CHARACTERIZATION OF SHOCK WAVE TURBULENT BOUNDARY LAYER INTERACTION USING PARTICLE IMAGE VELOCIMETRY

by

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A thesis submitted to the

Graduate School-New Brunswick
Rutgers, The State University of New Jersey
in partial fulfilment of the requirements
for the degree of
Master of Science

Graduate Program in Mechanical and Aerospace Engineering

written under the direction of
Dr. Tobias Rossmann

and approved by

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New Brunswick, New Jersey
January, 2012
ABSTRACT OF THE THESIS

Characterization of Shock Wave Turbulent Boundary Layer Interaction using Particle Image Velocimetry

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A method for the use of Particle Image Velocimetry in the Rutgers Supersonic Wind Tunnel is established and described. The use of ice crystals as PIV seed material is tested using an oblique shock as a step response to estimate the particle lag time and particle diameter. These values were found to be $2.8 \mu s$ and $0.43 \mu m$ respectively. The Stokes number was found to be $St = 0.11$.

The statistical properties of the undisturbed boundary layer are determined using PIV data and simplifying assumptions. The boundary layer thickness is $\delta = 15.6 \text{ mm}$ and the free stream velocity is $U_\infty = 621 \text{ m/s}$. This velocity corresponds to an approximate Mach number of 3.22. Using the Clauser method, the shear velocity is $u_\tau = 22.45 \text{ m/s}$. The log-law region has a last reliable point at height $y^+ \approx 500$ and extends to $y^+ \approx 3000$. Velocity fluctuation values and trends are in agreement with published results.

Investigation of streamwise XY planes of the boundary layer suggest the existence of coherent structures, particularly hairpin vortices. Galilean decomposition clearly shows vortex structures at height $y/\delta \approx 0.2$ at a convection velocity of $U_c/U_\infty = 0.75$. Investigating a spanwise XZ plane at height $y/\delta = 0.1$ shows coherent regions of high and low velocity which stretch beyond the field of view.
Shock Wave Turbulent Boundary Layer Interaction is investigated in the case of a 6 and 9 degree angle of attack flat plate used to generate an oblique shock which impinges upon the turbulent boundary layer. In the 9 degree case, an average separated region of the boundary layer exists to be $2.11\delta$ long in the streamwise direction and exhibits the characteristics of a strong interaction. Ahead of the separated region, a separation shock shows streamwise motion of about $\pm 0.25\delta$. The 6 degree case shows no boundary layer separation in the mean flow. A reflected shock exhibits reasonably steady behavior which is characteristic of a weak interaction. The sonic line shows that the subsonic layer dilates to a height of $y/\delta = 0.27$ and $y/\delta = 0.7$ for the 6 and 9 degree case respectively. In both interactions, the recovery of the boundary layer extends beyond the field of view in the streamwise direction.
Acknowledgements

First and foremost is to thank my advisor, Dr. Tobias Rossmann. From his guidance, I have achieved more than I would have thought to achieve in my college career. I am very grateful for the opportunities, challenges, and support he has provided me since the Fall semester of 2008. I also thank Dr. Doyle Knight and Dr. Jerry Shan for their roles in my thesis committee in addition to being part of the faculty of the MAE department who have all taught me much as both an undergraduate and graduate student.

John Petrowski deserves much recognition for his expertise in all things mechanical. I appreciate the time he spent teaching me how to use the machine shop as well as the maintenance performed which kept the wind tunnel operational.

I thank my fellow student and friend Yasin Abul-Huda who has helped in nearly every aspect of this study, from providing support and suggestions to yielding much of his time to help operate the wind tunnel since 2009. I also thank Thomas Maloney who has helped me run the wind tunnel countless times at all hours. I could have asked for no better people to have shared this research experience with.

Lastly, I thank my family for their enduring support throughout my entire college adventure, Sherry Fang for helping me to run the wind tunnel as well as for all of the positive encouragement she gave, and my friends for smiling and nodding whenever I spoke about my research.
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<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$u$</td>
<td>Streamwise Velocity</td>
</tr>
<tr>
<td>$v$</td>
<td>Vertical Velocity</td>
</tr>
<tr>
<td>$u'$</td>
<td>Streamwise Velocity Fluctuation</td>
</tr>
<tr>
<td>$v'$</td>
<td>Vertical Velocity Fluctuation</td>
</tr>
<tr>
<td>$u^+$</td>
<td>Streamwise Velocity, Wall Unit Scaling</td>
</tr>
<tr>
<td>$y^+$</td>
<td>Distance from Wall, Wall Unit Scaling</td>
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<tr>
<td>$u_\tau$</td>
<td>Shear/Friction Velocity</td>
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<td>$T$</td>
<td>Temperature</td>
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<tr>
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<td>Pressure</td>
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<tr>
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<tr>
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<td>Boltzman’s Constant</td>
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<td>$Re$</td>
<td>Reynolds Number</td>
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<td>Stokes Number</td>
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<td>Dynamic Viscosity</td>
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<td>Kinematic Viscosity</td>
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<tr>
<td>$\tau$</td>
<td>Shear Stress</td>
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<tr>
<td>$\delta$</td>
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<td>$\theta$</td>
<td>Momentum Thickness</td>
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<tr>
<td>$\delta^*$</td>
<td>Displacement Thickness</td>
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<td>$\lambda$</td>
<td>Mean Free Path</td>
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Subscripts

∞ Free Stream
0 Stagnation
e Boundary Layer Edge
w Wall
f Fluid
p Particle
d Diameter

Acronyms

SWTBLI Shock Wave Turbulent Boundary Layer Interaction
SWBLI Shock Wave Boundary Layer Interaction
PIV Particle Image Velocimetry
SPOF Symmetric Phase Only Filtering
VITA Variable Interval Time Average
1. Introduction

The purpose of this chapter is to briefly describe the primary principles, flow features, and imaging techniques which pertain to this study. Results from published works investigating supersonic turbulent boundary layers and shock wave turbulent boundary layer interaction are also discussed.

1.1 Background

1.1.1 Supersonic Turbulent Boundary Layer: Mean Flow Behavior

Incompressible laminar and turbulent boundary layers have been extensively studied for many years and the mean flow behavior is well known. The study of compressible turbulent boundary layers by Van Driest, Morkovin, Crocco, and others scientists, have developed and supported methods in which incompressible boundary layer theory can be applied to compressible boundary layers.

A turbulent boundary layer is considered to be made up of two separate layers called the inner and outer layers. The inner layer, also called the constant stress region, is part of the turbulent boundary layer which is found near to the wall and makes up roughly twenty percent of the total boundary layer thickness.

Incompressible Theory - Inner Layer

The inner layer (constant stress region) can be split into two regions from the equation:

\[ \tau_w = -\rho u' v' + \mu \frac{\partial \pi}{\partial y} \]  (1.1)
The first region closest to the wall within the inner layer, is the viscous sublayer. Within this region, viscous stresses dominate turbulent stresses since turbulence goes to zero as the wall is approached due to the no slip condition. For this viscous region, it follows that:

\[ \tau_w = \mu \frac{\partial \overline{u}}{\partial y} \]  

(1.2)

Integrated and simplified, it is then shown that:

\[ \frac{\overline{u}}{u_\tau} = \frac{yu_\tau}{\nu} \]  

(1.3)

where \( u_\tau \) is the friction velocity and is equal to:

\[ u_\tau = \sqrt{\frac{\tau_w}{\rho}} \]  

(1.4)

From Equation 1.3, the viscous length scale is \( \frac{\nu}{u_\tau} \) which then defines \( y^+ = y \left( \frac{\nu}{u_\tau} \right) \) as the viscous wall unit. The scaled velocity is defined as \( u^+ = \frac{\overline{u}}{u_\tau} \). Equation 1.3 then shows that the flow velocity within the viscous sublayer of the inner layer behaves linearly and that \( y^+ = u^+ \).

Moving away from the wall, viscous stresses become dominated by turbulent stresses and Equation 1.1 becomes:

\[ \tau_w = -\rho \overline{u'v'} \]  

(1.5)

The velocity profile within this second section of the constant stress region behaves logarithmically. Developed to describe the velocity profile for a zero pressure gradient incompressible boundary layer, the Law-of-the-Wall is what defines this logarithmic region. The Law-of-the-Wall is written as:

\[ \frac{\overline{u}}{u_\tau} = \frac{1}{\kappa} \log \left( \frac{yu_\tau}{\nu} \right) + C \]  

(1.6)
where $\kappa$ is von Kármán’s constant and $C$ is a constant of integration. These constants are generally accepted to have values of 0.41 and 5.0 respectively for a fully turbulent incompressible boundary layer. There is some debate as to whether or not these constants are independent of the Reynolds number; however, any variability is minimal and does not greatly change the interpretation of the data [23].

**Compressible Theory - Inner Layer**

To describe the velocity profile of compressible turbulent boundary layers using the Law-of-the-Wall, which was originally derived for an incompressible turbulent boundary layer, Van Driest velocity scaling is applied to compensate for compressibility effects. For an adiabatic wall and zero pressure gradient, the compressible version of the Law-of-the-Wall is written as:

$$\frac{u^*}{u_\tau} = \frac{1}{\kappa} \log \left( \frac{yu_\tau}{\nu_w} \right) + C^*$$

(1.7)

where:

$$u^+ = \frac{u^*}{u_\tau}, \quad y^+ = \frac{yu_\tau}{\nu_w}$$

$$u^* = \frac{u_e}{b} \sin^{-1} \left( \frac{2b^2 \frac{\pi}{u_e} - a}{\sqrt{\left(a^2 + 4b^2\right)}} \right)$$

$$a = \left( 1 + r \gamma - \frac{1}{2} \right) \frac{M_e^2 T_e}{T_w} - 1$$

$$b = \sqrt{r \left( \gamma - 1 \right) M_e^2 \frac{T_e}{T_w}}$$

$$r = .89$$

$$M_e = \sqrt{\gamma R T_e}$$

and $T_e$ is found using equation 3.4 where $u = u_e$. Using the assumption of an adiabatic wall, $a = 0$ and $T_w = T_{aw}$ where $T_{aw}$ is the adiabatic wall temperature given by equation 3.3.
It has been experimentally shown that for an adiabatic wall, the constant \( C^* \approx C \), and thus \( C^* \) is taken to have the same value as the incompressible constant \( C \), \( C^* = 5.0 \). [23]

**Outer Layer**

Beyond the constant stress region begins the outer layer. This region deviates from the logarithmic nature of the velocity profile to more so resemble the profile of a wake. Coles proposed Law-of-the-Wake describes this region as:

\[
\frac{\pi}{u_\tau} = \frac{1}{\kappa} \log \left( \frac{yu_\tau}{\nu} \right) + C + \frac{\Pi}{\kappa} w \left( \frac{y}{\delta} \right) \tag{1.8}
\]

where \( \Pi \) is the Coles parameter and \( w \left( \frac{y}{\delta} \right) \) is Coles wake function which is equal to \( w \left( \frac{y}{\delta} \right) = 2 \sin^2 \left( \frac{\pi y}{2\delta} \right) \). The Coles parameter defines the strength of the deviation from a logarithmic profile to that of a wake. This parameter has an accepted value of 0.55 for boundary layers in which \( Re_\theta > 5000 \).

**1.1.2 Shock Wave Turbulent Boundary Layer Interaction**

The study of Shock Wave Turbulent Boundary Layer Interaction (SWTBLI) has been an area of interest since the advancement to transonic and supersonic flight. There have been numerous experiments performed over several decades to further understand the flow phenomena of SWTBLI. While the general qualitative interaction region has been well documented, the quantitative details still present particular uncertainties that modern day researchers try to address. An example of such a problem that has received more recent attention is separation shock unsteadiness.

The mean physical structure of the boundary layer which undergoes interaction with an impinging oblique shock wave, can commonly resemble two distinct flow patterns. The observation of one flow pattern over the other primarily depends on the strength of the
oblique shock measured by the pressure rise across it. This pressure rise causes the boundary layer to respond adversely oftentimes to the point of separation from the surface.

**Weak Interaction**

![Diagram of Weak Shock Wave Boundary Layer Interaction](image)

Figure 1.1: Weak shock wave boundary layer interaction. Image from [5]

In Figure 1.1, a weak oblique shock wave penetrates into the boundary layer and branches into a series of compression waves as it meets the sonic line, under which the subsonic layer resides. The bending of the oblique shock within the supersonic portion of the boundary layer is due to the vertical Mach number gradient.

For a weak oblique shock, which does not induce boundary layer separation, the viscous sublayer and sonic line dilate from the wall. This dilation is caused by increasing pressure near the wall and upstream of the theoretical location of where the oblique shock would contact the wall in the absence of the boundary layer. This subsonic region of flow now acts as a flow disturbance and forces incoming flow to turn. This turning of flow is done through a series of compression waves that coalesce into a reflected shock. Weak expansion waves after the peak of the subsonic region, continue to turn the incoming flow around the disturbance and back towards the wall. Further on the boundary layer begins to gradually recover.
Strong Interaction

When the impinging oblique shock is strong enough to cause boundary layer separation, the general flow field is observed in Figure 1.2. Similar to the weak oblique shock interaction, the adverse pressure gradient applied from the impinging oblique shock severely dilates the viscous sublayer and thus raising the sonic line. Below the sonic line, the flow can be seen to separate and result with a region of reverse flow defined as the separation bubble. This separation bubble acts as a large flow disturbance, which results in the generation of coalescing compression waves to form a separation shock. This separation shock resides in front of the separation bubble and is known to be inherently unsteady.

After the incoming flow has been diverted upwards and around the separation bubble, expansion waves turn the flow back down towards the wall. Beyond the separation bubble, as the boundary layer begins to re-equilibrate, coalescing compression waves form a reattachment shock to once again have the flow travel parallel to the boundary layer surface.

The effects of SWTBLI are experienced in both external and internal applications with a common consequence of degraded performance and undesirable and complex flow characteristics. It is widely observed that the locally separated region of a supersonic
boundary layer exhibits oscillatory streamwise motion and/or pulsation at a low frequency. [5, 19, 24, 29]. The motion of a separation shock has been observed in studies of impinging oblique shocks, compression corners, over expanded nozzles, and normal shock boundary layer interaction. There is much debate and contradicting hypotheses as to the cause of the separation shock and bubble unsteadiness as well as the source of such quasi-periodic motion.

1.2 Literature Review

1.2.1 Supersonic Turbulent Boundary Layer

The statistical properties of supersonic turbulent boundary layers have become more commonly determined with the use of PIV by researchers for the past two decades. From PIV data, deducing the shear/friction velocity $u_\tau$ by fitting the measured streamwise velocity profile to the compressible Law-of-the-Wall profile (Equation 1.7) is commonly performed. This method, known as the Clauser method, can achieve by visual inspection an estimated friction velocity with error much less than 5% [13].

A trend found in fluctuation data of streamwise velocity normalized by the friction velocity, have been reported to show of a plateaued region between $0.4 < \frac{y}{\delta} < 0.6$ and $0.2 < \frac{y}{\delta} < 0.4$ in Mach 2 and Mach 3 flow respectively. The region of $\frac{y}{\delta} \lesssim 0.4$ is believed to contain large-scale coherent structures which may be the cause of large velocity fluctuations.

Spurred by the attached eddy hypothesis by Townsend, investigations for structures in turbulent boundary layers have further supported the existence of horseshoe and hairpin vortices. Head & Bandyopadhyay (1981) showed that the boundary layer is made up of many of such vortices which start at the wall and extend outwards [23]. It is noted that many studies have been performed at reasonably low Reynolds numbers ($Re_\theta < 10000$) and that for Reynolds number beyond this value, the exact effect of the boundary layer
structures is not well known. It has been observed however that as $Re_θ$ increases towards 10000, the vortices tend to stretch streamwise and shrink spanwise.

Figure 1.3: Representation of hairpin vortices annotated from [1]. The vector field corresponds to in plane velocity as recorded by particle image velocimetry. The figure shows that hairpin vortices projected onto a two-dimensional plane are represented by a vortex head above a VITA event.

The Variable Interval Time Averaging (VITA) technique is a method used to identify organized motions of coherent structures. It has been observed that low speed fluid near a wall undergoes a series of events which describe the action of bursting. A portion of fluid from the outer region disrupts low speed fluid found near a wall. This low speed fluid is subsequently ejected from near the wall and into the outer layer where it is then broken up and swept along by high speed fluid in the outer region. A VITA event is a region of flow in which ejection flow encounters sweeping flow [1]. Ejection flow is often seen when $u' < 0$ and $v' > 0$ while sweep flow is when $u' > 0$ and $v' < 0$. Figure 1.3 illustrates hairpin vortices and their velocity projection upon a two-dimensional plane. Ejection flow is seen as the region of slower speed fluid between the legs of the hairpin, while sweeping flow is
found away and behind the hairpin. The meeting of the ejection and sweep is labeled as a VITA event.

The technique of Galilean decomposition, performed on instantaneous Particle Image Velocimetry (PIV) images using varying convection velocities $U_c$, is known to reveal hairpin vortices throughout the supersonic boundary layer [15]. The hairpins have increasing inclination angles, as seen by a VITA event, as the convection velocity is increased. PIV performed in the planar spanwise direction (XZ plane) in both the logarithmic region and outer region, have also revealed structures which compliment those revealed by planar streamwise imaging (XY plane). Long, uniform, and coherent fluctuating velocity regions of positive and negative streamwise velocity represent the legs and space between hairpin vortices in the log-law region. These elongated regions have been shown to be $30\delta$ to $40\delta$ long in the streamwise direction at a height of $\frac{y}{\delta} = 0.2$ [8]. Little coherence is found in the outer region of the boundary layer which supports the notion that structures which were found in the logarithmic region have undergone bursting.

### 1.2.2 Shock Wave Turbulent Boundary Layer Interaction

Applications that experience supersonic flow, will often experience shock wave systems that come in contact and interact with a boundary layer formed on a surface. Commonly found occurrences of shock wave turbulent boundary layer interactions are found on transonic and supersonic aircraft, missiles, and other projectiles. Unfavorable consequences found in such applications can be generally attributed to the unsteady behavior associated with the boundary layer separation.

Upon separation, radical alteration of the flow field typically yields undesirable characteristics that can hinder design criteria. This consequence is often found along the body of an aircraft, a supersonic inlet, or an over expanded nozzle. SWTBLI increases efficiency losses for internal flows and increases drag on airfoils because of increased
levels of turbulence production and viscous dissipation [5]. For supersonic inlets, engine buzz or unstart phenomena are possible due to periodic pressure fluctuations which transmit downstream of the initial shock wave [3]. Aside from large pressure fluctuations, SWTBLI is frequently associated with an increase in localized heat transfer rates as well as premature structural fatigue [8]. Characterization and insight into the unsteady phenomena associated with SWTBLI (especially SWTBLI caused by an impinging oblique shock) is still an active area of study filled with much debate.

A multitude of researchers agree that low-frequency oscillation of the SWTBLI region is a reoccurring characteristic found in a variety of geometric configurations (diverging nozzle, impinging oblique shock, compression corner, etc). This low-frequency unsteadiness is reported to be a few orders of magnitude lower than the characteristic frequency associated with the incoming boundary layer defined as $\tau_f = \frac{U_\infty}{\delta}$ [11]. A corresponding Strouhal number $S_L \approx 0.03$ has been found on several occasions for different SWTBLI flows in which there was boundary layer separation [4, 6]. The current debate amongst researchers is in regards to the flow characteristic which is attributed to being the source of the low-frequency oscillation. There is much uncertainty as to whether such oscillations are driven by the incoming boundary layer turbulence or downstream effects from the separation shock.

PIV analysis of a SWTBLI region for a $Re_\theta = 49200$ Mach 2.07 flow using an 8 degree angle of attack flat plate as a shock generator found intermittent instantaneous separation but no separation in the mean. In this case, a maximum RMS streamwise velocity fluctuation of $\sqrt{u'}/U_\infty \approx 0.2$ was found to occur beneath the tip of the impinging oblique shock in the boundary layer. Turbulence anisotropy was present as RMS streamwise fluctuations were over three times as large as the RMS vertical velocity fluctuations. Large streamwise and vertical fluctuations in addition to Reynolds stress near the separation shock, suggests that vortical structures are being generated [10].
Time resolved PIV measurements were performed in the same facility with similar flow conditions as the previous case, and confirmed the existence and generation of vortices near the separation shock. Resolved vortical structures within the interaction region convect downstream at several hundred meters per second [24]. With a convection velocity of $U_c = 0.7U_\infty$, these structures are found to begin near the wall in the upstream boundary layer, pass through the separation shock, and finally convect upwards towards the impinging shock tip. These vortices are then distributed and dispersed normal to the wall and throughout the boundary layer downstream of the interaction [11].

Spanwise planar PIV parallel to the boundary layer surface at a height of $y/\delta = 0.2$ in a $Re_\theta = 35000$ Mach 2 wind tunnel revealed the existence of coherent structures $30\delta - 40\delta$ long as mentioned in the previous section. As these high and low velocity, and thus momentum, structures pass through the separation shock, shock motion is apparent. When the interaction region is in the presence of a high momentum structure, the separation locations are locally moved downstream. Conversely, as a low momentum structure is encountered, the separation locations are locally moved upstream [8]. This is strongly suggestive that these super structures may be mechanisms for the low-frequency oscillation of the separation shock less than 1kHz.

While it seems intuitive that the fluctuations of the incoming boundary layer causes fluctuations of the separation shock, there is also evidence that the separation bubble itself may cause shock motion. For SWTBLI in which there is no separation, the reflected shock wave is relatively stable [6]. In this case, because there is no separation in which to generate new structures downstream, any minor movement of the reflected shock must be caused by the incoming boundary layer turbulence. In the case of boundary layer separation, the separation shock and region is dramatically unsteady and exhibits motion despite no change.

\footnote{The names separation shock and reflected shock both refer to the shock wave residing in front of the interaction region generated from the impingement of the oblique shock. This shock is named respective to the occurrence of boundary layer separation.}
of the incoming boundary layer turbulence. This suggests that the separated region itself imposes new downstream conditions which are important to the upstream shock motion.

### 1.3 Imaging Techniques

#### 1.3.1 Schlieren

Schlieren is an imaging technique which serves to visualize transparent fluid flow. Density gradients found in the flow of air produce gradients in the index of refraction of light. By passing collimated light through such gradients, Schlieren imaging exploits the physics of light refraction to reconstruct the density gradients visually. By using a series of mirrors and optics the collimated light is focused onto a knife edge. The refracted rays, which bend towards or away from the knife edge, are then blocked out or passed through respectively. The passed light is captured using a camera and the resulting image represents the density gradients of the air being observed.

The knife edge can be strategically positioned in order to visualize the direction of desired density gradients in the flow. A knife edge which is placed vertically at the focal point of the collimated light will visualize horizontal density gradients. While, a knife edge placed horizontally will visualize vertical density gradients.

#### 1.3.2 Particle Image Velocimetry

Particle Image Velocimetry (PIV) is a measurement method for extracting instantaneous whole field velocity information of a flow under investigation. PIV uses a high powered laser to illuminate tracer particles entrained within a fluid flow. The laser beam is thinned in one direction and stretched in the other using laser mirrors and optics to form a laser sheet. The particles traveling within the area of the laser sheet undergo two rapid illuminations. The scattered light from the illuminated particles is collected using a high speed camera and
is stored as a single exposed image or a sequence of images (image pair). It is important that the tracer particles travel far enough so that their displacements can be determined yet short enough to minimize out of plane motion. Considering the previous statement, the time interval between the laser pulses is predetermined based upon the flow velocity as well as the level of magnification of the camera.

Distribution of the tracer particles into the flow is a process known as seeding. The general classification of high quality PIV images are ones in which there is a homogeneous distribution of tracer particles such that there is no evidence of flow structures upon visual inspection [18]. The particle density (particles per interrogation area) of a homogeneous distribution of particles is used as a means of defining various types of PIV. Low particle density requires the tracking of individual tracer particles. High density images also require special analysis as it is no longer possible to visually distinguish one particle from another on a single image. These two variations of PIV are given the names Particle Tracking Velocimetry and Laser Speckle Velocimetry respectively. Between these two extremes, a medium particle density distribution is one that permits the identification of individual particles while being no longer possible to visually distinguish the same particle on an image pair. This medium particle density is what allows for generalized cross-correlation methods to determine displacements.

Assuming proper seeding, illumination, and timing, the resulting images are then stored on a computer. In order to obtain a velocity field, the images are then broken down into small rectangular interrogation areas of a defined pixel size. The best estimation of velocity for a given interrogation area, is one in which all of the tracer particles confined to that interrogation area have travelled by the same amount. By means of cross-correlation, each interrogation area of each image pair is analyzed and results with a displacement vector. The process is repeated for all of the interrogation areas within each image pair. Finally, all displacements are divided by the known time between illuminations thus yielding velocity vector fields for each image pair.
One of the largest advantages of PIV when compared to other aerodynamic measurement devices, such as the pitot probe or hot-wire anemometer, is the non-intrusive quality. This quality is greatly appreciated in high-speed and supersonic flow analysis in which the presence of probes can generate shock waves and/or alter the flow under investigation. The ability to analyze large regions of a flow all at once is also very advantageous. By adjusting the time duration between laser pulses as well as the magnification of the camera, it is also possible to investigate flow at various resolutions. The use of multiple lasers and multiple high-speed cameras also make available the study of unsteady flow behavior through means of time-resolved data. Multiple cameras may also be used to derive out of plane motion in a method called stereo PIV.

1.4 Research Objectives

In this study, there are several prerequisite objectives that must be met prior to studying the effects of SWTBLI. Successfully employing a PIV setup is pivotal to permit any quantitative analysis.

The first objective will be to establish a PIV setup for the Rutgers Supersonic Wind Tunnel. A method for the collection of PIV data with the use of ice crystals as the seed material is to be attained through many trials and much experimentation. The feasibility of the flow tracing ability of the ice crystals will be determined.

Another objective is to numerically define the undisturbed turbulent boundary layer which develops on the test section floor. From these details, the results will be compared to published results as a method of validation. These results are necessary to provide knowledge of the incoming flow which will be seen when studying SWTBLI.

The last objective is to characterize the SWTBLI region from an impinging oblique shock wave using PIV. Observations of potential causes to unsteady effects, such as separation shock and separation bubble oscillation, are of particular interest. From the
characterization, the data will be used to support or contest the findings of published data of similar study.
2. Rutgers Supersonic Wind Tunnel

The objective of this chapter is to describe the facility and the setup of equipment used to perform the research of this study. The beginning sections aim to physically describe and illustrate how the equipment functions and how it is used to obtain the data. The later sections discuss the PIV analysis method and errors.

2.1 Overview

The Supersonic Wind Tunnel at Rutgers University is a blow-down wind tunnel, modularly designed such that interchangeable nozzles offer the ability to change the test section Mach number. For this presented research, a supersonic converging-diverging half nozzle is used to produce a Mach 3.45 flow for a maximum run time of 15 seconds.

Prior to operating the tunnel, air is compressed by a Mako four stage air compressor. The moisture in the air is removed with a Zander air dryer. The high pressure air is compressed to 13.8 MPa and is stored in four high pressure tanks which have a total volume of 8 m$^3$. To operate the wind tunnel, the high pressure air is throttled down to 1.38 MPa using a manual ball valve. The air flows through the ball valve and enters the Stagnation Chamber. Upon entering the stagnation chamber, the high pressure air reaches a blast plate and honeycomb which act to condition the flow before it passes through the nozzle. The flow is then accelerated through the supersonic nozzle before it reaches the test section at Mach 3.45 with a mass flow rate of 15 kg/s. Beyond the test section the flow passes through a diffuser and is exhausted into atmospheric conditions.

The test section cross sectional width and height is 0.15m by 0.15m respectively. The side and bottom walls contain glass windows to permit imaging and laser access. Optical
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tables are situated underneath the test section such that all laser, optical, and recording equipment can be securely fastened and aligned.

Figure 2.1: Rutgers Supersonic Wind Tunnel (not to scale)

2.2 Schlieren Setup

The schlieren system for this research is setup in the typical Z arrangement shown in Figure 2.2. The collimated light is provided by a Strobotac Electronic Stroboscope set at 1800 RPM (30 Hz). The light is reflected off of a series of flat mirrors and two parabolic mirrors before being focused upon a knife edge. Beyond the knife edge, the light is reflected off of a kinematic mirror and passes through two converging lenses which focus the image onto a CCD camera array. The camera then records the images onto a computer at 30 Hz. By matching the stroboscope and camera frequency, the flow appears frozen thus
allowing for the visualization of instantaneous density gradients. For the investigation of
shock wave boundary layer interaction, the knife edge was positioned horizontally in
order to best visualize the vertical density gradients found in the boundary layer.

Figure 2.2: Schlieren Setup

2.3 Particle Image Velocimetry

2.3.1 Setup

Equipment

The PIV in this research was performed using a New Wave Solo II Nd:YAG laser with an
energy output of 40 mJ per pulse. This energy output corresponded to a Q-switch delay of
225 μs and 200 μs for laser cavity 1 and laser cavity 2 respectively.
The pulsed laser beams are reflected perpendicularly into the test section by a round 532 nm 45 degree angled mirror. To create the laser sheet, the reflected laser beam is then passed through a 100 mm cylindrical lens which thins the laser beam unidirectionally, ultimately to a thickness of about 1 mm near the focus. The beam is also passed through a $-11.2$ mm and a $-25$ mm cylindrical lenses which expand the beam unidirectionally in the opposite direction. The use of these three lenses creates an intense uniform laser sheet.

![Figure 2.3: PIV setup](image)

The images were recorded using a 1200x1600 CCD LaVision Imager Pro high-speed camera with a double frame capture rate of 10 Hz. The minimum inter frame time interval is 2 µs. The first frame exposure time is specified by the manufacturer as 10 µs, however it was determined experimentally to be 5.25 µs. The second frame exposure time is 32 ms which allows for a large range of different speed flows. It was also determined experimentally that the camera has a hardware delay of 4.9 µs between when the camera is sent an electrical signal pulse to record the first image and when the first image is actually recorded.

The camera was fitted with a 62 mm diameter telephoto lens. For increased magnification, a +1 diopter close up lens was used along with the telephoto lens.
The PIV timing is controlled by an eight channel Labsmith LC880 pulse delay generator. This timing box is used to control the synchronization of the laser flash lamps, Q-switches, and camera. All of the hardware is connected to the timing box using BNC cables.

**Timing Logic**

A necessary step to achieve working PIV is to define a timing logic. This is done using the Labsmith pulse delay generator. Before proceeding with defining the logic, two assumptions are made. The first is that any delay between the timing box signal and the physical actions of the flash lamps firing or the Q-Switch opening are negligible. The second assumption is that when the Q-switch trigger pulse is received from the timing box, the laser light duration is assumed to be instantaneous. This assumption is reasonable since the laser duration is a few nano seconds.

When analysing the particle image pairs, the size of the interrogation area is recommended to be 4 times larger than that of the particle displacements [12]. Experimenting with various magnifications and time durations between laser pulses, which ultimately limits the displacement of particles in the recorded images, it was experimentally determined that for a given magnification, a free stream particle displacement of 7 to 10 pixels yielded a high percentage of valid full field correlation results. This pixel displacement, in conjunction with the recommendation from Keane et al., validates the usage of a 32 by 32 pixel interrogation area size. Using the predetermined pixel displacement as a starting point for the timing logic, it is then possible to work backwards to determine the proper time between the laser pulses. The magnification level, represented as pixels per unit length, must firstly be known.

To determine the level of magnification, a transparent grid is placed within the location of the laser sheet and the camera is then focused onto the transparent grid. Since the grid lines are of known dimensions, it is then possible to measure the pixels per grid unit.
For this research, the grid lines were .25in apart. When the image is recorded onto the computer, a measurement from grid line to grid line is made to determine the pixels per .25in. Then the following unit conversion is performed to determine the appropriate time between laser pulses to have a free stream particle displace a chosen distance in pixels.

\[ \Delta t [s] = \Delta Particle [pix] \times \frac{1}{\text{Zoom} \left[ \frac{\text{pix}}{\text{in}} \right]} \times \frac{.254 \text{ [m]} }{1 \text{ [in]}} \times \frac{1}{\sqrt{\gamma RT} \cdot M_\infty \left[ \frac{\text{m}}{\text{s}} \right]} \] (2.1)

\[ \gamma \] is the ratio of specific heats and is taken to be 1.4. \( R \) is the ideal gas constant taken as 287.03 \( \frac{J}{kg \cdot K} \) for dry air. \( T \) is the static temperature of 84.3 K found in the test section for a given Mach Number of 3.45 using:

\[ T [K] = T_0 \left( 1 + \frac{(\gamma - 1)}{2} M_\infty^2 \right)^{-1} \] (2.2)

Since the storage tanks for the high pressure air are kept outside, \( T_0 \) is taken to be the ambient temperature. Despite the ambient temperature changing throughout the seasons in which data was collected, for this calculation, \( T_0 \) is taken to be an average temperature of 285 K.
It was determined through experimentation that the maximum energy output of 40 mJ yielded the best results when performing image analysis and correlation. To achieve this energy output, laser 1 was set with a Q-switch delay of 225 µs while laser 2 was set with a Q-switch delay of 200 µs. The flash lamps of laser 1 are not delayed while the flash lamps of laser 2 have an offset delay which is set such that the time between the laser pulses is the time that was determined from Equation 2.1. Simple subtraction would yield the offset delay for laser 2 to be:

\[
\text{Offset Delay [µs]} = \Delta t [\mu s] + \text{Q-Switch 1 Delay [µs]} - \text{Q-Switch 2 Delay [µs]} \tag{2.3}
\]

Timing for the camera is lastly considered. It was experimentally determined that the first exposure frame is 5.25 µs. When working with high-speed flows, \(\Delta t\) is typically on the order of 1 µs or less. \(\Delta t\), or time between laser pulses, may be no shorter than the inter frame time of the camera of 2 µs. The laser 1 pulse is placed as far into the first exposure frame as possible to allow for short times between laser pulses given the limiting inter frame time. It was discovered that if placed too far into the first exposure, pixel *bleeding* would occur into the second recorded frame. It was experimentally found that the optimal
time within the 5.25 µs exposure was 5.11 µs. The laser 2 pulse is then ∆t after the laser 1 pulse, which falls within the exposure of the second frame as seen in Figure 2.6.

Aside from placing the laser 1 pulse 5.11 µs within the first exposure, the experimentally determined camera hardware delay of 4.9 µs must also be accounted for. A simple equation to determine the proper camera offset delay is:

\[
\text{Camera Offset Delay [µs]} = \text{Q-switch 1 delay [µs]} - (4.9 \text{ µs} + 5.11 \text{ µs}) \quad (2.4)
\]

![Figure 2.6: PIV timing logic](image)

### 2.4 Image Processing

Once the image pairs of the flow are recorded onto the computer, the standard means to process the images is to extract the displacement information with the use of cross-correlation algorithms. There are numerous commercial software available which will perform a multitude of pre-processing techniques as well as extensive algorithms to
extract displacements. Commercial software may implement advanced techniques to extract data, perform validation, and analyze data, however such software is very expensive and is not an option for this research.

The use of an open-source MATLAB code, PIVlab version 1.2\(^1\), was determined to be satisfactory in extracting displacement data from the recorded images. Originally, PIVlab version 1 used a direct cross-correlation approach with fixed interrogation areas and could achieve subpixel displacement accuracy using a three point subpixel Gaussian method. Halfway through the data collection for this research, the new version was released which uses a multi-pass approach that applies methods of window shifting and window deformation. This more advanced approach is known to be superior to the classic cross-correlation method and is considered to be a new standard when evaluating PIV images. Direct cross-correlation can result with error due to velocity gradients and loss of in-plane particles. By using window shifting and deformation, these errors are reduced. In addition to window shifting and deformation, the code for PIVlab was modified for this research to perform correlation using Symmetric Phase Only Filtering (SPOF).

Direct cross-correlation is the standard approach for determining particle displacements from particle image pairs. Figure 2.7 outlines the process in which interrogation areas, from within the first and second image of a particle image pair, are analysed to return a displacement value. In direct cross-correlation, the interrogation areas \((S_1(x, y)\) and \(S_2(x, y)\) as seen in figure 2.7) are first transformed into the frequency domain by performing the Fourier transform on the first interrogation area and taking the complex conjugate of the Fourier transform on the second interrogation area. In the frequency domain, each interrogation area then consists of amplitude and phase information \((S_1(p, q)\) and \(S_2(p, q)\)).

In Figure 2.7, \(W\) is called the windowing function which is used to improve correlation results. This is done by multiplying the complex conjugate of the Fourier

\(^1\)Developed by Dipl. Biol. William Thielicke and Prof. Dr. Eize J. Stamhuis. http://pivlab.blogspot.com/
transform of the second interrogation area with a particular windowing function to create a filter. Matched spatial filtering, which is essentially the same as direct cross-correlation, has a windowing function of $W = 1$. By using a windowing function of 1, multiplying the transformed interrogation areas, and taking the inverse Fourier transform of the result, the resulting correlation peak is constructed from the information of both the amplitude and phase portions of the transformed interrogation areas.

To improve upon direct cross-correlation, a windowing function other than $W = 1$ may be chosen. A windowing function of $W = \frac{1}{|S_2(p,q)|}$ removes amplitude information from influencing correlation results by setting all amplitude information to unity. By doing this, only phase information remains. This is beneficial since the phase signal contains the information of position while the amplitude contains size and shape information. By removing amplitude information, there is a focus on displacement which is shown by sharper correlation peaks and higher signal to noise ratios[28]. While this method has its advantages, it is shown that this choice of window function is not always an ideal improvement. Wernet shows that SPOF, which uses a windowing function of $W = \frac{1}{\sqrt{|S_1(p,q)|}\sqrt{|S_2(p,q)|}}$, optimizes correlation results from both amplitude and phase information in both transformed interrogation areas.

The use of SPOF yields larger signal to noise ratios and thinner Gaussian correlation peaks. In addition, SPOF also reduces peak locking effects when compared to traditional correlation methods. Biases can be introduced in each interrogation area due to different

Figure 2.7: Correlation process [28].
size particles moving at different speeds. These larger particles, which have larger particle image sizes, may weigh heavier on the correlation peak than smaller particle image sizes. The SPOF windowing function, which essentially equalizes the amplitude contribution of each particle to the correlation peak, reduces such bias from poly-dispersed seeding. SPOF also improves results from data which includes non-uniform seeding and surface flare effects. These benefits make it such that background subtraction and other image pre-processing effects are not necessary.

The method for removal of outliers is primarily performed based on the method presented by Westerweel and Scarano (2005) [27]. This method normalizes the median fluctuation (or residual: \( r_0 = U_0 - U_m \)) of the investigated velocity vector \( U_0 \), by the median of its eight nearest neighbor residuals \( r_m \) plus a conservative uncertainty \( \epsilon \) of 0.1 pixels. If this resulting value is above a threshold of 2, then the vector is rejected.

\[
    r_0^* = \left\{ \frac{|U_0 - U_m|}{r_m + \epsilon} \right\} < 2
\]  

In addition to this method, general thresholds are set on minimum and maximum \( U \) and \( V \) velocity for locations in the flow field in which the flow velocity is known a priori (i.e. boundary layer flow). Finally, all resulting vector maps are spot checked manually for outliers that may have made it through the afore mentioned processes.

### 2.5 Seeding Strategy

The theory of PIV remains the same for the multitude of flows which have different scales, speeds, and fluids. What does change, however, is the choice of tracer particles and the method in which they are seeded into the flow. The flow speed and fluid density is oftentimes used as the criteria for selecting the tracer particle material and size. Because of the density difference between the gas and the liquid or solid tracer particle, the size of the tracer particle must be considered. This consideration is discussed in more detail in the
following Section 2.6. Assuming the proper particle density and diameter (as the particles taken to be spherical) have been selected for performing PIV in a gas flow, the method of seeding the particles homogeneously and with a desired seeding density require particular care especially when the flow is supersonic.

Many commonly used solid tracer particles such as Titanium Dioxide (TiO$_2$) and Aluminium Oxide (Al$_2$O$_3$), are selected as the seed material for supersonic PIV that have diameters of 1 µm or less. These seeding materials are purchased from suppliers and come in the form of a dry powder. To properly seed the material into the flow, a fluidized bed is typically used to suspend the dense powder and to have the material behave as a collection of individual particles resembling that of a fluid. At such small sizes, frictional and electrostatic forces oftentimes cause agglomeration of particles. Because of this, it is customary to incorporate a cyclone separator which uses angular momentum to minimize agglomerated particles from entering the flow. Lastly, the mass flow rate of seeding material must be properly controlled as to obtain a desired seeding density.

To employ the use of micron to sub-micron sized solid particles is a costly endeavour as most equipment must be custom designed and fabricated to work sufficiently for an individual wind tunnel. There is even greater cost when designing the equipment for a supersonic wind tunnel which operates at high stagnation pressures. Furthermore, there are environmental and respiratory health concerns, especially when used in a blow-down wind tunnel. Because of these reasons, this solid seeding method was not an option for this study.

The use of ice crystals were experimentally used as the tracer particles to perform the PIV for this research. Liquid water was heated to 185°C under pressure up to 5.86 MPa in a 0.5 L tank using two Omegalux flexible heaters. The temperature of 185°C influences the water to atomize as small as possible when sprayed into the stagnation chamber which is at a pressure of 1.38 MPa. At 1.38 MPa, the temperature of saturated steam is 194.33°C.
$185^\circ$C was chosen to virtually eliminate the risk of flashing the water into steam in the event of a pressure fluctuation of the stagnation chamber, as well as for other safety reasons.

Two BETE-PJ40 fogging nozzles are used to spray the heated water into the stagnation chamber. The BETE-PJ40 nozzles are rated to produce a fog of droplets of sizes ranging from 1 µm to 50 µm. By heating the water near the boiling temperature, this promotes smaller droplet sizes, shifting the particle size distribution to be more near to 1 µm and below. As the small droplets travel through the supersonic nozzle, the rapid expansion and drop in temperature to 84.3 K freezes the droplets into small ice crystals. In addition, the regulated pressure behind the heated water as it is sprayed through two BETE-PJ40 fogging nozzles is used to control the seeding density.

Particle Image Velocimetry was yet to be performed in the Rutgers Supersonic Wind Tunnel prior to this research. The proper water mass flow rate through the nozzles, ice crystal size, and faithfulness to tracing the flow were unknown.

### 2.6 Particle Response Assessment

Particle Image Velocimetry measures the velocity of tracer particles, not the velocity of the fluid itself. The ultimate accuracy of PIV to measure the velocity of the fluid, is limited by how well the seed particles represent the instantaneous motions of the flow [16]. In flows that exhibit large velocity gradients, the particle velocity may lag behind the fluid velocity. Because of this fact, it is important to assess the particle response time. The frequency response of the seed particles limit the resolvability of measurements of turbulent motions based on their time scale. A commonly accepted method to measure seed particle lag is to consider the motion of a particle as it passes through an oblique shock wave.

As air passes across a shock, the change of velocity is considered to be instantaneous. This instantaneous change does not hold for a tracer particle because of inertia. The motion of the characteristically spherical particle, which undergoes a step response,
exponentially relaxes to the velocity of the fluid downstream of the shock. By setting up an experiment in which PIV is performed in the region of an oblique shock, the relaxation time and thus particle diameter may be derived from velocity data along a streamline which passes through the shock.

When assessing the particle response time and inferring an effective particle diameter \(d_p\) using PIV, it is common to make several simplifying assumptions. Assuming \(Re_d \ll 1\), incompressible flow such that \(M \ll 1\), and that the Knudsen number based on the particle diameter is small \(Kn_d \ll 1\), Stokes drag law can be applied [26]:

\[
F_d = 3\pi \mu_f U d_p \quad \text{or} \quad \frac{\partial U_p}{\partial t} = \frac{18 \mu_f U}{\rho_p d_p^2}
\]  

(2.6)

where \(U\) is the relative velocity of the fluid downstream of the shock and the tracer particle, \(U = (U_2 - U_p)\).

Using the equation of motion of a particle:

\[
U_2 = U_p + \frac{\partial U_p}{\partial t} \tau_p
\]  

(2.7)

where \(\tau_p\) is the relaxation time, it is found that:

\[
\tau_p = \frac{\rho_p d_p^2}{18 \mu_f}
\]  

(2.8)

To incorporate inertial, compressibility, and free-molecule effects, a modified Stokes drag law is commonly used [26]:

\[
\tau_p = \frac{\rho_p d_p^2}{18 \mu_f} \left(1 + 2.7 Kn_d\right)
\]  

(2.9)
where $Kn_d$ is the Knudsen number defined as the mean free path by the particle diameter.

$$Kn_d = \frac{\lambda}{d_p}$$

$$\lambda = \frac{kT}{\sqrt{2\pi d^2 P}}$$ \hspace{1cm} (2.10)

Solving the equation of motion for a step response where the fluid velocity upstream and downstream of the shock are $U_1$ and $U_2$ respectively, yields:

$$U_p = (U_1 - U_2) \exp\left(\frac{-t}{\tau_p}\right) + U_2$$ \hspace{1cm} (2.11)

or rewritten as:

$$\frac{(U_p - U_2)}{(U_1 - U_2)} = \exp\left(\frac{-t}{\tau_p}\right)$$ \hspace{1cm} (2.12)

At this point, plugging $\tau_p$ (one relaxation time) for $t$ into equation 2.12, the distance from the beginning of the step response to the point at which $U_p$ satisfies the above equation, is taken to be the relaxation distance. The relaxation distance of a particle is defined as the distance required for that particle to be reduced by $1/e = 0.368$ [16]. Determining the relaxation time experimentally is then found by dividing the relaxation distance by the average fluid velocity downstream of the shock $U_2$. Lastly, the effective particle diameter can be calculated using Equation 2.9.
Using the mean velocity from an ensemble size of 80, the particle relaxation time was assessed along 10 streamlines which passed through the oblique shock cast from a 9° angle of attack flat plate. The results are shown in Table 2.1.

The criterion for acceptable flow tracking is that the Stokes number $St \ll 1$ \cite{10}. For an integral time scale $\tau_f = \frac{\delta}{U_\infty}$, the Stokes number is equal to $St = \frac{\tau_p}{\tau_f} \ll 1$. For $\delta = 15.6$ mm and $U_\infty = 621$ m/s (shown in the next section), $St = 0.11$. This value higher than the results of Humble et al. and Urban and Mungal \cite{26} who reported $St = 0.06$. It is with this result that the ice crystals are said to yield results with accuracy reasonable for the scope of this study.

| $\rho_p$ | Ice Density | 917 kg/m³ |
| $\mu_f$ | Fluid Viscosity | 6.52 µPa·s |
| $\lambda$ | Mean Free Path | 133 nm |
| $\tau_p$ | Particle Response Time | $2.8 \pm .22$ µs |
| $d_p$ | Particle Diameter (modified Stokes) | $0.43 \pm 0.02$ µm |

Table 2.1: Particle Relaxation Data
2.7 Sources of Error

Seeding Evaluation

Figure 2.9, represents a single image collected based on the setup and seeding strategy discussed in the previous sections 2.3 and 2.5. From Figure 2.9 it is observed that the particle diameters are poly-dispersed. This is evident by the varying sizes of the particle image diameters within the image. This is most likely a consequence of the BETE-PJ40 sprayer nozzle and its wide range of droplet sizes despite heating the water.

Figure 2.9: Example of a recorded PIV image of Region 1 of SWTBLI as defined in section 4.2.1

The seed particles are also non-uniformly distributed throughout the boundary layer. It is assumed that the residence time of a single ice crystal in the test section is too small for the ice crystal to melt from the temperature of the boundary layer which increases towards the wall. It is more likely that angular momentum of vortices in the boundary layer cast
larger particles to the edges. This non-uniformity at times causes poor or no correlation in regions of low seeding density. Smaller or less dense seed particles would most likely have a positive impact on future results.

Optical aberration of the oblique shock wave causes a blurring of particles and increases the difficulty of proper correlation in that region. This aberration is caused by an apparent thickening of the shock as it interacts with the boundary layer formed on the side window of the test section. The correlation accuracy may be reduced in this region because of such blurring.

Systematic Error

Throughout this investigation, the laboratory equipment was shared amongst several students, who each required a different setup for their respective experiments. For this reason, prior to the collection of PIV data, the reassembly of the laboratory setup is scrutinized for consistency. Between each wind tunnel run, the camera is repositioned and realigned with the new imaging interrogation region. Great care is taken to reduce error due to misalignment.

The high pressure air, which drives the supersonic wind tunnel, is stored outside of the facility and is subject to the ambient temperature. The data for this research has been collected over several seasons in which the ambient temperature has changed. There is also a chance that the stagnation temperature changes between runs due to thermodynamic cooling of the air storage tanks from expansion during the run. There is typically 30 minutes between each run while the water/seed heats back to the proper temperature, but it is unknown if this duration is sufficient for the air in the storage tanks to return to the ambient temperature.

Lastly, the design stagnation pressure of the wind tunnel is 1.38 MPa and this is the value used in calculations. The wind tunnel is throttled by a manual ball valve in which the valve is quickly opened by hand and the stagnation pressure is read from a pressure gauge.
attached to the stagnation chamber. The operator keeps the stagnation pressure as close to 1.38 MPa as possible by manipulating the ball valve, however there may be fluctuations, undershoot, or overshoot of the stagnation pressure during the wind tunnel run. This type of variation should not pose unreasonable alteration to the flow nor nullify data during a wind tunnel run.

**Correlation Error**

The minimization of errors produced by correlation algorithms is a study by itself. The discussion of error is often left out of research articles, perhaps due to the difficulty in obtaining a quantifiable result. It is shown that generalized cross-correlation methods with sub pixel accuracy can result with a total error of 0.1 pixel [9]. Correlation methods which include window shifting and window deformation can reduce this error by an order of magnitude or more.

Velocity gradients found within an interrogation area will skew the velocity estimate by widening the correlation peak due to a larger averaging effect found in the interrogation area. Reducing the size of the interrogation area lessens the effect from velocity gradients, however this is not always an option because of seeding density, magnification, and minimum inter frame times. Window shifting, window deformation, and SPOF are used to minimize error due to velocity gradients and poly-dispersed non-homogeneous seeding.

For simple flows, where the mean flow is known a priori, an analytical bias error can be determined which can be used to correct for experimental error [14]. However in the present study, while the exact quantification of error is limited, the applied methods of window shifting, window deformation, and SPOF is believed to reduce the overall error due to correlation to less than the standard correlation error of 0.1 pixels.
Standard Error

The standard error of the mean is calculated to obtain 95% confidence bounds for mean flow velocities. It is calculated using:

\[ x' = \bar{x} \pm T_{\nu,p} \cdot S_\bar{x} \quad (2.13) \]

\[ S_\bar{x} = \sqrt{\frac{1}{(N-1)} \sum_{i=1}^{N} (x_i - \bar{x})^2} \quad (2.14) \]

where \( N \) is the number of samples, \( \bar{x} \) is the sample mean, \( T_{\nu,p} \) is the student’s T parameter for 95% confidence, and \( S_\bar{x} \) is the standard sample variation divided by the square root of the number of samples.
3. Undisturbed Boundary Layer Properties

The purpose of this chapter is to present the statistical properties of the undisturbed boundary layer formed on the floor of the test section. By characterizing the boundary layer prior to any perturbations, base conditions are established.

3.1 Introduction

For a boundary layer in a supersonic flow, the effect of compressibility causes the boundary layer to thicken. This occurs because of the decrease in density due to an increase of temperature across the boundary layer as the wall is approached.

Utilizing the Van Driest compressible boundary layer transformation (Equation 1.7), it is possible to analyze compressible boundary layers using incompressible boundary layer theory. Morkovin’s hypothesis and subsequently the strong and weak Reynolds analogies has been shown to hold well for flows where \( M_\infty < 5 \). Morkovin’s hypothesis suggests the usage of incompressible turbulence models for compressible flows with the notion that density fluctuations are small and do not effect the structure of turbulence. The strong Reynolds analogy states that for moderate rates of heat transfer at the wall, the total temperature can be assumed to remain the same across the boundary layer. The weak Reynolds analogy states that any fluctuations in the static pressure are small when compared to fluctuations in the density or temperature. By applying the strong and weak Reynolds analogies, the use of an extended Crocco relation and ideal gas law are used to
describe the temperature and density gradients based upon the velocity data obtained through PIV.

### 3.2 Analysis

PIV data of the undisturbed boundary layer was obtained as shown in Figure 2.3. The boundary layer found on the floor of the test section is considered to be fully developed in this field of view. To increase the size of the ensemble of the mean boundary layer profile, the data from all image pairs are averaged in the streamwise direction. From the size of imaging region, interrogation area, and step size shown in Table 3.1, the ensemble size represents well over two thousand boundary layer profiles.

In high-speed flows, the linear region of the boundary layer velocity profile is found very close to the wall. A limitation of PIV comes from the difficulty of obtaining data near solid surfaces in which surface flare (scattered laser light on solid surfaces) can corrupt image data and large velocity gradients can be a source of bias error. For this research, despite numerous attempts using high levels of magnification and cautious control of reflections and glare, the linear region could not be measured. Because of this inability to collect data in the linear region, the friction velocity cannot be determined using equations 1.2 and 1.4.

The Clauser method is an accepted practice for determining $u_\tau$ by fitting the ensemble averaged data to Equation 1.7. To transform the data to wall unit scaling, it is necessary to solve for $y^+$ and $u^+$ by approximating values of $u_\tau$. To find the viscosity at the wall where, $\nu_w = \frac{\mu_w}{\rho_w}$, it is first necessary to introduce a recovery factor and use the energy equation to solve for the static adiabatic temperature at the wall:

$$r = \frac{(T_{aw} - T_e)}{(T_0 - T_e)}$$  \hspace{1cm} (3.1)
\[ T_0 - T_e = \frac{u_e^2}{2c_p} \]  
\[ T_{aw} = T_e \left( 1 + r \frac{(\gamma - 1)}{2} M_e^2 \right) \]

The recovery factor \( r \) is taken to be a constant \( r = .89 \) which implies a turbulent Prandtl number not equal to unity. Since there is no direct way to measure the Mach number in the test section, the boundary layer edge static temperature, \( T_e \), is first approximated from the energy equation, Equation 3.2. The stagnation temperature is the outside ambient temperature since the high pressure storage tanks are located outside of the laboratory building. The specific heat of air \( c_p \) is temperature dependent, however it is reasonably assumed to be a constant of 1004 J kg\(^{-1}\) K\(^{-1}\) for simplicity. The Mach number can then be calculated by the standard definition \( M_e = \frac{U_e}{\sqrt{\gamma R T_e}} \).

The use of an extended Crocco relation is then used to solve for the temperature profile of the boundary layer [2]. In this relation, \( T_w \) is the temperature of the wall, and \( T_{aw} \) is given by equation 3.3.

\[ T(y) = T_w + (T_{aw} - T_w) \left( \frac{u(y)}{u_e} \right) + (T_e - T_{aw}) \left( \frac{u(y)}{u_e} \right)^2 \]

The density profile of the boundary layer can be solved using the ideal gas equation. The free stream static pressure is found from the isentropic relation shown in Equation 3.5a and plugging in the known stagnation pressure of \( P_0 = 1.38 \text{ MPa} \) and constants \( \gamma = 1.4 \) and \( R = 287.03 \frac{J}{kg\cdot K} \). The free stream Mach number is found from the measured free stream velocity and approximate free stream temperature deduced using the energy equation.
\[ P = P_0 \left( 1 + \frac{(\gamma - 1)}{2} M_\infty^2 \right) \left( \frac{-\gamma}{(\gamma - 1)} \right) \]  
(3.5a)

\[ \rho(y) = \frac{P}{RT(y)} \]  
(3.5b)

\[ \rho_w = \frac{P}{RT_w} \]  
(3.5c)

\( \rho_w \) is also solved from the ideal gas equation using the constant static pressure and stagnation temperature.

Lastly, solving for the kinematic viscosity at the wall, Sutherland’s formula for air is used to express the dynamic viscosity as a function of temperature:

\[ \mu_w = \mu_{ref} \left( \frac{T_{ref}}{T_w} \right) + 110.3 \left( \frac{T_w}{T_{ref}} \right)^{3/2} \]  
(3.6)

\( T_{ref} \) and \( \mu_{ref} \) are the reference temperature and dynamic viscosity which equal 273 K and 17.1 \( \mu Pa \cdot s \) respectively.

### 3.3 Results

#### 3.3.1 XY Plane Data

The ensemble size of the boundary layer is augmented by averaging the columns of all vector fields in all image pair results. This is done under the assumption that the boundary layer is fully developed within the field of view.
<table>
<thead>
<tr>
<th>Ensemble Size</th>
<th>2961</th>
</tr>
</thead>
<tbody>
<tr>
<td>Digital Resolution</td>
<td>24 ( \mu m ) pixel</td>
</tr>
<tr>
<td>Interrogation Area</td>
<td>Pass 1: 64x64, Pass 2: 32x32 pixels</td>
</tr>
<tr>
<td>Step Size</td>
<td>16 pixels (50% overlap)</td>
</tr>
</tbody>
</table>

Table 3.1: Boundary Layer PIV Properties

Figure 3.1: Van Driest transformed mean velocity profile
For an empirically determined friction velocity, $u_\tau$, the experimental data agrees well with Van Driest’s log law. At a magnification of $24 \frac{\mu m}{pixel}$, the closest wall resolution is $y^+ \approx 500$. This magnification of $24 \frac{\mu m}{pixel}$ is used to observe the SWTBLI discussed in the next chapter. This level of resolvability is comparable to the research performed by Souverein et al. in which the collected data was based on a first reliable measurement of $y^+ = 1000$ [24].

The error bars shown on Figure 3.1 represent the standard error of the mean at 95% confidence. As the wall is approached, the data deviates from the Van Driest log law. Near the wall, PIV may suffer from inaccuracy due to gradient bias, elevated levels of streamwise turbulence, as well as surface flare. The last reliable point of $y^+ \approx 500$ is determined by comparing the law-of-the-wall theoretical value to be within the confidence interval for this data point. The error bars decrease to nearly the size of the markers as the free stream is approached since the streamwise velocity fluctuations decrease. The statistical properties of the boundary layer are listed in Table 3.2 where the displacement thickness $\delta^*$ and
momentum thickness θ are defined as:

\[
\delta^* = \int_0^\delta \left( 1 - \frac{\rho u}{\rho_e u_e} \right) dy
\]  

(3.7)

\[
\theta = \int_0^\delta \frac{\rho u}{\rho_e u_e} \left( 1 - \frac{u}{u_e} \right) dy
\]  

(3.8)

The end of the logarithmic region occurs near \( y^+ \approx 3000 \) which corresponds to a \( \frac{y}{\delta} \approx 0.4 \). This extent of the logarithmic region is similar to that found by Ganapathisubramani and Lin et al. [7, 15].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_\infty )</td>
<td>3.22</td>
</tr>
<tr>
<td>( U_\infty )</td>
<td>621 m/s</td>
</tr>
<tr>
<td>( \delta )</td>
<td>15.6 mm</td>
</tr>
<tr>
<td>( \theta )</td>
<td>0.85 mm</td>
</tr>
<tr>
<td>( \delta^* )</td>
<td>4.70 mm</td>
</tr>
<tr>
<td>( Re_\theta )</td>
<td>78300</td>
</tr>
<tr>
<td>( T_0 )</td>
<td>285 K</td>
</tr>
<tr>
<td>( P_0 )</td>
<td>1.38 MPa</td>
</tr>
<tr>
<td>( \rho_e )</td>
<td>0.99 kg/m³</td>
</tr>
<tr>
<td>( \rho_w )</td>
<td>0.33 kg/m³</td>
</tr>
<tr>
<td>( \mu_e )</td>
<td>6.67 µPa s</td>
</tr>
<tr>
<td>( \mu_w )</td>
<td>17.7 µPa s</td>
</tr>
<tr>
<td>( u_\tau )</td>
<td>22.45 m/s</td>
</tr>
</tbody>
</table>

Table 3.2: Boundary Layer Statistical Properties
Figure 3.3: Boundary layer temperature profile

Turbulence Statistics

Figure 3.4: Streamwise velocity fluctuations normalized with Morkovin scaling.
It is seen that the streamwise velocity fluctuations trend towards zero as the free stream is approached. Conversely, as the wall is approached the fluctuations increase as a result of turbulence production near the wall. However, precisely at the wall turbulence must go to zero as the velocity goes to zero. This is not observed in the data because of the limited resolvability. There is a region from \( y/\delta \approx 0.4 \) to \( y/\delta \approx 0.6 \) where the trend of the fluctuations plateau, which is followed by a sharper decrease towards zero. This observation agrees with published results [7, 23, 10, 24]. An example of such data can be seen in Figure 3.7.

Vertical velocity fluctuations are less commonly measured because the data typically contains more scatter which limits conclusions [23]. Figure 3.5 agrees with this statement. It is seen that the data is very scattered which can be attributed to the highly turbulent nature throughout the boundary layer. Despite the scatter, the trend of the data resembles that shown in published results. A similar plateau region exists at \( y/\delta \approx 0.3 \) to \( y/\delta \approx 0.6 \) but at nearly 4 times less than that of the streamwise velocity fluctuation at this location.

Figure 3.5: Vertical velocity fluctuations normalized with Morkovin scaling
All three fluctuation results show large values which lie between $y/\delta \approx 0.2$ to $y/\delta \approx 0.6$. These results are suggestive of the existence of large scale structures which may cause such fluctuations.

**Galilean Decomposition**

To investigate the coherent structures which may have been captured in the instantaneous data, Galilean decomposition is performed. This technique is simple and is done by subtracting a percentage of the free stream velocity from the entire velocity field.
Mathematically:

\[ U_d = U - U_c \]  

(3.9)

where \( U_d \) is the decomposed vector velocity field. The resulting velocity field then represents fluid motion relative to a particular convection velocity \( U_c \).

Figure 3.8: Instantaneous boundary layer data decomposed by \( U_c/U_\infty = 0.85 \).

Subtracting \( U_c/U_\infty = 0.75 \) from several normalized instantaneous images, it is observed that structures and vortices span intermittently from \( \frac{y}{\delta} \approx 0.1 \) to \( \frac{y}{\delta} \approx 0.3 \). Figure 3.9a and Figure 3.9b show such decomposition performed. The presence of a few vortical structures centered near \( \frac{y}{\delta} = 0.2 \) are observed and are circled and enlarged to show detail. The presence of a dominant flow structure beneath the vortices may be considered to be a VITA event as an ejection flow region meets a sweeping flow region. This observation is supported by the Adrian et al. and Lin et al. who show that the hairpin has a vortex head located above a VITA event when viewed in a streamwise plane [1, 15].

Hairpin vortices are also seen occasionally at larger convection velocities particularly \( U_c/U_\infty = 0.85 \). In this case (Figure 3.8), such vortices are witnessed higher in the boundary layer around \( \frac{y}{\delta} = 0.4 \) to \( \frac{y}{\delta} = 0.5 \).
3.3.2 XZ Plane Data

To observe the boundary layer in the spanwise direction, the laser sheet was configured parallel to the test section floor at a height $y = 0.1\delta$. The streamwise velocity $U$ agreed
with results from the XY plane data of $U = 0.75U_\infty$. Velocity in the spanwise direction $W$ had a maximum of $W = 0.02U_\infty$.

**Galilean Decomposition**

Flow structures revealed by Galilean decomposition show alternating regions of high and low streamwise velocity compared to the mean velocity of $U = 0.75U_\infty$. In Figure 3.10, the contour plot and velocity vectors show the instantaneous streamwise velocity normalized by the free stream velocity, while the vectors are the Galilean decomposition with $U_c/U_\infty = 0.75$.

Ganapathisubramani et al. [8] observed similar coherent structures, as mentioned in Section 1.2. From the limited field of view, the full streamwise length of such structures are unknown however they are clearly coherent for at least $2\delta$ in the streamwise direction and roughly $0.7\delta$ in the spanwise direction. It may possible to infer that the regions which convect uniformly downstream are the legs of hairpin vortices. Likewise, the regions which appear to convect coherently upstream may be flow in between the legs of such hairpins.

It is also interesting to note the existence of spanwise vortices which appear at the shear layer formed between the high and low velocity regions. These vortices are best shown with a convection velocity of $U_c/U_\infty = 0.8$. Figure 3.10c,d show pairs of counter rotating vortices which have been circled. These structures have diameters which range from $0.1\delta$ to $0.4\delta$. 
Figure 3.10: Four instances of spanwise PIV at $y/\delta = 0.1$. Flow direction is from top to bottom. The contour map and vectors show instantaneous data decomposed by $U_c/U_\infty = 0.75$ and $U_c/U_\infty = 0.8$. 
4. Shock Wave Turbulent Boundary Layer Interaction

The purpose of this chapter is to characterize the flow field resulting from the interaction of an oblique shock impinging upon the boundary layer which has been characterized in the previous chapter. The impinging oblique shock wave is generated from a 6 and 9 degree angle of attack flat plate and the resulting interactions are compared. Figure 4.1 shows the 9 degree angle of attack flat plate mounted in the test section.

Setup

Figure 4.1: 9 degree angle of attack flat plate mounted in test section

Throughout this investigation, an angle of attack flat plate which spans 96% of the test section is used to generate the oblique shock. To reduce the risk of any accidental damage to the side windows in the event that the flat plate unexpectedly shifted during start up, shut down, or unstart, the flat plate was deliberately machined to not span the test
section completely. The intent is to keep the three-dimensional SWTBLI phenomena as two-dimensional as possible.

For the following sections, the schlieren setup is described in Section 2.2 and the particle image velocimetry setup and strategy is described in Sections 2.3 and 2.5.

4.1 Schlieren

Prior to quantifying the flow under investigation using PIV, a qualitative approach was taken in order to observe the general behavior of the shock wave turbulent boundary layer interaction region. Figure 4.2 shows the average of 300 instantaneous schlieren images of the interaction region. The images show the mean result from an angle of attack of the flat plate that is increased from 6 degrees to 9 degrees. The images have been enhanced to show detail using an unsharp mask filter.

The apparent Mach lines emanating from the bottom of the plate (more apparent in 4.2b-d) are generated from the minor surface roughness left over from the machining process and small sections of spray paint which have flaked off during the wind tunnel runs. The differences in levels of detail (darkness) from image to image is due to inconsistent knife edge cut-off. Nevertheless, the desired qualification of the interaction region is still achieved.

It can be seen that for an increasing angle of attack, the severity of the interaction of the oblique shock and the boundary layer also increases. It is seen that the impinging oblique shock wave is very well defined in the average images, while the reflected and separation shocks appears blurred. This blurring of the shock is an indication of unsteadiness due to shock motion. The separation shock motion is evident in the instantaneous images, and becomes more noticeable as the angle of attack is increased.
Figure 4.2: Schlieren images of a varying angle of attack flat plate. $a = 6^\circ$, $b = 7^\circ$, $c = 8^\circ$, $d = 9^\circ$

The depth at which the impinging shock penetrates the boundary layer decreases with increasing angle of attack. It is also seen that relative to the intersection point of the separation shock and impinging shock, the mean location of the separation shock moves further upstream with increasing angle of attack. This suggests an increase in the size of the separation bubble streamwise as well as normal to the floor.

The increase of the separation region is due to the increased static pressure rise across the oblique shock as the angle of attack is increased. This increased pressure across the shock produces a greater adverse pressure gradient upon the boundary layer which eventually leads to boundary layer separation. Figure 4.2a resembles the weak SWTBLI case shown in Figure 1.1, while Figure 4.2d resembles the strong SWTBLI case shown by Figure 1.2.
4.2 Particle Image Velocimetry

In this section, PIV data is analyzed for the interaction region of an impinging oblique shock generated by a 6 and 9 degree angle of attack flat plate. The mean flow and instantaneous data are analyzed and discussed.

4.2.1 XY Plane Data - 9 Degree Angle of Attack

Since the interaction region of SWTBLI is inherently three-dimensional, several streamwise assessments of the interaction were investigated. The locations of the laser
sheet for each trial is shown in Figure 4.4. For simplicity, the remainder of this report will refer to each XY plane sequentially as the following:

<table>
<thead>
<tr>
<th>Plane Number</th>
<th>Distance from Center of Test Section</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plane 1</td>
<td>-18mm</td>
</tr>
<tr>
<td>Plane 2</td>
<td>-13mm</td>
</tr>
<tr>
<td>Plane 3</td>
<td>-3mm</td>
</tr>
<tr>
<td>Plane 4</td>
<td>0mm</td>
</tr>
<tr>
<td>Plane 5</td>
<td>3mm</td>
</tr>
<tr>
<td>Plane 6</td>
<td>13mm</td>
</tr>
<tr>
<td>Plane 7</td>
<td>18mm</td>
</tr>
</tbody>
</table>

Each XY plane consists of three interrogation regions with an overlap of 0.25 in. These regions are shown in Figure 4.5. The numbers above the quarter inch grid designate the beginning and end point of each interrogation region in the streamwise direction. By breaking up the flow field into separate regions, the magnification and thus resolvability of the flow is increased. After the data is collected for each of the three regions, the data is then stitched together at the midpoint of each overlap region. This stitching of data reproduces the mean full field SWTBLI region.

![Image](image-url)

Figure 4.5: Schlieren image of 9 degree angle of attack flat plate with overlaying quarter inch grid showing the breakup of the three interrogation regions. The blue slanted lines represent the overlapped areas.

The PIV recording parameters are listed in Table 4.1 below.
Table 4.1: PIV parameters for XY plane SWTBLI data

Mean Flow Results

Appendix A.1 contains the mean full field interaction region for each plane. The results show streamlines plotted over filled contours of the streamwise and vertical velocities non-dimensionalized with the free stream streamwise boundary layer velocity of 621 m/s. The X and Y axis are scaled with the undisturbed boundary layer thickness $\delta = 15.6$ mm. On the X axis, the zero point represents the theoretical location where the impinging oblique shock would strike the wall in the absence of the boundary layer. The evident discontinuities present at $x/\delta \approx -2.5\delta$ and $x/\delta \approx -0.75\delta$ are the locations where the data is stitched together as described in Section 4.2.1.

All seven planes show similar flow characteristics as described in Section 1.1.2 and illustrated in Figure 1.2. It is observed that the location of the coalescing separation shock begins at about $-4.4\delta$ for each of the seven planes. This suggests that the impinging oblique shock is applying an even adverse pressure gradient across the span of the observable section of the boundary layer. In each plane, the intersection point of the separation shock and the impinging oblique shock are found at $x/\delta \approx -2.5$ and $y/\delta \approx 1.1$.

Using Equation 3.2 to approximate local temperature from the measured velocity, the sonic line is plotted where the local Mach number is unity. From this, it is observed that the subsonic region reaches a maximum height of $y/\delta \approx 0.7$ and extends well beyond the field.
of view in the streamwise direction. The sonic lines from all planes are plotted in Figure 4.6. It can be seen that while the general trend is preserved from plane to plane, the scatter of data increases moving towards the sonic line peak.

![Figure 4.6: Plot of the calculated sonic lines.](image)

Figure 4.7 shows the divergence of the full field expressed by the two-dimensional compressible continuity equation. This plot may be analysed similarly to that of a schlieren image. The intention is to better illustrate the locations of shocks and expansions as compared to simply observing velocity contour plots.

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = -\frac{1}{\rho} \left( u \frac{\partial \rho}{\partial x} + v \frac{\partial \rho}{\partial y} \right)
\] (4.1)

Using Equation 4.1 and Figure 4.7, negative values represent density increases across shocks while positive values represent density decreases across expansions. Downstream of the expansion fan near the peak of the sonic line, patches of small negative divergence may be the beginning of the weak compression waves which serve to gradually turn the flow parallel to the wall. Figure 4.7 also suggests that the temperature deduced by Equation 3.2 is a reasonable approximation justified from the observation that the impinging oblique shock wave terminates near the peak of the calculated sonic line as expected.
Within the interaction region of each plane, reverse flow is evident. The reverse flow reaches a maximum velocity of about $40 \text{ m/s}$ (in the mean) which is less than $0.1U_{\infty}$. In each case, the separation point occurs around $x/\delta = 3.35$ while the reattachment point is shown to vary from plane to plane. This discrepancy of the size, location, and nature of the reverse flow region suggests that the separation bubble is highly three-dimensional despite
that the flat plate spans 96% of the test section. To assess the three-dimensionality, plots of the streamwise velocity $U$ along fixed heights of $y = 0.03\delta, y = 0.25\delta, y = 0.5\delta, y = 0.75\delta$, and $y = 1.45\delta$ throughout the full field interaction are shown in Figure 4.9. The velocities are scaled with the free stream velocity found from the non-perturbed boundary layer data $U_\infty = 621 \text{ m/s}$.

Outside of the boundary layer, the velocities collapse well, which further supports that two-dimensionality of the impinging oblique shock is maintained. Plane 1 does however show a discontinuity and deviation after the first stitching point. This deviation may be explained by systematic error explained in Section 2.7. Along the fixed heights of $y = 0.03\delta$ through $y = 0.75\delta$, it is clear to see that the data exhibits larger scatter after passing through the separation shock. This is expected due to enhanced turbulence, turbulence production, and unsteady effects. Studying the figures in Appendix A.1, the strong interaction phenomena is preserved in general from plane to plane, however the streamlines show that in and around the separated region the planes differ.

Planes 1, 2 and 7 show the presence of a recirculation region centered near $x/\delta \approx 1.75$, $x/\delta \approx 2.0$, and $x/\delta \approx 1.75$ respectively. Planes 3 through 6 all show a large area of reverse flow, but do not exhibit the same obvious recirculation as seen in Planes 1, 2 and 7. Upon inspection of the streamlines, separation point, and reattachment point, it appears that there is a lack of symmetry about Plane 4, the center plane.

This lack of symmetry might be the effect of the low ensemble size as a result of the limitations of wind tunnel run times and the quantity of available seed per run. There is also the possibility that the interaction of the side wall boundary layers and the oblique shock cast from the angle of attack flat plate (which spans into these boundary layers) may have an effect on the interrogation regions of interest. Minor inaccuracies in the construction of the flat plate and mounting bracket may also produce non-symmetric flow effects. A crude oil streakline experiment is carried out in an attempt to gain insight of the effect of the
impinging oblique shock wave upon the side wall boundary layers. This experiment and results are discussed in Section 4.2.3.

Figure 4.9: Normalized streamwise velocity across the field of view at five fixed heights ($y = [1.45\delta, 0.75\delta, 0.5\delta, 0.25\delta, 0.03\delta]$). Height is labeled next to each corresponding group of data.

Discontinuities at the stitching locations of $x/\delta \approx -2.5$ and $x/\delta \approx -0.5$ show that run conditions may not have been precisely preserved during the collection of data. This inconsistency is most likely attributed to the procedure of manually throttling of the ball valve (discussed in Section 2.7) which is done to maintain a constant stagnation pressure throughout each wind tunnel run.
Appendix A.2 shows the average streamwise velocity of all 7 planes as a series of profiles taken throughout the entire field of view. Average 95% confidence error bars have also been placed accordingly. It is seen that the boundary layer is separated by $x = -3.3\delta$ and is reattached by $x = -1.19\delta$. It follows that the average separation bubble length is $2.11\delta$ or 31.9 mm and the average height of the separation bubble is $0.12\delta$ or 1.91 mm.

Velocity profiles from $x = -4.54\delta$ to $x = -3.92\delta$ show the boundary layer influenced by the adverse pressure gradient. As the velocity begins to stall near the wall, this velocity deficit is felt upwards through the boundary layer to about $y/\delta \approx 0.5$. Further on from $x = -3.92\delta$, the presence of the separated flow and dilation of the subsonic region reduces the velocity in the upper portion of the boundary layer producing a less full profile. This is seen to occur up to the reattachment point of $x = -1.19\delta$ where then the boundary layer begins to recover.

The largest standard error of ± 8% the free stream velocity is found at the origin of the separation shock where the highest streamwise fluctuation is found. Within the separation region, the error is 1.5% - 2.3% which corresponds to 9.32 m/s and 14.3 m/s respectively.

**Root Mean Square Fluctuations**

The root mean square of the fluctuations of Plane 4 is seen in Figure 4.10 which can be used as a reference for the descriptions below. The remaining planes can be seen in Appendix A.3. All fluctuation results have been non-dimensionalized with the free stream boundary layer velocity of 621 m/s. It should be noted that apparent large quantities of fluctuation near the edges of the field of view are likely due to undetected outliers plus limited size of the ensemble. In addition to the reduced quality of PIV correlation at the edges of the particle images, the vector validation algorithm has particular difficulty rejecting invalid vectors at the edges because of a reduced number of accurate nearest neighbors.
There appears points of vertical velocity fluctuations outside of the sonic line, particularly downstream from intersection of the impinging oblique shock and separation shock seen in Figure 4.10b. As was seen Figure 3.5, velocity fluctuation in the Y direction is subject to larger amounts of scatter. As it is generally known, turbulence is increased when passed through a shock wave. It is no surprise that this scatter of data is increased downstream of the oblique shock when compared to upstream fluctuation. These points are most likely explained by intensified turbulence, scatter, and the low ensemble size.

It is shown that in each plane, large amounts of streamwise velocity fluctuations occur along the ascending portion of the sonic line. Majority of these fluctuations reside on the subsonic side up to $y/\delta \approx 0.7$, and abruptly decay near the peak of the sonic line. In addition, the vertical velocity fluctuations are largest slightly beyond and underneath the peak of the sonic line. The presence of vertical velocity fluctuation found by the separation shock further emphasises separation shock unsteadiness. It is also observed that the average vertical velocity fluctuation is about three times less than that of the streamwise fluctuations. This observation suggests turbulent anisotropy. These results are
in agreement with Humble et al. and Souverein et al. [10, 24] who studied SWTBLI at Mach 2.1 and 1.69 respectively.

The non-dimensionalized Reynolds stress \( \frac{-u'v'}{u_\infty^2} \) shows positive stress after the separation shock. This initial increase may be attributed to increased boundary layer turbulence as it passes through the shock. Negative Reynolds stress is found along the ascending portion of sonic line which then transitions to positive stress along the descending portion of the sonic line. The change from large positive to negative Reynolds stress is an indication of the formation of large-scale eddies [10]. The latter positive region appears to extend well beyond the field of view as the boundary layer recovers.

**Instantaneous analysis**

Examples of some instantaneous results which best show the unsteady nature of SWTBLI in each interrogation region can be seen in Appendix A.4. Contour plots of the divergence are shown to illustrate the location of the separation shock while contour plots of the streamwise velocity show the unsteady nature of the separated region and overall flow.

The location of the separation shock was observed to fluctuate in the streamwise direction which is consistent with the previous observation made using schlieren imaging. The separation shock was shown in the mean flow to originate at \( x/\delta \approx -4.4 \). Inspection of instantaneous data shows the shock moves sporadically by \( x/\delta \approx \pm 0.25 \). Downstream of the separation shock, large amounts of turbulence and complex flow structure are observed.

The separation point of the flow appears to move with the location of the separation shock as expected. The severity of the separation region and the height at which it extends into the boundary layer varies significantly. In both cases, complex flow containing reverse flow, regions of high and low speed, and seemingly discontinuous flow as observed by streamlines. There are cases of complex flow extending from the floor to \( y/\delta = 0.7 \). There are other instances in which the complexity only extends to \( y/\delta = 0.3 \) with uniform flow.
above this point. In some instances the reverse flow is severe while in others there is only moderate reverse flow. Vortical flow is also observed soon after the separation shock in some instances (Figures A.15 and A.16).

Galilean decomposition of the instantaneous images of Region 1 shows vortices when decomposing both the streamwise and vertical velocity simultaneously. The size of these vortices vary from $0.25\delta$ to $0.4\delta$ and are found at a similar height to those revealed in the non-perturbed boundary layer of $y/\delta \approx 0.2$ to $y/\delta \approx 0.4$.
In Region 2, seen in Figure 4.12, shows a very large vortical motion residing near the peak of the subsonic region and extending nearly \(1.5\delta\) in the streamwise direction at a convection velocity of \(U_c/U_\infty = 0.8\). This large circulation is apparent in most instantaneous images, however it does change size, shape, and center location from instance to instance. Further information about this structure, such as time resolved data, may be beneficial towards learning about the effects it might have upon the low frequency oscillation of the separated region. Region 3, the recovering boundary layer, also contains vortical structures which appear at convection velocities less than that seen for the incoming boundary layer. These structures also appear more sporadically than those observed in the non-perturbed boundary layer.
Figure 4.12: Galilean decomposition of Region 2 at $U_c/U_\infty = 0.8$. 
Figure 4.13: Galilean decomposition of the recovering boundary layer. (a) $U_c/U_\infty = 0.6$. (b) $U_c/U_\infty = 0.35$.

4.2.2 XZ Plane Data - 9 Degree Angle of Attack Flat Plate

Data was collected in a spanwise XZ plane at a height of $y/\delta \approx 0.4$ near the front of the separation shock. At this height, the streamwise velocity corresponds to $U/U_\infty \approx$
The data was collected to investigate the instantaneous structure of the separation shock as turbulent structures pass through. The data was originally collected at $y/\delta \approx 0.4$ as a preliminary trial with the later intention of collecting full field spanwise data of the entire interaction region. Complications of entrained oil residue in the flow collecting on the surface of the test section floor window, corrupted image data and therefore full field spanwise data of the interrogation area was not achievable. Nevertheless, the original data at $y/\delta \approx 0.4$ does show interesting results.

Similar structures to those found at $y/\delta \approx 0.1$ are seen to pass through the separation shock. Structures with widths nearly $\approx 0.5\delta$ are seen at high speeds and nearly stagnated speeds. Portions of these finger like structures also show local regions of reverse flow. Examples of such flow is seen in Figure 4.14.
These finger-like protrusions might be the result of the region of flow between the legs of a hairpin vortex passing through the separation shock. Conversely, regions which seem to flow between these fingers are the high momentum legs of the hairpins which push the separation shock downstream. Such a result would support the results of Ganapathisubramani et al. in which the separation shock translated upstream and downstream based upon the impact of a low or high momentum structure respectively. [8].

### 4.2.3 Oil Streaklines

A simple experiment was performed where a thin bead of oil, which spanned the bottom of the test section, was spread upstream of the interaction region. After running the wind tunnel, the resulting streak lines were observed to note the effects of the side wall boundary layers on the interaction region of interest. A diagram of the experiment can be seen in Figure 4.15. In this diagram, the numbers 1, 2, and 3 correspond to the PIV regions shown in Figure 4.5.
From the results of the oil streakline experiment, it is observed that the boundary layers on the side wall separate which then divert flow towards the center of the test section. Figure 4.16 shows three photographs of the results which have been dewarped and annotated. Highlighted green lines are overlayed on several predominate oil streaklines to emphasize flow direction.

It is seen from Figure 4.16b and 4.16c, that streaklines emanating near the side windows, trend towards the center of the test section. It is clear that this effect is occurring on both side walls of the test section, however it is difficult to observe whether the effects are symmetric. Figure 4.16a shows the same result as Figure 4.16c but viewed from an angle looking into the test section through the left side window. From this figure, it can be seen that the oil streaklines trend towards PIV region 2, the region in which majority of the characteristic SWTBLI is occurring (as seen from the XY plane data).
Müller et al. conducted an oil streakline test upon a $24^\circ$ compression ramp in a Mach 2.5 flow in which the test section and ramp had a width of 300 mm. Figure 4.17 shows that side wall boundary layer separation protrudes into the flow near the separation point of the floor boundary layer. From the Figure and the known test section width, the size of the side wall separation into the streamwise flow is approximately 22.4 mm or 15% of half the test section width.

The interaction of this study operates at a higher Mach number than Müller et al. in addition to the generation the impinging oblique shock which may exaggerate side wall separation more than a compression ramp. Also, the Rutgers wind tunnel has a test section width of 150 mm, half the size of that used by Müller et al. This example, in addition to the oil streaklines presented above, serves to show that effects from the side wall boundary layers might produce undesirable three-dimensional flow into the region of interest.
4.2.4  XY Plane Data - 6 Degree Angle of Attack

Mean Flow Results

The flow field which results from a 6 degree angle of attack flat plate taken in the center of the test section (Plane 4), can be seen in Figure 4.20 and Figure 4.18. The setup and PIV recording properties are the same as the 9 degree interaction experiments seen in Table 4.1.
The results resemble that of the weak interaction shown in Figure 1.1 and also agree with the schlieren imaging.

(a)

![Normalized streamwise U velocity](image1)

(b)

![Normalized vertical V velocity](image2)

Figure 4.18: Interaction region resulting from a 6 degree angle of attack flat plate. (a) Normalized streamwise U velocity. (b) Normalized vertical V velocity. In each case the calculated sonic line is shown.
The oblique shock that penetrates the boundary layer applies a weaker adverse pressure gradient than that of the 9 degree angle of attack flat plate case. The dilation of the subsonic layer generates a reflected shock which is located at \( x/\delta \approx -1.75 \). The sonic line is pulled upwards from near the wall reaching a peak at \( y/\delta \approx 0.27 \) and slowly slopes back towards the wall extending beyond the field of view. From Figure 4.19, it is observed that the reflected shock foot resides all along the ascending portion of the sonic line as a series of compressions. Beyond the peak of the sonic line a weak expansion region turns the flow towards the wall, slowly contracting the recovering boundary layer. This expansion appears in close proximity to the reflected shock which results with the bending of the reflected shock beyond the boundary layer thickness.

When comparing the mean flow of this 6 degree case to that of the 9 degree case there are several notable differences. As expected, the adverse pressure gradient pulls the sonic line up from the floor however no reverse flow is apparent in the mean flow case. The peak of the sonic line occurs at \( y/\delta \approx 0.27 \), approximately 0.4\( \delta \) lower than the 9 degree interaction. In both the 6 and 9 degree case, the approximate linear slope of the ascending sonic line is roughly the same with a value of 0.3. As the sonic line is turned back towards the wall, the recovery is more gradual than that of the 9 degree case.
Figure 4.19: Divergence of 6 degree angle of attack interaction region.

Figure 4.20: Illustration of approximate locations of characteristic features of the weak SWTBLI region caused by a 6 degree impinging shock. The filled contour plot represents the vertical velocity.
**Root Mean Square Fluctuations**

The interaction region which results from a 6 degree angle of attack flat plate, shows fluctuations which resemble that of the 9 degree case as seen in from the figures in Figure 4.21. Similar to the 9 degree interaction, the highest amount of streamwise velocity fluctuation is found on the subsonic side of the ascending portion of the sonic line. Near the peak of the sonic line, the streamwise velocity fluctuations begin to decay very gradually and extend beyond the field of view. Interestingly, while the physical size of this region of streamwise fluctuations along the ascending portion of the sonic line may differ between the 6 and 9 degree interactions, the average amplitude near the sonic line is about the same at $\sqrt{u'^2 / U_\infty} \approx 0.24$.

(a)

![Graph showing streamwise velocity fluctuations](image-url)
Figure 4.21: 6 Degree Angle of Attack: (a) $\sqrt{u'^2/U_\infty}$, (b) $\sqrt{v'^2/U_\infty}$, (c) $u'v'/U_\infty^2$

In contrast to the vertical velocity fluctuations of the 9 degree case, the reflected shock is much less pronounced than the separation shock. This suggests that the level of unsteadiness is also less. Also, underneath the peak of the sonic line shows considerably less fluctuation than what was previously observed.
The contour plot of the Reynolds stress shows similar levels of positive stress in and around the reflected shock as compared to the 9 degree case. This may be representative of the turbulence from the incoming boundary layer interacting with the reflected/separated shock. In each case the incoming boundary layer turbulence should not differ. At the sonic line, both the 6 and 9 degree interactions exhibit negative values of Reynolds stress suggesting the presence of vortices. There is a considerable difference in the level of Reynolds stress past the peak of the sonic line. This difference may be the result of separated vs. non-separated flow. The turbulence production mechanisms of the non-separated flow should be considerably less than that of the separated case. This is most like attributed to the unsteady nature of the separation region.

**Instantaneous Analysis**

Analysis of the instantaneous images from a 6 degree angle of attack interaction region shows similar results to that of the mean flow. Some examples of instantaneous images are shown in Appendix A.3 (Region 1 has been left out as there is no interaction seen in this region). The reflected shock is very weak and hardly appears on the divergence plot. Because of this, there is no obvious movement of the reflected shock.

The perturbed boundary layer flow occasionally shows separation but most instances resemble the mean flow well. The fact that any separation is observed at all is still evidence that even with a considerably weak shock, unsteady affects are still apparent.

Performing Galilean decomposition on the instantaneous images of the 6 degree angle of attack interaction does not show the production of vortices near the reflected shock. However, similar to the 9 degree case, there is a large scale vortical motion which resides near the peak of the subsonic layer shown in Figure 4.22. This is seen with convection velocities between \( \frac{U_c}{U_\infty} = 0.7 \) and \( \frac{U_c}{U_\infty} = 0.8 \). The structure is nearly half the size of that seen in the 9 degree case, as one might expect from a more steady and weaker interaction. As the boundary layer recovers, small vortex structures appear sporadically.
and similarly to that seen in the 9 degree case at the same convective velocity.

Figure 4.22: Galilean decomposition of the 6 degree angle of attack interaction. (a)$U_c/U_\infty = 0.7$. (b)$U_c/U_\infty = 0.8$. 
5. Conclusions

5.1 Summary of Results

Particle Image Velocimetry

To evaluate the effectiveness of using ice crystals as the seed material for performing PIV, analysis of the particle response to a step change were performed across an oblique shock wave. The average particle response time was estimated through means of an exponential curve fit to yield $\tau_p = 2.8 \mu s$. This response time resulted with a Stokes number of $St = 0.11$ when scaled with the time scale of the boundary layer. This estimation of flow tracking ability is within reason for the scope of this study from the condition that $St << 1$.

Particle Image Velocimetry was performed for the first time in the Supersonic Wind Tunnel laboratory as the primary part of this study. To establish an operational PIV system, many tests and experiments were performed throughout the course of this research which were not reported. For completeness, several comments are made from such experiments about the PIV seeding system.

Qualitative results regarding the temperature requirement of the liquid water (seed material) under high pressure is that a high temperature of 175°C to 185°C is essential to the production of fine seed particles. Lower temperatures yielded a greater range of particle sizes as well as produced a more sparse distribution. These negative effects diminish the total of valid correlated vectors.

A proper water mass flow rate is also essential for producing quality particle images. The air pressure which forces the heated water into the stagnation chamber through the BETE nozzles, ultimately determines the flow rate. A pressure difference of $\Delta P =$
4.48 MPa creates a flow rate of about 0.07 L/s. This flow rate allows for 7 seconds worth of hot water during each wind tunnel run. Lower flow rates result with sparse seeding which reduces the yield of correlated vectors. Increased flow rates reduce the overall ensemble size due to the limited amount of hot water.

Lastly, it was experimentally shown that the ambient temperature at which the air storage tanks are subject to, ultimately decides the quality of the particle images. While there is no numerical or thermodynamic proof of this claim presented, during the course of this study the particle images and PIV results were on occasion of poor quality. On such occasions, the ambient temperature was the only variable of all of the run conditions (i.e. water temperature, flow rate, stagnation pressure, etc). It was concluded that an inflection point of PIV data of good quality verses PIV data of diminished or unusable quality, is present when the ambient temperature is between $65^\circ F$ to $70^\circ F$. Colder ambient temperatures than $65^\circ F$ yield the best particle images and thus the best correlation results. This conclusion restricts the availability of collecting data to seasonally as well as the time of day. A brief set of example images as well as corresponding ambient temperatures are shown below.

$39^\circ F$
$46^\circ F$

$59^\circ F$
$65^\circ F$

$70^\circ F$
The undisturbed boundary layer statistical data was produced by using simplifying assumptions provided by Crocco, Morkovin, and Van Driest. The statistical data can be seen in Table 3.1. The shear velocity was deduced using the Clauser method and was found to be $u_\tau = 22.45$ m/s. Using this value to scale the fluctuating statistics of $\frac{\overline{p_{w}^{T}u^{2}}}{\rho_{w}u_{\tau}^{2}}$ and $\frac{\overline{p_{w}^{T}v^{2}}}{\rho_{w}u_{\tau}^{2}}$, despite the data exhibiting some scatter, the general trends are consistent with those found in the literature review. It was concluded that the boundary layer is fully developed and fully turbulent with a Reynolds number with respect to the momentum thickness of $Re_{\theta} = 78300$ and a boundary layer thickness $\delta = 15.6$ mm.

Previous study performed in the laboratory has shown the free stream Mach number to be 3.45. In the experimental calculations, the mean free stream speed was determined to be $621 \, m/s$. This result does not correspond to a Mach number of 3.45 when approximating the free stream static temperature with the use of the energy equation. With such approximation the free stream Mach number is found to be 3.22.
From the area-Mach number relation equation:

\[
\frac{A}{A^*} = \frac{1}{M} \left[ \frac{2}{(\gamma + 1)} \left( 1 + \frac{(\gamma - 1)}{2} M^2 \right) \right]^{(\gamma+1)/2(\gamma-1)}
\] (5.1)

where \(A/A^*\) is the ratio of test section area to throat area and \(M\) is the Mach number in the test section assuming quasi-one-dimensional flow. Using the design Mach number of 3.45 and the known test section area of 152 mm results with a throat area of 23.5 mm. Plugging the throat area and an effective throat area of 152 mm - 2\(\delta^*\) (assuming the displacement thickness is equal on the top and bottom of the test section) back into the area-Mach number relation and solving for the test section Mach number yields \(M=3.38\). While this number has been reduced from 3.45, it still does not explain the measured value of 3.22. It is therefore recommended that further investigations into the source of the reduced Mach number from the design Mach number are performed.

Analyzing instantaneous images using Galilean decomposition showed the existence of coherent structures. Previous studies have shown that turbulent boundary layers consist of hairpin vortices which originate at the wall, rise up into the boundary layer where they are stretched in the streamwise direction, then burst near the boundary layer thickness. Instantaneous images from the XY plane data showed the existence of what may be evidence of such hairpin vortices. This evidence was seen in the form of what was considered to be a VITA event with a vortex head above the ejection flow region. The vortices were best observed with a convection velocity of \(U_c/U_\infty = 0.75\) at a height \(y/\delta \approx 0.2\) but were also seen with a convection velocity of \(U_c/U_\infty = 0.85\) and height \(y/\delta \approx 0.4\).

Structures were also clearly seen in the XZ plane data. Coherent regions of high and low velocity in the instantaneous images extended beyond the field of view, 2\(\delta\) in the streamwise direction at a height of \(y/\delta = 0.1\). This observation supports those made by Ganapathisubramani et al. and Lin et al. who both found long coherent structures of high
and low velocity residing in the boundary layer at \( y/\delta = 0.2 \). It was reported that these structures were nearly 40δ long in the streamwise direction. In addition to these structures, pairs of counter rotating spanwise vortices were also observed in the shear layers formed between high and low velocity regions.

**Shock Wave Turbulent Boundary Layer Interaction**

**9 Degree Angle of Attack**

The mean flow results of the interaction region from an impinging oblique shock cast from a 9 degree angle of attack flat plate, resembled that of the strong interaction seen in Figure 1.2. The calculated sonic line using the energy Equation (3.2) agreed well with the positioning of the separation shock and the termination point of the impinging oblique shock. In each XY plane, reverse flow was present at \( x/\delta = -3.3 \) and would at least extend over a length of \( \delta \) in the streamwise direction. On average the separation bubble had a length of 2.11δ and a height of 0.12δ. There was no symmetry from the center plane nor any distinguishable pattern of the points of separation or reattachment across the recorded planes.

It was suggested that the lack of symmetry from the center plane may be a combination of a few undesirable and uncontrollable consequences. First is the low ensemble size to characterize the mean flow. An ensemble size of less than 70 is common given the speed of the camera fixed at 10Hz and a water mass flow rate which expends all of the hot water/seed particles in about 7 seconds. Run conditions are subject to vary due to the manual process of throttling the pressure of the stagnation chamber as well as variation of the stagnation temperature either by season, time of day, or a thermodynamic cooling due to expansion during each wind tunnel run. This was evident by the apparent discontinuities seen in several of the stitching results. By a simple oil streakline experiment, an obvious interaction between the impinging oblique shock generated by the flat plate and the side wall boundary
layers was apparent. While there was no direct evidence that this side wall interaction affects the interrogation region of interest, the oil streak lines do suggest the possibility of such an occurrence.

Plots of the divergence of instantaneous images showed motion of the separated shock from the mean position of \( x/\delta \approx -4.4 \) with an amplitude estimated to be \( \pm 0.25 \delta \) by inspection. This corresponds to an unsteady shock motion of approximately 7.8 mm. This unsteady motion was supported by considerable vertical velocity fluctuations found in the region of the separation shock, shown particularly well in Appendix A.3 (Planes 5, 6 and 7).

Galilean decomposition of PIV Region 1 instantaneous images showed vortices after the separation shock which appeared similar to those found in the XY boundary layer investigation. These vortices were found at several different streamwise convection velocities, but all were revealed with a vertical convection velocity of \( V/U_\infty = 0.1 \). This vertical convection velocity suggests that vortices found in the boundary layer are being lifted up by the separation shock.

A large vortical motion was found in PIV Region 2 with a convection velocity of \( U_c/U_\infty = 0.8 \). While there are no direct conclusions drawn from the data as to the effect such a structure would have on the interaction region, it has been suggested in literature that periodic vortex shedding may have a part in the low frequency oscillatory motion of the separated region an order of magnitude lower than the characteristic frequency of the boundary layer \( U_\infty/\delta \) [6]. Study of the time resolved nature of this structure would be beneficial in seeing its effects upon the interaction region.

Large quantities of streamwise velocity fluctuation was seen in every case to originate at the separation shock foot and follow the ascending portion of the sonic line with majority of the fluctuations on the subsonic side. The fluctuations abruptly decay near the peak and continue to decay along the descending portion of the sonic line as the boundary layer recovers. The streamwise velocity fluctuations occured at approximately three times the
strength than the vertical velocity fluctuations. These results are in agreement with those found by Souverein et al. who also went on to show that the high levels of streamwise and vertical fluctuations in addition to high Reynolds stress found at the separation shock were evidence of developing vortical structures.

Average streamwise velocity profiles of all 7 planes were plotted throughout the interaction and showed the effects of the adverse pressure gradient and dilated subsonic region upon the boundary layer. The boundary layer profile quickly stalled to zero velocity and then separated. This velocity deficit was observed up to a height of $y \approx 0.3\delta$. As the boundary layer encountered the dilated subsonic region, majority of the boundary layer velocity was slowed and the profile became less full. The boundary layer then began to recover after $x \approx -1.7\delta$, slowly increasing in speed.

6 Degree Angle of Attack

The interaction region resulting from an impinging oblique shock generated by a 6 degree angle of attack flat plate was shown to resemble the SWTBLI weak interaction. In the mean, there is no separation or reverse flow present. The weak impinging oblique shock penetrated deeper into the boundary layer as was seen in the divergence plots. The height of the dilated sonic line only reached $y/\delta \approx 0.27$ as compared to $y/\delta \approx 0.7$ in the 9 degree case.

Instantaneous images showed little motion of the reflected shock but there was an occasional occurrence of separated reverse flow. The reduced shock motion was confirmed from the vertical velocity fluctuations not showing a pronounced shock feature ahead of the interaction region. Streamwise velocity fluctuations resembled the 9 degree case in strength and in shape but not in physical size. The strongest fluctuations originated near floor and followed the ascending portion of the sonic line. The recovery of the boundary layer appeared more gradual than the 9 degree case. This was seen from the slopes of the descending portion of the sonic lines. The Reynolds stress also appeared
similar in nature to the strong shock interaction, however the amount was less. This was no surprise as one would expect turbulence production to be less apparent with no separation and less unsteady behavior.

5.2 Future Work

The establishment of the PIV system, which provides full field velocity data with satisfactory accuracy and reliability, can be used as a primary quantitative measurement technique for a variety of future projects. However, given the ambient temperature dependent nature for obtaining quality PIV images concluded previously, the time frame for collecting data is restricted seasonally. It is therefore recommended that alternative seeding methods be explored. The use of ethanol for full tunnel seeding may be an improved alternative to water, however the integration of a solid particle seeding system is likely to provide the most improvement for future PIV results.

This study was unable to produce a symmetric flow field when analyzing the multiple planes recorded in the SWTBLI region. The purpose of such planes was to reconstruct and analyze the flow in a pseudo three-dimensional manner. The use of multiple cameras to perform stereo PIV would improve upon the investigation of the three-dimensional nature of the SWTBLI region. Stereo PIV may also permit further investigations into the undesirable effects upon the area of interest from the side wall boundary layer separation.

Despite the non-symmetric flow, the general flow field of a strong SWTBLI in which there was separation, was seen with good agreement to theory and published results. The unsteady effects and complex flow field which resulted from boundary layer separation were apparent. Control methods for boundary layer separation may be investigated using PIV and the results compared to the flow fields found in this study. Of particular interest, the use of venturi jet ejectors using high pressure air as the motive fluid to create a vacuum, may be explored as a control method for SWTBLI by means of boundary layer suction.
References


A. Plotted Results

A.1 Mean Results - 9 Degree Angle of Attack

Figure A.1: Plane 1: $-18$ mm from test section center. (a) Normalized streamwise $U$ velocity. (b) Normalized vertical $V$ velocity. In each case the calculated sonic line is shown.
Figure A.2: Plane 2: $-13 \text{ mm}$ from test section center. (a) Normalized streamwise $U$ velocity. (b) Normalized vertical $V$ velocity. In each case the calculated sonic line is shown.
Figure A.3: Plane 3: −3 mm from test section center. (a) Normalized streamwise $U$ velocity. (b) Normalized vertical $V$ velocity. In each case the calculated sonic line is shown.
Figure A.4: Plane 4: Test section center. (a) Normalized streamwise U velocity. (b) Normalized vertical V velocity. In each case the calculated sonic line is shown.
Figure A.5: Plane 5: 3 mm from test section center. (a) Normalized streamwise U velocity. (b) Normalized vertical V velocity. In each case the calculated sonic line is shown.
Figure A.6: Plane 6: 13 mm from test section center. (a) Normalized streamwise U velocity. (b) Normalized vertical V velocity. In each case the calculated sonic line is shown.
Figure A.7: Plane 7: 18 mm from test section center. (a) Normalized streamwise U velocity. (b) Normalized vertical V velocity. In each case the calculated sonic line is shown.
A.2 Velocity Profiles
Figure A.8: Normalized streamwise velocity profiles. Average 9 degree interaction of all planes (red triangle), 6 degree interaction (blue circle), Undisturbed boundary layer (black X)
A.3 Fluctuation Results

9 Degree Angle of Attack

(a)

(b)
Figure A.9: Plane 1: (a): $\sqrt{u'^2}/U_\infty$, (b): $\sqrt{v'^2}/U_\infty$, (c): $-u'v'/U_\infty^2$
Figure A.10: Plane 2: (a): $\sqrt{\overline{u'^2}}/U_\infty$, (b): $\sqrt{\overline{v'^2}}/U_\infty$, (c): $-\overline{u'v'}/U_\infty^2$
Figure A.11: Plane 3: (a): $\sqrt{u'^2}/U_\infty$, (b): $\sqrt{v'^2}/U_\infty$, (c): $-\overline{u'v'}/U_\infty^2$
Figure A.12: Plane 5: (a): $\sqrt{u'^2}/U_\infty$, (b): $\sqrt{v'^2}/U_\infty$, (c): $-u'v'/U_\infty^2$
Figure A.13: Plane 6: (a): $\sqrt{u'^2}/U_\infty$, (b): $\sqrt{v'^2}/U_\infty$, (c): $-u'v'/U_\infty^2$
Figure A.14: Plane 7: (a): $\sqrt{\overline{u'^2}}/U_\infty$, (b): $\sqrt{\overline{v'^2}}/U_\infty$, (c): $-\overline{u'v'}/U_\infty^2$
A.4 Instantaneous Results

Interrogation Region 1 - 9 Degree Angle of Attack

Figure A.15: Region 1 instantaneous result 1. (a) Divergence. (b) Contours of normalized streamwise velocity
Figure A.16: Region 1 instantaneous result 2. (a) Divergence. (b) Contours of normalized streamwise velocity.
Figure A.17: Region 1 instantaneous result 3. (a) Divergence. (b) Contours of normalized streamwise velocity
Figure A.18: Region 1 instantaneous result 4. (a) Divergence. (b) Contours of normalized streamwise velocity
Interrogation Region 2 - 9 Degree Angle of Attack

Figure A.19: Region 2 instantaneous result 1. Contours of normalized streamwise velocity

Figure A.20: Region 2 instantaneous result 2. Contours of normalized streamwise velocity
Figure A.21: Region 2 instantaneous result 3. Contours of normalized streamwise velocity

Figure A.22: Region 2 instantaneous result 4. Contours of normalized streamwise velocity
Interrogation Region 3 - 9 Degree Angle of Attack

Figure A.23: Region 3 instantaneous result 1. Contours of normalized streamwise velocity

Figure A.24: Region 3 instantaneous result 2. Contours of normalized streamwise velocity
Figure A.25: Region 3 instantaneous result 3. Contours of normalized streamwise velocity

Figure A.26: Region 3 instantaneous result 4. Contours of normalized streamwise velocity
Interrogation Region 2 - 6 Degree Angle of Attack

Figure A.27: Region 2 instantaneous result 1. (a) Divergence. (b) Contours of normalized streamwise velocity
Figure A.28: Region 2 instantaneous result 2. (a) Divergence. (b) Contours of normalized streamwise velocity
Figure A.29: Region 2 instantaneous result 3. (a) Divergence. (b) Contours of normalized streamwise velocity
Interrogation Region 3 - 6 Degree Angle of Attack

Figure A.30: Region 3 instantaneous result 1. Contours of normalized streamwise velocity

Figure A.31: Region 3 instantaneous result 2. Contours of normalized streamwise velocity
Figure A.32: Region 3 instantaneous result 3. Contours of normalized streamwise velocity