MULTI-OBJECTIVE IMPERFECT MAINTENANCE FOR DEPENDENT COMPETING RISK SYSTEMS WITH MULTIPLE DEGRADATION PROCESSES AND RANDOM SHOCKS

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A dissertation submitted to the

Graduate School-New Brunswick

Rutgers, the State University of New Jersey

In partial fulfillment of the requirements

For the degree of

Doctor of Philosophy

Graduate Program in Industrial and Systems Engineering

Written under the direction of

Dr. Hoang Pham

And approved by

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New Brunswick, New Jersey

May, 2012

ABSTRACT OF THE DISSERTATION

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Multiple competing risks are one of the important topics in reliability field, especially degradation processes and random shocks. This research aims to relax the independent assumption by considering that there exist dependent relationships not only among multiple degradation measures but also between degradation measure and random shocks. In reality, many systems have multiple components with more than one degradation measure which is dependent with each other due to their interplaying functions or common usage history. Independent assumption may underestimate system reliability estimation under many cases. Random shocks will also contribute to the system failure through two ways: (1) one is working directly on the degradation processes; (2) the other is causing immediate failure to the system.

We develop a new methodology to formulate the reliability prediction model for the

gradually degradating systems subject to multiple dependent competing risks of degradation processes and random shocks. Two kinds of random shocks are considered: (1) fatal shocks, which fail the system immediately; (2) non-fatal shocks, which exhibit two effects on the system degradation process, including sudden degradation increment and degradation rate acceleration. The dependency between degradation processes and random shocks are modulated by a time-scaled covariate factors while the dependency among degradation processes are fitted by copula method. Also the reliability and state probability estimation for the systems are derived under the research scope of multi-state system using both analytical and Monte Carlo simulation for the dependent competing-risk systems.

Different maintenance policy models involving imperefect preventive maintenance for this dependent model are introducted and compared with each other. Multi-objective optimization is applied to consider two important targets simultaneously in maintenance issues, including long-run expected cost rate and system availability.

ACKNOWLEDGEMENTS

I would never been able to finish my Ph.D dissertation without the guidance of my advisor and my committee members, help from my friends, and support from my family.

First, I would like to express my deep gratitude to my advisor, Dr Hoang Pham, for his excellent guidance and continuing encouragement during my whole Ph.D study, including not only my courseworks, but also my research.

I also wish to express my appreciation to the committee members, Dr. Elsayed A. Elsayed, Dr. W. Art Chaovalitwongse, Dr. Myong K. Jeong and Dr. Hongzhou Wang for their valuable suggestions on my thesis.

Many thanks go to my colleagues and faculty members of the department of Industrial and Systems Engineering for their assistant and support during my work.

Finally, I am deeply indebted to my loving family for their encouragement and moral support. They are always there to stand by me through the good and bad times.

DEDICATION

To my dear mother Shanling Wang, my dear father Zhengming Gou, my dear sister Peipei Wang

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Chapter 1

Introduction

1.1 The Importance of Competing Risks Model of Degradation and Random Shocks

Nowadays the products are developed to be more reliable with a longer lifetime and higher quality, so it is really a significantly challenge to obtain the sufficient and accurate time-to-failure data in a cost-effective manner prior to the product release. The design engineers may be unable to obtain the time-to-failure data of the new products by testing them under the normal operating environments, either because their lifetimes are too long, or the time period between design and product release is too short.

Two approaches are introduced to overcome these obstacles to obtain the information of the lifetime regarding the warranty periods and the reliability specifications of the products. One is Accelerated Life Testing (ALT); the other is degradation analysis. Furthermore, by combining the ALT and degradation analysis, a new approach, called Accelerated Degradation Testing (ADT), is created.

Degradation analysis involves the measurement of the degradation process of a product at various time points and this information is then used to estimate the eventual failure lifetime for the product. One of the major advantages of performing reliability analysis based on degradation data is that it relates the reliability analysis directly to the physics of failure mechanism. Cutting tools, hydraulic structures, brake linings, airplane engine compressor blades, corroding pipelines, and rotating, equipment, all of these structural systems suffer from increasing wear with usage and aging. In our real life, various physical deterioration processes can be observed, such as cumulative wear, crack growth, erosion, corrosion, fatigue, consumption, etc. The deteriorating processes of these systems might incur high cost due to production losses, delays and safety hazards if the resistance of a deterioration structure drops below certain applied failure threshold. In practice, because modern highly reliable products usually have very complex system structure, they may have multiple degradation measures. In other words, the product may have multiple components with its own degradation process and even one component can exhibit more than one degradation behavior.

However, degradation is only one mode of competing risks for the systems. Another important competing risk mode is random shock that represents the sudden impacts from the external operating environment towards the system itself. Therefore, the system failure mechanisms can be divided into two categories: one is catastrophic failure, in which units break down due to some sudden external shocks; the other is degradation failure, in which units fail to properly function because of the physical degradation.

In the catastrophic failure mode, the units break down as soon as the cumulative shock damage or some extreme shock exceeds certain predetermined failure threshold, while in the degradation failure mode, the units fail to work once the total degradation wear drops below some critical failure level. The applications with respect to the competing risks of degradation and random shocks are very common phenomenon in our practical life.

Practical Examples:

Rubidium standard (Quan and Kvam (2011)) is a frequency standard used to control the output frequency of cell phone stations, television stations, and global position systems (GPS) based on the hyperfine transition of electrons in rubidium-87 atoms. The degradation process of the rubidium's performance can be due to either the depletion of the discharge lamp or the consumption of the rubidium. Extremes of environmental factors will result in the shocks to rubidium standard, such as temperature, pressure, vibration and supply voltage.

Light-emitting diode (LED) lamp (Yang et al. (2010)) has been regarded as a potential substitute for the traditional fluorescence and incandescence lamps. Because LED which is the main component of LED lamps is highly reliable, it may experience very few failures during the life testing experiment. Therefore, degradation testing and analysis are a more efficient approach to obtain the information of LED's lifetime. But the LED has a complex system structure which may exhibit more than one degradation failure dominating the system reliability. Furthermore, the system's common usage history intends to cause a dependent relationship between the performance characteristics of the system degradation paths. Over-voltage and over-heating may also cause the failure of LED lamps.

The human body (Gross (1973)) is composed of a variety of biological systems, that is composed of the organs, that is composed of the tissues, that is composed of the cells. Each unit, from tiny invisible cells to the whole biological systems, may experience graceful degradation of its function until a certain age. Taking the human heart as an example, at the age of 40, the efficiency of the heart delivering blood to the body will begin to greatly reduce because of the gradual loss of elasticity of blood vessels. As a result, the arteries may harden or become blocked. On the other hand, many factors can contribute as a shock to the human body, such as non-normal living environment and illness. Diabetes which happens when too much blood sugar is in the blood for a long time can damage many parts of the human body, such as the heart, kidneys and blood vessels.

Li and Pham (2005a) develop a general model to estimate the reliability of multi-state systems subject to the multiple competing risks of two degradation processes and random shock, and later on Li and Pham (2005b) propose a condition-based inspection-maintenance model with thresholds. However, the two competing processes in these models are independent with each other. In our research, we will focus on the dependent relationships among multiple competing risks and consider various maintenance policies involving imperfect preventive maintenance under the multi-objective optimization scope for this complex dependent multiple competing risk model.

1.2 Dependent Relationship for Competing Risks of Degradation and Shocks

There exist two kinds of dependent relationships that we focus in our research regarding the competing risks. Now we will discuss them in detail as follows:

Dependency among multiple degradation measures

In real world, not only many systems have more than one component with its own degradation process, but also one single component may possess more than one degradation path. Therefore, a probabilistic mathematical model should be considered to describe and predict the reliability functions for the multiple degradation measures. The neglect of the relationships among the multiple degradation measures may underestimate the system reliability. In the Ph.D thesis of Wang (2003), by checking the dependent relationship from the simulation results of reliability estimation we can clearly see that the positive covariance between degradation measures will lead to higher reliability estimation than the independent case.

Dependency between degradation process and random shocks

Usually, there are two aspects to check the dependency between the degradation processes and random shocks:

A. Degradation towards shocks: Degradation will make the system more fragile to the random shocks. Fan et al. (2000) consider a multi-component system subjected to the non-homogeneous Poisson process shocks. In the model, aging will increase the magnitude of shock sizes, thereby resulting in a larger fatal probability.

B. Shocks towards degradation: When the random shocks occur, the degradation process may receive two types of impacts where the first type is a sudden increment jump and the second is degradation rate acceleration. Cha and Finkelstein (2009) extend the Brown-Proschan model by assuming that the random shocks will result in an immediate system failure with a probability $p(t)$, but accelerate the system aging process by certain random increment with probability $q(t)$.

The problem formulation is becoming more and more complicated when these dependencies among competing risks of degradation processes and random shocks cannot be ignored. Therefore, a more general analysis method for the dependent measures is required to be developed.

1.3 The Importance of Imperfect Preventive Maintenance

Perfect maintenance or repair assumes that the system will be "as good as new" upon the maintenance, but in real world this assumption is not so practical. Sometimes because of economic principle or resource availability, we only partially restore the system to a younger age with the system status between "as good as new" and "as bad as old". For that reason, a more realistic assumption considering that upon maintenance the system will be restored to an intermediate status between pre-maintenance conditions and "as good as new" should be studied, which is called imperfect maintenance. In the existing literatures of imperfect maintenance, various approaches of modeling the imperfect maintenance problems have been introduced. From the study of Wang and Pham (2006a), seven modeling approaches are summarized, shown as in Table 1.1.

Table 1.1: Summary of treatment methods for imperfect maintenance
(Wang and Pham (2006a))

Modeling method	References
(p, q) Rule	Chan and Downs (78), Helvic (80), Nakagawa (79,80,87), Brown and Proschan (82,83), Fontenot and Proschan (84), Lie and Chun (86), Yun and Bai (87), Bhattacharjee (87), Rangan and Grace (89), Srivastava and Wu (93), Wang and Pham (96,97), Lim et al. (98), Pham and Wang (00), Cha and Kim (01), Kvam et al. (02), Li and Shaked (03)
$(p(t), q(t))$ Rule	Beichelt (80,81), Block et al. (85,88), Abdel-Hameed (87), Whitaker and Samaniego (89), Makis and Jardine (91), Hollander et al. (92), Sheu et al. (95), Wang et al. (01), Wang and Pham (99,03)
Improvement Factor	Malik (79), Canfield (86), Lie and Chun (86), Jayabalan and Chaudhuri (92,95), Chan and Shaw (93), Suresh and Chaudhuri (94), Doyen and Gaudoin (04)
Virtual Age	Uemastsu and Nishida (87), Kijima (89), Makis and Jardine (93), Liu et al. (95), Gasmi et al. (03), Doyen and Gaudoin (04)
Shock Model	Bhattacharjee (87), Kijima and Nakagawa (91,92), Finkelstein (97)
(α, β) Rule or Quasi-renewal	Lam (88,96), Wang and Pham (96,97,99,06b), Pham and Wang (00,01), Yang and Lin (05), Wu and Clements-Croome (05)

Process	
Multiple (p, q) Rule	Shaked and Shanthikumar (86), Sheu and Griffith (92)

1.4 The Importance of Multi-objective Optimization in Maintenance

Maintenance optimization is a systematic process that attempts to balance the maintenance requirements and resources to identify the maintenance periodicity and appropriate maintenance technique should be conducted to achieve maintenance targets, such as safety control, component reliability, system availability and costs. Generally, in most literatures the maintenance optimization has two objectives, i.e. maximizing system availability (A_s) and minimizing system cost (C_s). There are three cases to model the maintenance optimization problems.

Case I: System Availability Maximization Problem:

$Max\ A_s$

Subject to $C_s \leq C_{\max}$

Case II: System Cost Rate Minimization Problem:

$Min\ C_s$

Subject to $A_s \geq R_s$

Case III: Simultaneously Maximizing Availability and Minimizing Cost Optimization Problem:

$Max\ A_s\ \ \&\ Min\ C_s$

Subject to $A_s \geq R_s$

$$C_s \le C_{max}$$

where R_s denotes the target system availability and C_{max} is the tolerable maximum cost.

The first two cases provide a one-dimension optimization for the maintenance issues while the case of multi-objective optimization show a multi-dimension optimization perspective for both engineers and customers to determine the criterions and specification for the product reliability that should be satisfied by Pareto frontier, in which all of the points are with the same resource-utility efficiency to achieve the maximized system availability and minimized expected maintenance cost rate. The multi-objective optimization in maintenance issues presents the alternative optimal solutions according to different customer preference and resource constraints. In recent years, many papers have begun to concentrate on the multi-objective maintenance optimization, such as Martorell S. et al. (2005, 2006), Quan et al. (2007), Okasha and Frangopol (2009) and Wang and Pham (2011).

1.5 Overview of the Thesis

The thesis is organized as follows. In Chapter 1, a general introduction of the research is presented. In Chapter 2, literature reviews are given. In Chapter 3, the objectives of the study are discussed and general probabilistic model is sketched. In Chapter 4, a two-process combination model for continuously degraded systems subject to cumulative effect from random shocks and degradation process with additive and multiplicative degradation path is developed. In Chapter 5, we propose a maintenance model involving imperfect maintenance actions based on the dependent competing risk model in Chapter 4, and derive the mathematical models for the system expected cost rate and asymptotic unavailability of maintained system by a multi-objective optimization using Genetic Algorithms (GA). In Chapter 6, a multiple dependent competing risk model is introduced for systems subject to two degradation processes and random shocks. The dependent structure of random shocks and degradation processes is modulated by a time-scaled covariate factor and the dependent behavior among various degradation processes is fitted by both constant and time-varying copulas. In Chapter 7, a generalized multi-state degradation model is presented based on the methodology propsed in Chapter 6, and the reliability and state probability estimation are derived by both analytical method and Monte Carlo simulation. In Chapter 8, a condition-based imperfect preventive maintenance model with thresholds is proposed for the dependent competing-risk systems, and four maintenance decision variables are determined by minimizing the system maintenance cost rate using Simulated Annealing. In Chapter 9, conclusion and future research are discussed. A representation of research flow chart in this thesis is illustrated in Figure 1.1.

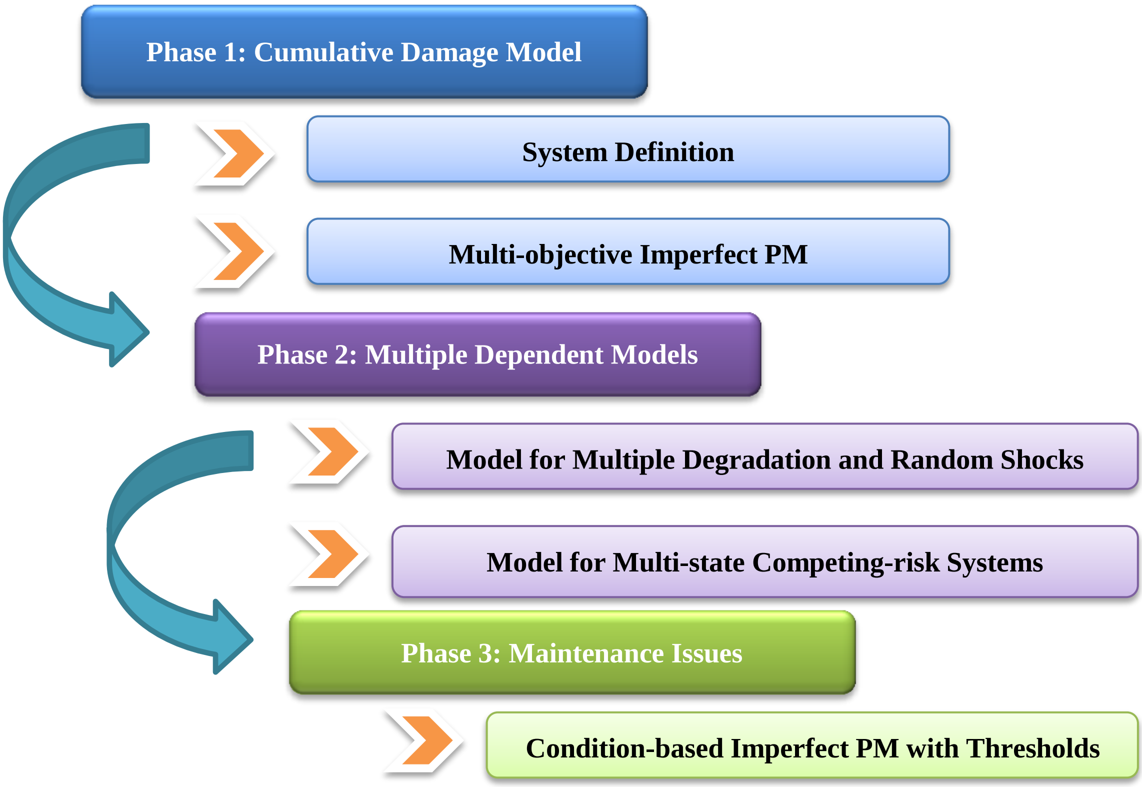


Figure 1.1: Research Flow Diagram

Chapter 2

Literature Review

2.1 Random Shock Model

Shock Models, one of the most importance subjects in reliability modeling, are usually defined by the time interval between two consecutive shocks, the damage size caused by individual random shock, and the system failure function. Usually, in shock models the system is subject to shocks of random magnitude occurring at random times. The basic setup in the shock models is pairs of the i.i.d two-dimension random variables $\{(A_n, B_n)\}_{n=0}^{\infty}$ where A_n denotes the magnitude of the *n*th shocks and B_n the time interval between the (*n*-1)st and *n*th shocks, or alternatively, the time between the *n*th and the (*n*+1)st shock, called model I and model II respectively. However, model II differs significantly from model I in the following two aspects: (1) in model II the magnitude A_n impacts the time interval B_n until the (*n*+1)st shock; (2) there exists a first shock at time zero in model II while in model I both A_0 and B_0 equal to zero. Shock models have been studies by a number of literatures, such as Esary and Marshall (1973), Gut and Husler (2005), Chen and Li (2008), Li and Kong (2007), Mallor and Omey (2001), and Bai et al. (2006), for the purpose of providing the mathematical formulations for modeling the system reliability in the random environments. Traditionally, three classic random shock models are widely used, listed as follows:

- Cumulative Shock Model

- Extreme Shock Model
- δ -Shock Model

Furthermore, some extensions including mixed shock model and run shock model are developed in recent years.

2.1.1 Cumulative Shock Model

Cumulative shock model means the system breaks down when the cumulative shock magnitude exceeds some given threshold. Let $T \geq 0$ denotes the lifetime of the system, $\{N(t), t \geq 0\}$ the counting process generated by the renewal sequences $\{B_n\}_{n=0}^{\infty}$, and $Z \subset R$ some prefixed critical region. A suitable representation of the cumulative shock model is indicated in the form of

$$\{T \leq t\} \Leftrightarrow \left\{ \sum_{i=0}^{N(t)} A_i \in Z \right\}$$

where A_i denotes the magnitude of the *n*th shocks. Thus, $F(t) = \Pr\{T \leq t\}$ and $\overline{F}(t) = 1 - F(t)$ denote the lifetime distribution and the survival function for the system respectively.

In term of the cumulative model, the properties of the lifetime distribution are studied in some early literatures. Esary and Marshall (1973) justify the properties of the lifetime distribution for a device subject to accumulative shocks governed by a Poisson process with a probability $\overline{P}_k$ of surviving the first *k* shocks. A-Hameed and Proschan (1973) extend the results from Esary and Marshall (1973) to a non-homogenous Poisson process in series system consisting of finite components and obtained the bound for the mean life of the device. Agrafiotis and Tsoukalas (1987) discuss the first passage time and asymptotic properties of a correlated cumulative shock model with the excess level increments for individual shock. In the model the shock interval and shock magnitude are correlated for given *n*. Gut and Husler (2005) put forward a generalized cumulative shock model in which only the summation of a suitable portion of the most recent shocks will contribute to the system stopping time, that is

$$\{T \le t\} \Leftrightarrow \min\left\{n : S_{k_n,n} = \sum_{j=n-k_n+1}^{n} X_j > t\right\}, t > 0$$

where X_j denotes the magnitude of the *j*th shock; $S_{k_n,n}$ the summation of the magnitude from the k_nth shocks to the *n*th shock.

In addition, some results of the maintenance policies for a system subjected to cumulative shocks have been studied by several researchers. Nakagawa and Kijima (1989) propose a periodic replacement policy with minimal repair at failure for the cumulative shock model in order to obtain the optimal solution for the time T^*, shock N^*, and damage Z^* at which time point the replacement is done. Qian et al. (2003) focus on the analysis of maintenance policies for an extended cumulative shock model with shocks occurring at a non-homogeneous Poisson process. The system will be maintained when cumulative shock do not exceeds the failure threshold, repaired when cumulative shock exceeds the threshold, and replaced at failure *N* or time *T*. Wortman et al. (1994) examine the maintenance strategy with the inspection time modulated by a renewal process for a non-self-announcing failure system subject to deterioration governed by random shocks. Chelbi and Ait-Kadi (2000) develop the expression of the time-stationary availability for a hidden failure system subject to the transient shocks with a predetermined inspection time in order to generate an optimal solution for the target availability level with limit resources.

2.1.2 Extreme Shock Model

Extreme shock model means that the system fails as soon as the magnitude of an individual shock goes into some given critical region $Z \subset R$, shown as:

$$\{T \le t\} \Leftrightarrow \left\{\min\{i : A_i \in Z\right\} \le N(t)\}$$

where T denotes the lifetime of the system, $N(t)$ the counting process of random shock occurring, and A_i the magnitude of the nth shocks.

In the early work on this model, Esary and Marshall (1973) study the properties of the maximum shock threshold underlying the assumption of homogeneous Poisson process, while Ross (1981) extends the analysis of Esary and Marshall (1973) to the non-homogeneous condition. One important setting of the extreme shock model is "general shock model" developed by Shanthikumar and Sumita (1983). The model extends the extreme shock model by considering a sequence of random shocks governed by a correlated pair of renewal sequences $\{X_n, Y_n\}$, where X_n denotes the magnitude of the nth shock and Y_n the time interval between two consecutive shocks.

Chen and Li (2008) analyze a deteriorating system subject to extreme shock with the deterioration process governed by both the external shocks and internal loading under the assumptions such that: (1) the magnitude of the random shock the system can bear will be decreasing with the numbers of repairs; (2) the repair time will be increasing upon each repair. Finally, an optimal replacement policy N*, at which failure number the system will be repaired, is determined by minimizing the long run average cost.

Cirillo and Husler (2011) propose an alternative Bayesian nonparametric perspective for the extreme shock models by using reinforced urn processes. Cha and Finkelstein (2011) combine the extreme shock models and cumulative shock models into a "combined shock model" to derive the survival probabilities for systems subject to nonhomogeneous Poisson process shocks.

2.1.3 δ -Shock Model

The δ-shock model in Li (1984) is defined as the system fails when the time lag between the two successive shocks falls into some critical region determined by a prefixed parameter δ. Therefore, a suitable representation for the δ-shock model can be given as

$$T \le t \Leftrightarrow \{\min\{i : B_i \in C(\delta), i \in N\} \le N(t)\}$$

where $C(\delta)$ is the critical region for the system, and B_i the time interval between the (n-1)st and nth shocks.

In the work of Li and Kong (2007), some useful results for the δ-shock model underlying homogeneous Poisson process are given, such as survival function, class properties and asymptotic behavior of the lifetime distribution. Moreover, the analytical survival function for the non-homogeneous Poisson process is also discussed.

Furthermore, Li and Zhao (2007) derive some useful properties for the reliability estimation of the coherent structure of series system, parallel system, and k-out-of n system, such as bounds for system mean lifetime and limiting probability. Lam and Zhang (2004) study a replacement policy for two systems embedded in the δ-shock model: one is deteriorating system with a non-decreasing threshold after repair times and geometrically increasing repair times; the other is improving system with decreasing threshold after repair and geometrically deceasing repair times. Rangan and Tansu (2008) extend the δ-shock model of Lam and Zhang (2004) by considering the random threshold failure in the context of renewal process. Tang and Lam (2006) study a δ-shock maintenance model for a deteriorating system with shocks occurring according to a renewal process, where the interarrival time of shocks follows a Weibull or gamma distribution. Because the system is deteriorating, the deadlock threshold for the δ-shock is geometrically non-decreasing after each repair, and the repair time is modulated by an increasing geometric process. Eryilmaz (2012) develops a generalized run-related δ-shock model in which the systems fail when the inter-arrival time of k consecutive shocks is less than certain threshold δ.

2.1.4 Some Extensions and Variations

Ross (1981) defines a general shock model with a damage function D_t in which $D_t(x_1,...,x_n,...,\mathbf{0})$ represents the damage at time t if exactly n shocks occur with the magnitude of $x_1,...,x_n$. By assuming $D_t(x_1,...,x_n,0)=\max(x_1,...,x_n,0)$ or $D_t(x_1,...,x_n,0)=\sum_{i=1}^{n}x_i$, the cumulative shock and extreme shock model are obtained respectively. Another realistic variation of shock model is provided by Fan et al. (2000) for the purpose of studying the lifetime distribution of the multi-component system that suffers from the interplay of aging and random shocks. In the model the probability that a shock with magnitude x arriving at time u is fatal to the system is given by $1-\exp[-\delta(a+u)-x]$, where a is the initial age of the system at time zero and δ is the system aging rate.

Igaki et al. (1995) generalize the cumulative and extreme shock model by considering a trivariate stochastic process including the magnitude of the shocks, the shock intervals, and the random system state. In the model, the magnitude of the shocks and the shock intervals are correlated with each other with a joint distribution affected by the transition of system state modulated by a Markov renewal process. Gut (2001) proposes a mixed shock model to combine the extreme model and cumulative shock model. In the model the system is supposed to break down either due to a cumulative effect of many small shocks or one single large shock, whichever comes first. Further realistic analysis of the model is stated by Gut and Husler (2005).

A new case, called the run shock model, is introduced by Mallor and Omey (2001), which models a system operating normally until *k* consecutive shocks with critical magnitude, which is expressed as

$$T \le t \Leftrightarrow \min\{n \mid A_{n-j} \in Z, j = 0,1,\ldots,k-1\} \le N(t)$$

where A_{n-j} represents the magnitude of the (*n-j*)th shock and *k* is some given positive integer.

One extension of the run shock model is proposed by Mallor and Santos (2003a), in which the system breaks down when the cumulative damage due to the shock in a critical *k* run, where all of shocks are critical and not contained in any of *k*+1 consecutive sequence, exceeds a fixed threshold *z*. This model is governed by three correlated variables, including the magnitude of shock, shock occurring interval and cumulative damage, respectively. Further analysis of the lifetime distribution and mean time to failure for the extended shock model is provided by Mallor and Santos (2003b). Moreover, Mallor et al. (2006) generalize the asymptotic behavior for the lifetime of the mixed shock model that is the combination of the accumulative shock model and run shock model.

In the work of Finkelstein and Zarudnij (2001), two types of non-cumulative shock processes are considered. The first model divides the shocks into three levels according to their magnitude, shocks with small level that are harmless to the system, shocks with intermediate level that lead to the system failure with some probability, and shocks with large level that result in the immediately system failure. The second model assumes that the system will not fail if two successive small shocks do not occur in a short time period. The system failure function and its exponential approximation are derived analytically. Bai et al. (2006) derive the asymptotic lifetime distribution for a new shock model based on a marked point process with cluster mark and then illustrate its application to an example with the insurance background.

2.2 Degradation Model

Degradation analysis involves the measurement of the degradation of a product at various time points and this information is used to estimate the eventual failure lifetime for the product. One of the major advantages of performing reliability analysis based on degradation data is that it relates the reliability analysis directly to the physics of failure mechanism. Many papers have been focused on the research and applications for the reliability degradation model, such as Lu and Meeker (1993), Yuan and Pandey (2009), Van Noortwijk et al. (1995), Nicolai et al. (2007), Huang and Dietrich (2005), Wang et al. (2009), and Chryssaphinou et al. (2011)

2.2.1 Methodology

Based upon the methodology, the studies of the degradation analysis can be divided into three main categories.

One of the most widely used methods to model the degradation data is called "General Path Model". Lu and Meeker (1993) introduce a two-stage method of estimating the parameters of the mixed-effect path model to describe the system degradation performance. The general path model can be represented as

$$x_{ij} = f(t;\phi,\theta_i) + \varepsilon_{ij} \tag{2.1}$$

where x_{ij} denotes the observation of degradation measure from the *j*th measurement on the *i*th unit; θ_i random effect parameter for the *i*th unit, follows normal distribution, that is, $\theta_i \sim N(\mu_\theta, \Sigma_\theta)$; ϕ the fixed-effect parameter.

The two steps for estimating the parameters in the general path model are given as:

- The 1st stage: for each sample unit, the degradation model is individually fitted to the sample path to obtain *n* estimates of the model parameters;
- The 2nd stage: the estimates of $\phi, \mu_\theta, and\ \Sigma_\theta$ are computed using the *n* estimates obtained by the 1st stage.

Yuan and Pandey (2009) elaborate the limitation of the linear regression for degradation analysis and then propose a general nonlinear mixed-effect (NLME) model to analyze and predict the degradation process. An alternative approach of two-stage maximum likelihood estimation for the general path model is proposed by Robinson and Crowder (2000), where a Bayesian estimation model is used to estimate the distribution function of failure time in the general path model. Wu and Shao (1999) develop the statistical inference using the ordinary least square and weighted least squares to estimate the parameters of degradation path in a nonlinear mixed-effect model.

The second approach to perform the degradation analysis is "Stochastic Process Model", such as Markov process and Brownian motion. Van Noortwijk et al. (1995) model a Bayesian failure model of degradation analysis, in which the average amounts of deterioration are l_1-isotropic, and then explicitly obtain the probability of preventive repairs per unit time, failure with and without inspection conditional on the average amount of degradation. Van Noortwijk and Pandey (2003) generalize a stochastic gamma process model for a stochastically deteriorating system. In the model, the system may fail when its deteriorating resistance drops below certain critical stress s. The degradation path is modulated by gamma distribution, and thus a suitable representation of the lifetime distribution is in the form of

$$F(t) = \Pr(T \le t) = \Pr(X(t) \ge r_0 - s\} = \int_{x=r_0-s}^{\infty} f_{X(t)}(x)dx = \frac{\Gamma(v(t),[r_0 - s]u)}{\Gamma(v(t))} \qquad (2.2)$$

where T is the lifetime for the system; r_0 is the initial resistance; s is the deterministic critical stress; and Γ is the incomplete gamma function.

The corresponding maintenance policies for the gamma deterioration model are provided by Zhao (2003), Pandey et al. (2005). Nicolai et al. (2007) compare three stochastic processes for degradation analysis, including Brownian motion with non-linear drift, gamma process with non-linear shape function and two-stage hit-and-grow (TSHG) process. After that, the parameter estimation of these three processes for the inspection data and expert data is provided. In Hsieh et al. (2009) a non-homogenous compound Poisson process is utilized to model the discrete degradation. Then the first passage time distribution, the likelihood estimation of the model parameters, and confident interval approximation are derived. Xue and Yang (1995) extend the 2-state reliability measure to the multi-state reliability by combing the Markov process and *s*-coherent system structure in order to derive the dynamic reliability for the multi-state system with complex structure, including the series systems and parallel systems.

Kharoufeh (2003) analytically derives the failure time distribution and the moments of the lifetime for a single-unit system with cumulative wear damage that is affected by the external environment by using the Markov additive process. Saassouh et al. (2007) propose a two-mode stochastically deteriorating model with a sudden change point in the degradation path, where the increments of deterioration follows a gamma law when the system is in the first mode, and the mean deterioration rate increases when it flips into the second mode. Based on the definition of the model, the decision rules for an online maintenance policy are determined to optimize the system performance from the angle of asymptotic unavailability.

Kharoufeh and Cox (2005) develop a degradation-based model for assessing the distribution and the moments of the lifetime function using the hybrid approach including two models: Model I describes a degradation process with the rate of degradation affected by the state of random environment governed by a homogeneous Markov process; Model II establishes a new path function by estimating the degradation rates using differential equation where the number of degradation status is approximated by K-nearest clustering method. Ebrahimi (2001) proposes a general stochastic model to estimate the reliability of the system in terms of a deterioration process with covariate. The survival and hazard functions are analytically derived for two semi-parametric models using the differential equation by taking the exponential form of the survival function.

The third method is "Statistical Method", including both parametric and nonparametric estimation. Huang and Dietrich (2005) present an extended graphical approach for degradation analysis by considering the ordering of degradation data. In the model, the degradation path is modulated by a truncated Weibull distribution and the analytical log likelihood expression is discussed. Bae et al. (2007) examine the relationship between the degradation path function and the lifetime distribution where the additive and multiplicative function is used to model the degradation path. The results verify that the degradation path will have a great influence on the form of the lifetime distribution in terms of the class properties of lifetime distribution and failure rate.

Zuo et al. (1999) extend the results of Yang and Xue (1995) from the s-normal process to the random process with a general distribution by using three approaches for degradation analysis. The first is the random process to fit the degradation process with a data free-distribution; the second is the traditional general path function; the third method is to use a multiple linear regression. Finally, a mixture model is introduced to model both hard failure and soft failure for the continuous state device. Bae and Kvam (2004) present a nonlinear random coefficients model to fit the degradation path for the non-monotonically deteriorating light displays, and then consider four different methods to approximate the log-likelihood estimation, including a first-order method, Lindstrom-bates algorithm, adaptive importance sampling, and adaptive Gaussian quadrature method.

2.2.2 Multi-state Degradation

For the multi-state degradation model, Chryssaphinou et al. (2011) propose a repairable multi-state degraded system composed of *m* components, whose sojourn time in any state is characterized by a discrete-time semi-Markov chain, and then the whole system is modeled as the paired process of semi-Markov chain. Soro et al. (2010) develop a continuous-time Markov model for evaluating the reliability performance measures of multi-state degraded system with imperfect preventive maintenance and minimal repairs. Eryilmaz (2010) derive the mean residual and mean past lifetime functions for multi-state *k*-out-of-*n* systems, in which the degradations follow a Markov process with discrete state spaces. Abou (2010) considers a number of alternative probabilistic models for multi-state systems with two failure modes by introducing multi-state operators that follow the associative and commutative laws.

Nourelfath and Ait-Kadi (2007) apply a Markov model to modulate the redundancy maintenance optimization problem with reliability constraints and minimal cost configuration for series-parallel multi-state system by considering the priorities between components. Ramirez-Marquez and Coit (2005) describe a new Monte-Carlo simulation methodology of reliability estimation process based on multi-state minimal cut vectors compared with the actual multi-state two-terminal reliability computation. Levitin and Lisnianski (2000) develop an imperfect preventive maintenance model for multi-state systems in order to obtain maintenance action sequences based on the system functioning levels using Genetic Algorithm to minimize the maintenance cost.

2.2.3 Maintenance Policy

Also increasing interest has been put upon the probabilistic models and mathematical formulations of the maintenance and replacement policies for the degradation or multi-state systems. Grall et al. (2002) propose a condition-based maintenance model including both the inspection and replacement policies based on a multi-level control-limit rule of a stochastically deteriorating system for the purpose of obtaining the optimal replacement threshold and inspection scheduling to minimize the long run expected cost. Wang et al. (2009) consider a novel maintenance model combing the condition-based replacement, periodical inspections, and (S, s) type provisioning policy, noted as (T, S, s, L_p) policy, where T denotes the inspection interval, S the maximum inventory level, s the reorder point, and L_p the replacement threshold. Furthermore, a simulation model is established to modulate the uncertain deterioration process and finally the maintenance scheduling is optimized to minimize the cost rate using genetic algorithm.

Kiessler et al. (2002) examine the limiting average availability of a hidden-failure deterioration system with the periodic inspections where deterioration rate is governed by a Markov model. Yang and Klutke (2000) characterize the properties of the lifetime distribution for the Levy degradation process and then illustrate the implement of the results to inspection scheduling for the maintained system with non self-announcing failures. Zhao (2003) presents a preventive maintenance policy for a deteriorating system with a critical reliability level to satisfy the preference of field managers with the imperfect PM effect modulated by a parameter of degradation ratio. Pandey et al. (2005) consider an age replacement policy for the gamma deterioration model where the component is replaced when the system fails or reaches a specific age, whichever occurs first.

Van Noortwijk and Frangopol (2004) describe two maintenance models of condition-based maintenance and reliability-based maintenance for the deteriorating civil infrastructures for the purpose of minimizing the life-cycle cost under the constraint of adequate reliability level. Delia and Rafael (2006) analyze the maintenance policies with two types of repair modes, including preventive and corrective repairs, and phase-type distributed repair times for a cold standby system subject to multi-stage degradation. A later work of Dellia and Rafael (2008) study a maintenance model with failure and inspection following arrival processes and two types of repair modes, minimal and perfect, distributed as different phase-type distributions for a deteriorating system suffering from both internal and external failures. Deloux et al. (2009) propose a maintenance policy that combines the statistical process control (SPC) and condition-based maintenance (CBM) for a continuously deteriorating system with two kinds of failure mechanisms (deteriorating and random shocks). The system failure is governed by deteriorating process as a function of the deterioration level and the system time but an associated failure acceleration factor due to stress is taken into account when the stress intensity exceeds some critical level.

2.3 Multiple Competing Risks of Degradation and Random Shocks

In real world, system typically deteriorates as a result of both the graceful degradation and discrete random shocks. However, most of the papers regarding the multiple competing risks of degradation and random shocks are under the assumption that these two processes are independent with each other. The incorrect independent assumption may underestimate the reliability and lifetime behaviors of the system. Therefore, on dealing with the relationship of competing risk of degradation and random shocks, the dynamic behaviors of the dependent structure between them may have a non-trivial impact on the reliability function estimation or on the maintenance and warranty policies for the deteriorating systems.

The dependent of these two processes can be exhibited in two aspects: (1) degradation process will make the system more vulnerable to the environment factors, such as temperature, pressure and random shocks; (2) random shocks will accelerate the degradation process with two modes, sudden jump or minor acceleration of the degradation rate. Furthermore, although in many studies of reliability model, the multiple degradation processes are assumed to have independent lifetimes, it may be more realistic to assume some sort of dependence among different degradation processes. For example, a system may have multiple components with its own degradation process or even one component may be subject to multiple degradations, in which the lower status of one degradation process could result in an increasing load on some of the other degradation processes. As a sequence, a more systematic probability model for the dynamic dependent structure underlying these two processes should be called for.

2.3.1 Independent Model for Degradation and Shocks

Sim and Endrenyi (1993) consider a Markov process to model the maintenance policy with periodically minimal maintenance and major maintenance after a number of minimal maintenances for a continuously operating system subject to degradation and Poisson failures. The optimal solution to minimize either cost rate or unavailability is derived. A probabilistic model of the reliability analysis for the deteriorating structural systems subject to Poisson shocks is introduced by Ciampoli (1998), where a stochastic differential equation is employed to model the degradation process in order to obtain the total damage of the system. A generalized Petri Net is proposed by Hosseini et al. (2000) to formulate a new condition-based maintenance model for a system subject to deterioration failures and Poisson failures. In order to maximize the system throughout, an optimal inspection policy based on minimal maintenance, major maintenance and major repairs is obtained. Van der Weibe et al. (2010) derive the reliability estimation and the optimal solution for calculating the discounted cost based on both condition-base and age-based policy for a maintained system that deteriorates due to both transient shocks and cumulative degradation process governed by a stochastic point process.

In practice, there is not only the system operating cost will increase with the system aging, but also the cost of time of the inspection, repair and replacement, Chiang and Yuan (2001) present a state-dependent maintenance policy $R_{i,j}(T, N, \alpha)$ for a continuously deteriorating system subject to degradation and fatal shocks using a continuous-time Markov process, where T is the system inspection interval, N is the system boundary for replacement, and α is the probability that repair will restore the system to a better state. Delia and Rafael (2006) examine the replacement policy for a Markovian degraded system submitted to internal or external failures with holding time on various system levels, external repair time and internal repair time, all of which follow the phase-type distribution. In the work by Kharoufeh et al. (2006), the lifetime distribution as well as the limiting availability for a periodically inspected single-unit system with hidden failure is explicitly derived by utilizing the Laplace-Stieltjes transform. The system is submitted to two failure mechanisms, the degradation wear that governed by its random environment characterized as a continuous Markov chain and random shocks modulated as a homogeneous Poisson process.

2.3.2 Dependent Model for Degradation and Shocks

Frostig and Kenzin (2009) derive the limiting average availability in a maintenance model for a hidden-failure system that suffers from the wear out and cumulative shock damage with a Poisson process. Two models are discussed: model I assumes the wear out process and shock will not receive any impact from the external environment; in model II, the shock magnitude, the shock rate and wear out process, all of them are dependent on the external environment modulated by a Markov process. Based on the approach proposed by Mori and Ellenwood (1994), Van Noortwijk et al. (2007) put forward a novel approach to combine two stochastic processes of deteriorating resistance and fluctuating load for the reliability analysis of a structural component. In the model, the deteriorating process is modulated as a gamma process and the random loading exceed is given by a generalized Pareto distribution with loading arriving according to a Poisson process. Chiodo and Mazzanti (2006) deal with the problem of the reliability function assessment for power system devices due to repeated shocks. The systems may survive under the condition that the individual stress load is less than the remaining degradation resistance.

Lehmann (2009) surveys two classes of degradation-threshold-shock models (DTS), including general DTS and DTS with covariates, where the system failure may be due to the competing risk of degradation and trauma. In the general DTS model, the traumatic failure is assumed to be modeled as a stochastic Poisson process with intensity factor that is governed by system aging level. In the DTS with covariates, a dynamic environment random variable is included in the model. Huynh et al. (2011) recently introduce a condition-based maintenance model for the degradation-threshold-shock (DTS) model to take the dependence between degradation process and shock process into account. Cha and Finkelstein (2009) extend the Brown-Proschan model by assuming that the random shocks will result in an immediate system failure with a probability $p(t)$, but accelerate the system aging process by certain random increment with probability $q(t)$. Finkelstein (2009) introduces a generalized Strehler-Mildvan model to estimate the first passage time of the survival function for the system subject to cumulative damage due to biological aging and sudden killing event. The asymptotic aging properties for the repairable system are discussed.

Satow et al. (2000) focus on the replacement policy for one single unit that suffers from cumulative damage due to aging process and shocks in order to obtain the optimal replacement level k^* which minimizes the expected cost rate. Klutke and Yang (2002) present a maintenance policy for the periodically inspected systems with non self-announcing failure, submitted to cumulative damage due to both graceful degradation and random shocks for the purpose of optimizing the system performance from the limiting average availability point of view. Sun et al. (2006) introduce an analytical model to quantitatively estimate the interactive failure and its failure rate based on three difference cases. A later work by Sun et al. (2009) develops an extended split system approach for the failure interactions. Simulated data is used to test the proposed model and the results indicate that the PM intervals for newly repair components with the presence of failure interactions will become shorter compared with the system without failure interactions.

Ye et al. (2011) propose a degradation-oriented single failure time model to capture two failure mechanisms of degradation and shocks using the Brown-Proschan model under the condition that only failure times and failure modes are recorded without the observable information of shock magnitude and degradation amount. Wang et al. (2011) consider a reliability estimation model for systems subject to three failure modes, including catastrophic failure, degradation failure, and failure due to shocks. In the model, the shocks have two effects on system performance: sudden increase of failure rate and direct change of degradation process.

2.3.3 Multiple Degradation Model

Li and Pham (2005a) focus on the reliability analysis for a generalized multi-state degradation system subject to multiple competing failure processes, consisting of two degradation processes and cumulative random shock. The paper assumes that all of these processes are independent, and any of them causes the system to failure according to the threshold values of each process. No repair or maintenance policies are considered. Based on the definition of multi-state degraded system in Li and Pham (2005a), a condition-based maintenance model is built by Li and Pham (2005b), where an average long-run cost rate function is minimized by Nelder-Mead downhill simplex method. The inter-inspection sequence is generated by a geometric sequence.

Wang and Coit (2004) propose a general model of predicting the reliability on the correlated multiple degradation processes and verify that the system reliability might be underestimated because of the incorrect independent assumption by the simulated data. A gamma-based state space model is studied by Zhou et al. (2009) to predict the lifetime for a multiple degradation processes with uncertain failure threshold using multivariate normal distribution, where expectation-maximization (EM) algorithm is utilized to estimate parameters of the model and Monte Carlo-based particle smoothing algorithm is used to deal with the expectation estimation of complete likelihood in step E of EM algorithm.

Pan and Balakrishnan (2011) introduce a reliability estimation model for a complex structure system with bivariate degradations involving two or more performance characteristics by utilizing the bivariate Brinbaun-Saunders distribution, and perform the Bayesian Markov chain Monte Carlo method to evaluate the accuracy of the reliability approximation. Wang and Pham (2012) recently develop a dependent competing risk model, in which the dependent structure of random shock and degradation is modulated by a time-scaled covariate factor, and the dependent structure among degradation processes is fitted by both constant and time-varying copulas, for a system subject to multi-degradation measures and random shocks, but without considering the maintenance issues.

Sari (2007) introduces a novel approach to address the problem that how to qualify the dependence relationship between two or more degraded performance characteristics (PC) in his Ph.D thesis. He applies the generalized linear model (GLM) to fit the degradation data and model the dependency between the PCs by using copula methods. In his later work, Sari et al. (2009) present a bivariate degradation model with constant stress to accommodate the dependency between more degradation measures distributed with different marginal functions. The experiment data of LED lamps is used to illustrate that the proposed model can provide better system estimation than independent assumption with a two-stage modeling process. Zhou et al. (2010) extended the Sari (2007) model by applying Bayesian MCMC approach to estimate the parameters of the gamma marginal degradation path functions and copulas together as a whole instead of treating the estimating separately.

2.3.4 Imperfect Preventive Maintenance

Cassady et al. (2005) explore the imperfect repair based on the Kijima's first virtue age model by validating the simulation results using 23 factorial experiment and converting reliability and maintainability parameters into coefficients of availability model using meta-models to determine the optimal replacement interval according to the system average cost. Liu and Huang (2010) apply the non-homogeneous continuous time Markov model (NHCTMM) to model the optimal replacement policy for the multi-state systems with the imperfect maintenance that utilizes the quasi-renewal process to describe the stochastic behavior of the multi-state aging element after each imperfect repair. Wang and Pham (2011) study a multi-objective maintenance optimization embedded within the imperfect PM and replacement for one single-unit system subject to the dependent competing risk of degradation wear and random shocks. Two decision variables for maintenance scheduling, the number of PMs to replacement and the initial PM interval, are determined by simultaneously maximizing the system asymptotic availability and minimizing the system cost rate using the fast elitist non-dominated Sorting Genetic Algorithm (NSGA-II). Satow and Kawai (2010) put forward an imperfect inspection with upper and lower inspection threshold for a bivariate failure distribution.

2.4 Copula Method

Only a few studies focus on the issue of multiple degradation processes according to the literature review of previous section. These two papers in Li and Pham (2005a, b) consider the reliability and maintenance model for two degradation processes and random shocks, but all of them are independent with each other. A traditional way to build correlated multiple degradation model is to utilize the tool of multivariate distribution which will create the restrictions on the same distribution of each marginal degradation path. Recently considerable attention has been paid to the dependence behavior between random variables modeled by copulas, which allow us to link the univariate marginal distributions to obtain a joint probability of the events. Compared with the traditional multivariate distribution, the most attractive advantages of the copula method are listed as follows: (a) the univariate marginal function can be modulated separately from their dependent structure; (b) the marginal probability can be drawn from different kinds of distributions without restriction; (c) the parameter coefficients of the copula model can be time-varying, not constant. Because of these advantages, copula model is a powerful alternative approach of the multivariate distribution to analyze the correlated multiple degradation processes. There are several aspects on copulas method could be worked with.

2.4.1 Theoretical Model

Embrechts and Puccetti (2010) provide analytical procedures to calculate the bounds on the distribution function of the sum of n dependent risks with overlapping margins, that is, the bounds for the sum $S = X_1 + ... + X_n$, where $X = (X_1, ..., X_n)$ belongs to

Frechet class of probability measure. Kojadinovic and Yan (2010) compare the asymptotic properties of three semi-parametric methods of estimating the parameters in copula models, which are maximum pseudo-likelihood estimation, method of moment estimator based on Spearman's rho, and method of moment estimator based on Kendall's tau. Monte Carlo simulation is used to examine the performance of the different estimators with finite samples and compute the asymptotic relative efficiency. Rodriguez-Lallena and Ubeda-Flores (2003) examine the properties of the conditional distribution of $H_1(\boldsymbol{X})$ given that the joint distribution of $\boldsymbol{X}$ is H_2, where H_1 and H_2 are the multivariate distribution functions for random vectors $X = (X_1, X_2, ..., X_n)$ with common univariate marginal distributions.

Chen and Fan (2006) derive the asymptotic properties of estimators for a class of copula-based semi-parametric stationary Markov models characterized by parametric copula functions and nonparametric margins. Hurlimann (2004) proposes a modified statistical method of inference functions of margins (IFM) characterized as two-step maximum likelihood estimations of univaraite marginal distributions and copulas, followed by minimizing the chi-square statistic of a bivariate version of the Pearson goodness-of-fit test to determine the dependence parameters for copula fitting in the bivariate cumulative returns. Abegaz and Naik-Nimbalkar (2008) introduce an alternative approach based on a copula-based Markov chain to investigate the conditional probability of distributions and utilize one- and two-stage statistical inference method to estimate the parameters. In addition, a parametric pseudo-likelihood ratio test is given to select the copula model for the two-stage estimation. Zezula (2009) illustrates how to use the special variance structure of Gaussian copulas to facilitate the parameter estimations under the condition that the data dimension is large.

2.4.2 Application

Van den Goorbergh et al. (2005) apply dynamic copula model to better-of-two-markets and worse-of-two-markets options on the S&P500 and NASDAQ to examine the dependent behavior of bivariate option pricing with association between the assets. The aim of the study in Zhang and Singh (2007) is to derive the bivariate joint distribution of rainfall frequency using four Archimedean copulas in order to determine the return periods. Based on the data from US stock, Fernandez (2008) verifies that using tail-dependency tests to select the copula model may be misleading especially when the data is featured with conditional volatility and series correlation. with the help of Monte Carlo simulation Al-Harthy et al. (2007) illustrate how the copula method can be suitable to model the dependencies in oil and gas evaluations and come to the conclusion that compared with some more commonly used approaches to model dependence the copula method can accurately detect the tail dependence structure of the variable distribution. Dalla Valle (2009) suggests a new methodology for studying the dependent relationships of operational risk management by combining the copula and Bayesian models computed using simulation methods, especially Markov chain Monte Carlo. Dakovic and Czado (2011) examine the point and interval estimates using joint maximum likelihood and semi-parametric models to estimate the parameters of a bivariate *t*-copula model in financial data.

Applications of copula methods in various fields can also be found in Ning (2010) for the dependence structure between the foreign exchange market and equity market, Roch and Alegre (2006) for daily equity returns, Ausin and Lopes (2010) for multivariate time series using time-varying copulas, Renard and Lang (2007) for design hydrology.

2.4.3 Special Case: Dependent Risk Model

A special case of copula application is dependent risk model, which can be used as a source of reference to construct the dependent competing risk model in reliability using copulas, for instance the studies in Kaishev et al. (2007), Bedford (2006), Lo and Wilke (2010), Cossette et al. (2008), and Embrechts et al. (2003).

Kaishev et al. (2007) establish a dependent multiple-decrement model to examine the dependencies among causes of death in order to analyze the impact of complete or partial elimination of causes of death on the survival function from competing risks using copulas. Bedford (2006) puts forward two different methods of non-parametric maximum likelihood and bilinear adjustment estimator to perform the quantile tests for copulas in competing risk problems. Cossette et al. (2008) derive the discounted penalty function via Laplace transform for a generalized Farlie-Bumbel-Morgenstern copula model in the presence of the associations between the claim sizes and interclaim time in a compound Poisson risk model. Another study of Cossette et al. (2002) presents two approaches of a class factor method and copula method to construct the dependent risk models for the insurance portfolio.

Embrechts et al. (2003) provide the properties and computational procedures of distributional bounds for the dependent risks functions using copulas. Ram and Singh (2008) put forward a mathematical modeling for a parallel redundant system with two *s*-independent repairable subsystems. In their model, the system performance characteristics, such as availability, mean time to failure, and expected profit, are derived under the "preemptive-resume repair discipline" by using a bivariate Gumbel-Hougaard family copula. Lo and Wilke (2010) develop a new copula graphic estimator applied to a model with multiple dependent competing risks, and apply the model to the data set of unemployment duration from Germany. In the work by Miladinovic and Tsokos (2009), a modified Gumbel failure model is used to study the system failure time, and Bayesian reliability estimates with five different parametric prior and one nonparametric kernel density prior are compared with each other for their effectiveness by using square error loss.

2.5 Multi-objective Maintenance

Many literature papers focus on maintenance optimization based on one objective. Vaurio (1995) develops advanced models to the general recursive equations for the availability and mission-failure probability of the standby structure system by considering different durations for testing and repairs, as well as various failure types, including start-up, standby and during mission failures, and two additional human errors. Zequeria and Berenguer (2006) study a maintenance policy considering three types of actions: minimal repairs, PM, and replacement. They study a system with two dependent competing failure modes of maintenance and non-maintenance by minimizing the system cost rate during an infinite time. In the model, the improvement factor for the failure rate upon PM actions depends on the time when the actions are performed. Cepin (2002) determines the optimal scheduling to improve the safety of equipment outages in nuclear power plants by minimizing the mean value of the selected time-dependent risk measure. Zhu et al. (2010) examine the maintenance model for a competing risk of degradation and sudden failure in which the unit is renewed when it reaches a predetermined degradation level, or comes to a sudden failure within the limit of a certain degradation threshold. Also, a PM is done at the scheduled time. The maintenance scheduling variables of degradation threshold and scheduled time to PM are determined by maximizing the system availability with the constraint of repair cost.

In terms of the multi-objective approach, Martorell et al. (2005) propose a new integrated Multi-Criteria Decision-making (IMCDM) method to determine the parameters in the technical specifications and maintenance (TSM) of safety-related equipment using a multi-objective Genetic Algorithm (GA) based on the reliability, availability, and maintenance (RAM) criterion. An example of an emergency diesel generator system illustrates the application and viability of the proposed method. Martorell et al. (2006) address the multi-objective problem of surveillance requirements at nuclear power plants with dependable variables of testing intervals (TI), and testing planning using a novel double-loop multiple objective evolutionary algorithms. In Quan et al. (2007), a new approach, which combines the preference with evolutionary algorithm by using utility theory to search the Pareto frontier rather than conducting a dominated Pareto search, was developed to find the optimal solutions for a multi-objective PM scheduling. Sanchez et al. (2009) put forward a GA based approach using distribution free tolerance intervals to address a multi-objective optimization of an unavailability and cost model embedded within the uncertainty of the imperfect maintenance. Okasha and Frangopol (2009) consider two strategies of selecting maintenance actions, maintenance scheduling, maintenance structural components for optimization programs to design and construct structural systems in terms of system reliability, redundancy, and life-cycle costs as criterion in the multi-objective GA. Two numerical examples are used to illustrate these two strategies. Marseguerra et al. (2004) introduce a multi-objective optimization approach to determining the optimal surveillance test interval (STI) based on GA search towards solutions of optimal performance with high assurance.

Chapter 3

Objectives of the Study

3.1 Research Objectives

In the scope of our research, we aim to focus on a generalization of reliability modeling framework to address the following two dependent relationship aspects, including: (1) the dependency between degradation processes and random shocks; (2) the dependency among multiple degradation processes, into a single model for a continuously degraded system subject to random shocks. Our objective is to develop the different maintenance policies involving imperfect preventive maintenance in terms of multi-objective optimization of simultaneously maximizing system asymptotic availability and minimizing expected maintenance cost rate based on the proposed multiple dependent competing risk models. More specifically, in our research we would like to achieve the following objectives:

1) Develop the reliability model for the dependent competing risk system subject to one single degradation process and random shocks
2) Study the multi-objective imperfect preventive maintenance policies for the system in objective 1
3) Extend the achieved model in objective 1 to multiple dependent competing risks with more than one degradation process and random shocks
4) Consider the probabilistic model in objective 3 under the scope of the

multi-state scenario

5) Modify the model in objective 4 to incorporate condition-based maintenance policy with threshold level for actions "doing nothing" and "imperfect maintenance". Maintenance cost rate is considered as the objective function.

3.2 Organization of the study

Our research is organized as follows:

In Chapter 1, we describe the motivation of the study and the overview of the thesis. In Chapter 2, an overall literature review on related topics of degradation processes, random shocks, competing risks and copula method are investigated. In Chapter 3, the objectives of the study are formulated and a general probabilistic model for the system subject to dependent competing risks from multiple degradation processes and random shocks is sketched.

In the following Chapters, the research will be carried out through three phases:

Phase 1-Cumulative damage model for competing risks with single degradation process and random shocks

In Chapter 4, a two-process combination model for the continuously degraded system subject to cumulative effect from random shocks and degradation processes with additive and multiplicative degradation path will be discussed. Two numerical examples are used to illustrate the application of proposed combination model for the additive and multiplicative degradation path. Based on the system definition, we then obtain a maintenance policy involving imperfect PM modulated by a common improvement factor to determine the pairs of optimal solutions (N*, T*), the number of imperfect PM till replacement and the imperfect PM interval, by minimizing the expected maintenance cost rate. Some extensions for the model are suggested for future research.

In Chapter 5, we study a multi-objective maintenance optimization embedded within the imperfect preventive maintenance (PM) for one single-unit system subject to the dependent competing risks of degradation wear and random shocks. We consider two kinds of random shocks in the system: 1) fatal shocks that will cause the system to fail immediately, and 2) nonfatal shocks that will increase the system degradation level by a certain cumulative shock amount. Also, an improvement factor in the form of quasi-renewal sequences is introduced to modulate the imperfect maintenance by raising the degradation critical threshold proportionally. Finally, the two decision variables for maintenance scheduling, the number of PMs to replacement, and the initial PM interval, are determined by simultaneously maximizing the system asymptotic availability, and minimizing the system cost rate using the fast elitist non-dominated Sorting Genetic Algorithm (NSGAII). Sensitivity analysis for two parameters, including imperfect PM degree, and quasi-renewal coefficient of imperfectPMinterval, is performed to provide insight into the behavior of the proposed maintenance policies. The comparison results show that the optimization solution is consistent between one-objective and multi-objective optimization, and the Pareto frontier for the maintenance optimization problem can provide alternative solutions according to customer preference and resource constraints.

Phase 2- Reliability estimation for dependent competing risks with multiple degradation and random shocks

In Chapter 6, we develop an *s*-dependent competing risk model for systems subject to multiple degradation processes and random shocks using time-varying copulas. The proposed model allows for a more flexible dependence structure between risks in which (a) the dependent relationship between random shocks and degradation processes is modulated by a time-scaled covariate factor, and (b) the dependent relationship among various degradation processes is fitted using the copula method. Two types of random shocks are considered in the model: fatal shocks, which fail the system immediately; and nonfatal shocks, which do not. In a nonfatal shock situation there are two impacts towards the degradation processes: sudden increment jumps, and degradation rate accelerations. The comparison results of the system reliability estimation from both constant and time-varying copulas are illustrated in the numerical examples to demonstrate the application of the proposed model. The modified joint distribution bounds in terms of Kendall's tau and Spearman's rho provide an improvement to Frechet-Hoeffding bounds for estimating the possible system reliability range.

In Chapter 7, a generalized reliability estimation method is developed for the multi-state system subject to multiple dependent competing risks with two degradation processes and random shocks. The status of the multi-state degraded system is determined by the Cartesian product of the two degradation measure levels. Then the system reliability estimation and state probability are derived by both analytical method and Monte Carlo simulation, which is applied to provide the approximate point estimation and confident interval to compare the results from analytical method. Finally, a numerical example is provided to illustrate the application of the proposed reliability estimation modeling with sensitivity analysis for the shock occurring rate.

Phase 3- Condition-based imperfect preventive maintenance for the repairable degraded system with multiple dependent competing risks

In Chapter 8, a condition-based maintenance model with imperfect preventive maintenance (PM) threshold for dependent competing-risk systems subject to multiple degradation processes and random shocks is studied. In the model, the dependent structure between degradation measures and random shocks is modulated by a time-scaled covariance while the dependent structure among degradation measures is linked using copula method. The four optimal maintenance decision variables will be determined by minimizing the expected long-run maintenance cost rate. They are the imperfect PM thresholds of the two degradation measures $\{L_1, L_2\}$, imperfect PM intervals $\{I\}$, and the imperfect PM number till replacement $\{N\}$. The system is continuously monitored till the imperfect PM threshold and then periodically inspected at each imperfect PM interval. A numerical example with sensitivity analysis in terms of imperfect PM degree β is discussed to illustrate the applications of the proposed maintenance model.

In our futher research, we also plan to extend the maintenance policies in Chapter 8 to incorporate the multi-objective maintenance optimization of system asymptotic availability and expected maintenance cost rate.

In Chapter 9, conclusion and future research are discussed.

3.3 Problem Assumption and Model Description

In our research, we will consider a generalized model for the dependent competing risk model with multiple degradation processes and random shocks. The system has n degradation processes each with k_i status, i=1, 2, .., n, and random shocks, all of which are interplaying with each other, as shown in Figure 3.1.

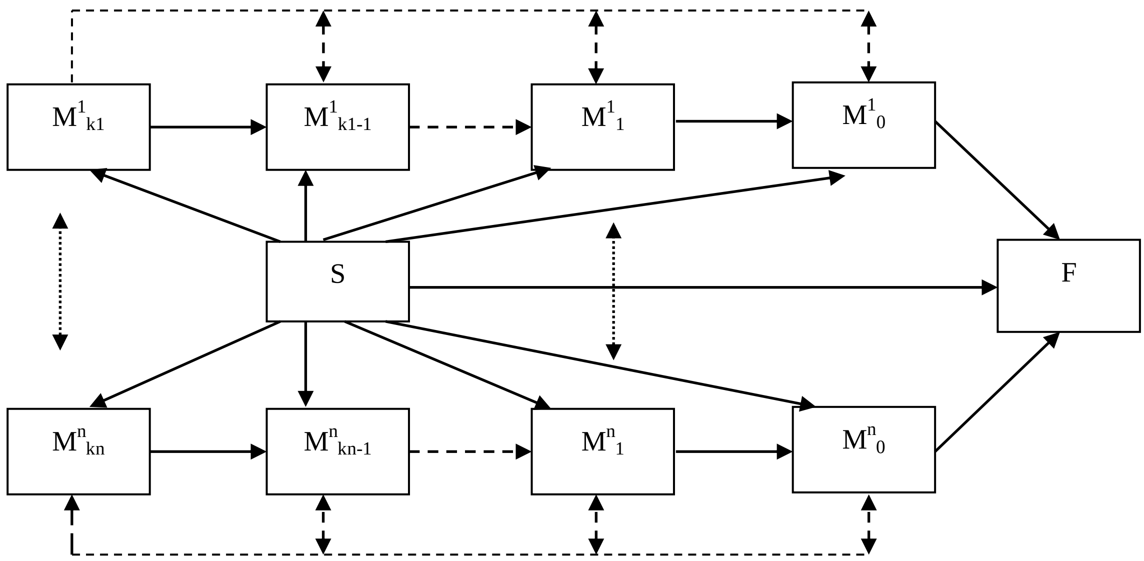


Figure 3.1: Flow diagram for the system subject to multiple competing risks

The basic assumptions of **system definition** are listed as follows:

A. The system is continuously deteriorating with multi-state. With the impact of degradation and random shocks, the system can either go to the next lower system status due to time wearing, or jump to other lower system statuses due to random shocks.
B. There exist two types of random shocks: one is fatal shock, resulting in system failure immediately, and another is non-fatal shock, resulting in two impacts on the system degradation processes of cumulative sudden jump and degradation rate acceleration.
C. The two processes of the degradation and random shocks are dependent with each other.
D. The multiple degradation processes are also dependent with each other.
E. The system has two failure mechanisms: one is shock failure due to sudden fatal shocks, and another is degradation failure due to the exceeding of certain critical threshold by any degradation path.

The basic assumptions of **system maintenance** issues are shown as follows:

A. Three types of maintenance actions are considered: imperfect preventive maintenance, minimum repairs and replacements.
B. The system undergoes different maintenance actions of “doing nothing”, “imperfect maintenance” and “replacement” according to the threshold levels determined by maintenance scheduling optimization.

C. Both the maintenance cost rate and system asymptotic availability is considered into the problems to form the multi-objective optimization programming.

In summary, this research will focus on the dependence, imperfect maintenance and multi-objective optimization.

Chapter 4

Imperfect Preventive Maintenance Policies for Two-process Cumulative Damage Model of Degradation and Random Shocks

4.1 Introduction

The failure of many units or systems, such as components, parts, machines can be generally classified into two kinds of failure modes: one is catastrophic failure in which units break down by some sudden external shocks; the other is degradation failure in which units fail to function due to the physical deterioration. There are a great number of such cases for this kind of the competing failure modes in our real life.

A. A battery supplies electric power by chemical reaction. It is gradually weaken by usage and finally turns out to be useless when the substances in the battery are exhaustive. On the other hand, overheating or over-voltage can also cause the damage of the battery.

B. An electronic component may have two kinds of failure modes: One failure mode is due to the overloading stress to the system caused by random voltage spikes; the other is due to wear-out process, which usually happens when it has run for many cycles.

In the catastrophic failure case the units break down as soon as cumulative shock damage or some extreme shock exceeds certain predetermined failure threshold, while in the degradation case the unit fails when the total degradation amount drops below a critical failure level. Therefore, it would be necessary to formulate probabilistic or stochastic models of such competing risk systems that outline the features of the two-process complex phenomenon combining degradation and random shocks. The reliability analysis for this model is a key step for the manufacturing engineers to make warranty plans and maintenance policies for the products featured by the two competing failure modes. Numerous mathematical formulations and probabilistic models are proposed to combine these two competing risk failure models.

Li and Pham (2005a) focus on the reliability analysis for a generalized multi-state degradation system subject to multiple competing failure processes, consisting of two degradation processes and random shocks. Based on the definition of such multiple competing failure system, a condition-based maintenance model is built by Li and Pham (2005b). Deloux et al. (2009) proposes a predictive maintenance policy, combining statistical process control (SPC) and condition-based maintenance (CBM), for a continuously deteriorating system with two failure mechanisms due to deterioration and random shocks. Based on the method of Mori and Ellenwood (1994), Van Noortwijk et al. (2007) put forward a novel approach to combine two stochastic processes of deteriorating resistance and fluctuating load for the reliability analysis of a structural component. Lehman (2009) surveys two classes of degradation-threshold-shock models (DTS), including general DTS and DTS with covariates.

A probabilistic procedure for the analysis of the deteriorating structural components and systems subject to Poisson shock is introduced by Ciampoli (1998). Chiodo and Mazzanti (2006) deal with the problem of the reliability function assessment for power system devices due to repeated shocks. A generalized Petri Net is proposed by Hosseini et al. (2000) to formulate a new condition-based maintenance model for a system subject to deterioration failures and Poisson failures. Klutke and Yang (2002) develop a maintenance policy for the hidden failure system that deteriorates due to the Poisson shock and constant rate degradation. Satow et al. (2000) study a replacement policy for a unit subject to cumulative damage which occurs by shock or aging. Peng et al. (2009b) develop a periodic inspection maintenance policy to minimize the impact of unscheduled failures and to minimize cost for micro-electro-mechanical systems (MEMS) devices subject to a normal-distributed shock loads and a linear degradation process.

This Chapter includes three parts to elaborate the problem from a continuously deteriorating system suffering from random shocks:

PART I: A two-process cumulative combination model of competing failure between degradation and random shock is introduced with additive and multiplicative degradation path.

PART II: Two numerical examples with sensitivity analysis for additive and multiplicative degradation path are used to illustrate the combination model.

PART III: Based on the definition of the model, an imperfect preventive maintenance policy is put forward to obtain the optimum pair (N^*, T^*) using numerical methods in order to minimize the expected total cost rate.

Different from the traditional competing model, a two-process cumulative model may be more suitable to describe the problem. It is because both of these damage works directly on the unit or system, and a cumulative damage can clearly reflect the system's status. Also, the perfect preventive maintenance is not so practical in real world. Therefore, we modulate the imperfect preventive maintenance by an improvement factor. By taking some assumptions to simplify the imperfect maintenance model, finally an optimum policy is obtained from the complex expected cost rate function.

4.2 A Combination Method for Degradation and Random Shock

Competing risk model is an important subject in reliability analysis, especially degradation and random shock. An abundant of papers (Li and Pham (2005a,b), Deloux et al. (2009), Van Noortwijk et al. (2007), Lehman (2009), Klutke and Yang (2002)) study the competing models for degradation and random shock, and also the maintenance policies for this kind of system. However, many of them assume they are two independent failure modes. In this Chapter, we put forward a different combination model in which the two processes have a cumulative impact on the system failure and the system deteriorates due to both degradation and random shocks.

4.2.1 Degradation Path Model

The path functions for degradation model make a great difference in various studies. One of the most widely used models, "General Path Model", is introduced by Lu and

Meeker (1993), which is a “two-stage” method of estimating parameters for the mixed-effect path model. Van Noortwijk and Pandey (2003) employ a stochastic gamma process model to account for both population and temporal variability associated with a degradation process. Zuo et al. (1999) present three statistical approaches for degradation analysis of continuous state devices.

In this research, basic additive and multiplicative models are considered to tolerate the item-to-item variation by including the random variable *X*.

Two kinds of degradation path function are shown as follows:

General additive degradation model

$$D(t; X, \theta) = \eta(t; \theta) + X \tag{4.1}$$

General multiplicative degradation model

$$D(t; X, \theta) = X \cdot \eta(t; \theta) \tag{4.2}$$

where $\eta(t;\theta)$ is a deterministic mean degradation path with fixed effect parameters θ for time $t \geq 0$ and *X* represents random variation around a mean degradation level with a cumulative distribution function (cdf) F_x and a probability density function (pdf) f_x.

The mean degradation path can either be monotonically decreasing or increasing, called decreasing degradation path (DDP) or increasing degradation path (IDP), respectively. Non-monotonic degradation model is also studied by several researchers, such as the non-monotonic degradation of light displays in Bae and Kvam (2004), but such case is seldom in real life.

For example, if *X* is Weibull-distributed, one simple example for multiplicative model is $D(t;X,\Theta)=\xi t$, where $\xi = X$ is the degradation rate with the Weibull cumulative distribution function

$$F_X(x)=1-\exp\left[-(\frac{x}{\theta})^r\right], \text{ for } \theta, r>0 \tag{4.3}$$

Another example for additive model is introduced by Fukada (1991) to describe the degradation process of electronic devices:

$$D(t;X,\Theta)=\theta_3\cdot\exp[-\exp\{(\theta_1 t)^{\theta_2}\}], \text{ for } \theta_1,\theta_2>0, \theta_3\geq 1 \tag{4.4}$$

By taking a log transformation, we obtain

$$\log D(t;X,\Theta)=\log\theta_3-\exp[(\theta_1 t)^{\theta_2}] \tag{4.5}$$

Therefore

$$X=\log\theta_3,\ \ \eta(t;\Theta)=-\exp[(\theta_1 t)^{\theta_2}] \tag{4.6}$$

From the definitions and examples we can see that these two models can be employed to cover a widely used degradation models in many studies. The random variable X can be used to increase the flexibility for the degradation mean path function to tolerate the item-to-item variation.

4.2.2 Random Shock Model

Shock models have been widely studied by Chen and Li (2008), Mallor and Santos (2003a,b), Mallor et al. (2006), Gut (2001), Li (1984), Li and Kong (2007), Li and Zhao (2007), and Bai et al. (2006) in order to provide mathematical formulations for modeling the system reliability in random environments. Traditionally, three principal models are considered: cumulative shock model, extreme shock models and run shock model. After that some extensions including mixed shock model and δ-shock model are developed. In this Chapter, traditional cumulative shock model is employed to simplify the calculation process, but mixed shock model can also be used in this combination method.

The random shocks occur in a homogeneous Poisson process with rate λ. Let the random variable $N(t)$ denote the number of shocks until time t with the probability

$$\Pr\{N(t)=n\}=\frac{(\lambda t)^n}{n!}e^{-\lambda t}\text{, } n\text{=0, 1, 2,...} \tag{4.7}$$

In addition, we denote the amount of damage caused by the jth shock by W_j, a sequence of nonnegative, independent and identically distributed random variable with a common distribution $G(x)=\Pr(W_j\le x)$ for all j. Let $S(t)$ denote the cumulative shock damage, given by a compound Poisson process $S(t)=\sum_{j=0}^{N(t)}W_j$. Assume $G^{(j)}(x)$ is the j-fold Stieltjes convolution with itself, it follows that

$$\Pr(S(t)=W_1+W_2+...+W_j\le x\}=G^{(j)}(x)\text{, } j\text{=0, 1, 2,...} \tag{4.8}$$

If $G(x)$ follows an exponential distribution with parameter μ, the compound Poisson process turns out to be a gamma distribution with parameters ($N(t)$, $1/\mu$).

4.2.3 Combination of degradation and Random Shock

We establish a combination model for degradation and random shocks, in which the system deterioration is cumulative due to both degradation and random shocks. In other words, the total damage for system deterioration is the cumulative effect of continuous degradation and sudden shocks, as shown in Figure 4.1.

The system failure is defined as that when the cumulative deterioration amount $M(t)$ drops below certain failure threshold, the system breaks down. Let R_0 denote initial system resistance, and R_l the critical failure threshold. The function of $M(t)$ can be represented as

$$M(t) = R_0 - D(t) - S(t) \tag{4.9}$$

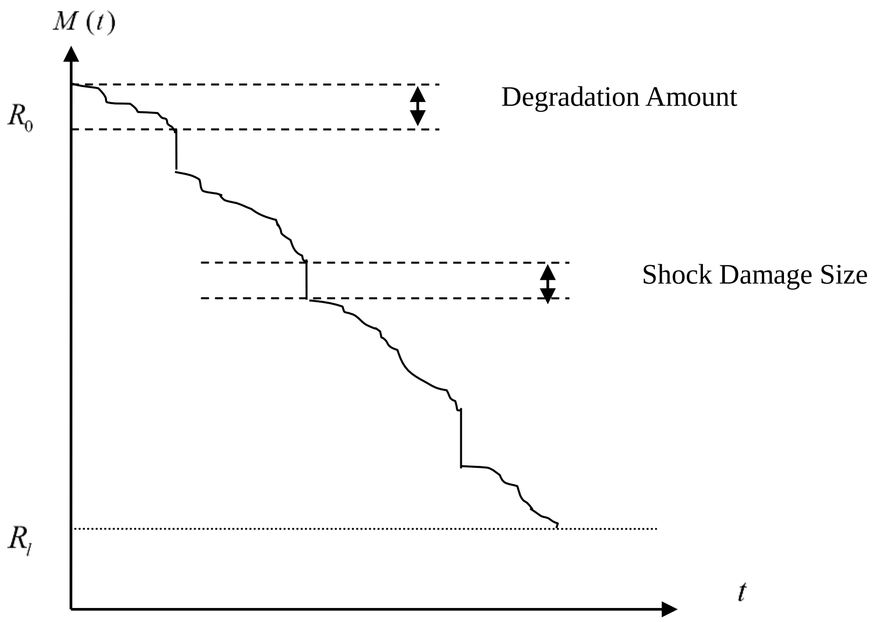


Figure 4.1: Combination of Degradation and Random Shocks

Therefore, the probability for the system to survive is given by

$$R(t) = P(R_0 - D(t) - S(t) > R_l) \tag{4.10}$$

If multiplicative model is used in the degradation path function, the reliability for the system is shown as follows

$$\begin{aligned} R(t) &= P(R_0 - X\eta(t,\theta) - \sum_{i=1}^{N(t)} W_i > R_l) \\ &= \sum_{n=0}^{\infty} P(X\eta(t,\theta) + \sum_{i=1}^{n} W_i < R_0 - R_l) P(N(t) = n) \\ &= \sum_{n=0}^{\infty} \frac{\exp(-\lambda t)(\lambda t)^n}{n!} \int_0^{R_0 - R_l} F_x(\frac{h}{\eta(t,\theta)}) dG^{(n)}(R_0 - R_l - h) \end{aligned} \tag{4.11}$$

If additive model is used in the degradation path function, the reliability for the system is shown as follows

$$R(t) = P(R_0 - X - \eta(t,\theta) - \sum_{i=1}^{N(t)} W_i > R_l)$$
$$= \sum_{n=0}^{\infty} P(X + \eta(t,\theta) + \sum_{i=1}^{n} W_i < R_0 - R_l) P(N(t) = n) \qquad (4.12)$$
$$= \sum_{n=0}^{\infty} \frac{\exp(-\lambda t)(\lambda t)^n}{n!} \int_0^{R_0 - R_l} F_x(h - \eta(t,\theta)) dG^{(n)}(R_0 - R_l - h)$$

Although the reliability formulas of both multiplicative and additive path are a little complex to calculate, we can obtain the approximate values by using numerical methods.

4.3 Numerical Example

The two examples in this section are to illustrate the model discussed in previous section: one for additive model, and another for multiplicative model.

4.3.1 Example I: Multiplicative Model

Case Study of a micro-actuator system designed by Sandia National Laboratory Tanner et al. (2000) is provided to illustrate the model. The linear degradation path of degradation amount is $X(t) = \varphi + \beta t$, in which φ=0 and the variable β is normal distributed with parameter $\mu_\beta = 8.4823 \times 10^{-9}$ and $\sigma_\beta = 6.0016 \times 10^{-10}$ by Peng et al. (2009a) (*t* is the number of revolutions in micro-engines rotation). Random shocks are assumed to follow a homogeneous Poisson process with rate $\lambda = 2.5 \times 10^{-5}$. Shock damage size are i.i.d. normal random variables, with $\mu_y = 1 \times 10^{-4}$ and $\sigma_y = 2 \times 10^{-5}$. The micro-engine fails when the total wear volume reaches a critical threshold, *H*, which is 0.00125 um^3 by Tanner et al. (2000), Peng et al. (2009a). From the definition of the lifetime distribution for multiplicative model, we can obtain the reliability function given by

$$R(t)=\sum_{n=0}^{\infty}\Phi\left[\frac{H-(u_{\beta}t+\varphi+nu_{y})}{\sqrt{\sigma_{\beta}^{2}t^{2}+n\sigma_{y}^{2}}}\right]\frac{\exp(-\lambda t)(\lambda t)^{n}}{n!} \tag{4.13}$$

where Φ is the cdf of standard normal distribution.

The software Mathematica 7.0 is used to plot the reliability distribution for this micro-actuator system as show in Figure 4.2. In the first 50000 cycles, the system reliability almost keeps one because no random shock happens and the cumulative system degradation amount is not large enough to cause the system failure. After that, with the continuous degradation and sudden shock, the system reliability begins to drop. Until the 180000th cycles the reliability for the system drops down to zero.

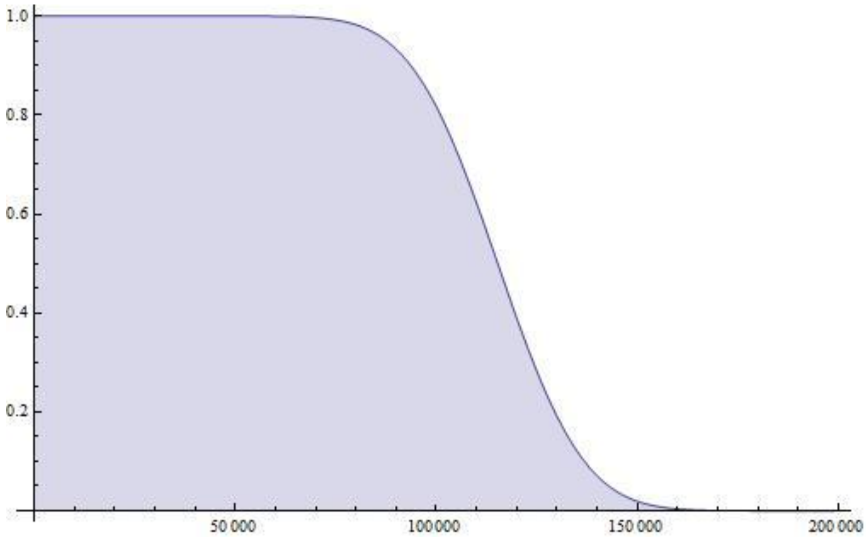


Figure 4.2: Reliability Distribution for Multiplicative Model

Sensitivity analysis for λ, u_{β}, u_{y}, H is investigated with regard to reliability distribution. From the graphs in Figure 4.3, we can see it clearly that except *H*, the system reliability is decreasing with the increasing of parameters λ, u_{β}, u_{y}. Compared with other parameters, *H* impacts the system reliability much more. The larger *H* is, the more reliability the system is. However, in real world, it is impossible to make *H* "large enough", since *H* depends on both the system physical characters and manufacturing technique.

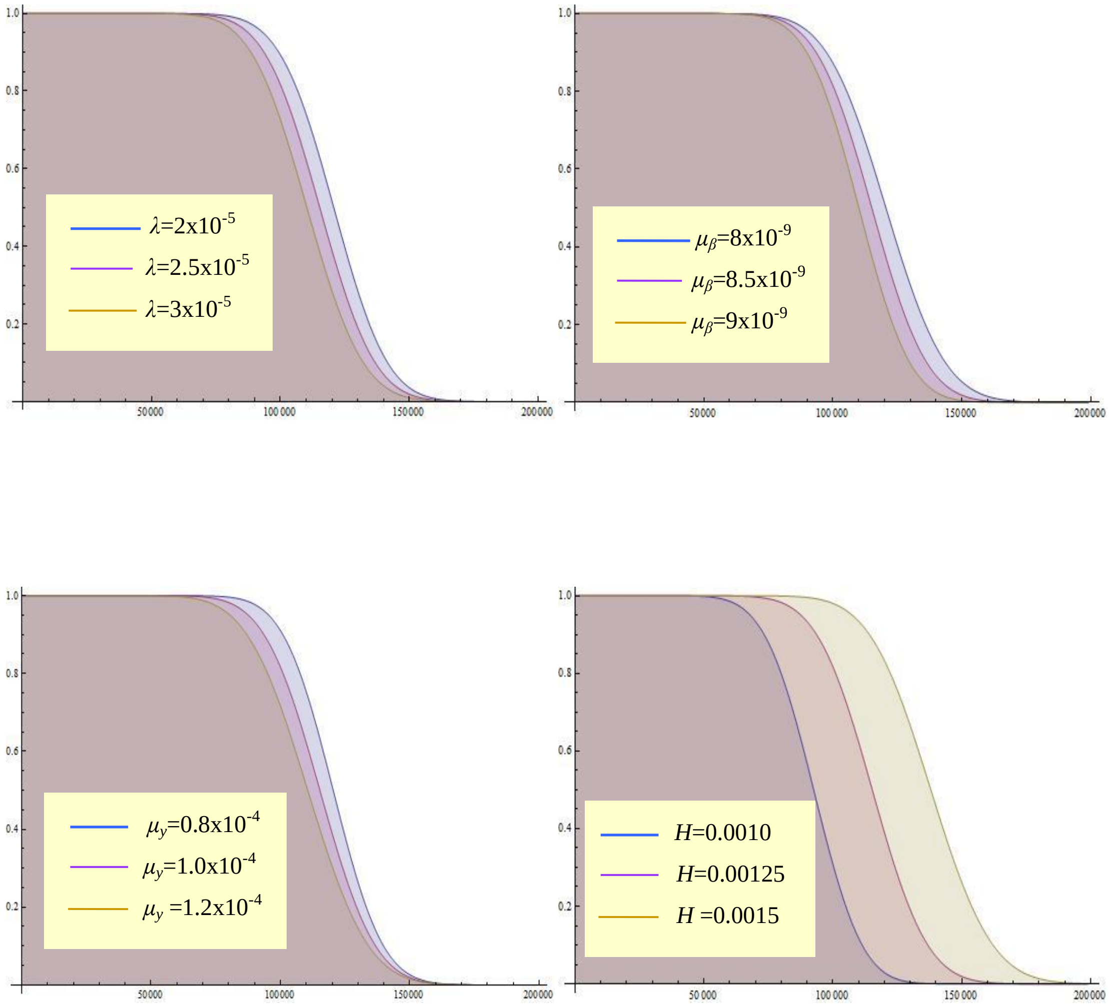


Figure 4.3: Sensitivity Analyses for λ, μ_β, μ_y, H

The lifetime for micro-actuator using different parameter λ is shown in Table 4.1. It is reasonable to see that the system reliability drops with the increasing of random shock intensity λ because intensely frequent random shock will make the system more vulnerable.

Table 4.1: Lifetime Table for Micro-actuator System for different λ

#	Time (Cycles $\times10^4$)	$\lambda = 2\times10^{-5}$	$\lambda = 2.5\times10^{-5}$	$\lambda = 3\times10^{-5}$
1	0.0	1.0000	1.0000	1.0000
2	6.0	0.9999	0.9996	0.9989
3	10.0	0.8935	0.8167	0.7265

4	10.5	0.8277	0.7279	0.6212
5	11.0	0.7404	0.6219	0.5063
6	11.5	0.6345	0.5059	0.3912
7	12	0.5167	0.3893	0.2850
8	12.5	0.3969	0.2818	0.1950
9	13	0.2857	0.1909	0.1248
10	13.5	0.1914	0.1204	0.0745
11	14	0.1189	0.0706	0.0413
12	14.5	0.0681	0.0383	0.0213
13	15.5	0.0175	0.0089	0.0045
14	16.5	0.0033	0.0016	0.0007
15	20	0.0000	0.0000	0.0000

4.3.2 Example II: Additive Model

The second case is in term of additive model with nonlinear degradation function. Assume the allowance degradation amount H equals to 500, that is the difference between the initial resistance and critical failure threshold. Once the cumulative damage exceeds this level, system breaks down. Let $N(t)$ denote the occurrence number of random shocks modulated by a homogeneous Poisson process with rate λ=0.1. The degradation function is described as $D(t)=A+\beta t^2$, where A is normal distributed with mean μ=5 and standard deviation σ=2.5, and β is constant degradation rate with value of 0.05. The random shock damage size is i.i.d exponential distributed with mean α=30. The reliability function is shown as

$$R(t)=\sum_{n=1}^{\infty}\frac{\exp(-\lambda t)(\lambda t)^n}{n!}\int_{\beta t^2}^{H}\Phi(\frac{h-\beta t^2-\mu}{\sigma})\gamma(H-h;n,\alpha)dh \\ +\exp(-\lambda t)\Phi(\frac{H-\beta t^2-\mu}{\sigma}) \tag{4.14}$$

where Φ is the cdf of standard normal distribution and γ is the pdf of gamma distribution.

Therefore, after plunging parameter values into this additive model, the reliability distribution is plotted by Mathematica 7.0 shown in Figure 4.4. With the continuous degradation and sudden shock, the system reliability decreases gradually. Until t=100, the system reliability drops nearly to zero.

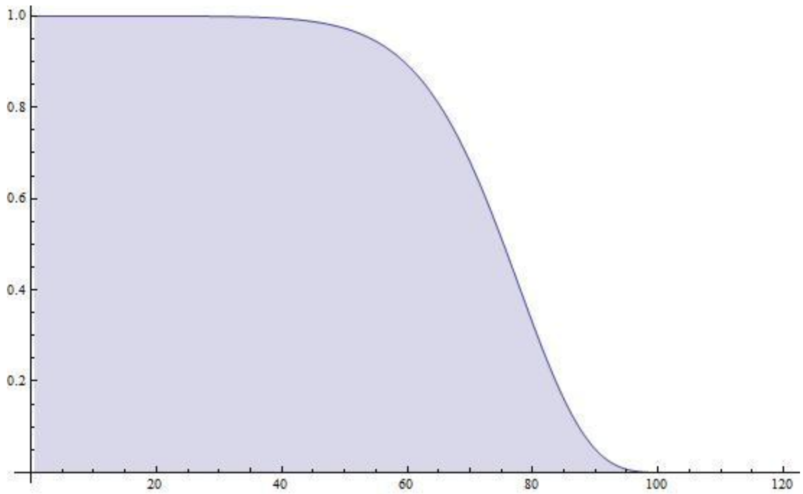


Figure 4.4: Reliability Distribution for Additive model

Sensitivity analysis for λ, β, μ, σ, α, H is investigated with regard to reliability distribution. From the graphs in Figure 4.5, we can see it clearly that the parameters μ,σ for A have little influence on reliability distribution. Because they reflect the variation from mean degradation path function, they could not be changed too much and should be confined in a reasonable small interval. Except H, the system reliability is decreasing with the increasing of parameters λ, β, α.

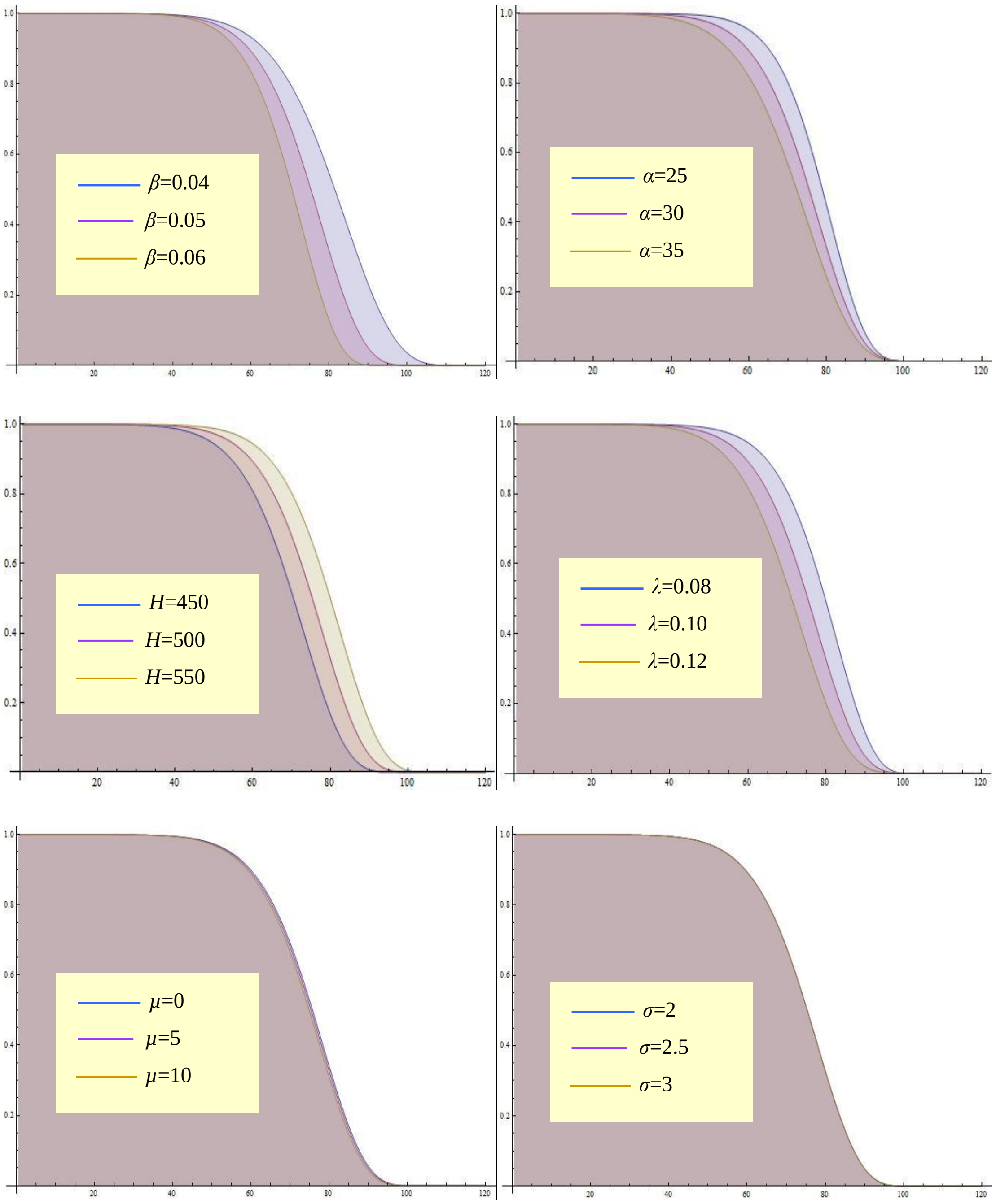


Figure 4.5: Sensitivity Analysis for *λ*, *β*, *μ*, *σ*, *α*, *H*

The lifetime for this additive model using different parameter α is shown in Table 4.2. Note that α denotes the mean damage size for each random shock. The larger the variable α is, the more random shock contributes to failure. Therefore reliability function is monotonically decreasing with the increasing of random variable α.

Table 4.2: Lifetime Table for Additive Model for different α

#	Time	$\alpha = 25$	$\alpha = 30$	$\alpha = 35$
1	0	1.0000	1.0000	1.0000
2	10	1.0000	1.0000	1.0000
3	20	1.0000	1.0000	0.9996
4	30	0.9999	0.9992	0.9971
5	40	0.9989	0.9948	0.9852
6	50	0.9918	0.9732	0.9415
7	60	0.9531	0.8933	0.8186
8	65	0.8997	0.8081	0.7101
9	70	0.8033	0.6808	0.5684
10	75	0.6510	0.5136	0.4049
11	80	0.4491	0.3272	0.2433
12	85	0.2374	0.1603	0.1133
13	90	0.0793	0.0508	0.0349
14	95	0.0104	0.0068	0.0048
15	100	0.0000	0.0000	0.0000

4.4 Imperfect Preventive Maintenance Model

Based on the combination model in section 4.2, an imperfect preventive maintenance (PM) policy with a common improvement factor is put forward to obtain the optimal policy pairs (N^*, T^*) minimizing the expected total cost rate.

4.4.1 Expected Cost Model

Considering the following maintenance policy shown in Figure 4.6:

- A sequence of imperfect preventive maintenance is done at fixed time interval T_n (n=1, 2, ..., N-1)
- If the unit fails between two consecutive PMs, minimum repair is done.
- A replacement is done at time T_N, that is, the unit is as good as new at time T_N.

We call the interval between the (n-1)st PM to the nth PM as period n. Let the random variable N_n denote the number of shocks in period n with the probability $\Pr(N_n = j\} = \frac{(\lambda T_n)^j}{j!}\exp(-\lambda T_n)$. In addition, we denote W_{nj} by the amount of damage caused by jth shock in period n. The degradation function is defined by $D(t) = \xi t$ just as the form in multiplicative model, where the random variable ξ is distributed with $F(x)$.

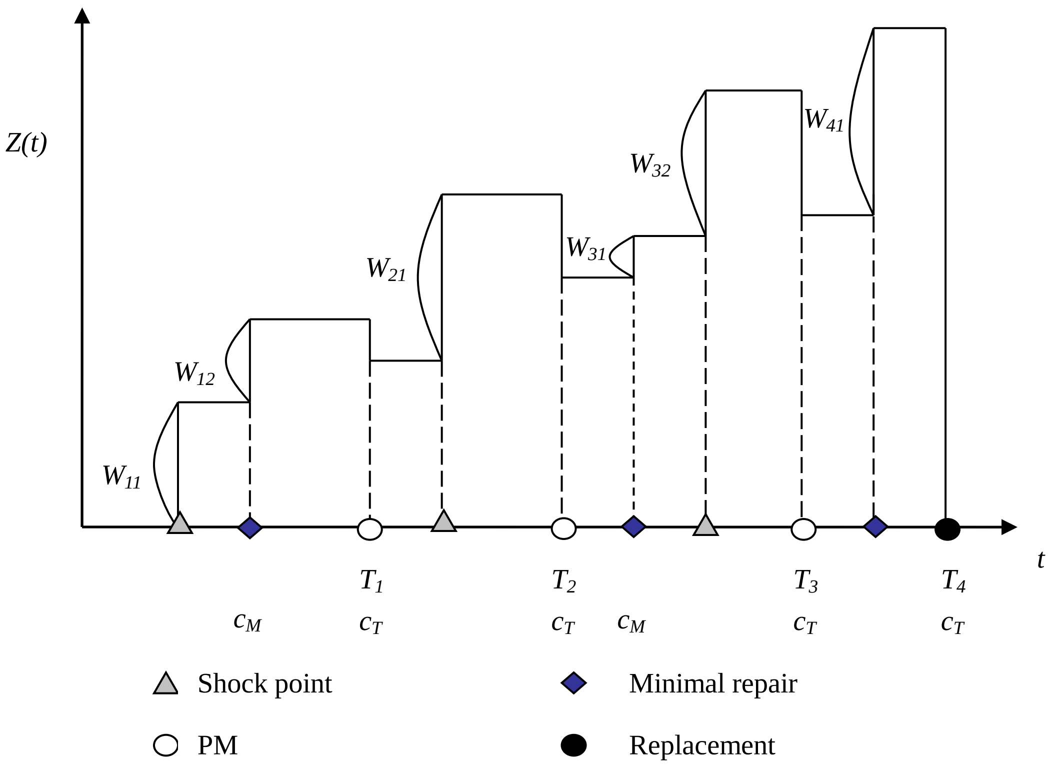


Figure 4.6: Imperfect PM Maintenance Model (Nakagawa (2007))

Here we introduce an improvement factor in imperfect PM: suppose that each PM can restore 100(1-a) % (0<a<1) of the total cumulative damage. If a=0, it is a perfect PM. Let Z_n the total damage at the end of period n, just prior to the nth PM, then the nth PM will restore the unit to aZ_n. If unit fails between the nth and the (n+1)st PM, it undergoes the minimum repair. However, it is assumed that the total damage is not reduced by the minimum repair and the repair time required for minimum repair is negligible. Thus, we have the relation

$$Z_n = aZ_{n-1} + \zeta T + \sum_{j=1}^{N_n} W_{nj} \tag{4.15}$$

Let L the critical the allowable degradation threshold determined by the different between system initial resistance and system failure critical level. Also assume c_T the cost associated with each PM, c_N the cost associated with replacement at Nth PM, and c_M the cost of minimum repair, with $c_N > c_T > c_M$ holding. Then the total cost in period n is

$$c(n) = c_T + c_M \sum_{j=1}^{N_n} P(aZ_{n-1} + \zeta T + W_{n1} + W_{n2} + ... + W_{nj} < L) \tag{4.16}$$

Every replacement at Nth PM can be viewed as a renewal process. According to the theory of renewal reward process, the cost rate can be expressed as E(C)/E(T). However, it is difficult to take the expectation for the total cost in this two-process cumulative damage model.

To simplify the problem, we assume if x is the magnitude of the total damage from degradation and shocks, the system fails with a probability of $p(x) = 1 - e^{-\theta x}$, which is increasing in x from 0 to 1. Assume $G^*(x)$ be the Laplace-Stieltjes transform of

$G(x)$, defined by $G^*(\theta) = \int_0^\infty e^{-\theta x} dG(x)$.

Therefore, the total cost in period n is given by

$$c(n) = c_T + c_M \sum_{j=1}^{N_n} p(aZ_{n-1} + \zeta T + W_{n1} + W_{n2} + ... + W_{nj}) \tag{4.17}$$

Similarly, the total cost in period N is

$$c(N) = c_N + c_M \sum_{j=1}^{N_N} p(aZ_{N-1} + \zeta T + W_{N1} + W_{N2} + ... + W_{Nj}) \tag{4.18}$$

By taking the expectation for the cost in period n, we have

$$\begin{aligned} E[c(n)] &= c_T + c_M E\left\{\sum_{j=1}^{N_n} p(aZ_{n-1} + \zeta T + W_{n1} + W_{n2} + ... + W_{nj})\right\} \\ &= c_T + c_M \sum_{i=1}^{\infty} \Pr\{N_n = i\} \times \sum_{j=1}^{i} E\left\{1 - \exp[-\theta(aZ_{n-1} + \zeta T + W_{n1} + W_{n2} + ... + W_{nj})]\right\} \end{aligned} \tag{4.19}$$

Let $B_n^*(\theta) = E\{\exp(-\theta Z_n)\}$

$$\begin{aligned} &E\left\{1 - \exp[-\theta(aZ_{n-1} + \zeta T + W_{n1} + W_{n2} + ... + W_{nj})]\right\} \\ &= 1 - B_{n-1}^*(\theta a) F^*(\theta T)[G^*(\theta)]^j \end{aligned} \tag{4.20}$$

Thus, because Nn has a homogeneous Poisson distribution with rate λ, we have

$$E[c(n)] = c_T + c_M \left[\lambda T - B_{n-1}^*(\theta a) F^*(\theta T) \frac{G^*(\theta)}{1 - G^*(\theta)} \left\{1 - e^{-\lambda T[1 - G^*(\theta)]}\right\}\right] \tag{4.21}$$

It remains to determine $B_{n-1}^*(\theta a)$.

We have

$$\begin{aligned} aZ_{n-1} &= a^2 Z_{n-2} + a\zeta T + a\sum_{j=1}^{N_{n-1}} W_{nj} = \sum_{j=1}^{n-1} a^j (\zeta T + \sum_{i=1}^{N_j} W_{ji}) \\ &= \zeta T \frac{a - a^n}{1 - a} + \sum_{j=1}^{n-1}\left(a^j \sum_{i=1}^{N_j} W_{ji}\right) \end{aligned} \tag{4.22}$$

so that,

$$B_{n-1}^{*}(\theta a)=E\{\exp(-\theta a Z_{n-1})\}=E\left[\exp\left\{-\theta\left[\zeta T\frac{a-a^{n}}{1-a}+\sum_{j=1}^{n-1}\left(a^{n-j}\sum_{i=1}^{N_j}W_{ji}\right)\right]\right\}\right]$$
$$=F^{*}(\theta\frac{a-a^{n}}{1-a}T)\exp\left\{-\lambda T\sum_{j=1}^{n-1}\left[1-G^{*}(\theta a^{j})\right]\right\} \tag{4.23}$$

Finally, we obtain the expectation of the total cost in period n

$$E[c(n)]=c_T+c_M\left[\begin{array}{l}\lambda T-\dfrac{G^{*}(\theta)}{1-G^{*}(\theta)}F^{*}(\theta T)F^{*}(\theta\dfrac{a-a^{n}}{1-a}T)\\ \times\exp\left\{-\lambda T\sum_{j=1}^{n-1}\left[1-G^{*}(\theta a^{j})\right]\right\}\left\{1-e^{-\lambda T[1-G^{*}(\theta)]}\right\}\end{array}\right] \tag{4.24}$$

Similarly, we can get the expectation for the total cost in period N.

Therefore, the expected cost rate until replacement is

$$c(N,T)=\frac{\sum_{n=1}^{N-1}E\{c(n)\}+E\{c(N)\}}{NT}$$
$$=\frac{(N-1)c_T+c_N+c_M\left\{\begin{array}{l}\sum_{n=1}^{N}\lambda T-\dfrac{G^{*}(\theta)}{1-G^{*}(\theta)}F^{*}(\theta T)\left\{1-e^{-\lambda T[1-G^{*}(\theta)]}\right\}\times\\ \sum_{n=1}^{N}\left[F^{*}(\theta\dfrac{a-a^{n}}{1-a}T)\exp\left\{-\lambda T\sum_{j=1}^{n-1}\left[1-G^{*}(\theta a^{j})\right]\right\}\right]\end{array}\right\}}{NT} \tag{4.25}$$
$$=\lambda c_M+\frac{(N-1)c_T+c_N-c_M\dfrac{G^{*}(\theta)}{1-G^{*}(\theta)}F^{*}(\theta T)B_N(T)}{NT}$$

where

$$B_N(T)=\left\{1-e^{-\lambda T[1-G^{*}(\theta)]}\right\}\sum_{n=1}^{N}\left[F^{*}(\theta\frac{a-a^{n}}{1-a}T)\exp\left\{-\lambda\phi_n T\right\}\right]$$
$$\phi_1=0,\phi_n=\sum_{j=1}^{n-1}\left[1-G^{*}(\theta a^{j})\right] \tag{4.26}$$

To simplify the formula, we define a function

$$Q_n(T)=c(n)-c_M\frac{G^{*}(\theta)}{1-G^{*}(\theta)}F^{*}(\theta T)F^{*}(\theta\frac{a-a^{n}}{1-a}T)\left\{1-e^{-\lambda T[1-G^{*}(\theta)]}\right\}\exp\left\{-\lambda T\phi_n\right\} \tag{4.27}$$

where $c(1)=c_N$, and $c(n)=c_T\,(n=2,3,...,N)$. Then

$$c(N,T) = \lambda c_M + \frac{1}{NT}\sum_{n=1}^{N} Q_n(T) \tag{4.28}$$

The optimization function is a mixed integer nonlinear programming (MINP) problem. The Numerical Nonlinear Global Optimization in Mathematica 7.0 provides the function NMinimize which implements several algorithms for finding global optimal solutions for constrained nonlinear function, which is not differentiable or continuous. Because there are integer variable to determine in this problem, differential evolution method will be employed. For everything else nonlinear, Nelder-Mead method is used.

4.4.2 Optimum Policies

In this section, we will try to find out

- optimum *N**(*T*) that minimizes *C*(*N**, *T*) for a fixed *T* by differential evolution
- optimum *T**(*N*) that minimizes *C*(*N*, *T**) for a fixed *N* by Nelder-Mead method
- optimum policy pair (*N**, *T**) by differential evolution

Before deriving the optimal policies, we assume that $G(x) = 1 - e^{-\mu x}$, $F(x) = 1 - e^{-\beta x}$ and the corresponding Laplace transform is $G^*(\theta) = \mu/(\theta + \mu)$, $F^*(\theta) = \beta/(\beta + \theta)$, where $\mu = 0.9$, $\theta = 0.1$ and $\beta = 10$. The Poisson process shock has an occurrence $\lambda = 0.35$. Let $c_M = 1$ and $c_T/c_N = 1 - a$.

4.4.2.1 Optimum Number *N**(*T*)

The optimum *N**(*T*) is obtained in order to minimize *C*(*N**, *T*) for a fixed *T*=25 by differential evolution method in Mathematica 7.0. The optimum value *N**(*T*) and expected cost rate for *a*=0.1-0.9 and $c_N/c_M = 3,5,8$ when *T*=25 are indicated in

Table 4.3. With the result from $c_N/c_M = 3$, we can observe that the optimum value increases as the variable *a* increases. However, this is not always the situation. When observing the results from $c_N/c_M = 5,8$, the optimum number is decreasing first and then goes to infinite. Therefore, the result indicates that $N^*(T)$ is not monotonically increasing with respect to *a*. When *a* is small, the total damage restore greatly by the imperfect PM and the system prefer to undergo only PM rather than replacement.

Table 4.3: Optimum number $N^*(T)$ and expected cost rate $C(N^*, T)$ when T=25

a	c_N/c_M=3		c_N/c_M=5		c_N/c_M=8	
	$N^*(T)$	$C(N^*,T)$	$N^*(T)$	$C(N^*,T)$	$N^*(T)$	$C(N^*,T)$
0.9	2	0.294463	3	0.336913	6	0.377654
0.8	2	0.297247	3	0.345052	5	0.396764
0.7	2	0.299680	2	0.351680	4	0.413643
0.6	2	0.301719	2	0.357719	4	0.428695
0.5	1	0.302056	2	0.363310	5	0.442167
0.4	1	0.302056	2	0.368392	∞	0.453014
0.3	1	0.302056	2	0.372896	∞	0.462646
0.2	1	0.302056	2	0.376740	∞	0.474082
0.1	1	0.302056	∞	0.379258	∞	0.487283

4.4.2.2 Optimum Number $T^*(N)$

The optimum $T^*(N)$ is obtained in order to minimize $C(N, T^*)$ for a fixed N by Nelder Mead method in Mathematica 7.0. The optimum time $T^*(N)$ and expected cost rate $C(N, T^*)$ for c_N/c_M=3, a=0.5 are obtained when N varies from 1 to 8. In Table 4.4 we obtain the optimum time $T^*(N)$ and expected cost rate $C(N, T^*)$ for c_N/c_M=3,

a=0.5 when N varies from 1 to 8. Comparing with various N from1 to 8, it seems that when N=1 and $T^*(N)$ =30.0427, the expected cost rate is minimized, that is $C(N, T^*)$=0.299986. It means just replacement is necessary to be performed instead of imperfect PM.

Table 4.4: Optimum time $T^*(N)$ and expected cost rate $C\ (N, T^*)$ when c_N/c_M=3, a=0.5

N	$T^*(N)$	$C(N, T^*)$
1	30.0427	0.299986
2	22.1375	0.302505
3	19.3886	0.307938
4	17.9690	0.312278
5	17.0971	0.315472
6	16.5060	0.317814
7	16.0789	0.319555
8	15.7562	0.320878

4.4.2.3 Optimum Pair (N^*, T^*)

The optimum pair (N^*, T^*) and minimum expected cost rate are obtained by differential evolution method in Mathematica 7.0 when c_N/c_M=3 and a=0.5. In Figure 4.7, we can see the three-dimension curve for various policy pairs (N, T; C).

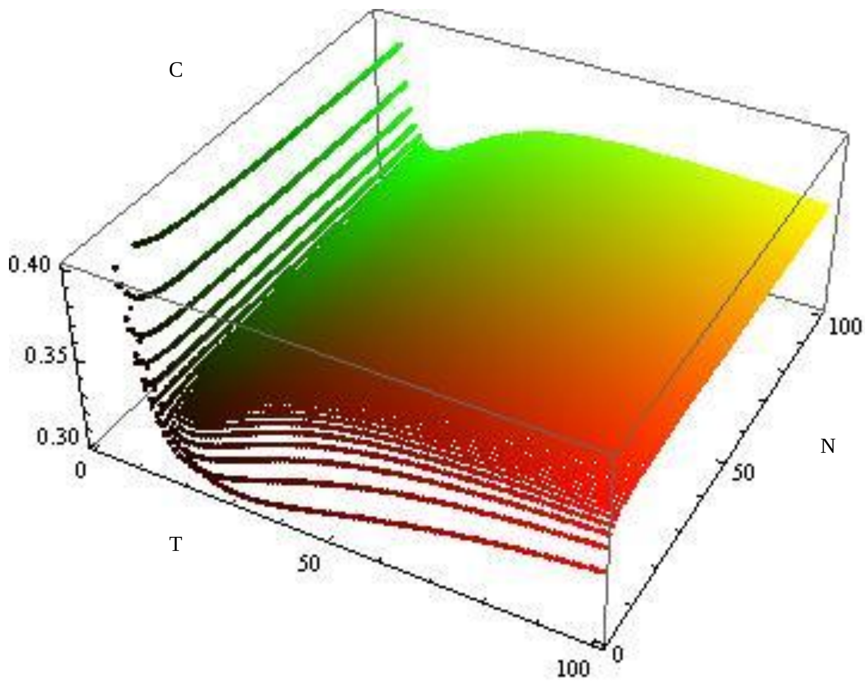


Figure 4.7: Policy pair (*N*, *T*) and expected cost rate

In order to avoid local minimum point, different scaling factor *s* in NMinimize option has been tested. Finally, the scaling factor is set to be equal to 0.1. Table 4.5 shows the first 5-step and last 5-step results from the differential evolution method in Mathematica 7.0. The results indicate that the policy pair of $N^*=1$, $T^*=30.0427$ gives us the minimum expected cost rate $C(T^*, N^*)=0.299986$, which is also consistent with the result from Table 4.4.

Table 4.5: Results from differential evolution algorithm when $c_N/c_M=3$, $a=0.5$

N	*T*	*C*
3	2.9524	0.750638
3	3.0763	0.724778
3	2.9378	0.753839
2	3.0122	0.815242
2	3.0122	0.815242
⋮	⋮	⋮

1	25.5918	0.301545
1	28.4421	0.300158
1	29.7933	0.299989
1	30.0427	0.299986
1	30.0427	0.299986

4.5 Conclusion

In this Chapter, a two-process cumulative combination model of competing failure between degradation and random shocks is introduced with additive and multiplicative degradation path. After that, two numerical examples with sensitivity analysis are used to illustrate the combination model. Finally, based on the definition of the model, an imperfect maintenance policy is put forward to obtain the optimum pair (*N**, *T**) by numerical methods.

Some extensions for this model are as follows:

First, in situations where the initial resistance R_0 is unknown, the reliability function for the multiplicative model can be rewritten as

$$R_T(t) = \int_{R_0=l_0}^{\infty} \left\{ \sum_{n=1}^{\infty} \frac{\exp(-\lambda t)(\lambda t)^n}{n!} \int_0^{R_0-R_l} F_x\left(\frac{h}{\eta(t,\theta)}\right) dG^{(n)}(R_0 - R_l - h) \right\} f_{R_0}(r_0) dr_0 \qquad (4.29)$$

where $f_{R_0}(r_0)$ is the probability density function of $R_0 \geq l_0$.

Second, the assumption of non-homogeneous Poisson process with rate function $\lambda(t,k)$ as the model for random shock, where k denotes the system status, is meaningful in real world, because the worse the system condition will make the system more vulnerable to the failure due to random shocks. Therefore, the reliability function for the multiplicative model can be rewritten as

$$R_T(t) = \sum_{n=1}^{\infty} \frac{\exp(-\int_0^t \lambda(\tau,k)d\tau)(\int_0^t \lambda(\tau,k)d\tau)^n}{n!} \int_0^{R_0-R_l} F_x(\frac{h}{\eta(t,\theta)})dG^{(n)}(R_0 - R_l - h) \quad (4.30)$$

How to define the *k* will be further investigated.

Chapter 5

A Multi-objective Optimization of Imperfect Preventive Maintenance Policy for Cumulative Competing Risk Systems with Hidden Failure

ACRONYMS

GA	Genetic Algorithm
IMCDM	Integrated Multi-Criteria Decision-Making
PM	Preventive Maintenance
NSGA	Non-dominated Sorting Genetic Algorithm
TSM	Technical Specifications and Maintenance
RAM	Reliability, Availability and Maintenance
TI	Testing Intervals
STI	Surveillance Test Interval

NOTATION

$v(t)$	occurring rate of random shock with nonhomogeneous Poisson Process
$q(t)$	nonfatal shock probability
$N(t)$	random shock number till time t
w_i	individual random shock magnitude
T_v	system degradation level
a	initial degradation level
δ	degradation rate
S	degradation failure threshold, exponentially distributed with μ
T_d	time associated with degradation failure
T_s	time associated with random shock failure
T_r	time associated with replacement

T_p	time associated with imperfect PM
T_i	length of the *i*th imperfect PM interval
β	quasi-renewal coefficient of imperfect PM interval
I_i	cumulative time interval till the *i*th imperfect PM
α	imperfect PM factor
C_s	cost associated with imperfect PM action following random shock failure
C_d	cost associated with imperfect PM action following degradation failure
C_w	cost associated with imperfect PM action without any type of failure
C_N	cost associated with the replacement
C_M	penalty cost per unit time associated with the idle time during the two consecutive imperfect PM
R_0	target reliability low bound

5.1 Introduction

Many systems or their components are subjected to the competing risks of degradation processes and random shocks. A battery supplies electric power by chemical reaction. It gradually weakens through usage, and becomes useless once the chemicals in the battery are exhausted. Also, overheating or over-voltage can cause damage in the battery.

On dealing with competing risks of the degradation processes and random shocks, some of the present papers assume that they are *s*-independent on each other, such as Li and Pham (2005a,b), Deloux et al. (2009), Mori and Ellingwood (1994), van Noortwijk et al. (2007), Lehmann (2009), and Ciampoli (1998). However, the dynamics of the dependent structure between these two processes are of importance, and should not be neglected (Singupurwalla (1995)). On the one hand, the degradation process will make the system more vulnerable to the random shocks. On the other hand, the random shocks will accelerate the system degradation processes by two types of impacts: sudden increment jump, and degradation rate acceleration. Therefore, there is also some interest in the correlation between these two kinds of processes as a function of time.

Kharoufeh et al. (2006) utilize the Laplace-Stieltjes transform to explicitly derive the lifetime distribution, as well as the limiting availability for a periodically inspected single-unit system with hidden failure, which is subject to the degradation wear due to its random environment characterized by a continuous Markov chain, and random shocks modulated by a homogeneous Poisson process. Ebrahimi (2001) proposes a general stochastic model to estimate the reliability of systems in terms of a deterioration process with covariates. Fan et al. (2000) consider the one-component system that suffers from non-homogeneous Poisson process shocks. In the system, aging will increase the magnitude of shock sizes, thereby resulting in a larger fatal probability. After that, an extension of that shock model to a multi-component system is examined. Satow and Kawai (2010) present an imperfect inspection with upper and lower inspection threshold for a bivariate failure distribution.

Many literature papers focus on maintenance optimization based on one objective. Vaurio (1995) develops advanced models to the general recursive equations for the availability and mission-failure probability of the standby structure system by considering different durations for testing and repairs, as well as various failure types, including start-up, standby and during mission failures, and two additional human errors. Zequeria and Berenguer (2006) study a maintenance policy considering three types of actions: minimal repairs, PM, and replacement. They studied a system with two dependent competing failure modes of maintenance and non-maintenance by minimizing the system cost rate during an infinite time. Cepin (2002) determines the optimal scheduling to improve the safety of equipment outages in nuclear power plants by minimizing the mean value of the selected time-dependent risk measure. Zhu et al. (2010) examine the maintenance model for a competing risk of degradation and sudden failure in which the unit is renewed when it reaches a predetermined degradation level, or comes to a sudden failure within the limit of a certain degradation threshold.

In terms of the multi-objective approach, Martorell et al. (2005) propose a new integrated Multi-Criteria Decision-making (IMCDM) method to determine the parameters in the technical specifications and maintenance (TSM) of safety-related equipment using a multi-objective Genetic Algorithm (GA) based on the reliability, availability, and maintenance (RAM) criterion. Martorell et al. (2006) address the multi-objective problem of surveillance requirements at nuclear power plants with dependable variables of testing intervals (TI), and testing planning using a novel double-loop multiple objective evolutionary algorithms. In Quan et al. (2007), a new approach, which combines the preference with evolutionary algorithm by using utility theory to search the Pareto frontier rather than conducting a dominated Pareto search, was developed to find the optimal solutions for a multi-objective PM scheduling.

Sanchez et al. (2009) put forward a GA based approach using distribution free tolerance intervals to address a multi-objective optimization of an unavailability and cost model embedded within the uncertainty of the imperfect maintenance. Okasha and Frangopol (2009) consider two strategies of selecting maintenance actions, maintenance scheduling, maintenance structural components for optimization programs to design and construct structural systems in terms of system reliability, redundancy, and life-cycle costs as criterion in the multi-objective GA. However, the multi-objective optimization embedded within the imperfect maintenance for the dependent competing risks of degradation processes and random shocks is a blank topic of great interest.

This chapter proposes a dependent competing risk model for a deteriorating system subject to shock processes, and a maintenance model involving imperfect maintenance actions. We derive the mathematical models for the system expected cost rate and asymptotic unavailability of the maintained system, and present a multi-objective optimization based on GAs. Three optimizations with sensitivity analysis under various cases are performed to compare the optimization results. The assumptions of this chapter are listed as follows.

A. The competing risks of degradation wear and random shocks are *s*-dependent on each other. There are two kinds of random shocks: 1) a fatal shock will cause the system failure immediately, and 2) a nonfatal shock will increase the system virtual age by certain cumulative shock loading.

B. An improvement factor is used to simulate the imperfect PM by modulating the system critical threshold with a quasi-renewal process.

C. System failure is hidden; that is, the system failure will be only detected at the time of the scheduled maintenance or replacement.

D. The cost associated with imperfect PM following no failure, degradation failure, and random shock failure can be varied.

5.2 Mathematical Model

5.2.1 System Description

Suppose we are interested in the reliability analysis for a single-unit system subjected to the compound Poisson process shock of magnitude *w* with a common distribution, and nonhomogenous rate $v(t)$. It is reasonable to assume that the shock rate is a non-homogeneous Poisson process with increasing occurrence rate because the system is more vulnerable to the shocks with the aging of the system.

In addition, assume that the *i*th shock occurring at time t_i can cause immediate failure of the system with probability $p(t_i)$, called fatal shock. Otherwise, it increases the system degradation level by the shock magnitude w_i with probability $q(t_i) = 1 - p(t_i)$, called nonfatal shock. Based on the above assumption, the system degradation level at time *t* can be expressed as

$$T_v = a + \delta t + \sum_{i=1}^{N(t)} w_i \tag{5.1}$$

where $a \geq 0$ is the initial degradation level for the system at time zero, and δ is the degradation rate.

Because some of the components or units may be used or refurbished, the system's initial degradation level needs not to be zero. System failure occurs when the degradation level reaches a certain critical threshold *S*, a random variable exponentially distributed with parameter μ. Therefore, the probability for the system to survive is given by

$$\begin{aligned} R(t) &= \prod_{i=1}^{N(t)} q(t_i) P(T_v < S) \\ &= \prod_{i=1}^{N(t)} q(t_i) P(a + \delta t + \sum_{i=1}^{N(t)} w_i < S) \\ &= \prod_{i=1}^{N(t)} q(t_i) \exp\left[-\mu(a + \delta t + \sum_{i=1}^{N(t)} w_i)\right] \\ &= \exp\left[-\mu(a + \delta t) - \mu \sum_{i=1}^{N(t)} w_i + \sum_{i=1}^{N(t)} \ln q(t_i)\right] \end{aligned} \tag{5.2}$$

Refer to the proof of Theorem 1 in Cha and Finkelstein (2009). The survival function of the system, and the corresponding failure rate are given by

$$R(t) = \exp\left\{-\mu(a + \delta t) - \int_0^t v(x)dx + M_W(-\mu)\int_0^t q(x)v(x)dx\right\}, \tag{5.3}$$

and

$$h(t) = \mu\delta + v(t) - M_W(-\mu)q(t)v(t) \tag{5.4}$$

where M_W is the moment generating function for the random variable *w*.

In this competing risk system, there are two kinds of failure mechanisms: one is degradation failure, and the other is random shock failure. The two processes of degradation and random shocks are dependent on each other because the degradation process will receive the cumulative loading from the random shocks. To examine these two failure mechanisms more clearly, we analyze the two processes separately.

Under the first process of degradation without considering the fatal shocks, the system survives until *t* with the probability

$$\begin{aligned} R_d(t) &= P(T_v < S) \\ &= \exp\left\{-\mu(a+\delta t) - \int_0^t v(x)q(x)dx + M_W(-\mu)\int_0^t v(x)q(x)dx\right\} \end{aligned} \tag{5.5}$$

$$F_d(t) = 1 - \exp\left\{-\mu(a+\delta t) - \int_0^t v(x)q(x)dx + M_W(-\mu)\int_0^t v(x)q(x)dx\right\}, \tag{5.6}$$

and

$$h_d(t) = \mu\delta + v(t)q(t) - M_W(-\mu)v(t)q(t) \quad . \tag{5.7}$$

Under the second process of random shock without considering the degradation process, the system survives until t with the probability

$$\begin{aligned} R_s(t) &= \prod_{i=1}^{N(t)} q(t_i) \\ &= \exp(-\int_0^t v(x)dx + \int_0^t v(x)q(x)dx) \end{aligned} \tag{5.8}$$

$$F_s(t) = 1 - \exp(-\int_0^t v(x)dx + \int_0^t v(x)q(x)dx), \tag{5.9}$$

and

$$h_s(t) = v(t) - v(t)q(t) . \tag{5.10}$$

From the above, we can see that the hazard rate of the competing risk system is equal to the summation of the hazard rate of the two processes,

$$h(t) = h_s(t) + h_d(t) . \tag{5.11}$$

In this competing risk system, the degradation failure happens under the condition that the system degradation level exceeds the critical threshold, but no fatal shocks happen until time t. That failure happens with the probability distribution

$$\begin{aligned} F_{T_d}(t) &= P(T_d \le t) = \int_0^t R_s(u)dF_d(u) \\ &= \int_0^t \left\{ \begin{aligned} &\exp(-\int_0^u v(x)dx + \int_0^u v(x)q(x)dx) \\ &\times\left[\mu\delta + v(u)q(u) - M_W(-\mu)v(u)q(u)\right] \\ &\times\exp\left\{-\mu(a+\delta u) - \int_0^u v(x)q(x)dx + M_W(-\mu)\int_0^u v(x)q(x)dx\right\} \end{aligned} \right\} du \end{aligned} \tag{5.12}$$

.

In this competing risk system, the random shock occurs when there exists a fatal shock, but the system degradation level stays below the critical threshold until time *t*. That failure happens with the probability distribution

$$\begin{aligned} F_{T_s}(t) &= P(T_s \le t) = \int_0^t R_d(u)dF_s(u) \\ &= \int_0^t \left\{ \begin{array}{l} \exp\left\{ -\mu(a+\delta u) - \int_0^u v(x)q(x)dx + M_W(-\mu)\int_0^u v(x)q(x)dx \right\} \\ \times\left[v(u) - v(u)q(u) \right] \\ \times \exp(-\int_0^u v(x)dx + \int_0^u v(x)q(x)dx) \end{array} \right\} du \end{aligned} \tag{5.13}$$

5.2.2 Imperfect Preventive Maintenace

Based on the induction of the reliability and hazard rate for the dependent competing risk model, we present a multi-objective optimization imperfect PM policy with a common improvement factor and hidden failure. Two decision variables for the maintenance policy, one is the imperfect PM interval, and another is the number of imperfect PM until replacement, are determined by simultaneously maximizing the system availability and minimizing the system expected cost rate. The imperfect PM interval sequence is modulated by a decreasing quasi-renewal process to capture the aging effect of the system.

5.2.2.1 Maintenance Planning

Consider the following maintenance policy as shown in Figure 5.1:

- A sequence of imperfect PM is done at the end of time interval T_n $(n = 1, 2, ..., N-1)$.
- If the system fails between two consecutive PMs, it will remain in the non-functioning condition until the next imperfect PM.

- After an imperfect PM, the system is not as good as new, which means the maintenance is not perfect.
- A replacement is done at the end of time interval T_N before the system reliability drops to a very low point R_0, so the unit is as good as new after replacement.

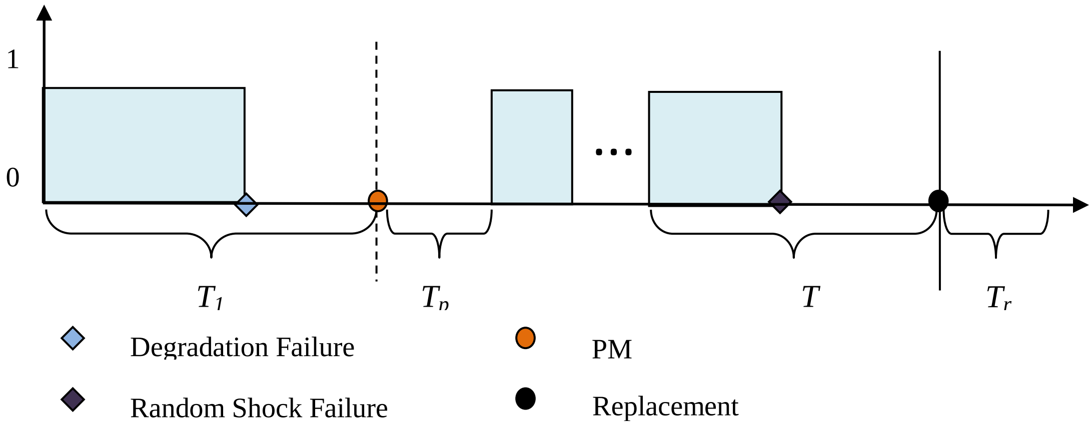


Figure 5.1: Imperfect PM Maintenance Model.

Because the system will be renewed once the replacement is performed, the time interval between two consecutive replacements is the renewal cycle time for the system. From the above relationship, it follows that the average accumulating imperfect PM interval for the system during one renewal cycle time is given by

$$I_N = \sum_{i=1}^{N} T_i = T_1 + \beta T_1 + \beta^2 T_1 + \beta^3 T_1 + \ldots + \beta^{N-1} T_1 = \frac{T_1(1-\beta^N)}{1-\beta} \quad (5.14)$$

Therefore, the decision variables in this maintenance policy are the PM interval T_1, and the PM number for replacement N.

5.2.2.2 Improvement Factor Method

Perfect maintenance or repair that assumes the system is as good as new after each PM or repair is not actually realistic. A more practical assumption is that, instead of perfect maintenance, an imperfect PM will restore the degradation process of the system to some younger system status between "as good as new" and "as bad as old". In the literatures, there are seven treatment methods for imperfect maintenance by Wang and Pham (2006a): (p, q) rule, ($p(t)$, $q(t)$) rule, improvement factor, virtual age, shock model, (α, β) rule or quasi-renewal process, and multiple (p, q) rule. To modulate the imperfect maintenance, an improvement factor is introduced into the model such that, upon each imperfect PM, the failure threshold of the system during the degradation process will be raised correspondingly such that the jump-up is proportional to the preceding threshold scaled by the improvement factor. In other words, the imperfect PM will improve the system "immunity" level.

Upon the first imperfect PM, it will restore the system critical threshold by a fraction of the immediately preceding one in the previous period. Therefore, the critical threshold in the 2nd period is indicated as S/α, $\alpha < 1$. Continuously, the critical threshold in the ith period equals

$$S_i(t) = \frac{S}{\alpha^{i-1}} \quad \text{for} \quad t \in (I_{i-1}, I_i]. \tag{5.15}$$

Because S is exponentially distributed with parameter μ, we have

$$S_i(t) \sim Exp(\mu\alpha^{i-1}) \quad \text{for} \quad t \in (I_{i-1}, I_i]. \tag{5.16}$$

It is obvious that we can obtain the reliability function and failure rate in the ith period as

$$R_i(t) = \exp\left\{-\mu\alpha^{i-1}(a+\delta t) - \int_0^t v(x)dx + M_W(-\mu\alpha^{i-1})\int_0^t q(x)v(x)dx\right\}, \tag{5.17}$$

and

$$h_i(t) = \mu\alpha^{i-1}\delta + v(t) - M_W(-\mu\alpha^{i-1})q(t)v(t) \tag{5.18}$$

for $t \in (I_{i-1}, I_i]$.

In addition, under the first process of degradation without considering the fatal shocks, the probability for the system to survive in (5.5), (5.6), and (5.7) is changed to

$$R_{di}(t) = \exp\left\{-\mu\alpha^{i-1}(a+\delta t) - \int_0^t v(x)q(x)dx + M_W(-\mu\alpha^{i-1})\int_0^t v(x)q(x)dx\right\}, \quad (5.19)$$

$$F_{di}(t) = 1-\exp\left\{-\mu\alpha^{i-1}(a+\delta t) - \int_0^t v(x)q(x)dx + M_W(-\mu\alpha^{i-1})\int_0^t v(x)q(x)dx\right\}, \quad (5.20)$$

and

$$h_{di}(t) = \mu\alpha^{i-1}\delta + v(t)q(t) - M_W(-\mu\alpha^{i-1})v(t)q(t) \quad (5.21)$$

for $t \in (I_{i-1}, I_i]$, respectively.

However, under the second process of random shock without considering the degradation, the probability for the system to survive remains the same.

Assume that the shock loading is exponentially distributed with parameter λ. Therefore we have

$$M_W(-\mu\alpha^{i-1}) = \frac{1}{1+\lambda\mu\alpha^{i-1}} \quad (5.22)$$

The reliability function and hazard function can be respectively simplified as

$$R_i(t) = \exp\left\{-\mu\alpha^{i-1}\delta(a+t) - \int_0^t v(x)dx + \frac{\int_0^t q(x)v(x)dx}{1+\lambda\mu\alpha^{i-1}}\right\}, \quad (5.23)$$

and

$$h_i(t) = \mu\alpha^{i-1}\delta + v(t) - \frac{q(t)v(t)}{1+\lambda\mu\alpha^{i-1}} \quad (5.24)$$

for $t \in (I_{i-1}, I_i]$.

Comparing the failure rate of the (*i*-1)st period with the *i*th period, we can see it clearly that the failure rate decreases after each imperfect PM because the failure rate is a decreasing function with variable *i*, and finally restores to the original system status upon each replacement, shown as in Figure 5.2.

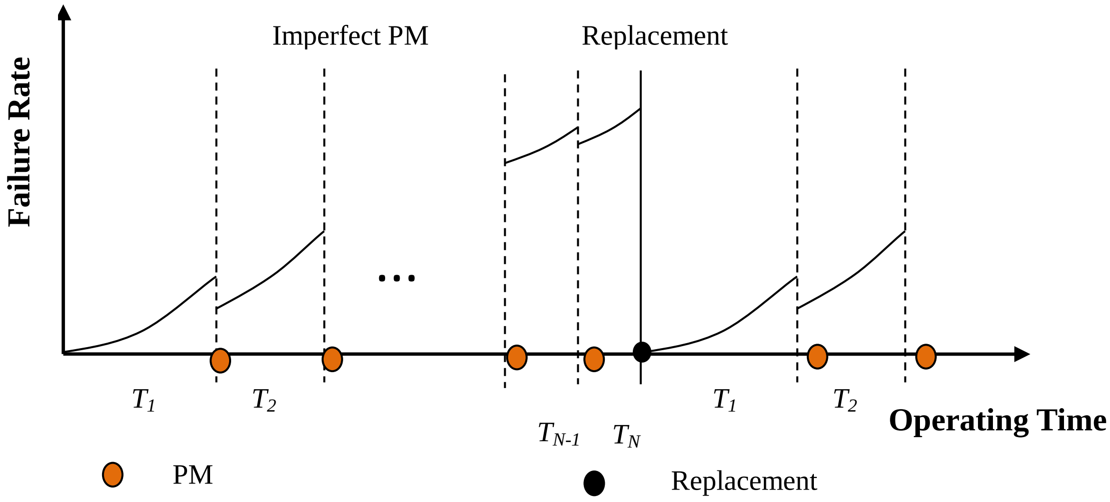


Figure 5.2: Failure Rate under the maintenance planning.

5.2.2.3 System Availability Optimization

In this periodically PM model, because the replacement that occurs at the *N*th period after *N*-1 imperfect PMs will restore the system to an "as good as new" condition, we can view the time interval between two consecutive replacements as the system renewal cycles to calculate the system availability.

The expected renewal cycle time is

$$E\left[CYCLE\right]=\frac{T_1(1-\beta^N)}{1-\beta}+(N-1)E[T_p]+E[T_r] \tag{5.25}$$

The expected up-time is

$$E\left[UP\right]=\frac{T_1(1-\beta^N)}{1-\beta}-E[\zeta] \tag{5.26}$$

where ζ is the cumulative idle time in the whole renewal cycle during the *N* period time.

Then $E[\zeta]$ is calculated as

$$E[\zeta]=\sum_{i=1}^{N}E\left[(I_i-T)I_{I_{i-1}<T\le I_i}\right]$$
$$=\sum_{i=1}^{N}\int_{I_{i-1}}^{I_i}(I_i-t)dF(t) \quad (5.27)$$

Therefore, we can obtain

$$E\left[UP\right]=\frac{T_1(1-\beta^N)}{1-\beta}-\sum_{i=1}^{N}\int_{I_{i-1}}^{I_i}(I_i-t)dF(t)$$
$$=\frac{T_1(1-\beta^N)}{1-\beta}-\sum_{i=1}^{N}\int_{I_{i-1}}^{I_i}\left\{\begin{array}{l}(I_i-t)\left[\mu\alpha^{i-1}\delta+v(t)-M_W(-\mu\alpha^{i-1})v(t)q(t)\right]\\ \times\exp\left[-\mu\alpha^{i-1}(a+\delta t)-\int_0^t v(x)dx+M_W(-\mu\alpha^{i-1})\int_0^t v(x)q(x)dx\right]\end{array}\right\}dt \quad (5.28)$$

The system asymptotic availability can be easily derived as

$$A(N,T_1)=\frac{\frac{T_1(1-\beta^N)}{1-\beta}-\sum_{i=1}^{N}\int_{I_{i-1}}^{I_i}\left\{\begin{array}{l}(I_i-t)\left[\mu\alpha^{i-1}\delta+v(t)-M_W(-\mu\alpha^{i-1})v(t)q(t)\right]\\ \times\exp\left[-\mu\alpha^{i-1}(a+\delta t)-\int_0^t v(x)dx+M_W(-\mu\alpha^{i-1})\int_0^t v(x)q(x)dx\right]\end{array}\right\}dt}{\frac{T_1(1-\beta^N)}{1-\beta}+(N-1)E[T_p]+E[T_r]}$$

(5.29)

or

$$\bar{A}(N,T_1)=\frac{(N-1)E[T_p]+E[T_r]+\sum_{i=1}^{N}\int_{I_{i-1}}^{I_i}\left\{\begin{array}{l}(I_i-t)\left[\mu\alpha^{i-1}\delta+v(t)-M_W(-\mu\alpha^{i-1})v(t)q(t)\right]\\ \times\exp\left[-\mu\alpha^{i-1}(a+\delta t)-\int_0^t v(x)dx+M_W(-\mu\alpha^{i-1})\int_0^t v(x)q(x)dx\right]\end{array}\right\}dt}{\frac{T_1(1-\beta^N)}{1-\beta}+(N-1)E[T_p]+E[T_r]}$$

(5.30)

5.2.2.4 Expected Cost Model

According to renewal reward process modeling, the expected cost rate can be expressed as the ratio of expected total cost in one cycle time to the expected duration of a cycle (Pham and Wang (2000)):

$$c(N,T_1)=\frac{E(C)}{E(T)}.$$

There are three kinds of possibilities in each PM interval: the failure may come from a fatal shock failure, it may come from a degradation failure, or the system can experience no failure. In the previous section, we assume that the imperfect PM times for each situation are the same because we assume that, no matter whether a failure happens or not, the regular inspection and routine maintenance will be needed. However, the imperfect PM cost associated with different system situations may vary. Without loss of generality, we can assume that $C_N > C_d > C_s > C_w$.

Based on the induction in Section 5.2.1, during the time period of $t \in (I_{i-1}, I_i]$, a degradation failure happens with the probability

$$P(I_{i-1} \le T_d \le I_i) = \int_{I_{i-1}}^{I_i} R_s(t) dF_d(t)$$
$$= \int_{I_{i-1}}^{I_i} \left\{ \begin{array}{l} \exp(-\int_{I_{i-1}}^{t} v(x)dx + \int_{I_{i-1}}^{t} v(x)q(x)dx) \\ \times\left[\mu\alpha^{i-1}\delta + v(t)q(t) - M_W(-\mu\alpha^{i-1})v(t)q(t)\right] \\ \times\exp\left\{-\mu\alpha^{i-1}(a+\delta t) - \int_0^t v(x)q(x)dx + M_W(-\mu\alpha^{i-1})\int_0^t v(x)q(x)dx\right\} \end{array} \right\} dt \quad (5.31)$$

And during the time period of $t \in (I_{i-1}, I_i]$, a random shock failure happens with the probability

$$P(I_{i-1} \le T_s \le I_i) = \int_{I_{i-1}}^{I_i} R_d(t) dF_s(t)$$
$$= \int_{I_{i-1}}^{I_i} \left\{ \begin{array}{l} \exp\left\{-\mu\alpha^{i-1}(a+\delta t) - \int_0^t v(x)q(x)dx + M_W(-\mu\alpha^{i-1})\int_0^t v(x)q(x)dx\right\} \\ \times\left[v(t) - v(t)q(t)\right] \\ \times\exp(-\int_{I_{i-1}}^{t} v(x)dx + \int_{I_{i-1}}^{t} v(x)q(x)dx) \end{array} \right\} dt . \quad (5.32)$$

The expected PM number following random shock failures during the one cycle time is

$$
\begin{aligned}
E[N_d] &= \sum_{i=1}^{N} P(I_{i-1} \le T_d \le I_i) \\
&= \sum_{i=1}^{N} \int_{I_{i-1}}^{I_i} \left\{ \begin{array}{l} \exp(-\int_{I_{i-1}}^{t} v(x)dx + \int_{I_{i-1}}^{t} v(x)q(x)dx) \\ \times\left[\mu\alpha^{i-1}\delta + v(t)q(t) - M_W(-\mu\alpha^{i-1})v(t)q(t) \right] \\ \times\exp\left\{ -\mu\alpha^{i-1}(a+\delta t) - \int_0^t v(x)q(x)dx + M_W(-\mu\alpha^{i-1})\int_0^t v(x)q(x)dx \right\} \end{array} \right\} dt
\end{aligned} \tag{5.33}
$$

The expected PM number following degradation failures during the one cycle time is

$$
\begin{aligned}
E[N_s] &= \sum_{i=1}^{N} P(I_{i-1} \le T_s \le I_i) \\
&= \sum_{i=1}^{N} \int_{I_{i-1}}^{I_i} \left\{ \begin{array}{l} \exp\left\{ -\mu\alpha^{i-1}(a+\delta t) - \int_0^t v(x)q(x)dx + M_W(-\mu\alpha^{i-1})\int_0^t v(x)q(x)dx \right\} \\ \times\left[v(t) - v(t)q(t) \right] \\ \times\exp(-\int_{I_{i-1}}^{t} v(x)dx + \int_{I_{i-1}}^{t} v(x)q(x)dx) \end{array} \right\} dt
\end{aligned} \tag{5.34}
$$

The expected PM numbers following no failures during the one cycle time is

$$E[N_w] = (N-1) - E[N_s] - E[N_d] \tag{5.35}$$

The expected cost rate in the whole renewal cycle can be expressed as

$$c(N,T_1) = \frac{C_M E\left[\zeta\right] + C_s E[N_s] + C_d E[N_d] + C_w E[N_w] + C_N}{\dfrac{T_1(1-\beta^N)}{1-\beta} + (N-1)E[T_p] + E[T_r]} \tag{5.36}$$

or

$$c(N,T_1)=\frac{\left\{\begin{array}{l} C_M\sum_{i=1}^{N}\int_{I_{i-1}}^{I_i}\left\{\begin{array}{l}(I_i-t)\left[\mu\alpha^{i-1}\delta+v(t)-M_W(-\mu\alpha^{i-1})v(t)q(t)\right]\\ \times\exp\left[-\mu\alpha^{i-1}(a+\delta t)-\int_0^t v(x)dx+M_W(-\mu\alpha^{i-1})\int_0^t v(x)q(x)dx\right]\end{array}\right\}dt\\ +C_s\sum_{i=1}^{N}\int_{I_{i-1}}^{I_i}\left\{\begin{array}{l}\exp\left\{-\mu\alpha^{i-1}(a+\delta t)-\int_0^t v(x)q(x)dx+M_W(-\mu\alpha^{i-1})\int_0^t v(x)q(x)dx\right\}\\ \times\left[v(t)-v(t)q(t)\right]\\ \times\exp(-\int_{I_{i-1}}^{t}v(x)dx+\int_{I_{i-1}}^{t}v(x)q(x)dx)\end{array}\right\}dt\\ +C_d\sum_{i=1}^{N}\int_{I_{i-1}}^{I_i}\left\{\begin{array}{l}\exp(-\int_{I_{i-1}}^{t}v(x)dx+\int_{I_{i-1}}^{t}v(x)q(x)dx)\\ \times\left[\mu\alpha^{i-1}\delta+v(t)q(t)-M_W(-\mu\alpha^{i-1})v(t)q(t)\right]\\ \times\exp\left\{-\mu\alpha^{i-1}(a+\delta t)-\int_0^t v(x)q(x)dx+M_W(-\mu\alpha^{i-1})\int_0^t v(x)q(x)dx\right\}\end{array}\right\}dt\\ +C_w\left[(N-1)-E[N_s]-E[N_d]\right]+C_N\end{array}\right\}}{\frac{T_1(1-\beta^N)}{1-\beta}+(N-1)E[T_p]+E[T_r]} \tag{5.37}$$

5.2.2.5 Maintenance Optimization

Maintenance optimization is a systematic process that attempts to balance the maintenance requirements and resources in order to identify the appropriate maintenance periodicity and technique. This approach should be conducted to achieve multiple maintenance targets, such as safety control, component reliability, system availability, and costs. Generally, the maintenance optimization system has two objectives: maximizing system availability (A_S), and minimizing system cost (C_S). The aim of the maintenance optimization problem in this chapter is to determine two decision variables: the number of imperfect PMs until replacement *N*, and the imperfect PM interval T_1. There are three cases to solve the maintenance optimization problem.

<u>Case I:</u> The general formulation to maximizing the system availability is

$Max\ A_s(N, T_1)$

Subject to $C_s \leq C_{\max}$.

Case II: The general formulation to minimizing the system cost rate is

$Min\ C_s(N, T_1)$

Subject to $A_s \geq R_s$.

Case III: The formulation of the multi-objective model to simultaneously maximize availability and minimize cost is

$Max\ A_s(N, T_1)$ and $Min\ C_s(N, T_1)$

Subject to $A_s \geq R_s$

$C_s \leq C_{\max}$

where the symbol R_s denotes the target system availability, and $C_{\max}$ is the tolerable maximum cost.

In this chapter, one assumption is that replacement is done at the end of time interval T_N before the system reliability drops to a very low point R_0. Therefore, without considering the constraints of the target system availability and tolerable maximum cost, the system reliability at the end of cycle time I_N should be kept above R_0:

$Max\ A_s(N, T_1)$ and $Min\ C_s(N, T_1)$

Subject to $R(I_N) \geq R_0$.

The optimization Toolboxes in Matlab R2010a are used to solve the maintenance optimization problems in these three cases. Under Case I and Case II, the optimization problem is a mixed-integer linear program. The "ga" function in Matlab is employed to optimize the system cost rate constrained to availability or system availability constrained to system cost by GA, which is a mixed-integer linear programming problem. Under the third formula, the "gamultiobj" function in Matlab is utilized to optimize the multi-objective function by using the controlled elitist algorithm, called the fast elitist Non-dominated Sorting Genetic Algorithm (NSGA-II).

5.3 Numerical Example

Assume that the shock loading is exponentially distributed with parameter λ. Then we have

$$M_W(-\mu\alpha^{i-1}) = \frac{1}{1+\lambda\mu\alpha^{i-1}}, \tag{5.38}$$

where $\mu = 0.01$, $\lambda = 5$, and $\alpha = 0.8$.

Let $v(x) = \theta x^{0.5}$, and $q(x) = \exp(-\eta x)$ with $\theta = 0.02$, $\eta = 0.015$, and quasi-renewal process coefficient for imperfect PM interval $\beta = 0.8$. The system replacement time $T_r = 0.2$, replacement cost $C_N = 250$, and the idle time penalty cost $C_m = 300$. Both the imperfect PM time and cost are proportional to the restoration degree α. Given that each percentage of the system restoration calls for extra time 0.001, and extra cost 3, we have the imperfect PM time

$T_p = 0.1 + 0.001 \times (1-\alpha) \times 100$,

and imperfect PM cost

$C_p = C_0 + 3 \times (1-\alpha) \times 100$,

where $C_0 = 40$ with PM without failure, C_0=60 with PM following random shock failure, and C_0=80 with PM following degradation failure.

Replacement should be done before the system is completely out of functioning, that is $R(I_N) \geq 0$. Because some decision variable pairs [*N*, *T1*] do not satisfy with that constraint, we assume that the system unavailability and cost rate for the corresponding infeasible solutions each equal some large value, such as $\overline{A} = 1$ and C=100, during the numerical analysis.

In Figures 5.3 and 5.4, we can examine the curve behavior of the system unavailability and cost rate by changing decision variable $N \in [1, 50]$ and $T_1 \in [1, 100]$ both with an increment step of 1. The minimized system unavailability is 0.0609 with a cost rate of 43.8741 when *N*=3, and T_1=5, while the minimized system cost rate is 38.0180 with an unavailability of 0.0723 when *N*=4, and T_1=8.

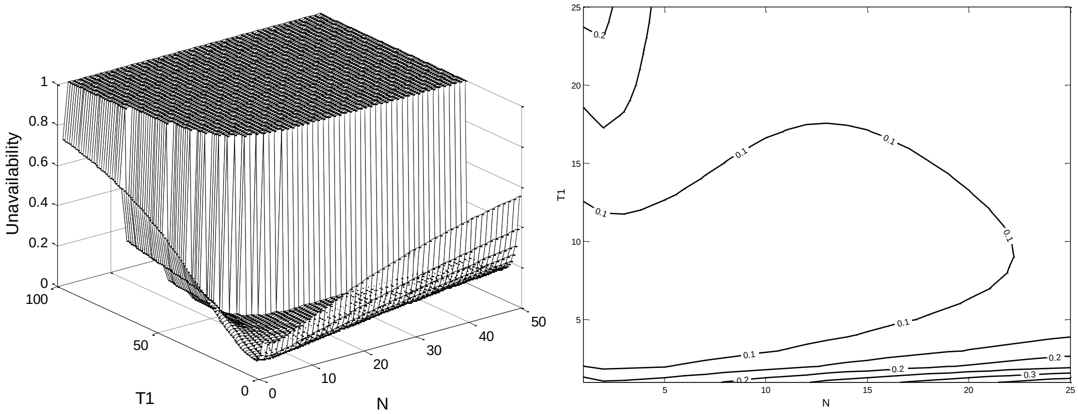


Figure 5.3: *N*-*T1*-Unavailability 3-D Plotting.

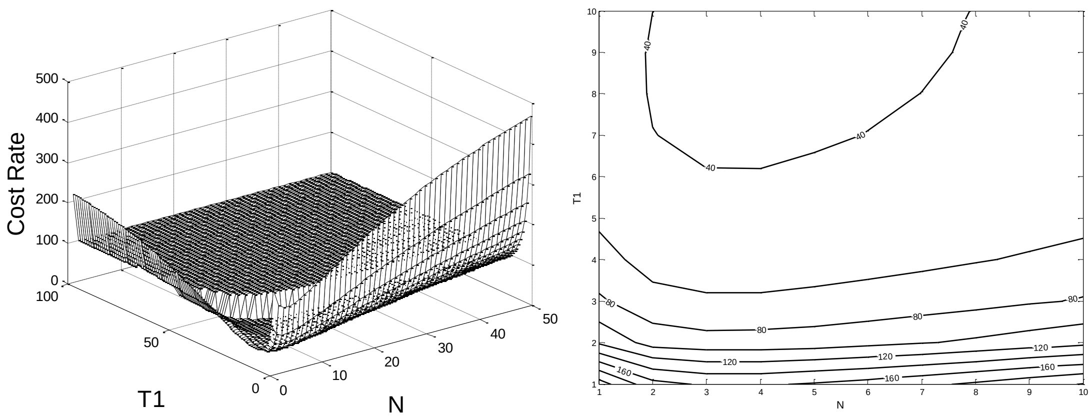


Figure 5.4: *N*-*T1*-Cost 3-D Plotting.

CASE I: Minimizing the System Unavailability

With the help of Matlab's optimization toolbox, GA is used to find the solution of this one-objective optimization problem in minimizing the system unavailability. The best solution of minimizing system unavailability is 0.0609 when the corresponding optimum pair of decision variables [N, T_1] is [3, 4.8829], but the system cost rate is a little bit high with a value of 44.4460. The result is reasonable because a shorter PM interval and earlier replacement are more likely to maintain the system in a high availability status.

From the results, we can obtain the system cycle time for each replacement without considering the imperfect PM time; the system operation time equals 11.9143. The curves of Figure 5.5 and Figure 5.6 are calculated from (5.24) and (5.25). The system reliability curve for that cycle time before replacement is shown in Figure 5.5, from which we can see that each imperfect PM will restore the system to a higher reliable status than the previous just before the imperfect PM, and the system reliability is maintained at a high level between 0.8635 and 1.0000. On the other hand, as indicated in Figure 5.6, the system failure rate during the cycle time before replacement has a sudden reduction upon each imperfect PM, ranging from 0.0100 to 0.0195.

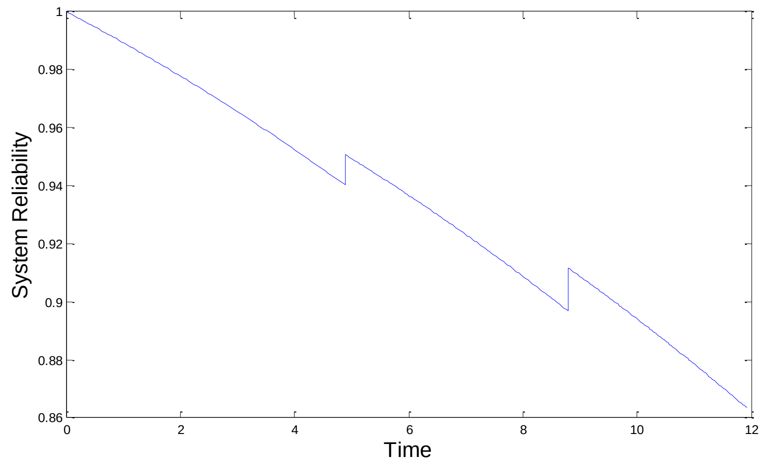


Figure 5.5: Reliability Curve in One Replacement Cycle Time under Case I.

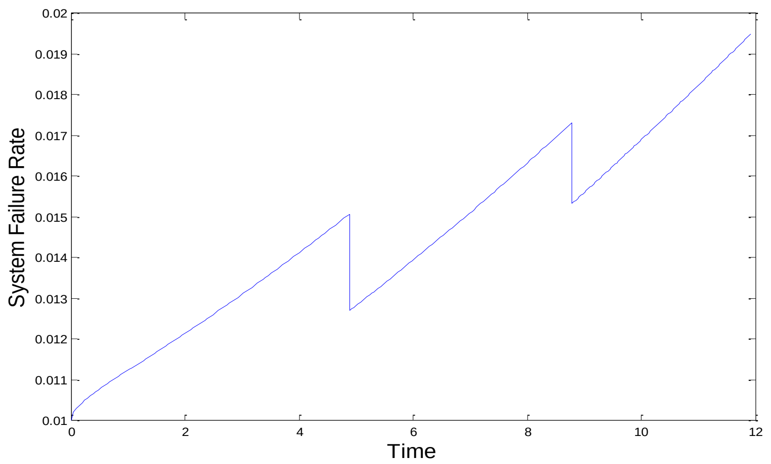


Figure 5.6: Failure Rate Curve in One Replacement Cycle Time under Case I.

CASE II: Minimizing the System Cost

Using the GA, we can obtain the optimal solution for this one-objective maintenance optimization problem in minimizing the system cost rate , that is, the minimum cost rate of the system is 38.0016 when the optimum pair of decision variables [N, T_1] is [4, 8.2151] with a high system unavailability of 0.0735.

The system cycle time for each replacement without considering maintenance time, that is the system operating time, equals 24.2507. Compared with the results in Case I, we can see the fact that the replacement cycle time is longer. Because the replacement cost is higher than the imperfect maintenance cost for α=0.8, the system will prefer a delayed replacement to decrease the cost rate. However, on the other hand, too long of a replacement cycle time will increase the idle times with the system reliability becoming lower, thereby resulting in a high cost rate. Therefore, the replacement cycle time is bounded within a certain region.

During the replacement cycle time, the system reliability curve in Figure 5.7 is becoming higher upon each imperfect PM level, ranging from 0.6296 to 1.0000. Compared with Case I, both the system availability and reliability in Case II is relatively lower than in Case I, but they offset the system resource to a much more economic cost rate. In other words, the system unavailability is increased by 20.69%, but at the same time the system cost rate is decreased by 15.50%. The system failure rate shown in Figure 5.8 has a sudden reduction upon each imperfect PM, ranging from 0.0100 to 0.0369.

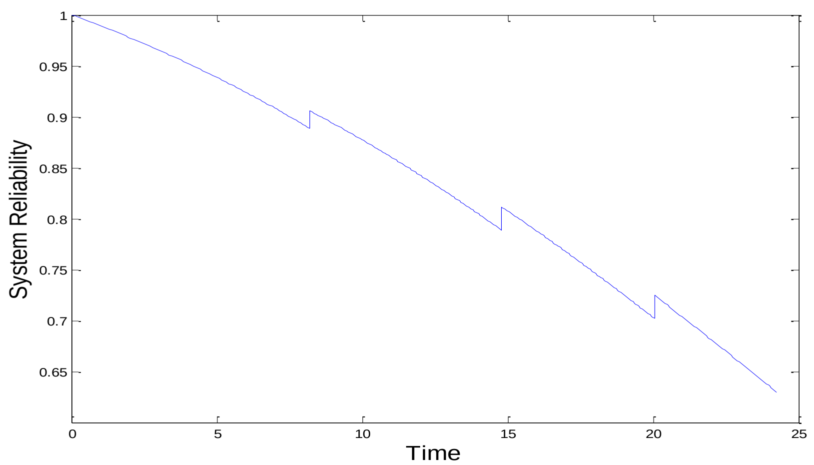


Figure 5.7: Reliability Curve in One Replacement Cycle Time under Case II.

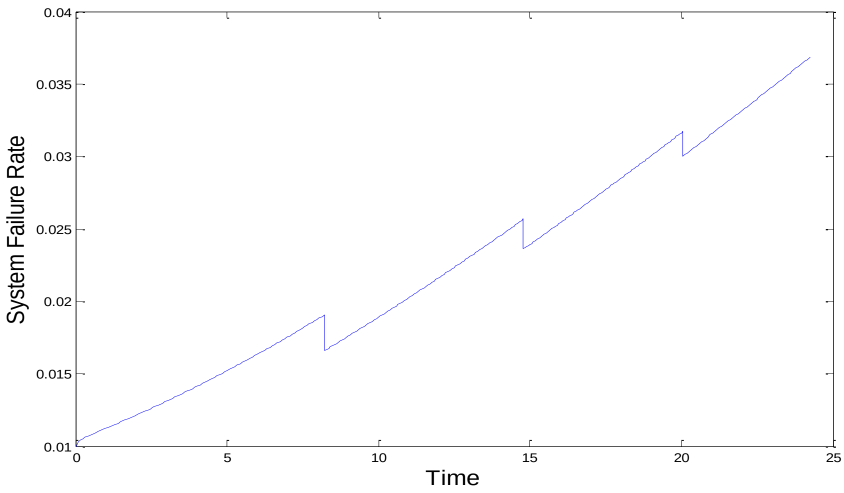


Figure 5.8: Failure Rate Curve in One Replacement Cycle Time under Case II.

CASE III: Multi-objective Optimization: Minimizing Unavailability and Cost Rate

The 'gamultiobj' function in Matlab's optimization toolbox is used to obtain the results of this multi-objective maintenance optimization problem with parameter settings of the population size 60 and Pareto frontier population fraction 0.7 to maintain a diverse population in each generation. The optimum solution is shown in Table 5.1, also including the comparison results from Cases I, and II. There are a total of forty points in this Pareto frontier, which simultaneously minimizes the system unavailability and cost rate indicated as in Figure 5.9.

Table 5.1: Comparison Results of Maintenance Optimization from Case I, II, III

Objective	**[N, T_1]**	**A**	**C**
Max A_s	[3, 4.8829]	0.9391*	44.4460
Min C_s	[4, 8.2151]	0.9265	38.0016*

	#	**[N, T_1]**	**A**	**C**	**#**	**[N, T_1]**	**A**	**C**
Max A_s	1	[3,4.8829]	0.9391*	44.4462	21	[3,5.7834]	0.9379	40.9509

&	2	[3,4.9145]	0.9391	44.2877	22	[3,5.7951]	0.9379	40.9175
Min C_s	3	[3,4.9403]	0.9391	44.1607	23	[3,5.8502]	0.9377	40.7633
	4	[3,4.9500]	0.9391	44.1135	24	[3,5.9583]	0.9374	40.4777
	5	[3,4.9901]	0.9391	43.9208	25	[3,6.0633]	0.9371	40.2198
	6	[3,5.0723]	0.9391	43.5409	26	[3,6.1097]	0.9369	40.1120
	7	[3,5.245]	0.9389	42.8017	27	[3,6.2065]	0.9366	39.8982
	8	[3,5.2793]	0.9389	42.664	28	[3,6.2472]	0.9365	39.8130
	9	[3,5.3658]	0.9388	42.3296	29	[3,6.3348]	0.9362	39.6379
	10	[3,5.4270]	0.9387	42.1039	30	[3,6.4322]	0.9358	39.4572
	11	[3,5.4309]	0.9387	42.0898	31	[3,6.5417]	0.9354	39.2701
	12	[3,5.4732]	0.9386	41.9390	32	[3,6.6045]	0.9351	39.1704
	13	[3,5.5112]	0.9385	41.8069	33	[3,6.7106]	0.9346	39.0138
	14	[3,5.5578]	0.9384	41.6496	34	[3,6.8721]	0.9339	38.8034
	15	[3,5.5742]	0.9384	41.5950	35	[3,6.9992]	0.9333	38.6601
	16	[3,5.6085]	0.9383	41.4834	36	[4,6.997]	0.9330	38.6113
	17	[3,5.6516]	0.9382	41.3466	37	[4,7.1269]	0.9324	38.4792
	18	[3,5.6567]	0.9382	41.3306	38	[3,7.2743]	0.9319	38.4124
	19	[3,5.7048]	0.9381	41.1826	39	[4,7.6066]	0.9300	38.1411
	20	[3,5.7438]	0.9380	41.0662	40	[4,8.2151]	0.9265	38.0016*

In Table 5.1, we can see that the corresponding optimization solutions in Case I & II with maximized system availability and minimized cost rate also belong to the points from the Pareto frontier (marked with *). The fact indicates that all of the three optimization problems have consistent solutions. Also we can see that the system availability and cost rate are two competing objectives; when one objective becomes better, the other objective will turn out to be worse.

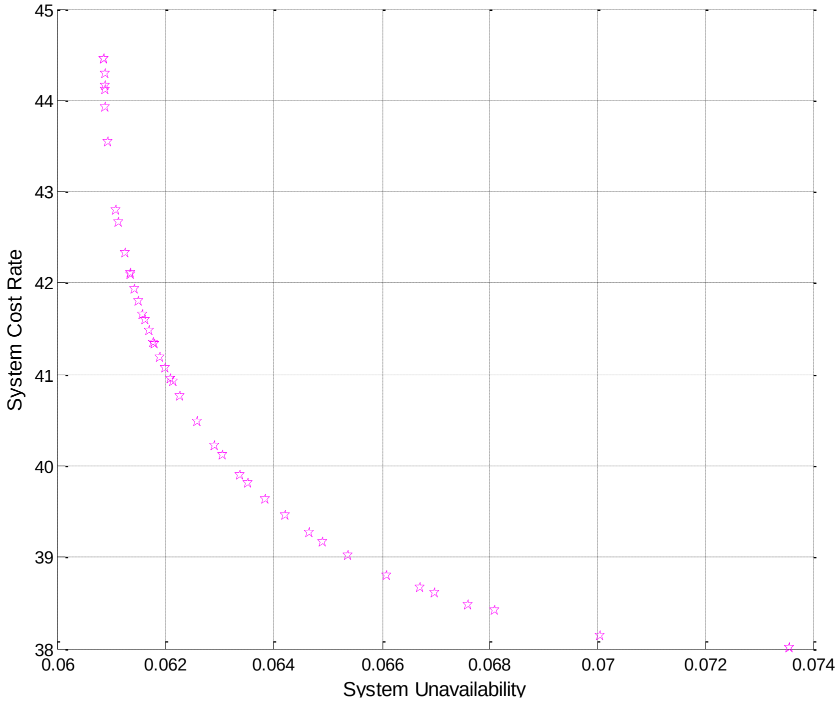


Figure 5.9: Pareto Frontier of Multi-objective Optimization under Case III.

The Pareto frontier shown in Figure 5.9 is useful to make the decisions for the maintenance scheduling problems. For instance, if we require the tolerable cost rate for the maintenance system should be no more than 39 according to the budget policy, then seven points in this Pareto frontier that satisfy the criteria can be chosen. However, within the budget plan, for the system availability, the higher, the better. Finally, the optimal pair of [3, 6.8721] with the corresponding system availability of 0.9339 and system cost rate of 38.8034 is the best choice under the budget constraint. There are also many maintenance problems in which more than one objective should be considered to allocate the system resources, so multi-objective optimization can provide alternative solutions according to different customer preference and system constraints.

5.4 Sensitivity Analysis

To examine the uncertainty for the optimization results, sensitivity analysis of two important parameters, including the restoration degree of imperfect PM α, and the quasi-renewal coefficient of imperfect PM β, is performed under three cases separately. The parameter α ranges from 0.1 to 0.9 while the parameter β ranges from 0.6 to 0.9.

Under Case I, only system unavailability is considered as the optimization objective. The results for sensitivity analysis under Case I are indicated in Table 5.2. When the parameter β is fixed with a value of 0.9, we can see that the optimized system unavailability is firstly increasing, and then decreasing with α ranging from 0.1 to 0.9. When α=0.3, the optimized system unavailability reaches a peak value of 0.0605. The smallest optimized system unavailability equals 0.0598 with α=0.1, which is the highest restoration degree. Also we can check the behavior of the optimum pair [N, T_1]. With the increasing of parameter α, the number of imperfect PM before replacement N is increasing, while the initial system imperfect PM interval is decreasing. On the other hand, when α is fixed at 0.9, the optimized system unavailability is decreasing with β ranging from 0.6 to 0.9. For the optimum pairs, with increasing β, the number of imperfect PM before replacement N is increasing while the initial system imperfect PM interval is decreasing.

Table 5.2: Sensitivity Analysis under Case I

α	β=0.9		β=0.8		β=0.7		β=0.6	
	$\overline{A}$	$[N, T_1]$	$\overline{A}$	$[N, T_1]$	$\overline{A}$	$[N, T_1]$	$\overline{A}$	$[N, T_1]$
0.9	0.0601	[3,4.2724]	0.0606	[3,4.7401]	0.0615	[3,5.2310]	0.0622	[2,5.3508]
0.8	0.0602	[3,4.4093]	0.0609	[3,4.8829]	0.0619	[2,5.1808]	0.0627	[2,5.4569]
0.7	0.0603	[3,4.5440]	0.0611	[3,5.0231]	0.0621	[2,5.2894]	0.0631	[2,5.5632]
0.6	0.0603	[3,4.6753]	0.0614	[3,5.1605]	0.0623	[2,5.3988]	0.0634	[2,5.6700]
0.5	0.0604	[3,4.8034]	0.0616	[2,5.2470]	0.0625	[2,5.5093]	0.0638	[2,5.7790]
0.4	0.0604	[3,4.9276]	0.0615	[2,5.3614]	0.0626	[2,5.6210]	0.0641	[2,5.8852]
0.3	0.0605	[2,5.2305]	0.0614	[2,5.4777]	0.0627	[2,5.7340]	0.0643	[2,5.9940]
0.2	0.0601	[2,5.3515]	0.0613	[2,5.5960]	0.0627	[2,5.8486]	0.0645	[2,6.1040]
0.1	0.0598	[2,5.4752]	0.0611	[2,5.7166]	0.0627	[2,5.9651]	0.0647	[2,6.2153]

Under Case II, only system cost rate is considered as the optimization objective. The results for sensitivity analysis under Case II are in Table 5.3. When β is fixed at 0.9, we can see the optimized system cost rate is decreasing with α ranging from 0.1 to 0.9. When α=0.1, and 0.2, the optimized system cost rate reaches the highest value of 44.0495 with optimum pair N=1, T_1=10.1018. When the restoration degree is high enough, the cost of imperfect PM is comparable with the replacement cost. As a result, the system may prefer an instant replacement instead of imperfect PM. With the increasing of the parameter α, both the number of imperfect PM before replacement and the initial imperfect PM interval are increasing. On the other hand, when α is fixed with the value of 0.9, the optimized cost rate is decreasing with β ranging from 0.6 to 0.9. For the optimum pairs, with the increasing of parameter β, the number of imperfect PM before replacement is increasing, while the initial imperfect PM interval is first decreasing and then increasing.

Table 5.3: Sensitivity Analysis under Case II

α	β=0.9		β=0.8		β=0.7		β=0.6	
	C	**[N, T_1]**	**C**	**[N, T_1]**	**C**	**[N, T_1]**	**C**	**[N, T_1]**
0.9	27.0753	[14,17.2784]	34.5334	[5,7.8104]	35.6160	[4,8.6962]	36.8280	[3,9.2769]
0.8	30.5293	[14,17.4945]	38.0016	[4,8.2151]	38.6357	[3,8.9860]	39.5944	[3,9.8643]
0.7	33.8170	[14,17.5441]	40.4618	[3,8.5615]	40.9658	[2,9.2728]	41.4068	[2,9.8619]
0.6	36.2126	[11,19.7229]	42.0959	[2,9.0648]	42.4133	[2,9.5872]	42.9339	[2,10.1333]
0.5	38.6764	[10,20.7841]	43.3986	[2,9.3521]	43.7868	[2,9.8866]	44.0495	[1,10.1018]
0.4	40.8908	[10,20.7864]	44.0495	[1,10.1018]	44.0495	[1,10.1018]	44.0495	[1,10.1018]
0.3	43.0184	[10,20.7866]	44.0495	[1,10.1018]	44.0495	[1,10.1018]	44.0495	[1,10.1018]
0.2	44.0495	[1,10.1018]	44.0495	[1,10.1018]	44.0495	[1,10.1018]	44.0495	[1,10.1018]
0.1	44.0495	[1,10.1018]	44.0495	[1,10.1018]	44.0495	[1,10.1018]	44.0495	[1,10.1018]

The results of the sensitivity analysis for both parameters α and β under Case III are shown in Figure 5.10, and Figure 5.11, separately, by varying α from 0.6 to 0.9 when β=0.8, and varying β from 0.6 to 0.9 when α=0.8 respectively. From the results in Figure 5.10, we can see that, with the increasing of α, both the system unavailability and cost rate have smaller values in the Pareto frontier. On the other hand, in Figure 5.11, a larger value for β also will result in a smaller combination of system unavailability and cost rate. However, by comparing these two sensitivity analyses in Figure 5.10 and 5.11, we can come to the conclusion that the parameter α seems to have a larger influence on the results than the parameter β because the change of α will move the curve much more than will changes in β upon the same percentage change of both parameters. Therefore, the restoration degree for the imperfect PM is a more important parameter than the quasi-renewal imperfect PM coefficient in determining the multi-objective solution in this numerical analysis. In Figure 5.11, we can observe the discontinuity behavior of the Pareto frontier when β=0.9, and α=0.8. The fact that the algorithm is unable to jump the discontinuity has to be ascribed to the intrinsically complex objective function structure or the incapability to find all points of the optimization (Rigoni and Poles (2005)).

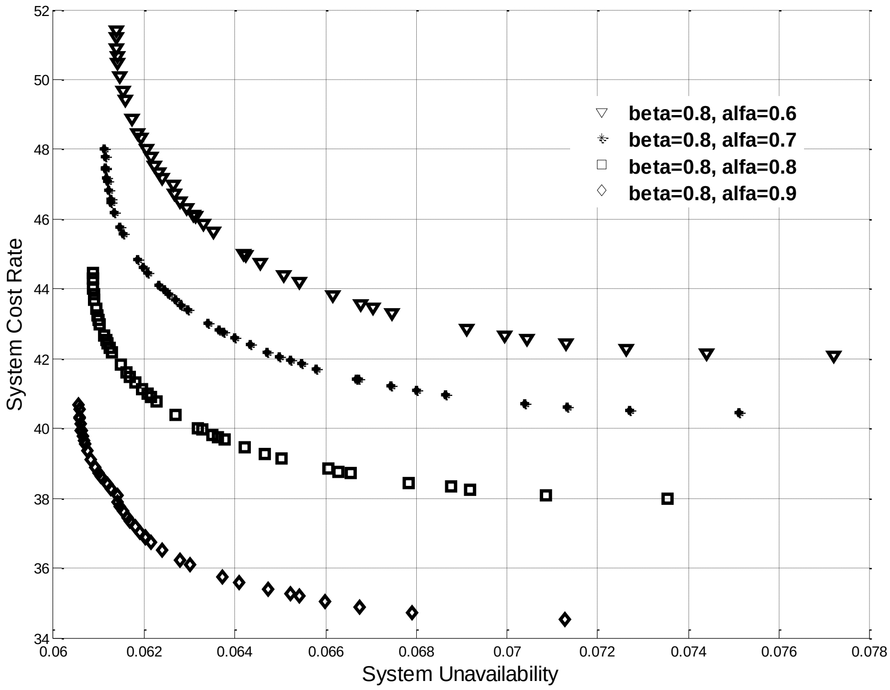


Figure 5.10: Sensitivity Analysis of variable α Under Case III.

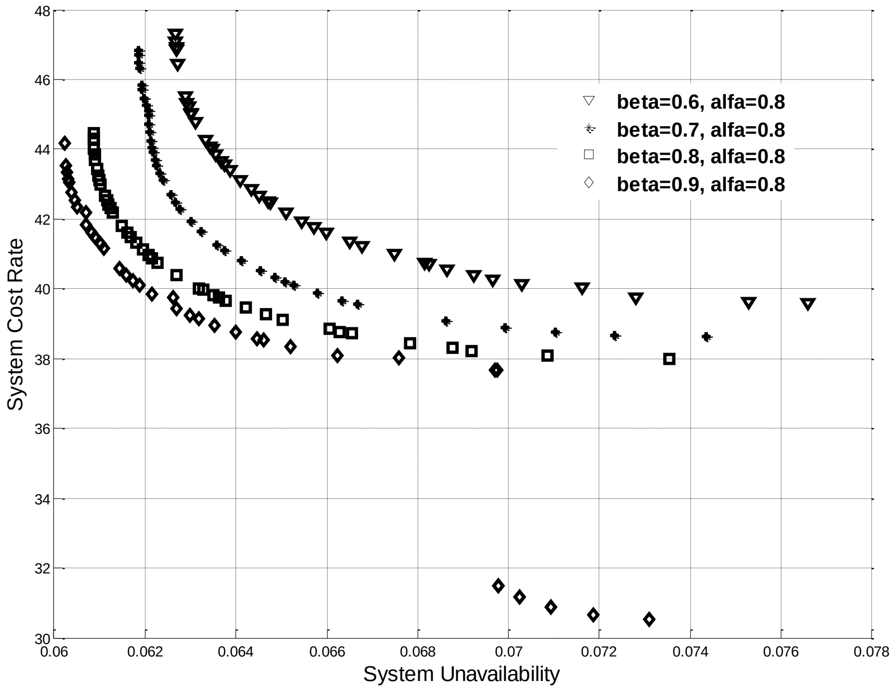


Figure 5.11: Sensitivity Analysis of variable *β* Under Case III.

5.5 Conclusion

This chapter contributes to the knowledge of the dependent relationships of the competing risk model of degradation processes and random shocks, and considers an imperfect PM model by raising the degradation critical threshold using a quasi-renewal process. We derive the mathematical models for the expected maintenance cost rate, and asymptotic unavailability of the maintained system. The optimal solution pairs of the number of imperfect PM before replacement, and the imperfect PM interval are obtained by a multi-objective procedure based on evolutionary metaheuristics.

Three optimizations under different cases are performed to compare the optimization results. Under Case I, only system asymptotic unavailability is considered as the objective function using GAs. Under Case II, only the system expected maintenance cost rate is taken into account as the objective function using GAs. Under Case III, we present a multi-objective optimization of both system asymptotic unavailability and maintenance cost rate using NSGA-II to find the Pareto optimal solutions. From the comparison results, we can see that the multi-objective outcomes in the Pareto frontier are consistent with the results from the one-objective under CASE I and CASE II. Furthermore, the multi-objective optimization illustrated in the Pareto frontier is a more useful tool for making the decisions of maintenance scheduling compared with the results from the single-objective optimization to provide alternative choices according to different customer preference and resource constraints.

The sensitivity analysis for two important parameters of the imperfect PM degree and quasi-renewal coefficient of imperfect PM interval is conducted to provide insight into the behavior of the proposed maintenance model. From the results of our sensitivity analysis, we can see that both these two parameters have a significant impact on the optimization results. However, in this numerical analysis, the imperfect PM degree seems to be more influential than the quasi-renewal coefficient of the imperfect PM interval.

In our future research, we will focus on the condition-based maintenance policies for multi-component systems embedded within the framework of dependent competing risk of degradation process and random shocks.

Chapter 6

Modeling the Dependent Competing Risks with Multiple Degradation Processes and Random Shocks Using Time-varying Copulas

ACRONYMS

NHPP	Nonhomogeneous Poisson process
MLE	Maximum likelihood estimation
AIC	Akaike information criterion
BIC	Bayesian information criterion
PQD	Positively quadrant dependent
CDF	Cumulative distribution function
DDP	Decreasing degradation path
IDP	Increasing degradation path
SJC	Symmetrised Joe-Clayton
DTS	Degradation threshold shocks

NOTATIONS

Θ_i	fixed effect parameter for the *i*th mean degradation path
X_i	random variable for the *i*th multiplicative degradation path
$F_{X_i}(x)$	CDF distribution for random variable X_i
$D_i(t; X_i, \Theta_i)$	multiplicative path function for the *i*th degradation process with random variable X_i, and parameter Θ_i
$\eta_i(t,\Theta_i)$	deterministic mean path for the *i*th degradation process with fixed effect parameters Θ_i
$M^{(i)}(t)$	the *i*th cumulative degradation wear up to time t
$\gamma_1^{(i)}$	coefficient of nonfatal shock number in the *i*th degradation process

$\gamma_2^{(i)}$	coefficient of cumulative nonfatal shock amount in the ith degradation process
$G(t, \gamma^{(i)})$	time-scaled covariate factor to accelerate the degradation rate with parameter $\gamma^{(i)}$
$l^{(i)}$	critical failure threshold for the ith degradation process
$T^{(i)}$	failure time of the ith degradation process
$f_i(t)$	failure rate of the ith degradation process
$R_i(t)$	marginal reliability function of the ith degradation process
$R(t)$	system reliability function
C	copula function
c	copula density function derived from the multivariate copula C
τ	Kendall's tau
$\underline{T}_\tau$	lower distribution bounds with Kendall's tau
$\overline{T}_\tau$	upper distribution bounds with Kendall's tau
ρ	Spearman's rho
$\underline{P}_\rho$	lower distribution bounds with Spearman's rho
$\overline{P}_\rho$	upper distribution bounds with Spearman's rho

6.1 Introduction

In reality, many systems or components are subjected to multiple degradation processes (Li and Pham (2005a), Pham et al. (1997)) as well as random shocks (Li and Pham (2005b)); either they can degrade in more than one way, or receive impacts from different types of random shocks. Therefore, the ability to consider multiple competing risk measures is required to examine the dependent structures between risks to derive the system reliability function more accurately. Assuming *s*-independence between degradation processes may underestimate system reliability (Wang (2003)). However, in this typical competing risk problem, there exist two kinds of the *s*-dependent structures that should be taken into consideration: (a) the *s*-dependent relationship between degradation processes and random shocks, and (b) the *s*-dependent relationship among various degradation processes.

A typical example for these *s*-dependent competing risks is the human body system (Gross (1973)). The human body is composed of a variety of biological systems, the organs, composed of the tissues, composed of the cells. Each unit in our body, from tiny invisible cells to the biological systems, may experience graceful degradation of its function until a certain age. Taking a human heart as an example, at the age of 40, the efficiency of the heart delivering blood to the body will begin to be greatly reduced because of the gradual loss of elasticity of blood vessels. As a result, the arteries may harden or become blocked. Also, many factors can contribute as a shock to the human body, such as non-normal living environment, and illness. Diabetes can damage many parts of the human body, such as the heart, kidneys, and blood vessels. Therefore, the human body is a complex system with correlated multiple organs and subsystems that contribute to the proper functioning of the physical mechanism.

As a consequence, a more systematic probability model combining these two dynamic dependent structures should be considered for multiple competing-risk systems. For the *s*-dependent models of degradation and random shocks, Finkelstein (2009) introduces a generalized Strehler-Mildvan model to estimate the first passage time of the survival function for the system subject to cumulative damage due to biological aging, and sudden killing events. Van Noortwijk et al. (2007) put forward a novel approach to combine two stochastic processes of deteriorating resistance and fluctuating load for the reliability analysis of a structural component. Ye et al. (2011) propose a degradation-oriented single failure time model to capture two failure mechanisms of degradation and shocks using the Brown-Proschan model under the condition that only failure times and failure modes are recorded without the observable information of shock magnitude and degradation amount. Wang and Pham (2011) recently study a multi-objective imperfect preventive maintenance optimization for one single-unit system subjected to the dependent competing risks of degradation wear and random shocks, by simultaneously maximizing the system asymptotic availability and minimizing the system cost rate.

For the multiple degradation models, Saassouh et al. (2007) consider a two-mode stochastically deteriorating model with a sudden change point in the degradation path, where the increments of deterioration follows a gamma law when the system is in the first mode, and the mean deteriorating rate increases when it flips into the second mode. Sari et al. (2009) present a bivariate degradation model with constant stress to accommodate the dependency between more degradation measures distributed with different marginal functions. Pan and Balakrishnan (2011) introduce a reliability estimation model for a complex structure system with bivariate degradations involving two or more performance characteristics by utilizing the bivariate Brinbaun-Saunders distribution.

Previous research has been focused on the reliability estimation of competing risks under either the dependent relationship between degradation and random shocks, or the dependent relationship among degradation processes. However, there are no studiesconsidering both types of dependent structures into one model. This chapter contributes to the knowledge of dependent competingrisk models by adding a time-scaled covariate factor governed by random shocks into the degradation paths, and also modulating the joint distribution of multiple degradation processes by linking the marginal functions using a copula method. The traditional way to build multiple degradation models is to utilize multivariate distributions, but this approach forces the limitation of a homogeneous distribution on each marginal degradation path. Recently, considerable attention has been paid to the *s*-dependence behavior between random variables modeled by copulas, which allows us to link the univariate marginal distributions to obtain a joint probability of the events.

Cossette et al. (2008) derive the discounted penalty function via Laplace transforms for a generalized Farlie-Bumbel-Morgenstern copula model in the presence of the associations between the claim sizes and inter-claim time in a compound Poisson risk model. Ram andSingh (2008) put forward a mathematical modeling for a parallel redundant system with two *s*-independent repairable subsystems by using a bivariate Gumbel-Hougaard family copula. Lo and Wilke (2010) develop a new copula graphic estimator applied to a model with multiple dependent competing risks, and apply the model to the data set of unemployment duration from Germany. In the work by Miladinovic and Tsokos (2009), a modified Gumbel failure model is used to study the system failure time, and Bayesian reliability estimates with five different parametric prior and one nonparametric kernel density prior are compared with each other for their effectiveness by using square error loss. Although copula is a flexible, powerful technique to build multivariate distributions widely used in various applications including economics, finance, and actuarial science, its application to model multiple competing risks in reliability are few. Kaishev et al. (2007) establish an *s*-dependent multiple-degradation model to examine the dependencies among causes of death to analyze the impact of complete or partial elimination of causes of death on the survival function from competing risks using copulas.

The main contribution of this chapter is the development of an innovative dependent competing risk model. (a) The modelconsiders two types of *s*-dependent structures into one model: the *s*-dependent relationship between degradation process and random shocks, and the *s*-dependent relationship among degradation processes. (b) We introduce a new method to handle the dependency between degradation processes and random shocks by adding a time-scaled covariate factor of nonfatal shocks into the cumulative degradation path functions. (c) Also, we employ the copula method which has several advantages over the directing methodto fit the joint distribution of competing random variables, and determine the modified bounds for the system reliability estimation.

6.2 Dependent Competing Risk Model

6.2.1 System Description

In this section, we describe the mathematical modeling for the *s*-dependent competing risks with multiple degradation processes and random shocks. The system has *m* degradation processes, each with k_i status, for *i* =1, 2,..., *m*, and one random shock, all of which are interplaying with each other, as shown in Figure 6.1. In this figure, S represents the random shocks, F represents the system failure, and $M_k^{(i)}$ represents the *k*th degradation status from the *i*th degradation path. A more specific example illustrated in Section 6.1 about the human body can be used to illustrate the model: the first degradation process is the human heart, the second degradation process is the blood vessels, and diabetes represents the random shocks.

The basic assumptions underlying our mathematical model are as follows.

1) The system is a multi-state deteriorating system. Initially, the system is in a good status $(M_{k_1}^{(1)}, M_{k_2}^{(2)}, ..., M_{k_m}^{(m)})$. As the time goes, it can go to the next degradation status due to aging, jump to another lower degradation status due to the cumulative amount of random shocks, or result in a system failure due to the fatal shock occurring.
2) We consider two kinds of random shocks in this system: (a) a fatal shock which causes the system to fail immediately, and (b) a nonfatal shock which has two impacts on the degradation process: sudden increment jump, and degradation rate acceleration.
3) The system is subject to two competing risks of degradation wear and random shocks, which are dependent oneach other, modulated by a time-scaled covariate factor.
4) The system has more than one degradation process, all of which are dependent

oneach other, linked by the copula method.

5) The system exhibits two types of failure mechanisms: fatal shock to the system; and although no fatal shock happens, the cumulative degradation amount of any degradation process exceeds a certain critical failure threshold.

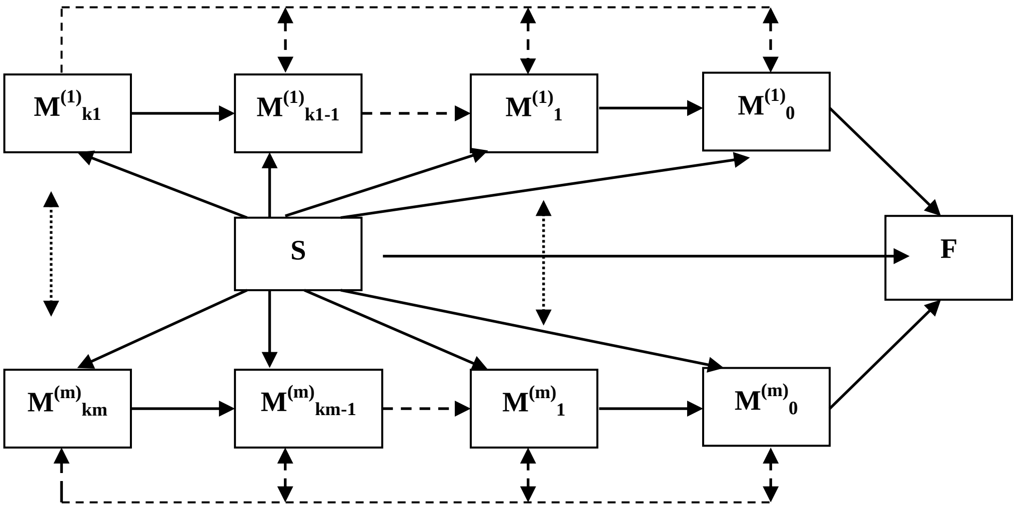


Figure 6.1: Flow diagram for the system subject to multiple competing risks.

6.2.2 Mathematical Model

6.2.2.1 Random Shock Model

Consider a system with multiple failure mechanisms which can be categorized as *m* degradation failures, and a fatal shock failure. We assume that the arrival of random shocks follows a homogeneous Poisson process $\{N(t), t \geq 0\}$ with occurring rate λ, where the random variable $N(t)$ denotes the number of random shocks until time t. Random shock loadings arriving at timesT_1, T_2,...,T_n are denoted as $\{w_{i1}, w_{i2},...,w_{in}\}$ for the ith degradation process i =1, 2,...,m, where n is the number of random shocks occurring during the degradation process. We assume that the sequence of random variables w_{ij} is nonnegative, s-independent, and identically distributed with a common distribution $Q_i(x)$ for all random shocks in the ith degradation process. Let $Q_i^{(j)}(x)$ for j=0,1,2,... be the j-fold Stieltjes convolution with $Q_i(x)$ itself. In other words, $Q_i^{(j)}(x) = P(w_{i1} + w_{i2} + ... + w_{ij} \le x)$.

The individual random shock could be fatal to the system at time point t with probability $p(t)$ so that the process of the fatal shocks occurring follows an NHPPwith a rate λ $p(t)$. However, if the random shock is nonfatal to the system, it will bring sudden increment jump and degradation rate acceleration to the degradation process. If fatal shock does not happen to the system until time t, there exists cumulative nonfatal shock damage to the ith degradation process, denoted by a compound Poisson process $S_i(t) = \sum_{j=0}^{N_2(t)} w_{ij}$. In that case, if $Q_i(x)$ follows an exponential distribution with mean μ, then the compound Poisson process leads to a gamma distribution, that is, $Q_i^{(j)}(x) \sim Gamma(N_2(t), \mu)$.

6.2.2.2 Degradation Path Function

The basic multiplicative path function for the ith degradation process is considered to tolerate the item-to-item variation by including the random variable X_i, and is

$$D_i(t; X_i, \theta_i) = X_i \cdot \eta_i(t; \theta_i) \tag{6.1}$$

The ith mean degradation path $\eta_i(t;\ \Theta_i)$ can be either monotonically decreasing, or monotonically increasing, called DDP, or IDP respectively. Non-monotonic degradation is also studied by several authors, such as the light displays in Bae and Kvam (2004), but such cases seldom appear in real life. For example, if X is Weibull-distributed, one simple example for multiplicative modeling is $D(t; X, \Theta) = \xi t$, where $\xi = X$ is the rate for the degradation path. More examples can be found in Bae et al. (2007), where we can see that such a basic degradation path function could be utilized to cover a wide range of degradation models.

6.2.2.3 Degradation Process Embedded with Random Shocks

In this section, we will introduce a new model to handle the dependent relationship of degradation and random shocks by using a time-scaled covariate factor. The degradation process will receive two kinds of impacts from the nonfatal random shocks: sudden increment jumps, and degradation rate acceleration. In other words, the nonfatal shock will affect the degradation process in such a way that the internal clock of the system will be accelerated by some degree. The *i*th degradation wear $M^{(i)}(t)$, including both aging with time and instantaneous damage induced by the random shocks, can be expressed as $M^{(i)}(t) = D_i(t) + S_i(t)$. The first term $D_i(t) = X_i \cdot \eta_i(t;\theta_i)$ is the multiplicative path function of the *i*th degradation process, and the second term $S_i(t) = \sum_{j=0}^{N_2(t)} w_{ij}$ reflects the sudden increment jump from the impact received by the nonfatal random shocks towards the *i*th degradation wear. Furthermore, to capture the extra effect of the degradation rate acceleration, we introduce a new item $G(t, \gamma^{(i)})$ into the *i*th degradation path function $D_i(t)$ by borrowing the original idea from the time-scaled model of accelerated life testing. The system time in the *i*th degradation paths $D_i(t)$ is scaled by an accelerated factor from t to $te^{G(t,\gamma^{(i)})}$. The *i*th cumulative degradation wear function can be defined as

$$M^{(i)}(t) = X_i \eta_i(te^{G(t,\gamma^{(i)})};\theta_i) + \sum_{j=1}^{N_2(t)} w_{ij} \tag{6.2}$$

where $G(t,\gamma^{(i)}) = \gamma_1^{(i)} N_2(t) + \gamma_2^{(i)} \sum_{j=1}^{N_2(t)} w_{ij}$, and the vector parameters $\gamma^{(i)}$ are unknown. Note that the first item in the above function $G(t, \gamma^{(i)})$ reflects the impact from the number of nonfatal shocks towards the *i*th degradation wear. Usually, we have $\gamma_1^{(i)} \geq 0$, and then the first term can be viewed as representing the fact that the degradation rate is more likely to increase with the nonfatal shock number. If $\gamma_1^{(i)} = 0$, it means that the degradation rate will not be affected by the nonfatal shock number. The second term $\gamma_2^{(i)} \sum_{j=1}^{N_2(t)} w_{ij}$ for $\gamma_2^{(i)} \geq 0$ is developed to modulate the situation that the cumulative nonfatal shock amount will contribute to an acceleration of the system degradation rate. If $\gamma_2^{(i)} = 0$, then no impact of cumulative nonfatal shock amount will happen to accelerate the degradation rate.

6.2.3 Reliability Estimation

6.2.3.1 Reliability Estimation Modeling

The measurements of the *m* degradation processes are random variables $M(t) = \{M^{(1)}(t), M^{(2)}(t), ..., M^{(m)}(t)\}$ at observation time point *t*. From the degradation failure definition, the system is considered to be failed if at least one of the *m* degradation processesreaches its corresponding critical failure threshold, which is known as $L = \{l^{(1)}, l^{(2)}, ..., l^{(m)}\}$. The catastrophic failures occur when the fatal shocks come to the system. Therefore, the system is in a working condition only when no fatal shock happens, and all of the individual degradation processes keep below their failure thresholds. Based on this definition, given the failure time of the *i*th degradation process $T^{(i)}$, the reliability of the multiple degradation system embedded with random shocks can be written as

$$\begin{aligned} R(t) &= \mathrm{P}[T^{(1)} > t, T^{(2)} > t, ..., T^{(m)} > t]\,\mathrm{P}[N_1(t) = 0] \\ &= \mathrm{P}[M^{(1)}(t) < l^{(1)}, M^{(2)}(t) < l^{(2)}, ..., M^{(m)}(t) < l^{(m)}]\,\mathrm{P}[N_1(t) = 0] \end{aligned} \tag{6.3}$$

If the multiple degradation failure mechanisms are assumed to be *s*-independent, the system reliability in (6.3) can be rewritten as

$$\begin{aligned} R(t) &= R^{(1)}(t) \times R^{(2)}(t) ... R^{(m)}(t) \times P[N_1(t) = 0] \\ &= \mathrm{P}[M^{(1)}(t) < l^{(1)}] \times P[M^{(2)}(t) < l^{(2)}] ... P[M^{(m)}(t) < l^{(m)}] \times \mathrm{P}[N_1(t) = 0] \end{aligned} \tag{6.4}$$

However, if the degradation failure mechanisms are not *s*-independent with each other, then we can easily see that (6.4) will not provide accuratesystem reliability estimation. In this case, as we considered in this chapter, a new approach for jointing the marginal reliability functions under different degradation distributions is needed. However, the traditional multivariate distributions lay the limitation of the same marginal functions. Therefore, instead of using multivariate distributions, the copula method is utilized to establish the *s*-dependent structure among various degradation measurements. There are many advantages of using the copula function approach in analyzing the dependence structure such as (a) copulas allow us to separately model the marginal behavior, and the dependence structure; (b) the copula function can provide us with the degree of the dependence, and also the structure of the dependence; (c) the univariate marginal function can be drawn from different distributions without restriction; and (d) copulas are invariant under strictly increasing and continuous transformation. Because of these advantages, the copula method is becoming a flexible, powerful technique to build ajoint distribution, especially when the marginal functions are known but complex, as in this problem.

6.2.3.2 Two-stage Statistical Inference for Copula Parameters

Let $\mathrm{X} = \{x_{1t}, x_{2t}, ..., x_{nt}\}_{t=1}^{T}$ be the data matrix, drawn from the marginal CDF $F_i(x_i)$ with copula *C*. The density function for the joint distribution can be obtained as

$$f(x_1, x_2, ..., x_n) = c(F_1(x_1), F_2(x_2), ..., F_n(x_n)) \cdot \prod_{i=1}^{n} f_i(x_i) \tag{6.5}$$

where $c(F_1(x_1), F_2(x_2), ..., F_n(x_n)) = \partial^n (C(F_1(x_1), F_2(x_2), ..., F_n(x_n))) / \partial F_1(x_1) \partial F_2(x_2) ... \partial F_n(x_n)$.

Therefore, the log-likelihood function is in the form of

$$l(\theta) = \sum_{t=1}^{T} \ln c(F_1(x_{1t}), F_2(x_{2t}), ..., F_n(x_{nt})) + \sum_{t=1}^{T} \sum_{i=1}^{n} \ln f_i(x_{it}) \tag{6.6}$$

where θ is composed of all the parameters from both marginal functions θ_1, and copula θ_2 .The maximum likelihood estimator is the value that maximizes $l(\theta; x)$; that is,

$$\widehat{\theta}_{MLE} = \max_{\theta \in \Theta} l(\theta) \tag{6.7}$$

If θ_1 is given, the estimation of the parameters θ_2 from the copula functioncan be performed as

$$\widehat{\theta}_2 = ArgMax_{\theta_2} \sum_{t=1}^{T} \ln c(F_1(x_{1t}), F_2(x_{2t}), ..., F_n(x_{nt}); \theta_2, \widehat{\theta}_1) \tag{6.8}$$

The detailed statistical inference of the copula method can be found in Nelsen (2006). In this chapter, we assume that the parameters for each marginal function θ_1 are already given. A two-stage MLE is used to perform the statistical inference for copulas: (a) the first stage is to calculate the marginal reliability probability of each degradation process with the given parameters in marginal functions; and (b) in the second stage, the MLE is used to estimate the parameters of the joint copula reliability function with the underlying of the dependent relationship between multiple degradation measures.

Stage 1: Marginal Degradation Reliability

The *i*th degradation process will survive totime *t* only if its cumulative degradation remains below its corresponding failure threshold $l^{(i)}$ conditioned on the event that there is no fatal shock. In other words, the conditional probability for the *i*th degradation process to survive is given by

$$
\begin{aligned}
R_i(t) &= P(M^{(i)}(t) < l^{(i)}) \\
&= \sum_{n=0}^{\infty} P(X_i \cdot \eta_i(te^{G(t,\gamma^{(i)})}) + S_i(t) < l^{(i)} \mid N_2(t) = n) P(N_2(t) = n) \\
&= \begin{bmatrix} P(X_i \cdot \eta_i(t) < l^{(i)}) P(N_2(t) = 0) \\ + \sum_{n=1}^{\infty} P(N_2(t) = n) \int_{z=0}^{l^{(i)}} P(X_i \cdot \eta_i(te^{\gamma^{(i)}{}_1 n + \gamma^{(i)}{}_2 z}) + z < l^{(i)}) dQ_i^{(n)}(z) \end{bmatrix} \\
&= \begin{bmatrix} exp(-\lambda \int_0^t q(u)du) F_{X_i}\left(\dfrac{l^{(i)}}{\eta_i(t)}\right) \\ + \sum_{n=1}^{\infty} \dfrac{exp(-\lambda \int_0^t q(u)du)(\lambda \int_0^t q(u)du)^n}{n!} \int_{z=0}^{l^{(i)}} F_{X_i}\left(\dfrac{l^{(i)} - z}{\eta_i(te^{\gamma^{(i)}{}_1 n + \gamma^{(i)}{}_2 z})}\right) dQ_i^{(n)}(z) \end{bmatrix}
\end{aligned}
\tag{6.9}
$$

where the function $q(u) = 1 - p(u)$.

The failure rate of the *i*th degradation process can be represented as

$$f_i(t) = -\frac{dR_i(t)}{dt} \tag{6.10}$$

which cannot be obtained in a closed form, but a numerical solution can be approximated.

Stage 2: System Reliability

Knowing the marginal distributions will not automatically lead to a unique copula fitting to link the joint distributions. Therefore, during the process of applying a copula to this problem, there are several issues that should be discussed: (a) which copula functions to use, (b) tail dependency, and (c) upper-lower bounds for the joint copula function. In the 1st stage, the analytical solutions for the marginal degradation reliability functions are obtained. If C is the joint copula of the marginal degradation distributions, the system reliability at time t in (6.3) can be expressed in terms of the copula method

$$R(t) = C\left(R_1(t), R_2(t), ..., R_m(t)\right)\exp(-\lambda\int_0^t p(u)du) \tag{6.11}$$

If the system has two degradation processes linked by bivariate Gumbel copula given as

$$C(v,z) = \exp\{-[(-\ln v)^{\alpha} + (-\ln z)^{\alpha})^{1/\alpha}\} \tag{6.12}$$

then we can obtain the system reliability function as

$$R(t) = \exp\{-[(-\ln R_1(t))^{\alpha} + (-\ln R_2(t))^{\alpha})^{1/\alpha}\}\exp(-\lambda\int_0^t p(u)du) \tag{6.13}$$

Using MLE, the full log-likelihood function of the multivariate copula with marginal reliability function can be expressed as

$$\sum_{j=1}^{m} \ln c\left(R_1(t_j^{(1)}), R_2(t_j^{(2)}), ..., R_m(t_j^{(m)})\right) \tag{6.14}$$

where c is the copula density function derived from the multivariate copula C.

The copula parameters can be estimated by maximizing the full log-likelihood function in (6.14). To check the goodness of the fit from the copula fitting, multiple criteria could be chosen such as log-likelihood, AIC, and BIC.

6.2.3.3 Reliability Bounds for Bivariate Degradation Processes

Based on the assumptions and derivation in the previous sections, we can obtain the marginal distributions for each degradation process embedded with random shocks, and the joint distribution of the system reliability function by the copula method. The Frechet-Hoeffding bounds (Nelsen (2006)) provide the limits for the possible copula functions based on the marginal distributions. If there exists a bivariate copula C for all $0 \le u, v \le 1$, then we have the Frechet-Hoeffding bounds

$$\max(u+v-1,0) \le C(u,v) \le \min(u,v)$$

where u, and v are the CDFs of marginal random variables.

When applying this bound to the bivariate degradation processes, we have the lower and upper bounds for the joint copula distribution at observation time t

$$\max(R_1(t)+R_2(t)-1,0) \le R(t) = C(R_1(t),R_2(t)) \le \min(R_1(t),R_2(t))$$

The Frechet-Hoeffding bounds can be further narrowed to provide a better limit for the system reliability estimation. Nelsen et al. (2001) suggest that the set of copulas with a common Kendall's tau ($-1 \le \tau \le 1$), the point-wise lower, and upper distribution bounds can be stated as

$$\underline{T}_{\tau}(u,v) = \max\left(0, u+v-1, \frac{1}{2}\left[(u+v) - \sqrt{(u-v)^2+1-\tau}\right]\right) \tag{6.15}$$

and

$$\overline{T}_{\tau}(u,v) = \min\left(u, v, \frac{1}{2}\left[(u+v-1) + \sqrt{(u+v-1)^2+1+\tau}\right]\right) \tag{6.16}$$

where $\tau = 4\iint_{I^2} C(v,z)dC(v,z) - 1 = 1 - 4\iint_{I^2} \frac{\partial C}{\partial v}(v,z)\frac{\partial C}{\partial z}(v,z)dvdz$ (6.17)

Instead of using Kendall's tau, for a set of copulas with a common value of Spearman's rho with $-1 \le \rho \le 1$, the point-wise lower, and upper distribution bounds can be written as

$$\underline{P}_{\rho}(u,v) = \max\left(0, u+v-1, \frac{u+v}{2} - \phi(u-v, 1-\rho)\right) \tag{6.18}$$

and

$$\overline{P}_{\rho}(u,v) = \min\left(u, v, \frac{u+v-1}{2} + \phi(u+v-1, 1+\rho)\right) \tag{6.19}$$

where

$$\rho = 12\iint_{I^2} C(u,v)dudv - 3 = 12\iint_{I^2} uvdC(u,v) - 3 \tag{6.20}$$

$$\phi(a,b) = \frac{1}{6}\left[\left(9b + 3\sqrt{9b^2 - 3a^6}\right)^{1/3} + \left(9b - 3\sqrt{9b^2 - 3a^6}\right)^{1/3}\right] \tag{6.21}$$

The proof of the upper and lower bounds in terms of Kendall's tau, and Spearman's rho can be found in Nelsen et al. (2001). Their paper also indicates that the newer lower bound with a positive Kendall's tau is an improvement over the Frechet-Hoeffding lower bound, while the new upper bound with a negative Kendall's tau is an improvement over the Frechet-Hoeffding upper bound. However, unlike the case of Kendall's tau, the lower bound will be improved when $\rho > -1/2$, and the upper bound will be improved when $\rho < 1/2$.

Therefore, the modified joint copula bounds at time *t* in this bivariate degradation can be expressed as

$$\underline{T}_{\tau}(R_1(t), R_2(t)) \le R(t) = C(R_1(t), R_2(t)) \le \overline{T}_{\tau}(R_1(t), R_2(t)) \tag{6.22}$$

for Kendall's tau, and

$$\underline{P}_{\tau}(R_1(t), R_2(t)) \le R(t) = C(R_1(t), R_2(t)) \le \overline{P}_{\tau}(R_1(t), R_2(t)) \tag{6.23}$$

for Spearman's rho.

6.3 Numerical Example

6.3.1 System Description

Consider a system subject to two degradation processes, and one random shock. The random shocks occur with rate $\lambda = 1/15$, and the fatal probability $p(t) = 1 - \exp(-\theta t)$, where $\theta = 0.0002$.For the first degradation path function, assume that $D(t; X, \theta) = \zeta t$, $X = \zeta$, and Weibull distributed (Zhang et al. (2010)) with CDF $F_X(x) = 1 - \exp[-(x/\mu)^k]$ for $\mu, k > 0$, where $\mu = 0.8$, and $k = 1$. The individual shock loading towards the first degradation process follows the *s*-normal distribution with parameter $\mu_w = 5$, and $\sigma_w = 3$. Assume the random variables in the time-scaled factor $\gamma_1^{(1)} = 0.05$, $\gamma_2^{(1)} = 0.008$, and critical failure threshold $l^{(1)}$=100 .For the second degradation path function, choose $D(t; X, \theta) = \theta_2 \log[\theta_1 + t]$ where $\theta_1 = 4.8$, and θ_2 is gamma distributed with pdf of $f_X(x) = x^{a-1}e^{-\frac{x}{b}} \big/ b^a \Gamma(a)$, $x \ge 0$, where $a = 50$, and $b = 0.01$. The individual shock loading towards the second degradation process follows the exponential distribution with mean $v = 0.1$. Assume $\gamma_1^{(2)} = 0.01$, $\gamma_2^{(2)} = 0.005$, and critical failure threshold $l^{(2)} = 3.5$.

According to (6.9) in Section 6.2, we can obtain the marginal reliability function for each degradation path, as shown in Figure 6.2(a), and the corresponding failure rate is indicated in Figure 6.2(b). From Figure 6.2, we can see that both of the degradation processes have a lifetime of approximately 450 days, and the first degradation process has a larger failure rate than the second one in the early time, but this situation is reversed from the cross time point of approximately 100 days.

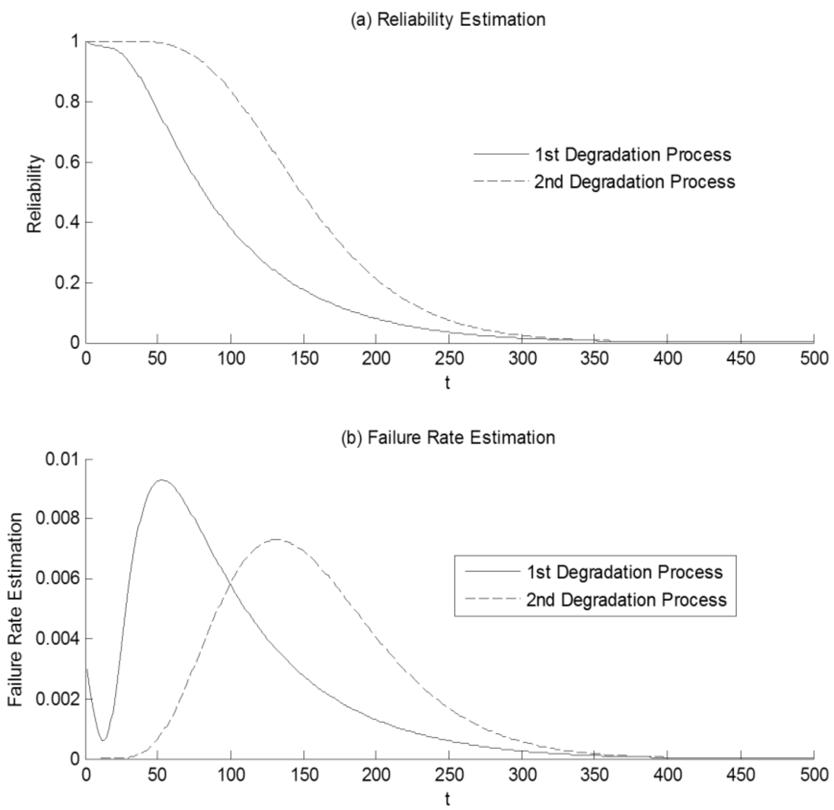


Figure 6.2: Marginal degradation function.

6.3.2 Multiple Processes with Constant Copulas

Given marginal degradation reliability functions derived by analytical methods, the copula method is applied to fit the dependent bivariate data with the help of the copula toolbox written by Andrew Patton using Matlab R2010a. To choose the most suitable fitting copula, we test both constant and time-varying copulas using the log-likelihood, AIC, and BIC as the criteria for goodness of fit. First of all, we test the results from the nine selected constant copulas, which are widely used in many other research fields such as finance. Comparingthe results shown in Table 6.1, we can see that, based on the log-likelihood, the Clayton copula gives us the highest likelihood, followed by SJC, and then Rotated Gumbel, but the performance of Rotated Clayton is the worst.

An important association measure in the copula method is tail dependence (Nelsen (2006)), which indicates the limiting probability that one margin value exceeds a certain threshold given that the other margin already exceeds the same threshold if that limiting probability exists. From Table 6.1, see that the three best fitting copulas all allow for non-zero low tail dependence. Furthermore, the fact that all of their tail dependenciesshare very similar values indicate the consistency of the fitting results.Acontour plot of the Clayton copula shown in Figure 6.3 depicts the bi-dimensional distribution with the estimated parameter $\alpha = 2.2932$, as fitted by this model.

Table 6.1: Results of constant copula fitting

	Copulas	*LL*	*Parameter Estimation*	*Lower Tail Dependence*	*Upper Tail Dependence*	*Ranking*
1	Normal	1216.75	0.7153	0	0	5
2	Clayton	1774.31	2.2932	0.7391	0	1
3	Rotated Clayton	120.56	0.2122	0	0.0381	9
4	Plackett	927.99	99.0243	0	0	6
5	Frank	629.96	9.3995	0	0	8
6	Gumbel	647.50	1.8075	0	0.5326	7
7	Rotated Gumbel	1723.76	2.2695	0.6428	0	3
8	Student's t	1405.20	[0.7600,3.1793]	0.4906	0.4906	4
9	Symmetrised Joe-Clayton	1766.46	[0.0930,0.7073]	0.7073	0.0930	2

6.3.3 Multiple Processes with Time-varying Copulas

After testing the results from constant copulas, we also check the performance of time-varying copulas. Three time-varying copulas are employed to fit the degradation joint distribution, including the time-varying Normal (or Gaussian) copula, time-varying Rotated Gumbel copula, and time-varying SJC copula. In Table 6.2, comparingthe fitting results from both constant and time-varying copulas, see that the ranking results from all three criteria are consistent with each other. The best copula fitting for the degradation joint distribution is the time-varying SJC copula with the highest log-likelihood of 1886.19, and AIC and BIC of -3772.40; followed by the time-varying Rotated Gumbel copula with a log-likelihood of 1882.95, and AIC andBIC of -3765.90; and then the Clayton copula with log-likelihood of 1774.31, and AIC and BIC of -3548.60. However, the Rotated Clayton is still the worst fitting among all the copulas, with a log-likelihood of 120.56 and AIC and BIC of-241.10.

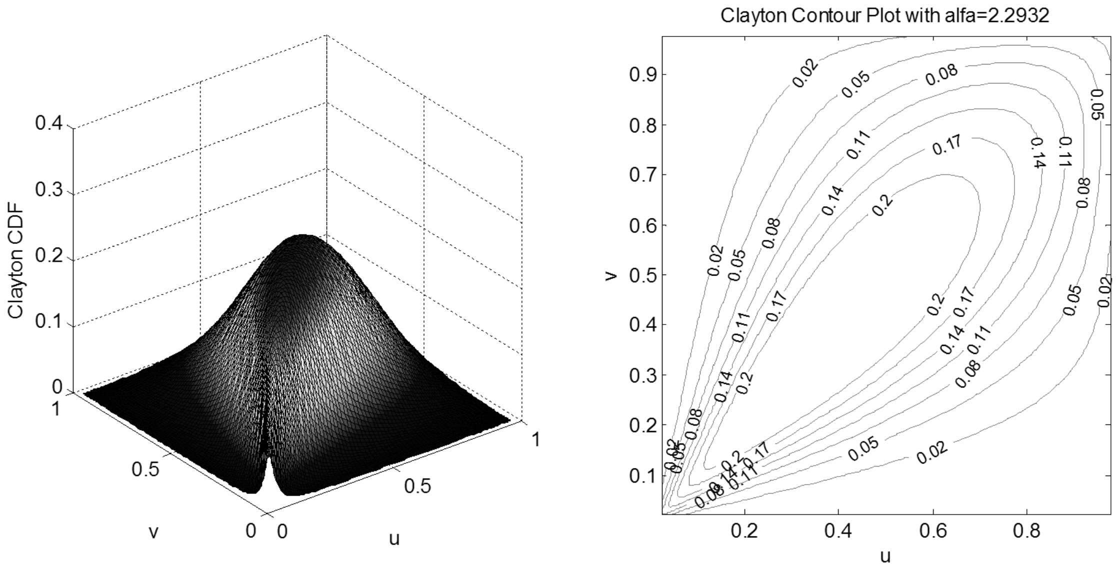


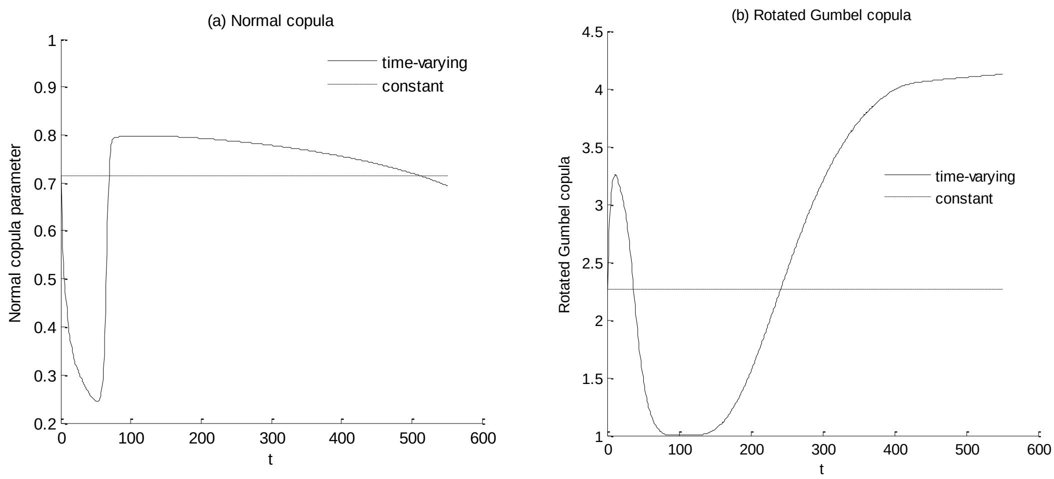
Figure 6.3: Clayton 3D Plot.

Table 6.2: Goodness of fit from constant vs. time-varying copulas

	Copulas	*LL*	*AIC*	*BIC*	*Ranking*
1	Normal	1216.75	-2433.50	-2433.50	8
2	Clayton	1774.31	-3548.60	-3548.60	3
3	Rotated Clayton	120.56	-241.10	-241.10	12
4	Plackett	927.99	-1856.00	-1856.00	9
5	Frank	629.96	-1259.90	-1259.90	11
6	Gumbel	647.50	-1295.00	-1295.00	10
7	Rotated Gumbel	1723.76	-3447.50	-3447.50	5
8	Student's t	1405.20	-2810.40	-2810.40	6

9	Symmetrised Joe-Clayton	1766.46	-3532.90	-3532.90	4
10	Time-varying Normal	1331.35	-2681.70	-2681.70	7
11	Time-varying Rotated Gumbel	1882.95	-3765.90	-3765.90	2
12	Time-varying SJC	1886.19	-3772.40	-3772.40	1

The parameter estimation curve for the time-varying copulas is indicated in Figure 6.4, where the solid lines come from the estimation of time-varying copulas, while the dashed lines come from the corresponding constant copulas. Compared with the results from their corresponding constant copulas, we can see that the more flexible parameter estimation of time-varying copulas provides a better goodness of fit no matter what criteriaare used. The performance of the time-varying Normal, the Rotated Gumbel, and the SJC copula increases 9.42%, 9.24%, and 6.78% respectively under the log-likelihood criteria.



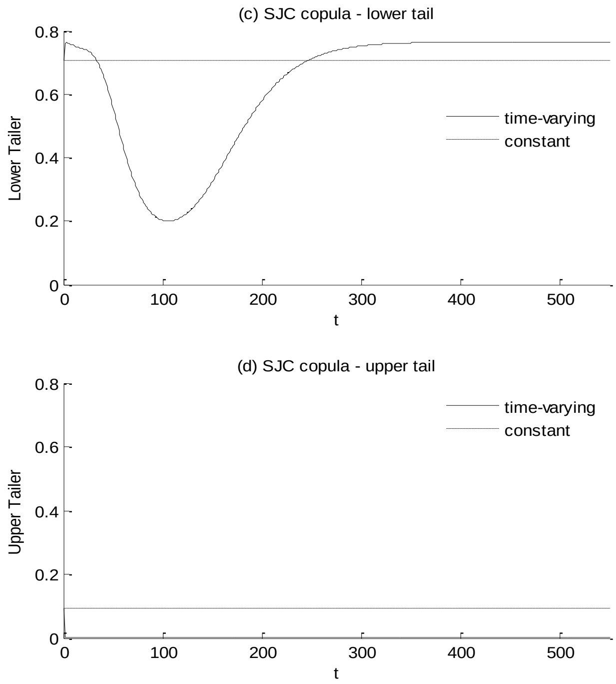


Figure 6.4: Parameter estimation curve for time-varying copulas.

6.3.4 Reliability Estimation

Plugging the marginal degradation reliability distributions into the fitted copulas, we can obtain the CDF behavior of the joint copula distribution, as shown in Figure 6.5, where the joint distribution curves among the best three copulas (time-varying SJC, time-varying Rotated Gumbel, and Clayton copula), maximum copula, minimum copula, and *s*-independent copula are compared with each other. For bivariate *s*-independent copula, $C(u, v)=uv$. The best fitting of the time-varying SJC copula lies between the time-varying Rotated Gumbel and Clayton copula, while all of these three best fitting copulas are bounded by the minimum and maximum Frechet-Hoeffding limits. The fact that all of these three best copulas overlap with the *s*-independent copula in the early time is because at the early system stage both of the two degradation processes are in a good status with high performance, and unlikely to receive impacts from the other degradation process. However, after a period of time, they all exhibit some degree of *s*-dependence due to the interplaying loadings from other degradation processes by deviating from the *s*-independent copula as indicated in Figure 6.5.

From the definition of the PQD (Nelsen (2006)), we can see that all of these three best fitting copulas hold the PQD property. Compared with the assumption that degradation processes are *s*-independent fromeach other, it may be more realistic to assume some sort of dependence among various degradation processes. For example, the low stage of one degradation process may result in an increasing loading on the other relative degradation processes. Therefore, in a two-degradation process, we may wish to establish a model in which the small degradation amount in the first degradation process tends to occur with small degradation amount in the second degradation process; that is, the two degradation processes are positively quadrant dependent.From Section 6.2.3.3, we know that, although Frechet-Hoeffding bounds are commonly used limits for copula functions, they can be further narrowed by either Spearman's rho (ρ), or Kendall's tau (τ). Using (6.22), and (6.23), we can obtain the corresponding modified joint copula Spearman's and Kendall's limits. As shown in Figure 6.5, Spearman's lower limit is a modified limit for the Frechet-Hoeffding lower bound. These modified upper and lower limits will contain all possible system reliability estimationsregardless of the copula functions, but with a given measure of association and marginal distributions.

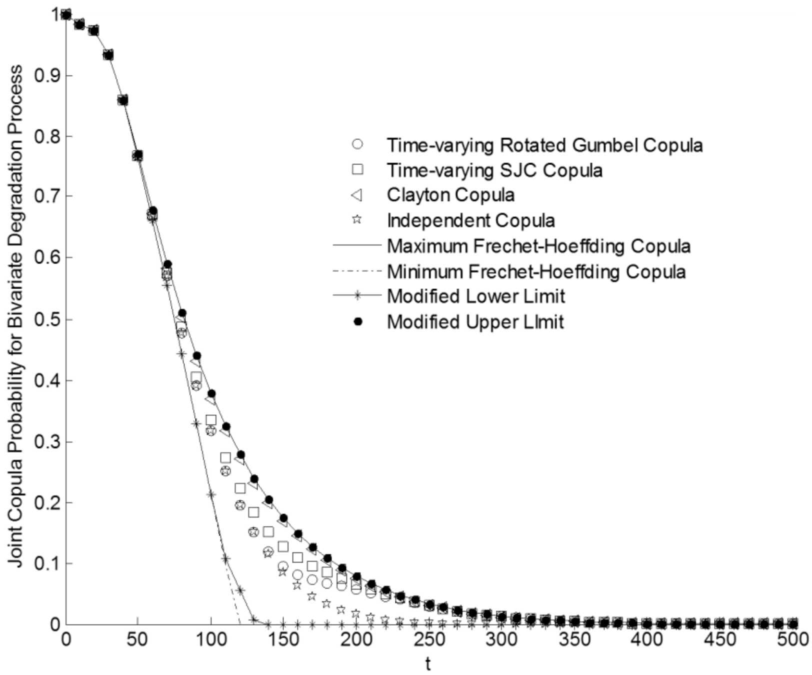


Figure 6.5: Comparison of the various joint copula probabilities.

Given the joint copula degradation fitting, we can derive the system reliability by multiplying the copula probability with the probability of no fatal shock occurring because these two events are *s*-independent of each other. As a sequence, the system reliability estimated by the best time-varying SJC copula is indicated in Figure 6.6 with the modified lower and upper limits, which can provide the system designers with the more accurate range for the system reliability in their design structure, and indicate the possible effects from the alternative plans of improving the system design. This information also can be used by decision makers to study the worst-case scenario to make the budget and system availability analysis for maintenance scheduling and warranty policy.

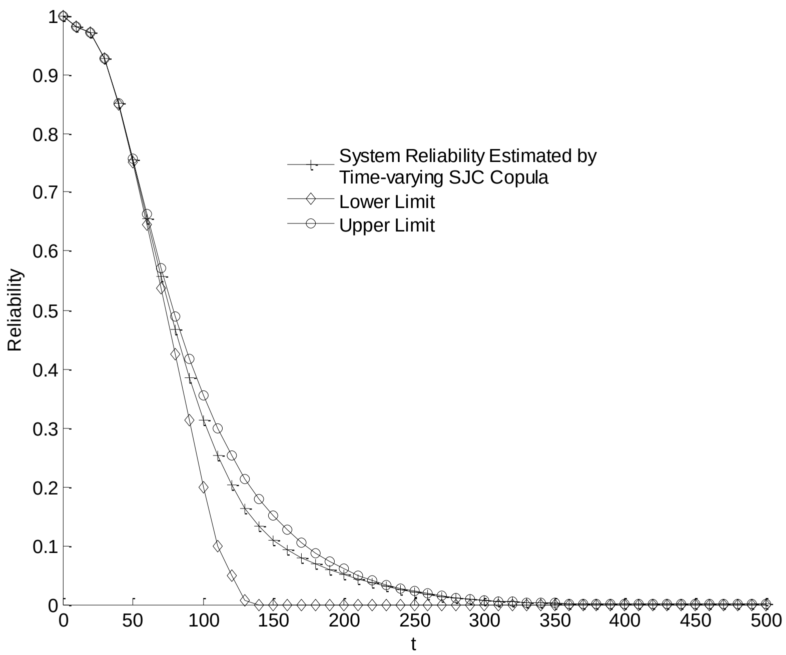


Figure 6.6: Best joint copula fitting with modified limits.

6.4 Conclusion

We employ the copula method to build a more flexible *s*-dependent competing model for a continuously degraded system in which (a) the dependent structure among different degradation processes is linked by both constant and time-varying copula methods, and (b) the *s*-dependent structure between random shocks and degradation processes is modulated by a time-scaled covariate factor that is used to absorb the extra nonfatal shock impact to accelerate the degradation rate.The two-stage statistical inference for copula parameters, and narrowed upper andlower distribution bounds,areapplied to provide a more robust estimation of the system reliability. Three criteria of log-likelihood, AIC (Pham (2006)), and BIC are used to check the goodness of fit of eleven copula functions' performance. Given the numerical solution to the proposed model, we can see that the time-varying copulas perform better than the constant copulas by allowing for time-changing parameter estimation. Furthermore, the modified distribution bounds give both the designers and decision makers more useful information aboutthe improvement plan for the system performance characteristics.

It is interesting for further research to study (a) the parameter estimation for the degradation function featured as cumulative degradation embedded with random shocks, and (b) the issue of developing a condition-based imperfect maintenance and repair policy for the proposed dependent multiple competing risk model.

Chapter 7

Copula Reliability Modeling of Multi-state Degraded Systems Subject to Multiple Dependent Competing Risks

7.1 Introduction

Traditional life testing is widely used for decades in manufacturing and industrial application by researchers and engineers to determine how well the critical components in a system will perform under different operating environment. Based on the estimation of failure time distribution for components or systems, some important decisions regarding product design, improvement and warranty plans can be made to satisfy the customer requirement. However, with the developing of modern technologies the systems are becoming more and more reliable so that it limits the availability of failure-time data. Therefore, sometimes when the systems are with high reliability, or when it is extremely expensive to test system failure, traditional life testing may not be so helpful. Recent technology advance in sensor monitoring systems have dramatically accelerated the research focus of degradation testing and degradation-based data analysis. Usually, the components or system will not fully fail suddenly, but can degrade with the aging and wear process. The system efficiency and performance may decrease with the increasing of the degradation levels until the degradation level of the system exceeds certain failure threshold. As a result, more attention should be paid to the system in terms of maintenance, availability, inspections as the system transits from a good system state to a bad system state.

For the multi-state degradation model, Chryssaphinou et al. (2011) propose a repairable multi-state degraded system composed of *m* components, whose sojourn time in any state is characterized by a discrete-time semi-Markov chain. Soro et al. (2010) develop a continuous-time Markov model for evaluating the reliability performance measures of multi-state degraded system with imperfect preventive maintenance and minimal repairs. Eryilmaz (2010) derives the mean residual and mean past lifetime functions for multi-state *k*-out-of-*n* systems, in which the degradations follow a Markov process with discrete state spaces. Li and Pham (2005a) present a methodology to generate system reliability and state probability for a generalized multi-state degraded system with multiple competing risks of two degradation processes and random shocks. Abou (2010) considers a number of alternative probabilistic models for multi-state systems with two failure modes by introducing multi-state operators that follow the associative and commutative laws. Nourelfath and Ait-Kadi (2007) apply a Markov model to modulate the redundancy maintenance optimization problem with reliability constraints and minimal cost configuration for series-parallel multi-state system by considering the priorities between components. Ramirez-Marquez and Coit (2005) describe a new Monte-Carlo simulation methodology of reliability estimation process based on multi-state minimal cut vectors compared with the actual multi-state two-terminal reliability computation.

In one system, due to the manufacturing specification, various components may behavior very differently, but at the same time the same usage history may also result in some dependent structures among the various degradation measures from components. The degraded status of the multi-state system is discretized into a finite number of states, which are characterized by the interplaying function of the degradation measure levels of components composed of systems. Furthermore, random shocks will contribute to the system in two ways: (a) fatal shock, which will fail the system immediately; and (b) non-fatal shock, which will not fail in this case but affect directly on system degradation path by cumulative shock loadings or degradation rate acceleration. The dependence of the degradation processes and random shocks can be exhibited in two aspects: on one hand, degradation process will make the system more vulnerable to the environment factors, such as temperature, pressure, and random shocks; on the other hand, the random shocks will accelerate the degradation process with two modes of sudden jump or minor change of degradation rate.

As a sequence, a more systematic probability model for the dynamic dependent structure underlying these two competing processes should be called for. In the work by Kharoufeh et al. (2007), they utilize the Laplace-Stieltjes transform to explicitly derive the lifetime distribution as well as the limiting availability for a periodically inspected single-unit system with hidden failure. Cha and Finkelstein (2009) extend the Brown-Proschan model by assuming that random shocks will result in an immediate system failure with a probability. But if the shocks are not extreme enough, it will increase the system aging process by certain random increment. Ebrahimi (2001) proposes a general stochastic model to estimate the reliability of the system in terms of a deterioration process with covariate. Wang and Pham (2012) recently develop a dependent competing risk model, in which the dependent structure of random shock and degradation is modulated by a time-scaled covariate factor, and the dependent structure among degradation processes is fitted by both constant and time-varying copulas, for a system subject to multi-degradation measures and random shocks.

This chapter extends the research of Li and Pham (2005a), Wang and Pham (2012) by considering a multi-state model for the dependent competing-risk systems subject to two degradation processes and random shocks. Two types of dependent structures are considered in the model using the method proposed in Wang and Pham (2012): (a) dependency between degradation and random shocks, modulated by a time-scaled covariate; (b) dependency among various degradation measures, linked by copula method. The status of the multi-state degraded system is determined by the Cartesian product of the two degradation measure levels (Li and Pham (2005a)). The system reliability and state probability are derived by both analytical method and Monte Carlo simulation, which is applied to provide the approximate point estimation and confident interval to compare the results from analytical method. Finally, a numerical example is provided to illustrate the application of the proposed model with sensitivity analysis for the occurring rate of the random shocks.

7.2 Mathematical Modeling

In this section, we describe the mathematical notation and reliability estimation model for the multi-state degraded systems with dependent competing risks, consisting of two degradation processes and random shocks. The basic assumptions under this proposed mathematical model are indicated as follows

a) The system is subject to three competing risks of two degradation processes and random shocks, which are pairwise dependents with each other.

b) The degradation process is monotonically increasing.

c) The system is a discrete multi-state degradation system, whose state is determined by the Cartesian product of the degradation levels of two degradation processes.

d) There exist two types of failure mechanisms in the system: one condition is that the fatal shocks comes to the system; another is that although there is no fatal shock occurring, the systems has degradation failure since one of the two degradation levels exceed its failure threshold.

7.2.1 Dependent Competing Risk

The system has three competing risks, including two degradation processes $\{M_1(t), M_2(t)\}_{t\geq 0}$ and random shocks characterized by a homogeneous Poisson process $\{N(t)\}_{t\geq 0}$. Assume $N(t)$ is the number of random shocks occurring at time t and random shocks arriving at a rate λ. The random shock loadings can be represented as the sequence of $\{w_{ij}\}$ distributed as a common distribution $Q_i(x)$ for all jth shock in the ith degradation path. The shocks are cumulative to a compound Poisson process which can be expressed as $S_i(t) = \sum_{j=0}^{N(t)} w_{ij}$. The random shock can be fatal at a probability of $p(t)$. Therefore, the fatal shock occurring rate $\lambda_1(t) = \lambda p(t)$.

In the model, basic multiplicative degradation path (Bae et al. (2007)) is used to model the degradation process in order to tolerate the item-to-item variation by including the random variable X_i. The formula for basic multiplicative function in the ith degradation path is shown as follows

$$D_i(t; X_i, \beta_i) = X_i \eta_i(t; \beta_i) \tag{7.1}$$

where $\eta_i(t; \beta_i)$ denotes a deterministic mean degradation path with fixed effect parameters β_i; the variable X_i represents the random variations around the mean degradation path, distributed as a common CDF $F_i(x)$. More detailed description and examples about multiplicative and additive degradation path functions can be found in Bae et al. (2007).

The total degradation wear in the ith path is a cumulative process with two parts: one is aging with time and usage and the other is instantaneous shock damage, which can be expressed as $M_i(t) = D_i(t) + S_i(t)$. The first term is the multiplicative function for the ith degradation path and the second term is the cumulative random shock loadings. Furthermore, in order to capture the extra effect of the degradation rate acceleration due to random shocks, we introduce a time-scaled covariate into the model by scaling the time t to $te^{G(t,\gamma^{(i)})}$ in the ith degradation path $D_i(t)$ where $G(t,\gamma^{(i)}) = \gamma_1^{(i)} N(t) + \gamma_2^{(i)} \sum_{j=1}^{N(t)} w_{ij}$. The first part in the formula reflects the impact received by degradation path from the number of random shocks and the second part indicates the possibility that the degradation rate can be accelerated by the amount of cumulative shock loadings. Therefore, the cumulative degradation wear in the ith degradation path can be expressed as follows:

$$M_i(t) = X_i \eta_i(te^{G(t,\gamma^{(i)})}; \theta_i) + \sum_{j=1}^{N(t)} w_{ij} \tag{7.2}$$

where $G(t,\gamma^{(i)}) = \gamma_1^{(i)} N(t) + \gamma_2^{(i)} \sum_{j=1}^{N(t)} w_{ij}$.

Assume the critical thresholds for these two degradation processes known as O= {O_1, O_2}, which can be defined as: failure threshold, maintenance threshold or system discrete degradation level threshold. The survival marginal probability function for the *i*th degradation process can be represented as in Wang and Pham (2011):

$$\begin{aligned} m_i(t) &= \mathrm{P}(M_i(t) < O_i) \\ &= \sum_{n=0}^{\infty} \mathrm{P}(X_i \cdot \eta_i(te^{G(t,\gamma^{(i)})}) + S_i(t) < O_i \mid N(t) = n)\,\mathrm{Pr}(N(t) = n) \\ &= \mathrm{P}(X_i \cdot \eta_i(t) < O_i) + \sum_{n=1}^{\infty} \mathrm{P}(N(t) = n) \int_{z=0}^{O_i} \mathrm{P}(X_i \cdot \eta_i(te^{\gamma^{(i)}{}_1 n + \gamma^{(i)}{}_2 z}) + z < O_i)\, dQ_i^{(n)}(z) \\ &= exp(-\lambda t) F_i\left(\frac{O_i}{\eta_i(t)}\right) + \sum_{n=1}^{\infty} \frac{exp(-\lambda t)(\lambda t)^n}{n!} \int_{z=0}^{O_i} F_i\left(\frac{O_i - z}{\eta_i(te^{\gamma^{(i)}{}_1 n + \gamma^{(i)}{}_2 z})}\right) dQ_i^{(n)}(z) \end{aligned} \tag{7.3}$$

where $Q_i^{(n)}(x)$ is the *n*-fold Stieltjes convolution with $Q_i(x)$ itself.

The difficulty for building a joint probability through two marginal functions in this problem is that marginal functions may follow the different distributions. Therefore, the traditional multivariate distributions, which lay the limitation on the same marginal functions, cannot be a solution to model these two dependent processes. Copula method comes to be a promising rescue for this problem due to its advantages over the multivariate distribution: (a) copula method allows to model the marginal behavior and dependence structure separately; (b) copula could provide both the structure and the degree of the dependency; (c) the univariate marginal function could be drawn from different distribution without restriction; and (d) copula is invariant with strictly increasing and continuous transformation. Given C_θ represents the copula function with parameter θ used in the model, the joint survival distribution for the two degradation paths with threshold O= {O_1, O_2} at time t can be expressed in terms of copula function as follows

$$J(t) = \Pr(M_1(t) < O_1, M_2(t) < O_2) = C_\theta\left(m_1(t), m_2(t)\right) \tag{7.4}$$

where θ is the parameter estimated by the copula fitting using the maximum likelihood estimation (MLE) method.

If the system has two degradation paths linked by bivariate Gumbel copula (Ram and Singh (2009), Ram and Singh (2010), Kumar and Singh (2008)), then we can obtain the joint distribution as follows

$$J(t) = \exp\{-[(-\ln m_1(t))^\theta + (-\ln m_2(t))^\theta)^{1/\theta}\} \tag{7.5}$$

The inference for the margins (IFM) method is applied to estimate the parameters for copula function. More detailed description about IFM method can be found in Nelson (2006), Cherubini et al. (2004).

7.2.2 Multi-state System Construction

This chapter considers some continuous probabilistic functions for the degradation paths and the system is degraded in a multi-state way where the system status is determined by the Cartesian product of the degradation levels of two dependent degradation processes with a finite number of discretized degradation levels. The methodology of constructing the multi-state degradation model is based on two-step procedures as in (Li and Pham (2005a)): in the 1st step, we discrete each system degradation process into a finite numbers of intervals by dividing various degradation level thresholds; in the 2nd step, we establish the system state space to combine the different degradation processes by using Cartesian product.

Step 1: Formulate the discrete state space sets for each degradation process

Assume that the sequences $\{A_1, A_2, .., A_{K1}\}$ are the degradation level thresholds associated with the 1st degradation process and $\{B_1, B_2, .., B_{K2}\}$ are the degradation level thresholds associated with the 2nd degradation process. The variables G_1 and G_2 are corresponding failure thresholds for the two degradation processes. Mathematically, the relationship between the discrete degradation states and their corresponding intervals are shown as follows

<u>Degradation Process 1</u>

$$
\begin{aligned}
&0 < M_1(t) \le A_{K_1} \Rightarrow state\ K_1 \\
&A_{K_1} < M_1(t) \le A_{K_1-1} \Rightarrow state\ (K_1-1)_1 \\
&\vdots \\
&A_2 < M_1(t) \le A_1 \Rightarrow state\ 1_1 \\
&G_1 = A_1 < M_1(t) \Rightarrow state\ 0_1
\end{aligned}
$$

<u>Degradation Process 2</u>

$$
\begin{aligned}
&0 < M_2(t) \le B_{K_2} \Rightarrow state\ K_2 \\
&B_{K_2} < M_2(t) \le B_{K_2-1} \Rightarrow state\ (K_2-1)_2 \\
&\vdots \\
&B_2 < M_2(t) \le B_1 \Rightarrow state\ 1_2 \\
&G_2 = B_1 < M_2(t) \Rightarrow state\ 0_2
\end{aligned}
$$

The 1st degradation process has K_1 level of states while the 2nd degradation process has K_2 level of states. Assume $\mathbf{\Omega}_1$ represents the state space for the 1st degradation process, that is, $\mathbf{\Omega}_1 = \{0,1,\ldots, K_1\}$, and $\mathbf{\Omega}_2$ represents the state space for the 2nd degradation process, that is, $\mathbf{\Omega}_1 = \{0,1,\ldots,K_2\}$. The state 0_i is a degradation failure state and the state K_i is an excellent system state. Therefore, the continuous degradation process is divided into discrete levels.

Step 2: Establish the system state using Cartesian product

The system state space is composed of K+2 states, $\mathbf{\Omega}_\mathrm{U} = \{K, \ldots, 1, 0, F\}$. For example, at a given time t when no fatal shock happens yet, suppose the 1st degradation process is in stage i_1 and the 2nd degradation process is in stage j_2, the system state is the output of a mapping relationship function f by Cartesian product from the input space domain, determined by a relationship matrix $\mathrm{H_c}$ as follows

$$H_c = \begin{array}{c} \\ 0_2 \\ 1_2 \\ \vdots \\ K_2 \end{array} \begin{array}{c} \begin{array}{cccc} 0_1 & 1_1 & \ldots & K_1 \end{array} \\ \begin{bmatrix} \times & 0 & \cdots & 0 \\ 0 & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & \cdots & K \end{bmatrix} \end{array}$$

The top row of H_c is the state from the 1st degradation process and the left column is the state from the 2nd degradation process. The system state is characterized as the function of two degradation states shown as in H_c, that is, $f(i_1, j_2) = k$, where k is the system state. It is important to note that the first element in matrix H_c does not exist since all of these three processes of two degradation processes and random shocks are competing risks, that is, the system will fail whichever process meets its failure criteria.

7.2.3 Reliability Estimation

In this section, the system reliability function and mean time to failure are derived analytically based on the system state analysis in section 7.2.2. The model can be used not only to evaluate the reliability for multi-state degraded systems but also to obtain the state probabilities for the systems being in various states. The probability for the system in a catastrophic failure state F, that is, fatal shock failure, can be expressed as

$$\begin{aligned} P_t(F) &= P\{M_1(t) \le G_1, M_2(t) \le G_2, N_1(t) > 0)\} \\ &= P\{M_1(t) \le G_1, M_2(t) \le G_2\} \times \left[1 - P\{N_1(t) = 0\}\right] \\ &= C_\theta(R_1(t;G_1), R_2(t;G_2)) \times \left[1 - e^{-\int_0^t \lambda_1(t)dt}\right] \end{aligned} \tag{7.6}$$

where G_1 and G_2 are corresponding failure thresholds for two degradation paths;

C_θ the copula functions with parameter θ; and

$$R_i(t;G_i) = exp(-\lambda t) F_i\left(\frac{G_i}{\eta_i(t)}\right) + \sum_{n=1}^{\infty} \frac{exp(-\lambda t)(\lambda t)^n}{n!} \int_{z=0}^{G_i} F_i\left(\frac{G_i - z}{\eta_i(te^{\gamma^{(i)}{}_1 n + \gamma^{(i)}{}_2 z})}\right) dQ_i^{(n)}(z).$$

The reliability function $R(t)$ is the sum of the probability of the system being in state $1,\ldots,K$, $R(t)=P\{system\ state \geq 1\} = \sum_{i=1}^{K} P_t(i)$

where $P_t(i)$ is the probability in state i at time t.

The system mean time to failure can be calculated as follows

$$\begin{aligned} E[T] &= \int_0^{\infty} P\{T>t\}dt \\ &= \int_0^{\infty} \left[P\{M_1(t)\leq G_1, M_2(t)\leq G_2\}\times P\{N_1(t)=0\}\right]dt \\ &= \int_0^{\infty} \left[C_\theta(R_1(G_1),R_2(G_2))\times e^{-\int_0^t \lambda_1(t)dt}\right]dt \end{aligned} \tag{7.7}$$

Therefore, the probability density function of the time to failure is computed as

$$\begin{aligned} f_T(t) &= -\frac{d}{dt}\left[P\{T>t\}\right] \\ &= -\frac{d}{dt}\left[P\{M_1(t)\leq G_1, M_2(t)\leq G_2\}\times P\{N_1(t)=0\}\right] \\ &= -\frac{d}{dt}\left[C_\theta(R_1(G_1),R_2(G_2))\times e^{-\int_0^t \lambda_1(t)dt}\right] \end{aligned} \tag{7.8}$$

Example: Assume that the system state spaces associated with the 1st degradation process and 2nd degradation process are, respectively, $\Omega_1=\{3_1,2_1,1_1,0_1\}$, and $\Omega_2=\{2_2,1_2,0_2\}$. The system state space is defined as $\Omega_U=\{3,2,1,0,F\}$ according to the given matrix H_c as follows (Li and Pham (2005a))

$$H_c = \begin{array}{c} \\ 0_2 \\ 1_2 \\ 2_2 \end{array}\begin{array}{c} \begin{array}{cccc} 0_1 & 1_1 & 2_1 & 3_1 \end{array} \\ \begin{bmatrix} X & 0 & 0 & 0 \\ 0 & 0 & 2 & 3 \\ 0 & 1 & 2 & 3 \end{bmatrix} \end{array}$$

Then we can obtain the equivalent classes as follows:

R_0= {(0_1, 1_2), (0_1, 2_2), (1_1, 0_2), (2_1, 0_2), (3_1, 0_2), (1_1, 1_2)}

R_1= {(1_1, 2_2)}

R_2= {(2_1, 1_2), (2_1, 2_2)}

$R_3 = \{(3_1, 1_2), (3_1, 2_2)\}$

and

$$R = \sum_{i=0}^{3} R_i$$
$$= \{(0_1,1_2),(0_1,2_1),(1_1,0_2),(2_1,0_2),(3_1,0_2),(1_1,1_2),(2_1,1_2),(3_1,1_2),(1_1,2_2),(2_1,2_2),(3_1,2_2)\}$$

The corresponding formulas for the system reliability and state probability functions based on this example are derived in next section.

7.3 Numerical Examples

The example in this section aims to illustrate the generalized multi-state model discussed in previous sections. Consider a system subject to three dependent competing risks for two degradation processes and random shocks. Assume the occurring rate of random shocks $\lambda = 1/15$ and the fatal shock probability is proportional to the time, that is, $t/500$. Therefore, the fatal shock occurring rate $\lambda_1(t) = t/7500$. For the first degradation path function, we choose $D(t; X, \theta) = \zeta t$, and $X = \zeta$ Weibull distributed with CDF of $F_X(x) = 1 - \exp[-(x/\mu)^k]$ where $\mu = 1.2$ and k=50. The individual shock loading towards the first degradation process follows the normal distribution with $\mu_w = 5$ and $\sigma_w = 1$. Assume random shock variables in a time-scaled factor $\gamma_1^{(1)} = 0.001$, $\gamma_2^{(1)} = 0.005$ and critical failure threshold G_1=100. For the second degradation path function, we choose $D(t; X, \theta) = \theta_2 \log[\theta_1 + t]$, where $\theta_1 = 2.7$ and θ_2 is gamma distributed with pdf $f_X(x) = x^{a-1} e^{-\frac{x}{b}} \big/ b^a \Gamma(a)$, $x \geq 0$, where a=400 and b=0.01. The individual shock loading follows the exponential distribution with mean $v = 1$. Assume random variables $\gamma_1^{(2)} = 0.06$, $\gamma_2^{(2)} = 0.01$ and critical failure threshold G_2=28. The degradation level thresholds are {A_1=30, A_2=60, A_3=100} for the 1st degradation path and {B_1=18, B_2=28} for the 2nd degradation path. This example follows the same structure of the system level matrix H_c as illustrated in section 7.2.3, and the corresponding system state graph corresponding to the matrix H_c is shown in Figure 7.1. The state {0, 1, 2, 3} is for the system in the stages of "failure", "bad", "fair" and "good".

The degradation path curves with simulated data are shown in Figure 7.2, from which we can see the impact from cumulative shock loadings and time-scaled factors towards the degradation paths. The fatal shock happens at time t=105.7471. The 2nd degradation path is still in a reliable state without passing the degradation failure threshold 28 while the 1st degradation path already crosses its corresponding failure threshold 100. Therefore, the system will fail due to the failure of the 1st degradation process prior to the fatal shock occurring.

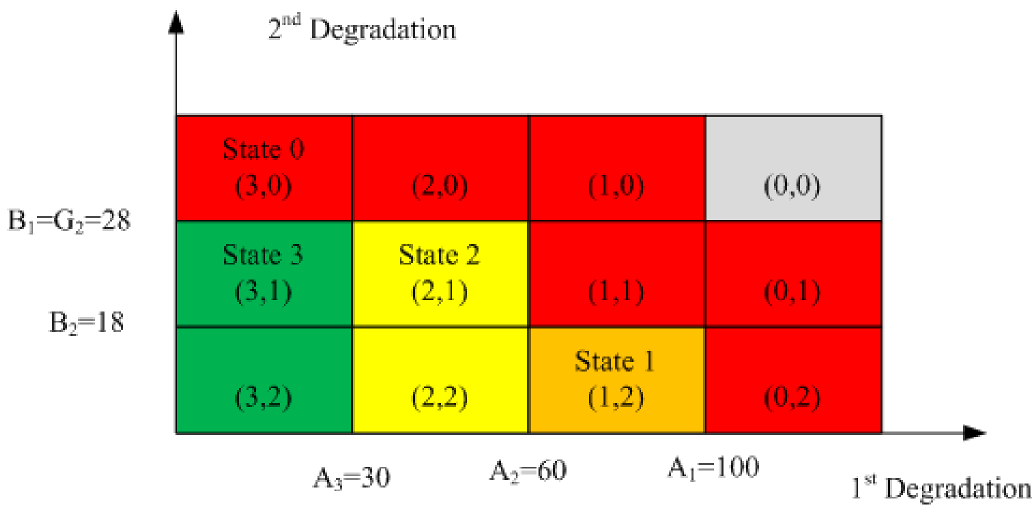


Figure 7.1: System State Graph.

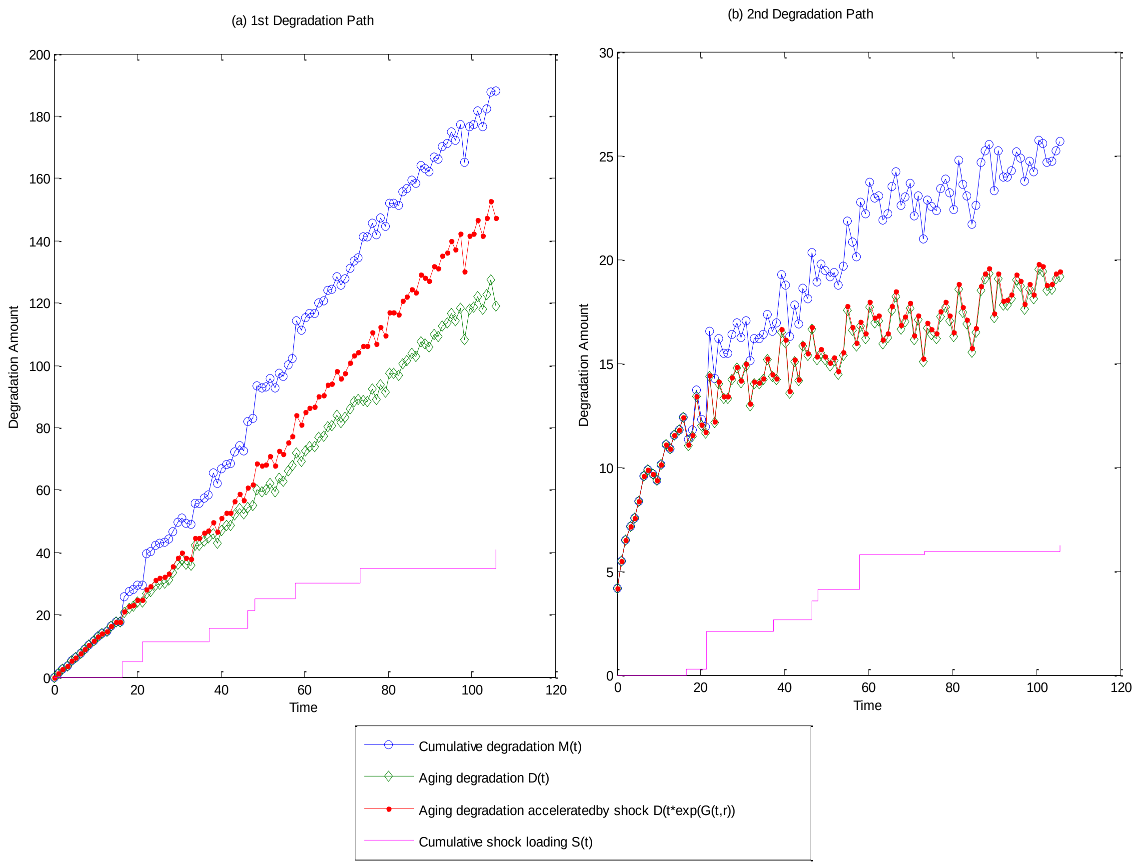


Figure 7.2: Simulated Degradation Paths.

According to the matrix H_c, the probability that the system is in state 0, which is degradation failure, is the sum of the probabilities of $f(0_1,1_2)$, $f(0_1,2_2)$, $f(1_1,0_2)$, $f(2_1,0_2)$, $f(3_1,0_2)$ and $f(1_1,1_2)$. In this chapter, we assume that the specific joint copula function for marginal degradation paths is known as Symmetrized Joe-Clayton (SJC) copula, which a potential copula for good fitness to allow for flexible dependency of both upper and lower tail based on the research of Wang and Pham (2012). The summation is derived as

$$
\begin{aligned}
P_t(0) &= P\{f(0_1,1_2)+f(0_1,2_2)+f(1_1,0_2) \\
&\quad +f(2_1,0_2)+f(3_1,0_2)+f(1_1,1_2)\}\times P\left\{N_1(t)=0\right\} \\
&= \begin{bmatrix} P(M_1(t)>A_1, M_2(t)\le B_1)+P(M_1(t)\le A_1, M_2(t)>B_1) \\ +P(A_2<M_1(t)\le A_1, B_2<M_2(t)\le B_1) \end{bmatrix}\times P\left\{N_1(t)=0\right\} \\
&= \begin{bmatrix} P(M_2(t)\le B_1)+P(M_1(t)\le A_1)+P(M_1(t)\le A_2, M_2(t)\le B_2) \\ -P(M_1(t)\le A_1, M_2(t)\le B_1)-P(M_1(t)\le A_2, M_2(t)\le B_1) \\ -P(M_1(t)\le A_1, M_2(t)\le B_2) \end{bmatrix}\times P\left\{N_1(t)=0\right\} \\
&= \begin{bmatrix} R_2(t;B_1)+R_1(t;A_1)+C_{\theta_1}(R_1(t;A_2),R_2(t;B_2))-C_{\theta_2}(R_1(t;A_1),R_2(t;B_1)) \\ -C_{\theta_3}(R_1(t;A_2),R_2(t;B_1))-C_{\theta_4}(R_1(t;A_1),R_2(t;B_2)) \end{bmatrix}\times e^{-\int_0^t \lambda_1(t)dt}
\end{aligned}
\tag{7.9}
$$

where $N_1(t)$ is number of fatal shock till time t; C_{θ_i} the SJC copula functions with parameter θ_i;

$$
\begin{aligned}
R_1(t;A_i) &= exp(-\lambda t)\left(1-\exp\left(-\left[\frac{A_i}{\mu t}\right]\right)\right) \\
&\quad +\sum_{n=1}^{\infty}\frac{exp(-\lambda t)(\lambda t)^n}{n!}\int_{z=0}^{G_i}\left(1-\exp\left(-\left[\frac{A_i-z}{\mu t e^{\gamma^{(i)}{}_1 n+\gamma^{(i)}{}_2 z}}\right]\right)\right)d\Phi\left[\frac{z-n\mu_w}{\sqrt{n}\sigma_w}\right];
\end{aligned}
$$

$$
\begin{aligned}
R_2(t;B_i) &= exp(-\lambda t)\frac{\gamma\left(a,\dfrac{B_i}{b\log(\theta_1+t)}\right)}{\Gamma(a)} \\
&\quad +\sum_{n=1}^{\infty}\frac{exp(-\lambda t)(\lambda t)^n}{n!}\int_{z=0}^{G_i}\frac{\gamma\left(a,\dfrac{B_i-z}{b\log(\theta_1+te^{\gamma^{(i)}{}_1 n+\gamma^{(i)}{}_2 z})}\right)}{\Gamma(a)}\,d\,\frac{\gamma\left(n,\dfrac{z}{v}\right)}{\Gamma(n-1)}.
\end{aligned}
$$

For each copula function in (7.9), because the threshold level combinations of {A_i, B_i} are varied, marginal degradation probabilities are different. Therefore, totally four SJC copulas with various covariate parameters θ_i are fitted. This is understandable for the reason that during each degradation level combination, the system is in different stages with various dependencies between two degradation paths. At early stage the dependency is weak since both of the two degradation paths are in a good state with high performance and unlikely to impact each other, while after a period of operating they intend to exhibit some degree of dependence due to common usage history and interplaying loadings. Figure 7.3 shows the probability for the system in state 0, where four curves are plotted versus time *t* (exact probability from analytical method, approximate mean probability from Monte Carlo simulation with 95% confidence interval (CI), exact probability from analytical method when λ=0.01). By default, $\lambda = 1/15$.

In Figure 7.3, we observe that the probability of the system being in state 0 is close to zero as *t* approaches to 200. However, a smaller random shock occurring rate results in a lower initial probability of being in state 0 and then higher probability after the approximate crossing point *t* = 76. The Monte Carlo simulation is almost consistent with the results from analytical method. This is a very encourage of our model results. The tiny differences between these two methods may be due to two reasons: (a) analytical method provides an exact probability with specific copula functions while the results from Monte Carlo simulation are only the approximate values; (b) the results from analytical method depend on the performance of the specific selected copula function. The advantage of the Monte Carlo simulation is that it is a simple method to address the complex probability problems and can provide the confidence intervals to check the performance of the simulation.

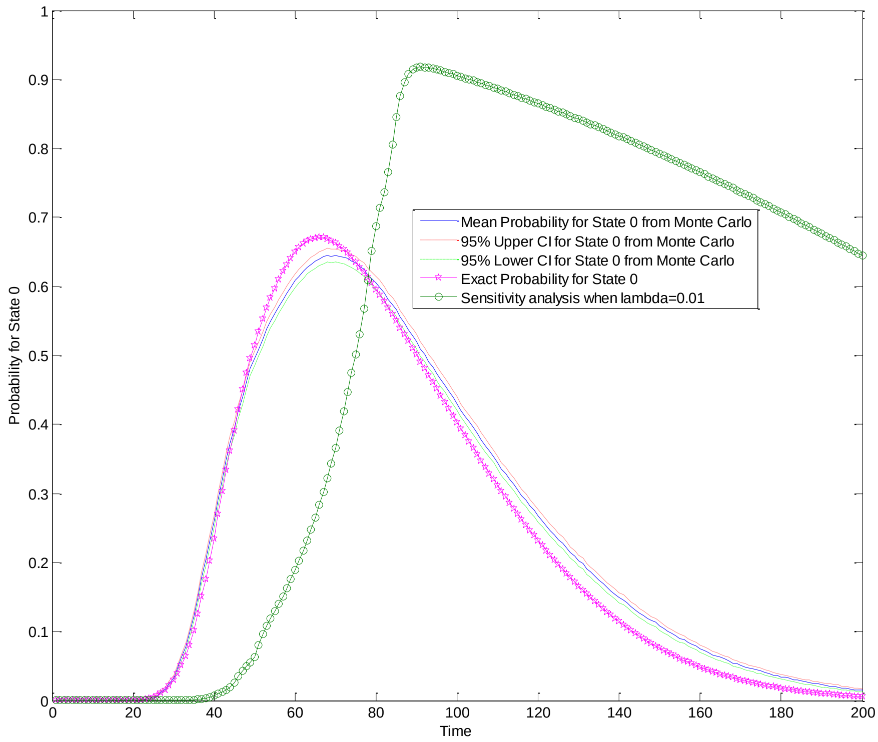


Figure 7.3: Probability plot for state 0.

The probability of the system being in state 1 is calculated as

$$\begin{aligned} P_t(1) &= P_t(f(1_1,2_2)) \\ &= P(A_2 < M_1(t) \le A_1, M_2(t) \le B_2, N_1(t) = 0) \\ &= \left[C_{\theta_4}(R_1(A_1), R_2(B_2)) - C_{\theta_1}(R_1(A_2), R_2(B_2))\right] \times e^{-\int_0^t \lambda_1(t)dt} \end{aligned} \tag{7.10}$$

Figure 7.4 shows the probability of being in state 1 as a function of time *t*, where line with circle marks represents the state probability when λ=0.01, and the line with star marks represents the probability when $\lambda = 1/15$. The time range of being in state 1 at $\lambda = 1/15$ is between 20 and 80 while time range at λ = 0.01 is between 40 and 90.

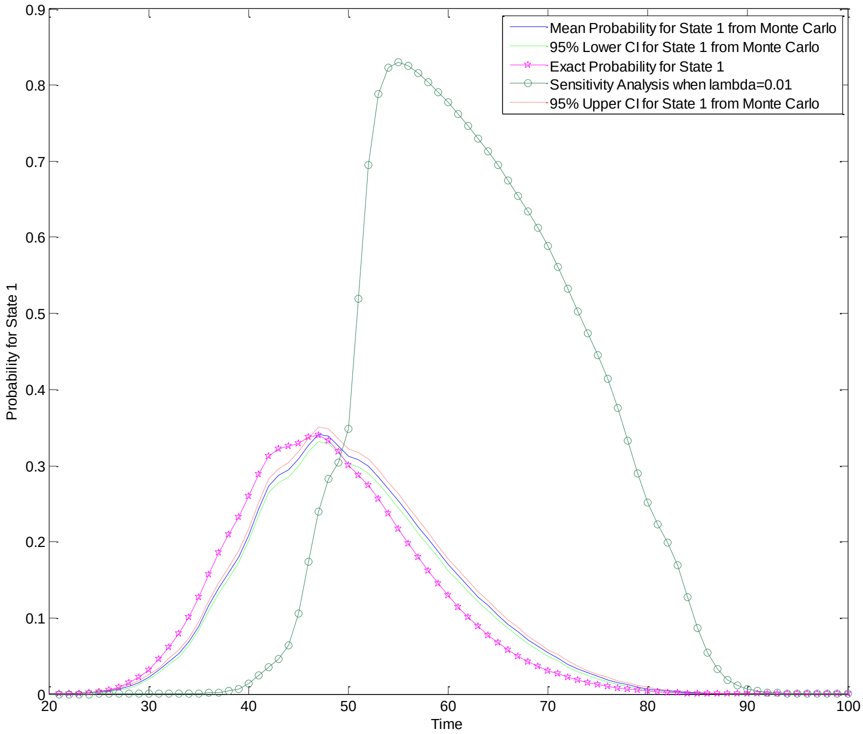


Figure 7.4: Probability plot for state 1.

We can also easily to derive the probability of the system being in state 2 as follows:

$$\begin{aligned}P_t(2) &= P_t(f(2_1,1_2), f(2_1,2_2)) \\ &= P(A_3 < M_1(t) \le A_2, M_2(t) \le B_1, N_1(t)=0) \\ &= \left[C_{\theta_3}(R_1(A_2),R_2(B_1)) - C_{\theta_5}(R_1(A_3),R_2(B_1))\right]\times e^{-\int_0^t \lambda_1(t)dt}\end{aligned} \tag{7.11}$$

Figure 7.5 shows the probability of being in state 2 as a function of time t where line with circle marks represents the state probability when $\lambda = 0.01$, and the line with star marks represents the probability when $\lambda = 1/15$. The results from both analytical method and Monte Carlo simulation match very well for probability in state 2. The non-differentiable behavior of the state probability in Figure 7.5 is because the results are from the subtraction of two copula probabilities.

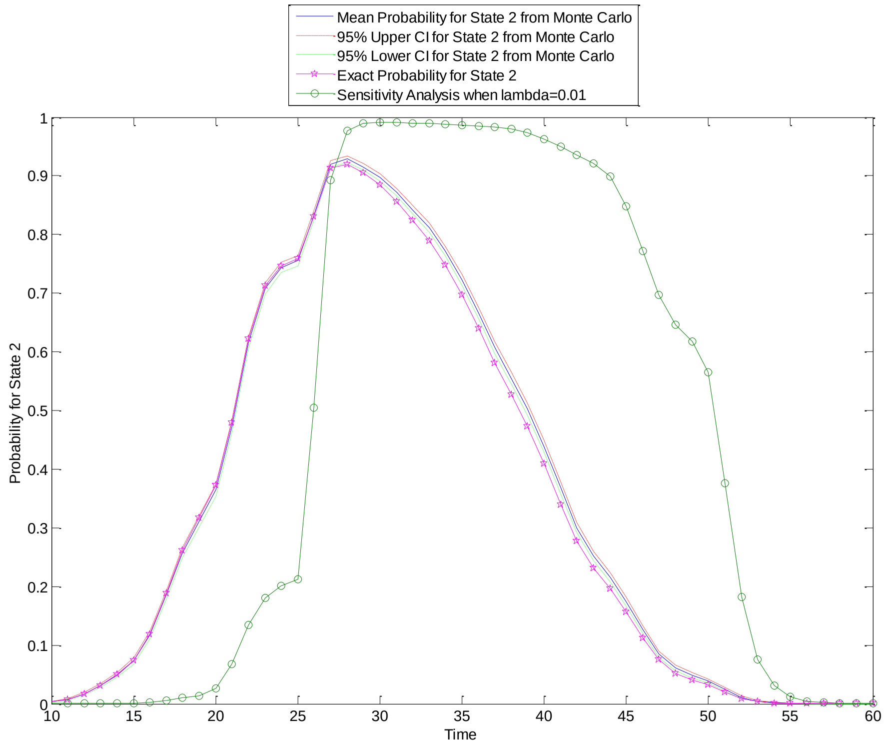


Figure 7.5: Probability plot for state 2.

Similarly, the probability of being in state 3 is obtained as

$$\begin{aligned} P_t(3) &= P_t(f(3_1,1_2), f(3_1,2_2)) \\ &= P(M_1(t) \le A_3, M_2(t) \le B_1, N_1(t) = 0) \qquad (7.12) \\ &= C_{\theta_5}(R_1(A_3), R_2(B_1)) \times e^{-\int_0^t \lambda_1(t)dt} \end{aligned}$$

Figure 7.6 shows the probability of being in state 3 as a function of time t where line with circle marks represents the state probability when $\lambda = 0.01$, and the line with star marks represents the probability when $\lambda = 1/15$. The results from both analytical method and Monte Carlo simulation match very well for probability in state 3. The time range of being in state 3 is almost the same from 0 to 30 for both rates $\lambda = 1/15$ and $\lambda = 0.01$.

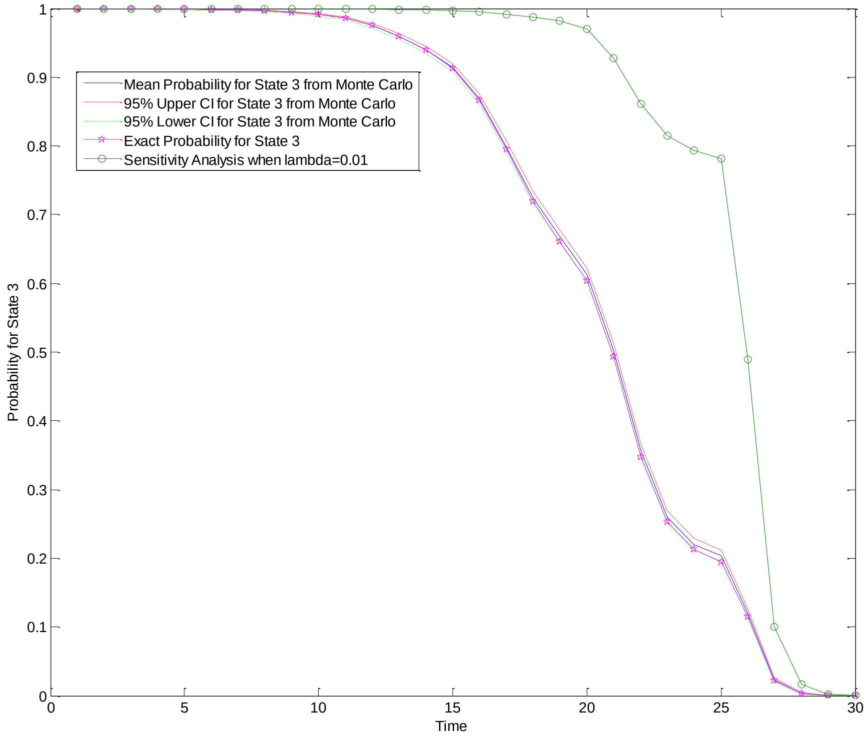


Figure 7.6: Probability plot for state 3.

The probability of being a catastrophic failure state *F* is given by

$$\begin{aligned} P_t(F) &= P\{M_1(t) \le A_1, M_2(t) \le B_1, N_1(t) > 0)\} \\ &= P\{M_1(t) \le A_1, M_2(t) \le B_1\} \times \left[1 - P\{N_1(t) = 0\}\right] \\ &= C_{\theta_2}(R_1(A_1), R_2(B_1)) \times \left[1 - e^{-\int_0^t \lambda_1(t)dt}\right] \end{aligned} \tag{7.13}$$

Figure 7.7 shows the probability of being in state *F* as a function of time *t*, where line with circle marks represents the state probability when λ=0.01, and the line with star marks represents the probability when $\lambda = 1/15$. With smaller occurring rate, the system has a lower chance to fail due to the fatal shocks but later on the system degradation will fail the system with a higher probability.

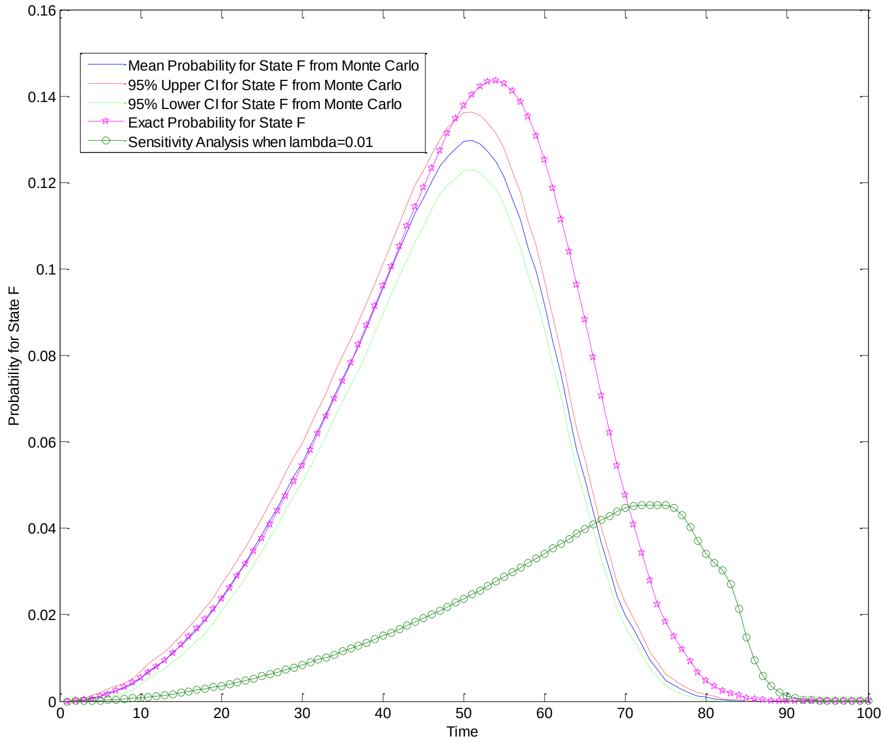


Figure 7.7: Probability plot for state *F*

Finally, the system reliability function $R(t)$ can be obtained as follows:

$$
\begin{aligned}
R(t) &= P\{system\ state \geq 1\} \\
&= \sum_{i=1}^{3} P\left\{ f(R_i) \right\} \\
&= \sum_{i=1}^{3} P_t(i) \\
&= \left[C_{\theta_4}(R_1(A_1), R_2(B_2)) - C_{\theta_1}(R_1(A_2), R_2(B_2)) + C_{\theta_3}(R_1(A_2), R_2(B_1)) \right] \times e^{-\int_0^t \lambda_1(t)dt}
\end{aligned}
$$

(7.14)

Figure 7.8 shows the system reliability as a function with time t for the rates $\lambda = 1/15$ and λ = 0.01. With a smaller shock occurring rate, the system has a higher reliability. The system reliability approaches to zero when t = 100.The system mean time to failure is 44.4946 when λ = 1/15 while the system mean time to failure is 69.0299 when λ = 0.01.

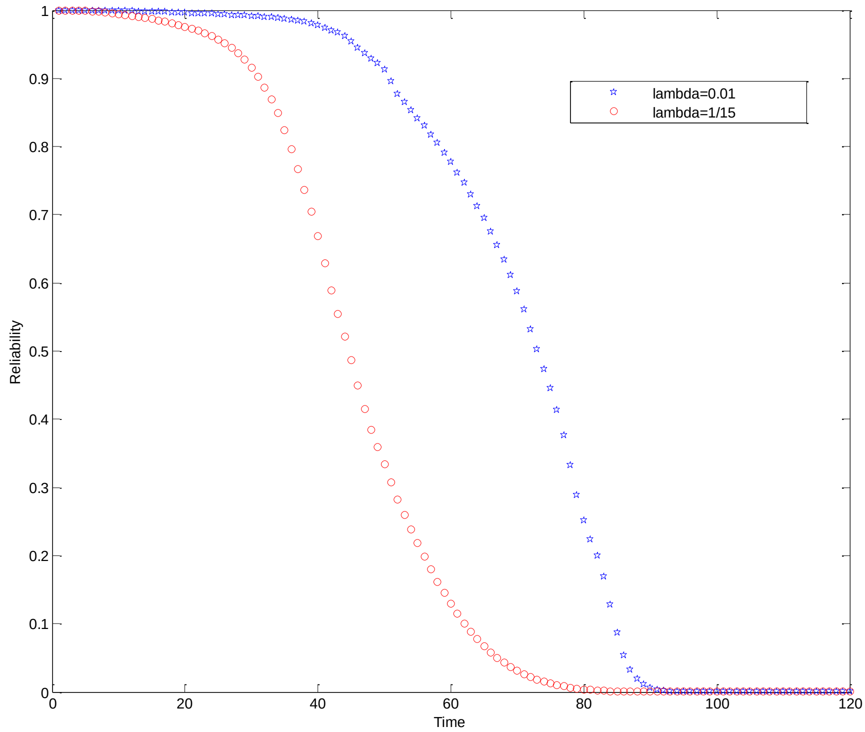


Figure 7.8: Reliability versus time

7.4 Conclusion

In this chapter, a generalized multi-state degradation model is developed to evaluate the reliability and state probability of dependent competing-risk system subject to multiple degradation processes and random shocks without considering the maintenance or repair issues. The results from analytical method and Monte Carlo simulated are compared with each other in the numerical example with the presence of the sensitivity analysis for the shock occurring rate. The fact that the results from both two methods are almost consistent indicates the good performance of copula method to capture the dependent relationship between competing risks. The differences between these two methods are mainly due to two reasons: (a) analytical method provides an exact probability with specific copula functions while the results from Monte Carlo simulation are only the approximate values; (b) the results from analytical method depend on the performance of the specific selected copula function. The advantage of the Monte Carlo simulation is that it is a simple method to address the complex probability problems and can provide the confident intervals to check the performance of the simulation.

Our further research will focus on: (a) Model application to real-life data, including the collection of degradation data, the statistical interference for the parameter estimation of the degradation path and copula functions, and (b) Maintenance cost and system availability for multi-component dependent competing-risk system with maintenance economic correlation.

Chapter 8

Condition-based Threshold-Type Imperfect Preventive Maintenance Policy for Dependent Competing-Risk Systems with Multiple Degradation Processes and Random Shocks

ACRONYMS

CBM	Condition-based Maintenance
PM	Preventive Maintenance
CM	Corrective Maintenance
IFM	The Inference for the Margins
MLE	Maximum Likelihood Estimation
PQD	Positively quadrant dependent

NOTATIONS

θ_i	fixed effect parameter for the *i*th mean degradation path
X_i	random variable for the *i*th multiplicative degradation path
$F_i(x)$	CDF distribution for random variable X_i
$D_i(t; X_i, \theta_i)$	multiplicative path function for the *i*th degradation path with random variable X_i and parameter θ_i
$\eta_i(t, \theta_i)$	deterministic mean for the *i*th degradation path with fixed effect parameters θ_i
$M_i(t)$	the *i*th cumulative degradation till time t
$\gamma_1^{(i)}$	coefficient of nonfatal shock number in the *i*th degradation path
$\gamma_2^{(i)}$	coefficient of cumulative nonfatal shock amount in the *i*th degradation path
$G(t, \gamma^{(i)})$	time-scaled covariate factor to accelerate the degradation rate with

	parameter $\gamma^{(i)}$
O_i	general threshold for the ith degradation path
m_i	marginal survival function for the ith degradation path with threshold O_i
$J(t)$	joint distribution for the degradation paths with threshold O_i
G_i	critical failure threshold for the ith degradation path
L_i	imperfect PM threshold for the ith degradation path
I	imperfect PM interval
N	maximum number of imperfect PM
$R_i(t)$	marginal reliability function of the ith degradation path
$R(t)$	system reliability function
$r_i(t)$	marginal survival function of the ith degradation path with imperfect PM threshold L_i
C	copula function
$\overline{C}$	survival copula function
β	imperfect PM degree
T	passage time for the imperfect PM threshold
T_f	system failure time
A_j	time point of system clock before the jth imperfect PM
B_j	time point of system clock after the jth imperfect PM
H_j	time point of system clock upon the jth imperfect PM
C_1	total maintenance cost in one cycle time
W_2	renewal cycle time
ζ	idle time in one cycle time
C_M	penalty cost per unit time associated with idle time
C_p	cost associated with an imperfect PM
C_C	CM action cost
f_c	fixed cost of purchasing monitoring device
v_c	variable cost per unit time of depreciation and operating expense

N_{p}	imperfect PM number
p_k	probability that degradation failure happens in the kth period
P_j	probability that there are a total number of j imperfect PMs in the renewal cycle
$F_T(t)$	distribution of the first passage time T for the imperfect PM threshold
$F_{T_f}(t)$	distribution of the system degradation failure time T_f

8.1 Introduction

Competing risk is an important issue in reliability field, especially degradation processes and random shocks. Li and Pham (2005a, b) study the reliability estimation model and develop a condition-based maintenance policy for periodically inspected systems subject to competing failures of two degradation processes and random shocks, but all of these processes are independent and maintenance is perfect to renew the system as good as new. However, in reality, many systems have multiple components with more than one degradation measure which is dependent with each other due to its interplaying function or common usage history. Independent assumption may underestimate the reliability of the system (Wang (2003)). On the other hand, random shocks can also contribute to the system failure in two ways: one is working directly on the degradation processes; the other is causing immediate failure to the system. This chapter aims to relax the independent assumption of Li and Pham (2005a, b) by considering dependent structures including not only dependency among multiple degradation measures but also between degradation measures and random shocks.

Lehmann (2009) surveys two classes of degradation-threshold-shock models (DTS) such as general DTS and DTS with covariates, where the system failure may be due to the competing risk of degradation and trauma. Compared with the time-based replacement policy, Huynh et al. (2011) recently introduce a condition-based maintenance model for the degradation-threshold-shock (DTS) model to take the dependence between degradation process and shock process into account. Deloux et al. (2009) propose a maintenance policy that combines the statistical process control (SPC) and condition-based maintenance (CBM) for a continuously deteriorating system with two kinds of failure mechanisms (deteriorating and random shocks).

Sun et al. (2006) introduce an analytical model to quantitatively estimate the interactive failure and its failure rate based on three difference cases. A later work by Sun et al. (2009) develops an extended split system approach for the failure interactions. Simulated data is used to test the proposed model and the results indicate that the PM intervals for newly repair components with the presence of failure interactions will become shorter compared with the system without failure interactions. Wang and Pham (2012) recently develop a dependent competing risk model, in which the dependent structure of random shock and degradation is modulated by a time-scaled covariate factor, and the dependent structure among degradation processes is fitted by both constant and time-varying copulas, for a system subject to multi-degradation measures and random shocks, but without considering the maintenance issues.

Furthermore, this chapter also extends the perfect PM to imperfect PM which makes the problem more practical. Cassady et al. (2005) explore the imperfect repair based on the Kijima's first virtue age model by validating the simulation results using 2^3 factorial experiment and converting reliability & maintainability parameters into coefficients of availability model using meta-models to determine the optimal replacement interval according to the system average cost. Liu and Huang (2010) apply the non-homogeneous continuous time Markov model (NHCTMM) to model the optimal replacement policy for the multi-state systems with the imperfect maintenance that utilizes the quasi-renewal process to describe the stochastic behavior of the multi-state aging element after each imperfect repair. Wang and Pham (2011) study a multi-objective maintenance optimization embedded within the imperfect PM and replacement for one single-unit system subject to the dependent competing risk of degradation wear and random shocks. Satow and Kawai (2010) put forward an imperfect inspection with upper and lower inspection threshold for a bivariate failure distribution.

In this chapter, we provide a literature review on related maintenance aspects including condition-based maintenance, imperfect preventive maintenance, independent and dependent competing risks, degradation and random shocks processes in section 8.1. We then discuss our proposed generalized condition-based maintenance model for dependent competing risk systems with multiple degradation processes and random shocks including assumptions. In section 8.3, a mathematical modeling for the optimal maintenance cost rate function will be derived analytically.

In section 8.4, a numerical example with sensitivity analysis will be used to illustrate the application of the proposed model. In section 8.5, a general conclusion will be discussed.

8.2 Model Description

In this chapter, an optimal maintenance strategy with condition-based imperfect PM thresholds is studied for a dependent competing risk system subject to two degradation paths measured by a pair of non-decreasing processes $\{M_1(t), M_2(t)\}_{t\geq 0}$ and random shocks characterized by a homogeneous Poisson process $\{N(t)\}_{t\geq 0}$. Generally, maintenance can be classified into two categories: condition-based or time-based. For the former, the action is performed when need arises, that is, one or more system performance indicators show that the system is going to fail or the system is seriously deteriorating. For the latter, the maintenance action is taken at some predetermined time points to improve the system functioning condition. Our model in this chapter focuses on the condition-based maintenance aspect because it is more effective to optimize system maintenance resources although it may require for periodic or continuous condition monitoring.

Two maintenance actions are considered in this model: the imperfect preventive maintenance (PM) restores the system condition to a younger state between "as good as new" and the current state instead of perfect PM with full restoration; the corrective maintenance renews the system "as good as new". Four decision variables will be determined optimally by minimizing the long-run expected system cost rate in this maintenance scheduling: maintenance threshold level for the first degradation process $\{L_1\}$, maintenance threshold level for the second degradation process $\{L_2\}$, imperfect PM interval $\{I\}$, and imperfect PM number till replacement $\{N\}$. Before any degradation level exceeds the imperfect PM thresholds, the maintenance strategy is doing nothing; otherwise, imperfect PM is taken periodically to bring back the system to a younger state between “as good as new” and the current state. Corrective maintenance (CM) is performed under two conditions: degradation failure between each inspection interval or maximum imperfect PM number till replacement, which occurs first. Our chapter aims to combine all of these factors into one model: condition-based maintenance with threshold, imperfect preventive maintenance and multiple dependent relationships between competing risks. Below is a list of some basic assumptions and notation will be used in the modeling.

Assumptions

A. Three non-decreasing processes of two degradation measures and one homogeneous Poisson process are pairwise dependent. The dependent relationships among these three processes are simulated using the method in Wang and Pham (2012): (1) the dependent structure between degradation measures and random shocks is modulated by a time-scaled covariance; (2) the dependent structure among degradation measures is linked by copula method.

B. The system is continuously monitored till imperfect PM threshold. This means that the system degradation level can be detected instantly. Afterwards, the system is switched to periodic inspection at each imperfect PM interval before replacement.

C. Monitoring cost should be taken into consideration, consisting of two parts: fixed cost of purchasing monitoring devices and variable cost of depreciation and operation expense.

D. An improvement factor is introduced into the model to reflect the effect of imperfect PM by rejuvenating the system clock back to a younger age.

E. Once any of the degradation level crosses the failure threshold $\{G_1, G_2\}$ between each inspection interval, the system will remain non-functioning and require a replacement at next inspection time, that is, the system suffers from hidden failure.

F. A CM action is more costly than an imperfect PM but the cost associated with imperfect PM is varied with the imperfect PM degree β.

G. Maintenance time is negligible.

8.3 Mathematical Modeling

8.3.1 Dependent Competing Risk Model

The system starts at a brand new condition subject to two degradation processes $\{M_1(t), M_2(t)\}_{t\geq 0}$ and a homogeneous Poisson process $\{N(t)\}_{t\geq 0}$. Assuming that the random shocks arrive with occurring rate λ and $N(t)$ denotes a random variable that represents the number of random shocks till time t. The sequence of random shock loadings w_{ij} is nonnegative, independent and identically distributed with a common distribution $Q_i(x)$ for all jth shock in ith degradation path.The random shocks have two impacts towards the degradation path: one is sudden increment jump cumulative to the degradation path characterized as a compound Poisson process; and the other is degradation rate acceleration which is reflected as a time-scaled covariate governed by shock number and cumulative shock loadings. Assume the cumulative shock loading till time t in ith degradation path is $S_i(t)$ for i = 1, 2 which can be represented as a compound Poisson process

$$S_i(t) = \sum_{j=0}^{N(t)} w_{ij} \tag{8.1}$$

The basic multiplicative degradation path in Bae et al. (2007) for the ith degradation process is used to model the degradation process in order to tolerate the item-to-item variation by including the random variable X_i, is given by

$$D_i(t; X_i, \theta_i) = X_i \eta_i(t; \theta_i) \tag{8.2}$$

where $\eta_i(t;\theta_i)$ denotes a deterministic mean degradation path with fixed effect parameters θ_i for time $t \geq 0$; the variable X_i represents the random variation around the mean degradation level, distributed with a common cumulative distribution function (CDF) $F_i(x)$. The mean degradation path can be either monotonically decreasing or increasing, called decreasing degradation path (DDP) or increasing degradation path (IDP), respectively. More examples of multiplicative degradation path with practical applications can be found in Bae et al. (2007). There also exist some studies for the cases of non-monotonic degradation, such as the light display in

Bae and Kvam (2004, 2006). However, such case is not in the research scope of this chapter.

The total degradation wear in the ith degradation path is cumulative effects from two aspects: one is aging with time and the other is instantaneous damage induced by random shocks, which can be expressed as $M_i(t) = D_i(t) + S_i(t)$. The first term is the multiplicative degradation function for the ith degradation path and the second term $S_i(t) = \sum_{j=0}^{N(t)} w_{ij}$ reflects the sudden increment jump from the impact received by the random shocks towards the ith degradation path. Furthermore, in order to capture the extra effect of the degradation rate acceleration, we introduce a new function $G(t, \gamma^{(i)})$ into the degradation path by borrowing the original idea from the time-scaled model of accelerated life testing. The aging amount in ith degradation process $D_i(t)$ is scaled by an accelerated factor from t to $te^{G(t,\gamma^{(i)})}$. Therefore, the cumulative degradation wear function in the ith degradation process can be defined as Wang and Pham (2011):

$$M_i(t) = X_i \eta_i (te^{G(t,\gamma^{(i)})}; \theta_i) + \sum_{j=1}^{N(t)} w_{ij} \tag{8.3}$$

where $G(t, \gamma^{(i)}) = \gamma_1^{(i)} N(t) + \gamma_2^{(i)} \sum_{j=1}^{N(t)} w_{ij}$. The form of the function $G(t, \gamma^{(i)})$ is assumed to be known but with some unknown parameter vector $\gamma^{(i)}$.

The first item in the above function $G(t, \gamma^{(i)})$ reflects the impact received from the number of random shocks towards the degradation process. Usually, we have $\gamma_1^{(i)} \geq 0$ and then the first term can be viewed as representing the fact that the degradation rate is more likely to increase with the random shocks number. The second term $\gamma_2^{(i)} \sum_{j=1}^{N(t)} w_{ij}$ for $\gamma_2^{(i)} \geq 0$ is developed to modulate the situation that the cumulative shock amount will contribute to an acceleration of the system degradation rate.

8.3.2 Joint Distribution Estimation by Copula Method

The measurements of the two degradation paths are random variables $M(t)=\{M_1(t),M_2(t)\}$ at observation time point *t*. Assume there exist various thresholds for these two degradation processes known as $O = \{O_1, O_2\}$, which may be failure threshold, maintenance threshold, or system discrete degradation level threshold. The survival marginal probability for the *i*th degradation process with threshold O_i can be represented as in Wang and Pham (2011)

$$\begin{aligned}
m_i(t) &= \Pr(M_i(t)<O_i)\\
&= \sum_{n=0}^{\infty}\Pr(X_i\cdot\eta_i(te^{G(t,\gamma^{(i)})})+S_i(t)<O_i \mid N(t)=n)\Pr(N(t)=n)\\
&= \Pr(X_i\cdot\eta_i(t)<O_i)+\sum_{n=1}^{\infty}\Pr(N(t)=n)\int_{z=0}^{O_i}\Pr(X_i\cdot\eta_i(te^{\gamma^{(i)}{}_1 n+\gamma^{(i)}{}_2 z})+z<O_i)dQ_i^{(n)}(z)\\
&= exp(-\lambda t)F_i\left(\frac{O_i}{\eta_i(t)}\right)+\sum_{n=1}^{\infty}\frac{exp(-\lambda t)(\lambda t)^n}{n!}\int_{z=0}^{O_i}F_i\left(\frac{O_i-z}{\eta_i(te^{\gamma^{(i)}{}_1 n+\gamma^{(i)}{}_2 z})}\right)dQ_i^{(n)}(z)
\end{aligned} \tag{8.4}$$

where $Q_i^{(n)}(x)$ is the *n*-fold Stieltjes convolution with $Q_i(x)$ itself.

A tool to joint marginal probability with different distributions is called for due to the dependency between various degradation paths. Therefore, the fact that the traditional multivariate distributions lay limitation on the same marginal functions makes copula method a promising rescue for this problem. There are many advantages of using copula method over the multivariate distribution such as: (1) copula allows us to model the marginal behavior and dependence structure separately; (2) copula could provide both the degree and the structure of the dependence; (3) the univariate marginal function can be drawn from different distribution without restriction; (4) copulas are invariant under strictly increasing and continuous transformation. If *C* is the copula of the marginal distributions, the joint survival distribution for the two degradation paths with threshold $O = \{O_1, O_2\}$ at time t can be expressed in terms of copula method as follows:

$$J(t) = \Pr(M_1(t) < O_1, M_2(t) < O_2) = C_\alpha\left(m_1(t), m_2(t)\right) \tag{8.5}$$

where α is the parameters estimated by the copula fitting using the maximum likelihood estimation (MLE).

If the system has two degradation paths linked by bivariate Gumbel copula (Ram and Singh (2009, 2010), Kumar and Singh (2008)) then we can derive the joint distribution as follows

$$J(t) = \exp\{-[(-\ln m_1(t))^\alpha + (-\ln m_2(t))^\alpha)^{1/\alpha}\} \tag{8.6}$$

The inference for the margins (IFM) method is used to estimate the parameters for copula function.The detailed description of IFM method can be found in Nelson (2006), Cherubini (2004).

8.3.3 Threshold-type Condition-based Maintenance

8.3.3.1 Maintenance Planning

A condition-based maintenance model is considered with imperfect PM threshold and critical failure threshold of the first degradation process $\{L_1, G_1\}$ and the second degradation process $\{L_2, G_2\}$. The system is continuously monitored till imperfect PM threshold so that the state of each degradation level can be recognized immediately. After that the system is inspected and maintained with imperfect PM periodically at equal time sequences $jI, j = 1, 2, ..., N$. The maximum number of imperfect PM till replacement $\{N\}$ means that there will be at most (N-1) imperfect PMs and at the next inspection time NT a replacement will be performed. According to the system degradation state detected, one of the following actions should be taken:

a. If both degradation values are below their imperfect PM thresholds, in other words, $\{M_1(t) < L_1\} \cap \{M_2(t) < L_2\}$, then the system is still in good condition. In this case, we do nothing but leave the system as it was.

b. If there exists any of the degradation processes falls into the imperfect PM zone, that is, $\{L_1 < M_1(t) < G_1\} \cup \{L_2 < M_2(t) < G_2\}$, then the system is called for an imperfect PM action which is done periodically at time $\{I, 2I, ..., jI, ...\}$.

c. If there exists any of the degradation process value is exceeding their corresponding critical threshold, that is, $\{M_1(t) > G_1\} \cup \{M_2(t) > G_2\}$, then the system is called for a CM action. In this case, the system has degradation failure and a CM is performed to renew the system.

d. If the system fails due to any degradation process between the inspection intervals, the system remains non-functioning and will be renewed as good as new by CM action at next inspection time point, which means the system suffers from hidden failure.

e. At each imperfect PM time point, if the system is found failure due to degradation, replacement is done to renew the system; otherwise imperfect PM will be performed till maximum imperfect PM number approaches.

We assume that after a CM action, system will restore as good as new, while upon each imperfect PM action, system will bring back to a younger state between "as good as new" and the current state, which depend on the imperfect PM degree β. Assume T

denote the time point for the first passage time when any of the degradation process level exceeds the imperfect PM critical threshold, I the imperfect PM interval and T_f the time point of the system degradation failure. Figure 8.1 shows the evolution of the system where $M_i(t)$ represents the ith degradation path level, L_i and G_i the imperfect PM and CM critical threshold for the ith degradation path, respectively, for i=1,2. Figure 8.2 shows the maintenance zone projected on the $M_1(t)$, $M_2(t)$ planes.

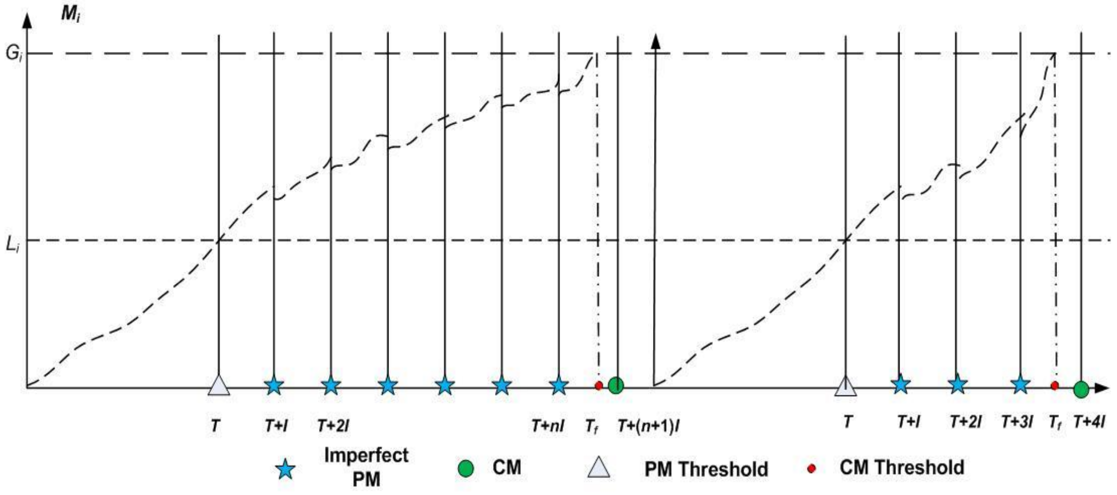


Figure 8.1: Evolution of the system condition

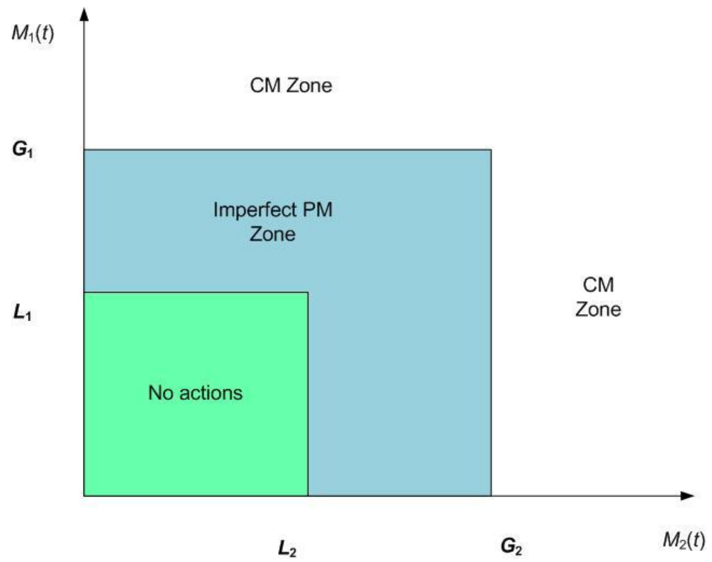


Figure 8.2: Maintenance zone projected on $M_1(t)$, $M_2(t)$

8.3.3.2 Imperfect PM Method

Perfect maintenance or repair that assumes the system is as good as new after each PM or repair is not actually realistic in practical world. A more practical assumption is that, instead of perfect maintenance, an imperfect PM will restore the degradation process of the system to some younger system status between “as good as new” and “as old as bad”. In the literatures, there are seven treatment methods for imperfect maintenance by Wang and Pham (2003): (p, q) rule (Li and Shaked (2003), Cha and Kim (2001), Brown and Proschan (1983)), ($p(t)$, $q(t)$) rule (Block et al. (1985), Sumita and Shanthikumar (1985)), improvement factor (Cheng and Chen (2003), Pascual and Ortega (2006), Kijima et al. (1988)), virtual age (Finkelstein (2010)), shock model (Wang and Pham (1996a)), (α, β) rule (Wang and Pham (1996b)) or quasi-renewal process (Rehmert and Nachlas (2009), Wang and Pham (1996c)), multiple (p, q) rule (Shaked and Shanthikumar (1986)). In order to model the imperfect PM, we introduce a time reduction factor β, that is, upon each imperfect PM the system time of the degradation process is reduced by the factor β proportion to the previous system time, as shown in Figure 8.3, where the system maintenance time is negligible. We assume that there are two clock time in the system: one is system clock which represents the age of the system physical mechanism; the other is actual clock which is the actual age of the system with the time.

In the biological body systems for example, as the age of twenty or younger the major organ systems usually are in a good condition, and people may not care much about human-body self-maintenance such as exercising and taking dietary supplement. We can view the system clock the same pace as the actual clock. However, as with the time goes, people will begin to pay more attention to their body aging process. So they will try to take some actions to maintain their body system, including a regular exercise, or a scheduled medical diagnose and treatment, which would help to slow down the aging process of the body system with some degree. Therefore, perhaps a twenty-eight years old person will have a younger body system condition with a system clock indicating an age of twenty-five because of the regular body maintenance. In this case, twenty-eight is the actual system age while twenty-five is the physical system clock. Of course this kind of maintenance cannot be a perfect maintenance.

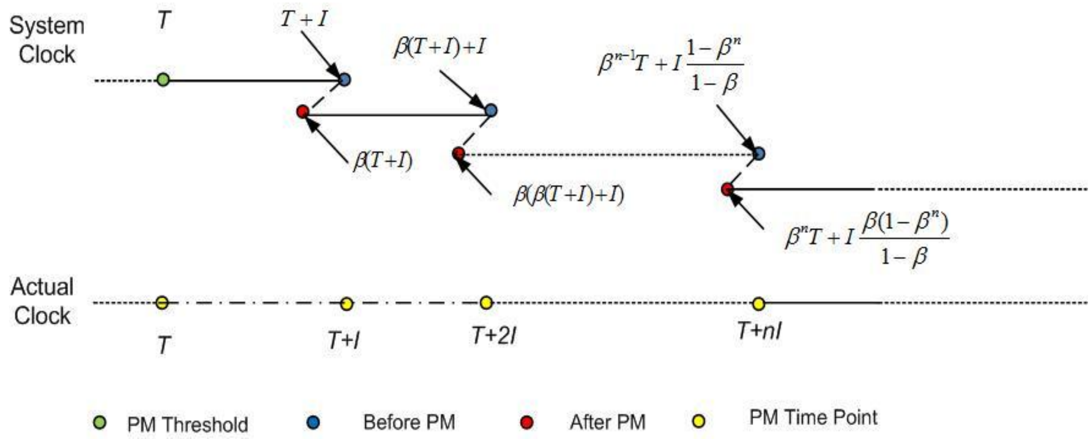


Figure 8.3: Imperfect preventive maintenance

Before the first imperfect PM of time period $[0,T+I)$, the system time is the same as the actual time. But with the restoration of imperfect PM, the system time is preceding to the actual time. Upon each imperfect PM, the system time is restored by a reduction factor of β ranging from 0 to 1. If β equals to 0, it is a perfect PM which can renew the system to the brand new condition. The imperfect PM continues until the system degradation failure interrupts and triggers a CM action or the maximum number of imperfect PM till replacement reaches.

As shown in Figure 3, before the first imperfect PM, the system time is $T+I$, but upon the imperfect PM, the system clock is restored to $\beta(T+I)$. If the system survives during the next imperfect PM interval, the same situation continues. The time point of system clock before the jth imperfect PM is equal to the time after the (j-1)st imperfect PM plus the imperfect PM interval. Therefore, before the second imperfect PM, the system time is $(\beta(T+I) + I)$, but upon the second imperfect PM, it is restored to $\beta(\beta(T+I)+I)$. Similarity, we can derive the formulation for the two sequences A_j and B_j. The sequence of A_j, which is the time point of system clock before the jth imperfect PM, can be represented as

$$A_j = \begin{cases} T & for \;\; j=0 \\ \beta^{j-1}T + I\dfrac{1-\beta^j}{1-\beta} & for \;\; j=1,2,...,N \end{cases} \tag{8.7}$$

The sequence of B_j, which is the time point of system clock after the jth imperfect PM, can be obtained

$$B_j = \beta A_j = \beta^j T + I\beta\frac{1-\beta^j}{1-\beta} \quad for \;\; j=0,1,2,...,N-1 \tag{8.8}$$

The sequence of H_j is the time point of the actual age clock upon the jth imperfect PM since the system maintenance time is negligible, given by

$$H_j = T + jI \qquad for \;\; j=0,1,2,...N \tag{8.9}$$

Without the imperfect PM, the system time should be t, while upon imperfect PM, thesystem time becomes t-H_j+B_j because the cumulative degradation restoration amount is the difference between the B_j and H_j. Therefore, according to the formula derivation in section 8.3.2, the system reliability in terms of copula function can be represented as

$$R(t)=\begin{cases}C_{\theta_1}(R_1(t),R_2(t)) & 0\le t<T\\ C_{\theta_1}(R_1(t-C_j+B_j),R_2(t-C_j+B_j)) & C_j\le t<C_{j+1}<T_f,\ j=0,2,\ldots,N-1\end{cases} \tag{8.10}$$

where

$$R_i(t)=exp(-\lambda t)F_i\left(\frac{G_i}{\eta_i(t)}\right)+\sum_{n=1}^{\infty}\frac{exp(-\lambda t)(\lambda t)^n}{n!}\int_{z=0}^{G_i}F_i\left(\frac{G_i-z}{\eta_i(te^{\gamma^{(i)}{}_1 n+\gamma^{(i)}{}_2 z})}\right)dQ_i^{(n)}(z)\,;$$

C_{θ_1} is the copula function with parameter θ_1 fitted by MLE;

T_f is the time point for the system degradation failure.

8.3.4 Average Long-Run Maintenance Cost Analysis

In this section an explicit expression for the average long-run cost rate is derived to optimize the condition-based maintenance policy through determining the imperfect PM critical threshold $\{L_1, L_2\}$, imperfect PM interval $\{I\}$ and imperfect PM number till replacement $\{N\}$.

8.3.4.1 Expected Maintenance Cost Rate within a Cycle

According to the renewal theory, the expected long-run maintenance cost rate during one cycle is given by

$$EC[L_1,L_2,I,N]=\frac{E[C_1]}{E[W_1]}$$

where C_1 is the total maintenance cost in one cycle time, W_1 is the renewal cycle time, and the variables L_1, L_2, I, N are parameters determined by maintenance optimization. Assume N_p be the number of imperfect PMs within one cycle time and $E[\xi]$ be the expected idle time in one cycle time when the event happens, that is, the system fails due to degradation failure from any of the two degradation processes during the inspection intervals so that the system will be idle until the next inspection time to require a CM action. Let us denote $E[T]$ the expected passage time for the imperfect PM threshold where T is the first passage time for the imperfect PM threshold.Therefore, the mean maintenance cost per cycle, $E[C_1]$, is given as

$$E[C_1] = C_m E[\zeta] + C_p E[N_p] + C_c + f_c + v_c E[T]$$

where C_m is the penalty cost per unit time associated with idle time, C_p is the cost associated with an imperfect PM, and C_c is the CM action cost. Without loss of generality, we assume that $C_c \geq C_p \geq C_m$. The monitoring cost here consists of two parts: fixed cost of purchasing monitoring device $\{f_c\}$ and variable cost per unit time of depreciation & operating expense $\{v_c\}$. Because the system is continuously monitored till the imperfect PM threshold T, the variable cost of monitoring cost is only taken into account during the period $[0,T)$.

The calculations for the maintenance cost rate function $EC[L_1, L_2, I, N]$ are now discussed:

1) Let $\mathrm{P}\{N_p = j\}$ be the probability that there are a total number of j imperfect PMs in the renewal cycle, for $j = 0,1,2,...,N$. Based on the assumptions in section 8.2.2.1, there exist j imperfect PMs during one cycle time if the degradation failure occurring within the time interval $(H_j, H_{j+1}]$. In other word, the imperfect PM will be terminated whenever the current inspection detects that the system suffers from the degradation failure in the previous operating interval while such situation is not revealed upon the previous inspections or the maximum inspection PM number till replacement approaches.

In order to obtain the probability for j imperfect PMs in one cycle, we need to study the probability that the degradation failure occurs in different time periods first. We can see that the total (N-1) imperfect PMs and one replacement can divide the time space into (N+2) periods, that is,

$$t = \begin{cases} [0,T) \\ [H_j, H_{j+1}) \quad for \;\; j = 0,1,2,...,N-1 \\ [H_N, \infty) \end{cases}$$

Suppose p_k the probability that the degradation failure happens in the kth period. If the degradation failure happens in the jth period, marked the event as E_1, the system should firstly survive in the previous (j-1) st periods, marked the event as E_2. So the survival probability is equal to $1-\sum_{k=0}^{j-1} p_k$. If we do not consider the imperfect PM, the probability space should be continuous, but due to the imperfect PM, the survival probability before (j-1)st imperfect PM is different from that after imperfect PM. Assume the event E_2: "the system survival before (j-1)st imperfect PM" and event E_2: "the system survival after (j-1)st imperfect PM". The probability for the system degradation failure in the jth period is

$$\begin{aligned} p_j &= \Pr(E_1) = \frac{\Pr(E_1 E_2^{"})}{\Pr(E_2^{"})} \Pr(E_2^{'}) \\ &= \frac{\Pr(M_1(B_{j-1}) < G_1, M_2(B_{j-1}) < G_2) - \Pr(M_1(A_j) < G_1, M_2(A_j) < G_2)}{\Pr(M_1(B_{j-1}) < G_1, M_2(B_{j-1}) < G_2)} \left[1 - \sum_{k=0}^{j-1} p_k \right] \end{aligned} \tag{8.11}$$

Therefore the probability can be derived as

$$\begin{cases} p_0 = 1 - \Pr(M_1(B_0) < G_1, M_2(B_0) < G_2) \\ p_1 = \Pr(M_1(B_0) < G_1, M_2(B_0) < G_2) - \Pr(M_1(A_1) < G_1, M_2(A_1) < G_2) \\ p_2 = \dfrac{\Pr(M_1(B_1) < G_1, M_2(B_1) < G_2) - \Pr(M_1(A_2) < G_1, M_2(A_2) < G_2)}{\Pr(M_1(B_1) < G_1, M_2(B_1) < G_2)} \Pr(M_1(A_1) < G_1, M_2(A_1) < G_2) \\ \vdots \\ p_j = \dfrac{\Pr(M_1(B_{j-1}) < G_1, M_2(B_{j-1}) < G_2) - \Pr(M_1(A_j) < G_1, M_2(A_j) < G_2)}{\Pr(M_1(B_{j-1}) < G_1, M_2(B_{j-1}) < G_2)} \left[1 - \sum_{k=0}^{j-1} p_k\right] \\ \vdots \\ p_N = \dfrac{\Pr(M_1(B_{N-1}) < G_1, M_2(B_{N-1}) < G_2) - \Pr(M_1(A_N) < G_1, M_2(A_N) < G_2)}{\Pr(M_1(B_{N-1}) < G_1, M_2(B_{N-1}) < G_2)} \left[1 - \sum_{k=0}^{N-1} p_k\right] \\ p_{N+1} = 1 - \sum_{k=0}^{N} p_k \\ \sum_{j=0}^{N+1} p_j = 1 \end{cases}$$

(8.12)

where A_j is the time point of the system clock before the jth imperfect PM, B_j is the time point of the system clock after the jth imperfect PM, G_i is the critical failure threshold for the ith degradation process, $M_i(t)$ is the cumulative amount of the ith degradation wear.

However, because T is the observation time for the first passage time of imperfect PM threshold, $\Pr(M_1(B_0) < G_1, M_2(B_0) < G_2) = 1$ and then $p_0 = 0$. Therefore, the probability for degradation failure in $[0, T)$ should be truncated from the probability space. Then we have

$$P_j = P\{N_p = j\} = {p_{j+1}} \Big/ {1 - p_0} \quad \text{for} \quad j = 0, ..., N-1. \tag{8.13}$$

Assume $P_N = {p_{N+1}} \Big/ {1 - p_0}$, it is obvious that $\sum_{j=0}^{N} P_j = 1$.

Therefore the expected number of imperfect PMs during one cycle time can be obtained as

$$E[N_p] = E\left[E[N_p \mid T = t]\right] = \int_0^\infty \left[\sum_{j=0}^{N-1} jP_j + (N-1)P_N\right] dF_T(t) \tag{8.14}$$

where the variable T is the first passage time of any degradation measure exceeding the corresponding imperfect PM threshold.

The distribution of the first passage time T is given by

$$F_T(t) = 1 - \Pr(T > t) = 1 - \Pr(M_1(t) < L_1, M_2(t) < L_2) \tag{8.15}$$

The expected maintenance threshold time $E[T]$ can be represented as

$$\begin{aligned} E[T] &= \int_0^\infty \Pr(T > t)dt = \int_0^\infty \Pr(M_1(t) < L_1, M_2(t) < L_2)dt \\ &= \int_0^\infty C_{\theta_2}(r_1(t), r_2(t))dt \end{aligned} \tag{8.16}$$

Therefore

$$\begin{aligned} &E[N_p] \\ &= \int_0^\infty \left\{ \begin{array}{l} \sum_{j=0}^{N-1} j \left[\begin{array}{l} \dfrac{\Pr(M_1(B_j) < G_1, M_2(B_j) < G_2) - \Pr(M_1(A_{j+1}) < G_1, M_2(A_{j+1}) < G_2)}{\Pr(M_1(B_j) < G_1, M_2(B_j) < G_2)\Pr(M_1(t) < G_1, M_2(t) < G_2)} \\ \left(1 - \sum_{k=0}^{j} p_k\right) \end{array} \right] \\ +(N-1)P_N \end{array} \right\} \\ &\quad d\left[1 - \Pr(M_1(t) < L_1, M_2(t) < L_2)\right] \\ &= \int_0^\infty \left\{ \sum_{j=0}^{N-1} j \left[\begin{array}{l} \dfrac{C_{\theta_1}(R_1(B_j), R_2(B_j)) - C_{\theta_1}(R_1(A_{j+1}), R_2(A_{j+1}))}{C_{\theta_1}(R_1(B_j), R_2(B_j))C_{\theta_1}(R_1(t), R_2(t))} \\ \left(1 - \sum_{k=0}^{j} p_k\right) \end{array} \right] + (N-1)P_N \right\} d\overline{C}_{\theta_2}(r_1(t), r_2(t)) \end{aligned}$$

$$(8.17)$$

where

$$R_i(t) = exp(-\lambda t)F_i\left(\frac{G_i}{\eta_i(t)}\right) + \sum_{n=1}^{\infty} \frac{exp(-\lambda t)(\lambda t)^n}{n!} \int_{z=0}^{G_i} F_i\left(\frac{G_i - z}{\eta_i(te^{\gamma^{(i)}{}_1 n + \gamma^{(i)}{}_2 z})}\right) dQ_i^{(n)}(z);$$

$$r_i(t) = exp(-\lambda t)F_i\left(\frac{L_i}{\eta_i(t)}\right) + \sum_{n=1}^{\infty} \frac{exp(-\lambda t)(\lambda t)^n}{n!} \int_{z=0}^{L_i} F_i\left(\frac{L_i - z}{\eta_i(te^{\gamma^{(i)}{}_1 n + \gamma^{(i)}{}_2 z})}\right) dQ_i^{(n)}(z);$$

C_{θ_1} is the copula function with parameter θ_1 fitted by MLE for failure threshold;

$\overline{C}_{\theta_2}$ is the survival copula function with parameter θ_2 fitted by MLE for passage time.

2) Assume T_f the system degradation failure timewith any of the two degradation processes crossing over its corresponding failure threshold $\{G_1, G_2\}$.When

$T_f \in (H_j, H_{j+1})$ for j=0, 1, 2,..., N-1, the system will be idle during the time interval $[T_f, H_{j+1})$. Let $E[\xi]$ denotes the expected idle time during the system failure and the next inspection time point, which can be given as

$$\begin{aligned} E[\zeta] &= E\left[E\left[\zeta \mid T = t\right]\right] \\ &= \int_0^\infty \sum_{j=0}^{N-1} E\left[\left((t+(j+1)I - T_f\right) 1_{t+jI<T_f\le t+(j+1)I}\right] dF_T(t) \\ &= \int_0^\infty \sum_{j=0}^{N-1} P_j \int_{t+jI}^{t+(j+1)I} \left[(t+(j+1)I-\mu\right] dF_{T_f}(B_j+\mu-(t+jI)) dF_T(t) \end{aligned} \tag{8.18}$$

where $F_{T_f}(\mu) = \Pr(T_f < \mu) = 1 - \Pr(M_1(\mu) < G_1, M_2(\mu) < G_2)$.

In the Equation (8.18), the probability is firstly conditioned on the first passage time T and then conditioned on the system degradation failure time T_f. Due to the difference between system clock and actual system age, actually the system clock at age T_f should be $B_j + T_f - (t+jI)$.

By taking the transformation of $zI = \mu - (t+jI)$, we obtain

$$E[\zeta] = \int_0^\infty \sum_{j=0}^{N-1} P_j \int_0^1 I\left(1-z\right) dF_{T_f}(B_j + zI) dF_T(t) \tag{8.19}$$

Therefire, the expected idle time can be expressed as (8.20)

$$\begin{aligned} E[\zeta] &= \int_0^\infty \left\{ \begin{array}{l} \sum_{j=0}^{N-1} \left\{ \begin{array}{l} \left[\begin{array}{l} \dfrac{\Pr(M_1(B_j)<G_1, M_2(B_j)<G_2) - \Pr(M_1(A_{j+1})<G_1, M_2(A_{j+1})<G_2)}{\Pr(M_1(B_j)<G_1, M_2(B_j)<G_2)\Pr(M_1(t)<G_1, M_2(t)<G_2)} \\ \left(1-\sum_{k=0}^{j} p_k\right) \end{array} \right] \\ \int_0^1 I\left(1-z\right) d\left(1-\Pr(M_1(B_j+zI)<G_1, M_2(B_j+zI)<G_2)\right) \end{array} \right\} \\ d\left[1-\Pr(M_1(t)<L_1, M_2(t)<L_2)\right] \end{array} \right\} \\ &= \int_0^\infty \left\{ \sum_{j=0}^{N-1} \left\{ \begin{array}{l} \left[\begin{array}{l} \dfrac{C_{\theta_1}(R_1(B_j), R_2(B_j)) - C_{\theta_1}(R_1(A_{j+1}), R_2(A_{j+1}))}{C_{\theta_1}(R_1(B_j), R_2(B_j)) C_{\theta_1}(R_1(t), R_2(t))} \\ \left(1-\sum_{k=0}^{j} p_k\right) \end{array} \right] \\ \int_0^1 I\left(1-z\right) d\overline{C}_{\theta_1}(R_1(B_j+zI), R_2(B_j+zI)) \end{array} \right\} d\overline{C}_{\theta_2}(r_1(t), r_2(t)) \right\} \end{aligned} \tag{8.20}$$

where the definition of the formulas in the function is the same as Equation (8.17);

$\overline{C}_{\theta_1}$ is the survival copula function with parameter θ_1 fitted by MLE for failure threshold.

3) The system is renewed whenever the maximum number of imperfect PMs until replacement approaches or the inspection detects that the system fails due to degradation failure during the previous operating period. Therefore, the expected cycle length $E[W_1]$ can be calculated as (8.21)

$$\begin{aligned}
E[W_1] &= E\left[E\left[W_1 \mid T=t\right]\right] \\
&= E\left[E\left[\sum_{j=0}^{N-1} E\left[W_1 \mid N_p = j\right] \mid T=t\right]\right] \\
&= \int_{t=0}^{\infty} \sum_{j=0}^{N} E\left[W_1 \mid N_p = j\right] dF_T(t) = \int_{t=0}^{\infty}\left[\sum_{j=0}^{N} P_j(t+(j+1)I)\right] dF_T(t) \\
&= \int_{t=0}^{\infty}\left[\sum_{j=0}^{N-1} P_j(t+(j+1)I) + (t+NI)P_N\right] dF_T(t) \\
&= \int_{t=0}^{\infty}\left\{\begin{array}{l}\sum_{j=0}^{N-1}\left[\begin{array}{l}\dfrac{\Pr(M_1(B_j)<G_1, M_2(B_j)<G_2) - \Pr(M_1(A_{j+1})<G_1, M_2(A_{j+1})<G_2)}{\Pr(M_2(B_j)<G_1, M_2(B_j)<G_2)\Pr(M_1(t)<G_1, M_2(t)<G_2)} \\ \left(1-\sum_{k=0}^{j} p_k\right)\end{array}\right](t+(j+1)I) \\ +(t+NI)P_N\end{array}\right\} \\
&\qquad d\left[1-\Pr(M_1(t)<L_1, M_2(t)<L_2)\right] \\
&= \int_{t=0}^{\infty}\left\{\begin{array}{l}\sum_{j=0}^{N-1}\left[\begin{array}{l}\dfrac{C_{\theta_1}(R_1(B_j),R_2(B_j)) - C_{\theta_1}(R_1(A_{j+1}),R_2(A_{j+1}))}{C_{\theta_1}(R_1(B_j),R_2(B_j))C_{\theta_1}(R_1(t),R_2(t))} \\ \left(1-\sum_{k=0}^{j} p_k\right)\end{array}\right](t+(j+1)I) \\ +(t+NI)P_N\end{array}\right\} d\overline{C}_{\theta_2}(r_1(t),r_2(t))
\end{aligned} \tag{8.21}$$

where the definition of the formulas in the function is the same as Equation (8.18).

8.3.4.2 Optimization Maintenance Cost Rate Policy

We determine the optimal imperfect PM interval time I, imperfect PM number till replacement N and PM threshold $\{L_1, L_2\}$ such that the long-run average maintenance cost rate $EC[L_1, L_2, I, N]$ is minimized. Mathematically, to minimize the following objective function (8.22):

$$EC[L_1,L_2,I,N]$$

$$= \frac{C_p \int_0^\infty \left\{ \sum_{j=0}^{N-1} j \left[\frac{C_{\theta_1}(R_1(B_j),R_2(B_j)) - C_{\theta_1}(R_1(A_{j+1}),R_2(A_{j+1}))}{C_{\theta_1}(R_1(B_j),R_2(B_j))C_{\theta_1}(R_1(t),R_2(t))} \left(1-\sum_{k=0}^{j} p_k\right) \right] + (N-1)P_N \right\} d\overline{C}_{\theta_2}(r_1(t),r_2(t))}{\int_{t=0}^\infty \left\{ \sum_{j=0}^{N-1} \left[\frac{C_{\theta_1}(R_1(B_j),R_2(B_j)) - C_{\theta_1}(R_1(A_{j+1}),R_2(A_{j+1}))}{C_{\theta_1}(R_1(B_j),R_2(B_j))C_{\theta_1}(R_1(t),R_2(t))} \left(1-\sum_{k=0}^{j} p_k\right) \right] (t+(j+1)I) + (t+NI)P_N \right\} d\overline{C}_{\theta_2}(r_1(t),r_2(t))}$$

$$+ \frac{C_m \int_0^\infty \left\{ \sum_{j=0}^{N-1} \left\{ \left[\frac{C_{\theta_1}(R_1(B_j),R_2(B_j)) - C_{\theta_1}(R_1(A_{j+1}),R_2(A_{j+1}))}{C_{\theta_1}(R_1(B_j),R_2(B_j))C_{\theta_1}(R_1(t),R_2(t))} \left(1-\sum_{k=0}^{j} p_k\right) \right] \int_0^1 I(1-z)d\overline{C}_{\theta_1}(R_1(B_j+zI),R_2(B_j+zI)) \right\} d\overline{C}_{\theta_2}(r_1(t),r_2(t)) \right\}}{\int_{t=0}^\infty \left\{ \sum_{j=0}^{N-1} \left[\frac{C_{\theta_1}(R_1(B_j),R_2(B_j)) - C_{\theta_1}(R_1(A_{j+1}),R_2(A_{j+1}))}{C_{\theta_1}(R_1(B_j),R_2(B_j))C_{\theta_1}(R_1(t),R_2(t))} \left(1-\sum_{k=0}^{j} p_k\right) \right] (t+(j+1)I) + (t+NI)P_N \right\} d\overline{C}_{\theta_2}(r_1(t),r_2(t))}$$

$$+ \frac{C_c + f_c + v_c \int_0^\infty C_{\theta_2}(r_1(t),r_2(t))dt}{\int_{t=0}^\infty \left\{ \sum_{j=0}^{N-1} \left[\frac{C_{\theta_1}(R_1(B_j),R_2(B_j)) - C_{\theta_1}(R_1(A_{j+1}),R_2(A_{j+1}))}{C_{\theta_1}(R_1(B_j),R_2(B_j))C_{\theta_1}(R_1(t),R_2(t))} \left(1-\sum_{k=0}^{j} p_k\right) \right] (t+(j+1)I) + (t+NI)P_N \right\} d\overline{C}_{\theta_2}(r_1(t),r_2(t))} \tag{8.22}$$

where

$$R_i(t) = exp(-\lambda t)F_i\left(\frac{G_i}{\eta_i(t)}\right) + \sum_{n=1}^{\infty} \frac{exp(-\lambda t)(\lambda t)^n}{n!} \int_{z=0}^{G_i} F_i\left(\frac{G_i - z}{\eta_i(te^{\gamma^{(i)}{}_1 n + \gamma^{(i)}{}_2 z})}\right) dQ_i^{(n)}(z);$$

$$r_i(t) = exp(-\lambda t)F_i\left(\frac{L_i}{\eta_i(t)}\right) + \sum_{n=1}^{\infty} \frac{exp(-\lambda t)(\lambda t)^n}{n!} \int_{z=0}^{L_i} F_i\left(\frac{L_i - z}{\eta_i(te^{\gamma^{(i)}{}_1 n + \gamma^{(i)}{}_2 z})}\right) dQ_i^{(n)}(z);$$

C_{θ_1} is the copula function with parameter θ_1 fitted by MLE for failure threshold;

$\overline{C}_{\theta_2}$ is the survival copula function with parameter θ_2 fitted by MLE for the first passage time;

$\overline{C}_{\theta_1}$ is the survival copula function with parameter θ_1 fitted by MLE for the first passage time.

Therefore, the condition-based maintenance optimization can be represented as

$$\begin{aligned} &Min \quad EC[L_1, L_2, I, N] \\ &s.t. \quad R(NI) \geq R_0 \end{aligned}$$

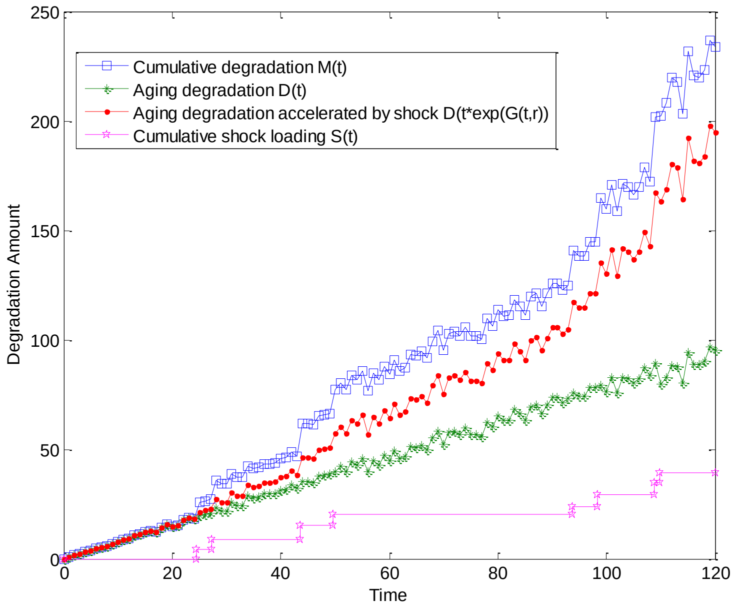
where the constraint means the system should be replaced before the system reliability drops to a very low critical point R_0 given the optimal combination of maximum imperfect PM and imperfect PM interval [N,I].

The above optimization function is a complex nonlinear mixed-integer programming with four dimensions parameters {L_1, L_2, I, N}, which is impossible to obtain the close-form optimum solution. The simulated annealing (SA) technique is one of the most popular meta-heuristic optimization methods for obtaining the optimal solutions of nonlinear constraint function, which do not require the derivation calculation of the objective function. The SA technique mimics the gradually cooling processing for the physical annealing of a material to increase its crystal size until it reaches a low enough temperature to solidify. Compared with other traditional optimization methods, the most advantage for SA techniques is that although it requires a long time to convergence, it has the capability to obtain almost near global or global optimum solutions for the combinatorial optimization problems in a large search space, especially when the search space is discrete.

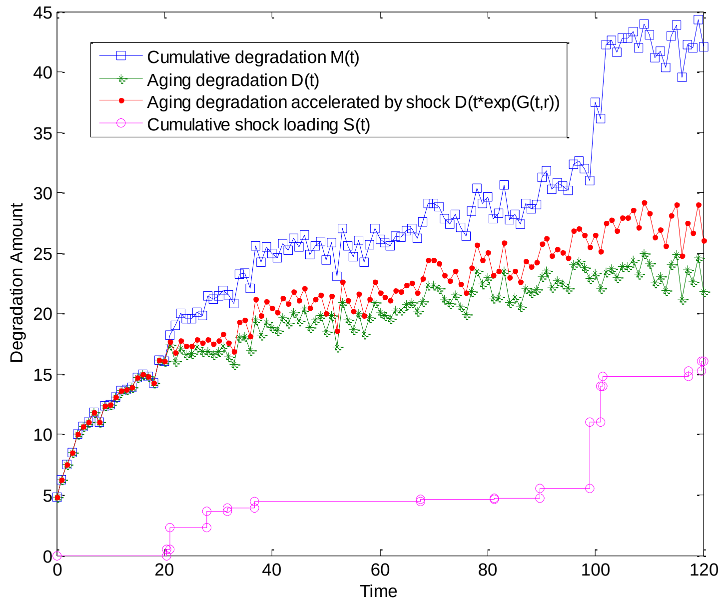
In this problem, because we cannot get the derivation form of the maintenance cost rate objective function due to its complexity and nonlinearity, and also the optimum solutions are constrained in some feasible bound area, such as integer, SA technique turns out to be a good optimization tool to fix this mixed-integer nonlinear optimization problems with constraints.The "simulannealbnd" function in global optimization toolbox is used to compute the optimum solutions in Matlab 7.10.0. Usually, the iterative procedure in SA algorithm is terminated when maximum number of iterations reaches or there is no significant improvement in the solution. We stop the iterations under the second condition.

8.4 Numerical Examples

A system with two degradation processes and one random shock is illustrated as numerical examples of the proposed condition-based maintenance model. Assume the occurring rate of random shock $\lambda = 1/15$. For the first degradation path function, choose $D(t; X, \theta) = \zeta t$, and $X = \zeta$ Weibull distributed with CDF of $F_X(x) = 1 - \exp[-(x/\mu)^k]$, where $\mu = 0.8$ and $k = 30$. The individual shock loading towards the first degradation process follows the normal distribution with parameter $\mu_w = 5$ and $\sigma_w = 1$. Assume random variables in the time-scaled factor $\gamma_1^{(1)} = 0.05$, $\gamma_2^{(1)} = 0.008$ and critical failure threshold G_1=100.For the second degradation path function, choose $D(t; X, \theta) = \theta_2 \log[\theta_1 + t]$, where $\theta_1 = 2.7$ and θ_2 is gamma distributed with pdf $f_X(x) = x^{a-1} e^{-\frac{x}{b}} \Big/ b^a \Gamma(a)$, $x \geq 0$, where $a = 500$ and $b = 0.01$. The individual shock loading towards the second degradation process follows the exponential distribution with mean $v = 1$. Assume random variables $\gamma_1^{(2)} = 0.06$, $\gamma_2^{(2)} = 0.01$ and critical failure threshold $G_2 = 28$. The time unit is week. The degradation path curves with simulated data can be shown in Figure 8.4, from which we can see the impact from cumulative shock loadings and time-scaled factors towards the degradation paths.



(a) 1st Degradation Path



(b) 2nd Degradation Path

Figure 8.4: Degradation Path for Two Degradation Processes

From the equations (8.4) and (8.5), we can obtain the marginal reliability function for these two degradation paths separately and the joint copula system reliability estimation without considering maintenance issue, as shown in Figure 8.5. We assume that the specific copula function fitting for the dependent modeling is known as Symmetrised Joe-Clayton (SJC) copula which is a potential good fitting model to allow for flexible dependency from both upper and lower tail based on the study of Wang and Pham (2012). In Figure 8.5, we also plot the behavior curves for independent copula and upper & lower Frechet-Hoeffding copula bounds in order to show a clear picture of the copula reliability estimation.

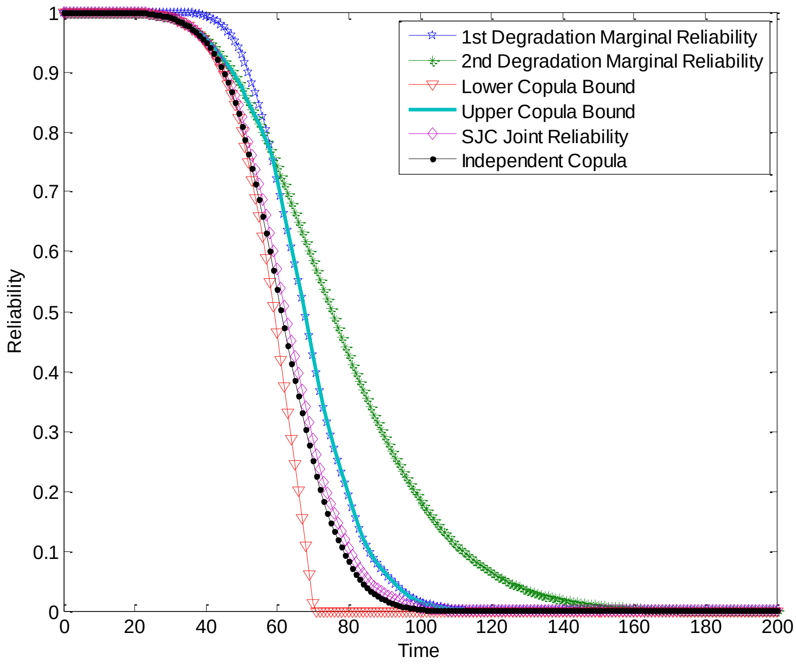


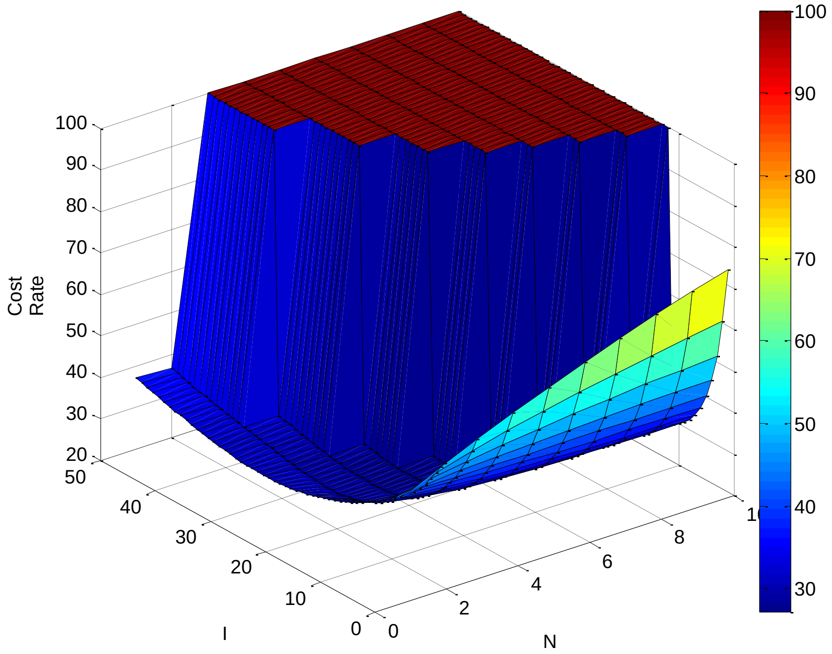
Figure 8.5: Marginal and Joint Reliability

Compared with the independent copula in Figure 8.5, we can see that both of the two degradation processes overlap with the independent copula at the early stage because they are in a good system status with high performance and unlikely to receive impacts from each other, but exhibit some degree of dependence due to interplaying loadings and common usage history after a period of operation time. Also the fitting copula holds the positively quadrant dependent (PQD) property, which means small (large) degradation amount in the first degradation process tend to occur with small (large) degradation amount in the second degradation process.

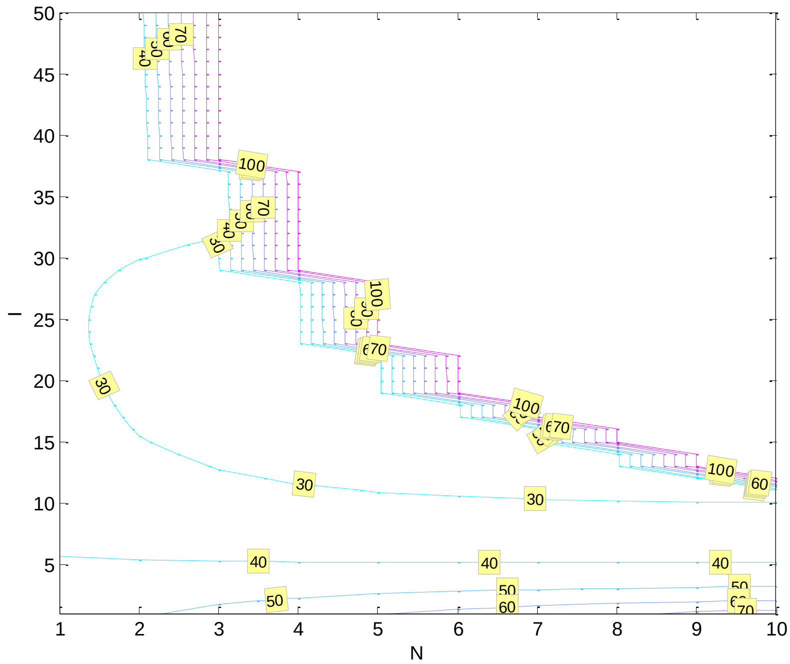
We assume both of the two degradation processes have multi-state degradation with four levels such as good, fair, bad and failure. The corresponding thresholds for the first degradation process are $\{a_1=25,\ a_2=50,\ a_3=75,\ a_4=100\}$ while those for the second degradation process are $\{b_1=7,\ b_2=14,\ b_3=21,\ b_4=28\}$. Maintenance thresholds $\{L_1, L_2\}$ can be chosen from these degradation thresholds $\{a_1, a_2, a_3\}$ and $\{b_1, b_2, b_3\}$. For example, the imperfect PM may be triggered by the combination thresholds $\{L_1=25,\ L_2=7\}$. Therefore, there are totally nine different combinations which the imperfect PM thresholds can be chosen from, and the final thresholds will be solved optimally by minimizing maintenance cost rate function.

Assume the imperfect PM degree β equals to 0.9 and the critical low reliability point R_0 equals to 0.001. Whenever the combination of maximum imperfect PM number till replacement and imperfect PM interval $[N, I]$ does not satisfy the constraint, the maintenance cost rate is set to a large enough value of 100. The cost associated with the CM action C_c is 1000, the penalty cost per unit time associated with the idle time C_m is 80, and the cost associated with the imperfect PM can be $C_p = C_0 + 100(1-\beta)C_v$, where C_0 is the fixed inspection fee and C_{v} is the imperfect PM fee associated with one percentage restoration. Assume C_0=100 and C_{v}=10. The fixed cost associated with purchasing monitoring devices f_{c} is 500 and the variable cost per unit time for device operating and depreciation v_{c} is 5.

We define that the searching domain is $N \in [1,10]$ for all integer and $I \in (0,50]$. Using the SA algorithm with the help of Matlab, we can obtain the minimized maintenance cost rate is 27.2084 with the optimal solution set is: {N*=6, I*=18.6232, L_1*=75, L_2*=21}. Under this optimal solution set, the expected maintenance threshold time $E[T]=36.4215$, the expected number of imperfect PMs $E[N_p]=1.4545$, the expected cycle time $E[W_1]=82.1313$ and the expected idle time $E[\zeta]=3.2707$. The contour plot of maintenance cost rate when L_1=75 and L_2=21 is shown in Figure 8.6, from which we can see that with the increasing of the imperfect PM interval from 1 to 50, the cost rate is first decreasing and then increasing, but too large value combination of imperfect PM interval and imperfect PM number will make the problem infeasible due to the dissatisfaction of the constraint of R_0. The optimal solutions indicate that based on the assumed cost structure, a larger imperfect PM threshold with a later system imperfect PM action would reduce the expected long-run maintenance cost rate. However, this is only the result from minimizing maintenance cost rate. If we take both the system availability and maintenance cost rate into account, smaller imperfect PM threshold with shorter imperfect PM interval may be better.



(a) *N*-*I*-Cost Rate 3-d Plot



(b) *N*-*I* Contour Plot of Cost Rate

Figure 8.6: Cost Rate Curve when {L_1=75, L_2=21}

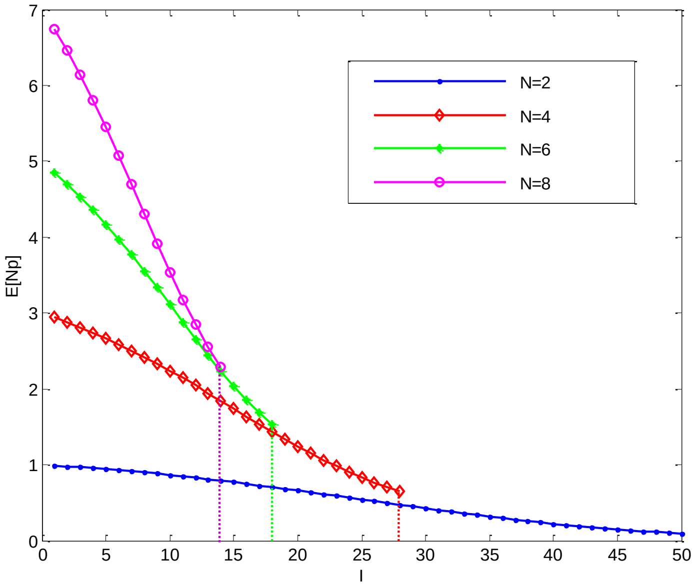
The relationship among maintenance cost components, imperfect PM interval and maximum imperfect PM number till replacementwhen N=2, 4, 6, 8, given L_1=75, L_2=21 is shown in Figure 8.7:

(a) The relationship between imperfect PM interval and expected imperfect PM number when *N*=2, 4, 6, 8 is shown in Figure 8.7(a). It is easy to see that the imperfect PM number will decrease with the imperfect PM interval, and increase with the maximum imperfect PM number till replacement.

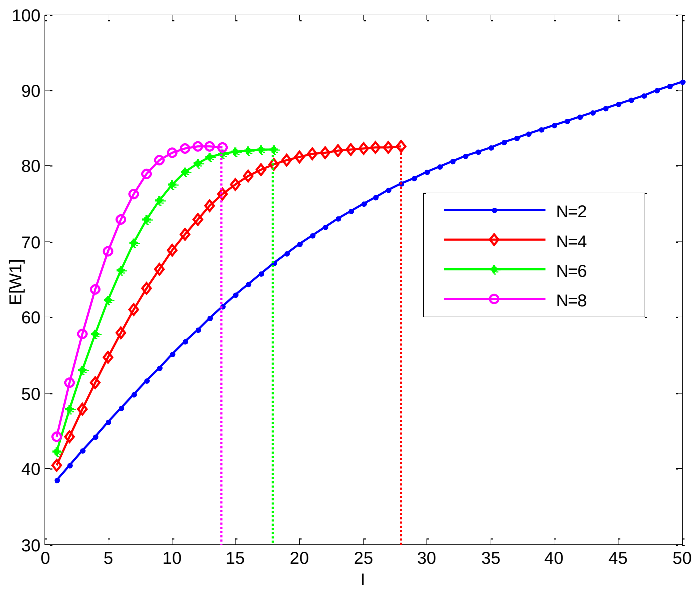
(b) The relationship between imperfect PM interval and expected cycle time when *N*=2, 4, 6, 8 is shown in Figure 8.7(b), from which we can see that the expected system cycle time is increasing with both the imperfect PM interval and maximum imperfect PM number till replacement.

(c) The relationship between imperfect PM interval and system expected idle time per cycle when *N*=2, 4, 6, 8 is shown in Figure 8.7(c). We can observe that the expected idle time per cycle will increase with the imperfect PM interval and is not so sensitive to the changing of the maximum imperfect PM number till replacement.

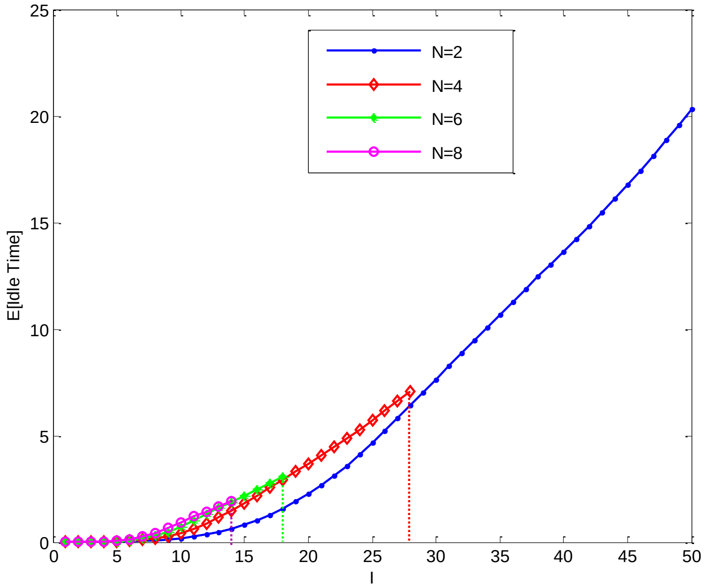
The dashed lines in these curves mean the consequent combination of *N* and *I* make the problem infeasible. Through the comparison of these three plotting, we can come to the conclusion that the imperfect PM interval is a critical parameter which will contribute to all of these three maintenance cost components, while the maximum imperfect PM number till replacement tends to have less influence towards the expected cycle time and expected idle time.



(a) $E[N\text{p}]$ vs. Imperfect PM Interval I



(b) $E[W_1]$ vs. Imperfect PM Interval I



(c) $E[\zeta]$ vs. Imperfect PM Interval I

Figure 8.7: Maintenance Cost Components vs. N and I when $\{L_1=75, L_2=21\}$

The optimization results from this sensitivity analysis with varied imperfect PM degree are provided in Table 8.1. From the comparison results, we can see that a minimized cost would prefer a larger imperfect PM threshold with a later imperfect PM, which is reasonable because to trigger the imperfect PM in a later time may save some maintenance cost under the condition that it will not significantly increase the penalty cost of idle time per cycle. The imperfect PM degree β=0.9 means that 10 percentage of system age will be restored by the imperfect PM. With the increasing of the imperfect PM degree (decreasing of the recovery percentage), we can see that: (a) The maintenance cost rate is not monotonic which is the result of balancing the decreasing of each imperfect PM cost and increasing the risk of idle time penalty; (b) The imperfect PM interval is decreasing; (c) The maximum number of imperfect PMs till replacement is decreasing and then increasing.

Table 8.1: Sensitivity analysis with the imperfect PM degree

Imperfect PM degree β	Minimized Cost Rate	Optimum Pairs			
		I*	N*	L_1*	L_2*
0.9	27.2084	18.6232	6	3	3
0.8	28.7932	20.0197	4	3	3
0.7	29.2464	21.3134	4	3	3
0.6	29.1440	22.7152	5	3	3
0.5	29.1669	26.4380	4	3	3

8.5 Conclusion

This chapter develops a condition-based maintenance threshold-type model with the presence of imperfect PM to restore the system degradation level by a time reduction improvement factor for complex systems subject to dependent competing risks of two degradation processes and random shocks. The dependent structure between degradation process and random shocks is characterized as an embedded time-scaled factor while the dependent structure among degradation processes is linked by copula method. The system is continuously monitored till the imperfect PM threshold and then inspected at each imperfect PM time point.

A numerical example of the proposed model for systems with multiple degradation processes consisting of four-level degradation stages and random shocks is illustrated. From the numerical results, we observe that to minimize the expected long run cost rate the system prefer a larger imperfect PM threshold with a later time to perform maintenance.Through the sensitivity analysis, with the increasing of the imperfect PM degree, the maintenance cost rate and maximum imperfect PM number till replacement is non-monotonically changing while the imperfect PM interval is decreasing.

The proposed condition-based maintenance model provides a generalized practical maintenance policy that can be used in many applications for complex competing risk systems in our daily life such as production machine, human body systems, military service systems, aircraft maintenance as well as others. Our further research interests are to investigate: (1) the multi-objective maintenance optimization to consider the system availability, repair spare inventory supply and profit gain; and (2) a maintenance policy for multi-component systems subject to dependent competing risks.

Chapter 9

Conclusion and Future Research

9.1 Concluding Remarks

The contribution of this thesis is to focus on the development of a generalized reliability estimation model with time-scaled covariate and copula methodology as well as various maintenance policies with imperfect preventive maintenance for dependent competing-risk system subject to multiple degradation processes and random shocks. Dependency, imperfect maintenance and multi-objective optimization are major concerns to both researcher in reliability field and practitioners in industry. This research aims to relax the assumptions for independency between degradation process and random shocks, as well as independency among multiple degradation processes by considering both of the dependent structures into one model.

First, we only consider one degradation process and random shocks.

A cumulative damage model for the degradation process and random shocks is proposed with both additive and multiplicative degradation path function. Then an optimum pair of $\{N^*, T^*\}$ is determined by a maintenance model with imperfect maintenance modulated by an improvement factor to minimize the long run cost rate. After that we extend the above model to a multi-objective maintenance optimization with hidden failure. However, for both models, random shocks and degradation processes have no interplaying with each other. Random shocks will increase the system degradation level by sudden jump but have no impact towards the degradation rate.

Second, we extend our model to multiple degradation processes and random shocks. There exist two types of random shocks in the system: (1) fatal shock, which causes the system to fail immediately; (2) non-fatal shock, which impacts the degradation processes in two ways of sudden increment jump and degradation rate acceleration. A generalized reliability estimation model for the dependent competing-risk systems subject to multiple degradation processes and random shocks is formulated as follows:

a) The dependency between degradation process and random shocks is modulated by the time-scale exponential covariate factors;

b) The dependency among different degradation processes is fitted by both constant and time-varying copula methods.

Three criterions of Log likelihood, AIC and BIC are applied to measure the goodness of the copula fitting from the maximum likelihood estimation. The results soundly shed light upon the application of copula methods to the multiple degradation analysis.

Next, we consider the multi-state model for the multiple competing-risk systems by taking the Cartersian product of two degradation process levels. The reliability and state probability estimation are derived by both analytical methods and Monte Carlo simulation with 95% Confident Intervals.

Finally, we move onto the condition-based preventive maintenance model for the dependent multiple competing-risk systems. The four optimal maintenance decision variables will be determined by minimizing the expected long-run maintenance cost rate, including the imperfect PM thresholds of the two degradation measures $\{L_1, L_2\}$, imperfect PM intervals $\{I\}$, and the imperfect PM number till replacement $\{N\}$. The threshold levels for system maintenance actions of “doing nothing” and “imperfect preventive maintenance” are determined by minimizing the expected long-run maintenance & monitoring cost rate.

9.2 Future Research

Our future research will focus on the following research problems that extend further research on our topics as follows:

Problem 1: Extend the proposed condition-based maintenance policies to incorporate the multi-objective maintenance optimization of maintenance cost rate, system availability and maintenance part inventory control.

Problem 2: Consider the multi-component system subject to dependent competing risks with different system configuration, such as parallel-series, series-parallel and *k*-out-of-*n* systems.

Problem 3: For the multi-component system, maintenance economic efficiency and correlation among various components should be considered into the maintenance optimization polices.

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