ESSAYS ON COORDINATION AND FINANCIAL MARKETS

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ABSTRACT OF THE DISSERTATION

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This dissertation is concerned with microeconomic models of equilibrium pricing in financial markets of varying organizational scope. The first two chapters are based on game-theoretic models characterized by a coordination problem. I begin at the industry level in Chapter 2 where I solve the *chicken-or-the-egg* problem of platform pricing. I show that the elasticity of demand is greater for the more valued side and hence a monopolist will charge that side a lower price. These results have implications on how exchanges admit traders and apply to more general two-sided markets. My findings are supported by experimental tests. Chapter 3 examines regulatory efficacy in manipulating a currency market with a peg. I solve for equilibrium speculator conduct and central bank intervention policy where the speculator population is discrete. Sudden equilibrium price collapses are related explicitly to fundamentals. The main findings are again supported by experimental tests. In the final chapter, I study a market where strategic behavior is set aside and market participants are instead assumed to be perfect delta hedgers who immediately exploit arbitrage opportunities. I numerically estimate the equilibrium price of an exotic option in such a market.

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Dedication

To the teachers at Haddonfield Memorial High School.

Table of Contents

List of Tables

List of Figures

Chapter 1

Introduction

This dissertation addresses three questions on financial industry organization, regulation and pricing.

Chapter 2 relates to the industrial organization of market platforms. A platform is a place where buyers and sellers meet. The New York Stock exchange is a platform. So is Microsoft Windows operating system, where software producers meet with software consumers. Each distinct group of participants is termed a *side* of the market. For example, the French Bourse traditionally consisted of sixty brokers who functioned purely as financial intermediaries connecting many investors. The Bourse was a *sixty-sided* platform. Similarly, the software writers and users are the two sides of the computer operating system market.

There is a certain ambiguity to the term *two-sided market* because a *two-sided market* is in fact comprised of two different markets. I define the *outer market* as the market for the platform itself. Microsoft and Mac both compete to produce computer operating system platforms. I define the *inner market* is the market which utilizes the platform. For example, the market for tax filing software is an *inner market* of the operating system two sided market. Chapter 2 is concerned with a monopolized outer market.

A basic analysis of demand is essential to answer this question. However this is not a trivial problem. The demand from one side of the market depends on that side's expectation of the demand from the other side of the market. To return to the exchange example above, even an exchange with superior quality will fail to attract many brokerage firms if there are not other brokerage firms on the exchange. But then how does such an exchange begin? This dilemma has been termed the *chicken-or-the-egg* problem of platform markets.

At the heart of the *chicken-or-the-egg-problem* of platform industry organization is a coordination problem with *indirect network externality.* Agents want to coordinate, not with their own group members, but with other group members. I study this coordination problem using a global-game theory and obtain analytical access demand curves. These yield novel insights regarding elasticities and cross-elasticities of demand with respect to prices and exogenous parameters as well as monopoly prices. In particular, demand is more price sensitive with respect the side of the market which is more desirable to the other side. Consequently, a monopolist platform lowers the price on that side and raises the price on the other. The predictions are borne out in an experimental test of the model. Laboratory behavior indicate an additional *own-price effect* whereby players place a relatively larger weight on own-price effects than the model predicts.

Chapter 3 studies a different type of coordination problem, this time at the regulatory level.

Some central bank policy makers believe that currency exchange rate stability is important for economic stability. One way to achieve currency exchange rate stability is to fix the exchange between the domestic currency and another major currency such as the dollar. Besides legal action, the only way to fix an exchange rate is to open a window which creates liquidity at that rate. To see why this has the intended effect, suppose a trade occurs at a different rate. One of the parties to that trade would be better off trading with the central bank instead. If the trade was at a price higher than the peg, then the buyer of the domestic currency would have been better off buying from the central bank instead. If the trade was at a price less than the peg, then the seller of the domestic currency would have been better off selling to the central bank instead. Therefore, such trades will not exist in equilibrium.

In order to provide such an exchange window, the central bank must have an adequate stock of foreign and domestic currency. In addition to satisfying regular demands, reserves serve the purpose of protecting against speculation. To understand this, assume that the peg is higher than the market price. If enough traders demand foreign currency at once, then the central bank may run out of foreign currency reserves and be unable to make the market at the pegged rate any longer. In the case where the peg is less than the market price, then if enough traders demand domestic currency at once, then the central bank may run out of domestic currency and be similarly unable to sustain the peg.[1]

There is a strategic element to trader behavior in a currency market with a pegged rate. Consider the first case where the peg is higher than the market rate. A similar argument follows immediately for the other case. Call the strategic traders *speculators*. *Speculators* recognize that if enough of them sell domestic currency in concert then their demand for foreign currency will overwhelm the central bank's reserves. If this happens then the peg will disappear and the market rate will reign. There are two key aspects to this game. First, the speculators bear an opportunity cost of capital for choosing to speculate. Second, speculators may experience capital gains if enough of them speculate together because they will possess foreign currency which will have appreciated in value. Hence, this is essentially a coordination game between the speculators.

The central bank may want to regulate such speculative behavior. For example, the central bank could tax capital outflows. This would decrease the value of holding the foreign currency and thereby raise the opportunity cost of selling domestic currency. However, since the central bank ostensibly has a better idea of the true value of the currency than the speculators do, their interventions may actually signal to the speculators information on the true potential payoff for crashing the peg. This information may affect speculator behavior and the central bank must therefore take care in its regulatory action.

Angeletos et al. (2006) have analyzed this problem and find that speculators play threshold strategies – sell domestic currency if and only if they sense the price crash will be great enough – and the central bank raises the speculator opportunity cost only for intermediate magnitudes of potential price crashes. I extend those results to a world where the population of speculators is a discrete set instead of a continuous interval

[1] As a caveat, the central bank is not as worried about the second scenario if is uses a fiat system since it may always just print more money to satisfy these demand spikes.

and test the extended model experimentally using the Heinemann et al. (2004) design. Preliminary results suggest that equilibrium strategies are played along with others. Estimated comparative statics of strategies offer mixed agreement with the predictions.

Where Chapter 2 dealt with financial market behavior at the industry level and Chapter 3 the regulatory level, Chapter 3 deals with equilibrium price formation within those markets.

I price a security which promises the buyer the right to purchase any one stock out of a basket of three stocks at any time prior to an expiration date called the expiry at a certain price called the strike. I name this security *cmax*. Along the way, I also price a security which promises the buyer the right to sell any one stock out of a basket of three stocks at any time prior to an expiration date at a certain price. I name this security *pmin*. For example, suppose a *cmax* security is bought on a basket consisting of IBM, Netflix, and 3M with a strike of 85 with an expiry one year in the future. If, after one year, the price of IBM is 70, the price of Netflix is 85, and the price of 3M is 95, then the purchaser of the option is in the money at expiry and will obviously select to buy the 3M stock for 85. Note that he has the option to exercise at any time prior to expiry too.

The proper price for this security rests on the insights of Black and Scholes. Assume a perfect market where all agents know all distributions of stock returns and have costless and instantaneous access to the markets for all stocks and credit. They may set up a perfect *delta hedge* consisting of one part underlying stock and and *delta* parts short the option where *delta* is defined as the derivative of the option price with respect to the underlying stock price. Such a portfolio is by definition insulated from stock price risk and therefore must receive a risk free return in equilibrium. Therefore, the option price is pinned to the stock price and the risk free rate. The precise relationship is referred to as the *Black-Scholes* formula.

The Black-Scholes formula is not an explicit equation for the option price. Instead, it characterizes the option price as a partial differential equation. However, numeral methods exist for approximating what the option price is for any set of fundamental parameters. I use *finite difference* techniques where the option price is estimated on a discrete grid representing all possible stock prices and time. The technique involves defining the option prices on the expiry plane in accordance with promised payoff function and then using Black Sholes formula to determine all other points iteratively. One shortcoming of this technique is that since the grid must be finite, artificial boundary conditions must be defined. These are usually chosen as part art, part science, using various limit arguments.

Chapter 2

Equilibrium Selection in Two-Sided Markets

2.1 Background

The definition of a two-sided market is not completely agreed on. The sufficient conditions that I use are:

1. There are two groups of buyers. Each group is termed a *side* of the market. Call them *Side A* and *Side B*.

2. Sellers produce two goods. Call them *Good A* and *Good B*. These sellers are termed *platforms*.

3. Buyers belonging to *Side A* are only willing to pay for a single unit of *Good A* and buyers belonging to *Side B* are only willing to pay for a single unit of *Good B*. The act of buying a good is referred to as *joining* the *platform*.

4. A Side A buyer's willingness to pay to join a platform is directly proportional to Side B's aggregate demand for that platform.

5. A Side B buyer's willingness to pay to join a platform is directly proportional to Side A's aggregate demand for that platform.

The definition extends straightforwardly to *multi-sided* markets.

The most instructive example is the market for heterosexual mingling. In this case, the *platforms* are nightclubs, a *side* of buyers is the male population and the other *side* of buyers is the female population. The two platform products are male access right and female access rights. Finally, a man's willingness to pay for access to a nightclub is directly related to the number of women present at that nightclub and a woman's willingness to pay for access to a nightclub is directly related to the number of men present at that nightclub.

Alternatively, consider a financial paper exchange. One side is the population of investment banks who write derivative contracts and another side is insurance companies who buy those contracts. Each side is willing to pay a fee to the exchange owner up to his expected gains from trade on these contracts. Furthermore, gains from trade are proportional to the number of participants on the other side due due to liquidity and diversification economies.

Another important example is the market for software operating systems. For example, Microsoft's Windows operating system offers software users and software writers a platform, Windows, through which to interact. The use of a platform instead of direct interaction greatly increases the gains from trade because Windows programming tools reduce costs for application writers and the consistent Windows interface increases the willingness to pay of consumers. A similar scenario occurs in the video game console market. Theoretically, a game producers could sell a software-hardware bundle to users. However, a centralized hardware platform such as Playstation or Nintendo obviates the need for application providers to sell a hardware component.

Finally, two-sided market theory has recently been applied to Internet regulation by the FCC. Broadband providers may charge fees for shuttling information between application providers and users. *Net-neutrality* advocates are concerned about the abuse of market power by these broadband providers, especially in cases where those firms are vertically integrated into the application market. The recent merger between Comcast, a broadband provider, and NBC, an application provider, led some to believe that the new conglomerate would harm competition in the upstream television market. Those against *net-neutrality* regulation believe that we do not understand efficient pricing in two-sided markets well enough to warrant explicit regulation. For example, as with most markets, it is not clear how or when price discrimination is welfare enhancing. For a more discussion, see Baker et al. (2011).

2.2 A Utility Model of Two-Sided Markets

I model a side of the market as a unit interval. An individual buyer who is a member of that side is a point on that unit interval. Each buyer has Lebesgue measure zero while the entire side has Lebesgue measure one. Denote $\mathcal{I}$ the set of sides, i an element of $\mathcal{I}$ and $-i$ an element of $\mathcal{I}\backslash i$. Denote $\mathcal{J}$ the set of platforms, j an element of $\mathcal{J}$ and $-j$ an element of $\mathcal{J}\backslash j$. Denote i' an arbitrary member of side i and j' an arbitrary member of side j. I restrict attention to a singleton set of platforms and two sides. That is, I model a monopolized two-sided market.

A side i buyer who joins the platform will enjoy the following utility:

$$u_{i'}(p_i, D_{-i}, \theta) = \theta + \lambda_i D_{-i} - p_i$$

where D_{-i}, platform demand from side $-i$, is the measure of side $-i$ agents who join the platform, p_i is the price charged by the platform to side i, θ is platform quality and λ_i is a utility rate per measure of side $-i$ agents who join the platform. I define demand more carefully below. For now, it should be understood as a number in the closed unit interval. Note that the seller may discriminate across groups because buyers are exogenously endowed with group membership and groups are assumed to care about only one of the products. For example, buyers who would like to use eBay for the purposes of selling have no use for a buyer registration. The opportunity cost of not joining a platform is normalized to zero.

Once buyers have made their joining decisions, they proceed to interact with one another. I do not model this process explicitly. Implicit in the preferences defined above is the notion that surplus is generated through their interaction and allocated according to some mechanism. The *inner market* is essentially a pure exchange economy. For my purposes, the nature of the inner market is fixed and summarized by the parameters λ_A, and λ_B. Consumer preferences with respect to the outer market can be interpreted as expectations of inner market gains from trade.

2.3 A Simple Game Theoretic Model

Since platform seekers care about what other platform seekers choose, analysis of two-sided markets lends itself well to game theory. Consider the following three three-stage *simple platform game*. In stage one, the platform chooses an investment level, ρ, and a set of prices (p_A, p_B) where p_A and p_B are the access fee prices payed by side A and side B buyers, respectively. In stage two, nature draws the quality of the platform according to $\Theta|\rho \sim \mathcal{N}(\rho, \sigma_\Theta^2)$ where $\mathcal{N}$ is the normal distribution. In stage three, agents observe the quality of the platform, the prices, and choose whether or not to join the platform, actions denoted J and NJ, respectively. Denote the strategy of agent i' as $s_{i'}(p_i, p_{-i}, \theta)$. Define side i ex-post[1] market demand $D_i(p_i, p_{-i}, \theta)$ as the measure of side i agents who join the platform. The payoff to a consumer who joins the platform is:

$$u_{i'}(p_i, p_{-i}, \theta) = \theta + \lambda_i D_{-i}(p_i, p_{-i}, \theta) - p_i \tag{2.1}$$

The payoff to a consumer who does not join the platform is zero. The platform's stage 1 expected payoff is:

$$\pi(p_A, p_B, \rho) = p_A \int_{-\infty}^{+\infty} D_A(p_A, p_B, \theta)\phi(\theta|\rho)d\theta + p_B \int_{-\infty}^{+\infty} D_B(p_A, p_B, \theta)\phi(\theta|\rho)d\theta - C(\rho) \tag{2.2}$$

where $C(\cdot)$ is a cost of investment function and $\phi(\cdot|\rho)$ is the conditional density of θ given ρ. I assume zero fixed cost and zero marginal cost for the platform for admitting and facilitating interaction among buyers.

Agent i' utility is bounded below by $\theta - p_i$ and above by $\theta + \lambda_i - p_i$. See the shaded region(s) in Figure 2.1. The lower bound obtains when an agent joins and no agents from the opposite side join and the upper bound obtains when all agents of the opposite side join. It follows then that if $\theta > p_i$, all i' will receive a positive payoff

[1] I call this market demand ex post because it gives the demand for a given realization of the platform quality. Ex ante demand is then the demand a platform expects given a choice of investment level, before quality has been realized.

independent of the actions of other agents. Similarly, if $\theta < p_i - \lambda_i$, all i' will receive a negative payoff independent of the actions of other agents. In other words, demand from side i will be 1 and 0 in those cases, respectively. However, for $p_i - \lambda_i < \theta < p_i$, the optimal action depends on what measure of $-i'$ also join. Define the i-window, $\mathcal{W}_i$, as $\mathcal{W}(p_i, \lambda_i) \equiv [p_i - \lambda_i, p_i]$.

Given a $\mathcal{W}_i$, we can say more about an agent i''s dominant strategies by looking at $\mathcal{W}_{-i}$. There are two main cases to consider: $\mathcal{W}_i \cap \mathcal{W}_{-i} = \emptyset$ and $\mathcal{W}_i \cap \mathcal{W}_{-i} \neq \emptyset$.

Consider the first case. Without loss of generality, assume that $p_i - \lambda_i > p_{-i}$. In other words, $\mathcal{W}_i$ is strictly greater than $\mathcal{W}_{-i}$. This is a world for members of side i are much less eager to join the platform because either the price they face is high or their marginal benefit from interacting with side $-i$ is small. Then for $\theta \in \mathcal{W}_i$, J_{-i} dominates NJ_{-i}. In other words, for quality ranges where side i may want to join, quality is sufficiently high for side $-i$ agents that they join for sure, independent of what side i agents choose. By iterated deletion of dominated actions, then, J_i dominates NJ_i. In other words, for quality levels where a naive side i player would be unsure about joining, the savvy side i player correctly anticipates that all side $-i$ agents will join. A symmetric argument holds when $\theta \in \mathcal{W}_{-i}$. NJ_i dominates J_i and, by iterated deletion of dominated actions, for NJ_{-i} dominates J_{-i}.

Consider the second case. Now there are platform qualities for which *both* sides of the market may or may not like to join depending on what measure of agents from the opposite side join. If $p_i > p_{-i}$, then for $\theta \in [p_{-i}, p_i]$, J_{-i} dominates NJ_{-i} and by iterated deletion of dominated actions, J_i dominates NJ_i. If $p_i - \lambda_i > p_{-i} - \lambda_{-i}$ then for $\theta \in [p_{-i} - \lambda_{-i}, p_i - \lambda_i]$, NJ_i dominates J_i and by iterated deletion of dominated actions, NJ_{-i} dominates J_{-i}.

With the above argument in mind, Proposition 1 characterizes all of the Nash equilibria of the stage three subgame of the *simple platform game*.

Proposition 1: All Nash equilibrium strategies of *the simple platform game* have the following form:

$$s_{i'}(p_i, p_{-i}, \theta) = \begin{cases} J_i & if\ \theta > max\{min\{p_i, p_{-i}\}, p_i - \lambda_i\}\ or\ \theta \in \mathcal{F} \\ NJ_i & else \end{cases} \tag{2.3}$$

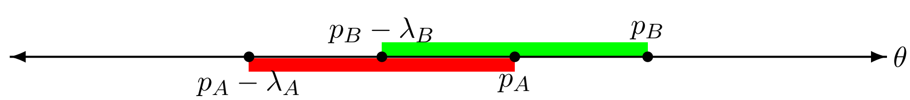
where $\mathcal{F} \subset \mathcal{W}_i \cap \mathcal{W}_{-i}$ if $\mathcal{W}_i \cap \mathcal{W}_{-i} \neq \emptyset$ and $\mathcal{F} = \emptyset$ if $\mathcal{W}_i \cap \mathcal{W}_{-i} = \emptyset$

The most important property of the Nash equilibria described in Proposition 1 is the arbitrariness of $\mathcal{F}$. It implies an infinite multiplicity of equilibria in the stage three subgame of the *simple platform game*. See Figure 2.1 for a graphical representation of Proposition 1.

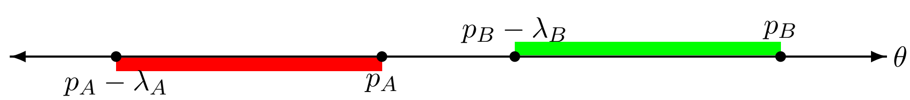
The microeconomic interpretation of this multiplicity of *Nash equilibria* is a multiplicity of both ex-post and ex-ante *demand curves*, that is, the demand curves which the monopolist expects given a choice of investment and the demand curves for given platform qualities, respectively. The ex post demand curve maps realized qualities levels into either zero or one. That is, participation by some positive fraction less than one of the agents from a given side can never occur in equilibrium. If agents who are "on" the platform are receiving a negative utility, then those agents are not best-responding and would rather be "off" the platform. If the agents who are "on" the platform are taking a positive utility, then all of the agents who are "off" the platform are not best-responding and would rather be "on" the platform as well. Consequently, the ex-post demand curve is discontinuous at at least one price and may actually increase in price over certain price regions. The ex-ante demand curves are derived by fixing an ex-post demand curve and integrating over the density distribution of quality given the investment level. This necessarily yields a smooth demand curve which may be upward sloping over some price regions. See Figure 2.2 for an illustration of all of possible ex-ante equilibrium demand curves.

By backward induction, the profit-maximizing monopolist chooses prices and quality to maximize his expected profit given the ex-ante demand curve implied by the stage three subgame Nash equilibrium strategies.

Figure 2.1: Equilibria of the *simple platform game* stage three subgame. $\mathcal{W}_A$ and $\mathcal{W}_B$ are shown in red and green, respectively, for two different parameterizations of the subgame. A side i agent will always play dominant strategies for $\theta \notin \mathcal{W}_i$ regardless of the parameterization. Case (a) depicts a game parameterized such that $\mathcal{W}_A \cap \mathcal{W}_B \neq \emptyset$. For $\theta < p_B - \lambda_B$, NJ_A and NJ_B are both dominant strategies. For $\theta > p_A$, J_A and J_B are both dominant strategies. For $\theta \in \mathcal{W}_A \cap \mathcal{W}_B$, players either coordination on joining or not joining. This is the region of multiple equilibria. Case (b) depicts a game parameterized such that $\mathcal{W}_A \cap \mathcal{W}_B = \emptyset$. For $\theta \in \mathcal{W}_A$, side A agents are pessimistic (believe no side B agents will join) and therefore do not join. For $\theta \in \mathcal{W}_B$, side B agents are optimistic (believe all side A agents will join) and therefore join.

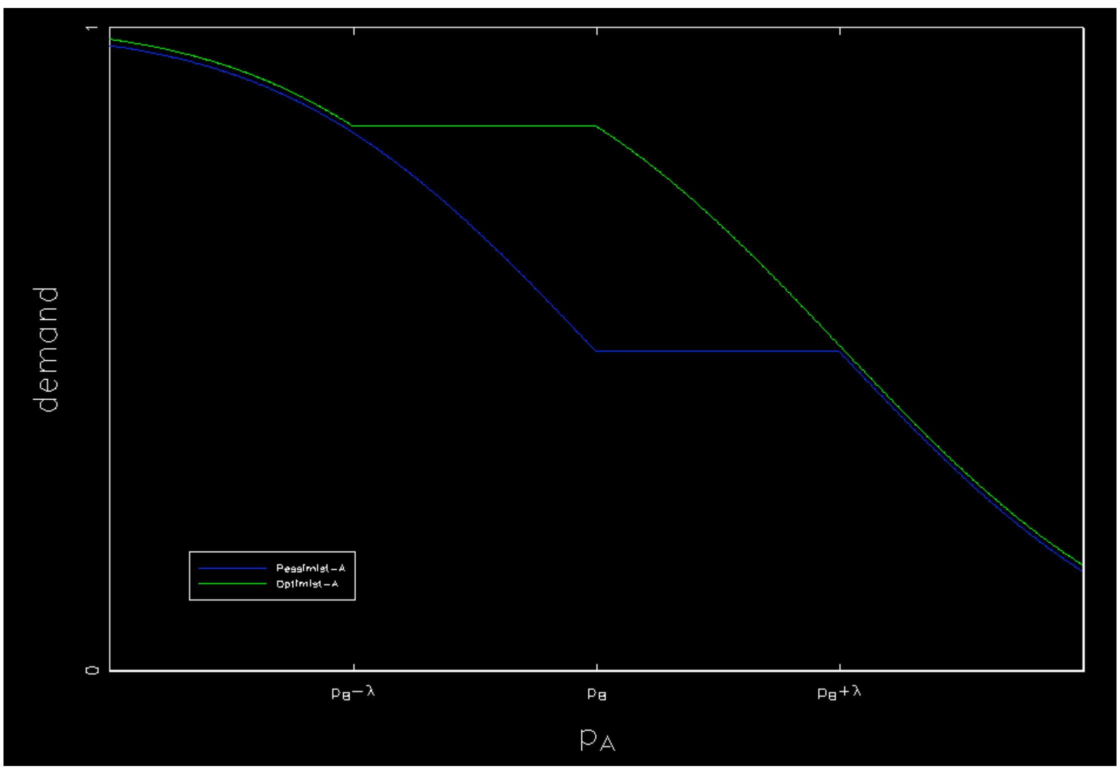


(a)



(b)

Figure 2.2: A plot of several possible ex ante demand curves for the *simple platform game*. The *optimistic* demand curve refers to side A demand in the *simple platform game* if all agents join the platform for all $\theta \in \mathcal{W}_A \cap \mathcal{W}_B$. The *pessimistic* demand curve refers to side A demand in the *simple platform game* if all agents do not join the platform for all $\theta \in \mathcal{W}_A \cap \mathcal{W}_B$. All functions between the two are feasible demand functions in the *simple platform game*. For $p_A \notin [p_B - \lambda_B, p_B + \lambda_A]$, all possible side A demand curves coincide. Note: The figure is sketched for the special case where $\lambda_A = \lambda_B$.



2.4 Literature Review and Contribution

Proposition 1 brings to light what has come to be known as the *chicken-or-the-egg* problem of two-sided markets: given a set of prices, both side's demands depend on beliefs about the other side's demand. This indeterminacy is similar to the *sunspot equilibria* problem studied in the bank run and currency crisis literatures.

A number of researchers have devised techniques to circumvent the chicken-or-the-egg problem. In an early precurser to the formal two-sided market literature, Caillaud and Jullien (2003) model the platform as a matching service. Each side is interested in finding a unique match from the other side. The platform provides greater value to each side when it has more members of the opposite side joined and available for search. For similar reasons as discussed in this model, there are multiple equilibria for many price systems. Caillaud and Jullien (2003) resolve this problem by restricting attention to those equilibrium demand curves which are downward sloping. Miscoordination is ignored.

In a similar model, Rochet and Tirole (2006) interpret the fixed benefit term as one component of a buyer's type which is distributed continuously over the population of buyers. In other words, the fixed benefit is a horizontal quality. When there are no side i agents on the platform, a positive measure of side $-i$ agents will find it dominant to join the platform, independent of the price. Similarly, when all side i agents are on the platform, a positive measure of side $-i$ agents have negative fixed benefits and find NJ dominant. Under some regularity conditions, a unique set of demands for any given set of prices will result.

Armstrong (2006) breaks with Rochet and Tirole (2006) by considering utility with no fixed benefit. He sidesteps the coordination problem by taking demand curves as given exogenously. He finds the intuitive result that, given a participation level on the other side, the socially efficient price is the marginal cost less the marginal indirect externality.[2] To derive monopoly prices, Armstrong (2006), models elasticity with an exogenous function and derives a Lerner condition.

[2] Efficient pricing is analyzed in greater detail by Bolt and Tieman (2006)

Hagiu (2006) avoids the coordination problem by assuming the sides enter the market sequentially. Sellers enter first. He analyzes two cases. In one, sellers expect all other buyers to enter and in the other sellers expect no other buyers to enter. These correspond to optimistic and pessimistic expectations of opposite side participation. This is akin to looking at the pareto dominant and pareto inferior equilibria of the *simple platform game* defined above.

The approach used in Hagiu (2006) is similar to the "rational expectations" approach used by Katz and Shapiro (1985). They look at a one-sided oligopoly market with direct network externalities. Consumers have unit demand and consumer surplus demand on the number of other consumers who have purchased the product. It is easy to see that individual demand curves are interdependent and there are therefore multiple equilibrium demand curves. For example, if no one buys, then no consumer has an incentive to buy. If everyone buys, then every consumer has an incentive to buy. Katz and Shapiro (1985) escape the equilibrium multiplicity problem by exogenizing buyers' expectation. Given expectations, Cournot first-order-conditions implicitly define the allocation of consumer demand across firms. The key to their approach is to restrict attention to such expectation-allocation pairs which are consistent. That is, consumers expectation of the network size of a firm is equal to the actual network size that results after the Cournot competition.

Weyl (2009) considers a more general demand system that allows for heterogeneity of both fixed and marginal benefit parameters across the population (I capture heterogeneity across groups). Seeing as my model is a special case of his, one would expect his results to apply to mine specifically. In particular, he makes the case for the coordination problem being unimportant. He suggests that the inverse demand curve is obtainable through the following argument. Fix the participation on side B and vary the price on side A. This traces out the side A demand. Then fix participation on side A and vary the price on side B to obtain side B demand. Together these half-demand curves can implement any participation levels. Like some of the papers mentioned above, this argument discretely sidesteps the coordination problem by assuming .

Weyl is essentially suggesting a two-stage game where in stage one, the platform sets one side's participation and in a second stage, he uses price to elicit demand on the other side. This argument can work for one side, but it is illogical to apply it to both sides simultaneously. Further, if the simultaneity problem was solved, preferences may be defined in such a way that some participation levels are not implementable. For example, in my formulation above, by fixing a participation level on side A, the platform will only be able to implement a demand of 0 or 1 on the other side. Finally, Weyl suggests any implementability problems can be overcome with the use of an "insulating tariff" whereby the platform posts prices which are contingent on the participation of the opposite. In this way, the platform fixes the utility level by offering a price schedule which depends on the participation of the other side. This is essentially what Armstrong is doing with his black-box demand functions above. The key to this argument is the idea that platforms do not charge at the door but instead charge *at the exit* after the actual demands are realized. Indeed, given preferences with unbounded support, if such a scheme were implementable, any demand could be implemented. However, it is doubtful that such pricing schemes could be implemented in reality. Even if they were, the question of what the demand curves are at the door remains unresolved. This is the case which I focus on here.

Standard coordination games consider a group of identical agents who face multiple paraeto-ranked equilibria.[3] Players actions are strategic complements. The coordination problem studied here is an interesting variant on classic coordination problems. Here, agents are placed into two groups. There is no complementarity in actions among players on the same side. However, actions are complementary across sides.

The central contribution of this paper is a resolution of the chicken-or-the-egg problem. In the *simple platform game*, the fixed benefit term, θ, was common knowledge to consumers. It is more realistic, however, to relax this assumption by assuming that agents have some noisy perception of the true fixed benefit term. More importantly, agents are unsure about *other* agents' beliefs on platform quality. I show that under this richer information structure, a unique equilibrium exists which pins down ex-ante

[3]See Cooper (1999) for an introduction to this voluminous literature.

and ex-post demand curves.

2.5 The Complete Model

Here I define the three-stage *complete platform game.* In stage one, the monopolist platform chooses an investment level $\rho > 0$ and a set of prices (p_A, p_B) where p_A and p_B are the access fee prices paid by side A and side B buyers, respectively. In stage two, nature draws the true quality of the platform according to $\Theta|\rho \sim \mathcal{N}(\rho, \sigma_\Theta)$ and agents' types according to $x_{i'}|\theta \sim \mathcal{N}(\theta, \sigma_x)$, where $\mathcal{N}$ indicates the normal distribution and σ_x is a level of noise which for ease of exposition is fixed for all buyer agents. In stage three, agents observe their type, $x_{i'}$, the investment level, ρ, the set of prices (p_A, p_B), and choose whether to join, $J_{i'}$, or not join, $NJ_{i'}$. Denote the strategy of agent i' as $s_{i'}(p_i, p_{-i}, x_{i'}, \rho)$. Define side i market demand $D_i(p_i, p_{-i}, \theta) \equiv \int_0^1 \int_{-\infty}^{+\infty} s_{i'}(p_i, p_{-i}, x_{i'}, \rho)\phi(x_{i'}|\theta)dx_{i'}di$. The payoff to a consumer who joins the platform is

$$u_i(p_i, p_{-i}, \theta) = \theta + \lambda_i D_{-i}(p_i, p_{-i}, \theta) - p_i \tag{2.4}$$

and the payoff to a buyer agent who does not join the platform is zero. Given a set of demand curves, the platform's stage one expected payoff is:

$$\pi(p_A, p_B, \rho) = p_A \int_{-\infty}^{+\infty} D_A(p_A, p_B, \theta)\phi(\theta|\rho)d\theta + p_B \int_{-\infty}^{+\infty} D_B(p_A, p_B, \theta)\phi(\theta|\rho)d\theta - C(\rho) \tag{2.5}$$

where $C(\cdot)$ is the cost of investment. Note that I assume zero fixed cost and zero marginal cost for the platform for admitting and facilitating interaction among buyers.

With respect to the *simple platform game*, the *complete platform game* represents a weakening of the common knowledge assumption on θ. Instead of it being publicly observed, agents receive private noisy signals which inform their prior belief on the distribution of the true platform quality. It is immediately clear that Nash equilibria of the *simple platform game* will not translate directly into Nash equilibria of the *complete platform.* In particular, in the former case, agents will coordinate their actions for $\theta \in \mathcal{F} \subset \mathcal{W}_A \cap \mathcal{W}_B$. This type of coordination is no longer possible when θ is not common knowledge. Strategies, instead, depend on the information set consisting of the private signal and the public market information. Agents must form beliefs on the payoff-relevant θ as well as other agents' beliefs on θ *ad infinitum.* This type of coordination problem with payoff uncertainty has been termed a *global game* by Carlsson and Van Damme (1993a).[4] I now present the main result of the paper.

Proposition 2: The stage three subgame has a Bayes Nash equilibrium consisting of the following strategies:

$$s_{i'}(p_i, p_{-i}, x_{i'}, \rho) = \begin{cases} J_{i'} & \text{if } x_{i'} > x_i^*(p_i, p_{-i}, \rho) \\ NJ_{i'} & \text{if } x_{i'} < x_i^*(p_i, p_{-i}, \rho) \end{cases} \tag{2.6}$$

$\forall i'$. If $\frac{\sigma_x^4}{\sigma_\Theta^4} \frac{\sigma_x^2+\sigma_\Theta^2}{\sigma_x^4+2\sigma_\Theta^2\sigma_x^2} \leq min\{\frac{2\pi}{\lambda_A^2}, \frac{2\pi}{\lambda_B^2}\}$, then it is a dominant strategy equilibrium and $(x_A^*(p_i, p_{-i}, \rho), x_B^*(p_i, p_{-i}, \rho))$ is unique. That is, all other strategies may be eliminated using iterated deletions of dominated strategies. Stage one ex ante demand functions are given by:

$$D_i(p_i, p_{-i}, \rho) = 1 - \Phi\left(\frac{x_i^*(p_i, p_{-i}, \rho) - \rho}{\sqrt{\sigma_\theta^2 + \sigma_x^2}}\right) \tag{2.7}$$

where Φ indicates the standard normal CDF. Furthermore, as signal noise vanishes, a simple algebraic solution for x_i^* obtains:

$$\lim_{\sigma_x \to 0} x_i^*(p_i, p_{-i}, \rho) = \begin{cases} p_i & \text{if } p_i < p_{-i} - \lambda_{-i} \\ \frac{\lambda_{-i}}{\lambda_i + \lambda_{-i}} p_i + \frac{\lambda_i}{\lambda_i + \lambda_{-i}} p_{-i} - \frac{\lambda_i \lambda_{-i}}{\lambda_i + \lambda_{-i}} & \text{if } p_{-i} - \lambda_{-i} \leq p_i \leq p_{-i} + \lambda_i \\ p_i - \lambda_i & \text{if } p_i > p_{-i} + \lambda_i \end{cases}$$

Define a platform *total price* as $\bar{p} \equiv p_i + p_{-i}$. The optimal stage one platform access fees and investment decision are given by:

$$\lim_{\sigma_x \to 0} p_M^* = \frac{\bar{p}^* + \lambda_M}{2}$$

$$\lim_{\sigma_x \to 0} p_{-M}^* = \frac{\bar{p}^* - \lambda_M}{2}$$

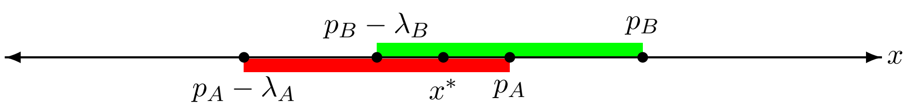
[4] For a lucid summary of the theory and application of global games, see Morris and Shin (2003)

where $\bar{p}^* = p^*_M + p^*_{-M}$, $M \equiv argmax_{i \in \{A,B\}}\{\lambda_i\}$, and $(\bar{p}^*, \rho^*)$ solves:

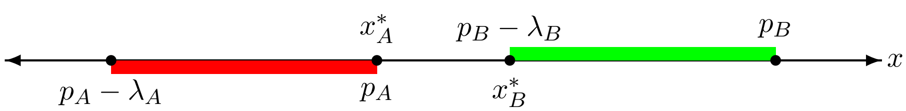
$$max_{(\bar{p},\rho)}\bar{p}\left(1 - \Phi\left(\frac{\frac{\bar{p}-\lambda_M}{2} - \rho}{\sigma_\theta}\right)\right) - C(\rho)$$

Proof 2: Appendix. See Figure 2.3 for a graphical representation of Proposition 2.

Figure 2.3: The unique equilibrium in the stage three subgame of the *complete platform game*. Case (a) depicts a game parameterized such that $\mathcal{W}_A \cap \mathcal{W}_B \neq \emptyset$. Proposition 2 proves that the unique equilibrium in this case is to join for $x_{i'} > x^*$. Case (b) depicts a game parameterized such that $\mathcal{W}_A \cap \mathcal{W}_B = \emptyset$. Proposition 2 proves that the unique equilibrium is for all side B agents to join for $x_{B'} > x^*_B \equiv p_B - \lambda_B$ and for all side A agents to join for $x_{A'} > x^*_A \equiv p_A$. Case (b) depicts a game parameterized such that $\mathcal{W}_A \cap \mathcal{W}_B = \emptyset$. In such cases, the equilibrium is essentially the same as in the *simple platform game*. This illustration applies to a *complete platform game* with very small noise. Where signal noise is large, the thresholds may deviate from the positions specified by the illustration in proportion to the magnitude of the signal noise.



(a)



(b)

Some important features of the result should be understood. First, the equillibria of the *complete platform game* and the *simple platform game* coincide where we would expect them too. Notice that in the *complete platform game* θ is almost but never quite common knowledge. As the signal becomes more precise, agents beliefs on θ, on other agents' beliefs on θ, etc., become increasingly accurate. In other words, common knowledge of θ is approached. In the *simple platform game*, on the other hand, θ is common knowledge outright. Hence, we would expect then that the unique equilibrium of the *complete platform game* to approach the equilibrium of the *simple platform game* as signal noise vanishes in some natural way. Were this not the case, the appeal of the *complete platform game* as a solution to chicken-or-the-egg problem illustrated in the *simple platform game* would be lost. Indeed, as signal noise vanishes, a measure one mass of agents will choose the same action in the equilibria of either game for some fixed θ and for $p_i \notin [p_{-i} - \lambda_{-i}, p_{-i} + \lambda_i]$. To see this, observe that ex post demand is given by:

$$\begin{aligned}
D_i(p_i, p_{-i}, \theta) &= \int_0^1 \int_{-\infty}^{+\infty} s_{i'}(p_i, p_{-i}, x_{i'}, \rho)\phi(x_{i'}|\theta)dx_{i'}di \\
&= \int_0^1 \int_{-\infty}^{+\infty} \mathbb{I}_{x_i > x^*}\phi(x_{i'}|\theta)dx_{i'}di \\
&= \int_0^1 \left[1 - \Phi\left(\frac{x^* - \theta}{\sigma_x}\right)\right] di \\
&= 1 - \Phi\left(\frac{x^* - \theta}{\sigma_x}\right)
\end{aligned}$$

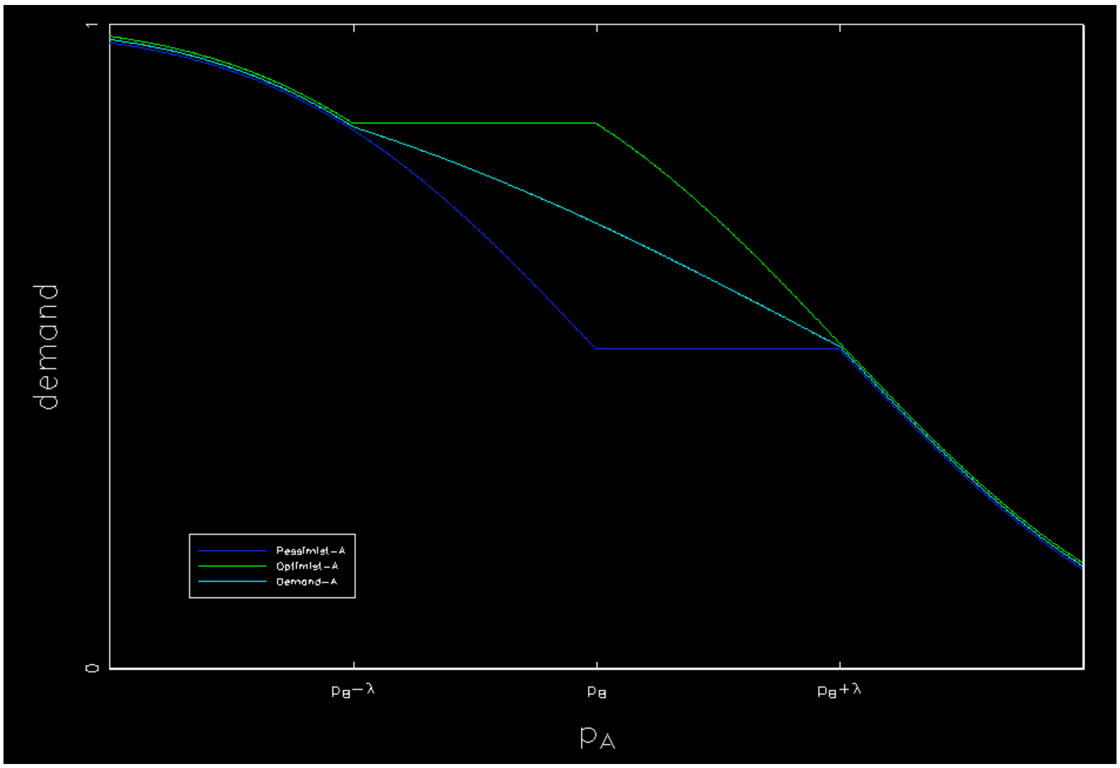
and that as signal noise goes to zero, ex post demand becomes a step function

$$\begin{aligned}
\lim_{\sigma_x \to 0} D_i(p_i, p_{-i}, \theta) &= \lim_{\sigma_x \to 0} \left\{1 - \Phi\left(\frac{x^* - \theta}{\sigma_x}\right)\right\} \\
&= \begin{cases} 1 & \text{if } x^* < \theta \\ 0 & \text{if } x^* > \theta \end{cases} \\
&= \begin{cases} 1 & \text{if } p_i < \theta \text{ and } p_i < p_{-i} - \lambda_{-i} \\ & \text{or } \frac{\lambda_{-i}}{\lambda_i + \lambda_{-i}} p_i + \frac{\lambda_i}{\lambda_i + \lambda_{-i}} p_{-i} - \frac{\lambda_i \lambda_{-i}}{\lambda_i + \lambda_{-i}} < \theta \text{ and } p_{-i} - \lambda_{-i} \le p_i \le p_{-i} + \lambda_i \\ & \text{or } p_i - \lambda_i < \theta \text{ and } p_i > p_{-i} + \lambda_i \\ 0 & \text{if } p_i > \theta \text{ and } p_i < p_{-i} - \lambda_{-i} \\ & \text{or } \frac{\lambda_{-i}}{\lambda_i + \lambda_{-i}} p_i + \frac{\lambda_i}{\lambda_i + \lambda_{-i}} p_{-i} - \frac{\lambda_i \lambda_{-i}}{\lambda_i + \lambda_{-i}} > \theta \text{ and } p_{-i} - \lambda_{-i} \le p_i \le p_{-i} + \lambda_i \\ & \text{or } p_i - \lambda_i > \theta \text{ and } p_i > p_{-i} + \lambda_i \end{cases}
\end{aligned}$$

which corresponds exactly to the equilibrium given by Proposition 1 for this interval.

Second, the *complete platform game* has a unique equilibrium for platform qualities where the *simple platform game* has multiple. When $\mathcal{W}_A \cap \mathcal{W}_B \neq \emptyset$, Proposition 2

Figure 2.4: The *refined* demand curve obtained for the *complete platform game*. For $p_A \in [p_B - \lambda_B, p_B + \lambda_A]$, the *refined* demand curves for each side are equal because the thresholds, $x^*(\cdot)$ are equal. For $p_A < p_B - \lambda_B$, side B *refined* demand becomes fully optimistic and therefore independent of p_A and side A *refined* demand becomes fully pessimistic. For $p_A > p_B + \lambda_A$ side B *refined* demand becomes fully pessimistic and therefore independent of p_A and side A *refined* demand becomes fully optimistic. Note: The figure is sketched for the special case where $\lambda_A = \lambda_B$.



implies that $x^*(p_i, p_{-i}, \rho) \in \mathcal{W}_A \cap \mathcal{W}_B$ whereas Proposition 1 is agnostic of equilibrium strategies. Furthermore, in this region, the equilibrium of the *complete platform game* is an intuitive compromise between the optimistic and pessimistic strategies of the *simple platform game* – agents will coordinate on joining the platform if and only if quality is high enough. See Figure 2.4 for an illustration of how the ex ante demand curve implied by Proposition 2 is a compromise between the optimistic and pessimistic demand curves from Proposition 1.

Third, notice that $\lim_{\sigma_x \to 0} x_i^*(p_i, p_{-i}, \rho) = \lim_{\sigma_x \to 0} x_{-i}^*(p_i, p_{-i}, \rho) \equiv x^*(p_i, p_{-i}, \rho)$ when

$\mathcal{W}_A \cap \mathcal{W}_B \neq \emptyset$. This says that for platforms where the chicken-or-the-egg problem exists, equilibrium strategies are the same for all platform buyers *independent* of side. This may sound counterintuitive at first. One could reasonably argue that if side A agents have a much higher marginal benefit of interaction than side B agents do, then side A agents should be more likely to join the platform. However, my counterintuitive result has sound intuition. Suppose $x_i^* \neq x_{-i}^*$. Since signal noise is small, types in $[min\{x_i^*, x_{-i}^*\}, max\{x_i^*, x_{-i}^*\}]$ are either joining when very few agents of the other side are joining or are not joining when very many of the other side are joining. In either case, no agents is best-responding. An interesting implication of this fact is that the platform should expect an even mix of agents when $\mathcal{W}_A \cap \mathcal{W}_B = \emptyset$. More specifically, there is ex-post *perfect coordination* always (miscoordiation for a measure zero mass of agents).

Fourth, Corollaries 1 and 2, below, give comparative statics for equilibrium behavior in the *complete platform game*. These are the real contribution of this refinement notion as such comparative statics are impossible in the *simple platform game* when the analyst is most interested in platforms where $\mathcal{W}_A \cap \mathcal{W}_B = \emptyset$.

Corollary 1:

$$\lim_{\sigma_x \to 0} \frac{\partial x_i^*(p_i, p_{-i}, \rho)}{\partial p_i} = \begin{cases} 1 & \text{if } p_i < p_{-i} - \lambda_{-i} \\ \frac{\lambda_{-i}}{\lambda_i + \lambda_{-i}} & \text{if } p_{-i} - \lambda_{-i} \leq p_i \leq p_{-i} + \lambda_i \\ 1 & \text{if } p_i > p_{-i} + \lambda_i \end{cases} \tag{2.8}$$

$$\lim_{\sigma_x \to 0} \frac{\partial x_i^*(p_i, p_{-i}, \rho)}{\partial p_{-i}} = \begin{cases} 0 & \text{if } p_i < p_{-i} - \lambda_{-i} \\ \frac{\lambda_i}{\lambda_i + \lambda_{-i}} & \text{if } p_{-i} - \lambda_{-i} \leq p_i \leq p_{-i} + \lambda_i \\ 0 & \text{if } p_i > p_{-i} + \lambda_i \end{cases} \tag{2.9}$$

Proof: Follows from Proposition 2 by differentiation.

Corollary 2:

$$\lim_{\sigma_x \to 0} \frac{\partial D_i(p_i, p_{-i}, \rho)}{\partial p_i} = \begin{cases} -\frac{1}{\sigma_\theta}\phi\left(\frac{x_i^*(p_i,p_{-i},\rho)-\rho}{\sigma_\theta}\right) & \text{if } p_i < p_{-i} - \lambda_{-i} \\ -\frac{1}{\sigma_\theta}\phi\left(\frac{x_i^*(p_i,p_{-i},\rho)-\rho}{\sigma_\theta}\right)\frac{\lambda_{-i}}{\lambda_i+\lambda_{-i}} & \text{if } p_{-i} - \lambda_{-i} \leq p_i \leq p_{-i} + \lambda_i \\ -\frac{1}{\sigma_\theta}\phi\left(\frac{x_i^*(p_i,p_{-i},\rho)-\rho}{\sigma_\theta}\right) & \text{if } p_i > p_{-i} + \lambda_i \end{cases} \tag{2.10}$$

$$\lim_{\sigma_x \to 0} \frac{\partial D_i(p_i, p_{-i}, \rho)}{\partial p_{-i}} = \begin{cases} 0 & \text{if } p_i < p_{-i} - \lambda_{-i} \\ -\frac{1}{\sigma_\theta}\phi\left(\frac{x_i^*(p_i,p_{-i},\rho)-\rho}{\sigma_\theta}\right)\frac{\lambda_i}{\lambda_i+\lambda_{-i}} & \text{if } p_{-i} - \lambda_{-i} \leq p_i \leq p_{-i} + \lambda_i \\ 0 & \text{if } p_i > p_{-i} + \lambda_i \end{cases} \tag{2.11}$$

Proof: Follows from Corollary 1 and the definition of ex ante demand.

Corollaries 1 and 2 show that the law of demand does in fact follow in this model of two-sided markets. Furthermore, the demand system is one of *complementary goods* with the elasticity of demand being greater with respect to the side which is valued more. Recall the computer operating system two-sided market example given above. Assume that the total gains from trade from the application consumer side of the market is greater than the total gains from trade from the application writer side of the market for a fully patronized platform. Corollary 2 says that market demand will be more elastic with respect to the application writer side price than to the application consumer side price. This suggests that, all else equal, operating system platforms will tend to charge the application consumer side more and the application writer side less.

A full treatment of the social welfare implications of market structure implied by this model is beyond the scope of this paper. See the Appendix for several preliminary results on this topic.

Finally, Proposition 2 offers predictions regarding the relationship between welfare and outlay allocation between the sides of the market. Because I have normalized the consumers populations to one per side, λ_i may be interpreted as the sum of all gains from trade taken by side i agents when all side i agents interact with all side $-i$ agents. Also, p_i may be interpreted as the *revenue* to the platform from side i agents. Recall the computer operating system example discussed above and assume for the moment that the market is monopolized and software consumers take a greater fraction of the total gains from trade on a fully patronized platform than the software producers. Proposition 2 says that this would be so if and only if the total revenue generated from the software consumer side is greater than the total revenue generated from the software writer side.

2.6 Experiment

2.6.1 Setup

It remains to be known how the *simple platform game* is played in the "real world." To probe this issue, I created a computerized *simple platform game* for human subjects to play. The game was written, networked, and implemented using the Z-Tree (Fischbacher (2007)) platform and experiments were carried out in the Gregory Wachtler Experimental Economics Laboratory at Rutgers University in October and November 2010.

This paper focussed on the chicken-or-the-egg problem of two sided market, a demand side phenomenon. Therefore, I focussed on the demand side of the market by choosing prices randomly across games while allowing players to respond freely to those prices. In essence, I tested the stage three sub-game of the *simple platform game.*

In order to operationalize an experimental version of the *simple platform game*, some simplifications were necessary. Of course, a continuum of players, or anything close to it, is impossible. To maximize the amount of data I had to work with, then, I filled the laboratory to capacity with twenty players. These players were Rutgers University undergraduate students. Next, the platform quality parameter was restricted to the set $\{1, ..., 25\}$. This set was chosen to simplify the cognitive load on the players. The price space was then restricted to the set $\{14, 19, 24\}$ and the valuation parameter space restricted to the set $\{1, 1.5\}$ to ensure the existence of the *chicken-or-the-egg* problem and enough variation in the predicted threshold.

The valuation parameters are interpreted slightly differently in this game than in the true *simple platform game.* In this game, valuation parameters are interpreted as the *per interaction* marginal benefit. Since there are twenty players in total and each can interact with up to ten other players on the platform, there are two hundred possible interactions on the platform. Therefore, the total gains from trade available to the players varied between two hundred and three hundred. In the pure *simple platform game*, a valuation parameter is interpreted as the total gains from trade earned by one side of the market when the platform is fully patronized by both sides. Under the parameterizations for this experimental version, this gives valuation parameters of one hundred or one hundred and fifty. The purpose of redefining the valuation parameters in the experimental version is, again, to reduce the cognitive load on the subjects by giving them a more intuitive notion of the value of interaction.

Each session consisted of twenty students playing a series of *simple platform games* on a single day. This usually took between one and two hours. Each session consisted of thirty six rounds. In each round, twenty five *simple platform games* were played simultaneously. One platform game was played for each platform quality in the platform quality space. The order of those qualities was randomly chosen each round. The purpose of choosing platform qualities in this pseudo-random fashion was to increase variation for statistical purpose. Since strategies are functions from the platform quality space and parameters into a join decision, another purpose of having players face all qualities levels at once is to see a player's entire strategy at that moment for given prices and valuation parameters.

Students were divided randomly into two groups, each representing a side of the platform. Since the *simple platform game* is static in nature, I wanted to minimize the dynamic effects that necessarily crop up when players play a static game repeatedly. Therefore, across games within a session, the side a player belonged to switched randomly. Also, within a session, a total of nine different price-valuation parameterizations were encountered by the players. All parameterizations were encountered once and then cycled through in the same order three additional times. Again, the purpose here is to preserve static effects and to minimize dynamic effects.

Players were incentivized by a monetary reward directly proportional to the utility they obtained in the games they played. Utility was summed over all games and multiplied by a reward factor which was payed out to students upon completion of the thirty-sixth game. A minimum payment of \$5 was guaranteed.

Each session began with a public reading of the instructions. Paper copies of the instructions were available to each player. After the instruction were read, players were given the opportunity to ask questions. Once all questions were answered, the first round was initiated.

In each round, players were presented first with a decision screen. The decision screen displays the price and valuation parameterization for that game at the header of the screen. Running down the center of the screen were twenty-five radio buttons, one for each of the twenty-five separate games that were being played. Recall that each of the twenty-five games is associated with a platform and that each of those platforms has an associated unique integer quality level between one and twenty-five. Each radio button gives the player two options, one for each action available to him: join and do not join. See Figure 2.5 for a screenshot of the decision screen.

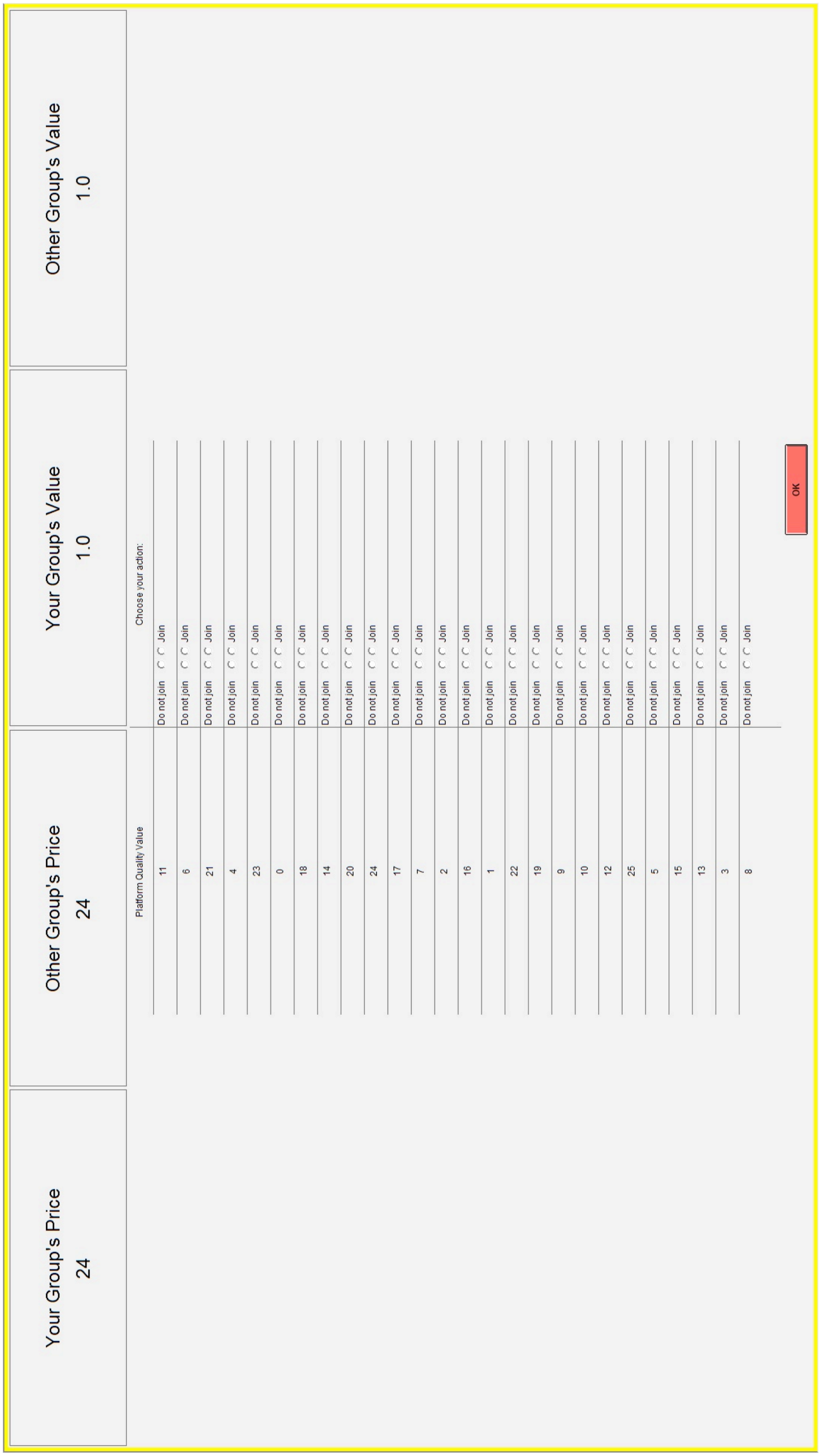


Figure 2.5: The decision screen.

There was no time limit for players to make their decisions. Players were free to revise their decisions as they wished until they clicked the OK button. After that, players' screens went blank. Once all players clicked their OK buttons, all players screens proceeded to the payoff screen. In the payoff screen each player could see, for each of the twenty-five games played, what his decision was, the number of players from his side who joined, the number of players from the other side who joined and the utility payoff taken from that game. See Figure 2.6 for a screenshot of the payoff screen.

Figure 2.6: The payoff screen.

Your Group's Price	Other Group's Price	Your Group's Value	Other Group's Value
24	24	1.0	1.0

Platform Quality Value	Your action was:	Number of Joiners from Your Group	Number of Joiners From Other Group	Your payoff:
11	Join	1	1	-12
6	Do not join	0	1	0
21	Do not join	0	1	0
4	Join	1	0	-20
23	Join	1	0	-1
0	Do not join	0	0	0
18	Do not join	0	1	0
14	Join	1	1	-9
20	Join	1	1	-3
24	Do not join	0	0	0
17	Do not join	0	0	0
7	Join	1	0	-17
2	Join	1	1	-21
16	Do not join	0	1	0
1	Do not join	0	1	0
22	Join	1	0	-2
19	Join	1	0	-5
9	Do not join	0	0	0
10	Do not join	0	1	0
12	Join	1	1	-11
25	Join	1	1	2
5	Do not join	0	0	0
15	Do not join	0	0	0
13	Join	1	0	-11
3	Join	1	1	-20
8	Do not join	0	1	0

2.6.2 Results

The analysis above assumed that players are utility maximizers and use iterated deletion of dominated strategies in selecting an ultimate strategy. To test that assumption, I begin by examining how frequently players chose dominated actions. I did this two ways depending on the definition of a dominated action. As discussed in detail in the analysis of the *simple platform game* above, when the *chicken-or-the-egg* problem exists, there are two orders of dominated strategies. First, a player will not join the platform if $\theta < p_i - \lambda_i$ and will join a platform if $\theta > p_i$. I call strategies inconsistent with these rules to be dominated level one. When $\mathcal{W}_A \cap \mathcal{W}_B \neq \emptyset$, however, players will not join for $min_{i \in \mathcal{I}} (p_i - \lambda_i) < \theta < max_{i \in \mathcal{I}} (p_i - \lambda_i)$ and will join for $min_{i \in \mathcal{I}} p_i < \theta < max_{i \in \mathcal{I}} p_i$. I call strategies inconsistent with these additional rules to be dominated level two. Note that a strategy which is undominated at level 2 must also be undominated at level 1.

See Figure 2.7 for the prevalence of dominated actions by period. All actions chosen in all sessions are first pooled. Each action is associated with a game and a corresponding platform quality level. I count the number of actions for which the associated quality level implies a certain action is dominant. This total is the demoninator of the ratio. The numerator is then the number of those actions for which the player did in fact choose the dominated action.

Frequency of dominated actions is low, less than 5% of the time for most rounds. It appears to be decreasing monotonically with noise as the session progresses. By far the period with the greatest prevalence of dominated actions is period one with over 20%.

Next, I check the prevalence of threshold strategies. That is, players should join the platform if and only if the platform quality is greater than a certain level. See Figures 2.8 and 2.9 for views of the complete decision histories of two players. These represent opposite extremes of observed player behaviors. I term players whose decision histories resemble Figure 2.9 *confused*. I term term players whose decision histories resemble Figure 2.8 *good*. Out of the eighty players on which this data is based, two were confused. Confused players are clearly not using threshold strategies while good players are (with few exceptions).

Figure 2.7: Number of dominated decisions relative to the number of dominated actions by period. Dominated decisions are taken here to indicate an instance when a player chooses a dominated action. The normalization is used to control for the fact that the number of dominated decisions should vary with the treatment depending on how many dominated actions theory predicts for those parameters. Level-1 dominated actions are dominated without any strategic consideration. Level-2 dominated actions are dominated when Level-1 dominated actions are eliminated.

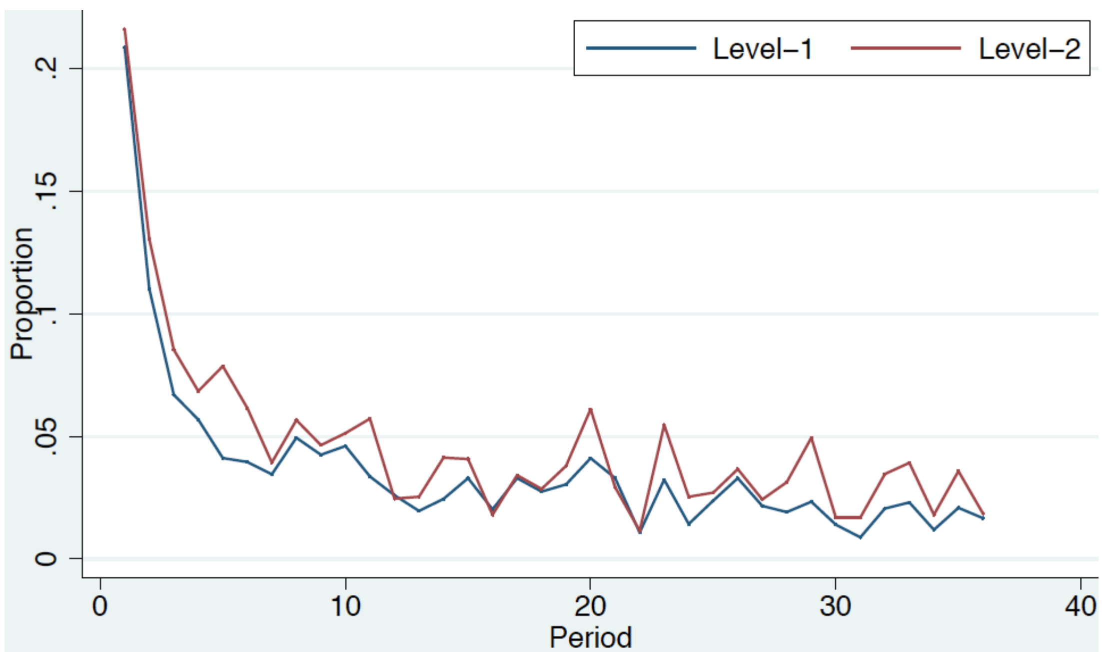


Figure 2.8: Typical decisions for a good player. The vertical axis indicates the discrete choice variable for whether or not to join. Here, a "1" indicates a join decision while a "0" indicates a do not join decision. The horizontal axis indicates the platform quality.

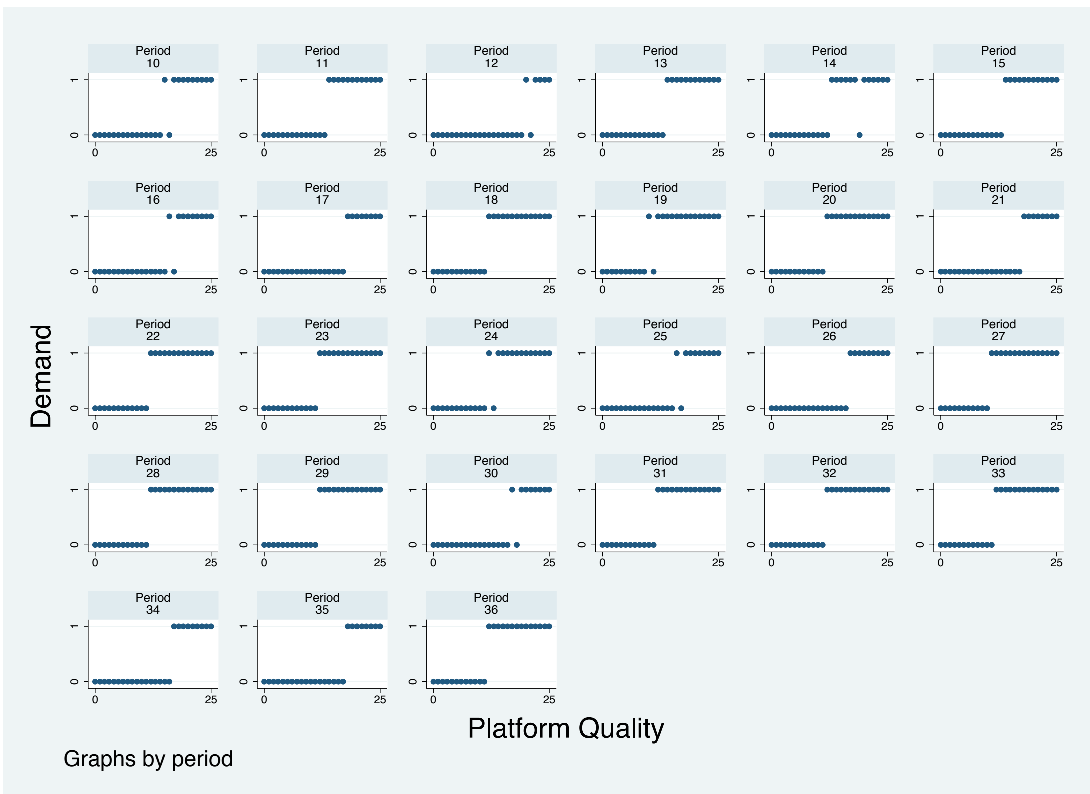
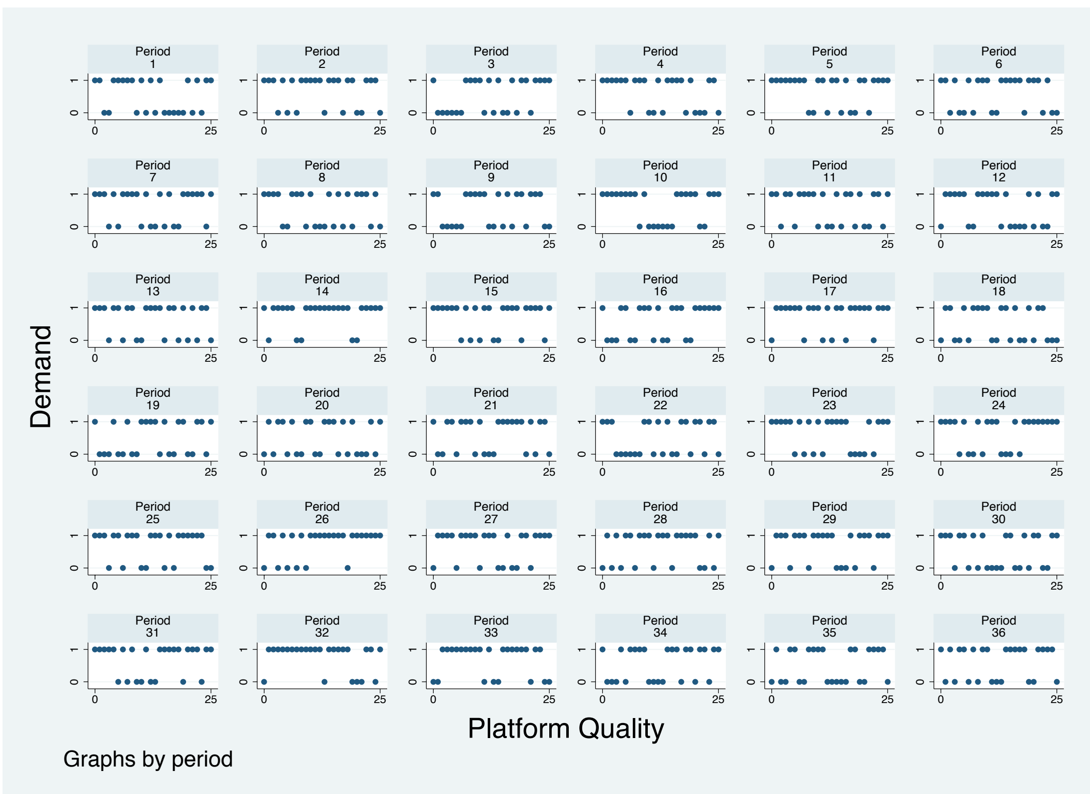


Figure 2.9: Typical decisions for a confused player. The vertical axis indicates the discrete choice variable for whether or not to join. Here, a "1" indicates a join decision while a "0" indicates a do not join decision. The horizontal axis indicates the platform quality.



The above analysis makes several additional predictions on the demand side behavior of rational agents in a two-sided market. I list them here in order of increasing novelty. *First*, the platform's goods are complementary and obey the law of demand. *Second*, Side A's demand and Side B's demand should always be equal.[5] *Third*, demand elasticity is greater with respect to the price of the *more desired* side of the market and, consequently, a monopolist platform will decrease the total price burden for that side.

To test these predictions, I use an econometric model. First, I exclude all the decisions of confused players from the analysis as well as all decisions made during rounds one through nine, the first cycle through all paramterizations faced within a session. With the remaining decision data, I assume that players are using *some* kind of threshold strategy. Following Heinemann et al. (2004) methodology, I model the join decision using a logit model with the join decision as the binary dependent variable and a linear combination of the prices and marginal benefit parameters as the independent latent variable.

$$Pr(join|parameters) = \frac{e^z}{1+e^z}$$

where:

$$\begin{aligned} z \equiv\, & Intercept + Quality \cdot X_{Quality} + OwnPrice19 \cdot X_{OwnPrice19} + \\ & OwnPrice24 \cdot X_{OwnPrice24} + OtherPrice19 \cdot X_{OtherPrice19} + \\ & OtherPrice24 \cdot X_{OtherPrice24} + OwnValueHi \cdot X_{OwnValueHi} + \\ & OtherValueHi \cdot X_{OtherValueHi} + OwnPrice19VLE \cdot X_{OwnPrice19VLE} + \\ & OwnPrice24VLE \cdot X_{OwnPrice24VLE} + OtherPrice19VLE \cdot X_{OtherPrice19VLE} + \\ & OtherPrice24VLE \cdot X_{OtherPrice24VLE} \end{aligned} \tag{2.12}$$

See Table 2.1 for variable definitions. See Table 2.2 for the results of the econometric regression where a individual-level random effect is included.

[5]This level of demand will vary in price and valuation parameters. The exception to this rule is for price and valuation parameters where the chicken-or-the-egg problem does not exist. See 2.2(b).

Table 2.1: Variables Used

Name	Nature	Description
$X_{Quality}$	continuous	Quality of the platform
$X_{OwnPrice19}$	dummy	=1 if own price is 19
$X_{OwnPrice24}$	dummy	=1 if own price is 24
$X_{OtherPrice19}$	dummy	=1 if other price is 19
$X_{OtherPrice24}$	dummy	=1 if other price is 24
$X_{OwnValueHi}$	dummy	=1 if own value is 1.5 instead of 1.0
$X_{OtherValueHi}$	dummy	=1 if other value is 1.5 instead of 1.0
$X_{OwnPrice19VLE}$	dummy	=1 if own price is 19 and own value is less than other value
$X_{OwnPrice24VLE}$	dummy	=1 if own price is 24 and own value is less than other value
$X_{OtherPrice19VLE}$	dummy	=1 if other price is 19 and own value is less than other value
$X_{OtherPrice24VLE}$	dummy	=1 if other price is 24 and own value is less than other value

Table 2.2: Econometric results

Coefficient	OLS Estimate	Z-score
Quality	0.871	83.09
OwnPrice19	-1.652	-24.89
OwnPrice24	-3.257	-44.24
OtherPrice19	-1.092	-16.95
OtherPrice24	-1.511	-23.01
OwnValueHi	0.366	6.06
OtherValueHi	0.168	2.78
OwnPrice19VLE	-0.558	-0.46
OwnPrice24VLE	0.520	4.47
OtherPrice19VLE	0.0627	0.54
OtherPrice24VLE	-0.504	-4.47
Intercept	-1.677	-10.52

2.6.3 Discussion

I define the "threshold" as the value the latent variable must take for the probability of joining to equal one-half. This value is obtained by solving:

$$\frac{e^z}{1+e^z} = 0.5$$

This equation implies $z = 0$. The solution is a restriction on the set of parameters which may be solved for quality to obtain a function for the quality threshold in terms of the prices and valuation parameters:

$$Intercept + Quality \cdot x^* + \beta \cdot NonQualityParameters = 0 \Leftrightarrow$$

$$x^* = \frac{-Intercept - \beta \cdot NonQualityParameters}{Quality}$$

where x^* is the threshold for a given set of parameters and β is a vector of coefficients for all non-quality parameters. This equation makes it clear that the threshold, and hence demand, moves in the opposite direction to the sign of the corresponding coefficient. Since the estimated coefficients directly imply estimates of the thresholds and I am only interested in the signs and relative magnitudes of the threshold comparative statics, a full marginal effects analysis is not required.

The first hypothesis stated that the demand system should be one of complementary goods. The fact that price effects are negative and that greater price changes lead to greater changes in the threshold are together strong support for the law of demand hypothesis. Furthermore, the fact the other price as well as own price coefficients are negative is supportive of the hypothesis of complentarity between the two goods.

The next hypothesis states that the thresholds should be the same between the two sides whenever the chicken-or-the-egg problem exists. The parameterizations chosen in these experiments guarantee that the problem does, in fact, exist always. One necessary condition for this hypothesis to be true is that if an own-price change of x implies an own-threshold change of y, then an other-price change of x must also imply an own-threshold change of y. Were this not so, then for some prices, each sides will change their threshold by different amounts and they must either have been playing different threshold to start with or be playing different thresholds afterwards. For example, if for a given set of prices each side joins the platform for signals greater than 10, Side A's threshold increases by 2 with respect to an own-price increase of 1 and Side B's threshold increases by 1 with respect to an other-price increases of 1, then after Side A's price increases by one, Side A's threshold will be 12 and Side B's will be 11. Clearly, thresholds are not always equal. Table 2.2 gives strong evidence that own price and other price effects are in fact different and therefore that sides do not choose the same thresholds. Instead, there is a overreaction with respect to own-price.

The final hypothesis states that the price elasticity is greatest with respect to the side which is most desired. The strong significance of the coefficients OwnPrice24VLE and OtherPrice24VLE suggest that this is in fact not the case. The large positive magnitude on OwnPrice24VLE shows that when the player's side's value is smaller, the effect of a change in his own price is mitigated. In contrast, the large negative magnitude on OtherPrice24VLE shows that when the player's side's value is smaller, the effect on a change in other price is exacerbated. These observations are in direct contradiction of my hypothesis.

2.7 Appendix

Proof of Proposition 2:

Step 1: In stage three, an agent observes the access fee prices, (p_A, p_B), his private signal, $x_{i'}$, and the investment level, ρ. x_i and ρ are not directly payoff relevant. They do, however, provide important information to agents. The payoff relevant factors that the agent must weight are the realized platform quality, θ, the fraction of agents who will join, D_{-i}, his price, p_i, and his valuation parameter, λ_i. The agent forms beliefs on θ according to Baye's rule using his information set. Let $\Phi_{A|b}(\cdot)$ and $\phi_{A|b}(\cdot)$ denote the CDF and PDF of random variable A conditional on a realization, b, of random variable B. Let $X_{i'}$ denote the random variable which generates the informative signal for agent i'. Let Θ denote the random variable which generates the platform quality. At the beginning of stage 3, an agent knows (x_i, ρ), $\phi_{X_{i'}|\theta}(\cdot)$ and $\phi_{\Theta|\rho}(\cdot)$ by assumption. I begin by rewriting these distributions in terms of the standardized normal distribution:

$$\begin{aligned}\Phi_{X_{i'}|\theta}(z) &= Pr(x_{i'} < z|\Theta = \theta, x_{i'} \sim X_{i'}) \\ &= Pr\left(\frac{x_{i'}-\theta}{\sigma_X} < \frac{z-\theta}{\sigma_X}|\Theta = \theta, x_{i'} \sim X_{i'}\right) \\ &= \Phi_N\left(\frac{z-\theta}{\sigma_X}\right)\end{aligned}$$

where Φ_N indicates the standard normal CDF. By differentiation:

$$\begin{aligned}\phi_{X_{i'}|\theta}(z) &= \frac{d\Phi_N(\frac{z-\theta}{\sigma_X})}{dz} \\ &= \frac{1}{\sigma_X}\phi_N\left(\frac{z-\theta}{\sigma_X}\right)\end{aligned}$$

Where ϕ_N indicates the standard normal PDF and σ_X indicates the standard deviation of the random variable $X_{i'}$. We similarly find that:

$$\phi_{\Theta|\rho}(z) = \frac{1}{\sigma_\Theta}\phi_N\left(\frac{z-\rho}{\sigma_\Theta}\right)$$

We can use these to derive $\phi_{\Theta|x_{i'},\rho}(\cdot)$ using Baye's rule:

$$\begin{aligned}\phi_{\Theta|x_{i'},\rho}(z) &= \frac{\phi_{X_{i'}|\theta,\rho}(z)\phi_{\Theta|\rho}(z)}{\phi_{X_{i'}|\rho}(z)} \\ &= \frac{\phi_{X_{i'}|\theta}(z)\phi_{\Theta|\rho}(z)}{\int_{-\infty}^{\infty}\phi_{X_{i'}|\theta}(z)\phi_{\Theta|\rho}(z)d\theta} \\ &= \frac{\phi_N(\frac{z-\theta}{\sigma_X})\phi_N(\frac{z-\rho}{\sigma_\Theta})}{\int_{-\infty}^{\infty}\phi_N(\frac{z-\theta}{\sigma_X})\phi_N(\frac{z-\rho}{\sigma_\Theta})d\theta} \\ &= \frac{1}{\sqrt{\frac{\sigma_X^2\sigma_\Theta^2}{\sigma_X^2+\sigma_\Theta^2}}}\phi_N\left\{\frac{z-\frac{\sigma_X^2\rho+\sigma_\Theta^2 x_{i'}}{\sigma_X^2+\sigma_\Theta^2}}{\sqrt{\frac{\sigma_X^2\sigma_\Theta^2}{\sigma_X^2+\sigma_\Theta^2}}}\right\}\end{aligned}$$

This distribution determines the agent's beliefs on the quality of the platform, given his information. Before I continue, it will be useful to define an agents beliefs on other agents signals.

$$\begin{aligned}
\phi_{X_i|x_{i'},\rho}(z) &= \int_{-\infty}^{\infty} \phi_{X_i|\theta}(z)\phi_{\Theta|x_{i'},\rho}(z)d\theta \\
&= \int_{-\infty}^{\infty} \frac{1}{\sigma_X}\phi_N\left(\frac{z-\theta}{\sigma_X}\right)\frac{1}{\sqrt{\frac{\sigma_X^2\sigma_\Theta^2}{\sigma_X^2+\sigma_\Theta^2}}}\phi_N\left(\frac{z-\frac{\sigma_X^2\rho+\sigma_\Theta^2 x_{i'}}{\sigma_X^2+\sigma_\Theta^2}}{\sqrt{\frac{\sigma_X^2\sigma_\Theta^2}{\sigma_X^2+\sigma_\Theta^2}}}\right)d\theta \\
&= \frac{1}{\sqrt{\frac{2\sigma_X^2\sigma_\Theta^2+\sigma_X^4}{\sigma_X^2+\sigma_\Theta^2}}}\phi_N\left(\frac{z-\frac{\sigma_X^2\rho+\sigma_\Theta^2 x_{i'}}{\sigma_X^2+\sigma_\Theta^2}}{\sqrt{\frac{2\sigma_X^2\sigma_\Theta^2+\sigma_X^4}{\sigma_X^2+\sigma_\Theta^2}}}\right)
\end{aligned}$$

An agent's payoff depends on the quality of the platform, the number of agents who join, the valuation he places on opposite side membership and the price his side pays:

$$u_{i'}(p_i, D_{-i}, \theta) = \theta + \lambda_i D_{-i}(p_i, p_{-i}, \theta) - p_i$$

Give our construction of the agents posterior of θ given his information, we may now write the expected payoff of joining:

$$\mathbb{E}[u_{i'}(p_i, D_{-i}, \theta | p_{-i}, x_{i'}, \rho)] = \frac{\sigma_X^2\rho + \sigma_\Theta^2 x_{i'}}{\sigma_X^2 + \sigma_\Theta^2} + \lambda_i \mathbb{E}[D_{-i}(p_i, p_{-i}, \theta)] - p_i$$

I consider now a Bayesian Nash Equilibrium solution concept to the simultaneous-move stage three game. All agents form the expected payoff function given their information as described above. Now I proceed to eliminate dominated actions.

Define the *optimist* function:

$$\bar{u}_{i'}(p_i, p_{-i}, x_{i'}, \rho) = \frac{\sigma_X^2\rho + \sigma_\Theta^2 x_{i'}}{\sigma_X^2 + \sigma_\Theta^2} + \lambda_i - p_i \tag{2.13}$$

This indicates the expected payoff to an agent i' who believes $D_{-i} = 1$.

Define $\underline{x}_i^0$:

$$\underline{x}_i^0 \equiv argmax_x\{\bar{u}_i(p_i, p_{-i}, x, \rho)\} \tag{2.14}$$

$$s.t.\ \bar{u}_i(p_i, p_{-i}, x, \rho) \leq 0$$

Claim:

$$argmax_x\{\bar{u}_i(p_i, p_{-i}, x, \rho)\}$$

$$s.t.\ \bar{u}_i(p_i, p_{-i}, x, \rho) \leq 0$$

is a singleton set.

Proof: $\bar{u}_i(p_i, p_{-i}, x, \rho)$ is strictly increasing in x:

$$\frac{\partial\{\bar{u}_i(p_i, p_{-i}, x, \rho)\}}{\partial x} = \frac{\partial\{\frac{\sigma_X^2\rho+\sigma_\Theta^2 x}{\sigma_X^2+\sigma_\Theta^2} + \lambda_A - p_i\}}{\partial x} = \frac{\sigma_\Theta^2}{\sigma_X^2+\sigma_\Theta^2} > 0 \ \forall x$$

$\frac{\sigma_X^2\rho+\sigma_\Theta^2 x}{\sigma_X^2+\sigma_\Theta^2} + \lambda_A - p_A = 0$ has a solution. The result follows.

For all side i consumers who observe $x_{i'} \leq \underline{x}_i^0$, NJ is a dominant action by definition of $\underline{x}_i^0$. Now I consider side $-i$ consumers. First, define the $\underline{x}$-optimist function:

$$\bar{u}_i(p_i, p_{-i}, x, \rho, \underline{x}) = \frac{\sigma_X^2\rho+\sigma_\Theta^2 x}{\sigma_X^2+\sigma_\Theta^2} + \lambda_i\left(1 - \Phi_N\left\{\frac{\underline{x} - \frac{\sigma_X^2\rho+\sigma_\Theta^2 x}{\sigma_X^2+\sigma_\Theta^2}}{\sqrt{\frac{2\sigma_X^2\sigma_\Theta^2+\sigma_X^4}{\sigma_X^2+\sigma_\Theta^2}}}\right\}\right) - p_i \tag{2.15}$$

The $\underline{x}$-optimist function gives the expected payoff to a consumer who believes that consumers from the other side will join the platform if and only if their signal is greater than $\underline{x}$.

Define $\underline{x}_{-i}^0$:

$$\underline{x}_{-i}^0 = argmax_x\{\bar{u}_{-i}(p_i, p_{-i}, x, \rho, \underline{x}_i^0)\}$$

$$s.t.\ \bar{u}_{-i}(p_i, p_{-i}, x, \rho, \underline{x}_i^0) \leq 0$$

Claim:

$$argmax_x\{\bar{u}_{-i}(p_i, p_{-i}, x, \rho, \underline{x}_i^0)\}$$

$$s.t.\ \bar{u}_{-i}(p_i, p_{-i}, x, \rho, \underline{x}_i^0) \leq 0$$

is a singleton set.

Proof:

$\bar{u}_{-i}(p_i, p_{-i}, x, \rho, \underline{x}_i^0)$ is strictly increasing in x:

$$\frac{\partial\{\bar{u}_{-i}(p_i,p_{-i},x,\rho,\underline{x}_i^0)\}}{\partial x} = \frac{\sigma_\Theta^2}{\sigma_X^2+\sigma_\Theta^2} + \lambda_i\frac{\frac{\sigma_\Theta^2}{\sigma_X^2+\sigma_\Theta^2}}{\sqrt{\frac{2\sigma_X^2\sigma_\Theta^2+\sigma_X^4}{\sigma_X^2+\sigma_\Theta^2}}}\phi_N\left\{\frac{z-\frac{\sigma_X^2\rho+\sigma_\Theta^2 x}{\sigma_X^2+\sigma_\Theta^2}}{\sqrt{\frac{2\sigma_X^2\sigma_\Theta^2+\sigma_X^4}{\sigma_X^2+\sigma_\Theta^2}}}\right\} > 0 \ \forall x.$$

$\bar{u}_{-i}(p_i, p_{-i}, x, \rho, \underline{x}_i^0) < 0$ for $x < \frac{p_{-i}-\frac{\sigma_X^2}{\sigma_X^2+\sigma_\Theta^2}-1}{\frac{\sigma_\Theta}{\sigma_X^2+\sigma_\Theta^2}}$

$$\bar{u}_{-i}(p_i, p_{-i}, x, \rho, \underline{x}_i^0) > 0 \text{ for } x > \frac{p_{-i}-\frac{\sigma_X^2}{\sigma_X^2+\sigma_\Theta^2}+1}{\frac{\sigma_\Theta}{\sigma_X^2+\sigma_\Theta^2}}$$

The result follows.

Claim: $\bar{u}_i(p_i, p_{-i}, x, \rho, \underline{x})$ is strictly decreasing in $\underline{x}$.

Proof: $\frac{\partial\{\bar{u}_i(p_i,p_{-i},x,\underline{x})\}}{\partial \underline{x}} = \frac{-\lambda_i}{\sqrt{\frac{2\sigma_X^2\sigma_\Theta^2+\sigma_X^4}{\sigma_X+\sigma_\Theta}}}\phi_N\left\{\frac{\underline{x}-\frac{\sigma_X^2\rho+\sigma_\Theta^2 x}{\sigma_X^2+\sigma_\Theta^2}}{\sqrt{\frac{2\sigma_X^2\sigma_\Theta^2+\sigma_X^4}{\sigma_X+\sigma_\Theta}}}\right\} < 0\ \forall \underline{x}$

Define the sequences $\{\underline{x}_i^k\}_{k=0}^{\infty}$ and $\{\underline{x}_{-i}^k\}_{k=0}^{\infty}$ recursively:

$$\underline{x}_i^k = argmax_x\{\bar{u}_i(p_i, p_{-i}, x, \rho, \underline{x}_{-i}^{k-1})\}$$

$$s.t.\ \bar{u}_i(p_i, p_{-i}, x, \rho, \underline{x}_{-i}^{k-1}) \leq 0$$

$$\underline{x}_{-i}^k = argmax_x\{\bar{u}_{-i}(p_i, p_{-i}, x, \rho, \underline{x}_i^{k-1})\}$$

$$s.t.\ \bar{u}_i(p_i, p_{-i}, x, \rho, \underline{x}_i^{k-1}) \leq 0$$

Claim: Sequences $\{\underline{x}_i^k\}_{k=0}^{\infty}$ and $\{\underline{x}_{-i}^k\}_{k=0}^{\infty}$ are strictly increasing.

Proof: The result follows immediately from the facts that $\bar{u}(p_i, p_{-i}, x, \rho, \underline{x})$ is strictly decreasing in $\underline{x}$ and strictly increasing in x for all i.

We now look for the limit of the sequence. If such a limit exists, it must satisfy the following conditions:

$$\bar{u}_i(p_i, p_{-i}, \underline{x}_i, \rho, \underline{x}_{-i}) = \frac{\sigma_X^2\rho + \sigma_\Theta^2\underline{x}_i}{\sigma_X^2+\sigma_\Theta^2} + \lambda_i\left(1-\Phi_N\left\{\frac{\underline{x}_{-i} - \frac{\sigma_X^2\rho+\sigma_\Theta^2\underline{x}_i}{\sigma_X^2+\sigma_\Theta^2}}{\sqrt{\frac{2\sigma_X^2\sigma_\Theta^2+\sigma_X^4}{\sigma_X^2+\sigma_\Theta^2}}}\right\}\right) - p_i = 0 \tag{2.16}$$

$$\bar{u}_{-i}(p_i, p_{-i}, \underline{x}_{-i}, \rho, \underline{x}_i) = \frac{\sigma_X^2\rho + \sigma_\Theta^2\underline{x}_{-i}}{\sigma_X^2+\sigma_\Theta^2} + \lambda_{-i}\left(1-\Phi_N\left\{\frac{\underline{x}_{i} - \frac{\sigma_X^2\rho+\sigma_\Theta^2\underline{x}_{-i}}{\sigma_X^2+\sigma_\Theta^2}}{\sqrt{\frac{2\sigma_X^2\sigma_\Theta^2+\sigma_X^4}{\sigma_X^2+\sigma_\Theta^2}}}\right\}\right) - p_{-i} = 0$$

Denote $(\underline{x}_i^*, \underline{x}_{-i}^*)$ a solution to the system.

By the implicit function theorem, we have $\frac{\partial \underline{x}_{-i}(\underline{x}_i)}{\partial \underline{x}_i} > 0$ and $\frac{\partial \underline{x}_i(\underline{x}_{-i})}{\partial \underline{x}_{-i}} > 0$. Also by the implicit function theorem, we have $\frac{\partial \underline{x}_i(\underline{x}_{-i})}{\partial \underline{x}_{-i}} < 1$ for $\frac{\sigma_x^4}{\sigma_\Theta^4}\frac{\sigma_x^2+\sigma_\Theta^2}{\sigma_x^4+2\sigma_\Theta^2\sigma_x^2} \leq \frac{2\pi}{\lambda_i^2}$ and $\frac{\partial \underline{x}_{-i}(\underline{x}_i)}{\partial \underline{x}_i} < 1$ for $\frac{\sigma_x^4}{\sigma_\Theta^4}\frac{\sigma_x^2+\sigma_\Theta^2}{\sigma_x^4+2\sigma_\Theta^2\sigma_x^2} \leq \frac{2\pi}{\lambda_{-i}^2}$. These two facts imply that if $\frac{\sigma_x^4}{\sigma_\Theta^4}\frac{\sigma_x^2+\sigma_\Theta^2}{\sigma_x^4+2\sigma_\Theta^2\sigma_x^2} \leq min\{\frac{2\pi}{\lambda_i^2}, \frac{2\pi}{\lambda_{-i}^2}\}$, then $(\underline{x}_i^*, \underline{x}_{-i}^*)$ is unique.

To summarize, any side i agent strategy that survives iterated deletion of dominated strategies will choose $NJ_{i'}$ for all $x_{i'} < \underline{x}_i^*$.

A symmetric argument concludes that there are unique $(\bar{x}_i^*, \bar{x}_{-i}^*)$ such that any side i agent strategy that survive iterated deletion of dominated strategies will choose $J_{i'}$ for all $x_{i'} > \bar{x}_i^*$ and $(\bar{x}_i^*, \bar{x}_{-i}^*)$ is determined by Equation ((2.16)) as well. Define $(x_i^*, x_{-i}^*) \equiv (\bar{x}_i^*, \bar{x}_{-i}^*) = (\underline{x}_i^*, \underline{x}_{-i}^*)$. I now restrict attention to the threshold values for infinitesimal private signal noise. After solving the system ((2.16)) for $(\underline{x}_i^*, \underline{x}_{-i}^*)$ explicitly, and taking the limit as private signal goes to zero, the following is obtained:

$$\lim_{\sigma_x \to 0} x_i^*(p_i, p_{-i}, \rho) = \begin{cases} p_i & \text{if } p_i < p_{-i} - \lambda_{-i} \\ \frac{\lambda_{-i}}{\lambda_i + \lambda_{-i}} p_i + \frac{\lambda_i}{\lambda_i + \lambda_{-i}} p_{-i} - \frac{\lambda_i \lambda_{-i}}{\lambda_i + \lambda_{-i}} & \text{if } p_{-i} - \lambda_{-i} \le p_i \le p_{-i} + \lambda_i \\ p_i - \lambda_i & \text{if } p_i > p_{-i} + \lambda_i \end{cases}$$

Step 2: Now consider Stage one. The monopolist platform chooses prices and investment to maximize his expected profits:

$$\pi(p_i, p_{-i}, \rho) = p_i \int_{-\infty}^{+\infty} D_i(p_i, p_{-i}, \theta)\phi(\theta|\rho)d\theta + p_{-i} \int_{-\infty}^{+\infty} D_{-i}(p_i, p_{-i}, \theta)\phi(\theta|\rho)d\theta - C(\rho)$$

Given the stage 3 subgame equilibrium, the expected profit function may be rewritten:

$$\pi(p_i, p_{-i}, \rho) = \left(1 - \Phi\left(\frac{x_i^*(p_i, p_{-i}, \rho) - \rho}{\sigma_\theta}\right)\right) p_i + \left(1 - \Phi\left(\frac{x_{-i}^*(p_i, p_{-i}, \rho) - \rho}{\sigma_\theta}\right)\right) p_{-i} - C(\rho)$$

where we focus attention on the limiting case of infinitesimal agent signal noise. Without loss of generality, assume $\lambda_i > \lambda_{-i}$.

Case 1: $p_i < p_{-i} - \lambda_{-i}$

$$\pi(p_i, p_{-i}, \rho) = \left(1 - \Phi\left(\frac{p_i - \rho}{\sigma_\theta}\right)\right) p_i + \left(1 - \Phi\left(\frac{p_{-i} - \lambda_{-i} - \rho}{\sigma_\theta}\right)\right) p_{-i} - C(\rho)$$

Case 2: $p_{-i} - \lambda_{-i} \le p_i \le p_{-i} + \lambda_i$

$$\pi(p_i, p_{-i}, \rho) = \left(1 - \Phi\left(\frac{\frac{\lambda_{-i}}{\lambda_i + \lambda_{-i}} p_i + \frac{\lambda_i}{\lambda_i + \lambda_{-i}} p_{-i} - \frac{\lambda_i \lambda_{-i}}{\lambda_i + \lambda_{-i}} - \rho}{\sigma_\theta}\right)\right) (p_i + p_{-i}) - C(\rho)$$

Case 3: $p_i > p_{-i} + \lambda_i$

$$\pi(p_i, p_{-i}, \rho) = \left(1 - \Phi\left(\frac{p_i - \lambda_i - \rho}{\sigma_\theta}\right)\right) p_i + \left(1 - \Phi\left(\frac{p_{-i} - \rho}{\sigma_\theta}\right)\right) p_{-i} - C(\rho)$$

Claim: The monopoly prices can neither fall into Case 1 nor Case 3.

Proof: To prove this, I require the following Lemma.

Lemma 4: $y'(a) < 0$ for the following implicit definition of y:

$$1 - \Phi(y) - (y + a)\phi(y) = 0$$

where Φ and ϕ are the CDF and PDF of the standard normal distribution.

Proof: By the implicit function theorem:

$$y'(a) = \frac{1}{\frac{d}{dy}\left[\frac{1-\Phi(y(a))}{\phi(y(a))}\right] - 1}$$

The result will follow if $\frac{d}{dy}\frac{1-\Phi(y(a))}{\phi(y(a))} < 0$. To see that this is true:
$\left[ln\phi(x)\right]'' = -\frac{1}{\sigma^2} \Rightarrow \phi(x)$ is log-concave $\Rightarrow [1-\Phi(x)]$ is log-concave$\Rightarrow \left[\frac{\phi(x)}{1-\Phi(x)}\right]' > 0 \Rightarrow$ $\left[\frac{1-\Phi(x)}{\phi(x)}\right]' < 0.\blacksquare$

Now consider Case 1. Write the Case 1 first order condition using the change of variables $\widetilde{p}_i \equiv \frac{p_i-\rho}{\sigma_\theta}$ and $\widetilde{p}_{-i} \equiv \frac{p_{-i}-\lambda_{-i}-\rho}{\sigma_\theta}$:

$$1-\Phi\left(\widetilde{p}_i\right) - \left(\widetilde{p}_i + \frac{\rho}{\sigma_\theta}\right)\phi\left(\widetilde{p}_i\right) = 0$$

$$1-\Phi\left(\widetilde{p}_{-i}\right) - \left(\widetilde{p}_{-i} + \frac{\lambda_{-i}+\rho}{\sigma_\theta}\right)\phi\left(\widetilde{p}_{-i}\right) = 0$$

Denote $(\widetilde{p}_i^*, \widetilde{p}_{-i}^*)$ the solution to the first order condition. By Lemma 4:

$$\frac{\lambda_{-i}+\rho}{\sigma_\theta} > \frac{\rho}{\sigma_\theta} \Rightarrow$$

$$\widetilde{p}_i^* > \widetilde{p}_{-i}^* \Leftrightarrow$$

$$p_i^* > p_{-i}^* - \lambda_{-i}$$

Therefore, the monopoly prices must satisfy: $p_i^* \not< p_{-i}^* - \lambda_{-i}$. By a symmetric argument, it may be shown that the prices may not fall into Case 3, that is, $p_i^* \not> p_{-i}^* + \lambda_i$

The preceding argument implies that the best response monopoly prices must fall into Case 2. Given a fixed total price, $\bar{p} \equiv p_i + p_{-i}$, and an investment level, ρ, the monopolist will choose (p_i, p_{-i}) to maximize demand. This is a constrained linear optimization problem whose solution is to set p_{-i} as low as possible and p_i as high as possible. Therefore, monopoly prices must satisfy $p_i^* \geq p_{-i}^* + \lambda_i$. But since we have eliminated Case 1 and Case 3, we must have $p_i^* = p_{-i}^* + \lambda_i$. The total price, $\bar{p}$, and ρ are then chosen to maximize the Case 2 profit, given that $p_i = \frac{\bar{p}+\lambda_i}{2}$, $p_{-i} = \frac{\bar{p}-\lambda_i}{2}.\blacksquare$

Proposition 3: In the limit as signal noise goes to zero, the socially optimal prices, (p_A^*, p_B^*), must satisfy:

$$\lambda_A p_B^* + \lambda_B p_A^* = -\frac{\lambda_A^2 + \lambda_B^2}{2}$$

$$p_A^* \in [p_B^* - \lambda_B, p_B^* + \lambda_A]$$

In addition, investment should be set to equate the expected marginal increase to the gains from trade to the marginal cost of investment.

Proof: Since fixed costs and marginal costs are zero, the social planner will set prices to maximize the total gains from trade on the platform. Gains are maximized when everyone joins the platform if and only if the quality of the platform is sufficiently high that when everyone is on, there is a positive surplus. This condition is met if and only if:

$$\theta + \lambda_A + \theta + \lambda_B > 0 \Leftrightarrow$$

$$\theta > -\frac{\lambda_A + \lambda_B}{2}$$

In the limit as signal noise goes to zero, this is equivalent to requiring $x_i^* = x_{-i}^* = -\frac{\lambda_A+\lambda_B}{2}$. Using Proposition 2, the result obtains.

Corollary 3: A social welfare maximizing set of prices must always have at least one negative price and if one price is positive, it must be for the side with the greater marginal benefit of interaction.

Proof: Follows immediately from Proposition 2.

Corollary 4: There always exists a set of social optimal prices such that the associated total price is less than the monopolist's equilibrium total price.

Proof: Follows immediately from Propositions 1 and 2.

Chapter 3

Experimental Comparative Statics of a Large Coordination Game with Endogenous Sunspot Equilibria

3.1 Background

Coordination games with strategic complementarity result in multiple pareto ranked equilibria. Carlsson and van Damme (1993b) show that a unique equilibrium exists in two-player binary coordination games when the common knowledge of payoffs assumption is relaxed is certain way. More specifically, consider the following game and information structure. Players do not observe x directly and instead receive a noisy signal x_i.

	α_2	β_2
α_1	x, x	$x, 0$
β_1	$0, x$	$4, 4$

$$x \sim \; U[\underline{x}, \bar{x}], \underline{x} < 0, \bar{x} > 4$$

$$x_i | x \sim U[x - \epsilon, x + \epsilon]$$

Such a game has been termed a "global game." See Carlsson and van Damme (1993b) for more detail. In the unique equilibrium, players choose a "threshold strategy" in which they play the risk dominant equilibrium conditional on their signal. For the game above this translates into playing α for signals greater than two and β for signals less than two. While this may appear to be a natural result, it could not be obtained with common knowledge of x. This result is particularly surprising as equilibrium refinement is usually associated with the strengthening of structure, not the weakening of it. Thus, "global games" offer both precise predictions and realism.

Since Carlsson and van Damme (1993b), the results have been generalized into richer settings (Morris and Shin (2001a), Frankel et al. (2003)). In their celebrated model, Morris and Shin (1998) apply the global games framework to a currency attack on a fixed exchange rate. Speculators are identified by points on the closed unit interval. After observing a noisy signal of the economy's state parameter, the speculator may attack the peg by short-selling the currency. After observing the measure of attackers, the government chooses between defending and abandoning the peg. The unique undominated strategy dictates threshold strategies: speculators attack if their signal suggests a poor enough state and the government abandons if it observes a poor enough state. Morris and Shin (1998) extract policy implications from their closed form solution, finding that the government may decrease the prior likelihood of crisis if it can increase the opportunity cost of attack.

Morris and Shin (1998) allude to the significance of the signaling structure in understanding currency attack. A series of papers have since extended the theory of signaling in global games (Angeletos et al. (2006), Angeletos and Werning (2006), Angeletos et al. (2007), Hellwig (2002), Metz (2003), (Morris and Shin (1999)), (Morris and Shin (2001b), Morris and Shin (2001a), Morris and Shin (2005)), (Heineman and Illing 2002)). These studies may be placed into two broad categories: those that consider exogenous signaling and those that consider endogenous signaling. Chief among the latter, Angeletos et al. (2006) extend the Morris and Shin (1998) currency attack setup by allowing the opportunity cost of attack faced by speculators to be a government policy choice. Whereas Morris and Shin (1998) use comparative statics to examine the cost of attack, Angeletos et al. (2006) recognize that bringing policy into effect may convey information to speculators which will impact their beliefs and optimal strategy. They obtain a full characterization of all possible government equilibrium strategies (Angeletos et al. (2007)). If the economy is sufficiently strong that fundamentals warrant the high peg or the economy is sufficiently weak that intervention is not necessary, then the government authorities will not intervene. However, for intermediate states of the economy, the authorities will intervene, but only to a single level. By choosing only a single intervention level (taxing capital outflows x percent), the authorities are able to conceal their information on the state. If the intervention level varied in a predictable way with the economic fundamentals, then agents would be able to back out what the fundamentals are and equilibrium multiplicity would be restored. Further, the bounds on this “intervention window” relate to the precision of speculator information and the degree of intervention in predictable ways. Surprisingly, Angeletos et al. (2006) find that higher policies are associated with more peg abandonment. This stands in stark contrast to Morris and Shin (1998). However, both agree that *any* intervention is more effective at reducing crisis than none.

The first major experimental work testing global games refinement theory is due to Heinemann et al. (2004). They use the Morris and Shin (1998) currency attack model to answer three main questions: do players use threshold strategies, how do the observed strategies compare to other solution concepts, and what is the role of information precision. They find that players do play with thresholds strategies which are more aggressive than the global games predictions. In other words, participants are more willing to take the risky action and hence coordinate on the payoff dominant equilibrium for a larger set of signals. Also, the likelihood of devaluation is greater when payoffs are common knowledge than when not. More importantly, the observed thresholds change with respect to the cost of attack in accordance with the global games solution. With private information, there is greater variability in individual thresholds than with common knowledge. The government may then transmit public information to increase the predictability of attack while raising the prior likelihood of one. Cornand (2006) extends the Heinemann et al. (2004) private information treatment by considering two new sub-treatments: one with an additional public signal and one with an additional private signal. Their results challenge the policy suggestions of Heinemann et al. (2004); adding public information to private information restores predictability and actually out-performs the treatment with two public signals in terms of predictability and likelihood of success. They recommend a single clear public signal from the government.

Currently, no experiment examines endogenous signaling in a global coordination game. Other studies have incorporated “cheap talk” into experimental coordination games (Cooper et al. (1992)). In our study, cheap talk may actually exacerbate the problem of equilibrium selection by providing more sunspots for players to coordinate on. Instead, we examine endogenous costly signaling in a global coordination game. Predictions are sharp and offer a boon to normative policy analysis of currency attack. It remains to be determined, however, whether or not these sharp predictions are theoretical artifacts or rooted in genuine behavioral phenomena. Heinemann et al. (2004) find that the Morris and Shin (1998) opportunity cost comparative static could indeed be observed experimentally. Angeletos and Werning (2006) extend the theoretical framework on which that finding is based. Our central goal, then, is to test these government signaling predictions offered by Angeletos and Werning (2006). Does the government respond to "aggressiveness of market expectations" (Angeletos and Werning (2006))? Will higher intervention levels have a helpful effect as suggested by Morris and Shin (1998) or a damaging effect as suggested by Morris and Shin (1998)? We use a modified version of the Heinemann et al. (2004) design and estimate the effects of signal precision, policy level, and endogeneity of attack cost on government and speculator behaviors. Angeletos and Werning (2006) note that a central bank may find itself in a "policy trap" whereby policy intervention initiates a self-defeating and self-fullfilling attack. We examine how government players deal with this dilemma and offer policy recommendations. Secondarily, we will offer a check and extension of the Heinemann et al. (2004) and Cornand (2006) findings on speculator behavior. In addition to treatments on informativeness of signals and cost of attack, our *endeogenity of policy* treatment will address the question - do speculators strategically consider the government's strategy in forming belief and choosing action?

3.2 Model

3.2.1 Setup

Our model follows Angeletos and Werning (2006) closely. The set of players is N speculators and a government. The government enjoys a pegged national currency value while speculators enjoy a depreciation of the peg if enough attack in concert.

More formally, their interactions are represented by a three-period game. The state of the economy is parametrized by θ with commonly known uniform prior. A high state indicates a strong economy where the market exchange rate is close to the peg. Conversely, a low state indicates a weak econmy and a disparity between the market rate and the peg. In period one, the government perfectly observes the state and chooses a policy level, $r \in \{\underline{r}, \bar{r}\} \subset (0,1)$, at a cost $C(r)$. We term $\underline{r}$ the *baseline* policy and any higher choice an *intervention* policy. From the point of view of a speculator, the government type is the state value. In period two, speculators publicly observe the policy, privately observe signals, $x_i = \theta + \xi_i; \xi_i \sim N(\theta, \sigma)$, and simultaneously choose whether to attack the peg or not. In the final period, the government observes the fraction of speculators who attack and chooses whether to defend or abandon the peg. If the peg is abandoned, the attacking speculators are successful. If the peg is defended, the attacking speculators are unsuccessful.

The government payoff to defending is $\theta - A - C(r)$ where A is the fraction of speculators who attack and $C(r)$ is a cost of intervention, continuous and increasing, with $C(\underline{r}) = 0$. The government payoff to abandoning is zero less any costs sunk on policy intervention. Speculator payoff is $1 - r$ for a successful attack, $-r$ for an unsuccessful attack and zero for stay.

	Abandon	*Defend*
Attack	$1 - r \quad , -C(r)$	$-r \quad , \theta - A - C(r)$
Stay	$0 \quad , -C(r)$	$0 \quad , \theta - A - C(r)$

3.2.2 Equilibrium

The government strategy defines the period one policy decision conditioned on the state, and the period three defense decision conditioned on the state and realized fraction of attackers. A speculator strategy is a map from policy-signal pairs into an attack decision. Speculator beliefs map policy-signal pairs into posterior probability distributions on θ. We use a perfect bayesian equilibrium solution concept — the government strategy must be a best response to speculator strategies, the speculators' strategies must best responses to one another as well as the government strategy given their beliefs and speculator beliefs must be consistent with the stategies. We assume all agents are risk neutral expected utility maximizers. Let Ψ and ψ denote the cumulative and probability density functions for a speculator's signal error term. Let X denote the vector of all signals and $A(X,r)$ the number of attackers. We suppress this dependence for notational simplicity. Let $D(\theta,A)=1$ denote the government's decision to abandon and $a(x,r)=1$ denote a speculator's decision to attack. $\underline{N\theta}$ denotes the greatest integer which is less than $N\theta$. All equilibria belong to one of the two following classes.

Class 1: Angeletos and Werning (2006) call these *inactive-policy* equilibria. Intuitively, speculators ignore the policy signal which leads the government to save on intervention costs by keeping the policy at baseline.

$$
\begin{aligned}
r(\theta) &= \underline{r} \qquad \forall\theta \\
a_i(x_i,r) &= \begin{cases} 1 & \text{if } x_i < \tilde{x} \\ 0 & \text{if } x_i > \tilde{x} \end{cases} \\
D(\theta,A) &= \begin{cases} 1 & \text{if } \theta < \mathrm{A} \\ 0 & \text{if } \theta > \mathrm{A} \end{cases} \\
A(X,r) &= \sum_{i=1}^{N} a_i(x,r)
\end{aligned}
$$

where $\tilde{x}$ solves:

$$
\int_{\mathbb{R}} \left[1 - Bin\left(\underline{N\theta} - 1, N-1, \Psi\left(\frac{\tilde{x}-\theta}{\sigma}\right)\right)\right] \frac{1}{\sigma}\left[\frac{\psi\left(\frac{\theta-\tilde{x}}{\sigma}\right)}{\Gamma}\right] d\theta = \underline{r}
$$

Class 2: Angeletos and Werning (2006) call these *active-policy* equilibria. They are indexed by a level of policy intervention. The intervention level may be interpreted as speculator aggressiveness in that their strategies tell them to ignore all (off-equilibrium) policy interventions up to that level. For some $r^* \in \left[\underline{r}, C^{-1}(1-\underline{r})\right]$:

$$r(\theta) = \begin{cases} r^* & \text{if } \theta \in [\underline{\theta}, \bar{\theta}] \\ \underline{r} & \text{else} \end{cases}$$

$$a_i(x_i, r) = \begin{cases} 1 & \text{if } x_i < \tilde{x} \\ 0 & \text{if } x_i > \tilde{x} \end{cases}$$

$$D(\theta, A) = \begin{cases} 1 & \text{if } \theta < \text{A} \\ 0 & \text{if } \theta > \text{A} \end{cases}$$

where $\tilde{x}$ solves:

$$\int\limits_{\mathbb{R}} \left[1 - Bin\left(\underline{N\theta} - 1, N-1, \Psi\left(\frac{\tilde{x}-\theta}{\sigma}\right)\right)\right] \frac{1}{\sigma}\left[\frac{\psi\left(\frac{\theta-\tilde{x}}{\sigma}\right)}{\Gamma + \Psi\left(\frac{\underline{\theta}-\tilde{x}}{\sigma}\right) - \Psi\left(\frac{\bar{\theta}-\tilde{x}}{\sigma}\right)}\right] d\theta = \underline{r}$$

and $\underline{\theta}$ and $\bar{\theta}$ satisfy:

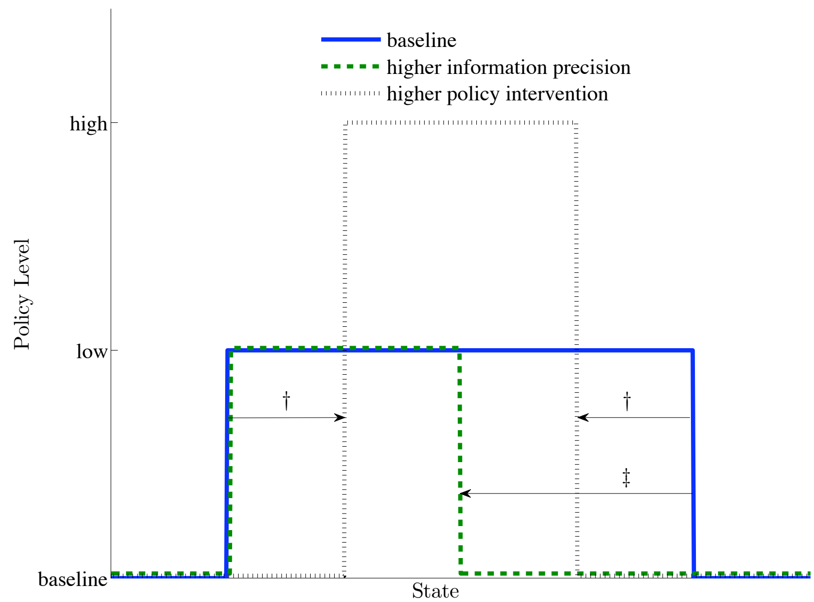
$$\underline{\theta} = C(\bar{r}) = E\left[A(X, \underline{r}) | \bar{\theta}\right]$$

Again, Γ is a normalization constant which depends on the state prior. See Figure 2 for a graphical depiction of *active-policy* equilibria along with the implied comparative static properties.

3.2.3 Explanation and Justification

A few important properties of these equilibria should be noted. First, the presence of endogenous government signaling restores equilibria multiplicity as compared to Morris and Shin (1998) – one for every possible choice of policy intervention. However, the speculator strategy with respect to a policy is uniquely determined. Second, the implicit formulation of $\tilde{x}$ says the expected payoff of attack must be at least as great as the cost. In other words, the marginal agent who is indifferent between attacking and staying receives the signal $\tilde{x}$. The first part of the left hand side gives the conditional probability of success for attacking given strategies and knowledge of the state. The second part of the left hand side is the posterior probability of the state given the informative signal. Recall that speculators earn one for success less the cost of attacking. Finally, Class 2

Figure 3.1: *Active-policy* equilibrium government strategy. (†) indicates the predicted comparative static with respect to magnitude of policy intervention level (G2). (‡) indicates the predicted comparative static with respect to precision of private signals (G4).



equilibria have government intervention only for intermediate types. We call this range the *intervention window*.

Are these equilibria reasonable? Consider Class 1. One can see that the speculator's strategy is independent of the cost of attack, r. The government prefers, then, to save on the cost of intervention by setting r as low as possible. Also, notice that the third period is a simple decision problem. This reduced the game to an N-person coordination game which meets the criteria for global games analysis. This in turn guarantees a unique dominance-solvable symmetric threshold strategy. Off equilibrium speculator beliefs on θ must be constructed carefully to maintain sequential rationality of speculators. There are many reasonable beliefs which do not rely on putting non-zero probability on dominated types.

The intuition behind Class 2 is similar. Given speculator strategies, the government can always calculate its expected payoff for a policy intervention. It will not intervene if it anticipates a successful attack. Speculators then coordinate on staying whenever a policy intervention takes place. Consequently, the government will never set more than one intervention level because the higher of the two will incur unnecessary cost. For low types, the cost of intervention is greater than the payoff. For high types, the government expects the attack to be unsuccessful and loses more paying the cost of intervention than he could possibly gain by reducing that attack size. We expect to see intervention only for intermediate types. As expected, speculator beliefs are now formed by the policy as well as the signal.

The proof that these are indeed equilibria closely parallels Angeletos and Werning (2006) Angeletos et al. (2007). We emphasize only the important departures.

Angeletos and Werning (2006) use an improper uniform prior on the state, θ. The information content is void and agents form posteriors which depend only on their signals. This fact is crucial to the uniqueness of equilibrium speculator strategy as discussed in Morris and Shin (2001b). For purposes of experimentation, we are forced to do without with this convenience. Consequently, uniqueness holds on a "case-by-case" basis. One sufficient condition is the uniqueness of $\tilde{x}$. This is guaranteed if speculator information is precise enough Frankel et al. (2003). Numerical simulation strongly suggests that this condition is satisfied for the calibration selected below. Should uniqueness fail, the above equilibrium remains intact.

Angeletos and Werning (2006) allow the government policy choice to take any value in $[\underline{r}, \bar{r}] \subset (0, 1)$. We chose to limit the number of policy choices in a game to two in order to simplify the subsequent data analysis. This limits the set of possible equilibria to one in Class 1 and one in Class 2. The sufficiency argument follows identically to Angeletos and Werning (2006). The only difference is that off-equilibrium speculator beliefs are simpler to handle here.

Angeletos and Werning (2006) consider a continuum of speculators. With this feature, the government can infer exactly the measure of agents who will attack in equilibrium by knowing the true state and the signal distributions. Abandoning or defending is therefore a forgone conclusion by period one. Consequently, speculators can infer the likelihood of success from their posterior beliefs on the state. In the experimental context considered here, the set of speculators is finite. The government can no longer infer exactly the number of speculators who will attack. However, the same proof technique applies if the government player is assumed to be an expected utility maximizer. The speculator threshold strategy can no longer be computed as described above. Instead, a speculator must consider his posterior on other speculators' signals. When he believes that a sufficient number are receiving low enough signals, then he will attack as well. Note that the period three decision is no longer a forgone conclusion but must be made after observing the number of speculators who actually attack.

3.3 Predictions

A number of predictions on government and speculator play can be derived from the equilibria described above. It is important to recognize that, conditioned on the policy intervention level, there is exactly on Class 1 equilibrium and exactly one Class 2 equilibrium[1]. Any other strategy besides those described is dominated.

Speculators

S1. *In treatments with baseline or exogenous policy, speculators will play undominated threshold strategies.* Speculators attack for signals suggesting a strong state and stay for signals suggesting a weak state. The strategy function will switch at one and only one critical signal in $[0, 1]$. However, in games with policy intervention, speculators coordinate on *stay.*

S2. *In treatments with baseline or exogenous policy, the expected fraction of attackers decreases in the state of the economy.* Equilibrium strategies predict speculators will coordinate on the same threshold. From the government's interim view, a greater state value implies few speculators are expected to receive a signal lower than their threshold. The prediction follows. Recall, the government abandons

[1] More precisely, these equilibria are unique up to a specification of off-equilibrium speculator beliefs

if the proportion of attackers is greater than the state of the economy. If the expected proportion of attackers is decreasing in the state, then the government's interim optimal strategy is monotone.[2]

S3. *In treatments with endogenous policy, speculators will not attack in response to intervention and the fraction of attackers decreases in the state of the economy for states not in the attack window.* In equilibrium, speculators interpret an intervention as a signal to stay. When no intervention is observed, speculators update beliefs based on their signal and the fact that the state must *not* be in the intervention window. A unique symmetric threshold strategy results and a logic similar to S2 follows.

S4. *The speculator threshold will decrease with respect to exogenous policy hikes.* That is, for higher *exogenous* policies, the speculators are more cautious. The speculator thresholds should decrease. Consequently, for higher exogenous policies, the government expects to abandon for a smaller set of states. This is precisely the comparative static result of Morris and Shin (1998) which was confirmed experimentally by Heinemann et al. (2004).[3]

Government

G1. *The government strategy is inverted U-shaped.* When the economy is strong, so few speculators attack that the cost of intervention is greater than the best case scenario of reducing the attack size to zero. When the economy is weak, the cost of intervention is greater than the value of the economy, making any effort to defend it useless. Only for intermediate states does the government have a chance to effectively manipulate public sentiment.

---

[2] This minor prediction is difficult to measure in an experimental scenario because for any state, it is possible for all the speculators to receive very low signals and attack. We observe *realized* fractions of attackers and have only one observation for each state.

[3] For endogenous policy hikes, there is no clear prediction on how the speculator threshold will change. The threshold strategy does not apply when interventions occur. When interventions do not occur, beliefs are formed identically to Class 1 equilibria, except now their beliefs are formed additionally by knowing what the state *is not*.

G2. *More "aggressive" policies yield a smaller set of government types that intervene.* In equilibrium, aggressive government policy is dual to aggressive speculator coordination. This only exacerbates the issues described above (G1). The window for states at which the government finds it beneficial to intervene becomes narrower from both sides.

G3. *More "aggressive" policies yield a larger set of government types who expect to abandon the peg.* A few properties of the equilibrium should be noted. Speculator threshold values are inside the policy window. Since the expected fraction of attackers decreases monotonically in the state, it follows that the government expects to abandon the peg for state values smaller than the lower window edge. By G2, this edge is increasing with the policy. The prediction follows. This prediction stands in total contrast to S4 where higher *exogenous* policies yield a *smaller* set of types at which the government expects to abandon the peg.

G4. *More precise information reduces the set of types who intervene.* By the argument for G1, the lower window edge is independent of noise. The upper edge, however, should collapse to the left edge with respect to precision to information precision. Intuitively, speculators who are better informed exacerbate the issues described under G1.

3.4 Experiment Design

Functional forms were chosen in order in maximize the power of the data to answer the predictions. In keeping with the Heinemann et al. (2004) design, we use fifteen participants. We choose the bounds on the state prior to be -0.2 and 1.2.[4] The policy space was chosen as $\{35, 55\}$ for low policy treatments and $\{35, 75\}$ for high policy treatments. Numerical simulation suggests that these offer the best compromise between detection of intervention and variation in predicted intervention window size across treatments.

The treatment variables are noise of information (σ), level of intervention policy ($\bar{r}$), and endogeneity of policy. The noise-policy interaction treatment was dropped.

[4]The prior support must intersect both dominance regions. See Morris and Shin (2005).

Within a session, one policy-noise combination is fixed. For rounds one through eight, policy is exogenous. The cost of attacking is given to the speculators at baseline level for rounds one through four and at the intervention level for round five through eight. In rounds nine through sixteen, policy is endogenous. One participant is chosen at random to play as the government instead of speculator for rounds nine through twelve and another for rounds thirteen through sixteen. See Tables 1 and 2 for a summary of the treatments.

Table 3.1: Treatment Design

	Low Precision	High Precision
Low Policy	Session 1	Session 3
High Policy	Session 2	n/a

Table 3.2: Sub-treatment within session by period.

1 – 4	5 – 8	9 – 12	13 – 16
Baseline	Exogenous Policy	Endogenous Policy	Endogenous Policy

Participants were recruited through the Rutgers University Department of Economics Human Subject Pool System. The database consists almost entirely of undergraduate students. Sessions were held in the Gregory Wachtler Experimental Economics Laboratory on July 7^{th}, 16^{th}, and 21^{st}, 2009. Students were guaranteed five dollars for showing up and were told that they could earn as much as fifty-three USD.

Instructions were read aloud and participants were asked to read along with their own copies. Participants were asked to solve questions testing their understanding of the game. The answers were then reviewed as a group.[5]

The game was played by computer using z-tree software (Fischbacher (2007)).

We used a modified version of the Heinemann et al. (2004) code. One session consists of sixteen separate rounds. Each round consists of ten independent simultaneously

[5] Instructions are available in an appendix.

played games. For rounds one through eight, each round consists of two stages. In the first stage, each game is viewed as a row that displays the player's private signal and cost of attack, and takes the speculator's choice of action for that game. When a speculator finishes making these decisions, he enters a blank wait screen. When all decisions are made, the program proceeds to the second stage where all participants view a results matrix for those ten games. This includes the true state, signal, policy, choice of action, number of attackers, success, and payoff. Once all players are ready, the game proceeds to the next round.

Rounds nine through sixteen insert an additional stage preceding the speculator decision stage. Here, one participant is chosen at random to assume the role of government for rounds nine through twelve and another for rounds thirteen through sixteen. The government player is similarly faced with ten independent games to which he must choose a policy level. Recall, the baseline policy is free to him and the intervention policy is costly. Each of these games corresponds exactly to a game that the speculators will subsequently play. When the government player finishes with these decisions, play proceeds to a speculator decision stage as described above except that the cost of attacking will vary by game as determined by the government's choices.

In all games, the computer chooses "Abandon" automatically if enough speculators attack to make this the government's rational choice.

Payoffs to speculators are scaled by a factor of 100 and the government by a factor of 50. This yields more intuitive payoffs for the players and equalizes average payoffs to government players and speculator players for any one round. At the end of a session, total earnings from all games are summed up and multiplied by a factor 0.05 to get the dollar payout. Students were payed this and their show-up fee after providing some personal information.

3.5 Results

3.5.1 Speculator Behavior

Our data strongly supports S1. 91.11% of all elicited speculator strategies are undominated threshold strategies.[6] The percentage is 80.00% in period one and ultimately rises to 100% in period eight.

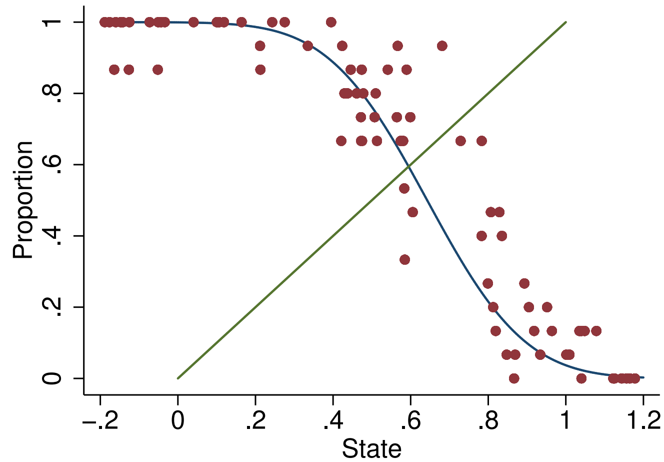
Figures 3.2, 3.3, 3.4 and 3.5 show fraction of attackers against government type for all games played. In games where the realized fraction of attackers is greater than the state, devaluation occurs. Otherwise, the peg stands. These games are represented by those point lying above and below the 45° line, respectively. Figures 3.2 and 3.4 offer support for S2. 3.3 and 3.5 offer support for S3. Hollow circle markers indicate games for which the government player chose intervention. By S3, we expect no speculators to attack for these games. Figure 3.4(b) offers the strongest support of this assertion. Figure 3.4(c) also appears supportive. On the other hand, Figure 3.6(b) suggests the contrary. Prediction S4 may be evaluated by reading Figures 3.2 and 3.4 from left to right. The point of inflection of the fitted curve is precisely the estimated speculator threshold for that treatment.[7] Regarding Figure 3.2, the low policy hike follows the prediction while the high policy hike does not. Figure 3.5 shows the fraction of attackers increased for some states and decreased for others.

We use a logit model to explain speculator attack decisions with signal and treatment variables as independents. The estimated coefficients (Table 3.4) are used to calculate the signal value for which the odds of attacking are one in any given treatment (Table

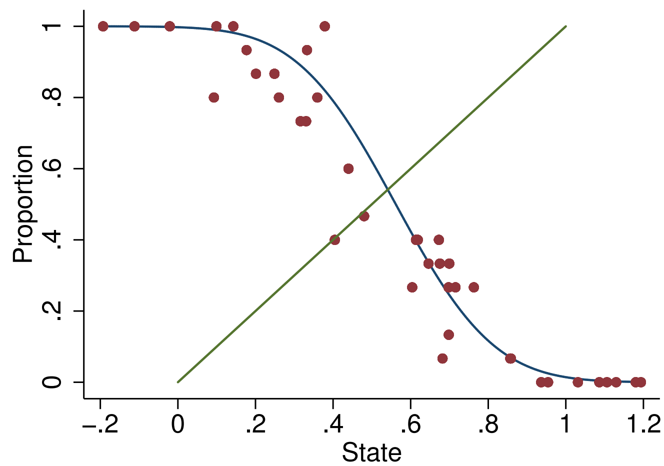
[6]Due to erratic government behavior, periods nine through sixteen were difficult to analyze while remaining faithful to the equilibrium. For instance, we should combine data across all periods and omit from the analysis those games for which intervention occurs. Unfortunately, since the government strategies are not as predicted, these omissions distort elicited speculator strategies beyond recognition. On the other hand, including all data for those periods explicitly ignores the equilibrium. Hence, we omit all speculator data from periods nine through sixteen in this section of the analysis.

[7]A speculator threshold is defined on the signal space, not the state space. However, the state for which the fitted curve is one half is interpreted as the state where one half of speculators attack and one half do not. The only state that can have this property is the one equal to the speculator threshold.

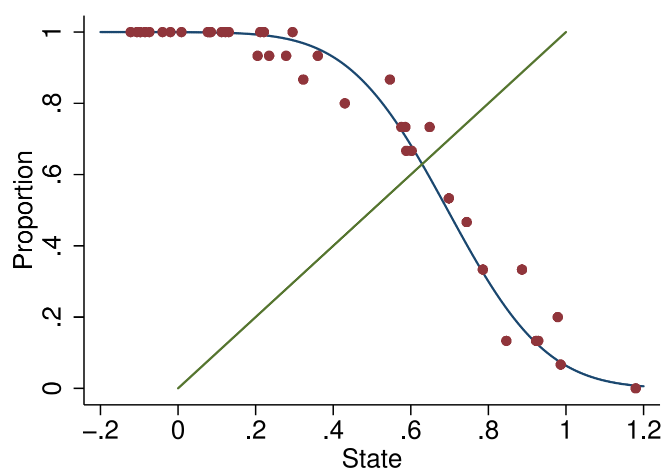
Figure 3.2: Low precision information. Exogenous policy.



(a) Baseline policy

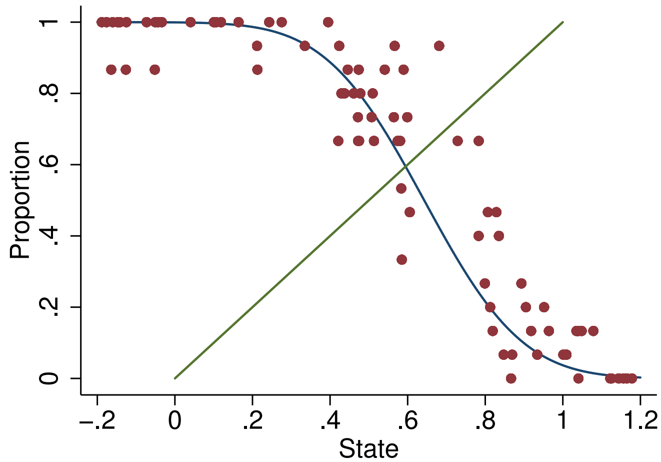


(b) Low policy

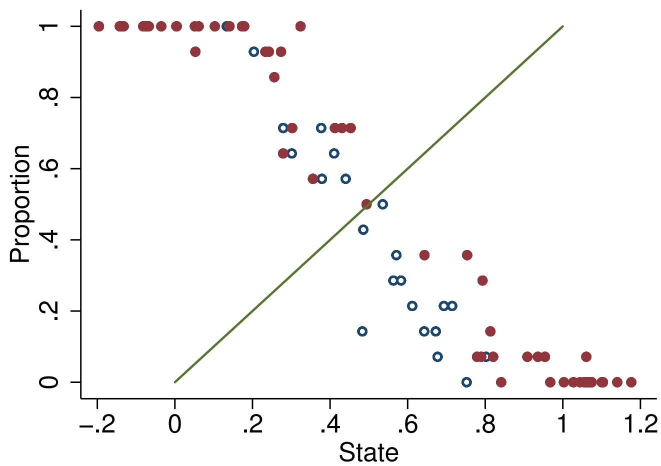


(c) High policy

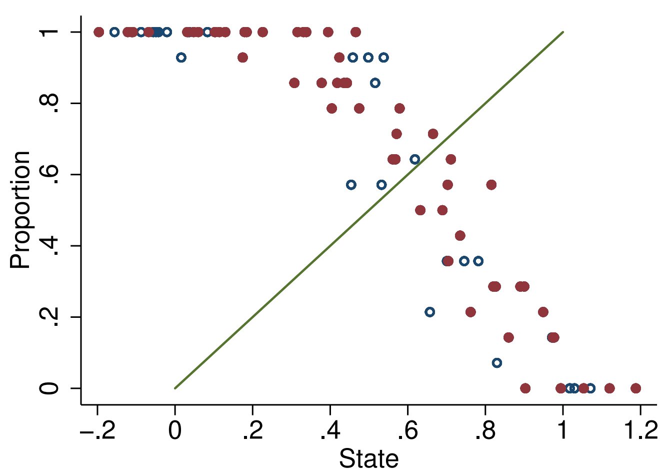
Figure 3.3: Low precision information. Endogenous policy.



(a) Baseline policy

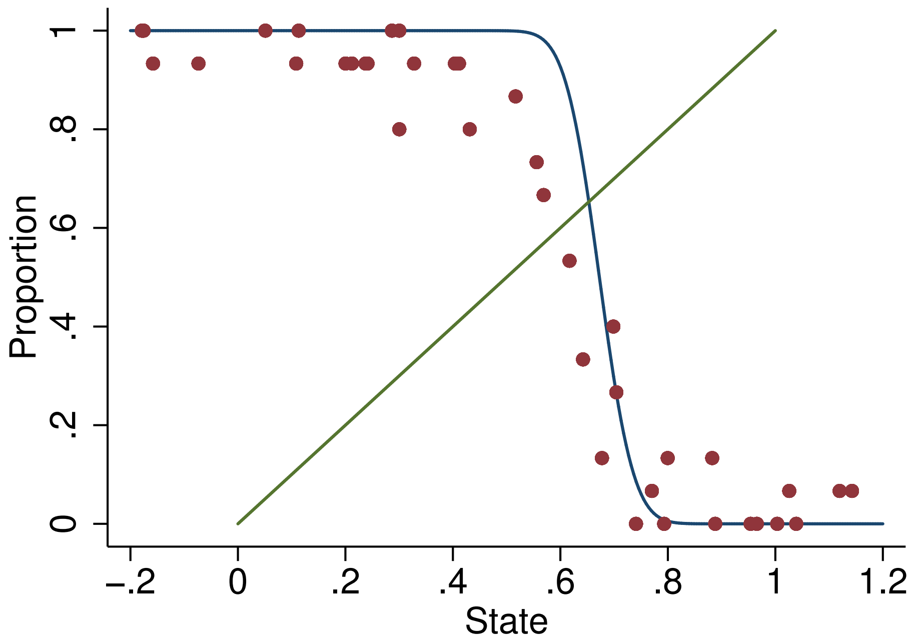


(b) Low policy

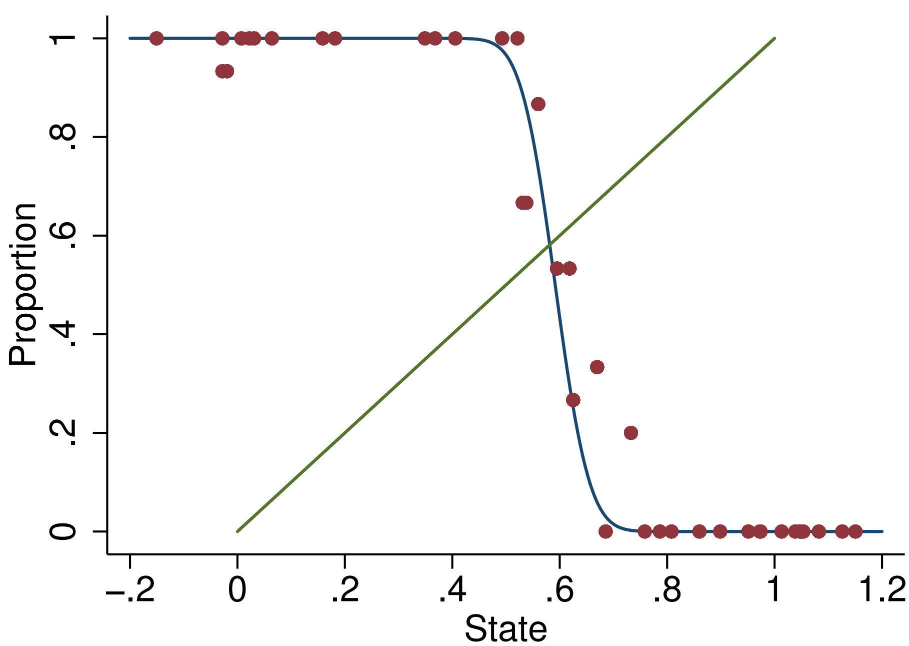


(c) High policy

Figure 3.4: High precision information. Exogenous policy.

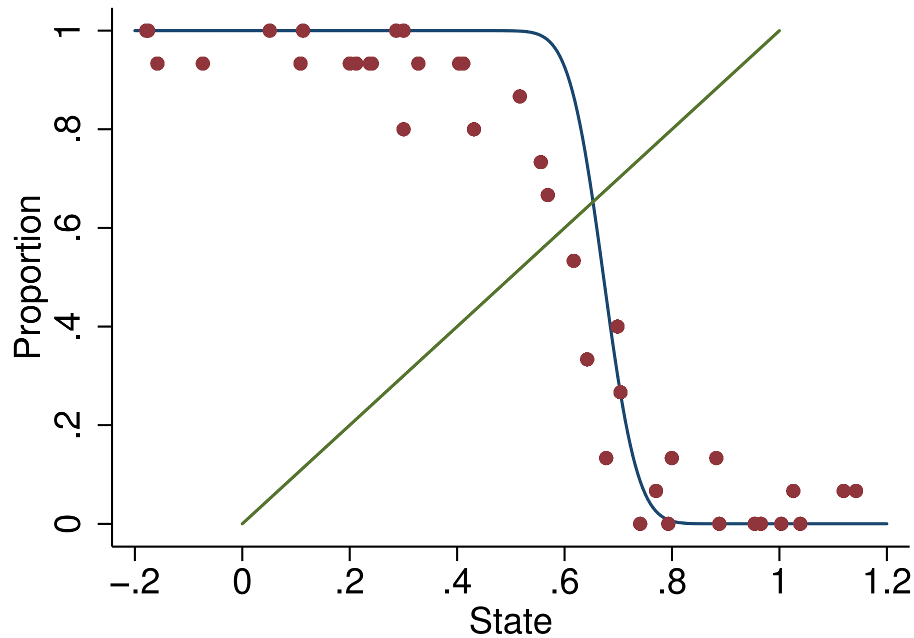


(a) Baseline policy

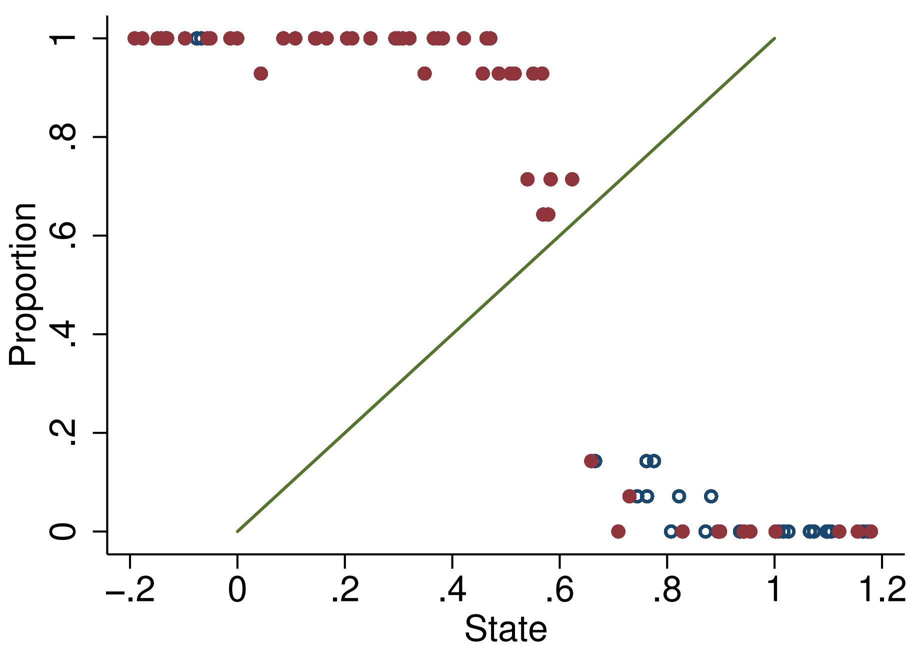


(b) Low policy

Figure 3.5: High precision information. Endogenous policy.



(a) Baseline policy



(b) Low policy

3.5). The decision to attack is coded as one.

$$ln\left(\frac{Pr(attack)}{Pr(stay)}\right) = \beta_o + \beta_x \cdot x + \beta_{info} \cdot info + \beta_{exo55} \cdot exo55$$
$$+\beta_{exo75} \cdot exo75 + \beta_{endo55} \cdot endo55 + \beta_{endo75} \cdot endo75$$

Table 3.3: Variables used in Speculator Attack Model

Name	Nature	Description
x	continuous	private state signal
info	dummy	=1 if private signal precision is high
exo55	dummy	=1 if policy is exogenously given as 55
exo75	dummy	=1 if policy is exogenously given as 75
endo55	dummy	=1 if a policy intervention to 55 is available
endo75	dummy	=1 if a policy intervention to 75 is available

Table 3.4: Estimation Results - Speculator Behavior

Coefficient	Expected Sign	Estimate	Standard Error
β_o	n/a	7.582244	0.2311552
β_x	–	-11.79868	0.32645
β_{info}	n/a	0.353733	0.1095056
β_{exo55}	–	-0.9553508	0.1417205
β_{exo75}	–	0.6200091	0.2094621
β_{endo55}	n/a	-1.030981	0.1290608
β_{endo75}	n/a	0.4469457	0.1517623

The signal coefficient is clearly negative as suggested by S1. The low policy treatment is significant in the predicted direction, offering support of S4. However, the high policy treatment shows more aggressive behavior by the speculators. This is unexpected. However, it should be noted that the high policy treatment was only played by

Table 3.5: Estimated Speculator Thresholds

Treatment	Info Precision	Policy Level	Policy Source	Threshold
1	low	baseline	n/a	0.64263469
2	low	low	endogenous	0.55525368
3	low	high	endogenous	0.68051567
4	low	low	exogenous	0.56166373
5	low	high	exogenous	0.69518369
6	high	baseline	n/a	0.6726154
7	high	low	endogenous	0.58523439
8	high	low	exogenous	0.59164444

one group. Further examination shows that this group played all games more aggressively than the other groups. When a session two dummy is included in the regression, the estimated coefficients become negative, as expected.

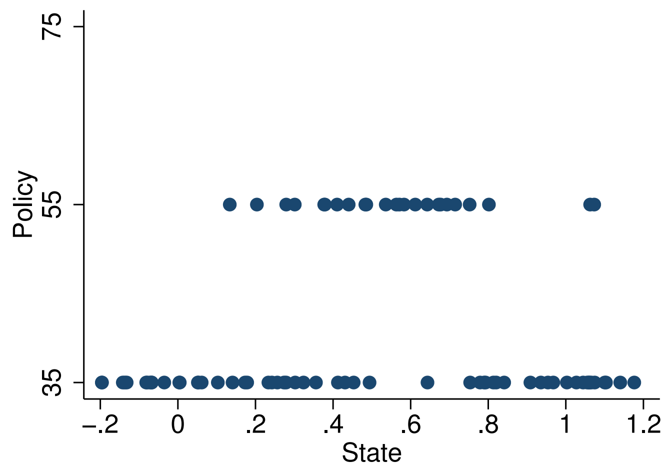
3.5.2 Government Behavior

The government data offers mixed support of G1 (Figure 3.6). In session one, both government players intervene for intermediate types with one exception (Figure 3.7(a)). Session two government players appear to be experimenting through all four periods of their tenure (Figure 3.7(b)). Session three government players appear to be using threshold strategies, similar to what we would expect from a speculator. In fact, some strategies are increasing while others are decreasing. Session three participant twelve is the only government player to choose a Class 1 equilibrium. The data do not appear to support G2 or G4.[8]

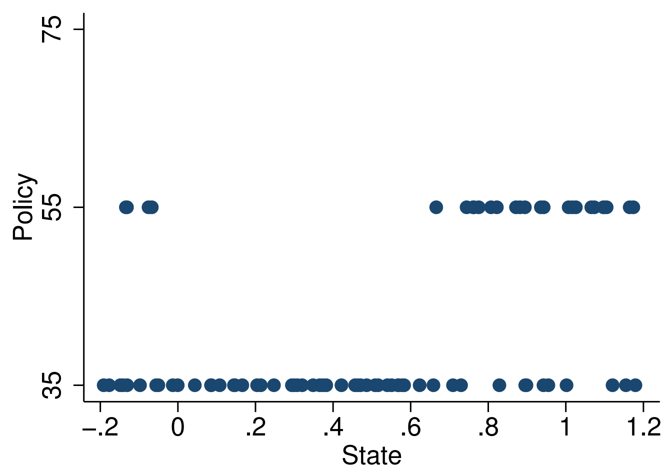
In search of some treatment effect on the intervention window placement, we consider the following econometric model. We assume governments play the equilibrium described above but calculate $\underline{\theta}$ and $\bar{\theta}$ with error. Call these miscalculations $\underline{\theta}^*$ and $\bar{\theta}^*$.

[8] Complete government choice data is available in an appendix.

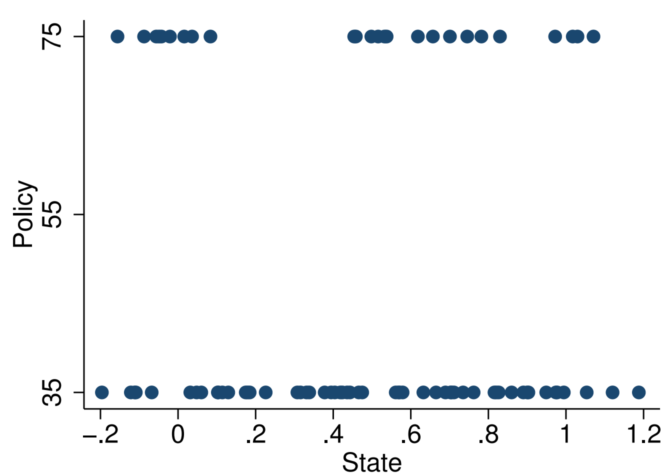
Figure 3.6: Government Behavior (information precision, policy).



(a) Baseline



(b) Low, high



(c) High, low

We assume a $\underline{\theta}^*$ and a $\bar{\theta}^*$ are drawn for each decision. For identification of all treatment effects, a reasonable criteria for separating non-intervening governments into a low and high type is necessary. We classify low non-interveners as those whose type is is less than the equilibrium $\underline{\theta}$ and high non-interveners as those whose type is greater than the equilibrium $\bar{\theta}$. Note that $\underline{\theta}$ and $\bar{\theta}$ are known and a function of the treatment. The log likelihood of one observation is given by:

$$\begin{aligned} lnL(\underline{\theta}, \bar{\theta}, \underline{\beta}_{info}, \underline{\beta}_{policy}, \bar{\beta}_{info}, \bar{\beta}_{policy}) &= intervention \cdot Pr\left(\underline{\theta}^* < \theta < \bar{\theta}^*\right) \\ &+ lowtypezero \cdot Pr\left(\underline{\theta}^* > \theta\right) \\ &+ hightypezero \cdot Pr\left(\theta > \bar{\theta}^*\right) \\ &+ midtypezero \cdot \left(1 - Pr\left(\underline{\theta}^* < \theta < \bar{\theta}^*\right)\right) \end{aligned}$$

where:

$$\underline{\theta}^* = \underline{\theta} + info \cdot \underline{\beta}_{info} + policy \cdot \underline{\beta}_{policy} + \epsilon$$

$$\bar{\theta}^* = \bar{\theta} + info \cdot \bar{\beta}_{info} + policy \cdot \bar{\beta}_{policy} + \epsilon$$

$$\epsilon \sim N(0, \sigma)$$

Table 3.6: Variables Used in Government Decision Model

Name	Nature	Description
θ	continuous	State of the economy
intervention	dummy	=1 if the government intervenes
lowtypezero	dummy	=1 if if the government does not intervene and $\theta < \underline{\theta}$
hightypezero	dummy	=1 if if the government does not intervene and $\theta > \bar{\theta}$
midtypezero	dummy	=1 if if the government does not intervene and $\underline{\theta} < \theta < \bar{\theta}$

The estimated coefficients are insignificant making an assessment of G2 and G4 difficult (Table 3.7).

The game appears to be cognitively difficult for the participants. A stripped down version may illuminate the intuition. Given that, there may still be a significant learning curve involved. Four periods of government play was chosen based on the Heinemann

Table 3.7: Estimation Results - Government Behavior

Coefficient	Expected Sign	Estimate	Standard Error
$\frac{\beta_{policy}}{\sigma}$	+	0.0121	0.24226880
$\frac{\beta_{info}}{\sigma}$	0	-0.1961	0.2486761
$\frac{\bar{\beta}_{policy}}{\sigma}$	−	0.0090	0.23798894
$\frac{\bar{\beta}_{info}}{\sigma}$	−	-0.1451	0.22799169

et al. (2004) finding that *speculators* needed four rounds to settle into a strategy. In future experiments, we plan to increase the government tenure length.

As discussed above, the equilibrium government strategy rests on the result that speculator do not attack in response to an intervention. Clearly, our experimental speculators do not play this way. The best government response, then, is to keep the policy at baseline for those states where speculators are successfully attacking. Given that, speculators will know that when the government does intervene, he must expect the speculators to loose. With this argument in mind, it seems plausible that the observed strategies are in the process of learning to play Class 1 equilibria. The second session two government player as well as the first session three government player appear to corroborate this argument. However, such a learning process would take many more periods than just four.

3.5.3 Conclusion

We successfully replicated the fundamental global games result of threshold strategies. The cost of attacking comparative static found in Heinemann et al. (2004) (S4) is not so clear. However, when a session two dummy is included in the speculator behavior regression, the result is obtained. In other words, that session seemed to have unusually aggressive players.[9]

[9] This group socialized more than other groups while waiting for the experiment to begin.

We were encouraged to see some government players choosing intervention for intermediate types. However, some government players spent their tenure experimenting with other possible strategies. It is possible that the cost of intervention is too high given that speculators are attacking in spite of intervention. This makes it difficult for the government players to find a set of states for which they can profitably intervene. In the future, the experiment will be redesigned to allow for more data on government decisions per session.

Our results draw attention to the fact that the existence of Class 2 equilibria rests critically on the result that players do not attack in response to intervention. In reality, speculators have inertia with respect to their behavior and do not react immediately to intervention. This suggests that a government may have trouble redirecting speculators into new equilibria through policy.

Instructions

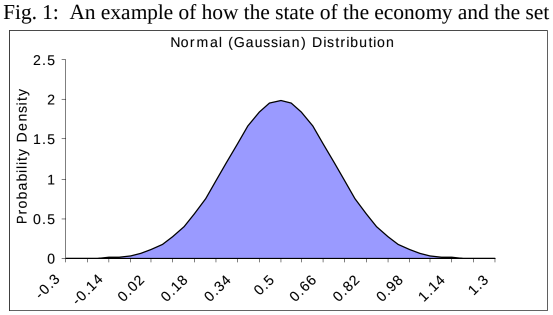
Introduction:
Thank you for participating in this economic experiment. From now until the end of the experiment, please do not communicate with other participants or use the internet. The data generated today and in other sessions will form the basis of a subsequent research paper. Your decisions will directly determine your cash earnings. No prior knowledge is required to play. The experiment should last approximately 60 minutes.

Background:
This experiment asks you to play a game with one another. There are two types of players: speculators and the government. Players earn money depending on their action and the policy that the government sets. There are sixteen separate rounds. In rounds one through eight, the government role is completely played by the computer. In rounds nine through sixteen, a player will be randomly chosen to make some decisions for the government. Each round consists of ten separate decision-problems that will be made simultaneously. Each decision-problem results in a payoff. After a round of decision-making, a screen will display to you your payoffs and other information for each decision-problem. By carefully examining these results, you will better understand how to play the game profitably. When you are satisfied with the results, press the "OK" button. When all players have pressed "OK," play will proceed to the next round. If you forget to press "OK," the experiment will wait for you indefinitely.

Speculator Information:
For each decision-problem, a random "state of the economy" is chosen. This is a number which lies between -0.2 and +1.2. All numbers in between have an equal probability of being chosen. For example, one decision-problem may have a "state" equal to 0.832 and another -0.111. This number represents how "strong" the economy is in the sense that a larger number indicates a stronger economy. The state is different for each decision problem *but is the same for all players*.

Speculators do not directly observe the true state of the economy. Instead, speculators receive a *private signal*, a random number that lies close to the true state, which only he may observe. In technical terms, the variability of this "noisy" signal is represented by a normal distribution centered at the true state with a standard deviation of 0.2. To illustrate how good your signal is, consider the following example (see Fig.1 and Tbl.1):

Fig. 1: An example of how the state of the economy and the set of signals may be chosen for one decision problem. State = 0.5.



Tbl. 1: An example of how the state of the economy and the set of signals may be drawn for one decision problem. State = 0.5.

Player#	Signal
1	0.06775
2	0.760354
3	0.312006
4	0.724471
5	-0.02311
6	0.520749
7	0.192932
8	0.619741
9	0.575511
10	0.295362
11	0.522006
12	0.503154
13	0.772818
14	0.629853
15	0.747465

With a signal, you should be able to formulate in your mind an idea of what the true state is. You should also be able to formulate in your mind an idea of what the other speculators' signals are. Please answer questions one and two.

Speculator Actions:
For each decision-problem, speculators have two choices of action: "attack" and "stay." In choosing "stay," speculators are guaranteed a payoff of zero for that decision-problem. This may be regarded as the "safe" action. In choosing "attack," speculators pay a "cost of attack" and receive 100 points only if enough other players also choose "attack" for that decision problem. This may be regarded as the "risky" action. The "cost of attack" is always the same for all players and for all decision-problems within a round but may change across rounds. For rounds one through four, the "cost of attack" is 35 points and for rounds five through eight, the "cost

of attack" is 55. For rounds nine through sixteen, the "cost of attack" will be chosen by the government (see below). You will be reminded of the "cost of attack" by the computer during the game. See Fig.2.

Fig. 2. A example speculator decision screen. Ten separate decision problems are shown.

Your Signal Is:	The cost of attacking is:	Choose an action:
0.783	35	Stay * * Attack
0.124	35	Stay * * Attack
1.307	35	Stay * * Attack
0.887	35	Stay * * Attack
0.547	35	Stay * * Attack
0.793	35	Stay * * Attack
-0.114	35	Stay * * Attack
-0.045	35	Stay * * Attack
0.521	35	Stay * * Attack
0.222	35	Stay * * Attack

Government Actions, Rounds 1-8:

For rounds one through eight, the computer will play the role of the government. The government observes the number of attackers and the true state of the economy and then chooses either to "keep" or "abandon." For "keep," the government receives a payoff equal to the difference between the value of the state and the fraction of attackers. For "abandon," the government receives a payoff of zero.

Payoff Examples

Example 1: If for one decision-problem the state is 0.2 and 5 out of 15 speculators attack, then the government chooses "abandon" because a payoff of zero is greater than the difference between the state and the fraction of attackers (0.2 – 0.33 < 0). The attack would be "successful" and the attacking players would be rewarded with 100 points minus the cost of attacking for that round. The non-attacking players receive a payoff of zero.

Example 2: If the state of the economy is 0.7 and one speculator out of 15 attacks, then the government would choose "keep" and receive a payoff of 0.7 – 0.07 = 0.63, which is greater than zero. Consequently, this attack would be "unsuccessful." The sole attacker will receive a negative payoff due to the "cost of attack" while the non-attacking speculators would receive a payoff of zero. For all intents and purposes, the state of the economy tells speculators the fraction of attacking speculators that is required for a successful attack in that decision problem.

Example 3: If the state of the economy is 0.551, then 55.1% of all speculators must choose "attack" in order to force the government to "abandon," resulting in a successful attack. Since there are 15 players in the game, this means that at least 9 players in total must attack together for success. If you chose to attack, you would hope that 8 additional players out of the remaining 14 are attacking. Recall that no one observes the true state but instead must rely on their noisy signal. Please solve questions three and four. You may use the Speculator Assistance Sheet.

Government Actions, Rounds 9-16:

For rounds nine through sixteen, a new stage will be inserted into the game where the government chooses the "cost of attack" faced by the speculators. We will term this choice the "policy." Two players will be chosen at random to play as the government instead of speculator – one player for rounds nine through twelve, and the other player for rounds thirteen through sixteen. For each decision-problem, the government player will observe the true state of the economy and choose the cost of attacking that the speculators will face in that decision problem (see Fig. 3).

Fig.3. A example government decision screen. Ten separate decision problems are shown.

The State is:	Choose a policy:
0.755542245	35 * * 55
-0.088121171	35 * * 55
1.155905322	35 * * 55
0.601947939	35 * * 55
0.913960441	35 * * 55
0.842193807	35 * * 55
0.2403411	35 * * 55
-0.066868523	35 * * 55
0.226040432	35 * * 55
0.333539034	35 * * 55

The government's payoff is determined in the following way. If the government chooses the smaller of the two policy levels, then his payoff will be equal to, as before, the state of the economy minus the fraction of attackers. If this number is negative, then the computer will automatically force the government to "abandon," resulting in a zero payoff to the government player for that decision-problem. If the government chooses the greater of the two policy levels, then his payoff is calculated in the same way as before, except that an intervention cost of 0.2 will be subtracted away. In this way, choosing the higher "policy" is not a trivial matter. Rather, the government must weight the costs and benefits of increasing the "policy." The actual number of points received by the government player will be multiplied by a factor of 50. This way, the number of points earned by the government will, on average, be comparable to the number of points earned by a speculator. Please answer question 5. You may use the Speculator Assistance Sheet.

The government chooses the policy before the speculators choose their actions. When he is finished, the experiment proceeds to the regular speculator choice screen as before but the cost of attacking will reflect the government's policy choice for that decision-problem. After the speculators make their decisions the results stage appears as normal.

Completing the Experiment:

After sixteen rounds have concluded, your payoffs from all decision-problems in all rounds will be summed up and converted into dollars at a rate of 50 cents per 100 points. Keep in mind that you have the opportunity to earn in upwards of 10,000 points in the entire session. After you have entered some personal information into a questionnaire and sign a receipt of payment, you will be given your grand payoff and will be free to leave.

Payoff Summary

Speculators:	"Successful"	100 – cost of attack.
	"Unsuccessful"	– cost of attack
Government:	"Keep"	state – fraction of attackers
	"Abandon"	0

Questions:

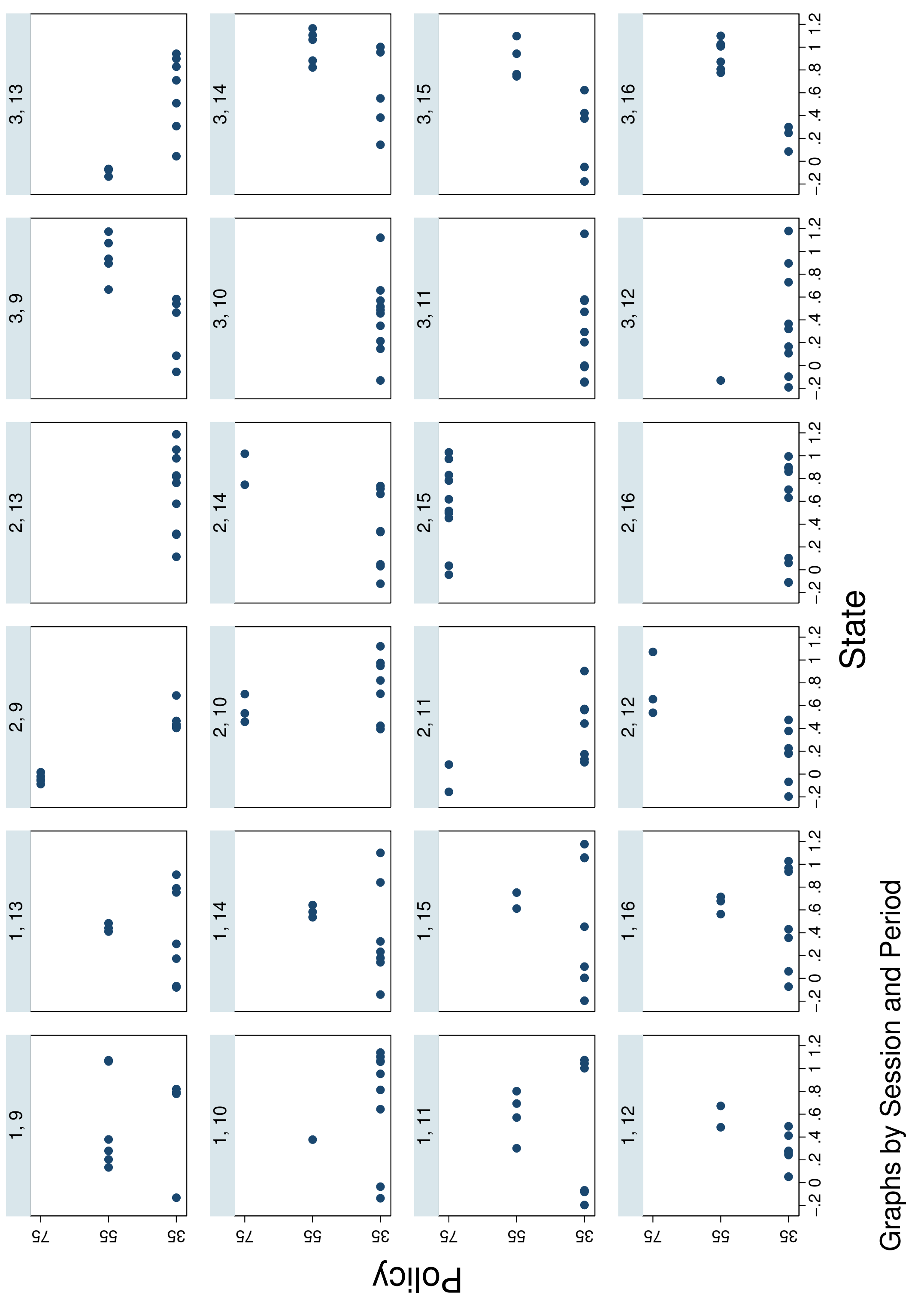
1. Your signal is 1.04.
   a. What do you guess is the true state of the economy?
   b. What do you guess is the average signal received by other players?

2. Your signal is -0.34.
   a. What do you guess is the true state of the economy?
   b. What do you guess is the average signal received by other players?

3. The state of the economy is 0.43. You and eight other players attack. The cost of attacking is given as 55. There are 15 speculators in total.
   a. Will the government abandon or keep?
   b. Is the attack successful?
   c. What is your payoff?
   d. What is the government payoff?

4. The state of the economy is 0.85. You stay while three other players attack. The cost of attacking is given as 35. There are 15 speculators in total.
   a. Will the government abandon or keep?
   b. Is the attack successful?
   c. What is your payoff?
   d. What is the payoff to the attacking players?
   e. What is the government's payoff?

5. The state of the economy is 0.3. The government has chosen a policy level of 55. 12 speculators attack. There are 14 speculators in total.
   a. Will the government abandon or keep?
   b. What are the attacking speculators' payoffs?
   c. What are the non-attacking speculators' payoffs?
   d. What is the government's payoff?


1, 9
1, 13
2, 9
2, 13
3, 9
3, 13
1, 10
1, 14
2, 10
2, 14
3, 10
3, 14
1, 11
1, 15
2, 11
2, 15
3, 11
3, 15
1, 12
1, 16
2, 12
2, 16
3, 12
3, 16
Policy
35
55
75
State
−.2 0 .2 .4 .6 .8 1 1.2
Graphs by Session and Period

Chapter 4

Pricing an American Call (Put) on the Max (Min) of Three Correlated Stocks

4.1 Introduction

The goal of this paper is to calculate the equilibrium price of an American call (put) option on the maximum (minimum) stock in a basket of three stocks when the underlying stock processes are correlated geometric brownian motion. I refer to them as the *cmax* and *pmin* options, respectively. This problem presents a number of interesting challenges. I delineate each.

4.1.1 American Style

There is a large literature, initiated by Black and Scholes (1973), which finds closed form prices of European style call and put options using elegant no-arbitrage arguments. However, there are no known closed form prices for call and put options with an early exercise option. This is unfortunate, because American options are quite common. Equality between a non-dividend paying American call and a non-divident paying European call is not helpful either since most American options are non-zero dividend.

4.1.2 Correlated Motion

As (4.2) shows, the presence of correlation in a rainbow option introduces a cross partial term. This term is difficult to deal with using *finite difference* (FD) techniques. Since my option is American, I am forced to use numerical methods.

4.1.3 Stability

An *explicit* FD analysis is conditionally stable, requiring a sufficiently large number of time steps relative to the spatial steps. This condition is made only more stringent as dimensionality increases. The answer to this problem is to use an implicit method. The Crank-Nicolson scheme is not practical as there is currently no method for dealing with a huge matrix inversion step. Alternating Direction Implicit (ADI) is one common technique which reduces the problem to a series of simple one dimensional problems.

4.1.4 Benchmarking

Even European version of the *cmax* and *cput* options are exotic and "off-the-shelf" formulae are not widely available. However, the academic literature does give hints on how such options could be priced. Broadie and Detemple (1997) (§7.1) finds necessary conditions that the closed form prices for *cmax pmin* must satisfy. Johnson (1987) offers a closed form price for the simpler European-style version of *cmax* and *pmin*. Implementing the Detemple-Brodie method for three-dimensions is unfortunately beyond the scope of this paper. The Johnson (1987) is opaque and contain errors. Fortunately, Ouwehand and West (2006) clarifies nicely.

4.1.5 Multidimensionality

With three stock processes to consider simulatenously, the computational requirement of ADI FD is enormous.

4.1.6 Boundary Conditions

For any FD approach, *explicit* or *implicit*, boundary conditions must be given to the option value grid. With multidimensional FD, there are many different boundaries to consider. In particular the *cmax* and *cmin* options possess subtle nuances which complicate the boundary definition.

4.2 Methodology

I begin by constructing a grid:

$$G^{\delta t,\delta S_1,\delta S_2,\delta S_3} := \{(t_k, S_i, S_j, S_h) : 0 \leq k \leq K, 0 \leq i \leq I, 0 \leq j \leq J, 0 \leq h \leq H\} \quad (4.1)$$

with $t_k := T - k\delta t$, $T := K\delta t$, $S_{min1} := 0$, $S_i := S_{min} + i\delta S_1$, $S_{max1} := I\delta S_1$, $S_{min2} := 0$, $S_j := S_{min3} + j\delta S_2$, $S_{max3} := J\delta S_2$, $S_{min3} := 0$, $S_h := S_{min3} + h\delta S_3$, $S_{max3} := H\delta S_1$, $S_{min3} := 0$, $V_i^k := V(T - k\delta t, i\delta S_1)$ for some chosen $\delta S_1, \delta S_2, \delta S_3$ and δt indicating the step size of the stock processes (the underlying) and time.

I refer to Wilmott (2006) for the Black Scholes PDE for an option price defined on three underlying securities which follow a jointly correlated geometric brownian motion:

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sum_{i=1}^{3}\sum_{j=1}^{3}\sigma_i\sigma_j\rho_{ij}S_iS_j\frac{\partial^2 V}{\partial S_i \partial S_j} + \sum_{i=1}^{3}(r - d_i)S_i\frac{\partial V}{\partial S_i} - rV = 0 \quad (4.2)$$

The ultimate goal is to obtain an estimate of $V(S_1, S_2, S_3, t)$. This is a second order partial differential equation in four variables. In particular, it is a convection-diffusion heat equation in three dimensions. The convection terms are the first order derivatives and the diffusion terms are the second order derivatives Duffy (2006). Furthermore, it is parabolic. I will now approximate Equation (4.2) with Set (4.1).

I use standard FD approximations of the derivative terms. For $0 < i < I, 0 < j <$

$J, 0 < h < H, 0 < k < K$ we have:

$$\begin{aligned}
\left(\frac{\partial V}{\partial t}\right)_{i,j,h}^{k} &\approx \frac{V_{i,j,h}^{k} - V_{i,j,h}^{k+1}}{\delta t} \\
\left(\frac{\partial V}{\partial S_1}\right)_{i,j,h}^{k} &\approx \frac{V_{i+1,j,h}^{k} - V_{i-1,j,h}^{k}}{2\delta S_1} \\
\left(\frac{\partial^2 V}{\partial S_1^2}\right)_{i,j,h}^{k} &\approx \frac{V_{i+1,j,h}^{k} - 2V_{i,j,h}^{k} + V_{i-1,j,h}^{k}}{\delta S_1^2} \\
\left(\frac{\partial V}{\partial S_2}\right)_{i,j,h}^{k} &\approx \frac{V_{i,j+1,h}^{k} - V_{i,j-1,h}^{k}}{2\delta S_2} \\
\left(\frac{\partial^2 V}{\partial S_2^2}\right)_{i,j,h}^{k} &\approx \frac{V_{i,j+1,h}^{k} - 2V_{i,j,h}^{k} + V_{i,j-1,h}^{k}}{\delta S_2^2} \\
\left(\frac{\partial V}{\partial S_3}\right)_{i,j,h}^{k} &\approx \frac{V_{i,j,h+1}^{k} - V_{i,j,h-1}^{k}}{2\delta S_3} \\
\left(\frac{\partial^2 V}{\partial S_3^2}\right)_{i,j,h}^{k} &\approx \frac{V_{i,j,h+1}^{k} - 2V_{i,j,h}^{k} + V_{i,j,h-1}^{k}}{\delta S_3^2} \\
\left(\frac{\partial^2 V}{\partial S_1 \partial S_2}\right)_{i,j,h}^{k} &\approx \frac{V_{i+1,j+1,h}^{k} - V_{i+1,j-1,h}^{k} - V_{i-1,j+1,h}^{k} + V_{i-1,j-1,h}^{k}}{4\delta S_1 \delta S_2} \\
\left(\frac{\partial^2 V}{\partial S_1 \partial S_3}\right)_{i,j,h}^{k} &\approx \frac{V_{i+1,j,h+1}^{k} - V_{i+1,j,h-1}^{k} - V_{i-1,j,h+1}^{k} + V_{i-1,j,h-1}^{k}}{4\delta S_1 \delta S_3} \\
\left(\frac{\partial^2 V}{\partial S_2 \partial S_3}\right)_{i,j,h}^{k} &\approx \frac{V_{i,j+1,h+1}^{k} - V_{i,j+1,h-1}^{k} - V_{i,j-1,h+1}^{k} + V_{i,j-1,h-1}^{k}}{4\delta S_2 \delta S_3}
\end{aligned} \tag{4.3}$$

4.2.1 Explicit Method

With the explicit method, we move backward in time. Begin with the future value of the option price at expiry. This is given by the specification of the option. Then determine the values the function must have taken one time period before given the restriction Equation (4.2). The FD approximation to (4.2) under the explicit scheme is:

$$\frac{V_{i,j,h}^{k}-V_{i,j,h}^{k+1}}{\delta t}+\underbrace{\frac{1}{2}\sigma_1^2S_1^2}_{a(i,j,h)}\frac{V_{i+1,j,h}^{k}-2V_{i,j,h}^{k}+V_{i-1,j,h}^{k}}{\delta S_1^2}+\underbrace{\frac{1}{2}\sigma_2^2S_2^2}_{b(i,j,h)}\frac{V_{i,j+1,h}^{k}-2V_{i,j,h}^{k}+V_{i,j-1,h}^{k}}{\delta S_2^2}$$

$$(4.4)$$

$$+\underbrace{\frac{1}{2}\sigma_3^2S_3^2}_{c(i,j,h)}\frac{V_{i,j,h+1}^{k}-2V_{i,j,h}^{k}+V_{i,j,h-1}^{k}}{\delta S_3^2}+\underbrace{\rho_{12}\sigma_1\sigma_2S_1S_2}_{d(i,j,h)}\frac{V_{i+1,j+1,h}^{k}-V_{i+1,j-1,h}^{k}-V_{i-1,j+1,h}^{k}+V_{i-1,j-1,h}^{k}}{\delta S_1\delta S_2}$$

$$+\underbrace{\rho_{13}\sigma_1\sigma_3S_1S_3}_{e(i,j,h)}\frac{V_{i+1,j,h+1}^{k}-V_{i+1,j,h-1}^{k}-V_{i-1,j,h+1}^{k}+V_{i-1,j,h-1}^{k}}{\delta S_1\delta S_3}$$

$$+\underbrace{\rho_{23}\sigma_2\sigma_3S_2S_3}_{f(i,j,h)}\frac{V_{i,j+1,h+1}^{k}-V_{i,j+1,h-1}^{k}-V_{i,j-1,h+1}^{k}+V_{i,j-1,h-1}^{k}}{\delta S_2\delta S_3}$$

$$+\underbrace{(r-d_1)S_1}_{g(i,j,h)}\frac{V_{i+1,j,h}^{k}-V_{i-1,j,h}^{k}}{2\delta S_1}+\underbrace{(r-d_2)S_2}_{h(i,j,h)}\frac{V_{i,j+1,h}^{k}-V_{i,j-1,h}^{k}}{2\delta S_2}+\underbrace{(r-d_3)S_3}_{i(i,j,h)}\frac{V_{i,j,h+1}^{k}-V_{i,j,h-1}^{k}}{2\delta S_3}$$

$$\underbrace{-r}_{j(i,j,h)}\ V_{i,j,h}^{k}=0$$

Rearrange to obtain the explicit scheme:

$$\begin{aligned}
v_{i,j,h}^{k+1}=&v_{i-1,j-1,h}^{k}\left(\frac{\delta t d(i,j,h)}{4\delta S_1\delta S_2}\right)+v_{i-1,j,h}^{k}\left(\frac{\delta t a(i,j,h)}{\delta S_2^2}-\frac{\delta t g(i,j,h)}{2\delta S_2}\right)+v_{i-1,j+1,h}^{k}\left(\frac{-\delta t d(i,j,h)}{4\delta S_1\delta S_2}\right) \qquad (4.5)\\
&+v_{i,j-1,h}^{k}\left(\frac{\delta t b(i,j,h)}{\delta S_2^2}-\frac{\delta t h(i,j,h)}{2\delta S_2}\right)\\
&+v_{i,j,h}^{k}\left(1-\frac{2\delta t a(i,j,h)}{\delta S_1^2}-\frac{2\delta t b(i,j,h)}{\delta S_2^2}-\frac{2\delta t c(i,j,h)}{\delta S_3^2}+\delta t j(i,j,h)\right)\\
&+v_{i+1,j+1,h}^{k}\left(\frac{\delta t d(i,j,h)}{4\delta S_1\delta S_2}\right)+v_{i+1,j-1,h}^{k}\left(\frac{-\delta t d(i,j,h)}{4\delta S_1\delta S_2}\right)+v_{i-1,j,h-1}^{k}\left(\frac{\delta t e(i,j,h)}{4\delta S_1\delta S_3}\right)\\
&+v_{i-1,j,h+1}^{k}\left(\frac{-\delta t e(i,j,h)}{4\delta S_1\delta S_3}\right)+v_{i,j,h-1}^{k}\left(\frac{-\delta t i(i,j,h)}{2\delta S_3}+\frac{\delta t c(i,j,h)}{\delta S_3^2}\right)\\
&+v_{i,j,h+1}^{k}\left(\frac{\delta t i(i,j,h)}{2\delta S_3}+\frac{\delta t c(i,j,h)}{\delta S_3^2}\right)+v_{i+1,j,h-1}^{k}\left(\frac{-\delta t e(i,j,h)}{4\delta S_1\delta S_3}\right)+v_{i+1,j,h+1}^{k}\left(\frac{\delta t e(i,j,h)}{4\delta S_1\delta S_3}\right)\\
&+v_{i,j-1,h-1}^{k}\left(\frac{\delta t f(i,j,h)}{4\delta S_2\delta S_3}\right)+v_{i,j-1,h+1}^{k}\left(\frac{-\delta t f(i,j,h)}{4\delta S_2\delta S_3}\right)\\
&+v_{i,j+1,h-1}^{k}\left(\frac{-\delta t f(i,j,h)}{4\delta S_2\delta S_3}\right)+v_{i,j+1,h+1}^{k}\left(\frac{\delta t f(i,j,h)}{4\delta S_2\delta S_3}\right)
\end{aligned}$$

Initial Conditions

The initial condition of *cmax* is:

$$V(i,j,h,0) = \max_{i \in \{1,2,3\}} (S_i - K)^+ \tag{4.6}$$

The initial condition of *pmin* is:

$$V(i,j,h,0) = \min_{i \in \{1,2,3\}} (K - S_i)^+ \tag{4.7}$$

These are the function values that V must take on the plane of the grid at option expiry.

Boundary Conditions

The boundary conditions are a delicate issue. In order for the iterative backtracking process to proceed, $V(i,j,h,k)$ must be defined whenever any stock is at a minimum or a maximum for all k, $0 < k < K$. To determine this condition, I use a typical Dirichlet condition which fixed the value of the option at the border (as opposed to fixing the first or second derivatives).

Consider for a moment the Black-Scholes price for a regular european call option. From Shreve (2004), the closed-form price is:

$$V(S,E,d,r,T,\sigma) = e^{-dT} S N(d_1) - E e^{-rT} N(d_2) \tag{4.8}$$

$$d_1 = \frac{log\left(\frac{S}{E}\right) + T(r - d + \frac{1}{2} \cdot \sigma^2)}{\sigma\sqrt{T}}$$

$$d_2 = \frac{log\left(\frac{S}{E}\right) + T(r - d - \frac{1}{2} \cdot \sigma^2)}{\sigma\sqrt{T}}$$

where S is the spot price, E is the strike price, σ is the volatility, r is the risk-free rate, T is the expiry, d is the dividend yield, and $N(\cdot)$ is the standard normal CDF.

Since $\lim_{S\to\infty} d_1 = +\infty$ and $\lim_{S\to\infty} d_2 = +\infty$, we have $\lim_{S\to\infty} V(S,E,d,r,T,\sigma) = e^{-dT}S - e^{-rT}K$.

This argument is traditionally used to set up the Dirichlet conditions for a vanilla call when estimated numerically. My security is much more exotic. However, I follow a similar train of argument and make the following conjecture to resolve this issue:

Conjecture 1: $\lim_{S_1(0)\to\infty} c_{max} = e^{-q_1\tau}S_1(0) - e^{-r\tau}K$

Argument: When the price of any of the three stocks is much large than the other two, *cmax* will look more and more like a call defined on one stock. Therefore, the argument given above applies equally well to the *cmax* security. Furthermore, the added value associated with the American style diminishes.

Conjecture 1 suggests that, when pricing *cmax*, $e^{q_1\tau}S_{max}(0) - e^{-r\tau}K$ is an appropriate boundary condition whenever a stock is at the maximum. It should be noted that there is no confusion as to which stock to use when more than one is at the max since the max is set to be the same for each stock.

However, the remaining boundary conditions for *cmax* are much less obvious. When no stock is at the maximum and at least one stock is at a minimum, then there are two cases to consider. In case one, one stock is at the minimum and two stock are in their respective interior regions. The appropriate value here is not trivial. The one low value stock may no longer be a factor in valuing that call, but two stock remain important. In fact, it is a complicated problem in it's own right – the question of this research paper for the case of two underlyings instead of three!

I use the early exercise price of the American option as my Dirichlet boundary condition for these cases. Clearly, this condition is a lower bound since the value of an option is at least as great as the present exercise value. I have tried several alternative assumptions and the final results are almost completely insensitive. In one case I set all the boundaries where not one stock is at the maximum to zero and I get exactly the same estimate. An alternative approach for future research might be to use closed form solutions available for the two stock case. I would need to create a routine which analyzes the closed form European values and then approximates the American value from there. Black (1975) suggests an approach where the American option value is approximated as the maximum between the value of early exercise today and the price of the European closed form today.

I use essentially the same arguments for defining the boundaries of the *pmin* grid.

Solution

With the initial conditions and the linearized Black-Scholes PDE, a series of linear equations are solved simultaneously to determine the option price for all possible stock prices the point in time one step before expiry. This process is repeated until the option price is determined at the present time for all stock prices. The explicit technique is conditionally stable. That is, errors may grow unboundedly for some set of grid parameters. To keep conditional stability, the time step must be sufficiently small. The results are shown in Table 4.1.

4.2.2 Alternatives to the Explicit Scheme

I do not use the *implicit* FD scheme but I discuss it briefly here because it motivates my use of ADI, discussed below. The implicit FD technique follows the exact same methodology as the explicit technique except Equation (4.4) is altered so that the derivative terms are backward looking instead of forward looking approximations. The same boundary conditions would be used. The resulting system of linear equations is more complicated to solve but offers numeric stability for all possible grids.

The Crank-Nicolson method is a compromise between the explicit and implicit schemes. It takes a linear combination of the scheme in Equation (4.4) and the modified version for the implicit scheme. The resulting system is numerically stable and much more accurate than either the explicit or implicit scheme. However, it is more computationally demanding than either method.

4.2.3 Example

I use the *Baoli-scheme* from Ge (2006). To begin, I alter the canonical form to fit with their notation.

Consider the following parabolic PDE:

$$\begin{aligned}\frac{\partial V}{\partial t} =& a_{11}\frac{\partial^2 V}{\partial S_1^2} + a_{22}\frac{\partial^2 V}{\partial S_2^2} + a_{33}\frac{\partial^2 V}{\partial S_3^2} \\ &+2a_{12}\frac{\partial^2 V}{\partial S_1 \partial S_2} + 2a_{23}\frac{\partial^2 V}{\partial S_2 \partial S_3} + 2a_{13}\frac{\partial^2 V}{\partial S_1 \partial S_3} \\ &+b_1\frac{\partial V}{\partial S_1} + b_2\frac{\partial V}{\partial S_2} + b_3\frac{\partial V}{\partial S_3} + cV\end{aligned} \tag{4.9}$$

An explicit FD approximation gives:

$$\frac{V_{i,j,h}^{n+1} - V_{i,j,h}^{n}}{\Delta t} = \sum_{x,y=1}^{3} a_{xy}\delta_{xy}V_{i,j,h}^{n} + \sum_{x=1}^{3} b_x\delta_x V_{i,j,h}^{n} + cV_{i,j,h}^{n} \tag{4.10}$$

An implicit FD approximation gives:

$$\frac{V_{i,j,h}^{n+1} - V_{i,j,h}^{n}}{\Delta t} = \sum_{x,y=1}^{3} a_{xy}\delta_{xy}V_{i,j,h}^{n+1} + \sum_{x=1}^{3} b_x\delta_x V_{i,j,h}^{n+1} + cV_{i,j,h}^{n+1} \tag{4.11}$$

Now, consider a more general Crank Nicolson scheme where θ weight is placed on the implicit scheme and $\bar{\theta} \equiv 1 - \theta$ is placed on the explicit scheme:

$$\frac{V_{i,j,h}^{k+1} - V_{i,j,h}^{k}}{\Delta t} = \sum_{x,y=1}^{3} \theta a_{xy}\delta_{xy}V_{i,j,h}^{k+1} + \sum_{x=1}^{3} \theta b_x\delta_x V_{i,j,h}^{k+1} + \theta V_{i,j,h}^{k+1} \tag{4.12}$$

$$+ \sum_{x,y=1}^{3} \bar{\theta} a_{xy}\delta_{xy}V_{i,j,h}^{k} + \sum_{x=1}^{3} \bar{\theta} b_x\delta_x V_{i,j,h}^{k} + \bar{\theta} cV_{i,j,h}^{k} \tag{4.13}$$

4.2.4 ADI

Since the Crank-Nicolson approach is prohibitively complex in the case of the *cmax* and *pmin* securities, I must search for an appropriate ADI scheme. See the Literature Review section below for a discussion of ADI. There are many ADI schemes available. Ge (2006) proposes the following ADI scheme where stock i is treated implicitly in the first step, stock j is treated implicitly in the second step and stock h is treated implicitly in the third step:

$$\frac{V_{i,j,h}^{k+\frac{1}{3}}-V_{i,j,h}^{k}}{\delta t} =\theta a_{11}\frac{V_{i+1,j,h}^{k+\frac{1}{3}}-2V_{i,j,h}^{k+\frac{1}{3}}+V_{i-1,j,h}^{k+\frac{1}{3}}}{\delta S_1^2}+\frac{1}{2}\bar{\theta}a_{22}\frac{V_{i,j+1,h}^{k}-2V_{i,j,h}^{k}+V_{i,j-1,h}^{k}}{\delta S_2^2} \tag{4.14}$$

$$+\frac{1}{2}\bar{\theta}a_{33}\frac{V_{i,j,h+1}^{k}-2V_{i,j,h}^{k}+V_{i,j,h-1}^{k}}{\delta S_3^2}+\theta a_{12}\frac{V_{i+1,j+1,h}^{k}-V_{i+1,j-1,h}^{k}-V_{i-1,j+1,h}^{k}+V_{i-1,j-1,h}^{k}}{4\delta S_1\delta S_2}$$

$$+\theta a_{13}\frac{V_{i+1,j,h+1}^{k}-V_{i+1,j,h-1}^{k}-V_{i-1,j,h+1}^{k}+V_{i-1,j,h-1}^{k}}{4\delta S_1\delta S_3}+\theta b_1\frac{V_{i+1,j,h}^{k+\frac{1}{3}}-V_{i-1,j,h}^{k+\frac{1}{3}}}{2\delta S_1}$$

$$+\frac{1}{2}\bar{\theta}b_2\frac{V_{i,j+1,h}^{k}-V_{i,j-1,h}^{k}}{2\delta S_2}+\frac{1}{3}\bar{\theta}b_3\frac{V_{i,j,h+1}^{k}-V_{i,j,h-1}^{k}}{2\delta S_3}+\frac{1}{3}\theta cV_{i,j,h}^{n+\frac{1}{3}}+\frac{1}{3}\bar{\theta}cV_{i,j,h}^{k}$$

$$\frac{V_{i,j,h}^{k+\frac{2}{3}}-V_{i,j,h}^{k+\frac{1}{3}}}{\delta t} =\frac{1}{2}\bar{\theta}a_{11}\frac{V_{i+1,j,h}^{k+\frac{1}{3}}-2V_{i,j,h}^{k+\frac{1}{3}}+V_{i-1,j,h}^{k+\frac{1}{3}}}{\delta S_1^2}+\theta a_{22}\frac{V_{i,j+1,h}^{k+\frac{2}{3}}-2V_{i,j,h}^{k+\frac{2}{3}}+V_{i,j-1,h}^{k+\frac{2}{3}}}{\delta S_2^2} \tag{4.15}$$

$$+\frac{1}{2}\bar{\theta}a_{33}\frac{V_{i,j,h+1}^{k}-2V_{i,j,h}^{k}+V_{i,j,h-1}^{k}}{\delta S_3^2}+\theta a_{12}\frac{V_{i+1,j+1,h}^{k}-V_{i+1,j-1,h}^{k}-V_{i-1,j+1,h}^{k}+V_{i-1,j-1,h}^{k}}{4\delta S_1\delta S_2}$$

$$+\theta a_{23}\frac{V_{i,j+1,h+1}^{k}-V_{i,j+1,h-1}^{k}-V_{i,j-1,h+1}^{k}+V_{i,j-1,h-1}^{k}}{4\delta S_2\delta S_3}+\frac{1}{2}\bar{\theta}b_1\frac{V_{i+1,j,h}^{k+\frac{1}{3}}-V_{i-1,j,h}^{k+\frac{1}{3}}}{2\delta S_1}$$

$$+\theta b_2\frac{V_{i,j+1,h}^{k+\frac{2}{3}}-V_{i,j-1,h}^{k+\frac{2}{3}}}{2\delta S_2}+\frac{1}{3}\bar{\theta}b_3\frac{V_{i,j,h+1}^{k}-V_{i,j,h-1}^{k}}{2\delta S_3}+\frac{1}{3}\theta cV_{i,j,h}^{k+\frac{2}{3}}+\frac{1}{3}\bar{\theta}cV_{i,j,h}^{k}$$

$$\frac{V_{i,j,h}^{k+1}-V_{i,j,h}^{k+\frac{2}{3}}}{\delta t} =\frac{1}{2}\bar{\theta}a_{11}\frac{V_{i+1,j,h}^{k+\frac{1}{3}}-2V_{i,j,h}^{k+\frac{1}{3}}+V_{i-1,j,h}^{k+\frac{1}{3}}}{\delta S_1^2}+\frac{1}{2}\bar{\theta}a_{22}\frac{V_{i,j+1,h}^{k+\frac{2}{3}}-2V_{i,j,h}^{k+\frac{2}{3}}+V_{i,j-1,h}^{k+\frac{2}{3}}}{\delta S_2^2} \tag{4.16}$$

$$+\theta a_{33}\frac{V_{i,j,h+1}^{k}-2V_{i,j,h}^{k}+V_{i,j,h-1}^{k}}{\delta S_3^2}+\theta a_{13}\frac{V_{i+1,j,h+1}^{k}-V_{i+1,j,h-1}^{k}-V_{i-1,j,h+1}^{k}+V_{i-1,j,h-1}^{k}}{4\delta S_1\delta S_3}$$

$$+\theta a_{23}\frac{V_{i,j+1,h+1}^{k}-V_{i,j+1,h-1}^{k}-V_{i,j-1,h+1}^{k}+V_{i,j-1,h-1}^{k}}{4\delta S_2\delta S_3}+\frac{1}{2}\bar{\theta}b_1\frac{V_{i+1,j,h}^{k+\frac{2}{3}}-V_{i-1,j,h}^{k+\frac{2}{3}}}{2\delta S_1}$$

$$+\frac{1}{2}\bar{\theta}b_2\frac{V_{i,j+1,h}^{k+\frac{2}{3}}-V_{i,j-1,h}^{k+\frac{2}{3}}}{2\delta S_2}+\theta b_3\frac{V_{i,j,h+1}^{k+1}-V_{i,j,h-1}^{k+1}}{2\delta S_3}+\frac{1}{3}\theta cV_{i,j,h}^{k+1}+\frac{1}{3}\bar{\theta}cV_{i,j,h}^{k}$$

SOR may be used to solve this system. I describe here how to solve for the first leg (4.14). To satisfy equation (4.23), the system may be written as:

$$\mathbf{M}_{jh}\mathbf{V}_{j,h}^{k+\frac{1}{3}}=\mathbf{q}_{jh} \tag{4.17}$$

where:

$$\mathbf{M_{jh}} = \begin{pmatrix} A1(1,j,h) & B1(1,j,h) & C1(1,j,h) & 0 & \cdots \\ 0 & A1(2,j,h) & B1(2,j,h) & C1(2,j,h) & \ddots \\ \ddots & \ddots & \ddots & \ddots & \\ \cdots & \cdots & A1(I-1,j,h) & B1(I-1,j,h) & C1(I-1,j,h) \end{pmatrix} \tag{4.18}$$

$$\begin{aligned} A1(i,j,h) &= 1 + \frac{2\delta t\theta a_{11}(i,j,h))}{\delta S_1^2} - \frac{\delta t\theta c(i,j,h)}{3} \\ B1(i,j,h) &= -\frac{\delta t\theta a_{11}(i,j,h)}{\delta S_1^2} - \frac{\delta t\theta b_1(i,j,h)}{2\delta S_1} \\ C1(i,j,h) &= -\frac{\delta t\theta a_{11}(i,j,h)}{\delta S_1^2} + \frac{\delta t\theta b_1(i,j,h)}{2\delta S_1} \end{aligned} \tag{4.19}$$

$$\mathbf{V_{jh}}^{k+\frac{1}{3}} = \begin{pmatrix} V_{0,j,h}^{k+\frac{1}{3}} \\ V_{1,j,h}^{k+\frac{1}{3}} \\ \vdots \\ V_{I-1,j,h}^{k+\frac{1}{3}} \\ V_{I,j,h}^{k+\frac{1}{3}} \end{pmatrix}, \mathbf{q_{jh}}^{k} = \begin{pmatrix} q_{jh,1} \\ q_{jh,2} \\ \vdots \\ q_{jh,I-2} \\ q_{jh,I-1} \end{pmatrix} \tag{4.20}$$

$$\begin{aligned} \mathbf{q_{jh,i}} &= V_{i-1,j-1,h}^{k} D1(i,j,h) + V_{i-1,j+1,h}^{k} E1(i,j,h) + V_{i-1,j,h-1}^{k} F1(i,j,h) \\ &+ V_{i-1,j,h+1}^{k} G1(i,j,h) + V_{i,j-1,h}^{k} H1(i,j,h) \\ &+ V_{i,j,h}^{k} I1(i,j,h) + V_{i,j+1,h}^{k} J1(i,j,h) \\ &+ V_{i,j,h-1}^{k} K1(i,j,h) + V_{i,j,h+1}^{k} L1(i,j,h) \\ &+ V_{i+1,j-1,h}^{k} M1(i,j,h) + V_{i+1,j+1,h}^{k} N1(i,j,h) \\ &+ V_{i+1,j,h-1}^{k} O1(i,j,h) + V_{i+1,j,h+1}^{k} P1(i,j,h) \end{aligned} \tag{4.21}$$

$$D1(i,j,h) = \left(\frac{\delta t\theta a_{12}}{4\delta S_1 \delta S_2}\right)$$
$$E1(i,j,h) = \left(\frac{-\delta t\theta a_{12}}{4\delta S_1 \delta S_2}\right)$$
$$F1(i,j,h) = \left(\frac{\delta t\theta a_{13}}{4\delta S_1 \delta S_3}\right)$$
$$G1(i,j,h) = \left(\frac{\delta t\theta a_{13}}{4\delta S_1 \delta S_3}\right)$$
$$H1(i,j,h) = \left(\frac{\delta t\bar{\theta} b_2}{4\delta S_2} + \frac{\delta t\bar{\theta} a_{22}}{2\delta S_2^2}\right)$$
$$I1(i,j,h) = \left(1 - \frac{\delta t\bar{\theta} a_{22}}{\delta S_2^2} - \frac{\delta t\bar{\theta} a_{33}}{\delta S_3^2} + \frac{\delta t\bar{\theta} c}{3}\right)$$
$$J1(i,j,h) = \left(\frac{\delta t\bar{\theta} a_{22}}{2\delta S_2^2} + \frac{\delta t\bar{\theta} b_2}{4\delta S_2}\right)$$
$$K1(i,j,h) = \left(-\frac{\delta t\bar{\theta} b_3}{4\delta S_3} + \frac{\delta t\bar{\theta} a_{33}}{2\delta S_3^2}\right)$$
$$L1(i,j,h) = \left(\frac{\delta t\bar{\theta} a_{33}}{2\delta S_3^2} + \frac{\delta t\bar{\theta} b_3}{4\delta S_3}\right)$$
$$M1(i,j,h) = \left(\frac{\delta t\theta a_{12}}{4\delta S_1 \delta S_2}\right)$$
$$N1(i,j,h) = \left(\frac{\delta t\theta a_{12}}{4\delta S_1 \delta S_2}\right)$$
$$O1(i,j,h) = \left(\frac{-\delta t\theta a_{13}}{4\delta S_1 \delta S_3}\right)$$
$$P1(i,j,h) = \left(\frac{\delta t\theta a_{13}}{4\delta S_1 \delta S_3}\right)$$

This may be rewritten as:

$$\begin{pmatrix} B1(1,j,h) & C1(1,j,h) & 0 & \cdots \\ A1(2,j,h) & B1(2,j,h) & C1(2,j,h) & \ddots \\ \ddots & \ddots & \ddots & \ddots \\ \cdots & \cdots & A1(I-1,j,h) & B1(I-1,j,h) \end{pmatrix} \cdot \begin{pmatrix} V_1^{k+\frac{1}{3}} \\ V_2^{k+\frac{1}{3}} \\ \vdots \\ V_{I-2}^{k+\frac{1}{3}} \\ V_{I-1}^{k+\frac{1}{3}} \end{pmatrix} \tag{4.22}$$

$$= \begin{pmatrix} q_{jh,1} \\ q_{jh,2} \\ \vdots \\ q_{jh,I-2} \\ q_{jh,I-1} \end{pmatrix} - \begin{pmatrix} A1(1,j,h) * V_0^{k+\frac{1}{3}} \\ 0 \\ \vdots \\ 0 \\ C1(I-1,j,h) * V_I^{k+\frac{1}{3}} \end{pmatrix}$$

I apply the SOR procedure discussed in the appendix to this system. For border condition for the imaginary steps, I interpolate between the two known border conditions using a weight of one-third accordingly.

Recall that the ADI scheme is a method to determine the option prices on the grid for the next time step. Since I am working in three space dimensions, this amounts to finding successive cubes. Furthermore, each leg of my ADI is a cube. The SOR procedure outlined is performed *for a given (j,h) pair* and will yield one column of the first cube leg. The procedure must be repeated *for each (j,h) pair* to ultimately assemble the entire cube leg.

The initial cube is saved as well as the first cube leg. Then a new procedure is initiated to determine the second cube leg. This time, the j dimension is treated implicitly while the cross terms, and i and h dimensions are treated explicitly.

4.3 Results

All of these results were obtained with the following parameterization: $\rho_{12} = \rho_{13} = \rho_{23} = 0.5, \sigma_1 = \sigma_2 = \sigma_3 = 0.3, S_1(0) = S_2(0) = S_3(0) = 100, r = 0.05, q_1 = q_2 = q_3 = 0.02$, $S_{min1} = S_{min2} = S_{min3} = 0$, $S_{max1} = S_{max2} = S_{max3} = 200$. Closed forms were obtained with Matlab using the solutions of Johnson (1987). See the Appendix for a discussion on this implementation. All explicit results are performed with 1000 times steps and 50 stock price steps. The explicit method did not converge for fewer time steps. The ADI method was very sensitive to the time steps and stock price steps. For all ADI results, I use 800 times steps. For the *cmax* results, I use $\delta S_1 = \delta S_2 = \delta S_3 = 5$. For any larger than 30, I have stability problems. For the *pmin* results, I use $\delta S_1 = \delta S_2 = \delta S_3 = 10$.

4.4 Benchmarking

The best benchmark would be a closed form solution for my exact problem. Broadie and Detemple (1997) has the most promising method for this but the procedure is complex and beyond the scope of this paper. A closed form solution does exist for the

Table 4.1: Numerical simulation results.

cmax						
European					American	
Strike	Closed Form	Explicit	Error (%)	ADI	Explicit	ADI
105	21.2882	21.2656	-0.106	20.3031	21.2695	20.669
110	18.2578	18.2578	0	17.0867	18.2611	17.3556
115	15.5759	15.5538	-0.1418859	13.8702	15.5567	14.0542
130	9.2862	9.27096	-0.1641145	8.70601	9.27237	8.72244
150	4.3397	4.31613	-0.5431251	4.8107	4.31658	4.81968
190	0.8174	0.570619	-0.301909714	0.90339	0.570622	0.904761
pmin						
European					American	
95	14.3568	14.317	-0.2772206	12.2913	14.5553	13.1036
90	11.1816	11.163	-0.1663447	9.78016	11.3463	10.3679
85	8.3849	8.37087	-0.1673246	7.26905	8.50684	7.70918
70	2.5798	2.5991	0.74812	2.71734	2.63593	2.7901
50	0.1452	0.157447	8.434573	0.353807	0.15891	0.358076
20	9.5276e-5	1.2514e-6	-98.69	0.000214	1.26331e-6	0.00020

European-style version of my problem problem, due to Johnson (1987). Since it is very easy to determine American style option prices once an FD scheme is set up for the European-style, it would be good to know that my scheme does replicate the closed form solutions to the European version. If they match, then it would be believable that the American estimates are accurate as well.

In the appendix below, I describe the method used by Stulz (1982), Johnson (1987), and Ouwehand and West (2006). The closed form bench marks are presented in the results section. I implement their solution using Matlab. I benchmark the Matlab code by successfully replicating the solution in Dang et al. (2010).

The results indicate that my explicit scheme is accurate. Accuracy decreases as the strike gets closer to the upper boundary. This may be happening because whenever the option is in the money, the value is being distorted by the boundary condition.

4.5 Literature Review

4.5.1 Alternating Direction Implicit (ADI)

ADI is a method for reducing the computational complexity of the Crank-Nicolson method. Instead of treating all stock prices explicitly and implicitly at once, ADI keeps all stocks explicit except for one. This scheme will be used to approximate the grid one-third of the way to the next grid plane. Then another stock is selected to be solved implicitly and the next third of the way to the next grid plane is estimated. This is repeated again for the third stock. At that point the next step on the grid is reached and each stock has been used implicitly for one-third of the time step. Keeping only one stock implicit reduces computational time tremendously but does sacrifice accuracy and stability.

ADI was first introduced by Peaceman and Rachford (1955). He solves two and three dimensional heat equation (diffusion terms, no convection terms) and finds an explicit-implicit solution a la Crank Nicolson. To overcome the difficulty of numerical estimation, he suggests a scheme whereby one dimension is treated implicitly first for a “half step” and then the other is treated implicitly in the next “half step.” Douglas and Rachford (1956) use a similar technique to solve the same problem. Douglas (1962) solves the heat equation on a three dimensional cube using a scheme which possesses better discretization error than the previous papers. Douglas and Gunn (1964) develop a scheme for solving a three dimensional PDE which is unconditionally stable.

ADI is difficult to implement when a cross partial derivative term is present in the PDE you are estimating. This is because several stock prices are referred to in a single term and it is not clear whether that term should be treated implicitly or explicit for any "leg" of the procedure. Therefore, there is great interest in designing ADI schemes which can handle cross derivative terms. Such a scheme would be helpful in the mathematical finance literature since the multivariate Black Scholes PDE includes cross terms whenever correlation is non-zero.

McKee and Mitchell (1970) designs an ADI scheme in two dimensions with a cross term and without convection terms which is unconditionally stable. Craig and Sneyd (1988) studies N-dimensional second order PDEs with diffusion and cross derivatives terms. For certain parameterizations, their scheme is unconditionally stable. With mixed derivative terms, the scheme is conditionally stable. They use a scheme whereby the cross partial term is kept completely explicit. This method is followed by many subsequent authors. Their method is a generalized version of Douglas (1962) and McKee and Mitchell (1970). Craig and Sneyd (1988) also suggest that their results are amenable to inclusion of any number of convection terms. However, a new stability requirement is introduced in the time step size. McKee et al. (1996) finds an unconditionally stable ADI scheme for a parabolic PDE in two dimentions which has both convection terms, diffusions terms, as well as a cross partial term. in't Hout and Welfert (2007) derive unconditional stability results for the Craig-Sneyd scheme for general two spatial-dimensional convection-diffusion equations. His result is a minor improvement over McKee et al. (1996). in't Hout and Welfert (2009) derives an unconditionally stable ADI scheme for multi space dimension diffusion equation with mixed derivative terms and no convection terms. Ge (2006) apply the Craig and Sneyd (1988) method to a convection-diffusion parabolic PDE in three dimensions with cross derivative terms. However, the stability analysis of their scheme is left unresolved. Craig and Sneyd

(1988) would suggest that it is indeed stable a large enough number of time steps.

4.5.2 SOR

Given a tridiagonal matrix, **M**, and a vector, **q** of the same dimension, SOR offers a routine for estimating the solution to the matrix equation **MV=q** for **V** without having to compute the inverse of **M**. This is very important in applied option pricing because **M** may be very large and demand too much computer processing power.
For:

$$\mathbf{M} = \begin{pmatrix} B_1^k & C_1^k & 0 & \cdots \\ A_2^k & B_2^k & C_2^k & \ddots \\ \ddots & \ddots & \ddots & \ddots \\ \cdots & \cdots & A_{I-1}^k & B_{I-1}^k \end{pmatrix}$$

the SOR method provides an iterative procedure for estimating **V**.

$$v_i^{(n+1)} = v_i^{(n)} + \frac{1}{Bi}(q_i - A_i v_{i-1}^{(n+1)} - B_i v_i^{(n)} - C_i v_{i+1}^{(n)}) \tag{4.23}$$

for $1 <\leq i \leq I$ and $A_1 \equiv 0$ and $C_{I-1} \equiv 0$

The SOR procedure begins with an initial guess for **v**, typically **q**. This guess is v^0. Then (4.23) is used to calculate $v^{(1)}$ and so on. The procedure is arrested when the following norm condition is satisfied:

$$||\mathbf{v}^{(n+1)} - \mathbf{v}^{(n)}|| < \epsilon$$

4.5.3 Pricing Options Defined on the max or min of a basket

Stulz (1982) is the first to address the simplest cases for basket options. He analyzes European call and put options on the maximum or minimum of two risky assets. Let V and H be the prices of the two underlying stocks. S_1 and S_2 follow a correlated geometric brownian motion with volatilities σ_1 and σ_2 respectively, correlation coefficients ρ and time to maturity, τ. Dividend yield is zero. Using an argument similar to Black and Scholes (1973), they derive the price for a call defined on the minimum of two risky assets. I refer to Ouwehand and West (2006) for an extension to the case with non-zero dividend yields as well as typo corrections. The dividend yield of asset i is denoted q_i.

$$\begin{aligned} c_{min} =& S_1(0)e^{-q_1\tau}\mathcal{N}_2\left(d_-^{2/1}, d_+^1, -\varrho_{12}\right) \\ & S_2(0)e^{-q_2\tau}\mathcal{N}_2\left(d_-^{1/2}, d_+^2, -\varrho_{21}\right) \\ & - Ke^{-r\tau}\mathcal{N}_2\left(d_-^1, d_-^2, \rho\right) \\ d_\pm^{i/j} =& \frac{\left(ln\frac{S_i(0)}{S_j(0)} + (q_j - q_i \pm \frac{1}{2}\sigma_{i/j}^2)\tau\right)}{\sigma_{i/j}\sqrt{\tau}} \\ d_\pm^i =& \frac{\left(ln\frac{S_i(0)}{K} + (r - q_i \pm \frac{1}{2}\sigma_i^2)\tau\right)}{\sigma_i\sqrt{\tau}} \\ \varrho_{ij} =& \frac{\sigma_i - \rho\sigma_j}{\sigma_{i/j}} \\ \sigma_{i/j}^2 =& \sigma_i^2 + \sigma_j^2 - 2\rho_{ij}\sigma_i\sigma_j \end{aligned} \tag{4.24}$$

where $N_2(\alpha, \beta, \theta)$ is the bivariate cumulative standard normal distribution with limits of integration given by α and β and correlation coefficient of θ. They then derive the price of a call defined on the maximum of two risky assets:

$$c_{max}(S_1(0), S_2(0), K, \tau) = c(S_1(0), K, \tau) + c(S_2(0), K, \tau) - c_{min}(S_1(0), S_2(0), K, \tau) \tag{4.25}$$

where $c(X, K, \tau)$ is the price of a European call option on asset X with an exercise price of K and time to expiration τ. Finally, the price of a put on the minimum of two risky assets:

$$\begin{aligned} p_{min}(S_1(0), S_2(0), K, \tau) =& e^{-r\tau}K - S_1(0)e^{-q_1\tau}(1 - \mathcal{N}(d_+)) \\ & - S_2(0)e^{-q_2\tau}\mathcal{N}(d_-) + c_{min}(S_1(0), S_2(0), K, \tau) \\ d_\pm =& \frac{ln\frac{f_1}{f_2} \pm \frac{1}{2}\sigma^2\tau}{\sigma\sqrt{\tau}} \\ f_i =& S_i e^{(r-q_i)\tau} \\ \sigma^2 =& \sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2 \end{aligned} \tag{4.26}$$

Johnson (1987) generalizes the results from Stulz (1982) to N-dimensions. I refer to Ouwehand and West (2006) for an extension to a model with non-zero dividend yields.

I write the solution for the case where N=3:

$$\begin{aligned} c_{max} =& S_1(0)e^{-q_1\tau}N_3\left(-d_-^{2/1}, -d_-^{3/1}, d_+^1, \rho_{23,1}, \rho_{24,1}, \rho_{34,1}\right) \\ &+S_2(0)e^{-q_2\tau}N_3\left(-d_-^{1/2}, -d_-^{3/2}, d_+^2, \rho_{13,2}, \rho_{14,2}, \rho_{34,2}\right) \\ &+S_3(0)e^{-q_3\tau}N_3\left(-d_-^{1/3}, -d_-^{2/3}, d_+^3, \rho_{12,3}, \rho_{14,3}, \rho_{24,3}\right) \\ &-Ke^{-r\tau}\left(1 - N_3\left(-d_-^1, -d_-^2, -d_-^3, \rho_{12}, \rho_{13}, \rho_{23}\right)\right), \rho_{12}, \rho_{13}\right) \end{aligned} \tag{4.27}$$

where

$$\rho_{ij,k} = \frac{\rho_{ij}\sigma_i\sigma_j - \rho_{ik}\sigma_i\sigma_k - \rho_{kj}\sigma_k\sigma_j + \sigma_k^2}{\sqrt{(\sigma_i^2 + \sigma_k^2 - 2\rho_{ik}\sigma_i\sigma_k)(\sigma_j^2 + \sigma_k^2 - 2\rho_{jk}\sigma_j\sigma_k)}} \tag{4.28}$$

$$\tag{4.29}$$

where ρ_{ij} is the correlation coefficient between securities i and j.

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