SIMPLIFIED PREDICTION EQUATION FOR ULTIMATE STRESS IN BEAMS PRESTRESSED WITH HYBRID TENDONS

by

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ABSTRACT OF THE THESIS

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The use of unbonded tendons is getting more widespread in post-tensioning industry; especially in rehabilitation and strengthening of existing damaged concrete members. The prediction of the stress at ultimate in unbonded tendon is important in calculating the capacity of structural members. This thesis presents a simplified prediction equation for concrete beams prestressed with hybrid (a combination of bonded and unbonded) tendons. The proposed equation is based on the Generalized Incremental Analysis (GIA) which uses the trussed-beam model developed by Ozkul et al. (2008) and Nassif et al. (2003).

The main objective of this research is to develop a simple, but accurate design equation for the prediction of the stress at ultimate in unbonded tendon. Most important parameters such as loading type, effective prestress of unbonded tendon, concrete strength, area of steel reinforcement and span-to-depth ratio are taken into consideration. The equation is applicable to beams prestressed with unbonded or hybrid, FRP or steel, external or internal tendons. For the validation of the proposed simplified equation, test
results available in the literature (199 beams) are collected. The results show that the proposed simplified equation exhibited very good accuracy for the calculation of stress at ultimate in unbonded tendon. The simplified equation is easy to use, accurate and applicable to any material type and combination of tendons.
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LIST OF SYMBOLS

\( A_{psB} \) Area of bonded prestressed reinforcement

\( A_{psU} \) Area of unbonded prestressed reinforcement

\( A_s \) Area of non-prestressed tensile reinforcement

\( A'_s \) Area of non-prestressed compressive reinforcement

\( b \) Width of the section

\( b_w \) Web thickness for flanged sections

\( c \) Neutral axis depth measured from extreme top concrete fiber

\( C \) Concrete Compressive Force

\( d_{psB} \) Depth of the bonded prestressed reinforcement measured from top concrete fiber

\( d_{psU} \) Depth of the unbonded prestressed reinforcement measured from top concrete fiber

\( d_s \) Depth of the non-prestressed tensile reinforcement measured from top concrete fiber

\( d'_s \) Depth of the non-prestressed compressive reinforcement measured from top concrete fiber

\( E_c \) Modulus of elasticity of concrete

\( e_{psU} \) Eccentricity of unbonded prestressed tendon measured from the centroid of section

\( E_{ps} \) Modulus of elasticity of related prestressed tendon

\( f \) Load Geometry Factor
$f_c$ Compressive strength of concrete

$f_{frpu}$ Tensile strength of FRP tendon

$f_{peU}$ Effective prestressing force in unbonded tendons after all losses

$f_{paU}$ Ultimate stress of unbonded tendons

$f_{pu}^F$ Stress in unbonded tendons calculated from force equations

$f_{pu}^E$ Stress in unbonded tendons calculated from energy equilibrium

$f_{pu}$ Tensile strength of prestressing tendons

$f_{py}$ Yield stress of prestressing tendons

$f_s$ Stress in non-prestressed tensile reinforcement

$f'_s$ Stress in non-prestressed compressive reinforcement

$f_y$ Yield stress of non-prestressed reinforcement

$h$ Height of the section

$h_f$ Flange thickness of section

$L$ Span length

$L_a$ Theoretical constant moment region length

$L_h$ Distance from support to the plastic hinge

$L_L$ Distance from support to applied point load

$L_p$ Plastic hinge length

$M$ Bending moment

$M_{pl}$ Moment at plastic hinge
\( P \)  Load applied on the beam
\( T \)  Tensile Force
\( \Sigma \)  Total
\( \beta \)  Stress block reduction factor (ACI 318-08)
\( \delta \)  Elongation of unbonded tendon length between end anchorages
\( \Delta \)  Midspan deflection
\( \Delta \varepsilon_{puU} \)  Strain change in unbonded prestressed tendon
\( \Delta f_{puU} \)  Stress change in unbonded prestressed tendon
\( \varepsilon_c \)  Strain in the concrete
\( \varepsilon_{cu} \)  Strain in the top fiber of concrete at ultimate
\( \varepsilon_{peU} \)  Effective prestrain in unbonded prestressing tendons
\( \varepsilon_{puU} \)  Ultimate strain in unbonded prestressing tendons
\( \phi \)  Curvature along the beam
\( \theta \)  Angle of rotation at support
Subscripts

$B$  Bonded prestressed tendon
$c$  Concrete
$E$  External work
$f$  Flange of a section
$F$  T section
$frp$  Fiber Reinforced Polymer (FRP) type of tendons
$I$  Internal energy
$nor$  Normalized equation
$PL$  Point Load
$pe$  Effective stress in prestressing tendon
$ps$  Prestressed tendon
$R$  Rectangular section
$s$  Non-prestressed reinforcement
$u$  Crushing of concrete at ultimate
$U$  Unbonded prestressed tendon
$w$  Web of the flange section

Superscripts

$E$  Energy equilibrium
$F$  Force equilibrium
CHAPTER I

INTRODUCTION

1.1 Overview

Prestressed concrete, in which prestressing reinforcement would put concrete under compressive stresses to overcome its weakness in tension and to improve its response to external loads, has several advantages over traditional reinforced concrete. With prestressing technology, entire section is made effective, longer spans, less deflection, better cracking control and corrosion resistance can be accomplished.

Two different methods were developed for prestressing concrete: pretensioning and post tensioning. In pretensioning, the strands in stressing bed are tensioned prior to casting of the concrete. After casting of the concrete, the strands are released from the tensioning bed and the tensile stress is transferred to concrete by the full bond between strand and concrete. Most pretensioned concrete elements are prefabricated in a factory and transported to the construction site. Pretensioning technology is used in the construction of bridges (precast, pretensioned I girders with spans up to 150 ft), parking structures, buildings and cylindrical structures such as water storage tanks.

In post-tensioning, after the casting of the concrete, the tendons are tensioned and then tendons are anchored. Post-tensioning can be applied by using internal bonded and/or unbonded (internal or external) tendons. The bonded tendons are put inside a duct which is then grouted to protect the tendons from corrosion. In unbonded tendons, the duct is filled with grease. Post-tensioning technology is used in the repair and
strengthening of the existing structures (use of external tendons), construction of segmental box girder bridges with longer spans, thinner slabs (especially in high rise buildings) and nuclear power plants.

The main focus of this study is to simplify the equation proposed by Ozkul et al. (2008) for use in calculating the stress in combination of bonded and unbonded or hybrid tendons. Hybrid tendons are used in incrementally prestressed concrete girders in which the girder with bonded tendons is pretensioned prior to the casting of the slab. After the slab is cast, the girder-slab system with unbonded tendons is post-tensioned (Han et al. 2003, Nassif et al. 2004). Using hybrid tendons would provide the benefit of achieving longer span lengths with same section depth or shallower section depths can be achieved in bridge technology. In addition, to improve the ductility of the unbonded FRP tendons, bonded steel tendons are added to the beams prestressed with unbonded tendon which is another important use of hybrid tendons.

1.2 Problem Statement

In beams prestressed with bonded tendons, the change in strain in the tendon at any section is equal to the change in strain in the adjacent concrete section. Therefore, the stress in the bonded tendons can easily be found by using force equilibrium and strain compatibility equations.

In comparison to beams prestressed with bonded tendons, for beams including unbonded tendons there is no strain compatibility between the tendon and concrete due to the lack of bond. Therefore, under the effect of applied load, the stress increase in an unbonded tendon $\Delta f_{psU}$ depends on the deformation of the whole member. Therefore, the
stress increase in the unbonded tendon cannot be estimated simply by using the strain compatibility and force equilibrium equations; the analysis of the whole member is required. Because of the complexity of the situation, it is difficult to develop an accurate and rational analytical model. In order to develop a deformation-based analysis, it is assumed that the strain is uniform along the section.

1.3 Research Significance

In order to compute the ultimate moment resistance of a beam prestressed with unbonded or hybrid tendons, the tendon stress at ultimate is required. In the past, researchers proposed many equations; however, most of these equations are not simple or practical enough to be used as a design equation. Moreover, the code equations are simple, but very conservative compared with experimental data. All code equations do not take into account different tendon materials. Therefore, there is a need for an accurate but simple equation for the prediction of the unbonded tendon stress at ultimate.

1.4 Objective and Scope of Work

The objective of this research is to develop an accurate, simple and rational equation for the prediction of the stress at ultimate that can be also applicable to beams prestressed with unbonded or hybrid, steel or FRP, external or internal tendons.

The scope of work can be explained as four matters:

1. To compare the proposed equation with generalized incremental analysis developed by Ozkul et al. (2008) and finite element analysis (FEA).
2. To perform a parametric study to investigate the effect of various parameters on the ultimate stress and stress increase in unbonded tendons.

3. To review the existing prediction equations from literature and also Code equations for the evaluation of the ultimate stress of unbonded tendons and to compare them with the experimental data for determining the accuracy of prediction equations.

4. To verify the accuracy and applicability of the proposed equation with experimental data.

1.5 Organization of the Thesis

The research work is described in six chapters:

Chapter 1 presents the introduction which includes the overview of the research, problem statement and objectives of the research.

Chapter 2 is the literature review which covers the previous studies done by different researches related to the stress in unbonded tendons. In addition, the code provisions for the calculation of the unbonded tendon stress at ultimate are summarized.

Chapter 3 presents the simplification of the equation for calculating the ultimate strength of beams with unbonded tendons. In this chapter, various alternatives for the ultimate strength equation are presented. The most accurate and simplest form is selected as the final form of the equation.

Chapter 4 is the parametric study part in which the effects of various parameters on the ultimate stress in unbonded tendons are investigated.
Chapter 5 is the results part which presents the comparison of proposed equation results with experimental ones. The results are shown for the total loading history and at ultimate for beams prestressed with unbonded or hybrid tendons.

Chapter 6 is the conclusion of this study which summarizes the research, observations and conclusions based on the results and finally presents some recommendations for the improvement of the current study.
CHAPTER II

LITERATURE REVIEW

2.1 Introduction

Since the early 1950’s, in order to evaluate the stress at ultimate in unbonded tendons, many experimental and analytical researches have been done. However, very few researches were implemented about the use of both bonded and unbonded (hybrid) tendons. Due to the increased use of the unbonded tendons there is still a need for more studies on this topic especially about the use of hybrid tendons.

In comparison with the analysis of beams and slabs with bonded tendons, the analysis of such members prestressed with unbonded tendons is more difficult. Because in members with unbonded tendons, there is no strain compatibility between the tendon and concrete due to the lack of bond. For that reason, the stress increase in an unbonded tendon is member dependent rather than section dependent. For the beams prestressed with bonded tendons, due to the bond between the concrete and the tendon, the strain increase in tendon is equivalent to the strain increase in the adjacent concrete at that level. Then, for these beams the stress in tendon can be determined by using the strain increase found from strain compatibility and the initial strain due to the effective prestress. On the other hand, for the beams prestressed with unbonded tendons, the strain in the tendon is assumed to be constant throughout the span. In order to find the strain increase in the unbonded tendon, the analysis of the deformation of the entire section is required.
Therefore, the evaluation of the stress in unbonded tendon at the ultimate limit state $f_{ptU}$ is a challenging and complicated process.

The stress of the unbonded tendon at the ultimate limit state is needed in order to calculate the moment capacity of the member. Therefore, a simple and accurate equation is required to estimate the stress change in the unbonded tendon. Many researchers have dealt with the design, analysis and the modeling of the prestressed concrete members with unbonded tendons to find an equation for $f_{ptU}$. In the following section, the studies related to this topic are summarized in two parts. After that, the code equations for the calculation of $f_{ptU}$ are presented.

### 2.2 Related Studies

The first part includes the theoretical and experimental investigations related to the beams prestressed with unbonded tendons. Many researches implemented to understand the behavior of members prestressed with unbonded tendons and to propose a design equation for the calculation of $f_{ptU}$. In the second part, the theoretical and experimental studies related to the beams prestressed with bonded and unbonded (hybrid) tendons are summarized. When the amount of researches done related to the unbonded tendons and hybrid tendons are compared, it can be easily observed that the studies about the use of hybrid tendons are very limited. In addition, most of the equations proposed for the beams with unbonded tendons, are not applicable for the beams with hybrid tendons. Moreover, there is no equation proposed particularly for the use of both bonded and unbonded tendons. This situation emphasizes the need for a unique, rational, simple and
accurate equation to calculate $f_{psU}$ which is applicable for different conditions such as beams prestressed with hybrid tendons.

### 2.2.1 Studies on Prestressing with Unbonded Tendons

Studies on prestressing with unbonded tendons were examined and summarized by Ozkul (2007) and Tanchan (2001). Therefore, only the studies after 2005 are summarized in this study.

Tanchan (2001) carried out an experimental and analytical study to investigate the behavior of high strength concrete (HSC) beams prestressed with unbonded tendons. For that purpose, the author conducted tests on nine rectangular HSC beams prestressed with unbonded tendons. The main variables were area of non-prestressed steel and prestressed steel, prestressing stress, concrete compressive strength and span-to-depth ratio. In order to search the effect of these variables on the stress increase in unbonded tendons at ultimate, the author performed finite element analysis and parametric study with the use of ABAQUS (2000).

Naaman (2005) proposed a rational method of analysis of beams prestressed or partially prestressed with unbonded tendons in three states: elastic uncracked state, elastic cracked state and ultimate limit state. For this purpose, strain reduction coefficients or bond coefficients, $\Omega$ were used to form a relationship between the case of bonded tendons and unbonded tendons. The values of the bond coefficients for the ultimate limit state $\Omega_u$ are based on a regression analysis. Therefore, $\Omega_u$ values should be considered approximate and additional research is needed for the more exact values of strain reduction coefficient at ultimate.
Lou and Xiang (2006) investigated the effect of the bonded tension reinforcement on the response beams prestressed with unbonded tendons by considering two parameters, namely the area and yield strength of the tension reinforcement. The authors developed a numerical model based on the finite element method to predict the entire nonlinear response of beams including unbonded tendons. In their finite element model, the arc-length method is used instead of conventional incremental load method. In each arc incremental step, the authors utilized the Newton-Raphson method for the iterative procedure. They used experimental test results to verify the validity of the proposed model. The authors concluded that unbonded beams without tension reinforcement represent very poor behavior. However, with the inclusion of minimum amount of tension reinforcement, the flexural behavior of equivalent beams is greatly improved.

Manisekar and Senthil (2006) reviewed the previous analytical investigation related to the ultimate stress in unbonded tendons. They examined the existing prediction equations suggested by various researchers and collected six experimental data sets to evaluate the accuracy of each equation. The authors found out that the stress increase is directly related to the formation of the plastic hinge for the beams prestressed with internal unbonded tendons. They also made a review of the external tendons. They pointed out that the analysis of external tendons is different from that of the internal unbonded tendons with the inclusion of the second order effects and frictional effects. Therefore, the authors concluded that the prediction procedure for beams with internal unbonded tendons adopted at present is not appropriate for the beams with external tendons.
Park et al. (2006) proposed a numerical model based on the finite element method to analyze the unbonded post-tensioned structures. In addition, they developed another model which represents the friction and bond effect at the interface of tendon and the concrete. The authors presented a numerical procedure for material nonlinear analysis of prestressed concrete structures by using these models. The proposed procedure can be used to predict the response of pretensioned and bonded or unbonded post-tensioned structures such as deformation, cracking throughout their service life.

Diep and Niwa (2006) reviewed existing prediction equations for the ultimate unbonded tendon stress and used experimental data of 120 beams for the validation of the equations. They observed that prediction equations recommended in the codes are too conservative and available prediction equations developed by researchers don’t provide good accuracy. The authors proposed a new prediction equation for the computation of the unbonded tendon stress at ultimate by using plastic region length concept together with the parametric study. The proposed equation can be observed as follows:

\[
\begin{align*}
  f_{ps} &= f_{pe} + E_{ps}E_{cu} \left( \frac{d_{ps}}{c_y} - 1 \right) \frac{L_u}{L} \\
  c_y &= \frac{0.95A_{ps}f_{py} + A_yf_y - A'_{ps}f_y' - 0.85f_c(b - b_w)h_f}{0.85\beta f_c b_w}
\end{align*}
\]

(2.1)

where,

Harajli (2006) presented a comprehensive evaluation of the main parameters that influence the stress in unbonded tendons at ultimate. The author mentioned that the plastic hinge length has a significant effect on the unbonded tendon stress. Therefore, by using a physical model and a large experimental database, the author developed an accurate expression for the equivalent plastic hinge length. Based on this expression,
three possible design alternatives were proposed for the calculation of the unbonded tendon stress. Based on his investigation, the author concluded that the 2005 ACI Code (ACI 318-05) is over conservative for calculating $\Delta f_{ps}$ and AASHTO-LRFD (2010) approach is more rational than the ACI Code. The proposed design alternatives can be observed in the following lines:

Alternative I:

$$f_{ps} = E_{ps} \varepsilon_{ps} \left( Q + \frac{1-Q}{1+\left( \frac{E_{ps} \varepsilon_{ps}}{K f_{py}} \right)^N} \right)$$

(2.2)

where,

$$\varepsilon_{ps} = \varepsilon_{pe} + \varepsilon_{cu} \left( \frac{d_p - c_e}{L_a / n_p} \right) \left( \frac{20.7}{f} + 10.5 \right)$$

Then, the stress in the unbonded tendon is calculated using the equation of Menegetto and Pinto (1973) with $N=7$ and $K=1$.

Alternative II:

$$f_{ps} = f_{pe} + E_{ps} \varepsilon_{cu} \left( \frac{d_p - c_e}{L_a / n_p} \right) \left( \frac{20.7}{f} + 10.5 \right) \leq f_{py}$$

(2.3)

Alternative III:

The stress $f_{ps}$ is presented as a combination of Alternatives I and II:

$$f_{ps} = \frac{f_{pe} + K_o E_{ps} \varepsilon_{cu} \left[ d_p - \frac{\rho_d f_y}{0.85 \beta f_{c}'} \right]}{1 + \frac{K_o E_{ps} \varepsilon_{cu} P_{p} d_p}{0.85 \beta f_{c}'} + \frac{1}{f_{c}'}}$$

(2.4)
\[ K_p = \phi_p \left( \frac{20.7}{f} + 10.5 \right) \frac{n_p}{L_u} \]

where, \( n_p \) is the number of plastic hinges, and \( f \) is a function of loading type which is 10 for one-point loading, 3 for two-point loading and 6 for uniform loading.

Ozkul (2007) presented a general method for the analysis of beams prestressed with unbonded tendons. The method is based on force equilibrium, compatibility of deflection and work energy principle. In addition, Ozkul (2007) performed finite element analysis. As a result, an equation for the calculation of the unbonded tendon stress is developed. Thirteen high strength concrete beams prestressed with unbonded tendons were tested. The results of these tests and available test data in literature are used to validate the accuracy of the methodology and the equation. It is concluded that both the general methodology and the equation predict the unbonded tendon stress accurately.

Ozkul et al. (2008) presented an experimental program and an analytical study for the investigation of the behavior of HSC (high strength concrete) beams prestressed with unbonded tendons. In the experimental program, 25 simply supported HSC beams were tested to failure. Their analytical study based on the trussed beam model that consists of concrete and unbonded tendon elements. In the analytical study, this model was solved by using two analysis approaches: finite element analysis (FEA) and generalized incremental analysis (GIA). Generalized incremental analysis is based on the force equilibrium, compatibility of equations and work energy principle and applicable at various loading stages. The authors, proposed the following equation for predicting the stress in beams prestressed with unbonded tendons:

\[ f_{ps} = f_{pe} + \frac{E_{ps}}{196} \left[ \frac{e \beta_2 f'_b b}{A_p f_p + A_{ps} f_{ps}} \right] \]

\[ k_1 \leq f_{py} \]  

\[ (2.5) \]
where,

\[ k_i = \left[ 1 - 2 \frac{L_h}{L} - \frac{L_p^2}{L_L L} \right]; L_h = \frac{L}{2} - \frac{L}{2f} - (0.5d + 0.05Z) \]

Experimental results were used to evaluate the accuracy of the proposed methodology and the equation. According to this evaluation, it was concluded that both the method and the equation presented in this study are accurate compared with experimental results.

Ellbody and Bailey (2008) conducted tests on two simply supported slabs to investigate the behavior of unbonded post-tensioned one-way concrete slabs. The authors proposed finite element model using ABAQUS (2006). Fine mesh 3D solid elements are used to model the unbonded slabs and the model includes the nonlinear material models for the tendon and concrete. After the verification of the model with the two test results, the authors implemented a parametric study with 28 slabs. They compared the design code British Standards, BS8110-1 (2002) result for ultimate load and deflection at midspan with the results obtained from finite element analysis. It is concluded that BS8110-1 (2002) results for ultimate load are conservative for unbonded post-tensioned one-way concrete slabs.

Du and Au (2008) proposed a method for the prediction of the ultimate stress in unbonded steel or FRP external tendons. This method is based on Pannell’s deformation-based model. In this method, the ratio of the equivalent plastic hinge length to the neutral axis depth \( \varphi \) is evaluated by using available three groups of test results. The authors suggested \( \varphi_{\text{steel}} = 10 \) for steel tendons for regular purpose design. For FRP tendons, this factor is expressed as a multiplication of the same factor for steel tendons: \( \varphi_{\text{FRP}} = \lambda \varphi_{\text{steel}} \).
According to their findings, if modulus of elasticity of FRP is less than that of steel, $\varphi_{FRP}$ is larger than $\varphi_{steel}$ and vice versa is also correct. After the determination of $\varphi$ from experimental results, the authors combined section analysis with deformation based-approach and proposed the following formula which is a modified version of AASHTO LRFD (1998) equation:

$$f_{ps} = f_{pc} + 6000\left(\frac{d_p - c}{l_e}\right) \leq (f_{py})_{steel} \text{ or } (f_{pu})_{FRP} \quad [MPa]$$

(2.6)

Where,

$$l_e = \frac{L}{(1 + N / 2)}$$

In the above equations, $L$ is the length of tendon between anchorages, $N$ is the number of support hinges required to form a failure mechanism and $c$ is the neutral axis depth. As can be seen from the representation of the equation, it is both applicable for steel and FRP tendons.

Au et al. (2009) performed a parametric study to investigate the effect of 11 various parameters on the flexural ductility of unbonded prestressed concrete beams. For that purpose, the authors developed a numerical method to analyze the behavior of prestressed concrete beams with unbonded tendons. The results obtained from numerical method are verified by comparison of the load deflection curves with some experimental results. In this study, the flexural ductility factor is expressed in terms of dimensionless curvature ductility factor. The curvature ductility factor $\mu_\phi$ is taken as the ratio of the ultimate curvature, $\phi_u$ to the curvature at first yield $\phi_y$. The effect of each parameter on the flexural ductility of beams with unbonded tendons is determined based on the
curvature ductility factor. The authors concluded that among 11 parameters examined, the depth of the prestressing tendons, depth to non-prestressed tension steel, concrete compressive strength, yield strength of non-prestressed tension steel and partial prestressing ratio have significant effect on the curvature ductility. In addition, the authors introduced some conversion factors to code equations used for the calculation of flexural ductility.

Vu et al. (2010) proposed a model for the calculation of the structural response of post-tensioned beams with unbonded tendons that can be applied at all cracking, serviceability and ultimate loading stages. The authors used non-linear macro finite element (M.F.E) for the computation of prestressed concrete serviceability after cracking and the concrete tension stiffening effect is taken into account in this computation. The M.F.E is a beam finite element defined mainly by the homogenous average inertia. The proposed model is validated by performing mechanical tests on two beams and using experimental results from literature. The authors concluded that the calculations of the bearing capacity and the deflection at failure based on the model are accurate when compared with the experimental results.

He and Liu (2010) proposed a methodology to evaluate the stress in unbonded tendons for externally or internally prestressed beams at the elastic and ultimate states. The methodology is based on a combination of force equilibrium and deformation compatibility equations. The main idea of the method is determining the relationship between the tendon stress increase, $\Delta f_{ps}$ and midspan deflection, $\delta_{mid}$ in elastic and inelastic states. Based on available experimental results, the authors found out that the tendon stress increase and midspan deflection have a linear relationship in linear elastic
range of beams. In their analysis, they concluded that $\Delta f_{ps}$ depends on tendon eccentricity, the neutral axis depth and the second-order effects. For externally prestressed beams, second order effects are taken into account by introducing two reduction factors: the stress reduction factor, $R_s$, and the depth reduction factor, $R_d$. For the plastic hinge length $L_p$, the expression proposed by Lee et al. (1999) is used:

$$\frac{1}{f} + \frac{1}{L/d_{ps}}.$$ Based on the methodology, the authors proposed a design formula which is applicable for both external and internal tendons and both steel and FRP tendons:

$$f_{ps,u} = f_{pe} + \Delta f_{ps,u} \leq 0.8 f_{pu} \quad (2.7)$$

$$\Delta f_{ps,u} = \kappa E_{ps} \frac{e_m}{c} R_s \frac{L_1}{L_2} \quad (2.8)$$

where,

$$\kappa = \begin{cases} 1.62 \times 10^{-3} [0.78 + 2.35 / (L / d_{ps})] & \text{for uniform or two-third-point loads} \\ 1.26 \times 10^{-3} [0.3 + 3 / (L / d_{ps})] & \text{for one-point loading} \end{cases}$$

In Equation (2.8), $R_s$ is the stress increment reduction factor which is equal to 1 for internal tendons, $c$ is the neutral axis depth and $L_1 / L_2$ is the ratio of the length of the loaded spans to the length of the total length of the tendon. The accuracy of the equation is verified by using experimental results of 89 internally prestressed and 9 externally prestressed beams. They concluded that the proposed equation does not lose much accuracy in comparison with the proposed methodology. In addition, it is mentioned that the second-order effect can be neglected for the externally prestressed beams with at least one deviator.
Zheng and Wang (2010) performed a moment-curvature nonlinear analysis method to estimate the stress increase in unbonded tendon at ultimate limit state. The authors considered the following parameters in their investigation: loading type, span to depth ratio $L/h$, prestressing reinforcement index $\beta_p$ and non-prestressing reinforcement index $\beta_s$. 35 beams from literature and 380 simulated beams are used to study the effect of these parameters. In conclusion, the authors proposed two sets of equations for simple and continuous beams. For simply supported beams $\Delta f_{ps}$ equations for different loading types can be observed as follows:

$$\Delta f_{ps} = (560 - 1449\beta_p - 837\beta_s)(0.86 + 2.4h/L)$$ for one point
$$\Delta f_{ps} = 663 - 1137\beta_p - 703\beta_s$$ for third point
$$\Delta f_{ps} = 631 - 1144\beta_p - 735\beta_s$$ for uniform

After the calculation of $\Delta f_{ps}$ by using the above equations, the ultimate stress can be calculated from well-known expression: $f_{ps} = f_{pe} + \Delta f_{ps}$. The authors compared the predicted values with the experimental results and concluded that the proposed equation has adequate precision. In addition, they included the comparisons of various design code equations and equations of various investigators with experimental results.

Yang and Kang (2011) proposed a method for the estimation of the stress at ultimate state in simply-supported beams post-tensioned with unbonded tendons. As a basis of their methodology, the authors introduced an equivalent-strain-distribution factor obtained using a nonlinear two-dimensional analysis. The developed nonlinear analysis model can be implemented at each loading stage by using a computer algorithm. Analytical parametric studies and nonlinear regression analysis are carried out to determine the strain reduction factor $\alpha_f$, which is a function of unbonded and bonded
steel reinforcement amount, span to depth ratio and tendon profile. After the determination of $\alpha_f$, the authors developed the following closed form equation for the calculation of unbonded tendon stress at ultimate state:

$$f_{ps} = \frac{-B_1 + \sqrt{B_1^2 - 4A_1C_1}}{2A_1} \leq f_{ps}$$  \hspace{1cm} (2.10)

where,

$$A_1 = A_p$$

$$B_1 = A_p f_y - A_p f_y - C_f - A_p f_{pe} + \alpha_f E_p \varepsilon_{cu} A_p$$

$$C_1 = \alpha_f E_p \varepsilon_{cu} (A_p f_y - A_p f_y - C_f - 0.85 f_y b_u \beta_d d_p) - f_{pe} (A_p f_y - A_p f_y - C_f)$$

The authors used 239 experimental test results from literature to verify the accuracy of the proposed equation. In addition, they compared code equations (ACI 318-08 and AASHTO, 1996) and various investigators’ equations with the experimental data. It is concluded that ACI 318-08 equation is very conservative, while AASHTO equation is unconservative for beams with span to depth ratio of 35 or less.

### 2.2.2 Studies on Prestressing with Unbonded and Bonded (Hybrid) Tendons

Studies on prestressing with a combination of unbonded and bonded (hybrid) tendons were examined and summarized by Charan (2006). Therefore, the studies that were explained before are not included in here.

Ghallab (2001) conducted an experimental and analytical study to investigate the behavior of the beams strengthened with external unbonded FRP tendons. For that purpose, the author tested thirteen prestressed beams, one with internal prestressing only, and the rest strengthened externally using Parafil Ropes Type G to failure. The author studied the effect of six factors on the behavior of the tested beams. Namely these factors...
are: the value of the external prestressing force and its eccentricity, deviator position, previous loading stage before strengthening, concrete strength and span to depth ratio. In addition, the investigator carried out analytical investigations to propose simple equations that can be used in the analysis of this beam types.

Ghallab and Beeby (2001) tested four prestressed beams, one with internal bonded prestressing steel only and three with internal bonded prestressing steel and external unbounded post-tensioned FRP-Parafil Ropes Type G tendons up to failure. For all four beams, non-prestressed mild steel was also used. The authors aim was to observe the behavior of the tested beams for examining the benefit of the use of Parafil Rope Type G for external prestressing and for evaluating its effect on the ultimate and service load behaviors. All beams were simply supported and two third-point loads were applied. The cracking patterns, load deflection responses, ultimate flexural responses, mode of failures and prestressing forces of the tested beams were compared. In their research, they concluded that using Parafil rope is a useful system for strengthening or rehabilitation of prestressed concrete structures. By providing a moderate amount external prestressing force, they observed a considerable improvement in the stiffness, cracking and ultimate flexural strength of the beams without any significant reduction in ductility.

Nassif et al. (2003) developed finite element model using ABAQUS software for the analysis of concrete beams prestressed with bonded and unbounded (hybrid) tendons. Their analysis is based on the idealized trussed-beam model which includes the beam element and unbounded tendon element. While developing the finite element model, nonlinear material behavior is considered. The analytical results are compared with the available test results in literature and the authors concluded that the proposed finite
element model gives accurate results for beams prestressed with bonded, unbonded or bonded and unbonded beams. In addition, the authors observed that ACI code equation for the estimation of stress in unbonded tendons is conservative and should be modified for the presence of non-prestressed reinforcement. Moreover, ACI equation is conservative and underestimates the ultimate capacity of beams prestressed with hybrid tendons.

Han et al. (2003) presented results of two full scale bridge girders prestressed with incrementally prestressed concrete (IPC) design concept. In the multistage prestressing in which prestressing forces applied several times at different loading stages, both bonded and unbonded internal tendons were used. The two beams are exactly identical except for the anchorage blocks; one with bracket type of anchorage and the other with coupler type. Both beams were simply supported and were tested under two loads at the third span. Based on the test results, the load deflection response, cracking patterns and the load strains of the beams were examined. The authors observed that the use of both bonded and unbonded tendons together in the same member and multistage prestressing of these tendons resulted in the improved overall performance of that member. With multistage prestressing technique, for the same girder depth longer spans can be achieved or for the same span length shallower girders can be used compared with those of traditional prestressed concrete girders. Therefore, they concluded that this type of bridge girders is an economical alternative to other types of prestressed concrete girders for long span bridges.

Ghallab and Beeby (2005) aimed to observe the effect of several factors related to prestressing systems, internal bonded prestressed steel, material properties, and beam
geometry on the increase in the ultimate stress in external Parafil ropes as well as external steel tendons in their study. In addition, they examined the accuracy of Eurocode, ACI and British Code (BS8110) equations for the ultimate stress in external tendons. For experimental study, they tested nine beams with internal bonded prestressing steel and external unbonded post-tensioned FRP-Parafil Ropes Type G tendons up to failure. All beams were tested under static loads; two loads at the third span or one load at the midspan. Seven beams tested previously by Ghallab (2001) were also used in this research. In addition to these beams, several beams externally strengthened were collected from literature. They found that the factors which influenced the ultimate stress in steel tendons had the same effect on the stresses in Parafil ropes. The authors also concluded that the Eurocode equation underestimates the ultimate stress in external tendons; the BS 8110 equation had good agreement with experimental results for Parafil rope, but it was not accurate enough for steel tendons. On the other hand, the ACI equation showed a low accuracy for the prediction of the external prestressing stress. Therefore, they recommended that more studies are needed to include factors such as tendon profile, effective tendon depth, number of deviators and to modify the ACI equation to be appropriate for high concrete strengths.

Du et al. (2011) tested nine beams up to failure under two point loading in order to study the effect of bonded and unbonded FRP tendons on the flexural capacity of the beams. In this investigation, three factors are taken into account: the bonding condition of CFRP tendons, the prestressing ratio and the location of CFRP tendons. Based on the test results, the authors concluded that the flexural capacity of the internally fully bonded beam is the highest and the externally prestressed beam without deviators is the lowest.
The flexural capacity of the beam with fully bonded tendon is 20% higher than the beam with internal unbonded tendon and 40% higher than the beam with external unbonded tendon without deviators. In addition, they observed that the ductility of the prestressed concrete beams can be improved by using both bonded and unbonded FRP tendons. The authors developed a method to calculate the nominal capacity of the beams with bonded and unbonded FRP tendons. The basis of this method is balanced reinforcement ratio which is used to decide the failure mode of the beam. The authors used strain reduction coefficient $\Omega_{u1}$ to account for the unbonded tendons and the depth reduction factor $R_u$ and strain reduction factor $\Omega_{u2}$ to consider the effect of external tendons. Then, these factors are combined with force equilibrium equations to develop moment capacity equations. The calculated values are compared with the experimental results and good accuracy is achieved.
2.3 Code Equations

In the following paragraphs, the code equations presented for the calculation of the unbonded tendons stress are shown. The examined Codes are ACI 318-08 (2008), AASHTO LRFD (2010), British Code (2001), Canadian Code–CSA (2006) and Eurocode (2004). However, please note that none of these code equations address the design case of using both bonded and unbonded tendons.

ACI 318-08 (2008) code equation which is based on the research performed by Mattock et al. (1971) and then modified by Mojtahedi and Gamble (1978) can be written as follows:

\[ f_{ps} = f_{pe} + \frac{f_{c}}{\mu \rho_{ps}} + 10,000 \text{ [psi]} \]

For \( \frac{L}{d_{ps}} \leq 35 \), \( \mu = 100 \) and \( f_{ps} \leq f_{pe} + 60,000 \) and \( f_{py} \)

For \( \frac{L}{d_{ps}} > 35 \), \( \mu = 300 \) and \( f_{ps} \leq f_{pe} + 30,000 \) and \( f_{py} \)

For beams prestressed with unbonded FRP tendons, ACI 440-04 (2004) recommended the following equation based on the work of Naaman et al. (2002):

\[ f_{ps} = f_{pe} + \Omega_u E_p \varepsilon_{cu} \left( \frac{d_p}{c} - 1 \right) \]

(2.12)

where,

\[ \Omega_u = \begin{cases} \frac{1.5}{(L / d_p)} & \text{for one point loading} \\ \frac{3}{(L / d_p)} & \text{for two point and uniform loading} \end{cases} \]
ACI 440-04 (2004) also mentioned that for the beams prestressed with external FRP tendons, the effective depth of the tendon $d_p$ should be replaced by $d_e$ in Equation (2.12):

\[ d_e = R_d d_p \]  \hspace{1cm} (2.13)

where,

\[ R_d = \begin{cases} 1.14 - 0.005(L / d_p) - 0.19(S_d / L) & \text{for one point loading} \\ 1.25 - 0.010(L / d_p) - 0.38(S_d / L) & \text{for third-point loading} \end{cases} \]

and $S_d$ is the spacing of the deviators and $L$ is the span length of the beam.

In the same manner, for the beams with external prestressing or a combination of internal and external prestressing, the strain reduction factor $\Omega_u$ needs to be changed in Equation (2.12) based on the work of Aravinthan and Mutsuyoshi (1997):

\[ \Omega_u = \begin{cases} \frac{0.21}{(L / d_p)} + 0.04 \left( \frac{A_{\text{int}}}{A_{\text{tot}}} \right) + 0.04 & \text{for one point loading} \\ \frac{2.31}{(L / d_p)} + 0.21 \left( \frac{A_{\text{int}}}{A_{\text{tot}}} \right) + 0.06 & \text{for third-point loading} \end{cases} \]  \hspace{1cm} (2.14)

where $A_{\text{int}}$ is area of internal prestressed reinforcement and $A_{\text{tot}}$ is area of internal and external prestressed reinforcement.

AASHTO LRFD (2010) recommended the following design equation:

\[ f_{ps} = f_{pe} + 900 \left( \frac{d_p - c}{l_e} \right) \leq f_{py} \]  \hspace{1cm} (2.15)

where,

$l_e$ = Effective Tendon Length
\[ l_i = \left( \frac{2l_i}{2 + N_i} \right) \]

\[ l_i = \text{Tendon Length between Anchorages} \]

\[ N_i = \text{Number of the Hinges at Supports} \]

\[ c = \text{Neutral Axis Depth} \]

\[
c = \frac{A_{ps}f_{ps} + A_yf_y - A'_{ps}f'_y - 0.85f_c\beta_i(b - b_w)h_f}{0.85f_c\beta_Ib_w}
\]

The British Code (2001) Equation can be seen as follows:

\[
f_{ps} = f_{pe} + 1015 \left( \frac{L}{d_{ps}} \right) \left( 1 - \frac{1.7f_{ps}A_{ps}}{f_{cs}b_d_{ps}} \right) \leq 0.7f_{ps} \quad [ksi]
\]

where,

\[ f_{cs} = \text{Cube Strength of Concrete} \approx \frac{f_c}{0.8} \]

Candian Code, CSA (2006) mentioned that the ultimate stress of unbonded tendons shall be taken as effective prestressing force unless a detailed deformation based analysis shows that a higher value can be used:

\[
f_{ps} = f_{pe}
\]

(2.17)

In Eurocode (2004), it is stated that if no detailed calculation is made, \( \Delta f_{ps} \) should be taken as 100MPa. As a result, Eurocode equation for \( f_{ps} \) is:

\[
f_{ps} = f_{pe} + 100 \quad [MPa]
\]

(2.18)
CHAPTER III

SIMPLIFICATION OF PREDICTION EQUATION

3.1 Introduction

In comparison to beams prestressed with bonded tendons, for beams including unbonded tendons there is no strain compatibility between the tendon and concrete due to the lack of bond. For that reason, $\Delta f_{preU}$ in an unbonded tendon is member dependent rather than section dependent. It means that the deformation of the whole member and the interaction between the unbonded tendon and concrete beam should be considered for the analysis of beams prestressed with unbonded or hybrid tendons. Therefore, $\Delta f_{preU}$ cannot be estimated simply by using the strain compatibility and section analysis; the analysis of the whole member is required.

Before passing to the analysis approaches for the beams prestressed with hybrid tendons, the main methodology behind the approaches is explained in this part. The trussed-beam model which is the structural idealization of a concrete beam prestressed with a straight internal unbonded tendon can be seen in Figure 3.1. This model was before presented in the researches by Tanchan (2001), Charan (2006) and Ozkul (2007). This model consists of two main elements: the concrete beam and the unbonded tendon. As can be seen from the figure, two spring elements are used to keep the distance between the concrete beam element and unbonded tendon element constant. At the ends of the beam, rigid connectors are used which remain perpendicular during loading. The bonded tendons are treated as rebar element with initial stress condition.
The trussed-beam model is examined and solved at various load levels. For that purpose, two methods of computation are used:

1. **Approach I: Finite Element Analysis (FEA):** Software Program ABAQUS (Version 6.8) is used for finite element analysis.

2. **Approach II: Generalized Incremental Analysis (GIA):** Mathematical derivation which is based on the nonlinear section analysis, deflections and conservation of the work-energy principle (Ozkul et al. 2008).

3. **Approach III: Closed Form Solution (Ozkul et al. 2008)**

These approaches are explained in the following section as a summary. For more details of the Approach I, Finite Element Analysis, please refer to the studies by Tanchan (2001), Charan (2006) and Ozkul (2007). In the same manner, for details for Approaches II and III Generalized Incremental Analysis and Closed Form Solution, please refer to the study by Ozkul (2007) and Ozkul et al. (2008), respectively. A summary of the methods of analysis can be seen in the following flowchart, Figure 3.2.
Figure 3.2: Summary of Methods of Analysis

METHODS OF ANALYSIS
(Based on Trussed-Beam Model)

Approach I

Finite Element Analysis (FEA)

Nonlinear Analysis of the Model using Software ABAQUS

Determination of Material Models

Incremental Loading Procedure-RIKS Algorithm

FEA Results

Approach II

Generalized Incremental Analysis (GIA) Ozkul et al. (2008)

An Incremental Loading Procedure used to Estimate Overall Beam Behavior

Nonlinear Section Analysis

Beam Deflections

Work-Energy Principle

GIA Results

Approach III

Closed Form Solution for $f_{pult}$ Ozkul et al. (2008)

Thesis Objective: Simplify Prediction Equation at Ultimate Limit State
3.2 Approach I: Finite Element Analysis (FEA)

In this study, the trussed beam model is analyzed using the finite element program, ABAQUS (Version 6.8). Previous researchers Tanchan (2001), Charan (2006) and Ozkul (2007) also did modeling by using trussed-beam model and ABAQUS software. For this study, similar procedure that these researchers explained has followed to include the beams that have hybrid tendons. The beams that have hybrid tendons were not modeled before. Therefore, this is an addition to previous work. Briefly, the model consists of two main elements, the concrete beam and unbonded tendon which are connected with rigid links. The bonded tendons are defined as rebar element with initial stress condition. The modified RIKS algorithm is used for the loading and analysis. The results obtained from finite element analysis are compared with the available experimental test results. The accuracy of the finite element results can be compared in Section 5.2.

3.3 Approach II: Generalized Incremental Analysis (GIA)

In GIA, energy principle which converts the vertical deformation of the beam to axial elongation in the unbonded tendon is used. As a result, with the combination of the classical section analysis, calculation of beam deflection by using moment area integration and conservation of energy principle, the stress in unbonded tendon can be calculated. For the details of GIA procedure, please refer to the work by Ozkul (2007) and Ozkul et al. (2008). The aim of this study is to simplify the equation for the prediction of ultimate stress in unbonded tendons. Therefore, in this study, a summary of the GIA is provided only for ultimate limit state.
For a typical rectangular section, Ozkul et al. (2008) applied force equilibrium \[ \sum C = \sum T \] and Whitney’s rectangular stress block, the initial estimate of the neutral axis depth \( c \) can be made:

\[
c_{(u)} = \frac{A_{pU} f_{pU} + A_{pB} f_{pB} + A_s f_y - A'_s f'_y}{0.85 \beta_1 f'_c b}
\]  

(3.1)

For known values \( c \) and top concrete strain \( \varepsilon_{cu} = 0.003 \) at ultimate, by using strain compatibility equations and stress-strain relationship of reinforcements, \( f'_{pU} \) is calculated as follows:

\[
f'_{pU(u)} = \frac{0.85 \beta_1 f'_c b c - A_s f_y + A'_s f'_y - A_{pB} f_{pB}}{A_{pU}}
\]  

(3.2)

Then, beam deflection is calculated by using moment area-integration method in which the moment of area under the curvature diagram is taken with respect to one of the supports. The deflection under point load is formulated as:

\[
\Delta_{PL(u)} = \frac{\phi_{(u)}}{2} \left[ (L - 2L_p) L - L_p^2 \right]
\]  

(3.3)

In the Equation (3.3), \( \phi_{(u)} \) is the curvature at ultimate state. Other undefined terms, and their formulas can be observed in Table 3-2. The load geometry factor \( f \) was defined by Harajli et al. (1991) and this factor was expressed as a function of loading type as shown in Table 3-2.

For the trussed-beam model shown in Figure 3.1, after yielding of non-prestressed steel and formation of large cracks, the beam curvature increases significantly with a large rotation known as formation of a plastic hinge. It is appropriate to perform the work-energy principle which basically states that the work done by the external force is
equal to the internal strain energy in a member. For a beam under third point loading as shown in Figure 3.1, the external energy created in the system at ultimate failure is explained as follows:

$$U_{E(u)} = \int_0^L P_{E(u)} \, dx = 2 \left( \frac{(P_{(u)} / 2) \Delta_{PL(u)}}{2} \right) = \left( \frac{P_{(u)}}{2} \right) \Delta_{PL(u)} \quad (3.4)$$

At ultimate limit state, two plastic hinge moments $M_{pl}$, are created in the beam is which is formulated as $M_{pl} = P_{(u)} L / 2 - A_{pu} \int_{\theta_{psU}}^{F_{psU}E} e_{psU} \, d x$. In addition, two moments are created by the unbonded prestressing force: $A_{psU} \int_{\theta_{psU}}^{F_{psU}E} e_{psU} \, d x$. The approximation $\theta_{psU} L \approx \Delta_{PL(u)}$ is made with the assumption of $\tan \theta_{psU} \approx \theta_{psU}$ (Ozkul et al. 2008). Then, the internal strain energy $U_{I(i)}$ is formulated:

$$U_{I(u)} = \left( \frac{P_{(u)} \Delta_{PL(u)}}{2} - A_{pu} \int_{\theta_{psU}}^{F_{psU}E} e_{psU} \, d x \theta_{psU} \right) - A_{psU} \int_{\theta_{psU}}^{F_{psU}E} e_{psU} \, d x \theta_{psU} + \left( A_{psU} \int_{\theta_{psU}}^{F_{psU}E} \delta_{psU} / 2 \right) \quad (3.5)$$

Now, by using energy principle, $U_{E(u)} - U_{I(u)} = 0$, the elongation in the unbonded tendon is formulated as: $\delta_{psU} = 4 e_{psU} \Delta_{PL(u)} / L$. Then, the strain in the unbonded tendon is calculated: $e_{psU} = e_{peU} + \delta_{psU} / L$. Finally, in order to obtain the unbonded prestressing force $f_{psU}$ from energy equilibrium, the stress corresponding to the strain $e_{psU}$ should be found by using the stress-strain relationship of the unbonded tendon.

The GIA procedure at ultimate limit state can be used to obtain a quadratic equation for $f_{psU}$. Based on GIA, Ozkul et al. (2008) derived the following equation:

$$f_{psU} = f_{peU} + \left( \frac{E_{psU}}{196} \right) \left( \frac{\beta f c b e_{psU}}{A_{psU} f_{psU} + A_t f_y} \right) \alpha_i \quad (3.6)$$

where,
\[ \alpha_i = \left(1 - \frac{2L_h}{L} - \frac{L_p}{LL_L}\right); \quad L_h = \frac{L}{2} - \frac{L}{2f} - \left(0.5d_{p, U} + 0.05L\right) \]

After normalizing Eq. (3.6), Ozkul et al. (2008) reduced this equation to the following closed form solution:

\[ f_{p, U} = f_{p, U} + \left(\frac{E_{p, U}}{196}\right) \left(\frac{\beta_1 f_c b e_{p, U}}{A_{p, U} f_{p, U} + A_s f_y}\right) \alpha_i \quad (3.7) \]

The main purpose of this study is to simplify Eq. (3.7) and also include the effect of bonded prestressing reinforcement in the equation. Therefore, a simple, accurate and rational equation should be derived for the prediction of \( f_{p, U} \) at ultimate limit state. In this part of the study, this derivation and selection of the final form of equation will be explained. In the following parts, different forms of equations are derived. Then, these equations are compared in terms of simplicity, accuracy, and ease of use.

### 3.4 Simplification of Prediction Equation

Equation (3.6) is written as a function of the eccentricity. This study will use two approaches. First approach is to simplify this equation as a function of eccentricity; while the second is to also simplify as a function of \( (d_{p, U} - c) \). To do so, two alternatives are considered: 1) By solving the quadratic equation obtained in Eq. (3.6), and (2) By using the normalized equation.
3.4.1 Quadratic Equation in Terms of Eccentricity

The stress in the unbonded tendon at ultimate limit state is expressed as follows:

\[ \varepsilon_{pu} = \varepsilon_{pe} + \frac{\delta}{L} \]  

(3.8)

After multiplying both sides of Equation (3.8) by \( E_{pu} \), the elongation in unbonded tendon, \( \delta = 4e_{pu} \Delta_{PL(u)} / L \) (obtained in Section 3.3) is inserted into the equation. As a result, the following form is obtained:

\[ f_{pu} = f_{pe} + \frac{4e_{pu} \Delta_{PL(u)}}{LL} \]  

(3.9)

Then, for the deflection under point load \( \Delta_{PL} \), Equation (3.3) is substituted into the equation:

\[ f_{pu} = f_{pe} + \frac{2(e_{pu})\phi_{pu} \left[(L - 2L_h) - L_p^2 \right]}{LL} \]  

(3.10)

In the above equation, the curvature value \( \phi_{pu} \) is equal to \( \varepsilon_{cu} / c \). For neutral axis depth \( c \), Equation (3.1) is substituted into the equation by ignoring non-prestressed compressive steel and by assuming rectangular section behavior:

\[ f_{pu} = f_{pe} + E_{pu} \left( \frac{2\varepsilon_{cu}}{LL} \right) \left( \frac{0.85 \beta \varepsilon_{pe}}{f_p} + \frac{\varepsilon_{pu}}{f_{pu}} \right) \left[ L_e (L - 2L_h) - L_p^2 \right] \]  

(3.11)

In order to obtain a quadratic equation in terms of \( e_{pu} \), \( e_{cu} \) can be taken as 0.003 and let the term \( E_{pu} \left( \frac{2\varepsilon_{cu}}{LL} \right)(0.85 \beta \varepsilon_{pe}/f_p) (L_e (L - 2L_h) - L_p^2) \) is equal to \( \eta \):

\[ (A_{pu}) f_{pu}^2 + (A_{pu}f_{pu} + A_s f_y - A_{pu} f_{pu}) f_{pu} + [-\eta - f_{pu} (A_{pu} f_{pu} + A_s f_y)] = 0 \]  

(3.12)

where,
\[ \eta = \frac{E_{psU} \beta \sqrt{b} e_{psU}}{196} \left( 1 - \frac{2L_h}{L} - \frac{L_p^2}{L^3} \right) \]

In order to solve the quadratic equation, the following well known mathematical equation can be used:

\[ f_{psU} = \frac{-b + \sqrt{(b^2 - 4ac)}}{2a} \]  

(3.13)

where,

\[ a = A_{psU} \]

\[ b = (A_{psB} f_{psB} + A_s f_y - A_{psU} f_{psU}) \]

\[ c = [-\eta f_{psU} (A_{psB} f_{psB} + A_s f_y)] \]

In order to put the equation in a more simple form, Equation (3.13) is expanded. Then, the final form of the quadratic equation in terms of eccentricity can be written as follows:

\[ f_{psU} = \frac{f_{psU} - \alpha_2}{2} + \sqrt{\left( \frac{f_{psU} + \alpha_2}{2} \right)^2 + \frac{E_{psU} \beta \sqrt{b} e_{psU}}{196 A_{psU}} \alpha_i} \]  

(3.14)

where,

\[ \alpha_i = \left( 1 - \frac{2L_h}{L} - \frac{L_p^2}{LL^3} \right); \quad L_h = \frac{L}{2} - \frac{L}{2f} - (0.5d_{psU} + 0.05L_L) \]

\[ \alpha_2 = \frac{A_s f_y + A_{psB} f_{psB}}{A_{psU}} \]

In Equation (3.14), for the calculation of the bonded tendon stress \( f_{psB} \), AASHTO equation for bonded tendons is used if the tendon is steel. If the bonded tendon is FRP, then the stress is taken as the ultimate stress of the bonded tendon \( f_{psB} \). The load
geometry factor $f$ is equal to 10 for one point loading, 6 for uniform loading and 3 for two-point loading, Table 3-2. In addition, for steel unbonded tendons linear elastic approximation is assumed. Therefore, the above equation should be limited to yield stress $f_{pyU}$ if the unbonded tendon is steel. If the unbonded tendon is FRP, the equation is limited to ultimate strength of FRP $f_{frpuU}$.

### 3.4.2 Normalized Equation in Terms of Eccentricity

As mentioned before, normalization of the equation is another option for obtaining an equation. For normalization, Equation (3.11) is rewritten in terms of normalized stress $f_{nor} = \frac{f_{puU}}{f_{peU}}$. It is known that the solution of the quadratic equation $f_{puU}$ is between $f_{peU}$ and $f_{paU}$. By using normalization and by considering the linear portion of the quadratic equation, Ozkul et al. (2008) proposed Eq. (3.7). After the inclusion of area of bonded prestressing reinforcement, the equation becomes:

$$f_{puU} = f_{peU} + \left( \frac{E_{puU}}{196} \right) \left( \frac{\beta f_{eU} e_{puU}}{A_{puU} f_{puU} + A_{puB} f_{puB} + A_{y} f_{y}} \right) \alpha_i$$

where,

$$\alpha_i = \left( 1 - \frac{2L_h}{L} - \frac{L^2_p}{LL_L} \right) ; \quad L_h = \frac{L}{2} - \frac{L}{2f} (0.5d_{puU} + 0.05L_L)$$

Equation (3.15) is limited to $f_{pyU}$ if the unbonded tendon is steel. If the unbonded tendon is FRP, the equation is limited to $f_{frpuU}$. 
3.4.3 Quadratic Equation in Terms of \((d_{pu} - c)\)

In GIA, the distance between unbonded tendon and concrete element is taken as eccentricity of the unbonded tendon, Figure 3.1. Therefore, all formulas in GIA are derived in terms of eccentricity. Based on the ongoing work of Nassif and Ozkul (2011), the distance between unbonded tendon and center of compression force can be taken as \((d_{pu} - c)\) and GIA can be implemented in terms of \((d_{pu} - c)\) instead of eccentricity \(e_{pu}\).

The authors found out that the elongation of the unbonded tendon at ultimate limit state is \(\delta_{(u)} = 2(d_{pu} - c)\theta_{(u)}\). By using this elongation formula, Equation (3.9) can be rewritten as follows:

\[
f_{pu} = f_{pe} + \frac{2(d_{pu} - c)\theta_{(u)}}{L}
\]  

(3.16)

In the same manner as done before for the case of the equation in terms of eccentricity \(e_{pu}\), all variables, \(\theta_{(u)}, \Delta_{PL}, c\), etc. are substituted into the Equation (3.16) and the following equation is obtained:

\[
f_{pu} = f_{pe} + E_{pu} \left( \frac{e_{cu}}{L_L} \right) \left( \frac{0.85 \beta f c d_{pu}}{A_{pu} f_{pu} + A_{pu} f_{pu} + A_{pu} f_{pu}} - 1 \right) \left[ L_L (L - 2L_h) - L_p^2 \right]
\]  

(3.17)

In order to obtain a quadratic equation, let the term

\[
E_{pu} \left( \frac{e_{cu}}{L_L} \right) (L_L (L - 2L_h) - L_p^2)
\]

be equal to \(\psi\) and ultimate strain of concrete \(e_{cu}\) can be taken as 0.003:

\[
(A_{pu} f_{pu})^2 + \left[ A_{pu} f_{pu} + A_{pu} f_{pu} + A_{pu} (\psi - f_{pu}) \right] f_{pu} \\
+ [(\psi - f_{pu})(A_{pu} f_{pu} + A_{pu} f_{pu}) - 0.85 \beta f c d_{pu} \psi] = 0
\]  

(3.18)

where,
\[
\psi = \frac{E_{ptU}}{333} \left( 1 - \frac{2L_p}{L} - \frac{L_p^2}{L_L L} \right)
\]

As done before the mathematical expression, Equation (3.13) is used to solve the quadratic equation, in which:

\[
a = A_{ptU}
\]

\[
b = [A_{psB} f_{psB} + A_s f_y + A_{ptU} (\psi - f_{ptU})]
\]

\[
c = [(\psi - f_{ptU}) (A_{psB} f_{psB} + A_s f_y) - 0.85 \beta f_c b d_{ptU} \psi]
\]

The quadratic form of the equation is derived. However, as can be seen it is not a simple formula for code purposes. It is possible to put this equation in a more simple form by expanding Equation (3.13). This form can be observed as follows:

\[
f_{ptU} = \frac{-A_s f_y - A_{psB} f_{psB}}{2A_{ptU}} - \frac{\psi}{2} + \frac{f_{ptU}}{2}
\]

\[
\frac{A_s f_y + A_{psB} f_{psB}}{2A_{ptU}} \left[ \frac{(A_s f_y + A_{psB} f_{psB})}{2A_{ptU}} \right] \left( \psi - f_{ptU} \right)^2
\]

\[
+ \left( \frac{\psi - f_{ptU}}{2} \right)^2 + \frac{0.85 \beta f_c b d_{ptU} \psi}{A_{ptU}}
\]

(3.19)

Now, let the term \( A_s f_y + A_{psB} f_{psB} \) is equal to \( \alpha_2 \), insert the term \( \psi \) and use \( \alpha_1 \) for the term \( 1 - \frac{2L_p}{L} - \frac{L_p^2}{L_L L} \). As a result, the final form of the equation can be written as follows:

\[
f_{ptU} = \frac{f_{ptU} - \alpha_2}{2} - \frac{E_{ptU} \alpha_1}{666} + \sqrt{\left( \frac{\alpha_2 - [(E_{ptU} \alpha_1 / 333) - f_{ptU}]^2}{2} \right)^2 + \frac{E_{ptU} \beta f_c b d_{ptU}}{392 A_{ptU} \alpha_1}}
\]

(3.20)

where,
\[ \alpha_i = \left( 1 - \frac{2L_h}{L} - \frac{L_p}{L_{L_L}} \right) ; \quad L_h = \frac{L}{2} - \frac{L}{2f} - (0.5d_{puU} + 0.05L_p) \]

\[ \alpha_z = \frac{A_s f_y + A_{puB} f_{puB}}{A_{puU}} \]

Equation (3.20) is limited to \( f_{puU} \) if the unbonded tendon is steel. If the unbonded tendon is FRP, the equation is limited to \( f_{fpuU} \).

3.4.4 Normalized Equation in Terms of \((d_{puU} - c)\)

It is known that the ultimate unbonded tendon stress value \( f_{puU} \) is between effective prestress \( f_{peU} \) and ultimate strength \( f_{puU} \) values. In other words, two roots of quadratic equation given in Equation (3.18) are between these two values \( f_{peU} \) and \( f_{puU} \). Therefore, as mentioned before, normalization of the equation is another option for obtaining an equation. For normalization, Equation (3.17) is rewritten in terms of normalized stress \( f_{nor} = \frac{f_{puU}}{f_{peU}} \).

For that purpose, both sides of this equation are divided to \( f_{peU} \):

\[ f_{nor} = f_{peU} + E_{puU} \left( \frac{\varepsilon_{cu}}{LL_p} \right) [L_p (L - 2L_h) - L_p^2] \times \left( \frac{0.85\beta_i b d_{puU}}{A_{puU} f_{peU} \left( f_{puU} + \frac{A_{puB} f_{puB} + A_s f_y}{A_{puU}} \right)} - \frac{1}{f_{peU}} \right) \]

\[(3.21)\]
Now, in the above equation the term \( f_{puU} + \frac{A_{pB} f_{pB} + A_y f_y}{A_{puU}} \) is divided to \( f_{puU} \) and the equation take the following final form:

\[
 f_{nor} = 1 + \left( \frac{w_1}{f_{nor} + w_2} - w_3 \right) 
\]

where,

\[
 w_1 = E_{puU} \left( \frac{E_{cu}}{LL_L} \right) [L_L (L - 2L_h) - L_p^2] \left( \frac{0.85 \beta_1 f_v b d_{puU}}{A_{puU} f_{puU}} \right) 
\]

\[
 w_2 = \left( \frac{A_{pB} f_{pB} + A_y f_y}{A_{puU} f_{puU}} \right) 
\]

\[
 w_3 = E_{puU} \left( \frac{E_{cu}}{LL_L} \right) [L_L (L - 2L_h) - L_p^2] 
\]

Equation (3.22) can be expressed as a linear equation:

\[
 f_{nor} = a + b(f_{nor}) 
\]

When \( f_{nor} = 1 \) in the right hand side of the above equation, \( a \) can be formulated:

\[
 a = (1 - b) + \left( \frac{w_1}{1 + w_2} - w_3 \right) 
\]

Equation (3.23) has a solution at \( f_{nor} = a / (1 - b) \) and can be rewritten as follow:

\[
 f_{nor} = 1 + \left( \frac{w_1}{(1 + w_2)(1 - b)} - \frac{w_3}{(1 - b)} \right) 
\]

After inserting the constants \( w_1 \), \( w_2 \) and \( w_3 \) into the above equation and both sides of the equation is multiplied by \( f_{puU} \), the following equation is obtained:
\[ f_{\text{pu}} = f_{\text{puU}} + E_{\text{puU}} \left( \frac{e_{\text{cu}}}{LL_t} \right) \left[ L_t (L - 2L_h) - L_p^2 \right] \]
\[
\times \left( \frac{0.85 \beta_1 f'_{yd} b d_{\text{puU}} - (A_{\text{puU}} f_{\text{puU}} + A_{\text{puB}} f_{\text{puB}} + A_s f_y)}{(1-b)(A_{\text{puU}} f_{\text{puU}} + A_{\text{puB}} f_{\text{puB}} + A_s f_y)} \right) \quad (3.25)
\]

In the above equation, \( b \) is always a negative term and \((1-b)\) is a correction on the effective prestress value \( f_{\text{puU}} \). In the above equation, instead of \((1-b)(A_{\text{puU}} f_{\text{puU}} + A_{\text{puB}} f_{\text{puB}} + A_s f_y)\), the term \((A_{\text{puU}} f_{\text{puU}} + A_{\text{puB}} f_{\text{puB}} + A_s f_y)\) can be used. This approximation will be proved with an example graphically. The term \((1-b)\) does not have an effect on \(A_s f_y\). The reason for that the non-prestressing steel is already yielded and the non-prestressing steel stress value cannot be larger than \(f_y\). In the same manner, if the bonded tendon is steel, it also already yielded. If the bonded tendon is FRP, it has no yielding point. However, before ultimate limit state or soon after ultimate limit state it reaches its ultimate strength \(f_{\text{puB}}\) and the bonded stress cannot exceed the ultimate strength value. As a result, the bonded tendon can be steel or FRP and the behavior of these two materials are different. Therefore, in the equation, the bonded tendon stress is preserved as \(f_{\text{puB}}\). In addition, ultimate strain of concrete \(e_{\text{cu}}\) is taken as 0.003 and let the term \(1 - \frac{2L_h}{L} \frac{L_p^2}{L_t L_h}\) be \(\alpha_i\). Now, Equation (3.25) is rearranged as follows:
\[
f_{\text{pu}} = f_{\text{puU}} + \frac{E_{\text{puU}}}{333} \left( \frac{0.85 \beta_1 f'_{yd} b d_{\text{puU}} - (A_{\text{puU}} f_{\text{puU}} + A_{\text{puB}} f_{\text{puB}} + A_s f_y)}{(A_{\text{puU}} f_{\text{puU}} + A_{\text{puB}} f_{\text{puB}} + A_s f_y)} \right) \alpha_i \quad (3.26)
\]
where,
\[ \alpha_i = \left(1 - \frac{2L_h}{L} \right) \left(1 - \frac{L_p^2}{LL_L} \right); \quad L_h = \frac{L}{2} \left(1 - \frac{0.5d_{pyU} + 0.05L_L}{2f} \right) \]

In addition, for steel unbonded tendons linear elastic approximation is assumed. Therefore, the finalized equation is limited to \( f_{pyU} \) if the unbonded tendon is steel. If the unbonded tendon is FRP, the equation is limited to \( f_{frpuU} \).

Now, a test beam from literature is taken to illustrate the normalization and to prove the condition of \((1-b)(A_{psU}f_{peU} + A_{psB}f_{psB} + A_yf_y) \approx (A_{psU}f_{puU} + A_{psB}f_{psB} + A_yf_y)\) graphically. One of the beams B1 tested by Jerrett et al. (1996) is used. This beam contains a combination of bonded and unbonded (hybrid) tendons. The rectangular beam has the dimensions 8in x 16in. The bonded tendons are steel, while the unbonded tendons are FRP. Other properties of the beam can be observed in the following table:

<table>
<thead>
<tr>
<th>Sectional Properties</th>
<th>( A_{psU} ) in(^2)</th>
<th>( A_{psB} ) in(^2)</th>
<th>( A_y ) in(^2)</th>
<th>( d_{psU} ) in</th>
<th>( d_{psB} ) in</th>
<th>( d_s ) in</th>
<th>( L ) in</th>
<th>( f'_c ) ksi</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.16</td>
<td>0.15</td>
<td>0</td>
<td>15.9</td>
<td>13.2</td>
<td>13.7</td>
<td>204</td>
<td>6.3</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Material Properties</th>
<th>( f_{peU} ) ksi</th>
<th>( f_{peB} ) ksi</th>
<th>( f_{puU} ) ksi</th>
<th>( f_{puB} ) ksi</th>
<th>( f_{pyU} ) ksi</th>
<th>( f_{pyB} ) ksi</th>
<th>( f_y ) ksi</th>
<th>( E_{psU} ) ksi</th>
</tr>
</thead>
<tbody>
<tr>
<td>169.7</td>
<td>162.4</td>
<td>298.8</td>
<td>288.6</td>
<td>---</td>
<td>202.9</td>
<td>69.9</td>
<td>21800</td>
<td></td>
</tr>
</tbody>
</table>

By using the above beam data, the constants for the normalized equation are found: \( w_1 = 1.826 \), \( w_2 = 1.438 \) and \( w_3 = 0.097 \). The stress value \( f_{psU} \) is between the effective prestressing \( f_{peU} \) and ultimate stress value \( f_{puU} \). If \( f_{psU} \) is normalized by
dividing it to $f_{peU}$, then $f_{nor}$ is between 1 and $f_{puU}/f_{peU}$ which is equal to 1.76 for this case.

This situation is illustrated by plotting $f_{nor}$ versus $f(f_{nor}) = 1 + \left( \frac{w_1}{f_{nor} + w_2} - w_3 \right)$:

![Graphical Solution for the Normalized Equation in terms of ($d_{puU} - c$)](image)

As can be seen in the above graph, the solution is between 1 and $f_{puU}/f_{peU} = 1.76$ which is $f_{nor} = 1.52$ for this case. The solution is the intersection of the two equations, $f(f_{nor}) = 1 + \left( \frac{w_1}{f_{nor} + w_2} - w_3 \right)$ and $f_{nor} = f_{nor}$. The normalized equation can be written in linear form, $f_{nor} = a + b(f_{nor})$ as explained before. For this example the linear equation is: $f_{nor} = 1.88 - 0.232f_{nor}$ and $a = 1.88$, $b = -0.232$. It is already mentioned that $b$ is a negative value. Now, it is proved with this example. As a result, the approximation of
(1 - b)(ApU PeU + ApB PsB + Ae f) \equiv (ApsUf + ApB PsB + Ae f) can be done since b is a negative value and the solution lies between f psU and f psU. This completes the proof.

3.4.5 Simplification of the Term $\alpha_1$

In the previous three parts, the quadratic and normalized form of equations were obtained in terms of $e psU$ and $(d psU - c)$. In other words, four equations were derived for the calculation of $f psU$; namely Eq's. (3.14), (3.15), (3.20) and (3.26). However, it can be easily observed that none of these equations are simple enough. The main purpose of this study is to propose an equation which is accurate and simple. Therefore, in order to get the final form of the equation, firstly these equations should be further simplified. For this simplification process, the term $\alpha_1$ included in all these equations needs to be modified. This expression $\alpha_1 = \left(1 - \frac{2L_h}{L} - \frac{L_p^2}{LL_L}\right)$ directly or indirectly includes: plastic hinge length $L_p$, the load geometry factor $f$, distance from support to applied load $L_L$ and effective depth of the unbonded tendon $d psU$. With the inclusion of $\alpha_1$, the plastic hinge length and loading type are considered in the equation. In order to simplify the equation, this term shall be written as a function of span to depth ratio $L / d psU$ and loading type. For that purpose, firstly the term $\alpha_1$ is simplified mathematically. Then, same simplification is implemented graphically.

For the mathematical simplification of the term $\alpha_1$, a beam under equal two-point loading is considered. In the following table, the definitions and equivalences used in the simplification process can be seen:
### Table 3-2: Definitions and Equivalences used for Simplification

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
<th>Equivalence</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L_p )</td>
<td>Plastic Hinge Length</td>
<td>( 0.5d_{psU} + 0.05L_L )</td>
</tr>
<tr>
<td>( L_L )</td>
<td>Distance from Support to Applied Point Load</td>
<td>( L_p + L_h ) or ( \frac{L-L_a}{2} )</td>
</tr>
<tr>
<td>( L_h )</td>
<td>Distance from Support to the Plastic Hinge</td>
<td>( L / 2 - L_a / 2 - L_p )</td>
</tr>
<tr>
<td>( L_a )</td>
<td>Theoretical Constant Moment Region Length</td>
<td>( L / f )</td>
</tr>
<tr>
<td>( L_a + 2L_p )</td>
<td>Equivalent Plastic Hinge Length</td>
<td>( L - 2L_h )</td>
</tr>
<tr>
<td>( L )</td>
<td>Span Length</td>
<td>( L_a + 2L_L ) or ( L_a + 2(L_p + L_h) )</td>
</tr>
<tr>
<td>( f )</td>
<td>Load Geometry Factor</td>
<td>( f = \begin{cases} 10 \text{ for one-point loading} \ 3 \text{ for two-point loading} \ 6 \text{ for uniform loading} \end{cases} )</td>
</tr>
</tbody>
</table>

The mathematical simplification process starts with using the equivalent term \((L_a + 2L_p)\) instead of \((L - 2L_h)\):

\[
\alpha_1 = 1 - \frac{2L_h}{L} - \frac{L_p^2}{LL_L} = \frac{1}{L L_L} [L_L (L - 2L_h) - L_p^2] = \frac{L_a + 2L_p}{L} - \frac{L_p^2}{LL_L} \tag{3.27}
\]

Then, the formula for plastic hinge length \( L_p = 0.5d_{psU} + 0.05L_L \) is substituted:

\[
\alpha_1 = \frac{L_a + d_{psU} + 0.1L_L}{L} - \frac{0.25d_{psU}^2 + 0.05d_{psU}L_L + 0.0025L_L^2}{LL_L} \tag{3.28}
\]

After that, the term \( \frac{L - L_a}{2} \) is put into Equation (3.28) instead of \( L_L \):

\[
\alpha_1 = \frac{0.95L_a + d_{psU} + 0.05L}{L} - \left( \frac{0.25d_{psU}^2}{L(L - L_a) / 2} + \frac{0.05d_{psU}L}{L} + \frac{0.0025(L - L_a) / 2}{L} \right) \tag{3.29}
\]

Now, the expression \( L_a = L / f \) is substituted into the above equation:
\[ \alpha_i = \frac{0.95(L/f) + d_{psU} + 0.05L}{L} - \left( \frac{0.5d_{psU}^2}{L(L-L/f)} + \frac{0.05d_{psU}}{L} + \frac{0.00125(L-L/f)}{L} \right) \]

\[ = \frac{0.95}{f^*} + \frac{1}{L/d_{psU}} + 0.05 - \frac{0.5}{(L/d_{psU})^2(1-1/f)} - \frac{0.05}{L/d_{psU}} - 0.00125\left(1-\frac{1}{f}\right) \]  

(3.30)

After necessary simplifications and rounding the digits into two, the following expression for \( \alpha_i \) in terms of \( L/d_{psU} \) and \( f \) is obtained:

\[ \alpha_i = 0.95 \left( \frac{1}{f} + \frac{1}{L/d_{psU}} \right) + 0.05 - \frac{0.5}{(L/d_{psU})^2(1-1/f)} \]  

(3.31)

For further simplification of above equation, experimental data set (199 beams) available in the literature is used. The second term of Equation (3.31) is 0.66% of the total expression as an average for 199 beams. Therefore, the second term is insignificant and can be ignored. As a result, only the first term remains. Instead of using the first term, \( \frac{1}{f} + \frac{1}{L/d_{psU}} \) is preferred to get a simpler and memorable expression for \( \alpha_i \). For 199 beams, 12% difference (average) is calculated when the term \( \frac{1}{f} + \frac{1}{L/d_{psU}} \) is used instead of \( 0.95 \left( \frac{1}{f} + \frac{1}{L/d_{psU}} \right) + 0.05 \). However, the important point is the effect of term \( \alpha_i \) on the calculation of \( f_{psU} \). In order to see the effect of changing the first term of Equation (3.31) on \( f_{psU} \) and \( \Delta f_{psU} \), the normalized equation in terms of eccentricity is used, Equation (3.15). In Equation (3.15), firstly \( \alpha_i \) is taken as \( 0.95 \left( \frac{1}{f} + \frac{1}{L/d_{psU}} \right) + 0.05 \) and the predictions are compared with experimental results of 199 beams. An average error of
6% and 35.5% are observed while calculating $f_{ptU}$ and $\Delta f_{ptU}$, respectively. Then, in Equation (3.15), $\alpha_i$ is taken as $\frac{1}{f} + \frac{1}{L / d_{ptU}}$ and the predictions are compared with experimental results. An average error of 6.2% and 33.7% are observed while calculating $f_{ptU}$ and $\Delta f_{ptU}$, respectively. Therefore, it is concluded that changing the term from $0.95\left(\frac{1}{f} + \frac{1}{L / d_{ptU}}\right) + 0.05$ to $\frac{1}{f} + \frac{1}{L / d_{ptU}}$ does not affect the predicted values of $f_{ptU}$ and $\Delta f_{ptU}$ significantly when the average percentage errors are compared. (Compare 6% and 6.2%; 35.5% and 33.7%). As a result, the term $\alpha_i = \left(1 - \frac{2L_h}{L} - \frac{L_p^2}{LL_L}\right)$ is mathematically simplified to $\alpha_i = \frac{1}{f} + \frac{1}{L / d_{ptU}}$.

After representing mathematical simplification of term $\alpha_i$, now the same simplified expression is obtained graphically. For that purpose, $\alpha_i$ versus $\frac{1}{L / d_{ptU}}$ graph is plotted by using the available data from literature. As mentioned before, 199 beams are collected from previous researches; 179 of them are prestressed with unbonded tendons and 20 of them are prestressed with hybrid tendons. Since $\alpha_i$ also stands for different loading types, it is appropriate to plot the graph for different loading types:
The above graph shows values of $\alpha_i$ versus $\frac{1}{L/d_{ptU}}$ for different loading types. For the representation of different loading types, the whole length of the member is considered as $L=12$. For example, two-point loading @4/12 means that the distance between one of the two point loads and support is $4/12$ or $L/3$. In the same manner, two-point loading @5/12 means that the distance between one of the two point loads and support is $5/12$ or $5L/12$. Same logic is valid for all other loading types shown in the figure. Over 199 beams, 55 of them are under one-point loading (single concentrated load at midspan), 125 of them are under two-point loading @4/12 (third point loading), 4 of them are under two-point loading @3/12, 4 of them are under two-point loading @5/12, 4 of them are under uniform loading, 1 of them is under two-point loading @4.2/12, 1 of them is under two-point loading @4.9/12, 4 of them are under two-point loading @5.2/12 and 1 of them is under two-point loading @5.3/12. In order to fit a linear line to the data...
points, enough data points need to be present. Therefore, linear curve fitting can be done only for data points of one-point loading, two-point loading @4/12, two-point loading @3/12 and two-point loading @5/12. As can be seen from Figure 3-4, the linear curve fitting equation is approximately \( \alpha_i = \frac{0.9}{L/d_{ptU}} + 0.1 \) for one point loading, 
\( \alpha_i = \frac{0.9}{L/d_{ptU}} + 0.2 \) for two-point loading @5/12, \( \alpha_i = \frac{0.9}{L/d_{ptU}} + 0.4 \) for two-point loading @4/12 and \( \alpha_i = \frac{0.9}{L/d_{ptU}} + 0.5 \) for two-point loading @3/12. It means that starting from single concentrated loading, as the distance between two point loads increases, \( \alpha_i \) also increases. Based on these linear fitting equations, a general expression is used:

\[ \alpha_i = \frac{0.9}{L/d_{ptU}} + \frac{1}{f} \]

In this equation, \( f \) is load geometry factor which was defined before in Table 3-2. This factor was used in the study of Harajli et al. (1991). As mentioned before, for one point loading \( f = 10 \), for equal two point loading @4/12 \( f = 3 \) and for uniform loading \( f = 6 \). For other loading types, \( f \) should be modified by using the expression \( f = L/L_a \). In this expression, \( L_a \) is the theoretical constant moment region length and \( L \) is the span length of the member. For example, for two-point loading @ 3/12, the distance between point load and support is \( L/4 \). The distance between two point loads, which is at the same time the theoretical constant moment region, is \( L/2 \). Therefore, \( f = 2 \) for two-point loading @ 3/12 and the equation becomes 
\[ \alpha_i = \frac{0.9}{L/d_{ptU}} + 0.5 \] which was already found based on Figure 3-4.
In the previous explanations, it was shown that the expression \( \frac{L_b - L_p}{L} \) can be simplified to: \( \frac{0.9}{L/d_{psU}} + \frac{1}{f} \) graphically. To get a more simple and memorable expression, instead of using multiplier 0.9 for the term \( \frac{1}{L/d_{psU}} \), 1.0 can be used. For 199 test beams, by changing \( \frac{0.9}{L/d_{psU}} + \frac{1}{f} \) to \( \frac{1}{L/d_{psU}} + \frac{1}{f} \), the term changes by 2.9% in average. As done in mathematical simplification part, in order to see the effect of changing the multiplier from 0.9 to 1.0 on \( f_{psU} \) and \( \Delta f_{psU} \), Equation(3.15) is used. In Equation(3.15), \( \alpha_i \) is taken as \( \frac{0.9}{L/d_{psU}} + \frac{1}{f} \) and the predictions are compared with experimental results of 199 beams. An average error of 6.32% and 33.71% are observed while calculating \( f_{psU} \) and \( \Delta f_{psU} \), respectively. When, \( \alpha_i \) is taken as \( \frac{1}{f} + \frac{1}{L/d_{psU}} \) on Equation(3.15), and it was shown that an average error of 6.24% and 33.67% are observed for \( f_{psU} \) and \( \Delta f_{psU} \). Therefore, it is concluded that changing the term from \( \frac{0.9}{L/d_{psU}} + \frac{1}{f} \) to \( \frac{1}{L/d_{psU}} + \frac{1}{f} \) does not affect the predicted values of \( f_{psU} \) and \( \Delta f_{psU} \) significantly when the average percentage errors are compared. (Compare 6.32% and 6.24% ; 3.71% and 33.67%). As a result, the term \( \alpha_i = \left(1 - \frac{2L_b}{L} - \frac{L_p}{LL_L}ight) \) is graphically simplified to \( \alpha_i = \frac{1}{f} + \frac{1}{L/d_{psU}} \).
The mathematical and graphical simplification of the term $\alpha_i$ was shown. As a result, the following simplified form of expression is found which stands for span-to-depth ratio and different loading types:

$$\alpha_i = \frac{1}{L / d_{pu}} + \frac{1}{f}$$  \hspace{1cm} (3.32)

After the simplification of the term $\alpha_i$, four more equations can be written. Before, two quadratic equations and two normalized equations were obtained in terms of $e_{pu}$ and $(d_{pu} - c)$. Now, in these equations the term $\alpha_i$ is simplified. As a result, there are totally 8 equations four of which are simplified. The tabular form of the equations and the selection of the final form of the equation for $f_{pu}$ will be shown.

### 3.4.6 Evaluation of Prediction Equations

There are totally 8 options for the selection of final form of the equation. Now, all these equations are put in tabular form. To show the accuracy of each equation, all of them are compared with the available test data collected from literature. As mentioned before, there are 199 beams; 179 of them are prestressed with unbonded tendons and 20 of them are prestressed with bonded and unbonded (hybrid) tendons. In the following table, correlation factor $R$ can be shown for $f_{pu}$ and $\Delta f_{pu}$ as a measure of accuracy of the equations. All equations seen in the table are limited to yield stress $f_{y\text{U}}$ if the unbonded tendon is steel. If the unbonded tendon is FRP, the equation is limited to ultimate strength of $f_{pu\text{U}}$ FRP.
### Table 3-3: Comparison of Prediction Equations

<table>
<thead>
<tr>
<th>Case</th>
<th>Number and Option</th>
<th>Equation</th>
<th>$R$ for Unbonded Beams (179)</th>
<th>$R$ for Hybrid Beams (20)</th>
<th>$R$ for All Beams (199)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Eq.s in terms of $e_{psU}$</td>
<td></td>
<td>$f_{psU}$</td>
<td>$\Delta f_{psU}$</td>
<td>$f_{psU}$</td>
</tr>
<tr>
<td>1</td>
<td>Quadratic Equation Eq. (3.14)</td>
<td>$f_{psU} = \frac{f_{peU} - \alpha_1}{2} + \left( \frac{f_{peU} + \alpha_1}{2} \right)^2 - \frac{E_{psU} \beta_1 f_y e_{psU}}{196 A_{psU}} \alpha_2$</td>
<td>0.96</td>
<td>0.86</td>
<td>0.96</td>
</tr>
<tr>
<td></td>
<td></td>
<td>where, $\alpha_1 = \left(1 - \frac{2L_h}{L} - \frac{L_p}{LL_f}\right)$; $\alpha_2 = \frac{A_y f_y + A_{psb} f_{psb}}{A_{psU}}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Simplified Form of Eq. (3.14)</td>
<td>$f_{psU} = \frac{f_{peU} - \alpha_1}{2} + \left( \frac{f_{peU} + \alpha_1}{2} \right)^2 - \frac{E_{psU} \beta_1 f_y e_{psU}}{196 A_{psU}} \alpha_2$</td>
<td>0.96</td>
<td>0.87</td>
<td>0.96</td>
</tr>
<tr>
<td></td>
<td></td>
<td>where, $\alpha_1 = \frac{1}{L/d_{psU}} + \frac{1}{f} ; \alpha_2 = \frac{A_y f_y + A_{psb} f_{psb}}{A_{psU}}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Normalized Equation Eq. (3.15)</td>
<td>$f_{psU} = f_{peU} + \frac{E_{psU}}{196} \left( \frac{\beta_1 f_y e_{psU}}{A_{psU} f_{psU} + A_{psb} f_{psb} + A_y f_y} \right) \alpha_1$</td>
<td>0.96</td>
<td>0.86</td>
<td>0.96</td>
</tr>
<tr>
<td></td>
<td></td>
<td>where, $\alpha_1 = \left(1 - \frac{2L_h}{L} - \frac{L_p}{LL_f}\right)$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Simplified Form of Eq. (3.15)</td>
<td>$f_{psU} = f_{peU} + \frac{E_{psU}}{196} \left( \frac{\beta_1 f_y e_{psU}}{A_{psU} f_{psU} + A_{psb} f_{psb} + A_y f_y} \right) \alpha_1$</td>
<td>0.96</td>
<td>0.87</td>
<td>0.96</td>
</tr>
<tr>
<td></td>
<td></td>
<td>where, $\alpha_1 = \frac{1}{L/d_{psU}} + \frac{1}{f}$</td>
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<td></td>
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</tr>
</tbody>
</table>
## Table 3-3 (continued): Comparison of All Possible Equations

<table>
<thead>
<tr>
<th>Case</th>
<th>Equation</th>
<th>( R ) for Unbonded Beams (179)</th>
<th>( R ) for Hybrid Beams (20)</th>
<th>( R ) for All Beams (199)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>Eq.s in terms of ( d_{psU} - c )</td>
<td>( f_{psU} = \frac{f_{psU} - \alpha_2}{2} - \frac{E_{psU} \alpha_1}{666} + \sqrt{\left( \frac{\alpha_2 - [(E_{psU} \alpha_1 / 333) - f_{psU}]}{2} \right)^2 + \frac{E_{psU} \beta f_{psU} d_{psU}}{392 A_{psU}} \alpha_1} )</td>
<td>0.95</td>
<td>0.86</td>
</tr>
<tr>
<td>6</td>
<td>Simplified Form of Eq. (3.20)</td>
<td>( f_{psU} = \frac{f_{psU} - \alpha_2}{2} - \frac{E_{psU} \alpha_1}{666} + \sqrt{\left( \frac{\alpha_2 - [(E_{psU} \alpha_1 / 333) - f_{psU}]}{2} \right)^2 + \frac{E_{psU} \beta f_{psU} d_{psU}}{392 A_{psU}} \alpha_1} )</td>
<td>0.95</td>
<td>0.85</td>
</tr>
</tbody>
</table>

where, \( \alpha_1 = \frac{1}{L / d_{psU} + 1} \); \( \alpha_2 = \frac{A_y f_y + A_{psB} f_{psB}}{A_{psU}} \)
### Table 3-3 (continued): Comparison of All Possible Equations

<table>
<thead>
<tr>
<th>Case</th>
<th>Number and Option</th>
<th>Equation</th>
<th>$R$ for Unbonded Beams (179)</th>
<th>$R$ for Hybrid Beams (20)</th>
<th>$R$ for All Beams (199)</th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$f_{psU}$</td>
<td>$\Delta f_{psU}$</td>
<td>$f_{psU}$</td>
</tr>
<tr>
<td>7</td>
<td>Norm. Equation Eq. (3.26)</td>
<td>$f_{psU} = f_{psU} + \frac{E_{psU}}{333} \left( \frac{0.85 \beta f_b d_{psU}}{(A_{psU} f_{psU} + A_{psB} f_{psB} + A_y f_y)} \right) \alpha_1$</td>
<td>0.95</td>
<td>0.86</td>
<td>0.97</td>
</tr>
<tr>
<td></td>
<td>Eq.s in terms of $d_{psU} - c$</td>
<td>where, $\alpha_1 = \left(1 - \frac{2L_h}{L} - \frac{I_p^2}{LL_h}\right)$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Simplif. Form of Eq. (3.26)</td>
<td>$f_{psU} = f_{psU} + \frac{E_{psU}}{333} \left( \frac{0.85 \beta f_b d_{psU}}{(A_{psU} f_{psU} + A_{psB} f_{psB} + A_y f_y)} \right) \alpha_1$</td>
<td>0.95</td>
<td>0.86</td>
<td>0.97</td>
</tr>
<tr>
<td></td>
<td></td>
<td>where, $\alpha_1 = \frac{1}{L/d_{psU}} + \frac{1}{f}$</td>
<td></td>
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</tbody>
</table>
The following comments are made based on the results shown in Table 3-3:

1. Results show that all of the equations are very close in terms of correlation coefficient and in return accuracy.

2. The correlation coefficient changes between 0.82 and 0.89 for $\Delta f_{psU}$ and 0.95 and 0.97 for $f_{psU}$. This result show that all of these equations are enough accurate.

3. For the simplification of the equations, the term $\alpha_i$ is changed. The results show that this change does not cause a decrease in the accuracy of the equations. Therefore, this simplification is reasonable.

4. The quadratic equations have mathematical basis. This makes their derivations reliable. In addition, $R$ values show that they are also accurate. However, the equations are very complicated and cannot be proposed for design purposes in a code.

5. The equations are categorized as the equations in terms of $e_{psU}$ and equations in terms of $(d_{psU} - c)$. The equations with $(d_{psU} - c)$ are longer than the equations with $e_{psU}$. In addition, even though the accuracy of equations are very close, still the equations with $e_{psU}$ are more accurate when all beams are considered. Therefore, the equations in terms of $e_{psU}$ are preferable.

The aim of this research is to get a simple and accurate equation for the calculation of $f_{psU}$ at ultimate. Based on the comments and the results shown in Table 3-3, it is appropriate to select Case 4, which is the simplified form of normalized equation in terms of $e_{psU}$. This is the best option compared with others since it is the simplest and the most accurate one with $R = 0.96$ for $f_{psU}$ and $R = 0.87$ for $\Delta f_{psU}$ for all beams.
3.5 Recommended Prediction Equation at Ultimate

The final form of the equation for the calculation of $f_{psU}$ at ultimate is selected and reasons for this preference were explained in Section 3.4.6. Now, the final form of the equation can be observed as follows:

$$f_{psU} = f_{peU} + \frac{E_{psU}}{196} \left( \beta f_e b e_{psU} \left( A_{psU} f_{puU} + A_{psB} f_{puB} + A_t f_y \right) \alpha_i \right) \leq f_{pyU} \text{ or } f_{frpuU}$$

(3.33)

where,

$$\alpha_i = \frac{1}{L/d_{psU} + 1} + \frac{f}{f}$$

In Equation (3.33), for the calculation of the bonded tendon stress $f_{psB}$, AASHTO equation for bonded tendons is used if the tendon is steel. If the bonded tendon is FRP, then the stress is taken as the ultimate stress of the bonded tendon $f_{puB}$. As explained before (Table 3-2), $f$ is equal to 10 for one point loading, 6 for uniform loading and 3 for two-point loading (third point loading). For other loading types, $f$ should be modified by using the expression $f = L/L_a$. Lastly, Equation (3.33) should be limited to the yield stress of the unbonded tendon $f_{pyU}$ if the unbonded tendon is steel. If the unbonded tendon is FRP, the equation is limited to ultimate strength of FRP $f_{frpuU}$.

If the beam does not include any bonded prestressed reinforcement; in other words if the beam is prestressed with only unbonded tendons, simply the area of bonded prestressed reinforcement should be equated to zero: $A_{psB} = 0$. As a result, a general form of equation, also with inclusion of non-prestressed compression reinforcement, is obtained:
A rational, simple and accurate equation is obtained for the estimation of the unbonded tendon stress at ultimate. Most important parameters such as span-to-depth ratio, area of reinforcements, loading type, effective prestress of unbonded tendon and concrete strength are taken into consideration while deriving the equation. This equation is applicable to beams prestressed with unbonded or bonded and unbonded (hybrid) tendons, FRP or steel tendons, external or internal tendons.

In Equation (3.33), \( \alpha \) includes two terms: \( \frac{1}{f} \) and \( \frac{1}{L/d_{puU}} \). For experimental data set (199 beams), the value of \( \frac{1}{L/d_{puU}} \) is 0.05 as average. In the next chapter on the parametric study, it will be shown that for beams in which \( L/d_{puU} \geq 15 \), the change in \( \Delta f_{puU} \) is not very significant (See Figure 4-1). Therefore, for beams that have \( L/d_{puU} \geq 15 \), \( \alpha \) can be simplified to \( \frac{1}{f} \) by ignoring the effect of \( \frac{1}{L/d_{puU}} \). As a result, the following equation is obtained:

\[
 f_{puU} = f_{puU} + \frac{E_{puU}}{196} \left( \frac{\beta_{e} f_{c} e_{puU}}{A_{puU} f_{puU} + A_{pB} f_{pB} + A_{s} f_{y}} \right) \left( \frac{1}{f} \right) \leq f_{puU} \text{ or } f_{fpU} \tag{3.35}
\]

For code purposes, another option is to simplify Equation (3.33) further by taking the \( \alpha \) factor as a constant. The code equations are simple and conservative equations. In order to obtain a simple and conservative equation, the term \( \alpha \) is taken as 0.2, based on the 199 experimental test data and by referring to Figure 3-4. Then, the following equation is obtained:
As a result, three equations are proposed: Equations (3.33),(3.35) and (3.36) can be named as Proposed Equations Alternative I, II and III, respectively. Alternatives I and II are more accurate than Alternative III since the different loading types are taken into account. However, Equation (3.35) has a restriction: it cannot be used for beams that have span-to-depth ratio smaller than 15. Equation (3.36), Alternative III is simpler than Alternatives I and II. Therefore, Equations (3.33) and (3.35) are proposed for getting more accurate results in designs and Equation (3.36) is proposed for code purposes.

Proposed equations are compared at Chapter 5, Results. The next chapter, paramedic study identifies the parameters that influence the prediction of stress at ultimate.
CHAPTER IV

PARAMETRIC STUDY

4.1 Introduction

In this part of the study, parametric study is implemented to investigate the effects of different parameters on the ultimate stress and stress increase in unbonded tendons. Before passing to that part, a summary of the parametric studies available in literature is provided.

4.2 Parametric Studies from Literature

The parametric studies carried out by different authors are summarized in terms of the effects of different terms on $\Delta f_{p_u}$ and $f_{p_u}$. In Table 4-1, the information related to the researches of Du and Tao (1985), Harajli and Hijazi (1991), Campbell and Chouinard (1991), Chakrabarti (1995), Allouche et al. (1999), Tanchan (2001), Diep and Niwa (2006) and Ozkul (2007) can be found. In Table 4-1, $\uparrow$ means increase, while $\downarrow$ stands for decrease and $-$ corresponds to no information is given. Please refer to the following definitions before looking into the table:

$c = $ Neutral Axis Depth

$d_{ps} = $ Effective Depth of Unbonded Prestressing Tendon

$f_{pe} = $ Effective Prestressing Force in Unbonded Tendons
$$PPR = \frac{A_{ps} \times f_{ps}}{A_{ps} \times f_{ps} + A_i \times f_y}$$

$q_{ps}$ = Reinforcement Index of Unbonded Tendon

$q_o$ = Combined Reinforcement Index, $\frac{\rho_{ps} \times f_{ps}}{f'_c} + \frac{\rho_s \times f_y}{f'_c}$

$w_{pp}$ = Reinforcement Index, $\frac{\rho_{ps} \times f_{ps}}{f'_c} + \frac{\rho_s \times f_y}{f'_c}$
Table 4-1: Summary of Parametric Studies from Literature

<table>
<thead>
<tr>
<th>Author</th>
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<th>Parameters</th>
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<th>Effect on $f_{psU}$</th>
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<td>↑</td>
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<td>↑</td>
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<tr>
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<td></td>
<td>$f'_c$</td>
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<td>↑</td>
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<tr>
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<td>←</td>
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<td>No Significant Effect</td>
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</tr>
<tr>
<td>Campbell and Chouinard (1991)</td>
<td>Experimental Results</td>
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<td></td>
<td></td>
<td>←</td>
<td>←</td>
</tr>
<tr>
<td>Chakrabarti (1995)</td>
<td>Experimental</td>
<td>$w_{pp}$</td>
<td>↓</td>
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<tr>
<td></td>
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<td>↑</td>
</tr>
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<td></td>
<td>$f'_c$</td>
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<td>↑</td>
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<td>←</td>
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<td>↑</td>
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<td>Effect on $f_{pSU}$</td>
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<td>-----------------</td>
<td>-------------------</td>
<td>------------</td>
<td>---------------------------</td>
<td>---------------------</td>
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<td>$\downarrow$</td>
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<td></td>
<td>$d_{ps}/c-1$, $f'<em>c/\rho</em>{ps}$, $(d_{ps}-c)/L$</td>
<td>$\uparrow$</td>
<td>$\uparrow$</td>
</tr>
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<td>Ozkul (2007)</td>
<td>Experimental Results</td>
<td>$A_s$, $A_{ps}$, $f_{pe}$</td>
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<td>$-$</td>
</tr>
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<td></td>
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<td>$f'_c$</td>
<td>$\uparrow$</td>
<td>$-$</td>
</tr>
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</table>
4.3 Parametric Study

In the parametric study, the beams that are used in the parametric study of Harajli and Hijazi (1991) are taken. All beams are rectangular sections with following dimensions: $b = 10\text{in}$, $h = 20\text{in}$. The beams include only unbonded tendons. The effective depths of unbonded tendons and non-prestressing reinforcement are $d_{psU} = 16\text{in}$ and $d_s = 18\text{in}$, respectively. The effect of following parameters on $\Delta f_{psU}$ and $f_{psU}$ are investigated in this study: span to depth ratio $L/d_{psU}$, concrete compressive strength $f'_c$, area of non-prestressing tension reinforcement $A_s$, yield stress of non-prestressing tension reinforcement $f_y$, area of compression reinforcement $A_s$, area of unbonded prestressing steel $A_{psU}$, effective stress of unbonded prestressing steel $f_{psU}$, ultimate stress of unbonded prestressing steel $f_{puU}$ and loading type. As a result 30 beams are investigated under eight groups, G1-G8. The parameters and related values can be seen in Table 4-2. For investigating the influence of loading type, all 30 beams are studied under one point loading, two-point loading and uniform loading. As result, totally 90 beams are used for parametric study. In order to examine the influence of span-to-depth ratio, $d_{psU}$ kept constant while span length is changed. For each beam, the investigated parameter versus ultimate stress in unbonded tendon $f_{psU}$ and the stress increase in unbonded tendon $\Delta f_{psU}$ graphs are plotted. These plots can be observed from Figure 4-1 to Figure 4-8 after Table 4-2. In these graphs; 1PT, 2PT and U mean one point loading, two point loading and uniform loading respectively. For the calculation of $\Delta f_{psU}$ and $f_{psU}$, GIA (in terms of $d_{psU} - c$) is used.
<table>
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<th>Concrete Beam</th>
<th>Non-Prestressing Steel (Tension and Compression)</th>
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<td>1.07</td>
<td>60</td>
</tr>
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<td>G8 245</td>
<td>245</td>
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</tr>
<tr>
<td>G8 270</td>
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</table>
Figure 4-1: Effect of Span-to-Depth Ratio on (a) $f_{pu}$ and (b) $\Delta f_{pu}$.

Figure 4-2: Effect of Concrete Compressive Strength on (a) $f_{pu}$ and (b) $\Delta f_{pu}$. 
Figure 4-3: Effect of Area of Non-prestressing Tension Reinforcement on (a) $f_{psU}$
and (b) $\Delta f_{psU}$

Figure 4-4: Effect of Yield Stress of Non-prestressing Tension Reinforcement on (a) $f_{psU}$
and (b) $\Delta f_{psU}$
Figure 4-5: Effect of Area of Non-prestressing Compression Reinforcement on (a) $f_{psU}$ and (b) $\Delta f_{psU}$

Figure 4-6: Effect of Area of Unbonded Prestressing Steel on (a) $f_{psU}$ and (b) $\Delta f_{psU}$
Figure 4-7: Effect of Effective Stress of Unbonded Prestressing Steel on (a) $f_{puU}$ and (b) $\Delta f_{puU}$

Figure 4-8: Effect of Ultimate Stress of Unbonded Prestressing Steel on (a) $f_{puU}$ and (b) $\Delta f_{puU}$
Results and Comments for Parametric Study Graphs (Figure 4-1 to Figure 4-8):

1) From Figure 4-1, it is observed that for all loading types, as the span-to-depth ratio $L/d_{ptU}$ increases, the increase in stress $\Delta f_{ptU}$ and in return, the stress $f_{ptU}$ decrease. In Figure 4-1(b), the results found by Harajli et al. (1991) using nonlinear analysis also shown. As can be seen, the results match with the findings of Harajli et al. (1991). As the $L/d_{ptU}$ ratio increases from 5 to 50, $\Delta f_{ptU}$ decreases by 24% for two-point loading, 36% for uniform loading, 67% for one point loading. In other words, the decrease in $\Delta f_{ptU}$ is more significant for members loaded with single concentrated load. It means that the effect of span-to-depth ratio increases as the length of the constant moment region decreases; as in the case of one point loading. On the other hand, as the $L/d_{ptU}$ ratio increases from 15 to 50, $\Delta f_{ptU}$ decreases by 8% for two-point loading, 14% for uniform loading, 37% for one point loading. This statement means that for span to depth ratios larger than fifteen $L/d_{ptU} \geq 15$, the drop in $\Delta f_{ptU}$ for all loading types is not very significant. Additionally, as $L/d_{ptU}$ ratio increases from 5 to 50, $f_{ptU}$ decreases by 7% for two-point loading, 9% for uniform loading, 13% for one point loading.

2) As can be seen in Figure 4-2, the increase in concrete compressive strength $f'_c$ leads to an increase in $\Delta f_{ptU}$ as well as $f_{ptU}$. This observation matches with the studies of Du and Tao (1985), Chakrabarti (1995), Tanchan (2001) and Ozkul (2007) which can be seen in Table 4-1. As $f'_c$ increases from 5 ksi to 9 ksi, $\Delta f_{ptU}$ increases by 46% for two-point loading, 48% for uniform loading, 51% for one
point loading. On the other hand, $f_{pu}$ value increases by 11% for two-point loading, by 8% for uniform loading and 4% for single concentrated load. Therefore, it is concluded that $f'_c$ has an important effect on ultimate stress increase and ultimate stress. Actually, this conclusion is expected since the increase in $f'_c$ directly leads to an increase in concrete compressive force. Therefore, based on force equilibrium, an increase in concrete compressive force causes an increase in unbonded tendon force. This means that unbonded tendon stress increases.

3) From Figure 4-3, it is observed that the increase in area of non-prestressing tension reinforcement $A_r$ causes a decrease in $\Delta f_{pu}$ together with $f_{pu}$. This finding matches with the results of Du and Tao (1985), Campbell and Chouinard (1991), Tanchan (2001) and Ozkul (2007) which can be seen in Table 4-1. As $A_r$ increases from 0.64 in$^2$ to 1.33 in$^2$, $\Delta f_{pu}$ decreases by 19% for two-point loading, 21% for uniform loading, 24% for one point loading. On the other hand, $f_{pu}$ value drops by 5% for two-point loading, by 4% for uniform loading and 2% for single concentrated load. Therefore, it is concluded that $A_r$ has an important effect on the ultimate stress increase values. This conclusion can be explained as follows: Area of non-prestressing steel $A_r$ is one of the terms used for the calculation of neutral axis depth $c$; as $A_r$ increases $c$ also increases. The increase in neutral axis depth causes a decrease in curvature, which in turn causes a decrease in deflection and as a result a drop in $\Delta f_{pu}$ as well as $f_{pu}$. 
4) The increase in yield stress of non-prestressing steel $f_y$ causes a reduction in $\Delta f_{pu}$ along with $f_{pu}$, Figure 4-4. With an increase from 40 ksi to 80 ksi for $f_y$, $\Delta f_{pu}$ decreases by 20% for two point loading, 22% for uniform loading and 25% for single concentrated load. On the other hand, $f_{pu}$ value drops by 5% for two-point loading, 4% for uniform loading and 2% for single concentrated load. This conclusion is something expected. Yield stress of non-prestressing steel $f_y$ is included in the equation of neutral axis depth $c$; as $f_y$ increases $c$ also increases. The increase in neutral axis depth causes a decrease in curvature, which in turn causes a decrease in deflection and as a result a drop in $\Delta f_{pu}$ and $f_{pu}$.

5) From Figure 4-5, it is observed that the increase in area of non-prestressing compression reinforcement $A'_s$ causes an increase in $\Delta f_{pu}$ together with $f_{pu}$. As $A'_s$ increases from 0.53 in$^2$ to 1.11 in$^2$, $\Delta f_{pu}$ increases by 20% for two-point loading, 21% for uniform loading, 23% for one point loading. On the other hand, $f_{pu}$ value ascends by 4% for two-point loading, by 3% for uniform loading and 1% for single concentrated load. Therefore, it is concluded that $A'_s$ has an important effect on the ultimate stress increase values. This conclusion can be explained as follows: Area of non-prestressing steel $A'_s$ is one of the terms used for the calculation of neutral axis depth $c$; as $A'_s$ increases $c$ decreases. The decrease in neutral axis depth causes an increase in curvature, which in turn causes an increase in deflection and as a result an increase in $\Delta f_{pu}$ as well as $f_{pu}$.
6) As can be seen in Figure 4-6, the increase in area of unbonded prestressing steel $A_{psU}$ causes a decrease in $\Delta f_{psU}$ and $f_{psU}$. This finding matches with the results of Du and Tao (1985), Tanchan (2001) and Ozkul (2007) which can be seen in Table 4-1. With an increase from 0.55 in$^2$ to 1.15 in$^2$ for $A_{psU}$, $\Delta f_{psU}$ drops by 51% for all loading types. On the other hand, $f_{psU}$ value decreases by 17% for two-point loading, by 13% for uniform loading and 5% for one point loading. Therefore, $A_{psU}$ has an important effect on the ultimate stress increase and ultimate stress values. In the same manner, if bonded prestressing reinforcement exists in a beam, it also has an important effect on $\Delta f_{psU}$ and $f_{psU}$. Therefore, the equations and/or analysis which do not consider the effect of bonded prestressing steel $A_{psB}$ lose accuracy substantially. This conclusion is something expected. Area of bonded and/or unbonded tendon is used for the calculation of neutral axis depth $c$; as $A_{psB}$ and/or $A_{psU}$ increase, $c$ also increases. The increase in neutral axis depth causes a decrease in curvature, which in turn causes a decrease in deflection and as a result a drop in $\Delta f_{psU}$ and $f_{psU}$.

7) The increase in effective prestressing force of unbonded prestressing steel $f_{peU}$ causes a significant reduction in $\Delta f_{psU}$, Figure 4-7. With an increase from 135 ksi to 189 ksi for $f_{peU}$, $\Delta f_{psU}$ decreases by 23% for two point loading, 25% for uniform loading types, and 28% for single concentrated load. Since $f_{psU}$ is equal to $f_{peU} + \Delta f_{psU}$, for sure the effective stress of unbonded tendon has an effect on $f_{psU}$ value. As $f_{peU}$ rises from 135 ksi to 189 ksi, $f_{psU}$ value increases by 21%
for two-point loading, by 25% for uniform loading and 33% for one point loading. Therefore, $f_{peU}$ has an important effect on the ultimate stress increase and ultimate stress values. This finding matches with the results of Chakrabarti (1995), Tanchan (2001) and Ozkul (2007) which can be seen in Table 4-1.

8) The increase in ultimate tensile strength of unbonded tendon $f_{puU}$ causes a reduction in $\Delta f_{puU}$, Figure 4-8. With an increase from 235 ksi to 270 ksi for $f_{puU}$, $\Delta f_{puU}$ decreases by 9% for two point loading, 10% for uniform loading types, and 12% for single concentrated load. As $f_{puU}$ increases, $f_{peU}$ also increases. Since $f_{puU}$ is equal to $f_{peU} + \Delta f_{puU}$, the increase in $f_{puU}$ and in turn $f_{peU}$ has an effect on $f_{puU}$ value. As $f_{puU}$ rises from 135 ksi to 189 ksi, $f_{puU}$ value increases by 8% for two-point loading, by 9% for uniform loading and 12% for one point loading.

9) As can be seen from Figure 4-1 to Figure 4-8, single concentrated loading gives lower values of $\Delta f_{puU}$ and $f_{puU}$ compared to two-point or uniform loadings. This situation is already observed in the researches of Harajli and Hijazi (1991), Allouche et al. (1999) and Tanchan (2001). The difference of the tendon stress for different loading types is directly related to the length of the constant moment region or the equivalent plastic hinge length. As this length increases, the unbonded tendon stress also increases. It means that under same conditions, the tendon stress is the highest for two-point loading, then for uniform loading and the lowest for one-point loading.
Now, all these findings can be summarized in the following table in terms of percentage (％) decrease ↓ and percentage (％) increase ↑. In this table, 1PT, 2PT and U mean one point loading, two point loading and uniform loading, respectively.

**Table 4-3: Summary of Results**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Effect on $\Delta f_{\text{psU}}$</th>
<th>Effect on $f_{\text{psU}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2PT</td>
<td>U</td>
</tr>
<tr>
<td>$L/d_{\text{psU}}$ ($\uparrow$ 900%)</td>
<td>↓ 24%</td>
<td>↓ 36%</td>
</tr>
<tr>
<td>$f_0$ ($\uparrow$ 80%)</td>
<td>↑ 46%</td>
<td>↑ 48%</td>
</tr>
<tr>
<td>$A_0$ ($\uparrow$ 108%)</td>
<td>↓ 19%</td>
<td>↓ 21%</td>
</tr>
<tr>
<td>$f_i$ ($\uparrow$ 100%)</td>
<td>↓ 20%</td>
<td>↓ 22%</td>
</tr>
<tr>
<td>$A_i$ ($\uparrow$ 109%)</td>
<td>↑ 20%</td>
<td>↑ 21%</td>
</tr>
<tr>
<td>$A_{\text{psU}}$ ($\uparrow$ 109%)</td>
<td>↓ 51%</td>
<td>↓ 51%</td>
</tr>
<tr>
<td>$f_{\text{psU}}$ ($\uparrow$ 40%)</td>
<td>↓ 23%</td>
<td>↓ 25%</td>
</tr>
<tr>
<td>$f_{\text{psU}}$ ($\uparrow$ 15%)</td>
<td>↓ 9%</td>
<td>↓ 10%</td>
</tr>
</tbody>
</table>
CHAPTER V

RESULTS

5.1 Introduction

In this chapter, the validity and also the applicability of GIA procedure, finite element analysis (FEA) and the proposed design equations (Alternatives I, II and III) are shown. For this validation process, the results obtained from GIA, FEA and the proposed equations are compared with the experimental results. The results are categorized under two parts: representation of the overall behavior of beams and ultimate strength of beams. To show the overall behavior of beams, applied load versus deflection graphs are plotted for the comparison of GIA, FEA, and experimental results at each limit state. On the other hand, the results obtained from GIA and the proposed equations are compared with experimental results for the calculation of unbonded tendon stress $f_{pu,U}$ and unbonded tendon stress increase $\Delta f_{pu,U}$ at ultimate limit state. In addition, the prediction equations proposed by other researchers are collected to compare the accuracy of the proposed design equations in Alternative I and II with the available equations from literature. Some of the code equations’ results for $f_{pu,U}$ are also compared with the experimental results and also the proposed design equation in Alternative III.
5.2 Comparison of Results for Prediction of Overall Response of Members

It is known that lots of researches have been implemented for the estimation of unbonded tendon stress at ultimate. However, for the prediction of the overall response of beams prestressed with unbonded tendons limited researches have been performed, namely: Balaguru (1981), Scordelis (1984), Harajli and Kanj (1991), Dall’ Asta and Dezi (1993), Roca and Mari (1993), Alkhairi and Naaman (1993), Figueiras and Povoas (1994), Pisani and Nicoli (1996), Moon and Burns (1997a), Ariyawardena and Ghali (2002), Nassif and Ozkul (2002), Grace and Singh (2003), Grace et al. (2004), Naaman (2005), Lou and Xiang (2006), Park et al. (2006), Ozkul et al. (2008), Vu, Castel and Francois (2010) and He and Liu (2010). Most of these studies were conducted using special purpose computer programs with some iterations or analytical models that are complex. The proposed models are not simple and difficult to implement. Another important point is that most of the models or procedures proposed by these authors for the prediction of the overall response of beams are valid for the beams prestressed with unbonded tendons. However, only the nonlinear computer program developed by Grace and Singh (2003) and Grace et al. (2004), is applicable to beams prestressed with a combination of bonded and unbonded (hybrid) tendons. On the other hand, GIA (developed by Ozkul et al. 2008) can easily be applied without the use computers and is applicable to beams prestressed unbonded or hybrid tendons.

Now, GIA (in terms of $d_{psUc} - c$) and FEA results are compared with experimental results. For that purpose, the load versus deflection graphs, Figure 5-1 through Figure 5-3, are plotted for the beams prestressed with hybrid tendons. These graphs include the
beams tested by Ghallab and Beeby (2005). In Figure 5-1 to Figure 5-3, the beams are analyzed at five key points by using GIA: cracking of concrete, yielding of non-presetressed reinforcement, yielding of bonded prestressed reinforcement, yielding of unbonded presetressed reinforcement and ultimate crushing of concrete. However, for some beams all of these five key points cannot be observed. The reason for that FRP tendons are used as unbonded tendons for all of these beams shown in the figures, and there is no yielding point of FRP tendon.

![Figure 5-1: Comparison of Load vs. Deflection Results of Beams Tested by Ghallab et al. (2005)](image-url)
Figure 5-2: Comparison of Load vs. Deflection Results of Beams Tested by Ghallab et al. (2005)

Figure 5-3: Comparison of Load vs. Deflection Results of Beams Tested by Ghallab et al. (2005)
As can be observed from the figures, GIA results show a very good correlation with experimental results at five limit states. The trends of the load-deflection curves are very similar for FEA and experimental results. Therefore, the whole load versus deflection relationship of a member can be estimated by using FEA. As a result, GIA and FEA are accurate and reliable methods to predict the service life, overall response of members prestressed with unbonded and bonded tendons.

5.3 Comparison of Results for Estimation of $f_{pu}$ and $\Delta f_{pu}$

In this part of the study, the results of proposed design equations Alt. I, II and III for $f_{pu}$ and $\Delta f_{pu}$ at ultimate are compared with the available experimental data results. Then, the equations for the calculation of the ultimate strength of unbonded tendons collected from various researches and codes are represented. In the same manner, the results of these equations for $f_{pu}$ and $\Delta f_{pu}$ at ultimate are compared with the experimental results. After that, the results of the comparisons are examined for highlighting the equations that have better accuracy then other existing prediction equations. Actually, the main aim of these comparisons is to show the accuracy and validity of proposed design equations. These comparisons are implemented for only unbonded tendons and a combination of bonded and unbonded (hybrid) tendons, separately. For creating the experimental database, beams tested by different authors are considered and all the related data including the measured values of are collected. For this effort, a wide range of selected beams prestressed with unbonded tendons and hybrid are taken into account. The experimental data obtained from literature includes 199

5.3.1 Results for Proposed Design Equations

As explained, by using the collected experimental data, the accuracy of the proposed equation in Alternative I Eq. (3.33), Alternative II Eq. (3.35) and Alternative III Eq. (3.36) and GIA (in terms of $d_{psUdc}$) are examined by comparing the predicted values with the experimentally measured values. For that purpose, six graphs were prepared for proposed equations and GIA, separately. For all graphs, Figure 5-4 to Figure 5-15, the experimental data set are categorized under two groups. One group of the data is the beams that include minimum area of non-prestressed reinforcement specified by ACI code 2008 for prestressed concrete beams ($A_t > A_{t, min} = 0.004A_t$). Other group of the data is the beams that do not include minimum area of non-prestressed reinforcement specified by ACI code 2008 ($A_t < A_{t, min} = 0.004A_t$). The beams that do not have required area of non-prestressed reinforcement show excessive deflection after application of load which causes stability problems in beams. Therefore, for these beams the accuracy of the
proposed equations and also GIA will decrease. This is the reason for splitting the experimental data set into two groups. First graphs, named as (a) for each graph from Figure 5-4 to Figure 5-15, stand for the comparison of predicted values of the ultimate unbonded tendon stress $f_U$ with experimentally measured values for beams with only unbonded tendons (179 beams). Second graphs, named as (b) for each graph from Figure 5-4 to Figure 5-15, represent the comparison of predicted values of $f_U$ with experimentally measured values for beams with hybrid tendons (20 beams). Third and fourth graphs, named as (c) and (d) for each graph, are plotted in the same manner and sequence for the comparison of stress increase in unbonded tendons $\Delta f_U$. Last graphs, named as (e) and (f) for each graph from Figure 5-4 to Figure 5-15, are prepared for the comparison of predicted values of $f_U$ and $\Delta f_U$ for all beams (199 beams). The graphs are separately examined for beams with unbonded tendons and hybrid beams to show the validity of the proposed equations and GIA for different kinds of beams. In addition, the graphs for $\Delta f_U$ are much more important and representative for the accuracy of a given equation or method since the main aim of the study is to calculate the stress increase in the unbonded tendon. After that point, $f_U$ can easily be calculated by using the expression of $f_U = f_U + \Delta f_U$ where $f_U$ is known from experimental data. Therefore, the rate of change of $f_U$ is smaller compared to $\Delta f_U$. In the graphs, diagonal line represents the perfect correlation between experimental and predicted values.
Figure 5-4: Comparison of Equation (3.33) for $f_{cut}$ with Beams Including
(a) Unbonded Tendons (b) Unbonded and Bonded Tendons

Figure 5-5: Comparison of Equation (3.33) for $\Delta f_{cut}$ with Beams Including
(c) Unbonded Tendons (d) Unbonded and Bonded Tendons
Figure 5-6: Comparison of Equation (3.33) for (e) $f_{p_{UL}}$ and (f) $\Delta f_{p_{UL}}$ with All Beams

Figure 5-7: Comparison of Equation (3.35) for $f_{p_{UL}}$ with Beams Including
(a) Unbonded Tendons  (b) Unbonded and Bonded Tendons
Figure 5-8: Comparison of Equation (3.35) for $\Delta f_{pu}$ with Beams Including
(c) Unbonded Tendons (d) Unbonded and Bonded Tendon

Figure 5-9: Comparison of Equation (3.35) for (e) $f_{pu}$ and (f) $\Delta f_{pu}$ with All Beams
Figure 5-10: Comparison Equation (3.36) for $f_{pu}$ with Beams Including
(a) Unbonded Tendons (b) Unbonded and Bonded Tendons

Figure 5-11: Comparison of Equation (3.36) for $\Delta f_{pu}$ with Beams Including
(c) Unbonded Tendons (d) Unbonded and Bonded Tendon
Figure 5-12: Comparison of Equation (3.36) for (e) $f_{psU}$ and (f) $\Delta f_{psU}$ with All Beams

Figure 5-13: Comparison of GIA for $f_{psU}$ with Beams Including (a) Unbonded Tendons (b) Unbonded and Bonded Tendons
Figure 5-14: Comparison of GIA for $\Delta f_{pu}$ with Beams Including (c) Unbonded Tendons (d) Unbonded and Bonded Tendons

Figure 5-15: Comparison of GIA for (e) $f_{pu}$ and (f) $\Delta f_{pu}$ with All Beams
It can be easily observed from Figure 5-6, Figure 5-9 and Figure 5-15, the proposed equations in Alternative I and II and GIA show nearly perfect correlation with the experimental data for both the beams with only unbonded tendons and bonded and unbonded (hybrid) tendons. Proposed equation in Alternative III also gives accurate results. The proposed equation in Alternative III does not give as accurate results as Alternatives I and II; however it gives conservative results, Figure 5-12. The reason for that is Equation Alternative III is simpler and developed for code purposes. As can be observed in Figure 5-12(e) and (f), most of the data points (80%) are under the perfect correlation line, which makes the equation conservative. The accuracy of the proposed equations decrease for the beams that do not have required area of non-prestressed reinforcement based on ACI Code 2008, Figure 5-6(f), Figure 5-9(f) and Figure 5-12(f). Same situation is also observed for GIA, Figure 5-14(c) and (d) and Figure 5-15(f). If those beams \( A_s < A_{min} = 0.004A_e \) are removed from the experimental data set, the accuracy of the proposed equations and GIA can be improved significantly.

The tendons are steel and internally prestressed for the beams with unbonded tendons. On the other hand, the unbonded tendons are FRP and externally prestressed for the beams with hybrid tendons (except one of the beams). Therefore, the proposed equations and GIA procedure are accurate for both the beams with steel and FRP tendons, internal and external tendons. For an equation, accuracy is not the only criteria. Another important issue is the derivation of the equation. The proposed equations in Alternative I, II and III are based on mathematical derivations; they are not empirical. Proposed equations are simple and easy to use; not complex. Briefly, all three proposed equations are accurate, simple and rational. In addition, they are applicable to beams with
only unbonded tendons and/or hybrid tendons, FRP and/or steel tendons, external and/or internal tendons.

5.3.2 Results for Equations from Literature

Various design equations have been proposed by different researchers to estimate the unbonded tendon stress at ultimate state. An important point is that the equations are valid for the beams with only unbonded tendons; not the beams with hybrid tendons. The proposed design equations are reviewed and summarized with inclusion of researchers and research years in Table 5-1. As can be seen from this table, totally 23 equations are collected from literature starting from 1962 to 2011. In the same manner as done in previous section (5.3.1) for the proposed design equation and GIA, the accuracies of the equations are investigated by comparing the predicted values with the experimentally measured values. The data set is not put under two groups \( A_s > A_{s_{\text{min}}} \) and \( A_s < A_{s_{\text{min}}} \) as done in previous part. The predicted values of the unbonded tendon stress \( f_{pu_{U}} \) and unbonded tendon stress increase \( \Delta f_{pu_{U}} \) are compared with the experimental values for the beams with only unbonded tendons (179), for the beams with unbonded and bonded tendons (20) and for all of the beams (199). As a result, six graphs are prepared for each equation that can be observed following Table 5-1, from Figure 5-16 to Figure 5-77. However, it is seen that for the equations of Allouche et al. (1999), Roberts-Wollmann et al. (2005), Diep and Niwa (2006) and Harajli-Alternatives I and II (2006), two graphs are shown for each. The reason for that these equations are not applicable to FRP tendons. Since the beams with unbonded and bonded tendons include unbonded FRP tendons, the graphs for them cannot be formed. Therefore, these equations are examined for only the
beams with unbonded tendons. In addition, for the equation proposed by Balaguru (1981), the deflection under point load is needed. However, this value is not given for all of the test beams. Therefore, the comparison graphs are not formed for this equation; it is just put into the table to show that it is also reviewed. If the equations do not include any term related to bonded tendons or the studies do not specifically mention anything related to beams with unbonded and bonded tendons, the bonded tendons are ignored for the calculation of \( f_{psU} \) and \( \Delta f_{psU} \) in hybrid beams. In the graphs, diagonal line represents the perfect correlation between experimental and predicted values.
Table 5-1: Equations from Literature to Estimate $f_{psU}$ at Ultimate

<table>
<thead>
<tr>
<th>Eq. No</th>
<th>Investigator(s) Research Year</th>
<th>Equation for $f_{psU}$</th>
</tr>
</thead>
</table>
| 1      | Warwaruk, et al., 1962        | If $f_{pe} \leq 0.6 f_{pu}$:  
$f_{ps} = f_{pe} + (30,000 - \frac{\rho_{ps}}{f_c} \times 10^{10}) \ [psi]$ |
| 2      | Pannell, 1969                 | $f_{ps} = \left[ \frac{\left( \frac{A_{ps}}{bd_{ps} f'_c} \right) + \lambda}{1 + \frac{\lambda}{\alpha}} \right] \rho_{ps} \leq (0.85 \times f_{pu})$  
$\lambda = \frac{\psi \rho_{ps} \epsilon_{cu} E_{ps} d_{ps}}{L f'_c}$, $\omega = 10.5$, $\alpha = 0.85 \beta_i$ |
| 3      | Mattock et al., 1971          | $f_{ps} = f_{pe} + \frac{1.4 f'_c}{100 \rho_{ps}} + 10,000 \leq f_{py}$ [psi] |
| 4      | Tam and Pannell, 1976         | $f_{ps} = \left[ \frac{\left( \frac{A_{ps}}{bd_{ps} f'_c} \right) + \lambda}{1 + \frac{\lambda}{\alpha}} \right] \rho_{ps}$  
$\lambda = \frac{\psi \rho_{ps} \epsilon_{cu} E_{ps} d_{ps}}{L f'_c}$; $\omega = 10.5$; $\alpha = 0.85 \beta_i$ |
| 5      | Mojtahedi and Gamble, 1978 (Current ACI 318-08 Equation) | $f_{ps} = f_{pe} + \frac{f'_c}{\mu \rho_{ps}} + 10,000$ [psi]  
For $\frac{L}{d_{ps}} \leq 35$, $\mu = 100$ and $f_{ps} \leq f_{pe} + 60,000$  
For $\frac{L}{d_{ps}} > 35$, $\mu = 300$ and $f_{ps} \leq f_{pe} + 30,000$ |
| 6      | Balaguru, 1981                | $f_{ps} = f_{pe} + E_{ps} [\alpha_1 (\delta / e) - \alpha_2 (\delta / e)^2]$  
$\alpha_1 = 0.000923 (e / l) + 5.108 (e / l)^2$ \quad 0 < e / l < 0.06  
$\alpha_2 = 11 \times 10^{-7} \exp (135e / l)$ \quad 0 < e / l < 0.04  
$\alpha_2 = 10^{-5} \exp (75e / l)$ \quad 0.04 < e / l < 0.06 |
<table>
<thead>
<tr>
<th>Eq. No</th>
<th>Investigator(s) Research Year</th>
<th>Equation for $f_{puU}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>Du and Tao, 1985</td>
<td>$f_{pu} = f_{pe} + 114 - \frac{278.46}{bd_s f_c} (A_s f_y + A_{ps, f_{pe}}) \text{ [ksi]}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(A_s f_y + A_{ps, f_{pe}}) \leq 0.30$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$0.55 f_{py} \leq f_{pe} \leq 0.65 f_{py}; f_{ps} \leq f_{py}$</td>
</tr>
<tr>
<td>8</td>
<td>Harajli, 1990</td>
<td>$f_{ps} = f_{pe} + \left(10,000 + \frac{f_c}{100 \rho_{ps}}\right) \left(0.4 + \frac{8}{L/d_{ps}}\right)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$f_{ps} \leq f_{py}; f_{ps} \leq f_{pe} + 60,000$</td>
</tr>
<tr>
<td>9</td>
<td>Harajli and Hijazi, 1991</td>
<td>If $f_{pe} \geq (0.5 \times f_{pu})$:</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Approach I:</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$f_{ps} = f_{pe} + \gamma_s f_{pu} \left(1 - \beta_0 \frac{c}{d_{ps}}\right) \leq f_{py}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$c = \frac{\mu_p A_{ps, f_{pu}} + A_s f_y - A_s f_y}{0.85 \beta f_c b + \beta_0 \gamma_s A_{ps} (f_{pu} / d_{ps})}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\gamma_s = \left(0.1 + \frac{2.0}{(L/d_{ps})}\right) \left(\frac{n}{n_o}\right); \beta_0 = 1.80; \mu_p = \frac{f_{pe}}{f_{pu}} + \gamma_s$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Approach II:</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$f_{ps} = f_{pe} + \gamma_s f_{pu} \left(1 - \gamma_p \frac{\rho_{ps} \mu_p f_{pu}}{f_c} + \frac{d_s}{d_{ps}} (\omega_s - \omega_c)\right) \leq f_{py}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\gamma_s = \left(0.1 + \frac{2.0}{(L/d_{ps})}\right) \left(\frac{n}{n_o}\right); \gamma_p = 2.10; \mu_p = \frac{f_{pe}}{f_{pu}} + \gamma_s$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$n_o = \text{number of loaded spans}; n = \text{total number of spans}$</td>
</tr>
<tr>
<td>10</td>
<td>Harajli and Kanj, 1991</td>
<td>$f_{ps} = f_{pe} + \gamma_0 f_{pu} \left(1 - 3 \frac{A_s f_y + A_{ps, f_{pe}}}{bd_{ps} f_c}\right)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\gamma_0 = \frac{n_o}{n} \left(0.12 + \frac{2.5}{L/d_{ps}}\right) \left(\frac{A_s f_y + A_{ps, f_{pe}}}{bd_{ps} f_c}\right) \leq 0.23$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>For simply supported beams, $n_o = n$</td>
</tr>
</tbody>
</table>
Table 5-1 (continued): Equations from Literature to Estimate $f_{psU}$ at Ultimate

<table>
<thead>
<tr>
<th>Eq. No</th>
<th>Investigator(s)</th>
<th>Equation for $f_{psU}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>Naaman and Alkhairi, 1991-Part 2</td>
<td>$f_{ps} = f_{pe} + \Omega_u E_{ps} E_{cu} \left( \frac{d_{ps}}{c} - 1 \right) \frac{L_1}{L_2} \leq 0.94 f_{py}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\Omega_u = \left( \frac{L}{d_{ps}} \right)$ for one point loading</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\Omega_u = \left( \frac{L}{d_{ps}} \right)$ for two point and uniform loading</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$c = \frac{-B_1 + \sqrt{B_1^2 - 4A_1C_1}}{2A_1}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$A_1 = 0.85f_c^b w_b \beta_1$ ; $C_1 = -A_{ps} E_{ps} E_{cu} \Omega_u d_{ps} (L_1 / L_2)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$B_1 = A_{ps} E_{ps} E_{cu} \Omega_u (L_1 / L_2) - f_{pe}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$+ A_s f'_y - A_s f_y + 0.85f_c^b (b - b_w)h_f$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(L_1 / L_2) = 1$ for simply supported beams</td>
</tr>
<tr>
<td>12</td>
<td>Chakrabarti, 1995</td>
<td>$f_{ps} = \frac{f_{pe} + 10,000 + A}{(1 - B)} \leq f_{py}$ [psi]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$A = \frac{f_c'}{100 \rho_s} \times \frac{d_p}{d_s} \times \frac{60,000}{f_y} \times (1 + \rho_s / 0.025) \leq 20,000$ psi</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$B = \frac{r \times f_c'}{100 \times \rho_p \times f_{pe}} \leq 0.25$ where, $r = \begin{cases} 1.0 &amp; \text{for } l / d_{ps} \leq 33 \ 0.8 &amp; \text{for } l / d_{ps} &gt; 33 \end{cases}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>For $\rho_s = 0$ and $l / d_{ps} &gt; 33$:</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$f_{ps} \text{(modified)} = f_{pe} + 0.65 \times \Delta f_{ps}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\Delta f_{ps} = \frac{(f_{pe} \times B) + 10,000 + A}{(1 - B)}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$f_{ps} \leq f_{pe} + 60,000$ for $l / d_{ps} \leq 33$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$f_{ps} \leq f_{pe} + 40,000$ for $l / d_{ps} &gt; 33$</td>
</tr>
<tr>
<td>Eq. No</td>
<td>Investigator(s) Research Year</td>
<td>Equation for $f_{puU}$</td>
</tr>
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<td>--------</td>
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</tr>
<tr>
<td>13</td>
<td>Lee et al., 1999</td>
<td>$f_{ps} = 10,000 + 0.8 f_{pe} + \frac{1}{15} \left( A_s - A_y \right) f_y + 80 \frac{d_c f_c}{d_{ps} \rho_{ps}} \left( \frac{1}{f} \frac{1}{L / d_{ps}} \right)$ [psi] $f_{ps} + 10,000 \leq f_{ps} \leq f_{py}$ $f = \left{ \begin{array}{ll} 10 &amp; \text{for one point loading} \ 3 &amp; \text{for two point and uniform loading} \end{array} \right.$</td>
</tr>
<tr>
<td>14</td>
<td>Allouche et al. 1999-Part 2</td>
<td>$f_{ps} = f_{pe} + \frac{1160}{l_e} (d_{ps} - c_y) \left[ 1 + \left( \frac{c_y}{d_{ps}} \right)^2 \right]$ [psi] $f_{pe} + 10(ksi) \leq f_{ps} \leq f_{py}$</td>
</tr>
<tr>
<td>15</td>
<td>Au and Du, 2004</td>
<td>$f_{ps} = f_{pe} + \frac{\varphi \times \varepsilon_{cu} \times E_{ps} (d_{ps} - c_{pe})}{l_e}$ $c_{pe} = \frac{A_{ps} f_{pe} + A_y f_y}{0.85 \beta f_y b}$ $\varphi = 9.3$ $\varepsilon_{cu} = 0.003$ $l_e = \text{length of tendon / number of plastic hinges for failure}$ $l_e = L$ for simply supported beams</td>
</tr>
<tr>
<td>16</td>
<td>Roberts-Wollmann et al., 2005</td>
<td>$f_{ps} = f_{pe} + 6200 \left( \frac{d_{ps} - c_y}{l_e} \right)$ [MPa] $l_e = \frac{L}{(1 + (N/2))}$ where $N$ is the number of support hinges</td>
</tr>
<tr>
<td>17</td>
<td>Diep and Niwa, 2006</td>
<td>$f_{ps} = f_{pe} + E_{ps} \varepsilon_{cu} \left( \frac{d_{ps}}{c_y} - 1 \right) \frac{L_o}{L}$ [MPa] $c_y = \frac{0.95 A_{ps} f_{py} + A_y f_y - A_s f_y - 0.85 f_c b_w (b - b_o) h_i}{0.85 \beta f_c b_w}$ $L_o = \frac{1 + q_o}{\beta l / d_{ps}} + q_o + m(-1)^k$ and $q_o = \frac{A_{ps} f_{pe}}{bd_{ps} f_c} + \frac{A_y f_y}{bd_s f_c}$ $k = 1$ &amp; $m = 0.02$ for one point loading $k = 2$ &amp; $m = 0.05$ for two point and uniform loading</td>
</tr>
</tbody>
</table>
Table 5-1 (continued): Equations from Literature to Estimate $f_{psU}$ at Ultimate

<table>
<thead>
<tr>
<th>Eq. No</th>
<th>Investigator(s)</th>
<th>Research Year</th>
<th>Equation for $f_{psU}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>Harajli, 2006</td>
<td></td>
<td>$\varepsilon_{ps} = \varepsilon_{pe} + \varepsilon_{cu} \left( \frac{d_p - c}{L_a / n_p} \right) \left( \frac{20.7}{f} + 10.5 \right)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$n_p = 1$ for simply supported members</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$10$ for one-point loading</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$3$ for two-point loading</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$6$ for uniform loading</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$f = \begin{cases} 1 &amp; \text{for simply supported members} \ 10 &amp; \text{for one-point loading} \ 3 &amp; \text{for two-point loading} \ 6 &amp; \text{for uniform loading} \end{cases}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Alternative I: $f_{ps} = E_{ps} \varepsilon_{ps} Q + \frac{1 - Q}{\left[ 1 + \left( \frac{E_{ps} \varepsilon_{ps}}{K f_{py}} \right)^N \right]^{1/N}}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$Q = \frac{f_{pu} - K f_{py}}{E_{ps} \varepsilon_{pu} - K f_{py}}$, $c = c_y = \frac{A_{ps} f_{py} + A_s f_y}{0.85 \beta_1 f_c b}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Alternative II: $f_{ps} = f_{pe} + \phi_{ps} E_{ps} \varepsilon_{cu} \left( \frac{d_p - c_y}{L_a / n_p} \right) \left( \frac{20.7}{f} + 10.5 \right) \leq f_{py} ; \phi_{ps} = 1.0$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Alternative III: $f_{ps} = \frac{f_{pe} + K_a E_{ps} \varepsilon_{cu} \left( \frac{d_p - \rho_s d_s f_y}{0.85 \beta_f f_c} \right)}{1 + K_a E_{ps} \varepsilon_{cu} \rho_p d_p / 0.85 \beta_1 f_c}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$K_a = \phi_{ps} \left( \frac{20.7}{f} + 10.5 \right) \frac{n_p}{L_a}$</td>
</tr>
</tbody>
</table>
Table 5-1 (continued): Equations from Literature to Estimate \( f_{psU} \) at Ultimate

<table>
<thead>
<tr>
<th>Eq. No</th>
<th>Investigator(s)</th>
<th>Research Year</th>
<th>Equation for ( f_{psU} )</th>
</tr>
</thead>
</table>
| 19     | Ozkul et al., 2008 | \[
f_{ps} = f_{pe} + \frac{E_{ps}}{196} \left( e \beta f_c b \right) k_i \leq f_{py}
\]
|        |                  | \[
k_i = \left[ 1 - 2 \frac{L_h}{L} \frac{L_p}{L} \right]
\] | \[
L_h = \frac{L}{2} - \frac{L}{2f} - (0.5d + 0.05Z)
\]
|        |                  | \[f = \begin{cases} 
3 & \text{for two-point loading} \\
6 & \text{for uniform loading}
\end{cases}\] | |
| 20     | Du and Au, 2009  | \[
f_{ps} = f_{pe} + 6000 \left( \frac{d_p - c}{l_e} \right) \leq (f_{py})_{steel} \text{ or } (f_{pu})_{FRP} \ [\text{MPa}]
\]
|        |                  | \[
l_e = \left( \frac{L}{1 + \frac{N}{2}} \right)\] | for simply supported beams, \( l_e = L \) |
| 21     | He and Liu, 2010 | \[
f_{ps,u} = f_{pe} + \Delta f_{ps,u} \leq 0.8 f_{pu}
\]
|        |                  | \[
\Delta f_{ps,u} = \kappa E_{ps} \frac{e_m}{c} R_s \frac{L}{L_2}
\]
|        |                  | \[
R_s = 1 \quad \text{for internally pretressed beams}
\] \[
(L_1 / L_2) = 1 \quad \text{for simply supported beams}
\] | \[
c = \frac{0.8 A_{ps} f_{pu} + A_s f_y - A_s f_y - 0.85 f_c b \beta (b - b_w) h_j}{0.85 f_c b h_w}
\]
|        |                  | \[
\kappa = \begin{cases} 
1.62 \times 10^{-3} [0.78 + 2.35 / (L / d_{ps})] \text{ uniform or } \frac{2}{3} \text{ p.loads} \\
1.26 \times 10^{-3} [0.3 + 3 / (L / d_{ps})] \text{ one-point loading}
\end{cases}
\]
| 22     | Zheng and Wang, 2010 | \[
f_{ps} = f_{pe} + \Delta f_{ps} \ [\text{MPa}]
\]
|        |                  | \[
\Delta f_{ps} = (560 - 1449 \beta_p - 837 \beta_s)(0.86 + 2.4h / L) \text{ for one point}
\] \[
\Delta f_{ps} = 663 - 1137 \beta_p - 703 \beta_s \text{ for third point}
\] \[
\Delta f_{ps} = 631 - 1144 \beta_p - 735 \beta_s \text{ for uniform}
\] | \[
\beta = \frac{f_{pe} A_{ps}}{f_c b d_{ps}}, \quad \beta_s = \frac{f_s A_s}{f_c b d_s}
\] |
Table 5-1 (continued): Equations from Literature to Estimate $f_{psU}$ at Ultimate

<table>
<thead>
<tr>
<th>Eq. No</th>
<th>Investigator(s) Research Year</th>
<th>Equation for $f_{psU}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>23</td>
<td>Yang and Kang, 2011</td>
<td>$f_{ps} = \frac{-B_1 + \sqrt{B_1^2 - 4A_1C_1}}{2A_1} \leq f_{py}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$A_1 = A_p$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$B_1 = A_p f_y - A_p f_y - C_f - A_p f_{pe} + \alpha_f E_p \varepsilon_{cu} A_p$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$C_1 = \alpha_f E_p \varepsilon_{cu} (A_p f_y - A_p f_y - C_f - 0.85 f_y b_c \beta\delta_p)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$C_f = 0$ for rectangular beam</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\alpha_i = 0.016 \xi \left( \frac{\omega_{pe} + \sqrt{(\omega_i - \omega_f)}}{L / d_p} \right)^{-0.46}$ one-point loading</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\alpha_f = \frac{2\alpha_2 L_{cr} + L_0}{2L_{cr} + L_0}$ two-point loading</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\alpha_u = 0.12 \xi \left( \frac{\omega_{pe} + \sqrt{(\omega_u - \omega_f)}}{L / d_p} \right)^{-0.21}$ uniform loading</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\xi = \begin{cases} 1 &amp; \text{straight tendon profile} \ 1.05 &amp; \text{harped tendon profile} \ 1.12 &amp; \text{draped tendon profile} \end{cases}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$L_0 = \text{constant moment region length}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$L_{cr} = \text{length between the cracked region end}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>and the applied loading point</td>
</tr>
</tbody>
</table>
Figure 5-16: Comparison of Equation by Warwaruk et al. (1962) for $f_{ptU}$ with Beams Including (a) Unbonded Tendons (b) Unbonded and Bonded Tendons

Figure 5-17: Comparison of Equation by Warwaruk et al. (1962) for $\Delta f_{ptU}$ with Beams Including (c) Unbonded Tendons (d) Unbonded and Bonded Tendons
Figure 5-18: Comparison of Equation by Warwaruk et al. (1962) for (e) $f_{pu,U}$ and (f) $\Delta f_{pu,U}$ with All Beams

Figure 5-19: Comparison of Equation by Pannell (1969) for $f_{pu,U}$ with Beams Including (a) Unbonded Tendons (b) Unbonded and Bonded Tendons
Figure 5-20: Comparison of Equation by Pannell (1969) for $\Delta f_{au}$ with Beams Including (c) Unbonded Tendons (d) Unbonded and Bonded Tendons

Figure 5-21: Comparison of Equation by Pannell (1969) for (e) $f_{pu}$ and (f) $\Delta f_{pu}$ with All Beams
Figure 5-22: Comparison of Equation by Mattock et al. (1971) for $f_{ptU}$ with Beams Including (a) Unbonded Tendons (b) Unbonded and Bonded Tendons

Figure 5-23: Comparison of Equation by Mattock et al. (1971) for $\Delta f_{ptU}$ with Beams Including (c) Unbonded Tendons (d) Unbonded and Bonded Tendons
Figure 5-24: Comparison of Equation by Mattock et al. (1971) for (e) $f_{ptU}$ and (f) $\Delta f_{ptU}$, with All Beams

$y = 8.12 + 0.91x$
$R = 0.93$

$y = 8.72 + 0.63x$
$R = 0.75$

Figure 5-25: Comparison of Equation by Tam and Pannell (1976) for $f_{ptU}$ with Beams Including (a) Unbonded Tendons (b) Unbonded and Bonded Tendons

$y = 50.97 + 0.66x$
$R = 0.9$

$y = 6.09 + 0.87x$
$R = 0.9$
Figure 5-26: Comparison of Equation by Tam and Pannell (1976) for $\Delta f_{pu}$ with Beams Including (c) Unbonded Tendons (d) Unbonded and Bonded Tendons

Figure 5-27: Comparison of Equation by Tam and Pannell (1976) for (e) $f_{pu}$ and (f) $\Delta f_{pu}$ with All Beams
Figure 5-28: Comparison of Equation by Du and Tao (1985) for $f_{pu}$ with Beams Including (a) Unbonded Tendons (b) Unbonded and Bonded Tendons

Figure 5-29: Comparison of Equation by Du and Tao (1985) for $\Delta f_{pu}$ with Beams Including (c) Unbonded Tendons (d) Unbonded and Bonded Tendons
Figure 5-30: Comparison of Equation by Du and Tao (1985) for (e) $f_{pu}$ and (f) $\Delta f_{pu}$ with All Beams

Figure 5-31: Comparison of Equation by Harajli (1990) for $f_{pu}$ with Beams Including (a) Unbonded Tendons (b) Unbonded and Bonded Tendons
Figure 5-32: Comparison of Equation by Harajli (1990) for $\Delta f_{psU}$ with Beams Including (c) Unbonded Tendons (d) Unbonded and Bonded Tendons

Figure 5-33: Comparison of Equation by Harajli (1990) for (e) $f_{psU}$ and (f) $\Delta f_{psU}$ with All Beams
Figure 5-34: Comparison of Equation by Harajli and Hijazi, Approach I (1991) for $f_{pu}$ with Beams Including (a) Unbonded Tendons (b) Unbonded and Bonded Tendons

Figure 5-35: Comparison of Equation by Harajli and Hijazi, Approach I (1991) for $\Delta f_{pu}$ with Beams Including (c) Unbonded Tendons (d) Unbonded and Bonded Tendons
Figure 5-36: Comparison of Equation by Harajli and Hijazi, Approach I (1991) for (e) $f_{puU}$ and (f) $\Delta f_{puU}$ with All Beams

Figure 5-37: Comparison of Equation by Harajli and Hijazi, Approach II (1991) for $f_{puU}$ with Beams Including (a) Unbonded Tendons (b) Unbonded and Bonded Tendons
Figure 5-38: Comparison of Equation by Harajli and Hijazi, Approach II (1991) for $\Delta f_{psU}$ with Beams Including (c) Unbonded Tendons (d) Unbonded and Bonded Tendons

Figure 5-39: Comparison of Equation by Harajli and Hijazi, Approach II (1991) for (e) $f_{psU}$ and (f) $\Delta f_{psU}$ with All Beams
Figure 5-40: Comparison of Equation by Harajli and Kanj (1991) for $f_{pu}$ with Beams Including (a) Unbonded Tendons (b) Unbonded and Bonded Tendons

Figure 5-41: Comparison of Equation by Harajli and Kanj (1991) for $\Delta f_{pu}$ with Beams Including (c) Unbonded Tendons (d) Unbonded and Bonded Tendons
Figure 5-42: Comparison of Equation by Harajli and Kanj (1991) for (e) $f_{ptU}$ and (f) $\Delta f_{ptU}$ with All Beams

Figure 5-43: Comparison of Equation by Naaman et al. (1991) for $f_{ptU}$ with Beams Including (a) Unbonded Tendons (b) Unbonded and Bonded Tendons
Figure 5-44: Comparison of Equation by Naaman et al. (1991) for $\Delta f_{ptu}$ with Beams Including (c) Unbonded Tendons (d) Unbonded and Bonded Tendons

Figure 5-45: Comparison of Equation by Naaman et al. (1991) for (e) $f_{ptu}$ and (f) $\Delta f_{ptu}$ with All Beams
Figure 5-46: Comparison of Equation by Chakrabarti (1995) for $f_{psU}$ with Beams Including (a) Unbonded Tendons (b) Unbonded and Bonded Tendons

Figure 5-47: Comparison of Equation by Chakrabarti (1995) for $\Delta f_{psU}$ with Beams Including (c) Unbonded Tendons (d) Unbonded and Bonded Tendons
Figure 5-48: Comparison of Equation by Chakrabarti (1995) for (e) $f_{psU}$ and (f) $\Delta f_{psU}$ with All Beams

Figure 5-49: Comparison of Equation by Lee et al. (1999) for $f_{psU}$ with Beams Including (a) Unbonded Tendons (b) Unbonded and Bonded Tendons
Figure 5-50: Comparison of Equation by Lee et al. (1999) for $\Delta f_{psU}$ with Beams Including (c) Unbonded Tendons (d) Unbonded and Bonded Tendons

Figure 5-51: Comparison of Equation by Lee et al. (1999) for (e) $f_{psU}$ and (f) $\Delta f_{psU}$ with All Beams
Figure 5-52: Comparison of Equation by Allouche et al. (1999) for (a) $f_{puU}$ and (b) $\Delta f_{puU}$ with Beams Including Unbonded Tendons

Figure 5-53: Comparison of Equation by Au and Du (2004) for $f_{puU}$ with Beams Including (a) Unbonded Tendons (b) Unbonded and Bonded Tendons
Figure 5-54: Comparison of Equation by Au and Du (2004) for $\Delta f_{pu}$ with Beams Including (c) Unbonded Tendons (d) Unbonded and Bonded Tendons

Figure 5-55: Comparison of Equation by Au and Du (2004) for (e) $f_{pu}$ and (f) $\Delta f_{pu}$ with All Beams
Figure 5-56: Comparison of Equation by Roberts-Wollmann et al. (2005) for (a) $f_{pU}$ and (b) $\Delta f_{pU}$ with Beams Including Unbonded Tendons

Figure 5-57: Comparison of Equation by Diep and Niwa (2006) for (a) $f_{pU}$ and (b) $\Delta f_{pU}$ with Beams Including Unbonded Tendons
Figure 5-58: Comparison of Equation by Harajli, Alternative I (2006) for (a) $f_{psU}$ and (b) $\Delta f_{psU}$ with Beams Including Unbonded Tendons

Figure 5-59: Comparison of Equation by Harajli, Alternative II (2006) for (a) $f_{psU}$ and (b) $\Delta f_{psU}$ with Beams Including Unbonded Tendons
Figure 5-60: Comparison of Equation by Harajli, Alternative III (2006) for $f_{pmU}$ with Beams Including (a) Unbonded Tendons (b) Unbonded and Bonded Tendons

Figure 5-61: Comparison of Equation by Harajli, Alternative III (2006) for $\Delta f_{pmU}$ with Beams Including (c) Unbonded Tendons (d) Unbonded and Bonded Tendons
Figure 5-62: Comparison of Equation by Harajli, Alternative III (2006) for (e) $f_{pu}$ and (f) $\Delta f_{pu}$ with All Beams

Figure 5-63: Comparison of Equation by Ozkul et al. (2008) for $f_{pu}$ with Beams Including (a) Unbonded Tendons (b) Unbonded and Bonded Tendons
Figure 5-64: Comparison of Equation by Ozkul et al. (2008) for $\Delta f_{ptU}$ with Beams Including (c) Unbonded Tendons (d) Unbonded and Bonded Tendons

Figure 5-65: Comparison of Equation by Ozkul et al. (2008) for (e) $f_{ptU}$ and (f) $\Delta f_{ptU}$ with All Beams
Figure 5-66: Comparison of Equation by Du and Au (2009) for $f_{pu}$ with Beams Including (a) Unbonded Tendons (b) Unbonded and Bonded Tendons

Figure 5-67: Comparison of Equation by Du and Au (2009) for $\Delta f_{pu}$ with Beams Including (c) Unbonded Tendons (d) Unbonded and Bonded Tendons
Figure 5-68: Comparison of Equation by Du and Au (2009) for (e) $f_{p_{TU}}$ and (f) $\Delta f_{p_{TU}}$ with All Beams

Figure 5-69: Comparison of Equation by He and Liu (2010) for $f_{p_{TU}}$ with Beams Including (a) Unbonded Tendons (b) Unbonded and Bonded Tendons
Figure 5-70: Comparison of Equation by He and Liu (2010) for $\Delta f_{pu}$ with Beams Including (c) Unbonded Tendons (d) Unbonded and Bonded Tendons

Figure 5-71: Comparison of Equation by He and Liu (2010) for (e) $f_{pu}$ and (f) $\Delta f_{pu}$ with All Beams
Figure 5-72: Comparison of Equation by Zheng et al. (2010) for $f_{psU}$ with Beams Including (a) Unbonded Tendons (b) Unbonded and Bonded Tendons

Figure 5-73: Comparison of Equation by Zheng et al. (2010) for $\Delta f_{psU}$ with Beams Including (c) Unbonded Tendons (d) Unbonded and Bonded Tendons
Figure 5-74: Comparison of Equation by Zheng et al. (2010) for (e) $f_{psU}$ and (f) $\Delta f_{psU}$ with All Beams

Figure 5-75: Comparison of Equation by Yang et al. (2011) for $f_{psU}$ with Beams Including (a) Unbonded Tendons (b) Unbonded and Bonded Tendons
Figure 5-76: Comparison of Equation by Yang et al. (2011) for $\Delta f_{psU}$ with Beams Including (c) Unbonded Tendons (d) Unbonded and Bonded Tendons

Figure 5-77: Comparison of Equation by Yang et al. (2011) for (e) $f_{psU}$ and (f) $\Delta f_{psU}$ with All Beams
As can be seen from Figure 5-18, Warwaruk et al. (1962) underestimates $f_{pU}$ and $\Delta f_{pU}$ for all kinds of beams. In other words, the equation is conservative. This equation is improved by Mattock et al. (1971) and it shows better accuracy for $f_{pU}$ and $\Delta f_{pU}$, (especially for $\Delta f_{pU}$) Figure 5-24.

Tam and Pannell (1976) considered the effect of non-prestressed reinforcement in their equation and by that way they improved the equation of Pannell (1976). However, from Figure 5-21 and Figure 5-27, it is observed that the accuracies of the equation of Pannell (1969) and Tam and Pannell (1976) are very close. For both equations, comparison of $f_{pU}$ is reasonable. However, most values of $\Delta f_{pU}$ are under the perfect correlation line which makes the equations conservative.

From Figure 5-28 to Figure 5-30, it is seen that the prediction equation of Du and Tao (1985) overestimates the values of $f_{pU}$ and $\Delta f_{pU}$ for both beams with only unbonded tendons, and hybrid tendons. Overestimation is not a wanted situation for a design equation because it will cause unsafe designs. The accuracy of the equation decreases in a significant amount especially for the calculation of $\Delta f_{pU}$ with beams including hybrid tendons, Figure 5-29(d).

The equation developed by Harajli (1990) shows good correlation for the calculation of $f_{pU}$ which is observed from Figure 5-31 to Figure 5-33. However, it underestimates the results for $\Delta f_{pU}$ for all kinds of beams. Harajli and Hijazi (1991) proposed two approaches in their research. Approach II shows better accuracy than Approach I when Figure 5-36 and Figure 5-39 are compared. However, still both of them do not show good accuracy for $\Delta f_{pU}$. From Figure 5-40 to Figure 5-42, it is observed that
the equation by Harajli and Kanj (1991) estimates $f_{puU}$ well. However, the values of $\Delta f_{puU}$ have a wide spread. In addition, when Figure 5-41(c) and Figure 5-41(d) are compared, it is shown that the accuracy of the equation drops abruptly for the calculation of $\Delta f_{puU}$ with the beams including hybrid tendons. This statement proves that the equation is not appropriate for the beams with unbonded and bonded (hybrid) tendons.

The equation by Naaman and Alkhairi (1991) gives good accuracy for $f_{puU}$ and $\Delta f_{puU}$ with beams including unbonded tendons, Figure 5-43(a) and Figure 5-44(c). However, the accuracy of the equation decreases for the case of the beams with hybrid tendons, Figure 5-43(b) and Figure 5-44(d). Actually, this observation does not only show that this equation is not applicable to beams with hybrid tendons. Another important point is that this equation cannot be used for FRP tendons since the beams with hybrid tendons include unbonded FRP tendons. As can be seen from Figure 5-43(b), some values of $f_{puU}$ are very high for the beams with hybrid tendons including FRP tendons and these high values are not acceptable. Because these values are even higher than ultimate strength of FRP tendons, $f_{puU}$.

The equation proposed by Chakrabarti (1995) show reasonable accuracy for $f_{puU}$, Figure 5-46(a) and (b). However, it has a wide disperse of data for $\Delta f_{puU}$, Figure 5-47(c) and (d). As a result, the equation shows good correlation for $f_{puU}$, Figure 5-48(e); however it underestimates the values of $\Delta f_{puU}$, Figure 5-48(f). As can be observed from Figure 5-51(e) and (f), the equation by Lee et al. (1999) demonstrates good correlation
for both $f_{psU}$ and $\Delta f_{psU}$. However, when Figure 5-50(d) is examined, it is observed that the accuracy of the equation is low for the calculation of $\Delta f_{psU}$ with hybrid tendons.

The equation proposed by Allouche et al. (1999) and Roberts-Wollmann et al. (2005) show reasonable comparison for $f_{psU}$, Figure 5-52(a) and Figure 5-56(a). However, they demonstrate wide scatter for prediction of $\Delta f_{psU}$, Figure 5-52(b) and Figure 5-56(b). In addition, these two equations are not applicable to the beams with unbonded FRP tendon which is a drawback. The accuracy of the equation developed by Au and Du (2004) is good for the calculation of $f_{psU}$, Figure 5-55(e). On the other hand, it underestimates the values of $\Delta f_{psU}$, Figure 5-55(f).

As can be seen from Figure 5-57(a) and (b), the equation by Diep and Niwa (2006) shows good correlation for $f_{psU}$ and underestimation for $\Delta f_{psU}$. In addition, this equation cannot be used for beams including unbonded FRP tendons. From Figure 5-58, Figure 5-59 and Figure 5-62, it is observed that the estimation of Alternative I, II and III by Harajli (2006) for $f_{psU}$ is good. However, all of these equations underestimate the values of $\Delta f_{psU}$. In addition, Alternative I and II are not applicable to beams including FRP tendons.

Figure 5-63(a) and Figure 5-64(c) show that the equation by Ozkul et al. (2008) predicts both $f_{psU}$ and $\Delta f_{psU}$ well for beams including only unbonded tendons. When the beams with hybrid tendons are taken into account, the accuracy of the equation decreases, Figure 5-63(b) and Figure 5-64(d). In addition, as can be seen from Figure 5-63(b), some values of $f_{psU}$ are very high because there is no limit of the equation for FRP tendons.
The equation by Du and Au (2009) shows good correlation for the calculation of $f_{pu}$, Figure 5-68(e). However, it demonstrates wide disperse of data for the calculation of $\Delta f_{pu}$ which is not good, Figure 5-68(f). From Figure 5-71(e) and (f), it can be said that He and Liu (2010) equation predicts $f_{pu}$ well and it underestimates $\Delta f_{pu}$.

The equation developed by Zheng and Wang (2010) shows good correlation for the calculation of $f_{pu}$ with all beams, Figure 5-74(e). However, since the data points are above the perfect correlation line, it means that the equation overestimates the results. When a design equation overestimates the results, it will end up with unreliable and unsafe designs. Therefore, this is not acceptable. In addition, the equation overestimates the results for $\Delta f_{pu}$ in a significant amount, Figure 5-74(f).

As can be seen from Figure 5-75(a) and Figure 5-76(c), it is easily observed that the equation by Yang and Kang (2011) show good correlation for both $f_{pu}$ and $\Delta f_{pu}$ with beams including unbonded tendons. However, this accuracy isn’t protected for the case of the beams with unbonded and bonded tendons. The accuracy of the equation decreases abruptly for beams with hybrid tendons for both $f_{pu}$ and $\Delta f_{pu}$, Figure 5-75(b) and Figure 5-76(d). As a result, the accuracy of the equation is not good when all beams are considered, Figure 5-77. In addition, when Figure 5-75(b) is examined, it is seen that some values of $f_{pu}$ are very high. This observation proves that the equation is not applicable to FRP tendons since there is no limit put for them in the equation (Remember that beams with hybrid tendons include unbonded FRP tendons). Moreover, the equation is too complicated and long. Therefore, it is not appropriate to use this equation for design purposes.
5.3.3 Results for Code Equations

Design equations are recommended by different codes to predict the unbonded tendon strength at ultimate. These code equations were summarized in Chapter 2, Literature Review. All of these equations are only valid for the beams prestressed with unbonded tendons. However, there is no available code equation that can be used for the beams with unbonded and bonded (hybrid) tendons. Some of the design code equations are summarized in Table 5-2. As can be seen from this table, 6 code equations are reviewed. Please note that, ACI 318-08 Code equation is used for the beams prestressed with unbonded steel tendons; while ACI 440-04 Code equation is used for the beams prestressed with unbonded FRP tendons. In the same manner as implemented in previous sections (5.3.1 and 5.3.2), the accuracies of the equations are examined by comparing the predicted values with the experimental values. For that purpose, the predicted values of $f_{pu}$ and $\Delta f_{pu}$ are compared with the experimental ones for the beams with only unbonded tendons (179-all of them are steel), for the beams with hybrid tendons (20-1 of them is steel, rest of the beams are FRP) and for all beams (199). As a result, six graphs are formed for each equation that is shown following Table 5-2, from Figure 5-78 to Figure 5-91. If the code equations do not include any term related to bonded tendons or the studies do not specifically mention anything related to beams with unbonded and bonded tendons, the bonded tendons are neglected for the calculation of $f_{pu}$ and $\Delta f_{pu}$ in hybrid beams. In the graphs, diagonal line represents the perfect correlation between experimental and predicted values.
### Table 5-2: Code Equations to Estimate $f_{psU}$ at Ultimate

<table>
<thead>
<tr>
<th>Eq. No</th>
<th>Code, Year</th>
<th>Design Equation for $f_{psU}$</th>
</tr>
</thead>
</table>
| 1      | ACI 318-08, 2008 | $rac{L}{d_{ps}} \leq 35: f_{ps} = f_{pe} + \frac{f'_c}{100 \rho_{ps}} + 10,000 \leq f_{ps} + 60,000 [psi]$  
          |             | $f_{ps} \leq f_{py}$          |
|        |             | $\frac{L}{d_{ps}} > 35: f_{ps} = f_{pe} + \frac{f'_c}{300 \rho_{ps}} + 10,000 \leq f_{pe} + 30,000 [psi]$ |
| 2      | ACI 440-04, 2004 | $f_{ps} = f_{pe} + \Omega_u E_p' e_{cu} \left( \frac{d_p}{c} - 1 \right)$  
          |               | $\Omega_u = \begin{cases} 
1.5 \left( \frac{L}{d_p} \right) & \text{for one point loading} 
\end{cases}$ |
|        |             | $c = \frac{-B_1 + \sqrt{B_1^2 - 4A_1C_1}}{2A_1}$  
          |               | $A_1 = 0.85 f'_c b_u \beta_1$  
          |               | $C_1 = -A_p E_p e_{cu} \Omega_u d_{ps} (L_1/L_2)$  
          |               | $B_1 = A_p (E_p e_{cu} \Omega_u (L_1/L_2) - f_{pe}) + A_s f'_y - A_s f_y$  
          |               | $+ 0.85 f'_c (b - b_u) h_f$ |
|        | For Beams Prestressed with Unbonded FRP Tendons | For beams with external or internal\&external prestressing, replace $d_p$ by $d_e$ and use following $\Omega_u$ :  
          |               | $d_e = R_d d_p$  
          |               | $R_d = \begin{cases} 
1.14 - 0.005 \left( \frac{L}{d_p} \right) - 0.19 \left( \frac{S_p}{L} \right) & \leq 1.0 \ \text{1pt loading} 
1.25 - 0.010 \left( \frac{L}{d_p} \right) - 0.38 \left( \frac{S_p}{L} \right) & \leq 1.0 \ \text{2pt loading} 
\end{cases}$  
          |               | $\Omega_u = \begin{cases} 
0.21 \left( \frac{A_{pint}}{A_{pot}} \right) + 0.04 & \text{1pt loading} 
2.31 \left( \frac{A_{pint}}{A_{pot}} \right) + 0.06 & \text{2pt loading} 
\end{cases}$ |
Table 5-2 (continued): Code Equations to Estimate $f_{psU}$ at Ultimate

<table>
<thead>
<tr>
<th>Eq. No</th>
<th>Code, Year</th>
<th>Design Equation for $f_{psU}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>AASHTO-LRFD, 2010</td>
<td>$f_{ps} = f_{pe} + 900 \left( \frac{d_p - c}{l_e} \right) \leq f_{py}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$l_e = \left( \frac{2l_i}{2 + N_s^l} \right)$, for simply supported beams $l_e = l_i$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$c = \frac{A_{ps} f_{ps} + A_s f_y - A_e^i f_y - 0.85 f_c^i \beta_i (b - b_w) h_f}{0.85 f_c^i \beta_l b_w}$</td>
</tr>
<tr>
<td>4</td>
<td>CSA Canadian Code, 2006</td>
<td>$f_{ps} = f_{pe}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>unless a detailed analysis accounting for deformations is used.</td>
</tr>
<tr>
<td>5</td>
<td>BS 8110 British Code, 2001</td>
<td>$f_{ps} = f_{pe} + \left( \frac{7000}{L} \left( 1 - \frac{1.7 f_{ps} A_{ps}}{f_{cu} d_{ps}} \right) \right) \leq 0.7 f_{pu} \ [MPa]$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$f_{cu} = \frac{f_c}{0.8}$</td>
</tr>
<tr>
<td>6</td>
<td>Eurocode, 2004</td>
<td>$f_{ps} = f_{pe} + 100 \ [MPa]$</td>
</tr>
</tbody>
</table>
Figure 5-78: Comparison of Design Equation in ACI 318-08 (2008) or ACI 440-04 (2004) for \( f_{psU} \) with Beams Including (a) Unbonded Tendons (b) Unbonded and Bonded Tendons

Figure 5-79: Comparison of Design Equation in ACI 318-08 (2008) or ACI 440-04 (2004) for \( \Delta f_{psU} \) with Beams Including (c) Unbonded Tendons (d) Unbonded and Bonded Tendons
Figure 5-80: Comparison of Design Equation in ACI 318-08 (2008) or ACI 440-04 (2004) for (e) $f_{pu}$ and (f) $\Delta f_{pu}$ with All Beams

$y = 11.91 + 0.52x$
$R = 0.57$

Figure 5-81: Comparison of Design Equation in AASHTO-LRFD (2010) for $f_{pu}$ with Beams Including (a) Unbonded Tendons (b) Unbonded and Bonded Tendons

$y = 51.92 + 0.73x$
$R = 0.83$

$y = 40.83 + 0.74x$
$R = 0.78$

$y = 47.63 + 0.67x$
$R = 0.91$

$y = 41.92 + 0.73x$
$R = 0.83$
Figure 5-82: Comparison of Design Equation in AASHTO-LRFD (2010) for $\Delta f_{psU}$ with Beams Including (c) Unbonded Tendons (d) Unbonded and Bonded Tendons

Figure 5-83: Comparison of Design Equation in AASHTO-LRFD (2010) for (e) $f_{psU}$ and (f) $\Delta f_{psU}$ with All Beams
Figure 5-84: Comparison of Design Equation in Canadian Code (2006) for $f_{pu}$ with Beams Including (a) Unbonded Tendons (b) Unbonded and Bonded Tendons

Figure 5-85: Comparison of Design Equation in Canadian Code (2006) for (c) $f_{pu}$ and (d) $\Delta f_{pu}$ with All Beams
Figure 5-86: Comparison of Design Equation in British Code (2001) for $f_{pct}$ with Beams Including (a) Unbonded Tendons (b) Unbonded and Bonded Tendons

Figure 5-87: Comparison of Design Equation in British Code (2001) for $\Delta f_{pct}$ with Beams Including (c) Unbonded Tendons (d) Unbonded and Bonded Tendons
Figure 5-88: Comparison of Design Equation in British Code (2001) for (e) $f_{puU}$ and (f) $\Delta f_{puU}$ with All Beams

Figure 5-89: Comparison of Design Equation in Eurocode (2004) for $f_{puU}$ with Beams Including (a) Unbonded Tendons (b) Unbonded and Bonded Tendons
Figure 5-90: Comparison of Design Equation in Eurocode (2004) for $\Delta f_{ptU}$ with Beams Including (c) Unbonded Tendons (d) Unbonded and Bonded Tendons

Figure 5-91: Comparison of Design Equation in Eurocode (2004) for (e) $f_{ptU}$ and (f) $\Delta f_{ptU}$ with All Beams
In the experimental data set, 179 beams are prestressed with only unbonded tendons and all of these tendons are steel. Therefore, for these beams, ACI 318-08 (2008) code equation is used. The rest 20 beams are prestressed with hybrid tendons and 19 of these beams include unbonded FRP tendons. Therefore, for these 19 beams, ACI 440-04 (2004) code equation which is applicable to FRP tendons is used. As can be seen in Figure 5-78 (a) and Figure 5-79 (c), ACI 318-08 (2008) underestimates $f_{pu}$ and $\Delta f_{pu}$ for the beams with only unbonded tendons. Actually, it is an expected result since code equations are generally conservative. However, ACI 440-04 (2004) equation overestimates $f_{pu}$ and $\Delta f_{pu}$, Figure 5-78 (b) and Figure 5-79 (d). This is not acceptable for a code equation, because a code equation should always give conservative results. From Figure 5-81 to Figure 5-83, it is observed that AASHTO-LRFD Equation (2010) also underestimates $f_{pu}$ and $\Delta f_{pu}$ for all kinds of beams. However, it shows better correlation for the calculation of $f_{pu}$ when compared with ACI code equation. On the other hand, same thing cannot be said for $\Delta f_{pu}$. Another important point is that the accuracies of both equations decrease for the beams with hybrid tendons when compared with the case of unbonded tendons.

Canadian Code Equation (2006) underestimates $f_{pu}$, Figure 5-85(c). On the other hand, $\Delta f_{pu}$ values are all zero, which is not possible in practical life, Figure 5-85(d). British Code Equation (2001) predicts $f_{pu}$ well as can be seen in Figure 5-86. However, it underestimates the results of $\Delta f_{pu}$ in a significant amount, especially for the beams with unbonded and bonded tendons, Figure 5-87. As a result, while it shows good
correlation for $f_{psU}$ with all beams, Figure 5-88(e) and it gives wide disperse of results for $\Delta f_{psU}$ with all beams, Figure 5-88(f).

Eurocode (2004) underestimates $f_{psU}$ as can be seen in Figure 5-91(e). However, the value of $\Delta f_{psU}$ is constant which is not proper, Figure 5-91(f). Therefore, Eurocode should include an equation for $\Delta f_{psU}$ based on calculation instead of recommendation.

## 5.4 Summary of Results

It was shown that both GIA and FEA are accurate and reliable methods to predict the overall response of members prestressed with unbonded and bonded tendons. Then, the results of the proposed equations in Alternative I, II and III, GIA (Ozkul et al. 2008), various prediction equations from literature and code equations are compared with experimental results to check the accuracy of each equation. Now, for the comparisons of proposed equations in Alternative I and II with the various prediction equations and proposed equation in Alternative III with code equations, Table 5-3 is prepared to represent the correlation factor $R$ of each equation found in the previous section, Section 5.3. The correlation factor which is a statistical quantity is a good indicator of accuracy of an equation. As the correlation factor increases, as expected the accuracy of the prediction equation also increases. The correlation factors are presented for beams with only unbonded tendons (179), with hybrid (20) tendons and all beams (199). In Table 5-3, some of the equations are not applicable to beams including hybrid tendons. These cases are mentioned with ‘N/A’. In the table, the column showing $R$ values for all beams are put in red color. Because this column is the most important one since the design
equation needs to be accurate for all kind of beams. In Table 5-3, the equations are categorized as the available equations in literature and code equations. Available equations in literature including proposed equations in Alternatives in I, II and code equations and proposed equation in Alternative III (developed for code purposes) are ranked separately based on accuracy. While doing this ranking, the accuracy of both $f_{psU}$ and $\Delta f_{psU}$ are taken into account. The graphs, Figure 5-16 to Figure 5-91, are plotted by ignoring the bonded tendons for the calculation of $f_{psU}$ and $\Delta f_{psU}$ in hybrid beams, if the equations do not include any term related to bonded tendons. Table 5-3 is prepared based on these graphs and this assumption. For these equations that do not include any term related to bonded tendons, Table 5-4 is prepared by including the term area of bonded tendon into the area of non-prestressed reinforcement. In Table 5-4 as done for Table 5-3, the available equations in literature including proposed equations in Alternative I, II and code equations and proposed equation in Alternative III are ranked separately based on accuracy. It is observed from Table 5-3 and Table 5-4 that the accuracy of proposed equation in Alternative I is the highest among other ones and Alternative II is following it. In the same manner, the proposed Equation in Alternative III (developed for code purposes) has the highest accuracy among other code equations. It should be noted that the correlation factor is not the only criteria for an equation to be acceptable. Other important issues are the simplicity and the basis of the equation. Both proposed equations are based on mathematical concepts and simple enough for design purposes. They are applicable to beams including unbonded or hybrid tendons and also steel or FRP tendons. All these advantages and results make the proposed equations in Alternative I, II and III acceptable and rational equations.
### Table 5-3: Summary of Correlation Factors for Different Equations

<table>
<thead>
<tr>
<th>No</th>
<th>Investigator(s) or Code</th>
<th>Year</th>
<th>$R$ for Unbonded Beams (179)</th>
<th>$R$ for Hybrid Beams (20)</th>
<th>$R$ for All Beams (199)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$f_{p,U}$</td>
<td>$\Delta f_{p,U}$</td>
<td>$f_{p,U}$</td>
</tr>
<tr>
<td>1</td>
<td>Alt. I- Eq.(3.33)</td>
<td>2011</td>
<td>0.96</td>
<td>0.87</td>
<td>0.96</td>
</tr>
<tr>
<td>2</td>
<td>Alt. II- Eq.(3.35)</td>
<td>2011</td>
<td>0.96</td>
<td>0.87</td>
<td>0.96</td>
</tr>
<tr>
<td>3</td>
<td>GIA (Ozkul et al.)</td>
<td>2008</td>
<td>0.95</td>
<td>0.86</td>
<td>0.98</td>
</tr>
<tr>
<td>4</td>
<td>Ozkul et al.</td>
<td>2008</td>
<td>0.96</td>
<td>0.86</td>
<td>0.88</td>
</tr>
<tr>
<td>5</td>
<td>Lee et al.</td>
<td>1999</td>
<td>0.96</td>
<td>0.85</td>
<td>0.86</td>
</tr>
<tr>
<td>6</td>
<td>He and Liu</td>
<td>2010</td>
<td>0.95</td>
<td>0.83</td>
<td>0.85</td>
</tr>
<tr>
<td>7</td>
<td>Harajli, Alternative II</td>
<td>2006</td>
<td>0.93</td>
<td>0.77</td>
<td>N/A</td>
</tr>
<tr>
<td>8</td>
<td>Mattock et al.</td>
<td>1971</td>
<td>0.94</td>
<td>0.79</td>
<td>0.87</td>
</tr>
<tr>
<td>9</td>
<td>Harajli, Alternative I</td>
<td>2006</td>
<td>0.93</td>
<td>0.74</td>
<td>N/A</td>
</tr>
<tr>
<td>10</td>
<td>Diep and Niwa</td>
<td>2006</td>
<td>0.93</td>
<td>0.72</td>
<td>N/A</td>
</tr>
<tr>
<td>11</td>
<td>Chakrabarti</td>
<td>1995</td>
<td>0.93</td>
<td>0.76</td>
<td>0.88</td>
</tr>
<tr>
<td>12</td>
<td>Zheng and Wang</td>
<td>2010</td>
<td>0.93</td>
<td>0.75</td>
<td>0.91</td>
</tr>
<tr>
<td>13</td>
<td>Harajli</td>
<td>1990</td>
<td>0.92</td>
<td>0.75</td>
<td>0.89</td>
</tr>
<tr>
<td>14</td>
<td>Harajli, Alternative III</td>
<td>2006</td>
<td>0.92</td>
<td>0.73</td>
<td>0.92</td>
</tr>
<tr>
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**AVAILABLE EQUATIONS IN LITERATURE**

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Table 5-4: Summary of Correlation Factors for Different Equations (Inclusion of the Term $A_{psB}$ into $A_r$)

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CHAPTER VI

SUMMARY AND CONCLUSIONS

6.1 Summary

The unbonded tendon stress at ultimate is needed to calculate the ultimate moment capacity of a member prestressed with unbonded or hybrid tendons. In addition, the use of unbonded tendons is getting more widespread in post-tensioning industry. Therefore, predicting the unbonded tendon stress is important in terms of cost and effectiveness of the design. Therefore, the main aim of this study was to develop an accurate, simple and rational equation for the computation of the unbonded tendon stress at ultimate, which is applicable to beams prestressed with unbonded or hybrid, steel or FRP and external or internal tendons.

This study presented a general overview of various studies dealing with prediction as well as code equations to calculate the stress in unbonded tendon. A simplified design equation was derived on the basis of the analysis method developed by Ozkul et al. 2008. Various forms of the equation were presented and the best option was selected based on simplicity, accuracy, and applicability. A parametric study was performed to study the effect of various parameters on the ultimate stress and stress increase in unbonded tendons. Results are presented in terms of load versus deflection to compare the accuracy of these proposed equations with those from GIA and FEA as well as the experimental data. Results show that both methods give very close results compared with those from
experimental data. The results also show that the proposed equations exhibited very good accuracy in comparison with other prediction equations and code equations.

6.2 Conclusions

Based on the results obtained in this study, the following conclusions can be made:

1. The ultimate stress in unbonded tendon is dependent on the parameters: 1) loading type, 2) effective stress of unbonded tendon, 3) ultimate stress of unbonded tendon, 4) concrete strength, 5) area of different reinforcements, 6) yield stress of non-prestressing reinforcement, 7) span-to-depth ratio. Any proposed prediction or design equation should take into account these parameters.

2. The majority of the prediction equations available in the literature are either empirical or complicated to be recommended as a design equation.


4. The accuracy of all prediction equations available in the literature as well as code equations decrease for the case of beams with hybrid tendons. Therefore, the effect of bonded prestressed reinforcement needs to be considered in the prediction equation.
5. The proposed equations in Alternative I, Eq. (3.33) and Alternative II, Eq. (3.35) are accurate and simple for calculating stress at ultimate in unbonded tendons having correlation factors $R = 0.96$ and $R = 0.87$ for Alternative I and $R = 0.96$ and $R = 0.86$ for Alternative II for $f_{psU}$ and $\Delta f_{psU}$, respectively. Equation in Alternative II is recommended for members with $L / d_{psU} \geq 15$.

Both equations are applicable to the beams with unbonded or hybrid tendons, different loading types (one-point, two-point, uniform) and beam geometries (rectangular, T, I, box sections), as well as different tendon profiles (straight, harped) and materials (steel, FRP).

6. Eq. (3.36), proposed in Alternative III, is simpler and more conservative than both Eq. (3.33) and Eq. (3.35). The correlation factors with experimental data are $R = 0.93$ and $R = 0.75$ for $f_{psU}$ and $\Delta f_{psU}$, respectively. Eq. (3.36) is applicable to the beams with unbonded or hybrid tendons, different loading types and beam geometries, as well as different tendon profiles and materials and is recommended for design purposes.
REFERENCES


