ESSAYS ON PRINCIPAL-AGENT MODELS

BY NADIDE BANU OLCAY

A dissertation submitted to the

Graduate School—New Brunswick

Rutgers, The State University of New Jersey

in partial fulfillment of the requirements

for the degree of

Doctor of Philosophy

Graduate Program in Economics

Written under the direction of

Professor Tomas Sjöström

and approved by

New Brunswick, New Jersey

May, 2012

ABSTRACT OF THE DISSERTATION

Essays on Principal-Agent Models

by Nadide Banu Olcay

Dissertation Director: Professor Tomas Sjöström

This dissertation consists of three chapters on principal-agent models. Chapter 2 studies a dynamic optimal contract design problem whereas chapter 3 is an empirical investigation of the incentive contracts in the market of top executives. Chapter 4 is a theoretical chapter exploring welfare impacts of the structure in the top-level bureaucracy.

In chapter 1, I consider a dynamic moral hazard model where the principal offers a series of short-run contracts. I study the optimal mix of two alternative instruments for incentive provision: a performance based wage (a "carrot") and a termination threat (a "stick"). Both carrot and stick, which act as substitutes for each other, are used more intensively as the agent approaches the end of her finite life. The sharing of the surplus from the relationship plays a key role: a termination threat is included in the optimal contract if and only if the agent's expected future gain from the relationship is sufficiently high, compared to the principal's expected future gain. Also, a termination threat is more likely to be optimal if output depends more on "luck" than on effort, if the discount factor is high, or if the agent's productivity is low.

Chapter 3 is an empirical study with a focus on the use of direct pay and forced turnover in executive contracting and how they depend on tenure and managerial ability. Managerial ability is proxied by the age at the time of promotion and by "reputation" which rely on media citations. I find that pay increases with tenure, yet there is no

strong evidence that termination threat follows a particular time pattern. A better reputation increases pay and decreases the likelihood of forced turnover. Managerial ability, as both proxied by age-at-promotion for insiders and by reputation for outsiders, decreases the likelihood of forced turnover.

Chapter 4 investigates the welfare implications of multiple principals in the highest level of bureaucracy. The existence of multiple principals generates a "common agency". The analysis reveals that the optimal hierarchy depends on the existence of "rents" from office that the principals enjoy: a single-principal model dominates common agency only if there are positive rents.

Acknowledgements

It is hardly possible to adequately thank my advisor, Prof. Tomas Sjöström, without whom this work would have simply been impossible. But as far as the words allow me to express my gratitude, I am profoundly thankful *for* providing compassionate supervision, guidance and inspiration on every stage of this thesis, which was leaving me in admiration of his inexhaustible energy and persistent enthusiasm; *for* persuading me to believe that my ideas are valuable and the confidence he placed in me; *for* his endless patience in helping me to write papers; and *for* his unflagged interest in my research even at the times of slow progress. Throughout my entire graduate life at Rutgers, it has been both a great experience to have such a positively challenging instructor and a blessing to have the chance to benefit from his sharp intellect, knowledge, intelligence and wisdom.

I am also grateful to Prof. Carolyn Moehling for her encouragement and guidance on my empirical chapter. Without her advice and her patience regarding my lack of experience on empirical work, this chapter would not be as it is. I am also thankful to Prof. Colin Campbell for providing valuable suggestions and discussions which helped to improve my papers. Prof. Kalyan Chatterjee, as the outsider reader of my dissertation committee, and Prof. Mark Killingsworth, are also gratefully acknowledged for their helpful comments.

My deepest gratitude to Prof. İsmail Sağlam for inspiring me towards studying economics at the graduate level. His research enthusiasm and his constant support to his students will always be an example for me to follow. I am also thankful to Prof. Fikret Adaman for his mentorship on my pursuits towards a graduate study.

I would like to specially thank Dorothy Rinaldi for her encouragement and support in my graduate life. She was more than an administrative staff for all of the graduate

students.

I would never reach to this point in my life without endless love and unconditional support of my parents. I also would like to thank my brother for especially facilitating my trips back home, and to Bengi for her constant moral support. A special thanks goes to my dear Esra who were always there for me even we are miles apart. Many thanks to my long-time friend, Oylum for sharing both the pain and the joy; to Başak for her patience and help at the rough times, and to Nil and Dilara, without whom my last year at Rutgers would be unthinkable. Meriç, Bahar, Gökçe, and many other friends at Rutgers, made me feel lucky to have been surrounded by them; thanks to all.

Dedication

To my mother, *Ayşe Selda Olcay*; and my father, *Gürol Olcay*.

Table of Contents

List of Tables

List of Figures

Chapter 1

Introduction

The principal-agent framework, which is the focus of analysis presented in this study, relies on a frequently observed real life phenomena: an economic agent's ("the principal") delegation of a task to another one ("the agent"), who has different objectives than the principal, becomes problematic for the principal when she does not have perfect information about the agent.

Incentive theory, via study of optimal contracts, attempts to find solutions to the problems which arise due to such conflicting interests in the presence of asymmetric information. It has been a long while since the economists realized the incentive problems, and these problems had been considered as a mere failure of the market mechanism until they were visited within the principal-agent paradigm.

This dissertation presents three chapters within this paradigm. Chapter 2 studies the optimal contracts in a dynamic principal-agent model while Chapter 3 is an empirical analysis of the employment contracts in the market of CEOs. The last chapter investigates the welfare implications of the two bureaucratic systems each of which has a different type of agency problem.

Chapter 2: Dynamic Incentive Contracts with Termination Threads

In this chapter I consider a dynamic principal-agent model which is applicable to employment contracts where the principal can use two different incentive instruments, incentive pay and threat of termination, both of which can be contingent on performance. Previous literature has extensively studied the provision of incentives only through one channel, i.e. incentive pay, but not much considered alternative channels. However, firing when the performance is not satisfactory can play an important role in providing incentives.

The idea that thread of termination is a powerful incentive devise is not peculiar to firm-employee relationships. Bank-borrower relationships where banks deny future loans to defaulters, or landlord-tenant relationships where the landlord reacts to poor crops by evicting the share-cropper, are other examples of the use termination as an incentive device.

I focus on the contracts which allow the use of the two incentive tools and study how the efficient mix of these devices changes through time. As a common real life observation, the increasing wage-profile through a worker's tenure on the same job inspired many researchers to understand the reasons behind. Lazear (1979), as a first attempt to explain the issue, develops a theoretical model where the firm fully commits to a long-term contract specifying all future wages at the start of the firm-worker relationship. In this setting, an upward sloping wage profile, which pays less than the worker's productivity in the early stages but promises to pay more towards the end of her career, is the efficient way to provide incentives for the firm since it completely solves the moral hazard problem.

How significant is the full-commitment assumptions in explaining upward sloping incentive schemes? In practice, committing to a long-term contract create problems on the firm's side. For that reason, employment contracts are mostly renewed in shorter terms after the performance of the employee is evaluated. Therefore, as a second attempt to explain the issue, Gibbons and Murphy (1992) studies the optimal contracts in the presence of short-term commitment.

In an adverse selection model, where the employee's true ability is not known, Gibbons and Murphy (1992) show that "career concerns" is the reason for increasing earnings profile. When the beliefs about her ability is updated each period after observing her performance, in the early stages of her career, the employee is already highly motivated to work hard. Then the optimal explicit incentives should be weak. However, in the later stages, career concerns is no longer important, hence the optimal incentives should be stronger. Gibbons and Murphy's (1992) model is helpful to explain a real life practice in employment contracts, yet the key point is the adverse selection and the

market's ability to update its beliefs about the employee.

I maintain Gibbons and Murphy's short-term commitment assumption. However, I drop the career concerns dimension but add the termination threat as an alternative incentive device and try to answer the following questions: what is the role of termination threat in a pure moral hazard model with limited commitment, and how does the optimal use of this threat interact with explicit wage incentive in a dynamic setting?

In order to study these questions, I consider a finite horizon principal-agent model: there is a fixed point in time when the relationship ends, where this assumption can be understood by the date of mandatory retirement that is used widely in practice. The model predicts that, first, the use of both incentive devices; i.e. termination probability contingent on poor performance and performance based wage increases as the agent is more senior in her tenure. Second, the two incentive mechanisms are used as substitutes for each other. The theory predictions are best understood when the results are interpreted in terms of how the surplus from the relationship is shared between the two parties. A termination threat is included in the optimal contract if and only if the agent's expected future surplus from the relationship is sufficiently high, compared to the principal expected future gain.

The main result, by relying on the share of surplus, is in the spirit of self-enforcing contract literature which studies the optimal contract design when there is an enforcement issue in the absence of verifiable outcomes. As a benchmark result in this area, MacLeod and Malcomson (1987) show that self enforcing contracts solve the enforcement problem only if the agreement can generate a surplus for at least one of the two parties.

In contrast to these models, I assume short-term performance-based contracts are enforceable. Also, since the agent has a finite life, the series of short-run contract is not stationary, in contrast to MacLeod and Malcomson (1987). Towards the end of the agent's "life", the expected future surplus from the relationship dwindles, and the termination threat becomes less effective. I show that even with short-term contracts and no "career concerns", the incentive wage and termination profiles are upward sloping.

That is, as the agent approaches the end of her life, the probability of termination if the quality of output is unacceptable, and the wage when the quality is acceptable, both increase.

Furthermore, for a given observed productivity (as a measure for "ability"), the model provides several predictions in terms of how the two incentive devices should be used. At a given point in time, higher ability increases pay and reduces probability of forced turnover. The key point here is that such ability change affects the share of surplus created by the relationship. As ability increases, principal's surplus increases relative to the agent's; therefore it becomes optimal to decrease probability of termination. However, to provide sufficient incentives to work hard, the decrease in termination probability is accompanied by the increase in incentive pay.

Chapter 3: Incentivizing CEOs via Pay and Forced Turnover: Do Tenure and Ability Matter?

This chapter is an empirical study of the employment contracts in the market for CEOs where short-term contracts are a common practice. The study is not a direct test of, but mainly inspired from the theoretical results of Chapter 2, with a focus on how the explicit incentives, in the form of total CEO pay, and the implicit incentives, i.e. the probability of forced CEO turnover, change with tenure and managerial ability.

Executive contract design, as a research field, has attracted the interest of many researchers mainly because of the need to optimally determine the contracts for the top-level employees, given that they constitute a significant part of the firm's compensation costs. For example, the empirical evidence presents that CEO compensation has risen dramatically beyond the rising levels of an average worker's compensation. While the average CEO pay at the largest companies in the US was 40 times that of the average worker a generation ago, in 2005, the pay of top American CEOs was over 400 times average earnings[1]. Boards, senior management teams, and shareholders are constantly struggling with how to design the right type of executive compensation plans.

Both the amount of the executive pay and the organizational role of executives

[1]The Economist, March 3rd, 2006.

increase the importance of executive compensation design. As a first step to approach this problem, the relationship between the executive and shareholders, who actually delegated their authority to the board of directors, has been identified as a principal-agent model. The design problem is to give the right incentives to the executives, i.e. motivating the CEO of the firm to act in the best interests of the shareholders. For example, if shareholders (the principals) desire to maximize their wealth as measured by the market value of the firm's common stock, stock options might be a way to encourage CEOs (the agents) to work in order to increase the firm's stock market value. Then the task of shareholders is actually to derive the optimal contracts that would induce results to increase the firm value.

The idea that separation of ownership and control in modern corporations can be viewed as an agency problem was fist suggested by Berle and Means (1932). Later Jensen and Meckling (1976) formalized this perspective. However, the modern literature on executive compensation research began in the early 1980s following the common acceptance of agency theory.

Executive contract design's direct link to agency theory increases this field's popularity beyond the fact of important roles of executives. As far as the empirical testing of contract theory is concerned, the executive market is a natural sample. However, executive compensation as a research field presents an extremely valuable laboratory for the theory for several reasons. First, the data for executive compensation is publicly available (through Standard and Poors' Compustat database, Forbes' Annual Compensation Surveys or Towers Perrin's Worldwide Total Remuneration surveys) where the compensation data is expressed in detail in terms of its components; salary, bonus (with respect to different accounting measures), stocks and options, restricted stock, etc. Second, it is easy to test the main predictions of contract theory that contracts offered by the principal affect the incentives of the agent. Given that the actions cannot be observed, the contract can be written in terms of the observed output rather than unobservable effort. Since it is easy to measure the performance of the CEO by just looking at the total earnings, profit or output of the firm, the hypothesis that contracts

affects incentives can directly be tested.

In this chapter, I empirically investigate the relationships of job tenure and managerial ability to the two incentive devices; pay and forced turnover. I also consider how ability and tenure interact in the provision of incentives. The dataset includes a sample of firms listed in S&P 1500 over the period 1998-2008. Data on executive compensation, firm performance and control variables, both at the CEO and firm level, is drawn from the COMPUSTAT database. Data for forced turnover is hand-collected by reading news items which are found through Factiva and Google web search. Two empirical proxies for managerial ability are used: media visibility, which attempts to capture reputational aspects of the "outsider" CEO's ability and is hand-collected through counting articles in top business journals; and the age when the "insider" CEO is promoted to the position.

The results present strong evidence that both instruments are used as an incentive device through the CEO's tenure. I also find that, as they are more senior, CEOs receive higher pay, but the results are inconclusive regarding the relationship between tenure-likelihood of forced turnover. The empirical proxy I use in the analysis, which is based on media visibility, performs well in explaining the positive relationship between pay and ability. In terms of the probability of forced turnover, my results are robust to the choice of empirical proxy: ability decreases probability of forced turnover.

Chapter 4: Common Agency within Bureaucracy

This chapter is related to the political economy literature and explores the welfare implications of the two structurally different bureaucratic systems both of each have an agency problem. It is conventional in political economy literature to model the relationship between voters and politicians in principal-agent framework where voters are identified as principals, and the politicians as their agents. Yet the agency problem is also applicable to multi-tiered governments, where policy implementation is complicated for the top-level principals, so that middle-level bureaucrats administers the policies that are decided by their principals, i.e. the top-level rulers. It was first Dixit (2006) pointing out this innate agency problem within bureaucracy. I develop his model further

and allow a multiplicity in the top-level bureaucracy where the middle-level bureaucrat is their common agent.

Standard economics literature presents a number examples relating to common agency framework. Laussel and Le Breton (1996) suggest a model in which a private firm (i.e. the agent) produces the public good and paid by the consumers (i.e. principals); whereas Martimort and Stole (2003) consider a common agency where the retailers (the principals) independently contract with the manufacturer (the agent) and the production level of one retailer directly affects the price faced by the other retailer.

Interestingly, relatively few attempts have been made to view the bureaucracy as a common agency problem. This study aims to fill the gap relating these two fields and investigate the impact of a multiplicity of principals, in the top level bureaucracy, on social welfare. A practical example for this model is where there exist multiple ministries responsible for the production of one type of public good and a municipality which has to take into account the concerns of different ministers while taking an action. Other examples include the case of administrative agencies who are basically responsible to the lawmakers, yet are practically influenced by the courts, media, and various interest groups, or as in European Union where several sovereign governments deal with a common entity in policy making.

I consider a simple common agency model which involves two top-level bureaucrats (the principals), a middle-level bureaucrat (the agent) who is in charge of producing public goods, and additionally a third party; the representative citizen who consumes the public good.

The key aspect of the model is that the principals value only one type of public good and hence contract with the common agent on the level of production of this public good. There is only one type of informational asymmetry at the bottom level of agency: the agent's effort, which determines the quality of the public goods, cannot be unobserved by the principals. However, the agent is compensated by the principal whose budget is determined by the citizen. Hence, there are two types of contracts, one is offered by a principal to the agent, and the other is offered by the citizen to a

principal. There is an additional informational asymmetry at the top level of agency: the contract between the agent and the principal, as well as the cost structure of the public good production cannot be observed by the citizen. There are rents from the office which can be enjoyed by the principals only when they succeed in inducing their agent to produce a high quality public good.

I show that the optimal system, from the citizen's point of view, directly depends on the existence of the rents. When there are positive rents, a single principal model is favorable: the citizen can always reduce the optimal transfer which is paid when both goods are supplied with high quality, by a fraction of the rent unless the rent is too large. The competition between the principals in common agency, however, limits the citizen's ability to reduce the cost of providing incentives in a similar manner. Therefore, single principal model yields a higher expected payoff to the citizen than a common agency. The two systems are equally welfare-efficient if there are no rents from the public office.

Chapter 2

Dynamic Incentive Contracts with Termination Threats

2.1 Introduction

The provision of incentives is the essence of economics. In attempts to deal with this issue, there has been a large theoretical literature built upon the principal-agent framework, in which the agent's incentives may not be aligned with those of the principal. The problem of the principal is then to find the optimal contract that would induce her agent to work in the interest of herself. The theory provides crucial implications for the optimal design of employment contracts.

An extensive agency literature (e.g., Holmstrom (1979), Grossman and Hart (1983), and Gibbons and Murphy (1992)) provides insight into the determinants of pay, but seldom considers alternative incentive mechanisms.[1] In particular, firms commonly provide incentives to workers through the threat of terminating the relationship if performance is not satisfactory.[2] Firing is a way of excluding agents from future benefits which would accrue if they retain their jobs. Under particular conditions, this is an efficient means of incentive provision, and the threat of dismissal can offset the need to provide incentives through other means, such as explicit cash compensation. In this paper we study optimal contracts with incentive pay and termination threats, both of which can be contingent on the agent's performance.

Given the abundant empirical evidence that incentives provided by employment contracts change through time, we consider the provision of incentives in a dynamic

[1] In the theoretical literature, two early exceptions are Shapiro and Stiglitz (1980) and Stiglitz and Weiss (1983), but they differ significantly from our model, as we discuss futher in the text.

[2] Subramanian (2002),Weisbach (1988), Warner, Watts and Wruck (1988), Murphy and Zimmerman (1993), Denis, Denis and Sarin (1997), Parrino (1997), and Goyal and Park (2002) point out the strong empirical support between performance and forced turnover.

model. We are interested in how the efficient mix of incentive devices changes through time. As far as wage incentives are concerned, a robust empirical finding is an upward sloping earning profile. By using a dynamic principal-agent model, our theory attempts to provide both explanations for such empirical findings and solutions to contract design when the relationship is repeated over a finite number of periods.

In his classic paper, Lazear (1979) provides a theoretical explanation for not only increasing wage schemes but also the frequent use of mandatory retirement policies. His argument is that a wage profile which pays less than the marginal productivity when the agent is young, but more when he gets old, solves the moral hazard problem over the agent's employment horizon.[3] Lazear (1979) argues that termination of employment should be mandatory, since marginal product is below the wage at later stages in the career, and the worker cannot be trusted to quit voluntarily at that time. The firm is imperfectly aware of the worker's outside alternative, but knows that social security starts at the age 65. Lazear's model shows how introducing dynamics can yield additional predictions into the basic agency set-up: optimal incentives must be stronger as the agent gets more senior. However his result relies on one crucial assumption: the firm can commit to a long-term contract which specifies all future wages as well as a mandatory retirement date.

I will use a pure moral hazard model to investigate optimal contracts when long-term commitment is not feasible. Particularly, I try to find answers to two main questions: what is the role of termination threat, and how does the optimal use of this threat interact with explicit wage incentives in a dynamic setting? We find that optimal use of both instruments becomes more intense as the agent is more senior. Moreover, the optimal contract uses the two devices (incentive pay and termination threat) as substitutes for each other.

The difficulty of committing to pre-determined incentive schemes challenges the validity of long-term contracts. Gibbons and Murphy (1992), for example, discard the

[3]Charmichael's (1983) accumulation of human capital explanation provides an alternative perspective to understand the gap between marginal product and the wage. Leigh (1984) empirically challenges Lazear's explanation for upward sloping wage profile and he finds wages rise in fact faster than marginal productivity.

full-commitment assumption and show that it is still possible to explain an upward time trend in wage profile. However, the key driver of their model is career concerns in the presence of adverse selection. When updated beliefs about ability and hence future wages depend on his observed performance, the worker has strong incentives to work hard when he is young. But as he gets old, his incentive to influence the employer's belief about his ability gets weaker, and wage incentives should increase. Gibbons and Murphy (1992) do not consider provision of incentives via termination threats. I will show that even in the absence of career concerns, the wage profile is upward sloping when the principal uses both termination threats and incentive pay.

Two recent examples of papers in which limited commitment is allowed together with termination option are Kwon (2005) and Subramanian (2002). Like my model, both of these models explore the optimal relationship between performance pay and termination threat when contracts are short-term.[4] But unlike my model, they assume an adverse selection problem exists. With adverse selection, incentive pay and termination threats are *complements*. The reason is that the more strongly pay depends on performance, the more likely it is that poor performance is caused by low ability rather than low effort. Firms providing high-powered monetary incentives therefore have more reason to fire their employees for poor performance. Thus, under adverse selection, the termination threat functions as a sorting mechanism that complements monetary incentives, rather than "economizes" on them as in my pure moral hazard model.

Hartzel (1998) studies incentive pay in a pure moral hazard problem with an *exogenous* probability of termination. He finds an inverse relationship between optimal pay and the probability of termination. In our paper, the termination threat is endogenous, i.e., optimally chosen as a part of the employment contract.

The idea that the threat of termination is a powerful incentive device is not peculiar to firm-employee relationships. Bank-borrower relationships where banks deny future loans to defaulters, or landlord-tenant relationships where the landlord reacts to poor crops by evicting the share-cropper, are other examples of the use termination as an

[4]Bac and Genc (2009) point out the same relationship between the two devices under short-term commitment and adverse selection but when performance is not contractable.

incentive device. In such models too, there is a trade-off between termination threats and other incentive devices: banks terminate the relationship rather than raise the interest rate, and landlords terminate the contract instead of lowering the share of the cropper.

Stiglitz and Weiss (1983) analyzed how, by threatening to cut off credit, the bank provides the borrower with incentives to make the right investments.[5] The optimal contracts have a "memory": past credit history affects current interest rate. This history acts as a reputation and can be used as a screening mechanism by the bank. The reputation causes the borrower's behavior to become intertemporally linked and, in equilibrium, the bank refuses credit to borrowers who fail to repay earlier loans.

Banerjee, Gertler and Ghatak (2002) studied the reform of agricultural tenancy laws. The law offered security of tenure to tenants and regulated the share of output that is paid as rent on farm productivity. They argued that a reform which relaxes the security of tenure increases efficiency if the regulated share of output, which in fact stands as the agent's outside option, is sufficiently high. Their result can be contrasted with classical labor contracts: optimal use of termination increases the principal's payoff if the agent's outside option is low enough. However, they do not consider the optimal mix of alternative instruments for incentive provision, and the dynamic interaction between them, which is the main focus of my paper.

Shapiro and Stiglitz (1984) showed that the termination threat provides insights for equilibrium unemployment in an efficiency wage model. When the agent's output is not contractible and monitoring is costly, they obtained a self enforcing contract that uses a termination threat to prevent shirking. But since they assume that the firing decision is contingent on performance but the wage is not, the efficiency wage model is open to criticism. In reality, firms frequently offer performance pay to its employees. Also, in the Shapiro-Stiglitz type models, the contract is stationary and does not provide insights into dynamic aspects of the use of the incentive tools.

---

[5]See Allen (1983) and Eaton and Gersovitz (1981) for other examples of termination contracts with limited commitment where in the case of sovereign debt the threat of not refinancing a country could be used to foster debt repayment.

As long-term commitment may be difficult to achieve in practice, a large literature has investigated the consequences of replacing a long-run contract with a series of short-run contracts. Chiappori et al. (1994) show that short-term contracts can replicate an optimal long-term contract provided the latter is *renegotiation proof*. Fudenberg, Holmstrom and Milgrom (1990) argue that the agent's perfect access to credit markets is key to the optimality of short-term contracts. Malcomson and Spinnewyn (1988), and Rey and Salanie (1996) show that a long-term contract dominates a sequence of short term contracts if it can commit either the principal or agent to a payoff in some future circumstance that is lower than what could be obtained from a short term contract negotiated if that circumstance occurs.

In our model, the principal offers short-term contracts that specify the wage and probability of termination as a function of the quality of the output produced by the agent. The quality of the output is assumed verifiable and hence contractible, but the agent's effort is not observed, hence not contractible. Long-term commitment is ruled out by assumption. The short-run contract specifies that the agent is fired if the observed quality is too low. Once the agent is fired, the principal cannot rehire her. We focus on a situation where the agent's outside option is not very good, so the termination threat can provide good incentives. Of course, termination is not costless to the principal, as he loses the expected future surplus from the relationship. The optimal short-run contract trades off the gain from providing current incentives against this future loss.

Even though there is no long-term commitment, the agent has rational expectations: by backward induction, she realizes that if she stays in the relationship, over the rest of the horizon she will get a surplus, as long as it is the interest of the principal to pay high wages in the future (even though there is no commitment to this). If the agent expects a lot of future surplus from continuing the relationship (so a termination threat provides high-powered incentives) but the principal's expected future surplus is not too high (so that the cost of firing the agent is not too high) then the benefit of a termination threat exceeds the cost. In this case, the optimal short-run contract

includes a termination threat. But if the principal's expected future gain from the relationship is high, while the agent does not expect a lot of future surplus, then the cost of a termination threat exceeds the benefit, and the optimal short-run contract does not include any termination threat.

As explained in the previous paragraph, the way the future surplus is expected to be shared plays a key role in our model. This is not surprising in view of the literature on of self-enforcing contracts. MacLeod and Malcomson (1987) showed that self enforcing contracts solve the enforcement problem when the performance is observable but not verifiable only if the agreement can generate a surplus for at least one of the two parties. The surplus in fact functions in the same way as the equilibrium unemployment in efficiency wage models such as Shapiro and Stiglitz (1984). MacLeod and Malcomson (1989), while keeping the assumption of unverifiable performance, show that performance-based contracts, either in the form of piece rate or an informally agreed-on bonus, could be made self enforcing if it is possible to generate a surplus from the employment.[6]

In contrast to these models, I assume short-term performance-based contracts are enforceable. I focus on the optimal mix of incentives, and show it depends on how the surplus is divided between the two parties. Since the agent has a finite life, the series of short-run contract is not stationary. Towards the end of the agent's "life", the expected future surplus from the relationship dwindles, and the termination threat becomes less effective. We show that even with short-run contracts and no "career concerns", the incentive wage and termination profiles are upward sloping. That is, as the agent approaches the end of her life, the probability of termination if the quality of output is unacceptable, and the wage when the quality is acceptable, both increase. Both the "stick" and the "carrot" are used more intensely for more senior workers.

To simplify the derivation of our results, we assume that the length of each period goes to zero and consider the model in continuous time. The literature on continuous

[6]Firm specific human capital, savings in costs of firing, reputations either on the side of the firm, where it will find it hard to attract workers if randomly fires its workers, or the agent who will again find it hard to find another job when dismissed, are some examples for generating surplus.

time principal-agent models was stimulated by the findings of Holmstrom and Milgrom (1987) who study a dynamic moral hazard problem in the absence of wealth effects or a changing economic environment. Their model generates a second-best solution and avoids the well-known drawback of its discrete-time counterpart where the first-order approach for the second-best solution may fail. Remarkably, their solution is simple: the optimal contract is linear in output. Among others, Schattler and Sung (1993), Sung (1995) and Ou-Yang (2003) further develop this result.

Later attempts to extend Holmstrom and Milgrom (1987) focus on calculating equilibria with wealth effects and changing economic conditions, which has been mostly addressed in discrete time formulations but presents a technical challenge for continuous time versions.[7] Sannikov (2006) created a continuous time model for which the solution can be characterized by an ordinary differential equation. DeMarzo and Sannikov (2004) show how this approach can work in an application to agency costs and capital structure. Cadenillas, Cvitanic, and Zapatero (2005) shows how the problem can be solved with full information, while Cvitanic and Zhang (2006) extend the Holmstrom and Milgrom (1987) model with adverse selection. Williams (2004) uses a general approach, including hidden savings, and is able to characterize the solution as a system of forward-backward stochastic differential equations. Westerfield (2006) develops an approach that uses the agent's continuation value as a state variable. However, these models do not deal with the commitment issue.

Only a few papers have studies short-term contracts in the continuous time limit. De Marzo and Sannikov (2006) study the optimal contract in a cash flow diversion model, but in their model there is adverse selection and the termination threat is used as a sorting mechanism. Our approach is more similar to Guriev and Kvasov (2005), who present a continuous time moral hazard problem where the contract is renegotiated at every point in time.

The presentation in this paper is organized as follows. In Section 2.2, we describe

[7] A significant number of works on continuous time principal agent models build on the vast literature of discrete-time long-term models. In two period models, Rogerson (1985) and Spear and Srivastava (1987) simplify the problem by using the agent's continuation value as a state variable, and Phelan and Townsend (1991) extend the dynamic analysis to many other types of problems.

the model with constant productivity and state our main theorem. Section 2.3 presents the proof of the main theorem. In Section 2.4, we make comparative statics analysis. Section 2.5 concludes.

2.2 The Model

There is one principal and one agent, both risk-neutral. The agent works during the time interval $[0,\, T]$. T can be thought of as an (exogenously determined) mandatory retirement date. We first set the model in discrete time, and then study the continuous-time limit of the discrete case, where the length of each period vanishes.

Suppose the length of each period in discrete time is Δ. Thus, there are T/Δ discrete periods during the interval $[0, T]$. The first period lasts from time 0 to time Δ, the second period from time Δ to time 2Δ, etc. With a slight abuse of terminology, we refer to the period which starts at time t as period t. Thus, period t lasts from time t to time $t + \Delta$. Payoffs received with one period's delay are discounted by the factor δ^{Δ}, where $\delta < 1$.

At the beginning of period t, the principal offers a short-term contract which is binding for this period only. As discussed below, the contract specifies the period's wage as a function of the quality of output produced during the period, and whether the agent is fired or can remain employed, also as function of the quality of the output. It is important to note that long-term contracts are not available, that is, the principal does not commit to a contract with the agent over the finite horizon.

If the agent accepts the contract at time t, then she chooses her effort level $e(t)$ for period t, normalized to either zero or one; $e(t) \in \{0,\, 1\}$. The agent's cost (disutility) of working hard ($e(t) = 1$) is $c\Delta$. (The cost of shirking is normalized to 0 without loss of generality.) The effort choice is unobservable to the principal. At the end of period t, output is realized with probability $\gamma\Delta$. The output is either high quality ("success") or low quality ("failure"). If output is realized, then the quality is observed by the principal.[8] The probability of success is determined by the agent's effort $e(t)$ during

[8]Equivalently, we could assume output is always realized at the end of each period, but the principal

period t. Specifically, the quality is high with probability $p_{e(t)}$ and low with probability $1 - p_{e(t)}$, where $0 < p_0 < p_1 < 1$. The value of high quality output to the principal is Y_H, and the value of low quality output is Y_L. Since output is realized with probability $\gamma\Delta$, and high effort raises the probability of high quality from p_0 to p_1, the principal considers high effort to be worth $y\Delta$, where

$$y \equiv \gamma(p_1 - p_0)\,(Y_H - Y_L)\,. \tag{2.1}$$

Thus, $y\Delta$ is the value of the expected increase in quality when effort is exerted, and y is interpreted as a measure of the agent's productivity. We will assume a lower bound on y:

Assumption 1 : $y \geq \frac{p_1 c}{p_1 - p_0}$

Assumption 1 guarantees that inducing effort is always profitable. To see the intuition, consider a hypothetical *one-shot* principal-agent game where the value of low effort is 0 and the value of high effort is y. If the principal pays nothing in case of failure, the minimum success wage she must pay to induce high effort is $w = c/(p_1 - p_0)$, and the principal's expected payoff is $y - p_1 w$ which is positive by Assumption 1. Our game is dynamic, but the conclusion is the same. That is, as long Assumption 1 holds, the principal finds it profitable to offer a contract that induces the agent to work hard (i.e., that satisfies the IC constraint derived below).

The period t contract specifies that at the end of period t, the agent receives a wage, which can depend on the quality of output observed this period. There is a limited liability constraint, so the wage must be non-negative. Without loss of generality, assume the principal does not pay anything when no output is realized, or when low output is realized (doing so would not be optimal). The period t contract also specifies whether or not the agent will be fired if low quality output is observed. (It will not be optimal to fire the agent when output is of high quality or if no output is realized.) Thus, a period t contract is a pair $(w(t), q(t)) \in \mathbb{R}^2_+$ where $w(t)$ is the "success wage"

---

only observes the quality with probability $\gamma\Delta$ (with probability $1 - \gamma\Delta$ no observation is made).

paid if high quality output is observed in period t, and $q(t) \in [0,1]$ is the probability that the agent is fired if low quality output is observed. Firing terminates the relationship between the principal and the agent: the agent will not be rehired.

If the agent is fired, she will earn her outside option $\bar{\omega}\Delta$ each period until time T. The value of the agent's outside option in period t is the discounted value of this future stream,

$$\bar{\omega}(t) = \sum_{j=1}^{J} \delta^{(j-1)\Delta} \bar{\omega}\Delta \tag{2.2}$$

where $J = T/\Delta - t$ is the number of remaining periods. For simplicity, we assume the principal cannot hire another agent before time T, and so will earn zero until time T if he fires the agent. We make one crucial assumption on the agent's outside option.

Assumption 2: $\bar{\omega} < \frac{p_0 c}{p_1 - p_0}$.

Assumption 2 will guarantee that the Individual Rationality (IR) constraint is not binding in equilibrium. To see the intuition consider again a hypothetical *one-shot* game where output is always realized. If the principal pays nothing for low quality, the minimum she must pay for high quality to induce high effort is $c/(p_1 - p_0)$, which in fact leaves the agent indifferent between exerting effort or not. Then if the agent accepts the contract, her expected payoff is $p_1 c/(p_1 - p_0) - c = \frac{p_0 c}{p_1 - p_0}$. If Assumption 2 holds, then this strictly exceeds $\bar{\omega}$ so the IR constraint is not binding. This will turn out to be the case also in our dynamic game. That is, if the principal pays a zero wage when the quality is low (or no output is realized), but if quality is high pays a wage sufficient to induce the agent to work hard, then Assumption 2 guarantees that the IR constraint is satisfied.

If the agent has not been fired before period t, then let $\pi(t)$ denote the principal's expected future discounted payoff, calculated at the beginning of time t. Recall that, under our assumptions, the principal always prefers to induce the agent to work hard. Therefore, at the end of period t, high quality output is observed with probability $p_1\gamma\Delta$, in which case the agent is paid $w(t)$. Low quality output is observed with probability $(1 - p_1)\gamma\Delta$, in which case the agent is fired with probability $q(t)$. The principal gets

zero until the end of time horizon if he fires the agent at the end of period t, but expects $\pi(t+\Delta)$ at the start of the next period, period $t+\Delta$, if the agent is not fired. Therefore, the principal's expected payoff is

$$\pi(t) = \Delta y - \gamma\Delta p_1 w(t) + [1 - q(t)(1-p_1)\gamma\Delta]\,\delta^{\Delta}\pi(t+\Delta) \tag{2.3}$$

The principal chooses the period t contract $(w(t), q(t))$ to maximize this expression subject to the agent's incentive compatibility (IC) and participation (IR) constraints. We assume y is large enough that $\pi(t) > 0$ holds for all t. Notice that at time t, the principal cannot influence $\pi(t+\Delta)$, because long-run contracts are not available.

Similarly, let $u(t)$ denote the agent's expected future discounted payoff, calculated at the beginning of period t, if she has not been fired so far. We derive the Incentive Compatibility (IC) constraint for period t. If she chooses high effort in period t, the agent's expected payoff is

$$u(t) = -c\Delta + \gamma\Delta p_1 w(t) + [1 - q(t)(1-p_1)\gamma\Delta]\,\delta^{\Delta}u(t+\Delta) + q(t)(1-p_1)\gamma\Delta\delta^{\Delta}\overline{\omega}(t+\Delta) \tag{2.4}$$

If instead she shirks in period t her expected payoff is

$$\gamma\Delta p_0 w(t) + [1 - q(t)(1-p_0)\gamma\Delta]\,\delta^{\Delta}u(t+\Delta) + q(t)(1-p_0)\gamma\Delta\delta^{\Delta}\overline{\omega}(t+\Delta) \tag{2.5}$$

The IC constraint says that the expression in equation (2.5) should not exceed the expression in equation (2.4). The IC constraint can be written as

$$w(t) + q(t)\delta^{\Delta}(u(t+\Delta)) - \overline{\omega}(t+\Delta)) \geq \frac{c/\gamma}{p_1 - p_0} \tag{2.6}$$

Now we derive the Individual Rationality (IR) constraint for period t. The IR constraint says that the incentive compatible contract must yield at least the agent's outside

option. Therefore, the IR constraint is

$$-c\Delta+\gamma\Delta p_1 w(t)+[1-q(t)(1-p_1)\gamma\Delta]\,\delta^{\Delta}u(t+\Delta))+q(t)(1-p_1)\gamma\Delta\delta^{\Delta}\overline{\omega}(t+\Delta)\geq\overline{\omega}(t) \tag{2.7}$$

Lemma 1 *If the IC constraint (2.6) is satisfied, then the IR constraint (2.7) holds with strict inequality.*

Proof. The IC constraint implies that $w(t)$ is at least

$$\frac{c/\gamma}{p_1-p_0}-q(t)\delta^{\Delta}(u(t+\Delta))-\overline{\omega}(t+\Delta))$$

Substituting this value of $w(t)$ into the left hand side of the IR constraint (2.7), we find that the left hand side of (2.7) becomes

$$\frac{p_0}{p_1-p_0}\Delta c+(1-q(t)\gamma\Delta)\delta^{\Delta}\left[u(t+\Delta))-\overline{\omega}(t+\Delta)\right]+\delta^{\Delta}\overline{\omega}(t+\Delta)$$

Note that, since IR must hold in every period, $u(t+\Delta)-\overline{\omega}(t+\Delta)\geq 0$. Moreover, $\overline{\omega}(t)=\bar{\omega}+\delta^{\Delta}\overline{\omega}(t+\Delta)$. Therefore, Assumption 2 guarantees that the left hand side of the IR constraint (2.7) strictly exceeds the right hand side. ■

Thus, from now on, we can disregard the IR constraint as it is always slack. In fact, that the IR constraint is slack is important. If the IR constraint holds with equality, then the agent is indifferent between staying or leaving the job. In this case, a termination threat does not really provide any incentive to work hard, so the principal may as well set $q(t)=0$. However, we are interested in how the principal trades off a stick (a termination threat) against a carrot (the wage) in providing incentives. For that reason, we are interested in the case where Assumption 2 holds, so that a termination threat is an effective incentive device.

Lemma 2 *For any t, the principal's optimal contract is such that the IC constraint holds with equality.*

Proof. If the IC constraint is slack, then either $q(t)>0$ or $w(t)>0$ or both. But the

principal can increase his profit by reducing $q(t)$ and/or $w(t)$ until there is equality in the IC constraint (2.6). Therefore, the IC constraint cannot be slack. ■

Thus, the IC constraint (2.6) binds in equilibrium:

$$w(t) = \frac{c/\gamma}{p_1 - p_0} - q(t)\delta^{\Delta}(u(t+\Delta)) - \overline{\omega}(t+\Delta)) \tag{2.8}$$

Substituting this $w(t)$ in equation (2.3), the principal's payoff at time t can be expressed as follows:

$$\pi(t) = \Delta\left[y - \frac{p_1 c}{p_1 - p_0}\right] + q(t)\gamma\Delta\delta^{\Delta}\Psi(t+\Delta) + \delta^{\Delta}\pi(t+\Delta) \tag{2.9}$$

where

$$\Psi(t) \equiv p_1\widehat{u}(t) - (1-p_1)\pi(t) \tag{2.10}$$

and $\widehat{u}(t) \equiv u(t) - \overline{\omega}(t) > 0$ denotes agent's *surplus* at time t.

Now we are ready to derive the optimal firing rule.

Proposition 3 *The optimal firing rule is as follows:*

$$q(t) = \begin{cases} 1 & \textit{if} \quad \Psi(t+\Delta) > 0 \\ 0 & \textit{if} \quad \Psi(t+\Delta) < 0 \\ [0,1] & \textit{if} \quad \Psi(t+\Delta) = 0 \end{cases}$$

Proof. The optimal firing rule is calculated by choosing $q(t)$ to maximize the principal's payoff, as given by equation (2.9). The proposition follows directly from equation (2.9). ■

The optimal firing rule requires a comparison of the benefit and cost of a termination threat. To see that, first observe in equation (2.8) that the success wage is decreasing in $q(t)$, i.e., holding cost of effort fixed, a higher punishment yields a lower success wage. In other words, there is substitutability between success wage and termination probability. The termination threat reduces the cost of satisfying the agent's IC constraint since effort can be induced by lowering $w(t)$ as much as $q(t)\widehat{u}(t+\Delta)$. In period t, the principal

is actually considering the *expected benefit* from firing, $p_1 q(t)\widehat{u}(t+\Delta)$, from imposing a nonzero probability of termination on the contract. On the other hand, the *expected cost* of imposing the termination threat is $(1-p_1)q(t)\pi(t+\Delta)$: since with probability $(1-p_1)q(t)$, the agent is fired when she fails and the principal loses the future stream of profits $\pi(t+\Delta)$. Therefore if the expected benefit of firing $p_1\widehat{u}(t)(t+\Delta)$ is greater than the expected cost $(1-p_1)\pi(t+\Delta)$, i.e. if $\Psi(t+\Delta)>0$, there are positive returns from imposing a nonzero termination probability. In that case, the optimal contract sets $q(t)=1$. If the costs are larger than the benefit, i.e. if $\Psi(t+\Delta)<0$, however, termination should not be used as an incentive device. The optimal contract sets $q(t)=0$ in that case. If the costs are equal to the benefit, the principal is indifferent between imposing termination threat or not. In sum, the expected *net benefit* from punishment, $\Psi(t+\Delta)$, determines the optimal level of punishment.

Notice that $\Psi(t+\Delta)>0$ is equivalent to

$$\widehat{u}(t) > \frac{1-p_1}{p_1}\pi(t).$$

Thus, Proposition 3 in fact captures one of the main insights of the model. Namely, the optimal short-run contract includes a termination threat if and only if the agent's expected future surplus from continuing the relationship, $\widehat{u}(t)$, is high enough relative to the principal's expected future surplus from it, $\pi(t)$.[9]

To further analyse the principal's problem, we assume Δ is very small. Otherwise, all formulas will depend on Δ in a complicated way, and apart from Proposition 3, it is hard to prove interesting results that are valid for all Δ. Thus, to avoid the complications, we take the limit as $\Delta \to 0$.

As $\Delta \to 0$ the IC condition (2.8) becomes

$$w(t) = -q(t)\widehat{u}(t) + \frac{c/\gamma}{p_1 - p_0} \tag{2.11}$$

[9]We have assumed for simplicity that the principal's outside option is worth zero, so there is no distinction between "surplus" and "payoff" for the principal. If instead the principal's outside option had been positive, we would (analogously to the agent) define the principal's surplus $\hat{\pi}(t)$ to be payoff from the relationship minus the outside option.

From equation (2.3), we calculate the time derivative of $\pi(t)$,[10]

$$\pi'(t) = \lim_{\Delta \to 0} \frac{\pi(t+\Delta) - \pi(t)}{\Delta} = \lim_{\Delta \to 0} [-y + \gamma p_1 w(t) + (\frac{1-\delta^{\Delta}}{\Delta} + \gamma(1-p_1)q(t)\delta^{\Delta})\pi(t+\Delta)]$$

$$= -y + \gamma p_1 w(t) + (r + \gamma(1-p_1)q(t))\pi(t) \tag{2.12}$$

$$= -y - \gamma p_1 q(t)\widehat{u}(t) + \frac{p_1 c}{p_1 - p_0} + (r + \gamma(1-p_1)q(t))\pi(t) \tag{2.13}$$

where the last equality uses equation (2.11), and where r is defined by $e^{-r} = \delta$.

Similarly, we calculate the time-derivative of $u(t)$, the agent's payoff from the incentive compatible contract. By using (2.11) to substitute for $w(t)$ in equation (2.4), we obtain

$$u'(t) = -\frac{p_0 c}{p_1 - p_0} + (\gamma q(t) + r)u(t) - \gamma q(t)\overline{\omega}(t)$$

From (2.2) we get that when $\Delta \to 0$,

$$\bar{\omega}(t) \to \frac{\bar{\omega}}{r}\left[1 - e^{-r(T-t)}\right] \tag{2.14}$$

From (2.14) we get

$$\bar{\omega}'(t) = -\bar{\omega} + r\bar{\omega}(t) \tag{2.15}$$

Finally, using (2.15) and definition of $\widehat{u}(t)$, we get

$$\widehat{u}'(t) = -\frac{p_0 c}{p_1 - p_0} + \overline{\omega} + (\gamma q(t) + r)\widehat{u}(t) \tag{2.16}$$

[10] We use the fact that $\lim_{\Delta \to 0} \frac{1-\delta^{\Delta}}{\Delta} = \lim_{\Delta \to 0}(-\delta^{\Delta} \ln \delta) = -\ln \delta$ and finally with a change of variable $\delta = e^{-r}$, we get $\lim_{\Delta \to 0} \frac{1-\delta^{\Delta}}{\Delta} = r$.

As $\Delta \to 0$, the optimal firing rule of Proposition 3 becomes:[11]

$$q(t) = \begin{cases} 1 & \text{if} \quad \Psi(t) > 0 \\ 0 & \text{if} \quad \Psi(t) < 0 \\ [0,1] & \text{if} \quad \Psi(t) = 0 \end{cases} \tag{2.17}$$

2.2.1 Dynamics of the Equilibrium

We have shown that the optimal contract at time t is determined by the sign of $\Psi(t)$. To understand the dynamics of the optimization problem, in this section we study the phase diagram in Figure 2.1, where for now, we assume $r = 0$. (We will show how $r > 0$ changes the dynamics in Figure 2.1 after analyzing the simplest case $r = 0$.)

In Figure 2.1, it is possible to see how a given point $(\pi(t), \hat{u}(t))$ determines the optimal firing rule at time t. Observe that $\Psi(t) = 0$ corresponds to the following line:

$$\widehat{u} = \frac{1 - p_1}{p_1}\pi \tag{2.18}$$

Then the firing rule, as expressed in equation (2.17), implies that on the line in (2.18), any $q(t) \in [0, 1]$ is optimal. Above the line, $\Psi(t) > 0$ holds, therefore $q(t) = 1$ is optimal; call this *region 1*. Below the line, $\Psi(t) < 0$ holds, therefore $q(t) = 0$ is optimal; call this *region 0*. In terms of Figure 2.1, the optimal dynamic contract corresponds to an *equilibrium path*; at time t the equilibrium path lies in *region* i if and only if $q(t) = i$ is optimal, and lies on the line $\widehat{u} = \frac{1-p_1}{p_1}\pi$ if and only if any $q(t) \in [0, 1]$ is optimal. Because the principal and agent earn nothing after time T, a necessary condition for the equilibrium path is:

$$(\pi(T), \widehat{u}(T)) = (0, 0).$$

Below we define what an equilibrium path implies in terms of Figure 2.1.

Definition 4 *A path* $(\pi(t), \hat{u}(t), q(t))$ *is an* equilibrium path *if it satisfies the terminal*

[11] The continuous time model has been derived by taking the limit $\Delta \to 0$ in the discrete time model. Alternatively, one could start with the continuous time model and derive the same optimal contract using continuous time dynamic programming.

condition $(\pi(T), \widehat{u}(T)) = (0, 0)$ *and if the following conditions hold for all* $t \in [0, T]$*:*

(i) $\pi'(t)$ *satisfies (2.13);*

(ii) $\hat{u}'(t)$ *satisfies (2.16);*

(iii) If $(\pi(t), \hat{u}(t))$ *lies in region 0, then* $q(t) = 0$;

(iv) If $(\pi(t), \hat{u}(t))$ *lies in region 1, then* $q(t) = 1$.

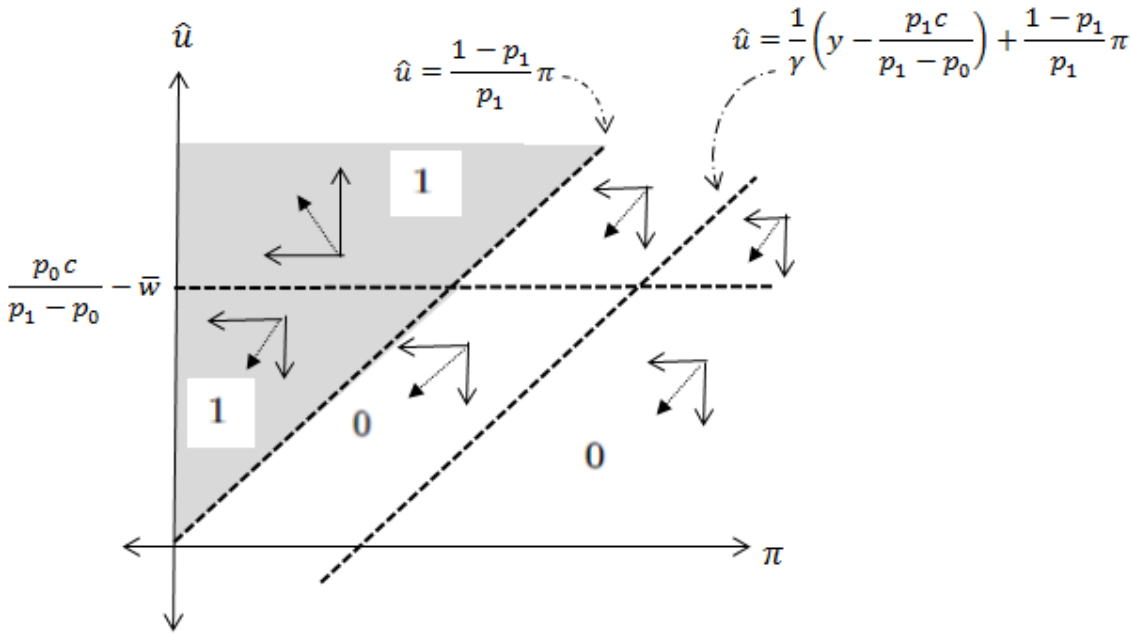


Figure 2.1: Dynamics of the Equilibrium ($r = 0$)

For simplicity, we suppress $q(t)$ and discuss the path of $(\pi(t), \hat{u}(t))$ in $(\pi, \hat{u})$ space. The optimal $q(t)$ is determined by $(\pi(t), \hat{u}(t))$ via conditions (iii) and (iv) in the definition of equilibrium path. Given any point $(\pi(t), \hat{u}(t))$ in region 0 or in region 1, we know $q(t)$ and thus we can find the motions $(\pi'(t), \hat{u}'(t))$ from the differential equations as derived in (2.13) and (2.16) after imposing $r = 0$.[12] These can be shown by a small vector attached to $(\pi(t), \hat{u}(t))$. If we do this for all points, we can join successive points in $(\pi, \hat{u})$ space to determine the whole path.

In Figure 2.1, the motions of $\pi(t)$ and $\hat{u}(t)$ are illustrated by small vector arrows,

[12]Some care is required on the line $\widehat{u} = \frac{1-p_1}{p_1}\pi$ since there, $q(t)$ can be anything between 0 and 1.

where a horizontal arrow points the direction of the motion of $\pi(t)$ and a vertical arrow points the direction of the motion of $\hat{u}(t)$. Then for *region 0*, i.e., below the line $\widehat{u} = \frac{1-p_1}{p_1}\pi$, the motions $\pi'(t)$ and $\hat{u}'(t)$ are obtained by imposing $q(t) = 0$ into the differential equations:

$$\begin{aligned} \pi'(t) &= -y + \frac{p_1 c}{p_1 - p_0} \\ \widehat{u}'(t) &= -\frac{p_0 c}{p_1 - p_0} + \overline{\omega} \end{aligned}$$

Assumptions 1 and 2 imply $\pi'(t) < 0$ and $\widehat{u}'(t) < 0$. Therefore both $\pi(t)$ and $\hat{u}(t)$ are decreasing in this region. Similarly in *region 1,* i.e., below the line $\widehat{u} = \frac{1-p_1}{p_1}\pi$, the motions are obtained by imposing $q(t) = 1$:

$$\pi'(t) = -y - \gamma p_1 \widehat{u}(t) + \frac{p_1 c}{p_1 - p_0} + \gamma(1 - p_1)\pi(t) \tag{2.19}$$

$$\hat{u}'(t) = -\frac{p_0 c}{p_1 - p_0} + \gamma \hat{u}(t) + \overline{\omega} \tag{2.20}$$

Equation (2.19) implies that the line

$$\hat{u} = \frac{1}{\gamma}(y - \frac{p_1 c}{p_1 - p_0}) + \frac{1 - p_1}{p_1}\pi,$$

which is parallel to the line in equation (2.18), determines the direction of the motion of $\pi(t)$; above (below) it we have $\pi'(t) < 0$ ($\pi'(t) > 0$), i.e. $\pi(t)$ is decreasing (increasing). Observe that this line lies in *region 0,* which means that in *region 1*, $\pi'(t) < 0$ always holds. The direction of the motion of $\hat{u}(t)$, however, is determined by the line

$$\hat{u} = \frac{1}{\gamma}(\frac{p_0 c}{p_1 - p_0} - \overline{\omega}).$$

Equation (2.20) implies $\hat{u}(t)$ is decreasing if $\hat{u} < \frac{1}{\gamma}(\frac{p_0 c}{p_1 - p_0} - \overline{\omega})$, increasing otherwise.

Hence given any point $(\pi(t), \hat{u}(t))$ in regions 0 or 1, the motion vectors of $\pi(t)$ and $\hat{u}(t)$ determine the subsequent change to proceed along the path passing through this point. In Figure 2.1, we observe that given any point $(\pi(t), \hat{u}(t))$, the motion of the path

is towards "south-east", except when $(\pi(t), \hat{u}(t))$ is in *region 1* and $\hat{u} > \frac{1}{\gamma}(\frac{p_0 c}{p_1 - p_0} - \overline{\omega})$. To determine which paths could constitute an equilibrium, first recall that the equilibrium path must satisfy the terminal condition. Therefore our equilibrium analysis requires characterizing the paths which are convergent to the origin in Figure 2.1. This implies that the path pointing to the "north-west" cannot be an equilibrium. Any path with motion pointing at "south-east" is a candidate for an equilibrium unless it crosses to region 1 when $\hat{u} > \frac{1}{\gamma}(\frac{p_0 c}{p_1 - p_0} - \overline{\omega})$. Because in that case, it would not converge to the origin.

With the help of phase diagram, we have studied the simplest case, $r = 0$. Now we consider the case where $r > 0$. Let $\Pi(t) \equiv e^{-rt}\pi(t)$ and $\hat{U}(t) \equiv e^{-rt}\hat{u}(t)$ denote the *present values* of π and u. In $(\Pi, \hat{U})$ space, the phase diagram (with $r > 0$) looks exactly like Figure 2.1, simply replacing π by Π on the horizontal axis and $\hat{u}$ by $\hat{U}$ on the vertical axis. See Figure 2.2. To see why this is true, first notice that $\hat{U}(t) = \frac{1-p_1}{p_1}\Pi(t)$ if and only if $\hat{u}(t) = \frac{1-p_1}{p_1}\pi(t)$. Therefore, just as before, the line

$$\hat{U} = \frac{1 - p_1}{p_1}\hat{\Pi}$$

separates Figure 2.2 into two regions, *region 0* where $q = 0$ is optimal and *region 1* where $q = 1$ is optimal. Second, when $q = 0$, the motions of $\widehat{u}(t)$ and $\pi(t)$ follow from

$$\begin{aligned} \pi'(t) &= -y + \frac{p_1 c}{p_1 - p_0} + r\pi(t) \\ \widehat{u}'(t) &= -\frac{p_0 c}{p_1 - p_0} + \overline{\omega} + r\widehat{u}(t) \end{aligned} \tag{2.21}$$

These equations imply

$$\begin{aligned} \Pi'(t) &= e^{-rt}\left(\pi'(t) - r\pi(t)\right) = e^{-rt}\left(-y + \frac{p_1 c}{p_1 - p_0}\right) \\ \hat{U}'(t) &= e^{-rt}\left(\widehat{u}'(t) - r\hat{u}(t)\right) = e^{-rt}\left(-\frac{p_0 c}{p_1 - p_0} + \overline{\omega}\right) \end{aligned}$$

Therefore,

$$\frac{\Pi'(t)}{\hat{U}'(t)} = \frac{\pi'(t)}{\hat{u}'(t)} \tag{2.22}$$

so the phase diagram in *region 0* is exactly the same in Figure 2.2 as in Figure 2.1.

Similarly when $q = 1$, the motions of $\widehat{u}(t)$ and $\pi(t)$ follow from

$$\begin{aligned} \widehat{u}'(t) &= -\frac{p_0 c}{p_1 - p_0} + \overline{\omega} + (\gamma + r)\widehat{u}(t) \\ \pi'(t) &= -y - \gamma p_1 \widehat{u}(t) + \frac{p_1 c}{p_1 - p_0} + (r + \gamma(1 - p_1))\pi(t) \end{aligned} \tag{2.23}$$

These equations imply

$$\begin{aligned} \Pi'(t) &= e^{-rt}\left(\pi'(t) - r\pi(t)\right) = e^{-rt}\left(-y - \gamma p_1 \widehat{u}(t) + \frac{p_1 c}{p_1 - p_0} + \gamma(1 - p_1)\pi(t)\right) \\ \hat{U}'(t) &= e^{-rt}\left(\widehat{u}'(t) - r\hat{u}(t)\right) = e^{-rt}\left(-\frac{p_0 c}{p_1 - p_0} + \overline{\omega} + \gamma \widehat{u}(t)\right) \end{aligned}$$

Therefore, (2.22) again holds: the phase diagram in *region 1* in Figure 2.2 is identical to the one in Figure 2.1.

Thus, the phase diagrams in Figure 2.1 (the case $r = 0$) and Figure 2.2 (the case $r > 0$) are identical, except for the labelling of the axes. Clearly, the terminal condition is also the same, $(\Pi(T), \hat{U}(T)) = (0, 0)$.

The phase diagram analysis ruled out paths that do not converge to the origin. However, it remains to characterize the equilibrium path. To complete our analysis, in the next section, we derive the explicit equations of the paths and find necessary conditions for a unique equilibrium path.

2.2.2 The Equilibrium Path: Main Theorem

In this section, using insights developed in the previous section, we investigate under what conditions the equilibrium path lies in *region i* at t and whether it lies in a given region for all t, or it crosses different regions over the contract horizon. We will show that there always exists a unique equilibrium path, which characterizes the sequence of optimal short-run contracts.

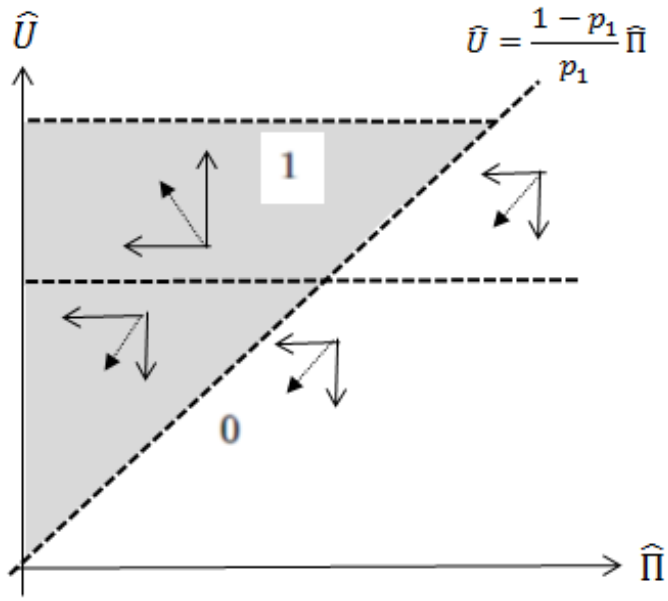


Figure 2.2: Dynamics of the Equilibrium ($r > 0$)

The optimal short-run contract optimally combines the two incentive tools, termination threat and success wage, and different circumstances will require different combinations. Will the principal prefer to use a termination threat (a "stick") or a high success wage (a "carrot") to provide incentives? From the agent's point of view, a termination threat provides only weak incentives if the agent does not expect to gain a lot from the relationship in the future. But if the relationship gives the agent a payoff which greatly exceeds her outside option, then the termination threat provides powerful incentives to work hard. From the principal's point of view, the advantage of the termination threat is that it stimulates effort even with a low success wage. In other words, the principal can reduce the compensation costs when termination threat is an available contracting tool. The drawback is that, if the agent is fired, the principal loses his future gains from the relationship.

This reasoning suggests that whether or not a termination threat is optimal depends on the relative sizes of the principal's and the agent's future gains. If the principal expects to gain a lot from the relationship but the agent thinks the relationship is not much better than her outside option, then the principal will not use a termination threat

(i.e. $q = 0$ at the optimum). If instead it is the agent who expects to gain a lot from the relationship (compared to her outside option), then the principal will use a termination threat (i.e. $q = 1$ at the optimum). More precisely, the key point is what fraction of the surplus generated by the relationship goes to the agent and the principal, respectively. This basic insight was already captured by Proposition 3. But Proposition 3 derived the optimal firing rule as a function of the endogenously determined payoffs $\widehat{u}(t)$ and $\pi(t)$. We will now consider how the optimal firing rule depends on the exogenously given parameters of the model. For this, we need to know how $\widehat{u}(t)$ and $\pi(t)$ depend on these parameters.

The principal's and the agent's payoffs in the dynamic relationship turn out to be closely connected to their payoffs in a hypothetical *one-shot* game with the same parameters as in our model. In the one-shot game, the principal gains y from high effort, and pays a success wage $\frac{c}{p_1 - p_0}$ to provide incentives. The principal's surplus from this one-shot game would be

$$S_P \equiv y - \frac{p_1 c}{p_1 - p_0}$$

while the agent's surplus (net of outside option) would be

$$S_A \equiv \frac{p_0 c}{p_1 - p_0} - \bar{\omega}$$

The total one-shot surplus would be

$$S \equiv y - c - \bar{\omega} = \underbrace{\left(y - \frac{p_1 c}{p_1 - p_0}\right)}_{S_P} + \underbrace{\left(\frac{p_0 c}{p_1 - p_0} - \bar{\omega}\right)}_{S_A} \tag{2.24}$$

Notice that S, S_P and S_A depend only on exogenously given parameters $(y, c, p_0, p_1, \bar{\omega})$. It turns out that in our *dynamic* model, these *one-shot* surpluses play a key role. Define

$$K(T) \equiv \frac{r + (1 - p_1)\gamma}{r + \gamma} \frac{1 - e^{-(r+\gamma)T}}{1 - e^{-(r+(1-p_1)\gamma)T}} \tag{2.25}$$

It is shown in the next section that $K(T) < 1$ for any $T > 0$. Our main result is the following.

Theorem 5 *(i) If*

$$S_A < (1 - p_1)S \tag{2.26}$$

then $q(t) = 0$ *and* $w'(t) = 0$ *for all* $t \in [0, T]$. *(ii) If*

$$S_A > \frac{1 - p_1}{K(T)} S \tag{2.27}$$

then $q(t) = 1$ *and* $w'(t) > 0$ *for all* $t \in [0, T]$. *(iii) If*

$$(1 - p_1)S < S_A < \frac{1 - p_1}{K(T)} S \tag{2.28}$$

then there is $t^* > 0$ *such that* $0 < q(t) < 1$, $q'(t) > 0$ *and* $w'(t) = 0$ *for* $t < t^*$, *and* $q(t) = 1$ *and* $w'(t) > 0$ *for* $t > t^*$.

The proof is in the next section, but here we provide an intuitive justification of Theorem 5. It can be shown that (2.26) holds if and only if productivity y exceeds some upper bound. Observe that in (2.10), it is only the principal's payoff, $\pi(t)$, which depends on y, and $\pi(t)$ is increasing in y. Since $\pi(t)$ directly determines the cost of imposing a termination threat, higher y means higher costs of termination threat. Therefore, when productivity y is large enough, the cost of imposing threat outweighs its benefit at all times, so optimality requires $q(t) = 0$ over the contract horizon. The best incentive device to induce high effort in that case is wage compensation: the optimal contract always pays the "efficiency wage" $w(t) = \frac{c/\gamma}{p_1 - p_0}$. Thus, if y is sufficiently high, optimal contract is static in terms of both devices where it pays a constant stream of wage combined with zero termination threat at all times. In other words, condition (2.26) defines an upper bound for the agent's share in total surplus: the optimal contract never imposes a positive termination threat if the agent's surplus in the one shot game is sufficiently low.

It can also be shown that condition (2.27) holds if and only if productivity y is below

some lower bound. If y is sufficiently low, costs of imposing a termination threat are low as compared to its benefits at all times. Then, at the optimum, the principal offers $q(t) = 1$ over the entire contract horizon. From the IC condition, optimal wage is set as low as possible at all times, $w(t) = -\hat{u}(t) + \frac{c/\gamma}{p_1 - p_0}$. Observe that when (2.27) holds, since $\hat{u}(t)$ is decreasing, optimal wage scheme is increasing through time: high effort could be implemented only by a success wage which increases over time. The reason is that as seniority increases, the agent's loss from getting fired decreases. Then incentives to work hard decreases through time. But termination threat cannot be adjusted to the agent's decreasing incentives simply because it is set at its maximum $q(t) = 1$ from the beginning. Therefore the only way to provide stronger incentives is to offer higher success wage as the agent gets older.

If condition (2.28) holds, productivity y is in a moderate range. Recall that $\pi(t)$ is the present value of future stream of profits from the relationship at time t. Therefore, since $\pi(t)$ determines the cost of imposing termination threat, we conclude that the cost is decreasing as the contract approach to the end of horizon. When productivity y is in a moderate range, costs are high at the start of contract horizon where the contract offers relatively lower termination threat. However as we move in time, the cost to the principal from losing his agent, decreases. Then the optimal contract gradually increases the termination threat while it keeps offering a fixed success wage. Finally the contract switches to severe threat regime when the costs from punishment decreases significantly at a critical t^*. In the rest of contract horizon, severe threat regime necessitates increasing success wage profile in order to, as explained above, give sufficient incentives to induce effort as the agent gets closer to the end of her career.

2.3 Proof of Theorem 5

We know that the optimal $q(t)$ is determined by the sign of Ψ, defined in (2.10). The evolution of Ψ is determined by

$$\Psi'(t) = p_1 \widehat{u}'(t) - (1 - p_1)\pi'(t) \tag{2.29}$$

If $q(t) = 0$ (respectively $q(t) = 1$), then the law of motions of $\widehat{u}(t)$ and $\pi(t)$ are defined in equations (2.21) (respectively (2.23)). This can be used to derive the evolution of Ψ under the hypothesis that $q(t) = 0$ (respectively $q(t) = 1$) for all t. We first consider if there can exist an equilibrium such that $q(t) = 0$ for all t.

2.3.1 Equilibrium path with $q(t) = 0$ for all t

If $q(t) = 0$ for all t, then $\widehat{u}(t)$ and $\pi(t)$ evolve according to equations (2.21), and we also know the terminal condition $\widehat{u}(T) = \pi(T) = 0$. By solving these differential equations, we find $\widehat{u}(t)$ and $\pi(t)$, under the hypothesis $q(t) = 0$. This will give us Ψ, by (2.10). For the sake of clarity, we will let Ψ_0 denote Ψ which has been obtained in this way, i.e., by assuming $q(t) = 0$ for all t. Notice that $\Psi_0(t)$ is the solution to the differential equation (2.29), if we use (2.21) to substitute for $\widehat{u}'(t)$ and $\pi'(t)$, and with the terminal condition $\Psi_0(T) = 0$. To verify that $(\widehat{u}(t), \pi(t))$ is truly an equilibrium path, since it was obtained under the hypothesis that $q(t) = 0$, we need to verify that $q(t) = 0$ is optimal. That is, the path must lie in *region 0*. From the analysis so far, we know that this requires $\Psi_0(t) < 0$ for all t, since this is the condition for $q(t) = 0$ to be optimal. Thus, our strategy is to first solve for $\Psi_0(t)$ and then check if $\Psi_0(t) < 0$ for all t.

Proposition 6 *The explicit solution for Ψ_0 is given by*

$$\Psi_0(t) = \frac{1}{r}(-(1-p_1)y - p_1\overline{\omega} + \frac{p_1(1-p_1+p_0)}{p_1-p_0}c)(1-e^{-r(T-t)}) \tag{2.30}$$

Proof. To derive $\Psi_0(t)$, we use the motions of $\widehat{u}(t)$ and $\pi(t)$ as expressed in equation (2.21) and find the general forms of $\widehat{u}(t)$ and $\pi(t)$. For some constants $D_0, D_1 > 0$,

$$\begin{aligned} u(t) &= \frac{p_0 c/r}{p_1 - p_0} + D_0 e^{rt} \\ \pi(t) &= \frac{1}{r}(y - \frac{p_1 c}{p_1 - p_0}) + D_1 e^{rt} \end{aligned}$$

The terminal conditions $\widehat{u}(T) = \pi(T) = 0$ fixes D_0 and D_1:

$$\begin{aligned} D_0 &= -\frac{p_0 c/r}{p_1 - p_0} e^{-rT} \\ D_1 &= \frac{1}{r}(y - \frac{p_1 c}{p_1 - p_0}) e^{-rT} \end{aligned}$$

Then

$$\begin{aligned} \widehat{u}(t) &= \frac{1}{r}(\frac{p_0 c}{p_1 - p_0} - \overline{\omega})(1 - e^{-r(T-t)}) \\ \pi(t) &= \frac{1}{r}(y - \frac{p_1 c}{p_1 - p_0})(1 - e^{-r(T-t)} \end{aligned}$$

Therefore, using the above expressions for $\widehat{u}(t)$ and $\pi(t)$, we get $\Psi_0(t)$:

$$\begin{aligned} \Psi_0(t) &= p_1 \widehat{u}(t) - (1 - p_1)\pi(t) \\ &= \frac{1}{r}(-(1 - p_1)y - p_1 \overline{\omega} + \frac{p_1(1 - p_1 + p_0)}{p_1 - p_0} c)(1 - e^{-r(T-t)}) \end{aligned}$$

■

Observe that by construction, we have $\Psi_0(T) = 0$. Also note that $\Psi_0(0) < 0$, and $\Psi_0'(t) > 0$ for all t, if

$$\frac{p_1(1 - p_1 + p_0)}{(1 - p_1)(p_1 - p_0)} c - \frac{p_1}{1 - p_1} \overline{\omega} < y \tag{2.31}$$

Therefore, since $\Psi_0(T) = 0$, we conclude that $\Psi_0(t) < 0$ for all t, i.e. the path $(\hat{u}(t), \pi(t))$ lies in *region 0*, if the condition in (2.31) holds. In Figure 2.3a this path is illustrated with a solid path on the phase diagram. By construction, there cannot be another such convergent path that always lies in *region 0*. Noticing that if (2.31) holds, $\Psi_0''(t) < 0$ for all t, in Figure 2.3b we illustrate these findings for $\Psi_0(t)$ with a solid curve. However if

$$y < \frac{p_1(1 - p_1 + p_0)}{(1 - p_1)(p_1 - p_0)} c - \frac{p_1}{1 - p_1} \overline{\omega}$$

then $\Psi_0(0) > 0$ and $\Psi_0'(t) < 0$ for all t. These, together with $\Psi_0(T) = 0$ imply that $\Psi_0(t) > 0$ for all t; such path is illustrated with a dashed path in Figure 2.3a and with

a dashed path in Figure 2.3b. Recall that $\Psi_0(t)$ is derived under the assumption that $q(t) = 0$ for all t. But $\Psi_0(t) > 0$ means $q(t) = 0$ is not optimal, so this does not give us an equilibrium path. In conclusion, when y is sufficiently high, i.e. when (2.31) holds, we have found a unique equilibrium path as expressed in equation (2.30), which lies in *region 0* at all times.

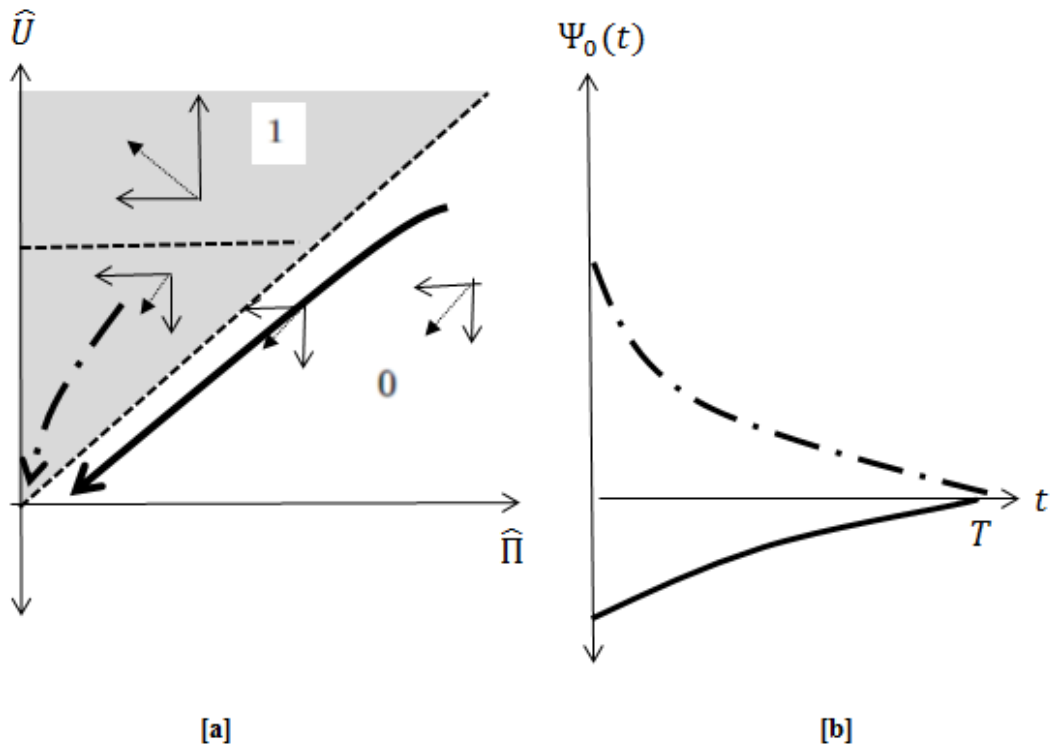


Figure 2.3: Equilibrium Path with $q(t) = 0$ for all t

2.3.2 Equilibrium path with $q(t) = 1$ for all t

If $q(t) = 1$ for all t, then $\widehat{u}(t)$ and $\pi(t)$ evolve according to equations (2.23), and we also know the terminal condition $\widehat{u}(T) = \pi(T) = 0$. By solving these differential equations, we find $\widehat{u}(t)$ and $\pi(t)$, under the hypothesis $q(t) = 1$. This will give us Ψ, by (2.10). For the sake of clarity, we will let Ψ_1 denote Ψ which has been obtained in this way, i.e., by assuming $q(t) = 1$ for all t. Notice that $\Psi_1(t)$ is the solution to the differential equation (2.29), if we use (2.23) to substitute for $\widehat{u}'(t)$ and $\pi'(t)$, and with the terminal

condition $\Psi_1(T) = 0$. To verify that $(\widehat{u}(t), \pi(t))$ is truly an equilibrium path, we need to verify that $q(t) = 1$ is optimal. That is, the path must lie in *region 1*. From the analysis so far, we know that this requires $\Psi_1(t) > 0$ for all t. Thus, our strategy is to first solve for $\Psi_1(t)$ and then check if $\Psi_1(t) > 0$ for all t.

Proposition 7 *The explicit solution for Ψ_1 is given by*

$$\Psi_1(t) = \frac{1}{\gamma + r}(\frac{p_0 c}{p_1 - p_0} - \overline{\omega})(1 - e^{-(r+\gamma)(T-t)}) - \frac{(1-p_1)(y - c - \overline{\omega})}{r + (1-p_1)\gamma}(1 - e^{-(r+(1-p_1)\gamma)(T-t)}) \tag{2.32}$$

Proof. To derive $\Psi_1(t)$ we use the of motions of $\widehat{u}(t)$ and $\pi(t)$ as defined by equation (2.23)

$$\widehat{u}'(t) = -\frac{p_0 c}{p_1 - p_0} + \overline{\omega} + (\gamma + r)\widehat{u}(t) \tag{2.33}$$

$$\pi'(t) = -y - \gamma p_1 \widehat{u}(t) + \frac{p_1 c}{p_1 - p_0} + (r + \gamma(1 - p_1))\pi(t) \tag{2.34}$$

Using equation (2.33), start with the general form, for some constant $M_0 > 0$:

$$\hat{u}(t) = \frac{1}{\gamma + r}(\frac{p_0 c}{p_1 - p_0} - \overline{\omega}) + M_0 e^{(r+\gamma)t} \tag{2.35}$$

Then equation (2.34) becomes

$$\pi'(t) = -y - \frac{\gamma p_1}{\gamma + r}(\frac{p_0 c}{p_1 - p_0} - \overline{\omega}) - \gamma p_1 M_0 e^{(r+\gamma)t} + \frac{p_1 c}{p_1 - p_0} + (r + \gamma(1 - p_1))\pi(t)$$

Therefore

$$\pi'(t) - (r + \gamma(1 - p_1))\pi(t) = -y - \frac{\gamma p_1}{\gamma + r}(\frac{p_0 c}{p_1 - p_0} - \overline{\omega}) + \frac{p_1 c}{p_1 - p_0} - \gamma p_1 M_0 e^{(r+\gamma)t}$$

Then the form of $\pi(t)$ can be derived by using the general form:

$$\pi(t) = \frac{1}{r + (1-p_1)\gamma}(y + \frac{\gamma p_1}{\gamma + r}(\frac{p_0 c}{p_1 - p_0} - \overline{\omega}) - \frac{p_1 c}{p_1 - p_0}) + M_0 e^{(r+\gamma)t} + M_1 e^{(r+(1-p_1)\gamma)t} \tag{2.36}$$

where M_0 and M_1 are the constants to be determined. We first impose $\widehat{u}(T) = 0$ in equation (2.35) and get

$$M_0 = -\frac{1}{\gamma + r}(\frac{p_0 c}{p_1 - p_0} - \overline{\omega})e^{-(r+\gamma)T}$$

Therefore the expression in equation (2.35) becomes

$$\widehat{u}(t) = \frac{1}{\gamma + r}(\frac{p_0 c}{p_1 - p_0} - \overline{\omega})(1 - e^{-(r+\gamma)(T-t)}) \tag{2.37}$$

Using (2.37), the expression in (2.36) becomes

$$\begin{aligned}\pi(t) &= \frac{1}{r + (1-p_1)\gamma}(y + \frac{\gamma p_1}{\gamma + r}(\frac{p_0 c}{p_1 - p_0} - \overline{\omega}) - \frac{p_1 c}{p_1 - p_0}) - \\ &\quad \frac{1}{\gamma + r}(\frac{p_0 c}{p_1 - p_0} - \overline{\omega})e^{-(r+\gamma)(T-t)} + M_1 e^{(r+(1-p_1)\gamma)t}\end{aligned}$$

Once again the terminal condition $\pi(T) = 0$ fixes M_1 :

$$M_1 = (-\frac{1}{r + (1-p_1)\gamma}(y + \frac{\gamma p_1}{\gamma + r}(\frac{p_0 c}{p_1 - p_0} - \overline{\omega}) - \frac{p_1 c}{p_1 - p_0}) - \frac{1}{\gamma + r}(\frac{p_0 c}{p_1 - p_0} - \overline{\omega}))e^{-(r+(1-p_1)\gamma)T}$$

Finally, the expression for $\pi(t)$ in equation (2.36) will be

$$\begin{aligned}\pi(t) &= \frac{1 - e^{-(r+(1-p_1)\gamma)(T-t)}}{r + (1-p_1)\gamma}(y + \frac{\gamma p_1}{\gamma + r}(\frac{p_0 c}{p_1 - p_0} - \overline{\omega}) - \frac{p_1 c}{p_1 - p_0}) - \\ &\quad \frac{(e^{-(r+\gamma)(T-t)} + e^{-(r+(1-p_1)\gamma)(T-t)})}{\gamma + r}(\frac{p_0 c}{p_1 - p_0} - \overline{\omega})\end{aligned}$$

Then, after some simplifications, $\Psi_1(t)$ can be written as

$$\begin{aligned}\Psi_1(t) &= p_1 \hat{u}(t) - (1-p_1)\pi(t) \\ &= \frac{1}{\gamma + r}(\frac{p_0 c}{p_1 - p_0} - \overline{\omega})(1 - e^{-(r+\gamma)(T-t)}) - \frac{(1-p_1)(y - c - \overline{\omega})}{r + (1-p_1)\gamma}(1 - e^{-(r+(1-p_1)\gamma)(T-t)})\end{aligned}$$

■

We will derive under what conditions $\Psi_1(t) > 0$ is required for an equilibrium path with $q(t) = 1$.

Recall that $K(T)$ was defined in equation (2.25). We need the following lemma.

Lemma 8 *$K(T) < 1$ for all $T > 0$.*

Proof. Consider

$$K'(T) = \frac{(r + (1-p_1)\gamma)e^{-(r+(1-p_1)\gamma)T}}{(r+\gamma)(1-e^{-(r+(1-p_1)\gamma)T})^2}((r+\gamma)e^{-\gamma p_1 T} - \gamma p_1 e^{-(r+\gamma)T} - (r + (1-p_1)\gamma))$$

It can be verified that

$$(r+\gamma)e^{-\gamma p_1 T} - \gamma p_1 e^{-(r+\gamma)T} - (r + (1-p_1)\gamma) < 0$$

for all $T > 0$, because the expression is decreasing in T and equal to 0 at $T = 0$. Then $K'(T) < 0$. Now observe that

$$\frac{1-e^{-(r+\gamma)T}}{1-e^{-(r+(1-p_1)\gamma)T}} \to \frac{r+\gamma}{r+(1-p_1)\gamma}$$

as $T \to 0$. This implies $K(T) \to 1$ as $T \to 0$. Therefore, since $K'(T) < 0$, we conclude that $K(T) < 1$ for all $T > 0$. ■

At this point, it is convenient to change variables and solve the differential equations "backwards". Therefore, let $\tau = T - t$ denote the time left before the horizon, and define $\Phi(\tau) \equiv \Psi_1(T - \tau)$. Explicitly,

$$\Phi(\tau) \equiv \frac{1}{\gamma + r}(\frac{p_0 c}{p_1 - p_0} - \overline{\omega})(1 - e^{-(r+\gamma)\tau}) - \frac{(1-p_1)(y - c - \overline{\omega})}{r + (1-p_1)\gamma}(1 - e^{-(r+(1-p_1)\gamma)\tau})$$

It turns out that, in order to check whether $\Psi_1(t) > 0$ holds, a very convenient method is to change variable from t to τ and study if it is true that $\Phi(\tau) > 0$. We need a preliminary result.

Lemma 9 *If $\tau^* > 0$ is such that $\Phi'(\tau^*) = 0$ then $\Phi''(\tau^*) < 0$, hence there can exist at most one such τ^*.*

Proof. Differentiating, we find that

$$\Phi'(\tau) = (\frac{p_0 c}{p_1 - p_0} - \overline{\omega})e^{-(r+\gamma)\tau} - (1-p_1)(y - c - \overline{\omega})e^{-(r+(1-p_1)\gamma)\tau}$$

and

$$\Phi''(\tau) = -\left[(r+\gamma)(\frac{p_0 c}{p_1 - p_0} - \overline{\omega})e^{-(r+\gamma)\tau} - (r+(1-p_1)\gamma)(1-p_1)(y - c - \overline{\omega})e^{-(r+(1-p_1)\gamma)\tau}\right]$$

It follows that if $\Phi'(\tau^*) = 0$ then

$$\Phi''(\tau^*) = -p_1\gamma(\frac{p_0 c}{p_1 - p_0} - \overline{\omega})e^{-(r+\gamma)\tau} < 0$$

■

Finally, Proposition 10 characterizes the conditions which determine the sign of $\Phi(\tau)$.

Proposition 10 *(i) If*

$$y < (\frac{K(T)p_0}{(1-p_1)(p_1-p_0)} + 1)c + (1 - \frac{K(T)}{1-p_1})\overline{\omega} \tag{2.38}$$

then $\Phi(\tau) > 0$ *for all* $\tau \in [0, T]$.

(ii) If

$$(\frac{K(T)p_0}{(1-p_1)(p_1-p_0)} + 1)c + (1 - \frac{K(T)}{1-p_1})\overline{\omega} < y < \frac{p_1(1-p_1+p_0)}{(1-p_1)p_1 - p_0}c - \frac{p_1}{1-p_1}\overline{\omega} \tag{2.39}$$

then there exists τ^* *such that* $\Phi(\tau) > 0$ *for all* $\tau \in [0, \tau^*)$ *and* $\Phi(\tau) < 0$ *for all* $\tau \in (\tau^*, T]$.

(iii) If

$$y > \frac{p_1(1-p_1+p_0)}{(1-p_1)p_1 - p_0}c - \frac{p_1}{1-p_1}\overline{\omega} \tag{2.40}$$

then $\Phi(\tau) < 0$ *for all* $\tau \in [0, T]$.

Proof. It can be checked that $\Phi(0) = 0$, that $\Phi(T) > 0$ is equivalent to (2.38), and that $\Phi'(0) > 0$ is equivalent to

$$y < \frac{p_1(1 - p_1 + p_0)}{(1 - p_1)p_1 - p_0}c - \frac{p_1}{1 - p_1}\overline{\omega} \tag{2.41}$$

(i) Notice that (2.38) implies (2.41) by Lemma 8. Hence, if (2.38) holds, then $\Phi(0) = 0$, $\Phi'(0) > 0$ and $\Phi(T) > 0$. Now Lemma 9 implies $\Phi(\tau) > 0$ for all $\tau \in [0, T]$.

(ii) If (2.39) holds, then $\Phi(0) = 0$, $\Phi'(0) > 0$ and $\Phi(T) < 0$. Now Lemma 9 implies there exists τ^* such that $\Phi(\tau) > 0$ for all $\tau \in [0, \tau^*)$ and $\Phi(\tau) < 0$ for all $\tau \in (\tau^*, T]$.

(iii) Notice that (2.40) implies (2.38) is violated, by Lemma 8. Hence, if (2.40) holds, then $\Phi(0) = 0$, $\Phi'(0) < 0$ and $\Phi(T) < 0$. Now Lemma 9 implies $\Phi(\tau) < 0$ for all $\tau \in [0, T]$. ■

Since $t = T - \tau$ and $\Phi(\tau) \equiv \Psi_1(T - \tau)$, it directly follows from Proposition 10 that $\Psi_1(t) > 0$ for all t if (2.38) holds but $\Psi_1(t) < 0$ for all t if (2.40) holds. However if (2.39) holds, there exists $t^* \equiv T - \tau^*$ such that $\Psi_1(t) < 0$ for all $t < t^*$, and $\Psi_1(t) > 0$ for all $t > t^*$. So we have found another equilibrium path, which lies in *region 1* at all times as far as (2.38) holds. In Figure 2.4a this is illustrated with a solid path on the phase diagram. Using Lemma 9, in Figure 2.4b, we summarize our findings for $\Psi_1(t)$ with a solid curve. By construction, there cannot be another such convergent path that always lies in *region 1.* Indeed, notice that (2.40) is equivalent to (2.31), which implies if (2.38) holds, (2.31) is violated; i.e. $\Psi_0(t)$ cannot give an equilibrium with $q(t) = 0$. Similarly, if (2.40) holds, hence (2.38) is violated, $\Psi_1(t)$ does not give an equilibrium with $q(t) = 1$, and as we have previously verified $q(t) = 0$ for all t is the unique equilibrium path in that case.

2.3.3 "Mixed" equilibrium path

For the intermediate case where (2.39) holds, i.e. both (2.38) and (2.31) are violated, Proposition 10 suggests that there always exists $t^* \in (0, T)$ where $\Psi_1(t) < 0$ for all $t < t^*$, and $\Psi_1(t) > 0$ for all $t > t^*$. Figure 2.5b illustrates $\Psi_1(t)$ for this case. When (2.39) holds, (2.31) is violated, then $\Psi_0(t)$ can never give an equilibrium with $q(t) = 0$.

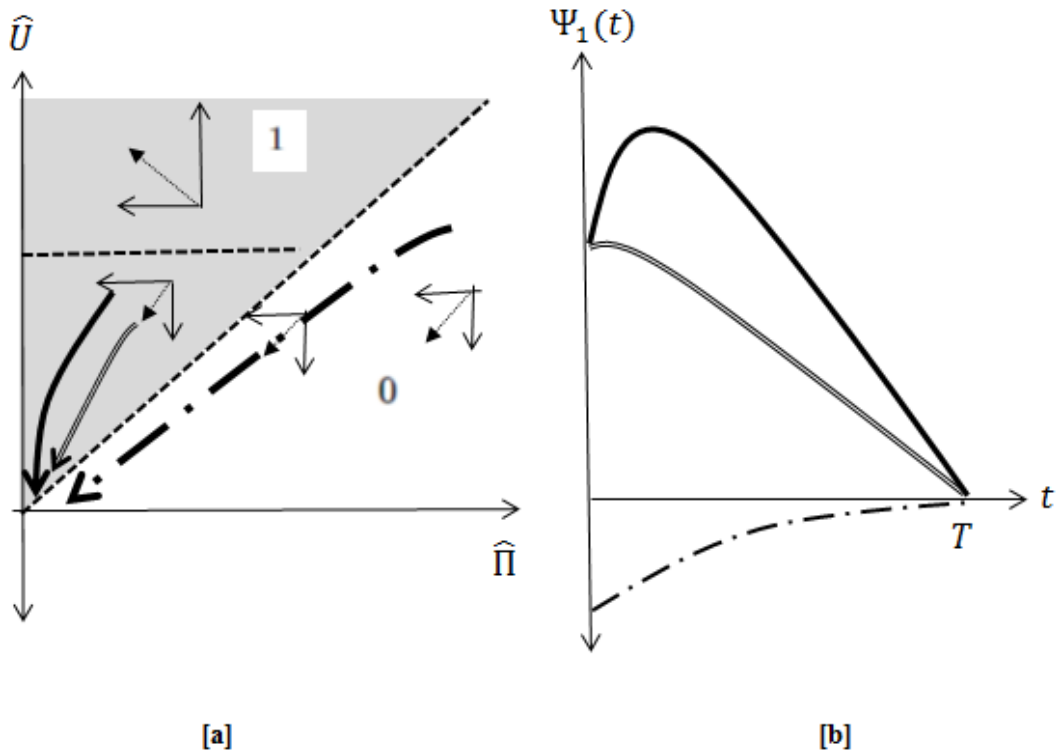


Figure 2.4: Equilibrium Path with $q(t) = 1$ for all t

But $\Psi_1(t) > 0$ for all $t > t^*$, $\Psi_1(t)$ is the unique equilibrium path for that time interval. However, it cannot give an equilibrium with $q(t) = 1$ for $t < t^*$ since $\Psi_1(t) < 0$ in that interval. Then the equilibrium path must satisfy $\Psi(t) = 0$ for all $t < t^*$, i.e. the equilibrium path lies on the line $\hat{u} = \frac{(1-p_1)}{p_1}\pi$ as illustrated in Figure 2.5b.

To find critical point in time, t^*, at which the optimal contract imposes a termination threat, we directly solve $\Psi_1(t^*) = 0$: since we solve for the equilibrium backwards and given that at the end of the contract horizon $\Psi_1(t)$ is optimal if (2.39) holds, it suffices to find where $\Psi_1(t) > 0$ no longer holds. Then equilibrium t^* follows from

$$\Psi_1(t^*) = \frac{1}{\gamma + r}(\frac{p_0 c}{p_1 - p_0} - \overline{\omega})(1 - e^{-(r+\gamma)(T-t^*)}) - \frac{(1-p_1)(y - c - \overline{\omega})}{r + (1-p_1)\gamma}(1 - e^{-(r+(1-p_1)\gamma)(T-t^*)}) = 0 \tag{2.42}$$

For future reference, the agent's and principal's payoffs at this critical t^* are computed using (2.37) and (2.36):

$$\hat{u}(t^*) = \frac{1}{\gamma + r}\left(\frac{p_0 c}{p_1 - p_0} - \overline{\omega}\right)\left(1 - e^{-(r+\gamma)(T-t^*)}\right) \tag{2.43}$$

$$\begin{aligned}\pi(t^*) \quad = \quad & \frac{1 - e^{-(r+(1-p_1)\gamma)(T-t^*)}}{r + (1-p_1)\gamma}\left(y + \frac{\gamma p_1}{\gamma + r}\left(\frac{p_0 c}{p_1 - p_0} - \overline{\omega}\right) - \frac{p_1 c}{p_1 - p_0}\right) \\ & \frac{\left(e^{-(r+\gamma)(T-t^*)} + e^{-(r+(1-p_1)\gamma)(T-t^*)}\right)}{\gamma + r}\left(\frac{p_0 c}{p_1 - p_0} - \overline{\omega}\right)\end{aligned}$$

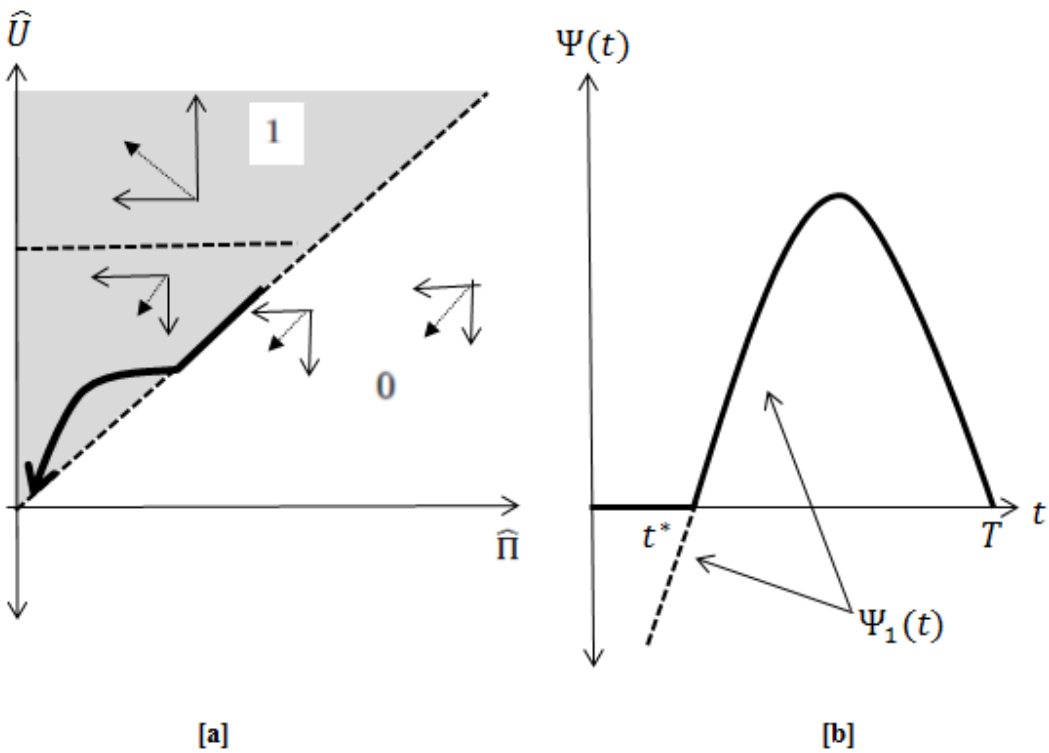


Figure 2.5: "Mixed" Equilibrium Path

Optimal Firing Rule for the "Mixed" Equilibrium

We showed that condition (2.39) produces a mixed equilibrium which requires $\Psi(t) = 0$ for all $t < t^*$ and $\Psi(t) = \Psi_1(t)$ for all $t > t^*$. Therefore, towards the end of horizon, the equilibrium path behaves exactly the same as the equilibrium path we derived in the previous section. The interesting part of the mixed equilibrium is that it requires $\Psi(t) =$

0 for $t < t^*$. From (2.17) where we derived the optimal firing rule: $q(t) \in [0,1]$ if $\Psi(t) = 0$. In this case, any $q(t)$ could be optimal, but we show in Proposition 11 that there is a unique $q(t)$ consistent with an equilibrium path. It also states the optimal wage and agent's continuation payoff on this time interval. Since the termination threat also determines optimal success wage and continuation payoffs in equilibrium, Proposition 11 also provides equilibrium paths of $w(t)$ and $\widehat{u}(t)$.

Proposition 11 *If* $\Psi(t) = 0$, *for all* $t \leq t^*$

(i) optimal firing rule is

$$q(t) = \frac{1}{\gamma \widehat{u}(t)}(-\frac{1-p_1}{p_1}y + \frac{1-p_1+p_0}{p_1-p_0}c - \overline{\omega}) \tag{2.44}$$

where $q(t)$ *increases on the interval* $(0, q(t^*))$

(ii) optimal wage is

$$\overline{w} = \frac{1}{\gamma}(\frac{1-p_1}{p_1}y + \overline{\omega} + c) \tag{2.45}$$

(iii) agent's continuation payoff follows

$$\widehat{u}(t) = \frac{1-p_1}{rp_1}(y - \frac{p_1 c}{p_1 - p_0})(1 - e^{-r(t^*-t)}) + \hat{u}(t^*)e^{-r(t^*-t)}) \tag{2.46}$$

where t^* *solves (2.42) and* $\hat{u}(t^*)$ *is defined in (2.43).*

Proof. (i) Since $\Psi(t) = 0$ corresponds to $\widehat{u}(t) = \frac{(1-p_1)}{p_1}\pi(t)$, we have

$$\frac{d\widehat{u}(t)}{d\pi(t)} = \frac{1-p_1}{p_1} \tag{2.47}$$

Using (2.13) and (2.16), (2.47) becomes

$$q(t)\widehat{u}(t) = \frac{1}{\gamma}(-\frac{1-p_1}{p_1}y + \frac{1-p_1+p_0}{p_1-p_0}c - \overline{\omega}) \tag{2.48}$$

Therefore (2.48) must hold when $\Psi(t) = 0$ in equilibrium. Since, whatever the functional form in equilibrium is, we know that $\hat{u}(t)$ is decreasing in t. Then keeping $q(t)\hat{u}(t)$ constant to satisfy the condition (2.48) for all $t \leq t^*$ implies that optimal $q(t)$ must

increase on this time interval. Therefore $q(t^*)$ is the highest optimal threat for $t < t^*$. To find $q(t^*)$, use $u(t^*)$, as calculated in (2.43), and equation (2.48). Then

$$q(t^*) = \frac{\frac{1}{\gamma}(-\frac{1-p_1}{p_1}y + \frac{1-p_1+p_0}{p_1-p_0}c - \overline{\omega})}{\frac{1}{\gamma+r}(\frac{p_0 c}{p_1-p_0} - \overline{\omega})(1 - e^{-(r+\gamma)(T-t^*)})} \tag{2.49}$$

(ii) From IC constraint, in equation (2.11), and optimal firing rule, as defined in equation (2.48), optimal $w(t)$ is in fact a constant:

$$\overline{w} = \frac{1}{\gamma}(\frac{1-p_1}{p_1}y + \overline{\omega} + c)$$

(iii) Using the optimal constant wage $\overline{w}$, optimal firing rule in equation (2.48), and the motion of the equilibrium continuation payoff of the agent as defined in equation (2.4), we can derive equilibrium $\hat{u}(t)$. For some constant $N_0 > 0$, we use the general form

$$\widehat{u}(t) = \frac{1-p_1}{rp_1}(y - \frac{p_1}{p_1-p_0}) + N_0 e^{rt}$$

To determine N_0, we use $\hat{u}(t^*)$, in (2.43), as a terminal condition.

$$N_0 = (\hat{u}(t^*) - \frac{1-p_1}{rp_1}(y - \frac{p_1}{p_1-p_0}))e^{-rt^*}$$

Therefore

$$\widehat{u}(t) = \frac{1-p_1}{rp_1}(y - \frac{p_1 c}{p_1-p_0})(1 - e^{-r(t^*-t)}) + \hat{u}(t^*)e^{-r(t^*-t)}$$

■

Hence there is an increasing termination threat on the time interval $[0, t^*]$ where $q(t) \in (0, q(t^*))$. After critical t^*, the maximum threat regime will be optimal, i.e. $q(t) = 1$ for all $t > t^*$. Since on the line, $\widehat{u}(t) = \frac{(1-p_1)}{p_1}\pi(t)$, condition (2.48) must hold and productivity y is constant, this means right hand side of the condition is also constant. Therefore left hand side being constant implies that from (2.11), a fixed wage will be optimal. Thus the optimal contract increases probability of firing while compensating the agent with a flat wage upto critical t^* : since the agent's incentive's to

work hard decreases as she gets more senior, the only way to adjust optimal incentives on this interval is to increase punishment. But after t^*, a fixed wage along with increasing termination threat profile is not sustainable, to give the sufficient incentives the optimal contract switches to increasing wage scheme along with maximum termination threat in the rest of horizon. Lemma 12 shows that optimal termination threat at critical t^* is in fact strictly less than 1.

Lemma 12 $q(t^*) < 1$.

Proof. First observe that $\Psi_1(t)$ is upward-sloping at $t = t^*$:

$$\Psi_1'(t^*) = -(\frac{p_0 c}{p_1 - p_0} - \overline{\omega})e^{-(r+\gamma)(T-t^*)} + (1-p_1)(y-c-\overline{\omega})e^{-(r+(1-p_1)\gamma)(T-t^*)} > 0 \quad (2.50)$$

Inequality (2.50) is equivalent to

$$\frac{(\frac{p_0 c}{p_1-p_0} - \overline{\omega})}{r + (1-p_1)\gamma} e^{-(r+\gamma)(T-t^*)} - \frac{(1-p_1)(y-c-\overline{\omega})}{r+(1-p_1)\gamma} e^{-(r+(1-p_1)\gamma)(T-t^*)} < 0 \quad (2.51)$$

At t^* we have

$$\Psi_1(t^*) = \frac{1}{\gamma + r}(\frac{p_0 c}{p_1 - p_0} - \overline{\omega})(1 - e^{-(r+\gamma)(T-t^*)}) - \frac{(1-p_1)(y-c-\overline{\omega})}{r+(1-p_1)\gamma}(1 - e^{-(r+(1-p_1)\gamma)(T-t^*)}) = 0 \quad (2.52)$$

Adding (2.51) and (2.52):

$$\begin{aligned} &\left[\frac{(\frac{p_0 c}{p_1-p_0} - \overline{\omega})}{r + (1-p_1)\gamma} e^{-(r+\gamma)(T-t^*)} - \frac{(1-p_1)(y-c-\overline{\omega})}{r+(1-p_1)\gamma} e^{-(r+(1-p_1)\gamma)(T-t^*)}\right] + \\ &\left[\frac{1}{\gamma+r}(\frac{p_0 c}{p_1-p_0} - \overline{\omega})(1 - e^{-(r+\gamma)(T-t^*)}) - \frac{(1-p_1)(y-c-\overline{\omega})}{r+(1-p_1)\gamma} e^{-(r+(1-p_1)\gamma)(T-t^*)}\right] \\ < \quad & 0 \end{aligned}$$

This can be simplified as follows:

$$\frac{1}{\gamma+r}\left(\frac{p_0 c}{p_1 - p_0} - \overline{\omega}\right) e^{-(r+\gamma)(T-t^*)} < \frac{(1-p_1)}{p_1\gamma}(y-c-\overline{\omega}) - \frac{(r+(1-p_1)\gamma)}{(\gamma+r)\,p_1\gamma}(\frac{p_0 c}{p_1-p_0} - \overline{\omega}) \quad (2.53)$$

But (2.53) implies

$$\begin{aligned}
\frac{1}{\gamma+r}\left(\frac{p_0 c}{p_1-p_0}-\overline{\omega}\right)\left(1-e^{-(r+\gamma)(T-t^*)}\right) &> \frac{1}{\gamma+r}\left(\frac{p_0 c}{p_1-p_0}-\overline{\omega}\right)-\frac{(1-p_1)}{p_1\gamma}(y-c-\overline{\omega}) \\
&\quad +\frac{(r+(1-p_1)\gamma)}{(\gamma+r)\,p_1\gamma}\left(\frac{p_0 c}{p_1-p_0}-\overline{\omega}\right) \\
&= \frac{1}{\gamma}\left(-\frac{1-p_1}{p_1}y+\frac{1-p_1+p_0}{p_1-p_0}c-\overline{\omega}\right)
\end{aligned}$$

Therefore, $q(t^*) < 1$. ■

2.3.4 Summary of the analysis

Consider $\Psi(t)$ as defined by (2.10). The above equilibrium analysis can be summarized as follows:

$$\Psi(t)=\begin{cases}
\Psi_1(t) & \text{if} \quad y<\left(\frac{K(T)p_0}{(1-p_1)(p_1-p_0)}+1\right)c+\left(1-\frac{K(T)}{1-p_1}\right)\overline{\omega} \\[2ex]
0 \text{ for } t\le t^* \text{ and } \Psi_1(t) \text{ for } t>t^* & \text{if} \quad \begin{array}{c}\left(\frac{K(T)p_0}{(1-p_1)(p_1-p_0)}+1\right)c+\left(1-\frac{K(T)}{1-p_1}\right)\overline{\omega}<y \\ <\frac{p_1(1-p_1+p_0)}{(1-p_1)(p_1-p_0)}c-\frac{p_1}{1-p_1}\overline{\omega}\end{array} \\[2ex]
\Psi_0(t) & \text{if} \quad y>\frac{p_1(1-p_1+p_0)}{(1-p_1)(p_1-p_0)}c-\frac{p_1}{1-p_1}
\end{cases}$$

Corresponding to $\Psi(t)$, we get the following termination threat over the contract horizon:

$$q(t)=\begin{cases}
1 & \text{if} \quad y<\left(\frac{K(T)p_0}{(1-p_1)(p_1-p_0)}+1\right)c+\left(1-\frac{K(T)}{1-p_1}\right)\overline{\omega} \\[2ex]
\in(0,q(t^*)) \text{ for } t\le t^* \text{ and } 1 \text{ for } t>t^* & \text{if} \quad \begin{array}{c}\left(\frac{K(T)p_0}{(1-p_1)(p_1-p_0)}+1\right)c+\left(1-\frac{K(T)}{1-p_1}\right)\overline{\omega} \\ <y \\ <\frac{p_1(1-p_1+p_0)}{(1-p_1)(p_1-p_0)}c-\frac{p_1}{1-p_1}\overline{\omega}\end{array} \\[2ex]
0 & \text{if} \quad y>\frac{p_1(1-p_1+p_0)}{(1-p_1)(p_1-p_0)}c-\frac{p_1}{1-p_1}
\end{cases}$$

Once optimal $q(t)$ is determined, optimal wage contract directly follows from the IC

constraint, expressed in equation (2.11). Specifically,

$$w(t) = \begin{cases} \frac{c/\gamma}{p_1 - p_0} & \text{if} \quad q(t) = 0 \\ -\hat{u}(t) + \frac{c/\gamma}{p_1 - p_0} & \text{if} \quad q(t) = 1 \\ \frac{1}{\gamma}(\frac{1-p_1}{p_1} y + \overline{\omega} + c) & \text{if} \quad q(t) \in (0,1) \end{cases} \tag{2.54}$$

The following proposition summarizes our findings so far.

Proposition 13 *The equilibrium path satisfies,*

(i) If

$$y < (\frac{K(T)p_0}{(1-p_1)(p_1-p_0)} + 1)c + (1 - \frac{K(T)}{1-p_1})\overline{\omega} \tag{2.55}$$

then

$$w(t) = -\hat{u}(t) + \frac{c/\gamma}{p_1 - p_0}$$

and $q(t) = 1$ *for all* $t \in [0, T]$, *where* $\hat{u}(t)$ *is defined in (2.37).*

(ii) If

$$(\frac{K(T)p_0}{(1-p_1)(p_1-p_0)} + 1)c + (1 - \frac{K(T)}{1-p_1})\overline{\omega} < y < \frac{p_1(1-p_1+p_0)}{(1-p_1)(p_1-p_0)}c - \frac{p_1}{1-p_1}\overline{\omega} \tag{2.56}$$

then for $t \leq t^*$ *we have*

$$w(t) = \frac{1}{\gamma}(\frac{1-p_1}{p_1} y + \overline{\omega} + c)$$

and

$$q(t) = \frac{1}{\gamma \widehat{u}(t)}(-(\frac{1-p_1}{p_1} y + \overline{\omega}) + \frac{1-p_1+p_0}{p_1-p_0} c)$$

where $\widehat{u}(t)$ *follows (2.46) and* t^* *solves (2.42); while for* $t > t^*$ *we have*

$$w(t) = -\hat{u}(t) + \frac{c/\gamma}{p_1 - p_0}$$

and $q(t) = 1$, *where* $\hat{u}(t)$ *is defined in (2.37).*

(iii) If

$$y > \frac{p_1(1-p_1+p_0)}{(1-p_1)(p_1-p_0)}c - \frac{p_1}{1-p_1}\overline{\omega} \tag{2.57}$$

then $w(t) = \frac{c/\gamma}{p_1 - p_0}$ *and* $q(t) = 0$ *for all* $t \in [0, T]$.

Figure 2.6 illustrates the optimal contract. Panels [a], [b] and [c] correspond to the cases in part (i), part (ii) and part (iii) of Proposition refContract2, respectively, where we illustrate the equilibrium path, termination threat and success wage for each case. In each panel, on the top we illustrate the equilibrium path, $\Psi(t)$ for a given range of productivity y. Given the equilibrium path on the top, the middle and lower rows show the components of the optimal $q(t)$ and $w(t)$, respectively.

Now we can complete the proof of Theorem 5. It is easy to check that the condition (2.31), which determines the cutoff for the punishment-free contract, can be rewritten as (2.26). If agent's productivity is sufficiently high, or agent's surplus in the one-shot game is sufficiently low, the optimal contract does not use termination threat at all. Therefore, the incentives to induce effort can only be provided by a high success wage through the contract horizon. Condition (2.38) can be re-expressed as condition (2.27), which defines a lower bound for the agent's expected surplus: the contract always imposes severe termination threat at all times if the agent's surplus in one shot game is not too low. But from (2.54) optimal wage is set as low as possible at every instant: $-\hat{u}(t) + \frac{c/\gamma}{p_1 - p_0}$. As explained before, success wage is upward sloping since as the agent's tenure increases, her incentives to work hard decreases: her expected loss in case of failure, $p_1\hat{u}(t)$, decreases. Hence the optimal incentives must be stronger as the agent gets more senior. Finally, the intermediate "mixed" case corresponds to condition (2.28). If productivity is in a moderate range, the optimal contract offers an increasing termination threat scheme, combined with a constant wage stream in the early phase of time horizon, which would be replaced by an increasing wage profile after critical t^*. As we verified previously, $q(t^*) < 1$, i.e. termination threat profile has a *jump* at t^*. For that reason, after t^*, success wage decreases for a limited time interval: $\overline{w} = q(t^*)\hat{u}(t^*) + \frac{c/\gamma}{p_1 - p_0} > -\hat{u}(t^*) + \frac{c/\gamma}{p_1 - p_0}$, as illustrated in panel [b] below. But this follows an increasing wage scheme until the end of time horizon. At the end of contract horizon, optimal wage certainly exceeds $\overline{w}$ since $w(T) = \frac{c/\gamma}{p_1 - p_0} > \frac{1}{\gamma}(\frac{1-p_1}{p_1}y + \overline{\omega} + c) > \overline{w}$, by condition (2.28).

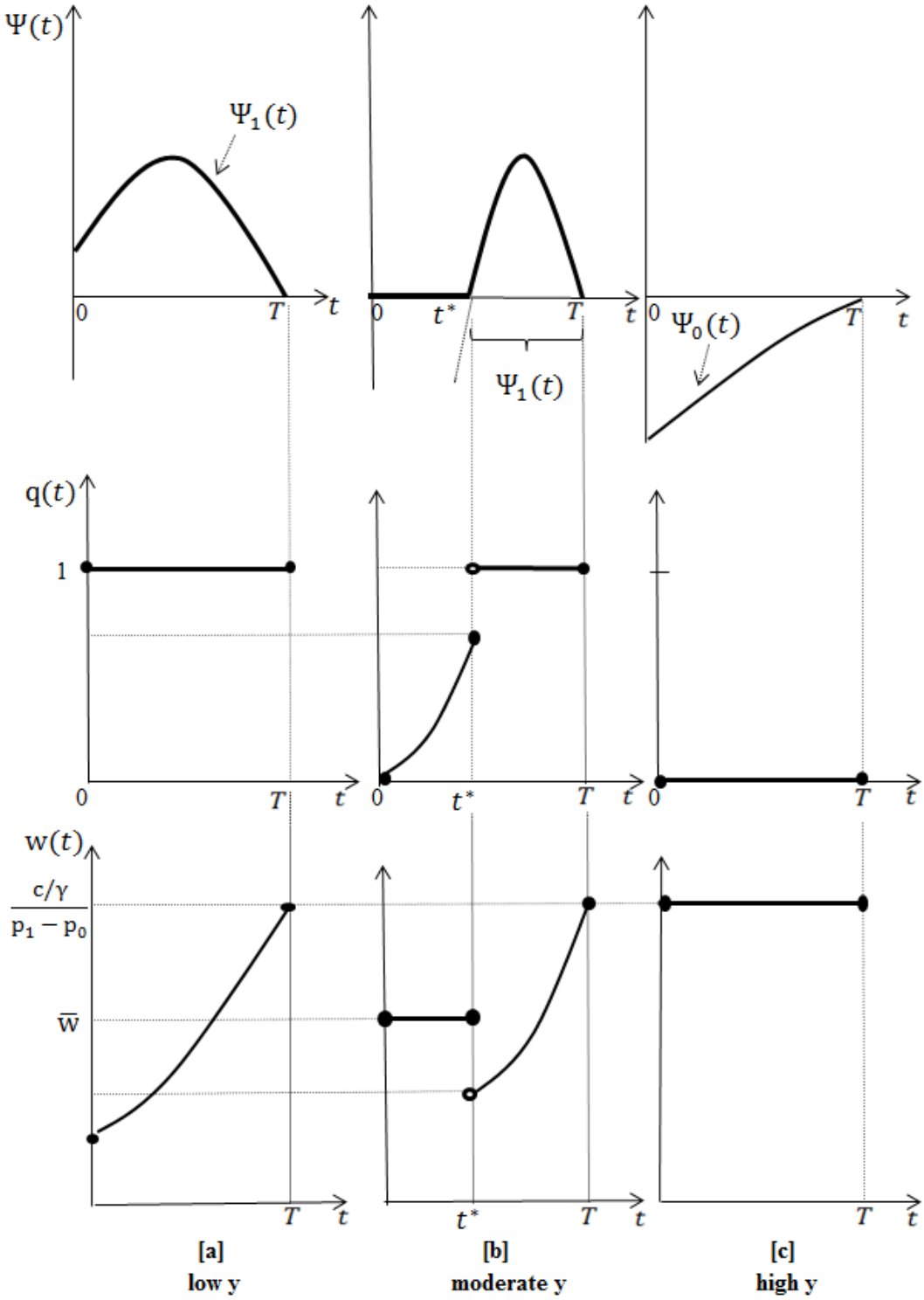


Figure 2.6: The Optimal Contract

2.4 Comparative Dynamics

In this section, we focus on how the change of parameters affect the nature of the equilibrium path. We are mainly interested in the changes in quality differential of agent's output, i.e. $Y_H - Y_L$; success probability when low effort is exerted, i.e. p_0; and discount rate, r. Observe from equation (2.1), where we define y, that a change in quality differential on agent's output, $Y_H - Y_L$, is in fact equivalent to a change in agent's productivity, holding $\gamma(p_1 - p_0)$ constant. The parameter p_0 (the probability of succeeding without trying) can be thought of as a measure of "luck", and r is a measure of impatience.

2.4.1 A change in quality differential

It is straightforward to see the effect of a change in $Y_H - Y_L$ on the optimal termination threat. Consider the lower and upper bounds for productivity y in Proposition 13. Since they are independent from the quality differential, it is easy to track how optimal $q(t)$ responds to a given change in the differential: as quality differential increases, termination threat (weakly) decreases. Below we verify this impact and also evaluate the change in $w(t)$ and illustrate how these results in fact follow from the impact on the equilibrium path in Figure 2.6.

To see the argument let us start assuming $Y_H - Y_L$, and hence y, is relatively low initially: $y = y_0$ such that (2.55) holds. Denoting the optimal contract by $(w_0(t), q_0(t))$ when $y = y_0$, we get

$$(w_0(t); q_0(t)) \ = (-\hat{u}_0(t) + c/\gamma(p_1 - p_0);\ 1), \quad \forall t \in [0, T) \tag{2.58}$$

From equation (2.32), clearly $\partial\Psi_1(t)/\partial y < 0$; i.e. as y increases, $\Psi_1(t)$ shifts down in a fashion as illustrated in panel [a] of Figure 2.7.[13] If the increase in y is sufficiently low such that (2.55) still holds, $\Psi_1(t)$ shifts down a little bit. Therefore the optimal

[13] From equation (2.32), we get $\Psi_1'(T) = -\frac{p_0 c}{p_1 - p_0} + \bar{\omega} + (1 - p_1)(y - c - \bar{\omega})$. Condition (2.55) implies $\Psi_1'(T) < 0$. As y increases such that (2.55) still holds, $\Psi_1'(T)$ gets less negative, i.e the function becomes flatter around T.

contract is still given in (2.58). Now suppose y increases to $y = y_1 > y_0$ such that (2.55) is violated but (2.56) holds. In other words, $\Psi_1(t)$ shifts down so we move to panel [b]. From the previous analysis, we know that if (2.56) holds, there exits t_1^* such that $\Psi_1(t_1^*) = 0$. Then the equilibrium path becomes:

$$(w_1(t), q_1(t)) = \begin{cases} (\overline{w}_1; \frac{1}{\gamma \hat{u}_1(t)}(-(\frac{1-p_1}{p_1}y_1 + \overline{\omega}) + \frac{1-p_1+p_0}{p_1-p_0}c)), & \forall t \in [0, t_1^*] \\ (-\hat{u}_1(t) + \frac{c/\gamma}{p_1-p_0}; 1), & \forall t \in (t_1^*, T) \end{cases} \tag{2.59}$$

where $\overline{w}_1 = \frac{1}{\gamma}(\frac{1-p_1}{p_1}y_1 + \overline{\omega} + c)$. A comparison of (2.58) and (2.59) leads to the following result.

Lemma 14 *$q_1(t) \leq q_0(t)$ but $w_1(t) \geq w_0(t)$ for all t.*

Proof. Obviously $q_1(t) = q_0(t)$ for all $t \in (t_1^*, T)$. Recall from Lemma 12 and Proposition 11 that $q_1(t_1^*) < 1$ and $q_1(t) < q_1(t_1^*)$, $\forall t \in [0, t_1^*]$. Therefore, $q_1(t) < q_0(t) = 1$ for all $t \in [0, t_1^*]$. Finally, we consider how $w(t)$ changes. First note that on (t_1^*, T), $\hat{u}(t)$ follows from (2.37) since $q = 1$ on this interval. Then $-\hat{u}_1(t) = -\hat{u}_0(t)$, i.e. $w_1(t) = w_0(t)$ for all $t \in (t_1^*, T)$. Also, from our previous analysis, we know that $\overline{w}_1 > -\hat{u}_1(t_1^*) + c/\gamma(p_1 - p_0)$. This implies $\overline{w}_1 > -\hat{u}_0(t_1^*) + c/\gamma(p_1 - p_0) = w_0(t_1^*)$. Since $w_0(t_1^*) > w_0(t)$ for all $t < t_1^*$, we conclude that $\overline{w}_1 > w_0(t)$. Therefore $w_1(t) \geq w_0(t)$ for all t. ■

Now suppose y increases to $y = y_2 > y_1$ such that (2.56) still holds. From the previous analysis, there exists t_2^* such that $\Psi_1(t_2^*) = 0$. But since $\Psi_1(t)$ keeps shifting down as illustrated in panel [b], $t_2^* > t_1^*$. Then the new equilibrium path is:

$$(w_2(t), q_2(t)) = \begin{cases} (\overline{w}_2; \frac{1}{\gamma \hat{u}_2(t)}(-(\frac{1-p_1}{p_1}y_2 + \overline{\omega}) + \frac{1-p_1+p_0}{p_1-p_0}c)), & \forall t \in [0, t_2^*] \\ (-\hat{u}_2(t) + \frac{c/\gamma}{p_1-p_0}; 1), & \forall t \in (t_2^*, T) \end{cases} \tag{2.60}$$

where $\overline{w}_2 = \frac{1}{\gamma}(\frac{1-p_1}{p_1}y_2 + \overline{\omega} + c)$.

Lemma 15 *$q_2(t) \leq q_1(t)$ but $w_2(t) \geq w_1(t)$ for all t.*

Proof. It is easy to check that $q(t)$ weakly decreases for all $t \in (t_1^*, T)$: even though $q_2(t) = q_1(t)$ for all $t \in (t_2^*, T)$, $q_2(t) < q_1(t)$ for all $t \in (t_1^*, t_2^*]$ by the same argument that we used above; the regime where $q < 1$ replaces $q = 1$ regime over this time interval. Obviously, agent's payoff is decreasing in $q(t)$. Since $q_2(t) \leq q_1(t)$ for all $t \in (t_1^*, T)$, then $\hat{u}_2(t_1^*) > \hat{u}_1(t_1^*)$. This means, from (2.49), $q_2(t_1^*) < q_1(t_1^*) < 1$. This, together with $q_2(t) < q_2(t_1^*)$ for all $[0, t_1^*]$ (since $q(t)$ must be upward sloping on $[0, t_2^*]$) implies that $q_2(t) < q_1(t)$ for all $t \in [0, t_1^*]$. To evaluate the impact on success wage, observe that $\hat{u}_1(t) = \hat{u}_2(t)$ for all $t \in (t_2^*, T)$. Then $w_1(t) = w_2(t)$ for all $t \in (t_2^*, T)$. But for all $t \in (t_1^*, t_2^*]$, $\overline{w}_2 > w_1(t)$: since

$$\overline{w}_2 > -\hat{u}_2(t_2^*) + \frac{c/\gamma}{p_1 - p_0} = -\hat{u}_1(t_2^*) + \frac{c/\gamma}{p_1 - p_0} > -\hat{u}_1(t_1^*) + \frac{c/\gamma}{p_1 - p_0} = w_1(t).$$

Therefore we conclude that $w_2(t) \geq w_1(t)$. ■

Finally suppose y increases to $y = y_3 > y_2$ such that (2.56) is violated and (2.57) holds. In other words, $\Psi_1(t)$ shifts down significantly where $\Psi_1(t) < 0$ for all t. Then Ψ_1 becomes irrelevant as we move to panel [c]. The new equilibrium path is:

$$(w_3(t); q_3(t)) \ = (\tfrac{c/\gamma}{p_1 - p_0};\ 0), \quad \forall t \in [0, T)$$

Lemma 16 *$q_3(t) < q_2(t)$ and $w_3(t) > w_2(t)$ for all t.*

Proof. The decrease in optimal threat is clear. We also observe that $\frac{c/\gamma}{p_1 - p_0} > \max\{\overline{w}_2, -\hat{u}_2(t) + \frac{c/\gamma}{p_1 - p_0}\}$ for all t. In other words, $w_3(t) > w_2(t)$ for all t. ■

Combining Lemmas 14, 15 and 16 yields the following result:

Theorem 17 *As the agent's productivity increases, holding $\gamma(p_1 - p_0)$ constant, the termination threat (weakly) decreases, whereas the success wage (weakly) increases. When it is defined, the critical t^* increases.*

Summarizing this analysis, we have found that the two contracting devices respond to an increase in quality differential in opposite ways: the termination threat decreases

while wage incentives increase. One straightforward intuition is that the *instantaneous* net benefit from punishment, i.e. $p_1\hat{u}(t) - (1 - p_1)\pi(t)$, decreases as productivity y increases. The principal does not want to lose her agent as his productivity increases, so she lowers the punishment level. At the same time, she increases the success wage to induce high effort. A similar intuition follows from our previous argument with one-shot surpluses. An increase in y holding $\gamma(p_1 - p_0)$ constant is equivalent to, using (2.24), an increase in S_P while S_A is constant, and hence an increase in S. Therefore, as agent's share from the total surplus in one-shot game decreases, the termination threat becomes less useful.

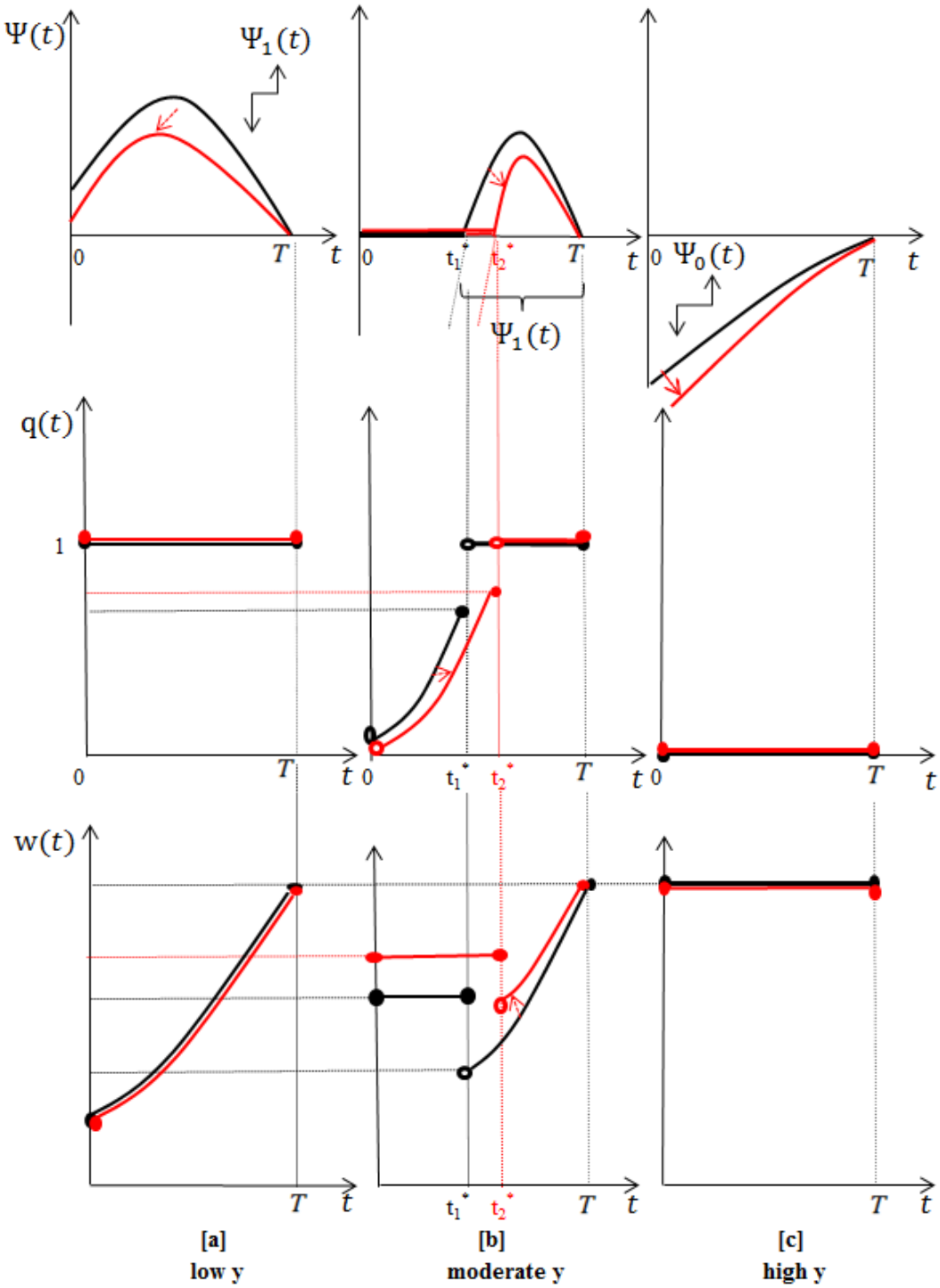


Figure 2.7: A Change in Quality Differential

2.4.2 A change in luck

To see the impact on the optimal contract with respect to p_0, we start with $p_0 = p_0^0$ such that (2.57) holds. We will gradually increase p_0 and consider how the contract in Figure 2.6 changes.

When (2.57) holds, $\Psi_0(t) < 0$ for all $t < T$. Therefore there is never any termination threat over the horizon. In panel [c] of Figure 2.8, the optimal contract is illustrated with black lines. Denote the equilibrium path when $p_0 = p_0^0$ by $(w_0(t), q_0(t))$:

$$(w_0(t); q_0(t)) \ = (-\hat{u}_0(t) + c/\gamma(p_1 - p_0^0);\ 0), \quad \forall t \in [0, T)$$

Note that right hand side of the inequality in (2.57) is increasing but left hand side is decreasing in p_0. Therefore as p_0 increases, (2.57) will eventually be violated. We also observe that $\partial\Psi_0(t)/\partial p_0 > 0$: the function moves up, keeping the end point constant, as p_0 increases. For sufficiently small increase in p_0, $\Psi_0(t)$ moves up a little and (2.57) still holds. In panel [c], we illustrate this impact. Hence $q(t)$ does not change. But since $\frac{c/\gamma}{p_1 - p_0}$ is increasing in p_0, $w(t)$ shifts up as we show in panel [c].

Now let $p_0 = p_0^1 > p_0^0$, such that (2.57) is violated but (2.56) holds. In that case, $\Psi_0(t) > 0$ for all t so Ψ_0 becomes irrelevant. From Proposition 13, there exists a critical t_1^* and the equilibrium path is as illustrated in panel [b]. Denote the optimal contract when $p_0 = p_0^1$ by $(w_1(t), q_1(t))$. Then we have:

$$(w_1(t), q_1(t)) = \begin{cases} (\overline{w}_1; \frac{1}{\gamma\hat{u}_1(t)}(-(\frac{1-p_1}{p_1}y(p_0^1) + \overline{\omega}) + \frac{1-p_1+p_0^1}{p_1-p_0^1}c)) & \forall t \in [0, t_1^*] \\ (-\hat{u}_1(t) + \frac{c/\gamma}{p_1-p_0^1}; 1), & \forall t \in (t_1^*, T) \end{cases} \tag{2.61}$$

where $\overline{w}_1 = \frac{1}{\gamma}(\frac{1-p_1}{p_1}y(p_0^1) + \overline{\omega} + c)$. Therefore $q(t)$ increases for all t, while the success wage, $w(t)$, decreases at all times. That is:

Lemma 18 *$q_0(t) < q_1(t)$ and $w_0(t) > w_1(t)$ for all t.*

Now assume a further increase in the luck parameter: $p_0 = p_0^2 > p_0^1$ such that (2.56) still holds and we again have mixed equilibrium where there exists t_2^* such that

$\Psi_1(t_2^*) = 0$. The optimal contract changes in the following way:

$$(w_2(t), q_2(t)) = \begin{cases} (\overline{w}_2; \frac{1}{\gamma \hat{u}_2(t)}(-(\frac{1-p_1}{p_1}y(p_0^2) + \overline{\omega}) + \frac{1-p_1+p_0^2}{p_1-p_0^2}c)) & t \in [0, t_2^*] \\ (-\hat{u}_2(t) + \frac{c/\gamma}{p_1-p_0^2}; 1) & \forall t \in (t_2^*, T) \end{cases} \tag{2.62}$$

where $\overline{w}_2 = \frac{1}{\gamma}(\frac{1-p_1}{p_1}y(p_0^2) + \overline{\omega} + c)$. To see how the equilibrium path in panel [b] changes, observe that $\partial\Psi_1(t)/\partial p_0 > 0$ and $\partial\Psi_1'(T)/\partial p_0 > 0$.[14] Then $\Psi_1(t)$ shifts up in the illustrated fashion. This verifies that $t_2^* < t_1^*$. In other words, the optimal contract enters the maximum termination-threat regime at an earlier point in time as p_0 increases: the agent will be exposed to the highest threat for a longer time interval at the end her career. Now we compare (2.61) and (2.62).

Lemma 19 *$q_2(t) \geq q_1(t)$ for all t.*

Proof. Clearly, $q_2(t) = q_1(t)$ for all $t \in (t_1^*, T)$. But $q_1(t) < q_1(t_1^*) < 1 = q_2(t)$ for all $t \in [t_2^*, t_1^*]$. To see the impact on $[0, t_2^*)$, we need to evaluate how optimal $q(t)$, as defined in (2.44), changes. Observe that $\partial y/\partial p_0 < 0$ and since $\hat{u}(t)$ follows from (2.46), $\partial\hat{u}(t)\partial p_0 < 0$. Then $\partial q(t)/\partial p_0 > 0$ so $q_2(t) > q_1(t)$ for all $t \in [0, t_2^*)$.This impact is illustrated in panel [b].[15] ■

Next, we compare $w_1(t)$ and $w_2(t)$ in (2.61) and (2.62). First note that $-\hat{u}(t) + c/\gamma(p_1 - p_0)$ is increasing in p_0.[16] Then $w_2(t) > w_1(t)$ for all $t \in (t_1^*, T)$. However, since

---

[14]Since $\frac{\partial y}{\partial p_0} < 0$,

$$\frac{\partial\Psi_1(t)}{\partial p_0} = \frac{p_1 c}{(p_1 - p_0)^2(\gamma + r)}(1 - e^{-(r+\gamma)(T-t)}) - \frac{(1-p_1)}{r + (1-p_1)\gamma}(1 - e^{-(r+(1-p_1)\gamma)(T-t)})\frac{\partial y}{\partial p_0} > 0$$

[15]It is easy to check that $\partial^2 q(t)/\partial p_0 \partial t > 0$ in that interval, i.e termination scheme gets steeper as illustrated in panel [b].

[16]Directly follows from:

$$\begin{aligned} \partial(-\hat{u}(t) + c/\gamma(p_1 - p_0))/\partial p_0 &= \frac{c}{(p_1 - p_0)^2}(-\frac{p_1}{\gamma + r}(1 - e^{-(r+\gamma)(T-t)}) + \frac{1}{\gamma}) \\ &> \frac{c}{(p_1 - p_0)^2}(-\frac{p_1}{\gamma + r} + \frac{1}{\gamma}) > 0 \end{aligned}$$

$\partial\overline{w}/\partial p_0 < 0$, $w_2(t) < w_1(t)$ for all $t \in [0, t_2^*)$. But the impact on $w(t)$ over the interval (t_2^*, t_1^*) is unclear.[17] Nevertheless, the comparison of $(w_1(t), q_1(t))$ and $(w_2(t), q_2(t))$ yields the following conclusion: whenever the optimal contract cannot adjust $q(t)$ to an increase in p_0 further (i.e. on the interval (t_1^*, T) where the threat was already set at the highest possible value), then high effort could be induced only with higher $w(t)$. However if $q(t)$ could be adjusted to an increase in p_0 (i.e. on the interval $[0, t_2^*)$), then cost minimizing contract decreases $w(t)$.

Finally let $p_0 = p_0^3 > p_0^2$ such that (2.56) is violated but (2.55) holds. Then $\Psi_1(t) > 0$ for all t and we have panel [a] in Figure 2.8. Then the optimal contract is

$$(w_3(t), q_3(t)) = (-\hat{u}_3(t) + c/\gamma(p_1 - p_0^3), 1); \forall t \in [0, T]$$

Obviously, $q(t)$ weakly increases. Thus, we have:

Lemma 20 *$q_3(t) = q_2(t)$ for all $[t_2^*, T)$, and $q_3(t) > q_2(t)$ for all $t \in [0, t_2^*)$.*

As p_0 keeps increasing, (2.55) still holds, i.e. $\Psi_1(t)$ shifts up as illustrated in panel [a]. Therefore $q(t)$ does not change. But since $-\hat{u}(t) + c/\gamma(p_1 - p_0)$ is increasing in p_0, $w(t)$ shifts up as illustrated.[18] Hence $w(t)$ increases at all times. Again, we have the same result: whenever it is not possible to adjust the optimal incentives via an increase $q(t)$, with respect to an increase in p_0, the optimal adjustment on the contract is to increase $w(t)$.

Theorem 21 *As the luck parameter p_0 increases, the termination threat (weakly) increases. When it is defined, the critical t^* decreases.*

It is also possible to show that $\partial^2 u(t)/\partial p_0 \partial t > 0$, i.e the upward sloping part of the wage scheme gets steeper as p_0 increases.

[17] Over the time interval $[t_2^*, t_1^*]$, the relationship between $w_2(t)$ and $\overline{w}_1$ is not clear. In panel [b], we illustrated a small shift in the increasing portion of the wage scheme: $w_2(t) < \overline{w}_1$ for all $[t_2^*, t_1^*]$. However if there is a larger shift, it would be possible to have $w_2(t) > \overline{w}_1$ for $t \in [t_3^*, t_1^*]$, but $w_2(t) < \overline{w}_1$ for all $t \in [t_2^*, t_3^*]$. It cannot be possible to have a huge shift such that $w_2(t) > \overline{w}_1$ for all $t \in [t_2^*, t_1^*]$: $\overline{w}_2 > w_2(t)$ at least for some $[t_2^*, t_2^* + \varepsilon]$; and $\overline{w}_1 > \overline{w}_2$. Therefore $\overline{w}_1 > w_2(t)$ holds at least over a small interval.

[18] Again since $\partial^2 u(t)/\partial p_0 \partial t > 0$, the wage scheme gets steeper when p_0 increases, as depicted in panel [a].

Summarizing, the termination threat is always increasing in p_0. Intuitively, as *luck* becomes more important, the incentives must be stronger to encourage effort, and the principal raises the termination threat. Again, in terms of one-shot surpluses, we observe from equation (2.24) that as p_0 increases, the agent's share from the total surplus increases: S_A increases as S decreases. Hence, the firing option becomes more attractive. We also showed critical t^* also increases as p_0 gets larger; i.e. the agent will be exposed to the maximum termination threat for a longer time interval.

Even though our analysis suggests that the impact on the optimal wage is not clear, for some cases, we have been able to show that if the principal can increase termination threat, as a response to an increase in p_0, optimal wage decreases. If the termination threat is already set at the maximum level, or the increase in p_0 is not enough to push productivity y down to a moderate range, then the principal does not have the option to adjust the termination threat. For such cases, the success wage is adjusted instead: higher p_0 increases the optimal success wage to provide stronger incentives to work hard.

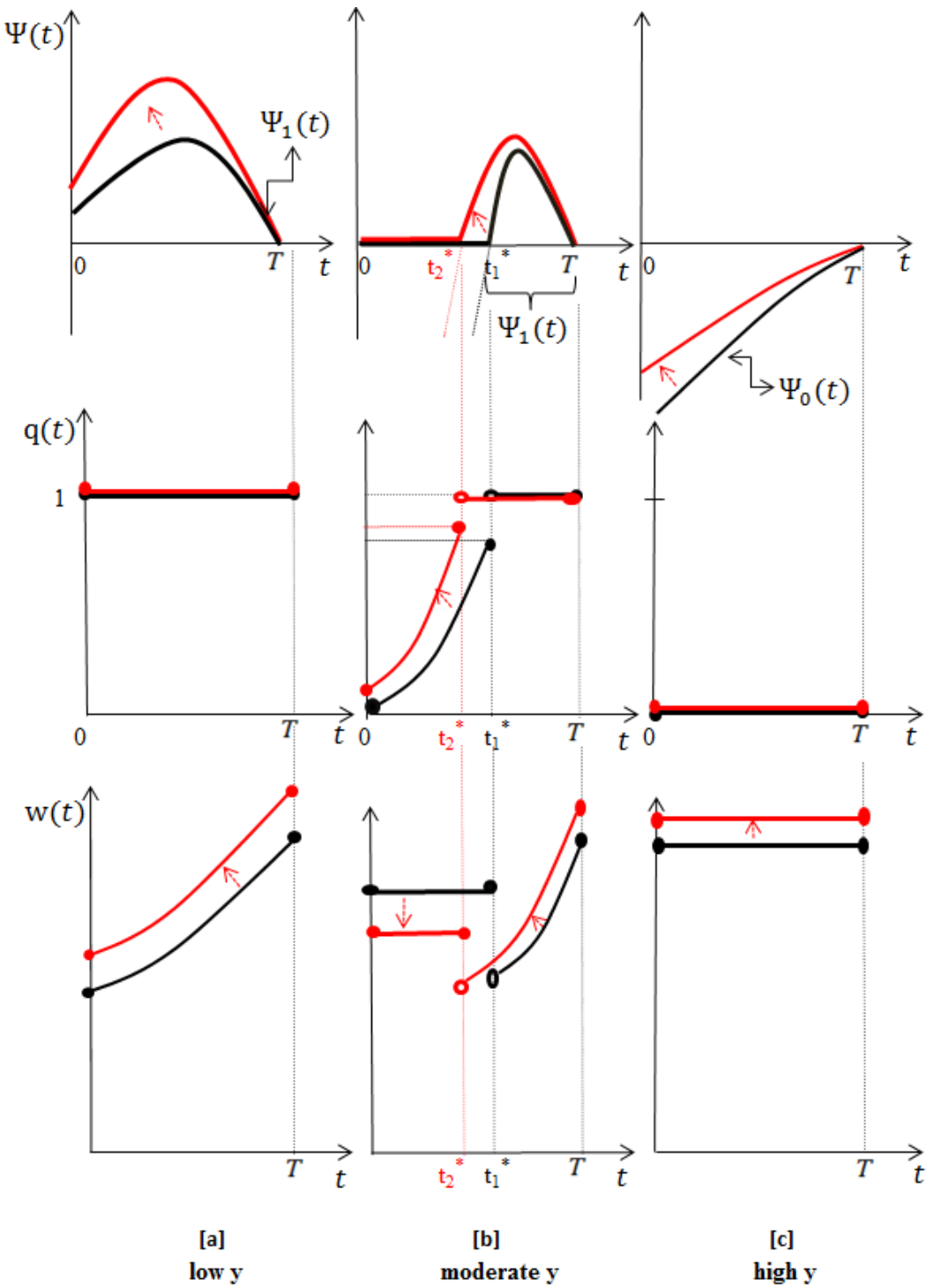


Figure 2.8: A Change in Luck

2.4.3 A change in discount rate

Finally, we consider how a change in r affects the equilibrium path. In this section, we do a partial analysis: first we will show how the optimal contract responds to a change in r when productivity (i.e., y) takes relatively high values (i.e. when (2.57) holds). Then we consider what happens when productivity is in a moderate range (i.e. when (2.56) holds) and finally when it takes relatively low values (i.e. when (2.55) holds).

a. High productivity

Suppose condition (2.57) holds. In this case, If y is high enough so that the optimal contract does not use any termination threat. It is straightforward to see that the optimal $q(t)$ is constant with respect to a change in r, because condition (2.57) does not depend on r. Observe from Proposition 13 that, when (2.57) holds, $w(t)$ does not depend on r either. The only impact is on $\Psi_0(t)$, that is derived in (2.30). Note that the sign of $\Psi_0(t)$ does not depend on r. In panel [c] of Figure 2.9, we illustrate the impact on $\Psi_0(t)$.[19]

Theorem 22 *If condition (2.57) holds, then the success wage increases as r increases.*

b. Moderate productivity

Suppose r increases from r_1 to $r_2 > r_1$, and suppose condition (2.56) holds both before and after the increase. We know from previous analysis that when (2.56) holds, there exists a mixed equilibrium where the path enters a maximum termination-threat regime at a critical t^*. Denote the corresponding critical point in time and the optimal contract when $r = r_i$ by t_i^* and $(w_i(t), q_i(t))$, respectively. Then when $r = r_1$, the equilibrium

[19] It can be checked that $\frac{1-e^{-r(T-t)}}{r}$ is decreasing in r .This implies, since (2.57) holds, $\Psi_0(t)$ increases, i.e., it shifts up, as r increases. But (2.57) implies that $\Psi_0(t) < 0$ will always holds regardless of the magnitude of the increase in r.

path is

$$(w_1(t), q_1(t)) = \begin{cases} (\overline{w}_1; \frac{1}{\gamma \widehat{u}_1(t)}(-(\frac{1-p_1}{p_1}y + \overline{\omega}) + \frac{1-p_1+p_0}{p_1-p_0}c)); & t \in [0, t_1^*] \\ (-\hat{u}_1(t) + \frac{c/\gamma}{p_1-p_0}; 1); & \forall t \in (t_1^*, T) \end{cases} \tag{2.63}$$

where $\overline{w}_1 = \frac{1}{\gamma}(\frac{1-p_1}{p_1}y + \overline{\omega} + c)$. When $r = r_2$, the equilibrium path is

$$(w_2(t), q_2(t)) = \begin{cases} (\overline{w}_1; \frac{1}{\gamma \widehat{u}_2(t)}(-(\frac{1-p_1}{p_1}y + \overline{\omega}) + \frac{1-p_1+p_0}{p_1-p_0}c)); & t \in [0, t_2^*] \\ (-\hat{u}_2(t) + \frac{c/\gamma}{p_1-p_0}; 1); & \forall t \in (t_2^*, T) \end{cases} \tag{2.64}$$

where $\overline{w}_2 = \frac{1}{\gamma}(\frac{1-p_1}{p_1}y + \overline{\omega} + c)$.

We first show that the critical time increases as r increases: $t_2^* > t_1^*$. In other words, the time interval where agent is exposed to maximum termination threat gets larger.

Lemma 23 $t_2^* > t_1^*$.

Proof. Define

$$\digamma(\theta) \equiv (\frac{p_0 c}{p_1 - p_0} - \overline{\omega})(\frac{((r+\gamma)\theta + 1)e^{-(r+\gamma)\theta} - 1}{(r+\gamma)^2} - \frac{((r+\gamma(1-p_1))\theta + 1)e^{-(r+\gamma(1-p_1))\theta} - 1}{(r+(1-p_1)\gamma)^2}) \tag{2.65}$$

Claim: $\digamma(\theta) > 0$ for all $\theta > 0$.

To prove this claim, consider

$$\digamma'(\theta) = (\frac{p_0 c}{p_1 - p_0} - \overline{\omega})\theta(-e^{-(r+\gamma)\theta} + e^{-(r+\gamma(1-p_1))\theta}) > 0$$

Since $\digamma(0) = 0$, then $\digamma(\theta) > 0$ for all $\theta > 0$. This proves the claim.

Now consider

$$\begin{aligned} \frac{\partial \Psi_1(t^*)}{\partial r} &= (\frac{p_0 c}{p_1 - p_0} - \overline{\omega})\frac{((r+\gamma)(T-t^*) + 1)e^{-(r+\gamma)(T-t^*)} - 1}{(r+\gamma)^2} - \\ & (1-p_1)(y - c - \overline{\omega})\frac{((r+\gamma(1-p_1))(T-t^*) + 1)e^{-(r+\gamma(1-p_1))(T-t^*)} - 1}{(r+(1-p_1)\gamma)^2} \end{aligned}$$

Let $\theta = T - t^* > 0$. Then (2.56) implies

$$\begin{aligned}
&\frac{\partial \Psi_1(t^*)}{\partial r} > (\frac{p_0 c}{p_1 - p_0} - \overline{\omega})[\frac{((r+\gamma)(T-t^*)+1)e^{-(r+\gamma)(T-t^*)} - 1}{(r+\gamma)^2} \\
&- \frac{((r+\gamma(1-p_1))\theta + 1)e^{-(r+\gamma(1-p_1))(T-t^*)} - 1}{(r+(1-p_1)\gamma)^2}] \\
= \quad & F(T-t^*) > 0
\end{aligned}$$

where the final inequality uses the claim.

Using (2.42) and the Implicit Function Theorem, we have

$$\frac{\partial t^*}{\partial r} = -\frac{\partial \Psi_1(t^*)/\partial r}{\partial \Psi_1(t^*)/\partial t^*}$$

Since $\partial \Psi_1(t^*)/\partial t^* > 0$ and $\partial \Psi_1(t^*)/\partial r > 0$ we conclude that $\frac{\partial t^*}{\partial r} < 0$. Therefore $t_2^* < t_1^*$.

■

We will show that the termination threat is weakly increasing in r. First we need a few preliminary results.

Definition 24 *Define*

$$\Omega(T) \equiv ((T-t)(\gamma+r)+1)e^{-(\gamma+r)(T-t^*)} - 1)$$

and

$$\Lambda(t^*) \equiv \frac{((T-t)(\gamma+r)+1)e^{-(\gamma+r)(T-t^*)} - (\gamma+r)(t^*-t) - 1}{(\gamma+r)^2}$$

Lemma 25 $\Omega(T) < 0$ *for all* $T > 0$.

Proof. $\Omega'(T) = -(\gamma+r)^2 e^{-(\gamma+r)(T-t^*)}(T-t) < 0$. Since $\Omega(0) = 0$, $\Omega'(T) < 0$ implies $\Omega(T) < 0$ for all $T > 0$. ■

Lemma 26 $\Lambda(t^*) > 0$, $\forall t^* < T$.

Proof. $\Lambda'(t^*) = \frac{\Omega(T)}{\gamma+r}$. By Lemma 25 , $\Lambda'(t^*) < 0$. Also since $\Lambda(T) = 0$, we conclude that $\Lambda(t^*) > 0$ for all $t^* < T$. ■

Note that $q_i(t)$ depends on the agent's continuation payoff $\widehat{u}_i(t)$, as defined in (2.46). We prove that $\hat{u}(t)$ is decreasing in r.

Lemma 27 $\frac{\partial \widehat{u}(t)}{\partial r} < 0$, *where* $\widehat{u}(t)$ *is defined in (2.46).*

Proof. Using (2.46), we get

$$\begin{aligned}\frac{\partial \widehat{u}(t)}{\partial r} &= -\frac{1-p_1}{r^2 p_1}(y - \frac{p_1 c}{p_1 - p_0})(1 - e^{-r(t^*-t)}) + \\ &\quad \frac{1-p_1}{r p_1}(y - \frac{p_1 c}{p_1 - p_0})(t^* - t)e^{-r(t^*-t)}\frac{\partial t^*}{\partial r} + (\frac{\partial \hat{u}(t^*)}{\partial r} - \hat{u}(t^*)(t^* - t))\frac{\partial t^*}{\partial r}e^{-r(t^*-t)})\end{aligned} \tag{2.66}$$

Since $\frac{\partial t^*}{\partial r} < 0$, the second term in equation (2.66) is negative. In the last term, we have:

$$\begin{aligned}&\frac{\partial \hat{u}(t^*)}{\partial r} - \hat{u}(t^*)(t^* - t) \\ =\ & \frac{1}{(\gamma + r)^2}[((T - t^*)(\gamma + r) + 1)e^{-(r+\gamma)(T-t^*)} - 1 - (\gamma + r)(t^* - t)e^{-(r+\gamma)(T-t^*)}] \\ =\ & \frac{((T - t)(\gamma + r) + 1)e^{-(\gamma+r)(T-t^*)} - (\gamma + r)(t^* - t) - 1}{(\gamma + r)^2}\end{aligned}$$

From Lemma 26, this expression is positive. This proves that the last term in equation (2.66) is negative. Therefore we conclude that $\frac{\partial \widehat{u}(t)}{\partial r} < 0$. ■

We are finally ready to prove that the termination threat increases.

Proposition 28 $q_2(t) \geq q_1(t)$.

Proof. Observe that for all $t \in [0, t_2^*]$, $q_1(t) = q_2(t)$. Also for all $t \in (t_2^*, t_1^*]$, $q_1(t) < q_2(t)$ since $q_1(t) < 1$ but $q_2(t) = 1$ on this time interval. Finally, since by Lemma 27, $\widehat{u}_2(t) < \widehat{u}_1(t)$, we conclude that $q_2(t) > q_1(t)$ for all $t \in [0, t_2^*]$. Therefore, $q_2(t) \geq q_1(t)$. ■

Unfortunately, we do not have an unambiguous result ofor $w(t)$. Consider $w_1(t)$ and $w_2(t)$ in (4.28) and (4.34). Observe that $w_1(t) = w_2(t)$ for all $t \in [0, t_2^*]$.But for all $t \in (t_1^*, t_2^*)$, the relationship between $w_1(t)$ and $w_2(t)$ is again unclear[20]. Finally, to

[20]For the time interval $[t_2^*, t_1^*]$, the relationship between $w_2(t)$ and $\overline{w}_1$ is not clear. This is again

compare $w_1(t)$ and $w_2(t)$ over the time interval (t_1^*, T), we determine how $\widehat{u}(t)$, which is derived in (2.37), changes as r changes:

$$\frac{\partial \widehat{u}(t)}{\partial r} = (\frac{p_0 c}{p_1 - p_0} - \overline{\omega}) \frac{((T-t)(\gamma + r) + 1)e^{-(r+\gamma)(T-t)} - 1}{(\gamma + r)^2} \tag{2.67}$$

Since

$$\frac{\partial \widehat{u}(t)}{\partial r} = (\frac{p_0 c}{p_1 - p_0} - \overline{\omega}) \frac{\Omega(T)}{(\gamma + r)^2},$$

by Lemma 25, we conclude that $\frac{\partial \widehat{u}(t)}{\partial r} < 0$. Then $\hat{u}_2(t) < -\hat{u}_1(t)$, i.e. $w_2(t) > w_1(t)$.

We summarize the discussion:

Theorem 29 *As long as condition (2.56) holds, the termination threat (weakly) increases, and the critical t^* decreases, as r increases.*

c. Low productivity

If (2.55) holds, $\Psi_1(t) > 0$ for all t. Therefore severe threat is optimal over the entire horizon. The impact over the equilibrium path $\Psi_1(t)$ can be found by considering:

$$\begin{aligned}
&\frac{\partial \Psi_1(t)}{\partial r} = (\frac{p_0 c}{p_1 - p_0} - \overline{\omega}) \frac{((r+\gamma)T + 1)e^{-(r+\gamma)(T-t)} - 1}{(r+\gamma)^2} \\
&-(1-p_1)(y - c - \overline{\omega}) \frac{((r + \gamma(1-p_1))(T-t) + 1)e^{-(r+\gamma(1-p_1))(T-t)} - 1}{(r + (1-p_1)\gamma)^2} \\
> \; & (\frac{p_0 c}{p_1 - p_0} - \overline{\omega})(\frac{((r+\gamma)(T-t) + 1)e^{-(r+\gamma)(T-t)} - 1}{(r+\gamma)^2} \\
&- \frac{((r + \gamma(1-p_1))\theta + 1)e^{-(r+\gamma(1-p_1))(T-t)} - 1}{(r + (1-p_1)\gamma)^2})
\end{aligned}$$

where the last inequality follows from the observation that (2.56) implies (2.57). Now let $\theta = T - t$ in (2.65). Then the last line in the above inequality is $\digamma(T-t)$. Since we have verified in the proof of Lemma 23 that $\digamma(.) > 0$, this implies $\frac{\partial \Psi_1(t)}{\partial r} > 0$.

---

because of the same reason we explained in the note 2.4.2. In panel [b], we illustrate a small shift in the upward sloping of the wage scheme: $w_2(t) < \overline{w}_1$ for all $[t_2^*, t_1^*]$. However if there is a larger shift, it would be possible to have $w_2(t) > \overline{w}_1$ for $t \in [t_3^*, t_1^*]$, but $w_2(t) < \overline{w}_1$ for all $t \in [t_2^*, t_3^*]$. It cannot be possible to have a huge shift such that $w_2(t) > \overline{w}_1$ for all $t \in [t_2^*, t_1^*] : \overline{w}_2 > w_2(t)$ at least for some $[t_2^*, t_2^* + \varepsilon]$; and $\overline{w}_1 = \overline{w}_2$. Therefore $\overline{w}_1 > w_2(t)$ holds at least over a small interval.

Therefore, as r increases, $\Psi_1(t)$ shifts up in a fashion as illustrated in panel [a] and the condition in (2.55) continues to hold. In other words, given that y is sufficiently low, severe threat will still be optimal at all times as r increases. But since, from (2.67), $\partial\hat{u}(t)/\partial r < 0$, the wage profile will shift up as depicted in panel [a].: optimal $w(t)$ increases at all times. Thus, we have the following:

Theorem 30 *If condition (2.55) holds, then the success wage increases as r increases.*

d. Summary of the effect of a change in r

The threat of termination always (weakly) increases as r increases. For the cases where the termination threat *strictly* increases, the intuition is that as the future becomes less important, the present value of the loss from getting fired decreases on the agent's side, i.e. incentives to work hard gets weaker. Therefore, the principal increases the threat, in order to induce high effort. However, if the principal does not have the opportunity to make such adjustment on the optimal threat because the threat is already maximal, then she offers higher wages to induce high effort as r increases.

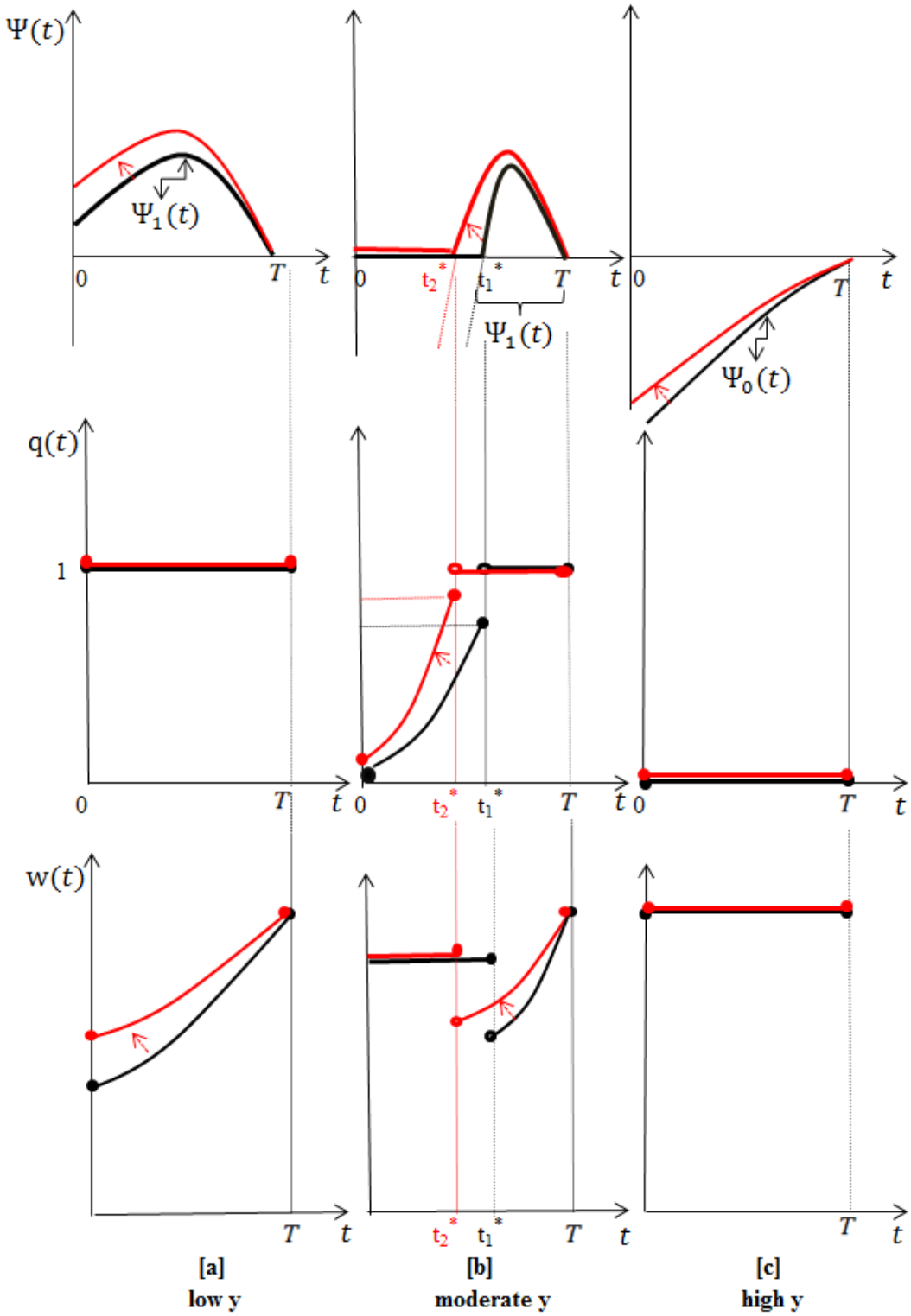


Figure 2.9: A Change in Discount Rate

2.5 Concluding Comments

Lazear (1979) showed that an increasing wage-schedule is the solution to the dynamic moral hazard problem when the principal can commit to a long-run contract. This important result explained real life practices including the frequent use of mandatory retirement policies. In contrast, we have studied the dynamics of alternative incentive tools in a moral hazard model with a fixed contract horizon when the principal's commitment ability is limited. We found conditions under which a termination threat is optimal. As in Lazear's model, the optimal incentives, in terms of both termination threat and performance wage, get stronger as the agent gets more senior. In particular, we obtain an upward sloping wage profile, even though the principal offers a sequence of short-run contracts.

Performance based firing is a common practise. In our model, the termination threat reduces the need for a performance based wage, and will be an efficient device under particular conditions. Specifically, the principal will use a termination threat if the agent expects a significant surplus from the relationship, relative to what the principal expects. The termination threat is more likely to be optimal when the agent's probability of success when shirking increases, or as the interest rate increases. However the termination threat is less desirable as the agent gets more productive. In that case, incentive provision is achieved by higher performance pay.

Chapter 3

Incentivizing CEOs via Pay and Forced Turnover: Do Tenure and Managerial Ability Matter?

3.1 Introduction

Boards, senior management teams, and shareholders are constantly struggling to design the right type of executive compensation plans. This is a typical agency problem: the executive (the agent) must be motivated to act in the best interests of the shareholders (the principals), who have delegated their authority to the board of directors. The theoretical principal-agent literature has mainly focused on the use of performance-related pay as an incentive device. However, incentives can also be provided by the threat of termination of employment following poor performance. These two instruments interact: a termination threat provides good incentives only if the expected future pay within the firm is significantly higher than the outside option.

As Lazear (1981) argues, principal-agent theory predicts that wages should increase with tenure to provide good incentives. On the other hand, it is less clear whether the optimal termination threat will also be increasing, i.e., if poor performance is more likely to result in termination for senior than for junior executives. Moreover, as Lazear's (1981) upward sloping wage schedule causes the senior executive's compensation to exceed her marginal product, the principal has the incentive to fire her even if she behaves well. Therefore, Lazear's (1981) optimal contract must involve a long-run commitment to not fire the executive unless her performance is bad. However, long-term contracts are not common in CEO labour markets; performance is mostly evaluated annually and contracts are renewed every year.

In Olcay (2012), I consider a dynamic principal-agent model with limited commitment to study the optimal contracts with incentive pay and termination threat. Even though the model is designed to study the incentive pay (i.e. pay-performance sensitivity), due its simplifying assumptions, the interpretation of its results regarding incentive pay is applicable to the total pay as well. The model predicts that the use of both incentive devices, incentive pay and termination threat, increases with tenure. A termination threat is included in the short-term contract if and only if the agent's expected future surplus from the relationship is sufficiently high compared to the principal's expected future gain. Moreover, at a given point in time, the observed productivity ("ability") of the agent affects the optimal use of the two incentive devices in opposite ways. The higher is the agent's ability, the higher should be the wage conditional on good performance, and the smaller should be the risk of termination conditional on bad performance.

The focus of our empirical study is first, to which extend pay and forced turnover are used as incentive devices for CEOs; second, whether there is a significant relationship of job tenure and managerial ability to these devices. We also consider how ability and tenure interact in the provision of incentives: does managerial ability influence the sensitivities of CEO pay and forced turnover to tenure?

Several previous theories also have implications for the relationships between tenure, managerial ability, and the two incentive devices, as well as tenure and ability impacts on the performance sensitivities of these devices. The most prominent are the learning theory of Murphy (1986), the carrer concerns model of Gibbons and Murphy (1992), and the entrenchment theory of Hermalin and Weisbach (1998). Table 3.1 summarizes the predictions of these theories. We start with discussing the predictions of each theory regarding the relationships of tenure with the two incentive devices.

Murphy (1986) proposes an adverse selection model where the executive's ability is not known to the shareholders. Then the optimal contract should be designed to facilitate the shareholder's learning about the executive's ability. First, pay-performance sensitivity decreases as the CEO's tenure increases: it is optimal to offer a contract

which is more sensitive to performance during the early years of tenure when the shareholders have a only a little information about the CEO's ability. As time passes, the need to tie pay directly to performance decreases as the shareholders learns more about the true ability by observing and assessing the CEO's performance. Second, as beliefs about ability are updated through time, pay level increases. Murphy (1986) does not provide direct predictions for the probability of forced turnover.

Gibbons and Murphy's (1992) model also has adverse selection, where a new CEO's true ability may not be perfectly known, and limited commitment. However, unlike Murphy (1986), the presence of a competitive executive market creates "career concerns" for the young executive whose true ability is not known. Therefore, in the early years of her tenure, the risk-averse executive is motivated to work hard to establish her reputation, but, at the same time, is reluctant to bear too much risk. This results in relatively a lower pay-performance sensitivity as compared to later years in her tenure where both board need to offset the reduction in her career concerns and risk-sharing becomes easier.

The adverse selection model in Gibbons and Murphy (1992) helps us to derive conclusions regarding the effect of performance on the likelihood of forced turnover. As Allgood and Fareel (2000) point out, since a board has high uncertainty about the CEO's ability at the start of her tenure, the board is less willing to punish when performance falls below the expected level. However, through time, learning leads to a decrease in the variance of expected performance and hence a performance that would not have triggered termination earlier may do so later. In other words, the impact of performance on the probability of forced turnover increases through tenure.

The entrenchment theory of Hermalin and Weisbach (1998) differs dramatically from standard principal-agent theories of optimal contracting. Not surprisingly, it produces quite different predictions about the relationship between tenure and the two incentive tools. If the executive gets more entrenched and dominates the board as she gets senior in her tenure, the board's ability to fire her in case of poor performance decreases. Moreover, the entrenched manager who dominates the board can in effect set her own

pay, and can make it less sensitive to firm performance (Bebchuk and Fried, 2004). Under the entrenchment theory, the longer the executive serves, the more entrenched she becomes, resulting in not only a higher level of pay and lower pay-performance sensitivity, but also in a lower likelihood of forced turnover and weaker performance-forced turnover relationship.

Regarding the relationship between managerial ability and incentive provision, the career concerns and entrenchment theories also present essential insights. In terms of explicit incentives, in the form of total pay, and ability, Gibbons and Murphy's (1992) career concerns theory makes an intuitive prediction. Since higher profits reveal good news about her ability, a CEO may negotiate for a raise after strong profits if she has enough bargaining power in the board. In this way, the CEO can capture the whole surplus created by the good news. But if low profits reveal bad news about the CEO's ability, the board may want to negotiate for a lower level of CEO pay. However, if the CEO has enough bargaining power in the presence of long-term contract, the shareholders have to bear the full negative surplus from bad news. A formalization of this perspective by Harris and Holmstrom (1982) suggests that when a CEO has enough bargaining power, shareholders suffer more from bad news about the CEO's ability than they benefit from good news. The entrenchment theory (Bebchuk, Fried, and Walker (2002) and Bebchuk and Fried (2003)) would explain the positive ability-pay relationship again by higher ability CEOs being more likely to get entrenched and increase their own pay. An alternative explanation is provided by a competitive talent assignment model of Gabaix and Landier (2008) suggesting that the most skilled CEOs are matched with the largest firms and earn the highest salaries.

Regarding the likelihood of forced turnover, in a career concerns model, since it relies on learning, a negative relationship between performance related turnover and ability arises: highly talented CEOs are identified as those whose performance is less uncertain and thereby less likely to be informative. Therefore, higher ability is associated with weaker performance-forced turnover relationship. Similarly, highly talented

CEOs would be immune to performance-related dismissals according to the entrenchment model (Bebchuk, Fried, and Walker (2002) and Bebchuk and Fried (2003)): CEOs of higher abilities are identified as more powerful and hence more likely to face a passive board of directors. In other words, for a higher ability and hence more entrenched CEO, probability of forced turnover is both weaker and less sensitive to performance.

To summarize, a number of theories have predicted a positive relationship between explicit incentives in the form of total compensation and tenure. The impact of tenure on the relationship between pay and performance is positive in carreer concerns model and Olcay (2012), but negative in learning and entrenchment models. Ability, on the other hand, effects total pay positively in all models. However, it affects pay-performance relationship negatively in entrenchment model whereas in Olcay (2012) it has the opposite impact. In terms of probability of forced turnover, tenure has a positive impact in Olcay (2012), negative impact in entrenchment model. Sensitivity of this device to performance is, however, positive in both carrer concerns and Olcay (2012), but negative in entrenchment model. On the other hand, all theories agree on their predictions concerning adverse impact of ability on the probability of forced turnover and its performance sensitivity. In Sections 3.3 and 3.4, we investigate these hypotheses empirically. The current theories provide no specific prediction on whether and how managerial ability affects tenure sensitivities of CEO pay and likelihood of CEO turnover, but I also explore this issue empirically. Future research on executive contract design may shed light on this issue from a theoretical perspective.

The empirical analysis in this paper is two fold. Analysis-I, in Section 3.3, focuses on CEO pay as an incentive tool in executive contracting and analyzes how it changes through tenure and managerial ability. The analysis of pay-to-performance sensitivity is left for future research. Analysis-II, in Section 3.4, concentrates on forced CEO turnover due to bad performance, and investigates how the use of this incentive device depends on tenure and managerial ability. Two empirical proxies for managerial ability are used: media visibility, which attempts to capture reputational aspects of the "outsider" CEO's ability and is hand-collected through counting articles in top business journals; and the

age when the "insider" CEO is promoted to the position. The idea is that higher media visibility and lower age at the time of promotion are both signals of higher managerial ability. Data on executive compensation, firm performance and control variables, both at the CEO and firm level, is drawn from the COMPUSTAT database. Data for forced turnover is hand-collected by reading news items which are found through Factiva and Google web search. The sample used in Analysis-I contains a larger data set with all firms in S&P 1500 (i.e. S&P 500-LargeCap, S&P 400- MidCap and S&P 600-SmallCap) between 1998-2008, except when the ability proxy is media visibility which is created only for firms in the S&P 500- LargeCap for the same period. The sample used in Analysis-II, however, is smaller and includes firms only in S&P 500-LargeCap over the period 1998-2008.

My empirical results suggest that CEO performance and the two devices; i.e. pay and threat of forced turnover, are stongly connected. In other words, these tools are indeed used as incentive mechanisms. Also, I find that, as they are more senior, CEOs receive higher pay. However, there is no support for whether the likelihood of forced turnover changes through a CEO's tenure. For outsider CEOs, reputation, as a measure for managerial ability, is shown to increase pay where the impact is strongest as more current reputational measures are used. At the same time, it decreases tenure sensitivity of pay only when it is proxied by the most current (i.e. short-term) reputational measure. As an alternative incentive tool for CEOs, the likelihood of forced turnover is decreasing in managerial ability where ability is proxied by age-at-promotion for insiders and by reputation for outsiders. Reputation is also found to a weaker impact on forced turnover as it measures by less current measures.

The existing empirical literature on the determinants of executive pay is large and we cannot give a full review here. Early studies concentrated on the relationship between CEO pay and company performance (Coughlan and Schmidt (1985), Murphy (1985, 1986), Jensen and Murphy (1990), Abowd (1990), Leonard (1990)). These articles show that firm performance is largely positively related to pay-performance sensitivity after controlling for risk. Others studies are focused on whether CEOs are rewarded for

performance which is measured relative to the market or industry (Antle and Smith (1986), Gibbons and Murphy (1990)).

A growing literature studies company financial performance following CEO turnover. An important early finding as noted by Coughlan and Schmidt (1985) and Warner, Watts and Wruck (1988), is that a firm's net of market performance and the probability of turnover are inversely associated. However, these studies document that managers are rarely fired due to poor performance even though the negative relationship between stock price performance and turnover following bad performance has generally been interpreted as evidence that boards fire poorly performing CEOs. Later research provides evidence for turnover rates that have become more sensitive to performance (Huson, Parrino and Starks 1998). This leads to the idea that termination, as an incentive tool, is becoming more important threat over the top executives.

A related finding by Weisbach (1988) is that the magnitude of the turnover-performance relation is strongest in companies dominated by independent outside directors. Parrino (1997), in an attempt to confirm the common use of relative performance evaluation for top executives, shows that the outside replacements are mostly used by the companies which perform relatively poorly as compared their peers. These studies indicate that poor firm performance is the single most important determinant of involuntary turnover.

Another line of empirical research diregards the theoretical implications of optimal contract design, and instead studies the relationship between forced turnover and the entrenchment effect of top managers' equity ownership. Murphy and Zimmerman (1993), Huson, Parrino, and Starks (2001), Denis, Denis, and Sarin (1997) and Hadlock and Lumer (1997), Goyal and Park (2002) and Dahya, Lonie, and Power (1998) investigate the impact of corporate governance on performance-turnover relationship. The probability of top executive turnover is found to be negatively related to the ownership stake of officers and directors. For example, Dahya, Lonie, and Power (1998) report that forced top management turnover is more frequent for executives who own less than 1% of their firms' equity.

The paper proceeds as follows. Section 3.2 describes the data sources and variables. Sections 3.3 and 3.4 present the empirical tests and results regarding pay and forced turnover, respectively. Section 3.5 concludes.

3.2 Data Description

In order to analyze the impact of CEO tenure and managerial ability on pay and the likelihood of forced turnover, I construct a dataset of the CEO labour market which contains detailed information on CEO turnovers. The dataset also includes empirical proxies for CEO ability. In this section, I explain how the dataset is constructed and the collection process for each of the variables used in this paper.

3.2.1 Sample Selection

The CEO succession data is hand-collected and includes all firms in S&P 500-LargeCap for the period, 1998-2008. ExecuComp provides information on the five top executives of all firms in the S&P 1500 (S&P LargeCap, S&P MidCap and S&P SmallCap). I recognize a turnover for each year in which the CEO is identified in ExecuComp changes.[1]

To identify forced departure, I carefully search news items on the Lexis-Nexis Academic Universe and Factiva, for each CEO change. Changes accompanied by poor performance are identified as forced turnover. These news items openly state that the prior stock price or accounting returns have been declining in the past quarters. Also for each succession in the sample, I hand collect exact announcement dates which are the earliest dates of the news about incumbent CEO departure and successor CEO appointment. All other news which explicitly mention that CEO has departed the company because she became a CEO at another firm, forced out for different reasons (for example improper use of corporate funds, violation of Security Exchange Act, etc.), resigned due to conflict with board, left due to health reasons, deceased or retired are

[1] To identify turnover, Parrino (1997), Huson, Parrino, and Stark (2001), Huson, Malatesta, and Parrino (2004), use Forbes surveys. Jenter and Kanaan (2006) also use ExecuComp for the period 1993-2001.

not included. My method of identifying turnover is similar to that of Weisbach (1988), Denis and Denis (1995).

By using news items, I form different categories for reasons of CEO departure. Table 3.2 gives information about the number of CEO departures for different reasons during 1998-2008. The sample consists of 542 CEO changes out of 4769 firm-year observations during 1998-2008. The total turnover rate is 11.4%.[2] Using this detailed information regarding CEO turnover as summarized in Table 3.1, I construct the forced turnover data in Table 3.5. Forced 1 is constructed using the news items in which it is explicitly stated that the CEO is terminated for poor performance. This yields 105 CEO changes. For the rest of classification I follow Parrino's (1997) method. Such classification is important in a study of forced CEO turnover since CEOs are seldomly fired openly due to poor performance. I classify the remaining changes as "potential forced departure" if the departing CEO is less than the normal retirement age which is the mean age for CEO departure. In our sample, this is around age 61. In Table 3.5, Forced 2 includes CEO changes if the reason of departure is announced either as retired (but excluding those announcements which explicitly state that the departure is not related with performance), or if the reason is not specified, or the news item specifies the departure as unexpected or abrupt, and for all these observations the CEO is below the normal retirement age. This yields 107 more observations for forced turnover. Forced 3 is the sum of Forced 1 and Forced 2.

In Table 3.2, 39% of all CEO changes are performance related. This finding is in line with a total forced turnover rate of 36%, as found by Milbourn (2009) whose sample covers 1993-2005 and firms in S&P 1500. However, the total forced turnover rate in our sample is relatively higher than the one reported by Huson, Parrino, and Starks (2001) who found a forced turnover rate of 23.4% for the sample of executives listed in Forbes only and over the period 1989-1994. Table 3.2, however, reveals that while the turnover data in the sample at hand does not show a particular time pattern, there is significant time variation both in the rate of forced turnover and in the rate of overall

[2]This is consistent with previous studies, 12.2% (Parrino1997), 9.3% (Denis et al. 1997).

turnovers.

3.2.2 Measures for Firm Performance and Firm Level Controls

I enhance the dataset with several measures of firm operating performance, in addition to various firm level controls which have been identified as important measures in the CEO labour market. All measures listed on this section are at calendar year-end. Descriptive statistics for these variables are summarized in Table 3.4.

I mainly use three different measures of firm performance all of which are provided by COMPUSTAT: return on assets (ROA), return on equity (ROE) and ratio of Net Income to the book value of assets (NIA). For each of the performance measure, I create industry average counterparts. This helps to filter out industry specific shocks in the analysis of forced turnover. In the COMPUSTAT database, each firm has been specified with a four-digit SIC code. To identify the industry, I use the first two digits of the SIC code. By using the overall sample in COMPUSTAT (over 20,000 firms), I calculate industry averages for the firms in the sample at hand.

The main set of firm controls includes firm size (logarithm of total assets) and firm's dividend yield. All measures for these variables are taken from COMPUSTAT. As discussed by the previous studies (for example Gabaix and Landier (2008) and Tervio (2007)), firm size is an important variable on CEO labour markets. The dividend yield of the firm is used to control for a firm's riskiness.

3.2.3 CEO Pay, Tenure and CEO Characteristics

In order to examine the effect of tenure on a CEO's level of pay, I use the natural logarithm of the dollar value of CEO's total pay (ExecuComp's TDC1). This is defined as the sum of salary, bonus, long-term incentive plans, restricted stock, and stock appreciation rights.

Previous research suggests that CEO pay is a function of CEO age (see, for example, Milbourn (2003) and Chevalier and Ellison (1999). Therefore age is included as a control variable in the analysis. Data for CEO tenure is created by using data items in

Execucomp. Finally, I classify CEOs who had been with their firms for one year or less at the time of their appointments as outsiders. [3]As Table 3.4 summarizes, the median CEO in our sample has 5 years of tenure, is 55 years old and has been promoted from inside.

3.2.4 Measures for CEO's power on Corporate Governance

To control for CEO's potential power on corporate governance, which might have a significant impact on pay level and likelihood of turnover, there are two different variables used in the analysis. Using data items in ExecuComp, I create two dummy variables: Board Member is equal to 1 if the CEO has served as a director during the calendar year, and Share Ownership is equal to 1 if the CEO holds more than 5% of the firm's total shares. Table 3.4 indicates that 3 % of the whole sample contain firm-year observations with the firm's CEO holding either 5 % or more of the total shares. Also 97% of the firm-year observations, contain CEOs who are a board member.

3.2.5 Measures for CEO Ability

I construct three different empirical proxies for CEO ability. The first two proxies are based on the number of articles in which the CEO's name and company affiliation appear in major US and global newspapers and business journals in a given calendar year. The third proxy uses the age of the CEO at the time of promotion. Below we discuss each in detail.

The first empirical proxy, Press, measures the media visibility of a CEO and attempts to capture how the CEO's ability is perceived. To create Press variable, I count the number of articles containing the CEO's name and company affiliation that appear in the major business newspapers. A list of these sources is listed in the Appendix B. A similar proxy is previously used by Milbourn (2003), Francis, Huang, Rajgopal and Zang (2004), and Rajgopal, Shevlin, and Zamora (2006)). However, since "reputation",

[3]ExecuComp has missing observations for the data item "Date Joined the Company" which determines whether the CEO is an outsider. For the sample at hand, it has only been possible to categorize 5915 (out of 11804) of the observations in our sample.

as implied by Press variable builds in time, I extend this proxy further and create three different proxies out of it: Press (1year), Press (3years) and Press (5years) refer to the number of articles during the previous year, last three years, and last five years prior to the current year. For example, in year 2005, Press (1 year) shows the article counts in year 2004, Press (3 years) shows the count between 2002-2004, finally Press (5years) between 1999-2004. With this extension, media visibility measures reputation in different terms; Press (1 year) refers to a relatively more current measure of reputation whereas Press (5 years) is a proxy for less current measure of reputation.

Using Lexis-Nexis database and restricting the search to the list of firms in Appendix B, I conduct text searches using both the CEO's last name and company name. The classification of media visibility variable uses the same methods as previously used by Milbourn (2003), Francis, Huang, Rajgopal and Zang (2004), and Rajgopal, Shevlin, and Zamora (2006)): CEOs with higher media visibility are more likely to be of higher ability.

Following Milbourn (2009), I construct a second empirical proxy, Good Press. This second measure aims to prevent any potential mistake that Press variable might cause since Press does not necessarily imply Good Press. Therefore I classify the articles in which CEO's name appear by connotation. Good Press only includes the number of articles with nonnegative tone. To do that, I first create a Bad Press variable, which includes the number of articles only with negative connation. To find the number of articles which Bad Press will include, I count the number of articles containing the CEO's name, company affiliation, and any of the words with a negative connotation that appear in the major U.S. and global business newspapers in a calendar year. The list of these negative key words, as included in Appendix B, is taken from Milbourn (2009).[4]Finally Good Press is defined by the difference between Press and Bad Press. As Milbourn (2009) points out, this overcomes the limitation of the standard approach (Milbourn (2003) and Rajgopal, Shevlin, and Zamora (2006)) which attempts to verify whether Press variable is highly correlated with Good Press only for a small and

[4]Milbourn (2009) created this list by randomly sampling 50 CEOs and reading articles about them.

randomly selected sample of CEOs. Similar to the previous studies, in the sample at hand, Good Press is highly correlated with Press (0.97). However in line with the cross sectional difference as Milbourn (2009) finds, the correlation is higher at the high end of the distribution of Press (0.81 for above median CEOs), and relatively lower at the low end of the distribution of Press (0.67 for above median CEOs). According to Table 3.4, the median CEO receives 6 positive media mentions during a year. This number is equal to 17 when media mentions are counted over three years and equal to 25 when counted over five years. However regardless of the tune of the mention, the median CEO name appears 8 times in the press over a year. Over 3 and 5 years, the median media visibility is equal to 20 and 29, respectively.

The final empirical proxy in our empirical analysis is Age-at-Promotion, which will be used in the regressions with insider CEOs only. The idea is that the firm has an opportunity to observe the performance of an insider executive for a significantly long time before she is promoted to CEO position. Therefore, the lower the insider CEO's age at the time of promotion, the higher her ability as perceived by the board, as compared to another insider CEO who has got the position at a later age. CEOs of higher ability will spend less time on the corporate ladder and given the potential reluctance of firms to promote younger executives due to well-known hurdles, a lower age at the time of promotion is indeed a good signal for higher ability. In this way, I intend to capture observed aspects of managerial ability, which cannot be achieved by our reputation proxy. To create this variable, I use ExecuComp's data items on current age and date when the executive became CEO and also use OneSource for missing data entries in ExecuComp. In our sample, asTable 3.4 indicates, the median CEO's age when she is promoted to the position is equal to 49.

Measuring managerial ability has in fact been a great challenge for empirical researchers. Among these, Hayes and Schaefer (1999) present an interesting approach to measure CEO ability. The method utilizes deriving implications for financial market responses to news of unexpected separations of the manager and firm. In contrast to such voluntary departures, they hypothesize that the average ability level of a sample of

managers who die suddenly should be lower than the managers who resign for a similar position at another firm. Hence differences in mean abnormal returns across these two groups can therefore be used to construct a measure of differences in managerial ability. However such analysis counts in infrequent events.

A recent method to create an empirical proxy for managerial ability is introduced by Demerjian et al. (2009). They use Data Envelopment Analysis (DEA) which yields a measure for managerial efficiency isolated from firm efficiency. DEA basically requires obtaining an efficiency score for the firm and then regressing it on variables that might affect efficiency other than managerial efficiency (e.g. market share). Finally, the residual out of the regression is treated as managerial efficiency. The advantage of DEA is that it can accomodate multiple inputs and outputs to measure efficiency. However, the results are highly sensitive to how the relative importance of the variables is specified and this cannot be statistically tested.

3.3 Analysis I: Relations of CEO Pay with Tenure and Managerial Ability

3.3.1 CEO Pay and Tenure

The theoretical background for the analysis in this section is based on the models as discussed in the Introduction. As explained, each model provides a consistent prediction in the sign or the slope of the relationship between CEO tenure and the level of CEO pay. Before presenting the results, first we discuss our empirical strategy which will be used to test the hypothesis on tenure-pay relationships.

a. Empirical Strategy

The econometric model specifications used in this section consider both linear and nonlinear relationships. The first specification assumes a linear relationship, hence uses the value of tenure (in years) whereas the second allows nonlinearity by including a quadratic term of this variable.

To deal with the potential problem of unobserved heterogeneity, I use a fixed-effects

(FE) model to study the relationship between tenure and pay. The baseline regression takes the following form:

$$\begin{aligned} w_{ijt} &= \beta_1 * Tenure_{ijt} + \beta_2 * (Tenure_{ijt})^2 + \beta_3 * Tenure_{ijt} * Performance + \\ &\quad \beta_4 * (Tenure_{ijt})^2 * Performance + \beta_5 * Performance + \beta_6 * Controls_{ijt} + \\ &\quad \alpha_{ij} + \epsilon_{ijt} \end{aligned} \tag{3.1}$$

where w_{ijt} is the log of annual total pay (TDC1) of CEO i, at firm j, in year t. The controls are both at the firm and at the CEO level and discussed in Section 3.2. Since in our data set, we do not observe CEOs working for more than one firm, in equation (3.1), α_{ij} is the time invariant and unobserved CEO-firm fixed effect (i.e. the "match effect") whereas ϵ_{ijt} is the idiosyncratic error term. Although the previously discussed theories do not suggest a particular prediction on the sign of $\widehat{\beta}_2$, they persistently point out a positive sign for $\widehat{\beta}_1$.

The advantage of the FE model is to eliminate the well-known bias created by Ordinary Least Squares (OLS) method. OLS model discards the unobserved differences across individuals that might yield incorrect estimates in the study of panel data. Even though it has limitations in investigating the effects of time invariant and observed individual characteristics, e.g. being an "outsider" CEO, FE model performs more efficiently, especially in the presence of time invariant unobserved heterogeneity which is correlated with the other explanatory variables, i.e. α_{ij} in equation (3.1). The validity of this approach regarding executive compensation literature was first pointed by Murphy (1985).[5] In this paper, employing a fixed-effect at the CEO-firm level helps us to interpret the results of the estimation as a measure of the relationship between pay and tenure for a given CEO-firm match.[6]

[5]Later examples with fixed-effects approach include Aggarwal and Samwick,(1999, 2003); Cichello, 2005.

[6]In line with fixed effects assumption, all regressions in this section report a correlation between vector of controls and individual fixed-effect.

It is important to note that by using equation (3.1), we investigate the determinants of total CEO pay. Since pay is in logs but the performance variable is in levels, $\widehat{\beta}_5$ helps us to calculate total pay-performance elasticities. Therefore, $\widehat{\beta}_3$ is an estimate of tenure impact on total pay-performance elasticities. As discussed in the introduction, several theories yield predictions regarding tenure impacts on pay-performance sensitivities. However, I leave calculating pay-performance sensitivities and testing related predictions for future work.[7]

b. Results

Using the econometric specification defined in equation (3.1), I run panel regressions to relate natural logarithm of CEO total pay to CEO tenure variables. I consider alternative measures for firm performance, namely ROA, ROE and NIA, and run the same regression three times with one performance measure at a time.[8] These measures are based on accounting returns which are widely used in the study of executive compensation literature due to lower noise they have as compared to stock prices.[9] In order to take into account potential common factors which affect all individuals in a given year, I add year dummies. Results of the regressions are presented in Table 3.5. Standard errors are corrected for both heterocedasticity and autocorrelation, and clustered at the CEO level.

In Table 3.5, the first specification considers Return on Assets (ROA) as a performance measure whereas the second and third use Return on Equity (ROE) and ratio of

[7]Calculating pay-performance sensitivity is a tedious task. The problem is, to get an accurate measure, sensitivities of both shares and options must be calculated. Shares sensitivity is a straightforward measure and is defined as the number of shares owned by the CEO, divided by total shares outstanding. However, options sensitivity is not easy to calculate. The difficulty arises because this sensitivity requires to calculate the prices of options outstanding which is not reported in the annual statements. A recent estimation technique to overcome this issue has been proposed by Core and Guay (2002).

[8]Companies are not uniform in terms of the use performance measures; the use of single performance measure is as common as the use of multiple performance measure. However as Murphy (1998) documents, even when multiple performance are used, they are "additive" and can be treated like separate plans. An example of additive incentive plan would be in which 80% of the cash compensation is based on net income and 20% is based on return on assets, with a separate schedule relating cash compensation to each performance measure.

[9]See Banker and Datar (1989).

Net Income to the book value of Assets (NIA), respectively. For each specification, I employ two sub-specifications: first assuming linearity in tenure-pay relationship, second allowing a potential non-linearity. The results of all regressions systematically report a positive and statistically significant relationship between level of CEO pay and tenure.

In terms of the control variables, as expected, the results in Table 3.5 suggest a positive and statistically significant relationship between firm performance indicators and CEO pay. Since the dependent variable, CEO pay, is in logarithms but all performance measures are in levels, it is possible to interpret the performance-pay elasticities using the fitted value of each performance measure evaluated at its sample median. For example, under non-linearity assumption, the related estimated coefficient implies that 1 % increase in ROA (ROE and NIA, respectively), corresponds to 0.06% increase in total pay (0.02% and 0.06%, respectively). [10] However, note that the cofficients of the interaction variables regarding tenure and performance in equation (3.1) is not significant.

The regression results, unsurprisingly, report that larger firms pay higher levels of pay; larger firms may employ better qualified and better paid managers (Rosen (1982); Kostiuk (1990)). Since firm size is defined as natural logarithm of sales, the estimated coefficient in Table 3.5, is directly interpreted as the elasticity of CEO pay to firm size. The results predict that, holding everything else constant, a firm that is 10% larger in size, depending on the econometric model specification, will pay its CEO about 20%-25% more.

CEO age has a marginally significant and positive impact on total pay. Calculating the elasticity of pay to age by evaluating the fitted value of age at sample median yields the following prediction: controlling for performance, tenure, firm size, risk and CEO's power in corporate governance, a CEO at the age of 55 will be paid 15% more relative to a CEO at the age of 50. One explanation for this finding might be that older CEOs

[10] These results could be contrasted with other studies which estimate pay-to-performance sensitivities using a change in share holder value as a performance variable. Pay-performance sensitivity, in a sense, represents the executive's "share" of value creation. Some classical examples are Jensen and Murphy (1990), Murphy (1993), and Yermack (1995). To illustrate a comparison with our results, Jensen and Murphy (1990), reports an estimated pay-to-performance sensitivity of 0.325% for a sample of Forbes executives from 1970-1988.

are more experienced and hence compensated by a higher level of pay. The other reason could be that older CEOs are likely to be more wealthier as compared to their younger peers; therefore they are more expensive to incentivize. Finally, I include controls for the CEO's corporate governance, i.e. Board Member and Share Ownership, both of which are defined in Section 2. However, the results report no significant relationship between these controls and CEO pay.

Figure 3.1 plots the relationship between log of total CEO pay when we assume a non-linear tenure-pay relationship, i.e. it illustrates the outcomes of Models 1, 2 and 3 under second specification, whose results are presented in the second column of each model in Table 3.5. Figure 3.1 is obtained in the following way: for each model, the predicted values for pay are obtained through the estimated coefficients of second specification in Table 3.5. Consistent with the previously discussed theoretical models, the figure suggests that tenure increases pay. Although the theory does not provide a particular prediction, observe that the impact of tenure on pay has a decreasing rate: pay is highly sensitive to tenure in first years but in later years the impact of tenure on pay decreases.

3.3.2 The Impact of Ability on CEO Pay and Tenure-Pay Relationship

This section focuses on how far managerial ability affects CEO pay and the tenure-pay relationship. All theories discussed in the Introduction agree on a common prediction: higher managerial ability is associated with higher pay regardless of whether information on ability is imperfect (as in learning and career concerns models) or not (as in Olcay (2012)), or whether no particular assumption is made on the nature of such information (as in "entrenchment" hypothesis).

Throughout this section, I employ the three empirical proxies introduced in Section 3.2: the two media visibility proxies, i.e. Press and Good Press, to capture the reputational aspect of managerial ability; and Age-at-Promotion which targets more observable dimensions of ability. As discussed earlier, the age of an insider CEO may function well as a proxy since the younger the CEO is at the age of promotion, the

higher her managerial ability will be as evaluated by the board. For outsider CEOs, however, we employ media visibility to proxy managerial reputation.

Before dicussing our empirical specification and results, we focus on the descriptive statistics for the top and the bottom quartiles of the sample, to get a first-hand idea about the relationships we are looking for. Table 3.6 focuses on the media visibility proxies, Good Press and Press, and contrasts the statistics of the two extremes for a given proxy. Table 3.7 replicates this statistical analysis with reference to Age-at-Promotion. In Table 3.6, Panels A, B and C summarizes information about sample median of each proxy measured over 1 year, 3 years and 5 years, respectively. The panels also include information about sub-sample median of the variables, such as tenure, compensation and performance. Regardless of the type of proxy, used to create the two extreme groups, median tenure is equal to 5 years. Not surprisingly, median CEO's performance is always higher in the top quartile.

Our analysis in this section also explores how far ability affects tenure sensitivity of pay. Therefore, in Figure 3.2 we provide a brief idea about the nature of the data where the two groups, namely the top and the bottom quartiles, are compared in terms of the percentage change in log total compensation as the median CEO's tenure increases from "less than 5 years" to "more than 5 years". The comparison suggests that median CEO at the top quartile experiences a higher percentage change in compensation as compared to median CEO at the bottom quartile, when ability is proxied by either Press (1 year) or Press (3 years), or Age-at-Promotion. However, in terms of the other proxy measures, we have the opposite conclusion.

a. Empirical Strategy

In purpose to explore the impact of managerial reputation over pay and tenure-pay relationship, I again use FE model where equation (3.1) is modified in the following way:

$$
\begin{aligned}
w_{ijt} &= \beta_1 * Tenure_{ijt} + \beta_2 * (Tenure_{ijt})^2 + \beta_3 * Ability_{it} + \\
&\quad \beta_4 * (Ability_{it} * Tenure_{ijt}) + \beta_5 * (Ability_{it} * Tenure_{ijt}) + \beta_6 * Controls_{ijt} + \\
&\quad \alpha_{ij} + \epsilon_{ijt} \qquad (3.2)
\end{aligned}
$$

In line with the previously discussed theories, we expect to find $\widehat{\beta}_3 > 0$. Even though our theoretical discussion does not provide a particular prediction, we also test whether ability has an impact on tenure-sensitivity of CEO pay, i.e. whether $\widehat{\beta}_4$ is statistically different than zero.

b. Results

We start with outsider CEOs and hence use our media visibility measures as proxies for ability in equation (3.2). The results of the regressions where we estimate equation (3.2) using FE method are presented in Tables 3.5-3.10 and Tables 3.11-3.13 where Good Press and Press is respectively employed. All specifications include year dummies whose coefficients are suppressed. Standard errors are clustered at CEO level. Table 3.5 employs the media visibility variable measured over 1 year ("short term") while Table 3.5 uses the same variable measured over 3 years ("medium term") and Table 3.10 over 5 years ("long term"). Furthermore, for each model, I run three separate regressions, each with a different performance measure. All specifications include firm and time controls whose estimated coefficients are suppressed. Regardless of whether Press or Good Press is used as an empirical proxy, the estimation results indicate that reputation, unless measured over a "long term", has a significant and positive effect on pay.

Focusing on the "short term" measure of reputation in Table 3.5, where proxy variable is Good Press, we find that one unit increase in the number of media mentions increases CEO pay by 1.22 % when ROA is used as a performance measure (by 1.19 % and 1.18 % when ROE and NIA is used as performance measure, respectively). As expected, the results in Table 3.11, where reputation is measured by total media

mentions (i.e. Press) report a lower impact of reputation on CEO pay: one more media mention will increase CEO pay by 0.48% when ROA is used as a performance measure (by 0.47% when either ROE or NIA is used as performance measure). The median CEO in our sample earns $6.7k. Therefore our results could be interpreted as follows: one more media mention for Good Press implies an increase in CEO pay by $80,000 on the average while for Press, it is associated with an increase of $40,000. On the other hand, the estimated coefficients of the interaction terms suggest a consistent finding across Tables 3.5-3.10 and Tables 3.11-3.13: reputation decreases the tenure sensitivity of CEO pay. More specifically, one more media mention, depending on the choice of performance measure, decreases the sensitivity by around 0.05 %.

The impact of reputation on the level of pay and tenure elasticity of pay, however, decreases as it is measured over a longer term. Examining the estimated coefficients for Good Press (3 years) in Table 3.5, we find that the sensitivity of pay to reputation is around 0.08 %. In other words, as the median CEO receives one more media mention, the total pay increases by 0.48% on the average. The alternative proxy measure, Press, again yields a slightly weaker reputation-pay relationship as reported in Table 3.12: one more media mention implies 0.38% increase in median CEO's pay.

To interpret the impact of this "medium term" measure over tenure elasticity of pay, first we observe that the estimated coefficient of tenure in both Table 3.5 and Table 3.12 is consistently insignificant in all speficiations. Therefore, we conduct significance tests to truly assess the regression results regarding the estimated coefficients of the interaction term. The related test results indicate that Good Press (3 years) and Tenure are jointly insignificant, then we conclude that our "medium term" measure of reputation does not have a significant impact on tenure elasticity of pay.[11] Finally, regardless of the choice of performance measure or the type of reputation proxy, i.e. Good Press or Press, we observe that a long term measure does not have a significant impact on pay or on tenure sensitivity of pay. Note that in Tables 3.5-3.12, ability as proxied by press variables does not have a significant impact when it interacts with

[11] Related p-values for F-test are 0.1719, 0.1897 and 0.1766 in Table 3.5 for model specifications 1, 2 and 3, respectively.

performance.

Finally, we employ our final proxy for managerial ability, Age-at-Promotion. Since this is a time-invariant variable, FE model cannot evaluate its impact on the dependent variable. Therefore, we employ OLS method to evaluate the effect of this fixed CEO characteristic. However, as mentioned earlier, this measure only proxies ability for insider CEOs. Table 3.14 reports the results where we find no significant relationship neither between ability and pay; nor between ability and tenure sensitivity of pay. As we discuss in the next section, however, Age-at-Promotion performs well in explaining likelihood of forced turnover and tenure-forced turnover relationship.

3.4 Analysis II: Relations of CEO Forced Turnover with Tenure and Managerial Ability

3.4.1 CEO Forced Turnover and Tenure

Our primary interest in this section is to explore how the likelihood of performance-related forced turnover changes through a CEO's tenure. As discussed in the Introduction, different theories would lead to different predictions on this relationship. Olcay (2012) and career concerns hypothesis (Gibbons and Murphy (1992)) both point out a positive relationship between CEO tenure and the likelihood of turnover following poor performance. However, the entrenchment hypothesis ((Bebchuk, Fried, and Walker (2002) and Bebchuk and Fried (2003)) predicts an inverse relationship if the CEO's corporate power improves over time as she stays at the same firm.

Figure 3.3 illustrates how the incidence of forced departures is distributed across time in our sample. The data shows that frequency of forced turnover significantly increases in the first 7 years of tenure. However after that, the frequency decreases.

a. Empirical Strategy

I employ a Cox semi parametric proportional hazard model to analyse the tenure impact on likelihood of forced turnover where I use several controls for commonly known determinants of turnover. The previous literature on the examination of the likelihood

of forced turnover includes a number of examples which extensively use logistic models (e.g Milbourn (2009), Subramaniyan, Chakrabortya and Sheikh (2002). However, given the nature of forced turnover data, logistic regressions might lead to inconsistent results. The main reason is the right-censored nature of the turnover data which logistic models cannot account for: by the end of 2008, which is the last year in our sample, there might be CEOs who have yet to leave their positions. The Cox model takes into account the fact that there can still be a risk of forced turnover in a given year even though the subject has not been exposed to failure event, i.e. forced turnover in our case, in that year. The advantage of the hazard model is that it provides the probability of forced turnover at a given point in time conditional on the fact that subject has survived up to this point. Moreover, the Cox model is semi-parametric and makes no assumption about the nature of the survival distribution.

The hazard function represents the probability of employment termination conditional on the duration lasting up to time t and takes the following form:

$$\lambda(t \mid X(t)) = \lambda_0(t)e^{\beta' X(t)} \tag{3.3}$$

where the X vector includes CEO and firm controls; CEO age, firm size and performance and several other controls for corporate governance. In equation (3.3), $\lambda_0(t)$ is the baseline hazard rate. Note that baseline hazard rate is a function of time only, it does not depend on the covariates. In other words, $\lambda_0(t)$ is the hazard rate for the covariate vector $X(t) = 0$. In order to estimate β, Cox (1972) model uses a semi-parametric technique. The technique defines the likelihood function as the sum of the probabilities that a CEO-firm relationship is forced to end at a particular time, given that one such event has occurred at time t.

b. Results

We start with a baseline specification using limited number of controls to estimate equation (3.3) where a Cox proportional hazard model is employed. The results are presented in Table 3.15. The failure event is defined as the incident of forced turnover,

which takes value 1 if there is forced turnover, and zero otherwise. In the estimations, I treat hazard rate as a dependent variable. Therefore, a positive coefficient implies a positive marginal impact on the hazard. In other words, if the estimated coefficient is positive for a covariate, this implies that the covariate induces a lower expected time as the CEO. Likewise, negative coefficients imply a longer expected time as CEO. Furthermore, the impact of a unit increase in a covariate over the hazard rate could simply be calculated by using $e^{\widehat{\beta}}$, where $\widehat{\beta}$ is the Cox estimated coefficient of the given covariate.

In all estimated models, the ratio of net income to the book of assets (NIA) is used as a performance variable. Including other alternative performance measures, namely Return on Equity (ROE) and Return on Assets (ROA), produce qualitatively similar results. In Table 3.11, Model 1 includes firm performance; but Model 2 includes industry-adjusted firm performance which is defined by the firm performance net of the industry average. For each firm, the industry is defined by using the 2-digit SIC code. Then the industry performance is calculated by averaging the performance of firms in the same industry with respect to the 2-digit SIC code. For each model in Table 3.15, I first specify a Cox model with firm performance measures only and without any controls. The second and third specification in each model includes controls, and controls with interaction terms, respectively. The other control variables are firm size, which is measured by natural logarithm of Net Sales, and CEO age. I also include industry performance when the firm performance is industry adjusted. For all specifications, standard errors are adjusted to incorporate the fact that multiple error terms can be attributed to each CEO, i.e. standard errors are clustured at the CEO level.

The results in Table 3.15 suggest that firm performance; whether adjusted or not, has a negative predicted coefficient. This is an expected result in fact: as performance increases, the hazard of forced turnover decreases. The size of the predicted coefficients also implies that this effect is economically significant. Regarding the second specification with controls, model estimates imply that CEO age and firm size have opposite effects on the hazard ratio, which is a consistent result across the two models. Holding

everything else constant, older CEOs are exposed to a lower hazard of forced turnover. However, the hazard increases as the firm size increases. Intuitively, larger firms attract a greater pool of CEOs. Therefore, for a large firm, it is much easier the replace a CEO if she does not perform well. On the other hand, the results show that industry performance is only marginally significant.

Table 3.15 also reports significant relations for the third specifications in each model where we allow interactions of age and firm size variables with the performance measures. Model 1 predicts that the interaction of firm performance with firm size is significant and positive. In other words, firm size increases the sensitivity of hazard of forced turnover to firm performance. However, Model 2 does not predict such significant result.

Finally, in order to increase the robustness of the Cox regression results, I take into account several other controls to capture CEO's power on corporate governance. For both models as described above, I include two additional variables: Board Member and Share Ownersip, both of which are defined in Section 3.2. Second, I control for whether the CEO is an outside hire. Table 3.16 presents the results. As compared to Table 3.15, observe that the levels of significance and estimated signs of variables, related with firm performance as well as firm size and age, do not change. An intuitive result in Table 3.16 is that, the hazard rate of forced turnover is inversely related with both measures of CEO power on corporate governance. Finally, the results indicate that being an outsider decreases the hazard rate of forced departure.[12]

The estimation results in Table 3.16 highlight the significance of the controls regarding CEO's corporate power. Therefore, to examine the time trend of survival probability, we depict the survival function of the last estimation where we have significant results in the absence of interaction terms. Figure 3.4a illustrates the estimated survival probability of Model 2 where the model includes adjusted firm performance as a measure. Model 1 produces a qualitatively similar estimated survival function. The

[12]Himmelberg and Hubbard (2000) consider this measure in their relative performance evaluation puzzle. Intuitively, the perceived ability level necessary to become a CEO as an outsider over an insider candidate with better knowledge of the firm's inner workings, is greater.

main conclusion from Figure 3.4a is that the survival function is downward sloping: as tenure increases, survival probability decreases. To investigate how tenure affects performance sensitivity of probability of survival, in Figures 3.4b we plot the estimated survival function at two different levels of performance, namely top 25% and bottom 25%. Figure 3.4c adds top 5% and bottom 5% as two other performance levels for a robustness check. We observe that as tenure increases, survival probability gets very close to zero for the lowest performer CEOs. This contradicts with the entrenchment theory which suggests as the CEO is more senior, she gets more entrenched and hence would be more likely to be retained even if is she performs poorly.

3.4.2 The Impact of Ability on Forced Turnover and Tenure-Forced Turnover Relationship

The theories discussed in the Introduction predict a common relationship between managerial ability and the likelihood of forced turnover. Specifically, the uniform prediction is that a CEO of higher ability will be exposed to a lower likelihood of forced termination. However, the theories differ regarding their assumptions on the information of ability. The entrenchment hypothesis does not make a particular assumption, but learning and career concerns models assume imperfect information of the board: true ability is not known but the "beliefs" on ability is updated each time the performance is observed. The model in Olcay (2012) assumes no such informational imperfection.

Before presenting the results of our hazard analysis, in Figure 3.5 we compare the forced turnover rates of the top and bottom quartiles of the sample population, where the categorization is made for a given empirical proxy. Regardless of the type of ability proxy employed, observe that the top quartile experiences a higher forced turnover rate as compared to the bottom quartile. However, the top quartile shows significant variation. As an example, when we focus on Good Press (1year) as an empirical proxy, among the first top 5% of the sample population, the forced turnover rate is 30%. But for the second top 5% and the third top 5% of the sample population the rates are 37% and 39%, respectively.

Empirical Strategy and Results

To analyse the impact of ability on the likelihood of turnover, again a Cox hazard model is used due to its advantages in the presence of censored data. Therefore, we estimate equation (3.3) while including ability as an additional covariate in the regression. I employ two set of regressions. First, I restrict the sample to outsider CEOs and use data on press variable as a proxy for reputation; and then focus on the subsample of insider CEOs while using Age-at-Promotion.

The results from the first set of regressions are summarized in Tables 3.17 and 3.18 where Good Press and Press are respectively used as a proxy. All standard errors are clustered at the CEO level. For each proxy, I employ three models: Model 1, including the short-term measure of the proxy; Models 2 and 3, containing the medium-term and the long-term measures, respectively. Moreover, each model has two specifications: one without any interaction terms, and one with where we allow an interaction of the ability proxy with the performance variable. Since firm performance, either adjusted or not, produces qualitatively similar results, I present only the estimation results with adjusted firm performance.

The results in Tables 3.17 and 3.18 suggest a significant impact for media visibility as a proxy for reputational aspect of managerial ability: higher media visibility implies lower estimated hazard of forced turnover. However, the estimated coefficient of a given Press variable in 3.18 is notably lower than its Good Press counterpart in Table 3.19. Not surprisingly, Good Press, which includes only "positive" reputation, decreases the hazard of forced turnover more than Press, whose small fraction accounts for "negative" reputation. A common result in Tables 3.17 and 3.18 is that reputation has the largest impact on the hazard of forced-turnover when it is proxied by the short-term measure. Furthermore, this impact monotonically decreases: reputation becomes less effective on the hazard of turnover as it is measured over a longer time window. Finally, as compared to Table 3.16, observe that the effects of firm performance, CEO age, industry performance and proxies for CEO corporate power, on hazard rate of forced turnover

do not change. However, interaction variables of ability and performance are consistently insignificant, i.e. at a given point in time ability does not significantly affect the performance-probability of forced turnover relationship.

Finally, we employ our last ability proxy, Age-at-Promotion to estimate the hazard of forced turnover. Table 3.19 reports the results where we consider two different models: one with firm performance and the other with adjusted firm performance as well as industry performance. Again the sample is restricted to insiders. In each model, we find that Age-at-Promotion is significantly and positively related to the hazard of forced turnover. Since higher Age-at-Promotion signals lower ability, our result implies that as ability increases, likelihood of forced turnover decreases. Also, we find no significant impact of ability regarding the relationship between performance-probability of forced turnover. Note that these conclusions reinforce our previous result where ability is proxied by reputation.

To sum, our analysis on ability-likelihood of forced turnover relationship supports the predictions of the previously discussed theoretical models: proxied either by reputation for outsider CEOs or by age of the CEO at the time of promotion for insiders, managerial ability decreases the likelihood of forced turnover. Moreover, the impact of reputation gets weaker as it is measured over longer terms; it is the short term measure which has the strongest effect on the likelihood of forced tunrover.

3.5 Concluding Comments

Both total pay and threat of dismissal in case of poor performance are common tools to provide incentives for top executives. This paper studies whether and how the two incentive tools are associated with tenure and managerial ability as well as the interaction between tenure and ability in terms of their impacts on these devices. I examine the predictions of several theories (namely, Murphy (1986), Gibbons and Murphy (1992), Hermalin and Weisbach (1998) and Olcay (2012)) to test the impacts of tenure and ability on both pay and likelihood of forced turnover.

Using a sample of firms in S&P 1500 and several controls at the CEO and firm level,

I find a positive relationship between tenure and total CEO pay, in support of all the theories discussed. However, the findings about time trend of survival probability rule out the entrenchment effects. This result, together with the strong connection of performance with the two devices, could be interpreted in favor of the optimal contracting models: incentives matter and firms use both instruments more strongly as the CEO is more senior in her tenure.

In order to explore the impacts of managerial ability on executive contracts, I use two different proxies. For outsider CEOs, a media visibility measure, aiming to capture reputational aspects of ability, is employed. But for insider CEOs, I use the age of the CEO at the time of promotion. The findings are parallel to the predictions of the theories which point out a common relationship between ability and the two incentive devices. Particularly, I find that reputation increases CEO pay where the impact weakens as the reputation is evaluated over a larger time window. However, regarding the likelihood of forced turnover, it has an inverse impact where the relationship similarly weakens as the reputation is evaluated over larger time windows. For insiders, ability, as proxied by the age at the time of promotion, is again significant in explaining its inverse relationship with the likelihood of forced turnover. These findings are in line with the theroretical literature on optimal contract design, specifically learning model (Murphy (1986)), career concerns model (Gibbons and Murphy (1992)), and Olcay (2012). Furthermore, in an attempt to inspire future theoretical work on executive contract design, I examined ability for its potential impacts on tenure sensitivity of the two contracting tools. The results suggest that only a short-term measure is shown to decrease the tenure sensitivity of pay. However, regarding forced turnover, our results are inconclusive.

Appendix A: Figures and Tables

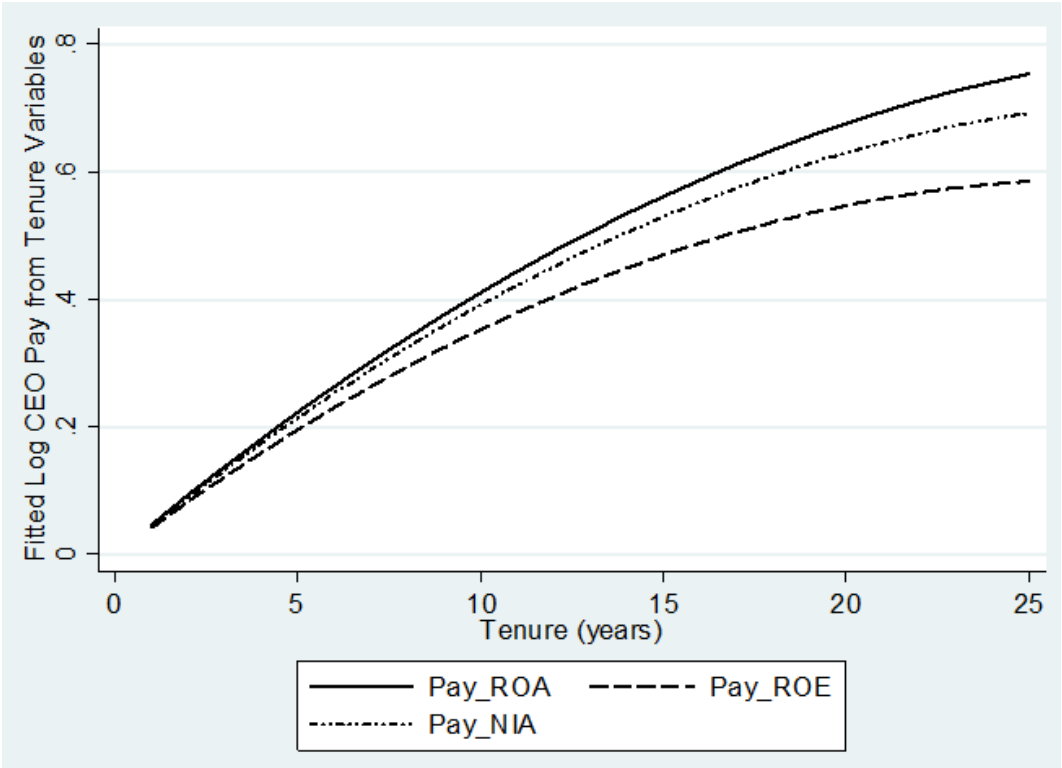


Figure 3.1: Tenure and CEO Pay

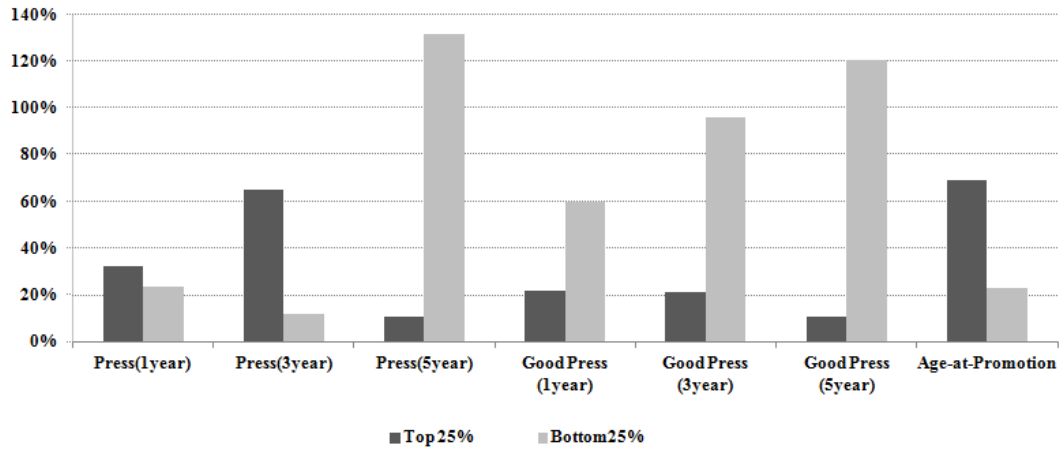


Figure 3.2: % Change in Median CEO Compensation as Tenure Increases (from "less than 5 years" to "more than 5 years")-Top 25% vs. Bottom 25%

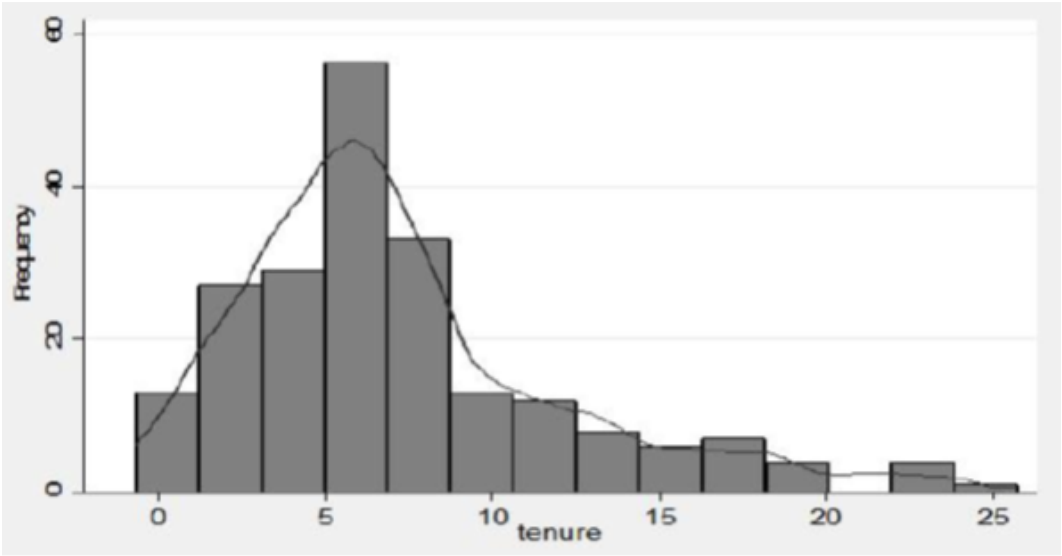
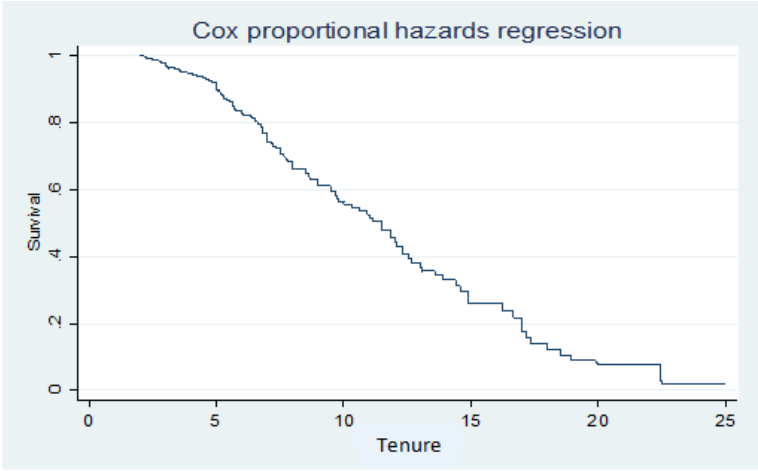
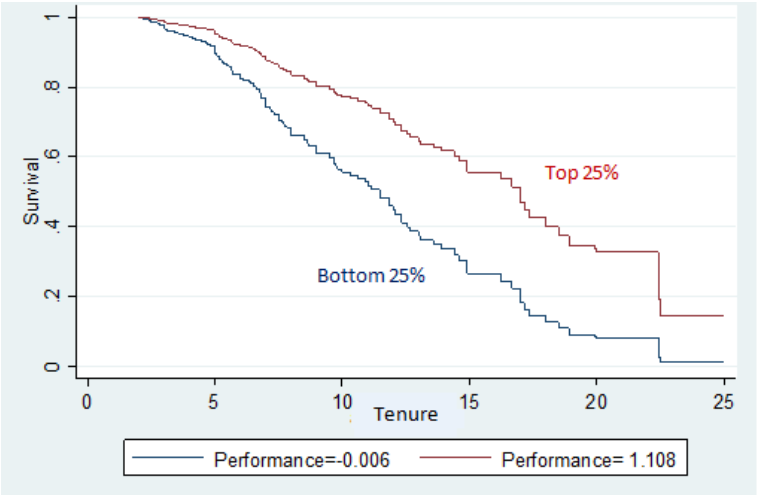


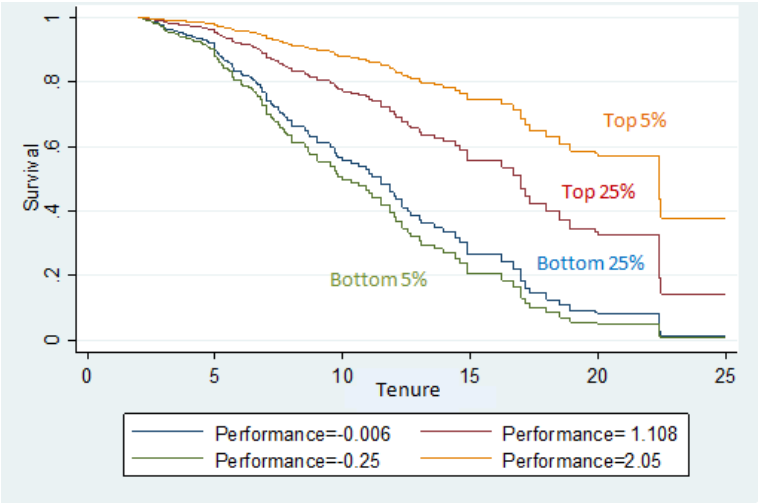
Figure 3.3: Incidence of Forced CEO Turnover



[a]



[b]



[c]

Figure 3.4: [**a**] Tenure and Likelihood of Forced CEO Turnover [**b**] Tenure and Likelihood of Forced CEO Turnover (Top 25% vs. Bottom 25% Performance) [**c**] Tenure and Likelihood of Forced CEO Turnover (At Top 5%, Top 25%, Bottom 25% and Bottom 5% Performance)

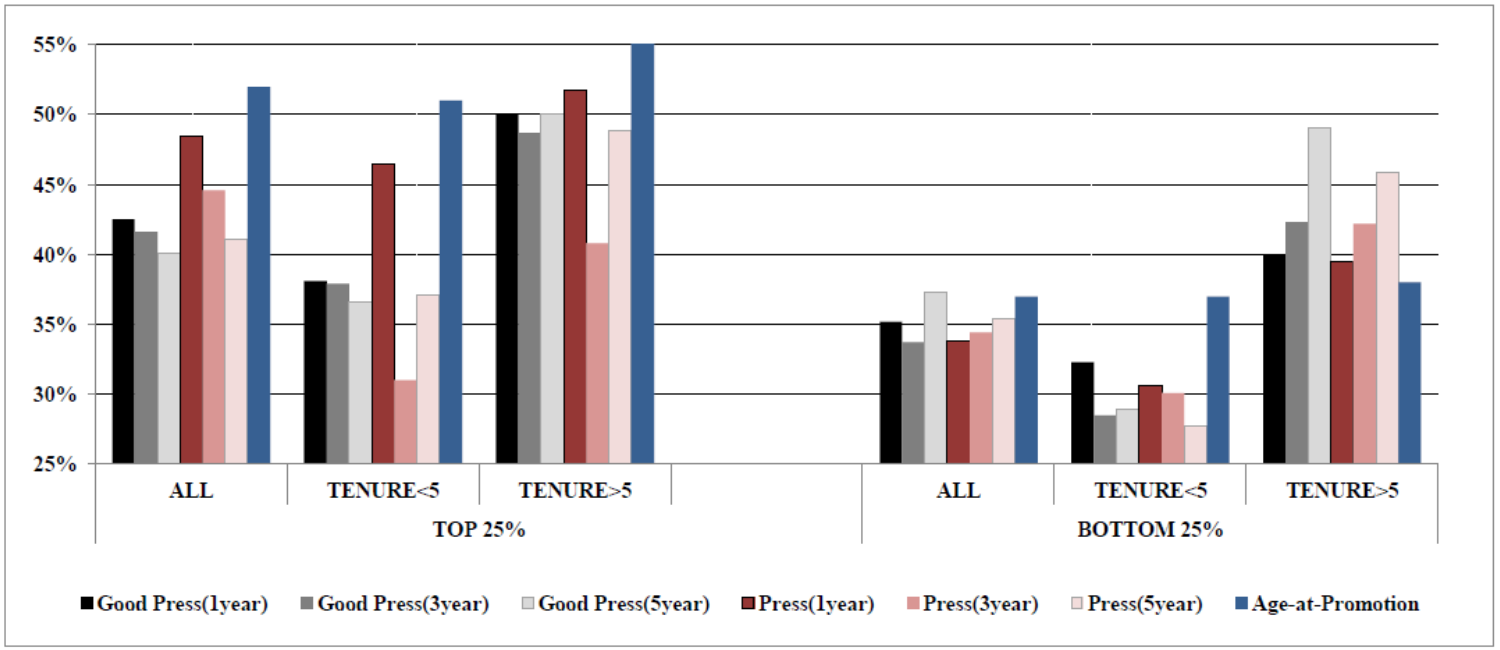


Figure 3.5: Incidence of Forced CEO Turnover–Top 25% vs. Bottom 25%

Table 3.1: Theoretical Predictions

	Learning		Career Concerns		Entrenchment		Olcay(2012)	
	Tenure	Ability	Tenure	Ability	Tenure	Ability	Tenure	Ability
Total CEO Pay	+	n/a	n/a	+	+	+	+	+
Pay-Performance	-	n/a	+	n/a	-	-	+	+
Probability of Forced Turnover	n/a	n/a	n/a	n/a	-	-	+	-
Probability of Forced Turnover-Perf.	n/a	n/a	+	-	-	-	+	-

Table 3.2: CEO Turnover

Year	Number of CEO Changes	Turnover Rate	Resign/ Retire after Bad Perf.	Resign 1	Resign 2	Resign 3	Retire 1	Retire 2	Merger/ Interim	Nothing
1998	43	11.2	11	1	1	2	12	10	3	3
1999	48	11.9	11	1	0	1	14	13	2	6
2000	58	13.8	18	3	2	4	10	13	2	6
2001	40	9.5	7	1	1	1	7	16	4	3
2002	44	10.3	11	3	2	1	7	11	3	6
2003	55	12.5	8	1	2	6	16	13	5	4
2004	63	14.0	9	0	5	7	18	20	0	4
2005	48	10.4	4	2	0	5	9	14	7	7
2006	63	13.3	10	4	6	3	21	13	2	5
2007	58	12.0	11	0	0	10	16	7	2	11
2008	22	5.5	5	1	0	2	5	5	2	2
TOTAL	542	11.4	105	17	19	42	135	135	32	57

Note: Resign/Retire Bad Per. consists of CEO changes after it is explicitly announced that the CEO change is due to bad performance. Resign1 is the cases where in the news it says the CEO suddenly quits and no further explanation is found. Resign2 is CEO changes after scandals (law suits due to affair with subordinates or violated such and such act), arrested or forced out for different reasons (such as conflict with the board or left the company for personal reasons.) Resign3 is the cases where CEO left the firm for good reason, health reasons or deceased. Retire1 is the instances where CEO has retired and joined the board as executive/non executive chairman afterwards. Retire2 includes CEO changes with explicit evidence that CEO has really retired or being retired is not related with performance. Merger/interim refers to CEO changes after merger or acquistion, or CEO had appointed on interim basis before. Nothing consists of cases where no news item related with CEO change is found.

Table 3.3: Forced CEO Turnover

	Number of CEO changes	Forced 1	% in All Changes	Forced 2	Forced 3	% in All Changes
1998	43	11	25.6	6	17	39.5
1999	48	11	22.9	9	20	41.7
2000	58	18	31.0	10	28	48.3
2001	40	7	17.5	3	10	25.0
2002	44	11	25.0	10	21	47.7
2003	55	8	14.5	10	18	32.7
2004	63	9	14.3	13	22	34.9
2005	48	4	8.3	8	12	25.0
2006	63	10	15.9	18	28	44.4
2007	58	11	19.0	12	23	39.7
2008	22	5	22.7	8	13	59.1
TOTAL	542	105	19.4	107	212	39.1

Note: FORCED1 consists of the observations in the category, Resign/Retire after Bad Perf. in Table 1. FORCED 2 is CEO changes below age of 60 and related with categories Retire1, Retire2 (excluding instances explicitly unrelated with performance), Resign1, Nothing. FORCED3 is the sum of FORCED1 and FORCED2.

Table 3.4: Descriptive Statistics

	N	Mean	Med.	Std. Dev.	Max	Min
CEO Characteristics and Pay						
Tenure	10276	7.8	5	7.5	42	1
Age	10246	55.1	55	7.3	85	34
Outsider	5915	0.35	0	0.47	1	0
Total CEO Pay ($thousand)	10659	8	8	1.09	13.3	5.5
Performance Measures and Firm Controls						
Size (log Total Assets, $mil.)	10019	7.4	7.3	1.5	12.9	1.9
Dividend Yield	10011	1.38	0.6	3.27	169.7	0.1
Return on Assets (ROA)	11024	0.054	0.049	0.079	0.59	-1.66
Return on Equity (ROE)	10933	0.12	0.14	0.36	16.09	-1.97
Net Income over Assets (NIA)	11034	0.05	0.05	0.087	0.98	-2.8
Industry Adjusted ROA	11034	0.037	0.019	0.14	2.07	-4.48
Industry Adjusted ROE	10933	0.1	0.044	0.82	16.8	-18.7
Industry Adjusted NIA	11034	0.03	0.01	0.18	2.08	-5.06
Corporate Governance						
Share Ownership	11804	0.03	0	0.19	1	0
Board Member	11804	0.97	1	0.12	1	0
CEO Ability						
Age-At-Promotion	3646	49.08	49	7.4	79	34
Good Press (1 year)	4781	23.6	6	65.6	1227	0
Good Press (3 years)	4781	57.7	17	155.6	3489	0
Good Press (5 years)	4781	81.5	25	221.8	4333	0
Press (1 year)	4781	29.5	8	84.8	1673	0
Press (3 years)	4781	73.7	20	199.4	4040	0
Press (5 years)	4781	104.2	29	286	4987	0

Table 3.5: CEO Performance, Pay and Tenure

	Model 1		Model 2		Model 3	
Variables	(1)	(2)	(1)	(2)	(1)	(2)
Tenure	0.028*	0.0510***	0.0254*	0.0486***	0.0273*	0.0477***
	(2.21)	(3.58)	(2.06)	(3.67)	(2.09)	(3.36)
$Tenure^2$		-0.000948**		-0.00102***		-0.000868**
		(-2.93)		(-3.38)		(-2.71)
ROA	0.907***	0.874***				
	(4.06)	(4.11)				
ROE			0.0763*	0.0424*		
			(2.05)	(2.07)		
NIA					0.082***	0.65***
					(4.32)	(4.28)
ROA∗Tenure	0.0479	0.0499				
	(1.35)	(0.62)				
ROA∗$Tenure^2$		-0.000760				
		(-0.27)				
ROE∗Tenure			0.0176	0.0120		
			(1.68)	(0.53)		
ROE∗$Tenure^2$				0.000180		
				(0.19)		
NIA∗Tenure					0.0557	0.0954
					(1.77)	(1.37)
NIA*$Tenure^2$						-0.00216
						(-0.92)
Age	0.0256	0.0248	0.0259*	0.0249	0.0268	0.0258
	(1.88)	(1.96)	(1.98)	(1.90)	(1.96)	(1.88)
Size	0.261***	0.207***	0.323***	0.256***	0.267***	0.211***
	(4.37)	(3.50)	(5.52)	(4.21)	(4.48)	(3.54)
Divyield	0.251	-0.00792	0.343	-0.0102	0.239	-0.00812
	(1.11)	(-0.98)	(1.49)	(-1.12)	(1.05)	(-1.01)
Share Ownership	-0.0297	-0.0323	-0.0319	-0.0368	-0.0251	-0.0296
	(-0.43)	(-0.49)	(-0.46)	(-0.56)	(-0.36)	(-0.45)
Board Member	-0.376	-0.203	-0.385	-0.201	-0.371	-0.203
	(-0.23)	(-0.26)	(-0.28)	(-0.30)	(-0.18)	(-0.20)
Number of Obs.	7568	7568	7508	7508	7568	7568
Number of CEOs	1930	1930	1922	1922	1930	1930
R^2	17%	21%	23%	25%	18%	21%

Table 3.6: Proxy: Media Visibility

	Press		Good Press	
	Top 25%	Bottom 25%	Top 25%	Bottom 25%
Panel A (1 year)				
Median Media Visibility	51	1	40	1
Tenure	5	5	5	5
CEO Pay(log total compensation)	9.84	8.3	9.23	8.4
ROA	0.068	0.052	0.058	0.053
ROE	0.170	0.142	0.161	0.146
NIA	0.058	0.052	0.058	0.053
Panel B (3 years)				
Median Media Visibility	129	3	101	3
Tenure	5	5	5	5
CEO Pay(log total compensation)	9.26	8.35	9.26	8.38
ROA	0.056	0.052	0.056	0.052
ROE	0.159	0.147	0.160	0.147
NIA	0.057	0.054	0.057	0.054
Panel C (5 years)				
Median Media Visibility	177	5	140	4
Tenure	5	5	5	4
CEO Pay(log total compensation)	9.26	8.36	9.25	8.38
ROA	0.058	0.051	0.058	0.051
ROE	0.161	0.144	0.161	0.144
NIA	0.058	0.052	0.058	0.052
Number of Obs.	1195	1195	1195	1195

Table 3.7: Proxy: Age-at-Promotion

	Top 25%	Bottom 25%
Age-at-Promotion	45	56
Tenure	9	3
CEO Pay(log total compensation)	8.69	8.71
ROA	0.058	0.04
ROE	0.146	0.142
NIA	0.059	0.047
Number of Obs.	2609	2609

Table 3.8: CEO Performance, Pay, Tenure and Reputation (Good Press (1 year))

Variables	(1)	(2)	(3)
Tenure	0.0790*	0.0716*	0.0791*
	(2.44)	(2.30)	(2.44)
Tenure^2	-0.00246**	-0.00234**	-0.00245**
	(-2.81)	(-2.70)	(-2.80)
ROA	1.972*		
	(2.43)		
ROE		0.321*	
		(2.07)	
NIA			1.801*
			(2.31)
Good Press (1 year)	0.0119*	0.0116*	0.0114*
	(2.44)	(1.86)	(2.25)
Good Press (1 year)*Tenure	-0.00153*	-0.00142*	-0.00149*
	(-2.18)	(-2.01)	(-2.08)
Good Press (1 year)*Tenure^2	0.0000397	0.0000366	0.0000385
	(1.76)	(1.74)	(1.67)
Good Press (1 year)*ROA	0.00369		
	(0.13)		
Good Press (1 year)*ROE		0.00664	
		(0.55)	
Good Press (1 year)*NIA			0.00583
			(0.21)
Size	0.280***	0.270***	0.286***
	(3.77)	(3.91)	(3.85)
Age	-0.00991	0.0000745	-0.0103
	(-0.53)	(0.00)	(-0.56)
Share Own.	0.973*	0.970**	0.970*
	(2.49)	(2.71)	(2.49)
Divyield	0.444	0.773	0.449
	(0.82)	(1.71)	(0.83)
No. of Obs.	524	513	524
No. of CEOs	125	123	125
R^2	21.0%	19.3%	21.4%

Note: This table reports panel regression of CEO pay levels (log of annual compensation) on various variables where CEO-firm fixed effects is used. Model 1 employs Return on Assets (ROA) as a performance measure. Models 2 and 3 uses Return on Equity (ROE) and the ratio of Net Income to the book value of assets (NIA) respectively. Reputation is proxied by Good Press (1 year), which counts the number of articles where CEO's name appears during 1 year prior to the current fiscal year. Size is proxied by natural log of Net Sales. Divyield is the firm's dividend yield. Share Ownership is a dummy which takes value 1 if the CEO's ownership of total shares is greater than 5%. Board Member is a dummy variable which is equal to one if the CEO served as a board member during the fiscal year. All specifications include year dummies. In parantheses, we present robust standard errors clustered by CEO. Level of significance are denoted by ***,**, and * for statistical significance at 1%, 5% and 10%, respectively.

Table 3.9: CEO Performance, Pay, Tenure and Reputation (Good Press (3 year))

Variables	(1)	(2)	(3)
Tenure	0.0609	0.0444	0.0611
	(1.44)	(1.12)	(1.44)
$Tenure^2$	-0.00207*	-0.00178	-0.00207*
	(-2.00)	(-1.76)	(-2.00)
ROA	1.874*		
	(2.17)		
ROE		0.261	
		(1.79)	
NIA			1.724*
			(2.09)
Good Press (3 years)	0.00462*	0.00402*	0.00439*
	(2.13)	(2.15)	(2.04)
Good Press (3 years)*Tenure	-0.000556*	-0.000488*	-0.000538*
	(-2.05)	(-2.01)	(-2.06)
Good Press (3 years)*$Tenure^2$	0.0000143	0.0000127	0.0000138
	(1.87)	(1.80)	(1.74)
Good Press (3 years)*ROA	0.00268		
	(0.45)		
Good Press (3 years)*ROE		0.00220	
		(0.87)	
Good Press (3 years)*NIA			0.00382
			(0.64)
Size	0.265**	0.247**	0.271**
	(2.95)	(3.04)	(3.02)
Age	-0.00716	0.00685	-0.00742
	(-0.30)	(0.34)	(-0.31)
Share Own.	0.930*	0.936**	0.929*
	(2.38)	(2.67)	(2.38)
Divyield	0.581	1.075*	0.597
	(0.90)	(2.06)	(0.92)
No. of Obs.	405	400	405
No. of CEOs	108	106	108
R^2	20.4%	18.9%	20.2%

Note: This table reports panel regression of CEO pay levels (log of annual compensation) on various variables where CEO-firm fixed effects is used. Model 1 employs Return on Assets (ROA) as a performance measure. Models 2 and 3 uses Return on Equity (ROE) and the ratio of Net Income to the book value of assets (NIA) respectively. Reputation is proxied by Good Press(3 years), which counts the number of articles where CEO's name appears during 3 years prior to the current fiscal year. Size is proxied by natural log of Net Sales. Divyield is the firm's dividend yield. Share Ownership is a dummy which takes value 1 if the CEO's ownership of total shares is greater than 5%. Board Member is a dummy variable which is equal to one if the CEO served as a board member during the fiscal year. All specifications include year dummies. In parantheses, we present robust standard errors clustered by CEO. Level of significance are denoted by ***,**, and * for statistical significance at 1%, 5% and 10%, respectively.

Table 3.10: CEO Performance, Pay, Tenure and Reputation (Good Press (5 years))

Variables	(1)	(2)	(3)
Tenure	0.0623	0.0424	0.0616
	(1.09)	(0.80)	(1.08)
$Tenure^2$	-0.00221	-0.00190	-0.00218
	(-1.76)	(-1.59)	(-1.74)
ROA	1.688		
	(1.30)		
ROE		1.487*	
		(2.24)	
NIA			1.554
			(1.22)
Good Press (5 years)	0.00157	0.00217	0.00143
	(0.88)	(1.17)	(0.77)
Good Press (5 years))*Tenure	-0.000224	-0.000260	-0.000212
	(-1.13)	(-1.29)	(-1.02)
Good Press (5 years)*$Tenure^2$	0.00000628	0.00000749	0.00000580
	(1.13)	(1.38)	(1.00)
Good Press (5 years)*ROA	-0.000138		
	(-0.03)		
Good Press (5 years)*ROE		-0.00193	
		(-1.02)	
Good Press (5 years)*NIA			0.00117
			(0.24)
Size	0.308**	0.262*	0.308**
	(2.74)	(2.43)	(2.74)
Age	0.00401	0.0169	0.00400
	(0.15)	(0.70)	(0.15)
Share Own.	0.947*	0.912*	0.949*
	(2.59)	(2.61)	(2.60)
Divyield	0.715	1.293	0.715
	(0.98)	(1.95)	(0.97)
No. of Obs.	306	305	306
No. of CEOs	81	80	80
R^2	19.6%	22.1%	19.6%

Note: This table reports panel regression of CEO pay levels (log of annual compensation) on various variables where CEO-firm fixed effects is used. Model 1 employs Return on Assets (ROA) as a performance measure. Models 2 and 3 uses Return on Equity (ROE) and the ratio of Net Income to the book value of assets (NIA) respectively. Reputation is proxied by Good Press(5 years), which counts the number of articles where CEO's name appears during 5 years prior to the current fiscal year. Size is proxied by natural log of Net Sales. Divyield is the firm's dividend yield. Share Ownership is a dummy which takes value 1 if the CEO's ownership of total shares is greater than 5%. Board Member is a dummy variable which is equal to one if the CEO served as a board member during the fiscal year. All specifications include year dummies. In parantheses, we present robust standard errors clustered by CEO. Level of significance are denoted by ***,**, and * for statistical significance at 1%, 5% and 10%, respectively.

Table 3.11: CEO Performance, Pay, Tenure and Reputation (Press (1 year))

Variables	(1)	(2)	(3)
Tenure	0.0775*	0.0703*	0.0774*
	(2.42)	(2.28)	(2.41)
$Tenure^2$	-0.00243**	-0.00231**	-0.00241**
	(-2.80)	(-2.69)	(-2.79)
ROA	1.988*		
	(2.49)		
ROE		0.327*	
		(2.12)	
NIA			1.837*
			(2.37)
Press (1 year)	0.00830*	0.00742*	0.00792*
	(2.34)	(2.15)	(2.11)
Press (1 year)*Tenure	-0.00111*	-0.00105	-0.00108*
	(-2.12)	(-1.92)	(-1.99)
Press (1 year)*$Tenure^2$	0.0000292	0.0000275	0.0000284
	(1.69)	(1.67)	(1.58)
Press (1 year)*ROA	0.00260		
	(0.12)		
Press (1 year)*ROE		0.00490	
		(0.53)	
Press (1 year)*NIA			0.00381
			(0.17)
Size	0.286***	0.274***	0.291***
	(3.84)	(3.99)	(3.93)
Age	-0.0103	-0.000128	-0.0106
	(-0.55)	(-0.01)	(-0.57)
Share Own.	0.985*	0.981**	0.980*
	(2.53)	(2.75)	(2.52)
Divyield	0.456	0.785	0.460
	(0.84)	(1.74)	(0.84)
Number of Obs.	524	513	524
Number of CEOs	125	123	125
R^2	21%	19%	21%

Note: This table reports panel regression of CEO pay levels (log of annual compensation) on various variables where CEO-firm fixed effects is used. Model 1 employs Return on Assets (ROA) as a performance measure. Models 2 and 3 uses Return on Equity (ROE) and the ratio of Net Income to the book value of assets (NIA) respectively. Reputation is proxied by Press(1 year), which counts the number of articles where CEO's name appears during 1 years prior to the current fiscal year. Size is proxied by natural log of Net Sales. Divyield is the firm's dividend yield. Share Ownership is a dummy which takes value 1 if the CEO's ownership of total shares is greater than 5%. Board Member is a dummy variable which is equal to one if the CEO served as a board member during the fiscal year. All specifications include year dummies. In parantheses, we present robust standard errors clustered by CEO. Level of significance are denoted by ***,**, and * for statistical significance at 1%, 5% and 10%, respectively.

Table 3.12: CEO Performance, Pay, Tenure and Reputation (Press (3 years))

Variables	(1)	(2)	(3)
Tenure	0.0610	0.0449	0.0612
	(1.44)	(1.13)	(1.45)
$Tenure^2$	-0.00207*	-0.00179	-0.00207*
	(-2.02)	(-1.78)	(-2.01)
ROA	1.913*		
	(2.19)		
ROE		0.272	
		(1.84)	
NIA			1.775*
			(2.12)
Press (3 years)	0.00367*	0.00332*	0.00351*
	(2.05)	(2.16)	(2.08)
Press (3 years)*Tenure	-0.000448*	-0.000406*	-0.000436*
	(-2.07)	(-2.50)	(-2.32)
Press (3 years)*$Tenure^2$	0.0000117	0.0000106	0.0000113
	(1.97)	(1.91)	(1.84)
Press (3 years)*ROA	0.00167		
	(0.36)		
Press (3 years)*ROE		0.00146	
		(0.73)	
Press (3 years)*NIA			0.00247
			(0.52)
Size	0.267**	0.249**	0.272**
	(2.96)	(3.06)	(3.03)
Age	-0.00742	0.00670	-0.00766
	(-0.31)	(0.33)	(-0.32)
Share Own.	0.924*	0.926**	0.922*
	(2.38)	(2.67)	(2.38)
Divyield	0.577	1.074*	0.592
	(0.89)	(2.05)	(0.91)
Number of Obs.	405	400	405
Number of CEOs	108	106	108
R^2	20%	19%	20%

Note: This table reports panel regression of CEO pay levels (log of annual compensation) on various variables where CEO-firm fixed effects is used. Model 1 employs Return on Assets (ROA) as a performance measure. Models 2 and 3 uses Return on Equity (ROE) and the ratio of Net Income to the book value of assets (NIA) respectively. Reputation is proxied by Press (3 years), which counts the number of articles where CEO's name appears during 3 years prior to the current fiscal year. Size is proxied by natural log of Net Sales. Divyield is the firm's dividend yield. Share Ownership is a dummy which takes value 1 if the CEO's ownership of total shares is greater than 5%. Board Member is a dummy variable which is equal to one if the CEO served as a board member during the fiscal year. All specifications include year dummies. In parantheses, we present robust standard errors clustered by CEO. Level of significance are denoted by ***,**, and * for statistical significance at 1%, 5% and 10%, respectively.

Table 3.13: CEO Performance, Pay, Tenure and Reputation (Press (5 years))

Variables	(1)	(2)	(3)
Tenure	0.0616	0.0422	0.0609
	(1.09)	(0.80)	(1.08)
Tenure^2	-0.00220	-0.00190	-0.00217
	(-1.77)	(-1.60)	(-1.75)
ROA	1.736		
	(1.33)		
ROE		1.517*	
		(2.27)	
NIA			1.618
			(1.26)
Press (5 years)	0.00122	0.00180	0.00112
	(0.91)	(1.28)	(0.80)
Press (5 years)*Tenure	-0.000178	-0.000217	-0.000169
	(-1.18)	(-1.42)	(-1.07)
Press (5 years)*Tenure^2	0.00000512	0.00000634	0.00000476
	(1.20)	(1.53)	(1.06)
Press (5 years)*ROA	-0.000580		
	(-0.16)		
Press (5 years)*ROE		-0.00183	
		(-1.22)	
Press (5 years)*NIA			0.000328
			(0.08)
Size	0.311**	0.264*	0.311**
	(2.76)	(2.46)	(2.76)
Age	0.00393	0.0169	0.00391
	(0.15)	(0.70)	(0.15)
Share Own.	0.935*	0.898*	0.937*
	(2.57)	(2.59)	(2.58)
Divyield	0.720	1.302	0.721
	(0.98)	(1.96)	(0.98)
Number of Obs.	306	305	306
Number of CEOs	81	80	80
R^2	19%	22%	19%

Note: This table reports panel regression of CEO pay levels (log of annual compensation) on various variables where CEO-firm fixed effects is used. Model 1 employs Return on Assets (ROA) as a performance measure. Models 2 and 3 uses Return on Equity (ROE) and the ratio of Net Income to the book value of assets (NIA) respectively. Reputation is proxied by Press (5 years), which counts the number of articles where CEO's name appears during 5 years prior to the current fiscal year. Size is proxied by natural log of Net Sales. Divyield is the firm's dividend yield. Share Ownership is a dummy which takes value 1 if the CEO's ownership of total shares is greater than 5%. Board Member is a dummy variable which is equal to one if the CEO served as a board member during the fiscal year. All specifications include year dummies. In parantheses, we present robust standard errors clustered by CEO. Level of significance are denoted by ***,**, and * for statistical significance at 1%, 5% and 10%, respectively.

Table 3.14: CEO Performance, Pay, Tenure and Ability (Age-at-Promotion)

Variables	Model 1	Model 2	Model 3
Tenure	-0.0433	-0.0506	-0.0430
	(-1.02)	(-1.19)	(-1.01)
$Tenure^2$	0.000399	0.000555	0.000390
	(0.26)	(0.36)	(0.25)
Age-At-Promotion	-0.00171	-0.00348	-0.00167
	(-0.29)	(-0.61)	(-0.28)
Age-At-Promotion*Tenure	0.00143	0.00157	0.00143
	(1.54)	(1.69)	(1.53)
Age-At-Promotion*$Tenure^2$	-0.0000202	-0.0000234	-0.0000200
	(-0.49)	(-0.57)	(-0.49)
ROA	0.224		
	(0.85)		
ROE		0.00129***	
		(6.06)	
NIA			0.305
			(1.12)
Size	0.439***	0.443***	0.439***
	(24.67)	(25.21)	(24.67)
Share Ownership	-0.464**	-0.450*	-0.465**
	(-2.63)	(-2.53)	(-2.63)
Dividend Yield	0.272*	0.297**	0.277*
	(2.37)	(2.66)	(2.40)
Number of Obs.	5977	5977	5977
Number of CEOs	1569	1569	1596
R^2	32.6	32.7	32.1

This table reports estimates from panel OLS regression of CEO pay levels (log of annual compensation) on various variables. Model 1 used Return on Assets (ROA) as a performance measure. Models 2 and 3 uses Return on Equity (ROE) and the ratio of Net Income to the book value of assets (NIA) respectively. Ability is proxied by the CEO's age at the time of promotion. Size is proxied by natural log of Net Sales. Divyield is the firm's dividend yield. Share Ownership is a dummy which takes value 1 if the CEO's ownership of total shares is greater than 5%. Board Member is a dummy variable which is equal to one if the CEO served as a board member during the fiscal year. All specifications include year dummies. In parantheses, we present robust standard errors clustered by CEO. Level of significance are denoted by ***,**, and * for statistical significance at 1%, 5% and 10%, respectively.

Table 3.15: CEO Performance and Forced Turnover

	Model 1			Model 2		
Variable	(1)	(2)	(3)	(1)	(2)	(3)
Firm Perf.	-0.67***	-0.845***	3.169			
	(-3.71)	(-4.11)	(0.63)			
Size		0.232**	0.268***		0.228**	0.272***
		(2.93)	(3.47)		(2.91)	(3.37)
Age		-0.034***	-0.034***		-0.034***	-0.036***
		(-3.37)	(-3.37)		(-3.38)	(-3.41)
Size*Firm Perf.			-0.812*			
			(-2.30)			
Age*Firm Perf.			0.0088			
			(0.16)			
Adj. Firm Perf.				-0.69***	-0.854***	4.505
				(-3.83)	(-4.16)	(1.02)
Industry Perf.				0.294	0.637*	6.556
				(1.10)	(2.35)	(1.38)
Size*Adj. Firm Perf.						-0.926
						(-1.57)
Age*Adj. Firm Perf.						-0.0002
						(-0.23)
Size*Ind. Perf.						-1.103
						(-1.78)
Number of Obs.	4269	4226	4226	4269	4226	4226
Number of CEOs.	934	915	915	934	915	915
Log Pseudolikelihood	-1162.4	-1039.2	-1038.1	-1161.7	-1039.1	-1037.7

Note: This table presents the estimates from Cox Proportational Hazard Model where hazard rate is the dependent variable. Failure event is defined as the performance related departure. Sample period covers 1998-2008. In Model 1, firm performance is measured by Return on Assets (ROA). Model 2 uses industry-adjusted ROA and 2-digit SIC industry ROA as a measure of industry performance. Size is proxied by natural lof of Net Sales. Robust clustered (within industry) standard errors are reported in parantheses. Level of significance are denoted by ***,**, and * for statistical significance at 1%, 5% and 10%, respectively.

Table 3.16: CEO Performance and Forced Turnover - More Controls

	Model 1		Model 2	
Variable	(1)	(2)	(1)	(2)
Outsider	-0.247*	-0.243*	-0.244*	-0.250*
	(-2.41)	(-2.37)	(-2.35)	(-2.41)
Firm Performance	-0.254***	-0.716*		
	(-5.12)	(-2.10)		
Size	0.0180*	0.0180*	0.0175*	0.0170
	(1.99)	(1.99)	(2.94)	(1.88)
Age	-0.00379**	-0.00382***	-0.00380**	-0.00377***
	(-3.26)	(-3.30)	(-3.27)	(-3.31)
Share Ownership	-13.82***	-12.43***	-12.90***	-13.02***
	(-34.05)	(-32.11)	(-32.73)	(-32.91)
Board Member	-0.254***	-0.260***	-0.253***	-0.157
	(-3.40)	(-3.70)	(-3.39)	(-1.05)
Board Member*Firm Perf.		0.00474		
		(1.42)		
Adjusted Firm Performance			-0.258***	-0.655
			(-5.01)	(-1.39)
Industry Performance			0.200***	2.437
			(3.57)	(0.86)
Board Member*Adj. Firm Perf.				0.396
				(0.86)
Board Mbr.*Industry Perf.				-2.639
				(0.86)
Number of Obs.	4226	4226	4226	4226
Number of CEOs.	915	915	915	915
Log Pseudolikelihood	-1022	-1021.9	-1021.5	-1020.7

Note:This table presents the estimates from Cox Proportational Hazard Model where hazard rate is the dependent variable. Failure event is defined as the performance related departure. Sample period covers 1998-2008. In Model 1, firm performance is measured by Return on Assets (ROA). Model 2 uses industry-adjusted ROA and 2-digit SIC industry ROA as a measure of industry performance. Size is proxied by natural lof of Net Sales. Outsider is a dummy which takes value 1 if the CEO is an external hire, 0 otherwise. Share Ownership is a dummy which takes the value 1 if the CEO's ownership is higher than 5% of the total shares of the firm; and 0 otherwise. Board Member is a dummy if the CEO served as a board director for the fiscal year. Robust clustered (within industry) standard errors are reported in parantheses. Level of significance are denoted by ***,**, and * for statistical significance at 1%, 5% and 10%, respectively.

Table 3.17: CEO Performance, Forced Turnover and Reputation (Good Press)

	Model 1		Model 2		Model 3	
Variables	(1)	(2)	(1)	(2)	(1)	(2)
Adj. Firm Perf.	-0.214*	-0.0809	-0.224*	-0.0451	-0.207	-0.118
	(-2.24)	(-0.45)	(-2.41)	(-0.23)	(-1.96)	(-0.68)
Industry Perf.	-0.487	-0.439	-0.489	-0.418	-0.193	-0.184
	(-1.43)	(-1.12)	(-1.45)	(-1.01)	(-0.88)	(-0.67)
Good Press(1y.)	-.0006***	-0.0002				
	(-3.85)	(-0.57)				
Good Press(1y.)*						
Adj. Firm Perf.		-0.00541				
		(-1.17)				
Good Press(1y.)*						
Ind. Perf.		-0.00423				
		(-0.75)				
Good Press(3y.)			-.00019**	-0.00006		
			(-3.64)	(-0.53)		
Good Press(3y.)*						
Adj. Firm Perf.				-0.00147		
				(-1.32)		
Good Press(3y.)*						
Ind. Perf.				-0.0008		
				(-0.63)		
Good Press(5y.)					-0.00010*	-0.00007
					(-2.25)	(-1.08)
Good Press(5y.)*						
Adj. Firm Perf.						-.000006
						(-1.03)
Good Press(5y.)*						
Ind. Perf.						-0.0004
						(-0.64)
Age	-0.00394*	-0.0039*	-0.0038*	-0.0038*	-0.0042*	-0.0043*
	(-2.28)	(-2.26)	(-2.28)	(-2.16)	(-2.31)	(-2.30)
Firm Size	0.0454	0.0449	0.0444	0.0434	0.0346	0.0339
	(1.92)	(1.76)	(1.86)	(1.70)	(1.56)	(1.45)
Share Ownership	-2.550***	-3.787	-3.822	-3.821	-3.839	-3.841
	(-17.82)	(.)	(.)	(.)	(.)	(.)
Board Member	-0.46***	-0.43***	-0.46***	-0.44***	-0.46***	-0.45***
	(-7.02)	(-7.21)	(-7.16)	(-7.68)	(-8.30)	(-7.74)
Number of Obs.	559	559	490	490	372	372
Pseudo Log.	-101.68	-101.23	-101.91	-101.28	-80.64	-80.39

Note: This table presents the estimates from Cox Proportational Hazard Model where hazard rate is the dependent variable. Failure event is defined as the performance related departure. Sample period covers 1998-2008. Firm performance is measured by Return on Equity (ROA). Size is proxied by natural lof of Net Sales. Outsider is a dummy which takes value 1 if the CEO is an external hire, 0 otherwise. Share Ownership is a dummy which takes the value 1 if the CEO's ownership is higher than 5% of the total shares of the firm; and 0 otherwise. Board Member is a dummy if the CEO served as a board director for the fiscal year. Press1, Press 3 and Press 5 count the number of articles in which CEO's names appeared during the last 1 year, 3 years, and 5 years prior to the fiscal year. The number only includes articles with positive or neutral connotations. Robust clustered (at the CEO level) standard errors are reported in parantheses. Level of significance are denoted by ***,**, and * for statistical significance at 1%, 5% and 10%, respectively.

Table 3.18: CEO Performance, Forced Turnover and Reputation (Press)

	Model 1		Model 2		Model 3	
Variables	(1)	(2)	(1)	(2)	(1)	(2)
Adj. Firm Perf.	-0.211*	-0.0926	-0.223*	-0.0496	-0.207	-0.123
	(-2.23)	(-0.55)	(-2.41)	(-0.26)	(-1.94)	(-0.72)
Industry Perf.	-0.488	-0.457	-0.489	-0.421	-0.195	-0.187
	(-1.44)	(-1.17)	(-1.45)	(-1.02)	(-0.88)	(-0.69)
Press(1y.)	-.0005***	-0.0001				
	(-4.40)	(-0.60)				
Press(1y.)*						
Adj. Firm Perf.		-0.0044				
		(-1.18)				
Press(1y.)*						
Ind. Perf.		-0.003				
		(-0.75)				
Press(3y.)			-.0001***	-0.00005		
			(-4.24)	(-0.59)		
Press(3y.)*						
Adj. Firm Perf.				-0.0012		
				(-1.27)		
Press(3y.)*						
Ind. Perf.				-0.0007		
				(-0.64)		
Press(5y.)					-.00009*	-.00006
					(-2.30)	(-1.24)
Press(5y.)*						
Adj. Firm Perf.						-.000005
						(-0.98)
Press(5y.)*						
Ind. Perf.						-0.00035
						(-0.62)
Age	-0.0039*	-0.004*	-0.0038*	-0.0038*	-0.0042*	-0.0043*
	(-2.31)	(-2.32)	(-2.29)	(-2.18)	(-2.33)	(-2.32)
Firm Size	0.0460	0.0456	0.0448	0.0440	0.0353	0.0349
	(1.93)	(1.78)	(1.86)	(1.72)	(1.58)	(1.49)
Share Own.	-2.502***	-2.435***	-3.822	-3.822	-3.840	-3.841
	(-17.53)	(-17.45)	(.)	(.)	(.)	(.)
Board Mbr.	-0.46***	-0.43***	-0.47***	-0.44***	-0.47***	-0.46***
	(-7.07)	(-7.09)	(-7.34)	(-7.65)	(-8.41)	(-7.64)
Number of Obs.	559	559	490	490	372	372
Pseudo Log	-101.61	-101.18	-101.85	-101.25	-80.55	-80.27

Note: This table presents the estimates from Cox Proportational Hazard Model where hazard rate is the dependent variable. Failure event is defined as the performance related departure. Sample period covers 1998-2008. Firm performance is measured by Return on Equity (ROA). Size is proxied by natural lof of Net Sales. Outsider is a dummy which takes value 1 if the CEO is an external hire, 0 otherwise. Share Ownership is a dummy which takes the value 1 if the CEO's ownership is higher than 5% of the total shares of the firm; and 0 otherwise. Board Member is a dummy if the CEO served as a board director for the fiscal year.Press1,Press 3 andPress 5 count the number of articles in which CEO's names appeared during the last 1 year, 3 years, and 5 years prior to the fiscal year. Robust clustered (at the CEO level) standard errors are reported in parantheses. Level of significance are denoted by ***,**, and * for statistical significance at 1%, 5% and 10%, respectively.

Table 3.19: CEO Performance, Forced Turnover and Ability (Age-at-Promotion)

	Model 1		Model 2	
Variables	(1)	(2)	(1)	(2)
Age-At-Promotion	0.0213*	0.0314*	0.0196*	0.0266*
	(2.19)	(1.97)	(2.08)	(2.12)
Adjusted Firm Perf.			-0.00106*	-0.613
			(-2.32)	(0.95)
Firm Perf.	-0.199	0.707		
	(-0.93)	(0.74)		
Industry Perf.			-0.187	
			(-0.85)	
Age-At-Promotion*Adj. Firm Perf.				-0.0178
				(-1.35)
Age-At-Promotion*Firm Perf.		-0.0198		
		(-0.92)		
Age-At-Promotion*Adj. Firm Perf.				-0.0223
				(-0.55)
Size	0.00890	0.00889	0.0102	0.00891
	(0.76)	(0.75)	(0.84)	(0.74)
Share Ownership	-14.36	-9.323***	-9.770***	-10.19***
	(.)	(-26.96)	(-28.19)	(-28.75)
Board Member	0.438	-0.296	-2.699	0.695
	(.)	(-1.40)	(.)	(.)
Number of Obs.	1565	1565	1565	1565
Number of CEOs	364	364	364	364
Log-pseudo likelihood	-182.76	-182.6	-182.64	-182.38

Note: This table presents the estimates from Cox Proportational Hazard Model where hazard rate is the dependent variable. Failure event is defined as the performance related departure. Sample period covers 1998-2008 and internally-hired CEOs. In Model 1, firm performance is measured by Return on Assets (ROA). Model 2 uses industry-adjusted ROA and 2-digit SIC industry ROA as a measure of industry performance. Size is proxied by natural lof of Net Sales. Share Ownership is a dummy which takes the value 1 if the CEO's ownership is higher than 5% of the total shares of the firm; and 0 otherwise. Board Member is a dummy if the CEO served as a board director for the fiscal year. Ability is proxied by the age of CEO at the time of promotion. Robust clustered (at the CEO level) standard errors are reported in parantheses. Level of significance are denoted by ***,**, and * for statistical significance at 1%, 5% and 10%, respectively.

Appendix B

Publications included in the search to construct the media visibility variables:

Businessweek, Dow Jones News Service, Financial Times, Forbes, Fortune, International Herald Tribune, Los Angeles Times, The Economist, The New York Times, The Wall Street Journal, The Wall Street Journal Asia, The Wall Street Journal Europe, The Washington Post, USA Today

Our Good Press proxy excludes from the total count articles containing the following keywords:

scandal or investigat* or (cut w/2 jobs) or resign* or (force* w/3 quit) or dismiss* or demote* or demotion or accuse* or critici* or allegation* or indict* or arrest* or guilty or fraud or litigation or abrasive or excessive pay or overpaid or perquisites or (force* w/3 step down) or under scrutiny or under pressure or law suit or class action or in trouble

Chapter 4

Common Agency within Bureaucracy

4.1 Introduction

Almost everyone would agree that the state has a significant role in providing primary education, health care, security of persons and property, some physical infrastructure, and governance. Economists realize the inefficiency of private provision of public goods, which renders governmental intervention inevitable in these areas. However, suboptimal provision of public goods by the government reduces private effort and incentives, and causes economic failure. In such cases, from a public economics point of view, the state "fails" in its basic purpose. Since the success or failure of government policies have crucial impacts on the real economy and social welfare, the solutions to improve the performance of the public sector has been an important source of debate. Among several factors that might affect the efficiency of the public sector, the design of state bureaucracies is particularly important. This issue has not only been investigated by economists, but also by political scientists.

One common design problem, also inherent in American bureaucratic system, is to find the best organizational structure in which different bureaucratic actors are responsible for different political actions. This is known as "separation of powers" in the political science literature. Persson and Tabellini (2000) rationalize separation of powers by arguing that it increases the voters' ability to impose accountability on politicians and limit the equilibrium rents. In their model, an executive proposes a tax rate to the legislature, which in turn announces the level of public good spending, as well as a rent allocation that the executive might veto. This arrangement represents a specific separation of the political powers between the president and Congress in a

presidential democracy, or between the different standing committees in a congressional setting. This kind of separation is also helpful in explaining the existence of different ministries in a parliamentary setting. A practical example is the following. Suppose that there exist two ministries responsible for the production of one type of public good. This can be the minister of energy attempting to have the local municipality built a power plant on the one hand, and the minister for environment who tries to provide a certain level of environmental quality on the other. The case of a municipality which has to take into account the concerns of different ministers while taking an action is an example of a system with multi-principals, a so-called "common agency".

In a common agency, a single agent works for several top-level bureaucrats, the principals, each with different objectives. A part of the analysis in this paper considers a common agency game within the bureaucracy. For a long time, political economists have recognized the relevance of principal-agent modeling by identifying the voters as principals and the politicians as their agents. Yet the agency problem is also applicable to multi-tiered governments, where policy implementation is complicated for the top-level principals, so that middle-level bureaucrats administers the policies that are decided by their principals, i.e. the top-level rulers. A close look to the literature reveals that relatively few attempts have been made to view the bureaucracy as a common agency problem. This study aims to fill the gap relating these two fields and investigate the impact of such multiplicity of principals in top level bureaucracy on social welfare.

Our common agency model involves two top-level bureaucrats (the principals), a middle-level bureaucrat (the agent) who is in charge of producing public goods, and additionally a third party, the representative citizen who consumes the public good. The key aspect of the model is that the principals value only one type of public good and hence contract with the common agent on the level of production of this public good. There is moral hazard: the agent's effort cannot be unobserved by the principals. The effort choice of the agent determines the probability distribution of the quality of the public goods. The agent is compensated by the principal whose budget is determined by the citizen. Hence, there are two types of contracts, one is offered by a principal to

the agent, the other is offered by the citizen to a principal. The contract between the agent and the principal, however, cannot be observed by the citizen. This might lead to "corruption" in equilibrium. However, the analysis indicates that as long as the citizen is perfectly informed about the cost structure of the agent, there is no possibility of corruption.

The model provides several interesting results. First of all, under full information, single principal model weakly dominates common agency. However, if the citizen has imperfect information about the cost parameters, the results might be different and are directly related to the existance of the rents from office. We assume that if the principal succeeds in inducing her agent to produce a high quality public good, she can stay in the office and enjoy a rent, which is not transferable. When there are positive rents, a single principal model is favorable from the point of the citizen. One reason is the cost advantage of single-principal model: the citizen can always reduce the optimal transfer which is paid when both goods are supplied with high quality, by a fraction of the rent unless the rent is too large. The competition between the principals in common agency, on the other hand, limits the ability of the citizen to reduce the cost of providing incentives in a similar manner. Therefore, single principal model yields a higher expected payoff to the citizen than a common agency. If there are no rents from the public office, the two systems are equally welfare-efficient.

Dixit (2006) attempts to explain state failure by considering the innate agency problem within the government. The idea is similar to the one presented here, in the sense that the bureaucracy is viewed as a principal-agent model. Yet, his model does not consider a common agency. His analysis is essentially two-fold. First, he measures the degree of benevolence of both top-level and middle-level bureaucrat. Second, he incorporates the degree of the observability of the actions and the cost structure of the agent. The question is whether the agency problem in dealing with bureaucrats is worse for a "predatory" ruler than for a "benevolent" ruler. In this context, interactions arise endogenously: as informational limitations become more severe, it gets more likely that a predatory ruler hires selfish bureaucrats on the one hand. On the other hand, a

collaborative relationship between a benevolent ruler and a "caring" bureaucrat arises. The informational constraints of the principals yield different results for different degrees of unobservability, therefore a case-specific analysis is vital to predict state failure.

Common agency problems frequently occur in real life, like the case of administrative agencies who are basically responsible to the lawmakers, yet are practically influenced by the courts, media, and various interest groups. Or, like in European Union where several sovereign governments deal with a common entity in policy making. Dixit et al. (1997), in a common agency framework, considers a political process of economic policy making. They analyze a model in which a subset of all citizens are allowed to lobby the government and promise contributions in return for policy favors. The existence of multiple principals introduces new issues of whether it is possible to achieve an efficient outcome for all parties. In a complete information environment, they prove that the principals' Nash equilibrium in truthful strategies implements an efficient allocation. The model in Grossman and Helpman (2001) which studies the competition among different special interest groups for political influence is also relevant to this context. The competition is modeled as one in which each of several special interest groups confronts the policy maker with offers of campaign contributions in exchange for various policies. By imposing some assumptions on the schedules, they are able to characterize an equilibrium containing a set of contribution schedules that are mutually best responses (considering the anticipated behavior of the policymaker).

The common agency problem as an example of multi-contracting has been studied in various contexts in the literature and it is possible to contrast the existing models in several dimensions. The first dimension in common agency modeling is "intrinsic" or "delegated" common agency. Under the intrinsic common agency assumption, the agent is forced to either accept the whole set of contracts at once, or refuse all of them. In a delegated common agency setting, however, the agent can pick a subset of offers she receives.

A second dimension is related with the existence of "direct" versus "indirect" externalities. An example of a delegated common agency game with indirect externalities is

introduced in Laussel and Le Breton (1996). They consider a model which consists of a private firm producing the public good for n consumers compensating the producer with monetary contributions. Their results show that efficiency can be sustained with truthful Nash equilibria in which no user has an incentive to free ride. Martimort and Stole (2003), on the other hand, consider a typical example of common agency with direct externalities, with two retailers distributing the output of a common manufacturer and competing in the same final market. The retailers (i.e., principals) independently contract with the manufacturer (i.e. the agent) and the production level of one retailer directly affects the price faced by the other retailer. Martimort and Moreira (2006) emphasize inefficiency in public good production due to the existance of a common agency. In their common agency environment, it is not the contributors who try to underestimate their valuations for the public good but it is the common agent who has an incentive to claim that the others have a lower level of willingness to pay than what they actually have. The problem of the principals is then to assess the agent's market information which, in turn, creates inefficient outcomes. Mezzetti (1997) presents a slightly different model where a common agent is responsible to accomplish complimentary tasks for the two principals. The result favors common agency: even though the principals' payoffs are higher under cooperation, the total surplus is greater in independent contracting, which could be explained by a reduction in specialization due to the principals' cooperation.

The presentation of the paper is as follows. Next section describes the model and derives the equilibria for single-principal and common agency models. Section 4.3 is a comparative assessment of the two bureaucratic systems in terms of citizen's welfare. The comparison of the two regimes under a full information model is provided in Section 4.4. Section 4.5 concludes.

4.2 The Model

Assume that there are two public goods both of which are produced by the 'agent', i.e., the lower level bureaucrat. The quality of the public good could be either high or low;

for good i, $q_i^k \in \{q_i^h, q_i^l\}$. For simplicity, q_i^l is normalized to zero. While the quality can be observed, the effort spent by the agent to produce the public good is unobservable. The effort exerted by the agent determines the cost of production according to the function

$$C(e_1, e_2) = \delta(e_1 + e_2 + \gamma e_1 e_2)$$

where e_i denotes the effort spent in the production of public good i. We assume $\gamma \in [0, 1]$ and $\delta > 0$. There is limited liability and the agent should be as well of as she could be if she declines the contract, i.e. she should be paid enough such that she gets at least her reservation payoff. With no loss in generality, the reservation payoff of the agent is normalized to zero.

The citizen values the consumption of the public good of both types, yet at different degrees. For all $k \in \{h, l\}$, the utility she derives is $\alpha\ q_1^k$ $((1-\alpha)q_i^k)$ if the public good 1 (2) is of k quality. The parameter $\alpha \in [0, 1]$ indicates the taste of the citizen over the two public goods.

The principal's utility depends on the net monetary transfer, i.e. the difference between what she receives from the citizen and the quantity she pays to the agent, plus the rent from the public office. If the principal succeeds in inducing her agent to produce high quality public good of which she is in charge, she enjoys the rent; otherwise she has to resign.

The timeline is defined as follows. First the citizen decides on the monetary transfer she would pay for good i of quality k. Given the contract offered by the citizen, the principal (principals, in common agency) chooses the transfers to be paid to the agent. The agent chooses his effort supply if he accepts the contract. Finally the quality levels of the public goods and hence payoffs are realized.

We will start with considering agent's problem but first we make a simplifying assumption about agent's choice set of effort.

Assumption: $e_i \in \{0, 1\}$, $i = 1, 2$.

4.2.1 Agent's Problem

If the agent accepts the contract, he decides on how much to work. The probability of the good being high or low quality is determined by the effort choice of the agent in the following way: if he exerts e_i in task i, the probability that he will produce a high quality public good type i (i.e. the "success" in task i) is e_i. We assume that the success in task i and the success in task j are independent events, i.e. the probability of the event with "success in both tasks" is e_1e_2; the probabilities of the other events, i.e. meaning "success only in one task" and "success in none of the two tasks" are determined in a similar manner.

If he succeeds in both tasks, the award is t^*, where $t^* \geq t_1 + t_2$. He receives only t_i when he achieves high quality in task i. He will be paid zero otherwise.

The agent will maximize

$$EU_A(e_1, e_2) = e_1e_2(t^* - t_1 - t_2) + e_1t_1 + e_2t_2 - \delta(e_1 + e_2 + \gamma e_1e_2) \tag{4.1}$$

The term $(t^* - t_1 - t_2)$ in (4.1) can be interpreted as a *bonus* payment. If the contract pays a nonzero bonus, then it means the reward paid when both tasks are completed with success is higher than the sum of individual payments that are paid only one success is observed.

Given the choice set of effort, the agent considers his expected payoff in (4.1) to determine his optimal effort. For example, he will supply (1,1) if only if the expected payoff of supplying (1,1) is greater than supplying (1,0), (0,1) or (0,0). Denoting his optimal effort choice with (e_1^*, e_2^*), his strategy is characterized as follows, given t^*, t_1 and t_2 :

(i) Set $(e_1^*, e_2^*) = (1, 1)$ if and only if

$$t^* - t_1 \geq \delta(1 + \gamma) \tag{4.2}$$

$$t^* - t_2 \geq \delta(1 + \gamma) \tag{4.3}$$

$$t^* \geq \delta(2+\gamma) \tag{4.4}$$

Equations (4.2)-(4.4) hold if and only if

$$t^* \geq \max\{t_1 + \delta(1+\gamma), t_2 + \delta(1+\gamma), \delta(2+\gamma)\}$$

(ii) Set $(e_1^*, e_2^*) = (1, 0)$ if and only if

$$\delta(1+\gamma) > t^* - t_1 \tag{4.5}$$

$$t_1 > t_2 \tag{4.6}$$

$$t_1 \geq \delta \tag{4.7}$$

Equations (4.5)-(4.7) hold if and only if

$$t_1 > \max\{t_2, t^* - \delta(1+\gamma)\} \qquad \text{and} \qquad t_1 \geq \delta$$

(iii) Set $(e_1^*, e_2^*) = (0, 1)$ if

$$\delta(1+\gamma) > t^* - t_2 \tag{4.8}$$

$$t_2 > t_1 \tag{4.9}$$

$$t_2 \geq \delta \tag{4.10}$$

Equations (4.8)-(4.10) hold if and only if

$$t_2 > \max\{t_1, t^* - \delta(1+\gamma)\} \qquad \text{and} \qquad t_2 \geq \delta.$$

(iv) Set $(e_1^*, e_2^*) = (0, 0)$ if and only if

$$\delta > t_1 \tag{4.11}$$

$$\delta > t_2 \tag{4.12}$$

$$\delta(2 + \gamma) > t^* \tag{4.13}$$

Equations (4.11)-(4.13) hold if and only if

$$\delta > \max\{t_1, t_2, \frac{t^*}{2 + \gamma})$$

The agent is indifferent between (1,0) and (0,1) if and only if

$$t_1 = t_2 \geq \delta$$

$$t^* - t_1 < \delta(1 + \gamma)$$

$$t^* - t_2 < \delta(1 + \gamma) \tag{4.14}$$

Finally, the agent will accept the contract only when he receives at least his reservation payoff. Then his strategy also takes into account his participation constraint:[1]

$$EU_A(e_1^*, e_2^*) \geq 0 \tag{4.15}$$

[1]It is obvious that the reservation payoff coincides with the payoff that the agent would receive by exerting (0,0). Hence if (1,1) or (1,0) or (0,1) is optimal, then the payoff from this effort choice yields at least the reservation payoff; i.e participation constraint is satisfied. If (0,0) is the optimal effort choice, the agents exactly receives the reservation payoff; i.e. participation constraint is again satisfied.

4.2.2 Equilibrium in Single-Principal Model

a. Principal's Problem

The upper level bureaucracy is identified with the "principal" in this model. As the principal can offer a bonus contract, the citizen can also pay the principal a bonus if both tasks are accomplished with high quality, i.e. the contract offered by the citizen promises to pay $T^* \geq T_1 + T_2$ when high quality is observed in both tasks. It pays only T_i when public good i is high quality and nothing when both goods are of low quality. The rent from the public office, denoted as R, could be extracted only when both goods are of high quality. Therefore the expected payoff of the principal can be expressed as:

$$EU_P(t_1, t_2, t^*) = e_1 e_2(R + T^* - T_1 - T_2 - (t^* - t_1 - t_2)) + e_1(T_1 - t_1) + e_2(T_2 - t_2) \quad (4.16)$$

Again the term $T^* - T_1 - T_2$ has a similar interpretation with the bonus in agent's contract. Given the contract, (T_1, T_2, T^*) offered by the citizen, the principal chooses (t_1, t_2, t^*) to maximize her expected payoff in (4.16) subject to the agent's strategy as derived in (4.2)-(4.14). She knows that the agent should be paid at least $t^* = \delta(2 + \gamma)$ to exert (1,1), $t_i = \delta$ to supply either (1,0) or (0,1), and zero to choose (0,0). Therefore she determines her strategy, (t_1, t_2, t^*), in the following way:

(i) Offer $t^* = \delta(2 + \gamma)$, and any $t_1, t_2 < \delta$ if and only if

$$R + T^* - \delta(2 + \gamma) \geq \max\{T_1 - \delta, T_2 - \delta, 0\} \quad (4.17)$$

(ii) Offer $t_1 = \delta$, any $t_2 < t_1$ and $t^* < \delta(2 + \gamma)$ if and only if

$$T_1 > \max\{R + T^* - \delta(1 + \gamma), T_2\} \text{ and } T_1 \geq \delta \quad (4.18)$$

(iii) Offer any $t^* < \delta(2 + \gamma)$, and $t_2 = \delta$, and any $t_1 < t_2$ if and only if

$$T_2 > \max\{R + T^* - \delta(1 + \gamma), T_1\} \text{ and } T_2 \geq \delta \quad (4.19)$$

(iv) Offer any $t^* < \delta(2+\gamma)$, and $t_1 = t_2 = \delta$, if and only if

$$T_1 = T_2 = T \geq \delta \text{ and } R + T^* - \delta(1+\gamma) < T \tag{4.20}$$

(v) Offer any $t^* < \delta(2+\gamma)$ and any $t_1, t_2 < \delta$ if and only if

$$T_1, T_2 < \delta \text{ and } \ R + T^* - \delta(2+\gamma) < 0 \tag{4.21}$$

To understand her strategy consider (i), for example: the contract (t_1, t_2, t^*) such that $t^* = \delta(2+\gamma)$ and any $t_1, t_2 < \delta$ implements the outcome (1,1). It is optimal to implement (1,1) if and only if the payoff from this outcome, i.e. $R + T^* - \delta(2+\gamma)$, is greater than the payoff from the contracts that would implement the outcomes (1,0), (0,1) and (0,0); i.e. $T_1 - \delta$, $T_2 - \delta$ and zero respectively. Therefore, the principal optimally implements (1,1) if and only if $R+T^*-\delta(2+\gamma) \geq \max\{T_1-\delta, T_2-\delta, 0\}$. Similarly, the contracts in (ii), (iii), (iv) and (v) implements the outcome (1,0), (0,1), either (1,0) or (0,1), and (0,0), respectively, whereas the conditions in (4.18)-(4.21) are both sufficent and necessary to determine the optimality of implementing the corresponding outcomes.

Since the citizen cannot observe the transfers between the principal and the agent, there is a potential welfare loss that might arise due to corruption. In other words, if the citizen does not know how much is transffered from the upper level to the lower level, there might occur *leakages* in the system.

Definition 31 *If $t_i < T_i$ or $t^* < T^*$, the principal is said to be "corrupt".*

It follows that corruption takes place in equilibrium in case (i) if $T^* > \delta(2+\gamma)$, in cases (ii) if $T_1 > \delta$, in case (iii) if $T_2 > \delta$ and in case (iv) $T_1 = T_2 > \delta$, and finally in case (v) for any δ where this case applies.

b. Citizen's Problem

If the agent succeeds in task 1, the citizen enjoys high quality public good that she values at $\alpha \in [0,1]$. Her utility from consumption is then equal to αq_1^h , where q_1^h is the quality level of high quality public good type 1. Similarly, her consumption is valued at $(1-\alpha)q_2^h$ if a high quality public good type 2 is produced. If the agent fails in a task, a low quality public good with quality level zero is realized.

The expected payoff of the citizen can be expressed as follows:

$$EU_C^{SP} = e_1(\alpha q_1^h - T_1) + e_2((1-\alpha)q_2^h - T_2) - e_1 e_2(T^* - T_1 - T_2) \tag{4.22}$$

Although the principal and the agent symmetrically have private information on the cost parameter δ, the citizen is not perfectly informed. But she knows that δ is uniformly distributed on $[0, \overline{\delta}]$, with $\overline{\delta} \geq \max\ \{\alpha q_1^h,\ (1-\alpha)q_2^h\}$. Then the citizen constructs her strategy by taking the uncertainty over δ into account. In doing so, she considers two things.

First, as the analysis of the previous section suggets, the transfers T_1, T_2 and T^* determine the feasibility of the outcomes: outcome (1,0) is feasible if and only if $\delta < T_1$. From the citizen's point of view, the probability of this event is then $F(T_1)$. However, outcome (1,1) is feasible if and only if $\delta(2+\gamma) < T^*$; i.e. with probability $F(\frac{T^*}{2+\gamma})$. Therefore, for given a contract option such that $T_1 < T_2 < \frac{T^*}{2+\gamma}$, with probability $F(T_1)$, any outcome is feasible, but with probability $F(\frac{T^*}{2+\gamma}) - F(T_2)$ only outcomes (1,1) and (0,0) are feasible, and so on.

Second, given a set of feasible outcomes for a given δ, the citizen considers the principal's strategy, as derived in equations (4.17)-(4.21), to predict which outcome will be implemented. Then for a contract option (T_1, T_2, T^*), the citizen can easiliy compute her expected payoff in the following way:

Example 32 *Consider a contract option,* (T_1, T_2, T^*), *where* $\frac{T^*}{2+\gamma} < T_2 < T_1$. *In Figure 4.1, we illustrate how the set of feasible outcomes changes as* δ *changes. Obviously, if* $\delta > T_1$, δ *will be too large relative to the principal's budget and hence the principal*

cannot induce any outcome except the outcome (0,0). Therefore, with probability $1 - F(T_1)$, *outcome (0,0) will be implemented. Note that this is the only range of* δ *where this outcome is implemented since any other outcome, when feasible, yields a (weakly) greater payoff to the principal. On the other hand, with probability* $F(T_1) - F(\frac{T^*}{2+\gamma})$, *we have* $\frac{T^*}{2+\gamma} < \delta \leq T_1$ *and outcome (1,0) will be implemented since: i) when* $T_2 < \delta \leq T_1$, *outcomes (1,0) and (0,0) are both feasible but (1,0) will be implemented since it yields a higher payoff to the principal, ii) when* $\frac{T^*}{2+\gamma} < \delta \leq T_2$, *outcome (0,1) is also feasible but since it is enough to offer* δ *to induce either one of the two outcomes, (1,0) and (0,1),* $T_2 < T_1$ *implies inducing (1,0) yields a higher payoff to the principal. Finally, with probability* $F(\frac{T^*}{2+\gamma})$, *we have* $\delta \leq \frac{T^*}{2+\gamma}$*: any outcome is feasible. By the same argument, outcome (0,0) yields the lowest payoff to the principal and outcome is (1,0) preferred over outcome (0,1). Then to determine whether outcome (1,1) or outcome (1,0) will be implemented, the citizen takes into account the principal's incentive compatibility. Recall that the principal's payoffs are* $R + T^* - \delta(2+\gamma)$, *and* $T_1 - \delta$ *if he implements (1,1) and (1,0), respectively. In other words, when* $\delta \leq \frac{T^*}{2+\gamma}$, *the principal compares* δ *with* $\frac{R+T^*-T_1}{1+\gamma}$ *to determine which outcome will yield a higher expected payoff to him. Then there are two options:*

Option 1: $\frac{T^*}{2+\gamma} \leq \frac{R+T^*-T_1}{1+\gamma}$.*Then* $\delta \leq \frac{T^*}{2+\gamma}$ *implies* $\delta \leq \frac{R+T^*-T_1}{1+\gamma}$*;i.e. the outcome (1,1) yields a higher payoff to the principal. In other words, with probability* $F(\frac{T^*}{2+\gamma})$, *outcome (1,1) will be implemented. Therefore, the citizen's total expected payoff is:*

$$F(\frac{T^*}{2+\gamma})(\alpha q_1^h + (1-\alpha)q_2^h) + [F(T_1) - F(\frac{T^*}{2+\gamma})]\alpha q_1^h$$

Option 2: $\frac{R+T^*-T_1}{1+\gamma} < \frac{T^*}{2+\gamma}$. *Then if* $\delta \leq \frac{R+T^*-T_1}{1+\gamma}$, *the outcome (1,1) will be implemented, but if* $\frac{R+T^*-T_1}{1+\gamma} < \delta \leq \frac{T^*}{2+\gamma}$, *outcome (1,0). Therefore, since as discussed above, (1,0) is also implemented when* $\frac{T^*}{2+\gamma} < \delta \leq T_1$, *we conclude that the principal implements outcome (1,1) with probability* $F(\frac{T^*}{2+\gamma})$, *but (1,0) with probability* $F(T_1) - F(\frac{R+T^*-T_1}{1+\gamma})$. *Then the expected payoff of the citizen is:*

$$F(\frac{R+T^*-T_1}{1+\gamma})(\alpha q_1^h + (1-\alpha)q_2^h) + [F(T_1) - F(\frac{R+T^*-T_1}{1+\gamma})]\alpha q_1^h$$

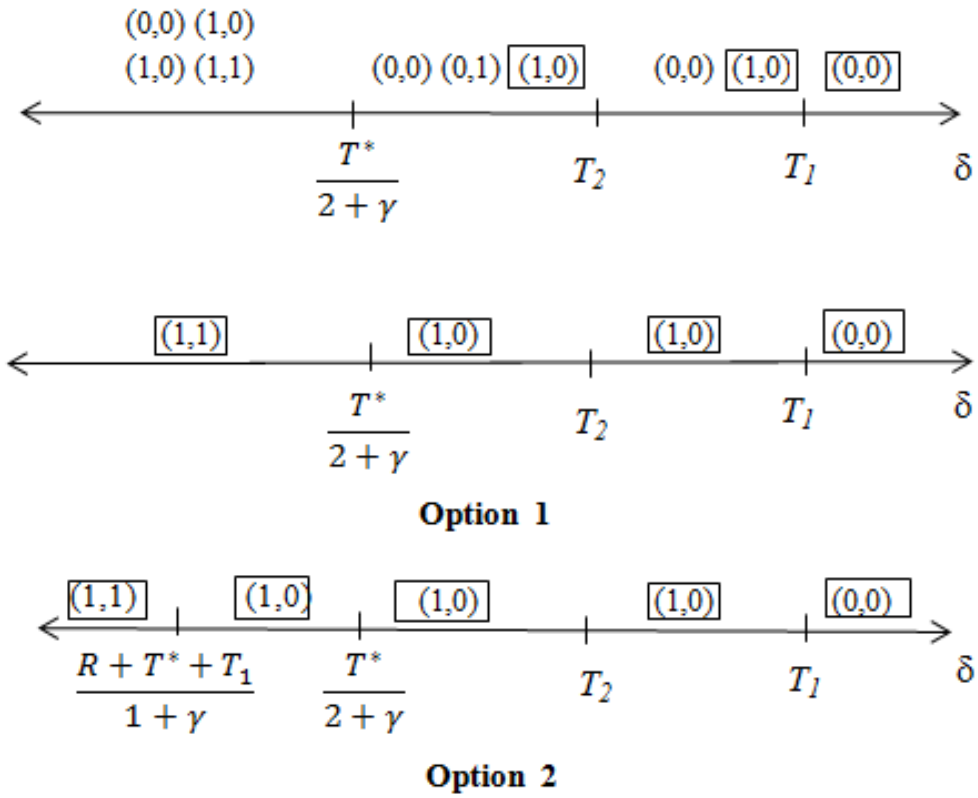


Figure 4.1: Example: Equilibrium Outcomes for a Given Contract Option

As illustrated, for a given contract option (T_1, T_2, T^*), it is the relationship between T_1, T_2 and $\frac{T^*}{2+\gamma}$ that determines the set of feasible outcomes for different ranges of δ. Then, the citizen takes into account how the principal will behave for a given δ, when he is offered a contract (T_1, T_2, T^*), and calculate her expected payoff from the contract accordingly. Therefore, as a first step in determining the citizen's strategy, we will consider the possible contract options, each of which speficies a particular form for this relationship, and derive their optimality conditions. For example, we first ask when a conract option (T_1, T_2, T^*), where $\frac{T^*}{2+\gamma} < T_2 < T_1$, is optimal? Next, we solve for the optimal transfers T_1, T_2 and T^* that maximizes the citizen's expected payoff. Of course, the citizen has many options to consider. But we first show that some options are always suboptimal. Particularly, as Lemma 33 suggests, T^* cannot be too high at the optimum.

Lemma 33 *It is not optimal to set* $T_1, T_2 \leq \frac{T^*}{2+\gamma}$.

Proof. Assume that the citizen offers (T_1, T_2, T^*) where $T_1, T_2 \leq \frac{T^*}{2+\gamma}$. Obviously, if $\delta > \frac{T^*}{2+\gamma}$, the principal cannot afford to induce any outcome, but (0,0). Hence, the citizen expects to receive a zero payoff with probability $1 - F(\frac{T^*}{2+\gamma})$. Observe that $T_1 \leq \frac{T^*}{2+\gamma}$ implies $T_1 \leq \frac{R+T^*-T_1}{1+\gamma}$. Therefore, when both outcomes (1,1) and (1,0) are feasible, i.e. when $\delta \leq T_1$, $\delta \leq \frac{R+T^*-T_1}{1+\gamma}$ holds. Hence, the principal strictly prefers implementing outcome (1,1) over (1,0). Similarly, $T_2 \leq \frac{T^*}{2+\gamma}$ implies $T_2 \leq \frac{R+T^*-T_2}{1+\gamma}$, and by the same reasoning, the principal strictly prefers outcome (1,1) over (0,1) when both are feasible, i.e. when $\delta \leq T_2$. Then we conclude that the principal implements the outcome (1,1) when $\delta \leq \frac{T^*}{2+\gamma}$. In other words, the citizen receives $\alpha q_1^h + (1-\alpha) q_2^h$ with probability $F(\frac{T^*}{2+\gamma})$. Depending on the citizen's taste over the two public goods, there are three possible cases. We show that under neither of these cases a contract option with $T_1, T_2 \leq \frac{T^*}{2+\gamma}$ is optimal:

Case 1: $\alpha q_1^h > (1-\alpha) q_2^h$. Fix T^*. The citizen is willing to pay at most $\alpha q_1^h + (1-\alpha) q_2^h$ when both goods are of high quality. Therefore, optimal T^* satisfies $T^* \leq \alpha q_1^h + (1-\alpha) q_2^h$. Since $\alpha q_1^h > (1-\alpha) q_2^h$, at the optimum we have $T^* < 2\alpha q_1^h$. This implies $\frac{T^*}{2+\gamma} < \alpha q_1^h$. Then, we can always find $\widetilde{T_1}$, $\widetilde{T_1} > T_1$, such that $\frac{T^*}{2+\gamma} < \widetilde{T_1} \leq \alpha q_1^h$ and $\widetilde{T_1} < \frac{R+T^*-\widetilde{T_1}}{1+\gamma}$. This means that the citizen continues to receive $\alpha q_1^h + (1-\alpha) q_2^h$ for all $\delta \leq \frac{T^*}{2+\gamma}$ (since we have fixed T^*). But now she is receiving an additional αq_1^h when $\frac{T^*}{2+\gamma} < \delta \leq \widetilde{T_1}$, by paying $\widetilde{T_1} \leq \alpha q_1^h$. Therefore, $T_1 \leq \frac{T^*}{2+\gamma}$ is not optimal if $\alpha q_1^h > (1-\alpha) q_2^h$.

Case 2: $(1-\alpha) q_2^h > \alpha q_1^h$. Similar to Case 1: the citizen can always improve her payoff by offering $\widetilde{T_2}$, $\widetilde{T_2} > T_2$, such that $\frac{T^*}{2+\gamma} < \widetilde{T_2} \leq (1-\alpha) q_2^h$ and $\widetilde{T_2} < \frac{R+T^*-\widetilde{T_2}}{1+\gamma}$. Therefore, $T_2 \leq \frac{T^*}{2+\gamma}$ is not optimal if $(1-\alpha) q_2^h > \alpha q_1^h$.

Case 3: $\alpha q_1^h = (1-\alpha) q_2^h \equiv q$. Then $T^* < 2q$ must hold at the optimum. Again fix T^*. Hence, we can always find $\widetilde{T_i}$, $\widetilde{T_i} > T_i$ for $i \in \{1, 2\}$, such that $\frac{T^*}{2+\gamma} < \widetilde{T_i} \leq q$ and $\widetilde{T_i} < \frac{R+T^*-\widetilde{T_i}}{1+\gamma}$. This means that the citizen continues to receive $\alpha q_1^h + (1-\alpha) q_2^h$ if $\delta \leq \frac{T^*}{2+\gamma}$ (since we have fixed T^*), but now she is receiving an additional q if $\frac{T^*}{2+\gamma} < \delta \leq \widetilde{T_i}$, by paying $\widetilde{T_i} \leq q$. Therefore, $T_2, T_1 \leq \frac{T^*}{2+\gamma}$ is not optimal if $\alpha q_1^h = (1-\alpha) q_2^h$. ■

Lemma 33 rules out the contract options which set T^* too large relative to T_1 and

T_2. Then, in equilibrium, the contract sets at least one of the two transfers above $\frac{T^*}{2+\gamma}$, i.e. optimality requires

$$\max(T_1, T_2) > \frac{T^*}{2+\gamma} \tag{4.23}$$

There are several possible contract options which satisfy condition (4.23). However, note that it is the particular relationship between T_1 and T_2 which matters: for example, a conract option with $T_1 > T_2$ assigns zero probability to the outcome (0,1) and hence, holding everything else constant, yields the same expected payoff to the citizen regardless of whether $\frac{T^*}{2+\gamma} < T_2 < T_1$ or $\frac{T^*}{2+\gamma} < T_2 < T_1$. For that reason, we classify the remaining contract options as those which set either $T_1 > T_2$, or $T_2 > T_1$, or $T_1 = T_2$. We start with deriving an optimality condition for the contract options which set $T_1 > T_2$.

Lemma 34 *If* $\alpha q_1^h > (1-\alpha)q_2^h$, *optimal contract sets* $T_1 > T_2$.

Proof. Assume that $\alpha q_1^h > (1-\alpha)q_2^h$ and the citizen offers T_1 and T_2 such that

$$T_1 = \widehat{T_1} \text{ and } T_2 = \widehat{T_2}, \widehat{T_1} > \widehat{T_2} \tag{4.24}$$

By Lemma 33,

$$\frac{T^*}{2+\gamma} < \widehat{T_1}$$

should hold at the optimum. Consider the implementable outcomes: with probability $1 - F(\widehat{T_1})$, outcome (0,0) will be implemented. Note that the principal always strictly prefers outcome (1,0) over (0,1) when both are feasible since $T_1 > T_2$ and hence the former yields a strictly higher payoff. Then, under the contract option in (4.24), outcome (0,1) will never be implemented. Furthermore, when both outcomes, (1,0) and (1,1) are feasible, i.e. when $\delta \leq \frac{T^*}{2+\gamma}$, $\frac{R+T^*-\widehat{T_1}}{1+\gamma}$ determines which outcome will be implemented. There are two options:

Option 1: $\frac{R+T^*-\widehat{T_1}}{1+\gamma} < \frac{T^*}{2+\gamma}$. In that case, with probability $F(\widehat{T_1}) - F(\frac{R+T^*-\widehat{T_1}}{1+\gamma})$, outcome (1,0) will be implemented and the citizen receives αq_1^h. With probability

$F(\frac{R+T^*-\widehat{T_1}}{1+\gamma})$, outcome (1,1) will be implemented and the citizen will receive $\alpha q_1^h + (1-\alpha)q_2^h$. Now fix that T^*. First let $(T_1, T_2) = (\widetilde{T_1}, \widetilde{T_2})$, where $\widetilde{T_2} = \widehat{T_1}$ and $\widetilde{T_1} = \widehat{T_2}$. Note that $\widetilde{T_2} > \widetilde{T_1}$. Since T^* is fixed, the citizen still receives $\alpha q_1^h + (1-\alpha)q_2^h$ with probability $F(\frac{T^*}{2+\gamma})$. However, she receives $(1-\alpha)q_2^h$ with probability $F(\widetilde{T_2}) - F(\frac{R+T^*-\widetilde{T_2}}{1+\gamma}) = F(\widehat{T_1}) - F(\frac{R+T^*-\widetilde{T_1}}{1+\gamma})$. Observe that, since $\alpha q_1^h > (1-\alpha)q_2^h$, the contract with $(T_1, T_2) = (\widetilde{T_1}, \widetilde{T_2})$ yields a lower payoff. Second, let $(T_1, T_2) = (\widetilde{T_1}, \widetilde{T_2})$ where $\widetilde{T_2} = \widetilde{T_1} = \widehat{T_1}$. Then, the principal will be indifferent in implementing the two outcomes, (1,0) and (0,1), with probability $F(\widehat{T_1}) - F(\frac{R+T^*-\widehat{T_1}}{1+\gamma})$, but implements (1,1) with probability $F(\frac{R+T^*-\widehat{T_1}}{1+\gamma})$. However, since $\alpha q_1^h > (1-\alpha)q_2^h$, the citizen is not indifferent between the two outcomes and setting $T_1 > T_2$ yields a higher payoff.

Option 2: $\frac{R+T^*-\widehat{T_1}}{1+\gamma} \geq \frac{T^*}{2+\gamma}$. In that case, with probability $F(\widehat{T_1}) - F(\frac{T^*}{2+\gamma})$, outcome (1,0) will be implemented and the citizen receives αq_1^h . With probability $F(\frac{T^*}{2+\gamma})$, outcome (1,1) will be implemented so the citizen will receive $\alpha q_1^h + (1-\alpha)q_2^h$. Again fix that T^* and let $(T_1, T_2) = (\widetilde{T_1}, \widetilde{T_2})$, where $\widetilde{T_2} = \widehat{T_1}$ and $\widetilde{T_1} = \widehat{T_2}$. Note that $\widetilde{T_2} > \widetilde{T_1}$. Since T^* is fixed, the citizen still receives $\alpha q_1^h + (1-\alpha)q_2^h$ with probability $F(\frac{T^*}{2+\gamma})$. However she will receive $(1-\alpha)q_2^h$ with probability $F(\widetilde{T_2}) - F(\frac{T^*}{2+\gamma}) = F(\widehat{T_1}) - F(\frac{T^*}{2+\gamma})$. Observe that, since $\alpha q_1^h > (1-\alpha)q_2^h$, the contract with $(T_1, T_2) = (\widetilde{T_1}, \widetilde{T_2})$ yields a lower payoff. Next, let $(T_1, T_2) = (\widetilde{T_1}, \widetilde{T_2})$ where $\widetilde{T_2} = \widetilde{T_1} = \widehat{T_1}$. Again, such contract yields (1,1) with probability $F(\frac{T^*}{2+\gamma})$, but will leave the principal indifferent implementing the two outcomes, (1,0) and (0,1) with probability $F(\widehat{T_1}) - F(\frac{T^*}{2+\gamma})$. However, the citizen is not indifferent between the two outcomes and setting $T_1 > T_2$ yields a higher payoff.

■

Intuitively, if the citizen values the public good type 1 more than the other one, she pays more for that good in equilibrium. But how much will be paid in this case? Proposition 35 defines the optimal contract.

Proposition 35 *If $\alpha q_1^h > (1-\alpha)q_2^h$, the optimal contract (T_1, T_2, T^*) is*

$$\begin{cases} \left(\frac{2(2+\gamma)\alpha q_1^h + (1-\alpha)q_2^h}{4\gamma+7}, 0, \frac{(2+\gamma)(\alpha q_1^h + 2(1-\alpha)q_2^h)}{4\gamma+7} \right) & \text{if} \quad R > \frac{(3+2\gamma)\alpha q_1^h - (1-\alpha)q_2^h}{4\gamma+7} \\ \left(\frac{(3+2\gamma)R + \alpha q_1^h + (1-\alpha)q_2^h}{2(2+\gamma)}, 0, \frac{\alpha q_1^h + (1-\alpha)q_2^h - R}{2} \right) & \text{if} \quad \frac{(1+\gamma)\alpha q_1^h - (1-\alpha)q_2^h}{3+2\gamma} \leq R \leq \frac{(3+2\gamma)\alpha q_1^h - (1-\alpha)q_2^h}{4\gamma+7} \\ \left((\frac{\alpha q_1^h}{2}, 0, \frac{\alpha q_1^h + (1-\alpha)q_2^h - R}{2}) \right) & \text{if} \quad R < \frac{(1+\gamma)\alpha q_1^h - (1-\alpha)q_2^h}{3+2\gamma} \end{cases}$$

Proof. Assume that $\alpha q_1^h > (1-\alpha)q_2^h$. By Lemma 34 and Lemma 33, optimal contract sets $T_2 < T_1$ and $\frac{T^*}{2+\gamma} < T_1$. Then, outcome (0,1) will never be implemented: since outcome (0,1) is feasible only if outcome (1,0) is feasible and from the principal's perspective, it is enough to offer δ to induce either of the two outcomes. Then, $T_2 < T_1$ implies implementing (1,0) yields a higher payoff. Regarding the two outcomes, (1,0) and (1,1), both are feasible if $\delta \leq \frac{T^*}{2+\gamma}$. When this is the case, as illustrated in Example 32, how large $\frac{R+T^*-T_1}{1+\gamma}$ relative to δ is important to determine which outcome will be implemented. Therefore, in Step 1 below, we consider different contract options and in each we determine the best contract. In Step 2, we characterize the optimal contract.

Step 1. There are two options.

Option 1: Suppose that the citizen offers T^* and T_1 such that $\frac{T^*}{2+\gamma} \leq \frac{R+T^*-T_1}{1+\gamma}$. The feasible outcomes and the induced outcome for a given range of δ are as illustrated in Option 1 in Figure 4.1. If $\delta \leq \frac{T^*}{2+\gamma}$, $\delta \leq \frac{R+T^*-T_1}{1+\gamma}$ also holds, i.e. the principal strictly prefers outcome (1,1) over outcome (1,0) when both are feasible. If $\frac{T^*}{2+\gamma} < \delta \leq T_1$, however, the only feasible outcomes are (1,0) and (0,0), and the former outcome will be implemented since it yields a higher payoff to the principal. If $\delta > T_1$, the principal will implement (0,0).Therefore, the contract induces the outcome (1,1) with probability $F(\frac{T^*}{2+\gamma})$, (1,0) with probability $F(T_1) - F(\frac{T^*}{2+\gamma})$, and (0,0) with probability $1 - F(T_1)$. Clearly, we set $T_2 = 0$ at the optimum. Optimal (T_1, T^*) maximizes the citizen's

expected payoff[2]:

$$EU_C^{SP} = F(\frac{T^*}{2+\gamma})(\alpha q_1^h + (1-\alpha)q_2^h - T^*) + [F(T_1) - F(\frac{T^*}{2+\gamma})](\alpha q_1^h - T_1)$$

such that

$$\frac{T^*}{2+\gamma} \leq \frac{R + T^* - T_1}{1+\gamma}$$

Let $\mathfrak{L}(T^*, T_1, \lambda)$ denote the Lagrangian function where λ is a Lagrange multiplier.

$$\begin{aligned}\mathfrak{L}(T^*, T_1, \lambda) &= F(\frac{T^*}{2+\gamma})(\alpha q_1^h + (1-\alpha)q_2^h - T^*) + \\ &\quad [F(T_1) - F(\frac{T^*}{2+\gamma})](\alpha q_1^h - T_1) + \lambda((2+\gamma)R + T^* - (2+\gamma)T_1)\end{aligned}$$

and

$$(2+\gamma)R + T^* - (2+\gamma)T_1 \geq 0, \quad \lambda \geq 0 \quad \text{and} \quad \lambda((2+\gamma)R + T^* - (2+\gamma)T_1) = 0$$

First assume that $(2+\gamma)R+T^*-(2+\gamma)T_1 = 0$, i.e. the constraint is binding. Using the Kuhn-Tucker Conditions we get

$$(T_1, T^*) = (\frac{(3+2\gamma)R + \alpha q_1^h + (1-\alpha)q_2^h}{2(2+\gamma)}, \frac{\alpha q_1^h + (1-\alpha)q_2^h - R}{2}) \tag{4.25}$$

and

$$\lambda = \frac{((3+2\gamma)\alpha q_1^h - (1-\alpha)q_2^h - R(4\gamma+7))}{\overline{\delta}(2+\gamma)} \tag{4.26}$$

If

$$\frac{(3+2\gamma)\alpha q_1^h - (1-\alpha)q_2^h}{4\gamma+7} \geq R, \tag{4.27}$$

then (4.26) implies $\lambda \geq 0$, hence (4.25) and (4.26) is a solution to the maximization

[2] SP stands for "single principal" where as CA denotes "common agency".

problem. However, if

$$\frac{(3+2\gamma)\alpha q_1^h - (1-\alpha)q_2^h}{4\gamma+7} < R, \tag{4.28}$$

then (4.26) implies $\lambda < 0$, which violates the Kuhn-Tucker conditions. Then the constraint does not bind in that case. The solution is to set $\lambda = 0$. The conditions yield

$$(T_1, T^*) = (\frac{2(2+\gamma)\alpha q_1^h + (1-\alpha)q_2^h}{4\gamma+7}, \frac{(2+\gamma)(\alpha q_1^h + 2(1-\alpha)q_2^h)}{4\gamma+7}) \tag{4.29}$$

Call the contract in (4.25) by *Type-1,* and the contract in (4.29) by *Type-2.* Then, restricting the contract set to *Option 1*, the best contract is *Type-1* if $\frac{(3+2\gamma)\alpha q_1^h - (1-\alpha)q_2^h}{4\gamma+7} \geq R$, and it is *Type-2* if $\frac{(3+2\gamma)\alpha q_1^h - (1-\alpha)q_2^h}{4\gamma+7} < R$. The citizen's expected payoffs from contracts *Type-1* and *Type-2* are respectively:

$$EU_C^{SP1} = \frac{(\alpha q_1^h + (1-\alpha)q_2^h)^2 + 2R((3+2\gamma)\alpha q_1^h - (1-\alpha)q_2^h) - (4\gamma+7)R^2}{4(2+\gamma)\overline{\delta}} \tag{4.30}$$

and

$$EU_C^{SP2} = \frac{(\gamma+2)(\alpha q_1^h)^2 + \alpha(1-\alpha)q_1^h q_2^h + ((1-\alpha)q_2^h)^2}{(4\gamma+7)\overline{\delta}} \tag{4.31}$$

Option 2. Suppose that the citizen offers T_1 and T^* such that $\frac{R+T^*-T_1}{1+\gamma} \leq \frac{T^*}{2+\gamma}$. The feasible outcomes and the induced outcome for a given range of δ are the same as illustrated in Option 2 in Figure 4.1. Therefore, the outcome (1,1) will be implemented with probability $F(\frac{R+T^*-T_1}{1+\gamma})$; (1,0) with probability $[F(T_1) - F(\frac{R+T^*-T_1}{1+\gamma})]$ and (0,0) with probability $[1 - F(T_1)]$. T^* and T_1 will be chosen to maximize the expected payoff

$$EU_C^{SP} = F(\frac{R+T^*-T_1}{1+\gamma})(\alpha q_1^h + (1-\alpha)q_2^h - T^*) + [F(T_1) - F(\frac{R+T^*-T_1}{1+\gamma})](\alpha q_1^h - T_1)]$$

such that

$$\frac{T^*}{2+\gamma} \geq \frac{R+T^*-T_1}{1+\gamma}$$

The Lagrangian function for this optimization problem is

$$\mathfrak{L}(T^*, T_1, \lambda) = F(\frac{R + T^* - T_1}{1+\gamma})(\alpha q_1^h + (1-\alpha)q_2^h - T^*)$$
$$+[F(T_1) - F(\frac{R + T^* - T_1}{1+\gamma})](\alpha q_1^h - T_1) - \lambda((2+\gamma)R + T^* - (2+\gamma)T_1)$$

The Kuhn-Tucker conditions for this constrained maximization problem are:

$$\begin{aligned} \frac{1}{\bar{\delta}}(2T_1 - 2T^* + (1-\alpha)q_2^h - R) - (1+\gamma)\lambda &= 0 \\ \frac{1}{\bar{\delta}}(R + (1+\gamma)\alpha q_1^h + 2T^* - 2(2+\gamma)T_1 - (1-\alpha)q_2^h) + (2+\gamma)(1+\gamma)\lambda &= 0 \end{aligned}$$

and

$$(2+\gamma)R + T^* - (2+\gamma)T_1 \leq 0, \lambda \geq 0, \quad \text{and} \quad \lambda((2+\gamma)R + T^* - (2+\gamma)T_1) = 0$$

Assume that $(2+\gamma)R + T^* - (2+\gamma)T_1 = 0$, i.e that the constraint is binding. Then we solve for payoff maximizing (T^*, T_1, λ). Since this case coincides with the case in Option 1 when we have a binding constraint, payoff maximizing (T_1, T^*) is the same as derived in (4.25), i.e. contract *Type-1* is optimal. Using the Kuhn-Tucker conditions, we get

$$\lambda = \frac{(3+2\gamma)R + (1-\alpha)q_2^h - (1+\gamma)\alpha q_1^h}{(1+\gamma)(2+\gamma)\bar{\delta}} \tag{4.32}$$

Observe that if

$$R \geq \frac{(1+\gamma)\alpha q_1^h - (1-\alpha)q_2^h}{3+2\gamma}, \tag{4.33}$$

then (4.32) implies $\lambda \geq 0$. Therefore (T^*, T_1, λ), as defined in (4.25) and (4.32) solves the maximization problem. But if

$$R < \frac{(1+\gamma)\alpha q_1^h - (1-\alpha)q_2^h}{3+2\gamma}, \tag{4.34}$$

then (4.32) implies $\lambda < 0$, violating Kuhn-Tucker conditions. Then the constraint does not bind, i.e $(2+\gamma)R + T^* - (2+\gamma)T_1 < 0$. We simply set $\lambda = 0$. Then the conditions

yield

$$(T_1, T^*) = (\frac{\alpha q_1^h}{2}, \frac{\alpha q_1^h + (1-\alpha)q_2^h - R}{2}) \tag{4.35}$$

Call the contract in (4.35) by *Type-3.* Then, focusing on *Option 2,* the best contract is *Type-1* if $\frac{(1+\gamma)\alpha q_1^h - (1-\alpha)q_2^h}{3+2\gamma} \geq R$, and it is *Type-3* if $\frac{(1+\gamma)\alpha q_1^h - (1-\alpha)q_2^h}{3+2\gamma} < R$. The citizen's expected payoffs from contract *Type-1* is as calculated in (4.30), and from *Type-3* is:

$$EU_C^{SP3} = \frac{(\gamma+1)(\alpha q_1^h)^2 + 2R(1-\alpha)q_2^h + ((1-\alpha)q_2^h)^2 + R^2}{4(\gamma+1)\overline{\delta}} \tag{4.36}$$

Step 2. Denote $\frac{(1+\gamma)\alpha q_1^h - (1-\alpha)q_2^h}{3+2\gamma} \equiv R_1$ and $\frac{(3+2\gamma)\alpha q_1^h - (1-\alpha)q_2^h}{4\gamma+7} \equiv R_2$. Clearly, $R_2 < R_1$. In Figure 4.2, we compare the expected payoffs from contracts *Type-1*, *Type-2* and *Type-3*, i.e. EU_C^{SP1}, EU_C^{SP2} and EU_C^{SP3}, respectively. The optimal contract maximizes the expected payoff of the citizen. Therefore, if we restrict ourselves to the class contracts in Option 1: $EU_C^{SP1} < EU_C^{SP2}$ for all R, however, contract *Type-2* is not feasible for $R \leq R_1$ since it violates the constraint, therefore *Type-2* is optimal for all $R > R_1$. Restricting our attention to the class of contracts in Option 2: $EU_C^{SP1} < EU_C^{SP3}$ for all R, however, contract *Type-3* is not feasible for $R \geq R_2$ since it violates the constraint, therefore *Type-3* is optimal for all $R < R_2$. For all R such that $R_2 \leq R \leq R_1$, optimal contract is *Type-1,* since *Type-2* and *Type-3* are not feasible. ■

Observe that optimal T^* decreases as R increases. Intuitively, as the rents from office, that could be earned only when success in both tasks occurs, increases, the optimal incentives provided by T^* decreases. In other words, it becomes cheaper to induce success in both tasks when the reward from keeping the office increases. However, optimal T_1 increases as R increases. Since as R gets larger, T^* decreases at the optimum and for the citizen, it is payoff increasing to offer a higher T_1 to increase the likelihood of outcome (1,0).

Next, we show that if the citizen values the public good type 2 more than the other one, she pays more for that good in equilibrium.

Lemma 36 *If* $(1-\alpha)q_2^h > \alpha q_1^h$, *optimal contract sets* $T_2 > T_1$.

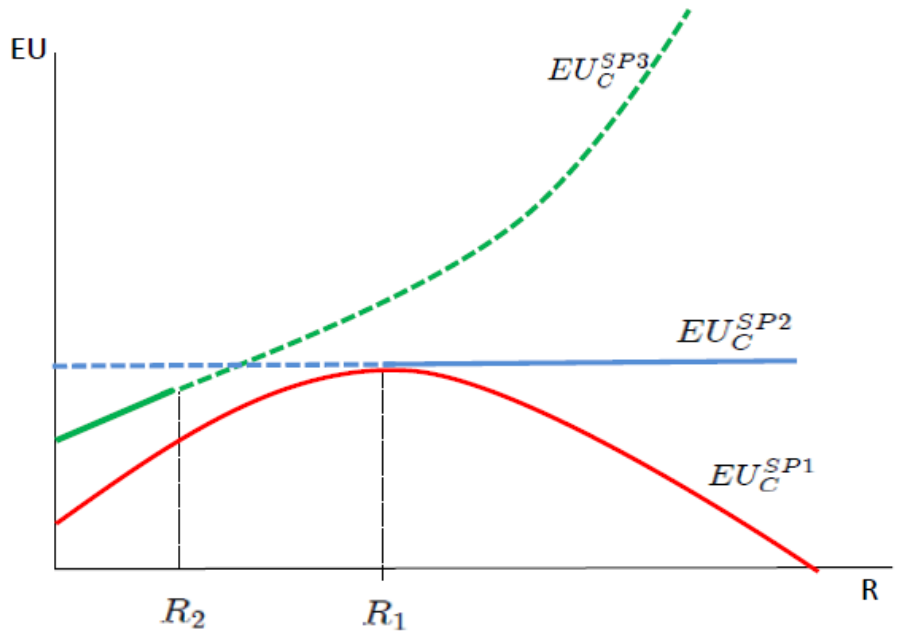


Figure 4.2: Proof of Proposition 35

Proof. Assume that the citizen offers T_1 and T_2 such that

$$T_1 = \widehat{T_1} \text{ and } T_2 = \widehat{T_2}, \widehat{T_2} > \widehat{T_1} \tag{4.37}$$

By Lemma 33,

$$\frac{T^*}{2+\gamma} < \widehat{T_2}$$

should hold. Consider the implementable outcomes: with probability $1 - F(\widehat{T_2})$, outcome (0,0) will be implemented. Note that the principal always strictly prefers outcome (0,1) over (0,1) when both are feasible since $T_2 > T_1$ and hence the former yields a strictly higher payoff. Then, under the contract option in (4.37), outcome (1,0) will never be implemented. Furthermore, when both outcomes, (0,1) and (1,1) are feasible, i.e. when $\delta \leq \frac{T^*}{2+\gamma}$, $\frac{R+T^*-\widehat{T_2}}{1+\gamma}$ determines which outcome will be implemented. There are two options:

Option 1: $\frac{R+T^*-\widehat{T_2}}{1+\gamma} < \frac{T^*}{2+\gamma}$. In that case, with probability $F(\widehat{T_2}) - F(\frac{R+T^*-\widehat{T_2}}{1+\gamma})$, outcome (0,1) will be implemented and the citizen receives $(1-\alpha)q_2^h$. With probability

$F(\frac{R+T^*-\widehat{T_2}}{1+\gamma})$, outcome (1,1) will be implemented and the citizen will receive $\alpha q_1^h + (1-\alpha)q_2^h$. Now fix that T^*. First let $(T_1, T_2) = (\widetilde{T_1}, \widetilde{T_2})$, where $\widetilde{T_1} = \widehat{T_2}$ and $\widetilde{T_2} = \widehat{T_1}$. Note that $\widetilde{T_1} > \widetilde{T_2}$. Since T^* is fixed, the citizen still receives $\alpha q_1^h + (1-\alpha)q_2^h$ with probability $F(\frac{T^*}{2+\gamma})$. However, she receives αq_1^h with probability $F(\widetilde{T_1}) - F(\frac{R+T^*-\widetilde{T_1}}{1+\gamma}) = F(\widehat{T_2}) - F(\frac{R+T^*-\widehat{T_2}}{1+\gamma})$. Observe that, since $(1-\alpha)q_2^h > \alpha q_1^h$, the contract with $(T_1, T_2) = (\widetilde{T_1}, \widetilde{T_2})$ yields a lower payoff. Second, let $(T_1, T_2) = (\widetilde{T_1}, \widetilde{T_2})$ where $\widetilde{T_2} = \widetilde{T_1} = \widehat{T_2}$. Then, the principal will be indifferent in implementing the two outcomes, (1,0) and (0,1), with probability $F(\widehat{T_2}) - F(\frac{R+T^*-\widehat{T_2}}{1+\gamma})$, but implements (1,1) with probability $F(\frac{R+T^*-\widehat{T_2}}{1+\gamma})$. However, since $(1-\alpha)q_2^h > \alpha q_1^h$, the citizen is not indifferent between the two outcomes and setting $T_2 > T_1$ yields a higher payoff.

Option 2: $\frac{R+T^*-\widehat{T_2}}{1+\gamma} \geq \frac{T^*}{2+\gamma}$. In that case, with probability $F(\widehat{T_2}) - F(\frac{T^*}{2+\gamma})$, outcome (0,1) will be implemented and the citizen receives $(1-\alpha)q_2^h$. With probability $F(\frac{T^*}{2+\gamma})$, outcome (1,1) will be implemented so the citizen will receive $\alpha q_1^h + (1-\alpha)q_2^h$. Again fix that T^* and let $(T_1, T_2) = (\widetilde{T_1}, \widetilde{T_2})$, where $\widetilde{T_1} = \widehat{T_2}$ and $\widetilde{T_2} = \widehat{T_1}$. Note that $\widetilde{T_1} > \widetilde{T_2}$. Since T^* is fixed, the citizen still receives $\alpha q_1^h + (1-\alpha)q_2^h$ with probability $F(\frac{T^*}{2+\gamma})$. However she will receive αq_1^h with probability $F(\widetilde{T_1}) - F(\frac{T^*}{2+\gamma}) = F(\widehat{T_2}) - F(\frac{T^*}{2+\gamma})$. Observe that, since $(1-\alpha)q_2^h > \alpha q_1^h$, the contract with $(T_1, T_2) = (\widetilde{T_1}, \widetilde{T_2})$ yields a lower payoff. Next, let $(T_1, T_2) = (\widetilde{T_1}, \widetilde{T_2})$ where $\widetilde{T_2} = \widetilde{T_1} = \widehat{T_2}$. Again, such contract yields (1,1) with probability $F(\frac{T^*}{2+\gamma})$, but will leave the principal indifferent implementing the two outcomes, (1,0) and (0,1) with probability $F(\widehat{T_2}) - F(\frac{T^*}{2+\gamma})$. However, the citizen is not indifferent between the two outcomes and setting $T_2 > T_1$ yields a higher payoff. ■

Similar to a contract option which sets $T_1 > T_2$ and hence remove the possibility of outcome (0,1) to arise in equilibrium, a contract with $T_2 > T_1$ annihilates the implementibility of one of the outcomes: the principal will never implement the outcome (1,0) in equilibrium. Likewise, as Proposition 37 suggests, if $(1-\alpha)q_2^h > \alpha q_1^h$, optimal transfers, (T_1, T_2) is just symmetric to the transfers in Proposition 35.

Proposition 37 *If* $(1-\alpha)q_2^h > \alpha q_1^h$, *the optimal contract* (T_1, T_2, T^*) *is*

$$\begin{cases} \left(0, \frac{2(2+\gamma)(1-\alpha)q_2^h+\alpha q_1^h}{4\gamma+7}, \frac{(2+\gamma)(2\alpha q_1^h+(1-\alpha)q_2^h)}{4\gamma+7} \right) & \text{if} \quad R > \frac{(3+2\gamma)(1-\alpha)q_2^h-\alpha q_1^h}{4\gamma+7} \\ \left(0, \frac{(3+2\gamma)R+\alpha q_1^h+(1-\alpha)q_2^h}{2(2+\gamma)}, \frac{\alpha q_1^h+(1-\alpha)q_2^h-R}{2} \right) & \text{if} \quad \frac{(1+\gamma)(1-\alpha)q_2^h-\alpha q_1^h}{3+2\gamma} \leq R \leq \frac{(3+2\gamma)(1-\alpha)q_2^h-\alpha q_1^h}{4\gamma+7} \\ \left((0, \frac{(1-\alpha)q_2^h}{2}, \frac{\alpha q_1^h+(1-\alpha)q_2^h-R}{2}) \right) & \text{if} \quad R < \frac{(1+\gamma)(1-\alpha)q_2^h-\alpha q_1^h}{3+2\gamma} \end{cases}$$

So far we have studied two cases where the citizen's taste over the two public goods is asymmetric. We have showed that the optimal contract offers more for the public good that the citizen values more. Now we consider the remaining situation where the citizen is indifferent between the two puclic goods, i.e. $\alpha q_1^h = (1-\alpha)q_2^h$.

Denote $\alpha q_1^h = (1-\alpha)q_2^h \equiv q^h$. In two steps, we will show that the citizen is indifferent in offering $T_1 = T_2$ or $T_1 \neq T_2$, where (T_1, T_2) satisfies $\max(T_1, T_2) > \frac{T^*}{2+\gamma}$ (by Lemma (33)). First, we restrict our attention to the class of contracts which set $T_1 = T_2$, and the derive the optimal contract under this restriction. Next, we compare the maximized expected payoff of the citizen under contracts $T_1 = T_2$ and $T_1 \neq T_2$.

Proposition 38 *If* $\alpha q_1^h = (1-\alpha)q_2^h \equiv q^h$, *and* $T_1 = T_2$, *then the payoff maximizing contract is*

$$(T_1, T_2, T^*) = \begin{cases} \left(\frac{(5+2\gamma)q^h}{4\gamma+7}, \frac{(5+2\gamma)q^h}{4\gamma+7}, \frac{3(2+\gamma)q^h}{4\gamma+7} \right) & \text{if} \quad R > \frac{2(1+\gamma)q^h}{4\gamma+7} \\ \left(\frac{2q^h+(3+2\gamma)R}{2(2+\gamma)}, \frac{2q^h+(3+2\gamma)R}{2(2+\gamma)}, \frac{2q^h-R}{2} \right) & \text{if} \quad \frac{\gamma q^h}{3+2\gamma} \leq R \leq \frac{2(1+\gamma)q^h}{4\gamma+7} \\ \left((\frac{q^h}{2}, \frac{q^h}{2}, \frac{2q^h-R}{2}) \right) & \text{if} \quad R < \frac{\gamma q^h}{3+2\gamma} \end{cases}$$

Proof. Denote $T_1 = T_2 = T$. By Lemma (33), it is never payoff maximizing to set $T \leq \frac{T^*}{2+\gamma}$. Then, a payoff maximizing contract sets $T > \frac{T^*}{2+\gamma}$. Hence with probability $1 - F(T)$, outcome (0,0) will be implemented and the citizen receives zero. However,

with probability $F(T)-F(\frac{T^*}{2+\gamma})$, the principal is indifferent between implementing (1,0) and (0,1), and the citizen receives q^h in either case. With probability $F(\frac{T^*}{2+\gamma})$, cost of agent's effort is low enough: $\delta < \frac{T^*}{2+\gamma}$, hence outcome (1,1) is also feasible. Then in that case,as before, which outcome will be implemented depends on $\frac{R+T^*-T}{1+\gamma}$. Therefore, as in the proof of Proposition (35), the payoff maximizing contract is derived in two steps.

Step 1. There are two options:

Option 1: Suppose that the citizen chooses T^* and T such that $\frac{T^*}{2+\gamma} \leq \frac{R+T^*-T}{1+\gamma}$. In that case, with probability $F(\frac{T^*}{2+\gamma})$, the outcome will be (1,1) and receives $2q^h$ with probability $F(T)-F(\frac{T^*}{2+\gamma})$. Then, the associated Lagranian function will be:

$$\begin{aligned}\mathfrak{L}(T^*,T,\lambda) &= F(\frac{T^*}{2+\gamma})(2q^h-T^*)+ \\ &\quad [F(T)-F(\frac{T^*}{2+\gamma})](q^h-T)+\lambda((2+\gamma)R+T^*-(2+\gamma)T)\end{aligned}$$

The Kuhn-Tucker conditions are

$$\begin{aligned}\frac{1}{\bar{\delta}}((2+\gamma)q^h-2(2+\gamma)T+T^*)-(2+\gamma)^2\lambda &= 0 \\ \frac{1}{\bar{\delta}}(q^h-2T^*+T)+(2+\gamma)\lambda &= 0\end{aligned}$$

$$(2+\gamma)R+T^*-(2+\gamma)T \geq 0,\ \lambda \geq 0 \quad \text{and} \quad \lambda((2+\gamma)R+T^*-(2+\gamma)T)=0$$

First assume that $(2+\gamma)R+T^*-(2+\gamma)T=0$, i.e. the constraint is binding. Using the Kuhn-Tucker Conditions, we solve for (T^*,T,λ) :

$$(T,T^*)=(\frac{(3+2\gamma)R+2q^h}{2(2+\gamma)},\frac{2q^h-R}{2}) \tag{4.38}$$

$$\lambda=\frac{1}{\bar{\delta}(2+\gamma)}(2(1+\gamma)q^h-R(4\gamma+7)) \tag{4.39}$$

If

$$R \leq \frac{2(1+\gamma)q^h}{4\gamma+7}$$

then (4.39) implies $\lambda \geq 0$, hence (4.38) and (4.39) is a solution to the maximization problem. Call the contract in (4.38) by *Type-1**. The citizen's expected payoff under contract *Type-1** is:

$$EU_C^{SP1^*} = \frac{4(q^h)^2 + 4(1+\gamma)Rq^h - (4\gamma+7)R^2}{4(2+\gamma)\overline{\delta}} \tag{4.40}$$

However, if

$$\frac{2(1+\gamma)q^h}{4\gamma+7} < R \tag{4.41}$$

then (4.39) implies $\lambda < 0$, which violates the Kuhn- Tucker conditions. Then, the constraint does not bind and the solution is $\lambda = 0$. The Kuhn-Thucker conditions yield

$$(T, T^*) = (\frac{(5+2\gamma)q^h}{4\gamma+7}, \frac{3(2+\gamma)q^h}{4\gamma+7}) \tag{4.42}$$

Call the contract in (4.38) by *Type-2**. The citizen's expected payoff is under contract *Type-2** is then:

$$EU_C^{SP2^*} = \frac{(\gamma+4)(q^h)^2}{(4\gamma+7)\overline{\delta}} \tag{4.43}$$

Option 2: Suppose that the citizen chooses T^* and T such that $\frac{R+T^*-T}{1+\gamma} \leq \frac{T^*}{2+\gamma}$. In that case, the outcome (1,1) will be implemented with probability $F(\frac{R+T^*-T}{1+\gamma})$ and the citizen receives $2q^h$. However, with probability $[F(T) - F(\frac{R+T^*-T}{1+\gamma})]$, the principal is indifferent between implementing (1,0) and (0,1); but the citizen receives q^h in either case. With probability $1-F(T)$, the outcome (0,0) will be implemented and the citizen receives zero. Then the Lagrangian function for the optimization problem is:

$$\begin{aligned}\mathfrak{L}(T^*, T, \lambda) &= F(\frac{R+T^*-T}{1+\gamma})(2q^h - T^*) + [F(T) - F(\frac{R+T^*-T}{1+\gamma})](q^h - T)] - \\ &\quad \lambda((2+\gamma)R + T^* - (2+\gamma)T)\end{aligned}$$

The Kuhn-Tucker conditions for this constrained maximization problem are:

$$\begin{aligned}\frac{1}{\bar{\delta}}(2T_1 - 2T^* + (1-\alpha)q_2^h - R) - (1+\gamma)\lambda &= 0\\ \frac{1}{\bar{\delta}}(R + \gamma q^h + 2T^* - 2(2+\gamma)T) + (2+\gamma)(1+\gamma)\lambda &= 0\end{aligned}$$

$$(2+\gamma)R + T^* - (2+\gamma)T_1 \leq 0, \lambda \geq 0 \quad \text{and} \quad \lambda((2+\gamma)R + T^* - (2+\gamma)T) = 0$$

Again start with the case where the contraint is binding: $(2+\gamma)R+T^*-(2+\gamma)T = 0$.Then using the Kuhn-Tucker Conditions, we solve for (T^*, T, λ). Payoff maximizing (T^*, T) coincides with contract *Type-2**, hence the citizen's payoff is as calculated in (4.43), and optimal λ is:

$$\lambda = \frac{(3+2\gamma)R - \gamma q^h}{(2+\gamma)\bar{\delta}} \tag{4.44}$$

Observe that if $R \geq \frac{\gamma q^h}{3+2\gamma}$, then (4.44) implies $\lambda \geq 0$. In that case (T^*, T, λ), as defined in (4.38) and (4.44), solves the maximization problem. But if $R < \frac{\gamma q^h}{3+2\gamma}$,then (4.44) implies $\lambda < 0$,which violates the Kuhn-Tucker conditions. Then the constraint does not binding and the solution is $\lambda = 0$.Then, the conditions yield

$$(T, T^*) = (\frac{q^h}{2}, \frac{2q^h - R}{2}) \tag{4.45}$$

Call the contract in (4.45) by *Type-3**. Then the payoff to the citizen under contract *Type-3** is:

$$EU_C^{SP3^*} = \frac{(\gamma+2)(q^h)^2 + 2Rq^h + R^2}{4(\gamma+1)\bar{\delta}}. \tag{4.46}$$

Step 2. Denote $\frac{\gamma q^h}{3+2\gamma} \equiv R_1^*$ and $\frac{2(1+\gamma)q^h}{4\gamma+7} \equiv R_2^*$. Clearly, $R_2^* < R_1^*$. To find the optimal contract, we use a similar method as used in the proof of Proposition 35 and compare the expected payoffs from contracts *Type-1**, *Type-2** and *Type-3**, i.e $EU_C^{SP1^*}, EU_C^{SP2^*}$ and $EU_C^{SP3^*}$, respectively. The method produces a similar figure to that of Figure 4.2, where the relative position of the curves remain the same, and R_1 and R_2 are replaced with R_1^* and R_2^*. First restrict the choice set to the class of contracts in Option 1: $EU_C^{SP1^*} < EU_C^{SP2^*}$ for all R, however, contract *Type-2** is not feasible for $R \leq R_1^*$

since it violates the constraint, therefore *Type-2** is optimal for all $R > R_1^*$. Restricting our attention to the class of contracts in Option 2: $EU_C^{SP1^*} < EU_C^{SP3^*}$ for all R, however, contract *Type-3** is not feasible for $R \geq R_2^*$ since it violates the constraint, therefore *Type-3** is optimal for all $R < R_2^*$. For all R such that $R_2^* \leq R \leq R_1^*$, optimal contract is *Type-1**, since *Type-2** and *Type-3** are not feasible. ■

Note that if we let $\alpha q_1^h = (1-\alpha)q_2^h \equiv q^h$ in Proposition 35, the equilibrium transfers, (T_1, T^*) of contract *Type-1,* as derived in (4.25), are equal to those under contract *Type-1**, as derived in (4.38). The only difference is that contract *Type-1* sets $T_2 = 0$ whereas contract *Type-1** sets $T_1 = T_2$. However, observe that, under the condition $\alpha q_1^h = (1-\alpha)q_2^h \equiv q^h$, the resulting payoffs of the contracts *Type-1* and *Type-1**, as calculated in (4.30) and (4.43), respectively. Similarly, contracts *Type-2* and *Type-3* are equivalent to contracts *Type-2** and *Type-3**, respectively, hence yield the same expected payoff to the citizen. Therefore, we conclude that the citizen is indifferent between offering a contract with $T_1 = T_2$ or $T_1 > T_2$ when she is indifferent between consuming the two high quality public goods. Similarly, it is easy to show that the citizen is indifferent between offering a contract with $T_1 = T_2$ or with $T_2 > T_1$ by comparing the expected payoffs of the contracts in Proposition 37 and in Proposition 38. Therefore, we have the following result.

Proposition 39 *If the citizen values the consumption of high quality public good of either type by q^h, the optimal contract is $(T_1, T_2) \in \{(T,0),(0,T),(T,T)\}$ and*

$$(T, T^*) = \begin{cases} \left(\frac{(5+2\gamma)q^h}{4\gamma+7}, \frac{3(2+\gamma)q^h}{4\gamma+7} \right) & \textit{if} \quad R > \frac{2(1+\gamma)q^h}{4\gamma+7} \\ \left(\frac{2q^h+(3+2\gamma)R}{2(2+\gamma)}, \frac{2q^h-R}{2} \right) & \textit{if} \quad \frac{\gamma q^h}{3+2\gamma} \leq R \leq \frac{2(1+\gamma)q^h}{4\gamma+7} \\ \left(\frac{q^h}{2}, \frac{2q^h-R}{2} \right) & \textit{if} \quad R < \frac{\gamma q^h}{3+2\gamma} \end{cases}$$

4.2.3 Equilibrium in Common Agency

In this section, the deviation from the previous model is that there are two principals who are repsonsible for a single task. They simultaneously offer a contract to the agent: t_i is paid by principal i if success in task i (i.e. high quality of public good type i) is realized, nothing is paid otherwise. Given (t_1, t_2), the agent chooses optimal effort (e_1^*, e_2^*), where $e_i \in \{0, 1\}$ $\forall i \in \{0, 1\}$. The agent's strategy, (e_1^*, e_2^*) is derived in Section 4.2.1, but note that unlike in the previous model where a single principal has an option to offer a *bonus* contract, in common agency there is no such option: the agent earns $t^* = t_1 + t_2$ if he succeeds in both tasks. Therefore, we modify agent's strategy, as derived in (4.2)-(4.14), by substituting $t^* = t_1 + t_2$:

$$(e_1^*, e_2^*) = \begin{cases} (1,1) & \text{if} \quad \min\{t_1, t_2\} \geq \delta(1+\gamma) \\ (1,0) & \text{if} \quad t_2 < \min\{\delta(1+\gamma), t_1\} \text{ and } t_1 \geq \delta \\ (0,1) & \text{if} \quad t_1 < \min\{\delta(1+\gamma), t_2\} \text{ and } t_2 \geq \delta \\ \text{either } (1,0) \text{ or } (0,1) & \text{if} \quad t_1 = t_2 = \delta \\ (0,0) & \text{if} \quad \delta > \max\{t_1, t_2\} \end{cases}$$

Note that the agent is indifferent between (1,0) and (0,1) if $t_1 = t_2 = \delta$. Figure 4.3 indicates the effort choices of the agent, given the contract offers by the principals. The pair (.,.) in given region corresponds to the optimal effort choice of the agent. If both tranfers are sufficiently high, he will work hard for both tasks. If he is not paid enough for both tasks, then he will shirk in both. For the remaining cases where only one of the tranfers is large enough to compensate the agent; he works only for the task that pays enough.

a. Principal's Problem

The principal i receives T_i from the citizen and offers t_i to the agent, which will be paid only if public good type i is produced with high quality. If the agent is successful in

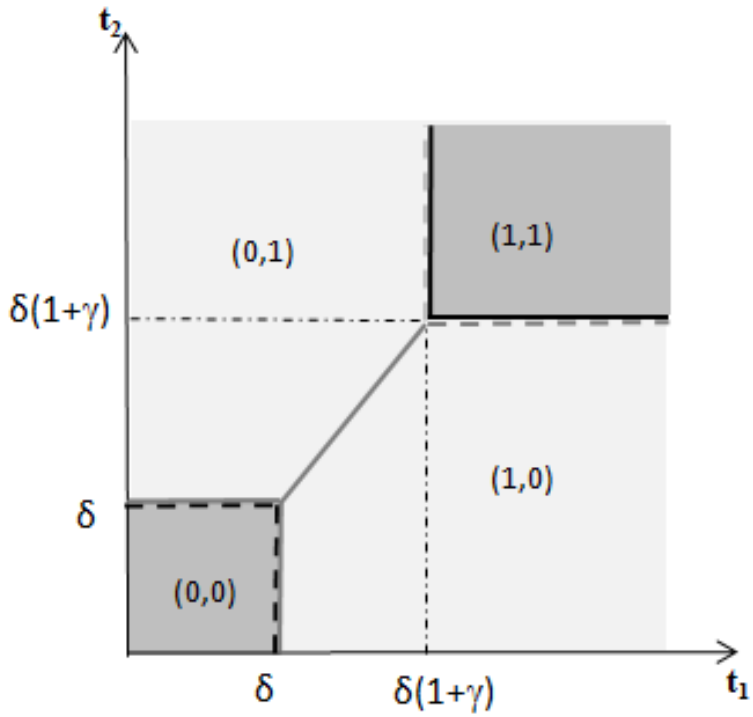


Figure 4.3: Agent's Effort Choice in Common Agency

task i, principal i enjoys the rent from office, $R/2$. Therefore, the expected utility of the principal i can be expressed in the reduced form:

$$EU_{P_i}(t_i) = e_i(\frac{R}{2} + T_i - t_i)$$

The problem of principal i is to choose $t_i \leq T_i$ to maximize EU_{P_i}. However, principal i's strategy also takes into account principal $-i$'s offer, t_{-i} for a given T_{-i}. Therefore, (t_1^*, t_2^*) will be an equilibrium in which the principals best-respond to each other. To characterize the principals' strategies, we consider several cases, each with a given (T_1, T_2).

Case 1: $T_i \geq \delta(1+\gamma),\ \forall i \in \{1,2\}$. This is the only case where the outcome (1,1) follows: if $T_i < \delta(1+\gamma)$, but $T_{-i} \geq \delta(1+\gamma)$, principal i's offer $t_i \leq T_i$ will be too small to induce the agent's effort in both tasks. Therefore payoff maximizing transfers are: $t_i^* = \delta(1+\gamma)$, $\forall i \in \{1,2\}$.

Case 2: $T_i \geq \delta(1+\gamma)$ but $T_{-i} < \delta(1+\gamma)$. Principal i can attract the agent with any $t_i = T_{-i} + \epsilon$, then she will offer $t_i = T_{-i} + \epsilon, \epsilon \in [0, T_i - T_{-i})$. We assume that the equilibrium best responses are $t_i^* = T_{-i}$ and $t_{-i}^* = 0$, and that as a part of the agent's strategy he picks (1,0) when $i = 1$ and (0,1) when $i = 2$.

Case 3: $T_{-i} < \delta$ but $T_i \geq \delta$. The principal $-i$'s budget is too small to attract agent's effort in task $-i$. Then, principal i's best response is to offer $t_i^* = \delta$. The agent will pick (1,0) if $i = 1$ and (0,1) if $i = 2$.

Case 4: $T_i < \delta$, $\forall i \in \{1, 2\}$.None of the principals can compensate the agent's effort; the outcome (0,0) follows and with no loss generality, we assume that the optimal offers are $t_i^* = 0$,$\forall i \in \{1, 2\}$.

Case 5: $\delta \leq T_i < \delta(1+\gamma)$, $\forall i \in \{1, 2\}$, and $T_i > T_{-i}$. The principal i offers $t_i = T_{-i} + \epsilon, \epsilon \in [0, T_i - T_{-i}]$. Again the same reasoning as in Case 2 applies: the best responses are $t_i^* = T_{-i}$ and $t_{-i}^* = 0$ and as a part of his strategy the agent chooses (1,0) for $i = 1$ and (0,1) for $i = 2$.

Case 6: $\delta \leq T_i < \delta(1+\gamma)$, $\forall i \in \{1, 2\}$ and $T_i = T_{-i}$. Competition would wipe away the surplus in the total budget: the equilibrium transfers are $t_i^* = T_i$, $\forall i \in \{1, 2\}$, and the agent is indifferent between spending effort in task 1 or 2.

It is easy to check that (t_1^*, t_2^*) in all cases above satisfy the agent's participation constraint in equation (4.15).[3]

As in the previous section, we have a similar definition for corruption where the principal pockets some portion of his budget.

Definition 40 *If $t_i < T_i$, principal i is said to be "corrupt".*

It is clear that corruption is likely in all cases except in case 4 and 6. In case 1, if $T_i > \delta(1+\gamma)$, both principals are corrupt if $T_i > \delta(1+\gamma), \forall i \in \{1, 2\}$; only principal i if $T_i > \delta(1+\gamma)$, $T_{-i} = \delta(1+\gamma)$. In cases 2, 3 and 5, principal i is corrupt if $T_i > \delta$.

Figure 4.6represents the equilibrium transfers, (t_1^*, t_2^*) for a given (T_1, T_2), as well as the corresponding outcome that will be implemented. The equilibrium outcome is:

[3]Again the same argument applies here. The participation constraint will be satisfied in all cases. See footnote 1.

$$
\begin{array}{lll}
(1,1) & \text{if} & \min(T_1, T_2) \geq \delta(1+\gamma) \\
(1,0) & \text{if} & T_1 \geq \delta \text{ and } T_2 \leq \delta(1+\gamma) \\
(0,1) & \text{if} & T_2 \geq \delta \text{ and } T_1 \leq \delta(1+\gamma) \\
\text{either } (1,0) \text{ or } (1,1) & \text{if} & \delta(1+\gamma) > T_1 = T_2 \geq \delta \\
(0,0) & \text{if} & \max(T_1, T_2) < \delta
\end{array}
\tag{4.47}
$$

Note that if $\delta(1+\gamma) > T_1 = T_2 \geq \delta$, the total budget is not sufficient to induce agent's effort on both tasks and competition between the principals results in $(t_1^*, t_2^*) = (\delta, \delta)$, where the agent will be indifferent spending effort either on task 1 or task 2. This case represented by the line segment $I_1 I_2$ in Figure 4.6.

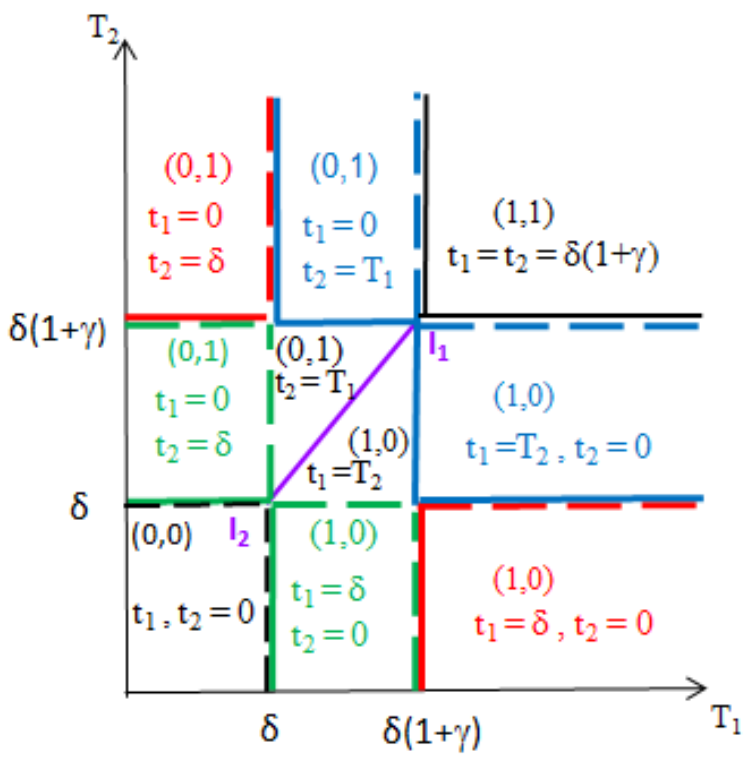


Figure 4.4: Equilibrium in Common Agency

b. Citizen's Problem

As in the single-principal model, if the agent succeeds in task 1, the citizen enjoys a high quality public good that she values $\alpha \in [0, 1]$, and receives αq_1^h utility, where q_1^h

is the quality level of high quality public good type 1. Similarly she receives $(1-\alpha)q_2^h$ utility if a high quality public good type 2 is produced. If the agent fails in a task, a low quality public good with quality level zero is realized.

The expected payoff of the citizen can be expressed as follows:

$$EU_C^{CA} = e_1(\alpha q_1^h - T_1) + e_2((1-\alpha)q_2^h - T_2) \tag{4.48}$$

The optimal contract (T_1, T_2) maximizes the expression in (4.48). Because of the assumption that the citizen is asymmetrically informed about the cost parameter δ, she constructs her strategy by taking into account the possible outcomes that will arise as a result of her contract offers. In doing so, she considers the principals' strategies and the resulting outcome for a given (T_1, T_2) as derived in (4.47). We use this insight in deriving the optimal contract in Proposition (41).

Proposition 41 *Optimal contract is* $(T_1, T_2) = (\frac{\alpha q_1^h}{2}, \frac{(1-\alpha)q_2^h}{2})$, *and the citizen receives:*

$$EU_C^{CA} = \frac{(\gamma+1)(\max(\alpha q_1^h, (1-\alpha)q_2^h))^2 + (\min(\alpha q_1^h, (1-\alpha)q_2^h)^2}{4(\gamma+1)\overline{\delta}}$$

Proof. Step 1. There are three contract options.

Option 1: Suppose the citizen offers (T_1, T_2) such that $T_1 > T_2$. Then $T_1 > \frac{T_2}{\gamma+1}$. From (4.47), the outcome will be (0,0) if $\delta > T_1$; (1,0) if $\frac{T_2}{\gamma+1} < \delta \leq T_1$, and (1,1) if $\delta \leq \frac{T_2}{\gamma+1}$. Figure 4.5 illustrates the equilibrium outcome as δ changes. Then the citizen's expected payoff is:

$$EU_C^{CA}(T_1, T_2) = F(\frac{T_2}{\gamma+1})(\alpha q_1^h + (1-\alpha)q_2^h - T_1 - T_2) + (F(T_1) - F(\frac{T_2}{\gamma+1}))(\alpha q_1^h - T_1)$$

First order conditions yield

$$(T_1, T_2) = (\frac{\alpha q_1^h}{2}, \frac{(1-\alpha)q_2^h}{2}) \tag{4.49}$$

The citizen's expected payoff from contract in equation (4.49) is:

$$EU_C^{CA}(T_1, T_2) = \frac{(\gamma+1)(\alpha q_1^h)^2 + ((1-\alpha)q_2^h)^2}{4(\gamma+1)\overline{\delta}} \tag{4.50}$$

Option 2: Suppose the citizen offers (T_1, T_2) such that $T_2 > T_1$. Then $T_1 > \frac{T_2}{\gamma+1}$. From (4.47), the outcome will be (0,0) if $\delta > T_1$; (0,1) if $\frac{T_1}{\gamma+1} < \delta \leq T_2$, and (1,1) if $\delta \leq \frac{T_1}{\gamma+1}$. Figure 4.5 illustrates the equilibrium outcome as δ changes. Then the citizen's expected payoff is:

$$EU_C^{CA}(T_1, T_2) = F(\frac{T_1}{\gamma+1})(\alpha q_1^h + (1-\alpha)q_2^h - T_1 - T_2) + (F(T_2) - F(\frac{T_1}{\gamma+1}))((1-\alpha)q_2^h - T_2)$$

Using first order conditions, the solution is again (4.49). The citizen's expected payoff is:

$$EU_C^{CA2} = \frac{(\gamma+1)((1-\alpha)q_2^h)^2 + (\alpha q_1^h)^2}{4(\gamma+1)\overline{\delta}} \tag{4.51}$$

Option 3: Suppose the citizen offers (T_1, T_2) such that $T_1 = T_2 \equiv T$. From (4.47), the outcome will be (0,0) if $\delta > T$; (1,0) or (0,1) if $\frac{T}{\gamma+1} < \delta \leq T$, and (1,1) if $\delta \leq \frac{T}{\gamma+1}$. Figure 4.5 illustrates the equilibrium outcome as δ changes. Then the citizen's expected payoff is:

$$EU_C^{CA}(T_1, T_2) = F(\frac{T}{\gamma+1})(\alpha q_1^h + (1-\alpha)q_2^h - 2T) + (F(T) - F(\frac{T}{\gamma+1}))(\frac{\alpha q_1^h + (1-\alpha)q_2^h)}{2} - T) \tag{4.52}$$

First order conditions yield

$$T = \frac{\alpha q_1^h + (1-\alpha)q_2^h}{2} \tag{4.53}$$

The citizen's expexted payoff from this contract is:

$$EU_C^{CA3}(T_1, T_2) = \frac{(\gamma+2)(\alpha q_1^h + (1-\alpha)q_2^h)^2}{16(\gamma+1)\overline{\delta}} \tag{4.54}$$

Step 2. The contract in (4.53) is not optimal if $\alpha q_1^h \neq (1-\alpha)q_2^h$: if $\alpha q_1^h > (1-\alpha)q_2^h$, the expected payoff in equation (4.54) is less than in equation (4.50); and if $(1-\alpha)q_2^h >$

αq_1^h the expected payoff in equation (4.54) is less than in equation (4.51). But if $(1-\alpha)q_2^h = \alpha q_1^h$, the contracts in (4.49), (4.52) and (4.53) are identical, and hence the expected payoffs in (4.50), (4.51) and (4.54) are equal. Note that the contracts in (4.49) and (4.52) are identical. However, if $\alpha q_1^h > (1-\alpha)q_2^h$, the condition in contract Option 2 is violated, therefore the equilibrium payoff of the citizen is as derived using Option 1 in (4.50). Similarly, if $(1-\alpha)q_2^h > \alpha q_1^h$, the equilibrium payoff of the citizen is as derived using Option 2 in (4.51). ■

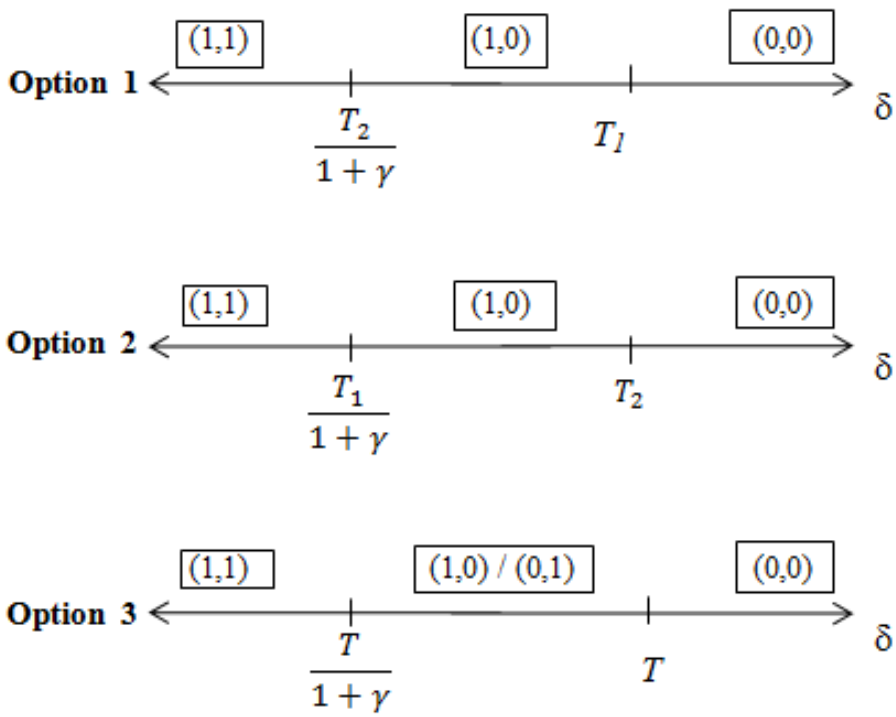


Figure 4.5: Proof of Proposition 41

Comparing the two bureaucratic regimes, the total transfer which is paid when both type of public goods are of high quality, i.e. $T_1 + T_2$, is higher in common agency than the corresponding transfer in single-principal model, i.e. T^* : consider for example the case where $\alpha q_1^h > (1-\alpha)q_2^h$, and compare the sum of equilibrium transfers, $T_1 + T_2$, in Proposition 41 with the optimal T^* for different ranges of R in Proposition 35. Observe that, regardless of R, optimal T^* in Proposition 35 is always lower than optimal $T_1 + T_2$ in Proposition 41. In other words, common agency overpays to achieve high quality in both type of public goods at the same time. The reason is that, in single-principal model

the citizen is able to decrease optimal T^* by a fraction of the benefit from keeping the office, R. However, this advantage dissappears in common agency since best responses of the principals do not depend on R as they compete to attract the agent's effort. Such competition, in turn, increases the agent's bargaining power, and requires a higher total compensation to induce his effort in both tasks. Therefore, common agency is not cost efficient in terms of achieving high quality for both public goods.

Focusing on the optimal transfer which is paid only when one type of public good is produced with high quaility, we observe that common agency offers less (considering the nonzero transfer) except when R is too low and hence the optimal transfers are equal in the two regimes. The reason is, as R increases, the citizen reduces optimal T^* since it becomes less costly to incentivize the principal to induce her agent spent effort in both tasks. This helps the citizen to offer a higher T_1 (when the citizen values public good type 1 more) or T_2 (when the citizen values public good type 2 more) in equilibrium to increase her expected payoff.

4.3 A Comparative Assessment: One Principal or Many?

So far we have analyzed the equilibrium separately in the two bureaucratic systems. Now we are ready to compare the efficiency of the two systems. We define the *efficient* bureaucratic system as the one which maximizes the citizen's welfare. Therefore, we compare the expected payoffs of the citizen in the two systems to identify the efficient regime.

Theorem 42 ***If there are positive rents from public office, a single-principal model is more efficient than common agency. If there are no such rents, the two sytems are equally efficient.***

Proof. Assume without loss of generality, $\alpha q_1^h > (1-\alpha)q_2^h$. Let $\Phi(R) \equiv EU_C^{SP} - EU_C^{CA}$, where EU_C^{SP} and EU_C^{CA} denote the equilibrium payoffs to the citizen in single-principal model and common agency, respectively. EU_C^{CA} is calculated in (4.50). But EU_C^{SP}

depends on R. If

$$R > \frac{(3+2\gamma)\alpha q_1^h - (1-\alpha)q_2^h}{4\gamma+7}$$

EU_C^{SP} is as calculated in (4.31). Then

$$\Phi(R) = \frac{(1+\gamma)(\alpha q_1^h)^2 + 4\alpha q_1^h(1-\alpha)q_2^h - 3((1-\alpha)q_2^h)^2}{4(1+\gamma)(7+4\gamma)\overline{\delta}}$$

Since $\alpha q_1^h > (1-\alpha)q_2^h$, $\Phi(R) > 0$. However, if

$$\frac{(1+\gamma)\alpha q_1^h - (1-\alpha)q_2^h}{3+2\gamma} < R < \frac{(3+2\gamma)\alpha q_1^h - (1-\alpha)q_2^h}{4\gamma+7}$$

then EU_C^{SP} is given in (4.30). Therefore

$$\Phi(R) = \frac{2R(\gamma+1)((3+2\gamma)\alpha q_1^h - (1-\alpha)q_2^h) - ((1+\gamma)\alpha q_1^h - (1-\alpha)q_2^h)^2 - (4\gamma+7)(\gamma+1)R^2}{4(\gamma+1)(\gamma+2)\overline{\delta}}$$

and

$$\Phi'(R) = \frac{2(\gamma+1)((3+2\gamma)\alpha q_1^h - (1-\alpha)q_2^h) - 2(4\gamma+7)(\gamma+1)R}{4(\gamma+1)(\gamma+2)\overline{\delta}}$$

Note that $\Phi'(R) > 0$ and since

$$\Phi(R)(\frac{(1+\gamma)\alpha q_1^h - (1-\alpha)q_2^h}{3+2\gamma}) = \frac{(\gamma+1)(\gamma+2)\alpha q_1^h + (4\gamma+5)(\gamma+2)(1-\alpha)q_2^h}{4\overline{\delta}(\gamma+1)(\gamma+2)(3+2\gamma)} > 0$$

for all $R < \frac{(3+2\gamma)\alpha q_1^h - (1-\alpha)q_2^h}{4\gamma+7}$, we conclude that

$$\Phi(R) > 0, \forall R \in [\frac{(1+\gamma)\alpha q_1^h - (1-\alpha)q_2^h}{3+2\gamma}, \frac{(3+2\gamma)\alpha q_1^h - (1-\alpha)q_2^h}{4\gamma+7}]$$

Finally, if

$$R < \frac{(1+\gamma)\alpha q_1^h - (1-\alpha)q_2^h}{3+2\gamma}$$

then EU_C^{SP} is as calculated in (4.36) and we have

$$\Phi(R) = \frac{R^2 + 2R(1-\alpha)q_2^h}{4(\gamma+1)\overline{\delta}}$$

Therefore $\Phi(R) > 0$, $\forall R > 0$. However, $\Phi(R) = 0$ when $R = 0$. ■

We conclude that, from the citizen's point of view, a single-principal model is favourable over a common agency unless there are no extra benefits to the principal(s) from keeping the public office. In a single-principal model, the rents from office, accruing only when high quality is achieved in both tasks, helps the citizen to decrease the optimal transfer paid in that instance. Common agency, on the other hand, creates a welfare loss by relatively overpaying the principals in total when they both succeed in producing high quality public good. The point is that since the principals compete to receive the agent's effort, their best responses are independent of the rent from public office and the citizen takes the best responses of the principals into account when determining the optimal transfers. Therefore, the citizen loses the opportunity to reduce the optimal transfer paid when high quality occurs in both tasks by fraction of the rent. If there are no rents, the cost advantage of the single-principal model in providing incentives disappears; i.e. the cost to incentivize the single principal to produce high quality in both tasks increases. In that case, the optimal tranfers under the two regimes are equal. Therefore, the two bureaucratic systems yield the same payoff to the citizen in equilibrium.

4.4 An Extension: Full Information

We finally remove the informational asymmetry between the citizen and the principal(s) and compare the efficiency of the two systems under the case where δ is no longer private information of the bureaucratic agents.

4.4.1 Equilibrium in Single-Principal Model

The optimal contract, (T_1, T_2, T^*) maximizes the citizen's expected payoff as expressed in (4.22). Recall that the citizen receives $\alpha q_1^h + (1-\alpha)q_2^h$ when both goods are of high quality (i.e. when outcome (1,1) is realized) and the cost of inducing this outcome is $\delta(2+\gamma)$. Similarly, she receives αq_1^h when only public good type 1 is high quality (i.e. when outcome (1,0) is realized) and $(1-\alpha)q_2^h$ only public good type 2 is high quality (i.e. when outcome (0,1); but the cost of inducing either outcome is δ. Therefore, the optimal contract directly follows from comparing the citizen's payoff from these cases:

Case 1: Offer $T^* = \delta(2+\gamma)$ and $T_1, T_2 < \delta$ if and only if inducing the outcome (1,1), i.e. $\alpha q_1^h + (1-\alpha)q_2^h - \delta(2+\gamma)$, yields a higher payoff than inducing (1,0) or (0,1), which would respectively yield $\alpha q_1^h - \delta$ and $(1-\alpha)q_2^h - \delta$. In other words, this contract is optimal if and only if

$$\min\{\alpha q_1^h, (1-\alpha)q_2^h\} \geq \delta(1+\gamma).$$

Case 2: Offer $T_1 = \delta$, $T_2 < T_1$ and $T^* < \delta(2+\gamma)$ if and only if

$$\begin{aligned} \min\{\alpha q_1^h, \delta(1+\gamma)\} &\geq (1-\alpha)q_2^h \\ \text{and } \alpha q_1^h &\geq \delta. \end{aligned}$$

Case 3: Offer $T_2 = \delta$, $T_1 < T_2$ and $T^* < \delta(2+\gamma)$ if and only if

$$\begin{aligned} \min\{(1-\alpha)q_2^h, \delta(1+\gamma)\} &\geq \alpha q_1^h \\ \text{and } (1-\alpha)q_2^h &\geq \delta. \end{aligned}$$

Case 4: Offer $T_1, T_2 < \delta$, $T^* < \delta(2+\gamma)$ if and only if

$$\max\{\alpha q_1^h, (1-\alpha)q_2^h\} \leq \delta.$$

Then the citizen's equilibrium payoff in single-principal model with full-information

is

$$EU_C^{SP} = \begin{cases} \alpha q_1^h + (1-\alpha)q_2^h - \delta(2+\gamma) & \text{if} \quad \min\{\alpha q_1^h, (1-\alpha)q_2^h\} \geq \delta(1+\gamma) \\ \alpha q_1^h - \delta & \text{if} \quad \begin{array}{c} \min\{\alpha q_1^h, \delta(1+\gamma)\} \geq (1-\alpha)q_2^h \\ \text{and } \alpha q_1^h \geq \delta \end{array} \\ (1-\alpha)q_2^h - \delta & \text{if} \quad \begin{array}{c} \min\{(1-\alpha)q_2^h, \delta(1+\gamma)\} \geq \alpha q_1^h \\ \text{and } (1-\alpha)q_2^h \geq \delta \end{array} \\ 0 & \text{if} \quad \max\{\alpha q_1^h, (1-\alpha)q_2^h\} \leq \delta \end{cases}$$

4.4.2 Equilibrium in Common Agency

The optimal contract in common agency, (T_1, T_2) maximizes the citizen's payoff as derived in (4.48). The difference from the single-principal model is that T_i is paid only when principal i is successful. Also the cost of inducing outcome (1,1) is $2\delta(1+\gamma)$, but δ for inducing either outcome (1,0) or (0,1). The citizen's strategy is then as follows.

Case 1: Offer $T_1 = T_2 = \delta(1+\gamma)$, if and only if

$$\min\{\alpha q_1^h, (1-\alpha)q_2^h\} \geq \delta(1+2\gamma)$$

Case 2: Offer $T_1 = \delta$, $T_2 < T_1$, if and only if

$$\begin{aligned} \min\{\alpha q_1^h, \delta(1+2\gamma)\} &\geq (1-\alpha)q_2^h \\ \text{and } \alpha q_1^h &\geq \delta \end{aligned}$$

Case 3: Offer $T_2 = \delta$, $T_1 < T_2$, if and only if

$$\begin{aligned} \min\{(1-\alpha)q_2^h, \delta(1+2\gamma)\} &\geq \alpha q_1^h \\ \text{and } (1-\alpha)q_2^h &\geq \delta \end{aligned}$$

Case 4: Offer $T_1, T_2 < \delta$, if and only if

$$\delta \geq \max\{\alpha q_1^h, (1-\alpha)q_2^h\}.$$

Then the citizen's equilibrium payoff in common agency with full-information is

$$EU_C^{SP} = \begin{cases} \alpha q_1^h + (1-\alpha)q_2^h - 2\delta(1+\gamma) & \text{if} \quad \min\{\alpha q_1^h, (1-\alpha)q_2^h\} \geq \delta(1+2\gamma) \\ \alpha q_1^h - \delta & \text{if} \quad \begin{array}{c} \min\{\alpha q_1^h, \delta(1+2\gamma)\} \geq (1-\alpha)q_2^h \\ \text{and } \alpha q_1^h \geq \delta \end{array} \\ (1-\alpha)q_2^h - \delta & \text{if} \quad \begin{array}{c} \min\{(1-\alpha)q_2^h, \delta(1+\gamma)\} \geq \alpha q_1^h \\ \text{and } (1-\alpha)q_2^h \geq \delta \end{array} \\ 0 & \text{if} \quad \delta \geq \max\{\alpha q_1^h, (1-\alpha)q_2^h\} \end{cases}$$

Figure 4.6 represents the preferences of the citizen over the bureaucratic systems. When both αq_1^h and $(1-\alpha)q_2^h$ are greater than $\delta(1+2\gamma)$ or less than $\delta(1+\gamma)$ the citizen is indifferent between the two systems. These regions are represented by I in the figure. For other cases a single principal system is preferable. These are denoted with SP. Thus, under certainty, a single- principal model weakly dominates a common agency.

4.5 Concluding Remarks

The political economy literature points out the agency problem between the voters as principals and the politicians as their agents: the voters faces the problem of designing the best incentive scheme for elected politicians who might not be acting in voters' interest when there are informational assymetries. Dixit (2006) incorporates this conventional modelling of political agency problem with the one which exists within bureaucracy. Our model takes this three-tier structure one step further by allowing a multiplicity in the middle-tier; i.e. a common agency. One practical example for such

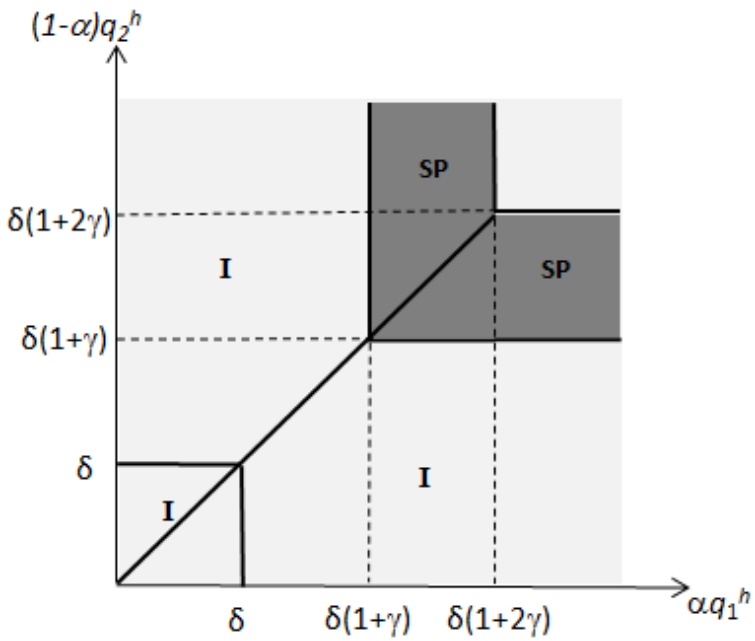


Figure 4.6: Equilibrium in Common Agency: Full Information

breaucratic system is the case of a municipality which has to take into account the concerns of different ministers while taking an action. We attempt to explore the welfare dimensions of a common agency within bureaucracy in comparison to a single-principal model where there is only one principal acting as a top-level bureaucrat.

The model involves two principals at the top level of bureaucracy and, at the lower level, their common agent who is in charge of producing public goods. The public goods, which could be either high or low quality depending on the agent's unobservable effort, are consumed by the citizen. There are two different type of contracts; one is offered by a principal to the agent, the other is offered by the citizen to a principal. The problem that the contract between the agent and the principal being unobserved by the citizen creates room for corruption. However, we show that as long as the citizen has perfect information about the cost structure of the agent, she can prevent corruption in equilibrium.

We first consider the model with a single principal. The single-principal model allows a bonus contract in equilibrium where the citizen pays a higher transfer to the principal, when both goods are of high quality, than the sum of transfers when only

one type of good is high quality. We show that the optimal contract offered by the citizen depends on the rents from public office which accrues to the principal only when she succeeds in inducing her agent to produce both public goods with high quality. Futhermore, as the rents get larger, the citizen offers a lower bonus transfer but a higher transfer which is paid only when one type of good is of high quality. Common agency, however, produces higher costs to the citizen since the optimal transfer, i.e. the sum of transfers paid when both goods are high quality, cannot be reduced in the same way as in a single-principal model. Therefore, since a common agency has a cost disadvantahe, a single-principal model is favourable from the citizen's point of view. However, if there are no rents from public office, then the single-principal model loses its cost advantage in providing incentives. In that case, the two regimes are equally preferrable.

References

Abowd, J. M., "Does Performance-based Compensation Affect Corporate Performance?", *Industrial and Labor Relations Review*, (1990) Vol. 43, No. 3, pp. 52-73.

Aggarwal, R. and Samwick, A., "The other side of the tradeoff: The impact of risk on executive compensation", *Journal of Political Economy*, (1999) Vol.107, pp. 65–105.

Aggarwal, R. and Samwick, A. A., "Performance Incentives within Firms: The Effect of Managerial Responsibility", *Journal of Finance*, American Finance Association, (2003) Vol. 58, No. 4, pp. 1613-1650.

Allen, F.,"Credit Rationing and Repayment Incentives", *Review of Economic Studies*, (1983) Vol. 40, No. 4, pp. 639-646.

Allgood, S. and Farrell, K. A., "The Effect of CEO Tenure on the Relationship between Firm Performance and Turnover", University of Nebraska, Finance Department Faculty Publications (2000).

Antle, R. and Smith, A. "An Empirical Investigation of the Relative Performance Evaluation of Corporate Executives", *Journal of Accounting Research*, (1986) Vol. 24, No.1, pp. 1-39.

Bac, M. and Genc, S., "The French First Employment Contract: Efficient Screening Device or Kleenex Contract?", *Journal of Economics*, (2009) Vol. 91, No.1, pp. 77-88.

Banerjee, A. V., Gertler P. J. and Ghatak, M., "Empowerment and Efficiency: Tenancy Reform in West Bengal", *Journal of Political Economy*, (2002) Vol. 110, No. 21, pp. 239-280.

Banker, R., and Datar, S., "Sensitivity, precision, and linear aggregation of signals for performance evaluateon". *Journal of Accounting Research*, No. 27, pp.

21-39.

Bebchuk, L. A. and J. M. Fried, "Pay Without Performance: The Unfulfilled Promise of Executive Compensation", Harvard University Press, Cambridge, MA, (2004).

Benjamin E. H. & M. S. Weisbach, "Endogenously Chosen Boards of Directors and Their Monitoring of the CEO", Working Papers, University of California at Berkeley, Haas School of Business, (1996).

Berle, A. A., Jr.; Means, G., "The Modern Corporation and Private Property", New York: Macmillan, (1932).

Cadenillas, A., J. Cvitani´c, and F. Zapatero, "Dynamic Principal-Agent Problems with Perfect Information", Working Paper, SSRN (2005).

Charmichael, L., "Firm Specific Human Capital and Promotion Ladders", *Bell Journal of Economics*, (1983) Vol. 14, No.1, Spring. pp. 251-258.

Chevalier, K. and Ellison R.,"Are Some Mutual Fund Managers Better Than Others? Cross-Sectional Patterns in Behavior and Performance", *Journal of Finance*, American Finance Association, (1999) Vol. 54(3), pp. 875-899.

Chiappori, P.A., Macho-Stadler, I., Rey, P. and Salanie, B., "Repeated Moral Hazard: the Role of Memory, Commitment, and the Access to Credit Markets", *European Economic Review*, (1994) Vol. 38, pp. 1527-1553.

Cichello, M., "The impact of firm size on pay-performance sensitivities", *Journal of Corporate Finance*, (2005) Vol. 11(4), pp. 609-627.

Core, J. and Guay, W.,"Estimating the Value of Employee Stock Option Portfolios and Their Sensitivities to Price and Volatility", *Journal of Accounting Research*, (2002) Vol. 40, pp. 613-630.

Coughlan, A. and Schmidt, R., "Executive compensation, management turnover, and firm performance: An empirical investigation", *Journal of Accounting and Economics,* (1985) 7 (1–3) pp. 43–66.

Cox, D. R., "Regression Models and Life-Tables", *Journal of the Royal Statistical Society*, (1972) Series B (Methodological) Vol. 34, No. 2, pp. 187–220.

Cvitani´c, J., and J. Zhang, "Optimal Compensation with Adverse Selection and Dynamic Actions", Working Paper, SSRN (2006).

Dahya, J., A.A. Lonie and D.M. Power.,"Ownership structure, firm performance and top executive change: An analysis of UK firms", Journal of Business Finance and Accounting, (1998) Vol. 25 , pp. 1089–1117.

DeMarzo, P. M., and Y. Sannikov, "A Continuous-Time Agency Model of Optimal Contracting and Capital Structure", Working Paper, Stanford University (2004).

Demerjian, P. Lev, B. and McVay, S., "Quantifying Managerial Ability: A New Measure and Validity Tests", Working Paper (2009).

Denis, D. J. , Denis, D. K. and Sarin, A., "Ownership structure and top executive turnover", *Journal of Financial Economics*, (1997) Vol. 45, pp.193-221.

Dixit, A., "Predatory States and Failing States: An Agency Perspective", No. 131 Working Papers, Princeton University, Department of Economics, Center for Economic Policy Studies (2006).

Dixit, A., G. M. Grossman and E. Helpman, "Common Agency and Coordination: General Theory and Application to Government Policy Making", *Journal of Political Economy,* (1997) Vol. 105, No. 4, pp. 752-769.

Eaton, J. and Gersovitz, M., "Debt with Potential Reputation: Theoretical and Empirical Analysis", *Review of Economic Studies*, (1987) Vol. 48, pp. 289–309.

Edmans, A., X. Gabaix, T. Sadzik, and Y. Sannikov, "Dynamic Incentive Accounts", Working Paper, SSRN (2009).

Falato, T. A., Li, D. and Milbourn, T., "The Role of CEO Talent on Pay, Firm Performance and Turnover", Working Paper (2009).

Francis, J., A. H. Huang, S. Rajgopal and A. Y. Zhang, "CEO reputation and reporting quality." Contemporary Accounting Research, (2008) Vol. 25, No. 1.

Gabaix, X. & Landier, A., "Why has CEO pay increased so much?", *Quarterly Journal of Economics*, (2008) Vol. 123, pp. 49-100.

Gibbons, R. and K. Murphy, "Optimal Incentive Contracts in the Presence of Career Concerns: Theory and Evidence", *Journal of Political Economy*, (1992) Vol.

100, No. 3.

Goyal, V. K. and Park, C. W., "Board leadership structure and CEO turnover", *Journal of Corporate Finance*, (2002) No. 8, pp. 49–66.

Grossman, S. and Hart, O., "An Analysis of The Principal-Agent Problem", *Econometrica*, (1983) Vol. 51, No. 1, pp. 7-45.

Grossman, G. M. and E. Helpman., "Special Interest Politics", The MIT Press: Cambridge, Massachusetts: London, England (2001).

Guriev, S. and Kvasov, D.,"Contracting on Time", *American Economic Review*, (2005) Vol. 95, No. 5. pp.1369-1385.

Hadlock, C. J., and G. B. Lumer, "Compensation, turnover and top management incentives: Historical evidence", *Journal of Business*, (1997) Vol. 70, pp. 153-187.

Hartzell, J.,"The Impact of Turnover in Executive Compensation", NYU Stern, Working Paper Series (1998).

Hayes, R. and Schaefer, S., "How much are differences in managerial ability worth?", *Journal of Accounting and Economics*, (1999) No. 27, pp. 125-148.

Holmstrom, B., "Moral Hazard and Observability", *The Bell Journal of Economics,* (1979) Vol. 10, No.1 Spring, pp.74-91.

Holmstrom, B. and P. Milgrom, "Aggregation and Linearity in the Provision of Intertemporal Incentives", *Econometrica*, (1987) Vol. 55, pp. 303-328.

Huson, M., R. Parrino, R. and L. Starks, L., "Internal monitoring mechanisms and CEO turnover: A long term perspective", *Journal of Finance*, (2001) No. 56, pp. 2265–2297.

Jensen, M. C..; Meckling, W. H., "Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure", *Journal of Financial Economics*, (1976) Vol. 3, No. 4, pp. 305–360.

Jensen, M. and Murphy, K. J., "Performance pay and top management incentives", *Journal of Political Economy*, (1990) Vol. 98, No. 2 , pp. 225–264.

Kostiuk, P., "Firm Size and Executive Compensation", *Journal of Human Resources,* (1990) Vol.25, No.1, pp. 90-105.

Kwon, I., "Threat of Dismissal: Incentive or Sorting?", *Journal of Labor Economics*, (2005) Vol. 23, No. 4, pp. 797-838.

Laussel, D. and Le Braton, M. "Efficient Private Production of Public Goods under Common Agency", *Games and Economic Behavior*, (1998) Vol. 25, pp. 194-218.

Lazear, E.,"Why Is There Mandatory Retirement?", *Journal of Political Economy*, (1979) Vol. 87, pp. 1261-84.

Leigh, D. E., "Why is there mandatory retirement?: An empirical re-examination", *The Journal of Human Resources*, (1984) Vol. 19, No. 4, pp. 512-531.

Leonard, J., "Executive pay and firm performance", *Industrial and Labor Relations Review,* (1990) Vol. 43, pp.13-29.

Martimort, D. and L. Stole, "Contractual Externalities and Common Agency Equilibria", *Advances in Theoretical Economics*, (2003), Vol. 3, No. 1, pp. 1037-1037.

Martimort, D. and Moreira, H., "Common Agency With Informed Principles". Graduate School of Economics, Getulio Vargas Foundation (Brazil) in its series Economics Working Papers (Ensaios Economicos da EPGE), No. 551 (2006).

MacLeod. W.and Malcomson, J., "Involuntary Unemployment in Dynamic Contract Equilibria", *European Economic Review*, (1987) Vol. 31, pp. 427-435.

MacLeod, W. and J. Malcomson, "Implicit Contracts, Incentive Compatibility, and Involuntary Unemployment", *Econometrica*, (1989) Vol. 57, No. 2.

Malcomson, W. and Spinnewyn, F., "The Multiperiod Principal Agent Problem", *Review of Economic Studies*, (1988) Vol. 55, pp. 391-407.

Milbourn, T. "CEO Reputation and Stock Based Compensation", *Journal of Financial Economics*, (2003) No. 68, pp. 233-262.

Mezzetti, C., "Common Agency With Horizontally Differentiated Principles". *The RAND Journal of Economics*, (1997) Vol.28, No.2 , pp. 323-345.

Murphy, K., "Incentives, learning, and compensation: a theoretical and empirical investigation of managerial labor contracts", *The RAND Journal of Economics*, (1986) Vol. 17, No.1.

Murphy, K., "Corporate Performance and Managerial Remuneration: An Empirical Analysis", *Journal of Accounting and Economics*, (1985) 7(1-3), pp. 11-42.

Murphy, K., "Executive compensation", *Handbook of Labor Economics*, in: O. Ashenfelter & D. Card (ed.), Handbook of Labor Economics, Elsevier, Edition 1, Vol.3, Chapter 38, pp. 2485-2563 (1999).

Murphy, K. J. and Zimmerman, J., "Financial performance surrounding CEO turnover", *Journal of Accounting and Economics*, (1993) Vol. 16, pp. 273–315.

Ou-Yang, H., "Optimal Contracts in a Continuous-Time Delegated Portfolio Management Problem", *Review of Financial Studies*, (2003) Vol. 16, No. 1, pp. 173–208.

Parrino, R., "CEO turnover and outside succession: A cross-sectional analysis", *Journal of Financial Economics*, (1997) Vol. 46, pp. 165–197.

Persson, T. and G. Tabellini. *Political Economics: Explaining Economic Policy*, The MIT Press: Cambridge, Massachusetts: London, England (2000).

Phelan, C. and R. Townsend, "Computing Multi-period, Information-Constrained Optima", *Review of Economic Studies*, (1991) Vol. 58, pp. 853-881.

Rajgopal, S., Shevlin, T., and Zamora, V., "CEOs' Outside Employment Opportunuties and the Lack of Relative Performance Evaluation in Compensation Contracts", *Journal of Finance*, (2006) Vol. LXI, No. 4, August, pp. 1813-1844.

Rey, P. and & Salanie, B., "On the Value of Commitment with Asymmetric Information", *Econometrica*, (1996) Vol. 64, No.6, pp. 1395-1414.

Rogerson, W. P., "The First Order Approach to Principal-Agent Problems", *Econometrica*, (1985) Vol. 53, No. 6, pp. 1357–1368.

Rosen, S., "Authority, Control, and the Distribution of Earnings" *The Bell Journal of Economics*, The RAND Corporation, (1982) Vol. 13(2), pp. 311-323.

Shapiro, C. and Stiglitz, J.E., "Equilibrium unemployment as a worker discipline device", *American Economic Review*, (1984) June, 74, pp. 433-444.

Spear, S., and S. Srivastava, "On Repeated Moral Hazard with Discounting", *Review of Economic Studies*, (1987) Vol. 54, pp. 599-607.

Stiglitz, J.E. and Weiss, A., "Incentive Effects of Termination: Applications to

the Credit and Labor Markets", *American Economic Review*, (1983) Vol. 71, No. 3, June, pp. 393-410.

Subramanian, N., Chakraborty, A., Seikh, S., "Performance Incentives, Performance Pressure and Turnover", Working Paper, SSRN (2002).

Shumway, T., "Forecasting bankruptcy more accurately: a simple hazard model", *Journal of Business*, (2001) Vol. 74, pp. 101-124.

Tervio, M., "Differences that CEOs Make: An Assignment Model Approach", *American Economic Review*, (2007) Vol. 98, No. 3, pp. 642-668.

The Economist, March 3rd, 2005.

Warner, J., Watts, R. and Wruck, K., "Stock prices and top management changes", *Journal of Financial Economics*, (1988) Vol. 20, pp. 461–492.

Weisbach, M., "Outside directors and CEO turnover", *Journal of Financial Economics*, (1988) Vol. 20, No. 1–2, pp. 461–560.

Westerfield, M. M., "Optimal Dynamic Contracts with Hidden Actions in Continuous Time", USC, Working Paper (2006).

Williams, N., "On Dynamic Principal-Agent Problems in Continuous Time", Working Paper, Princeton University (2004).

Yermack, D., "Do corporations award CEO stock options effectively?", *Journal of Financial Economics,* (1995) Elsevier, Vol. 39(2-3), pp. 237-269.

Vita

Nadide Banu Olcay

Colleges attended, with dates, subjects, degrees:

2005-2012 Ph. D. in Economics, Rutgers University, New Brunswick

2003-2005 M. A. in Economics, Boğaziçi University, Istanbul Turkey.

1999-2003 B.A. in Economics, Boğaziçi University, Istanbul Turkey.

Teaching and Fellowships:

2005-2006 Graduate School Fellowship, Rutgers University

2006-2009 Teaching Assistant, Department of Economics, Rutgers University

2010-2011 Part-time Lecturer, Department of Economics, Rutgers University