ESSAYS ON THE NEW KEYNESIAN MODEL

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This dissertation discusses the application, estimation and extension of the New Keynesian macroeconomic model.

Chapter 2 estimates the exchange rate pass-through using Bayesian estimation techniques for Korea and the US. The degree of exchange rate pass-through for Korea is found to increase after the Asian crisis. The nominal rigidity parameter for Korean goods in the foreign countries increased from before the Asian crisis to after the Asian crisis. This suggests that although the important two assumptions on price setting, PCP and LCP, do not hold in this paper, importers in Korea have a stronger tendency to follow the PCP assumption and importers in U.S. have a stronger tendency to follow the LCP for Korean goods after the Asia Crisis. This is consistent with the notion that tradable prices are set in the currency that is most stable.

Chapter 3 introduces the asset market segmentation into an otherwise standard New
Keynesian model, and shows that the Taylor principle continues to be essential for macroeconomic stability. In particular, central banks should change nominal interest rates more than one-for-one in response to sustained increases in the inflation rate to guarantee a unique equilibrium. This finding reverses the results of previous studies that argued the Taylor principle, under asset market segmentation, no longer provides an important criterion for the stability properties of interest rate rules.

Chapter 4 estimates and compares three models, the New Keynesian model with market segmentation, the New Keynesian model with market segmentation and without money, and the standard New Keynesian model with capital, using Bayesian approaches. The posterior mean of parameter fraction of traders for the New Keynesian model with market segmentation is more similar with Landon-Lane and Occhino (2008) than one for the other models. The value of log marginal likelihood, used in evaluating the three models, for the New Keynesian model with market segmentation is the most high of three models.
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Dedication

To my wife, Hye Rang Park, my daughter, Haeryung Lee, my parents-in-law, and my parents.
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Chapter 1

Introduction

In the past, it was common that the Keynesian macroeconomics was modeled by combining ad hoc macroeconomic variables derived from separate equations. So the Keynesian model has been criticized due to the lack of microfoundation for their macro-theory. Modern macroeconomics has developed the dynamic general stochastic equilibrium (DSGE) system studying the aggregate decision of rational agents resulted from individual agent’s optimization. The DSGE macroeconomics is started from seminal works of Lucas (1975) and Kydland and Prescott (1977) studying the real business cycles. The development of DSGE model has inspired a lot of energy to the Keynesian model. The classical assumption of perfectly flexible prices, however, has been shown not to be held in many empirical studies. New Keynesian general equilibrium models have been developed that incorporate both optimizing behavior by individual agents and nominal rigidities. This allows for the study of the effects of economic policy without having to make the strong assumption of perfectly flexible prices.

The New Keynesian successes the thought of the Keynesian and overcomes their weakness, ad-hoc models, by adopting DSGE systems. Monetary economics has been one of the most active fields in macroeconomics for the past several years and the New Keynesian model has provided very useful frameworks to analyze monetary policies. The important
features of the New Keynesian model are as follows. First, each firm produced a differentiated good in imperfect competition in the goods market. Firms are faced with the monopolistic competition. Second, there are nominal rigidities, sticky prices or wages. Each firm cannot adjust their price every period. Third, there is the short-run non-neutrality of monetary policy resulted from the existence of the nominal rigidity.

Vissing-Jorgensen (2002) shows that only 44.1% of households participated into the stock market in 1994. All households do not have the same portfolio. Mulligan and Martin (2000), as of 1989, 59 percent of the U.S. population did not hold interest bearing assets. Mankiw (2000) argues that two canonical models, the Barro-Ramsey model and the overlapping generations Diamond-Samuelson model, are not appropriate to analyze fiscal policies. He suggests three reasons for that. First, some agent follows the permanent-income hypothesis and the others follow the simple rule-of-thumb of consuming their current income. This means that consumers following the rule-of-thumb cannot smooth their consumption over time. Second, many households have net worth near zero. This means that many households do not have financial assets to smooth the intertemporal consumption assumed in a lot modern macroeconomic models. Third, much wealth is concentrated on a few people and bequests are an important factor in great wealth accumulation. This causes some households to depart from the permanent-income hypothesis. These finding may demonstrate that asset markets are segmented.

The New Keynesian model and the market segmentation model have separately developed to offer powerful frameworks to studying macroeconomics over the past decade. Gali et al. (2004), Nakajima (2006) and Lama and Medina (2007), however, have tried to combine features from the New Keynesian model and the market segmentation model to analyze monetary policies in these days. The introduction of the market segmentation in

\footnote{Two canonical models assume that all households can smooth their consumption over time by using asset markets.}
the New Keynesian model can allow the money which is neglected in the standard New Keynesian model to play an important role in the New Keynesian model with market segmentation. The money can make affects on determinacy, the liquidity effect, and responses of the consumption and the output to exogenous shocks in the model. The most important contribution of this dissertation is to show that the money may be an important factor in the New Keynesian model when the markets segmentation is incorporated in the New Keynesian model.

This dissertation is organized as follows. Chapter 2 applies the open economy model of the New Keynesian model to Korea and estimates structural parameters for Korea using the Bayesian method. Chapter 3 discusses how the market segmentation is incorporated in the New Keynesian model and how conditions of interest rate rules, such like the Taylor rule, for determinacy are changed in the New Keynesian model with market segmentation. Chapter 4 estimates and evaluates three models, the New Keynesian model with market segmentation, the New Keynesian model with market segmentation and without money, and the standard New Keynesian model with capital, using the Bayesian approach.

1.1 Literature Review


Clarida et al. (2000) show with quantitative precision that the difference in monetary policies between pre- and post-1979, the year Paul Volcker was selected as Chairman of the Board of Governors of the Federal Reserve System, may make macroeconomic performances
considerably different. They discuss that if the coefficient of inflation in the feedback rule is less than 1, the policy in itself may make the economy unstable and the coefficient for the pre-Volcker rule was always less than 1.\textsuperscript{2}

Woodford (2001) discusses that the Taylor principle satisfies the determinacy and eliminate the self-fulfilling spiral of inflation in the neo-Wicksellian model and shows that the Taylor rule can be a proper tool to make the inflation and the output gap stable.

Carlstrom and Fuerst (2005) discuss how introduce of capitals and the investment affects the determinacy in the discrete time model. In the Calvo sticky price model, if the monetary authority uses forward-looking rules, the investment results in the very tight condition for determinacy. In their calibrate, the existence of investment may let the determinacy originally impossible in a forward-looking rule.\textsuperscript{3} In a current-looking Taylor rules, introduce of investment does not affect the condition for determinacy, an aggressive Taylor rule.

Akay (2010) discusses that since CIA constraints may generate the local real determinacy if the intertemporal elasticity of substitution in consumption is sufficiently low, the CIA model is rare used in the New Keynesian literature. The intuition is as following, if there is the sunspot driven increase in current consumption which in turn results in the increase in expected inflation and then the rise in nominal interest rate, this will lead the future consumption to reduce. If the intertemporal elasticity of substitution in consumption is sufficiently high, the future consumption will drop considerably and the initial increase in expected inflation will be offset. If the intertemporal elasticity of substitution in consumption is sufficiently low, the initial increase in expected inflation will be self-fulfilling

\textsuperscript{2}The intuition on this is as follows. If the coefficient is less than 1, the increase in the expected inflation induces the decrease in the real rate which, in turn, simulates the aggregate demand and then leads to the increase in inflation. This is the self-fulfilling of expected inflation.

\textsuperscript{3}We notice that Clarida et al. (2000) show that the New Keynesian model without investment requires the condition for determinacy to be aggressive in forward-looking rules.
and then the sunspot equilibrium will be supported. He also shows that the introduction of the adjustment cost of investment and the persistence of habit in consumption into the New Keynesian model with CIA constraints will lessen ranges for indeterminacy.

Alvarez et al. (2001) show that the market segmentation model with the exchange economy can make the interest rate policies such as the Taylor rule, in which inflation rates can be reduced from the increase in interest rates in short-term, to harmony with the quantity theory of money. The model is set on the CIA constraint and generates the liquidity effect, which will disappear in their model otherwise.

Alvarez et al. (2002) analyze the effects of money injection through an open market operation on interest rates and exchange rates. They assumed that asset markets are endogenously segmented, that is, agents should pay a fixed cost in order to transfer money between the asset market and the good market. They insist that the market segmentation model can generate two features of interest rates which standard CIA models do not explain well, the negative co-movement between expected inflation and real interest rate and the liquidity effect in the short-term. Also, their model can successfully produce persistent real effects.

Landon-Lane and Occhino (2008) introduce production and the limited participation model into an exogenous segmented market model. The introduction of production economy allow them to analyze the effect of monetary policy on the output. They find that the segmented market friction significantly improves the statistical out-of-sample forecast performance of the model, and helps generate delayed and realistic impulse response functions to monetary policy shocks.

Alvarez et al. (2003) and Chiu (2007) introduce the inventory-theoretic model to explain dynamics of money, the velocity of money, and prices and then they can produce staggered

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4This fixed cost causes agents to trade infrequently bonds and money. Therefore, all households do not necessarily participate into asset markets every period.
prices. In a model of Alvarez et al. (2003), households hold the inventory of money in the
bank account to pay their expenditure of consumption for N periods. They show that in
the long-run since the stock of money increases relative to consumption, the velocity of
money remains constantly and the level of prices increases as much as the supply of money
but in the short-run the exogenous increase in money supply leads the endogenous decrease
in the aggregate velocity of money and then the price level will respond sluggishly. Also,
since increase in money supply takes some time to operating this way through money
inventories, the sluggish respond of prices will be persistent. Their model produce some
quantity results that the velocity of money is relatively stable and the price level increase
with the growth of M2/Consumption, which it is similar with data, in the long-run and
that the volatility of the velocity of money from the model is highly correlated with one
from data in the short-run. The responses of money, prices, and the velocity of money
from the model are similar with ones from VAR literature. Chiu (2007) endogenizes a
decision of agents on a time of transfer of money between asset markets and goods markets
by introducing a fixed cost in transferring money. His endogenous inventory theoretical
model can capture the long-run positive correlation between money growth and the velocity
of money that the exogenous inventory theoretical model of Alvarez et al. (2003) cannot

5Agents have an opportunity to transfer money between the brokerage account, in which agents hold
interest bearing assets in asset markets, and the bank account, in which they hold money for consumption
in goods markets, only once every N periods.

6To understand, we have to consider the individual velocity of money. Households transferring recently
funds from their brokerage account to their bank account tend to spend slowly their money in order to
smooth their consumption for remaining periods before they obtain an opportunity to transfer money.
Therefore, they have the relatively low individual velocity of money. Households, however, expecting to
replenish their money balance in the bank account soon have a tendency to spend relatively quickly their
money in their bank account. Hence, they have the relative high individual velocity of money. The
aggregate velocity of money depends on a distribution of holding money at any points. If the monetary
authority increase money supply through the open market operation, the money injections is absorbed only
by households having an opportunity to participate in the asset market at current. Therefore, the stock
of money of households absorbing exclusively money injections will more increase and since they have a
tendency to spend money more slowly than the average, the aggregate velocity of money will reduce.
Singh et al. (2004) cast a question: *are sticky prices (i.e., frictions in good markets) more relevant in emerging markets than frictions in asset markets?* They study the optimal monetary policy in the emerging market because the friction in asset markets is prevalent in emerging markets and argue that the choice of the exchange rate regime, fixed and flexible, should depend on not only the kind of shocks but also the kind of restrictions, goods markets and asset markets. They argue that there are state-contingent rules to achieve the first-best equilibrium in which non-traders can smooth their consumption. Intuitually, the monetary authority can increase the rate of money growth (the rate of devaluation) to increase (decrease) the real balance of traders (non-traders) and then to increase (decrease) consumption of traders (non-traders) when there are positive output shocks. Therefore, the monetary authority can smooth non-traders’ consumption. In the segmented market model, policies by the monetary authority make an effect on traders and non-traders asymmetrically. They, however, show that since the monetary authority is required to respond contemporaneous shocks, the implement of the state contingent rules to achieve the first-best equilibrium is difficult in practice. So they study the non-state contingent rules to perform the first-best equilibrium. They suggest a time invariant money growth rule and a fixed rate of devaluation. They also conclude that non-state contingent rules based on money supply is dominate ones based on exchange rate in terms of welfare so the flexible exchange rate regime may be better than the pegged exchange rate regime. They, however, also do not consider sticky prices and a production economy.

Gali et al. (2004) introduce capital stocks to distinguish Ricardians who can smooth their consumption and non-Ricardians who cannot. Their key result is that the coefficient of inflation in the Taylor rule may be a increasing function of the weight of non-Ricardians. Hence, if the weight of non-Richardians is sufficiently high, we may require too aggressive
Taylor rule. They also argue that, in the forward-looking rule, if the weight of non-Richardians is sufficiently high, we cannot obtain ranges for determinacy.

Nakajima (2006) generates the liquidity effect by introducing the market segmentation into the New Keynesian model with velocity shocks and shows that the liquidity effect can affect the determinacy of the Taylor rule, the passive Taylor rule. He suggest that the monetary authority should pay attention to the liquidity effect when the nominal interest rate is used as the goal of monetary policy.
Chapter 2
Estimating Small Open Economy DGSE Models with Incomplete Pass-Through in Korea

2.1 Introduction

In many empirical studies the classical assumption of perfectly flexible prices has not been held. So the New Keynesian models incorporating both optimizing behavior of agents and nominal rigidities have been developed to study the effects of economic policy. This New Keynesian approach in small open economies is referred to as New Open Economy Macroeconomics (NOEM henceforth) models. In modeling nominal rigidities in the NOEM, an important question is whether export/import prices are sticky in the currency of the producers, or in the currency of the destination market. There exist two competing assumptions on this, the local vs. producer currency pricing assumption. Obstfeld and Rogoff (1995) assume that prices of exported/imported goods are set in the producers currency, which is referred to as producer currency pricing (PCP henceforth). Alternatively, Betts and Devereux (2000) postulate that producers preset prices of their goods in the currency of the markets where they sell their goods, which is referred to as local currency pricing (LCP henceforth). These assumptions play an important role on the issues such as the international monetary transmission mechanism and on the design of optimal stabilization policies because results of optimal monetary policy and international transmission are affected differently by assumption of PCP or LCP. To answer to a question that realistically
which assumption is adapted, we should investigate pass-through elasticity. Therefore, the estimation of the pass-through is important when studying optimal stabilization policy and exchange rate regime optimality in the New Keynesian open economy general equilibrium model.

The PCP assumption on export/import pricing assumes that the law of one price holds for every tradable good; that is, the exchange rate pass-through is complete. Many empirical literatures, however, shows the law of one price does not hold in the real world. For example, Campa and Goldberg (2005) estimates the exchange rate pass-through into the import prices of 23 OECD countries and finds incomplete pass-through in the short run. This implies that both PCP and LCP are significantly rejected in the short-run. In the long run, however, PCP is more feasible than LCP for many types of imported goods.

To analyze the optimal monetary policy under incomplete pass-through, many economists use a New Keynesian dynamic stochastic general equilibrium (DSGE) model. To incorporate the incomplete pass-through in a DSGE model, the endogenous pass-through form has been studied by Devereux et al. (2004), Justiniano and Preston (2004) and Leigh and Lubik (2005). Devereux et al. (2004) shows that countries with relatively low volatility of money growth will have relatively low rates of exchange rate pass-through, while countries with relatively high volatility of money growth will have relatively high rate of exchange rate pass-through. According to Monacelli (2005), the optimal monetary policy in an open economy is essentially different from the one in the closed economy under incomplete pass-through. Endogenous cost-push shocks result in productivity-driven deviations from the law of one price and the optimal commitment policy asks for more stable nominal and real exchange rate volatility under incomplete pass-through. In the endogenous incomplete pass-through model, Calvo (1983)-type price setting parameter, which measuring the magnitude of nominal rigidities in the import sectors, explains the degree of pass-through.
In Korea, most of work on the pass-through is related to study the direct relation between import and exchange rate. Many papers studying the exchange rate pass-through by using Korean data showed the incomplete pass-through of exchange rate. For example, Lee and Kim (2007) estimate the pass-through of exchange rate in Korea with Campa and Goldbergs methodology. There are few papers that study the optimal monetary policy in a small open economy setting for Korea. Kim et al. (2004) showed by using a variant of Gali and Monacelli (2005) that a fixed exchange rate regime is more superior to the discretion if the monetary authority does not win the public confidence. Yang et al. (2005) studied the dynamics of the inflation targeting with parameters estimated by Elekdag et al. (2006) through impulse response functions under incomplete pass-through in Korea. Jung (2005) analyzed the effect of monetary policy in DGSE model with RMSAE (relative mean square approximation error) by Watson (1993). Elekdag et al. (2006) estimated with Bayesian estimation techniques a small open economy with the financial accelerator mechanism and the balance sheet channel by using Korean data.

The Bayesian method has been used extensively in studying and estimating NOEM or DSGE models. It is, however, not easy to find work in the literature studying with Bayesian techniques New Keynesian DSGE model with endogenous pass-through for Korean economy. Therefore, the attempt to estimate structural parameters for Korean economy using the Bayesian approach in the small-scale two country model with incomplete pass-through will be meaningful. This paper aims to estimate using Bayesian methods a small-scale two country NK-DSGE model for Korea that is based on the small-scale two country model in Lubik and Schorfheide (2006). It is hoped that the estimates and inferences about structural parameters from such a model will add insight to appropriate monetary policy for Korea. In our framework, the exchange rate pass-through is endogenous and it will be governed by Calvo parameter in import sectors, denoted by $\theta_F$ and $\theta^*_H$. If these parameters' values are 0, then the exchange rate pass-through is complete and the economy follows the
PCP assumption. This work will separately estimate the small-scale two country models under the incomplete pass-through; Korea-U.S. Also, this paper will divide the total sample period (1987:I 2005:IV) into two sections, period 1 (before Asia Crisis; 1987:I 1996:IV) and period 2 (after Asia Crisis; 1999:I 2005:IV). This is because the Korean exchange rate regime had been the fixed exchange rate regime until 1997, but after Asia Crisis the regime has changed to the floating exchange rate regime. Therefore, the division of time will be useful to analyze the effect the change in exchange rate regime in Korea had on the exchange rate pass through.

This paper is organized as follows. We set Small Open Economy DGSE Models with Incomplete Pass-Through in section 2. We report results of Bayesian approaches to the model in section 3. We conclude in section 5.

2.2 Model

This paper adopts the small-scale two-country NOEM model without capital accumulation. This economy consists of the households consuming goods and supplying labor, firms producing monopolistically competitive goods, a monopolistic importer charging a mark-up on their imported good, and government controlling monetary policy without issuing money. Households in both countries will consume both goods which are produced in the Home and Foreign market. Each household in Home (Foreign) country supplies their labor in order to produce Home (Foreign) goods. In this model, the pass-through is endogenous. It means that local retailers import differentiated goods for which the law of one price holds at the dock and charge a mark-up on these goods in the market in which they sell the goods. This means that the law of one price will not hold in the short-run, but will
hold in the long-run.\footnote{This assumption is set in order to explain the empirical results of Campa and Goldberg (2005), the PCP and LCP hypothesis is rejected in the short run but PCP is feasible in the long run.} Therefore, there is endogenous deviation from purchasing power parity (PPP) via price-setting importers in the short-run, but not in the long-run in this model.

Monopolistically competitive producers, however, preset prices in their own currency at the domestic and foreign countries. Also, the real side, preferences and technologies, is symmetric across the two countries, but the nominal side is asymmetries across the two countries in this model. The nominal rigidities of domestic and import goods will be set differently across countries and there will be a distinction between the monetary policy rule in the Home and Foreign countries. This model assumes the complete international asset market in which risks will be efficiently shared and our model can have a manageable reduced form. The economic activities associated with Home (Korea) is denoted H, Foreign (US) F. The activities in Foreign with respect to location are indexed by ”*”. For example, $C_F (C^*_H)$ represent the consumption of the foreign (home) produced good in the Home (Foreign) country.

### 2.2.1 Domestic Households

The domestic economy is populated by a continuum of households consuming Dixit-Stiglitz aggregates of domestic ($C_H$) and imported ($C_F$) goods. This model assumes that they have an intertemporal utility function with habit persistence:

$$E_0 \left[ \sum_{t=0}^{\infty} \beta^t \left( \frac{(C_t/A_{Wt})^{1-\tau}}{1-\tau} - N_t \right) \right],$$

(2.1)

where $C_t = C_t - h\gamma C_{t-1}$, $0 \leq h \leq 1$, represents the effect consumption of the household. There is habit persistence in this effective consumption with habit persistence parameter...
h. $\gamma$ is the steady state growth rate of $A_{Wt}$. The household has a time separable utility function in which period utility is separable in consumption and labor supplied. Households have a constant relative risk aversion set of preferences over consumption, with their coefficient of relative risk aversion given by $\tau > 0$. Labor supplied is equal to $N_t$. $A_{Wt}$ is a non-stationary world-wide technology shock, and $0 < \beta < 1$ is the discount factor.

Period $t$ consumption, $C_t$, is a composite consumption index

$$C_t = \left[ (1 - \alpha)^{1/\eta} C_{Ht}^{(\eta - 1)/\eta} + \alpha C_{Ft}^{(\eta - 1)/\eta} \right]^{\eta/(\eta - 1)},$$

where $0 \leq \alpha \leq 1$ is the import share and $\eta > 0$ measures the elasticity of substitution between domestic and foreign goods. Households optimal allocation of aggregate consumption depends on the standard demand function: $C_{Ht} = (1 - \alpha) (P_{Ht}/P_t)^{-\eta} C_t$ and $C_{Ft} = \alpha (P_{Ft}/P_t)^{-\eta} C_t$, where $P_t = \left[ (1 - \alpha) P_{Ht}^{1-\eta} + \alpha P_{Ft}^{1-\eta} \right]^{1/(1-\eta)}$ is the consumer price index (CPI) and $P_{Ht}$ and $P_{Ft}$ are domestic and foreign goods price indices.\(^2\)

The budget constraint is the following:

$$P_{Ht} C_{Ht} + P_{Ft} C_{Ft} + E[Q_{t,t+1} D_{t+1}] \leq W_t N_t + D_t - T_t,$$

\(^{(2.2)}\)

where $W_t$ is the nominal wage, $Q_{t,t+1}$ is the stochastic discount factor, $D_t$ is payments from a portfolio of assets, and $T_t$ is the lump-sum taxes. All households are able to consume identical consumption in all $t$ under the initial financial assets that allow them to have the same initial intertemporal budget constraints because all agents value consumption streams identically and have the same price. And they will invest portfolios of financial asset in order to continue to have identical intertemporal budget constraint in all states.

The households problem is to maximize the intertemporal utility function subject to

\(^2\)For detail of demand functions, see the appendix A.1.1.
the budget constraints for all $t$. Thus the final intratemporal optimality condition is given by:

$$A^{1-\tau} W_t \lambda_t = C^\tau - h \gamma \beta E_t \left[ (A W_t C_{t+1} / A W_{t+1})^{-\tau} (A W_t / A W_{t+1}) \right],$$

(2.3)

where $\lambda_t^{-1} = W_t / P_t$ is the marginal utility of income.

Intertemporal optimization implies

$$Q_{t,t+1} = \beta (\lambda_{t+1} / \lambda_t) \cdot (P_t / P_{t+1}),$$

(2.4)

so that taking expectation in (4), we can obtain the nominal interest rate:

$$1 / R_t = \beta E_t [(\lambda_{t+1} / \lambda_t) \cdot (P_t / P_{t+1})].$$

(2.5)

### 2.2.2 International Risk-sharing

Since households in the domestic and foreign have identical preferences, the F.O.C analogous to equation 4) should also hold in the foreign country. Since the stochastic discount factor will be equalized in equilibrium, the complete asset market assumption implies perfect risk sharing in both of domestic and foreign:

$$\beta (\lambda_{t+1} / \lambda_t) \cdot (P_t / P_{t+1}) = Q_{t,t+1} = \beta (\lambda^*_t / \lambda^*_t) \cdot (P^*_t / P^*_{t+1}) \cdot (e_t / e_{t+1}).$$

(2.6)

This implies that the uncovered interest rate parity condition, $E_t Q_{t,t+1} [R_t - R^*_t (e_{t+1} / e_t)] = 0$, holds.

### 2.2.3 Domestic Producers

A continuum of monopolistically competitive producers, indexed by $j(\in [0, 1])$, produces domestic differentiated goods and Calvo-type price setting is assumed. Therefore, a fraction
$1 - \theta_H$ of goods prices are set optimally, while $0 < \theta_H < 1$ of goods prices will be set according to the steady state inflation rate $\pi$. The optimal price $\tilde{P}_t$ will be common value to all firms because each supplier that can reset their price in period $t$ faces the same decision problem. Therefore, the Dixit-Stiglitz price index will evolve according to the following equation:

$$P_{Ht} = \left[ \int_0^1 P_{Ht}^{1-\omega}(j) dj \right]^{1/(1-\omega)} = \left[ (1 - \theta_H) \tilde{P}_{Ht}^{1-\omega} + \int_0^1 \theta_H (P_{H,t-1}(j) \pi)^{1-\omega} dj \right]^{1/(1-\omega)},$$

so that

$$P_{Ht} = \left[ (1 - \theta_H) \tilde{P}_{Ht}^{1-\omega} + \theta_H (P_{H,t-1} \pi)^{1-\omega} \right]^{1/(1-\omega)}, \quad (2.7)$$

where $\omega > 1$ is the elasticity of substitution between types of differentiated domestic or foreign goods.

Each firm $j$ has a following demand function:

$$Y_{Ht}(j) = \left( \frac{P_{Ht}(j)}{P_{Ht}} \right)^{-\omega} (C_{Ht} + G_{Ht} + C_{Ht}^*), \quad (2.8)$$

where $G_{Ht}$ is the government expenditure on domestic good. Also each firm has a linear production technology, $Y_{Ht}(j) = A_{W_t} A_{Ht} N_t(j)$. Thus, the firms price-setting problem in period $t$ is to maximize

$$E_T \left[ \sum_{t=T}^{\infty} \theta_{Ht}^{t-T} Q_{T,t} Y_{Ht}(j) \left[ P_{Ht}(j) \pi^{t-T} - P_{Ht} MC_{Ht} \right] \right], \quad (2.9)$$

with respect to $P_{HT}(j)$ subject to (8), where $MC_{Ht} = W_t / P_t$ is real marginal cost function for each firm. The factor $\theta_{Ht}^{t-T}$ is the probability that the firm will not set newly its price

\[3\] For details of the monopolistic producer’s demand function, see the appendix A.1.2.
in the next $t - T$ periods. The first order condition is

$$E_T \left[ \sum_{t=T}^{\infty} \theta_H^{t-T} Q_{T,t} Y_{Ht}(j) \left[ \bar{P}_{HT}(j) \pi^{t-T} - (\omega/\omega - 1) P_{Ht} MC_{Ht} \right] \right] = 0$$

and the solution to this problem results in the familiar Phillips-curve.

2.2.4 Incomplete Pass-Through and Retail Forms

Since we assume that local monopolistically competitive retailers import differentiated goods for which law of one price holds at the border and charge a mark-up on these goods in their domestic market, PPP can deviate in the short-run. So the law of one price gap can be defined as:

$$\psi_{Ft} = e_t P_{Ft}^* / P_{Ft}. \quad (2.10)$$

If PPP holds, then $\psi_{Ft} \equiv 1$. The fact that importers charge a mark-up to consumers implies that the exchange rate pass-through is imperfect. And under incomplete pass-through the law of one price does not hold. Importers also set their price under Calvo-type price-setting. A fraction $0 < \theta_F < 1$ of retailers will adjust prices according to the steady state inflation rate $\pi$ and a fraction $1 - \theta_F$ of importers will charge their prices optimally. The importers demand function is

$$C_{Ft}(j) = \left( \bar{P}_{Ft}(j) / P_{Ft} \right)^{-\omega} C_{Ft}. \quad (2.11)$$

---

4By Monacelli (2005), although charging mark-up on imported goods results in deviations from the law of one price in the short-run, the law of one price can hold in the long-run as complete pass-through is reached asymptotically.
The importers optimization problem is to maximize

$$E_T \left[ \sum_{t=T}^{\infty} \theta_F^{t-T} Q_{T,t} C_{Ft}(j) \left[ \tilde{P}_{Ft}(j) \pi^{t-T} - \epsilon_t P^*_F(j) \right] \right],$$  (2.12)

with respect to $\tilde{P}_{Ft}(j)$ subject to (11).

This problem's F.O.C is

$$E_T \left[ \sum_{t=T}^{\infty} \theta_F^{t-T} Q_{T,t} C_{Ft}(j) \left[ \tilde{P}_{Ft}(j) \pi^{t-T} - \left( \omega/(\omega - 1) \right) \epsilon_t P^*_F(j) \right] \right]$$

and the solution to this problem will derive an aggregate supply curve for import prices.

In this model the parameter $\theta_F$ explains the degree of pass-through. If $\theta_F = 0$, the simple law of one price will hold, that is, the PCP assumption is hold. The law of one price gap for the specific good is the marginal cost of purchasing imports.

### 2.2.5 Government

Government will operate the monetary policy described by an interest rate feedback rule:

$$R_t = \tilde{R}_t^{1-\rho_R} R_{t-1}^{\rho_R} \exp(\varepsilon_{Rt}),$$  (2.13)

where $\varepsilon_{Rt}$ is a monetary policy shock and $\tilde{R}_t$ is the nominal target rate. There are two specifications for $\tilde{R}_t$, output gap rule specification and output growth rule specification. First, government reacts to inflation, deviations of output from potential output and (nominal) exchange rate:

$$\tilde{R}_t = r \bar{\pi} (\pi_t/\bar{\pi})^{\varphi_1} \left( Y_t/Y_t \right)^{\varphi_2} e^{\varphi_3} : \text{ output gap rule specification.}$$  (2.14)

Second, government responds to inflation, deviations of output growth its equilibrium
steady-state γ and exchange rate:

\[ \hat{R}_t = r \tilde{\pi} (\pi_t / \tilde{\pi})^{\varphi_1} (Y_t / Y_{t-1})^{\varphi_2} e^{\varphi_3} : \text{ output growth rule specification.} \quad (2.15) \]

The parameter \( r \) is the steady-state real interest rate, \( \tilde{\pi} \) is the target inflation rate, which correspond with the steady-state inflation rate in equilibrium, and \( \hat{Y}_t \) is the potential output without nominal rigidities.

### 2.2.6 Real Exchange Rate and Terms of Trade

The real exchange rate is defined as \( s_t = e_t P_t^* / P_t \). The terms of trade, the price of domestic goods in terms of the price of imports, is defined respectively as \( q_t = P_{Ht} / P_{Ft} \) and \( q_t^* = P_{Ft}^* / P_{Ht}^* \) in domestic and foreign economy. When pass-through is perfect, home and foreign terms of trade correspond inversely. By the definition of the real exchange rate, \( \psi_{Ft}/q_t = \psi_{Ht}^*/q_t^* \).

### 2.2.7 Equilibrium

A symmetric equilibrium will be considered in this model. All consumers start with identical initial asset that allowing them to face the same budget constraint in every period. So they can make the same decision on consumption. Since all producers in domestic country face the identical price-setting problem, they will set a common price \( P_{Ht} \). And all importers and foreign firms will also choose a common price \( P_{Ft} \) and \( P_{Ft}^* \) similarly so that \( j \) subscript can be omitted. In the equilibrium, all market will be cleared. Here is Goods market clearing conditions in domestic and foreign economy:

\[ Y_{Ht} = C_{Ht} + G_t + C_{Ht}^* \quad \text{and} \quad Y_{Ft}^* = C_{Ft}^* + G_t^* + C_{Ft}, \quad (2.16) \]
where $G_t$ and $G^*_t$ are the government expenditure in the domestic and foreign economies respectively. The both governments in domestic and foreign economy will run a balanced budget each period, $G_t = T_t$ and $G^*_t = T^*_t$ in equilibrium.

### 2.2.8 Linearization

To study the dynamics around steady state, we take log-linear approximation. $\hat{x}_t = \log x_t - \log \bar{x}_t$ represents log-deviation from the steady state of the variable $x_t$ and $\bar{x}$ denotes the steady state of variable $x$. From the domestic firms price-setting problem we can obtain a forward-looking Phillips-curve:

$$\hat{\pi}_{Ht} = \beta E_t \hat{\pi}_{H,t+1} + \kappa_H \hat{m}_c t,$$

(2.17)

where $\kappa_H = (1 - \theta_H)(1 - \theta_H \beta)/\theta_H > 0$ and $\hat{m}_c t = -\hat{\lambda}_t - \alpha \hat{q}_t - \hat{A}_{Ht}$. The marginal utility of income, $\hat{\lambda}_t$, can be linearized as following:

$$-\hat{\lambda}_t = (\tau/(1 - h \beta)) \hat{C}_t - (h \beta/(1 - h \beta)) E_t \left[ \tau \hat{C}_{t+1} + \hat{z}_{t+1} \right],$$

(2.18)

and the law of motion for the habit stock is as following:

$$(1 - h) \hat{C}_t = \hat{c}_t - h \hat{c}_{t-1} + h \hat{z}_t,$$

(2.19)

where $\hat{z}_t = \triangle \hat{A}_{Wt}$.

Thus the world-wide shock, $A_{Wt}$, changes the intertemporal consumption trade-off due to the habit persistence dynamics, so the Euler equation becomes the following:

$$-\hat{\lambda}_t = -E_t \hat{\lambda}_{t+1} - \left( \hat{R}_t - E_t \hat{\pi}_{t+1} \right) + E_t \hat{z}_{t+1}.$$

(2.20)
Using this we can obtain an aggregate supply curve for import prices from the price-setting problem of importers:

\[
\hat{\pi}_{Ft} = \beta Et \hat{\pi}_{F,t+1} + \kappa_F \hat{\psi}_{Ft},
\]  

(2.21)

where \( \kappa_F = (1 - \theta_F)(1 - \theta_F \beta)/\theta_F \). This means that the import price inflation will rise when an imports world price exceeds the local currency price of the same good. The parameter \( \theta_F \) measures the exchange rate pass-through. We can derive the CPI-inflation from the definition:

\[
\hat{\pi}_t = \alpha \hat{\pi}_{Ft} + (1 - \alpha) \hat{\pi}_{H,t},
\]  

(2.22)

and the terms of trades evolve according to:

\[
\hat{q}_t = \hat{q}_{t-1} - \hat{\pi}_{Ft} + \hat{\pi}_{Ht}.
\]  

(2.23)

Since the law of one price does not hold, the real exchange rate can be written under incomplete pass-through as

\[
\hat{s}_t = \hat{e}_t + \hat{p}_t^* - \hat{p}_t = \hat{\psi}_{Ft} - (1 - \alpha) \hat{q}_t - \alpha \hat{q}_t^*,
\]  

(2.24)

where \( \hat{p}_t^* = (1 - \alpha) \hat{p}_{Ft}^* + \alpha \hat{p}_{Ht}^* \) and \( \hat{p}_t = \alpha \hat{p}_{Ft} + (1 - \alpha) \hat{p}_{Ht} \). From the equation (20) we can find that there are two source of deviation from aggregate PPP, \( \hat{\psi}_{Ft} \) and \(-(1 - \alpha) \hat{q}_t - \alpha \hat{q}_t^*\). Here the term \((1 - \alpha) \hat{q}_t - \alpha \hat{q}_t^*\) represents the heterogeneity of consumption baskets between the home and foreign. The deviation from the law of one price, \( \hat{\psi}_{Ft} \), induces the deviation from PPP. Thus we can obtain an important result that the volatility of the real exchange rate can be explained by the law of one price gap under incomplete pass-through. From
(20), we can derive nominal exchange rate dynamics:

\[ \Delta \hat{e}_t = \hat{\pi}_t - \hat{\pi}^*_t + \Delta \hat{s}_t. \]  

(2.25)

The uncovered interest rate parity condition is:

\[ \hat{R}_t - \hat{R}^*_t = E_t \Delta \hat{e}_{t+1}. \]  

(2.26)

The linearized international risk-sharing condition which explains the relation between marginal utility of income between home and foreign is:

\[ \hat{\lambda}_t = \hat{\lambda}^*_t - \hat{s}_t. \]  

(2.27)

The good market clearing condition is:

\[ \hat{y}_{Ht} = \hat{c}_t + \hat{g}_t - \left( \alpha / \tau \right) \hat{s}_t - \alpha (1 - \alpha) \eta (\hat{q}_t - \hat{q}^*_t). \]  

(2.28)

From linearizing the interest rate feedback rule, equation (13), we can obtain the Taylor-type monetary policy rule:

\[ \hat{R}_t = \rho_R \hat{R}_{t-1} + (1 - \rho_R) \left[ \varphi_1 \hat{\pi}_t + \varphi_2 (\Delta \hat{y}_t + \hat{z}_t) + \varphi_3 \Delta \hat{e}_t \right] + \varepsilon_{R,t}. \]  

(2.29)
There are 5 equations depicting the disturbance of the exogenous autoregressive shocks:

\[
\hat{z}_t = \rho z \hat{z}_{t-1} + \varepsilon_{zt},
\]
\[
\hat{A}_{Ht} = \rho A \hat{A}_{H,t-1} + \varepsilon_{At},
\]
\[
\hat{A}^{*}_{Ft} = \rho A \hat{A}^{*}_{F,t-1} + \varepsilon_{A^{*}t},
\]
\[
\hat{g}_t = \rho G \hat{g}_{t-1} + \varepsilon_{Gt},
\]
\[
\hat{g}^{*}_t = \rho G^{*} \hat{g}^{*}_{t-1} + \varepsilon_{G^{*}t},
\]

where \(z_t\) captures exogenous fluctuations of the technology growth rate. The domestic block of the economy is given by equation (17)-(29) with the 17 unknowns \(\{\hat{c}_t, \hat{y}_{Ht}, \hat{R}_t, \hat{R}^{*}_t, \hat{\pi}_t, \hat{\pi}^{*}_t, \hat{\pi}_{Ht}, \hat{\pi}_{Ft}, \hat{s}_t, \hat{\psi}_{Ft}, \hat{\rho}_t, \hat{\lambda}_t, \hat{\lambda}^{*}_t, \hat{g}_t, \hat{z}_t\}\). The log-linearized DSGE model in this paper will consist of both 22 equations with endogenous variables of the both home and foreign economy and 5 exogenous processes.\(^5\)

### 2.2.9 Model Solution

Given these exogenous disturbance process the log-linearized DSGE model can be expressed as a linear rational expectations (LRE) model solved by the methods described in Sims (2002). And before we estimate the DSGE model, we should find a solution to the LRE. The log-linearized DSGE model in this paper can be expressed by:

\[
\Gamma_0(\vartheta)x_{t+1} = \Gamma_1(\vartheta)x_t + \Gamma_\varepsilon(\vartheta)\varepsilon_{t+1} + \Gamma_\eta(\vartheta)\eta_{t+1},
\]

where \(x_t\) is the vector of unobservable model variables, \(\varepsilon_t\) is the vector of the exogenous processes, \(\varepsilon_t = 0\) and \(E\varepsilon_t\varepsilon'_t = \Sigma_t\), and \(\eta_t\) is the rational expectations forecast errors,

\(^5\)For the difference equation system for this model, see the appendix A.2.
\( \eta_{t+1} = \hat{x}_{t+1} - E_t \hat{x}_{t+1} \). The matrices \( \Gamma \) explain the dynamics of the exogenous shock processes and \( \vartheta \) represent the structural parameters of the model. Then the LRE system will have the following solution by Sims method:

\[
x_t = \Phi_1 x_{t-1} + \Phi_2 \varepsilon_t,
\]

(2.32)

where \( \varepsilon_t \) iid\( N(0, Q) \). From the equation (32), we can express the log-linearized model as state-space model:

\[
X_t = A(\vartheta) + Bx_t; \quad \text{measurement equation, (2.33)}
\]

where \( X_t \) is an vector of observable variables with output growth, inflation, and nominal interest rate for the both home and foreign country and the exchange rate, \( A(\vartheta)_{7 \times 1} \) is the mean of \( X_t \), \( E X_t = A(\vartheta) \), and \( B_{7 \times 27} \) just choose elements of \( x_t \). In this model, \( A(\vartheta) = [\gamma, \tilde{\pi}, \tilde{r} + \tilde{\pi} + 4\gamma, \gamma, \tilde{\pi}, \tilde{r} + \tilde{\pi} + 4\gamma, 0]' \).

2.3 Data and Bayesian Approach

2.3.1 Data

This model chose seven quarterly variables, home and foreign real GDP per capita growth rate, home and foreign CPI inflation, home and foreign interest rate, nominal depreciation rate and estimate three two-country model, Korea-US. For real GDP per capita of Korea and the US, the economically active population (national concept; POP) from OECD are used and Real seasonally adjusted GDP, CPI, and government 3-year yield bond interest rate are from Bank of Korea for Korea and from Federal Reserve Bank of S.t Louis for

\[ ^6 \text{See an appendix C of this dissertation for detail on a general procedure using Bayesian methods to estimate DSGE models.} \]
the U.S. Especially, for interest rates of Korea, public-housing bond interest rate is used. Exchange rate of Won/Dollar is found in BOK. Per capita output growth rate is defined as $100 \times \ln(\frac{GDP_t}{POP_t})/\ln(\frac{GDP_{t-1}}{POP_{t-1}})$. Annualized inflation based on CPI is defined as $400 \times \ln(\frac{CPI_t}{CPI_{t-1}})$. Nominal interest rate is annualized. Nominal depreciation rate is defined as $100 \times \ln(\frac{E_t}{E_{t-1}})$. The total samples period is from 1987:I to 2005:IV (2002:IV in Euro case). This period will be divided into two sections, period 1 (1987:I 1996:IV, before Asia Crisis) and period 2 (1999:I 2005:IV, after Asia Crisis).

2.3.2 Prior

The prior represents our subjective beliefs about $\theta$ before observing the data. Also, the prior can summarize information that is not used in the estimation sample $X$. In principle, priors reflect a researchers beliefs from economic theories before the researcher observes the data, but, in practice, most priors are obtained from pre-sample evidence or micro-econometric results. Also, some estimation values of macro parameters from other countries can be an alternative source for the prior. This paper adopts the prior of Lubik and Schorfheide (2006). In Lubik and Schorfheide (2006), priors are based on a pre-sample of observations of U.S. and Euro from 1970:I to 1882:IV. Table 1.1 summarizes the prior distribution used in the analysis.

2.3.3 Empirical Study

In this paper, we use Dynare to estimate the small open New Keynesian model. To run Monte Carlo Markov Chain algorithm, we have to obtain a mode of the posterior and a

---

7Because there is no quarter population of Korea, and the U.S., we use the interpolation to obtain the quarterly population data.

8Public-housing bond interest rate have been used widely many literatures in Korea.

9For detail on priors, see Lubik and Schorfheide (2006).
Hessian computed at the mode of the posterior.\textsuperscript{10} In this paper, to compute $\text{var}(\mu|X) = \Sigma$, an inverse of the Hessian, we use an alternative method which is not used in Dynare. The way is to begin with $\Sigma = c\Omega = cI_{32}$ where $c$ is a scalar to control the acceptance probability and try and find a value for $c$ which does not imply completely useless values for the acceptance rate, says, 0.00001 or 0.99999. From this value of $c$, we can a very rough estimate of $\Omega$. Then we set $\Sigma = c\Omega$ and find a new $c$ which generate a little more reasonable acceptance rate. With the new $c$, we get a new $\Omega$. We will iterate this process until we find a good value of $\Sigma$.\textsuperscript{11} To estimate the model, we obtain 100,000 draws and the last 50,000 draws are only used to calculate the feature of the posterior.

If the Calvo parameter for imported goods, $\theta_F$ and $\theta^*_H$, governing the exchange rate pass-through, is zero, the PCP assumption will hold in home and foreign countries. If the Calvo parameter for the imported goods, however, is one, the LCP assumption will hold in both home and foreign countries. From the result, Table 1.2, we can confirm the fact that the exchange rate pass-through is incomplete thus ruling out both assumption in the short run.

Table 1.2 and Table 1.3 show the posterior moments of the structural parameters when data from Korea and US in period 1 (before the Asia Crisis) and period 2 (after the Asia Crisis), respectively, are used to construct the likelihood function. The parameter characterizing the nominal rigidity on the domestically produced Korean good in the Korean market, $\theta_H$, is estimated to be about 0.35 and 0.36 in period 1 and 2 respectively. The 5th and 95th percentiles are from 0.27 to 0.56 in period 1 and from 0.23 to 0.50 in period 2 respectively. The nominal rigidity in price of Korean goods in Korean market is not affected by the Asia Crisis and is very low relative to the nominal rigidity in price of U.S.

\textsuperscript{10}For this, appendix C.

\textsuperscript{11}In repeating it, we can find a $\Omega$ converged. For detail on this procedure, see Geweke (2005).
goods, $\theta^*_F$, in U.S., 0.97 and 0.90 in period 1 and 2 respectively. The posterior mean of $\theta_H$ is lower than one reported by Elekdag et al. (2005) who estimate the Calvo parameter for Korea and report that a median value is 0.505 and the 5th and 95th percentiles lie between 0.48 and 0.53.\textsuperscript{12} The result of nominal rigidity of U.S. good in the U.S., $\theta^*_F$, is very high relative to other literatures; Schorfheide (2005) reports 0.55 in the period 1960-1977. Lubik and Schorfheide (2006) report 0.66 in the period, 1983-2002.

The Calvo parameter of the imported goods in Korea and U.S., $\theta_F$ and $\theta^*_H$, governs the exchange rate pass-through. The nominal rigidity on the price of US goods in Korea, $\theta_F$, decreases from about 0.53 in period 1 to about 0.45 in period 2, that is, the degree of exchange rate of pass-through increases. This result reflects the fact that after the Asia Crisis Korean exchange rate regime had been changed gradually from fixed rate regime to floating rate regime. Therefore, it seems that the price of imported goods is more affected from the exchange rate fluctuation, the nominal rigidity is less fixed and the exchange rate pass through is growing gradually. The 5th and 90th percentiles are from 0.18 to 0.71 in period 1 and from 0.24 to 0.71 in period 2 respectively. From this observation, we can analogize that won/dollar exchange rate has a larger effect on the inflation in Korea in period 2 than in period 1. The Calvo parameter of the Korean goods in the U.S., $\theta^*_H$, increases from about 0.66 to about 0.70. This suggests that importers in both countries have a stronger tendency to preset price of imported goods in dollars after the Asia Crisis than before the Asian crisis. This is consistent with Devereux et al. (2004), who claim that imported goods prices have tendency to be preset in the currency of a country that has the more stable money growth. Another prediction of Devereux et al. (2004) is that as the exchange rate pass-through is increasing, the elasticity of substitution between domestic and foreign goods, $\eta$, is decreasing. The elasticity of substitution between domestic and

\textsuperscript{12}Elekdag et al. (2005) did not divide the Calvo parameter into the Calvo parameters for home goods and for foreign goods.
foreign goods is increasing from about 0.59 in period 1 to about 1.16 in period 2 and the exchange rate pass-through is increasing, that is, the Calvo parameter is decreasing, in Korea. The fact that the elasticity of substitution between domestic and foreign goods is high suggests that Korea output is sensitive to changes in the terms of trade. So we can infer that incomplete pass-through can be important in driving output in Korea because the terms of trade is related to the law of price gap.

The policy rule coefficients for the Korea do not dramatically change after the Asia Crisis. The inflation policy rule parameter $\varphi_1$ increased from about 1.50 to about 1.55 while $\varphi_2$ decreased from 0.50 to 0.30 and $\varphi_3$ decreased from 0.1 to 0.09. The policy rule in Korea is largely responding to deviations of inflation from its target rate. But the policy rule in the U.S. has strong responses to both inflation and output growth movements. The policy rule coefficient for the U.S., $\varphi_1^*$ and $\varphi_2^*$, decrease from 1.63 to 1.39 and from 1.34 to 1.11 respectively while $\varphi_3^*$ is hardly changed at about 0.08. Habit formation, $h$, is low, 0.25 in period 1 and 0.31 in period 2. After the Asia Crisis the former times consumption gets more important. The quarter-to-quarter percentage growth rate of $A_{Wt}$ in steady state, $\gamma^{(A)}$, and steady state inflation rate, $\pi^{(A)}$, are decreasing from 0.78 and 7.0 in period 1 to 0.45 and 3.08 in period 2, respectively. This result coincides that Korea have been experienced low growth rate and inflation rate after the Asia Crisis.

Dynare shows “MCMC univariate diagnostics” which is the main source of feedback to gain confidence with results. Figure 1.1 displays MCMC univariate diagnostics of some variables for the “Before Asian Crisis” and the “After Asian Crisis.” The horizontal axis is the number of Random walk H-M iterations and the vertical axis is the measure of the parameter moments, with the first, corresponding to the measure at the initial value of the

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13 Elekdag et al. (2005) reported that the median value of inflation policy rule parameter is 2.238, the median value of output policy rule parameter is 0.018, and median value of the exchange rate policy rule parameter is 0.054.
Random walk M-H iterations. If chains are relatively constant and converge and results between the various chains are close, the results from MCMC univariate diagnostics can be sensible. This figure display two chains in each case are close and stable.

2.4 Conclusion

Although many Korean literatures have studied a monetary policy of Korea with NOEM models, it is not easy to find a literature to study New Keynesian DSGE model with Bayesian approach. And many literature related to pass-through have focused on the relationship between export and export prices. Recent many literatures, however, have studied the New Keynesian DSGE model under incomplete pass-through. Thus the attempt of this work to analyze the New Keynesian DSGE model with Korean data will be able to contribute to provide valuable information to policymakers in Korea. The study of the exchange rate pass-through is important because we should understand the properties of the pass-through to analyze the predicted volatility of real exchange rate, the international transmission mechanism of monetary shocks, and the optimal monetary policy. Therefore, this paper can offer basic information to researches to study these issues for Korea.

This paper estimates the parameters of a small-scale two-country model with Bayesian estimation techniques based on Korea and the U.S. From the empirical result we can find that all both countries experience the incomplete pass-through. In estimating the nominal rigidity the degree of nominal rigidities in domestic and import sector of Korea, $\theta_H$ and $\theta_F$, are less than one of the U.S. And the domestic sector Calvo parameter, $\theta_H$, is always less than the import sector, $\theta_F$. The nominal rigidity parameter for imported goods in Korea, $\theta_F$, decreases from period 1 to period 2. This reflects that the price of imported goods in Korea get more affected from the exchange rate fluctuation, the nominal rigidity is less fixed and the exchange rate pass-through is growing gradually since the Korean exchange
rate regime changed from the fixed rate regime to the floating rate regime. The elasticity of substitution between domestic and foreign goods, \( \eta \), increases from period 1 to period 2 in Korea. This suggests that incomplete pass-through can play an important role in driving output in Korea because Korea output is sensitive to changes in the terms of trade which is related to the law of price gap. The nominal rigidity parameter for Korean goods in the U.S., \( \theta^*_H \), increases from period 1 to period 2. This suggests that although the important two assumptions on price setting, PCP and LCP, do not hold in this paper, importers in Korea have a stronger tendency to follow the PCP assumption and importers in the U.S. have a stronger tendency to follow the LCP for Korean goods after the Asia Crisis. We can find that there is a dramatic decrease in the estimated steady state inflation rate, \( \pi^{(A)} \), and the quarter-to-quarter percentage growth rate of \( A_{Wt} \), \( \gamma^{(A)} \), from the before section to the after section. We can think this of natural result because Korea has experienced low economic growth rate and low inflation rate after 1997.

This paper’s results are consistent with Devereux et al. (2004) in that the import firm in both countries will tend to pre-set their prices in the country that has the more stable money growth and the exchange rate pass-through is higher the lower is the elasticity of substitution between domestic and foreign goods.
<table>
<thead>
<tr>
<th>Name</th>
<th>Density</th>
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<th>standard deviations</th>
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<tr>
<td>$\theta_F$</td>
<td>Beta</td>
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Table 2.1: Prior Distribution
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Table 2.2: Posterior Distribution for “Before the Asian Crisis:1987:1 1996:IV”
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</tr>
<tr>
<td>$\sigma_Z$</td>
<td>Inv Gamma</td>
<td>0.5841</td>
<td>0.0485</td>
<td>0.3397</td>
<td>0.8162</td>
<td>0.50</td>
<td>4.00</td>
</tr>
<tr>
<td>$\sigma_E$</td>
<td>Inv Gamma</td>
<td>3.5402</td>
<td>0.3408</td>
<td>3.0050</td>
<td>4.0569</td>
<td>3.50</td>
<td>4.00</td>
</tr>
</tbody>
</table>

Table 2.3: Posterior Distribution for “After the Asian Crisis:1999:1 2005:IV”
Figure 2.1: MCMC univariate diagnostics
Chapter 3

Determinacy in the New Keynesian Model with Market Segmentation

3.1 Introduction

There are two interesting empirical facts in the macroeconomic field. First, prices are sticky and second, asset markets are segmented. These two empirical facts have been separately developed to provide powerful frameworks for a monetary economic model in the New Keynesian model and the market segmentation model, respectively, over two decades. The exogenous market segmentation model has two features that the standard New Keynesian model does not capture.\footnote{The exogenous market segmentation means that there are two types of households, traders, who can participate in the asset market, and non-traders, who cannot participate in the asset market. Traders and non-traders cannot switch sides. In here, we assume that the market segmentation means the exogenous market segmentation.} First, traders can smooth their consumption but non-traders cannot smooth their consumption.\footnote{According to Mulligan and Martin (2000), as of 1989, 59 percent of the U.S. population did not hold interest-bearing assets. Mankiw (2000) argues that there are three reasons why two canonical models, the Barro-Ramsey model and the overlapping generations Diamond-Samuelson model, are not appropriate for policy analysis. First, some agent follows the permanent-income hypothesis and the others follow the simple rule-of-thumb of consuming their current income. Second, many households have near-zero net worth. Third, much wealth is concentrated in the hands of a few people and bequests are an important factor in great wealth accumulation.} Second, the market segmentation model can generate the liquidity effect.\footnote{Edmond and Weill (forthcoming) provide intuition on how the segmented market model can generate the liquidity effect in the short run and the Fisher effect in the long run.} The market segmentation model, however, is yet to
explain the dynamics of inflation and interest rates on the output, which the standard New Keynesian model can illustrate, because the latter assumes an exchange economy. Some economists have tried to incorporate the market segmentation model into the standard New Keynesian model since the early 2000s because the individual disadvantages of the two models can thus be overcome.

Gali et al. (2004), Nakajima (2006), Gali et al. (2007), and Lama and Medina (2007) have tried to introduce the market segmentation model into the New Keynesian model to analyze the monetary or fiscal policies implemented at that time. We find an interesting assertion from Gali et al. (2004) and Nakajima (2006). In Gali et al. (2004), Ricardians, or traders, can smooth their consumption but non-Ricardians, or non-traders, cannot smooth their consumption. They, however, cannot generate the liquidity effect because their economy does not have money. Nakajima (2006), however, produces only the liquidity effect. In his model, since there are no capital stocks in the closed economy, both traders and non-traders cannot smooth their consumption over time. This difference between these models produces contrasting results with regard to the Taylor rule. Gali et al. (2004) argue that the presence of non-Ricardians, or non-traders, calls for a more aggressive interest rule for determinacy than the Taylor principle suggests. Nakajima (2006), however, argues that if the liquidity effect exists, the Taylor rule should be passive for determinacy.

From the mid-1980s to early 2000s, the U.S. economy experienced remarkably reduced fluctuations in terms of both output and inflation. Many economists have called this striking feature of the economy “the Great Moderation.” Bernanke (2004) addressed that

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4Landon-Lane and Occhino (2008) try to introduce the production economy into the exogenous market segmentation model, but they do not assume sticky prices.

5Gali et al. (2007) study the fiscal policy under Gali et al. (2004). Lama and Medina (2007) study the monetary policy in an open economy by introducing the two features of the exogenous market segmentation into the New Keynesian model; their approach, however, follows the Ramsey approach and hence they do not study the issue of determinacy.
improved performance of the monetary policy might have dampened the fluctuations.\textsuperscript{6} Many economists have argued that the change in the monetary policy after Volcker was appointed as the chairman of the Federal Reserve Board had improved the economic performance and had stabilized the variability in both output and inflation.\textsuperscript{7} \textsuperscript{8} Given the historical lessens learned, we want to avoid fluctuations in some variables such as inflation and output from self-fulfilling expectations. The issue of the stabilization of interesting economic variables can be reduced to the issue of seeking unique solution to the macroeconomic models. Therefore, when the monetary authority implements monetary policies to stabilize the macroeconomic variables, it should significantly consider determinacy.

Woodford (2001) discusses that the Taylor principle satisfies determinacy and eliminates the self-fulfilling spiral of inflation in the neo-Wicksellian model, and shows that the Taylor rule can be a proper tool to stabilize inflation and output gap. Carlstrom and Fuerst (2005) discuss how the introduction of capital and investment affects determinacy in a discrete-time model. In the Calvo sticky price model, if the monetary authority uses forward-looking rules, the investment results in a very tight condition for determinacy. In their calibration, the existence of investment may yield determinacy that was originally deemed impossible in a forward-looking rule. In a current-looking Taylor rule, the introduction of investment does not affect the condition for determinacy, as in an aggressive Taylor rule. Dupor (2001), however, shows that the introduction of capital may yield a passive Taylor rule in the continuous-time model. Clarida et al. (2000) argue that the Taylor rule satisfied the Taylor principle in the Great Moderation period and Lubik and Schorfheide (2004) show


\textsuperscript{7}For details, see Clarida et al. (2000).

that the Taylor rule satisfying the Taylor principle increases determinacy in the Great Moderation period. Davig and Leeper (2007) argue that the macroeconomic fluctuations can be more frequent when the Taylor principle is not satisfied since monetary policy results in more the impact of fundamental shocks.

This paper starts by asking the following question: is the Taylor principle still a good criterion for uniqueness in the New Keynesian model with the market segmentation having the liquidity effect and the difference in consumption smoothing between traders and non-traders? This paper’s key results are as follows. First, the Taylor rule does not need to be too aggressive or passive. The Taylor rule can satisfy the Taylor principle in the New Keynesian model with market segmentation. Second, the threshold of the coefficient of inflation in the Taylor rule that is the minimum for satisfying determinacy is constant regardless of the proportion of traders. These mean that the contrasting influences of the liquidity effect and the difference in consumption smoothing may be offset. Third, the behaviors of non-traders in response to exogenous shocks are very different from those of traders. This can result in impulse response of output and consumption to exogenous shocks being hump shaped.

This paper is organized as follows. We set the New Keynesian model with the market segmentation in section 2. We analyze the equilibrium and the linearization of the economy in section 3. We discuss the main results of this paper in section 4. We conclude in section 5.

3.2 Model

We combine two models, the New Keynesian model in the supply side and the market segmentation model in the demand side. In terms of demand, households are assumed to be divided into two types, traders, who can participate into the financial market and can
accumulate the capital, and non-traders, who cannot participate into the financial market and cannot accumulate the capital. Therefore, the asset market is exogenously segmented as in Alvarez et al. (2001). For consumption, both traders and non-traders are restricted by the cash-in-advance (CIA) constraint. The market segmentation can yield the liquidity effect. In this economy, only traders can accumulate the capital and this distinguishes traders from non-traders in that only the traders can smooth their consumption over time. Money is supplied through the open market operation, and hence the money injection is absorbed only by traders. Traders and non-traders do not choose their labor supply. In terms of supply, we assumed that there are two types of firms, the intermediate goods firms and final goods firms. The intermediate goods firm produces differentiated intermediate goods in a monopolistic competition market and follows the Calvo (1983) pricing setting. The final goods firms produce one final good in a perfect competition market.

### 3.2.1 Households

#### 2.1.1 Traders

We assume that there are two types households, traders and non-traders, who live infinitely, as mentioned above. A fraction of households $\lambda$ can accumulate their own physical capital and can participate in the asset market to trade bonds issued by the government, and hence can smooth their consumption over time. They receive profits from the intermediate goods firms and a lump-sum transfer from the government, and provide their labor to intermediate goods firms. We refer to such households as traders. Since only traders can participate in the asset markets, they absorb all of the money injected by the monetary authority. Let a superscript “$T$” denote the traders’ variables. A representative trader

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9 We adopt the exogenous market segmentation due to simplicity, for comparison with previous papers, and because as stated in the second footnote, Mulligan and Martin (2000) and Mankiw (2000) support the existence of non-traders.
maximizes the lifetime utility,

\[ E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{(C_t^T)^{1-\sigma}}{1-\sigma} - \frac{(N_t^T)^{1+\varphi}}{1+\varphi} \right), \]

subject to the CIA constraint\(^{10, 11}\)

\[ P_t C_t^T + R_t^{-1} B_{t+1} = M_{t-1}^T + B_t + T_t, \tag{3.1} \]

where \(B_t\) denotes the risk-free one-period bonds issued by the government that are carried from period \(t-1\) to period \(t\), \(P_t\) is the price level in period \(t\), \(C_t^T\) is the traders’ consumption in period \(t\), \(R_t\) is the nominal return on the bonds in period \(t\), \(M_{t-1}^T\) is the nominal money balance of traders carried from period \(t-1\) to period \(t\), and \(T_t\) is the lump-sum transfer from the government in period \(t\); and subjected to the budget constraint

\[ M_t^T + P_t I_t^T = W_t N_t^T + R_t^k K_t^T + \frac{\Pi_t}{\lambda}, \tag{3.2} \]

where \(I_t^T\) is the investment in period \(t\), \(K_t^T\) is the capital stock in period \(t\), \(N_t^T\) is the work hours supplied by the traders, \(W_t\) is the nominal wage, \(R_t^k\) is the nominal rental, and \(\Pi_t\) is the return from the immediate goods firms.\(^{12}\) The traders’ capital accumulation equation

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\(^{10}\) We assume that the equality in the CIA condition always holds.

\(^{11}\) In general, in the market segmentation model, the velocity shock is introduced to capture the relation between money and prices. For simplicity, we assume that there is no velocity shock.

\(^{12}\) In this model, the investment is not restricted by the CIA constraint. Wang and Wen (2006) argue that the output’s response to monetary shocks becomes more persistent with the degree of capital constraint by the CIA condition. Instead, for output persistence, we introduce adjustment costs in this model. For more details on the relation between the capital restricted by the CIA condition and the output persistence, see Wang and Wen (2006).
is

\[ K^T_{t+1} = (1 - \delta)K^T_t + \phi \left( \frac{I^T_t}{K^T_t} \right) K^T_t. \]  

(3.3)

\[ \phi' > 0, \quad \phi'' \leq 0, \quad \phi' (\delta) = 1, \quad \phi (\delta) = \delta, \]

where \( \phi(I^T_t/K^T_t)K^T_t \) denotes the capital adjustment cost and \( \delta \) is the rate of depreciation.

The first-order conditions for the traders’ optimization problem are as following, Euler equation,\(^{13}\)

\[ \frac{1}{R_t} = \beta E_t \left\{ \left( \frac{C^T_{t+1}}{C^T_t} \right)^{-\sigma} \left( \frac{P_t}{P_{t+1}} \right) \right\}, \]  

(3.4)

and Tobin \( q \), the (real) shadow value of capital, given as

\[
Q_t = E_t \left\{ \frac{1}{R_{t+1}} \left( \frac{R^k_{t+1}}{P_{t+1}} + Q_{t+1} \left( (1 - \delta) + \phi_{t+1} - \left( \frac{I^T_{t+1}}{K^T_{t+1}} \right) \phi'_{t+1} \right) \right) \frac{P_{t+1}}{P_t} \right\} \]

(3.5)

\[ Q_t = \frac{1}{\phi'_t}. \]

The elasticity of the investment-capital ratio with respect to \( Q \) is assumed as

\[ -\frac{1}{\phi''(\delta) \delta} \equiv \eta. \]

We do not derive the equation of labor supply by traders because we will assume that labor hours are assumed to be determined by firms.\(^{14}\)

2.1.2 Non-traders

\(^{13}\)In this economy, only traders can obtain the Euler equation. Therefore, the real interest rate will be related only to traders’ consumption.

\(^{14}\)This assumption is from Gali et al. (2007) and is made for simplicity. Under this assumption, the real wage is determined to equal the marginal rate of substitution at all times, and it is optimal for households to supply labor as much as firms’ demand.
The remaining fraction of households $1 - \lambda$ can consume only with their wage and cannot participate in the asset market and cannot accumulate capital. Therefore, they cannot smooth their consumption over time and their existence violates the persistent income hypothesis. We will call them non-traders. A superscript “N” denotes the non-traders’ variables. A representative non-trader maximizes the lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{(C_t^N)^{1-\sigma}}{1-\sigma} - \frac{(N_t^N)^{1+\varphi}}{1+\varphi} \right),$$

subject to the CIA constraint

$$P_t C_t^N = M_{t-1}^N,$$

where $C_t^N$ is the non-traders’ consumption in period $t$ and $M_{t-1}^N$ is the non-traders’ money balance carried from period $t - 1$ to period $t$; and subject to the budget constraint

$$M_t^N = W_t N_t^N,$$

where $W_t$ is the nominal wage and $N_t^N$ is the labor supply from non-traders. From the CIA constraint, the consumption of non-traders is

$$C_t^N = \frac{W_{t-1}}{P_{t-1}} \frac{P_{t-1}}{P_t} N_{t-1}^N. \quad (3.6)$$

This equation will provide us with a logic to explain the non-traders’ behavior in response to exogenous shocks. Further, non-traders’ labor will be determined by the firms’ labor demand.

2.1.3 Wage Schedule
Let us assume that there exists a schedule for wage determination

\[ E_t \frac{W_t}{P_{t+1}} = H(C_{t+1}, N_t), \tag{3.7} \]

where \( H_C > 0 \) and \( H_N > 0 \). Under this schedule, each firm will hire labor equally across households regardless of the household’s type. Therefore, \( N_t = N^T_t = N^N_t \) for all \( t \). For consistency with balanced growth, \( H \) can be expressed as \( E_t C_{t+1}^{\sigma} N_t^\varphi \).\textsuperscript{15}

3.2.2 Firms

2.2.1 Final Goods Firms

We assume that final good firms produce one final good by using a continuum of intermediate goods as inputs in the perfect competitive markets. The final goods’ price is flexible. The final good production function is a constant elasticity of substitution (CES) bundler, and is given as

\[ Y_t = \left[ \int_0^1 Y_t(i)^{\frac{\epsilon - 1}{\epsilon}} d[\frac{1}{\epsilon}] \right]^\frac{\epsilon}{\epsilon - 1}, \]

where \( \epsilon > 1 \) is the elasticity of substitution in production and \( Y_t(i) \) is the amount of intermediate good \( i \) required as input.

Given \( P_t \) and \( P_t(i) \), we can obtain the demand for a good \( i \) from the zero-profit problem for the final goods firms, as follows;

\[ Y_t(i) = Y_t \left( \frac{P_t}{P_t(i)} \right)^\epsilon. \]

By incorporating the demand for a good \( i \) into the function of \( Y_t \), we can obtain the

\textsuperscript{15}For more details, see Gali et al. (2007).
final good pricing rule as

$$P_t = \left[ \int_0^1 P_t(i)^{1-\epsilon} di \right]^{\frac{1}{1-\epsilon}}.
$$

### 2.2.2 Intermediate Goods Firms

We assume that there is a continuum of intermediate goods firms indexed by $i \in [0, 1]$ producing differentiated goods and that these differentiated goods are used as inputs in the production of the final good. A firm $i$’s production function follows the Cobb-Douglas production function as follows:

$$Y_t(i) = A_t K_t(i)^\alpha N_t(i)^{1-\alpha}.
$$

(3.8)

Every intermediate goods firm wants to minimize its total cost in every period, and hence,

$$\min_{K_t(i), N_t(i)} R^k_t K_t(i) + W_t j N_t(i)
$$

subject to $Y_t(i) = A_t K_t(i)^\alpha N_t(i)^{1-\alpha}$. The first-order condition for this problem is

$$\frac{(1-\alpha)R^k_t}{\alpha W_t} = \frac{N_t(i)}{K_t(i)}.
$$

(3.9)

We can obtain the demand functions for the labor and capital of a firm $i$ by incorporating the above equation into the production function as

$$N_t(i) = \frac{Y_t(i)}{A_t} \left( \frac{R^k_t (1-\alpha)}{W_t \alpha} \right)^\alpha,
$$

(3.10)

$$K_t(i) = \frac{Y_t(i)}{A_t} \left( \frac{R^k_t (1-\alpha)}{W_t \alpha} \right)^{\alpha-1}.
$$

(3.11)
Further, we can write the real marginal cost for this firm by incorporating these equations into the total cost function as

$$RMC_t = \frac{W_t/P_t}{(1-\alpha)A_t} \left( \frac{(R^k_t/P_t) \left( 1 - \alpha \right)}{(W_t/P_t) \alpha} \right)^\alpha. \tag{3.12}$$

In monopolistic competitive markets, the intermediate goods firms cannot change their price every period as in the Calvo (1983). A fraction of all intermediate goods firms $\theta$, price stickiness, cannot adjust their price while the remaining fraction $1-\theta$ can optimally choose their price in every period. The firms that can adjust their price in period $t$ are selected randomly. The optimization problem for the intermediate firms in period $t$ is the choice of the optimal price $P_t^*$ to maximize

$$\max_{P_t^*} E_t \sum_{j=0}^{\infty} \theta^j Q_{t,t+j} Y_{t+j} \left( \frac{P_{t+j}}{P_t^*(i)} \right)^\epsilon \left[ P_t^* (j) - \frac{W_{t+j}}{(1-\alpha)A_{t+j}} \left( \frac{R_{t+j}^k(1-\alpha)}{W_{t+j}\alpha} \right)^\alpha \right],$$

subject to $Y_{t+j}(i) = A_{t+j}K_{t+j}(i)^\alpha N_{t+j}(i)^{1-\alpha}$.

We can obtain the optimal price $P_t^*$ by solving this problem as

$$P_t^* = \frac{\epsilon}{\epsilon - 1} \frac{E_t \sum_{j=0}^{\infty} (\beta\theta)^j Y_{t+j} W_{t+j} A_{t+j} \left( \frac{R_{t+j}^k(1-\alpha)}{W_{t+j}\alpha} \right)^\alpha}{E_t \sum_{j=0}^{\infty} (\beta\theta)^j Y_{t+j}},$$

where $\frac{\epsilon}{\epsilon - 1}$ is the mark-up. We can express the price level by combining the final goods pricing rule and the Calvo pricing rule as

$$P_t = [\theta P_{t-1}^{1-\epsilon} + (1-\theta)(P_t^*)^{1-\epsilon}]^\frac{1}{1-\epsilon}.$$
3.2.3 Government

The government can inject money into the economy through the open-market operation, and hence, the government’s budget constraint is

\[ M_t + E_t R_t^{-1} B_{t+1} \frac{\lambda}{\lambda} = M_{t-1} + \frac{B_t}{\lambda} + \frac{T_t}{\lambda}. \] (3.13)

From this budget equation, we can see that the injected money is absorbed only by traders through the trading of bonds in the asset market. The growth rate of money is

\[ \log\left(\frac{M_t}{M_{t-1}}\right) = \mu_t. \]

The government can implement monetary policies. We assume that there are two alternative monetary policies, the simple Taylor rule and the controlling of the money growth rate, given as

\[ r_t = r + \phi_r \pi_t + \phi_y \hat{y}_t + v_t, \] (3.14)

\[ \mu_t = \rho \mu_{t-1} + \varepsilon_t^\mu, \] (3.15)

where \( r_t (\equiv R_t - 1) \) is the nominal interest rate, \( r \) is the nominal interest rate in the steady state, \( \hat{y}_t \) is the deviation from their steady state, and \( \pi_t \) is the inflation rate. We assume the zero steady state inflation rate.
3.3 Equilibrium

3.3.1 Clearing condition

The clearing condition for consumption is

\[ C_t = \lambda C_t^T + (1 - \lambda) C_t^N. \] (3.16)

The clearing condition for the capital market is

\[ \int_0^1 K_t(i) di = K_t = \lambda K_t^T, \quad I_t = \lambda I_t^T. \] (3.17)

The clearing condition for the labor market is

\[ \int_0^1 N_t(i) di = N_t = \lambda N_t^T + (1 - \lambda) N_t^N = N_t^T = N_t^N. \] (3.18)

The clearing condition for the money market is

\[ M_t = \lambda M_t^T + (1 - \lambda) M_t^N = P_t C_t, \] (3.19)

The above implies that the quantity theory holds for this economy.

The clearing condition for the goods market is

\[ Y_t = C_t + I_t \] (3.20)
The profit is as follows:

\[
\Pi_t = P_t \int_0^1 \Pi_t(i) \, di = \int_0^1 P_t(i) Y_t(i) \, di \quad \text{and} \quad \int_0^1 Y_t(i) \, di.
\]

\[
= P_t Y_t - \frac{W_{t+j}}{(1-\alpha) A_{t+j}} \left( \frac{R_{t+j}^k (1-\alpha)}{W_{t+j} \alpha} \right)^\alpha \int_0^1 Y_t(i) \, di.
\]

(3.21)

### 3.3.2 Linearization

First, we have to derive the steady state values of the variables. The appendix B in the end of this dissertation will present the process for the steady state values of the variables. The small characters with a hat denote the variables' deviation from the steady state. After linearizing, we can derive a difference system to analyze the dynamics and determinacy of this economy.

From equation (3), we can get the following linearized form for the capital accumulation:

\[
\hat{k}_{t+1} = (1-\delta) \hat{k}_t + \delta \hat{i}_t.
\]

(3.22)

Equation (4) provides us with the Euler equation:

\[
\hat{c}_t^T = E_t \hat{c}_{t+1}^T + \frac{1}{\sigma} (E_t \pi_{t+1} - \hat{r}_t).
\]

(3.23)

In this economy, we have to note that the Euler equation holds only for the traders, and the IS curve in the New Keynesian model with the market segmentation will be different from that in the standard New Keynesian model. We can derive a linearized form for given Tobin q:

\[
\hat{q}_t = \beta E_t \hat{q}_{t+1} + \chi_q E_t \hat{r}_{t+1}^k - E_t (\hat{r}_{t+1} - \pi_{t+1}),
\]

(3.24)

where \(\chi_q \equiv 1 - \beta (1-\delta)\). Note that \(\hat{q}_t\) depends on \(\hat{r}_{t+1}\) and not \(\hat{r}_t\). We can also obtain an
alternate form for Tobin $q$ from its definition:

$$
\hat{q}_t = \frac{1}{\eta} \hat{i}_t - \frac{1}{\eta} \hat{k}_t.
$$

From equation (6), the non-traders’ current consumption is

$$
\hat{c}^N_t = \hat{w}_{t-1} - \pi_t + \hat{n}_{t-1}.
$$

We can obtain the labor demand for intermediate goods firms from equation (7) as

$$
\varphi \hat{n}_t + \sigma E_t \hat{c}_{t+1} = \hat{w}_t - E_t \pi_{t+1}.
$$

A linearized form for the aggregate production function is

$$
\hat{y}_t = \hat{a}_t + \alpha \hat{k}_t + (1 - \alpha) \hat{n}_t.
$$

We can derive a linearized form for the first-order condition for the intermediate goods firms from equation (9) as

$$
\hat{r}^k_t - \hat{w}_t = \hat{n}_t - \hat{k}_t.
$$

From equation (10) and (11), the demands for labor and capital are as follows:

$$
\hat{n}_t = \hat{y}_t - \hat{a}_t + \alpha (\hat{r}^k_t - \hat{w}_t),
$$

$$
\hat{k}_t = \hat{y}_t - \hat{a}_t + (\alpha - 1) (\hat{r}^k_t - \hat{w}_t).
$$

---

16Actually, in this equation, the elasticity of labor supply should be replaced with the elasticity of wages with respect to hours. In the case of perfect competition in labor markets, however, the elasticity of labor supply is equal to the elasticity of wages with respect to hours because real wages should be the marginal rate of substitution.
We note that since equation (29) and (30) can be derived from equation (27) and (28), we do not use these equations to derive the difference equation system.

A linearized form for the real marginal cost function can be derived from equation (12) as

$$\hat{mc}_t = (1 - \alpha)\hat{w}_t + \alpha\hat{r}_k^t - \hat{a}_t = -\hat{y}_t + n_t + \hat{w}_t = -\hat{y}_t + \hat{k}_t + \hat{r}_k^t. \quad (3.31)$$

The second equality is derived from equation (28) and (29). The Taylor rule is

$$\hat{r}_t = \phi_\pi\pi_t + \phi_y\hat{y}_t + \nu_t. \quad (3.32)$$

From the definition of $\mu_t$, $\mu_t$ can be expressed as follows:

$$\mu_t = \hat{m}_t + \pi_t - \hat{m}_{t-1} = \hat{c}_t + \pi_t - \hat{c}_{t-1}. \quad (3.33)$$

The current consumption can be written as

$$\hat{c}_t = \lambda\tau_c\hat{c}_t^T + (1 - \lambda\tau_c)\hat{c}_t^N, \quad (3.34)$$

where $\tau_c \equiv \frac{CT}{C}$.  

A linearized form for the aggregate capital stock is

$$\hat{k}_t = \hat{k}_t^T. \quad (3.35)$$
The aggregate output has two components, consumption and investment, given as

\[
\hat{y}_t = \tau_y \hat{c}_t + (1 - \tau_y)\hat{i}_t, \tag{3.36}
\]

where \(\tau_y \equiv \frac{\bar{C}}{\bar{Y}}\).

The optimal price, given by equation (13), provides us with an equation to describe the dynamics of inflation:

\[
\pi_t = \beta E_t \pi_{t+1} + \kappa \hat{\mu}_t, \tag{3.37}
\]

where \(\kappa \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}\). The exogenous shocks are given as follows:

\[
\begin{align*}
\hat{\alpha}_t &= \rho_a \hat{\alpha}_{t-1} + \varepsilon_t^\alpha \\
\hat{\nu}_t &= \rho_v \hat{\nu}_{t-1} + \varepsilon_t^\nu \\
\hat{\mu}_t &= \rho_\mu \hat{\mu}_{t-1} + \varepsilon_t^\mu
\end{align*}
\]

### 3.4 Dynamics

To study the determinacy and dynamics of this economy, we have to derive a difference equation system by combining equations (22)-(37). After a tedious arithmetic operation, we derived the following difference equation system for this economy:

\[
A_E x_{t+1} = B x_t, \tag{3.38}
\]

where \(x_t \equiv [\hat{k}_t, \hat{n}_t, \hat{c}_t, \pi_t, \hat{h}_t]'\) and all elements in matrices \(A\) and \(B\) are functions of the structural parameters of this model.\(^{17}\) To obtain a unique solution for the difference system, since there are two predetermined variables, \(\hat{k}_t\) and \(\hat{h}_t\), three eigenvalues of \([A^{-1}B]\) should

\(^{17}\)We drop the terms relating to exogenous shocks since we only analyze determinacy.
lie outside and two eigenvalues of \([A^{-1}B]\) should lie inside the unit circle.\(^{18}\) The appendix B contains details about derivation of this system.

### 3.4.1 Calibration

For the baseline calibration, we will use the basic values from Gali et al. (2004). In this model, we assume that the time preference \(\beta\) is 0.99. This means that the real interest rate in the asset market in the steady state is about 4% annually. The elasticity of substitution in production \(\epsilon\) is assumed to be 6, which is the value consistent with 0.2 of the mark-up in the steady state. The elasticity of output with respect to capital \(\alpha\) is assumed to be 1/3. The rate of depreciation is assumed to be 0.025, implying an annual rate of 10%. The parameter prescribing the stickiness of prices \(\theta\) is set to 0.75, implying that the average duration is \((1 - 0.75)^{-1}\) quarters or one year. The elasticity of investment-capital ratio with respect to \(Q\), given as \(\eta\), is set as 1. The proportion of traders in the population \(\lambda\) is assumed to be 0.5 as in Campbell and Mankiw (1990). In the literature, 1 is set as the elasticity of substitution with regard to consumption, \(1/\sigma\), and the elasticity of labor supply in general. If \(\phi\) is set as 1, the elasticity of labor supply is the unit Frisch elasticity of labor supply. In the Taylor rule, we will assume that \(\phi_\pi = 1.5\) and \(\phi_y = 0.5\). \(\rho_a, \rho_v,\) and \(\rho_{\mu}\) are set as 0.5.

### 3.4.2 Determinacy

The literature on the market segmentation has argued that the liquidity effect, the short run negative relationship between the money supply and the nominal interest rate, may arise because the injection of money is absorbed exclusively by only some agents in this

\(^{18}\)For more details, see Blanchard and Kahn (1980).
If there exists only money, but not capital, in the New Keynesian model with market segmentation, the economy can generate the liquidity effect and in this economy the current non-traders’ consumption will be affected by the last period labor income, \( n_{t-1} + w_{t-1} \), and the current inflation, \( \pi_t \). Suppose that a sunspot boosting current consumption happens, then the monetary authority will increase the interest rate to eliminate the sunspot shock. In the normal economy the authority’s action will decrease the total consumption, but the action will force for non-traders to increase their consumption because of decrease in the current inflation in this economy. If the fraction of traders is sufficiently high, this economy will react similarly with the standard New Keynesian model to the policy. If, however, the fraction of non-traders is sufficiently high, the policy to increase the interest rate will amplify the effect of the sunspot shock since the increase in non-traders’ consumption may exceed the decrease in traders’ consumption. Therefore, if the fraction of non-traders is sufficiently high, the monetary authority may decrease the interest rate to eliminate the effect of the sunspot shock. So, the passive Taylor rule can guarantee the determinacy if there is only money in the New Keynesian model with market segmentation.\(^{19}\)

\(^{19}\)The money supply has two effects. First, the expected inflation effect. This means that the positive money supply will increase the expected inflation. Second, the segmentation effect. This means that the positive money supply will decrease the real interest rate because the increase in the real balance of traders will simulate the consumption of traders. If the segmentation effect dominates the expected inflation effect, the liquidity effect can be happened. And as the fraction of traders decreases, the liquidity effect will increase.

\(^{20}\)Nakajima (2006) argues that under the liquidity effect, a passive interest rate rule implies the unique equilibrium. His intuition is as follows: let \( \hat{r}_t = \phi_\pi \pi_t \) be the Taylor rule and let the output be fixed. If the real interest rate is constant, the Euler equation, \( \hat{r}_t = E_t \pi_{t+1} \), can be reduced to the Fisher effect, and then \( E_t \pi_{t+1} = \phi_\pi \pi_t \). In this economy, as \( \pi_t \) is a non-predetermined variable, it requires \( \pi_t > 1 \) for local determinacy. This implies that \( |\pi_{t+1}| > |\pi_t| \) is required for determinacy. When, however, the liquidity effect exists, the Fisher effect does not hold. Therefore, the active Taylor rule no longer guarantees \( |\pi_{t+1}| > |\pi_t| \). The low interest rate can support the high expected inflation under the liquidity effect. Hence, the Taylor rule may become passive to guarantee \( |\pi_{t+1}| > |\pi_t| \).
Gali et al. (2004) discuss that the existence of non-traders, referred to as non-Ricardian or the rule-of-thumb consumers in their paper, in the standard New Keynesian model with capital may affect the Taylor rule for local determinacy. They insist that when the fraction of traders \( \lambda \) is sufficiently low, the Taylor rule becomes even more aggressive for the unique equilibrium. Their intuition is as follows. If there are no non-traders in the economy and sunspot shocks increase in labor hours, then the consumption of households increases. By increasing the interest rate, the initial consumption boom can be eliminated. If, however, the fraction of non-traders is sufficiently high and so is the increase in labor hours by sunspot shocks, the consumption of traders and non-traders will increase. The increase in the interest rate will reduce only the consumption of traders because non-traders’ consumption is affected by labor income. If the increase in the consumption of non-traders is greater than the decrease in the consumption of traders, the initial consumption boom can be sustained. To eliminate the consumption boom, the monetary authority should increase the interest rate more than usual. Therefore, the Taylor rule becomes more aggressive.

There exists an interesting phenomenon. Both Nakajima (2006) and Gali et al. (2004) assume the New Keynesian model with the exogenous market segmentation. While Nakajima (2006) assumes the market segmentation with cash and without capital, Gali et al. (2004) study the market segmentation with capital and without cash. This implies that while Nakajima (2006) can obtain the liquidity effect but not the differences in consumption smoothing between traders and non-traders, Gali et al. (2004) can obtain the phenomenon that traders can smooth their consumption and non-traders cannot, but cannot obtain the liquidity effect. This difference results in the contrasting findings of these two papers. As non-traders increase, Nakajima (2006) argues that under the liquidity effect, the passive Taylor rule can generate determinacy, and while Gali et al. (2004) point out that the presence of non-traders may require the Taylor rule to be more aggressive for determinacy.
The exogenous market segmentation model, however, has the features, the liquidity effect and the difference in the consumption smoothing. As such, we can raise a question about determinacy: should the Taylor rule be too aggressive or too passive in the New Keynesian model with market segmentation? This paper will show that the Taylor rule can satisfy the Taylor principle.

Figure 2.1 shows the ranges for the parameters \((\phi_y, \phi_\pi)\) pertaining to determinacy under the current Taylor rule with the values of the structural parameters assumed above. The range for determinacy from this paper shows that a too aggressive or passive Taylor rule may not be required in our model. However, this figure does not guarantee that the Taylor rule satisfies the Taylor principle at this stage. From Figure 2.1, we can expect that the contrasting influences on determinacy from the liquidity effect and the difference in consumption smoothing may be offset under our baseline calibration. This figure is similar to the one of the standard New Keynesian model. An interesting observation from Figure 2.1 is that as the coefficient of output in the Taylor rule increases, the coefficient of inflation rate in the Taylor rule increases. This is very different from the standard New Keynesian model in which \(\phi_\pi\) and \(\phi_y\) have a negative relationship because of the standard Phillips curve.\(^{21}\) In this model, the real marginal cost is affected by the expected inflation because of the cash-in-advance constraint. In the long run, the permanent increase in inflation will increase the real marginal cost, this will increase prices then decrease the aggregate demand. Therefore, the inflation can be related negatively to the output in the long-run.

If there is an permanent increase in the inflation rate of size \(d\pi\) from the steady state,

\(^{21}\)In the standard New Keynesian model, the slope should be negative for the Taylor principle to be satisfied from the long-run positive relation between the inflation and the output. For more details, see Gali (2008).
then we can rewrite the Taylor rule as (the Appendix contains the details)

\[ d\hat{r} = \phi_{\pi}d\pi + \phi_{y}d\hat{y} = \frac{c_1\phi_{\pi} - c_2\phi_{y}}{c_1 - \phi_{y}}d\pi. \]

If the Taylor principle is satisfied, the coefficient of \( d\pi, \frac{c_1\phi_{\pi} - c_2\phi_{y}}{c_1 - \phi_{y}}, \) will be greater than 1 and can be rearranged as

\[ \phi_{\pi} > 1 + \frac{c_2 - 1}{c_1} \phi_{y}. \]

The value of \( \frac{c_2 - 1}{c_1} \) is about 0.37 from the baseline calibration, and this value exactly coincides with the slope of the relation between \( \phi_{\pi} \) and \( \phi_{y} \) in Figure 1. The ranges satisfying \( \frac{c_1\phi_{\pi} - c_2\phi_{y}}{c_1 - \phi_{y}} > 1 \) are consistent with Figure 1. Therefore, we can say that the Taylor rule satisfies the Taylor principle in the New Keynesian model with market segmentation.

Figure 2.2 shows the ranges of the parameters (\( \lambda, \phi_{\pi} \)) for determinacy under baseline calibration. As you can see in the Appendix, the fraction of traders \( \lambda \) does not affect the tangent of the relationship between \( \phi_{\pi} \) and \( \phi_{y} \) in Figure 2.1. This may mean that the threshold of the coefficient of inflation in the Taylor rule cannot be influenced by the proportion of traders. Figure 2.2 confirms this guess. The intuition behind the result is as follows. If \( \lambda \), the proportion of traders, is sufficiently high, the liquidity effect in this economy will weaken or will be dominated by the Fisher effect, and this economy will become close to the New Keynesian model with capital accumulation and without the market segmentation. As papers on the determinacy in the New Keynesian model point out, the existence of capital stock will not have an effect on the determinacy condition of the current Taylor rule under discrete-time conditions. Therefore, an economy with a sufficiently large \( \lambda \) can have a constant value of the threshold of \( \phi_{\pi} \) in response to the change in the proportion of traders \( \lambda \). This finding is consistent with the one in Gali et al. (2004).

If, however, the proportion of traders is low enough to make the liquidity effect strong and
the consumption smoothing effect weak, an economy may experience conflicting influences from the liquidity effect that require the Taylor rule to be passive for determinacy and the weaken consumption smoothing effect that requires the Taylor rule to be more aggressive for determinacy. As \( \lambda \) is decreasing, the liquidity effect may be further strengthened to allow the Taylor rule to be passive and the consumption smoothing effect become more weak to allow the Taylor rule to be more aggressive. As a result, the effect of the increased liquidity may almost be dramatically offset by the effect of the decreased consumption smoothing. Therefore, the Taylor rule may not have to be too passive or aggressive to achieve a unique solution to the system from equation (38). The result is considerably different from that in Nakajima (2006) where the Taylor rule can be passive for determinacy and from that in Gali et al. (2004) where the Taylor rule can be too aggressive when the proportion of non-traders is increasing. The exogenous market segmentation model has both features, the liquidity effect and the difference in consumption smoothing. Hence, we can expect that the New Keynesian model with market segmentation including these features may yield the Taylor principle where the interest rate should respond more one for one to the movement of inflation rates. Figure 2.2 visually shows this intuition.

Figure 2.3 shows the ranges of the parameters \((\lambda, \phi_\pi)\) for determinacy when \(\sigma\), the reciprocal of the intertemporal elasticity of substitution, changes from 2 to 4 and 5. When \(\sigma=2\), the top panel in Figure 2.3, there is no difference from Figure 2.2. When, however, \(\sigma=4\), the middle panel in Figure 2.3, the ranges for indeterminacy increase a little when the proportion of traders is too low. Moreover, you can see that when \(\sigma = 5\), the bottom panel in Figure 2.3, the range for indeterminacy increases considerably for a sufficiently low proportion of traders, \(\lambda\). From this, we can know that as the intertemporal elasticity of substitution of consumption decreases, indeterminacy increases.\(^{22}\) The intuition behind

\(^{22}\)Akay (2010) argues that the CIA model is not frequently used in the New Keynesian model because of this finding. Gali et al. (2004), however, show this finding without the CIA constraint. Therefore, we think
this is as follows. If current consumption is increased by sunspot shocks, then this will increase the expected inflation. If the intertemporal elasticity of substitution of consumption is sufficiently low, we cannot expect that the next consumption is sharply reduced by the increased expected inflation. Therefore, the initial consumption boom can be sustained. If there exists capital, since capital plays the role of a consumption buffer, the initial consumption boom by sunspot shocks cannot be sustained. From this, we get that the capital can be a factor to eliminate indeterminacy from the low intertemporal elasticity of the substitution of consumption.

In Figure 2.4, the top panel on the left side shows the ranges of the parameters \((\varphi, \phi_\pi)\) for determinacy under the baseline calibration. As you can see in the Appendix, as the elasticity of labor supply \(\varphi\) increases, the tangent of the relationship between \(\phi_\pi\) and \(\phi_y\) in Figure 2.1 will decrease, and hence, the threshold of \(\phi_\pi\) will also decrease. The first panel of Figure 2.4 shows this. The elasticity of labor supply, however, does not change the sign of the tangent. The top panel on the right side of Figure 2.4 shows the ranges of parameters \((\theta, \phi_\pi)\) for determinacy under the baseline calibration. As you can see in the Appendix, as the price stickiness \(\theta\) increases, the tangent of the relationship between \(\phi_\pi\) and \(\phi_y\) in Figure 2.1 remains constant around 1.25 and falls sharply after \(\theta\) reaches 0.75. When \(\theta\) is around 0.86 and 0.87, the sign of the slope changes from positive to negative. Then, the relation between \(\phi_\pi\) and \(\phi_y\) becomes negative. The bottom panel on the left side of Figure 2.4 shows the ranges of the parameters \((\alpha, \phi_\pi)\) for determinacy under the baseline calibration. The threshold of \(\phi_\pi\) will decrease slowly around \(\alpha=0.7\) because the change in the elasticity of output with respect to capital causes a moderate reduction in the tangent of the relationship between \(\phi_\pi\) and \(\phi_y\) in Figure 2.1. However, after around \(\alpha=0.7\), the tangent rapidly falls and the sign of the tangent changes at around \(\alpha=0.77\) or 0.78. The bottom panel on the
right side of Figure 2.4 shows the ranges of parameters \((\beta, \phi)\) the determinacy under the baseline calibration. If the time preference is less than about 0.5, the threshold of \(\phi\) is 0. Following this, as the time preference \(\beta\) increases, the threshold of \(\phi\) will rise because the tangent increases. At about \(\beta=0.95\) or 0.96, the sign of the tangent changes from negative to positive. From Figure 2.4, we can find that parameters \(\theta\), \(\alpha\), and \(\beta\) can change the sign of the slope. The values of \(\theta\), \(\alpha\), and \(\beta\), however, that affect the sign of the tangent are unreasonable. Therefore, under reasonable these parameters space, we can deduce that the relationship between \(\phi\) and \(\phi\) will be positive and that this paper’s results remain unchanged. From the above figures, we can see that the range for determinacy in this economy, the New Keynesian model with market segmentation including the two features, in response to the changes in the values of structural parameters is more stable than that from Gali et al. (2004).

### 3.4.3 Impulse response

In this section, we will show the impulse response functions of this model to analyze the dynamic properties of the economy. Figure 2.5 shows the impulse response function to the technology shock \(a_t\) under the baseline calibration with \(\rho_a = 0.5\). A 1% technological improvement will increase the output, the first panel from the left, by about 0.7 or 0.8%, and will decrease the nominal interest rate, the fifth panel from the right, by about 0.6%. The consumption, the second panel from the right, will go up about 0.8%. The traders’ consumption, the third panel from the left, rises by about 1%. The non-traders’ consumption, the third panel from the right, goes up by about 0.5% because the positive technology shock reduces the inflation rate by about 0.4%. In the next period, the consumption of non-traders declines because of the decrease in the demand for labor, the second panel from the left, by about 0.5% and the real wages, the fourth panel from the
right, by about 0.7% because of the positive technology shock. Since the technology shock sharply decreases non-traders’ consumption, the output and consumption do not respond persistently in this economy. The behavior of non-traders can be confirmed by equation (25) in which the current consumption of non-traders depends on the last period’s real wage, the last period’s labor supply, and the current inflation.

Figure 2.6 shows the impulse response function to the positive interest shock $v_t$ under the baseline calibration with $\rho_v = 0.5$. If there is a contractionary monetary policy shock, $\varepsilon_t^\nu > 0$, of 1%, the nominal rate, the fifth panel from the right, will go up by about 0.3% and this will reduce directly only the consumption of traders, the third panel from the left, by about 0.7%. Their consumption, however, will rebound promptly because of a drop in the inflation rate and investment and the rise in the return on bonds. You can see that this contractionary monetary policy shock makes the impulse response of output, the first panel from the left, and consumption, the second panel from the right, hump-shaped. It seems that this is due to the hump-shaped non-traders’ consumption impulse response, the third panel from the left. When there is a positive interest shock, the consumption of non-traders will increase immediately because of a decline in the inflation rate, the fifth panel from the left. The shock, however, will dwindle the demand for labor, the second panel from the left, and the real wage, the fourth panel from the right, and this will then sharply decrease the non-traders’ consumption by about 1.7% in the next period; this situation will sustain for eight or nine quarters.

Figure 2.7 shows the impulse response function to the positive money growth shock $\mu_t$ under the baseline calibration with $\rho_\mu = 0.5$. The existence of the liquidity effect is an important aspect in this figure because the standard New Keynesian model does not generate the liquidity effect.\footnote{See Gali (2008).} When there is an expansionary monetary policy shock,
$\varepsilon_{t}^{\mu} > 0$, both output, the first panel from the left, and inflation, the fifth panel from the left, will increase by about 0.5%. The nominal rate, the fifth panel from the right, declines by about 0.6% when the shock appears, confirming the existence of the liquidity effect. As, however, time passes, the nominal rate overshoots a little. This implies that although the liquidity effect prevails against the Fisher effect in the short run, the Fisher effect will dominate the liquidity effect in the long run. The liquidity effect will raise the traders' consumption, the third panel from the left. Although the increase in the inflation rate owing to the expansionary monetary policy shock reduces the non-traders' consumption, the third panel from the right, the rise in the demand of labor, the second panel from the left, and real wages, the fourth panel from the right, allows the non-traders' consumption to increase in the future. Therefore, non-traders' consumption is hump-shaped. This will make the impulse responses of output and the consumption, the second panel from the right, hump-shaped. From Figure 6 and Figure 7, we can induce that the hump-shaped behavior of the non-traders can generate the hump-shaped response of output and consumption to exogenous shocks.

From these impulse response functions, we can say that monetary policies can asymmetrically affect the consumption of traders and non-traders. Therefore, when the monetary authority implements monetary policies, it should pay attention to the different behaviors of traders and non-traders in response to the policy. Further, this fact motivates us to study the monetary policy and fiscal policy in the New Keynesian model with market segmentation.

Finally, we will provide a sketch on the system and the ranges for determinacy in the forward-looking Taylor rule in the Appendix without any explanation. Figure 2.8 shows that the ranges for determinacy under the forward-looking Taylor rule are not different from the ones under the current Taylor rule. We can think that the logic in the current
Taylor rule can be similarly applied to the forward-looking Taylor rule.

3.5 Conclusion

We study the Taylor rule’s condition for determinacy in the New Keynesian model with market segmentation. The standard New Keynesian model supports the Taylor principle. Papers, however, introducing the market segmentation into the New Keynesian model insists that the market segmentation can result in a departure from the Taylor principle for the determinacy. Nakajima (2006) and Gali et al. (2004) adopt only one of the two features of the market segmentation model, the liquidity effect and the difference in the consumption smoothing, respectively. Nakajima (2006) suggests that under the liquidity effect in the New Keynesian model with market segmentation, the Taylor rule is required to be passive for determinacy, but Gali et al. (2004) propose that the difference in smoothing consumption between traders and non-traders calls for the Taylor rule to be more aggressive for determinacy if the proportion of traders is sufficiently low. The general exogenous market segmentation model, however, has both features that have contrasting influences. If we want to introduce the exogenous market segmentation into the New Keynesian model, we should consider the two features. We set the New Keynesian model with market segmentation including two features. The contrasting influences from the two features are offset in this model, as we predict. The following are the results of this paper. First, the Taylor rule can satisfy the Taylor principle in the New Keynesian model with market segmentation. Second, the coefficients of the output gap and inflation are positively related. Third, although the proportion of traders is sufficiently low, the threshold of the coefficient of inflation in the Taylor rule is constant regardless of the proportion of traders since influence of the liquidity effect is offset by that of the lessened consumption smoothing. Fourth, the intertemporal elasticity of substitution of consumption $1/\sigma$ is sufficient low,
the range for indeterminacy in this economy will increase dramatically when the proportion of traders $\lambda$ is sufficiently low. However, capital can eliminate indeterminacy from the low intertemporal elasticity of substitution of consumption. Fifth, asymmetric behaviors are exhibited by traders and non-traders. As such, the impulse response of output and consumption can be hump-shaped.

While there are many papers studying the fiscal policy in the New Keynesian model with the rule-of-thumb consumers, or non-traders, and studying monetary policies in the market segmentation model without the sticky price assumption and the endogenous production, the studies on the monetary policies using the New Keynesian model with market segmentation including the two features, the liquidity effect and the difference in consumption smoothing, are rare. As we have seen above, the behaviors of traders in response to a monetary policy are very different from those of non-traders. This finding yields that the monetary authority should consider the existence of non-traders when they implement monetary policies. Therefore, we may have motivation to study monetary or fiscal policies in the New Keynesian model with market segmentation. We can apply this model to study the monetary and fiscal policies in the near future. Further, this model can be applied to analyze the policies on economic polarization between the rich (traders) and the poor (non-traders).
Figure 3.1: Determinacy Ranges for the Current Taylor Rule
Figure 3.2: Determinacy Ranges for $(\lambda, \phi_h)$ under the current Taylor Rule
Figure 3.3: Determinacy Ranges for the current Taylor Rule: $\sigma = 2, 4, \text{ and } 5$
Figure 3.4: Determinacy Ranges under the current Taylor Rule
Figure 3.5: Impulse Response to the Technology Shock
Figure 3.6: Impulse Response to the Interest Shock
Figure 3.7: Impulse Response to the Money supply Shock
Figure 3.8: Determinacy Ranges for the Forward-looking Taylor Rule
Chapter 4

Bayesian Estimation of the New Keynesian Model with Market Segmentation

4.1 Introduction

For the last ten year the Bayesian approach has been centered in estimating DSGE (Dynamic Stochastic General Equilibrium) models. Since the likelihood function of DSGE models is covered with many local maxima, local minima, and almost flat surfaces from the scarcity of data and the flexibility of DSGE models, the maximum likelihood approach has been in trouble of dealing with DSGE models. Therefore the calibration approach has been populated since 1980’s because of the trouble in evaluating the likelihood of DSGE models. The development of the Filtering theory, such like the Kalman Filter, however, has made the evaluation possible. Also, MCMC (Monte Carlo Markov Chain) facilitates the exploration and characterization of the likelihood function of DSGE models. Today, the econometric analysis on DSGE models is starting from the perspective of Bayesian. If so, what makes the Bayesian method to prevail over the other conventional econometric methods in analyzing DSGE models? First, the prior from micro-based evidences and economic theories can provide economists with useful and powerful information. Second, the Bayes theory is an optimal information processing rule, Zellner (1988). Bayesian methods satisfy the Likelihood Principle that all information in a sample are included in their likelihood function. Third, the Bayesian method can overcome problems from the lack of data
in macroeconomic models. Forth, MCMC can deal with the likelihood function of DSGE models with a lot maxima and minima and flat surfaces. Fifth, the Bayesian method can solve the misspecification and identification of DSGE models.¹

Structural empirical estimation of DSGE models suffers from two serious problems; 1) model misspecification due to the model being small and stylized and 2) parameter identification due to the flatness of the likelihood function for these large scale models. Bayesian methods to estimate DSGE model have therefore become popular since the Bayesian approach has many advantages in dealing naturally with these problems. Adolfson et al. (2005) and Smets and Wouters (2003) estimated with Bayesian technique the DSGE model with sticky price and wage for the Euro area. Justniano and Preston (2004) estimate a variety of specification of a small open economy model with Bayesian approach. A two-country setting plays an important role in the theoretical study of open economy DSGE models Among papers estimate a two-country DSGE model are those by de Walque and Wouters (2004) and Lubik and Schorfheide (2006) who use US and Euro data. Landon-Lane and Occhino (2008) estimate the market segmentation model with endogenous production economy. Smets and Wouters (2007) provide us with basic information on the Bayesian estimation of DSGE models. They employ seven exogenous shocks to eliminate the singularity problem in estimating DSGE model using Bayesian methods. Laforte (2007) compares and estimates three mechanisms of a DSGE model of the U.S. economy, the Calvo pricing model, the Wolman pricing model, and the sticky information model, using the Bayesian method. Lubik and Schorfheide (2006) provide us with a basic model to analyze the Korean economy. Wouters and Smets (2005) estimate a DSGE model for both U.S. and the euro.

Some economists have tried to combine the market segmentation model, a kind of

¹For detail on the Bayesian method for DSGE models, see An and Schorfheide (2007) Fernandez-Villaverde (2009), Fernandez-Villaverde et al. (2009)
CIA monetary model, with the standard New Keynesian model since 2000. When the market segmentation is introduced into the standard New Keynesian model, the money and the capital can play a critical role for determinacy. As you can see in Gali et al. (2004), the introduction of capital in the New Keynesian model with market segmentation requires the Taylor rule to be more aggressive. In Nakajima (2006), the introduction of money in the New Keynesian model with market segmentation requires the Taylor rule to be passive. However, when the money and capital are introduced in the New Keynesian model with market segmentation, the Taylor rule should satisfy the Taylor Principle for determinacy. The objective of this paper is to estimate the New Keynesian model with market segmentation including both money and capital developed in the chapter 2 and to compare it to the New Keynesian model with market segmentation including only capital, such like Gali et al. (2004), and the standard New Keynesian model with capital. As you can see, the New Keynesian model with market segmentation including both money and capital supports the Taylor Principle and produces the hump-shaped impulse response function of output and consumption that Gali et al. (2004) and Nakajima (2006) does not. We can see that in the New Keynesian model with market segmentation including the money and the capital, the fraction of traders who can participate in the asset market do not affect the determinacy of the model. If so, the question of this paper is if the New Keynesian model with markets segmentation including both money and capital has more better theoretical explanation on the real economy than the New Keynesian model with market segmentation such like Gali et al. (2004) and Nakajima (2006), does the New Keynesian model with market segmentation including both money and capital have more better empirical performance than the conventional New Keynesian model with market segmentation including either money or capital?

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3In this paper, henceforth, the term of the New Keynesian model with market segmentation will be
The organization of this paper is as follows; the section 2 will introduce the New Keynesian model with market segmentation, the section 3 will explain estimation results, and the section 4 concludes. In addition, we will provide the procedure on Bayesian estimation on DSGE models in an Appendix.

4.2 Model

From the side of demand, the asset market is exogenously segmented in this economy, so there are two types of households, traders and non-traders. Traders can participate in the asset market and accumulate the capital but non-traders cannot both. Therefore, traders can smooth their consumption but non-traders cannot. Since money injections is supplied through the open market operation in this model, all money injections are absorbed by only traders and this results in the liquidity effect that the increase in money induces the decrease in the interest rate in the short-run. Also traders receive profits from the intermediate goods firm. Labors from traders and non-traders are determined by the intermediated goods firm for simplicity. The only source of non-traders’ consumption is the labor income. Non-traders do not solve their maximizing lifetime utility problem because the labor is determined by the intermediated goods firms in this economy. The consumption of traders and non-traders will be restricted by the CIA (Cash in Advance) restriction. From the side of supply, this economy follows the standard New Keynesian model with capital. The price setting procedure follows the Calvo price setting. The production markets are divided into two sectors, the final goods market and the intermediate goods market. The intermediate goods are produced by labors and capitals and used as inputs in producing the final goods. The market of intermediate goods is monopolistic competitive but the market of final goods is perfect competitive. In this paper, a superscript “T” means

refereed as the New Keynesian model with market segmentation including both money and capital.
variables for traders and a superscript “N” variables for non-traders.\(^4\)

### 4.2.1 Linearization of the model

In this subsection, we will provide our linearized model. We can get the following linearized forms for this economy as follows: the capital accumulation:

\[
\hat{k}_{t+1} = (1 - \delta)\hat{k}_t + \delta \hat{i}_t, \tag{4.1}
\]

where \(\delta\) is the rate of depreciation.

The Euler equation for traders:

\[
\hat{c}_t^T = E_t \hat{c}_{t+1}^T + \frac{1}{\sigma} (E_t \pi_{t+1} - \hat{r}_t), \tag{4.2}
\]

where \(1/\sigma\) is the elasticity of substitution with regard to the consumption. In this economy, since the Euler equation holds only for the traders, the IS curve in this model will be different from that in the standard New Keynesian model.

We can derive a linearized form for given Tobin \(q\):

\[
\hat{q}_t = \beta E_t \hat{q}_{t+1} + \chi_q E_t \hat{r}_{t+1}^k - E_t (\hat{r}_{t+1} - \pi_{t+1}), \tag{4.3}
\]

where \(\chi_q \equiv 1 - \beta (1 - \delta)\) where \(\beta\) is the time preference.

We can also obtain an alternate form for Tobin \(q\) from its definition:

\[
\hat{q}_t = \frac{1}{\eta} \hat{i}_t - \frac{1}{\eta} \hat{k}_t,
\]

where \(\eta\) is the elasticity of investment-capital ratio with respect to Tobin \(p\).

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\(^4\)For more detail for this model, see the chapter 2 in this dissertation.
The non-traders’ current consumption is

\[ \hat{c}_t^N = \hat{w}_{t-1} - \pi_t + \hat{n}_{t-1}. \] (4.4)

We can obtain the labor demand for intermediate goods as following

\[ \varphi \hat{n}_t + \sigma E_t \hat{c}_{t+1} = \hat{w}_t - E_t \pi_{t+1}, \] (4.5)

where \( \varphi \) is the elasticity of labor supply.

A linearized form for the aggregate production function is

\[ \hat{y}_t = \hat{a}_t + \alpha \hat{k}_t + (1 - \alpha) \hat{n}_t, \] (4.6)

where \( \alpha \) is the elasticity of output with respect to capital.

We can derive a linearized form for the first-order condition for the intermediate goods firms as

\[ \hat{r}_t^k - \hat{w}_t = \hat{n}_t - \hat{k}_t. \] (4.7)

A linearized form for the real marginal cost function can be derived as

\[ \hat{mc}_t = (1 - \alpha) \hat{w}_t + \alpha \hat{r}_t^k - \hat{a}_t = -\hat{y}_t + \hat{n}_t + \hat{w}_t = \hat{y}_t + \hat{k}_t + \hat{r}_t^k. \]

The Taylor rule is

\[ \hat{r}_t = \phi_\pi \pi_t + \phi_y \hat{y}_t + v_t. \] (4.8)

The current consumption can be written as

\[ \hat{c}_t = \lambda \tau_c \hat{c}_t^T + (1 - \lambda \tau_c) \hat{c}_t^N, \] (4.9)
where \( \tau_c \equiv \frac{C}{C^T} \) and \( \lambda \) is the fraction of traders.

A linearized form for the aggregate capital stock is

\[
\hat{k}_t = \hat{k}_t^T.
\]

The aggregate output has two components, consumption and investment, given as

\[
\hat{y}_t = \tau_y \hat{c}_t + (1 - \tau_y) \hat{i}_t,
\]

where \( \tau_y \equiv \frac{C}{Y} \).

The optimal price provides us with an equation to describe the dynamics of inflation:

\[
\pi_t = \beta E_t \pi_{t+1} + \kappa \hat{m}c_t,
\]

where \( \kappa \equiv \frac{(1-\theta)(1-\beta \theta)}{\theta} \) and \( \theta \) is the stickiness of prices. The exogenous shocks are given as follows:

\[
\hat{a}_t = \rho_a \hat{a}_{t-1} + \varepsilon^a_t,
\]
\[
\upsilon_t = \rho_{\upsilon} \upsilon_{t-1} + \varepsilon^\upsilon_t.
\]

To study dynamics and Bayesian estimation of this economy, we have to derive a LRE (Linear Rational Expectations) system for the log-linearized DSGE model with equations from 1) to 11) and the exogenous shocks as following,

\[
\Gamma_0 x_{t+1} = \Gamma_1 x_t + c + \Psi \epsilon_{t+1} + \Pi \eta_{t+1},
\]

where \( x_t \equiv [\hat{y}_t, \hat{k}_t, \hat{n}_t, \hat{c}_t, \hat{c}_T, \hat{c}_N, \hat{w}_t, \pi_t, \hat{r}_t, \hat{r}_k, \hat{z}_t, \hat{a}, \upsilon_t] \), \( \epsilon_t \equiv [\epsilon^a_t, \epsilon^\upsilon_t] \), \( \eta_t \equiv [\eta^1_t, \eta^2_t, \eta^3_t, \eta^4_t] \), \( \rho_a, \rho_{\upsilon} \) and \( \rho_{\hat{m}} \) are the persistence parameters, \( \tau_y \), \( \lambda \), and \( \kappa \) are kept constant. All elements in \( \Gamma_0 \) and \( \Gamma_1 \) functions of basic structural parameters. \( \eta_{t+1}(\equiv \hat{x}_{t+1} - E\hat{x}_t, \hat{x}_t \in \mathbf{x}_t) \) denotes the rational expectations forecast errors. We will follow Sims
methods to solve the difference equation system.\textsuperscript{5} The solution to the LRE system, (12), is a following transition equation,

\[ x_{t+1} = Fx_t + G\epsilon_{t+1}. \] (4.13)

To exploit the Kalman filter evaluating the likelihood function of the DSGE model, we have to consider another equation, a measurement equation, relating the model variables \( x_t \) to a vector of observables \( X_t \) as following,

\[ X_t = H'x_t + \nu_t, \quad \nu_t \sim N(0,R). \] (4.14)

In this measurement equation, \( X_t \) is composed of de-trended per output growth, de-trended per consumption growth, demeaned inflation, demeaned nominal interest rates, de-trended per labor supply and de-trended per work hours for the U.S. \( H' \) does not depend on the structural parameter as it merely selects elements of \( x_t \). \( \nu_t \) is the measurement error vector.\textsuperscript{6} In the appendix C, we will provide a general procedure to estimate DSGE models using Bayesian methods.

\textbf{4.3 Empirical Results}

Bayesian analysis is system-based and adapts the solved DSGE model to an aggregate time series vector. The objective of estimation from Bayesian view is to make a conditional stochastic statement on parameterization on this model. The estimation on DSGE models is based on the likelihood from the structure of the DSGE model and observed data and

\textsuperscript{5}For detail, see Sims (2002).

\textsuperscript{6}There exists the singularity problem from any DSGE model generating a rank-deficient covariance matrix for \( X_t \). To avoid the singularity problem, we make the number of structural shocks to equal the number of observable variables or add measurement errors in the observation equation.
the prior providing additional information in estimating deep parameters. The production of the likelihood function and the prior can generate the posterior distribution depicting our belief after observing data by the Bayes’ rule as

\[ P(\mu|X) \sim L(X|\mu)P(\mu), \]

where \( P(\mu|X) \) is the posterior, \( L(X|\mu) \) the likelihood function, \( P(\mu) \) the prior and \( \mu \) the vector of structural parameters.

To estimate the New Keynesian model with market segmentation, we collect time series data about output, consumption, inflation, interest rates, labor supply, and work hours from FRED 2 database maintained by the Federal Reserve Bank of St. Louis from 1954:III to 2010:IV. In this paper, the definition of data variables will follow Smets and Wouters (2007). For estimation to our model, output, consumption, labor supply and work hours are de-trended and inflation and interest rates are demeaned.

4.3.1 Prior

The prior captures a pre-sample belief on structural parameters. One of benefits of Bayesian methods is to be able to exploit the pre-sample information. In estimating macroeconomic models, firstly microeconomic evidences can provide with sensible priors and it is natural for economists use the prior from micro evidences. Fernandez-Villaverde (2009) argues that

\[ \text{Output is defined as } \ln(GDPC96/LNSindex) \times 100, \text{ consumption } \ln((PCEC/GDPDEF)/LNSindex) \times 100, \text{ inflation } \ln(GDPDEF/GDPDEF(-1)) \times 100, \text{ interest rates } FFR/4, \text{ labor supply } \ln(PRS85006023/LNSindex) \times 100, \text{ and work hours } \ln(PRS85006103/GDPDEF) \times 100. \text{ GDPC96 is the seasonally adjusted real gross domestic product. GDPDEF is the seasonally adjusted gross domestic product-implicit price deflator (2005=100). PCEC is the seasonally adjusted personal consumption expenditure. CE16OV is the seasonally adjusted civilian employment. FFR is the annually federal funds rate. LNSindex is CE16OV(t)/CE16OV(2005Q3) for } t = 1954Q1,...,2010Q10. \text{ PRS85006023 is the seasonally adjusted non-farm business average weekly hours index (2005=100), and PRS85006103 the seasonally adjusted non-farm business hourly compensation index (2005=100).} \]
some estimation values of macro parameters from other countries can be an alternative source for the prior because of an default belief that one of many important differences between economics and the other social science is that agents, individuals, among nations is basically identical and behavioral differences among them can be explained by a difference of relative prices. In this paper the prior distribution of parameters is from different papers, An and Schorfheide (2007), Fernandez-Villaverde (2009), Adolfson et al. (2005), Laforte (2007), Landon-Lane and Occhino (2008), Smets and Wouters (2007), and Lubik and Schorfheide (2006). Especially, the prior for the key structural parameter $\lambda$, the fraction of traders, is from Landon-Lane and Occhino (2008). They use evidence from the Survey of Consumer Finances of 1992 in order to choose a prior for the fraction of traders.\footnote{To detail for procedure to derive this prior, see Landon-Lane and Occhino (2008).} Also, the prior for the price stickiness, $\theta$, is from Lubik and Schorfheide (2006). They explain that the prior for the parameter is chosen based on evidence on the average frequency of price changes.\footnote{For detail, see Lubik and Schorfheide (2006)} In this paper, the prior means of measurement errors are from data and the prior standard deviations are arbitrarily chosen 1 and infinity. In here, $\beta = 0.99$, $\delta = 0.025$, $\eta = 1$, and $\epsilon = 6$ are fixed parameters. The marginal prior distributions for the parameters of the New Keynesian model with market segmentation are summarized in Tables 1.

### 4.3.2 Bayesian Estimation

To implement the estimation using Bayesian approach, we use the Dynare. We have 100,000 draws and 2 blocks from the Random walk Metropolis-Hastings and discard first 50,000 draws for estimate the parameters. In Dynare, Chris Sims’s program, csminwel, is used to compute the mode of the posterior. The acceptance rates are that the block 1 is 0.2838 and the block 2 is 0.2821 for the New Keynesian model with market segmentation (NKMS),
the block 1 is 0.2986 and the block 2 is 0.2957 for the New Keynesian model with market segmentation without money (NKMSnomoney), and the block 1 is 0.2983 and the block 2 is 0.2975 for the standard New Keynesian model with capital (NKcapital), respectively. Usually, the appropriate acceptance rate is from 0.2 to 0.4. Too high acceptance rate might not allow your draws from Markov Chain Monte Carlo (MCMC) to visit the tails of your target distribution and too low acceptance rate might get stuck draws in a subspace of the parameter range. The appendix at the end of this chapter will provide us with the details for Bayesian computation.

Table 3.2, 3.3, and 3.4 and Figure 3.1, 3.2, and 3.3 show results of the Bayesian estimation of three models. Basically, These Tables report posterior means and posterior standard deviation. The posterior standard deviation is from the inverse Hessian computed at the posterior mode. Also, the 5th and 95th percentiles of the posterior distribution are reported and the posterior and the prior will be compared numerically. These Figures show visually information of the posterior and the prior. The prior mean and standard deviation of the inverse of the elasticity of substitution with regard to consumption, $\sigma$, are 1 and 0.5, respectively. The 5th to 95th percentiles of prior is from 0.3428 to 1.9367. The posterior means and standard deviation of this parameter are 4.4023 for NKMS, 1.7558 for NKMSnomoney, and 4.5114 for NKcapital. This parameter can affect determinacy of DSGE models. If the value of this parameter is too high, about over 5, many DSGE models can be affected by indeterminacy. If, however, there exists capitals, indeterminacy may be disappeared dramatically. Fortunately, all values of this parameter for each model do not generate indeterminacy.

The prior mean of standard deviation of the elasticity of labor supply, $\varphi$, are 1 and 0.5, respectively. The values of this parameter are very different among three models. NKMS is about 3, NKMSnomoney is about 0.7, and NKcapital is about 5.2. The posterior standard
deviation of this parameter for NKMS and NKcapital are more broader than one of prior. The posterior results of the elasticity of output with respect to capital, $\alpha$, for the three models are similar with each other, the posterior means are about $1/3$ and the posterior standard deviations are about 0.0187. In case of NKMS, it seems that data do not identify this parameter.

It seems that posterior means for the coefficients for inflation, $\phi_\pi$ and output gap, $\phi_y$, in the Taylor rule for the three models satisfy the Taylor Principle. The posterior means for these parameters lie from about 1.33 to about 2.0. Especially, in the case of NKMS, the values of the coefficients of inflation and output gap, $\phi_\pi = 1.6$ and $\phi_y = 0.5236$, are similar with the standard calibration values, $\phi_\pi = 1.5$ and $\phi_y = 0.5$ and the posterior variances for these parameters are less than the prior variance, see the forth row in Figure 3.1.

The posterior mean of the fraction of traders, $\lambda$, is 0.1445 and the posterior standard deviation is 0.0764. This value is very similar with Landon-Lane and Occhino (2008) in which the posterior mean is 0.216. The 5th and 95th percentiles for NKMS is from 0.019 to 0.25 and the interval for Landon-Lane and Occhino (2008) is from 0.119 to 0.300. The posterior mean, however, of this parameter for NKMSnomoney is 0.9727 and it means that the New Keynesian with market segmentation and without money model is very similar with the standard New Keynesian model with capital. The posterior means of the price stickiness are 0.8844 for NKMS, 0.6927 for NKMSnomoney, and 0.75 for NKcapital and the 5th and 95th percentiles are from 0.8683 to 0.9028 for NKMS, from 0.6483 to 0.7378 for NKMSnomoney, and from 0.7284 to 0.7716. In Smets and Wouters (2007), the posterior mean of this parameter is 0.65 and the 5th and 95th percentiles are from 0.56 to 0.74. In Fernandez-Villaverde et al. (2009), with the prior of Smets and Wouters (2007), the posterior mean of this parameter is 0.82 and the 5th and 95th percentile are from 0.78 to 0.87.
The posterior means of AR (auto-regression) coefficients of the technology shock and the monetary shock, $\rho_a$ and $\rho_\nu$, are 0.5635 and 0.8168 for NKMS, 0.9022 and 0.5065 for NKMSnomoney, and 0.7922 and 0.5209 for NKcapital, respectively. NKMS has long memory for the technology shock and NKMSnomoney and NKcapital has long memory for the monetary shock. By Smets and Wouters (2007), the high persistence of AR processes of shocks can explain most of the forecast error variance of the real variables at long horizons.

Dynare shows “MCMC univariate diagnostics” which is the main source of feedback to gain confidence with results. If chains are relatively constant and converge and results between the various chains are close, the results from MCMC univariate diagnostics can be sensible. Figure 3.4 display MCMC univariate diagnostics of exogenous shocks and measurement error shocks for three models. The horizontal axis is the number of Random walk H-M iterations and the vertical axis is the measure of the parameter moments, with the first, corresponding to the measure at the initial value of the Random walk M-H iterations. From this figure, we can say that we can obtain stationary posterior distributions since two chains in each case are close and stable.

Figure 3.5 displays the impulse response to the technology shock for three models.\textsuperscript{11} The important feature of this figure is that the impulse responses of aggregate consumption and non-traders’ consumption to the technology shock are hump-shaped.\textsuperscript{12} The impulse

\textsuperscript{10}If the prior standard deviations get large, what will happen to the posterior means and standard deviations? When the prior standard deviation of exogenous shocks and measurement error get infinite, there is no big difference from results of the origin prior for three models. But, this result cannot say strongly that our models are not sensitive to changes in the prior. We will remain the sensitive analysis as near future works.

\textsuperscript{11}To implement the impulse response, we use the mean of parameters.

\textsuperscript{12}Chapter 2 explains why the hump-shaped impulse responses of consumption and output can be generated in the New Keynesian model with market segmentation.
responses of consumption and output for NKMSnomoney and NKcapital are very similar and are not hump-shaped. The posterior means of AR coefficients of the technology shock for NKMSnomoney and NKcapital are 0.90 and 0.79 so their impulse responses of consumption and output to technology shock have long memory. Figure 3.6 displays the impulse response to the monetary shock for three models and the impulse responses of aggregate consumption and traders’ and non-traders’ consumption are hump-shaped. Since the posterior mean of AR coefficients of the monetary shock for NKMS is 0.82, the impulse responses for NKMS have long memory.

Table 3.5 reports the log marginal likelihood to compare the three models. Values of the log date density are -2316.20 for NKMS, -2458.46 for NKMSnomoney, and -2443.05 for NKcapital. From these values, we can say that NKMS is more probably than other two models, NKMSnomoney and NKcapital, in probability.

4.4 Conclusion

This paper estimate and compare three models, New Keynesian model with market segmentation, New Keynesian model with market segmentation and without money, and standard New Keynesian model with capital, using Bayesian approaches. In this paper, the important structural parameter is the fraction of traders, $\lambda$, and the posterior means and standard deviations are 0.1445 and 0.0764 for the New Keynesian model with market segmentation and 0.9727 and 0.0149 for the New Keynesian model with market segmentation and without money. The posterior means of the New Keynesian model with market segmentation is more similar with Landon-Lane and Occhino (2008), 0.216, than one of the New Keynesian model with market segmentation and without money. The 5th and 95th percentiles for the New Keynesian model with market segmentation is from 0.019 to 0.25 and the interval for Landon-Lane and Occhino (2008) is from 0.119 to 0.300. The log
marginal likelihood is used in evaluating the three models. The value of the log marginal likelihood for the New Keynesian model with market segmentation is the most high of three models.

Although the discussion on optimal monetary and fiscal policies for the three models is beyond this paper, in the near future, we can deal with it. The topic can be interested because of asymmetric behaviors of traders and non-traders to technology and monetary shocks.
<table>
<thead>
<tr>
<th>Name</th>
<th>Domain</th>
<th>Density</th>
<th>means</th>
<th>standard deviations</th>
<th>(5 and 95 percentile)</th>
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<td>$R^+$</td>
<td>Gamma</td>
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<td>0.5</td>
<td>$(0.3428, 1.9367)$</td>
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<tr>
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<td>Gamma</td>
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<tr>
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<td>Beta</td>
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<td>0.02</td>
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<td>Gamma</td>
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<td>0.25</td>
<td>$(1.1137, 1.9338)$</td>
</tr>
<tr>
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<td>$(0.1723, 0.9664)$</td>
</tr>
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<tr>
<td>$\sigma_n^\nu$</td>
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<td>Gamma</td>
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<td>1</td>
<td>$(1.9944, 5.2479)$</td>
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Table 4.1: Prior Distribution

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<thead>
<tr>
<th>Name</th>
<th>Density</th>
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<th>s.d</th>
<th>percentile</th>
<th>prior m.</th>
<th>prior s.d.</th>
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<td>0.02</td>
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<td>0.25</td>
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</table>

Table 4.2: Posterior Distribution for NKMS
### Table 4.3: Posterior Distribution for NKMSnomoney

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<tr>
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<th>means</th>
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<th>percentile</th>
<th>prior m.</th>
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<td>(0.3428, 1.9367)</td>
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<tr>
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<td>Gamma</td>
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<tr>
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<td>1</td>
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### Table 4.4: Posterior Distribution for NKcapital

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<th>Name</th>
<th>Density</th>
<th>means</th>
<th>s.t.</th>
<th>percentile</th>
<th>prior m.</th>
<th>prior s.d.</th>
<th>percentile</th>
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<td>0.02</td>
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Table 4.5: Log marginal likelihood
Figure 4.1: Posterior and Prior for NKMS
Figure 4.2: Posterior and Prior for NKMSnomoney
Figure 4.3: Posterior and Prior for NKcapital
Figure 4.4: MCMC univariate diagnostics

a) NKMS

b) NKMSnomoney

c) NKcapital
Figure 4.5: Impulse Response to the Technical shock
Figure 4.6: Impulse Response to the Interest shock
In modeling nominal rigidities in the NOEM (New Open Economy Macroeconomics), the assumption that plays a key role is whether export/import prices are sticky in the currency of the producers, or in the currency of the destination market. To answer this question, we should estimate pass-through elasticity. Chapter 2 estimates the exchange rate pass-through in a small open economy DSGE model for Korea and the U.S. employing Bayesian estimation techniques. The empirical results show that Korea and the U.S. experience incomplete exchange rate pass-through of varying degrees. Main results are that the price of imported goods in Korea gets more affected from the exchange rate fluctuation, and the nominal rigidity becomes less fixed after Asian Crisis than before Asian Crisis. The elasticity of substitution between domestic and foreign goods increases from the Before Asian Crisis to the After Asian Crisis in Korea. This suggests that incomplete pass-through can play an important role in driving output in Korea because Korea output is sensitive to changes in the terms of trade which is related to the law of price gap. The nominal rigidity parameter for Korean goods in the U.S. increases from the Before Asian Crisis to the After Asian Crisis. This suggests that although the important two assumptions on price setting, PCP and LCP, do not hold in this paper, importers in Korea have a stronger tendency to follow the PCP assumption and importers in the U.S. have a stronger tendency to follow the LCP for Korean goods after the Asia Crisis.

The objectives of Chapter 3 are to incorporate the market segmentation model in the
New Keynesian model, and then to answer a following question: is the Taylor principle still a good criterion for the uniqueness in the New Keynesian model with market segmentation? Main results of my paper are that the Taylor rule can satisfy the Taylor principle for a unique solution to the model and that there exists asymmetric behavior between traders, who can participate in the asset market, and non-traders, who cannot. The first finding reverses the results of previous studies that argued the Taylor principle, under asset market segmentation, no longer provide an important criterion for the stability properties of interest rate rules. This finding is regardless of the fraction of traders. The second finding provides another resource to generate the hump-shaped impulse response of consumption and output to exogenous shocks.

Chapter 4 estimate and compare three models, New Keynesian model with market segmentation, New Keynesian model with market segmentation and without money, and standard New Keynesian model with capital, using Bayesian approaches. The posterior mean of the fraction of traders for the New Keynesian model with market segmentation is similar with Landon-Lane and Occhino (2008). The log marginal likelihood is used in evaluating the three models. The value of the log data density for the New Keynesian model with market segmentation is the most high of three models.
Appendix A

Chapter 2

A.1 Deriving Equations

A.1.1 Demand function

To obtain the standard demand function, the representative agent should solve the following problem for all $t$:

$$\max_{C_H, C_F} C = \left[ (1 - \alpha)^{1/\eta} C_H^{(\eta - 1)/\eta} + \alpha^{1/\eta} C_F^{(\eta - 1)/\eta} \right]^{\eta/(\eta - 1)},$$

subject to

$$Z = P_H C_H + P_F C_F,$$

where $Z$ is the total expenditure and is given. For simplicity, we omit all $t$ in the formulation. Then F.O.C is

$$\left( \frac{P_H}{P_F} \right)^\eta = \left( \frac{1 - \alpha}{\alpha} \right) \left( \frac{C_F}{C_H} \right).$$

Therefore,

$$C_H = \frac{(1 - \alpha)Z}{P_H \left[ (1 - \alpha)P_H^{-\eta} + \alpha P_F^{-\eta} \right]},$$

$$C_F = \frac{\alpha Z}{P_F \left[ (1 - \alpha)P_H^{-\eta} + \alpha P_F^{-\eta} \right]}.$$

It is the demand function that maximizes $C$ given the expenditure, $Z$.

The consumption-based price index $P$ is the minimum expenditure

$$Z = P_H C_H + P_F C_F.$$
such that $C = \left[(1 - \alpha)^{1/\eta}C_H^{(\eta-1)/\eta} + \alpha^{1/\eta}C_F^{(\eta-1)/\eta}\right]^{\eta/(\eta-1)} = 1$, given $P_H$ and $P_F$, that is the consumption price index $P$ solves the following problem:

$$\min_{C_H, C_F} P_H C_H + P_F C_F,$$

s.t. $C = \left[(1 - \alpha)^{1/\eta}C_H^{(\eta-1)/\eta} + \alpha^{1/\eta}C_F^{(\eta-1)/\eta}\right]^{\eta/(\eta-1)} = 1$.

From the first optimization problem, the aggregate consumption $C$ can be

$$C = \left[(1 - \alpha)^{1/\eta} \left(\frac{(1-\alpha)Z}{P_H^\eta[(1-\alpha)P_H^{1-\eta}+\alpha P_F^{1-\eta}]}\right)^{(\eta-1)/\eta}
+ \alpha^{1/\eta} \left(\frac{\alpha Z}{P_F^\eta[(1-\alpha)P_H^{1-\eta}+\alpha P_F^{1-\eta}]}\right)^{(\eta-1)/\eta}\right]^{\eta/(\eta-1)}.$$

Since $P$ is the consumption-based price index minimize the expenditure $Z$ such that $C=1$,

$$\left[(1 - \alpha)^{1/\eta} \left(\frac{(1-\alpha)Z}{P_H^\eta[(1-\alpha)P_H^{1-\eta}+\alpha P_F^{1-\eta}]}\right)^{(\eta-1)/\eta}
+ \alpha^{1/\eta} \left(\frac{\alpha Z}{P_F^\eta[(1-\alpha)P_H^{1-\eta}+\alpha P_F^{1-\eta}]}\right)^{(\eta-1)/\eta}\right]^{\eta/(\eta-1)} = 1.$$

Then $P_t = \left[(1 - \alpha)P_H^{1-\eta} + \alpha P_F^{1-\eta}\right]^{1/(1-\eta)}$. And $Z/P$ is the ratio of spending to the minimum price of a unit of the consumption index, $C = Z/P$. Therefore,

$$C_H = \frac{(1-\alpha)Z}{P_H^\eta[(1-\alpha)P_H^{1-\eta}+\alpha P_F^{1-\eta}]} = \frac{(1-\alpha)PC}{P_H^\eta[(1-\alpha)P_H^{1-\eta}+\alpha P_F^{1-\eta}]} = (1 - \alpha) \left(\frac{P_H}{P_F}\right)^{-\eta} C,$$

and

$$C_F = \frac{\alpha Z}{P_F^\eta[(1-\alpha)P_H^{1-\eta}+\alpha P_F^{1-\eta}]} = \frac{\alpha PC}{P_F^\eta[(1-\alpha)P_H^{1-\eta}+\alpha P_F^{1-\eta}]} = \alpha \left(\frac{P_F}{P_H}\right)^{-\eta} C.$$
A.1.2 Monopolistic domestic firm’s demand function

To obtain each monopolistic domestic firms demand function, first we should solve the following problem in home:

$$\max_{C_H(j)} C_H = \left[ \int_0^1 C_H(j)^{\omega-1}/\omega \, dj \right]^{\omega/(\omega-1)},$$

s.t. $Z_H = \int_0^1 P_H(j)C_H(j)\,dj$: nominal budget constraint, where $Z_H$ is any fixed total nominal expenditure on home goods. Then F.O.C is

$$C_H^{(\omega-1)/\omega}C_H(j)^{-1/\omega} = \mu P_H(j)/P_H,$$

$\mu$ is the Lagrangian multiplier. From F.O.C, we can derive the following equation for any two goods $j$ and $j^*$:

$$C_H(j^*) = C_H(j) \left[ \frac{P_H(j)}{P_H(j^*)} \right]^\omega.$$  

Substitute the F.O.C into the nominal budget constraint then the budget constraint can be expressed

$$Z_H = \int_0^1 P_H(j^*)C_H(j^*)\,dj^* = P_H(j)^\omega C_H(j) \int_0^1 P_H(j^*)^{1-\omega} \, dj^* = P_H(j)^\omega C_H(j) P_H^{1-\omega},$$

where price index for goods produced in home is $P_H = \left[ \int_0^1 P_H(j^*)^{1-\omega} \, dj^* \right]^{1/(1-\omega)}$. Then the demand for $j$ in home country is

$$C_H(j) = \left[ \frac{P_H(j)}{P_H} \right]^{-\omega} \left( \frac{Z_H}{P_H} \right) = \left[ \frac{P_H(j)}{P_H} \right]^{-\omega} C_H.$$

Since the producer preset their good prices in their own currency at the domestic and foreign market, that is, $P_H(j)/P_H = P_H^*(j)/P_H^*$, the world demand consumption for good
j can take the following form:

\[ Y_H(j) = \left[ \frac{P_H(j)}{P_H} \right]^{-\omega} \quad Y_H = \left[ \frac{P_H(j)}{P_H} \right]^{-\omega} (C_H + G_H + C_H^*). \]

We can derive the retailers demand function, equation (11), with the analogous method above.

**A.2 Difference equation system for the model**

1. Domestic firms price setting:

   \[ \hat{\pi}_H = \beta E_t \hat{\pi}_{H,t+1} + \kappa_H \hat{m}c_t, \]

   where \( \kappa_H = (1 - \theta_H)(1 - \theta_H \beta)/\theta_H > 0 \) and \( \hat{m}c_t = -\hat{\lambda}_t - \alpha \hat{q}_t - \hat{A}_{H,t} \).

2. Domestic habits:

   \[ (1 - h) \hat{C}_t = \hat{c}_t - h \hat{c}_{t-1} + h \hat{z}_t, \]

   where \( \hat{z}_t = \Delta \hat{A}_{W,t} \).

3. Domestic Marginal utility of consumption:

   \[-\hat{\lambda}_t = \left( \tau/(1 - h\beta) \right) \hat{C}_t - \left( h\beta/(1 - h\beta) \right) E_t \left[ \tau \hat{C}_{t+1} + \hat{z}_{t+1} \right]. \]

4. Domestic importers’ price setting:

   \[ \hat{\pi}_F = \beta E_t \hat{\pi}_{F,t+1} + \kappa_F \hat{\psi}_F, \]

   where \( \kappa_F = (1 - \theta_F)(1 - \theta_F \beta)/\theta_F \).
5. Definition of CPI

\[ \hat{\pi}_t = \alpha \hat{\pi}_F t + (1 - \alpha) \hat{\pi}_H t. \]

6. Terms of Trade Dynamics:

\[ \hat{q}_t = \hat{q}_{t-1} - \hat{\pi}_F t + \hat{\pi}_H t. \]

7. Real Exchange Rate Dynamics:

\[ \hat{s}_t = \hat{\psi}_F t - (1 - \alpha) \hat{q}_t - \alpha \hat{q}_t^*. \]

8. Nominal Depreciation Rate:

\[ \Delta \hat{e}_t = \hat{\pi}_t - \hat{\pi}_t^* + \Delta \hat{s}_t. \]

9. Home goods market clearing:

\[ \hat{y}_{Ht} = \hat{c}_t + \hat{g}_t - (\alpha/\tau) \hat{s}_t - \alpha(1 - \alpha) \eta(\hat{q}_t - \hat{q}_t^*). \]

10. Risk Sharing Condition:

\[ \hat{\lambda}_t = \hat{\lambda}_t^* - \hat{s}_t. \]

11. Monetary Policy Rule:

\[ \hat{R}_t = \rho_R \hat{R}_{t-1} + (1 - \rho_R)[\varphi_1 \hat{\pi}_t + \varphi_2 (\Delta \hat{y}_t + \hat{s}_t) + \varphi_3 \Delta \hat{e}_t] + \varepsilon_{R,t}. \]

12. UIP equation:

\[ \hat{R}_t - \hat{R}_t^* = E_t \Delta \hat{e}_{t+1}. \]
13. Foreign firms price setting

\[ \hat{\pi}^*_{Ft} = \beta E_t \hat{\pi}^*_{H,t+1} + \kappa_{H} \hat{m}c_t, \]

where \( \kappa_{F} = (1 - \theta_{F})(1 - \theta_{F} \beta)/\theta_{F} > 0 \) and \( \hat{m}c_t^* = -\hat{\lambda}_t^* - \alpha^* \hat{q}_t^* - \hat{A}_{Ft}. \)

14. Foreign habits

\[ (1 - h) \hat{C}_t^* = \hat{c}_t^* - h\hat{c}_{t-1} + h\hat{z}_t, \]

where \( \hat{z}_t = \triangle \hat{A}_{Wt}. \)

15. Foreign Marginal utility of consumption:

\[ -\hat{\lambda}_t^* = (\tau/(1 - h\beta)) \hat{C}_t - (h\beta/(1 - h\beta)) E_t [\tau \hat{C}_{t+1}^* + \hat{z}_{t+1}] . \]

16. Foreign importers’ price setting:

\[ \hat{\pi}^*_{Ht} = \beta E_t \hat{\pi}^*_{H,t+1} + \kappa_{H} \hat{\psi}_{Ht}, \]

where \( \kappa_{H} = (1 - \theta_{H})(1 - \theta_{H} \beta)/\theta_{H}. \)

17. Definition of CPI in Foreign:

\[ \hat{\pi}^*_t = \alpha \hat{\pi}^*_H + (1 - \alpha) \hat{\pi}^*_{F,t}. \]

18. Foreign Terms of Trade Dynamics:

\[ \hat{q}_t^* = \hat{q}_{t-1}^* - \hat{\pi}^*_F + \hat{\pi}^*_H. \]
19. Foreign Real Exchange Dynamics:
\[ \hat{s}_t = \hat{\psi}_H^* - (1 - \alpha)\hat{q}_t^* - \alpha \hat{q}_t. \]

20. Foreign goods market clearing:
\[ \hat{y}_F^* = \hat{c}_t^* + \hat{g}_t^* - (\alpha/\tau)\hat{s}_t - \alpha(1 - \alpha)\eta(\hat{q}_t - \hat{q}_t^*). \]

21. Foreign Monetary policy rule
\[ \hat{R}_t^* = \rho R^* \hat{R}_{t-1}^* + (1 - \rho R^*)[\varphi_1^* \hat{\pi}_t^* + \varphi_2^*(\Delta \hat{y}_t^* + \hat{z}_t) + \varphi_3^* \Delta \hat{e}_t] + \varepsilon_{R,t}. \]

22. Euler Equation
\[ -\hat{\lambda}_t = -E_t\hat{\lambda}_{t+1} - \left(\hat{R}_t - E_t\hat{\pi}_{t+1}\right) + E_t\hat{z}_{t+1}. \]

There are 5 equations depicting the disturbance of the exogenous autoregressive shocks:
\[
\begin{align*}
\hat{z}_t &= \rho_z \hat{z}_{t-1} + \varepsilon_{zt}, \\
\hat{A}_H^t &= \rho_A \hat{A}_{H,t-1} + \varepsilon_{At}, \\
\hat{A}_F^* &= \rho_A \hat{A}_{F,t-1} + \varepsilon_{A^*t}, \\
\hat{g}_t &= \rho_G \hat{g}_{t-1} + \varepsilon_{Gt}, \\
\hat{g}_t^* &= \rho_{G^*} \hat{g}_{t-1}^* + \varepsilon_{G^*t}.
\end{align*}
\]
Appendix B
Chapter 3

B.1 Steady state

\( A_t = \bar{A} = 1, \; M_t/M_{t-1} = 1, \; P_t/P_{t-1} = 1, \; \bar{I} = \delta \bar{K}. \)

Assuming zero inflation steady state.

From equation (3.4):

\[
\bar{R} = \frac{1}{\beta}
\]

From the definition of the Tobin \( q \) and \( \bar{I} = \delta \bar{K} \):

\[
\bar{Q} = 1
\]

From equation (3.5), \( r^k \) is real capital rates:

\[
1 = \beta (\bar{r}^k + (1 - \delta)) \\
\bar{r}^k = \frac{1}{\beta} - (1 - \delta)
\]

From equation (3.6), \( w \) is real wage rates:

\[
\bar{C}^N = \bar{w} \bar{N}
\]
From Gali et al. (2007), the relationship between the intertemporal consumption marginal rate of substitution and the real wage is

\[ \sigma \bar{C} + \varphi \bar{N} = \bar{w}. \]

From equation (3.9):

\[ \frac{\bar{r}^k (1 - \alpha)}{\bar{w} \alpha} = \gamma = \frac{\bar{N}}{K} \Rightarrow \bar{K} = \frac{1}{\gamma} \bar{N} \]

From equation (3.12)

\[
\bar{RMC} = \frac{\epsilon - 1}{\epsilon} = \frac{\bar{w}}{1 - \alpha} \left( \frac{\bar{r}^k (1 - \alpha)}{\bar{w} \alpha} \right)^\alpha
\]

\[ \bar{w} = \left( \frac{(\epsilon - 1)(1 - \alpha)_{\alpha \alpha}}{\epsilon (\bar{r}^k)^\alpha} \right)^{1/(1 - \alpha)} \]

From equation (3.8)

\[ \bar{Y} = K^\alpha N^{1 - \alpha} = \gamma^{1 - \alpha} \bar{K}. \]

From equation (3.20):

\[
\dot{\bar{Y}} = \dot{\bar{C}} + \dot{\bar{I}} \\
\dot{\bar{C}} = \bar{Y} - \delta \bar{K} = (1 - \delta \gamma^{\alpha - 1}) \bar{Y} \\
\dot{\bar{C}} = (1 - \delta \gamma^{\alpha - 1}) \gamma^{-\alpha} \bar{N} \\
\dot{\bar{C}} = (1 - \delta \gamma^{\alpha - 1}) \gamma^{-\alpha} (\beta \bar{w} \bar{C}^{-\sigma})^{1/\varphi} \\
\dot{\bar{C}} = \left( (1 - \delta \gamma^{\alpha - 1}) \gamma^{-\alpha} (\beta \bar{w})^{1/\varphi} \right)^{\varphi/(\varphi + \sigma)}. \]

From the value of \( \dot{\bar{C}} \), the value of \( \bar{N} \) is

\[ \bar{N} = \frac{1}{(1 - \delta \gamma^{\alpha - 1}) \gamma^{-\alpha} \dot{\bar{C}}}. \]
Then, the value of $\bar{K}$, $\bar{Y}$, $\bar{C}^N$, and $\bar{C}^T$ are, respectively,

\begin{align*}
\bar{K} &= \frac{1}{\gamma} \bar{N}, \\
\bar{Y} &= \bar{K}^\alpha \bar{N}^{1-\alpha}, \\
\bar{C}^N &= \bar{w} \bar{N}, \\
\bar{C}^T &= \frac{1}{\lambda} \left( \bar{C} - (1 - \lambda) \bar{C}^N \right).
\end{align*}

Also, we can obtain the steady state value of variables remained.

### B.2 Dynamics

1 From equations (3.27) and (3.36):

\begin{align*}
\hat{y}_t &= \alpha \hat{k}_t + (1 - \alpha) \hat{n}_t = \tau_y \hat{c}_t + (1 - \tau_y) \hat{i}_t \\
\Rightarrow \hat{i}_t &= \frac{\alpha}{1 - \tau_y} \hat{k}_t + \frac{1 - \alpha}{1 - \tau_y} \hat{n}_t - \frac{\tau_y}{1 - \tau_y} \hat{c}_t
\end{align*}

Incorporating this equation into equation (3.22):

\begin{align*}
\hat{k}_{t+1} &= (1 - \delta) \hat{k}_t + \delta \left( \frac{\alpha}{1 - \tau_y} \hat{k}_t + \frac{1 - \alpha}{1 - \tau_y} \hat{n}_t - \frac{\tau_y}{1 - \tau_y} \hat{c}_t \right) \\
\hat{k}_{t+1} &= \left( 1 - \delta \right) + \frac{\delta \alpha}{1 - \tau_y} \hat{k}_t \ + \frac{\delta(1 - \alpha)}{1 - \tau_y} \hat{n}_t - \frac{\delta \tau_y}{1 - \tau_y} \hat{c}_t \quad \text{(B.1)}
\end{align*}

1We drop terms of shocks because the term is not needed in solving the uniqueness of the system, equation (3.38).
From equation (3.31):

\[ \hat{m}c_t = -\dot{y}_t + \hat{n}_t + \tilde{w}_t \]

\[ = -(\alpha \hat{k}_t + (1 - \alpha)\hat{n}_t) + \hat{n}_t + \tilde{w}_t - E_t \pi_{t+1} + E_t \pi_{t+1} \]

\[ = -\alpha \hat{k}_t + \alpha \hat{n}_t + \varphi \hat{n}_t + \sigma E_t \hat{c}_{t+1} + E_t \pi_{t+1} \]

\[ = -\alpha \hat{k}_t + (\alpha + \varphi)\hat{n}_t + \sigma E_t \hat{c}_{t+1} + E_t \pi_{t+1}. \]

Incorporating this equation into equation (3.37):

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa (\alpha \hat{k}_t + (\alpha + \varphi)\hat{n}_t + \sigma E_t \hat{c}_{t+1} + E_t \pi_{t+1}) \]

\[ \pi_t = (\beta + \kappa) E_t \pi_{t+1} - \alpha \kappa \hat{k}_t + \kappa (\alpha + \varphi)\hat{n}_t + \kappa \sigma E_t \hat{c}_{t+1} \]

\[ \kappa \sigma E_t \hat{c}_{t+1} + (\beta + \kappa) E_t \pi_{t+1} = \alpha \kappa \hat{k}_t - \kappa (\alpha + \varphi)\hat{n}_t + \pi_t. \] (B.2)

From equation (3.34)

\[ \hat{c}_t = \lambda \tau_c \hat{c}_t + (1 - \lambda \tau_c) \hat{c}_t^N \]

\[ \hat{c}_t^T = \frac{1}{\lambda \tau_c} \hat{c}_t - \frac{1 - \lambda \tau_c}{\lambda \tau_c} \hat{c}_t^N \]

From equation (3.25):

\[ \hat{c}_t^N = \dot{w}_{t-1} - \pi_t + \hat{n}_{t-1} \]

\[ = \sigma \hat{c}_t + \varphi \hat{n}_{t-1} + \hat{n}_{t-1} \]

\[ = \sigma \hat{c}_t + (1 + \varphi)\hat{n}_{t-1}. \]

This equation into equation on \( \hat{c}_t^T \), (3.23):

\[ \hat{c}_t^T = \frac{1}{\lambda \tau_c} \hat{c}_t - \frac{1 - \lambda \tau_c}{\lambda \tau_c} (\sigma \hat{c}_t + (1 + \varphi)\hat{n}_{t-1}) \]

\[ = \frac{1 - \sigma (1 - \lambda \tau_c)}{\lambda \tau_c} \hat{c}_t - \frac{(1 - \lambda \tau_c)(1 + \varphi)}{\lambda \tau_c} \hat{n}_{t-1} \]

\[ = \frac{\chi_c}{\lambda \tau_c} \hat{c}_t - \frac{\chi_c}{\lambda \tau_c} \hat{n}_{t-1}, \]
where $\chi_c \equiv 1 - \sigma(1 - \lambda \tau_c)$ and $\chi_n \equiv 1 - \sigma(1 - \chi_c)(1 + \chi_c)$. Putting this equation into (3.23):

$$\frac{\chi_c}{\lambda \tau_c} c_t - \frac{\chi_n}{\lambda \tau_c} n_{t-1} = E_t \frac{\chi_c}{\lambda \tau_c} c_{t+1} - \frac{\chi_n}{\lambda \tau_c} n_t + \frac{1}{\sigma}(E_t \pi_{t+1} - \hat{r}_t)$$

$$\sigma \chi_c C_t - \sigma \chi_n \hat{n}_{t-1} = \sigma \chi_c E_t \hat{c}_{t+1} - \sigma \chi_n \hat{n}_t + \lambda \tau_c (E_t \pi_{t+1} - \hat{r}_t)$$

$$\sigma \chi_n E_t \hat{c}_{t+1} + \lambda \tau_c E_t \pi_{t+1} = \sigma \chi_c C_t + \sigma \chi_n \hat{n}_t + \lambda \tau_c \hat{r}_t - \sigma \chi_n \hat{n}_{t-1}$$

$$\sigma \chi_n E_t \hat{c}_{t+1} + \lambda \tau_c E_t \pi_{t+1} = \sigma \chi_c \hat{c}_t + \sigma \chi_n \hat{n}_t + \lambda \tau_c (\phi \pi \pi_t + \phi_\pi (\alpha \hat{k}_t + (1 - \alpha) \hat{n}_t)) - \sigma \chi_n \hat{n}_{t-1}$$

$$\sigma \chi_n E_t \hat{c}_{t+1} + \lambda \tau_c E_t \pi_{t+1} = \lambda \tau_c \phi \pi_\pi \hat{k}_t + (\lambda \tau_c \phi \pi (1 - \alpha) + \sigma \chi_n) \hat{n}_t$$

$$+ \sigma \chi_c \hat{c}_t + \lambda \tau_c \phi \pi \pi_t - \sigma \chi_n \hat{n}_{t-1} \quad \text{(B.3)}$$

From equation (3.31):

$$\mu c_t = -y_t + \hat{n}_t + \hat{w}_t = -y_t + \hat{k}_t + \hat{r}_t$$

$$\Rightarrow \hat{n}_t - E_t \pi_{t+1} + E_t \pi_{t+1} = \hat{k}_t + \hat{r}_t$$

$$\Rightarrow \hat{r}_t = \hat{k}_t + (1 + \varphi) \hat{n}_t + \sigma E_t c_{t+1} + E_t \pi_{t+1}$$

$$\hat{r}_t = \phi \pi \pi_t + \phi \pi (\alpha \hat{k}_t + (1 - \alpha) \hat{n}_t)$$

Putting $\hat{r}_t = \frac{\alpha}{1 - \tau_y} \hat{k}_t + \frac{1 - \alpha}{1 - \tau_y} \hat{n}_t - \frac{\tau_y}{1 - \tau_y} \hat{c}_t$ into the definition of Tobin $q$:

$$\hat{q}_t = \frac{1}{\eta} \left( \frac{\alpha}{1 - \tau_y} \hat{k}_t + \frac{1 - \alpha}{1 - \tau_y} \hat{n}_t - \frac{\tau_y}{1 - \tau_y} \hat{c}_t - \hat{k}_t \right)$$

$$= \frac{1}{\eta} \left( \frac{\alpha - 1 + \tau_y}{1 - \tau_y} \hat{k}_t + \frac{1 - \alpha}{1 - \tau_y} \hat{n}_t - \frac{\tau_y}{1 - \tau_y} \hat{c}_t \right)$$

$$= \psi_1 \hat{k}_t + \psi_2 \hat{n}_t - \psi_3 \hat{c}_t$$

$$\psi_1 = \frac{\alpha - 1 + \tau_y}{\eta (1 - \tau_y)} \quad \psi_2 = \frac{1 - \alpha}{\eta (1 - \tau_y)} \quad \psi_3 = \frac{\tau_y}{\eta (1 - \tau_y)}$$
Putting $\dot{r}_t^k$ and $\dot{q}_t$ into equation (3.24):

$$\psi_1 \dot{k}_t + \psi_2 \dot{n}_t - \psi_3 \dot{c}_t = \beta E_t \left( \psi_1 \dot{k}_{t+1} + \psi_2 \dot{n}_{t+1} - \psi_3 \dot{c}_{t+1} \right)$$

$$+ \chi_q E_t \left( -\dot{k}_{t+1} + (1 + \varphi) \dot{n}_{t+1} + \sigma E_t c_{t+2} + E_t \pi_{t+2} \right) - E_t (\dot{r}_{t+1} - \pi_{t+1})$$

$$(\beta \psi_1 - \chi_q - \phi_y \alpha) E_t \dot{k}_{t+1} + (\beta \psi_2 + \chi_q (1 + \varphi) - \phi_y (1 - \alpha)) E_t \dot{n}_{t+1}$$

$$- \beta \psi_3 E_t \dot{c}_{t+1} - (\phi_\pi - 1) E_t \pi_{t+1} + \sigma \chi_q E_t c_{t+2} + \chi_q E_t \pi_{t+2}$$

$$= \psi_1 \dot{k}_t + \psi_2 \dot{n}_t - \psi_3 \dot{c}_t$$

From equations (B.2) and (B.3), we can get $E_t \dot{c}_{t+2}$ and $E_t \dot{\pi}_{t+2}$, respectively,

$$E_t \pi_{t+2} = d_k \dot{k}_{t+1} - d_n E_t \dot{n}_{t+1} - d_c E_t \dot{c}_{t+1} + d_\pi E_t \pi_{t+1} + d_t \dot{n}_t$$

$$E_t \dot{c}_{t+2} = f_k \dot{k}_{t+1} - f_n E_t \dot{n}_{t+1} + f_c E_t \dot{c}_{t+1} + f_\pi E_t \pi_{t+1} - f_t \dot{n}_t$$

$$d_k = \frac{\chi_c a_k - \kappa b_k}{\chi_c (\beta + \kappa) - \kappa \pi c}$$

$$d_n = \frac{\chi_c a_n + \kappa b_n}{\chi_c (\beta + \kappa) - \kappa \pi c}$$

$$d_c = \frac{\kappa b_c}{\chi_c (\beta + \kappa) - \kappa \pi c}$$

$$d_\pi = \frac{\chi_c - \kappa b_n}{\chi_c (\beta + \kappa) - \kappa \pi c}$$

$$f_k = \frac{a_k - (\beta + \kappa) d_k}{\sigma \kappa}$$

$$f_n = \frac{a_n - (\beta + \kappa) d_n}{\sigma \kappa}$$

$$f_c = \frac{(\beta + \kappa) d_c}{\sigma \kappa}$$

$$f_\pi = \frac{1 - (\beta + \kappa) d_\pi}{\sigma \kappa}$$

$$a_k = \alpha \kappa$$

$$a_n = \kappa (\alpha + \varphi)$$

$$b_k = \lambda \tau_c \phi_y \alpha$$

$$b_n = \lambda \tau_c \phi_y (1 - \alpha) + \sigma \chi_n$$

$$b_c = \sigma \chi_c$$

$$b_\pi = \lambda \tau_c \phi_\pi$$

$$b_t = \sigma \chi_n$$
Incorporating these equations into the above equation:
\[

t (\beta \psi_1 - \chi_q (1 - \sigma f_k - d_k) - \phi_y \alpha) E_t \hat{k}_{t+1} \\
+ (\beta \psi_2 + \chi_q ((1 + \varphi) - \sigma f_n - d_n) - \phi_y (1 - \alpha)) E_t \hat{n}_{t+1} \\
- (\beta \psi_3 - \chi_q (\sigma f_c - d_c)) E_t \hat{c}_{t+1} + (\chi_q (\sigma f_\pi + d_\pi) - (\phi_\pi - 1)) E_t \pi_{t+1} \\
= \psi_1 \hat{k}_t + (\psi_2 + \chi_q (\sigma f_t - d_t)) \hat{n}_t - \psi_3 \hat{c}_t \\
\text{(B.4)}
\]

From equations (B.1)-(B.4), we can obtain a dynamic difference equation system for this model:

\[
Ax_{t+1} = Bx_t \\
\text{where} \\
x_t \equiv [\hat{k}_t, \hat{n}_t, \hat{c}_t, \pi_t, \hat{h}_t]' \\
\hat{h}_t \equiv n_{t-1} \\
A = \begin{bmatrix}
    a_{11} & 0 & 0 & 0 & 0 \\
    0 & 0 & a_{23} & a_{24} & 0 \\
    0 & 0 & a_{33} & a_{34} & 0 \\
    a_{41} & a_{42} & a_{43} & a_{44} & 0 \\
    0 & 0 & 0 & 0 & a_{55}
\end{bmatrix}, \\
B = \begin{bmatrix}
    b_{11} & b_{12} & b_{13} & 0 & 0 \\
    b_{21} & b_{22} & 0 & b_{24} & 0 \\
    b_{31} & b_{32} & b_{33} & b_{34} & b_{35} \\
    b_{41} & b_{42} & b_{43} & 0 & 0 \\
    0 & b_{52} & 0 & 0 & 0
\end{bmatrix}
\]
To unique solution to this difference equation system, three eigenvalues of the matrix $[A^{-1}B]$ should be out of unit circle.

### B.3 The response of interest rates to inflation

From equation (3.32)

$$d\hat{r} = \phi_{\pi} d\pi + \phi_{y} d\hat{y},$$  \hspace{1cm} (B.5)

From equation (3.31)

$$d\hat{m}c = -d\hat{y} + d\hat{n} + d\hat{w} = -d\hat{y} + (1 + \varphi) d\hat{n} + \sigma d\hat{e} + d\pi.$$  

Putting this equation into the Phillipse curve equation, the equation (3.37),

$$(\beta + \kappa - 1) d\pi = \kappa d\hat{y} - \kappa(1 + \varphi) d\hat{n} - \kappa \sigma d\hat{e}.$$  \hspace{1cm} (B.6)
From equation (3.27)

\[ d\hat{y} = \alpha d\hat{k} + (1 - \alpha)d\hat{n} \]  
\[ \text{(B.7)} \]

From equation (3.36)

\[ d\hat{y} = \tau_y d\hat{c} + (1 - \tau_y)d\hat{k} \]  
\[ \text{(B.8)} \]

because \( d\hat{t} = d\hat{k} \).

From equation (3.24)

\[ \chi_q d\hat{r}^k - d\hat{r} + d\pi = 0 \]

because \( d\hat{q} = 0 \). And from the equation (3.31)

\[ d\hat{r}^k = -d\hat{k} + (1 + \varphi)d\hat{n} + \sigma d\hat{c} + d\pi. \]

Combining two equations on \( d\hat{r}^k \) above

\[ \chi_q \left( -d\hat{k} + (1 + \varphi)d\hat{n} + \sigma d\hat{c} + d\pi \right) - d\hat{r} + d\pi = 0 \]

\[ \chi_q \left( -d\hat{k} + d\hat{y} - \frac{\beta + \kappa - 1}{\kappa}d\pi + d\pi \right) - d\hat{r} + d\pi = 0 \]

\[ -\chi_q d\hat{k} + \chi_q d\hat{y} - \left( \chi_q \frac{\beta - 1}{\kappa} - 1 \right) d\pi - d\hat{r} = 0 \]  
\[ \text{(B.9)} \]

where the second equality is from equation (B.7).

From equation (B.7)

\[ d\hat{k} = \frac{1}{\alpha}d\hat{y} - \frac{1 - \alpha}{\alpha}d\hat{n}. \]  
\[ \text{(B.10)} \]
Putting this equation into equation (B.8)

\[
\frac{dy}{\alpha} = \tau y d\hat{c} + (1 - \tau y) \left( \frac{1}{\alpha} d\hat{y} - \frac{1 - \alpha}{\alpha} d\hat{n} \right)
\]

\[
\left(1 - \frac{1 - \tau y}{\alpha}\right) \frac{dy}{\alpha} = \tau y d\hat{c} - \frac{(1 - \tau y)(1 - \alpha)}{\alpha} d\hat{n}
\]

\[
d\hat{c} = a_1 d\hat{y} + a_2 d\hat{n}.
\]

(B.11)

where \( a_1 \equiv \frac{\alpha - 1 + \tau y}{\tau y \alpha} \) and \( a_2 \equiv \frac{(1 - \tau y)(1 - \alpha)}{\tau y \alpha} \). Putting this equation into the equation (B.9) after equation (B.12) is replaced into equation (B.10)

\[
(\beta + \kappa - 1)d\pi = \kappa d\hat{y} - \kappa (1 + \varphi) d\hat{n} - \kappa \sigma (a_1 d\hat{y} + a_2 d\hat{n})
\]

\[
(\beta + \kappa - 1)d\pi = \kappa (1 - \sigma a_1) d\hat{y} - \kappa (1 + \varphi + \sigma a_2) d\hat{n}
\]

\[
d\hat{n} = b_1 d\hat{y} - b_2 d\pi,
\]

(B.12)

where \( b_1 \equiv \frac{1 - \sigma a_1}{1 + \varphi + \sigma a_2} \) and \( b_2 \equiv \frac{\beta + \kappa - 1}{\kappa (1 + \varphi + \sigma a_2)} \).

Putting equation (B.10) into the equation (B.9) after equation (B.12) is replaced into equation (B.10)

\[-\chi q \left( \frac{1}{\alpha} d\hat{y} - \frac{1 - \alpha}{\alpha} (b_1 d\hat{y} - b_2 d\pi) \right) + \chi q d\hat{y} - \left( \chi q \frac{\beta - 1}{\kappa} - 1 \right) d\pi - d\hat{r} = 0\]

\[
d\hat{r} = c_1 d\hat{y} + c_2 d\pi,
\]

(B.13)

where \( c_1 \equiv -\chi q \left( \frac{1}{\alpha} - \frac{(1 - \alpha)b_1}{\alpha} - 1 \right) \) and \( c_2 \equiv -\left( \chi q \frac{(1 - \alpha)b_2}{\alpha} + \chi q \frac{(\beta - 1) - \kappa}{\kappa} \right) \). Then \( d\hat{y} = \frac{1}{c_1} d\hat{r} - \frac{c_2 d\pi}{c_1} \) and put this into equation (B.5)

\[
d\hat{r} = \frac{c_1 \phi \pi - c_2 \phi y}{c_1 - \phi y} d\pi.
\]

(B.14)

If \( \frac{c_1 \phi \pi - c_2 \phi y}{c_1 - \phi y} > 1 \), the Taylor rule in this model satisfies the Taylor Principle.
B.4 The forward-looking interest rate rules.

Equation (B.3) can be rearranged as

\[ \sigma \chi c E_t \hat{c}_{t+1} + \lambda \tau c E_t \pi_{t+1} = \sigma \chi n \hat{n}_t + \sigma \chi c \hat{c}_t + \lambda \tau c \hat{r}_t - \sigma \chi n \hat{n}_{t-1} \]  (B.15)

From equations (B.2) and (B.15), we can get \( E_t \hat{c}_{t+2} \) and \( E_t \hat{\pi}_{t+2} \), respectively,

\[
\begin{align*}
E_t \hat{\pi}_{t+2} &= h_k \hat{k}_{t+1} - h_n E_t \hat{n}_{t+1} - h_c E_t \hat{c}_{t+1} + h_r E_t \hat{r}_{t+1} - h_l \hat{n}_t \\
E_t \hat{c}_{t+2} &= j_k \hat{k}_{t+1} - j_n E_t \hat{n}_{t+1} + j_c E_t \hat{c}_{t+1} + j_r E_t \hat{r}_{t+1} - j_l \hat{n}_t
\end{align*}
\]

\[
\begin{align*}
h_k &= \frac{\chi \alpha_k}{\chi c (\beta + \kappa) - \kappa \lambda \tau c} \quad & h_n &= \frac{\chi \alpha_n + \kappa g_n}{\chi c (\beta + \kappa) - \kappa \lambda \tau c} \\
h_c &= \frac{\kappa g_c}{\chi c (\beta + \kappa) - \kappa \lambda \tau c} \quad & h_r &= \frac{\chi c (\beta + \kappa) - \kappa \lambda \tau c}{\chi c (\beta + \kappa) - \kappa \lambda \tau c} \\
h_l &= \frac{\kappa \lambda r}{\chi c (\beta + \kappa) - \kappa \lambda \tau c} \quad & h_i &= \frac{\chi c (\beta + \kappa) - \kappa \lambda \tau c}{\chi c (\beta + \kappa) - \kappa \lambda \tau c} \\
j_k &= \frac{a_k - (\beta + \kappa) h_k}{\sigma \kappa} \quad & j_n &= \frac{a_n - (\beta + \kappa) h_n}{\sigma \kappa} \\
j_c &= \frac{(\beta + \kappa) h_c}{\sigma \kappa} \quad & j_r &= \frac{1 - (\beta + \kappa) h_r}{\sigma \kappa} \\
j_l &= \frac{(\beta + \kappa) h_l}{\sigma \kappa} \quad & j_i &= \frac{(\beta + \kappa) h_i}{\sigma \kappa} \\
a_k &= \alpha \kappa \quad & a_n &= \kappa (\alpha + \varphi) \\
g_n &= \sigma \chi n \quad & g_c &= \sigma \chi c \\
b_r &= \chi \tau c \quad & b_l &= \sigma \chi n
\end{align*}
\]
Equation (B.4) can be expressed as

\begin{align*}
(\beta \psi_1 - \chi_q(1 - \sigma j_k - h_k)) E_t \hat{k}_{t+1} \\
+ (\beta \psi_2 + \chi_q((1 + \phi) - \sigma j_n - h_n)) E_t \hat{n}_{t+1} \\
-(\beta \psi_3 - \chi_q(\sigma j_c - h_c)) E_t \hat{c}_{t+1} + (\chi_q(\sigma j_r + h_r) + 1) E_t \pi_{t+1} \\
+ (\chi_q(\sigma j_r - h_r) - 1) \hat{r}_{t+1} \\
= \psi_1 \hat{k}_t + (\psi_2 + \chi_q(\sigma j_l - h_l)) \hat{n}_t - \psi_3 \hat{c}_t
\end{align*}

(B.16)

The forward-looking interest rate rule is

\begin{equation}
\hat{r}_t = \phi_\pi E_t \pi_{t+1} + \phi_y E_t \hat{y}_{t+1}.
\end{equation}

(B.17)

We can derive a difference equation system from the equations (B.1), (B.2), (B.15), (B.16), (B.17), and \( \hat{h}_t = \hat{n}_{t-1} \).
Appendix C

General Procedure of Bayesian Estimation on DSGE model

The Bayesian method is a rule to change your belief by using data. This approach basically follows the Bayes’ theorem,

\[ P(A|B) = \frac{P(B \cap A)}{P(B)}, \]

where \( A \) and \( B \) are events and \( P(B) \neq 0 \). In here, a probability \( P(A) \) is to measure strength of belief on a statement that a event \( A \) is true. This is a subjective view of the probability. This view, however, can allow us to use some words such as “very probable” or “highly unlikely.” Hence, we can reach an inference, which the conventional econometrics cannot, such that a theory \( A \) is not very probable while a theory \( B \) is highly likely. This is big attraction of the Bayesian algorithm. Although Bayesian inferences are not objective but subjective, Bayesian ensures that a way that people change their belief should follow a probability law, especially, the Bayes’ theorem.

The Bayes’ theorem can be translated into

\[ p(\mu|X) = \frac{p(X|\mu)p(\mu)}{p(X)}, \]

\[ p(\mu|X) \sim p(X|\mu)p(\mu), \]

where \( p(\mu|X) \) is the posterior, \( p(X|\mu) \) the likelihood function, \( p(\mu) \) the prior, \( p(X) \) the
marginal distribution or predictive distribution of data, $\mu$ the vector of structural parameters and $X$ a sample space.\(^1\)

C.1 The likelihood

If $x \in X$ is data that you collect, $p(x|\mu)$ can be called the likelihood function of $\mu$. We will consider only a kernel of likelihood function in a procedure to obtain posterior distributions, that is, we want only a kernel of posterior for inferences. This fact says us that different likelihood functions can derive an identical posterior distribution and can result in an identical inference. Therefore, the Bayesian inference will follow the likelihood principle.\(^2\)

C.2 Prior

Priors say your subjective belief on $\mu$ in a distribution and provide a basic of Bayesian inference with the likelihood function. That is, the prior reflects subjective opinions or provides us with information derived from data not included in estimation samples. Therefore, the prior can be a natural device for economists. Most of economists will think rationally that many behavioral parameters will have probable values in reasonable rages. These belief will be from well-educated economic intuition and the prior introduces researchers the belief as pre-sample information. In estimating macroeconomic models, firstly microeconomic evidences can provide with sensible priors and it is natural for economists use the prior from micro evidences. Some estimation values of macro parameters from other countries can be an alternative source for the prior because of an default belief that one of many

\(^1\)This appendix summarizes Koop (2003), Lancaster (2004), An and Schorfheide (2007) a lecture note of Alexander Kriwoluzky, and a lecture note of Juan F Rubio Ramirez.

\(^2\)By Lancaster (2004), the definition of the likelihood principle is as following; this states that likelihoods that are proportional should lead to the same inferences (given the same prior). Notice that the data that might have been observed by the two investigators are quite different. What matters for Bayesian inference are the data that were observed; the data might have been seen but were not are irrelevant.
important differences between economics and the other social science is that agents, individuals, among nations is basically identical and behavioral differences among them can be explained by a difference of relative prices. Therefore, when we deal with emerging economies and data are very limited, the pre-sample information from the other country will be very useful.

If a likelihood function can assign same probabilities to some parameters $\mu \in \mathcal{M}$, $\mathcal{M}$ is a parameter space, for all data, that is, $p(x|\mu_1) = p(x|\mu_2)$, $\mu_1$ and $\mu_2 \in \mathcal{M}$, then we will do not change our belief on $\mu_1$ and $\mu_2$ and $\mu_1$ and $\mu_2$ is said to be observationally equivalent. That there are these flat spots of parameters for all observable data in a likelihood means that we cannot identify parameters. The identification problem occurs. Since flat spots on likelihoods function do not have a unique maximum and a second-differential matrix on non-identified spots become typically singular, the maximum likelihood inference gets in trouble. Bayesian, however, do not maximize a likelihood function so it does not matter in the Bayesian view.

C.3 Posterior

The posterior says your belief on parameters given the prior on parameters and the likelihood function from data. In the frequentist econometrics, the counter part of the posterior is a table of estimates of parameters with their estimated standard errors. In the posterior, our interest is to obtain scalar functions of parameter vector $\mu$. That is to want marginal distributions related with $\mu$ and to mean to include integrals of $p(\mu|x)$ on all elements except for interested parameter for the marginal distribution. If these distributions are simple, then it is derived analytically or asymptotically, but generally the distribution is

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3By Lancaster (2004), the definition of the identification is as following: a value $\mu_a$ of a parameter is identified if there is no other value $\mu_b$ such that $p(x|\mu_a) = p(x|\mu_b) \forall x \in X$, $X$ is a sample space. The model is identified if all the parameter values are identified, in which case the parameter $\mu$ is said to be identified.
very complicated, so we can do this job and find some features of the required distribution by using samples simulated from required distributions by a computer. Therefore, a remained problem in implementing the Bayesian algorithm is how to sample from the required distribution.

There are four case to sample from the required distribution. A distribution from which we want to sample is “target distribution.” The first case is that the target distribution is available. The second case is that although the target distribution is not a standard form, this distribution can be sampled from available distributions. The first and second case are simply implemented by computer programs. The third case is that although the target distribution is not in a standard form, it may be possible to sample from the target distribution by combining samples from available distributions with some tricks computationally. In this case, the rejection sampling and the importance sampling are used. The forth case is that the target distribution is not in a standard form and the rejection or importance sampling are not easily exploited. In this case, one solution is to construct a stochastic process satisfying as following: 1) have a stationary distribution, 2) converge the stationary distribution, and 3) the stationary distribution is the target distribution. We can use Markov Chain Monte Carlo (MCMC) method for this case. Sample from MCMC are typically not independent draws from the target distribution but each draw are considered to be from the target distribution. Before exploring MCMC method, we will illustrate a general procedure to derive the likelihood function for DSGE models.

C.4 Evaluating the likelihood function for DSGE models

Since most of DSGE models do not have “paper and pencil” solutions, we should feature equilibrium dynamics of DSGE models by numerical approaches. If exogenous shocks are
given, the log-linearized DSGE model can be expressed as a linear rational expectations (LRE) model and the LRE model can be solved by the methods described in Sims (2002). A LRE model for a log-linearized DSGE model can be expressed by:

\[ \Gamma_0(\mu)\mathbf{y}_{t+1} = \Gamma_1(\mu)\mathbf{y}_t + \Gamma_\varepsilon(\mu)\varepsilon_{t+1} + \Gamma_\eta(\mu)\eta_{t+1}, \]

where \( \mathbf{y}_t \) is the vector of unobservable model variables, \( \varepsilon_t \) is the vector of the exogenous processes, \( E\varepsilon_t = 0 \) and \( E\varepsilon_t\varepsilon'_t = \Sigma_t \), and \( \eta_t \) is the rational expectations forecast errors, \( \eta_{t+1}^y = \hat{y}_{t+1} - E_t\hat{y}_{t+1} \). The matrices \( \Gamma \) explain the dynamics of the exogenous shock processes and \( \mu \) represent the structural parameters of the model. Then the LRE system will have the following solution by Sims method:

\[ \mathbf{y}_t = \Phi_1\mathbf{y}_{t-1} + \Phi_2\varepsilon_t; \text{ state transition equation} \]

where \( \varepsilon_t \sim iidN(0, Q) \). Then we can express the log-linearized model as state-space model:

\[ \mathbf{X}_t = A(\mu) + B\mathbf{y}_t + \nu_t, \quad \nu_t \sim N(0, R); \text{ measurement equation,} \]

where \( \mathbf{X}_t \) is an vector of observable variables, \( A(\mu) \) is the mean of \( \mathbf{X}_t \), \( EX_t = A(\mu) \), and \( B \) does not depend on the structural parameter as it merely selects elements of \( \mathbf{y}_t \). \( \nu_t \) is the measurement error vector.

The next step is to obtain a likelihood function of a DSGE model not having analytical solution. To evaluate the likelihood function for DSGE models, we can use the state-space representation and filtering theories. In here, we will evaluate the likelihood function using the Kalman filter that is an algorithm for calculating linear least forecasts of the state vector \( \mathbf{y} \) on the basis of data observed through date \( t \), \( E_{t-1}[\mathbf{y}_t] = \hat{\mathbf{y}}_t \) and calculates these forecasts recursively. To set up the a likelihood function, we will derive a conditional
forecast error, and a mean squared error matrix, or covariance matrix, through the repeated Kalman filter algorithm. At the end of iteration step we will express $\hat{y}_{t+1}$ and $\Sigma_{y,t+1} = E[(y_{t+1} - \hat{y}_{t+1})(y_{t+1} - \hat{y}_{t+1})']$.

The Kalman filter algorithm can be summarized as following; if the state vector and its mean square error matrix are given, or given yesterday’s expectations of the state today, we can i) forecast the observable variables, ii) compute the corresponding forecast error and update your beliefs about the state today, and iii) given the updated beliefs form expectations about the state tomorrow, their associated forecast error and use those again to forecast the observable variables in i).

Innovation representation: We can define forecast error as

\[ s_t = X_t - \hat{X}_t = X_t - B\hat{y}_t = B(y_t - \hat{y}_t) + \nu_t, \]

and

\[ \Sigma st = B\Sigma_{yt}B' + R. \]

Updating: The optimal forecast of $\hat{y}_{t+1}$ is then given by

\[ \hat{y}_{t+1} = \Phi_1\hat{y}_t + K_t s_t, \]

where $K_t$ denotes the Kalman gain and is given by

\[ K_t = \Phi_1\Sigma_{yt}B'(B\Sigma_{yt}B' + R)^{-1}. \]

Its corresponding mean square error is updated by

\[ \Sigma_{y,t+1} = \Phi_1\Sigma_{yt}\Phi_1' + \Phi_2Q\Phi_2' - K_tB\Sigma_{yt}\Phi_1'. \]
It is necessary to give an initial condition for $y_0$ and $\Sigma y_0$ in order to start the Kalman Filter. The process for $y_t$ is typically assumed to be covariance stationary. The unconditional mean is the solution to $E[y_{t+1}] = \Phi_1 E[y_t]$. The unconditional variance is the solution to the Lyapunov equation, $\Sigma_y = \Phi_1 \Sigma y \Phi_1' + \Phi_2 Q \Phi_2'$. Therefore, $y_0 = 0$, and $\Sigma y_0 = [I - (\Phi_1 \otimes \Phi_1)]^{-1} \text{vec}(\Phi_2 Q \Phi_2')$.

The Kalman filter iteration follows as following:

At the $i$-th step, given $\hat{y}_{i-1}$, $\Sigma y, i - 1$, and $X_i$, compute:

i) $s_i = X_i - B \hat{y}_{i-1}$,

ii) $K_i = \Phi_1 \Sigma y, i - 1 B' (B \Sigma y, i - 1 B' + R)^{-1}$,

iii) $\hat{y}_i = \Phi_1 \hat{y}_{i-1} + K_i s_i$,

iv) $\Sigma y, i = \Phi_1 \Sigma y, i - 1 \Phi_1' + \Phi_2 Q \Phi_2' - \Phi_1 \Sigma y, i - 1 B' (B \Sigma y, i - 1 B' + R)^{-1} B \Sigma y, i \Phi_1'$,

v) Go back to i).

We want to derive a function that tells us how likely the time series is, i.e., the realization of the data, give a vector of deep parameters (and a DSGE model structure). Assume that the initial state and the sequence of innovations $\{\varepsilon_t, \nu_t\}_{t=1}^T$ are multivariate Gaussian. The distribution of $X_t$ conditional on $X^{t-1} = X_{t-1}, \ldots, X_1$ is given by

$$X_t | X^{t-1}, \mu \sim N \left( B \hat{x}_t, (B \Sigma_y B' + R) \right).$$

Then, a conditional likelihood function is

$$\mathcal{L}(X_t | X^{t-1}, \mu) = (2\pi)^{-n/2} |\Sigma_{x_t}|^{-0.5} \times \exp \left\{ -0.5 (X_t - B \hat{y}_t)' \Sigma_{x_t}^{-1} (X_t - B \hat{y}_t) \right\},$$

where $n$ is the number of observable variables. The likelihood associated with the sample
\( \mathbf{X} \) is given by the product of the conditional likelihood functions,

\[
\mathcal{L}(\mathbf{X}_t|\mu) = \prod_{t=1}^{T} \mathcal{L}(\mathbf{X}_t|\mathbf{X}_{t-1}, \mu).
\]

Correspondingly, the log likelihood \( l = \log(\mathcal{L}) \) is given by

\[
l(\mathbf{X}|\mu) = \sum_{t=1}^{T} l(\mathbf{X}_t|\mathbf{X}_{t-1}, \mu).
\]

We will evaluate and maximize the likelihood function and compute the Hessian at the maximum. The log likelihood is evaluated the following way;

i) compute the constant: \( C = -(Tn/2) \log(2\pi) \),

ii) at every iteration step \( i \), given \( s_i \) and \( \Sigma_{s_i} \), add up \( C \) and compute:

\[
l(\mathbf{X}_t|\mathbf{X}_{t-1}, \mu) = -0.5 \log |\Sigma_{s_i}| - 0.5 \left( s_i^t \Sigma_{s_i}^{-1} s_i \right).
\]

The Hessian \( \mathcal{H} \) as a matrix, which \( i, j \) - th element is defined as \( \mathcal{H}_{i,j} = \frac{\partial^2 l(\mathbf{X}|\mu)}{\partial \mu_i \partial \mu_j} \),

\( i, j = 1, 2, \cdots, k \), \( k \) is the number of structural parameters. The inverse of the negative Hessian is a consistent estimator of the variance covariance matrix of a maximum likelihood estimator.

### C.5 Bayesian computations

We will evaluate the posterior around the distribution mode which will provide us with initial values in MCMC. We want to describe the posterior distribution \( p(\mu|\mathbf{X}) \), and then, the posterior moments will be

\[
E[g(\mu)] = \frac{\int g(\mu)p(\mu|\mathbf{X})d\mu}{\int p(\mu|\mathbf{X})d\mu}.
\]
Since this integrations are (usually) difficult to deduct analytically, to solve this we will simulate the distribution and compute the statistic of interest given sample.

In this stage, our aim is to find a kernel corresponding to a stationary distribution, in here, a posterior, that we want to target, because we want to sample from the target distribution. MCMC is a method to sample from the target distribution by constructing a markov chain converging to the stationary distribution. We should find an appropriate transition kernel, $\mathcal{K}(x,y)$, converging a target distribution, the posterior distribution, as a stationary distribution.

### C.5.1 Metropolis-Hastings

Let $p(y)$ be a target distribution, $y \in \mathcal{M}$ may be vector valued, and $q(y|x)$ be a sequence of proposal density, $x \in \mathcal{M}$. Draws from $q(y|x)$ can be rejected in this method, the families $q(y|x)$ are not transition kernels. The state of chain will be unchanged in this case. We can construct a kernel with the target distribution, $p(y)$, as a stationary distribution by using the following algorithm.

**Metropolis-Hasting algorithm**

1. Choose an initial value, $y_0$, and set $j = 0$.
2. Draw $y^*$ from $q(\cdot | y_j)$.
3. Calculate the ration $r(y_j, y^*) = \frac{p(y^*) q(y_j | y^*)}{p(y_j) q(y^* | y_j)}$.
4. If $r \geq 1$ set $y_{j+1} = y^*$; otherwise set $\begin{cases} y_{j+1} = y^* \text{ with probability } r, \\ y_{j+1} = y_t \text{ with probability } 1-r. \end{cases}$
5. Increase $t$ by one and then proceed to step 2.

Let $\rho(y_j, y^*) = \min\left( \frac{p(y^*) q(y_j | y^*)}{p(y_j) q(y^* | y_j)}, 1 \right)$ which is a probability that a $y^*$ is accepted. This expression’s rational is that the chain is most likely to accept $y$’s that are probable relative to the current value of the chain. If the function $q(y|x)$ is symmetric in $y$ and $x$, the Metropolis-Hasting algorithm change to the Metropolis method.
C.5.2 Random walk Metropolis-Hastings

When a good proposal density for the posterior cannot be found, the Random walk Metropolis-Hastings (M-H) algorithm is very useful. In the Random walk Metropolis-Hastings (M-H) algorithm, we choose \( q(y|x) = q(y - x) \), with \( q \) is a multivariate density. We do not need to approximate the posterior, so formally, draws are generated according to \( y_j^* = y_{j-1} + \epsilon_j \) where \( \epsilon \) is a random perturbation, or a increment random variable, of the current state of the chain and \( j = 0, 1, 2, \ldots, J \). Note that \( y_j^* \) and \( y_{j-1} \) enter symmetrically in the random walk chain, then this means the acceptance probability is

\[
\rho(y_{j-1}, y_j^*) = \min \left( \frac{p(y = y_j^* | X)}{p(y = y_{j-1} | X)}, 1 \right),
\]

and it can be clearly seen that the random walk chain tends to move towards regions of higher posterior probability. Commonly and conveniently the choice of density for \( \epsilon_j \) is the multivariate Normal. Therefore, the density of \( y_j^* \) is \( N(y_{j-1}, \Sigma) \). In this step, we have to select \( \Sigma \) so that the acceptance probability tends to be neither too high nor too low. If the acceptance probability is very small, candidate draws tends to move far out in the tail of the posterior in regions which the posterior indicates are quite improbable. If the acceptance probability, however, is very high, candidate draws tend to be very close to one another the acceptance probability will be near one.

We can set \( \Sigma = c\Sigma_\mu \) where \( c \) is a scalar and \( \Sigma_\mu \) is an estimate of the posterior covariance matrix of \( \mu \in \mathcal{M} \), a vector of parameters. \( c \) is related to the acceptance rate of chains and you can choose \( c \) for the acceptance rate to be from about 0.25 to about 0.45. \( c \) and the acceptance rate is related negatively. Then, we have to select \( \Sigma_\mu \), an estimate of \( \text{var}(\mu | X) \). An approach is to set \( \Sigma_\mu \) equal to \( \text{var}(\hat{\mu}_{ML}) \), the estimate of the variance of the maximum likelihood estimate. The posterior distribution of \( \mu \) is assumed to be asymptotically normal. And then construct a Gaussian estimation around the posterior mode with a scaled version.
of the asymptotic covariance matrix as covariance matrix for the proposal distribution. It will generate a sequence of dependent draws from the posterior distribution that can be averaged to approximate posterior moments.

We have to implement the Random walk M-H for estimation of our DSGE model as following;

Step 0. Read data,
Step 1. Set arbitrarily an initial value for \( \mu_0 \) and \( J \).
Step 2. Solve the DSGE model and obtain the posterior mode and the inverse of the Hessian computed at the mode.
   a) Given \( \mu_0 \) evaluate prior \( p(\mu_0) \),
   b) Given \( \mu_0 \) solve the DSGE model using Sims methods,
   c) Evaluate the likelihood function of the DSGE model, \( p(x|\mu_0) \),
      using the Kalman filter,
   d) Obtain the posterior mode \( \hat{\mu} \) and the inverse of the Hessian \( \Sigma_\mu \) computed at the mode.
Step 3. Implement the Random walk M-H.
   Start by drawing \( \mu_0 \) from \( N(\hat{\mu}, c^2\Sigma_\mu) \).
   a) Draw \( \mu^* \) from \( N(\mu_{j-1}, c^2\Sigma_\mu) \), for \( j = 1, 2, 3, \ldots, J \),
   b) Accept \( \mu^*(\mu_j = \mu^*) \) with probability \( \min[1, r(\mu_{j-1}, \mu^*|x)] \)
      and reject \( (\mu_j = \mu_{j-1}) \) otherwise,
      \[ r(\mu_{j-1}, \mu^*|x) = \frac{p(\mu^*|x)}{p(\mu_{j-1}|x)} \],
   c) If \( j \leq J \) then \( j \rightarrow j + 1 \) and go to a).
Step 4. Approximate the posterior expected value of a function \( g(\mu) \) by \( 1/J \sum_{j=1}^{J} g(\mu_j) \).
C.6 Marginal data density

The Bayesian framework provides us with a very useful tool to evaluate relatively the model fit. To compare models, the posterior odds are usually used. To compute the posterior odds, we have to know posterior probabilities and the key object in the calculation of posterior probabilities is the marginal data density \( p(X) = \int p(X|\mu)p(\mu)d\mu \). And, the computation of the marginal data density is the practical trouble in implementing posterior odds. In here, we consider the modified harmonic mean estimator of Geweke (1999).

We want to compute \( p(X) = \int p(X,\mu)d\mu \) for the model comparison. If \( \int f(\mu)d\mu = 1 \), harmonic mean estimators are based on the identity,

\[
\frac{1}{p(X)} = \frac{\int f(\mu)d\mu}{\int p(X|\mu)p(\mu)d\mu} = \int \frac{f(\mu)}{p(X|\mu)p(\mu)}p(\mu|X)d\mu = E\left[ \frac{f(\mu)}{p(X|\mu)p(\mu)} \right].
\]

Conditional on the choice of \( f(\mu) \) an obvious estimator is

\[
\hat{p}(X) = \left[ \frac{1}{J} \sum_{j=1}^{J} \frac{f(\mu^{(j)})}{p(X|\mu^{(j)})p(\mu^{(j)})} \right]^{-1},
\]

where \( \mu^{(j)} \) is drawn from the posterior \( p(\mu|X) \). To make the numerical approximation efficient, Geweke (1999) suppose to use the density of a truncated multivariate normal distribution,

\[
f(\mu) = \tau^{-1}(2\pi)^{-d/2}|V_{\mu}|^{-1/2}\exp\left[ -0.5(\mu - \bar{\mu})'V_{\mu}^{-1}(\mu - \bar{\mu}) \right] \times F\left\{ (\mu - \bar{\mu})'V_{\mu}^{-1}(\mu - \bar{\mu}) \leq F_{\chi^2_d}^{-1} \right\},
\]

where \( \bar{\mu} \) and \( V_{\mu} \) are the posterior mean and covariance matrix computed from the output of the posterior simulator, \( d \) is the dimension of the parameter vector, \( F_{\chi^2_d}^{-1} \) is the cumulative density function of a \( \chi^2 \) random variable with \( d \) degrees of freedom, \( \tau \in (0,1) \), and \( F(x \leq \)
a) is the indicator function that is 1 if $x \geq a$ and zero otherwise.

In comparing models by Bayesian methods, a model’s performance get better as the log data density of the model is increasing.
References


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