Description: Making general sense of<br>$\mathbf{a}(\mathbf{x}+\mathrm{y})=\mathbf{a x}+\mathbf{a y}$<br>Parent Tape: Early Algebra Ideas<br>About Binomial Expansion, Stephanie's<br>Interview One of Seven<br>Date: 1995-11-08<br>Location: Harding Elementary School<br>Researcher: Professor Carolyn Maher

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| 1 | R1 | Okay. Now. Do you see - Do you see how I convinced myself that that would always work? |
| :---: | :---: | :---: |
| 2 | Stephanie | Um hm. |
| 3 | R1 | Do you see how I was trying to think if I was this little kid - you know how we would persuade a little kid that why if I doubled $x$, you get two $x$. What does doubling $x$ mean? |
| 4 | Stephanie | Um hm. Right. |
| 5 | R1 | Right? So why does this work? Multiplying $w$ plus $l$ times five? |
| 6 | Stephanie | Right? |
| 7 | R1 | Are you convinced that will always work? That I - did I convince you that this is always going to work when I multiply by five. |
| 8 | Stephanie | Well, yeah. 'Cause I - uh, I mean I always thought, you know. |
| 9 | R1 | I mean someone saying that's not a rule but - I'm saying that what $w$ plus $l$ means five times - it means that you have it five times. |
| 10 | Stephanie | Yeah. |
| 11 | R1 | Alright, does that make sense? |
| 12 | Stephanie | Yeah. |
| 13 | R1 | So suppose I asked you to convince me that why the Distributive Law works for eight times $w$ plus the expression $w$ plus $l$. |
| 14 | Stephanie | It's the same reason that it works for two. |
| 15 | R1 | Right. What's that reason? |
| 16 | Stephanie | That you're simply taking that number and adding it with the same number the amount of times it's telling you. |
| 17 | R1 | Okay. So the same time as two and the same time as five. |
| 18 | Stephanie | Um hm. |
| 19 | R1 | And you would do eight the way - okay so that's not too bad if you know the number here: two or five or eight. So you think it will work for any number? Two, five, eight, eleven? Will it work the same way? |


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| 20 | Stephanie | I think so. |
| :---: | :---: | :---: |
| 21 | R1 | Sixteen? - One million? |
| 22 | Stephanie | Yeah. |
| 23 | R1 | Okay. So if I say any number... |
| 24 | Stephanie | Okay. |
| 25 | R1 | Say "a". |
| 26 | Stephanie | Well that was - what you had here. |
| 27 | R1 | So how would you convince me that it would work for say any number "a"? |
| 28 | Stephanie | That... |
| 29 | R1 | If you have $(x+y) a$ times? How would how would you reason it in your head? How would you think about it? |
| 30 | Stephanie | That you're taking any number. |
| 31 | R1 | Um hm. |
| 32 | Stephanie | And you're adding it with itself. |
| 33 | R1 | Um hm. |
| 34 | Stephanie | as many times as $a$ is. |
| 35 | R1 | Okay. |
| 36 | Stephanie | Like... |
| 37 | R1 | Yeah. |
| 38 | Stephanie | I'm trying. |
| 39 | R1 | Okay. That's interesting. That's pretty neat. So um. Why don't you write that down? That's really kinda nice Stephanie what you just said. I just want to be sure you get a chance to think about it. You're going to try now to tell me what this is. $[a(x+y)=$ ?] Okay? Why the distributive law works here. I'm interested in.. |
| 40 | Stephanie | Um. |
| 41 | R1 | Think about that for a minute and write it and I'll have a glass of water. |
| 42 | Stephanie | I - it just... |
| 43 | R1 | Yeah just say it, just write what you've said. |
| 44 | Stephanie | Oh what I just said before. |
| 45 | R1 | Sure, yeah. |
| 46 | Stephanie | That [Stephanie writes "Your taking any number and adding it of itself the amount of times that the variable a equals"] repeats what she wrote. |

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| 47 | R1 | Okay. So what do you end up with when you've done that? When you've added $(x+y)$ to itself $a$ times? |
| :---: | :---: | :---: |
| 48 | Stephanie | Um. |
| 49 | R1 | What do you end up with when you add $(x+y)$ ? |
| 50 | Stephanie | That - |
| 51 | R1 | Do you end up with $a$ times $(x+y)$ ? |
| 52 | Stephanie | Well- |
| 53 | R1 | Is there another way you could say that? |
| 54 | Stephanie | Well - I would - I don't - $(x+y)$. |
| 55 | R1 | See, I don't want to end up where I started. If I'm adding $(w+l)$ to itself five times, I didn't end up with this $[5(w+l)]$. Do you see? |
| 56 | Stephanie | Um hm yeah. I understand. I just - I don't know how many times $a$ is. |
| 57 | R1 | But can you write an expression that says how many times in general? Without knowing it? - See when you knew it was two times, you knew how to write it. When you knew it was five times you knew how to write it. |
| 58 | Stephanie | We just - |
| 59 | R1 | And we conjectured when it was eight times how to write it. |
| 60 | Stephanie | It would just be like $(x+y)=(x+y)-\mathrm{I}$ don't know cause like here I don't have the - |
| 61 | R1 | Okay. Well, how many times are you going to do this now? |
| 62 | Stephanie | As many times as $a$ is. |
| 63 | R1 | Okay. So write that down. $(x+y)$ as many times as $a$. Okay now. |
| 64 | Stephanie | Oh, do you want me to say just $(x+y)$ as many times as $a$ ? |
| 65 | R1 | Sure. [Stephanie writes.] |
| 66 | Stephanie | Okay. |
| 67 | R1 | Okay. Can you imagine this in your head? |
| 68 | Stephanie | Yeah. |
| 69 | R1 | You got $(x+y)$ as many times as $a$. |
| 70 | Stephanie | Okay. |
| 71 | R1 | Can you imagine that? |


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| 72 | Stephanie | Well yeah. But I'm just imagining like any number. |
| :---: | :---: | :---: |
| 73 | R1 | Tell me what's in your head when you see this. |
| 74 | Stephanie | Just like rows of $x$ 's and - |
| 75 | R1 | Rows of $x$ 's - how many $x$ 's would you end up with when you're all done? |
| 76 | Stephanie | Um. A lot. |
| 77 | R1 | How many? If you're doing it - if you have $(x+y) a$ times? |
| 78 | Stephanie | $a$ amount of $x$ 's. |
| 79 | R1 | Right. And how many $y$ 's? |
| 80 | Stephanie | $a$ amount - like |
| 81 | R1 | So why don't you write that down? Isn't that what it's going to be? How would you write $a$ amount of $x$ 's and $a$ amount of $y$ 's? |
| 82 | Stephanie | Um. Could I just write you would end up with $a$ amount of $x$ 's? |
| 83 | R1 | Yeah. I'd like to see how you would write that. [Stephanie writes: "a amount of $x$ 's $+a$ amount of y's".] |
| 84 | Stephanie | It's just like - like - It looks like this, only it's - |
| 85 | R1 | Okay. So can you write it in a simple form? The $a$ amount of $x$ 's and $a$ amount of $y$ 's? What does $a$ times the expression $(x+y)$ now equal? If we want to replace this question mark, how could you write $a$ amount of $x$ 's and $a$ amount of $y$ 's? |
| 86 | Stephanie | $a x+a y$ ? |
| 87 | R1 | Doesn't that make sense? |
| 88 | Stephanie | Um hm. |
| 89 | R1 | Which is what you conjectured before. |
| 90 | Stephanie | Yeah. |
| 91 | R1 | Based on - Does that make sense? |
| 92 | Stephanie | Yeah. |
| 93 | R1 | You really believe it, right? |
| 94 | Stephanie | Yeah. |

