| Description: Clip 5 of 9: Building (a+b) cubed and identifying the pieces<br>Parent Tape: Early Algebra Ideas About Binomial Expansion, Stephanie's Interview Four of Seven<br>Date: 1996-02-21<br>Location: Harding Elementary School<br>Researcher: Professor Carolyn Maher | Transcriber(s): Aboelnaga, Eman<br>Verifier(s): Yedman, Madeline<br>Date Transcribed: Fall 2010<br>Page: 1 of 6 |
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| 0:00 | 1 | R1 | 'Cause I'm thinking here, see, [*replaces $a^2b$ piece horizontally in original model; adds new a squared b piece on top of a cubed piece vertically, then moves it vertically on top of $a^2b$ piece*] can you make a cube with these pieces? Can you build a cube? |
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| | 2 | Stephanie | That, like, all that length? |
| | 3 | R1 | With this as a base [*indicating model*]? Just build it, without worrying about what they are. Can you just make it, can you put the puzzle together? [*Stephanie attempts to put pieces together to create cube*] |
| | 4 | Stephanie | I don't know if there's enough, like [*hesitates*], no. Not a – well [*resumes rearranging pieces, succeeds at assembling cube*]. Oh. There. |
| | 5 | R1 | My goodness. That's pretty neat. Now. |
| | 6 | Stephanie | Oh boy… |
| | 7 | R1 | What kind of question might you be asking? You've done a really nice job, saying what all those pieces are, and what it was coming up what, one layer of it, you know? |
| | 8 | Stephanie | Mhm. |
| | 9 | R1 | You did all those components of the first layer, that's very lovely. And then you went up $b$, right? |
| | 10 | Stephanie | Mhm. |
| | 11 | R1 | So I'm kind of interested in [*pause*] you know, you had- you ended up with an $a$ squared $b$, and an $a$ squared $b$. |
| | 12 | Stephanie | Yeah. |
| | 13 | R1 | An $ab$ squared, but you ended up with this [*pointing to paper with work from before*] before that, with this [*showing work from previous; work before simplifying; accidentally knocking over cube*] whoops. What did I do, I destroyed it. I don't wanna put it together the way you didn't have it? Do you remember what you did? Was it like this? [*reassembling cube*] |
| | 14 | Stephanie | Yes. |
| | 15 | R1 | I don't know if they belong in those places or not [*reassembling cube*] That's something we can think about, maybe they do, maybe they don't, I haven't thought about it. But, we know where the $a$ cubed is. |
| | 16 | Stephanie | Yes. |

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| | 17 | R1 | That's this little piece. |
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| | 18 | Stephanie | Yes. |
| | 19 | R1 | I mean, are all of these pieces there? [*Indicating terms on paper and pieces of cube*] |
| | 20 | Stephanie | Probably. |
| | 21 | R1 | This is $a$ plus $b$, here [*indicating cube*].  Should we- How can we figure that out? |
| | 22 | Stephanie | Well, we already have, we have this piece [*going to write on paper*] |
| | 23 | R1 | Let's get another piece of paper [*gets another sheet of paper*]. We already have the $a$ cubed piece. |
| | 24 | Stephanie | We have $a$ cubed [*writes terms on paper*], we have $a$ cubed-squared $b$, we have $ab$ squared, and we have another $a$ squared $b$. And I guess, on the base level [*pulling apart a piece of the cube*], does that count? [*Drops some pieces, reassembles cube*] |
| | 25 | R1 | That was all those pieces- you- |
| | 26 | Stephanie | Yeah, so it doesn't.  So like, we have these four [*pointing to paper*] pieces… With just this layer. |
| | 27 | R1 | Hmm.  Just the bottom layer. |
| | 28 | Stephanie | Yeah. |
| | 29 | R1 | Mhm.  And [*returning to previous work on paper, before simplified*], according to this thing we needed three $a$ squared $b$, you only had one.  You need $3ab$ squared, you only had one. Right? |
| | 30 | Stephanie | Well we have two $a$ squared $b$.  [*pause*] Don't we? |
| | 31 | R1 | Hmm.  I guess we do. Right. |
| | 32 | R1 | We have an $a$ squared $b$, we have two $a$ squared $b$. [*places old and new work next to each other*] I don't know, is this the right way to think about this? It's interesting. [*pause*] What's a $b$ cubed? |
| | 33 | Stephanie | $b$ cubed? Um… [*deconstructs cube, picks up $ab^2$ piece from bottom layer*] That's $b$ squared [*puts cube back together*]. And that's gonna be… [*pauses*] |
| | 34 | R1 | You said this was $b$ squared? Over here, right? [*removes piece, pointing to bottom layer of cube*] |
| | 35 | Stephanie | Yeah, that was $b$ squared. |

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| | 36 | R1 | What was $b$? Show me $b$.  What was the length $b$? |
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| | 37 | Stephanie | $b$ is like this, [*running finger along edge of $ab^2$ piece*] or this [*running finger along $b^3$ piece*], so I guess it's going up another $b$, so… But it's already $ab$ squared, but there's no $ab$ cubed. |
| | 38 | R1 | Well, [*pulling out $ab^2$ piece, pointing to tracing on paper*] this was $b$ squared, right? And then when you went up one it became $ab$ squared. That was this piece [*replacing $ab^2$ piece*]. |
| | 39 | Stephanie | Yes. |
| | 40 | R1 | Right? Isn't that right? |
| | 41 | Stephanie | Yes. |
| | 42 | R1 | [*moving piece in and out of place*] 'Cause you went up $a$.  So you went to $ab$ squared. |
| | 43 | Stephanie | Mhm. |
| | 44 | R1 | So, what's this [*places $b^3$ piece on tracing on paper*]? |
| | 45 | Stephanie | Well that's $b$. That's going up $b$.  Like, that would be going up $b$ [*pointing along edge of $b^3$ piece*]. |
| | 46 | R1 | So. |
| | 47 | Stephanie | So I guess that would be $b$ cubed. |
| | 48 | R1 | So tell me why that's $b$ cubed. |
| | 49 | Stephanie | 'Cause you're going up, like, you already have $b$ squared and you're going up another $b$. |
| | 50 | R1 | Okay, so this piece is $b$ cubed [*picks up piece*] |
| | 51 | Stephanie | Okay. |
| | 52 | R1 | Yeah, I think so.  'Cause if you're telling me this is $b$, and this is $b$, and this is $b$ [*points to edges*], does that look like a cube? |
| | 53 | Stephanie | Yeah. |
| | 54 | R1 | That looks like a $b$ cubed, and that looks like an $a$ cubed [*pointing at pieces*] $a$-$a$-$a$. So we know the $a$ and the $b$ cubed. That's pretty good. |
| | 55 | Stephanie | So we have $b$ cubed [*writes on paper*]. |
| | 56 | R1 | Now do you believe you can find all these pieces in here [*pointing at previous paper with terms before simplifying*]? What's your conjecture at this point? |
| | 57 | Stephanie | I don't- Probably. |
| | 58 | R1 | Okay, you kind of think that's a reasonable thing to pursue. That's why I think we should stop. |

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| | 59 | Stephanie | Okay… |
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| | 60 | R1 | I think you know enough. W- If you think about it, [*picks up pieces from cube*] you know, you can give names to some of these, right? Right? |
| | 61 | Stephanie | Yes. |
| | 62 | R1 | K. So what did you call this one again [*holding up ab²*]? |
| | 63 | Stephanie | $b$ squared [*pauses*]. Didn't I? It was- yeah that was $b$ squared. |
| | 64 | R1 | Which part is $b$ squared? The whole piece? |
| | 65 | Stephanie | Well this is $b$, and this is $b$ [*pointing to edges*] |
| | 66 | R1 | Is this solid $b$ squared, or- |
| | 67 | Stephanie | Oh, it's flat $b$ squared. |
| | 68 | R1 | Flat is $b$ squared, but when you- |
| | 69 | Stephanie | $a$- $ab$? |
| | 70 | R1 | So it's $ab$ squared. |
| | 71 | Stephanie | Okay. |
| | 72 | R1 | Does that make sense? This is $ab$ squared. Isn't that interesting? We can think of this piece as $ab$ squared. |
| | 73 | Stephanie | Okay. |
| | 74 | R1 | Okay. So, it might help you to write this down, or draw pictures, anything you need to remind yourself of what pieces you know and that you believe. Because remember, you're the one who gave them all these names here [*pointing at tracings on paper*], should I move this for a minute [*slides cube off of tracing, Stephanie writes on new paper "Blue piece- ab²*] |
| | 75 | Stephanie | Okay [*continues writing*]. White is $b$ cubed. Yellow is $a$ squared [*pauses, corrects "2" with "3" on paper*] cubed. |
| | 76 | R1 | Why did you change it? |
| | 77 | Stephanie | Because I was talking about the paper, instead of the yellow. |
| | 78 | R1 | Okay, good. So when you think about the paper, it's the two dimensions, and when you think of the actual block- |
| | 79 | Stephanie | Mhm- |
| | 80 | R1 | You have to think of three dimensions. |
| | 81 | Stephanie | Mhm. And the green was [*writes*] $a$ squared $b$. |
| | 82 | R1 | Okay. So. [*pauses*] The green one is $a$ squared $b$ [*gathers green pieces*], how many of those do you have? |
| | 83 | Stephanie | Three. Three $a$ squared $b$. |

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| | 84 | R1 | And what's the blue one [*picking up blue piece*] |
| | 85 | Stephanie | Oh, so we have 3$a$ squared $b$ [*pointing to original paper with simplified work*] |
| | 86 | R1 | Oh. |
| | 87 | Stephanie | [*Crosses out two $a^2b$ terms on newer paper, rewrites "3$a^2b$" instead*] And we have $a$ cubed [*writes*] and we have $b$ cubed, and we have $ab$ cubed- squared- [*looks at pieces*] we have 3$ab$ squared [*writes*] |
| | 88 | R1 | So, why are these $ab$ squared [*picks up blue piece*] |
| | 89 | Stephanie | Because, it's like, $a$ up, $b$ over [*pointing to edges of piece*] |
| | 90 | R1 | Believe that, absolutely. Okay. |
| | 91 | Stephanie | So that's it, we have all the pieces. |
| | 92 | R1 | So you believe… |
| | 93 | Stephanie | Yeah. |
| | 94 | R1 | You're absolutely convinced? |
| | 95 | Stephanie | Yes. |
| | 96 | R1 | You can explain that to your teacher? |
| | 97 | Stephanie | Yeah, kind of. |
| | 98 | R1 | And to Melanie? |
| | 99 | Stephanie | Yes. |
| | 100 | R1 | Kind of? Or- If you think about- this is really cool. This was a nice problem, Ethel. But you should've given us more pieces. To throw us off. |
| | 101 | R2 | Should've made them all the same color, too. |
| | 102 | R1 | Should've made them all the same color? That would have been very hard [*laughs*]. It's nicer to have them different colors, don't you think? So next time you can make them a little harder. Okay, so you believe that the quantity $a$ plus $b$ squared means $a$ plus $b$ three times [*points at paper*]. You'd have to think about this a lot until you have the $a$ cubed piece, you have the $a$ squared $b$ piece three times- |
| | 103 | Stephanie | Mhm. |
| | 104 | R1 | You have the $a$ $b$ squared piece |
| | 105 | Together | three times. |
| | 106 | R1 | And you have the $b$ cubed piece three times. |
| | 107 | Stephanie | Yes. |

| | 108 | R1 | And when you build it all up, I'm going to ask you to do it one more time… |
| --- | --- | --- | --- |
| | 109 | Stephanie | Okay. |
| | 110 | R1 | Okay [*Stephanie builds*]. You're gonna have a cube? |
| | 111 | Stephanie | Um, yeah. |
| | 112 | R1 | And what are the length, width, and height of that cube? |
| | 113 | Stephanie | [*pause; building*] The length is *ab*, the width is *ab*, and the height is *ab*. |
| | 114 | R1 | *ab*? |
| | 115 | Stephanie | What? |
| | 116 | R1 | *ab*? Show me. |
| | 117 | Stephanie | *a* plus *b*. |
| | 118 | R1 | *a* plus *b*. And what's the *a*, and what's the *b*? |
| | 119 | Stephanie | Well, this can, this is the *a* [*pointing to top of $a^2b$ piece*] and this is the *b* [*running finger along remaining edge of cube*]. |
| | 120 | R1 | Okay, and so you have to keep separate the linear measure and then the two dimension and then three? It's easy to- |
| | 121 | Stephanie | Okay. |
| | 122 | R1 | It's easy to… That's interesting. So, um, you have a way of doing *a* plus *b* quantity cubed, you have a model that I think I want you to think about a little bit more. |
| | 123 | Stephanie | Okay- |
| | 124 | R1 | Okay, and then, what do you think I'd ask [*scene cut*] |
| | 125 | Stephanie | *a* plus *b* to the fourth? |
| | 126 | R1 | Yeah. |
| | 127 | Stephanie | Okay. |
| | 128 | R1 | And you could even anticipate what I would ask you after that. |
| | 129 | Stephanie | Yeah. |
| | 130 | R1 | Uh huh. You might work them out, and look at them, and study them a little bit. |
| | 131 | Stephanie | Okay, [*scene skips*] |