

<b>Description: Night Session – Pascal’s Identity</b> <b>Parent Tape:</b> <b>Date: 1999-05-12</b> <b>Location: David Brearley High School</b> <b>Researcher: Professor Carolyn Maher</b>	<b>Authors: Uptegrove, Elizabeth B.</b> <b>Verified: Poprik, Brad</b> <b>Date Transcribed: 2003</b> <b>Page: 1 of 54</b>
--	---

Line	Time	Name	Transcript	Coding and Explanation
1.	00:00:00	–	[Romina, Jeff, and Michael are sitting at a table talking with Researcher 1. At the start, the display says 02:01:01.27. No sounds can be heard. Researcher 2 walks across the room, opens the door, and goes out. The camera shows the rest of the room. Researcher 2 returns.]	
2.	00:01:05	R1:	-mentioned some of what went on. I have, I don't have a clue. Can you sort of tell me about it and how some of you suggested it's connected to other things you had done? I'm really curious. Feel free to use the board and show me and tell me.	
3.	00:01:16	Jeff:	Well.	
4.	00:01:17	R1:	But I don't know what went on.	
5.	00:01:17	Romina:	When, when we came up with that thing that almost was like the Pascal's Triangle. What was it with <i>e</i> ? What were we doing?	
6.	00:01:21	Jeff:	It was, um-	
7.	00:01:22	Romina:	Ten, ten percent of a hundred.	
8.	00:01:25	Jeff:	Is that what it was?	
9.	00:01:26	Romina:	Yeah.	
10.	00:01:28	Jeff:	Um, I'm not sure. I don't know.	
11.	00:01:31	Romina:	Can we have a calculator? Are we allowed to have one?	
12.	00:01:33	R1:	Sure. Hope you know where they are.	
13.	00:01:36	R4:	Yeah.	
14.	00:01:37	R1:	You may want to have them around anyway.	
15.	00:01:43	Jeff:	This could be our first time ever using calculators.	
16.	00:01:45	Romina:	Yeah.	
17.	00:01:46	R1:	Wow, first time.	
18.	00:01:46	Jeff:	We usually-	
19.	00:01:46	R1:	You never use them?	
20.	00:01:47	Romina:	Thank you.	
21.	00:01:48	R3:	You're welcome.	
22.	00:01:52	Michael:	Are there any games on this?	
23.	00:01:58	Jeff:	We said one times or one minus.	
24.	00:01:59	Romina:	What, what am I doing?	
25.	00:02:00	Jeff:	Oh that's good right there.	

<b>Description: Night Session – Pascal’s Identity</b> <b>Parent Tape:</b> <b>Date: 1999-05-12</b> <b>Location: David Brearley High School</b> <b>Researcher: Professor Carolyn Maher</b>	<b>Authors: Uptegrove, Elizabeth B.</b> <b>Verified: Poprik, Brad</b> <b>Date Transcribed: 2003</b> <b>Page: 2 of 54</b>
--	---

Line	Time	Name	Transcript	Coding and Explanation
26.	00:02:01	Romina:	Yeah. [Romina laughs.]	
27.	00:02:02	Jeff:	One. Was it one minus one over a hundred?	
28.	00:02:02	Romina:	Hm.	
29.	00:02:06	Jeff:	Hundred raised to-	
30.	00:02:11	Romina:	OK, why am I- Oh, okay. I didn’t know what I did wrong.	
31.	00:02:14	Jeff:	Oh.	
32.	00:02:17	Romina:	Wasn’t it like, weren’t we doing this?	
33.	00:02:18	Jeff:	Yeah, that’s what it was.	
34.	00:02:20	Romina:	Yeah.	
35.	00:02:21	Michael:	That’s this- [unintelligible; chair is moving].	
36.	00:02:23	Romina:	OK, this is scary. Look when we were- cause we were discussing like percentages. And, uh, like an increase and we did a hundred and we took ten percent of it and that’s one two one, that’s one three three one, and, you know, that is-	
37.	00:02:38	Jeff:	Yeah, we kept going it-	
38.	00:02:39	Romina:	It doesn’t come out yeah. After a while it goes.	
39.	00:02:41	Jeff:	It kind of makes you think. After a while it stops, but we were, uh-	
40.	00:02:44	Romina:	We really thought that was it, look.	
41.	00:02:45	Jeff:	We were into it.	
42.	00:02:48	R1:	Oh. So what does it mean?	
43.	00:02:50	Jeff:	Uh, we didn’t, we didn’t know.	
44.	00:02:51	Romina:	We didn’t know because then it stops, though.	
45.	00:02:53	Jeff:	Yeah, but it was interesting for, for a while.	
46.	00:02:55	Romina:	While it was going on it was very-	
47.	00:02:57	Jeff:	We were kind of, uh-	
48.	00:02:59	Michael:	Are we going to [Inaudible.]?	
49.	00:03:00	Jeff:	But um, what was the question? What were you-	
50.	00:03:02	Romina:	We wanted to know what we did in class today.	
51.	00:03:04	Jeff:	Um, we were looking a lot at, at working at $e$ and, and the equation for it.	
52.	00:03:09	Michael:	And how it, how it, how it connects with $\ln$ and-	
53.	00:03:11	Jeff:	Yeah, um-	
54.	00:03:11	Romina:	And we were also trying to find like, you know how we had when we had $a$ plus $b$	

<b>Description: Night Session – Pascal’s Identity</b> <b>Parent Tape:</b> <b>Date: 1999-05-12</b> <b>Location: David Brearley High School</b> <b>Researcher: Professor Carolyn Maher</b>	<b>Authors: Uptegrove, Elizabeth B.</b> <b>Verified: Poprik, Brad</b> <b>Date Transcribed: 2003</b> <b>Page: 3 of 54</b>
--	---

Line	Time	Name	Transcript	Coding and Explanation
			to the $n$ ? We want to know what- And we had like numbers before it when we got to big numbers, we want to know, you figure out what the numbers were, like in front of the $a$ , you know, cubed.	
55.	00:03:24	Jeff:	You know, that's, that's like, um-	
56.	00:03:25	R1:	You could use the board too.	
57.	00:03:26	Jeff:	Uh, we just- Like if you were looking, if we were looking for like $a$ plus $b$ -	
58.	00:03:33	Romina:	To the tenth.	
59.	00:03:33	Jeff:	To the tenth say, um, obviously it- Was the first one ten? Was it //one $a$ to the tenth and //then ten-	
60.	00:03:39	Michael:	//No it's one, yeah.	
61.	00:03:40	Romina:	// $b$ . Oh no, you're right. Sorry.	
62.	00:03:42	Jeff:	Ten $a$ to the ninth $b$ to the first, right?	
63.	00:03:45	Romina:	Mm hm.	
64.	00:03:45	Jeff:	And then how to find out //this number.	
65.	00:03:47	Romina:	//What the next one was.	
66.	00:03:48	Michael:	It's forty-five.	
67.	00:03:49	Jeff:	And it was forty-five but we were working on how to figure it out when we were doing it. We knew it was the choose thing, whatever that means. The- You do a forty- What was it? Ten choose two?	
68.	00:03:58	Michael:	Yeah.	
69.	00:03:58	Romina:	Uh-huh.	
70.	00:03:58	Jeff:	You know what I'm talking about? Like, uh, was it N-C-R- actually that's supposed to be lower case. Two- is that how you do it? Right?	
71.	00:04:05	Michael:	Yeah, it's one of these things like that.	
72.	00:04:06	Jeff:	And that equals forty-five and that's the answer. You know. I'm not, we're not really sure how all this works but it's like, what is that, if-	
73.	00:04:13	Romina:	We, we learned that, we learned that with her.	
74.	00:04:15	Jeff:	Yeah. Yeah the- Yeah, we, we went, we went over that, remember that? With the total-	
75.	00:04:19	Romina:	We tried to go over that. [Romina laughs.]	

<b>Description: Night Session – Pascal’s Identity</b> <b>Parent Tape:</b> <b>Date: 1999-05-12</b> <b>Location: David Brearley High School</b> <b>Researcher: Professor Carolyn Maher</b>	<b>Authors: Uptegrove, Elizabeth B.</b> <b>Verified: Poprik, Brad</b> <b>Date Transcribed: 2003</b> <b>Page: 4 of 54</b>
--	---

Line	Time	Name	Transcript	Coding and Explanation
76.	00:04:20	Jeff:	If you have ten different, what was it? Ten different things.	
77.	00:04:24	Michael:	You have-	
78.	00:04:25	Romina:	//Ten high. //Ten high.	
79.	00:04:26	Jeff:	//Ten high. How many-	
80.	00:04:26	Romina:	//How many would have two reds, only two reds.	
81.	00:04:27	Jeff:	//How many would have two, two reds.	
82.	00:04:29	R1:	One more time.	
83.	00:04:31	Jeff:	If you had towers// of ten high.	
84.	00:04:32	Michael:	//If you had like towers.	
85.	00:04:32	R1:	Towers.	
86.	00:04:35	Jeff:	If you have towers with ten high //and two colors.	
87.	00:04:35	Michael:	//How many different places can you put two reds in there?	
88.	00:04:36	Jeff:	Yeah.	
89.	00:04:37	Romina:	Yeah.	
90.	00:04:37	Jeff:	And like <i>a</i> would be one color and <i>b</i> would be blue, um, <i>b</i> would be the other color. Then how many would you have, <i>a</i> being two in the whole thing? And that would be forty-five and that's, that's what this number would be.	
91.	00:04:50	R1:	And these towers are how tall?	
92.	00:04:52	Jeff:	Ten tall.	
93.	00:04:53	Romina:	Ten.	
94.	00:04:54	Jeff:	That'd be the ten there.	
95.	00:04:54	Romina:	Mm hm.	
96.	00:04:54	Jeff:	The two would be the two colors and then, right?	
97.	00:04:58	Michael:	No.	
98.	00:04:58	Romina:	No, two of one color.	
99.	00:04:59	Jeff:	No, ten would be the two of the one color and the two is implied that there's two, only two colors? Or-	
100.	00:05:04	Michael:	The two is the-	
101.	00:05:04	Romina:	It's only <i>a</i> plus <i>b</i> .	
102.	00:05:06	Jeff:	Yeah but in the, when you write this, I mean is it implied that there's only two colors?	

<b>Description: Night Session – Pascal’s Identity</b> <b>Parent Tape:</b> <b>Date: 1999-05-12</b> <b>Location: David Brearley High School</b> <b>Researcher: Professor Carolyn Maher</b>	<b>Authors: Uptegrove, Elizabeth B.</b> <b>Verified: Poprik, Brad</b> <b>Date Transcribed: 2003</b> <b>Page: 5 of 54</b>
--	---

Line	Time	Name	Transcript	Coding and Explanation
103.	00:05:10	Romina:	I believe it is but-	
104.	00:05:12	Jeff:	Is that, is it implied?	
105.	00:05:14	Romina:	I, I'll go with the yeah. I don't know. [Romina laughs.]	
106.	00:05:16	Michael:	Uh, You talking about this?	
107.	00:05:17	Jeff:	Yeah.	
108.	00:05:18	Michael:	//No, It, it,	
109.	00:05:18	Romina:	//Is that like-	
110.	00:05:19	Jeff:	Is that one, the only one works for-	
111.	00:05:20	Michael:	It's just like you have ten things where, how many different places can you put these two? That's all.	
112.	00:05:25	Jeff:	Yeah, I know but-	
113.	00:05:25	Michael:	You know what I'm saying?	
114.	00:05:25	Jeff:	But if there's, oh, yeah, two. All right, I see what you're saying.	
115.	00:05:25	Michael:	That's all.	
116.	00:05:28	Jeff:	There could be a hundred colors but it would still-	
117.	00:05:31	Michael:	Yeah you pick two things out of those ten.	
118.	00:05:32	Jeff:	Yeah.	
119.	00:05:33	Michael:	How many different places can you put them?	
120.	00:05:34	Jeff:	Put them. All right. All right.	
121.	00:05:35	Michael:	Forty-five, I think,	
122.	00:05:37	R1:	So, so you're saying that's forty-five and what if I wanted eight red? Eight red ones or eight a's?	
123.	00:05:41	Jeff:	Then it would be ten-	
124.	00:05:41	Michael:	Um.	
125.	00:05:42	Romina:	Ten choose eight.	
126.	00:05:43	Jeff:	Choose eight, yeah.	
127.	00:05:44	Michael:	A smaller number.	
128.	00:05:45	Jeff:	Because //that would be how many different spots can you move those eight of one color in the tower of ten.	
129.	00:05:47	Romina:	//Now how do you-	
130.	00:05:50	Michael:	It's forty-five also.	

<b>Description: Night Session – Pascal’s Identity</b> <b>Parent Tape:</b> <b>Date: 1999-05-12</b> <b>Location: David Brearley High School</b> <b>Researcher: Professor Carolyn Maher</b>	<b>Authors: Uptegrove, Elizabeth B.</b> <b>Verified: Poprik, Brad</b> <b>Date Transcribed: 2003</b> <b>Page: 6 of 54</b>
--	---

Line	Time	Name	Transcript	Coding and Explanation
131.	00:05:51	R1:	Why?	
132.	00:05:52	Romina:	Like how do //you, how do you, how do you do that on a calculator?	
133.	00:05:53	R1:	//How'd you do that so fast Michael?	
134.	00:05:53	Jeff:	Um.	
135.	00:05:54	Michael:	No, I just like did it all in my head, that's all.	
136.	00:05:54	Jeff:	You go to, uh, math.	
137.	00:05:56	R1:	Tell us how you did it.	
138.	00:05:57	Michael:	Um.	
139.	00:05:57	Jeff:	Probability.	
140.	00:05:58	Michael:	There's a button that-	
141.	00:06:00	Jeff:	N-C-R.	
142.	00:06:00	Michael:	Take ten, that button then eight.	
143.	00:06:02	Romina:	Then math.	
144.	00:06:03	Michael:	And it comes out forty-five.	
145.	00:06:05	Jeff:	Why is that the case?	
146.	00:06:07	Romina:	Hm.	
147.	00:06:09	Michael:	Well if you take like on the-	
148.	00:06:10	Romina:	Well because-	
149.	00:06:12	Michael:	You know how on Pascal's Triangle.	
150.	00:06:13	Romina:	That's like the two.//You have eight left over.	
151.	00:06:14	Jeff:	//Oh, cause you could switch them all around. Is that, is that, I guess you're counting. //You got, you got, yeah	
152.	00:06:17	R1:	I don't know. Tell me.	
153.	00:06:17	Michael:	Cause then you would have-	
154.	00:06:18	Romina:	//Is that the same thing as that because, like, the eight left over to get to the ten, right?	
155.	00:06:18	Michael:	//It'll be- It would be the same thing.	
156.	00:06:22	Jeff:	Exactly.	
157.	00:06:23	Romina:	It's like almost switching colors.	
158.	00:06:23	Jeff:	Yeah.	
159.	00:06:24	Romina:	It'd be like two of the other color.	

<b>Description: Night Session – Pascal’s Identity</b> <b>Parent Tape:</b> <b>Date: 1999-05-12</b> <b>Location: David Brearley High School</b> <b>Researcher: Professor Carolyn Maher</b>	<b>Authors: Uptegrove, Elizabeth B.</b> <b>Verified: Poprik, Brad</b> <b>Date Transcribed: 2003</b> <b>Page: 7 of 54</b>
--	---

Line	Time	Name	Transcript	Coding and Explanation
160.	00:06:25	Jeff:	And then, and then, yeah, exactly.	
161.	00:06:26	R1:	Say that one more time, Romina.	
162.	00:06:28	Romina:	It'd be two of the other color instead of, like say you started with red for this two. That was for the reds and then when you//make red eight.	
163.	00:06:33	Michael:	//That would be the other eight.	
164.	00:06:34	Romina:	The, like, say the blues have two.	
165.	00:06:36	Jeff:	And it's seven. And then obviously three should be the same as that.	
166.	00:06:39	Romina:	Yeah. Yeah.	
167.	00:06:47	R1:	So, you're pressing the calculator, you have a new command that gets you those numbers.	
168.	00:06:53	Romina:	We know how to do it, I mean it's not-	
169.	00:06:54	R1:	But if you didn't have the calculator?	
170.	00:06:57	Romina:	We'd write them out.	
171.	00:06:57	Jeff:	You'd have to write them all out.	
172.	00:06:58	Michael:	Well, Bob-	
173.	00:07:00	R1:	Because Alex wants to know how you do that without a calculator.	
174.	00:07:03	Jeff:	Well, I obviously if the calculator-	
175.	00:07:04	R1:	Can you, can you help him understand that?	
176.	00:07:06	Jeff:	Well we would make a, say, tower of ten.	
177.	00:07:10	Michael:	Can I say something? //All right, um-	
178.	00:07:10	Romina:	//I don't know.	
179.	00:07:11	Jeff:	Bob. Yeah go for it.	
180.	00:07:12	Michael:	No, I'm talking, I just wanted to say that Bob Sidley had like an actual formula to write the equals and-	
181.	00:07:17	Jeff:	Do we know, do we know what it is? Or-	
182.	00:07:19	Michael:	I think so. I don't know if I can remember it.	
183.	00:07:19	R1:	Why don't I leave you a few minutes and think about explaining this to us.	
184.	00:07:24	Michael:	It depends on-	
185.	00:07:24	Jeff:	Well it's not, it's not that hard to explain.	
186.	00:07:25	Michael:	You remember it? I forgot it.	
187.	00:07:26	R1:	OK, I'll stay then.	

<b>Description: Night Session – Pascal’s Identity</b> <b>Parent Tape:</b> <b>Date: 1999-05-12</b> <b>Location: David Brearley High School</b> <b>Researcher: Professor Carolyn Maher</b>	<b>Authors: Uptegrove, Elizabeth B.</b> <b>Verified: Poprik, Brad</b> <b>Date Transcribed: 2003</b> <b>Page: 8 of 54</b>
--	---

Line	Time	Name	Transcript	Coding and Explanation
188.	00:07:28	Alex:	Well, actually no.	
189.	00:07:29	Romina:	It's not right.	
190.	00:07:30	Michael:	All right.	
191.	00:07:31	R1:	He has a bad memory.	
192.	00:07:32	Michael:	I got to like do trial and error to see if I can figure it out what it was.	
193.	00:07:35	R1:	OK.	
194.	00:07:35	Jeff:	All right, say that's ten? Then, um, you would just have to find- Say you had, uh, one was one color and two was the other color.	
195.	00:07:43	Romina:	Why, why don't you show her how to do it for like three. Show them how we can get all of them. That you have to draw the tower. [ Romina laughs.]	
196.	00:07:48	Jeff:	You have two colors. And out of this tower of three you'd have to find out all the different places you could put those two colors in. So you could put it there and there. Or you could put it, uh, there and there. Or, am I missing any? Yes, I am.	
197.	00:08:04	R1:	I understand.	
198.	00:08:05	Romina:	You could just do like-	
199.	00:08:06	Jeff:	Yeah.	
200.	00:08:06	Romina:	Do you want to go for another one?	
201.	00:08:07	Jeff:	No, go for it.	
202.	00:08:08	Romina:	No, you could just do, you could do like our blue, blue, blue.	
203.	00:08:12	Jeff:	You gonna write every one?	
204.	00:08:14	Romina:	Well, there wasn't that many. No I'm just like giving you an example.	
205.	00:08:15	Jeff:	Yeah.	
206.	00:08:16	Romina:	And then you just kind of move it through. And that's how we figure them out when we have to write them out.	
207.	00:08:23	R1:	So you're saying there's a way of getting these without the calculator.	
208.	00:08:30	Jeff:	Yeah. And there's a, there's a formula that somebody-	
209.	00:08:34	Romina:	Not too-	
210.	00:08:34	Jeff:	-had come up with but I don't know, I don't know how it, how it goes. I'm really not sure.	
211.	00:08:38	Romina:	I've seen it.	



<b>Description: Night Session – Pascal’s Identity</b> <b>Parent Tape:</b> <b>Date: 1999-05-12</b> <b>Location: David Brearley High School</b> <b>Researcher: Professor Carolyn Maher</b>	<b>Authors: Uptegrove, Elizabeth B.</b> <b>Verified: Poprik, Brad</b> <b>Date Transcribed: 2003</b> <b>Page: 9 of 54</b>
--	---

Line	Time	Name	Transcript	Coding and Explanation
212.	00:08:39	Jeff:	I don't remember it.	
213.	00:08:40	Romina:	Yeah, there's some-	
214.	00:08:41	Michael:	Yeah.	
215.	00:08:41	Romina:	Something to that effect.	
216.	00:08:42	Michael:	It was this guy.	
217.	00:08:43	Romina:	That's it?	
218.	00:08:43	Michael:	Yeah.	
219.	00:08:44	Jeff:	It's this right here?	
220.	00:08:45	Michael:	Yeah.	
221.	00:08:46	R1:	Why don't you show us up here, Michael.	
222.	00:08:48	Michael:	Oh, man. I, I didn't come up with this, so don't ask me why [unintelligible, chair moving]	
223.	00:08:49	R1:	It doesn't matter that you came up with it.	
224.	00:08:52	Michael:	If you would have like $n$ choose $x$ .	
225.	00:09:02	Romina:	That's on, that's on the division, $n$ to the $x$ , or is that just like your-	
226.	00:09:05	Jeff:	That $n$ to the $x$ ?	
227.	00:09:07	Michael:	No, that's, that's choose to the, that's how you write it I think. I think that's how you write it.	
228.	00:09:08	Romina:	That's just, that's what it is?	
229.	00:09:11	R1:	Do you want an equals sign there?	
230.	00:09:13	Michael:	No. That's, that's not in- Yeah. Yeah, I could do that. Times $x$ . That, that would be the number.	
231.	00:09:24	R1:	OK. Hi, Ankur. Come on in.	
232.	00:09:25	Ankur:	Hi. Sorry I'm late.	
233.	00:09:27	R1:	We're glad you're here.	
234.	00:09:29	Jeff:	Didn't you go with them?	
235.	00:09:30	Ankur:	No, I didn't go with them. I went with Steve.	
236.	00:09:34	Jeff:	That's dirty.	
237.	00:09:34	R1:	Hi, did you eat?	
238.	00:09:36	Ankur:	No.	

<b>Description: Night Session – Pascal’s Identity</b> <b>Parent Tape:</b> <b>Date: 1999-05-12</b> <b>Location: David Brearley High School</b> <b>Researcher: Professor Carolyn Maher</b>	<b>Authors: Uptegrove, Elizabeth B.</b> <b>Verified: Poprik, Brad</b> <b>Date Transcribed: 2003</b> <b>Page: 10 of 54</b>
--	--

Line	Time	Name	Transcript	Coding and Explanation
239.	00:09:37	R1:	Are you hungry? Yes.	
240.	00:09:39	Ankur:	Yeah I guess so. But it’s all right. It’s all right.	
241.	00:09:39	Romina:	You can, uh-	
242.	00:09:39	R1:	I’ll tell you what. I, I-.	
243.	00:09:41	Michael:	I hate stopping-	
244.	00:09:58	Jeff:	All right, what are we going to do?	
245.	00:09:59	Michael:	Oh. Oh yeah, um-	
246.	00:10:01	Romina:	What, what does that get? That gets you, like-	
247.	00:10:03	Michael:	That gives you that choose thing.	
248.	00:10:04	Jeff:	That gives you-	
249.	00:10:05	Michael:	I don't, I don't know what it means.	
250.	00:10:08	Romina:	I was working with him one day when he brought that up but he lost me.	
251.	00:10:11	Jeff:	That was the day that me and, and my table were doing the, uh, finding the square roots without a calculator.	
252.	00:10:16	Michael:	Yeah, but he did that before like-	
253.	00:10:18	Romina:	Not with me.	
254.	00:10:19	Michael:	-in, in class when he was talking about choosing.	
255.	00:10:19	Jeff:	Who?	
256.	00:10:20	Romina:	I was in your group.	
257.	00:10:21	Jeff:	Oh, gee, you got an eyelash.	
258.	00:10:24	Michael:	No, in class when he was talking about choosing. He figured it out. And	
259.	00:10:27	Jeff:	All right, um, well you figure, say you do, uh, say you're doing three, right? So that would be three times two times one. That would be each space, I imagine.	
260.	00:10:40	Romina:	//And $x$ is how many, how many you want of the color?	
261.	00:10:41	Michael:	//Yeah, I guess that would be how many combinations.	
262.	00:10:43	Jeff:	Yeah, that would, because that would give you the total number of-	
263.	00:10:45	Romina:	Yeah. Yeah that’d be it, yeah.	
264.	00:10:46	Jeff:	Total number of combinations?	
265.	00:10:48	Michael:	Oh I guess, yeah.	
266.	00:10:50	Jeff:	All right. So then that would be, say, six factorial. Divided by. That would be- Why would you- //why, where does that work?	

<b>Description: Night Session – Pascal’s Identity</b> <b>Parent Tape:</b> <b>Date: 1999-05-12</b> <b>Location: David Brearley High School</b> <b>Researcher: Professor Carolyn Maher</b>	<b>Authors: Uptegrove, Elizabeth B.</b> <b>Verified: Poprik, Brad</b> <b>Date Transcribed: 2003</b> <b>Page: 11 of 54</b>
--	--

Line	Time	Name	Transcript	Coding and Explanation
267.	00:10:58	Michael:	//No wait. //This is, that would be-	
268.	00:11:00	Romina:	//Come on, x is the number we want and x is like the number we want to get, like the choose number.	
269.	00:11:05	Michael:	That's the, yeah, choosing number..	
270.	00:11:07	Jeff:	And-	
271.	00:11:08	Romina:	So I guess this would-	
272.	00:11:10	Michael:	He was telling me like this was-	
273.	00:11:12	Romina:	That would like take away all the, all the ones we would choose.	
274.	00:11:15	Michael:	He said something about repeats. One would take away //the repeats.	
275.	00:11:15	Jeff:	//Yeah, this would this would take away the repeats, right?	
276.	00:11:17	Michael:	I guess.	
277.	00:11:19	Romina:	And will this, and this will give you-	
278.	00:11:20	Michael:	And this will, this will take away the-	
279.	00:11:21	Jeff:	This will take away all the other ones?	
280.	00:11:22	Michael:	The other ones that, that you don't care where they are.	
281.	00:11:22	Romina:	Like the ones that are higher than-	
282.	00:11:24	Jeff:	Yeah. Yeah.	
283.	00:11:25	Michael:	You only care about, you only care about the two that are moving. Not the other-	
284.	00:11:26	Jeff:	Yeah, exactly.	
285.	00:11:28	Michael:	Not the other, uh, four. It's just-	
286.	00:11:29	Jeff:	And then, and X-	
287.	00:11:30	Romina:	That makes sense.	
288.	00:11:32	Jeff:	Yeah. That make, let's see if it works. So say-	
289.	00:11:35	Romina:	Where's factorial on this?	
290.	00:11:36	Jeff:	Where's the exclamation point?	
291.	00:11:37	Michael:	Math.	
292.	00:11:39	Jeff:	Ah.	
293.	00:11:39	Michael:	Probability 4.	
294.	00:11:41	Romina:	What are we doing, three or six?	
295.	00:11:41	Michael:	Just hit four.	
296.	00:11:42	Jeff:	All right.	

<b>Description: Night Session – Pascal’s Identity</b> <b>Parent Tape:</b> <b>Date: 1999-05-12</b> <b>Location: David Brearley High School</b> <b>Researcher: Professor Carolyn Maher</b>	<b>Authors: Uptegrove, Elizabeth B.</b> <b>Verified: Poprik, Brad</b> <b>Date Transcribed: 2003</b> <b>Page: 12 of 54</b>
--	--

Line	Time	Name	Transcript	Coding and Explanation
297.	00:11:43	Michael:	I mean would you-	
298.	00:11:45	Jeff:	I don't even know why I did that. That was stupid. Uh, quit, all right. Six divide- Mm. Six divided by-	
299.	00:11:53	Romina:	Where is the little, where is that? I don't know.	
300.	00:11:56	Jeff:	Uh.	
301.	00:11:57	Michael:	Are you doing. why are you doing the six only? Oh, because three factorial is six, right?	
302.	00:12:01	Jeff:	You got it. That's not six factorial. Um, divided by three minus, what's X? We don't know?	
303.	00:12:09	Michael:	Do two.	
304.	00:12:10	Jeff:	Minus two.	
305.	00:12:14	Michael:	Times-	
306.	00:12:16	Romina:	You're a lot farther on that than I am.	
307.	00:12:20	Michael:	You have to put a parenthesis around that whole thing, too. Later on that.	
308.	00:12:24	Jeff:	Then times.	
309.	00:12:24	Michael:	No, you got to, at the beginning of that and the end of this thing.	
310.	00:12:31	Michael:	Get rid of that one there.	
311.	00:12:35	Jeff:	Don't we have to close that in, though?	
312.	00:12:36	Michael:	No, you don't have to close that.	
313.	00:12:37	Jeff:	Oh. All right. So do I have to delete that other one? No.	
314.	00:12:39	Michael:	No, leave that like that. Two.	
315.	00:12:42	Jeff:	So divided by two factorial. Um. Let's see.	
316.	00:12:50	Michael:	Let's see. And do it the other way and it should come out.	
317.	00:12:53	Jeff:	What way, what was the other way?	
318.	00:12:55	Romina:	Did, did it work? Four.	
319.	00:13:00	Michael:	No, it's three.	
320.	00:13:01	Romina:	Oh.	
321.	00:13:01	Jeff:	Oh, that's all right.	
322.	00:13:03	Michael:	Yeah, it works.	
323.	00:13:04	Jeff:	All right. Yeah that's the case then. All right, so when Ankur's done eating, you can explain all this. And we can explain it to Ankur? Cause you know that's	

<b>Description: Night Session – Pascal’s Identity</b> <b>Parent Tape:</b> <b>Date: 1999-05-12</b> <b>Location: David Brearley High School</b> <b>Researcher: Professor Carolyn Maher</b>	<b>Authors: Uptegrove, Elizabeth B.</b> <b>Verified: Poprik, Brad</b> <b>Date Transcribed: 2003</b> <b>Page: 13 of 54</b>
--	--

Line	Time	Name	Transcript	Coding and Explanation
			coming. Ankur missed the beginning, so explain it to him.	
324.	00:13:15	Michael:	There wasn’t really a beginning, though.	
325.	00:13:16	Jeff:	It doesn't matter. We're still going to have to. I didn't know. I forgot that we were, uh, we have to explain things.	
326.	00:13:23	Michael:	I thought it was just recess or something. [Romina laughs.]	
327.	00:13:25	Jeff:	Yeah, I know. I'm like that. And, uh-	
328.	00:13:34	Ankur:	I’m not normally in this class and everyone else is.	
329.	00:13:37	Alex:	You can go ahead and sit anywhere you want. //That's probably a good spot.	
330.	00:13:37	Romina:	//I saw you out of the corner of my eye. You confused me. I'm like what is he doing?	
331.	00:13:44	Romina:	I drove him.	
332.	00:13:45	Jeff:	Ah.	
333.	00:13:46	Romina:	I drove him here. I drove Jeff here.	
334.	00:13:48	Ankur:	By yourself?	
335.	00:13:48	Jeff:	Yeah righ.	
336.	00:13:49	Romina:	Well, my dad. [Inaudible]	
337.	00:13:50	Jeff:	All right, um. Do you want to hear our explanation of, of this and why this works?	
338.	00:14:00	R1:	That would be really great, but it would help us enormously if you would use the board.	
339.	00:14:02	Michael:	I wrote that, so you don't have to.	
340.	00:14:04	R1:	Would you mind, Jeff?	
341.	00:14:05	Jeff:	All right, I need help because-	
342.	00:14:06	R1:	They'll help you.	
343.	00:14:07	Jeff:	I don't want to get stuck. I got stuck up there last time by myself and I was looking like an idiot. All right, the reason why this works- We have no chalk. Right there in the side. All right, the reason why this works, first you get all the total number of this will cover all the total possibilities of your tower.	
344.	00:14:26	Romina:	//And factorial. [Inaudible]	
345.	00:14:27	Jeff:	//Say in terms of a tower. The factorial right here. So say you're doing towers of three. You have the three factorial and that'll cover all of the different combinations that you could put three in with two colors. All right? [There is a	

<b>Description: Night Session – Pascal’s Identity</b> <b>Parent Tape:</b> <b>Date: 1999-05-12</b> <b>Location: David Brearley High School</b> <b>Researcher: Professor Carolyn Maher</b>	<b>Authors: Uptegrove, Elizabeth B.</b> <b>Verified: Poprik, Brad</b> <b>Date Transcribed: 2003</b> <b>Page: 14 of 54</b>
--	--

Line	Time	Name	Transcript	Coding and Explanation
			pause. Jeff waves his hands.] Yes?	
346.	00:14:45	Ankur:	Sounds good.	
347.	00:14:48	R1:	Why don't you go through and when you're all done I'll ask my question. Just go.	
348.	00:14:51	Michael:	Like, you should use the explanation like she used. Like the people on the line. That's better because you have like the first one. Then you have-	
349.	00:14:57	Romina:	Two spaces.	
350.	00:14:58	Jeff:	All right, I'll do people on the line.	
351.	00:14:59	Michael:	Two spaces. You have two people left so that's times two.	
352.	00:15:00	Jeff:	All right, say we're doing us three right here.	
353.	00:15:01	Michael:	Yeah, on the line.	
354.	00:15:02	Jeff:	This, us three, um-	
355.	00:15:04	Romina:	There's three different people to fill in the first spot.	
356.	00:15:07	Jeff:	Yeah. Then there's, after, then once one goes there, there's only two people left to fill in this spot. And then-	
357.	00:15:10	Romina:	So you multiply three and two.	
358.	00:15:12	Jeff:	Three times two and then once, once someone goes in the other, there's only one person left. And they get the last spot, so that's times the one.	
359.	00:15:18	Romina:	And that's everyone.	
360.	00:15:19	Jeff:	That make more sense?	
361.	00:15:20	R1:	Well I'm, I didn't mind your other example here.	
362.	00:15:23	Jeff:	Yeah, I, I just like the okay through the way so I could move- You know, steady progress.	
363.	00:15:27	R1:	Um. But I guess, so why are you multiplying?	
364.	00:15:30	Romina:	We don't like that question.	
365.	00:15:30	Jeff:	Ah.	
366.	00:15:31	R1:	You don't like that question.	
367.	00:15:32	Romina:	No. That, that one gets us all the time.	
368.	00:15:34	R1:	Why aren't you adding?	
369.	00:15:35	Jeff:	Uh, because you don't add. It's just, you don't do it. [Romina laughs]. There's no adding going on it any where anymore. That's like out of style. [Romina laughs.]	
370.	00:15:41	R1:	That's not the answer.	

<b>Description: Night Session – Pascal’s Identity</b> <b>Parent Tape:</b> <b>Date: 1999-05-12</b> <b>Location: David Brearley High School</b> <b>Researcher: Professor Carolyn Maher</b>	<b>Authors: Uptegrove, Elizabeth B.</b> <b>Verified: Poprik, Brad</b> <b>Date Transcribed: 2003</b> <b>Page: 15 of 54</b>
--	--

Line	Time	Name	Transcript	Coding and Explanation
371.	00:15:42	Jeff:	I know that doesn't, that doesn't work. Um, you do it because, uh-	
372.	00:15:52	Michael:	I can't help you on this one.	
373.	00:15:53	Jeff:	Yeah, I know.	
374.	00:15:54	Romina:	Yeah, we're-	
375.	00:15:56	Michael:	That's a good question.	
376.	00:15:57	R1:	OK, I'll leave you to tell me.	
377.	00:15:58	Michael:	Why do you multiply?	
378.	00:15:59	R1:	You'll figure that out.	
379.	00:16:00	Romina:	We never know this one.	
380.	00:16:02	Jeff:	Yeah it's like the //eternal question.	
381.	00:16:03	Ankur:	//Yeah it's cause, if, if you have three things, there's three things you put here, right?	
382.	00:16:03	Romina:	Mm hm.	
383.	00:16:09	Ankur:	There's red, white and blue. And then there's only-	
384.	00:16:09	Romina:	Uh, are we [Inaudible.].	
385.	00:16:10	Ankur:	-two things.	
386.	00:16:12	Michael:	//And if there's two more-	
387.	00:16:12	Ankur:	//Out of that two-	
388.	00:16:13	Romina:	//We're doing just two colors. We're doing two colors.	
389.	00:16:14	Jeff:	Yeah, just do- No, we're- Yeah.	
390.	00:16:16	Michael:	//If you have like three things, right	
391.	00:16:17	Romina:	//To explain it, maybe you want to do three different colors?	
392.	00:16:18	Jeff:	No. Yeah, all right, maybe we can do that. All right, how you saying this?	
393.	00:16:22	Ankur:	There's red, white and blue, right?	
394.	00:16:25	Romina:	OK.	
395.	00:16:26	Ankur:	You take, if red goes over here, that means you only have, with red there could go either go white and blue.	
396.	00:16:32	Romina:	Mm hm.	
397.	00:16:33	Ankur:	Like it's each one of those three goes with two more. You know what I mean? There's three things-	
398.	00:16:40	Michael:	You could see how you got this.	

<b>Description: Night Session – Pascal’s Identity</b> <b>Parent Tape:</b> <b>Date: 1999-05-12</b> <b>Location: David Brearley High School</b> <b>Researcher: Professor Carolyn Maher</b>	<b>Authors: Uptegrove, Elizabeth B.</b> <b>Verified: Poprik, Brad</b> <b>Date Transcribed: 2003</b> <b>Page: 16 of 54</b>
--	--

Line	Time	Name	Transcript	Coding and Explanation
399.	00:16:41	Ankur:	-here and then there's two things here.	
400.	00:16:39	Michael:	You can say you have-	
401.	00:16:40	Jeff:	All right, yeah.	
402.	00:16:41	Ankur:	Each one of those, those three goes with //two other.	
403.	00:16:42	Jeff:	//Those three things go with-	
404.	00:16:43	Romina:	//Oh OK, like with our line thing.	
405.	00:16:44	Ankur:	//So it's three times two.	
406.	00:16:45	Jeff:	All right.	
407.	00:16:45	Romina:	Like our line thing.	
408.	00:16:47	Michael:	Or you could say like you have two more colors to add on. So you could do, you could make these into two different combinations.	
409.	00:16:52	Ankur:	Yeah.	
410.	00:16:53	Michael:	So that's two.	
411.	00:16:53	Jeff:	Yeah. That's- Yeah, that's why. All right.	
412.	00:16:54	Michael:	That's like times. That's why you multiply.	
413.	00:16:55	Ankur:	That's how you-	
414.	00:16:56	Michael:	That's just why. All right? Don't ask us anymore.	
415.	00:16:59	Jeff:	All right, so then, all right. Uh, //Researcher 1.	
416.	00:17:00	Romina:	//Researcher 1. [Romina laughs.]	
417.	00:17:03	Jeff:	All right, I think we're good with this.	
418.	00:17:06	R1:	I'll stay here. Explain it to me on the board.	
419.	00:17:07	Jeff:	All right, the reason- here, Ankur.	
420.	00:17:10	Ankur:	Just do it; you're right there. You're standing.	
421.	00:17:11	Romina:	You could just say it.	
422.	00:17:13	Jeff:	Um, just do it with three colors?	
423.	00:17:15	Ankur:	Yeah.	
424.	00:17:16	Jeff:	All right, say you have three colors, red, white and blue. Uh, here you do it.	
425.	00:17:21	Ankur:	Yeah, one of those colors goes in the first.	
426.	00:17:22	Jeff:	All right.	
427.	00:17:23	Ankur:	One of those colors goes in the first spot.	
428.	00:17:24	Jeff:	So, say you have your three spots. Say red goes in the first one, all right? Then	



<b>Description: Night Session – Pascal’s Identity</b> <b>Parent Tape:</b> <b>Date: 1999-05-12</b> <b>Location: David Brearley High School</b> <b>Researcher: Professor Carolyn Maher</b>	<b>Authors: Uptegrove, Elizabeth B.</b> <b>Verified: Poprik, Brad</b> <b>Date Transcribed: 2003</b> <b>Page: 17 of 54</b>
--	--

Line	Time	Name	Transcript	Coding and Explanation
			you could do-	
429.	00:17:28	Ankur:	Either one of them-	
430.	00:17:29	Romina:	Draw the line to the white and the blue.	
431.	00:17:31	Ankur:	One, one color goes in the first spot, so there's two colors left. So there's three different colors that can go in the first spot and each of those colors can go with two other colors.	
432.	00:17:39	Jeff:	Two other ones. So this is either going to be a white and blue or a blue and a white. Right? And then- Or the white could to the first thing and this is going to be one of the two other colors or the blue's going to go here and it's going to be, the other two are going to be the combination either way of the other one. So that's why you multiply.	
433.	00:17:57	Romina:	Make that a B.	
434.	00:17:57	Jeff:	I used to have a Band-Aid on and now I can get the chalk to stick to my finger.	
435.	00:18:00	Michael:	It is impressive, huh? [Romina laughs.]	
436.	00:18:03	R1:	What does this have to do with the towers and what you were showing me about $a$ plus $b$ to the $n$ ?	
437.	00:18:11	Michael:	Well, you want- you asked us why you multiply.	
438.	00:18:12	R1:	And why, why they would be-	
439.	00:18:13	Ankur:	We just answered why we multiply.	
440.	00:18:14	Michael:	Yeah.	
441.	00:18:15	Jeff:	Yeah. We're not there yet.	
442.	00:18:16	R1:	OK.	
443.	00:18:17	Romina:	We're still working on that.	
444.	00:18:19	Jeff:	Yeah, all right.	
445.	00:18:20	Michael:	All right. So that's why you multiply.	
446.	00:18:22	Jeff:	All right. Moving on. So that's why that's three factorial. So that's, all right, that's good.	
447.	00:18:24	Romina:	That's all your combinations right there.	
448.	00:18:27	Jeff:	Yeah. All right. All right, now we're going to put that number over, um, $n$ minus $x$ , um-	
449.	00:18:41	Romina:	Factorial.	

<b>Description: Night Session – Pascal’s Identity</b> <b>Parent Tape:</b> <b>Date: 1999-05-12</b> <b>Location: David Brearley High School</b> <b>Researcher: Professor Carolyn Maher</b>	<b>Authors: Uptegrove, Elizabeth B.</b> <b>Verified: Poprik, Brad</b> <b>Date Transcribed: 2003</b> <b>Page: 18 of 54</b>
--	--

Line	Time	Name	Transcript	Coding and Explanation
450.	00:18:42	Michael:	Explain that part.	
451.	00:18:44	Jeff:	All right. This, the $n$ would be the number you were-	
452.	00:18:49	Michael:	You want to, you know you’re choosing from. Like, let’s say two. This is three choose two. You want to know how many different places you could put those three.	
453.	00:18:55	Jeff:	Yeah. So that’s where the $n$ comes in.	
454.	00:18:57	Michael:	So you-	
455.	00:18:58	Jeff:	So you’re going to take a number so that would be, that would be three and the same reason the three’s up there, it’s coming down here.	
456.	00:19:03	Michael:	Minus, minus the $x$ .	
457.	00:19:04	Jeff:	Minus-	
458.	00:19:06	Michael:	Then it’ll give you one.	
459.	00:19:07	Jeff:	Wait, the reason you subtract, that’s why you’re raising the, how come the $x$ is there? Because you’re raising it to two, um. That’s it. Right?	
460.	00:19:17	Michael:	Right.	
461.	00:19:17	Jeff:	That’s why it’s there.	
462.	00:19:18	Romina:	And then-	
463.	00:19:18	Jeff:	The $x$ .	
464.	00:19:18	Romina:	Multiply-	
465.	00:19:19	Jeff:	And then that subtracted will give you, will give you-	
466.	00:19:24	Michael:	If this was, if this was a higher number like five choose two, you, that $n$ minus $x$ would be like a three and-	
467.	00:19:30	Jeff:	And the factorial-	
468.	00:19:32	Michael:	Those-	
469.	00:19:32	Jeff:	-will eliminate all the other ones-	
470.	00:19:34	Michael:	Yes, and those-	
471.	00:19:34	Jeff:	-that you don’t want.	
472.	00:19:35	Michael:	Those three, it doesn’t, you don’t want to know where, It doesn’t matter where they are. That’s why you want, you want, you know, eliminate them. Because you only, you’re only worrying about the two. How many different combinations that you could put those two in. Cause it’s five choose two. You’re only worried about like	

<b>Description: Night Session – Pascal’s Identity</b> <b>Parent Tape:</b> <b>Date: 1999-05-12</b> <b>Location: David Brearley High School</b> <b>Researcher: Professor Carolyn Maher</b>	<b>Authors: Uptegrove, Elizabeth B.</b> <b>Verified: Poprik, Brad</b> <b>Date Transcribed: 2003</b> <b>Page: 19 of 54</b>
--	--

Line	Time	Name	Transcript	Coding and Explanation
			the, the two. Like I said, the people on the line. Five people on the line, you want to know how many different places you could put those, those two people.	
473.	00:19:59	Jeff:	All the other ones where-	
474.	00:20:00	Michael:	Now there's going to be, there's going to be a lot of, a lot of repeats because you're also going to count by those other three people where they're going to be and you're not worried about those other three people.	
475.	00:20:08	Jeff:	So that only makes-	
476.	00:20:10	Michael:	So that's, that's why you would divide, to get rid of the, to get rid of them.	
477.	00:20:12	Jeff:	To subtract them.	
478.	00:20:15	Michael:	No, that's divide. Why divide that, $n$ minus $x$ ?	
479.	00:20:18	Jeff:	Oh, that's the way. All right. All right. OK, all right.	
480.	00:20:22	Michael:	And so, you say the next part. I don't know. [Inaudible.]	
481.	00:20:24	Jeff:	All right. And, why, why do we want-	
482.	00:20:26	Michael:	I don't know. I don't, I, no it's times, huh.	
483.	00:20:28	Romina:	Times.	
484.	00:20:29	Jeff:	Actually, that was supposed to be another one in there. Why is, why-	
485.	00:20:37	Romina:	Didn't, didn't you guys say something about repeats?	
486.	00:20:40	Michael:	Yeah, that's what Bob said. I don't know. I don't trust that kid.	
487.	00:20:41	Romina:	That gets like the repeats out.	
488.	00:20:44	Michael:	But it worked. It works. That's all.	
489.	00:20:45	Jeff:	All right, we don't know where the, the final $x$ comes from.	
490.	00:20:47	R1:	Why don't you, um-	
491.	00:20:47	Michael:	Why don't we think about it?	
492.	00:20:48	Jeff:	Work on it?	
493.	00:20:49	R1:	You need to work out a piece of the problem and see if you can tell me. Ankur's not convinced. He's looking at me, not being convinced.	
494.	00:20:55	Jeff:	What, what um-	
495.	00:20:56	Michael:	You're not convinced, Ankur?	
496.	00:20:57	Jeff:	Like I, I mean how-	
497.	00:20:59	R1:	Are you convinced, Ankur, about this?	

<b>Description: Night Session – Pascal’s Identity</b> <b>Parent Tape:</b> <b>Date: 1999-05-12</b> <b>Location: David Brearley High School</b> <b>Researcher: Professor Carolyn Maher</b>	<b>Authors: Uptegrove, Elizabeth B.</b> <b>Verified: Poprik, Brad</b> <b>Date Transcribed: 2003</b> <b>Page: 20 of 54</b>
--	--

Line	Time	Name	Transcript	Coding and Explanation
498.	00:21:00	Jeff:	Yeah but say, all right, say we're doing five choose two, right, with this. Then we go five factorial. Which is what?	
499.	00:21:07	Michael:	That'll give you all the combinations they can put everybody in.	
500.	00:21:09	Jeff:	Uh, twenty times three.	
501.	00:21:11	Ankur:	OK. Sixty.	
502.	00:21:12	Jeff:	Would be sixty times two.	
503.	00:21:14	Ankur:	One-twenty.	
504.	00:21:14	Jeff:	One-twenty? That would be; it's one-twenty, right, Romina?	
505.	00:21:18	Romina:	Yeah.	
506.	00:21:20	Jeff:	We're faster than the calculator, around here. [Romina laughs.] We're good like that. So that'd be one-twenty.	
507.	00:21:24	Michael:	And, and if you're doing choose two, obviously there's going to be a lot of times where those two are going to be in the same spot as the other three are going to be-	
508.	00:21:30	Romina:	What are you doing, five choose two?	
509.	00:21:31	Michael:	-you know, I guess moving around different spots.	
510.	00:21:31	Jeff:	Yeah.	
511.	00:21:31	Michael:	That's why you want to get rid of the, the $n$ minus $x$ thing.	
512.	00:21:35	Jeff:	Yeah, we got, that makes sense.	
513.	00:21:36	Michael:	Yeah, that, that makes sense to you?	
514.	00:21:37	Jeff:	That, that part right here, is this all good? Up to this point? Do you understand why this is all happening?	
515.	00:21:44	R1:	I'm waiting for the whole thing.	
516.	00:21:47	Michael:	Whole thing? Oh we're not done with that yet.	
517.	00:21:49	Jeff:	Then, um, then you multiply. Well, at this point here you have three.	
518.	00:21:54	Romina:	That's six.	
519.	00:21:54	Jeff:	Yeah, it's six. So you have one-twenty over six times five factorial.	
520.	00:22:03	Romina:	No isn't it-	
521.	00:22:03	Michael:	Oh I think its the repeats-	
522.	00:22:04	Jeff:	Or-	
523.	00:22:04	Michael:	Would, would be like-	
524.	00:22:05	Romina:	Isn't it three factorial, two factorial?	

<b>Description: Night Session – Pascal’s Identity</b> <b>Parent Tape:</b> <b>Date: 1999-05-12</b> <b>Location: David Brearley High School</b> <b>Researcher: Professor Carolyn Maher</b>	<b>Authors: Uptegrove, Elizabeth B.</b> <b>Verified: Poprik, Brad</b> <b>Date Transcribed: 2003</b> <b>Page: 21 of 54</b>
--	--

Line	Time	Name	Transcript	Coding and Explanation
525.	00:22:07	Jeff:	Three factorial. Oh two, oh, it's act-, all right, yeah. Two.	
526.	00:22:10	Michael:	Yeah, I guess the, the x-	
527.	00:22:12	Jeff:	That's the number you were raising-	
528.	00:22:14	Michael:	That x.	
529.	00:22:15	Jeff:	-and, and five choose x, say and there was-	
530.	00:22:15	Michael:	That's what. Since you- Mm hm.	
531.	00:22:16	Jeff:	And this was-	
532.	00:22:18	Ankur:	I get it. I get it. I get it. I get it. [Romina laughs.]	
533.	00:22:21	Michael:	I, I got it now.	
534.	00:22:23	Jeff:	Like that.	
535.	00:22:25	Michael:	All right, then the last number would be-	
536.	00:22:26	Jeff:	Because this just gives you the number.	
537.	00:22:28	Michael:	You have- Yeah.	
538.	00:22:29	Jeff:	You're going to multiply by the number.	
539.	00:22:29	Michael:	Those, those, you want to get rid of those. The, all the combinations that the three are moved around and those, those two aren't.	
540.	00:22:33	Jeff:	Yeah, they-	
541.	00:22:35	Michael:	But then those two themselves will be repeat-	
542.	00:22:36	Jeff:	Yeah-	
543.	00:22:37	Michael:	You will be mixed up.	
544.	00:22:38	Jeff:	Be repeating that's what you- that's why you	
545.	00:22:39	Michael:	That's why you want to get rid of that, too.	
546.	00:22:40	Jeff:	Exactly. And then, so that would be just two.	
547.	00:22:42	Michael:	Yeah.	
548.	00:22:43	Jeff:	So it would be one-twenty divided by twelve and you get ten. Is that what it is?	
549.	00:22:55	Michael:	Yeah it is. Do you get like why we divide by the $n$ minus $x$ and the, the $x$ ? You know, you, you get that?	
550.	00:23:07	R3:	I don't get that. Could you [Inaudible.]?	
551.	00:23:07	Michael:	You don't get that?	
552.	00:23:08	R1:	Ankur, did you have that?	
553.	00:23:09	Jeff:	What, what part don't, don't..	

<b>Description: Night Session – Pascal’s Identity</b> <b>Parent Tape:</b> <b>Date: 1999-05-12</b> <b>Location: David Brearley High School</b> <b>Researcher: Professor Carolyn Maher</b>	<b>Authors: Uptegrove, Elizabeth B.</b> <b>Verified: Poprik, Brad</b> <b>Date Transcribed: 2003</b> <b>Page: 22 of 54</b>
--	--

Line	Time	Name	Transcript	Coding and Explanation
554.	00:23:10	R1:	I wonder if Ankur has that? I wonder if Ankur could explain.	
555.	00:23:11	Romina:	I don't think the $x$ [Inaudible.].	
556.	00:23:15	Michael:	All right. The top thing, the $n$ to the, the $n$ to the, uh, factorial was going to give you how many?	
557.	00:23:21	Romina:	That's all the combinations.	
558.	00:23:22	Michael:	That's every single combination.	
559.	00:23:23	Romina:	I got that. That I got.	
560.	00:23:24	Michael:	Right? Now you're, you're only worried about them, those two people in that line. So there's going to be some instances where those two people are going to be in the same place and those three-	
561.	00:23:32	Jeff:	Are the ones changing.	
562.	00:23:33	Michael:	Will be, you know, will be switch, you know, changing.	
563.	00:23:34	Jeff:	And that's-	
564.	00:23:35	Michael:	So that's, that would be the, the three factorial. You want to, you want to get rid of that. You want to get rid of them.	
565.	00:23:40	Ankur:	Wait, say that again.	
566.	00:23:41	Romina:	Hold on. Well, we-	
567.	00:23:41	Michael:	Don't worry about that three, we're doing like five.	
568.	00:23:43	Romina:	No, we're doing this one so the two-	
569.	00:23:43	Ankur:	All right, so you have the five minus two, is that what you're explaining on there?	
570.	00:23:46	Romina:	Five minus two, that's-	
571.	00:23:46	Michael:	So you have the hundred and twenty different combinations.	
572.	00:23:46	Ankur:	Yeah.	
573.	00:23:47	Jeff:	Total.	
574.	00:23:49	Michael:	All right. But you don't think like when those two people are going to be in these two spots-	
575.	00:23:52	Jeff:	And everyone else is changing.	
576.	00:23:54	Michael:	-not those other three.	
577.	00:23:54	Jeff:	And those are, those are, those make no difference because all we're worried about are where those two people are.	
578.	00:23:56	Romina:	Oh like when, oh, oh, okay, okay, okay.	

<b>Description: Night Session – Pascal’s Identity</b> <b>Parent Tape:</b> <b>Date: 1999-05-12</b> <b>Location: David Brearley High School</b> <b>Researcher: Professor Carolyn Maher</b>	<b>Authors: Uptegrove, Elizabeth B.</b> <b>Verified: Poprik, Brad</b> <b>Date Transcribed: 2003</b> <b>Page: 23 of 54</b>
--	--

Line	Time	Name	Transcript	Coding and Explanation
579.	00:23:58	Michael:	All right, those two people are going to be moving around and it- you know, they're like-	
580.	00:23:59	Jeff:	These people are going to stay the same and every, all the three people, they're just going-	
581.	00:24:00	Michael:	-the two people staying in the same place. So that's why you get rid of that.	
582.	00:24:02	Jeff:	You know, going nuts.	
583.	00:24:02	Michael:	But then those two people themselves could switch places too.	
584.	00:24:06	Ankur:	Yeah. [Ankur nods.]	
585.	00:24:07	Michael:	You know what I'm saying?	
586.	00:24:07	Ankur:	Um-huh.	
587.	00:24:08	Michael:	Or if-	
588.	00:24:08	Ankur:	So then you got to get rid of those, too.	
589.	00:24:08	Michael:	-there were three that could go on.	
590.	00:24:10	Jeff:	So that's why you get rid of the three.	
591.	00:24:11	Ankur:	That's why you do the $x$ factorial	
592.	00:24:12	Michael:	Then you get rid of the, you know-	
593.	00:24:14	Jeff:	The other one.	
594.	00:24:15	Ankur:	Yeah, so you get rid of those.	
595.	00:24:16	Romina:	OK.	
596.	00:24:17	Jeff:	And then, then-	
597.	00:24:17	Romina:	Oh, there you go. That makes sense.	
598.	00:24:19	Michael:	Because you're not worried about every, each person.	
599.	00:24:20	Romina:	Just the two.	
600.	00:24:21	Michael:	Just worry about two, right.	
601.	00:24:22	Jeff:	Just those two. Exactly.	
602.	00:24:23	Romina:	Yeah, we all have, I got it. I'm good.	
603.	00:24:24	Michael:	Extension?	
604.	00:24:26	R1:	Ankur? Can you explain this because poor Researcher 3 is trying to understand this, and she's not following Michael.	
605.	00:24:36	Ankur:	Something like, I understood it but-	
606.	00:24:39	Jeff:	Just go through it dude.	

<b>Description: Night Session – Pascal’s Identity</b> <b>Parent Tape:</b> <b>Date: 1999-05-12</b> <b>Location: David Brearley High School</b> <b>Researcher: Professor Carolyn Maher</b>	<b>Authors: Uptegrove, Elizabeth B.</b> <b>Verified: Poprik, Brad</b> <b>Date Transcribed: 2003</b> <b>Page: 24 of 54</b>
--	--

Line	Time	Name	Transcript	Coding and Explanation
607.	00:24:40	Ankur:	All right. The top number is five factorial, that's the total number of possibilities for, for five, for five people.	
608.	00:24:45	Michael:	One twenty	
609.	00:24:46	Ankur:	And then the five minus two comes, comes in where you're not worried about everyone, you're just worried about two people at a time. So we need to subtract the five minus two. Those get, that gives you and you do factorial, that gives you all the possibilities of just two people, right?	
610.	00:25:05	Michael:	No, that gives you	
611.	00:25:05	Romina:	Three people.	
612.	00:25:06	Jeff:	No, three extras.	
613.	00:25:07	Michael:	The three that you don't, you're not worried about.	
614.	00:25:08	Jeff:	That's going to eliminate everyone except the two people you're worried about.	
615.	00:25:12	Ankur:	OK. Everyone except the two people you're worried about. And then the $x$ factorial eliminates, except the-	
616.	00:25:18	Michael:	When the two people-	
617.	00:25:19	Romina:	Two people, yeah.	
618.	00:25:20	Ankur:	Yeah. When the two people are switched back and forth when you have the same ones over again. [Romina laughs].	
619.	00:25:25	Jeff:	OK, [Inaudible.].	
620.	00:25:26	R3:	It's, it's getting better. It's getting better. So they switch back and forth you're saying and with your fingers. I think I'm getting switch back- So could you give me an example?	
621.	00:25:38	Ankur:	Like when you have, when you have like person $A$ and, over here.	
622.	00:25:41	Michael:	You want to stand up and show them?	
623.	00:25:41	Ankur:	And person $B$ over here. And then you have person $B$ and person $A$ .	
624.	00:25:42	Michael:	You want to be in a line and we'll show them?	
625.	00:25:43	R1:	Michael, start from the beginning very slow.	
626.	00:25:45	Michael:	All right. You have five people.	
627.	00:25:46	R1:	Stand up and show us.	
628.	00:25:47	R3:	Stand up and show us.	
629.	00:25:50	Jeff:	All right, I'm going to sit in your seat cause I can't see.	



<b>Description: Night Session – Pascal’s Identity</b> <b>Parent Tape:</b> <b>Date: 1999-05-12</b> <b>Location: David Brearley High School</b> <b>Researcher: Professor Carolyn Maher</b>	<b>Authors: Uptegrove, Elizabeth B.</b> <b>Verified: Poprik, Brad</b> <b>Date Transcribed: 2003</b> <b>Page: 25 of 54</b>
--	--

Line	Time	Name	Transcript	Coding and Explanation
630.	00:25:51	Michael:	I'm going to write it nice and clear so you all can see. All right. You got five people, in a line. You agree with me that's how many different combinations you can put those five people.	
631.	00:26:10	R3:	That part I understand.	
632.	00:26:11	Michael:	All right.	
633.	00:26:11	R3:	I understood the multiplication that you showed.	
634.	00:26:14	Michael:	Now in, you're only worried, you want to know how many different places you can put those two people. All right? So, in all the combinations you're going to have, they're going to be repeated a lot. A lot. When you have like, the two people in a certain place and you know, those three. If the three are, are like this. And then one of them switches, that's another combination. And you get a lot of repeats like that.	
635.	00:26:37	R3:	Oh, I see. OK.	
636.	00:26:39	Michael:	So by eliminating that, you eliminate the combinations that repeat by the three people moving around.	
637.	00:26:45	R3:	Uh-hum.	
638.	00:26:45	Michael:	Then let's say you just have those two people in, in any given combination. If, if one, if this guy switches the place with this guy it's the, they're different combinations, but in this we're not worried about where they are. We just, you understand?	
639.	00:27:00	R3:	Mm hm.	
640.	00:27:02	Michael:	That's why we get rid of the, the two factorial to, to, uh, eliminate the amount like as many times as you could, as many combinations as you could put those two people. Right? Like the three would, would be to eliminate the combinations you could put those three people that you're not worried about. Then the two, they would repeat because those people too, they move around. They, they could, they move around in the, in the line also. And then when, when, when you're done with all that, you just get, um, you get how many places you can just put that two. Like you're not worried if, like you don't care who they are. You don't care like if this guy has a switch with this guy. You understand like why you would eliminate, how that eliminates.	

<b>Description: Night Session – Pascal’s Identity</b> <b>Parent Tape:</b> <b>Date: 1999-05-12</b> <b>Location: David Brearley High School</b> <b>Researcher: Professor Carolyn Maher</b>	<b>Authors: Uptegrove, Elizabeth B.</b> <b>Verified: Poprik, Brad</b> <b>Date Transcribed: 2003</b> <b>Page: 26 of 54</b>
--	--

Line	Time	Name	Transcript	Coding and Explanation
641.	00:27:41	R1:	OK. I don't want to think of people. I want to think of the tower now. Isn't that what Jeff said? And now I'm thinking of towers that are five tall?	
642.	00:27:50	Jeff:	Yeah. You can, we just-	
643.	00:27:51	R1:	And we're talking of those that have two reds?	
644.	00:27:54	Jeff:	Yeah. Well. [Inaudible.]	
645.	00:27:54	R1:	Explain it to me with that.	
646.	00:27:55	Jeff:	All right. Say, say we're doing, we're doing towers that were, were five tall. Towers of five tall with two different colors in it. Then that's the total amount of possibilities is the five factorial that you could have. All right, in, with, with five high with the combinations. So that's where, that's the five factorial on top. Then the three factorial on the bottom would be five different, five different spots minus the two spots that you're concerned about, leaving you with the three other spots-	
647.	00:28:26	Romina:	You could say-	
648.	00:28:26	Jeff:	-that you don't care about. That's going to eliminate all of them.	
649.	00:28:29	Romina:	That's like, if you say like the reds. Let's say reds are our two colors that they stay in the same place, and like-	
650.	00:28:34	Jeff:	Reds.	
651.	00:28:34	Romina:	They're. Like yeah, the two stay in the same place and then the other three are just switching while they're in staying in the same place.	
652.	00:28:39	Jeff:	Yeah, they're staying in the same spot.	
653.	00:28:40	Romina:	But we're not concerned with them.	
654.	00:28:41	Jeff:	That's why you're not concerned with those.	
655.	00:28:43	Michael:	It's going to repeat like six times.	
656.	00:28:44	Jeff:	Yeah. So that's where the three factorial comes from, and you're multiplying that by the two factorial. Those are what you're-	
657.	00:28:50	Romina:	That's to say like the first place and the third place and then they just switch.	
658.	00:28:51	Michael:	Yeah, like- this way	
659.	00:28:52	Jeff:	Exactly.	
660.	00:28:54	Michael:	They just don't have a name on them so the, they're the same thing.	
661.	00:28:56	Romina:	Yeah.	
662.	00:28:57	Jeff:	And then that's where the bottom number comes from and then you divide them by	

<b>Description: Night Session – Pascal’s Identity</b> <b>Parent Tape:</b> <b>Date: 1999-05-12</b> <b>Location: David Brearley High School</b> <b>Researcher: Professor Carolyn Maher</b>	<b>Authors: Uptegrove, Elizabeth B.</b> <b>Verified: Poprik, Brad</b> <b>Date Transcribed: 2003</b> <b>Page: 27 of 54</b>
--	--

Line	Time	Name	Transcript	Coding and Explanation
			each other and that gives you what we're looking for.	
663.	00:29:04	R1:	OK, so I think I follow what you said. But why were we doing this?	
664.	00:29:09	Jeff:	Uh, you, we don't-	
665.	00:29:11	Michael:	We were talking about-	
666.	00:29:12	Romina:	We want, you wanted us to explain choose.	
667.	00:29:13	Michael:	The choose that we, all right, whoa-	
668.	00:29:15	Romina:	Which goes back to Pascal's Triangle and see where $a$ plus $b$ -	
669.	00:29:19	Michael:	Yeah.	
670.	00:29:19	Romina:	-to the $n$ . And we could figure out the beginning number.	
671.	00:29:20	Michael:	All right. Over here, you wanted, the $a$ plus $b$ to the $n$ thing, you wanted to know how we got the choose thing. What does that mean?	
672.	00:29:24	Jeff:	Yeah, how we got the third number.	
673.	00:29:25	Romina:	Yeah.	
674.	00:29:26	Jeff:	And that's how we got off to, to here.	
675.	00:29:30	R1:	OK, so what did that have to do with what you did in class today?	
676.	00:29:32	Romina:	That's how we would get the number.	
677.	00:29:33	Jeff:	We were looking at, we were doing this in class today. That's what we were doing. We were looking at $a$ plus $b$ -	
678.	00:29:37	Romina:	We're going to be-	
679.	00:29:38	Michael:	It was like in Pascal's Triangle things go like, by that. Like this choose this. Like, um, if you go to the one, three, three, one part of it, it would be, um-	
680.	00:29:47	R1:	Show me on the board, Michael.	
681.	00:29:49	Jeff:	Go get 'em, Mike.	
682.	00:29:52	Michael:	This would be like, all right, this would be like three choose one. How many different places you put that one, that one guy. There's only one place. There's only, oh, I'm wrong. What am I doing?	
683.	00:30:18	Romina:	That's when you only have like, it's all one color.	
684.	00:30:20	Michael:	No, there, there's a way it has something to do with- I think that would be three choose zero, I guess. No. All right, and then the next one would be three choose three. Obviously three different places.	

<b>Description: Night Session – Pascal’s Identity</b> <b>Parent Tape:</b> <b>Date: 1999-05-12</b> <b>Location: David Brearley High School</b> <b>Researcher: Professor Carolyn Maher</b>	<b>Authors: Uptegrove, Elizabeth B.</b> <b>Verified: Poprik, Brad</b> <b>Date Transcribed: 2003</b> <b>Page: 28 of 54</b>
--	--

Line	Time	Name	Transcript	Coding and Explanation
685.	00:30:32	R1:	Three choose what? What was the next one?	
686.	00:30:34	Michael:	Three choose one. The next would be three choose two, which we just figured that out. There's three. And last one is three choose three. You can only put those three people in those three places. You can't, you know, no more places to put them.	
687.	00:30:48	R1:	OK so that's really interesting. That's really very interesting. So you've put something else together. I have another question. You could write more rows of that triangle.	
688.	00:30:58	Michael:	Yeah.	
689.	00:30:59	R1:	And now you're telling me you can write them as the choose way, you've called that. So can you take, let's say another row or two and show me the addition rule and what it looks like with your new notation.	
690.	00:31:17	Michael:	You're talking about the addition rule when you	
691.	00:31:18	R1:	For a particular, for a particular row.	
692.	00:31:20	Michael:	Add this and this and go like that?	
693.	00:31:21	R1:	Sure, or three and three, six. Show me what that looks like with that new notation. Do you understand my question?	
694.	00:31:29	Michael:	Uh, I don't really.	
695.	00:31:29	Romina:	I don't understand.	
696.	00:31:29	Ankur:	Instead of writing three you write-	
697.	00:31:31	R1:	Write your next row, Michael. Now some time ago you, you had a reason. You explained to me-	
698.	00:31:45	Michael:	Why you add.	
699.	00:31:46	R1:	Why you add.	
700.	00:31:47	Michael:	Yeah.	
701.	00:31:48	R1:	You remember that? You, might, might be useful for folks who haven't heard it to hear it whatever way you want to explain it.	
702.	00:31:53	Michael:	I don't think I can explain it too good. Um.	
703.	00:31:55	R1:	Um, you know, however you want to explain it. You've had it a few ways.	
704.	00:32:00	Michael:	Um, I can't, I can't remember too well. I know why you add, if I explain it, I don't	

<b>Description: Night Session – Pascal’s Identity</b> <b>Parent Tape:</b> <b>Date: 1999-05-12</b> <b>Location: David Brearley High School</b> <b>Researcher: Professor Carolyn Maher</b>	<b>Authors: Uptegrove, Elizabeth B.</b> <b>Verified: Poprik, Brad</b> <b>Date Transcribed: 2003</b> <b>Page: 29 of 54</b>
--	--

Line	Time	Name	Transcript	Coding and Explanation
			think anyone will understand.	
705.	00:32:13	R1:	Try.	
706.	00:32:15	Michael:	I didn't. Didn't I tell you guys like last time I came here?	
707.	00:32:18	Jeff:	Well, go for it, dude, just-	
708.	00:32:20	Romina:	You could try.	
709.	00:32:20	Michael:	You don't have that paper, do you? You can just hand them, hand that out.	
710.	00:32:21	Romina:	You started talking about toppings. I think something-	
711.	00:32:24	Michael:	Hand that out instead.	
712.	00:32:25	Jeff:	Just-	
713.	00:32:27	Michael:	Um, all right. If, all right, let's go to, let's go to this one. This would be like three different places I guess. And um-	
714.	00:32:37	Jeff:	Which one are we looking at?	
715.	00:32:38	Michael:	That one right there. You have three-	
716.	00:32:41	Jeff:	That would be $a$ plus $b$ to the third.	
717.	00:32:42	Michael:	All right, let's say you have like, here's a number, all right? Zero means no toppings. One would, this, one would be-	
718.	00:32:51	Romina:	It would be, one's a topping.	
719.	00:32:51	Michael:	One would be a topping. So first category is everything with no toppings. And that's, you can't make, that's, that's your number for that one.	
720.	00:33:01	Michael:	Next would be- There's all the, the ones that have one topping.	
721.	00:33:12	Jeff:	Right, you got to make that zero at the end. You messed up.	
722.	00:33:14	Michael:	What?	
723.	00:33:14	Jeff:	Last one should be a hundred, not a hundred and one.	
724.	00:33:15	Michael:	I knew that. There's your, um, your three choose one. And there's three different combinations you could put that. Um, I can go on forever doing this.	
725.	00:33:25	Michael:	But, um, when you have a new, when you add another place, another topping-	
726.	00:33:34	Jeff:	That could be one or the other, one or the other, one or the other	
727.	00:33:36	Michael:	So it could be one or the other.	
728.	00:33:37	Michael:	It could be a zero or a one, a zero or a one, a zero or a one.	
729.	00:33:38	Jeff:	Yeah. All right.	

<b>Description: Night Session – Pascal’s Identity</b> <b>Parent Tape:</b> <b>Date: 1999-05-12</b> <b>Location: David Brearley High School</b> <b>Researcher: Professor Carolyn Maher</b>	<b>Authors: Uptegrove, Elizabeth B.</b> <b>Verified: Poprik, Brad</b> <b>Date Transcribed: 2003</b> <b>Page: 30 of 54</b>
--	--

Line	Time	Name	Transcript	Coding and Explanation
730.	00:33:39	Michael:	So all these threes would either move up a step onto the next category and, uh, have two toppings.	
731.	00:33:47	Michael:	Or they might stay behind and still only have one if they have the zero.	
732.	00:33:52	Michael:	So three, three will get a topping, and go to this one.	
733.	00:33:56	Michael:	And three won’t, will stay.	
734.	00:33:58	Michael:	And obviously this guy’s going to get a topping. That’s why you add this one.	
735.	00:34:03	Jeff:	Uh-huh.	
736.	00:34:03	Michael:	So now this guy’s going to have, without toppings. You’re going to add a topping onto him. That’s going to be one topping. These three with one topping won’t get one so, you know-	
737.	00:34:14	Jeff:	That’s their four.	
738.	00:34:15	Michael:	You put, you can put them in the same category as this one.	
739.	00:34:17	Jeff:	Yeah.	
740.	00:34:17	Michael:	That’s four.	
741.	00:34:17	Jeff:	Those are your four.	
742.	00:34:18	Michael:	And you know-	
743.	00:34:19	Ankur:	Three.	
744.	00:34:19	Jeff:	Those three.	
745.	00:34:20	Michael:	The three that had two toppings won’t get any.	
746.	00:34:23	Jeff:	Yeah. So they’ll go to [Inaudible.].	
747.	00:34:23	Michael:	And you could put them in together with the ones that did get something. That’s why you would add. Keep on adding.	
748.	00:34:28	R3:	What do you mean by toppings?	
749.	00:34:29	Michael:	Pizza toppings.	
750.	00:34:30	R3:	Um, for example-	
751.	00:34:31	Michael:	Like here you would have a choice of three different ones. Here you would have a choice of five and like the ones would be like the mushrooms, the peppers, the whatever, just by going like- The one would indicate you have it or not.	
752.	00:34:46	R1:	OK. OK. I remember.	

<b>Description: Night Session – Pascal’s Identity</b> <b>Parent Tape:</b> <b>Date: 1999-05-12</b> <b>Location: David Brearley High School</b> <b>Researcher: Professor Carolyn Maher</b>	<b>Authors: Uptegrove, Elizabeth B.</b> <b>Verified: Poprik, Brad</b> <b>Date Transcribed: 2003</b> <b>Page: 31 of 54</b>
--	--

Line	Time	Name	Transcript	Coding and Explanation
753.	00:34:48	Michael:	You remember.	
754.	00:34:49	R1:	I remember this. But now I don’t want to think of the numbers in that triangle, I want to think of those as chooses. So for example, let’s just take this row. One, three, three, one.	
755.	00:35:11	Michael:	Mm hm.	
756.	00:35:13	R1:	All right. If I wrote these as chooses the way you’re writing them-	
757.	00:35:19	Michael:	Three choose zero, three choose one.	
758.	00:35:20	R1:	This is three choose zero.	
759.	00:35:21	Michael:	Yeah.	
760.	00:35:22	R1:	This is three choose one.	
761.	00:35:23	Jeff:	Choose one. Same thing.	
762.	00:35:24	R1:	Three choose-	
763.	00:35:25	Michael:	Two and three choose, then three choose, three choose three.	
764.	00:35:28	R1:	Right.	
765.	00:35:29	Jeff:	So that’s how you get it. It’s like the same thing, cause like three and zero is like three and three, right? And then three two.	
766.	00:35:32	R1:	OK, so-	
767.	00:35:34	Michael:	You want us to write the triangle looking like that?	
768.	00:35:36	R1:	I would, I would, I would like you to do that and then tell me what the general rule is.	
769.	00:35:41	Jeff:	All right.	
770.	00:35:42	R1:	With this notation. Do you understand my question? I’ll leave you to work on that. So, so I’d like you to write out some of the rows with the triangle, and then I’d like-	
771.	00:35:51	Michael:	So to use it like, like that. Like the next one would be, uh, four choose zero.	
772.	00:35:55	Jeff:	Yeah and-	
773.	00:35:56	Romina:	Four choose -	
774.	00:35:56	Michael:	The four choose zero then //four choose one, four choose two-	
775.	00:35:57	Jeff:	//Four choose one, four choose two.	
776.	00:35:58	Ankur:	Four choose three.	
777.	00:36:00	Michael:	We’re in a bad place.	

<b>Description: Night Session – Pascal’s Identity</b> <b>Parent Tape:</b> <b>Date: 1999-05-12</b> <b>Location: David Brearley High School</b> <b>Researcher: Professor Carolyn Maher</b>	<b>Authors: Uptegrove, Elizabeth B.</b> <b>Verified: Poprik, Brad</b> <b>Date Transcribed: 2003</b> <b>Page: 32 of 54</b>
--	--

Line	Time	Name	Transcript	Coding and Explanation
778.	00:36:02	R1:	Right. You probably want to use this.	
779.	00:36:03	Michael:	Yeah.	
780.	00:36:03	R1:	So that people can read it.	
781.	00:36:04	Michael:	Um.	
782.	00:36:05	Alex:	Ask them your question one more time.	
783.	00:36:06	R1:	OK, so I'd like you to rewrite your triangle if you like.	
784.	00:36:09	Michael:	From top to bottom?	
785.	00:36:10	R1:	Top to bottom.	
786.	00:36:11	Romina:	Do you want the ones and like-	
787.	00:36:13	Jeff:	All right. So what-	
788.	00:36:14	R1:	I want everything-	
789.	00:36:14	Jeff:	What would-	
790.	00:36:14	R1:	I want everything written in this form. Do you understand?	
791.	00:36:16	Ankur:	Uh-huh. [Ankur nods.]	
792.	00:36:17	Michael:	That's, that's easy.	
793.	00:36:18	R1:	And then I would like the general row.	
794.	00:36:19	Jeff:	Is that one?	
795.	00:36:19	R1:	What would the general row look like? Where you have towers?	
796.	00:36:24	Romina:	That's a zero, no that's zero choose zero	
797.	00:36:27	Ankur:	X high.	
798.	00:36:28	R1:	Something like that.	
799.	00:36:29	Jeff:	All right, well that's [Inaudible]	
800.	00:36:30	R1:	Ankur understands. So he can tell you.	
801.	00:36:37	Romina:	See, like that?	
802.	00:36:38	Michael:	So it would be, um, like N over, not two over.	
803.	00:36:42	Ankur:	Well, it would be-	
804.	00:36:43	Michael:	N choose-	
805.	00:36:44	Ankur:	It would be-	
806.	00:36:46	Romina:	Well, and N, make N like your height or something.	
807.	00:36:49	Jeff:	All right, so say-	



<b>Description: Night Session – Pascal’s Identity</b> <b>Parent Tape:</b> <b>Date: 1999-05-12</b> <b>Location: David Brearley High School</b> <b>Researcher: Professor Carolyn Maher</b>	<b>Authors: Uptegrove, Elizabeth B.</b> <b>Verified: Poprik, Brad</b> <b>Date Transcribed: 2003</b> <b>Page: 33 of 54</b>
--	--

Line	Time	Name	Transcript	Coding and Explanation
808.	00:36:50	Romina:	N equals height.	
809.	00:36:52	Jeff:	Well that would-	
810.	00:36:52	Ankur:	Well, write the X. Write $a$ plus $b$ to the whatever it is next to it.	
811.	00:36:57	Jeff:	Yeah.	
812.	00:36:58	Ankur:	You know what I mean?	
813.	00:36:59	Jeff:	Yeah. So right. That would be $a$ plus $b$ to the-	
814.	00:37:00	Michael:	This would be nothing, you know, it would be adding.	
815.	00:37:02	Jeff:	Yeah, zero, one, two. So $a$ plus $b$ to the second.	
816.	00:37:05	Romina:	Well, it'd be like $N$ over $N$ minus, but what?	
817.	00:37:07	Jeff:	Yeah, well, $a$ plus $b$ to the second, so it would be if, or $a$ plus $b$ to the $n^{\text{th}}$ .	
818.	00:37:13	Romina:	To the-	
819.	00:37:14	Ankur:	No, all you need is like-	
820.	00:37:14	Romina:	$n$ is factorial.	
821.	00:37:14	Jeff:	It'd be $n$ , $n$ over-	
822.	00:37:16	Michael:	$n$ , fa-	
823.	00:37:18	Jeff:	$n$ mi-	
824.	00:37:18	Romina:	No, that's just like- No, it's not right. I'm just saying like-	
825.	00:37:21	Jeff:	It would be-	
826.	00:37:23	Romina:	You would have to multiply it.	
827.	00:37:24	Jeff:	$n$ over-	
828.	00:37:28	Michael:	Well, if you had an $n$ , it would be, uh-	
829.	00:37:30	Ankur:	To the height of the tower which is $n$ , right?	
830.	00:37:32	Michael:	You'd have a bunch of $n$ 's.	
831.	00:37:33	Jeff:	Yeah, and it'd be over, just z-	
832.	00:37:34	Michael:	There'd be $n$ plus one $n$ 's going this way.	
833.	00:37:37	Jeff:	Yeah. If-	
834.	00:37:38	Michael:	All right?	
835.	00:37:38	Jeff:	it would be $n$ over 0.	
836.	00:37:39	Michael:	So if $n$ was three, you'd have four $n$ 's going this way.	
837.	00:37:42	Jeff:	Yeah.	
838.	00:37:42	Michael:	And the bottom numbers would be just going from 0 to-	

<b>Description: Night Session – Pascal’s Identity</b> <b>Parent Tape:</b> <b>Date: 1999-05-12</b> <b>Location: David Brearley High School</b> <b>Researcher: Professor Carolyn Maher</b>	<b>Authors: Uptegrove, Elizabeth B.</b> <b>Verified: Poprik, Brad</b> <b>Date Transcribed: 2003</b> <b>Page: 34 of 54</b>
--	--

Line	Time	Name	Transcript	Coding and Explanation
839.	00:37:44	Jeff:	Just-	
840.	00:37:45	Michael:	To-	
841.	00:37:45	Jeff:	Yeah. Well, yeah.	
842.	00:37:46	Michael:	0 to $n$ .	
843.	00:37:50	Jeff:	Exactly.	
844.	00:37:51	Michael:	To $n$ .	
845.	00:37:51	Jeff:	To $n$ . Whatever $n$ equals.	
846.	00:37:53	Romina:	Is there a way to write that, you know how to write over times [Inaudible.]?	
847.	00:37:58	Ankur:	I guess.	
848.	00:37:59	Jeff:	Yeah, so how do you, yeah, wait, now that makes sense but, so it would be $n$ over 0 to the $n^{\text{th}}$ . And whatever-	
849.	00:38:08	Michael:	Zero, what are you talking about?	
850.	00:38:09	Jeff:	Wherever you're looking for.	
851.	00:38:09	Ankur:	What are you talking about, 0 to the $n$ ?	
852.	00:38:11	Michael:	0 minus $n$ ? That would be negative.	
853.	00:38:13	Jeff:	No, not minus, like that's to whatever $n$ is. $n$ over 0, $n$ over 1.	
854.	00:38:18	Romina:	1.	
855.	00:38:19	Jeff:	Not divided by like $n$ , 1, $n$ , uh, 2, $n$ , 3.	
856.	00:38:25	Michael:	That was-	
857.	00:38:26	Jeff:	All the way until $n$ could be over $n$ . You know what I'm saying?	
858.	00:38:28	Michael:	Yeah.	
859.	00:38:29	Jeff:	Not, not divided by. I was using bad, uh, bad looking things there. But-	
860.	00:38:34	Michael:	Each of those would be a number-	
861.	00:38:35	Jeff:	Yeah, it's what, 0 to $n$ .	
862.	00:38:37	Ankur:	And $n$ represents the height of the tower?	
863.	00:38:39	Romina:	The height of the tower, yup.	
864.	00:38:42	Michael:	Yeah, $n$ , $n$ represents-	
865.	00:38:43	R1:	Do you want that divided sign here?	
866.	00:38:45	Michael:	No.	
867.	00:38:45	R1:	On that one?	
868.	00:38:46	Jeff:	No.	

<b>Description: Night Session – Pascal’s Identity</b> <b>Parent Tape:</b> <b>Date: 1999-05-12</b> <b>Location: David Brearley High School</b> <b>Researcher: Professor Carolyn Maher</b>	<b>Authors: Uptegrove, Elizabeth B.</b> <b>Verified: Poprik, Brad</b> <b>Date Transcribed: 2003</b> <b>Page: 35 of 54</b>
--	--

Line	Time	Name	Transcript	Coding and Explanation
869.	00:38:46	Ankur:	No. Cross that off.	
870.	00:38:46	Romina:	No.	
871.	00:38:46	Jeff:	I was using it to separate, and that was, that’s a habit of mine, it looks bad.	
872.	00:38:49	Michael:	Oh, sorry about that. It would be, uh, as many, it’s like height of the tower with two colors. You have two numbers.	
873.	00:38:59	Jeff:	Yeah. How do you, how are you, can you write that to get this?	
874.	00:39:04	Romina:	Like that’s what I meant. Like I didn’t mean factorial. I meant like when we used four first and like three first. I don’t know how to write that, though.	
875.	00:39:10	R1:	So you go 0, 1, 2, 3, dot, dot, dot, up to $n$ .	
876.	00:39:16	Jeff:	Yeah.	
877.	00:39:16	Michael:	Mm hm.	
878.	00:39:17	R1:	Can we get one in the middle there, like $n$ choose $r$ ?	
879.	00:39:22	Jeff:	Like how would you just go right to $n$ choose 3? Or $n$ choose $r$ ? Like what- [Researcher 1 nods.]	
880.	00:39:29	Michael:	What are you talking about?	
881.	00:39:30	Romina:	Like instead of using 0, 1, 2, 3.	
882.	00:39:31	Jeff:	$r$ being any number on the bottom.	
883.	00:39:35	R1:	Because you said $n$ choose $x$ up there.	
884.	00:39:37	Jeff:	Yeah.	
885.	00:39:37	R1:	//I just picked what I wanted-	
886.	00:39:38	Michael:	//Oh, you want uh, you want to do that.	
887.	00:39:39	Jeff:	Yeah, so, so it would be-	
888.	00:39:40	Michael:	Um-	
889.	00:39:40	Ankur:	$n$ choose-	
890.	00:39:44	Michael:	It would be $n$ .	
891.	00:39:49	Jeff:	Wouldn’t that just be $n$ choose $r$ for whatever $r$ you wanted? Whatever number you wanted up to, as long as it didn’t exceed $n$ ?	
892.	00:39:59	Michael:	This, this is different than that. Isn’t it? Like this, these are just like a list of numbers. That’s, that’s just giving you one of these numbers.	
893.	00:40:05	Jeff:	Uh, you know all that, but I’m saying, if you wanted to write $n$ choose to get a certain number, wouldn’t it just be $n$ choose $r$ ? Like that? And then as long as $r$	

<b>Description: Night Session – Pascal’s Identity</b> <b>Parent Tape:</b> <b>Date: 1999-05-12</b> <b>Location: David Brearley High School</b> <b>Researcher: Professor Carolyn Maher</b>	<b>Authors: Uptegrove, Elizabeth B.</b> <b>Verified: Poprik, Brad</b> <b>Date Transcribed: 2003</b> <b>Page: 36 of 54</b>
--	--

Line	Time	Name	Transcript	Coding and Explanation
			doesn’t exceed $n$ or it’s less than 0 like $r$ -	
894.	00:40:15	Ankur:	Wouldn’t that equal that?	
895.	00:40:16	Romina:	Yeah, wouldn’t it?	
896.	00:40:16	Michael:	I guess you could write one of those.	
897.	00:40:18	Romina:	Yeah. Isn’t it supposed to equal that?	
898.	00:40:18	Michael:	Right there.	
899.	00:40:19	Ankur:	That’s- that is.	
900.	00:40:19	Romina:	It’s the same thing.	
901.	00:40:21	Ankur:	That does.	
902.	00:40:24	Michael:	You could do that. It’s a lot of-	
903.	00:40:26	R1:	OK, so you’ve written out three rows and then you wrote out the $n^{\text{th}}$ row.	
904.	00:40:33	Michael:	The reason why, 0, 1, 2, 3 is that number is always going to be that number. It’s not, it’s never going to change.	
905.	00:40:35	R1:	[Researcher 1 walks to the board.] OK. I’ll buy that. But something in here could be an $n$ choose $r$ . Right? Something in here could be an $n$ choose $r$ .	
906.	00:40:41	Romina:	Mm hm.	
907.	00:40:42	R1:	That’s what I heard you say, Jeff?	
908.	00:40:43	Jeff:	Yes.	
909.	00:40:43	R1:	Sort of a general one in here, $n$ choose $x$ .	
910.	00:40:46	Jeff:	That’s what-	
911.	00:40:47	R1:	Whatever you choose to use.	
912.	00:40:47	Jeff:	Yeah, that’s what that is. So, yeah.	
913.	00:40:49	R1:	OK. OK, so this is my question to you. You’ve written out two rows and you have the third one there.	
914.	00:40:55	Jeff:	Mm hm.	
915.	00:40:56	R1:	Maybe somebody will come up here and write these up nicely.	
916.	00:40:59	Jeff:	Is that what you want?	
917.	00:41:01	R1:	Yes. Because then I want to ask, I want; after you do that I have a question to ask you. Thanks.	
918.	00:41:06	Michael:	You want to erase those?	
919.	00:41:17	Jeff:	You want to make that the line so bad. I know.	

<b>Description: Night Session – Pascal’s Identity</b> <b>Parent Tape:</b> <b>Date: 1999-05-12</b> <b>Location: David Brearley High School</b> <b>Researcher: Professor Carolyn Maher</b>	<b>Authors: Uptegrove, Elizabeth B.</b> <b>Verified: Poprik, Brad</b> <b>Date Transcribed: 2003</b> <b>Page: 37 of 54</b>
--	--

Line	Time	Name	Transcript	Coding and Explanation
920.	00:41:19	Michael:	No, don’t do that.	
921.	00:41:30	Ankur:	How far do you want him to go?	
922.	00:41:34	Michael:	One more.	
923.	00:41:34	Jeff:	I want to, uh. You want one more for good measure?	
924.	00:42:02	Michael:	No. Don’t worry about it.	
925.	00:42:03	R1:	Go to the $n^{\text{th}}$ one, then.	
926.	00:42:06	Jeff:	Wouldn’t that just be-	
927.	00:42:07	R1:	Dot, dot, dot.	
928.	00:42:08	Jeff:	N zero	
929.	00:42:10	Michael:	Dot, dot, dot, N to the N.	
930.	00:42:20	R1:	And the last one, Jeff. Is the last one N N?	
931.	00:42:24	Michael:	Yeah.	
932.	00:42:25	Romina:	Mm hm.	
933.	00:42:25	Jeff:	Yeah.	
934.	00:42:26	R1:	Do you want to put it at the end?	
935.	00:42:28	Michael:	Yeah, put it at the end, make it nice.	
936.	00:42:30	R1:	What’s the middle one there? What would you, how would you show the middle one?	
937.	00:42:31	Jeff:	Uh, actually, you could put N, X.	
938.	00:42:33	R1:	OK. N choose X, N choose N.	
939.	00:42:40	Jeff:	Those are dots because you can’t really make a dot. Now you can.	
940.	00:42:44	R1:	OK, now, now, show me, show me, while you’re up there, Jeff, just show me, uh, an addition rule of Pascal’s Triangle. Let’s say from, give me an example from the third, fourth row to the fifth row.	
941.	00:42:55	Jeff:	Fourth row to this?	
942.	00:42:57	R1:	Fourth row to the fifth.	
943.	00:42:59	Michael:	The three to the four.	
944.	00:43:00	Jeff:	Oh, fourth row. All right. Um.	
945.	00:43:02	R1:	Show me that three plus three is six. Which ones would it be?	
946.	00:43:07	Jeff:	That would, like you’re saying from here [3 choose 1] to here [3 choose 2] going to	

<b>Description: Night Session – Pascal’s Identity</b> <b>Parent Tape:</b> <b>Date: 1999-05-12</b> <b>Location: David Brearley High School</b> <b>Researcher: Professor Carolyn Maher</b>	<b>Authors: Uptegrove, Elizabeth B.</b> <b>Verified: Poprik, Brad</b> <b>Date Transcribed: 2003</b> <b>Page: 38 of 54</b>
--	--

Line	Time	Name	Transcript	Coding and Explanation
			there [4 choose 2]?	
947.	00:43:10	Michael:	Uh-huh.	
948.	00:43:10	R1:	OK, show me. How would you draw your little arrow to shows that?	
949.	00:43:15	Michael:	This one and that one.	
950.	00:43:16	Jeff:	Yeah, is that it? Is that all, so that’s all you want?	
951.	00:43:18	Michael:	Yeah.	
952.	00:43:18	R1:	Is that true? Do you believe that?	
953.	00:43:20	Jeff:	Yeah.	
954.	00:43:20	Michael:	Yeah, I believe so.	
955.	00:43:21	R1:	You all believe that?	
956.	00:43:22	Romina:	Yeah.	
957.	00:43:22	Michael:	Uh-huh.	
958.	00:43:22	R1:	No one could persuade you otherwise?	
959.	00:43:23	Ankur:	No.	
960.	00:43:23	Michael:	No.	
961.	00:43:25	R1:	OK, so you’re saying three choose one, plus //three choose two equals four choose two. Right?	
962.	00:43:27	Jeff:	//Three choose two should equal four choose two.	
963.	00:43:30	Romina:	Look at all the numbers are added up.	
964.	00:43:32	R1:	OK. So what's four choose two plus four choose three?	
965.	00:43:35	Jeff:	Four choose two plus four choose three? That would be, [Michael laughs.] that would be five-	
966.	00:43:40	Michael:	Oh, five-	
967.	00:43:41	Ankur:	Five choose-	
968.	00:43:43	Michael:	Five choose three.	
969.	00:43:44	Ankur:	Yeah.	
970.	00:43:46	Michael:	Right?	
971.	00:43:47	Ankur:	Yeah.	
972.	00:43:48	Jeff:	Yeah.	
973.	00:43:48	R1:	I don't know if Romina's convinced.	
974.	00:43:50	Jeff:	Why is it five choose three?	

<b>Description: Night Session – Pascal’s Identity</b> <b>Parent Tape:</b> <b>Date: 1999-05-12</b> <b>Location: David Brearley High School</b> <b>Researcher: Professor Carolyn Maher</b>	<b>Authors: Uptegrove, Elizabeth B.</b> <b>Verified: Poprik, Brad</b> <b>Date Transcribed: 2003</b> <b>Page: 39 of 54</b>
--	--

Line	Time	Name	Transcript	Coding and Explanation
975.	00:43:52	R1:	Yeah, I don't think Jeff is either.	
976.	00:43:52	Jeff:	Is this here-	
977.	00:43:53	Romina:	Yeah, I don't really-	
978.	00:43:53	Ankur:	Because it's, it's always the one on the right.	
979.	00:43:55	Michael:	Because, see, this guy gets another topping, I guess, so he turns, he would be a two.	
980.	00:44:01	Jeff:	Uh huh.	
981.	00:44:02	Michael:	Whatever it is in here. And this guy doesn't, so it stays two.	
982.	00:44:03	Jeff:	Ah, it doesn't, so that's two.	
983.	00:44:04	Michael:	So-	
984.	00:44:05	Jeff:	It wasn't that.	
985.	00:44:06	Michael:	Because he's moving up, this bottom number's going to change.	
986.	00:44:09	Jeff:	Oh, all right.	
987.	00:44:09	R1:	Explain that one more time, Michael, please.	
988.	00:44:10	Jeff:	Here.	
989.	00:44:11	Michael:	Um, wherever this guy goes, wherever this guy goes he's going to get another topping because he's moving this way.	
990.	00:44:15	Romina:	Um-hm.	
991.	00:44:15	Jeff:	So that turns it into a two.	
992.	00:44:16	Michael:	So this bottom number's going to change to two.	
993.	00:44:19	Michael:	This guy's not going anywhere. Cause the bottom number stays the same.	
994.	00:44:21	Michael:	So it's going to be five. Because you know the next one's going to be five and it, it has to be a two because- You understand why you add? All right. Good.	
995.	00:44:33	Romina:	I'm with you.	
996.	00:44:34	R1:	OK, so that's really very interesting. Let me ask you to explain that to Brian for a minute, but we'll let him eat first. Did you eat, Brian?	
997.	00:44:40	Brian:	No.	
998.	00:44:40	R1:	Just help yourself. You can watch us.	
999.	00:44:43	Jeff:	We don't get another break?	
1000.	00:44:45	R1:	All right, Brian, just eat. You can.	
1001.	00:44:46	Brian:	I don't think you want to know what I went through.	
1002.	00:44:48	Ankur:	Well at least you got a tux.	

<b>Description: Night Session – Pascal’s Identity</b> <b>Parent Tape:</b> <b>Date: 1999-05-12</b> <b>Location: David Brearley High School</b> <b>Researcher: Professor Carolyn Maher</b>	<b>Authors: Uptegrove, Elizabeth B.</b> <b>Verified: Poprik, Brad</b> <b>Date Transcribed: 2003</b> <b>Page: 40 of 54</b>
--	--

Line	Time	Name	Transcript	Coding and Explanation
1003.	00:44:49	R1:	We're glad you're here.	
1004.	00:44:50	Brian:	Neither did I. I didn't.	
1005.	00:44:52	Ankur:	I didn't either.	
1006.	00:44:52	Romina:	What happened?	
1007.	00:44:52	Ankur:	[Inaudible.] what happened to my coat.	
1008.	00:44:53	Brian:	The coat is like fit for a midget. [Break in tape.]	
1009.	00:44:57	Alex:	Keep going.	
1010.	00:44:57	Michael:	All right.	
1011.	00:44:59	R1:	[Side conversation.] OK, sure, why not.	
1012.	00:45:00	Alex:	OK. Good.	
1013.	00:45:01	Jeff:	All right. Well, all right.	
1014.	00:45:02	Ankur:	[Inaudible.] you remember.	
1015.	00:45:04	Jeff:	All right, we're looking at, we're looking at this right here. You guys got to pay attention to it.	
1016.	00:45:08	R1:	Erase it better, Jeff, before you start, because-	
1017.	00:45:09	Ankur:	Yeah, remember and you didn't pay me back for like three months.	
1018.	00:45:12	Brian:	I had it the whole time.	
1019.	00:45:14	Ankur:	Yeah, but it cancelled out.	
1020.	00:45:15	Jeff:	All right. Say we have this row right here. We got um, $N$ choose 0. And over here we have $N$ choose $X$ . And then over here we have $N$ choose $N$ . All right? Then this right here would be- Oh, we're explaining the general addition, the addition rule using this type of, to fill out the triangle. Using chooses to fill out the triangle and this here would be $N$ choose $X$ plus one and then $N$ , $N$ choose $X$ plus two and so on to whatever $N$ equals. Right there'd be dot dot- I didn't, I didn't leave enough room. And this here would be $X$ minus one and then-	
1021.	00:46:02	Ankur:	You did that one man.	
1022.	00:46:03	Jeff:	What?	
1023.	00:46:04	Ankur:	Nothing.	
1024.	00:46:05	Jeff:	That'd be $X$ minus two and so on each way. Right? So it'd be that.	
1025.	00:46:10	Ankur:	Can I see the row above that?	



<b>Description: Night Session – Pascal’s Identity</b> <b>Parent Tape:</b> <b>Date: 1999-05-12</b> <b>Location: David Brearley High School</b> <b>Researcher: Professor Carolyn Maher</b>	<b>Authors: Uptegrove, Elizabeth B.</b> <b>Verified: Poprik, Brad</b> <b>Date Transcribed: 2003</b> <b>Page: 41 of 54</b>
--	--

Line	Time	Name	Transcript	Coding and Explanation
1026.	00:46:12	Jeff:	And the row above this would be $N$ minus one, right? Yeah.	
1027.	00:46:17	Michael:	Mm hm.	
1028.	00:46:19	Jeff:	Um, choose zero. This again would be $N$ , $N$ minus one choose $X$ and then-	
1029.	00:46:29	Michael:	$N$ minus one.	
1030.	00:46:30	Jeff:	$N$ minus one, $N$ minus one. That's a one. Um, how do you want me to, to- Where do you want me to go from here?	
1031.	00:46:40	R1:	Well, you know, um, Brian wasn't here, so you might want to give him some background to what you've been doing.	
1032.	00:46:46	Jeff:	Start at the beginning? We did, we worked for an hour and a half getting to this point. Explaining this, doing this. All right, um.	
1033.	00:46:54	R1:	But Brian's a quick study.	
1034.	00:46:54	Brian:	That's what I am.	
1035.	00:46:56	Jeff:	All right. We did, uh, this is Pascal's Triangle using-	
1036.	00:47:02	Brian:	The whole choose thing.	
1037.	00:47:03	Jeff:	-the choose situation. That's what this is.	
1038.	00:47:04	Michael:	You know how choose works, like one, three, three, one.	
1039.	00:47:06	Brian:	Yeah.	
1040.	00:47:07	Jeff:	Yeah.	
1041.	00:47:07	Michael:	Three choose zero, three choose one-	
1042.	00:47:08	Brian:	One, four, six-	
1043.	00:47:09	Michael:	Yeah. It's all like chooses of something.	
1044.	00:47:11	Jeff:	All right. So, um, I don't- Um, how would you like to, uh, how do you want to do this? How do you want to-	
1045.	00:47:19	Michael:	We're just-	
1046.	00:47:20	Jeff:	Well, tell him what we did.	
1047.	00:47:21	Michael:	-replacing the three in the chooses by $N$ 's and $X$ 's.	
1048.	00:47:24	Jeff:	Yeah, exactly. And rather doing, like, uh, rather- Say this is the, uh-	
1049.	00:47:29	Michael:	If $N$ was three.	
1050.	00:47:30	Jeff:	Yeah, say if $N$ was the third row, it would be three choose zero. That would give you one.	
1051.	00:47:36	Ankur:	Like, you know how it's one, three, three, one. Three choose zero gives you one.	

<b>Description: Night Session – Pascal’s Identity</b> <b>Parent Tape:</b> <b>Date: 1999-05-12</b> <b>Location: David Brearley High School</b> <b>Researcher: Professor Carolyn Maher</b>	<b>Authors: Uptegrove, Elizabeth B.</b> <b>Verified: Poprik, Brad</b> <b>Date Transcribed: 2003</b> <b>Page: 42 of 54</b>
--	--

Line	Time	Name	Transcript	Coding and Explanation
1052.	00:47:38	Jeff:	Three choose one.	
1053.	00:47:39	Michael:	That’d be three.	
1054.	00:47:39	Jeff:	That would give you the three. The three choose two. That would give you the other three. That’s equal to three and then three choose three. That equals the other one. And like that’s filling out this part of the triangle and so on. And that’s what, that’s what we’re doing now. We went, other stuff we did we did the whole, we found that equation to find out choose.	
1055.	00:48:01	Michael:	What choose means.	
1056.	00:48:02	Jeff:	Yeah, we did all that.	
1057.	00:48:03	Romina:	And choose.	
1058.	00:48:04	Jeff:	But you missed out on all that. That’s the choose equation.	
1059.	00:48:05	Romina:	That’s the choose equals.	
1060.	00:48:08	Jeff:	And we spent time explaining. That’s what we spent the bulk, bulk of the thing, trying to figure out how to explain that. And-	
1061.	00:48:14	Brian:	What’s that little exclamation point?	
1062.	00:48:15	Michael:	//Factorial.	
1063.	00:48:16	Romina:	//Factorial.	
1064.	00:48:16	Ankur:	//Factorial.	
1065.	00:48:16	Jeff:	Factorial.	
1066.	00:48:17	Brian:	That’s what it is?	
1067.	00:48:17	Romina:	Yeah.	
1068.	00:48:17	Jeff:	Yeah.	
1069.	00:48:18	Brian:	All right.	
1070.	00:48:18	Jeff:	It was really excited, like $N!$ [Michael laughs]	
1071.	00:48:20	Romina:	You want to know what this is? That’s all the combinations. [Romina points to her paper; refer to Figure J18.] That’s minusing. You know how like they’re saying-	
1072.	00:48:26	Brian:	Yeah.	
1073.	00:48:26	Romina:	-three choose two.	
1074.	00:48:27	Brian:	Yeah.	
1075.	00:48:27	Romina:	We don’t care about the three, so that’s like when the threes are switching, not the	

<b>Description: Night Session – Pascal’s Identity</b> <b>Parent Tape:</b> <b>Date: 1999-05-12</b> <b>Location: David Brearley High School</b> <b>Researcher: Professor Carolyn Maher</b>	<b>Authors: Uptegrove, Elizabeth B.</b> <b>Verified: Poprik, Brad</b> <b>Date Transcribed: 2003</b> <b>Page: 43 of 54</b>
--	--

Line	Time	Name	Transcript	Coding and Explanation
			twos. And that's when the twos are like in the first place and the third place, and they just switch and nothing else moves.	
1076.	00:48:35	Brian:	So this-	
1077.	00:48:35	Romina:	It's basically the same thing.	
1078.	00:48:35	Brian:	Is this, is that this over this?	
1079.	00:48:37	Michael:	Yeah.	
1080.	00:48:38	Romina:	It's $N$ , $N$ factorial over $N$ minus $X$ factorial times $X$ factorial.	
1081.	00:48:45	Michael:	And that equals $N$ choose $X$ .	
1082.	00:48:46	Romina:	Like this is when the- the things we don't- No, I'm just saying these are the things that we don't care about when they- they switch and this is when the things we do care about, just switch in the same place and everything stays the same.	
1083.	00:48:57	Brian:	All right.	
1084.	00:48:58	Romina:	And that's all of them. [Romina laughs.]	
1085.	00:49:00	Ankur:	The Reader's Digest version.	
1086.	00:49:01	Romina:	Yeah.	
1087.	00:49:01	R1:	What was that, Ankur?	
1088.	00:49:02	Ankur:	No, I just said like the Reader's Digest version or something. [Romina laughs.]	
1089.	00:49:05	R1:	The Reader's Digest version?	
1090.	00:49:07	Jeff:	Yeah. So where, where do you want to go with, with this?	
1091.	00:49:10	R1:	Well, I want you to show me how the addition rule works in general.	
1092.	00:49:14	Jeff:	All right. Well that's not much of a problem-	
1093.	00:49:16	R1:	So you showed me what $N$ minus one choose $X$ -	
1094.	00:49:17	Michael:	Go from, go from, go from $N X$ and $N X$ plus one.	
1095.	00:49:19	Jeff:	Wait, this is, this is //[Inaudible]	
1096.	00:49:21	Ankur:	Yeah, add that in terms of $X$ . Like below it, you know what I mean?	
1097.	00:49:23	Michael:	Add these two. What are these two going to equal?	
1098.	00:49:26	Jeff:	All right, well that's gonna be-	
1099.	00:49:27	Michael:	We want the next-	
1100.	00:49:28	Jeff:	// $N$ plus one over-	
1101.	00:49:30	Michael:	// $N$ plus one over-	
1102.	00:49:30	Ankur:	$X$ plus one.	

<b>Description: Night Session – Pascal’s Identity</b> <b>Parent Tape:</b> <b>Date: 1999-05-12</b> <b>Location: David Brearley High School</b> <b>Researcher: Professor Carolyn Maher</b>	<b>Authors: Uptegrove, Elizabeth B.</b> <b>Verified: Poprik, Brad</b> <b>Date Transcribed: 2003</b> <b>Page: 44 of 54</b>
--	--

Line	Time	Name	Transcript	Coding and Explanation
1103.	00:49:33	Jeff:	$X$ plus one?	
1104.	00:49:33	Michael:	$N$ .	
1105.	00:49:34	Ankur:	Yeah. I think. Uh-huh.	
1106.	00:49:37	Jeff:	That’s what these two are going to come into?	
1107.	00:49:39	Ankur:	Mm hm.	
1108.	00:49:40	Jeff:	Right?	
1109.	00:49:41	Michael:	Yeah.	
1110.	00:49:41	Ankur:	Yeah.	
1111.	00:49:40	Jeff:	And that’s cause-	
1112.	00:49:41	R1:	Can you write it, can you write it as an equation? Just like you wrote three plus three equals six.	
1113.	00:49:46	Jeff:	Um, that would-	
1114.	00:49:48	Ankur:	$N$ plus, just that plus that.	
1115.	00:49:50	R1:	Why don't you do it on the side?	
1116.	00:49:51	Jeff:	Just $N$ . Oh, would it be-	
1117.	00:49:51	Michael:	Oh, $N$ choose $X$ .	
1118.	00:49:52	Jeff:	$N$ choose $X$ , um, plus-	
1119.	00:49:53	Ankur:	Plus.	
1120.	00:49:54	Jeff:	- $N$ choose $X$ plus one.	
1121.	00:49:57	Michael:	Equals that.	
1122.	00:50:00	Jeff:	Plus one, equals that right there.	
1123.	00:50:02	R1:	//[Inaudible]	
1124.	00:50:04	Jeff:	Then, well, that's, that's because this would be gaining an $X$ and going into the $X$ plus one.	
1125.	00:50:14	Michael:	Yeah.	
1126.	00:50:15	Jeff:	And this would be losing an $X$ .	
1127.	00:50:16	Michael:	No, no, not losing, not getting anything.	
1128.	00:50:16	Ankur:	Staying the same.	
1129.	00:50:17	Romina:	No.	
1130.	00:50:18	Ankur:	It's not getting anything.	

<b>Description: Night Session – Pascal’s Identity</b> <b>Parent Tape:</b> <b>Date: 1999-05-12</b> <b>Location: David Brearley High School</b> <b>Researcher: Professor Carolyn Maher</b>	<b>Authors: Uptegrove, Elizabeth B.</b> <b>Verified: Poprik, Brad</b> <b>Date Transcribed: 2003</b> <b>Page: 45 of 54</b>
--	--

Line	Time	Name	Transcript	Coding and Explanation
1131.	00:50:18	Jeff:	That would be staying the same and that's-	
1132.	00:50:19	Ankur:	That's, yeah, the plus that.	
1133.	00:50:20	Jeff:	-is the $X$ plus one.	
1134.	00:50:22	Michael:	And the top numbers have changed because you have more.	
1135.	00:50:24	Jeff:	Because you're adding more things.	
1136.	00:50:25	Ankur:	One more.	
1137.	00:50:25	Jeff:	One more-	
1138.	00:50:27	Michael:	Topping or-	
1139.	00:50:27	Jeff:	Place	
1140.	00:50:28	R1:	Say it so Brian can follow it because he wasn't here for the earlier pizza discussion.	
1141.	00:50:31	Michael:	He follows, you can follow it?	
1142.	00:50:32	Brian:	I can just sit in the back and watch.	
1143.	00:50:33	R1:	Go ahead, Brian. Don't be easy on them, Brian, make them work.	
1144.	00:50:35	Jeff:	What, what we're doing is the next line of the triangle- Remember how today in class you know how the other triangle was one, two-	
1145.	00:50:40	Brian:	Yeah.	
1146.	00:50:41	Jeff:	-three, that whole row there? Well, that's the increase in $N$ , and then the $X$ plus one. If you added another topping onto your whole. Say we're doing pizzas.	
1147.	00:50:50	Brian:	All right.	
1148.	00:50:51	Jeff:	If you add another topping onto it?	
1149.	00:50:53	Romina:	You know how we get the triangle and how we go one two one and add those two together.	
1150.	00:50:56	Brian:	Yeah.	
1151.	00:50:56	Jeff:	Yeah.	
1152.	00:50:57	Romina:	That's what we're doing right there.	
1153.	00:50:57	Jeff:	Yeah. Well, that's what we're doing.	
1154.	00:50:58	Ankur:	We're just adding it.	
1155.	00:50:58	Michael:	You know why, do you know why we add, though?	
1156.	00:50:58	Brian:	That's all you're all doing?	
1157.	00:50:59	Romina:	That's all we're doing.	
1158.	00:51:02	Jeff:	We, we were explaining why you add.	

<b>Description: Night Session – Pascal’s Identity</b> <b>Parent Tape:</b> <b>Date: 1999-05-12</b> <b>Location: David Brearley High School</b> <b>Researcher: Professor Carolyn Maher</b>	<b>Authors: Uptegrove, Elizabeth B.</b> <b>Verified: Poprik, Brad</b> <b>Date Transcribed: 2003</b> <b>Page: 46 of 54</b>
--	--

Line	Time	Name	Transcript	Coding and Explanation
1159.	00:51:03	Brian:	All right, keep going.	
1160.	00:51:03	Jeff:	And why you do it, is it cause when you add another topping like onto it, this one- Say the toppings were one and zero.	
1161.	00:51:10	Brian:	Uh huh.	
1162.	00:51:11	Jeff:	If it gets a topping, that's why it goes up to the $X$ plus one. And since it doesn't get anything, it'll stay the same. And in this one, it's staying the same, right?	
1163.	00:51:20	Michael:	Yeah.	
1164.	00:51:21	Jeff:	And that's why it's going there. Like saying that's the zero.	
1165.	00:51:25	Brian:	OK.	
1166.	00:51:26	Jeff:	And going to there. Make sense?	
1167.	00:51:28	Brian:	Yes. It actually does.	
1168.	00:51:30	Jeff:	So, so that would be the general addition rule in this case? That's it?	
1169.	00:51:34	R1:	Are you impressed?	
1170.	00:51:35	Jeff:	Impressed?	
1171.	00:51:37	R1:	Mm hm.	
1172.	00:51:37	Michael:	Not really.	
1173.	00:51:37	Jeff:	Not really. I don't think we did anything that spectacular.	
1174.	00:51:42	Michael:	Yeah, that's all.	
1175.	00:51:43	R1:	Well, you might be.	
1176.	00:51:44	Ankur:	Nothing more than we ever did before.	
1177.	00:51:45	R1:	You might pick up a probability book in-	
1178.	00:51:46	Jeff:	Is this all in-	
1179.	00:51:47	R1:	-freshman college and see if you recognize this.	
1180.	00:51:51	Jeff:	I mean, I don't know. It just, just seems like-	
1181.	00:51:52	Romina:	We just talked	
1182.	00:51:53	R1:	If someone said to you, why does this work and this is a rule and you've shown me things with factorials, you can probably write those in factorial notations. I bet you could. In fact, I wish someone would do it on the board on the right there. Write that addition statement using factorial notations.	
1183.	00:52:11	Jeff:	All right. Um, you want to do that? Want to do it?	
1184.	00:52:14	Michael:	Just that thing real quick?	

<b>Description: Night Session – Pascal’s Identity</b> <b>Parent Tape:</b> <b>Date: 1999-05-12</b> <b>Location: David Brearley High School</b> <b>Researcher: Professor Carolyn Maher</b>	<b>Authors: Uptegrove, Elizabeth B.</b> <b>Verified: Poprik, Brad</b> <b>Date Transcribed: 2003</b> <b>Page: 47 of 54</b>
--	--

Line	Time	Name	Transcript	Coding and Explanation
1185.	00:52:15	Jeff:	We're writing this right here?	
1186.	00:52:16	R1:	Sure.	
1187.	00:52:16	Jeff:	The addition rule in factorial notation?	
1188.	00:52:19	R1:	That's another form, isn't it?	
1189.	00:52:20	Jeff:	Yeah.	
1190.	00:52:22	R1:	Brian would like to know that, I know he would.	
1191.	00:52:25	Romina:	Bless you. [Someone says Thanks]	
1192.	00:52:27	Brian:	Right.	
1193.	00:52:27	Jeff:	I'm thrilled	
1194.	00:52:27	Ankur:	Oh, yeah.	
1195.	00:52:28	Michael:	That whole thing plus-	
1196.	00:52:31	Ankur:	Plus.	
1197.	00:52:35	Michael:	Aw this is gonna be a pain.	
1198.	00:52:39	Michael:	No.	
1199.	00:52:40	Ankur:	No, it's just N.	
1200.	00:52:41	Jeff:	Yeah, N factorial.	
1201.	00:52:42	Michael:	I just, I just saw that. Um.	
1202.	00:52:48	Ankur:	Over, just do everything it is.	
1203.	00:52:50	Michael:	$N$ minus $X$ .	
1204.	00:52:53	Ankur:	$X$ , parenthesis.	
1205.	00:52:54	Michael:	Plus one.	
1206.	00:52:58	Ankur:	Yeah. And then add and do the $X$ factorial. Put that all in parentheses.	
1207.	00:53:04	Jeff:	It's not an $X$ , it's not $X$ . Yeah, there you go. There you go.	
1208.	00:53:10	Ankur:	No, it's not the top.	
1209.	00:53:12	Michael:	Yeah, the whole thing.	
1210.	00:53:13	Ankur:	Plus one? Do you have that plus one on the bottom?	
1211.	00:53:18	Michael:	Yeah. Equals. Um. [Michael laughs.] Um, this whole thing on the bottom, um.	
1212.	00:53:30	Ankur:	It's the same, it's the same thing. Just copy it.	
1213.	00:53:33	Jeff:	Yeah.	
1214.	00:53:34	Ankur:	$N$ .	
1215.	00:53:35	Jeff:	$N$ .	

<b>Description: Night Session – Pascal’s Identity</b> <b>Parent Tape:</b> <b>Date: 1999-05-12</b> <b>Location: David Brearley High School</b> <b>Researcher: Professor Carolyn Maher</b>	<b>Authors: Uptegrove, Elizabeth B.</b> <b>Verified: Poprik, Brad</b> <b>Date Transcribed: 2003</b> <b>Page: 48 of 54</b>
--	--

Line	Time	Name	Transcript	Coding and Explanation
1216.	00:53:35	Ankur:	Minus $X$ .	
1217.	00:53:36	Jeff:	Minus $X$ plus, exactly. You know how like intimidating this equation must be, like if you just pick up a book and look at that? There you go. Yeah.	
1218.	00:53:57	Michael:	There you go. That’s what you want, I think.	
1219.	00:54:03	R1:	Do you all agree?	
1220.	00:54:04	Jeff:	Yeah. I got chalk all over my pants like Dr. Zabrower.	
1221.	00:54:11	Michael:	That means like-	
1222.	00:54:12	Jeff:	That’s-	
1223.	00:54:13	Michael:	It’s too confusing?	
1224.	00:54:14	R3:	Is that the same thing?	
1225.	00:54:15	Michael:	Yeah.	
1226.	00:54:15	Ankur:	It is the same thing.	
1227.	00:54:17	R3:	It is?	
1228.	00:54:17	Michael:	Yeah. $N$ .	
1229.	00:54:17	Ankur:	As that. Yeah.	
1230.	00:54:18	Michael:	This thing, all right, you see how that is that?	
1231.	00:54:20	R1:	Mm hm.	
1232.	00:54:22	Michael:	You know how- I’ll go up there again.	
1233.	00:54:27	Jeff:	We just wrote out the, yeah, exactly, we wrote out the equation, how to find $N$ choose, exactly.	
1234.	00:54:33	Michael:	That’s, that’s, I guess that’s what you want.	
1235.	00:54:37	Jeff:	Yeah. It’s exactly- We just wrote, we instead of writing-	
1236.	00:54:39	Michael:	You agree with this? Right? So we just wrote, we wrote that-	
1237.	00:54:45	Jeff:	We wrote it in the, in the form.	
1238.	00:54:45	Ankur:	In that form.	
1239.	00:54:45	Michael:	It still doesn’t look, it doesn’t look too good.	
1240.	00:54:47	Jeff:	Yeah. It looks kind of mean.	
1241.	00:54:49	Michael:	We wrote that like that.	
1242.	00:54:53	R1:	Did you all very carefully check that arithmetic?	
1243.	00:54:55	Michael:	You think we’re wrong?	



<b>Description: Night Session – Pascal’s Identity</b> <b>Parent Tape:</b> <b>Date: 1999-05-12</b> <b>Location: David Brearley High School</b> <b>Researcher: Professor Carolyn Maher</b>	<b>Authors: Uptegrove, Elizabeth B.</b> <b>Verified: Poprik, Brad</b> <b>Date Transcribed: 2003</b> <b>Page: 49 of 54</b>
--	--

Line	Time	Name	Transcript	Coding and Explanation
1244.	00:54:57	Ankur:	What, you found an error?	
1245.	00:54:58	Jeff:	All right. Well what’s, what, go to the, uh, write the regular equation down.	
1246.	00:55:02	Romina:	Here’s a-	
1247.	00:55:02	Ankur:	There it is, right there.	
1248.	00:55:02	R1:	Why don’t you get a piece of paper and-	
1249.	00:55:04	Jeff:	Where is it?	
1250.	00:55:05	Ankur:	It’s right above $N$ over $X$ .	
1251.	00:55:05	Michael:	Oh, yeah. Never mind.	
1252.	00:55:06	Jeff:	All right.	
1253.	00:55:06	Romina:	You found it?	
1254.	00:55:06	Jeff:	Yeah.	
1255.	00:55:06	Ankur:	The first one.	
1256.	00:55:14	Michael:	There you go.	
1257.	00:55:17	Jeff:	Yeah, all right.	
1258.	00:55:19	R1:	You sure?	
1259.	00:55:21	Michael:	Yeah, I’m sure. You got anything else? Yeah, I guess.	
1260.	00:55:24	R1:	Did you check it?	
1261.	00:55:26	Michael:	What do you mean? Is it wrong?	
1262.	00:55:29	R1:	Now that, that’s really, really very frightening.	
1263.	00:55:32	Michael:	Yeah.	
1264.	00:55:32	R1:	What do you think? Is that foreboding?	
1265.	00:55:35	Jeff:	I guess.	
1266.	00:55:36	R1:	I wonder if there’s a way of simplifying it.	
1267.	00:55:39	Jeff:	Of what?	
1268.	00:55:39	Michael:	Simplifying it. Hey!	
1269.	00:55:40	Ankur:	Yeah, you could [Inaudible.]; that’s simplifying.	
1270.	00:55:42	Jeff:	Yeah that’s, that’s pretty-	
1271.	00:55:44	R1:	That’s a way to simplify it. But you know I see $N$ plus one parenthesis minus parenthesis $X$ plus one. That looks like that could be a little simpler. See that $N$ plus one parenthesis that Michael just put there.	

<b>Description: Night Session – Pascal’s Identity</b> <b>Parent Tape:</b> <b>Date: 1999-05-12</b> <b>Location: David Brearley High School</b> <b>Researcher: Professor Carolyn Maher</b>	<b>Authors: Uptegrove, Elizabeth B.</b> <b>Verified: Poprik, Brad</b> <b>Date Transcribed: 2003</b> <b>Page: 50 of 54</b>
--	--

Line	Time	Name	Transcript	Coding and Explanation
1272.	00:55:59	Michael:	Yeah.	
1273.	00:56:00	R1:	Minus the expression $X$ plus one. Suppose you distributed that minus one.	
1274.	00:56:07	Jeff:	So you want, all right, so- All right.	
1275.	00:56:10	Michael:	Why would you want to do that?	
1276.	00:56:10	Jeff:	So distributing, say over there, right? You’d have, you’d have $N$ plus one minus $X$ minus one factorial?	
1277.	00:56:19	Romina:	Mm hm.	
1278.	00:56:20	Jeff:	Um, that would be in, in parenthesis.	
1279.	00:56:24	Michael:	Oh yeah, yeah, there you go.	
1280.	00:56:24	Jeff:	And then, well, that-	
1281.	00:56:27	Romina:	Why don't you get another piece of paper?	
1282.	00:56:31	Jeff:	So, all right, so it'd be $N$ plus one factorial divided by, um, $N$ plus one in parentheses minus $X$ minus one factorial. All right? And then, well, that's, that's pretty much all you can do there. Then $X$ plus one factorial, so you could actually can, you can cancel out? Can you cancel that out? The $X$ , minus $X$ minus one and the $X$ plus one? Or-	
1283.	00:57:04	R1:	That's what I'm asking you to think about. Not right, not now necessarily, but, um-	
1284.	00:57:06	Jeff:	Yeah, can you, I mean, can you cross out factorials or is that the first factorial on the bottom of the one all the way to the right? Does that affect, that's affecting the $N$ plus one too, so can you, are you allowed to cross out like that? Cross these both out?	
1285.	00:57:20	R1:	What that’s a good question. What do you all think?	
1286.	00:57:22	Jeff:	Well, can we throw in numbers and see?	
1287.	00:57:25	Romina:	Would we be able to cross out the $N$ plus ones?	
1288.	00:57:27	Jeff:	Well then what are you left with?	
1289.	00:57:29	Romina:	Yeah. Yeah. It doesn't-	
1290.	00:57:30	Jeff:	Factorial divided by factorial?	
1291.	00:57:33	Michael:	Now wouldn't that just be, uh-	
1292.	00:57:35	Jeff:	Now I'm saying you could.	
1293.	00:57:36	Michael:	But now you're talking about simplifying, wouldn't that just be, uh-	
1294.	00:57:38	Jeff:	Yeah.	

<b>Description: Night Session – Pascal’s Identity</b> <b>Parent Tape:</b> <b>Date: 1999-05-12</b> <b>Location: David Brearley High School</b> <b>Researcher: Professor Carolyn Maher</b>	<b>Authors: Uptegrove, Elizabeth B.</b> <b>Verified: Poprik, Brad</b> <b>Date Transcribed: 2003</b> <b>Page: 51 of 54</b>
--	--

Line	Time	Name	Transcript	Coding and Explanation
1295.	00:57:39	Romina:	I don't, would that, this whole thing be-	
1296.	00:57:41	Jeff:	Yeah then it would be nothing, right?	
1297.	00:57:42	Ankur:	Plus one.	
1298.	00:57:43	Romina:	Yeah.	
1299.	00:57:44	Jeff:	Then that would cross out and that would cross out.	
1300.	00:57:45	Romina:	You get two factorials.	
1301.	00:57:47	Ankur:	You can't do that.	
1302.	00:57:47	Michael:	You know that	
1303.	00:57:48	Jeff:	Yeah.	
1304.	00:57:47	Michael:	She's talking about simplifying, and you just like, you know, put that negative in there and it would be just $N$ minus $X$ ?	
1305.	00:57:56	Jeff:	Where? Where's this at?	
1306.	00:57:57	Michael:	Right at $N$ minus. minus, that one right there.	
1307.	00:57:59	Romina:	The one all the way to the side.	
1308.	00:58:01	Jeff:	Oh yeah, and then the, all right, so you, so you do that, $N$ minus $X$ factorial.	
1309.	00:58:01	Michael:	That. That could be-	
1310.	00:58:01	Jeff:	$N$ minus. Yeah exactly.	
1311.	00:58:04	Michael:	Uh, I'm not too good with my uh-	
1312.	00:58:07	Jeff:	Simplification.	
1313.	00:58:08	Michael:	Yeah.	
1314.	00:58:08	Jeff:	Yeah, because that, it would be- You got the plus one.	
1315.	00:58:11	Michael:	I'm just wondering. Wouldn't you, wouldn't that equal $N$ plus one minus $X$ minus one?	
1316.	00:58:19	Jeff:	Yes, then the plus one and the minus one-	
1317.	00:58:19	Michael:	Are gone.	
1318.	00:58:19	Jeff:	So it would be $N$ minus $X$ factorial.	
1319.	00:58:20	Michael:	$N$ minus $X$ so-	
1320.	00:58:21	Jeff:	It'd be $N$ minus $X$ factorial, um, times $X$ plus one factorial? Right? Yeah.	
1321.	00:58:34	Michael:	A little simpler. I still don't like it though.	
1322.	00:58:37	Jeff:	Then, but then you could cross out, OK, could you cross out?	

<b>Description: Night Session – Pascal’s Identity</b> <b>Parent Tape:</b> <b>Date: 1999-05-12</b> <b>Location: David Brearley High School</b> <b>Researcher: Professor Carolyn Maher</b>	<b>Authors: Uptegrove, Elizabeth B.</b> <b>Verified: Poprik, Brad</b> <b>Date Transcribed: 2003</b> <b>Page: 52 of 54</b>
--	--

Line	Time	Name	Transcript	Coding and Explanation
1323.	00:58:39	Michael:	Which are you talking about?	
1324.	00:58:40	Jeff:	Up, no, the bottom and the top.	
1325.	00:58:42	Romina:	The top.	
1326.	00:58:42	Jeff:	Oh, that's plus one. All right, my bad, I wasn't even paying attention.	
1327.	00:58:45	Michael:	Anything else to simplify?	
1328.	00:58:49	Jeff:	Well, if $X$ equals negative one, just-	
1329.	00:58:51	Ankur:	And can't you do that on the other side too?	
1330.	00:58:51	Michael:	Um.	
1331.	00:58:51	Romina::	Um.	
1332.	00:58:54	Jeff:	That would be, um-	
1333.	00:58:56	Ankur:	It would be $N$ minus one.	
1334.	00:58:56	Jeff:	$N$ minus $X$ minus one factorial. No.	
1335.	00:59:01	Michael:	No, it'll still be the same number.	
1336.	00:59:02	Jeff:	Yeah. And it'll be $X$ plus one.	
1337.	00:59:03	Michael:	You want us to do that, do that too? Or don't even bother.	
1338.	00:59:05	Jeff:	Factorial.	
1339.	00:59:08	R1:	I'm, I'm impressed that twenty of ten you're doing this arithmetic. Um, you know, of course the next thing to do is to learn how to do the algebra of factorials so that you indeed could do the addition.	
1340.	00:59:23	Michael:	[Inaudible.].	
1341.	00:59:23	Jeff:	[Inaudible.] the factorial.	
1342.	00:59:24	R1:	Would you like to know how to do that? Would you like to know how to do the algebra of factorials? I bet you know how to do a little bit already. I'll just show you one thing that I know you know and I'll leave you to think about this because everyone is getting tired, but let's just take something like this, right? Six choose two, right? And you know, you, you told me you could write that how? As-	
1343.	00:59:55	Michael:	Um, six factorial over-	
1344.	00:59:57	R1:	Six factorial.	
1345.	00:59:59	Michael:	Three fact, four factorial times two factorial.	
1346.	01:00:03	R1:	Times two factorial, right?	
1347.	01:00:05	Romina:	Mm hm.	

<b>Description: Night Session – Pascal’s Identity</b> <b>Parent Tape:</b> <b>Date: 1999-05-12</b> <b>Location: David Brearley High School</b> <b>Researcher: Professor Carolyn Maher</b>	<b>Authors: Uptegrove, Elizabeth B.</b> <b>Verified: Poprik, Brad</b> <b>Date Transcribed: 2003</b> <b>Page: 53 of 54</b>
--	--

Line	Time	Name	Transcript	Coding and Explanation
1348.	01:00:06	R1:	And you know what six factorial is, right? Six times five.	
1349.	01:00:11	Michael:	Times one-twenty.	
1350.	01:00:12	Jeff:	Thirty. Yeah.	
1351.	01:00:13	R1:	I'm not going to do that though. I don't like to. I don't like to do multiplication. I'm very lazy. I'm just going to write six times five times four factorial. Is that okay?	
1352.	01:00:21	Jeff:	That's, that's simplifying is great, then you can- [Students all talk at once.]	
1353.	01:00:24	R1:	But can I do that?	
1354.	01:00:26	Romina:	Yeah.	
1355.	01:00:26	Michael:	And then you could cross out the four factorials and-	
1356.	01:00:27	Romina:	Oh.	
1357.	01:00:28	R1:	Oh, then I can cross out the four factorials.	
1358.	01:00:28	Jeff:	Oh, all right, that makes sense.	
1359.	01:00:29	R1:	Right?	
1360.	01:00:31	Jeff:	So you just get thirty divided by, you get thirty divided by two.	
1361.	01:00:33	R1:	Yeah. Look at all the time that will save you in an SAT question.	
1362.	01:00:35	Jeff:	That'd be big.	
1363.	01:00:37	R1:	But, but if you think about this-	
1364.	01:00:39	Jeff:	She broke, she broke it down farther.	
1365.	01:00:40	Romina:	Oh yeah she just-	
1366.	01:00:42	Jeff:	Like rather than say you have six factorial-	
1367.	01:00:43	Ankur:	Mm hm.	
1368.	01:00:43	Jeff:	She broke it down until she got a number that she got that she wanted.	
1369.	01:00:45	Romina:	She had two numbers.	
1370.	01:00:47	Jeff:	That matched the number on the bottom.	
1371.	01:00:48	Ankur:	All right. Yeah.	
1372.	01:00:50	Jeff:	Then you end up like with the two factorial and then cross out and that's thirty over the two factorial and that's two. So it's just fifteen..	
1373.	01:00:51	Michael:	But then it would probably be even longer than that. Cause if $N$ is a big number-	
1374.	01:00:55	R1:	Does it matter?	
1375.	01:00:59	Michael:	-you'd have to write, you would have to write $N$ times $N$ minus one times $N$ minus-	

<b>Description: Night Session – Pascal’s Identity</b> <b>Parent Tape:</b> <b>Date: 1999-05-12</b> <b>Location: David Brearley High School</b> <b>Researcher: Professor Carolyn Maher</b>	<b>Authors: Uptegrove, Elizabeth B.</b> <b>Verified: Poprik, Brad</b> <b>Date Transcribed: 2003</b> <b>Page: 54 of 54</b>
--	--

Line   Time       Name       Transcript

Coding and Explanation