</td>
<td>150</td>
</tr>
<tr>
<td>lk_{-}9,l</td>
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<td>2.1</td>
<td>0.72</td>
<td>150</td>
</tr>
<tr>
<td>lk_{-}10,l</td>
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<td>1.32</td>
<td>0.62</td>
<td>100</td>
</tr>
<tr>
<td>lk_{-}11,l</td>
<td>50</td>
<td>1.25</td>
<td>0.73</td>
<td>100</td>
</tr>
<tr>
<td>lk_{-}12,l</td>
<td>52</td>
<td>1.21</td>
<td>0.78</td>
<td>100</td>
</tr>
<tr>
<td>lk_{-}13,l</td>
<td>75</td>
<td>1.12</td>
<td>0.58</td>
<td>100</td>
</tr>
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</table>
Table C.12 Parameters in the Average Link Travel (Shift) Time Function of Land carrier $l$

\[
ATT^{lk_{l1,l}}(f^{lk_{l1,l}}) = ast^{lk_{l1,l}} \left[ 1 + \theta_0 \left( \frac{f^{lk_{l1,l}}}{cap^{lk_{l1,l}}} \right)^{\theta_1} \right] + att^{lk_{l1,l}} \left[ 1 + \theta_2 \left( \frac{f^{lk_{l1,l}}}{cap^{lk_{l1,l}}} \right)^{\theta_3} \right]
\]

<table>
<thead>
<tr>
<th>Link</th>
<th>$ast$ (hour)</th>
<th>$att$ (hour)</th>
<th>$\theta_0$</th>
<th>$\theta_1$</th>
<th>$\theta_2$</th>
<th>$\theta_3$</th>
<th>$cap$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$lk_{j1,l}$</td>
<td>4.2</td>
<td>-</td>
<td>1.2</td>
<td>2</td>
<td>-</td>
<td>-</td>
<td>300</td>
</tr>
<tr>
<td>$lk_{j2,l}$</td>
<td>5.1</td>
<td>-</td>
<td>3.2</td>
<td>2</td>
<td>-</td>
<td>-</td>
<td>200</td>
</tr>
<tr>
<td>$lk_{j3,l}$</td>
<td>3.7</td>
<td>-</td>
<td>3.2</td>
<td>2</td>
<td>-</td>
<td>-</td>
<td>200</td>
</tr>
<tr>
<td>$lk_{j4,l}$</td>
<td>-</td>
<td>2.8</td>
<td>-</td>
<td>-</td>
<td>1.3</td>
<td>2</td>
<td>300</td>
</tr>
<tr>
<td>$lk_{j5,l}$</td>
<td>-</td>
<td>6.5</td>
<td>-</td>
<td>-</td>
<td>1.6</td>
<td>2</td>
<td>150</td>
</tr>
<tr>
<td>$lk_{j6,l}$</td>
<td>-</td>
<td>5.4</td>
<td>-</td>
<td>-</td>
<td>1.8</td>
<td>2</td>
<td>150</td>
</tr>
<tr>
<td>$lk_{j7,l}$</td>
<td>-</td>
<td>7.6</td>
<td>-</td>
<td>-</td>
<td>1.6</td>
<td>2</td>
<td>150</td>
</tr>
<tr>
<td>$lk_{j8,l}$</td>
<td>-</td>
<td>6.8</td>
<td>-</td>
<td>-</td>
<td>3.4</td>
<td>2</td>
<td>150</td>
</tr>
<tr>
<td>$lk_{j9,l}$</td>
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<td>-</td>
<td>-</td>
<td>3.6</td>
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<td>150</td>
</tr>
<tr>
<td>$lk_{j10,l}$</td>
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<td>-</td>
<td>-</td>
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<tr>
<td>$lk_{j11,l}$</td>
<td>-</td>
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<td>-</td>
<td>-</td>
<td>3.6</td>
<td>2</td>
<td>100</td>
</tr>
<tr>
<td>$lk_{j12,l}$</td>
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<td>-</td>
<td>-</td>
<td>2.8</td>
<td>2</td>
<td>100</td>
</tr>
<tr>
<td>$lk_{j13,l}$</td>
<td>-</td>
<td>5.2</td>
<td>-</td>
<td>-</td>
<td>2.8</td>
<td>2</td>
<td>100</td>
</tr>
</tbody>
</table>
Table C.13 Parameters in the Average Link Operating Cost Function of the Colluded Land Carriers

\[ AOC_{lk,l,j} (f_{lk,l,j}) = \tau_o + \tau_1 \left( \frac{f_{lk,l,j}}{\text{cap}_{lk,l,j}} \right) + \tau_2 \left( \frac{f_{lk,l,j}}{\text{cap}_{lk,l,j}} \right)^2 \]

<table>
<thead>
<tr>
<th>Link</th>
<th>(\tau_o)</th>
<th>(\tau_1)</th>
<th>(\tau_2)</th>
<th>Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>lk_{l1,l}</td>
<td>35</td>
<td>1.23</td>
<td>0.54</td>
<td>1,200</td>
</tr>
<tr>
<td>lk_{l2,l}</td>
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<td>2.2</td>
<td>0.65</td>
<td>1,200</td>
</tr>
<tr>
<td>lk_{l3,l}</td>
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<td>1.08</td>
<td>0.55</td>
<td>1,000</td>
</tr>
<tr>
<td>lk_{l4,l}</td>
<td>45</td>
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<td>0.36</td>
<td>1,200</td>
</tr>
<tr>
<td>lk_{l5,l}</td>
<td>60</td>
<td>1.68</td>
<td>0.68</td>
<td>600</td>
</tr>
<tr>
<td>lk_{l6,l}</td>
<td>68</td>
<td>1.36</td>
<td>0.72</td>
<td>600</td>
</tr>
<tr>
<td>lk_{l7,l}</td>
<td>57</td>
<td>1.29</td>
<td>0.76</td>
<td>600</td>
</tr>
<tr>
<td>lk_{l8,l}</td>
<td>116</td>
<td>1.7</td>
<td>0.48</td>
<td>500</td>
</tr>
<tr>
<td>lk_{l9,l}</td>
<td>122</td>
<td>2.1</td>
<td>0.72</td>
<td>500</td>
</tr>
<tr>
<td>lk_{l10,l}</td>
<td>120</td>
<td>1.32</td>
<td>0.62</td>
<td>1,000</td>
</tr>
<tr>
<td>lk_{l11,l}</td>
<td>47</td>
<td>1.25</td>
<td>0.73</td>
<td>500</td>
</tr>
<tr>
<td>lk_{l12,l}</td>
<td>44</td>
<td>1.21</td>
<td>0.78</td>
<td>500</td>
</tr>
<tr>
<td>lk_{l13,l}</td>
<td>68</td>
<td>1.12</td>
<td>0.58</td>
<td>800</td>
</tr>
</tbody>
</table>
Table C.14 Parameters in the Average Link Travel (Shift) Time Function of the Colluded Land Carriers

\[
ATT^{lk_{-j,l}}(f^{lk_{-j,l}}) = ast^{lk_{-j,l}} \left( 1 + \theta_0 \left( \frac{f^{lk_{-j,l}}}{cap^{lk_{-j,l}}} \right)^{\theta_1} \right) + att^{lk_{-j,l}} \left( 1 + \theta_2 \left( \frac{f^{lk_{-j,l}}}{cap^{lk_{-j,l}}} \right)^{\theta_3} \right)
\]

<table>
<thead>
<tr>
<th>Link</th>
<th>ast (hour)</th>
<th>att (hour)</th>
<th>( \theta_0 )</th>
<th>( \theta_0 )</th>
<th>( \theta_2 )</th>
<th>( \theta_3 )</th>
<th>cap</th>
</tr>
</thead>
<tbody>
<tr>
<td>lk_j1_l</td>
<td>4.2</td>
<td>-</td>
<td>1.2</td>
<td>2</td>
<td>-</td>
<td>-</td>
<td>1,200</td>
</tr>
<tr>
<td>lk_j2_l</td>
<td>5.1</td>
<td>-</td>
<td>3.2</td>
<td>2</td>
<td>-</td>
<td>-</td>
<td>1,200</td>
</tr>
<tr>
<td>lk_j3_l</td>
<td>3.7</td>
<td>-</td>
<td>3.2</td>
<td>2</td>
<td>-</td>
<td>-</td>
<td>1,000</td>
</tr>
<tr>
<td>lk_j4_l</td>
<td>-</td>
<td>2.8</td>
<td>-</td>
<td>-</td>
<td>1.3</td>
<td>2</td>
<td>1,200</td>
</tr>
<tr>
<td>lk_j5_l</td>
<td>-</td>
<td>6.5</td>
<td>-</td>
<td>-</td>
<td>1.6</td>
<td>2</td>
<td>600</td>
</tr>
<tr>
<td>lk_j6_l</td>
<td>-</td>
<td>5.4</td>
<td>-</td>
<td>-</td>
<td>1.8</td>
<td>2</td>
<td>600</td>
</tr>
<tr>
<td>lk_j7_l</td>
<td>-</td>
<td>7.6</td>
<td>-</td>
<td>-</td>
<td>1.6</td>
<td>2</td>
<td>600</td>
</tr>
<tr>
<td>lk_j8_l</td>
<td>-</td>
<td>6.8</td>
<td>-</td>
<td>-</td>
<td>3.4</td>
<td>2</td>
<td>500</td>
</tr>
<tr>
<td>lk_j9_l</td>
<td>-</td>
<td>7.2</td>
<td>-</td>
<td>-</td>
<td>3.6</td>
<td>2</td>
<td>500</td>
</tr>
<tr>
<td>lk_j10_l</td>
<td>-</td>
<td>7.8</td>
<td>-</td>
<td>-</td>
<td>3.4</td>
<td>2</td>
<td>1,000</td>
</tr>
<tr>
<td>lk_j11_l</td>
<td>-</td>
<td>4.2</td>
<td>-</td>
<td>-</td>
<td>3.6</td>
<td>2</td>
<td>500</td>
</tr>
<tr>
<td>lk_j12_l</td>
<td>-</td>
<td>4.4</td>
<td>-</td>
<td>-</td>
<td>2.8</td>
<td>2</td>
<td>500</td>
</tr>
<tr>
<td>lk_j13_l</td>
<td>-</td>
<td>5.2</td>
<td>-</td>
<td>-</td>
<td>2.8</td>
<td>2</td>
<td>800</td>
</tr>
</tbody>
</table>
Table C.15 Parameters in the Inverse Demand and Supply Functions

\[ IS(s_x) = \chi_0 + \chi_1 \times s_x \]

<table>
<thead>
<tr>
<th>$\alpha_0$</th>
<th>$\alpha_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,080</td>
<td>0.02</td>
</tr>
</tbody>
</table>

\[ ID(d_y) = \rho_0 - \rho_1 \times d_y \]

<table>
<thead>
<tr>
<th>$\lambda_0$</th>
<th>$\lambda_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2,588</td>
<td>0.03</td>
</tr>
</tbody>
</table>

Table C.16 Parameters in the Average Shipper Link Travel Time Cost Function

\[ ATT^{lk-s}(f^{lk-s}) = att^{lk-s} \left[ 1 + \kappa_0 \left( \frac{f^{lk-s}}{cap^{lk-s}} \right)^{\kappa_1} \right] \]

<table>
<thead>
<tr>
<th>Link</th>
<th>$att$ (hour)</th>
<th>$\kappa_0$</th>
<th>$\kappa_1$</th>
<th>Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>lk_s1</td>
<td>434.4</td>
<td>0.15</td>
<td>1.8</td>
<td>1,250</td>
</tr>
<tr>
<td>lk_s2</td>
<td>15.3</td>
<td>0.15</td>
<td>1.8</td>
<td>1,250</td>
</tr>
<tr>
<td>lk_s3</td>
<td>14.6</td>
<td>0.15</td>
<td>1.8</td>
<td>1,250</td>
</tr>
<tr>
<td>lk_s4</td>
<td>400.8</td>
<td>0.15</td>
<td>1.8</td>
<td>800</td>
</tr>
<tr>
<td>lk_s5</td>
<td>12.8</td>
<td>0.15</td>
<td>1.8</td>
<td>800</td>
</tr>
<tr>
<td>lk_s6</td>
<td>12.3</td>
<td>0.15</td>
<td>1.8</td>
<td>800</td>
</tr>
<tr>
<td>lk_s7</td>
<td>415.2</td>
<td>0.15</td>
<td>1.8</td>
<td>1,200</td>
</tr>
<tr>
<td>lk_s8</td>
<td>13.4</td>
<td>0.15</td>
<td>1.8</td>
<td>1,000</td>
</tr>
<tr>
<td>lk_s9</td>
<td>13.3</td>
<td>0.15</td>
<td>1.8</td>
<td>600</td>
</tr>
</tbody>
</table>
APPENDIX D. RESULTS OF THE SHIPPER AND CARRIER MODEL

Table D.1 Flow and Marginal Transportation cost of Ocean Carriers under the Competitive Game

<table>
<thead>
<tr>
<th>Path</th>
<th>Ocean carrier 2</th>
<th>Ocean carrier 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Flow (TEUs)</td>
<td>Marginal cost ($)</td>
</tr>
<tr>
<td>$p_{h_{o1,o}}$</td>
<td>27</td>
<td>610</td>
</tr>
<tr>
<td>$p_{h_{o2,o}}$</td>
<td>10</td>
<td>610</td>
</tr>
<tr>
<td>$p_{h_{o3,o}}$</td>
<td>37</td>
<td>610</td>
</tr>
<tr>
<td>$p_{h_{o4,o}}$</td>
<td>22</td>
<td>610</td>
</tr>
<tr>
<td>$p_{h_{o5,o}}$</td>
<td>12</td>
<td>610</td>
</tr>
<tr>
<td>$p_{h_{o6,o}}$</td>
<td>40</td>
<td>610</td>
</tr>
<tr>
<td>$p_{h_{o7,o}}$</td>
<td>17</td>
<td>610</td>
</tr>
<tr>
<td>$p_{h_{o8,o}}$</td>
<td>40</td>
<td>610</td>
</tr>
<tr>
<td>$p_{h_{o9,o}}$</td>
<td>25</td>
<td>610</td>
</tr>
<tr>
<td>$p_{h_{o10,o}}$</td>
<td>22</td>
<td>610</td>
</tr>
<tr>
<td>$p_{h_{o11,o}}$</td>
<td>18</td>
<td>610</td>
</tr>
<tr>
<td>Total</td>
<td>270</td>
<td>-</td>
</tr>
</tbody>
</table>
Table D.1 (Continued) Flow and Marginal Transportation cost of Ocean Carriers under the Competitive Game

<table>
<thead>
<tr>
<th>Path</th>
<th>Ocean carrier 4</th>
<th></th>
<th>Ocean carrier 5</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Flow (TEUs)</td>
<td>Marginal cost ($)</td>
<td>Flow (TEUs)</td>
<td>Marginal cost ($)</td>
</tr>
<tr>
<td>ph_{o1,o}</td>
<td>23</td>
<td>607</td>
<td>21</td>
<td>606</td>
</tr>
<tr>
<td>ph_{o2,o}</td>
<td>6</td>
<td>607</td>
<td>4</td>
<td>606</td>
</tr>
<tr>
<td>ph_{o3,o}</td>
<td>33</td>
<td>607</td>
<td>31</td>
<td>606</td>
</tr>
<tr>
<td>ph_{o4,o}</td>
<td>18</td>
<td>607</td>
<td>17</td>
<td>606</td>
</tr>
<tr>
<td>ph_{o5,o}</td>
<td>9</td>
<td>607</td>
<td>7</td>
<td>606</td>
</tr>
<tr>
<td>ph_{o6,o}</td>
<td>36</td>
<td>607</td>
<td>35</td>
<td>606</td>
</tr>
<tr>
<td>ph_{o7,o}</td>
<td>13</td>
<td>607</td>
<td>12</td>
<td>606</td>
</tr>
<tr>
<td>ph_{o8,o}</td>
<td>35</td>
<td>607</td>
<td>34</td>
<td>606</td>
</tr>
<tr>
<td>ph_{o9,o}</td>
<td>19</td>
<td>607</td>
<td>17</td>
<td>606</td>
</tr>
<tr>
<td>ph_{o10,o}</td>
<td>16</td>
<td>607</td>
<td>13</td>
<td>606</td>
</tr>
<tr>
<td>ph_{o11,o}</td>
<td>12</td>
<td>607</td>
<td>9</td>
<td>606</td>
</tr>
<tr>
<td>Total</td>
<td>220</td>
<td>-</td>
<td>200</td>
<td>-</td>
</tr>
</tbody>
</table>
### Table D.2 Flow and Marginal Transportation cost of Ocean Carriers under the Collusive Game

<table>
<thead>
<tr>
<th>Path</th>
<th>Flow (TEUs)</th>
<th>Marginal cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ph_{o1}$</td>
<td>127</td>
<td>611</td>
</tr>
<tr>
<td>$ph_{o2}$</td>
<td>36</td>
<td>611</td>
</tr>
<tr>
<td>$ph_{o3}$</td>
<td>165</td>
<td>611</td>
</tr>
<tr>
<td>$ph_{o4}$</td>
<td>100</td>
<td>611</td>
</tr>
<tr>
<td>$ph_{o5}$</td>
<td>99</td>
<td>611</td>
</tr>
<tr>
<td>$ph_{o6}$</td>
<td>44</td>
<td>611</td>
</tr>
<tr>
<td>$ph_{o7}$</td>
<td>117</td>
<td>611</td>
</tr>
<tr>
<td>$ph_{o8}$</td>
<td>184</td>
<td>611</td>
</tr>
<tr>
<td>$ph_{o9}$</td>
<td>130</td>
<td>611</td>
</tr>
<tr>
<td>$ph_{o10}$</td>
<td>120</td>
<td>611</td>
</tr>
<tr>
<td>$ph_{o11}$</td>
<td>108</td>
<td>611</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>1,230</strong></td>
<td><strong>-</strong></td>
</tr>
</tbody>
</table>

### Table D.3 Flow and Marginal Transportation Cost of Port Terminal Operators under the Ocean Carrier Competitive game

<table>
<thead>
<tr>
<th>Port terminal operator</th>
<th>Path</th>
<th>Link</th>
<th>Throughput (TEUs)</th>
<th>Marginal cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>$ph_{p4}$</td>
<td>$lk_{p7} \rightarrow lk_{p11}$</td>
<td>200</td>
<td>233</td>
</tr>
<tr>
<td>2</td>
<td>$ph_{p5}$</td>
<td>$lk_{p8} \rightarrow lk_{p9} \rightarrow lk_{p10} \rightarrow lk_{p11}$</td>
<td>120</td>
<td>233</td>
</tr>
<tr>
<td>2</td>
<td><strong>Total</strong></td>
<td></td>
<td><strong>320</strong></td>
<td><strong>-</strong></td>
</tr>
<tr>
<td>3</td>
<td>$ph_{p6}$</td>
<td>$lk_{p12} \rightarrow lk_{p13}$</td>
<td>162</td>
<td>257</td>
</tr>
<tr>
<td>3</td>
<td>$ph_{p7}$</td>
<td>$lk_{p14} \rightarrow lk_{p15}$</td>
<td>149</td>
<td>257</td>
</tr>
<tr>
<td>3</td>
<td>$ph_{p8}$</td>
<td>$lk_{p16} \rightarrow lk_{p17} \rightarrow lk_{p18}$</td>
<td>157</td>
<td>257</td>
</tr>
<tr>
<td>3</td>
<td><strong>Total</strong></td>
<td></td>
<td><strong>468</strong></td>
<td><strong>-</strong></td>
</tr>
</tbody>
</table>
Table D.4 Flow and Marginal Transportation Cost of Port Terminal Operators under the Ocean Carrier Collusive Game

<table>
<thead>
<tr>
<th>Port terminal operator</th>
<th>Path</th>
<th>Link</th>
<th>Throughput (TEUs)</th>
<th>Marginal cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Port terminal operator 2</td>
<td>$ph_{p4}$</td>
<td>$lk_{p7} \rightarrow lk_{p11}$</td>
<td>163</td>
<td>224</td>
</tr>
<tr>
<td></td>
<td>$ph_{p5}$</td>
<td>$lk_{p8} \rightarrow lk_{p9} \rightarrow lk_{p10} \rightarrow lk_{p11}$</td>
<td>97</td>
<td>224</td>
</tr>
<tr>
<td></td>
<td><strong>Total</strong></td>
<td></td>
<td><strong>260</strong></td>
<td><strong>-</strong></td>
</tr>
<tr>
<td>Port terminal operator 3</td>
<td>$ph_{p6}$</td>
<td>$lk_{p12} \rightarrow lk_{p13}$</td>
<td>188</td>
<td>272</td>
</tr>
<tr>
<td></td>
<td>$ph_{p7}$</td>
<td>$lk_{p14} \rightarrow lk_{p15}$</td>
<td>170</td>
<td>272</td>
</tr>
<tr>
<td></td>
<td>$ph_{p8}$</td>
<td>$lk_{p16} \rightarrow lk_{p17} \rightarrow lk_{p18}$</td>
<td>184</td>
<td>272</td>
</tr>
<tr>
<td></td>
<td><strong>Total</strong></td>
<td></td>
<td><strong>542</strong></td>
<td><strong>-</strong></td>
</tr>
</tbody>
</table>

Table D.5 Flow and Marginal Transportation Cost of Land Carriers

<table>
<thead>
<tr>
<th>Land carrier O - D</th>
<th>Path</th>
<th>Link</th>
<th>Throughput (TEUs)</th>
<th>Marginal cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k2$ - $w1$</td>
<td>$ph_{l4,l}$</td>
<td>$lk_{l2,l} \rightarrow lk_{l3,l}$</td>
<td>47</td>
<td>233</td>
</tr>
<tr>
<td></td>
<td>$ph_{l5,l}$</td>
<td>$lk_{l2,l} \rightarrow lk_{l9,l}$</td>
<td>33</td>
<td>233</td>
</tr>
<tr>
<td></td>
<td><strong>Total</strong></td>
<td></td>
<td><strong>80</strong></td>
<td><strong>-</strong></td>
</tr>
<tr>
<td>$k3$ - $w1$</td>
<td>$ph_{l6,l}$</td>
<td>$lk_{l3,l} \rightarrow lk_{l10,l}$</td>
<td>39</td>
<td>222</td>
</tr>
<tr>
<td></td>
<td>$ph_{l7,l}$</td>
<td>$lk_{l3,l} \rightarrow lk_{l11,l} \rightarrow lk_{l13,l}$</td>
<td>23</td>
<td>222</td>
</tr>
<tr>
<td></td>
<td>$ph_{l8,l}$</td>
<td>$lk_{l3,l} \rightarrow lk_{l21,l} \rightarrow lk_{l13,l}$</td>
<td>16</td>
<td>222</td>
</tr>
<tr>
<td></td>
<td><strong>Total</strong></td>
<td></td>
<td><strong>78</strong></td>
<td><strong>-</strong></td>
</tr>
</tbody>
</table>