Description: Clip 7 of 9: Explaining each piece of the geometric model of (a+b) cubed as it relates to the terms in the algebraic expansion to observers **Parent Tape: Early Algebra Ideas About Binomial Expansion, Stephanie's Interview Four of Seven** **Date: 1996-02-21** **Location: Harding Elementary School** **Researcher: Professor Carolyn Maher**	**Transcriber(s): Aboelnaga, Eman** **Verifier(s): Yedman, Madeline** **Date Transcribed: Fall 2010** **Page: 1 of 8**

0:00	1	Stephanie	Okay. So then it would be [*resumes writing*] 2*ab* times *a* plus *b*, which is, *a* times 2*ab* is 2 *a* squared *b*. And *b* times 2*ab* is 2*a b* squared. Ahem. Plus… um… *b* squared times *a* plus *b*, which would be *a* times *b* squared is *ab* squared, plus *b* times *b* squared, which is *b* to the third. And that can be simplified. [*pause*] That can be [*writes*] *a* s- cubed plus you can – ahem- *a* squared *b* plus 2*a* squared *b* is [*writes*] 3*a* squared *b*. Plus 2*a* squared- 2*ab* squared plus *ab* squared is 3*ab* squared plus *b* to the third. And that's [*turns paper to show work*]- can't be simplified anymore, so that's the same thing as- um- [*writes*] *a* plus *b* quantity cubed. And then- ahem- we- [*pause, flips through papers*] So then if you were gonna use these [*places Algebra blocks on table*] to show this, um, we'd start out with the two dimensional figure, which was [*retrieves paper*] *a* plus *b* quantity squared.
	2	R4	So *a* plus *b* quantity squared is a two dimensional?
	3	Stephanie	Yes.
	4	R4	Even though you showed that, right there? [*indicating 10x10x1 block used earlier*]
	5	Stephanie	Yeah.
	6	R4	That's two dimensional?
	7	Stephanie	Well, no-
	8	R4	Okay.
	9	Stephanie	But I was just… Cause you know, there's nothing else to use, to show it… so *that*, *a* squared [*pointing at drawing*], *ab*, *b* squared, *ab*, makes up *a* plus *b* quantity squared. So, um, if you took like, if this was *ab* [*places a^2b piece on picture*], if this fit there and that fit there [*places base layer of Algebra cubes on drawing*]. There you built it up-
	10	R4	How-How is that-
	11	Stephanie	Like-
	12	R4	How is that *a* squared *b*? That, the green with the- *a* squared *b*? How is that *a* squared *b*? And the other one is *ab* squared? How do they differ? 'Cause I was back there, I couldn't really see what you were doing with Dr. Maher…
	13	Stephanie	Wait, which one's *ab*?

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	14	R4	This one's- you said this was *a* squared *b*?
	15	Stephanie	Oh.
	16	R4	How's that- How did you determine that that was *a* squared *b*?
	17	Stephanie	Oh, because, um, [*removes ab^2 piece to show drawing*] this is *b* squared, and you built it up *a*, like, 'cause, it's- this [*indicating height*] is *a*.
	18	R4	Okay.
	19	Stephanie	Like this piece and this piece, so you built it up *a*, so it would be *a* squared- *b* squared.
	20	R4	Oh, okay. Alright.
	21	Stephanie	But. Here, this piece [*moving a^2b piece*] is *a- ab*, [*moving ab^2 piece*] this is *b* squared. So this piece [*ab^2 piece*] would be- Um- Like this piece here [*picks up a^2 b piece*] 'Cause it's *ab*, if you built it up *a*, it would be *a* squared *b*. And this piece [*picks up a^3 piece*] 'cause this piece is *a* squared, and you build it up *a*, it would be *a* cubed.
	22	R4	Oh, building it up, okay.
	23	Stephanie	Yeah, you built it up.
	24	R4	Now I understand what you mean by build it up, okay. I wasn't sure.
	25	Stephanie	Yeah, that's why. And so, [*rearranging pieces*] like, you know that this piece is *a* squared *b*.
	26	R4	Why is that *a* squared *b*, show me-
	27	Stephanie	Okay-
	28	R4	So if I were to-
	29	R3	*a* doesn't have a color, *a* has just the dimension, the height.
	30	Stephanie	Yeah…
	31	R3	That's the problem with it. That's why it's so hard to visualize.
	32	Stephanie	Yeah.
	33	R4	'Cause if I were to show my class I want to be able to explain it to them.
	34	Stephanie	So like, this is *a*- 'cause on like, one- two dimensional, if this is.. this would be *ab* [*referring back to $(a+b)^2$ drawing*].
	35	R4	Okay.
	36	Stephanie	And it you build it up *a*, it would become *a* squared *b* [*placing*

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			a^2b piece on drawing, indicating dimensions]. Okay? And this piece [*picks up piece*], so we know that this piece is *a* squared *b* [*places piece back on paper on the side, picks up a^3 piece, points to a^2 region of tracing*]. This piece is *a* squared. If you build it *a* [*places a^3 piece on tracing*], it's *a* cubed.
	37	R4	Alright, okay.
	38	Stephanie	So this piece-
	39	R3	It's *a* squared *b* like that, isn't it? [*places a^2b piece on table vertically*] This is the square [*pointing at base of piece*], and it's *b* high. Isn't that it?
	40	Stephanie	Well…
	41	R3	'Cause *a* is- has no color, it's just this centimeter [*indicating base of piece*]. This is the square. [*places piece on table*] I mean, how are you gonna call this *a* squared *b*? [*pauses*] Unless [*picks piece up*], that's the square, and *b*'s the height. [*Stephanie picks up piece*]
	42	R4	You're building it up *a*…
	43	Stephanie	Yeah, because…
	44	R1	But Terry wants to build it up a different way.
	45	R3	No, I want to see how this is- I see when you sort of come together, but with the- the square one, the *a'*s and *b'*s each had color, and you could clearly see that the square of *a*- it was a square of *a*- you had an edge that was *a* plus *b*, and then you ended up with a square that was the color of *a*, and a square that was the color of *b*, it was very clear. Then suddenly something here is happening. I've only got *a* and *b*, and I'm cubing them, so I know I'd have a cube, and where is that other color- you've suddenly got all these colors. [*pause*] Something has happened to the v-
	46	Stephanie	I don't understand what you're saying though. Like-
	47	R3	I'm saying that I am very- I mean I see where you're making the model, and I see you've got something [*rearranging Algebra blocks*] that's got a 4 by 4 by 4 cube, and you've got a 1 by 1 by 1 cube-
	48	Stephanie	Mhm.

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	49	R3	And then you've got three of these things [a^2b *pieces*], and three of these things [ab^2 *pieces*], you can build it all up into a cube. But the colors are confusing me. They're not helping me.
	50	Stephanie	But the colors don't, like-
	51	R3	But they did before
	52	Stephanie	No, that was just because it helped me remember, that the green piece- but the color itself has, like, nothing to do with it. It could be purple- and- it doesn't make a difference [*pause*]
	53	R3	Right.
	54	Stephanie	But I just wrote it down in colors, that way it helped me remember that this piece was *a* squared *b*. But why is it- do you wanna know, like…
	55	R3	[*hesitates, rearranges papers, pulls out paper with* $(a+b)^2$ *drawing*] Over here, you had *a*'s and *b*'s. Okay, two things. And you modeled it with two colors [*pause*]. Right?
	56	Stephanie	I mo- I didn't model it with-
	57	R3	Well actually, you didn't. I guess I (inaudible)
	58	Stephanie	I didn't model it with colors.
	59	R3	Well…
	60	R1	See I guess that, um…
	61	R3	There's something wrong with the model.
	62	R1	The color can get in the way. Because, um, if you take the 10 by 10 by 10 cube, right
	63	Stephanie	This one?
	64	R1	The reason it ends up being a 10 by 10 by 10.
	65	R3	It's really a 10 by 10 by 1.
	66	R1	Exactly. Well that one is a 10 by 10 by 1.
	67	R3	We're treating it like a flat, but it's really a-
	68	R1	But in what Stephanie's building, she didn't call that one, she called that *a*.
	69	R3	Okay. Alright.
	70	R1	See you referred to- you referred to the little yellow cube as a unit cube, but Stephanie's referring to the little yellow cube as an *a* by *a* by *a*.
	71	R3	So-

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	72	R1	See the difference?
	73	R3	-is this 4 of it? [*referring to b^3 piece*]
	74	R1	No-
	75	R3	Or is it not 4 of it?
	76	R1	-it doesn't matter.
	77	R3	Well, yeah, but, you can't do it both ways. I don't think- it's confusing-
	78	R1	Let's ask Stephanie the question. That- I think she called that *a* by *a* by *a*, the yellow. Is that right?
	79	Stephanie	Yeah. The yellow is *a* cubed.
	80	R1	*a* cubed?
	81	Stephanie	Yes.
	82	R1	So the length, width, and height are *a*, which is what (inaudible) used in her model. So she's thinking in terms of length *a*, which happens to be the length, width, and height of that yellow cube.
	83	Stephanie	Mhm.
	84	R1	And she's thinking of *b*, which happens to be…
	85	Stephanie	*b* is like, like [*picks up a^2b piece*] this high- it's almost like 2*a*. 'Cause, like, [*picks up a^3 piece*] the thing is they're all-
	86	R1	*b* is like 2*a*, exactly.
	87	Stephanie	*b* is like 2*a.*
	88	R1	In this case, right.
	89	R3	You mean 2*a* cubed.
	90	Stephanie	'Cause like, [*looks at a^3 piece and a^2b piece*]
	91	R1	No, *b* is linear. *a* and *b* are linear. [*pause*] It's length *a*, and length *b*-
	92	R3	But if this [*picks up a^3 piece*] is linear, then this is a square. Then suddenly-
	93	R1	How can you help Dr. Pearl with that? She's seeing that as a square. How can you help her with that? You may have to go to the picture, you know, 'cause that's exactly where, what's helped before.
	94	Stephanie	[*moves pieces, gets paper with drawing on it*] This [*emphasizing a portion of side length on square*] like, is a un- this is *a* long. This piece right here is *a* long. Okay? And this [*emphasizing*

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			corresponding side length on other side] is *a* long. And so it's *a* squared. We're saying that this is *a* cubed [*indicating a^3 piece*]. We're saying that this is *a* long [*pointing to edge of cube*], by *a* long [*pointing to other edge of cube*] y-you know? Length, width, and height; they're all *a*. [*pause*] Okay? [*pause*] This [*referring back to the drawing*] is *b* like, um, this is *b* long by *b* long [*redrawing segments on sides of b x b square in $(a+b)^2$ model*]. Okay? So we're saying- and this is *b* cubed- s- or well [*mumbles to self; picks between Algebra block pieces*] this is *b* cubed [*choosing b^3 piece*] and they're saying that this is *b* u- they're all *b*. We're not saying that like [*pauses, picks up a^3 piece again*]- this isn't *a* [*indicating whole cube*] this is *a* [*indicating side length of cube*] this little piece, this unit is *a*. Okay? [*pauses*] Okay. So-
	95	R4	And the white, that's all *b*? The-
	96	Stephanie	The white is-
	97	R4	'Cause I think we're looking at the whole–
	98	Stephanie	-*b* by *b* by-
	99	R4	-cubes, and that's
	100	Stephanie	-yeah-
	101	R4	throwing us off
	102	Stephanie	yeah.
	103	R4	Okay.
	104	Stephanie	So, this is *a* squared [*looking at a^2 box in drawing, placing a^3 piece on top*] you build it up *a* units, and it would be *a* cubed.
	105	R4	Okay.
	106	Stephanie	So this piece is *a* cubed [*picks up a^3 piece*]. Okay.
	107	R4	Mhm.
	108	Stephanie	This is *ab* [*pointing to ab rectangle in picture, holding a^2b piece in hand*], and you're- and if you build it up *a*, it's *a* units, it's *a* squared *b*. So this is *a* squared *b* [*picking up piece again*]. K?
	109	R1	Or *a* squared *b* times. See how you have the *a* squared *b* times? You have another *a* squared and another *a* squared.
	110	Stephanie	Yes.
	111	R4	Mhm.

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	112	R1	Or *a* squared *b* times.
	113	Stephanie	This is [*cough*] *b* squared [*points at b^2 part of diagram on paper*]. If you build it up *a* units, it's *ab* squared. Okay? So this piece is *ab* squared. [*pause*] And, um-
	114	???	Where's the other piece?
	115	Stephanie	Oh, this is like the same thing [*places second a^2b piece on diagram*]. It's *a* squared *b*. So you know that this piece is *a* squared *b* [*picks up a^2b piece*], this piece is- no [*picking up ab^2 piece*]- yeah. This piece is *a* squared *b* [*picks up a^2b piece*], this piece is *ab* squared [*picks up ab^2*], and this piece is *a* cubed [*picks up a^3 piece*].
	116	R1	Right.
	117	Stephanie	Alright [*places all pieces on table*]. And so then what we-
	118	???	(inaudible)
	119	Stephanie	[*Stephanie reaches for paper from earlier with $(a+b)^3$ expanded and simplified*] –oops- [*knocks into table*] –was- find out if we had, like, all the pieces that were here, and so if you build, um, and then [*reaches for Algebra blocks, drops one*]–oops- if we build this up, like if you keep building like that, like this is *ab* cubed [*placed ab^2 piece on diagram*], *a* cubed *b* [*places a^2b piece on diagram, then a^3*]-um *a* squared *b*, *a* cubed [*places a^2b piece*], *a* squared *b*, [*places b^3 piece on top of ab^2*] and you build it up. If you built [*removes b^3, ab^2 pieces, points to b^2 part of diagram, holding b^3 piece*] *b* squared up *b* times- *b* units, it would become *b* to the third. So this piece is *b* cubed. So you have every piece here [*referring back to the paper with $(a+b)^3$ work on it*]. You have *a* cubed [*picks up a^3 piece, places it down; picks up a^2b piece*], you have, um [*pauses*], what is that? *a* squared *b* [*places piece down, picks up ab^2 piece*] you have *ab* squared [*places piece down, picks up b^3 piece*] and you have, um, *b* cubed [*places piece down, gathers all a^2b pieces*]. And you have three of these, so that becomes 3*a* squared *b* [*gathers ab^2 pieces*], and you have three of these, so it becomes 3*b*- 3*ab* squared, and you have your *a* cubed and your *b* cubed. And that makes up the problem. And you can build that into like [*pauses, assembles pieces into cube*].
	120	R4	And it doesn't matter which way you put the colors?

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	121	Stephanie	No, because the colors don't matter. It's the [*points to edge of cube*] units.
	122	R4	I have to tell you, I find that very interesting, because I- I know what *a* plus *b* quantity cubed, uh, raised to the third power is, but I never saw it like that.
	123	Stephanie	Yeah.
	124	R4	And why is it 3*a* squared *b*, and 3*ab* squared- that- th- I- I find that totally interesting.