

Description: Clip 3 of 9: Describing the volume of $(a+b)$ cubed using squared materials Parent Tape: Early Algebra Ideas About Binomial Expansion, Stephanie's Interview Four of Seven Date: 1996-02-21 Location: Harding Elementary School Researcher: Professor Carolyn Maher	Transcriber(s): Aboelnaga, Eman Verifier(s): Yedman, Madeline Date Transcribed: Fall 2010 Page: 1 of 9
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0:00	1	R1	The color's not going to show a little bit, but okay. Okay. So, um, a plus b quantity cubed, you said, means
	2	Stephanie /R1	a plus b times a plus b times a plus b
	3	R1	Okay, so you used it three times as a factor. And then when you actually used your distributive property
	4	Stephanie	Mhm.
	5	R1	and then simplified
	6	Stephanie	Mhm.
	7	R1	you ended up with
	8	Stephanie	a cubed plus $3 a$ squared b plus $3 ab$ squared plus b cubed.
	9	R1	Um, so that's, um, that's very interesting. Now, you have, this you- you first said has these two pieces.
	10	Stephanie	Mhm.
	11	R1	Mhm. [<i>points to line with $(a^2 + 2ab + b^2)(a + b)$ on paper while speaking</i>] This piece, and this piece. Right? [<i>Stephanie nods.</i>] Now, you have a way of thinking about this piece [<i>R1 points to $(a^2 + 2ab + b^2)$</i>], don't you?
	12	Stephanie	Oh, you mean like this? Like what we did here? [<i>Pulls paper from beginning and points to box representation of square with side $a + b$</i>]
	13	R1	Is that a way of thinking about it?
	14	Stephanie	Yeah.
	15	R1	Okay, so if you had to say in words how you were thinking about this piece [<i>points to paper</i>], right?
	16	Stephanie	Yeah...
	17	R1	Which you happen to have made a picture, and you've actually shown me-
	18	Stephanie	Mhm.
	19	R1	How would you say, in words?
	20	Stephanie	Like what this piece is? [<i>Points at $(a^2 + 2ab + b^2)$</i>]
	21	R1	Yes.
	22	Stephanie	Well, it's a plus b quantity squared.
	23	R1	Right, but if you were to talk about this picture, what did you

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			make a picture of, what's it showing?
	24	Stephanie	Oh, I made a picture of a square.
	25	R1	Of a square, tell me more about that square.
	26	Stephanie	Oh, a square and-with sides measuring ab - a plus b .
	27	R1	Okay.
	28	Stephanie	And, um, [pauses]
	29	R1	That's good enough, right?
	30	Stephanie	Yeah.
	31	R1	So you've made a square with side length a plus b [Stephanie nods] and this piece represents the area of that square.
	32	Stephanie	Mhm.
	33	R1	Right? Okay, and what you've said here- so we know we have this piece, but we have it a plus b times, don't we?
	34	Stephanie	Mhm.
	35	R1	And this piece a plus b times, [points at paper] cause we're finding the product.
	36	Stephanie	Yeah.
	37	R1	Can you conjecture what that might look like?
	38	Stephanie	What that might look like... [pauses, thinking]
	39	R1	We're going back to this piece [points to $a^2 + 2ab + b^2$ on paper] a plus b times. Now remember, when you can't make sense of something with letters, try to imagine-
	40	Stephanie	With numbers?
	41	R1	-if you're doing it with numbers. Sometimes that's a useful way to think about it. So you might not want to think about it as a plus b .
	42	Stephanie	Alright. [nods]
	43	R1	But you might. But you know what this piece [points at paper-speaking inaudible]
	44	Stephanie	But you want me to show like, you how that would look if it was a plus b times? Like, how, a plus b quantity squared would look a plus b times?
	45	R1	Do you have any idea?
	46	Stephanie	[pauses, thinking] Do you want me to show it on a cube?
	47	R1	I don't know-

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	48	Stephanie	Is that what you want me to do?
	49	R1	I don't know, can you do it on a cube?
	50	Stephanie	I don't know, I don't-
	51	R1	Can you show it on a cube, do you think? Did you play with any of that stuff?
	52	Stephanie	You could show it. [<i>pauses</i>]
	53	R1	That's an interesting thing- [<i>gets bag and pours contents, Algebra blocks, on table</i>] Alright. [<i>rearranges blocks on table with Stephanie</i>] I'm interested if you could show it with this.
	54	Stephanie	Alright.
	55	R1	I don't know if you can or not, I haven't tried it, but I'm not really- I honestly haven't done it myself. This is something that Ethel brought, and um, Ethel seemed to think these are useful, and, so the question is, um, can you [<i>rearranges Algebra blocks, counts them</i>] 1, 2, 3, 4, 5, 6, 7, 8 pieces [<i>points to count terms written on paper</i>] Hm. That's interesting.
	56	Stephanie	Um, I mean I guess the side would be like ab by ab by ab - a plus b , by a plus b , by a plus b , but...
	57	R1	That's reasonable.
	58	Stephanie	And that's really what this is. [<i>points at paper, pauses</i>]
	59	R1	And you could make your own, you could use your own things to make- you don't have to-
	60	Stephanie	Well, I'm just like-
	61	R1	I don't have a clue where to start, frankly.
	62	Stephanie	It's-
	63	R1	We could work on this together.
	64	Stephanie	-it's the same thing. I mean, you could use, this is just smaller than like, one of these... [<i>reaches to get something from below table, puts 10x10x10 cube on table</i>] You know? But like...
	65	R1	[<i>Gets something from below table, places a 'flat' a 10x10x1 box on table, rearranges pieces on table</i>] Okay. So we have this piece, and we have that piece.
	66	Stephanie	Mhm.
	67	R1	I like to do this, in case you want to- [<i>gets paper, places 'flat' on table, traces perimeter on paper</i>] label it in any way. I'm

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			going to do this for you. Probably not gonna have it as neat as you're gonna have it- already I have corners that are rounder than they're supposed to be, right? Is that good enough?
	68	Stephanie	Yeah that's fine.
	69	R1	Alright?
	70	Stephanie	Mhm.
	71	R1	I wonder what you think about it-
	72	Stephanie	Well, you see, the thing is, that [<i>coughs</i>] that's like, two-dimensional.
	73	R1	Okay, so we can think of this as two-dimensional [<i>picks up flat</i>]
	74	Stephanie	Yeah, but-
	75	R1	So what part would that be? [<i>Pulls out paper with work on it</i>]
	76	Stephanie	I guess that would be this part [<i>points at paper</i>] a squared plus a plus- Oh! Okay, it would be this [<i>points at paper</i>]
	77	R1	Oh, Okay, well can you, kind of draw it [<i>pulls out paper with traced square</i>]
	78	Stephanie	Yeah [<i>draws (a+b)² diagram on new square drawing</i>]
	79	R1	Okay, great. So we can think of this [<i>places 10x10x1 box on picture</i>] that way.
	80	Stephanie	Mhm.
	81	R1	Okay, [<i>removes box</i>] so we know that piece.
	82	Stephanie	It's a plus b number of times, and this is a plus b, like right here [<i>points at side of drawn square</i>], like this, side-
	83	R1	This length is-
	84	Stephanie	Yeah- is a plus b [<i>pauses</i>]. So I don't know what that means...
	85	R1	I guess, okay, this side is a plus b [<i>points at paper</i>]
	86	Stephanie	Yes.
	87	R1	This side is a plus b [<i>points</i>]
	88	Stephanie	Mhm.
	89	R1	These are different, [<i>pulls 10x10x10 cube and 10x10x1 box together</i>] in what way?
	90	Stephanie	[<i>picks up cube</i>] This, like, this is a cube.
	91	R1	What's the difference?
	92	Stephanie	It's got three dimensions.

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	93	R1	Okay, what are they?
	94	Stephanie	Length, width, and depth.
	95	R1	Okay, what is the length?
	96	Stephanie	Length [<i>points along edge of cube</i>]
	97	R1	Remember, remember you called it-in terms of <i>a plus b</i> .
	98	Stephanie	Well then it's just like, oh! [<i>laughing</i>] It's <i>a plus b</i> . The length is like, <i>a plus b</i> , the width is <i>a plus b</i> , the height is <i>a plus b</i> .
	99	R1	So.
	100	Stephanie	So. It's just...
	101	R1	[<i>picks up 10x10x1 box, turns it on side to show it is same height as 10x10x10 cube</i>] So, I think that's true, the height is <i>a plus b</i> .
	102	Stephanie	Yeah, it's all <i>a plus b</i> .
	103	R1	So, is that helpful?
	104	Stephanie	I don't know, I mean I already... I just don't know how to show this [<i>paper</i>] that <i>a plus b</i> , length times.
	105	R1	Well, [<i>points at paper</i>] you showed this.
	106	Stephanie	Yeah, I mean that's just <i>a plus b</i> , I don't know- I- I mean, that's like...
	107	R1	Okay, let me ask you a question; forget <i>a plus b</i> for a minute.
	108	Stephanie	Okay.
	109	R1	What is this really [<i>points at 10x10x1 box</i>]? What do we consider this?
	110	Stephanie	Oh, it's a hundred units. Like, the area? Or like-
	111	R1	It's a hundred, well, h- how- where did a hundred come from?
	112	Stephanie	Well,
	113	R1	Is it the area? The area is what? The area is a hundred?
	114	Stephanie	Mhm.
	115	R1	A hundred what?
	116	Stephanie	Square units.
	117	R1	So we can think of this as a hu- where did that come from?
	118	Stephanie	[<i>Runs pen along sides of box</i>] 'Cause it's ten units here, and ten square units here and ten times ten is a hundred.
	119	R1	Do you believe it? Do you believe there are a hundred square units here?

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	120	Stephanie	Yeah.
	121	R1	You absolutely-
	122	Stephanie	[<i>laughing</i>] Yes.
	123	R1	-believe it?
	124	Stephanie	Yes [<i>nodding</i>]
	125	R1	I couldn't convince you that, that that's not true [<i>Stephanie shakes head</i>]. Okay, so you know that.
	126	Stephanie	Yes.
	127	R1	Okay. Now, [<i>points at box's edges</i>] this has length ten, width ten, right-
	128	Stephanie	Mhm.
	129	R1	It has area, a hundred square units, I could have said it had [<i>runs finger along edge of box</i>] <i>a</i> is 4, <i>b</i> is 6...
	130	Stephanie	Mhm [<i>nods</i>] It wouldn't make-
	131	R1	<i>a</i> is 2...
	132	Stephanie	Yeah, it wouldn't make a difference.
	133	R1	<i>a</i> is two and a half... [<i>Stephanie laughs</i>] Now. What about this one [<i>points to cube</i>]
	134	Stephanie	[<i>Pause</i>] In numbers, or like...
	135	R1	Yeah, numbers, if you're dealing with...
	136	Stephanie	Well, um. The area would be, um, one thousand? The area would be a thousand.
	137	R1	Area?
	138	Stephanie	Well, oh, the volume.
	139	R1	What do you mean by volume?
	140	Stephanie	Um, length times width times height.
	141	R1	But I don't know what that means.
	142	Stephanie	Well it's the three dimensions of the cube.
	143	R1	What does the thousand mean. I know what the hundred means [<i>points to box</i>], I can count-
	144	Stephanie	There's a thousand little, like [<i>picks up little one-unit cube</i>] units. Square units in there. Like, you could fill it up with a thousand square units.
	145	R1	How do I know that? Can you- do you see that? I only see [<i>points to cube's faces</i>] 10, 20, 30 40, 50, [<i>picks up, points at</i>

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			<i>bottom</i>] 60.
	146	Stephanie	You only see 60?
	147	R1	I'm sorry, 600.
	148	Stephanie	Okay.
	149	R1	<i>[touches top face]</i> 100.
	150	Stephanie	Yes.
	151	R1	<i>[points to side]</i> And I see another hundred.
	152	Stephanie	Mhm.
	153	R1	<i>[points to other sides]</i> And this would be...
	154	Stephanie	Okay.
	155	R1	And you're telling me there are a thousand.
	156	Stephanie	Yes.
	157	R1	So why?
	158	Stephanie	Okay. Well it's 10 high, right? <i>[picks up cube, compares to box]</i> If it was just th- one of these, <i>[indicating box]</i> is like the same as like, it's one. Like one part.
	159	R1	A hundred.
	160	Stephanie	A hundred. And you know that like <i>[touching side of cube]</i> this, is the same as that <i>[indicating box]</i> .
	161	R1	So that's a hundred, okay. So what-
	162	Stephanie	It's the same thing, if I took this off it would be one hundred <i>[indicating side of cube]</i> , and you know that there's ten of them <i>[pointing to each layer of cube along the edge]</i> . So you see that ten of them would make this- ten high? You know?
	163	R1	So if I took one of them off, I would get one hundred.
	164	Stephanie	Yes.
	165	R1	If I peeled another off...
	166	Stephanie	It would be two hundred.
	167	R1	<i>[tapping top of cube]</i> 300, 400, 500 <i>[trails off]</i> . That's the way you think about getting a thousand.
	168	Stephanie	Yeah.
	169	R1	I peel- How many times would I peel them off?
	170	Stephanie	Ten.
	171	R1	Ten times? Why ten?
	172	Stephanie	<i>[coughs]</i> 'Cause that's how high it is. That's how many fill it

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			up.
	173	R1	Oh. How wide is it?
	174	Stephanie	Ten.
	175	R1	And how long is it?
	176	Stephanie	Ten.
	177	R1	Can you use that same argument to show me what a plus b is times that [<i>points at paper</i>]? Only now we're calling this a plus b .
	178	Stephanie	Well it's a plus b high, it's a plus b long, and it's a plus b wide-
	179	R1	Mhm.
	180	Stephanie	So, um [<i>picks up 10x10x1 box</i>] if this is a plus b squared, you see that like, [<i>pointing at 10x10x10 cube</i>], this, if you t- took this off, it would be a plus b squared, and you need to take a plus b amount of these off to get a plus b cubed.
	181	R1	Does that make sense?
	182	Stephanie	A little bit.
	183	R1	A little bit, not quite [<i>Stephanie laughs</i>], a little fuzzy yet?
	184	Stephanie	Yeah, 'cause it's harder using letters-
	185	R1	You bet.
	186	Stephanie	Especially when there's like, a plus b . It's not just like a number of...
	187	R1	Yeah, right. So it's easy to think of it as we're thinking of ten. [<i>points to edge of cube</i>] Peeling it off ten times. But if I said I'm peeling it off six and then four times to make my ten. Or eight then two times to make my ten, or seven then three times to make my ten, a and then b times to make my ten, is that easier to see?
	188	Stephanie	Yeah. It's easier to like-
	189	R1	Harder to think in those abstract-
	190	Stephanie	Yeah-
	191	R1	-terms, in those symbols. It really is. [<i>Stephanie nods</i>] A lot of people don't think - they just do things. They don't try to think and imagine what it means. But, um, every now and then you oughta try to think about- 'cause it's elusive, it's gonna come,

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			and it's gonna go, and it's gonna come and it's gonna go. Which is very interesting. Okay well that's something to think about some more, and- so, I can think of the- Tell me again how I can think of a plus b quantity cubed, one more time.
	192	Stephanie	You want like this [<i>points to paper</i>] or you want me to show you like here [<i>points to cube</i>].
	193	R1	Well, what- show me here first [<i>points to cube</i>], then we're going to try to break it down [<i>rearranges papers</i>] to pieces.
	194	Stephanie	Alright, well, if this is a plus b , like this side is a plus b [<i>uses box</i>] and this side is a plus b , then there are a plus b squared number of pieces in here. Do you believe that?
	195	R1	I believe that. And I even believe that it is a squared plus $2ab$ plus b squared.
	196	Stephanie	Yes. So-
	197	R1	You've convinced me of that.
	198	Stephanie	So, if I were- there's a plus b , like, rows of these. If I took a plus b number of, like, this [<i>indicates box</i>], it would make that- it would fill that up [<i>indicates cube</i>].
	199	R1	Okay.
	200	Stephanie	If I took off one of these [<i>indicates box</i>], you see if I took this first row off, right here, I'd have a plus b squared, of- a plus b squared number-
	201	R1	a plus b quantity squared
	202	Stephanie	Yeah, and so I'd have to take up a plus b number of those, to like, fill it up [<i>indicates cube</i>], or something?
	203	R1	Okay, so,